ASTRONOMY FOR SURVEYORS

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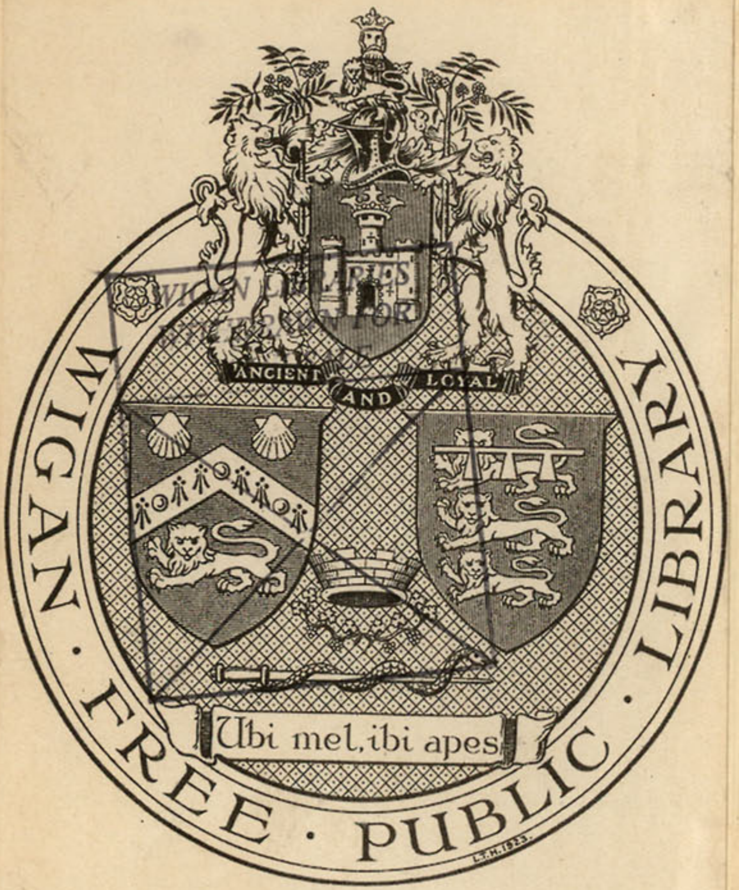
V. K. RICE-OXLEY
AND
W. V. SHEARER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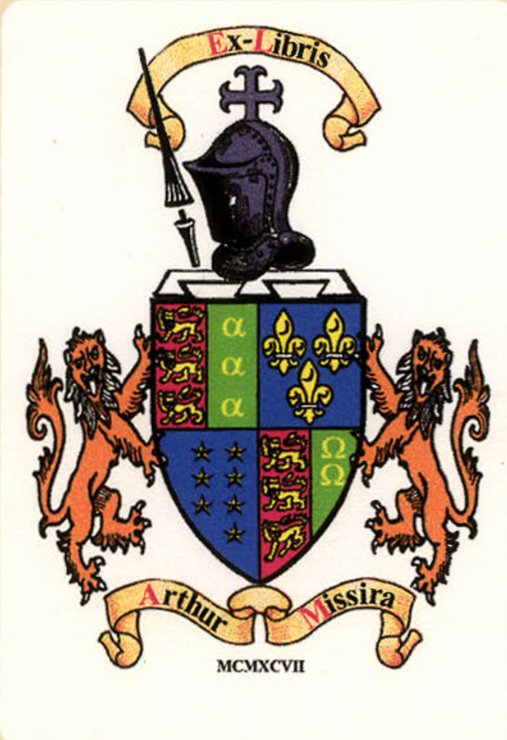
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BY

M. K. RICE-OXLEY
A.C.G.I., A.M.I.C.E.
ASSISTANT PROFESSOR OF CIVIL ENGINEERING, CITY AND GUILDS (ENGINEERING) COLLEGE
IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY, SOUTH KENSINGTON

AND

W. V. SHEARER
B.Sc., Whit.Schol., A.M.I.C.E., M.I.Mech.E.
LECTURER ON FIELD ASTRONOMY, ETC., CITY AND GUILDS (ENGINEERING) COLLEGE
IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY, SOUTH KENSINGTON

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PREFACE

IN this book the authors have endeavoured to produce a text-book which will be of use alike to the student and to the practical surveyor. No attempt has been made to deal with geodesy, on which subject several excellent treatises are available, but the work is confined to an explanation of the principal methods of making such astronomical observations and computations as will require to be made by geodetic and topographical surveyors, prospectors, and explorers and others interested in the application of astronomy to surveying.

These methods can be applied without a detailed theoretical analysis of the underlying principles involved, but the authors have thought it well to include a certain amount of such analysis for the benefit of those who wish to understand the theoretical as well as the practical side of the subject.

The authors are indebted to Sir Henry Lyons, F.R.S., Dr. J. Ball of the Egyptian Desert Survey, and Dr. H. Knox-Shaw, for many helpful suggestions.

M. K. R.-O.
W. V. S.

October 1928

ASTRONOMY FOR SURVEYORS

CHAPTER I

GENERAL

Plane Surveying.—In plane surveying it is assumed that the mean surface of the earth within the area surveyed is a horizontal plane, i.e. a plane normal to the direction of gravity as indicated by a plumb line, and no account is taken of the fact that, owing to the curvature of the earth's mean surface, the direction of a plumb line at one point on such a survey is not, in normal circumstances, parallel to the direction of a plumb line at another point on the same survey.

This assumption is justified where the area of the survey is small, and errors arising from it may be neglected. However, it is obvious that in the survey of large areas or in the setting out of inter-state boundaries or long lengths of railway and other lines the curvature of the earth must be taken into account and its surface can no longer be considered as a horizontal plane.

Geodetic Surveying.—Surveying which takes the earth's curvature into account is known as Geodetic surveying. In general, the area over which the earth's surface may be considered as a plane for survey work may be taken as about 20 square miles; for an area of 20 to 20,000 square miles it may be considered as part of a sphere, and as part of a spheroid for an area of over 20,000 square miles.

Geodetic surveying necessitates the determination of not only the relative positions of a number of points on the earth's surface but also their absolute position, referred to certain definite axes, and for this latter purpose recourse must be had to astronomical observations.

It will also be apparent that in plane surveying it is sometimes necessary to know the absolute position on the earth's surface of one or more stations in the survey, as, for example, to fix the location of a proposed line of railway in a country where pegs would be quickly obliterated by tropical growth or other circumstances.

Field Astronomy.—Observations for these and similar purposes, and the interpretation of their results, come under the heading of Field Astronomy and deal *inter alia* with the determination of true meridian, latitude, longitude, and time. The principles involved are the same as those in daily use in aerial and marine navigation, but, the circumstances being different, the methods are naturally somewhat different and different instruments are used.

Earth's Relation to other Heavenly Bodies.—In order to understand fully the methods used in Field Astronomy it is necessary to have a clear conception of the relation in which the earth stands, as regards position and motion, to the other heavenly bodies, viz. the sun, moon, stars, and planets, as observations on these bodies supply the data for computations by which the required results are obtained.

The apparent movements of these bodies having been observed from the remotest times, the conception of what may be called the mechanism of the system has been deduced and so firmly established that there is no doubt as to its correctness, and the results of every observation are consistent with it.

The solar system consists of the sun with a number of planets, of which the earth is one, moving around it, some of which planets have attendant satellites. The

distances separating the bodies forming the solar system, though great, are as nothing compared with the distances which separate the solar system from the stars. The latter, therefore, form a background on which the sun, moon, and planets appear to move in consequence of their own movement and that of the earth from which we view them.

As before mentioned, the apparent movements of the bodies forming the solar system have been studied from the earliest times, and from the results of the mass of observations Kepler (*b.* 1571) deduced the following laws governing their movements.

Kepler's Laws.—1. Each planet moves in an elliptical orbit of its own round the sun, the latter being in the plane of the ellipse and in one focus thereof.

2. The radius vector, or line joining the centre of a planet to that of the sun, describes equal areas in equal times. Thus (Fig. 1) if the planet moves in a given time from E_1 to E_2, and from E_3 to E_4 in the same length of time, the area A_1 will be equal to the area A_2.

3. The square of the time that each planet takes to complete its orbit varies as the cube of the semi-axis major of that orbit.

From these laws Newton (*b.* 1642) deduced the law of gravitation, namely, that each planet is acted on by a force of attraction towards the sun, the magnitude of this force being inversely proportional to the square of the distance of the planet from the sun.

From the second law it will be seen that the angular velocity of a planet varies inversely as the square of its distance from the sun. Let the planet, distant R from the sun, move from E_5 to E_6 (Fig. 1) in a small interval of time dt with linear velocity v and angular velocity ω around S; then since dt is small the path of the planet from E_5 to E_6 may be considered as straight and of length vdt.

The area described by the radius vector in the time

dt is the area of the triangle E_5SE_6, which is $\frac{1}{2}vdtR \sin \theta$, where θ is the angle between the radius vector and the tangent to the curve at E_5; but the component of the velocity of the planet at right angles to the radius vector is $v \sin \theta$, and this is equal to $R \times \omega$.

Therefore the area of the triangle $= \frac{1}{2}R^2\omega dt$; but for any given interval of time dt the area of the triangle described by the radius vector is, by Kepler's second law, constant, therefore $R^2\omega$ is constant, or, in other words, ω varies inversely as R^2.

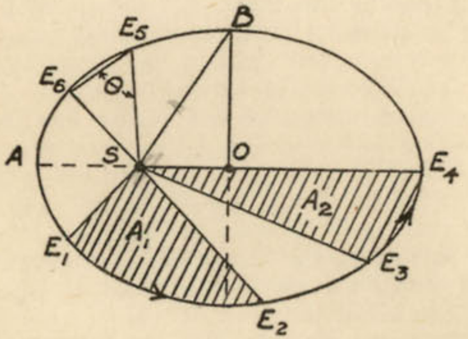

FIG. 1.

The foregoing is a statement, necessarily brief, of the laws which govern the movements of the planets in their orbits around the sun. The same laws apply, *mutatis mutandis*, to the movements of a satellite, such as the moon, around its primary, the earth.

The Earth's Orbit.—As in the case of the other planets, the earth's orbit around the sun is an ellipse. The plane of this ellipse is called the *ecliptic*; the eccentricity, e, of the ellipse is small, as is therefore also its ellipticity, E, since, as shown below, the ellipticity is equal to one-half the square of the eccentricity.

In the ellipse (Fig. 1), SB = OA

and the eccentricity $e = \frac{OS}{OA}$

$$= \frac{\sqrt{a^2 - b^2}}{a} = \sqrt{\left(1 - \frac{b}{a}\right)\left(1 + \frac{b}{a}\right)}$$

where OA $= a$, and OB $= b$,

or $e^2 = \left(1 - \frac{b}{a}\right)\left(1 + \frac{b}{a}\right)$; but the ellipticity $E = \frac{a-b}{a} = 1 - \frac{b}{a}$;

therefore $e^2 = E\left(1 + \frac{b}{a}\right)$.

If the eccentricity be decreased, b approaches more nearly in value to a, and in the limit $e^2 = 2E$.

The eccentricity of the earth's orbit is 0·01679, or about $\frac{1}{60}$, and the ellipticity therefore about $\frac{1}{7200}$.

In the ellipse (Fig. 1), $\frac{SE_4}{SA} = \frac{OA + OS}{OA - OS} = \frac{1 + \frac{OS}{OA}}{1 - \frac{OS}{OA}} = \frac{1+e}{1-e}$.

The ratio of the greatest and least distances of the earth from the sun is therefore $\frac{1 + 0{\cdot}01679}{1 - 0{\cdot}01679}$ or 1·0341 : 1.

It will be seen (Fig. 2) that the earth is nearest to the sun at A; it is in this position on 1 January and is then said to be in *perihelion*, the sun being in *perigee*. The earth is farthest from the sun when at C on 2 July, and is said to be then in *aphelion* and the sun in *apogee*.

The line joining A to C is called the *apse line* and the points themselves, situated at the ends of the major axis, the *apses* of the orbit.

The movement of the earth in its orbit is accompanied by a rotation about its own axis, the earth making approximately 365¼ turns in the period of describing the orbit, i.e. in one year. The points at which the axis of rotation meets the surface of the earth are termed the *terrestrial poles*, North and South respectively.

The *terrestrial equator* is the plane at right angles to the axis of rotation and passing through the centre of the earth.

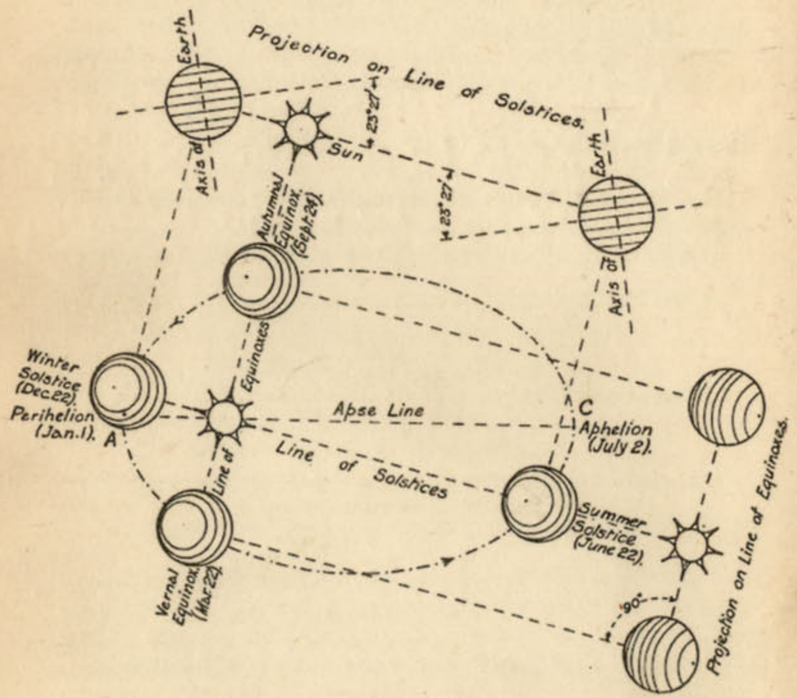

FIG. 2.

The *terrestrial meridian* of any place on the earth is the plane passing through that place and containing the earth's axis. The angle between the terrestrial meridians of two places is called the *difference of longitude* between the places.

Obliquity of the Ecliptic.—The plane of the ecliptic is not coincident with that of the equator; the angle between the two planes is called the *obliquity of the ecliptic*, and has a value of about 23° 27′, which is subject to a small and slow periodic change. The axis of the earth is obviously inclined to the plane of the ecliptic at an angle of 90° minus the obliquity, i.e. at about 66° 33′, and remains practically parallel to itself during the period of describing the orbit.

On or about 22 June and 22 December, i.e. about nine days before the earth in its orbit reaches aphelion and perihelion respectively, the plane containing the axis of the earth and the radius vector, i.e. the line joining the earth and sun, is perpendicular to the plane of the ecliptic (Fig. 2), the radius vector then making an angle with the plane of the equator equal to the obliquity of the ecliptic. The instants at which this is the case are known as the *summer solstice* and *winter solstice* respectively.

On or about 22 March and 24 September the axis of the earth is at right angles to the radius vector; the sun is therefore in the plane of the earth's equator and, as will be seen, day and night are then of practically equal duration all over the world. The corresponding positions of the earth and sun are therefore known as the *equinoxes*, *vernal* and *autumnal* respectively.

The line joining the equinoxes is obviously at right angles to the line of solstices.

Celestial Sphere.—The movement of the earth around the sun and the rotation of the former on its axis having been considered briefly, the conception of the celestial sphere may now be introduced.

It may be stated here that any plane containing the centre of a sphere cuts the surface of the sphere in what is called a *great circle*, and any other plane, i.e. not containing the centre of the sphere, cuts the surface in what is called a *small circle*. In the case of the celestial sphere any great circle passing through the zenith is called a *vertical circle*, the zenith being, as explained later, the point vertically above the observer.

It is to be noted that the earth's orbit, large as it is, is but a point compared with the distances separating the solar system from the stars, since the mean distance of the sun from the earth is only about 1/125000 that of the nearest star, so that straight lines from a star to all points on the earth, at all positions in its orbit, may for all practical purposes be considered parallel.

The angular positions of the stars, and not their linear distances from the earth, are observed, and hence it is convenient to consider them as fixed upon the inner surface of an imaginary sphere, called the *celestial sphere*, at the centre of which the observer is stationed. It is obvious that as a result of the rotation of the earth the observer's view of the celestial sphere appears exactly the same as if the earth were stationary and the celestial sphere rotating around the same axis, with the same angular velocity, but in the opposite direction, i.e. from East to West. It is clear that the points called the *North and South celestial poles*, where the axis of the earth if produced would meet the celestial sphere, would appear stationary and the stars would preserve their angular distances from these points and from each other constant, appearing to travel along concentric paths or *diurnal circles* with uniform angular velocity.

Rotation of the Earth on its Axis.—The earth may be regarded as a nearly spherical body rotating on an axis, the rotation being capable of experimental proof apart from astronomical observations, and the axis of rotation remaining nearly parallel to itself during the earth's revolution round the sun.

It must here be observed that the figure of the earth is not a true sphere but very nearly an oblate spheroid. If it were a true sphere, the resultant attraction of the sun and moon would pass through the centre of the earth and could have no effect on the direction of the earth's

axis, and the position of the equinoxes would remain fixed. The effect of the equatorial protuberance is that there is a disturbing couple which would tend to put the axis perpendicular to the ecliptic if the earth were stationary, but as the earth is in rotation gyrostatic action consequently causes the axis to describe a cone, the axis of which is perpendicular to the plane of the orbit. If the earth were hollow its axis would describe this cone in a

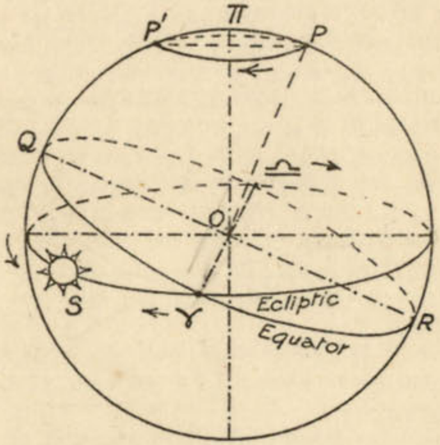

Fig. 3.

comparatively short period, but since it is loaded with a heavy interior the rotation in the cone is slow and takes about 25,800 years.

In Fig. 3 is shown the ecliptic, of which π is the pole, and in consequence of the conical movement of the earth's axis the celestial pole P is not fixed relative to the stars, but describes a small circle PP′P on the celestial sphere in the period of about 25,800 years above mentioned. The plane QR of the equator is always perpendicular to OP. The intersection ♈O♎ of the planes of the equator and ecliptic therefore also rotates in the same direction about the point O, causing the point of intersection ♈ to move towards the left and the other point of intersection ♎ to move towards the right, as shown by the arrows adjacent to these points. This slow movement of these points of intersection is known as *precession*. The apparent movement of the sun in the ecliptic being in the direction of the arrow at S, which may be called the forward direction, the point ♈, known as the *First Point of Aries*, has a retrograde movement; consequently the sun S arrives at ♈ slightly before it, the sun. has completed an entire circuit of its orbit relative to the stars; the equinox thus precedes its position in each revolution relative to its former position. When the sun in its orbital movement reaches the point ♈ it is in the plane of the equator and crossing it from South to North, i.e. the point ♈ is the vernal equinox, about 21 March; and when at the point ♎, the *First Point of Libra*, the sun is again crossing the plane of the equator, now from North to South, i.e. the point ♎ is the autumnal equinox, about 21 September; and the line ♈♎ is the line of equinoxes.

The point ♈ is highly important, as it is used as a reference point for cataloguing the positions of stars in the system in general use.

The phenomena arising from the rotation of the earth about its axis may now be considered.

Let Fig. 4 represent the earth with an observer situated at A. The observer has a movement of translation with the earth in its orbit round the sun, and also a movement of rotation about the axis PP′. The former movement, owing to the dimensions of the orbit being small compared with the distance to the nearest star, has little effect on the appearance presented by the celestial sphere, but the daily rotation about the axis PP′ causes the celestial sphere to appear to have a movement of rotation in the opposite direction about the axis PP′ produced to meet the celestial sphere at the celestial poles. There

fore the points where PP′ produced cut the celestial sphere appear to be fixed points.

If a line Ap of infinite length be assumed drawn through

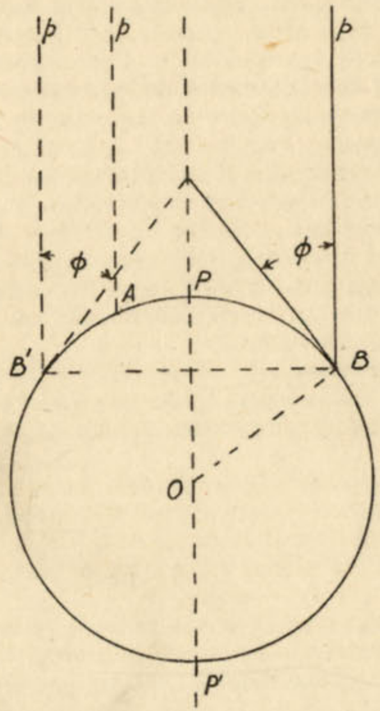

Fig. 4.

A parallel to PP′ it will cut the celestial sphere in the same point as the latter produced, since the sphere is of infinite radius, and to the observer at A the celestial sphere will appear to rotate about the axis Ap; likewise, to an observer at B the celestial sphere will appear to rotate about B*p*, *p* being one point for all possible positions of the observer at any time of the day or year, although he cannot see the whole celestial sphere at any one time, since the solid earth below him hides one half. For example, the view of an observer at B is bounded by his *horizon*, which is a tangent at B to the sphere.

Altitude of the Celestial Pole.—Obviously only one of the celestial poles is visible to the observer at B, unless he is on the equator, and it will be at an angle of elevation ϕ above his horizon, i.e. its *altitude* will be ϕ and it will be directly above the north point of his horizon for the northern hemisphere. Twelve hours later the observer will have been carried to B′ by the rotation of the earth, and the celestial pole will still be at the same altitude ϕ, and similarly for any intermediate time, i.e. the altitude of the pole will remain fixed.

Celestial Equator.—Fig. 5 represents the appearance presented by the celestial sphere to the observer at O, the celestial pole P being elevated at an angle PON above his horizon.

The *celestial equator* QR is the great circle, perpendicular to PP′, in which the earth's equatorial plane, when produced, cuts the celestial sphere. The line EW is the intersection of the plane of the equator with that of the horizon.

Zenith.—The point Z on the celestial sphere vertically above the observer is the *zenith*, OZ being perpendicular to the horizon plane NESW. OZ is the direction of a plumb line at O. ZS_2 is the *zenith distance* of the star S_2.

Meridian Plane.—The plane containing ON, OP, and OZ is a vertical plane and is called the *meridian plane*, i.e. the meridian plane is a vertical plane passing through the pole and the zenith, and therefore also passing through the North and South points of the horizon.

Since the meridian plane contains both the lines OP and OZ, it is perpendicular to each of the planes QERW

and NESW, i.e. to the plane of the celestial equator and to the plane of the horizon, and therefore to their intersection EW. From this it follows that the line EW is at right angles to the line NS. The vertical circle EZW is known as the *prime vertical*.

The line NOS in which the meridian plane cuts the

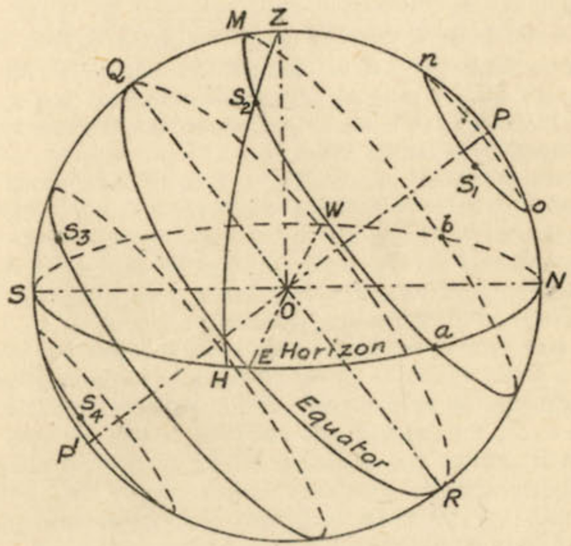

FIG. 5.

plane of the horizon is called the *meridian line*, and is the line of true or geographical North and South.

Diurnal Circles.—The celestial sphere then appears to rotate in a period of 24 hours about the axis POP′, and each star describes a circle having P as its centre. Any star not actually on the celestial equator describes a small circle of the sphere, and the paths of all stars are small parallel circles of which P is the pole; except that those on the celestial equator have a path which is a great circle.

The circle described by a star such as S_1 in the neighbourhood of the pole P is entirely above the observer's horizon NESW, and the star never sets, but would always be visible to him if atmospheric conditions were favourable and its light were not obscured by that of the sun.

The circle described by a star such as S_2 lies partly above and partly below the horizon, and the star rises and sets daily, rising at *a* in the North-East and setting at *b* in the North-West, being above the horizon for the greater part of the 24 hours.

A star situated exactly in the plane of the equator rises in a due easterly direction at E and sets in a due westerly direction at W, being above the horizon for 12 hours.

A star such as S_3 south of the equator is obviously above the horizon for a few hours only, its path being for the greater part below the horizon.

A star such as S_4 has its path entirely below the horizon and is never visible to an observer whose horizon is NESW.

Culmination or Transit.—If the path of a visible star, such as S_2, for example, be followed it will be seen that when it crosses the meridian NPZS, as defined above, at M, it reaches its greatest altitude above the horizon. Crossing the meridian is known as *culminatiom*, *transit*, or *meridian passage*.

Stars in the neighbourhood of the pole, the paths of which lie entirely above the horizon, evidently cross the meridian above the horizon twice in the 24 hours, once above the pole P and once below it. The term culmination is limited to the upper crossing of the meridian or *upper transit*, e.g. the upper transit of S_1 is at *n*, the *lower transit* at *o*.

In the figure, S_2 represents the position of a star at any instant in its daily path. The latter is described with uniform velocity, as the earth rotates with uniform angular

velocity, and the position of the star with reference to the horizon and with reference to the meridian is continually changing, in fact such change is apparent to a casual observer in an hour or two.

Altitude and Azimuth.—The angular elevation of the star above the horizon, viz. the vertical angle S_2OH (Fig. 5) or the arc S_2H of the vertical circle ZS_2H, is called the *altitude* of the star; the horizontal angle PZS_2, or the arc NH, is called the *azimuth* of the star, being the angle between the meridian plane and the vertical plane containing the star. Both the altitude and the azimuth refer to a particular instant, since their values change as the star moves in its diurnal circle. The maximum altitude is the altitude at transit, and is known as the *meridian altitude*. The instrument in general use in field astronomy is the theodolite, which is adapted for the measurement of these vertical and horizontal angles. Azimuth is always measured in the clockwise direction, and usually from the elevated pole.

CHAPTER II

TIME

Hour Angle.—As the earth rotates with uniform angular velocity, and as the stars maintain their positions relative to one another over long periods with only minute changes, the interval between successive transits of any star would be the same as for any other star, if the earth's axis kept parallel to itself, i.e. if the position of the pole were constant.

The diurnal movement of any selected star might clearly be used for the measurement of time, e.g. the time of the star's upper transit over the meridian might be called oh om os and the interval to the next transit be divided into 24 hours. A clock set to read oh om os at one transit and having 24 equal hour spaces on its dial could then be so rated as to read oh om os again at the next upper transit. The time at any instant between the transits would be measured by what is called the *hour angle* of the star, which may be defined as follows:

Let S_1 (Fig. 6) represent the position of a star a short time, say an hour or two, after it has crossed the meridian. The great circle passing through S_1 and containing the axis OP rotates with uniform velocity about the axis OP, and at the instant of the star's transit this great circle coincides with the meridian PZS, and the time according to the notation indicated above would be oh om os.

When the star is at S_1 the spherical angle ZPS_1 is its hour angle at that instant, and obviously bears the same ratio to 360° as the time elapsed since its transit bears to 24 hours of the kind shown by the clock; in fact, the

clock can be regarded as having an hour hand synchronizing with the arc PS_1 and showing oh om os when the star is on the meridian, and supplemented by a minute and a second hand to define the exact position of the hour hand at any instant.

For the sake of uniformity, the hour angle is always

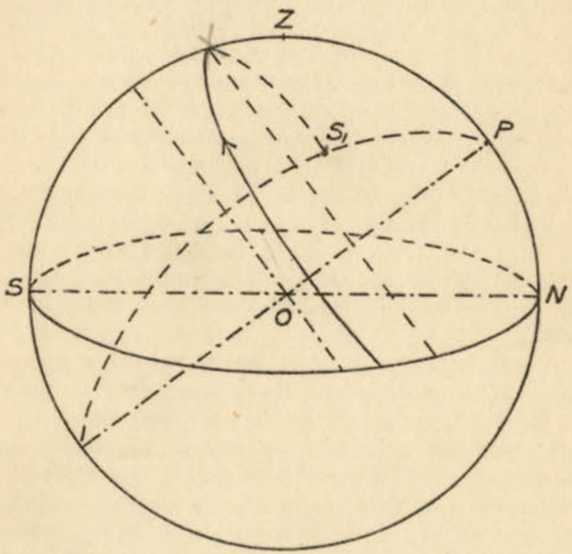

Fig. 6.

reckoned westwards from the meridian, but for the purposes of computation it is sometimes convenient to state it the shortest way from the meridian, provided that it is expressed so that there can be no ambiguity.

Sidereal Day.—A day of the length described above would be very nearly what is called a *sidereal day*; there is, however, a small difference, due to the fact already mentioned that the direction of the axis of the earth, owing to precession, does not remain absolutely unchanged.

The variation in the direction of the earth's axis introduces a corresponding slight deviation from the otherwise strictly circular path of a star, and consequently an inequality in the intervals between successive transits of a star and in the length of the day so measured; the extent of such deviation would depend on the position of the selected star on the celestial sphere. For the measurement of a day of absolutely uniform length it becomes necessary to select, in place of the transits of any star, the transits of a point so situated on the celestial sphere that the interval of time between successive transits is not affected by the change in the direction of the earth's axis, small though such change may be. It may now be stated that the point selected to define by its transits the sidereal day is one of the two points of intersection of the equator and ecliptic, viz. the First Point of Aries, ♈, of Fig. 3, which point, owing to the precessional movement of the pole P, moves slowly along the ecliptic with a motion corresponding to that of the pole P in the small circle PP′P. The intervals between successive transits of this point are so nearly equal that they may for all practical purposes be treated as being exactly so. The selected point, ♈, therefore makes a complete circuit of the ecliptic in the same period, about 25,800 years, as the pole P takes to describe its path (Fig. 3), which, being due to the conical movement of the earth's axis, is also circular.

Sun's Apparent Diameter.—The earth's orbit round the sun has already been stated to be an ellipse with the sun in one focus. The sun being in the plane of the earth's orbit, the sun's centre as seen from the earth appears in consequence to describe a great circle on the celestial sphere, and as the distance from the earth to the sun varies, the sun's apparent angular diameter likewise varies, being greatest where the distance is least, and vice versa. The apparent or angular semi-diameter is given for each day of the year in the *Nautical Almanac*, and the method of dealing with it in observations on the sun is explained on page 84 of this book.

The *Nautical Almanac*, published by the Admiralty for three years in advance, contains, in common with similar foreign publications, particulars of the varying positions of celestial bodies and other data required for the reduction of observations, and is in constant use for field astronomy and all astronomical work. An abridged edition, which will be found sufficient in many cases for field astronomy, is also published. Further details of the information given in the *Nautical Almanac* will be found in Appendix II.

Whitaker's Almanack gives, in necessarily restricted form, some useful astronomical information.

Declination and Right Ascension.—Referring to Fig. 7, which shows the apparent diurnal path of a star S_1, let S_1 represent the position of a star at an instant. Join ZS_1 by a great circle and produce it to cut the horizon at A. The star's altitude above the horizon is measured by the arc AS_1, which is the elevation which must be given to the telescope of the theodolite in order to bring the star to the cross wires. The star's altitude is, however, in general changing continuously. The angle between the meridian plane ZPN and the vertical circle ZS_1A has already been defined as the star's azimuth at the instant, which is also in general changing continuously as the star describes its diurnal circle. The co-ordinates altitude and azimuth therefore depend on time and on the position of the observing station

In order that the position of a body on the celestial sphere may be catalogued, it is necessary to select planes of reference which are independent of the location of the observing station, and as far as possible independent also of time.

The planes of reference selected are (1) the equator and (2) the plane passing through the poles and through the First Point of Aries, ♈.

The angular distance from the equator is called the *declination*, which may be either North or South. PS_1M

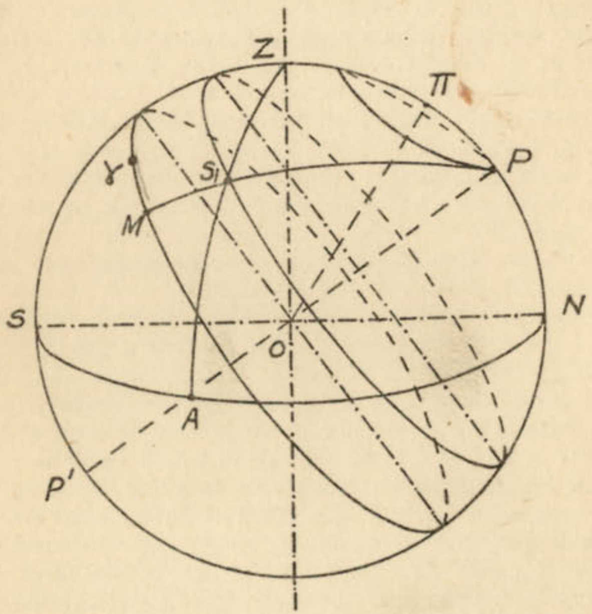

Fig. 7.

(Fig. 7) being a great circle, the arc MS_1, expressed in angular measure, is the declination of S_1. The great circle PS_1M is called the star's *declination circle* or *hour circle*, and it accompanies the star during its diurnal rotation. The arc PS_1, also expressed in angular measure, is the *polar distance*, and is the complement of the declination.

The angular distance of M from the plane P♈P′, i.e. the

arc ♈M, is known as the ***right ascension*** (R.A.) of the star, S_1, or of any other star on the great circle passing through P and M.

The R.A. is always measured eastwards from ♈, and is always expressed, not in degrees, minutes, and seconds of arc, but in hours, minutes, and seconds of time, 360° corresponding to 24 hours, since a complete revolution of the celestial sphere with reference to the earth takes 24 hours.

The angular distances, such as ♈M, measuring the right ascension, are accordingly converted into hours, minutes, and seconds at the rate of 15° per hour, and catalogued in that form as right ascensions.

It is to be noted that owing to precession the pole P is not an absolutely fixed point on the celestial sphere, but is moving slowly in a small circle around π. Consequently the plane of the equator, and with it ♈, is also moving with respect to the stars, i.e. both the declination and the R.A. are affected by a gradual slow change. The co-ordinates of stars are given in the *Nautical Almanac* for every tenth day—in some cases for every day.

Sidereal Time.—The approximate sidereal day has already been referred to. A sidereal day is strictly defined as the interval between successive transits of ♈, and a clock showing sidereal time should mark 0h 0m 0s when ♈ is on the meridian. Stars succeed one another across the meridian in the order of their right ascensions, and as the latter are measured from ♈ the declination circle of ♈ synchronizes with the hour hand of the sidereal clock, and the sidereal time when any star is on the meridian is the same as the star's R.A.

The sidereal time at any instant is the hour angle of ♈ reckoned westwards from the meridian.

Fig. 8 represents the earth in plan with an observer at any point *a* in the plane APA′ and a star S in his meridian. The same star as seen by an observer at *b* in the plane BPB′ would not be in his meridian, but would be at an hour angle of θ, and the angle aPb, the difference in longitude between the points a and b, is also equal to θ.

The sidereal times at a and b would differ by $\frac{\theta \text{ (in degrees)}}{15}$ hours.

If R.A. be the right ascension of the star, the *L.S.T.* (*local sidereal time*) at a for the instant represented, viz. when the star is on the meridian, is also R.A.

And the L.S.T. at b = R.A. + $\frac{\theta^\circ}{15}$

or L.S.T. = R.A. + hour angle.

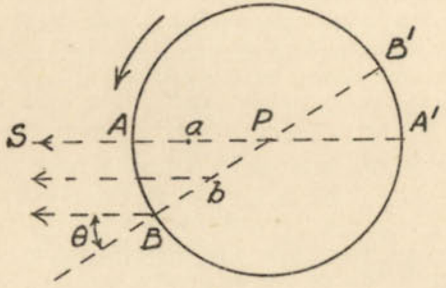

FIG. 8.

The observer will in all cases be provided with a clock, from which he knows the approximate local time at any instant, but the clock will in practically all cases have an error, i.e. it will be either fast or slow, and in order to determine its error, the L.S.T. can be ascertained by an observation for the hour angle of a suitable star.

The hour angle cannot be directly observed with a theodolite, but the altitude of the star can be observed and the clock read at the same instant.

The declination being known, the hour angle can be computed by the methods of spherical trigonometry, and this added to the R.A. gives the L.S.T.

If a sidereal clock is used, the difference between the L.S.T. thus ascertained and the clock reading is the clock error; if a mean time clock is used, its readings must be corrected to reduce them to sidereal time by methods which will be explained later.

In the above remarks the meridian is to be regarded as a plane rotating with uniform angular velocity about the axis PP'.

Using the ordinary conception of longitude, it is seen that if the difference of longitude expressed in degrees, minutes, and seconds between two places be converted into hours, minutes, and seconds at the rate of one hour to 15°, the result expresses the difference between the local sidereal times at the two places.

Apparent or Solar Time.—The sun as seen from the earth appears to describe a great circle of the heavens in the course of the year, viz. the ecliptic, but as the actual angular velocity of the earth about the sun is not uniform, the apparent angular velocity of the sun is also not uniform. In addition, the ecliptic is inclined to the equator.

The effect of these two circumstances on the solar day, i.e. the day as measured by the sun, can be readily seen.

In Fig. 9 the outer circle represents the earth as seen from a point in the direction of the axis produced, P, the north pole, being in the centre of the circle.

The sun, if in the direction of the dotted line 1, i.e. if anywhere in the plane containing the axis of the earth and the dotted line 1, would be on the meridian of any point such as A in that plane. When the earth has rotated 360° relatively to the celestial sphere, the same part of the celestial sphere will again be on the meridian of A; but the sun has in the meantime moved to a position indicated by the dotted line 2, and the earth has still to rotate through a small angle before the meridian of A will overtake the sun, i.e. before it will again be *apparent* *noon* at A, apparent noon being the instant at which the centre of the true sun is on the meridian. The angular distance between the lines 1 and 2 represents movement in right ascension; any change in the declination of the sun would make no difference to its meridian passage. Now since the earth is rotating with uniform angular velocity, if the sun were moving with uniform angular velocity in right ascension the intervals between its successive meridian passages for any one place would always

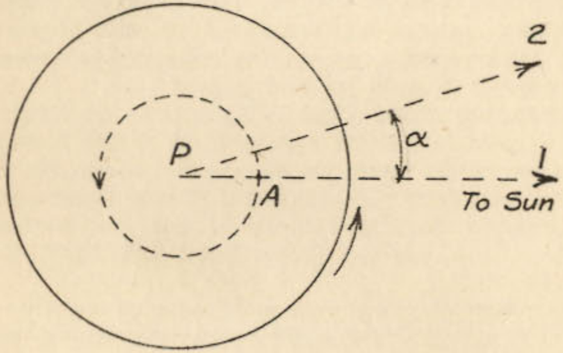

FIG. 9.

be equal, and the sun could be used as a time measurer with perfect accuracy and great simplicity. However, this is not, in fact, the case. The two circumstances just mentioned, viz. the fact, firstly, that the apparent movement of the sun on the celestial sphere is not performed with uniform angular velocity, and secondly that, even if it were so performed, the fact that the ecliptic is inclined to the equator, would cause the movement of the sun in right ascension to be variable.

Accordingly, the intervals between successive meridian passages of the sun are not equal, and if the sun is to be

used as an exact measurer of time for civil purposes, allowance has to be made for this.

Mean Time.—The sun completes its apparent path in the ecliptic relative to the First Point of Aries in a definite interval of time, called a *tropical year*, which is found to be 366·24222 sidereal days, i.e. the earth has rotated 366·24222 times relative to ♈ in a tropical year, or the point ♈ on the celestial sphere appears to have made 366·24222 revolutions during one tropical year. Now as the sun has, during the same period, described its orbit once in the opposite direction, the earth has rotated 366·24222 − 1, i.e. 365·24222 times relative to the sun, and the tropical year accordingly contains 365·24222 mean solar days, which is therefore the period of the annual return of the seasons.

The actual days as measured by successive meridian passages of the true sun are not equal in length, owing to its unequal motion in right ascension. Accordingly, an imaginary body called the *mean sun* is introduced, having a uniform motion in R.A. equal to the mean motion of the true sun in R.A.

In order that the mean sun may never at any time of the year differ greatly in R.A. from the true sun, its relation to the true sun is fixed as follows:

When the true sun is in perigee, a body called the *dynamical mean sun* is assumed to be started off along the ecliptic with a uniform angular velocity equal to the mean angular velocity of the true sun. The dynamical mean sun would, owing to the orbit being symmetrical about the major axis, again coincide with the true sun at apogee. However, the time intervals between successive meridian passages of the dynamical mean sun would not be equal, in spite of its uniform angular velocity, on account of the inclination of its orbit to the equator. At the instant when the dynamical mean sun passes through the First Point of Aries, a second imaginary body, called the *astronomical mean sun*, is assumed to be started off from ♈ along the celestial equator with the same angular velocity as the dynamical mean sun. The motion of the astronomical mean sun in R.A. is accordingly uniform, and the intervals between successive meridian passages thereof are equal.

