

# Spiral

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In mathematics, a **spiral** is a curve which emanates from a point, moving farther away as it revolves around the point.

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Cutaway of a nautilus shell showing the chambers arranged in an approximately logarithmic spiral

## Spirals and helices

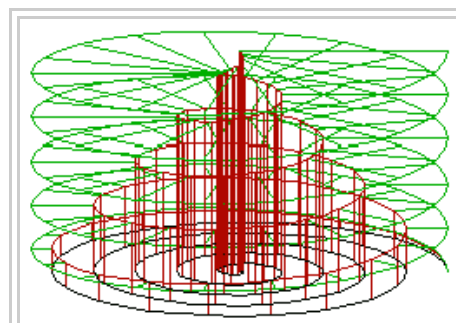
Two major definitions of "spiral" in a respected American dictionary are:<sup>[1]</sup>

- a.** A curve on a plane that winds around a fixed center point at a continuously increasing or decreasing distance from the point.
- b.** A three-dimensional curve that turns around an axis at a constant or continuously varying distance while moving parallel to the axis; a helix.

Definition **a** describes a planar curve, that extends in both of the perpendicular directions within its plane; the groove on one side of a record closely approximates a plane spiral (and it is by the finite width and depth of the groove, but *not* by the wider spacing between than within tracks, that it falls short of being a perfect example); note that successive loops *differ* in diameter. In another example, the "center lines" of the arms of a spiral galaxy trace logarithmic spirals.

Definition **b** includes two kinds of 3-dimensional relatives of spirals:

- A conical or volute spring (including the spring used to hold and make contact with the negative terminals of AA or AAA batteries in remote controls), and the vortex that is created when water is draining in a sink is often described as a spiral, or as a conic helix.
- Quite explicitly, **b** also includes a cylindrical coil spring and a strand of DNA, both of which are quite helical, so that "helix" is a more *useful* description than "spiral" for each of them; in general, "spiral" is



An Archimedean spiral, a helix, and a conic spiral

seldom applied if successive "loops" of a curve have the same diameter.<sup>[1]</sup>

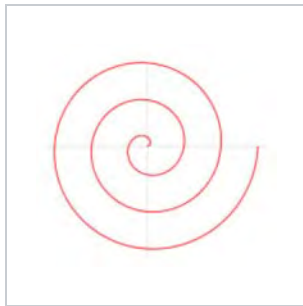
In the side picture, the black curve at the bottom is an Archimedean spiral, while the green curve is a helix. The curve shown in red is a conic helix.

## Two-dimensional spirals

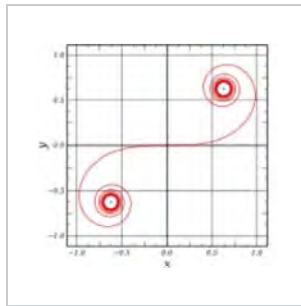
A two-dimensional spiral may be described most easily using polar coordinates, where the radius  $r$  is a monotonic continuous function of angle  $\theta$ . The circle would be regarded as a degenerate case (the function not being strictly monotonic, but rather constant).

Some of the most important sorts of two-dimensional spirals include:

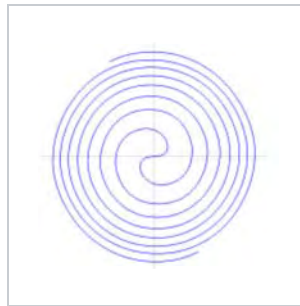
- The Archimedean spiral:  $r = a + b \cdot \theta$  (see also:Involute)
- The Euler spiral, Cornu spiral or *clothoid*
- Fermat's spiral:  $r = \theta^{1/2}$
- The hyperbolic spiral:  $r = a/\theta$
- The lituus:  $r = \theta^{-1/2}$
- The logarithmic spiral:  $r = a \cdot e^{b\theta}$ ; approximations of this are found in nature
- The Fibonacci spiral and golden spiral: special cases of the logarithmic spiral
- The Spiral of Theodorus: an approximation of the Archimedean spiral composed of contiguous right triangles
- The involute of a circle, used twice on each tooth of almost every modern gear



