A TEXTBOOK

ON

MINING ENGINEERING

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

ARITHMETIC
FORMULAS
GEOMETRY AND TRIGONOMETRY
GASES MET WITH IN MINES
MINE VENTILATION
MINE SURVEYING AND MAPPING

WITH PRACTICAL QUESTIONS AND EXAMPLES

1907

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61

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NOTICE.

This set of volumes differs from those printed for the first edition by reason of the fact that Volumes VIII and IX, the Tables and Formulas and Key Volumes, have been placed under one cover, thus making eight volumes in the set instead of nine, as formerly. Otherwise there are no changes, except the correction of a few typographical errors and the removal of the volume numbers from the title pages and backs.

PREFACE TO FIRST EDITION.

The first seven of the eight volumes of this set contain all the Instruction and Question Papers of the Full Mining Course. While the individual Papers are not exhaustive in their treatment of the subjects included in the Course, yet they give in clear, concise language just the information necessary to the practical man, and the subjects are so arranged as to avoid repetition.

These Papers present much matter that has never before been included in any single set of volumes and are thoroughly practical in their nature. Time and effort have not been spared in their preparation, and when considered as a whole they form a treatment of the entire subject that is more thoroughly up to date, is expressed in clearer language, and is more complete in detail than is contained in any other single set of books thus far published. The ground covered by the advanced Papers may be considered as embraced in four divisions—Drawing and Surveying, Mine Mechanical, Coal Mining, and Metal Mining.

The Drawing and Surveying Division contains the Papers on Geometrical Drawing and Mine Surveying and Mapping. The combination of the practical drawing and mapping work with the theory as presented in the Papers gives the student a very practical and comprehensive knowledge of the subject.

The Mine Mechanical Division covers the entire ground from Elementary Mechanics, Pneumatics, and Hydraulics to the practical consideration of mining machinery and the selection, arrangement, and maintenance of the same. The Papers treating on electrical subjects have been especially prepared and give the latest practice in the application of electricity to mining. The Paper on Percussive and Rotary Boring is the best treatment of the subject in print.

The Coal Mining Division provides thorough instruction in the art of coal mining, including Geology, Prospecting, Gases Met With in Mines, the opening up and operating of mines, together with the preparation of coal for the market. The ground is covered in a clearer and more thorough manner than in any other work on this subject.

The Metal Mining Division takes up the matters of special interest to metal miners, beginning with the preliminary subjects of Blowpiping, Mineralogy, Geology, Assaying, etc., which enable the prospector to recognize or test minerals for their value. The succeeding Papers treat of the laying out and operating of metal mines according to the most approved methods and include the many variations met with in practice. This division also includes Papers upon the arrangement of the surface plant, the framing of buildings, ore dressing, and the amalgamation of gold and silver ores.

All the Papers in this Course occur also in our Complete Coal Mining and Metal Mining Courses, this being a combination of the two last-named Courses. Hence, in order that the plates used in printing the Papers of the two Courses named might be used in printing the same Papers in this Course, we have so numbered the pages that each Paper or part is independent of the others; that is, each

Paper or part begins with page 1. In order to make the indexes intelligible, we have given each Paper and part a number. This number has been placed on the headline of each page opposite the page number, and to distinguish it from the page number, it has been preceded by the printer's section mark, §. Consequently a reference such as § 36, page 12, would be readily found by looking at the inside edges of the headlines until § 36 is found, and then through § 36 until page 12 is found.

The Question Papers have been given the same section numbers as the corresponding Instruction Papers, and are grouped together at the end of the volumes containing the Instruction Papers to which they refer.

The volumes of the present Course, the full mining, are eight in number:

Volume I (§§ 1–10) contains the Instruction and Question Papers on Arithmetic, Formulas, Geometry and Trigonometry, Gases Met With in Mines, Mine Ventilation, and Mine Surveying and Mapping.

Volume II (§§ 11–21) contains the Instruction and Question Papers on Economic Geology of Coal, Prospecting for Coal, Shafts, Slopes, and Drifts, Methods of Working Coal Mines, Mechanics, Steam and Steam Boilers, Steam Engines, Air and Air Compression, Hydromechanics and Pumping.

Volume III (§§ 22–27) contains the Instruction and Question Papers on Mine Haulage, Hoisting and Hoisting Appliances, Surface Arrangements of Bituminous Mines, Surface Arrangements of Anthracite Mines, Percussive and Rotary Boring, and Compressed-Air Coal-Cutting Machinery.

Volume IV (§§ 28–33) contains the Instruction and Question Papers on Dynamos and Motors, Electric Hoisting and Haulage, Electric Pumping, Signaling, and Lighting, and Electric Coal-Cutting Machinery.

Volume V (§§ 34–39) includes the instruction and Question Papers on Blowpiping, Mineralogy, Assaying, Geology, Prospecting, and Placer and Hydraulic Mining.

Volume VI (§§ 40–43) contains the Instruction and Question Papers on Preliminary Operations at Metal Mines, Metal Mining, Surface Arrangements at Metal Mines, and Ore Dressing and Milling.

Volume VII contains the Instruction Paper on Geometrical Drawing and the Plates that go with it. The Plates used in the Paper on Mine Surveying and Mapping are also included in this Volume.

Volume VIII contains the principal rules, tables, and formulas given in the various Instruction Papers. The letters used in the formulas are defined, and the formulas and tables are conveniently arranged for reference, so that the student is saved the labor and time of hunting them out in the Instruction Paper. This volume also contains the answers to the questions and the solutions to the examples in the Question Papers. Whenever it has been deemed inadvisable to answer a question, a reference to the proper article in the Instruction Paper has been given, the reading of which will enable the student to answer the question himself.

INTERNATIONAL CORRESPONDENCE SCHOOLS.

CONTENTS.

INDEX.

NOTE.—All items in this index refer first to the section (see the Preface) and then to the page of the section. Thus, "Air 5 15" means that air will be found on page 15 of section 5.

N.

O.

ARITHMETIC.

(PART 1.)

DEFINITIONS.

1. Arithmetic is the art of reckoning, or the study of numbers.

2. A **unit** is *one*, or a single thing, as *one*, *one* car, *one* prop, *one* drill.

3. A **number** is a unit, or a collection of units, as *one*, *three* rails, *five* wheels.

4. The **unit of a number** is one of the collection of units which constitutes the number. Thus, the unit of *twelve* is *one*, of *twenty* picks is *one* pick.

5. A **concrete number** is a number applied to some particular kind of object or quantity, as three *mules* five *shovels*, ten *tons*.

6. An **abstract number** is a number that is not applied to any object or quantity, as *three*, *five*, *ten*.

7. Like numbers are numbers which express units of the *same kind*, as six *drills* and ten *drills*, two *feet* and five *feet*.

8. Unlike numbers are numbers which express units of *different kinds*, as ten *rooms* and eight *entries*, seven *drivers* and five *trappers*.

NOTATION AND NUMERATION.

9. Numbers are expressed in three ways: (1) By words; (2) by figures; (3) by letters.

10. Notation is the art of expressing numbers by figures or letters.

11. Numeration is the art of reading the numbers which have been expressed by figures or letters.

12. The **Arabic notation** is the method of expressing numbers by figures. This method employs ten different **figures** to represent numbers, viz.:

Figures	0	1	2	3	4	5	6	7	8	9
Names	*naught, cipher, or zero.*	*one*	*two*	*three*	*four*	*five*	*six*	*seven*	*eight*	*nine*

The first character (0) is called **naught, cipher,** or **zero,** and, when standing alone, has no value.

The other nine figures are called **digits,** and each one has a value of its own.

Any whole number is called an **integer.**

13. As there are only *ten figures* used in expressing numbers, *each figure* must express a different value at different times.

14. The value of a figure depends upon its *position* in relation to others.

15. Figures have **simple** values and **local** or **relative** values.

16. The **simple** value of a figure is the value it expresses when standing alone.

17. The **local** or **relative** value is the *increased* value it expresses by having other figures placed on its right.

For instance, if we see the figure 6 standing alone, thus. 6

we consider it as *six units*, or simply **six.**

Place another 6 to the *left* of it; thus. 66

The original figure is still *six units*, but the second one is *ten times* 6, or 6 **tens.**

If a third 6 be now placed still one place further to the *left*, it is increased in value *ten times* more, thus making it 6 **hundreds** 666

A fourth 6 would be 6 **thousands** 6666

A fifth 6 would be 6 **tens of thousands,** or **sixty thousand** 66666

A sixth 6 would be 6 **hundreds of thousands** . 666666

A seventh 6 would be 6 **millions** 6666666

The entire line of seven figures is read *six millions, six hundred sixty-six thousands, six hundred sixty-six.*

18. The **increased value** of each of these figures is its *local* or *relative* value. Each figure is *ten times* greater in value than the one immediately on its *right*.

19. The **cipher** (0) has no value itself, but it is useful in determining the place of other figures. To represent the number *four hundred five*, two digits only are necessary, one to represent *four hundred*, and the other to represent *five units;* but if these two digits are placed together, as 45, the 4 (being in the second place) will mean 4 *tens*. To mean 4 *hundreds*, the 4 should have two figures on its right, and a cipher is therefore inserted in the place usually given to tens, to show that the number is composed of *hundreds* and *units* only, and that there are no tens. *Four hundred five* is therefore expressed as 405. If the number were *four thousand and five*, two ciphers would be inserted; thus, 4005. If it were *four hundred fifty*, it would have the cipher at the right-hand side to show that there were no units, and only *hundreds* and *tens;* thus, 450. *Four thousand and fifty* would be expressed 4050, the first cipher indicating that there are no hundreds and the second that there are no units.

NOTE.—When speaking of the figures of a number by referring to them as first figure, second figure, etc., always begin to count at the *left*. Thus, in the number 41,625, 4 is the first figure, 6 the third figure, 5 the fifth or last figure, etc.

20. In *reading* figures, it is usual to *point off* the number into groups of three figures each, beginning with the right-hand or **units** column, a comma (,) being used to point off these groups.

Billions.			*Millions.*			*Thousands.*			*Units.*		
Hundreds of Billions.	**Tens of Billions.**	**Billions.**	**Hundreds of Millions.**	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds of Units.	Tens of Units.	Units.
4	3	2,	1	9	8,	7	6	5,	4	3	2

In pointing off these figures, begin at the right-hand figure and count—*units*, *tens*, *hundreds;* the next group of three figures is *thousands*, therefore, we insert a comma (,) before beginning with them. Beginning at the figure 5, we say *thousands*, *tens of thousands*, *hundreds of thousands*, and insert another comma; we next read *millions*, *tens of millions*, *hundreds of millions*, and insert another comma; we then read *billions*, *tens of billions*, *hundreds of billions.*

The entire line of figures would be read: *Four hundred thirty-two billions*, *one hundred ninety-eight millions*, *seven hundred sixty-five thousands*, *four hundred thirty-two.* When we thus *read* a line of figures it is called **numeration,** and if the **numeration** be changed back to *figures*, it is called **notation.**

For instance, the writing of the figures,

72,584,623,

would be the **notation,** and the **numeration** would be *seventy-two millions*, *five hundred eighty-four thousands*, *six hundred twenty-three.*

21. NOTE.—It is customary to leave the *s* off the words millions, thousands, etc., in cases like the above, both in speaking and writing; hence, the above would usually be expressed, seventy-two million, five hundred eighty-four thousand, six hundred twenty-three.

22. The four fundamental processes of Arithmetic are **addition, subtraction, multiplication,** and **division.** They are called fundamental processes, because all operations in Arithmetic are based upon them.

ADDITION.

23. **Addition** is the process of finding the sum of two or more numbers. The sign of addition is $+$. It is read *plus*, and means *more.* Thus, $5 + 6$ is read *5 plus 6*, and means that 5 and 6 are to be added.

24. The sign of equality is $=$. It is read *equals* or *is equal to.* Thus, $5 + 6 = 11$ may be read *5 plus 6 equals 11.*

25. *Like numbers* can be added; *unlike numbers* can not be added. Thus, 6 dollars *can* be added to 7 dollars and the sum will be 13 dollars, but 6 dollars *can not* be added to 7 feet.

26. The following table gives the sum of any two numbers from 1 to 12:

TABLE 1.

1 and 1 are 2	2 and 1 are 3	3 and 1 are 4	4 and 1 are 5
1 and 2 are 3	2 and 2 are 4	3 and 2 are 5	4 and 2 are 6
1 and 3 are 4	2 and 3 are 5	3 and 3 are 6	4 and 3 are 7
1 and 4 are 5	2 and 4 are 6	3 and 4 are 7	4 and 4 are 8
1 and 5 are 6	2 and 5 are 7	3 and 5 are 8	4 and 5 are 9
1 and 6 are 7	2 and 6 are 8	3 and 6 are 9	4 and 6 are 10
1 and 7 are 8	2 and 7 are 9	3 and 7 are 10	4 and 7 are 11
1 and 8 are 9	2 and 8 are 10	3 and 8 are 11	4 and 8 are 12
1 and 9 are 10	2 and 9 are 11	3 and 9 are 12	4 and 9 are 13
1 and 10 are 11	2 and 10 are 12	3 and 10 are 13	4 and 10 are 14
1 and 11 are 12	2 and 11 are 13	3 and 11 are 14	4 and 11 are 15
1 and 12 are 13	2 and 12 are 14	3 and 12 are 15	4 and 12 are 16
5 and 1 are 6	6 and 1 are 7	7 and 1 are 8	8 and 1 are 9
5 and 2 are 7	6 and 2 are 8	7 and 2 are 9	8 and 2 are 10
5 and 3 are 8	6 and 3 are 9	7 and 3 are 10	8 and 3 are 11
5 and 4 are 9	6 and 4 are 10	7 and 4 are 11	8 and 4 are 12
5 and 5 are 10	6 and 5 are 11	7 and 5 are 12	8 and 5 are 13
5 and 6 are 11	6 and 6 are 12	7 and 6 are 13	8 and 6 are 14
5 and 7 are 12	6 and 7 are 13	7 and 7 are 14	8 and 7 are 15
5 and 8 are 13	6 and 8 are 14	7 and 8 are 15	8 and 8 are 16
5 and 9 are 14	6 and 9 are 15	7 and 9 are 16	8 and 9 are 17
5 and 10 are 15	6 and 10 are 16	7 and 10 are 17	8 and 10 are 18
5 and 11 are 16	6 and 11 are 17	7 and 11 are 18	8 and 11 are 19
5 and 12 are 17	6 and 12 are 18	7 and 12 are 19	8 and 12 are 20
9 and 1 are 10	10 and 1 are 11	11 and 1 are 12	12 and 1 are 13
9 and 2 are 11	10 and 2 are 12	11 and 2 are 13	12 and 2 are 14
9 and 3 are 12	10 and 3 are 13	11 and 3 are 14	12 and 3 are 15
9 and 4 are 13	10 and 4 are 14	11 and 4 are 15	12 and 4 are 16
9 and 5 are 14	10 and 5 are 15	11 and 5 are 16	12 and 5 are 17
9 and 6 are 15	10 and 6 are 16	11 and 6 are 17	12 and 6 are 18
9 and 7 are 16	10 and 7 are 17	11 and 7 are 18	12 and 7 are 19
9 and 8 are 17	10 and 8 are 18	11 and 8 are 19	12 and 8 are 20
9 and 9 are 18	10 and 9 are 19	11 and 9 are 20	12 and 9 are 21
9 and 10 are 19	10 and 10 are 20	11 and 10 are 21	12 and 10 are 22
9 and 11 are 20	10 and 11 are 21	11 and 11 are 22	12 and 11 are 23
9 and 12 are 21	10 and 12 are 22	11 and 12 are 23	12 and 12 are 24

This table should be carefully committed to memory. Since 0 has no value, the sum of any number and 0 is the number itself; thus, 17 and 0 are 17.

27. For *addition*, place the numbers to be added directly under each other, taking care to place *units* under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on.

When the numbers are thus written, the *right-hand figure of one number is placed directly under the right-hand figure* *of the number above it*, thus bringing the unit figures of all the numbers to be added in the same vertical line. Proceed as in the following examples:

28. Example.—What is the sum of 131, 222, 21, 2, and 413?

Solution.—

```
       131
       222
        21
         2
       413
       ---
sum    789  Ans.
```

Explanation.—After placing the numbers in proper order, begin at the bottom of the right-hand or *units* column, and add, mentally repeating the different sums. Thus, three and two are five and one are six and two are eight and one are nine, the sum of the numbers in *units* column. Place the 9 directly beneath as the first or *units* figure in the sum.

The sum of the numbers in the next or *tens* column equals 8 *tens*, which is the second or *tens* figure in the sum.

The sum of the numbers in the next or *hundreds* column equals 7 *hundreds*, which is the third or *hundreds* figure in the sum.

The sum or answer is 789.

29. Example.—What is the sum of 425, 36, 9,215, 4, and 907?

Solution.—

```
        425
         36
       9215
          4
        907
       ----
         27
         60
       1500
       9000
      -----
sum   10587  Ans.
```

Explanation.—The sum of the numbers in the first or units column is seven and four are eleven and five are sixteen and six are twenty-two and five are twenty-seven, or 27 units; i. e., two tens and seven units. Write 27 as shown.

The sum of the numbers in the second or tens column is six tens, or 60. Write 60 underneath 27 as shown. The sum of the numbers in the third or hundreds column is 15 hundreds, or 1,500. Write 1,500 under the two preceding results as shown. There is only one number in the fourth or thousands column, 9, which represents 9,000. Write 9,000 under the three preceding results. Adding these four results, the sum is 10,587, which is the sum of 425, 36, 9,215, 4, and 907.

NOTE.—It frequently happens, when adding a long column of figures, that the sum of two numbers, one of which does not occur in the addition table, is required. Thus, in the first column above, the sum of 16 and 6 was required. We know from the table that $6 + 6 = 12$; hence, the first figure of the sum is 2. Now, the sum of any number less than 20 and of any number less than 10 must be less than 30, since $20 + 10 = 30$; therefore, the sum is 22. Consequently, in cases of this kind, add the first figure of the larger number to the smaller number, and, if the result is greater than 9, increase the second figure of the larger number by 1. Thus, $44 + 7 = ?$ $4 + 7 = 11$; hence, $44 + 7 = 51$.

30. The addition may also be performed as follows:

```
      425
       36
     9215
        4
      907
    -----
sum 10587 Ans.
```

EXPLANATION.—The sum of the numbers in units column is 27 units, or 2 tens and 7 units. Write the 7 units as the first or right-hand figure in the sum. Reserve the two tens and add them to the figures in tens column. The sum of the figures in the tens column plus the 2 tens reserved and carried from the units column is 8, which is written down as the second figure in the sum. There is nothing to carry to the next column, because 8 is less than 10. The sum of the numbers in the next column is 15 hundreds, or 1 thousand and 5 hundreds. Write down the 5 as the third or hundreds figure in the sum and carry the 1 to the next column. $1 + 9 = 10$, which is written down at the left of the other figures.

The second method saves space and figures, but the first is to be preferred when adding a long column.

31. EXAMPLE.—Add the numbers in the column below:

SOLUTION.—

	890
	82
	90
	393
	281
	80
	770
	83
	492
	80
	383
	84
	191
sum	3899 Ans.

EXPLANATION.—The sum of the digits in the first column equals 19 units, or 1 ten and 9 units. Write down the 9 and carry 1 to the next column. The sum of the digits in the second column + 1 is 109 tens, or 10 hundreds and 9 tens. Write down the 9 and carry the 10 to the next column. The sum of the digits in this column plus the 10 reserved is 38. The entire sum is 3,899. Ans.

32. Rule.—I. *Begin at the right, add each column separately, and write the sum, if it be only one figure, under the column added.*

II. *If the sum of any column consists of two or more figures, put the right-hand figure of the sum under that column, and add the remaining figure or figures to the next column.*

33. Proof.—*To prove addition, add each column from top to bottom. If you obtain the same result as by adding from bottom to top, the work is probably correct.*

EXAMPLES FOR PRACTICE.

34. Find the sum of

(*a*) 104 + 203 + 613 + 214.
(*b*) 1,875 + 3,143 + 5,826 + 10,832.
(*c*) 4,865 + 2,145 + 8,173 + 40,084.
(*d*) 14,204 + 8,173 + 1,065 + 10,042.

Ans. { (*a*) 1,134. (*b*) 21,676. (*c*) 55,267. (*d*) 33,484.

(*e*)	10,832 + 4,145 + 3,133 + 5,872.	Ans.	(*e*) 23,982.
(*f*)	214 + 1,231 + 141 + 5,000.		(*f*) 6,586.
(*g*)	123 + 104 + 425 + 126 + 327.		(*g*) 1,105.
(*h*)	6,354 + 2,145 + 2,042 + 1,111 + 3,333.		(*h*) 14,985.

1. If mine No. 6 produced 7,018 tons in the first week in January, 7,236 tons in the second week, 6,348 tons in the third week, and 2,543 tons in the fourth week, how many tons were produced in the entire month? Ans. 23,145 tons.

2. A company received during one month 384 R. R. cars for mine No. 3, 785 R. R. cars for mine No. 8, and 1,056 R. R. cars for mine No. 11. What was the entire allowance for the month?
Ans. 2,225 R. R. cars.

3. A week's record of coal burned in an engine room is as follows: Monday, 1,800 pounds; Tuesday, 1,655 pounds; Wednesday, 1,725 pounds; Thursday, 1,690 pounds; Friday, 1,648 pounds; Saturday, 1,020 pounds. How much coal was burned during the week?
Ans. 9,538 pounds.

SUBTRACTION.

35. In Arithmetic, **subtraction** is the process of finding how much greater one number is than another.

The greater of the two numbers is called the **minuend.**

The smaller of the two numbers is called the **subtrahend.**

The number left after subtracting the subtrahend from the minuend is called the **difference** or **remainder.**

36. The sign of subtraction is —. It is read **minus,** and means *less*. Thus, 12 — 7 is read *12 minus 7*, and means that 7 is to be taken from 12.

37. EXAMPLE.—From 7,568 take 3,425.

SOLUTION.—

minuend	7568
subtrahend	3425
remainder	4143 Ans.

EXPLANATION.—Begin at the right-hand or *units* column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the entire remainder.

38. When there are more figures in the *minuend* than in the *subtrahend*, and when some figures in the minuend are *less* than the figures directly under them in the subtrahend, proceed as in the following example:

EXAMPLE.—From 8,453 take 844.

SOLUTION.—

minuend	8453
subtrahend	844
remainder	7609 Ans.

EXPLANATION.—Begin to subtract at the right-hand or *units* column. We can not take 4 from 3, and must, therefore, borrow 1 from 5 in *tens* column and annex it to the 3 in *units* column. The 1 *ten* = 10 *units*, which added to the 3 in *units* column = 13 *units*. 4 from 13 = 9, the first or *units* figure in the remainder.

Since we borrowed 1 from the 5, only 4 remains; 4 from 4 = 0, the second or *tens* figure. We can not take 8 from 4, and must, therefore, borrow 1 from 8 in *thousands* column. Since 1 *thousand* = 10 *hundreds*, 10 *hundreds* + 4 *hundreds* = 14 *hundreds*, and 8 from 14 = 6, the third or *hundreds* figure in the remainder.

Since we borrowed 1 from 8, only 7 remains, from which there is nothing to subtract; therefore, 7 is the next figure in the remainder or answer.

The operation of borrowing is performed by mentally placing 1 before the figure following the one from which it is borrowed. In the above example the 1 borrowed from 5 is placed before 3, making it 13, from which we subtract 4. The 1 borrowed from 8 is placed before 4, making 14, from which 8 is taken.

39. EXAMPLE.—Find the difference between 10,000 and 8,763.

SOLUTION.—

minuend	10000
subtrahend	8763
remainder	1237 Ans.

EXPLANATION.—In the above example we borrow 1 from the second column and place it before 0, making 10; 3 from

10 = 7. In the same way we borrow 1 and place it before the next cipher, making 10; but as we have borrowed 1 from this column and have taken it to the units column, only 9 remains, from which to subtract 6; 6 from 9 = 3. For the same reason we subtract 7 from 9 and 8 from 9 for the next two figures, and obtain a total remainder of 1,237.

40. Rule.—*Place the subtrahend (or smaller number) under the minuend or larger number, in the same manner as for addition, and proceed as in Arts.* ***37, 38,*** *and* ***39.***

41. Proof.—*To prove an example in subtraction, add the remainder to the subtrahend. The sum should equal the minuend. If it does not, a mistake has been made, and the work should be done over.*

Proof of the above example:

subtrahend	8763
remainder	1237
minuend	10000 Ans.

EXAMPLES FOR PRACTICE.

42. From

(*a*) 94,278 take 62,574.		(*a*) 31,704.
(*b*) 53,714 take 25,824.		(*b*) 27,890.
(*c*) 71,832 take 58,109.		(*c*) 13,723.
(*d*) 20,804 take 10,408.		(*d*) 10,396.
(*e*) 310,465 take 102,141.	Ans.	(*e*) 208,324.
(*f*) (81,043 + 1,041) take 14,831.		(*f*) 67,253.
(*g*) (20,482 + 18,216) take 21,214.		(*g*) 17,484.
(*h*) (2,040 + 1,213 + 542) take 3,791.		(*h*) 4.

1. The total weight of a mine car loaded with coal is 4,326 pounds, and the empty car weighs 1,564 pounds. What is the weight of the coal? Ans. 2,762 lb.

2. The output of a mine is 23,586 tons per month, of which 2,178 tons are taken for royalty. How many tons per month are left for the operator? Ans. 21,408 tons.

3. Two mines shipped 250,860 tons of coal in one year; one shipped 86,732 tons. How many tons did the other ship? Ans. 164,128 tons.

4. An entire mine produced 21,600 tons of coal in a month. The east and west sections furnished 10,680 tons, and the north section 4,332 tons. How many tons did the south section furnish?

Ans. 6,588 tons.

5. The total cost of erecting steam plant was $2,675. The engine cost $900; the boiler, $775; the fittings and connections, $225. The remainder was expended in erecting the engine house. What was the cost of the engine house? Ans. $775.

MULTIPLICATION.

43. To **multiply** a number is to *add* it to itself a certain number of times.

44. **Multiplication** is the process of multiplying one number by another.

The number thus added to itself, or the number to be multiplied, is called the **multiplicand.**

The number which shows how many times the *multiplicand* is to be taken, or the number by which we *multiply*, is called the **multiplier.**

The result obtained by multiplying is called the **product.**

45. The sign of multiplication is $\times$. It is read *times* or *multiplied by*. Thus, 9×6 is read *9 times 6*, or *9 multiplied by 6.*

46. It matters not in what order the numbers to be multiplied together are placed. Thus, 6×9 is the same as 9×6.

47. In the following table, the product of any two numbers (neither of which exceeds 12) may be found:

TABLE 2.

1 times 1 is 1	2 times 1 are 2	3 times 1 are 3
1 times 2 are 2	2 times 2 are 4	3 times 2 are 6
1 times 3 are 3	2 times 3 are 6	3 times 3 are 9
1 times 4 are 4	2 times 4 are 8	3 times 4 are 12
1 times 5 are 5	2 times 5 are 10	3 times 5 are 15
1 times 6 are 6	2 times 6 are 12	3 times 6 are 18
1 times 7 are 7	2 times 7 are 14	3 times 7 are 21
1 times 8 are 8	2 times 8 are 16	3 times 8 are 24
1 times 9 are 9	2 times 9 are 18	3 times 9 are 27
1 times 10 are 10	2 times 10 are 20	3 times 10 are 30
1 times 11 are 11	2 times 11 are 22	3 times 11 are 33
1 times 12 are 12	2 times 12 are 24	3 times 12 are 36
4 times 1 are 4	5 times 1 are 5	6 times 1 are 6
4 times 2 are 8	5 times 2 are 10	6 times 2 are 12
4 times 3 are 12	5 times 3 are 15	6 times 3 are 18
4 times 4 are 16	5 times 4 are 20	6 times 4 are 24
4 times 5 are 20	5 times 5 are 25	6 times 5 are 30
4 times 6 are 24	5 times 6 are 30	6 times 6 are 36
4 times 7 are 28	5 times 7 are 35	6 times 7 are 42
4 times 8 are 32	5 times 8 are 40	6 times 8 are 48
4 times 9 are 36	5 times 9 are 45	6 times 9 are 54
4 times 10 are 40	5 times 10 are 50	6 times 10 are 60
4 times 11 are 44	5 times 11 are 55	6 times 11 are 66
4 times 12 are 48	5 times 12 are 60	6 times 12 are 72
7 times 1 are 7	8 times 1 are 8	9 times 1 are 9
7 times 2 are 14	8 times 2 are 16	9 times 2 are 18
7 times 3 are 21	8 times 3 are 24	9 times 3 are 27
7 times 4 are 28	8 times 4 are 32	9 times 4 are 36
7 times 5 are 35	8 times 5 are 40	9 times 5 are 45
7 times 6 are 42	8 times 6 are 48	9 times 6 are 54
7 times 7 are 49	8 times 7 are 56	9 times 7 are 63
7 times 8 are 56	8 times 8 are 64	9 times 8 are 72
7 times 9 are 63	8 times 9 are 72	9 times 9 are 81
7 times 10 are 70	8 times 10 are 80	9 times 10 are 90
7 times 11 are 77	8 times 11 are 88	9 times 11 are 99
7 times 12 are 84	8 times 12 are 96	9 times 12 are 108
10 times 1 are 10	11 times 1 are 11	12 times 1 are 12
10 times 2 are 20	11 times 2 are 22	12 times 2 are 24
10 times 3 are 30	11 times 3 are 33	12 times 3 are 36
10 times 4 are 40	11 times 4 are 44	12 times 4 are 48
10 times 5 are 50	11 times 5 are 55	12 times 5 are 60
10 times 6 are 60	11 times 6 are 66	12 times 6 are 72
10 times 7 are 70	11 times 7 are 77	12 times 7 are 84
10 times 8 are 80	11 times 8 are 88	12 times 8 are 96
10 times 9 are 90	11 times 9 are 99	12 times 9 are 108
10 times 10 are 100	11 times 10 are 110	12 times 10 are 120
10 times 11 are 110	11 times 11 are 121	12 times 11 are 132
10 times 12 are 120	11 times 12 are 132	12 times 12 are 144

This table should be carefully committed to memory.

Since 0 has no value, the product of 0 and any number is 0.

48. To multiply a number by one figure only:

EXAMPLE.—Multiply 425 by 5.

SOLUTION.— *multiplicand* 425
multiplier 5
product 2125 Ans.

EXPLANATION.—For convenience, the *multiplier* is generally written *under* the *right-hand figure* of the *multiplicand.* On looking in the multiplication table, we see that 5 × 5 are 25. Multiplying the *first figure* at the *right* of the *multiplicand,* or 5, by the *multiplier* 5, it is seen that 5 times 5 units are 25 units, or 2 tens and 5 units. Write the *5 units* in *units* place in the *product*, and *reserve* the 2 tens to add to the product of tens. Looking in the multiplication table again, we see that 5 × 2 are 10. Multiplying the *second figure* of the multiplicand by the multiplier 5, we see that 5 times 2 tens are 10 tens, and 10 tens plus the 2 tens reserved, are 12 tens, or 1 hundred plus 2 tens. Write the *2 tens* in *tens* place, and reserve the 100 to add to the product of hundreds. Again, we see by the multiplication table that 5 × 4 are 20. Multiplying the *third* or *last figure* of the multiplicand by the multiplier 5, we see that 5 times 4 hundreds are 20 hundreds, and 20 hundreds plus the 1 hundred reserved, are 21 hundreds, or 2 thousands plus 1 hundred, which we write in *thousands* and *hundreds* places, respectively.

Hence, the product is 2,125.

This result is the same as adding 425 five times. Thus,

425
425
425
425
425
sum 2125 Ans.

EXAMPLES FOR PRACTICE.

49. Find the product of

(*a*)	61,483 × 6.		(*a*)	368,898.
(*b*)	12,375 × 5.	Ans.	(*b*)	61,875.
(*c*)	10,426 × 7.		(*c*)	72,982.
(*d*)	10,835 × 3.		(*d*)	32,505.

(*e*) 98,376 × 4.	Ans.	(*e*) 393,504.
(*f*) 10,873 × 8.		(*f*) 86,984.
(*g*) 71,543 × 9.		(*g*) 643,887.
(*h*) 218,734 × 2.		(*h*) 437,468.

1. If two men can drive a heading 5 feet in one day, how far can they drive it in 389 days? Ans. 1,945 ft.

2. If a string team can haul 8 cars a trip, how many cars can it haul in 169 trips? Ans. 1,352 cars.

3. A stationary engine makes 5,520 revolutions per hour. Running 9 hours a day, 5 days in the week, and 5 hours on Saturday, how many revolutions would it make in 4 weeks? Ans. 1,104,000 revolutions.

50. To multiply a number by two or more figures:

EXAMPLE.—Multiply 475 by 234.

SOLUTION.—

```
    multiplicand     475
      multiplier     234
                    ----
                    1900
                   1425
                   950
                  ------
         product  111150  Ans.
```

EXPLANATION.—For convenience, the multiplier is generally written *under* the multiplicand, placing units under units, tens under tens, etc.

We *can not* multiply by 234 at one operation; we must, therefore, multiply by the *parts* and then *add* the **partial products.**

The parts by which we are to multiply are 4 units, 3 tens, and 2 hundreds. 4 times 475 = 1,900, the first partial product; 3 times 475 = 1,425, the second partial product, the *right-hand figure of which is written directly under the figure multiplied by*, or 3; 2 times 475 = 950, the third partial product, the right-hand figure of which is written directly under the figure multiplied by, or 2.

The sum of these three partial products is 111,150, which is the *entire product.*

51. Rule.—I. *Write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.*

II. *Begin at the right and multiply each figure of the multiplicand by each successive figure of the multiplier, placing the right-hand figure of each partial product directly under the figure used as a multiplier.*

III. *The sum of the partial products will equal the required product.*

52. Proof.—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results agree, the work is correct.*

53. When there is a *cipher* in the *multiplier*, multiply the entire multiplicand by it; since the result will be zero, place a cipher under the cipher in the multiplier. Thus,

(*a*)	(*b*)	(*c*)	(*d*)
0	2	15	708
× 0	× 0	× 0	× 0
0 Ans.	0 Ans.	0 Ans.	0 Ans.

(*e*)	(*f*)	(*g*)
3114	4008	31264
203	305	1002
9342	20040	62528
62280	120240	3126400
632142 Ans.	1222440 Ans.	31326528 Ans.

In examples (*e*), (*f*), and (*g*), we multiply by 0 as directed above; then multiply by the next figure of the multiplier and place the first figure of the product alongside the 0, as shown.

EXAMPLES FOR PRACTICE.

54. Find the product of

(*a*)	3,842 × 26.	Ans. (*a*)	99,892.
(*b*)	3,716 × 45.	(*b*)	167,220.
(*c*)	1,817 × 124.	(*c*)	225,308.
(*d*)	675 × 38.	(*d*)	25,650.
(*e*)	1,875 × 33.	(*e*)	61,875.
(*f*)	4,836 × 47.	(*f*)	227,292.

(*g*)	5,682 × 543.	(*g*)	3,085,326
(*h*)	3,257 × 246.	(*h*)	801,222.
(*i*)	2,875 × 302.	(*i*)	868,250.
(*j*)	17,819 × 1,004.	(*j*)	17,890,276.
(*k*)	38,674 × 205.	Ans. (*k*)	7,928,170.
(*l*)	18,304 × 100.	(*l*)	1,830,400.
(*m*)	7,834 × 10.	(*m*)	78,340.
(*n*)	87,543 × 1,000.	(*n*)	87,543,000.
(*o*)	48,763 × 100.	(*o*)	4,876,300.

1. If two miners can dig 12 tons of coal in one day, how many tons can they dig in 18 days? Ans. 216 tons.

2. If it requires 18 mine cars to load one R. R. car, how many mine cars will be required to fill 2,437 R. R. cars? Ans. 43,866 mine cars.

3. The output of a mine is 1,236 tons per day; what is its output for a month of 26 days? Ans. 32,136 tons.

4. A Stanley header drives 36 feet of entry in one day. How many feet of entry will it drive in 289 days? Ans. 10,404 ft.

DIVISION.

55. Division is the process of finding how many times one number is contained in another of the same kind.

The number to be *divided* is called the **dividend.**

The number by which we *divide* is called the **divisor.**

The number which shows how many times the *divisor is contained in the dividend* is called the **quotient.**

56. The sign of division is ÷. It is read *divided by.* 54 ÷ 9 is read *54 divided by 9.* Another way to write *54 divided by 9* is $\frac{54}{9}$. Thus, 54 ÷ 9 = 6, or $\frac{54}{9} = 6$.

In both of these cases 54 is the dividend, and 9 is the divisor.

Division is the *reverse* of **multiplication.**

57. To divide when the divisor consists of but one figure, proceed as in the following example:

EXAMPLE.—What is the quotient of 875 + 7?

divisor dividend quotient

SOLUTION.— 7)875(125 Ans.

```
  7)875(125  Ans.
    7
    --
    17
    14
    --
     35
     35
     --
remainder 0
```

EXPLANATION.—7 is contained in 8 *hundreds* 1 *hundred* times. Place the one as the first or *left-hand figure* of the quotient. Multiply the divisor 7 by the 1 *hundred* of the quotient, and place the product 7 *hundreds* under the 8 *hundreds* in the dividend, and subtract. Beside the remainder 1, bring down the next or *tens* figure of the quotient, in this case 7, making 17 *tens;* 7 is contained in 17, 2 times. Write the 2 as the *second figure* of the quotient. Multiply the divisor 7 by the 2 in the quotient, and subtract the product from 17. Beside the remainder 3, bring down the next or *units* figure of the dividend, in this case 5, making 35 *units.* 7 is contained in 35, 5 times, which is placed in the quotient. Multiplying the divisor by the last figure of the quotient, 5 times $7 = 35$, which subtracted from 35, under which it is placed, leaves 0. Therefore, the quotient is 125. This method is called **long division.**

58. In **short division,** only the divisor, dividend, and quotient are written, the operations being performed mentally.

dividend

divisor 7) $8\,{}^{1}7\,{}^{3}5$

quotient 1 2 5 Ans.

The mental operation is as follows: 7 is contained in 8, once and one remainder; imagine 1 to be placed before 7 making 17; 7 is contained in 17, 2 times and 3 over; imagine 3 to be placed before 5 making 35; 7 is contained in 35, 5 times. These partial quotients, placed in order as they are found, make the entire quotient 125.

The small figures are placed in the example given to better illustrate the explanation; they are never written when actually performing division in this way.

59. If the divisor consists of *two or more* figures, proceed as in the following example:

EXAMPLE.—Divide 2,702,826 by 63.

```
               divisor   dividend    quotient
SOLUTION.—        63)2702826(42902  Ans.
                     252
                     ---
                      182
                      126
                      ---
                       568
                       567
                       ---
                         126
                         126
                         ---
                           0
```

EXPLANATION.—As 63 is not contained in the first two figures, 27, we must use the first three figures, 270. Now, by trial, we must find how many times 63 is contained in 270; 6 is contained in the first two figures of 270, 4 times. Place the 4 as the first or left-hand figure in the quotient. Multiply the divisor 63 by 4, and subtract the product 252 from 270. The remainder is 18, beside which we write the next figure of the dividend, 2, making 182. Now, 6 is contained in the first two figures of 182, 3 times, but on multiplying 63 by 3, we see that the product 189 is too great, so we try 2 as the second figure of the quotient. Multiplying the divisor 63 by 2, and subtracting the product 126 from 182, the remainder is 56, beside which we bring down the next figure of the dividend, making 568; 6 is contained in 56 about 9 times. Multiply the divisor 63 by 9 and subtract the product 567 from 568. The remainder is 1, and bringing down the next figure of the dividend, 2, gives 12. As 12 is smaller than 63, we write 0 in the quotient and bring down the next figure, 6, making 126. 63 is contained in 126, 2 times without a remainder. Therefore, 42,902 is the quotient.

60. Rule.—I. *Write the divisor at the left of the dividend, with a line between them.*

II. *Find how many times the divisor is contained in the lowest number of the left-hand figures of the dividend that will contain it, and write the result at the right of the dividend, with a line between, for the first figure of the quotient.*

III. *Multiply the divisor by this quotient; write the product under the partial dividend used, and subtract, annexing to the remainder the next figure of the dividend. Divide as before, and thus continue until all the figures of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend, and proceed as before.*

V. *If there be a remainder at last, write it after the quotient, with the divisor underneath.*

61. Proof.—*Multiply the quotient by the divisor, and add the remainder, if there be any, to the product. The result will be the dividend.*

```
                      divisor dividend quotient
Thus,                    63)4235(67 14/63  Ans.
                            378
                            ---
                             455
                             441
                             ---
            remainder         14

Proof,      quotient          67
            divisor           63
                             ---
                             201
                            402
                            ----
                            4221
            remainder         14
                            ----
            dividend        4235
```

EXAMPLES FOR PRACTICE.

62. Divide the following:

(*a*) 126,498 by 58.		(*a*) 2,181.
(*b*) 3,207,594 by 767.		(*b*) 4,182.
(*c*) 11,408,202 by 234.		(*c*) 48,753.
(*d*) 2,100,315 by 581.		(*d*) 3,615.
(*e*) 969,936 by 4,008.	Ans.	(*e*) 242.
(*f*) 7,481,888 by 1,021.		(*f*) 7,328.
(*g*) 1,525,915 by 5,003.		(*g*) 305.
(*h*) 1,646,301 by 381.		(*h*) 4,321.
(*i*) 1,486,968 by 371.		(*i*) 4,008.

1. At a certain mine there were 1,674 tons of coal loaded in one day. How many mine cars must have been dumped if each held 2 tons? Ans. 837 cars.

2. If one miner can dig 6 tons of coal in one day, how many miners will be required to dig 1,326 tons in one day? Ans. 221 miners.

3. There were 2,344 tons of coal loaded at a mine in one day. If 1,172 cars were required to load this coal, how many tons were put in each car? Ans. 2 tons.

4. How many R. R. cars will a siding 1,792 feet long hold, the cars each being 32 feet long? Ans. 56 R. R. cars.

5. A tail-rope system hauls 50 cars a trip. How many trips must be made to load 60 R. R. cars holding 30 tons each, if each mine car holds 2 tons? Ans. 18 trips.

CANCELATION.

63. **Cancelation** is the process of shortening operations in division by casting out equal factors from both dividend and divisor.

64. The **factors** of a number are those numbers which, when *multiplied* together, *equal the given number*. Thus, 5 and 3 are factors of 15, since $5 \times 3 = 15$. Likewise, 8 and 7 are the factors of 56, since $8 \times 7 = 56$.

65. A **prime number** is one which can not be divided by any number except itself and 1. Thus, 2, 3, 11, 29, etc., are prime numbers.

66. A **prime factor** is any factor that is a prime number.

Any number that is not a prime is called a **composite** number, and may be produced by multiplying together its prime factors. Thus, 60 is a composite number, and is equal to the product of its prime factors, $2 \times 2 \times 3 \times 5$.

Numbers are said to be **prime to each other** when no two of them can be divided by any number except 1; the numbers themselves *may* be either prime or composite. Thus, the numbers 3, 5, and 11 are prime to each other, so also are 22, 25, and 21—all composite numbers.

67. *Canceling equal factors from both dividend and divisor does not change the quotient.*

Canceling a factor in both dividend and divisor is the same as *dividing them both by the same number*, and this, evidently, *does not change the quotient*.

Write the numbers forming the *dividend* above a horizontal line, and those forming the *divisor* below it; then cancel the equal factors.

68. EXAMPLE.—Divide $4 \times 45 \times 60$ by 9×24.

SOLUTION.—Placing the dividend over the divisor, and canceling

$$\frac{\cancel{4} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\cancel{9} \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}} = \frac{50}{1} = 50. \quad \text{Ans.}$$

EXPLANATION.—The 4 in the dividend and 24 in the divisor are both divisible by 4, since 4 divided by 4 equals 1, and 24 divided by 4 equals 6. Cross off the four and write the 1 over it; also, cross off the 24 and write the 6 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times 60}{9 \times \underset{6}{\cancel{24}}}.$$

60 in the dividend and 6 in the divisor are divisible by 6, since 60 divided by 6 equals 10, and 6 divided by 6 equals 1. Cross off the 60 and write 10 over it; also, cross off the 6 and write 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times \overset{10}{\cancel{60}}}{9 \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}}.$$

Again, 45 in the dividend and 9 in the divisor are divisible by 9, since 45 divided by 9 equals 5, and 9 divided by 9 equals 1. Cross off the 45 and write the 5 over it; also, cross off the 9 and write the 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}}.$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number except 1, without a remainder, it is impossible to cancel further.

Multiply all the uncanceled numbers in the dividend together, and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in the dividend equals $5 \times 1 \times 10 = 50$; the product of all the uncanceled numbers in the divisor equals $1 \times 1 = 1$.

$$\text{Hence, } \frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}} = \frac{1 \times 5 \times 10}{1 \times 1} = 50. \quad \text{Ans.}$$

It is usual to omit the 1's when canceling them, instead of writing them as above.

69. Rule.—I. *Cancel the common factors from both the dividend and divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

EXAMPLES FOR PRACTICE.

70. Divide

	Ans.
(*a*) $14 \times 18 \times 16 \times 40$ by $7 \times 8 \times 6 \times 5 \times 3$.	(*a*) 32.
(*b*) $3 \times 65 \times 50 \times 100 \times 60$ by $30 \times 60 \times 13 \times 10$.	(*b*) 250.
(*c*) $8 \times 4 \times 3 \times 9 \times 11$ by $11 \times 9 \times 4 \times 3 \times 8$.	(*c*) 1.
(*d*) $164 \times 321 \times 6 \times 7 \times 4$ by $82 \times 321 \times 7$.	(*d*) 48.
(*e*) $50 \times 100 \times 200 \times 72$ by $1,000 \times 144 \times 100$.	(*e*) 5.
(*f*) $48 \times 63 \times 55 \times 49$ by $7 \times 21 \times 11 \times 48$.	(*f*) 105.
(*g*) $110 \times 150 \times 84 \times 32$ by $11 \times 15 \times 100 \times 64$.	(*g*) 42.
(*h*) $115 \times 120 \times 400 \times 1,000$ by $23 \times 1,000 \times 60 \times 800$.	(*h*) 5.

FRACTIONS.

71. A **fraction** is a part of a whole number: *One-half, one-third, two-fifths* are fractions.

72. *Two* numbers are required to express a fraction; one is called the **numerator,** and the other the **denominator.**

73. The numerator is placed above the denominator, with a line between them; as, $\frac{2}{3}$. 3 is the *denominator*, and shows into how many *equal parts* the *unit* or *one* is divided. The *numerator* 2 shows how many of these equal parts are taken or considered. The denominator also indicates the names of the parts.

$\frac{1}{2}$ is read one-half.
$\frac{3}{4}$ is read three-fourths.
$\frac{3}{8}$ is read three-eighths.
$\frac{5}{16}$ is read five-sixteenths.
$\frac{29}{47}$ is read twenty-nine-forty-sevenths.

74. In the expression "$\frac{3}{4}$ of an apple," the denominator 4 shows that the apple is to be (or has been) cut into 4 *equal parts*, and the numerator 3 shows that *three of these parts*, or *fourths*, are taken or considered.

If each of the parts, or fourths, of the apple were cut in *two equal pieces*, there would then be twice as many pieces as before, or $4 \times 2 = 8$ pieces in all; one of these pieces would be called one-eighth, and would be expressed in figures as $\frac{1}{8}$. Three of these pieces would be called three-eighths, and written $\frac{3}{8}$. The words three-fourths, three-eighths, five-sixteenths, etc., are abbreviations of three one-fourths, three one-eighths, five one-sixteenths, etc. It is evident that the *larger the denominator*, the greater is the number of parts into which anything is divided; consequently, the parts themselves are smaller, and the value of the fraction is less for the same number of parts taken. In other words, $\frac{7}{9}$, for example, is smaller than $\frac{7}{8}$, because if an object be divided into 9 parts, the parts are smaller than if the same object had been divided into 8 parts; and, since $\frac{1}{9}$ is smaller than $\frac{1}{8}$, it is clear that 7 one-ninths is a smaller amount than 7 one-eighths. Hence, also, $\frac{3}{8}$ is less than $\frac{3}{4}$.

75. The **value** of a fraction is the numerator divided by the denominator; as, $\frac{4}{2} = 2$, $\frac{6}{2} = 3$.

76. The line between the numerator and denominator means *divided by*, or $\div$.

$\frac{3}{4}$ is equivalent to $3 \div 4$.
$\frac{5}{8}$ is equivalent to $5 \div 8$.

77. The numerator and denominator of a fraction, when considered together, are called the **terms** of a fraction.

78. The *value* of a fraction whose numerator and denominator are equal is 1.

$\frac{4}{4}$, or four-fourths, $= 1$.
$\frac{8}{8}$, or eight-eighths, $= 1$.
$\frac{64}{64}$, or sixty-four-sixty-fourths, $= 1$.

79. A **proper fraction** is a fraction whose numerator is *less* than its denominator. Its value is *less* than 1; as, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{16}$.

80. An **improper fraction** is a fraction whose numerator *equals* or is *greater* than the denominator. Its value is 1 or *more* than 1; as, $\frac{4}{4}$, $\frac{9}{8}$, $\frac{43}{32}$.

81. A **mixed number** is a whole number and a fraction united. $4\frac{2}{3}$ is a mixed number, and is equivalent to $4 + \frac{2}{3}$. It is read *four and two-thirds.*

REDUCTION OF FRACTIONS.

82. **Reduction of fractions** is the process of changing their form without changing their value.

83. *A fraction is reduced to higher terms by multiplying both terms of the fraction by the same number.* Thus, $\frac{3}{4}$ is reduced to $\frac{6}{8}$ by multiplying both terms by 2.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}.$$

The value is not changed, since $\frac{3}{4} = \frac{6}{8}$. For, suppose that an object, say an apple, is divided into 8 equal parts. If these parts be arranged into 4 piles, each containing 2 parts, it is evident that each pile will be composed of the same amount of the entire apple as would have been the case had the apple been originally cut into 4 equal parts. Now, if one of these piles (containing 2 parts) be removed, there will be 3 piles left, each containing 2 equal parts, or 6 equal parts in all, i. e., six-eighths. But, since one pile, or one quarter, was removed, there are three-quarters left. Hence, $\frac{3}{4} = \frac{6}{8}$. The same course of reasoning may be applied to any similar case. Therefore, multiplying both terms of a fraction by the same number does not alter its value.

84. To reduce a fraction to an equal fraction having a given denominator:

EXAMPLE.—Reduce $\frac{7}{8}$ to an equal fraction having 96 for a denominator.

SOLUTION.—Both the numerator and the denominator must be multiplied by the same number in order not to change the value of the fraction. The denominator must be multiplied by some number which will, in this case, make the product 96; this number is evidently $96 \div 8 = 12$, since $8 \times 12 = 96$. Hence, $\frac{7 \times 12}{8 \times 12} = \frac{84}{96}$. Ans.

85. Rule.—*Divide the given denominator by the denominator of the given fraction, and multiply both terms of the fraction by the result.*

EXAMPLE.—Reduce $\frac{3}{4}$ to 100ths.

SOLUTION.— $100 \div 4 = 25$; hence, $\frac{3 \times 25}{4 \times 25} = \frac{75}{100}$. Ans.

86. *A fraction is reduced to lower terms by dividing both terms by the same number.* Thus, $\frac{8}{10}$ is reduced to $\frac{4}{5}$ by dividing both terms by 2.

$$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}.$$

That $\frac{8}{10} = \frac{4}{5}$ is readily seen from the explanation given in Art. **83**; for, multiplying both terms of the fraction $\frac{4}{5}$ by 2, $\frac{4 \times 2}{5 \times 2} = \frac{8}{10}$, and if $\frac{4}{5} = \frac{8}{10}$, $\frac{8}{10}$ must equal $\frac{4}{5}$. Hence, *dividing both terms of a fraction by the same number does not alter its value.*

87. A fraction is reduced to its *lowest terms* when its numerator and denominator can not both be divided by the same number without a remainder; as, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{11}{24}$, $\frac{8}{15}$. In other words, the numerator and denominator are prime to each other.

EXAMPLES FOR PRACTICE.

88. Reduce the following:

(*a*) $\frac{7}{16}$ to 128ths.			(*a*) $\frac{56}{128}$.
(*b*) $\frac{24}{132}$ to its lowest terms.			(*b*) $\frac{2}{11}$.
(*c*) $\frac{64}{1000}$ to its lowest terms.	Ans.		(*c*) $\frac{8}{125}$.
(*d*) $\frac{5}{7}$ to 49ths.			(*d*) $\frac{35}{49}$.
(*e*) $\frac{13}{16}$ to 10,000ths.			(*e*) $\frac{8125}{10000}$.

---

89. To reduce a whole number or mixed number to an improper fraction:

EXAMPLE.—How many fourths in 5?

SOLUTION.—Since there are 4 fourths in 1 ($\frac{4}{4} = 1$), in 5 there will be 5×4 fourths, or 20 fourths; i. e., $5 \times \frac{4}{4} = \frac{20}{4}$. Ans.

EXAMPLE.—Reduce $8\frac{3}{4}$ to an improper fraction.

SOLUTION.—$8 \times \frac{4}{4} = \frac{32}{4}$. $\frac{32}{4} + \frac{3}{4} = \frac{35}{4}$. Ans.

90. Rule.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product, and place the denominator under the result. If it is desired to reduce a whole number to a fraction, multiply the whole number by the denominator of the given fraction, and write the result over the denominator.*

EXAMPLES FOR PRACTICE.

91. Reduce to improper fractions:

(*a*) $4\frac{1}{8}$.		(*a*) $\frac{33}{8}$.
(*b*) $5\frac{1}{9}$.		(*b*) $\frac{46}{9}$.
(*c*) $10\frac{2}{10}$.	Ans.	(*c*) $\frac{102}{10}$.
(*d*) $37\frac{3}{4}$.		(*d*) $\frac{151}{4}$.
(*e*) $50\frac{4}{5}$.		(*e*) $\frac{254}{5}$.
(*f*) Reduce 7 to a fraction whose denominator is 16.		(*f*) $\frac{112}{16}$.

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92. To reduce an improper fraction to a whole or mixed number:

EXAMPLE.—Reduce $\frac{21}{4}$ to a mixed number.

SOLUTION.—4 is contained in 21, 5 times and 1 remaining (see Art. **75**); as this is also divided by 4, its value is $\frac{1}{4}$. Therefore, $5 + \frac{1}{4}$, or $5\frac{1}{4}$ is the number. Ans.

93. Rule.—*Divide the numerator by the denominator, the quotient will be the whole number; the remainder, if there be any, will be the numerator of the fractional part of which the denominator is the same as the denominator of the improper fraction.*

EXAMPLES FOR PRACTICE.

94. Reduce to whole or mixed numbers:

(*a*) $\frac{145}{6}$.		(*a*) $24\frac{1}{6}$.
(*b*) $\frac{185}{3}$.		(*b*) $61\frac{2}{3}$.
(*c*) $\frac{701}{6}$.	Ans.	(*c*) $116\frac{5}{6}$.
(*d*) $\frac{149}{3}$.		(*d*) $49\frac{2}{3}$.
(*e*) $\frac{76}{19}$.		(*e*) 4.
(*f*) $\frac{125}{25}$.		(*f*) 5.

95. A **common denominator** of *two* or *more fractions* is a number which will contain (i. e., which may be divided by) the denominator of each of the given fractions without a remainder. The **least common denominator** is the least number that will contain each denominator of the given fractions without a remainder.

96. To find the least common denominator:

EXAMPLE.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

SOLUTION.—We first place the denominators in a row, separated by commas.

2)	4,	3,	9,	16
2)	2,	3,	9,	8
3)	1,	3,	9,	4
	1,	1,	3,	4

$2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator. Ans.

EXPLANATION.—Divide the numbers by some prime number that will divide at least two of them without a remainder (if possible), bringing down to the row below those denominators which will not contain the divisor without a remainder. Dividing each of the numbers by 2, the second row becomes 2, 3, 9, 8, since 2 will not divide 3 and 9 without a remainder. Dividing again by 2, the result is 1, 3, 9, 4.

Dividing the third row by 3, the result is 1, 1, 3, 4. Since the remaining numbers are prime to each other, we cease dividing further. The product of all the divisors and of the numbers prime to each other, is $2 \times 2 \times 3 \times 3 \times 4 = 144$, which is the required least common denominator.

97. Example.—Find the least common denominator of $\frac{4}{9}$, $\frac{5}{12}$, $\frac{7}{18}$.

Solution.—

$$\begin{array}{r|l} 3 & 9,\ 12,\ 18 \\ \hline 3 & 3,\ \ 4,\ \ 6 \\ \hline 2 & 1,\ \ 4,\ \ 2 \\ \hline & 1,\ \ 2,\ \ 1 \end{array}$$

$3 \times 3 \times 2 \times 2 = 36$. Ans.

98. To reduce two or more fractions to fractions having a common denominator:

Example.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to fractions having a common denominator.

Solution.—The common denominator is any number which will contain 3, 4, and 2. The *least* common denominator is 12, because it is the smallest number which can be divided by 3, 4, and 2 without a remainder.

$$\frac{2}{3} = \frac{8}{12}, \quad \frac{3}{4} = \frac{9}{12}, \quad \frac{1}{2} = \frac{6}{12}.$$

Reducing $\frac{2}{3}$ (see Art. **84**), 3 is contained in 12, 4 times. By multiplying both numerator and denominator of $\frac{2}{3}$ by 4, we find

$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$. In the same way we find $\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{2} = \frac{6}{12}$.

99. Rule.—*Divide the common denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

100. Reduce to fractions having a common denominator:

(*a*) $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*) $\frac{3}{16}$, $\frac{3}{4}$, $\frac{7}{32}$.
(*c*) $\frac{7}{8}$, $\frac{7}{88}$, $\frac{10}{11}$.
(*d*) $\frac{3}{5}$, $\frac{5}{8}$, $\frac{11}{40}$.
(*e*) $\frac{4}{10}$, $\frac{6}{40}$, $\frac{9}{20}$.
(*f*) $\frac{7}{15}$, $\frac{17}{30}$, $\frac{11}{30}$.

Ans. {
(*a*) $\frac{6}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*) $\frac{6}{32}$, $\frac{24}{32}$, $\frac{7}{32}$
(*c*) $\frac{77}{88}$, $\frac{7}{88}$, $\frac{80}{88}$.
(*d*) $\frac{24}{40}$, $\frac{25}{40}$, $\frac{11}{40}$.
(*e*) $\frac{16}{40}$, $\frac{6}{40}$, $\frac{18}{40}$.
(*f*) $\frac{14}{30}$, $\frac{17}{30}$, $\frac{11}{30}$.

ADDITION OF FRACTIONS.

101. *Fractions cannot be added unless they have a common denominator.* We cannot add $\frac{3}{4}$ to $\frac{7}{8}$ as they now stand, since the denominators represent parts of different sizes. Fourths cannot be added to eighths.

Suppose we divide an apple into 4 equal parts, and then divide 2 of these parts into two equal parts. It is evident that we shall have 2 one-fourths and 4 one-eighths. Now, if we add these parts, the result is $2 + 4 = 6$ something. But what is this something? It is not fourths, for six fourths are $1\frac{1}{2}$, and we had only 1 apple to begin with; neither is it eighths, for six eighths are $\frac{3}{4}$, which is less than 1 apple. By reducing the quarters to eighths, we have $\frac{2}{4} = \frac{4}{8}$, and adding the other 4 eighths, $4 + 4 = 8$ eighths. This result is correct, since $\frac{8}{8} = 1$. Or, we can, in this case, reduce the eighths to quarters. Thus, $\frac{4}{8} = \frac{2}{4}$; whence, adding $2 + 2 = 4$ quarters, a correct result since $\frac{4}{4} = 1$.

Before adding, fractions should be reduced to a common denominator, preferably the *least* common denominator.

102. EXAMPLE.—Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$.

SOLUTION.—The *least common denominator*, or the *least number* which will contain all the denominators, is 8.

$$\tfrac{1}{2} = \tfrac{4}{8},\ \tfrac{3}{4} = \tfrac{6}{8},\ \tfrac{5}{8} = \tfrac{5}{8}.$$

EXPLANATION.—As the denominator tells or indicates the *names* of the parts, the numerators only are added in order to obtain the total number of parts indicated by the denominator. Thus, 4 one-eighths plus 6 one-eighths plus 5 one-eighths =

$$\tfrac{4}{8} + \tfrac{6}{8} + \tfrac{5}{8} = \tfrac{4+6+5}{8} = \tfrac{15}{8} = 1\tfrac{7}{8}.\quad \text{Ans.}$$

103. EXAMPLE.—What is the sum of $12\frac{3}{4}$, $14\frac{5}{8}$, and $7\frac{5}{16}$?

SOLUTION.—The least common denominator in this case is 16.

$$\begin{aligned} 12\tfrac{3}{4} &= 12\tfrac{12}{16} \\ 14\tfrac{5}{8} &= 14\tfrac{10}{16} \\ 7\tfrac{5}{16} &= \ \ 7\tfrac{5}{16} \\ \hline \textit{sum} &= 33 + \tfrac{27}{16} = 33 + 1\tfrac{11}{16} = 34\tfrac{11}{16}.\quad \text{Ans.} \end{aligned}$$

The sum of the fractions $= \frac{27}{16}$ or $1\frac{11}{16}$, which added to the sum of the whole numbers $= 34\frac{11}{16}$.

Example.—What is the sum of 17, $13\frac{3}{16}$, $\frac{9}{32}$, and $3\frac{1}{4}$?

Solution.—The least common denominator is 32. $13\frac{3}{16} = 13\frac{6}{32}$, $3\frac{1}{4} = 3\frac{8}{32}$.

$$\begin{array}{r} 17 \\ 13\frac{6}{32} \\ \frac{9}{32} \\ 3\frac{8}{32} \\ \hline \textit{sum}\ 33\frac{23}{32} \end{array} \quad \text{Ans.}$$

104. Rule.—I. *Reduce the given fractions to fractions having the least common denominator, and write the sum of the numerators over the common denominator.*

II. *When there are mixed numbers and whole numbers add the fractions first, and if their sum is an improper fraction, reduce it to a mixed number, and add the whole number with the other whole numbers.*

EXAMPLES FOR PRACTICE.

105. Find the sum of

(*a*) $\frac{4}{6}$, $\frac{7}{24}$, $\frac{3}{8}$.	Ans.	(*a*) $1\frac{7}{18}$.
(*b*) $\frac{2}{3}$, $\frac{5}{15}$, $\frac{24}{45}$.		(*b*) $1\frac{8}{15}$.
(*c*) $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$.		(*c*) $1\frac{3}{16}$.
(*d*) $\frac{5}{6}$, $\frac{11}{72}$, $\frac{13}{18}$.		(*d*) $1\frac{51}{72}$.
(*e*) $\frac{10}{11}$, $\frac{6}{33}$, $\frac{23}{66}$.		(*e*) $1\frac{29}{66}$.
(*f*) $\frac{23}{45}$, $\frac{11}{15}$, $\frac{14}{45}$.		(*f*) $1\frac{5}{9}$.
(*g*) $\frac{4}{11}$, $\frac{7}{22}$, $\frac{14}{22}$.		(*g*) $1\frac{7}{22}$.
(*h*) $\frac{3}{7}$, $\frac{14}{49}$, $\frac{2}{7}$.		(*h*) 1.

1. An entry is driven $\frac{1}{8}$ mile to the north, then $\frac{1}{5}$ mile to the east, and finally $\frac{1}{3}$ mile to the south. What is the total length of the entry?
Ans. $\frac{79}{120}$ mile.

2. In going from the working face to the tipple a mine car travels $\frac{11}{13}$ mile in the mine, $\frac{14}{39}$ mile on an inclined plane, and $\frac{23}{26}$ mile on a tramway. What is the length of the entire road? Ans. $2\frac{7}{78}$ miles.

3. First east heading gets $\frac{1}{6}$ of the entire amount of air produced by the fan at the drift mouth, second east gets $\frac{1}{8}$, first west gets $\frac{1}{9}$, and second west gets $\frac{1}{10}$ of it. What amount of the air do these four headings get? Ans. $\frac{181}{360}$.

SUBTRACTION OF FRACTIONS.

106. *Fractions can not be subtracted without first reducing them to a common denominator.* This can be shown in the same manner as in the case of addition of fractions.

EXAMPLE.—Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

SOLUTION.—The common denominator is 16.

$$\frac{3}{8} = \frac{6}{16}. \quad \frac{13}{16} - \frac{6}{16} = \frac{13-6}{16} = \frac{7}{16}. \quad \text{Ans.}$$

107. EXAMPLE.—From 7 take $\frac{3}{8}$.

SOLUTION.—$1 = \frac{8}{8}$; therefore, since $7 = 6 + 1$, $7 = 6 + \frac{8}{8} = 6\frac{8}{8}$, or $6\frac{8}{8} - \frac{3}{8} = 6\frac{5}{8}$. Ans.

108. EXAMPLE.—What is the difference between $17\frac{9}{16}$ and $9\frac{15}{32}$?

SOLUTION.—The common denominator of the fractions is 32. $17\frac{9}{16} = 17\frac{18}{32}$.

$$\begin{array}{rl} \textit{minuend} & 17\frac{18}{32} \\ \textit{subtrahend} & 9\frac{15}{32} \\ \hline \textit{difference} & 8\frac{3}{32} \quad \text{Ans.} \end{array}$$

109. EXAMPLE.—From $9\frac{1}{4}$ take $4\frac{7}{16}$.

SOLUTION.—The common denominator of the fractions is 16. $9\frac{1}{4} = 9\frac{4}{16}$.

$$\begin{array}{rll} \textit{minuend} & 9\frac{4}{16} \text{ or } & 8\frac{20}{16} \\ \textit{subtrahend} & 4\frac{7}{16} & 4\frac{7}{16} \\ \hline \textit{difference} & 4\frac{13}{16} & 4\frac{13}{16} \quad \text{Ans.} \end{array}$$

EXPLANATION.—As the fraction in the subtrahend is greater than the fraction in the minuend, it can not be subtracted; therefore, *borrow* 1, or $\frac{16}{16}$, from the 9 in the minuend and add it to the $\frac{4}{16}$; $\frac{4}{16} + \frac{16}{16} = \frac{20}{16}$. $\frac{7}{16}$ from $\frac{20}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, 8 remains; 4 from $8 = 4$; $4 + \frac{13}{16} = 4\frac{13}{16}$.

110. EXAMPLE.—From 9 take $8\frac{3}{16}$.

SOLUTION.—

$$\begin{array}{rll} \textit{minuend} & 9 \text{ or } & 8\frac{16}{16} \\ \textit{subtrahend} & 8\frac{3}{16} & 8\frac{3}{16} \\ \hline \textit{difference} & \frac{13}{16} & \frac{13}{16} \quad \text{Ans.} \end{array}$$

EXPLANATION.—As there is no fraction in the minuend from which to take the fraction in the subtrahend, borrow 1, or $\frac{16}{16}$, from 9. $\frac{3}{16}$ from $\frac{16}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, only 8 is left. 8 from $8 = 0$.

111. Rule.—I. *Reduce the fractions to fractions having a common denominator. Subtract one numerator from the other and place the remainder over the common denominator.*

II. *When there are mixed numbers, subtract the fractions and whole numbers separately, and place the remainders side by side.*

III. *When the fraction in the subtrahend is greater than the fraction in the minuend, borrow 1 from the whole number in the minuend and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.*

IV. *When the minuend is a whole number, borrow 1; reduce it to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and place it over that fraction for subtraction.*

EXAMPLES FOR PRACTICE.

112. Subtract

(*a*) $\frac{10}{24}$ from $\frac{11}{12}$.	Ans.	(*a*) $\frac{1}{2}$.
(*b*) $\frac{7}{14}$ from $\frac{17}{28}$.		(*b*) $\frac{3}{28}$.
(*c*) $\frac{4}{30}$ from $\frac{5}{10}$.		(*c*) $\frac{11}{30}$.
(*d*) $\frac{15}{35}$ from $\frac{45}{70}$.		(*d*) $\frac{3}{14}$.
(*e*) $\frac{15}{16}$ from $\frac{57}{48}$.		(*e*) $\frac{1}{4}$.
(*f*) 13$\frac{1}{4}$ from 30$\frac{1}{2}$.		(*f*) 17$\frac{1}{4}$.
(*g*) 12$\frac{1}{8}$ from 27.		(*g*) 14$\frac{7}{8}$.
(*h*) 5$\frac{1}{4}$ from 30.		(*h*) 24$\frac{3}{4}$.

1. A team driver is paid \$2.17$\frac{1}{2}$ a day, and a gathering driver \$1.66$\frac{3}{4}$ a day. What is the difference in their wages for one day? Ans. \$.50$\frac{3}{4}$.

2. The yardage for the main entries is \$1.57$\frac{1}{4}$, and for the butt entries it is \$.55$\frac{3}{8}$. What is the difference in the price per yard for both entries? Ans. \$1.01$\frac{7}{8}$.

3. A miner's expenses for oil and powder were \$3.72$\frac{1}{8}$. If the oil cost \$.87$\frac{1}{2}$, what did the powder cost? Ans. \$2.84$\frac{5}{8}$.

MULTIPLICATION OF FRACTIONS.

113. *In multiplication of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

114. *Multiplying the numerator or dividing the denominator multiplies the fraction.*

EXAMPLE.—Multiply $\frac{3}{4}$ by 4.

SOLUTION.— $$\tfrac{3}{4} \times 4 = \frac{3 \times 4}{4} = \tfrac{12}{4} = 3. \quad \text{Ans.}$$

$$\text{Or} \quad \tfrac{3}{4} \times 4 = \frac{3}{4 \div 4} = \tfrac{3}{1} = 3. \quad \text{Ans.}$$

The word "of" when placed between two fractions, or between a fraction and a whole number, means the same as $\times$, or times. Thus,

$$\tfrac{3}{4} \text{ of } 4 = \tfrac{3}{4} \times 4 = 3.$$

$$\tfrac{1}{8} \text{ of } \tfrac{5}{16} = \tfrac{1}{8} \times \tfrac{5}{16} = \tfrac{5}{128}.$$

EXAMPLE.—Multiply $\frac{3}{8}$ by 2.

SOLUTION.— $$2 \times \tfrac{3}{8} = \frac{3 \times 2}{8} = \tfrac{6}{8} = \tfrac{3}{4}. \quad \text{Ans.}$$

$$\text{or} \quad 2 \times \tfrac{3}{8} = \frac{3}{8 \div 2} = \tfrac{3}{4}. \quad \text{Ans.}$$

115. EXAMPLE.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION.— $$\tfrac{4}{16} \times \tfrac{7}{8} = \frac{4 \times 7}{16 \times 8} = \tfrac{28}{128} = \tfrac{7}{32}. \quad \text{Ans.}$$

or, by cancelation, $$\frac{\not{4} \times 7}{\underset{4}{\not{16}} \times 8} = \frac{7}{4 \times 8} = \tfrac{7}{32}. \quad \text{Ans.}$$

116. EXAMPLE.—What is $\frac{4}{8}$ of $\frac{3}{4}$ of $\frac{16}{32}$?

SOLUTION.— $$\frac{\not{4} \times 3 \times \not{16}}{8 \times \not{4} \times \underset{2}{\not{32}}} = \frac{3}{8 \times 2} = \frac{3}{16}. \quad \text{Ans.}$$

117. EXAMPLE.—What is the product of $9\frac{3}{4}$ and $5\frac{5}{8}$?

SOLUTION.—$9\frac{3}{4} = \frac{39}{4}$; $5\frac{5}{8} = \frac{45}{8}$.

$$\tfrac{39}{4} \times \tfrac{45}{8} = \frac{39 \times 45}{4 \times 8} = \tfrac{1755}{32} = 54\tfrac{27}{32}. \quad \text{Ans.}$$

118. EXAMPLE.—Multiply $15\frac{7}{8}$ by 3.

SOLUTION.—

$$\begin{array}{r} 15\tfrac{7}{8} \\ \underline{3} \\ 47\tfrac{5}{8} \end{array} \quad \text{or} \quad \begin{array}{l} 15\tfrac{7}{8} \\ \underline{\;\;3} \\ 45 + \tfrac{21}{8} = 45 + 2\tfrac{5}{8} = 47\tfrac{5}{8}. \quad \text{Ans.} \end{array}$$

119. Rule.—I. *Divide the product of the numerators by the product of the denominators. All factors common to the numerators and denominators should first be cast out by cancelation.*

II. *To multiply one mixed number by another, reduce them both to improper fractions.*

III. ***To multiply a mixed number by a whole number, first multiply the fractional part by the multiplier, and if the product is an improper fraction, reduce it to a mixed number, and add the whole number part to the product of the multiplier and whole number.***

EXAMPLES FOR PRACTICE.

120. Find the product of

(a) $7 \times \frac{3}{16}$.	Ans.	(a) $1\frac{5}{16}$.
(b) $14 \times \frac{5}{16}$.		(b) $4\frac{3}{8}$.
(c) $\frac{21}{32} \times \frac{5}{14}$.		(c) $\frac{15}{64}$.
(d) $\frac{16}{27} \times 4$.		(d) $2\frac{10}{27}$.
(e) $\frac{19}{18} \times 7$.		(e) $7\frac{7}{18}$.
(f) $17\frac{18}{21} \times 7$.		(f) 125.
(g) $\frac{105}{224} \times 32$.		(g) 15.
(h) $\frac{15}{28} \times 14$.		(h) $7\frac{1}{2}$.

1. A trapper is paid $\frac{7}{8}$ of a dollar a day. What will be his wages for 23 days? Ans. $20\frac{1}{8}$ dollars.

2. A contractor's entire earnings were 16 yards of rock tunnel at $37\frac{1}{7}$ dollars per yard, and his expenses were $442\frac{1}{8}$ dollars. What were his net earnings? Ans. $152\frac{9}{56}$ dollars.

3. A ratchet and two drills cost 24 dollars; one drill cost $2\frac{1}{8}$ dollars and the other $3\frac{13}{16}$ dollars. What was the cost of the ratchet? Ans. $18\frac{1}{16}$ dollars.

DIVISION OF FRACTIONS.

121. *In division of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

122. *Dividing the numerator or multiplying the denominator, divides the fraction.*

EXAMPLE.—Divide $\frac{6}{8}$ by 3.

SOLUTION.—When *dividing* the *numerator*, we have

$$\frac{6}{8} \div 3 = \frac{6 \div 3}{8} = \frac{2}{8} = \frac{1}{4}. \text{ Ans.}$$

When *multiplying* the *denominator*, we have

$$\frac{6}{8} \div 3 = \frac{6}{8 \times 3} = \frac{6}{24} = \frac{1}{4}. \text{ Ans.}$$

EXAMPLE.—Divide $\frac{3}{16}$ by 2.

SOLUTION.— $\frac{3}{16} \div 2 = \frac{3}{16 \times 2} = \frac{3}{32}$. Ans.

EXAMPLE.—Divide $\frac{14}{32}$ by 7.

SOLUTION.— $\frac{14}{32} \div 7 = \frac{14 \div 7}{32} = \frac{2}{32} = \frac{1}{16}$. Ans.

123. To **invert** a fraction is to *turn it upside down;* that is, make the numerator and denominator change places.

Invert $\frac{3}{4}$ and it becomes $\frac{4}{3}$.

124. EXAMPLE.—Divide $\frac{9}{16}$ by $\frac{3}{16}$.

SOLUTION.—1. The fraction $\frac{3}{16}$ is contained in $\frac{9}{16}$ 3 times, for the denominators are the same, and one numerator is contained in the other 3 times. 2. If we now *invert* the *divisor* $\frac{3}{16}$, and multiply, the solution is

$$\frac{9}{16} \times \frac{16}{3} = \frac{\overset{3}{\cancel{9}} \times \cancel{16}}{\cancel{16} \times \cancel{3}} = 3. \quad \text{Ans.}$$

This brings the same quotient as in the first case.

125. EXAMPLE.—Divide $\frac{3}{8}$ by $\frac{1}{4}$.

SOLUTION.—We can not divide $\frac{3}{8}$ by $\frac{1}{4}$, as in the first case above, for the *denominators* are *not* the same; therefore, we must solve as in the second case.

$$\frac{3}{8} \div \frac{1}{4} = \frac{3}{8} \times \frac{4}{1} = \frac{3 \times \cancel{4}}{\underset{2}{\cancel{8}} \times 1} = \frac{3}{2} \text{ or } 1\tfrac{1}{2}. \quad \text{Ans.}$$

126. EXAMPLE.—Divide 5 by $\frac{10}{16}$.

SOLUTION.—$\frac{10}{16}$ inverted becomes $\frac{16}{10}$.

$$5 \times \frac{16}{10} = \frac{\cancel{5} \times \overset{8}{\cancel{16}}}{\underset{\cancel{2}}{\cancel{10}}} = 8. \quad \text{Ans.}$$

127. EXAMPLE.—How many times is $3\frac{3}{4}$ contained in $7\frac{7}{16}$?

SOLUTION.— $3\frac{3}{4} = \frac{15}{4}$; $7\frac{7}{16} = \frac{119}{16}$.

$\frac{15}{4}$ inverted equals $\frac{4}{15}$.

$$\frac{119}{16} \times \frac{4}{15} = \frac{119 \times \cancel{4}}{\underset{4}{\cancel{16}} \times 15} = \frac{119}{60} = 1\tfrac{59}{60}. \quad \text{Ans.}$$

128. **Rule.**—*Invert the divisor, and proceed as in multiplication.*

129. We have learned that a line placed between two numbers indicates that the number above the line is to be divided by the number below it. Thus, $\frac{18}{3}$ shows that 18 is to be divided by 3. This is also true if a fraction or a fractional expression be placed above or below a line.

$\dfrac{9}{\frac{3}{8}}$ means that 9 is to be divided by $\frac{3}{8}$; $\dfrac{3 \times 7}{\frac{8 + 4}{16}}$ means that 3×7 is to be divided by the value of $\dfrac{8 + 4}{16}$.

$\dfrac{\frac{1}{4}}{\frac{3}{8}}$ is the same as $\frac{1}{4} \div \frac{3}{8}$.

It will be noticed that there is a heavy line between the 9 and the $\frac{3}{8}$. This is necessary, since otherwise there would be nothing to show as to whether 9 was to be divided by $\frac{3}{8}$, or $\frac{9}{3}$ was to be divided by 8. Whenever a heavy line is used, as shown here, it indicates that *all above the line* is to be divided by *all below it*.

EXAMPLES FOR PRACTICE.

130. Divide

(*a*) 15 by $6\frac{3}{7}$.		(*a*) $2\frac{1}{3}$.
(*b*) 30 by $\frac{6}{8}$.		(*b*) 40.
(*c*) 172 by $\frac{4}{5}$.		(*c*) 215.
(*d*) $\frac{14}{18}$ by $1\frac{7}{16}$.	Ans.	(*d*) $\frac{112}{207}$.
(*e*) $\frac{192}{8}$ by $14\frac{3}{8}$.		(*e*) $1\frac{77}{115}$.
(*f*) $\frac{142}{27}$ by $17\frac{1}{3}$.		(*f*) $\frac{71}{234}$.
(*g*) $\frac{14}{18}$ by $\frac{145}{72}$.		(*g*) $\frac{56}{145}$.
(*h*) $\frac{128}{18}$ by $72\frac{1}{3}$.		(*h*) $\frac{64}{651}$.

1. If a track-layer laid 47 yards of track in $9\frac{7}{8}$ hours, how many yards of track did he average per hour? Ans. $4\frac{60}{79}$ yards.

2. A blacksmith can sharpen a pick in $\frac{1}{3}$ of a minute. If he works for 4 hours, how many picks can he sharpen? Ans. 720 picks.

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131. Whenever an expression like one of the three following is obtained, it may always be simplified by transposing the denominator from *above* to *below* the line, or from *below* to *above*, as the case may be, taking care, however, to indicate that the denominator when so transferred is a multiplier.

1. $\dfrac{\frac{3}{4}}{9} = \dfrac{3}{9 \times 4} = \frac{3}{36} = \frac{1}{12}$; for, regarding the fraction above the heavy line as the numerator of a fraction whose denominator is 9, $\dfrac{\frac{3}{4} \times 4}{9 \times 4} = \dfrac{3}{9 \times 4}$, as before.

2. $\frac{9}{\frac{3}{4}} = \frac{9 \times 4}{3} = 12$. The proof is the same as in the first case.

3. $\frac{\frac{5}{9}}{\frac{3}{4}} = \frac{5 \times 4}{3 \times 9} = \frac{20}{27}$; for, regarding $\frac{5}{9}$ as the numerator of a fraction whose denominator is $\frac{3}{4}$, $\frac{\frac{5}{9} \times 9}{\frac{3}{4} \times 9} = \frac{5}{\frac{3 \times 9}{4}}$; and

$$\frac{\frac{5}{\frac{3 \times 9}{4}} \times 4}{\frac{3 \times 9}{4} \times 4} = \frac{5 \times 4}{3 \times 9} = \frac{20}{27}, \text{ as above.}$$

This principle may be used to great advantage in cases like $\frac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times 4\frac{1}{2} \times 5\frac{1}{6}}$. Reducing the mixed numbers to fractions, the expression becomes $\frac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times \frac{9}{2} \times \frac{31}{6}}$. Now transferring the denominators of the fractions and canceling,

$$\frac{1 \times 310 \times 27 \times 72 \times 2 \times 6}{40 \times 9 \times 31 \times 4 \times 12} = \frac{1 \times \overset{\cancel{10}}{\cancel{310}} \times \overset{3}{\cancel{27}} \times \overset{\overset{3}{\cancel{6}}}{\cancel{72}} \times \cancel{2} \times \overset{3}{\cancel{6}}}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{40}} \times \cancel{9} \times \cancel{31} \times \cancel{4} \times \underset{2}{\cancel{12}}} =$$

$$\frac{27}{2} = 13\frac{1}{2}.$$

Greater exactness in results can usually be obtained by using this principle than can be obtained by reducing the fractions to decimals. The principle, however, should not be employed *if a sign of addition or subtraction occurs either above or below the dividing line.*

DECIMALS.

132. Decimals are *tenth* fractions; that is, the parts of a unit are expressed on the scale of ten, as *tenths*, *hundredths*, *thousandths*, etc.

133. The denominator, which is always 10 or a multiple of 10, as 10, 100, 1,000, etc., is not expressed as it would be in common fractions, by writing it under the numerator, with a line between them; as, $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$. The denominator is always understood, the numerator consisting of the figures on the right of the *unit* figure. In order to distinguish the unit figure, a period (.), called the **decimal point,** is placed between the unit figure and the next figure on the right. The decimal point may be regarded in two ways: first, as indicating that the number on the right is the numerator of a fraction whose denominator is 10, 100, 1,000, etc.; and second, as a part of the Arabic system of notation, each figure on the right being 10 times as large as the next succeeding figure, and 10 times as small as the next preceding figure, serving merely to point out the unit figure.

134. The *reading* of a decimal number depends upon the number of decimal places in it, or the number of figures to the *right* of the unit figure.

The first figure to the right of the unit figure expresses *tenths.*

The second figure to the right of the unit figure expresses *hundredths.*

The third figure to the right of the unit figure expresses *thousandths.*

The fourth figure to the right of the unit figure expresses *ten-thousandths.*

The fifth figure to the right of the unit figure expresses *hundred-thousandths.*

The sixth figure to the right of the unit figure expresses *millionths.*

Thus:

$.3 = \frac{3}{10} = 3$ tenths.
$.03 = \frac{3}{100} = 3$ hundredths.
$.003 = \frac{3}{1000} = 3$ thousandths.
$.0003 = \frac{3}{10000} = 3$ ten-thousandths.
$.00003 = \frac{3}{100000} = 3$ hundred-thousandths.
$.000003 = \frac{3}{1000000} = 3$ millionths.

The first figure to the right of the unit figure is called the *first* **decimal place**; the second figure, the *second decimal place*, etc. We see in the above that the *number of decimal places in a decimal equals the number of ciphers to the right of the figure 1 in the denominator of its equivalent fraction.* This fact kept in mind will be of much assistance in reading and writing decimals.

Whatever may be written to the *left* of a *decimal point* is a whole number. The decimal point *merely separates the fraction on the right from the whole number on the left.*

When a whole number and decimal are written together, the expression is a *mixed number*. Thus, 8.12 and 17.25 are mixed numbers.

The relation of decimals and whole numbers to each other is clearly shown by the following table:

hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.	decimal point.	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	ten-millionths.	hundred-millionths.
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9

The figures to the *left* of the decimal point represent *whole numbers;* those to the *right* are *decimals.*

In both the decimals and whole numbers, the *units* place is made the starting point of notation and numeration. Both whole numbers and decimals *decrease* on the scale of *ten* to the *right*, and *increase* on the scale of *ten* to the *left*. The *first* figure to the *left* of units is *tens*, and the *first* figure to the *right* of units is *tenths*. The *second* figure to the *left* of units is *hundreds*, and the *second* figure to the *right* is *hundredths*. The *third* figure to the *left* is *thousands*, and the *third* to the *right* is *thousandths*, and so on; the *whole* numbers on the *left* and the *decimals* on the *right*. The figures equally distant from units place correspond in name, the *decimals* having the ending *ths*, to distinguish them from *whole* numbers. The following is the numeration of the number in the above table: Nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one, and twenty-three million, four hundred fifty-six thousand, seven hundred eighty-nine hundred millionths.

The decimals increase to the *left*, on the scale of *ten*, the same as whole numbers; for, beginning at say 4-thousandths in the table, the next figure to the left is *hundredths*, which is ten times as great, and the next *tenths*, or ten times the *hundredths*, and so on through both decimals and whole numbers.

135. *Annexing or taking away a cipher at the right of a decimal does not affect its value.*

$.5$ is $\frac{5}{10}$; $.50$ is $\frac{50}{100}$, but $\frac{5}{10} = \frac{50}{100}$; therefore, $.5 = .50$.

136. *Inserting a cipher between a decimal and the decimal point divides the decimal by 10.*

$$.5 = \frac{5}{10};\ \frac{5}{10} \div 10 = \frac{5}{100} = .05.$$

137. *Taking away a cipher from the left of a decimal multiplies the decimal by 10.*

$$.05 = \frac{5}{100};\ \frac{5}{100} \times 10 = \frac{5}{10} = .5.$$

138. In some cases it is convenient to express a mixed decimal fraction in the form of a common (improper) fraction. To do so it is only necessary to write the entire number, omitting the decimal point, as the numerator of the fraction, and the denominator of the decimal part as the denominator of the fraction. Thus, $127.483 = \frac{127483}{1000}$; for $127.483 = 127\frac{483}{1000} = \frac{127000+483}{1000} = \frac{127483}{1000}$.

ADDITION OF DECIMALS.

139. Addition of decimals is similar in all respects to addition of whole numbers—units are placed under units, tens under tens, etc.; this, of course, brings the decimal points in line directly under one another. Hence, in placing the numbers to be added, it is only necessary to take care that the *decimal points are in line*. In adding whole numbers the right-hand figures are always in line; but in adding decimals, the right-hand figures will not be in line unless each decimal contains the same number of figures.

whole numbers	*decimals*	*mixed numbers*
342	.342	342.032
4234	.4234	4234.5
26	.26	26.6782
3	.03	3.06
sum 4605 Ans.	*sum* 1.0554 Ans.	*sum* 4606.2702 Ans.

140. A decimal, as .342, ought really to be expressed as 0.342, but it is quite customary to omit the cipher on the left of the decimal point, though many authors use it.

EXAMPLE.—What is the sum of 242, .36, 118.725, 1.005, 6, and 100.1 ?

SOLUTION.—

```
     242.
        .36
     118.725
        1.005
        6.
     100.1
     -------
sum  468.190   Ans.
```

141. Rule.—*Place the numbers to be added so that the decimal points will be directly under each other. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

EXAMPLES FOR PRACTICE.

142. Find the sum of

		Ans.
(*a*)	.2143, .105, 2.3042, and 1.1417.	(*a*) 3.7652.
(*b*)	783.5, 21.473, .2101, and .7816.	(*b*) 805.9647.
(*c*)	21.781, 138.72, 41.8738, .72, and 1.413.	(*c*) 204.5078.
(*d*)	.3724, 104.15, 21.417, and 100.042.	(*d*) 225.9814.
(*e*)	200.172, 14.105, 12.1465, .705, and 7.2.	(*e*) 234.3285.
(*f*)	1,427.16, .244, .32, .032, and 10.0041.	(*f*) 1,437.7601.
(*g*)	2,473.1, 41.65, .7243, 104.067, and 21.073.	(*g*) 2,640.6143.
(*h*)	4,107.2, .00375, 21.716, 410.072, and .0345.	(*h*) 4,539.02625.

1. By carefully measuring the six sides of a tract of land it was found that the first side measured 537.683 feet, the second 87.36 feet, the third 836.391 feet, the fourth 732.129 feet, the fifth 237.261 feet, and the sixth 523.689 feet. What is the exact distance around the property ? Ans. 2,954.513 feet.

2. The area of a circle is 3.1416 square feet, the area of a square

74.326 square feet, and the area of a triangle 83.56 square feet. What is the total area of the three figures? Ans. 161.0276 sq. ft.

3. A certain airway is made up of three sections, varying in size, so that it requires 9.786 pounds per square foot to pass the desired quantity of air through the first section, 7.86 pounds to pass it through the second section, and 5.63 pounds to pass it through the third section. What is the total pressure for the airway? Ans. 23.276 lb. per sq. ft.

4. The exact weight of a car is .7839 ton, and the weight of the coal which it contains is 1.983 tons. What is the total weight of the loaded car? Ans. 2.7669 tons.

SUBTRACTION OF DECIMALS.

143. As in subtraction of whole numbers, units are placed under units, tens under tens, etc., bringing the decimal points under each other, as in addition of decimals.

EXAMPLE.—Subtract .132 from .3063.

SOLUTION.—

minuend	.3063	
subtrahend	.132	
difference	.1743	Ans.

144. EXAMPLE.—What is the difference between 7.895 and .725?

SOLUTION.—

minuend	7.895	
subtrahend	.725	
difference	7.170 or 7.17	Ans.

145. EXAMPLE.—Subtract .625 from 11.

SOLUTION.—

minuend	11.000	
subtrahend	.625	
difference	10.375	Ans.

146. **Rule.**—*Place the subtrahend under the minuend, so that the decimal points will be directly under each other. Subtract as in whole numbers, and place the decimal point in the remainder, directly under the decimal points above.*

When the figures in the decimal part of the subtrahend extend beyond those in the minuend, place ciphers in the minuend above them, and subtract as before.

EXAMPLES FOR PRACTICE.

147. From

	Ans.
(*a*) 407.385 take 235.0004.	(*a*) 172.3846.
(*b*) 22.718 take 1.7042.	(*b*) 21.0138.
(*c*) 1,368.17 take 13.6817.	(*c*) 1,354.4883.
(*d*) 70.00017 take 7.000017.	(*d*) 63.000153.
(*e*) 630.630 take .6304.	(*e*) 629.9996.
(*f*) 421.73 take 217.162.	(*f*) 204.568.
(*g*) 1.000014 take .00001.	(*g*) 1.000004.
(*h*) .783652 take .542314.	(*h*) .241338.

1. The average weight of a cubic foot of anthracite coal is 93.5 pounds, and of bituminous coal 84 pounds. What is the difference in the weights of a cubic foot of anthracite and bituminous coal? Ans. 9.5 pounds.

2. A 2-foot bar composed of 1 foot of iron and 1 foot of steel was heated until its entire length became 2.00234799 feet. What was the expansion of the steel, if the iron expanded .001258 of a foot? Ans. .00108999 foot.

3. A meter is 39.370432 inches long and a decimeter is 3.9370432 inches long. What is the difference in the lengths of the meter and decimeter? Ans. 35.4333888 in.

MULTIPLICATION OF DECIMALS.

148. In multiplication of decimals, we do not place the decimal points directly under each other, as in addition and subtraction. We pay no attention for the time being to the decimal points. Place the multiplier under the multiplicand, so that the *right-hand* figure of the one is under the *right-hand* figure of the other, and proceed exactly as in multiplication of whole numbers. After multiplying, *count the number of decimal places in both multiplicand and multiplier, and point off the same number in the product.*

EXAMPLE.—Multiply .825 by 13.

SOLUTION.— *multiplicand* .825
multiplier 13
2475
825
product 10.725 Ans.

In this example there are three decimal places in the multiplicand and none in the multiplier; therefore, 3 decimal places are pointed off in the product.

149. EXAMPLE.—What is the product of 426 and the decimal .005?

SOLUTION.— *multiplicand* 426
multiplier .005
product 2.130 or 2.13 Ans.

In this example there are 3 decimal places in the multiplier and none in the multiplicand; therefore, 3 decimal places are pointed off in the product.

150. It is *not* necessary to multiply by the ciphers on the *left* of a decimal; they merely determine the number of decimal places. Ciphers to the *right* of a decimal should be omitted, as they only make more figures to deal with, and do not change the value.

151. EXAMPLE.—Multiply 1.205 by 1.15.

SOLUTION.— *multiplicand* 1.205
multiplier 1.15

6025
1205
1205

product 1.38575 Ans.

In this example there are 3 decimal places in the multiplicand, and 2 in the multiplier; therefore, 3 + 2, or 5, decimal places must be pointed off in the product.

152. EXAMPLE.—Multiply .232 by .001.

SOLUTION.— *multiplicand* .232
multiplier .001
product .000232 Ans.

In this example we multiply the multiplicand by the digit in the multiplier, which makes 232 in the product, but since there are 3 decimal places in each, the multiplier and the multiplicand, we must prefix 3 ciphers to the 232, to make 3 + 3, or 6, decimal places in the product.

153. Rule.—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply as in whole numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.*

EXAMPLES FOR PRACTICE.

154. Find the product of

(*a*) .000492 × 4.1418.	Ans.	(*a*) .0020377656.
(*b*) 4,003.2 × 1.2.		(*b*) 4,803.84.
(*c*) 78.6531 × 1.03.		(*c*) 81.012693
(*d*) .3685 × .042.		(*d*) .015477.
(*e*) 178,352 × .01.		(*e*) 1,783.52.
(*f*) .00045 × .0045.		(*f*) .000002025.
(*g*) .714 × .00002.		(*g*) .00001428.
(*h*) .00004 × .008.		(*h*) .00000032.

1. If it costs .743 of a dollar to ship one ton of coal from the mines to the city, what will it cost to ship 4,376.58 tons?
Ans. 3,251.79894 dollars.

2. An operator pays 7.563 cents per ton as royalty. Supposing his mine ships 1,853.65 tons a month, what is his royalty per month?
Ans. 14,019.15495 cents.

3. A meter is 3.2808992 feet long. What is the length of 8.31 meters?
Ans. 27.264272352 feet.

4. If a steam pump delivers 2.39 gallons of water per stroke and runs at 51 strokes a minute, how many gallons of water would it pump in 58.5 minutes?
Ans. 7,130.565 gallons.

DIVISION OF DECIMALS.

155. In division of decimals we pay no attention to the decimal point until after the division is performed. The *number of decimal places in the dividend must equal, or be made to equal by annexing ciphers, the number of decimal places in the divisor. Divide exactly as in whole numbers. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend, and point off as many decimal* *places in the quotient as there are units in the remainder thus found.*

EXAMPLE.—Divide .625 by 25.

SOLUTION.—

```
        divisor dividend quotient
           25 ) .625 ( .025  Ans.
                 50
                 ---
                 125
                 125
                 ---
       remainder   0
```

In this example there are no decimal places in the divisor, and 3 decimal places in the dividend; therefore, there are 3 minus 0, or 3, decimal places in the quotient. One cipher has to be prefixed to the 25, to make the 3 decimal places.

156. EXAMPLE.—Divide 6.035 by .05.

SOLUTION.—

```
        divisor dividend quotient
           .05 ) 6.035 ( 120.7  Ans.
                 5
                 --
                 10
                 10
                 ---
                   35
                   35
                   --
       remainder    0
```

In this example we divide by 5, as if the cipher were not before it. There is one more decimal place in the dividend than in the divisor; therefore, one decimal place is pointed off in the quotient.

157. EXAMPLE.—Divide .125 by .005.

SOLUTION.—

```
        divisor dividend quotient
          .005 ) .125 ( 25  Ans.
                 10
                 ---
                  25
                  25
                  --
       remainder   0
```

In this example there are the same number of decimal places in the dividend as in the divisor; therefore, the quotient has no decimal places, and is a whole number.

158. EXAMPLE.—Divide 326 by .25.

SOLUTION.—

```
        divisor  dividend  quotient
          .2 5 ) 3 2 6.0 0 ( 1 3 0 4   Ans.
                 2 5
                 ---
                   7 6
                   7 5
                   ---
                     1 0 0
                     1 0 0
                     -----
       remainder         0
```

In this problem two ciphers were annexed to the dividend, to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

159. EXAMPLE.—Divide .0025 by 1.25.

SOLUTION.—

```
          1.2 5 ) .0 0 2 5 0 ( .0 0 2   Ans.
                        2 5 0
                        -----
       remainder            0
```

EXPLANATION.—In this example we are to divide .0025 by 1.25. Consider the dividend as a whole number, or 25 (disregarding the two ciphers at its left, for the present); also, consider the divisor as a whole number, or 125. It is clearly evident that the dividend 25 will not contain the divisor 125; we must, therefore, annex one cipher to the 25, thus making the dividend 250. 125 is contained twice in 250, so we place the figure 2 in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only four decimal places in the dividend; but one cipher was annexed, thereby making $4+1$, or 5, decimal places. Since there are 5 decimal places in the dividend and 2 decimal places in the divisor, we must point off $5-2$, or 3, decimal places in the quotient. In order to point off 3 decimal places, two ciphers must be prefixed to the figure 2, thereby making .002 the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

160. Rule.—I. *Place the divisor to the left of the dividend, and proceed as in division of whole numbers; in the* *quotient, point off as many decimal places as the number of decimal places in the dividend exceed those in the divisor, prefixing ciphers to the quotient, if necessary.*

II. *If in dividing one number by another there be a remainder, the remainder can be placed over the divisor, as a fractional part of the quotient, but it is generally better to annex ciphers to the remainder, and continue dividing until there are 3 or 4 decimal places in the quotient, and then if there still be a remainder, terminate the quotient by the plus sign (+), which shows that it can be carried further.*

161. EXAMPLE.—What is the quotient of 199 divided by 15?

SOLUTION.—

```
            15)199(13 + 4/15  Ans.
               15
               --
                49
                45
                --
   remainder     4

Or,   15)199.000(13.266+  Ans.
         15
         --
          49
          45
          --
           40
           30
           --
           100
            90
           ---
            100
             90
            ---
   remainder 10
```

$13\frac{4}{15} = 13.266+$

$\frac{4}{15} = .266+$

162. It frequently happens, as in the above example, that the division will never terminate. In such cases, decide to how many decimal places the division is to be carried, and carry the work one place further. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign (−), thus indicating that the quotient is not quite as large as indicated; if the figure thus obtained is less than 5, write the plus sign (+) after the quotient, thus indicating that the number is slightly greater than as indicated. In the last example, had it been desired to obtain the answer correct to four decimal places, the work would have been carried to five places, obtaining 13.26666, and the answer would have been given as 13.2667−. This remark applies to any other calculation involving decimals, when it is desired to omit some of the figures in the decimal. Thus, if it is desired to retain three decimal places in the number .2471253, it would be expressed as .247+; if it was desired to retain five decimal places, it would be expressed as .24713−. Both the + and − signs are frequently omitted; they are seldom used outside of Arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not *quite* exact.

EXAMPLES FOR PRACTICE.

163. Divide

	Ans.
(*a*) 101.6688 by 2.36.	(*a*) 43.08.
(*b*) 187.12264 by 123.107.	(*b*) 1.52.
(*c*) .08 by .008.	(*c*) 10.
(*d*) .0003 by 3.75.	(*d*) .00008.
(*e*) .0144 by .024.	(*e*) .6.
(*f*) .00375 by 1.25.	(*f*) .003.
(*g*) .004 by 400.	(*g*) .00001.
(*h*) .4 by .008.	(*h*) 50.
(*i*) 177.6 by 2.4.	(*i*) 74.
(*j*) .98 by .7.	(*j*) 1.4.

1. A man received $27.25 for driving 19.75 yards of entry. What did he receive per yard? Ans. $1.38.

2. A cistern has 52,845 pounds of water in it. How many cubic feet of water does it contain if 1 cubic foot of water weighs 62.5 pounds? Ans. 846 cu. ft.

3. An atmosphere is equal to 14.7 pounds per square inch. Under how many atmospheres are men working in a caisson where the pressure is 30.87 pounds per square inch? Ans. 2.1 atmospheres.

4. A pump delivers 12.13 gallons at each stroke. How many strokes must it make in order to deliver 1,285.78 gallons? Ans. 106 strokes.

TO REDUCE A FRACTION TO A DECIMAL.

164. EXAMPLE.— $\frac{3}{4}$ equals what decimal?

SOLUTION.— $\frac{4)\,3.0\,0}{.7\,5}$, or $\frac{3}{4}$ = .75. Ans.

EXAMPLE.—What decimal is equivalent to $\frac{7}{8}$?

SOLUTION.—

8) 7.0 0 0 (.8 7 5
6 4
6 0
5 6 or $\frac{7}{8}$ = .875. Ans.
4 0
4 0
0

165. Rule.—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

EXAMPLES FOR PRACTICE.

166. Reduce the following common fractions to decimals:

(*a*) $\frac{15}{32}$.
(*b*) $\frac{7}{8}$.
(*c*) $\frac{21}{32}$.
(*d*) $\frac{51}{64}$.
(*e*) $\frac{4}{25}$.
(*f*) $\frac{5}{8}$.
(*g*) $\frac{10}{200}$.
(*h*) $\frac{4}{1000}$.

Ans.
(*a*) .46875.
(*b*) .875.
(*c*) .65625.
(*d*) .796875.
(*e*) .16.
(*f*) .625.
(*g*) .05.
(*h*) .004.

---

167. To reduce inches to decimal parts of a foot:

EXAMPLE.—What decimal part of a foot is 9 inches?

SOLUTION.—Since there are 12 inches in 1 foot, 1 inch is $\frac{1}{12}$ of a foot, and 9 inches is $9 \times \frac{1}{12}$, or $\frac{9}{12}$ of a foot. This, reduced to a decimal by the above rule, shows what decimal part of a foot 9 inches is.

1 2) 9.0 0 (.7 5 of a foot. Ans.
8 4
6 0
6 0
0

168. Rule I.—*To reduce inches to decimal parts of a foot, divide the number of inches by 12.*

II. *Should the resulting decimal be an unending one and it is desired to terminate the division at some point, say, the fourth decimal place, carry the division one place further, and if the fifth figure is 5 or greater, increase the fourth figure by 1. Omit the signs + and −.*

EXAMPLES FOR PRACTICE.

169. Reduce to the decimal part of a foot:

(*a*) 3 in.		(*a*)	.25.
(*b*) $4\frac{1}{2}$ in.		(*b*)	.375.
(*c*) 5 in.	Ans.	(*c*)	.4167.
(*d*) $6\frac{5}{8}$ in.		(*d*)	.5521.
(*e*) 11 in.		(*e*)	.9167.

TO REDUCE A DECIMAL TO A FRACTION.

170. EXAMPLE.—Reduce .125 to a fraction.

SOLUTION.— $.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$. Ans.

EXAMPLE.—Reduce .875 to a fraction.

SOLUTION.— $.875 = \frac{875}{1000} = \frac{35}{40} = \frac{7}{8}$. Ans.

171. Rule.—*Under the figures of the decimal, place 1 with as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms by dividing both numerator and denominator by the same number.*

EXAMPLES FOR PRACTICE.

172. Reduce the following to common fractions:

(*a*) .125.		(*a*)	$\frac{1}{8}$.
(*b*) .625.		(*b*)	$\frac{5}{8}$.
(*c*) .3125.		(*c*)	$\frac{5}{16}$.
(*d*) .04.	Ans.	(*d*)	$\frac{1}{25}$.
(*e*) .06.		(*e*)	$\frac{3}{50}$.
(*f*) .75.		(*f*)	$\frac{3}{4}$.
(*g*) .15625.		(*g*)	$\frac{5}{32}$.
(*h*) .875.		(*h*)	$\frac{7}{8}$.

---

1. A car holds .987 of a ton of coal. Determine the number of hundredweights and pounds it holds. Ans. 19 cwt. 74 lb.

2. The outside diameter of a pipe is 6.382 inches, and the thickness of the iron of which it is made is .121 of an inch. What is the inside diameter in inches and fraction of an inch? Ans. $6\frac{7}{50}$ inches.

3. A spring is made up of four flat steel bars. The thicknesses of the bars are, respectively, .5 inch, .25 inch, .125 inch, and .0625 inch. What is the entire thickness of the spring, expressed as a fraction of an inch? Ans. $\frac{15}{16}$ inch.

173. To express a decimal approximately as a fraction having a given denominator:

174. EXAMPLE.—Express .5827 in 64ths.

SOLUTION.— $.5827 \times \frac{64}{64} = \frac{37.2928}{64}$, say $\frac{37}{64}$.

Hence, $.5827 = \frac{37}{64}$, nearly. Ans.

EXAMPLE.—Express .3917 in 12ths.

SOLUTION.— $.3917 \times \frac{12}{12} = \frac{4.7004}{12}$, say $\frac{5}{12}$.

Hence, $.3917 = \frac{5}{12}$, nearly. Ans.

175. Rule.—*Reduce 1 to a fraction having the given denominator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required.*

EXAMPLES FOR PRACTICE.

176. Express

(*a*) .625 in 8ths.	Ans.	(*a*) $\frac{5}{8}$.
(*b*) .3125 in 16ths.		(*b*) $\frac{5}{16}$.
(*c*) .15625 in 32ds.		(*c*) $\frac{5}{32}$.
(*d*) .77 in 64ths.		(*d*) $\frac{49}{64}$.
(*e*) .81 in 48ths.		(*e*) $\frac{39}{48}$.
(*f*) .923 in 96ths.		(*f*) $\frac{89}{96}$.

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177. The sign for dollars is $. It is read *dollars.* $25 is read *25 dollars.*

Since there are 100 cents in a dollar, 1 cent is 1-one-hundredth of a dollar; the first two figures of a decimal part of a dollar represent *cents.* Since a mill is $\frac{1}{10}$ of a cent, or $\frac{1}{1000}$ of a dollar, the third figure represents *mills.*

Thus, $25.16 is read *twenty-five dollars and sixteen cents;* $25.168 is read *twenty-five dollars, sixteen cents, and eight mills.*

178. The **vinculum** ——, **parenthesis** ( ), **bracket** [ ], and **brace** { } are called **symbols of aggregation,** and are used to include numbers which are to be considered together; thus, $13 \times \overline{8 - 3}$, or $13 \times (8 - 3)$, shows that 3 is to be taken from 8 before multiplying by 13.

$$13 \times (8 - 3) = 13 \times 5 = 65. \quad \text{Ans.}$$

$$13 \times \overline{8 - 3} = 13 \times 5 = 65. \quad \text{Ans.}$$

When the vinculum or parenthesis is not used, we have

$$13 \times 8 - 3 = 104 - 3 = 101. \quad \text{Ans.}$$

179. In any series of numbers connected by the signs +, −, ×, and ÷, the operations indicated by the signs must be performed in order from left to right, *except* that no addition or subtraction may be performed if a sign of multiplication or division *follows* the number on the *right* of a sign of addition or subtraction, until the indicated multiplication or division has been performed. In all cases the sign of multiplication takes the precedence, the reason being that when two or more numbers or expressions are connected by the sign of multiplication, the numbers thus connected are regarded as factors of the product indicated, and not as separate numbers.

EXAMPLE.—What is the value of $4 \times 24 - 8 + 17$?

SOLUTION.—Performing the operations in order from left to right, $4 \times 24 = 96$; $96 - 8 = 88$; $88 + 17 = 105$. Ans.

180. EXAMPLE.—What is the value of the following expression: $1,296 \div 12 + 160 - 22 \times 3\frac{1}{2} = ?$

SOLUTION.—$1,296 \div 12 = 108$; $108 + 160 = 268$; here we cannot subtract 22 from 268 because the sign of multiplication *follows* 22; hence multiplying 22 by $3\frac{1}{2}$, we get 77, and $268 - 77 = 191$. Ans.

Had the above expression been written $1,296 \div 12 + 160 - 22 \times 3\frac{1}{2} \div 7 + 25$, it would have been necessary to have divided $22 \times 3\frac{1}{2}$ by 7 before subtracting, and the final result would have been $22 \times 3\frac{1}{2} = 77$; $77 \div 7 = 11$; $268 - 11 = 257$; $257 + 25 = 282$. Ans. In other words, it is necessary to perform *all* of the multiplication or division included between the signs + and −, or − and +, before adding or subtracting. Also, had the expression been written $1,296 \div 12 + 160 - 24\frac{1}{2} \div 7 \times 3\frac{1}{2} + 25$, it would have been necessary to have multiplied $3\frac{1}{2}$ by 7 before dividing $24\frac{1}{2}$, since the sign of multiplication takes the precedence, and the final result would have been $3\frac{1}{2} \times 7 = 24\frac{1}{2}$; $24\frac{1}{2} \div 24\frac{1}{2} = 1$; $268 - 1 = 267$; $267 + 25 = 292$. Ans.

It likewise follows that if a succession of multiplication and division signs occurs, the indicated operations must not be performed in order, from left to right—the multiplication must be performed first. Thus, $24 \times 3 \div 4 \times 2 \div 9 \times 5 = \frac{1}{5}$. Ans. In order to obtain the same result that would be obtained by performing the indicated operations in order, from left to right, symbols of aggregation must be used. Thus, by using two vinculums, the last expression becomes $24 \times \overline{3 \div 4} \times \overline{2 \div 9} \times 5 = 20$, the same result that would be obtained by performing the indicated operations in order, from left to right.

EXAMPLES FOR PRACTICE.

181. Find the values of the following expressions:

			Ans.
(*a*)	$(8 + 5 - 1) \div 4$.	(*a*)	3.
(*b*)	$5 \times 24 - 32$.	(*b*)	88.
(*c*)	$5 \times 24 \div 15$.	(*c*)	8.
(*d*)	$144 - 5 \times 24$.	(*d*)	24.
(*e*)	$(1,691 - 540 + 559) \div 3 \times 57$.	(*e*)	10.
(*f*)	$2,080 + 120 - 80 \times 4 - 1,670$.	(*f*)	210.
(*g*)	$(\overline{90 + 60} \div 25) \times 5 - 29$.	(*g*)	1.
(*h*)	$\overline{90 + 60} \div 25 \times 5$.	(*h*)	1.2.

ARITHMETIC.

(PART 2.)

PERCENTAGE.

182. Percentage is the process of calculating by *hundredths.*

183. The *term* **per cent.** is an abbreviation of the Latin words *per centum*, which mean *by the hundred.* A certain per cent. of a number is the number of hundredths of that number which is indicated by the number of units in the per cent. Thus, 6 per cent. of 125 is $125 \times \frac{6}{100} = 7.5$; 25 per cent. of 80 is $80 \times \frac{25}{100} = 20$; 43 per cent. of 432 pounds is $432 \times \frac{43}{100} = 185.76$ pounds.

184. The **sign** of per cent. is %, and is read *per cent.* Thus, 6% is read *six per cent.;* $12\frac{1}{2}$% is read *twelve and one-half per cent.*, etc.

When expressing the per cent. of a number to use in calculations, it is customary to express it decimally instead of fractionally. Thus, instead of expressing 6%, 25%, and 43% as $\frac{6}{100}$, $\frac{25}{100}$, and $\frac{43}{100}$, it is usual to express them as .06, .25, and .43.

Per Cent.	Decimal.	Fraction.	Per Cent.	Decimal.	Fraction.
1%.....	.01	$\frac{1}{100}$	150 %....	1.50	$\frac{150}{100}$ or $1\frac{1}{2}$
2%.....	.02	$\frac{2}{100}$ or $\frac{1}{50}$	500 %....	5.00	$\frac{500}{100}$ or 5
5%.....	.05	$\frac{5}{100}$ or $\frac{1}{20}$	$\frac{1}{4}$%....	.0025	$\frac{\frac{1}{4}}{100}$ or $\frac{1}{400}$
10%.....	.10	$\frac{10}{100}$ or $\frac{1}{10}$	$\frac{1}{2}$%....	.005	$\frac{\frac{1}{2}}{100}$ or $\frac{1}{200}$
25%.....	.25	$\frac{25}{100}$ or $\frac{1}{4}$	$1\frac{1}{2}$%....	.015	$\frac{1\frac{1}{2}}{100}$ or $\frac{3}{200}$
50%.....	.50	$\frac{50}{100}$ or $\frac{1}{2}$	$8\frac{1}{3}$%....	$.08\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
75%.....	.75	$\frac{75}{100}$ or $\frac{3}{4}$	$12\frac{1}{2}$%....	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
100%.....	1.00	$\frac{100}{100}$ or 1	$16\frac{2}{3}$%....	$.16\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
125%.....	1.25	$\frac{125}{100}$ or $1\frac{1}{4}$	$62\frac{1}{2}$%....	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$

The preceding table will show how many per cent. can be expressed either as a decimal or as a fraction.

185. The names of the different elements used in percentage are: the *base*, the *rate per cent.*, the *percentage*, the *amount*, and the *difference*.

186. The **base** is the number on which the per cent. is computed.

187. The **rate** is the number of hundredths of the base to be taken.

188. The **percentage** is the part, or number of *hundredths*, of the base indicated by the rate; or, the percentage is the result obtained by multiplying the base by the rate.

Thus, when it is stated that 7% of \$25 is \$1.75, \$25 is the base, 7% is the rate, and \$1.75 is the percentage.

189. The **amount** is the sum of the base and percentage.

190. The **difference** is the remainder obtained by subtracting the percentage from the base.

Thus, if a man has \$180, and he earns 6% more, he will have, altogether, $\$180 + \$180 \times .06$, or $\$180 + \$10.80 = \$190.80$. Here \$180 is the base; 6%, the rate; \$10.80, the percentage, and \$190.80, the *amount*.

Again, if an engine of 125 horsepower uses 16% of it in overcoming friction and other resistances, the amount left for obtaining useful work is $125 - 125 \times .16 = 125 - 20 = 105$ horsepower. Here 125 is the base; 16%, the rate; 20, the percentage, and 105, the *difference*.

191. From the foregoing it is evident that to find the percentage, the base must be multiplied by the rate. Hence,

Rule.—*To find the percentage, multiply the base by the rate expressed decimally.*

EXAMPLE.—Out of a lot of 300 tons of coal, 76% were sold. How many tons were sold?

SOLUTION.— 76%, the rate, expressed decimally, is .76; the base is 300; hence, the number of tons sold, or the percentage, is, by the above rule,

$$300 \times .76 = 228 \text{ tons. Ans.}$$

Expressing the rule as a formula,

$$percentage = base \times rate.$$

192. When the percentage and rate are given, the base may be found by dividing the percentage by the rate. For, suppose that 12 is 6%, or $\frac{6}{100}$, of some number; then 1%, or $\frac{1}{100}$, of the number is 12 ÷ 6, or 2. Consequently, if 2 = 1%, or $\frac{1}{100}$, 100%, or $\frac{100}{100}$, = 2 × 100 = 200. But, since the same result may be arrived at by dividing 12 by .06, for 12 ÷ .06 = 200, it follows that

Rule.—*When the percentage and rate are given, to find the base, divide the percentage by the rate, expressed decimally.*

Formula, *base = percentage ÷ rate.*

EXAMPLE.—Bought a certain number of tons of coal and sold 76% of it. If I sold 228 tons, how many tons did I buy?

SOLUTION.—Here 228 is the percentage, and 76%, or .76, is the rate; hence, applying the rule,

228 ÷ .76 = 300 tons. Ans.

193. When the base and percentage are given, to find the rate, the rate may be found, expressed decimally, by dividing the percentage by the base. For, suppose that it is desired to find what per cent. 12 is of 200. 1% of 200 is 200 × .01 = 2. Now, if 1% is 2, 12 is evidently as many per cent. as the number of times that 2 is contained in 12, or 12 ÷ 2 = 6%. But the same result may be obtained by dividing 12, the percentage, by 200, the base, since 12 ÷ 200 = .06 = 6%. Hence,

Rule.—*When the percentage and base are given, to find the rate, divide the percentage by the base, and the result will be the rate, expressed decimally.*

Formula, *rate = percentage ÷ base.*

EXAMPLE.—Bought 300 tons of coal and sold 228 tons. What per cent. of the total number of tons was sold?

SOLUTION.—Here 300 is the base and 228 is the percentage; hence, applying rule, rate = 228 ÷ 300 = .76 = 76%. Ans.

EXAMPLE.—What per cent. of 875 is 25?

SOLUTION.—Here 875 is the base and 25 is the percentage; hence, applying rule, 25 ÷ 875 = .02$\frac{6}{7}$ = 2$\frac{6}{7}$%. Ans.

PROOF.— 875 × .02$\frac{6}{7}$ = 25.

EXAMPLES FOR PRACTICE.

194. What per cent. of

(*a*) 360 is 90?		(*a*)	25%.
(*b*) 900 is 360?		(*b*)	40%.
(*c*) 125 is 25?		(*c*)	20%.
(*d*) 150 is 750?	Ans.	(*d*)	500%.
(*e*) 280 is 112?		(*e*)	40%.
(*f*) 400 is 200?		(*f*)	50%.
(*g*) 47 is 94?		(*g*)	200%.
(*h*) 500 is 250?		(*h*)	50%.

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195. The amount may be found, when the base and rate are given, by multiplying the base by 1 plus the rate, expressed decimally. For, suppose that it is desired to find the amount when 200 is the base and 6% is the rate. The percentage is $200 \times .06 = 12$, and, according to definition, Art. **189,** the amount is $200 + 12 = 212$. But the same result may be obtained by multiplying 200 by $1 + .06$, or 1.06, since $200 \times 1.06 = 212$. Hence,

Rule.—*When the base and rate are given, to find the amount, multiply the base by 1 plus the rate, expressed decimally.*

Formula, *amount* = *base* × (1 + *rate*).

EXAMPLE.—If a man earned $725 in a year, and the next year 10% more, how much did he earn the second year?

SOLUTION.—Here 725 is the base and 10% is the rate, and the amount is required. Hence, applying the rule,

$725 \times 1.10 = \$797.50$. Ans.

196. When the base and rate are given, the difference may be found by multiplying the base by 1 minus the rate, expressed decimally. For, suppose that it is desired to find the difference when the base is 200 and the rate is 6%. The percentage is $200 \times .06 = 12$; and, according to definition, Art. **190,** the difference $= 200 - 12 = 188$. But the same result may be obtained by multiplying 200 by $1 - .06$, or .94, since $200 \times .94 = 188$. Hence,

Rule.—*When the base and rate are given, to find the difference, multiply the base by 1 minus the rate, expressed decimally.*

Formula, *difference* = *base* × (1 − *rate*).

EXAMPLE.—Bought 300 tons of coal, and sold all but 24% of it. How many tons were sold?

SOLUTION.—Here 300 is the base, 24% is the rate, and it is desired to find the difference. Hence, applying the rule,

$$300 \times (1 - .24) = 228 \text{ tons. Ans.}$$

197. When the amount and rate are given, the base may be found by dividing the amount by 1 plus the rate. For, suppose that it is known that 212 equals some number increased by 6% of itself. Then it is evident that 212 equals 106% of the number (base) that it is desired to find. Consequently, if $212 = 106\%$, $1\% = \frac{212}{106} = 2$, and $100\% = 2 \times 100 = 200 =$ the base. But the same result may be obtained by dividing 212 by $1 + .06$, or 1.06, since $212 \div 1.06 = 200$. Hence,

Rule.—*When the amount and rate are given, to find the base, divide the amount by 1 plus the rate, expressed decimally.*

Formula, *base* = *amount* ÷ (1 + *rate*).

EXAMPLE.—The theoretical discharge of a certain pump, when running at a piston speed of 100 feet per minute, is 278,910 gallons per day of 10 hours. Owing to leakage and other defects, this value is 25% greater than the actual discharge. What is the actual discharge?

SOLUTION.—Here 278,910 equals the actual discharge (base) increased by 25% of itself. Consequently, 278,910 is the amount; 25% is the rate, and, applying rule,

actual discharge = 278,910 ÷ 1.25 = 223,128 gallons. Ans.

198. When the difference and rate are given, the base may be found by dividing the difference by 1 minus the rate. For, suppose that 188 equals some number less 6% of itself. Then, 188 evidently equals $100 - 6 = 94\%$ of some number. Consequently, if $188 = 94\%$, $1\% = 188 \div 94 = 2$, and $100\% = 2 \times 100 = 200$. But the same result may be obtained by dividing 188 by $1 - .06$, or .94, since $188 \div .94 = 200$. Hence,

Rule.—*When the difference and rate are given, to find the base, divide the difference by 1 minus the rate, expressed decimally.*

Formula, *base* = *difference* ÷ (1 — *rate*).

EXAMPLE.—Bought a certain number of tons of coal and sold 76% of it. If there were 72 tons left unsold, how many tons did I buy?

SOLUTION.—Here 72 is the difference and 76% is the rate. Applying rule,

$$72 \div (1 - .76) = 300 \text{ tons. Ans.}$$

EXAMPLE.—The theoretical number of foot-pounds of work per minute required to operate a boiler feed-pump is 127,344. If 30% of the total number actually required be allowed for friction, leakage, etc., how many foot-pounds are actually required to work the pump?

SOLUTION.—Here the number actually required is the base; hence, 127,344 is the difference, and 30% is the rate. Applying the rule,

$$127{,}344 \div (1 - .30) = 181{,}920 \text{ foot-pounds. Ans.}$$

199. EXAMPLE.—A certain air-stack produces a ventilating pressure of 2.76 inches of water. By increasing the height 20 feet, the pressure was increased to 3 inches of water. What was the gain per cent.?

SOLUTION.—Here it is evident that 3 inches is the amount and that 2.76 inches is the base. Consequently, $3 - 2.76 = .24$ inch is the percentage, and it is required to find the rate. Hence, applying the rule given in Art. **193**,

$$\text{gain per cent.} = .24 \div 2.76 = .087 = 8.7\%. \text{ Ans.}$$

200. EXAMPLE.—A certain air-stack produces a ventilating pressure of 3 inches of water. The stack being injured by a storm, the pressure was reduced to 1.2 inches of water. What was the loss per cent.?

SOLUTION.—Here it is evident that 1.2 inches is the difference (since it equals 3 inches diminished by a certain per cent. loss of itself) and 3 inches is the base. Consequently, $3 - 1.2 = 1.8$ inches is the percentage. Hence, applying the rule given in Art. **193**,

$$\text{loss per cent.} = 1.8 \div 3 = .60 = 60\%. \text{ Ans.}$$

201. To find the gain or loss per cent.:

Rule.—*Find the difference between the initial and final values; divide this difference by the initial value.*

EXAMPLE.—If a man buys a house for $1,860, and some time afterwards builds a barn for 25% of the cost of the house, does he gain or lose, and how much per cent., if he sells both house and barn for $2,100?

SOLUTION.—The cost of the barn was $\$1{,}860 \times .25 = \$465$; consequently, the initial value, or cost, was $\$1{,}860 + \$465 = \$2{,}325$. Since he sold them for $2,100, he lost $\$2{,}325 - \$2{,}100 = \$225$. Hence, applying rule,

$$225 \div 2{,}325 = .0968 = 9.68\% \text{ loss. Ans.}$$

EXAMPLES FOR PRACTICE.

202. Solve the following:

			Ans.	
(*a*)	What is $12\frac{1}{2}$% of $900?		(*a*)	$112.50.
(*b*)	What is $\frac{4}{5}$% of 627?		(*b*)	5.016.
(*c*)	What is $33\frac{1}{3}$% of 54?		(*c*)	18.
(*d*)	101 is $68\frac{3}{4}$% of what number?		(*d*)	$146\frac{10}{11}$.
(*e*)	784 is $83\frac{1}{3}$% of what number?		(*e*)	940.8.
(*f*)	What % of 960 is 160?		(*f*)	$16\frac{2}{3}$%.
(*g*)	What % of $3,606 is $$450\frac{3}{4}$?		(*g*)	$12\frac{1}{2}$%.
(*h*)	What % of 280 is 112?		(*h*)	40%.

1. A steam plant consumed an average of 3,640 pounds of coal per day. The engineer made certain alterations which resulted in a saving of 250 pounds per day. What was the per cent. of coal saved?
Ans. 7%, nearly.

2. If the speed of an engine running at 126 revolutions per minute should be increased $6\frac{1}{2}$%, how many revolutions per minute would it then make? Ans. 134.19 revolutions.

3. A hydraulic ram, when the valves were in perfect condition, discharged 190.4 gallons of water per hour. A little sand got under the valve and reduced the discharge 15%. What amount of water did it then discharge per hour? Ans. 161.84 gal.

4. If I lend a man $1,100, and this is $18\frac{1}{2}$% of the amount that I have on interest, how much money have I on interest? Ans. $5,945.95.

5. A test showed that an engine developed 190.4 horsepower, 15% of which was consumed in friction. How much power was available for use? Ans. 161.84 H.P.

6. By adding a condenser to a steam-engine, the power was increased 14%, and the consumption of coal per horsepower per hour was decreased 20%. If the engine could originally develop 50 horsepower, and required $3\frac{1}{2}$ pounds of coal per horsepower per hour, what would be the total weight of coal used in an hour, with the condenser, assuming the engine to run full power? Ans. 159.6 pounds.

DENOMINATE NUMBERS.

203. **A denominate number** is a concrete number, and may be either simple or compound, as 8 quarts, 5 feet, 10 inches, etc.

204. **A simple denominate number** consists of units of but one denomination, as 16 cents, 10 hours, 5 dollars, etc.

205. A **compound denominate number** consists of units of two or more denominations of a similar kind, as 3 yards 2 feet 1 inch ; 34 square feet 57 square inches.

206. In **whole numbers** and in **decimals** the *law* of increase and decrease is on the scale of 10, but in **compound** or **denominate numbers** the scale varies.

MEASURES.

207. A **measure** is a *standard unit*, established by *law* or *custom*, by which *quantity* of any kind is measured. The *standard unit* of **dry measure** is the Winchester bushel ; of **weight,** the pound ; of **liquid measure,** the gallon, etc.

208. Measures are of six kinds :

1. Extension.
2. Weight.
3. Capacity.
4. Time.
5. Angles.
6. Money or value.

MEASURES OF EXTENSION.

209. **Measures of extension** are used in measuring lengths, distances, surfaces, and solids.

LINEAR MEASURE.

TABLE 3.

			Abbreviation.	in.	ft.	yd.	rd.	fur.	mi.
12	inches (in.)	= 1 foot . .	ft.						
3	feet	= 1 yard . .	yd.	36 =	3 =	1			
5.5	yards	= 1 rod . . .	rd.	198 =	$16\frac{1}{2}$ =	5.5 =	1		
40	rods	= 1 furlong	fur.	7,920 =	660 =	220 =	40 =	1	
8	furlongs	= 1 mile . .	mi.	63,360 =	5,280 =	1,760 =	320 =	8 =	1

SURVEYOR'S LINEAR MEASURE.

TABLE 4.

7.92 inches	= 1 link	li.
25 links	= 1 rod	rd.
4 rods / 100 links	= 1 chain	ch.
80 chains	= 1 mile	mi.

mi.		ch.		rd.		li.		in.
1	=	80	=	320	=	8,000	=	63,360

210. The linear unit, generally used by surveyors, is **Gunter's chain,** which is equal to 4 rods, or 66 feet.

211. An **engineer's chain,** used by civil engineers, is 100 feet long, and consists of 100 links. In computations, the links are written as so many hundredths of a chain.

SQUARE MEASURE.

TABLE 5.

144 square inches (sq. in.)	=	1 square foot	sq.ft.
9 square feet	=	1 square yard	sq.yd.
30¼ square yards	=	1 square rod	sq.rd.
160 square rods	=	1 acre	A.
640 acres	=	1 square mile	sq.mi.

sq.mi.		A.		sq.rd.		sq.yd.		sq.ft.		sq.in.
1	=	640	=	102,400	=	3,097,600	=	27,878,400	=	4,014,489,600

SURVEYOR'S SQUARE MEASURE.

TABLE 6.

625 square links	=	1 square rod	sq.rd.
16 square rods	=	1 square chain	sq.ch.
10 square chains	=	1 acre	A.
640 acres	=	1 square mile	sq.mi.
36 square miles (6 mi. square)	=	1 township	Tp.

sq.mi.		A.		sq.ch.		sq.rd.		sq.li.
1	=	640	=	6,400	=	102,400	=	64,000,000

CUBIC MEASURE.

TABLE 7.

1728 cubic inches (cu. in.)	=	1 cubic foot	cu.ft.
27 cubic feet	=	1 cubic yard	cu.yd.
128 cubic feet	=	1 cord	cd.
24¾ cubic feet	=	1 perch	P.

cu.yd.		cu.ft.		cu.in.
1	=	27	=	46,656

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

TABLE 8.

16 ounces (oz.)	=	1 pound	lb.
100 pounds	=	1 hundredweight	cwt.
20 cwt., or 2,000 lb.	=	1 ton	T.

T.		cwt.		lb.		oz.
1	=	20	=	2,000	=	32,000

212. The ounce is divided into halves, quarters, etc. Avoirdupois weight is used for weighing coarse and heavy articles. One avoirdupois pound contains 7,000 grains.

LONG TON TABLE.

TABLE 9.

16 ounces	=	1 pound	lb.
112 pounds	=	1 hundredweight	cwt.
20 cwt., or 2,240 lb.	=	1 ton	T.

213. In all the calculations throughout this and the succeeding volumes, 2,000 pounds will be considered one ton, unless the long ton (2,240 pounds) is especially mentioned.

TROY WEIGHT.

TABLE 10.

24 grains (gr.)	=	1 pennyweight	pwt.
20 pennyweights	=	1 ounce	oz.
12 ounces	=	1 pound	lb.

lb.		oz.		pwt.		gr.
1	=	12	=	240	=	5,760

214. Troy weight is used in weighing gold and silverware, jewels, etc. It is used by jewelers.

MEASURES OF CAPACITY.

LIQUID MEASURE.

TABLE 11.

4 gills (gi.)	=	1 pint	pt.
2 pints	=	1 quart	qt.
4 quarts	=	1 gallon	gal.
31½ gallons	=	1 barrel	bbl.
2 barrels, or 63 gallons	=	1 hogshead	hhd.

hhd.		bbl.		gal.		qt.		pt.		gi.
1	=	2	=	63	=	252	=	504	=	2,016

DRY MEASURE.

TABLE 12.

2 pints (pt.)	=	1 quart	qt.
8 quarts	=	1 peck	pk.
4 pecks	=	1 bushel	bu.

bu.		pk.		qt.		pt.
1	=	4	=	32	=	64

MEASURE OF TIME.

TABLE 13.

60 seconds (sec.)	=	1 minute	min.
60 minutes	=	1 hour	hr.
24 hours	=	1 day	da.
7 days	=	1 week	wk.
365 days, 12 months	=	1 common year	yr.
366 days	=	1 leap year.	
100 years	=	1 century.	

NOTE.—It is customary to consider one month as 30 days.

MEASURE OF ANGLES OR ARCS.

TABLE 14.

60 seconds (″)	=	1 minute	′.
60 minutes	=	1 degree	°.
90 degrees	=	1 right angle or quadrant	∟.
360 degrees	=	1 circle	cir.

cir.		°		′		″
1	=	360°	=	21,600′	=	1,296,000″

MEASURE OF MONEY.

UNITED STATES MONEY.

TABLE 15.

10 mills (m.)	=	1 cent	ct.
10 cents	=	1 dime	d.
10 dimes	=	1 dollar	$.
10 dollars	=	1 eagle	E.

E.		$		d.		ct.		m.
1	=	10	=	100	=	1,000	=	10,000

MISCELLANEOUS TABLE.

TABLE 16.

12 things are 1 dozen.
12 dozen are 1 gross.
12 gross are 1 great gross.
2 things are 1 pair.
20 things are 1 score.
1 league is 3 miles.
1 fathom is 6 feet.
1 meter is nearly 39.37 inches.
1 hand is 4 inches.
1 palm is 3 inches.
1 span is 9 inches.
24 sheets are 1 quire.
20 quires, or 480 sheets, are 1 ream.
1 bushel contains 2,150.4 cubic in.
1 U. S. standard gallon (also called a wine gallon) contains 231 cubic in.
1 U. S. standard gallon of water weighs 8.355 pounds, nearly.
1 cubic foot of water contains 7.481 U. S. standard gallons, nearly.
1 British imperial gallon weighs 10 pounds.

It will be of great advantage to the student to carefully memorize all of the above tables.

REDUCTION OF DENOMINATE NUMBERS.

215. Reduction of denominate numbers is the process of changing their denomination without changing their value. They may be changed from a higher to a lower denomination or from a lower to a higher—either is reduction. As,

2 hours = 120 minutes.
32 ounces = 2 pounds.

216. Principle.—Denominate numbers are changed to *lower* denominations by *multiplying*, and to *higher* denominations by *dividing*.

To reduce denominate numbers to lower denominations:

217. EXAMPLE.—Reduce 5 yd. 2 ft. 7 in. to inches.

SOLUTION.—

```
 yd.    ft.    in.
   5     2      7
   3
 ---
  15 ft.
   2 ft.
 ---
  17 ft.
  12
 ---
  34
 17
 ---
 204 in.
   7 in.
 ---
 211 inches.  Ans.
```

EXPLANATION.—Since there are 3 feet in 1 yard, in 5 yards there are 5 × 3, or 15 feet, and 15 feet plus 2 feet = 17 feet. There are 12 inches in a foot; therefore, 12 × 17 = 204 inches, and 204 inches plus 7 inches = 211 inches = number of inches in 5 yards 2 feet and 7 inches. Ans.

218. EXAMPLE.—Reduce 6 hours to seconds.

SOLUTION.—

```
    6    hours.
   60
  ---
  360    minutes.
   60
 ----
21600 seconds.  Ans.
```

EXPLANATION.—As there are 60 minutes in one hour, in six hours there are 6 × 60, or 360 minutes; as there are no minutes to add, we multiply 360 minutes by 60, to get the number of seconds.

219. In order to avoid mistakes, if any denomination be omitted, represent it by a cipher. Thus, before reducing 3 rods 6 inches to inches, insert a cipher for yards and a cipher for feet; as,

rd.	yd.	ft.	in.
3	0	0	6

220. Rule.—*Multiply the number representing the highest denomination by the number of units in the next lower required to make one of the higher denomination, and to the product add the number of given units of that lower denomination. Proceed in this manner until the number is reduced to the required denomination.*

EXAMPLES FOR PRACTICE.

221. Reduce

			Ans.
(*a*)	4 rd. 2 yd. 2 ft. to ft.	(*a*)	74 ft.
(*b*)	4 bu. 3 pk. 2 qt. to qt.	(*b*)	154 qt.
(*c*)	13 rd. 5 yd. 2 ft. to ft.	(*c*)	231.5 ft.
(*d*)	5 mi. 100 rd. 10 ft. to ft.	(*d*)	28,060 ft.
(*e*)	8 lb. 4 oz. 6 pwt. to gr.	(*e*)	48,144 gr.
(*f*)	52 hhd. 24 gal. 1 pt. to pt.	(*f*)	26,401 pt.
(*g*)	5 cir. 16° 20′ to minutes.	(*g*)	108,980′.
(*h*)	14 bu. to qt.	(*h*)	448 qt.

To reduce lower to higher denominations:

222. EXAMPLE.—Reduce 211 in. to higher denominations.

SOLUTION.—

```
12 ) 211 in.
      3 ) 17 ft. + 7 in.
           5 yd. + 2 ft.   Ans.
```

EXPLANATION.—There are 12 inches in 1 foot; therefore, 211 divided by 12 = 17 feet and 7 inches over. There are 3 feet in 1 yard; therefore, 17 feet divided by $3 = 5$ yards and 2 feet over. The last quotient and the two remainders constitute the answer, 5 yards 2 feet 7 inches.

223. EXAMPLE.—Reduce 15,735 grains Troy weight to higher denominations.

SOLUTION.—

```
24)15735 gr.(655 pwt.
   144
   ---
    133
    120
    ---
     135
     120
     ---
      15 gr.

20)655 pwt.(32 oz.
   60
   --
    55
    40
    --
    15 pwt.

12)32 oz.(2 lb.
   24
   --
    8 oz.
```

EXPLANATION.—There are 24 grains in 1 pennyweight, and in 15,735 grains there are as many pennyweights as 24 is contained in 15,735, or 655 pennyweights and 15 grains remaining. There are 20 pennyweights in 1 ounce, and in 655 pennyweights there are 32 ounces and 15 pennyweights remaining. There are 12 ounces in 1 pound, and in 32 ounces there are 2 pounds and 8 ounces remaining. The last quotient and the three remainders constitute the answer, 2 pounds 8 ounces 15 pennyweights 15 grains.

The above problem is worked out by long division, because the numbers are too large to solve easily by short division. The student may use either method.

224. Rule.—*Divide the number representing the denomination given by the number of units of this denomination required to make one unit of the next higher denomination. The remainder will be of the same denomination, but the quotient will be of the next higher. Divide this quotient by* *the number of units of its denomination required to make one unit of the next higher. Continue until the highest denomination is reached, or until there is not enough of a denomination left to make one of the next higher. The last quotient and the remainders constitute the required result.*

EXAMPLES FOR PRACTICE.

225. Reduce to units of higher denominations:

(*a*) 7,460 sq. in.; (*b*) 7,580 sq. yd.; (*c*) 148,760 cu. in.; (*d*) 7,896 cu. ft. to cd.; (*e*) 17,651″; (*f*) 1,120 cu. ft. to cd.; (*g*) 8,000 gi.; (*h*) 36,450 lb.

Ans.
- (*a*) 5 sq. yd. 6 sq. ft. 116 sq. in.
- (*b*) 1 A. 90 sq. rd. 17 sq. yd. 4 sq. ft. 72 sq. in.
- (*c*) 3 cu. yd. 5 cu. ft. 152 cu. in.
- (*d*) 61 cd. 88 cu. ft.
- (*e*) 4° 54′ 11″.
- (*f*) 8 cd. 96 cu. ft.
- (*g*) 3 hhd. 61 gal.
- (*h*) 18 T. 4 cwt. 50 lb.

ADDITION OF DENOMINATE NUMBERS.

226. EXAMPLE.—Find the sum of 3 cwt. 46 lb. 12 oz.; 8 cwt. 12 lb. 13 oz.; 12 cwt. 50 lb. 13 oz.; 27 lb. 4 oz.

SOLUTION.—

T.	cwt.	lb.	oz.	
0	3	46	12	
0	8	12	13	
0	12	50	13	
0	0	27	4	
1	4	37	10	Ans.

EXPLANATION.—Begin to add at the right-hand column: $4 + 13 + 13 + 12 = 42$ ounces; as 16 ounces make 1 pound, 42 ounces $\div 16 = 2$ and a remainder of 10 ounces, or 2 pounds and 10 ounces. Place 10 ounces under ounce column, and add 2 pounds to the next or pound column. Then, $2 + 27 + 50 + 12 + 46 = 137$ pounds; as 100 pounds make a hundredweight, $137 \div 100 = 1$ hundredweight and a remainder of 37 pounds. Place the 37 under the pounds column, and add 1 hundredweight to the next or hundredweight column. Next, $1 + 12 + 8 + 3 = 24$ hundredweight.

20 hundredweight make a ton; therefore $24 \div 20 = 1$ ton and 4 hundredweight remaining. Hence, the sum is 1 ton 4 hundredweight 37 pounds 10 ounces. Ans.

227. EXAMPLE.—What is the sum of 2 rd. 3 yd. 2 ft. 5 in.; 6 rd. 1 ft. 10 in.; 17 rd. 11 in.; 4 yd. 1 ft.?

SOLUTION.—

	rd.	yd.	ft.	in.	
	2	3	2	5	
	6	0	1	10	
	17	0	0	11	
	0	4	1	0	
	26	$3\frac{1}{2}$	0	2	
or	26	3	1	8	Ans.

EXPLANATION.—The sum of the numbers in the first column = 26 inches, or 2 feet and 2 inches remaining. The sum of the numbers in the next column plus 2 feet = 6 feet, or 2 yards and 0 feet remaining. The sum of the next column plus 2 yards = 9 yards, or $9 \div 5\frac{1}{2} = 1$ rod and $3\frac{1}{2}$ yards remaining. The sum of the next column plus 1 rod = 26 rods. To avoid fractions in the sum, the $\frac{1}{2}$ yard is reduced to 1 foot and 6 inches, which added to 26 rods 3 yards 0 feet and 2 inches = 26 rods 3 yards 1 foot 8 inches. Ans.

228. EXAMPLE.—What is the sum of 47 ft. and 3 rd. 2 yd. 2 ft. 10 in.?

SOLUTION.—When 47 ft. is reduced it equals 2 rd. 4 yd. 2 ft., which can be added to 3 rd. 2 yd. 2 ft. 10 in. Thus,

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	6	$1\frac{1}{2}$	1	10	
or	6	2	0	4	Ans.

229. Rule.—*Place the numbers so that like denominations are under each other. Begin at the right-hand column, and add. Divide the sum by the number of units of this denomination required to make one unit of the next higher. Place the remainder under the column added, and carry the quotient to the next column. Continue in this manner until the highest denomination given is reached.*

EXAMPLES FOR PRACTICE.

230. What is the sum of

(*a*) 25 lb. 7 oz. 15 pwt. 23 gr.; 17 lb. 16 pwt.; 15 lb. 4 oz. 12 pwt.; 18 lb. 16 gr.; 10 lb. 2 oz. 11 pwt. 16 gr.?

(*b*) 9 mi. 13 rd. 4 yd. 2 ft.; 16 rd. 5 yd. 1 ft. 5 in.; 16 mi. 2 rd. 3 in.; 14 rd. 1 yd. 9 in.?

(*c*) 3 cwt. 46 lb. 12 oz.; 12 cwt. 9½ lb.; 2¼ cwt. 21⅝ lb.?

(*d*) 10 yr. 8 mo. 5 wk. 3 da.; 42 yr. 6 mo. 7 da.; 7 yr. 5 mo. 18 wk. 4 da.; 17 yr. 17 da.?

(*e*) 17 tons 11 cwt. 49 lb. 14 oz.; 16 tons 47 lb. 13 oz.; 20 tons 13 cwt. 14 lb. 6 oz.; 11 tons 4 cwt. 16 lb. 12 oz.?

(*f*) 14 sq. yd. 8 sq. ft. 19 sq. in.; 105 sq. yd. 16 sq. ft. 240 sq. in.; 42 sq. yd. 28 sq. ft. 165 sq. in.?

Ans. {
(*a*) 86 lb. 3 oz. 16 pwt. 7 gr.
(*b*) 25 mi. 47 rd. 1 ft. 5 in.
(*c*) 18 cwt. 2 lb. 14 oz.
(*d*) 78 yr. 1 mo. 3 wk. 3 da.
(*e*) 65 tons 9 cwt. 28 lb. 13 oz.
(*f*) 167 sq. yd. 136 sq. in.

SUBTRACTION OF DENOMINATE NUMBERS.

231. EXAMPLE.—From 21 rd. 2 yd. 2 ft. 6½ in., take 9 rd. 4 yd. 10¼ in.

SOLUTION.—

rd.	yd.	ft.	in.	
21	2	2	6½	
9	4	0	10¼	
11	3½	1	8¼	Ans.

EXPLANATION.—Since 10¼ inches cannot be taken from 6½ inches, we must borrow 1 foot, or 12 inches, from the 2 feet in the next column and add it to the 6½. $6\frac{1}{2} + 12 = 18\frac{1}{2}$. $18\frac{1}{2}$ inches $- 10\frac{1}{4}$ inches $= 8\frac{1}{4}$ inches. Then, 0 foot from the 1 remaining foot $= 1$ foot. 4 yards cannot be taken from 2 yards; therefore, we borrow 1 rod, or 5½ yards, from 21 rods and add it to 2. $2 + 5\frac{1}{2} = 7\frac{1}{2}$; $7\frac{1}{2} - 4 = 3\frac{1}{2}$ yards. 9 rods from 20 rods $= 11$ rods. Hence, the remainder is 11 rods 3½ yards 1 foot 8¼ inches. Ans.

To avoid fractions as much as possible, we reduce the ½ yard to inches, obtaining 18 inches; this added to 8¼ inches, gives 26¼ inches, which equals 2 feet 2¼ inches. Then, 2 feet $+ 1$ foot $= 3$ feet $= 1$ yard, and 3 yards $+ 1$ yard $= 4$ yards. Hence, the above answer becomes 11 rods 4 yards 0 feet 2¼ inches.

232. EXAMPLE.—What is the difference between 3 rd. 2 yd. 2 ft. 10 in. and 47 ft.?

SOLUTION.—47 ft. = 2 rd. 4 yd. 2 ft.

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	0	$3\frac{1}{2}$	0	10	
or		3	2	4	Ans.

To find (approximately) the interval of time between two dates:

233. EXAMPLE.—How many years, months, days, and hours between 4 o'clock P.M. of June 15, 1868, and 10 o'clock A.M., September 28, 1891?

SOLUTION.—

yr.	mo.	da.	hr.	
1891	8	28	10	
1868	5	15	16	
23	3	12	18	Ans.

EXPLANATION.—Counting 24 hours in 1 day, 4 o'clock P.M. is the 16th hour from the beginning of the day, or midnight. On September 28, 8 months and 28 days have elapsed, and on June 15, 5 months and 15 days. After placing the earlier date under the later date, subtract as in the previous problems. Count 30 days as 1 month.

234. Rule.—*Place the smaller quantity under the larger quantity, with like denominations under each other. Beginning at the right, subtract successively the number in the subtrahend in each denomination from the one above, and place the differences underneath. If the number in the minuend of any denomination is less than the number under it in the subtrahend, one must be borrowed from the minuend of the next higher denomination, reduced and added to it.*

EXAMPLES FOR PRACTICE.

235. From

(*a*) 125 lb. 8 oz. 14 pwt. 18 gr. take 96 lb. 9 oz. 10 pwt. 4 gr.
(*b*) 126 hhd. 27 gal. take 104 hhd. 14 gal. 1 qt. 1 pt.
(*c*) 65 T. 14 cwt. 64 lb. 10 oz. take 16 T. 11 cwt. 14 oz.
(*d*) 148 sq. yd. 16 sq. ft. 142 sq. in. take 132 sq. yd. 136 sq. in.

(*e*) 100 bu. take 28 bu. 2 pk. 5 qt. 1 pt.
(*f*) 14 mi. 34 rd. 16 yd. 13 ft. 11 in. take 3 mi. 27 rd. 11 yd. 4 ft. 10 in.

Ans.
(*a*) 28 lb. 11 oz. 4 pwt. 14 gr.
(*b*) 22 hhd. 12 gal. 2 qt. 1 pt.
(*c*) 49 T. 3 cwt. 63 lb. 12 oz.
(*d*) 16 sq. yd. 16 sq. ft. 6 sq. in.
(*e*) 71 bu. 1 pk. 2 qt. 1 pt.
(*f*) 11 mi. 7 rd. 5 yd. 9 ft. 1 in.

MULTIPLICATION OF DENOMINATE NUMBERS.

236. EXAMPLE.—Multiply 7 lb. 5 oz. 13 pwt. 15 gr. by 12.

SOLUTION.—

lb.	oz.	pwt.	gr.	
7	5	13	15	
			12	
89	8	3	12	Ans.

EXPLANATION.—15 grains × 12 = 180 grains. 180 ÷ 24 = 7 pennyweights and 12 grains remaining. Place the 12 in the grain column and carry the 7 pennyweights to the next. Now, 13 × 12 + 7 = 163 pennyweights ; 163 ÷ 20 = 8 ounces and 3 pennyweights remaining. Then, 5 × 12 + 8 = 68 ounces; 68 ÷ 12 = 5 pounds and 8 ounces remaining. Then, 7 × 12 + 5 = 89 pounds. The entire product is 89 pounds 8 ounces 3 pennyweights 12 grains. Ans.

237. Rule.—*Multiply the number representing each denomination by the multiplier, and reduce each product to the next higher denomination, writing the remainders under each denomination, and carrying the quotient to the next, as in Addition of Denominate Numbers.*

238. NOTE.—In multiplication and division of denominate numbers, it is sometimes easier to reduce the number to the lowest denomination given before multiplying or dividing, especially if the multiplier or divisor is a decimal. Thus, in the above example, had the multiplier been 1.2, the easiest way to multiply would have been to reduce the number to grains; then, multiply by 1.2, and reduce the product to higher denominations. For example, 7 lb. 5 oz. 13 pwt. 15 gr. = 43,047 gr. 43,047 × 1.2 = 51,656.4 gr. = 8 lb. 11 oz. 12 pwt. 8.4 gr. Also, 43,047 × 12 = 516,564 gr. = 89 lb. 8 oz. 3 pwt. 12 gr., as above. The student may use either method.

EXAMPLES FOR PRACTICE.

239. Multiply

(*a*) 15 cwt. 90 lb. by 5; (*b*) 12 yr. 10 mo. 4 wk. 3 da. by 14; (*c*) 11 mi. 145 rd. by 20; (*d*) 12 gal. 4 pt. by 9; (*e*) 8 cd. 76 cu. ft. by 15; (*f*) 4 hhd. 3 gal. 1 qt. 1 pt. by 12.

Ans.
(*a*) 79 cwt. 50 lb.
(*b*) 180 yr. 11 mo. 2 wk.
(*c*) 229 mi. 20 rd.
(*d*) 112 gal. 2 qt.
(*e*) 128 cd. 116 cu. ft.
(*f*) 48 hhd. 40 gal. 2 qt

DIVISION OF DENOMINATE NUMBERS.

240. Example.—Divide 48 lb. 11 oz. 6 pwt. by 8.

Solution.—

	lb.	oz.	pwt.	gr.	
8)	48	11	6	0	
	6 lb.	1 oz.	8 pwt.	6 gr.	Ans.

Explanation.—After placing the quantities as above, proceed as follows: 8 is contained in 48 six times without a remainder. 8 is contained in 11 ounces once with 3 ounces remaining. $3 \times 20 = 60$; $60 + 6 = 66$ pennyweights; 66 pennyweights $\div 8 = 8$ pennyweights and 2 remaining; 2×24 grains $= 48$ grains; 48 grains $\div 8 = 6$ grains. Therefore, the entire quotient is 6 pounds 1 ounce 8 pennyweights 6 grains. Ans.

Example.—A silversmith melted up 2 lb. 8 oz. 10 pwt. of silver, which he made into 6 spoons; what was the weight of each spoon?

Solution.—

	lb.	oz.	pwt.		
6)	2	8	10		
		5 oz.	8 pwt.	8 gr.	Ans.

Explanation.—Since we cannot divide 2 pounds by 6, we reduce it to ounces. 2 pounds $= 24$ ounces, and 24 ounces $+ 8$ ounces $= 32$ ounces; 32 ounces $\div 6 = 5$ ounces and 2 ounces over. 2 ounces $= 40$ pennyweights. 40 pennyweights $+ 10$ pennyweights $= 50$ pennyweights, and 50 pennyweights $\div 6 = 8$ pennyweights and 2 pennyweights over. 2 pennyweights $= 48$ grains, and 48 grains $\div 6 = 8$ grains. Hence, each spoon contains 5 ounces 8 pennyweights 8 grains. Ans.

241. EXAMPLE.—Divide 820 rd. 4 yd. 2 ft. by 112.

SOLUTION.—

```
           rd.   yd. ft. rd. yd. ft.  in.
      112)820    4  2 ( 7   1   2  5.143  Ans.
          784
          ---
           36 rd. rem.
          5.5
          ---
          180
         180
         ----
         198.0 yd.
           4
         ---
     112)202 yd. ( 1 yd.
         112
         ---
          90 yd. rem.
           3
         ---
         270 ft.
           2 ft.
         ---
     112)272 ft. ( 2 ft.
         224
         ---
          48 ft. rem.
          12
          --
          96
         48
         ---
     112)576 in. ( 5.1428 + in., or 5.148 in.
         560
         ---
          160
          112
          ---
           480
           448
           ---
            320
            224
            ---
             960
             896
             ---
              64
```

EXPLANATION.—The first quotient is 7 rods with 36 rods remaining. $5.5 \times 36 = 198$ yards; 198 yards + 4 yards = 202 yards; 202 yards ÷ 112 = 1 yard and 90 yards remaining. $90 \times 3 = 270$ feet; 270 feet + 2 feet = 272 feet; 272 feet ÷ 112 = 2 feet and 48 feet remaining; $48 \times 12 = 576$ inches; 576 inches ÷ 112 = 5.143 inches, nearly. Ans.

The preceding example is solved by long division, because the numbers are too large to deal with mentally. Instead of expressing the last result as a decimal, it might have been expressed as a common fraction. Thus, $576 \div 112 = 5\frac{16}{112} = 5\frac{1}{7}$ inches. The chief advantage of using a common fraction is that if the quotient be multiplied by the divisor, the result will always be the same as the original dividend.

242. Rule.—*Find how many times the divisor is contained in the first or highest denomination of the dividend. Reduce the remainder (if any) to the next lower denomination, and add to it the number in the given dividend expressing that denomination. Divide this new dividend by the divisor. The quotient will be the next denomination in the quotient required. Continue in this manner until the lowest denomination is reached. The successive quotients will constitute the entire quotient.*

EXAMPLES FOR PRACTICE.

243. Divide

(*a*) 376 mi. 276 rd. by 22; (*b*) 1,137 bu. 3 pk. 4 qt. 1 pt. by 10; (*c*) 84 cwt. 48 lb. 49 oz. by 16; (*d*) 78 sq. yd. 18 sq. ft. 41 sq. in. by 18; (*e*) 148 mi. 64 rd. 24 yd. by 12; (*f*) 100 tons 16 cwt. 18 lb. 11 oz. by 15; (*g*) 36 lb. 18 oz. 18 pwt. 14 gr. by 8; (*h*) 112 mi. 48 rd. by 100.

Ans. {
(*a*) 17 mi. $41\frac{7}{11}$ rd.
(*b*) 113 bu. 3 pk. 1 qt. $\frac{1}{2}$ pt.
(*c*) 5 cwt. 28 lb. $3\frac{1}{16}$ oz.
(*d*) 4 sq. yd. 4 sq. ft. $2\frac{5}{18}$ sq. in.
(*e*) 12 mi. 112 rd. 2 yd.
(*f*) 6 tons 14 cwt. 41 lb. $3\frac{14}{15}$ oz.
(*g*) 4 lb. 8 oz. 7 pwt. $7\frac{3}{4}$ gr.
(*h*) 1 mi. $38\frac{22}{25}$ rd.

1. If 12 mine cars were dumped to load a railroad car with coal, what was the average weight of coal in each, if the total weight of coal in the railroad car was 28 tons 17 cwt. 32 lb. Ans. 2 tons 8 cwt. 11 lb.

2. A turnout 65 yd. 0 ft. 6 in. long will hold exactly 23 mine cars. What is the length of each car? Ans. 2 yd. 2 ft. 6 in.

3. A shaft is 286 ft. 3 in. deep, and it is timbered down to the bed rock, a distance equal to $\frac{2}{7}$ the depth of the shaft. How much of the shaft is timbered? Ans. 81 ft. $9\frac{3}{7}$ in.

4. A boiler shell which is 16 ft. $3\frac{1}{16}$ in. long is made up of 3 sheets.

If the lap at each of the two middle seams is $2\frac{1}{4}$ in., what is the length of each sheet? Ans. 5 ft. $6\frac{25}{48}$ in.

5. In a return tubular boiler the heating surface is divided as follows: Outside shell, 98 sq. ft. 9.8 sq. in.; heads, 5 sq. ft. $4\frac{1}{2}$ sq. in.; tubes, 683 sq. ft. 10.75 sq. in. What is the total area of the heating surface in square feet and square inches? Ans. 786 sq. ft. 25.05 sq. in.

6. In a mine having an endless-rope haulage system 1 mi. 2 rd. 4 yd. 1 ft. long, 5 trips are attached to the main or outgoing band. What is the distance the trips are apart? Ans. 64 rd. 3 yd. $\frac{1}{5}$ ft.

INVOLUTION.

244. **Involution** is the process of multiplying a number by itself one or more times. The product obtained by multiplying a number by itself is called a **power** of that number.

Thus, the *second* power of 3 is 9, since 3×3 are 9.

The *third* power of 3 is 27, since $3 \times 3 \times 3$ are 27.

The *fifth* power of 2 is 32, since $2 \times 2 \times 2 \times 2 \times 2$ are 32.

245. An **exponent** is a *small figure* placed to the *right* and a little *above* a number to show to what *power* it is to be raised, or how many times the number is to be used as a factor, as the small figures ^{2}, ^{3}, and ^{5} below:

$$3^2 = 3 \times 3 = 9.$$
$$3^3 = 3 \times 3 \times 3 = 27.$$
$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32.$$

246. The **root** of a number is that number which, used the required number of times as a factor, produces the number. In the above cases, 3 is a root of 9, since 3×3 are 9. It is also a root of 27, since $3 \times 3 \times 3$ are 27. Also, 2 is a root of 32, since $2 \times 2 \times 2 \times 2 \times 2$ are 32.

247. The *second* power of a number is called its **square.**

Thus, 5^2 is called the **square** of 5, or 5 *squared*, and its value is $5 \times 5 = 25$.

248. The *third* power of a number is called its **cube.**

Thus, 5^3 is called the *cube* of 5, or 5 *cubed*, and its value is $5 \times 5 \times 5 = 125$.

249. To find any power of a number:

EXAMPLE.—What is the third power, or cube, of 35?

SOLUTION.—To find the cube of a number, use the number three times as a factor.

$$35 \times 35 \times 35,$$

$$\text{or}\quad \begin{array}{r} 35 \\ \underline{35} \\ 175 \\ \underline{105} \\ 1225 \\ \underline{35} \\ 6125 \\ \underline{3675} \\ \end{array}$$

$$\textit{cube} = 42875 \quad \text{Ans.}$$

EXAMPLE.—What is the fourth power of 15?

SOLUTION.— $15 \times 15 \times 15 \times 15,$

$$\text{or}\quad \begin{array}{r} 15 \\ \underline{15} \\ 75 \\ \underline{15} \\ 225 \\ \underline{15} \\ 1125 \\ \underline{225} \\ 3375 \\ \underline{15} \\ 16875 \\ \underline{3375} \\ \end{array}$$

$$\textit{fourth power} = 50625 \quad \text{Ans.}$$

250. EXAMPLE.— $1.2^3 =$ what?

SOLUTION.— $1.2 \times 1.2 \times 1.2,$

$$\text{or}\quad \begin{array}{r} 1.2 \\ \underline{1.2} \\ 1.44 \\ \underline{1.2} \\ 1.728 \end{array} \quad \text{Ans.}$$

EXAMPLE.—$.47^5$ = what?

SOLUTION.— $.47 \times .47 \times .47 \times .47 \times .47$.

```
or           .47
             .47
            ----
             329
            188
           -----
           .2209
             .47
          ------
           15463
           8836
         -------
         .103823
             .47
         -------
          726761
         415292
       ---------
       .04879681
             .47
       ---------
        34157767
       19518724
     -----------
     .0229345007  Ans.
```

251. EXAMPLE.—What is the third power, or cube, of $\frac{3}{8}$?

SOLUTION.—$\left(\frac{3}{8}\right)^3 = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3 \times 3}{8 \times 8 \times 8} = \frac{27}{512}$. Ans.

252. Rule.—I. *To raise a whole number, or a decimal, to any power, use it as a factor as many times as there are units in the exponent.*

II. *To raise a fraction to any power, raise both the numerator and denominator to the power indicated by the exponent*

EXAMPLES FOR PRACTICE.

253. Raise the following to the powers indicated:

	Ans.
(*a*) 85^2.	(*a*) 7,225.
(*b*) $\left(\frac{12}{13}\right)^2$.	(*b*) $\frac{144}{169}$.
(*c*) 6.5^2.	(*c*) 42.25.
(*d*) 14^4.	(*d*) 38,416.
(*e*) $\left(\frac{3}{4}\right)^3$.	(*e*) $\frac{27}{64}$.
(*f*) $\left(\frac{5}{6}\right)^3$.	(*f*) $\frac{125}{216}$.
(*g*) $\left(\frac{7}{2}\right)^3$.	(*g*) $\frac{343}{8}$.
(*h*) 1.4^5.	(*h*) 5.37824.

EVOLUTION.

254. **Evolution** is the reverse of involution. It is the process of finding the root of a number which is considered as a power.

255. The **square root** of a number is that number which, when used twice as a factor, produces the number.

Thus, 2 is the square root of 4, since 2×2, or 2^2, $= 4$.

256. The **cube root** of a number is that number which, when used three times as a factor, produces the number.

Thus, 3 is the cube root of 27, since $3 \times 3 \times 3$, or 3^3, $= 27$.

257. The **radical sign** $\sqrt{\ }$, when placed before a number, indicates that some root of that number is to be found.

258. The **index** of the root is a *small figure* placed *over* and to the *left* of the *radical sign*, to show what root is to be found.

Thus, $\sqrt[2]{100}$ denotes the *square root* of 100.

$\sqrt[3]{125}$ denotes the *cube root* of 125.

$\sqrt[4]{256}$ denotes the *fourth root* of 256, and so on.

259. When the square root is to be extracted, the index is generally omitted. Thus, $\sqrt{100}$ indicates the square root of 100. Also, $\sqrt{225}$ indicates the square root of 225.

SQUARE ROOT.

260. The *largest* number that can be written with *one* figure is 9, and $9^2 = 81$; the *largest* number that can be written with *two* figures is 99, and $99^2 = 9,801$; with *three* figures 999, and $999^2 = 998,001$; with *four* figures 9,999, and $9,999^2 = 99,980,001$, etc.

In *each* of the above it will be noticed that the square of the number contains just *twice* as many figures as the number.

In order to find the square root of a number, the first step is to find how many figures there will be in the root. This is done by pointing off the number into *periods* of *two* figures each, *beginning at the right.* The number of periods will indicate the number of figures in the root.

Thus, the square root of 83,740,801 must contain 4 figures, since, pointing off the periods, we get 83'74'08'01, or 4 periods; consequently, there must be 4 figures in the root. In like manner, the square root of 50,625 must contain 3 figures, since there are (5'06'25) 3 periods.

261. EXAMPLE.—Find the square root of 31,505,769.

```
                                              root
SOLUTION.—   (a)    5             3 1'5 0'5 7'6 9 ( 5 6 1 8   Ans.
                    5        (b)  2 5
                  -----           ---
             (d)  1 0 0      (c)    6 5 0
                      6             6 3 6
                  -------           -----
                  1 0 6      (e)      1 4 5 7
                      6               1 1 2 1
                  -------             -------
                  1 1 2 0               3 3 6 6 9
                        1               3 3 6 6 9
                  -------               ---------
                  1 1 2 1                       0
                        1
                  ---------
                  1 1 2 2 0
                          3
                  ---------
                  1 1 2 2 3
```

EXPLANATION.—Pointing off into periods of two figures each, it is seen that there are four figures in the root. Now, find the largest single number whose square is less than or equal to 31, the first period. This is evidently 5, since $6^2 = 36$, which is greater than 31. Write it to the right, as in long division, and also to the left, as shown at (*a*). This is the first figure of the root. Now, multiply the 5 at (*a*) by the 5 in the root, and write the result under the first period, as shown at (*b*). Subtract, and obtain 6 as a remainder.

Bring down the next period 50, and annex it to the remainder 6, as shown at (*c*), which we call the **dividend.** Add the root already found to the 5 at (*a*), getting 10, and annex a cipher to this 10, thus making it 100, which we call the **trial divisor.** Divide the dividend (*c*) by the trial divisor (*d*), and obtain 6, which is *probably* the next figure of the root. Write 6 in the root, as shown, and also add it to 100, the trial divisor, making it 106. This is called the **complete divisor.**

Multiply this by 6, the second figure in the root, and subtract the result from the dividend (*c*). The remainder is 14, to which annex the next period, making it 1,457, as shown at (*e*), which we call the **new dividend.** Add the second figure of the root to the complete divisor 106, and annex a cipher, thus getting 1,120. Dividing 1,457 by 1,120, we get 1 as the next figure of the root. Adding this last figure of the root to 1,120, multiplying the result by it, and subtracting from 1,457, the remainder is 336.

Annexing the next and last period, 69, the result is 33,669. Now, adding the last figure of the root to 1,121, and annexing a cipher as before, the result is 11,220. Dividing 33,669 by 11,220, the result is 3, the fourth figure in the root. Adding it to 11,220, and multiplying the sum by it, the result is 33,669. Subtracting, there is no remainder; hence, $\sqrt{31,505,769} =$ 5,613. Ans.

262. The square of any number wholly decimal always contains twice as many figures as the number squared. For example, $.1^2 = .01$; $.13^2 = .0169$; $.751^2 = .564001$, etc.

263. It will also be noticed that the number squared is always less than the decimal. Hence, if it be required to find the square root of a decimal, and the decimal has not an even number of figures in it, annex a cipher. The best way to determine the number of figures in the root of a decimal is to begin at the decimal point, and, going towards the *right*, point off the decimal into periods of two figures each. Then, if the last period contains but one figure, annex a cipher.

264. EXAMPLE.—What is the square root of .000576 ?

			root
SOLUTION.—	2	.00'05'76(	.024 Ans.
	2	4	
	40	176	
	4	176	
	44	0	

EXPLANATION.—Beginning at the decimal point, and pointing off the number into periods of two figures each, it is seen that the first period is composed of ciphers; hence, the first figure of the root must be a cipher. The remaining portion of the solution should be perfectly clear from what has preceded.

265. If the number is not a perfect power, the root will consist of an interminable number of decimal places. The result may be carried to any required number of decimal places by annexing periods of two ciphers each to the number.

266. EXAMPLE.—What is the square root of 3? Find the result to five decimal places.

```
                                           root
SOLUTION.— 1          3.00'00'00'00'00(1.73205+   Ans.
           1          1
           --         --
           20         200
            7         189
           --         ---
           27          1100
            7          1029
           ---         ----
           340            7100
             8            6924
           ---            ----
           343             1760000
             3             1732025
           ----            -------
           3460              27975
              2
           ----
           3462
              2
           ------
           346400
                5
           ------
           346405
```

EXPLANATION.—Annexing five periods of two ciphers each to the right of the decimal point, the first figure of the root is 1. To get the second figure, we find that, in dividing 200 by 20, it is 10. This is evidently too large.

Trying 9, we add 9 to 20, and multiply 29 by 9, the result is 261, a result which is considerably larger than 200; hence, 9 is too large. In the same way it is found that 8 is also too large. Trying 7, 7 times 27 is 189, a result smaller than 200; therefore, 7 is the second figure of the root. The next two figures, 3 and 2, are easily found. The fifth figure in the root is a cipher, since the trial divisor 34,640 is greater than the new dividend 17,600. In a case of this kind, we annex another cipher to 34,640, thereby making it 346,400, and bring down the next period, making the 17,600, 1,760,000. The next figure of the root is 5, and as we now have five decimal places, we will stop.

The square root of 3 is, then, 1.73205 +. Ans.

267. Example.—What is the square root of .3 to five decimal places?

Solution.—

		root
5	.30'00'00'00'00(	.54772 + Ans.
5	25	
100	500	
4	416	
104	8400	
4	7609	
1080	79100	
7	76629	
1087	247100	
7	219084	
10940	28016	
7		
10947		
7		
109540		
2		
109542		

Explanation.—In the above example, we annex a cipher to .3, making the first period .30, since every period of a decimal, as was mentioned before, must have two figures in it. The remainder of the work should be perfectly clear.

268. If it is required to find the square root of a mixed number, begin at the decimal point, and point off the periods both ways. The manner of finding the root will then be exactly the same as in the previous cases.

269. EXAMPLE.—What is the square root of 258.2449?

SOLUTION.—

1	2'58.24'49(16.07 Ans.
1	1
20	158
6	156
26	22449
6	22449
3200	0
7	
3207	

EXPLANATION.—In the above example, since 320 is greater than 224, we place a cipher for the third figure of the root, and annex a cipher to 320, making it 3,200. Then, bringing down the next period 49, 7 is found to be the fourth figure of the root. Since there is no remainder, the square root of 258.2449 is 16.07. Ans.

270. Proof.—*To prove square root, square the result obtained. If the number is an exact power, the square of the root will equal it; if it is not an exact power, the square of the root will very nearly equal it.*

271. Rule.—I. *Begin at units place, and separate the number into periods of two figures each, proceeding from left to right with the decimal part, if there is any.*

II. *Find the greatest number whose square is contained in the first or left-hand period. Write this number as the first figure in the root; also, write it at the left of the given number.*

Multiply this number at the left by the first figure of the root, and subtract the result from the first period; then annex the second period to the remainder.

III. *Add the first figure of the root to the number in the first column on the left, and annex a cipher to the result; this is the trial divisor. Divide the dividend by the trial divisor* *for the second figure in the root, and add this figure to the trial divisor to form the complete divisor. Multiply the complete divisor by the second figure in the root, and subtract this result from the dividend. (If this result is larger than the dividend, a smaller number must be tried for the second figure of the root.) Now bring down the third period, and annex it to the last remainder for a new dividend. Add the second figure of the root to the complete divisor, and annex a cipher for a new trial divisor.*

IV. *Continue in this manner to the last period, after which, if any additional places in the root are required, bring down cipher periods, and continue the operation.*

V. *If at any time the trial divisor is not contained in the dividend, place a cipher in the root, annex a cipher to the trial divisor, and bring down another period.*

VI. *If the root contains an interminable decimal, and it is desired to terminate the operation at some point, say, the fourth decimal place, carry the operation one place further, and if the fifth figure is 5 or greater, increase the fourth figure by 1 and omit the sign +.*

272. Short Method.—If the number whose root is to be extracted is not an exact square, the root will be an interminable decimal. It is then usual to extract the root to a certain number of decimal places. In such cases, the work may be greatly shortened as follows: Determine to how many decimal places the work is to be carried, say 5, for example; add to this the number of places in the integral part of the root, say 2, for example, thus determining the number of figures in the root, in this case $5 + 2 = 7$. Divide this number by 2 and take the next higher number. In the above case, we have $7 \div 2 = 3\frac{1}{2}$; hence, we take 4, the next higher number. Now extract the root in the usual manner until the same number of figures have been obtained as was expressed by the number obtained above, in this case 4. Then form the trial divisor in the usual manner, but omitting to annex the cipher; divide the last remainder by the trial divisor, as in long division, obtaining as many figures of the quotient as there are remaining figures of the root, in this case 7 − 4 = 3. The remainder so obtained is the remaining figures of the root.

Consider the example in Art. **267.** Here there are 5 figures in the root. We therefore extract the root to 3 places in the usual manner, obtaining .547 for the first three root figures. The next trial divisor is 1,094 (with the cipher omitted), and the last remainder is 791. Then, 791 ÷ 1,094 = .723, and the next two figures of the root are 72, the whole root being .54772 +. Always carry the division one place further than desired, and if the last figure is 5 or greater, increase the preceding figure by 1. This method should not be used unless the root contains five or more figures.

NOTE.—If the last figure of the root found in the regular manner is a cipher, carry the process one place further before dividing as described above.

EXAMPLES FOR PRACTICE.

273. Find the square root of

(*a*) 186,624.		(*a*) 432.
(*b*) 2,050,624.		(*b*) 1,432.
(*c*) 29,855,296.		(*c*) 5,464.
(*d*) .0116964.		(*d*) .1081 +.
(*e*) 198.1369.		(*e*) 14.0761.
(*f*) 994,009.	Ans.	(*f*) 997.
(*g*) 2.375 to four decimal places.		(*g*) 1.5411.
(*h*) 1.625 to three decimal places.		(*h*) 1.275.
(*i*) .3025.		(*i*) .55.
(*j*) .571428.		(*j*) .7559 +.
(*k*) .78125.		(*k*) .8839.

CUBE ROOT.

274. In the same manner as in the case of square root, it can be shown that the periods into which a number is divided, whose cube root is to be extracted, must contain three figures, except that the first or left-hand period of a whole or mixed number may contain one, two, or three figures.

275. Example.—What is the cube root of 375,741,853,696?

Solution. —

(1)	(2)	(3) *root*
7	49	375'741'853'696(7216 Ans.
7	98	343
14	14700	32741
7	424	30248
210	15124	2493853
2	428	1557361
212	1555200	936492696
2	2161	936492696
214	1557361	0
2	2162	
2160	155952300	
1	129816	
2161	156082116	
1		
2162		
1		
21630		
6		
21636		

Explanation.—Write the work in three columns as follows: On the right place the number whose cube root is to be extracted, and point it off into periods of three figures each. Call this column (3). Find the largest number whose cube is less than or equal to the first period, in this case 7. Write the 7 on the right, as shown, for the first figure of the root, and also on the extreme left at the head of column (1). Multiply the 7 in column (1) by the first figure of the root 7, and write the product 49 at the head of column (2). Multiply the number in column (2) by the first figure of the root 7, and write the product 343 under the figures in the first period Subtract and bring down the next period, obtaining 32,741 for the dividend. Add the first figure of the root to the number in column (1), obtaining 14, which call the *first correction.* Multiply the first correction by the first figure of the root, add the product to the number in column (2), and obtain 147. Add the first figure of the root to the first correction, and obtain 21, which call the *second correction.* Annex *two* ciphers to the number in column (2), and obtain 14,700 for the trial divisor; also, annex *one* cipher to the second correction, and obtain 210. Dividing the dividend by the trial divisor, we obtain $\frac{32741}{14700} = 2+$, and write the 2 as the second figure of the root. Add the 2 to the second correction, and obtain 212, which, multiplied by the second figure of the root, and added to the trial divisor, gives 15,124, the complete divisor. This last result, multiplied by the second figure of the root and subtracted from the dividend, gives a remainder of 2,493. Annexing the third period, we obtain 2,493,853 for the new dividend. Adding the second figure of the root to the number in column (1), we get 214 as the new first correction; this, multiplied by the second figure of the root and added to the trial divisor, gives 15,552. Adding the second figure of the root to the first new correction gives 216 as the second new correction. Annexing two ciphers to the number in column (2) gives 1,555,200, the new trial divisor. Annexing one cipher to the second new correction gives 2,160. Dividing the new dividend by the new trial divisor, we obtain $\frac{2493853}{1555200} = 1+$, and write 1 as the third figure of the root. The remainder of the work should be perfectly clear from what has preceded.

276. In extracting the cube root of a decimal, proceed as above, taking care that each period contains *three* figures. Begin the pointing off at the decimal point, going towards the right. If the last period does not contain three figures, annex ciphers until it does.

277. EXAMPLE.—What is the cube root of .009129329?

SOLUTION.—

2	4	root
2	8	.009'129'329(.209
4	120000	8
2	5481	1129329
600	125481	1129329
9		0
609		

EXPLANATION.—Beginning at the decimal point, and pointing off as shown, the largest number whose cube is less than 9 is seen to be 2; hence, 2 is the first figure of the root. When finding the second figure, it is seen that the trial divisor 1,200 is greater than the dividend; hence, write a cipher for the second figure of the root; bring down the next period to form the new dividend; annex two ciphers to the trial divisor to form a new trial divisor; also, annex one cipher to the 60 in column (1). Dividing the new dividend by the new trial divisor, we get $\frac{1129329}{120000} = 9+$, and write 9 as the third figure of the root. Complete the work as before.

278. EXAMPLE.—What is the cube root of 78,347.809639?

SOLUTION.—

		root
4	16	78'347.809'639 (42.79
4	32	64
8	4800	14347
4	244	10088
120	5044	4259809
2	248	3766483
122	529200	493326639
2	8869	493326639
124	538069	0
2	8918	
1260	54698700	
7	115371	
1267	54814071	
7		
1274		
7		
12810		
9		
12819		

Explanation.—Since we have a mixed number, begin at the decimal point and point off periods of three figures each, in both directions. The first period contains but two figures, and the largest number whose cube is less than 78 is 4; consequently, 4 is the first figure of the root. The remainder of the work should be perfectly clear. When dividing the dividend by the trial divisor for the third figure of the root, the quotient was 8 +; but, on trying it, it was found that 8 was too large, the complete divisor being considerably larger than the trial divisor. Therefore, 7 was used instead of 8.

279. **Example.**—What is the cube root of 5 to five decimal places?

Solution.—

		root
1	1	5.000'000'000'000'000(1.70997+
1	2	1
2	300	4000
1	259	3913
30	559	87000000
7	308	78443829
37	8670000	8556171000
7	45981	7889992299
44	8715981	666178701000
7	46062	614014317973
5100	876204300	52164383027
9	461511	
5109	876665811	
9	461592	
5118	87712740300	
9	3590839	
51270	87716331139	
9		
51279		
9		
51288		
9		
512970		
7		
512977		

Explanation.—In the preceding example we annex five periods of ciphers, of three ciphers each, to the 5 for the decimal part of the root, placing the decimal point between the 5 and the first cipher. Since it is easy to see that the next figure of the root will be 5, we increase the last figure by 1, obtaining 1.70998 for the correct root to 5 decimal places. Ans.

280. Example.—What is the cube root of .5 to four decimal places?

Solution.—

		root
7	49	.500'000'000'000(.7937+
7	98	343
14	14700	157000
7	1971	150039
210	16671	6961000
9	2052	5638257
219	1872300	1322743000
9	7119	1321748953
228	1879419	994047
9	7128	
2370	188654700	
3	166579	
2373	188821279	
3		
2376		
3		
23790		
7		
23797		

Explanation.—In the above example we annex two ciphers to the .5 to complete the first period, and three periods of three ciphers each. The cube root of 500 is 7; this we write as the first figure of the root. The remainder of the work should be perfectly plain from the explanations of the preceding examples.

281. Example.—What is the cube root of .05 to four decimal places ?

Solution.—

		root
3	9	.050'000'000'000(.3684 +
3	18	27
6	2700	23000
3	576	19656
90	3276	3344000
6	612	3180032
96	388800	163968000
6	8704	162685504
102	397504	1282496
6	8768	
1080	40627200	
8	44176	
1088	40671376	
8		
1096		
8		
11040		
4		
11044		

282. Proof.—*To prove cube root, cube the result obtained. If the given number is an exact power, the cube of the root will equal it; if not an exact power, the cube of the root will very nearly equal it.*

283. Rule.—I. *Arrange the work in three columns, placing the number whose cube root is to be extracted in the third or right-hand column. Begin at units place, and separate the number into periods of three figures each, proceeding from the decimal point towards the right with the decimal part, if there is any.*

II. *Find the greatest number whose cube is not greater than the number in the first period. Write this number as the first figure of the root; also, write it at the head of the first column. Multiply the number in the first column by the first figure in the root, and write the result in the second* *column. Multiply the number in the second column by the first figure of the root; subtract the product from the first period, and annex the second period to the remainder for a new dividend; add the first figure of the root to the number in the first column for the first correction. Multiply the first correction by the first figure of the root, and add the product to the number in the second column. Add the first figure of the root to the first correction to form the second correction. Annex one cipher to the second correction, and two ciphers to the last number in the second column; the last number in the second column is the trial divisor.*

III. *Divide the dividend by the trial divisor to find the second figure of the root. Add the second figure of the root to the number in the first column, multiply the sum by the second figure of the root, and add the result to the trial divisor to form the complete divisor. Multiply the complete divisor by the second figure of the root, subtract the result from the dividend in the third column, and annex the third period to the remainder for a new dividend. Add the second figure of the root to the number in the first column to form the first correction; multiply the first correction by the second figure of the root, and add the product to the complete divisor. Add the second figure of the root to the first correction to form the second correction. Annex one cipher to the second correction, and two ciphers to the last number in the second column to form the new trial divisor.*

IV. *If there are more periods to be brought down, proceed as before. If there is a remainder after the root of the last period has been found, annex cipher periods, and proceed as before. The figures of the root thus obtained will be decimals.*

V. *If the root contains an interminable decimal, and it is desired to terminate the operation at some point, say the fourth decimal place, carry the operation one place further, and if the fifth figure is 5 or greater, increase the fourth figure by 1 and omit the sign +.*

284. Art. **272** can be applied to cube root (or any other root) as well as to square root. Thus, in the example, Art. **279**, there are to be $5 + 1 = 6$ figures in the root. Extracting the root in the usual manner to $6 \div 2 = 3$, say 4 figures, we get for the first four figures 1,709. The last remainder is 8,556,171, and the next trial divisor, with the ciphers omitted, is 8,762,043. Hence, the next two figures of the root are $8,556,171 \div 8,762,043 = .976$, say .98. Therefore, the root is 1.70998.

ROOTS OF FRACTIONS.

285. If the given number is in the form of a fraction, and it is required to find some root of it, the simplest and most exact method is to reduce the fraction to a decimal and extract the required root of the decimal. If, however, the numerator and denominator of the fraction are perfect powers, extract the required root of each separately, and write the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

286. Example.—What is the square root of $\frac{9}{64}$?

Solution.— $\sqrt{\frac{9}{64}} = \frac{\sqrt{9}}{\sqrt{64}} = \frac{3}{8}$. Ans.

287. Example.—What is the square root of $\frac{5}{8}$?

Solution.—Since $\frac{5}{8} = .625$, $\sqrt{\frac{5}{8}} = \sqrt{.625} = .7906$. Ans.

288. Example.—What is the cube root of $\frac{27}{64}$?

Solution.— $\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$. Ans.

289. Example.—What is the cube root of $\frac{1}{4}$?

Solution.—Since $\frac{1}{4} = .25$, $\sqrt[3]{\frac{1}{4}} = \sqrt[3]{.25} = .62996 +$. Ans.

290. Rule.—*Extract the required root of the numerator and denominator separately; or, reduce the fraction to a decimal and extract the root of the decimal.*

EXAMPLES FOR PRACTICE.

291. Find the cube root of

(*a*) $\frac{27}{512}$.
(*b*) 2 to five decimal places.
(*c*) 4,180,769,192.462 to five decimal places.
(*d*) $\frac{87}{125}$.
(*e*) $\frac{3}{8}$.
(*f*) 513,229.783302144 to three decimal places.

Ans.
(*a*) $\frac{3}{8}$.
(*b*) 1.25992+.
(*c*) 1,610.96238.
(*d*) .8862+.
(*e*) .7211+.
(*f*) 80.064.

TO EXTRACT OTHER ROOTS THAN THE SQUARE AND CUBE ROOTS.

292. Example.—What is the fourth root of 256?

Solution.— $\sqrt{256} = 16.$

$\sqrt{16} = 4.$

Therefore, $\sqrt[4]{256} = 4.$ Ans.

In this example, $\sqrt[4]{256}$, the index is 4, which equals 2×2. The root indicated by 2 is the square root; therefore, the square root is extracted twice.

293. Example.—What is the sixth root of 64?

Solution.— $\sqrt{64} = 8.$

$\sqrt[3]{8} = 2.$

Therefore, $\sqrt[6]{64} = 2.$ Ans.

In this example, $\sqrt[6]{64}$, the index is 6, which equals 2×3. The root indicated by 3 is the cube root; therefore, the square and cube roots are extracted in succession.

294. Rule.—*Separate the index of the required root into its factors (2's and 3's), and extract successively the roots indicated by the several factors obtained. The final result will be the required root.*

295. Example.—What is the sixth root of 92,873,580 to two decimal places?

Solution.— $6 = 3 \times 2$. Hence, extract the cube root, and then extract the square root of the result. $\sqrt[3]{92,873,580} = 452.8601$, and $\sqrt{452.8601} = 21.28+$. Ans.

296. It matters not which root is extracted first, but it is probably easier and more exact to extract the cube root first.

EXAMPLES FOR PRACTICE.

297. Extract the

(*a*) Fourth root of 100.
(*b*) Fourth root of 3,049,800,625.
(*c*) Sixth root of 9,474,296,896.

Ans. { (*a*) 3.16227+. (*b*) 235. (*c*) 46.

RATIO.

298. Suppose that it is desired to compare two numbers, say 20 and 4. If we wish to know how many times larger 20 is than 4, we divide 20 by 4 and obtain 5 for the quotient; thus, $20 \div 4 = 5$. Hence, we say that 20 is 5 times as large as 4, i. e., 20 contains 5 times as many units as 4. Again, suppose we desire to know what part of 20 is 4. We then divide 4 by 20 and obtain $\frac{1}{5}$; thus, $4 \div 20 = \frac{1}{5}$, or .2. Hence, 4 is $\frac{1}{5}$ or .2 of 20. This operation of comparing two numbers is termed *finding the ratio* of the two numbers. Ratio, then, is a comparison. It is evident that the two numbers to be compared must be expressed in the same unit; in other words, the two numbers must both be abstract numbers or concrete numbers of the same kind. For example, it would be absurd to compare 20 horses with 4 birds, or 20 horses with 4. Hence, **ratio** may be defined as a comparison between two numbers of the same kind.

299. A ratio may be *expressed* in three ways; thus, if it is desired to compare 20 and 4, and express this comparison as a ratio, it may be done as follows: $20 \div 4$; $20 : 4$, or $\frac{20}{4}$. All three are read *the ratio of 20 to 4*. The ratio of 4 to 20 would be expressed thus: $4 \div 20$; $4 : 20$, or $\frac{4}{20}$. The first method of expressing a ratio, although correct, is seldom or never used; the second form is the one oftenest met with, while the third is rapidly growing in favor, and is likely to supersede the second. The third form, called the fractional form, is preferred by modern mathematicians, and possesses great advantages to students of Algebra and of higher mathematical subjects. The second form seems to be better adapted to arithmetical subjects, and is one we shall ordinarily adopt. There is still another way of expressing a ratio, though seldom or never used in the case of a simple ratio like that given above. Instead of the colon, a straight vertical line is used; thus, $20 \mid 4$.

300. The **terms** of a ratio are the two numbers to be compared; thus, in the above ratio, 20 and 4 are the terms. When both terms are considered together, they are called a **couplet;** when considered separately, the first term is called the **antecedent,** and the second term the **consequent.** Thus, in the ratio 20 : 4, 20 and 4 form a couplet, and 20 is the antecedent, and 4 the consequent.

301. A ratio may be **direct** or **inverse.** The *direct ratio* of 20 to 4 is 20 : 4, while the *inverse ratio* of 20 to 4 is 4 : 20. The direct ratio of 4 to 20 is 4 : 20, and the inverse ratio is 20 : 4. An inverse ratio is sometimes called a **reciprocal** ratio. The **reciprocal** of a number is 1 divided by the number. Thus, the reciprocal of 17 is $\frac{1}{17}$; of $\frac{3}{8}$ is $1 \div \frac{3}{8} = \frac{8}{3}$; i.e., the reciprocal of a fraction is the fraction inverted. Hence, the inverse ratio of 20 to 4 may be expressed as 4 : 20, or as $\frac{1}{20} : \frac{1}{4}$. Both have equal values; for, $4 \div 20 = \frac{1}{5}$, and $\frac{1}{20} \div \frac{1}{4} = \frac{1}{20} \times \frac{4}{1} = \frac{1}{5}$.

302. The term **vary** implies a ratio. When we say that two numbers vary as some other two numbers, we mean that the ratio between the first two numbers is the same as the ratio between the other two numbers.

303. The **value** of a ratio is the result obtained by performing the division indicated. Thus, the value of the ratio 20:4 is 5, it is the quotient obtained by dividing the antecedent by the consequent.

304. By expressing the ratio in the fractional form, for example, the ratio of 20 to 4 as $\frac{20}{4}$, it is easy to see, from the laws of fractions, that if both terms be multiplied, or both divided by the same number, it will not alter the value of the ratio. Thus,

$$\frac{20}{4} = \frac{20 \times 5}{4 \times 5} = \frac{100}{20}; \text{ and } \frac{20}{4} = \frac{20 \div 4}{4 \div 4} = \frac{5}{1}.$$

305. It is also evident, from the laws of fractions, that multiplying the antecedent or dividing the consequent multiplies the ratio; and dividing the antecedent or multiplying the consequent divides the ratio.

306. When a ratio is expressed in words, as the ratio of 20 to 4, the first number named is always regarded as the antecedent and the second as the consequent, without regard to whether the ratio itself is direct or inverse. *When not otherwise specified, all ratios are understood to be direct.* To express an inverse ratio, the simplest way of doing it is to express it as if it were a direct ratio, with the first number named as the antecedent, and then transpose the antecedent to the place occupied by the consequent and the consequent to the place occupied by the antecedent; or if expressed in the fractional form, invert the fraction. Thus, to express the inverse ratio of 20 to 4, first write it 20 : 4, and then, transposing the terms, as 4 : 20; or as $\frac{20}{4}$, and then inverting as $\frac{4}{20}$. Or, the reciprocals of the numbers may be taken, as explained above. To **invert** a ratio is to transpose its terms.

EXAMPLES FOR PRACTICE.

307. What is the value of the ratio of

(*a*)	98 to 49 ?	(*a*)	2.
(*b*)	\$45 to \$9 ?	(*b*)	5.
(*c*)	$6\frac{1}{4}$ to $\frac{1}{2}$?	(*c*)	$12\frac{1}{2}$.
(*d*)	3.5 to 4.5 ?	(*d*)	$.77\frac{7}{9}$.
(*e*)	The inverse ratio of 76 to 19 ?	(*e*)	$\frac{1}{4}$.
(*f*)	The inverse ratio of 49 to 98 ?	Ans. (*f*)	2.
(*g*)	The inverse ratio of 18 to 24 ?	(*g*)	$1\frac{1}{3}$.
(*h*)	The inverse ratio of 9 to 15 ?	(*h*)	$1\frac{2}{3}$.
(*i*)	The ratio of 10 to 3, multiplied by 3 ?	(*i*)	10.
(*j*)	The ratio of 35 to 49, multiplied by 7 ?	(*j*)	5.
(*k*)	The ratio of 18 to 64, divided by 9 ?	(*k*)	$\frac{1}{32}$.
(*l*)	The ratio of 14 to 28, divided by 5 ?	(*l*)	$\frac{1}{10}$.

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308. Instead of expressing the value of a ratio by a single number, as above, it is customary to express it by means of another ratio in which the consequent is 1. Thus, suppose that it is desired to find the ratio of the weights of two pieces of iron, one weighing 45 pounds and the other weighing 30 pounds. The ratio of the heavier to the lighter is then 45 : 30, an inconvenient expression. Using the fractional form, we have $\frac{45}{30}$. Dividing both terms by 30, the consequent, we obtain $\frac{1\frac{1}{2}}{1}$ or $1\frac{1}{2} : 1$. This is the same result as obtained above, for $1\frac{1}{2} \div 1 = 1\frac{1}{2}$, and $45 \div 30 = 1\frac{1}{2}$.

309. A ratio may be squared, cubed, or raised to any power, or any root of it may be taken. Thus, if the ratio of two numbers is 105 : 63, and it is desired to cube this ratio, the cube may be expressed as $105^3 : 63^3$. That this is correct is readily seen; for, expressing the ratio in the fractional form, it becomes $\frac{105}{63}$, and the cube is $\left(\frac{105}{63}\right)^3 = \frac{105^3}{63^3} = 105^3 : 63^3$. Also, if it is desired to extract the cube root of the ratio $105^3 : 63^3$, it may be done by simply dividing the exponents by 3, obtaining 105 : 63. This may be proved in the same way as in the case of cubing the ratio. Thus, $105^3 : 63^3 = \left(\frac{105}{63}\right)^3$, and $\sqrt[3]{\left(\frac{105}{63}\right)^3} = \frac{105}{63} = 105 : 63$.

310. Since $\left(\frac{105}{63}\right)^3 = \left(\frac{5}{3}\right)^3$, it follows that $105^3 : 63^3 = 5^3 : 3^3$ (this expression is read: the ratio of 105 cubed to 63 cubed equals the ratio of 5 cubed to 3 cubed), and, hence, that the antecedent and consequent may both be multiplied or both divided by the same number, irrespective of any indicated powers or roots, without altering the value of the ratio. Thus, $24^2 : 18^2 = 4^2 : 3^2$. For, performing the operations indicated by the exponents, $24^2 = 576$ and $18^2 = 324$. Hence, $576 : 324 = 1\frac{7}{9}$, or $1\frac{7}{9} : 1$. Also, $4^2 = 16$ and $3^2 = 9$; hence, $16 : 9 = 1\frac{7}{9}$, or $1\frac{7}{9} : 1$, the same result as before. Also, $24^2 : 18^2 = \frac{24^2}{18^2} = \left(\frac{24}{18}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{4^2}{3^2} = 4^2 : 3^2$.

The statement may be proved for roots in the same manner. Thus, $\sqrt[3]{24^3} : \sqrt[3]{18^3} = \sqrt[3]{4^3} : \sqrt[3]{3^3}$. For the $\sqrt[3]{24^3} = 24$ and $\sqrt[3]{18^3} = 18$; and, $24 : 18 = 1\frac{1}{3}$ or $1\frac{1}{3} : 1$. Also, $\sqrt[3]{4^3} = 4$ and $\sqrt[3]{3^3} = 3$; $4 : 3 = 1\frac{1}{3}$ or $1\frac{1}{3} : 1$.

NOTE.—If the numbers composing the antecedent and consequent have different exponents, or if different roots of those numbers are indicated, the operations described in Art. **310** cannot be performed. This is evident; for, consider the ratio $4^2 : 8^3$. When expressed in the fractional form, it becomes $\frac{4^2}{8^3}$, which cannot be expressed either as $\left(\frac{4}{8}\right)^2$ or as $\left(\frac{4}{8}\right)^3$, and, hence, cannot be reduced as described above.

PROPORTION.

311. **Proportion** is an equality of ratios, the equality being indicated by the double colon (::) or by the sign of equality (=). Thus, to write in the form of a proportion the two equal ratios, 8 : 4 and 6 : 3, which both have the same value 2, we may employ one of the three following forms:

$$8 : 4 :: 6 : 3 \qquad (1)$$
$$8 : 4 = 6 : 3 \qquad (2)$$
$$\frac{8}{4} = \frac{6}{3} \qquad (3)$$

312. The first form is the one most extensively used, by reason of its having been exclusively employed in all the older works on mathematics. The second and third forms are being adopted by all modern writers on mathematical subjects, and, in time, will probably entirely supersede the first form. In this paper we shall adopt the second form, unless some statement can be made clearer by using the third form.

313. A proportion may be *read* in two ways. The old way to read the above proportion was—*8 is to 4 as 6 is to 3*; the new way is—*the ratio of 8 to 4 equals the ratio of 6 to 3.* The student may read it either way, but we recommend the latter.

314. Each ratio of a proportion is termed a **couplet**. In the above proportion, 8 : 4 is a couplet, and so is 6 : 3.

315. The numbers forming the proportion are called **terms;** and they are numbered consecutively from left to right, thus:

$$\overset{\textit{first}}{8} : \overset{\textit{second}}{4} = \overset{\textit{third}}{6} : \overset{\textit{fourth}}{3}$$

Hence, in any proportion, the ratio of the first term to the second term equals the ratio of the third term to the fourth term.

316. The first and fourth terms of a proportion are called the **extremes,** and the second and third terms, the **means.** Thus, in the foregoing proportion, 8 and 3 are the extremes and 4 and 6 are the means.

317. A **direct proportion** is one in which both couplets are direct ratios.

318. An **inverse proportion** is one which requires one of the couplets to be expressed as an inverse ratio. Thus, 8 is to 4 inversely as 3 is to 6 must be written $8 : 4 = 6 : 3$; i. e., the second ratio (couplet) must be inverted.

319. Proportion forms one of the most useful sections of Arithmetic. In our grandfathers' Arithmetics, it was called "The rule of three."

320. Rule I.—*In any proportion, the product of the extremes equals the product of the means.*

Thus, in the proportion,

$$17 : 51 = 14 : 42.$$

$17 \times 42 = 51 \times 14$, since both products equal 714.

321. Rule II.—*The product of the extremes divided by either mean gives the other mean.*

EXAMPLE.—What is the third term of the proportion $17 : 51 = \quad : 42$?

SOLUTION.—Applying rule II, $17 \times 42 = 714$, and $714 \div 51 = 14$. Ans.

322. Rule III.—*The product of the means divided by either extreme gives the other extreme.*

EXAMPLE.—What is the first term of the proportion $\quad : 51 = 14 : 42$?

SOLUTION.—Applying rule III, $51 \times 14 = 714$, and $714 \div 42 = 17$. Ans.

323. When stating a proportion in which one of the terms is unknown, represent the missing term by a letter, as x. Thus, the last example would be written,

$$x : 51 = 14 : 42$$

and for the value of x we have $x = \frac{51 \times 14}{42} = 17$.

324. If the same (addition and subtraction excepted) operations be performed upon *all* of the terms of a proportion, the proportion is not thereby destroyed. In other words, if all of the terms of a proportion be (1) multiplied or (2) divided by the same number; (3) if all the terms be raised to the same power; if (4) the same root of all the terms be taken, or (5) if both couplets be inverted, the proportion still holds. We will prove these statements by a numerical example, and the student can satisfy himself by other similar ones. The fractional form will be used, as it is better suited to the purpose. Consider the proportion $8 : 4 = 6 : 3$. Expressing it in the third form, it becomes $\frac{8}{4} = \frac{6}{3}$. What we are to prove is that, if any of the five operations enumerated above be performed upon all of the terms of this proportion, the first fraction will still equal the second fraction.

1. Multiplying all the terms by any number, say 7, $\frac{8 \times 7}{4 \times 7} = \frac{6 \times 7}{3 \times 7}$; or $\frac{56}{28} = \frac{42}{21}$. Now $\frac{56}{28}$ evidently equals $\frac{42}{21}$, since the value of either ratio is 2, and the same is true of the original proportion.

2. Dividing all the terms by any number, say 7, $\frac{8 \div 7}{4 \div 7} = \frac{6 \div 7}{3 \div 7}$; or $\frac{\frac{8}{7}}{\frac{4}{7}} = \frac{\frac{6}{7}}{\frac{3}{7}}$. But $\frac{8}{7} \div \frac{4}{7} = 2$, and $\frac{6}{7} \div \frac{3}{7} = 2$ also, the same as in the original proportion.

3. Raising all the terms to the same power, say the cube, $\frac{8^3}{4^3} = \frac{6^3}{3^3}$. This is evidently true, since $\frac{8^3}{4^3} = \left(\frac{8}{4}\right)^3 = 2^3 = 8$, and $\frac{6^3}{3^3} = \left(\frac{6}{3}\right)^3 = 2^3 = 8$ also.

4. Extracting the same root of all the terms, say the cube root, $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \frac{\sqrt[3]{6}}{\sqrt[3]{3}}$. It is evident that this is likewise true, since $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \sqrt[3]{\frac{8}{4}} = \sqrt[3]{2}$, and $\frac{\sqrt[3]{6}}{\sqrt[3]{3}} = \sqrt[3]{\frac{6}{3}} = \sqrt[3]{2}$ also.

5. Inverting both couplets, $\frac{4}{8}=\frac{3}{6}$, which is true, since both equal $\frac{1}{2}$.

325. If both terms of either couplet be multiplied or both divided by the same number, the proportion is not destroyed. This should be evident from the preceding article, and also from Art. **304.** Hence, in any proportion, equal factors may be canceled from the terms of a couplet, before applying rules II or III. Thus, the proportion $45:9 = x:7.1$, we may divide both terms of the first couplet by 9 (that is, cancel 9 from both terms), obtaining $5:1 = x:7.1$, whence $x = 7.1 \times 5 \div 1 = 35.5$. (See note in Art. **310.**)

326. The principle of all calculations in proportion is this: *Three of the terms are always given, and the remaining one is to be found.*

327. EXAMPLE.—If 4 men can earn $25 in one week, how much can 12 men earn in the same time?

SOLUTION.—The required term must bear the same relation to the given term of the same kind as one of the remaining terms bears to the other remaining term. We can then form a proportion by which the required term may be found.

The first question the student must ask himself in every calculation by proportion is:

"What is it I want to find?"

In this case it is dollars. We have two sets of men, one set earning $25, and we want to know how many dollars the other set earns. It is evident that the *amount* 12 men earn bears the same relation to the *amount* that 4 men earn as 12 men bears to 4 men. Hence, we have the proportion, the amount 12 men earn is to $25 as 12 men is to 4 men; or, since either extreme equals the product of the means divided by the other extreme, we have

The amount 12 men earn : $25 = 12 men : 4 men,

or the amount 12 men earn $= \frac{\$25 \times 12}{4} = \$75$. Ans.

Since it matters not which place x or the required term occupies, the problem could be stated as any of the following forms, the value of x being the same in each:

(*a*) \$25 : the amount 12 men earn = 4 men : 12 men; or the amount 12 men earn = $\frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

(*b*) 4 men : 12 men = \$25 : the amount 12 men earn; or the amount that 12 men earn = $\frac{\$25 \times 12}{4}$, or \$75, since either extreme equals the product of the means divided by the other extreme.

(*c*) 12 men : 4 men = the amount 12 men earn : \$25; or the amount that 12 men earn = $\frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

328. If the proportion is an inverse one, first form it as though it were a direct proportion, and then invert one of the couplets.

EXAMPLES FOR PRACTICE.

329. Find the value of x in each of the following:

			Ans.
(*a*)	\$16 : \$64 :: x : \$4.	(*a*)	x = \$1.
(*b*)	x : 85 :: 10 : 17.	(*b*)	$x = 50$.
(*c*)	24 : x :: 15 : 40.	(*c*)	$x = 64$.
(*d*)	18 : 94 :: 2 : x.	(*d*)	$x = 10\frac{4}{9}$.
(*e*)	\$75 : \$100 = x : 100.	(*e*)	$x = 75$.
(*f*)	15 pwt. : x = 21 : 10.	(*f*)	$x = 7\frac{1}{7}$ pwt.
(*g*)	x : 75 yd. = \$15 : \$5.	(*g*)	$x = 225$ yd.

1. If 75 pounds of lead cost \$2.10, what would 125 pounds cost at the same rate? Ans. \$3.50.

2. If A does a piece of work in 4 days and B does it in 7 days, how long will it take A to do what B does in 63 days? Ans. 36 days.

3. The circumferences of any two circles are to each other as their diameters. If the circumference of a circle 7 inches in diameter is 22 inches, what will be the circumference of a circle 31 inches in diameter? Ans. $97\frac{3}{7}$ inches.

INVERSE PROPORTION.

330. In Art. **318**, an inverse proportion was defined as one which required one of the couplets to be expressed as an inverse ratio. Sometimes the word *inverse* occurs in the statement of the example; in such cases the proportion can be written directly, merely inverting one of the couplets. But it frequently happens that only by carefully studying the conditions of the example can it be ascertained whether the proportion is direct or inverse. When in doubt, the student can always satisfy himself as to whether the proportion is direct or inverse by first ascertaining what is required, and stating the proportion as a direct proportion. Then, in order that the proportion may be true, if the first term is smaller than the second term, the third term must be smaller than the fourth; or if the first term is larger than the second term, the third term must be larger than the fourth term. Keeping this in mind, the student can always tell whether the required term will be larger or smaller than the other term of the couplet to which the required term belongs. Having determined this, the student then refers to the example, and ascertains from its conditions whether the required term is to be larger or smaller than the other term of the same kind. If the two determinations agree, the proportion is direct; otherwise, it is inverse, and one of the couplets must be inverted.

331. EXAMPLE.—If A's *rate* of doing work is to B's as 5 : 7, and A does a piece of work in 42 days, in what time will B do it?

SOLUTION.—The required term is the number of days it will take B to do the work. Hence, stating as a direct proportion,

$$5 : 7 = 42 : x.$$

Now, since 7 is greater than 5, x will be greater than 42. But, referring to the statement of the example, it is easy to see that B works faster than A; hence it will take B a less number of days to do the work than A. Therefore, the proportion is an inverse one, and should be stated

$$5 : 7 = x : 42,$$

from which $x = \frac{5 \times 42}{7} = 30$ days. Ans.

Had the example been stated thus: The time that A requires to do a piece of work is to the time that B requires, as 5 : 7; A can do it in 42 days, in what time can B do it? it is evident that it would take B a longer time to do the work than it would A; hence, x would be greater than 42, and the proportion would be direct, the value of x being $\frac{7 \times 42}{5}$ = 58.8 days.

EXAMPLES FOR PRACTICE.

332. Solve the following:

1. If a pump which discharges 4 gal. of water per min. can fill a tank in 20 hr., how long will it take a pump discharging 12 gal. per min. to fill it? Ans. $6\frac{2}{3}$ hr.

2. If a pump discharges 90 gal. of water in 20 hr., in what time will it discharge 144 gal.? Ans. 32 hr.

3. If 50 cu. ft. of air weigh 4.2 pounds when the absolute temperature is 562°, what will be the absolute temperature when the same volume weighs 5.8 pounds, the pressure being the same in both cases? Ans. 407°, very nearly.

CAUSES AND EFFECTS.

333. Many examples in proportion may be more easily solved by using the principle of *cause and effect*. That which may be regarded as producing a change or alteration in something, or as accomplishing something, may be called a **cause,** and the change or alteration, or thing accomplished, is the **effect.**

334. *Like causes produce like effects.* Hence, when two causes of the same kind produce two effects of the same kind, the ratio of the causes equals the ratio of the effects; in other words, the first cause is to the second cause as the first effect is to the second effect. Thus, in the question, if 3 men can lift 1,400 pounds, how many pounds can 7 men lift? we call 3 men and 7 men the *causes* (since they accomplish something, viz., the lifting of the weight), the number of pounds lifted, viz., 1,400 pounds and x pounds, are the effects. If we call 3 men the first cause, 1,400 pounds is the first effect; 7 men is the second cause, and x pounds is the second effect. Hence, we may write

1st cause		*2d cause*		*1st effect*		*2d effect*
3	:	7	=	1,400	:	x,

whence $x = \dfrac{7 \times 1,400}{3} = 3,266\frac{2}{3}$ pounds.

The principle of cause and effect is extremely useful in the solution of examples in compound proportion, as we shall now show.

COMPOUND PROPORTION.

335. All the cases of proportion so far considered have been cases of **simple proportion**; i. e., each term has been composed of but one number. There are many cases, however, in which two or all of the terms have more than one number in them; all such cases belong to **compound proportion.** In all examples in compound proportion, both causes or both effects or all four consist of more than two numbers. We will illustrate this by an

EXAMPLE.—If 40 men earn $1,280 in 16 days, how much will 36 men earn in 31 days?

SOLUTION.—Since 40 men earn something, 40 men is a cause, and since they take 16 days in which to earn something, 16 days is also a cause. For the same reason, 36 men and 31 days are also causes. The effects, that which is earned, are 1,280 dollars and x dollars. Then, 40 men and 16 days make up the first cause, and 36 men and 31 days make up the second cause. $1,280 is the first effect and $$x$ is the second effect. Hence, we write

$$\begin{matrix} \textit{1st cause} & & \textit{2d cause} & & \textit{1st effect} & & \textit{2d effect} \\ \begin{matrix}40\\16\end{matrix} & : & \begin{matrix}36\\31\end{matrix} & = & 1{,}280 & : & x. \end{matrix}$$

Now, instead of using the colon to express the ratio, we shall use the vertical line (see Art. **299**), and the above becomes

$$\begin{matrix}40\\16\end{matrix} \left| \begin{matrix}36\\31\end{matrix} = 1{,}280 \right| x.$$

In the last expression, the product of all of the numbers included between the vertical lines must equal the product of all the numbers without them; i. e., $36 \times 31 \times 1{,}280 = 40 \times 16 \times x$.

$$\text{Or } x = \frac{36 \times 31 \times \overset{\overset{2}{\cancel{80}}}{\cancel{1280}}}{\cancel{40} \times \cancel{16}} = \$2{,}232. \text{ Ans.}$$

336. The above might have been solved by canceling factors of the numbers in the original proportion. For if any number within the lines has a factor common to any number without the lines, that factor may be canceled from both numbers. Thus,

$$\begin{matrix}\cancel{40}\\\cancel{16}\end{matrix} \left| \begin{matrix}36\\31\end{matrix} = \overset{\overset{2}{\cancel{80}}}{\cancel{1280}} \right| x,$$

16 is contained in 1,280, 80 times. Cancel 16 and 1,280, and

write 80 above 1,280. 40 is contained in 80, 2 times. Cancel 40 and 80 and write 2 above 80. Now, since there are no more numbers that can be canceled, $x = 36 \times 31 \times 2 =$ \$2,232, the same result as was obtained in the last article.

337. Rule.—*Write all the numbers forming the first cause in a vertical column, and draw a vertical line; on the other side of this line write in a vertical column all of the numbers forming the second cause. Write the sign of equality to the right of the second column, and on the right of this form a third column of the numbers composing the first effect, drawing a vertical line to the right; on the other side of this line, write, for a fourth column, the numbers composing the second effect. There must be as many numbers in the second cause as in the first cause, and in the second effect as in the first effect; hence, if any term is wanting, write x in its place. Multiply together all of the numbers within the vertical lines, and also all those without the lines (canceling previously, if possible), and divide the product of those numbers which do not contain x by the product of the others in which x occurs, and the result will be the value of x.*

EXAMPLE.—If 40 men can dig a ditch 720 feet long, 5 feet wide, and 4 feet deep in a certain time, how long a ditch 6 feet deep and 3 feet wide could 24 men dig in the same time?

SOLUTION.—Here 40 men and 24 men are the causes and the two ditches are the effects. Hence,

$$\begin{array}{c|ccc|l} \cancel{40} & 24 & = & \begin{array}{c}\cancel{3}\\ \cancel{18}\\ \cancel{720}\\ 5\\ 4\end{array} & \begin{array}{l}x\\ \cancel{3}\\ \cancel{6}\end{array} \end{array} \text{ whence, } x = 24 \times 5 \times 4 = 480 \text{ feet. Ans.}$$

EXAMPLE.—The volume of a cylinder varies directly as its length and directly as the square of its diameter. If the volume of a cylinder 10 inches in diameter and 20 inches long is 1,570.8 cubic inches, what is the volume of another cylinder 16 inches in diameter and 24 inches long?

SOLUTION.—In this example, either the dimensions or the volumes may be considered the causes; say we take the dimensions for the causes. Then, squaring the diameters,

$$\begin{array}{c|ccc|c} \begin{array}{c}10^2\\ 20\end{array} & \begin{array}{c}16^2\\ 24\end{array} & = & 1{,}570.8 & x, \end{array} \text{ or } \begin{array}{c|ccc|c} \begin{array}{c}100\\ \cancel{20}\\ 5\end{array} & \begin{array}{c}256\\ \cancel{24}\\ 6\end{array} & = & 1{,}570.8 & x; \end{array}$$

whence, $x = \dfrac{256 \times 6 \times 1{,}570.8}{5 \times 100} = 4{,}825.4976$ cubic inches. Ans.

EXAMPLE.—If a block of granite 8 ft. long, 5 ft. wide, and 3 ft. thick weighs 7,200 lb., what will be the weight of a block of granite 12 ft. long, 8 ft. wide, and 5 ft. thick?

SOLUTION.—Taking the weights as the effects, we have

$$\begin{array}{c|c} & 4 \\ \not{8} & \not{1}\not{2} \\ \not{5} & \not{8} \\ \not{3} & \not{5} \end{array} = 7{,}200 \left| \; x, \text{ or } x = 4 \times 7{,}200 = 28{,}800 \text{ pounds. Ans.} \right.$$

EXAMPLE.—If 12 compositors in 30 days of 10 hours each set up 25 sheets of 16 pages each, 32 lines to the page, in how many days 8 hours long can 18 compositors set up, in the same type, 64 sheets of 12 pages each, 40 lines to the page?

SOLUTION.—Here compositors, days, and hours compose the causes, and sheets, pages, and lines the effects. Hence,

$$\begin{array}{c|c} 3 & \not{3} \\ \not{1}\not{2} & \not{1}\not{8} \\ & \\ \not{3}\not{0} & x \\ \not{3} & \\ 10 & \not{8} \end{array} = \begin{array}{c|c} \not{5} & 2 \\ \not{2}\not{5} & \not{6}\not{4} \\ \not{4} & \\ \not{1}\not{6} & \not{1}\not{2}, \text{ or } x = 3 \times 10 \times 2 = 60 \text{ days. Ans.} \\ & \not{4} \\ \not{3}\not{2} & \not{4}\not{0} \\ & \not{5} \end{array}$$

338. In examples stated like the second in Art. **337**, should an inverse proportion occur, write the various numbers as in the preceding examples, and then transpose those numbers which are said to vary inversely from one side of the vertical line to the other side.

EXAMPLE.—The centrifugal force of a revolving body varies directly as its weight, as the square of its velocity, and inversely as the radius of the circle described by the center of the body. If the centrifugal force of a body weighing 15 pounds is 187 pounds when the body revolves in a circle having a radius of 12 inches, with a velocity of 20 feet per second, what will be the centrifugal force of the same body when the radius is increased to 18 inches and the speed is increased to 24 feet per second?

SOLUTION.—Calling the centrifugal force the effect, we have

$$\begin{array}{c|c} 15 & 15 \\ 20^2 & 24^2 \\ 12 & 18 \end{array} = 187 \left| \; x. \right.$$

Transposing 12 and 18 (since the radii are to vary inversely) and squaring 20 and 24,

$$\begin{array}{c|c} \not{1}\not{5} & \not{1}\not{5} \\ & 2 \\ 25 & \not{3}\not{6} \\ \not{4}\not{0}\not{0} & \not{5}\not{7}\not{6} \\ \not{1}\not{8} & 12 \end{array} = 187 \left| \; x, \text{ or } x = \frac{12 \times 2 \times 187}{25} = 179.52 \text{ pounds. Ans.} \right.$$

EXAMPLES FOR PRACTICE.

339. Solve the following by compound proportion :

1. If 12 men dig a trench 40 rods long in 24 days of 10 hours each, how many rods can 16 men dig in 18 days of 9 hours each ?
Ans. 36 rods.

2. If a piece of iron 7 ft. long, 4 in. wide, and 6 in. thick weighs 600 lb., how much will a piece of iron weigh that is 16 ft. long, 8 in. wide, and 4 in. thick ? Ans. 1,828$\frac{4}{7}$ lb.

3. If 24 men can build a wall 72 rods long, 6 feet wide, and 5 feet high in 60 days of 10 hours each, how many days will it take 32 men to build a wall 96 rods long, 4 feet wide, and 8 feet high, working 8 hours a day ? Ans. 80 days.

4. The horsepower of an engine varies as the mean effective pressure, as the piston speed, and as the square of the diameter of the cylinder. If an engine having a cylinder 14 inches in diameter develops 112 horsepower when the mean effective pressure is 48 pounds per square inch and the piston speed is 500 feet per minute, what horsepower will another engine develop if the cylinder is 16 inches in diameter, piston speed is 600 feet per minute, and mean effective pressure is 56 pounds per square inch ? Ans. 204.8 horsepower.

5. Referring to the second example in Art. **337**, what will be the volume of a cylinder 20 inches in diameter and 24 inches long ?
Ans. 7,539.84 cubic inches.

6. Knowing that the product of $3 \times 5 \times 7 \times 9$ is 945, what is the product of $6 \times 15 \times 14 \times 36$? Ans. 45,360.

7. An engine of 15 horsepower can pump out $\frac{3}{5}$ of the water contained in a sump by working 8 hours per day for 9 days. In how many days of 12 hours could an engine of 16 horsepower perform the remainder of the work ? Ans. 3$\frac{3}{4}$ days.

8. If 8 men can make a mining (bearing in) 112 feet long, 1 foot high at the front, and 4 feet deep in a longwall face in 7 hours, how many men will be required to make a mining 80 feet long in the same face in 5 hours, if the mining must be 1.3 feet at the front and 5 feet deep ? Ans. 13 men.

FORMULAS.

340. The term **formula,** as used in mathematics and in technical books, may be defined as *a rule in which symbols are used instead of words;* in fact, a formula may be regarded as a shorthand method of expressing a rule. Any formula can be expressed in words, and when so expressed it becomes a rule.

Formulas are much more convenient than rules; they show at a glance all the operations that are to be performed; they do not require to be read three or four times, as is the case with most rules, to enable one to understand their meaning; they take up much less space, both in the printed book and in one's note-book, than rules; in short, whenever a rule can be expressed as a formula, the formula is to be preferred.

As the term "quantity" is a very convenient one to use, we will define it. In mathematics, the word **quantity** is applied to anything that it is desired to subject to the ordinary operations of addition, subtraction, multiplication, etc., when we do not wish to be more specific and state exactly what the thing is. Thus, we can say "two or more numbers," or "two or more quantities"; the word quantity is more general in its meaning than the word number.

341. The signs used in formulas are the ordinary signs indicative of operations, and the signs of aggregation. All these signs are explained in arithmetic, but some of them will here be explained in order to refresh the student's memory.

The signs indicative of operations are six in number, viz.: $+$, $-$, $\times$, $\div$, $|$, $\sqrt{}$.

Division is indicated by the sign $\div$, or by placing a straight line between the two quantities. Thus, 25 | 17, 25 / 17, and $\frac{25}{17}$ all indicate that 25 is to be divided by 17. When both quantities are placed on the same horizontal line, the straight line indicates that the quantity on the left is to be divided by that on the right. When one quantity is below the other, the straight line between indicates that the quantity above the line is to be divided by the one below it.

The sign ($\sqrt{}$) indicates that some root of the quantity to the right is to be taken; it is called the **radical sign.** To indicate what root is to be taken, a small figure, called the **index,** is placed within the sign, this being always omitted when the square root is to be indicated. Thus, $\sqrt{25}$ indicates that the square root of 25 is to be taken; $\sqrt[3]{25}$ indicates that the cube root of 25 is to be taken; etc.

The signs of aggregation are four in number; viz., ——, ( ), [ ], and { }, respectively called the **vinculum,** the **parenthesis,** the **brackets,** and the **brace;** they are used when it is desired to indicate that all the quantities included by them are to be subjected to the same operation. Thus, if we desire to indicate that the sum of 5 and 8 is to be multiplied by 7, and we do not wish to actually add 5 and 8 before indicating the multiplication, we may employ any one of the four signs of aggregation as here shown: $\overline{5+8} \times 7$, $(5+8) \times 7$, $[5+8] \times 7$, $\{5+8\} \times 7$. The vinculum is placed above those quantities which are to be treated as one quantity and subjected to the same operations.

While any one of the four signs may be used as shown above, custom has restricted their use somewhat. The vinculum is rarely used except in connection with the radical sign. Thus, instead of writing $\sqrt[3]{}(5+8)$, $\sqrt[3]{}[5+8]$, or $\sqrt[3]{}\{5+8\}$ for the cube root of 5 plus 8, all of which would be correct, the vinculum is nearly always used, $\sqrt[3]{5+8}$.

In cases where but one sign of aggregation is needed (except, of course, when a root is to be indicated), the parenthesis is always used. Hence, $(5 + 8) \times 7$ would be the usual way of expressing the product of 5 plus 8, and 7.

If two signs of aggregation are needed, the brackets and parenthesis are used, so as to avoid having a parenthesis within a parenthesis, the brackets being placed outside. For example, $[(20 - 5) \div 3] \times 9$ means that the difference between 20 and 5 is to be divided by 3, and this result multiplied by 9.

If three signs of aggregation are required, the brace, brackets, and parenthesis are used, the brace being placed outside, the brackets next, and the parenthesis inside. For example, $\{[(20 - 5) \div 3] \times 9 - 21\} \div 8$ means that the quotient obtained by dividing the difference between 20 and 5 by 3 is to be multiplied by 9, and that after 21 has been subtracted from the product thus obtained, the result is to be divided by 8.

Should it be necessary to use all four of the signs of aggregation, the brace would be put outside, the brackets next, the parenthesis next, and the vinculum inside. For example, $\{[(\overline{20 - 5} \div 3) \times 9 - 21] \div 8\} \times 12$.

As stated in arithmetic, when several quantities are connected by the various signs indicating addition, subtraction, multiplication, and division, the operation indicated by the sign of multiplication must always be performed first. Thus, $2 + 3 \times 4$ equals 14, 3 being multiplied by 4, before adding to 2. Similarly, $10 \div 2 \times 5$ equals 1, since 2×5 equals 10, and $10 \div 10$ equals 1. Hence, in the above case, if the brace were omitted, the result would be $\frac{1}{4}$, whereas, by inserting the brace, the result is 36.

Following the sign of multiplication comes the sign of division in order of importance. For example, $5 - 9 \div 3$ equals 2, 9 being divided by 3 before subtracting from 5. The signs of addition and subtraction are of equal value; that is, if several quantities are connected by plus and minus signs, the indicated operations may be performed in the order in which the quantities are placed.

There is one other sign used, which is neither a sign of aggregation nor a sign indicative of an operation to be performed; it is (=), and is called the sign of *equality;* it means that all on one side of it is exactly equal to all on the other side. For example, $2 = 2$, $5-3 = 2$, $5\times(14-9) = 25$

342. Having called particular attention to certain signs used in formulas, the formulas themselves will now be explained. First, consider the well-known rule for finding the horsepower of a steam-engine, which may be stated as follows:

Divide the continued product of the mean effective pressure in pounds per square inch, the length of the stroke in feet, the area of the piston in square inches, and the number of strokes per minute, by 33,000; the result will be the horsepower.

This is a very simple rule, and very little, if anything, will be saved by expressing it as a formula, so far as clearness is concerned. The formula, however, will occupy a great deal less space, as we shall show.

An examination of the rule will show that four quantities (viz., the mean effective pressure, the length of the stroke, the area of the piston, and the number of strokes) are multiplied together, and the result is divided by 33,000. Hence, the rule might be expressed as follows:

$$\text{Horsepower} = \begin{matrix}\text{mean effective pressure}\\ \text{(in pounds per square inch)}\end{matrix} \times \begin{matrix}\text{stroke}\\ \text{(in feet)}\end{matrix} \times$$
$$\begin{matrix}\text{area of piston}\\ \text{(in square inches)}\end{matrix} \times \begin{matrix}\text{number of strokes}\\ \text{(per minute)}\end{matrix} \div 33{,}000.$$

This expression could be shortened by representing each quantity by a single letter; thus, representing horsepower by the letter "H," the mean effective pressure in pounds per square inch by "P," the length of stroke in feet by "L," the area of the piston in square inches by "A," the number of strokes per minute by "N," and substituting these letters for the quantities that they represent, the above expression would reduce to

$$H = \frac{P \times L \times A \times N}{33{,}000}$$

a much simpler and shorter expression. This last expression is called a *formula.*

The formula just given shows, as we stated in the beginning, that a formula is really a shorthand method of expressing a rule. It is customary, however, to omit the sign of multiplication between two or more quantities when they are to be multiplied together, or between a number and a letter representing a quantity, it being always understood that, when two letters are adjacent with no sign between them, the quantities represented by these letters are to be multiplied. Bearing this fact in mind, the formula just given can be further simplified to

$$H = \frac{P L A N}{33,000}.$$

The sign of multiplication, evidently, can not be omitted between two or more numbers, as it would then be impossible to distinguish the numbers. A near approach to this, however, may be attained by placing a dot between the numbers which are to be multiplied together, and this is frequently done in works on mathematics when it is desired to economize space. In such cases it is usual to put the dot higher than the position occupied by the decimal point. Thus, $2 \cdot 3$ means the same as 2×3; $542 \cdot 749 \cdot 1,006$ indicates that the numbers 542, 749, and 1,006 are to be multiplied together.

It is also customary to omit the sign of multiplication in expressions similar to the following: $a \times \sqrt{b+c}$, $3 \times (b+c)$, $(b+c) \times a$, etc., writing them $a\sqrt{b+c}$, $3(b+c)$, $(b+c)a$, etc. The sign is not omitted when several quantities are included by a vinculum, and it is desired to indicate that the quantities so included are to be multiplied by another quantity. For example, $3 \times \overline{b+c}$, $\overline{b+c} \times a$, $\sqrt{b+c} \times a$, etc., are always written as here printed.

343. Before proceeding further, we will explain one other device that is used by formula makers and which is apt to puzzle one who encounters it for the first time—it is the use of what mathematicians call *primes* and *subs.*, and what printers call *superior* and *inferior* characters. As a rule, formula makers designate quantities by the initial letters of the names of the quantities. For example, they represent volume by v, pressure by p, height by h, etc. This practice is to be commended, as the letter itself serves in many cases to identify the quantity which it represents. Some authors carry the practice a little further, and represent all quantities of the same nature by the same letter throughout the book, always having the same letter represent the same thing. Now, this practice necessitates the use of the primes and subs. above mentioned, when two quantities have the same name but represent different things. Thus, consider the word *pressure* as applied to steam, at different stages between the boiler and the condenser. First, there is *absolute* pressure, which is equal to the gauge pressure in pounds per square inch plus the pressure indicated by the barometer reading (usually assumed in practice to be 14.7 pounds per square inch, when a barometer is not at hand). If this be represented by p, how shall we represent the gauge pressure? Since the absolute pressure is always greater than the gauge pressure, suppose we decide to represent it by a capital letter, and the gauge pressure by a small (lower-case) letter. Doing so, P represents absolute pressure, and p, gauge pressure. Further, there is usually a "drop" in pressure between the boiler and the engine, so that the initial pressure, or pressure at the beginning of the stroke, is less than the pressure at the boiler. How shall we represent the initial pressure? We may do this in one of three ways and still retain the letter p or P to represent the word pressure: First, by the use of the prime mark; thus, p' or P' (read *p prime* and *P major prime*) may be considered to represent the initial gauge pressure, or the initial absolute pressure. Second, by the use of sub. figures; thus, p_1 or P_1 (read *p sub. one* and *P major sub. one*). Third, by the use of sub. letters; thus, p_i or P_i (read *p sub. i* and *P major sub. i*). In the same manner p'' (read *p second*), p_2, or p_r might be used to represent the gauge pressure at release, etc. The sub. letters have the advantage of still further identifying the quantity represented; in many instances, however, it is not convenient to use them, in which case primes and subs. are used instead. The prime notation may be continued as follows: p''', p^{iv}, p^{v}, etc.; it is inadvisable to use superior figures, for example, p^1, p^2, p^3, p^a, etc., as they are liable to be mistaken for exponents.

The main thing to be remembered by the student is that *when a formula is given in which the same letters occur several times, all like letters having the same primes or subs. represent the same quantities, while those which differ in any respect represent different quantities.* Thus, in the formula

$$t = \frac{w_1 s_1 t_1 + w_2 s_2 t_2 + w_3 s_3 t_3}{w_1 s_1 + w_2 s_2 + w_3 s_3},$$

w_1, w_2, and w_3 represent the weights of three different bodies; s_1, s_2, and s_3, their specific heats; and t_1, t_2, and t_3, their temperatures; while t represents the final temperature after the bodies have been mixed together. It should be noted that those letters having the *same* subs. refer to the same bodies. Thus, w_1, s_1, and t_1 all refer to one of the three bodies; w_2, s_2, t_2, to another body; etc.