The astronomical mean sun thus defined is generally referred to simply as the *mean sun.*

Mean time is measured by the hour angle of the mean sun, the instant when the mean sun crosses the meridian above the horizon of any station being *local mean noon* for that station or *local mean time* (*L.M.T.*) 12h 00m 00s.

The instant when the centre of the true sun is on the meridian of a station is *local apparent noon* (*L.A.N.*) of that station, and the hour angle of the true sun, measured westwards from the meridian, gives *local apparent time* (*L.A.T.*), it being observed that when the hour angle is zero the L.A.T. is 12h 00m 00s.

At any instant the local mean time is the hour angle of the mean sun, measured westwards from the meridian, +12 hours, the instant when the mean sun is on the meridian being mean noon, or 12 hours L.M.T.

Equation of Time.—In general the hour angles of the true sun and of the mean sun will differ, i.e. the L.M.T. and L.A.T. will not agree. The difference between them, expressed in minutes and seconds of time, is called the *equation of time.* The equation of time is to be regarded as a correction to be applied to L.A.T. to derive the L.M.T. at any instant, or by applying it with the opposite sign to L.M.T. the L.A.T. is obtained. The equation of time has to be applied in all cases of observations on the sun. Being the difference between the hour angles of two bodies, each of which has a definite position on the celestial sphere, the equation of time is consequently the same at any given instant for all parts of the earth.

Longitude.—The difference of longitude between two places is the angular distance between their terrestrial meridians. The meridian of Greenwich is taken as the zero of longitudes, longitudes being expressed either as hours, minutes, and seconds of time, or degrees, minutes, and seconds of arc, east or west of Greenwich.

From the foregoing it will be seen that if two places differ in longitude by say 30°, i.e. if their terrestrial meridians are 30° apart, the difference between their local times, whether sidereal or mean, is 2 hours, for the reason that the time interval between the meridian passages of the First Point of Aries at the two places is two sidereal hours, and the time interval between the meridian passages of the mean sun is two mean time hours.

Standard Meridian Time.—To avoid the confusion which would result from the use of the local mean times of a large number of places, the world has been divided into a number of *time zones,* each of which uses a mean time differing from that of Greenwich, or *G.M.T.*, by an integral number of half-hours, according to the longitude of the place or country concerned. For instance, G.M.T. is used in the British Isles, France, Belgium, Spain; Germany and various other countries use the time of the meridian 15° E., i.e. one hour ahead of Greenwich, and so on.

Each country thus uses for ordinary civil purposes what is called *standard meridian time,* being the L.M.T. of the most convenient meridian differing from that of Greenwich by a multiple of half an hour. The standard time adopted in various countries is given in Appendix IV; a somewhat shorter list can be found in the *N.A.*

Examples.—A place is situated in longitude 40° E. of Greenwich. Find the G.M.T. (Greenwich mean time) when the L.M.T. is 3h 30m 00s P.M.

It should be noted that in all cases the day begins at midnight, noon being 12h 00m 00s, consequently the given time is L.M.T. 15h 30m 00s.

Now 40° of longitude when converted into time at the rate of 15° per hour corresponds to 2h 40m 00s, which is the difference between the L.M.T. and the G.M.T. at any instant. The place being east of Greenwich, the L.M.T. is ahead of the G.M.T., since the earth turns from west to east; therefore to obtain the G.M.T. the difference of longitude must be subtracted from the L.M.T.

The conversion, therefore, is as follows:

	h.	m.	s.
L.M.T. =	15	30	00
long. E. =	2	40	00
G.M.T. =	12	50	00

The following example involves both the equation of time and the longitude.

Find the L.A.T. corresponding to L.M.T., 4 March 1928, 8h 5m A.M., in longitude 60° 45′ W.

	h.	m.	s.
L.M.T., 4 March =	8	05	00
long. W. =	4	03	00
G.M.T. =	12	08	00

Equation of time (E) at G.M.N., from *N.A.*, = 11m 52s·05 (decreasing)
Variation in 1 hour = 0·554s

Therefore E at 12h 08m 00s $= 11\text{m } 52\text{s}{\cdot}05 - \left(0{\cdot}554 \times \frac{8}{60}\right)$

= 11m 52s to be added to apparent time or subtracted from mean time.

	h.	m.	s.
But G.M.T. =	12	08	00
E =		11	52
G.A.T. =	11	56	08
long. W. =	4	03	00
L.A.T. =	7	53	08

Conversion of Time.—The fundamental ideas of time—sidereal, apparent solar, and mean—having been discussed, it becomes necessary to show how the time at any instant, expressed in one of the three systems named, can be converted into either of the other systems.

The necessity for such conversion arises from the fact

that star observations depend on sidereal time, and sun observations depend on apparent time, whereas the chronometer in use may be either a mean time or a sidereal time chronometer.

The conversion of sidereal to mean time and vice versa will be dealt with first.

It has already been stated (p. 25) that the tropical year contains 366·24222 sidereal days, and therefore 365·24222 mean solar days.

The actual interval of time is the same, viz. a tropical year, in both systems of time measurement, but the numbers expressing it differ in the two systems, the ratio of the numbers being 366·24222 : 365·24222.

This ratio is therefore the ratio between the numbers expressing any time interval in sidereal or mean time.

Let M be the expression of a time interval in mean time.

Let S be the expression of the same time interval in sidereal time.

$$\text{Then } \frac{M}{S} = \frac{365{\cdot}24222}{366{\cdot}24222} = 0{\cdot}99726957$$

$$\text{and } \frac{S}{M} = \frac{366{\cdot}24222}{365{\cdot}24222} = 1{\cdot}00273791$$

The mean time clock thus loses on the sidereal clock, the rate of the sidereal clock relative to the mean time clock being 1·00273791 : 1. This rate when reduced to minutes and seconds shows that in 1 hour of mean time the sidereal clock registers 1h 00m 09s·8565, i.e. the sidereal clock gains 9·8565 sidereal seconds on the mean time clock in 1 hour of mean time.

Correspondingly, in 1 hour as registered by the sidereal clock the mean time clock will register 9·8296 mean time seconds less than 1 hour.

The quantity 9·8565 seconds represents what is called the *acceleration* of the sidereal clock per M.T. hour, and

9·8296 seconds represents what is called the ***retardation*** of the M.T. clock per S.T. hour.

The *N.A.* gives for each day of the year the *Greenwich sidereal time at Greenwich mean noon* (*S.T.G.M.N.*). This figure thus forms the starting-point for conversion of mean time to sidereal time at that or any other place for which S.T. is required.

The first step in all time conversions is to apply the difference of longitude to the given local time and thus obtain the corresponding Greenwich time of the given kind. The conversion of this corresponding Greenwich time into the required kind of time for the longitude of Greenwich is then made, and finally the difference of longitude is applied to obtain the required local time. The latitude of the station has no effect on the local time, which depends on the longitude only.

Example.—A station is situated in longitude 24° 40′ 30″ E. and latitude 25° N. Find the L.S.T. when the L.M.T. is 4h 30m P.M. on 30 December 1927.

	h.	m.	s.
24° 40′ 30″ =	1	38	42
L.M.T. =	16	30	00
long. E. =	1	38	42
G.M.T. =	14	51	18
M.T. interval from G.M.N. =	2	51	18
From *N.A.*, S.T.G.M.N. =	18	31	43·40
acceleration in 2h =			19·713
,, ,, 51m =			8·378
,, ,, 18s =			0·049
G.S.T. =	21	23	29·54
long. E. =	1	38	42
L.S.T. =	23	02	11·54

Example.—At a station in longitude 15m 45s W. the L.M.T. is 9h 35m 40s A.M. on 19 October 1928. Find the L.S.T. at the instant.

The difference of longitude being here expressed in time, it can be applied directly.

	h.	m.	s.
L.M.T. =	9	35	40
long. W. =		15	45
G.M.T. =	9	51	25
M.T. interval from G.M.N., 18 Oct. =	21	51	25
acceleration for 21h =		03	26·986
,, ,, 51m =			8·378
,, ,, 25s =			0·068
S.T. interval from G.M.N., 18 Oct. =	21	55	00·432
From *N.A.*, S.T.G.M.N., 18 Oct. =	13	46	54·14
G.S.T. =	35	41	54·572

The clock reads only to 24h, so 24h is subtracted from the foregoing G.S.T. to give

	h.	m.	s.
G.S.T. =	11	41	54·57
long. W. =		15	45
L.S.T. =	11	26	09·57

Instead of working from the previous G.M.N., as has been done above, the G.S.T. at the following G.M.N. might have been used, as the time interval to that instant is shorter, thus:

	h.	m.	s.
L.M.T. =	9	35	40
long. W. =		15	45
G.M.T. =	9	51	25
M.T. interval to G.M.N., 19 Oct. =	2	08	35
acceleration for 2h =			19·713
,, ,, 8m =			1·314
,, ,, 35s =			0·096
S.T. interval to G.M.N., 19 Oct. =	2	08	56·123
From *N.A.*, S.T.G.M.N., 19 Oct. =	13	50	50·690
G.S.T. =	11	41	54·567
long. W. =		15	45
L.S.T. =	11	26	09·57 as before.

Example.—At a station in longitude 15m 45s W., the L.S.T. is 9h 35m 40s, the date being 19 October 1928. Find the corresponding G.M.T.

	h.	m.	s.
L.S.T. =	9	35	40
long. W. =		15	45
G.S.T. =	9	51	25
S.T.G.M.N., 19 Oct. =	13	50	50·69
S.T. interval to G.M.N. =	3	59	25·69
retardation for 3h =			29·489
,, ,, 59m =			9·666
,, ,, 25s =			0·068
,, ,, 0·69s =			0·002
Total retardation =			39·225
S.T. interval to G.M.N. =	3	59	25·69
M.T. interval to G.M.N. =	3	58	46·465
G.M.T. =	8	01	13·54

It will be found, as already stated, that in all cases the most direct method of performing any time conversion is by at once converting the given local time into the corresponding Greenwich time by application of the longitude with the appropriate sign. Methods are sometimes used in which the L.S.T. at L.M.N. is deduced from the G.S.T. at G.M.N. given in the *Nautical Almanac*; such methods have little to recommend them and should be avoided.

In connection with what has been stated so far the following points should be borne in mind:

1. The true sun moves along the ecliptic (inclined at about 23° 27′ to the equator) with a variable motion in R.A., the want of uniformity being due to two causes, viz., firstly the unequal velocity of the earth in its orbit, and secondly the inclination of that orbit to the equator.

2. The mean sun moves along the equator with a uniform motion in R.A.

3. The equation of time, i.e. the difference between apparent solar time and mean solar time, is the same all over the earth at any instant. It is the difference between the hour angles of the mean sun and the true sun.

4. The difference between L.S.T. and L.M.T. is the same all over the earth at any instant, being the difference between the hour angle of the First Point of Aries and that of the mean sun, plus or minus 12 hours, as L.S.T. is measured from the upper transit of ♈, whereas the upper transit of the mean sun is 12 hours L.M.T.

For observations on the sun it is usually necessary to know both the sun's declination and the equation of time at the instant.

Interpolation of "N.A." Values.—It has already been shown (p. 7) that the sun is in the plane of the earth's equator at the vernal equinox, i.e. on or about 22 March; at that time the sun's declination is therefore 00° 00′ 00″. Again, at the autumnal equinox, i.e. on or about 24 September, the sun is also in the plane of the earth's equator, and its declination is therefore again 00° 00′ 00″; but at the summer solstice, i.e. on or about 22 June, the line joining the earth to the sun makes an angle with the plane of the equator equal to the obliquity of the ecliptic (Fig. 2), so that the sun's declination is then about 23° 27′ N.; at the winter solstice, i.e. on or about 22 December, the sun's declination is, for the same reason, about 23° 27′ S. The sun's declination thus changes during the year between the values of 23° 27′ N. and 23° 27′ S.

The *N.A.* gives the declination and the equation of time both for mean noon and for apparent noon at Greenwich for every day of the year, with the variation in one hour at G.A.N. in each of these quantities. The variation in one hour from G.M.N. may be taken as being the same as the given variation in one hour from G.A.N. The declination or the equation of time at any other instant of Greenwich time can be readily derived.

For example, suppose that an observation on the sun has been taken at a station in longitude 2h 05m 00s East, 20 October 1928, at 16h 30m 20s standard meridian time

—2 hours East—and it is required to find the sun's declination at the instant of the observation.

		h	m	s	
S.M.T. of observation . . .		16	30	20	
Longitude of standard meridian, E. .		2	00	00	
G.M.T. of observation . . .		14	30	20	
	i.e.	2	30	20	P.M.

By reference to the *Nautical Almanac* it is found that on 20 October 1928 the sun's apparent declination at G.M.N. is 10° 21′ 16″·7 S. (the explanation of the use of the word "apparent" in the precept or heading to this column is given in Appendix II), and that the variation in 1 hour at G.A.N. = 53″·86.

When the interval between Greenwich noon and the instant of observation is short, it will be sufficiently accurate for most purposes to multiply the variation in one hour by the length of the interval, in hours, from Greenwich noon, to obtain the total variation which has occurred since Greenwich noon.

Thus: variation in 1 hour = 53″·86

„ 2 hours = 107″·72
„ 30 mins. = 26″·93
„ 20 secs. = 0″·30

Total variation since G.M.N. = 134″·95 = 2′ 14″·9
Declination at G.M.N. = 10° 21 16·7 S.

Declination at instant of observation = 10° 23′ 31″·6

This method is only approximate, as the rate of variation is not constant. If a more accurate value is wanted, the procedure is as follows:

The quantity 53″·86 is the actual variation in 1 hour at G.A.N.; but what is required is the mean rate of variation over the period separating the instant of observation from G.M.N., as this mean rate multiplied by the length of the interval (2h 30m 20s in this case) gives the total variation in declination since G.M.N. to be applied to the declination at G.M.N. in order to obtain the declination at the instant of observation. The mean rate of variation over the period of 2h 30m 20s elapsed since G.M.N. may be taken as being the variation at the middle of the said interval, i.e. at G.M.T. 1h 15m 10s P.M.

The declination of the sun is increasing, and the
Variation in 1 hour at G.A.N. 20 Oct. is 53″·86 from *N.A.*
„ „ „ 21 Oct. is 53″·46 „ „

Therefore change in rate in 24 hours 0″·40 (decreasing)
„ „ „ 1h 15m 10s 0″·021

Hourly variation at G.A.N. 53″·86
Therefore hourly variation at 1h 15m 10s = 53″·84
Total variation in 2h 30m 20s = 134″·9 or 2′ 14″·9
Declination at G.M.N., from *N.A.*, 10° 21′ 16″·7

Therefore declination at instant of observation, 10° 23′ 31″·6

The result is in this case the same as that obtained by the approximate method, but had the observation been made at another time of the year, when the hourly variation was changing more rapidly, the results would have shown a small difference.

Even this method is not mathematically strictly accurate, but the error will amount at the most to a fraction of a second of arc, and sun observations are not relied on for observations requiring such a degree of accuracy. That the error is small may be seen by computing, from the sun's declination at G.M.N. of one day, the declination for the following G.M.N. The result will be found to differ but slightly from the declination given in the *N.A.* for that day. A precisely similar method is used in interpolating for the value of the equation of time at any instant of G.M.T. other than G.M.N. or G.A.N.

Example.—To find the equation of time (E) at local apparent time 3h 45m 20s P.M. in longitude 95° W. on 31 October 1928.

	h.	m.	s.
L.A.T. =	3	45	20 P.M.
long. W. =	6	20	00
G.A.T. =	10	05	20 P.M.

		m.	s.
From *N.A.*, variation in equation of time in 1 hr. at G.A.N.	=		0·099
∴ " " " 10h "	=		0·99
" " " 5m "	=		0·008
" " " 20s "	=		0·000
From *N.A.*, equation of time at G.A.N. (increasing)	=	16	19·27
∴ required value of E	=	16	20·27

Alternatively:

		s.
From *N.A.*, variation in E in 1 hour at G.A.N., 31 Oct.	=	0·099
" " " " 1 Nov.	=	0·065
∴ change in variation in 24 hours	=	0·034

∴ variation in 1 hr. at middle of interval, viz. at 5 P.M., is

$$0{\cdot}099-\left(\frac{5}{24}\times 0{\cdot}034\right) = 0{\cdot}092 \text{ s.}$$

		m.	s.
∴ variation in 10h	=		0·92
" 5m	=		0·008
" 20s	=		0·000
E at G.A.N.	=	16	19·27
∴ required value of E	=	16	20·20

The difference in the results obtained by the two methods is in this case only in the second place of decimals.

CHAPTER III

SPHERICAL TRIGONOMETRY

In observations for the purposes of field astronomy the body observed, for example, a star or the sun, gives a line of sight of indefinite length, the origin of this line of sight being the centre of the imaginary celestial sphere.

To fix the direction of this line of sight, two others, also of indefinite length, are observed or deduced, one in the direction of the zenith Z, the other in the direction of the pole P, both also having their origin at O (Fig. 10).

Sides and Angles of Spherical Triangles.—Thus OZ, OS, and OP define three planes passing through one point O and inclined to each other, and to obtain the desired results from the observations it is necessary to determine the plane trigonometrical ratios which obtain between the angles at which these three planes are inclined to each other, as measured in planes perpendicular to their lines of intersection. As already stated, it is assumed in astronomical work that the points S, Z, and P are on the surface of an imaginary celestial sphere, and therefore it is convenient to assume that the three intersecting planes are bounded by a spherical surface having its centre at the common point of origin O. The three planes therefore cut the bounding sphere in great circles, the radius of which is the same as that of the sphere. The arcs of these great circles on the surface of the bounding sphere, namely the arcs PS, SZ, and PZ, form the *sides of the spherical triangle* PZS, but these arcs or sides are measured by the angles, namely, POS, SOZ, and ZOP, which they subtend at the centre, O, of the sphere.

The angles between the three planes, measured in planes perpendicular to their lines of intersection, are called the *angles of the spherical triangle.* In astronomical work they are usually denoted by the capital letters P, Z, and S, indicating the points at which the lines of intersection of the planes cut the bounding sphere; thus the angle SZP is the angle between the planes SOZ and ZOP, the angle ZPS is the angle between the planes ZOP and POS, and the angle PSZ is the angle between the planes POS and SOZ.

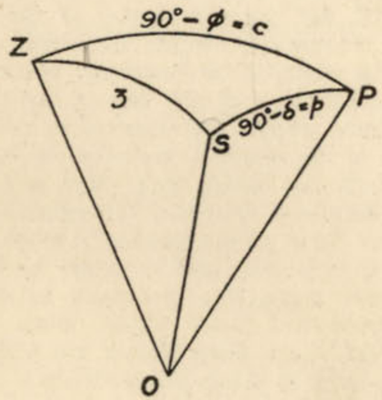

FIG. 10.

Bearing in mind the definitions already given, and referring to Fig. 10, which may be compared with Fig. 7,

Angle SZP is the azimuth
,, ZPS ,, hour angle $=t$
,, PSZ ,, parallactic angle
Side PS ,, codeclination or polar distance $=90°-\delta=p$
,, SZ ,, zenith distance $=z$
,, PZ ,, colatitude $=90°-\phi=c$.

Some of these quantities being given or observed, it becomes necessary to determine one or more of the others

in order to obtain the desired results, and to this end spherical trigonometry is made use of.

The formulæ necessary for the solution of a spherical triangle may be obtained by the methods of analytical geometry. This will necessitate making use of the relationship between the co-ordinates of a point, as referred to two different pairs of axes in the same plane, which relationship may, as a preliminary, be seen by reference to Fig. 11, where P is a point whose co-ordinates with reference to the axes OX and OY are x and y, and with reference to the

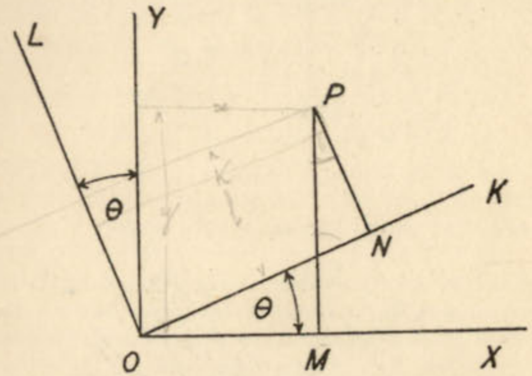

FIG. 11.

axes OK and OL are k and l, the axes OX and OY being in the same plane as OK and OL, and being in fact the axes OX and OY rotated through an angle θ.

The relationship between the co-ordinates is:

$$k = ON = OM \cos\theta + PM \sin\theta = x\cos\theta + y\sin\theta$$
$$l = PN = PM \cos\theta - OM \sin\theta = y\cos\theta - x\sin\theta$$

Now let ABC (Fig. 12) represent a spherical triangle on the surface of a sphere the radius of which is r and the centre of which is at O. Take three rectangular axes OX, OY, OZ such that the side AB of the spherical triangle lies in the plane OX, OY, and let the co-ordinates of C be x,

y, and z. Let OX′ and OY′ be the position of the axes OX and OY when rotated through an angle c in the plane OX, OY, and let x', y', z be the co-ordinates of C with reference to OX′, OY′, OZ. Draw CN perpendicular to the plane OX′, OY′, and CS perpendicular to OX and CS′ perpendicular to OX′. Join S′S.

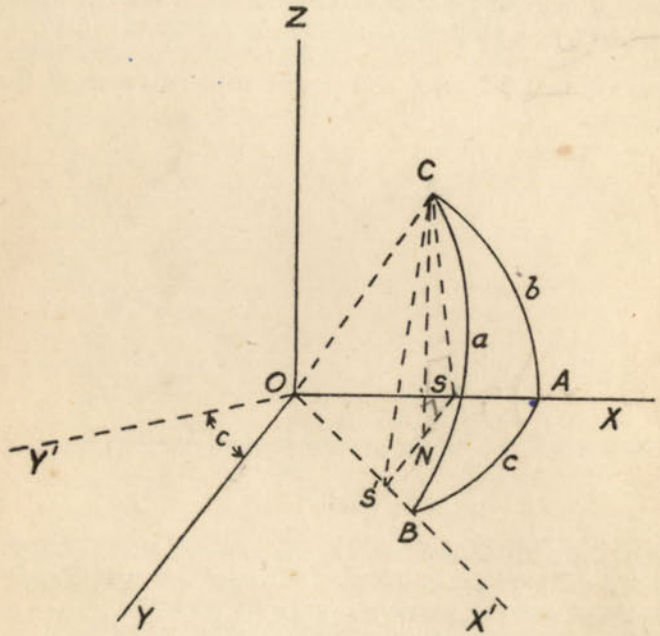

FIG. 12.

Then $x = \mathrm{OS} = r \cos \mathrm{COS} = r \cos b$
$y = \mathrm{SN} = \mathrm{CS} \cos \mathrm{A} = r \sin b \cos \mathrm{A}$
$z = \mathrm{CN} = \mathrm{CS} \sin \mathrm{A} = r \sin b \sin \mathrm{A}$;

but, as shown above, $x' = x \cos c + y \sin c$,
$y' = y \cos c - x \sin c$,
$z' = z$.

Further, $x' = \mathrm{OS'} = r \cos a$
$y' = -\mathrm{S'N} = -\mathrm{CS'} \cos \mathrm{B} = -r \sin a \cos \mathrm{B}$
$z' = \mathrm{CS'} \sin \mathrm{B} = r \sin a \sin \mathrm{B}$.

Substituting in the three previous equations,
$r \cos a = r \cos b \cos c + r \sin b \cos \mathrm{A} \sin c$
$-r \sin a \cos \mathrm{B} = r \sin b \cos \mathrm{A} \cos c - r \cos b \sin c$
$r \sin a \sin \mathrm{B} = r \sin b \sin \mathrm{A}$.

Therefore, dividing by r,

$$\cos a = \cos b \cos c + \sin b \sin c \cos \mathrm{A} \quad . \quad . \quad . \quad . \quad (1)$$
$$\sin a \cos \mathrm{B} = \cos b \sin c - \sin b \cos c \cos \mathrm{A} \quad . \quad . \quad (2)$$
$$\sin a \sin \mathrm{B} = \sin b \sin \mathrm{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Dividing equation (2) by $\sin b$ gives

$$\cos c \cos \mathrm{A} = \sin c \cot b - \frac{\sin a \cos \mathrm{B}}{\sin b}$$
$$= \sin c \cot b - \sin \mathrm{A} \cot \mathrm{B} \quad . \quad . \quad . \quad (4)$$

In the triangle ABC, for simplicity each angle has been made less than 90°, and the point N therefore falls between OX and OX′, thus giving y and y' opposite signs, i.e. $y = \mathrm{SN}$, and $y' = -\mathrm{SN}$.

Fundamental Equations.—The equations (1), (2), (3), and (4) are the fundamental formulæ of spherical trigonometry, and are true for every spherical triangle, whatever be the values of the sides and angles, and therefore may be applied to the special case of a triangle with one angle, such as A, a right angle, in which case $\sin \mathrm{A} = 1$ and $\cos \mathrm{A} = 0$.

Right-Angled Triangles.—Substitution of these values in equations (1), (2), and (3) gives

$\cos a = \cos b \cos c$ from equation (1),

$\cos \mathrm{B} = \dfrac{\tan c}{\tan a}$ „ „ (1) and (2),

$\tan \mathrm{B} = \dfrac{\tan b}{\sin c}$ „ „ (2) and (3),

$\sin \mathrm{B} = \dfrac{\sin b}{\sin a}$ „ „ (3).

To apply the equations obtained above, it is only necessary to substitute the astronomical nomenclature for that employed, for generality, in these equations; thus in Fig. 13, if Z is the zenith, P the pole, and S a star, the great circles forming the meridian circle, the declination circle, and the vertical circle passing through these points form a spherical triangle ZPS identical with the spherical triangle ABC, and the equations for the latter can be applied to the astronomical triangle. Thus from equation (1)

$$\cos a = \cos b \ \cos c + \sin b \ \sin c \ \cos A,$$

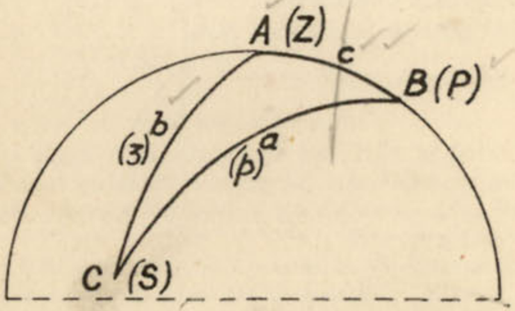

FIG. 13.

and applying this to the astronomical triangle, the equation becomes

$$\cos z = \cos c \cos p + \sin c \sin p \cos P.$$

Similarly, $\cos p = \cos z \cos c + \sin z \sin c \cos Z.$

Equation (1) can therefore be applied to any case in which two sides and the included angle, or three sides, are given.

"Cot" Formula.—Equation (4), viz.

$$\cos c \cos A = \sin c \cot b - \sin A \cot B,$$

is sometimes known as the *cot formula*, and may be memorized as follows. If any four consecutive parts, i.e. sides and angles, of the spherical triangle be taken, either clock-

wise or anti-clockwise, such as angle B, side *c*, angle A, side *b*, i.e. B, *c*, A, *b*, then cos (inner side) cos (inner angle)

=sin (inner side) cot (other side)

—sin (inner angle) cot (other angle).

Therefore, applying this to the four consecutive parts B, *c*, A, *b*,

$$\cos c \cos A = \sin c \cot b - \sin A \cot B.$$

In applying this rule, the first of the four consecutive parts may be a side; thus for the four consecutive parts *b*, C, *a*, B the equation becomes

$$\cos a \cos C = \sin a \cot b - \sin C \cot B.$$

Some of the applications of these equations for practical purposes will now be given for reference.

Formulæ for Time Observations.—For time observations such as described later in Chapter VII the fundamental formula

$$\cos z = \cos c \ \cos p + \sin c \ \sin p \ \cos P$$

can be modified to obtain the desired results.

The quantity observed is zenith distance, $ZS = z$

The *N.A.* gives the declination δ from which $PS = 90° - \delta = p$

A known quantity is the latitude ϕ from which $PZ = 90° - \phi = c$

i.e. each of the three sides of the spherical triangle PZS (Fig. 13) is known, and the angle ZPS is to be computed.

The above-mentioned fundamental formula gives

$$\cos P = \frac{\cos z - \cos c \ \cos p}{\sin c \ \sin p} = \frac{\cos z - \sin \phi \ \sin \delta}{\cos \phi \ \cos \delta}$$

$$= \cos z \ \sec \delta \ \sec \phi - \tan \delta \ \tan \phi \quad . \quad . \quad . \quad (5)$$

The two terms of the right-hand side of the equation are computed separately by log tables, and their difference gives the natural cosine of the angle ZPS. The log of this natural cosine is then taken and the corresponding angle looked up in the tables of log cosines to obtain the value of ZPS.

A formula more suitable for logarithmic computation may be derived as follows:

$$\cos z = \cos c \cos p + \sin c \sin p \cos P$$

and $$\cos P = 1 - 2\sin^2\frac{P}{2}$$

$$1 - 2\sin^2\frac{P}{2} = \frac{\cos z - \cos c \cos p}{\sin c \sin p}$$

$$2\sin^2\frac{P}{2} = 1 - \frac{\cos z - \cos c \cos p}{\sin c \sin p}$$

$$= \frac{\sin c \sin p + \cos c \cos p - \cos z}{\sin c \sin p}$$

$$= \frac{\cos(p-c) - \cos z}{\sin c \sin p}$$

$$= \frac{-2\sin\frac{p-c+z}{2} \cdot \sin\frac{p-c-z}{2}}{\sin c \sin p}$$

$$\therefore \quad 2\sin^2\frac{P}{2} = \frac{2\sin\frac{p+z-c}{2} \cdot \sin\frac{z+c-p}{2}}{\sin p \sin c}$$

Let $$2s = p + c + z;$$

then $$s = \frac{p+c+z}{2}$$

and $$s - p = \frac{z+c-p}{2}$$

and $$s - c = \frac{z+p-c}{2}$$

$$2\sin^2\frac{P}{2} = \frac{2\sin(s-c)\sin(s-p)}{\sin c \sin p}$$

or $$\sin\frac{P}{2} = \sqrt{\frac{\sin(s-c)\sin(s-p)}{\sin c \cdot \sin p}} \quad . \quad . \quad . \quad (6)$$

This form of the equation is suitable for direct logarithmic computation, as p, c, and z are all known. The form given in equation (5) has, however, certain advantages, as, if observations are to be made for several days at one station, the values of tan ϕ and sec ϕ remain constant, and the values of tan δ and sec δ change but slightly for any particular star.

An analogous expression for cos $\frac{P}{2}$ can also be found, as follows:

Since $$\cos z = \cos p \cos c + \sin p \cdot \sin c \cdot \cos P$$

$$\cos P = \frac{\cos z - \cos p \cos c}{\sin p \cdot \sin c}$$

$$2\cos^2\frac{P}{2} - 1 = \frac{\cos z - \cos p \cdot \cos c}{\sin p \cdot \sin c}$$

$$2\cos^2\frac{P}{2} = \frac{\cos z - (\cos p \cos c - \sin p \sin c)}{\sin p \sin c}$$

$$= \frac{\cos z - \cos(p+c)}{\sin p \sin c}$$

$$= \frac{-2\sin\frac{p+c+z}{2} \sin\frac{z-p-c}{2}}{\sin p \sin c}$$

$$= \frac{2\sin s \sin(s-z)}{\sin p \sin c}$$

$$\therefore \cos\frac{P}{2} = \sqrt{\frac{\sin s \sin(s-z)}{\sin p \sin c}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The value of $\frac{P}{2}$ may thus be found from either its sine or cosine, and consequently also from its tangent, as

$$\tan\frac{P}{2} = \frac{\sin\frac{P}{2}}{\cos\frac{P}{2}} = \sqrt{\frac{\sin(s-c)\sin(s-p)}{\sin s \sin(s-z)}} \quad . \quad . \quad (8)$$

Formula for Determination of Latitude by Observation of Polaris.—In the method to be described on page 115 for the determination of latitude by observation of Polaris, the fundamental equation, No. (1), is reduced to the form

$$\sin h = \sin (h+x) . \cos p + \cos (h+x) . \sin p . \cos \text{ZPS},$$

and it is required to find the value of x in terms of h and p.

Since p and x are both small, Taylor's theorem may be applied :

$$\sin (h+x) = \sin h + x \cos h - \frac{x^2}{\underline{|2}} \sin h - \frac{x^3}{\underline{|3}} \cos h \quad . \quad . \quad . \quad (a)$$

$$\cos (h+x) = \cos h - x \sin h - \frac{x^2}{\underline{|2}} \cos h + \frac{x^3}{\underline{|3}} \sin h \quad . \quad . \quad . \quad (b)$$

The substitution of these values in the above equation for $\sin h$ gives

$$\sin h = \cos p \times (a) + \sin p \times (b) \times \cos t,$$

t being the angle ZPS ; or

$$\sin h = \left(1 - \frac{p^2}{\underline{|2}} + \frac{p^4}{\underline{|4}} \dots\right)\left(\sin h + x \cos h - \frac{x^2}{\underline{|2}} \sin h - \frac{x^3}{\underline{|3}} \cos h \dots\right)$$

$$+\left(p - \frac{p^3}{\underline{|3}} + \frac{p^5}{\underline{|5}} \dots\right)\left(\cos h - x \sin h - \frac{x^2}{\underline{|2}} \cos h + \frac{x^3}{\underline{|3}} \sin h \dots\right) \cos t$$

$$= \sin h + x \cos h + p \cos t . \cos h - \frac{x^2}{\underline{|2}} \sin h - px \cos t \sin h$$

$$- \frac{p^2}{\underline{|2}} \sin h \quad . \quad . \quad .$$

$$\therefore -x \cos h = p \cos t \cos h - \frac{x^2}{\underline{|2}} \sin h - px \cos t \sin h - \frac{p^2}{\underline{|2}} \sin h \dots$$

$$\therefore x = -p \cos t + \frac{x^2}{\underline{|2}} \tan h + px \cos t . \tan h + \frac{p^2}{\underline{|2}} \tan h + \dots$$

The first approximation is $x = -p \cos t$.

Substituting this value for x in succeeding terms,

$$\begin{aligned} x &= -p \cos t + \tfrac{1}{2}p^2 \cos^2 t . \tan h - p^2 \cos^2 t . \tan h + \tfrac{1}{2}p^2 \tan h \\ &= -p \cos t - \tfrac{1}{2}(p^2 \cos^2 t . \tan h - p^2 \tan h) \\ &= -p \cos t + \tfrac{1}{2}p^2 \tan h\,(1 - \cos^2 t) \\ &= -p \cos t + \tfrac{1}{2}p^2 \sin^2 t . \tan h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9) \end{aligned}$$

Computation of the Position of a Star for a Prearranged Instant.—When it is required to compute the position of a star at a prearranged instant for observation in daylight or twilight, as described on page 149, the following quantities are known :

The polar distance	$(90°-\delta) = p =$ PS (Fig. 13)
The colatitude	$(90°-\phi) = c =$ PZ
The angle ZPS	(L.S.T.−R.A. for a west star, or 24−(L.S.T.−R.A.) for an east star)

i.e. in the triangle ZPS the two sides PZ and PS are known, and also the included angle ZPS.

The quantities required to be computed are :

the zenith distance . .	z
the azimuth angle . .	PZS

z can be found conveniently by the fundamental formula

$$\cos z = \cos p \cos c + \sin p \sin c \cos \text{P}$$

By formula No. (3) :

$$\frac{\sin z}{\sin \text{P}} = \frac{\sin p}{\sin \text{PZS}}$$

and

$$\sin \text{PZS} = \frac{\sin p \sin \text{P}}{\sin z}$$

In this equation the quantities p, P, and z are known, z having just been computed, and therefore sin PZS can be computed, but as $\sin \text{A} = \sin (180° - \text{A})$ there will be a doubt as to which of the two possible values of PZS is the correct one ; in fact, one of the two values will be the angle PZS, and the other the supplement of that angle. It will frequently happen that the surveyor will know from his acquaintance with the stars which of the two values is the correct one. The two values will, of course, correspond to two azimuths equally inclined to the prime vertical, one being north and the other south of it.

However, a definite single solution for the angle PZS can be found by alternative methods.

z having been computed as above, on page 44 it was shown that

$$\sin\frac{P}{2}=\sqrt{\frac{\sin(s-p)\sin(s-c)}{\sin p\sin c}}$$

and likewise it may be shown, or by analogy it may be seen, that

$$\sin\frac{Z}{2}=\sqrt{\frac{\sin(s-z)\sin(s-c)}{\sin z\sin c}}\quad . \quad . \quad . \quad (10)$$

further,

$$\cos\frac{Z}{2}=\sqrt{\frac{\sin s\ \sin(s-p)}{\sin z\ \sin c}}\quad . \quad . \quad . \quad . \quad (11)$$

from either of which $\frac{Z}{2}$ may be found; and as its value will never exceed 90°, the value of PZS is obtained without ambiguity; but as the angle PZS may be measured either eastwards or westwards from P, it is necessary to determine from the star's R.A. whether it is east or west of the meridian.

Formula for Azimuth Determination.—In observations for azimuth as described in Chapter VIII, the difference of azimuth between a terrestrial reference mark and a heavenly body is observed at a known instant of time; it is then necessary to compute the azimuth of the heavenly body at that instant. In the preceding paragraph methods for this computation have been given, but an additional method largely used is as follows:

In the triangle PZS (Fig. 13), applying the cot formula to the four consecutive parts Z, c, P, and p,

$$\cos c\cos P=\sin c\cot p-\sin P\cot Z$$

$$\therefore\ \cot Z=\frac{\sin c\cot p-\cos c\cos P}{\sin P}$$

Multiplying numerator and denominator by $\sin p$ gives

$$\cot Z=\frac{\sin c\cos p-\cos c\sin p\cos P}{\sin p\ \sin P}$$

Let $k\sin x=\sin p\ \cos P$

and $k\cos x=\cos p$;

then $\tan x=\frac{\sin p\ \cos P}{\cos p}=\tan p\cos P$,

and
$$\cot Z=\frac{k\sin c\ \cos x-k\cos c\sin x}{\sin p\ \sin P}$$
$$=\frac{k\sin(c-x)}{\sin p\ \sin P}$$

Multiplying numerator and denominator by $\cos P$ gives

$$\cot Z=\frac{k\sin(c-x)\cos P}{\sin p\ \sin P\ \cos P}$$
$$=\frac{\sin(c-x)\cot P}{\sin x}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Accordingly, to compute the azimuth angle Z the subsidiary angle x is first determined from the equation $\tan x=\tan p\cos P$; Z is then computed from equation (12).

Formula for Latitude by Circum-Meridian Altitudes.—In observations for latitude by the method described in Chapter VI, known as circum-meridian altitudes, it is necessary to compute the difference between the altitude of a star a few minutes before or after its transit and its altitude at transit, i.e. its meridian altitude.

The known quantities are the star's declination and therefore its polar distance, its altitude at an instant, and its hour angle at that instant. The star is describing a small circle about the pole, and it is required, as stated above, to find an expression, or an approximate expression, for the difference between its altitude at the instant of observation and its meridian altitude, the instant of observation being separated by only a short interval of time from the transit.

Let h_m be the meridian altitude of the star.
,, h be the altitude when the star is at a small hour angle t before or after transit.
,, z_m and z be the zenith distances corresponding to h_m and h respectively.

Equation (I) gives $\cos z = \cos p \cos c + \sin p \sin c \cos t$,
or, $\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t$,
and h, δ, and t being known, ϕ could be computed.

This equation holds for all values of t however great, but for small values of t a more convenient approximate expression can be found, as follows:

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \left(1 - 2 \sin^2 \frac{t}{2}\right)$$

$$\therefore \sin h = \cos (\phi - \delta) - 2 \cos \phi \cos \delta \sin^2 \frac{t}{2}$$

$$\text{and } \phi - \delta = z_m = 90° - h_m$$

$$\therefore \sin h = \sin h_m - 2 \cos \phi \cos \delta \sin^2 \frac{t}{2}$$

$$\text{or, } \sin h_m - \sin h = 2 \cos \phi \cos \delta \sin^2 \frac{t}{2}$$

$$\therefore 2 \cos \frac{h_m + h}{2} \cdot \sin \frac{h_m - h}{2} = 2 \cos \phi \cos \delta \sin^2 \frac{t}{2}$$

Now if the altitude be observed near the meridian,

$$\frac{h_m + h}{2} = h \text{ (approximately)},$$

and $\sin \frac{1}{2}(h_m - h) = \frac{1}{2}(h_m - h)$ (approximately, as $h_m - h$ is small)

then $$\cos h \times \tfrac{1}{2}(h_m - h) = \cos \phi \cos \delta \sin^2 \frac{t}{2}$$

and $$h_m - h = \frac{\cos \phi \cos \delta}{\cos h} \cdot 2 \sin^2 \frac{t}{2}$$

In this expression h_m and h are in radians, and the difference $h_m - h$ is also in radians. In order that the difference may be in seconds of arc the expression for $h_m - h$ must be multiplied by the numbers of seconds in one radian, or it may be expressed thus:

$$(h_m - h)'' = \frac{\cos \phi \cos \delta}{\cos h} \cdot \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}$$

The above is the approximate expression for the difference of altitude of a star when at an hour angle t from its meridian altitude, provided the hour angle be small. The expression is known as *the reduction to the meridian.*

Formula for Calculation of Azimuth from Latitudes and Difference in Longitude.—To find the remaining angles when two sides and an included angle are known. From equation (II)—

$$\tan \frac{Z}{2} = \sqrt{\frac{\sin (s-z) \sin (s-c)}{\sin s \sin (s-p)}}$$

This may be written in general terms

$$\tan^2 \frac{A}{2} = \frac{\sin (s-c) \sin (s-b)}{\sin s \sin (s-a)}$$

But $$\tan \tfrac{1}{2}(A+B) = \frac{\tan \frac{1}{2}A + \tan \frac{1}{2}B}{1 - \tan \frac{1}{2}A \tan \frac{1}{2}B}$$

Substituting in this equation for tan $\frac{1}{2}$A and tan $\frac{1}{2}$B their values obtained from the above equation, viz.

$$\tan^2\frac{A}{2}=\frac{\sin(s-c)\sin(s-b)}{\sin s\sin(s-a)}\text{ and reducing, gives}$$

$$\tan\tfrac{1}{2}(A+B)=\left(\frac{\sin(s-c)\sin s}{\sin(s-a)\sin(s-b)}\right)^{\frac{1}{2}}\left(\frac{\sin(s-b)+\sin(s-a)}{\sin s-\sin(s-c)}\right)$$

$$=\frac{2\sin\frac{2s-b-a}{2}\cos\left(\frac{a-b}{2}\right)}{2\sin\frac{c}{2}\,.\,\cos\frac{2s-c}{2}}\,.\cot\frac{1}{2}C$$

$$=\frac{\sin\frac{c}{2}\,.\,\cos\frac{a-b}{2}}{\sin\frac{c}{2}\,.\,\cos\frac{a+b}{2}}\,.\cot\frac{1}{2}C$$

$$=\frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\,.\cot\frac{1}{2}C \quad . \; . \; . \; . \; . \; . \; . \quad (13)$$

$$\text{Likewise }\tan\tfrac{1}{2}(A-B)=\frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\,.\cot\frac{1}{2}C \quad . \; . \; . \quad (14)$$

CHAPTER IV

INSTRUMENTS

Transit Theodolite.—The observer in the field will in most cases be restricted to the use of one form of theodolite only, namely the ordinary portable transit theodolite, and a more complete equipment, such as a geodetic theodolite for azimuth, a zenith telescope for latitude, or a prismatic astrolabe will not be available on account of expense and questions of transport. These instruments will therefore not be referred to, as all the observations described in this book can be made with the ordinary portable transit theodolite.

All sizes of transit theodolite from 3 in. to 12 in. are employed for the purpose of field astronomy, but, bearing in mind the degree of accuracy required and the question of portability, the 5-in. or 6-in. micrometer instrument, reading direct to 10 secs. and by estimation to 1 sec., is probably the most useful.

A good engineer's vernier theodolite may be used for field astronomy, but micrometer reading of the circles instead of by verniers is very desirable, as is also the provision of a sensitive spirit level on the T-arm carrying the vertical circle micrometers or verniers. In addition, a striding level should be provided to ensure that the horizontal axis is horizontal, or to measure its departure from the horizontal.

Various accessories not always found with the transit theodolite will be required, such as :

(*a*) A diagonal eyepiece for use when the altitude exceeds about 45° ; and in this connection it may be noted that the proportions of the instrument should be such that it can be transited without the focus of the eyepiece having to be altered. In observing with the diagonal eyepiece the prism inverts the image, so that with the ordinary Ramsden type of eyepiece the image appears inverted, but correct as regards right and left.

(*b*) Means for illuminating the field of view. A small electric lamp is best for this purpose, the rays being projected through a lens in the hollow trunnion axis on to a small mirror which reflects them on to the wires. The lamp-socket is attached to one of the standards. A small oil lamp should be available in case of failure of the dry battery used for the electric lamp. If the instrument is not provided with a hollow trunnion, a paper reflector may be attached in front of the object glass. It is bent over the glass and provided with an opening to enable the light from the star to enter the telescope, the lamp being placed so that its light is reflected down the tube. Another method of illuminating the wires is to place a blob of white wax on the object glass, and the light from a lamp shining on this will be diffused down the telescope tube.

(*c*) Sighting points fixed to the upper and lower sides of the telescope tube to give a line of sight parallel to the line of collimation, to facilitate pointing to a star.

(*d*) A dark glass to be attached to the eyepiece when observing the sun.

Mechanical Condition.—In order that satisfactory results may be obtained with an instrument such as that indicated above, it is essential that it should be kept in good mechanical condition and in a satisfactory state of adjustment. The tripod should be examined carefully from time to time in order to see that there is no shake in it. The nuts and screws of the metal-work should be tightened up and also the shoe fastenings, the shoes being sharpened up if necessary. When placing the instrument on its tripod it may be turned in the direction of unscrewing until a slight click is heard indicating that the threads have come to the correct position where they will engage on screwing up. When carrying the instrument, tighten all clamps so that no movement can take place; the clamps should also be tightened after the instrument has been put into its case with the clamps loose. If the instrument is on its tripod but not in actual use, it should be covered with a waterproof hood.

When handling a theodolite it should be lifted by the tribrach and not by the standards or micrometer arms. In general, care and cleanliness are essential for keeping the instrument in order.

If the instrument does not revolve smoothly on its axes, these should be cleaned with a piece of fine wash-leather and a light oil.

If the focusing tube of the telescope appears to move harshly, a piece of grit has probably got on to the tube bearing; this should be attended to at once, as the damage will increase.

The slow motion tangent screws should be cleaned periodically with an old toothbrush and petrol or paraffin, and replaced with vaseline or tallow on the threads.

The graduations on the horizontal and vertical circles should be cleaned when necessary with a piece of clean soft rag and light oil after first dusting off all grit with a camel-hair brush. If the graduations require re-blacking a little black oil paint from a tube should be smeared over the divisions and cleaned off with a piece of soft tissue paper.

The lenses should always be kept clean and their surfaces free from moisture and dust.

If the inner surfaces of the object glass require cleaning, it should be taken to pieces very carefully and the glasses replaced in exactly the same position relatively to each other; most object glasses have a slot and a mark on the edge to facilitate assembling. Care must be taken not to screw the counter-cell too tightly or the glass may become strained. There is usually a mark on the outside edge of the object glass cell so that it can be screwed back into the telescope in the original position.

The focusing lenses of internal focusing telescopes can be cleaned by removing the object glass and the diaphragm; each side of the glass is then cleaned in turn with a small stick round which a piece of wash-leather is wrapped.

A glass diaphragm or graticule can usually be cleaned with a camel-hair brush, but if not it should be cleaned in the same way as a lens. If the ruled lines of the diaphragm appear too faint, they can be blackened in with a piece of soft tissue paper that has first been rubbed with a very soft blacklead pencil, the surface being then cleaned with the newly torn edge of a piece of card.

If the wire of a web diaphragm is broken, it may be repaired as follows with a spider line. A cardboard frame is made and a spider placed on one edge. The frame is then gently shaken so that the spider hangs from it, the fibre being wound up with the turns wide apart and the end secured in a notch. To fix the wire the diaphragm ring is taken out, the old wire removed, and the varnish cleaned from the notches with spirits of wine and warm water. A clean uniform piece of line is then selected and two little balls of wax attached to the ends, one of the balls being held while the other hangs freely in order to untwist the fibre. The line is then cleaned by rubbing it gently with a paint brush dipped in clean water. The diaphragm ring is placed on a block of wood and the line stretched across the ring and adjusted by means of a pin to be exactly in the notches, the wax balls hanging freely on either side. One end of the line is fixed by a small drop of shellac, and when this has hardened the other end is fixed likewise, care being taken that the line remains taut. Sealing-wax dissolved in spirits of wine may be used instead of shellac.

Requirements in the Non-adjustable Parts.—In spite

of attention to the mechanical and optical condition of the instrument, defects will develop by constant use, and owing to these and imperfections in workmanship the ideal requirements of a theodolite are only partially fulfilled, and there will always be instrumental errors due to defects in different parts of the instrument.

A particular routine in observing may be adopted for reducing or eliminating the effect of small defects in the non-adjustable parts, whilst provision is made for adjusting other parts to eliminate the more serious errors.

Requirements in the non-adjustable parts are as follows:

1. The whole instrument should be stable. To test for this, set up the instrument and sight a well-defined point, fixing all clamps lightly. Apply a firm but gentle lateral pressure to the eyepiece. The intersection of the cross wires will probably leave the sighted point, but should return to it on removing the pressure.

2. The inner and outer vertical axes should have the same geometrical axis of rotation, namely the vertical axis of the instrument. To test for this, set up the instrument, and with the lower plate clamped bring the level on the vertical circle T-arm parallel to two foot-screws, and bring the bubble to the centre of its run by means of the foot-screws alone. Turn through 90°, and bring the bubble, if necessary, to the centre of its run by the third foot-screw. Repeat until the bubble is central in these two positions. Turn through 180° from the original position; if the bubble has left the centre of its run, bring it half-way back by the foot-screws and half-way by the T-arm clip-screws. Repeat until the bubble remains central in all positions. Now loosen the bottom horizontal plate, and clamp the top and turn the plates round. If the bubble does not remain central, the outer axis of rotation is not vertical, and therefore not parallel to the inner axis. If the error is large, the instrument must be sent for repair.

3. The division of the horizontal and vertical circles

should be accurate. The accuracy of the divisions cannot be readily tested, but with modern methods of division the errors are unlikely to be large, and their effect can be reduced to any desired extent by repeating the measurement on different parts of the horizontal circle and taking the average of the results. In the case of the vertical circle, since this is fixed to the telescope this procedure is not possible, and the only precaution that can be taken to lessen the resulting error is to read both verniers as explained later.

4. The zero readings of the verniers should be at the opposite ends of the same diameter. To test, read both verniers in any position and note the difference between the readings. Move the verniers, and again note the difference. A constant difference other than 180° indicates that the required condition is not fulfilled. Error from this source will arise if one vernier is used for orienting and the other for reading the angle. Either vernier will give the correct measurement if the same vernier is used for orienting and reading the angle.

5. The centre of graduation of the horizontal circle should be in the vertical axis and that of the vertical circle in the horizontal axis. To test, read both verniers in any position and note the difference, then rotate the verniers through about 90° and again ascertain the difference in their readings. If the difference is constant there is no eccentricity. If there is a difference, the error is eliminated by averaging the values of the angle given by each vernier.

In Fig. 14 let O be the centre of the graduated circle and O_1 the centre of the vernier circle or axis of rotation. When the line of sight is in the position CO_1D the vernier readings at A and B differ by 180°. If the line of sight is now turned through an angle α into the position EF, the vernier at A moves through an angle α to G and that at B through the same angle to H, but neither vernier records the angle α. The reading at H is BOH or (α—error),

and that at G is (180° + AOG) = (180° + α + error), since $OHO_1 = OGO_1$ = error (e). The readings differ, therefore, by 180° + twice the error. If both verniers are read, the mean is $\frac{(\alpha - e) + (180° + \alpha + e)}{2}$, i.e. the error due to eccentricity of the circle is eliminated.

6. The plane of the graduated edge of the horizontal circle should be at right angles to the vertical axis, and

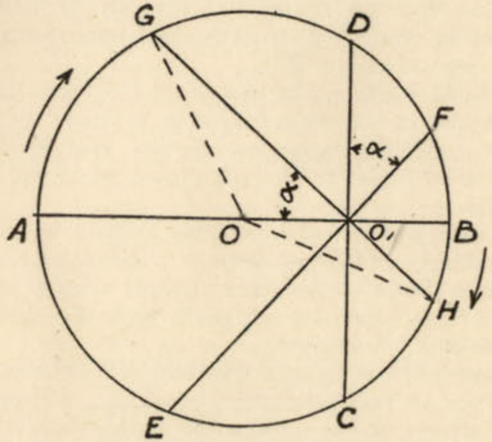

FIG. 14.

that of the vertical circle at right angles to the horizontal axis. This condition is easily satisfied by the instrument maker within limits of error which will have no appreciable effect on the readings. If the condition is not satisfied, for example, in the case of the horizontal circle, then when the vertical axis is vertical the horizontal circle is oblique and an observed horizontal angle is not measured in the horizontal plane; the error will, however, be so small as to be negligible.

7. The relation between the greatest magnification of the telescope and the pitch of the tangent screw should be such that a just perceptible movement of the tangent screw will cause a just perceptible movement of the wires across an object observed. A similar relation should exist between the magnification of the micrometer lenses and the pitch of the micrometer screws. To test for this condition, centre the cross wires on a well-defined point and read the vernier or micrometer with all clamps fixed. Move the vernier or micrometer through its least count, or a fraction of it, if discernible, and note if the line of sight has moved perceptibly.

Adjustable Parts.—The above are the more important requirements in the non-adjustable parts of the transit theodolite, and the question of the examination and elimination of errors in the adjustable parts will now be considered.

The adjustment of the portable transit theodolite as used for field astronomy does not differ to any great extent from that of the ordinary transit theodolite, except that a striding level is supplied with the former for horizontal axis adjustment.

Parallax.—If the image formed by the objective is not in the plane of the cross wires, the image will appear to move relatively to the cross wires as the observer moves his eye, and accurate sighting will be impossible, as the line of sight may be made to appear to intersect different points according to the position of the eye. This state of affairs is known as eyepiece parallax and is eliminated as follows :

1. It is first necessary to focus the eyepiece for distinct vision of the cross wires ; to do this, point the telescope to the sky or turn the focusing screw until no object can be distinguished in the field, and adjust the eyepiece until the wires are sharply defined and on closing the eye and reopening it the wires appear perfectly distinct at once without the eye having to accommodate itself. If

necessary, light may be reflected down the telescope tube with a piece of white paper to illuminate the cross wires. The position of the eyepiece for correct focus depends on the individual's eyesight.

2. It is next necessary to bring the image of the object into the plane of the cross wires. To satisfy this condition, sight the telescope on to the object and, keeping the eye on the cross wires, turn the focusing screw until the image appears sharply defined and until, on moving the eye slightly, there is no apparent movement of the wires over the image since they are in the same plane. The position of any particular object glass for correct focus depends only on the distance of the object observed, and is independent of the observer's eyesight.

Adjustment of the Horizontal Plate Levels.—Set up the instrument and bring the level attached to the vertical circle T-arm parallel to two foot-screws, and bring the bubble to the centre of its run by these screws. Turn through 90°, and bring the bubble to the centre by the third foot-screw. Repeat until the bubble is central in both these positions. Now turn through 180° from the first position, and if the bubble has left the centre of its run bring it back half by the foot-screws and half by the clip-screws. Repeat until the bubble remains in the centre of its run when the horizontal plate is turned into any position. The vertical axis being now vertical, the bubbles on the levels of the horizontal plate should be brought to the centres of their run by their own adjusting screws alone, when the axes of the tubes will be at right angles to the vertical axis.

The T-arm level is used as being more sensitive than the levels on the horizontal plate, but its use may be dispensed with as follows :

Set up the instrument and bring the longer level on the horizontal plate parallel to two foot-screws, and bring the bubble to the centre of its run by these screws. Turn through 90°, and bring the bubble to the centre of its run by the third foot-screw. Repeat until the bubble is central in both these positions. Now turn through 180° from the first position, and if the bubble has left the centre of its run bring it back half-way by the foot-screws and half by its own adjusting screws. Repeat until the bubble remains at the centre of its run when the plate is turned to any position. If the bubble of the shorter level on the horizontal plate is not at the centre of its run, bring it to the centre by means of its own adjusting screws alone.

Adjustment of the Striding Level.—The object of the striding level, as already stated, is to test the horizontality or otherwise of the horizontal axis of the telescope. The V-foot of each leg of the level gives two points of support on the horizontal axis; it is necessary that the level axis, i.e. the horizontal line tangential to the surface of the centred bubble, which lies in the vertical plane through the axis of the bubble tube, shall be parallel to the line joining the points midway between these two points of support in each V.

To adjust, place the striding level in position on the pivots of the horizontal axis and bring the bubble to the centre of its run by the foot-screws. Test for wind by rocking the level slowly backwards and forwards. If the bubble does not remain at the centre of its run, then the bubble axis and the horizontal axis of the telescope lie at an angle to each other when projected upon a horizontal plane, and the striding level is said to have wind. Correct this wind by the lateral adjusting screws at the end of the bubble tube.

The second step is to level the instrument as before and place the striding level on the horizontal axis, and if the bubble is not exactly at the centre of its run bring it central with the foot-screws. Lift off the striding level and carefully replace it end for end. If the bubble is not now in the centre of its run, bring it half-way back by means of its vertical adjusting screw. Re-level

with the foot-screws and repeat until the adjustment is perfect.

Adjustment of the Horizontal Axis.—When the instrument is levelled up the vertical axis is truly vertical, and it is desirable, but not absolutely necessary, as explained in Chapter V, that the horizontal axis of the telescope shall then be truly horizontal, i.e. at right angles to the vertical axis.

If the level on the horizontal plate is but little inferior in sensitivity to the striding level, as is often the case, the adjustment may be made as follows, after having adjusted the plate and striding levels as already described.

Make the vertical axis truly vertical, and place the striding level in position. If the bubble of this level is not at the centre of its run, bring it to the centre by means of the adjusting screws alone which control the trunnion support in one standard.

If the vertical axis is not made truly vertical, then the amount by which the striding bubble is displaced on rotating the telescope in azimuth through 180° must be corrected half by means of the trunnion adjusting screws and half by the foot-screws, and the adjustment continued until the bubble remains in the centre of its run.

In the case of a theodolite not provided with a striding level, the horizontal axis of the telescope may be adjusted after the collimation adjustment (which is described later) as follows:

Level up the instrument and sight on to a well-defined object at a considerable altitude, such as the top of a lightning conductor, and clamp the horizontal circles. Now depress the telescope, and note the exact reading of the vertical wire on a staff laid horizontally on the ground in front of the telescope as far from it as convenient. If the horizontal axis is horizontal, the vertical wire should intersect the staff truly vertically below the object sighted, since the collimation adjustment will have put the line of collimation at right angles to the horizontal axis. Loosen the top plate and turn it approximately 180°, and again sight the observed object at high altitude; this obviously will necessitate transiting the telescope. Clamp the horizontal plates and again depress the telescope, and take the reading of the vertical wire on the staff. If the horizontal axis is horizontal, the reading will be the same as before. If not, the mean of the readings is the one which is truly vertically beneath the observed object, and the trunnion axis is adjusted until on sighting the observed object and depressing the telescope this true reading is given by the vertical wire.

Adjustment of the Vertical Circle Zero Reading.— It is obvious that the vertical circle reading should be zero when the line of sight is horizontal. This condition, however, is not absolutely essential if the mean of face-right and face-left readings is taken, as a reversal of face will cause a reversal in sign of the error, which will consequently be eliminated, as may be seen from an examination of Fig. 15. It is, however, preferable to adjust the instrument so that the reading is zero when the line of sight is horizontal, and this adjustment is carried out as follows:

Level up the instrument by means of the foot-screws and horizontal plate levels, and having brought the bubble of the T-arm level to the centre of its run by the clip-screws, take the mean of the vertical circle readings when the horizontal wire intersects a well-defined point as far away as possible. Transit the telescope in order to reverse face, and on turning the telescope in azimuth through 180°, having again adjusted the bubble of the T-arm level, bring the horizontal wire to the same point as observed previously by means of the vertical circle tangent screw, and take the mean of the vertical circle readings. The mean of the face-right and face-left readings is the true reading. Set the telescope by means of the tangent screw to give this reading; this will cause the horizontal wire to move off the observed point, and it is brought back to this point by means of the T-arm clip-screws. This eliminates the error,

and the vertical circle reading will be zero when the angle of elevation of the line of sight is zero, i.e. when the line of sight is horizontal. The bubble of the level attached to

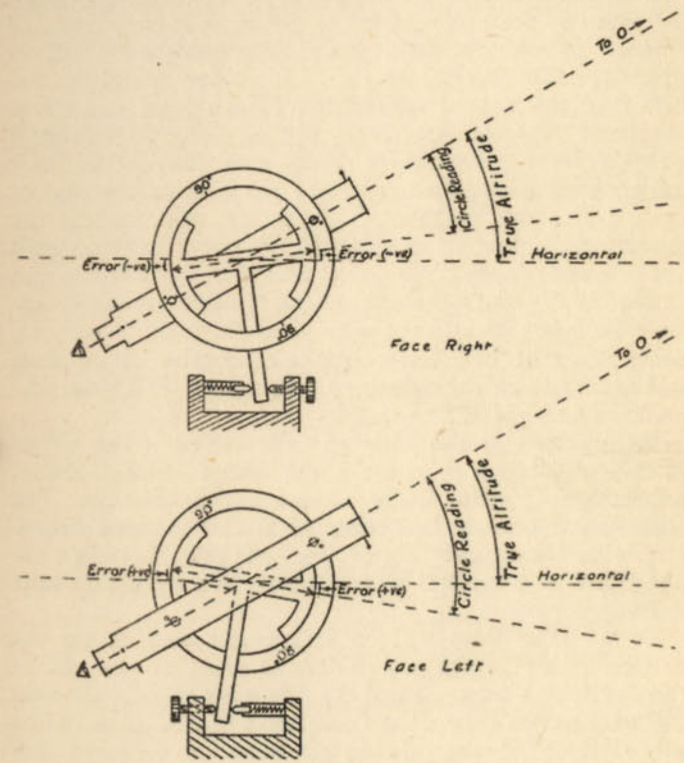

FIG. 15.

the T-arm will not now be at the centre of its run; bring it to the centre by the adjusting screws at the end of the bubble tube.

Owing to the difficulty of making the above-described adjustment perfectly, it will nearly always be found that the F.R. and F.L. readings on one and the same point, even with the T-arm level truly centred for each pointing, will differ slightly. The difference of either reading from the mean of the two readings is the *error of collimation in altitude*, or what is called frequently *index error*. It is eliminated by reversal of face. It is due partly to the fact that the line of collimation (which joins the intersection of the cross wires to the optical centre of the object glass) does not coincide exactly with the optical axis (which joins the optical centre of the eyepiece to that of the object glass), and partly to the fact that, even if the line of collimation did so coincide, the circle reading would not be exactly zero if the optical axis were truly horizontal. In the case of pointings made only on one face, the correction for collimation in altitude may be applied, this correction being constant and known for a particular instrument, and equal to half the difference between the F.R. and F.L. readings on a point.

Adjustment of the Line of Collimation.—The object of this adjustment is to make the line of sight or line of collimation at right angles to the horizontal axis. The adjustment is made by means of the diaphragm screws, and it is convenient to test first whether the wires are horizontal and vertical respectively when the instrument is levelled up.

To do this, level up the instrument and clamp the vertical circle. Sight a well-defined point at one end of the horizontal wire. Turn the telescope slightly so that the wire moves across the point until the latter is at the other end of the wire. If the wire is still on the point, it is horizontal; if not, loosen the diaphragm screws and rotate the diaphragm around the longitudinal axis of the telescope, and test again until the desired horizontality is obtained. The vertical wire is fixed relatively to the horizontal wire, and should therefore be truly vertical when sighted to a plumb line suspended in front of the object glass.

Next, level up the instrument on fairly level ground midway between two staffs placed horizontally at the same

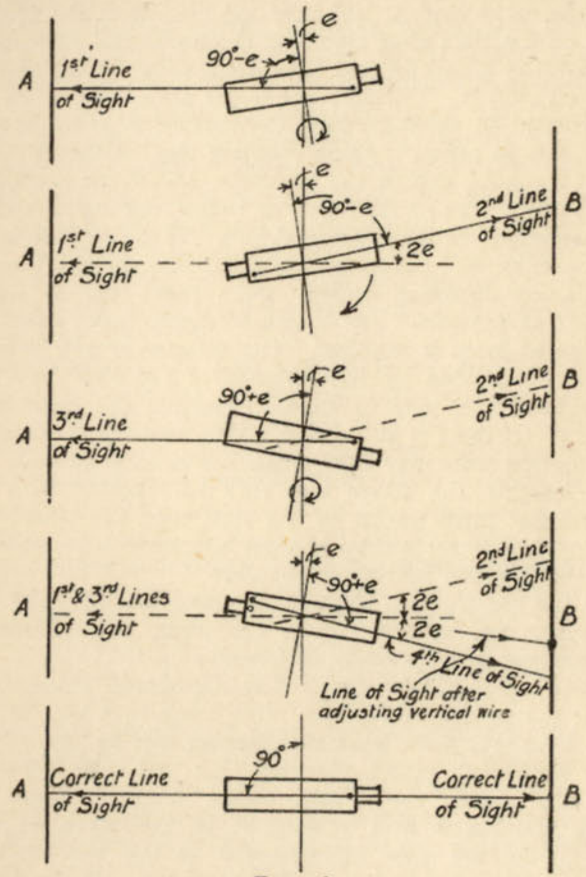

Fig. 16.

level (Fig. 16), the staffs being as far as convenient from the instrument, and as nearly as possible in the plane of the horizon of the instrument to obviate errors due to incorrect levelling.

Sight on to staff A, and take the reading of the vertical wire on the staff after clamping the horizontal movement. Transit the telescope, and take the reading of the vertical wire on staff B. Unclamp the upper plate, and rotate the instrument in azimuth until the vertical wire reading on staff A is as before. Again, keeping the horizontal movement clamped, transit the telescope, and if the reading of the vertical wire on staff B is as before, the line of sight is perpendicular to the horizontal axis. If the reading is not the same, correct one-quarter of the difference with the side diaphragm-adjusting screws, and repeat till as nearly perfect as possible. The length of sights being equal, no change of focus is required. One-quarter of any residual difference divided by the distance of the staff from the instrument will give in angular measure the collimation error *c*. If there is any defect in the focusing mechanism, collimation error may arise when the focus is altered. To test for this, the above test may be repeated with the instrument much nearer to one staff than the other; and if there is an error it may, in some instruments, be adjusted by lateral screws bearing on the slide.

If the construction of the instrument be such that the telescope can be reversed in the wyes, the collimation adjustment may be made as follows:

Level up the instrument. The clip-screws being slack, point the intersection of the cross wires to a well-defined object on which an accurate pointing can be got. With the horizontal plates clamped, lift out the telescope carefully from the wyes, rotate it 180° about its longitudinal axis and replace it in the wyes, so that the pivots are now interchanged in the bearings and the telescope directed towards the object. If the object is not now intersected by the vertical wire, correct half the discrepancy by the diaphragm screws and again cause it to intersect the object by means of the horizontal plate tangent screw. Repeat the test to check the adjustment.

With regard to all the above adjustments it may be said that in general the routine of the observations should be such that, although the instrument has been carefully adjusted, any residual instrumental errors are eliminated as far as possible; and to this end the observations must consist of an equal number of face-right and face-left sights, and the circle readings must be obtained on all the verniers or micrometers concerned. It must, however, be remembered that in the case of observations such as those of equal altitude where the aim is to preserve constant conditions of the instrument, a change of face is not required.

Micrometers.—It has already been stated that the theodolite employed should preferably have micrometers for reading the horizontal and vertical angles. The micrometer consists essentially of a compound microscope with a micrometer box mounted between the objective and eyepiece at such a distance from each that movable wires in the micrometer box can be brought into the focal plane of each lens system. The mechanical arrangements of the micrometer box vary slightly on different instruments, but it consists essentially of a metal box into which the eyepiece and objective tube are screwed on opposite sides. The box contains a slide which can be moved laterally by turning a graduated drum (3, Fig. 17). Across the opening of the movable slide are stretched two closely spaced vertical wires, and the slide is acted on by two light springs to prevent backlash when it is moved by turning the graduated drum. The distance of the objective from the vertical wires, and the distance of the microscope as a whole from the graduated circle which it is desired to read, are so arranged that the image of the circle graduations is formed in the plane of the wires, and the width of the image of one graduated circle division is such that an exact number of turns of the graduated drum will carry the vertical wires exactly across this division, i.e. from one graduation to the next. Thus fractional parts of a graduated circle division can be measured. In reality, the fractional part is measured on a chord (since the vertical wires are caused to move in a straight line) instead of on an arc, but the error is not appreciable.

There must obviously be a line of reference from which

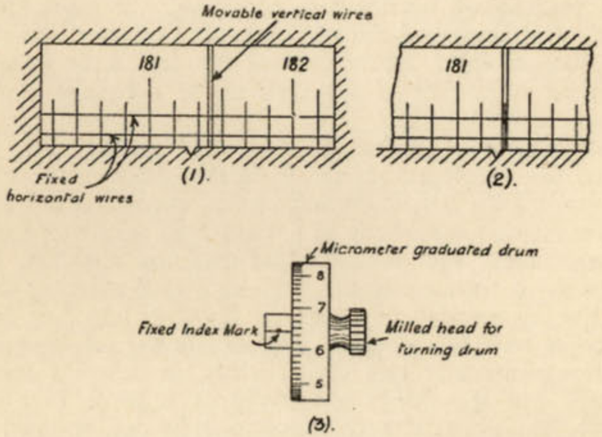

FIG. 17.

to obtain the readings; this point is usually the imaginary line midway between the vertical wires when they occupy the centre of the field of view, the reading of the graduated drum being zero. This point is located approximately by a V-shaped notch in front of the plane of the wires as shown in Fig. 17, and this notch gives the approximate reading.

Least Count of a Micrometer.—The graduated circle of the theodolite should first be examined to determine the value of one division: this will usually be 10 minutes.

On turning the graduated revolving drum, the number of turns required to move the vertical wires from the image of one graduation to the next is measured, the fact that there are two vertical wires instead of one facilitating the centring over a graduation. If n turns are required, and the circumference of the drum is divided into p divisions, then the value of each division on the drum is $\frac{1}{np}$ of a circle division.

In 5-in. and 6-in. theodolites with the circles graduated to 10 minutes, n is usually 1, and the graduated drum has 60 divisions, so that the value of each of these 60 divisions is 10 seconds.

To Read the Micrometer.—Referring to Fig. 17, in order to obtain the micrometer reading, it will be observed that the notch reading is between 181° 10′ and 181° 20′. The vertical wires are moved laterally until they lie one on each side of the image of a circle graduation adjacent to the notch, e.g. 181° 20′, it being immaterial on which side of the notch the selected graduation is situated.

The minutes and seconds are now read off on the graduated drum; in the figure this reads 6′ + between 20″ and 30″, or by estimation 24″, so that the complete reading is 181° 16′ 24″.

The same procedure is then followed for the graduation on the other side of the notch, the drum being revolved in the opposite direction when bringing the wires to the graduation, i.e. in the final centring of the wires over the graduations the direction of motion of the wires should be contrary in the two cases. The mean of the two drum readings is taken as the micrometer reading.

Adjustment for Parallax and Focus.—It is seldom that this adjustment is required, but it applies to every micrometer microscope and should be carried out in conjunction with the adjustment for run which is referred to below. A piece of white paper should first be placed on the graduated circle under the microscope and illuminated by the reflector, and the vertical wires then brought into sharp focus by adjusting the eyepiece. As in the case of the eyepiece of the theodolite, this is an adjustment depending on the eyesight of the observer. The paper is now removed, and if the graduations of the circle are not distinct the microscope must be moved bodily nearer to or away from the circle. To do this, the clamp-screw of the microscope socket must be eased. One of the graduations of the circle is then brought to the centre of the field of view and the vertical wires moved to lie one on each side of it by turning the graduated drum. If on moving the eye to the left or right there is no apparent movement of the wires relative to the graduation, there is no parallax; but if there is movement, the microscope must be moved slightly away from or to the circle until such movement is eliminated.

Taking Runs.—As stated above, an exact number of turns (usually 1) of the revolving drum should carry the vertical wires exactly across one division of the graduated circle, and this should be tested occasionally, the test being known as *taking runs.* If it is found that, on moving the wires exactly across one circle division, the reading of the revolving drum shows that exactly one complete revolution of the drum has not been made, i.e. the first and second readings differ, the difference between the readings and the nominal value of the space is known as the *run of the micrometer.*

In testing, the mean of two or three runs should be taken, and if the error is not more than 2 or 3 seconds it can be neglected.

The microscope can be adjusted for errors of run, but the adjustment is a delicate one for field purposes, and if the degree of accuracy required warrants it the effect of the error may be eliminated by applying a correction, but this is seldom necessary for field astronomy purposes except for the most precise work.

To eliminate errors due to backlash in the parts of

the micrometer, back and forward micrometer readings should be taken and averaged as explained. This will also reduce the error arising from lack of precision in centring the wires on the graduations.

Timekeepers.—The most precise portable timekeeper either for mean or sidereal time is the box chronometer, such as employed on board ship and in observatories. Such an instrument is suitable for field astronomical purposes if it can be kept stationary, but is too delicate to maintain a constant rate during transport.

The best timekeeper for field use is a good watch with lever escapement, and a useful accessory is a good stop-watch registering to 0·01 second.

A chronometer watch differs from a chronometer in the following details:

1. The fusee is eliminated.
2. A lever escapement is used.
3. A different form of compensation balance is used,

but the general working principles are the same. The lever escapement, as compared with that of a chronometer, gives two impulses to the latter's one, and with this escapement it is practically impossible to stop a watch by any such turning about as would stop one with a chronometer escapement; it also has the advantage of being self-starting. The lever escapement action involves sliding friction in the mechanism, viz. between the escape wheel teeth and the pallets, which have to be oiled, and this is a disadvantage, as the thickening of the oil must in time affect the going of the watch.

However, a good chronometer watch, with careful and intelligent handling, is capable of giving results practically equal to those of a chronometer. Such a watch is tested for position error by the makers, as it is not slung in gimbals like a chronometer, its going being observed with the figures XII, III and IX successively uppermost, and the results compared with each other and with the results when horizontal, dial uppermost. A good quality watch will have practically no position error; but, to reduce the wear on the mechanism as far as possible, and to give its time-keeping qualities the best scope, it should be kept horizontal with the dial uppermost, and it should be wound carefully at the same hour each day. The watch should be exposed as little as possible to change of temperature or atmospheric humidity.

Watch Error.—In spite of the precautions suggested above, the watch will not always register correct time, owing to various imperfections, i.e. there will always be an error. The *watch error* at any instant is the difference between the correct time and the time as indicated by the watch, and this error will not, as a rule, maintain a constant value.

It is required of a good watch that, when subject to a particular set of conditions, it shall go at a uniform speed so that its error changes uniformly.

Watch Rate.—The daily change of error is known as the *watch rate*, and it may be either a gaining or a losing one.

The rate will differ according as the watch is stationary (i.e. *standing rate*) or travelling (i.e. *travelling rate*).

Determination of Standing Rate.—The standing rate is determined by obtaining, at the place where the watch is situated, not less than two sets of time signals or observations, as explained in Chapter VII, separated by an interval of a few days, each time observation being made as far as possible in the same manner.

Example.—A time observation on 12 June at about 23h 10m G.M.T. showed the watch to be 10m 19s·6 slow, and at the same place on 17 June at about 22h 30m the watch was 10m 54s·8 slow. Find the watch rate.

	d.	h.	m.
June	17	22	30
June	12	23	10

Interval between observations = 4 23 20 = 4·972 days.
Change of error = 10m 54s·8 − 10m 19s·6 = 35s·2 lost.

$$\therefore \text{ rate} = \frac{35 \cdot 2}{4 \cdot 972} = 7\text{s}\cdot 08 \text{ losing, daily.}$$

The constancy of the rate may be checked by taking time observations every second night for a period of say seven to ten days.

Determination of Travelling Rate.—The conditions under which the watch will be travelling will be as follows:

1. Between two stations, the difference of longitude of which is known.
2. Returning to the starting station.
3. Not returning to the starting station, and longitude *en route* not being known.

In case 1 the watch error is determined at starting and on arrival by time observations or from wireless time signals, and any change of error is obviously then partly due to the difference between the local times of the two places, i.e. to the difference in longitude. This difference being known, the change in error due to this is allowed for, and the remaining difference is divided by the time interval between the observations to get the rate.

Travelling rate conditions include the normal stationary period at night, but if a halt be made for several days the period of the halt must be allowed for in computing the travelling rate if the watch is kept stationary during the halt.

In case 2 the calculation is performed as for standing rate, since the time observations are taken at the same place.

In case 3 the travelling rate must be determined before the journey is commenced, and the calculations are as for standing rate, but during the interval between the observations for error the watch must be carried on a daily march under as nearly as possible the same conditions as those likely to be met with on the journey, and of the same average daily duration.

Example.—Before commencing a survey tour a time observation showed the chronometer watch to be 4m 13s·8 fast. At the end of the tour the error was found to be 0m 55s slow, the time interval being 11·34 days.

The difference in longitude was 2° 07′ 19″·6, and the tour eastwards. Find the travelling rate.

Difference of longitude expressed in time

$$=\frac{2° \ 07' \ 19''{\cdot}6}{15}=8\text{m } 29\text{s}{\cdot}3,$$

and since the journey was eastwards the difference in longitude makes the watch 8m 29s·3 slower than it apparently was.

Apparent change of error = (4m 13s·8 + 0m 55s) = 5m 8s·8 lost.

∴ change of error due to rate

$$=(8\text{m } 29\text{s}{\cdot}3-5\text{m } 8\text{s}{\cdot}8)=3\text{m } 20\text{s}{\cdot}5 \text{ gained.}$$

$$\therefore \text{ travelling rate}=\frac{200{\cdot}5}{11{\cdot}34}=17\text{s}{\cdot}68 \text{ gaining.}$$

Wireless Receiving Sets.—As described in Chapter IX, wireless time signals are broadcast from various stations in different parts of the globe, and there are now few localities in which the surveyor with a comparatively small receiving set cannot pick up a time signal from one or other of these stations. Constant improvements are being made in receiving sets, and an intending purchaser is well advised to consult a firm of reputable manufacturers as to the best type for his particular locality. In general, the manufacturers will require to know what transmitting stations are likely to be made use of, the nature of the country to be surveyed, the means of transport available, and facilities for maintenance or renewal of dry batteries or accumulators. The manufacturer, with this information at hand, can then design the receiving set with due regard to the wave length of the signals available, the strength of signal required so as not to be unduly interfered with by atmospherics, the best type of aerial and earth, the best source of power for the valves, and the desirable size and weights of the various components.

CHAPTER V

ASTRONOMICAL AND INSTRUMENTAL CORRECTIONS TO OBSERVATIONS

UNDER the heading of astronomical corrections to observations are included two corrections which have, in certain cases, to be applied to the observed positions of heavenly bodies in order to obtain their true positions. These two corrections are (*a*) *astronomical refraction*, (*b*) *geocentric parallax*. They are independent of instrumental corrections and would exist even if the instruments used were in perfect adjustment and free from any errors.

Astronomical Refraction.—In passing from one medium to another of different density, light is, in general, deflected or refracted, on account of the fact that the velocity of light differs in media of different density, being, with few exceptions, less the greater the density of the medium. If light is passing from a medium in which its velocity is v_1 into another medium in which its velocity is v_2, and if the incident ray makes an angle i with the normal to the surface of separation at the point of incidence, and if r is the angle between the refracted ray and the said normal

$$\frac{\sin i}{\sin r}=\frac{v_1}{v_2}$$

the ratio $\frac{v_1}{v_2}$ is the *coefficient of refraction*, and is usually denoted by the symbol μ.

Further, the plane containing the incident and refracted rays contains the normal to the surface of separation at the point of incidence, or, in other words, the refracted ray always lies in the plane containing the incident ray, and the normal to the surface of separation at the point of incidence.

The atmosphere is extremely rarefied in the upper regions, its density increasing continuously in the downward direction owing to the increased pressure due to the superincumbent layers; this increase is approximately in geometric progression. Accordingly a ray of light in passing from one point to another at a different height above the earth's surface is traversing a medium of varying density and is, therefore, refracted. Since the deflection

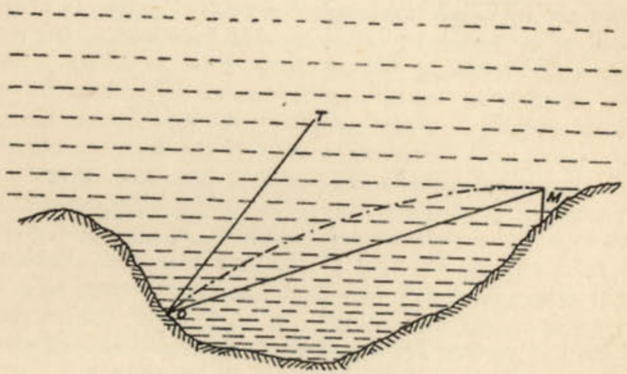

FIG. 18.

is towards the normal when the second medium is denser than the first, if the observer is at a lower level than the object observed, the light reaching him has been deflected towards the normal to the surface of separation, that is, it has been deflected towards the vertical.

Thus the rays by which an object M (Fig. 18) is seen at O have not travelled in the straight line MO, but they have been deflected or refracted continuously as the atmospheric density increases downwards and have followed a curved path as shown by the curved dotted line MO. The tangent, OT, at O to this curved line is the direction

in which the object M is seen by an observer at O, being the direction in which the rays meet his eye.

The apparent altitude of M as seen from O is greater than its true altitude by the amount of the atmospheric refraction. To obtain the true altitude the apparent altitude has to be 'cleared of refraction,' or the apparent altitude is 'affected with refraction.'

If O were observed from M, the path of the ray of light would also be the curved dotted line OM, and the true altitude would still be less than the apparent altitude. It may happen in hot climates that the lower strata of the atmosphere, on account of their proximity to the heated ground, are less dense than the strata for some distance above them, and in this case the true altitude will be greater than the apparent altitude; this is unlikely to apply to astronomical observations, but may be the cause of obscure errors in levelling operations.

In the case of light which reaches the earth from a celestial body, the rays undergo the entire refraction due to passing from the vacuous surrounding space through the increasingly dense strata of air to the observer. If the atmospheric density were a function only of height above sea-level, and if a sufficient portion of the earth's surface, with the overlying strata of air, might be considered plane surfaces, the question of refraction would be comparatively simple. But the question is complicated by the curvature of the earth's surface and the corresponding curvature of the atmospheric strata above it; however, even under these conditions the amount of refraction for any given altitude would be the same for all points of the compass. But the density is by no means a function only of the height above the earth's surface, but is constantly varying at any one point, due to temperature changes and to the movements of the air, and the amount of refraction may, therefore, vary in different compass directions. Astronomical or atmospheric refraction thus introduces a serious difficulty into astronomical work.

As far as possible, observations should be so arranged in pairs that the effect of an error in the assumed refraction may be eliminated.

It will readily be seen that if it be assumed that the atmospheric density is a function only of the height above sea-level, the refraction will be entirely in a plane containing the plumb line and only altitudes of an observed body will be affected, the refraction having no effect on azimuth. While this assumption is not always correct, it is nearly so in most cases; any effect of refraction on azimuth would be eliminated by taking a number of observations under different meteorological conditions, as it is not possible to allow for it otherwise.

There are several formulæ for refraction, based partly on theoretical considerations as to the rate of change of density with altitude, and partly on the results of observations for the values of the constants which appear in the formulæ.

Depending as it does on the density of the atmosphere, refraction is a function of those quantities which affect the density, viz. the atmospheric temperature and pressure, increasing with increase of barometric pressure, and decreasing with increase of temperature. Its value is very nearly proportional to the tangent of the zenith distance for zenith distances up to about 60°, being, of course, zero at the zenith itself.

A formula which gives its value accurately enough for most purposes is

$$\text{refraction} = 58''{\cdot}2 \cot h,$$

to be subtracted from the observed altitude h.

The most accurate refraction tables are those of Bessel, which are given in various mathematical tables with the method of applying the corrections for pressure and temperature. An abbreviated table of mean refraction with the corrections to be applied for pressure and temperature is given in Appendix V.

Geocentric Parallax.—The term parallax is applied in general to the apparent displacement in the position of an object resulting from a change in position of the point from which it is viewed. Quantitatively, it is the apparent change in the direction of the body resulting from such change in the view-point. Direction is measured by reference to a line assumed to be fixed in space; the nearer an object is, the greater will be its parallax for a given change in the position of the view-point. If an object at a moderate distance be seen against a more distant background, any movement of the observer except in the direction of the line joining him to the object, will cause a parallactic displacement of the object relative to the background, which displacement will be the greater the nearer the object is to the observer; and if the distance of the background be known, the amount of the apparent displacement due to a known movement of the observer provides a means of determining the distance of the object from the observer. Further, if the directions of the object as seen from two view-points at a known distance apart can be compared by referring them to a definite unvarying direction in space, the distance of the object can equally well be determined. Such parallactic displacements form the basis of all determinations of distances of inaccessible objects, both in terrestrial and in astronomical work; for instance, the distance of the sun from the earth is determined by a comparison of its apparent position as seen from two widely separated stations on the earth, and the distances of stars are determined by comparison of their apparent positions when observed from opposite points of the earth's orbit, i.e. from two points about 185,000,000 miles apart.

Geocentric parallax, as its name implies, is the angular difference between the directions of a celestial body as seen by an observer on the earth, and its position as it would be seen from the centre of the earth. All celestial bodies, with the exception of those forming the solar system, are at such vast distances that even when observed from opposite points of the earth's orbit the apparent change in their directions (annual parallax), due even to this large displacement of the observer, does not in any case amount to one second of arc, consequently their geocentric parallax is absolutely negligible. This is, however, not the case with the sun, the moon, or the planets, which have an appreciable geocentric parallax. Their *apparent places*,

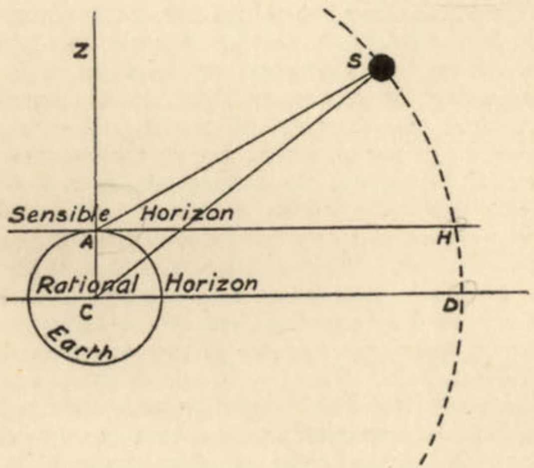

FIG. 19.

tabulated in the *N.A.*, are as they would be if seen from the centre of the earth, which is the most convenient form in which to express them, as observations taken at any part of the earth can readily be reduced to the corresponding values for the centre of the earth, as will presently be shown, and this reduction or correction is to be applied to all observations on the sun, moon, or planets. In Fig. 19 the circle represents the earth, assumed spherical, and S a celestial body, seen from A in the direction AS. Its zenith

distance is the angle ZAS, and its altitude is the angle HAS. If it could be seen from C and referred to the same zenith Z, its zenith distance would be the angle ZCS, and its altitude the angle DCS, and the geocentric parallax is ZAS – ZCS, i.e. the angle ASC.

The geocentric parallax is accordingly the angle ASC, which is the angle subtended at S by the radius joining A to the centre C of the earth.

This angle is zero for an object in the zenith, and to obtain its value at any zenith distance

$$\frac{AC}{CS} = \frac{\sin ASC}{\sin CAS} = \frac{\sin \text{parallax}}{\sin \text{zenith distance}}$$

$$\therefore \quad \sin \text{parallax} = \frac{AC}{CS} \times \sin z.$$

The parallax for the observer at A is therefore a maximum when $\sin z = 1$, i.e. when $z = 90°$, and its value for that case is termed the *horizontal parallax*. Accordingly

$$\sin (\text{horizontal parallax}) = \frac{AC}{CS}$$

and $\qquad \sin \text{parallax} = \sin (\text{hor. parallax}) \times \sin z.$

The horizontal parallax is in all cases a small angle, and the above may therefore be written

parallax in secs. of arc
= hor. parallax in secs. of arc × sin z.

In the case of the sun, the horizontal parallax is given on page 1 of the *N.A.* for every tenth day of the year, to two places of decimals. It varies between about 8″·66 and 8″·95, the latter value corresponding to the earth's perihelion on 1 January: a mean of 8″·80 may be taken for field work.

The effect of parallax on a body at a measurable distance from the earth is to make its apparent zenith distance greater than its true zenith distance as seen from the earth's centre, and referred to the same zenith. The correction is, therefore, additive to apparent altitudes, and of opposite sign to refraction, which is subtractive from apparent altitudes. For a spherical earth, geocentric parallax would have no effect on the azimuth of a heavenly body.

In the foregoing, a spherical earth was assumed. As the true figure of the earth departs to some extent from a sphere, the results are not strictly applicable; in fact, geocentric parallax causes an error in both altitude and azimuth, in general, but the parallax in azimuth is so small as to be negligible in field work. Further, as the correction for geocentric parallax is only applicable to members of the solar system, and in particular to the sun and moon, which are not relied on for precise work, it is unnecessary to consider parallax in azimuth, which is only the small component, due to the ellipticity of the earth's figure, of the total parallax, whereas the parallax in altitude is an appreciable quantity and is always to be corrected for in sun or moon observations.

The apparent displacement, due to parallax of any kind, whether geocentric or otherwise, is always towards the point from which the observer moves. For a spherical earth the assumed transference of the observer from the surface of the earth to its centre therefore displaces the observed body towards his zenith; for a spheroidal earth the transference of the view-point from the earth's surface to its centre would cause an apparent displacement towards the geocentric zenith, which is at an angular distance from the geographical zenith equal to the reduction of latitude. This will be referred to later in Chapter VI. The displacement is therefore mainly in altitude, but partly also in azimuth, except when the observed body is on the meridian, when there is no parallax in azimuth.

Correction for Sun's Semi-Diameter.—In observations on the sun, which presents a disc the apparent diameter of which varies during the year from about 31′ 30″ to 32′ 35″, it is usual to make the observation on the limb or

edge of the disc, which is brought into contact with the horizontal wire for an altitude observation, or with the vertical wire for an azimuth observation. The angle subtended at the earth's centre by the sun's radius is known as the *sun's semi-diameter*, and its value obviously varies inversely as the distance of the sun from the earth; when the sun is nearest to the earth, i.e. on or about 1 January, the semi-diameter is a maximum, when the sun is farthest

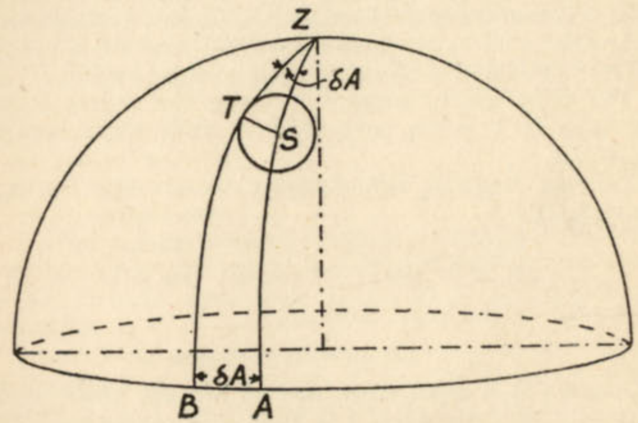

FIG. 20.

from the earth, i.e. on or about 2 July, the semi-diameter is a minimum, hence

$$\frac{\text{sun's minimum semi-diameter}}{\text{sun's maximum semi-diameter}} = \frac{\text{sun's maximum distance}}{\text{sun's minimum distance}}.$$

The latter ratio was shown on page 5 to be 1·0341 : 1. The sun's semi-diameter is given in the *Nautical Almanac* for G.M.N. for every day of the year to 0·01 of a second of arc, and in the abridged edition to one-tenth of a minute of arc. The altitude of the centre of the disc is clearly equal to the altitude of the upper or lower limb, diminished by or increased by the semi-diameter, which is therefore the correction to be applied to an observed altitude of the limb, to obtain the altitude of the centre.

The azimuth of the centre differs, however, from the azimuth of the limb by a quantity which depends not only on the semi-diameter, but also on the sun's altitude, as will be seen from consideration of Fig. 20 in which S is the centre of the small circle representing the sun's disc, ZSA is a vertical circle through S, ZTB is a vertical circle tangential at T to the limb of the disc, and ST is part of a great circle and is therefore perpendicular to ZT.

The difference in azimuth between the centre S and the limb at T is the angle SZT or AZB, which may be called δA.

Let the angular semi-diameter be s. Then in the triangle ZTS

$$\frac{\sin ST}{\sin SZT}=\frac{\sin SZ}{\sin STZ} \therefore \frac{\sin s}{\sin \delta A}=\frac{\sin z}{\sin 90^\circ}$$

where SZ $=z$, the zenith distance.

$$\therefore \quad \sin \delta A = \sin s \, . \operatorname{cosec} z,$$

or, since s is a small angle, and as azimuth observations will only be done when z is large and consequently δA small,

$$\delta A = s \, . \operatorname{cosec} z$$
$$= s \, . \sec h, \text{ if } h \text{ be the altitude.}$$

The correction to the horizontal circle reading, if a limb has been observed for azimuth, is therefore $\pm\ s \, . \sec h$.

Instrumental Errors.—When all the relative adjustments discussed in Chapter IV have been made as nearly perfect as possible, the conditions as to perpendicularity of axes to one another will still not be fulfilled with mathematical accuracy, but slight residual errors will always be present, which can in most cases be eliminated either by making observations on both faces of the instrument, or by the application of appropriate corrections to the readings of the circles; the nature and amount of such corrections are determined from the bubble tube readings, which indicate the magnitude of the errors, by methods explained in the following pages. Even if the instrument itself were in perfect adjustment, there would still in all probability be a slight error in the setting up and levelling, as a result of which the vertical axis, instead of being truly vertical, would be inclined at a small angle to the vertical, producing a corresponding deviation of the other axes from their correct positions. The effect of these errors on both horizontal and vertical angular measurements will now be considered. The errors will be treated in the following order:

(*a*) Collimation error,

(*b*) The horizontal axis not being perpendicular to the vertical axis,

(*c*) The vertical axis not being set up truly vertically;

and it will be assumed in each case that the instrument is free of errors other than the particular one under consideration.

(*a*) *Collimation error.*—This exists when the line joining the intersection of the wires to the optical centre of the object glass is not perpendicular to the horizontal axis, but is inclined to one side or the other of the true perpendicular. When this is the case, the rotation of the telescope on the horizontal axis causes the line of sight to generate a conical instead of a plane surface, the intersection of which with the celestial sphere will be a small circle parallel to the vertical great circle which it ought to sweep out. Thus, in Fig. 21, let HR represent the horizontal axis of the theodolite. If there were no collimation error the line of sight would sweep out the vertical circle AZA′, but a collimation error will cause it to sweep out a small circle BZ′B′ parallel to the great circle AZA′. The angular distance AB is the collimation error c referred to on page 68.

The magnitude of the errors in the measured values of horizontal and vertical angles arising from an error of collimation is determined as follows :

Let S represent an object the image of which is observed to be on the intersection of the cross wires ; its true zenith distance is ZS, but its observed zenith distance is ZS′ or the angle ZHS′, where SS′ is part of the great circle through H, S, and R, and is therefore perpendicular to ZA.

In azimuth observations on a star, the quantity which

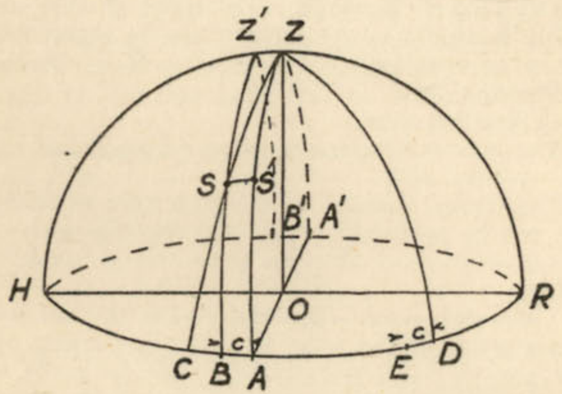

FIG. 21.

it is usually desired to determine is the comparative azimuth of the object referred to that of a fixed terrestrial reference object, that is, the horizontal angle between the fixed terrestrial point and the object. Such terrestrial point will usually be chosen as near to the horizon as possible ; it may be supposed for the present purpose to be on the horizon and represented by E. In order to measure the horizontal circle reading between E and the object S, the telescope is first directed towards the point E so that the image of E appears on the vertical wire.

On account of the collimation error the true axis of collimation does not intersect E, but cuts the horizon at a point D at an angular distance $c = \mathrm{AB}$ along the horizon from E. The instrument is then rotated until the image of S is brought to the intersection of the cross wires, which will necessitate a rotation about the vertical axis until the true axis of collimation reaches the vertical circle ZA, when the vertical wire will intersect the object S on the telescope being tilted. The horizontal angle as measured is therefore DZA or the arc DA, which is equal to the arc EB. The observed difference in azimuth is therefore the arc DA or the angle DZA, or EZB, whereas the true difference in azimuth is the angle EZS or the arc EC.

Consequently the error in the value of the measured horizontal angle between E and S is $\mathrm{EC} - \mathrm{EB} = \mathrm{BC} = \mathrm{AC} - \mathrm{AB}$.

Then from the triangle SZS′

$$\frac{\sin \mathrm{ZS}}{\sin \mathrm{ZS'S}} = \frac{\sin \mathrm{SS'}}{\sin \mathrm{SZS'}}$$

or
$$\frac{\sin z}{\sin 90^\circ} = \frac{\sin c}{\sin \mathrm{AC}} \qquad \therefore \quad \sin \mathrm{AC} = \frac{\sin c}{\sin z}$$

and as c and AC are small angles, $\mathrm{AC} = \dfrac{c}{\sin z} = c \operatorname{cosec} z$.

$\therefore$ the error in the measured value of the horizontal angle, viz. $\mathrm{AC} - \mathrm{AB}$, is $c \operatorname{cosec} z - c$, or $c(\operatorname{cosec} z - 1)$, i.e. $c(\sec h - 1)$, where $c \operatorname{cosec} z = \mathrm{AC}$ and $c = \mathrm{DE} = \mathrm{AB}$.

The correction to be applied to the horizontal circle reading for a collimation error c'' is $c'' \sec h$ for an object at an altitude h. The correction must also be applied to the horizontal circle reading on the fixed reference object ; it will be c'' for a reference object on the horizon.

If, as in Fig. 21, the line of sight lies to the observer's right of the true axis of collimation, each reading of the horizontal circle requires to be increased to obtain the true reading, i.e. the correction is $+c$ cosec z, whereas if the line of sight be to the left of the true axis of collimation the correction is $-c$ cosec z, z being in each case the zenith distance of the object observed, i.e. of the reference mark or the star.

So far as collimation error is concerned, no error would exist in the values of measured horizontal angles between objects at equal altitudes, as the correction would be the same for both, and one pointing on either face to each object would give the true horizontal angle between them. If the objects are at different altitudes, the error in the measured horizontal angle is obviously eliminated by taking readings of the angle both F.R. and F.L., as the errors will be equal, but of opposite signs in the two cases.

For the error in observed zenith distances, from the triangle SZS′,

$$\cos ZS = \cos ZS' \cos SS' + \sin ZS' \sin SS' \cos ZS'S$$

or $\cos z = \cos z' \cos c$, since $\cos ZS'S = \cos 90° = 0$

z' being the observed zenith distance.

If, then, c be a small angle, $\cos c$ is very nearly 1, and $\cos z = \cos z'$ or $z = z'$, that is, a small collimation error produces no appreciable effect on measured zenith distances or altitudes.

(*b*) *Error due to the horizontal axis not being perpendicular to the vertical axis.*—In Fig. 22 let H′R′ represent the horizontal axis, inclined at an angle b to the true horizon HR. If the telescope be rotated about H′R′ the intersection of the cross wires will sweep out the great circle AZ′A′, the angular distance from the true zenith Z to Z′ being b. Let S represent the position of an object the image of which is on the cross wires. As in treating of collimation error, it will be supposed that it is desired to measure the difference in azimuth between S and a fixed reference mark, which may be supposed to lie on the true horizon. Draw ZSC, the vertical circle through Z and S cutting the horizon at C.

If the azimuth of A referred to that mark be θ, then the correct azimuth of S referred to the same mark is θ + AZS, but on account of the tilt of the axis the observed azimuth of S will be the same as that of A, viz. θ. The error in the horizontal circle reading for the point S is therefore AZS, or the arc AC.

In the spherical triangle ASC, from the four consecutive parts SC, SCA, CA, CAS,

$$\cos CA \cos SCA = \sin CA \cot SC - \sin SCA \cot CAS.$$

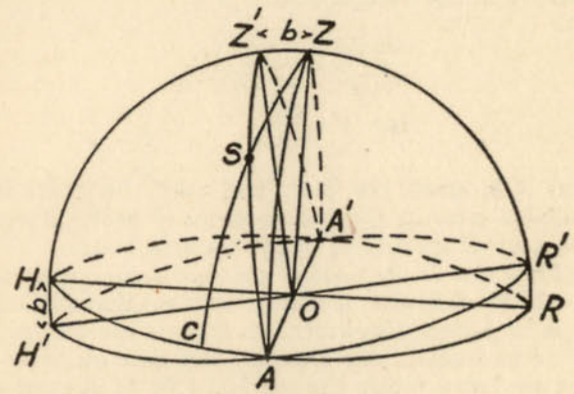

FIG. 22.

But SCA = 90° and CAS = 90° − b and SC = altitude = h.

$\therefore \quad \sin CA \cot h = \cot (90° - b).$

$\therefore \quad \sin CA \cot h = \tan b.$

or $\quad \sin CA = \tan b \tan h.$

Therefore, since CA and b are small,

$$CA'' = b'' \tan h.$$

That is to say, if the horizontal axis be inclined at an angle b to the true horizon, and if the telescope be directed to a point S at an altitude h, the reading of the horizontal circle differs from the reading which would be obtained if the horizontal axis were truly horizontal by an amount $b \tan h$, which is the correction to be applied to the circle reading: positive if the end of the trunnion axis on the observer's left is the higher, and negative if lower. The value of b is measured by means of the striding level, as explained on page 100.

For the error in observed zenith distances,

let $z = ZS$, the true zenith distance,

and $z' = Z'S$, the observed zenith distance.

Then, from the triangle ASC,

$$\frac{\sin SA}{\sin 90^\circ} = \frac{\sin SC}{\sin (90^\circ - b)}$$

$$\cos z' = \frac{\cos z}{\cos b}$$

and as b is small $\cos b = 1$ and $z = z'$, i.e. there is no appreciable error in the measurement of zenith distances due to a small inclination of the horizontal axis.

(*c*) *Error due to the vertical axis not being set up vertically.*—In the following it will be assumed that the instrument is in perfect adjustment as regards the perpendicularity of its axes to one another, but that owing to the setting up being faulty the vertical axis is inclined at a small angle v to the true vertical. In Fig. 23 the position of the true zenith is indicated by Z, while Z′ at an angular distance v from Z is the point where the vertical axis, as set up, cuts the celestial sphere. If the instrument had been set up accurately, its horizontal circle would have met the celestial sphere in the true horizon FKC, but owing to the inclination of the vertical axis the horizontal circle is tilted and cuts the celestial sphere in the great circle F′KC′ inclined at the small angle v to the horizon FKC and intersecting it in the horizontal diameter KK′.

Let P represent an elevated point which is to be observed.

Draw the great circles ZPD and Z′PEH. The true azimuth of P referred to K and its true zenith distance are respectively KD and ZP, whereas its azimuth referred to K and its zenith distance, both measured by the instrument as set up, are KE and Z′P respectively. The errors in the observed azimuth and zenith distance are therefore KE−KD and Z′P−ZP respectively.

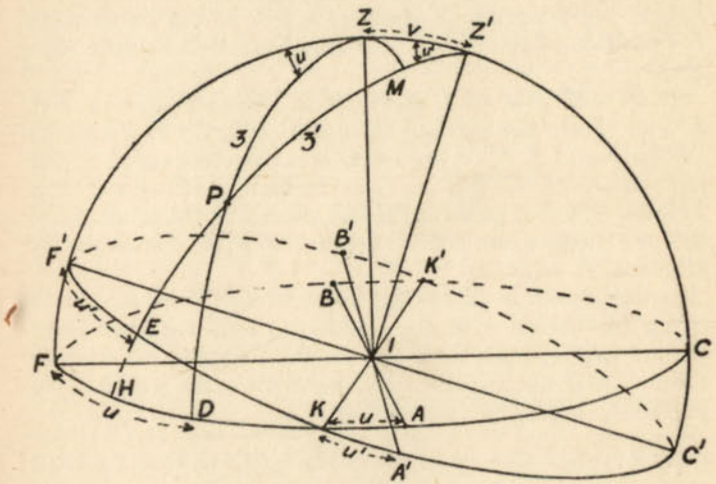

Fig. 23.

The error in azimuth, viz. KE−KD, will be dealt with first, and it is required to find an expression for the magnitude of this error. In order to do so, the position of the horizontal axis of the instrument will be considered. If the setting up had been accurate, the position of the horizontal axis in order to sight P would have been AB, in the plane FKC, and such that DA = 90°; the actual position of the horizontal axis when P is sighted is, however, A′B′, in the plane F′KC′, and such that EA′ = 90°. If F′C′

represent the line of greatest inclination in the plane F′KC′, then F′K and FK will each be 90°, i.e.

$$FD + DK = 90°, \text{ and } DK + KA = 90°, \therefore FD = KA$$
$$F'E + EK = 90°, \text{ and } EK + KA' = 90°, \therefore F'E = KA'.$$

The horizontal axis A′B′ has therefore a definite position in the plane F′KC′, such that KA′ = F′E, and it is inclined at a small angle to the horizon, which angle may be called b as before. The maximum value of b, viz. v, will occur when the position of the point P is such that A′B′ coincides with F′C′.

Draw ZM part of a great circle perpendicular to Z′P. A′ and B′ are the poles of the great circle Z′PEH, and the inclination of A′B′ to the horizon is therefore equal to the inclination of Z′PEH to the vertical, i.e. b = arc ZM. Further, M is the point of Z′PEH nearest to the true zenith, and is therefore the highest point of Z′PEH, consequently HM = 90°.

It was shown in discussing the error DH in azimuth due to an inclination b of the horizontal axis that the error DH = b tan h, where h = PD. In the right-angled triangle KEH, applying the cot formula to the four consecutive parts HK, HKE, KE, and KEH,

$$\cos KE . \cos HKE = \sin KE . \cot HK - \sin HKE . \cot KEH;$$

and as KEH = 90°, and HKE = v,

$$\cos KE . \cos v = \sin KE . \cot HK.$$

As v is small, cos v may be taken as 1, and then

$$\tan KE = \tan HK, \text{ or } KH = KE.$$

Therefore the error in the measured azimuth, viz. KE − KD, is equal to

$$KH - KD = DH = b \tan h.$$

The error in azimuth is therefore in this case, as in the

case where the inclination is due to non-perpendicularity of the axis, also $b \tan h$, where b is the inclination of the horizontal axis and h is the true altitude of P, viz. PD, which differs by only a small quantity, negligible for the evaluation of the correction, from its measured altitude PE.

The inclination b of the horizontal axis is measured in seconds of arc by means of the striding level, as will be explained presently, and it is a matter of indifference whether such inclination be due to error of adjustment or error of setting up. The correction $b \tan h$ is to be applied with the appropriate sign to the actual circle reading, in order to eliminate the errors due both to faulty adjustment and to faulty setting up. The sign of the correction depends upon which end of the axis is the higher. If, as in Fig. 23, the end of the axis on the observer's right be the higher, and if, as is usual, the horizontal circle is so graduated that the readings increase from K towards E, the correction is negative, as KE is greater than KD; conversely, if the left-hand end of the axis be the higher, the correction is positive. A change of face of the instrument will not alter the inclination of the horizontal axis due to this cause, and the error cannot be eliminated by that procedure; but the actual inclination of the axis is to be measured for the pointing on each face, and the corresponding correction applied to each circle reading, when the mean of the corrected F.R. and F.L. readings will give the true reading of the azimuth circle. In general, the inclinations of the axis will differ for the F.L. and F.R. pointings, as part of the error will be due to the horizontal axis not being perpendicular to the vertical axis; but the error, to whatever cause it be due, is eliminated by the procedure described above.

When the horizontal angle between two points, whether at the same or at different altitudes, is being measured on one face of the theodolite, the difference between the readings of the horizontal circle will not in general give the true horizontal angle between the points; but in all cases the correction $b \tan h$ is to be applied to the circle reading for each pointing.

As regards zenith distance, it has already been shown that index error or collimation in altitude is eliminated by reversal of the instrument, and as it is assumed that, except when special reasons for the contrary exist, all observations will be made on both faces, it may be supposed that the index error is zero. The vertical circle will then read zero zenith distance when the telescope is directed towards Z′, and the observed zenith distance of P will be Z′P, while the true zenith distance is ZP; the error is therefore Z′P−ZP, and if the instrument be reversed, the second reading will also be affected with the same error, which is not eliminated by the reversal. The point M, being, as has been shown, the highest point of the circle Z′PEH, is the point from which zenith distances ought to be measured; they would then be affected with only the very slight error PZ−PM, which was shown, in treating of inclination of the horizontal axis, to be negligible. M in Fig. 23 corresponds in fact to Z′ of Fig. 22, in discussing which it was shown that ZS = Z′S, consequently, in Fig. 23, ZP = PM.

The error in the observed zenith distance is therefore zero if measured from M, and Z′M if measured from Z′.

If altitudes are being read, the true altitude of P is PD, the difference between which and PH is negligible. If a spirit level were attached to the vernier arm with its axis parallel to the plane of the vertical circle, the centre of its bubble would occupy the position corresponding to M in Fig. 23; and if the diameter joining the zero points of the verniers were at right angles to IM, the circle reading would give the altitude correctly, apart from index error. The diameter joining the zero points will not usually be exactly at right angles to IM, but its inclination to that position is given by the displacement of the bubble. In order to make this clear, the use of the spirit level will now be explained.

Spirit Level.—The spirit level consists essentially of a glass tube, the bore of which is either barrel-shaped or curved axially to a circular arc of large radius, and sealed at the ends after being nearly filled with a liquid such as alcohol, ether, or petroleum ether. The glass tube is fixed in a tubular brass casing, open at one side to allow the upper side of the glass tube to be seen, in such a way that when the level is in use the plane of the arc is vertical with the convex side of the arc upwards. The vacant space, termed the bubble, always occupies the highest part of the tube, which is graduated on its upper side; the graduation corresponding to the highest part of the tube can consequently be read. Various systems of graduation are in use, the most usual having a central zero. Any alteration in the inclination of the longitudinal axis of the level causes a displacement of the bubble, the amount of such displacement being proportional to the change of inclination and also to the radius of curvature of the tube. Thus if a change of inclination of i radians causes the bubble to move a distance d along the tube, the radius of which is r, $d = r \times i$.

If the change of inclination be i seconds of arc, as 1 radian = 206265″

$$d = \frac{r \times i}{206265}, \text{ and } i'' = \frac{206265 \times d}{r}$$

The displacement for a change of inclination of one second of arc is therefore $\frac{r}{206265}$; the radius r is thus determined for any required degree of sensitivity; the greater the radius the greater the sensitivity, i.e. the smaller the change of inclination corresponding to one division of the level scale.

Usually for the ordinary 5-inch theodolite the scale divisions are about one-tenth of an inch long, and the value of one level division may be from 10 to 20 seconds of arc, being, as already explained, the angle subtended at the centre of curvature of the arc by the length between adjacent graduation marks.

A level may be an extremely sensitive instrument for indicating changes of inclination, but its readings are not always reliable for quantitative determinations of inclination, owing to temperature changes causing distortion of the tube, with a consequent change in the value of a level division.

The bubble being usually of considerable length, the position of its centre is derived from the scale readings at the ends of the bubble; with a central zero, the distance of the centre of the bubble from the zero is evidently half the difference between the scale readings at the bubble ends, provided that the two ends of the bubble are on opposite sides of the zero, as ought to be the case.

The method of determining an inclination by means of the level will now be explained. For convenience it will be supposed that the inclination of a surface is to be determined, and that each end of the level tube is supported by a foot resting on that surface. In an ideal level the zero of the graduations would be exactly at the middle point of the arc of the tube and the feet would be of exactly equal length, and consequently the two ends of the tube would be equidistant from the points of contact of the feet with any surface to which the level were applied. If such a level were placed on a horizontal plane surface, the centre of the bubble would coincide with the zero of the graduations, and would do so, however the feet were moved about on the surface, even if the level were reversed end for end. Such a state of affairs cannot be realized in practice, and it becomes necessary to inquire how the non-fulfilment of these conditions affects the level, and how its readings are used to measure the inclination of a surface. Referring to Fig. 24, it will be assumed that the feet AC and BD are of unequal height, that the zero O of the graduations does not coincide with the middle point O′ of the arc CD, and also that the line AB on the surface to which the level is applied is inclined at an angle i to the horizontal.

The centre of the bubble occupies the highest point E of the tube.

Let V be the value in seconds of arc of one division of the level scale.

On account of the inclination i of AB, the bubble is displaced through $\frac{i}{V}$ scale divisions from the position it would have if AB were horizontal.

The displacement due to the inequality of the feet may be called d divisions.

If these two causes of displacement were removed, the centre of the bubble would be at O′, the mid-point of CD. Consequently $O'E = \frac{i}{V} + d$.

Let the scale readings of the right and left ends of the

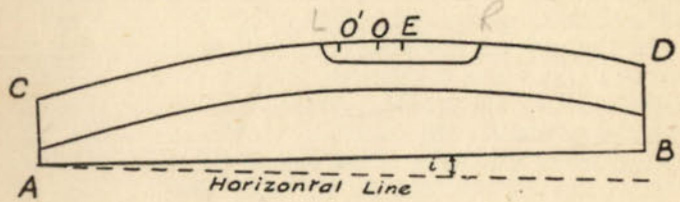

FIG. 24.

bubble be R and L respectively; the distance OE is then $\frac{R-L}{2}$.

$$O'E = O'O + OE$$

$$\therefore \quad \frac{i}{V} + d = OO' + \frac{R-L}{2} \text{ and } R-L = \frac{2i}{V} + 2d - 2(OO')$$

If the level be now reversed end for end, the bubble will, on account of the fact that AB is not horizontal, change its position. Let the readings of the right and left ends, as reversed, be now R′ and L′.

The above expression for R−L will still apply if the necessary changes of sign be applied to such quantities as are reversed, and the equation becomes

$$R'-L'=\frac{2i}{V}-2d+2(OO')$$

Adding the two expressions,

$$\Sigma R-\Sigma L=\frac{4i}{V}, \text{ or } i=\frac{\Sigma R-\Sigma L}{4}\times V.$$

The inclination of the line AB is therefore obtained by applying the level direct and then reversed, and evaluating $\frac{\Sigma R-\Sigma L}{4}\times V$, which eliminates both the error due to the zero not being at the centre of the tube and also that due to the line CD not being parallel to AB.

In the case of the theodolite, the rotation about the vertical axis, which is necessary in order to take both face right and face left readings, reverses the bubble tube end for end in exactly the same way as assumed above, and consequently the tilting of the axis in the direction of the line of sight is obtained by the same method, from the bubble readings.

The value of a division of the level scale is best determined by a level-trier, in which the level is rested in a frame one end of which is supported by a micrometer screw of known pitch, so that any desired inclination can be given to the frame and to the level resting in it, and the value of one division thus found.

It can be determined in the field, in the case of the level on the miscroscope arm, in the following way. Set up the theodolite, bring the bubble carefully to the centre of the graduations, make a careful pointing on any object, and take the vertical circle readings. Now displace the bubble through any convenient number of divisions by means of the clip-screws; this will throw the cross wires off the object. Bring the cross wires back to the object by the

tangent screw; this will change the reading. The difference between the new reading and the original reading is the angle through which the level has been tilted for a bubble displacement of a number of divisions known from the readings of its ends, and the angle of tilt for one division is readily obtained.

The procedure for eliminating the effect of an inclination of the vertical axis on observed zenith distances affords an example of the use of a spirit level for measuring inclinations.

The inclination of the axis may be regarded as being the resultant of two component inclinations, one in the plane of the vertical circle and one at right angles thereto. The effect of the latter component is to produce a slight tilting of the horizontal axis, equal in amount to the said component, and therefore never greater than the setting-up error of the vertical axis.

The error in observed zenith distances due to this cause has already been shown, in dealing with inclination of the horizontal axis, to be negligible. There remains, therefore, to be considered only the component of the inclination of the vertical axis in a plane parallel to the vertical circle.

In Fig. 25, let OZ′ represent the projection of the vertical axis on the plane of the vertical circle, OZ the true vertical, and OP the direction of the line of sight to an object P, and ROH the position of the vernier or microscope arm carrying the reading microscopes.

Reversal of the instrument brings the line of sight into the position OP′, and in order to bring P again on to the cross wires the telescope has to be rotated about its horizontal axis through the angle P′OP = 2(Z′OP). The microscope arm will now be in the position R′OH′ as it also has rotated about the axis OZ′, and the reading of the vertical circle will have changed by an amount indicating a zenith distance Z′OP, differing from the true zenith distance by ZOZ′. If then ZOZ′ were measured and applied as a correction to the observed zenith distance, the true zenith distance would be obtained. A level attached to any portion of the alidade, with its axis parallel to the vertical circle, would effect this measurement; the level carried by the microscope arm is usually more sensitive than a plate level, and is therefore used for the purpose. As explained in connection with the spirit level, the formula $\frac{\Sigma R - \Sigma L}{4} \times V$ determines the amount of the

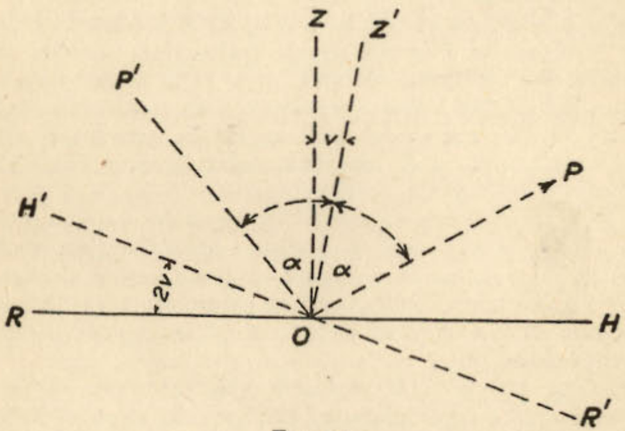

FIG. 25.

tilting of the vertical axis in the direction of the line of sight.

The ends of the bubble are in this case usually referred to as eye end E and object end O, and the angle of tilt in the direction of the object observed is $\frac{\Sigma O - \Sigma E}{4} \times V$, the correction being additive to altitudes and subtractive from zenith distances when O is greater than E, and vice versa.

Accordingly, in the observing of an altitude on both

faces of the instrument, the application of the level correction eliminates error due to faulty setting up of the vertical axis, and as the change of face also eliminates index or collimation in altitude, it may be taken that an altitude so observed is the true altitude, apart from flexure of the telescope tube, which will be referred to in dealing with latitude observations.

It is advisable, however, for each pointing to bring the bubble as near as possible to the centre of its run by means of the clip screws to avoid large level corrections, which may be unreliable. This virtually transfers the correction for the bubble displacement to the circle readings. The adjustment of the bubble should be done before the pointing on the object, and the bubble ends ought to be read immediately after the pointing and before the circle readings are taken, lest any further displacement due to temperature changes should occur.

Summary of Corrections.—For the practical work of observing, the foregoing corrections may be summarized as follows:

ASTRONOMICAL CORRECTIONS:

Refraction, applicable to all observed altitudes, $-58''\cdot 2 \cot h$ for values of h from 35° to 90°, where h is the observed altitude. See Appendix V.

Parallax, applicable to observed altitudes of bodies of the solar system, particularly the sun, for which it is $+8''\cdot 8 \operatorname{cosec} h$.

Semi-diameter, applicable to sun observations both in altitude and azimuth.

In altitude, $\pm s$ as given for each day in the *N.A.*

In azimuth, $\pm s \,.\, \sec h$, where h is the altitude and s the semi-diameter.

INSTRUMENTAL CORRECTIONS:

Collimation in azimuth.—In azimuth, correction $\pm c'' \sec h$, necessity for correction eliminated by reversal of face.

Collimation in altitude, or index error.—In altitude, eliminated by reversal of face.

Horizontal axis error.—In altitude, resulting error neglible.

In azimuth, correction $\pm b \tan h$ to whatever cause due, where b is the inclination of the axis as measured by the striding level.

Vertical axis error.—In altitude, error eliminated by reversal of face and application of T-arm level correction.

In azimuth, correction included in $b \tan h$ as above.

CHAPTER VI

LATITUDE AND LONGITUDE

Latitude and Longitude.—The co-ordinates used in defining the position of a point or station on the earth are its *latitude* and *longitude*. As a first approximation, the form of the earth may be assumed to be that of a sphere, as shown in Fig. 26, in which PP′ represents the axis of rotation, P and P′ being the poles, and A a station situated on the great circle PAP′.

AT, the tangent plane at A, represents the horizon at that point, and it has been shown that the angle TAp or ϕ is the altitude of the elevated celestial pole at A. As the two lines AO and OQ are respectively at right angles to the lines AT and Ap enclosing the angle ϕ, the angle AOQ also is equal to ϕ. This angle ϕ, being (for the assumed spherical earth) both the altitude of the celestial pole, viz. the angle TAp, and the angular distance of the station A from the equator, viz. the angle AOQ, is called the *latitude* of the station A, and is either north or south according to whether the station A is north or south of the equator. A plane parallel to the equator and passing through A cuts the surface of the sphere in a small circle called a *parallel of latitude*. For purposes of computation north and south latitudes must be given opposite signs; usually north latitudes are treated as positive, and south latitudes as negative.

The *longitude* of A is the angular distance between its terrestrial meridian plane QAR and the meridian plane of Greenwich, i.e. the longitude of the station A is the angle GPA (Fig. 26, *b*). Longitudes are, for astronomical work, usually stated in hours, minutes, and seconds of

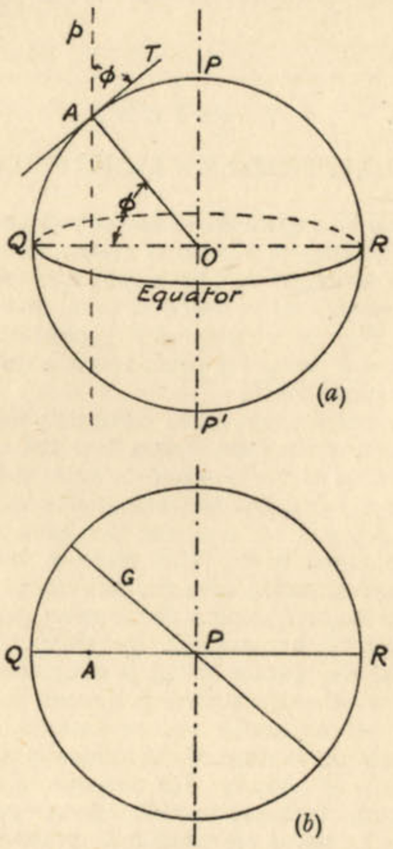

FIG. 26.

time East or West of Greenwich; if stated in degrees, minutes, and seconds of angle they are converted into

hours, minutes, and seconds of time by reckoning 15° of angle to one hour of time. The conversion is most conveniently done by division or multiplication by 15, as the case requires, thus:

Example.—Convert 84° 45′ 30″ difference of longitude into time:

$$\begin{array}{r} 5\,)\,84^\circ \quad 45' \quad 30'' \\ \hline 3\,)\,16 \quad 57 \quad 06 \\ \hline 5\text{h} \quad 39\text{m} \quad 02\text{s} \end{array}$$

Example.—Express 3h 35m 42s difference of longitude in angular measure:

$$\begin{array}{r} 3\text{h} \quad 35\text{m} \quad 42\text{s} \\ 5 \\ \hline 17 \quad 58 \quad 30 \\ 3 \\ \hline 53^\circ \quad 55' \quad 30'' \end{array}$$

The difference of longitude between two meridians expressed as above in time represents the mean time interval between the passage of the mean sun across the two meridians, or the sidereal time interval between the passage of a star across the two meridians; further, it represents the difference between the hour angles of any heavenly body at the two places at any given instant.

Figure of the Earth.—The actual circumstances are, however, not quite so simple, careful measurements of the earth by geodetic methods having shown that the form of the earth is not quite spherical, but approximates closely to an oblate spheroid, i.e. the figure formed by the revolution of an ellipse about its minor axis, the polar diameter being the minor axis, and the equatorial diameter the major axis.

The most recent determinations of the dimensions of the earth give the following result :

The semi-axis major = 20,925,871 feet.
" " " minor = 20,855,721 "

The radius R for a given latitude ϕ may be taken as R = semi-axis major $(1 - C \sin^2 \phi)$, where C is the compression or ellipticity and is equal to :

$$\frac{\text{semi-axis major} - \text{semi-axis minor}}{\text{semi-axis major}}$$

and consequently has the value $\frac{1}{298 \cdot 3}$

This departure from the spherical form does not affect the definition of longitude, which remains as for the assumed spherical earth, viz., the angular distance between two meridian planes, one passing through the place, the other through Greenwich, but the question of latitude becomes slightly more complicated, as will be shown.

If a non-rotating homogeneous fluid earth be imagined, of which the particles are subject only to their mutual gravitational forces, such an earth would assume a spherical form and the direction of the plumb line at any point of it would be radial, the attraction on the plumb bob being directed towards the centre. Now if such a sphere could be rotated, the particles of the earth as well as those of the plumb line would be affected by centrifugal force. It can be shown mathematically that for a rotating fluid mass, of which the particles are subject only to gravitational forces amongst themselves, one figure of equilibrium is a spheroid, of which the minor axis is the axis of rotation, corresponding to the polar axis of the earth. The ocean surface of such an earth would, owing to rotation, assume a spheroidal form, i.e. all meridian sections would become ellipses, having the polar diameter as their common minor axis. The plumb line will not be directed towards the centre of the ellipse, as both the centrifugal force and the altered

disposition of the particles of the earth will operate to cause it to be directed towards a point such as N (Fig. 27) on the polar axis, but the plumb line will still be normal to the free liquid surface, which latter is, in fact, an equipotential surface, while the plumb line takes the direction of a line of force and is therefore normal to the liquid surface.

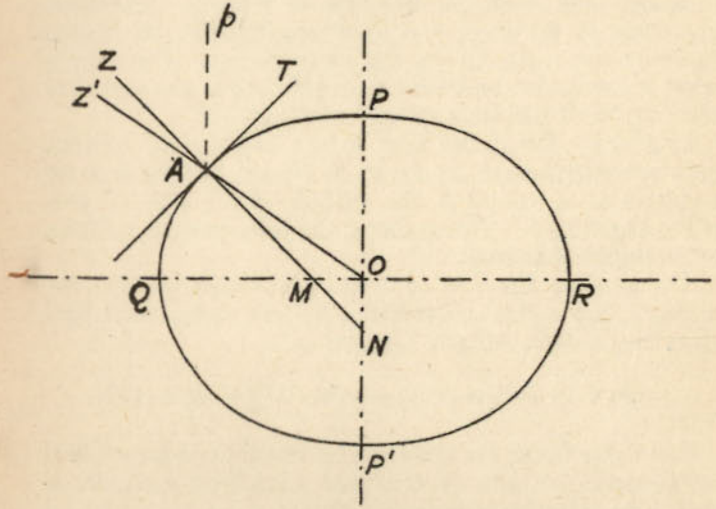

FIG. 27.

Fig. 27 represents a meridian section of such a rotating body, PP′ being the polar axis and A any station on the surface. The horizon plane at any point will be the tangent plane to the free liquid surface, i.e. to the spheroid and the plumb line will be the normal to the surface at the point. AT is a tangent at A to the ellipse and represents the horizon at A, while AN, perpendicular to AT, is the normal and represents the direction of the plumb line at A ; NA produced gives Z, the zenith of A. OA produced gives Z′, the *geocentric zenith*. If a spirit level were placed at A, its bubble would take up a position as near to Z as the tube would permit, the liquid in the tube being virtually a part of the free liquid surface of the earth. Although the earth as now existing has not a surface all of which is liquid, the foregoing represents fairly accurately the existing state of the earth as regards mean surface, rotation, and resultant direction of gravity. However, according to the most precise determinations, the actual figure of the earth, i.e. of the mean surface, is not quite what it should be for a fluid mass rotating in the period of one day, but it is at least approximately so.

Latitude.—Returning now to the question of latitude on a spheroidal earth, as shown in Fig. 27, and considering a station at any point A, the altitude of the celestial pole is the angle TAp. This is called the *astronomical, geodetic, or geographical latitude*.

The angular distance of A from the equator, i.e. the angle AOQ, is called the *geocentric latitude*, and is clearly less than the geodetic latitude TAp, because

$$\text{angle TA}p = \text{angle AMQ} = \text{angle AOQ} + \text{angle OAM}.$$

The angle OAM, the quantity by which the astronomical or geodetic latitude exceeds the geocentric latitude, is called the *reduction to geocentric latitude*.

The latitudes in general use and shown on maps are geodetic not geocentric, and correspond in general to the angle TAp, i.e. to the altitude of the celestial pole, but there may be a slight apparent difference at any station arising from a local deviation of the plumb line due to irregularity in the distribution of matter in the earth in the neighbourhood.

The objects of work in field astronomy are in all cases purely terrestrial, i.e. observations are not done with a view to investigations in astronomy, but they have for their object:

Determination of co-ordinates (latitude and longitude of a station),
" " meridian or azimuth of a line,
" " local deviations of the plumb line.

Determination of Latitude by Meridian Altitudes.—

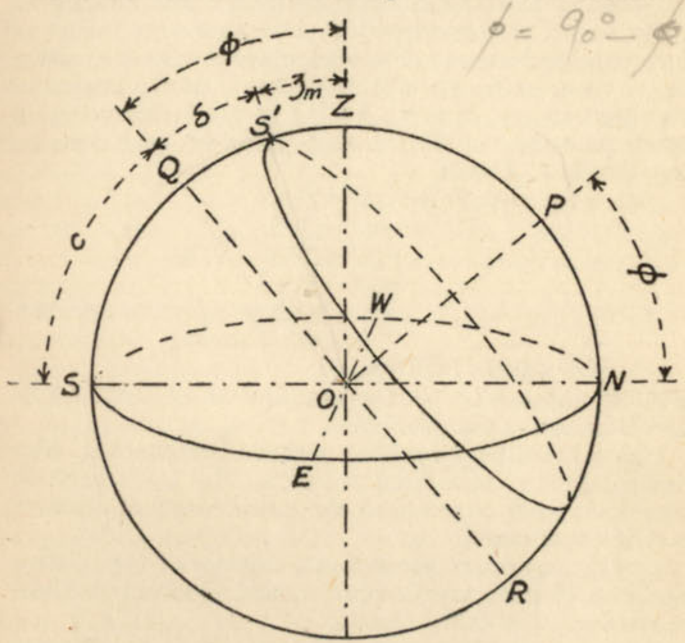

FIG. 28.

The determination of latitude to a moderate degree of accuracy is one of the simpler operations in field astronomy and, as a knowledge of the latitude is usually required before the results of any other observations can be computed, it is also one of the first operations to be carried out on arrival at a station.

Fig. 28 represents the celestial sphere as seen by an observer at O. P is the elevated pole, NESW the observer's horizon, Z his zenith, and SZPN his meridian plane.

The latitude, being the altitude of the elevated pole, is the angle PON.

As there is no star precisely at P it becomes necessary, in order to find the latitude, to deduce the altitude of P by means of observations on other heavenly bodies of known polar distance. The most obvious method of doing so is by measuring the altitude of a star at the instant of its culmination or transit; for if δ be the declination of a star culminating at S′, the angle QOS′ $=\delta$, and if h_m be its altitude at transit,

$$\begin{aligned}\text{then the angle PON} &= 90°-\text{POZ}\\ &= 90°-\text{QOS}\\ &= 90°-(\text{S}'\text{OS}-\text{S}'\text{OQ})\\ &= 90°-h_m+\delta\\ &= z_m+\delta, \text{ where } z_m \text{ is the meridian zenith distance,}\\ \text{or } 90°-\text{angle PON} &= h_m-\delta.\end{aligned}$$

But 90°−angle PON is the complement of the latitude and is known as the *colatitude.*

Hence colatitude = meridian altitude − declination. The declination is to be treated as positive if it is of the same name (i.e. north or south) as the latitude, and as negative if of different name.

In any particular case a rough diagram of the relative positions of pole, zenith, equator, and observed star will obviate any possibility of error.

Procedure in Observing.—If a star is to be observed on the meridian with a theodolite, as the star is on the meridian only for an instant, it cannot be observed both F.R. (face right) and F.L. (face left) at that instant. The best procedure is to observe the star alternately F.R. and F.L. for some minutes before and after the previously computed clock time of transit. The pair of consecutive observations giving the highest mean is taken, and the said mean treated as the observed meridian altitude.

The observation is quickly carried out, and this method has the advantage that the approximate latitude is obtained quickly with a minimum of computation. The declination of the star is taken from the *N.A.*, from the section giving *apparent places of stars*, in which section the R.A. and declination are given for every tenth day throughout the year, or in the case of circumpolar stars for every day of the year, at the instant of upper transit at Greenwich. The declination at the instant of observation is got by interpolation from the tabulated declinations, which need not be done with great accuracy, as a single meridian observation, i.e. a pair of pointings F.R. and F.L., with an ordinary theodolite would not be expected to give a result correct to within a few seconds of arc, more precise methods being available if greater accuracy is desired.

The star's observed altitude has to be corrected for refraction.

The observation may be carried out on the sun, in which case the declination at the instant of observation has to be obtained by the method given on page 33.

The alternate F.R. and F.L. pointings should, in the case of the sun, be made alternately on the upper and lower edges or limbs of the disc, as this eliminates the correction for semi-diameter and also tends to eliminate errors arising from the difficulty of making a perfect contact with the horizontal wire of the theodolite, as it is likely that these errors on upper and lower limbs will balance one another.

For the sun, the correction for parallax will have to be applied.

Balancing of Observations.—Any error in the assumed refraction will affect the computed latitude by the amount of the said error. This error can be partially eliminated or reduced, in the case of star observations, by observing two stars, one north and one south of the zenith and at as nearly as possible the same altitude. The errors in the deduced latitudes arising from the error in the assumed refraction will then be equal, but of opposite sign, in the two cases, and the mean of the two deduced latitudes will be free of refraction errors.

For, let z_1 and z_2 be the observed zenith distances affected with refraction;
r_1 and r_2 the assumed refractions;
r the error in refraction being equal in the two cases, provided that z_1 and z_2 are nearly equal;
δ_1 and δ_2 the declinations at the moment of observation;

then (z_1+r_1-r) and (z_2+r_2-r) are the true zenith distances, and the latitude is $(z_1+r_1-r)+\delta_1$, or $-(z_2+r_2-r)+\delta_2$;

or, taking half the sum, latitude $=\dfrac{(z_1+r_1)+\delta_1-(z_2+r_2)+\delta_2}{2}$.

Now the latitude deduced from the first star observed is

$$\phi_1 = z_1 + r_1 + \delta_1,$$

and the latitude deduced from the second star is

$$\phi_2 = -(z_2 + r_2) + \delta_2$$

and the mean is $\dfrac{(z_1+r_1)+\delta_1-(z_2+r_2)+\delta_2}{2}$ which is the expression just deduced for the true latitude, i.e. the refraction error has been eliminated by observing two stars, one north and one south of the zenith and of nearly equal zenith distances.

If a constant instrumental error exists in the altitudes given by the instrument, such as an index error, this has, of course, been eliminated by the described procedure, as each star has been observed on both faces; but if two stars, one north and one south of the zenith, are to be observed, it will be sufficient to observe each star on one and the same face, and to take the mean of the deduced latitudes; in this case each of the deduced latitudes will be affected with the index error, but with opposite signs in the two cases; the mean will then be the correct latitude.

It frequently happens that latitudes determined from observations north and south of the zenith differ by an appreciable amount, however carefully made. This is accounted for by droop or flexure, which is present in most theodolites. Its effect is not eliminated by reversal of face, but is, however, eliminated by balancing observations on both sides of the zenith.

The stars for observing should be selected beforehand, and should be such that the interval separating their transit is not more than 10 or 15 minutes, if possible, so that there may be little or no change in the refraction for the two observations.

This method can be relied on to give a result correct to within a few seconds with a field theodolite reading to 20″. For more accurate work a special instrument called the *zenith telescope* having a micrometer eyepiece is used; two stars which transit at nearly the same time and at very nearly equal altitudes, one north and one south of the zenith, are observed at transit on one face of the instrument, the difference in their altitudes being measured by the micrometer eyepiece without reference to circle readings, from which difference the latitude can readily be deduced. The method is known as *Talcott's method*, having been first used by Captain Talcott.

Latitude from Polaris.—An alternative method for finding latitude is by an altitude of the Pole Star. This method has the advantage that it can be adopted at any time when the Pole Star (Polaris) can be seen in the theodolite, without waiting for the transit of any particular star. The method consists in observing the altitude of Polaris and noting the clock time of the observation, from which the star's hour angle can be deduced, and the latitude computed by an approximate formula, applicable only to stars of small polar distance, which is given below.

In observing, the usual procedure for determining a star's altitude and the corresponding clock time is followed, viz. a pointing on one face, two on the other face, and finally one on the original face, the clock time being recorded for each of the four pointings. The micrometer or vernier readings for each pointing are corrected for level error, and the mean of the four altitudes is taken as being the observed altitude at the mean of the clock times. The level correction may, of course, equally well be applied to the mean of the circle readings, in one operation, which is simpler.

This observed altitude is corrected for refraction to obtain the true altitude at the instant of observation; the clock time is corrected for the known error of the clock, and the star's hour angle deduced from the right ascension given in the *N.A.* and the longitude, which will be known at least approximately.

Let z be the true zenith distance of Polaris,
p ,, polar distance of Polaris,
c ,, colatitude of the station,
t ,, hour angle.

The latitude ϕ of the place of observation will differ from the altitude h of Polaris by a quantity which may be called x, and which will in no case exceed the polar distance p of Polaris.

Then $$\phi = h + x.$$

Then in the triangle ZPS (Fig. 13) from equation (1)

$$\cos z = \cos p \cos c + \sin p \sin c \cos \text{ZPS},$$
$$\text{or } \sin h = \sin \phi \cos p + \cos \phi \sin p \cos \text{ZPS}$$
$$= \sin (h+x) \cos p + \cos (h+x) \sin p \cos \text{ZPS}.$$

The angle ZPS will be equal either to t or $-t$.

$$\therefore \quad \sin h = \sin (h+x) \cos p + \cos (h+x) \sin p \cos t.$$

The solution of this equation is given on page 47, where it is shown that

$$x = -p \cos t + \tfrac{1}{2}(p \sin t)^2 \tan h,$$

which is the correction to be applied to the altitude h to obtain the latitude ϕ.

In this equation the angle p is in circular measure, i.e. in radians, and the resulting value of the correction x will also be in radians, which is inconvenient.

To obtain the value of x in seconds of arc, let n be the number of seconds of arc in one radian. Then, since 1″ is a very small angle, sin 1″ = numerical value of 1″ in radians $= \frac{1}{n}$.

$$\therefore \quad \frac{x''}{n} = -\frac{p''}{n} \cos t + \tfrac{1}{2}\left(\frac{p''}{n}\right)^2 \sin^2 t \,.\, \tan h.$$

$$\therefore \quad x'' = -p'' \cos t + \tfrac{1}{2}(p'' \sin t)^2 \tan h \sin 1''.$$

The expression for the latitude ϕ then becomes

$$\phi = h - p \cos t + \tfrac{1}{2}(p \sin t)^2 \tan h \,.\, \sin 1'',$$

where p is in seconds of arc. The angle t is most conveniently taken the shortest way from upper transit; e.g. if the computed hour angle is 18h 25m 35s, the value of t used in the formula will be the equivalent in angular measure of 24h−18h 25m 35s, i.e. 5h 35m 25s × 15, or 83° 51′ 15″.

It can be shown that in the case of Polaris, the polar distance of which is about 1° 5′, the error due to admitting the approximation as used in deriving the above expression for x cannot amount to as much as 1″ of arc. The formula can therefore be used for any latitude determination in which greater accuracy is not required, but in low latitudes the uncertainty of the amount of refraction at the low altitude of Polaris in such latitudes renders the method unsuitable for accurate work. It is, however, susceptible of considerable accuracy in latitudes higher than about 35° N. The method is less useful for field work in the southern hemisphere, as the stars in the neighbourhood of the south celestial pole are not sufficiently bright to be picked up readily.

Example—

OBSERVATION ON POLARIS FOR LATITUDE

Clock Times. P.M.	Face	Level. E	Level. O	Vertical Circle.				Mean of Circle.
h m s								
4 32 30	FL	20·8	22·8	51° 26′ 51″	26′ 56″	25′ 22″	25′ 27″	51° 26′ 09″
4 36 33	FR	22·0	21·7	51 27 09	27 12	28 31	28 37	51 27 52
4 38 54	FR	22·0	21·7	51 27 47	27 49	29 15	29 21	51 28 33
4 42 32	FL	20·0	24·0	51 29 42	29 40	28 08	28 09	51 28 55
4 | 150′ 29″		84·8	90·2					4 | 111′ 29″
4h 37m 37s·2								51° 27′ 52″·2

The mean of the circle readings, viz. 51° 27′ 52″·2, is affected with the level error, and a level correction, as explained on page 102, has to be applied to obtain the observed altitude.

The value of a level division for the particular theodolite used was 10″; accordingly the level correction is

$$\frac{90{\cdot}2-84{\cdot}8}{8}\times 10'' = 6''{\cdot}7$$

and is positive for altitudes, since ΣO is greater than ΣE.

The correction for refraction, taken from tables of mean refraction, is 45″ (for a temperature of 50° F. and pressure of 29·6 in.).

Accordingly, mean circle reading is 51° 27′ 52″·2
level correction + 00 00 6·7
refraction − 00 00 45

True altitude = 51° 27′ 13″·9

Next, taking the mean clock reading, the hour angle of Polaris has to be evaluated for that instant.

A correction for the clock error on G.M.T. is applied, and the hour angle computed thus:

	h	m	s
Clock (mean time) =	4	37	37·2
error, fast =	0	00	37
G.M.T. =	4	37	00·2
S.T.G.M.N. =	14	50	56·2
add for S.T. =	00	00	39·426
	00	00	06·078
G.S.T. =	19	28	41·904
long. west =	00	00	41·5
L.S.T. =	19	28	00·4
R.A. =	1	36	31
Hour angle, H.A. =	17	51	29·4 West
or =	6	08	30·6 East

The value of the R.A. is that given in the *N.A.* for the date of the observation. The declination, from the same source, is 88° 55′ 04″, therefore the polar distance, p, is 1° 04′ 56″ = 3896″.

t = 6h 08m 30s·6 or 92° 07′ 39″	log $p = \bar{3}{\cdot}59062$	log $p = \bar{3}{\cdot}59062$
	log cos $t = \bar{2}{\cdot}56965\ n$	log sin $t = \bar{1}{\cdot}99970$
	log 1st correction = $2{\cdot}16027\ n$	$\bar{3}{\cdot}59032$
		$\times 2 = \bar{7}{\cdot}18064$
	1st correction = 144″·6+	log tan $h = 0{\cdot}09867$
	= 2′ 24″·6+	log sin 1″ = $\bar{6}{\cdot}68557$
		log ½ = $\bar{1}{\cdot}69897$
		= 1·66385
		2nd correction = 46″·1+

h = 51° 27′ 13″·9
1st correction = +2′ 24″·6
2nd " = + 46″·1

= 51° 30′ 24″·6

which is the latitude deduced from the observation.

It will be noticed that the letter n is written after the value of log cos t. This is to indicate that the quantity, viz. cos t, of which it is the logarithm, is negative.

It may here be observed that to obtain a result correct to one second, five-figure logarithms are, in general, sufficient, but where greater accuracy is required, seven-figure logarithms should be used.

Use of Pole Star Tables.—Both the *Nautical Almanac* and the corresponding American publication, the *American Ephemeris*, give Pole Star tables for facilitating the reduction of observations on Polaris for latitude, by dispensing with the computation.

The Tables in both publications are based on the equation given on page 117, but they are constructed somewhat differently. In the *N.A.*, the value of the 1st correction, viz. $-p \cos t$, is given in Table I for values of t increasing by 2 minutes of S.T. from 0h to 24h. The quantities given in this table have been computed for a mean value of p; p is, however, not in fact constant, as the position of the celestial pole varies on account of precession and nutation.

The quantities in Table I, therefore, require a small correction, given in Table III and called the 3rd correction, to reduce them to the true values corresponding to the actual values of the polar distance and R.A. of Polaris on the date of the observation.

The 2nd correction is given in Table II.

The quantities for the 3rd correction would be negative in some cases; but in order that the tabular correction may in all cases be positive, Table III has been constructed in such a way that the observed altitude is to be reduced by 1 minute of arc before application of the corrections.

Application of the tables to the observation just computed, which was taken on 4 November 1927, gives

Altitude reduced by 1 minute	51°	26′	13″·9
1st correction (for L.S.T. 19h 28m 00s)	0	2	16
2nd ,,	0	0	46
3rd ,,	0	1	09
Latitude	51	30′	24″·9

The tables do not give such accuracy as the computation, but even if the degree of accuracy is not sufficient for the purpose of the observation, they afford a ready means of checking the computation for any large error.

An observation on Polaris for latitude ought, if convenient, to be combined with a meridian observation of a star south of the Zenith for the reasons already explained, viz. that errors in the assumed refraction are largely reduced and any instrumental errors eliminated.

For the reduction of the observation it is necessary to know the L.S.T. in order to compute the hour angle, t, of the star at the instant of observation. The clock will give the standard meridian time, whether sidereal or mean, of the instant of observation; the longitude will be known at least approximately, and may be applied to the standard meridian time to obtain the local time.

A small error in the assumed local time will not affect the computed latitude to an appreciable extent, but in any case the computed latitude will be sufficiently near the true latitude to enable the observer to take observations for local sidereal time by the methods to be described in the chapter on time determinations, and the longitude will then be found from the difference between the local time and the standard meridian time. This longitude may then be used for a more accurate determination of latitude. In fact, the observations form a series of successive approximations, and this applies to most, if not to all astronomical observations, the number of the observations in the series increasing with the accuracy required.

Latitude by Circum-Meridian Altitudes.—The method and principle of finding the latitude by the meridian altitude of a star have already been explained. It is possible, however, to obtain a more accurate and more reliable determination by observing a series of altitudes of a star, with the corresponding clock times, for a few minutes before and after the transit or culmination of the star, and applying a correction to each altitude to reduce or correct it to the altitude the star would have at its transit, the hour angle corresponding to each observed altitude being known. In other words the altitude of the star is observed at a known small hour angle, and a correction corresponding to the hour angle is applied, the amount of the correction representing the difference between the star's altitude at the instant of observation and its meridian altitude.

From each separate pointing on the star, with the corresponding known hour angle, the latitude can be computed, and the mean of the series is taken as being the latitude deduced from the observation.

The expression for the amount of the correction, or the *reduction to the meridian*, is given on page 51 and is

$$\frac{\cos\phi\cos\delta}{\cos h}\times\frac{2\sin^2\frac{t}{2}}{\sin 1''}$$

The factor $\frac{2\sin^2\frac{t}{2}}{\sin 1''}$ depends only on the hour angle t, and its value can be tabulated for values of t advancing by one or more seconds. Such a table will be found in Appendix No. VI.

In the factor $\frac{\cos\phi\cos\delta}{\cos h}$, δ is known, and h is known from the observation. ϕ is, however, not known, being in fact the latitude which it is desired to obtain. An approximate value of ϕ is, however, obtained by treating h as a meridian altitude, from which ϕ is derived by the formula $\phi-\delta=90°-h$. The best procedure is therefore to take the pair of consecutive pointings which give the highest altitude, to treat this highest altitude as h in the factor $\frac{\cos\phi\cos\delta}{\cos h}$, and the value of ϕ is then got from $\phi-\delta=90°-h$.

Accordingly for all the pointings the value of $\frac{\cos\phi\cos\delta}{\cos h}$ is constant.

In proceeding to determine a latitude by this method, a star is selected which will transit at a convenient time, and at a convenient altitude, say about 30° to 50°. Pointings on the star are taken from about 10 minutes before transit to about the same interval after transit, and the altitudes and corresponding clock times booked. The pointings are best taken alternately face right and face left, but face may be changed after each two pointings if preferred.

If h_1 be the first observed altitude, after correction for refraction, the hour angle being then t_1, the corresponding meridian altitude h is

$$h=h_1+\frac{\cos\phi\cos\delta}{\cos h}\times\frac{2\sin^2\frac{t}{2}}{\sin 1''}$$

$\frac{\cos\phi\cos\delta}{\cos h}$ is constant and may be represented by A.

$\frac{2\sin^2\frac{t}{2}}{\sin 1''}$ depends only on the hour angle t_1 and may be represented by m_1.

$$\therefore\quad h=h_1+\mathrm{A}m_1$$

Similarly $h=h_2+\mathrm{A}m_2$

and $h=h_3+\mathrm{A}m_3$

or $h=h_n+\mathrm{A}m_n$

The meridian altitude corresponding to each pointing is thus computed, and if there were no instrumental or observing errors these meridian altitudes ought all to be equal. Their mean is taken as being the true meridian altitude, and from it the latitude ϕ is deduced by the formula $\phi-\delta=90°-h$. Such a series of circum-meridian altitudes gives a more accurate result than a single observation on the meridian, or a pair of observations one on each side of the meridian.

Any pointing which gives a value of h differing greatly from the mean may be rejected as unreliable. The values of m, i.e. $\frac{2\sin^2\frac{t}{2}}{\sin 1''}$ are, as already stated, most conveniently taken from tables (Appendix VI), t being for each pointing the actual hour angle at the instant of pointing.

Accordingly, if a star is being observed, t is the sidereal interval between the instant of pointing and the transit. If a mean time clock is being used, the clock time of transit is first computed, and the clock time of each pointing booked. The intervals obtained by subtraction will be mean time intervals, from which the corresponding sidereal intervals must be computed or taken from tables, to obtain the value of t for each pointing. It will usually be sufficiently accurate to perform the reduction to sidereal intervals by adding 10 seconds per hour to the mean time intervals, the accurate quantity being 9·8565 seconds per hour. An equal number of pointings should, if possible, be taken before and after transit; if this be done, the effect of a small unknown error of the clock on local time will be reduced.

If the observation is being done on the sun, the clock time of transit, being local apparent noon, L.A.N., is first computed. The clock will probably be a mean time clock, and the sun's hour angle for any pointing will then be given directly by the difference between the clock time of the transit and the clock time of the pointing. It should be mentioned here that the sun's hour angle at an instant separated by a mean time interval of t from transit is not exactly t, as the equation of time has, in general, changed during the interval t, but the change in the few minutes interval will in all cases be so small as to be negligible in field work.

The declination to be taken in the case of the sun is, for each pointing, the actual declination at the instant of that pointing, but it will be sufficiently accurate to take for all the pointings the value of the declination at the mean of the times of the pointings. The pointings should preferably be done half on the upper limb and half on the lower limb, as any error in making the contact between the limb and the horizontal wire will probably be of the same kind in the two cases, and the error will cancel out in taking the mean.

Observation for Latitude by Circum-Meridian Altitudes.

STAR OBSERVED γ^2 CETI

R.A. 2h 39m 33s·5
δ 2° 55′ 54″·2 N.

Clock (M.T.)	Face	Level E	Level O	Vertical Circle		Mean Altitude Readings	Level	Observed Altitude
P.M. h m s				° ′ ″	′ ″	° ′ ″	″	° ′ ″
6 27 38	R	7·0	6·5	47 56 00	56 00	42 04 00	− 5	42 03 55
6 29 52	R	7·2	6·0	47 55 00	55 30	42 04 45	−12	42 04 33
6 33 32	L	7·1	6·0	42 07 00	07 30	42 07 15	−11	42 07 04
6 36 09	L	7·6	6·0	42 07 30	08 00	42 07 45	−16	42 07 29
6 39 39	R	6·2	7·0	47 53 30	54 00	42 06 15	+ 8	42 06 23
6 41 29	R	6·8	6·8	47 53 30	54 00	42 06 15	0	42 06 15
6 44 10·5	L	7·0	6·2	42 06 20	06 40	42 06 30	− 8	42 06 22
6 46 15	L	7·0	6·1	42 05 00	05 30	42 05 15	− 9	42 05 06

Value of 1 division of level = 20″.

	h m s
L.S.T. of transit =	2 39 33·5
Long. W. =	37·1
G.S.T. of transit =	2 40 10·6
S.T.G.M.N. =	20 02 24·3
Sidereal interval =	6 37 46·3
M.T. interval, 6h =	5 59 01·023
37m =	36 53·938
46s =	45·874
0s·3 =	·299
G.M.T. =	6 36 41·134
Clock fast =	1 15
Clock time of transit =	6 37 56

Take for $\frac{\cos\phi\cos\delta}{\cos h}$ best pair of F.R. and F.L. readings:

F.L. = 42° 07′ 29″
F.R. = 42 06 23

2 | 84 13 52

42 06 56   log cos $\phi = \bar{1}\cdot80040$

Refraction = 01 04   „ cos $\delta = \bar{1}\cdot99943$

h = 42 05 52 $= \bar{1}\cdot79983$

δ = 2 55 54   „ cos $h = \bar{1}\cdot87040$

c = 39 09 58   log A $= \bar{1}\cdot92943$

ϕ = 50° 50′ 02″   A = ·850

Clock	Face	M.T. int. to transit	S.T. int. to transit	m	Am	Altitude h	h+Am
h m s		m s	m s	"	"	° ′ ″	° ′ ″
6 27 38	R	10 18	10 19·7	209·3	178	42 03 55	42 06 53
6 29 52	R	8 04	8 05	128·3	109	42 04 33	42 06 22
6 39 39	R	1 43	1 43	5·8	4·9	42 06 23	42 06 28
6 41 29	R	3 33	3 33·6	24·9	21·1	42 06 15	42 06 36
6 33 32	L	4 24	4 24·7	38·2	32·5	42 07 04	42 07 36
6 36 09	L	1 47	1 47	6·2	5·2	42 07 29	42 07 34
6 44 10	L	6 14	6 15	76·7	65·2	42 06 22	42 07 27
6 46 15	L	8 19	8 20·3	136·4	115·9	42 05 06	(42 07 02)

	°	′	″
Mean F.R. =	42	06	34·7
,, F.L. =	42	07	32·3
2	84	14	07·1
	42	07	03·5
Refraction =		01	04
	42	05	59·5
δ =	02	55	54·2
c =	39	10	05·3
ϕ =	50	49	54·3

The meridian altitude (42° 07′ 02″) has been rejected, as deviating considerably from the other values. The advantage of computing the meridian altitude separately for each pointing is that a poor pointing such as the one for 6h 46m 15s can be rejected as deviating too far from the mean of all the pointings. Provided, however, that there is no particular reason to suspect that any pointing is a poor one, the reduction of the observation can be shortened somewhat by taking a mean value of the altitude readings, applying to it the corresponding level and refraction corrections, and adding to it the quantity A multiplied by the mean value of m for all the pointings. This procedure is correct mathematically, but does not permit of the rejection of a poor pointing.

For

$$h_m = h_1 + Am_1$$
$$h_m = h_2 + Am_2$$
$$h_m = h_3 + Am_3, \text{ etc.}$$
$$h_m = \frac{h_1 + h_2 + h_3 + \ldots}{\text{No. of pointings}} + A \cdot \frac{m_1 + m_2 + m_3 + \ldots}{\text{No. of pointings}}$$

i.e. h_m = mean value of h + A × mean value of m

or $h_m = h_0 + Am_0$

Applying this procedure to the present observation :

Clock (M.T.)	Face	Level E	Level O	Mean Circle Reading	M.T. int. to transit	S.T. int. to transit	m
h m s				° ′ ″	m s	m s	"
6 27 38	R	7·0	6·5	42 04 00	10 18	10 19·7	209·3
6 29 52	R	7·2	6·0	04 45	8 04	8 05	128·3
6 33 32	L	7·1	6·0	07 15	4 24	4 24·7	38·2
6 36 09	L	7·6	6·0	07 45	1 47	1 47	6·2
6 39 39	R	6·2	7·0	06 15	1 43	1 43	5·8
6 41 29	R	6·8	6·8	06 15	3 33	3 33·6	24·9
6 44 10·5	L	7·0	6·2	06 30	6 14	6 15	76·7
6 46 15	L	7·0	6·1	05 15	8 19	8 20·3	136·4
		55·9	50·6	8 | 48 00			8 | 625·8
				42° 06′ 00″			m_0 = 78·2

Level correction $= \frac{55{\cdot}9 - 50{\cdot}6}{16} \times 20 = -6''{\cdot}6$ A = ·85

Am_0 = 66″·5

Refraction =		−1	04
Total correction =		−1	10·6
Mean observed altitude =	42	06	00
h_0 =	42	04	49·4
Am_0 =			66·5
$h_0 + Am_0$ =	42	05	55·9
δ =	2	55	54·2
c =	39	10	01·7
ϕ =	50°	49′	58″·3

The reduction by this method is accordingly somewhat shorter, but the longer method, viz., by reducing each pointing separately, is to be preferred if time permits of its use.

Methods for determining longitude are given in Chapter IX.

CHAPTER VII

DETERMINATION OF LOCAL TIME

General.—The determination of local time is frequently necessary in the operations of field astronomy and navigation. The determination of the longitude of an observing station resolves itself into a determination of the difference between local time and Greenwich or other standard meridian time, as explained. Conversely, if the longitude of a station is known accurately, an observation for local time will, when corrected for longitude of the station, give the Greenwich time at the instant of observation, and the difference between this and the chronometer time of the observation gives the error of the clock on Greenwich time. It is to be understood that in all cases what is meant by determination of time is in reality determination of the clock error on local time, and a chronometer or clock is an essential part of the equipment.

Time by a Single Altitude.—The method in general use in field astronomy consists in measuring with a theodolite the altitude of a star and noting the corresponding clock time. At any definite station, any star reaches a particular altitude at two definite instants of local sidereal time, one instant when the altitude is increasing and the other when the altitude is decreasing. The transit of the star occurs at the middle of the interval between these two instants, and the hour angles of the star at the two instants are such that their sum is 24 hours.

According to equation (5),

$$\cos P = \cos z \sec \phi \sec \delta - \tan \phi \tan \delta$$

z is determined by the observation,
ϕ is known for the station,
and δ is known from the *N.A.*
P is then computed by one of the methods given on pages 43 to 45.

Selection of Stars.—The selection of suitable stars for time observations is based on the following considerations. It is obviously necessary that the star should be changing its altitude rapidly, in order that the instant of its intersection by the horizontal wire of the telescope may be noted with precision; a star the altitude of which is changing slowly will remain for an appreciable time intersected by the wire and is unsuitable for time observations. Further, as the latitude of the station enters into the formula by which the hour angle is determined, and as the latitude may not be known with exactitude, stars should be selected such that an error in the assumed latitude will produce the least possible error in the local time deduced from an observation of the star's altitude. It will now be shown that these conditions are best fulfilled by stars in the prime vertical.

The first condition as regards suitability for time observations is, as stated, that the star's altitude, and therefore also its zenith distance, should be changing rapidly. The rate of change of zenith distance is $\frac{dz}{dt}$, which is therefore to be a maximum.

From equation No. 1,

$$\cos z = \cos p \cos c + \sin p \sin c \cos P.$$

The star's polar distance p is practically constant for long periods, and may be treated as constant for the purpose of any one observation. The colatitude c is constant for any one station. The variables are the angle P and the zenith distance z. P changes uniformly with the time, i.e. $\frac{dP}{dt}$ is constant, being in fact 15° per hour, or 15″ of arc per sidereal second in the case of a star.

Differentiating the above equation with respect to t,

$$-\sin z \frac{dz}{dt} = 0 - \sin p \,.\, \sin c \,.\, \sin P \,.\, \frac{dP}{dt},$$

$$\sin z \frac{dz}{dt} = \sin p \sin c \sin P \times \text{a constant},$$

$$\frac{dz}{dt} = \frac{\sin p \sin c \sin P}{\sin z} \times \text{constant};$$

$$\text{but } \frac{\sin p}{\sin Z} = \frac{\sin z}{\sin P}, \text{ or } \sin p \sin P = \sin z \sin Z,$$

$$\therefore \frac{dz}{dt} = \frac{\sin z \,.\, \sin Z \,.\, \sin c}{\sin z} \times \text{constant},$$

$$= \sin Z \sin c \times \text{constant}.$$

The value of the constant $\frac{dP}{dt}$ is 15″ of arc per second of time; accordingly, if δt represent a short interval of time, in seconds, and δz the change in zenith distance, in seconds of arc, during that interval,

$$\delta z'' = 15 \sin Z \sin c \;\delta t \text{ in seconds of time.}$$

As $\sin c$ is constant, $\frac{dz}{dt}$ is obviously a maximum when $\sin Z$ has its maximum value, i.e. when $Z = 90°$, i.e. when the star is on the prime vertical.

The zenith distance, and therefore also the altitude, of a star is therefore changing most rapidly when the star is on the prime vertical.

From the equations it is seen that all stars in the same vertical circle have the same rate of change of altitude, and stars in the prime vertical have the greatest rate of change of altitude.

If the above expression be written

$$\delta t \text{ in seconds of time} = \frac{\delta z''}{15 \sin Z \sin c}$$

it is clear that a small error in the measurement of z produces least effect on the resulting value of t when $\sin Z$ is a maximum, i.e. when $Z = 90°$.

Taking now the second condition for suitability for time observations, viz. that the effect of a small error in the assumed latitude on the resulting computed time is to be a minimum, we have, as before,

$$\cos z = \cos p \cos c + \sin p \sin c \cos P.$$

Assuming now that the zenith distance z has been correctly observed, z and p are constant, and the variables are c and P.

Differentiating with respect to c,

$$0 = -\cos p \sin c + \sin p\left(\cos c \cos P - \sin c \sin P\frac{dP}{dc}\right)$$

$$\text{or} \quad \cos p \sin c - \sin p \cos c \cos P = -\sin p \sin c \sin P\frac{dP}{dc} \ldots (a)$$

Applying the cot formula to the four consecutive parts p, P, c, Z,

$$\cos c \cos P = \sin c \cot p - \sin P \cot Z.$$

$$\therefore \quad \cos c \cos P = \sin c \frac{\cos p}{\sin p} - \sin P \cot Z$$

$$\therefore \quad \sin P \cot Z = \frac{\sin c \,.\, \cos p - \cos c \,.\, \sin p \,.\, \cos P}{\sin p}$$

$$\therefore \quad \sin c \cos p - \cos c \sin p \cos P = \sin p \sin P \cot Z.$$

$\therefore$ from equation (a) above,

$$\sin p \sin P \cot Z = -\sin p \sin c \sin P\frac{dP}{dc}$$

$$\text{or} \qquad \frac{dP}{dc} = -\frac{\cot Z}{\sin c},$$

which is a minimum, and is zero, when $\cot Z = 0$.

Now the meaning of $\frac{dP}{dc}$ being a minimum is that for a small change in c, the resulting change in P is a minimum, and as P is the hour angle (east or west), the resulting computed value of the local time has the least error when cot $Z=0$, i.e. when $Z=90°$. The error will, in fact, be 0 if Z is exactly 90°, i.e. if the star is observed on the prime vertical. The error is clearly a minimum when sin c has its maximum possible value, viz. 1, when $c=90°$, i.e. when the observing station is on the equator.

Accordingly, for both the reasons mentioned, the best stars to select for time observations are those on or near the prime vertical.

Balancing by East and West Stars.—Apart from observing errors, an error in the assumed value for the refraction will have a direct effect on the corrected altitude, and therefore also on the deduced local time. For instance, if an east star is being observed, and too high a value be assumed for the refraction, the corrected altitude will be lower than the true altitude, and the computed hour angle, measured eastwards from the meridian, will be too great. The deduced instant of observation will therefore be earlier than the true instant of observation, being in fact the instant corresponding to a lower altitude than the true one. If now a west star be observed at about the same altitude, and if, as is likely, too high a value be assumed for the refraction in this case also, the computed hour angle, measured westwards from the meridian, will also be too great. The deduced instant of observation will therefore, corresponding as it does to too great an hour angle, be later than the true instant of observation. An error in the assumed refraction is therefore largely eliminated by taking an observation on both an east and a west star; and this should be done in all cases, if convenient. Instrumental errors such as droop are also eliminated by this procedure.

An example of a time observation, taken on 19 January 1928, in latitude 51° 29′ 57″ N., on east and west stars, with a 6-in. micrometer theodolite reading direct to 10″, and by estimation to 1″, altitude level reading 10″ per division, using a sidereal clock, follows:

WEST STAR—VEGA (α LYRAE) R.A. 18h 34m 27s·8
δ 38° 42′ 47″ N.

Sidereal Clock.	Face	Level. E	Level. O	Vertical Circle.				Mean.
h m s								
23 18 49·5	FL	22·2	20·0	40° 10′ 55″	10′ 51″	9′ 44″	9′ 48″	40° 10′ 19″·5
23 21 54·5	FR	22·3	22·0	39 43 48	43 36	42 10	42 18	39 42 58 ·4
23 24 23	FR	21·8	23·0	39 21 20	21 24	19 54	19 53	39 20 37 ·5
23 27 55·5	FL	23·2	21·1	38 48 04	48 01	49 09	49 15	38 48 37
4 | 93 02·5								4 | 158 02 32 ·4
23 23 15·6								39° 30′ 38″·1

$10'' \times \left(\frac{89·5-86·1}{8}\right) =$ −4″·2 level
−1′ 10″ refraction
−1′ 14″·2 | −1 14 ·2

$h=$ 39° 29′ 24″
$z=$ 50 30 36
$c=$ 38 30 03
$p=$ 51 17 13

$2s=$ 140 17 52
$s=$ 70 08 56
$s-p=$ 18 51 43
$s-c=$ 31 38 53

$$\text{Sin}\,\frac{P}{2}=\sqrt{\frac{\sin(s-p)\sin(s-c)}{\sin p \sin c}}$$

logs

$\sin(s-p)$	$\bar{1}·5095908$	$\sin p$	$\bar{1}·8922548$
$\sin(s-c)$	$\bar{1}·7199111$	$\sin c$	$\bar{1}·7941575$
$\frac{1}{\sin p \sin c}$	$0·3135877$		$\bar{1}·6864123$
	2 | $\bar{1}·5430896$		

log sin $\frac{P}{2}=\bar{1}·7715448$

$\frac{P}{2}=36° \, 13' \, 26''$

$P=72° \, 26' \, 52''$

	h	m	s
$t=$	4	49	47·5
R.A. =	18	34	27·8
L.S.T.	23	24	15·3
Clock	23	23	15·6
Slow	00	00	59·7 on L.S.T.

EAST STAR (ι TAURI) R.A. 4h 58m 47s·7
δ 21° 29′ 20″ N.

Sidereal Clock.	Face	Level. E	Level. O	Vertical Circle.				Mean.
h m s								
00 44 50·5	L	23·1	21·1	33° 13′ 49″	13′ 46″	12′ 47″	12′ 53″	33° 13′ 18″·7
00 49 36	R	22·6	22·0	33 57 47	57 51	58 21	58 20	33 58 05
00 53 07·5	R	22·8	22·0	34 30 45	30 50	30 14	30 13	34 30 30 ·5
01 00 28·5	L	21·1	23·3	35 38 01	37 59	36 44	36 40	35 37 21
4 | 3 28 02·5								4 | 137° 19′ 15″·2
00 52 00·6								34° 19′ 48″·8

$10'' \times \left(\frac{89 \cdot 6 - 88 \cdot 4}{8}\right) = -1''\cdot 5$ level

$-1' 23''$ refraction

$-1' 24''\cdot 5$

— 1 24 ·5

$h = 34° 18' 24''\cdot 3$
$z = 55\ 41\ 35\ \cdot 7$
$c = 38\ 30\ 03$
$p = 68\ 30\ 40$

$2s = 162° 42' 18''\cdot 7$

$s = 81\ 21\ 09\cdot 3$

$s - p = 12\ 50\ 29\cdot 3$

$s - c = 42\ 51\ 06\cdot 3$

logs
sin $(s-p)$ $\bar{1}\cdot 3468501$ sin p $\bar{1}\cdot 9687111$
sin $(s-c)$ $\bar{1}\cdot 8325752$ sin c $\bar{1}\cdot 7941575$

$\frac{1}{\sin p \sin c}$ 0·2371314 $\bar{1}\cdot 7628686$

2 | $\bar{1}\cdot 4165567$

log sin $\frac{P}{2}$ $\bar{1}\cdot 7082783$

$\frac{P}{2} = 30° 43' 09''\cdot 5$
P = 61 26 19
h m s
= 4 05 45·3 East

$t = 19\ 54\ 14\cdot 7$ West
R.A. = 4 58 47·7

L.S.T. = 00 53 02·4
Clock = 00 52 00·6

Slow = 00 1 01·8

m s
West star = 00 59·7 slow
East star = 01 01·8 ,,

Mean 1 00·7 ,, on L.S.T.

Note.—The star being faint in twilight accounts for the delay in picking up for each pointing.

CHAPTER VIII

DETERMINATION OF AZIMUTH

General.—In making a survey, other than a compass survey, covering a small extent of ground, it is usual to observe the *magnetic bearing* of one line of the survey, i.e. the inclination or direction of that line referred to the magnetic north and south line. This magnetic bearing is not used except to plot on the map of the ground the direction of magnetic north; generally a line is also drawn inclined to the direction of magnetic north at an angle equal to the *declination* or *variation* of the compass, and therefore showing the direction of geographical north. This declination or variation is not constant, but for any one place it is subject to periodic changes, both of short and of long periods; further, its amount differs at different places, the lines of equal declination being curved in a rather irregular manner. In London, the magnetic compass pointed due north about the year 1657; the declination then became westerly, and reached a maximum of about 24° in 1816; since then it has been decreasing, the declination being at present (1928) about 14° W., with an annual decrease of about 10′. Apart from this long-period change, the declination has also changes of shorter periods, of respectively eleven years, one year, and one day. Admiralty charts show the amount of the declination for the area covered by the chart, together with its rate of annual change.

In view of the perturbations to which it is subject, the magnetic compass is not to be regarded as a means of determining direction with precision, and the direction of the geographical meridian deduced from it by correction for declination is liable to considerable error. In addition there is, in all probability, an error due to the magnetic axis of the needle not coinciding with its axis of figure.

For the purposes for which a survey of a small area is usually undertaken, an accurate determination or plotting of a north and south line is unnecessary, and the accuracy of the survey itself is not affected.

When a survey over an extended area is undertaken, the directions with reference to one another of the various lines forming the network of the survey are determined by angular measurements, as, for example, in triangulation or traversing. The lengths of the lines are either computed or measured, and, their lengths and relative angular positions being known, the network can be plotted, due regard being paid to the figure of the earth. For the details of the method, the reader is referred to any standard work on Geodetic Surveying.

It will obviously be necessary to fix the absolute position of this network on the earth's surface by the determination of the latitude and longitude of one point or more, and by the determination of the directions of one line or more from such a point referred, for convenience, to the geographical meridian of that point. Such a direction is the azimuth of the line and is the horizontal angle between the meridian plane through the point and the vertical plane at the point and containing the line. Methods for the determination of azimuth for this and other purposes will now be described.

It will be clear on reference to Fig. 5 that for any definite latitude, at any definite instant of local time, both the altitude and azimuth of any star have definite values, and vary continuously with the time; consequently if any one of the three quantities time, altitude, or azimuth be known, the remaining two can be computed by solving the spherical triangle concerned; it being remarked that for any value of the altitude above the horizon there are two

corresponding values for the time, at equal intervals before and after the time of transit of the star.

It is thus possible to compute the azimuth of a star from a knowledge of either the L.S.T. or the star's altitude, and either quantity may be used as the basis of an observation.

The observation in either case consists essentially in observing the intersection of the star by the vertical wire of the theodolite, taking the reading of the horizontal circle and comparing it with the reading of that circle when the vertical wire of the telescope is directed to the terrestrial object the azimuth of which is to be determined.

To provide the data necessary for the computation of the star's azimuth at the instant of intersection, either the clock time of the intersection or the altitude at intersection has to be noted, as has been explained. According to whichever of these procedures is adopted, the method may be called *azimuth by hour angle* or *azimuth by altitude*.

The former method is to be preferred in general, for the reason that the observation is simpler; the computation is, however, rather longer. As it is probable that the observation will have to be done after dark, it is necessary to arrange for the illumination of the terrestrial object the azimuth of which is to be determined; this object is usually called the *reference mark* (*R.M.*) or *reference object* (*R.O.*). Any convenient box having a vertical slit about $\frac{1}{8}$ in. to $\frac{1}{2}$ in. wide, according to the distance at which it is to be placed, and a lamp which can be set inside the box, is all that is necessary. The wires of the diaphragm will, of course, also have to be illuminated, as in other night observations. A great deal of this inconvenience may be avoided, and the observations taken while there is still sufficient daylight to point accurately on the R.M., by making a rough determination of the meridian by sun observations of the same kind as the star observations under discussion. The sun observation, although less accurate than star observations, will give a result of a fair degree of accuracy, by means of which the star can be found and observed before dark, and greater accuracy obtained than from the preliminary sun observation. In the determination of azimuth it is assumed that the latitude and longitude of the observing station are known.

Azimuth by a Star at Elongation. — An azimuth observation may be made on any star, but there are certain circumstances which will influence the observer in his selection. The most obvious of these is the condition that at the time of the observation the star should not be moving rapidly in azimuth, otherwise a small error in the assumed time of the observation may produce a considerable error in the computed azimuth. It is therefore advisable to select a star which has the slowest possible motion in azimuth. Further, as the latitude of the observing station enters into the computation, the star selected should be such that an error in the assumed latitude produces a minimum error in the resulting azimuth. It will now be explained that certain stars, at two particular points in their diurnal paths, have no motion at all in azimuth, their motion being at those points entirely vertical.

Referring to Fig. 29, which represents the celestial sphere, the apparent daily path of a star S in the vicinity of the elevated celestial pole P is the small circle shown as an ellipse. The star will always be in an approximately northerly or southerly direction, according to whether the latitude is north or south; if its motion be followed with the telescope of the theodolite from the instant of its lower transit, when it has no motion in altitude, the star will be seen to be moving eastwards in azimuth and upwards in altitude. When the star has reached a position such as S, the vertical plane containing the axis of the telescope is in the position ZS in Fig. 29 (*b*), i.e. the azimuth of the star is the angle PZS. As the motion of the star progresses, its azimuth increases, i.e. the angle PZS increases, until after the lapse of some hours from the instant of lower transit the telescope will have been turned in azimuth until the line ZS is tangential at *e* to the ellipse representing the

path of the star. At that instant the star has no motion in azimuth, its motion being entirely vertical. The star is

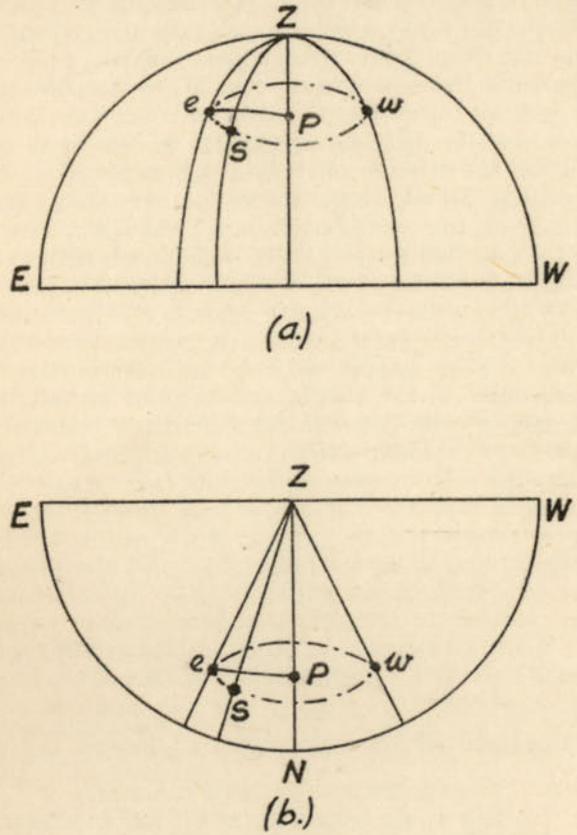

FIG. 29.

said to be then at *eastern elongation*. Similarly, the other tangent from Z to the ellipse at *w* gives the azimuth of the star at *western elongation*; and if it were possible to see the star during the whole sidereal day, the telescope of the theodolite would have to be swung about the vertical axis through an angle limited by the two tangents to the ellipse. For some time before and after each elongation, the motion of the star in azimuth is very slow: a star at or near elongation is therefore a very suitable object for observing for determination of true meridian, as during the interval required to take pointings on the star on both faces of the theodolite the azimuth of the star will not have changed appreciably. It will easily be seen that only stars fulfilling certain conditions will elongate at all. If a star be so far from the pole that the point Z falls within the ellipse representing the plan of its path, it will be impossible to draw a tangent to the ellipse from the point Z. This will be the case if the star's polar distance is greater than the co-latitude; the star's upper and lower transits will then be on opposite sides of the zenith, and in order to follow the star's motion with the theodolite during a sidereal day the telescope would have to be turned completely round the vertical axis. If the polar distance of the star were equal to the colatitude, the star would pass through the zenith at upper transit, and the point Z in the plan would lie on the ellipse representing the plan of the star's diurnal circle.

The hour angle at elongation is Z*Pe*. To determine its value, consider the spherical triangle of which the plan is PZ*e*. Z*e* being tangential to the circle described by the star, the angle P*e*Z is 90°. Applying the four-part or cot formula to the four consecutive parts ZP, ZP*e*, P*e*, and P*e*Z,

$$\cos Pe \cos ZPe = \sin Pe \cot ZP - \sin ZPe \cot PeZ.$$

Now $Pe = p$, the polar distance of the star,

$ZPe = t$, the hour angle of the star at elongation,

$ZP = c$, the colatitude,

and $PeZ = 90°$,

$\therefore \cos p \cos t = \sin p \cot c - 0.$

$\therefore \cos t = \tan p \cot c$

$= \cot \delta \tan \phi.$

From this equation the hour angle of the star at elongation is computed, and if this hour angle be applied with positive and negative signs to the R.A. of the star, which is the L.S.T. of its upper transit, the results are respectively the L.S.T. of western and eastern elongation.

The computation is made beforehand; suitable stars for both northern and southern latitudes are given in the *N.A.*, in which the apparent places of a number of close circumpolar stars are given, for both north and south celestial poles. In the northern hemisphere a very suitable star for azimuth observations is *Polaris*.

For some minutes before and after elongation the motion of the star in azimuth is so small that for approximate work it may be neglected. The rate of motion in azimuth obviously depends on the latitude of the station and on the declination of the star, being less in low than in high latitudes, and less for stars near the pole than for stars further from the pole.

The expression for the azimuth of the star at elongation is easily obtained by consideration of the spherical triangle PZ*e* in Fig. 29, Z*e*P being in this case a right angle.

$$\frac{\sin ZP}{\sin ZeP} = \frac{\sin Pe}{\sin PZe}$$

or

$$\frac{\sin c}{\sin 90°} = \frac{\sin p}{\sin PZe}$$

$$\therefore \quad \sin PZe = \frac{\sin p}{\sin c} = \cos \delta \sec \phi.$$

This expression for azimuth at elongation is identical with the correction for the sun's semi-diameter in azimuth given on page 86, viz.

$$\sin \delta A = \sin s \operatorname{cosec} z,$$

where s corresponds to the star's polar distance $= 90° - \delta$, and z corresponds to the colatitude $90° - \phi$;

whence $\sin \delta A = \sin (90° - \delta) \operatorname{cosec} (90° - \phi)$

$= \cos \delta \sec \phi.$

In making the observation the star is observed on the vertical wire, and it is of course necessary to take pointings on the star both F.R. and F.L. in order to eliminate such instrumental errors as are eliminated by that procedure. Consequently it is probable that neither the F.R. nor the F.L. pointing will have been made at the precise instant of elongation, but as the rate of change of azimuth is very slow for several minutes before and after that instant, the observation will give a result correct to within a few seconds of arc. If the instrument be provided with a striding level, it should be used for this as for all azimuth observations, to obtain the correction $b \tan h$ corresponding to the inclination of the horizontal axis.

To avoid the necessity for refocusing the telescope when the latter is redirected from the star to the R.M. or vice versa, the R.M. ought in all azimuth observations to be at as great a distance as can be arranged. This will also reduce the error due to inaccurate centring of the instrument over the observing station, as even a slight inaccuracy in setting up the instrument over the station may produce an appreciable difference in the azimuth of a near R.M. compared with its azimuth from the true position of the observing station. The R.M. should also preferably be near the horizon to obviate the necessity of applying the striding level correction to the horizontal circle readings on the R.M., which correction, viz. $b \tan h$, is zero on the horizon. The line of sight to the R.M. ought to be well clear of any buildings or other objects likely to cause lateral refraction due to local variations of temperature.

As at any instant other than that of elongation the angle PZS is less than at the instant of elongation, the readings of the horizontal circle will all lie on the one side of the true reading corresponding to the instant of elongation. When the computed azimuth at elongation is applied to the observed horizontal circle readings on the star to obtain the circle reading corresponding to the direction of the meridian, a small error in the latter will result, but the error will usually be so small as to be negligible provided that the polar distance of the star is small, and that all the pointings are made within a few minutes before or after elongation. If greater accuracy be required, a correction corresponding to the reduction to the meridian in latitude observations, and based on similar considerations, may be applied, but for most purposes the correction is unnecessary.

Azimuth by a Star at any Hour Angle. — Observations for azimuth may be made on a star at any hour angle, in which case it is necessary to have a knowledge of the clock error on L.S.T. as well as of the latitude. In the chapter on spherical trigonometry, on page 47 it was shown how the true azimuth of a star may be computed for any instant of L.S.T.

The procedure in observing is as follows. The telescope is directed, say F.R., to the reference mark, the azimuth of which is to be determined, and the reading of the horizontal circle is taken. The telescope is then directed to the star, and the latter is brought by the tangent screws to such a position that it will cross the vertical wire, near the horizontal wire, in a few seconds. The booker is warned to stand by at the clock, the observer removes his hand from the instrument, and at the instant when the star is intersected by the vertical wire he gives the signal 'Up' to the booker, who notes the clock time and then prepares to book the level and horizontal circle readings which the observer calls out to him. The observer then applies the striding level to the horizontal axis and gives the readings of the bubble ends, left and right; he then reverses the striding level end for end, and gives the corresponding left and right readings of the bubble ends. He then gives the readings of the horizontal circle, followed by an approximate reading to the nearest minute of the vertical circle, to be used in conjunction with the striding level readings for the determination of the correction. This completes the F.R. observation, and he changes face and points to the star F.L., allowing the star to make its intersection as before and giving the readings, followed by a pointing F.L. to the R.M.

The four pointings, face right and face left on star and R.M., may be treated as one observation, on the assumption that the mean of the horizontal circle readings on the star, each duly corrected for the inclination of the horizontal axis, corresponds to the circle reading at the mean of the noted clock times, which is equivalent to assuming that the star's motion in azimuth is uniform.

The computation is done by the formula given on page 49, viz.

$$\cot Z = \frac{\sin (c-x) \cot P}{\sin x}$$

where $\tan x = \tan p \, . \cos P$.

The most suitable stars for azimuth observations by this method are those the azimuth of which is changing most slowly, viz. stars at or near elongation, or in any case stars as far from the meridian as possible. As the altitude does not enter into the observation, the result is free from any possibility of error due to refraction.

The method may be applied to the sun, in which case the pointings F.R. and F.L. will be made to the leading and following limbs respectively, or vice versa, to eliminate the correction *s sec h* for the difference in azimuth between the sun's centre and the limb. The elimination is not perfect, as in the interval between the two pointings on the sun the altitude, and consequently also *s sec h*, will have changed, but the error may be neglected except for precise work. The sun's declination should be taken for the mean of the times of the two pointings on the sun.

An example of an azimuth observation by this method is given on page 147.

Azimuth by Altitude.—For an azimuth observation by this method, both the altitude of the star and the horizontal angle between it and the reference mark have to be observed. It is therefore necessary to observe the star at the intersection of the cross wires. The procedure in observing is as follows. A reading on the R.M. is first taken on one face of the theodolite; the telescope is then pointed towards the star so that the image of the latter is approaching the horizontal wire. The vertical wire is then brought on to the star and moved with it in azimuth by means of the tangent screw of the upper motion. At the instant when the image of the star is on the horizontal wire the observer ceases to move the tangent screw and calls 'Up' to the booker, who notes the clock time and records the level and circle readings given to him by the observer. Although it is advisable to book the clock time, it is not essential, because the observed altitude of the star provides the data for a time computation, as has been shown; in fact, the observation of both horizontal and vertical angles supplies the data for both a time and an azimuth computation, and a knowledge of the longitude of the station is not required. The observation may with advantage be done when the star is near elongation, and when therefore its motion in azimuth is small, or, for a star which does not elongate, when the star is on or near the prime vertical. Both these conditions may be covered by the statement that in all cases the star should be as far from the meridian as possible.

This method may also be applied to the sun, provided the longitude and local time be known so that the sun's declination at the instant may be found, in which case for the F.R. and F.L. pointings contact with the limbs of the image should be made with the sun in opposite quadrants of the cross wires, which will eliminate the semi-diameter so far as altitude is concerned, and will practically eliminate it also in azimuth. A repetition of the observation, in which the leading and following limbs are observed in the reverse order, will further reduce the error.

For approximate work, to the order of about 1′ of arc, the observation may be done on the sun's centre instead of on the limbs. Either the vertical or the horizontal wire is brought to the sun's centre by estimation, with the centre of the image approaching the other wire. The motion is followed by the one tangent screw, keeping the centre of the sun's image on the wire until the image is bisected by the other wire, when the pointing is complete. The bisection of the disc can be done with greater accuracy than might be supposed, giving a result sufficiently accurate for many purposes.

An example of an azimuth determination from a star by this method is given on page 151.

The method of determining the azimuth by hour angle has the advantage, as compared with the method by altitude, that the star has to be observed on only one wire, the instant of its intersection by which can be observed with great accuracy, and the clock time noted. As an erroneous value for the clock error will, of course, affect the resulting computed azimuth of the star, the azimuth observation ought to be accompanied by time observations on east and west stars, one taken before and one after the observation for azimuth.

In determining azimuth by altitudes, the image of the star has to be followed with one of the wires by means of the tangent screw; this necessitates the observer having his hand on the instrument, whereby a possible source of inaccuracy is introduced. In sun observations, it is always somewhat difficult to make a perfect contact simultaneously with two limbs on the two cross wires; consequently, if the time be known accurately, it is preferable to make the observation on the vertical wire only, on the two limbs in succession, and to base the computation for azimuth on the sun's hour angle, as already explained.

Example of an Azimuth Observation on the Sun (by Hour Angle).

Date: 16 December 1926. Latitude of Station: 51° 29′ 57″ N.
Value of 1 level division: 12″. Longitude of Station: 41s·5 W.
M.T. Clock fast on G.M.T.: 1m 39s·2.

Formula: $\cot Z = \dfrac{\sin(c-x)\cot P}{\sin x}$, where $\tan x = \tan p \cdot \cos P$.

Object	Face	Clock	Striding Level L	Striding Level R	Horizontal Circle Micrometers				Mean
		h m s			° ′ ″	′ ″	′ ″	′ ″	° ′ ″
R.M.	L				7 41 06	41 06	41 00	40 54	7 41 01·5
|o	L	14 23 54	10·5, 11·7	9·8, 8·8	43 32 05	32 04	32 00	31 52	43 32 00·2
o|	L	14 27 13·5	11·5, 11·7	9·4, 9·0	44 47 48	47 51	47 52	47 45	44 47 49
R.M.	L				7 41 02	41 01	40 56	40 52	7 40 57·7
R.M.	R				7 41 17	41 13	41 20	41 22	7 41 18
o|	R	14 35 47·5	9·8, 9·9	11·3, 11·1	46 37 43	37 37	37 30	37 27	46 37 34·2
|o	R	14 38 11	10·0, 10·4	11·0, 10·6	46 35 02	34 58	34 54	34 53	46 34 56·7
R.M.	R				7 41 22	41 22	41 26	41 27	7 41 24·2

Mean altitude (for striding level correction): 8°.

The angles, etc., entered above are the readings booked at the time of the observation, and it will usually be convenient to enter them at the time into a special book kept for the purpose of recording observations. The taking of means and the computation and application of level corrections may with advantage be done in the same book. The method of applying the level correction to the circle readings has already been explained, but will be repeated here in full for convenience.

In the present case, as the altitude was only about 8°, the level correction is small, and the value of tan h has been taken as constant for all the pointings.

For the first F.L. pointing on the sun,

$$b = \frac{\Sigma L - \Sigma O}{4} \times V = \frac{22{\cdot}2 - 18{\cdot}6}{4} \times 12'' = 10''{\cdot}8\,;$$

and as the left-hand end of the axis is the higher, the correction is positive. Tan 8°=0·14, and the correction=10″·8×0·14=+1″·5. This procedure has been followed for each pointing on the sun, with the following result:

FACE LEFT—

Object	Mean hor. Circle	b	tan h	b tan h	Corrected Circle	
	° ′ ″	″		″	° ′ ″	
R.M.	7 40 59·6	..	..	..	7 40 59·6	being the mean of 2 pointings.
|o	43 32 00·2	+10·8	0·14	+1·6	43 32 01·8	Mean
o|	44 47 49	+14·4	0·14	+2·0	44 47 51	44° 09′ 56″·4

Mean angle from R.M. to sun's centre: 36° 28′ 56″·8.

FACE RIGHT—

Object	Mean hor. Circle	b	tan h	b tan h	Corrected Circle	
	° ′ ″	″		″	° ′ ″	
R.M.	7 41 21·1	..	..	..	7 41 21·1	being the mean of 2 pointings.
o|	46 37 34·2	−8·1	0·14	−1·1	46 37 33·1	Mean
|o	46 34 56·7	−3·6	0·14	−0·5	46 34 56·2	46° 36′ 14″·6

Mean angle from R.M. to sun's centre: 38° 54′ 53″·5.

COMPUTATION FOR HOUR ANGLE

	F.L. h	m	s	F.R. h	m	s
Mean of clock times .	14	25	33·7	14	36	59·2
Clock fast . . .		1	39·2		1	39·2
G.M.T. . . .	14	23	54·5	14	35	20·0
Equation of time . .		4	30·8		4	30·6
G.A.T. . . .	14	28	25·3	14	39	50·6
Longtitude W. . .			41·5			41·5
L.A.T. . . .	14	27	43·8	14	39	09·1
Hour angle, W. . .	2	27	43·8	2	39	09·1
Angle P (ZPS) . .	36°	55′	57″·0	39°	47′	16″·5

The sun's declination for each of the above instants of G.A.T. has next to be found by interpolation from the value given in the *N.A.* for G.A.N., with the variation in 1 hour. This has been done by the method shown on page 33; the computation is continued as follows:

	F.L.	F.R.
δ	23° 18′ 25″·9 S.	23° 18′ 27″·2 S.
p	113 18 25 ·9	113 18 27 ·2
P	36 55 57 ·0	39 47 16·5
log tan p . . .	0·36571 n	0·36570 n
log cos P . . .	$\bar{1}$·90273	$\bar{1}$·88560
log tan x . . .	0·26844 n	0·25130 n
x	118° 19′ 22″	119° 16′ 39″
c	38 30 03	38 30 03
$c-x$	−79 49 19	−80 46 36
log sin $(c-x)$ . .	$\bar{1}$·99311 n	$\bar{1}$·99435 n
log cot P . . .	0·12395	0·07945
	0·11706 n	0·07380 n
log sin x . . .	$\bar{1}$·94462	$\bar{1}$·94065
log cot Z . . .	0·17244 n	0·13315 n
∴ sun's azimuth from south	33° 54′ 45″	36° 21′ 05″
Angle R.M. to sun .	36 28 56 ·8	38 54 53 ·5
R.M. east of south .	2 34 11 ·8	2 33 48 ·5
Mean	2° 34′ 00″	

∴ azimuth of R.M. from south=357° 26′ 00″

The approximate azimuth of the R.M. having been found, a suitable star can be selected for a more precise determination of azimuth, its altitude and azimuth computed for a convenient time, and the star observed before dark. The selection of a star for observation and the computation of its altitude and azimuth for a convenient prearranged time can, of course, be done at any time beforehand, so that when the sun observation has been carried out and computed it is only necessary to set the theodolite to the star's computed altitude and azimuth and to await the prearranged time, when the star will be found in the field of the telescope. The computations for finding the star can be done as shown on page 150.

As regards the selection of a star, if it is to be observed before dark it ought to be of not less than the second magnitude, and preferably of the first, so that it may be seen without difficulty. In the present instance α Arietis was selected, and its altitude and azimuth computed for 4 P.M. G.M.T. as follows:

Example: Computation of Altitude and Azimuth of α Arietis at 4 p.m. G.M.T. (16h 00m) on 16 December 1926 in Latitude 51° 29′ 57″ N. and Longitude 41s·5 W.

	h/°	m/′	s/″	
R.A.	2h	03m	02s·8	$\cos z = \cos p \cos c + \sin p \sin c \cos P$
δ	23°	07′	04″·6 N.	logs:
p	66°	52′	55″·4	$\cos p$ $\bar{1}$·59398 $\sin p$ $\bar{1}$·96364
S.T.G.M.N.	17h	37m	28s·9	$\cos c$ $\bar{1}$·89354 $\sin c$ $\bar{1}$·79416
				$\cos P$ $\bar{1}$·60244
Sid. Int. to 4 P.M.	4	00	39·4	$\bar{1}$·48752
				$\bar{1}$·36024
G.S.T.	21	38	08·3	antilogs 0·30727 + 0·22921
Long. W.			41·5	
L.S.T.	21	37	26·8	
R.A.	2	03	02·8	nat. $\cos z$ 0·53648
Hour angle	19	34	24·0	log. $\cos z$ $\bar{1}$·72955
P(ZPS)	4	25	36	z 57° 33′ 20″
or	66°	24′	00″	h 32° 26′ 40″

Using formula No. 11:

$$\cos \frac{Z}{2} = \sqrt{\frac{\sin s \sin (s-p)}{\sin z \sin c}}$$

				logs:			
$z =$	57°	33′	20″				
$c =$	38	30	03	$\sin s$	$\bar{1}$·99517	$\sin z$	$\bar{1}$·92630
$p =$	66	52	55	$\sin (s-p)$	$\bar{1}$·40115	$\sin c$	$\bar{1}$·79416
$2s =$	162	56	18	$\frac{1}{\sin z \sin c}$	0·27954		$\bar{1}$·72046
$s =$	81	28	09		2 | $\bar{1}$·67586		
$s-p =$	14	35	14		$\bar{1}$·83793		

$$\therefore \frac{Z}{2} = 46° \ 29' \ 05''$$

$$\text{or } Z = 92° \ 58' \ 10''$$

Example on Determination of Azimuth by Altitude.

Date: 4 March 1928. Station: Lat. 50° 49′ 58″ N.
Star: α Tauri. Long. 37s W.

Object	Face	Clock	Level E	Level O	Vertical Circle		Mean Alt.	Horizontal Circle		Mean
		P.M.			° ′ ″	′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″
R.M.	FR	..	..	..	..	..	..	323 28 20	28 00	323 28 10
Star	FR	9 19 50	6·0	6·0	54 37 30	37 30	35 22 30	167 06 00	05 20	167 05 40
Star	FL	9 23 12	6·2	5·8	34 53 40	53 50	34 53 45	167 53 00	52 20	167 52 40
R.M.	FL	..	..	..	..	..	..	323 27 30	28 00	323 27 45
						2	70 16 15			

$$\text{Level correction} = \frac{12{\cdot}2 - 11{\cdot}8}{4} \times 20'' = -2'' \qquad 35\ 08\ 07{\cdot}5$$

Refraction	−1 21
Level	− 2
Corrected alt.	35 06 44·5
∴ $z =$	54 53 15·5

$$\text{Formula } \sin \frac{Z}{2} = \sqrt{\frac{\sin (s-z) \sin (s-c)}{\sin z \sin c}}$$

	h	m	s	logs:			
R.A.	4	31	46·6				
δ	16°	21′	56″·9 N.	$\sin (s-z)$	$\bar{1}$·68498	$\sin z$	$\bar{1}$·91277
p	73	38	03·1	$\sin (s-c)$	$\bar{1}$·84702	$\sin c$	$\bar{1}$·80043
z	54	53	15·5	$\frac{1}{\sin z \sin c}$	0·28680		$\bar{1}$·71320
c	39	10	02		2 | $\bar{1}$·81880		
$2s$	167	41	20·6	$\sin \frac{Z}{2}$	$\bar{1}$·90940		
s	83	50	40·3	$\frac{Z}{2}$	54° 15′ 50″		
$s-z$	28	57	24·8	Z	108° 31′ 40″		
$s-c$	44	40	38·3				

$$\text{Angle between star and R.M.} = \frac{156° \ 22' \ 30'' + 155° \ 35' \ 05''}{2} = 155° \ 58' \ 48''$$

Angle between star and north = 108° 31′ 40″

∴ Angle from north to R.M. = 47° 27′ 08″

i.e. R.M. is 47° 27′ 08″ E. of N.

Convergence of Meridians.—Let AP in Fig. 30 be the meridian through a point A on the terrestrial sphere, and let BP be the meridian through a second point B, not necessarily in the same latitude as A, and let AB be a great circle passing through A and B; then the azimuth of AB from A is given by the spherical angle PAB $=\alpha$, and the azimuth of A from B is (360°−spherical angle PBA), i.e. $\beta+180°$.

The azimuth of B from A, i.e. angle α, differs from the

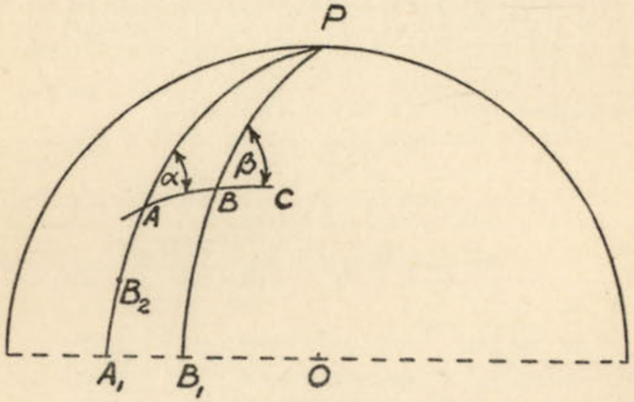

Fig. 30.

angle β, which is the (azimuth of A from B−180°), unless AB is an equatorial arc, or A and B lie on the same meridian, or the latitude of A is equal to that of B but of opposite sign. This difference is due to the fact that the great circles, of which the meridians AP and BP are arcs, are not parallel, but converge and intersect one another at the pole P; and this difference between the angles α and β is called the Convergence of the Meridians. The line of sight of a theodolite traces out a vertical plane when the telescope is turned about its horizontal axis, provided of course that the instrument is in accurate adjustment. This vertical

plane, if the earth be regarded as a sphere, is a plane passing through the centre of the earth so that a straight line set out by a theodolite is in reality the arc of a great circle on the earth's surface. It follows from what has been said above that, owing to the convergence of the meridians, the azimuth of such a straight line is not, in general, constant. The convergence of the meridians can be seen on any map

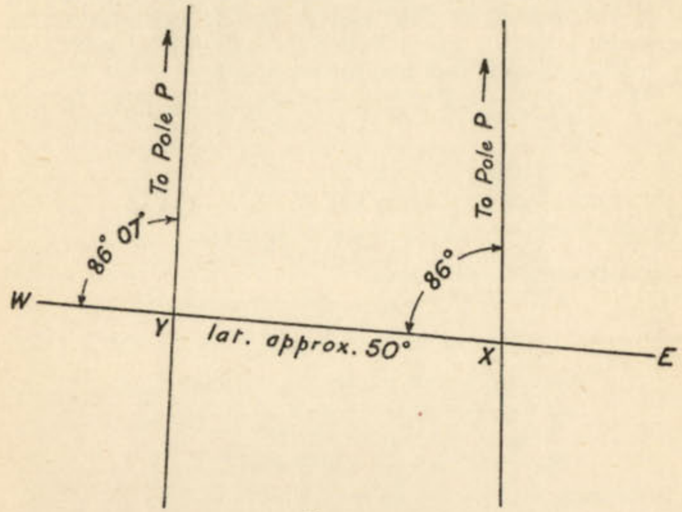

Fig. 31.

of a moderately large area, and even on the area represented by the 6-inch Ordnance Survey of England sheets, on which the longitude is given on the top and bottom margins. The true north and south line through any point is not a line parallel to the margin, but one drawn through the point and intercepting equal longitudes at the top and bottom of the sheet.

The effect of the convergence of meridians may be seen in the following example. Suppose in Fig. 31 XY is a survey line running approximately east to west and in approximate latitude 50°, and about 370 chains in length. If the azimuth of XY at X had been found astronomically to be 274°, i.e. N. 86° W., at Y the azimuth of XY produced as determined astronomically would be about 273° 53′, i.e. N. 86° 07′ W., the difference of 07′ being due to the convergence of the meridians XP and YP.

Formula for Convergence of Meridians.—In Fig. 30, P is the terrestrial pole and A and B two stations in latitudes L and L_1 respectively; the azimuth of AB being α, it is required to find the convergence θ.

PAB is a spherical triangle in which the angle PBA = $180° - \beta$, and

$$\frac{\sin(180°-\beta)}{\sin\alpha} = \frac{\sin \text{PA}}{\sin \text{PB}}, \text{ or } \frac{\sin\beta}{\sin\alpha} = \frac{\cos \text{L}}{\cos \text{L}_1}$$

Let $\quad \text{L}_1 - \text{L} = l$; then $\beta - \alpha = \theta$.

Substituting these values,

$$\frac{\sin(\alpha+\theta)}{\sin\alpha} = \frac{\cos(\text{L}_1 - l)}{\cos \text{L}_1}$$

or $$\frac{\sin(\alpha+\theta) - \sin\alpha}{\sin(\alpha+\theta)+\sin\alpha} = \frac{\cos(\text{L}_1-l) - \cos\text{L}_1}{\cos(\text{L}_1-l)+\cos\text{L}_1}$$

Therefore $$\frac{\sin\frac{\theta}{2}\cos\left(\alpha+\frac{\theta}{2}\right)}{\cos\frac{\theta}{2}\sin\left(\alpha+\frac{\theta}{2}\right)} = \frac{\sin\frac{l}{2}\sin\left(\text{L}_1-\frac{l}{2}\right)}{\cos\frac{l}{2}\cos\left(\text{L}_1-\frac{l}{2}\right)}$$

or $$\frac{\tan\frac{\theta}{2}}{\tan\frac{l}{2}} = \tan\left(\alpha+\frac{\theta}{2}\right)\tan\frac{(\text{L}+\text{L}_1)}{2}, \text{ since } \text{L}_1 - \frac{l}{2} = \frac{\text{L}+\text{L}_1}{2}$$

The above holds good whatever θ and l are, but if θ and l are small, then

$$\tan\frac{\theta}{2} = \frac{\theta}{2}, \quad \tan\frac{l}{2} = \frac{l}{2}, \quad \text{and} \tan\left(\alpha+\frac{\theta}{2}\right) = \tan\alpha,$$

except when α is approximately 90° or 270°.

Therefore $$\theta = l\tan\alpha\tan\frac{\text{L}+\text{L}_1}{2}$$

θ and l will be in the same units, and if L and L_1 are of equal magnitude but opposite sign, $\frac{\text{L}+\text{L}_1}{2} = 0$ and $\theta = 0$.

When A and B are on the equator, both α and β are obviously 90°, and θ is therefore zero.

This formula should not be used for values of α greater than 75°, or when the azimuth is within 15° of 90° or 270°, as tan α becomes indefinitely large as α approaches 90°.

Alternative Formula for Convergence.—In some cases the difference of longitude d of the two stations A and B may be known instead of the difference in latitude, in which case the cot formula may be applied to the spherical triangle PAB in order to determine θ, as follows:

$$\cos \text{AP} \cos d = \sin \text{PA} \cot \text{PB} - \sin d \cot\alpha$$

or $$\sin \text{L} \cos d = \cos \text{L} \tan \text{L}_1 - \sin d \cot\alpha$$

or $$\sin d \cot\alpha = \cos\text{L}\tan\text{L}_1 - \sin\text{L}\left(1 - 2\sin^2\frac{d}{2}\right)$$

$$= \frac{\sin(\text{L}_1-\text{L})}{\cos\text{L}_1} + 2\sin\text{L}\sin^2\frac{d}{2}$$

$$= \sin l \sec \text{L}_1 + 2\sin\text{L}\sin^2\frac{d}{2}$$

If d and l are small,

$$2\sin\text{L}\sin^2\frac{d}{2} = \tfrac{1}{2}d^2\sin\text{L},$$

which can be neglected, and then

$$d = l\tan\alpha\sec\text{L}_1.$$

But it has already been shown that

$$\theta = l\tan\alpha\tan\frac{\text{L}+\text{L}_1}{2}$$

Therefore $\theta = d \tan \frac{(L+L_1)}{2} \cos L_1$

$$= d \sin \frac{(L+L_1)}{2} \cdot \frac{\cos L_1}{\cos \frac{(L+L_1)}{2}}$$

But $\frac{\cos L_1}{\cos \frac{(L+L_1)}{2}} = \frac{\cos L_1}{\cos \left(L_1 - \frac{l}{2}\right)} = 1$, if l is small.

Therefore $\theta = d \sin \frac{L+L_1}{2}$

This formula therefore requires only the knowledge of the mean latitude and the difference in longitude.

If A and B are on one meridian, then $d=0$ and $\theta=0$.

It may readily be determined whether α is greater than β or vice versa from the relationship

$$\frac{\sin \beta}{\sin \alpha} = \frac{\cos L}{\cos L_1}$$

Another convenient form of the result $\theta = d \sin \frac{L+L_1}{2}$ expresses the convergence in terms of the *departure* between A and B, i.e. their distance apart measured in an east and west direction. The parallel of mid-latitude is a circle of radius $R \cos \frac{L+L_1}{2}$, where R is the radius of the assumed spherical earth. Therefore, if X denotes the departure,

$$d \text{ (in circular measure)} = \frac{X}{R \cos \frac{(L+L_1)}{2}}$$

$$\therefore \theta = \frac{X}{R \cos \frac{(L+L_1)}{2} \sin 1''} \cdot \sin \frac{(L+L_1)}{2}$$

$$= \frac{X \tan \frac{L+L_1}{2}}{R \sin 1''}$$

It is to be noticed that the formulæ for convergency have been derived on the supposition that the earth is a true sphere, and are not, in fact, exact owing to the spheroidal shape of the earth, but results obtained from their use are sufficiently accurate except for refined geodetic work.

The following example will illustrate the application of the above principles.

Example.—The terminal points A and B of a survey line are in latitude 51° 10′ 30″ and 51° 13′ 46″ respectively. The longitude of A is 2° 00′ 00″ W. and that of B 0° 09′ 30″ W. Find the great circle distance AB, and the azimuth of B from A and of A from B. Referring to Fig. 30 :

$$PA = 38° \ 49' \ 30''$$
$$PB = 38° \ 46' \ 14''$$
$$APB = 1° \ 50' \ 30''$$

To obtain the azimuth of B from A it is necessary to calculate the angle PAB, i.e. α.

This can be done by means of equations (13) and (14) for a spherical triangle, viz. :

[See Fig. 30.] $\tan \frac{1}{2}(B+A) = \frac{\cos \frac{1}{2}(b-a)}{\cos \frac{1}{2}(b+a)} \cot \frac{P}{2}$

and $\tan \frac{1}{2}(B-A) = \frac{\sin \frac{1}{2}(b-a)}{\sin \frac{1}{2}(b+a)} \cot \frac{P}{2}$

where $b-a = 00° \ 03' \ 16''$

$b+a = 77° \ 35' \ 44''$ and $\frac{P}{2} = 55' \ 15''$

Therefore $\frac{b-a}{2} = 00° \ 01' \ 38''$ and $\frac{b+a}{2} = 38° \ 47' \ 52''$

$\log \cos \frac{1}{2}(b-a) = \bar{1}{\cdot}9999999$

$\log \cot \frac{P}{2} = 1{\cdot}7939047$

$1{\cdot}7939046$

$\log \cos \frac{1}{2}(b+a) = \bar{1}{\cdot}8917396$

$\log \tan \frac{1}{2}(B+A) = 1{\cdot}9021650$

$\frac{1}{2}(B+A) = 89° \ 16' \ 54''$

$\log \sin \frac{1}{2}(b-a) = \bar{4}{\cdot}6768009$

$\log \cot \frac{P}{2} = 1{\cdot}7939047$

$\bar{2}{\cdot}4707056$

$\log \sin \frac{1}{2}(b+a) = \bar{1}{\cdot}7969720$

$\log \tan \frac{1}{2}(B-A) = \bar{2}{\cdot}6737336$

$\frac{1}{2}(B-A) = 2° \ 42' \ 04''$

and $\frac{1}{2}(B+A) = 89° \ 16' \ 54''$

Therefore $B = 91° \ 58' \ 58''$

$A = 86° \ 43' \ 50''$

The azimuth of B from A is consequently 86° 34′ 50″, and of A from B (360° 00′ 00″ − 91° 58′ 58″), or 268° 01′ 02″.

The azimuth of A from B can also be determined by calculating the convergence of the meridians and applying this convergence to the value of the azimuth of B as determined from A, as follows:

Let β (i.e. 180° 00′ 00″ − B) $= \alpha + \theta$,

where θ is the convergence then from the equation

$$\theta = d \sin\left(\frac{(L + L_1)}{2}\right)$$

where d is the difference in longitude between the stations A and B, and L, L_1, their respective latitudes

$$\theta = 110'{\cdot}5 \sin 51^\circ\ 12'\ 08''$$
$$= 86'\ 12''$$

But $\alpha = 86^\circ\ 34'\ 50''$

Therefore $\beta = 86^\circ\ 34'\ 50'' + 1^\circ\ 26'\ 12''$
$$= 88^\circ\ 01'\ 02''$$

Therefore the angle PBA $= 180^\circ\ 00'\ 00'' - 88^\circ\ 01'\ 02''$
$$= 91^\circ\ 58'\ 58''$$

i.e. the azimuth of A from B is 268° 01′ 02″, as before.

To obtain the great circle distance AB, the formula

$$\frac{\sin AB}{\sin APB} = \frac{\sin PB}{\sin PAB}$$ may be used

$$\sin AB = \frac{\sin PB \sin APB}{\sin PAB}$$

logs:

sin PB	$\bar{1}{\cdot}7967153$
sin APB	$\bar{2}{\cdot}5070136$
	$\bar{2}{\cdot}3037289$
sin PAB	$1{\cdot}9992261$
sin AB	$\bar{2}{\cdot}3045028$

$$\therefore AB = 1^\circ\ 09'\ 19''$$

CHAPTER IX

DETERMINATION OF LONGITUDE—WIRELESS TIME SIGNALS

General. — The difference of longitude between two places has been defined as the angular distance between their terrestrial meridians, the meridian of Greenwich being taken as the zero of longitudes.

Longitudes are expressed either in hours, minutes, and seconds of time or in degrees, minutes, and seconds of arc, east or west of Greenwich; and the difference of longitude of two places is the difference, at any instant, of their local mean or sidereal times.

Formerly the method of determining the longitude of a place was to determine the local time of some celestial phenomenon, such as the occultation of a certain star by the moon, or its angular distance from the moon, and by comparing this with the Greenwich time of the same circumstances the difference, at a certain instant, in the times was obtained, and hence the longitude. Such a method may be called absolute, and the celestial phenomena made use of may be, amongst others, moon culminations, lunar distances, and occultations of stars by the moon.

Lunar distances were given in the *N.A.* up to 1913, but they are not now of sufficient use to justify their retention, and are omitted.

The surveyor interested in absolute methods can find these described in the earlier editions of various treatises, but further reference to such methods will not be made here, as the introduction of wireless telegraph time signals has revolutionized the determination of longitude; and even if such signals are not made use of, a relative method of some kind depending on the connection of two places, one of known longitude, either by measurement or by astronomical observations at both places, is made use of.

Relative Methods in General Use.—1. *Triangulation*, i.e. a place of known latitude and longitude is connected by triangulation to the place the longitude of which is to be determined. The azimuth of one ray from the initial station being observed and the triangles solved, having regard to the true shape of the earth, the unknown longitude can be calculated.

2. *Latitudes and azimuths.*—If A and B be two mutually visible stations the latitudes of which are known or can be determined astronomically, and if the azimuth of AB be known or determined, then it is clear that the length of AB and the difference in longitude between A and B can be calculated; or, if only an approximation is required, these values may be obtained graphically. For the calculations involved, the reader is referred to any standard work on Geodetic Surveying.

3. *By transport of chronometers.*—In principle this method of determining the difference of longitude is extremely simple. If the error of the chronometer on local time at one place is determined, and the chronometer transported to a second place where the error on the local time is again determined, then, assuming that the chronometer has neither gained nor lost, the difference in the errors will be the difference between the local times of the two places, and will therefore be a measure of the difference in longitude. In practice, no chronometer can be relied on neither to gain nor to lose, and the travelling and standing rate of the chronometer must be determined and allowed for as explained in Chapter IV. Where great accuracy is required, at least five chronometers should be carried and the difference in longitude determined from each, the mean value being adopted finally.

4. *Longitude by telegraphic signals over wires.*—For ordinary field work purposes with chronometers the following procedure will meet requirements.

The errors and rates of chronometers at two stations A and B are determined, and the observer at station A sends a telegraphic signal at a known time by his chronometer, the observer at B noting the time of receipt by his chronometer; the difference of longitude of stations A and B can then be computed by calculating the correct local times at which the signal was despatched and received, which will give the difference in the correct local times at the same instant, and therefore the difference in longitude.

In order to minimize errors, the following amplification of the above should be arranged for:

(*a*) The error of the chronometer at both stations should be determined before and after the exchange of signals, the exact times at which the signals were exchanged being found by interpolation.

(*b*) A series of signals should be sent, in order to reduce any error in sending and receiving a single signal.

(*c*) In order to prevent bias, a mean time chronometer can be used at one station and a sidereal at the other.

(*d*) A series of signals should be sent in each direction (i.e. E. to W. and W. to E.) so that any error due to lag in the instruments will be eliminated, since the difference in longitude will be in the one case too large and in the other case too small, the mean giving the true result.

For sending the signals a sounder is the most satisfactory instrument, the armature making a distinct tap when the key is depressed and again when it is released.

The observer at each station computes his local mean or sidereal time at which the various signals were sent and received, and the difference between the means of the correct local times of receiving and sending gives the difference of longitude in time.

The computations are similar to those employed when wireless time signals are made use of.

5. *Longitude by wireless time signals.*—Before describing the application of wireless time signals to the determination of longitude, it will be necessary to describe the nature of these signals, and the procedure for determining from them the chronometer error.

Wireless Time Signals.—The improvements in the transmission and reception of wireless time signals in recent years have simplified the surveyor's work considerably, especially for the determination of longitude, which formerly was one of the most difficult and troublesome observations which the surveyor was called upon to make; for, even in the rare event of its being possible to connect up with overland telegraph wires to an observatory or other station as previously mentioned in order to compare the beat of the travelling chronometer with that of the observatory or station at the other end, errors would be introduced due to time lag in the line itself and in the necessary relays, magnets, etc. Now, however, nearly every nation transmits time signals at scheduled times, and there are few places on the globe where the time signal from one or other of these transmitting stations cannot be obtained with certainty with a simple type of apparatus. In the tropics, atmospherics may cause considerable trouble at times, but it will rarely be impossible to get one or more signals during the course of the day. In general, the times of transmission of W.T. (wireless telegraph) time signals are given in Standard Mean Time, but sidereal time is used in the case of some rhythmic time signals.

Methods of Transmission.—A great number of W.T. time signals are operated automatically by precise mechanism connected to the standard clock at an observatory which, through electrical contacts, controls the emitting apparatus of the station which is broadcasting the signals. These signals may be relied upon, and should be correct to a tenth of a second.

At smaller stations and those not connected to an observatory the signals are sent by hand, the operator

obtaining the time from the standard clock at the station, which is checked periodically by astronomical observations or reliable wireless signals from another station. Signals sent by hand should be correct to 0·25 second.

Some of the more powerful stations broadcast short and long wave length signals simultaneously.

Systems of Signals.—There is still a lack of uniformity in the systems used for the transmission of time signals, but most countries use one or other of the four systems described below.

Some stations, however, use individual systems; for example, Daventry at present (1928) transmits time signals consisting of six dots representing successive seconds, the final dot, which is made at 10h 30m, 16h 00m, and 22h 00m, being the time signal. The wave length employed is 1604·3 metres.

(*a*) *The International System.*—This is also known as the *Onogo* system from the sequence of Morse letters used in the time code. In this system the transmission takes three minutes, as follows:

1st Minute.—The letter X (– · · –) sent from 0 to 45 seconds every five seconds; six seconds' silence, followed by the letter O (– – –), each dash being of one second's duration, commencing at the 55th, 57th, and 59th seconds.

2nd Minute.—The letter N (– ·) sent every ten seconds, commencing at the 8th, 18th, 28th, 38th, and 48th seconds, each dot being given at every ten, i.e. at the 10th, 20th, 30th, and 40th seconds; five seconds' silence, followed again by the letter O (– – –) as in the first minute.

3rd Minute.—A series of the letter G (– – ·) sent every ten seconds, commencing at the 6th, 16th, 26th, and 46th seconds, each dot being given at every ten; five seconds' silence, followed by the final signal, the letter O (– – –) as in the second minute.

In the transmission of these signals each dash = 1 second and each dot = 0·25 second.

The particular signal that is taken as the exact time signal varies; sometimes the end of the last dash in the letter O (– – –) representing an even minute is taken, whilst Germany takes the dot of the letters N (– ·) and G (– – ·) as representing the exact time, but either method can be employed with confidence.

The above system is employed by Ceylon, Germany, India, Java, South Australia, and Western Australia on wave lengths which range from 600 metres to 18,900 metres.

(*b*) *The New International System.*—In 1925 the system described above was amended by substituting six dots sent at the 55th, 56th, 57th, 58th, 59th, and 60th seconds of each minute for the three one-second dashes that commence

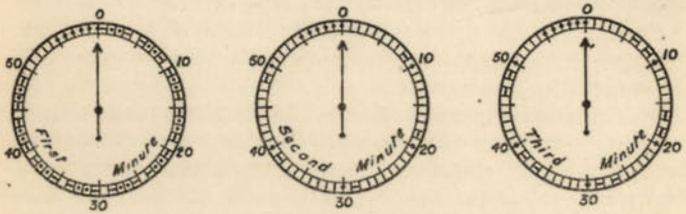

FIG. 32.

at the 55th, 57th, and 59th seconds of the three minutes in system (*a*) which constitute the time signals.

The New International System is employed by Brazil, France, Russia, Portuguese East Africa, and Victoria.

A diagrammatic representation of this system is given here (Fig. 32).

In systems (*a*) and (*b*) call or warning signals, which also serve to identify the transmitting station, are broadcast prior to the actual time signals.

(*c*) *The United States System.*—In this system the duration of the signals is five minutes, and warning signals are dispensed with. The signals consist of the transmission of a dot (·) for every second, omitting the 29th, 55th, 56th, 57th, 58th, and 59th during each of the first four minutes; in the fifth minute the dot is omitted at the 29th, 50th, 51st,

52nd, 53rd, 54th, 55th, 56th, 57th, 58th, and 59th seconds, and at the 60th second a one-second dash (–) is sent, the beginning of which is the exact time signal. As a rule this is followed by the call signal of the station and the letters O.K. which signify that the signal is correct.

This system is used by the Hawaiian Islands, Panama Canal Zone, and the United States of America.

The same system is used by Chile, except that a dot (·) replaces the final one-second dash (–) of the last minute.

(*d*) *The Rhythmic or Vernier System.*—If greater accuracy, say of the order of a hundredth part of a second, than that afforded by the above described systems is required, Rhythmic or Vernier time signals are available. Such accuracy would be out of proportion for ordinary field purposes, but it may be convenient to make use of these signals, which are therefore described here. These signals were first transmitted by the Eiffel Tower, and it was followed by stations in French Indo-China, Germany, and Russia. The principle of this system is as follows:

The number of signals sent is about 300 and the length of the transmission approximately five minutes, the transmission being automatic. A pendulum clock is arranged to send a series of beats, represented by dots, at the rate of 50 every 49 seconds, the interval between consecutive dots being approximately 0·98 second. Every 60th interval is denoted by a dash (–), representing two beats, in order to facilitate counting. By counting the number of intervals which occur between the first dot (or beat) of the pendulum and that dot (or beat) which coincides with a beat of the chronometer, a chronometer comparison is obtained, i.e. the chronometer time corresponding to the time of the first signal is obtained by subtracting from the chronometer time of coincidence the time interval represented by the number of intervals multiplied by the length of each interval. Disregarding the coincidences at the half seconds, it should be possible to obtain six comparisons by coincidences at the whole seconds during one transmission, and the mean of the deduced chronometer times for the time of the first signal is taken. It is to be observed that signals sent out on this system are sent out at no stipulated time, and in order to utilize them it is necessary to listen in for the times of the first and last signals, as observed at an observatory, which are broadcast some time later, in some cases on the following day.

(*e*) *The New International System of Rhythmic Time Signals.*—This is a modification which began in 1926 of the previous system and consists of 306 signals transmitted in 300 seconds of mean time. At the beginning of each of the five minutes a dash (—) is transmitted; this dash is followed by 60 dots (..... etc.), the final signal at the sixth minute being signalled by a dash (—), commencing at the beginning of this minute. There are therefore 305 intervals in 300 seconds of mean time, the length of each interval is therefore 300/305 of a mean time second or 0·9836 mean time seconds.

This system is used at Bordeaux, Eiffel Tower (Paris), La Fayette, Saigon, Rugby, Moscow, and Leningrad. At the present time (1928) the final dash, i.e. the 306th signal from Rugby, is given at 10h 00m 00s and at 18h 00m 00s on a wave length of 18,740 metres.

One of the chief advantages of the new system over the old is that the signals are transmitted at a definite instant of mean time, as accurately as the time is then known at the controlling observatory. This has the added advantage that the signals are available as ordinary, i.e. not rhythmic, signals. For most purposes a comparison obtained by disregarding the dots and using the commencement of the dashes only (given at the exact minute) will be sufficiently accurate. However, if it is desired to make use of the greater accuracy obtainable from the rhythmic coincidences it is not necessary actually to count the signals. The nearest whole second on the chronometer preceding the beginning of each dash is booked, and the chronometer time of the coincidence following each dash

Transmitting Station : Eiffel Tower. Observing Station : South Kensington.

Date : 9 May 1928.

	First Minute of Signal	Second Minute of Signal	Third Minute of Signal	Fourth Minute of Signal	Fifth Minute of Signal
	h m s	h m s	h m s	h m s	h m s
G.M.T. of commencement of signal.	9 31 00	9 32 00	9 33 00	9 34 00	9 35 00
Chronometer time of commencement of dash (A) . . .	9 31 02	9 32 02	9 33 02	9 34 02	9 35 02
Observed chronometer times of coincidences . . .	9 31 22 9 31 24	9 32 22 9 32 24	9 33 22 9 33 25	9 34 23 9 34 24	9 35 25
Mean observed chronometer time to nearest whole second (B) .	9 31 23	9 32 23	9 33 24	9 34 24	9 35 25
Corresponding signal number of coincidence (B–A) . .	21	21	22	22	23
Interval in seconds corresponding to above signal number .	20·66	20·66	21·64	21·64	22·62
G.M.T. of coincidence . . .	9 31 20·66	9 32 20·66	9 33 21·64	9 34 21·64	9 35 22·62
Chronometer time of coincidence .	9 31 23	9 32 23	9 33 24	9 34 24	9 35 25
Chronometer error . . .	Fast 2·34	0 00 2·34	0 00 2·36	0 00 2·36	0 00 2·38

The mean chronometer error is, therefore, 2·35 seconds fast.

is also booked. The difference in seconds of these times gives the number of the coincident rhythmic signal from the commencement of the corresponding dash, which is transmitted at a known instant of time.

The signal number of coincidence from the dash being known, the number of intervals, n, is likewise known and is, in fact, the same as the signal number. Each interval being as stated above 0·9836 second in duration the time elapsed from the commencement of the dash to the time of coincidence is $n \times 0{\cdot}9836$ seconds of mean time, which quantity if subtracted from the clock time of coincidence gives the exact clock time corresponding to the known time of the commencement of the dash, and hence the chronometer error is determined for each dash and the mean error is taken.

The example on previous page will make this procedure clear.

In the *Admiralty List of Wireless Signals* may be found, *inter alia*, particulars of the transmitting stations of over thirty countries throughout the world which transmit time signals. The particulars given include the latitude and longitude of the station, the wave length or lengths employed, the system used, and the times of transmission with full details of the procedure. The particulars given may vary from year to year, so that those contemplating the reception of signals should obtain the latest information from this or a similar publication.

Longitude by Means of W.T. Signals.—The computation for determining the longitude, the clock error having been obtained by means of wireless signals, is as in the example below. As long as possible before the time signal is to be received, the chronometer should be placed on a table convenient to the theodolite, and a set of time observations should be taken just before and just after the receipt of the signal. The procedure and computations for these time observations are described elsewhere.

The following example gives the result of an observation for longitude :

An observer with a S.T. chronometer received the Daventry wireless time signal at 16h 00m G.M.T., and his chronometer time was 23h 49m 31s·8.

The results of time observations gave the following :

Before Signals		After Signals	
Star	Chron. Error	Star	Chron. Error
	m s		m s
α Lyrae W.	00 59·7 slow	ζ Cygni	00 58·9 slow
ι Tauri E.	1 01·8 ,,	β Tauri	1 02·65 ,,
Mean	1 00·75 ,,	Mean	1 00·77 ,,

The chronometer had therefore no appreciable rate, and was 1m 00s·76 slow on L.S.T.

	h	m	s
Chronometer S.T. time of receipt of signal =	23	49	31·8
Chronometer error =	00	1	00·76
Correct L.S.T. of receipt of signal =	23	50	32·5
G.M.T. of signal =	16	00	00·0
Corresponding G.S.T. of signal (from *N.A.*) =	23	51	14·0
G.S.T. – L.S.T. =	00	00	41·5

from which, longitude = 10′ 22″·5 W.

The calculations for the chronometer error from the time observation before the arrival of the signal are given on page 133.

It is obvious that for ordinary field purposes a time observation shortly, say about half an hour, before the arrival of the signal is sufficient unless there is any reason to suspect that the chronometer has a very appreciable rate.

APPENDIX I

EFFECT OF ERRORS IN ASSUMED VALUES

It is evident that all the quantities which the observer will make use of in his computation to obtain the final desired result cannot be known or determined with absolute accuracy, but all are liable to errors due to various causes. The effects of such errors for any particular magnitude of error can obviously be determined for any given case by a comparison of the results obtained by varying one of the quantities by a definite amount equal to the error likely to occur. Alternatively, from the general expression employed for the computation of any particular observation, the effect of an error δq in one quantity q may be determined by differentiation.

For convenience of reference, the formulæ expressing the effect of the errors most likely to occur in the more common observations are given below:

Time Observations by Star Altitudes.—(*a*) The observed quantity is the altitude.

(*b*) Latitude is assumed to be known.

The effect of an error in each of these will be considered in turn.

(*a*) *Effect of an Error in the Observed Altitude.*—This was shown on page 130 to be

$$\delta t^s = \frac{\delta z''}{15 \sin Z \cdot \sin c}$$

where $\delta z''$ is the error in seconds of arc in the observed altitude and δt^s is the error in the resulting time in seconds.

e.g. if $\delta z''$ be $10''$
and $Z = 60°$
" $\phi = 40°$ or $c = 50°$

Then $\delta t^s = \dfrac{10''}{15 \sin 60° \sin 50°} = 1{\cdot}00$ secs.

(*b*) *Effect of an Error in the Assumed Latitude.*—This was shown on page 131 to be

$$\delta P'' = -\delta c'' \frac{\cot Z}{\sin c}$$

where $\delta c''$ is the error in the assumed latitude in seconds of arc

or δP in seconds of time $= \dfrac{-\delta c'' \cdot \cot Z}{15 \sin c}$

e.g. if $\phi = 40°$ or $c = 50°$
$\delta c'' = 10''$
$Z = 75°$

then δP (in secs. of time) $= \dfrac{-10'' \cot 75°}{15 \sin 50°} = 0{\cdot}233$ secs.

Azimuth Observations by Altitude.—In this case the azimuth is found by computation from the three sides of the triangle, p, c, and z, of which p, the polar distance, got from the *N.A.*, may be taken as correct; but the assumed colatitude c and the observed zenith distance may both be in error, and the effect of an error in each of them will be considered in turn.

(*a*) *Effect of an Error in the Assumed Latitude.*—This may readily be found from the fundamental formula

$$\cos p = \cos z \cos c + \sin z \cdot \sin c \cdot \cos Z,$$

from which the formulæ (10) and (11) on page 48 are derived.

By differentiating the above formula with respect to c,

$$0 = -\cos z \sin c + \sin z \left(\cos c \cdot \cos Z - \sin c \cdot \sin Z \cdot \frac{dZ}{dc}\right)$$

$$\cos z \cdot \sin c = \sin z \left(\cos c \cdot \cos Z - \sin c \sin Z \cdot \frac{dZ}{dc}\right)$$

$$\therefore \quad -\cot z \cdot \sin c + \cos c \cdot \cos Z = \sin c \cdot \sin Z \cdot \frac{dZ}{dc}$$

$$\therefore \quad \sin c \cdot \sin Z \cdot \frac{dZ}{dc} = \cos c \cdot \cos Z - \sin c \cdot \cot z.$$

Now applying the cot formula to the consecutive parts z, Z, c, and P,

$$\cos c \cdot \cos Z = \sin c \cdot \cot z - \sin Z \cdot \cot P$$

$$\therefore \quad \sin Z \cdot \cot P = \sin c \cdot \cot z - \cos c \cdot \cos Z$$

$$= -\sin c \cdot \sin Z \cdot \frac{dZ}{dc}$$

$$\therefore \quad \frac{dZ}{dc} = -\frac{\cot P}{\sin c}$$

$$\therefore \quad \delta Z'' = -\delta c'' \cdot \frac{\cot P}{\sin c}$$

e.g. if $\delta c'' = 10''$
and $P = 5 \text{ hours} = 75°$
" $\phi = 20°$ or $c = 70°$

$$\delta z'' = -10'' \times \frac{\cot 75°}{\sin 70°} = -2''\cdot 85$$

(*b*) *Effect of an Error in the Observed Zenith Distance.*—From the fundamental formula

$$\cos p = \cos z \cos c + \sin z \sin c \cdot \cos Z$$

by differentiation with respect to z, c and p being constant,

$$0 = -\cos c \cdot \sin z + \sin c\left(\cos z \cos Z - \sin z \sin Z \cdot \frac{dZ}{dz}\right)$$

$$\text{or} -\cos c \cdot \sin z + \sin c \cdot \cos z \cdot \cos Z = \sin c \cdot \sin z \cdot \sin Z \cdot \frac{dZ}{dz}$$

The left-hand side of this equation can by spherical trigonometry be shown to be $= -\sin p \cdot \cos S$.

$$\therefore \quad -\sin p \cdot \cos S = \sin c \cdot \sin z \cdot \sin Z \cdot \frac{dZ}{dz}$$

$$\therefore \quad \frac{dZ}{dz} = -\frac{\sin p}{\sin c} \cdot \frac{\cos S}{\sin z \cdot \sin Z}$$

$$= -\frac{\sin Z}{\sin S} \cdot \frac{\cos S}{\sin z \cdot \sin Z}$$

$$= -\frac{\cot S}{\sin z}$$

$$\therefore \quad \delta Z'' = -\delta z'' \cdot \cot S \cdot \operatorname{cosec} z.$$

$\delta Z''$ will be a maximum when $S = 0°$ or $180°$ and is zero when $S = 90°$, i.e. when the star is at elongation; and for a star which does not elongate, $\delta Z''$ is a minimum when S is as nearly as possible $90°$.

Azimuth Observations by Hour Angle.—In this case the azimuth is computed from the hour angle obtained from the clock, the latitude being known. Both these quantities are liable to be in error. The effect of an error in the latitude has already been considered.

Effect of an Error in the Hour Angle.—Applying the cot formula to the four consecutive parts Z, c, P, and p,

$$\cos c \cdot \cos P = \sin c \cdot \cot p - \sin P \cdot \cot Z,$$

in which c and p are constant.

Differentiating with respect to P,

$$-\cos c \sin P = 0 - \cos P \cdot \cot Z + \sin P \cdot \operatorname{cosec}^2 Z \frac{dZ}{dP}$$

$$\sin P \cdot \operatorname{cosec}^2 Z \cdot \frac{dZ}{dP} = \cos P \cdot \cot Z - \cos c \cdot \sin P$$

$$\frac{dZ}{dp} = \cot P \cdot \cos Z \cdot \sin Z - \frac{\cos c \cdot \sin P}{\sin P \cdot \operatorname{cosec}^2 Z}$$

$$= \cot P \cdot \cos Z \cdot \sin Z - \cos c \cdot \sin^2 Z$$

$$= \sin Z(\cot P \cdot \cos Z - \cos c \cdot \sin Z)$$

$$= \frac{\sin Z}{\sin P}(\cos P \cdot \cos Z - \cos c \cdot \sin Z \cdot \sin P).$$

The quantity in brackets can, by spherical trigonometry, be shown to be $= -\cos S$

$$\therefore \quad \frac{dZ}{dP} = -\frac{\sin Z \cdot \cos S}{\sin P}$$

$$\therefore \quad \frac{dZ}{dP} = -\frac{\sin p}{\sin z} \cdot \cos S$$

$$\text{or} \quad \delta Z'' = -\frac{\sin p}{\sin z} \cdot \cos S \cdot \delta P'',$$

which, for any given value of z, is a minimum when $S = 90°$ or $270°$, i.e. when the star is at elongation.

APPENDIX II

THE 'NAUTICAL ALMANAC'

General.—The *Nautical Almanac* was first published in 1767 by the Board of Longitude, but is now published, for three years in advance, by His Majesty's Stationery Office, by order of the Admiralty. In addition to the complete edition, an abridged edition for the use of seamen is also issued. In the latter edition, the places of the sun and stars are not given to the same degree of accuracy as in the former; the number of stars given is smaller, and various other information is omitted as unlikely to be required for navigational purposes. A careful study of the quantities tabulated in the *N.A.* as regards such points as their maximum and minimum values with their dates of occurrence, and the relationship between the quantities given in the various columns, will be well repaid by the clearer conception of the mechanism of the celestial sphere which such study will give the observer.

Referring to the unabridged edition, the first page gives particulars regarding obliquity of the ecliptic, precession, nutation, horizontal parallax of the sun, aberration, etc.

Page I for each month gives the sun's apparent R.A. and declination at the instant of Greenwich apparent noon, with the variation in one hour of each of these quantities. The use of the word *apparent* in the precepts or headings to these columns is explained later. In brief, it means that the position of the sun as indicated by the co-ordinates given includes the apparent displacement necessarily resulting from the fact that the observer sees the sun from

a moving earth, and also that the position is referred to the true position of the equinox and of the equator at the instant and not to their mean positions.

The variations in one hour are to be applied as explained in Chapter II.

It should be observed that at apparent noon, as the sun's centre is on the meridian, his apparent R.A. is also the hour angle of the First Point of Aries, and therefore also the sidereal time at apparent noon.

On the same page, in an adjoining column, is given the sidereal time of the semi-diameter passing the meridian, i.e. the sidereal interval between the transit of the limb and the transit of the centre, which is sometimes useful, as it enables the observer to determine readily beforehand the time when either limb of the sun is on the meridian, or, for a rough determination of the L.A.T. if the meridian be known from previous observations.

It will be noticed that except for a slight discrepancy the interval decreases with decreasing declination, that is, as the sun approaches the equator. That this should be so is obvious from elementary considerations, as the time interval concerned depends mainly on the angular distance between the two declination circles touching the sun's limb and passing through the sun's centre respectively, which angle increases with increasing declination, being equal to the semi-diameter multiplied by the secant of the declination.

The angular distance between the two declination circles referred to is not exactly equivalent to the time of the semi-diameter passing the meridian, on account of the sun's motion during the interval of passing the meridian, which prolongs the said interval by several seconds. The time of the semi-diameter passing the meridian is, of course, independent of the latitude, and varies only slightly with the longitude.

The last two columns on page I of each month give the Equation of Time and its variation in 1 hour. The tabulated quantities are to be applied to apparent time to obtain mean time; the precept at the head of the column states whether it is additive or subtractive, i.e. whether the sun is behind the mean time clock or ahead of it. The variation in one hour is to be applied as explained in Chapter II.

Page II for each month gives, for G.M.N., the sun's apparent R.A. and declination and the semi-diameter. It will be observed that the semi-diameter is a maximum on or about 1 January, when the sun is nearest to the earth, that is, in perigee. The minimum occurs on or about 1 July.

Column 5 gives the Equation of Time at G.M.N., and column 6 the Sidereal Time at G.M.N. The latter quantity being the hour angle of the First Point of Aries, and the apparent R.A. of the sun given in column 2 being the angular distance of the sun from the First Point of Aries, their difference is the hour angle of the true sun when the mean sun is on the meridian which is the Equation of Time at G.M.N. Accordingly, if the Equation of Time be applied with the correct sign to the sun's apparent R.A., both quantities being for mean noon, the result is the sidereal time at mean noon, as may be seen by an inspection of the tabulated values in the *N.A.*

The sidereal time at G.M.N. being the hour angle of the First Point of Aries when the mean sun is on the meridian, is also the R.A. of the mean sun at G.M.N. In the abridged *N.A.* the R.A. of the mean sun is given in the column headed R.A.M.S. for two-hour intervals; the value for 12h, or mean noon, agrees with the sidereal time at G.M.N. given in the complete edition.

The S.T.G.M.N. is used in all cases of conversion of M.T. into S.T. and vice versa. In the interval from one G.M.N. to the next, i.e. in 24 M.T. hours, the sidereal clock gains 3m 56s·555, as will be seen by subtracting the S.T.G.M.N. for one day from its value for the next day.

Pages III and IV for each month contain quantities not generally necessary for surveying purposes, such as the longitude and latitude of the sun and moon (their co-ordinates measured along the ecliptic from the First Point of Aries and at right angles to the ecliptic), also the mean time of the transit of the First Point of Aries, and do not

call for any reference so far as the subject of Field Astronomy is concerned. They are given in the *N.A.* for observatory work.

Pages V to XII for each month give the moon's position in R.A. and declination for each hour of every day of the month, and are useful in the case of observation on the Moon, but the surveyor will rarely have occasion to use them.

The positions of the planets, in apparent R.A. and declination, which follow are occasionally of use in field work.

The portion of the *N.A.* which gives the places of stars is necessary for field work. The *mean places* for the beginning of the year are first given, but the quantities which the surveyor uses are the *apparent places*, i.e. as affected with aberration and measured from the true equinox and equator, which are given for every day of the year for a number of circumpolar stars, for the instant of their upper transit at Greenwich, and for every tenth day in the case of other stars, the values at intermediate dates being found by interpolation. The stars are arranged in the order of their R.A.

The tables given for the conversion of time intervals, mean into sidereal and vice versa, are useful.

Every few years the *N.A.* gives in an appendix the *Derivation* of the tabulated quantities, showing how they have been obtained.

The quantities given on pages I and II of the *Nautical Almanac* for each month for the R.A. and declination are, of course, *geocentric*, i.e. they correspond to the position of the sun as it would be seen from the centre of the earth, which differs in general from its position as seen by an observer on the earth's surface by the amount of the sun's parallax, as explained in Chapter V, where it was shown that the observed altitude after being cleared of refraction has to be corrected for parallax to obtain the true altitude, referred to the rational horizon.

Aberration.—The use of the word 'apparent' in the precepts or headings to the columns for R.A. and declination requires explanation. The earth is moving in its orbit round the sun with a mean velocity of about 18 miles per second; the view-point from which an observer on the earth sees the celestial bodies is consequently not stationary, and this movement of the view-point has an effect on the apparent direction in which celestial bodies are seen, because the velocity of light (about 180,000 miles per second) has to be compounded with the velocity of the earth in its orbit to obtain the relative velocity.

Suppose an observer to be moving in the direction AB

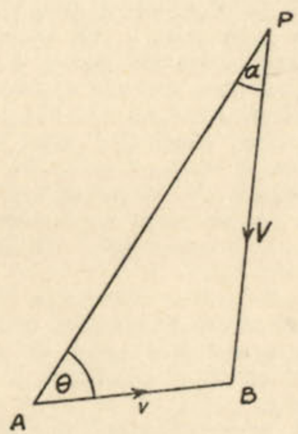

FIG. 33.

with a velocity v, and a particle to be moving in the direction PB with a velocity V (Fig. 33). If the lengths AB and PB be taken proportional to the velocities v and V respectively, the particle P and the observer A will reach the point B at the same instant. When the observer is at the point A, the particle appears to him to lie in the direction AP, and if the motion of the observer and the motion of the particle are uniform, the particle P will always appear to the observer to lie in a direction parallel to AP, until it strikes him at B. In fact, PA represents

in magnitude and direction the velocity of P relative to the observer.

If a ray of light from a star be substituted for the particle P, the same argument still holds, and the observer, when at the point B, will be receiving light from the star the true direction of which is PB, while its apparent direction is parallel to PA. In other words, the effect of the motion of the observer combined with that of the light reaching him is to cause an apparent displacement of the star towards the point in the direction of which the observer is moving. This effect is known as the aberration of light, and the consequent apparent displacement of a heavenly body is referred to in general as *aberration*. The actual motion of the observer is the resultant of his motion due to the earth's rotation combined with the movement of the earth in its orbit. The component due to the earth's rotation, known as diurnal aberration, is comparatively small (the velocity at the earth's equator being only about 0·3 mile per second), and is negligible for most purposes; but the component due to the earth's velocity of about 18 miles per second in its orbit produces an appreciable effect on the positions of the heavenly bodies, which is taken into account in preparing the values of the *apparent right ascension* and *apparent declination* given in the columns of the *N.A.* The apparent R.A. and declination are in each case affected with aberration, i.e. the position of the sun as given includes the apparent displacement due to aberration.

The amount of the aberration in the case of the sun is about 20″.

Referring again to Fig. 33, let the angle APB $=\alpha$, i.e. α is the angle between the true direction PB and the apparent direction PA, or it is the apparent displacement due to aberration.

Since $$\frac{v}{V}=\frac{\sin\alpha}{\sin\theta}$$

$$\sin\alpha=\frac{v}{V}\sin\theta$$

The aberration α being a small angle, α in circular measure may be substituted for sin α.

$$\therefore \alpha = \frac{v}{V} \times \sin \theta.$$

$$\therefore \alpha \text{ in seconds of arc} = \frac{v \sin \theta}{V \sin 1''}.$$

$\frac{v}{V}$ is called the *constant of aberration.*

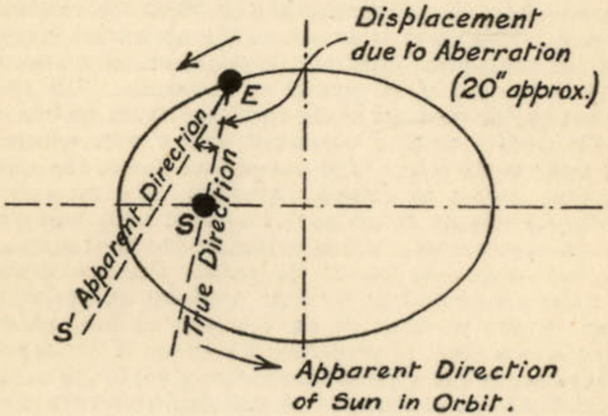

FIG. 34.

The maximum value of α occurs when $\theta = 90°$ and $\sin \theta = 1$, in which case $\sin \alpha = \frac{v}{V}$

If the observer's motion be at right angles to the true direction of the observed body, $\tan \alpha = \frac{v}{V}$.

To consider the effect of aberration on the sun's apparent position on the celestial sphere, let Fig. 34 represent the earth's orbit, S being the sun and E the position of the earth at an instant.

If the earth be moving in the direction shown by the arrow against E, the direction of the apparent annual

motion of the sun in the ecliptic is also that shown by the arrow. The true direction of the sun from the earth is ES, but owing to aberration the *apparent direction* is ES′. The direction of the apparent annual motion of the sun on the celestial sphere being that of the arrows, the effect of aberration, displacing the apparent direction of the sun from ES to ES′, is to cause the sun to lag behind its true position on the ecliptic. The aberration, depending as it does on the velocity of the earth in its orbit, varies slightly, being a maximum at perihelion and a minimum at aphelion ; its mean value is about 20″·5. The apparent displacement due to aberration is in the case of the sun in the plane of the ecliptic.

Equation of the Equinoxes.—In addition to aberration, there is another reason why the R.A. and declination of the sun given in the *N.A.* are called apparent R.A. and declination. This is to be found in the circumstance that the First Point of Aries, from which right ascensions are measured, has a motion which is not uniform along the ecliptic. The precession of the equinoxes was referred to and briefly explained in Chapter I, where it was stated that the sun's attraction on the equatorial protuberance produces a couple tending to bring the plane of the earth's equator into coincidence with the plane of the ecliptic, but that in consequence of the earth's rotation the effect is that the pole of the equator moves slowly in a circle about the pole of the ecliptic. Precession is, however, in reality rather more complicated. At the equinoxes, for instance, the sun being in the plane of the earth's equator, the tilting couple referred to as causing the precession disappears entirely. The precession due to this cause, therefore, ceases at the equinoxes. But the moon has also an effect in producing precession, similar to that of the sun, and due to the attraction of the moon on the equatorial protuberance. The moon does not move in the ecliptic, but in a plane inclined at about 5° to it, and the nodes of the orbit, i.e. the points where the orbit cuts the ecliptic, are not fixed but move backwards along the ecliptic, making a complete revolution in about 18 years and 7 months. The motion is accordingly somewhat complicated, but the result, so far as the motion of the First Point of Aries is concerned, is that that point moves along the ecliptic with a non-uniform motion.

The actual point of intersection of the ecliptic and the equator at any instant is the *true equinox*. An imaginary point moving along the ecliptic with a uniform velocity equal to the mean velocity of the true equinox is the *mean equinox*, and the correction to be applied to the position of the true equinox to obtain the position of the mean equinox is called the *equation of the equinoxes*, in the same way that the equation of time is the correction to be applied to the position of the true sun to obtain the position of the mean sun (so far as R.A. is concerned at least).

As the position of the true sun gives apparent time, so the apparent R.A. and declination at any instant are measured from the true equinox and equator at that instant.

APPENDIX III

NOMENCLATURE AND MAGNITUDE OF STARS—STAR CHARTS—HINTS ON OBSERVING—OUTFIT REQUIRED

Nomenclature.—It is necessary that the observer shall be able to identify the star observed, whether a previously selected star is observed or one which appears suitable at the time of observation. To this end the following remarks on the nomenclature, magnitude, and location of stars will be of help to the observer.

The ancients divided the stars into groups or constellations, and these have been extended in later times for convenience in designating any particular star. The names of the groups in some cases are based on a real or fancied resemblance to some animal or inanimate object as outlined by the stars in the group, as, for example, the constellations of Cygnus (the Swan), Scorpio (the Scorpion), Crux (the Cross). In order to distinguish the stars forming any particular group from one another they are each assigned a letter of the Greek alphabet, beginning with α for the brightest; these letters indicating the degree of brightness refer only to the particular constellation, and do not compare the brightness of stars in different constellations; and, further, the magnitude is not in all cases according to the order of the letter. For example, α Leonis is the brightest star in the constellation of Leo, β Aquilæ is the second brightest in the constellation of Aquila, but α Canis Majoris is a brighter star than α Ursæ Minoris (Polaris). If the 24 letters of the Greek alphabet do not suffice for any one constellation, the Roman alphabet is employed when they are exhausted. The fainter stars in a constellation are designated by numerals, increasing in the order of their right ascensions, but these are seldom necessary for survey work. Stars are sometimes referred to by quoting their index number in one of the various star catalogues which have been compiled. Thus B.A.C. 7504 is the star so numbered in the British Association Catalogue.

Many of the more important stars have had special names allotted to them thus: α Ursæ Minoris (Polaris), α Tauri (Aldebaran), α Geminorum (Castor), α Aurigæ (Capella), α Canis Majoris (Sirius), etc.

Magnitude.—For further convenience the brightness or *magnitude* of a star is indicated in the *Nautical Almanac* by a number: the smaller the number, the brighter the star. The magnitude of Polaris is given as 2·1, the brightness of a star of magnitude m being approximately $2\frac{1}{2}$ times that of a star of magnitude $(m+1)$. Ten stars are brighter than the first magnitude, indicated by unity, and their magnitude is indicated by decimal fractions; thus, the magnitude of α Centauri is 0·9, indicating a brightness 0·1 of a magnitude greater than the unit.

Sirius and Canopus have magnitudes of −1·4 and −1·0 respectively, i.e. their brightness is 2·4 and 2·0 magnitudes greater than a star of unit magnitude. Knowing the magnitude of the various stars, the observer can select one which, *ceteris paribus*, is most suitable for observation.

Stars up to the 6th magnitude are visible to the naked eye under favourable conditions, but for theodolite work those brighter than the 4th magnitude are best.

Star Charts.—In order to help to identify the stars, numerous Star Charts have been prepared; knowing the R.A. and δ of any star, its position is plotted on a celestial map or globe, R.A. circles and declinations being made use of instead of longitude circles and latitudes as used for plotting the position of a place on a terrestrial map or globe. From these charts the *relative* positions of the various stars are seen at once, their position in the celestial sphere being found by referring them to imaginary lines joining well-defined stars or groups of stars. The seven

bright stars of the constellation Ursa Major, for instance, form good guides to determine the position of other stars.

Two star charts (Figs. 35 and 36) are given at the end of the book, for the northern and southern hemispheres respectively. The principal constellations are shown, which may be used as a guide for the location of any required star, bearing in mind that the chart is merely a diagram representing the declinations and right ascensions, and is not a true projection of the celestial sphere; for example, three stars in a straight line on the chart do not in general lie on a great circle in the sky. The charts are prepared from the view-point of an observer at the centre of the celestial sphere. The following points concerning the relative positions of some of the various stars may be noted on the charts as an aid to the observer in locating any desired star, it being understood that when a straight line between two stars on the celestial sphere is referred to the portion of a great circle passing through these stars is meant.

Northern Hemisphere.—1. The two stars α and β Ursæ Majoris are known as the Pointers. The line joining β to α Ursæ Majoris when produced passes very near to the Pole Star, which is at the end of the tail of Ursa Minor (the Little Bear). The seven bright stars of Ursa Major form a group called the Plough.

2. If the curve joining the stars forming the handle of the Plough be continued away from the Pole, it will pass through α Boötis (Arcturus), about 30° away from η Ursæ Majoris.

3. If the line of the Pointers be produced to the Pole and an imaginary line be traced perpendicular to this line, and on the opposite side of it to the Plough, it will pass through α Aurigæ (Capella), about 45° from the Pole Star.

4. If this perpendicular line be traced on the same side as the Plough from the Pole Star it will pass near α Lyræ (Vega), about 50° from the Pole Star.

5. The diagonal α and γ Ursæ Majoris when produced passes close to α Virginis (Spica), about 30° beyond α Boötis (Arcturus).

6. The line through the Pointers, if produced about 60° past the Pole Star, forms one side of the Great Square of Pegasus, passing as it does through β and α Pegasi.

7. The Great Square of Pegasus is conspicuous, with one side produced through certain stars in the constellation of Andromeda, ending in α Persei.

8. α Lyræ (Vega) and α Aquilæ (Altair) form a nearly isosceles triangle with α Ophiuci.

9. The constellation of Cygnus (the Swan) lies between α Lyræ (Vega) and the Great Square of Pegasus.

10. α Boötis and α Virginis (Spica) form an equilateral triangle with β Leonis (Denebola).

Southern Hemisphere.—1. The constellation of Scorpio (the Scorpion), with three bright stars in both the head and body with a tail of smaller stars, is very conspicuous.

2. α Scorpii (Antares) forms a right-angled triangle with α Ophiuci and α Serpentis, the right angle being at the last-named.

3. The three stars in the Belt of Orion point to α Tauri (Aldebaran) and in the opposite direction to α Canis Majoris (Sirius). The magnitude of the last-named is −1·6, it being the brightest star in the heavens.

4. α Canis Majoris (Sirius) and β Orionis (Rigel) form an equilateral triangle with α Columbæ.

5. α Canis Majoris (Sirius) and β Geminorum form an isosceles triangle with α Leonis (Regulus).

6. α Leonis (Regulus), α Canis Minoris (Procyon), and α Canis Majoris (Sirius) form a trapezium with α Hydræ.

7. α Canis Minoris (Procyon) forms an equilateral triangle with α Orionis (Betelgeuse) and α Canis Majoris (Sirius).

8. Two stars α and β Centauri point to the constellation of Crux (Southern Cross) which is formed by four stars α, β, γ, and δ. The line $\gamma\alpha$ Crucis points to the South Pole of the celestial sphere, so that when the Cross is upright it is on the meridian. The South Pole lies about 30° from the centre of the Cross.

9. The line from β Orionis (Rigel) to α Columbæ if produced passes through α Argus (Canopis), β Argus, and β Centauri.

10. The line from α Argus (Canopus) to α Eridani if produced passes through α Piscis Australis (Fomalhaut).

The eye should be accustomed to estimate correctly an angular distance, as, for example, the angular distance between the two pointers of Ursa Major, viz. about $5\frac{1}{2}°$; the length of the Swan, i.e. from α to β Cygni, is about 24°, and the depth of the Southern Cross, i.e. from α to γ Crucis, about 6°.

A star chart may be prepared on the supposition that the observer is at the centre of the celestial sphere, as in fact he is, and in this case the stars will appear in their proper position when the chart is viewed; or the observer may be assumed to be outside the celestial sphere, vertically over his position on the earth, in which case the E. and W. points on the chart will correspond with the true E. and W. points, but the relative positions of the stars will be reversed when the chart is viewed. To avoid confusion, the position of a star should be considered by its R.A., and δ shown on the chart.

Hints on Observing.—Whenever possible, star observations should be done while there is still sufficient daylight to permit of the observer and the booker going about their work by its aid, and for the observer to see the cross wires clearly without artificial illumination thereof. A torch may be used without inconvenience for reading the levels and circles.

If star observations in daylight are to be done, the star must be sufficiently bright to be seen by the theodolite in use, i.e. probably of the 1st magnitude; but even stars of the 2nd and 3rd magnitude may be seen in the theodolite while there is still enough daylight to read circles, book angles, etc. For daylight observations the azimuth and altitude of the selected star are to be worked out beforehand for a convenient clock time: an example of such working out is given on page 151. It is obviously necessary, in order that the theodolite may be set to the computed azimuth, that the azimuth of the R.M. should be known; it may be determined in the earlier part of the day by a sun observation, with an accuracy greater than necessary.

It need hardly be stated that star observations are in general more accurate than sun observations, but a good observer can get surprisingly good results from the sun. In low latitudes the twilight is of short duration, but in higher latitudes the long twilight makes the above remarks particularly applicable.

In malarial districts it may be necessary to rely on sun observations, as mosquitoes may make night work difficult.

The theodolite should be supported on well-driven pegs when it is necessary to have the observing station on yielding ground, and should at all times be kept well sheltered and free from temperature differences. The theodolite should be set up about half an hour before any observing is to be done, so that all the parts may, as far as possible, have reached a steady temperature before the instrument is used. The eyepiece and the reading microscopes should be focused for the observer's eye, so that no delay may occur between the different pointings. For star observations in daylight it is necessary to set the object glass to stellar focus, as quite a small difference in the focus will render the image of a star invisible. It may be focused on the sun (not forgetting to use a dark glass on the eyepiece) or on a distant object. Observations in windy weather should, for obvious reasons, be avoided if possible.

Outfit Required.—In Chapter IV the type of theodolite suitable for observation has been mentioned, and in the same chapter time-keepers have been discussed. In addition to these the observer will require the following kit :

Barometer, for which a watch aneroid is suitable.

Thermometer, which may be combined with the aneroid.

Electric torch with refills ; also an oil lamp, if there is any chance of dry batteries not being available.

The Nautical Almanac for the current year.

Tables of Logarithms.

Star Charts.

A lamp for the R.M. for azimuth observations, consisting of a lamp placed in a box with a vertical slit, the width of slit varying from $\frac{1}{8}$ of an inch for a sight $\frac{1}{4}$ of a mile in length to $\frac{1}{2}$ an inch at one mile.

Forms for booking angles.

Forms for computations.

A small portable folding table is convenient for the booker.

Pegs and mallet.

APPENDIX IV

STANDARD TIME OF VARIOUS COUNTRIES

Time Zones

Fast on Greenwich Time—

12 hrs. Fiji Islands.
11½ ,, Chatham Is., New Zealand.
11 ,, Caroline Is., East Santa Cruz, Marshall Is., Nauru, New Caledonia, New Hebrides.
10 ,, Caroline Is. (west of 154° E.), Lord Howe Is., Marianne Is. (except Guam), New Guinea, New South Wales, Queensland, Tasmania, Victoria.
9½ ,, Northern Territory of Australia, part of New South Wales, South Australia.
9 ,, Corea, Japan, Jap (Caroline Is.).
8 ,, East China, Formosa, Hong Kong, Labuan, Macao, North Borneo, Philippine Is., Port Arthur, Portuguese Timor, West Australia.
7½ ,, Sarawak.
7 ,, Federated Malay States, French Indo-China, Hoihau, Siam, Straits Settlements.
6½ ,, Andaman Is., Burma, Nicobar Is.
5½ ,, Ceylon, India (except Calcutta and Portuguese India), Laccadive Is.
5 ,, Archipelago, Chagos, Portuguese India.
4 ,, Mauritius, Reunion, Seychelles.
3 ,, Eritrea, French Somaliland, Iraq, Italian Somaliland, Madagascar, Russia, Tanganyika.
2½ ,, Kenya, Uganda.

2 hrs. Bulgaria, Cyprus, Egypt, Esthonia (except Reval), Finland, Greece, Latvia, Palestine, Portuguese East Africa, Rhodesia, Rumania, Syria, Turkey, Union of South Africa, Western Russia. } *East European Time.*

1 hour Angola, Albania, Austria, Belgian Congo, Cameroons, Czecho-Slovavakia, Denmark, French Equatorial Africa, Hungary, Italy.
Libya, Lithuania, Luxemburg, Malta, Nigeria, Norway, Poland, Portuguese West Africa, Sardinia, Sicily, South-West Africa, Sweden, Switzerland, Tunis, West Africa, Yugo-Slavia. } *Mid-European Time.*

Greenwich Time—

Algeria, Balearic Is., Belgium, Channel Is., Corsica, Dahomey, Faroe Is., France, Gibraltar, Gold Coast (January to August), Great Britain, Irish Free State, Ivory Coast, Morocco, Northern Ireland, Portugal, Princes Is., Spain, St. Thomas I., Togoland.

Slow on Greenwich Time—

1 hour Ascension I., Canary Is., French Guinea, Iceland, Liberia, Madeira, Mauritania, Portuguese Guinea, Senegal, Sierra Leone.
2 hrs. Azores, Cape Verde Is., Fernando I., Trinidad I.
3 ,, Eastern Brazil.
3½ ,, Uruguay.
4 ,, Argentine, Barbados, Brazil (Central), Canada (E. of 67° long.), French Guiana, Grenada, Guadeloupe, Leeward Is., Martinique, Nova Scotia, Porto Rico, Prince Edward I., St. Lucia, St. Pierre, St. Vincent, Tobago, Trinidad. } *Maritime, Atlantic, or Inter-Colonial Time.*

$4\frac{1}{2}$	,,	Venezuela.	
5	,,	Bahamas, Brazil (Western), Canada (from 67th to 89th meridian), Cuba, Haiti, Jamaica, Panama, Peru, U.S.A. (Eastern States of).	*Eastern Time.*
6	,,	Canada (Central parts), Costa Rica, Florida, Honduras, Mexico (parts of), U.S.A. (Central parts of).	*Central Time.*
7	,,	Canada (West of 103rd meridian), Mexico, U.S.A. (Mountain States of).	*Mountain Time.*
8	,,	British Columbia, California, Nevada, Oregon, Washington.	*Pacific Time.*
9	,,	Yukon, Sitka.	
10	,,	Alaska, Austral Is., Low Archipelago, Marquesas, Society Is.	
$10\frac{1}{2}$	,,	Hawaiian Is.	
$11\frac{1}{2}$	,,	Western Samoa.	

In Canada and the United States five standard times are used, these territories being divided into zones approximately $7\frac{1}{2}°$ on either side of five central meridians. Brazil is divided into three zones in which the times 3, 4, and 5 hours slow on Greenwich are standard respectively, and the Soviet Government have made a similar arrangement for Russian Territory in Europe and Asia. In the Tonga Is., the time 12h 20m fast on Greenwich is used; though the longitude of these islands is about 11h 40m west of Greenwich, the Date Line passes to the east of them.

The standard time of Aden and British Somaliland is 2h 59m 54s fast on Greenwich, that of Calcutta 5h 53m 20s·8 fast, Holland 0h 19m 32s·1 fast, and of Reval 1h 38m 57s fast.

The Date or Calendar Line.—The line where the change of date occurs as adopted by the British Admiralty is a modification of the 180th meridian, and is drawn so as to include islands of any one group on the same side of the line or is modified for political reasons.

The line joins up the following eight points:

1.	Lat.	60° S.	Long.	180°
2.	,,	$51\frac{1}{2}°$ S.	,,	180°
3.	,,	$45\frac{1}{2}°$ S.	,,	$172\frac{1}{2}°$ W.
4.	,,	$15\frac{1}{2}°$ S.	,,	$172\frac{1}{2}°$ W
5.	,,	5° S.	,,	180°
6.	,,	48° N.	,,	180°
7.	,,	$52\frac{1}{2}°$ N.	,,	170° E.
8.	,,	65° N.	,,	169° W.

then through the centre of Bering Strait to a point Lat. 70° N., Long. 180°.

APPENDIX V

MEAN REFRACTION

OF CELESTIAL OBJECTS FOR TEMPERATURE 50°, AND PRESSURE 29·6 INCHES

Alt.	Refr.	Alt.	Refr.	Alt.	Refr.	Alt.	Refr.	Alt.	Refr.	Alt.	Refr.
° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″
0 0	33 0	4 30	10 48	9 0	5 48	17 0	3 04	26 0	1 56	45 30	0 56
5	32 10	35	10 39	5	5 45	10	3 03	10	1 55	46 0	0 55
10	31 22	40	10 29	10	5 42	20	3 01	20	1 55	30	0 54
15	30 36	45	10 20	15	5 39	30	2 59	30	1 54	47 00	0 53
20	29 50	50	10 11	20	5 36	40	2 57	40	1 53	30	0 52
25	29 06	55	10 02	25	5 34	50	2 55	50	1 52	48 0	0 51
30	28 23	5 0	9 54	30	5 31	18 0	2 54	27 0	1 51	30	0 50
35	27 41	5	9 46	35	5 28	10	2 52	15	1 50	49 0	0 49
40	27 00	10	9 38	40	5 25	20	2 51	30	1 49	30	0 49
45	26 20	15	9 30	45	5 23	30	2 49	45	1 48	50 0	0 48
50	25 42	20	9 23	50	5 20	40	2 47	28 0	1 47	30	0 47
55	25 05	25	9 15	55	5 18	50	2 46	15	1 46	51 0	0 46
1 0	24 29	30	9 08	10 0	5 15	19 0	2 44	30	1 45	30	0 45
5	23 54	35	9 01	10	5 10	10	2 43	45	1 44	52 0	0 44
10	23 20	40	8 54	20	5 05	20	2 41	29 0	1 42	30	0 44
15	22 47	45	8 47	30	5 00	30	2 40	20	1 41	53 0	0 43
20	22 15	50	8 41	40	4 56	40	2 38	40	1 40	30	0 42
25	21 44	55	8 34	50	4 51	50	2 37	30 0	1 38	54 0	0 41
30	21 15	6 0	8 28	11 0	4 47	20 0	2 35	20	1 37	55 0	0 40
35	20 46	5	8 21	10	4 43	10	2 34	40	1 36	56 0	0 38
40	20 18	10	8 15	20	4 39	20	2 32	31 0	1 35	57 0	0 37
45	19 51	15	8 09	30	4 34	30	2 31	20	1 33	58 0	0 35

Alt.	Refr.	Alt.	Refr.	Alt.	Refr.	Alt.	Refr.	Alt.	Refr.	Alt.	Refr.
50	19 25	20	8 03	40	4 31	40	2 29	40	1 32	59 0	0 34
55	19 00	25	7 57	50	4 27	50	2 28	32 0	1 31	60 0	0 33
2 0	18 35	30	7 51	12 0	4 23	21 0	2 27	20	1 30	61 0	0 32
5	18 11	35	7 45	10	4 20	10	2 26	40	1 29	62 0	0 30
10	17 48	40	7 40	20	4 16	20	2 25	33 0	1 28	63 0	0 29
15	17 26	45	7 35	30	4 13	30	2 24	20	1 26	64 0	0 28
20	17 04	50	7 30	40	4 09	40	2 23	40	1 25	65 0	0 26
25	16 44	55	7 25	50	4 06	50	2 21	34 0	1 24	66 0	0 25
30	16 24	7 0	7 20	13 0	4 03	22 0	2 20	20	1 23	67 0	0 24
35	16 04	5	7 15	10	4 00	10	2 19	40	1 22	68 0	0 23
40	15 45	10	7 11	20	3 57	20	2 18	35 0	1 21	69 0	0 22
45	15 27	15	7 06	30	3 54	30	2 17	20	1 20	70 0	0 21
50	15 09	20	7 02	40	3 51	40	2 16	40	1 19	71 0	0 19
55	14 52	25	6 57	50	3 48	50	2 15	36 0	1 18	72 0	0 18
3 0	14 36	30	6 53	14 0	3 45	23 0	2 14	30	1 17	73 0	0 17
5	14 20	35	6 49	10	3 43	10	2 13	37 0	1 16	74 0	0 16
10	14 04	40	6 45	20	3 40	20	2 12	30	1 14	75 0	0 15
15	13 39	45	6 41	30	3 38	30	2 11	38 0	1 13	76 0	0 14
20	13 34	50	6 37	40	3 35	40	2 10	30	1 11	77 0	0 13
25	13 20	55	6 33	50	3 33	50	2 09	39 0	1 10	78 0	0 12
30	13 06	8 0	6 29	15 0	3 30	24 0	2 08	30	1 09	79 0	0 11
35	12 53	5	6 25	10	3 28	10	2 07	40 0	1 08	80 0	0 10
40	12 40	10	6 22	20	3 26	20	2 06	30	1 07	81 0	0 9
45	12 27	15	6 18	30	3 24	30	2 05	41 0	1 05	82 0	0 8
50	12 15	20	6 15	40	3 21	40	2 04	30	1 04	83 0	0 7
55	12 03	25	6 11	50	3 19	50	2 03	42 0	1 03	84 0	0 6
4 0	11 51	30	6 08	16 0	3 17	25 0	2 02	30	1 02	85 0	0 5
5	11 40	35	6 05	10	3 15	10	2 01	43 0	1 01	86 0	0 4
10	11 29	40	6 01	20	3 12	20	2 00	30	1 00	87 0	0 3
15	11 18	45	5 58	30	3 10	30	1 59	44 0	0 59	88 0	0 2
20	11 08	50	5 55	40	3 08	40	1 58	30	0 58	89 0	0 1
25	10 58	55	5 52	50	3 06	50	1 57	45 0	0 57	90 0	0 0

CORRECTION OF MEAN REFRACTION

Ap. Alt.	Height of the Thermometer															
	20°	24°	28°	32°	36°	40°	44°	48°	52°	56°	60°	64°	68°	72°	76°	80°
° ′	′ + ″	′ + ″	′ + ″	′ + ″	′ + ″	′ + ″	′ + ″	′ + ″	′ − ″	′ − ″	′ − ″	′ − ″	′ − ″	′ − ″	′ − ″	′ − ″
0 0	2 40	2 18	1 55	1 33	1 11	51	31	10	10	29	48	1 7	1 25	1 43	2 1	2 19
0 10	2 32	2 12	1 49	1 28	1 8	48	29	10	9	27	45	1 4	1 21	1 38	1 54	2 12
0 20	2 25	2 5	1 44	1 24	1 4	46	28	9	9	26	44	1 1	1 17	1 33	1 49	2 5
0 30	2 18	1 59	1 39	1 20	1 1	44	26	9	8	25	41	58	1 13	1 28	1 43	1 59
0 40	2 11	1 53	1 34	1 16	58	42	25	8	8	24	39	55	1 10	1 24	1 38	1 53
0 50	2 5	1 48	1 29	1 12	55	40	24	8	8	23	37	52	1 6	1 20	1 34	1 48
1 0	1 59	1 43	1 25	1 9	53	38	23	8	7	21	36	50	1 3	1 17	1 30	1 43
1 10	1 53	1 38	1 21	1 6	50	36	22	7	7	20	34	48	1 0	1 13	1 26	1 38
1 20	1 48	1 33	1 17	1 3	48	34	21	7	6	19	32	45	57	1 9	1 21	1 33
1 30	1 43	1 29	1 14	1 0	46	32	20	7	6	18	31	43	54	1 6	1 18	1 29
1 40	1 39	1 25	1 11	57	44	31	18	6	6	18	30	41	52	1 4	1 15	1 25
1 50	1 35	1 21	1 8	55	42	30	17	6	6	17	28	39	50	1 1	1 11	1 21
2 0	1 31	1 18	1 5	53	39	29	17	6	5	16	27	37	48	58	1 8	1 18
2 20	1 23	1 11	1 0	48	37	26	16	5	5	15	25	35	44	54	1 3	1 11
2 40	1 17	1 6	55	44	34	24	14	5	5	14	23	32	41	50	58	1 6
3 0	1 11	1 1	51	41	32	22	13	4	4	13	21	30	38	46	54	1 1
3 20	1 6	57	47	38	29	21	13	4	4	12	20	28	35	43	50	57
3 40	1 2	53	44	36	28	20	12	4	4	11	18	26	33	40	47	53
4 0	58	49	41	33	26	18	11	4	4	10	17	24	31	37	44	50
4 30	53	45	38	31	24	17	10	3	3	9	16	22	28	34	40	45
5 0	48	41	35	28	22	16	9	3	3	9	14	20	26	31	36	40
5 30	45	38	32	26	20	14	9	3	3	8	13	19	24	29	34	38
6 0	41	35	30	24	19	13	8	3	2	7	12	17	22	26	31	35
6 30	38	33	28	22	17	12	7	2	2	7	11	15	20	24	29	33

7	36	31	26	21	16	12	7	2	2	6	10	14	19	23	27	31
8	32	27	23	19	15	10	6	2	2	5	9	13	16	20	24	27
9	28	24	20	16	13	9	5	2	2	5	8	11	14	18	21	24
10	26	22	18	15	12	8	5	2	1	4	7	10	13	16	19	22
11	23	20	17	14	11	8	5	2	1	4	7	9	12	15	18	20
12	21	18	15	13	10	7	4	1	1	4	6	9	11	13	16	18
13	20	17	14	12	9	7	4	1	1	3	6	8	10	12	15	17
14	18	16	13	11	8	6	4	1	1	3	5	7	9	11	14	16
15	17	15	12	10	8	6	3	1	1	3	5	7	9	11	13	15
16	16	14	12	9	7	5	3	1	1	3	5	6	8	10	12	14
17	15	13	11	9	7	5	3	1	1	3	4	6	8	9	11	13
18	14	12	10	8	6	5	3	1	1	2	4	6	7	9	10	12
19	13	11	9	8	6	4	3	1	1	2	4	5	7	8	10	11
20	13	11	9	7	6	4	2	1	1	2	4	5	6	8	9	11
25	10	8	7	6	5	3	2	1	1	2	3	4	5	6	7	8
30	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7
35	7	6	5	4	3	2	1	0	0	1	2	3	3	4	5	6
40	6	5	4	3	3	2	1	0	0	1	2	2	3	3	4	5
45	5	4	3	3	2	2	1	0	0	1	1	2	2	3	3	4
50	4	3	3	2	2	1	1	0	0	1	1	2	2	2	3	3
55	3	3	2	2	2	1	1	0	0	1	1	1	2	2	2	3
60	3	2	2	2	1	1	1	0	0	0	1	1	1	2	2	2
65	2	2	2	1	1	1	0	0	0	0	1	1	1	1	2	2
70	2	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1
80	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1
90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Height of the Barometer.				− 28·26	− 28·56	− 28·85	− 29·15	− 29·45	+ 29·75	+ 30·05	+ 30·35	+ 30·64	+ 30·93			

USE OF TABLES.—In the above table of Mean Refractions the value is given for the temperature 50° F., and pressure 29·6 inches.
The refraction, for example, for an altitude of 32° 10′, is found opposite to it in the next column, and is 1′ 30″.

Apparent altitude =	32°	10′	00″
Mean refraction =	0	01	30
True altitude =	32°	08′	30″

When the temperature and pressure differ from the above values the correction involved can be obtained from the table of Correction of Mean Refraction, thus:

Correction for altitude 32° 10′ and temperature 69° =	−0°	00′	04″
Correction for altitude 32° 10′ and pressure 30·35 =	+0	00	02
Correction for both =	−0	00	02
Mean refraction =	0	01	30
True refraction =	0	01	28
Apparent altitude =	32	10	00
Therefore true altitude =	32°	08′	32″

The above tables are accurate enough for most practical surveying purposes, but where greater accuracy is desired Bessel's Refractions may be used. Tables of these can be found in Chambers's *Seven Figure Mathematical Tables*, from which publication the above tables have been taken.

[APPENDIX VI.

APPENDIX VI

REDUCTION TO THE MERIDIAN IN CIRCUM-MERIDIAN OBSERVATIONS FOR LATITUDE

$$m = \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}$$

t	0m	1m	2m	3m	4m	5m	6m	7m	8m	9m	10m	11m	12m
s	"	"	"	"	"	"	"	"	"	"	"	"	"
0	0.0	2.0	7.8	17.7	31.4	49.1	70.7	96.2	125.7	159.0	196.3	237.5	282.7
2	0.0	2.1	8.1	18.1	31.9	49.7	71.5	97.1	126.7	160.2	197.6	239.0	284.3
4	0.0	2.2	8.4	18.5	32.5	50.4	72.3	98.0	127.8	161.4	198.9	240.4	285.8
6	0.0	2.4	8.7	18.9	33.0	51.1	73.1	99.0	128.8	162.6	200.3	241.9	287.4
8	0.0	2.5	8.9	19.3	33.5	51.7	73.9	99.9	129.9	163.8	201.6	243.3	289.0
10	0.1	2.7	9.2	19.7	34.1	52.4	74.7	100.8	130.9	165.0	202.9	244.8	290.6
12	0.1	2.8	9.5	20.1	34.6	53.1	75.5	101.8	132.0	166.2	204.2	246.3	292.2
14	0.1	3.0	9.8	20.5	35.2	53.8	76.3	102.7	133.1	167.4	205.6	247.7	293.8
16	0.1	3.1	10.1	20.9	35.7	54.5	77.1	103.7	134.2	168.6	206.9	249.2	295.4
18	0.2	3.3	10.4	21.4	36.3	55.1	77.9	104.6	135.3	169.8	208.3	250.7	297.0
20	0.2	3.5	10.7	21.8	36.9	55.8	78.8	105.6	136.3	171.0	209.6	252.2	298.6
22	0.3	3.7	11.0	22.3	37.4	56.5	79.6	106.6	137.4	172.2	211.0	253.6	300.2
24	0.3	3.8	11.3	22.7	38.0	57.3	80.4	107.5	138.5	173.5	212.3	255.1	301.8
26	0.4	4.0	11.6	23.1	38.6	58.0	81.3	108.5	139.6	174.7	213.7	256.6	303.5
28	0.4	4.2	11.9	23.6	39.2	58.7	82.1	109.5	140.7	175.9	215.1	258.1	305.1
30	0.5	4.4	12.3	24.0	39.8	59.4	83.0	110.4	141.8	177.2	216.4	259.6	306.7
32	0.6	4.6	12.6	24.5	40.3	60.1	83.8	111.4	143.0	178.4	217.8	261.1	308.4
34	0.6	4.8	12.9	25.0	40.9	60.8	84.7	112.4	144.1	179.7	219.2	262.6	310.0
36	0.7	5.0	13.3	25.4	41.5	61.6	85.5	113.4	145.2	180.9	220.6	264.1	311.6
38	0.8	5.2	13.6	25.9	42.1	62.3	86.4	114.4	146.3	182.2	222.0	265.7	313.3
40	0.9	5.4	14.0	26.4	42.8	63.0	87.3	115.4	147.5	183.5	223.4	267.2	315.0
42	1.0	5.7	14.3	26.9	43.4	63.8	88.1	116.4	148.6	184.7	224.8	268.7	316.6
44	1.1	5.9	14.7	27.4	44.0	64.5	89.0	117.4	149.7	186.0	226.2	270.3	318.3
46	1.2	6.1	15.0	27.9	44.6	65.3	89.9	118.4	150.9	187.3	227.6	271.8	319.9
48	1.3	6.4	15.4	28.3	45.2	66.0	90.8	119.5	152.0	188.5	229.0	273.3	321.6
50	1.4	6.6	15.8	28.8	45.9	66.8	91.7	120.5	153.2	189.8	230.4	274.9	323.3
52	1.5	6.8	16.1	29.4	46.5	67.6	92.6	121.5	154.3	191.1	231.8	276.4	325.0
54	1.6	7.1	16.5	29.9	47.1	68.3	93.5	122.5	155.5	192.4	233.2	278.0	326.7
56	1.7	7.3	16.9	30.4	47.8	69.1	94.4	123.6	156.7	193.7	234.7	279.5	328.4
58	1.8	7.6	17.3	30.9	48.4	69.9	95.3	124.6	157.8	195.0	236.1	281.1	330.0
60	2.0	7.8	17.7	31.4	49.1	70.7	96.2	125.7	159.0	196.3	237.5	282.7	331.7

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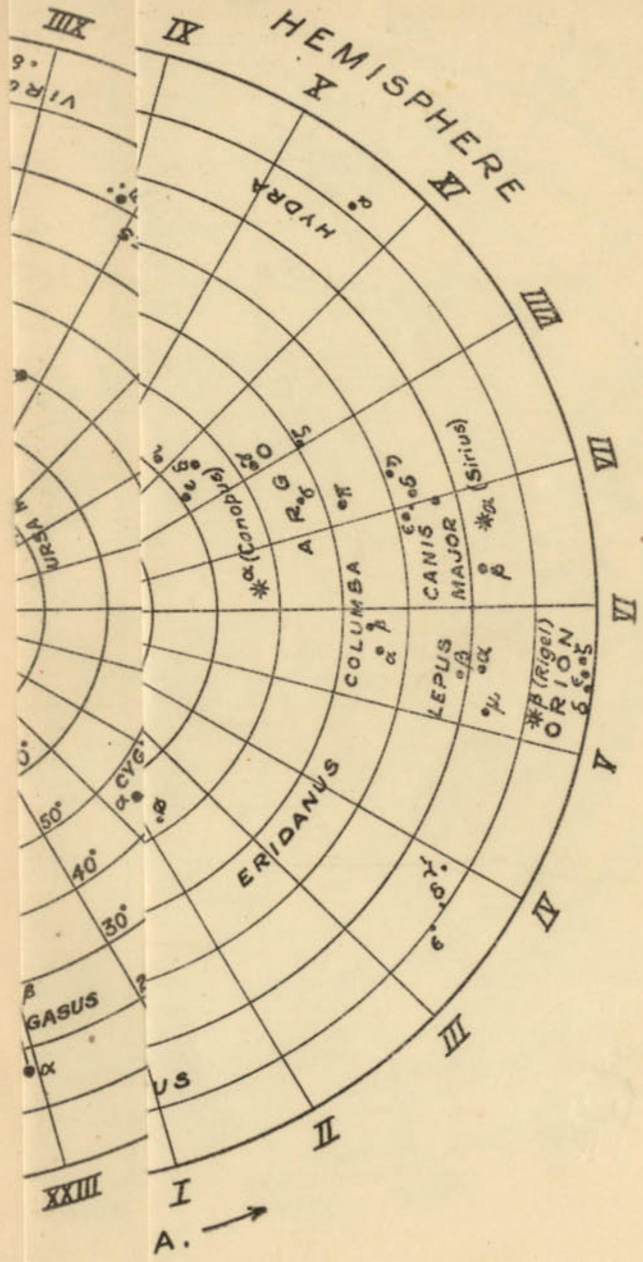

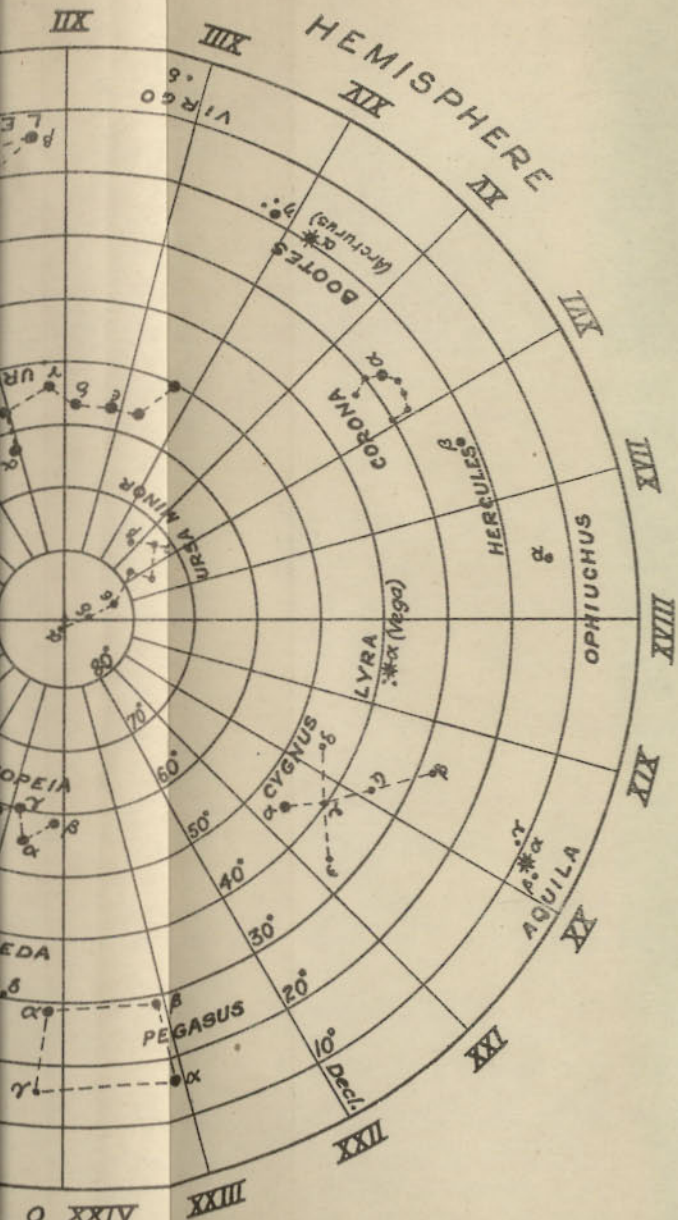
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FIG. 35.

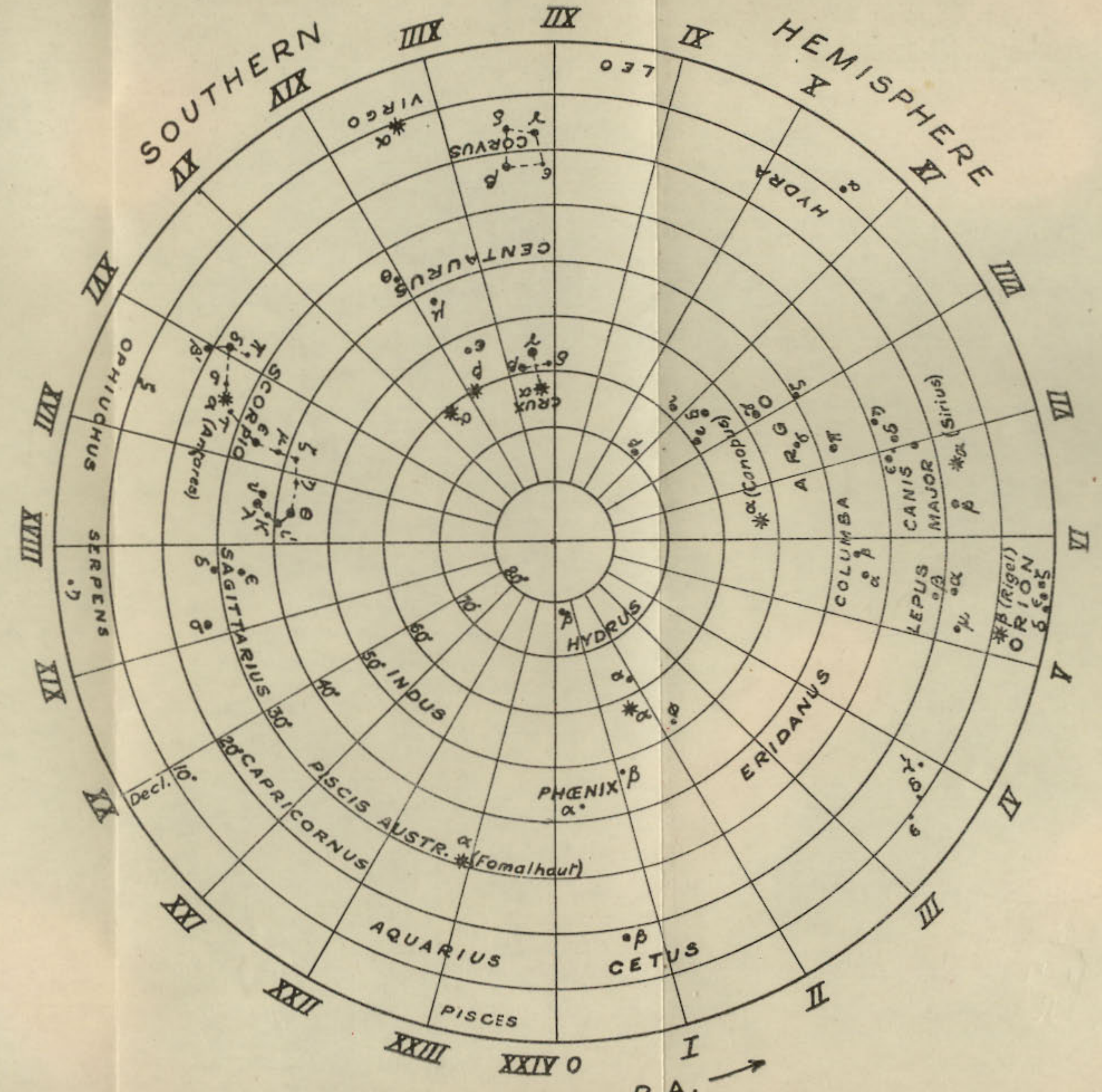
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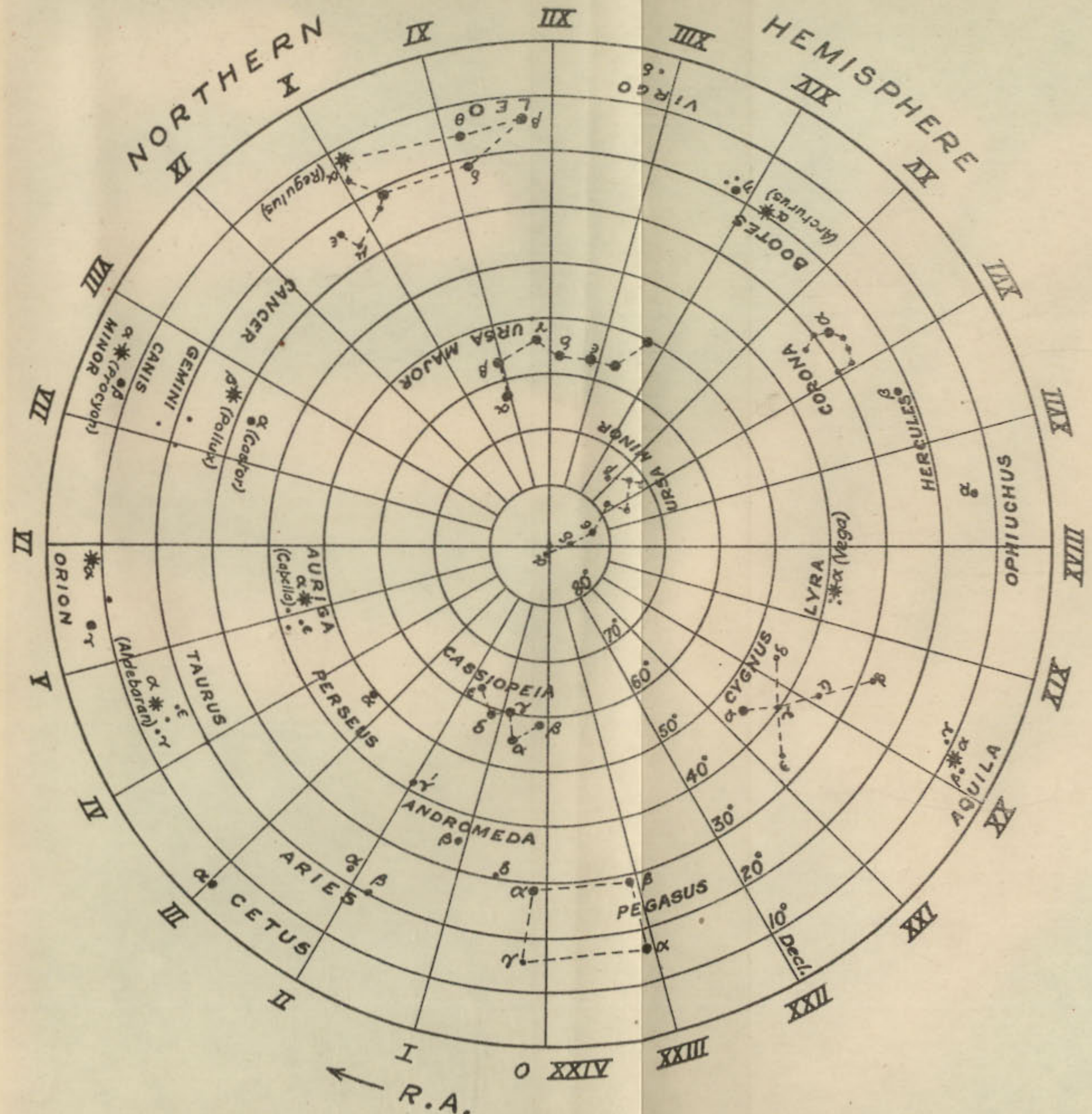
FIG. 36.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URSA MINOR
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GEMINI
β (Pollux)
α (Castor)
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AURIGA
α (Capella)
TAURUS
α (Aldebaran)
PERSEUS
CASSIOPEIA
CYGNUS
AQUILA
ANDROMEDA
ARIES
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PEGASUS

FIG. 35

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