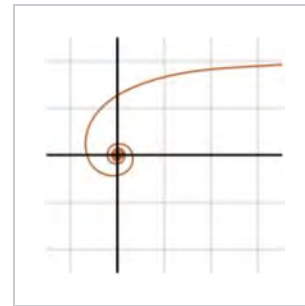
Archimedean spiral



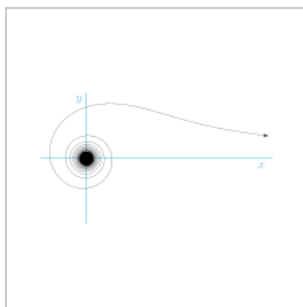
Cornu spiral



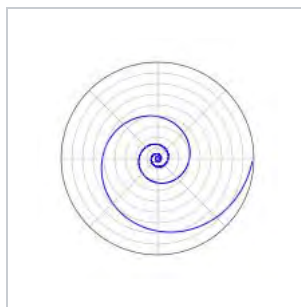
Fermat's spiral



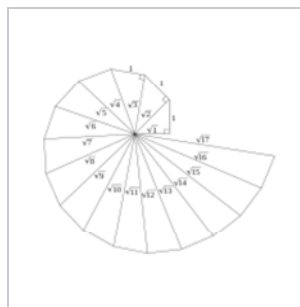
hyperbolic spiral



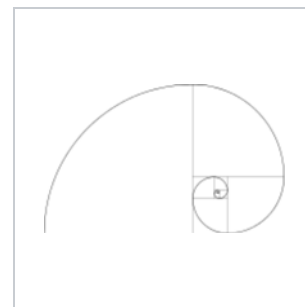
lituus



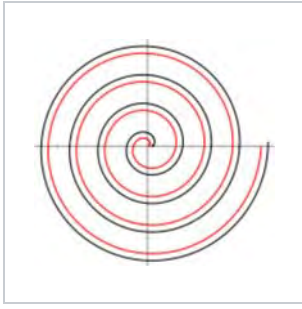
logarithmic spiral



spiral of Theodorus



Fibonacci Spiral  
(golden spiral)



The involute of a circle (black) is not identical to the Archimedean spiral (red).

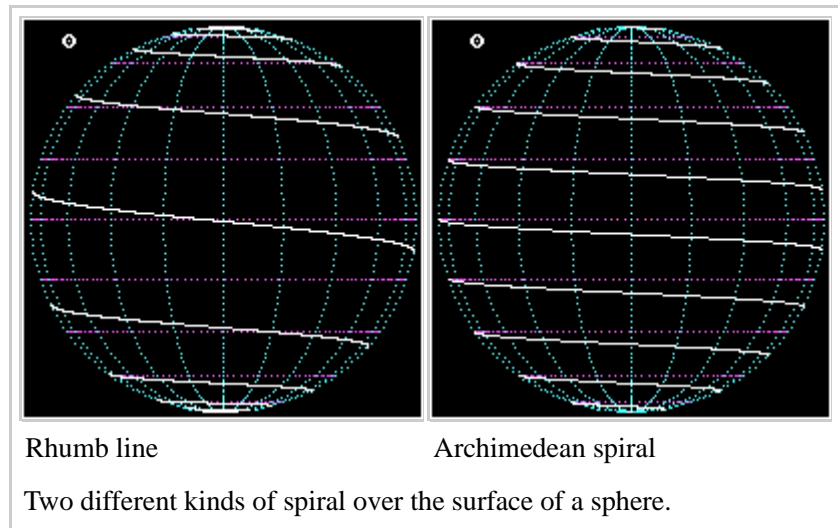
## Three-dimensional spirals

For simple 3-d spirals, a third variable,  $h$  (height), is also a continuous, monotonic function of  $\theta$ . For example, a conic helix may be defined as a spiral on a conic surface, with the distance to the apex an exponential function of  $\theta$ .

The helix and vortex can be viewed as a kind of three-dimensional spiral.

For a helix with thickness, see spring (math).

A rhumb line (also known as a loxodrome or "spherical spiral") is the curve on a sphere traced by a ship with constant bearing (e.g., travelling from one pole to the other while keeping a fixed angle with respect to the meridians). The loxodrome has an infinite number of revolutions, with the separation between them decreasing as the curve approaches either of the poles, unlike an Archimedean spiral which maintains uniform line-spacing regardless of radius.



Rhumb line

Archimedean spiral

Two different kinds of spiral over the surface of a sphere.

## In nature

The study of spirals in nature has a long history. Christopher Wren observed that many shells form a logarithmic spiral; Jan Swammerdam observed the common mathematical characteristics of a wide range of shells from *Helix* to *Spirula*; and Henry Nottidge Moseley described the mathematics of univalve shells. D'Arcy Wentworth Thompson's *On Growth and Form* gives extensive treatment to these spirals. He describes how shells are formed by rotating a closed curve around a fixed axis, the shape of the curve remains fixed but its size grows in a geometric progression. In some shell such as *Nautilus* and ammonites the generating curve revolves in a plane perpendicular to the axis and the shell will form a planar discoid shape. In others it follows a skew path forming a helico-spiral pattern. Thompson also studied spirals occurring in horns, teeth, claws and plants.<sup>[2]</sup>

A model for the pattern of florets in the head of a sunflower was proposed by H Vogel. This has the form

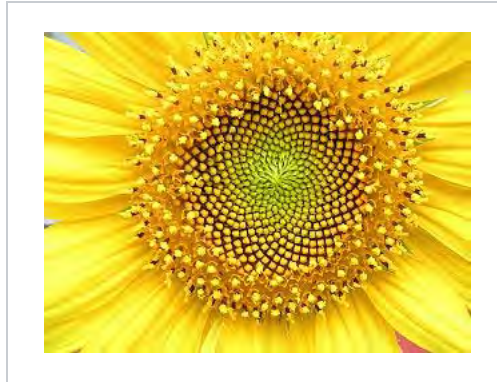
$$\theta = n \times 137.5^\circ, r = c\sqrt{n}$$

where  $n$  is the index number of the floret and  $c$  is a constant scaling factor, and is a form of Fermat's spiral. The angle  $137.5^\circ$  is the golden angle which is related to the golden ratio and gives a close packing of florets.<sup>[3]</sup>

Spirals in plants and animals are frequently described as whorls. This is also the name given to spiral shaped fingerprints.



The center Galaxy of Cat's Eye Nebula



Sunflower head displaying florets in spirals of 34 and 55 around the outside.

## As a symbol

A spiral like form has been found in Mezine, Ukraine, as part of a decorative object dated to 10,000 BCE.

The spiral and triple spiral motif is a Neolithic symbol in Europe (Megalithic Temples of Malta). The Celtic symbol the triple spiral is in fact a pre-Celtic symbol.<sup>[4]</sup> It is carved into the rock of a stone lozenge near the main entrance of the prehistoric Newgrange monument in County Meath, Ireland. Newgrange was built around 3200 BCE predating the Celts and the triple spirals were carved at least 2,500 years before the Celts reached Ireland but has long since been incorporated into Celtic culture.<sup>[5][5]</sup> The triskelion symbol, consisting of three interlocked spirals or three bent human legs, appears in many early cultures, including Mycenaean vessels, on coinage in Lycia, on staters of Pamphylia (at Aspendos, 370–333 BC) and Pisidia, as well as on the heraldic emblem on warriors' shields depicted on Greek pottery.<sup>[6]</sup>



The Newgrange entrance slab

Spirals can be found throughout pre-Columbian art in Latin and Central America. The more than 1,400 petroglyphs (rock engravings) in Las Plazuelas, Guanajuato Mexico, dating 750-1200 AD, predominantly depict spirals, dot figures and scale models.<sup>[7]</sup> In Colombia monkeys, frog and lizard like figures depicted in petroglyphs or as gold offering figures frequently includes spirals, for example on the palms of hands.<sup>[8]</sup> In Lower Central America spirals along with circles, wavy lines, crosses and points are universal petroglyphs characters.<sup>[9]</sup> Spirals can also be found among the Nazca Lines in the coastal desert of Peru, dating from 200 BC to 500 AD. The geoglyphs number in the thousands and depict animals, plants and geometric motifs,

including spirals.<sup>[10]</sup>

Spiral shapes, including the swastika, triskele, etc., have often been interpreted as solar symbols. Roof tiles dating back to the Tang Dynasty with this symbol have been found west of the ancient city of Chang'an (modern-day Xian).

Spirals are also a symbol of hypnosis, stemming from the cliché of people and cartoon characters being hypnotized by staring into a spinning spiral (one example being Kaa in Disney's *The Jungle Book*). They are also used as a symbol of dizziness, where the eyes of a cartoon character, especially in anime and manga, will turn into spirals to show they are dizzy or dazed. The spiral is also found in structures as small as the double helix of DNA and as large as a galaxy. Because of this frequent natural occurrence, the spiral is the official symbol of the World Pantheist Movement.<sup>[11]</sup> The spiral is also a symbol of the dialectic process and Dialectical monism.



This Petroglyph with a spiral figure carved into it was made by the Hohokams, a Native American tribe in the United States, over 1000 years ago.

## In art

The spiral has inspired artists throughout the ages. Among the most famous of spiral-inspired art is Robert Smithson's earthwork, "Spiral Jetty", at the Great Salt Lake in Utah.<sup>[12]</sup> The spiral theme is also present in David Wood's Spiral Resonance Field at the Balloon Museum in Albuquerque, as well as in the critically acclaimed Nine Inch Nails 1994 concept album *The Downward Spiral*. The Spiral is also a prominent theme in the anime Gurren Lagann, where it represents a philosophy and way of life. It also central in Mario Merz and Andy Goldsworthy's work.

## See also

- Megalithic Temples of Malta
- Hypogeum of Hal-Saflieni
- Celtic maze (straight-line spiral)
- Fibonacci number
- Patterns in nature
- Seashell surface
- Spirangle
- Spiral vegetable slicer
- Stairs
- Triskelion

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  12. Israel, Nico (2015), *Spirals : the whirled image in twentieth-century literature and art*, New York Columbia University Press, pp. 161–186, ISBN 978-0-231-15302-7

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## External links

- *SpiralZoom.com* (<http://SpiralZoom.com>), an educational website about the science of pattern formation, spirals in nature, and spirals in the mythic imagination.
- Spirals by Jürgen Köller (<http://www.mathematische-basteleien.de/spiral.htm>)
- Spirals (<http://eol.org/collections/10796>) – an Encyclopedia of Life collection with examples of spirals in nature.
- Archimedes' spiral transforms into Galileo's spiral. Mikhail Gaichenkov, OEIS (<http://oeis.org/A202407>)
- Educational webpage connecting spirals to nature, art and patterns. (<http://interactive-earth.com/resources/general-activities/pattern-generator-sine-cosine/about.html>)



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