It is very easy to apply the above formula when the values of the quantities represented by the different letters are known. All that is required is to substitute the numerical values of the letters, and then perform the indicated operations. Thus, suppose that the values of w_1, s_1, and t_1 are, respectively, 2 pounds, .0951, and 80°; of w_2, s_2, and t_2, 7.8 pounds, 1, and 80°; and of w_3, s_3, and t_3, $3\frac{1}{4}$ pounds, .1138, and 780°; then, the final temperature t is, substituting these values for their respective letters in the formula,

$$t = \frac{2 \times .0951 \times 80 + 7.8 \times 1 \times 80 + 3\frac{1}{4} \times .1138 \times 780}{2 \times .0951 + 7.8 \times 1 + 3\frac{1}{4} \times .1138} =$$

$$\frac{15.216 + 624 + 288.483}{.1902 + 7.8 + .36985} = \frac{927.699}{8.36005} = 110.97°.$$

In substituting the numerical values, the signs of multiplication are, of course, written in their proper places; all the multiplications are performed before adding, according to the rule previously given.

344. The student should now be able to apply any formula involving only algebraic expressions that he may meet with, and which does not require the use of logarithms for its solution. We will, however, call his attention to one or two other facts that he may have forgotten.

Expressions similar to $\dfrac{160}{\dfrac{660}{25}}$ sometimes occur, the heavy line indicating that 160 is to be divided by the quotient obtained by dividing 660 by 25. If both lines were light it would be impossible to tell whether 160 was to be divided by $\dfrac{660}{25}$, or whether $\dfrac{160}{660}$ was to be divided by 25. If this latter result were desired, the expression would be written $\dfrac{\dfrac{160}{660}}{25}$. In every case the heavy line indicates that all above it is to be divided by all below it.

In an expression like the following, $\dfrac{160}{7+\dfrac{660}{25}}$, the heavy line is not necessary, since it is impossible to mistake the operation that is required to be performed. But, since $7+\dfrac{660}{25}=\dfrac{175+660}{25}$, if we substitute $\dfrac{175+660}{25}$ for $7+\dfrac{660}{25}$, the heavy line becomes necessary in order to make the resulting expression clear. Thus,

$$\frac{160}{7+\dfrac{660}{25}}=\frac{160}{\dfrac{175+660}{25}}=\frac{160}{\dfrac{835}{25}}.$$

Fractional exponents are sometimes used instead of the radical sign. That is, instead of indicating the square, cube, fourth root, etc., of some quantity, as 37, by $\sqrt{37}$, $\sqrt[3]{37}$, $\sqrt[4]{37}$, etc., these roots are indicated by $37^{\frac{1}{2}}$, $37^{\frac{1}{3}}$, $37^{\frac{1}{4}}$, etc. Should the numerator of the fractional exponent be some quantity other than 1, this quantity, whatever it may be, indicates that the quantity affected by the exponent is to be raised to the power indicated by the numerator; the denominator is *always* the index of the root. Hence, instead of writing $\sqrt[3]{37^2}$ for the cube root of the square of 37, it may be written $37^{\frac{2}{3}}$, the denominator being the index of the root; in other words, $\sqrt[3]{37^2} = 37^{\frac{2}{3}}$. Likewise, $\sqrt[5]{(1 + a^2 b)^3}$ may also be written $(1 + a^2 b)^{\frac{3}{5}}$, a much simpler expression.

345. We will now give several examples showing how to apply some of the more difficult formulas that the student may encounter.

1. The area of any segment of a circle that is less than (or equal to) a semicircle is expressed by the formula

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r - h),$$

in which A = area of segment;
π = 3.1416;
r = radius;
E = angle obtained by drawing lines from the center to the extremities of arc of segment;
c = chord of segment;
and h = height of segment.

Example.—What is the area of a segment whose chord is 10 inches long, angle subtended by chord is 83.46°, radius is 7.5 inches, and height of segment is 1.91 inches?

Solution.—Applying the formula just given,

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r - h) = \frac{3.1416 \times 7.5^2 \times 83.46}{360} - \frac{10}{2}(7.5 - 1.91) =$$
$$40.968 - 27.95 = 13.018 \text{ square inches, nearly. Ans.}$$

2. The area of any triangle may be found by means of the following formula, in which A = the area, and a, b, and c represent the lengths of the sides:

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2\,b}\right)^2}.$$

Example.—What is the area of a triangle whose sides are 21 feet, 46 feet, and 50 feet long?

Solution.—In order to apply the formula, suppose we let a represent the side that is 21 feet long; b, the side that is 50 feet long; and c, the side that is 46 feet long. Then substituting in the formula,

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2} = \frac{50}{2}\sqrt{21^2 - \left(\frac{21^2 + 50^2 - 46^2}{2 \times 50}\right)^2}$$
$$= \frac{50}{2}\sqrt{441 - \left(\frac{441 + 2,500 - 2,116}{100}\right)^2} = 25\sqrt{441 - \left(\frac{825}{100}\right)^2}$$
$$= 25\sqrt{441 - 8.25^2} = 25\sqrt{441 - 68.0625} = 25\sqrt{372.9375}$$
$$= 25 \times 19.312 = 482.8 \text{ square feet, nearly. Ans.}$$

The operations in the above examples have been extended much farther than was necessary; it was done in order to show the student every step of the process. The last formula is perfectly general, and the same answer would have been obtained had the 50-foot side been represented by a, the 46-foot side by b, and the 21-foot side by c.

3. The Rankine-Gordon formula for determining the least load in pounds that will cause a long column to break is

$$P = \frac{SA}{1 + q\frac{l^2}{G^2}},$$

in which P = load (pressure) in pounds;
S = ultimate strength (in pounds per square inch) of the material composing the column;
A = area of cross-section of column in square inches;
q = a factor (multiplier) whose value depends upon the shape of the ends of the column and on the material composing the column;
l = length of column in inches;
and G = least radius of gyration of cross-section of column.

The values of S, q, and G^2 are given in printed tables in books in which this formula occurs.

EXAMPLE.—What is the least load that will break a hollow wrought-iron column whose outside diameter is 14 inches; inside diameter, 11 inches; length, 20 feet, and whose ends are flat?

SOLUTION.—For steel, $S = 150,000$. and $q = \frac{1}{25,000}$ for flat-ended steel columns; A, the area of the cross-section, $= .7854\,(d_1^2 - d_2^2) = .7854\,(14^2 - 11^2)$, d_1 and d_2 being the outside and inside diameters, respectively; $l = 20 \times 12 = 240$ inches; and $G^2 = \frac{d_1^2 + d_2^2}{16} = \frac{14^2 + 11^2}{16}$. Substituting these values in the formula,

$$P = \frac{SA}{1 + q\frac{l^2}{G^2}} = \frac{150{,}000 \times .7854\,(14^2 - 11^2)}{1 + \frac{1}{25{,}000} \times \frac{240^2}{\frac{14^2 + 11^2}{16}}} =$$

$$\frac{150{,}000 \times 58.905}{1 + .1163} = \frac{8{,}835{,}750}{1.1163} = 7{,}915{,}211 \text{ pounds. Ans.}$$

4. Example.—When $A = 10$, $B = 8$, $C = 5$, and $D = 4$, what is the value of E in the following:

(a) $E = \sqrt[3]{\frac{BCD}{A\left(2 + \frac{D^2}{C^2}\right)}}$? (b) $E = \frac{A - \frac{3}{4}D + \frac{4B^2}{A + C}}{A - \sqrt{\frac{2B^2}{A + 22}}}$?

Solution.—(a) Substituting,

$$E = \sqrt[3]{\frac{8 \times 5 \times 4}{10\left(2 + \frac{4^2}{5^2}\right)}}.$$

To simplify the denominator, square the 4 and 5, add the resulting fraction to 2, and multiply by 10. Simplifying, we have

$$E = \sqrt[3]{\frac{160}{10\left(2 + \frac{16}{25}\right)}} = \sqrt[3]{\frac{160}{10 \times \frac{66}{25}}} = \sqrt[3]{\frac{160}{\frac{660}{25}}} = \sqrt[3]{\frac{200}{33}}.$$

Reducing the fraction to a decimal before extracting the cube root,

$$E = \sqrt[3]{6.0606} = 1.823. \text{ Ans}$$

(b) Substituting,

$$E = \frac{10 - \frac{3}{4} \times 4 + \frac{4 \times 8^2}{10 + 5}}{10 - \sqrt{\frac{2 \times 8^2}{10 + 22}}} = \frac{10 - 3 + \frac{4 \times 64}{15}}{10 - \sqrt{\frac{2 \times 64}{32}}} =$$

$$\frac{7 + 17.066+}{10 - \sqrt{4}} = \frac{24.066+}{8} = 3.008+. \text{ Ans.}$$

345$_1$.—In the preceding pages, the unknown quantity has always been represented by the single letter at the left of the sign of equality, while the letters at the right have represented known values from which the required values could be found. It is possible, however, to find the value of the quantity represented by any letter in a formula, if the values represented by all the others are known. For example, let it be required to find how many strokes per minute an engine having a piston area of 78.54 sq. in. must make in order to develop 60 horsepower, if the mean effective pressure is 40 lb. per sq. in., and the length of stroke $1\frac{1}{4}$ ft. By substituting the given values in the formula $H = \frac{PLAN}{33,000}$, we have

$$60 = \frac{40 \times 1\frac{1}{4} \times 78.54 \times N}{33,000},$$

in which N, the number of strokes, is to be found.

But it is evident that the expression on the right of the sign of equality is equal to $\frac{40 \times 1\frac{1}{4} \times 78.54}{33,000} \times N$, a fraction whose numerator is composed of 3 factors. Reducing the numerator to a single number by performing the indicated multiplications, we obtain, after canceling,

$$60 = \frac{119}{1,000} \times N = .119\,N.$$

If 60 equals .119 N, then N equals 60 divided by .119; hence,

$$N = \frac{60}{.119} = 504.2 \text{ strokes per minute.}$$

The method of procedure is essentially the same when the unknown quantity occurs in the denominator of a formula. Thus, in the formula $f = \frac{m\,v^2}{r}$, suppose that $f = 375$, $m = 1.25$, and $v = 60$. Then, substituting,

$$375 = \frac{1.25 \times 60^2}{r} = \frac{4,500}{r}.$$

But if 375 equals 4,500 divided by r, then $375 \times r = 4,500$; hence, r must equal 4,500 divided by 375, or $r = \frac{4,500}{375} = 12.$

EXAMPLES FOR PRACTICE.

Find the numerical values of x in the following formulas, when $A = 9$, $B = 8$, $d = 10$, $e = 3$, and $c = 2$:

1. $x = \frac{d + c^2}{d^2 - 40}$. Ans. $x = \frac{7}{30}$.

2. $x = \frac{\frac{1}{2}(A + e)}{c\,e}$ Ans. $x = 1\frac{1}{2}$.

3. $x = \sqrt{\dfrac{d^3}{2c}} + \sqrt{AB^2}.$ Ans. $x = 29.$

4. $x = \dfrac{Ae}{\sqrt{16Bc}} + \dfrac{5}{16}.$ Ans. $x = 2.$

5. $x = (c + 2e)\left(\sqrt[3]{B} - \dfrac{1}{c}\right) + \dfrac{e^2 - c^2}{e^2 + c^2}.$ Ans. $x = 12\frac{5}{13}.$

6. $x = \sqrt{\dfrac{Bcd}{A\left(2 + \dfrac{d^3}{e^2}\right)}}.$ Ans. $x = .396 +$

GEOMETRY AND TRIGONOMETRY.

GEOMETRY.

668. Geometry is that branch of mathematics which treats of the properties of lines, angles, surfaces, and volumes.

LINES AND ANGLES.

669. A **point** indicates position only. It has neither length, breadth, nor thickness.

670. A **line** has only one dimension: length.

671. A **straight line,** Fig. 2, is one that does not change its direction throughout its whole length. A straight line is also frequently called a **right line.**

FIG. 2.

672. A **curved line,** Fig. 3, changes its direction at every point.

FIG. 3.

673. A **broken line,** Fig. 4, is one made up wholly of straight lines lying in different directions.

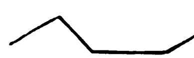


FIG. 4.

674. Parallel lines, Fig. 5, are equally distant from each other at all points.

FIG. 5.

675. A line is **perpendicular** to another when it meets that line so as not to incline towards it on either side, Fig. 6.

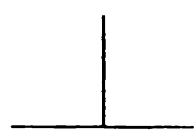


FIG. 6.

676. A **horizontal line** is a line parallel to the horizon, or water level, Fig. 7.

677. A **vertical line,** Fig. 7, is a line perpendicular to a horizontal line; consequently, it has the direction of a plumb line.

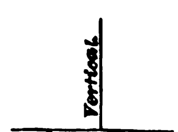


FIG. 7.

678. When two lines cross or cut each other, as in Fig. 8, they are said to **intersect**, and the point at which they intersect is called the **point of intersection**, as *A*.

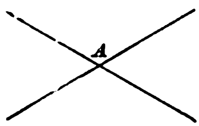


FIG. 8.

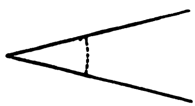


FIG. 9.

679. An **angle**, Fig. 9, is the opening between two lines which intersect or meet; the point of meeting is called the **vertex** of the angle.

680. In order to distinguish one line from another, two of its points are given if it is a straight line, and as many more as are considered necessary if it is a broken or curved line. Thus, in Fig. 10, the line *A B* would mean the straight line included between the points *A* and *B*. Similarly, the straight line between *C* and *B*, or between *B* and *D*, would be called the line *C B*, or the line *B D*. The broken line made up of the lines *A B* and *C B*, or *A B* and *B D*, would be called the broken line *C B A* or *A B C*, and *A B D* or *D B A*, according to the point started from.

681. To distinguish angles, name a point on each line, and the point of their intersection, or vertex of the angle. Thus, in Fig. 10, the angle formed by the lines *A B* and *C B* is called the angle *A B C* or the angle *C B A*: the letter at the vertex is always placed in the middle. The angle formed by the lines *A B* and *B D* is called the angle *A B D* or the angle *D B A*.

When an angle stands alone so that it cannot be mistaken for any other angle, only the vertex letter need be given; thus, the angle *O*, or the angle *P*, etc.

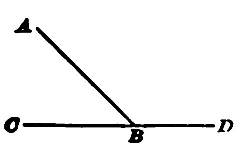


FIG. 10.

682. If one straight line meets another straight line at a point between its ends (see Figs. 10 and 11), two angles, *A B C* and *A B D*, are formed, which are called **adjacent angles.**

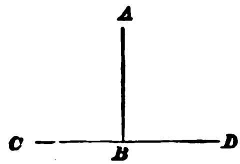


FIG. 11.

683. When these adjacent angles, *A B C* and *A B D*, are equal, they are called **right angles**; see Fig. 11.

684. An **acute angle** is less than a right angle. *A B C*, Fig. 12, is an acute angle.

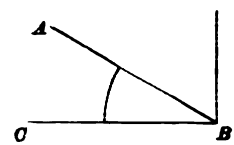


FIG. 12.

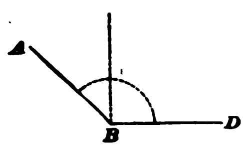


FIG. 13.

685. An **obtuse angle** is greater than a right angle. *A B D*, Fig. 13, is an obtuse angle.

686. When two straight lines intersect, they form four angles about the point of intersection. Thus, in Fig. 14, the lines *A B* and *C D*, intersecting at the point *O*, form four angles, *B O D*, *D O A*, *A O C*, and *C O B*, about the point *O*. The angles which lie on the *same* side of one straight line, as *D O B* and *D O A*, are **adjacent angles.** The angles which lie *opposite* each other are called **opposite angles.** Thus, *A O C* and *D O B*, also *D O A* and *B O C*, are opposite angles.

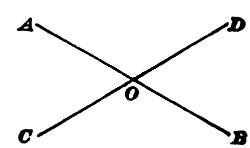


FIG. 14.

When one straight line intersects another straight line, as in Fig. 14, the opposite angles are equal. Thus, $D O B = A O C$, and $D O A = B O C$.

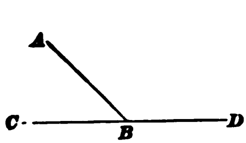


FIG. 15

When one straight line meets another straight line at a point between its ends, the sum of the two adjacent angles *A B D* and *A B C*, Fig. 15, equals two right angles.

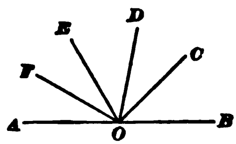


FIG. 16.

687. If a number of straight lines on the same side of a given straight line meet at the same point, the sum of all the angles formed is equal to two right angles. Thus, in Fig. 16, $C\,O\,B + D\,O\,C + E\,O\,D + F\,O\,E + A\,O\,F =$ two right angles.

688. If a straight line intersects another straight line, so that the adjacent angles are equal, the lines are *said to be* ***perpendicular*** *to each other.* In such a case, four right angles are formed about the point of intersection. Thus, in Fig. 17, $B\ O\ C = C\,O\,A$; hence, $B\,O\,C$, $C\,O\,A$, $A\,O\,D$, and $D\,O\,B$ are right angles. From this it is seen that *four right angles* are all that can be formed about a given point.

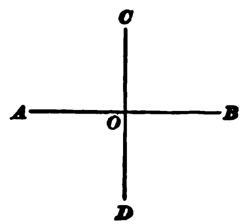


FIG. 17.

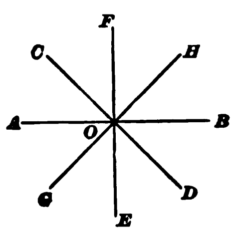


FIG. 18.

689. Through a given point any number of straight lines may be drawn; and the sum of all the angles formed about the point of intersection equals four right angles. Thus, in Fig. 18, $H\,O\,F + F\,O\,C + C\,O\,A + A\,O\,G + G\,O\,E + E\,O\,D + D\,O\,B + B\,O\,H =$ four right angles.

EXAMPLE.—In a fly-wheel with 12 arms, what part of a right angle is included between the center lines of any two arms, the arms being spaced equally?

SOLUTION.—Since there are 12 arms, there are 12 angles. The sum of all the angles equals four right angles. Hence, one angle equals $\frac{1}{12}$ of four right angles, or $\frac{4}{12} = \frac{1}{3}$ of one right angle.

690. A perpendicular drawn from a point over or under a given straight line is the shortest distance from the point to the line, or to the line extended. Thus, if A, Fig. 19, is the given point, and $C\ D$ the given line, then the perpendicular $A\ B$ is the shortest distance from A to $C\,D$.

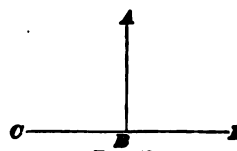


FIG. 19.

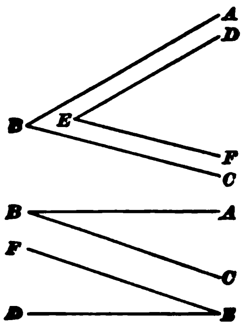


FIG. 20.

691. If two angles have their sides parallel and lie in the same or in opposite directions, they are equal. Thus, if the side $A\ B$, Fig. 20, is parallel to the side $D\ E$, and if the side $B\ C$ is parallel to the side $E\ F$, then the angle $E =$ the angle B.

692. If two sides of an angle are perpendicular to two sides of another angle, the two angles are equal. Thus, if $D\,E$ and $G\ H$, Fig. 21, are perpendicular to $B\,A$, and $E\ F$ and $H\,K$ are perpendicular to $B\ C$, then will angle $E =$ angle $B =$ angle H.

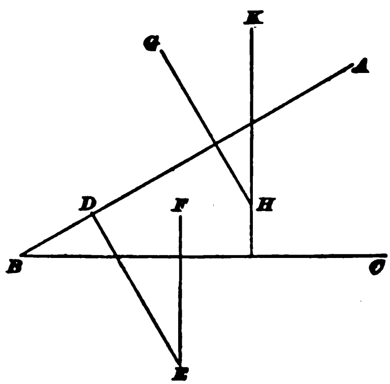


FIG. 21.

EXAMPLES FOR PRACTICE.

1. In a pulley with five arms, what part of a right angle is included between the center lines of any two arms? Ans. $\frac{4}{5}$ of a right angle.

2. If one straight line meets another straight line so as to form an angle equal to $1\frac{1}{3}$ right angles, what part of a right angle does its adjacent angle equal? Ans. $\frac{2}{3}$ of a right angle.

3. If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, what part of a right angle is contained in each angle? Ans. $\frac{1}{3}$ of a right angle.

PLANE FIGURES.

693. A **surface** has only two dimensions. *length* and *breadth*.

694. A **plane surface** is a flat surface. If a straight edge be laid on a plane surface, every point along the edge of the straight edge will touch the surface, no matter in what direction it is laid.

695. A **plane figure** is any part of a plane surface bounded by straight or curved lines.

696. When a plane figure is bounded by **straight** lines only, it is called a **polygon.** The bounding lines are called the **sides,** and the broken line that bounds it (or the whole distance around it) is called the **perimeter** of the polygon.

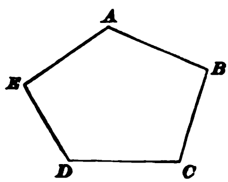


FIG. 22.

697. The angles formed by the sides are called the **angles** of the polygon. Thus, $A\ B\ C\ D\ E$, Fig. 22, is a polygon. $A\ B$, $B\ C$, etc., are the **sides;** $E\ A\ B$, $B\ C\ D$, etc., are the **angles,** and the broken line $A\ B\ C\ D\ E\ A$ is the **perimeter.**

698. Polygons are classified according to the number of their sides: One of three sides is called a **triangle;** one of four sides, a **quadrilateral;** one of five sides, a **pentagon;** one of six sides, a **hexagon;** one of seven sides, a **heptagon;** one of eight sides, an **octagon;** one of ten sides, a **decagon;** one of twelve sides, a **dodecagon,** etc.

699. **Equilateral polygons** are those in which the sides are all equal. Thus, in Fig. 23, $A\ B = B\ C = C\ D = D\ A$; hence, $A\ B\ C\ D$ is an equilateral polygon.

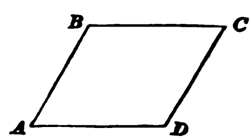


FIG. 23.

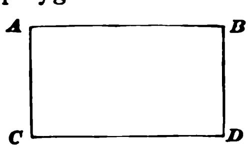


FIG. 24.

700. An **equiangular polygon** is one in which all of the angles are equal. Thus, in Fig. 24, angle $A =$ angle $B =$ angle $D =$ angle C; hence, $A\ B\ D\ C$ is an equiangular polygon.

701. A **regular polygon** is one in which all of the sides and all of the angles are equal. Thus, in Fig. 25, $A\ B = B\ D = D\ C = C\ A$, and angle $A =$ angle $B =$ angle $D =$ angle C; hence, $A\ B\ D\ C$ is a regular polygon.

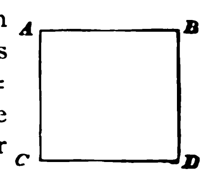
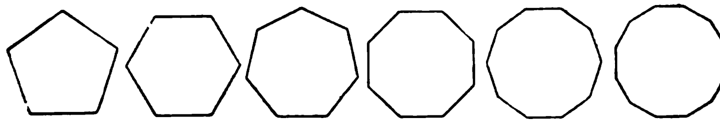


FIG. 25

702. Other regular polygons are shown in Fig. 26.



Pentagon. Hexagon. Heptagon. Octagon. Decagon. Dodecagon.

FIG. 26.

703. The sum of all the interior angles of any polygon equals two right angles, multiplied by a number which is two less than the number of sides in the polygon. Thus, $A\ B\ C\ D\ E\ F$, Fig. 27, is a polygon of six sides (hexagon), and the sum of all the interior angles, $A + B + C + D + E + F =$ two right angles $\times$ 4 ($= 6 - 2$), or 8 right angles.

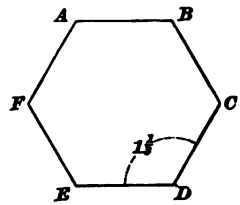


FIG. 27.

EXAMPLE.—If the above figure is a regular hexagon (has equal sides and equal angles), how many right angles are there in each interior angle?

SOLUTION.—$6 - 2 = 4$. Two right angles $\times 4 = 8$ right angles $=$ the total number of right angles in the polygon; and as there are six equal angles, we have $8 \div 6 = 1\frac{1}{3}$ right angles $=$ the number of right angles in each interior angle. Ans.

THE TRIANGLE.

704. Triangles are divided into four classes: **Isosceles** triangles, **scalene** triangles, **right-angled** triangles, and **oblique-angled** triangles.

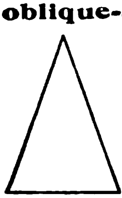


FIG. 28.

705. An **isosceles** triangle, Fig. 28, is one having two of its sides equal.

706. When the three sides are equal, as in Fig. 29, it is called an **equilateral** triangle.

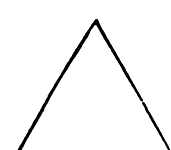


FIG. 29.

FIG. 30.

707. A **scalene** triangle, Fig. 30, is one having no two of its sides equal.

708. A **right-angled** triangle, Fig. 31, is any triangle having one right angle. The side opposite the right angle is called the **hypotenuse.** Among mathematicians, a right-angled triangle is now termed a **right triangle.**

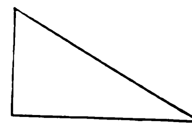


FIG. 31.

FIG. 32.

709. An **oblique** triangle, Fig. 32, is one which has no right angles.

710. The **base** of any triangle is the side upon which the triangle is supposed to stand.

711. The **altitude** of any triangle is a line drawn from the vertex of the angle opposite the base perpendicular to the base, or to the base extended. Thus, in Figs. 33 and 34, $B D$ is the *altitude* of the triangles $A B C$.

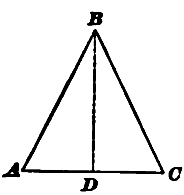


FIG. 33.

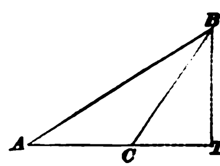


FIG. 34.

In an isosceles triangle, the angles opposite the equal sides are equal. Thus, in Fig. 35, $A B = B C$; hence, angle C = angle A.

In any isosceles triangle, if a perpendicular be drawn from the vertex opposite the unequal side to that side, it bisects (cuts in halves) the side. Thus, $A C$, Fig. 35, is the unequal side in the isosceles triangle $A B C$; hence, the perpendicular $B D$ bisects $A C$, or $A D = D C$.

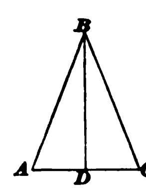


FIG. 35.

If two angles of any triangle are equal, the triangle is isosceles.

712. In any **triangle,** the sum of the three angles equals two right angles. Thus, in Fig. 36, the sum of the angles at A, B, and $C =$ two right angles; that is, $A + B + C =$ two right angles. Hence, if any two angles of a triangle are given, the third may be found by subtracting the sum of the two from two right angles. Suppose that $A + B = 1\frac{7}{10}$ right angles; then, C must equal $2 - 1\frac{7}{10} = \frac{3}{10}$ of a right angle.

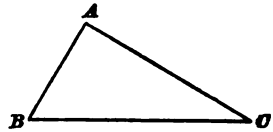


FIG. 36.

713. In any **right-angled** triangle there can be but one right angle, and since the sum of all the angles equals two right angles, it is evident that the sum of the two acute angles must equal a right angle. Therefore, if in any right-angled triangle one acute angle is known, the other can be found by subtracting the known angle from a right angle. Thus, in Fig. 37, $A\ B\ C$ is a right-angled triangle, right-angled at C. Then, the angles $A + B =$ one right angle. If $A = \frac{3}{4}$ of a right angle, $B = 1 - \frac{3}{4} = \frac{1}{4}$ of a right angle.

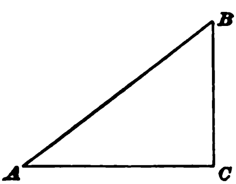


FIG. 37.

714. In any right-angled triangle, the square described on the hypotenuse is equal to the sum of the squares described upon the other two sides. If $A\ B\ C$, Fig. 38, is a right-angled triangle, right-angled at B, then the square described upon the hypotenuse $A\ C$ is equal to the sum of the squares described upon the sides $A\ B$ and $B\ C$; consequently, if the lengths of the sides $A\ B$ and $B\ C$ are known, we can find the length of the hypotenuse by adding the squares of the lengths of the sides $A\ B$ and $B\ C$, and then extracting the square root of the sum.

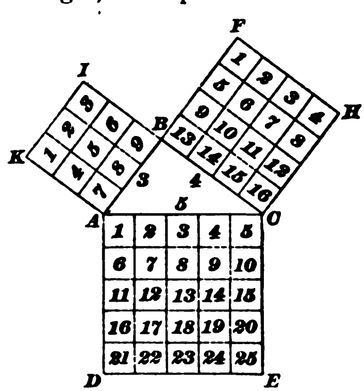


FIG. 38

Example.—If $AB = 3$ inches, and $BC = 4$ inches, what is the length of the hypotenuse AC?

Solution.— $3^2 = 9$; $4^2 = 16$; adding,
$9 + 16 = 25$. $\sqrt{25} = 5$.
Therefore, $AC = 5$ inches. Ans.

If the hypotenuse and one side are given, the other side can be found by subtracting the square of the given side from the square of the hypotenuse, and then extracting the square root of the remainder.

Example.—The side given is 3 inches, the hypotenuse is 5 inches; what is the length of the other side?

Solution.— $3^2 = 9$; $5^2 = 25$. $25 - 9 = 16$, and $\sqrt{16} = 4$ inches. Ans.

Example.—If, from a church steeple which is 150 feet high, a rope is to be attached to the top, and to a stake in the ground 85 feet from its foot (the ground being supposed to be level), what must be the length of the rope?

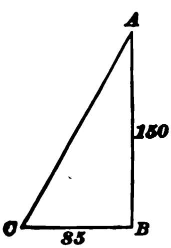


Fig. 39.

Solution.—In Fig. 39, AB represents the steeple 150 feet high; C, a stake 85 feet from the foot of the steeple, and AC, the rope. Here we have a right-angled triangle, right-angled at B, and AC is the hypotenuse.

The square of $AC = 85^2 + 150^2 = 7{,}225 + 22{,}500 = 29{,}725$.

Therefore, $AC = \sqrt{29{,}725} = 172.4$ feet, nearly. Ans.

715. Two triangles are **equal** when the *sides* of one are equal to the sides of the other.

716. Two triangles are **similar** when the *angles* of one are equal to the angles of the other. *The corresponding sides of similar triangles are proportional.*

For example, in the triangles ABC and abc, Fig. 40, side ac is perpendicular to AC; the side ab to AB, and side cb to BC; hence, angle A = angle a, since the sides of one are perpendicular to the sides of the other. In like manner, angle B = angle b, and angle C = angle c. The two triangles are, therefore, similar,

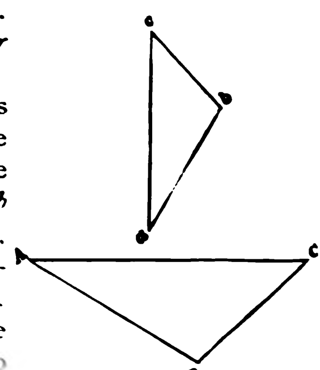


Fig. 40.

and their corresponding sides are proportional. That is, any two sides of one triangle are to each other as the two corresponding sides of the other triangle; or, one side of one triangle is to the corresponding side of the other as another side of the first triangle is to the corresponding side of the second. The following are examples of the many proportions that may be written. In this case, the corresponding sides of the two triangles are the ones that are perpendicular to each other:

$$AB : BC = ab : bc,$$
$$AB : AC = ab : ac,$$
$$BC : bc = AB : ab,$$
$$AC : ac = BC : bc, \text{ etc.}$$

Example.—The sides of a triangle are 18 inches and 21 inches, and the base is 24 inches long; what are the lengths of the sides of a similar triangle whose base is 8?

Solution.—Since the sides are proportional, we have the proportions $24 : 8 = 21 : x$, and $24 : 8 = 18 : x$. From the first, $x = 7$ and from the second, $x = 6$. Ans.

717. If a straight line is drawn through two sides of a triangle parallel to the third side, it divides those sides proportionally. Thus, in Fig. 41, let the line DE be drawn parallel to the side BC in the triangle ABC. Then,

$$AD : DE = AB : BC.$$

It is to be noticed, also, that the triangles ADE and ABC are similar, and their sides are proportional. The proportion $AD : DB = AE : EC$ is a useful one.

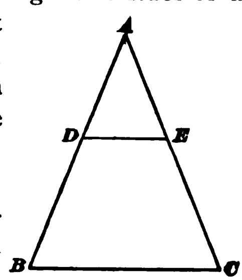


Fig. 41.

Example.—In the last figure, if $AE = 14$, $AD = 12$ and $EC = 9$, what does DB equal?

Solution.—From the proportion $AD : DB = AE : EC$, $12 : DB = 14 : 9$, whence $DB = 7\frac{5}{7}$. Ans.

Example.—The base of a right-angled triangle is 12 inches, and its altitude 40 inches. How wide is the triangle at 24 inches from the base?

Solution.—Since the triangle is right-angled, the length of the perpendicular side equals the altitude, or 40 inches. By drawing a line parallel to the base, and 24 inches above it, a second and similar triangle will be found whose corresponding side $= 40 - 24$, or 16 inches, and the length of whose base is the required width. Hence, $40 : 12 = 16 : x$, or $x = 4.8$ inches. Ans.

EXAMPLES FOR PRACTICE.

1. How many right angles are there in one of the interior angles of a regular heptagon? Ans. $1\frac{3}{7}$ right angles.

2. The angle at the vertex of an isosceles triangle equals one-half a right angle. What do the other angles equal? Ans. $\frac{3}{4}$ of a right angle.

3. One of the acute angles of a right-angled triangle equals $\frac{5}{8}$ of a right angle. What is the size of the other acute angle? Ans. $\frac{3}{8}$ of a right angle

4. If the two sides about the right angle in a right-angled triangle are 52 and 39 feet long, how long is the hypotenuse? Ans. 65 feet.

5. A ladder 65 feet long reaches to the top of a house when its foot is 25 feet from the house. How high is the house, supposing the ground to be level? Ans. 60 feet.

6. In a triangle $A\ B\ C$, side $A\ B = 32$ feet, $B\ C = 34$ feet and $A\ C = 48$ feet. If side $A\ B$ of a similar triangle is 72 feet long, what are the lengths of the other two sides? Ans. $A\ C = 108$ feet; $B\ C = 76.5$ feet.

7. The base of a right-angled triangle is 24 inches, and its altitude, 72 inches. At what distance from the top is the triangle 16 inches wide? Ans. 48 inches.

THE CIRCLE.

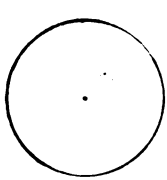


Fig. 42.

718. A **circle,** Fig. 42, is a plane figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.**

719. The **diameter** of a circle, $A\ B$, Fig. 43, is a straight line passing through the center and terminated at both ends by the circumference.

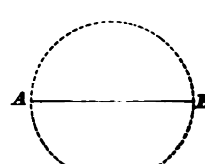


Fig. 43.

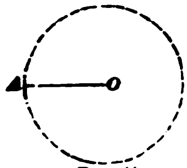


FIG. 44.

720. The **radius** of a circle, *O A*, Fig. 44, is a straight line drawn from the center to the circumference. It is equal in length to one-half the diameter. The plural of radius is **radii.** All radii of any circle are equal in length.

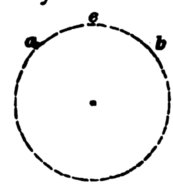


FIG. 45.

721. An **arc** of a circle, *a e b*, Fig. 45, is any part of its circumference.

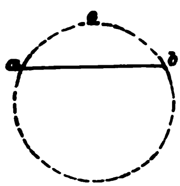


FIG. 46.

722. A **chord** is a straight line joining any two points in a circumference; or, it is a straight line joining the extremities of an arc.

Thus, in Fig. 46, *a b* is the *chord* of the arc *a e b*.

723. A **segment** of a circle is the space included between the arc and its chord.

724. A **sector** of a circle is the space included between an arc and two radii drawn to the extremities of the arc. Thus, in Fig. 47, the space included between the arc *A B* and the radii *O A* and *O B* is a sector of the circle.

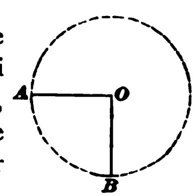


FIG. 47.

725. Two circles are equal when the *radius or diameter* of one equals the *radius or diameter* of the other.

Two arcs are equal when the *radius and chord* of one equals the *radius and chord* of the other.

726. If *A D B C*, Fig. 48, is a circle in which two diameters *A B* and *C D* are drawn at right angles to each other, then, *A O D*, *D O B*, *B O C* and *C O A* are right angles. The circumference is thus divided into four equal parts; each of these parts is called a **quadrant.**

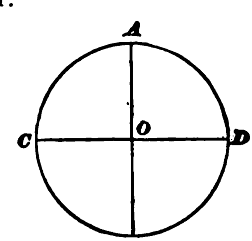


FIG. 48.

727. In Geometry, **angles** are measured by the number of right angles, or parts of a right angle, which they contain; since, in the circle, a right angle intercepts a quadrant, an angle is also measured by the number of quadrants, or parts of a quadrant, that it intercepts.

728. An angle at the center is measured by its intercepted arc.

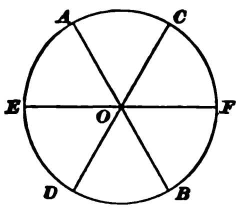


FIG. 49.

EXAMPLE.—If a circle is divided into six equal sectors, how many quadrants, or parts of a quadrant, are contained in the angle of each sector?

SOLUTION.—In Fig. 49, *A C F B D E* is a circle divided into six equal sectors. The sum of all the quadrants in the circle is 4. Hence, $4 \div 6 = \frac{2}{3}$ of a quadrant in each sector. Ans.

729. An **inscribed** angle is one whose vertex lies on the circumference of a circle, and whose sides are chords. It is measured by *one-half* the intercepted arc. Thus, in Fig. 50, *A B C* is an inscribed angle, and it is measured by one-half the arc *A D C*.

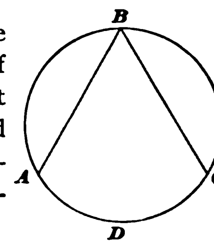


FIG. 50.

EXAMPLE.—If in the figure the arc $ADC = \frac{2}{5}$ of the circumference, what is the measurement of the inscribed angle *A B C*?

SOLUTION.—Since the angle is an inscribed angle, it is measured by one-half the intercepted arc, or $\frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$ of the circumference. The whole circumference contains four quadrants; hence, $4 \times \frac{1}{5} = \frac{4}{5}$ of a quadrant, or $\frac{4}{5}$ of a right angle; therefore, the measurement of the angle *A B C* is $\frac{4}{5}$ of a quadrant.

730. If a circle is divided into halves, each half is called a **semicircle,** and each half circumference is called a **semicircumference.**

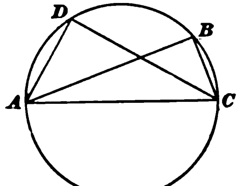


FIG. 51

731. Any angle that is inscribed in a semicircle and intercepts a semicircumference, as *A B C*, or *A D C*, Fig. 51, is a right angle, since it is measured by one-half a semicircumference, or by a quadrant.

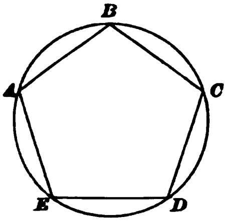


FIG. 52.

732. An **inscribed** polygon is one whose vertexes lie on the circumference of a circle, and whose sides are chords, as *A B C D E*, Fig. 52.

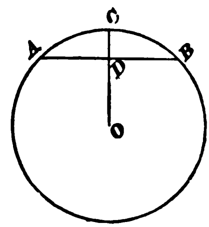


FIG. 53.

733. If, in any circle, a radius be drawn perpendicular to any chord, it bisects (cuts in halves) the chord. Thus, if the radius *O C*, Fig. 53, is perpendicular to the chord *A B*, *A D* = *D B*.

EXAMPLE.—If a regular pentagon is inscribed in a circle, and a radius is drawn perpendicular to one of the sides, what are the lengths of the two parts of the side, the perimeter of the pentagon being 27 inches?

SOLUTION.—A pentagon has five sides, and since it is a regular pentagon, all the sides are of equal lengths: the perimeter of the pentagon, which is the distance around it, and equals the sum of all the sides, or 27 inches. Therefore, the length of one side $= 27 \div 5 = 5\frac{2}{5}$ inches. Since the pentagon is an inscribed pentagon, its sides are chords, and as a radius perpendicular to a chord bisects it, we have $5\frac{2}{5} \div 2 = 2\frac{7}{10}$ inches, for the length of each of the parts of the side, cut by a radius perpendicular to it. Ans.

734. If a straight line be drawn perpendicular to any chord at its middle point, it must pass through the center of the circle.

Through any three points not in the same straight line, a circumference can be drawn. Let *A*, *B* and *C*, Fig. 54, be any three points. Join *A* and *B*, and *B* and *C*, by straight lines. At the middle point of *A B*, draw *H K* perpendicular to *A B*; at the middle point of *B C* draw *E F* perpendicular to *B C*. These two perpendiculars intersect at *O*. With *O* as a center, and *O B* as a radius, describe a circle; it will pass through *A*, *B*, and *C*.

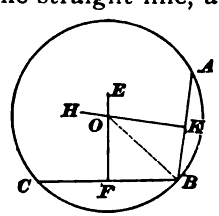


FIG. 54.

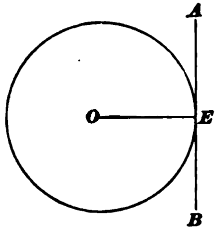


FIG. 55.

735. A **tangent** to a circle is a straight line which touches the circle at one point only ; it is always perpendicular to a radius drawn to that point. Thus, in Fig. 55, $A\ B$ drawn perpendicular to the radius $O\ E$ at its extremity E, is a *tangent* to the circle.

If a straight line is perpendicular to a radius at its extremity, it is tangent to the circle.

736. If two circles intersect each other, the line joining their centers bisects at right angles the line joining the two points of intersection. If the two circles, whose centers are O and P, Fig. 56, intersect at A and B, the line $O\ P$ bisects at right angles the line $A\ B$; or $A\ C = B\ C$.

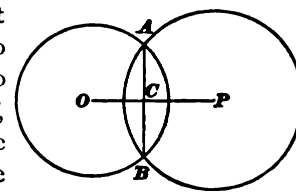


FIG. 56.

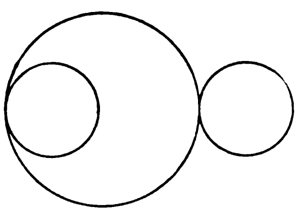


FIG. 57.

737. One circle is said to be **tangent** to another circle when they touch each other at one point only, as in Fig. 57. This point is called the **point of contact,** or the **point of tangency.**

738. When two or more circles are described from the same center, as in Fig. 58, they are called **concentric circles.**

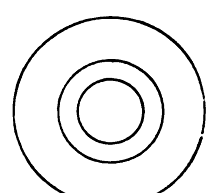


FIG. 58.

739. If, from any point on the circumference of a circle, a perpendicular be let fall upon a given diameter, this perpendicular will be a mean proportional between the two parts into which it divides the diameter.

If $A B$, Fig. 59, is the given diameter, and C any point on the circumference, then is the perpendicular $C D$ a mean proportional between $A D$ and $D B$, or $A D : C D = C D : D B$.

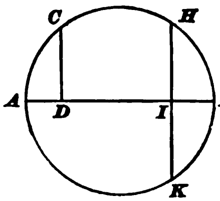


FIG. 59.

Therefore, $\overline{CD}^2 = A D \times D B$,

and $CD = \sqrt{A D \times D B}.$

EXAMPLE.—If $HK = 30$ feet, and $IB = 8$ feet, what is the diameter of the circle, HK being perpendicular to $A B$?

SOLUTION.— 30 feet ÷ 2 feet = 15 feet = IH. And

$$BI : IH = IH : IA, \text{ or } 8 : 15 = 15 : IA.$$

Therefore, $IA = \frac{15^2}{8} = \frac{225}{8} = 28\frac{1}{8}$ feet, and $IA + IB = 28\frac{1}{8} + 8 = 36\frac{1}{8}$ feet $= A B$, the diameter of the circle. Ans.

EXAMPLE.—The diameter of the circle $A B$ is $36\frac{1}{8}$ feet, and the distance $B I$ is 8 feet; what is the length of the line HK?

SOLUTION.—As the diameter of the circle is $36\frac{1}{8}$ feet, and as BI is 8 feet, IA is equal to $36\frac{1}{8} - 8 = 28\frac{1}{8}$ feet. Hence, $BI : IH = IH : IA$, or $8 : IH = IH : 28\frac{1}{8}$.

Therefore, $IH = \sqrt{8 \times 28\frac{1}{8}} = 15$ feet, and as $HK = IH + IK$, or $2\,IH$, $HK = 15 \times 2 = 30$ feet. Ans.

EXAMPLES FOR PRACTICE.

1. If a circle is divided into ten equal sectors, what part of a quadrant is contained in the angle of each sector? Ans. $\frac{2}{5}$ of a quadrant.
2. An angle inscribed in a circle intercepts one-fourth of the circumference. What is the size of the angle? Ans. $\frac{1}{2}$ of a right angle.
3. The perimeter of a regular inscribed octagon is 100 inches long. If a radius is drawn perpendicular to one of the sides, what are the lengths of the two parts of the side? Ans. $6\frac{1}{4}$ inches.
4. If, in Fig. 59, the diameter $A B = 32\frac{1}{2}$ feet, and the distance $IB = 8$ feet, what is the length of the chord HK? Ans. 28 feet.
5. In the same figure, if the distance BI is 6 inches, and HK 18 inches, what is the diameter of the circle? Ans. 19.5 inches.

TRIGONOMETRY.

740. Trigonometry is that branch of mathematics which treats of the solution of triangles.

Every triangle has six parts—three **sides** and three **angles**. If any three parts are given, one of them being a side, the other three can be found. The process of finding the unknown parts from the given parts is called the **solution** of the triangle.

741. In Trigonometry, the circumference of every circle is supposed to be divided into 360 equal parts, called **degrees**; every degree is subdivided into 60 equal parts, called **minutes**, and every minute is again divided into 60 equal parts, called **seconds**. Degrees, minutes and seconds are denoted by the symbols °, ′, ″. Thus, the expression 37° 14′ 44″, is read 37 degrees, 14 minutes and 44 seconds.

Since one degree is $\frac{1}{360}$ of any circumference, it follows that the length of an arc of one degree will be different in circles of different diameters, but the proportion of the length of an arc of one degree to the whole circumference will always be the same, viz., $\frac{1}{360}$ of the circumference.

Hence, in two given circles the length of an arc of 1° will be proportional to the two radii. Thus, if $A\ O\ B$, Fig. 60, is an angle of 1° on the larger circle, it is also 1° on the smaller concentric circle, and the length of the arc $A\ B$ is to the length of the arc $C\ D$ as the radius $O\ B$ is to the radius $O\ D$; or, arc $A\ B$: arc $C\ D = O\ B : O\ D$.

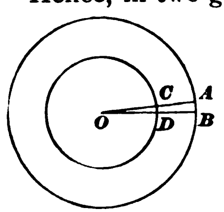


FIG. 60.

EXAMPLE.—If the arc $C D = 2$ inches, radius $O D = 5$ inches, and radius $O B = 9$ inches, what is the length of the arc $A B$?

SOLUTION.— $A B : 2 = 9 : 5$, or $A B = \frac{9 \times 2}{5} = 3\frac{3}{5}$ inches. Ans.

742. In Trigonometry, the arcs of circles are used to measure angles. All angles are supposed to have their vertexes at the center O of the circle (see Fig. 61), one side of the angle lying to the right of O, and coinciding with the horizontal diameter, as $O\ B$.

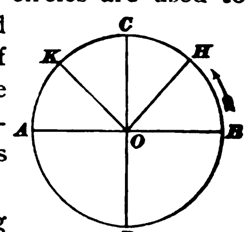


FIG. 61.

The point B on the arc is the starting point in measuring an angle; the angle is supposed to increase by moving around the circumference in the direction indicated by the arrow, until the number of degrees, minutes and seconds in the angle have been measured off on the arc. Suppose that it stops at the point H; draw $O H$, and $H O B$ will be the angle. If K had been the stopping point, $K O B$ would have been the angle.

EXAMPLE.—A given angle equals 51°; to lay it off trigonometrically on a circle: Describe a circle with any convenient radius, and draw the horizontal diameter $A B$. From the point B, count off 51° in the direction of the arrow. If the 51° stops at the point H, draw $O H$, and $H O B$ will equal 51°, and will be the required angle.

743. Since a quadrant is a fourth part of a circle, the number of degrees in a quadrant is one-fourth of 360°, or 90°. Hence, a right angle always contains 90°.

EXAMPLE.—The earth turns completely around on its axis once in one day; through how many degrees does it turn in one hour?

SOLUTION.—In one day there are 24 hours, and since the earth turns through 360° in 24 hours, in one hour it will turn through $360° \div 24 = 15°$. Ans.

744. In adding two angles together, seconds are added to seconds, minutes to minutes, and degrees to degrees; so, also, in subtracting two angles, seconds are subtracted from seconds, minutes from minutes, and degrees from degrees.

EXAMPLE.—Add 75° 46′ 17″ and 14° 27′ 34″.

SOLUTION.—

$$\begin{array}{r} 75° \; 46' \; 17'' \\ \underline{14° \; 27' \; 34''} \\ 89° \; 73' \; 51'' \end{array}$$

Since 73′ = 1° 13′, the 1° is added to the 89°, and the sum is then written 90° 13′ 51″. Ans.

EXAMPLE.—What is the difference between 126° 14′ 20″ and 45° 28′ 13″?

SOLUTION.—

$$\begin{array}{r} 126° \; 14' \; 20'' \\ \underline{45° \; 28' \; 13''} \\ 7'' \end{array}$$

Since 28′ cannot be taken from 14′, 1° (= 60′) is taken from 126° and added to the 14′, and the above is written:

$$\begin{array}{r} 125° \; 74' \; 20'' \\ \underline{45° \; 28' \; 13''} \\ 80° \; 46' \; 7''. \end{array} \text{ Ans.}$$

EXAMPLE.—Subtract 49° 36′ 14″ from 90°.

SOLUTION.—Since 1° = 60′ and 1′ = 60″, we can write, 90° = 89° 59′ 60″, and

$$\begin{array}{r} 89° \ 59' \ 60'' \\ \underline{49° \ 36' \ 14''} \\ 40° \ 23' \ 46''. \end{array} \text{ Ans.}$$

EXAMPLE.—Add 83° 15′ 39″ and 96° 44′ 21″.

SOLUTION.—

$$\begin{array}{r} 83° \ 15' \ 39'' \\ \underline{96° \ 44' \ 21''} \\ 179° \ 59' \ 60'' \end{array}$$

Since 60″ = 1′, add 1′ to 59′ making it 60′; since 60′ = 1°, add 1 to 179°, making it 180°.

Therefore, 83° 15′ 39″ + 96° 44′ 21″ = 180°. Ans.

EXAMPLES FOR PRACTICE.

1. Add 43° 0′ 59″ and 10° 59′ 40″. Ans. 54° 0′ 39″.
2. From 180° 12′ 20″ subtract 3° 12′ 56″. Ans. 176° 59′ 24″.
3. From 84° take 83° 14′ 10″, and to the result add 14′ 10″. Ans. 1°.

THE TRIGONOMETRIC FUNCTIONS.

745. A **function** of a quantity is another quantity depending upon the first one for its value. The circumference of a circle, for example, is a function of the diameter, because the length of the circumference depends upon the length of the diameter.

746. In Trigonometry, the number of degrees contained in an angle is often denoted by certain lines, called the **trigonometric functions,** whose lengths depend upon the arcs which measure the angles. These lines are the characteristic quantities of trigonometry.

The principal trigonometric functions are the *sine*, *cosine*, and *tangent*.

747. The **sine** of an angle is the line drawn from the point where one side of the angle cuts the circumference perpendicular to the other side. Thus, *A C*, Fig. 62, is the sine of the angle *B O A*. In making calculations, the word *sine* is abbreviated to *sin*, and instead of writing sine of *B O A*, or sine of 34° 15′, write ***sin B O A***, and ***sin 34° 15′***.

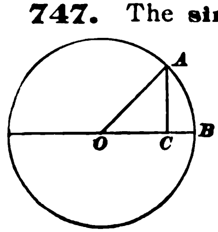


FIG. 62.

748. The **cosine** of an angle is the distance from the foot of the sine to the center of the circle. Thus, in the above figure, $O\ C$ is the cosine of the angle $B\ O\ A$. The word *cosine* is abbreviated to *cos.* Thus, the cosine of $B\ O\ A$ is written **cos *B O A*.**

749. If a tangent is drawn at the right extremity of the horizontal diameter of a circle, which forms one side of an angle, and the other side of the angle is prolonged to meet it, the distance intercepted by the two sides of the angle is called the **tangent** of that angle. Thus, in Fig. 63, $D\ B$ is the tangent of $B\ O\ A$. The word *tangent* is abbreviated to *tan.* Thus, the tangent of $B\ O\ A$ is written **tan *B O A*.**

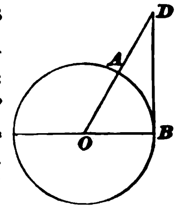


FIG. 63.

750. If a tangent is drawn from the upper extremity of a vertical diameter of a circle, whose horizontal diameter forms one side of an angle, and the other side of the angle is prolonged until it meets this tangent, the distance intercepted on this tangent between the extremity of the vertical diameter and the prolonged line is called the **cotangent** of that angle. Thus, $E\ F$, Fig. 64, is the cotangent of the angle $B\ O\ A$. The word *cotangent* is abbreviated to *cotan.* Thus, the cotangent of $B\ O\ A$ is written **cotan *B O A*.**

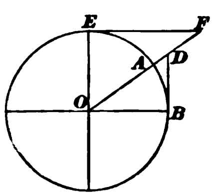


FIG. 64.

These abbreviations must always be pronounced in full. Thus, cos 14° 22′ 46″, is pronounced *cosine of fourteen degrees, twenty-two minutes and forty-six seconds;* tan 45° is pronounced *tangent of forty-five degrees.*

EXAMPLE.—Given, an angle = 60°, to draw its sine, cosine, tangent, and cotangent.

SOLUTION.—Describe any circle, $H\ E\ B\ G$, Fig. 65, and draw the horizontal diameter $H\ B$, and the vertical diameter $G\ E$. From B, mark off $B\ A$ = 60°, and draw $O\ A$; then, $B\ O\ A$ = 60°. At A, draw $A\ C$ perpendicular to $O\ B$. $A\ C$ = sin 60°, and $O\ C$ = cos 60°.

Draw a tangent at B, and prolong $O\ A$ until $O\ A$ intersects the tangent at D; then $D\ B$ = tan 60°.

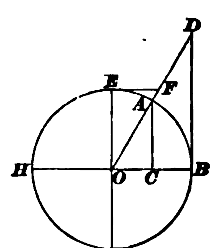


FIG. 65.

Draw a tangent at E, and prolong it until it meets $O A$ prolonged at F; then, $E F =$ cotan 60°. Ans.

751. The sine, cosine, tangent, and cotangent of the *same* angle in circles of different radii are proportional to the radii. In a circle whose radius is 3, for example, they are three times as great as the sine, cosine, tangent, and cotangent of the *same* angle in a circle whose radius is 1. Thus, in Fig. 66, the angles $B O A$ and $F O H$ are the same; but if the radius $O B$ is three times as long as the radius $O F$, then, by the principle of similar triangles (see Art. **716**), it can easily be shown that the sine $A C$ is three times as long as the sine $H K$; that the cosine $O C$ is three times as long as the cosine $O K$; that the tangent $D B$ is three times as long as the tangent $E F$, and that the cotangent $R N$ is three times as long as the cotangent $L M$. Hence, we have the two important principles:

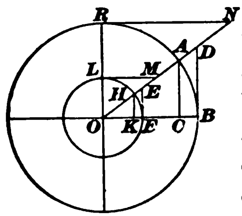


FIG. 66.

If the values of the sine, cosine, tangent, and cotangent are known for any angle in a circle whose radius is 1, their values for the **same** *angle in any other circle can be obtained by multiplying their known values in a circle whose radius is 1, by the radius of the required circle.*

Conversely, if the values of the sine, cosine, tangent, and cotangent are known for an angle in a circle whose radius is other than 1, their values for the **same** *angle in a circle whose radius is 1 can be obtained by dividing the known values by the radius of the given circle.*

EXAMPLE.—Sin 28° 21′ to a radius 1 = .47486; what is the sine of the same angle to a radius $4\frac{1}{2}$?

SOLUTION.— $.47486 \times 4\frac{1}{2} = 2.13687$. Ans.

EXAMPLE.—If tan 19° 35′ to a radius 5 = 1.77880, what does the tangent of the same angle to a radius 1 equal?

SOLUTION.— $1.77880 \div 5 = .35576$. Ans.

752. To facilitate calculations, tables of the trigonometric functions are employed. These tables give the sine, cosine, tangent, cotangent, etc., of the degrees and minutes in a circle whose radius is 1. Hence, when the sine, cosine, tangent, or cotangent is given for an angle in a circle whose radius is greater or less than 1, its value must be found for a circle whose radius is 1, before the table can be used.

For example, suppose we have measured the sine of a certain angle belonging to a circle whose radius is 20 feet, and found that it measures 9.4972 feet, and wanted to find how many degrees the angle contained. In this case the sine of the angle for a circle whose radius is 20 feet = 9.4972 feet. By dividing 9.4972 by 20, we have $\frac{9.4972}{20} = .47486$, which is the sine of the *same* angle in a circle whose radius is 1; and by looking in a table containing the natural sines of angles for different degrees and minutes, we find opposite .47486, 28° 21′, which is the size of the angle. The method of using the table of natural sines, cosines, tangents, and cotangents will be explained later.

753. The *application* of the trigonometric functions is made in right-angled triangles. Thus, in Fig. 67, $A\,B\,C$ is a right-angled triangle, right-angled at C. If A be taken as a center, and $A\,B$ as a radius, and an arc $D\,B\,F$ be described, $B\,C$ will be the *sine* of the angle $B\,A\,C$, and $A\,C$ will be the *cosine* of $B\,A\,C$. If, with the same center A, but with $A\,C$ as a radius, an arc $C\,H\,E$ be described, $B\,C$ will be the *tangent* of $B\,A\,C$.

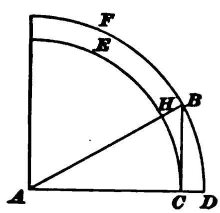


FIG. 67.

754. To show the method of using the sine, cosine and tangent, the following cases will be considered:

CASE I.—Suppose we have a right-angled triangle $A\,B\,C$, Fig. 68, right-angled at C, and that we know the lengths of $A\,B$ and $B\,C$, and wish to find the length of $A\,C$, and the angles A and B.

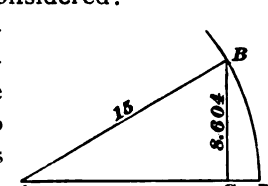


FIG. 68.

As $A\,C\,B$ is a right-angled triangle, we can calculate $A\,C$ by subtracting the square of $B\,C$ from the square of $A\,B$, and extracting the square root of the remainder, or $A\,C = \sqrt{\overline{A\,B}^2 - \overline{B\,C}^2}$. (See Art. **714.**) But we should not know the angles A and B. Hence, we proceed as follows: With A as a center, and $A\,B$ as a radius, describe the arc $B\,D$; then, $B\,C$ is the sine of the angle A, and $A\,B$ is the radius of the circle. Suppose that $A\,B = 15$ feet, and $B\,C = 8.604$ feet. Now, reduce $B\,C$ to a circle whose radius is 1, by dividing 8.604 by 15, or $\frac{8.604}{15} = .5736 =$ sine of angle A, in a circle whose radius is 1.

Looking in a table of natural sines, we find opposite .5736, 35°; hence, angle $A = 35°$. Now, observe from the figure that $A\,B$ is the hypotenuse, and $B\,C$ is the side opposite the angle we wish to find; then, since we divided $B\,C$ by $A\,B$, to reduce the sine to a circle whose radius is 1, we have

Rule 1.—$\frac{\textit{side opposite}}{\textit{hypotenuse}} =$ *sine of angle in a circle whose radius is 1.*

Since angle $A +$ angle $B = 90°$; angle $B = 90° - 35° = 55°$. Having found the angle B, the side $A\,C$ can be easily found by referring to a table of natural sines, in which the sine of 55° will be found to be equal to .81915.

From rule **1**, we have $\frac{\text{side opposite}}{\text{hypotenuse}} =$ sine of angle, or since B is an angle of 55°, and $A\,C$ is the side opposite, $\frac{A\,C}{15} = .81915$, or $A\,C = .81915 \times 15 = 12.29$ feet. We now know all of the sides and angles.

Since $A\,C = .81915 \times 15 =$ sine of the angle $B \times$ hypotenuse, and $A\,C =$ side opposite, we have,

Rule 2.—*Side opposite = hypotenuse × sine.*

CASE II.—Suppose that in a right-angled triangle, $A\,B\,C$, Fig. 69, right-angled at C, the side $A\,C = 14$, and the hypotenuse $A\,B = 18$, had been given, to find $B\,C$, and the angles A and B. $B\,C$ is called the **side opposite** the angle A, and $A\,C$ is called the **side adjacent**. With A as a center,

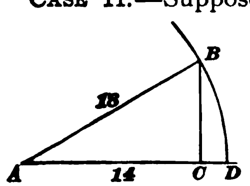


FIG. 69.

and $A B$ as a radius, describe the arc $B D$. Then $A C$ is the cosine of the angle A to a radius $A B = 18$; cos A to a radius $1 = 14 \div 18 = .77778$. Referring to a table of natural cosines, .77778 will be found to be the cos of 38° 56′ 32″.

Hence, the angle $A = 38° 56' 32''$. To find B, subtract A from 90°.

$$\begin{array}{r} 90° = 89° \ 59' \ 60'' \\ 38° \ 56' \ 32'' \\ \hline 51° \ \ 3' \ 28'' \end{array}$$

Therefore, $B = 51° 3' 28''$.

But since $14 \div 18 = A C \div A B$, we have

Rule 3.— $\dfrac{\textit{side adjacent}}{\textit{hypotenuse}} = \textit{cosine}.$

$B C$ is side opposite the angle A ; therefore, since side opposite = hypotenuse × sine (rule **2**), $B C = 18 \times \sin 38° 56' 32'' = 18 \times .62853 = 11.31$.

Since cosine $= \dfrac{\text{side adjacent}}{\text{hypotenuse}}$ (rule **3**), we have

Rule 4.—*Side adjacent = hypotenuse × cosine.*

Case III.—In Fig. 70, $A C B$ is a right-angled triangle, right-angled at C. Let $B C = 9$, and $A C = 16$. To find the hypotenuse $A B$, and angles A and B. In the preceding cases, the hypotenuse was given ; A was taken as a center, and the hypotenuse $A B$ as a radius. But since, in the present case, the hypotenuse is unknown, we cannot use it as in rules **1**, **2**, **3**, and **4**. So we take the same center A, and $A C$ as a radius, and describe the arc $C D$. Then, $B C$ is the tangent of the angle A to a radius 16, and $9 \div 16 = .5625$, or tangent A to a radius 1. In a table of natural tangents we find .5625 opposite 29° 21′ 28″.

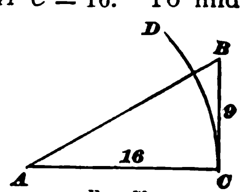


Fig. 70.

Therefore, $A = 29° 21' 28''$, and $B = 90° - 29° 21' 28'' = 60° 38' 32''$.

To find $A B$, we use rule **2**, side opposite = hypotenuse × sine, or $9 = \sin 29° 21' 28'' \times$ hypotenuse. In the table of natural sines, we find that sin 29° 21′ 28″ = .49026. 9 = .49026 × hypotenuse, or hypotenuse $= \frac{9}{.49026} = 18.36$.

Since $B\,C = 9$, or side opposite, $A\,C = 16$, or side adjacent, and $9 \div 16 = B\,C \div A\,C$, or tangent, we have

Rule 5.—*Tangent* $= \frac{\textit{side opposite}}{\textit{side adjacent}}$; also,

Rule 6.—*Side opposite = tangent × side adjacent.*

We also have

Rule 7.—*Cotangent* $= \frac{\textit{side adjacent}}{\textit{side opposite}}$; and

Rule 8.—*Side adjacent = cotangent × side opposite.*

By means of these eight rules, we can find the side and angles of any triangle, when three of its parts are given, one part being a side.

They should be *memorized*, and for convenience are again given here.

Rule 1.—*Sine* $= \frac{\textit{side opposite}}{\textit{hypotenuse}}$.

Rule 2.—*Side opposite = hypotenuse × sine.*

Rule 3.—*Cosine* $= \frac{\textit{side adjacent}}{\textit{hypotenuse}}$.

Rule 4.—*Side adjacent = hypotenuse × cosine.*

Rule 5.—*Tangent* $= \frac{\textit{side opposite}}{\textit{side adjacent}}$.

Rule 6.—*Side opposite = side adjacent × tangent.*

Rule 7.—*Cotangent* $= \frac{\textit{side adjacent}}{\textit{side opposite}}$.

Rule 8.—*Side adjacent = cotangent × side opposite.*

To these may be added two more:

Rule 9.—*Hypotenuse* $= \frac{\textit{side opposite}}{\textit{sine}}$.

Rule 10.—*Hypotenuse* $= \frac{\textit{side adjacent}}{\textit{cosine}}$.

When solving triangles by means of these rules, and the hypotenuse is given or required, use rule **1**, **2**, **3**, **4**, **9**, or **10**; but, when the two sides about the right angle are given, use rules **5** and **6** or **7** and **8**.

EXAMPLES FOR PRACTICE.

1. Given, an angle = 45°, to draw its sine, cosine and tangent.

2. Given, the tangent of an angle in a circle 3 inches in diameter = 2 inches. Draw a figure, and from it find the sine of the same angle in a circle whose radius is twice as great.

3. The cosine of an angle in a circle whose radius is 1 inch is .9654 inch. (*a*) What is the cosine of the same angle in a circle whose radius measures $3\frac{1}{2}$ inches? (*b*) What in a circle whose radius is .63 inch?

Ans. { (*a*) 3.3789 in. (*b*) .6082 in.

TRIGONOMETRIC TABLES.

755. In the foregoing rules, the sines, cosines, tangents, and cotangents are to be taken from printed tables, which have previously been referred to. There are two kinds of tables in general use ; the first is a table of natural sines, cosines, tangents, etc., and gives their actual values for a circle whose radius is 1. In other tables, logarithms are used. The first is the table that we shall employ, and the method of using it will now be explained.

756. Given, an angle, to find its sine, cosine, tangent, and cotangent :

EXAMPLE.—Let it be required to find the sine, cosine, tangent and cotangent of an angle of 37° 24'.

SOLUTION.—Look in the table of **natural sines** along the tops of the pages and find 37°. The left-hand column is marked ('), meaning that the minutes are to be sought in that column, and begin with 0, 1, 2, 3, etc., up to 60. Glancing *down* this column until 24 is found, find opposite this 24 in the column marked *sine*, and headed 37°, the number .60738; then .60738 = sin 37° 24' to a radius 1. In exactly the same manner, find in the column marked *cosine*, and headed 37°, the number .79441, which corresponds to cos 37° 24'; or cos 37° 24' = .79441. So, also, find in the column marked *tangent*, and headed 37°, and opposite 24', the number .76456; whence, tan 37° 24' = .76456. Finally, find in the column marked *cotangent* and headed 37°, and opposite 24', the number 1.30795; whence, cotan 37° 24' = 1.30795.

In most of the tables published, the angles run only from 0° to 45°, at the heads of the columns; to find an angle greater than 45°, *look at the bottom of the page and glance upwards*, ***using the extreme right-hand column to find minutes,*** which begin with 0 at the bottom and run upwards, 1, 2, 3, etc., up to 60.

EXAMPLE.—Find the sine, cosine, tangent, and cotangent of 77° 43.

SOLUTION.—Since this angle is greater than 45°, look along the bottom of the tables, until the column marked *sine*, and headed 77°, is found. Glancing up the column of minutes on the *right*, until 43 is found, find opposite 43' in the column marked *sine* at the bottom, and headed 77°, the number .97711; this is the sine of 77° 43', or sin 77° 43' =.97711. Similarly, in the column marked *cosine*, and headed 77°, find opposite 43', in the right-hand column, the number .21275; this is the cosine of 77° 43', or cos 77° 43' = .21275. So, also, find that 4.59283 is the tangent of 77° 43', or tan 77° 43' = 4.59283. Finally, find in the same manner, that the cotangent of 77° 43', or cotan 77° 43' = .21773.

Let it be required to find the sine of 14° 22' 26".

EXPLANATION.—The sine of 14° 22' 26", lies between the sine 14° 22' and 14° 23'. Sin 14° 22' = .24813; sin 14° 23' = .24841; difference = .00028. Hence, there are 28 parts between sin 14° 22' and sin 14° 23', or 28 parts corresponding to a difference of 1'. Now, since 1' = 60", the number of parts between sin 14° 22' and sin 14° 22' 26", that is, the number of parts corresponding to a difference of 26", must be $\frac{26}{60} \times 28 = 12.1$. Since .1 is less than .5, omit it, and we have 12 as the number of parts to be added to the sine of 14° 22' to produce the sine of 14° 22' 26".

Therefore, sin 14° 22' 26" = .24813 + .00012 = .24825.

By referring to the table of sines, cosines, tangents, and cotangents, it will be observed that, as the angles increase in size, the sines and tangents increase, while the *cosines and cotangents decrease*. In the above example, therefore, had it been required to find the *cosine* or the *cotangent* of 14° 22' 26", the correction for the 26" would have been *subtracted* from the cosine or the cotangent of 14° 22', instead of added to it. From the foregoing, we have, to find the sine, cosine, tangent, or cotangent of an angle containing seconds, the following rule:

Rule 11.—*Find in the table the sine, cosine, tangent, or cotangent corresponding to the degrees and minutes of the angle.*

For the seconds, find the difference of the values of the sine, cosine, tangent, or cotangent, taken from the table between which the seconds of the angle fall; multiply this difference *by a fraction whose numerator is the number of seconds in the given angle, and whose denominator is 60.*

If sine or tangent, add this correction to the value first found; if cosine or cotangent, subtract the correction.

EXAMPLE.—Find the sine, cosine, tangent, and cotangent of 56° 43′ 17″.

SOLUTION.—Sin 56° 43′ = .83597. Sin 56° 44′ = .83613. Since 56° 43′ 17″ is greater than 56° 43′ and less than 56° 44′, the value of the sine of the angle lies between .83597 and .83613; the difference = .83613 − .83597 = .00016; multiplying this by the fraction $\frac{17}{60}$, .00016 × $\frac{17}{60}$ = .00005, nearly, which is to be *added* to .83597, the value first found, or .83597 + .00005 = .83602. Hence, sin 56° 43′ 17″ = .83602. Ans.

Cos 56° 43′ = .54878; cos 56° 44′ = .54854; the difference = .54878 − .54854 = .00024, and .00024 × $\frac{17}{60}$ = .00007, nearly. Now, since the *cosine* is desired, we must *subtract* this correction from cos 56° 43′ or .54878; subtracting, .54878 − .00007 = .54871. Hence, cos 56° 43′ 17″ = .54871. Ans.

Tan 56° 43′ = 1.52332; tan 56° 44′ = 1.52429; the difference = .00097, and .00097 × $\frac{17}{60}$ = .00027, nearly. Since the tangent is desired, we must *add*, giving 1.52332 + .00027 = 1.52359. Hence, tan 56° 43′ 17″ = 1.52359. Ans.

Cotan 56° 43′ = .65646; cotan 56° 44′ = .65604; the difference = .00042, and .00042 × $\frac{17}{60}$ = .00012, nearly. Since the cotangent is desired, we must *subtract*, giving .65646 − .00012 = .65634. Hence, cotan 56° 43′ 17″ = .65634. Ans.

757. Given, the sine, cosine, tangent, or cotangent, to find the angle corresponding:

EXAMPLE.—The sine of an angle is .47486; what is the angle*?

SOLUTION.—Consulting the table of natural sines, glance down the columns marked *sine*, until .47486 is found opposite 21′, in the left-hand column, and under the column headed 28°. Therefore, the angle whose sine = .47486 is 28° 21′, or sin 28° 21′ = .47486. Ans.

EXAMPLE.—Find the angle whose cosine is .27032.

SOLUTION.—Looking in the columns marked *cosine*, at the top of the page, it is not found; hence, the angle is greater than 45°; consequently, looking in the columns marked *cosine* at the bottom of the page, it is found opposite 19′, in the *right-hand* column of minutes, and in the column headed 74° at the bottom. Therefore, the angle whose cosine is .27032, is 74° 19′, or cos 74° 19′ = .27032. Ans.

*NOTE.—Whenever the sine, cosine, tangent, or cotangent of an angle is given, and no radius is specified, the radius is always understood to be 1.

EXAMPLE.—Find the angle whose tangent is 2.15925.

SOLUTION.—On searching a table of natural tangents, it is found to belong to an angle greater than 45°, so it must be looked for in the column marked *tangent* at the bottom. It is found opposite 9', in the right-hand column of minutes, and in the column headed 65° at the bottom. Therefore, tan 65° 9' = 2.15925. Ans.

EXAMPLE.—Find the angle whose cotangent is .43412.

SOLUTION.—From the table of natural cotangents, it is found that this value is less than the cotangent of 45°, so it must be found in the column marked *cotangent* at the bottom. Looking there, it is found in the column headed 66° at the bottom, and opposite 32', in the right-hand column of minutes. Therefore, the angle whose cotangent is .43412, is 66° 32', or cotan 66° 32' = .43412. Ans.

Let it be required to find the angle whose sine is .42531.

EXPLANATION.—Referring to the table of sines, this number is found to lie between .42525, the sine of 25° 10', and .42552, the sine of 25° 11'. The difference between these two numbers = .42552 − .42525 = .00027, or 27 parts; the difference between .42525, the sine of 25° 10', and .42531, the sine of the given angle, = .42531 − .42525 = .00006, or 6 parts. Therefore, the angle whose sine is .42531, is greater than 25° 10' by 6 of the 27 parts between sin 25° 10' and sin 25° 11'. Hence, the angle whose sine = .42531 = 25° $10\frac{6}{27}$'.

Since 1' = 60", $\frac{6}{27}$ minute = $\frac{6}{27} \times 60 = 13.3''$. Therefore, the angle whose sine is .42531 = 25° 10' 13.3".

In this case, the correction is added for the cosine and cotangent, as well as for the sine and tangent, because for any difference between the given sine, cosine, tangent, or cotangent, and the one belonging to the angle next lower, the size of the *angle* always *increases* a corresponding amount.

758. To find the angle corresponding to a given sine, cosine, tangent, or cotangent, whose exact value is not contained in the table:

Rule 12.—*Find the difference of the two numbers in the table between which the given sine, cosine, or tangent falls, and use the number of parts in this difference as the denominator of a fraction.*

Find the difference between the number belonging to the **smaller angle,** *and the given sine, cosine, tangent, or cotangent, and use the number of parts in the difference just found as the numerator of the fraction mentioned above. Multiply this fraction by 60, and the result will be the number of seconds to be added to the* **smaller angle.**

EXAMPLE.—Find the angle whose sine = .57698.

SOLUTION.—Looking in the table of natural sines, in the columns marked *sine*, it is found between .57691 = sin 35° 14′ and .57715 = 35° 15′. The difference between them is .57715 − .57691 = .00024, or 24 parts. The difference between the sine of the smaller angle, or sin 35° 14′ = .57691, and the given sine, or .57698, is .57698 — .57691 = .00007, or 7 parts.

Then, $\frac{7}{24} \times 60 = 17.5''$, and the angle = 35° 14′ 17.5″, or sin 35° 14′ 17.5″ = .57698. Ans.

EXAMPLE.—Find the angle whose cosine is .27052.

SOLUTION.—Looking in the table of cosines, it is found to belong to a greater angle than 45° and, hence, must be sought for in the columns marked *cosine*, at the bottom of the page. It is found between the numbers .27060 = cos 74° 18′ and .27032 = cos 74° 19′. The difference between the two is .27060 — .27032 = .00028, or 28 parts. The cosine of the *smaller angle*, or 74° 18′, is .27060, and the difference between this and the given cosine is .27060 — .27052 = .00008, or 8 parts.

Hence, $\frac{8}{28} \times 60 = 17.14''$, nearly, and the angle whose cosine is .27052 = 74° 18′ 17.14″, or cos 74° 18′ 17.14″ = .27052. Ans.

EXAMPLE.—Find the angle whose tangent is 2.15841.

SOLUTION.—2.15841 falls between 2.15760 = tan 65° 8′, and 2.15925 = tan 65° 9′. The difference = 2.15925 − 2.15760 = .00165, or 165 parts. 2.15841 — 2.15760 = .00081, or 81 parts. $\frac{81}{165} \times 60 = 29.5''$, nearly, and the angle whose tangent is 2.15841 = 65° 8′ 29.5″, or tan 65° 8′ 29.5″ = 2.15841.

EXAMPLE.—Find the angle whose cotangent is 1.26342.

SOLUTION.— 1.26342 falls between 1.26395 = cotan 38° 21′, and 1.26319 = cotan 38° 22′. The difference = 1.26395 − 1.26319 = .00076. 1.26395 − 1.26342 = .00053. $\frac{53}{76} \times 60 = 41.9$, nearly, and the angle whose cotangent is 1.26342 = 38° 21′ 41.9″, or cotan 38° 21′ 41.9″ = 1.26342.

EXAMPLES FOR PRACTICE.

1. Find (*a*) the sine, (*b*) cosine and (*c*) tangent of 48° 17′.

Ans. { (*a*) .74644. (*b*) .66545. (*c*) 1.12172.

2. Find the (*a*) sine, (*b*) cosine, and (*c*) tangent of 13° 11′ 6″.

Ans. { (*a*) .22810. (*b*) .97364. (*c*) .23427.

3. Find the (*a*) sine, (*b*) cosine, and (*c*) tangent of 72° 0′ 1.8″.

Ans. { (*a*) .95106. (*b*) .30901. (*c*) 3.07777.

4. (*a*) Of what angle is .26489 the sine? (*b*) Of what is it the cosine?

Ans. { (*a*) 15° 21′ 37.2″. (*b*) 74° 38′ 22.8″.

5. (*a*) Of what angle is .68800 the sine? (*b*) Of what the cosine? (*c*) Of what the tangent?

Ans. { (*a*) 43° 28′ 20″. (*b*) 46° 31′ 40″. (*c*) 34° 31′ 40.5″.

THE SOLUTION OF TRIANGLES.

759. As previously stated, every triangle has six parts, three sides and three angles, and if any three parts are given, one of them being a side, the other three may be found.

In right-angled triangles, it is only necessary to know *two* parts in addition to the right angle, one of which must be a side.

Rules **1** to **12** are sufficient for solving all cases.

RIGHT-ANGLED TRIANGLES.

The method of solving right-angled triangles has already been explained, but a few additional examples will be considered. There are two cases:

Case I.—*When the two given parts are a side and an angle:*

EXAMPLE.—In Fig. 71, the hypotenuse $A B$ of the right-angled triangle $A C B$ is 24 feet, and the angle A is 29° 31′, to find the sides $A C$ and $B C$, and the angle B.

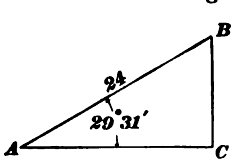


FIG. 71.

NOTE.—When working examples of this kind, construct the figure, and mark the known parts. This is a great help in solving the example. Hence, in the figure draw the angle A to represent an angle of 29° 31′, and complete the right-angled triangle $A C B$, right-angled at C, as shown. Mark the angle A and the hypotenuse, as is done in the figure.

SOLUTION.—Angle $B = 90° - 29° 31' = 60° 29'$. To find $A C$, use rule **4**; viz., $A C$, or side adjacent = hypotenuse × cosine = $24 \times \cos 29° 31' = 24 \times .87021 = 20.89$ feet, nearly.

To find BC, use the same rule; thus, $BC = 24 \times \cos 60° 29' = 24 \times .49268 = 11.82$ feet, nearly. To find BC, rule **2** could also have been used, viz., side opposite = hypotenuse × sine, or $BC = 24 \times \sin 29° 31' = 24 \times .49268 = 11.82$ feet, nearly.

Ans. $\begin{cases} \text{Angle } B = 60° 29'. \\ \text{Side } AC = 20.89 \text{ ft.} \\ \text{Side } BC = 11.82 \text{ ft.} \end{cases}$

EXAMPLE.—One side of a right-angled triangle, ABC, Fig. 72, is 37 feet 7 inches long; the angle opposite is 25° 33′ 7″. How long is the hypotenuse, the side adjacent, and what is the other angle?

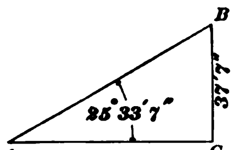


FIG. 72.

SOLUTION.—Angle $B = 90° - 25° 33' 7'' = 64° 26' 53''$.

To find the hypotenuse, use rule **9**,

$$\text{hypotenuse} = \frac{\text{side opposite}}{\text{sine}}.$$

Since the side opposite is given in feet and inches, both must be reduced to feet, or both to inches.

7 in. $= \frac{7}{12}$ of a foot $= .583 +$ of a foot, and $BC = 37.583$ ft.

Therefore, the hypotenuse $= \dfrac{37.583}{\sin 25° 33' 7''} = \dfrac{37.583}{.43133} = 87.133$ feet $=$ 87 ft. 2 in., nearly.

To find the side AC, use rule **4**, side adjacent = hypotenuse × cosine $= 87.133 \times \cos 25° 33' 7'' = 87.133 \times .90219 = 78.61$ feet $= 78$ ft. $7\frac{1}{4}$ in., nearly.

Ans. $\begin{cases} \text{Angle } B = 64° 26' 53''. \\ AC = 78 \text{ ft. } 7\frac{1}{4} \text{ in.} \\ AB = 87 \text{ ft. } 2 \text{ in.} \end{cases}$

The work involved in finding the sine and cosine of 25° 33′ 7″, in the above example, is as follows: Sin 25° 33′ = .43130; sin 25° 34′ = .43156; difference = .00026; $.00026 \times \frac{7}{60} = .00003$. Hence, sin 25° 33′ 7″ = .43130 + .00003 = .43133.

Cos 25° 33′ = .90221; cos 25° 34′ = .90208; difference = .00013; $.00013 \times \frac{7}{60} = .00002$, nearly. Hence, cos 25° 33′ 7″ = .90221 − .00002 = .90219.

Case II.—*When two sides are given:*

EXAMPLE.—In the right-angled triangle, ABC, Fig. 73, right-angled at C, $AC = 18$, and $BC = 15$, to find AB and the angles A and B.

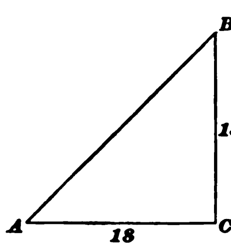


FIG. 73.

SOLUTION.—Since the hypotenuse is not given, use rule **5**, viz.:

$$\text{Tangent } A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{15}{18} = .83333.$$

To find the angle whose tangent is .83333, we have: Tangent of next less angle = .83317 = tan 39° 48′; of the next greater angle = .83366; difference = .00049. The difference between .83317, the tangent of the smaller angle, and .83333, the given tangent, $= .83333 - .83317 = .00016$. Hence, $\frac{16}{49} \times 60 = 19.6''$, and the angle whose tangent is $.83333 = 39° \ 48' \ 19.6'' =$ angle A.

Angle $B = 90° - 39° \ 48' \ 19.6'' = 50° \ 11' \ 40.4''$.

To find the hypotenuse $A\,B$, use rule **9.**

$$\text{Hypotenuse} = \frac{\text{side opposite}}{\text{sine}} = \frac{15}{\sin 39° \ 48' \ 19.6''} = \frac{15}{.64018} = 23.43.$$

Ans. $\begin{cases} \text{Angle } A = 39° \ 48' \ 19.6''. \\ \text{Angle } B = 50° \ 11' \ 40.4''. \\ A\,B = 23.43. \end{cases}$

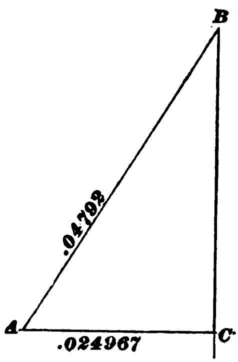


FIG. 74.

EXAMPLE.—In the right-angled triangle, $A\,B\,C$, Fig. 74, right-angled at C, $A\,C =$ .024967 mile, and $A\,B = .04792$ mile; to find the other parts.

SOLUTION.—To find angle A, use rule **3.**

$$\text{Cosine } A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{.024967}{.04792} = .52101.$$

The angle whose cosine is $.52101 = 58° \ 36' =$ angle A. Angle $B = 90° - 58° \ 36' = 31° \ 24'$.

To find side $B\,C$, use rule **6.**

Side opposite $A =$ side adjacent $\times$ tan A, or $B\,C = .024967 \times 1.63826 = .0409$ mile.

Ans. $\begin{cases} \text{Angle } A = 58° \ 36'. \\ \text{Angle } B = 31° \ 24'. \\ B\,C = .0409 \text{ mile}. \end{cases}$

EXAMPLE.—In the right-angled triangle, $A\,B\,C$, Fig. 75, right-angled at C, $A\,B = 308$ feet, and $B\,C = 234$ feet; to find the other parts.

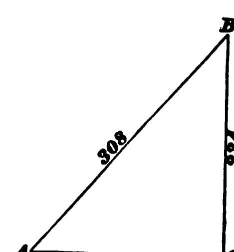


FIG. 75.

SOLUTION.—To find angle A, use rule **1.**

$$\text{Sine } A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{234}{308} = .75974.$$

The angle whose sine is $.75974 = 49° \ 26' \ 28''$, nearly, $=$ angle A. Angle $B = 90° - 49° \ 26' \ 28'' = 40° \ 33' \ 32''$.

To find $A\,C$, use rule **8.**

Side adjacent $A =$ cotan $A \times$ side opposite, or $A\,C = .85586 \times 234 =$ 200.27 feet.

Ans. $\begin{cases} \text{Angle } A = 49° \ 26' \ 28''. \\ \text{Angle } B = 40° \ 33' \ 32''. \\ A\,C = 200.27 \text{ feet}. \end{cases}$

EXAMPLES FOR PRACTICE.

1. In a right-angled triangle $A B C$, right-angled at C, the hypotenuse $A B = 40$ inches, and angle $A = 28° 14' 14''$. Solve the triangle.

Ans. $\begin{cases} \text{Angle } B = 61° 45' 46''. \\ A C = 35.24 \text{ in.} \\ B C = 18.92 \text{ in.} \end{cases}$

2. In a right-angled triangle $A B C$, right-angled at C, the side $B C =$ 10 feet 4 inches. If angle $A = 26° 59' 6''$, what do the other parts equal?

Ans. $\begin{cases} \text{Angle } B = 63° 0' 54''. \\ A B = 22 \text{ ft. } 9\tfrac{1}{4} \text{ in., nearly.} \\ A C = 20 \text{ ft. } 3\tfrac{1}{4} \text{ in., nearly.} \end{cases}$

3. In a right-angled triangle, $A B C$, the hypotenuse $A B = 60$ feet, and the side $A C = 22$ feet. Solve the triangle.

Ans. $\begin{cases} \text{Angle } A = 68° 29' 22.2''. \\ \text{Angle } B = 21° 30' 37.8''. \\ B C = 55.82 \text{ ft.} \end{cases}$

4. In a right-angled triangle $A B C$, right-angled at C, side $A C =$ .364 foot and side $B C = .216$ foot. Solve the triangle.

Ans. $\begin{cases} \text{Angle } A = 30° 41' 7.5''. \\ \text{Angle } B = 59° 18' 52.5''. \\ A B = .423 \text{ ft.} \end{cases}$

OBLIQUE TRIANGLES.

760. When three parts of *any* triangle are given, one being a side, the remaining parts can be found by means of right-angled triangles, by drawing a perpendicular from one angle to the opposite side. The parts of these triangles can then be computed, and from them the parts of the required triangle can be found.

761. CAUTION.—*When dividing the triangle into two right-angled triangles, care must be taken that the perpendicular be so drawn that one of the right-angled triangles will have two known parts besides the right angle; otherwise the triangle cannot be solved.*

Case I.—*When the three known parts are a side and two angles, or an angle and two sides:*

EXAMPLE.—In Fig. 76, the angle $A = 46° 14'$, the angle $B = 88° 24' 11''$, and the side $A B = 21$ in.; to find $A C$, $B C$, and the angle C.

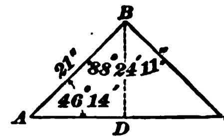


FIG. 76.

SOLUTION.—Since the sum of all the angles of any triangle = 2 right angles, or 180°, we can find the angle C by adding the two known angles, and subtracting their sum from 180°.

$$88° 24' 11'' + 46° 14' = 134° 38' 11''.$$
$$180° - 134° 38' 11'' = 45° 21' 49'' = C.$$

From the vertex B, draw $B D$ perpendicular to $A C$. The triangle $A B C$ is now divided into two right-angled triangles, $A D B$ and $B D C$.

In the right-angled triangle $A D B$, the angle A, the right angle D, and the hypotenuse $A B$ are known; to find $B D$ and $A D$. Use rule **2.** Side opposite or $B D = 21 \times \sin 46° 14' = 21 \times .72216 = 15.17$ in., nearly.

Use rule **4.** Side adjacent, or $A D = 21 \times \cos 46° 14' = 21 \times .69172$, or $A D = 14.53$ in., nearly.

In the right-angled triangle $B D C$, the angle C and the side opposite, or $B D$, are known; to find $B C$ and $D C$.

Use rule **9.** Hypotenuse, or $B C = \frac{B D}{\sin 45° 21' 49''} = \frac{15.17}{.71158} =$ 21.32 in., nearly.

Use rule **4.** $C D$, or side adjacent $= 21.32 \times \cos 45° 21' 49'' = 21.32 \times .70261 = 14.98$ in.

Since $A D + D C = A C$, we have $14.53 + 14.98 = 29.51$ in. $= A C$.

Ans. $\begin{cases} A C = 29.51 \text{ in.} \\ B C = 21.32 \text{ in.} \\ \text{Angle } C = 45° 21' 49''. \end{cases}$

If, in the above example, the angle C had been given instead of the angle A, the dividing line should have been drawn from the angle A to the side $B C$, as in the following example:

EXAMPLE.—In the triangle $A B C$, Fig. 77, given, $A B = 18''$, angle $B = 60°$, and angle $C = 38° 42'$; to find the other three parts.

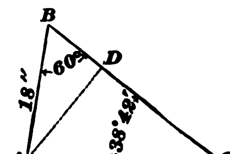


FIG. 77.

SOLUTION.—In the triangle $A B C$, we have angle $A = 180° - (60° + 38° 42') = 81° 18'$.

From the vertex A, draw the line $A D$ perpendicular to $B C$, thus forming the right-angled triangle $A B D$, in which two parts (the side $A B$ and angle B) are known besides the right angle.

Begin with the right-angled triangle $A B D$. To find $B D$, use rule **4.** $B D = 18 \text{ in.} \times \cos 60° = 18 \times .5 = 9$ in. To find $A D$, use rule **2.** $A D = 18 \text{ in.} \times \sin 60° = 18 \times .86603 = 15.59$ in.

In the right-angled triangle $A D C$, $A D$ and the angle C are known.

To find AC, use rule **9.** $AC = \frac{AD}{\sin C} = \frac{15.59}{.62524} = 24.93$ in.

To obtain DC, use rule **4.** $DC = AC \times \cos C = 24.93 \times .78043 = 19.46$ in.

Since $BC = BD + DC$, $BC = 9 + 19.46 = 28.46$ in.

Ans. $\begin{cases} \text{Angle } A = 81° 18'. \\ AC = 24.93 \text{ in.} \\ BC = 28.46 \text{ in.} \end{cases}$

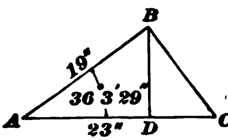


FIG. 78.

EXAMPLE.—In Fig. 78, $AB = 19$ in., $AC = 23$ in., and the included angle $A = 36° 3' 29''$; to find the other two angles and the side BC.

SOLUTION.—From the vertex B, draw BD perpendicular to AC, forming the two right-angled triangles ADB and BDC. In the right-angled triangle ADB, AB is known and also the angle A. Hence, by rule **2,**

$BD = 19 \times \sin 36° 3' 29'' = 19 \times .58861 = 11.18$ in., nearly.

By rule **4,** $AD = 19 \times \cos 36° 3' 29'' = 19 \times .80842 = 15.36$ in.

$AC - AD = 23 - 15.36 = 7.64$ inches $= DC$.

In the right-angled triangle BDC, the two sides BD and DC, about the right angle, are known; hence (rule **5**), $\tan C = \frac{BD}{DC} = \frac{11.18}{7.64} = 1.46335$, and angle $C = 55° 39' 10''$.

Applying rule **9,** $BC = \frac{BD}{\sin 55° 39' 10''} = \frac{11.18}{.82564} = 13.54$ in.

Angle $B = 180° - (36° 3' 29'' + 55° 39' 10'') = 180° - 91° 42' 39'' = 88° 17' 21''$.

Ans. $\begin{cases} \text{Angle } C = 55° 39' 10''. \\ \text{Angle } B = 88° 17' 21''. \\ \text{Side } BC = 13.54 \text{ in.} \end{cases}$

The following example presents a case where the perpendicular must be drawn outside of the triangle in order to form a right-angled triangle two of whose parts are known, besides the right angle:

EXAMPLE.—Given, the triangle ABC, Fig. 79, in which $AB = 88$ ft. 6 in., $BC = 57$ ft., and angle $A = 35° 0' 38''$, to find the other parts.

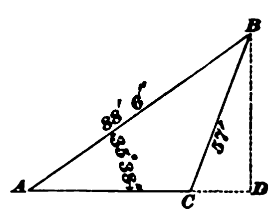


FIG. 79.

SOLUTION.—From the vertex B, draw the line BD perpendicular to the base AC extended, forming the right-angled triangles ADB and CDB.

In the right-angled triangle ADB, AB and angle A are known, to find AD and BD.

By rule **4,** $AD = 88$ ft. 6 in. $\times \cos 35° 0' 38'' = 88.5$ ft. $\times .81905 = 72.49$ ft.

By rule **2**, $BD = 88$ ft. 6 in. $\times \sin 35° \; 0' \; 38'' = 88.5 \times .57373 =$ 50.78 ft.

Now, in the right-angled triangle CDB, BC and BD are known, to find angle BCD and side CD.

By rule **1**, $\sin BCD = \frac{BD}{BC} = \frac{50.78}{57} = .89088$; whence, angle $BCD = 62° \; 59' \; 4.3''$.

By rule **4**, $CD = 57 \times \cos 62° \; 59' \; 4.3'' = 57 \times .45423 = 25.89$ ft.

We now have the data necessary for obtaining the required parts of the triangle ABC. Since the angle $BCD = 62° \; 59' \; 4.3''$, the adjacent angle $ACB = 180° - 62° \; 59' \; 4.3'' = 117° \; 0' \; 55.7''$. Also, angle $ABC = 180° - (35° \; 0' \; 38'' + 117° \; 0' \; 55.7'') = 180° - 152° \; 1' \; 33.7'' = 27° \; 58' \; 26.3''$. Since $AD = 72.49$ ft. and $CD = 25.89$ ft., $AC = 72.49 - 25.89 = 46.6$ ft. $= 46$ ft. $7\frac{3}{16}$ in., nearly.

Ans. { Angle $C = 117° \; 0' \; 55.7''$.
Angle $B = 27° \; 58' \; 26.3''$.
Side $AC = 46$ ft. $7\frac{3}{16}$ in.

Case II.—*When the three sides are given, to find the angles.*

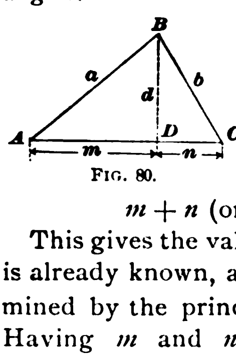


Fig. 80.

This case is solved by drawing a line from the vertex of the angle opposite the longest side, perpendicular to that side, as BD in Fig. 80. The parts m and n of the side AC are then determined from the following proportion:

$$m + n \text{ (or } AC) : a + b = a - b : m - n.$$

This gives the value of $m - n$. The value of $m + n = AC$ is already known, and from the two, m and n may be determined by the principles of arithmetic, as explained below. Having m and n, therefore, the right-angled triangles ABD and CBD may be solved.

Having found the value of $m - n$ and knowing the value of $m + n$, the values of m and n may be determined as follows: It is a principle of arithmetic that if the sum of two numbers and their difference be given, *the greater of the two numbers is equal to one-half their sum plus their difference, and the less of two numbers is equal to one-half their sum minus their difference.* For example, suppose that the sum of two numbers is 22 and their difference is 8. Then, the greater number is $(22 + 8) \div 2 = 15$; and the less number is $(22 - 8) \div 2 = 7$. Therefore, letting m be the greater number and n the less number, $m + n$ represents their sum and $m - n$ their difference; whence,

$$m = \frac{(m + n) + (m - n)}{2},$$

$$n = \frac{(m + n) - (m - n)}{2}.$$

EXAMPLE.—Given, a triangle whose sides are 17 ft. 3 in., 21 ft., and 32 ft. long. Find the angles.

SOLUTION.— $m + n$, the longest side, $= 32$ ft.

$a + b$, the sum of the two shorter sides, $= 17.25 + 21 = 38.25$ ft.

$a - b$, the difference of the two shorter sides, $= 3.75$ ft. Hence,

$$32 : 38.25 = 3.75 : m - n, \text{ or } m - n = \frac{38.25 \times 3.75}{32} = 4.48 \text{ ft.}$$

Then, $m = \dfrac{(m + n) + (m - n)}{2} = \dfrac{32 + 4.48}{2} = 18.24$ ft.;

and $n = \dfrac{(m + n) - (m - n)}{2} = \dfrac{32 - 4.48}{2} = 13.76$ ft.

Now, referring to the last figure, we have, in the triangle ADB side $a = 21$ and $m = 18.24$ ft.; whence, by rule **3**, $\cos A = \dfrac{18.24}{21} = .86857$, or $A = 29° 42' 25.7''$.

In triangle CBD, side $b = 17.25$ and $n = 13.76$ ft.; whence, by rule **3**, $\cos C = \dfrac{13.76}{17.25} = .79768$, or $C = 37° 5' 26.7''$.

Angle $ABC = 180° - (29° 42' 25.7'' + 37° 5' 26.7'') = 113° 12' 7.6''$.

Ans. { Angle $A = 29° 42' 25.7''$. Angle $B = 113° 12' 7.6''$. Angle $C = 37° 5' 26.7''$.

EXAMPLES FOR PRACTICE.

1. Given, an oblique triangle ABC, in which side $AB = 21$ feet, angle $A = 22° 10' 16''$, and angle $B = 78° 24' 24''$. Find the other parts.

Ans. { Angle $C = 79° 25' 20''$. $AC = 20.93$ ft. $BC = 8.06$ ft.

2. Given, a triangle ABC, in which $AB = 32$ inches, angle $B = 54° 16'$, and angle $C = 58° 18' 9''$. Find the other parts.

Ans. { Angle $A = 67° 25' 51''$. $AC = 30.53$ in. $BC = 34.73$ in.

3. In a triangle ABC, $AB = 20$ feet 6 inches, $BC = 16$ feet, and angle $B = 46° 40' 42''$. Find the values of the other parts.

Ans. { Angle $A = 50° 42' 51''$. Angle $C = 82° 36' 27''$. $AC = 15.04$ ft.

4. In the triangle $A\,B\,C$, $A\,C = 100$ feet, $B\,C = 60$ feet, and angle $A = 20°$. Solve the triangle.

Ans. $\begin{cases} \text{Angle } B = 34° \; 45' \; 7.5''. \\ \text{Angle } C = 125° \; 14' \; 52.5''. \\ A\,B = 143.26 \text{ ft.} \end{cases}$

5. In a triangle $A\,B\,C$, $A\,B = 98$ inches, $B\,C = 140$ inches, and $A\,C = 210$ inches. Compute the angles A, B and C.

Ans $\begin{cases} A = 34° \; 2' \; 52.5''. \\ B = 122° \; 52' \; 40.2''. \\ C = 23° \; 4' \; 27.3''. \end{cases}$

MENSURATION.

762. Mensuration is that part of Geometry which treats of the measurement of lines, surfaces and solids.

MENSURATION OF PLANE SURFACES.

763. The **area** of a surface is expressed by the number of unit squares it will contain.

764. A **unit square** is the square having the unit for its side. For example, if the unit is 1 inch, the unit square is the square whose sides measure 1 inch in length, and the area would be expressed by the number of square inches that the surface contains. If the unit were 1 foot, the unit square would measure 1 foot on each side, and the area would be the number of square feet that the surface contains, etc.

765. The square that measures 1 inch on a side is called a **square inch,** and the one that measures 1 foot on a side is called a **square foot.** Square inch and square foot are abbreviated to sq. in. and sq. ft., or to □″ and □′.

THE TRIANGLE.

766. Rule.—*The area of any triangle equals one-half the product of the base and the altitude.*

EXAMPLE.—What is the area of a triangle whose base is 18 feet, and altitude 7 feet 9 inches?

SOLUTION.—9 inches = $\frac{9}{12}$ of a foot = $\frac{3}{4}$ of a foot. $18 \times 7\frac{3}{4} = 139\frac{1}{2}$, and $\frac{1}{2}$ of $139\frac{1}{2} = 69\frac{3}{4}$ square feet. Ans.

THE QUADRILATERAL.

767. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

768. There are four kinds of parallelograms: the **square,** the **rectangle,** the **rhombus** and the **rhomboid.**

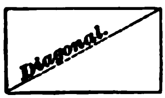


FIG. 81.

769. A **rectangle,** Fig. 81, is a parallelogram whose angles are all right angles.

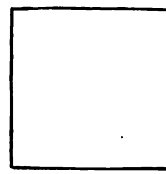


FIG. 82.

770. A **square,** Fig. 82, is a rectangle, all of whose sides are equal.

FIG. 83.

771. A **rhomboid,** Fig. 83, is a parallelogram whose opposite sides only are equal, and whose angles are not right angles.

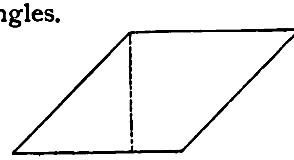


FIG. 84.

772. A **rhombus,** Fig. 84, is a parallelogram having equal sides, and whose angles are not right angles.

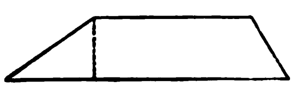


FIG. 85.

773. A **trapezoid,** Fig. 85, is a quadrilateral which has only two of its sides parallel.

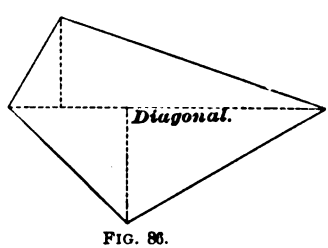


FIG. 86.

774. A **trapezium,** Fig. 86, is a quadrilateral having no two sides parallel.

775. The **altitude** of a parallelogram, or of a trapezoid, is the perpendicular distance between the parallel sides.

776. A **diagonal** is a straight line drawn from the vertex of any angle of a quadrilateral to the vertex of the angle opposite; a diagonal divides the quadrilateral into two triangles.

A diagonal divides a parallelogram into two *equal* and *similar* triangles.

777. To find the area of a parallelogram:

Rule.—*The area of any parallelogram equals the product of the base and the altitude.*

EXAMPLE.—What is the area of a parallelogram whose base is 12 feet and altitude $7\frac{1}{2}$ feet?

SOLUTION.—Area $= 12 \times 7\frac{1}{2} = 90$ square feet. Ans.

778. To find the area of a trapezoid:

Rule.—*The area of a trapezoid equals one-half the sum of the parallel sides multiplied by the altitude.*

EXAMPLE.—What is the area of a trapezoid whose parallel sides are 9 feet and 15 feet, and whose altitude is 6 feet 7 inches?

SOLUTION.—6 feet 7 inches $= 6\frac{7}{12}$ feet. $\frac{9+15}{2} = 12$. Area $= 12 \times 6\frac{7}{12} = 79$ square feet. Ans.

779. To find the area of an irregular figure bounded by straight lines:

Rule.—*Divide the figure into triangles, and find the area of each triangle separately. The sum of the areas of all the triangles will be the area of the figure.*

EXAMPLE.—The diagonal of a trapezium is 15 feet. The altitudes drawn from the vertexes of the two triangles to this diagonal as a base are 6 feet 8 inches and 4 feet 9 inches, respectively. What is the area of the trapezium?

SOLUTION.—8 inches $= \frac{8}{12}$ of a foot $= \frac{2}{3}$ of a foot. $\frac{15 \times 6\frac{2}{3}}{2} = 50$ square feet = area of one triangle.

9 inches $= \frac{9}{12}$ of a foot $= \frac{3}{4}$ of a foot. $\frac{15 \times 4\frac{3}{4}}{2} = 35.63$ square feet = the area of the other triangle.

$50 + 35.63 = 85.63$ square feet = the area of the trapezium. Ans.

THE CIRCLE.

780. To find the circumference or diameter of a circle:

Rule.—*The circumference of a circle equals the diameter multiplied by 3.1416.*

Rule.—*The diameter of a circle equals the circumference divided by 3.1416.*

EXAMPLE.—What is the circumference of a circle whose diameter is 15 inches?

SOLUTION.—$15 \times 3.1416 = 47.12$ inches, circumference. Ans.

EXAMPLE.—What is the diameter of a circle whose circumference is 65.973 inches?

SOLUTION.—$65.973 \div 3.1416 = 21$ inches diameter. Ans.

781. To find the length of an arc of a circle:

Rule.—*The length of an arc of a circle equals the circumference of the circle of which the arc is a part, multiplied by the number of degrees in the arc, and divided by 360.*

EXAMPLE.—What is the length of an arc of 24°, the radius of the arc being 18 inches?

SOLUTION.—$18 \times 2 = 36$ inches = the diameter of the circle. $36 \times 3.1416 = 113.1$ inches, the circumference of the circle of which the arc is a part.

$$113.1 \times \frac{24}{360} = 7.54 \text{ inches, the length of the arc. Ans.}$$

782. To find the area of a circle:

Rule.—*Square the diameter, and multiply by .7854.*

EXAMPLE.—What is the area of a circle whose diameter is 15 inches?

SOLUTION.—$15^2 = 225$. $225 \times .7854 = 176.72$ sq. in. Ans.

783. Given, the area of a circle to find its diameter:

Rule.—*Divide the area by .7854 and extract the square root of the quotient.*

EXAMPLE.—The area of a circle = 17,671.5 square inches. What is its diameter in feet?

SOLUTION.— $\sqrt{\frac{17,671.5}{.7854}} = 150$ inches.

$\frac{150}{12} = 12\frac{1}{2}$ feet, the diameter. Ans.

784. To find the area of a sector:

Rule.—*Divide the number of degrees in the arc of a sector by 360. Multiply the result by the area of the circle of which the sector is a part.*

EXAMPLE.—The number of degrees in the angle formed by drawing radii from the center of a circle to the extremities of the arc of the circle is 75°. The diameter of the circle is 12 inches; what is the area of the sector?

SOLUTION.— $\frac{75}{360} = \frac{5}{24}$. $12^2 \times .7854 = 113.1$ square inches.

$\frac{5}{24} \times 113.1 = 23.56$ square inches, the area. Ans.

785. To find the area of a segment of a circle:

Rule.—*Draw radii from the center of the circle to the extremities of the arc of the segment; find the area of the sector thus formed, subtract from this the area of the triangle formed by the radii and the chord of the arc of the segment, and the result is the area of the segment.*

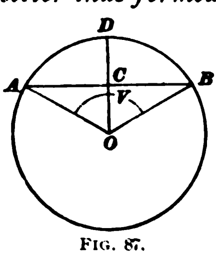


FIG. 87.

In problems requiring the area of the segment, the chord $A B$, Fig. 87, may be given, or the height of the segment $C D$, or the angle V; if any one of these three be given, and the radius of the circle is known, the area can be found.

Also, if any two are given, the radius can be found.

EXAMPLE.—If the diameter of the circle is 10 inches, and the chord of the segment is 7 inches, what is the area of the segment?

SOLUTION.—In the above figure, suppose that the chord $A B = 7$ inches, and the diameter = 10 inches; draw $O A$, $O B$, and a radius perpendicular to the chord, thus dividing $A B$ into two equal parts (see Art. **733**). The triangle $A O B$ is now divided into two equal right-angled triangles, ACO and OCB, in which the hypotenuse = radius, or $\frac{10}{2} = 5$, and one side, $AC = BC = \frac{7}{2}$, or $3\frac{1}{2}$.

$$\text{Sin } COB = \frac{CB}{OB} = \frac{3\frac{1}{2}}{5} = .70000, \text{ and angle } COB = 44° \ 26', \text{ nearly.}$$

Angle $AOB = 44° \ 26' \times 2 = 88° \ 52'$. $CO = OB \times \cos COB = 5 \times .71407 = 3.57''$.

$$\text{Area of sector} = 10^2 \times .7854 \times \frac{88\frac{52}{60}}{360} = 19.39 \text{ sq. in., nearly.}$$

$$\text{Area of triangle} = \frac{7 \times 3.57}{2} = 12.5 \text{ sq. in., nearly.}$$

$19.39 - 12.5 = 6.89$ sq. in., the area of the segment. Ans.

EXAMPLE.—Given, the chord of the arc of a segment = 7 inches, and the height of the segment = 1.43 inches, to find the radius.

SOLUTION.—Suppose that in Fig. 88, $ACBE$ is a circle struck with the required radius, that the chord $AB = 7$ inches, and that the height CD of the segment = 1.43 inches. Join C with A and B, and the right-angled triangle $ADC = BDC$.

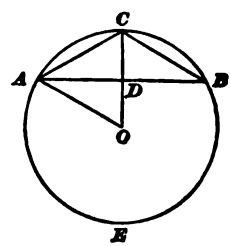


FIG. 88.

$$\text{Tan } CBD = \frac{CD}{BD} = \frac{1.43}{3.5} = .40857.$$

Angle $CBD = 22° \ 13\frac{1}{3}'$, nearly.

Since CBD or its equal CBA is an inscribed angle (see Art. **729**), it is measured by one-half the intercepted arc AC; hence, the number of degrees in arc $AC = 22° \ 13\frac{1}{3}' \times 2 = 44° \ 27'$, or the number of degrees in the angle AOC.

In the right-angled triangle ADO,

$$AO = \frac{\text{side opposite}}{\sin AOD} = \frac{AD}{\sin AOC} = \frac{3.5}{.70029} = 5 \text{ inches, nearly} \quad \text{Ans.}$$

THE ELLIPSE.

786. An **ellipse** is a plane figure bounded by a curved line, to any point of which the sum of the distances from two fixed points within, called the **foci**, is equal to the sum of the distances from the foci to any other point on the curve.

In Fig. 89, let A and B be the foci, and let C and D be any two points on the perimeter. Then, according to the above definition, $AC + CB = AD + DB$, and both these sums are also equal to the major axis FE.

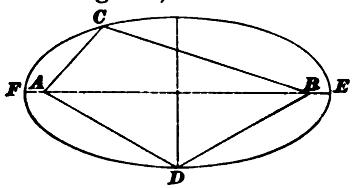


FIG. 89.

787. The long diameter is called the **major axis**; the short diameter, the **minor axis.**

788. To find the perimeter of an ellipse: There is no exact method, but the following is close enough for most cases:

Rule.—*Multiply the major axis by 1.82, and the minor axis by 1.315. The sum of the results will be the perimeter.*

EXAMPLE.—What is the perimeter of an ellipse whose axes are 10 and 4 inches?

SOLUTION.— $10 \times 1.82 = 18.2$ inches. $4 \times 1.315 = 5.26$ inches. $18.2 + 5.26 = 23.46$ inches, or the perimeter. Ans.

789. To find the exact area of an ellipse:

Rule.—*The area of an ellipse is equal to the product of its two diameters multiplied by .7854.*

EXAMPLE.—What is the area of an ellipse whose diameters are 10 and 6 inches?

SOLUTION.— $10 \times 6 \times .7854 = 47.12$ square inches, area. Ans.

EXAMPLES FOR PRACTICE.

1. What is the area in square feet of a rhombus whose base is 84 inches, and whose altitude is 3 feet? Ans. 21 sq. ft.
2. One side of a room is 16 feet long. If the floor contains 240 square feet, what is the length of the other side? Ans. 15 ft.
3. How many square feet in a board 12 feet long, 18 inches wide at one end and 12 inches wide at the other end? Ans. 15 sq. ft.
4. How many square yards of plastering will be required for the ceiling and walls of a room 10×15 feet, and 9 feet high? The room contains one door $3\frac{1}{2} \times 7$ feet, three windows $3\frac{1}{4} \times 6$ feet, and a base-board 8 inches high. Ans. 53.5 sq. yd.

5. What is the area of a triangle whose base is 10 feet 6 inches long, and whose altitude is 18 feet? Ans. 94.5 sq. ft.

6. The area of a triangle is 16 square inches. If the altitude is 4 inches, what does the base measure? Ans. 8 in.

7. The upper side of a trapezium is 16 inches long, and the lower side 14 inches. If the figure be divided into two triangles by a diagonal whose altitudes, drawn from their vertexes to the two given sides as bases, are 17 and 3 inches, respectively, what is the area of the trapezium? Ans. 157 sq. in.

8. Find the area of a circle 2 feet 3 inches in diameter.
Ans. 3.976 sq. ft.

9. A carriage-wheel was observed to make 71$\frac{5}{8}$ turns while going 300 yards. What was its diameter? Ans. 4 ft., nearly.

10. Required, the diameter of a circle whose area is 2,004 square inches. Ans. 50.51 in.

11. Required, the area of a regular pentagon inscribed in a circle whose diameter is 20 inches. Ans. 237.77 sq. in.

12. The number of degrees in the angle formed by drawing radii from the center of a circle to the extremities of the arc of the circle is 84 degrees. The diameter of the circle is 17 inches; what is the area of the sector? Ans. 52.96 sq. in.

13. Given, the chord of the arc of a segment = 24 inches, and the height of the segment = 6.5 inches, to find (*a*) the diameter of the circle, and (*b*) the area of the segment.
Ans. { (*a*) 28.654 in.
(*b*) 109.87 sq. in.

14. (*a*) What is the perimeter of an ellipse whose axes are 15 and 9 inches, and (*b*) what is the area?
Ans. { (*a*) 39.14 in.
(*b*) 106.03 sq. in.

790. Rule.—*The area of any plane figure may be found by dividing it into triangles, quadrilaterals, circles or parts of circles, and ellipses, finding the area of each part separately and adding them together.*

EXAMPLE.—What is the area of a flat circular ring, Fig. 90, whose outside diameter equals 10 inches, and whose inside diameter equals 4 inches?

The area of the large circle = $10^2 \times .7854 =$ 78.54 square inches; the area of the small circle $= 4^2 \times .7854 = 12.57$ square inches.

$78.54 - 12.57 = 65.97$ square inches, or the area. Ans.

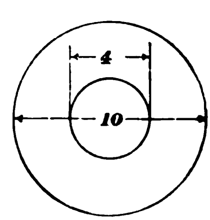


FIG. 90.

EXAMPLE.—What is the exact area in square inches of Fig. 91?

SOLUTION.—Divide the figure into rectangles, triangles, and parts of a circle, as shown by the dotted lines, then the total area equals 8-inch circle − 4-inch circle − segment AB + rectangle $ABGF$ + 2 times the triangle CDE + 2 times the triangle RST + 2 times the rectangle $DESR$ + rectangle $HIKL$ + 2 times the rectangle $LMNP$ + 2 times the triangle MON.

$$8^2 \times .7854 = 50.27 \square''.$$
$$4^2 \times .7854 = 12.57 \square''.$$

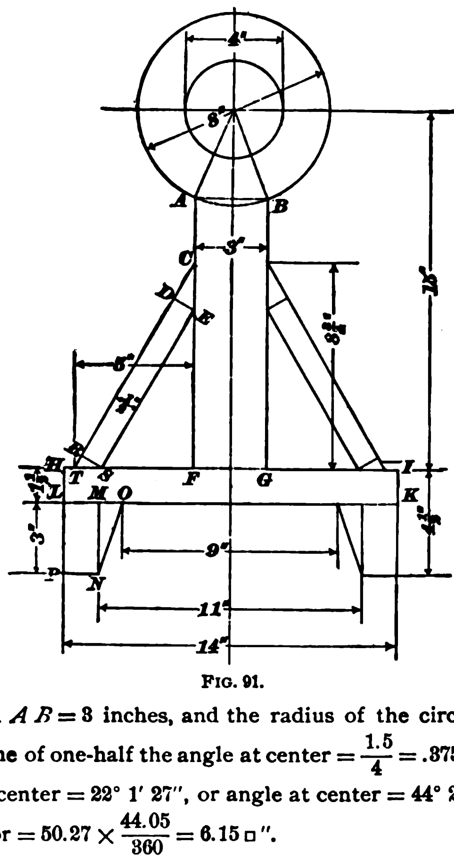


FIG. 91.

The chord $AB = 3$ inches, and the radius of the circle $= 4$ inches; hence, the sine of one-half the angle at center $= \frac{1.5}{4} = .375$, and one-half the angle at center $= 22° \ 1' \ 27''$, or angle at center $= 44° \ 2' \ 54'' = 44.05°$.

Area of sector $= 50.27 \times \frac{44.05}{360} = 6.15 \square''$.

The altitude of the triangle $= 4 \times \cos 22° \ 1' \ 27'' = 3.71$ inches.

The area of the triangle $= \frac{3.71 \times 3}{2} = 5.56 \square''$.

The area of the segment $= 6.15 - 5.56 = 0.59 \square''$.

The area of the rectangle $ABGF = (15 - 3.71) \times 3 = 33.87 \square''$.

In the triangle CDE, $\tan C = \frac{5}{8\frac{3}{4}} = .57143 = \frac{DE}{CD} = \frac{.5}{CD}$; hence, $CD = \frac{.5}{.57143} = .875$ inch.

The area of the triangle $CDE = \frac{.875 \times .5}{2} = .22 \square''$, nearly.

$.22 \times 2 = .44 \square'' =$ twice the area of the triangle CDE. Since in the triangle RST, RS is perpendicular to CR and TS is perpendicular to CF, the angle $S =$ angle C; hence, $\tan S = .57143 = \frac{RT}{SR} = \frac{RT}{.5}$; therefore, $RT = .57143 \times .5 = .29$ inch, nearly.

Area $RST = \frac{.29 \times .5}{2} = .07 \square''$, nearly.

Twice the area of the triangle $RST = .07 \times 2 = .14 \square''$.

Since $\tan C = .57143$, $C = 29° 44' 42''$.

In the rectangle $DESR$, $DR = CT - (CD + RT)$. But $CT = \frac{5}{\sin 29° 44' 42''} = \frac{5}{.49614} = 10.08$ in.

$CD + RT = .875 + .29 = 1.16$. $DR = 10.08 - 1.16 = 8.92$. $8.92 \div 2 = 4.46 \square'' =$ the area of $DESR$.

Twice the area of the rectangle $DESR = 4.46 \times 2 = 8.92 \square''$.

The area of the rectangle $HIKL = 14 \times 1\frac{1}{2} = 21 \square''$.

The area of the rectangle $LMNP = \left(\frac{14 - 11}{2}\right) \times 3 = 1\frac{1}{2} \times 3 = 4\frac{1}{2} \square''$; and $4\frac{1}{2} \times 2 = 9 \square''$.

The area of the triangle $MON = \left(\frac{11 - 9}{2} \times 3\right) \div 2 = 1.5 \square''$.

Twice the area of the triangle $MON = 1.5$ inches $\times 2 = 3 \square''$.

Then, $50.27 + 33.87 + 0.44 + 0.14 + 8.92 + 21 + 9 + 3 = 126.64 \square''$. $12.57 + 0.59 = 13.16 \square''$. $126.64 - 13.16 = 113.48 \square''$.

Therefore, the area of the figure $= 113.48 \square''$. Ans.

THE MENSURATION OF SOLIDS.

791. A **solid,** or body, has three dimensions: length, breadth, and thickness. The sides which enclose it are called the **faces,** and their intersections are called **edges.**

792. The **entire surface** of a solid is the area of the whole outside of the solid, including the ends.

793. The **convex surface** of a solid is the same as the entire surface, except that the areas of the ends are not included.

794. The **volume** of a solid is expressed by the number of times it will contain another volume, called the unit of volume. Instead of the word *volume*, the expression **cubical contents** is frequently used.

THE PRISM AND CYLINDER.

795. A **prism** is a solid whose ends are equal polygons and parallel to each other, and whose sides are parallelograms.

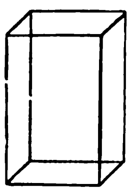


FIG. 92.

796. A **parallelopipedon,** Fig. 92, is a prism whose bases (ends) are parallelograms.

797. A **cube,** Fig. 93, is a parallelopipedon whose faces and ends are squares.

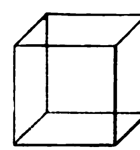


FIG. 93.

798. The cube, whose edges are equal to the unit of length, is taken as the unit of volume when finding the volume of a solid.

Thus, if the unit of length is 1 inch, the unit of volume will be the cube whose edges measure 1 inch, or 1 *cubic inch;* and the number of cubic inches the solid contains will be its volume. If the unit of length is 1 foot, the unit of volume will be one *cubic foot*, etc. Cubic inch, cubic foot, and cubic yard are abbreviated to cu. in., cu. ft., and cu. yd., respectively.

799. Prisms take their names from their bases. Thus, a *triangular prism* is one whose bases are triangles; a *pentagonal prism* is one whose bases are pentagons, etc.

800. A **cylinder,** Fig. 94, is a round body of uniform diameter with circles for its ends.

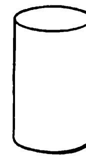


FIG. 94.

801. A **right prism,** or **right cylinder,** is one whose center line (axis) is perpendicular to its base. In this subject all of the solids will be considered as having their center lines perpendicular to their bases.

802. The **altitude** of a prism or cylinder is the perpendicular distance between its two ends.

803. To find the area of the convex surface of any right prism, or right cylinder:

Rule.—*Multiply the perimeter of the base by the altitude.*

EXAMPLE.—In a right prism whose base is a square, one side of which is 9 inches, and whose altitude is 16 inches, what is its convex area?

SOLUTION.— $9 \times 4 = 36$ = the perimeter of the base.
$36 \times 16 = 576$ □', or the convex area. Ans.

To find the entire area, add the areas of the two ends to the convex area:

EXAMPLE.—What is the entire area of the parallelopipedon mentioned in the last question?

SOLUTION.—The area of one end $= 9^2 = 81$ □'. $81 \times 2 = 162$ □', or the area of both ends. $576 + 162 = 738$ □', the entire area of the parallelopipedon. Ans.

EXAMPLE.—What is the entire area of a right cylinder whose base is 16 inches in diameter, and whose altitude is 24 inches?

SOLUTION.— $16 \times 3.1416 = 50.27$ inches, or the perimeter (circumference) of the base. $50.27 \times 24 = 1{,}206.48$ □', the convex area.
$16^2 \times .7854 \times 2 = 402.12$ □', the area of the ends.
$1{,}206.48 + 402.12 = 1{,}608.6$ □', the entire area. Ans.

804. To find the volume of a right prism, or cylinder:

Rule.—*The volume of any right prism or cylinder equals the area of the base multiplied by the altitude.*

If the given prism is a cube, the three dimensions are all equal, and the volume equals the cube of one of the edges.

EXAMPLE.—What is the volume of a rectangular prism whose base is 6×4 inches, and whose altitude is 12 inches?

SOLUTION.—The base of a rectangular prism is a rectangle. Hence, $6 \times 4 = 24$ □′, the area of the base. $24 \times 12 = 288$ cubic inches, or the volume. Ans.

EXAMPLE.—What is the volume of a cube whose edge is 9 inches?

SOLUTION.— $9^3 = 9 \times 9 \times 9 = 729$ cubic inches, the volume. Ans.

EXAMPLE.—What is the volume of a cylinder whose base is 7 inches in diameter, and whose altitude is 11 inches?

SOLUTION.— $7^2 \times .7854 = 38.48$ □′, the area of the base. $38.48 \times 11 = 423.28$ cubic inches, the volume. Ans.

THE PYRAMID AND CONE.

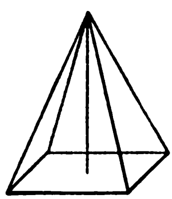


FIG. 95.

805. A **pyramid**, Fig. 95, is a solid whose base is a polygon, and whose sides are triangles uniting at a common point, called the **vertex.**

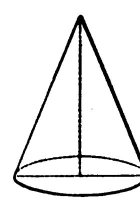


FIG. 96.

806. A **cone**, Fig. 96, is a solid whose base is a circle, and whose convex surface tapers uniformly to a point called the **vertex.**

807. The **altitude** of a pyramid or cone is the perpendicular distance from the vertex to the base.

808. The **slant height** of a *pyramid* is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height of a *cone* is any straight line drawn from the vertex to the circumference of the base.

809. To find the area of a right pyramid or right cone:

Rule.—*The convex area of a right pyramid or cone equals the perimeter of the base multiplied by one-half the slant height.*

EXAMPLE.—What is the convex area of a pentagonal pyramid, if one side of the base measures 6 inches, and the slant height equals 14 inches?

SOLUTION.—The base of a pentagonal pyramid is a pentagon, and, consequently, it has five sides.

$6 \times 5 = 30$ inches, or the perimeter of the base.

$30 \times \frac{14}{2} = 210 \square''$, or the convex area. Ans.

EXAMPLE.—What is the entire area of a right cone whose slant height is 17 inches, and whose base is 8 inches in diameter?

SOLUTION.— $8 \times 3.1416 = 25.1328$ inches, the perimeter.

$25.1328 \times \frac{17}{2} = 213.63 \square''$, the convex area.

$8^2 \times .7854 = \underline{50.27} \square''$, the area of base.

sum $= 263.90 \square'' =$ the entire area. Ans.

810. To find the volume of a right pyramid or cone:

Rule.—*The volume of a right pyramid or cone equals the area of the base multiplied by one-third of the altitude.*

EXAMPLE.—What is the volume of a triangular pyramid, one edge of whose base measures 6 inches, and whose altitude is 8 inches?

SOLUTION.—Draw the base as shown in Fig. 97; it will be an equilateral triangle, all of whose sides are 6 inches long.

Draw a perpendicular, BD, from the vertex to the base; it will divide the base into two equal parts, and will be the altitude of the triangle.

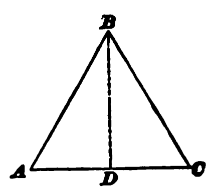


FIG. 97.

In the right-angled triangle BDA, the hypotenuse $BA = 6$ inches, and side $AD = \frac{AC}{2} = \frac{6}{2} = 3$ inches, to find the other side:

$BD = \sqrt{6^2 - 3^2} = 5.2$ inches, nearly.

Area of $BAC = \frac{6 \times 5.2}{2} = 15.6 \square''$, the area of the base.

$15.6 \times \frac{8}{3} = 41.6$ cubic inches, the volume. Ans.

EXAMPLE.—What is the volume of a cone whose altitude is 18 inches, and whose base is 14 inches in diameter?

SOLUTION.— $14^2 \times .7854 = 153.94 \square''$, the area of the base.

$153.94 \times \frac{18}{3} = 923.64$ cubic inches, the volume. Ans.

THE FRUSTUM OF A PYRAMID OR CONE.

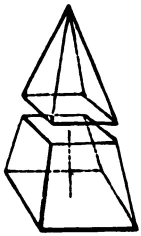


FIG. 98.

811. If a pyramid be cut by a plane parallel to the base, as in Fig. 98, so as to form two parts, the lower part is called the **frustum** of the pyramid.

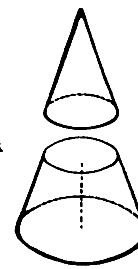


FIG. 99.

812. If a cone be cut in a similar manner, as in Fig. 99, the lower part is called the **frustum** of the cone.

813. The upper end of the frustum of a pyramid or cone is called the **upper base,** and the lower end the **lower base.** The **altitude** of a frustum is the perpendicular distance between the bases.

814. To find the convex area of a frustum of a right pyramid or right cone:

Rule.—*The convex area of a frustum of a right pyramid or right cone equals one-half the sum of the perimeters of its bases multiplied by the slant height of the frustum.*

EXAMPLE.—Given, the frustum of a triangular pyramid, in which one side of the lower base measures 10 inches, one side of the upper base measures 6 inches, and whose slant height is 9 inches ; find the convex area.

SOLUTION.— 10 inches $\times$ 3 = 30 inches, the perimeter of the lower base.

6 inches $\times$ 3 = 18 inches, the perimeter of the upper base.

$\frac{30 + 18}{2} = 24$ inches, or one-half the sum of the perimeters of the bases. $24 \times 9 = 216 \square''$, the convex area. Ans.

EXAMPLE.—If the diameters of the two bases of a frustum of a cone are 12 inches and 8 inches, respectively, and the slant height is 12 inches, what is the entire area of the frustum?

SOLUTION.— $\frac{(12 \times 3.1416) + (8 \times 3.1416)}{2} \times 12 = 376.99$ □", the area of the convex surface.

$8^2 \times .7854 = 50.27$ □".
$12^2 \times .7854 = 113.1$ □".

$113.1 + 50.27 = 163.37$ □", the area of the two ends.

$376.99 + 163.37 = 540.36$ □", the entire area of the frustum. Ans.

815. To find the volume of the frustum of a pyramid or cone:

Rule.—*Add the areas of the upper base, the lower base, and the square root of the product of the areas of the two bases; multiply this sum by one-third of the altitude.*

EXAMPLE.—Given, a frustum of a hexagonal pyramid, each edge of the lower base measuring 8 inches, and each edge of the upper base 5 inches, and whose altitude is 14 inches; what is its volume?

SOLUTION.—A hexagonal pyramid is one whose base is a hexagon, or six-sided polygon. Divide the base into 6 equal triangles, as shown in Fig. 100. To find the altitude of one of the triangles, proceed as follows:

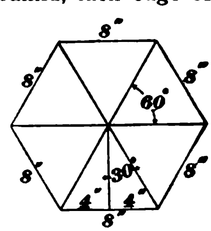


FIG. 100.

The angle at the vertex of one of the triangles will be $\frac{360}{6} = 60°$, and the angle on each side of the perpendicular to the base (or altitude) will be $\frac{60}{2} = 30°$.

The altitude $= \frac{4}{\tan 30°} = \frac{4}{.57735} = 6.93$ inches, nearly.

The area of the triangle $= \frac{8 \times 6.93}{2} = 27.72$ □".

$27.72 \times 6 = 166.32$ □" = the area of the hexagon, or the area of the lower base.

In a similar way, find the area of the upper base to be 64.97 □". Then, applying the rule, $166.32 + 64.97 + \sqrt{166.32 \times 64.97} = 166.32 + 64.97 + 103.95 = 335.24$.

$335.24 \times \frac{14}{3} = 1{,}564.45$ cubic inches = the volume. Ans.

EXAMPLE.—What is the volume of a frustum of a cone whose upper base is 8 inches, the lower base is 12 inches in diameter, and whose altitude is 15 inches?

SOLUTION.—The area of the upper base $= 8^2 \times .7854 = 50.27 \square''$. The area of the lower base $= 12^2 \times .7854 = 113.1 \square''$.

The square root of their product $= \sqrt{50.27 \times 113.1} = 75.4$.
$50.27 + 113.1 + 75.4 = 238.77$.

$238.77 \times \frac{15}{3} = 1{,}193.85$ cubic inches, the volume. Ans.

THE SPHERE.

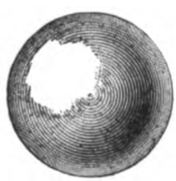


FIG. 101.

816. A **sphere**, Fig. 101, is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center.

The word **ball** is commonly used instead of sphere.

817. To find the area of the surface of a sphere:

Rule.—*The area of the surface of a sphere equals the square of the diameter multiplied by 3.1416.*

EXAMPLE.—What is the area of the surface of a sphere whose diameter is 14 inches?

SOLUTION.—

$$14^2 \times 3.1416 = 14 \times 14 \times 3.1416 = 615.75 \square'', \text{ the area. Ans.}$$

818. To find the volume of a sphere:

Rule.—*The volume of a sphere equals the cube of the diameter multiplied by .5236.*

EXAMPLE.—What is the weight of a lead cannon ball 12 inches in diameter, a cubic inch of lead weighing .41 pound?

SOLUTION.—$12 \times 12 \times 12 \times .5236 = 904.78$ cubic inches, or the volume of the ball. $904.78 \times .41 = 370.96$ pounds. Ans.

819. If any solid be sliced in pieces, whose adjacent surfaces are flat, any piece is called a **plane section** of the solid.

Plane sections are divided into three classes: Longitudinal sections, cross-sections, and right sections. A **longitudinal section** is any plane section taken lengthwise through the solid. Any other plane section is called a **cross-section.** If the surface exposed by taking a plane section of a solid is perpendicular to the center line of the solid, the section is called a **right section.** The surface exposed by any longitudinal section of a cylinder is a rectangle. The surface exposed by a right section of a cube is a square; of a cylinder or cone, a circle; an oblique cross-section of a cylinder is an ellipse. The lower half of a right section of a cone or pyramid is called a frustum of the cone or pyramid.

THE CYLINDRICAL RING.

820. To find the convex area of a cylindrical ring:

A **cylindrical ring** is a cylinder bent to a circle. The **altitude** of the cylinder before bending is the same as the length of the dotted center line *D*. Fig. 102.

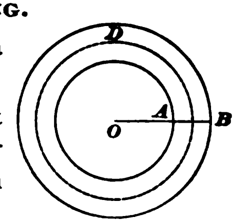


FIG 102.

821. The **base** will correspond to a cross-section on the line *A B* drawn from the center *O*. Hence, to find the convex area, multiply the circumference of an imaginary cross-section on the line *A B* by the length of the center line *D*.

EXAMPLE.—If the outside diameter of the ring is 12 inches, and the inside diameter is 8 inches, what is its convex area?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12+8}{2} = 10$, and $10 \times 3.1416 = 31.416$ inches, the length of the center line.

The radius of the inner circle is 4 inches, of the outside circle 6 inches; therefore, the diameter of the cross-section on the line *A B* is 2 inches. Then, $2 \times 3.1416 = 6.2832$ inches, and $6.2832 \times 31.416 = 197.4$ □", or the convex area. Ans.

822. To find the volume of a cylindrical ring:

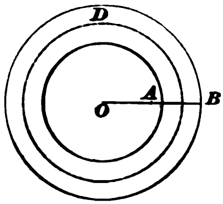


FIG. 103.

The volume will be the same as that of a cylinder whose altitude equals the length of the dotted center line D, Fig. 103, and whose base is the same as a cross-section of the ring on the line A B, drawn from the center O. Hence, to find the volume of a cylindrical ring, multiply the area of an imaginary cross-section on a line A B, by the length of the center line D.

EXAMPLE.—What is the volume of a cylindrical ring whose outside diameter is 12 inches, and whose inside diameter is 8 inches?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12 + 8}{2} = 10$.

$10 \times 3.1416 = 31.416$ inches, the length of the center line.

The radius of the outside circle = 6 inches, of the inside circle = 4 inches; therefore, the diameter of the cross-section on the line *A B* = 2 inches.

Then, $2^2 \times .7854 = 3.1416$ □", the area of the imaginary cross-section.

And $3.1416 \times 31.416 = 98.7$ cubic inches, the volume. Ans.

EXAMPLES FOR PRACTICE.

1. Find the weight of an iron bar 16 feet long and 2 inches in diameter, the weight of iron being taken at .28 pound per cubic inch.
Ans. 168.89 lb.

2. What is the area of the entire surface of a hexagonal prism 12 inches long, each edge of the base being 1 inch long?
Ans. 77.196 sq. in.

3. What is the volume of a triangular pyramid, one edge of whose base measures 3 inches, and whose altitude is 4 inches? Ans. 5.2 cu. in.

4. Find the volume of a cone whose altitude is 12 inches and the circumference of whose base is 31.416 inches. Ans. 314.16 cu. in.

5. A round tank is 8 feet in diameter at the top (inside) and 10 feet at the bottom. If the tank is 12 feet deep, how many gallons will it hold, there being 231 cubic inches in a gallon? Ans. 5,734.2 gallons.

6. Required, the area of the convex surface of the frustum of a square pyramid whose altitude is 16 inches, one side of the lower base being 28 inches long, and of the upper base 10 inches.
Ans. 1,395.18 sq. in.

7. What is the volume of a sphere 30 inches in diameter?
Ans. 14,137.2 cu. in.

8. How many square inches in the surface of the sphere of example 7? Ans. 2,827.44 sq. in.

9. Required, the area of the convex surface of a circular ring, the outside diameter of the ring being 10 inches and the inside diameter $7\frac{1}{2}$ inches. Ans. 107.95 sq. in.

10. Find the cubical contents of the ring in the last example. Ans. 33.734 cu. in.

11. The volume of a sphere is 606.132 cubic inches; required, the area of the convex surface of a cone whose slant height is 10 inches and the diameter of whose base is the same as the diameter of the sphere. Ans. 164.934 sq. in.

12. What is the volume of the frustum of example 6? Ans. 6,208 cu. in.

PROJECTIONS.

823. If perpendiculars be drawn from the extremities of a line, as $A B$, Fig. 104 or Fig. 105, to another line, as $H K$, as shown in the figures, that portion of $H K$ included between the foot of each perpendicular is called the **projection** of $A B$ upon $H K$. Thus, $C D$ is the projection of $A B$ upon $H K$, the point C is the projection of the point A upon $H K$, and the point D is the projection of the point B upon $H K$.

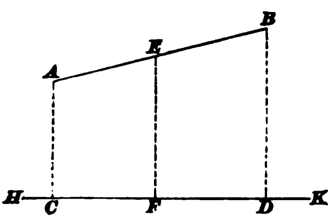


FIG. 104.

The projection of any point of $A B$, as E, can be found by drawing a perpendicular from E to $H K$, and the point where this perpendicular intersects $H K$ is its projection; in this case the point F is the projection of the point E upon $H K$.

It makes no difference whether the line is straight or curved, the method of finding the projection is exactly the same.

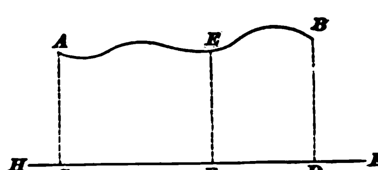


FIG. 105.

From this it is seen that the projection of the hypotenuse of a right-angled triangle upon the base, as $A\ D$ (see Fig. 106), is the cosine of the angle which the hypotenuse makes with the base, and the projection of the hypotenuse upon the side opposite, or $B\ D$, is the sine of the angle which the hypotenuse makes with the base.

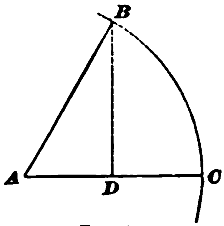


FIG. 106.

In a similar way, a surface is projected upon a flat surface.

Thus, it is desired to project the irregular surface $a\ b\ d\ c$, Fig. 107, upon the flat surface $A\ B\ D\ C$. Draw the lines $a\ a'$, $b\ b'$ perpendicular to the flat surface; join the points a' and b' where these perpendiculars intersect the flat surface $A\ B\ D\ C$ by a straight line $a'\ b'$, and $a'\ b'$ is the projection of $a\ b$ upon $A\ B\ D\ C$. The projection of the surface $a\ b\ d\ c$ upon the plane $A\ B\ D\ C$ is, in this case, the quadrilateral $a'\ b'\ d'\ c'$.

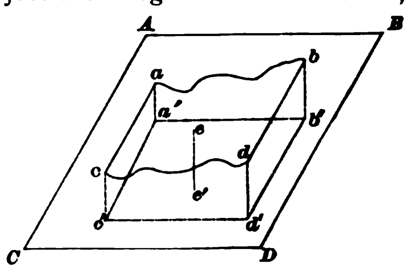


FIG. 107.

SYMMETRICAL AND SIMILAR FIGURES.

824. An **axis of symmetry** is any line so drawn that, if the part of the figure on one side of the line be folded on this line, it will coincide exactly with the other part, point for point and line for line. Thus, in Fig. 108, if the upper half be folded over on the diameter $C\ D$, it will coincide exactly with the lower half; also, if the part on the right of the diameter $A\ B$ be folded over on $A\ B$, it will coincide exactly with the part on the left of this line.

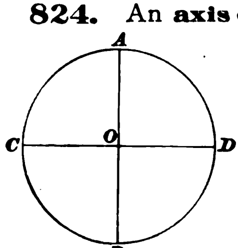


FIG. 108.

It is evident from the above that a circle may have any number of axes of symmetry. In certain cases, however, a figure may be symmetrical with regard to only one axis. Thus, the isosceles triangle $A B C$, Fig. 109, is symmetrical with regard to the axis $B D$, because the part $B C D$ would coincide with the part $B A D$, if folded over on the line $B D$; but no other axis of symmetry could be drawn. A rectangle has two axes of symmetry at right angles to each other. A hexagon has six axes of symmetry.

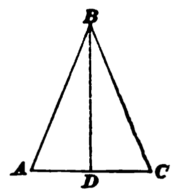


FIG. 109.

825. Similar figures are those which are alike in form. As in the case of triangles, which have been considered, two figures, to be similar, must have their corresponding sides in proportion, and the angles of one equal to the corresponding angles of the other. Any two circles are similar.

826. The areas of two similar figures are to each other as the squares of any one dimension. Thus, a parallelogram 10 inches long and 4 inches wide contains 40 square inches. A similar parallelogram 20 inches long would be 8 inches wide, and would contain 160 square inches, while the two areas would be to each other as the squares of the corresponding sides of the parallelograms. That is,

$$40 : 160 = 10^2 : 20^2,$$
$$\text{or } 40 : 160 = 4^2 : 8^2.$$

EXAMPLE.—A circle 10 inches in diameter contains 78.54 square inches; what is the area of one 12 inches in diameter?

SOLUTION.—Let x = the area of the larger circle. Then,

$$78.54 : x = 10^2 : 12^2, \text{ or } x = \frac{78.54 \times 144}{100} = 113.0976 \text{ sq. in. Ans.}$$

827. The cubical contents (and weights) of **similar solids** are to each other as the *cubes* of any one dimension.

EXAMPLE.—If a cast iron ball 9 inches in diameter weighs 100 pounds, what would one 15 inches in diameter weigh?

SOLUTION.—$100 : x = 9^3 : 15^3$, or $x = \frac{100 \times 3,375}{729} = 462.96$ pounds, the weight of the larger ball. Ans.

EXAMPLE.—A regular hexagon has sides 5" long; how much greater will the area of another regular hexagon be whose sides are 30" long?

SOLUTION.—$30 \div 5 = 6$, or the length of a side of a 30" hexagon is 6 times as great as the length of a side of a 5" hexagon; the area will be $6^2 = 36$ times as great. Ans.

This example may also be solved by letting 1 represent the area of the 5" hexagon. Then, $1 : x = 5^2 : 30^2$, or $x = \frac{900}{25} = 36$.

GASES MET WITH IN MINES.

CHEMISTRY.

COMPOSITION OF MATTER.

828. Chemistry is that branch of science which treats of the composition of substances and the alterations they undergo in their composition by a change in the kind, number, and relative position of their atoms.

829. Mass and Volume.—The mass of a body is the amount of matter contained in it.

The volume of a body is the space which it occupies. If a body of irregular shape be plunged into a cylindrical jar of water, the rise of the water in the jar, multiplied by the area of its cross-section, will give the exact volume of the body. *Volume is always equal to displacement.*

830. Density is compactness of mass, and has reference to the amount of matter in a given volume of a body. Thus, there is more matter in a cubic foot of iron than in a cubic foot of water; therefore, we say iron is more dense than water; likewise, carbonic acid gas is more dense than air.

831. Specific Gravity.—The specific gravity of any body whatever—solid, liquid, or gas—is the measure of its density. And, in order to measure anything, we must have a standard, or unit, of measure. The standard, or unit, by which we measure the density of all solids and liquids alike is water. In like manner, the unit of measure for all mine gases is air. The chemist uses hydrogen gas as his unit of measure for gases.

Our units of measure, then, are as follows:

For solids and liquids, 62.5 lb. = weight 1 cu. ft. water.

For gases, .0766 lb. = weight 1 cu. ft. air (temperature, 60° F.; barometer, 30″).

For example, if we wish to measure the density of iron, we must first know the weight of 1 cubic foot of the iron, and we then find how many times our unit of measure is contained in this weight, which will give us the density (specific gravity) of the iron. Thus, we know the weight of a certain kind of iron is 480 pounds per cubic foot, and we wish to determine its specific gravity. Applying our measure of the density of solids, we find that $\frac{480}{62.5} = 7.68$ is the specific gravity of this iron.

The student must notice carefully that the specific gravity of a body is always the ratio between the weights of equal volumes of the body and of the unit or standard. For this reason, if we take any equal volume of the body and of the unit, or standard, and divide the weight of the one by the weight of the other, we will obtain the same ratio or specific gravity. Now, if we take any irregular piece of coal or other substance, and having first weighed it in the air, we then weigh this same piece of coal in water, the coal will be buoyed up by the weight of the water which it displaces. Hence, the amount the coal loses, when weighed in water, is the same as the weight of its own volume of water. Therefore, it is evident that if we divide the weight of any solid when weighed in air by the loss of weight when weighed in water, we shall obtain the same ratio, which is the specific gravity of the substance.

From the foregoing, we have the following rule to find the specific gravity of any solid:

Rule.—*Divide the weight of the substance in air by the difference between its weight in air and its weight in water; the quotient will be the specific gravity of the substance.*

Let W = weight of the substance in air;

W_1 = weight of the substance in water;

Sp. Gr. = specific gravity of the substance.

Then, $$\text{Sp. Gr.} = \frac{W}{W - W_1} \qquad \textbf{(1.)}$$

EXAMPLE.—The weight of a piece of coal in the air is 7.62 lb.; when weighed in water, it weighs only 1.62 lb. What is its specific gravity?

SOLUTION.— $\frac{7.62}{(7.62 - 1.62)} = \frac{7.62}{6} = 1.27$. Ans.

Or, if we know the weight of 1 cubic foot of a substance—solid, liquid, or gas—and we divide its weight per cubic foot by our unit of measure, the result will be the same as the specific gravity. Thus, we have the following rule for finding the specific gravity of any solid, liquid, or gas when the weight per cubic foot is given.

Rule.—(*a*) ***For Solids or Liquids.***—*Divide the weight per cubic foot of the solid or liquid by the unit weight of the standard (weight of 1 cubic foot of water, 62.5 pounds); the quotient will be the specific gravity of the solid or liquid.*

(*b*) ***For Gases.***—*Divide the weight per cubic foot of the gas by the unit weight of the standard for gases (weight of 1 cubic foot of air, temperature 60° F., barometer 30 inches); the quotient will be the specific gravity of the gas.*

Let w = weight of 1 cubic foot of the solid, liquid, or gas. Then, we have

(*a*) For solids or liquids,

$$\text{Sp. Gr.} = \frac{w}{62.5}. \qquad \textbf{(2.)}$$

(*b*) For gases,

$$\text{Sp. Gr.} = \frac{w}{.0766}. \qquad \textbf{(3.)}$$

EXAMPLE.—Take the weight of a cubic foot of mercury as 850 pounds (at normal temperature and pressure), and find its specific gravity.

SOLUTION.— $\frac{850}{62.5} = 13.6$. Ans.

EXAMPLE.—If the weight of a cubic foot of carbonic acid gas at a temperature of 60° F. and a pressure of 30" of mercury is .117129 pound, what is its specific gravity?

SOLUTION.— $\frac{.117129}{.0766} = 1.5291$. Ans.

832. To make plain the true relation of the terms mass, volume, density, and specific gravity, let us suppose we have 1,000 cubic feet of air at the ordinary atmospheric pressure. If this pressure is increased to two atmospheres, the volume will be reduced to one-half of what it was. The mass, however, will not be changed, because the quantity of matter is still the same; but the density of the air is doubled, and the specific gravity is doubled, since each cubic foot of air contains twice the mass that it contained before compression, while the reduced volume of 500 cubic feet of air contains the same mass that was contained in the 1,000 cubic feet.

The practical use to which the specific gravity of a body is applied is to calculate the weight of a given volume of the substance. For example, the weight of a cubic foot of water is 62.5 pounds, and if the specific gravity of a sample of bituminous coal is 1.27, then the weight of a cubic foot of bituminous coal will be $62.5 \times 1.27 = 79.375$ pounds.

Rule.—(*a*) ***For Solids or Liquids.***—*Multiply the weight of one cubic foot of water (62.5) by the specific gravity of the solid or liquid; the product will be the weight of one cubic foot of the solid or liquid.*

(*b*) ***For Gases.***—*Multiply the weight of one cubic foot of air (.0766 pound), temperature 60° F., barometer 30", by the specific gravity of the gas; the product will be the weight of one cubic foot of the gas.*

(*a*) For solids or liquids,

$$w = 62.5 \times \text{Sp. Gr.} \quad \textbf{(4.)}$$

(*b*) For gases,

$$w = .0766 \times \text{Sp. Gr.} \quad \textbf{(5.)}$$

EXAMPLES FOR PRACTICE.

1. What is the weight of a cubic foot of anthracite coal having a specific gravity of 1.55? Ans. 96.875 pounds.

2. Find the weight of 100 cubic yards of earth having a specific gravity of 1.75. Ans. 147.656 tons.

3. What is the weight of 200 cubic feet of carbonic acid gas at a

temperature of 60° F. and a barometer pressure of 30 inches, the specific gravity of the gas being 1.5291 (see formula **5**)?

Ans. 23.4258 pounds.

4. Find the weight of 500 cubic feet of marsh-gas at a temperature of 60° F. and a pressure due to 30 inches of barometer, the gas having a specific gravity of 0.559. Ans. 21.41 pounds, nearly.

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833. Matter.—Matter is the substance of which the universe consists. It is indestructible and subject to changes of form under different conditions of heat and pressure; therefore, we find it assuming all the three forms common to matter; namely, the gaseous, liquid, and solid. In all the conditions in which we find matter, it consists of atoms and molecules.

834. Atoms and Molecules.—*Atoms.*—An atom is the smallest conceivable division of matter; and, hence, an atom is always simple in its character.

Molecules.—A molecule is formed by the chemical union of two or more atoms. The atoms composing a molecule may be *like* or *unlike;* and, hence, the molecule may be either simple or compound.

The force that binds *atoms* together to form a molecule is a chemical force, which we call *affinity.*

The *force* that binds *molecules* together to form *mass* is a molecular force, which we call *attraction.*

Affinity binds *atoms* together.

Attraction unites *molecules.*

835. Elements.—An elementary body consists of a simple substance that can not be analyzed or reduced to parts that have other properties than those peculiar to itself. An element is a substance, or form of matter, composed wholly of *like* atoms. Thus, *hydrogen* is an element, because it is composed only of hydrogen atoms. For the same reason, oxygen, nitrogen, carbon, iron, lead, silver, gold, etc., are all elements. Table 17 comprises the most of the elements now known.

836. Compounds.—Any substance or form of matter that is composed of *unlike* atoms is a compound. Two classes of compounds exist, viz.:

(*a*) **Chemical compounds,** in which the combining atoms unite in definite, fixed proportions, according to chemical laws, which give to the atoms of each element certain combining powers. For example, water is a chemical compound, being always formed by the union of *two* atoms of hydrogen to *one* atom of oxygen.

In like manner, when *one* atom of carbon unites with *one* atom of oxygen, carbonic oxide gas is formed; but when *one* atom of carbon unites with *two* atoms of oxygen, carbonic acid gas is produced. These two gases have very different properties.

Again, when *one* atom of carbon unites with *four* atoms of hydrogen, marsh-gas results; but when *two* atoms of carbon unite with the *four* atoms of hydrogen, olefiant gas (ethene) is produced.

These are all examples of chemical compounds, as are also salt, blue vitriol, nitric acid, etc., for they are all formed by the chemical union of dissimilar atoms.

(*b*) **Mechanical mixtures** are not *true compounds*, as they are composed more properly of *unlike molecules*, in place of *unlike atoms*. The molecules of the different substances forming the mixture may be present in any proportions, and the mixture will have properties varying with the proportions of the ingredients.

The atmosphere about us is a good example of a mechanical mixture, as we shall see later, for it consists principally of oxygen and nitrogen gases, mixed in a free state (having no chemical bond of union). The proportion of these two gases in the atmosphere is quite constant, being approximately *one* of oxygen to *four* of nitrogen.

Solutions of different salts in water are examples of mechanical mixtures; the strength of the solution or mixture will vary with the amount of salt dissolved.

TABLE 17.

Name.	Symbol.	Atomic Weight.	Name.	Symbol.	Atomic Weight.
* **Aluminium,**	*Al.*	27.4	**Mercury,**	*Hg.*	200.0
* Antimony,	*Sb.*	122.0	* Molybdenum,	*Mo.*	96.0
* Arsenic,	*As.*	75.0	Nickel,	*Ni.*	58.0
Barium,	*Ba.*	137.0	Niobium (Columbium, Cb.),	*Nb.*	94.0
Beryllium (Glucinum, Gl.),	*Be.*	9.2	***Nitrogen,***	*N.*	14.0
Bismuth,	*Bi.*	210.0	Osmium,	*Os.*	200.0
Boron,	*B.*	11.0	***Oxygen,***	*O.*	16.0
Bromine,	*Br.*	80.0	Palladium,	*Pd.*	106.0
Cadmium,	*Cd.*	112.0	* *Phosphorus,*	*P.*	31.0
Cæsium,	*Cs.*	133.0	Platinum,	*Pt.*	197.4
Calcium,	*Ca.*	40.0	**Potassium,**	*K.*	39.1
* ***Carbon,***	*C.*	12.0	Rhodium,	*Ro.*	104.0
Cerium,	*Ce.*	91.3	Rubidium,	*Rb.*	85.4
Chlorine,	*Cl.*	35.5	Ruthenium,	*Ru.*	104.0
* Chromium,	*Cr.*	52.2	Samarium,	*Sm.*	150.0
Cobalt,	*Co.*	60.0	Scandium,	*Sc.*	44.9
* Columbium (Niobium, Nb.),	*Cb.*	94.0	* Selenium,	*Se.*	79.0
Copper,	*Cu.*	63.4	* ***Silicon,***	*Si.*	28.0
Decipium,	*Dp.*	159.0	**Silver,**	*Ag.*	108.0
Didymium,	*D.*	95.0	**Sodium,**	*Na.*	23.0
Erbium,	*E.*	112.6	Strontium,	*Sr.*	88.0
Fluorine,	*F.*	19.0	* ***Sulphur,***	*S.*	32.0
Gallium,	*Ga.*	69.8	* Tantalum,	*Ta.*	182.0
Glucinum (Beryllium, Be.),	*Gl.*	9.2	* Tellurium,	*Te.*	128.0
* **Gold,**	*Au.*	197.0	Terbium,	*Tb.*	75.4
* **Hydrogen,**	*H.*	1.0	Thallium,	*Tl.*	204.0
Indium,	*In.*	113.4	Thorium,	*Th.*	118.4
Iodine,	*I.*	127.0	* Tin,	*Sn.*	118.0
Iridium,	*Ir.*	198.0	* Titanium,	*Ti.*	50.0
Iron,	*Fe.*	56.0	* Tungsten,	*W.*	184.0
Lanthanum,	*La.*	92.0	* Uranium,	*U.*	120.0
Lead,	*Pb.*	207.0	Vanadium,	*V.*	51.3
Lithium,	*Li.*	7.0	Ytterbium,	*Yb.*	173.0
Magnesium,	*Mg.*	24.0	Yttrium,	*Y.*	89.0
* Manganese,	*Mn.*	55.0	Zinc,	*Zn.*	65.0
			Zirconium,	*Zr.*	89.6

* Sometimes basic, sometimes acid.

NOTE.—Heavy-faced type indicates the elements of most importance to the student of this subject. Basic elements are printed in common type; acid elements in italics

837. Dissociation.—This part of our subject would not be complete without some particular reference to the mode of union and disunion of atoms. The affinities which exist between the atoms of the different elements vary very much. In some cases, the affinity is so slight as to render the compound very unstable, and dissociation of the atoms will ensue from the least cause. Examples of this are the various fulminators and some of the detonating explosives. On the other hand, the affinities between atoms of certain other elements, as oxygen and hydrogen, or oxygen and carbon, are very strong, and their union is very apt to be accompanied with the manifestation of considerable energy, either in the form of heat or mechanical work.

In this development of energy, incident to the dissociation of atoms, lies one of the most important principles in the chemistry of mining.

838. Atomic Weights.—By this we mean the weights of the atoms of all the elementary bodies. These weights are expressed in terms of the weight of the hydrogen atom; that is, the lightest known element in nature; for example, it is said the atomic weight of nitrogen is 14, meaning to say that an atom of nitrogen is 14 times as heavy as an atom of hydrogen, and so on with the atomic weights of the other elements.

Table 17 gives the most of the known elements, with their respective symbols and atomic weights.

Atomic weight means only *relative weight*. It does not mean pounds, or ounces, or grammes, or any other denomination in particular. For example, since, from analysis, we know that water is a chemical compound formed by the union of *two* atoms of hydrogen with *one* atom of oxygen, the **molecular weight** (weight of 1 molecule) of water (H_2O) will be as follows:

Hydrogen (H_2), 2 atoms (2×1) $= 2$
Oxygen (O), 1 atom $= 16$

Water (H_2O), 1 molecule $= 18 =$ molecular weight.

Then, we readily see that hydrogen forms $\frac{2}{18} = \frac{1}{9}$ of the weight of water, and oxygen forms $\frac{16}{18} = \frac{8}{9}$ of the weight of the same.

In the same way, the atomic weight of carbon being 12 and that of hydrogen 1, we have for the weight of a molecule (called *molecular weight*) of marsh-gas (CH_4):

Carbon (C), 1 atom	$= 12$	
Hydrogen (H_4), 4 atoms (4×1)	$= 4$	
Marsh-gas (CH_4), 1 molecule	$= 16$	= molecular weight.

We see, therefore, that the carbon forms $\frac{12}{16} = \frac{3}{4}$, or 75 per cent. by weight of the marsh-gas, and hydrogen forms $\frac{4}{16} = \frac{1}{4}$, or 25 per cent. of the same.

This makes plain that the atomic weights of the various elements are only relative, hydrogen being taken as unity.

EXAMPLES.—(*a*) What per cent. of the weight of carbonic oxide gas is pure oxygen?

SOLUTION.—Carbonic oxide gas (CO) contains one atom of carbon and one atom of oxygen, by weight, carbon 12 parts and oxygen 16 parts (see Table 17), 12 and 16 being the relative weights of the atoms, or the atomic weights, of carbon and oxygen. The molecular weight, therefore, of carbonic oxide gas is $12 + 16 = 28$, of which oxygen forms $\frac{16}{28} = \frac{4}{7} = 57.143$ per cent., nearly. Ans.

(*b*) What weight of carbonic acid gas will be produced in burning 100 pounds of coal containing 90 per cent. of carbon?

SOLUTION.— 90% of 100 = 90 pounds of carbon.

Carbon (C) atomic weight (Table 17) 1 atom	12
Oxygen (O_2) atomic weight (Table 17) 2 atoms (16×2)	32
Carbonic acid gas (CO_2), molecular weight	44

Percentage of carbon $= \frac{12}{44} = \frac{3}{11} = 27.27\%$. Then, 90 pounds of carbon is $\frac{3}{11}$ of the weight of carbonic acid gas formed, and $\frac{1}{11} = \frac{1}{3}$ of $90 = 30$ pounds, and $\frac{11}{11}$ or the whole weight of gas formed $= 30 \times 11 = 330$ pounds. Ans.

(*c*) If the specific gravity of carbonic acid gas is 1.5291, what volume of gas, at a temperature of 60° F., and a barometric pressure equal to 30", will be produced, in the last problem, by burning the 100 pounds of coal, which we found yields 330 pounds of this gas?

SOLUTION.—The specific gravity of the gas being 1.5291, one cubic foot at the temperature and pressure given will weigh $.0766 \times 1.5291 = .117129$ pounds; and 330 pounds of the gas will contain as many cubic feet as $\frac{330}{.117129} = 2{,}817.4$ cubic feet. Ans.

EXAMPLES FOR PRACTICE.

1. What percentage of the weight of marsh-gas (CH_4) is pure hydrogen (a molecule of marsh-gas containing *one* atom of carbon and *four* atoms of hydrogen)? Ans. 25%.

2. If the specific gravity of marsh-gas (Table 19) is 0.559, what weight of hydrogen will be contained in 1,000 cubic feet of this gas, at a temperature of 60° F., barometer 30"? Ans. 10.705 lb., nearly.

3. If all of the hydrogen in example 2 were to unite with oxygen (in the proportion of *two* atoms of hydrogen to *one* atom of oxygen) to form water, what weight of water would be produced? Ans. 96.345 lb.

4. What weight of carbon is contained in 100 cubic feet of carbonic oxide gas (CO), a molecule of this gas containing *one* atom of carbon and *one* atom of oxygen, temperature = 60° F., barometer 30"? Ans. 3.1745 lb.

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839. Symbols.—To facilitate the writing of chemical equations, each of the elements is expressed in writing by a symbol, as given in Table 17. In like manner, a chemical compound is expressed by the symbols of its constituent elements. Thus, water, composed of *two* atoms of hydrogen and *one* atom of oxygen, is expressed by the symbol H_2O. In the same manner, we write CO for carbonic oxide gas, CO_2 for carbonic acid gas, and CH_4 for carbureted hydrogen or marsh-gas; the number of atoms of each element, when more than one, being denoted by the little subscript figure following its symbol. When it is desired to express more than one *molecule*, we write the figure indicating the number before the formula of the molecule. Thus, four molecules of carbonic acid gas are written $4\,CO_2$, and in these four molecules there are *four* atoms of carbon and *eight* atoms of oxygen.

These symbols are, in most cases, the first letter of the name of the element itself. For example, the symbol for oxygen is O, for hydrogen it is H; and such symbols are always capital letters. When the initial letters of the names of different elements are the same, one of the elements is designated by its initial letter; but each of the others has some additional letter to distinguish it, and this extra letter, in each case, is a small letter. Thus, the symbol for carbon is C; for chlorine, Cl; for chromium, Cr; for cadmium, Cd, etc. Others of the elements have for their symbols an abbreviation of their Latin names; as, for example, the symbol for iron is Fe (Latin, *ferrum*), and silver, Ag (Latin, *argentum*). In writing chemical symbols, be careful that you do not use capitals and small letters indiscriminately, for the meaning may thereby be greatly changed. Thus, were we to write CO (carbonic oxide), a deadly gas would be meant, while if it had been made Co, it would represent the chemical symbol for cobalt.

840. Chemical Equations.—An equation expresses equality. We may have a numerical equation; as, for example,

$$2 + 4 = 3 \times 2.$$

The sign of equality divides the two equal members of an equation, always showing that they are equal to each other.

A *chemical equation* is used to show the arrangement and grouping of atoms, *before* and *after* a reaction. We must remember that *matter may be changed in its form, but can not be destroyed;* hence, the grouping of the atoms will be different after the reaction from what it was before the reaction took place; but the number and kind of atoms will be equal before and after such reaction. For example, when sulphuric acid (H_2SO_4) acts upon metallic zinc (Zn), the hydrogen of the acid is replaced by the zinc, and the result is that a salt (sulphate of zinc) is formed and a gas (hydrogen) is set free. This reaction is expressed by the following chemical equation:

$$Zn + H_2SO_4 = ZnSO_4 + 2H.$$

841. Atomic volume is relative volume, as atomic weight is relative weight. It has been ascertained that, with few exceptions, the specific gravities of simple gases, when referred to hydrogen as unity, are equal to their respective atomic weights; as, for example, the density (specific gravity) of nitrogen, referred to hydrogen, is 14; and, referring to Table 17, we see that its atomic weight is, likewise, 14. From the foregoing has been deduced the following law:

First Law of Volume.—(*a*) *Like volumes of simple gases contain the same number of atoms.*

(*b*) *The atoms of all simple gases are of the same size.*

It has been further ascertained that the densities of compound gases are, with few exceptions, equal to one-half of their *molecular* weights; as, for example, the density of carbonic acid gas (CO_2), referred to hydrogen as unity, is 22; its molecular weight (sum of the atomic weights of its elements) is $12 + (2 \times 16) = 44$. (See Table 17.) Hence, we see that its density is one-half of its molecular weight. From this the following law has been deduced:

Second Law of Volume.—*The molecules of compound gases occupy twice the volume of an atom of hydrogen gas.*

The exceptions to these general laws of volume are very few, and not important to our subject. The foregoing rules do not refer to solids or liquids.

These two *laws of volume* make it possible to determine the volume of gases resulting from any given chemical reaction. For example, when carbonic oxide gas burns, it unites with the oxygen of the air according to the equation

$$CO + O = CO_2.$$

But the molecule CO (1 atom C and 1 atom O) is, according to the second law of volume, equal to the size of *two* atoms of hydrogen gas; and, according to the first law of volume, the atom O, with which it unites, is equal in size to *one* atom of hydrogen gas. Hence, the volumes of CO and O, which unite, are to each other as 2 : 1. In the same manner, we find the volume of CO_2 formed is equal to the original volume of CO. In other words, *two* volumes of carbonic oxide gas, mixed with *one* volume of oxygen, and exploded, will form only *two* volumes of carbonic acid gas.

In like manner, when ammonia (NH_3) is decomposed in a tube, by electric sparks, it is found that *two* volumes NH_3 yield *one* volume N and *three* volumes H, or ***four*** volumes of the simple gases.

EXAMPLE.—Determine the volume of carbonic acid gas (CO_2) resulting from the explosion of 500 cubic feet of marsh-gas (CH_4), at equal temperatures and pressures.

SOLUTION.—Assuming that all of the carbon (C) of the marsh-gas is converted into carbonic acid gas (CO_2), we find

1 *molecule* CH_4 *yields* 1 *molecule* CO_2.

By the second law of volume, each of these molecules occupies twice the volume of an atom of hydrogen gas, and they are, therefore, equal to each other. Hence, 500 cubic feet of marsh-gas yield 500 cubic feet of carbonic acid gas, under the assumed conditions (equal temperatures and pressures). Ans.

EXAMPLE.—How many cubic feet of oxygen have been consumed in the formation of the 500 cubic feet of carbonic acid gas of the previous example?

SOLUTION.—*Two atoms* of oxygen are consumed in the formation of each *molecule* of carbonic acid gas (CO_2). These *two* atoms, according to the first law of volume, are of the same size or volume as *two* atoms of *hydrogen* gas; likewise, according to the second law of volume, the *molecule* of carbonic acid gas formed occupies *twice* the volume of an atom of *hydrogen* gas. Hence, the volume of the oxygen consumed is equal to the volume of the gas formed (500 cu. ft.). Ans.

EXAMPLES FOR PRACTICE.

In the following examples, assume constant temperature and pressure:

1. How many cubic feet of oxygen will be consumed in the formation of 100 cubic feet of carbonic oxide gas (CO)? Ans. 50 cu. t.

2. If the hydrogen in 100 cubic feet of ammonia gas were set free, what volume would it make? Ans. 150 cu. ft.

3. The formula for ethene, or olefiant gas, is C_2H_4; what volume of oxygen will be required to convert 100 cubic feet of this gas into CO_2 and H_2O? Ans. 300 cu. ft.

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842. Constitution of Matter.—In order to rightly understand the relation of force to matter, we must consider the latter as made up of minute particles, which we have already termed atoms and molecules.

The union of atoms produces molecules, and the union of molecules produces mass.

The *atoms* forming the molecules of a substance may be *like* or *unlike*. When they are *like*, the substance is *elementary;* when unlike, it is *compound*.

The *molecules* of any homogeneous mass are always alike.

843. Molecular Forces.—The force which unites *atoms* is *affinity;* it is a chemical force.

The *molecules* of all matter are acted upon by two opposite or contrary forces; viz., the force of *attraction* and the force of *repulsion*. The former of these two forces acts to bind the molecules together, the latter to drive them apart. The *attractive* principle or force exists in every molecule of a mass, to draw it towards every other molecule; it is an inherent force, peculiar to all matter to a greater or less extent.

The *repulsive* force existing between the molecules of a mass is what may be termed an *imposed* force. It is not common to the mass, but is an induced or applied force. This repulsive force is largely the result of heat or the temperature of the mass.

844. Heat Unit.—We measure the quantity of heat by what is termed the *thermal*, or *heat*, *unit*. The British thermal unit is the amount of heat which will raise the temperature of one pound of water one degree of the Fahrenheit scale. Table 18 gives, in round numbers, the number of British thermal units produced by the burning of one pound of different solids and gases in oxygen.

TABLE 18.

Substance.	British Thermal Units (per Pound).
Hydrogen gas (H)	62,000
Marsh-gas (CH_4)	23,500
Carbonic oxide gas (CO)	4,300
Anthracite coal	15,230
Bituminous coal	14,400
Coke	12,600
Wood (ordinary)	5,000

AIR AND GASES.

THE ATMOSPHERE.

845. Composition.—An analysis of the atmosphere about us shows it to consist of a *mixture* of oxygen and nitrogen, with varying amounts of carbonic acid gas and ammonia. The oxygen and nitrogen are always free or uncombined, and are present in the proportions given below.

	By Volume.	By Weight.
Nitrogen,	79.3	77
Oxygen,	20.7	23
	100.0	100

The amounts of the other ingredients are changing all the time, due to local causes. Thus, the air of a crowded room, or a mine, or the air in the vicinity of large factories, may show a high percentage of carbonic acid gas; while, again, the air of an open field, just after a shower, may show scarcely a trace of this gas; while the proportions of oxygen and nitrogen will show no practical variation. The proportions of these two elements existing in the atmosphere, by volume, is in the ratio of about four volumes of nitrogen to one volume of oxygen. The oxygen and nitrogen are in a *free* state; that is, they are mechanically *mixed* in this proportion throughout the entire atmosphere, and are not chemically *combined*.

The atmosphere is essential to all animal and vegetable life. Its oxygen, which forms about one-fifth of its volume, enters into a large number of chemical reactions, and supports combustion in every form.

PHYSICAL PROPERTIES OF AIR AND GASES.

846. Weight of Air.—It was supposed by the ancients that air had no weight, and it was not until about the year 1650 that it was proven that air really has weight. A cubic inch of air, under ordinary conditions, weighs .31 grain, nearly. The ratio of the weight of air to water is about

1 : 774; that is, air is only $\frac{1}{774}$ as heavy as water. If a vessel, made of light material, be filled with a gas lighter than air, so that the total weight of the vessel and gas is less than the air which they displace, the vessel will rise. It is on this principle that balloons are made.

Since air has weight, it is evident that the enormous quantity of air that constitutes the atmosphere must exert a considerable pressure upon the earth. This is easily proven by taking a long glass tube closed at one end and filling it with mercury. If the finger be placed over the open end so as to keep the mercury from running out, and the tube be inverted and placed in a glass of mercury, as shown in Fig. 110, the mercury in the tube will fall, then rise, and, after a few oscillations, will come to rest at a height above the top of the mercury in the glass equal to about 30 inches. This height will always be the same under the same atmospheric conditions. Now, if the atmosphere has weight, it must press upon the upper surface of the mercury in the glass with equal intensity upon every square unit, except upon that part of the surface occupied by the tube. In order that there will be equilibrium, the weight of the mercury in the tube must be equal to the pressure of the air upon an area of the upper surface of the mercury in the glass, equal to the area of the inside of the tube. Suppose that the area of the inside of the tube is 1 square inch, then, since mercury is 13.6 times as heavy as water, and a cubic inch of water weighs .03617 pound, the weight of the mercurial column is $.03617 \times 13.6 \times 30 = 14.7574$ pounds.

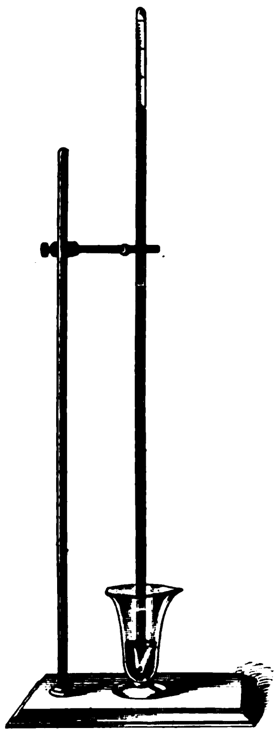

FIG. 110.

The actual height of the mercury is a little less than 30 inches, and the actual weight of a cubic inch of distilled water is a little less than .03617 pound. When these considerations are taken into account, the average weight of the mercurial column at the level of the sea under normal conditions is 14.69 pounds, or, practically, 14.7 pounds. Since this weight, exerted upon 1 square inch of the liquid in the glass, just produced equilibrium, it is plain that the pressure of the outside air is 14.7 pounds upon every square inch of surface.

847. Vacuum.—The space between the upper end of the tube and the upper surface of the mercury is called a *Toricellian vacuum*, or simply a *vacuum*, meaning that it is an entirely empty space, and does not contain any substance, solid, liquid, or gaseous. If there was a gas of some kind there, no matter how small the quantity might be, it would expand, filling the space, and its tension would cause the column of mercury to fall and become shorter, according to the amount of gas or air present. The space is then called a *partial vacuum*. If the mercury fell 1 inch, so that the column was only 29 inches high, we would say, in ordinary language, that there were *29 inches of vacuum*. If it fell 8 inches, we would say that there were 22 inches of vacuum; if it fell 16 inches, we would say that there were 14 inches of vacuum, etc. Hence, when the vacuum-gauge of a condensing-engine shows 26 inches of vacuum, there is enough air in the condenser to produce a pressure of $\frac{30 - 26}{30} \times 14.7 = \frac{4}{30} \times 14.7 = 1.96$ pounds per square inch.

If the tube had been filled with water instead of mercury, the height of the column of water to balance the pressure of the atmosphere would have been $30 \times 13.6 = 408$ inches = 34 feet. This means that if a tube be filled with water, inverted, and placed in a dish of water in a manner similar to the experiment made with the mercury, the height of the column of water would be 34 feet.

848. The **barometer** is an instrument used for measuring the pressure of the atmosphere. There are two kinds in general use—the mercurial barometer and the aneroid barometer. The *mercurial barometer* is shown in Fig. 111. The principle is the same as the inverted tube, shown in Fig. 110. In this case, the tube and cup at the bottom are protected by a brass or iron casing. Near the top of the tube is a graduated scale which can be read to $\frac{1}{1000}$ of an inch by means of a vernier. Attached to the casing is an accurate thermometer for determining the temperature of the outside air at the time the barometric observation is taken. This is necessary, since mercury expands when the temperature is increased, and contracts when the temperature falls; for this reason a standard temperature is assumed, and all barometer readings are reduced to this temperature. This standard temperature is usually taken at 32° F., at which temperature the average height of the mercurial column at sea-level is 30 inches. Another correction is made for the altitude of the place above sea-level, and a third correction for the effects of capillary attraction.

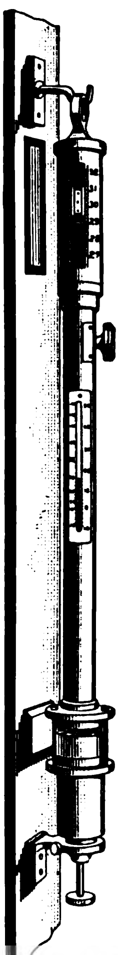

FIG. 111.

In Fig. 112 is shown a cut of an *aneroid barometer*. These instruments are made in various sizes, from the size of a watch up to an 8 or 10 inch face. They consist of a cylindrical box of metal with a top of thin, elastic, corrugated metal. The air is removed from the box. When the atmospheric pressure increases, the top is pressed inwards, and when it is diminished, the top is pressed outwards by its own elasticity, aided by a spring beneath. These movements of the cover are transmitted and multiplied by a combination of delicate levers, which act upon an index-hand, and cause it to move either to the right or left, over a graduated scale. These barometers are self-correcting (compensated) for variations in temperature. They are very portable, occupying but a small space, and are so delicate that they are said to show a difference in the atmospheric pressure when transferred from the table to the floor. The mercurial barometer is the standard. With air, as with water, the lower we get, the greater the pressure, and the higher we get, the less the pressure. At the level of the sea, the height of the mercurial column is about 30 inches; at 5,000 feet above the sea, it is 24.7 inches; at

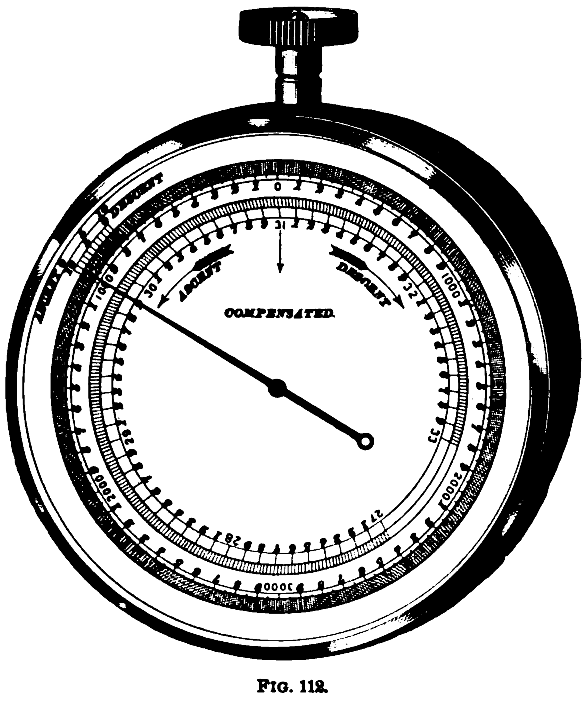

FIG. 112.

10,000 feet above the sea, it is 20.5 inches; at 15,000 feet, it is 16.9 inches; at 3 miles, it is 16.4 inches, and at 6 miles above the sea-level, it is 8.9 inches.

849. Density of Air.—The density and weight of a cubic foot of air vary with the altitude; that is, a cubic foot of air at an elevation of 5,000 feet above the sea-level will not weigh as much as a cubic foot at sea-level. This is proven conclusively by the fact that at a height of $3\frac{1}{2}$ miles the mercurial column measures but 15 inches, indicating that half the weight of the entire atmosphere is below that. It is known that the height of the earth's atmosphere is at least 50 miles; hence, the air just before reaching the limit must be in an exceedingly rarefied state. It is by means of barometers that great heights are measured. The aneroid barometer has the heights marked on the dial, so that they can be read directly. With the mercurial barometer, the heights must be calculated from the reading.

850. Atmospheric Pressure.—The atmospheric pressure is everywhere present, and presses all objects in all directions with equal force. If a book is laid upon the table, the air presses upon it in every direction with an equal average force of 14.7 pounds per square inch. It would seem as though it would take considerable force to raise a book from the table, since, if the size of the book were 8 inches by 5 inches, the pressure upon it is $8 \times 5 \times 14.7 = 588$ pounds; but there is an equal pressure beneath the book to counteract the pressure on the top. It would now seem as though it would require a great force to open the book, since there are two pressures of 588 pounds each, acting in opposite directions, and tending to crush the book; so it would but for the fact that there is a layer of air between each leaf acting upwards and downwards with a pressure of 14.7 pounds per square inch. If two metal plates be made as perfectly smooth and flat as it is possible to get them, and the edge of one be laid upon the edge of the other, so that one may be slid upon the other, and thus exclude the air, it will take an immense force, compared with the weight of the plates, to separate them. This is because the full pressure of 14.7 pounds per square inch is then exerted upon each plate, with no counteracting equal pressure between them.

If a piece of flat glass be laid upon a flat surface that has been previously moistened with water, it will require considerable force to separate them; this is because the water excludes the air between the flat surface and glass, and any attempt to separate these causes a partial vacuum between the glass and the surface, thereby reducing the counter pressure beneath the glass.

851. Tension of Gases.—In Fig. 110 the space above the column of mercury was said to be a vacuum, and that if any gas or air was present it would expand, its tension forcing the column of mercury downwards. If enough gas is admitted to cause the mercury to stand at 15 inches, the tension of the gas is evidently $\frac{14.7}{2} = 7.35$ pounds per square inch, since the pressure of the outside air of 14.7 pounds per square inch balances only 15 inches instead of 30 inches of mercury; that is, it balances only half as much as it would if there were no gas in the tube; therefore, the tension (pressure) of the gas in the tube is 7.35 pounds. If more gas is admitted, until the top of the mercurial column is just level with the mercury in the cup, the gas in the tube has then a tension equal to the outside pressure of the atmosphere. Suppose that the bottom of the tube is fitted with a piston, and that the total length of the inside of the tube is 36 inches. If the piston be shoved upwards so that the space occupied by the gas is 18 inches long instead of 36 inches, the temperature remaining the same as before, it will be found that the tension of the gas within the tube is 29.4 pounds. It will be noticed that the volume occupied by the gas is only half that in the tube before the piston was moved, while the pressure is twice as great, since $14.7 \times 2 = 29.4$ pounds. If the piston be shoved up, so that the space occupied by the gas is only 9 inches instead of 18 inches, the temperature still remaining the same, the pressure will be found to be 58.8 pounds per square inch. The volume has again been reduced one-half, and the pressure increased two times, since $29.4 \times 2 = 58.8$ pounds. The volume now occupied by the gas is 9 inches long, whereas, before the piston was moved, it was 36 inches long; as the tube was assumed to be of uniform diameter throughout its length, the volume is now $\frac{9}{36} = \frac{1}{4}$ of its original volume, and its pressure is $\frac{58.8}{14.7} = 4$ times its original pressure. Moreover, if the temperature of the confined gas remains the same, the pressure and volume will always vary in a similar way. The law which states these effects is called *Mariotte's law.*

852. Mariotte's Law.—*The temperature remaining the same, the volume of a given quantity of gas varies inversely as the pressure.*

The meaning of this is: If the volume of the gas is diminished to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, etc., of its former volume, the tension will be increased 2, 3, 5, etc., times, or, if the outside pressure be increased 2, 3, 5, etc., times, the volume of the gas will be diminished to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, etc., of its original volume, the temperature remaining constant. It also means that if a gas is under a certain pressure, and the pressure is diminished to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{10}$, etc., of its original pressure, that the volume of the confined gas will be increased 2, 4, 10, etc., times—its tension decreasing at the same rate.

Suppose 3 cubic feet of air to be under a pressure of 60 pounds per square inch in a cylinder fitted with a movable piston; then the product of the volume and pressure is $3 \times 60 = 180$. Let the volume be increased to 6 cubic feet, then the pressure will be 30 pounds per square inch, and $30 \times 6 = 180$ as before. Let the volume be increased to 24 cubic feet, it is then $\frac{24}{3} = 8$ times its original volume, and the pressure is $\frac{1}{8}$ of its original pressure, or $60 \times \frac{1}{8} = 7\frac{1}{2}$ pounds, and $24 \times 7\frac{1}{2} = 180$, as in the two preceding cases. It will now be noticed that if a gas be enclosed within a confined space, and allowed to expand without losing any heat, *the product of the pressure and the corresponding volume for one position of the piston is the same as for any other position of the piston.* If the piston was to compress the air, the rule would still hold good.

Let p = pressure for one position of the piston;
p_1 = pressure for any other position of the piston;
v = volume corresponding to the pressure p;
v_1 = volume corresponding to the pressure p_1.

Then, $$p v = p_1 v_1; \qquad (6.)$$

also, $$p_1 = \frac{p v}{v_1}; \qquad (7.)$$

and $$v_1 = \frac{p v}{p_1}. \qquad (8.)$$

Knowing the volume and the pressure for any position of the piston and the volume for any other position, the pressure may be calculated by formula **7**, or if the pressure is known for any other position, the volume may be calculated by formula **8.**

EXAMPLE.—If 1.875 cubic feet of air be under a pressure of 72 pounds per square inch, (*a*) what will be the pressure when the volume is increased to 2 cubic feet? (*b*) to 3 cubic feet? (*c*) to 9 cubic feet?

SOLUTION.—(*a*) $p_1 = \frac{p v}{v_1} = \frac{72 \times 1.875}{2} = 67\frac{1}{2}$ pounds per square inch. Ans.

(*b*) $p_1 = \frac{72 \times 1.875}{3} = 45$ pounds per square inch. Ans.

(*c*) $p_1 = \frac{72 \times 1.875}{9} = 15$ pounds per square inch. Ans.

EXAMPLE.—If 10 cubic feet of air have a tension of 5.6 pounds per square inch, (*a*) what is the volume when the tension is 4 pounds? (*b*) 8 pounds? (*c*) 25 pounds? (*d*) 100 pounds?

SOLUTION.—(*a*) $v_1 = \frac{p v}{p_1} = \frac{5.6 \times 10}{4} = 14$ cubic feet. Ans.

(*b*) $v_1 = \frac{5.6 \times 10}{8} = 7$ cubic feet. Ans.

(*c*) $v_1 = \frac{5.6 \times 10}{25} = 2.24$ cubic feet. Ans.

(*d*) $v_1 = \frac{5.6 \times 10}{100} = .56$ cubic foot. Ans.

As a necessary consequence of Mariotte's law, it may be stated that *the density of a gas varies directly as the pressure and inversely as the volume;* that is, *the density increases as the pressure increases, and decreases as the volume increases.*

This is evident, since if a gas has a tension of two atmospheres, or $14.7 \times 2 = 29.4$ pounds per square inch, it will weigh twice as much as the same volume would if the tension was one atmosphere, or 14.7 pounds per square inch. For, let the volume be increased until it is twice as great as the original volume, the tension will then be one atmosphere. The total weight of the gas has not been changed, but there are now 2 cubic feet for every 1 cubic foot of the original volume, and the weight of one cubic foot now is only half as great as before. Thus, the density decreases as the volume increases, and as an increase of pressure causes a decrease of volume, the density increases as the pressure increases.

Let D be the density corresponding to the pressure p and volume v, and D_1 be the density corresponding to the pressure p_1 and volume v_1; then,

$$p : D :: p_1 : D_1, \text{ or } p D_1 = p_1 D, \qquad \textbf{(9.)}$$

and

$$v : D_1 :: v_1 : D, \text{ or } v D = v_1 D_1. \qquad \textbf{(10.)}$$

Since the weight is proportional to the density, the weights may be used in place of the densities in formulas **9** and **10.** Thus, let W be the weight of a quantity of air or other gas whose volume is v and pressure is p; let W_1 be the weight of the same quantity when the volume is v_1 and pressure is p_1; then,

$$p : W :: p_1 : W_1, \text{ or } p W_1 = p_1 W, \qquad \textbf{(11.)}$$

$$v : W_1 :: v_1 : W, \text{ or } v W = v_1 W_1. \qquad \textbf{(12.)}$$

EXAMPLE.—The weight of 1 cubic foot of air at a temperature of 60° F. and under a pressure of 1 atmosphere (14.7 pounds per square inch) is .0763 pound; what would be the weight per cubic foot if the volume was compressed until the tension was 5 atmospheres, the temperature still being 60° ?

SOLUTION.—Using formula **11,**

$p : W :: p_1 : W_1$, or $1 : .0763 :: 5 : W_1$, or $W_1 = .3815$ lb. Ans.

EXAMPLE.—If in the last example the air had expanded until the tension was 5 pounds per square inch, what would have been its weight per cubic foot ?

SOLUTION.—Here $p = 14.7$, $p_1 = 5$, and $W = .0763$. Hence, using the same formula, $14.7 : .0763 :: 5 : W_1$, or $W_1 = .02595$ lb. Ans.

EXAMPLE.—If 6.75 cubic feet of air at a temperature of 60° F., and a pressure of one atmosphere, are compressed to 2.25 cubic feet (the temperature still remaining 60° F.), what is the weight of a cubic foot of the compressed air ?

SOLUTION.—Using formula **12**,

$$v : W_1 :: v_1 : W, \text{ or } 6.75 : W_1 :: 2.25 : .0763,$$

or $$W_1 = \frac{.0763 \times 6.75}{2.25} = .2289 \text{ lb. Ans.}$$

853. Relation of Temperature to Volume.—In all that has been said before, it has been stated that the temperature was constant; the reason for this will now be explained: Suppose 5 cubic feet of air to be confined in a cylinder whose area is 10 square inches, placed in a vacuum so that there will be no pressure due to the atmosphere, and the cylinder be fitted with a piston weighing, say, 100 pounds. The tension of the gas will be $\frac{100}{10} = 10$ pounds per square inch. Suppose that the temperature of the air is 32° F., and that it is heated until the temperature is 33° F., or the temperature is increased 1°; it will be found that the piston has risen a certain amount, and, consequently, the volume has increased, while the pressure is the same as before, or 10 pounds per square inch. If more heat is applied until the temperature of the gas is 34° F., it will be found that the piston has again risen and the volume again increased, while the pressure still remains the same. It will be found that for every increase of temperature, there will be a corresponding increase of volume. The law which expresses this change is called *Gay-Lussac's law*.

854. Gay-Lussac's Law.—*If the pressure remains constant, every increase of temperature of 1° F. produces in a given quantity of gas an expansion of $\frac{1}{491}$ of its volume at 32° F.*

If the pressure remains constant, it will also be found that every decrease of temperature of 1° F. will cause a decrease of $\frac{1}{491}$ of the volume at 32° F.

Let v = volume of gas before heating;
v_1 = volume of gas after heating;
t = temperature corresponding to volume v;
t_1 = temperature corresponding to volume v_1.

Then, $$v_1 = v\left(\frac{459 + t_1}{459 + t}\right). \qquad (\mathbf{13.})$$

That is, *the volume of gas after heating (or cooling) equals the original volume multiplied by 459 plus the final temperature, divided by 459 plus the original temperature.*

EXAMPLE.—When 5 cubic feet of air at a temperature of 45° are heated under constant pressure up to 177°, what is its new volume?

SOLUTION.—Applying formula **13**,

$$v_1 = v\left(\frac{459 + t_1}{459 + t}\right) = 5 \times \left(\frac{636}{504}\right) = 6.309 \text{ cu. ft.}$$

Suppose that a certain volume of gas is confined in a vessel so that it can not expand; in other words, suppose that the piston of the cylinder before mentioned to be fastened so that it can not move. Let a gauge be placed on the cylinder so that the tension of the confined gas can be registered. If the gas is heated, it will be found that for every increase of temperature of 1° F. there will be a corresponding increase of $\frac{1}{491}$ of the tension at 32° F.; that is, the volume remaining constant, the tension increases $\frac{1}{491}$ of the tension at 32° F. for every degree rise of temperature.

Let p = the original tension;
t = the corresponding temperature;
t_1 = any higher temperature;
p_1 = corresponding tension.

Then, $$p_1 = p\left(\frac{459 + t_1}{459 + t}\right). \qquad \textbf{(14.)}$$

That is, *if a certain quantity of gas is heated from $t°$ to $t_1°$, the volume remaining constant, the resulting tension p_1 will be equal to the original tension multiplied by 459 plus the final temperature, divided by 459 plus the original temperature.*

EXAMPLE.—If a certain quantity of air is heated under constant volume from 45° to 177°, what is the resulting tension, the original tension being 14.7 pounds per square inch?

SOLUTION.—Using formula **14**,

$$p_1 = p\left(\frac{459 + t_1}{459 + t}\right) = 14.7 \times \left(\frac{636}{504}\right) = 18.55 \text{ lb. per sq. in.}$$

855. Absolute Zero.—According to the modern and now generally accepted theory of heat, the atoms and molecules of all bodies are in an incessant state of vibration. The vibratory movement in the liquids is faster than in the solids; it is faster in the gases than in either of the others. Any increase of heat increases the vibrations, and a decrease of heat decreases them. From experiments and calculations based upon higher mathematics, it has been concluded that at 459° below zero on the Fahrenheit scale, or at 273° below zero on the Centigrade scale, all these vibrations cease. This point is called the *absolute zero*, and all temperatures reckoned from this point are called the *absolute temperatures*. The point of absolute zero has never been reached nor closely approached, the lowest recorded temperature being 360° F. below zero, but, nevertheless, it has a meaning, and is used in many formulas, being nearly always denoted by T. Ordinary temperatures are denoted by t. When the word temperature alone is used, the meaning is the same as ordinarily used, but when absolute temperature is specified, 459° F. must be added to the temperature. The absolute temperature corresponding to 212° F. is 459° + 212° = 671° F. If the absolute temperature is given, the ordinary temperature may be found by subtracting 459° from the absolute temperature. Thus, if the absolute temperature is 520° F., the temperature is 520° − 459° = 61° F.

Let $P =$ pressure per square inch;
$V =$ volume of air in cubic feet;
$T =$ absolute temperature;
$W =$ weight.

Then,
$$P = \frac{.37052\ W\,T}{V}; \qquad (15.)$$

$$V = \frac{.37052\ W\,T}{P}; \qquad (16.)$$

$$T = \frac{P\,V}{.37052\ W}; \qquad (17.)$$

$$W = \frac{P\,V}{.37052\ T}. \qquad (18.)$$

NOTE.—The constant .37052 is the reciprocal of the weight, in pounds, of 1 cubic foot of air at 1° absolute temperature (Fahr.), and a pressure of 1 pound per square inch.

EXAMPLE.—If 40 cubic feet of air weigh 3.5 pounds, and have a temperature of 82°, what is the pressure (tension) in pounds per square inch?

SOLUTION.— $P = \frac{.37052\,W\,T}{V} = \frac{.37052 \times 3.5 \times 541}{40} = 17.539$ lb. per sq. in. Ans.

EXAMPLE.—What is the volume in cubic feet of a certain quantity of air having a tension of 17.539 pounds per square inch, a temperature of 80°, and which weighs 3.5 pounds?

SOLUTION.— $V = \frac{.37052\,W\,T}{P} = \frac{.37052 \times 3.5 \times 541}{17.539} = 40$ cu. ft. Ans.

EXAMPLE.—If 40 cubic feet of air having a tension of 17.539 pounds per square inch weigh 3.5 pounds, what is the temperature?

SOLUTION.— $T = \frac{P\,V}{.37052\,W} = \frac{17.539 \times 40}{.37052 \times 3.5} = 541°$, nearly. Hence, $541° - 459° = 82°$. Ans.

EXAMPLE.—If 40 cubic feet of air have a tension of 17.539 pounds per square inch, and a temperature of 82°, (*a*) what is its weight? (*b*) what is its weight per cubic foot?

SOLUTION.—(*a*) $W = \frac{P\,V}{.37052\,T} = \frac{17.539 \times 40}{.37052 \times 541} = 3.5$ lb. Ans.

(*b*) $3.5 \div 40 = .0875$ lb. per cu. ft. Ans.

856. Mixing of Gases.—If two liquids which do not act chemically upon each other are mixed together and allowed to stand, it will be found that after a time the two liquids have separated, and the heavier has fallen to the bottom. If two vessels containing gases of different densities be put in communication with each other, the gases will mingle freely together till the mixture is uniform in each vessel. If one vessel be above the other, and the heavier gas be in the lower vessel, the same result will occur. The greater the difference of the densities of the gases, the quicker a uniform mixture will be formed, assuming that no chemical action takes place between the gases. When the gases have the same temperature and pressure, the pressure of the mixture will be the same; this is evident, since the total volume has not been changed, and unless the volume or temperature changes, the pressure can not change. This property of the mixing of gases is a very valuable one, since, if they acted like liquids, carbonic acid gas (the result of combustion), which is 1½ times as heavy as air, would remain next to the earth, instead of dispersing into the atmosphere, the result being that no animal life could exist.

Mixtures of Equal Volumes of Gases Having Unequal Pressures.—*If two gases having the same volume and temperature, but different pressures, be mixed in a vessel whose volume equals one of the equal volumes of the gas, the pressure of the mixture will be equal to the sum of the two pressures, provided that the temperature remains the same as before.*

EXAMPLE.—Two vessels containing 3 cubic feet of gas, each at a temperature of 60°, and at a pressure of 40 pounds and 25 pounds per square inch, respectively, are placed in communication with each other, and all the gas is compressed into one vessel. If the temperature of the mixture is also 60°, what is the pressure?

SOLUTION.—According to the law just given, the pressure will be 40 + 25 = 65 lb. per sq. in.

857. Mixture of Two Gases Having Unequal Volumes and Pressures.—

Let v and p be the volume and pressure of one of the gases.
Let v_1 and p_1 be the volume and pressure of the other gas.
Let V and P be the volume and pressure of the mixture.

Then, if the temperature remains the same,

$$P = \frac{p\,v + p_1\,v_1}{V}. \qquad (19.)$$

$$V = \frac{p\,v + p_1\,v_1}{P}. \qquad (20.)$$

EXAMPLE.—Two gases of the same temperature, having volumes of 7 cubic feet and 4½ cubic feet, and tensions of 25 pounds and 18 pounds per square inch, respectively, are mixed together in a vessel whose volume is 10 cubic feet. The temperature remaining the same, what is the resulting pressure?

SOLUTION.— $P = \frac{p\,v + p_1\,v_1}{V} = \frac{(25 \times 7) + (18 \times 4\frac{1}{2})}{10} = \frac{256}{10} = 25.6$ lb. per sq. in. Ans.

EXAMPLE.—What must be the volume of a vessel which will hold two gases whose volumes are 7 cubic feet and 4½ cubic feet, and whose tensions are 25 pounds and 18 pounds per square inch, respectively, in order that the pressure may be 25.6 pounds per square inch, the temperature remaining the same throughout?

SOLUTION.— $V = \frac{p\,v + p_1\,v_1}{P} = \frac{(25 \times 7) + (18 \times 4\frac{1}{2})}{25.6} = 10$ cu. ft. Ans.

GASES COMMON TO MINES.

858. The gases met with in mines are comparatively few in number, but a thorough knowledge of their properties and the manner of their detection is most important. The following are the gases most commonly occurring in mines, considered in the order of their importance, as dangerous to life and health.

Marsh-gas (carbureted hydrogen) (CH_4). Sp. Gr., 0.559.

Marsh-gas (mixed with air)—**Firedamp** (Saturated). Sp. Gr., 0.96.

Carbonic oxide gas (CO)—**White damp.** Sp. Gr., 0.967.

Carbonic acid gas (CO_2)—**Black damp.** Sp. Gr., 1.5291.

Carbonic acid gas....(CO_2) *Carbonic oxide gas*...(CO) *Nitrous oxide gas*...(N_2O) *Nitrogen* (*free*).......(N) *Hydrogen* (*free*)......(H) *Watery vapor*......(H_2O)	**After-damp.** (Composition very variable.)

Sulphureted hydrogen (H_2S)—**Stink damp.** Sp. Gr., 1.1912.

Ethene, or **olefiant gas** (C_2H_4). Sp. Gr., 0.973.

The two latter gases are rarely found in mines, and when present are only in limited volumes.

859. Marsh-gas (CH_4).—This is the most disastrous, in its effects, of any of the gases known to mining.

(*a*) **Occurrence.**—Pure marsh-gas (CH_4), Sp. Gr., 0.559, exists, or *has existed*, as an occluded gas, to a greater or less extent, in all coal formations, and is a product of the early metamorphism of vegetable matter under the water and superimposed strata, by which all air was excluded. Marsh-gas is often seen rising in bubbles from the bottom of stagnant pools; it is always the product of the decomposition of vegetable matter, away from air, and in the presence of water. When such decomposition of vegetable matter occurs in a *dry* place, away from air, ethene, or olefiant gas (C_2H_4), which is richer in carbon than marsh-gas, is formed.

Marsh-gas transpires from the pores, foliations, and crevices of a freshly exposed face of a gaseous coal-seam. It may also issue from the floor or roof of the seam, as stratigraphical or other conditions may have rendered these adjacent strata more pervious to the gas than the coal-seam itself. It may transpire from the entire face, or may issue in a stronger flow from a crevice or *feeder*. It may even find vent as a *blower* of gas, under great pressure. Natural marsh-gas never occurs in a pure state, but is always mixed with other gases. The mode of occurrence of these gases will be further explained in the study of *Diffusion*, *Occlusion*, and *Transpiration*.

(*b*) **Properties.**—Marsh-gas is a combustible gas, burning with a bluish flame, but it will not support combustion. It is slightly more than one-half as heavy as air of the same temperature and pressure. Upon first transpiring from the face, or issuing from the fissures of a formation, the gas diffuses rapidly in the air, until its limit of diffusion is reached in a confined space, as in the still air of a mine.

Marsh-gas is the lightest of the hydrocarbons, a molecule of marsh-gas consisting of *one* atom of carbon united to *four* atoms of hydrogen. It is an odorless, colorless, and tasteless gas. It does not poison the animal system; a person may breathe with impunity air containing a large percentage of the gas for a considerable time.

One of the most important properties of marsh-gas, to the miner, is that which it possesses of not igniting immediately upon contact with flame. Ignition of the gas takes place only after the lapse of an appreciable period of time, which, although but a fraction of a second, is sufficient to render the use of many detonating explosives safe in presence of firedamp, the detonation in this class of explosives being instantaneously followed by a period of extreme cold.

(*c*) **Detection of Marsh-Gas.**—On account of the rapid diffusibility of marsh-gas (Table 19), its detection in the mine is practically the detection of firedamp. (See Detection of Firedamp.)

860. Firedamp.—Any explosive mixture of marsh-gas and air is termed *firedamp*.

(*a*) **Occurrence.**—On account of the high diffusive power of marsh-gas, *firedamp* is formed very rapidly wherever marsh-gas issues from the coal or strata. This diffusion takes place upon the outer envelope of the gas in contact with the air. A considerable body of the gas, owing to its lighter weight and warmer temperature, ascends and flows along the roof, collecting in cavities and convenient places for lodgment. The specific gravity of this diffusing gas approaches that of air, and its subsequent diffusion is slow. For this reason, we look for firedamp in the cavities of the roof and the higher working-places of the mine.

(*b*) **Properties.**—The explosive limits of firedamp mixtures can not be closely defined, as such conditions as the purity of the marsh-gas, and the pressure to which the firedamp is subjected, vary the explosive points slightly. However, under ordinary conditions, when 1 part of marsh-gas mixes with 5½ parts of air, the combination is at its lowest explosive limit. As the proportion of air is increased, the explosive violence grows steadily greater till it reaches a maximum, when the mixture is in the proportion of 9½ parts of air to 1 of gas. From this point, as the proportion of air is increased, the explosive violence grows more and more feeble till the mixture consists of 13 parts of air to 1 of gas, when explosion ceases altogether.

The percentage of marsh-gas in an atmosphere of fire-damp, when at its lowest explosive limit, is calculated thus:

Relative volume of gas,	1
Relative volume of air,	5.5
Relative volume of mixture,	6.5

and $\frac{1 \times 100}{6.5} = 15.38$ per cent. of marsh-gas.

In like manner, for the higher explosive limit, we have

Relative volume of gas,	1
Relative volume of air,	13
Relative volume of mixture,	14

and $\frac{1 \times 100}{14} = 7.14$ per cent. of marsh-gas.

The presence in firedamp of $\frac{1}{7}$ of its volume of carbonic acid gas will render it inexplosive.

The effect of an increase of pressure upon the explosive range of gas is to extend it. A mixture of marsh-gas and air that is below or above the explosive limits, is often rendered explosive by an increase of pressure. This may often occur in proximity to a blast when the air of the workings would otherwise be safe.

The effect of suspended coal-dust in the air is to widen the explosive range. This is probably due to the increase of temperature incident to the burning of the gases distilled from the dust.

(*c*) **Detection of Firedamp.**—The detection of this gas in the mine is to be entrusted to the most experienced men only, for it is fraught with danger to all in the mine. Many devices have been invented for the purpose of detecting the presence of gas, as well as to determine at the same time the approximate percentage of the mixture of gas and air. Any machine to be of practical value in this line, must be capable of making the test promptly and safely at the point of danger, and of revealing the presence of $\frac{1}{2}$ per cent. of gas.

We shall refer more particularly to the means at our disposal for detecting firedamp later, and shall describe the various forms of lamps in common use. We will state here, that, at the present time, no machine or device for testing has given satisfaction equal to the safety-lamp, which is prompt and always at hand. The lamp is elevated cautiously to the place where gas is suspected, care being taken to keep the lamp in an upright position, that its flame may not approach the gauze of the lamp. If gas is present, it will enter the lamp with the air, and will burn when present in large quantities, filling the whole lamp with flame. If the percentage of gas in the air is small, however, say two per cent., its presence is manifested only by a small blue tip to the flame of the lamp, which may be seen more distinctly by screening the eyes from the brighter portion of the flame with the hand.

An experienced and careful observer will detect, with the ordinary lamp, a percentage of the gas as low as 2 per cent. It is, however, often desirable to detect the presence of smaller quantities than this in the air of dusty mines, where the coal-dust is highly inflammable. For this purpose, specially constructed lamps are used. In the use of the lamp for the detection of presence of gas, care must be taken to make no quick movement; especially is this needful in case of flaming in the lamp. The lamp must be immediately removed from the gas, but not so quickly as to blow the flame through the gauze. This requires much self-possession on the part of the observer.

861. White Damp (*CO*).—The "white damp" of the mines is carbonic oxide gas. It is a dangerous gas, because of its harmful effects and its unsuspected presence.

(*a*) **Occurrence.**—Carbonic oxide gas is a product of the incomplete combustion of carbonaceous fuel, the supply of air being limited. Thus, it is produced largely by the slow combustion of coal in the gob, by mine fires, and by the explosion of powder.

(*b*) **Properties.**—This gas is a colorless, odorless, and tasteless gas. It is somewhat lighter than air at the same temperature and pressure. It burns with a pale, violet flame, like that which may be seen at any time over a freshly fed anthracite fire. It is very poisonous to the system when inhaled, being rapidly absorbed by the blood, and it acts as a narcotic, producing drowsiness or stupor, followed by acute pains in the head, back, and limbs, and afterwards by delirium. If the victim of this gas is not rescued soon, death will inevitably result.

Carbonic oxide gas has the widest explosive range of any gas except hydrogen. When 1 volume of the gas is mixed with about 6.7 volumes of air, the lowest explosive mixture is obtained. From this point it continues to be explosive until the proportion of gas is increased to the extent of 1 volume of gas to every 1.6 volumes of air. It is this property of carbonic oxide gas which makes it such an active agent in the transmission of the flame of a mine explosion from one point in the mine workings to another seemingly isolated point. Under ordinary conditions, however, this gas is not present in sufficient quantity to yield an explosive mixture.

(*c*) **How Detected.**—Carbonic oxide gas may be detected in the mine workings by its effect upon the flame of an ordinary lamp. The flame is much brighter and reaches upwards, and it is thus lengthened out into a more or less slim, quivering taper with a bluish tip, which may be seen more clearly by screening the eyes from the brighter portion of the flame with the hand.

862. Black Damp (CO_2).—The "black damp" of the mines, or, as it is often called, "choke-damp," is carbonic acid gas. It is not as dangerous as either of the preceding gases, because its presence in the mine workings is at once manifested by the dimness of the lamps.

(*a*) **Occurrence.**—This gas is always a product of combustion in the presence of a plentiful supply of air. It is produced by the burning of lamps, breathing of men and animals, decomposition and decay, and is a later product of all explosions of powder and gas. The principal source, however, is water from the coal and strata which hold it in solution, and from which it escapes as the water evaporates.

(*b*) **Properties.**—This is a colorless and odorless gas, but it possesses a distinctly sharp taste in the mouth when breathed. It is one-half again as heavy as air at the same temperature and pressure, and, therefore, collects near the floor and in the low places in the mine. It is incombustible, and, when present in the air to any considerable extent, extinguishes lamps. It acts as a narcotic, and produces, after a time, headache and nausea, causing death by suffocation.

(*c*) **How Detected.**—The presence of carbonic acid gas is readily detected by the flame of a lamp becoming reduced in size, and, when more gas is present, by its extinguishment; by lime-water, which, when exposed to the gas, becomes milky in appearance; and by damp, blue litmus paper, which becomes red when exposed in an atmosphere containing carbonic acid gas. The flame becomes reduced in size, and, when more gas is present, is extinguished altogether. Being heavier than air, it must be sought for at the floor of the entries and in the low parts of the mine.

863. (*a*) **Traces of Sulphureted Hydrogen Gas** (H_2S).—This gas, though not commonly occurring in troublesome quantities, is yet a very dangerous gas to meet. It is heavier than air, having a specific gravity of 1.1912. It is violently explosive when mixed with air of about seven times its volume. The gas is very poisonous when inhaled. In small quantities in the air, it produces derangement of the system; when inhaled in larger quantities, it rapidly produces unconsciousness and prostration. The smell of the gas affords the best index of its presence, which has given rise to its being termed "*stink damp*" by the miners, for it smells like rotten eggs.

(*b*) **Ethene, or Olefiant Gas** (C_2H_4).—This gas occurs in varying amounts as a constituent of marsh-gas. It is this gas which causes the flame of marsh-gas to burn with some luminosity. It is a product of the dry decomposition of vegetable matter and the distillation of coal.

OTHER PROPERTIES OF GASES.

864. Diffusion.—*All* gases which do not act chemically on each other, especially air and gases *of different densities*, when in proximity, tend to *diffuse* into each other; that is to say, their molecules pass freely among each other, and tend to form a complete intermixing of the two gases. This property is called *diffusion*, and is caused by the lack of equilibrium between the molecular vibrations of the two masses; so that the molecules of the two masses tend to thoroughly intermingle. (See Art. **856.**)

865. Rate of Diffusion.—The diffusion of gases takes place much more rapidly in a moving current than in still air. The relative rates or velocities of the diffusion of the gases into each other are in the inverse ratio of the square roots of their densities. For example, taking the density of air as 1, then the density of hydrogen gas, by Table 19, is .0693, and the square root of .0693 by the table is .2632; therefore, the relative velocity of the diffusion of hydrogen gas into air will be $\frac{1}{\sqrt{.0693}} = \frac{1}{.2632} = 3.7987$. This corresponds with the results given in the third column of the table; and the use of the table may be understood in this way. In all cases, divide 1 by the square root given in the second column of the table, and the quotient will be the relative velocity of the diffusion of the gas in question into air. For example, the square root of the density of marsh-gas is given in the second column as .7477; then, $\frac{1}{.7477} =$ $1.3375 =$ the relative velocity of the diffusion of marsh-gas into air. The annexed table of densities shows the comparative rates of diffusion of the various gases and air into a vacuum:

TABLE 19.

Gas.	Density, or Specific Gravity.	Square Root of Density.	$\frac{1}{\sqrt{\text{Density}}}$.	Velocity of Diffusion. Air = 1.
Hydrogen	0.0693	0.2632	3.7987	3.830
Marsh-gas	0.5590	0.7477	1.3375	1.344
Carbonic oxide	0.9670	0.9834	1.0169	1.015
Nitrogen	0.9713	0.9855	1.0147	1.014
Oxygen............	1.1057	1.0515	0.9510	0.949
Sulphureted hydrogen..............	1.1912	1.0914	0.9163	0.950
Carbonic acid......	1.5291	1.2366	0.8087	0.812

The values given in the last column of this table were obtained by experimenting with the gases, and agree quite closely with the calculated values given in the preceding column. From the last column we see that 1,344 volumes of marsh-gas will diffuse in the same time as 1,000 volumes of air or 812 volumes of carbonic acid gas.

866. Occlusion of Gases.—A gas is *occluded* (hidden) when it exists in the pores of a solid mass. A familiar example of the occlusion of gases is found in the coal-seams, where gases often exist in large quantities and are a source of danger in mining.

The conditions which have held these gases in the coal and adjoining strata, till set free by the penetration of mine workings, are largely a close coal and an impervious roof and floor. The kind and amount of gases occluded in different coal-seams, and even in different parts of the same seams, vary much, and alter, to a large extent, the character of the coal enclosing them.

The gases most commonly occluded in coal-seams are marsh-gas, nitrogen, carbonic acid gas and traces of oxygen, carbonic oxide, ethene, and some other hydrocarbons.

The relative percentages of these gases vary largely, even in freshly mined coals.

867. Pressure of Occluded Gases.—The pressure of occluded gases has been shown, by a number of experiments in England, France, and Belgium, to reach as high as 10 and 16 atmospheres; and, in exceptional cases, 32 atmospheres has been the recorded pressure. Whatever degree of exactness these experiments may have, they serve to show, at least, the enormous pressures under which occluded gases may be projected from a newly exposed face. In some instances, this flow of natural gas from certain veins has furnished fuel for extensive steam plants. In general, the tapping of a gaseous seam relieves the pressure, after a limited time, by the escape of the gas.

The pressure of occluded gases is often manifested in a newly exposed face of coal by a sharp cracking and hissing sound, throwing the splintered coal with considerable violence into the face of the miner.

868. Transpiration of Gases from Coal.—When a coal-seam containing occluded gases is being worked, the pressure on the gas drives it outwards from the coal, and often from the roof and floor of the seam. The regular emission of gas from a solid mass in which it was contained is called *transpiration*.

869. Feeders and Blowers.—Wherever a cavity, crevice, or fissure exists in proximity to or in connection with a gaseous seam, it becomes charged with the occluded gases of the seam, under the same pressure. A dangerous reservoir of gas is thus formed, which may at any moment be pierced or tapped by the pick or drill of the miner and discharge its contents into the mine workings. Such *cavities*, *crevices*, or *fissures* charged with gas are termed "*feeders*," and, when tapped, the stream of gas issuing from them is called a "*blower*." According to the size of the internal reservoir of gas, such a blower may continue to discharge its gas, with practically no abatement, for a long time.

870. Outbursts.—In the working of the seams of some localities, the presence of occluded gases is frequently manifested by a violent outburst at the working face. These outbursts often take place without warning, and produce an effect similar to that of an explosion, throwing down the coal in large quantities.

The cause is due to a *feeder* finding access to a more or less vertical crevice or cleat behind the working face of the coal-seam. Its pressure thus becomes distributed over a considerable area of coal, and exerts a powerful localized force. This is due to a pressure on a large area being made to act on a small one with multiplied force.

Fig. 113 represents a dangerous pocket of gas lying beneath an impervious stratum of close-grained rock, which has prevented its escape. The gas is under enormous pressure, incident to the subsidence and contraction of the strata. The cleats or vertical fissures shown in the coal-seam are "face cleats," the entry or gangway being driven "end on." The pressure of the gas causes the foliated shale to rest heavily on the timbers, and later a fissure occurs in this strata, which opens a communication for the gas with the face cleats of the coal. The pressure of the gas may thereby be distributed over a large area of the rib, with the result just described.

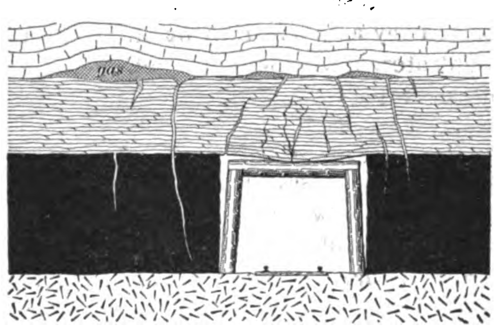

FIG. 113.

There are well-authenticated, although seemingly incredible, cases upon record where headings and chutes have been completely blocked by a compacted mass of from 15 to 20 tons of fine coal, thus thrown from the face without the slightest warning. In other instances, the outburst may be accompanied by a subterranean pounding, or "bumping," as the miners term it, or by a sudden report, similar to that of a blast. This pounding or "bumping" sometimes continues at intervals for two or three days prior to the outburst. By far the larger number of violent outbursts are of marsh-gas; although instances are recorded of very violent outbursts of carbonic acid gas.

871. Calculation of the Initial Force of an Explosion.—The *force* of an explosion depends upon the expansive power of the gases resulting from the explosion. The expansive power of these gases depends upon their relative volumes *before* and *after* the explosive reaction has taken place. The *initial force* of an explosion is the force developed at the moment of ignition. To calculate the initial force of an explosion, we must first determine the *relative* or *atomic volume* (Art. **841**) of the resulting gases. Thus, in the *complete* explosion of marsh-gas (CH_4), the reaction which takes place is expressed by the following equation:

$$CH_4 + 4\,O + 16\,N = CO_2 + 2\,H_2\,O + 16\,N,$$

or 1 atom of carbon + 4 of hydrogen + 4 of oxygen + 16 of nitrogen = 25 atoms; and 1 atom of carbon + 2 of oxygen + 4 of hydrogen + 2 of oxygen + 16 of nitrogen = 25 atoms.

We notice in this *complete* explosion, the maximum explosive energy must be developed, because *all* of the carbon of the marsh-gas is converted immediately into carbonic acid gas, and *all* of the hydrogen into watery vapor, both of which are dead or inert products, having given out their energy. We shall also see later that this occurs when the marsh-gas forms 9.38 per cent. of the firedamp.

By observing the above equation, we see that for each molecule of CH_4 there is produced *one* molecule of CO_2 and *two* molecules of $H_2\,O$. *Four* atoms of O are consumed in the reaction, which are derived from the air and represent a volume of air containing approximately 4 atoms of O and 16 atoms of N. (More accurately, the 4 atoms of O represent 20.7 per cent. of the entire air used; see Art. **845.**)

Now, determining the atomic volume of the gases before and after the reaction, we find the volume of the firedamp (marsh-gas and air) is the same as the volume of the carbonic acid gas, watery vapor, and nitrogen produced; thus, the molecule of marsh-gas occupies the same space as the molecule of carbonic acid gas; the *two* molecules of watery vapor occupy the same space as the *four* atoms of oxygen. (See Second Law of Volume, Art. **841.**) The nitrogen is unchanged by the reaction.

Before Explosion.				After Explosion.			
Gas.	Symbol.		Atomic or Relative Volumes.	Gas.	Symbol.		Atomic or Relative Volumes.
Marsh-gas.	CH_4	1 molecule.	2	Carb. acid gas.	CO_2	1 molecule.	2
Air.	O	4 atoms.	4	Watery vapor.	$2\,H_2O$	2 molecules.	4
	N	Free nitrogen.	15.32	Nitrogen.	N	Free nitrogen.	15.32
		Total volume,	21.32			Total volume,	21.32

By observing the above table, we see that the column of relative volumes shows 2 volumes of marsh-gas and $4 + 15.32 = 19.32$ volumes of air, which is in the ratio of 1 volume of marsh-gas to 9.66 volumes of air, the firedamp being at its maximum explosive point. We observe, also, by the table, that 4 volumes of oxygen are consumed in the complete explosion of 2 volumes of marsh-gas. We have previously learned (Art. **845**) that oxygen forms 20.7 per cent. of the volume of the air; the remaining 79.3 per cent. being nitrogen in a free state. Hence, to find the relative volume of air per 2 volumes of gas, we write the following proportion: $20.7 : 4 :: 100 : x = 19.32$ volumes of air, or, for 1 volume of gas, we have $20.7 : 2 :: 100 : x =$

9.66 volumes of air. The entire relative volume of firedamp concerned in the reaction is, then, the sum of the relative volumes (21.32 volumes), as by the table.

The percentage of pure marsh-gas in this mixture, or body of firedamp, is found thus, $\frac{2 \times 100}{21.32} = 9.38$ per cent. of marsh-gas. There being no change in the atomic volume of these gases before and after explosion, the expansive power produced by the combustion of the mixture can be found from the increase in temperature from 60° F. (normal) to 1,200° F. (temperature of ignition for marsh-gas). Thus, the total pressure of a confined gas is always proportional to its absolute temperature. The absolute temperature is the temperature above absolute zero, which is 459° below the Fahrenheit zero. Hence, to transform Fahrenheit temperature to absolute temperature we add 459 degrees. Then, knowing the pressure of the atmosphere to be 14.7 at the normal temperature 60° F., we write the simple proportion

$$459 + 60 : 459 + 1{,}200 :: 14.7 : x,$$
$$\text{or, } 519 : 1{,}659 :: 14.7 : x = 47.0 \text{ pounds, nearly.}$$

Therefore, the absolute pressure (or the pressure above vacuum), after the explosion, is practically 47 pounds, and the ruptive pressure is 47.0 − 14.7 (or the atmospheric pressure) = 32.3 pounds per square inch.

872. Calculation of the Weight of a Gas.—*The weight of any gas, at a given pressure and temperature, is equal to the weight of an equal volume of air, at the same pressure and temperature, multiplied by the specific gravity of the gas.*

Let W = weight in pounds;
V = volume in cubic feet;
B = barometric pressure in inches;
D = specific gravity of the gas—found in Table 19;
T = absolute temperature.

Then, $$W = \frac{1.3253\,V\,B\,D}{T}. \qquad \textbf{(21.)}$$

EXAMPLE.—What is the weight of 100 cubic feet of carbonic acid gas at a pressure of 31 inches of mercury and a temperature of 32° F.?

SOLUTION.— $W = \frac{1.3253 \times 100 \times 1.5291 \times 31}{459 + 32} = 12.7947$ lb. Ans.

NOTE.—The constant 1.3253 is the weight, in pounds, of 1 cubic foot of air at 1° absolute temperature (Fahr.) and 1 inch barometric pressure.

EXAMPLES FOR PRACTICE.

1. Find the weight of 200 cubic feet of marsh-gas (CH_4), at a temperature of 70° and a pressure of 30 inches of mercury. Ans. 8.4028 lb.

2. Referring to example 1, what is the weight of the hydrogen gas, in this amount of marsh-gas? Ans. 2.1007 lb.

3. In case of an explosion of firedamp in which 8.4028 pounds (example 1) of marsh-gas were concerned, all of its hydrogen combining with the oxygen of the air to form water (H_2O), (*a*) what would be the weight of watery vapor resulting from the explosion? (*b*) What would be the weight of oxygen consumed in this part of the reaction? (See Art. **838.**)

Ans. { (*a*) 18.9063 lb.
(*b*) 16.8056 lb.

4. Referring to example 3, if all of the carbon of the 8.4028 pounds (example 1) of marsh-gas, combined with the oxygen of the air to form carbonic acid gas (CO_2), (*a*) what weight of carbonic acid gas would result from the explosion? (*b*) What would be the weight of oxygen consumed in *this* part of the reaction? (See Art. **838.**)

Ans. { (*a*) 23.1077 lb.
(*b*) 16.8056 lb.

5. Referring, now, to examples 3 and 4, (*a*) what is the total weight of oxygen consumed in the reaction? (*b*) Determine the total weight of air consumed in the reaction, incident to the explosion. (See Art. **845.**)

Ans. { (*a*) 33.6112 lb.
(*b*) 146.1356 lb.

6. What volume of dry air is required to completely explode 200 cubic feet of marsh-gas? (See Arts. **841** and **845.**)

Ans. 1,932.3 cu. ft.

7. Referring to example 6, if this volume of air (1,932.3 cubic feet) is consumed in the complete combustion of 200 cubic feet of marsh-gas, (*a*) what per cent. of the mixture (firedamp) does the marsh-gas form? (*b*) What volume of firedamp was exploded?

Ans. { (*a*) 9.38 per cent.
(*b*) 2,132.3 cu. ft.

COMBUSTION.

873. Combustion, in its broadest sense, refers to chemical union, attended with *heat*, sometimes with *light* and *flame*. Combustion always results in a complete transformation of the body acted upon, and of the gas which *supports the combustion*, forming other gases.

874. Oxidation.—Oxygen gas is the great supporter of combustion; and the process of combustion is then called *oxidation*. This gas has a strong affinity for carbon and hydrogen; and, thus, we have formed two of the most commonly occurring compounds, *water* and *carbonic acid gas*. The *first* of these is as truly an essential to animal life as the *second* is an inevitable result of the same.

There are numerous other illustrations of true combustion, however, than those in which *oxygen* plays a part. For example, the burning of a lighted taper in an atmosphere of *chlorine* gas; or, the explosion of equal quantities of *hydrogen* and *chlorine* gases.

875. Temperature of Combustion.—Combustion may take place at any temperature; that is to say, the oxidation is often carried on slowly and at a low temperature, and the body just as truly destroyed or consumed as when the action is stronger and the temperature high enough to produce flame.

The process is then spoken of as **slow combustion,** because the action is slower, or less energetic than in **active combustion,** when flame is produced. The consuming of the animal tissues of the body is an example of *slow combustion*. The disintegration of fine coal, in the gob heaps and goaves of the mine, is followed, in time, by a slow oxidation of the coal and the formation of carbonic oxide and carbonic acid gases. This slow oxidation is as truly a form of combustion as when the coal is burned at a higher temperature and flame results.

We conclude, then, that a high *temperature* is not an essential to slow combustion. The chemical activity of any combustion will determine its initial temperature; on the other hand, the various products of combustion have varying heat capacities (specific heats), and thereby absorb varying amounts of the initial temperature, the remaining difference being the *sensible* heat of the combustion.

876. Spontaneous combustion is a term applied to the sudden bursting forth of flame, or active combustion, in a body, caused by the internal generation of heat in the body itself. Spontaneous combustion is the result of slow combustion, or chemical action, the developed heat gradually increasing till ignition takes place. The production of carbonic oxide gas (CO) within the confines of the body, where the supply of air is limited, greatly assists ignition.

EXPLOSIVES AND EXPLOSIONS.

877. Explosives.—This term refers to any chemical compound or mechanical mixture that is capable, under certain conditions of heat or shock, of exerting a powerful ruptive pressure. The common form of explosives in use is an intimate, mechanical mixture of chemical compounds, such as will readily give rise to dissociation of their respective atoms, and the rapid formation of gases, under the proper conditions. These gases, from the temperature incident to the explosion, possess an enormous expansive force, resulting in a ruptive pressure of many tons upon the square inch.

On account of the great importance of explosives in mining, and on account of their being the direct cause of a large number of mine accidents, affecting many who are in no wise to blame for their occurrence, a careful study of their nature and use is needful. We will consider, in order, the conditions incident to the explosion of a charge in a drill-hole, for the purposes of blasting.

The chief factor which determines the strength of an explosive is the rapidity of its combustion. There are two modes of propagation of the combustion of explosives, giving rise to two general classes; in the first class the propagation being slow, while in the second it is extremely rapid.

(*a*) *Explosives that deflagrate.*
(*b*) *Explosives that detonate.*

878. Deflagration is a form of combustion dependent upon the thermal conductivity of the mass. The combustion is propagated from particle to particle of the mass, much as heat travels from one end of an iron rod to the other. All the black powders furnish examples of *deflagration*. The ignition of such a powder at one point is transmitted throughout the mass with a speed dependent upon the combustibility of the powder and its thermal conductivity. Each atom burns independently, and exerts no further influence upon the surrounding atoms, except as the heat of its burning is communicated to them.

879. Detonation.—This form of combustion, unlike deflagration, is transmitted, with almost lightning rapidity, to every particle of the mass. The detonation of a single particle seems to exert a wave-like compression throughout the mass that causes a like detonation of the entire body. Its speed of propagation is estimated at 16,400 feet per second; so that any explosion by *detonation* is, practically, an instantaneous development of the entire expansive energy of the mass. Nitroglycerine is an example of such an explosive.

880. Action of Explosives.—The theory of the action of explosives is, in outline, as follows:

Chemical action, incident to the ignition of a charge, assisted by *heat* and *pressure*, transforms the solid explosive compound or mixture into gaseous products, developing in such transformation or *combustion* a definite number of heat units.

881. Chemical Reaction.—When a charge of powder is exploded in a drill-hole, the combustion which takes place is supported by the oxygen of the niter in the powder. This salt is a powerful oxidizer, and gives up its oxygen to the sulphur and carbon. A large number of gases are formed, chief among which are nitrogen, carbonic acid gas, and carbonic oxide gas. It is impossible to give any accurate analysis of the gases resulting from any one explosion. The gaseous products vary according to the pressure under which ignition takes place, and will vary, therefore, in each individual charge. Any chemical equation, therefore, expressing the reaction which takes place must be only approximate.

882. Black Powders.—The better grades of the black powders are formed by the intimate mixture of 2 molecules of niter, 1 of sulphur, and 3 of fine charcoal, making the following proportions by weight:

Niter (saltpeter), (potassium nitrate)	(KNO_3)	= 74.83%
Sulphur	(S)	= 11.84%
Carbon (fine charcoal)	(C)	= 13.33%
		100.00

In practice, however, the proportions are usually taken as follows:

Niter	75 parts.
Sulphur	10 parts.
Carbon	15 parts.
	100 parts.

883. Blasting Powder.—It is common practice, in the manufacture of *blasting* powders, to increase the amount of carbon or charcoal, while the amount of the niter is decreased. In blasting powders, the following proportions are more commonly used, although this practice varies in different localities:

Niter	66 parts.
Sulphur	10.5 parts.
Carbon	23.5 parts.
	100 parts.

Cheaper grades of blasting powders are often made by substituting sodium nitrate $(Na\,NO_3)$ for the potassium nitrate, either in part or wholly; but such substitution produces a very inferior powder. The sodium salt absorbs moisture when exposed to even the slightest dampness, and thereby causes such powders to lose much of their strength.

884. Size of Grain.—The size of the grain is an important factor in determining the rapidity with which the powder acts. The finer the grain, the quicker the action; and *vice versa*, the coarser the grain, the slower the action. A little study will show the need of a careful application of this principle, on the part of the miner. For example, *gunpowder* is a fine-grained powder; what is desired is the *rapid* movement of a *small* mass; hence, its action must be very rapid, and the stock of the gun must be strong enough to resist the inertia of the bullet.

On the other hand, in all kinds of blasting, a *slower movement* of a *larger mass* is desired; hence, we employ a slower powder, one whose whole expansive energy will not be developed in a single flash. We must consider, also, that the blasting of different materials requires a different action in the powder, according to the character of the material blasted. Thus, the blasting of rock requires a quicker powder than the blasting of coal, while a soft, laminated shale will yield more completely to a very slow powder. The need for this adaptation of size is obvious and reasonable.

Fig. 114 shows, approximately, the four sizes of black powder in most common use in coal-mining. These sizes

FIG. 114.

are adapted to different grades of work and a varying hardness in the coal. The smaller sizes are adapted to a hard, brittle coal, while the larger sizes are, on the other hand, adapted to a softer and tougher coal. The smaller sizes are likewise adapted to *narrow work* (entry work) and to *shooting on the solid*, while the coarser grades yield better results in *breast* and *pillar* work, where the resisting forces are not so great. The nature of the coal, the class of work, and the judgment of the miner must determine the size of powder best adapted for his use. Many miners, however, use very poor judgment in this respect, and reap a reward in the decrease of their net earnings.

885. "Blown-out" Shot.—This term is applied to any blast whose energy is expended upon the air, instead of being converted into mechanical work. The intensity of the projected flame is augmented by the high temperature and pressure resulting from the unyielding character of the walls enclosing the charge.

886. "Windy" Shot.—This term refers to a blast whose energy is, in part or wholly, expended upon the air. It differs from a *blown-out* shot only in the absence of the high temperature and pressure of the projected flame.

887. Causes.—The causes giving rise to the above are numerous, and may be summarized as follows:

(*a*) The shot may be too deeply laid.

1. The angle of a *gripped* shot may be too large; that is, the hole may be drilled at an angle so great that the charge will lie too deep.

2. The depth of a hole may locate the charge too much upon the *solid* (back of the *cutting* or *mining*).

3. The projecting bottoms and tops of the seam may arch the resistance in such a manner as not to allow the charge an opportunity to do its work.

(*b*) 1. The tamping (stemming) may be insufficient for the charge exploded or the sectional area of the hole.

2. The tamping may be of such an inflammable and gaseous nature as to become a dangerous factor in lengthening out the flame of the blast by the gases distilled from it under the flame of the blast.

(*c*) 1. The *solid*, in the region of the charge, may be creviced or fissured naturally, or by a former blast.

2. The coal may "seam out."

(*d*) 1. Too strong (fine-grained) a powder may be employed, which results in blowing the tamping and giving vent to the flame and gases of the blast before the inertia of the mass has been overcome.

2. Too coarse a powder and too heavy a charge of it may result in a considerable amount of partly burned and burning powder being thrown out upon the air, to expend its energy in expansion instead of in mechanical work. A like result will always be produced by an *excessive* charge of any size powder.

3. A mixture of different grades of powder will nearly always result in a considerable portion of the charge being thrown upon the air, partly burned or burning. The mixing of a small amount of gunpowder with blasting powder, for the purpose of "making it stronger," is a pernicious act, and would justify the discharge of the man found guilty of so doing.

(*e*) A drill-hole of too large a diameter, as compared with the amount of the charge, will result, in the majority of cases, in the projection of the charge, because the large sectional area of the hole brings an undue pressure upon the tamping.

(*f*) 1. A succession of two or more blasts, fired in a limited working place, may produce an effect similar in every respect to that produced by a *windy shot*. It is caused by the firing of the carbonic oxide gas and the suspended dust of the first shot, by the flame of the second.

2. A like result obtains very often when a heavy blast is fired in too close proximity to accumulations of dust.

In general, if the hole is "gripped" too strongly, or the charge itself located too deeply upon the solid, a "blown-out" shot will result from the unyielding nature of the walls, and a flame of great intensity will be projected from the bore of the hole when the tamping or stemming has yielded.

If the charge is too heavy for the work to be accomplished, a "*blown-out*," but more properly called, a "*windy*" *shot*, will result. The temperature of the flame will be normal in this case, but the danger arises from the projection and explosion of a considerable amount of the charge upon the air after rupture has taken place. The energy of a portion of the charge is thus bestowed upon the air instead of being converted into mechanical work, by breaking down the coal.

888. Flameless Explosives.—From our previous study, we readily perceive the dangers incident to blasting in mine workings. So numerous are the conditions which render the use of explosives in a mine dangerous, that it has often been a matter of serious consideration whether the use of any form of explosives should be tolerated in mines known to be gaseous. It is recognized that the *flame* incident to the explosion is the dangerous factor, and many attempts have been made to so alter the composition of the explosive as to yield gaseous products which were not inflammable. This result has only been realized in part. Nevertheless, explosives have been produced in the combustion of which a very limited flame results; and the use of such explosives renders mining more safe. These are mostly formed by a mixture of nitrated compounds (ammonia nitrate and nitro-benzine, or nitro-naphthalene). For the most part, they are detonators, and are exploded by a fulminating cap.

DETONATING EXPLOSIVES.

889. The detonating explosives are divided into three general classes, viz.:

(*a*) Such as have *glycerine* for a base, as *nitroglycerine*, *dynamite*, *carbonite*, *stonite*, and *ardeerite*.

(*b*) Such as are formed from *cotton*, as *guncotton*, *tonite*, and *potentite*.

Gelignite and gelatine-dynamite (blasting gelatine) are formed by mixing nitroglycerine with guncotton, in varying proportions.

(*c*) Such as have ammonia nitrate for a base (called the Sprengel class, after their inventor), as *Roburite*, *Securite*, *Ammonite*, *Oxonite* (*Rack-a-rock*), *Panclastite*, *Bellite*, and *Hellhoffite*.

890. Nitroglycerine is a heavy, oily liquid, formed by the action of a mixture of strong nitric and sulphuric acids upon glycerine. It is a chemical compound, and as such differs from most other explosives. The dissociation of atoms takes place instantaneously throughout its mass, and thus affords one of the most powerful explosives known.

Its specific gravity is 1.6. It freezes at 40° F. Heated to 360° F., it either burns or explodes. One volume of nitroglycerine exploded yields 1,298 volumes of gas. Nobel places the temperature of the explosion at 3,270° F., and states that the *expanded* gases of the explosion will occupy 10,384 times the original volume, which will develop a ruptive pressure of 76.322 tons per square inch, under ordinary conditions.

Nitroglycerine, when frozen, will not explode by any ordinary cause; but an elevation of temperature makes its handling dangerous. It is readily exploded by a smart blow, when spread upon a flat surface; but a bottle of the liquid may be smashed to pieces, at times, without causing an explosion. When nitroglycerine has become sour and impure, spontaneous decomposition is developed, forming gas and oxalic acid, which often results in a disastrous explosion, especially when the liquid is contained in a tightly-stoppered vessel.

Nitroglycerine is rendered more safe for blasting purposes and for transportation, by its being employed in the form of *dynamite.*

891. Dynamite.—This explosive is nitroglycerine, absorbed by any porous substance. There are different grades of dynamite, differing by the varying amount of nitroglycerine absorbed. They are rated as follows, the percentages varying according to the different brands :

Grade No. 1, from 50 to 70 per cent. nitroglycerine.
Grade No. 2, from 33 to 50 per cent. nitroglycerine.
Grade No. 3, from 27 to 30 per cent. nitroglycerine.
Grade No. 4, from 20 to 25 per cent. nitroglycerine.

The principal brands in use are "Hercules," "Atlas," and

"Ætna." The dynamite cartridges consist of strong paper shells, previously dipped in melted paraffine, and filled with the explosive. They are usually 8 inches long and of the following diameters and weights:

Diameter $\frac{7}{8}$ inch	Weight about 4 ounces.
Diameter 1 inch	Weight about 5 ounces.
Diameter $1\frac{1}{4}$ inches......	Weight about 8 ounces.
Diameter $1\frac{1}{2}$ inches......	Weight about 12 ounces.
Diameter $1\frac{3}{4}$ inches......	Weight about 15 ounces.
Diameter 2 inches......	Weight about $1\frac{1}{4}$ pounds.
Diameter 3 inches......	Weight about 3 pounds.
Diameter 4 inches......	Weight about 5 pounds.

The weight of any dynamite cartridge may be calculated by means of the following simple rule:

Rule.—*Multiply the square of the diameter of the cartridge by its length, all in inches, and take $\frac{5}{8}$ of the product; the result will be the weight of the cartridge in ounces.*

Let W = weight of cartridge (ounces);
d = diameter of cartridge (inches);
l = length of cartridge (inches).

Then, $W = \frac{5}{8} l d^2$. **(22.)**

An average No. 2 grade of this explosive will yield an initial ruptive pressure of 24 tons per square inch.

Safe methods of using dynamite are explained further on, in the section on Shafts, Slopes, and Drifts.

Other forms of dynamite have been invented and brought forward from time to time. These mostly consist of nitroglycerine, in smaller quantities, absorbed in various waste products, as cork shavings, sawdust, etc. In the original dynamite, the absorbent was an infusorial earth found in northern Germany, which absorbed three times its own weight of nitroglycerine. The forms of dynamite referred to above are known as *carbonite*, *stonite*, and *ardeerite*.

892. Guncotton (nitro-cotton) is a product similar in all respects to nitroglycerine, being formed by the action of a mixture of strong nitric and sulphuric acids upon ordinary cotton, or cellulose, wood-pulp, paper, or rags. In appearance, guncotton resembles ordinary cotton; 100 parts, by weight, of cotton should form 183 parts of guncotton; but, on account of more or less incomplete action, and a solution of a portion of the guncotton, before the whole mass has been converted, in practice 100 parts of cotton yield only from 160 to 178 parts of guncotton.

	Exploded in Free Air.	Detonated Under Pressure.
Carbonic oxide gas........	30 parts	40
Carbonic acid gas.........	20 parts	25
Marsh-gas.................	10 parts	Trace
Nitrogen dioxide	9 parts	None
Nitrogen..................	8 parts	15
Hydrogen..................	None	20
Aqueous vapor	23 parts	None

Guncotton is exploded by percussion. In some cases, it has been known to explode with violence when heated to 110° F., although other instances are recorded where the temperature has been raised to 200° F. without an explosion taking place. It has been known to be exploded by the heat of the sun's rays. It is liable to decompose, which often results in spontaneous combustion. Exploded, it yields a gaseous product consisting of 100 parts, as shown in the above table.

As will be readily seen, from its gaseous products, it is not adapted for use in mine workings. Its explosive force, as compared with an equal weight of gunpowder, is as 4.5 to 1.

893. *Tonite* and *potentite* are forms of guncotton to which nitrates of potassium, or barium have been added.

894. *Gelatine-dynamite*, or blasting-gelatine, and also gelignite, are mixtures of nitroglycerine and guncotton, on the supposition that a more perfect combustion is thereby obtained. A honey-colored, gelatinous mass is obtained, which does not freeze as readily as nitroglycerine or dynamite, and withstands the action of water better. It is more liable to explosion from a sudden blow than is dynamite. Its gaseous products prevent its general use in mining.

895. Sprengel Explosives.—What may very properly be called the *Sprengel explosives*, after their inventor, are the highly nitrated compounds formed by varying mixtures of nitrate of ammonium, $(NH_4) NO_3$, which contains 60 per cent. of its weight of oxygen, with other nitrated compounds, as nitro-naphthalene, nitro-benzol, etc. The explosives belonging to this class are of recent invention and are not well known; but, on account of the property which they all possess, to a greater or less extent, of suppressing the *flame* of their explosion, they will eventually find an important application in mining (Art. **888**). The most important and best known of these are *roburite*, *securite*, *ammonite*, *oxonite*, called also *rack-a-rock*, and *bellite*. The first three of these are alluded to as exceedingly safe and powerful explosives, by G. W. Wilkinson and other competent authorities.

896. Comparison of Explosives.—The value of an explosive lies in its being instantly convertible into gaseous products, having a high temperature and being incombustible. The explosive that embodies these qualifications to the highest degree is the strongest. However, except in very gaseous mines, high explosives are not used in coal-mining, because they shatter the coal and make too much small coal and slack. The less powerful and slower black powder is used, as it breaks down the coal in larger lumps.

(*a*) It is necessary, in order to secure the greatest rending force in an explosive, that its transformation into the gaseous state should be *instantaneous* and *complete*.

(*b*) The higher the temperature developed in the explosion, the greater will be the expansive force of the gaseous products.

(*c*) The more incombustible the gaseous products, the less flame will be produced by the explosion, and the more security will attend its use in gaseous mines.

The following table will be of use to the mining student, in making comparisons between some of the more common explosives in use in mines.

TABLE 20.

Explosive.	Temperature of the Explosion (F.).	Products of Explosion.		Ruptive Pressure. (Pounds per Sq. In.)
		Combustible.	Incombustible.	
Blasting powder..	2,000° to 3,600°	42 per cent.	58 per cent.	12,400 to 20,500
Nitroglycerine ...	5,740°	0 per cent.	100 per cent.	152,640
Dynamite	5,280°	0 per cent.	100 per cent.	48,000
Blasting-gelatine..	5,830°	46 per cent.	54 per cent.	....
Guncotton	4,800°	61 per cent.	39 per cent.	90,000 to 100,000
Tonite	4,800°	8 per cent.	92 per cent.	....
Roburite	3,800°	0 per cent.	100 per cent.	....
Ammonite	......	0 per cent.	100 per cent	....
Securite..........	......	0 per cent.	100 per cent.	....
Carbonite	......	41 per cent.	59 per cent.	....

Table 21 gives the temperature of combustion of some of the more important gases relative to mining chemistry.

TABLE 21.

Gases.	Temperature of Combustion (F.).
Marsh-gas..................	1,220°
Ordinary illuminating gas ..	1,198°
Carbonic oxide gas.........	1,184°
Hydrogen.................	1,148°

897. Character of Mine Explosions.—Many conditions influence and determine the character of an explosion of mine gases. The term *explosion*, in its present application. is broadened to include any type of rapid combustion of mine gases in the air-passages or workings, from a quiet burning, sweeping the roof of the passage and advancing at a moderate velocity, to a wild hurricane of fire, dust, and débris, propelled at an inconceivable speed by the expansive energies caused by the ignition of the gases in the air. The conditions that thus determine the character of an explosion of mine gases are, briefly, as follows:

(*a*) The **proportionate mixing of the gases** and their affinities for each other when excited by heat produces the violence of their dissociation and recombination in other forms as compounds.

For example, the explosive mixture may be air charged with marsh-gas, in such proportions as to develop its maximum violence; or, on the other hand, a large proportion of carbonic oxide gas may be produced as a result of a local explosion of marsh-gas, and this gaseous mixture may burn quietly along the roof of a passageway without exploding. Again, these conditions may suddenly change, and the slow burning at any moment develop explosive violence by contact with another body of gas.

(*b*) The **oxygen of the air** being the ever-ready means to dissociation, the abundance of its supply in the air of the workings determines largely the chemical activities.

(*c*) **Coal-dust,** suspended in the air of mine workings, acted upon by the flame of an explosion, distils **carbonic oxide gas.** This gas has the effect of lengthening the flame, which feeds upon it, and thereby propagates an otherwise local explosion.

(*d*) The physical surroundings of an explosion of mine gases, such as the size of the working places, and all the conditions which hinder the free expansion of the gases, affect the **pressure** and **temperature** of the explosion. These are important factors in determining the products of the explosion and the extent of the flame.

898. Causes of Ignition.—These are many. The ignition of an explosive mixture of gases requires some cause that will raise its temperature to the point of ignition. In the case of firedamp, however, this temperature must be maintained for a certain fraction of a second, or the gas fails to ignite. This is a very important point, as upon it depends the security of detonating explosives.

For example, the initial temperature of the explosion of dynamite is 5,280° F. (Table 20); but so rapid is the propagation of the combustion in the dynamite, that this temperature is only maintained for a time not exceeding $\frac{1}{1000}$ of a second, when its heat is converted into mechanical work, the temperature falling simultaneously with the expansion far below the point of ignition for firedamp (1,220° F.), and thus failing to ignite this dangerous mixture. The interval of time necessary for the ignition of firedamp is probably due to the absorption of heat by the watery vapor formed by the dissociation, and which must be converted into steam at a high temperature before ignition of the gaseous products can take place.

In the case of the ignition of a body of gas (firedamp), the cause is usually the flame of a naked lamp, or a defective safety-lamp, or the flame incident to blasting.

In the case of an explosion in a *non-gaseous* mine, the gases which enter into the explosion are derived from the distillation of the coal-dust suspended in the air, and, in a measure, also from the fine coal pulverized by the crushing force of the blast. In this latter case, the cause of ignition is plainly the projected flame of a "*blown-out*" shot, which has a volume and intensity sufficient for the conversion of a large body of suspended dust into gas.

899. Temperature of an Explosion.—In any explosion whatever, whether it be a body of gas in the mine workings, or a charge of powder, or other explosive, in a drill-hole, the primary or initial temperature of the reaction is determined from the *heat units*, stored in the original constituents of the explosive mixture, and the *specific heats* or *heat capacities* of the resulting products of the explosion. This *temperature of ignition* may be calculated from the principles of thermal chemistry, and is always a ***fixed*** temperature, as far as the explosive is concerned.

The *temperature of the explosion*, on the other hand, is determined or influenced by other causes, and is always, to a greater or less extent, lowered by external causes; as, for example, (*a*) loss of heat, by conduction, before the full development of the explosion; (*b*) loss of heat, by absorption due to expansion, before the full development of the explosion.

It will be readily seen that these losses are larger, the slower the progress of the combustion. Thus, in the case of a deflagrating charge, as of *black powder*, whose temperature of *ignition* is 3,632° F., the temperature of *explosion*, depending upon the strength of the resisting walls, is lowered to a practical 2,000° F. In the case of the quiet burning of a body of firedamp, diluted below the explosive point, or the burning of a trail of carbonic oxide gas, left in a passageway at times by the quick advance of an explosion, and fed later by fresh air from rooms or chambers, the *effective* temperature of the burning is often far below the actual temperature of ignition of these gases (firedamp 1,220°, carbonic oxide gas, 1,184°), on account of the absorption of the heat of ignition by the freely expanding gases.

900. Coal-Dust.—This discussion would not be complete without some special reference to the influence of this dangerous factor, present to a greater or less extent in many coal-mine explosions.

The presence of coal-dust suspended in the air of mine workings, and acted upon by a flame of sufficient volume and intensity, gives rise to *two* practical effects, viz.:

(*a*) Elongation and propagation of the flame.

(*b*) Widening of the explosive range of firedamp.

These effects have been described, (*a*), Art. **897** (*c*), and (*b*), Art. **860** (*b*).

The *facts* in regard to any kind of dust, and its influence upon the character of an explosion, are the following:

(*a*) The dust must be combustible, or it has, comparatively, no effect.

(*b*) The finer the dust and the more inflammable its nature, the quicker and fiercer will be its combustion.

(*c*) The free suspension of the dust in the air, and its complete combustion, are greatly assisted by a strong air current.

(*d*) The coal-dust (fine and larger particles) is heated to incandescence by the flame of the burning gases, distilling combustible gas, which adds to the flame, thereby transmitting the explosion through the airways.

(*e*) The incandescent carbon has the power to convert any carbonic acid gas (CO_2) with which it comes in contact into combustible carbonic oxide gas (CO).

The above facts are the results of practical experience, derived from actual observation of such occurrences, guided by an intelligent knowledge of the chemical possibilities, as demonstrated by experiment. The dust of anthracite coal is not susceptible of explosion under the prevailing conditions, being less friable and requiring a higher temperature to distil its gases.

901. Reducing Liability to Explosion.—The liability to accident by explosion can be reduced only by removing, as far as it is possible to do so, the causes and conditions which lead to such explosions. The incipient conditions of a mine explosion are, with rare exceptions, found in the following:

(*a*) A body of marsh-gas, collected in some cavity or recess of the roof or disused heading; or issuing suddenly from the working face, as a *feeder* or an *outburst*, and becoming transformed into a body of firedamp by its mixture with the air of the workings; and the presence and contact of the flame of a naked light, or a defective safety-lamp, or the projected flame of a blast, or, as sometimes occurs, the flame of a safety-lamp blown through its gauze by the force of the current, or the force of a blast, to which it has been inadvertently exposed.

(*b*) The presence of a considerable quantity of fine coal, in the form of dust, in close proximity to a working face where blasting is performed; and the projection of the flame of a *blown-out shot* of such volume and intensity as to effect the raising of a cloud of the dust, and to convert the same into an incandescent volume generating combustible gas. This action has been proven, by the convincing results of experiments which leave no room to doubt, that the presence of marsh-gas, while it stimulates and strengthens the explosion, is by no means essential to it.

(*c*) The successive and quick firing of several shots, in a close working place, may precipitate an explosion, from the firing of considerable volumes of carbonic oxide gas, produced in the discharge of the first shots.

SAFETY-LAMPS.

DESCRIPTION OF LAMPS.

902. General Description.—A safety-lamp is a lamp of special construction. In appearance it very much resembles a small lantern, which it is. The flame is completely enclosed in wire gauze or in glass and wire-gauze casings, which prevent its contact with an outside body of gas. Its use serves two purposes; viz., *first*, protection in gaseous workings where an open light would cause serious results, by the ignition of the gas; and, *second*, to indicate to the miner the presence of gas.

903. Principle of the Safety-Lamp.—From our previous study of combustion (Art. **873**), we have learned that the temperature of the burning gases must not fall below the point of ignition of those gases, or the flame will be extinguished. Whenever this temperature is not reached at the initial points of reaction, there can be no ignition, and hence no flame.

In safety-lamps, the isolation of the flame is secured through the cooling effect of the wire gauze surrounding it. The gauze permits the passage of air or gas, but flame is extinguished when it comes in contact with the cool gauze. (See Art. **904.**)

904. Effect of Cooling.—Flame is the result of gases burning at a white heat. The temperature of ignition for each gas, in air, is a fixed point capable of calculation, and expresses the number of heat units evolved in the reaction, less the heat units absorbed and rendered latent by the products of the combustion. The proximity of any cooling surface whatever to a flame, has the effect of reducing the temperature of the reacting gases. The molecular vibrations of the cooler surface are so sluggish that the heat of the reaction in the flame is converted into *molecular work*, raising the temperature of the cool surface to a certain extent, but extinguishing the flame in its immediate proximity. This phenomenon may readily be observed by presenting a flame to a cool surface, when it will be seen that the flame does not touch the surface, but is separated by a thin layer of gas that does not burn, because it has been cooled below the point of ignition. This will continue as long as the surface remains cool. For the same reason, one may put a very cold hand, for a moment, into the flame of a fire without burning the hand or feeling the heat.

In the case of a flame impinging against a cool wire gauze, or other perforated surface, the conditions are very favorable for the cooling of the gases of the flame below the point of ignition, as they pass through the small openings. The gases are divided into minute streams or jets, by the meshes of the gauze, and cooled instantly, being thereby extinguished.

905. Temperature of Flame.—The cooling and extinguishing of a flame is greatly assisted by the air-currents pouring towards it, and diffusing among the gaseous molecules. This action of the air isolates, as it were, each burning hydrocarbon particle. Each separate particle is thus surrounded by an envelope which renders more possible the cooling of the particle, because its temperature is somewhat below the temperature of ignition at the center. This has led to the rather indefinite and often misleading phrase, "*temperature of the flame.*" We can not rightly speak of the *temperature of flame* except in a general way, because it is not a definite quantity, but depends wholly upon conditions of which we have no gauge, and we find a different effective temperature in different parts of the same flame.

906. Requirements of a Good Lamp.—Safety-lamps, as previously stated, are used for two separate purposes, and this has given rise to two types of lamps, differing quite widely in their construction. They are:

(*a*) Lamps for general mining use.

(*b*) Lamps for testing for gas.

The requirements a good lamp must possess, for the purposes of general use in a mine, are the following:

(*a*) Safety in strong currents.

(*b*) Minimum liability to accident.

(*c*) Maximum illuminating power.

(*d*) Diffusion of light upwards.

(*e*) Simplicity of construction, and security of fastenings or lock.

907. Davy Lamp.—Fig. 115 shows a perspective view of this lamp. Fig. 116 is a sectional view of the same lamp. The Davy lamp admits air freely through the lower part of the gauze, as shown by the arrows at *a a*; while the products of combustion pass out through the upper portion of the gauze cylinder *b b* and the gauze plate *c* at the top of the lamp. This free passage of the gas-charged air in and out of the lamp ensures a good cap, and has made the Davy lamp a favorite with fire bosses, notwithstanding the danger that is always present in the unbonneted Davy lamp of the flame of the lamp being communicated to the outside gas, either through flaming in the lamp or from exposure to a current. The lamp is not safe when exposed to a current of a greater velocity than 6 feet per second. When gas is present in a thin stratum at the roof, its presence will

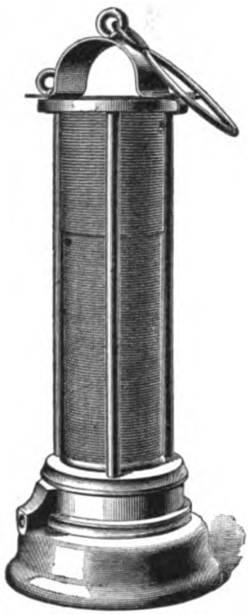

FIG. 115.

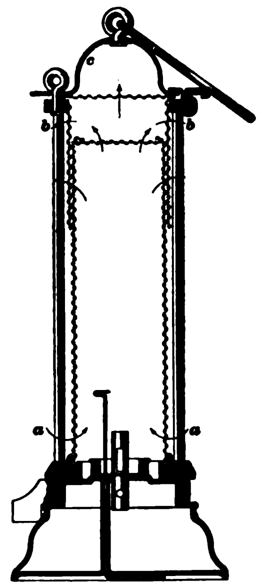

FIG. 116.

not be revealed by the Davy lamp. In the hands of a careful and experienced man, this lamp will detect the presence of gas in quantities as low as 3 per cent.

908. Stephenson Lamp.—This lamp consisted of a glass chimney surmounted by a perforated copper cap and surrounded by a perforated copper shield. The lamp gave a poor light, and was immediately supplanted by the Davy lamp with gauze covering.

909. Geordy Lamp.—This lamp, so called after George Stephenson, its inventor, was a combination of the glass chimney of the original Stephenson lamp and the gauze of the Davy lamp. It was regarded for a considerable time as a thoroughly reliable and safe lamp. It gave a better light than the Davy lamp, and for a while came into quite extensive use. It was quite susceptible to gas, and made a good lamp for testing, because the gas-cap could be more easily distinguished through the glass than through the gauze, although the caps were not as high as in the Davy lamp. The supply, or feed, was more restricted, and entered the lamp below the flame and passed out through the gauze above the glass chimney.

910. Clanny Lamp.—This lamp was designed to secure greater protection for the flame, combined with a better light, than was provided in the Geordy lamp. A perspective view of the Clanny lamp is shown in Fig. 117, and a

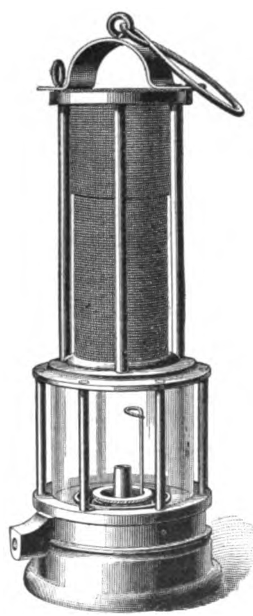

FIG. 117.

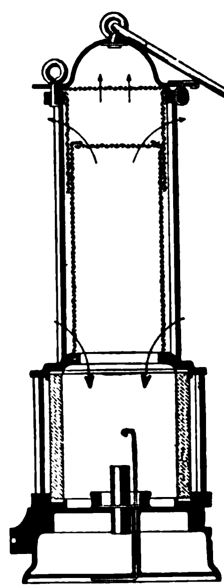

FIG. 118.

section of the same in Fig. 118. The air, instead of being admitted below the flame, as in the Geordy lamp, is admitted through the lower portion of the gauze cylinder, just above the glass, and descends, within the lamp, to the flame.

The lamp, while it may present some points of protection of the flame against strong currents, does not make a good lamp, either for testing or for general purposes of illumination. The glass is apt to become dimmed by the smoke of the flame, owing to the interference of the downward and upward currents above the flame. A considerable percentage of gas may be present in the air before its presence will be revealed by this lamp. The lamp loses its protective qualities whenever sufficient gas is present to produce flaming.

911. Evan Thomas Lamp.—This is an improvement upon the Clanny lamp, in two points; viz., the air drawn in at *a* (Fig. 119) is conducted downwards between the two

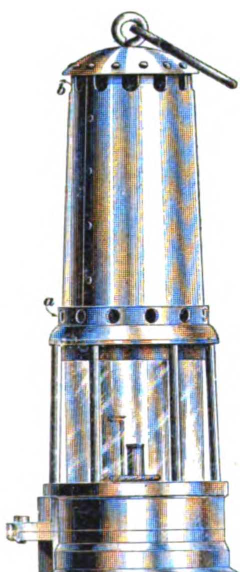

FIG. 119.

FIG. 120.

glass chimneys with which the lamp is provided, and enters the lamp below the flame. The upper gauze of the lamp is provided with a sheet-iron bonnet, which is a great protection in case of flaming or inner explosion. The downward current of cool air serves, also, to keep the glasses cool, and increases their power to transmit light; a heated glass always impairs the transmission of light.

Fig. 120 shows another type of this lamp, designed for the use of fire bosses. The inner glass cylinder is replaced by a cylinder of gauze; in other respects the principle of the lamps is the same. At the time of the invention of this lamp, the study and designing of lamps, with respect to securing greater protection, received renewed attention, and resulted in bringing forward various devices having this end in view.

912. Marsaut Lamp.—The principal feature of this lamp, a perspective view of which is shown in Fig. 121 and a section of the same in Fig. 122, is the multiple-gauze

FIG. 121.

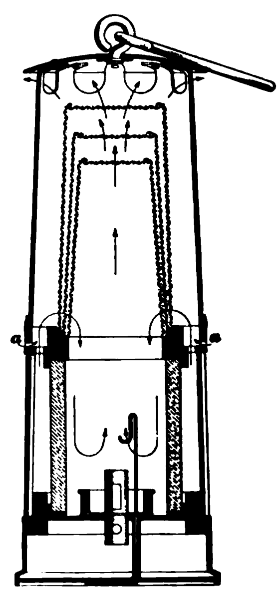

FIG. 122.

chimneys. The lamp shown in the figure (Fig. 122) has three of these gauze chimneys, one over the other, and an outer bonnet of sheet iron. This lamp is adapted for use in strong currents. The air enters the lamp above the glass chimney; and much that has been said in reference to the Clanny lamp in this respect is applicable to the Marsaut.

913. Mueseler Lamp.—This lamp, of which Fig. 123 shows a perspective view and Fig. 124 a section, presents an

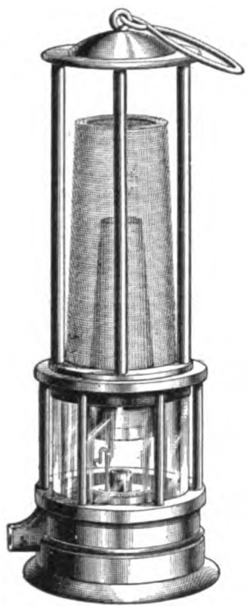

FIG. 123.

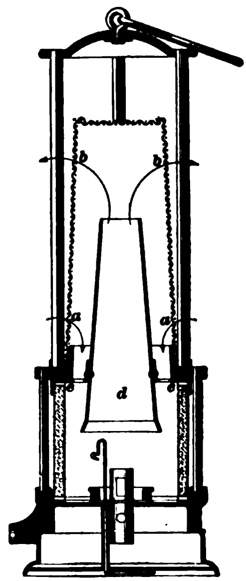

FIG. 124.

important departure. It is the first lamp introducing a feature calculated to increase its draft, and thereby improve its illuminating power. In this lamp is provided a central tube or chimney *d* of sheet iron, conical in shape, and held in position by a horizontal, perforated diaphragm of sheet iron *e e*, at the junction of the gauze and glass cylinders. The air enters the lamp through the gauze at *a a*, and, passing through the perforations of the diaphragm, is drawn down under the expanded mouth of the central chimney and in close proximity to the flame. The draft of this chimney increases to a considerable degree the illuminating power of the lamp, while the central tube adds very largely also to the security of the lamp against currents and inner explosions, the latter seldom being communicated outside of the lamp. It is not a lamp adapted to the detection of gas, but it has been known to withstand a current of 100 feet per second.

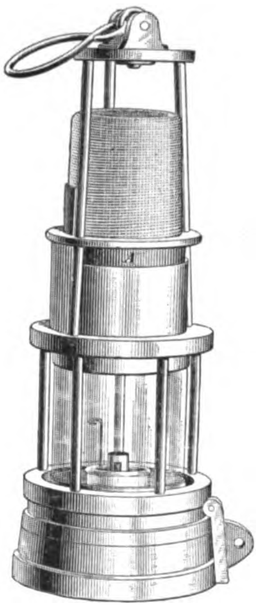

FIG. 125.

914. Howat's Deflector.—This consists of an annular ring *A* (Fig. 125), so arranged as to deflect the entering current of air downwards upon the flame. It has been fitted to the Marsaut lamp, with a marked improvement in the illuminating power of that lamp.

915. Ashworth-Hepplewhite-Gray Lamp.—Among the lamps of special design for testing for gas, that shown in Fig. 126 is perhaps the most convenient, and combines in one lamp many of the best features. The air, when the lamp is being used for testing, enters the tops of the four standards, as shown at *a a*, and, passing down the standards, enters the lamp below the flame, thereby producing the best conditions for yielding a good gas-cap. The glass chimney *c c* is made slightly conical, tapering towards the top; the same conical shape is, also, given to the gauze chimney *g*, above the glass. The gauze chimney is bonneted. The conical shape of the glass assists the

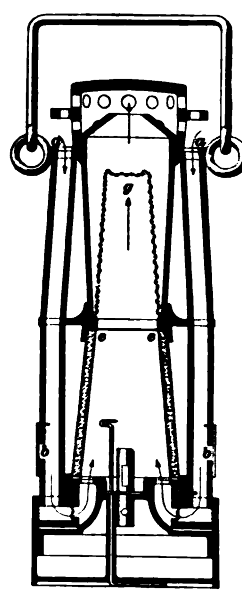

FIG. 126.

upward diffusion of the light, and makes the inspection of the roof easier; while the same shape in the gauze chimney renders the lamp more safe and secure against an inner explosion of gas being communicated outside of the lamp. The air being drawn into the lamp through the top of the standards, makes it possible with this lamp to detect a thin stratum of gas near the roof. When not in use for testing, the air may be admitted at the bottom of the standards, at *b b*, by moving a little shutter that closes them.

916. Pieler Lamp.—This lamp, shown in Fig. 127, is a gauze lamp, similar to a Davy lamp, but burns alcohol instead of oil, in order to render the observance of the gas-caps easier. In its safest form, the gauze is bonneted by a sheet-iron bonnet, the gas-caps being observed through a glass window. This window is very apt to become dimmed with smoke and moisture, and impair the observance of the caps. The lamp is provided with a shield *c*, surrounding the flame, and the latter is adjusted so that its tip does not extend above the top of the shield.

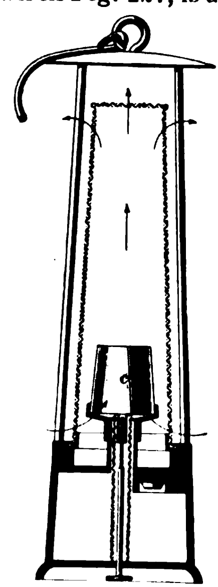

FIG. 127.

This lamp was designed by the inventor to yield a standard flame which would always present a certain height and volume, and yield flame-caps of a uniform height, for given percentages of gas. The following table was prepared by him to show the percentage of gas corresponding to different heights of flame-caps.

¼ per cent. of gas yields a cap 1.25 inches high.
½ per cent. of gas yields a cap 2.00 inches high.
1 per cent. of gas yields a cap 3.50 inches high.
1½ per cent. of gas yields a cap 4.75 inches high.
1¾ per cent. of gas { cap reaches the top of the lamp, and beyond this percentage of gas the lamp fills with flame.

The heights of these caps are measured from the top of the shield *c*. The lamp flames easily, in a mixture containing more than 1¾ per cent. of gas, and is, therefore, a source of danger, and requires great care and caution in its use. Any variation in the strength of the alcohol varies the height of the flame. The flame is, therefore, not strictly a standard flame.

917. The Illuminating Power of Safety-Lamps.—The amount of light given off by any safety-lamp is much less than that of the ordinary naked light used in mines. Table 22 gives the comparative illuminating power of some of the various lamps described. The light of a sperm candle is taken as 1, or unity.

TABLE 22.

Name of Lamp.	Illuminating Power of Lamp, with a Candle Taken as 1, or Unity.
Davy	0.16
Geordy	0.10
Clanny	0.20
Mueseler	0.35
Evan Thomas	0.45
Marsaut, 3 gauzes	0.45
Marsaut, 2 gauzes	0.55
Howat's Deflector	0.65
Ashworth-Hepplewhite-Gray	0.65 (about)

918. Flame-Caps or Gas-Caps.—By experiment, it has been determined that the presence of carbonic acid gas, even to the extent of 5 per cent., has no effect upon the flame-cap.

It has also been ascertained that the height of the flame-cap changes with the size and height of the flame itself, and also with the oil used to produce the flame.

919. The Oil Used in Safety-Lamps.—All the lamps described, with the exception of the improved Ashworth-Hepplewhite-Gray and the Pieler lamps, are constructed to burn either vegetable or seal oil. In the last two, light mineral oils are burned.

According to the English Mine Commission, the safest oils to use are vegetable oils, such as rape, made from rape-seed, and colza, made from cabbage-seed, and seal oil. None of these are explosive. Petroleum used alone is liable to explode, and should be avoided.

In point of brilliancy, the flame of a lamp burning seal oil is superior to one burning either rape or colza oil, and the wick is less liable to become charred.

By addition of one part of petroleum to two parts of rape or seed oil, the light is increased.

Many of the oils in common use in safety-lamps have a tendency to encrust the wick and thereby lower the flame. Sometimes petroleum or benzine has been added to the oil, which reduces this tendency, and yields a better flame for testing purposes. Alcohol yields a hotter and less luminous flame and a much higher cap. In some cases, a hydrogen flame has been used for testing purposes. The hydrogen is compressed into a small steel cylinder attached to the lamp, and is burned in the lamp at the mouth of a small tube. This apparatus gives a standard flame for testing, but it can not always be conveniently obtained.

920. Locks for Safety-Lamps.—All safety-lamps should be securely locked, and in such a manner as to preclude the possibility of being tampered with. Screw-pins are not an adequate protection.

The best lock, for security and cheapness, is the lead-plug lock, shown in Fig. 128.

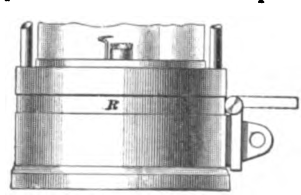

FIG. 128.

On the right-hand side of the oil-vessel of the lamp a pin projects, with a hole in it. Around the bottom of the top part of the lamp there is a thin, movable, metal ring *R*, and to this ring is fixed a hinged latch.

The ring is turned around, until the latch drops over the pin. A small plug of soft lead is put through the hole in the pin, to prevent the hinged latch from being lifted, and this lead plug is punched flat at both ends, to prevent it from being pulled out. The plugs are cast in a mold, at the colliery; and, as they are cut to pieces, in the lamp room, when the lamp is returned to be cleaned, they are collected and remelted, and the lead is used over and over again. To prevent tampering with the lead plug, it is punched up at both ends, with a punch containing a letter of the alphabet. These letters are interchangeable, and it is usual to use a new letter each day, so that the workmen can not counterfeit them.

Machines are used for locking these lamps, and other machines for cleaning them. Safety-lamps should be thoroughly cleansed at the close of every shift and put in readiness for another day.

TESTING FOR FIREDAMP IN MINES.

921. The Fire Boss.—The duties devolving upon a fire boss are of a very serious nature. In his hands is often placed the safety of every man in the mine. A simple oversight upon his part may result in the most appalling catastrophe.

The safety-lamp, at the present time, is the only practical means at the disposal of this man for the detection of firedamp. According to the good condition and sensitiveness of the lamp, and the experience of the man, his report of the condition of each working place and section of the mine under his charge is more or less accurate. That the fire boss should be a careful, painstaking, and conscientious man is readily seen upon a little reflection. Suppose, for example, a current of 50,000 cubic feet of air per minute is being furnished to a certain section of a gaseous mine. In this section, perhaps, the fire boss may detect a small percentage of gas in the current, say $\frac{1}{2}$ of one per cent. The total flow of gas is then $\frac{1}{2}$ of $\frac{1}{100}$ of $50,000 = 250$ cubic feet per minute. If a door is left open upon the airway, or a fall occurs so as to reduce the current, say, to 3,000 cubic feet of air per minute, this gas will render the reduced current explosive in a very short time, and only prompt and decisive action on the part of the fire boss will avert a catastrophe.

922. Testing by Lamp.—The use of the lamp, for the purpose of testing for gas, depends upon the observing of the height of a pale, bluish tip, or cap, to the flame of the lamp. If the lamp flame is too bright, a small gas-cap can not be seen, as the eye will be blinded by the light of the flame.

For this reason the non-luminous, alcohol or hydrogen flames are better adapted for observing the gas-cap. The body of the flame should be screened from the eye while taking an observation. This is sometimes effected by holding the hand between the flame and the eye, or by interposing a metallic screen, as in the Pieler lamp.

The flame of the lamp is usually lowered to a small size when testing, and it is always best to adopt a uniform size of flame, to ensure uniform results. No quick movement must be made. In case of flaming in the lamp, coolness and presence of mind are necessary to remove the lamp carefully from the gaseous body. A quick movement will precipitate an explosion by the forcing of the inner flame through the hot gauze.

A good lamp for testing purposes will have a free admission of air, preferably below the flame. The background of the flame, or the gauze through which the flame is observed, should present no reflecting surfaces, as any reflection interferes seriously with the sensitiveness of the observation.

The lamp is, thus far, the most practical means at our disposal for gas-testing in mines. The percentages of gas in the air, determined by its use, are necessarily only approximately accurate; but the determination is made at once at the point where the gas has accumulated, and the value of this approximate knowledge can not be disputed.

923. Testing by Machines.—Undoubtedly the Shaw Gas-Testing Machine, a view of which is shown in Fig. 129, is the most accurate, simple, and complete mechanical device for this purpose known. Its use, however, is restricted largely by its lack of portability. On account of this, it can not replace the method of testing for gas, at the working faces, by means of the safety-lamp.

The machine consists of two cylinders, or pump-barrels, *A* and *B*, constructed of such relative size, and so connected

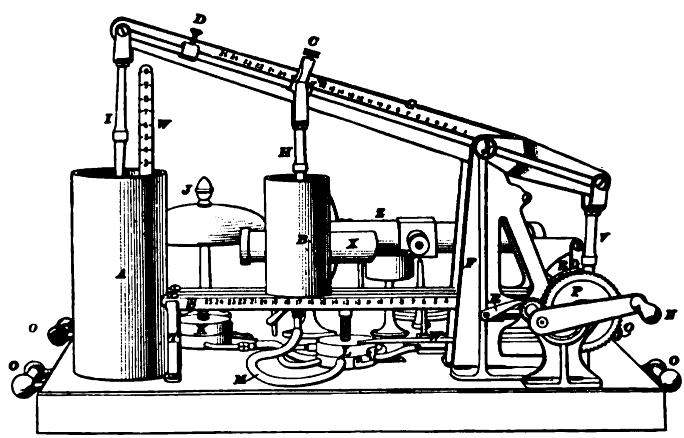

FIG. 129.

to a common lever *G*, as to pump relative quantities of gas and air into an ignition-chamber *Z*. One of the pump-cylinders *A* is stationary and pumps air, while the other cylinder *B* is so arranged as to be movable, and can be set to pump any proportionate amount of gas. Thus, it is easy to so arrange these two cylinders that a definite mixture of gas and air will be pumped into the ignition-chamber. The beam, or lever, is graduated to read the percentage of gas pumped.

The ignition-chamber *Z* is a cylinder having a loose piston. The mixture pumped into this chamber strikes first against the piston, at the left-hand end of the cylinder, and, filling the cylinder, is expelled through an igniting nozzle at the opposite end. If the mixture is explosive, its ignition and explosion drives the piston forcibly against the gong *J*, at the end of the cylinder. The ignition is accomplished by a small gas-jet, or other flame burning at the discharge orifice of the chamber.

MINE VENTILATION.

(PART 1.)

INTRODUCTORY.

GRAVITATION.

924. As a knowledge of gravitation and the laws of falling bodies is necessary in the study of mine ventilation, these subjects will be briefly treated before the principles governing the flow of air are discussed.

925. All bodies in the universe exert a certain attractive force on every other body, which tends to draw the bodies together. This attractive force is called **gravitation.**

If a body is held in the hand, a downward pull is felt, and if let go of will fall to the ground. This pull is commonly called *weight*, but it really is the attraction between the earth and the body.

926. Force of gravity is a term used to denote the attraction between the earth and bodies upon or near its surface. It always acts in a straight line between the center of the body and the center of the earth. The force of gravity varies at points on the earth's surface.

It is slightly less on the top of a high mountain than at the level of the sea. For this reason the weight of a body also varies. But if the weight of a body at any place be divided by the force of gravity at that place, the result is called the *mass* of the body.

927. The **mass of a body** is the measure of the actual amount of matter that it contains, and is *always the same.*

If the mass of the body is represented by m, its weight by W, and the force of gravity at the place where the body is weighed by g, we have

$$\text{mass} = \frac{\text{weight of body}}{\text{force of gravity}}, \text{ or } m = \frac{W}{g}. \qquad \textbf{(23.)}$$

928. Law of Gravitation:

The force of attraction by which one body tends to draw another body towards it is directly proportional to its mass, and inversely proportional to the square of the distance between their centers.

929. Laws of Weight:

Bodies weigh most at the surface of the earth. Below the surface, the weight decreases as the distance to the center decreases.

Above the surface, the weight decreases as the square of the distance increases.

ILLUSTRATION.—If the earth's radius is 4,000 miles, a body that weighs 100 pounds at the surface will weigh nothing at the center, since it is attracted in every direction with equal force. At 1,000 miles from the center it will weigh 25 pounds, since

$$4{,}000 : 1{,}000 = 100 : 25.$$

At 2,000 miles from the center it will weigh 50 pounds, since

$$4{,}000 : 2{,}000 = 100 : 50.$$

At 3,000 miles from the center it will weigh 75 pounds, and at the surface, or 4,000 miles from the center, it will weigh 100 pounds. If carried still higher, say 1,000 miles from the surface, or 5,000 miles from the center of the earth, it will weigh 64 pounds, since

$$5{,}000^2 : 4{,}000^2 = 100 : 64.$$

At 4,000 miles from the surface it will weigh 25 pounds, since

$$8{,}000^2 : 4{,}000^2 = 100 : 25.$$

930. Formulas for Gravity Problems:

Let W = weight of body at the surface;
w = weight of a body at a given distance above or below the surface;
d = distance between the center of the earth and the center of the body;
R = radius of the earth = 4,000 miles.

Formula for weight when the body is below the surface,

$$wR = dW. \qquad (24.)$$

Formula for weight when the body is above the surface,

$$wd^2 = WR^2. \qquad (25.)$$

EXAMPLE.—How far below the surface of the earth will a 25-pound ball weigh 9 pounds?

SOLUTION.—Use formula **24**, $wR = dW$.
Substituting the values of R, W, and w, we have

$$9 \times 4{,}000 = d \times 25;$$

or $d = \dfrac{9 \times 4{,}000}{25} = 1{,}440$ miles from the center. Ans.

EXAMPLE.—If a body weighs 700 pounds at the surface of the earth, at what distance above the earth's surface will it weigh 112 pounds?

SOLUTION.—Use formula **25**, $wd^2 = WR^2$.
Substituting the values of R, W, and w, we have

$$112 \times d^2 = 700 \times 4{,}000^2;$$

or $d = \sqrt{\dfrac{700 \times 4{,}000^2}{112}} = 10{,}000$ miles.

Therefore, $10{,}000 - 4{,}000 = 6{,}000$ miles above the earth's surface. Ans.

EXAMPLE.—The top of Mt. Hercules was said to be 32,000 feet, say 6 miles, above the level of the sea. If a body weighs 1,000 pounds at sea-level, what would it weigh if carried to the top of the mountain?

SOLUTION.— $wd^2 = WR^2$; or, $w \times 4{,}006^2 = 1{,}000 \times 4{,}000^2$;

whence, $w = \dfrac{4{,}000^2 \times 1{,}000}{4{,}006^2} = 997$ pounds. Ans.

EXAMPLES FOR PRACTICE.

1. How much would 1,000 tons of coal, weighed at the surface, weigh one mile below the surface? Ans. 1,999,500 lb.

2. How much would the coal in example 1 weigh one mile above the surface? Ans. 1,999,000 lb., nearly.

3 How far above the earth's surface would it be necessary to carry a body in order that it may weigh only half as much ?

Ans. 1,656.854 miles, nearly.

4. A man weighs 160 pounds at the surface; how much will he weigh 50 miles below the surface ? Ans. 158 lb.

5. If a body weighs 100 pounds 400 miles above the earth's surface, how much will it weigh at the surface ? Ans. 121 lb.

NOTE.—Use 4,000 miles as the radius of the earth.

FALLING BODIES.

931. If a leaden ball and a piece of paper are dropped from the same height, the ball will strike the ground first. This is not because the leaden ball is the heavier, but because the resistance of the air has a greater retarding effect upon the paper than upon the ball. If we placed this same leaden ball and a piece of paper in a glass tube, Fig. 130, from which all of the air has been exhausted, it would be found that, when the tube was inverted, both would drop to the bottom in exactly the same time. This experiment proves that it was only the resistance of the air that caused the ball to reach the ground first in the former experiment. This resistance of the air may be nearly equalized by making the two bodies of the same shape and size. For example, if a wooden and an iron ball, having equal diameters, were dropped from the same height, they would strike the ground at almost exactly the same instant, although the iron ball might be ten times as heavy as the wooden ball.

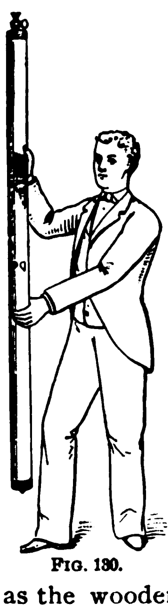

FIG. 130.

Suppose there were several leaden balls, as shown in Fig. 131, at *a*; it is obvious that if they were dropped together, all would strike the ground at the same time. If the balls were melted together into one ball, as *b*, they would still fall together, and strike the ground in the same time as before.

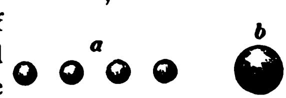

FIG. 131.

Since a number of horses can not run a mile in less time than a single horse, so 100 pounds can fall no farther in a given time than 1 pound can.

932. Acceleration is the rate of increase of velocity. If a force acts upon a body free to move, then, according to the first law of motion, it will move forever with the same velocity unless acted upon by another force.

Suppose that, at the end of one second, the same force were to act again, the velocity at the end of the second second would be twice as great as at the end of the first second. If the same force were to act again, the velocity at the end of the third second would be three times that at the end of the first second. So, if a constant force acts upon a body free to move, the velocity of the body at the end of any time will be the velocity at the end of one second, multiplied by the number of seconds.

This constant force is called a **constant accelerating force,** or **constant retarding force,** according as the velocity is constantly *increased* or *decreased.*

If a body is dropped from a high tower, the velocity with which it approaches the ground will be constantly increased or accelerated; for the attraction of the earth, or force of gravity, is constant and acts upon the body as a constant accelerating force. It has been found by careful experiments that this force of gravity, or constant accelerating force on a freely falling body, is equivalent to giving the body a velocity of 32.16 feet in one second; it is always denoted by g. As was mentioned before, g varies at different points on the earth, being 32.0902 at the equator and 32.2549 at the poles. Its value for this latitude (about 41° 25′ north) is very nearly 32.16, and this value should always be used in solving problems. It has also been found by experiment that a freely falling body starting from rest will have fallen 16.08 feet at the end of the first second; 64.32 feet at the end of the second second; 144.72 feet at the end of the third second; 257.28 feet at the end of the fourth second, etc., all of which are shown in the diagram, Fig. 132.

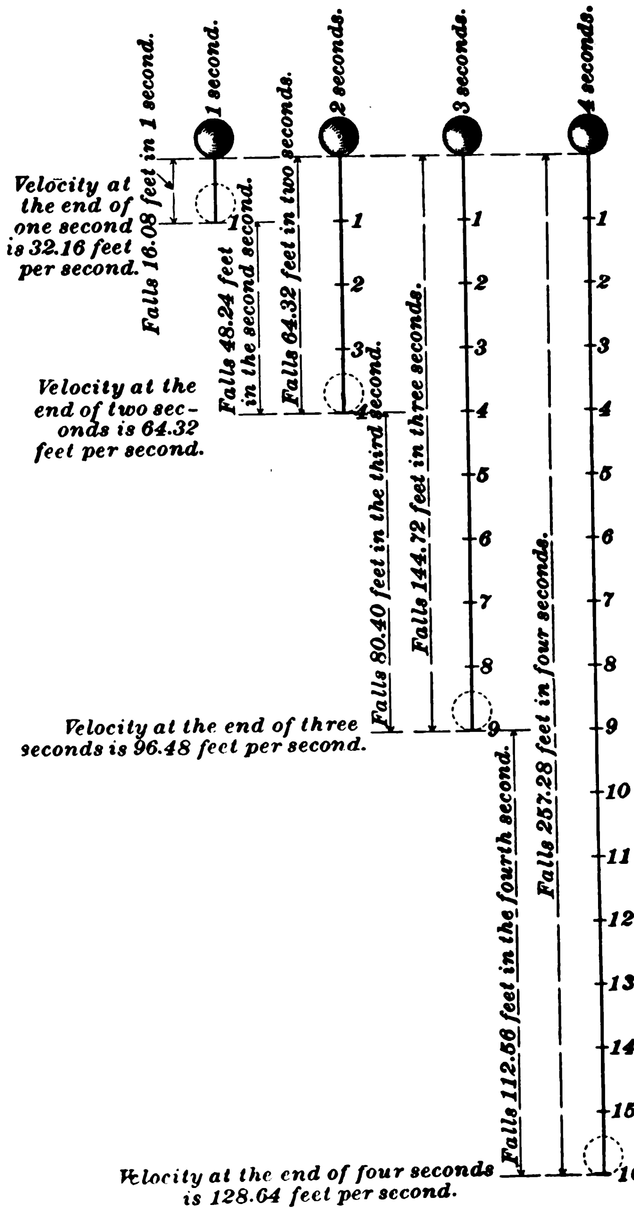
1 second.
2 seconds.
3 seconds.
4 seconds.
Falls 16.08 feet in 1 second.
Velocity at the end of one second is 32.16 feet per second.
Falls 48.24 feet in the second second.
Falls 64.32 feet in two seconds.
Velocity at the end of two seconds is 64.32 feet per second.
Falls 80.40 feet in the third second.
Falls 144.72 feet in three seconds.
Velocity at the end of three seconds is 96.48 feet per second.
Falls 112.56 feet in the fourth second.
Falls 257.28 feet in four seconds.
Velocity at the end of four seconds is 128.64 feet per second.

FIG. 132.

Since $\frac{64.32}{16.08} = 4 = 2^2$; $\frac{144.72}{16.08} = 9 = 3^2$; $\frac{257.28}{16.08} = 16 = 4^2$, and 2^2, 3^2, 4^2 are the squares of the number of seconds during which the body falls, it is easy to see that the space through which a body free to move will fall in a given time is equal to 16.08 multiplied by the square of the time in seconds.

Since $16.08 = \frac{32.16}{2} = \frac{1}{2} g$, the space $= \frac{1}{2} g \times$ square of time in seconds.

933. Formulas for Falling Bodies:

Let g = force of gravity = constant accelerating force due to the attraction of the earth;
t = number of seconds the body falls;
v = velocity at the end of the time t;
h = distance that a body falls during the time t.

$$v = g t. \qquad \textbf{(26.)}$$

That is, the velocity acquired by a freely falling body at the end of t seconds equals 32.16, multiplied by the time in seconds.

EXAMPLE.—What is the velocity of a body after it has fallen 4 seconds, assuming that the air offered no resistance?

SOLUTION.—Using formula **26**,

$$v = g\,t = 32.16 \times 4 = 128.64 \text{ feet per second. Ans.}$$

$$t = \frac{v}{g}. \qquad \textbf{(27.)}$$

That is, the number of seconds during which a body must have fallen to acquire a given velocity equals the given velocity in feet per second, divided by 32.16.

EXAMPLE.—A falling body has a velocity of 192.96 feet per second; how long had it been falling at that instant?

SOLUTION.—Using formula **27**,

$$t = \frac{v}{g} = \frac{192.96}{32.16} = 6 \text{ seconds. Ans.}$$

$$h = \frac{v^2}{2 g}. \qquad \textbf{(28.)}$$

That is, the height from which a body must fall to acquire a given velocity equals the square of the given velocity, divided by 2 × 32.16.

EXAMPLE.—From what height must a stone be dropped to acquire a velocity of 24,000 feet per minute?

SOLUTION.— 24,000 ÷ 60 = 400 feet per second. Using formula **28**,

$$h = \frac{v^2}{2g} = \frac{400^2}{2 \times 32.16} = \frac{160,000}{64.32} = 2,487.56 \text{ feet. Ans.}$$

$$v = \sqrt{2gh}. \qquad \textbf{(29.)}$$

That is, the velocity that a body will acquire in falling through a given height equals the square root of the product of twice 32.16 and the given height.

EXAMPLE.—A body falls from a height of 400 feet; what will be its velocity at the end of its fall?

SOLUTION.—Using formula **29**,

$$v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 400} = 160.4 \text{ feet per second. Ans.}$$

$$h = \tfrac{1}{2}gt^2. \qquad \textbf{(30.)}$$

That is, the distance a body will fall in a given time equals 32.16 ÷ 2, multiplied by the square of the number of seconds.

EXAMPLE.—How far will a body fall in 10 seconds?

SOLUTION.—Using formula **30**,

$$h = \tfrac{1}{2}gt^2 = \tfrac{1}{2} \times 32.16 \times 10^2 = 1,608 \text{ feet. Ans.}$$

$$t = \sqrt{\frac{2h}{g}}. \qquad \textbf{(31.)}$$

That is, the time it will take a body to fall through a given height equals the square root of twice the height, divided by 32.16.

EXAMPLE.—How long will it take a body to fall 4,116.48 feet?

SOLUTION.—Using formula **31**,

$$t = \sqrt{\frac{2 \times 4,116.48}{32.16}} = 16 \text{ seconds. Ans.}$$

A body thrown vertically upwards starts with a certain velocity called the **initial velocity.** In this case gravity acts as a constant retarding force. The formulas given above will also apply in this case.

EXAMPLE.—If a cannon-ball is shot vertically upwards with an initial velocity of 2,000 feet per second, (*a*) how high will it go? (*b*) How long a time must elapse before it reaches the earth again?

SOLUTION.—(*a*) Using formula **28**,

$$h = \frac{v^2}{2g} = \frac{2{,}000^2}{2 \times 32.16} = 62{,}189 \text{ feet, nearly, } = 11.778 \text{ miles. Ans.}$$

To find the time it takes to reach a height of 62,189 feet, use formula **27**.

$$t = \frac{v}{g} = \frac{2{,}000}{32.16} = 62.19 \text{ seconds.}$$

Since it will take the same length of time to fall to the ground, the total time will be $62.19 \times 2 = 124.38$ seconds $= 2$ minutes 4.38 seconds. Ans.

THE NECESSITY OF VENTILATION.

934. Ventilation is the replacing of the foul air contained in an enclosed space by fresh air from the atmosphere.

To a person accustomed to working out of doors the necessity of ventilation is not apparent. He breathes, and the foul gas exhaled from his lungs dissipates into the ocean of atmosphere about him, leaving no trace behind, so rapidly is it diluted by the ever-moving air around him. When he descends into a mine, the case is widely different. Here, unless assisted by artificial means, the air-currents move very slowly or not at all. Poisonous gases from the workings must be diluted by fresh air; the men require a certain amount of fresh air to sustain life; the lamps require a certain amount in order that they may burn and give forth light; the horses or mules require still more; air or an air-current is required for other purposes. The result of all this is that unless a constant supply of fresh air is being circulated through the mine, it very soon becomes impossible for men or animals to live in it—much less work there.

The science of mine ventilation may be comprised under three general headings:

1. The quantity of air required.
2. The laws governing the flow of air through mines.
3. The means for inducing the flow of air through mines.

THE QUANTITY OF AIR REQUIRED.

935. The question as to what amount of air is necessary in mines does not admit of an exact answer. No two mines present the same conditions, and what is an ample provision of air in one mine is inadequate in another.

As each man requires a certain amount of pure air at every breath, it has been the rule in the past to select one man as the unit of calculation, and to allow so many cubic feet for every man employed underground. Some writers have made additional allowances for the mules and lamps.

Any estimate based on these lines is mere guesswork. The amount of air necessary for the support of life and the combustion of lights is insignificant in comparison with the other requirements.

A man requires a quantity of air which varies according to the exertion he is making, and this quantity, for a miner, may be estimated at 28 cubic feet per hour, or half a cubic foot per minute. A lamp consumes about the same quantity, and a mule about six times as much as a man.

A considerable quantity of air is required to render harmless the gas transpiring from the coal. If this gas were given off regularly, a correct estimate of the quantity of air required to dilute and render it harmless could be arrived at; but, owing to sudden outbursts, this can not be done.

A shallow mine is more likely to have had the gas drained off by the nearness of the seam to the surface, and is, therefore, not likely to require so much air for the removal of gas, in working, as a deep mine.

A change in the barometer has a decided influence upon the ventilation. A low barometer indicates a lighter weight of the air, and this, by reducing the pressure, assists in the freer admission of standing gas from the goaves and disused workings, and makes necessary an increased quantity of air to remove this gas. Heated air requires more fresh air to reduce the temperature and make the atmosphere of the mine healthy and comfortable for the workmen.

At the same time, it is necessary to remember that, although the current should be sufficiently strong to enable it to be felt if the face is turned towards it, it must still not be so strong as to chill those who enter it while sweating.

936. The laws relating to the ventilation of coal-mines in the different States of the Union require, with two exceptions, a minimum of 100 cubic feet of air per man per minute. The Anthracite Mine Law of Pennsylvania fixes 200 cubic feet per man per minute as the minimum. The law of the State of Maryland fixes no minimum, but requires that the mine "shall be in a healthful condition for the men working therein." The English Mines Regulation Act of 1887 requires "sufficient to dilute and render harmless all noxious gases."

937. Instead of attempting to fix the quantity required at so much per man, it would be better to class the mines in each district into groups, having reference to the number of men employed, the area of the workings, the output, the nature of the coal, the depth of the workings from the surface and the general conditions regarding the amount of gas evolved, etc., and to make an average estimate of the volume required for the mines of each group.

In such a classification, the increase of the ventilation would be in accordance with the importance of the different requirements. These requirements may be summarized as follows :

The total quantity of air required should increase—

1. With the maximum number of men employed;
2. With the maximum number of mules in use;
3. With the maximum quantity of explosives used;
4. With the maximum daily output;
5. With the depth of the seam from the surface,
6. With the thickness of the seam;
7. With the extent of the live workings;
8. With the extent of the gob.

The volume of air to be allowed for these causes can be determined only after careful and exhaustive research, but, if determined, it would ensure safety much more certainly than the minimum system at present in vogue.

In the largest and most gaseous mine in the anthracite region of Pennsylvania, the average quantity provided per man per minute ranges from 200 to 700 cubic feet.

THE LAWS GOVERNING THE FLOW OF AIR.

THE THEORETICAL VELOCITY OF AIR.

938. The **theoretical velocity** of air is the velocity at which the air enters the downcast shaft, and before it is subject to the resistance of friction due to the sides of the mine passages. It is a purely theoretical quantity and of little practical use. To produce a flow of air between the upcast and downcast shafts, the pressure, or weight, of the column of air in the downcast must be greater than the pressure, or weight, of the column of air in the upcast.

939. Suppose that Fig. 133 represents a section of a mine in which the downcast shaft *A B* and the upcast shaft *D C* have the same height. The air can be caused to flow from *A* to *D* by creating a difference of pressure or of weight in the columns of air in the two shafts, that in the shaft *D C* being less than that in the shaft *A B*.

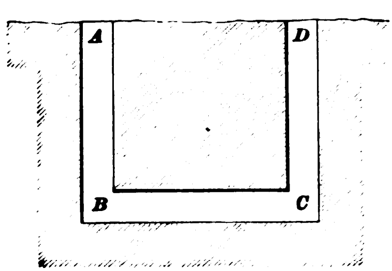

FIG. 133.

This difference of pressure, or weight, of the air columns can be created in two ways: (1) By increasing the density, or pressure, of the air in the shaft *A B*. (2) By expanding the air, or decreasing the pressure, in the shaft *D C*. Each of these methods results in destroying the equilibrium, or equality of pressure, or weight, in the shafts.

Without entering here into a description of the methods for producing the difference of pressure, it may be stated that the ventilation is accomplished, according to the first of the above methods, by the use of a blowing-fan or by a waterfall, and, according to the second method, by means of a furnace, exhaust-fan, or steam-jet.

940. To find the theoretical velocity of air in a mine, due to the difference in the pressures in the upcast and downcast shafts, we have the following formula, in which

v = velocity of the air in feet per second;
F = the constant force represented by difference of pressure in pounds per square foot;
w = weight of a cubic foot of air;
g = acceleration due to gravity = 32.16 ft.

$$v = \sqrt{\frac{2gF}{w}}. \qquad (32.)$$

941. The Motive Column.—That portion of the downcast column of air which represents the difference between the weights of the air columns in the downcast and the upcast shafts is called the **motive column.** The excess of weight in the air in the downcast over that in the upcast is what causes the flow of air up the upcast. Hence, if we subtract the pressure per square foot at the bottom of the upcast from the pressure per square foot at the bottom of the downcast, and divide the difference by the weight of a cubic foot of air in the downcast, we have the length of the motive column, or the column whose weight overcomes the balance and causes the current to move up the upcast. For example, in

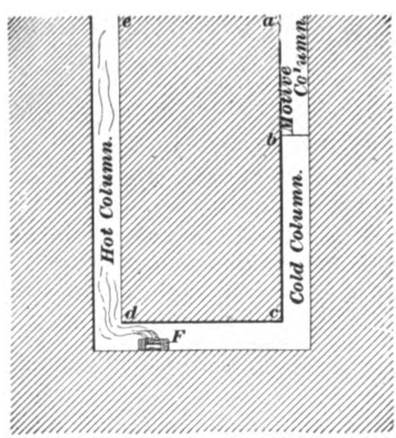

FIG. 134.

Fig. 134, the long hot column *c d* is equal in weight to the short cold column *b c*, and they balance each other, but the column *a b* of cold air is resting on *b c* and destroys the balance, and causes a current to flow; hence, *a b* is the motive column.

942. The length of the motive column may be found by means of one of the following formulas, in which

W = the weight of a cubic foot of air in the downcast shaft;
p = the pressure of the downcast shaft;
p_1 = the pressure in the upcast shaft;
t_1 = the average temperature of the air in the downcast shaft;
t = the average temperature of the air in the upcast shaft;
D = the depth of the upcast shaft in feet;
M = the length of the motive column in feet;
G = the water-gauge in inches. (See Art. **1058.**)

Then,

$$M = \frac{p - p_1}{W}. \qquad \textbf{(33.)}$$

$$M = \frac{5.2\,G}{W}. \qquad \textbf{(34.)}$$

$$M = \frac{D\,(t - t_1)}{459 + t}. \qquad \textbf{(35.)}$$

EXAMPLE.—If the temperature of the air in the downcast shaft is 40° F., and in the upcast shaft 120° F., what is the height of the motive column, the depth of the upcast shaft being 200 feet?

SOLUTION.—Applying formula **35**,

$$M = \frac{D(t - t_1)}{(459 + t)} = \frac{200\,(120 - 40)}{(459 + 120)} = 27.63 \text{ feet.}$$

PROOF.—The proof of the accuracy of this conclusion may be found as follows: The weight of a cubic foot of air in the downcast shaft is .07968, and the weight of a cubic foot of air in the upcast shaft is .06867; then the entire weight of the upcast shaft column is $.06867 \times 200 = 13.73400$ pounds, and the weight of the portion of the downcast column that balances the weight of the upcast column is $200 - 27.63 = 172.37$, and $172.37 \times .07968 = 13.734$ pounds.

In formula **35** it is assumed that the temperatures of the downcast and outer air are the same. There is no material error involved in this assumption, and the height of the motive column so obtained is practically correct, since any increase in temperature due to the depth is partly, if not wholly, neutralized by the moisture in the shaft absorbing heat from the air.

PRESSURE AND RESISTANCES.

943. When the word "pressure" is used in mine ventilation, it means the force that produces a movement of the air through the workings, and is called the **ventilating pressure.** The velocities of air-currents depend upon differences in pressures, the greater the difference the greater the velocity of the current. It should, therefore, be remembered that it is not the gross pressure at the beginning of an air-current that produces its velocity, but rather the difference between the gross pressures at both ends, which is the *ventilating pressure.* A difference of pressure of one pound per square foot will produce a current of wind in the open air having a velocity of about 19 miles per hour.

944. The ventilating pressure may be expressed in pounds (in which case it is called the **total pressure**), in pounds per square foot, in inches of water-gauge, or in feet of motive column. Unless otherwise stated, it will be expressed in pounds per square foot. Should it be necessary to express it in inches of water-gauge, it may be easily converted into pounds per square foot by multiplying the number of inches of water-gauge by 5.2.

In order to avoid the long term "ventilating pressure," and also to make the language conform to other books pertaining to the subject of mine ventilation, the word *pressure* only will be used, except when it is thought best to use the full term.

The resistances met with in mines may be divided into three classes: First, the resistance due to friction; second, the resistance due to changing the direction of the current,

i. e., bends; third, the resistance due to contracting or enlarging the airway.

The most important of these resistances is that due to friction, and is the first that will be considered.

THE THREE LAWS OF FRICTION.

945. As the result of many experiments, the truth of the three following laws, called the **three laws of friction,** has been firmly established.

946. First Law.—*When the velocity remains the same, the total pressure required to overcome friction varies directly as the extent of the rubbing surface.*

947. By **rubbing surface** is meant the entire area touched by the air in passing through the airway. The cross-section of the airway may be a square, a rectangle, a trapezoid, or a circle. If the cross-section is a square, the perimeter equals the length of one of the sides of the cross-section multiplied by 4; if a rectangle or trapezoid, the perimeter equals the sum of all the sides, and if a circle, the perimeter equals the diameter multiplied by 3.1416. Having found the perimeter, the rubbing surface may be found by multiplying the perimeter by the length of the airway.

948. The first law states that if the rubbing surface be increased, the pressure must be increased in the same proportion in order to pass the air with the same velocity. In other words, if the rubbing surface be increased $1\frac{1}{2}$, 2, 3, 4, etc., times, the pressure must also be increased $1\frac{1}{2}$, 2, 3, 4, etc., times in order to pass the same quantity of air.

In applying this law, it does not matter whether the pressure per square foot or the total pressure is considered, if in the first case the sectional area remains the same.

EXAMPLE.—Suppose that a certain airway passes 10,000 cubic feet of air per minute; what must be the increase in pressure in order to pass the same amount through an airway whose cross-section has the same area, but whose rubbing surface is 1.6 times as great?

SOLUTION.—Since the rubbing surface is increased 1.6 times, while the other factors (velocity, quantity, sectional area, etc.) remain the same, it follows that, according to the first law of friction, the pressure must also be increased 1.6 times. Ans.

949. The form of the cross-section of the airway exerts a considerable influence on the amount of rubbing surface, as the following examples will show:

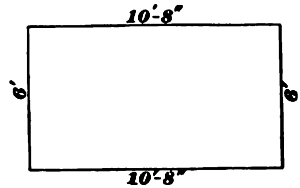

FIG. 135.

EXAMPLE 1.—Find (*a*) the rubbing surface and (*b*) the area of the cross-section of an airway 1,000 feet long having a rectangular cross-section, whose sides are 10 feet 8 inches long by 6 feet high. (See Fig. 135.)

SOLUTION.—(*a*) The rubbing surface equals the perimeter multiplied by the length; or, since 10 ft. 8 in. $= 10\frac{2}{3}$ ft., $(10\frac{2}{3} + 6 + 10\frac{2}{3} + 6) \times 1,000 = 33\frac{1}{3} \times 1,000 = 33,333\frac{1}{3}$ sq. ft. Ans.

(*b*) Area $= 10\frac{2}{3} \times 6 = 64$ sq. ft. Ans.

EXAMPLE 2.—Suppose that in the preceding example the rectangular section had been 16 feet wide and 4 feet high, what would have been the rubbing surface and area?

SOLUTION.—The rubbing surface $= (16 + 4 + 16 + 4) \times 1,000 = 40 \times 1,000 = 40,000$ sq. ft., and the area $= 16 \times 4 = 64$ sq. ft. Ans.

In this example the cross-sectional area is the same as in example 1, while the rubbing surface is $\frac{1}{6}$ greater. Had the sides been 32 feet and 2 feet, the sectional area would have been 64 square feet, as above, but the rubbing surface would have been $(32 + 2 + 32 + 2) \times 1,000 = 68,000$ square feet, or 2.07 times as much as in example 1. Hence, to pass the same quantity of air, the pressure would require to be increased 1.07 times.

950. It is easy to see that the more oblong the rectangle is the more rubbing surface there is for the same sectional area, and it is evident that the perimeter of a square section is less than that of a rectangular section having the same area. Thus, the perimeter of the square, Fig. 136, is but 32 feet, while that of the rectangle in Fig. 135 is $33\frac{1}{3}$ feet, and that of the rectangle in example 2 is 40 feet. If

Fig. 136 represents a section of an airway 1,000 feet long,

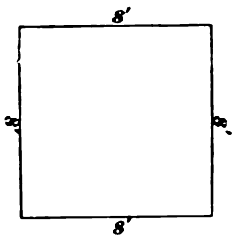

FIG. 136.

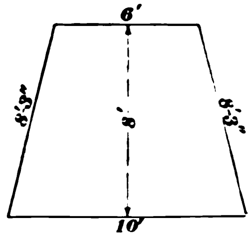

FIG. 137.

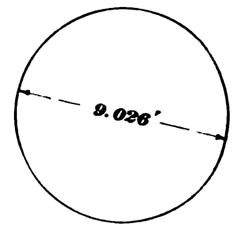

FIG. 138.

the rubbing surface is 32 × 1,000 = 32,000 square feet.

EXAMPLE 3.—Suppose an airway to have a trapezoidal cross-section like that shown in Fig. 137, and to be 1,000 feet long; what is the rubbing surface and sectional area?

SOLUTION.—The rubbing surface (since 8 ft. 3 in. = $8\frac{1}{4}$ ft.) equals $(10 + 8\frac{1}{4} + 6 + 8\frac{1}{4}) \times 1{,}000 = 32{,}500$ sq. ft., and sectional area $= \frac{6 + 10}{2} \times 8 = 64$ sq. ft. Ans.

EXAMPLE 4.—A circular airway is 9.026 feet in diameter and 1,000 feet long; what is its rubbing surface and sectional area?

SOLUTION.—The rubbing surface equals 3.1416 times the diameter multiplied by the length $= 3.1416 \times 9.026 \times 1{,}000 = 28.36 \times 1{,}000 = 28{,}360$ sq. ft., and the sectional area $= 9.026^2 \times .7854 = 64$ sq. ft.

It will be noticed that in all of the above examples the area of the cross-section is 64 square feet, while the rubbing surface varies from 28,360 to 68,000 square feet.

The results obtained above are all combined in the following table, and show at a glance the effect produced by varying the forms of the cross-section, the sectional area and the length of the airway being the same in all.

951. Table 23 shows that for a given sectional area the circular airway has the least rubbing surface, and that the square airway comes next, while, with rectangular airways, that which most nearly approaches the square form has the least rubbing surface. Hence, for economy in ventilation, circular airways are best; but, since they are seldom practicable, owing to other considerations, square airways should be used whenever it is possible to do so. If rectangular or trapezoidal airways are absolutely necessary, they should, in so far as it is practicable, approach the square form.

TABLE 23.

Form of Section.	Dimensions of Section.	Length in Feet.	Perimeter in Feet.	Rubbing Surface in Square Feet.	Sectional Area in Square Feet.
Circular	9.026′ diam.	1,000	28.36	28,360	64
Square	8′ × 8′	1,000	32	32,000	64
Trapezoidal	(10′ and 6′) × 8¼′	1,000	32½	32,500	64
Rectangular	10′ 8″ × 6′	1,000	33⅓	33,333	64
Rectangular	16′ × 4′	1,000	40	40,000	64
Rectangular	32′ × 2′	1,000	68	68,000	64

It is evident that increasing the length of the airway will also increase the rubbing surface. Hence, if two airways have the same perimeter and area, but different lengths, and the air is transmitted with the same velocity in each, the pressures will be in direct proportion to the lengths.

EXAMPLE.—Two airways are each 6 feet high and 9 feet wide; consequently their areas of section and perimeters are equal. The length of one of these airways is 1,800 feet, and the pressure indicated by the water-gauge is 1.72 inches; the length of the other airway is 2,700 feet. If the velocity is the same in both these airways, what should be the height of the water-gauge for the airway 2,700 feet in length?

SOLUTION.—Since the areas, perimeters, and velocities are the same, the heights of the water-gauges will be directly as the lengths of the airways, or $1.72 : x :: 1,800 : 2,700$; whence, $x = 2.58$ in. Ans.

952. Second Law.—*When the velocities and rubbing surfaces remain the same, the pressures required to force air through the passages of a mine increase and decrease inversely as the sectional areas of the passages increase or decrease.*

953. The second law states that if the velocity remains the same and the rubbing surfaces are equal, the pressure per square foot will increase as the sectional area decreases;

or, the pressure per square foot will decrease as the sectional area increases; that is, if the sectional area be reduced to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{10}$, etc., of the original sectional area, the pressure per square foot must be increased, 2, 4, 6, 10, etc., times, respectively, to pass the air with the same velocity, the rubbing surface being the same in both cases. Or, if the sectional area be increased 2, 4, 6, 10, etc., times the original sectional area, the pressure per square foot may be, respectively, reduced to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{10}$, etc., of the original pressure per square foot required to pass the air with the same velocity, the rubbing surface being the same in both cases.

EXAMPLE.—Suppose that the pressure per square foot required to pass air at a given velocity is .02 inch per square foot in a square airway 8 feet high and 8 feet wide. What pressure per square foot will be required to pass air at the same velocity through a circular airway whose perimeter is the same as that of the square one, namely, 32 feet, and whose length is the same as that of the square one?

SOLUTION.—According to the second law of mine friction, when the velocities are the same, the pressures vary inversely as the sectional areas of the airways. If the perimeter be divided by 3.1416, the quotient will be the diameter of the section of the circular airway; and the square of this diameter multiplied by .7854 is the area of the cross-section of the circular airway in square feet; hence,

$$\text{area} = \left(\frac{32}{3.1416}\right)^2 \times .7854 = 81.49 \text{ square feet.}$$

Then the pressure required is found by the proportion 81.49 : 64 :: .02 : x; or, $x = .0157$ inch of water-gauge. Ans.

954. The second law only applies to *pressure per square foot*, for the total pressure remains the same, as it should; since, when the rubbing surface and velocity remain the same, the total resistance (= total pressure) must also remain the same, no matter what the sectional area may be. Thus, in the above example, the total pressure for the square airway is $64 \times .02 = 1.28$ lb., and for the circular airway, $81.49 \times .0157 = 1.28$ lb.

EXAMPLE.—(*a*) An 8′ × 10′ rectangular airway is 5,000 feet long; what must be the length of a similar airway, 6′ × 8′, having the same rubbing surface? (*b*) If a pressure of .5 pound per square foot is required to pass the air through the 8′ × 10′ airway with a certain velocity, what pressure per square foot is required to pass the air through the 6′ × 8′ airway with the same velocity?

SOLUTION.—(*a*) The rubbing surface of the $8' \times 10'$ airway is $(8 + 10 + 8 + 10) \times 5{,}000 = 180{,}000$ sq. ft. The perimeter of the $6' \times 8'$ airway is $6 + 8 + 6 + 8 = 28$ ft. Consequently, the length of the $6' \times 8'$ airway is $180{,}000 \div 28 = 6{,}428\frac{4}{7}$ ft. Ans.

(*b*) Since, according to the second law of friction, the pressures per square foot vary inversely as the sectional areas, when the rubbing surfaces and velocities remain the same, and the sectional areas are $8 \times 10 = 80$ sq. ft., and $6 \times 8 = 48$ sq. ft., $80 : 48 :: x : .5$; or, $x = .8\frac{1}{3}$ lb. per square foot. Ans.

Here again the total pressures are the same, since $80 \times .5 = 40$ lb., and $48 \times .8\frac{1}{3} = 40$ lb.

955. Third Law.—*The pressure required to overcome friction in an airway varies as the squares of the velocities when the rubbing surface and the areas of section are the same; and the pressures required to overcome friction vary as the squares of the velocities multiplied by the rubbing surfaces per square foot of section in all airways.*

956. If the velocity be increased $1\frac{1}{2}$, 2, 3, 5, etc., times, the rubbing surface remaining the same, the pressure must be increased $(1\frac{1}{2})^2$, 2^2, 3^2, 5^2, etc., or $2\frac{1}{4}$, 4, 9, 25, etc., times, respectively; and if the velocity be reduced $1\frac{1}{2}$, 2, 3, 5, etc., times, the rubbing surface remaining the same, the pressure must be reduced $(1\frac{1}{2})^2$, 2^2, 3^2, 5^2, etc., or $2\frac{1}{4}$, 4, 9, 25, etc., times, respectively.

If the sectional area and rubbing surface both remain the same, the pressure per square foot will also vary directly as the square of the velocity.

EXAMPLE.—Suppose that in the last example the velocity was 400 feet per minute, and that it was desired to increase it to 450 feet per minute, what would be the total pressure required?

SOLUTION.—Since the pressures vary directly as the squares of the velocities, $400^2 : 450^2 :: 40 : x$; or, $x = 50\frac{5}{8}$ lb. Ans.

EXAMPLE.—In the above example, what would be the pressure per square foot, were the velocity increased from 400 to 450 feet per minute in the $6' \times 8'$ airway?

SOLUTION.—The pressure per square foot was found to be $.8\frac{1}{3}$ pound; hence, according to the third law, since the sectional area and rubbing surface remain the same, $400^2 : 450^2 :: 8\frac{1}{3} : x$; or $x = 1.055$ lb. per square foot. Ans.

THE COEFFICIENT OF FRICTION.

957. By means of the three laws of friction, and by the aid of other laws which can be deduced from them, and which will be given later, it is possible, when all of the data for one airway and a part of the data for another airway are known, to calculate the remaining data for the second airway; or, if all the data for an airway are known, to calculate the effect produced by varying the pressure, velocity, etc. But in order to calculate the pressure required to force the air (overcome the resistances) through a given airway, to calculate the pressure required to pass a certain quantity per minute through a given airway, and to calculate the horsepower, etc., it is necessary to know the *coefficient of friction.*

958. *The* **coefficient of friction** *is that amount of the total ventilating pressure which is required to overcome the resistance offered by one square foot of rubbing surface when the velocity is 1 foot per minute.*

959. For example, this may be further explained by stating that the coefficient of friction is equivalent to the pressure required to overcome the friction in an airway one-quarter of a foot long, 1 foot square in section, and through which the air is passing with a velocity of 1 foot per minute.

Since the total pressure may be expressed in pounds, or as so many feet of motive column, having a cross-section equal to the sectional area of the airway, the coefficient of friction may also be expressed as a fraction of a pound or a factor of the motive column in feet. In the various works treating on mine ventilation, the coefficient is usually expressed in pounds, and will be so expressed throughout this discussion.

The coefficient of friction then becomes a unit which, multiplied by the rubbing surface in square feet (according to the first law), and again multiplied by the square of the velocity in feet per minute (according to the third law), will give the total ventilating pressure in pounds.

960. The coefficient of friction varies somewhat for different mines, according to the degree of smoothness of the rubbing surface, and probably to a slight extent on account of the character of the material forming the sides of the airway. Different experimenters have obtained values which show considerable variation in their results; but the value most commonly used is that determined by J. J. Atkinson, and is the one which will be used in this discussion. This unit is known to be too high, but since every change in direction, owing to bends, and every reduction or enlargement of the passageway, etc., entails extra losses which are very difficult to calculate, it will be more convenient to use Atkinson's coefficient and disregard the extra losses. By so doing, the entire air-course is treated as if it were a straight airway, and the calculations are greatly simplified. The value of Atkinson's coefficient of friction is .0000000217 pound. In other words, the pressure required to overcome the resistance offered by 1 square foot of rubbing surface when the velocity is 1 foot per minute, is that part of a pound represented by 217 divided by 1 followed by 10 ciphers, or .0000000217, expressed decimally.

961. EXAMPLE.—(*a*) What is the total pressure required to overcome the frictional resistances of a $6' \times 8'$ airway, 12,750 feet long, if the velocity is 480 feet per minute? (*b*) What is the pressure per square foot? (*c*) What should the water-gauge read?

SOLUTION.—(*a*) According to the foregoing statements, the total pressure is equal to the continued product of the coefficient of friction, the rubbing surface, and the square of the velocity; hence, since the rubbing surface $= 28 \times 12{,}750 = 357{,}000$ sq. ft., total pressure $= .0000000217 \times 357{,}000 \times 480^2 = 1{,}784.89$ lb. Ans.

(*b*) The pressure per square foot equals the total pressure divided by the sectional area $= \frac{1{,}784.89}{8 \times 6} = 37.18$ lb. per square foot. Ans.

(*c*) Since 1 inch of water-gauge represents a pressure of 5.2 pounds per square foot, 37.18 pounds represent $\frac{37.18}{5.2} = 7.15$ in. Ans.

962. To express the foregoing by means of formulas, let

$P =$ total ventilating pressure in pounds;

$p =$ ventilating pressure in pounds per square foot;

a = sectional area of airway in square feet;
k = coefficient of friction = .0000000217;
s = total rubbing surface in square feet;
v = velocity of air in airway in feet per minute;
o = perimeter of airway in feet;
l = length of airway in feet;
W = water-gauge in inches.

Throughout this subject the letters as printed above will always represent the same quantities.

$$P = p\,a. \qquad \textbf{(36.)}$$

That is, the total pressure equals the pressure per square foot multiplied by the sectional area of the airway.

EXAMPLE.—If the sectional area of the airway is 56 square feet, and the pressure per square foot is 8.46 pounds, what is the total pressure?

SOLUTION.—Applying formula **36**,

$$P = p\,a = 8.46 \times 56 = 473.76 \text{ lb. Ans.}$$

$$P = k\,s\,v^2. \qquad \textbf{(37.)}$$

That is, the total pressure equals the continuea product of the coefficient of friction, the rubbing surface, and the square of the velocity.

EXAMPLE.—An airway 6′ × 6′ and 5,000 feet long passes air with a velocity of 340 feet per minute; what is the total ventilating pressure?

SOLUTION.—Applying formula **37**, $s = 6 \times 4 \times 5{,}000 = 120{,}000$ sq. ft., and $v = 340$. Hence, $P = k\,s\,v^2 = .0000000217 \times 120{,}000 \times 340^2 =$ 301 lb., nearly. Ans.

$$p = \frac{k\,s\,v^2}{a}. \qquad \textbf{(38.)}$$

That is, the pressure per square foot equals the continued product of the coefficient of friction, the rubbing surface, and the square of the velocity, divided by the sectional area of the airway.

EXAMPLE.—What is (*a*) the pressure per square foot in the last example? (*b*) the water-gauge?

SOLUTION.—(*a*) Substituting in formula **38**, $a = 6 \times 6 = 36$ sq. ft., and

$$p = \frac{.0000000217 \times 120{,}000 \times 340^2}{36} = 8.36 \text{ lb. Ans.}$$

(*b*) Since $p = 5.2\,W$, $W = \frac{p}{5.2} = \frac{8.36}{5.2} = 1.61$ in., nearly. Ans.

$$s = \frac{P}{k\,v^2} = \frac{p\,a}{k\,v^2}. \qquad \textbf{(39.)}$$

That is, the rubbing surface equals the total pressure divided by the coefficient of friction multiplied by the square of the velocity; or, it equals the pressure per square foot multiplied by the sectional area divided by the product of the coefficient of friction and the square of the velocity.

EXAMPLE.—A gangway is 8′ × 8′; if the water-gauge shows ¾ inch and the velocity of the air is 280 feet per minute, what is the rubbing surface?

SOLUTION.—The pressure per square foot is $p = 5.2\,W = 5.2 \times \frac{3}{4} = 3.9$ lb. per square foot; the sectional area is $8 \times 8 = 64$ sq. ft.

Hence, substituting in formula **39**,

$$s = \frac{p\,a}{k\,v^2}; \text{ or, } s = \frac{3.9 \times 64}{.0000000217 \times 280^2} = 146{,}713 \text{ sq. ft. Ans.}$$

$$v = \sqrt{\frac{p\,a}{k\,s}}. \qquad \textbf{(40.)}$$

That is, the velocity in feet per minute equals the square root of the pressure in pounds per square foot multiplied by the sectional area in square feet, divided by the product of the coefficient of friction and the rubbing surface in square feet.

EXAMPLE.—In the last example suppose that the rubbing surface was known to be 146,713 square feet, and it was desired to find the velocity. Show how you would find it.

SOLUTION.—Substituting the different values in formula **40**,

$$v = \sqrt{\frac{p\,a}{k\,s}} = \sqrt{\frac{3.9 \times 64}{.0000000217 \times 146{,}713}} = 280 \text{ ft. per min. Ans.}$$

When the total rubbing surface and the perimeter are known, the length of the airway may be found by means of the formula

$$l = \frac{s}{o}. \qquad \textbf{(41.)}$$

That is, the length of the airway is equal to the rubbing surface divided by the perimeter.

EXAMPLE.—The perimeter of an airway is 32 feet, and the rubbing surface is 146,713 feet; what is the length of the airway?

SOLUTION.—Applying formula **41**,

$$l = \frac{s}{o} = \frac{146,713}{32} = 4,585 \text{ ft. Ans.}$$

As before stated, the rubbing surface equals the product of the length and the perimeter; or,

$$s = l\,o. \qquad \textbf{(42.)}$$

THE QUANTITY OF AIR DISCHARGED.

963. Since a certain quantity of air is required to pass along the airway in order to secure the proper amount of ventilation, it is necessary to know how much air can be passed with a given velocity; or, knowing the quantity required, it is necessary to calculate the velocity, and from that to determine the pressure. If the velocity and sectional area are known, the quantity may be determined by the following formula, in which q = the quantity in cubic feet per minute:

$$q = a\,v. \qquad \textbf{(43.)}$$

That is, the quantity of air discharged in cubic feet per minute through a given airway is equal to the area of the section in square feet multiplied by the velocity in feet per minute.

964. A little consideration will show that formula **43** must be true; for, suppose that the sectional area is 1 square foot and the velocity is 1 foot per minute; then it is perfectly evident that the quantity discharged in 1 minute is 1 cubic foot. If the velocity be increased 2, 3, 4, etc., times, the number of cubic feet discharged will also be, respectively, 2, 3, 4, etc., times the original quantity; that is, the velocity will be 2, 3, 4, etc., feet per minute, and the quantity 2, 3, 4, etc., cubic feet per minute. Likewise, if the velocity remains at 1 cubic foot per minute, but with the area increased 2, 3, 4, etc., times, the quantity will be increased to 2, 3, 4, etc., cubic feet per minute. Consequently, if the area and velocity are both changed, the change in quantity must be the product of the two; that is, if the area be increased from 1 square foot to, say, 26 square feet, and the velocity increased from 1 foot per minute to 1,000 feet per minute, the quantity will be increased from 1 cubic foot to $26 \times 1{,}000 = 26{,}000$ cubic feet per minute.

EXAMPLE.—A circular airway has a diameter of 9.026 feet, and the velocity of the air is 330 feet per minute; what is the quantity passing in cubic feet per minute?

SOLUTION.—Applying formula **43**, $a = 9.026^2 \times .7854 = 64$ sq. ft. Hence, $q = a\,v = 64 \times 330 = 21{,}120$ cu. ft. per minute. Ans.

965. If the quantity to be discharged and the sectional area are known, and it is required to find the velocity, use the following formula:

$$v = \frac{q}{a}. \qquad \textbf{(44.)}$$

That is, the velocity in feet per minute equals the quantity passing in cubic feet per minute divided by the sectional area in square feet.

EXAMPLE.—A circular airway has a diameter of 9.026 feet; what must be the velocity in order to pass 21,120 cubic feet per minute?

SOLUTION.—The sectional area was found to be 64 square feet in the last example. Hence, substituting in formula **44**,

$$v = \frac{q}{a} = \frac{21{,}120}{64} = 330 \text{ ft. per minute. Ans.}$$

966. The size of the airway usually depends upon other considerations than the quantity and velocity; but, in order to render the subject more complete, the following formula is given:

$$a = \frac{q}{v}. \qquad \textbf{(45.)}$$

That is, the sectional area equals the quantity in cubic feet per minute divided by the velocity in feet per minute.

967. Formulas **43**, **44**, and **45** may be combined with formulas **28** to **32**, so that the pressure (or velocity) may be determined at once, when the quantity and other needful data are known; but the simplest way is to calculate the velocity by formula **40**, and then substitute the value obtained in formula **43** to find the quantity; or to calculate the velocity by formula **44,** and substitute in formula **37** or **38** to find the pressure. The formulas are given, however, in Table 24, and are there denoted by the letters **a, b, c,** etc., to distinguish them from the numbered formulas, which are considered to be more important.

EXAMPLE.—What is the total ventilating pressure required to pass 21,120 cubic feet of air per minute through an 8′ × 8′ air-course 6,000 feet long?

SOLUTION.—The sectional area $= 8 \times 8 = 64$ sq. ft. $= a$. The rubbing surface $= 8 \times 4 \times 6{,}000 = 192{,}000$ sq. ft. $= s$.

By formula **44,**

$$v = \frac{q}{a} = \frac{21{,}120}{64} = 330 \text{ ft. per minute.}$$

Therefore, applying formula **37,**

$$P = k\,s\,v^2 = .0000000217 \times 192{,}000 \times 330^2 = 453.72 \text{ lb. Ans.}$$

WORK AND POWER.

968. Work is equal to resistance in pounds multiplied by the space in feet through which the resistance is overcome. That is, suppose that it takes a force (pressure) of 25 pounds to move a certain body; then, if the resistance is uniform, as, for example, in lifting a weight, and the body is moved through a distance of 36 feet, the work done is $25 \times 36 = 900$ foot-pounds. Since time is not mentioned in the above definition, it follows that work is independent of the time; that is, no matter whether it takes 1 second or 1 year to move the body 36 feet, the work done is 900 foot-pounds.

Now, in order to compare the work done by different machines, time must be considered. Hence, the amount of work done in overcoming a resistance of 1 pound, through a space (distance) of 1 foot in 1 minute, is called the **unit of power.** The power of a machine is, then, the number of foot-pounds of work which it can perform in 1 minute, and this number divided by 33,000 is called the **horsepower of the machine.**

The power required to produce the proper ventilative effects may be easily calculated when the total pressure and the velocity are known, or when the pressure per square foot and the quantity passed in cubic feet per minute are known. Thus, the total pressure represents the force required to overcome the resistances, and the velocity in feet per minute represents the space (distance) passed through in 1 minute; consequently, the product of the total pressure, P, and the velocity in feet per minute equals the work per minute, or the power. That is, representing the number of units of power by u,

$$u = P v. \qquad \textbf{(46.)}$$

Likewise, since $P = p\,a$, $u = p\,a\,v$; but, according to formula **43**, $a\,v = q$; hence,

$$u = p\,q. \qquad \textbf{(47.)}$$

By dividing formulas **46** and **47** by 33,000, the horsepower may be found. Letting H represent the horsepower,

$$H = \frac{u}{33{,}000} = \frac{P v}{33{,}000} = \frac{p\,a\,v}{33{,}000} = \frac{p\,q}{33{,}000}. \qquad \textbf{(48.)}$$

EXAMPLE.—What horsepower is required to pass the air in the last example?

SOLUTION.—The total pressure was found to be 453.72 pounds, and the velocity 330 feet per minute. Hence, by formula **48**,

$$H = \frac{P v}{33{,}000} = \frac{453.72 \times 330}{33{,}000} = 4.537 \text{ H. P., nearly. Ans.}$$

EXAMPLE.—If the water-gauge reading is 1.9 inches, and the quantity of air passing is 20,000 cubic feet per minute, what horsepower is required?

SOLUTION.—The pressure per square foot $= 5.2 \times 1.9 = 9.88$ lb. Therefore, applying formula **48**,

$$H = \frac{p\,q}{33{,}000} = \frac{9.88 \times 20{,}000}{33{,}000} = 6 \text{ H. P., nearly. Ans.}$$

969. From formula **48**, several other important formulas may be derived by a simple transposition of the terms. If the horsepower, sectional area, and the velocity of the air are known, and it is desired to find the ventilating pressure in pounds per square foot, the following formula may be used:

$$p = \frac{33{,}000\,H}{a\,v}. \qquad \textbf{(49.)}$$

Or, if the horsepower and the quantity of air to be passed per minute are known, and p is required,

$$p = \frac{33,000\,H}{q}. \qquad \textbf{(50.)}$$

970. If it be required to ascertain the quantity which a certain horsepower will cause to pass with a given pressure, it may be found by formula **51**,

$$q = \frac{33,000\,H}{p}. \qquad \textbf{(51.)}$$

Similarly, the velocity may be found by formula **52**, when the horsepower and total pressure, or the horsepower, pressure per square foot, and sectional area are known.

$$v = \frac{33,000\,H}{P} = \frac{33,000\,H}{p\,a}. \qquad \textbf{(52.)}$$

EXAMPLE.—It is required to pass 20,000 cubic feet of air per minute. (*a*) What is the pressure per square foot if only 6 horsepower are required? (*b*) What is the water-gauge reading?

SOLUTION.—(*a*) Since only the horsepower and quantity are given, formula **50** must be used. Substituting,

$$p = \frac{33,000\,H}{q} = \frac{33,000 \times 6}{20,000} = 9.9 \text{ lb. per square foot. Ans.}$$

(*b*) Since $p = 5.2\,W$, $W = \frac{p}{5.2} = \frac{9.9}{5.2} = 1.9$ in., very nearly. Ans.

EXAMPLE.—Had the sectional area in the above example been 50 square feet, what would the velocity have been?

SOLUTION.—This example may be solved in two ways. By formula **44**,

$$v = \frac{q}{a} = \frac{20,000}{50} = 400 \text{ ft. per minute. Ans.}$$

By formula **52**,

$$v = \frac{33,000\,H}{p\,a} = \frac{33,000 \times 6}{9.9 \times 50} = 400 \text{ ft. per minute. Ans.}$$

971. Formulas **46** to **52** may be combined with formulas **37** to **42** to produce other formulas, which will shorten the work to some extent in certain cases; but the student will find it a better plan, as a rule, to calculate the pressure, velocity, or whatever he needs, by using one of the formulas from **36** to **44,** and then substituting in one of the later formulas.

A number of these combination formulas will be given in Table 24, and the student may use them if he so chooses. One of these combination formulas is so important that it will now be given.

Multiplying both sides of formula **37** by v, $Pv = ksv^3$; but by formula **46,** $Pv = u$; hence,

$$u = ksv^3. \qquad \textbf{(53.)}$$

That is, the power in foot-pounds per minute equals the continued product of the coefficient of friction, the rubbing surface, and the **cube** *of the velocity.*

Likewise, since $u = pq$ (formula **47**), $pq = ksv^3$; or,

$$q = \frac{ksv^3}{p}. \qquad \textbf{(54.)}$$

EXAMPLE.—An air-course has a length of 4,752 feet; its perimeter is 30 feet, and its sectional area is 50 square feet. (*a*) What quantity of air will it pass at a velocity of 400 feet per minute? (*b*) What power will be required? The pressure is 9.9 pounds per square foot.

SOLUTION.—(*a*) This question is most easily solved by means of formula **43,** but to show the reliability of formula **54,** it will be solved both ways. By formula **43,**

$q = av = 50 \times 400 = 20{,}000$ cu. ft. per minute. Ans.

By formula **54,** since $s = 4{,}752 \times 30 = 142{,}560$ sq. ft.

$$q = \frac{ksv^3}{p} = \frac{.0000000217 \times 142{,}560 \times 400^3}{9.9} = 20{,}000 \text{ cu. ft., nearly. Ans.}$$

(*b*) Substituting in formula **53,**

$$u = ksv^3 = .0000000217 \times 142{,}560 \times 400^3 =$$

198,000 ft.-lb. per minute, nearly, $= \frac{198{,}000}{33{,}000} = 6$ H. P. Ans.

972. There is one more combination formula which is chiefly valuable on account of the deductions which may be made from considerations of it, and which will now be given in order that the student may be able to answer a question sometimes asked at examinations for mine foremen's certificates. But, in order that an intelligent understanding may result, it is necessary to digress here and explain a certain geometrical law.

973. Two figures are **similar** when the smaller may be so placed within the larger that their perimeters shall be parallel throughout their entire lengths, and their corresponding sides proportional. Thus, if in Fig. 139 the perimeters of the two trapezoids are parallel when one is placed within the other, and $AD : ad :: BC : bc$ and the same relation is also true of any other two sides, as $AD : ad :: DC : dc$, then the two trapezoids are *similar*. Equilateral triangles, squares, and circles are always similar.

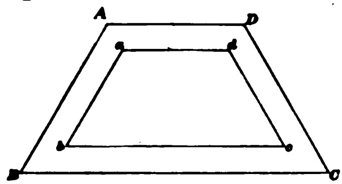

FIG. 139.

Now, it is proved in geometry that the areas of similar figures are to each other as the squares of any side, the squares of their perimeters, or the squares of any line similarly placed in them, as, for example, a diagonal or diameter. Also, that the volumes of similar solids are to each other as the cubes of similarly placed lines in them. Likewise, if *any* two similar figures are varied according to some power of similarly placed lines, all similar figures will vary according to the same powers of their similarly placed lines. For example, if the volume of a certain prism is 21 cubic inches and the length of a certain line in it is 2 inches, what will be the volume of a similar prism if the length of a similarly placed line in it is 3 inches? Since the volumes of similar solids are to each other as the cubes of their similar lines, $2^3 : 3^3 :: 21 : x$; or, $x = \frac{21 \times 27}{8} = 70.875$ cubic inches.

974. Returning now to the subject of ventilation, consider a mine having a square cross-section, and represent the length of a side by d. Then the area is d^2 and the perimeter is $4d$. According to formula **43,** $q = av$; but, since for this case $a = d^2$, $q = d^2 v$. By formula **40,** $v = \sqrt{\frac{pa}{ks}} = \sqrt{\frac{pd^2}{ks}}$, since $a = d^2$. By formula **42,** $s = lo$, and since for this case $o = 4d$, $s = l \times 4d = 4ld$. Substituting this value of s in the above formula for v, $v = \sqrt{\frac{p\,d^2}{k \times 4\,l\,d}} = \sqrt{\frac{p\,d}{4\,k\,l}}$. Substituting this value of v in the expression giving the value of q,

$$q = d^2 \times \sqrt{\frac{p\,d}{4\,k\,l}}; \text{ or, } q = \sqrt{\frac{p\,d^5}{4\,k\,l}}. \qquad \textbf{(55.)}$$

975. It must be remembered that formula **55** applies only to square airways. A consideration of it shows that if two square airways have the same length and pressure per square foot, the quantities of air which they will pass will be to each other as the square roots of the fifth powers of the lengths of their sides.

Also, if two square airways of different lengths are required to pass the same quantity of air with the same pressure per square foot, the lengths of the sides will be to each other as the fifth roots of the lengths of the airways.

Now, since squares are similar figures, the two statements just made will also apply to any two airways whose cross-sections are similar figures.

976. The following example is a question asked at an examination held at Pittsburg, in March, 1893:

EXAMPLE.—If 15,000 cubic feet of air per minute are passing through an airway 4,000 feet in length, and 6 feet by 8 feet in section, what should be the dimensions of the section of another airway of precisely the same form (i. e., a similar section) to pass the same quantity of air, the length, however, being 3,000 feet, instead of 4,000 feet, as in the former case?

SOLUTION.—Since the pressure is not stated, it is evidently intended to be the same in both cases. Then, according to the above statements, the lengths of similar sides are to each other as the fifth roots of the lengths. Hence, $6 : x :: \sqrt[5]{4,000} : \sqrt[5]{3,000}$;

or, $$x = 6 \times \frac{\sqrt[5]{3,000}}{\sqrt[5]{4,000}} = 6\sqrt[5]{\frac{3,000}{4,000}} = 6 \times .944 = 5.644 \text{ ft.}$$

Now, since the sections are similar, the sides are proportional; hence, $6 : 5.664 :: 8 : x$; or, $x = \frac{5.664 \times 8}{6} = 7.552$ ft. Therefore, the section is $5.664' \times 7.552'$. Ans.

977. From formula **55** there may also be deduced the proposition that if two square airways have the same length and pass the same quantities of air, the lengths of the sides of the airway will then vary *inversely* as the fifth roots of the pressures. This statement applies, of course, to airways of similar sections. Conversely, the pressures vary inversely as the fifth powers of the lengths of the sides.

A method of finding the fifth roots of numbers will be given hereafter. (See Art. **1000.**)

978. Other Resistances.—All that is necessary for the calculation of the resistance of the flow of air through a straight airway has now been given, and all that remains to be considered, so far as appertains to the flow of air, are those effects produced by bends, contractions, or enlargements of the sections, and splits. Each of the foregoing results in a change in the velocity of the flowing air, and, consequently, in a change in the ventilating pressure. The losses due to bends are considerable, particularly a bend of 90° or greater. Where a bend is absolutely necessary, the corners should be rounded (if practicable) to as large a radius as possible, if it is desired to reduce the mine resistance to a minimum. There is no reliable formula for calculating the resistance due to bends, but they certainly reduce the velocity to a great extent, especially a bend of 90° or greater. If the reduction or enlargement of the sectional area is slight compared with the sectional area of the airway, the consequent loss of velocity may be disregarded entirely. In any case, it is a difficult matter to decide how much to allow for such loss. The losses due to regulators and to splits will be treated separately in a later section. Since, as before mentioned, the coefficient of friction, .0000000217, is very high, much above what would actually be obtained in practice for a straight airway, the losses due to bends, enlargements, and contractions may be neglected altogether without any material error, the airway being calculated as if it were straight and of uniform section throughout. This statement applies, of course, to slight enlargements or contractions.

979. Formulas.—Formulas **36** to **54,** inclusive, and others which are not quite as important, are given in Table 24, so as to be convenient for reference. The formulas not previously given are denoted by the letters **a, b, c,** etc. A specimen calculation is also worked out with each formula. To prevent any misconception, the letters and their meanings are repeated below:

a = sectional area of airway in square feet;
H = horsepower;
k = coefficient of friction = .0000000217;
l = length of airway in feet;
o = perimeter of airway in feet;
p = ventilating pressure in pounds per square foot;
P = total ventilating pressure in pounds;
q = quantity of air in cubic feet per minute;
s = rubbing surface in square feet;
u = units of power in foot-pounds per minute;
v = velocity in feet per minute;
W = water-gauge in inches of water.

To render the formulas more convenient for reference, they are not given in sequence according to their numbers, but are classified according to the letters whose values it is desired to find, the letters having the meaning given above.

The basis for the calculations is an airway 5 feet wide by 4 feet high and 2,000 feet long, the velocity to be 500 feet per minute.

TABLE 24.

Formulas.		Specimen Calculations.
To find the area:		
$a = \frac{P}{p}$.	(a.)	$a = \frac{195.3}{9.765} = 20$ sq. ft. Ans.
$a = \frac{k s v^2}{p}$.	(a'.)	$a = \frac{.0000000217 \times 36{,}000 \times 500^2}{9.765} = 20$ sq. ft. Ans.
$a = \frac{q}{v}$.	(45.)	$a = \frac{10{,}000}{500} = 20$ sq. ft. Ans.
$a = \frac{u}{p v}$.	(b.)	$a = \frac{97{,}650}{9.765 \times 500} = 20$ sq. ft. Ans.
$a = \frac{33{,}000 H}{p v}$.	(c.)	$a = \frac{33{,}000 \times 2.959}{9.765 \times 500} = 20$ sq. ft. Ans.
$a = \frac{k s v^2 q}{u}$.	(d.)	$a = \frac{.0000000217 \times 36{,}000 \times 500^2 \times 10{,}000}{97{,}650} = 20$ sq. ft. Ans.
To find the horsepower:		
$H = \frac{u}{33{,}000}$.	(48.)	$H = \frac{97{,}650}{33{,}000} = 2.959$ horsepower. Ans.
$H = \frac{P v}{33{,}000}$.	(48.)	$H = \frac{195.3 \times 500}{33{,}000} = 2.959$ horsepower. Ans.
$H = \frac{p q}{33{,}000}$.	(48.)	$H = \frac{9.765 \times 10{,}000}{33{,}000} = 2.959$ horsepower. Ans.

TABLE 24—*Continued.*

Formulas.		Specimen Calculations.
$H = \frac{p a v}{33,000}$.	(48.)	$H = \frac{9.765 \times 20 \times 500}{33,000} = 2.959$ horsepower. Ans.
To find the coefficient of friction:		
$k = \frac{P}{s v^2}$.	(e.)	$k = \frac{195.3}{36,000 \times 500^2} = .0000000217$ lb. per sq. ft. per minute. Ans.
$k = \frac{p a}{s v^2}$.	(f.)	$k = \frac{9.765 \times 20}{36,000 \times 500^2} = .0000000217$ lb. per sq. ft. per minute. Ans.
$k = \frac{u}{s v^3}$.	(g.)	$k = \frac{97,650}{36,000 \times 500^3} = .0000000217$ lb. per sq. ft. per minute. Ans.
$k = \frac{p q}{s v^3}$.	(h.)	$k = \frac{9.765 \times 10,000}{36,000 \times 500^3} = .0000000217$ lb. per sq. ft. per minute. Ans.
To find the length of the airway:		
$l = \frac{s}{o}$.	(41.)	$l = \frac{36,000}{18} = 2,000$ ft. Ans.
To find the perimeter of the airway:		
$o = \frac{s}{l}$.	(i.)	$o = \frac{36,000}{2,000} = 18$ ft. Ans.

TABLE 24—*Continued.*

Formulas.		Specimen Calculations.
To find the total pressure:		
$P = p\,a.$	**(36.)**	$P = 9.765 \times 20 = 195.3$ lb. Ans.
$P = k\,s\,v^2.$	**(37.)**	$P = .0000000217 \times 36,000 \times 500^2 = 195.3$ lb. Ans.
$P = \frac{u}{v}.$	**(j.)**	$P = \frac{97,650}{500} = 195.3$ lb. Ans.
$P = \frac{33,000\,H}{v}.$	**(k.)**	$P = \frac{33,000 \times 2.959}{500} = 195.3$ lb. Ans.
$P = \frac{k\,s\,q^2}{a^2}.$	**(l.)**	$P = \frac{.0000000217 \times 36,000 \times 10,000^2}{20^2} = 195.3$ lb. Ans.
To find the pressure in pounds per square foot:		
$p = \frac{P}{a}.$	**(m.)**	$p = \frac{195.3}{20} = 9.765$ lb. per square foot. Ans.
$p = \frac{k\,s\,v^2}{a}.$	**(38.)**	$p = \frac{.0000000217 \times 36,000 \times 500^2}{20} = 9.765$ lb. per square foot. Ans.
$p = \frac{u}{q}.$	**(n.)**	$p = \frac{97,650}{10,000} = 9.765$ lb. per square foot. Ans.
$p = \frac{33,000\,H}{a\,v}.$	**(49.)**	$p = \frac{33,000 \times 2.959}{20 \times 500} = 9.765$ lb. per square foot. Ans.
$p = \frac{33,000\,H}{q}.$	**(50.)**	$p = \frac{33,000 \times 2.959}{10,000} = 9.765$ lb. per square foot. Ans.

TABLE 24—*Continued.*

Formulas.		Specimen Calculations.
$p = \frac{k s v^3}{q}$.	(o.)	$p = \frac{.0000000217 \times 36{,}000 \times 500^3}{10{,}000} = 9.765$ lb. per square foot. Ans.
$p = \frac{k s q^2}{a^3}$.	(o′.)	$p = \frac{.0000000217 \times 36{,}000 \times 10{,}000^2}{20^3} = 9.765$ lb. per square foot. Ans.
$p = 5.2\, W$.		$p = 5.2 \times 1.87788 = 9.765$ lb. per square foot. Ans.
To find the quantity of air passing in cubic feet per minute:		
$q = a v$.	(43.)	$q = 20 \times 500 = 10{,}000$ cu. ft. per minute. Ans.
$q = \frac{u}{p}$.	(p.)	$q = \frac{97{,}650}{9.765} = 10{,}000$ cu. ft. per minute. Ans.
$q = \frac{33{,}000\, H}{p}$.	(51.)	$q = \frac{33{,}000 \times 2.959}{9.765} = 10{,}000$ cu. ft. per minute. Ans.
$q = \frac{k s v^3}{p}$.	(54.)	$q = \frac{.0000000217 \times 36{,}000 \times 500^3}{9.765} = 10{,}000$ cu. ft. per minute. Ans.
$q = a \sqrt{\frac{p a}{k s}}$.	(q.)	$q = 20 \sqrt{\frac{9.765 \times 20}{.0000000217 \times 36{,}000}} = 10{,}000$ cu. ft. per minute. Ans.
To find the rubbing surface in square feet:		
$s = \frac{P}{k v^2}$	(39.)	$s = \frac{195.3}{.0000000217 \times 500^2} = 36{,}000$ sq. ft. Ans.
$s = \frac{p a}{k v^2}$.	(39.)	$s = \frac{9.765 \times 20}{.0000000217 \times 500^2} = 36{,}000$ sq. ft. Ans.

TABLE 24—*Continued.*

Formulas.		Specimen Calculations.
$s = l\,o.$	(42.)	$s = 2.000 \times 18 = 36.000$ sq. ft. Ans.
$s = \frac{u}{k\,v^3}.$	(r.)	$s = \frac{97\ 650}{0000000217 \times 500^3} = 36\ 000$ sq. ft. Ans.
$s = \frac{p\,q}{k\,v^3}.$	(s.)	$s = \frac{9.765 \times 10.000}{.0000000217 \times 500^3} = 36{,}000$ sq. ft. Ans.
To find the units of power in foot-pounds per minute:		
$u = P\,v.$	(46.)	$u = 195.3 \times 500 = 97.650$ ft.-lb. per minute. Ans.
$u = p\,q.$	(47.)	$u = 9.765 \times 10.000 = 97{,}650$ ft.-lb. per minute. Ans.
$u = 33{,}000\,H.$	(t.)	$u = 33{,}000 \times 2.959 = 97{,}650$ ft.-lb. per minute. Ans.
$u = p\,a\,v.$	(u.)	$u = 9.765 \times 20 \times 500 = 97{,}650$ ft.-lb. per minute. Ans.
$u = k\,s\,v^3.$	(53.)	$u = .0000000217 \times 36{,}000 \times 500^3 = 97{,}650$ ft.-lb. per minute. Ans.
$u = \frac{k\,s\,q^3}{a^3}$	(u′.)	$u = \frac{.0000000217 \times 36{,}000 \times 10{,}000^3}{20^3} = 97{,}650$ ft.-lb. per minute. Ans.
To find the velocity in feet per minute:		
$v = \sqrt{\frac{p\,a}{k\,s}}.$	(40.)	$v = \sqrt{\frac{9.765 \times 20}{.0000000217 \times 36{,}000}} = 500$ ft. per minute. Ans.
$v = \sqrt{\frac{P}{k\,s}}.$	(v.)	$v = \sqrt{\frac{195.3}{.0000000217 \times 36{,}000}} = 500$ ft. per minute. Ans.

TABLE 24—*Concluded.*

Formulas.		Specimen Calculations.
$v = \frac{q}{a}$.	(44.)	$v = \frac{10,000}{20} = 500$ ft. per minute. Ans.
$v = \frac{u}{P}$.	(w.)	$v = \frac{97,650}{195.3} = 500$ ft. per minute. Ans.
$v = \frac{u}{p\,a}$.	(x.)	$v = \frac{97,650}{9.765 \times 20} = 500$ ft. per minute. Ans.
$v = \frac{33,000\,H}{P}$.	(y.)	$v = \frac{33,000 \times 2.959}{195.3} = 500$ ft. per minute. Ans.
$v = \frac{33,000\,H}{p\,a}$.	(y'.)	$v = \frac{33,000 \times 2.959}{9.765 \times 20} = 500$ ft. per minute. Ans.
$v = \sqrt[3]{\frac{u}{k\,s}}$.	(z.)	$v = \sqrt[3]{\frac{97,650}{.0000000217 \times 36,000}} = 500$ ft. per minute. Ans.
$v = \sqrt[3]{\frac{p\,q}{k\,s}}$.	(z'.)	$v = \sqrt[3]{\frac{9.765 \times 10,000}{.0000000217 \times 36,000}} = 500$ ft. per minute. Ans.
To find the water-gauge:		
$W = \frac{p}{5.2}$.		$W = \frac{9.765}{5.2} = 1.87788$ in. Ans.

NOTE.—The water-gauge is calculated to five decimal places, so that it will correspond to the other values; two places are sufficient in practice.

LAWS OF VENTILATION.

980. In order to ascertain the effects produced by varying the airway or by varying the quantity, velocity, etc., of the air, it is generally easier to make use of one of the following *laws* than to solve by means of one of the foregoing formulas. The laws are also useful for comparing the results obtained from two airways. Letting p, q, v, s, etc., represent, respectively, the pressure, quantity, velocity, rubbing surface, etc., before the change, and p_1, q_1, v_1, s_1, etc., the same things after the change, the laws may be stated as follows:

(1) The pressure varies directly as the extent of the rubbing surface; i. e., $p : p_1 :: s : s_1$, or $P : P_1 :: s : s_1$.

(2) The pressure varies directly as the density* of the air; i. e., $p : p_1 :: w : w_1$, or $P : P_1 :: w : w_1$.

(3) The pressure varies directly as the square of the quantity; i. e., $p : p_1 :: q^2 : q_1^2$, or $P : P_1 :: q^2 : q_1^2$.

(4) The pressure varies directly as the square of the velocity; i. e., $p : p_1 :: v^2 : v_1^2$, or $P : P_1 :: v^2 : v_1^2$.

(5) The pressure varies directly as the length of the airway; i. e., $p : p_1 :: l : l_1$, or $P : P_1 :: l : l_1$.

(6) The pressure varies directly as the length of the perimeter; i. e., $p : p_1 :: o : o_1$, or $P : P_1 :: o : o_1$.

(7) The pressure per square foot varies inversely as the area of the airway; i. e., $p : p_1 :: a_1 : a$.

(8) The quantity varies directly as the square root of the pressure; i. e., $q : q_1 :: \sqrt{p} : \sqrt{p_1}$, or $q : q_1 :: \sqrt{P} : \sqrt{P_1}$.

(9) The quantity varies directly as the cube root of the power; i. e., $q : q_1 :: \sqrt[3]{u} : \sqrt[3]{u_1}$, or $q : q_1 :: \sqrt[3]{H} : \sqrt[3]{H_1}$.

(10) The quantity varies inversely as the square root of the rubbing surface; i. e., $q : q_1 :: \sqrt{s_1} : \sqrt{s}$.

(11) The velocity varies directly as the square root of the pressure; i. e., $v : v_1 :: \sqrt{p} : \sqrt{p_1}$, or $v : v_1 :: \sqrt{P} : \sqrt{P_1}$.

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* By density is meant the *weight of a cubic foot in pounds.*

(12) The velocity varies directly as the square root of the area; i. e., $v : v_1 :: \sqrt{a} : \sqrt{a_1}$.

(13) The velocity varies inversely as the square root of the length of the airway; i. e., $v : v_1 :: \sqrt{l_1} : \sqrt{l}$.

(14) The velocity varies inversely as the square root of the rubbing surface; i. e., $v : v_1 :: \sqrt{s_1} : \sqrt{s}$.

(15) The power varies directly as the cube of the quantity; i. e., $u : u_1 :: q^3 : q_1^3$, or $H : H_1 :: q^3 : q_1^3$.

(16) The rubbing surface varies inversely as the square of the quantity; i. e., $s : s_1 :: q_1^2 : q^2$.

(17) The rubbing surface varies inversely as the square of the velocity; i. e., $s : s_1 :: v_1^2 : v^2$.

(18) The sectional area varies directly as the square of the velocity; i. e., $a : a_1 :: v^2 : v_1^2$.

(19) The length of the airway varies inversely as the square of the velocity; i. e., $l : l_1 :: v_1^2 : v^2$.

(20) The length of the airway varies inversely as the square of the quantity; i. e., $l : l_1 :: q_1^2 : q^2$.

For similar airways, let d equal the length of a side; then,

(21) The quantity varies directly as the square root of the fifth power of the length of the side; i. e., $q : q_1 :: \sqrt{d^5} : \sqrt{d_1^5}$.

(22) The pressure varies inversely as the fifth power of the length of the side; i. e., $p : p_1 :: d_1^5 : d^5$.

(23) The length of the side varies inversely as the fifth root of the pressure; i. e., $d : d_1 :: \sqrt[5]{p_1} : \sqrt[5]{p}$.

(24) The length of the side varies directly as the fifth root of the square of the quantity; i. e., $d : d_1 :: \sqrt[5]{q^2} : \sqrt[5]{q_1^2}$.

To the above laws may also be added another:

(25) If equal quantities of air pass through two airways, the velocities will vary inversely as the areas; i. e., $v : v_1 :: a_1 : a$.

F. I.—21

PRACTICAL PROBLEMS.

981. To illustrate the application of the foregoing laws and formulas, a series of practical examples such as are asked at examinations for mine foremen, together with their solutions, will now be given. By paying particular and careful attention to the statements of the examples and the solutions following them, the student should then be able to work similar ones without trouble. The above twenty-five laws should be carefully memorized, so that the student will not be obliged to refer to them.

1. What quantity of air is passing down a shaft 12 feet in diameter when the current has a velocity of 325 feet per minute?

SOLUTION.—Since the diameter is specified, the shaft is evidently circular. Applying formula **43**,

$$q = a v = 12^2 \times .7854 \times 325 = 36,756.72 \text{ cu. ft. per minute. Ans.}$$

2. Where the airway is 12 feet wide at the bottom, 10 ft. 4 in. wide at the top, and 6 ft. 6 in. high, and the velocity of the air is 340 feet per minute, what is (*a*) the sectional area of the airway, and (*b*) the quantity of air passing per minute?

SOLUTION.—(*a*) The section is a trapezoid; hence,

$$\text{area} = \frac{10\frac{1}{3} + 12}{2} \times 6\frac{1}{2} = 72\tfrac{7}{12} \text{ sq. ft.},$$

since 4 in. $= \frac{1}{3}$ ft., and 6 in. $= \frac{1}{2}$ ft. Ans.

(*b*) Applying formula **43**,

$$q = a v = 72\tfrac{7}{12} \times 340 = 24,678\tfrac{1}{3} \text{ cu. ft. per minute. Ans.}$$

3. If a shaft 8 ft. by 24 ft. in section is the intake, and the fan is exhausting 160,000 cubic feet of air per minute, what is the velocity of the air-current in the shaft?

SOLUTION.—Applying formula **44**,

$$v = \frac{q}{a} = \frac{160,000}{8 \times 24} = 833\tfrac{1}{3} \text{ ft. per minute. Ans.}$$

4. The section of an airway is a right-angled triangle, 10 feet wide at the base and $7\frac{1}{2}$ feet high; what quantity of air is passing when the velocity is 280 feet per minute?

SOLUTION.—Area of section $= \frac{10 \times 7.5}{2} = 37.5$ sq. ft. Then, applying formula **43**,

$$q = a v = 37.5 \times 280 = 10,500 \text{ cu. ft. per minute. Ans.}$$

5. An air-course is 500 yards long, 6 feet high, and 7 feet wide; what is (*a*) its sectional area, (*b*) its perimeter, and (*c*) its rubbing surface?

SOLUTION.—(*a*) Sectional area $a = 6 \times 7 = 42$ sq. ft. Ans.

(*b*) Perimeter $o = 6 \times 2 + 7 \times 2 = 26$ ft. Ans.

(*c*) Applying formula **42**,

$$s = l\,o = 500 \times 3 \times 26 = 39{,}000 \text{ sq. ft. Ans.}$$

6. The rubbing surface is 25,000 sq. ft. and the perimeter 50 ft.; what is the length?

SOLUTION.—Applying formula **41**,

$$l = \frac{s}{o} = \frac{25{,}000}{50} = 500 \text{ ft} \quad \text{Ans.}$$

7. When the water-gauge is 1.85 in., what pressure per square foot does it indicate?

SOLUTION.— $p = 5.2\,W = 5.2 \times 1.85 = 9.62$ lb. per square foot. Ans.

8. What is the total ventilating pressure of an airway 6 feet by 7 feet, the water-gauge being .5 of an inch?

SOLUTION.—Pressure per square foot $= 5.2 \times .5 = 2.6$ lb.; area $= 6 \times 7 = 42$ sq. ft. Applying formula **36**,

$$P = p\,a = 2.6 \times 42 = 109.2 \text{ lb. Ans.}$$

9. What quantity of air is passing through an airway 7 feet high by 7 feet wide when the velocity of the current is 300 feet per minute?

SOLUTION.—Applying formula **43**,

$$q = a\,v = 7 \times 7 \times 300 = 14{,}700 \text{ cu. ft. per minute. Ans.}$$

10. If 80,000 cubic feet of air are required per minute in a mine, and the shaft velocity must not exceed 800 feet per minute, what is the smallest sectional area that the shaft may have?

SOLUTION.—Using formula **45**,

$$a = \frac{q}{v} = \frac{80{,}000}{800} = 100 \text{ sq. ft. Ans.}$$

11. Suppose a gangway 10 feet by 10 feet and 1,000 feet long, in which the air has a velocity of 450 feet per minute, and the pressure as indicated by the water-gauge is 2 pounds; what is (*a*) the water-gauge reading, (*b*) the quantity of air passing per minute, and (*c*) the horse-power?

SOLUTION.—(*a*) Water-gauge $= W = \frac{2}{5.2} = .38$ in. Ans.

(*b*) Applying formula **43**,

$$q = a\,v = 10 \times 10 \times 450 = 45{,}000 \text{ cu. ft. per minute. Ans.}$$

(*c*) Using formula **48**,

$$H = \frac{p\,q}{33{,}000} = \frac{2 \times 45{,}000}{33{,}000} = 2.727 \text{ H. P.} \quad \text{Ans.}$$

12. If you have two airways under the same pressure, one 6 feet wide, 6 feet high, and 5,000 feet long, the other 8 feet wide, 4½ feet high, and 5,000 feet long, which will pass the greater quantity of air, and why?

SOLUTION.—Since the pressure and length remain the same, it is evident that the airway having the smaller perimeter will pass the greater quantity, since the rubbing surface will be less; perimeter of first airway $= 6 \times 4 = 24$ ft.; of the second airway, $8 \times 2 + 4\frac{1}{2} \times 2 = 25$ ft. Representing by 1 the amount passed by the first airway, and applying law (**10**), $1 : q_1 :: \sqrt{25} : \sqrt{24}$, or $q_1 = .98$; i. e., the second airway will pass 98% of the amount passed by the first airway. Ans.

13. The pressure producing ventilation is 7.8 pounds per square foot; what is the water-gauge?

SOLUTION.— $W = \frac{p}{5.2} = \frac{7.8}{5.2} = 1.5 = 1\frac{1}{2}$ in. Ans.

14. When the quantity of air passing is 60,000 cubic feet, with a water-gauge of 1.5 inches, what are the units of power producing ventilation?

SOLUTION.—Pressure $= p = 5.2\,W = 5.2 \times 1.5 = 7.8$ lb. per square foot. Using formula **47**,

$$u = p\,q = 7.8 \times 60{,}000 = 468{,}000 \text{ ft. lb. per minute.} \quad \text{Ans.}$$

15. How many horsepower are represented by 468,000 units of power?

SOLUTION.—Using formula **48**,

$$H = \frac{u}{33{,}000} = \frac{468{,}000}{33{,}000} = 14.18 \text{ H. P., nearly.} \quad \text{Ans.}$$

16. With a water-gauge of $\frac{6}{10}$ of an inch, the quantity of air passing is 24,000 cubic feet per minute; what water-gauge will be required to pass 36,000 cubic feet per minute?

SOLUTION.—Since the water-gauge and pressure are directly proportional to each other, law (**3**) may be applied; or,

$$24{,}000^2 : 36{,}000^2 :: .6 : x; \text{ whence, } x = 1.35 \text{ in. of water.} \quad \text{Ans.}$$

17. If 16,500 cubic feet of air are passing per minute with a pressure of 4.68 pounds per square foot, what quantity will pass with a pressure of 6.24 pounds per square foot?

SOLUTION.—Applying law (**8**),

$$16{,}500 : q_1 :: \sqrt{4.68} : \sqrt{6.24}; \text{ or, } q_1 = 19{,}052 \text{ cu. ft. per minute.} \quad \text{Ans.}$$

18. If 3 horsepower pass 15,000 cubic feet of air per minute, what horsepower would be required to double the quantity?

SOLUTION.—Applying law (**15**),

$$3 : H_1 :: 15{,}000^3 : (15{,}000 \times 2)^3; \text{ or, } H_1 = 24 \text{ H. P.} \quad \text{Ans.}$$

19. Is there any disadvantage or loss in having the air travel at a high speed?

SOLUTION.—There is a very decided loss; for, according to the third law of friction, the pressure varies as the square of the velocity. If, therefore, the velocity is to be doubled, the pressure must be increased as the *square of two;* that is, four times. If the velocity is to be trebled, the pressure must be increased as the square of three; that is, nine times.

20. If 32,000 cubic feet of air are passing through an airway 6′ × 5′, under a pressure of 3.6 pounds per square foot, what pressure is necessary in an airway 9′ × 5′ to pass the same quantity?

SOLUTION.—Call the resistance of the first airway A, and that of the second one B, and call the required pressure x; then, $A : B :: 3.6 : x$, or $x = \frac{B}{A} \times 3.6$, because the pressures vary directly as the resistances.

$$x = \frac{\left(\frac{32{,}000}{45}\right)^2 \times \frac{28}{45} \times 3.6}{\left(\frac{32{,}000}{30}\right)^2 \times \frac{22}{30}}; \text{ or, by cancelation, } x = \frac{\frac{28}{45^3} \times 3.6}{\frac{22}{30^3}}.$$

Further, $x = \frac{30^3}{22} \times \frac{28}{45^3} \times 3.6 = \frac{2^3}{11} \times \frac{14}{3^3} \times 3.6 = 1.3575$ lb., the required pressure per square foot. Ans.

21. If a pressure of 3.2 pounds per square foot produces a velocity of 560 feet per minute, what pressure is required to produce a velocity of 700 feet per minute in the same airway?

SOLUTION.—Applying law (**4**),

$$3.2 : p_1 :: 560^2 : 700^2; \text{ whence, } p_1 = 5 \text{ lb. per square foot.} \quad \text{Ans.}$$

22. If 24,000 cubic feet are passing through an airway having a rubbing surface of 75,000 square feet, what quantity will pass if the rubbing surface is increased to 100,000 square feet, the increase of rubbing surface being due to the lengthening of the airway?

SOLUTION.—Applying law (**10**),

$$24{,}000 : q_1 :: \sqrt{100{,}000} : \sqrt{75{,}000}; \text{ or, } q_1 = 20{,}785 \text{ cu. ft.} \quad \text{Ans.}$$

23. If in an airway 1,200 feet long the air has a velocity of 400 feet per minute under a pressure of 3 pounds per square foot, what must the pressure be to maintain the same velocity if the length of airway is increased to 1,800 feet?

SOLUTION.—Applying law (**5**).

$3 : p_1 :: 1,200 : 1,800$; or, $p_1 = 4.5$ lb. per square foot. Ans.

24. If the air passes with a velocity of 600 feet per minute through an airway whose sectional area is 64 square feet, what will the velocity be if the area is decreased to 48 square feet, the pressure remaining constant?

SOLUTION.—Applying law (**12**).

$600 : v_1 :: \sqrt{64} : \sqrt{48}$; or, $v_1 = 519.6$ ft. per minute. Ans.

25. Two circular airways of the same length have diameters of 3 feet and 4 feet, respectively; if a pressure of 5 pounds per square foot will force the air through the 4-foot airway, what pressure is required to pass the same quantity through the 3-foot airway?

SOLUTION.—Law (**22**) must be applied to this case, and since a circle has no sides, the perimeter or diameter may be used in the proportion. Hence, $5 : p_1 :: 3^5 : 4^5$; or, $5 : p_1 :: 243 : 1,024$; whence, $p_1 = 21.07$ lb. per square foot. Ans.

26. If 10,000 cubic feet of air pass per minute through a circular airway 12 feet in diameter, how many cubic feet per minute will pass through an airway 6 feet in diameter and having the same length, the pressure being the same in both cases?

SOLUTION.—Applying law (**21**),

$10,000 : q_1 :: \sqrt{12^5} : \sqrt{6^5}$; or, $q_1 = 1,768$ cu. ft. per minute. Ans.

982. Remarks.—By aid of the foregoing laws and formulas, the student can calculate any problem relating to the flow of air which does not involve splits or regulators. Many of the formulas would be unnecessary if the student had even a slight knowledge of algebra. For example, formulas **a′**, **f**, **39** and **40** are all derived from formula **38** by simply transposing terms; and formula **38** is, in turn, derived from formula **37** by substituting for P its value $p\ a$. If the student has no knowledge of algebra, he should memorize all of the numbered formulas.

983. In working examples, the student should proceed as follows: Consider example 11, Art. **981.** First ascertain what is required. In this example we want the water-gauge, the quantity, and the horsepower. The water-gauge is easily obtained, since p, the pressure per square foot, is given. To find q, the quantity, we look in Table 24, and find that there are five formulas by which the value of q may be obtained.

We can not use **p** or **51**, because we do not know the values of u or H; but we can use any one of the three remaining formulas, for we know, or can readily find, the values of a, v, s, and k, which are given in the example. Such being the case, we naturally use the one which will require the least amount of labor, and that is, evidently, formula **43.** To find H, the horsepower, we refer again to Table 24, and find four different forms of formula **48**, any one of which may be used, since u, in the first form, may be found by means of formula **47.** We again use the easiest one, which is the one used in the solution, since the values of p and q are both known.

Examples solved like Example 18, Art. **981**, are worked in a similar manner. First find what is wanted (in this case H_1) and what is given (in this case H, q, and q_1); then look in the list of laws (Art. **980**) for one giving the relation between the horsepower and the quantity (in this case law **15**).

INFLUENCE OF A STACK UPON THE MOTIVE COLUMN.

984. The erection of a stack over the upcast shaft has the same effect as increasing the effective depth of the shaft, or, in other words, increasing the height of the motive column. The quantity of air in circulation is thereby increased according to the proportion

$$\sqrt{D} : \sqrt{D + h} :: q_1 : q_2;$$

or,

$$q_2 = q_1 \sqrt{\frac{D + h}{D}},$$

and

$$q_1 = q_2 \sqrt{\frac{D}{D + h}}, \qquad (\mathbf{55_1.})$$

in which q_1 = quantity of air in circulation per minute without the stack;

q_2 = quantity of air in circulation per minute after the stack is erected;

D = depth of shaft;

h = height of stack.

EXAMPLE.—The depth of a certain furnace shaft is 225 feet, the height of the stack over it is 31 feet; to what will a circulation of 24,000 cu. ft. per minute be reduced if the stack is blown down?

SOLUTION.—Applying formula **55₁**,

$$q_1 = 24,000\sqrt{\frac{225}{225 + 31}} = 24,000 \times \frac{15}{16} = 22,500 \text{ cu. ft. per minute. Ans.}$$

SPLITTING THE AIR.

985. By splitting is here meant dividing the ventilating current into two or more currents, each of which circulates in a separate district of the mine; any currents thus formed is commonly called a **split.**

The benefits to be derived from the splitting of the air-current may be stated as follows:

(*a*) A larger volume of air may be circulated in a mine with the same power.

(*b*) Fresher and purer air is supplied at the working face in each district, and the velocity of the current traversing the face is moderate.

(*c*) Each district has its own circulation, which is readily controlled, and may be increased or decreased as occasion may require.

(*d*) An explosion or a windy shot occurring in one district is not as often transmitted throughout the mine.

The student will realize at once the importance of a thorough knowledge of this portion of the subject. For example, the means employed for the ventilation of a new mine, whether fan or furnace, are often found after a year or two of rapid development to be inadequate to the present need. This difficulty is easily overcome in most cases by judicious splitting of the ventilating current.

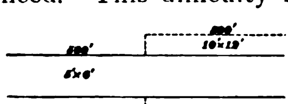


FIG. 140.

986. In order to study the effects of splitting, consider Fig. 140, which represents an airway 1,000 feet long and 5' × 6' in section. Suppose that 12,000 cubic feet per minute are passing; then, the velocity is $\frac{12,000}{5 \times 6} = 400$ feet per minute. The rubbing surface is $(2 \times 5 + 2 \times 6) \times 1,000 = 22,000$ square feet. Hence, by formula **53**, $u = k s v^3 = .0000000217 \times 22,000 \times 400^3 = 30,553.6$ foot-pounds per minute.

Suppose that midway in its length the airway were to be enlarged, as shown by the dotted lines. The velocity in the large airway must now be greatly reduced, since the quantity discharged is the same as before (assuming that the velocity in the small airway remains 400 feet per minute), and the area of the section being larger, the velocity must be less. By formula **44**, the velocity $= v = \frac{q}{a} = \frac{12,000}{10 \times 12} = 100$ feet per minute. Now, since the rubbing surface of the small airway is just one-half of what it was before, the power required to force the air through it must evidently be one-half, or $\frac{30,553.6}{2} = 15,276.8$ foot-pounds per minute. The power required to force the air through the large airway is (see formula **47**) $u = p\,q = \frac{k s v^2}{a} \times q =$

$$\frac{.0000000217 \times [(2 \times 10 + 2 \times 12) \times 500] \times 100^3}{10 \times 12} \times 12,000 =$$

477.4 ft.-lb. per minute. Hence, the total power $= 15,276.8 + 477.4 = 15,754.2$ foot-pounds per minute, while in the former case, 30,553.6 foot-pounds per minute were required. This shows that by enlarging the airway, as shown, the same quantity may be passed with a greatly reduced power, or the quantity may be greatly increased with the same power. The quantity that will pass with the same power is easily found by applying law **15.** Thus,

$$15,754.2 : 30,553.6 :: 12,000^3 : q_1^3,$$

or

$$q_1 = 14,965 \text{ cu. ft. per min.}$$

Since, before the enlargement, 12,000 cubic feet were discharged, the gain is $14,965 - 12,000 = 2,965$ cubic feet per minute, or very nearly 25 per cent.

The above calculation shows that in the case just considered the small airway requires $\frac{15,276.8}{477.4} = 32$ times as much power as the large one, and that if the small airway be decreased in length, this proportion of 32 to 1 will also decrease. Consequently, in cases like the above, it is best to decrease the length of the small airway as much as possible.

986_1. To split the air-current to advantage requires that the *main* airways, both *intake* and *return*, as also the *downcast* and *upcast* shafts, shall be of ample sectional area. A wise provision should always be made in this respect, since the entire circulation for all the districts must pass through these main airways. Were it not for this crowding of all the circulation into these main conduits for a short distance, the volume of air produced, when the power remains constant, would be always proportional to the number of primary splits. That is to say, we would obtain double the quantity of air with the same power when we double the number of splits, and likewise for any number of primary splits the quantity would be in proportion to the number of splits. But since all the circulation must be crowded through the main airway at a high velocity till the point is reached where the first split is made, we do not obtain an increase of quantity in the same proportion as we increase the number of splits. The increase in quantity will always be in a less ratio. To make this clear, let us take for illustration some practical examples.

EXAMPLE 1.—Find the power (foot-pounds per minute) that will circulate 24,000 cubic feet of air per minute in a mine, under the following conditions: the air to be circulated in one continuous current through an airway 16,000 feet long, including the return, the size of all the airways to be 6′ × 10′ throughout the mine.

SOLUTION.—Using formula **u′**,

$$u = \frac{k\,s\,q^3}{a^3} = \frac{.0000000217 \times 2\,(6 + 10) \times 16,000 \times 24,000^3}{(6 \times 10)^3} = 711,066 \text{ ft.-lb.}$$

per minute. Ans.

EXAMPLE 2.—Find the power (foot-pounds per minute) that will circulate 24,000 cubic feet of air through a mine under the following conditions: Referring to Fig. 141, the air is divided at the foot of the downcast *d* into four splits, each 3,600 feet long and 6 × 10 feet in section, and finally united at the foot of the upcast *u*. The shafts are each 800 feet deep and are also 6′ × 10′ in section. It will be noticed that the size and total length of airways are the same as in Example 1.

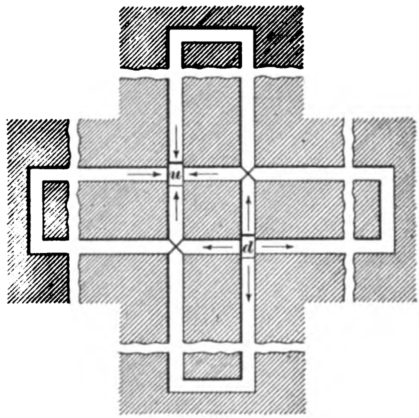


FIG. 141.

SOLUTION.—It is evident that the total power is equal to the sum of the powers absorbed in the different passages. Using formula **u**′, the power absorbed in the two shafts is

$$u = \frac{k\,s\,q^3}{a^3} = \frac{.0000000217 \times 2\,(6+10) \times 2 \times 800 \times 24{,}000^3}{(6 \times 10)^3} = 71{,}107 \text{ ft.-lb.}$$

per minute, nearly.

Using the same formula and noting that, since the splits are equal, the quantity of air passing through each will be $\frac{24{,}000}{4} = 6{,}000$ cu. ft., the power absorbed by the splits is

$$u = \frac{k\,s\,q^3}{a^3} = \frac{.0000000217 \times 2\,(6+10) \times 4 \times 3{,}600 \times 6{,}000^3}{(6 \times 10)^3} = 10{,}000 \text{ foot-}$$

pounds per minute, nearly.

The total power is therefore 71,107 + 10,000 = 81,107 ft.-lb. per min. Ans.

This result shows that in this case it required, after splitting, but 11.4 per cent. of the power originally required to pass the same quantity of air through the mine.

EXAMPLE 3.—Determine the quantity of air that the power used in Example 1 will pass through a mine having the same conditions as those in Example 2.

SOLUTION.—Applying law **15**,

$$81{,}107 : 711{,}066 = 24{,}000^3 : q_1^3;$$

or, $q_1 = \sqrt[3]{\frac{711{,}066}{81{,}107}} \times 24{,}000 = 49{,}488$ cubic feet per minute, nearly. Ans.

Comparing the result in Example 3 with the quantity circulated in Example 1, it will be seen that, by splitting as in

Example 2, over twice as much air is passed through the mine with the same power.

986₂. In all cases of equal splitting at a distant point in the mine, the increased quantity of air put in circulation by the original power may be found by the formula

$$q_1 = \frac{n\,q}{\sqrt[3]{1 + \frac{l}{L}(n^3 - 1)}}. \qquad \textbf{(55_2.)}$$

in which q = original quantity;
q_1 = increased quantity;
n = number of splits;
l = length of airway from beginning of intake to point of splitting;
L = total length of original airway.

EXAMPLE.—In a drift mine 3,500 feet long, 50,000 cubic feet of air per minute are circulated in a continuous current. What quantity will the same power circulate if three splits are made at a point on the intake 1,000 feet from the drift mouth ?

SOLUTION.—Applying formula **55₂**,

$$q_1 = \frac{n\,q}{\sqrt[3]{1 + \frac{l}{L}(n^3 - 1)}} = \frac{3 \times 50{,}000}{\sqrt[3]{1 + \frac{1{,}000}{3{,}500}(3^3 - 1)}} = 73{,}706 \text{ cu. ft.} \quad \text{Ans.}$$

986₃. If the method of ventilating a mine be changed from a continuous current to a number of splits, the total quantity of air that the original power will pass through the splits can be found by the following formula, in which q_t = total quantity passing through the splits; q = quantity passing through original airway; a_t = total area of splits; a = area of original airway; s_t = total rubbing surface of splits; and s = rubbing surface of original airway. If desired, the quantities passing through the separate splits can then be found by the method used in Art. **992.**

$$q_t = \frac{a_t\,q}{a}\sqrt[3]{\frac{s}{s_t}}. \qquad \textbf{(55_3.)}$$

EXAMPLE.—If a certain power circulates 80,000 cu. ft. of air per min. through an airway 9′ × 6′ in section and 9,400 ft. long, what quantity will it pass through the following splits which are substituted for the original airway? Split *A*, 9′ × 6′ in section and 5,400 ft. long; split *B*, 8′ × 5 in section and 3,600 ft. long; split *C*, 6′ × 6′ in section and 3,000 ft. long.

SOLUTION.—Using formula **55**,

$$q_t = \frac{(9 \times 6 + 8 \times 5 + 6 \times 6) \times 80,000}{9 \times 6} \times$$

$$\sqrt[3]{\frac{2(9+6) \times 9,400}{2(9+6) \times 5,400 + 2(8+5) \times 3,600 + 2(6+6) \times 3,000}} =$$

183,206, say, 183,200 cu. ft. per min. Ans.

987. To realize the benefits which may be obtained by splitting, consider Fig. 142, which is a simple, practicable case. *D* is the downcast and *U* the upcast shaft. Imagine the airways *A I* and *K J* to be removed. Then the air will flow down the shaft *D* and along the airway *D B C H E F G U*

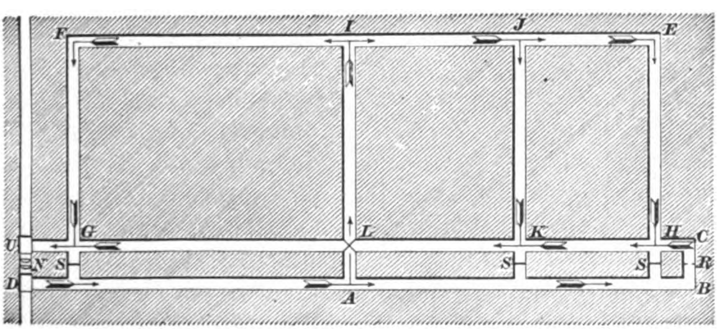


FIG. 142.

and up the upcast shaft. Suppose that the distance *D B* were 3,000 feet and the distance *A I*, 1,500 feet. The total distance traveled by the air from the foot of the downcast to the foot of the upcast would then be about 9,000 feet. Before the air reached *F* it would be foul and heavy with mine gases, carbonic acid gas, and other impurities, and it would be nearly impossible to work there. By splitting the air, this condition of things is remedied to a great extent. Thus, by splitting at *A*, fresh air from *D* passes along *A I* and is again split at *I*; a part going to *F* and another part towards *E*. At *J*, there is another split, a certain proportion going to *K* and the remainder to *E*, and from *E* to *H*. *D B* is called the **main airway** and *U C* the **main return.** After splitting at *A*, that portion of the air which does not pass along *L I* continues along the main airway to the point *C*, where it passes into the main return and flows directly to the upcast at *U*. In order to accomplish the result described, a **bridge** is necessary at *L* to keep the fresh air from mingling with the return air; stoppings must be introduced at *S*, *S*, *S*, and a regulator must be placed at *R*. A **regulator** is an arrangement by which the sectional area of an airway can be reduced; it is virtually an increase of resistance to the movement of the flowing air. Only the reasons for using it will be mentioned here, as it will be described fully later.

Air or any other fluid will also travel along the path of least resistance, and always tends towards equilibrium. Now, suppose that it requires a greater power to force a certain quantity of air along the combined paths *A I F G U*, *I J K L*, and *J E H K*, than along the path *A B C U*; then more air will go towards *B* than towards *I*. But it is the exact opposite of this that is required; in other words, it is necessary that more air should flow towards *I* than towards *B*. By interposing a sufficiently great resistance at *R*, the greater volume of air will be forced to flow towards *I*.

This, then, is the principal object of splitting—*to supply the workings with fresh air.* Splitting must not, however, be carried to too great an extent, since every split reduces the velocity very rapidly, and the current will soon become too feeble to sweep out the noxious gases.

988. The first split, or, in fact, any split in the main airway, is variously called a **main split,** a **primary split,** or a **split of the first degree,** as at *A*. The second split, as at *I*, is called a **secondary split,** or a **split of the second degree.** The split at *J* is called a **tertiary split,** or a **split of the third degree.** It should be noted that the degree of the split does not refer to the number of splits, though it happens so in the above case. The air coming along the main airway is divided at *A*, a part going to *I*; this part is again divided, a part going to *J*, and this last part is once more divided. Where two returns unite, as at *G*, *K*, and *H*, they are called **junctions.**

989. Unequal Splitting.—It was stated, when describing the necessity and action of a regulator, that the air always tended towards equilibrium. By this was meant that when the air had adjusted itself to the conditions governing its flow, a certain proportion would go one way and another proportion the other way, and no matter what the quantity passing might be, these proportions would always be preserved, provided there were no alterations in the lengths or sectional areas of the airways. To take a very simple illustration, suppose that in Fig. 143, *D* is the downcast and *U* the upcast. Then, it is evident, from what has been previously stated, that more air will flow through *D C U* than through *D B U*, since *D C U* is shorter, has less rubbing surface, and, consequently, offers less resistance than *D B U*, the same sectional area and perimeter being assumed for both airways. In this case the air is split at *D*, and whenever there is a split in which one airway receives a greater quantity than the other, it is called an **unequal split.**

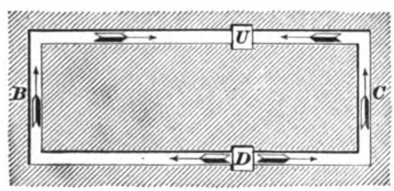


FIG. 143.

990. In every case of splitting, whether equal or unequal, *the pressure per square foot is the same in both splits.*

In order to explain this apparently inconsistent statement, one of the most important pertaining to the science of mine ventilation, it is necessary to digress for a time from the main subject.

In Fig. 144, let *A B D C* represent a vessel filled with a fluid, say water, for convenience, having two columns, *A B* and *C D*, fitted with pistons, as shown, and communicating by the passageway *B D*. Suppose that the area of the smaller piston be 1 square foot and of the larger, 5 square feet; then, in order that there shall be equilibrium, that is, in order that the level of the water in both vessels shall be the same,

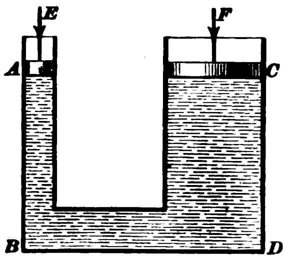


FIG. 144.

the pressure per square foot on the piston at *A* must be equal to the pressure per square foot on the piston at *C*. This follows from **Pascal's law,** which states that *in the case of any fluid* (gas or liquid), *pressure is transmitted undiminished in all directions, whether downwards, upwards, or sideways.* If a force *E* of 5 pounds acts upon the piston *A*, a force of 5 pounds per square foot (since the area of *A* is 1 square foot) will be transmitted *upwards* against the piston *C*. Hence, to prevent *C* from moving upwards, a downward force *F* of $5 \times 5 = 25$ pounds must be applied to *C*. Moreover, it matters not what the areas of the pistons *A* and *C* are, the pressure per square foot must be the same on both pistons in order that they shall not move.

991. The same result obtains in a case like Fig. 145. Here a force *E* acts upon the piston *A*, and the pressure per square foot on *A* is transmitted with equal intensity to all parts of the surfaces touched by the water. This is exactly analogous to a split in which *A G* represents the downcast shaft and *C G* and *G H* the splits.

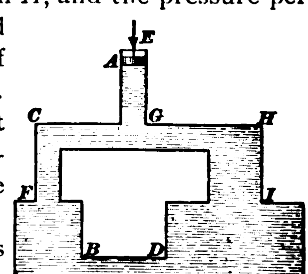


FIG. 145.

As stated above, Pascal's law is true for either liquids or gases. It has to be modified somewhat when applying it to the case of air in motion, since it is then true only when the motion is uniform. But to secure uniformity of motion, the resistance must be uniform, a condition which is practically always the case in mine ventilation. Since the airway up to the split requires a certain pressure to overcome the resistance it offers, this pressure should be deducted from the reading of the water-gauge, and the remainder treated as the pressure in each split.

992. Resuming now the subject of unequal splitting, consider Fig. 146. Let *D* and *U* represent the downcast and upcast shafts, respectively. Four unequal splits are here represented. The upcast and downcast shafts are 15' × 10' and 600 feet deep; the airway *D A U* is 5' × 8' and 2,000 feet long; the airway *D B U* is 6' × 9' and 1,500 feet long; the airway *D C U* is 7' × 9' and 3,000 feet long, and the airway *D E U* is 8' × 10' and 1,800 feet long. Suppose that the velocity of the air in the shafts is 700 feet per minute and that it is required to find the pressure per square foot, the quantity passed by each split, and the horsepower required to circulate the air.

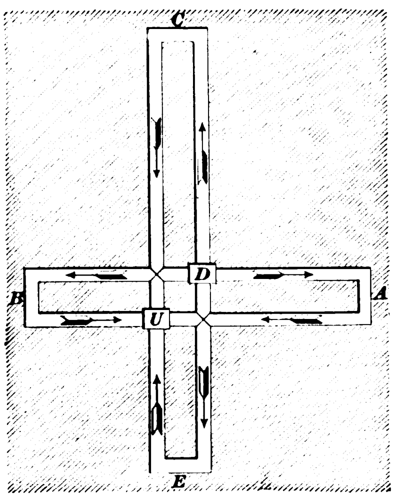


FIG. 146.

It is first necessary to find the total quantity of air passing through the shaft. This evidently equals, using formula **43**, $q = a\,v = 15 \times 10 \times 700 = 105{,}000$ cubic feet per minute.

The pressure per square foot required to pass this through the two shafts is

$$p = 2 \times \frac{k\,s\,v^2}{a} =$$

$$\frac{2 \times .0000000217 \times (2 \times 15 + 2 \times 10) \times 600 \times 700^2}{15 \times 10} =$$

4.2532 lb. per square foot.

Before the pressure per square foot for the splits can be found, it is necessary to calculate the quantities passing through each split. In order not to confuse the student, only the steps necessary for the calculation will here be given.

Let q_1, a_1 and s_1, q_2, a_2 and s_2, q_3, a_3 and s_3, q_4, a_4 and s_4 represent the quantity, sectional area, and rubbing surface in the splits *D A U*, *D B U*, *D C U*, and *D E U*, respectively. Then calculate the following expressions:

$$\sqrt{\frac{a_1^3}{s_1}} = \sqrt{\frac{40^3}{52,000}} = 1.1094, \text{ since } a_1 = 5 \times 8 = 40 \text{ and } s_1 = (2 \times 5 + 2 \times 8) \times 2,000 = 52,000.$$

$$\sqrt{\frac{a_2^3}{s_2}} = \sqrt{\frac{54^3}{45,000}} = 1.8706, \text{ since } a_2 = 6 \times 9 = 54 \text{ and } s_2 = (2 \times 6 + 2 \times 9) \times 1,500 = 45,000.$$

$$\sqrt{\frac{a_3^3}{s_3}} = \sqrt{\frac{63^3}{96,000}} = 1.6139, \text{ since } a_3 = 7 \times 9 = 63 \text{ and } s_3 = (2 \times 7 + 2 \times 9) \times 3,000 = 96,000$$

$$\sqrt{\frac{a_4^3}{s_4}} = \sqrt{\frac{80^3}{64,800}} = 2.8109, \text{ since } a_4 = 8 \times 10 = 80 \text{ and } s_4 = (2 \times 8 + 2 \times 10) \times 1,800 = 64,800.$$

$$sum = 7.4048$$

Dividing each of the above results by their sum, and multiplying by the total quantity passing through the shaft, 105,000 cubic feet per minute, the results thus obtained will be the quantities of air passing through the different splits. Thus,

$$q_1 = \frac{1.1094}{7.4048} \times 105,000 = 15,731 \text{ cu. ft. per minute in } D\,A\,U.$$

$$q_2 = \frac{1.8706}{7.4048} \times 105,000 = 26,525 \text{ cu. ft. per minute in } D\,B\,U.$$

$$q_3 = \frac{1.6139}{7.4048} \times 105,000 = 22,885 \text{ cu. ft. per minute in } D\,C\,U.$$

$$q_4 = \frac{2.8109}{7.4048} \times 105,000 = 39,859 \text{ cu. ft. per minute in } D\,E\,U.$$

$$sum = 105,000 \text{ cu. ft. per minute.}$$

Now, find the velocities by applying formula **44.**

$$v_1 = \frac{15,731}{40} = 393.3 \text{ ft. per minute in } D\,A\,U.$$

$$v_2 = \frac{26,525}{54} = 491.2 \text{ ft. per minute in } D\,B\,U.$$

$$v_3 = \frac{22,885}{63} = 363.3 \text{ ft. per minute in } DCU.$$

$$v_4 = \frac{39,859}{80} = 498.2 \text{ ft. per minute in } DEU.$$

Since the pressure is the same for each split, it is necessary to find it for one only. Hence,

$$p_1 = \frac{k s_1 v_1^2}{a_1} = \frac{.0000000217 \times 52,000 \times 393.3^2}{40} = 4.3637 \text{ lb. per}$$
square foot.

The total ventilating pressure per square foot is $4.2532 + 4.3637 = 8.6169$, say 8.62, pounds per square foot.

By formula **48**, the horsepower =

$$H = \frac{pq}{33,000} = \frac{8.62 \times 105,000}{33,000} = 27.43 \text{ horsepower, nearly.}$$

Examples similar to the above may be solved in the same way.

REGULATORS.

993. A regulator is shown in Fig. 147, and consists principally, as will be noticed, of a sliding shutter moving

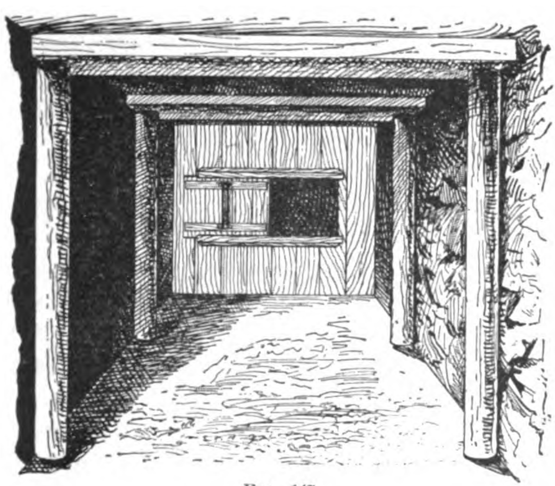


FIG. 147.

in grooves. By means of this shutter, the width of the opening may be adjusted so as to cause a greater or less quantity of air to pass through the airway.

In order to clearly understand the effect produced by a regulator, it is necessary to consider once more what determines the ventilating pressure per square foot. By formula **38**, $p = \frac{k s v^2}{a}$, and since the value of neither k nor a changes for the same airway through the introduction of a regulator, they may be neglected in *comparing* the results obtained by changing s and v. Now, if p represents the ventilating pressure per square foot in the spilts, it should be evident from what has been stated before that the mere introduction of a regulator in any split will not change the value of p in that split, provided the quantity of air passing through the other splits be not increased, since, if p were increased for one split, it would have to be increased a like amount also for the other splits, in order to restore the equilibrium according to Pascal's law, and this would increase the quantity of air in the other splits. But the introduction of a regulator in any split reduces the quantity of air passing through that split, and, as a consequence, reduces the velocity. Hence, if p is to remain the same, s must be increased, or some device must be used which will produce the same effect as increasing s; this device is the regulator itself. The conclusion is now evident: *the regulator is equivalent to lengthening the airway.*

994. Since, by formula **47**, the power $= u = p q$, and p remains the same after the regulator has been placed in the split, while q is reduced in consequence of the reduction of the quantity of air in the split containing the regulator, it is evident that less power will be required than before the regulator was introduced. Hence, if the power remains the same, both the velocity and the pressure will be increased throughout the mine, and the other splits will pass more air than before and *at a higher pressure*. This last is a very important feature in the case of gaseous mines, and will now be explained.

995. In Fig. 148, let *D* be the downcast and *U* the upcast shaft. The air is split at *A*, as shown. Suppose that the shafts are 8′ × 14′ and 500 feet deep, and that all of the airways are 9′ × 12′ and of the following lengths: *D A* = 1,320 feet; *A B C U* = 2,640 feet = ½ mile, and *A E U* = 10,560 feet = 2 miles.

Suppose that the water-gauge in one of the return airways near *U* indicates, say, 1.53 inches, then the quantity of air passing in each split may readily be found. Since $p = 5.2\ W$, $p = 5.2 \times 1.53 = 7.956$ lb. per square foot. A certain amount of this is absorbed in overcoming the resistance of *D A*, while the remainder urges the air through

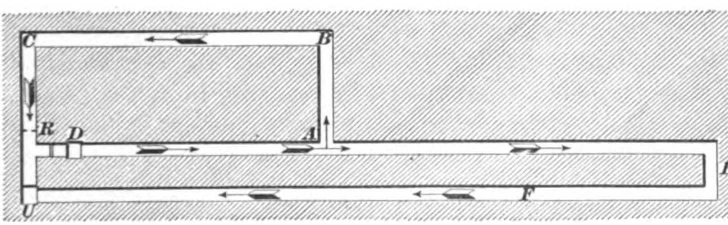


FIG. 148.

A B C U and *A E U*. In order to find what proportion of the pressure is expended in *D A*, and what proportion in the splits, it is first necessary to find the relative velocities in the two splits. Representing by v_1 and v_2 the velocities of air in *A B C U* and *A E U*, respectively, and applying law **(13)**,

$$v_1 : v_2 :: \sqrt{2} : \sqrt{.5};\ \text{or,}\ v_1 = v_2 \times \frac{\sqrt{2}}{\sqrt{.5}} = v_2 \times \sqrt{4} = 2\,v_2.$$

Now, representing by p_1 the pressure in the splits; by p, the pressure required to pass the air through the airway *D A*, and by v, the velocity in *D A*, we have, by applying formula **38**,

$$p = \frac{k\,s\,v^2}{a},\ \text{and}\ p_1 = \frac{k\,s_1\,v_1^2}{a};$$

in other words, $$p : p_1 :: \frac{k\,s\,v^2}{a} : \frac{k\,s_1\,v_1^2}{a};$$

or, $$p : p_1 :: s\,v^2 : s_1\,v_1^2.$$

996. Since both splits have the same sectional area, and the velocity in the short one is twice that in the long one, it is evident that the short split passes twice the quantity that the long one does, or the short split passes two-thirds and the long split one-third of the total quantity coming along the airway DA. If v is the velocity in DA, it is evident (since the sectional areas are equal) that $v_1 = \frac{2}{3}v$, and $v_2 = \frac{1}{3}v$. Then, since $p : p_1 :: s v^2 : s_1 v_1^2$, $p : p_1 :: s v^2 : s_1 (\frac{2}{3}v)^2$; or, substituting the values of s and s_1, $p : p_1 :: (2 \times 9 + 2 \times 12) \times 1{,}320 \times v^2 : (2 \times 9 + 2 \times 12) \times 2{,}640 \times \frac{4}{9} v^2$; whence, $p = 1\frac{1}{8} p_1$.

Now, since $p + p_1 = 7.956$, $1\frac{1}{8} p_1 + p_1 = 7.956$, or $2\frac{1}{8} p_1 = 7.956$, and $p_1 = 3.744$ lb. per square foot = pressure for the splits. Also, $p = 1\frac{1}{8} p_1 = 1\frac{1}{8} \times 3.744 = 4.212$ lb. per square foot = pressure for DA.

Applying now formula **40**, the velocity in $AEU = v_2 =$

$$\sqrt{\frac{p_1 a}{k s}} = \sqrt{\frac{3.744 \times 9 \times 12}{.0000000217 \times (2 \times 9 + 2 \times 12 \times 2 \times 5{,}280)}} =$$

205 ft. per minute, very nearly.

Hence, $v_1 = 2 v_2 = 2 \times 205 = 410$ ft. per minute, and $v = 3 v_1 = 615$ ft. per minute. The quantity passing through $DA = q = a v = 9 \times 12 \times 615 = 66{,}420$ cu. ft. per minute. The quantity passing through the short split is $66{,}420 \times \frac{2}{3} = 44{,}280$ cu. ft. per minute, and through the long split, $66{,}420 \times \frac{1}{3} = 22{,}140$ cu. ft. per minute.

Applying formula **44** to find the velocity in the shaft,

$$v_s = \frac{q}{a_s} = \frac{66{,}420}{8 \times 14} = 593.04 \text{ ft. per minute,}$$

letting v_s, a_s, and p_s be the velocity, area, and pressure for the shaft, respectively. Remembering that there are two shafts, the pressure required to drive the air through them is

$$p_s = \frac{k s_s v_s^2}{a_s} =$$

$$\frac{.0000000217 \times (2 \times 8 + 2 \times 14 \times 500 \times 2) \times 593.04^2}{8 \times 14} =$$

2.998 lb. per square foot.

Consequently, the total pressure per square foot required to move the air is $7.956 + 2.998 = 10.954$ lb. per square foot; the power $= p\,q = 10.954 \times 66{,}420 = 727{,}565$ ft. lb. per minute, and the horsepower $= \frac{727{,}565}{33{,}000} = 22.05$ H. P.

997. It will be noticed that the velocity of the air in the long split *A E U* is very low, being but 205 feet per minute, and should the grade be an upward one, or even should there be no grade at all, it will be very difficult, if not impossible, to drive out any mine gas that may collect at *E*. To increase the power sufficiently to accomplish this would be a *very* costly method; but by putting a regulator at *R*, the quantity of air going through the short split may be so much reduced that with the same power a sufficient quantity of air may be driven through the long split as to dislodge the mine gases at *E*. If necessary, all of the air going through the short split may be shut off and the whole ventilative power of the mine applied to the long split. This is the most important result achieved by the regulator.

998. Suppose, however, that it was desired to ascertain the area of the regulator opening, in order to have the short split pass the same quantity of air that the long split passes. Taking the velocity in the long split as 205 feet per minute, that in the short split will then be 205 feet also, and the pressure required may be found by means of law (**4**) as follows: $p : p_1 :: v^2 : v_1^2$, or $3.744 : p_1 :: 410^2 : 205^2$; whence, $p_1 = .936$ lb. per square foot = pressure required to send the air through the split *A B C U* at a speed of 205 feet per minute. But the actual pressure is 3.744; hence, the regulator must offer a resistance of $3{,}744 - .936 = 2.808$ lb. per square foot. Assuming the regulator to have been adjusted properly, a water-gauge placed in it will show a difference of pressure between the two sides of the regulator of 2.808 lb. per square foot $= \frac{2.808}{5.2} = .54$ in. of water.

The area of the opening may now be calculated by aid of the following formula:

$$A = \frac{.0004\,q}{\sqrt{W}}, \qquad \textbf{(56.)}$$

in which A = area of opening in square feet;
q = quantity of air in cubic feet per minute which it is desired to pass through the opening;
W = difference of pressure in inches of water on the two sides of the regulator.

Substituting in formula **56** the values previously found,

$$A = \frac{.0004 \times 22,140}{\sqrt{.54}} = 12.05 \text{ sq. ft.}$$

The total quantity of air now going through the mine is 22,140 + 22,140 = 44,280 cu. ft. per minute, or two-thirds of the quantity which went through before the regulator was introduced; and since the pressure per square foot remains the same as before, the horsepower required is but two-thirds of that previously required. Hence, if the horsepower be increased to its former value, the quantity will also be increased, but not to the same amount as before, since any increase in the quantity increases the velocity, which necessarily increases the frictional resistances—in other words, the ventilating pressure. The calculation will not be gone through with here to show just how much the ventilating pressure will be increased, as it is of no particular value to the student, and might tend to confuse him. He should, however, be able to see that the ventilating pressure and the velocity are both increased by the introduction of a regulator, and this is what is required to drive out the gas.

999. One more advantage obtained by splitting the air will now be noticed, and it is one of great value.

Fig. 149 represents a system of splits in which *FA* represents the fresh, or main, airway, and *RA* the return airway. The student will notice that when two arrow-heads are joined to one tail, there is a split, and when two tails are joined to one head there is a junction. Suppose that in the left-hand half of the mine represented in the figure, gas were to accumulate in one of the farther workings, and the air had not sufficient pressure to drive it out. By shutting off the air in the other half of the mine, the entire power of

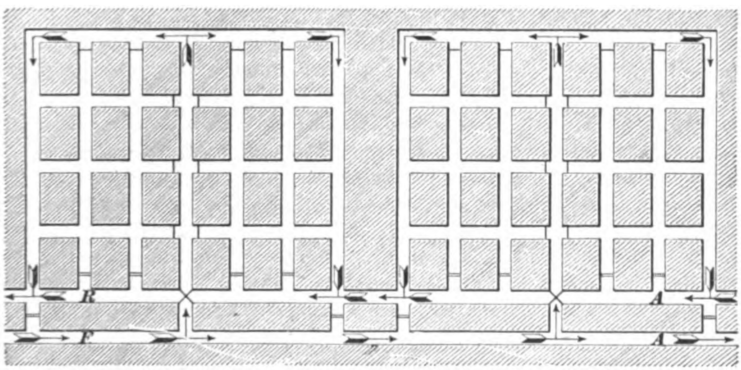


FIG. 149.

the ventilation may be employed to increase the pressure of the air in the left-hand half and drive out the gas. This is termed "sweeping out the mine," and is one of the greatest advantages obtained by splitting.

THE FIFTH ROOT.

1000. By aid of Table 25 the fifth root of any number may be found correctly to four figures. The arithmetical method of extracting the fifth root is very long and laborious. Since four figures are sufficient for all practical purposes in problems pertaining to mine ventilation, it was thought better to give the table than to give the rule generally used. The method of using the table will be exhibited by examples.

EXAMPLE.—Extract the fifth root of 1,264.782.

SOLUTION.—Only the first five figures of the number are required when using the table. When the sixth figure is 5, or greater, increase the fifth figure by 1, and omit the remaining figures. Doing so, the question becomes $\sqrt[5]{1,264.8} = ?$ Looking in column 4 of the table for the nearest number *smaller* than the given number, it is found to be 1,158.6, opposite the number 4.1 in column 1, and 4.1 are the first two figures of the root. To find two more figures of the root, proceed as follows: 1,264.8 − 1,158.6 = 106.2. Divide this remainder by the number in column 3 in the same row as the two numbers previously found, in this case 141.3, and obtain two figures of the quotient. If the second figure is greater than 5, increase the first figure by 1 and neglect the second figure. Should the second figure be a 5, obtain three figures of the quotient, and if the *third* figure is 5, or greater, increase the first figure by 1, and neglect the other two. Thus, 106.2 ÷ 141.3 = .751, and the number to be used is .7, since the third figure is less than 5. It is necessary to obtain three figures of this quotient only when the *second figure is a 5.* Now, multiply this quotient, .7 in this case, by the number in column 2 and in the same row as the three previous numbers found in the table, and add the result to the number found in column 3. Thus, 6.89 × .7 = 4.823, and 141.3 + 4.823 = 146.123. Finally, divide the difference found above (106.2) by 146.123; the result will be the next two figures of the root. Thus, 106.2 ÷ 146.123 = .727, or .73. Hence, the entire root to four figures is 4.173. Ans.

EXAMPLE.—Find the fifth root of 45,261.

SOLUTION.—Only the numerical work is given; the student should read the explanation given above in connection with the work. 45,261 − 44,371 = 890. 890 ÷ 2,610 = .34, or .3. 61.4 × .3 = 18.42. 2,610 + 18.42 = 2,628.42. 890 ÷ 2,628.42 = .338, say .34. Hence, $\sqrt[5]{45,261}$ = 8.534. Ans.

If the number is wholly decimal, take the first five figures to the right of the decimal point (annexing ciphers if necessary to make five figures) and treat the number as if it were a whole number with five figures.

EXAMPLE.— $\sqrt[5]{.664}$ = ?

SOLUTION.—Annexing two ciphers to make the necessary five figures, $\sqrt[5]{.664} = \sqrt[5]{.66400}$. Whence, 66,400 − 65,908 = 492. 492 ÷ 3,582 = .13. 77.9 × .1 = 7.79. 3,582 + 7.79 = 3,589.79. 492 ÷ 3,589.79 = .137, or .14. Hence, $\sqrt[5]{.664}$ = .9214. Ans.

EXAMPLE.— $\sqrt[5]{42,675,830}$ = ?

SOLUTION.—Begin at units place and point off the number into periods of **five** figures each. Thus, 426′75830. Retain the first five figures, beginning with the left, the result is 426′76. Regarding the division mark for the present as a decimal point, proceed as in the preceding examples. 426.76 − 391.35 = 35.41. 35.41 ÷ 59.3 = .59, or .6. 3.59 × .6 = 2.154. 59.3 + 2.154 = 61.454. 35.41 ÷ 61.454 = .576, or .58. Hence, the figures of the root are 3358. The position of the decimal point may be determined from the statement that *there must be as many figures in the integral part of the root as there are periods in the integral part of the number whose root is to be found.* Since there are two such periods in the above number, $\sqrt[5]{42,675,830} = 33.58$. Ans.

Had the number been 4,267,583,000,000, the number of periods would have been three, and the fifth root,

$$\sqrt[5]{426'75830'00000} = 335.8.$$

It will be a good exercise for the student to prove the following:

$$\sqrt[5]{426,758.3} = 13.37; \ \sqrt[5]{4,267,583} = 21.19;$$

$$\sqrt[5]{426,758,300} = 53.22, \text{ and } \sqrt[5]{4,267,583,000} = 84.34.$$

1001. If it is absolutely necessary for the student to extract the fifth root without the aid of a table, he may do so in the following manner:

$\sqrt[5]{4,267,583} = ?$

1. Point off the number into periods, as above directed, obtaining in this case 42'67583.

2. Find a number expressed by one figure whose fifth power is next less than the number expressed by the first period. It will aid the student, in finding the first figure of the root, if he will remember that if the first period contains but one figure, the first figure of the root must be 1; if but two figures, the first figure of the root can not be greater than 2; if but three figures, the first figure of the root is either 2 or 3; if but four figures, the first figure of the root can not be greater than 6, and if the first period contains five figures, the first figure of the root may be 6, 7, 8, or 9. Try 2 for the first figure of the root of the above number and raise it to the fifth power; the result is $2^5 = 32$. Since 32 is less than 42, the first figure of the root is 2.

3. To find the second figure, subtract the fifth power of the first figure from the first period and annex the second period to the remainder, or, if there is no second period, bring down five ciphers. Performing the operation on the above number, $42 - 32 = 10$; annexing the second period, the result is 1,067,583.

4. Raise the first figure of the root to the fourth power,

multiply the result by 5, and annex four ciphers. Annex four ciphers to the cube of the first figure, and add the result to the last result. Thus, $2^4 \times 5 = 80$; annexing four ciphers $= 800,000$. 2^3 with four ciphers annexed $= 80,000$, and $800,000 + 80,000 = 880,000$.

5. Divide the result obtained in 3 by the result obtained in 4, and the quotient will *very probably* be the second figure of the root. Thus, $1,067,583 \div 880,000 = 1 +$, and the first two figures of the root are 21.

6. Raise the first two figures of the root to the fifth power and subtract the result from the given number whose root is to be found, annexing five ciphers to the given number if it contains but one period. Thus, $21 \times 21 = 441$; $441 \times 21 = 9,261$, the cube; $9,261 \times 21 = 194,481$, the fourth power, and $194,481 \times 21 = 4,084,101$. Hence, $4,267,583 - 4,084,101 = 183,482$.

7. Multiply the fourth power of the first two figures (obtained in 6) by 5, and divide the remainder obtained in 6 by the result, and obtain two figures of the quotient. If the second figure of the quotient is greater than 5, increase the first figure by 1 and neglect the second figure; otherwise, use only the first figure. Should the second figure be 5, obtain three figures of the quotient, and if the *third* figure is 5, or greater, increase the first figure by 1. Thus, $21^4 = 194,481$ (see 6), and $194,481 \times 5 = 972,405$; then, $183,482 \div 972,405 = .18 +$ or .2.

8. Multiply the cube of the first two figures of the root (obtained in 6), with a cipher annexed, by the number found in 7, and add the result to 5 times the fourth power (obtained in 7). Thus, $21^3 = 9,261 = 92,610$, with a cipher annexed. $92,610 \times .2 = 18,522$. $972,405 + 18,522 = 990,927$.

9. Divide the remainder obtained in 6 by the result obtained in 8 and carry the quotient to three *decimal* places. If the third figure of the decimal is 5 or greater, increase the second figure by 1. These two figures of the quotient are the third and fourth figures of the root. Thus, $183,482 \div 990,927 = .185$, say .19. Hence the figures of the root

are 2119, and since there are two periods, $\sqrt[5]{4,267,583} = 21.19$. Ans.

NOTE.—The method outlined above is exactly what is accomplished by means of Table 25, but the work is very much more laborious. It is, however, the simplest known method of finding the fifth root of numbers.

EXAMPLE.— $\sqrt[5]{9} = ?$

SOLUTION—2. Since there is but one figure, the first figure of the root is 1.

3. $9 - 1^5 = 8$, since $1^5 = 1$. Annexing five ciphers gives 800,000.

4. $1^4 \times 5$ with four ciphers annexed = 50,000 ; 1^3 with four ciphers annexed = 10,000 ; the sum = 50,000 + 10,000 = 60,000.

5. 800,000 ÷ 60,000 = 13 +. This result is much too high, since the quotient thus obtained (which is the probable second figure of the root) should not exceed 9. Now, remembering that the fifth power of 2 is 32, it is evident that $\sqrt[5]{9}$ must be considerably less than 1.9, which nearly equals 2. Trying 1.6, the fifth power is $1.6^5 = 10.48576$, which is also too high, but quite close ; hence, 1.5 is probably the correct number to use, and the first two figures of the root are 15.

6. 15 × 15 = 225; 225 × 15 = 3,375; 3,375 × 15 = 50,625, and 50,625 × 15 = 759,375. 900,000 − 759,375 = 140,625.

7. $15^4 \times 5$ = 50,625 × 5 = 253,125 ; 140,625 ÷ 253,125 = .555, or .6.

8. 15^3 with a cipher annexed = 33,750 ; 33,750 × .6 = 20,250, and 253,125 + 20,250 = 273,375.

9. 140.625 ÷ 273,375 = .514, say .51. Hence, $\sqrt[5]{9} = 1.551$. Ans.

TABLE 25.

1	2	3	4	1	2	3	4
1.0	.100	.5000	1.0000	5.6	17.6	491.7	5,507.3
1.1	.133	.7321	1.6105	5.7	18.5	527.8	6,016.9
1.2	.173	1.037	2.4883	5.8	19.5	565.8	6,563.6
1.3	.220	1.428	3.7129	5.9	20.5	605.9	7,149.2
1.4	.274	1.921	5.3782	6.0	21.6	648.0	7,776.0
1.5	.338	2.531	7.5938	6.1	22.7	692.3	8,446.0
1.6	.410	3.277	10.486	6.2	23.8	738.8	9,161.3
1.7	.491	4.176	14.199	6.3	25.0	787.6	9,924.4
1.8	.583	5.249	18.896	6.4	26.2	838.9	10,737
1.9	.686	6.516	24.761	6.5	27.5	892.5	11,603
2.0	.800	8.000	32.000	6.6	28.7	948.7	12,523
2.1	.926	9.724	40.841	6.7	30.1	1,007	13,501
2.2	1.06	11.71	51.536	6.8	31.4	1,069	14,539
2.3	1.22	13.99	64.363	6.9	32.9	1,133	15,640
2.4	1.38	16.59	79.626	7.0	34.3	1,201	16,807
2.5	1.56	19.53	97.656	7.1	35.8	1,271	18,042
2.6	1.76	22.85	118.81	7.2	37.3	1,344	19,349
2.7	1.97	26.57	143.49	7.3	38.9	1,420	20,731
2.8	2.20	30.73	172.10	7.4	40.5	1,499	22,190
2.9	2.44	35.36	205.11	7.5	42.2	1,582	23,730
3.0	2.70	40.50	243.00	7.6	43.9	1,668	25,355
3.1	2.98	46.18	286.29	7.7	45.7	1,758	27,068
3.2	3.28	52.43	335.54	7.8	47.5	1,851	28,872
3.3	3.59	59.30	391.35	7.9	49.3	1,948	30,771
3.4	3.93	66.82	454.35	8.0	51.2	2,048	32,768
3.5	4.29	75.03	525.22	8.1	53.1	2,152	34,868
3.6	4.67	83.98	604.66	8.2	55.1	2,261	37,074
3.7	5.07	93.71	693.44	8.3	57.2	2,373	39,390
3.8	5.49	104.3	792.35	8.4	59.3	2,489	41,821
3.9	5.93	115.7	902.24	8.5	61.4	2,610	44,371
4.0	6.40	128.0	1,024.0	8.6	63.6	2,735	47,043
4.1	6.89	141.3	1,158.6	8.7	65.9	2,864	49,842
4.2	7.41	155.6	1,306.9	8.8	68.1	2,998	52,773
4.3	7.95	170.9	1,470.1	8.9	70.5	3,137	55,841
4.4	8.52	187.4	1,649.2	9.0	72.9	3,281	59,049
4.5	9.11	205.0	1,845.3	9.1	75.4	3,429	62,403
4.6	9.73	223.9	2,059.6	9.2	77.9	3,582	65,908
4.7	10.4	244.0	2,293.5	9.3	80.4	3,740	69,569
4.8	11.1	265.4	2,548.0	9.4	83.1	3,904	73,390
4.9	11.8	288.2	2,824.8	9.5	85.7	4,073	77,378
5.0	12.5	312.5	3,125.0	9.6	88.5	4,247	81,537
5.1	13.3	338.3	3,450.3	9.7	91.3	4,426	85,873
5.2	14.1	365.6	3,802.1	9.8	94.1	4,612	90,392
5.3	14.9	394.5	4,182.0	9.9	97.0	4,803	95,099
5.4	15.7	425.2	4,591.7	10.0	100.0	5,000	100,000
5.5	16.6	457.5	5,032.8				

MINE VENTILATION.

(PART 2.)

THE PRODUCTION OF VENTILATING CURRENTS.

VARIOUS SYSTEMS OF INDUCING CURRENTS.

1002. Ventilating Currents.—The motion of air-currents in mines is caused by a difference in pressure between the two ends of the current, or, in other words, a difference in pressure between the downcast and upcast. The direction of the flow is always from the higher towards the lower pressure.

In the case of ventilation produced by exhaust-fans or furnaces, the higher pressure is the normal pressure of the atmosphere, and the lower pressure is that produced in the fan-drift, or at the bottom of the furnace-shaft. In the case of a blowing-fan, the higher pressure consists of the atmospheric pressure plus the pressure exerted by the fan, and the lower pressure is the atmospheric pressure at the top of the upcast. A waterfall in the downcast shaft produces motion in a current on the same principle as a blowing-fan, and a steam-jet in the upcast acts on the same principle as the exhaust-fan. However, it must be borne in mind that neither of the two latter methods is as efficient as a fan. These facts show clearly that the object of all artificial ventilating appliances must be to provide the required difference of pressure. Current motion may, therefore, be caused by either of two methods: (*a*) methods of compression, by means of which the air in the downcast is raised to a pressure greater than the atmosphere, or (*b*) methods of exhaustion, by means of which the pressure of the air in the upcast is made less than the pressure of the atmosphere.

1003. The Laws of Current Motion.—As a current of air for mine ventilation begins and ends in the atmosphere, it is necessary that a ventilator be applied to produce a terminal depression for the current to fall into, and a subsequent compression to finally force it out into the

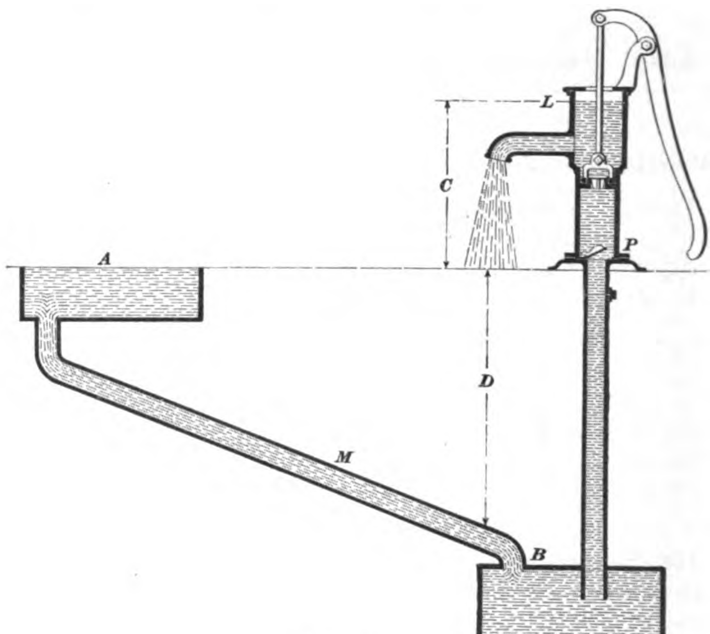


FIG. 150.

atmosphere. In Fig. 150 a pump is used to illustrate what has been expressed in words. The water in the cistern *A* is subject to the pressure of the atmosphere, and falls into a depression at *B*, through the pipe *M*. The depression at *B* is created by the pump in the same manner as a fan creates a depression. The excess of pressure in the atmosphere over that exerted at *B* is the measure of depression. It is this depression that causes the water to flow from *A* to *B*, and a similar depression that causes the air in a mine to flow from the top of the downcast into the upcast. In the illustration, the pump *P* produces the depression. The lifting of the piston reduces the pressure of the atmosphere on the water in *B*, and the falling water pressed by the atmosphere at *A* rushes in to fill up the void. Without this depression, the water would naturally rise in the pump to the level of *A*, but would remain at rest and not flow out. Therefore, further energy is required to cause it to rise high enough to flow out of the nozzle. In the same way a fan must not only cause a depression, so as to cause the air to flow into the fan-drift, but it must also exert energy to force the air out into the atmosphere. In the case of the pump, to raise the water to *L*, and enable it to flow out of the nozzle, energy equal to a fall from *L* to the nozzle is required. This fall overcomes the friction and the delivery pressure, and is similar to the compression required in an exhaust-fan for it to throw the mine current out of its chimney. The illustration shows clearly that *D* is the measure of the depression below the pressure of the atmosphere, and that *C* is the measure of the compression above the atmosphere; further, it explains the principles of the double fall, or the fall from the atmospheric pressure at *A* to the depression at *B*, and the fall from the pressure above the atmosphere at *L* to the atmospheric pressure at *A*.

1004. How Ventilating Currents Are Produced.—The means by which ventilating currents are produced are all included under the following heads:

(*a*) Ventilation by natural heat.
(*b*) Ventilation by artificial heat.
(*c*) Ventilation by waterfalling.
(*d*) Ventilation by mechanical agencies.
(*e*) Ventilation by a steam-jet.
(*f*) Ventilation by a water-jet.

1005. Natural ventilation is produced in a mine when the top of the upcast and the top of the downcast are at different elevations, or, in other words, when one is some distance up a hill and the other at or near the base. A natural ventilating current is only set in motion when the temperature of the outer air and that of the walls of the mine passages is different. This method of ventilation differs from all others in one important respect, namely, the direction of the current is reversed in summer from what it is in winter. In summer, when the external air is hotter than the walls of the mine passages, the warm air descends the deeper shaft, and in so doing is cooled by the absorption of heat by the walls of the shaft. This cooled column thus becomes a heavier one than the one parallel to it, shown in (*a*), Fig. 151. In this figure, *a b* is the shaft and *m* is the mine. The cooled air column in *a b*, being heavier than the external column *d c*, causes the air to flow from *b* to *d*. The direction of flow in winter is illustrated in (*b*), Fig. 151.

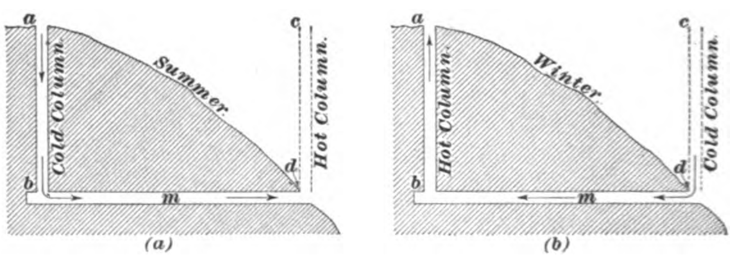


FIG. 151.

As the walls of the mine passages are warmer than the external air, the column of air in the shaft *a b* is warmer than the parallel column of the external air *c d*. Therefore, the external column being heavier forces the warmer, lighter column up the shaft, and causes the current to flow from *d* to *b*.

In the event of the external air having the same temperature as the walls of the mine passages, there is no flow of air-current, because one column balances the other.

1006. Ventilation by artificial heat is produced by a furnace fire situated at the bottom of the upcast shaft. This fire heats the column of air in the upcast shaft and makes it less dense and lighter than the column of cold air in the downcast. The weight of a cubic foot of air in either shaft is calculated by formula **57**,

$$W = \frac{1.3253 \times B}{(459 + t)}; \qquad (57.)$$

B = the barometic pressure in inches of mercury;
t = Fahrenheit temperature of air in the shaft;
W = weight of a cubic foot of air.

EXAMPLE.—The downcast shaft of a mine is 600 feet deep, the mean barometric pressure in the shaft is 30 inches, and the mean temperature of the air in the shaft is 62° F. What is the average weight of a cubic foot of air in this shaft?

SOLUTION.—Applying formula **57**,

$$W = \frac{1.3253 \times 30}{(459 + 62)} = .07631 \text{ lb. Ans.}$$

EXAMPLE.—The upcast shaft of the same mine is 600 feet deep; the mean barometric pressure is the same (30 inches), and the mean temperature of the air in the shaft is 196° F. What is the average weight of a cubic foot of air in this shaft?

SOLUTION.—Applying formula **57**,

$$W = \frac{1.3253 \times 30}{(459 + 196)} = .0607 \text{ lb. Ans.}$$

The two foregoing examples show that the air in the upcast shaft is lighter than that in the downcast.

Now, if the height of each column is taken at 600 feet, the ventilating pressure per square foot can be found by multiplying the weight of air per cubic foot by the height of the column in feet. Thus,

Downcast	$= .07631 \times 600 = 45.786$ lb.
Upcast	$= .0607 \times 600 = 36.420$ lb.
Difference, or ventilating pressure per sq. ft.	$=$ 9.366 lb.

EXAMPLES FOR PRACTICE.

1. The downcast shaft of a mine is 437 feet deep, the mean barometric pressure is 30.25 inches, and the mean temperature of the air in the shaft is 67° F. What is the weight of a column of air in this shaft, having a base of 1 square foot? Ans. 33.8081 lb.

2. The downcast shaft of a mine is 1,147 feet deep, the mean barometric pressure is 29.9 inches, and the mean temperature of the air in

the shaft is 50° F. What is the weight of a column of air in this shaft, having a base of 1 square foot? Ans. 89.29395 lb.

3. The upcast shaft of a mine is 347 feet deep, the mean barometric pressure is 30 inches, and the mean temperature of the air in the shaft is 187° F. What is the weight of a column of air in this shaft, having a base of 1 square foot? Ans. 21.35785 lb.

4. The upcast shaft of a mine is 1,170 feet deep, the mean barometric pressure is 29.5 inches, and the mean temperature of the air in the shaft is 160° F. What is the weight of a column of air in this shaft, having a base of 1 square foot? Ans. 73.8972 lb.

EFFECT OF TEMPERATURE ON VOLUME.

1007. The volume of a given quantity of air varies directly as its absolute temperature, the barometric pressure and weight remaining the same. This principle is expressed in formula **58**,

$$T = \frac{V}{v} \times (459 + t), \qquad \textbf{(58.)}$$

in which T = absolute temperature of greater volume;
V = greater volume;
v = lesser volume;
and t = given temperature of lesser volume in Fahrenheit degrees.

EXAMPLE.—If the volume of a given quantity of air is 35,672 cubic feet when its temperature is 57° F., what must its temperature be to increase the volume to 51,756 cubic feet, supposing the atmospheric pressure and weight to remain the same?

SOLUTION.—Applying formula **58**, $\frac{51,756}{35,672} \times (459 + 57) = 748.65°$, or the absolute temperature necessary for increasing the volume. Now, having the absolute temperature, it is necessary to reduce it to Fahrenheit temperature. This can be readily done by subtracting 459 from the absolute temperature. Then, 748.65° − 459° = 289.65° F. Ans.

EXAMPLES FOR PRACTICE.

1. If the volume of a given quantity of air when its temperature is 60° F. is 46,732 cubic feet, what must its temperature be to increase the volume to 65,000 cubic feet, supposing the atmospheric pressure and weight to remain the same? Ans. 262.88° F.

2. If the volume of a given quantity of air when its temperature is 160° F. is 65,000 cubic feet, what must its temperature be when the volume is 50,000 cubic feet, supposing the atmospheric pressure and weight to remain the same? Ans. 17.1° F.

THE MOTIVE COLUMN.

1008. The ventilating pressure can be found directly through the medium of the motive column. This motive column is the short column of air whose weight provides the ventilating pressure. If the length of this motive column is subtracted from the length of the downcast column, the weight of the remaining portion of the downcast column is equal to the weight of the upcast column. This is explained by Fig. 152, in which *U* is the upcast column and *D* is the downcast column; the furnace is shown at *F*, and *M C* is the motive volume. This motive column, then, is a column of air in the downcast shaft whose weight is equal to the excess of weight of the cold column over that of the hot one. The most convenient way to find the length of the motive column is by what is called Nicholas Wood's formula, which is based on the law that the weights of the columns are inversely as their absolute temperatures. This law can be expressed by formula **59**,

FIG. 152.

$$M = \frac{(t - t_1)}{(459 + t)} \times D, \qquad \textbf{(59.)}$$

in which t = higher temperature, or that of the upcast;
t_1 = lower temperature, or that of the downcast;
D = depth of the shaft in feet;
and M = motive column.

Now suppose a case in which the mean temperature of the downcast shaft is 58° F., and the mean temperature of the furnace shaft is 186° F., and the depth of the shafts is 800 feet. By formula **59,** the length of the motive column in this case will be equal to $\frac{(186 - 58)}{(459 + 186)} \times 800 = 158.75$ feet.

If, then, the average weight of a cubic foot of air in the downcast shaft is calculated by formula **57,** and is found to be .077 pound, the ventilating pressure can be found by the following formula:

$$p = \frac{(t - t_1)}{(459 + t)} \times .077 \times D, \qquad \textbf{(60.)}$$

in which p = ventilating pressure in pounds per square foot;
t = higher temperature;
t_1 = lower temperature;
and D = depth of shaft.

Thus, $\frac{(186 - 58)}{(459 + 186)} \times .077 \times 800 = 12.224$ pounds per square foot, the ventilating pressure required.

EXAMPLE.—The ventilating shafts of a mine are each 800 feet deep, the temperature of the downcast column is 58° F., and that of the upcast column is 202° F. What is the weight of a column of air in the downcast shaft 1 square foot in the base, and what is the weight of a column of equal length in the upcast shaft? Show by formula **60** that the difference between the weights of the two columns is equal to the weight of the motive column, the mean barometric pressure in the two shafts being 30.5 inches.

SOLUTION.—The weight of a cubic foot of air in the downcast shaft is, by formula **57,** equal to $\frac{1.3253 \times 30.5}{459 + 58} = .078185$ pound, and, by formula **57,** the weight of a cubic foot in the upcast shaft is found to be

$$\frac{1.3253 \times 30.5}{459 + 202} = .061152 \text{ lb. Ans.}$$

Having found the weight of a cubic foot of air in each shaft, the weight of a column with a base of 1 square foot in each shaft is found by multiplying the weight per cubic foot of air in each shaft by the depth of the shaft; therefore, the weight of the downcast column equals $.078185 \times 800 = 62.548$ pounds, and the weight of the upcast column equals $.061152 \times 800 = 48.922$ pounds. The difference between the respective weights of the columns $= 62.548 - 48.922 = 13.626$ pounds.

By formula **60**, if the weight of the cubic foot of air is taken at .078, as the barometer is high, $p = \frac{(202 - 58)}{(459 + 202)} \times .078 \times 800 = 13.594$ pounds.

It will be observed in this connection that the result secured by using formula **60** is a little less than that found by using the weights of the columns, but the difference arises entirely from the fact that the weight of a cubic foot in the downcast column is actually .078185 pound instead of .078; had the weight of a cubic foot of air been taken at .078185 pound, the answers would have agreed more closely.

EXAMPLES FOR PRACTICE.

NOTE.—The weight of 1 cubic foot of air at a temperature of 62° F., and a barometric pressure of 30 inches, is equal to .076 pound. This is close enough for the weight of a cubic foot of air for use in the following examples.

1. The ventilating shafts of a mine are each 950 feet deep, the temperature of the downcast column is 60° F., and that of the upcast is 230° F. (*a*) What is the length of the motive column? (*b*) What is the difference in the weights of the ventilating columns per square foot of area?

Ans. { (*a*) 234.4 ft.
(*b*) 17.814 lb.

2. The ventilating shafts of a mine are each 760 feet deep, the temperature of the downcast is 52° F., and that of the upcast is 280° F. (*a*) What is the length of the motive column? (*b*) What is the difference in the weights of the ventilating columns per square foot of area?

Ans. { (*a*) 234.5 ft.
(*b*) 17.82 lb.

VENTILATING BY FURNACES.

THE CONSTRUCTION OF FURNACES.

1009. As the furnace is still used in some regions for the ventilation of small mines where the output does not justify the erection of a ventilating fan, a few facts concerning its construction and use should be known. The object of a furnace is to produce a motive column by rarefying the air in the upcast shaft with heat. In shallow mines, however, where an efficient motive column can not be obtained, the fan is much more efficient and economical. In spite of this, the furnace is still used. Therefore, its construction must be described.

Fig. 153 is an illustration of a type of furnace quite generally used. It is important, in building a furnace, to construct it so as to keep the excessive heat of the fire from the coal on its flanks, and from the rock above it. In Fig. 153, *L*, *L* show the sides in the coal-seam. The drifts *D*, *D* provide for the isolation of the heat from the coal. Immediately above the fire is a double arch, and as the inner one is subject to constant variations of temperature, ribs of brick are run between the inner and the outer arch to prevent collapse, and to keep the air-space so widely open that a current of air may freely pass through it and keep the heat from the roof. The importance of this arrangement is due to the fact that in cases where the roof stone contains water, the crown arch is continually buckling with the pressure produced by steam, and this causes the top stone to break and fall. *P* is the ash-pit, and *G* is the bearing-up bar, or front fire-grate girder. At *B* are seen the fire-bars that conjointly make up the fire-grate surface. The furnace arch is generally semicircular, and the height from the fire-bars to the under surface of the arch is generally 1¼ times the width of the fire-grate surface. The dimensions of the furnace are determined on the basis of the amount of work it is intended to perform, and when the breadth of the furnace is found, all the other dimensions are deduced from it.

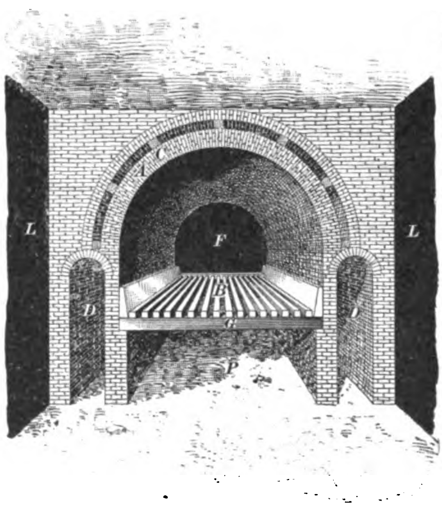


FIG. 153.

The length of the furnace-bars should not exceed 5 feet, and as this dimension is uniform for all furnaces, the important dimension required for constructing a furnace is its breadth. The area of the fire-grate surface varies inversely as the square root of the depth of the furnace-shaft. Before the width of a furnace can be determined, the amount of air necessary for the efficient ventilation of the mine must be fixed, and, in addition to this, the ventilating pressure in inches of water-gauge must be approximately known. From these factors, the horsepower of the required furnace can be calculated by dividing the product of the volume of air in cubic feet per minute and the pressure in pounds per square foot, by 33,000.

EXAMPLE.—Suppose a case in which the quantity of air required is 120,000 cubic feet per minute, and the probable mine resistance for that quantity is 2 inches of water-gauge; what horsepower is required in the ventilation?

SOLUTION.— $\frac{120,000 \times 2 \times 5.2}{33,000} = 37.8$ H. P. Ans.

EXAMPLES FOR PRACTICE.

1. A mine is ventilated by an air-current of 200,000 cubic feet per minute, and the water-gauge reading is 2.1 inches; what horsepower is exerted in moving the air? Ans. 66.18 H. P.

2. A mine is ventilated by an air-current of 125,000 cubic feet per minute, and the water-gauge reading is 3.5 inches; what horsepower is exerted in moving the air? Ans. 68.94 H. P.

GRATE SURFACE.

1010. The fire-grate surface required is found by the following formula:

$$s = \frac{34}{\sqrt{D}}, \qquad \textbf{(61.)}$$

in which D = depth of the furnace-shaft in feet;
34 = a constant number proved by many experiments;
and s = square feet of fire-grate surface required per horsepower of the ventilation.

EXAMPLE.—The depth of the shaft is 400 feet, and the horsepower required in the ventilation is 37.8; what area of fire-grate is required?

SOLUTION.—By applying formula **61**, $\frac{34}{\sqrt{400}} = 1.7$ square feet of fire-grate surface required per horsepower. Since 37.8 horsepower is required, the fire-grate surface should be $37.8 \times 1.7 = 64.26$ sq. ft. Ans.

A grate surface of the size calculated in the above example will efficiently ventilate a mine 400 feet deep, with 120,000 cubic feet of air per minute, circulated against a resistance equal to 2 inches of water-gauge. Again, if the bars of the fire-grate are 5 feet long, the breadth of the furnace, in feet, will in this case be equal to $\frac{64.26}{5} = 12.85$ feet.

EXAMPLE.—Let a furnace-shaft be 900 feet deep, and the ventilating current be equal to 200,000 cubic feet per minute, with a mine resistance equal to 2 inches of water-gauge; what must be the breadth of the furnace when the length of the fire-bars is taken at 5 feet?

SOLUTION.—The horsepower required is equal to $\frac{200,000 \times 2 \times 5.2}{33,000} =$ 63 H. P. The fire-grate surface per horsepower, by use of formula **61**, is found to equal $\frac{34}{\sqrt{900}} = 1.133$ square feet; and, therefore, the square feet of fire-grate surface required are equal to $63 \times 1.133 = 71.379$ square feet; and, if the length of the fire-bars be taken at 5 feet, the breadth of the furnace is equal to $\frac{71.379}{5} = 14.28$ ft. Ans.

An examination of the two examples will show that, notwithstanding the fact that the horsepower required in the latter case is so much greater than in the former, yet the fire-grate surface is very little increased, owing to the greater depth of the shaft.

EXAMPLES FOR PRACTICE.

1. What grate surface will be required to produce a current of 200,000 cubic feet per minute, with a water-gauge of 2.1 inches, if the furnace-shaft is 900 feet deep? Ans. 74.98 sq. ft.

2. What width of furnace will be required to produce a current of 100,000 cubic feet per minute, with a water-gauge of 2 inches, if the shaft is 625 feet deep, and the grate-bars of the furnace are 5 feet long? Ans. 8.57 ft.

REMARKS ON FURNACE VENTILATION.

1011. Where furnace ventilation is practised and the return air contains inflammable gas, it is often necessary to feed the furnace with fresh air and use the heated gases from the fire to heat and rarefy the upgoing column of return air from the mine. In Fig. 154 the heated air from the furnace is marked *H*, and is seen to pass up that portion of the shaft at *S*. Again, nothing but the return air *R* is seen to pass the dumb drift *P*, as shown by the arrows. The return air unites and mixes with the heated gases of the fire at the junction of the dumb drift with the shaft. The furnace receives its supply of fresh air at *A*, as shown in section and plan. In the plan, the fresh air to feed the furnace is indicated by the arrow at *A*, and in the section the return air from the mine is seen to enter the dumb drift at *R*. The object of this drift is to isolate the return air from the flaming gases of the furnace. The junction of the dumb drift with the shaft should not occur at a less elevation than 150 feet above the furnace, and in some cases where bituminous coal is burned, safety is not secured until the junction takes place at an elevation of 300 feet. As this is equal to the depth of many shafts, and more than the maximum depths of others where furnaces are used, it is clear that, at its best, the furnace does not afford a safe means for the ventilation of a gaseous mine.

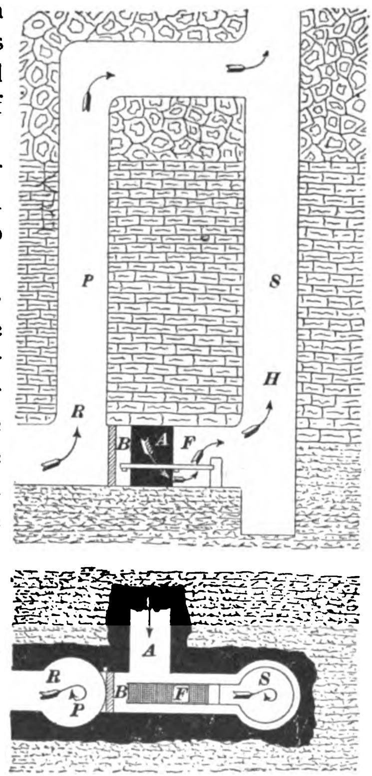


FIG. 154.

VENTILATION BY WATERFALLING.

1012. In some regions, waterfall ventilation is important, because it is cheap and very efficient. It often occurs, however, through oversight, that this agency is not adopted, although the conditions for its use are highly favorable. In Fig. 155 is shown what is called a **trompe**, or **waterfall ventilator,** in common use in many parts of the world, for the ventilation of such coal and metal mines as have the shaft bottom or lower level situated above the drainage level of the district. The trompe is a rectangular tube made of wood, and has an area of section equal to from 4 to 6 square feet. The length is regulated by the prevailing conditions, but the greater the length the better. The water delivered into the trompe generally comes from a neighboring stream and is conducted by a spout or trough *L*. Here the water is first divided into small streams by passing it through perforations in the top plate *G*. These water threads are broken up by their inert force into drops that fall in succession from one to another of a series of sloping shelves that are called dashboards, as shown by *D*, *D*, *D*, *D*, etc. The water, on striking the upper board, rebounds, and the spray that is thus produced rains on to the under ones, and so on from one to another until the trompe becomes a vertical tube enclosing a shower of fine water-drops that produce a powerful, energetic blast of air. As the drops fall they produce a partial vacuum in their rear and a compression on their under

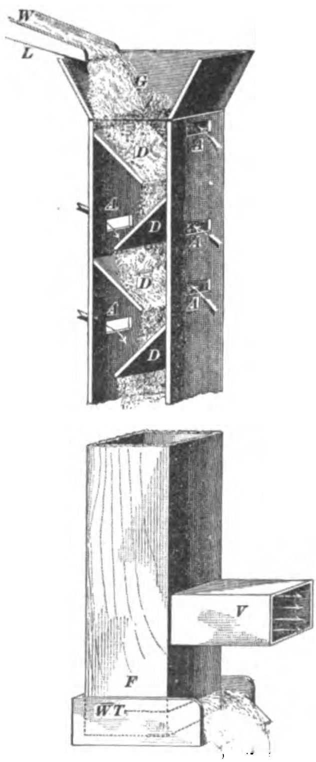


FIG. 155.

side, which causes an inrush of air into the ports *A*, *A*, *A*, etc. The water ultimately falls into a trap *W T*, where it overflows, and the air then blows out of the horizontal delivery branch of the ventilator at *V*. The trompe is used in the downcast shaft, and, therefore, acts as a blower to propel a current through the galleries of the mine.

1013. Where copious ventilation is required, an entire shaft is made to act as a large trompe, as in the case illustrated by Fig. 156. Here, however, instead of using a perforated plate, a brush mat is provided for breaking up the water into spray, as shown between the buntons in the middle of the shaft. The water-flow is here conducted by a trough *W* into the spray-maker, where it is broken into drops and made to rain down the shaft in a rapid shower. This rain provides a powerful ventilation, and can be used with great advantage with a fall of from 100 to 200 feet. The arrangement is also cheap and economical where a copious mountain stream is available all the year round. So considerable is the pressure produced in this way, that waterfalls have been used to produce an air-blast for smelting iron in cupola furnaces. In some cases where water is available and can be used for ventilating mines, a vertical shaft to be used as a trompe is sunk on the side of a mountain at sufficient elevation above the top of the downcast. The bottom of the trompe is made a little below the top of the downcast shaft. The air from the fall is conducted into the downcast shaft by a drift, and to confine the air to the flow of the mine, the top of the shaft is covered with a trap-lid. The drift connecting the trompe and the downcast is extended past the downcast to the surface. The

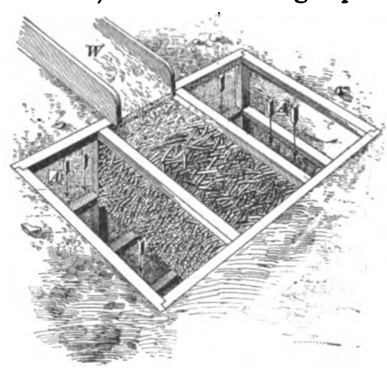


FIG. 156.

water falling down the trompe is collected in a small sump, trapped into a trough in the drift, whence it flows past the downcast and runs away. Where the flow of water is copious, 200,000 or 300,000 cubic feet of air per minute can in this manner be supplied for the ventilation of a large mine.

VENTILATION BY STEAM-JET AND WATER-JET.

1014. Ventilation by Steam-Jet.—Sometimes a current of air is set in motion with a steam-jet projected into a channel along which the current moves, but economical results have not been obtained in this way.

1015. Ventilation by a Water-Jet.—Sometimes a jet of sprayed water is projected along the path of a current to produce ventilation. This method has not been very successful for producing large volumes of air, except when used as a waterfall, as previously described. But it has been applied with comparatively good results for producing a local current in a cheap and efficient way. It is only necessary, however, for the student to know that such means are used for setting currents of air in motion.

MECHANICAL VENTILATORS.

PRINCIPLES GOVERNING THE ACTION OF FANS.

1016. Chief among the mechanical ventilators of mines is the centrifugal fan, and it is, therefore, important that its principles of construction and mode of action should be understood. The fan is really a valveless pump, its blades taking the place of the pump-piston, and, so far as the exhausting and blowing out of the mine air is concerned, the fan and the pump act in the same way.

To set a fluid in motion, the ventilating fan, like the pump, must overcome three distinct causes of resistance; to make clear how these causes originate, an air-pump such as shown in Fig. 157 is used. The first cause of resistance is that due to the friction of the mine. To show clearly its distinctive individuality, the pump is so contrived that no air can enter it without passing down the tube *A B* in the vessel at the left side of the figure; and to create an artificial resistance, the vessel just referred to is seen to be half filled with water, so that before any air can get inside of this closed vessel, two things must happen. First, the pump-piston *G* must move upwards, and, as a result of this movement, a depression of the air pressure will occur below the piston and above the water in the closed vessel. This

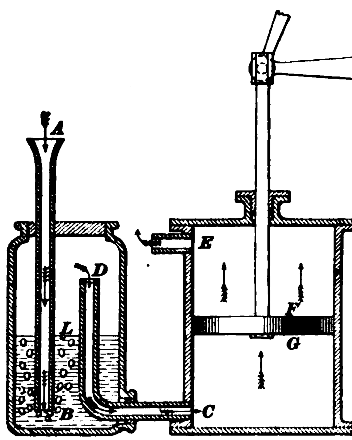


FIG. 157.

having taken place, the external air will by its greater pressure force the water down and out of the bottom end of the pipe *A B*, and it will then bubble up through the water and enter the pipe *D C* on its way into the cylinder, as shown by the arrow.

1017. The Depressions Produced by the Working of a Fan.—To make the depression required, the piston must move and produce a depression that will so reduce the pressure of the air in the cylinder and the vessel, that it will require the weight of the water the air displaces added to the pressure of the air within the vessel to equal the atmospheric pressure outside.

For example, suppose that the depth of the water through which the air must be forced is equal to 2 inches, or a pressure of 10.4 pounds per square foot. Then, if the outside pressure is equal to 2,116 pounds per square foot, the inside pressure can not be more than $2,116 - 10.4 = 2,105.6$ pounds. The piston has here made such a depression as is found in a fan drift, and that is the equivalent of what is called the mine resistance, or the pressure required to set a current of air in motion through a mine. The use of a fan is to make this depression. The first, or principal depression, and the equivalent of the force required to produce it, is represented in Fig. 157 by a weight $M R$ hung on the opposite end of the beam that turns on the center pin S. The entire use of the vessel $D L B$ is to generate an artificial resistance to imitate a mine resistance.

1018. The second cause of resistance is that due to the force required to set the air in motion in the cylinder through the port $D C$. Air can not be set in motion out of the vessel $D L B$ without a depression in the cylinder at $G C$, for air-currents move only from a higher to a lower pressure. Therefore, the piston must move sufficiently to make a displacement not only equal to the depression the weight $M R$ would produce, but, in addition, a depression equal to that which the weight $I M$ would produce. This means that the depression within the cylinder at $G C$ must be equal to $I M + M R = I R$, while the depression within the vessel $D L$ will only be equal to $M R$. The depression $I M$ is the one which represents the depression required for the entry of air into a fan.

1019. From what has been stated, the student can see that two depressions must be made by the action of the fan. The first one is provided to cause the fall of a current from the atmosphere through the mine into the fan drift, and the second one is provided for the air in the fan drift to fall into the fan. As has been shown, the sum of these depressions is equal to the pressure represented by the weights $I M$ and

MR. In addition to these, however, there is a third resistance, OI, which is the pressure required for the air to fall out of the fan into the atmosphere. The piston can not force the air out of the upper end of the cylinder without a difference between the inside and the outside pressure; altogether, then, the sum of the pressures required for a fan to do its work is equal to OR, or $OI + IM + MR$.

1020. The centrifugal ventilating fan furnishes the best agent for the economical ventilation of mines, for two reasons: first, it is safer than the furnace; and, second, its efficiency is uniformly the same for deep and for shallow mines; whereas the efficiency of the furnace is very small for shallow mines, and is not much greater than the fan in the ventilation of deep ones. From all points of view, then, the centrifugal fan is the best ventilating machine in use.

COMPARISON OF FAN AND FURNACE.

1021. The underlying principles of the modes of action of the fan and the furnace are so different as to require particular notice. The ventilating pressure produced by the furnace is the result of the difference in the weights of the ventilating columns; whereas the ventilating pressure produced by a fan is the result of a difference in the total pressures upon two shafts. For example, if a pair of shafts are 1,200 feet in depth, and are ventilated by a furnace with the temperature of the downcast column 62° F., and that of the upcast 135° F., then by formula **57** the weight of a cubic foot of each can be found as follows: In the downcast, the weight of a cubic foot of air is equal to $\frac{1.3253 \times 30}{(459 + 62)} = .0763$ pound (see formula **57**), and the weight of a cubic foot of air in the upcast is equal to $\frac{1.3253 \times 30}{(459 + 135)} =$ .0669 pound. (See formula **57.**) The difference in the weights is, therefore, equal to $.0763 - .0669 = .0094$ pound. The pressure, per square foot, producing ventilation under the given conditions of depth and heat, is, according to formula **60**,

$$\frac{(135 - 62)}{(459 + 135)} \times .0763 \times 1{,}200 = 11.252 \text{ pounds.}$$

1022. Fan ventilation is not produced, like furnace ventilation, by a difference in the weights of the ventilating columns. If, in the case of fan ventilation in the same shafts, the weight of a cubic foot of air in the downcast is equal to .0763 pound, the ventilating pressure of the fan is equal to that of the furnace, and the temperature of the upcast column is the same as that of the downcast one, namely, 62° F.; then, by taking the pressure of the atmosphere at 2,116 pounds per square foot, the weight of a cubic foot in the upcast shaft can be found by the following formula:

$$w = \frac{P - p}{P} \times W, \qquad \textbf{(62.)}$$

in which w = weight of 1 cubic foot of air in the upcast;
P = atmospheric pressure per square foot, or 2,116 pounds;
p = ventilating pressure in pounds per square foot;
W = weight of 1 cubic foot of air in the downcast.

Hence, in the case under consideration,

$$\frac{(2{,}116 - 11.252)}{2{,}116} \times .0763 = .075894 \text{ pound. Ans.}$$

The difference in the weight of a cubic foot of air in the downcast and of a cubic foot in the upcast is, therefore, equal to $.0763 - .075894 = .000406$ pound. That is, the weights of the upcast and downcast columns are practically the same.

To produce a ventilating pressure of 11.252 pounds per square foot with a furnace and with a fan, the following curious differences occur:

Differences in the weights of a cubic foot of air:

Furnace ventilation, .0094 pound.
Fan ventilation, .000406 pound.

Difference in the total pressures upon the ventilating columns, in the given examples:

For furnace ventilation, none; because the motion of the current is caused by a difference of weight in the two columns.

For fan ventilation, 11.252 pounds; direct pressure, with practically no difference in the weights of the two columns.

Plainly stated, the facts are these: The furnace rarefies the air by heat, and the air flows because the rarefied column is lighter than the other column. The fan, by exhaustion or compression, makes the total pressure upon the top of one column greater or less than the pressure upon the top of the other column; so that, although the weights of the two columns are practically the same, the difference in pressure on the tops of the columns produces the flow.

1023. From what has been explained, it is easy to infer that the mode of action that characterizes the centrifugal fan is that of producing differences of pressure between the air entering and leaving a mine, and entering and leaving a fan. For example, if the absolute pressure of the air within the fan drift were not below the external pressure of the air entering a mine, the air could not be set in motion. It is clear, then, that the prime object of the fan is to make a depression at one end of the mine so that the greater pressure at the other will set the air in motion. Further, after the air has passed through the passages of a mine and has reached the fan drift, it can not enter the fan unless the pressure within the fan is less than that of the air in the drift. Therefore, a fan, to produce a ventilating current, must make a provision for two distinct depressions, one to cause the air-current to flow towards the fan and one to cause the current of air to enter the fan itself. Again, air can not leave a fan unless its pressure is raised above that of the external air. Then, if the air leaving is at a pressure greater than that of the atmosphere, it is clear that the work to be done is equal to that of producing a motive pressure equal to the sum of two negative pressures and the positive one. That is, it is necessary in this case to make a depression equal to about 10 pounds on the square foot to overcome the mine resistance, and a further depression of about 2 pounds on the square foot is required to cause the air to enter the fan. The sum of these depressions becomes 10 + 2 pounds, or 12 pounds, on the square foot below the pressure of the atmosphere. Again, as the fan must make a compression for blowing the air out, say to 2 pounds on the square foot, the work of making the depression and the compression is altogether equal to $10 + 2 + 2 = 14$ pounds on the square foot.

1024. The above principles of action are explained by Fig. 158, in which a funnel and the pipe *A B* represent the

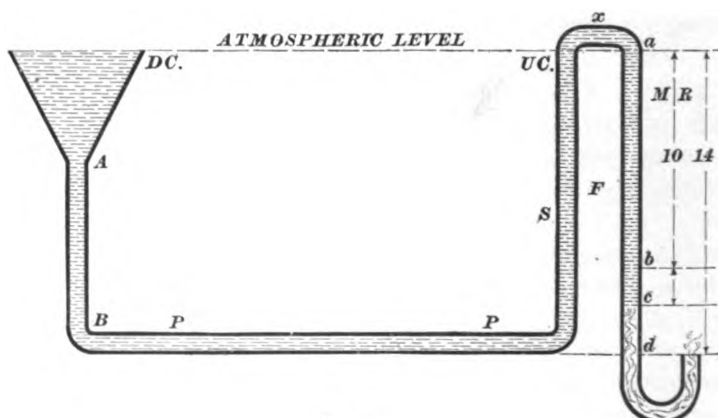


FIG. 158.

downcast shaft *D C*, and the pipe *S* forming one limb of the siphon represents the upcast shaft *U C*, while the descending limb *F* represents the depression and compression of a fan. It may be seen that, if at any moment the pressures or weights of the columns *D C* and *U C* are equal, the fluid will not flow through the pipes, because the atmospheric pressure will not force it through the elbow *x*; but as soon as a portion of it fills the descending limb *F*, a depression takes place in the column *S* and the fluid falls from *D C* to *U C*. Now, let us apply this principle to show

how an exhaust-fan produces ventilation. The horizontal pipe *PP* takes the place of the galleries in a mine, and the columns *A B* and *S* take the place of the shafts, thus making *A B* the downcast and *S* the upcast shaft. The falling limb of the siphon *F* represents the exhaust-fan. It now becomes important that, through the medium of this diagram, the two depressions and the single compression generated by the exhausting-fan be studied. In the first place, the fan represented by *F* makes a depression into which the air of the mine falls. If the mine resistance is nearly equal to 2 inches of water-gauge, or 10 pounds on the square foot, it can be graphically shown, as in that portion of the diagram at the right-hand side of the figure, that *a b* is proportional to the depression required to overcome the mine resistance. Again, the depression required for the air to enter the fan is represented by *b c*; hence, for the air to flow through the mine and fall into the fan, a depression must be made equal to *a c*. Further, a pressure above the atmosphere is required to blow the air out of the fan, as shown at *c d*. The total amount of pressure then required to cause the air to flow through the mine and fall into the fan, and to blow it out, is $a\,b + b\,c + c\,d = a\,d$.

CALCULATION OF VELOCITIES AND PRESSURES.

1025. In calculating the resistance due to the flow of air through mine passages only, the well-known formula $p = \frac{k\,s\,v^2}{a}$ is used; but the pressure required to blow air into a fan, and blow it out, must be found in a different way, because the conditions that originate the resistance are different. For example, the greater portion of the resistance met by a current flowing through a mine is generated by the rubbing surfaces of the airways; but there are no rubbing surfaces to produce resistances when air moves through an orifice that practically has no length, as in the case of the port of entry into a fan and the port of discharge out of it.

There are resistances that are peculiar to orifices that have no length, such as the *vena contracta*. Now, but for this interference with the movements of fluids, air at a pressure of 2,116 pounds on the square foot would rush into a vacuum with a velocity whose square would be equal to 1,800,000. After this number has been corrected for the resistance due to the *vena contracta*, it is reduced from 1,800,000 to 685,600.

1026. The *vena contracta* is that resistance due to the divergent and convergent movements of the particles of a fluid moving through an orifice. To make this clear, a reference to Fig. 159 will show that the air particles *a*, *c*, *e*, *f*, and *h* are all converging towards *O*, the center of a fan orifice. As a consequence, their velocities in lines parallel to each other, and perpendicular to the plane of the orifice, are quite different; for example, in the time that the particle *c* requires to move to *O*, *e* moves to *d*, and *a* to *b*, or *f* to *g*, and *h* to *i*. Again, while *a* moves to *b*, the same particle is tending to move from *b* to *O*; since the latter movement is prevented, the direction of the particle is deflected, and this contracts the neck of the inverted cone of the inflow, and still further promotes the resistance at entry. Another cause of resistance is found in the whirl set up by the converging particles deflecting each other near the orifice, as at *D*, and this is the result of the velocities of the particles increasing as they accumulate in a reducing space, as *o p*, *q r*, or *n m*. These

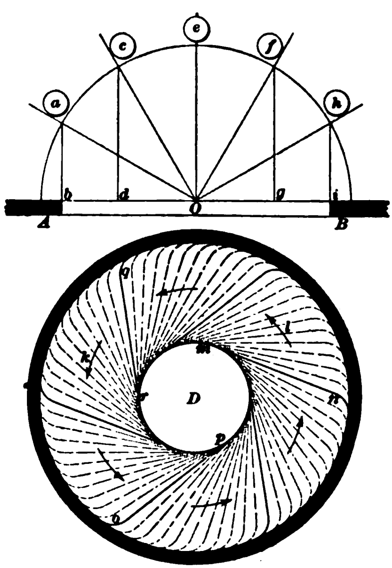


FIG. 159.

facts are still more clearly exemplified in Fig. 160. Here the lateral pressure has produced the contraction *D D*, and it is curious to observe that when the lateral pressure is relieved, the column swells out again to the full size of the orifice *A B*, and still further expands beyond the orifice, as at *E E*. Now, the result of these resistances is that the mean velocity of the inflow is reduced proportionately from 1 to .62;

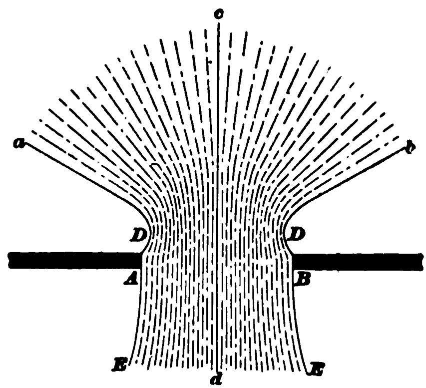


FIG. 160.

for air this is called the coefficient of the inflowing velocity, or the *vena contracta*. It may thus be seen that when the particles of a current flow along converging lines, the channel of the stream is constricted, and the general velocity is reduced.

1027. If it requires an atmospheric pressure of 2,116 pounds per square foot to cause air to blow through an orifice into a vacuum with a mean velocity whose square is 685,600, then the square of the velocity for any other pressure is readily found, when it is remembered that the squares of the velocities of air-currents vary directly as the pressures. This principle may be stated by the following formula :

$$v = \sqrt{685,600 \times \frac{p}{2,116}}, \qquad \textbf{(63.)}$$

in which p = given pressure in pounds per square foot,
and v = velocity in feet per second.

Suppose the pressure is equal to 3 inches of water-gauge, or 15.6 pounds on the square foot; then, by formula **63**, $v = \sqrt{685,600 \times \frac{15.6}{2,116}} = 71.095$, the velocity in feet per second. It should be observed, however, that 685,600 and

2,116 are constant numbers, and that they can be eliminated by substituting a single constant.

Dividing 685,600 by 2,116, and extracting the square root of the quotient, formula **63** becomes

$$v = 18\sqrt{p}. \qquad \textbf{(64.)}$$

EXAMPLE.—Required the velocity with which air will move through an orifice under a pressure of 15.6 pounds per square foot.

SOLUTION.—Applying formula **64**,

$v = 18 \times \sqrt{15.6} = 71.095$, the velocity in feet per second. Ans.

The pressure required to blow air through an orifice when the mean velocity is given can be found by the formula

$$p = \left(\frac{v}{18}\right)^2. \qquad \textbf{(65.)}$$

EXAMPLE.—If air is blowing through an orifice, such as the entry into a fan, with a velocity of 71.095 feet per second, what will be the pressure or depression required?

SOLUTION.—Applying formula **65**,

$$p = \left(\frac{71.095}{18}\right)^2 = 15.6 \text{ lb. per square foot. Ans.}$$

The following equations show how formulas **64** and **65** are obtained:

$$\sqrt{685,600 \times \frac{p}{2,116}} = 18 \times \sqrt{p} = v,$$

$$\frac{v^2}{685,600} \times 2,116 = \left(\frac{v}{18}\right)^2 = p. \quad \text{Ans.}$$

1028. The following examples will illustrate the use of the formulas given:

EXAMPLE.—The mean velocity of the air blowing into the orifice of entry of a fan is 25.5 feet per second; what is the depression in pounds per square foot required to give this velocity?

SOLUTION.—By formula **65**, the pressure will be equal to $\left(\frac{25.5}{18}\right)^2 =$ 2.006 lb. per sq. ft. Ans.

EXAMPLE.—The pressure by which air is blown into a fan is 2.006 pounds per square foot; what is the mean velocity of the entering air?

SOLUTION.—By formula **64**, $v = 18\sqrt{2.006} = 25.5$ ft. per sec. Ans.

EXAMPLE.—The orifice for the entry of air into a fan is 10 feet in diameter, and the pressure of the external atmosphere is 2,116 pounds per square foot; where the air reaches its maximum depression within the fan, its pressure is equal to 2,104.5 pounds per square foot; and the pressure within the fan drift is equal to 2,106 pounds per square foot. (*a*) What is the pressure that is effective in blowing the air out of the drift into the fan? (*b*) What is the pressure in pounds per square foot that is equal to the mine resistance? (*c*) What is the quantity of air entering the fan in cubic feet per minute?

SOLUTION.—(*a*) If the maximum pressure within the fan is 2,104.5 pounds per square foot, and the maximum pressure in the fan drift is 2,106 pounds per square foot, the effective pressure blowing air into the fan is equal to $2,106 - 2,104.5 = 1.5$ lb. Ans.

(*b*) The pressure in pounds per square foot required to overcome the mine resistance is equal to $2,116 - 2,106 = 10$ lb. per sq. ft. Ans.

(*c*) The velocity of the air entering the fan in feet per second can be found by formula **64**; therefore, it is $18 \times \sqrt{1.5} = 22.0446$ feet per second. If the diameter of the orifice of entry is 10 feet, the area of the orifice in square feet will be equal to $10^2 \times .7854 = 78.54$ square feet; and as 60 times the velocity in feet per second is equal to the velocity in feet per minute, the following is the quantity of air entering the fan: $78.54 \times 22.0446 \times 60 = 103,883$ cu. ft. per min. Ans.

EXAMPLE.—The depression necessary for air to enter a fan is 1.5 pounds per square foot. The orifice of entry is 10 feet in diameter, and the area of the orifice of discharge is $\frac{3}{4}$ that of the orifice of entry. If, as has been shown, it requires 1.5 pounds per square foot to blow the air into the fan, what pressure per square foot will be required to blow it out?

SOLUTION.—The pressure to blow the air out will be inversely proportionate to the squares of the given areas. If the orifice of entry is 10 feet in diameter, then the area in square feet will be $10^2 \times .7854 = 78.54$ square feet, and the area of the orifice of discharge will be equal to $\frac{3}{4}$ of $78.54 = 58.905$ square feet; then the pressure required to blow the air out of the fan will be equal to

$$\left(\frac{78.54}{58.905}\right)^2 \times 1.5 = 2.666 \text{ lb. per sq. ft. Ans.}$$

EXAMPLE.—The pressure required to overcome the frictional resistance of the air-currents in a mine is 12 pounds per square foot, and the quantity of air entering the fan is 150,000 cubic feet per minute. (*a*) What pressure is required to blow the air through an orifice of entry which is 12 feet in diameter? (*b*) What pressure will be required to blow the air out of the fan if the orifice of discharge has an area equal to $\frac{3}{4}$ of the area of the orifice of entry? (*c*) What will be the range of pressure between the pressure of discharge and the maximum depression within the fan?

SOLUTION.—(*a*) To find the pressure required to blow the air into the fan, first find the velocity of the entering air in feet per second; that is, divide the quantity in feet per minute by 60 times the area of the orifice of entry, and the quotient will be the velocity in feet per second; thus, $v = \frac{150,000}{12^2 \times .7854 \times 60} = 22.1048$ feet per second. The pressure blowing the air into the fan is, by formula **65**, $p = \left(\frac{22.1048}{18}\right)^2 =$ 1.50809 lb. Ans.

(*b*) The area of the orifice of entry is $12^2 \times .7854 = 113.0976$ square feet, and as the orifice of discharge is $\frac{4}{5}$ of the area of the orifice of entry, it will be $113.0976 \times \frac{4}{5} = 90.478$ square feet.

The volume of air in cubic feet per minute leaving the fan is exactly the same as that entering it; if the areas of entry and discharge are different, the velocities must be inversely proportional to the areas, because the velocity must be greater through a small area than through a large one. In this example the velocity through the large area is to that of the small one as 1 is to $\frac{113.0976}{90.478}$. Again, the pressures vary as the squares of the velocities, and, therefore, the pressure required to blow the air out is

$$\left(\frac{113.0976}{90.478}\right)^2 \times 1.50809 = 2.35639 \text{ lb. per sq. ft. Ans.}$$

(*c*) The total range of pressure between the pressure of discharge and the maximum depression within the fan can be found as follows:

Mine resistance	12.00000 pounds.
Blowing-in pressure.........	1.50809 pounds.
Blowing-out pressure........	2.35639 pounds.
Total	15.86448 pounds. Ans.

EXAMPLE.—If a pressure of 2 pounds per square foot is required to blow air out of a fan which has an orifice of discharge equal to 95 square feet, what depression will be required to blow air into the same fan when the orifice of entry has an area of 120 square feet?

SOLUTION.—As in the above example, the required pressure will be the ratio of the squares of the given areas multiplied by the given pressure; or $p = \left(\frac{95}{120}\right)^2 \times 2 = 1.253472$ lb. per sq. ft., the pressure required to blow air into the fan. Ans.

EXAMPLES FOR PRACTICE.

1. The velocity of air blowing through an orifice is 45 feet per second; what pressure per square foot is required to give this velocity?
Ans. 6.25 lb. per sq. ft.

2. When the pressure required to blow air through an orifice is equal to 5 pounds per square foot, what velocity will be produced?
Ans. 40.2492 ft. per sec.

3. The velocity of air blowing through an orifice is equal to 180 feet per second; what pressure per square foot will be required to give this velocity? Ans. 100 lb. per sq. ft.

4. With what velocity can air be blown through an orifice under a pressure of 120 pounds per square foot? Ans. 197.18 ft. per sec.

5. If it requires a depression of 1.5 pounds per square foot for air to blow into the port of entry of a fan that is 12 feet in diameter, what pressure would be required to blow the air out through a port of discharge that is 10 feet in diameter? Ans. 3.1104 lb.

6. If it requires a pressure of 4 pounds per square foot to blow air through the port of discharge of a fan that has an area of 90 square feet, what pressure will be required for air to enter the same fan when the port of entry has an area of 150 square feet?
Ans. 1.44 lb. per sq. ft.

7. What will be the total pressure of the air just within the port of entry of a fan when the atmospheric pressure is 2,116 pounds per square foot, the mine resistance is equal to 10.4 pounds per square foot, and the depression necessary for the air to blow into the fan is 1.2 pounds per square foot? Ans. 2,104.4 lb. per sq. ft.

1029. The total range of pressure by which a ventilating fan does its work extends from the maximum depression within the fan to the maximum compression without it

For example, suppose the following are the totals of the pressures:

Mine resistance.........	10.0 pounds.
Blowing-in pressure.....	1.5 pounds.
Blowing-out pressure ...	2.0 pounds.
Total range of pressure..	13.5 pounds.

The limits of the total range of pressure arise in this way: If the total pressure of the external atmosphere is 2,116 pounds per square foot, this constitutes an actual depression into which the air from the fan is blown; consequently, the maximum pressure in this example is $2,116 + 2 = 2,118$, and the minimum pressure within the fan is 2,118 − 13.5 = 2,104.5 pounds per square foot. Now, 2,118 − 2,104.5 = 13.5 pounds, as previously shown.

EXAMPLE.—What pressure per square foot will be required to blow 150,000 cubic feet of air per minute into a fan (*a*) when the orifice of entry is equal to 10 feet in diameter, and (*b*) when the orifice of entry is equal to 5 feet in diameter?

SOLUTION.—(*a*) The velocity in feet per second of the air passing through the orifice 10 feet in diameter is found as follows:

$$\frac{150{,}000}{10^2 \times .7854 \times 60} = 31.831 \text{ feet per second. By formula } \mathbf{65},$$

$$p = \left(\frac{31.831}{18}\right)^2 = 3.1272 \text{ lb. per sq. ft. Ans.}$$

(*b*) In the same manner the velocity of the air entering the orifice 5 feet in diameter is found to be $\frac{150{,}000}{5^2 \times .7854 \times 60} = 127.323$ feet per second, and the required pressure, by formula **65**, is equal to $\left(\frac{127.323}{18}\right)^2 = 50.035$ lb. per sq. ft., the pressure per square foot required to blow 150,000 cubic feet of air per minute through an orifice 5 feet in diameter. Ans.

The pressure required to blow 150,000 cubic feet of air per minute through an orifice 5 feet in diameter is 16 times greater than the pressure required to blow air through an orifice 10 feet in diameter.

For, to blow equal quantities through unequal areas in equal times, the pressures vary inversely as the fourth powers of the diameters of the orifices. To prove the statement, let the quantity be 150,000 cubic feet of air per minute, and let the pressure for an orifice 10 feet in diameter be 3.1272 pounds per square foot; then the pressure per square foot required to blow the same volume of air per minute through an orifice 5 feet in diameter is equal to $(\frac{10}{5})^4 \times 3.1272 = 50.035$ pounds, as in the above example.

DIMENSIONS OF THE PORTS OF A VENTILATING FAN.

1030. To obtain the best results with the ventilating fan, the depression necessary for the entry of air should, if possible, not exceed one pound per square foot. Hence, the velocity should not exceed 18 feet per second; for, by formula

64, $v = 18 \times \sqrt{p}$, and $18 \times \sqrt{1} = 18$. Now, 18 feet per second is equal to $18 \times 60 = 1{,}080$ feet per minute. Using this velocity, the diameter of the port of entry may be found by the following formula:

$$d = .0343 \sqrt{q}, \qquad \textbf{(66.)}$$

where d is the diameter of the port of entry and q is the quantity of air flowing per minute through *one* port of entry. If there are two ports, that is, if the fan receives air on both sides, q is obtained by dividing the total quantity per minute by 2.

EXAMPLE.—What should be the theoretical diameter of the port of entry of a fan to pass 200,000 cubic feet of air per minute?

SOLUTION.—Using formula **66**,

$$d = .0343 \sqrt{q} = .0343 \sqrt{200{,}000} = 15.355 \text{ ft. Ans.}$$

1031. The area of the throat of a fan must be equal to the area of the curved surface of an imaginary cylinder whose diameter is equal to that of the port or ports of entry; and, as the length of this cylinder is exactly equal to the breadth of the fan-blades, it is important that the relationship of this area to that of the port of entry should be fully understood. The breadth of the blades or the length of the imaginary cylinder just referred to is found as follows:

Let d = diameter of port of entry;
b = breadth of blades.

Then the curved surface of the imaginary cylinder is

$$3.1416\, d\, b$$

the area of the port of entry is

$$.7854\, d^2$$

Therefore, $3.1416\, d\, b = .7854\, d^2$;

or, $$b = \tfrac{1}{4} d. \qquad \textbf{(67.)}$$

This formula is applied when there is but one port of entry. When there are two ports of entry, $b = \frac{1}{2}\, d$.

EXAMPLE.—What should be the width of blade of a fan which is to deliver 160,000 cu. ft. of air per minute, there being one port of entry?

SOLUTION.—Using formula **66**,

$$d = .0343\ \sqrt{160,000} = 13.72 \text{ ft.}$$

Now, applying formula **67**,

$$b = \tfrac{1}{4} d = \tfrac{1}{4} \times 13.72 = 3.43 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE.—If 160,000 cubic feet are delivered per minute and there are two ports of entry, what should be the diameter of each port of entry and the width of the blade?

SOLUTION.— $q = \frac{160,000}{2} = 80,000.$ Using formula **66**,

$$d = .0343\ \sqrt{80,000} = 9.7 \text{ ft.} \quad \text{Ans.}$$

$$b = \tfrac{1}{2} d = 4.85 \text{ ft.} \quad \text{Ans.}$$

The area of the port of discharge in an ideal fan should not be less than .81 of the area of the port of entry. It is true that few fans will work satisfactorily when this port is so large, but such fans can not give best results, because when the area of the discharge port is too much restricted, the excessive pressure required to blow out the air is much greater than it should be. Again, if there is not enough constriction in the port of discharge, there is bound to be excessive vibration of the air in the fan, necessitating the employment of a shutter. Therefore, .81 is far above the average proportion in many fans, but it is an ideal that should be sought for.

EXAMPLE.—The area of the port of entry of a fan is equal to 150 square feet, and the area of the port of discharge is .6 of this; then, if a pressure of 1 pound per square foot is required to blow the air into the fan, what pressure will be required to blow it out?

SOLUTION.—The pressures for blowing in and blowing out are inversely proportional to the squares of the areas; therefore,

$$\left(\frac{150}{150 \times .6}\right)^2 \times 1 = 2.77 \text{ lb. per sq. ft.} \quad \text{Ans.}$$

EXAMPLE.—If the area of the port of entry of a ventilating fan is equal to 150 square feet, and the area of the port of discharge is equal to .81 of the area of port of entry, and if a pressure of 1 pound per square foot is required to blow the air into the fan, what pressure will be required to blow it out?

SOLUTION.—The area of the port of discharge will be $150 \times .81 = 121.5$ square feet, and the pressure to blow the air out of the fan will be

$$\left(\frac{150}{121.5}\right)^2 \times 1 = 1.52415 \text{ lb. per sq. ft. Ans.}$$

EXAMPLE.—The area of the port of entry of a ventilating fan is 150 square feet, and the area of the port of discharge is .5 of the area of the port of entry. If it requires 1 pound of depression to blow the air in, what compression will be required to blow it out?

SOLUTION.—The area of the port of discharge will be $150 \times .5 = 75$ square feet, and the pressure to blow the air out will, therefore, be

$$\left(\tfrac{150}{75}\right)^2 \times 1 = 4 \text{ lb. per sq. ft. Ans.}$$

THE MANOMETRIC EFFICIENCY.

1032. Manometric efficiency is that percentage of the total pressure generated by a ventilating fan that is efficient in blowing the ventilating current through a mine. What is here called the total pressure consists of three additive quantities:

1. The mine resistance M in pounds per square foot, as measured with the water-gauge.
2. The depression I required for the air to enter a fan.
3. The pressure O required to blow the air out of a fan.

Let A = area of port of entry;
a = area of port of discharge;
C = manometric efficiency.

Then the pressure O is given by the following formula:

$$O = \frac{A^2}{a^2} \times I. \qquad \textbf{(68.)}$$

The percentage of manometric efficiency C is found by formula **67**, where

$$C = \frac{100\,M}{M + I + O}. \qquad \textbf{(69.)}$$

EXAMPLE.—What is the manometric efficiency of a ventilating fan when the mine resistance is 2.5 inches of water-gauge, the depression at the port of entry of the fan is 2 pounds per square foot, the area of the port of entry is 100 square feet, and the area of the port of discharge is 60 square feet?

SOLUTION.—By formula **68**, $O = \frac{100^2}{60^2} \times 2 = 5.555$ lb.

Again, by formula **69**, $C = \frac{100 \times 13}{(13 + 2 + 5.555)} = 63$ per cent. Ans.

EXAMPLE.—Required the percentage of useful effect or manometric efficiency of a ventilating fan when the mine resistance is equal to 12 pounds per square foot, the depression required for blowing air into the fan is equal to 1 pound per square foot, and the compression for blowing the air out is equal to 1.5 pounds per square foot ?

SOLUTION.—The total range of pressure is:

Mine resistance..................	12.0 pounds.
Blowing-in pressure.............	1.0 pound.
Blowing-out pressure............	1.5 pounds.
Total	14.5 pounds.

The pressure required for mine resistance alone is 12 pounds; therefore, the efficiency of the fan, in so far as the ventilating of the mine is concerned, is $\frac{12}{14.5} \times 100 = 82.7586$ per cent. Ans.

The last two examples are given to show the importance of making the ports of entry and discharge sufficiently large to prevent needless waste in the working of a ventilating fan.

EXAMPLE.—The mine resistance is equal to 10 pounds per square foot, the blowing-in pressure is equal to 2 pounds per square foot, and the area of the port of discharge is so small that the blowing-out pressure is 8 pounds per square foot; what is the percentage of useful effect, or manometric efficiency of the fan as a ventilator ?

SOLUTION.—The total range of pressure is equal to the following sum:

Mine resistance	10 pounds.
Blowing-in pressure	2 pounds.
Blowing-out pressure.............	8 pounds.
Total.........................	20 pounds.

Therefore, the efficiency of the fan as a ventilator is equal to $\frac{10}{20} \times 100 =$ 50 per cent. Ans.

EXAMPLE.—A fan is exhausting from a mine 180,000 cubic feet of air per minute, and the area of the port of intake is 60 square feet; what is the pressure required for blowing the air into the fan ?

SOLUTION.—First find the mean velocity of the entering air in feet per second as follows:

$$\frac{180{,}000}{60 \times 60} = 50 \text{ feet per second};$$

therefore, by formula **65**, the required pressure is $(\frac{50}{18})^2 = 7.716$ lb. per sq. ft. Ans.

EXAMPLES FOR PRACTICE.

1. What is the manometric efficiency of a fan when the mine resistance is equal to 15 pounds per square foot, the blowing-in pressure is 2 pounds per square foot, and the blowing-out pressure is 5 pounds per square foot? Ans. 68.18 per cent.

2. The manometric efficiency of a fan is 70 per cent., and the mine resistance is 3 inches of water-gauge; what are (*a*) the blowing-in and (*b*) the blowing-out pressures when the area of the port of discharge is .6 of the area of the port of entry? Ans. { (*a*) 1.7672 lb. per sq. ft. (*b*) 4.918 lb. per sq. ft. }

CENTRIFUGAL FORCE.

1033. The tendency of every body in motion is to move in a straight line, unless the body is acted upon by some force which causes it to deviate from the straight line. In the case of a body attached to a string and moving in a circle, the deviating force is the pull exerted by the string, and is called **centripetal force.** The so-called **centrifugal force** is that force which is equal and opposite to the centripetal force; in other words, it is a reaction. Centrifugal force can not cause motion; hence, if the string were cut, the centripetal force would no longer act, and the body, being then free to move, would start off in the direction of a straight line tangent to the circle, as shown by the line $c\,g$ in (*a*), Fig. 161. Centrifugal force is manifested under two conditions. In the first case there is a uniform deflection and a uniform velocity with a constant radius, as, for instance, when a body is made to rotate with a uniform velocity at the outer extremity of a constant radius. The centrifugal force can be found by the formula

$$f = \frac{w\,v^2}{R\,g}, \qquad \textbf{(70.)}$$

in which w = weight in pounds;
v = velocity in feet per second;
R = radius in feet;
g = acceleration due to gravity, or 32.16;
and f = force in pounds pulling the body towards the center of its revolution.

This is a case like that in which a pound weight might be made to revolve on the end of a string, and should the string at any moment be cut, the weight would simply move off in a line tangent to the curve.

In the second case, the centrifugal force, the velocity, and the radius are constantly increased, and correspond to the centrifugal force developed by a ventilating fan, which develops a velocity outwards from the center of revolution. To explain the difference, take a case in which a pound weight is made to revolve within a tube instead of being attached to the end of a string. Now, if the tube moves with the same speed as the string, the pound weight will move outwards along the inside of the tube. The velocity thus acquired will cause the weight to move in a path situated between the tangent to the curve and the prolongation of the radius at the point of disengagement, instead of in a path tangential to the curve of the outer circle. The body revolving on the end of the fixed radius has, at the moment of disengagement, acquired only sufficient centrifugal force to make it describe a path tangential to its circle; whereas, the body moving through the tube has, in addition to the centrifugal force of the former body, acquired a force due to an increased outward acceleration.

The force acquired in the second case is calculated by the formula

$$f = \frac{w v^2}{3.1416 g}, \qquad \textbf{(71.)}$$

in which the factors are the same as in formula **70,** except that the constant 3.1416 is substituted for R.

1034. In (a), Fig. 161, both of the above cases are illustrated. The radius of the circle described by the first body revolving is $o\ e$, or $o\ h$, and if this body is by any means disengaged, as, for example, by the breaking of a string, at the instant it is passing the point e, the body will reach the point g at the moment it should have arrived at h, having moved along the line $e\ g$, which is tangent to the curve $e\ h$. If, however, the same body is made to revolve in a tube and also to commence its outward journey at the center o, by the time it reaches e it will have acquired a high outward velocity that the fixed body can not possess. Therefore, at the moment of disengagement it will move off in the path $e\ f$, and will arrive at f in as short a time as the

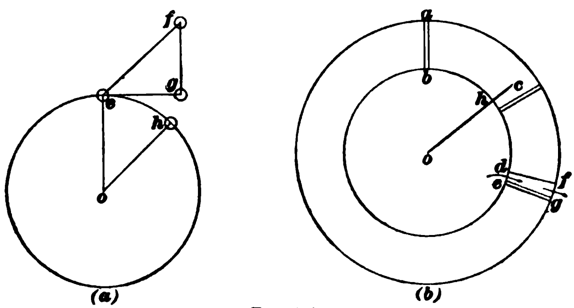


FIG. 161.

first body will require to reach g. The result is that the velocity developed by a continual deflection and acceleration is much greater in amount than that due to a body revolving with a uniform velocity on the outer extremity of a radius of constant length.

DEFINITIONS OF TERMS.

1035. The centrifugal force developed by a fan is calculated by the following formula :

$$f = \frac{w\,l\,v^2}{3.1416\,g}. \qquad \textbf{(72.)}$$

Before giving examples that involve the use of formula **72**, the meaning and use of its factors must be explained. For example, the velocity is obtained from the length of the radius of gyration, and this is obtained by adding to the radius of the port of entry a fraction of the radial length of the blades, the latter being found by multiplying the radial length by .6. The radial length of the blades is found by the formula

$$l = \frac{D - d}{2}, \qquad \textbf{(73.)}$$

in which D = diameter of the fan;
d = diameter of the port of entry;
and l = radial length of the blades.

EXAMPLE.—The diameter of a fan is 30 feet, and the diameter of the port of entry is 12 feet. What is the radial length of the blade?

SOLUTION.—By formula **73**, the radial length of the blades is $\frac{30 - 12}{2} = 9$ ft. Ans.

1036. In (*b*), Fig. 161, is given a graphic illustration of the terms in question. For example, taking *o* as the center of a fan, *c* is situated at the center of gyration, and *o c* is the radius of gyration. As *o h* is the radius of the port of entry, $o\,h + h\,c$ (or, what is the same thing, $o\,h + a\,b \times .6$) is equal to the radius of gyration. This is expressed by the formula

$$r = \frac{d}{2} + .6\,l, \qquad \textbf{(74.)}$$

in which r is the radius of gyration. When air is flowing along the blades of a fan, all the air on the blades, from the periphery of the port of entry to the outer periphery of the fan itself, is subject to centrifugal force, and, as the blades may be 9 or 10 feet long, to find the weight of air subject to centrifugal force, the weight of a cubic foot of air is multiplied by l, the length of the blades; this is the meaning of the expression in formula **72**, where $w\,l$ occurs as two of the factors.

Now, it must be clear that the velocity of the moving air at *b* is much less than it is at *a*; and, therefore, the mean of the squares of the velocities, multiplied by the weights, occurs at the center of gyration *c*, for a stream of air lies on every blade, as that shown at *d e g f*. An example will make this clear:

EXAMPLE.—A ventilating fan is 30 feet in diameter, the radial length of the blades is 9 feet, and the length of the radius of gyration is 11.4 feet; (*a*) what is the mean velocity generating centrifugal force when the fan is making 50 revolutions per minute? (*b*) What is the total pressure produced? (*c*) What is the quantity of air passed per minute?

SOLUTION.—The velocity generating centrifugal force in feet per second will be equal to $\frac{11.4 \times 2 \times 3.1416 \times 50}{60} = 59.69$ feet per second, the required velocity. From this velocity, the total pressure in pounds per square foot to produce the two depressions already noticed and the compression for blowing out can be found by formula **72.** In the case for which the velocity has been calculated, the length of the blade is 9 feet; if the average weight of a cubic foot of air is taken at .076 pound, formula **72** gives $f = \frac{.076 \times 9 \times 59.69^2}{3.1416 \times 32.16} = 24.11$ pounds per square foot. Next, suppose that the mine resistance in a case like this is equal to 3 inches of water-gauge, or 15.6 pounds per square foot, and the depression within the fan is 4 pounds per square foot below that in the fan drift, and that the pressure per square foot for blowing out is 4.51 pounds above the atmosphere. These figures yield the factors for calculating the quantity of air that this fan is exhausting out of the mine in cubic feet per minute. By formula **64,** $v = 18\sqrt{p} = 18 \times \sqrt{4} = 36$ feet per second, the mean velocity of the air entering the fan. Next, the port of entry, which is circular, is $30 - 2l$, or $30 - 18 = 12$ feet in diameter, and its area is $12^2 \times .7854 = 113.0976$ square feet. 36 is the mean velocity in feet per second, and, therefore, $36 \times 60 = 2,160$, the velocity in feet per minute. If the area found be multiplied by the mean velocity in feet per minute, the result will be the quantity of air exhausted by the fan in cubic feet per minute, as $113.0976 \times 2,160 = 244,290.8$ cubic feet of air per minute. Ans.

EXAMPLE.—A ventilating fan is 28 feet in diameter, and the diameter of the port of entry is 10 feet; what is the radial length of the blades?

SOLUTION.—By formula **73,**

$l = \frac{D - d}{2}$; then, $\frac{28 - 10}{2} = 9$ feet, the radial length of the blades. Ans.

EXAMPLE.—A ventilating fan is 28 feet in diameter, and the orifice of entry is 10 feet in diameter; what is the length of the radius of gyration?

SOLUTION.—By formula **74,**

$$r = \frac{d}{2} + .6 \times l;$$

or, $r = \frac{10}{2} + (.6 \times 9) = 10.4$ ft., the radius of gyration. Ans.

EXAMPLE.—The radius of gyration for a ventilating fan is 9.5 feet, and the length of blade is 7.5 feet; the diameter of the orifice of entry is 10 feet; the angular velocity is 50 revolutions per minute; what is the total range of the fan's ventilating pressure?

SOLUTION.—The velocity per second is equal to $\frac{9.5 \times 2 \times 3.1416 \times 50}{60} =$ 49.742 feet; then, by formula **72**,

$$f = \frac{w\, l\, v^2}{3.1416\, g} = \frac{.076 \times 7.5 \times 49.742^2}{3.1416 \times 32.16} = 13.96 \text{ lb.},$$

the total ventilating pressure. Ans.

1037. The Center of Gyration.—The calculations thus far given are based on the assumption that the blades of the fan lie longitudinally in radial lines, but in many cases this does not occur. Therefore, it is necessary to be able to make the expressions adaptable for fans in which blades make different angles with the radii. Now, when the blade makes an angle with the radius, its efficient length is practically shortened in the proportion of the cosine of the angle. For example, suppose a blade makes an angle of 45° with the radius projected from the circumference of the port of entry; then the efficient length of the blade is only .7 of its actual length, as shown in Fig. 162, in which $A\,B$ is the radius, $C\,D$ the actual length of the blade, and $C\,B$ the efficient length of the blade. Again, when the blades make an angle with the radii, the efficient angular velocity, that is, the number of revolutions, is practically reduced in the proportion of the cosine of the angle which the blades make with the radii. For, suppose a case in which the fan is making 50 revolutions per minute. The 50 would be reduced by the cosine of the angle, because where the blades decline from the radii, the relative efficiency of the velocity is only for an angle of 45°, which is equal to 50 × .7 = 35 revolutions per minute. To make this allowance, the expressions may be simplified by multiplying the number of revolutions by the square of the cosine; formula **72** thus becomes

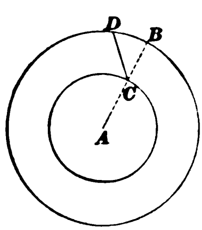


FIG. 162.

$$f = \frac{w\,l\,v^2\,(\cos a)^2}{3.1416\,g}, \qquad \textbf{(75.)}$$

in which a = angle the blades make with the radii in every case;

f = total pressure per square foot;

v = velocity of the center of gyration in feet per second;

l = actual length of the fan blades;

w = weight of a cubic foot of air, or .076;

g = acceleration due to gravity, or 32.16;

and 3.1416 = a constant.

If, for example, the blades of a fan are each 8 feet long, and are so set as to make an angle of 30° with the radii, and the velocity of the center of gyration is 60 feet per second, the total pressure, by use of formula **75**, is calculated as follows:

$$f = \frac{3{,}600 \times .076 \times 8 \times .86603^2}{3.1416 \times 32.16} = 16.25 \text{ lb., total pressure.}$$

1038. The importance of being able to calculate the mean velocity of a current of air flowing outwards along the blades of a fan can not be overlooked, because, when this velocity is not known, the formula adopted only secures a rough approximation of what a ventilating fan is capable of doing. To understand the matter, the student must first have clear views concerning how air moves in its passage through a fan, and the best way of obtaining this is to consider in order the three components of the resultant motion.

1039. *First.*—The angular velocity is that due to the revolution of the fan. As all the particles in a current of air flowing along the blades of a fan make a revolution round the common center of motion in the same time, it is necessary to explain how the angular velocity affects the sum of the work of a fan. Angular velocity means the same thing as revolutions per minute or per second, for the angular velocity increases or decreases directly as the number of revolutions increases or decreases.

1040. *Second.*—While all the particles of air within a fan have the same angular velocity, the linear velocities are directly proportional to their radial distances from the center of motion; for example, all the particles marked *6*, *7*, *8*, *9*, *10*, *11*, *12*, in the upper portion of Fig. 163, have the same angular velocity; that is, they all make one revolution in the same time, but their linear velocities are different, for they are all describing circles of different lengths. The linear velocity of *6* is proportional to the length of the arc *a b*. The linear velocity of *12* is proportional to the length of the arc *m n*, and the same relationship holds true with all the other numbers.

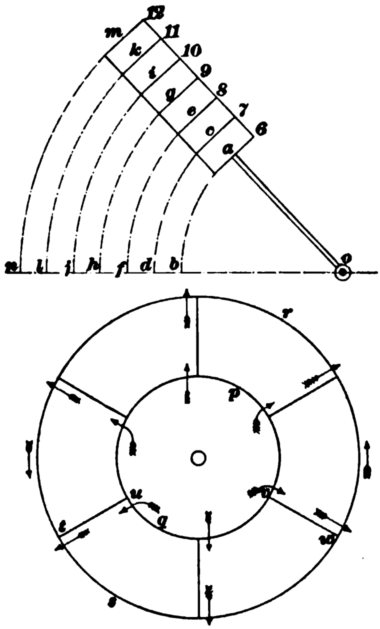


FIG. 163.

1041. *Third.*—What is called radial motion is the flow of the stream of air along the blades of a fan. There are many mistaken ideas in regard to this motion. For example, the common idea is that the currents of air entering a fan diffuse between the diverging blades, and, therefore, the velocity reduces as the divergence increases. A little consideration, however, shows the error of this conclusion. The air-currents, as a whole, flow through a fan with an unvarying velocity, for the weight of the air leaving a fan can never exceed the weight of the air entering it, but the particles making up the currents have different velocities in the direction of the fan's motion. Again, the increasing effect of the centrifugal force, as the stream advances to the circumference of the fan, tends more and more to prevent any increase in the depth of the flow; indeed, it tends rather to reduce it. This is fully proved on watching the contraction that takes place in a river where it contracts its flow as the result of the effect of the centrifugal force that is generated by a bend in the channel. The flow of the air is such as indicated by the arrows along the blade of the fan, as seen in the lower portion of Fig. 163. The fan is supposed to be turning in the direction of the hands of a watch, and the inner circular space within the fan blade is the port of entry, or the orifice by which all air enters a fan.

1042. The Evase Chimney.—Fig. 164 is an illustration of four important points. The first is the fan drift illustrated by the round tube I; the second is the fan-drift depression, as shown by the water-gauge D; the third is the water-gauge G, that shows the depression that occurs within the fan; and the fourth is the greater pressure or compression of the air at discharge, as shown by the water-gauge P. These points have all been explained. E is the evase chimney used to reduce the amount of waste that occurs in blowing air out of a fan into the external atmosphere. In the case of the Guibal fan, for instance, where the orifice of entry is relatively small and the velocity of discharge is relatively high, if something is not done to reduce the high velocity, the blast of air striking the comparatively still air of the atmosphere causes a rebound that produces great resistance. The evase chimney is used to reduce this needless loss of energy in the discharged air. To show the efficiency of an evase chimney, assume the orifice of discharge from the casing to the chimney

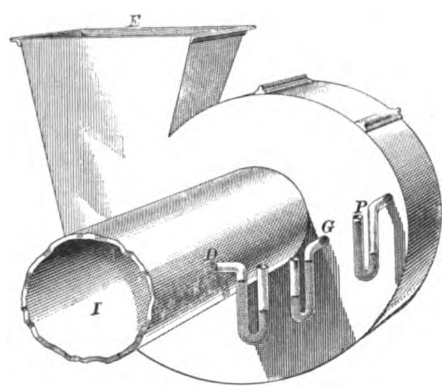


FIG. 164.

to be equal to 50 square feet, and the area of the top of the evase chimney to be equal to 200 square feet. Now, the velocity of discharge at the top of the evase chimney will only be one-fourth of the velocity of the air blowing through the orifice of discharge of the fan, and as the resistances that arise when air at a high velocity strikes still air are proportionate to the squares of the velocities, the resistance due to the air leaving the top of the evase chimney is to the resistance of the higher velocity as 50^2 is to 200^2 or as 1 is to 16.

EXAMPLES FOR PRACTICE.

1. What is the centrifugal force in pounds due to a 5-pound weight revolving on the end of a rigid radius under the following conditions: Length of radius, 6 feet; number of revolutions per second, 10?
Ans. 3,682.7 lb.

2. What is the pressure per square foot due to a stream of water flowing through a pipe that is rotating on one of its ends, the length of the pipe being 5 feet and making 2 revolutions per second?
Ans. 4,395.9 lb. pressure per sq. ft.

3. A ventilating fan is 25 feet in diameter, and the port of entry is 10 feet in diameter. What is the radial length of the blades?
Ans. 7.5 ft.

4. A fan is 25 feet in diameter, and the diameter of the port of entry is 10 feet. What is the length of the radius of gyration?
Ans. 9.5 ft.

5. A ventilating fan is 25 feet in diameter, the port of entry is 10 feet in diameter, and the radius of gyration is 9.5 feet. What is the velocity in feet per second of the center of gyration when the fan makes 45 revolutions per minute? Ans. 44.7678 ft. per sec.

6. The velocity of the center of gyration of a ventilating fan is 44.7678 feet per second, and the radial length of the blades is 7.5 feet. What is the total pressure generated by the fan to overcome the mine resistance, and to set the air in motion into and out of itself?
Ans. 11.3032 lb. per sq. ft.

TYPES OF FANS.

1043. Centrifugal fans are used for blowing and exhausting. Exhausting fans are in most general use, though there are many advocates and users of the blowing-fan. So far as mechanical efficiency is concerned, exhaust-fans and blowing-fans are practically equal.

The general principles of each are the same, except in reverse order. The exhaust-fan draws from the mine and discharges into the outer atmosphere, while the blowing-fan draws from the outer atmosphere and discharges into the mine.

1044. Prominent Types of Fans.—The most prominent types of centrifugal ventilators now in use in mining countries are four in number, the principal representatives of which are (*a*) *Waddle*, (*b*) *Schiele*, (*c*) *Guibal*, (*d*) *Capell.* These four will be described as representing the main features of all other forms, which are modifications of these original types.

1045. The Waddle Fan.—The characteristic features of the *Waddle* fan, Fig. 165, are the *curvatures of its blades*—backwards from the direction of their motion; their tapered form, tapering towards the circumference, and the tight box sides, which revolve with the blades. The blades leave the orifice of intake radially, but curve backwards from the direction of their motion till they are almost tangential at the circumference. The blades are so tapered from the orifice of intake to the circumference that the breadths of the blades at

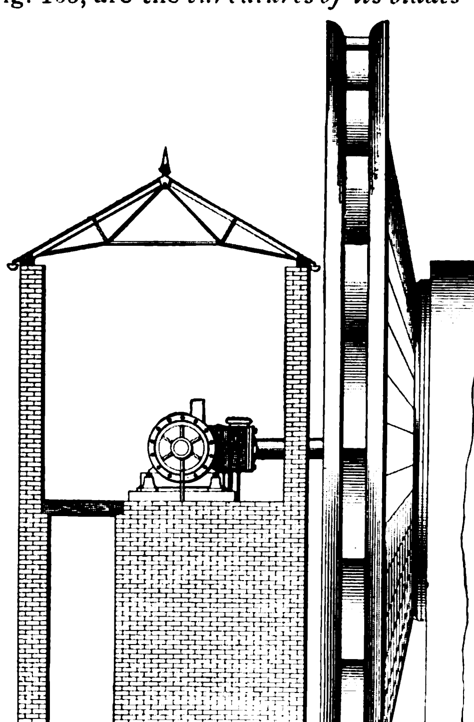


FIG. 165.

different distances from the center of the fan-wheel vary inversely as the radial lengths of the points at which the breadths are measured.

1046. The **Schiele fan,** Fig. 166, very much resembles the Guibal fan in its mode of action, although its construction is in some important respects quite different. In the Schiele fan a disk takes the place of the spider wheel in the Guibal fan, and this makes necessary the duplication of the blades, for they are attached on opposite sides of the disk. The disk makes a complete partition within the fan, and, therefore, the supply of air to the blades must come from two ports of entry, one for each set of blades. The Schiele fan

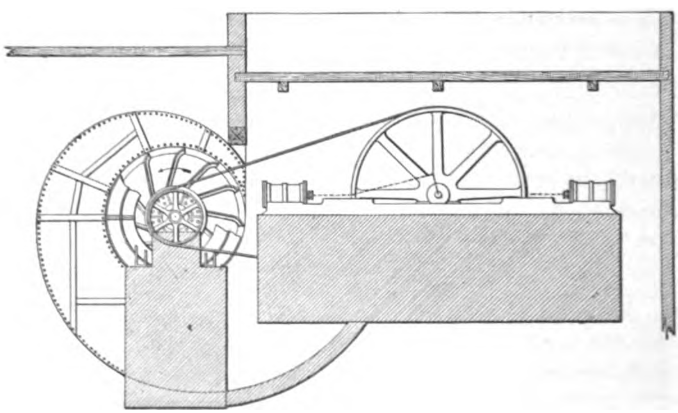


FIG. 166.

is a fast-running one, and, therefore, to do the same amount of work its diameter is only about one-third that of a Guibal fan of the same capacity. By reference to the figure, it will be seen that this fan is set within a spiral casing surrounding the circumference and leading to an expanding or evase chimney. The student must notice carefully the effect of the spiral casing surrounding the circumference of the Schiele fan, as it is a most important factor in fan construction. It provides a uniformly increasing sectional area about the fan, to accommodate the flow from each compartment. The velocity of the air is thus made uniform all around the circumference, and each compartment furnishes its proportion of air in a continuous flow. In exhausting, the discharge from the casing is conducted to the expanding chimney, where its velocity is much reduced, before it is finally thrown out upon the atmosphere. In force or blower fans, the expanding casing should connect with the mine passages by a uniformly expanding fan drift.

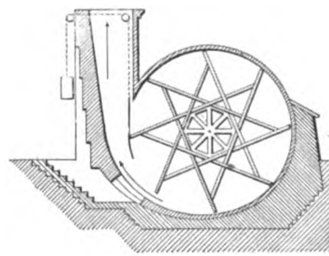


FIG. 167.

1047. The Guibal Fan.—This fan revolves within a closed case and delivers its air into an involute chamber, which gradually expands into the evase chimney. The fan is illustrated by Fig. 167.

1048. The Capell Fan.—This fan is constructed somewhat after the type of the Guibal fan, with considerable additions and improvements, such as is illustrated by Fig. 168. All the centrifugal fans in use, however, may be put into two groups; namely, closed and open fans. Among those just noticed, the Waddle is an open fan; the Schiele, the Guibal, and the Capell are closed fans.

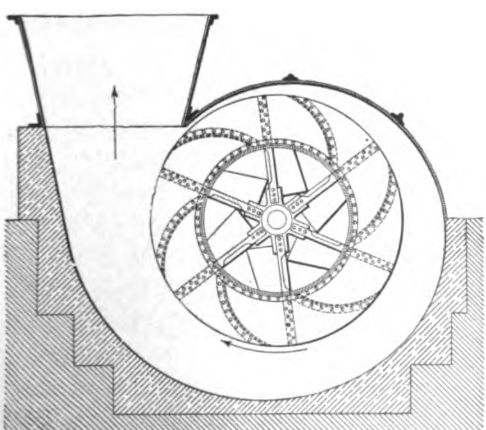


FIG. 168.

PRACTICAL VENTILATION.

QUANTITY, VELOCITY, AND CONDUCTION OF AIR.

1049. Use of the Air-Current.—The necessity for the air-current in mines is threefold, viz: (1) to furnish fresh air to the men and animals in the mine; (2) to sufficiently dilute and render harmless the poisonous and explosive gases of the workings; (3) to remove these by sweeping them from their lodging places or the cavities in which they are lodged.

1050. Efficient Ventilation.—The efficient or thorough ventilation of a mine is dependent upon three essential elements:

(*a*) Volume of the current.
(*b*) Velocity of the current.
(*c*) Manner of conducting the current.

1051. Volume of Air Required.—The quantity of air required to be furnished per minute to the workings of a mine is usually fixed by the law of the state or country in which the mine is located. The amount specified is usually 100 cubic feet per minute per man, and 500 cubic feet per minute per mule, in non-gaseous mines. In gaseous mines this amount is increased to 150 cubic feet per man, and, in some cases, as in the anthracite mines of Pennsylvania, 200 cubic feet per minute are required by law. This method of fixing the amount of air required is purely arbitrary, and often not adapted to the existing conditions, as the workings in a thin seam will have an abundance of air and, perhaps, too high a velocity, while in a thicker seam the velocity will be too low for proper ventilation.

1052. Velocity of the Current.—This is a very important factor in the ventilation of a mine, as upon it largely depends the removal of the mine gases. A body of firedamp or of marsh-gas which is exuding or has collected in some cavity of the roof, or at the face of some rise heading, will require a current having a certain velocity to dilute it and drive it from its position. In like manner, the heavier gases, as carbonic acid gas, settling towards the dip workings and low places of the pit, can not be carried out if the velocity of the current is too low. The current velocity should not be permitted to fall below 3 or 4 feet per second at any working face.

1053. On the other hand, too high a velocity of the ventilating current is always objectionable, and, in gaseous mines, dangerous. The Anthracite Mine Law of Pennsylvania provides that all air-passages shall be of a sufficient area to allow the passing of 200 cubic feet of air per man per minute at a velocity not exceeding 450 feet per minute; and this velocity may be taken as a safe maximum limit in gaseous mines. It must be remembered that a safety-lamp carried against a current is virtually subjected to a velocity equal to that of the current plus the velocity with which the lamp is carried against the current. In non-gaseous mines, the velocity of the main intake may be anywhere from 10 to 20 feet per second without causing serious annoyance. In many cases it exceeds this amount. It should not fall below 3 or 4 feet per second in any airway in the mine.

1054. Conducting the Current.—The thorough ventilation of the working face in any mine is dependent to a large extent upon the manner in which the doors, stoppings, brattices, and overcasts, or bridges, are erected. These must not leak. **Doors** must be hung with a fall sufficient to close behind a passing car, and must be fitted tightly to a substantial frame. Where the ventilating pressure is light, a door is sometimes hung to swing both ways, to save trapping. Ordinarily, however, this can not be done, and the door must then be hung to open against the current. Canvas flaps are sometimes nailed to the bottom of a door to prevent leakage and allow of good clearance. Double doors are often used in cross-cuts near the shaft bottom between the main airway and the return. The object of double doors is to prevent the momentary stoppage of the ventilating current when a car is passing through the cross-cut. The doors are set from 6 to 8 yards apart, or farther, if the length of the cross-cut will permit. **Stoppings** are commonly built of a double wall of slate, with a few inches of space between. This space is filled completely to the roof with fine sand or clay from the surface, or with dirt taken from the roads. Stoppings upon the main air-courses of large mines are often plastered with clay, or laid up with brick instead of slate walls. **Brattices** are partitions, usually of wood or canvas (brattice-cloth), for the purpose of dividing the airway for a short distance near the face into an intake and a return. A **curtain** is a heavy canvas hung across an airway or the mouth of a room to partially turn the air. A curtain serves as a regulator, inasmuch as it permits a portion of the air-current to pass it, while the remaining portion is forced into another passageway.

1055. Regulators are contrivances for regulating the division of the air between two or more airways. The usual form of regulator is shown in Fig. 147, and consists of a wooden brattice built across the airway and having an opening provided with a shutter, by which the size of the opening may be increased or decreased, and thus any desired division of the air secured. **Bridges** are built in airways for the purpose of crossing the air-currents. A bridge may be an **overcast**, as shown in Fig. 169, in which C is the coal-seam, G is the main airway, and R is an overcast cut out of the roof for the purpose of conducting a current of air over or under another current. If the cross-current is conducted under the main airway, it is called an **undercast**. In this case the *bridge* forms the floor of the main airway and the roof of the cross-cut. The dis-

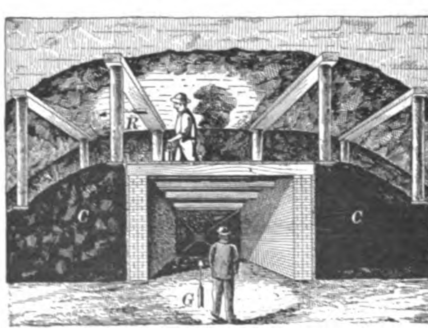


FIG. 169.

advantages of the *undercast* are: (1) water is apt to close the passage; (2) the bridge is more difficult to keep air-tight, being subject to travel of mules and cars; (3) if the cross-current is the intake, the air is made unwholesome by the dust of travel being sifted down through the bridge floor. The bridge floor is usually made by laying a double thickness of plank upon cross-stringers of railroad iron or oak, and covering the whole with dirt from the roadways.

1056. Fig. 170 is an illustration of the practical arrangement of stoppings, brattices, and curtains. *A B* are a

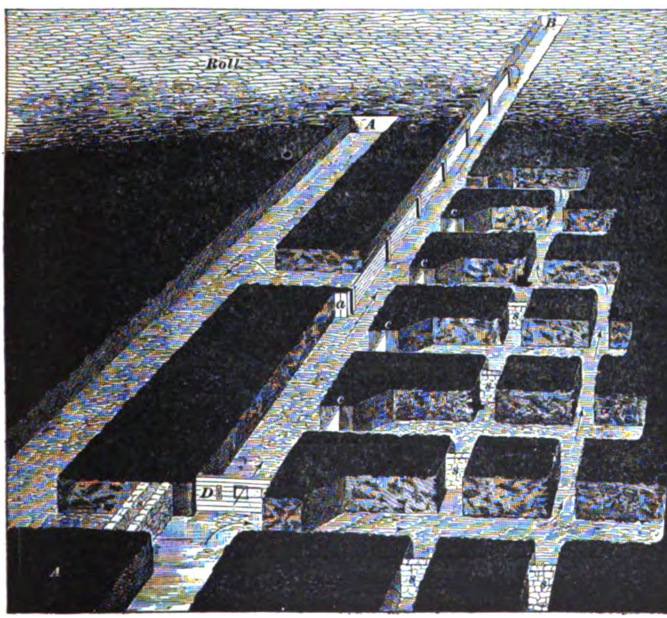


FIG. 170.

pair of entries which struck a roll. The entry *A* was abandoned for the time, and *B* was driven ahead as a fore-winning or prospect entry. For this purpose a temporary brattice was erected from the outside rib of the last cross-cut at *A*, towards the face of the entry *B*, by setting a row of props two feet from the rib of the entry, and nailing brattice-cloth or light boards to them. The air-current is then forced to travel towards the face before it can return to the cross-cut behind the brattice. An entry stopping and room stoppings are shown at *s*, *s*, *s*. Curtains are hung at the mouths of all the rooms, except the outside and inside ones, which are left open. A curtain or a door is then hung upon the entry at *D*, just inside the mouth of the first room, which deflects the current into the face of the rooms. A curtain at *D* will usually accomplish this, but if the room workings are extensive, a door should be used and an opening left in it, or at one side of it, sufficient for the ventilation of the portion of entry thus cut off from the current. This example will serve to illustrate a practical method of conducting a current of air through a mine. It is sometimes necessary to deflect the current so that it will sweep a particular cavity of the roof or point of the entry, where a dangerous body of gas would otherwise collect. Wherever this is necessary, attention must be given to it, as no system of ventilation will be efficient unless the air is made to brush the gas from all of its lodging places. The *volume* of the current may be sufficient, and it may travel at the required *velocity*, yet the ventilation of the working places will be poor, unless the air is properly conducted and sweeps the entire face and roof.

INSTRUMENTS.

INSTRUMENTS FOR MEASURING THE RESISTANCE OF AIRWAYS.

1057. The instruments for measuring the resistance of air are:

(*a*) *Pressure.*—Water-gauge and manometer.

(*b*) *Velocity.*—Anemometer.

The student has learned that the intake pressure of an airway is always greater than the return pressure. The difference of pressure is measured usually by means of a water-column in one arm of a bent glass tube. This instrument is called a **water-gauge.**

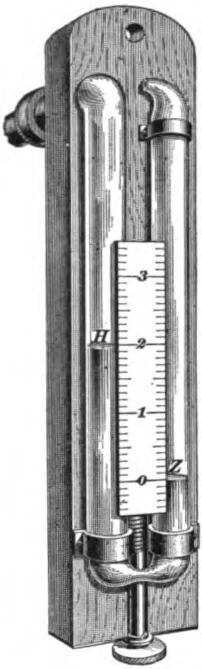


FIG. 171.

1058. The Water-Gauge.—Fig. 171 shows the usual form of water-gauge in use in the mines. The scale is divided into inches and decimals of an inch and is movable, in order that its zero can be adjusted to the lower water-level by means of the thumb-screw below. The tube is bent into the form of a letter U, both arms being open at the top to the free admission of air. The left-hand arm at the top, however, is cemented into a brass tube which makes a square bend, passing through the wooden base to which it is secured, so that the open tube can be connected with the opposite air-current to measure the difference in pressure.

Fig. 172 shows the water-gauge in position upon a mine door or brattice, in a cross-cut between the intake and return airways. *C* is the intake and *D* the return of the mine. One of the open ends of the gauge-tube *A* is thus subject to the intake pressure, while the other end is acted upon by the return pressure.

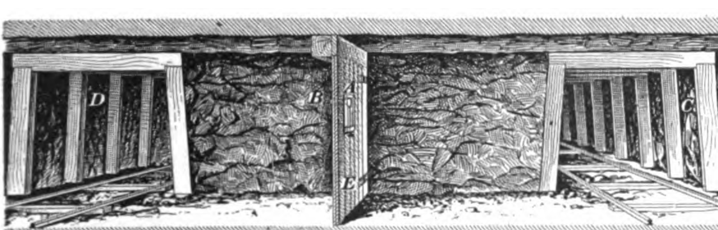


FIG. 172

These pressures being unequal, their difference will be equal to the weight of water which is unbalanced in the gauge.

When the pressure of the air upon each end of the gauge is equal, the water will stand at the same height in each arm; if, now, the pressure be increased upon one end, the level of the water in that arm will sink, while it rises in the other an equal amount. Suppose that the intake pressure over that of the return is sufficient to cause the water-level to sink 1 inch in the arm open to the intake. The level of the water in the other arm will rise 1 inch. Now, by moving the scale until its zero corresponds exactly with the lower water-level, the reading of the upper level will be 2 inches. This will represent 2 inches of water-column, balanced only by the ventilating pressure of the mine.

1059. To calculate the pressure per square foot of area which supports this water-column: The weight of 1 cubic foot of water (62.5 pounds) corresponds to a pressure of 62.5 pounds per square foot for 12 inches of water-column; and 1 inch of water-column or water-gauge will, therefore, be equivalent to $\frac{62.5}{12} = 5.2$ pounds pressure per square foot. Hence, to calculate the pressure p in pounds per square foot of sectional area, when the water-gauge W is given in inches, the formula $p = 5.2\,W$ is used.

That is, *the unit of ventilating pressure or the pressure upon each square foot of the sectional area of an airway, in pounds, is 5.2 times the reading of the water-gauge in inches.*

1060. The reading of the water-gauge must always be taken between the intake and the return current, and as near the mouth of the return current as possible, in order that it shall express the full resistance of the mine.

1061. The Anemometer.—This is a wind-gauge for measuring the velocity of a current in an airway by timing the revolutions of the vanes. Fig. 173 shows the most reliable form of anemometer. A is a wind-wheel whose revolutions are indicated by the registering dials at B. These dial-hands are so geared to one another and to the spindle, or axle, of the vane, that the divisions upon each dial represent 10 revolutions of the next preceding dial-hand, and each division of the large circle corresponds to 1 revolution of the vane. There are 100 divisions in the large circle; and, therefore, 1 revolution of the large hand denotes 100 revolutions of the vane. One revolution of the dial-hand *c* denotes

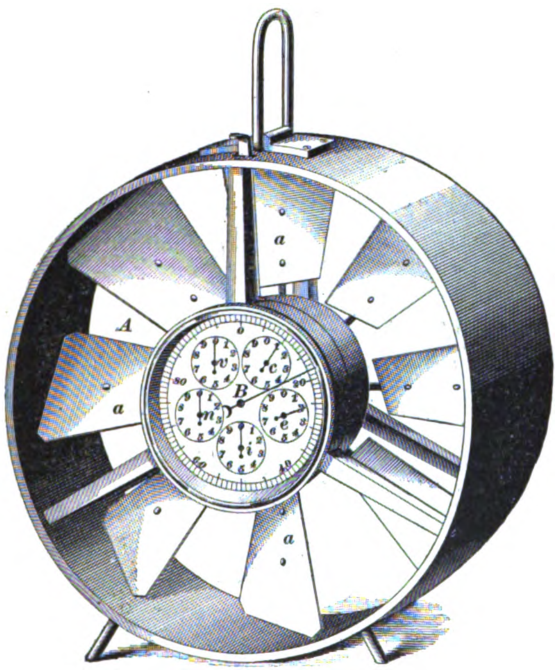


FIG. 173.

10 revolutions of the large hand *B*, or 1,000 revolutions of the vane. Thus, the dials in Fig. 173 register 2,118 revolutions of the vane.

1062. The vanes are inclined at such an angle that 1 revolution of the vane corresponds to 1 foot of travel of the air in the airway. There is a disconnecting device shown near the handle by which the registering dials can be instantly thrown out of gear, which makes it possible for the operator to take more accurate readings One important point to be borne in mind, in taking careful measurements of the current passing in an airway, is that the velocity is not the same in all parts of the passage. The friction of the current upon the sides and top and bottom of the airway retards the air nearest to these surfaces. As a consequence, the air rolls, as it were, in whirling circles upon these surfaces. The velocity of the current is, therefore, greatest at the center of the passageway and least in the corners.

1063. In Fig. 174 is illustrated a common method of obtaining a fair average reading for the entire area of the entry. The passageway is divided into 9 equal squares by the imaginary vertical and horizontal lines *a a*, *a a*, etc. The anemometer is held in each of the outer squares for the same length of time, say 15 seconds, moving it from one to another in regular succession, and is then held in the center square for a period as long as that of the other 8 squares combined, thereby occupying $\frac{8 \times 15}{60} \times 120 = 240$ seconds, or 4 minutes. Suppose, when this has been carefully done, the reading of the anemometer is as shown in Fig. 173 (2,118); then the average velocity for the entire area of the airway would be $\frac{2,118}{4} = 529\frac{1}{2}$ feet per minute. The quantity of air q passing in the airway per minute is calculated by the formula

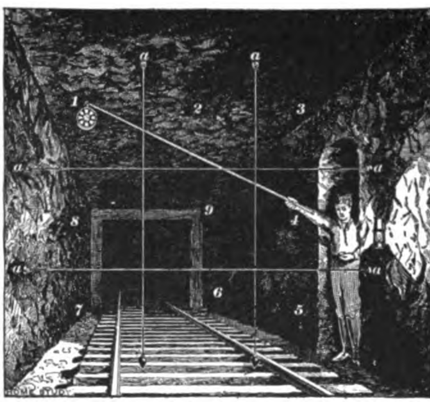


FIG. 174.

$$q = a\,v.$$

When using the anemometer, the operator should endeavor, as far as possible, not to obstruct the passageway or contract its area by his body. He should stand to one side of the center of the passage and make allowance for his body, especially when the sectional area of the airway is small. The anemometer should be held at right angles to the direction of the current. Consideration must also be given to the fact that an air-current, like a water-current, moves in channels. The velocity of the air will always be found very much greater along the rib of an airway which corresponds to the outer circle of a bend. The instrument held by this rib will often show a good velocity, while close to the other rib there is scarcely sufficient motion to obtain a reading. In fact, velocity measurements should be taken, if possible, where the passage is straight and smooth.

INSTRUMENTS FOR MEASURING DENSITY OF AIR.

1064. The instruments for determining the density of air are :

(*a*) The *barometer*, for measuring atmospheric pressure.

(*b*) The *thermometer*, for finding the temperature of the air.

The density of air depends mainly upon two factors, barometric pressure and temperature. When these are known, the weight of 1 cubic foot of air is easily determined by formula **57.**

1065. The Barometer.—This instrument has been previously described in another paper, and will only be referred to here. Its use is important in all mining operations, where mines are opened on an extensive scale. It is often found in the offices of large mining companies, connected with a continuous self-registering apparatus that records the barometric height for every hour.

1066. The Thermometer.—The temperature of the air is determined by the thermometer, which is a glass tube having a small bulb blown upon the lower end. The bulb and a portion of the tube are filled with mercury, the upper end being closed, after boiling the mercury, to expel the air. The tube is attached firmly to a base, as shown in Fig. 175, which is then graduated so as to mark the degrees of temperature by the expansion of the mercury, which rises and falls in the stem.

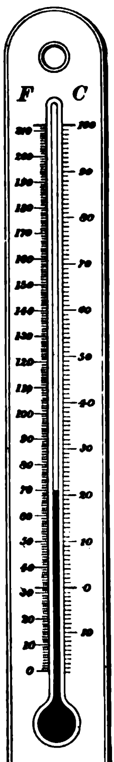


FIG. 175.

There are two thermometer scales in common use, the Fahrenheit scale, marked F in the figure, and the Centigrade scale, marked C. They differ principally in the location of the zero mark.

The **Centigrade scale** is a decimal scale. Its zero is marked by the freezing-point of water, while the boiling-point is marked 100°.

The **Fahrenheit scale** is the scale most used in America and in England. The freezing-point of water is marked 32° above zero, and the boiling-point 212° above zero.

1067. By comparing these two scales, it can be seen that 100° on the Centigrade scale correspond to $212 - 32 = 180°$ on the Fahrenheit, or 5 degrees C. = 9 degrees F. Hence, to convert any Centigrade reading into the corresponding Fahrenheit reading, use the following formula:

$$F = \tfrac{9}{5} C + 32, \qquad \textbf{(76.)}$$

in which F is the Fahrenheit reading and C the Centigrade reading. That is, $\tfrac{9}{5}$ *of any Centigrade reading, plus 32, is equal to the corresponding Fahrenheit reading, attention being paid to plus and minus readings, when above or below zero, respectively.*

To convert any Fahrenheit reading into the corresponding Centigrade reading, use the following formula:

$$C = \tfrac{5}{9}(F - 32), \qquad \textbf{(77.)}$$

in which the letters have the same meaning as in formula **76.** That is, *from the given Fahrenheit reading subtract 32, and take $\tfrac{5}{9}$ of the remainder; the result will be the corresponding Centigrade reading.*

NOTE.—In each of the two preceding rules, all readings, of either scale, above zero are plus, and all below zero are minus. A few examples will make the method clear.

EXAMPLE.—Convert 50° C. into the corresponding Fahrenheit reading.

SOLUTION.—Using formula **76**,

$$F = \frac{9}{5} \times 50 + 32 = 122° \text{ F. Ans.}$$

EXAMPLE.—Convert $-10°$ C. into the corresponding Fahrenheit reading.

SOLUTION.—Using formula **76**,

$$F = (\tfrac{9}{5} \times -10) + 32 = -18 + 32 = 14° \text{ F. Ans.}$$

EXAMPLE.—Convert $-30°$ C. into the corresponding Fahrenheit reading.

SOLUTION.—Using formula **76**,

$$F = (\tfrac{9}{5} \times -30) + 32 = -54 + 32 = -22° \text{ F. Ans.}$$

EXAMPLE.—Convert $-4°$ F. into the corresponding Centigrade reading.

SOLUTION.—Using formula **77**,

$$C = \tfrac{5}{9}(-4 - 32) = \tfrac{5}{9} \times -36 = -20° \text{ C. Ans.}$$

The student will notice that in these formulas, 32 is added and subtracted algebraically; that is, when the signs are like, the quantities are added together, their sum having the same sign; but when the signs are unlike, the lesser quantity is subtracted from the greater, and the remainder takes the sign of the greater. Thus, in the solution of example 2, where $+32$ is added to -18, subtract 18 from 32, and the remainder, 14, takes the plus sign. In example 3, subtract 32 from 54, and the remainder, 22, takes the minus sign.

EXAMPLES FOR PRACTICE.

1. What temperature Fahrenheit corresponds to 100° C.?
   Ans. 212° F.

2. Convert 290° Centigrade into the corresponding Fahrenheit reading. Ans. 554° F.

3. What reading upon the Centigrade scale corresponds to 5° Fahrenheit? Ans. $-15°$ C.

4. Convert $-40°$ Fahrenheit into the corresponding Centigrade reading. Ans. $-40°$ C.

AIR COLUMNS.

1068. In speaking of the *natural* means at hand for producing ventilation, *heated air columns* have been treated. It is important to notice that since air has weight, *all* columns of air exert a downward pressure upon the area of their base equal to the weight of the column. Also, the downward pressure of air columns creates an *equal* pressure in every direction upon the air of each level of the airway. That is, the pressure throughout each level section is the same; but if the elevations of the sections are different, the pressure in each section will be different. The pressure per square foot of sectional area in the airway due to any air column is equal to the weight of a column of air of the same height and having a uniform section of 1 square foot from bottom to top.

1069. Air columns may be vertical, as in the case of the furnace shaft *B A*, Fig. 176, or inclined, as in the case of the slope air column *C E*. In either case, the pressure at the base upon 1 square foot of sectional area, due to the weight of the air column, is calculated from its *vertical height*. In the figure, the depth of the shaft *D* is equal to the vertical height of the slope, and for the same temperature the pressures per square foot of sectional area, due to these two air columns, are equal.

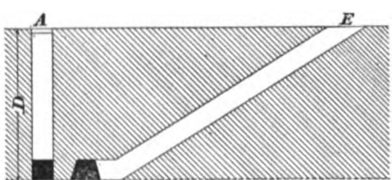


FIG. 176.

1070. As explained previously, the difference in the weight of two air columns connected by an airway at their bases causes the air to flow through the airway, from the heavier towards the lighter column. It can readily be seen that the weight of one of these columns is **positive,** while that of the other is **negative,** so far as concerns the motion of the air-current. The weight of the *positive* column always acts in the direction in which the current moves in proportion to its excess of weight over the negative column. Thus, the excess of weight of the positive air column causes the flow of the current.

1071. It is a matter of common observation in mines that **rise workings** are more difficult to ventilate than **dip workings,** when the bottoms of the shafts are on the dip side of the workings. The reason for this is found in the fact that the intake air, or that flowing to the rise, is always cooler and denser than the return air, which has become heated by the higher temperature of the workings, and which must flow down grade to the upcast. The effect of this will be to *retard* the ventilation of the workings.

On the other hand, if the intake runs to the *dip*, as in dip workings, and the continuous flow of the air is upward to the upcast, there will result a *heavy* positive column and a light *negative* column; the combined effect of these will *assist* the ventilation.

The influence of dips and rises in mine workings is thus seen to be a powerful one, in fact often completely controlling the ventilation of the workings. For this reason, seams having any considerable inclination should be so ventilated that as far as practicable the course of the current will be towards the rise. This is known as **ascensional ventilation,** and is an important consideration in the ventilation of all mines.

1072. As previously explained, the weight of the **motive column** is the excess of weight producing a flow of air; hence, it is the algebraic sum of the weights of all the air columns, positive and negative. It is often convenient to reduce the various factors in any ventilation to a single *motive column*, which at once expresses the height of air column whose weight produces the ventilating pressure.

1073. An essential point, in regard to all air columns, is the *density* of the air which forms it. Whatever affects the density of the air affects the weight of the column. Temperature, pressure, moisture, presence of gases, all affect the weight of the air column, to a greater or less degree. In nice calculations, it would be right to consider all these factors for each column respectively. In ordinary calculations, however, it is customary to consider the temperature of the column and the barometric pressure only, ignoring the mine pressure, the amount of moisture in the air, and the presence of gases. The latter, excepting moisture, are often very important factors. Mine pressure may affect the motive column to the amount of 2½ per cent., always increasing the density of the intake air. The presence of carbonic acid gas in the upcast current, or in the return current of any ascensional ventilation, acts to reduce the motive column, and may amount to as much as 20 per cent.

THE BEST METHODS OF VENTILATING GASEOUS AND NON-GASEOUS MINES.

1074. Ventilation of Flat, Non-Gaseous Seams.—Fig. 177 shows a plan of underground workings in a non-gaseous mine, worked upon the pillar and chamber method. The seam lies flat, or nearly so. The main feature of the ventilation here shown is the manner of splitting the air at each pair of cross-entries, so as to give to each pair a separate current. Or, one split of the air may be made to ventilate two pairs of entries near the face of the workings. On account of the expense, overcasts are never put in at any pair of cross-entries until the development warrants the outlay. It is evident, from observing the plan, that each overcast saves either a door or a stopping, and always leaves the main road free of doors. It must be remembered, however, that the practical limit to splitting an air-current is the velocity of the divided current, which must not fall below 3 or 4 feet per second in non-gaseous mines, and 5 or 6 feet per second where gas is given off. For example, if the sectional areas of the airways are each, say, 50 square feet, and there is only 15,000 cubic feet of air passing upon the main airway, this current can not be split, as its velocity now is but 5 feet per second, and another split would reduce it below the limit.

1075. Another important feature, in the practical ventilation of a mine, is the location of the stables. They must be situated close to the bottom of the shaft, so that the mules can be easily rescued in case of accident, and where the daily feed and refuse can be readily handled. It is also essential that they be ventilated by a separate split of fresh air, as shown in Fig. 177, and that the return

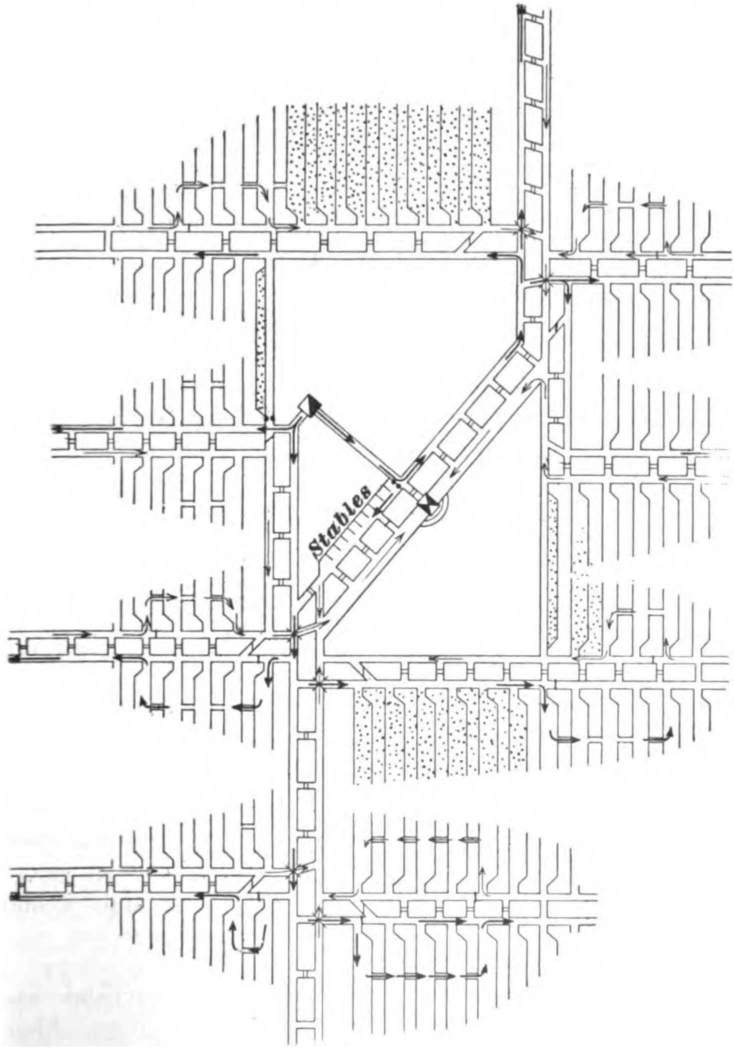


FIG. 177.

current from the stables should pass through a regulator, to prevent excessive drafts, and thence directly out to the upcast shaft, and not contaminate the air of the mine by mixing with it. In Fig. 177 the feed and refuse are handled through the door shown in the first cross-cut, next to the shaft. The mules enter the stable at the other end, which requires no door.

1076. Another important feature to be considered, in the ventilation of all mines, is the arrangement of the haulage roads with respect to the air-currents. It will be observed in Fig. 177 that the main haulage roads are made the *return* airways of the mine. This is the better plan in all non-gaseous mines, for two principal reasons: (*a*) freedom from dust upon the *intake;* (*b*) freedom from ice in the hoisting shaft in winter.

1077. Ventilation of Gaseous Seams.—In gaseous mines, the haulage is always of necessity done in the intake airways, to lessen the liability of explosion. If the mine represented in Fig. 177 were gaseous, it would be necessary to reverse the current, and cause it to circulate in a direction opposite to that shown, making the *hoisting* shaft the *downcast.* This is usually done by means of an exhaust-fan placed at the mouth of the *upcast* shaft. The doors throughout the mine would all require to swing in the opposite direction, but in other respects the arrangements in the two cases are identical, except only in respect to the velocity of the current, as already explained.

In very fiery mines, the main roads, or gangways, are often driven triple instead of double. This method is called the "Triple Entry System," a section of which is shown in Fig. 178.

The middle entry is always made the intake and haulage road, the two outer entries being the return for each side of the mine, respectively. The *exhaustive* system of ventilation must be used in this case as in every other case where the *haulage road* is made the *intake;* otherwise, a door would have to be placed upon the haulage road, and there should then be two doors to prevent the stoppage of the current while the trip is passing. Such doors have been introduced into the hoisting shafts in certain cases. They were made to work automatically and reciprocally, the one opening after the other had closed, each time the cage passed. This arrangement is complicated, and should never be used where it can be avoided.

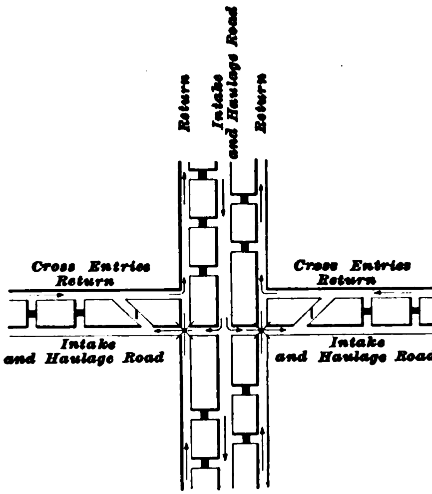


FIG. 178.

The chief advantage of the triple-entry system is that it furnishes a separate return for each side of the mine, and in case of an explosion, there is more complete isolation of the affected district.

1078. Ventilation of Inclined Seams.—The main point to be considered, in the ventilation of all inclined seams, is that it shall be *ascensional.* In other words, the *intake* air being, as a rule, cooler than that of the return, should be conducted at once to the lowest portion of the workings, whence, as it gradually absorbs heat from the mines, it tends to rise. The natural heat of the mine, as has been previously explained, always creates a small *motive column*, which *assists* the ventilation when the cooler intake runs to the dip, and its return, to the rise. In general, the cool outside air *falls naturally* to the lowest place in the mine, and as it becomes heated in its passage through the workings, it *rises naturally* to the higher portions. As far as it is practicable to do so, this principle of ascensional ventilation must be applied in conducting an air-current through workings in an inclined seam.

1079. Fig. 179 shows the ventilation of the workings of an inclined seam opened by a slope. The *intake* is a shallow shaft near the mouth of the slope, while the return is the main haulage road leading to the mouth of the slope. The first four pairs of entries, it will be observed, are ventilated by a separate split. Each of these splits is conducted through the lower entry of the pair, directly to the face of the entries, and after passing through the inside cross-cut, it enters the last room, being deflected, if necessary, by a canvas hung on the entry. The current traverses the working face of each room by passing through the cross-cuts in

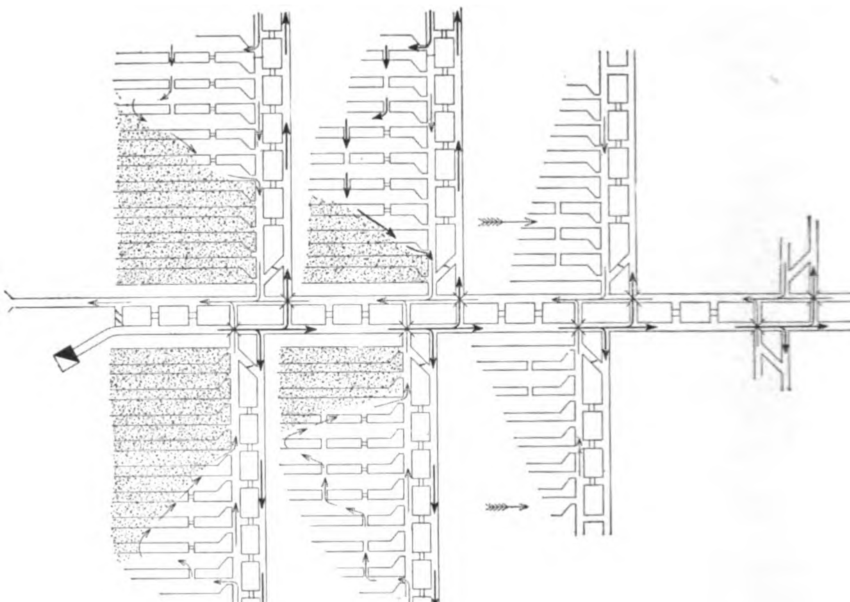


FIG. 179.

the room-pillars, and along the edge of the gob where the pillars are being drawn back.

If the seam represented in Fig. 179 were a gaseous seam, the direction of the current should be reversed, the main haulage roads being made the intake airways for each respective section of the mine. This is best done by making the fan at the air-shaft exhaust.

The types of ventilation, or the manner of conducting the air through a mine, illustrated in these general plans, cover all cases of practical ventilation. The successful application of the principles involved in any one particular case will, however, often depend, owing to varying conditions, upon the skill and ingenuity of the man in charge.

1080. Manner of Handling a Body of Gas.—The removal of a body of gas that has accumulated in an idle room or unused chamber requires great precaution if the gas is fiery. This should never be attempted until all the men who are in the neighborhood of the outgoing air-current have been withdrawn. Brattices of canvas should then be so hung as to direct into the workings where the gas is lodged the main portion of the air traveling along the airway. Or, if the gas has accumulated in some cavity of the roof, the current must be made to sweep this cavity by a brattice erected in the entry beneath it. Great care is necessary not to ignite the gas. Only the most reliable safety-lamps should be employed, and they should be placed in fresh air, at a safe distance away, and carefully watched.

The practice, already alluded to, of passing a gaseous current over a furnace by first diluting it with a sufficient quantity of fresh air direct from the downcast, is a dangerous one. Indeed, best mining practice to-day does not tolerate a furnace in a mine that yields gas.

1081. Entrance of a Mine After an Explosion.—This part of the subject will be considered only as it depends upon the restoration of the ventilating current in the airways and workings. The call for volunteers and their organization into two or three rescuing parties, each under its own efficient leader, is followed immediately by the adoption of such measures as a hasty examination shows to be necessary to restore ventilation.

If an exhaust-fan was in use at the mine opening previous to the explosion, it will generally be found to be less injured by the force of the explosion than would be the case with a blower-fan, depending, of course, upon the location of the initial explosions with respect to the upcast and downcast shafts. The force of an explosion is usually exerted in the direction of the intake opening. Any injury to the fan that incapacitates it for use must be speedily remedied. The original course of the ventilating current should not be altered, except upon the most urgent demands.

As quickly as the intake current begins to enter the mine, the men should follow, equipped with good lamps, picks, shovels, saws, axes, sledges, brattice-cloth, and boards. They proceed at once to follow the air, rebuilding doors and stoppings or erecting a temporary line of brattice for conducting the air around a fall. The main point to be borne in mind is that no effectual advance can be made ahead of the air.

1082. Mine Fires.—This term applies to any form of slow or rapid combustion taking place in the passages or workings of a mine. Mine fires are a dangerous element in *any* mine. In gaseous mines in particular is this the case, the presence of a fire being an imminent source of peril.

The chief **causes** leading to mine fires are: (*a*) *spontaneous combustion*, as it is called, or combustion from natural causes, arising from the storing of slack and fine coal in the gob; (*b*) *ignition of the coal* by a gas-feeder fired by the flame of a blast; (*c*) *ignition of a door-frame, brattice, or timbers* by a naked lamp. Whatever the cause, a fire in the workings or passages of a mine should receive prompt attention. Its presence is manifested not more by the heat developed than by the peculiar odor imparted to the air.

1083. Treatment of Mine Fires.—The treatment of mine fires will be considered with particular reference to the ventilating current. The manner of treatment is divided into four classes, according to the stage of development the fire has reached, viz.:

(*a*) Direct method, with hose and water or portable chemical fire-extinguishers.

(*b*) Loading out in mine cars.

(*c*) Isolating from the air by special stoppings, so built as to effectually prevent air leakage.

(*d*) Flooding.

1084. In its incipient stages, a mine fire can be extinguished by the direct method, and a gob fire especially can readily be loaded out in mine cars; but it frequently happens that the fire assumes larger proportions before it is detected.

The first method of treatment is a simple one, and needs no explanation except to state that efficient chemical fire-extinguishers are on the market, and that the simple methods of operating them are fully explained by the manufacturers.

1085. When it becomes necessary to build stoppings for the isolation of a mine fire, much care is needed, both in the location of the stoppings and in the order in which they are erected. The places chosen as the locations of stoppings should be, as far as practicable, in the narrowest openings available in the solid coal. The affected area should be completely shut off and sealed to the access of air. The stoppings must be well built, and of the quickest available material. Care must always be taken to begin sealing off a fire at its side next the return air and work towards the intake, sealing the intake opening last. The reason for this is that, by closing the return-air side first, the gases set free by the fire drive back the fresh air after that stopping is made, and when the intake is properly closed there is no chance for imprisoned pure air to initiate an explosion. It also affords a better opportunity for the dilution of the gases with the air of the ventilating current. Care, however, must be taken to avoid breathing the carbonic oxide, or white damp, incident to mine fires, as it is the most poisonous gas known.

In a gaseous mine it would be unsafe to proceed in any other way than that above described. Violent explosions have been known to result from neglect of these precautions. From the moment of the sealing of the inlet end, if this end be sealed first, the gases generated by the fire, and otherwise, increase in volume and move slowly towards the outlet, where they have free access to the airway in a dangerous form.

When these stoppings have remained closed for a sufficient period, and it is necessary, in order to work the coal, that they be opened, it must be done with the utmost caution, and in the reverse order to that in which they were sealed. The stopping at the intake end is thus the first one to be taken down, as it was the last one put up. Careful search must at once be made to discover any smouldering remains of the fire, and for days after the place must be closely watched.

1086. Flooding a mine, in order to extinguish a fire, is only considered as a last resort. At times, certain portions of the mine which alone are affected are shut off and flooded. It then becomes necessary to construct dams sufficiently strong to withstand the pressure due to the water. The construction of these dams is treated particularly in Methods of Working.

MINE SURVEYING AND MAPPING.

(PART 1.)

MINE SURVEYING.

INTRODUCTION.

1087. Mine surveying is the art of making such measurements as will determine the relative positions of two or more points, either on the surface or in the mine, or both on the surface and in the mine. It consists in measuring, with a view of drawing, on paper, plans and sections of the underground workings of a mine, the mine buildings, and other improvements, the natural features on the surface, and the mine concession or lease.

1088. Trustworthy plans and sections are of great value in giving a view of all the important features of the property, in affording data to assist in the further development of the mine, and in avoiding expensive litigation due to trespass on adjoining properties. Accidents and loss caused by driving in a wrong direction, or into old workings where gas or water is met in great quantities, are also avoided by good surveys and reliable plans.

1089. Mine surveying, until recent years, did not keep pace with the advancement in other branches of surveying, and it is to be regretted that mine surveys in some regions are still made with instruments which have long been set aside as too inaccurate for surface surveys.

1090. The great value of our mineral deposits and the enormous extent to which they are being worked, together with the desire of avoiding trouble as above mentioned, have necessitated a high degree of perfection in mine surveying. Indeed, the law, in most cases, requires that the operator or superintendent of a mine shall make, or cause to be made, by a competent person, an accurate map or plan of the mine or mines of which he has charge. This has also tended to improve and advance the art of mine surveying.

MEASURING DISTANCES.

1091. The most suitable and accurate instrument for measuring distances in mine surveys is the **steel tape.** It is made of a ribbon of tempered steel, varying in length from 50 to 500 feet, in width from .08 to .5 inch, and in thickness from .015 to .04 inch. At each fifth foot a small piece of brass wire is soldered across the tape, the white solder extending about an inch on each side of the wire. In the latter is filed a small notch which marks the exact spot where the fifth foot ends. The distances from the zero-point of the tape are marked upon the solder by counter-sunk figures. The tape is wound upon a simple wooden reel, about one foot in diameter, which can be held in one hand and turned by the other. Two D-shaped brass handles, which can be detached, accompany the tape, and are carried upon the reel.

1092. The work of measuring in a mine is greatly facilitated by having a steel tape sufficiently long to reach from the neck of the chamber, or room, to the face—a distance usually of about 300 feet. The first five feet should be graduated into feet and tenths of a foot. Such a tape will enable one to measure any distance correctly to tenths of a foot, and quite approximately to hundredths, because a point lying between two tenth-divisions can be very closely estimated.

The method of measuring the distance between two points with a tape graduated in this manner is as follows: Measure off as many 300-foot lengths as the distance will permit; and, to measure the remaining distance, hold that 5-foot mark which records a distance greater than the remaining distance, but most nearly to it, at the point marking the end of the last 300-foot length, and then read the distance on the graduated 5-foot length. Then this distance, plus the number of 5-foot lengths minus 1, plus the number of 300-foot lengths, will be the total distance required.

1093. **Chains,** which are still somewhat used, are made of steel wire links, and are either 50 or 100 feet in length. Each tenth foot is marked by a brass tag. That at the end of the first ten feet has one prong, that at twenty feet has two prongs, etc. Intermediate feet are determined by counting the links from the preceding tag, each link being one foot in length. Fractional parts of a foot are measured by a small instrument graduated to tenths and hundredths of a foot. Thus, in measuring a distance of 40.75 feet, the 75 hundredths of a foot are determined by measuring from the 40-foot mark on the chain to the point by means of the small graduated stick or piece of steel. To make the calculations more simple, fractional parts of a foot are not measured as inches, but as tenths and hundredths of a foot. Thus, 37 feet 6 inches, in surveying work, is always recorded as 37.5 feet. Similarly, 53 feet 9 inches is 53.75 feet.

1094. The term **chainmen** is applied to the men whose special duty is to measure distances between survey stations. To measure distances, two men are required, a **fore-chainman** and a **hind-chainman.** The hind-chainman has the more responsible position. They should be provided with eleven pins, pointed on one end and bent into a ring at the other, and made of tempered steel wire $\frac{1}{8}$ inch in diameter. The pins are usually from 12 to 15 inches long, and are used to mark the end of the chain or tape in measuring a line.

1095. Supposing the chain to be tied up, the fore-chainman throws it out in the direction opposite to that in which the chaining is to be done, gives the hind-chainman a pin, takes nine in his left hand, and the end of the chain and one pin in his right hand, and draws the chain in the direction of the line. The hind-chainman examines the chain as it passes, to see that there are no kinks or bent links, or, if a tape be used, to see that there are no loops in it.

When the fore-chainman has gone the proper distance, he stops, rests his right elbow on his right knee, and extends his right hand (in which he holds the handle of the pin) as far as possible from his body, so that the hind-chainman may have an unobstructed view of the pin and the farther end of the line. The fore-chainman must keep the chain straight and taut, and obey the signals of the hind-chainman.

The hind-chainman places his end of the chain at the point of beginning, and, by placing himself behind the point, with a motion of his arm directs the fore-chainman where to place his pin. For example, if the pin ought to be moved a considerable distance to the right, the right arm is held far out from that side of his body; if it should be moved only a little, the arm is held nearly vertical. The signal thus indicates both the direction and the amount of motion required. As the pin approaches the proper position, the arm comes more nearly vertical. When the pin is at the proper place, the hind-chainman calls out "*stick!*" and the fore-chainman then brings his left hand to bear upon the top of the pin, and forces it *vertically* into the ground. After the pin is set, he should test it to see that the pin, at the surface of the ground, is just in contact with the front face of the handle. When the position of the pin is satisfactory to the fore-chainman, he calls out "*stuck!*" and at this signal the hind-chainman loosens his end of the chain, and both move forward the length of the chain.

The fore-chainman should keep his eye steadily on the farther end of the line, so that he may keep near the line. When the hind-chainman reaches the pin already set, he calls "*halt!*" and the fore-chainman prepares to set a pin. After the fore-chainman has placed his pin in line, the hind-chainman holds his end of the chain against the pin, at the same time placing his hand on the top of the pin to hold it firm, and calls out "*stick!*" and at the reply "stuck," he removes his pin and the work proceeds as before.

When the fore-chainman has set his last pin, he calls "*tally!*" and the hind-chainman comes up, and gives the fore-chainman the ten pins which he has, both men counting them to be sure that none have been lost. The hind-chainman then makes note of the tally, and the work proceeds in the usual manner.

When the fore-chainman reaches the end of the line, he stops, holds his end of the chain against it, calls "*stuck!*" and the hind-chainman then comes forward and counts the distance beyond the last pin. Each tally represents ten chains; each pin held by the hind-chainman (not including the one in the ground) represents a chain, and the feet just counted make up the total distance. Notice that the pin last set is not counted. It should always remain in the ground until the distance is recorded.

1096. Since, in steel tapes, the handle extends beyond the end graduation, the fore-chainman should grasp the handle of the tape in his *left* hand, rest his left elbow on his left knee, and hold the pin in his *right* hand, instead of as previously directed in using the chain.

1097. Owing to the frequent dips, rises, and turns in the entries of a mine, the stations, which will be explained later, are usually so close together that a tape 300 feet long will reach from one to the other, thereby obviating the need of pins. Where the tape is too short to reach from station to station, chalk-marks are often used instead of pins to mark the successive lengths of the tape.

1098. In chaining, it is just as important to hold the chain or tape horizontally as it is to keep it in line between the two points whose distance apart is being measured. In chaining up hill (see Fig. 180), the fore-chainman draws the chain out to its full length, as in any case, and then returns to within such a distance from the hind-chainman that when the chain is drawn out to that length horizontally, it shall not be too high to be held conveniently. The hind-chainman holds his end of the chain carefully over the point, or station, by means of the plumb-line, while he directs the fore-chainman in the usual manner.

The point fixed in this manner by the fore-chainman must not be marked by a regular pin, but by a small peg, or nail. At the order "stick," the fore-chainman does not go forward immediately, but waits until the hind-chainman comes up and takes the chain at the precise point held to the ground. This point is now held above the peg by the hind-

FIG. 180.

chainman, who uses the plumb as before, and alines the fore-chainman, who has taken hold of the chain a few links farther on and is holding it to the ground. These short distances are not recorded, the end of a full chain being marked by a regular pin. In measuring down a hill, the reverse of the foregoing operation is used. In fact, if convenient, all measurements should be made with the fore-chainman down the hill, as he can easily hold his end of the chain in the air and drop the plumb-bob, while the hind-chainman holds his end firmly on the ground. Measuring down a hill is not only easier, but is more accurate than measuring up.

1099. Chainmen should be *intelligent* men who fully realize the importance of accuracy in measuring distances. The correctness of the map to be made depends as much on accurate measurements as on the correct reading of the angles. They should also understand that, in general, it is not the *actual* distance between two points, either in the mine or on the surface, that is required, but that it is the *horizontal projection* of this actual distance which is required to make the mine map; for instance, it is not the actual distance *A B*, Fig. 180, that is required, but the horizontal projection of this distance, which is *A C*, that must be known to make the map. They should remember, while chaining, *that the shortest distance between two points is measured on a straight line.*

1100. In measuring up or down a *regular* grade or slope in a mine, it is best to measure the distance on the grade, and multiply this by the cosine of the angle of dip or rise, to find the distance to be recorded, or the length of the horizontal projection of the slope. Errors are less liable to occur by this method than by the ordinary method of measuring up or down hill as previously explained.

COMPASS SURVEYING.

THE COMPASS.

1101. The compass may be either a *pocket compass* or a *surveyor's compass*, and may be used by holding it in the hand, or supporting it with a **Jacob's staff** or with a **tripod.** The Jacob's staff is convenient for use in making outside surveys, but, as a rule, is useless in the mine.

Owing to the inaccuracy of the compass, due to yearly and daily variations, and its sensitiveness to local attraction from iron in various forms, it is not used to make the survey from which the general map of the mine is constructed. Its use is desirable, however, as with it chambers can be turned off and driven in any desired direction, and, in general, any work may be done with it where absolute accuracy is not required.

1102. The **pocket compass** consists merely of a magnetic needle swinging freely on a pivot in the center, both being enclosed in a metallic or wooden box with a glass face; below the glass is a circle graduated into 360 degrees. These compasses are made with and without folding sights. The folding sights are accurately placed at the north and south sides of the compass-box, that on the north end having a slot in which is fixed a vertical wire, and that on the south end having merely a fine slot with several small circular holes along it, to which the eye is applied in sighting.

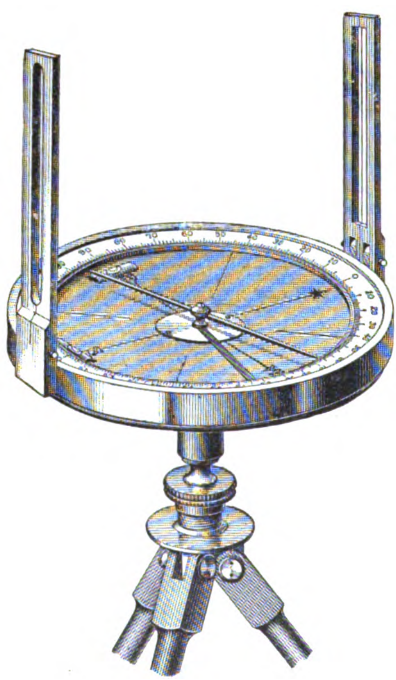


Fig. 181.

1103. The **surveyor's compass** is similar to the pocket compass with folding sights, but larger (see Fig. 181). It is arranged for use on a tripod or a Jacob's staff by having a spindle with a ball end, which fits accurately into a socket that can be screwed either on the tripod or on the Jacob's staff. It is also supplied with two level tubes set at right angles to each other. The ball and socket joint enables the compass-box to be moved in any direction, so that it can readily be leveled. It is also arranged to swing horizontally on its axis, and it can be clamped in any position.

TO ADJUST THE COMPASS.

1104. The Levels.—First bring the bubbles to the center of the bubble tubes by pressing with the hand on different parts of the plate, and then turn the compass-box half way around; i. e., revolve it through 180°; should the bubbles run to the ends of the tubes, it would indicate that those ends to which the bubbles run are highest; lower them by tightening the screws immediately under those ends and by loosening those under the other ends until, by estimation, the error is half removed; level the plate again, and repeat the operation until the bubbles remain in the center during an entire revolution of the compass.

1105. The Needle.—This is adjusted in the following manner: Having the eye nearly in the same horizontal plane with the graduated rim of the compass circle, bring one end of the needle in line with any prominent division of the circle, as the zero or ninety-degree mark, using a small splinter of wood or a slender iron wire, and notice if the other end corresponds with the degree on the opposite side; if it does, the needle is said to *cut* opposite degrees. If the needle does not cut opposite degrees, apply a small wrench about one-eighth of an inch below the point of the center pin, and bend the pin until the ends of the needle are brought into line with the opposite degrees. Then, holding the needle in the same position, turn the compass half way around, and note whether it now cuts opposite degrees; if not, correct half the error by bending the needle, and the remainder by bending the center pin.

The operation must be repeated until perfect reversion is secured in the first position. This being obtained, it may be tried on another quarter of the circle; if any error is there manifested, the correction must be made in the center pin only, the needle being already straightened by the previous operation. When again made to cut, it should be tried on the other quarters of the circle, and corrections made in the same manner until the error is entirely removed and the needle will reverse in every point of the divided circle.

TO USE THE COMPASS.

1106. In using the compass, the surveyor should keep the *south* end towards his person and read the bearings from the *north* end of the needle. In the surveyor's compass he will observe that the positions of the E and W letters on the face of the compass are reversed from their natural positions, in order that the direction of the sight may be correctly read.

The compass circle being graduated to half-degrees, a little practice will enable the surveyor to read the bearings to quarters—estimating with his eye the space bisected by the point of the needle.

The compass is usually divided into quadrants, and zero is placed at the north and south ends. Ninety degrees is placed at the east and west marks, and the graduations increase right and left from the zero marks to 90°.

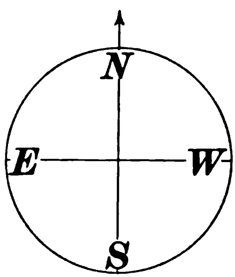


FIG. 182.

1107. In Fig. 182, the positions of the four quadrant points are shown as they are marked upon the circle of a surveyor's compass. It will be observed that the east and west points have changed places. The line of sight of a surveyor's compass is always along the graduated north and south line, the graduated circle being rigidly connected with the sights, while the needle swings free.

Suppose a sight is taken with a surveyor's compass in the direction shown by the position of the north and south line

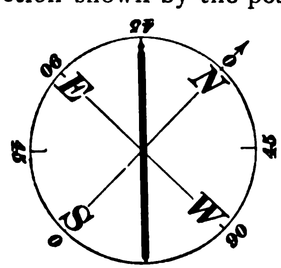


FIG. 183.

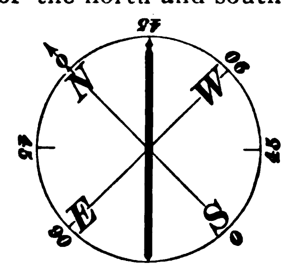


FIG. 184.

in Fig. 183, the observer standing (as he always should) at the south end of the compass circle. The needle, having been swinging free, would maintain its position, pointing towards the north. The compass circle, being fixed rigidly to the sights, would have moved with them, and the four quadrant points would be in the position shown in Fig. 183. The angle through which the compass is supposed to have been turned is 45 degrees, which will be the graduation upon the compass circle to which the needle points. The sight having been taken to the right of north, it is plain (aside from the illustration) that its direction is northeast. From this direction and the knowledge that the deflected angle is 45 degrees, we would expect, by "reading the needle," to obtain the correct direction, N 45° E. This reading can be obtained only by transposing the east and west points from the positions they occupy in nature to those shown in Figs. 182 and 183.

1108. Fig. 184 illustrates the position of the surveyor's compass when a sight is taken 45 degrees to the left of north. We know this reading must be north 45 degrees west (N 45° W). Fig. 184 shows the needle pointing north, with the 45-degree mark of the graduated circle opposite the north end of the needle. The correct reading can be indicated on the compass circle only when the east and west points occupy the transposed positions shown in the figure.

THE VERNIER.

1109. The **vernier** is a device for reading divisions of a scale smaller than those into which the scale is divided. A careful study of Figs. 185 to 190 will give the student a good knowledge of the principles of the vernier.

1110. If the scale A, Fig. 185, is divided into ten equal parts, and the scale A', whose length is equal to nine of the parts of the scale A, is also divided into ten equal parts, it is evident that each division on the scale A' is equal to

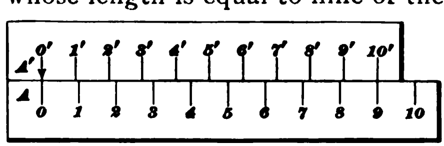


FIG. 185.

nine-tenths of a division on the scale A. For convenience, the scale A will be considered as the true scale, while the scale A' will be called the *vernier scale*. It is evident that five divisions on the vernier scale will be equal to $5 \times \frac{9}{10} = \frac{45}{10} = 4\frac{1}{2}$ divisions on the true scale. If the zero-points, or graduations, of the two scales be set opposite each other, the point *5'* on the vernier scale will be exactly at $4\frac{1}{2}$, or midway between *4* and *5*, on the true scale. It is, therefore, evident that if the zero-point of the vernier scale be set exactly midway between *0* and *1* of the true scale, the point *5'* on the vernier scale will be exactly opposite the point *5* on the true scale, as shown in Fig. 186. We thus know that, in order to place the zero-point of the vernier scale exactly half way between the points *0* and *1* of the true scale, we have only to set the point *5'* of the vernier scale opposite the point *5* of the true scale.

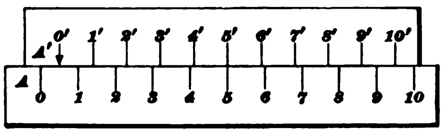


FIG. 186.

Similarly, if we wish to set the zero-point of the vernier scale at one-tenth of a division from the zero-point of the true scale, we have only to set the point *1'* on the vernier scale opposite the point *1* on the true scale. For, if the points *1* and *1'* be set exactly opposite each other, it is plain that the zero-point of the vernier will be at a distance of nine-tenths of a division from these points, or one-tenth of a division from the zero-point of the true scale, as shown in Fig. 187. If we wish to set the zero-point of the vernier scale at seven-tenths of a division from the zero-point of the true scale, we simply set the point *7'* on the vernier scale opposite the point *7* on the true scale, as shown in Fig. 188. For the zero-point of the vernier scale will then be at a distance of $7 \times \frac{9}{10} = 6\frac{3}{10}$ divisions from the point *7*, or $7 - 6\frac{3}{10} = \frac{7}{10}$ of a division from the zero-point of the true scale.

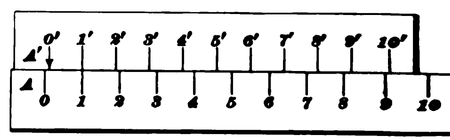


FIG. 187.

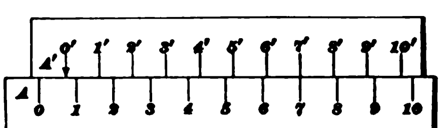


FIG. 188.

But, in order that this principle shall hold, it is not necessary for the zero-point on the vernier scale to be opposite the division adjacent to the zero-point on the true scale;

it may be opposite any division. In such a case, for the fractional part of the division of the true scale at which the zero-point of the vernier scale is situated, that graduation

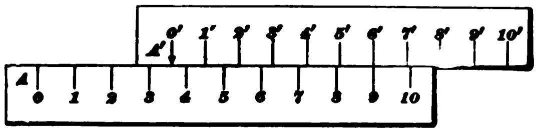


FIG. 189.

of the vernier scale is read which is opposite *any* graduation of the true scale. Thus, in Fig. 189, the zero-point of the vernier scale is between the graduations *3* and *4* of the true scale, while the graduation *6'* of the vernier scale is directly opposite a graduation on the true scale. The zero of the vernier scale is thus known to be at a distance of $3\frac{6}{10}$ divisions from the zero-point of the true scale. Similarly, in Fig. 190, the zero-point of the vernier scale reads $1\frac{4}{10}$ on the true scale. It will be found that for *any* position of the vernier scale, the position of its zero-point, with reference to the graduations of the true scale between

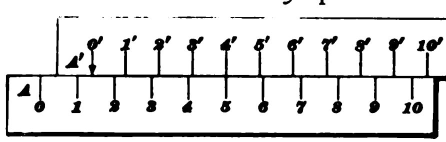


FIG. 190.

which it is situated, will be correctly indicated by that graduation on the vernier scale which is directly opposite a graduation on the true scale. The number of the graduation of the vernier scale which coincides with a graduation of the true scale will be the distance, in tenths of a division of the true scale, of the zero-point of the vernier scale beyond its adjacent and next lower graduation of the true scale.

1111. There are two kinds of verniers, *direct* and *retrograde*. In the **direct vernier** the divisions on the vernier scale are smaller than the divisions on the fixed, or true, scale; while in the **retrograde vernier** the divisions on the vernier scale are larger than those on the fixed scale.

1112. Magnetic Variation.—The direction in which a magnetic needle points is termed a **magnetic meridian,**

and the angle that the magnetic meridian forms with the **true meridian,** which is a line running *truly* north and south, is termed the **declination** of the needle. This deviation of the magnetic needle from a true meridian is not constant, but increases or decreases a very sensible amount in a series of years. For this reason, in running over the lines of a tract of land from field notes of some years' standing, the surveyor would be obliged to make an allowance, both perplexing and uncertain, in the bearing of every line, were it not for the use of a vernier.

1113. To Turn Off the Variation.—If the surveyor has a vernier attached to his compass, he can set the compass to any variation, by moving the vernier to either side (and with it, of course, the compass circle attached), by placing his instrument on some well-defined line of the old survey, and turning the *tangent screw* (slow-motion screw) until the needle of his compass indicates the same bearing as that given in the old field notes of the original survey. Then, by screwing up the clamping nut underneath the vernier, he can run all the other lines from the old field notes without further alteration.

1114. Figs. 191 and 192 show the retrograde form of a compass vernier. To read the position of the vernier in Fig. 191, we notice that the zero-point on this vernier, or inner scale, lies *to the right* of the zero-point on the outer scale. The graduations on the outer scale here *increase*

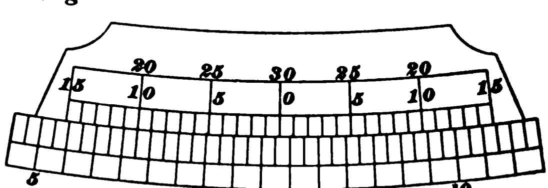


FIG. 191.

numerically from its zero-point *towards the right;* hence, we read the number of degrees in the *same* direction until we reach the zero-point of the vernier. Beginning at zero on the outer scale, we pass $3\frac{1}{2}$ degrees *to the right* before we encounter the zero of the vernier, and the next graduation beyond the zero of the vernier is the 4-degree mark, which shows at once that the reading is between 3½ degrees (3° 30′) and 4 degrees; that is, it is 3° 30′ plus a certain number of minutes yet unknown. This number of minutes is read upon the vernier. To obtain this reading, we begin at the zero-point of the vernier and look along the graduation lines *towards the left*, which is in the *opposite* direction from that followed on the outer circle, to find a coinciding line on the outer scale. By a **coinciding line** we mean a line on the outer scale which is a direct continuation of a line on the vernier scale. This line is found to be the eighth one from the zero-mark, and indicates 8 minutes, which must be added to the 3° 30′ read on the outer circle, giving 3° 30′ + 08′ = 3° 38′ as the correct reading.

1115. To read the position of the vernier in Fig. 192, we notice that the zero-point on this vernier lies *to the left* of the zero-point on the outer scale. The graduations on this scale *increase* numerically from its zero-point *towards the left*; hence, we read the number of degrees in the *same* direction until we reach the zero-point of the vernier. Beginning at zero on the outer scale, we pass 3½ degrees *to the left* before we encounter the zero of the vernier, and the next graduation beyond the zero of the vernier is the 4-degree mark, which shows at once that the reading is between 3½

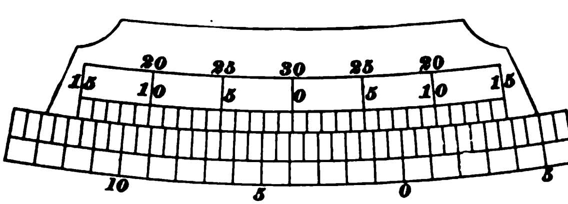


FIG. 192.

degrees (3° 30′) and 4 degrees; that is, it is 3½ degrees plus a certain number of minutes yet unknown. This number of minutes is read upon the vernier. To obtain this reading, we begin at the zero-point of the vernier and look along the graduation lines *towards the right*, which is in the *opposite* direction from that followed on the outer circle, to find a coinciding line. By doing this, however, we find that the extreme right-hand graduation on the vernier is reached without coming to a coinciding line. We then begin at the *extreme left end* of the vernier, where, it will be noticed, the graduation mark reads *15*, just as at the other end, and read *from the left end towards the zero* of the vernier, counting the minutes as shown by the *top* row of figures. It will be seen that the 26th line coincides with a line on the outer circle; hence, 26 minutes must be added to the reading previously obtained on the outer circle, giving $3° 30' + 26' = 3° 56'$ as the position of the zero of the vernier in Fig. 192.

AZIMUTHS AND BEARINGS.

1116. The **true azimuth** of a line is the angle the line makes with a true meridian measured from the north point as zero, around in the direction N-E-S-W to N. As the angle is limited only by the full circumference, the value of the azimuth varies from 0° to 360°. Thus, in Fig. 193, an azimuth measured from the point *N* as *0°* around to the right, passes *90°* at *E*, *180°* at *S*, *270°* at *W*, and reaches *360°* at *N*.

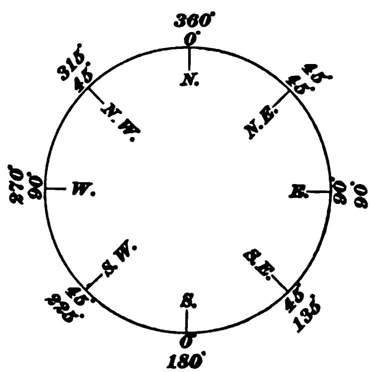


FIG. 193.

The azimuth of a line referred to a given line is the angle formed by the line, with the prolongation of the given line, or a parallel to it, drawn through the angular point, the measuring being made around to the right. Thus, in Fig. 194, the azimuth of *B D* referred to *A B* is the angle *F B D*; the azimuth of *D E* referred to *A B* is the angle *H D E*.

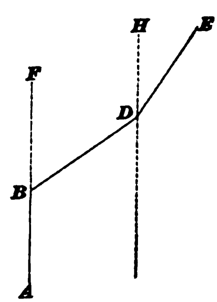


FIG. 194.

1117. The **true bearing** of a line is the angle the line makes with a true meridian, and the **magnetic bearing** is the angle it makes with a magnetic meridian, the measurement in both cases being made from the N and S points as 0°, to the E and W points as 90°. Bearings may be referred to any line, called the **assumed meridian,** in which case, however, it should be remembered that they are determined from their azimuths with respect to the same line, as will be explained later. Being limited in size to a quadrant, bearings vary in value from 0° to 90°. Thus, in Fig. 193, the bearing of any line, between *N* and *E* or *N* and *W* is greater than *0*°, and less than *90*°, and the bearing of any line between *S* and *E* or *S* and *W* is likewise greater than *0*°, and less than *90*°. Because there are four quadrants, the position of the line or bearing is definitely established only when the angle in degrees and minutes is accompanied by the letters of the quadrant points between which it lies. Thus, lines bearing N 45° E and S 45° W go in exactly *opposite* directions, while two others bearing N 45° E and S 45° E, respectively, make an angle of 90° with each other.

TO DETERMINE A TRUE MERIDIAN BY MEANS OF THE POLAR STAR.

1118. Of the bright stars in the northern heavens, the nearest to the pole is the first star in the tail of the Little Bear. This star is called the **Polar star,** and may be located, from the fact that it is in line with Alpha and Beta of the Great Bear (see Fig. 195), and at a distance from Alpha of about 5 times the distance between Alpha and Beta, measuring in the direction from Beta to Alpha.

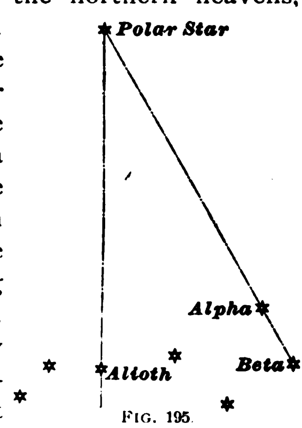


FIG. 195.

The Polar star very nearly coincides with the North pole, being less than 1° 17′ from it. It revolves about the pole, and twice in each revolution it is exactly in the true meridian; that is, in a vertical plane passing through the true pole. We can tell when the Polar star is in the true meridian by its relative position to another star called Alioth, which is in the tail of the Great Bear, or Great Dipper, as it is sometimes called, and nearest the four stars that form the quadrilateral in that constellation.

1119. In order to determine the true meridian, suspend from a firmly embedded stick, inclined either to the east or west, a plumb-line, allowing the plumb-bob to rest in a pail of water to lessen the vibrations. It is a good plan, where convenient, to suspend the plumb-line from a stick projecting from an east or west window in the higher stories of a house. Then, south of the plumb-line, at a distance not farther than where neither of the stars (Polar star and Alioth) can be seen above the string, place a smooth board horizontally upon two stakes, driven into the ground so as to form a sort of bench, running east and west. Upon this place one of the compass sights, or anything having a small hole in it to look through. Adjust the sight upon the board until it comes exactly in line with the plumb-line and Polar star, and keep it in line by moving the eye and sight along the board in the opposite direction to that in which the Polar star moves, until the star Alioth also falls exactly back of the string. At this instant the sight and plumb-line will be in the true meridian, or very nearly so. The true meridian will be more accurately located if the sight is taken upon the North star about 17 minutes after the star Alioth and the North star fall directly behind the plumb-line. The plumb-line and the sight may be left till morning, when points marking the true meridian should be fixed in the ground.

1120. The transit, which will be described later, may also be used to determine the true meridian. It is set up at some suitable place, directed to, and held upon, the North star until 17 minutes after the star Alioth and the North star fall directly behind the vertical cross-wire, when the line of sight will coincide with the true meridian. Points should be fixed immediately, a lamp being used to illuminate the cross-wires. It is a great convenience to know about the time the above stars and the point from which the observation is to be taken are in the same meridian plane. The student must bear in mind that in this method of determining the true meridian, the North star is sighted to while above the true north.

Table 26 gives the time at which the North star passes the meridian *above* the pole for every tenth day in which the North star will be visible at night and within almost every latitude of the United States.

TIME OF NORTH STAR PASSING THE MERIDIAN.

TABLE 26.

Months.	1st Day.	11th Day.	21st Day.
January	6.30 P. M.	5.51 P. M.	5.11 P. M.
August	4.33 A. M.	3.53 A. M.	3.14 A. M.
September	2.31 A. M.	1.52 A. M.	1.12 A. M.
October...........	12.34 A. M.	11.50 P. M.	11.11 P. M.
November	10.28 P. M.	9.48 P. M.	9.09 P. M.
December.........	8.30 P. M.	7.50 P. M.	7.11 P. M.

HOW TO MAKE A COMPASS SURVEY.

1121. The accompanying illustration, Fig. 196, will give the student an idea of the actual operation of making a survey with the compass. The point A is a **station,** marked by a stake, driven into the ground, with a tack in its head, the plumb-bob attached to the compass being made to hang directly over the tack. When this has been done, the compass plate is leveled. The **foresight-man** is sent forward to station B, upon which he holds a flagpole. The north side of the compass is turned towards B, and the observer looks through the uprights and turns the compass plate until the vertical hair in the upright on the north side **cuts,** or coincides with, the flagpole held vertically on *B*. When this has been done, the compass will be in the position shown at *A* in Fig. 196. The top of the page being north, the bottom south, the

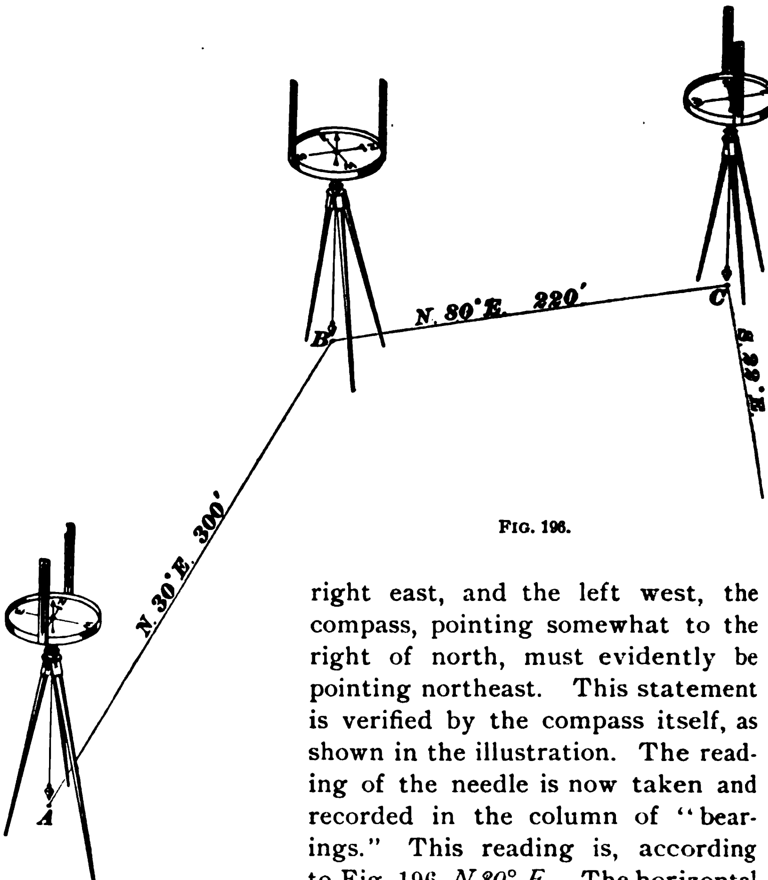


FIG. 196.

right east, and the left west, the compass, pointing somewhat to the right of north, must evidently be pointing northeast. This statement is verified by the compass itself, as shown in the illustration. The reading of the needle is now taken and recorded in the column of "bearings." This reading is, according to Fig. 196, *N 30° E*. The horizontal distance from *A* to *B* is also measured, and is recorded in the column of "distances." This distance is 300 feet.

The instrument is now moved forward to *B*, where it is plumbed and leveled as it was at *A*. A sight (called **back-sight**) is then taken to the flagpole held on the tack at *A*. In this position, the needle reading in degrees and fractional parts of a degree should be the same as it was at *A*, but the *direction* should be diametrically opposite. In other words, the backsight should read S 30° W. This precaution of backsighting is taken to detect any local attraction that may exist at either station. If the two readings very nearly agree, they may be considered correct; if there is a wide divergence in the two, it is probable that there is local magnetic attraction at one or the other of the stations, and neither one can be depended upon unless further investigation is made to prove at which point it exists. One method that can be followed is to take readings on the line of sight at intermediate distances; as, for instance, at 100 feet and at 200 feet from *A*. Should the bearings at both of these intermediate points agree with the bearing at either *A* or *B*, such agreement would show which of the two is correct, and which should be adopted.

The foresight-man is sent forward to station *C*, upon which he holds the flagpole, and the compass, properly set over *B*, is sighted upon it. Its position when so sighted is shown at *B* in the figure.

The needle is read and its bearing, *N 80° E*, is recorded in the proper column in the note-book. The horizontal distance from *B* to *C* is likewise measured and recorded; it is found to be 220 feet.

The instrument is then moved forward to *C*, plumbed and leveled, and a backsight taken upon *B*. If the forward and backsights at *A* and *B*, respectively, have disagreed for the first course, and the forward and backsights at *B* and *C*, respectively, have agreed for the second course, it is evident that the attraction is at *A*, and the backsight on that station from *B* can be relied upon. The same operations are repeated through the entire survey.

COMPASS WORK IN THE MINES.

1122. Every mine official should be thoroughly familiar with the surveyor's compass, so as to be able to use it for various purposes and under all conditions that arise in mining work. It is true that in some mines nearly all the work is done without the use of compass lines, in which case the fact is at once apparent upon examination of the map of the workings. The map makes it equally plain that the work could have been done much better had the entries and chambers been driven on established lines. It is so done in nearly every mine in charge of thoroughly competent officials.

Probably the simplest operation for which the compass is used is in the location of two points to serve as a guide in driving an opening in a particular direction. Local conditions make it necessary to vary the usual method to suit the special case.

1123. The entry shown at the bottom of Fig. 197 was driven first, and when it reached the point where Sta. 101 is located, it was found necessary to change its direction.

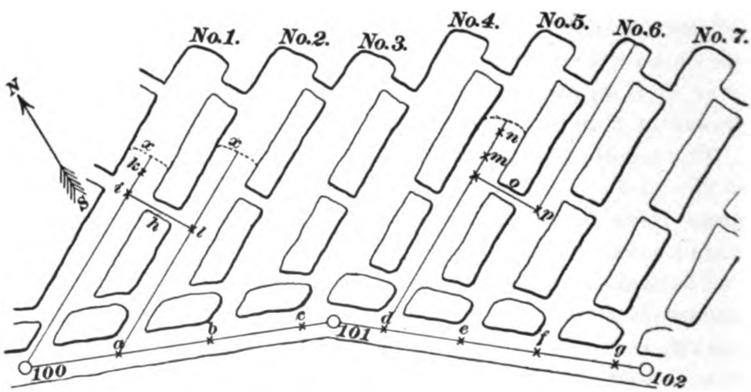


FIG. 197.

Stas. 100, 101, and 102 were put in by the regular surveyor with the use of the transit, which makes the bearing of their connecting lines known.

The mine foreman, having examined this section of the mine, decides upon the proper angle which the center lines of the chambers must make with the line of dip of the seam. He then takes a tracing of the entry, which is made for his use by the mine surveyor, on which the north and south line (meridian) is marked, and draws the center lines of the breasts in the direction decided upon, and as far apart as the nature of the seam requires.

Having been told by the surveyor that the direction of the line from Sta. 100 to Sta. 101 is S 67° 20′ E, and having found with the aid of a protractor that the center lines he has drawn on the tracing run N 62° E, he is prepared to go into the mines and put up points for the center lines. Arriving at Sta. 100, he sets the compass directly under it (the stations are supposed to be in the roof), levels it, unclamps (lowers) the needle, and sights to Sta. 101. Upon reading the needle, after it has settled, he finds that it indicates a bearing of S 67° 15′ E, which is practically the same as the bearing found by the transit survey, and shows that no local attraction is influencing the needle.

A chamber being desired opposite Sta. 100, the compass plate is turned until the north end of the needle points to N 62° E. An assistant is then directed to hold his light against the roof, in line with the compass sights, through which the foreman is looking and guiding the position of the light. The point in line having been found and marked with a piece of chalk carried for the purpose, a hole about 1 inch in diameter is drilled into the roof and a wooden plug, tapering to a slightly larger diameter, driven firmly into it. An ordinary nail is held against the plug, and the light is held back of the nail, so that it can be seen through the compass sights. The nail is moved as the foreman looking through the sights may direct, and when it "coincides" with the vertical hair in the sight it is driven into the plug to within ½ inch of its head. This completes the operation.

As this point is necessarily close to Sta. 100, it may be advisable to locate another point on the lower (right-hand) side of the entry, as more accurate results are obtained when the sight points are farther apart; besides, it will then be unnecessary for the miner to make use of Sta. 100, for in doing so he might injure or destroy it. This second point, being in a direction exactly opposite the first one, requires the compass to be turned through 180°, or a half circle, when its needle will read S 62° W. When the compass is in this position, the point is put in the roof in exactly the same way that the first one was.

1124. Let us take another case, where the operation is not quite so simple, as, for instance, where the needle is influenced by local attraction. Suppose that the compass when set up at Sta. 100 and sighted to Sta. 101 had given a needle-reading of S 58° 45′ E, instead of S 67° 20′ E, the proper bearing furnished by the mine surveyor. Evidently, it would not be right to turn the compass until the needle gives a reading of N 62° E, for that would necessitate its being turned through a greater angle than in the previous case, and certainly both can not be correct. The remedy, however, is simple. The reading S 58° 45′ E is 8° 35′ to the right of the true reading, since 67° 20′ − 58° 45′ = 8° 35′. This correction must be made to the bearing of the center line of the chamber. A line passing 8° 35′ to the right of one running N 62° 00′ E will have a bearing of 62° 00′ + 8° 35′ = N 70° 35′ E. Consequently, if the compass is turned until the north end of the needle points to N 70° 35′ E (N 70° 30′ E is near enough), the sight points can be put in and will be correct.

The local attraction influencing the needle may deflect it in the opposite direction; that is, the bearing from Sta. 100 to Sta. 101, as shown by the compass-needle, might be S 73° 30′ E. In this case there is a difference of 6° 10′ between the true and observed bearing, since 73° 30′ − 67° 20′ = 6° 10′, the needle being deflected to the left that amount. Correcting the bearing of the center line of the chamber by 6° 10′ to the left makes it N 55° 50′E, since 62° 00′ − 6° 10′= 55° 50′.

1125. Other chambers are to be started from this entry, the center lines of all to be parallel. For this purpose, the compass is set under Sta. 100 and sighted to Sta. 101, no notice whatever being taken of the needle-reading. It is necessary to calculate the distance that must be laid off *on the entry* between the center lines, for it will be noticed that this distance is not the same as that measured at right angles to the center lines. These calculations will be explained later. The points *a*, *b*, and *c* are put in on the line of sight between Stas. 100 and 101, at distances apart as determined by calculation, and the operation of putting in sight points at each of them is exactly the same as already described for Sta. 100. It must not be taken for granted, however, that there will be nothing at these points to deflect the needle if no attraction existed at Sta. 100. As the compass is set up at each one of these points, the precaution must be taken to **backsight** to Sta. 101 or to Sta. 100, when the needle-reading should be N 67° 20′ E, or S 67° 20′ W, respectively.

1126. At Sta. 101, the direction of the entry changes, its bearing to Sta. 102 being S 50° E. On this line of sight, the points *d*, *e*, *f*, and *g* are located, from which sight points are to be put in, but *d*, *e*, *f*, and *g* will not be the same distance apart that *a*, *b*, and *c* are, because of the change in direction of the entry. This change in direction, however, does not change the compass work; that is, if the compass set at *d*, for example, and sighted to Sta. 102 gives a needle-reading of S 50° E, it must, as before explained, be turned until the needle reads N 62° E, in order to properly put in the sight points. In case the needle is deflected from its proper position at any one of the points *d*, *e*, *f*, or *g*, the correction is exactly the same in kind and application as previously explained.

1127. The work of extracting the coal from the chambers having progressed until the faces reach a point *x*, Fig. 197, it becomes necessary to put in new sight points near the face, in order that the proper direction may be continued, the first points having become unreliable guides because of the distance. The compass is again set up at Sta. 100, sighted to Sta. 101, and a needle-reading taken. This is done to determine the presence or absence of local attraction, for the progress of events since the first points were put up may have brought such attracting objects as sheet iron, iron rails, etc., near this spot. The fact that the needle showed the proper reading upon one occasion is, therefore, no guarantee that it will do likewise upon any other. Whether there is attraction or not, the compass is then turned until the sights point in the proper direction. In case there is attraction, a check upon the correctness of this direction is had by suspending a plumb-line from the point previously put in near the mouth of the chamber. The compass sights must show this line to be in the newly determined direction.

A sight is now taken towards the face of the chamber, and a point *i* is located in the roof in front of (preferably) the **heading,** or **cross-hole** *h*. The usual plug is driven at *i*, and a nail is driven into this plug exactly in line with the compass sights. A second point *k* is put in nearer the face in exactly the same way.

The points *i* and *k* could have been put up by setting the compass at Sta. 100 and sighting to the point previously put in near the mouth of the chamber to give the proper direction.

In the same way, new points can be put up in each of the other chambers by setting the compass at the points *a*, *b*, *c*, etc. Suppose, however, that a fall of roof has occurred in the entry at the second chamber, which has carried down the point *a*, as well as the two sight points previously located there. With the compass set at Sta. 100, a new point could be put in at exactly the same place occupied by the destroyed point *a*, and the points put in the chamber from it. However, the location of the point *i* in the first chamber opposite the cross-hole *h* makes the reestablishment of the point *a* superfluous.

The compass is set up at *i* and a sight taken upon Sta. 100, when, if there is no local attraction, the needle-reading will be S 62° W. The compass sights are then turned to the left through an angle of 90°, which will give a needle-reading of S 28° E, if the previous reading was S 62° W. If there *is* local attraction at *i*, so that the backsight-reading to Sta. 100 is *not* S 62° W, the compass is turned through an angle of 90° just the same. If the center lines of the chambers are 45 feet apart, that distance is laid off through the cross-hole *h*, and the point *l* is put in the roof in the line of sight. The point *l* so established is on the center line of the second chamber, and with the compass set up there, two points are put in near the face of the chamber in the usual manner.

1128. The preceding explanation shows the manner in which work should be done if the results must be as nearly correct as it is possible to make them. A beginner should follow this method precisely, and experience will teach him where and to what extent the careful process may be modified. A few suggestions in this direction are here given.

Suppose, in chamber *No. 5*, the two new sight points *m* and *n* were placed beyond the cross-hole *o*. A chalk-mark is made on the center line opposite this cross-hole, and a 45-foot measurement taken from the point so marked through the cross-hole, and as nearly at a right angle to the center line as can be estimated. The point *p*, so determined in *No. 6* chamber, is marked with chalk, and the compass set over it. A check upon the accuracy of this operation is made by sighting to the chalk-mark in *No. 5* chamber; the needle-reading then obtained should approximate N 28° E. It is evident that the point *p* will be nearer the correct center line of *No. 6* chamber the nearer the measurement comes to being at right angles. If it is *not* at right angles, the two center lines will be too close together. Let us see how much of an error in judging the right angle must be made to bring the center lines 1 foot too close; that is, to make them 44 feet apart instead of 45 feet.

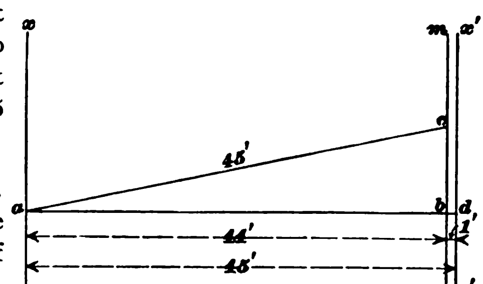


FIG. 198.

1129. In Fig. 198, $x\ y$ and $x'\ y'$ are the center lines of two adjoining chambers, the distance $a\ d$ between them being 45 feet. The line $m\ n$, parallel to $x'\ y'$, is distant from it a length $b\ d$ equal to 1 foot. The length

$a\ c$ shows the erroneous position of the 45-foot measurement, which brings the center lines 1 foot too close together.

In the triangle $a\ b\ c$, right-angled at b, we have

$$\cos c\,a\,b = \frac{a\,b}{a\,c}.$$

But $a\,b = a\,d - b\,d = 45 - 1 = 44$ feet, and $a\,c = 45$ feet.

Then, $\cos c\,a\,b = \frac{a\,b}{a\,c} = \frac{44}{45} = .97777$, which corresponds nearly to an angle of 12° 06′. (See table of Natural Cosines.)

In the triangle $a\,b\,c$ we also have

$b\,c = \sqrt{\overline{a\,c}^2 - \overline{a\,b}^2} = \sqrt{45^2 - 44^2} = \sqrt{2{,}025 - 1{,}936} = \sqrt{89} =$ 9.4 feet.

It is not likely that any one capable of doing compass work would be so lacking in judgment as to make an error of 12° 06′, or of 9.4 feet, in performing so simple a piece of work as that described.

1130. The distance between the chambers, or rooms, measured along the entry, may be determined by the following formula:

$$D = \frac{p}{\sin A}, \qquad \textbf{(78.)}$$

where D = the required distance; p = the perpendicular, or right-angle distance between the chambers, which is always known, being determined by the foreman, as previously stated; and A is the angle formed by the intersection of the center lines of the entry and the chamber.

Applying formula **78,** the distance between the center lines of the chambers, measured along the entry between Stas. 100 and 101, Fig. 197, can be found. The bearing of the line $a\ b$ is S 67° 20′ E, and that of the line $a\ l$ is N 62° E, making the angle $b\ a\ l$ equal to 50° 40′. The perpendicular distance between the rooms is 45 feet. Substituting these values in formula **78,** we have

$$D = \frac{45}{\sin 50° \ 40'} = \frac{45}{.77347} = 58.18 \text{ ft., nearly.} \quad \text{Ans.}$$

LATITUDES AND DEPARTURES.

1131. The **latitude** of a point is its distance north or south of some "parallel of latitude," or line running east and west. The **departure** of a point is its distance east or west of some meridian, or line running north and south. It is the same as the longitude of the point.

In compass surveying, the *magnetic meridian* is the line from which the departures, or longitudes, of points are measured. The distance which one end of a line is due north or south of the other end is the difference of latitude of the two ends of the line, and is called the **northing** or **southing**; or, simply, the *latitude* of the end considered.

The distance which one end of a line is due east or west of the other end is the difference in longitude of the two ends of the line, and is called the **easting** or **westing**; or, simply, the *departure*.

The meaning of these statements will be made clearer by consulting Fig. 199.

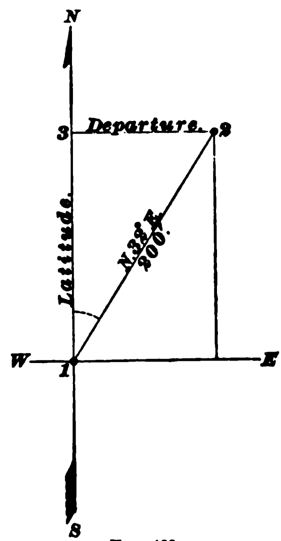


FIG. 199.

1132. To show how to find the latitude and departure of any line or course whose direction or bearing from any meridian is known, take an example:

Station.	Bearing.	Distance.
1–2	N 32° E	200′

In the accompanying diagram, Fig. 199, the direction of the compass-needle is represented by the north and south line, the parallel of latitude by the east and west line, and the given course by the line *1–2*, whose direction is N 32° E, and length is 200 feet.

The *latitude*, then, according to the definition, will be represented by the line *1–3*, the distance which the point *2* (one end of the line) is due north of the point *1*, the other end.

The *departure*, according to the definition, will, similarly, be represented by the line *2–3*.

The question of finding the latitude and departure resoives itself into calculating two sides of a right-angled triangle, one angle of which, besides the right angle, is known, and also the length of one side, the hypotenuse. See Fig. 200.

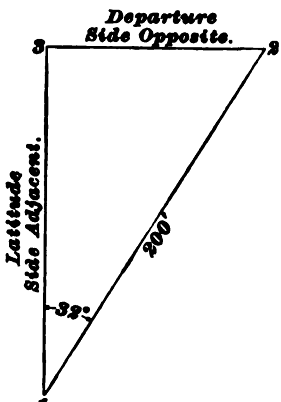


Fig. 200.

Departure = side opposite = sine × hypotenuse = sine 32° × 200 ft. Since sine 32° = .52992, departure = .52992 × 200 ft. = 105.984 ft.

Latitude = side adjacent = cosine × hypotenuse = cosine 32° × 200 ft. Since cosine 32° = .84805, latitude = .84805 × 200 ft. = 169.61 ft.

1133. The latitudes and departures of any distance for any bearing can be found by the method given above, with the use of a table of natural sines and cosines. But to facilitate these calculations, which are of so frequent occurrence, the Traverse Tables have been prepared.

1134. Following are the notes of a compass survey, which we will traverse, in order to show how to apply the preceding principles to actual surveys:

Station.	Bearing.	Distance.
1–2	N 67° 45′ E	223.8
2–3	N 16° 30′ E	198.2
3–4	N 53° 30′ W	197.5
3–5	S 22° 15′ E	430.3
5–1	N 73° 45′ W	441.7
4–6	S 58° 45′ W	378.1
6–7	S 11° 30′ E	406.8
7–1	N 34° 15′ E	245.0

1135. As the latitude of a course may be either north or south, and the departure either east or west, four columns must be added to the preceding form of notes, headed north, south, east, west, in which the calculated values are placed. It is also customary to determine the latitude and departure of each station with reference to the first station of the survey, for which four more columns for the *total* north, south, east, and west are necessary. In Art. **1137** will be found a table giving the form of the calculated notes.

1136. It may be well to call attention to the fact that when the bearing is *less* than 45° the departure is *less* than the latitude, and when the angle of the course is *greater* than 45° the departure is *greater* than the latitude. This shows at a glance whether a mistake has been made in writing the latitude for the departure and the departure for the latitude, which error is sometimes made in placing these numbers in the columns headed N, S, E, and W. Having done this correctly, as shown in the preceding notes, the latitude and the departure of each station show its distance north or south and east or west *of the preceding station.* From these we can calculate the latitude and departure of each station with reference to Sta. 1.

1137. In Fig. 201, the lines *2 a* and *2 b* show the distance that Sta. 2 is north and east of Sta. 1. Similarly, the lines *3 c* and *2 c* show the distance that Sta. 3 is north and east of Sta. 2. At the same time the lines *3 d* and *3 e* show the distance that Sta. 3 is north and east of Sta. 1. But $3\,d = 2\,a + 3\,c$, and $3\,e = 2\,b + 2\,c$. Since $2\,a = 84.7$ and $3\,c = 190.0$, then $3\,d = 2\,a + 3\,c = 84.7 + 190.0 = 274.7$ ft., the *total* latitude (in this case northing) of Sta. 3, which number is placed in the "total N" column opposite Sta. 3 in the first column at the left. Also, $2\,b = 207.1$ and $2\,c = 56.3$, then $3\,e = 2\,b + 2\,c = 207.1 + 56.3 = 263.4$, which is the total departure (in this case easting) of Sta. 3, and is placed in the "total E" column opposite Sta. 3.

The lines *4 f* and *3 f* show the distance that Sta. 4 is north and west of Sta. 3, and $4g$ and $4h$ show the distance it is north and east of Sta. 1. But $4g = 3d + 4f$, and $4h = 3e - 3f$. Since $3d = 274.7$ and $4f = 117.5$, then $4g = 3d + 4f = 274.7 + 117.5 = 392.2$, the total latitude (in this case northing) of Sta. 4 with reference to Sta. 1, which is placed in the "total N" column opposite Sta. 4. Also, $3e = 263.4$ and $3f = 158.8$, so that $4h = 3e - 3f = 263.4 - 158.8 = 104.6$ ft., the total departure (in this case easting) of Sta. 4, which is placed in the "total E" column opposite Sta. 4.

Station.	Bearing.	Distance.	N.	S.	E.	W.	Total.			
							N.	S.	E.	W.
1–2	N 67° 45′ E	223.8	84.7		207.1		84.7		207.1	
2–3	N 16° 30′ E	198.2	190.0		56.3		274.7		263.4	
3–4	N 53° 30′ W	197.5	117.5			158.8	392.2		104.6	
3–5	S 22° 15′ E	430.3		398.3	162.9			123.6	426.3	
5–1	N 73° 45′ W	441.7	123.6			426.3	0	0	0	0
4–6	S 58° 45′ W	378.1		196.1		323.2	196.1			218.6
6–7	S 11° 30′ E	406.8		398.6	81.1			202.5		137.5
7–1	N 34° 15′ E	245.0	202.5		137.5		0	0	0	0

The lines $5k$ and $3k$ show the distance that Sta. 5 is south and east of Sta. 3, and $5l$ and $5m$ show how far it is south and east of Sta. 1. But $5l = 5k - 3d$, and $5m = 3e + 3k$. Since $5k = 398.3$ and $3d = 274.7$, then $5l = 5k - 3d = 398.3 - 274.7 = 123.6$ ft., the total latitude (in this case southing) of Sta. 5, which is placed in the "total S" column opposite Sta. 5. Also, $3e = 263.4$ and $3k = 162.9$, hence $5m = 3e + 3k = 263.4 + 162.9 = 426.3$ ft., the total departure (in this case easting) of Sta. 5, which is placed in the "total E" column opposite Sta. 5.

The lines $6n$ and $4n$ show the distance that Sta. 6 is south and west of Sta. 4, and $6p$ and $6r$ show how far north and west it is from Sta. 1. But $6p = 4g - 6n$, and $6r = 4n - 4h$. Since $4g = 392.2$ and $6n = 196.1$, then $6p = 4g - 6n = 392.2 - 196.1 = 196.1$ ft., the total latitude (in

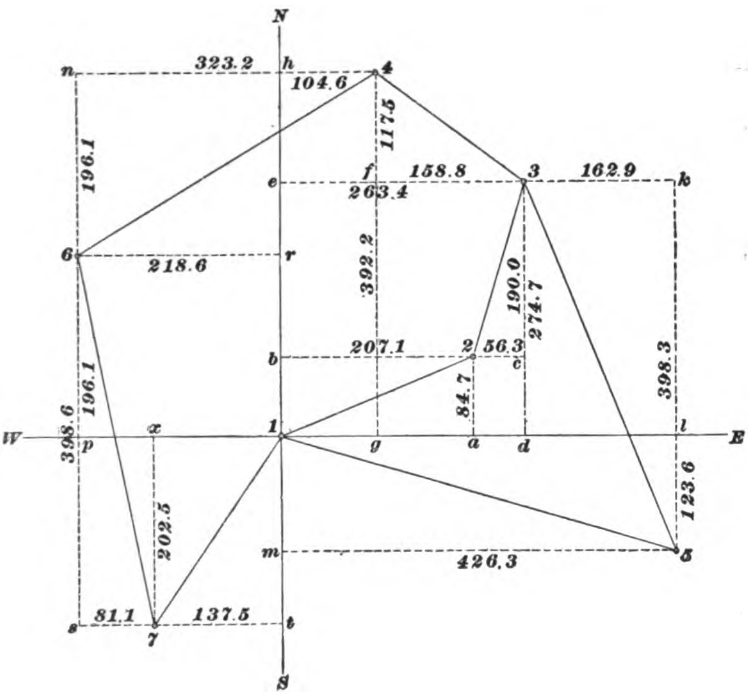


FIG. 201.

this case northing) of Sta. 6, which is written in the "total N" column opposite Sta. 6. Also, $4n = 323.2$ and $4h = 104.6$, so that $6r = 4n - 4h = 323.2 - 104.6 = 218.6$ ft., the total departure (in this case westing) of Sta. 6, which is written in the "total W" column opposite Sta. 6.

The lines $6s$ and $7s$ show how far south and east Sta. 7 is from Sta. 6, and $7x$ and $7t$ show how far south and west Sta. 7 is from Sta. 1. But $7x = 6s - 6p$, and $7t = 6r - 7s$.

Since $6s = 398.6$ and $6p = 196.1$, $7x = 6s - 6p = 398.6 - 196.1 = 202.5$ ft., the total latitude (in this case southing) of Sta. 7, which value is written in the "total S" column opposite Sta. 7. Also, $6r = 218.6$ and $7s = 81.1$, so that $7t = 6r - 7s = 218.6 - 81.1 = 137.5$ ft., the total departure (in this case westing) of Sta. 7, which value is written in the "total W" column opposite Sta. 7.

1138. The preceding explanation should be readily understood in connection with Fig. 201. In practice, however, these calculations are (or certainly should be) made before the survey is platted; hence, there is no figure to guide and check the correctness of the work. On this account care must be exercised in calculating the total latitudes and departures where more than one station has been located from the same station. For example, Stas. 4 and 5 were both located from Sta. 3. *The error that is likely to be made is this:*

Sta. 3.........	Total north	274.7	Total east	263.4
Sta. 4.........	north	+ 117.5	west	− 158.8
	Total north	392.2	Total west	104.6
Sta. 5.........	south	− 398.3	east	− 162.9
	Total south	6.1	Total east	58.3

Here Sta. 5 is shown with reference to Sta. 4, *which is wrong*, for Sta. 5 is not located from Sta. 4. Obvious as this error is, it is one that can be avoided only by continually exercising care.

MAPPING.

PLATTING TO SCALE.

1139. The courses having been measured and their bearings determined, by means of the chain or steel tape and compass, it is now necessary to represent the work upon paper, or to make a map. To do this, the student must learn how to represent a line upon paper according to some scale, and how to lay off angles. For these purposes two plates will be given, in order that the student will get sufficient practice to enable him to do this work accurately and intelligently. The details of making a complete map, however, will not be given until the student studies Transit Surveying. The plates for the three sections of Mine Surveying and Mapping are placed in the volume of Geometrical Drawing.

1140. Platting to Scale.—To reproduce a surveyed area on paper, it is necessary to draw each line a certain number of times smaller than its real size, the amount of the reduction depending generally upon the area to be platted. For example, the Anthracite Mine Law of Pennsylvania requires the maps of the mine workings to be drawn to a scale of 100 feet to the inch. This means that every one hundred feet of actual measurement obtained by the survey shall be represented on the map by a distance of one inch. Thus, if two stations of a survey are 250 feet apart, these stations, as located on the map, must be 250 divided by 100 = 2.5 inches apart.

1141. The length any line must be drawn upon a map to represent a measured line to a certain scale is obtained by dividing the measured distance by the number of feet that one inch on the map shall represent. Thus:

50 ft. to a scale of 100 ft. to an inch = $\frac{50}{100}$ = .5 in.;
100 ft. to a scale of 200 ft. to an inch = $\frac{100}{200}$ = .5 in.;
75 ft. to a scale of 50 ft. to an inch = $\frac{75}{50}$ = 1.5 in.;
600 ft. to a scale of 300 ft. to an inch = $\frac{600}{300}$ = 2 in.;
etc., etc., etc.

Similarly, the measured distance in inches and decimal parts of an inch between two points on a map or other drawing, multiplied by the number of feet to the inch, gives the actual distance, in feet, between the points. Thus:

2.4 in. to a scale of 50 ft. to an inch = 2.4 × 50 = 120 ft.;
1.8 in. to a scale of 100 ft. to an inch = 1.8 × 100 = 180 ft.;
2 in. to a scale of 200 ft. to an inch = 2 × 200 = 400 ft.;

3.3 in. to a scale of 300 ft. to an inch = $3.3 \times 300 = 990$ ft.; etc., etc., etc.

As an illustration, suppose we had a measured distance of 70 feet to lay off to a scale of 100 feet to an inch. By taking a scale whose inches are divided into 10 equal parts, as shown in Fig. 202, and laying off a distance *B A* equal to 7 of the tenth-divisions, we would be laying off the required distance. For, 70 feet to a scale of 100 feet to an inch = $\frac{70}{100} = .7$ in.

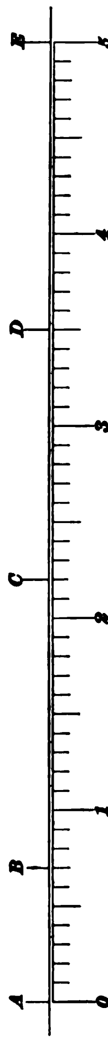


FIG. 202.

Again, if we had 110 feet to lay off to a scale of 50 feet to an inch, that distance would have to be made equal to $\frac{110}{50} = 2.2$ in., which, in Fig. 202, is the distance *A C*.

Suppose we had a line *A D* given (Fig. 202), which we knew had been drawn to a scale of 75 feet to an inch, how many feet would it represent? Applying the tenth scale, we find it measures 3.5 inches; therefore, the distance *A D* represents $3.5 \times 75 = 262.5$ ft.

Again, if the line *A E*, in the same illustration, had been drawn to a scale of 30 feet to an inch, and measurement showed the length *A E* to be 5 inches, then it would represent an actual distance of $5 \times 30 = 150$ ft.

1142. It must be remembered that only the approximate distance can be obtained by scaling a line on a drawing and multiplying this length by the scale of the drawing. For instance, in the last example given, the actual distance might be 150.25 feet, yet the line *A E* would measure 5 inches, as previously, for so small a length as .25 foot can not be represented on a scale of 30 feet to the inch. In general, the greater the number of feet represented by one inch, the less the accuracy with which a distance can be determined by scaling it from the drawing.

PLATTING ANGLES.

PLATE I.

1143. Plate I (see Geometrical Drawing) contains six angle lines, three of which are included in Fig. 1 and three in Fig. 2. The three lines ***a***, ***b***, and ***c*** under Fig. 1 will be drawn to a scale of 200 feet to the inch, platting the angles with a protractor, the use of which was fully explained in Geometrical Drawing.

The student will plat these lines according to the following directions, being careful to give to each line approximately the same position which it occupies in the plate. This statement also applies to all the plates which are to be drawn from this Instruction Paper. In these examples, distances are expressed in stations of 100 feet each. The direction of each line is referred to that of the immediately preceding line, which line is produced and the angle recorded as being to the right (R) or left (L) of that line.

NOTES FOR LINE ***a***.

Stations.	Angles.
25 + 84	End of Line
21 + 94	L 32° 35′
15 + 53	R 44° 10′
11 + 72	L 60° 30′
5 + 25	L 25° 15′
0	

In practical office work, the lines produced are drawn lightly in pencil, and erased as soon as the angles are laid off. In the lines ***a*** and ***b***, Fig. 1, the lines produced are dotted and the angles written in dotted arcs, in order that the student may clearly and fully understand the method. The dimensions of all the plates, the directions for drawing the border lines, and the order of sending in the plates or other drawings of the three sections of this subject are the same as for the plates on Geometrical Drawing. The notes for line ***a*** in Fig. 1 are as shown.

1144. The starting-point *A* of the line is numbered 0. The first angle turned is at Sta. 5 + 25, which we denote by

B. Locating the starting-point *A* about three-fourths of an inch from the lower and left-hand border lines, we draw a straight line, giving it the same direction as that given to it in the engraving. Scale off from *A*, to a scale of 200 feet to the inch, the first course 525 feet in length, locating the point *B*. Produce *A B* to *C*, being sure to make *B C* a little greater than the diameter of the protractor. At Sta. 5 + 25, *B*, an angle of 25° 15′ is turned to the left. Now, placing the center of the protractor on the point *B*, with the zero-point on the line *B C*, lay off the angle 25° 15′ to the left of *B C*, marking the point of angle measurement *D* with a needle-point. Through the points *B* and *D* draw a straight line. The angle *C B D* is 25° 15′, and the line *B D* is the direction of the next course. The second angle, 60° 30′, is turned to the left at Sta. 11 + 72. The length of the second course is found by subtracting 525 from 1,172, giving a difference of 647 feet. Produce *B D* and scale off the second course 647 feet, locating the point *E* at Sta. 11 + 72. Produce *B E* to *F*, and lay off to the left of *E F* the angle 60° 30′, locating the point *G*. Join *E* and *G*. The angle *F E G* is 60° 30′, and the line *E G* is the direction of the next course.

1145. The third angle is *R* 44° 10′, and is turned at Sta. 15 + 53. The length of the third course is found by subtracting 1,172 from 1,553, giving a difference of 381 feet. Produce *E G*, and scale off from *E* the distance 381 feet, locating the point *H* at Sta. 15 + 53. Produce *E H* to *K*, and to the right of *H K* lay off the given angle 44° 10′, locating the point *L*. The line joining the points *H* and *L* forms with *H K* an angle of 44° 10′, and gives the direction of the next course. The next angle is L 32° 35′, and turned at Sta. 21 + 94. The length of the course is found by subtracting 1,533 from 2,194, giving a difference of 641 feet. Produce *H L*, and scale off from *H* the distance 641 feet, locating the point *M* at Sta. 21 + 94. Produce *H M* to *N*, and to the left of *M N* lay off the given angle 32° 35′, locating the point *O*. Draw *M O*. The angle *N M O* is 32° 35′, and *M O* is in the direction of the next and last course of line *a*, whose length is found by subtracting 2,194 from 2,584. The difference is 390 feet. We produce the line *M O*, and from *M* scale off the last course of 390 feet, locating the point *P* at Sta. 25 + 84. At each angular point in the line an arc is described, giving the measurement of the angle.

The student will in a similar manner plat the following notes for the lines *b* and *c*, Plate I, Fig. 1 :

NOTES FOR LINE *b*.

Stations.	Angles.
23 + 10	End of Line
16 + 35	R 25° 10′
12 + 82	L 15° 15′
8 + 50	L 30° 40′
4 + 40	R 15° 20′
0	

NOTES FOR LINE *c*.

Stations.	Angles.
28 + 60	End of Line
21 + 46	R 34° 30′
17 + 09	R 53° 28′
11 + 96	L 25° 10′
5 + 33	R 21° 10′
0	

1146. To Lay Off an Angle By Chords.—This is done by means of a table of chords in which the lengths of chords for all angles from 0° to 90° are given in terms of a radius 1. A radius of any convenient length may be assumed, and the corresponding chord length obtained by multiplying the length of the chord given in terms of radius 1 by the length of the assumed radius. Thus, let it be required to lay off from a given line an angle of 40° 10′ to the left. Let *A B*, Fig. 203, be the given line. Produce *A B* to *C*, making *B C* = 400 feet, the length of the assumed radius.

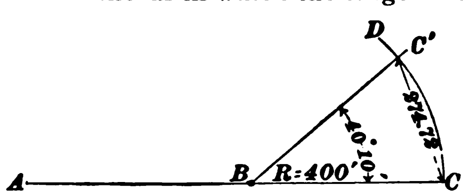


FIG. 203.

From a table of chords, we find the chord of an angle of 40° 10′ in terms of a radius 1 is .6868. Multiplying this chord by 400 feet, the length of the assumed radius, we have 274.72 feet, the length of the required chord. From *B* as a center, with a radius *B C* = 400 feet, describe to the left of *B C* the indefinite arc *C D*, being sure that the length of *C D* is slightly greater than the length of the required chord, and from *C* as a center, with a radius of 274.72 feet, describe an arc intersecting the arc *C D* in *C′*. Through *B* and *C′* draw a straight line. The angle *C B C′* is 40° 10′, the required angle. This method of platting angles is more accurate, though less rapid, than platting with a protractor.

NOTE.—The table of chords used for the calculations given in this paper may be found in Trautwine's Pocket Book, a very useful book to all surveyors. If the student does not possess a copy, he may easily find the required chord from his table of sines by multiplying the sine of half the given angle by 2. Thus, the chord of 40° 10′ = 2 sin (40° 10′ ÷ 2) = 2 sin 20° 05′ = 2 × .34339 = .68678 = .6868, using but four places, the same as given in the table.

1147. Fig. 2, Plate I, contains three examples in the lines ***a***, ***b***, and ***c***, in which the angles are laid off by chords. The notes for line ***a*** are given in the following table :

The first course *A B* is 360 feet in length, which the student will draw to a scale of 200 feet to the inch. The starting-point *A* is numbered 0, and *B*, the end of the first course, 3 + 60. At *B* an angle of 30° 30′ is laid off to the right. Produce *A B* 400 feet, which we assume to be the length of the radius in calculating chord lengths for laying off angles, and locate the point *C*. Then from *B* as a center, with a radius of 400 feet, describe the indefinite arc *C C′* on the right side of the radius *B C*, being sure that the arc shall contain at least 30° 30′.

NOTES FOR LINE ***a***.

Stations.	Angles.
25 + 80	End of Line
20 + 38	L 37° 20′
15 + 18	L 31° 08′
9 + 13	R 39° 26′
3 + 60	R 30° 30′
0	

We find in a *table of chords* that the chord of 30° 30′ = .5261, which, multiplied by 400 feet, the length of the assumed radius, gives 210.44 feet, the length of the required chord. From *C* as a center, with a radius of 210.44 feet, describe an arc intersecting the arc *C C'* in the point *E*. A line joining *B* and *E* will form with the radius *B C* an angle *C B E* = 30° 30', the required angle. The next angle, R 39° 26', is turned at Sta. 9 + 13, making the length of the second course 553 feet.

Denote Sta. 9 + 13 by *F*. Produce *B F* 400 feet to *G*. From *F* as a center, with a radius *F G* of 400 feet, describe to the right of *F G* the indefinite arc *G G'*, being sure that the arc shall contain at least 39° 26'. The chord of 39° 26' to a radius of 1 is .6747, which, multiplied by 400 feet, gives 269.88 feet, the length of the required chord. From *F* as a center, with a radius of 269.88 feet, describe an arc intersecting the arc *G G'* in *H*. A line joining *F* and *H* will form with the radius *F G* the angle *G F H* = 39° 26', the required angle. The next angle, viz., L 31° 08', is turned at Sta. 15 + 18, making the length of the third course 605 feet. Call Sta. 15 + 18, *K*. Produce *F K* 400 feet to *L*. From *K* as a center, with a radius *K L* of 400 feet, describe to the left of *K L* the indefinite arc *L M*. The chord of 31° 08' is .5367, which, multiplied by 400 feet, gives 214.68 feet, the length of the required chord. From *L* as a center, with a radius of 214.68 feet, describe an arc intersecting the arc *L M* in the point *N*. Join *K* and *N*, forming with *K L* the angle *L K N* = 31° 08'. The next angle, viz., L 37° 20', is turned at Sta. 20 + 38, making the length of the fourth course 520 feet. Call Sta. 20 + 38, *O*. Produce *K O* 400 feet to *P*. From *O* as a center, with a radius *O P*, describe the indefinite arc *P Q*. The chord of 37° 20' is .6401, which, multiplied by 400 feet, gives 256.04 feet, the length of the required chord. From *P* as a center, with a radius of 256.04 feet, describe an arc intersecting the arc *P Q* in *R*. Join *O* and *R*, forming with *O P* the angle *P O R* = 37° 20'. The end of the line *S* is at Sta. 25 + 80, making the length of the last course 542 feet. In a similar manner, plat the notes for lines ***b*** and ***c***, which follow.

NOTES FOR LINE *b*.

Stations.	Angles.
22 + 40	End of Line
16 + 50	L 18° 20′
8 + 60	R 25° 14′
3 + 25	R 8° 10′
0	

NOTES FOR LINE *c*.

Stations.	Angles.
25 + 34	End of Line
19 + 94	L 51° 22′
14 + 81	R 21° 20′
10 + 38	R 39° 18′
4 + 13	L 64° 30′
0	

1148. To Lay Off an Angle By Its Bearing.—By this method of laying off angles, the direction of each line is referred to the magnetic meridian, or a north and south line. In platting a land or railroad survey, a line giving the direction of the magnetic meridian is drawn through each station at which a bearing is taken.

PLATE II.

1149. Plate II contains five angle lines, the angles of the three lines given in Fig. 1 being platted by *magnetic bearings*, and those in Fig. 2 by *tangents*. In Fig. 1, line ***a***, the distances are given in stations of 100 feet each; in lines ***b*** and ***c***, the distances are given in chains. The student will draw line ***a*** to a scale of 200 feet to the inch, and lines ***b*** and ***c*** to a scale of 2 chains (132 feet) to the inch. The notes of line ***a*** are given in this article.

Let A be the starting-point of the line, which we number station 0. Let the arrow NS give the direction of the magnetic meridian. Through A draw a meridian AB parallel to NS. The bearing of the first course is N 10° 15′ E. From the meridian passing through A, lay off this bearing angle with a protractor. The first course is 375 feet. Draw a line through *A* having the given bearing and scale the distance 375 feet. This will bring us to Sta. 3 + 75, which we denote by the letter *C*, where a bearing of N 60° E is taken. The end of this course is at Sta. 6 + 90. The length of the second course will, therefore, be the difference between 6 + 90 and 3 + 75, which is 315 feet. Through *C* draw a meridian *C D*, from which lay off the bearing angle of 60° and draw a line marking the second course. Scaling the distance 315 feet, we reach Sta. 6 + 90, which we call *E*. Here a bearing N 83° 30′ E is taken. Through *E* draw a meridian *E F*, and from it lay off the bearing N 83° 30′ E. The end of this course is at Sta. 10 + 40. Its length will, therefore, be the difference between 10 + 40 and 6 + 90, which is 350 feet.

NOTES FOR LINE a.

Stations.	Bearings.
28 + 15	End of Line
23 + 55	S 45° 00′ E
18 + 92	S 70° 45′ E
14 + 20	N 80° 30′ E
10 + 40	S 81° 20′ E
6 + 90	N 83° 30′ E
3 + 75	N 60° 00′ E
0	N 10° 15′ E

Scale off this distance from *E*, locating Sta. 10 + 40, which we call *G*. The bearing at *G* is S 81° 20′ E. Through *G* draw the meridian *G H*. As the bearing is S E, the meridian will fall below the station, from which lay off the bearing S 81° 20′ E, and draw a line in the direction of this course. The next bearing is taken at Sta. 14 + 20. The length of the course is, therefore, the difference between 14 + 20 and 10 + 40, which is 380 feet. Call Sta. 14 + 20, *K*. Through *K* draw the meridian *K L*. The bearing here is N 80° 30′ E. From the meridian *K L*, lay off this bearing and draw a line in the direction of the course. In a similar manner locate the remaining stations and lay off the remaining bearings of the line. The bearing of each course should be distinctly written above it, the letters reading in the same direction in which the line is measured.

1150. The notes for the lines ***b*** and ***c*** are as follows:

NOTES FOR LINE *b*.

Stations.	Bearings.	Distances.
1	N 40½° E	4.22 chains.
2	N 65¼° E	6.75 chains.
3	S 75½° E	8.70 chains.
4	S 45¼° E	6.60 chains.
5	S 20¼° W	5.18 chains.

NOTES FOR LINE *c*.

Stations.	Bearings.	Distances.
1	S 47° E	6.60 chains.
2	N 20¼° E	8.80 chains.
3	S 80° E	4.32 chains.
4	S 20° E	6.54 chains.
5	N 65½° E	7.48 chains.

1151. The regular 100-foot stationing is used in railroad and highway surveying, but in land surveying the lengths of the courses are given in surveyor's chains. As the fractional parts of chains are given decimally, the length of each course is readily scaled on the plat with a decimal scale. The notes of line ***b*** are platted as follows: The starting-point is called Sta. 1, and is so marked on the plat. Call Sta. 1, *A*. Through *A* draw a meridian *A B*, and from it lay off the first bearing, N 40½° E. Lay off the first course, which is 4.22 chains in length, to a scale of 2 chains to the inch, locating Sta. 2. Call Sta. 2, *C*. Through *C* draw a meridian *C D*, and lay off the given bearing, N 65¼° E. The course with this bearing is 6.75 chains in length, which scale off, locating Sta. 3. In similar manner plat the remainder of line ***b***, and also line ***c***. Mark distinctly each course, giving its direction and length, being careful that the figures and letters shall read in the same direction in which the line is being run.

1152. To Lay Off an Angle By Its Tangent.—In laying off an angle by its tangent, the line from which the angle is turned is prolonged to a distance equal to the length of the assumed radius. The length of the tangent of the given angle is then found in terms of the assumed radius and the tangent platted. A line joining the angular point with the extremity of the calculated tangent will give the direction of the required line, which is then measured to the given scale.

Let *A B* in Fig. 204 be the given line, from which an angle of 30° 15′ is to be laid off to the right at the point *B*. Produce *A B* to *C*, making *B C* = 400 feet, the length of the assumed radius. The tangent of 30° 15′, in terms of a radius 1, is .58318, which, multiplied by 400 feet, the length of the assumed radius, gives 233.27 feet, the length of the required tangent. At *C*, erect a perpendicular to *B C* 233.27 feet in length, equal to the calculated tangent. Denote the end of this tangent by *C′*. Join *B* and *C′*. The angle *C B C′* = 30° 15′, the given angle, and the line *B C′* is the required line.

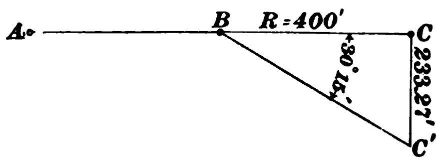


FIG. 204.

1153. The notes in this article, which are platted in Plate II, Fig. 2, the student will plat to a scale of 200 feet to the inch.

The notes of line *a* are platted as follows: Having adjusted the paper to the drawing-board and drawn a meridian *N S*, fix the starting-point *A*, which number 0. The first course is 500 feet in length, which plat by drawing a meridian *A B* through Sta. 0, and scale off 400 feet, equal to the length of the assumed radius *A C*. The bearing of the first course is N 43° 10′ E. The tangent of 43° 10′ is .93797, which, multiplied by 400 feet, the length of the radius, gives

375.19 feet, the length of the required tangent. Erect a right perpendicular to *A B* at *C*, and on this perpendicular scale off the tangent 375.19 feet, calling the extremity of the tangent *D*. Draw *A D*. The angle *C A D* will be 43° 10′ The first course is 500 feet in length, which scale off on the line *A D* at 200 feet to the inch, locating Sta. 5 + 00 at *E*. The angle at Sta. 5 + 00 is 43° 30′ to the right. Produce *A E*, and scale off the radius *E F* = 400 feet. The

NOTES FOR LINE A.

Stations.	Angles.	Bearings.
25 + 00	End of Line	
19 + 97	L 40° 10′	N 78° 45′ E
13 + 22	R 32° 15′	S 61° 05′ E
5 + 00	R 43° 30′	N 86° 40′ E
0		N 43° 10′ E

tangent of 43° 30′ = .94896, which, multiplied by 400 feet, gives 379.58 feet, the length of the required tangent. Erect a right perpendicular to *E F* at *F*, and scale off the tangent 379.58 feet, locating the point *G*. Draw *E G*. The angle *F E G* is 43° 30′, and the line *E G* the required line, the bearing of which is N 86° 40′ E. Produce *E G* to *H*, making *E H* = 1,322 − 500 = 822 feet in length.

The line changes direction again at Sta. 13 + 22, where an angle of 32° 15′ is turned to the right. Denote Sta. 13 + 22 by *H*. Produce *E H* 400 feet, equal to the assumed radius, calling its extremity *K*. The tangent of 32° 15′ = .63095, which, multiplied by 400 feet, gives 252.38 feet, the length of the required tangent. Erect a right perpendicular to *H K* at *K*, and on that perpendicular scale off the tangent 252.38 feet, locating the point *L*. Join *H* and *L*. The angle *K H L* is 32° 15′, and the bearing of *H L* is S 61° 05′ E.

The line changes direction again at Sta. 19 + 97. Call this station *M*. The angle at this point is 40° 10′ to the left. Produce *H M* 400 feet to *N*, and at *N* erect a left perpendicular to *M N*. The tangent of 40° 10′ is .84407, which, multiplied by 400 feet, gives 337.63 feet, the length of the required tangent. On the perpendicular to *M N*, scale off this tangent, locating the point *O*. Join *M* and *O*. The

NOTES FOR LINE *b*.

Stations.	Angles.	Bearings.
27 + 47	End of Line	
20 + 97	R 42° 20′	S 34° 25′ E
13 + 73	R 49° 10′	S 76° 45′ E
7 + 63	L 62° 15′	N 54° 05′ E
0		S 63° 40′ E

end of the line is Sta. 25 + 00. The length of the last course is readily found by subtracting 19 + 97 from 25 + 00. The difference, 503 feet, is scaled off on *M O*, locating the point *P*, the end of the line. The bearing of *M P* is N 78° 45′ E. In a similar manner plat the notes of line ***b***.

TO PLAT A SURVEY.

1154. The platting of a survey made with a compass consists in drawing on paper the lines and angles which have been measured on the ground. The angles are laid off and the lines drawn to scale. There are two methods of doing this, which are at present in general use; namely, platting by bearings and platting by latitudes and departures.

1155. Platting By Bearings.—This method consists in laying out the courses in a manner similar to that shown in Plate II, Fig. 1.

As an example of platting by this method, take the survey notes given in Art. **1159,** and proceed as follows:

With a fine needle, mark a point at a convenient spot on the paper to represent Sta. 1 (Fig. 205), and about the point draw a small circle. The location of the point must be so chosen that the plat will not run off the paper, which can be readily done if the person doing the platting has a knowledge of the survey itself. If he has not, a study of the

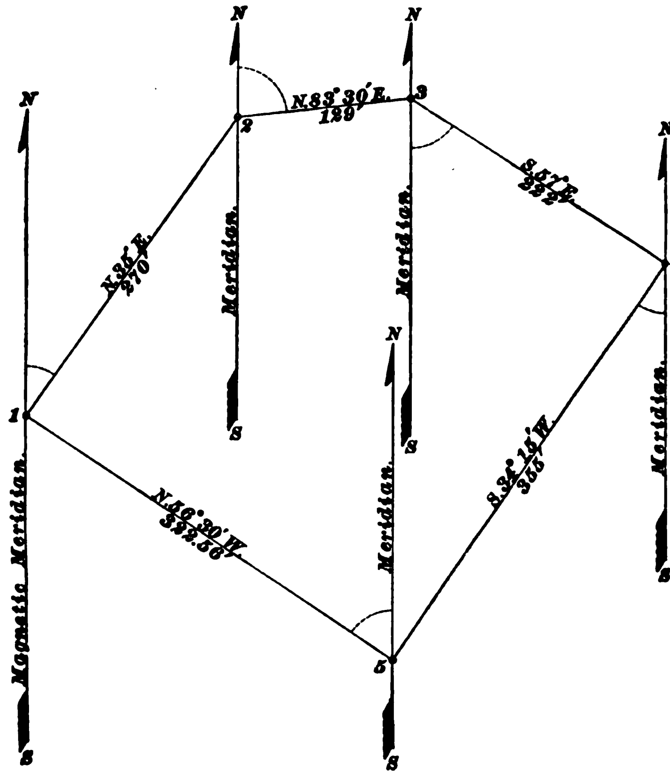


FIG. 205.

survey notes will show the approximate position for the point.

With a T square or a triangle, draw a line through the point to represent the direction of the compass-needle, or magnetic meridian. To draw the first course, N 35° E, place the protractor in the proper position, and at 35° from the north end make a dot with a lead-pencil. Through this dot and Sta. 1 draw a straight line, its approximate length being known from the measured distance to Sta. 2 (270 feet). The direction, or bearing, of this line is then N 35° E, and on it the length of the first line, or course, is laid off to scale from Sta. 1. This length is 270 feet, which drawn to a scale of 100 feet = 1 inch = $\frac{270}{100}$ = 2.7 inches from Sta. 1 to Sta. 2.

Through Sta. 2 draw another meridian; lay off the angle of the second course, N 83° 30′ E, as before, and set off the length of this course, 129 feet, from *2* to *3*.

Proceed in like manner for each course. When the last course, *5–1*, is platted, it should end precisely at the starting-point, Sta. 1, as the survey would do if it were a closed survey, as of a field. If the plat does not "close," or "come together," it shows an error has been made either in the original survey or in the work of platting.

1156. Platting By Latitudes and Departures.—In order to apply this method, the survey must first be traversed. **Traversing** a survey means simply finding the latitudes and departures of the different courses, as explained before.

The latitude = distance × cosine of bearing.

The departure = distance × sine of bearing.

1157. If the survey is a continuous one around a tract, ending at the place of beginning, the sum of the northings should equal the sum of the southings; and the sum of the eastings should equal the sum of the westings. Or, in other words, the sum of all the latitudes north should equal the sum of all the latitudes south; and the sum of all the departures east should equal the sum of all the departures west. It is evident that by coming back to the place of beginning, the surveyor has traveled the same distance north as he has south, and the same distance east as he has west. But if sights have been taken from stations of the main survey to stations or objects off the regular lines of the survey, care must be taken not to include their latitudes and departures in the additions necessary to prove a tie, or to determine the position of other points in the main survey.

1158. The most accurate way to construct a map is to traverse the survey and place all stations on it by the traversed distances, or to at least put a number of the principal stations on the map by the traversed distances, and use the protractor only to plat the intermediate stations.

Station.	Bearing.	Distance.	Latitude. N.	Latitude. S.	Departure. E.	Departure. W.	Total Latitude. N.	Total Latitude. S.	Total Depart'e. E.	Total Depart'e. W.
1–2	N 35° E	270.0	221.0		155.0		221.0		155.0	
2–3	N 83° 30′ E	129.0	15.0		128.0		236.0		283.0	
3–4	S 57° E	222.0		121.0	186.0		115.0		469.0	
4–5	S 34° 15′ W	355.0		293.0		200.0		178.0	269.0	
5–1	N 56° 30′ W	322.56	178.0			269.0	0	0	0	0
			414.0	414.0	469.0	469.0				

1159. Where platting is done from the traverse notes, the map is divided very carefully into squares, the length of the sides of the square depending upon the scale to which the survey is to be platted. For a scale of 100 feet = 1 inch, squares whose sides are 10 inches are most convenient. Two sides of each square are meridian lines, while the other two are east and west lines.

The accompanying example will illustrate platting by use of the traversed distances:

1160. There are two methods by which the survey can be platted by latitudes and departures, the first (and poorer one) being in some respects similar to the method of platting bearings, as each new station is located by first drawing a meridian line through the preceding one and locating the new station from it.

1161. Begin platting the survey, Fig. 206, by choosing a convenient point for Sta. 1. Then, draw the meridian through the point as already described. On this meridian, lay off due north the latitude of the course *1–2* to the required scale of 100 feet = 1 inch. The distance to be laid off (see the notes in the accompanying table) equals $\frac{221}{100}$ = 2.21 inches. Then, on the line passing due east through Sta. 1, lay off a distance equal to the departure of the course. This distance would be = $\frac{155}{100}$ = 1.55 inches. Complete the parallelogram, as shown in the figure, and draw the diagonal *1–2*, which is the required line.

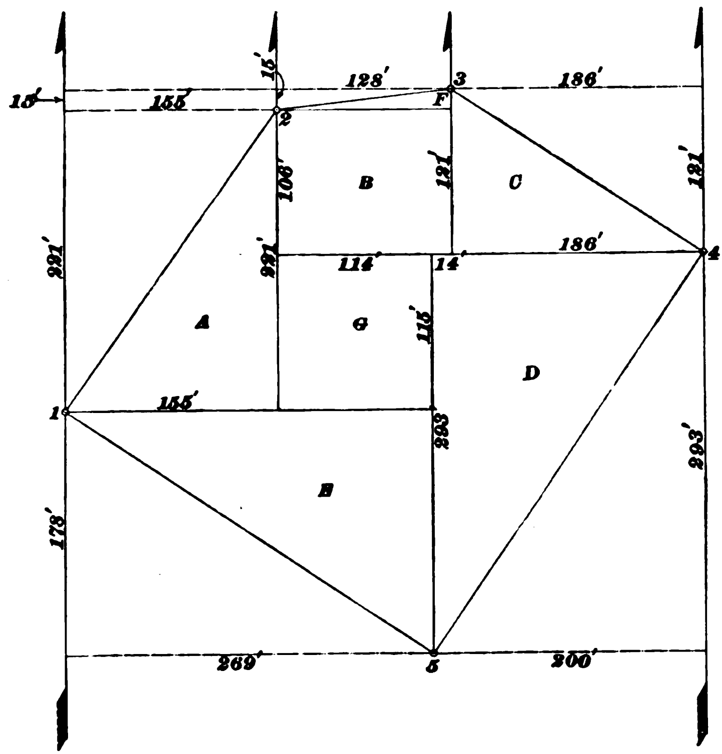


FIG. 206.

Then pass a meridian through Sta. 2, and also a due east and west line. On the meridian, lay off the north latitude, 15 feet, to scale = $\frac{15}{100}$ = .15 inch; and on the east and west line, the east departure, 128 feet, to scale = $\frac{128}{100}$ = 1.28 inches. Complete the parallelogram, as shown, and draw the diagonal *2–3*, which is the required line. And so on for every other case until the plat is completed.

1162. The second method of platting by latitudes and departures gives results that are as accurate, if not more so, than can be obtained by any other method. Here the *total* latitudes and departures are employed; consequently, each station is located independently of every other one (except the first), and no errors can be carried from one location to another.

Select Sta. 1, as described for Fig. 206, and draw a meridian line through it, and another line at right angles to it,

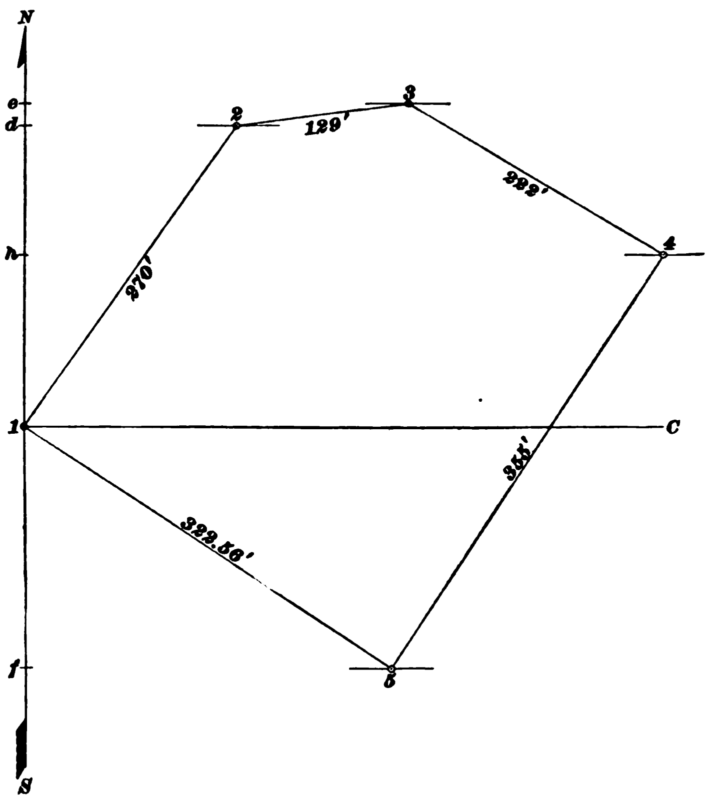


FIG. 207.

running due east and west. To locate Sta. 2, Fig. 207, measure on the meridian line passing through Sta. 1 a distance (see "notes") $1\,d = \frac{221}{100} = 2.21$ inches. Then lay your scale upon the map *above* the point *d*, its zero-mark upon the meridian line. Place the parallel ruler along the east and west line *1 C*, and then move it until its edge touches the point *d*. Draw a faint line along the edge of the parallel ruler at a distance of *about* $\frac{155}{100}$ = 1.55 inches from the point *d*, the approximate location of Sta. 2 being shown by the scale placed upon the map. Then remove the parallel ruler, and, placing the zero of the scale carefully at the point *d*, mark off on the faint line just drawn a point *exactly* 1.55 inches from *d*. This point is Sta. 2. To check the correctness of the work, measure the distance from *1* to *2*, and if no mistake has been made it will scale 2.7 inches, representing $2.7 \times 100 = 270$ feet.

1163. By drawing a short, faint line, as described, we have but very little erasing to do with a rubber. If, instead of placing the scale as described, we draw a line of indefinite length eastward from *d*, considerable erasing would have to be done after Sta. 2 is located. On a mine map which is somewhat soiled from frequent handling, the "spotted" effect produced by the cleaner parts where erasing has been done is very unsightly, and should be avoided.

To locate Sta. 3, (1) measure off *1 e* (Fig. 207) equal to $\frac{236}{100}$ = 2.36 inches; (2) place the scale on map above *e*, zero of scale on meridian line; (3) place the parallel ruler along *1 C*; (4) move parallel ruler until its edge passes through *e*; (5) draw a faint line about $\frac{283}{100}$ = 2.83 inches from *e*; (6) remove the parallel ruler; (7) place zero of scale at *e* and locate Sta. 3 on the faint line at exactly 2.83 inches from *e*; (8) measure distance from *2* to *3* as a check, which should scale $\frac{129}{100}$ = 1.29 inches. Similarly, for Sta. 4, measure *1 h* = $\frac{115}{100}$ = 1.15 inches, and locate Sta. 4 on the faint line at a distance of $\frac{469}{100}$ = 4.69 inches from *h*. Then apply the scale to check the distance *3–4* equal to $\frac{222}{100}$ = 2.22 inches.

1164. The Closing Line of a Survey.—In mining work, it is frequently necessary to drive an entry or other opening to connect two working places. For this purpose a survey is made from one place to the other, and when the notes have been traversed, the length and the bearing of the connecting line are calculated. To illustrate this point, take the survey given in Art. **1159,** and suppose the survey to have ended with Sta. 5. When the latitudes and departures have been calculated for the several courses, the notes will be as follows:

Station.	Bearing.	Distance.	Latitude.		Departure.	
			N.	S.	E.	W.
1–2	N 35° E	270.0	221.0		155.0	
2–3	N 83° 30′ E	129.0	15.0		128.0	
3–4	S 57° E	222.0		121.0	186.0	
4–5	S 34° 15′ W	355.0.		293.0		200.0
			236.0	414.0	469.0	200.0

1165. From these notes it is evident that Sta. 5 is 414.0 – 236.0 = 178.0 feet south and 469.0 – 200.0 = 269.0 feet east of Sta. 1. The calculation of the course and distance from Sta. 5 to Sta. 1 is then simply the solution of a right-angled triangle, as shown in Fig. 208.

The distance from Sta. 5 to Sta. 1 in a straight line is the hypotenuse of a right-angled triangle whose base is 269 feet and whose height is 178 feet. Then, $178^2 + 269^2$ = square of distance *1* to *5*, or 31,684 + 72,361 = 104,045. Then, the square root of 104,045 = 322.56 +. We now have the three sides of a triangle, and we want the angle at Sta. 1.

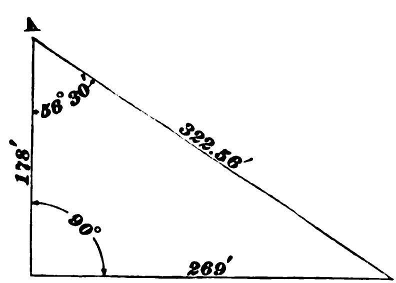


FIG. 208.

$$\text{Sine of angle } A = \frac{269}{322.56}$$

= .83395, which is the sine of the angle which the line joining Stas. 5 and 1 makes with the meridian. By consulting a table of natural sines, we find that it is the sine of an angle of 56° 30′, or, more exactly, a trifle more than 56° 30′, but as the difference is less than half a minute, we call it 56° 30′. Now, this means that Sta. 5 is 56° 30′ southeast of Sta. 1; therefore, to connect Sta. 5 with Sta. 1, the opening must be driven a distance of 322.56 ft. on a course N 56° 30′ W.

1166. To Find the Area of Ground Surveyed.—There are several methods of calculating the areas of surveys. Only one, and, perhaps, the most accurate one, need be described. This method depends upon dividing the land into various triangles, rectangles, or trapezoids, as the case may be, calculating the area of each one separately, and taking the sum of the results as the total area.

In order to proceed with this method, it is first necessary and essential to accurately plat the surveys to scale, by means of latitudes and departures, as has been shown. It will then be found easy and convenient to divide the survey into triangles and rectangles; viz., *A*, *B*, *C*, *D*, *E*, *F*, *G*, Fig. 206.

To calculate these various areas, it must be borne in mind that:

The area of a triangle = its base × ½ the altitude.

The area of a rectangle = its base × altitude.

The area of the triangle $A = \frac{155 \times 221}{2} = 17{,}127.5$ sq. ft.

The area of the triangle $C = \frac{186 \times 121}{2} = 11{,}253.0$ sq. ft.

The area of the triangle $D = \frac{200 \times 293}{2} = 29{,}300.0$ sq. ft.

The area of the triangle $E = \frac{269 \times 178}{2} = 23{,}941.0$ sq. ft.

The area of the triangle $F = \frac{128 \times 15}{2} = 960.0$ sq. ft.

The area of the rectangle $B = 128 \times 106 = 13{,}568.0$ sq. ft.

The area of the rectangle $G = 115 \times 114 = 13{,}110.0$ sq. ft.

Total area $A + B + C + D + E + F + G = 109{,}259.5$ sq. ft.

In a similar manner, the area of any irregular piece of ground can be calculated.

It very frequently happens that, instead of all the subdivisions of the plat having the regular form of triangles and rectangles, some are found which are trapezoids. This, however, does not alter the method in the slightest degree. The area of a trapezoid, being equal to one-half the sum of its parallel sides multiplied by its altitude, can be easily calculated.

MINE SURVEYING AND MAPPING.

(PART 2.)

TRANSIT SURVEYING.

THE TRANSIT.

DESCRIPTION OF THE TRANSIT.

1167. The **transit** is an instrument for measuring horizontal and (when furnished with a vertical circle) vertical angles. It is the only instrument that should be used for measuring angles in any survey where great accuracy is desired. The advantages of a transit over a vernier compass are mainly due to the use of a telescope. By its use, angles can be measured either vertically or horizontally, and, as the vernier is used throughout, extreme accuracy is secured.

1168. The verniers on a transit differ from those on a compass, particularly in the manner of reading them, though the principle of construction is the same in both. Where the full circle (or *limb*) of a transit is graduated to half degrees (30 minutes), 30 spaces on the vernier are equal to 29 half-degree spaces on the limb. Consequently, each vernier division is *smaller* than the smallest division on the limb, a circumstance which is characteristic of the *direct* vernier used on transits.

1169. The illustration, Fig. 209, shows a first-class transit, in which *a* is a dust and rain guard to object-slide of telescope. The object-slides of all telescopes are necessarily exposed to flying dust, grit, etc.; this settles on the slide and is carried into the main tube of the telescope, rapidly wearing the tube and the slide, destroying both the accurate projection of the object-glass in a straight line and the true position of the line passing through the intersection of the cross-hairs and the optical center of the object-glass. This line is called the **line of collimation.** Dampness and rain are also admitted inside the telescope, dimming the glasses and settling on the cross-wires. This arrangement *a* is a perfect safeguard against all these evils. The parts lettered *f* and *g* are also dust-guards to the leveling-screws *K*. The vernier plate tangent screw, or slow-motion screw *b*, has also a dust and rain guard.

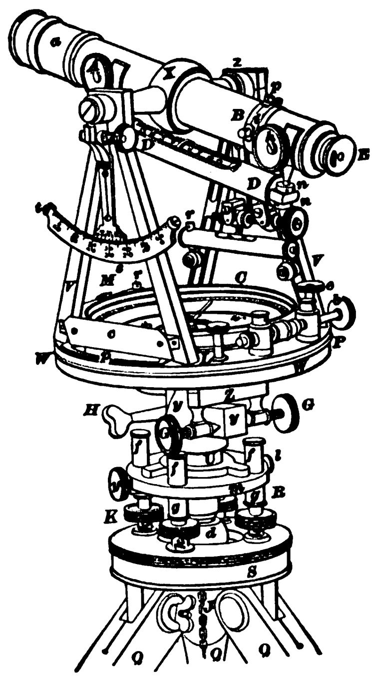


FIG. 209.

Ivory reflectors *c* are placed over the glass windows of the two verniers *p*. The second vernier *p* and its ivory reflector *c* are on the opposite side of the horizontal plate *P*, between the standards *V*, and of course can not be shown in the figure.

The lower ends of the leveling-screws *K* are so made as to be segments of a sphere. They work and are concealed in the sockets *k* which rest on the plate *S*. By this means, while using the leveling-screws *K*, a smooth, equable motion is obtained without indenting the plate on which they rest, and they also facilitate the use of the plummet-shifting plate *d*. By means of this plate the plummet of the transit can be set precisely over a point on the ground after approximately setting the plummet by means of the tripod legs. The plummet is suspended from the center of the instrument at *J*.

The wooden tripod legs *Q* are attached to the brass checks of the tripod head on which the plate *S* is screwed. Each leg is tightened or loosened by merely turning the threaded wing nut near *J*. A long level-tube *D D* is placed under the telescope for leveling, etc.

The graduated vertical arc *s* has two rows of figures, the one for angles of elevation and the other for angles of depression. The telescope can be set perfectly level by means of the long level-tube *D D* and the milled-headed tangent screw on the standard of the telescope near *n*. To read an angle of depression, the vernier arm is made to touch the screw *t* at the left-hand end of the vertical arc *s*; the zero (0°) lines of the vertical arc and the vernier arm will then agree. *Then clamp the vernier arm to the axis of the telescope by means of its milled-headed screw.* On depressing the telescope, the *vernier* arm will show the angle passed over.

The ring *B* surrounding the telescope carries four capstan-headed screws, one of which is shown at *p* and another at *i*; the remaining two can not be shown, for one is diametrically opposite *p*, and the other diametrically opposite *i*. The diametrical lines joining these capstan-headed screws are perpendicular to each other. These capstan-headed screws are to be used in adjusting the cross-wires of the telescope to the optical center of the telescope. The small holes in the head of the screw *p* are for the purpose of admitting a small steel pin by which the screw can be turned, in case adjustment is necessary. The capstan-headed nuts *n* are used to adjust the long level-tube *D D*. One of these nuts must be loosened before the other is tightened.

The eyepiece of the telescope is shown at *E* and the axis of the telescope at *X*. One end *z* of the axis is provided with an adjustable block moved by a screw. This is for the adjustment of the cross-wires, so that they can be made to trace a vertical line as the telescope is revolved about its axis. The screw *L* is for *raising* or *lowering* the compass-needle. *C* is the compass-box, and *e* is a clamp-screw for binding the two plates together. After such binding, the upper, or vernier, plate can be moved very slowly by means of the tangent screw *b*. The clamp-screw *H* is used to clamp the lower socket.

The transit head is moved in the lower socket by the tangent or set-screws *G*. The large screw-cap *S* joins the instrument to the tripod. The capstan-headed screws *r* are used in adjusting the two levels *M*. The curved piece seen projecting *over* the level *M* at the front of the compass-plate is intended as a guard to protect the glass level-tube in *M* from accident.

To bring the line of sight to bear precisely upon any point of an object seen in the field of view of the telescope, two fine lines, called cross-hairs, are placed with their intersection at the common focus of the object-glass and eyepiece. The intersection of these cross-hairs can be seen through the eyepiece, and appears to be in the same position as that of the image of the distant object.

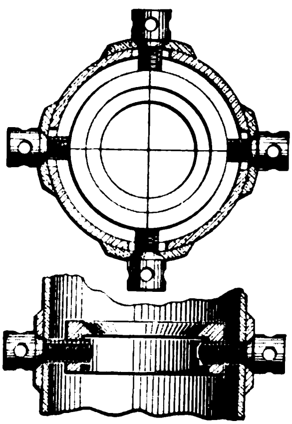


FIG. 210.

The cross-hairs are fastened to a thick brass ring *B*, placed within the telescope and held in position by capstan-headed screws let into this ring. See Fig. 210. They are placed at right angles to each other, the one being vertical and the other horizontal. The ring, together with the cross-hairs, can be moved by the capstan-headed screws. The cross-hairs are either of platinum wire, drawn very fine, or of spiders' threads. Platinum wire is best, as it is not affected by changes of temperature.

ADJUSTMENTS OF THE TRANSIT.

1170. The use of a transit tends to disarrange some of its parts, which detracts from the accuracy of its work, but in no way injures the instrument itself. Correcting this disarrangement of parts is called **adjusting the transit.**

1171. First Adjustment.—*To make the level-tubes parallel to the vernier plate.*

Plant the feet of the tripod firmly in the ground. Turn the instrument until one of the levels is parallel to a pair of opposite leveling-screws; the other level will be parallel to the other pair. Bring the bubble in each tube to the middle with the pair of leveling-screws to which the tube is parallel. Next turn the vernier plate half way around, that is, revolve it through an angle of 180°. If the bubbles have remained in the middle of the tubes, the levels are in proper adjustment. If they have not remained so, but have moved towards either end, bring them half way back to the middle of the tubes by means of the capstan-headed screws attached to the tubes, and the rest of the way back by the leveling-screws. Again turn the vernier plate through 180°, and if the bubbles do not remain at the middle of the tubes, repeat the correction. Sometimes the adjustment is made by one trial, but usually it is necessary to repeat the operation.

Each level must be adjusted separately.

1172. Second Adjustment.—*To make the line of collimation perpendicular to the horizontal axis that supports the telescope.*

With the instrument firmly set at *A*, Fig. 211, and carefully leveled, sight to a pin or tack set at a point *B*, about 400 feet distant and on level or nearly level ground.

Reverse the telescope, that is, turn it over on its axis until it points in the opposite direction, and set a point at about the same distance, which will be at *D*, for example, if this

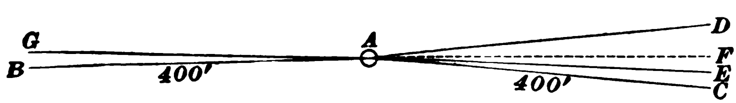


FIG. 211.

adjustment needs correction. Unclamp the vernier plate, and, without touching the telescope, revolve the instrument about its vertical axis sufficiently far to take another sight upon the point *B*. Then turn the telescope on its axis and locate a third point, as at *C*. Measure the distance *C D*, and at *E*, one-fourth of the distance from *C* to *D*, set the pin or tack. Move the cross-hairs, by means of the capstan-headed screws, until the vertical hair exactly covers the pin at *E*, being careful to move it in the opposite direction from that in which it appears it should be moved. Having done this, and then having reversed the telescope, the line of sight will not be at the point *B*, but at *G*, a distance from *B* equal to *C E*. Again sight to *B*, then reverse, and the pin will be at *F* in the same straight line with *A B*. It may be necessary to repeat the operation to secure an exact adjustment.

1173. Third Adjustment.—*To make the horizontal axis of the telescope parallel to the vernier plate, so that the line of collimation will revolve in a vertical plane.*

Sight to some point *A*, Fig. 212, at the top of a building, so that the telescope will be elevated at a large angle. Depress the telescope, and set a pin on the ground below at a point *B*. Loosen the clamp, turn over the telescope, and turn the plate around sufficiently far to take an approximately accurate sight upon the point *A*. Then, clamp the instrument, and again take an exact sight to the point *A*. Next, depress the telescope, and set another pin on the ground, which will come at *C*. The distance *B C* is double the error of adjustment. Correct the error by raising or lowering one end of the telescope axis by means of a small screw placed in the standard for that purpose. The amount the screw must be turned is determined only by repeated trials.

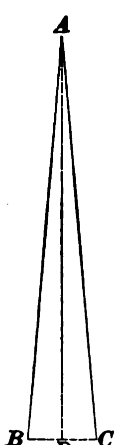


FIG. 212.

1174. Fourth Adjustment.—*To make the axis of the attached level of the telescope parallel to the line of collimation.*

Drive two stakes at equal distances from the instrument and in exactly opposite directions. Level the plate carefully, and clamp the telescope in a horizontal position, or as nearly so as possible. Sight to a rod placed alternately upon each stake, and have the stakes driven down until the rod-reading is the same on both stakes. When this condition is reached, the heads of the stakes are at the same level. Then, move the instrument beyond one stake, and set it up so that it will be in line with both stakes. Level the plate again, and elevate or depress the telescope so that, when a sight is taken to the rod held on first one stake and then on the other, the reading will be alike on both. In this position, the line of collimation is level, and the bubble in the level attached to the telescope should stand in the center of the bubble-tube. If it does not, bring it to the center by turning the nuts at the ends of the tube, being careful at the same time to keep the telescope in the position that gives equal rod-readings on both stakes.

HOW TO READ THE VERNIER.

1175. The vernier of a transit is direct reading, 30 divisions on the vernier scale being equal to 29 half-degree graduations on the limb.

1176. To read the vernier in Fig. 213, we look first for the zero of the vernier, and find it is between the 15° and

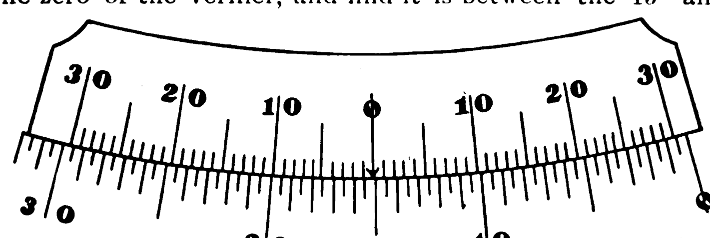


FIG. 213.

$15\frac{1}{2}°$ graduation of the limb. The outer graduations increase from 0° on the right to 10°, 20°, 30°, etc., on the left; that is, the increase is from *right to left*. Consequently, we start at the zero of the vernier and read *towards the left* until a graduation is reached which exactly coincides with a graduation on the limb. This graduation is found to be the sixth one from the zero, and indicates six minutes, since the vernier is divided into thirty minutes on each side of zero. These six minutes, being an *increase* from the 15°

graduation on the limb *towards* the 15½° graduation, must be added, and we obtain 15° + 06′ = 15° 06′ as the proper reading.

1177. To read the position of the vernier in Fig. 214, we look first for the zero of the vernier, and find it is between the 67° 30′ and 68° graduation on the limb. The

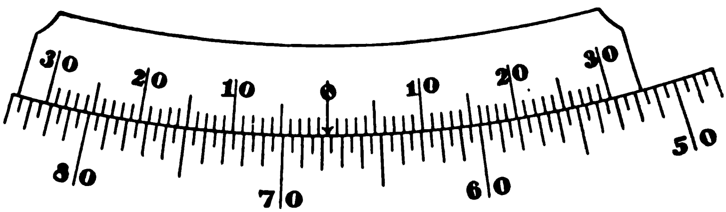


FIG. 214.

outer graduations increase from 50° on the right to 60°, 70°, 80°, etc., towards the left; that is, the increase is from *right to left.* Consequently, starting at the zero of the vernier, we also read *towards the left* until a graduation on the vernier is reached which exactly coincides with a graduation on the limb. This graduation is found to be the thirteenth one from the zero, and indicates thirteen minutes. These thirteen minutes, being an *increase* from the 67° 30′ graduation on the limb towards the 68° graduation, must be added, and we obtain 67° 30′ + 13′ = 67° 43′ as the proper reading.

1178. To read the position of the vernier in Fig. 215, we look first for the zero of the vernier, and find it is be-

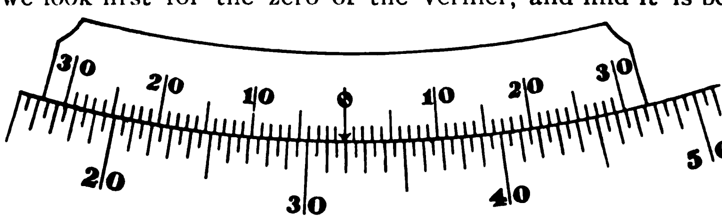


FIG. 215.

tween the 31° 30′ and 32° graduation of the limb. The outer graduations read from 20° on the left to 30°, 40°, 50°, etc., on the right; that is, they increase numerically from *left to right.* Consequently, starting at the zero of the vernier, we also read *towards the right* until a graduation on the vernier is reached which exactly coincides with a graduation on the limb. This graduation is found to be the twenty-eighth one from the zero, and indicates twenty-eight minutes. These twenty-eight minutes, being an *increase* from the 31° 30′ graduation on the limb towards the 32° graduation, must be added, and we obtain 31° 30′ + 28′ = 31° 58′ as the proper reading.

1179. To read the position of the vernier in Fig. 216, we look first for the zero of the vernier, and find it is between the 74° and 74° 30′ graduation of the limb. The outer graduations read from 60° on the left to 70°, 80°, and 90° on the right; that is, they increase numerically from *left to right*. Starting at the zero of the vernier, we also

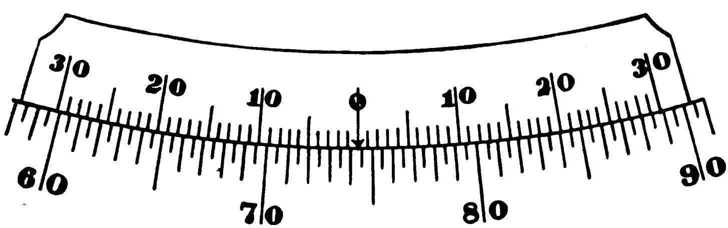


FIG. 216.

read *towards the right* until a graduation on the vernier is reached which exactly coincides with a graduation on the limb. This graduation is found to be the twenty-third one from the zero, and indicates twenty-three minutes. These twenty-three minutes, being an *increase* from the 74° graduation on the limb towards the 74° 30′ graduation, must be added, and we obtain 74° + 23′ = 74° 23′ as the correct reading.

MEASUREMENT OF HORIZONTAL ANGLES.

1180. A horizontal angle is an angle measured in a horizontal plane from one point to two other points, which may be, but usually are not, in a horizontal plane.

In measuring horizontal angles, it does not matter whether the telescope has to be elevated or depressed, since the telescope revolves on its axis in a vertical plane, and the angles *measured* are always the *horizontal projections* of the true angles.

Place the axis of the instrument exactly over the point at which the angle is to be measured. This is effected by means of a plumb-bob suspended from the center of the sliding center plate which fits into the head of the tripod. Level the instrument. Place the zero of the vernier near the zero of the fixed plate, and fasten the clamp-screw *c*, Fig. 209, of the vernier plate, thus binding the vernier plate and the fixed plate together. Turn the screw *b* until the zeros coincide *exactly*. Loosen the lower clamp *H*, thus permitting the instrument to be turned about its spindle; direct the telescope upon one of the objects very nearly, and without wasting time in trying to secure perfect bisection of the cross-hairs, tighten this lower clamp *H*, and make perfect bisection with the tangent screws *G*, *G*. This having been done, loosen the clamp-screw *c* of the vernier plate and direct the telescope upon the other object. Tighten *c*, and make the cross-hairs bisect this object by turning the tangent screw *b*. The angle sought is the angle passed over by the zero of the vernier plate, and is read off the fixed plate from its zero-point to the point that coincides with the zero of the vernier.

1181. *To find with the transit the azimuths of several successive courses with a given first course.*

Let *A B*, Fig. 217, be the given first course. Place the instrument at *B*, and carefully level it. Make the zero of the vernier coincide with the zero of the plate, and clamp the vernier plate to the horizontal limb by the clamp-screw *c*, Fig. 209. Direct the telescope (bubble-tube upwards) upon *A*, and clamp the limb by turning *H*. Turn the tangent screws *G*, *G* until the cross-hairs exactly bisect *A*. Reverse the telescope by revolving it upon

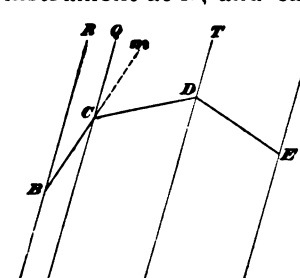


FIG. 217.

its horizontal axis, and it will point in the direction of *B R*, the prolongation of *A B*; unclamp the vernier plate by turning *e*, and direct the telescope to *C*. Clamp *e*, and make the bisection by turning *b*. The reading will be the angle *R B C* (the azimuth of *B C* with *A B*), and is equal to the number of degrees on the plate passed over by the zero of the vernier. Keep the vernier plate clamped, and remove the instrument to *C*, reverse the telescope on its *horizontal* axis so the bubble-tube is on top, loosen the lower clamp *H*, and sight to *B*. The fixed plate has now its zero-point in the direction of *Q P*, parallel to *A R*, as it had at *B*. Revolve the telescope on its horizontal axis so that it points in the direction *C m*; now loosen the vernier plate by the clamp-screw *e*, and direct the telescope on the point *D*. The reading will be the angle *Q C D*, which *C D* makes with *P Q*, or with its parallel *A R*, and is the azimuth of *C D* with *A B*. Proceed in like manner with any number of successive courses.

If this process be continued around to the point *A*, forming a closed polygon, and a sight be taken from *A* to *B*, the reading of the instrument will be, if the work be correct, the same as it was at the start, viz., 0°. Again, had a sight been taken from *E* to *B*, and the transit moved to *B* and sighted to *C*, the reading should be the same as it was when the instrument was first set up at *B* and directed to *C*. Surveyors take advantage of this principle to check the transit work, considering a **close in arc,** as it is termed, to be within one minute for good work.

The course *A B*, with respect to which the azimuths are taken, is called the **meridian of the survey.**

1182. In finding the *area* of a piece of ground, it is *not* necessary to know the *magnetic bearings* of the courses. It is sufficient to have the *bearings* of the several successive courses *with respect to one of the courses taken as a meridian.* These may be found from the azimuths, determined as above, in the following manner:

Let us suppose that the course taken to start with is a north and south line. We will consider this course as the meridian to which the azimuths of all succeeding courses will be referred.

Let the course *A B*, Fig. 218, be taken as this meridian. The azimuths of the succeeding courses are found by the transit, by the method already described, to be as follows:

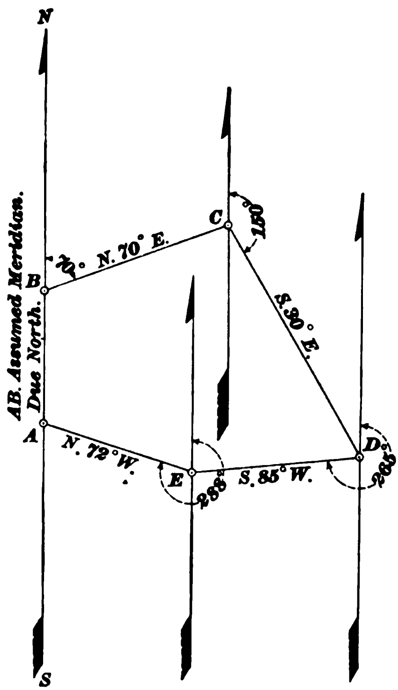


FIG. 218.

The azimuth of *A B* referred to *A B* = 0°.

The azimuth of *B C* referred to *A B* = 70°, angle turned to the right.

The azimuth of *C D* referred to *A B* = 150°, angle turned to the right.

The azimuth of *D E* referred to *A B* = 265°, angle turned to the right.

The azimuth of *E A* referred to *A B* = 288°, angle turned to the right.

In order, now, to reduce these azimuths to the bearings of the courses, we must bear in mind that these azimuths were obtained by turning off angles to the right by means of the vernier. In Fig. 219, the vernier is conceived as moving in the direction of the hands of a watch. It is plainly seen from it that:

When the azimuth is 0°, the course is due north.

When the azimuth is between 0° and 90°, the course lies in a N E direction.

When the azimuth is exactly 90°, the course is due east.

When the azimuth is between 90° and 180°, the course lies in a S E direction.

When the azimuth is exactly 180°, the course is due south.

When the azimuth is between 180° and 270°, the course lies in a S W direction.

When the azimuth is exactly 270°, the course is due west.

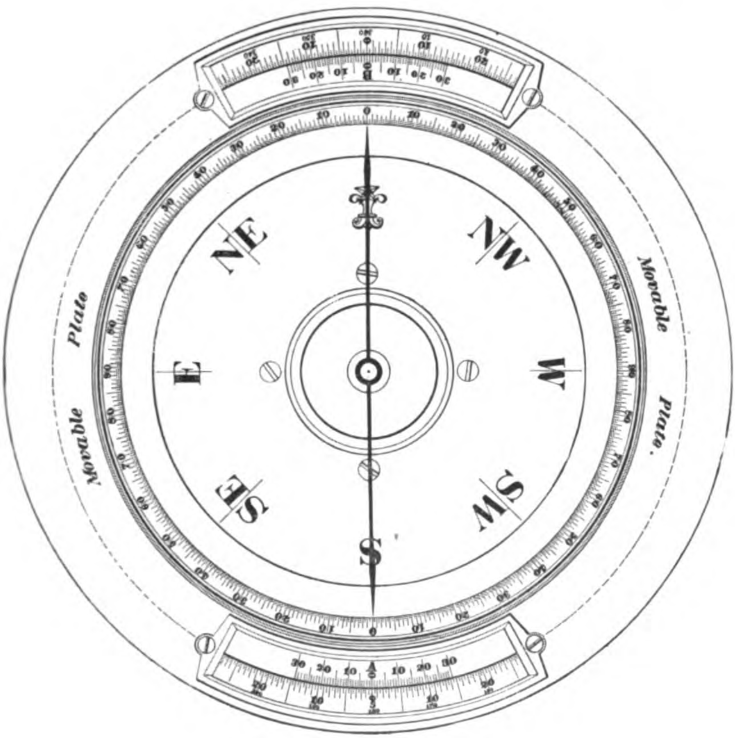


FIG. 219.

When the azimuth is between 270° and 360°, the course lies in a N W direction.

When the azimuth is exactly 360° or 0°, the course is due north, as before.

1183. Consulting Fig. 218, with *A B* as the meridian, the azimuth of *B C*, 70°, is also its bearing N 70° E.

The azimuth of *C D* = 150°, and its bearing with *A B* = 180° − 150° = S 30° E.

The azimuth of *D E* = 265°, and its bearing with *A B* = 265° − 180° = S 85° W.

The azimuth of $E A = 288°$, and its bearing with $A B = 360° - 288° =$ N 72° W.

Station.	Azimuth with $A B$.	Bearing with $A B$.
A	0°	North
B	70°	N 70° E
C	150°	S 30° E
D	265°	S 85° W
E	288°	N 72° W

Hence, we see, Fig. 218:

When the azimuth is less than 90°, the course has the same angle with the bearing in the N E direction.

When the azimuth lies between 90° and 180°, the azimuth subtracted from 180° gives the bearing of the course in a S E direction.

When the azimuth lies between 180° and 270°, the bearing is obtained by subtracting 180° from the azimuth, and the course is S W.

When the azimuth lies between 270° and 360°, the bearing is obtained by subtracting the azimuth from 360°, and the course is N W.

In the above, the meridian $A B$ was assumed to be a north and south line. It may or may not be. Suppose it is also desired to find the magnetic bearings of the courses from their azimuths. Then, if $A B$ is not really a north and south line, its magnetic bearing must be obtained by the compass, and then the magnetic bearings of the succeeding courses can be calculated by adding or subtracting the magnetic bearing of $A B$ to the azimuths of the given courses, according as the magnetic bearing of $A B$ is N E or N W of the meridian, or north and south line.

Above, $A B$ was assumed to be the meridian. Now, suppose $A B$, Fig. 220, was in reality a line bearing N 20° E.

Then, by adding this 20° to the azimuths of the successive courses when referred to *A B* as the meridian, we obtain the

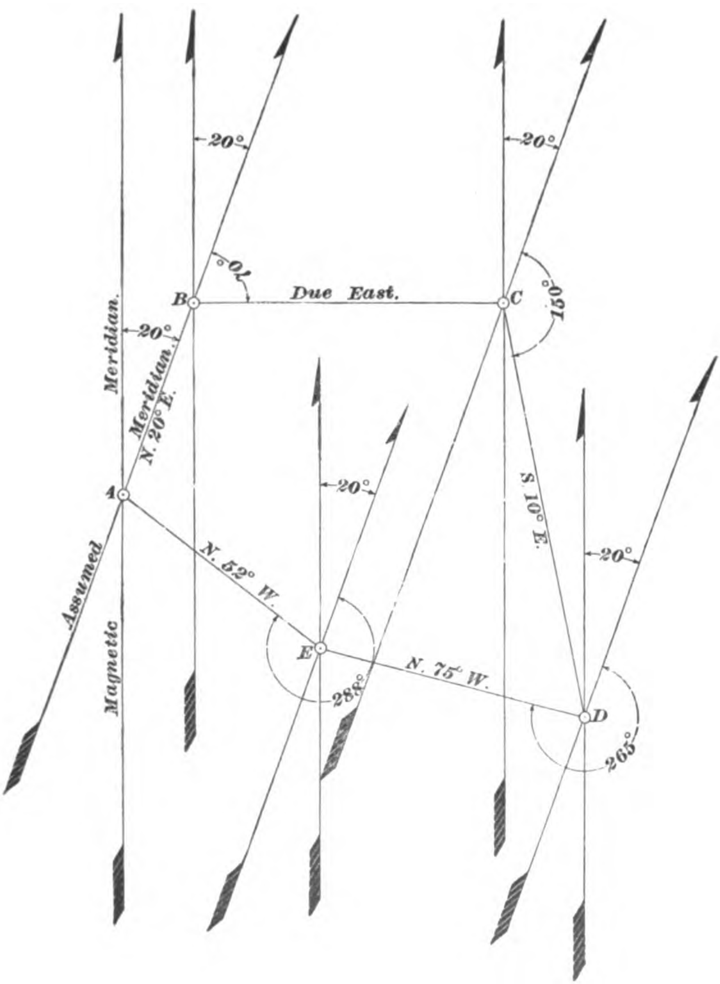


FIG. 220.

azimuths of the several courses referred to the *magnetic meridian*. Their magnetic bearings, or bearings when referred to this magnetic meridian, are as follows:

Station.	Azimuth with *A B*.	Azimuth with Magnetic Meridian.	Bearing with Magnetic Meridian.
A	0°	0° + 20°	N 20° E
B	70°	70° + 20°	Due East
C	150°	150° + 20°	S 10° E
D	265°	265° + 20°	N 75° W
E	288°	288° + 20°	N 52° W

If the magnetic bearing of *A B* were N 20° W, then the 20° would have to be subtracted from the various azimuths, and the bearings calculated as suggested above would become:

Station.	Azimuth with *A B*.	Azimuth with Magnetic Meridian.	Bearing with Magnetic Meridian.
A	0°	0° − 20°	N 20° W
B	70°	70° − 20°	N 50° E
C	150°	150° − 20°	S 50° E
D	265°	265° − 20°	S 65° W
E	288°	288° − 20°	S 88° W

OBSTACLES TO ALINEMENT.

1184. Fig. 221 shows a method of prolonging a line, as *A B*, when an obstacle, such as a house, rock, or tree, is

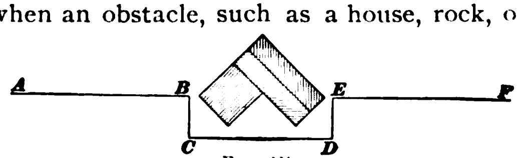


FIG. 221.

encountered. The instrument is set up at *B*, and a line is turned off at right angles to *A B* towards *C*, and the distance accurately measured. From *C* a line on the original course, or parallel with *A B*, is run towards *D*. From *D* a line is turned off at right angles towards *E*, which must be exactly the same distance from *D* as *C* is from *B*. Then, the point E being in exact line with A and B, the line $E F$ is run as far as desired on the same course as the line from A to B.

1185. Fig. 222 shows a method of prolonging the line $A B$ by means of an equilateral triangle. The instrument is set up at the point B, and an angle $D B C$ equal to 60° is turned off, and the line $B C$ laid off any convenient distance.

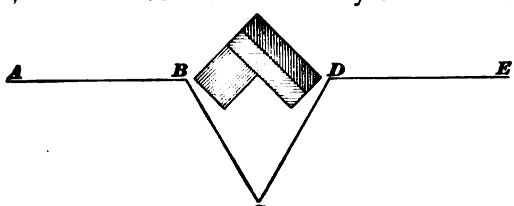


FIG. 222.

The line $C D$ is then laid off, equal to $B C$, and making an angle of 60° with it. Finally, at D the line $D E$ is laid off, making an angle of 60° with the line $C D$ prolonged. The line $D E$ will be the prolongation of the line $A B$.

TRANSIT AND SIDE NOTES.

1186. **Transit notes** comprise all angular measurements and the total distances between stations; they are kept by the transitman. **Side notes** include all linear measurements usually accompanied with a graphic sketch, especially where such sketch is more effective than written remarks for the draftsman. These notes are best kept by a special noteman who goes along with the chainmen.

There are a great many ways of recording the notes of a mine survey, each engineer having a favorite method. Whatever the method, it should be such a one as may be readily understood by any mine surveyor who might have occasion to use the notes. Completeness and neatness are the essential points to be observed.

1187. The best method of keeping transit and side notes is where each set of notes is kept in a separate book. This method is imperative where two men do the work at the same time. Each mine should have a separate set of books for ordinary and special work, in order that the engineer will have reference notes in a portable form. With a special book for each mine, no index is needed to find a certain survey, as the book for the proper mine can be taken, the date looked up, and the notes found.

1188. Form for Transit Notes.—It is impossible to give a general form for transit notes, as forms must vary according to conditions and requirements. Only such a form will be given here as will enable the student to understand the principles involved, whereby he can read other forms at sight, and even devise suitable forms for special requirements. By paying strict attention to the various forms used throughout this paper, the student will have no difficulty in understanding the need and advisability of having different forms for different work.

Suppose the transit is set up at *b*; with backsight to *a*; foresight to *c*; the azimuth, reading from *b* to *c*, 16° 38′; bearing N 16° 12′ E; and the distance *b c* is 532.87 feet measured on a pitch of + 15° 28′. Then a good form will be:

Station.	Azimuth.	Bearing.	Vertical Angle.	Actual Distance.	Remarks.
a					Sta. *b* is on
b–*c*	16° 38′	N 16° 12′ E	+ 15° 28′	532.87	west side of
					Lift No. 4,
					Eureka Mine.

1189. Form for Side Notes.—Side notes, quite unlike transit notes, can have a special form for all mine work, although engineers differ in opinion as to which is the best general form.

The taking of side notes in an ordinary outside survey is secondary to the instrumental work; while in underground work of ordinary character, the lines of the survey are skeletons upon which the side notes are built. The side notes are, therefore, of the highest value, and the forms for taking them should embrace the salient features of the underground work, so that the draftsman can reproduce them faithfully, even if he may never have been inside. The form shown in Fig. 223 is given as one of the best in use.

Here the object is to sketch approximately the underground workings, and to place the numbers of the stations and measurements upon the sketch in an intelligible manner. The number of a station is placed in a rectangle, and the distance of that station from the preceding one is placed immediately under the rectangle. The numbers in the center of the passage give the distance measured along the survey line from the preceding station, at which side measurements were taken. These measurements are recorded on each side of the central measurement.

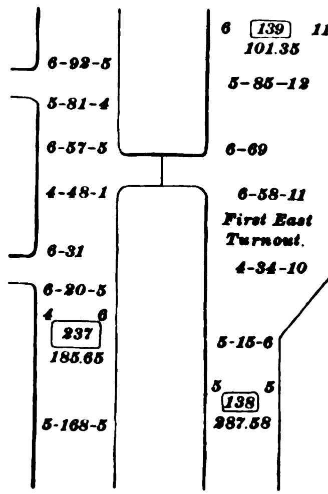


FIG. 223.

1190. When the corners of the pillars are rounded off at the intersection of two passageways, it is a good plan to note as the corner of each pillar the point of intersection of the two lines which represent the general direction of the two adjacent sides of the pillar. Thus, in Fig. 224 are shown two intersecting headings, or entries, *h*, with the corners of the pillars *p* rounded off. The points *c* which are taken as the corners of the pillars *p* are usually determined by the chainman in a practical way. He first stands in line with one side of the pillar, and then moves along this line until he gets in line with the adjacent side of the same pillar, and thus locates the corner.

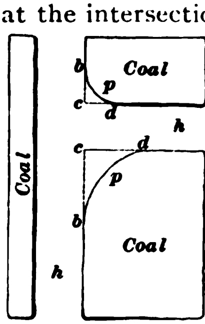


FIG. 224.

1191. For the transit work and the chaining, the following form of notes will answer for all purposes when the elevations are not carried with the transit:

Station.	Azimuth.	Magnetic Needle.	Bearing.	Vertical Angle.	Distance.

The magnetic needle readings are taken merely to check the azimuth readings. The bearings are determined from the azimuth readings in the office. Following are notes of an actual survey, with the magnetic needle and azimuth columns omitted:

Station.	Bearing.	Vertical Angle.	Distance.
1176–1177	S 40° 48′ W		115.20
1177–1253	S 13° 32′ E	+ 2° 40′	53.15
1253–1254	S 82° 39′ E		88.46
1254–1255	S 65° 08′ E	+ 0° 30′	144.68
1255–1256	S 79° 19′ E		122.81
1256–1268	N 79° 31′ W		86.60
1268–1269	S 9° 34′ W	+ 21° 59′	37.91
1269–1270	N 82° 37′ W		57.71
1270–1271	N 72° 25′ W	− 2° 04′	67.35
1270–face	S 0° 30′ E	+ 25° 15′	65.00
1271–1272	N 52° 48′ W	− 4° 18′	63.64
1271–1273	S 31° 32′ W	+ 19° 02′	89.06
1273–face	S 1° 45′ E	+ 37° 30′	65.00
1272–1274	N 88° 44′ W	+ 2° 29′	43.45
1272–1275	S 17° 58′ W	+ 16° 47′	105.38
1275–face	S 10° 10′ W	+ 30° 58′	95.00
1274–1276	N 87° 19′ W	− 1° 15′	46.77
1274–1277	S 19° 38′ W	+ 16° 38′	112.15
1277–face	S 15° 36′ W	+ 30° 45′	110.00
1276–1278	N 48° 33′ W	− 6° 10′	46.86
1278–1279	N 42° 12′ W	− 1° 20′	42.21
1278–1280	S 23° 31′ W	+ 16° 33′	121.73
1280–face	S 10° 32′ E	+ 25° 00′	100.00
1279–1281	N 12° 13′ E	− 1° 08′	44.09

TRAVERSED NOTES OF THE SURVEY.

Station.	Bearing.	Hor. Distance.	Latitude.		Departure.		Total Latitude.		Total Departure.	
			North.	South.	East.	West.	North.	South.	East.	West.
1176–1177	S 40° 48′ W	115.20		87.21		75.27		87.21		75.27
1177–1253	S 13° 32′ E	53.09		51.62	12.42			138.83		62.85
1253–1254	S 82° 39′ E	88.46		11.32	87.73			150.15	24.88	
1254–1255	S 65° 08′ E	144.67		60.84	131.26			210.99	156.14	
1255–1256	S 79° 19′ E	122.81		22.77	120.68			233.76	276.82	
1256–1268	N 79° 31′ W	86.60	15.76			85.15		218.00	191.67	
1268–1269	S 9° 34′ W	35.15		34.66		5.84		252.66	185.83	
1269–1270	N 82° 37′ W	57.71	7.42			57.23		245.24	128.60	
1270–1271	N 72° 25′ W	67.31	20.33			64.17		224.91	64.43	
1271–1272	N 52° 48′ W	63.46	38.37			50.55		186.54	13.88	
1271–1273	S 31° 32′ W	84.19		71.76		44.03		296.67	20.40	
1272–1274	N 88° 44′ W	43.41	.96			43.40		185.58		29.52
1272–1275	S 19° 58′ W	100.89		95.97		31.12		282.51		17.24
1274–1276	N 87° 19′ W	46.76	2.19			46.71		183.39		76.23
1274–1277	S 19° 38′ W	107.46		101.21		36.11		286.79		65.63
1276–1278	N 48° 33′ W	46.59	30.84			34.92		152.55		111.15
1278–1279	N 42° 12′ W	42.20	31.26			28.35		121.29		139.50
1278–1280	S 23° 31′ W	116.69		107.00		46.56		259.55		157.71
1279–1281	N 12° 13′ E	44.08	43.08		9.33			78.21		130.17

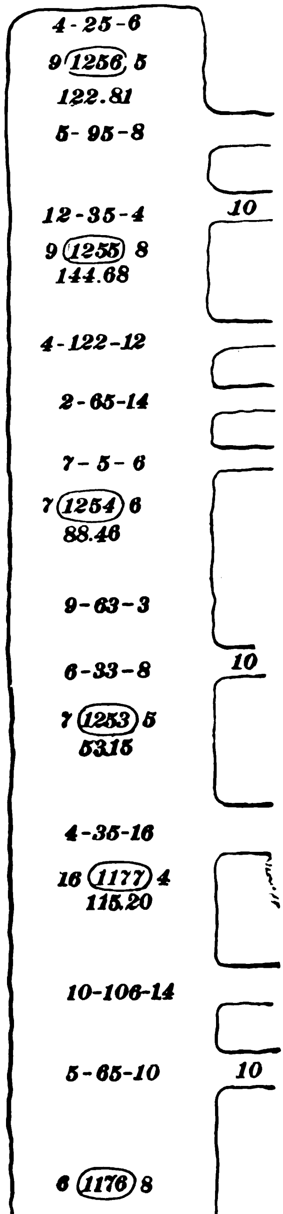


FIG. 225.

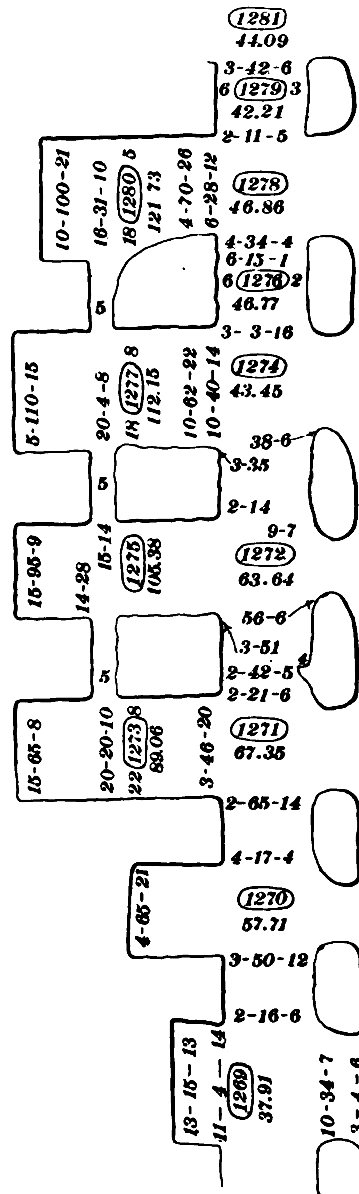


FIG. 226.

1192. It will be noticed that some vertical angles are recorded with the plus (+) sign and others with the minus (−) sign. The plus angles indicate that the measurements were made *up* the pitch, and the minus angles that they were made *down* the pitch. Both are, of course, reduced to a horizontal distance in the same way, namely, by multiplying the measured distance by the natural cosine of the angle of inclination. This reduction must be made for each inclined measurement before latitudes and departures can be calculated, for in these only horizontal distances can be used. Where no vertical angle is recorded, the measurement was made horizontally.

The traversed notes, except for the *face sights* (such as 1270–face, etc.), are given in Art. **1191.** It is well not to traverse a face sight, but to plat it with the protractor, which requires less work and is quite accurate. The total latitudes and departures are here calculated with reference to Sta. 1176 as the initial station of the survey, which is not, of course, the initial station in the original survey.

1193. Fig. 225 shows the side notes of the gangway. The chutes leading up into the breasts or chambers are of a uniform width of 10 feet; hence, this distance is recorded only occasionally.

Where distances have not been measured horizontally, it must be remembered that, before the side notes can be platted, the distances which locate them must first be reduced to horizontal distances. For example, the side notes, