## Facts

1. The great year is a period of 24,000 years; 12,000 years ascending into a higher age, and 12,000 years descending into a lower age. The ancient Egyptians, Sumerians, Megalithic builders, Indians and other ancient cultures knew it as the precession of the equinox.
2. Just as the day is one spin of the Earth on its axis, and the year is one revolution of the Earth around the Sun, so too is one great year based on a celestial motion: one orbit of our Sun around its binary center of mass.
3. Just as the day has light and darkness and the year has summer and winter, so too does the great year have its golden age and dark age.
4. Just as the time of a day can be told by the position of the Sun in the sky, and the time of year can be told by the position of the Sun on the horizon, so too can the time of the great year be told by the position of the Sun relative to the 12 ancient constellations of the zodiac. The daily time system of 24 periods (hours) made of two 12 period sections (AM and PM), left to us by ancient cultures on both hemispheres, is a microcosm of the great year principal.
5. The precession of the equinox (through the zodiac) acts as the clock of the great year. The moving equinox (relative to inertial space) is driven by the curved motion of the Sun through space.
6. Precession is not caused by lunisolar forces, nor does the Earth act like a top relative to the Sun. Precession is a direct result of the Suns binary motion which produces a commensurate change in Earth orientation, a.k.a. precession.
7. The low point, or dark age of the great year is when the Sun and its companion were at apoasis, their furthest point apart. This last occurred in 499AD. We are now about 1500 years into the ascending phase and consciousness is expanding.
8. The high point, of the next golden age, will occur at periapsis when the Sun and its companion are closest to each other. This last occurred in 11,500 BC and will next occur in 12,500 AD.
9. The expansion of consciousness in the ascending cycle is due to the Suns binary motion bringing the Earth closer to the beneficial influence of the magnetic center. The contraction of consciousness in the descending phase is due to the Suns motion carrying us away from the magnetic center.
10. The magnetic center or grand center is thought to be in the region of Alcyon in the Pleiades. There is significant folklore and myth related to this star and this constellation.
11. The exact mechanics that draw us closer to the region, and the mechanism of influence upon consciousness are not well understood in the current age. However, it is known that most stars are binary or multiple systems, most stars are in constant motion and that consciousness is affected by light on the retina, negative ions in the atmosphere, radiation and other subtle forces.
12. Both the ascending and descending cycle are made of four ages analogous to the four seasons: the two poles are the dark age or winter of man, called the iron age by the Greeks and Kali yuga by the Indians; and the age of enlightenment or summer of man, called theGolden age or Satya yuga. Following is a chart of the ages and their respective lengths. We are currently in the 304th year of the Dwapara yuga.
13. The archeological record supports the construct of a great year: The evidence of advanced civilizations in ancient Sumeria, Egypt, India, Mesoamerica, Megalithic Europe, and many more still being discovered, is overwhelming. The fact they all declined into a world wide dark age is obvious and evidence of a descending cycle. The renaissance, agricultural, industrial, information and biotech revolutions all evidence an ascending consciousness.
14. Just as the first day of spring is not always warmer than the last day of winter, so too do the ages unfold slightly unevenly. The current era carries many paradigms of the recent material dark age while mankind struggles towards the light of a higher age. A dangerous time is passing when mankind has access to higher technologies but still harbors the material thoughts of a lower age.
15. Precession now equates to one cycle in 25,770 years only because we are near apoapsis, the slow part of the elliptical orbit. According to Keplers laws, as the Sun moves towards periapsis, the rate will continue to accelerate and will average near 24,000 years.
16. The Yuga cycle of India is incorrectly interpreted by many to consist of divine years resulting in rising and falling ages on a scale so vast as to be meaningless to man. Such longer cycles may exist, however, precession, the great year and the original meaning of the yugas, referred to the cyclical movement of the equinox through the zodiac; a period equaling 24,000 years.
17. Hamlets Mill, the epic work by Hertha von Dechend, professor of the history of science at the University of Frankfurt, and Giorgio de Santillana, professor of the history and philosophy of science at M.I.T., shows the link between myth and astronomy. They describe myth and folklore as the scientific language of yore, and point out that precession was the number one topic of discussion in ancient science and was known to ancient cultures around the world.
18. The current paradigm of history was developed by British and French explorers, investigating and writing about Egyptian and megalithic ruins on the heels of Darwins book: Origin of the Species. Their Judeo Christian backgrounds, over extrapolation of Darwinian principles, and lack of information on the astronomical alignments of the structures they were investigating, led them to conclude the structures were built by primitive man. This paradigm has continued to the present.
19. New technology is driving new discoveries of what is currently considered anomalous archaeological evidence. The dating of the sphinx to more than twice its heretof ore acknowledged age, the finding of a huge new 9000 year old civilzation in the Bay of Cambay, and other exciting discoveries will eventually cause the current paradigm to fall and allow the acceptance of the great year.
20. Better knowledge of the great year concept and construct, its cosmological basis and its link
with consciousness, will change the way man views: history, archeology, paleontology, anthropology, solar system mechanics, religion, spirituality and consciousness and a number of other subjects of major importance. Establishing the solar system as a new moving reference frame, and the great year as a new system of long cycle chronology will also be quite helpful to the scientific community. Fun times ahead in the ascending age.

## What is The Great Year?

The Great Year, is the term that some ancient civilizations use to describe the slow precession of the equinox through the twelve houses of the ancient zodiac, a period that takes about 24,000 years. Different cultures refer to this cycle by different names including: the Platonic year, Perfect year, Yuga cycle, Ages of Man or just the equinoctial cycle, but one thing is clear, it was known to virtually every ancient culture throughout the globe. In their epic work Hamlets Mill Giorgio de Santillana and Hertha von Dechend document the great year tale and point out it was the number one topic woven into myths and folklore around the ancient world.

Why were our ancestors so fascinated by this subject that they memorized stories that were passed down for thousands of years and built megalithic structures on every continent to monitor this movement? We think it is because the tales are true! That is, as the Sun curves through space carrying the Earth with it, our bodies and our planet move to a region where they are affected by different cosmic forces that indirectly result in the rise and fall of civilization. As mans consciousness expands and contracts, and the cycle plays out, just like a solar year with its seasons, it results in great ages of enlightenment and dark ages of misery. Indeed, the archaeological record shows a broad decline of ancient civilizations beginning about 5000 years ago, a long world wide dark age and then finally a rise in consciousness with the renaissance continuing to the present day. Were the tales and myths and stone henges really just for amusement and farming? Or is Hamlets Mill correct: folklore is the scientific language of ancient times, and they were trying tell us of the dark days to come, and trying desparetly to preserve knowledge in the pyramids and megaliths and temples so carefully aligned to the heavens incorporating sophisticated mathematical principles.

This is the story of the Great Year and new scientific evidence to support it. Recent solar system studies seem to indicate that precession is indeed caused by a curving motion of our sun through space. While not yet widely accepted, if true it a startling finding confirming the wisdom of the ancients.

## The Golden Age

The last and longest period, spanning 4800 years in the half cycle, is the age of Truth and Enlightenment. This is considered to be the period when human intellect can comprehend everything, including God, and the intelligence of man will be highly advanced. In this age, man will work in harmony with the Divine Plan. It is virtually impossible to comprehend at or current stage of development what our consciousness in the Golden Age will be like. However, one can imagine that outwardly man will live in perfect harmony with nature and the world will resemble a garden of immense proportions. Some of the myths and fables about Eden, Shangi La and the beauty and peace of the last Golden Age still linger.

## The Silver Age

The Mental age as it is commonly referred to lasts for a period of 3600 years. During this period human intellect can comprehend the divine magnetism resulting in advanced human traits such as telepathic communication and perfect memory. Today we think writing is a sign of intelligence, and we believe the simple hieroglyphs evolved into more complex writing. The fact is they devolved into writing, as this is a skill that is superfluous in the higher ages when one can virtually remember and know all. Thankfully, our planet is soon to evolve so quickly that at our current level of awareness we can hardly comprehend our state of consciousness in just a few decades to come, not to mention in the next Yuga. Look back 150 years who could have guessed what it would be like in the year 2000? Not even Jules Verne saw the possibility of gigabit computing, wireless communication, or the hopeful and sometimes scary, possibilities of biotechnology. But most of the upcoming changes will be in consciousness and lifestyle. It is impossible to convey a knowledge of the next cycle to the masses in this cycle, but many highly conscious individuals will intuit its enormous possibilities.

## The Bronze Age

Termed as the Electrical or Atomic age, it lasts for a total period of 2,400 years of the Daiva Yuga. This is the period when human intellect can then comprehend fine matter, subtle forces and their attributes, characterized by electrical and atomic-energy developments: the age of telegraphy, radio, airplanes, and other space-annihilators. In the descending period, moving away from the spiritual age there is less material development than in the ascending age where the materialistic influence is still strong. The ties to the prior cycle are slow to change. Thus at the beginning of the ascending Bronze age, man is still very materially oriented but beginning to discover subtle particles and forces and understanding more about energy on both a human and worldly level. It is a very exciting time of discovery but the biggest revelations are soon to be the realization our bodies are simply vehicles made of subtle energies operating in a very real matrix of energy and ideas.

## The Iron Age

Referred to as the dark and materialistic age, it lasts for a period of 1,200 years in the half cycle. In this age, human intellect is unable to comprehend anything beyond the gross material world that we live in. The descending Iron age lasted from about 702BC to 499AD (a period of great destruction) and the ascending dark age ended about1699AD. Like the seasons they do not stop and start abruptly but after a few hundred years the difference in ages can be well identified.

## About The Film

The Great Year is a compelling documentary that explores the possibility that the fall of ancient civilizations around the globe, and the rise of modern civilization, might be related to our Suns motion around a companion star. The film examines evidence that ancient civilizations may have known of this celestial cycle and that our Sun may indeed display the characteristics of binary motion.

Just as the Earths spin on its axis causes day and night and our planets annual orbit around the Sun is responsible for the ongoing cycle of the seasons, what if there is some greater celestial cycle, lasting thousands of years, slowly influencing the rise and fall of civilization across the globe? Where is the evidence? What could be the cause?

To many ancient cultures, the answers lie in the stars. In their view, time and civilization did not progress ever forward in a strict linear path, but moved in a cyclical pattern, with human civilization and consciousness rising and falling as great ages came and went. To the ancient Mayans, we are entering the time of the Fifth Sun; Hindu and Vedic scholars spoke of the Yuga Cycle a great circular progression of ages; and in ancient Greece, Plato taught of a large cycle of time which would slowly return us to a Golden Age. He called this cycle: The Great Year.

The Great Year investigates the common thread in these beliefs and looks back into time seeking answers to the questions that still loom over science today. How far back into history do humankinds roots really go? What did the ancients know about the stars and their movements and what can we learn from them? Why was the Precession of the Equinox universally revered? Many of these cultures spoke of an unseen sun that drives this movement of the stars across the sky over thousands of years and causes great ages to rise and fall. Could there be an unseen binary partner to our Sun? The Great Year examines this theory and finds growing scientific evidence to support it.

What makes The Great Year so compelling is that it reveals a startling truth embodied in the number one ancient mystery: the Precession of the Equinox. By showing the cutting edge scientific evidence that challenges the current theory, this film is sure to set off debates in the scientific, archaeological, and astronomical communities.

This provocative film, narrated by James Earl Jones, is accompanied by 18 minutes of animation and a moving original musical score. The message behind the film may be the beginning of a whole new way to look at time and history, and just might set off a new scientific movement to find our Suns binary companion.

## About The COMPANY

At The Great Year we study ancient mysteries and folklore as well as lost civilizations and their architectures in an effort to determine if ancient beliefs have a basis in modern science. We look for astronomical evidence or scientific studies that might support the ancient beliefs, and when confirmed, produce papers, films, books or specialized presentations to communicate our findings.

The aim of The Great Year is to investigate new theories related to human history and the rise of civilization. As a production company The Great Year backs film and literary projects that explore the beliefs held by ancient civilizations and commonalities in the mythologies of cultures throughout the ancient world. Similarly, The Great Year looks to develop features that research the depth of scientific knowledge held by the people of ancient times, as indicated in their writings or by their architecture and other recovered artifacts, and draw comparisons to the body of knowledge that humankind holds today.

These films and books are driven by the idea that there is great value to be gained from learning exactly how ancient societies throughout the world built and sustained themselves, how sophisticated they were, both in academic knowledge and technology, and how they preserved their culture, both materially and spiritually.

The Great Year is dedicated to the exploration of theories that aim to expand our understanding of human history and knowledge, specifically in the fields of archaeoastronomy and ancient beliefs in grand cycles of time.

## Commentaries

Read The Latest Commentary On Hamlet's Mill, One of the Most Ground Breaking Studies of Ancient Beliefs Ever Written. Click Here

## New Astronomical Discovery sheds light on Hamlets Mill and ties Precession of the Equinox to Indian Yuga Cycle and Platonic Year

Hamlet's Mill is the much acclaimed work by Giorgio de Santillana, professor of history and science at MIT and and Hertha von Dechend, professor of history and science at University of Frankfurt. These highly respected scholars completed Hamlets Mill in 1969 and the worlds of archaeology and anthropology have never been the same.

The main idea behind the book, which includes a detailed study of myths and folklore from around the ancient world, is that myths are the scientific language of yore. More importantly, Hamlets Mill clearly shows that most of the universal myths, especially those that are duplicated from culture to culture and often do not make sense to the uninitiated, are about astronomical phenomena. At the time of publication, this was a surprising fact to most traditional archaeologists and anthropologists who assumed ancient man was too primitive to understand complex astronomy. The truth is most traditional archaeologists today understand the subtleties of astronomy even less than the ancients! Fortunately, in addition to Hamlets Mill there have been a number of excellent books published on the topic of archaeoatronomy and hence the field is recently growing in acceptance.

One of the main themes of the book is that the phenomenon known as precession of the equinox (the Earths changing orientation to inertial space as measured at the time of the equinox) was well understood in the ancient world. Moreover it seemed to be a favorite topic of ancient peoples. Many of the old myths compared the Earth to a large mill, whereby the equinoctial point of the earth slowly precesses through the zodiac, thus clearly describing the main observable of precession.

The brillance of de Santillana and von Dechend was in understanding the significance of this cross cultural myth (that it is a scientific language dealing with astronomy), and pointing out the universal importance the ancients attached to the phenomenon of precession. But they were unable to discover why the ancients put so much emphasis on precession. However, in the thirty plus years since Hamlets Mill was published there have been a number of scholars that have done original research on solar system enigmas and the precession phenomena. Recent papers and published work by the Binary Research Institute, Italian scientist Carlo Santagata, and the Sirius Resesarch Group out of Canada seem to show that precession is due to the Suns motion not lunisolar forces as originally postulated by Newton. If these findings are correct it is clear there may be an even larger importance attached to the mythic Mill story than that envisioned by de Santillana and von Dechend. It indicates the underlying reason why the ancients were so adamant about preserving knowledge of precession is that it is not just due to a wobbling earth, but to a motion of the solar system as fundamental as the spinning motion that produces a day and the revolving motion that produces a year: it is the celestial underpinning of a huge chronology system as reliable as the clock or the calendar. Moreover, ancient cultures associated this cyclical motion with a gradual rise and fall of civilization over eons of time. They implied that the celestial motion had some biological and psychological effect upon mankind, equivalent to the day and night effect of a spinning earth and the seasonal effect of a tilted earth revolving round the Sun. Examine these
words by mythic Amlodhi, owner of a great Mill, to paraphrase: In those ancient times it ground out peace and plenty. Later, however, in decaying days, it ground out only salt. Now at the bottom of the sea, it grinds rock and sand, and has created a vast whirlpool, Maelstrom, which leads to the land of the dead.

De Santillana and von Dechend try gallantly to figure out the significance of the Mill but only scratch the surface. The Mill of course is the precession of the equinox, which the authors understood. What they could not understand, because it is was not yet known to western science, is that precession (in the most likely new theory) is caused by the Sun traveling around a companion star, which causes the earth to geometrically precess relative to inertial space. It is this celestial motion that is the Mill behind the Yuga cycle or Platos Great Year, whereby it is said that mankind and civilization itself waxes and wanes between the ages of enlightenment (the Golden Age) and the dark ages. Just as the spinning Earth causes night and day, and the tilted Earth going round the Sun causes the seasons, so too does the grand cycle, and the indirect influence of our binary companion cause a type of long term seasonality which effects everything on earth just as much as night or day or the seasons. At the time that Amlodhi is telling his story the world is near the bottom of the Kali yuga, or darkest age (approx 500AD), the world is at the bottom of the sea so to speak. He related that it had been in a declining phase for many many years, that once it had ground out peace and plenty (in the higher ages), then salt a less valuable but still useful commodity, and finally rock and sand something of no value. He was describing the 12,000-year descending Yuga phase when things gradually deteriorate on Earth, and was telling the story from the point of view of someone near the bottom of the cycle.

The great ancient civilizations of Mesopotamia, ancient Egypt, ancient India, the little understood megalithic culture, the lost Mayan civilization, etc. all apparently knew of the Earths third motion and its significant implications. They left us hints of their knowledge in the structure of their buildings and temples that reveal an unusual awareness of celestial mechanics and enable a calculation of the precessing equinox. These and other ancient, higher age civilizations, also left us the many myths that de Santillana and von Dechend so carefully document. If the ancients could talk in our modern language they would probably say:

People of the modern earth awaken, there is nothing to fear, we are moving out of the darkest age into the light of a higher age. Just as a day waxes and wanes due to the spinning motion of the earth, bringing with the rising sun vitality to every man, and just as the tilted Earth experiences the splendor of the seasons simply by revolving around the Sun, and this brings annual renewal to plants and animals and all things living, so too is there an even larger cycle, also based on a celestial movement which also brings renewal to the planet. It is the Earths third motion. The first motion of the earth takes 24 hours, we call it a day and it only involves one body, the earth spinning on its own axis. The second motion takes a year and involves two bodies, the Earth and the Sun around which the Earth revolves. The third motion, involves three bodies and takes longer still. Yes the pole star does change, it was Thuban, it is now Polaris and will eventually be Vega (when you will not need these words). Yes, the earth does change orientation to inertial space at the current rate of about 50 arc seconds annually. But it will speed up as we approach our companion star. This phenomena is caused by the Earth slowly changing its orientation as the Sun and solar system gently curve through space. Just as the very short cycle of a day is caused by one body motion, and the intermediate cycle of the year is caused by two body motion, you will soon
see the Earths changing orientation to space is caused by a three body motion: the Sun in 24,000 years revolving around its companion which turns the Earth as it goes.

You might wonder why I say precession is a 24,000-year cycle when the current rate is only 50.29 arc seconds of movement per year, which equates to a cycle time now computed at about 25,770 years in 2002. It is because the Suns orbit around its binary center of mass is an elliptical orbit as are all orbits of bodies in motion. Because we are now leaving the farthest point from our companion star, apoapsis, we are slowly accelerating, this is why the precession rate seems to increase each year. Essentially the orbit follows Keplers laws and we speed up and slow down depending on whether the two masses are moving farther apart (slowing) or closer together (speeding up). The rate was near 25,920 years at the bottom of the cycle, and it will go much faster as we near periapsis, the Golden Age. Overall, it will take about 24,000 years to return to the same position as today.

It is a microcosm of this complete cycle that the Mesopotamian, Egyptian and Incan cultures used to divide a day on which our modern time system is based. Just as the day at the time of equinox averages 12 hours of ascending light (am) and 12 hours of descending light (pm) so does it mimic the Great Year made of 12,000 years ascending, and 12,000 years descending. This time keeping system was one more way in which the ancients sought to impart higher knowledge to the inevitable lower age to follow.

We left apoapsis, the farthest point from our binary companion, about 1500 years ago. This always coincides with the low point of the dark ages, which most recently was around 498AD. Like the solstice the planet takes a while to notice the effects of the ascending phase. Progress was slight during the ascending Kali Yuga but you might have noticed a quickening in the world, at the time of the renaissance when we entered the dwapara yuga, the electrical age. Think about that word renaissance, it does not describe a strictly Darwinian world that is linearly evolving; it means a renewal of something that once was. This is the nature of the Yuga cycle, whereby the Ages of Man; Taurus, Pisces, Aquarius, etc. represent the changing seasons of the Great Year.

The world was once populated with many great ancient civilizations the wonders of which modern archaeology has barely discovered. They are beginning to realize the farther back we go towards the last great golden age the more amazing the level of culture. The Greeks were more artistic and more civil than the Romans, the last great civilization to die out before the depths of the dark age. The Egyptians were more advanced than the Greeks who had access to the library of Alexandria and profound astronomical knowledge. The Akkadian culture more advanced than the Babylonians, and the people of Sumeria more advanced than any of the Mesopotamian civilizations that came after it. Do you see the trend? Civilization became darker as we left our companion star, just as the day grows darker as we leave noon, or the year grows darker and less productive as we leave the summer solstice. It is not meant to hurt us, it is a renewal process, based on a celestial cycle. It is the way the universe works; moons revolving round planets, planets revolving round stars, stars taking companions and revolving round them, and all of these producing an ebb and flow of tides, and light and darkness and powerful forces man is yet to become acquainted with. Ancient man, in the higher ages knew these things and much more. Seeing the age was waning, memories fading, he tried to preserve knowledge in myths and folklore, the alignment of great stones, the shafts, geometry and orientation of pyramids, but most
of the knowledge was drowned out in the maelstrom of the darkest age.

Soon huge new discoveries of ancient man will be made and once again the time of civilized man will be pushed back. In this great electrical age, the Dwapara Yuga, we are rapidly developing the technology to find things under the sea, to date things to their true period, to piece together and understand things that were incomprehensible only a few hundred years ago. Mankind is slowly awakening again.

The Yuga cycle today is largely misinterpreted due to errors that crept into the calculation at the beginning of the last Kali yuga. Consequently, the yugas are now thought to be absurdly long periods of time which are of little value to man and do not conform to the archaeological record. The error is in their multiplication as divine years. Without this error each yuga falls neatly into the equinoctial cycle of 24,000 years.

So do not fear the future, for the times you are in may still seem dark but the dawning of a higher age is at hand. Enjoy every day, every season, and every life as you incarnate throughout the Great Year.

Listed below are links that can be used as references to further explain some of the basic concepts discussed on the website.

NEW - On Newton's Paradox
http://www.binaryresearchinstitute.org/pdf/gravi.pdf

## Binary Star Systems

http://www.aspsky.org/mercury/mercury/9904/murphy3.html

## Binary Star Properties

http://www.astronomynotes.com/starprop/s10.htm http://www.gettysburg.edu/academics/physics/clea/A102ol13.html

## Precession of the Equinoxes

http://www.crystalinks.com/precession.html
http://www.accessnewage.com/articles/astro/ageaq1.htm

## Current LuniSolar Model

http://scienceworld.wolfram.com/physics/PrecessionoftheEquinoxes.html
http://www.seds.org/~spider/spider/ScholarX/coord_ch.html

## Properties of Earth Motion

http://www.crh.noaa.gov/fsd/astro/season.htm

## The Seasons

http://www.rog.nmm.ac.uk/leaflets/seasons/equinox.html

## Angular Momentum

http://www.rog.nmm.ac.uk/leaflets/seasons/equinox.html
http://www.ubfellowship.org/archive/science/jupiter.htm

## Sheer Edge

http://www.astro.lsa.umich.edu/users/garyb/WWW/KBO/
http://www.astro.lsa.umich.edu/users/rhiannon/currentresearch.html
http://www.discover.com/apr_01/breaksolar.html
http://www.academicpress.com/inscight/10252000/grapha.htm

## Sidereal vs Solar Time

http://csep10.phys.utk.edu/astr161/lect/time/timekeeping.html http://astrosun.tn.cornell.edu/courses/astro201/sidereal.htm

## Comet Paths

http://www.discover.com/oct_01/featplanets.html
http://www.earthfiles.com/earth317.htm

## Acceleration of Rate of Precession

http://scienceworld.wolfram.com/physics/PrecessionoftheEquinoxes.html KBO - KX76
http://spaceflightnow.com/news/n0108/24kbo/
http://www.space.com/scienceastronomy/solarsystem/big_kuiper_object_010702.html

## Discovery of Ruins in the Gulf of Cambay, India

These ruins could significantly push back the timeline for the development of civilization, and may even indicate the possible existence of sophisticated cultures several thousand years ago. If this is the case, it would be an indication of a long term decline in civilization between 9,500 BCE and the events leading into the dark ages of the early AD period.

## The Age of the Sphinx

Robert Schoch and John Anthony West have presented compelling evidence over the past decade showing that the Sphinx could actually pre-date dyanastic Egypt. The Great Sphinx shows signs of water erosion, which date the structure to at least $10,500 \mathrm{BCE}$. While still heavily debated, these findings point strongly to the existence of earlier civilization.

## Discovery of Ancient Civilization Hamoukar, Syria

Dating of this city indicates that it is over 6,000 years old, making it rival Sumer as the oldest confirmed civilization. Researchers at this site have found that what appears to have been a relatively complex society existed without any major form of writing, long thought to be a precursor to organized civilization.

## Discovery of Ancient City Near Caral, Peru

The ruins at Caral show that civilization existed at least a thousand years earlier than previously thought in the Americas, casting doubt on conventional theory.

## Mithraic Beliefs: The Hypercosmic Sun

Hindu Tradtion is not alone in the belief that there is a dual to the sun driving precession. The Mithraic religion, and early rival to Christianity, was centered around precession and the belief in a "Hypercosmic Sun" beyond view that drove the stars.

## THE HAMOUKAR EXPEDITION

## HAMOUKAR - EARLY CITY IN NORTHEASTERN SYRIA

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## INTRODUCTION

A man was mowing the lawn the other day in Damascus. You don't think of lawn mowing there, especially if your experience of the country has been in the summer, fall, or winter when everything is dry and brown. But right now, in early spring (April 2000), there is green in much of the country. For a while it seemed that this would be the third year of drought in Syria, but there has been rain off and on, especially over the last two weeks. The pattern is not uniform. As you go along the roads, you notice that the fields on one side are much greener, with wheat and barley at four to six inches in height. On the other side, the fields
 are spotty, reaching only two inches of height at most, and then again will come a green area.

I was in Syria in late March in order to begin the process of building a mudbrick house on the site
of Hamoukar, a new focus of research for the Oriental Institute. I arrived with a plan that had been drawn up by John Sanders from ideas I had sketched out. My ideas came partly from an old plan that was created for Nippur by Carl Haines but had not been carried out. I also had in mind the Nippur house that was built by Jim Knudstad, with some changes that would reflect the different environmental conditions in northeastern Syria. John was thinking, also, of Assyrian palace plans, and the house we have decided to build has something of Sargon and Ashurbanipal in it, and they were people who knew about rain and cold, as well as the heat of summer.


When you work in the Syrian Jezira, the upper Khabur river basin, you are very soon made aware of the critical role of rain. The history of occupation in this area, as shown best by the research of the Oriental Institute's Tony Wilkinson, is an episodic one. Centuries of settlement based on rain-fed agriculture have been followed by centuries of abandonment, with the area given over to nomadic pastoralism. The Khabur River, with its several branches, is not a major water course, and during most of the year is represented by dry wadis. But most of the important sites lie alongside those branches, which could supply water enough to support large towns.

To the north, within easy visual distance, is the southwestern-most ridge of the Taurus/Zagros Mountains. On clear days you can see more distant and higher ridges in Turkey and Iraq, with snow covering them for much of the year. Rain and snow, falling in those mountains, feed the Euphrates and its tributaries the Balikh and Khabur, as well as the Tigris and its tributaries. The snow, melting in the spring, is the source for the irrigation in Syria and Iraq, but since there is still very little flow irrigation in the Khabur basin, the mountain runoff has not been as immediately important there. The rain in Turkey does recharge the groundwater of northern Syria. With economic development in both countries, however, the groundwater level is dropping very fast. Syrian villages, which used to draw water from wells dug down 20 m , now must drill artesian wells reaching as deep as 200 m . Irrigation for cotton, a major cash crop in the area, is lowering the level even of artesian water.

## SYRIAN-AMERICAN EXPEDITION TO TELL HAMOUKAR

When we began the first season of excavation at Hamoukar (September - November 1999), we wondered how the site got its water. This tell, which is a very big one by Syrian standards, does not sit on a branch of the Khabur, but between two rather modest wadis. Located about five miles from the Iraqi border, just west of a major border crossing point called Yarubiyah, ancient Hamoukar lay on a major traffic artery from Nineveh to Aleppo. That position must have had a great deal to do with its size and function at various times in the past.

The site was noticed by a number of archaeologists in the 1920s and 1930s, and some proposed that it might be Washshukanni, the still-unlocated capital of the Mitanni empire (c. 1500 bc ). If the people who proposed that identification had examined the pot sherds on the surface of the tell, they would not have made it. There is no evidence of an occupation of Hamoukar at the time of the Mitannians.

Over the years, several excavators have been interested in Hamoukar because it does have plenty of evidence for the Uruk period (c. 3200 bc ), the time when the
 first cities were developed in southern Iraq and colonies were established in Syria. Remains of the Uruk period have been excavated at Tell Brak, to the south of Hamoukar, and at a number of other sites in Syria, especially along the Euphrates.


I first stepped onto Hamoukar in April 1999. I had rented a car in Damascus to go to the eastern part of Syria to look at possible sites to dig. After stopping at Raqqa, on the Euphrates, and being shown a number of very interesting sites, I went on to Deir az-Zor. There, I was shown a couple of sites, but nothing as interesting as those I had already seen. I then drove on to Hassekeh, the provincial capital for the Upper Khabur area. The local director of the Antiquities Service, Abdul Messieh Bagdo, joined me, and we examined another five or six sites, all of which were possible candidates for excavation. I asked him if we could also go to Hamoukar, although I had been told that the site was not going to be given to anyone at this time. I was being shown sites that were in danger because of development projects or illegal digging, and these would have priority.

It was clear, even from a distance, that Hamoukar was a large site, several times larger than anything I had yet seen on the trip. We drove onto the tell and walked up to the top, which has a small modern cemetery. Looking around, I could see that a large part of the southern and eastern slopes were occupied by modern mudbrick houses. But even before I had a chance to look at the pottery on the surface of the mound, rain began to pour down. We ducked under the porch of a nearby house and waited for a bit, but it soon became obvious that this was going to be a serious rain. I thought we should get the car off the tell fast, before we became stuck.

After I got back to Chicago, I wrote a letter to Prof. Dr. Sultan Muhesen, Director General of Antiquities and Museums, with a list of the sites, in order of preference. At the end of the letter, I added a line to point out that Hamoukar was the most endangered mound I had seen, due to the large settlement already covering a large part of it, with a paved road and shops. Whichever site we were given, we would be forming a joint Syrian-American expedition, with personnel and other
 resources. Assuming that we would be given the first or second choice on my list, I began planning for work on a relatively modest scale. I was greatly surprised when I received Dr. Muhesen's letter informing me that we were being given permission for Hamoukar. I then had to rethink my entire approach to the work.

I arrived in Syria in late August and flew up to Qamishli on a Syrian Airways Tupolev jet in an hour and twenty minutes. The same trip by road would have taken at least ten hours. From

Qamishli, I went by taxi to Hassekeh where I was met by my co-director, Muhammad Maktash (Director of the Raqqa Museum). After consulting with the local Antiquities officials, who had already been looking into possible housing for the expedition, Muhammad and I hired another taxi to take us to Hamoukar for a couple of days. There was no place large enough for the expedition in Hamoukar village, but there appeared to be one or two houses in Yarubiyah, eight kilometers to the east. This town, most often referred to by local people by its old name, Tell Kochek, is the border outpost mentioned earlier. We found a brand new house, not quite finished, and rented it. Although a bit small, it would do. I flew back to Damascus to meet the rest of the staff while Muhammad returned to Raqqa to buy equipment to furnish the house.

Clemens Reichel and Jason Ur flew in a few days earlier than the rest, so I took them on a day trip to Beirut, where they had never been. It is easy to go to Beirut (Americans can get visas at the border or the airport), and the road trip takes only three hours, even with a customs check. To watch a city remake itself after fifteen years of war is fascinating. Blasted buildings stand next to brand new ones, and the entire downtown is a large open space, where hundreds of destroyed buildings have been removed and used to create two new peninsulas in the Mediterranean. Archaeological sites, dug after the clearance, are open for now, but some are already under new high rises. People are busily chatting on cell phones, using ATM machines, spending hours in Internet cafes, and dressing in the latest European styles. Even after a long civil war, Beirut is still on to the latest craze.

Two days later, I sent Clemens and Jason in a rented minibus with lots of baggage and equipment for the long ride to Yarubiyah. A couple of days later, the rest of the crew flew with me to Qamishli to find a house full of furniture and even a cook in place. Muhammad Maktash had done a great job in getting us established.

I assumed that this first season would be pretty uneventful, a lot of preliminary steps and a shaking-out operation. We needed to get used to the area, to the local digging conditions, to the local workmen, and to one another. Some of the staff had not been on a dig with me. An exception was John Sanders, whom I pried away from his computers at the Oriental Institute to use his skills as an archaeological architect. John and I have worked together since 1972. Peggy Sanders, a superb artist, was able to join us for the end of the season to draw objects. I also induced Judith Franke to leave her position as Director of the Dickson Mounds Museum to come dig, once more, in the Near East. She and I had last worked together at Nippur in 1973. Tony Wilkinson came to join us for a few days, taking a break from another fieldwork commitment in Syria. Clemens and Jason both had previous field experience in Syria and Turkey, but Brigitte Watkins and Carrie Hritz were going to be in Syria for the first time.

## CONTOUR MAP AND SURFACE SURVEY



We began work by doing a contour map of the site. John Sanders and Carrie Hritz did this in about ten very full days. This map was the basis for the work done by Jason Ur, who was in charge of the surface collection. The picking up and recording of sherds on the surface can give a very good preliminary idea of the size and shape of settlement at a site through time. Unless digging proves otherwise, the Uruk settlement is not as big as other scholars have thought, being only about 13 hectares (c. 32 acres). The site was at its biggest in the third millennium, reaching 102 hectares, or more than 250 acres. It was then abandoned, with people dispersing to form small villages around the site. During the Neo-Assyrian period, c. 800 bc, there was a small village on the mound, and another in the Seleucid period, c. 200 bc. Finally, during the early Islamic period (c. 700 ad ), the last ancient occupants built on the top of the mound.

The surface sherds indicated some particularly interesting places for digging; for instance, there is certainly an area of pottery production on the eastern edge of the site, with stacks of bowls fused by over-firing. But probably the most important result of the surface collection was the confirmation of a very large, low settlement to the south of the main tell. Tony Wilkinson had spotted, on an aerial photograph, some light areas among the fields in that direction and suggested we investigate them. Jason's search among the fields showed that these lighter areas
 were, in fact, cultural remains, datable by sherds to the early fourth millennium. If the entire area is one site, it is a very large one, more than 250 hectares ( 500 acres, plus). That size would make it a major city and we cannot believe that a city existed at this early period. I assume that what we have here is a relatively small village or a set of villages that shifted position over several hundred years. We will not know for certain until we put in some pits next season.

## GEOMORPHOLOGICAL TRENCHES

The satellite photographs also led to another operation. More than 100 m out from the mound, on the northern and eastern sides (see map on page 7), there is visible on the photographs a dark, curving line that one would be tempted to identify as a city wall. When you are on the site, however, you can see nothing that rises as a city wall would. In fact, there is the opposite effect - a long, curving, dip in the middle of the fields. We hired a backhoe to cut a series of trenches (Area D) from the edge of the mound out across that dip. Tony Wilkinson came to the site for two days to examine, sample, and record the vertical faces of the trenches. His preliminary conclusions are that we may have a city wall and a moat right up against the tell. In the area beyond, there are
some bits of evidence of pottery firing but no houses. And the dip reflects an ancient ditch or wadi that carried water during the third millennium bc. Right after Tony left, we paid the operator of the machine to fill in the holes so that the farmers could continue to work their fields.

## EXCAVATIONS

Our excavations in the first season were restricted to three trenches, A, B, and C. Area A was a step trench, 60 m long by 3 m wide, run from south to north down the face of the mound. Area B was located farther to the south, in a place where surface sherds were mainly Uruk in type. Area C was in the middle of a group of thirteen abandoned houses at the northeastern corner of the site. (Those abandoned houses represent a story of straying sheep, murder, revenge, law suits, monetary judgments, governor's decrees, and demolition ... but that's a long story for another time.)


Area A. The step trench, supervised by Clemens Reichel and Brigitte Watkins, was located on the steep northern slope of the mound in an area that appeared unusually smooth and clean. We didn't realize it at the time, but we were cutting through the village's "ski slope," a place where the kids slide down the muddy surface in winter, riding big pieces of cardboard, metal sheets, large metal serving trays, and anything else that will serve. Judging by the same kind of bare strips running down other tells, this sport is pretty widespread in the region.

The reason for doing a step trench is that you can get a very good idea of all of the occupations in a tell without having to make a very deep, vertical shaft through it. The problem with a vertical shaft is that it gets smaller and smaller as you go down because you have to keep leaving the edges undug in order to make a stairway to get in and out. Thus, a pit that starts out at the top as $10 \& a p ; 10 \mathrm{~m}$ will be only $3 \& a p ; 3 \mathrm{~m}$ when you get 7 m down. On a tell like Hamoukar, 18 m high, such a pit would give you less than half the history of the site. A step trench, cutting down in progressive stages along the edge of the mound, can give you a much more representative sample of the occupations. Usually there is a meter or so of disturbed soil washed down from above, with a mixed
 group of sherds, but the greatest part of each step will be undisturbed deposit.

In our step trench, the bottom of each step was about 4 to 5 m below the next one above. We did not reach the bottom of the slope, although we were fairly close to the track that runs along the edge of the mound. We will probably have to put in one or two more steps to reach virgin soil. The deepest level we have reached has evidence of an occupation dating to the early fourth millennium. These layers are cut by a huge trench, perhaps a moat of a later time, but our exposure is too narrow to tell for sure. Up the slope in the next step we encountered some house walls, all of mudbrick, running up to the bottom of a huge mudbrick wall measuring at least 4 m in width and 4 in height. It would be tempting to call this a city wall, but we need to expand the exposure to make certain. The pottery associated with this wall, termed local Late Chalcolithic in Syria, is datable to some time in the mid-fourth millennium bc. Above the level of the big wall we exposed three building levels that could be dated to the late Uruk period, some time around 3200 bc. The pottery
here is mainly Uruk in type, with beveled rim bowls and other items that are native to southern Iraq. Certainly, at this time there was a southern Mesopotamian presence at Hamoukar.

Above the late Uruk houses we found several layers of mudbrick buildings datable by the pottery to the third millennium bc. The successive buildings have pavements of baked bricks and the latest one also has thick plasters of clay finished off with a lime plaster. This series of buildings looks to me to be more than just private houses.

In the earliest levels, as in the Uruk and third millennium layers, we found ancient wells, completely filled in.

Directly on top of the uppermost third millennium building was constructed a building of the early Islamic period (c. 700 ad ). The next period of occupation is the present-day village, and its cemetery, the nearest grave of which lies no more than 10 m from the north end of the step trench.

Area C. Among the abandoned houses at the northeastern corner of the site, we sank a 2 \≈ 2 m pit designed to assess the occupational history of this low part of the mound. Carrie Hritz was in charge of this operation. Just below the surface, she encountered a mudbrick wall that could be dated to about the eighth century, as was already indicated by surface sherds. About a meter lower, the southeast corner of the pit almost exactly coincided with the corner of a mudbrick building that had a buttress
 which was decorated by two small niches. To the north, the buttress ended in a doorway, leading toward the east. The door jamb, the buttress, the corner, and the southern wall were all coated with a white lime plaster. The niched buttress indicates that this building was not a private house but was most probably a temple. Sherds gave a date in the late third millennium, the equivalent of the Akkadian period, when the kings of Akkad in southern Mesopotamia expanded their empire into this region. I should add that the pottery is not southern Mesopotamian, but local, with types well known from other sites such as Tell Brak which was certainly occupied by the Akkadians. With its potential for elucidating a critical period of Mesopotamian history, it is obvious that this area is a prime candidate for an expanded exposure next season.

Area B. We chose as an area for broader expansion a place where Uruk sherds were abundant on the surface. Judith Franke and Abdul Salama supervised this operation, opening a series of 5 m squares running from east to west. The easternmost square turned out to be a puzzle, with masses of red clay and very few objects, even sherds. We finally concluded that we were in a solid mudbrick platform or wall that we cannot date securely as yet.


Farther upslope, we exposed a group of houses with unimpressive mudbrick walls. But the objects and sherds from these houses were extraordinarily numerous. Ash was everywhere, making it difficult to distinguish undisturbed layers of ashy debris on beaten earth floors from ashes in intrusive pits. There were clearly huge, ragged pits as well as narrow, neater pits cutting down into the buildings from levels that have eroded away. These pits had in them Uruk sherds as well as locally made items. The ashy debris from the houses themselves had no Uruk material at all. The Uruk pits must relate to a level that is eroded away or exists at the very top of the slope, which we have not yet excavated. Within the houses, that can be
dated to the mid-fourth millennium by the local Late Chalcolithic pottery, we have determined the source of the ashes. In one room, there are the remains of four, and possibly five, successively used ovens. These ovens are built of mudbricks, which have become partially fired through use. The shape of the ovens is something like an igloo, ovoid in plan and with a domed mudbrick roof. The ovens were used for a variety of cooking activities, probably for bread baking and beer making, as well as for the cooking of meats. In the debris within and around the ovens, we have recovered many animal bones as well as an abundance of charred grains, including wheat, barley, and oats. Dr. Amr al-Azm, a professor at the University of Damascus and a member of our team, is studying the plant remains and will be able to give us more detailed information on them in the near future.

Besides ovens, we also discovered two ancient wells in Area B. Like the ancient wells exposed in the step trench, the two wells here answer the question of how the ancient inhabitants got their water. Like the modern villagers, the ancient people dug down to the water table wherever they needed water. The modern villagers report that, until the water level began to drop over the last few years, the water in their wells was "sweet."

The pottery found in the Area B houses is dominated by large cooking pots, called casseroles by archaeologists working in this part of Syria. But there are also a variety of smaller vessels and even very fine wares, usually in the same shapes as the larger vessels. The ability of the local potters was extraordinary; some of the fine wares are as thin as the shell of an ostrich egg.

In the houses, we found fragments of bone figurines that have
 been termed "eye idols," because of their huge eyes (and absence of other features of the head). The most complete example was recovered from a baby grave. Figurines of this type, which may in fact have been representatives of people, not deities, were discovered at Tell Brak in the 1930s and have been used as a marker for the mid-fourth millennium.


Most important as artifacts that inform us about the nature of the society that created the cooking establishment at Area B are the more than eighty stamp seals, fifteen seal impressions, and many beads found there. Most of the finds were from one pit, probably a grave. Here were found thousands of beads of bone, faience, shell, and stone, some of such a small size that I assume they were meant to be sewn onto clothing in patterns, rather than being worn as jewelry. The stamp seals are mostly of bone, carved into the shapes of animals, with incised lines or figurative scenes on the bases. One of our larger seals is in the form of a leopard, with its spots indicated by tiny dowels sunk into drilled holes. On its lower surface is a row of horned animals. There is an equally well-made seal in the form of a horned animal (with horns broken off), with horned animals in file, once again. But the larger seals are much less common than other, smaller seals in the shapes of animals. The most common shape of the smaller seals is that of a lion, but we also have a pair of lions, lion heads joined at the back, ibexes, bears, dogs, rabbits, fish, and birds. There is also a major type in the form of a rectangle with grooves on one face and incised hatchings on the stamping surface. Very similar bone artifacts found at Tell Brak in the 1930s were called amulets. At Hamoukar, we termed them stamp seals because we have found in the houses lumps of clay and bitumen with the impressions of scenes with animals, very similar to
those we have on the larger stamp seals.
We have not, as yet, recovered a piece of clay with an impression made by any smaller seals with simple incision or cross-hatching, but we still call them seals because we have one type that includes both the figurative scene and the incised hatching. This type, in the form of a duck with its head turned over its back, occurs in three sizes. The smallest one has only hatching, but the other two both have scenes of animals. If the two larger items were used as seals, the smaller one should have been as well. The difference between the stamp surfaces, with figurative scenes on the larger two and incised lines on the smallest, must lie not in a difference of function but in the users. We would propose that the larger, more elaborate seals with figurative scenes were held only by the few people who had greater authority, while the smaller incised seals were used by many more people who were sealing as members of a large group with less authority. The difference would be something like the signature of the Director of Customs, used only by him, as compared to rubber stamps that say "US Customs," which can be used by hundreds of employees of that bureau.

Those last lines imply a degree of complexity at ancient Hamoukar that might seem remarkable because it is so early (fourth millennium). But seals, especially when found to have been used on clay or bitumen stoppers or as door-locks, are prime evidence of some kind of system of accounting or responsibility. They need not point to a bureaucracy, but could be the marks of ownership or responsibility over specific goods or duties performed. They may in fact imply a level of complexity that we would relate to state formation.


All the evidence from Area B points to the making of food on a scale that is far more than that needed for household consumption. It is on an institutional or industrial scale that one usually associates with a state. The possible existence of a city wall at the same time as the cooking establishment at Area B makes us think that this part of Syria had developed early kingdoms before the coming of the Uruk people. That is an important suggestion because, in general, it is normally proposed that civilization and the earliest states developed at about 3500 bc in southern Iraq, specifically in ancient Sumer. It is also usually proposed that this southern core area drew upon a less developed periphery in neighboring regions. It has been recognized for some years that Mesopotamian colonies were established in Syria, but the exact dating of the first establishments and their relationship to the local people is still being debated.


It has been suggested that after the contact with the Uruk people, local Syrian and Turkish kingdoms were stimulated into development. But now, our evidence and the evidence from a few other sites in Syria and Turkey seems to show that more complex societies were evolving not just in southern Iraq, but simultaneously in a number of areas. A few scholars, working in Turkey and Syria, are beginning to suggest that maybe civilization began not in southern Mesopotamia, but in their area. Before we start revising our textbooks, however, we should remember that there was an even earlier contact with people who had come from southern Mesopotamia. During the Ubaid period, c. 4500 bc , southern pottery and other artifacts reached as far west as the Mediterranean and at least as far south as Dhahran, Saudi Arabia. In some places, not just objects, but entire buildings were, in a sense, transplanted. At Tepe Gawra in northern Iraq there is a group
of buildings that is clearly "at home" in Sumer. I would suggest that we need to reconsider our ideas about the beginnings of civilization, pushing them farther back, from the Uruk period into the Ubaid. This would mean that the development of kingdoms (early states) occurred before writing was invented and before the appearance of several other criteria that we think of as marking "civilization." Regardless of the level of development in the Ubaid period, the movement of people and objects from the south of Mesopotamia to the neighboring areas set up vital linkages between people hundreds of miles distant from one another. It was these linkages that made possible the transmission of ideas and people that underlay the joint development of complex societies.

This set of important issues can be addressed very effectively at Tell Hamoukar. At Nippur and other major sites, where the early city levels are meters below later remains, we cannot touch upon such questions without years of preliminary digging away of the debris of later periods. Here, with the fourth millennium and Uruk levels lying right under the surface on part of the mound, and third millennium material, which may relate to the Akkadian conquest of eastern Syria, in evidence at the surface in many parts of the site, our work at Hamoukar promises to be extremely rewarding. I cannot hope that in every season we will find the wealth of objects that we recovered this year, but the site does seem to be an unusually productive one.

Maybe you will be able to visit us in future, taking tea in the shaded porches of our new dig house, for which the bricks are being made as you read this. We will be back digging at Hamoukar in the fall, and you will be hearing about our work soon after.

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RETURN TO NEWS AND NOTES<br>RETURN TO HAMOUKAR<br>RETURN TO ARCHAEOLOGY<br>RETURN TO ORIENTAL INSTITUTE

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http://www-oi.uchicago.edu/OI/PROJ/HAM/NN_Sum00/NN_Sum00.html

# On Newton's paradoxes 

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#### Abstract

Reading the Principia [1] it is easy to realize that Newton, without being aware of the fact, has corrected the law of the fall of bodies formulated by Galilei. In the present paper we examine the consequences of such a fact and show that they affect the entire Celestial Mechanics, being related to the very small 43 " per century of the perihelion shift of Mercury as well as to the substantial 50 " per year of the grand phenomenon of the lunisolar precession.


key word: Galileo - Newton - Le Verrier - Einstein - General Relativity - Perihelion of Mercury - Law Hall's law - Precession.

## 1 Introduction

The greatest scientific work of all times, Philosophiae Naturalis Principia Mathematica [1], by Isaac Newton, was published for the first time in July 1687, in a few copies. In the following three centuries it got some criticism, but in practice it remained unequalledly unaltered, since the nature of all the great contributions produced in that age is exclusively mathematical. But one of the few points that became subjects of discussions was the distinction between absolute and relative motions. Actually Newton states that when a motion gives rise to inertial forces then it is absolute. On this subject, in the Scolio IV of Definition VIII, he describes the well-known example of the bucket of water, hanged by means of a long thread and rotating. It is known that the water, after some time, tends to rise towards the edge of the container and so Newton deduces that this is a case of absolute motion. The archbishop Berkeley expressed its opinion against this conclusion. His reasoning was that if all the universe rotated with opposite speed around the bucket, and this was still, the same phenomenon would occur. In more recent times, E. Mach [2] agreed with the criticism of Berkeley and in this connection textually stated: Try to hold still the Newtonian bucket, and to rotate the sky of the stars and to verify the absence of centrifugal forces. Obviously we do not know the possible reply of Newton to objections of this kind, but we can certainly say that, making recourse to his 3rd principle of the Dynamics, Newton surely would reply that both the bucket and the universe will rotate in opposite directions with speeds respectively proportional to the inverse of their masses, or their moments of inertia ${ }^{1}$. We will not consider further objections of

[^0]this kind, however we can conclude that the Principle of Action and Reaction, when it can be applied without ambiguity, allows us to state exactly what occurs in the physical world, even in cases concerning the Galileian relativity.
On the contrary, as we will see in great detail, a careful study of the Principia will show many other contradictions in Newton thoughts, and his use of some non-orthodox procedures: contradictions and procedures that are still accepted today.
It is well-known that the 3rd experimental Kepler Law states that for all the planets of the solar system the ratio $\left(\mathrm{d}^{3} / \mathrm{T}^{2}\right)$ is rigorously constant.
Newton, when he considers the two-body problem, corrects this law with great opportunity and coherence by introducing the masses of the planets in it, and concludes that
$$
\frac{d^{3}}{T^{2}}=\frac{G M}{4 \square^{2}} I^{1}+\frac{m}{M}=
$$
where the meaning of the symbols adopted is evident.
Actually, Newton - differently from Galileo, who thinks that the bodies can only undergo the effect of the Earth gravitation but do not act on the Earth - extends the active gravitational effect to all the bodies in the universe, assuming that this effect is proportional to each single mass. Unfortunately, this powerful generalization - that has great implications, as we will see - in practice is completely voided by the procedure used in the solution of the two-body problem, when we pass from the inertial reference frame centered on the barycentre of the two masses (or the reference frame anchored to the fixed stars), to the non-inertial reference frame anchored to the central mass. Therefore, even if he recognizes that our planetary system, including the Sun, rotates around a barycentre different from the center of the Sun, Newton actually imposes an unjustified and unjustifiable heliocentrism that substitutes the controversial and fought geocentrism that, in very dark times ${ }^{2}$, caused the death of Giordano Bruno and the abjuration of Galileo Galilei.
We will see that the removal of this unjustifiable Newtonian heliocentrism directly affects both the very small 43 "- 44 " per century of the shift of the perihelion of Mercury and the substantial 50 " per year of the most remarkable and grand phenomenon of the lunisolar precession, and thus affects in a substantial way the entire Celestial Mechanics.

## 2 Some Preliminary Calculations

As a consequence of Newton's universal theory of gravitation, two celestial bodies, at a distance $d$, with masses M and m (Fig. 1),

in an inertial reference frame centered on the barycentre K of the two masses (or in an inertial reference frame anchored to the fixed stars), are affected by the force

[^1]\[

$$
\begin{equation*}
\mathrm{F}=\mathrm{G} \frac{\mathrm{Mm}}{\mathrm{~d}^{2}} \tag{1}
\end{equation*}
$$

\]

Using the symbols $\mathrm{x}_{\mathrm{M}}$ and $\mathrm{x}_{\mathrm{m}}$ to denote the distances of the two masses from the baricentre K , we have

$$
\begin{equation*}
x_{M}=\frac{d}{M+m} m \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
x_{M}=\frac{d}{M+m} M . \tag{3}
\end{equation*}
$$

In the hypothesis of circular orbits, from the two obvious relations ${ }^{3}$

$$
\begin{align*}
\frac{M V_{M K}^{2}}{x_{M}} & =G \frac{M m}{d^{2}}  \tag{4}\\
\frac{m V_{m K}^{2}}{x_{m}} & =G \frac{M m}{d^{2}}
\end{align*}
$$

we have the speeds $V_{M K}$ and $V_{m K}$ of the two masses in the frame $K$ :

$$
\begin{equation*}
V_{M K}=\sqrt{\frac{G m^{2}}{d(M+m)}} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
V_{m \mathrm{k}}=\sqrt{\frac{G M^{2}}{d(M+m)}} \tag{6}
\end{equation*}
$$

In the reference frame centered on the primary mass $M$ we will have that the speed of the secondary mass is given by

$$
\begin{equation*}
v_{m M}=\sqrt{\frac{G m^{2}}{d(M+m)}}+\sqrt{\frac{G M^{2}}{d(M+m)}}=\sqrt{\frac{G M}{d} f^{1}+\frac{m}{M}}= \tag{7}
\end{equation*}
$$

We obtain the same conclusions by evaluating first the Newtonian force in the non-inertial reference frame centered on the primary mass. The accelerations relative to K are

$$
\begin{equation*}
\mathrm{a}_{\mathrm{MK}}=\frac{\mathrm{Gm}}{\mathrm{~d}^{2}} \tag{8}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{a}_{\mathrm{mk}}=\frac{\mathrm{GM}}{\mathrm{~d}^{2}} \tag{9}
\end{equation*}
$$

Therefore, in the system centered on M, we have

$$
\begin{equation*}
a_{m M}=\frac{G M}{d^{2}}+\frac{G m}{d^{2}}=\frac{G M}{d^{2}} A+\frac{m}{M} G \tag{10}
\end{equation*}
$$

from which it follows that ${ }^{4}$

$$
\begin{equation*}
F_{m M}=\frac{G M m}{d^{2}}=1+\frac{m}{M} \tag{11}
\end{equation*}
$$

From (11) we have

[^2]\[

$$
\begin{equation*}
m \frac{V_{m M}^{2}}{d}=\frac{G M m}{d^{2}}=1+\frac{m}{M}= \tag{12}
\end{equation*}
$$

\]

and therefore the speed of the mass $m$ in the reference frame centered on the primary mass is given by

$$
V_{m M}=\sqrt{\frac{G M m}{d^{2}}=1+\frac{m}{M}}
$$

i.e. we have found again relation (7). From this relation we also find the $3^{\text {rd }}$ Kepler's law, revisited by Newton, that is

$$
\begin{equation*}
\frac{4 \square^{2} d^{3}}{\mathrm{~T}^{2}}=\mathrm{GM}=1+\frac{\mathrm{m}}{\mathrm{M}}= \tag{13}
\end{equation*}
$$

It is evident that, if we consider the fall of terrestrial bodies, from (11), using $\mathrm{a}_{\mathrm{m}}$ to denote their acceleration relative to the Earth surface and R to denote the radius of the Earth, we have

$$
\mathrm{ma}_{\mathrm{m}}=\frac{\mathrm{GMm}}{\mathrm{R}^{2}} \mathrm{H}^{-1}+\frac{\mathrm{m}}{\mathrm{M}}=
$$

from which it follows that

$$
\begin{equation*}
a_{m}=\frac{G M}{R^{2}} f^{1}+\frac{m}{M}=g=1+\frac{m}{M}= \tag{14}
\end{equation*}
$$

From (14) we immediately deduce that the law of Galilei for the fall of bodies can be obtained only when the gravitational effects exerted by these bodies are neglected; in the same way, from (13) we obtain the 3rd Kepler's law, in the form in which it was originally stated, only when the gravitational mass of the generic orbiting planet is considered equal to zero.
In a similar way, when a pendulum is considered, we find

$$
\begin{equation*}
\mathrm{T}=2 \square \sqrt{\frac{1}{\mathrm{~g} \mathrm{~F}^{1}+\frac{\mathrm{m}}{\mathrm{M}}}} \tag{15}
\end{equation*}
$$

so its period is also a function of the mass of the pendulum. Since in this case, considering for example a pendulum having a mass equal to 1 Kg , we find that $\mathrm{m} / \mathrm{M}=1.67 \mathrm{e}-25$, we can realize the reason why Newton never appreciated that the conscious and consistent correction of the 3rd Kepler's law, that anyway controls the continuous fall of the planets around the Sun, automatically implies even the correction of Galilei's law [3] and the pendulum's law. In fact, Newton, contradicting completely himself, textually states (Prop. VI \{ Theorem VI):
The fall of all the bodies on the Earth (taking into account the unequal delay caused by the very law resistance of the air) occurs in equal times, as already observed by others; and it is possible to verify with great precision the equality of these times in pendulums. I have made experiments with pendulums made of gold, silver, lead, etc. ....
In the following he states:
Let us imagine, in fact, that these terrestrial bodies were rosen up to the Moon's orbit, and together with the Moon, deprived of any movement, were let free so that all of themfall in the same time on the Earth; then, because of the previous reasoning, it is certain that they, together with the Moon, would cover equal spaces in equal times.
... And for the same reason, the planets that revolve around the Sun, letting them fall fromequal distances fromthe Sun, would cover, during their fall towards the Sun, equal spaces in equal times. (Even the planets whose masses are contained in his 3rd Kepler's law). In Prop. X \{ Theorem X, concerning the same subject, he states (Newton's tube) ... the bodies fall within the tube freedy and without any appreciable resistance; a piece of gold and a very light feather, let free at the same time, fall with equal speed, and even if they cover an height of four, six, or else eight feet, fall contemporaneously on the bottom, as we can learn fromthe experience.

We must admit that, watching with our eyes that a feather and a piece of gold fall, in a vacuum tube, with the same speed is a very impressive fact ${ }^{5}$.

## 3 Criticism of the Two-Body Problem

After having analysed the possible trajectories that a body can follow if it is attracted by a fixed center even with a law proportional to the inverse of the squared distance, Newton faces the twobody problem, i.e. the basis of the modern theory of planetary perturbations. In Prop. LVII. Theorem XXI he textually states:
If two bodies attract each other with forces of any kind and simultaneously rotate around the common center of gravity, I say that because of the effect of these forces a trajectory can be covered, around one or the other body not in motion, that is similar and equal to the trajectory that the bodies so moved cover mutually one around the other.
Fig. 2, that reproduces the original figure in the Principia, shows graphically the concept expressed by Newton.


Fig. 2

On the left hand side, we see the bodies S and P that rotate around the common center C ; it is evident that Newton considers the inertial reference frame anchored to the baricentre or to the fixed stars. On the right hand side, in small letters, the same figure shows the situation like it is perceived by the non-inertial observer centered in the Sun, who, according to Newton, can be considered absolutely fixed even if it is accelerated by the secondary mass. Actually, the figure on the right can be obtained from the figure on the left by tracing in the latter the straight line parallel to the line T R and passing by S . This assumption, that, as we already said, is the basis of modern Celestial Mechanics, gives rise to the following procedure for the solution of the two-body problem.
We consider the Sun as it was absolutely fixed, by thinking to the mass of the system ( $\mathbf{M}+\mathbf{m}$ ) as it was completely concentrated in the center of the Sun, and assuming that the secondary mass $m$ was completely negligible [4]. This also implies that the major semi-axis of the orbit relative to the non-inertial reference frame is always fixed.
Let us refer now to Fig. 3. In this figure solid lines represent the two orbits covered by the masses M and m around the baricentre K. Furthermore, there is a frame made up by broken lines, whose intersections can be seen as a schematic representation of the fixed stars scenario. Thus it is evident that the mass m , for example, after half an orbit will be found in the point 1 , while the mass M will be found on the right of its original position at a distance of about three units from the point where it was before. On the other hand, if we accept the solution by Newton, after the said time the mass $m$ will be found in the point 2 , characterized by another star constellation, while the mass M is still in

[^3]its original position. Thus, if we accept the classical solution, the orbits covered in the two reference frames are completely different. In fact, while in the frame anchored to K the mass m covers the circumference with radius $\mathrm{x}_{\mathrm{m}}$, the same mass in the system anchored to S covers a circumference with radius $\mathrm{x}_{\mathrm{m}}(1+\mathrm{m} / \mathrm{M})=\mathrm{d}$. On the other hand, when, for example in geometry, one constructs the transformation formulae that relate one reference frame to the other, the guide rule is that the trajectory remains the same. We can certainly state that, with or without the presence of the fixed stars, the observer anchored to the mass M, contrary to the observer centered in K, can measure the acceleration of the mass M : this acceleration characterizes and distinguishes the two reference frames. This is well-known to the astronauts when they, during their aptitude tests, are strongly accelerated in pods rotating at high speed.


Thus a problem arises: the determination of a procedure that enables the observer centered on the primary mass to reconstruct the real and unique orbit covered by the secondary mass $m$ in the reference frame anchored to the fixed stars, and to recognize, among the other things, that he has an acceleration.
A solution that satisfies the previous requirements can be the following.
If we admit for the moment, according to Newton, that the force in the reference frame centered on the primary mass is given by the relation

$$
F_{m M}=\frac{G M m}{d^{2}}=1+\frac{m}{M} ;
$$

we deduce that, when the mass $m$ is negligible compared with $M$, the reference frame centered on the primary mass is practically inertial and therefore, according to Newton, can be considered fixed. From this we deduce that this infinitesimal or fictitious mass, in the reference frame centered on the primary mass M , has a speed equal to

$$
\begin{equation*}
V_{m M_{0}}=\sqrt{\frac{G M}{d}} . \tag{16}
\end{equation*}
$$

Now let us assume that the mentioned infinitesimal mass, after a prefixed time $\quad \mathrm{T}$, due to the speed given by (16), starting from its initial position $m$, describes the arc $m-m "$, with radius $d$ and center M, that can be seen in Fig. 3. Once this operation has been done, since in the given time the mass M moved to $\mathrm{M}^{\prime}$, it is necessary to rotate, at the same time, both the semiaxis M-m" and the small piece of the orbit covered by the arc m-m", by the angle Mm"M'. This is needed because, contemporaneously, the abovementioned semi-axis should coincide with (or should be parallel to) the real semi-axis $\mathrm{M}^{\prime}-\mathrm{m}$ ' and the described arc should better approximate the orbit covered by the real mass m around the baricentre. It is evident that the rotation needed to obtain that M was very
close to M' has the revolution direction of the orbiting mass.
In Fig. 4 this procedure is repeatedly applied in the first quadrant. This construction shows that the consecutive arcs covered by the mass $m$ approximate the orbit covered by $m$ around the baricentre, and that, contemporaneously, the broken line covered by the primary mass, due to the repeated rotations of the semiaxis, approximates the orbit actually covered by M around the barycentre ${ }^{6}$. It is evident that, choosing infinitesimal times, both the said arcs and the broken line will tend to envelope, and coincide exactly with, the two orbits around K.
Thus we can say that the non-inertial observer centered on the mass M can retrace the real orbit described by the secondary mass as long as he does the following simple operations:

- first he has to find the orbit described by an infinitesimal mass $m$ around the mass M ; in such a case he con consider himself, in every respect, like an observer fixed in the space, according to Newton; in this way he obtains, for example, an ellipse whose major semiaxis is fixed in the space;
- subsequently he imposes to the previous orbit a rotation such that its major semiaxis would be rotated, in the direction of the revolution, by an angle that we will evaluate.


Fig. 4

In the provisional hypothesis that the point m" of Fig. 3 is very close to the point m', like we can deduce both from Fig. 3 and Fig. 4, after a complete revolution of the mass m, the mass M has covered the whole circumference of radius $\mathrm{x}_{\mathrm{M}}$, so the mentioned angle, expressed in radiants, would be equal to

$$
\begin{equation*}
\square=\frac{2 \square x_{M}}{d}=2 \square \frac{m d}{(M+m) d}=2 \square \frac{m}{(M+m)} . \tag{17}
\end{equation*}
$$

Actually the angle, as we will see soon, is exactly one half of this value. To see this, let us note that since the arcs described by the masses are respectively proportional to the speed of each mass, they can represent the speed of the masses.
We have already said that the speed of the mass m , considered infinitesimal, is equal to

$$
\begin{equation*}
V_{m M_{0}}=\sqrt{\frac{G M}{d}} . \tag{18}
\end{equation*}
$$

Instead, the speed of the real mass $m$ in the reference frame anchored to the baricentre is given by

[^4]$$
V_{m k}=\sqrt{\frac{G M^{2}}{d(M+m)}}=\sqrt{\frac{G M}{d}} \sqrt{\frac{M}{(M+m)}}=\sqrt{\frac{G M}{d}} \sqrt{\frac{1}{1+\frac{m}{M}}},
$$

Taking into account that the ratio $(\mathrm{m} / \mathrm{M})$ is very close to zero, this speed is also given by

$$
\left.\sqrt{\frac{G M}{d}} \sqrt{\frac{1}{\mathrm{~B}^{1}+\frac{\mathrm{m}}{\mathrm{M}}}} \square \sqrt{\frac{\mathrm{GM}}{\mathrm{~d}}} \sqrt{1 \square \frac{\mathrm{~m}}{\mathrm{M}}} \square \sqrt{\frac{\mathrm{GM}}{\mathrm{~d}}}=1 \square \frac{1}{2} \frac{\mathrm{~m}}{\mathrm{M}}\right]
$$

and thus we have

$$
\begin{equation*}
v_{m k} \square \sqrt{\frac{G M}{d}} \square \frac{1}{2} \frac{m}{M} \sqrt{\frac{G M}{d}} . \tag{19}
\end{equation*}
$$

This speed of the secondary mass $m$, in the baricentre system, is less than the speed given by (18), and is represented in Fig. 5 by the arc $\mathrm{m}-\mathrm{m}$ ', because the point m ' is relative to the orbit described in the K reference frame. Instead, the speed of the real mass m in the reference frame of the fixed mass M is given by ${ }^{7}$

$$
V_{m M}=\sqrt{\frac{\mathrm{GM}}{\mathrm{~d}}}=1+\frac{\mathrm{m}}{\mathrm{M}}-\sqrt{\frac{\mathrm{GM}}{\mathrm{~d}}}=1+\frac{\mathrm{m}}{\mathrm{M}} G \sqrt{\frac{\mathrm{GM}}{\mathrm{~d}}}=1+\frac{1}{2} \frac{\mathrm{~m}}{\mathrm{M}}=
$$

or

$$
\begin{equation*}
V_{m M}=\sqrt{\frac{G M}{d}}+\frac{1}{2} \frac{m}{M} \sqrt{\frac{G M}{d}} \tag{20}
\end{equation*}
$$



Fig. 5
and is represented by the arc m-m'"'; it coincides with the mass given by the Newtonian procedure, indeed the straight line $\mathrm{M}-\mathrm{m}^{\prime \prime \prime}$ ' is parallel to the straight line $\mathrm{M}^{\prime}$ ' m '. By comparison of these relations we deduce that the speed of the mass m , considered negligible, in the inertial reference frame centered on the mass M, given by (18), is represented by the arc $\mathrm{m}-\mathrm{m}$ "'. Indeed the considerations above allow us to deduce that the arc $m "-m "$ is equal to the arc $m ">m$ ""' and both represent the speed $\frac{1}{2} \frac{m}{M} \sqrt{\frac{G M}{d}}$, which implies that the $\operatorname{arc} m^{\prime \prime}-m^{\prime} "$, , that is equal to the $\operatorname{arc} M-M^{\prime}$,

[^5]represents the speed $\frac{m}{M} \sqrt{\frac{G M}{d}}$. This implies that the rotation angle that must be applied to the orbit covered by the negligible mass to make its major semiaxis parallel ${ }^{8}$ to the semiaxis relative to the baricentre of the masses, is given by the angle M-m"' a that is exactly equal to one half of the angle $\mathrm{Mm}^{\prime \prime} \mathrm{M}^{\prime}$. In fact these two angles defines the two arcs Ma and Mb whose ratio is $1 / 2$. Therefore we conclude that, after a complete revolution of the infinitesimal mass around M , which at the same time is considered fixed, we must rotate the major semiaxis of this orbit by the angle given by
\[

$$
\begin{equation*}
\square=\square \frac{m}{M+m} \tag{21}
\end{equation*}
$$

\]

This relation implies immediately that, when the secondary mass is negligible, i.e. in the case when the reference frame centered on the primary mass can be considered completely inertial, there is no need to apply any rotation to the semiaxis of the orbit determined in such a way; only in this case the perihelion of the orbit is fixed in the space.
We can obtain the same result in a more straightforward way. According to Newton, the force in the primary mass reference frame is given by

$$
\begin{equation*}
F=G \frac{M m}{d^{2}}=1+\frac{m}{M} \beta \tag{22}
\end{equation*}
$$

so the speed of the secondary mass $m$, in the primary mass frame, is equal to

$$
\begin{equation*}
V_{m M}=\sqrt{\frac{G M}{d}} f^{1}+\frac{m}{M}-\square \sqrt{\frac{G M}{d}}+\frac{1}{2} \frac{m}{M} \sqrt{\frac{G M}{d}} . \tag{23}
\end{equation*}
$$

It is clear that, if the orbiting mass m is negligible compared with the central mass, the reference frame centered on the primary mass M is rigorously inertial and the secondary mass describes a fixed orbit in the space. Its speed will be obviously given by

$$
V_{\mathrm{mm}}=\sqrt{\frac{\mathrm{GM}}{\mathrm{~d}}} .
$$

When the secondary mass $m$ is not negligible, its speed will be given by (23). It is also evident that the difference between the above-mentioned speeds

$$
\begin{equation*}
\square V=\sqrt{\frac{G M}{d}}=1+\frac{m}{M}-\sqrt{\frac{G M}{d}} \square \frac{1}{2} \frac{m}{M} \sqrt{\frac{G M}{d}} \tag{24}
\end{equation*}
$$

is due completely to the gravitational action of the secondary mass on the primary mass, that only produces a recoil of the mass M . More precisely, we have

$$
\begin{equation*}
\frac{d V_{m M}}{d m}=\frac{1}{2} \frac{m}{M} \sqrt{\frac{G M}{d=1+\frac{m}{M}}} \tag{25}
\end{equation*}
$$

which implies

$$
\begin{equation*}
\square V=\frac{1}{2} \frac{m}{M} \sqrt{\frac{G M}{d=1+\frac{m}{M}}} . \tag{26}
\end{equation*}
$$

The length of the arc covered by the real mass $m$ with the speed given by (26) on the circumference of radius d during a whole period of revolution T is given by

$$
\begin{equation*}
s=\frac{1}{2} \frac{m}{M+m} \sqrt{\frac{G M}{d=1+\frac{m}{M}}} \sqrt{\frac{4 \square^{2} d^{3}}{G M} 1+\frac{m}{M}}=\square \frac{m}{M+m} d \tag{27}
\end{equation*}
$$

[^6]and thus
$$
\square=\square \frac{\mathrm{m}}{\mathrm{M}+\mathrm{m}} .
$$

All the above is schematically shown in Fig. 6.


Fig. 6
When the secondary mass m has covered a complete orbit, the primary mass M assumes the position $\mathrm{M}^{\prime}$, with the major semiaxis rotated by the angle $\square$.
It is easy to realize that, operating in such a way in the case of circular orbits, the secondary mass, in the reference frame centered on M , will cover exactly the orbit that the real mass m covers around K with the speed given by (18). At the same time the circular orbit itself will be characterized by a slide speed, in the same direction of the revolution, given by (26). In conclusion the secondary mass will have a total speed equal to which is equal to the speed of the real mass in the Newtonian reference frame. In fact we have

$$
\begin{equation*}
V_{m M}=\sqrt{\frac{G M}{d}} f^{1}+\frac{m}{M}-\sqrt{\frac{G M}{d}}+\frac{1}{2} \frac{m}{M} \sqrt{\frac{G M}{d}} . \tag{28}
\end{equation*}
$$

We can conclude that the speed of the secondary mass in the reference frame centered on the fixed primary mass of Newton can be reinterpreted by considering two parts:

- the so-called Galileian part represented by the formula $\sqrt{G M / d}$, which does not depend on the orbiting mass and generates an orbit fixed in the space;
- and the speed part given by the relation $(m /(2 M) \sqrt{G M / d}$, which causes the rotation of the orbit, which is proportional to the secondary mass.


## 4 Formula for the Inertial $\square$ Non-Inertial Reference Frame Transformation

When Newton, still very young, had the genial intuition that the acceleration of the Moon and the acceleration of the bodies on the Earth's surface were connected by the relation of the inverse square of distance, he needed to check his idea with an experimental test. Unfortunately the data available to him were not sufficiently precise and he was forced to leave his attempt. Much time later, having obtained more reliable astronomic data, he made another attempt. Contrary to his original expectations, he found that there is not a perfect equivalence between his hypothesis and
the reality, since, for a real equality the law should be of the type

$$
\begin{equation*}
\mathrm{F}=\frac{\text { Const. }}{\mathrm{d}^{2.016731516}} \tag{29}
\end{equation*}
$$

instead of an exponent of d exactly equal to 2 , as he assumed. This fact was a problem also for Cotes, author of the original introduction to the Principia. Indeed, Cotes, to avoid possible criticism, states that it is true that in the case of the Moon the law of the inverse square is not completely satisfied, but it is also true that the deviation can be justified by considering the perturbations caused mainly by the Sun. This deviation is given by Newton in an incomplete way, justifying only a part of it. Just in this context, continuing his research in this direction, with one of the most valuable pages of his masterpiece, and using also his so-called fluxion calculus (today's differential calculus), he shows the following (Prop. XLV \{ Problem XXXI).
The exponent of $d$ in the equation (29), for an elliptical orbit with small eccentricity, is a function of the forward shift or backdating of the periastron or, as it was usual to say at the time, of the apses of the orbit, according to the relation

$$
\begin{equation*}
\left.3 \square \frac{360^{\circ}}{360^{\circ}+\square^{\circ}}\right]^{2}=3 \square \frac{2 \square}{2 \square+\square^{\mathrm{rad}}} \|^{2} . \tag{30}
\end{equation*}
$$

Obviously, if the forward shift or backdating is zero, the exponent of $d$ is exactly equal to 2 . Corollary 1 of Prop. XLV, always concerning the Moon, states textually that The centripetal force, thus, decreases according to a proportion slightly greater than the square, but 59: 75 times closer to the square than to the cube Even here we see the care expressed by Cotes in the preface of the Principia.
But we have to say that even the perihelia, or the apses of the planets, are not fixed but have a considerable shift in the same direction of the revolution of the planets around the Sun. In particular, the lunar perigee has a more remarkable shift than the perihelia of all planets. This fact is already a question point. Let us observe that, while the ratio between the mass of Jupiter, the biggest planet of our system, and the Sun is equal to $1 / 1040$, the ratio between the mass of the Moon and the mass of the Earth is much higher, being equal to about $1 / 81$; this let us forecast, on the basis of the previous arguments, a strong recoil of the Earth because of the gravitational mass of the Moon. Having said that, taking into account that the apses of the Moon have a forward shift equal to $3^{\circ} 3^{\prime}$ in a complete revolution of about 27 days, then, in such a case, we find that the exponent of $d$ coincides with the exponent given by (29). Newton, with the well-known two-body procedure, which disregards the movement of the primary mass, finds for all the planets and therefore even for the moon, an orbit absolutely fixed in the space. Consequently he must explain, by means of other causes, a discrepancy of the exponent of the mean radius whose value is 0.01646 , just equal to all the observed $3^{\circ} 3^{\prime}$. He attributes this value to the influence of the Sun on the lunar orbit, but, as we have already said, he is only able to justify a part of the said discrepancy (Prop. LXVI \{ Theorem XXVI etc.). Many years after, the French astronomer and mathematician Clairaut (1747) stated officially that the lunar motions are incompatible with the Newtonian theory of gravitation [7], obtaining even the agreement of Euler, and suggested the existence of an additional force besides the Newtonian force, inversely proportional to the cube of the distance. In a second instance, even more sensationally, he denied himself. When he took part in the Competition announced by the Academy of St. Petersburgh he showed that what he had previously stated was not true, maybe only with the purpose to win. And still today, the Moon has some completely unexplainable motions [4, p. 222].

In view of the previous considerations, the Earth, because of the lunar mass, must move back ${ }^{9}$. This backward movement can be interpreted by the terrestrial observer, as already said, like a forward shift of the lunar perigee. Therefore we have

[^7]\[

$$
\begin{align*}
& 3 \square \frac{2 \square}{2 \square+\square^{\mathrm{rad}}} \|^{2}=\frac{1}{-1+\frac{1}{2} \frac{m}{M+m}}  \tag{31}\\
& \square 3 \square 1 \square \frac{1}{2} \frac{m}{M+m} A^{2} \square 3 \square 1 \square \frac{m}{M+m}=2+\frac{m}{M+m} \square 2+\frac{m}{M}
\end{align*}
$$
\]

and so we obtain

$$
\begin{equation*}
F_{M}=G \frac{M m}{d^{2+\frac{m}{M+m}}} \square G \frac{M m}{d^{2+\frac{m}{M}}} \tag{32}
\end{equation*}
$$

or

$$
\begin{equation*}
F_{M} \square=G \frac{M m}{d^{2.012161826}} \tag{33}
\end{equation*}
$$

with a residual, equal to 0.045697 , that must be justified by other causes. Formula (32) can be seen like the transformation of Newton's law when the observer anchored to the inertial reference frame of the mass baricentre is substituted by the observer centered on the primary mass.

## 5 The Shift of Mercury Perihelion

In this case (32) becomes

$$
\begin{equation*}
F_{\mathrm{s}}=\mathrm{G} \frac{\mathrm{Mm}}{\mathrm{~d}^{2.000000165}} \tag{34}
\end{equation*}
$$

which in practice is the same as the correction of Newton's law previously proposed by the astronomer Hall. He evidently starts from the assumption that the shift of Mercury perihelion is equal to 43 '" per century and, using Newton's relation, concludes that the exponent of $d$ is equal to 2.00000016 for all the planets.

Actually, according to (21), we deduce that the apparent forward shift of the perihelion of Mercury due to the real backward shift of the Sun, along a revolution period of 88 days, is equal to

$$
\square^{\circ}=180^{\circ} \frac{\mathrm{m}}{\mathrm{M}+\mathrm{m}}=180^{\circ} \square 0.000000165=0.000029728
$$

In connection with the shift of the perihelion of Mercury we must do some considerations about some inconsistencies contained in the calculations of Le Verrier and Newcomb. The meticulous French astronomer applies literally the theory of arbitrary constants by Laplace, father of the modern theory of perturbations. This theory, among other things, has also the purpose of determining the unknown masses of some planets starting from the distortions that these masses induce on the Keplerian orbit (fixed in the space) of another planet. Since Mercury has no satellites, he was forced to deduce the approximate amount of its mass by means of a number of indirect checks. In its work [4] he textually states:
Dans plusieurs recherches, j'ai reduit cette masse a 1/3000000 (of the solar mass), en consideration des pertubations qu'elle a fait eprouver a la comete d'Encke, dans son passage au perihelie, en 1838. Mais, suivant M. Encke, le masse de Mercure serait encore plus faible, et egal ea 1/5000000 de la masse du Soleil. Nous conclurons donc seulement que cette masse est fort petite, et qu'elle ne peut avoir aucune influence sensible sur le calcul du grand axe de l'orbite ${ }^{10}$.
At this point we need a clarification. The data that an astronomer can use to determine the characteristics of an orbit are, among the others, the period of revolution and the mass of a planet.

[^8]Knowing these data it is possible to calculate the semi-axis a of the orbit of a planet through the well-known relation

$$
\begin{equation*}
\mathrm{T}=\sqrt{\frac{4 \square^{2} a^{3}}{\mathrm{GM} D+\frac{\mathrm{m}}{\mathrm{M}}}}, \tag{35}
\end{equation*}
$$

The astronomers write down (35) in the following form

$$
\begin{equation*}
\mathrm{n}=\mathrm{a}^{\square 3 / 2}(1+\mathrm{m})^{1 / 2} \sqrt{\mathrm{k}} \tag{36}
\end{equation*}
$$

assuming that n is the arc, expressed in sexagesimal seconds, that the planet covers in 24 hours, that the mass of the Sun is equal to 1 , and that m is equal to the ratio of the mass of the planet and the mass of the Sun. Furthermore they assume that a is the major semi-axis of the planet orbit, that the Earth-Sun distance is equal to 1 , and that $\sqrt{\mathrm{K}}=3548.18760696510$ of sexagesimal arc [6, p. 198]. When Le Verrier states that the mass of Mercury is negligible compared with the mass of the Sun, he just wants to state that no appreciable errors are done. This undemonstrated certainty makes explicit reference to the following statement by Laplace [5].
We have determined, [5, Chapiter V], the arbitrary constant quantities, so that the mean motion and the equation of the center may not be changed by the mutual action of the planets. Now we have, in the elliptical hypothesis,

$$
\frac{1+m}{a^{3}}=n^{2}
$$

the mass of the Sun being put equal to unity. Hence we obtain

$$
\begin{equation*}
a=n^{\square 2 / 3}(1+m)^{1 / 3} \square n^{\square 2 / 3}=1+\frac{1}{3} m \tag{37}
\end{equation*}
$$

for the semi-transverse axis, which must be used in the elliptical part of the radius vector. If we suppose, in conformity to the principles assumed in (4078-4079, etc), that

$$
\mathrm{a} \square \mathrm{n}^{\square 2 / 3}
$$

(thus the gravitational masses of the planets are neglected) we must increase a , $\mathrm{a}^{\prime}$, etc., in the calculation of the elliptical part of the radius vector by the quantities (1/3)m etc.; but this augmentation is only sensible in the orbits of Jupiter and Saturn.
Thus, according to Laplace, in the case of Mercury, it is possible to neglect its mass. Let us see if this is true. We will demonstrate that this gratuitous assumption is completely false and noxious. From (36) we have

$$
\frac{d n}{d m} \square \frac{1}{2} a^{\square 3 / 2}=1 \frac{1}{2} m=\sqrt{k}
$$

from which it follows

$$
\begin{equation*}
\square \mathrm{n}=\frac{1}{2} \mathrm{ma}^{\square 3 / 2} \mathrm{~m}^{\square} \frac{1}{2} \mathrm{~m} \sqrt{\mathrm{k}} \square \frac{1}{2} \mathrm{ma}^{\square 3 / 2} \sqrt{\mathrm{k}} \tag{38}
\end{equation*}
$$

Assuming the following values [6] [8]:

$$
a=\frac{57.91 e l 1}{146.467 e \mathrm{l} 1}=0.3874433
$$

$$
\mathrm{m}=0.000000165
$$

(38) gives the following value

$$
\square \mathrm{n}=0.0012138^{\prime} \text { '. }
$$

This is the angle subtended by the arc of the orbit, expressed in sexagesimal seconds, for a mass variation equal to m , in 24 hours. In a century we have

$$
\square=0.0012138 \square 36524=44.33^{\prime \prime} .
$$

To be clearer, given Newton's results, let us suppose that Mercury covers a circular orbit around the Sun (Fig. 7). Let us admit, for the moment, that Mercury has a completely negligible mass. As a
consequence it runs along its orbit with the following speed:

$$
\begin{equation*}
V_{1}=\sqrt{\frac{\mathrm{GM}}{\mathrm{~d}}}=\sqrt{\frac{6.67 \mathrm{e} \square 8 \square 1989 \mathrm{e} 30}{57.91 \mathrm{ell}}}=4786339.784[\mathrm{~cm} / \mathrm{sec}] \tag{39}
\end{equation*}
$$

On the other hand, if we take into account even its mass, we will have:

$$
\begin{equation*}
V_{2}=\sqrt{\frac{\mathrm{GM}}{\mathrm{~d}}} \sqrt{\left[1+\frac{\mathrm{m}}{\mathrm{M}}\right.}=4786339.784 \square \sqrt{1.000000165}[\mathrm{~cm} / \mathrm{sec}] \tag{40}
\end{equation*}
$$



Thus there is a speed difference equal to

$$
\square \mathrm{V}=0.39442[\mathrm{~cm} / \mathrm{sec}]
$$

These two hypothetical bodies that run on the same orbit with distinct speeds, after a period of 88 days, equal to the period of revolution of Mercury, are separated by a distance given by the arc

$$
s=0.39442 \square 88 \square 24 \square 3600=2992254.144[\mathrm{~cm}]
$$

After a century they will be separated by the arc

$$
\mathrm{S}=\frac{2998854.144}{88} 36524=1244660781[\mathrm{~cm}]
$$

which is equivalent to an angle centered in the Sun equal to

$$
\square^{\mathrm{rad}}=\frac{1244660781}{57.91 \mathrm{ell}}=0.00021493
$$

or else equal to

$$
\square^{\prime \prime}=\frac{0.00021493 \square 180}{\square}=44.33^{\prime \prime} .
$$

This is equivalent to say that, in view of the theory of Newton, to neglect or to not know the secondary mass implies, in the case of Mercury, an error practically equal to the supposed forward shift ${ }^{11}$.
We already stated at the beginning of this argument that Le Verrier assumes that Mercury has a mass (expressed in grams) comprised in the interval

$$
\square \mathrm{m}=\left[\begin{array}{ll}
3.978 \mathrm{e} 26, & 6.63 \mathrm{e} 26
\end{array}\right] .
$$

[^9]If we make all the calculations with these values, we have an uncertainty in the determination of the planet equal to 37 " in a century, practically equal to the 38 " that he, with great resonance, communicated to the scientific community of his age, obviously thinking to the influence of the unexisting planet Vulcan.
The work by Newcomb is only slightly different; he, at the end of the calculations, deduces for Mercury a mass comprised between the limits

$$
\frac{1 \pm 0.35}{7900000}
$$

i.e.

$$
\square \mathrm{m}=[3.4 \mathrm{e} 26, \quad 1.64 \mathrm{e} 26]
$$

that implies an uncertainty per century equal to 24 ". To completely appreciate what we have just said, we must first state the following. Actually, even with the two-body procedure of Newton, the secondary mass $m$ is taken into account. In fact, as we already said, from Fig. 3 we deduce that the fictitious orbit that the mass m covers around M , assumed fixed, has a radius consequently given by

$$
\begin{equation*}
\mathrm{d}=\mathrm{x}_{\mathrm{m}}=1+\frac{\mathrm{m}}{\mathrm{M}} \tag{41}
\end{equation*}
$$

where $\mathrm{x}_{\mathrm{m}}$ is a radius that can be called the real radius of the orbit covered by m , since it is covered around the baricentre K. Thus, when we neglect the mass of the planet, the length of the semi-axis of the orbit does not change. On the other hand, nothing prevents us to think that the radius of the orbit of two masses is fixed, if one is equal tom and the other is zero. In such a case the increase of the radius neglected by Le Verrier becomes an arc increase orthogonal to it ${ }^{12}$. The latter becomes a localization error of the planet on its orbit, if we want to stay still in the context of the two-body problem like it was originally set by Newton, that coincides with the missing shift of Mercury's perihelion. Therefore, if the astronomical calculations are done with these approximations, we are forced to accept the consequent and strong implications with the greatest caution. On the other hand, if we admit the physical existence of the recoil, we have that the above-mentioned radius increase or speed increase become completely a real backward shift of the central mass or an apparent forward shift of the perihelion of the orbit considered, in case we still would continue to think of a central mass fixed in the space.

## 6 Integration of the Equation and the Orbital Spin

In polar coordinates we can write

$$
\mathrm{m}
$$

Setting, as usual, $u=1 / r$, we have

$$
\begin{equation*}
\frac{\mathrm{d}^{2} u}{\mathrm{~d} \square^{2}}+\mathrm{u} \square \frac{\mathrm{GMm}}{\mathrm{~h}^{2}} u^{\frac{m}{M}}=\frac{\mathrm{GM}}{\mathrm{~h}^{2}} \mathrm{u}^{\square} \tag{42}
\end{equation*}
$$

In case of an ellipse with small eccentricity, we introduce a parameter $u_{0}$ (taking into account that $u$ $=1 / \mathrm{r}$ ) very close to the inverse of the mean radius of the orbit. We can write:

$$
\begin{equation*}
\mathrm{u}^{\square}=\left(\mathrm{u} \square \mathrm{u}_{\mathrm{o}}+\mathrm{u}_{\mathrm{o}}\right)^{\square}=\mathrm{u}_{\mathrm{o}}^{\square} \stackrel{\square}{\square} \square \frac{\mathrm{u}}{\mathrm{u}_{\mathrm{o}}} \boldsymbol{B} \tag{43}
\end{equation*}
$$

Since $u_{o}$ is very close to $u$, the term $\left(1-u / u_{o}\right)$ is very small and almost equal to zero. Then, with a good approximation, we have

[^10]from which it follows ${ }^{13}$
\[

$$
\begin{equation*}
\frac{\mathrm{d}^{2} \mathrm{u}}{\mathrm{~d} \square^{2}}+\mathrm{u} \square \frac{\mathrm{GM}}{\mathrm{~h}^{2}}\left(\mathrm{u}_{0}^{\square} \square \square \mathrm{u}_{0}^{\square}+\square \mathrm{u}_{o}^{\square \square \mathrm{u}} \mathrm{u}\right) . \tag{45}
\end{equation*}
$$

\]

Setting

$$
\begin{gather*}
a=\frac{G m}{h^{2}} u_{0}^{\square}, \quad b=\square \frac{G M}{h^{2}} \nabla u_{0}^{\square}  \tag{46}\\
c=\frac{G M}{h^{2}} \Delta u_{0}^{\square \square 1}=\frac{G M}{h^{2} u_{0}} \frac{m}{M} u_{0}^{m / M},  \tag{47}\\
d=a+b \tag{48}
\end{gather*}
$$

we finally have, for an elliptical orbit with small eccentricity, as mentioned above,

$$
\begin{equation*}
\frac{d^{2} u}{d \square^{2}}+u \square d+c u \tag{49}
\end{equation*}
$$

hose solution is the following:

$$
\begin{equation*}
\mathrm{u}=\frac{\mathrm{d}}{\mathrm{c} \square 1} \mathrm{C}_{1} \sin (\sqrt{1 \square \varnothing} \square)+\mathrm{C}_{2} \cos (\sqrt{1 \square \varnothing} \square)=\frac{\mathrm{d}}{\mathrm{c} \square 1}+\mathrm{C} \cos (\sqrt{1 \square \varnothing \square \square \square}) \tag{50}
\end{equation*}
$$

or, in a simpler form:

$$
\begin{equation*}
u=\frac{d}{c \square 1}+C \cos (\sqrt{1 \square \varnothing})=A+C \cos (\sqrt{1 \square \varnothing}) . \tag{51}
\end{equation*}
$$

Using polar coordinates we find

$$
\begin{equation*}
r=\frac{p}{1+\square \cos (\sqrt{1 \square \varnothing})} \tag{52}
\end{equation*}
$$

Differentiating and equating to zero, we deduce that after a complete revolution

$$
\begin{equation*}
(\sqrt{1 \square \varnothing} \square)=2 \square . \tag{53}
\end{equation*}
$$

Since we have

$$
\begin{equation*}
c=\frac{G M}{h^{2} u_{0}} \frac{m}{M} u_{0}^{m / M}=\frac{G M}{\left(r_{0} v_{0}\right)^{2}} r_{0} \frac{m}{M} u_{0}^{m / M}=(1) \frac{m}{M} u_{0}^{m / M}=\frac{m}{M} u_{0}^{m / M} \tag{54}
\end{equation*}
$$

and, from (53),

$$
\begin{equation*}
\square=\frac{2 \square}{\sqrt{1 \square c}}=\frac{2 \square}{\sqrt{1 \square \frac{m}{M} u_{o}^{m / M}}} \square 2 \square \square^{1}+\frac{1}{2} \frac{m}{M} u_{o}^{m / m}= \tag{55}
\end{equation*}
$$

and since we also have (in the case of Mercury or the other planets)

$$
u_{0}^{m / M}=u_{o}^{0.000000065} \square 1
$$

we find from (55) that, after a complete revolution, the rotation angle of the axis of the orbit is

$$
\square=2 \square+\frac{1}{2} \frac{\mathrm{~m}}{\mathrm{M}}=2 \square+\square \frac{\mathrm{m}}{\mathrm{M}}
$$

and thus it is increased of the quantity

$$
\begin{equation*}
\square \square=\square \frac{\mathrm{m}}{\mathrm{M}} \tag{56}
\end{equation*}
$$

Then (52) can also be written as

[^11]\[

$$
\begin{equation*}
r=\frac{p}{1+\square \cos =\sqrt{1 \square \frac{\mathrm{~m}}{\mathrm{M}}} \square} . \tag{57}
\end{equation*}
$$

\]

Thus the orbit is characterized by a spin given by

$$
\begin{equation*}
\frac{d \square}{d t}=\&=\frac{\square}{T} \frac{m}{M+m}=\frac{\square}{\sqrt{\frac{4 \square^{2} d^{3}}{G M}} \frac{m}{M+m}+\frac{m}{M}-}=\frac{1}{2} \frac{m}{M} \sqrt{\frac{G M}{d^{3} G^{1}+\frac{m}{M}}} . \tag{58}
\end{equation*}
$$

In the case of the Sun-Mercury pair, in a century, we have a rotation of the major semi-axis of the orbit towards the point $\square$ that, in sexagesimal seconds, is approximately equal to

$$
\frac{\square^{\mathrm{rad}}}{\mathrm{sec}}=\frac{0.3285 \mathrm{e} 27}{2 \square 1989 \mathrm{e} 30} \sqrt{\frac{6.67 \mathrm{e} \square 8 \square 1989 \mathrm{e} 30}{(57.11 \mathrm{e} 1)^{3} \square+\frac{0.3285 \mathrm{e} 27}{1989 \mathrm{e} 30}}} \square 6.96 \mathrm{e} \square 14
$$

or

$$
\frac{\square^{\prime \prime}}{100}=6.9692 e \square 14 \square \frac{180}{\square} \square 3600 \square 36524 \square 86400 \square 45^{\prime \prime}
$$

On the other hand, in the case of the Sun-Jupiter pair, just in 1 year, we have

$$
\begin{gathered}
\frac{\square^{\mathrm{rad}}}{\mathrm{sec}}=\frac{1897.1 \mathrm{e} 27}{2 \square 1989 \mathrm{e} 30} \sqrt{\frac{6.67 \mathrm{e} \square 8 \square 1989 \mathrm{e} 30}{(778.34 \mathrm{e} 1)^{3} \square \cap+\frac{1897.1 \mathrm{e} 27}{1989 \mathrm{e} 30}}} \square 7.99 \mathrm{e} \square 12 \\
\square^{\prime \prime} / 1=7.9955 \mathrm{e} \square 12 \square \frac{180}{\square} \square 3600 \square 365.24 \square 86400 \square 52^{\prime \prime} .
\end{gathered}
$$

Finally, let us observe that, if we find the force law from (57), we have:

$$
F_{M}=\square \frac{m m^{2}}{p} u^{2}=1 \square \frac{m}{M}+\frac{m}{M} p u=\square \frac{m h^{2}}{p} u^{2}=1+\frac{m}{M} p u
$$

This relation becomes

$$
\begin{equation*}
F_{M}=\square \frac{m^{2}}{p} u^{2} A^{1}+\frac{m}{M} p u=\square \frac{G M m}{d^{2}} \square \frac{m}{M} p \frac{G M m}{d^{3}}+. .+ \tag{59}
\end{equation*}
$$

and therefore the observer centered in the primary mass sees the action of an additional force inversely proportional to the cube of the distance on the secondary mass (isn't it that one of Clairaut ?). If we assume that the constant p and the variabled are almost the same, we obtain again Newton's formula:

$$
F_{M}=\square \frac{G M m}{d^{2}} \square \frac{m}{M} p \frac{G M m}{d^{3}}=\square \frac{G M m}{d^{2}}=1+\frac{m}{M}
$$

## 7 On the Theory of Planetary Perturbations

According to Newton's theory, we deduce that, if the Moon would fall on the Earth, even the Earth, affected by the lunar gravity, would move towards the Moon. Thus an earthly observer would have an acceleration and therefore a measurable movement compared with the fixed stars, that certainly are not involved in the said phenomenon. The same thing continuously occurs for the Sun that, due to the different planetary masses, undergoes continuous accelerations that force it to move in the same direction of the planets, around the common center of gravity. Thus an observer centered on a given planet should see the Sun that moves backward with its own motion on the relevant ecliptic, thus going towards the $\square$ point with a speed that is a function of the secondary mass considered. The
following table shows the various backward movements, expressed in sexagesimal seconds, calculated by means of one of the previous formulae, that each planet causes to the Sun (whose mass is assumed equal to 1989 e 30 [g] [8]), or the apparent forward shift of the perihelion of the planet, in a century.

| Planet | Distance | Mass | Revolution | Backward shift |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{cm}[\square \mathrm{ell}]$ | $\mathrm{g}[\square \mathrm{e} 27]$ | days or years | per century |
| Mercury | 57.91 | 0.3285 | 88 | $44{ }^{\prime}{ }^{\prime}$ |
| Venus | 108.21 | 4.8714 | 224.7 | 258 |
| Earth | 149.467 | 5.976 | 365.24 | 195 |
| Mars | 227.94 | 0.645 | 687 | 11 |
| Jupiter | 778.34 | 1897.1 | 11.86 | 5206 |
| Saturn | 1427 | 567.7 | 29.46 | 627 |
| Uranus | 2869.6 | 86.7 | 84.01 | 34 |
| Neptune | 4496.6 | 105.2 | 164.8 | 21 |
| Pluto | 5947 | 0.00989 | 247.7 | 0.0013 |

It is immediately evident that Jupiter causes approximately, in a year, a $52^{\prime \prime}$, backward shift, and the Earth a 2" backward shift. The difference is equal to 50 ", and is, strangely, exactly same that should be due to the lunisolar precession. Before we describe a deeper study of this subject, let us give an insight to the theory of planetary perturbations. In the following table we show, for each planet, the perihelion shift measured starting from the पpoint, temporarily considered fixed in the space, along a century [6].

| Planet | Perihelion forward shift <br> in seconds of arc per century |
| :---: | :---: |
| Mercury | $5600.821^{\prime \prime}$ |$|$| Venus | $6064.415^{\prime}$, |
| :---: | :---: |
| Earth | $6190.672^{\prime \prime}$ |
| Mars | $6627.197^{\prime \prime}$ |
| Jupiter | $5799.664^{\prime \prime}$ |
| Saturn | $7053.272^{\prime \prime}$ |
| Uranus | $5344.819^{\prime \prime}$ |
| Neptune | $5129.875^{\prime \prime}$ |

The reference frame adopted to study the motion of the planets of the solar system has its origin in he center of the Sun, obviously considered fixed in the space, and an axis directed in the $\square$ point, that is one of the intersections of the ecliptic and the equatorial plane. The major semi-axis of the ellipses covered by the various planets is not fixed compared with the $\square$ point, but it has a forward shift in the direction of a backward revolution, covering the amount shown in the table. So let us see the way in which these forward shifts are justified in the Classical Celestial Mechanics.
Stating in advance that in a year there is a relative movement, between the Sun and the $\square$ point, equal to approximately 50 ' per year, and that this movement is explained by Newton as if it was a consequence, given the assumed fixed position of the Sun, of the whole phenomenon of the lunisolar precession, we deduce that the theory of planetary perturbations must explain, for every planet, the residual forward shift shown in the following table.

| Planet | Residual forward shift <br> in seconds of arc per century |
| :---: | :---: |
| Mercury | 580 " |
| Venus | 45 |
| Earth | 1170 |
| Mars | 1607 |
| Jupiter | 779 |
| Saturn | 2033 |
| Uranus | 324 |
| Neptune | 109 |

It is well-known that the Newtonian theory is able to explain only 537 "of the whole shift of the perihelion of Mercury. The orbit of Venus is almost circular, and therefore it is difficult to appreciate its forward shift. Perhaps for Mars a tenth of seconds cannot be explained. On the other hand if we accept the present version of the backward shift, the following residuals should still be explained:

| Planet | Exponent | Residual forward shift <br> in seconds of arc per century |
| :---: | :---: | :---: |
| Mercury | $2,000.000 .165$ | $536^{\prime}{ }^{14}$ |
| Venus | $2,000.002 .449$ | $-213^{14}$ |
| Earth | $2,000.003 .004$ | 975 |
| Mars | $2,000.000 .324$ | 1596 |
| Jupiter | $2,000.953 .795$ | 779 |
| Saturn | $2,000.285 .149$ | 1406 |
| Uranus | $2,000.043 .589$ | 290 |
| Neptune | $2,000.052 .790$ | 21 |

Although such a work can be done in a distinct article, we can already forecast a model that could easily give quite correct results. Actually we can admit, in a first instance, that the Sun is forced to rotate around the Sun-Jupiter baricentre with a given speed. Therefore we can assume, as a first scheme for calculations, that the system is formed by two masses, of which the primary is forced to rotate around the said center. However we must observe that the residuals that should be explained are, in both cases, approximately the same. But we must still re-examine the whole question of the precession, which we will briefy consider in the following section.

## 8 On the Problem of the Lunisolar Precession

There is no doubt that Newton cannot attribute all the shift (experimentally observed by the astronomers and measured by the above-mentioned 50 '" of arc per year) between the Sun and the $\square$ point to the one and only phenomenon of the precession. This is due to the fact that the Sun must move towards the —point, even by a small distance, because of the gravitational force exerted by the planets. On the other hand, Newton himself, in the third book of the Principia, textually admits (Prop. XII. Theorem XII) that
The Sun has a continuous motion, but never moves at a great distance fromthe common center of gravity of all the bodies.

[^12]And, if this is true, the Sun must necessarily move, on the ecliptic traced by itself, in a backward direction, towards the $\square$ point by a distance that still has to be determined. On the other hand, in the hypothesis that the demonstration of Newton on the cause of the precession was completely wrong, the fact that Jupiter would force the Sun to move towards the $\square$ point exactly by $\left(52^{\prime \prime}-2^{\prime \prime}\right)=50^{\prime \prime}($ which is the result of a calculation due to the independent determination of the masses of the Sun, Jupiter, and the Earth) should and would perhaps be the greatest success of Newton's theory.
But should the explanation of Newton about the precession be completely excluded?
A first fact is certain.
The whole 50 '" per year cannot be attributed completely to the equatorial bulge of the Earth. This item is sure and undisputable. Then in what proportions must the above-mentioned shift be distributed between the real motion of the Sun and the said effect?
This is the problem.
On the other hand it is clear that, if the precession was completely due to the motion of the Sun, the formula

$$
\begin{equation*}
F=\frac{G M m}{d^{2+\frac{m}{M}}} \tag{60}
\end{equation*}
$$

would be absolutely exact. If, instead, the precession should be attributed completely to the equatorial bulge, the exact formula would be:

$$
\begin{equation*}
\mathrm{F}=\frac{\mathrm{GMm}}{\mathrm{~d}^{2}} \tag{61}
\end{equation*}
$$

So it seems clear that in an intermediate case the gravitation law should be of the type

$$
F=\frac{G M m}{d^{2+\square \frac{\mathrm{m}}{\mathrm{M}}}}
$$

with a coefficient $\quad][0,1]$ that can be determined even by repeated attempts. Another important point that should be taken into account is that our criticism of the two-body problem assumes, among the other things, that gravitational actions are instantaneous and that there is perfect equality between gravitational mass and inertial mass.
So it seems reasonable to enumerate the objections that can be moved against the interpretation of the precession done by Newton.

- The first is the already said objection concerning the proper motion of the Sun on the ecliptic, caused by the planets.
- The second is that the precession is variable and its time law, given by Newcomb's relation (1900) ${ }^{15}$, is found to be [9]

$$
\begin{equation*}
50.2564+0.000222(\text { year }-1900) \tag{63}
\end{equation*}
$$

Thus, since 1900 till today, it is increased by 0,023 ". This increase is completely incompatible with the invariability of the masses of the Moon and the Earth (Newton (Prop. XXXIX. Problem XX) attributes $\square 41$ " to the action of the Moon and $\square 9$ " to the action of the Sun on the terrestrial bulge ${ }^{16}$ ). Instead, it is compatible with the evident variable composition of the different backward shifts that affect the Sun because of every planet. Actually, this composition is a function of the relative positions of the single planets (conjunctions, oppositions, quadratures, etc.). In fact, this study, never attempted, should enlighten the phenomenon, clarify the connections between the said

[^13]variabilities, and contemporaneously give a way to determine the speed of the gravitational perturbation. This possibility follows because a foreseeable variation of the previous shift due to a particular configuration of the planets could show its effects after some time. The physicists W. Cruttendon and V. Dayes [9] think that the explanation of Newton is completely inconsistent and that the said 50 " per year are totally due to the proper motion of the Sun because it could be one of the component stars of a binary system. And the Sun and Jupiter do they form a binary system ?

- Another subtle consideration has been suggested by K. Homann [10]. He completely denies any validity of Newton's explanation, and attributes the said 50 " to the orbital motion of the Sun around a star of the Sirius group.
- Actually, Newton's explanation bases on a similitude between the motion of the lunar nodes and the effect of lunisolar actions on the terrestrial bulge. We immediately note that this hypothesis is strongly forced: contrary to the equatorial bulge, the Moon is not strictly anchored to the Earth. Its orbit could be orthogonal to the ecliptic plane without affecting the terrestrial globe. On the contrary, if the Earth and the Moon were connected in a rigid structure, then only in that case the action of the Sun on the Moon would actually cause the wellknown gyroscopic effect. The Moon covers an orbit slightly inclined compared with the ecliptic plane. The straight line characterized by the intersection of the lunar orbit projected on the celestial sphere and the ecliptic plane determines the said nodes N and N '. These nodes advance clockwise (just as would do the $\square$ point) for the different actions that the Sun exerts on the Moon when it lies above or below the ecliptic plane.
The subject in the final item deserves a deepest consideration; so, to see the relative deductions in a better way, let us consider Fig. 8:


Fig. 8
where we show the orbit of the baricentre G around the Sun (the arc nGn ) and the orbit of the Moon and the Earth around the common barycentre G. All these orbits are covered clockwise. The intersection nodes of the orbit of the Moon and the orbit of the Earth around the Sun, or better, of the baricentre G around the Sun, are marked by the letter n . In the same figure we have also shown the ecliptic (circumference of radius GS), i.e. the projection on the celestial sphere of the apparent trajectory that the Sun covers around G, and the orbit of the Moon (ellipse characterized by the arc N'LN; we have an ellipse because, although the lunar orbit is practically circular, this circumference is inclined by $5^{\circ} 9$ ' compared with the circumference that represents the ecliptic) also projected on the celestial sphere, like it can be seen by an observer centered in the baricentre G. (Here it is clearly evident that we could make our reasoning on the nodes nn , instead of the nodes $\mathrm{NN}^{\prime}$ ). The lunar nodes N and $\mathrm{N}^{\prime}$, taken into account that the lunar orbit is assumed to be fixed - as
required by the two - body problem in Newton's formulation -, move clockwise, as can be seen in the figure, and go towards the Earth that, on the contrary, is forced to move counterclockwise by the mass of the Moon around the common barycentre G. Even in this case it is quite evident that we cannot consider the Earth as it was absolutely fixed and therefore attribute the whole motion of the nodes to the nodes themselves, like we did for the motion of the $\square$ point. Instead we have seen that the central body can be considered fixed provided we attribute a clockwise rotation equal to $\square=180^{\circ} \mathrm{m} / \mathrm{M}$ to the axis of the orbit, and, just as in the case of the Moon, given its exceptional mass compared with the mass of the Earth, the said forward shift is at a maximum and equal to $2.18685^{\circ}$ in 27.32166 days. On the other hand, the mass of the Moon has been determined just by means of an astronomical detection of the small ellipse that the Earth covers around the common center of gravity [11, p. 335] and thus it is impossible to deny that a proper motion of the Earth towards the said nodes exists. Instead, according to Newton, the whole forward shift of the nodes -a clockwise motion that covers a whole arc of $360^{\circ}$ in 18.6 years and thus, in a whole period of revolution of the Moon, is equal to $1.4478^{\circ}$, should be attributed to the action of the Sun that, in the attempt to make coincident the lunar orbit and the ecliptic plane, causes the said clockwise motion of the nodes. Newton imagines a set of moons that form a ring ${ }^{17}$ and thus extends the motion of the nodes to such a ring. Subsequently he thinks to a real solidification of these moons and thus imagines that this solidified ring takes the place of the equatorial bulge of the Earth. Finally Newton thinks that this ring is firmly anchored to the spherical earthly globe to which it transmits, without any reduction, the motion of the above-mentioned ring. Assigning to the Moon a force 4 times greater than the force of the Sun (assumed equal to 1 ), he establishes that the precession phenomenon has to be attributed to the Moon ( $\square 40$ ") and to the Sun ( $\square 10$ "). Furthermore, the Moon should exhibit a forward shift of the perigee, completely independent from the clockwise motion of the nodes, that covers $360^{\circ}$ in 8.85 years and corresponds to a forward shift of the periastron equal to $3.0429^{\circ}$, always in a period of revolution equal to 27.32166 days. This forward shift, in this case, would be due only and exclusively to the other action of the Sun on the Moon that, in such a case, sometimes reduces and sometimes increases the action of the terrestrial gravitation. All of this by thinking that the Earth is absolutely fixed. On the other hand, it is wellknown that Newton, even in the case of the precession, adjusts with suitable fudgefactors [12, p. 22 and following] the parameters of the problem until there is coincidence between the theoretical solution and the experimental data. Westfall [13] has shown that Newton, even in this very special circumstance, manipulates many times the inclination of the equator on the ecliptic plane, the density of the Earth, and the ratio between the lunar attraction and the solar attraction in order to obtain that the final result is equal to the 50 " known in its age (today the accepted value is 50.4 " ${ }^{18}$. Certainly we cannot exclude a priori that, because of the equatorial bulge, the $\square$ point moves towards the Sun covering an unknown distance, as well as we cannot deny that the Sun is pushed towards the same point, because of the gravity of all planets and in particular of Jupiter. The problem is the determination of the subdivision of the relative motions between the abovementioned causes. Unfortunately the theory of the precession is founded on assumptions that are not certain and that, opportunely changed according to various criteria, imply a shift of the $\square$ point that is largely variable. A typical example is the work of the famous mathematician Daniel Bernoulli, who, in unsuspicious times, starting from some experimental observations, according to a report by d'Alembert [14], came to the conclusion that the ratio between the forces that the Moon and the Sun exert on the equatorial bulge is $5 / 2$. Leaving unaltered all the other hypotheses of Newton, included

[^14]the solidification of the ring of moons with the terrestrial globe, this implies that only 35 " of the total 50 " experimentally observed can be justified. If the opinion of Bernoulli had survived ${ }^{19}$, Celestial Mechanics would already have serious problems of survival in the ancient 1700 because it would have to justify much larger forward shifts of the perihelia of the various planets.
Thus it is easy to imagine that the parameters relative to the precession were always chosen in such a way to assign the entire shift only to the point $\square$ On the basis of these remarks, leaving a thorough examination to another paper, we can only conclude that only precise experimental measurements, opportunely set out, can establish with certainty both the absolute motion of the Sun and the $\square$ point, even taking into account the subtle remarks by K. Homann concerning the non-decrease of the precession [10].

## 9 On the General Relativity Principle

We want to clarify a common way of thinking according to which the truthfulness of Galilei's law on the fall of bodies is directly related to the equality of the inertial mass and the gravitational mass. Actually the reverse is true.
If Galilei's law is not verified then there can be the perfect equality of the inertial and gravitational masses. Even this commonplace can be retraced up to Newton (the independence of the oscillation times of pendulums from the masses used). In fact, if it is true that the inertial mass and the gravitational mass are identical or proportional, then it is true Newton's law for the fall of bodies, i.e.

$$
\begin{equation*}
a_{m}=\frac{G M}{R^{2}}=1+\frac{m}{M} A=g=1+\frac{m}{M} \tag{64}
\end{equation*}
$$

and thus Galilei's law is not verified. Einstein, whose only purpose was the extension of the validity range of the Special Relativity, made recourse to Galilei's law, a law that no researcher ever cared to verify fully ${ }^{20}$, starting from Galilei himself, and that maybe none will care to verify. Actually, according to the historian Di Trocchio [12, pag. 14], in 1978 two researchers, C.B. Adler and B. Coulter decided to replicate Galilei's experiment (from the Tower of Pisa) and verified that the two balls reach the Earth with a very small delay. This delay does not justify completely the hypothesis of Aristotle according to which a body having double weight should reach the Earth with double speed, but however the heavier body falls first. Always Di Trocchio states that the Aristotelians could modify their theory because of this result. In this context we must observe that Adler and Coulter did not take into account the possible revision of the said experiment according to the consequences of the theory of Newton ${ }^{21}$. It is interesting to note that, given (64), the abovementioned researchers should have appreciated no difference with the naked eye, given the extreme smallness of the corrective term. If this difference is even perceivable without special devices, according to Newton, we must take into account other causes like, for example, the nonhomogeneity of the subsoil (concentrated masses) that amplify the phenomenon. In any case, if we accept Newton's theory, we must say that the Aristotelians were fully right when they stated that a body with double weight reach the Earth with double speed, since the mentioned very small speeds

[^15]sum up and are masked by the common identical speed, much greater, that the Earth gives in an equal way to all the bodies. In fact, given (64), while the acceleration g is the (Galileian) acceleration that the Earth exerts on all the falling bodies in the same way, the acceleration (gm/M) is the one that the generic falling body exerts on the Earth as a function of its own different mass.
Einstein, before the ultimate formulation of General Relativity, using the famous gedanken experiment of the lift in a gravitational field, deduced brilliantly, on the basis of the identity principle of inertia and gravitation, or better according to Galilei's law, the deviation of a light ray caused by a celestial body. But maybe, if we make a deepest analysis, since any photon has its own mass $\mathrm{h} \square \mathrm{C}^{2}$, a ray of light in a gravitational field, besides verifying Einstein's relation for that part of $\mathrm{a}_{\mathrm{m}}$ that we can briefly call the Galileian part, should also exhibit the same behaviour that it shows throughout a prism, just because perhaps the gravitational masses measure with extreme precision the smallest differences of the weight, due to the part represented by ( $\mathrm{gm} / \mathrm{M}$ ). And also the clocks should measure different times if they have different masses. Thus there is not one single spacetime, but many, each being a function of the secondary mass. Concerning the forward shift of Mercury perihelion a clarification is needed. According to General Relativity, a great gravitational mass like the Sun changes the geometry of space-time, so that an infinitesimal orbiting mass undergoes a real forward shift of the perihelion ${ }^{22}$. In the Newtonian language, this means that Newton's force is accompanied by an additional force that is proportional to the fourth power of d . According to the present version, this would be instead an apparent forward shift of Mercury's perihelion due to a real backward shift of the Sun, that should be attributed to the active gravitational mass of Mercury. If, even here, we used the language of non-Euclidean geometry, we could say that the curvature of the three-dimensional physical space around a mass is an exclusive function of the peripheral mass. Only when this mass is infinitesimal we would recover the traditional geometry (Lobacewsky). But it seems sufficiently evident that, whatever the gravity law would be, in the transformation from an inertial reference frame to another reference frame anchored to the central mass, we must take into account the remarks of Section 3. On the other hand we must admit that the observing window of Kepler, from which Newton deduced the law of gravitation, is only a very modest neighbourhood to explore, if it is compared, for example, with the radius of a galaxy. So the pretence of an extension of a law deduced in such a restricted context could be very hazardous; it is sufficient to think to the inner singularity of Newton's law (and Coulomb's law) for masses (or charges) when their distance goes to zero.

## 10 Conclusions

Certainly, due to the extension of the above-mentioned subjects, the present paper cannot be exhaustive. The lack of further analytical developments that take into account the mobility of the central mass forces this paper to be only the statement of a fundamental problem, that arises from the unaware correction made by Newton to the experimental (?) law of Galileo Galilei. On the other hand, it is well-known that the two-body problem is the basis of any perturbative theory, so one of its variants is a source of further analytical complications that must be overcome if we want to establish the possible internal consistence of the searched correction. The fact that the astronomers have always thought of a Sun absolutely fixed in the space, with all the relevant and consequential mistakes, contrary to the vain Newtonian assumption that the secondary mass, because of its gravity, moves the central mass, is more and more demonstrated by the extreme fact that one of the gravitational perturbative theories distributes on the entire Keplerian orbit the whole mass of the planet, nullifying completely the recoil of the central mass.
If we accept the validity of relation (64), we can understand that if Galilei really had executed the

[^16]famous experiment of the Tower of Pisa, even with modern devices, he would have never ascertained the infinitesimal difference given by the said formula. So, maybe, a precise experiment executed with that goal could give surprising results. Indeed, if it is true that the heaviest body falls first [12], this is even more surprising and conflicts with the extreme smallness of the difference that should be detected according to (64). In fact, unless the excessive closeness to the Earth surface would imply radical modifications, Galilei's law should be true only if the extreme smallness of the difference to detect would be true. But it is clear that, admitting the validity of Newton's law, the situation is completely different at the solar system scale, where the masses can be considered like points. In such a context, and mostly in the case of the Earth\{Moon and Sun\{Jupiter pairs, the m/M ratio is not negligible and gives rise to an undeniable shift of the primary mass, contrary to any statement by Newton in primis and by the mathematicians and astronomers Laplace, Clariaut, Euler, Tisserand, Newcomb, etc., until today.

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## 11 Notes about the Author

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# A Thousand Blazing Suns page 3 

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## Binaries to the Rescue

Before the density of stars in the core of a globular cluster can reach infinity, core collapse will be reversed, primarily by the formation of binary stars in the core. Binary stars can be formed in two ways, each of which requires the dense stellar environment found in collapsed-core clusters.

## ${ }^{\circ}$ Tidal-capture binaries

In one scenario, two stars pass within three stellar radii of each other. Tides raised on each star cause both stars to slow down and be captured in orbit around each other. This process of binary formation is known as tidal capture.

## ${ }^{\circ}$ Three-body binaries

In the other scenario, three single stars pass very near one another. The outcome is that two stars become a binary and the third star is saddled with the excess energy. Because this process uses three stars, the binary formed is called a three-body binary. Typically the two stars of a three-body binary are loosely bound, and most will eventually be disrupted by the close passage of another star. Calculations show that one in ten thousand of these binaries will survive, however, and then drive the post-collapse evolution of the cluster.

Once formed, binary stars can reverse core collapse by transferring energy to passing single stars: When a single star passes close to a binary, the binaryÕs orbit will shrink, causing a loss of gravitational potential energy. The single star benefits from this loss and gains kinetic energy. This three-body interaction causes both the binary and the single star to speed away from their mutual center of mass, and this increase in orbital speed within the cluster causes stars to move out of the core, lowering its density and reversing core collapse.


The central density of a globular cluster versus time. Note the rapid rise in the central density with the onset of core


A collapsed-core globular cluster has surface brightness that rises all the way to the center of the cluster. This plot
collapse. Once started, the process of collapse accelerates rapidly, causing the density of stars in the core to become up to ten billion times that of our solar neighborhood.
During collapse binary stars form that reverse the collapse and cause the core to undergo a series of oscillations. Plot courtesy of author.
shows how the surface density of red giants and neutron stars increases into the cluster center. Because of mass segregation the number of neutron stars at the center of a collapsed-core cluster far exceeds that of the giants and main sequence stars. Because of this mass segregation, most low-mass x-ray binaries and millisecond pulsars tend to be found near a cluster's center. Plot courtesy of author.

What if binary stars were present when the cluster first formed? In our neighborhood of the Milky Way, the majority of stars are binary systems, so we should expect to find primordial binary stars in globular clusters. Observations indicate that 10 to 20 percent of stars in globular clusters are binaries. If binary stars are already present, enough energy can be extracted from them to temporarily stave off core collapse. Such a cluster is likely to be in a quasi-equilibrium phase, in which it is using its binary stars but not undergoing full-blown core collapse. Even in this case the density of stars in the cluster core will be much higher than in an uncollapsed cluster. Eventually any primordial binaries in the cluster core will be used up and the core will fall into deep collapse. Then the scenarios we discussed earlier, in which new binaries form and reverse core collapse, come into play.

## The Masses of Stars

## Chapter index in this window - Chapter index in separate window

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To determine the masses of stars, Kepler's third law is applied to the motions of binary stars---two stars orbiting a common point. The greater the combined mass of the two stars, the greater the gravity acceleration is, and, therefore, the smaller their orbital period. A majority of the several hundred billion stars in the Galaxy are in a system with two or more stars orbiting each other. Usually the binary stars are spectroscopic binary stars. A spectroscopic binary system is two stars orbiting a common point at too great a distance away from us to resolve the two stars individually, but whose binary nature is indicated in the periodic shift of their spectral lines as they orbit around each other. Spectroscopic binary stars are used because (a) there are a lot more far away stars than nearby ones and (b) more importantly, you can easily measure their speeds from the doppler shifted lines.

Spectroscopic binary. binary too far away to resolve individual stars but whose binary nature is detected by periodic shifts in its spectral lines.


Newton's form of Kepler's third law gives the combined mass of the two stars: $($ mass $1+$ mass 2$)=$ (separation distance) $\left.)^{3 /(o r b i t a l ~ p e r i o d) ~}\right)^{2}$ if you use solar mass units, the A.U. for the distance unit between the stars, and the time unit of years for the orbital period. The total distance between the two stars is used in Kepler's third law, but their individual distances from the common point they orbit is used to determine the stars' individual masses.

# The center of mass is proportionally closer to the larger mass. 

Since stars have about the same mass (within a factor of 20), they both orbit around a common point, called the center of mass, that is significantly different from one of the star's center. The center of mass $($ C.M. $)$ is the point where $($ mass star 1$) \times($ C.M. distance 1$)=($ mass star 2$) \times($ C.M. distance 2$)$, or the point they would be balanced upon if the stars were on a stellar seesaw (it is the "x" in the figure below). The massive star is proportionally closer to the center of mass than the low-mass star and the massive star also moves proportionally slower than the low-mass star so its spectral lines have a smaller doppler shift.


Newton's Law of Gravity explains why this is. The gravitational force exerted by the massive star causes a large acceleration in the motion of the low-mass star, so the low-mass star moves faster and has a larger orbit. The weaker gravity exerted by the low-mass star produces a smaller acceleration on the massive star, so the massive star's orbital speed is less and its orbit is smaller. Think about how you could also explain this using Newton's second and third laws of motion.

The distance travelled by an object $=$ velocity $\times$ the time it takes. The distance travelled by the star is just the circumference of the orbit $=2 \pi \times$ the radius of a circular orbit and something similar for an elliptical orbit. Therefore, each star's C.M.-distance $r=$ the star's velocity $\times$ the star's orbital period $/$ $(2 \pi)$. This allows you to use the easily measured velocity in Kepler's third law and in the center of mass relations. The doppler shifts of the spectral lines are used to construct a radial velocity curve---a plot of the radial velocity (line of sight velocity) vs. time. The low-mass star will move proportionally faster than the massive star.

Uncertainty arises, though, if the binary orbital plane is inclined to our line of sight by an angle amount $i$. In that very common case, the radial velocity $=$ total velocity $\times \operatorname{Sin}($ inclination angle). The orbit's inclination angle $i$ ranges from $i=0^{\circ}$ for a face-on orbit (viewing the orbit from directly above the system) to $i=90^{\circ}$ for an edge-on orbit (viewing the orbit along its plane). The inclination angle can be approximately determined from the plot of radial velocity vs. time. If the binary is an eclipsing binary, then you know that $i=90^{\circ}$ because you see them periodically pass in front of each other. Eclipsing binaries also allow us to accurately determine the diameters of stars (discussed in the next section). The radial velocity measurement technique has also been used to find planets around other stars and to locate black holes from the doppler shifts they produce in the visible stars they orbit around.


Remember these rules:

1. Stars stay on the opposite side of the center of mass from each other.
2. The massive star moves slower than the low-mass star.
3. The center of mass is also the point where mass $1 \times$ velocity $1=$ mass $2 \times$ velocity 2

Using the distance of the center of mass from each star, you can proportion out the total mass to each star. Here are the steps to figure out each star's mass:

1. Find the total mass (mass star A + mass star B) from Kepler's 3rd law.
2. Find the proportion of each star's mass to the total mass from the center of mass: (mass star $\mathrm{A}) /($ mass star B$)=($ C.M. distance B $) /(\mathrm{C} . \mathrm{M}$. distance A$)$ or $($ mass star A$) /($ mass star B$)=$ (velocity star B)/(velocity star A). Note which star's values are on top of the fraction and which are on the bottom! Simplify the fraction down as far as possible.
3. If you set the mass of star $A=($ mass of star $B) \times($ the fraction of the previous step) and substitute this for the mass of star A in the first step (Kepler's 3rd law step), you will find star

B's mass $=$ the total mass/ $(1+$ the fraction from step 2$)$.
4. Star A's mass $=$ star B's mass $\times$ (the fraction from step 2 ).
5. Check that the proportions add up to the total mass!
(By the way, you can use this proportion idea in cooking if you need to have a 32 -ounce mixture and the recipe calls for 3 parts sugar to 2 parts flour or if a recipe is for 6 people but you need to serve 8 people.) The masses of different types of stars are summarized in the Main Sequence Star Properties table below.

## How do you do that?

Use the observed velocities in the figure below to find the individual masses of the stars. The stars have a measured period of $4 / 3$ years and a separation distance of 4 A.U.


## Step 1:

Kepler's third law says the total mass $=43 /(4 / 3)^{2}=64 /(16 / 9)=36$ solar masses.

## Step 2:

I will let the massive star be "star A". So (mass star A) $/($ mass star B) $=400 / 100=4$.

## Step 3:

mass star $B=36$ solar masses $/(1+4)=7.2$ solar masses.

## Step 4:

mass star $\mathrm{A}=$ mass star $\mathrm{B} \times 4=28.8$ solar masses.
Step 5: Check:
28.8 solar masses +7.2 solar masses does equal 36 solar masses. This step makes sure you did not make an arithmetic error in the previous steps. If the sum does not equal the value in step 1 , then re-check your math!


## Vocabulary

## center of mass

radial velocity curve

## spectroscopic binary

## Formulae

- Center of mass distance: mass star $\mathrm{A} /$ mass star $\mathrm{B}=$ distance $\mathrm{B} /$ distance A , where the distances are each measured from the center of mass. Notice which star's distance is in the top of the fraction!
- separation distance $=$ distance $\mathrm{A}+$ distance B , where the distances are each measured from the center of mass.
- Center of mass velocity: mass star A / mass star B = velocity star B / velocity star A. Notice which star's velocity is in the top of the fraction!
- Kepler's 3rd law: (mass star A + mass star B) $=(\text { separation distance })^{3} /(\text { orbital period })^{2}$ if use solar mass units, A.U. for the distance unit between the stars, and the time unit of years for the orbital period.


## Review Questions

1. How the masses of stars found? What kind of star systems are used and which famous law of orbital motion is used?
2. How many times closer to the center of mass is the massive star than the low-mass star?
3. How do you use the radial velocity curve to find the mass proportions and separation distance? How much faster or slower does the low-mass star move than the massive star?
4. Three binary systems with a separation of 2 A.U. between the two stars in each system. System (1) has the two stars orbiting the center of mass in 1 year, system (2) has the two stars orbiting the center of mass in 5 months, and system (3) has the two stars orbiting the center of mass in 2 years. Put the binary systems in the correct order by increasing total mass (least massive first and ignore the inclination angle $i$ ).
5. Star A is 0.2 A.U. from the center of mass and its companion star B is 0.6 A.U. from the center of mass. Which star is more massive?
6. If the two stars in the previous question have orbital periods of 0.35777 years, what are the individual masses of the two stars? (Hint: find their combined mass from Kepler's third law and
then use their relative center of mass distances to find how many times more massive one star is than the other.)
7. Use the radial velocity curve graph in the text above. Assume that star A reaches a velocity of 90 kilometers/second and star B reaches only 10 kilometers/second. If the separation distance $=$ 10 A.U., and the orbital period = 10 years, what is the combined mass of the two stars? From the center of mass relation also find (star A mass)/(star B mass) and their individual masses.
8. Which star system(s) would you be able to measure the radial velocity: (a) stars orbiting in a plane that is along our line of sight $\left(i=0^{\circ}\right)$; (b) stars orbiting in a plane that is perpendicular to our line of sight (face-on, $i=90^{\circ}$ ); (c) stars orbiting in a plane with $i=30^{\circ}$.
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## ASTRO 101/102

## OUTLINE OF LECTURE 13 WHAT BINARY STARS TELL US:

 STELLAR MASSES
## I: Introduction: Binary Stars and Stellar masses

- Though most stars look like single points of light, the majority of stars are in double or even multiple star systems, where a pair (or several) stars orbit each other under their mutual gravitational attraction.
- All but the closest and most widely separated of these binary star systems look like single points of light, because we can't resolve the individual stars at such large distances.
- Binary stars are of fundamental importance not only because most stars are binary, but also because they provide us with our primary means of measuring stellar masses.
- We can only measure the mass of an object if we watch the gravitational force it exerts on another object. We used this principle to measure the mass of Jupiter and of the sun using Kepler's 3rd law. We need to measure the period, P (in years), and semimajor-axis, A (in AU), of an orbiting body, then note that the mass (in solar masses) is simply:
$\mathbf{M}=\mathbf{A}^{\mathbf{3}} / \mathbf{P}^{\mathbf{2}}$

II: Ways in which we detect binary stars

- Visual Binary stars
o Close enough to us (or far enough from each other) to be resolved into two objects
- We can watch them move around each other, given enough time.
- A good example is Mizar, the middle star of the handle of the Big Dipper, which is really two stars, Mizar A and Mizar B--you can "split" Mizar and see both components using a good pair of binoculars. Castor, in the constellation of Gemini, is a double with two components separated by only a few seconds of arc.
- Inclination angle of orbit to our line of sight is not obvious, and remains ambiguous without other evidence.
- Sometimes only one of the two components of a binary is bright enough ot be detected, but the presence of another star can be inferred from the motion of the brighter component, which will appear to be orbiting an unseen companion. Such a system is called an astrometric binary.
- There are stars that appear close to each other that are, in fact, just chance alignments along the line of sight. They don't orbit around one another. We term these "false" binaries optical double stars.


## - Spectroscopic binary stars

o We detect binarity in most stars by noting that the absorption lines in their spectra regularly
shift back and forth in wavelength--we are seeing the Doppler Shift produced by the star as the radial velocity component of its orbital motion changes.
o Sometimes the spectra of both stars of the pair can be seen shifting back and forth, so that the absorption lines sometimes seem double (when one star is traveling away and the other approaching), and sometimes single (when both stars are traveling across our line of sight.

- Sometimes no doppler shift can be seen, but the star clearly shows the spectra of two different types of stars (say a hot one and a cool one); these are called composite spectrum binaries. Since the changing doppler shift can't be detected, we can't tell anything about the orbit from these stars.


## - Eclipsing binary stars

- If the plane of the orbit lies in our line of sight, we notice eclipses every time one star passes in front of another, blocking its light.
- The light curve, the plot of brightness versus time over an orbit of the binary, can tell us about the sizes of the stars, their relative brightness, the exact inclination of the orbit, and the size of the orbit, as well as many other physical details of the system.
III: The range of stellar masses, and the relation between stellar mass and luminosity.
- In general the hotter stars, O and B , are more massive stars, the cooler stars less massive (we'll see soon that this applies to "normal" stars which are called "main sequence" stars).
- The most massive stars are about 50 times the mass of the sun. The lowest mass stars that burn hydrogen to produce energy (very cool M stars) have a bit more than 0.08 times the mass of the sun.
- Astronomers would like to find stars that are less massive than 0.08 solar masses---called "brown dwarfs"---because such stars could not sustain nuclear fusion reactions in their core. They might be generating their energy through gravitational contraction. Since brown dwarfs would be very faint, the search for them has been difficulty, but a few firm identifications have been made recently.
- For most stars show that the relation between mass and luminosity is roughly a power law:

L is proportional to $\mathrm{M}^{\sim 3}$,
so the most massive stars are much more luminous than the least massive stars, and have shorter life expectancies.

## NEW VOCABULARY

## PRECESSION OF THE EQUINOX

## 26,000 Year Precession Period



The Earth's rotation axis is not fixed in space. Like a rotating toy top, the direction of the rotation axis executes a slow precession with period of 26,000 years for the entire ecliptic of our planetary bodies to travel around our sun, a trip of $\mathbf{3 6 0}$ degrees. Each one of the 12 signs of the zodiac takes about 2100 years for our solar system to pass through. Every 72 years we actually move backward 1 degree. After 2100 years we move out of one age and into another.

The precession is like a star clock that helps us date the rotations of earth in our solar system through our galaxy.

At the time of the birth of Christ we were moving out of the Age of Aries which was the Roman Empire into the Age of Pisces. That happened around 60 BC. The early Christians were aware of this and used as their symbol the $\mathbf{2}$ fishes going in opposite directions.

Since the rotation axis is precessing in space, the orientation of the Celestial Equator also precesses with the same period. This means that the position of the equinoxes is changing slowly with respect to the background stars. This precession of the equinoxes means that the right ascension and declination of objects changes very slowly over a 26,000 year period. This effect is negligibly small for casual observing, but is an important correction for precise observations.

## HISTORY

The Babylonians possibly knew already that the rotation of the stellar constellations was subject to change, but Hipparchus was, in the 2nd century B.C., the first astronomer who
gave a description of this phenomenon. It lasted until 1543, however, before Copernicus associated this change with a changing direction of the rotation axis of the Earth. Because of the gravitational forces of the sun and the moon on the equatorial bulge of the rotating earth, taking into account $t$ the angle of 23.439 degrees between the rotation axis of the earth and the normal vector to the plane in which the earth orbits around the sun (the ecliptic), the rotation axis moves with respect to a space-fixed reference frame.

This motion is called precession and proceeds in about $\mathbf{2 5 , 8 0 0}$ years along a cone with a half apex angle of 23.439 degrees, which causes the vernal equinox to move along the ecliptic by 50.291 arcseconds per year. This precession of the equinoxes affects the length of a tropical year as well as the length of a sidereal day. Because the directions to the sun and the moon vary and because of gravitational forces from the planets, the true rotation axis wobbles around the precession cone. De largest of these nutations amounts to $\mathbf{9 . 2}$ arcseconds in $\mathbf{1 8 . 6}$ years and is caused by the moon.

Precession was the third-discovered motion of the Earth, after the far more obvious daily rotation and annual revolution. Precession is caused by the gravitational influence of the Sun and the Moon acting on the Earth's equatorial bulge. To a much lesser extent, the planets exert influence as well.

## POLAR MOTION:

Euler predicted in 1758 that the rotation axis would also show a motion with respect to an earth-fixed reference frame. Eventually, in 1891, it was Chandler who determined the periodf this free polar motion from some 50 years of observations of the geographical latitude of atronomical observatories. The Chandler period ( 435 days) deviates from the Euler period ( $\mathbf{3 0 4}$ days) because of the non-rigidity and the inhomogeneous mass distribution of the earth. The radius of the Chandler wobble of the rotation pole is about $\mathbf{6}$ metres.

In 1899 the ILS (International Latitude Service) was established to coordinate the observations of the rotation pole. Using these observations, the forced polar motions that had been predicted by Thompson in 1876 could be confirmed. These motions are caused by the gravitational forces of sun and moon as well as by geophysical processes within the atmosphere, the oceans and the interior of the earth. They amount to about 3 metres at the earth's poles.

Since the end of last century, it has been assumed that the rotation pole also shows a secular drift of about 10 cm per year in the direction towards Ellesmere Island, possibly caused by post-glacial and tectonic uplifts. In 1962 the ILS was superseded by the IPMS (International Polar Motion Service) and in 1988 the IPMS and the Earth Rotation Section of the BIH (Bureau International de l'Heure) were combined to form the IERS (International Earth Rotation Service; Central Bureau at the Paris Observatory).

## VARIATIONS IN LENGTH OF THE DAY:

The Babylonians as well as the Greeks assumed that the earth was at rest at the center of the
universe. Some astronomers claimed that the earth itself rotated on its axis, Heraclides in the 4rd century B.C. the first, but in general the astronomers adhered to the arguments of Aristotle (4rd century B.C.) and Ptolemy (2nd century A.D.), which had 'proved' that the earth could not move. Only in the 16th century did Copernicus come up with convincing arguments for the opposite case.

The period of rotation (length-of-day) of the earth, however, was assumed to be constant until well into this century, apart from a secular change. In 1754 Kant predicted that friction with the tidal forces on earth would cause a deceleration of the earth's rotation, but it took more than a century before Ferrel and Delaunay could confirm this effect. The secular decrease of the rotation rate causes $n$ increase of the length-of-day of about 2 milliseconds per century. This value can be determined by comparing the observations of eclipses of the sun and the moon by the Babylonians, Greeks, Arabs and Chinese with computed eclipses when using a constant rotation rate. At present also fossiles and paleomagnetic data are used to determine the increase in length-of-day.

Not until 1875 the surmise was raised by Newcomb that also the rate of rotation would be subject to irregularities. Only in 1936 was this confirmed by the determination of a seasonal.

Space-geodetic techniques:
Since the advent of the modern space-geodetic positioning techniques, VLBI (Very Long Baseline Interferometry), LLR (Lunar Laser Ranging) and SLR (Satellite Laser Ranging) in the sixties and GPS (Global Positioning System) in the past years, an ever-increasing number of variations in both nutation and polar motion and length-of-day have been found. From space-geodetic observations, the irregularities in the earth's rotation are now routinely determined at intervals of 1 day and even a few hours, with precisions which are less than 0.5 milliarcsecond for nutation and polar motion and less than 0.05 millisecond for length-of-day or UT, corresponding to 1.5 cm at the earth's surface.

## AGE OF AQUARIUS

This is the dawning of the Age of Aquarius celebrated in the musical Hair. It is a period when according to astrological mysticism and related hokum there will be unusual harmony and understanding in the world. We could certainly use a dose of harmony and understanding in this old world; unfortunately, it is unlikely to come because of something as irrelevant as the position of the vernal equinox with respect to the constellations of the Zodiac.


## Vernal Equinox

It takes approximately 80 years to do a transition of this size. Which places us about two thirds through this transition. We will be fully into Aquarius, one degree, around the year 2013. Interestingly enough, the Mayan Calendar and many of the ancient dating systems on the planet actually end the cycle of this age in the year 2012.

Signs of Aquarius: Paradigm shifts in the medical profession, from alapathic medicine to more holistic medicine. We see more equality between men and women emerging. Women are coming into their own and to be treated as equals in jobs, ownership of property, voting rights, etc.

The Age of Aquarius is also about technology and information spreading. It's about space travel and computers. It's about things moving quickly and rapidly. Time has been speeding up because we are moving into a section of the sky that is creating a vibrational awakening on the planet.

We are awakening our spiritual potential and our innate gifts as spiritual beings. This includes the awakening of clairvoyance, clairaudience, telepathy, and healing abilities. These innate gifts have been dormant in us for the last 2,000 year. Now we are going to realize our full spiritual potential.

We are presently ending a century, a millineum, and now 180 degrees around the star clock of the procession of the equinoxes. That was the time of the great flood. That was the Age of Leo....Many ancient Star clocks depict the age of Leo The Flood happened somewhere in the beginning to the middle of Leo. There are many stories of the flood in different cultures around the world.

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# Has the Age of Aquarius Arrived? Precession of the Equinoxes 

by Naomi Bennett

Until recently, the Greek Hipparchus, was given the credit for discovering the procession of the equinoxes. This 25,920 cycle is the foundation for the Age of Pisces which by modern measurement puts each age as 2,160 years long. The procession causes the fixed stars to rise and set against the earth's solar years so that at the first day of spring, which is the spring equinox, the constellation of Aries no longer rises but the constellation of Pisces does now. In contrast, the Egyptians started their year at the summer solstice when the constellation Orion rose with the sun to mark the beginning of the flooding of the Nile.

With more insightful study of ancient myths and Egyptian temples, texts and myths, it now appears that the Egyptians priests knew and measured the procession back to the old kingdom and there are hints that this knowledge went back to the Age of Gemini which ended somewhere near $6,140 \mathrm{BC}$.

In 1895, John Lockyer, the British Astronomer Royal, wrote The Dawn of Astronomy where he visited and measured most of the Egyptian temples to prove their alignment to fixed stars and their realignment every 400 years to compensate for the precession. He was highly disregarded in his time but recognized for his original thinking in the 70's.

Schwaller de Lubicz showed that the Zodiac of Dendera in the Temple of Hathor (around 100 BC ) was actually a time clock measurement that marked the procession of the equinoxes from the Age of Pisces back to the Age of Taurus which started in 4380 BC by his calculations. Schwaller estimated the Age of Pisces to begin near 60 BC and the Age of Aquarius to begin at 2100 AD .

Dr. Giorgio de Santillana, historian and author of Hamlet's Mill, traced all the ancient myths and legends around the world to show they had a foundation in the description of the procession. Jane B. Sellers then extended Santillana's concept in The Death of the Gods in Ancient Egypt by analyzing Egyptian myth to show the consistent reference to the procession of the stars in their religious beliefs in the Osiris- Horus myths. These myths were the center of their theology, they empowered the pharaohs, and determined their death beliefs and practices.

The Orion Mystery by Robert Bauval (an engineer) and Adrian Gilbert use the procession of the equinoxes to add proof that the Great Pyramid and the King's Chamber were used to send the dead Pharaoh's spirit to the Orion constellation. By use of modern astronomy software they calculate the Age of Aquarius to begin approximately $2,070 \mathrm{AD}$ when a half precession cycle of 12,920 shows the Orion constellation at it's highest declination of -1 degree 50 seconds and it's maximum altitude at the Meridian at 58 degrees 11 seconds. It's opposite point at $10,300 \mathrm{BC}$ would have been the beginning of the Age of Leo when Zeta Orionis would have a
lowest declination of -48 degrees 53 seconds and an altitude at the meridian of 11 degrees 8 seconds. (Footnote 1)

This last calculation hints that the Egyptians might have considered that the concept of Great Return could begin at the Age of Leo when the stars in Orion are at its lowest declination. Jane Sellers stated that "I am convinced that for ancient man, the numbers 72...2160, 25,920 all signified the concept of the Eternal Return." Footnote 2. It takes 72 years for the precession to move 1 degree, 2,160 years for one Age, and 25,920 for the entire cycle to repeat.

Robert Hand in the essay, The Age and Constellation of Pisces, uses the date of 221 AD as the date when the topical and sidereal zodiacs where in alignment which was deduced by Fagan and Bradley. From this point he calculates the first star in Pisces to cross the vernal point at 111 BC , which would place the Age of Aquarius to begin near 2,060 AD. Footnote 3.

So, from three very different calculations, we have the Age of Aquarius beginning from 2,060 to 2,100 AD.

Yet, why do I sense that we have already entered the Age of Aquarius? Since the invention of mass production, global use of electricity, the car, phone, television, NASA, invention of technology to shrink the globe with computers, and satellite transmission of voice and data like the Internet. These are all related to Aquarius. Since the advent of psychology with Freud, Jung and Adler, the Church as been displaced as a prime spiritual force for many. This century brought the decline of Church power over most governments. They can only advise and recommend now. This is a decline in the force of Pisces and the Great Religions. Since we are within 1 degree of a new age, are we sliding into the change gradually over a century or two? Is the presence of Uranus and soon Neptune in 1998 into Aquarius a preview to the future or a reaching out to pull the Age more into Now?

These questions are not answerable by absolute fact but by observation. So far, Aquarius hasn't been the sign of brotherly love but of individual rebels with extreme ideas like the Oklahoma City bombing the first month that Uranus entered Aquarius, the militia, the Unabomber and Rabin's assassin. Since January of this year, major scientific discoveries have been made. The Universe has quadrupled in size, Jupiter's atmosphere was too cool, the oldest stars are 6 billions years too old by current theory. Scientific sacred cows are falling and should continue for the next 7 years.

Kepler believed that the great conjunctions of Jupiter and Saturn in the sign of Pisces truly noted the full beginning of the Age. But that was before the discovery of Uranus, Neptune and Pluto. What are our modern measurements now? Could Uranus and Neptune be adequate significators? Uranus will reenter Aquarius in 2080 just in time for the New Age. Aquarius's ruler will be in its own sign at entry. Personally, I like this idea best.

Footnote 1. The Orion Mystery, by Robert Bauval and Adrian Gilbert, Mandarin Books, 1995. Footnote 2. The Death of Gods in Ancient Egypt, Jane B. Sellers, Penquin Books, 1992.
Footnote 3. Essays on Astrology, Robert Hand, Whitford Press, 1982.

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# Changes of Celestial Coordinates 

## - Proper Motion

- Precession
- Nutation
- Aberration
- Parallax
- Refraction

Several effects lead to the notion that measured coordinates slightly deviate from those given, e.g., in catalogs, and that these measurements change with time, location and observing conditions. The only "real" effect, i.e. physical change of relative position, is the proper motion of solar system objects and stars, which causes the objects to come into another direction (and distance) with respect to Earth.

Effects on Earth's motion leads to coordinate effects which don't change any direction from the starlight coming, but of the measurement (or the coordinate system) only: precession and nutation. The motion of Earth with a periodically varying direction of velocity is also responsible for the aberration of light, an apparent deviation of stars from their position with annual periodicity. Finally, nearby objects in the solar system show positional and daily parallax effects, and similarly, parallax can be measured for nearby stars. Besides proper motion, this is the only effect discussed here showing up, e.g., in photographs. At last, the Earth's atmosphere makes stars to appear in another position than they are, this is called refraction.

Historically, precession was already discovered around 130 BC by ancient Greek astronomer Hipparchus when he compared his observations with those of predecessors from about 200 years earlier. Edmond Halley discovered the proper motion of stars in 1718 when comparing his observations with Ptolemy's catalog (based on Hipparchus' observations), James Bradley detected the aberration of light in 1725-26 (published 1729) and the nutation of Earth in 1747, while the parallax of fixed stars was not detected before 1838 when Friedrich Wilhelm Bessel discovered it for 61 Cygni.

## Proper Motion

Stars are not really fixed in space but move according to their space velocity and the gravitational field in their environment. As a star changes its absolute position in space, it will slowly change the direction in which it appears to be from Earth (which also changes its position as the Solar System moves through space). This will be visible as a continuously growing displacement of the star from its original position.

Only the tangential component of the relative motion of a star shows up in proper motion, which is
measured in arc seconds per year or per century; the radial component (which changes the distance) can be measured with much higher accuracy in the Doppler shift of spectral lines visible in the spectra of stars.

The star with the largest observed proper motion is 9.7 mag Barnard's Star in Ophiuchus with 10.27 "/y (arc seconds per year). According to F. Schmeidler, only about 500 stars are known to have proper motions of more than $1 \mathrm{k} / \mathrm{y}$.

## Precession

Precession of the Earth's polar axis is caused by the gravitational pull of the Sun and the Moon on the equatorial bulge of the flattened rotating Earth. It makes the polar axis precess around the pole of the ecliptic, with a period of 25,725 years (the so-called Platonic year). The effect is large enough for changing the equatorial coordinate system significantly in comparatively short times (therefore, Hipparchus was able to discover it around 130 B.C.). Sun and moon together give rise to the lunisolar precession p 0 , while the other planets contribute the significantly smaller planetary precession p 1 , which sum up to the general precession p ; numerical values for these quantities are (from Schmeidler; $t$ is the time in tropical years from 2000.0):

$$
\begin{aligned}
& \mathrm{p} 0=50.3878^{\prime \prime}+0.000049^{\prime \prime} \star \mathrm{t} \\
& \mathrm{p} 1=-0.1055^{\prime \prime}+0.000189^{\prime \prime} \star \mathrm{t} \\
& \mathrm{p}=50.2910^{\prime \prime}+0.000222^{\prime \prime} \star \mathrm{t}
\end{aligned}
$$

These values give the annual increase of ecliptical longitude for all stars.
The effect on equatorial coordinates is formally more complicated, and approximately given by

$$
\begin{aligned}
& \text { p_RA }=m+n * \sin R A * \tan \text { Dec } \\
& \text { p_Dec }=n * \cos R A
\end{aligned}
$$

where the constants $m$ and $n$ are the precession components given by

$$
\begin{aligned}
\mathrm{m} & =+46.124 \mathrm{n}+0.000279 \mathrm{k} \star \mathrm{t} \\
& =3.0749 \mathrm{~s}+0.0000186 \mathrm{~s} * \mathrm{t} \\
\mathrm{n} & =+20.043 \mathrm{~m}-0.000085 \prime * \mathrm{t} \\
& =1.3362 \mathrm{~s}-0.0000056 \mathrm{~s} * \mathrm{t}
\end{aligned}
$$

Precession will shift the North Celestial Pole even closer to Polaris (Alpha Ursae Minoris) until 2115, and around 14,000 AD, Vega (Alpha Lyrae) will be an extremely bright polar star. About 5000 years ago, closest to the celestial pole about 2850 BC, Thuban (Alpha Draconis) has been the pole star.

## Nutation

As Earth's axis precesses around the pole of the ecliptic, this motion is superimposed by small periodic fluctuations called nutation. Ths nutation is caused by the motion of Lunar orbital nodes, which is retograde and has a period of 18.60 tropical years. Due to this effect, the celestial poles follow small ellpises with a semimajor axis of 9.202", which is called constant of nutation.

## Aberration

As Earth moves through space, starlight seems to come from a different direction as if Earth were at rest; this effect is called the (annual) aberration of light. It causes stars to be displaced along an ellipse of $\mathrm{a}=\mathrm{k}=20.496^{\prime \prime}$ semimajor axis (this is the aberration constant), with semiminor axis depending on ecliptical latitude be: $\mathrm{b}=\mathrm{k} * \sin$ be.

Similarly, slower Earth's rotation causes the diurnal aberration of light with aberration constant 0.31".

## Parallax

Daily and annual motion of an observer due to Earth's rotation and revolution causes diurnal and annual parallax, a slight displacement of objects which are not too far away compared to Earth's diameter or the diameter of Earth's orbit, respectively.

## Diurnal parallax:

Daily displacement is maximal at the equator, so that the parallax pi is maximal there. Its amount (equatorial-horizontal parallax) pi0 is given by sin pi0 $=\mathrm{R} / \mathrm{d}$ for a body at distance d, with equatorial Earth radius R. For other geographical latitudes B, the right hand side of this relation is to be multiplied by a factor $\cos B$.

For the Moon: pi0 = 3422.44" = 57' 2.44" (almost 1 deg)
For the Sun: pi0 = 8.794"

## Annual parallax:

In this case, Earth's orbit is the baseline for the parallax construction. The effect is very small, as stars are far away, the nearest being Alpha Centauri with an annual parallax of 0.772 " (distance 4.3 light years). While almost always unimportant for coordinate measurements, this effect is of vital importance for the determination of distances in the universe.

## Refraction

When light passes from one medium to another medium of different density (e.g., from the vacuum to Earth's atmosphere), the speed of light in the medium is changed (light is slower in the denser medium), causing the wavelengths of light to bend at different angles. Refraction in Earth's atmosphere of light coming from a celestial object causes the object to appear in a slightly shifted position than it actually is; more acurately, objects near the horizon appear "lifted" to slightly higher altitudes. This effect even lifts objects from up to 35 arc minutes below horizon to above the apparent horizon, so that they can be seen, and it makes objects apparently rise a bit earlier and set a bit later.

## - Back to Astronomical Coordinates Main Page

## Hartmut Frommert (spider@seds.org)

[SEDS]

# National Weather Service Forecast ojifice Sioux Falls, SD 



The Earth has an elliptical orbit around our Sun. This being said, the Earth is at its closest point distance wise to the Sun in January (called the Perihelion) and the furthest in July (the Aphelion). But this distance change is not great enough to cause any substantial difference in our climate. This is why the Earth's 23.5 degree tilt is all important in changing our seasons. Near June 21st, the summer solstice, the Earth is tilted such that the Sun is
positioned directly over the Tropic of Cancer at 23.5 degrees north latitude. This situates the northern hemisphere in a more direct path of the Sun's energy. What this means is less sunlight gets scattered before reaching the ground because it has less distance to travel through the atmosphere. In addition, the high sun angle produces long days. The opposite is true in the southern hemisphere, where the low sun angle produces short days Furthermore, a large amount of the Sun's energy is scattered before reaching the ground because the energy has to travel through more of the atmosphere. Therefore near June 21st, the southern hemisphere is having its winter solstice because it "leans" away from the Sun.

Advancing 90 days, the Earth is at the autumnal equinox on or about September 21st. As the Earth revolves around the Sun, it gets positioned such that the Sun is directly over the equator. Basically, the Sun's energy is in balance between the northern and southern hemispheres. The same holds true on the spring equinox near March 21st, as the Sun is once again directly over the equator.

Lastly, on the winter solstice near December 21st, the Sun is positioned directly over the Tropic of Capricorn at 23.5 degrees south latitude. The southern hemisphere is therefore receiving the direct sunlight, with little scattering of the sun's rays and a high sun angle producing long days. The northern hemisphere is tipped away from the Sun, producing short days and a low sun angle.

What kind of effect does the earth's tilt and subsequent seasons have on our length of daylight (defined as sunrise to sunset). Over the equator, the answer is not much. If you live on or very close to the equator, your daylight would be basically within a few minutes of 12 hours the year around. Using the northern hemisphere as a reference, the daylight would lengthen/shorten during the summer/winter moving northward from the equator. The daylight difference is subtle in the tropics, but becomes extremely large in the northern latitudes. Where we live in the mid latitudes, daylight ranges from about 15 hours around the summer solstice to near nine hours close to the winter solstice. Moving to the arctic circle at 66.5 degrees north latitude, the Sun never sets from early June to early July. But around the winter solstice, the daylight only lasts slightly more than two hours. There becomes a profound difference in the length of daylight heading north of the arctic circle. Barrow, Alaska at slightly more than 71 degrees north latitude, lies just less than 300 nautical miles north of the arctic circle. Barrow sees two months of total darkness, as the Sun never rises for about a month on each side of the winter solstice. On the other hand, Barrow also has total light from mid May to early August. And what about the north pole, or 90 degrees north latitude? The Sun rises in the early evening near the spring equinox and never sets again until just after the autumnal equinox, or six months of light. Conversely, after the Sun sets in the mid morning just after the autumnal equinox, it will not be seen

Miscellaneous

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again until the following spring equinox, equating to six months of darkness.
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## The Jupiter Problem

A review of the Urantia Book's model of the origin of the solar system and problems related to retrograde motion and angular momentum.

By Dick Bain

## Introduction

The author of Paper 57 in "The Urantia Book" informs us that our solar system formed from material pulled out of our sun by a passing dark giant of space, Angona. (Pg657D.) This theory of origin, known to astronomers as the catastrophic or dualistic theory, was proposed independently by Thomas Crowder Chamberlin and Forest Ray Moulton in the early part of this century.(1) Another source (2) says that the theory was first suggested early in this century by astronomer Sir James Jean and geophysicist Sir Harold Jefferies.

The astronomic community eventually rejected this theory for several reasons, one being that such an encounter would be quite rare. In fact, we are told on page 466A that most planets did not have such an origin. The Encyclopedia Britannica gives an additional reason for rejection of the catastrophic theory: "....acquired a more mature understanding of the behavior of gases under astrophysical conditions.

This perspective led to the realization that hot gases stripped from a stellar atmosphere would simply dissipate in space; they would not condense to form planets." (1) It seems to me that the idea in "The Urantia Book" sounds more reasonable; some of the material pulled out would fall back into the sun, some would be captured by the body passing by the sun, but some material would stay in orbit. Perhaps this orbiting material formed a disc around our sun, and from this disc the planets of our solar system formed.

## The Problem of Angular Momemtum

There was another problem found with the catastrophic theory, namely the distribution of angular momentum in the solar system. Angular momentum is a measure of the speed of rotation of a body around a center and it's distance from that center of rotation. Though the sun has $99.9 \%$ of the solar system's mass, it has less than $.5 \%$ of its angular momentum. Jupiter, with only a fraction of a percent of the mass in the solar system has about $99 \%$ of the angular momentum of the solar system. This situation would not be expected if the solar system had a catastrophic origin.
Significantly however, this unexpected distribution of angular momentum is a problem as well for the other major theory of planetary formation, the nebular or monistic theory.

In the eighteenth century, the philosopher Immanual Kant proposed that our planetary system coalesced from a cloud or nebula of dispersed particles. About twenty years later, the mathematician LaPlace proposed that a cloud of dust and gases around a sun would form into rings from which planets would coalesce. (1) In fact, this idea of ring formation is mentioned in "The Urantia Book" on Page 170C. The author does not specifically say that the rings form into planets, but the entry is under the heading "THE ORIGIN OF SPACE BODIES," so that planetary
formation from the rings is intimated. Astronomers are now finding many young stars with discs of dust and gas around them, and this tends to support the idea that planets form from such rings. But in the case of our solar system, the nebular hypothesis has problems other than that of the distribution of angular momentum.

## The Problem of Retrograde Motion

One of the unusual features found in our system is retrograde motion (or more correctly, retrograde rotation) of two planets, and some moons of several planets. If a planetary system formed from a uniform disc of material, we would expect the planets and their satellites to all lie in the same plane and rotate in the same direction. If a planet rotates in the opposite direction from the others, that phenomenon is an example of a type of retrograde motion.

There are two planets, Venus and Uranus, that exhibit retrograde rotation in our solar system. Astronomers have not found an explanation to account for this retrograde motion that is satisfactory to everyone. The problem of retrograde motion in our solar system is mentioned on Page 657B where the Life Carrier author tells us, "Retrograde motion in any astronomic system is always accidental and always appears as a result of the collisional impact of foreign space bodies. Such collisions may not always produce retrograde motion, but no retrograde ever appears except in a system containing masses which have diverse origins." According to the author, the masses which caused the retrograde motion were captured by our sun from the passing Angona system. And in addition to the problems already mentioned, the nebular hypothesis now has a Jupiter problem.

A recent article in _Science News_ (3) reported that a team from MIT examined 20 nearby, sunlike stars one to ten million years old and reported that even these very young stars did not have enough molecular hydrogen in their vicinity to form a planet the size of Jupiter. The researchers conclude that either a planet like Jupiter would have to form very quickly before the hydrogen was lost, or more likely there is only a small chance of such planets forming in the first place. If, on the other hand, material were pulled out from our sun as claimed in "The Urantia Book", there would be plenty of material to form the two gas giant planets Jupiter and Saturn.

The catastrophic origin hypothesis/Angona theory may still have more strikes against it than the nebular hypothesis, but it looks like the score is beginning to even up. Perhaps early in the third millennial inning astronomers will resurrect the catastrophic hypothesis and come to the same conclusion as the author of Paper 57. And 2001 isn't so far off, is it?
(1) Encyclopedia Britannica Macropedia, 1993
(2) Preston Cloud (1978) _Cosmos, Earth and Man_, Yale University Press
(3) "The Importance of Being Jupiter", Science News, Feb. 18, 1995

A service of
The Urantia Book Fellowship

## Lynne Allen

## Current Research

## Observational Limits on the Distant Kuiper Belt

The Kuiper Belt is an exciting new avenue for studying the formation and evolution of our solar system. A 'fossil record' of the history of the giant planets has been preserved in the distribution, particularly the dynamical distribution, of Kuiper Belt Objects (KBOs). The Kuiper Belt beyond 50 AU provides a strong test of different theories relating to the birth environment and early history of the solar system. While the first KBO was detected almost a decade ago, there are still only about 500 known KBOs today.

For my doctoral research, I have conducted a deep, relatively wide field, Kuiper Belt survey, finding 33 KBOs and 1 Centaur. These range from 45 to 300 km in diameter, with distances between 20 and 53 AU from the Sun. While most KBOs have been discovered in individual images from surveys with magnitude limits of $\mathrm{R}<24$, my survey used a pencil-beam technique to reach much fainter, to $\mathrm{R}<25.5$. I have also covered a much larger area, $\sim 2.4$ square degrees, than previous pencil-beam surveys which typically are $\ll 1$ square degree.

It is a significant observational challenge to reach magnitudes fainter than $\mathrm{R} \sim 24$ when the targets are moving objects. A simple average of multiple images results in trailing losses as the object gets smeared out by its own motion. This is overcome by using a shift-and-add technique, similar to that used in other deep "pencil-beam" KB surveys (Tyson etal 1992; Cochran etal 1995; Gladman etal 1998; Luu \& Jewitt 1998; Chiang \& Brown 1999), where each image is registered to the rate of motion of a potential KBO in the field and then averaged. Thus, the KBO appears to remain motionless while the stars are trailed. In order to search over the thousands of potential rates required for our data, I have written software that quickly shifts and adds the images at each KBO motion vector and automatically finds moving objects. The candidates are confirmed by a second set of observations of the same field spaced a week apart, which also allows an accurate determination of the KBO distance and a preliminary orbit estimate.

While our knowledge of the Kuiper Belt interior to 50 AU has grown tremendously over the past decade, little is yet known about the region beyond 50 AU . Inside 48 AU, the Kuiper Belt has been significantly depleted over the age of the solar system as the gravitational perturbations from the giant planets have extensively sculpted this inner region (cf. Malhotra, Duncan \& Levison 2000). Current estimates of the surface density within 48 AU are approximately two orders of magnitude less than the primordial surface density necessary to form the largest observed KBOs in the $10^{8}$
years before growth was curtailed in the region by giant planet perturbations (Stern \& Colwell 1997; Kenyon \& Luu 1997). It is also about two orders of magnitude less than the surface density expected if the mass of the giant planets is smoothly extrapolated into this area (cf.
Weidenschilling 1977). Beyond 50 AU , these gravitational perturbations would have been negligible, so an unperturbed distant Kuiper Belt would contain a much higher density of objects. At present, fewer than 10 KBOs out of the entire known sample are at distances greater than 50 AU , and none of these are on unperturbed, circular orbits. With my survey's faint limiting magnitude it is possible to detect smaller, more common KBOs to greater distances, thus measuring the density and extent of the outer Kuiper Belt.

My initial survey data covered 1.3 square degrees with an average limiting magnitude of $\mathrm{R}=25.4$. With this faint magnitude limit, it is possible to compare the density of objects beyond 55 AU to physically similar objects between 30 and 50 AU without relying on uncertain models of the size distribution of KBOs. Although this survey could have detected objects with diameters larger than 160 km at distances up to 65 AU over much of the field, I did not find any KBOs beyond 53 AU , and none in the circular orbits expected of a primordial Kuiper Belt beyond 48 AU . Comparing this absence of KBOs in an annulus from 55 to 65 AU with the seven KBOs found between 40 and 50 AU in the same survey indicates the volume density in the outer region is at most $68 \%$ the volume density of the inner region, at a $95 \%$ confidence level (Allen, Bernstein \& Malhotra 2001). Several theories have been proposed to explain the absence of KBOs beyond 50 AU found in this and other surveys, each having distinctly different implications for the history of the outer solar system.

One possibility is that a dense, primordial Kuiper Belt does exist beyond 50 AU , but is dynamically cold, thus appearing as a very thin disk of small objects (Hahn 2000). Almost all previous deep survey fields have been focused on the ecliptic plane. This would allow a thin disk to have evaded detection, as it could be expected to be inclined by up to a few degrees from the ecliptic. To evaluate this possibility, I carried out a second set of observations of fields in and near the invariable plane (the angular momentum plane of the solar system), the most likely location for this primordial dense disk.

These survey fields were strategically located to rule out as many locations (inclinations and nodes) as possible for a distant cold disk. These fields again failed to discover any KBOs beyond 50 AU brighter than $\mathrm{R}=25$, although 10 new objects were discovered at distances between 30 and 50 AU . By projecting the classical KBO population observed at 40 AU onto a plane to estimate the expected population of the thin disk, I can rule out a cold disk located in the invariable plane. Over $90 \%$ of possible cold disk orientations with inclinations less than 1.0 degree, relative to the invariable plane, are also ruled out at the $95 \%$ confidence level.

This places severe constraints on any possible distant cold disk, suggesting that either the primordial solar nebula was smaller than currently expected or the Kuiper Belt was dynamically
excited by some source other than just the giant planets. These other sources could be perturbations from giant planetesimals or planetary embryos scattered through the belt or perturbations from a close stellar encounter early in the history of the solar system (Ida etal 2000). Further work with faint surveys will be necessary before a small solar nebula can be distinguished from a perturbed distant Kuiper Belt, but this information will help us place our solar system in the wider context of planet formation.


Figure 1: Distance vs. Diameter for KBOs found in my 1998 and 1999 deep surveys. The curves indicate lines of constant magnitude, corresponding to the $50 \%$ magnitude limits for the 1998 and 1999 survey fields. The two rectangles indicate the 'inner' and 'outer' regions mentioned above. After measuring the detection efficiency with artificial moving objects, this limits the volume density of $\mathrm{D}>160 \mathrm{~km}$ KBOs from 55 to 65 AU to be $1 / 3$ less than that of the region from 40 to 50 AU , at $95 \% \mathrm{CL}$.


Figure 2: Deep survey fields locations, in the invariable plane. A to G are my initial survey fields (1998, 1999), H through L (2001, near the center of the plot) are fields specifically located to detect any cold distant Kuiper Belt disk. The dark solid line is the ecliptic plane, the lighter lines are the distant disks with inclination of 1 degree which are not ruled out by these fields. If the distant disk is at least as populous as the classical KBOs and appears extremely thin ( $<1$ degrees)
on the sky, over $90 \%$ of all possible distant cold disk locations are ruled out at the $95 \%$ CL.

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## Timekeeping

Historically, the regular motion of objects in the sky served as the basis for timekeeping. The diurnal motion of the sky caused by the rotation of the Earth on its axis defined the day, the year was defined by the motion of the Earth on its orbit about the Sun, and the month was defined in relation to the revolution of the Moon about the Earth. Although precise modern timekeeping is done electronically, many of the details and the terminology of timekeeping remain rooted in its astronomical heritage.

## Sidereal Time and Solar Time

In using the sky for timekeeping, we must define a reference point to determine when a cycle of the required motion has been completed. If we choose a reference point afixed to the celestial sphere, the corresponding time is being referenced to the distant stars and is termed sidereal time. If instead we choose the Sun as the reference point, the corresponding time is called solar time (or tropical time).

Technically, the sidereal time is defined as the length of time since the vernal equinox has crossed the local celestial meridian. An equivalent definition of the sidereal time is the right ascension of any star presently located on the local celestial meridian. Thus, if the star Sirius is presently on your celestial meridian, the sidereal time is $\mathbf{6}$ hours and 45 minutes because we saw earlier that Sirius is located at 6 hr 45 min right ascension on the celestial sphere. Generally our everyday (civil) time is referenced to the (average) motion of the Sun, not the vernal equinox. Thus, sidereal time generally does not coincide with the everyday (wall clock) time. To be precise, the sidereal time agrees with the solar time only at the autumnal equinox; at any other time, they differ (they are exactly 12 hours apart at the time of the vernal equinox).

## Sidereal Days and Solar Days

The sidereal day is defined to be the length of time for the vernal equinox to return to your celestial meridian. The solar day is defined to be the length of time for the Sun to return to your celestial meridian. The two are not the same, as illustrated in the following animation.


Because the Earth is in motion on its orbit around the Sun in the course of a day, the Earth must turn about 4 minutes longer each day ( 3 minutes and 56 seconds, to be exact) to bring the Sun back to the celestial meridian than to bring the vernal equinox back to the celestial meridian. Thus, the solar day is $\mathbf{3}$ minutes and 56 seconds longer than the sidereal day. It is this almost 4 minute per day discrepancy that causes the difference in sidereal and solar time, and is responsible for the fact that different constellations are everhead at a given time of day during the Summer than in the Winter.

## Time Zones and Universal Time

As a matter of civil convenience, the Earth is divided into various time zones. The time for many astronomical events is given in Universal Time (UT), which is (approximately) the local time for Greenwich, England---the Greenwich Mean Time or GMT. The conversion from UT to local zone time may be made using this map or this set of links. Alternatively, here is a clickable Java applet illustrating the world's timezones.


## Sidereal Time

Sidereal time is time kept with respect to the distant stars.


Because the Earth moves in its orbit around the Sun, the Earth must rotate more than 360 degrees in one solar day .

- A solar day lasts from when the Sun is on the meridian at a point on Earth until it is next on the meridian. A solar day is exactly 24 hours (of solar time). Because of the Earth's revolution, a solar day is slightly longer than a sidereal day. In every day life, we use solar time.
- The Earth must rotate an extra 0.986 degrees between solar crossings of the meridian. Therefore in 24 hours of solar time, the Earth rotates 360.986 degrees.
- Because the stars are so distant from us, the motion of the Earth in its orbit makes an negligible difference in the direction to the stars. Hence, the Earth rotates 360 degrees in one sidereal day .
- A sidereal day lasts from when a distant star is on the meridian at a point on Earth until it is next on the meridian. A sidereal day lasts 23 hours and 56 minutes (of solar time), about 4 minutes less than a solar day.
- Can you figure out where this value comes from?

Likewise the Earth moves in its orbit around the Sun, the Moon must rotate more than 360 degrees in one synodic lunar month .

- A synodic month is the time from new Moon to new Moon. Because of the Earth's revolution around the Sun, the Moon must orbit more than 360 degrees until it is again in direct line with the Sun.
- Can you figure out where this value comes from?


## Sidereal Lunar Month


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## New object dethrones Ceres as largest minor planet <br> BY JEFF FOUST <br> SPACEFLI GHT NOW <br> Posted: August 24, 2001

European astronomers have confirmed that an object in the outer solar system discovered earlier this year is the largest minor planet in the solar system, bigger than both the asteroid Ceres and Pluto's moon Charon.

A team of German, Finnish, and Swedish astronomers used a unique "virtual telescope" to study 2001 KX76 and found that the Kuiper Belt object (KBO) is at least 1,200 kilometers in diameter, significantly larger than Ceres, the largest asteroid. The results were announced Thursday by the European Space Agency and the European Southern Observatory.


This image is a colour composite image, based on three exposures with the Wide Field Imager (WFI) at the MPG/ESO 2.2-m telescope at the La Silla Observatory. Photo: ESA, ESO, Astrovirtel \& Gerhard Hahn (German Aerospace Center, DLR)

2001 KX76 was discovered in May by a group of American astronomers, who found the object in images taken by a four-meter telescope in Chile. When the discovery was announced July 2, astronomers estimated the object to be between 960 and 1,270 kilometers in diameter, based on the object's brightness, estimates of its albedo, or reflectivity, and the initial orbit calculated for it.

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In an effort to refine the object's orbit, European astronomers used Astrovirtel, a new virtual telescope that allows astronomers to search through archived images from several observatories to see if 2001 KX76 could be located in any of its images. The object was located in several of Astrovirtel's images, dating back to 1982.

Those images allowed astronomers to calculate a more accurate orbit for 2001 KX76. With that new orbit -- which shows that the KBO is slightly more distant from the Sun than the previous calculations -astronomers recalculated the size of the object. If 2001 KX76 has an albedo of 7 percent, the same as that measured earlier this year for another KBO, Varuna, then 2001 KX76 has a diameter of 1,200 kilometers. If it has a slightly darker albedo of 4 percent, a common value for comets, then the object is 1,400 kilometers across.

By comparison, Ceres, which has been considered the largest minor planet in the solar system, has a diameter of only 932 kilometers. 2001 KX76 is also at least as large as Charon, Pluto's only moon, which is no more than 1,200 kilometers across.


In this schematic diagram the relative sizes of the largest Kuiper Belt Objects (KBO) are illustrated. The newly discovered object, 2001 KX76 (diameter about 1200 km ), is the largest known KBO and is even larger than Pluto's moon, Charon. For comparison Pluto's diameter is about 2300 km . Photo: ESA, ESO, Astrovirtel \& Gerhard Hahn (German Aerospace Center, DLR)

The discovery of 2001 KX 76 is more ammunition in the debate regarding the classification of Pluto, the smallest and most distant planet. As new discoveries close the gap in size between the largest KBOs and Pluto, 2,275 km across, some planetary scientists have argued that Pluto should either be jointly classified as a planet and KBO or be stripped of planet status altogether.

Pluto appears to share a number of characteristics with objects in the Kuiper Belt, a disk of icy bodies beyond the orbit of Neptune. Of the more than 400 KBOs discovered since the first, 1992 QB1, was found nine years ago, a sizeable fraction have similar orbits to Pluto, locked in an orbital resonance with Neptune such that they complete two orbits of the Sun in the time it takes Neptune to make three. Pluto also appears to have a similar composition to many KBOs based on spectroscopic studies of the belt.

The recalculation of the size and orbit of 2001 KX76 also marks a major step forward for virtual observatories, which are only now

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beginning to harness the vast archives of data collected by telescopes for additional uses such as this. "These observations were originally made for a completely different project," noted team leader Gerhard Hahn.

The research also involved collaboration between amateur and professional astronomers, with the key calculations of the object's orbit performed by German amateur astronomer Arno Gnaedig on his home computer. "The Web and the access to 'virtual observatories' means that amateur astronomers -- located far from any 'real' professional telescopes -- can also make important contributions," he noted.

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Big Moon-Sized Object Found Beyond Neptune
By SPACE.com Staff
posted: 03:03 pm ET
02 July 2001

Pluto has suffered another demotion with scientists discovering a very bright, icy body possibly even larger than the tiny ice-planet's moon Charon and orbiting the Sun in the same celestial neighborhood.
The object, called 2001 KX76, orbits beyond Neptune in the Kuiper Belt of early solar system ice-and-rock smidgeons.
"This object is intrinsically the brightest Kuiper Belt Object found so far," says Lowell Observatory Director Robert Millis, leader of the survey team. And brightness is how astronomers calculate size.

Under one assumption about the object's reflectivity (albedo), $2001 \mathrm{KX76}$ is probably 788 miles ( 1,270 kilometers) in diameter, making it 44 miles larger across than Charon and even bigger than Ceres, the largest known asteroid.
"2001 KX76 is so exciting because it demonstrates that significant bodies remain to be discovered in the Kuiper Belt," Millis said.
"We have every reason to believe that objects ranging up to planets as large or larger than Pluto are out there waiting to be found. Until the Kuiper Belt has been thoroughly explored, we cannot pretend to know the extent or the content of the Solar System."

## Varuna verity

The truth is that 2001 KX76 could be a bit smaller than Charon. It all depends on how reflective such objects are.

For instance, earlier this year, a Kuiper Belt Object (KBO) called 20000 Varuna was announced with an estimated diameter of 558 miles ( 900 kilometers), based on a calculated reflectivity of 7 percent.

Applying this albedo to 2001 KX76 gives it a diameter of only 595 miles ( 960 kilometers).

## Reddish and resonant

2001 KX76 was discovered in the course of the Deep Ecliptic Survey, a NASA-funded search for KBOs.

The team spotted 2001 KX76 in deep digital images of the southern sky taken with the 4-meter Blanco Telescope at Cerro Tololo on May 22 by James L. Elliot of the Massachusetts Institute of Technology (MIT) and Lawrence H. Wasserman of Lowell Observatory.
2001 KX76 currently is just over 4 billion miles ( 6.4 billion kilometers) from the Sun. Its orbit is inclined by approximately 20 degrees with respect to the orbital plane of the major planets, but the detailed shape of its orbit remains uncertain.

Available evidence suggests that the newly discovered KBO may be in an orbital resonance with Neptune, orbiting the Sun three times for each time that Neptune completes four orbits.
The object has a distinctly reddish color typical of many primitive bodies in the outer Solar System.

## KBOs



The orbit of 2001 KX76 keeps it in the Kuiper Belt, beyond Pluto.

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The existence of the Kuiper Belt was postulated by J. A. Fernandez and by M. Duncan, T. Quinn, and S. Tremaine in the 1980s to explain the origin of short-period comets.

These comets move around the Sun in the same direction as the planets and are found in orbits that are tipped only modestly with respect to the ecliptic plane.

These researchers showed that short-period comets could not have originated from the more distant spherical Oort Comet Cloud as originally believed. They predicted that a second, more flattened reservoir of "proto-comets" must lie beyond the orbit of Neptune.

The first Kuiper Belt Object was found in 1992 by David Jewitt and Jane Luu of the University of Hawaii. Since then, astronomers have found over 400 KBOs, but tens of thousands likely remain to be discovered.

These objects are believed to be remnants from the formation of the Solar System, and consequently are among the most primitive and least-evolved objects available for study by planetary astronomers.

## The Deep Ecliptic Survey

About one-quarter of the known KBOs have been found by the Deep Ecliptic Survey Team.
Much more precise measurement of KBO diameters will be possible with NASA's upcoming Space Infrared Telescope Facility (SIRTF) mission, due for launch in 2002.

The survey team's research is supported by the NASA Planetary Astronomy Program through grants to Lowell Observatory and MIT. The team includes astronomers from Lowell Observatory, MIT and the Large Binocular Telescope Observatory.

## SPACE.com TOP STORIES

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Lost City 'Could Rewrite History'<br>The City is Believed to Predate the Harappan Civilisation<br>By BBC News Online's Tom Housden

The remains of what has been described as a huge lost city may force historians and archaeologists to radically reconsider their view of ancient human history.

Marine scientists say archaeological remains discovered 36 metres ( 120 feet) underwater in the Gulf of Cambay off the western coast of India could be over 9,000 years old.

The vast city - which is five miles long and two miles wide - is believed to predate the oldest known remains in the subcontinent by more than 5,000 years. The site was discovered by chance last year by oceanographers from India's National Institute of Ocean Technology conducting a survey of pollution.

Using sidescan sonar - which sends a beam of sound waves down to the bottom of the ocean they identified huge geometrical structures at a depth of 120 ft .

Debris recovered from the site - including construction material, pottery, sections of walls, beads, sculpture and human bones and teeth has been carbon dated and found to be nearly 9,500 years old.

## Lost Civilisation

The city is believed to be even older than the ancient Harappan civilisation, which dates back around 4,000 years.

Marine archaeologists have used a technique known as sub-bottom profiling to show that the buildings remains stand on enormous foundations. The whole model of the origins of civilisation will have to be remade from scratch Graham Hancock Author and film-maker Graham Hancock who has written extensively on the uncovering of ancient civilisations - told BBC News Online that the evidence was compelling:
"The [oceanographers] found that they were dealing with two large blocks of apparently man made structures.
"Cities on this scale are not known in the archaeological record until roughly 4,500 years ago when the first big cities begin to appear in Mesopotamia.
"Nothing else on the scale of the underwater cities of Cambay is known. The first cities of the historical period are as far away from these cities as we are today from the pyramids of Egypt," he said.

## Chronological Problem

This, Mr Hancock told BBC News Online, could have massive repercussions for our view of the ancient world. Harappan remains have been found in India and Pakistan "There's a huge chronological problem in this discovery. It means that the whole model of the origins of civilisation with which archaeologists have been working will have to be remade from scratch," he said. However, archaeologist Justin Morris from the British Museum said more work would need to be undertaken before the site could be categorically said to belong to a 9,000 year old civilisation. "Culturally speaking, in that part of the world there were no civilisations prior to about 2,500 BC. What's happening before then mainly consisted of small, village settlements," he told BBC News Online.

Dr Morris added that artefacts from the site would need to be very carefully analysed, and pointed out that the C14 carbon dating process is not without its error margins. It is believed that the area
was submerged as ice caps melted at the end of the last ice age 9-10,000 years ago.
Although the first signs of a significant find came eight months ago, exploring the area has been extremely difficult because the remains lie in highly treacherous waters, with strong currents and rip tides. The Indian Minister for Human Resources and ocean development said a group had been formed to oversee further studies in the area.
"We have to find out what happened then ... where and how this civilisation vanished," he said.
research: archaeologY: the sphinx

## The Date of the Great Sphinx of Giza

by Associate-Professor Robert Schoch (conference paper, 1999)
Many recent Egyptologists have attributed the carving of the Great Sphinx of Giza to the Old Kingdom Pharaoh Khafre (Chephren), ca. 2500 B.C. On the basis of a number of lines of geological, seismological, Egyptological, and related evidence, I have come to the conclusion that the structure commonly known as the Great Sphinx was built in stages (originally it may not have even been a Sphinx). Initial carving of the core body of the Sphinx is estimated to have taken place during the period of approximately 7,000 to 5,000 B.C. The Sphinx has subsequently been reworked and refurbished many times over the succeeding millennia -- including, probably, during the reign of Khafre. In particular, the rump or rear of the Sphinx was carved out much later than the core body, and the head of the Sphinx has been recarved.

My geological work suggests that Khafre merely restored the Sphinx. The body of the Sphinx, carved from the local bedrock and thus sitting in the bottom of an artificial hollow (ditch), and the walls of the Sphinx enclosure exhibit well developed precipitation induced ( $\mathrm{p}-\mathrm{i}$ ) weathering (characterized by a rolling and undulating vertical profile) not typically seen on Old Kingdom Giza Plateau structures (which exhibit primarily wind induced weathering marked by a more angular profile with soft layers removed by wind abrasion) also excavated from the Mokattam limestone. This deep p-i weathering of the Sphinx is interpreted as predating the current arid regime of the area, and thus indicates that the body of the Sphinx predates Old Kingdom times by perhaps several millennia. Though we continue to refine our knowledge of the details of the paleoclimatic history of the Giza Plateau over the last 10,000 years, we already know enough to associate certain dominant modes of weathering with certain parts of that climatic history. Portions of the Sphinx predate Old Kingdom times. The temples adjacent to the Sphinx are built of limestone core blocks taken from the ditch quarried out to form the body of the Sphinx. These core blocks are faced with Aswan granite attributed to Khafre, but the core blocks were already deeply weathered when the granite facing was originally applied. The first of several ancient repair campaigns to the weathered body of the Sphinx was done with typical Old Kingdom style masonry, but the core body of the Sphinx was already deeply weathered when this earliest repair work was carried out [1]. Corroborative evidence includes low-energy seismic refraction data that records up to $100 \%$ deeper weathering below the original floor of the Sphinx enclosure as compared to weathering seen in the identical limestones in an area presumably quarried during Khafre's time in the rear of the Sphinx enclosure. Seismic investigations have also uncovered a series of cavities or chambers under the Sphinx [2].

## References

[1] see Robert M. Schoch, 1992, KMT, A Modern Journal of Ancient Egypt, vol. 3, no. 2, pp. 52-59, 66-70; Robert M. Schoch with Robert Aquinas McNally, 1999, Voices of the Rocks, New York: Harmony Books.
[2] see Thomas Dobecki and Robert M. Schoch, 1992, Geoarchaeology, vol. 7, no. 6, pp. 527-544.

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Front Page Thursday, 26 April, 2001, 19:12 GMT 20:12 UK
World Oldest city in the Americas


Caral was built long before the huge stone structures of Mexico
An ancient city in what is now Peru was built at the same time as the great pyramids of Egypt, archaeologists have revealed.

New evidence indicates the desert site at Caral, on the slopes of the Andes, was built between $2,600 \mathrm{BC}$ and $2,000 \mathrm{BC}$.

This date pushes back the emergence of the first complex society in the New World by nearly 800 years.

And it suggests that the people behind the project were advanced enough to organise the labour needed to create the architectural wonder of the day.

Caral is one of 18 sites in central Peru's Supe Valley, which stretches eastward from the
Pacific coastline, up the slopes of the Andes.

## Earth pyramids

All the inland settlements once had architecture on a grand scale, including the six huge platform mounds seen at Caral.

Because of its size and complexity, archaeologists had thought Caral was built about $1,500 \mathrm{BC}$.

But carbon dating of plant samples found at the site add another 1,000 years or so to this figure.
That puts Caral in the same period as the great pyramids of Egypt, and long before the huge stone structures of Mexico.
"What we're learning from Caral is going to rewrite the way we think about the development of early Andean civilisation," said

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study leader Jonathan Haas of the Field Museum in Chicago, US.

The Peruvian-American archaeological team says the pyramids and irrigation system show an organised society in which masses of people were paid, or compelled, to work on centralised projects.

This suggests that power and wealth were held by an elite group at a time when, in most of the Americas, people were still hunting and gathering in much smaller communities.


The pyramids are buried under a layer of windblown and sand

## No pots

"The size of a structure is really an indication of power," said Haas.
"It means that leaders of the society were able to get their followers to do lots of work."

What is surprising to archaeologists is that the city was created by a society that had yet to invent pottery or cultivate grain.

Its people grew peppers, beans, avocadoes and potatoes - all of which they roasted, having no pots to boil them in.

They also ate lots of anchovies, which may have been used in dried form as a kind of currency, as grain was later.

The research is published in the journal Science.

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In This Section

# MITHRAS AND THE HYPERCOSMIC SUN David Ulansey 

In Studies in Mithraism, John R.Hinnells, ed. (Rome: "L'Erma" di Brettschneider, 1994) pp. 257-64.

One of the most perplexing aspects of the Mithraic mysteries consists in the fact that Mithraic iconography always portrays Mithras and the sun god as separate beings, while-- in stark contradiction to this absolutely consistent iconographical distinction between Mithras and the sun-- in Mithraic inscriptions Mithras is often identified with the sun by being called "sol invictus," the "unconquered sun." It thus appears that the Mithraists somehow believed in the existence of two suns: one represented by the figure of the sun god, and the other by Mithras himself as the "unconquered sun." It is thus of great interest to note that the Mithraists were not alone in believing in the existence of two suns, for we find in Platonic circles the concept of the existence of two suns, one being the normal astronomical sun and the other a so-called "hypercosmic" sun located beyond the sphere of the fixed stars.

In my book The Origins of the Mithraic Mysteries I have argued that the god Mithras originated as the personification of the force responsible for the newly discovered cosmic phenomenon of the precession of the equinoxes. Since from the geocentric perspective the precession appears to be a movement of the entire cosmic sphere, the force responsible for it most likely would have been understood as being "hypercosmic," beyond or outside of the cosmos. It will be my argument here that Mithras, as a result of his being imagined as a hypercosmic entity, became identified with the Platonic "hypercosmic sun," thus opening up the way for the puzzling existence of two "suns" in Mithraic ideology.

The most important source for our knowledge of the Platonic tradition of the existence of two suns is the Chaldaean Oracles, the collection of enigmatic sayings generated late in the second century C.E. by a father and son both named Julian. These oracular sayings were, as is well known, seized upon by Porphyry and later Neoplatonists as constituting a divine revelation. For our purposes, the most important element in the Chaldaean teachings is that of the existence of two suns. As Hans Lewy says,

The Chaldaeans distinguished between two fiery bodies: one possessed of a noetic nature and the visible sun. The former was said to conduct the latter. According to Proclus, the Chaldaeans call the "solar world" situated in the supramundane region "entire light." In another passage, this philosopher states that the supramundane sun was known to them as "time of time...."[1]

As Lewy showed definitively in his study, the Chaldaean Oracles were the product of a Middle Platonic milieu, since they are permeated by concepts and images known from Platonizing thinkers ranging from Philo to Numenius. It is thus likely that the Chaldaean concept of a hypercosmic sun is at least partly derived from the famous solar allegories of Plato's Republic, in which the sun is used as a symbol for the highest of Plato's Ideal Forms, that of the Good. In Book VI of the Republic (508Aff.) Plato compares the sun to the Good, saying that as the sun is the source of all illumination and understanding in the visible world (the horatos topos), the Good is the supreme source of being and understanding in the world of the forms (the noetos topos or "intelligible world"). Plato then amplifies this image in his famous allegory of the cave at the beginning of Book VII of the Republic. In this famous passage, Plato symbolizes normal human life as life in a cave, and then describes the ascent of one of the cave-dwellers up out of the cave where he sees for the first time the dazzling light of the sun outside the cave.

Thus in Book VI of the Republic we see the image of the sun used as a metaphor for the Form of the Good--the source of all being which exists in the "intelligible world" beyond the ordinary "visible world" of human experience--and then in Book VII, in the allegory of the cave, this same image of the sun is used even more concretely to symbolize that which exists outside of the normal human world represented by the cave.

In addition, as has often been noted, there seems to have been a connection in Plato's imagination between his allegory in Book VII of the Republic of the ascent of the cave dweller to the sunlit world outside the cave and his myth in the Phaedrus of the ascent of the soul to the realm outside of the cosmos where "True Being" dwells. The account in the Phaedrus reads:

For the souls that are called immortal, so soon as they are at the summit [of the heavens], come forth and stand upon the back of the world: and straightway the revolving heaven carries them round, and they look upon the regions without. Of that place beyond the heavens none of our earthly poets has yet sung, and none shall sing worthily. But this is the manner of it, for assuredly we must be bold to speak what is true, above all when our discourse is upon truth. It is there that true being dwells, without colour or shape, that cannot be touched; reason alone, the soul's pilot, can behold it, and all true knowledge is knowledge thereof. [2]

As R. Hackforth says,

No earlier myth has told of a hyperouranios topos [place beyond the heavens], but this is not the first occasion on which true Being, the ousia ontos ousa, has been given a local habitation. In the passage of Rep. VI which introduces the famous comparison of the Form of the Good to the sun we have a noetos topos contrasted with a horatos (508C): but a spatial metaphor is hardly felt there.... A truer approximation to the hyperouranios topos occurs in the simile of the cave in Rep. VII, where we are plainly told that the prisoners' ascent into the light of day symbolises ten eis ton noeton tes psyches anodon (517B); in fact, the noetos topos of the first simile has in the second developed into a real spatial symbol. [3]

Paul Friedländer agrees with Hackforth completely in seeing a connection in Plato's mind between the ascent from the cave in the Republic and the ascent to the "hypercosmic place" in the Phaedrus:

> The movement "upward"... had found its fullest expression in the allegory of the cave in the Republic. [Now in the Phaedrus]... the dimension of the "above" is stated according to the new cosmic co-ordinates. For the "intelligible place" (topos noetos) in the Republic (509D, 517B) now becomes "the place beyond the heavens" (topos hyperouranios)...[4]

What is, of course, important to see here is that there exists already in Plato the obvious raw material for the emergence of the idea of the "hypercosmic sun": when the prisoners escape the cave in the Republic what they find outside it is the sun, but if Hackforth and Friedländer are correct the vision of what is outside the cave in the Republic is linked in Plato's mind with the vision of what is outside the cosmos in the myth recounted in the Phaedrus. It would therefore be a natural and obvious step for a Platonist to imagine that what is outside the cosmic cave of the Republic--namely, the sun, the visible symbol of the highest of the Forms and of the source of all being--is also what is to be found outside the cosmos in the "hypercosmic place" described in the Phaedrus.

An intermediate stage in the development of the concept of the "hypercosmic sun" between Plato and the Chaldaean Oracles can be glimpsed in Philo's writings, for example in the following passage from De Opificio Mundi:

> The intelligible as far surpasses the visible in the brilliancy of its radiance, as sunlight assuredly surpasses darkness.... Now that invisible light perceptible only by mind...is a supercelestial constellation [hyperouranios aster], fount of the constellations obvious to sense. It would not be amiss to term it "all-brightness," to signify that from which sun and moon as well as fixed stars and planets draw, in proportion to their several capactiy, the light befitting each of them...[5]

Here we see Philo referring to the existence in the intelligible sphere of a
"hypercosmic star" (hyperouranios aster) which he links with the image of sunlight, and which he sees as the ultimate source of the light in the visible heavens.[6] Philo's formulation here is, of course, strikingly similar to the Chaldaean concept of the hypercosmic sun, the description of which by Lewy we should recall here: "The Chaldaeans distinguished between two fiery bodies: one possessed of a noetic nature and the visible sun. The former was said to conduct the latter. According to Proclus, the Chaldaeans call the 'solar world' situated in the supramundane region 'entire light.'"[7]

The trajectory we have been tracing from Plato through Middle Platonism to the Chaldaean Oracles continues beyond the time of the Chaldaean Oracles into early Neoplatonism, for we find the concept of the existence of two suns clearly spelled out in the writings of Plotinus, in a context that makes it clear that for Plotinus one of these suns was "hypercosmic." In chapter 2, paragraph 11 of his fourth Ennead, Plotinus speaks of two suns, one being the normal visible sun and the other being an "intelligible sun." According to Plotinus,
> ...that sun in the divine realm is Intellect-- let this serve as an example for our discourse-- and next after it is soul, dependent upon it and abiding while Intellect abides. This soul gives the edge of itself which borders on this [visible] sun to this sun, and makes a connection of it to the divine realm through the medium of itself, and acts as an interpreter of what comes from this sun to the intelligible sun and from the intelligible sun to this sun... [8]

What is especially interesting for us is that in the same third chapter of the fourth Ennead, a mere six paragraphs after the passage just quoted, Plotinus explicitly locates the intelligible realm-- which he has just told us is the location of a second sun-- in the space beyond the heavens. The passage reads:

One could deduce from considerations like the following that the souls when they leave the intelligible first enter the space of heaven. For if heaven is the better part of the region perceived by the senses, it borders on the last and lowest parts of the intelligible. [9]

As A.H. Armstong says of this passage, "There is here a certain 'creeping spatiality'... [Plotinus'] language is influenced, perhaps not only by the 'cosmic religiosity' of his time, but by his favorite myth in Plato's Phaedrus (246D6-247E6)."[10] In any event, we here find Plotinus in the third chapter of the fourth Ennead first positing the existence of an "intelligible sun" besides the normal visible sun, and then locating the intelligible realm spatially in the region beyond the outermost boundary of the heavens.

Finally, to return to the Chaldaean Oracles, the fact that the Chaldaean concept of the "hypercosmic sun" was at least sometimes taken in a completely literal and spatial sense is shown by a passage from the Platonizing Emperor Julian's Hymn to Helios. According to Julian, in certain unnamed mysteries it is taught that "the sun travels in
the starless heavens far above the region of the fixed stars."[11] Given the fact that Julian's thinking was steeped in the Neoplatonic philosophy of Iamblichus who was deeply committed to the Chaldaean Oracles as a source of divinely inspired knowledge, and given the fact that the doctrine of the "hypercosmic sun" is an established teaching of the Chaldaean Oracles, it is virtually certain, as Robert Turcan points out in his remarks about this passage, that Julian is referring here to the teaching of the Chaldaean Oracles.[12] The passage from Julian, therefore, shows that the "hypercosmic sun" of the Chaldaean Oracles was understood as being "hypercosmic" not in a merely symbolic or metaphysical sense, but rather in the literal sense of being located physically and spatially in the region beyond the outermost boundary of the cosmos defined by the sphere of the fixed stars.

Our discussion thus far has shown that in the late second century C.E. there is found in the Chaldaean Oracles the doctrine of the existence of two suns: one the normal, visible sun, and the other a "hypercosmic"sun. The evidence from Julian shows that the "hypercosmic" nature of this second sun was understood as meaning that it was literally located beyond the outermost sphere of the fixed stars. The fact that the Chaldaean Oracles emerged out of the milieu of Middle Platonism suggests that the doctrine of the "hypercosmic sun" found in the Oracles did not develop overnight, but that it has roots in the Platonic tradition, most likely, as we have seen, going back ultimately to Plato himself: specifically, to the allegory in the Republic of the ascent beyond the world-cave to the sunlit realm outside and the related myth of the Phaedrus describing the ascent of the soul towards its ultimate vision of the hyperouranios topos, the "hypercosmic place" beyond the heavens. An intermediate stage between Plato and the Chaldaean Oracles is found in Philo's reference to the "hypercosmic star" which is the source of the light of the visible heavenly bodies, and slightly later than the Chaldaean Oracles we find Plotinus making reference to two suns, one of them being in the intelligible realm which he places spatially beyond the heavens.

We may say, therefore, that it is likely that there existed in Middle Platonic circles during the second century C.E. (and probably much earlier as well) speculations about the existence of a second sun besides the normal, visible sun: a "hypercosmic" sun located in that "place beyond the heavens" (hyperouranios topos) described in Plato's

## Phaedrus.

We see here, of course, a striking parallel with the Mithraic evidence in which we also find two suns, one being Helios the sun-god (who is always distinguished from Mithras in the iconography) and the other being Mithras in his role as the "unconquered sun." On the basis of my explanation of Mithras as the personification of the force responsible for the precession of the equinoxes this striking parallel becomes readily explicable. For as we have seen, the "hypercosmic sun" of the Platonists is located beyond the sphere of the fixed stars, in Plato's hyperouranios topos. But if my theory about Mithras is correct (namely, that he was the personification of the force responsible for the precession of the equinoxes) it follows that Mithras--as an entity capable of moving the entire cosmic sphere and therefore of
necessity being outside that sphere--must have been understood as a being whose proper location was in precisely that same "hypercosmic realm" where the Platonists imagined their "hypercosmic sun" to exist. A Platonizing Mithraist (of whom there must have been many-- witness Numenius, Cronius, and Celsus), therefore, would almost automatically have been led to identify Mithras with the Platonic "hypercosmic sun," in which case Mithras would become a second sun besides the normal, visible sun. Therefore, the puzzling presence in Mithraic ideology of two suns (one being Helios the sun-god and the other Mithras as the "unconquered sun") becomes immediately understandable on the basis of my theory about the nature of Mithras.

Finally, the line of investigation which I have pursued here can also allow me to provide a simple and convincing interpretation for two further puzzling elements of Mithraic iconography. First, all the various astronomical explanations of the tauroctony which scholars are currently advancing (including my own) agree that the bull in the tauroctony is meant to represent the constellation Taurus. However, the constellation Taurus as seen in the night sky faces to the left while the bull in the tauroctony always faces to the right. How can this apparent discrepancy be explained? On the basis of my theory this question has an obvious answer. For although it is the case that the constellation Taurus as seen from the earth (i.e., from inside the cosmos) faces to the left, it is also the case that on ancient (and modern) star-globes which depict the cosmic sphere as it would be seen from the outside the orientation of the constellations is naturally reversed, with the result that on such globes (like the famous ancient "Atlas Farnese" globe) Taurus is always depicted facing to the right exactly like the bull in the tauroctony. This shows that the Mithraic bull is meant to represent the constellation Taurus as seen from outside the cosmos, i.e. from the "hypercosmic" perspective, which is, of course, precisely the perspective we should expect to find associated with Mithras if my argument in this paper is correct.[13]

Second, the line of investigation I have pursued here can also provide a simple and convincing interpretation of the iconographical motif known as the "rock-birth" of Mithras, in which Mithras is shown emerging out of a rock. As is well known, Porphyry, quoting Eubulus, explains in the Cave of the Nymphs that the Mithraic cave in which Mithras kills the bull and which the Mithraic temple imitates was meant to be an image of the cosmos (De Antro. 6). Of course, the hollow Mithraic cave would have to be an image of the cosmos as seen from the inside. But caves are precisely hollows within the rocky earth, which suggests the possibility that the rock out of which Mithras is born is meant to represent the cosmos as seen from the outside. Confirmation of this interpretation is provided by the fact that the rock out of which Mithras is born is often shown entwined by a snake, a detail which unmistakably evokes the famous Orphic motif of the snake-entwined cosmic egg out of which the cosmos was formed when the god Phanes emerged from it at the beginning of time.[14] It thus seems reasonable to conclude that the rock in the Mithraic scenes of the "rock-birth" of Mithras is a symbol for the cosmos as seen from the outside, just as the cave (the hollow within the rock) is a symbol for the

I would argue, therefore, that the "rock-birth" of Mithras is a symbolic representation of his "hypercosmic" nature. Capable of moving the entire universe, Mithras is essentially greater than the cosmos, and cannot be contained within the cosmic sphere. He is therefore pictured as bursting out of the rock that symbolizes the cosmos (not unlike the prisoner emerging from the cosmic cave described by Plato in Rep. VII), breaking through the boundary of the universe represented by the rock's surface and establishing his presence in the "hypercosmic place" indicated by the space into which he emerges outside of the rock.

And, to conclude, in this context it is no accident that in the "rock-birth" scenes Mithras is almost always shown holding a torch; for having established that his proper place is outside of the cosmos, Mithras has become identified with the "hypercosmic sun": that light-giving being which dwells, as Proclus says,
in the supermundane (worlds) [en tois hyperkosmiois]; for there exists the "solar world (and the) whole light..." as the Chaldaean Oracles say and which I believe.[15]

## NOTES

[1] Hans Lewy, Chaldaean Oracles and Theurgy (Paris: Études Augustiniennes, 1978) pp. 151-2.
[2] 247B-C; trans. R. Hackforth, Plato's Phaedrus (Cambridge: Cambridge University Press, 1952) pp. 71,78.
[3] Ibid., pp. 80-1.
[4] Paul Friedländer, Plato I: An Introduction (New York: Pantheon Books, 1958) p. 194.
[5] VIII.31; trans. F.H. Colson, Philo (London: William Heinemann, 1929) vol. 1, p. 25.
[6] Philo often speaks of God using expressions such as the "intelligible sun" (noetos helios [Quaest. in Gen. IV.1; see Ralph Marcus, trans., Philo Supplement 1: Questions and Answers on Genesis (Cambridge: Harvard University Press, 1953) p. 269, n.l]) or similar expressions involving light and illumination located in the intelligible realm; for references see Pierre Boyancé, Études sur le songe de Scipion (Paris: E. de Boccard, 1936) pp. 73-4; Lewy, Chaldaean Oracles, p. 151, n. 312; David Runia, Philo of Alexandria and the Timaeus of Plato (Leiden: E.J. Brill,
1986) p. 435 and n. 143. Boyancé (p. 73-4) quite reasonably argues that such expressions were identical in Philo's mind with the hyperouranios aster ("hypercosmic star") of De Opificio Mundi VIII.31.
[7] For a superb discussion of the broader context in which the development of the concept of the "hypercosmic sun" most likely occured, see Boyancé, Études, pp. 65-77. Recently A.P. Bos has argued that the story of the ascent to the sunlit world outside of the cave in Plato's Republic was explicitly connected by Aristotle with Plato's image in the Phaedrus of the ascent of the soul to the "place beyond the heavens," and that this connection played a central role in one of Aristotle's lost dialogues whose major elements were then preserved and utilized by Plutarch in his De Facie. See A.P. Bos, Cosmic and Meta-Cosmic Theology in Aristotle's Lost Dialogues (Leiden: E.J. Brill, 1989): the argument is complex and the book should be read as a whole, but see esp. pp. 67-8, 182. The development of the concept of the "hypercosmic sun" also must, of course, be seen in the context of the evolution of the "solar theology" described by Franz Cumont in his La théologie solaire du paganisme romain (Paris: Librairie Kliensieck, 1909). A very important and intriguing argument is made for the presence of a tradition of a "hypercosmic sun" in Orphic circles by Hans Leisegang, "The Mystery of the Serpent," in Joseph Campbell, ed., The Mysteries (Princeton: Princeton University Press, 1955) pp. 194-261. The Greek magical papyri and the Hermetic corpus provide numerous examples of solar imagery in which the sun is in various ways symbolically elevated to at least the summit of the cosmos if not explicitly to a "hypercosmic" level. Finally, Hermetic, Gnostic, and Neoplatonic texts all betray an almost obsessive concern with enumerating and distinguishing the various cosmic spheres and levels, and especially with establishing where the boundary lies between the cosmic and the hypercosmic realms (the hypercosmic realm being identified by the Hermetists and Neoplatonists with the "intelligible world" and by the Gnostics with the "Pleroma"). This concern with establishing the boundary between the cosmic and the hypercosmic must have fed into speculations about the "hypercosmic sun," and--intriguingly--one of the clearest symbolic formulations of this boundary between the cosmic and the hypercosmic is found in the religious system of the Chaldaean Oracles (exactly, that is, in the system in which we find explicitly formulated the image of the "hypercosmic sun"), where the figure of Hecate is understood as the symbolic embodiment of precisely this boundary (on the image of Hecate in the Chaldaean Oracles see now Sarah Iles Johnston, Hekate Soteira [Atlanta: Scholars Press, 1990]).
[8] IV, 3.11.14-22; trans. A.H. Armstrong, Plotinus (Cambridge, Mass., 1984) vol. 4, pp. 71-73.
[9] IV.3.17.1-6; ibid, pp. 87-89.
[10] Ibid., p. 88, n. 1.
[11] Or. 4.148A; trans. W. C. Wright, Julian (Cambridge: Harvard University Press,
1962) p. 405.
[12] Robert Turcan, Mithras Platonicus (Leiden: E.J. Brill, 1975) p. 124. Julian was well acquainted with the Chaldaean Oracles: see Polymnia Athanassiadi-Fowden, Julian and Hellenism (Oxford: Oxford University Press, 1981) pp. 143-53. Roger Beck has recently suggested that Julian is referring here to the Iranian cosmology in which the sun and moon are located beyond the stars (Planetary Gods and Planetary Orders in the Mysteries of Mithras [Leiden: E.J. Brill, 1988], pp. 2-3, n.2). However, Julian's intimate association with Iamblichus and the Chaldaean Oracles, in which the doctrine of the "hypercosmic sun" is well established, renders the possibility that Julian is referring to the Iranian tradition highly unlikely. As Hans Lewy says, "There seems to be no connection between [Julian's teaching] and Zoroaster's doctrine according to which the sun is situated above the fixed stars" (Chaldaean Oracles, p. 153, n. 317). However, it is certainly true that the existence of the Iranian cosmology placing the sun beyond the stars could easily have provided some additional motivation for the emergence of the identification between the "Persian" Mithras and the Platonic "hypercosmic sun" for which I have argued here. On the Iranian cosmology see M.L. West, Early Greek Philosophy and the Orient (Oxford: Oxford University Press, 1971), pp. 89-91; Walter Burkert, "Iranisches bei Anaximandros," Rheinisches Museum 106 (1963) pp. 97-134.
[13] It should be noted that the fact that the bull in the tauroctony faces to the right renders untenable Roger Beck's suggestion that the tauroctony is a picture of the night sky as seen by an observer on earth at the time of the setting of the constellation Taurus ("Cautes and Cautopates: Some Astronomical Considerations," Journal of Mithraic Studies 2.1 [1977] p. 10; Planetary Gods and Planetary Orders in the Mysteries of Mithras [Leiden: E.J. Brill, 1988] p. 20), since such an observer would see Taurus facing to the left. The fact that the bull in the tauroctony faces right is explicable only if we understand the tauroctony as the creation of someone who had in mind an astronomical star-globe showing the cosmic sphere as seen from the outside, and not-- as Beck argues-- an image of the sky as seen from the earth.
[14] That the rock from which Mithras is born was identified with the Orphic cosmic egg is in fact proven beyond doubt, as is well known, by the striking similarity between the Mithraic Housesteads monument (CIMRM 860), which shows Mithras being born out of an egg (which is thus identified with the rock from which he is usually born), and the famous Orphic Modena relief showing Phanes breaking out of the cosmic egg (CIMRM 695). In connection with this Orphic-Mithraic syncretism, Hans Leisegang, "Mystery of the Serpent" (above, n. 8), esp. pp. 201-215, has collected a fascinating body of material--including among other things the Modena relief and the passage from Julian which I have discussed above--supporting the contention that the breaking of the Orphic cosmic egg is linked directly with the concept of the "hypercosmic." Leisegang's discussion as a whole provides strong support for my general argument in this paper.
[15] Chaldaean Oracles Frag. 59 (= Proclus, In Tim. III.83.13-16); trans. Ruth

Majercik, The Chaldaean Oracles (Leiden: E.J. Brill, 1989) p. 73. The sun was often imagined in antiquity as a torchbearer, as for example in SVF 1:538: "Cleanthes... used to say... that the sun is a torchbearer" (cited in Jean Pépin, "Cosmic Piety," in Classical Mediterranean Spirituality [New York: Crossroad, 1986] p. 425); a fragment from Porphyry: "In the mysteries of Eleusis, the hierophant is dressed as demiurge, the torchbearer as the sun..." (also cited in Pepin, "Piety," p. 429); and of course Lucius in Apuleius' Golden Ass XI.24: "In my right hand I carried a lighted torch... thus I was adorned like unto the sun...." (trans. W. Adlington, Apuleius The Golden Ass [London: William Heinemann, 1928] p. 583).

## Back to The Cosmic Mysteries of Mithras

## Back to Home of David Ulansey


[^0]:    ${ }_{1}$ Certainly the observers on a train and on the quay of a station have no way to ascertain who is moving and who is still. But, with absolute certainty, we can apply the 3rd Principle of the Dynamics to deduce that the relative speed V, measured between the two groups of observers, is distributed among them according to the obvious relations

    $$
    V_{m}=V \frac{M}{M+m}
    $$

    and

    $$
    V_{M}=V \frac{m}{M+m}
    $$

    where $m$ and $M$ are respectively the mass of the train and the mass of the Earth shell that has undergone the recoil. And if, during the starting phase, the train or the mass M have undergone some plastic deformations, even from these it is possible to deduce the said speeds. We will see that the previous ratio between the interacting masses has a very important role even in gravitational phenomena, since even here the 3rd Principle of the Dynamics holds.

[^1]:    ${ }^{2}$ These times are not finished, each age has its own dark times.

[^2]:    ${ }^{3}$ Note that, according to the 3rd Principle of the Dynamics, it is assumed that the forces acting on the two masses are identical.
    ${ }^{4}$ Analogously, an observer anchored to the secondary mass can measure the force acting on the primary mass:

    $$
    \mathrm{F}_{\mathrm{Mm}}=\frac{\mathrm{GMm}}{\mathrm{~d}^{2}}=1+\frac{\mathrm{M}}{\mathrm{~m}}=
    $$

[^3]:    5 If is true that for a common weight the rate $\mathrm{m} / \mathrm{M}=1.67 \mathrm{e}-25$ is extremely short is also true that in the solar system things are totally different. In fact for the couple Sun-Mercury $\mathrm{m} / \mathrm{M}=1.65 \mathrm{e}-7$; for the couple Sun-Jupiter $\mathrm{m} / \mathrm{M}=1 \mathrm{e}-3$ and for couple Heart-Moon $\mathrm{m} / \mathrm{M}=1 \mathrm{e}-2$ !

[^4]:    ${ }^{6}$ In Fig. 4 we show only the rotations of the various vector radii and not the rotations of the various arcs: this has been done to clarify the figure. On the other hand, in Fig. 3 we represent with broken lines two circumferences: the lower one is the circumference characterized by the arc m" 2 , rotated by the angle Mm" ${ }^{\prime}$ '.

[^5]:    ${ }^{7}$ This speed is the sum of the speeds of the two masses in the K frame.

[^6]:    ${ }^{8}$ It would be coincident if m"' coincided with m'".

[^7]:    ${ }^{9}$ Note that the mass of the Moon has been determined just by taking into account that the Earth-Moon system orbits around the common baricentre.

[^8]:    ${ }^{10}$ We have already seen (see Fig. 3, too) that the two-body procedure of Newton requires that the secondary mass describes around the Sun, considered fixed, an orbit having a mean radius greater than the radius of the orbit covered around the common baricentre, and exactly equal to the latter amplified by the coefficient $(1+\mathrm{m} / \mathrm{m})$.

[^9]:    ${ }^{11}$ Actually it is a residual shift of the perihelion of Mercury that cannot be explained by Newton's theory.

[^10]:    ${ }^{12}$ Laplace (see note no. 2251) states: If this quantity were an arc of the planet's orbit, perpendicular to the radius vector, it would subtend only an angle of 3. 6", when viewed from Sun (in the case of Uranus).

[^11]:    ${ }^{13}$ Osserviamo lo sviluppo in serie del binomio di Newton troncato al primo termine.

[^12]:    ${ }^{14}$ We have already said that the orbit of Venus is almost circular. In such a case, the apparent forward shift due to the real backward shift of the Sun, according to what we have already said in Section 3, is masked by the intrinsic speed of the planet, since it is difficult to highlight the slide speed of the orbit itself.

[^13]:    ${ }^{15}$ There are certainly more precise relations, but the content is the same.
    ${ }^{16}$ Thus, if we assume that the force exerted by the Sun on the equatorial bulge is equal to 1 , the analogous force by the Moon is 4 times stronger.

[^14]:    ${ }^{17}$ A ring of moons around the Earth will cause a null recoil, as it is easy to verify. Jupiter and Saturn, in opposition, would cause a residual recoil of the Sun equal to 45 ".
    ${ }^{18}$ Newton uses such tricks even for the verification of the law of the inverse square distance, as well as for the speed of sound in the air. And we can imagine that if Newton had known that a part of the movement of the $\square$ point could be attributed to the Sun, he would have adjusted everything in order to obtain a perfect result with his calculations.

[^15]:    ${ }^{19}$ It is clear that at the time none suspected the existence of the real shift of the Sun that any gravitational theory allows to calculate, as it is easy to verify.
    ${ }^{20}$ There were some unofficial tests that appear to have established that the heaviest body falls first [12].
    ${ }^{21}$ Note that, in many fundamental points, the theory of Galilei is not reread and reinterpreted by the following theory of Newton, but superimpose and survives to the latter, notwithstanding the existence of strong contradictions. An example is just the fall law, while the other concerns relative motions. The last problem is temporarily set apart even because at present both the gravity and the electric field seem to be exclusively positional fields -another hypothesis undemonstrated but considered certain and expected.

[^16]:    ${ }^{22}$ But if, in the context of General Relativity, we also consider the mass of Mercury, given its smallness, the results should not be much different from the Newtonian results and then we would have 43 " for the relativistic forward shift plus 44 " for the recoil!

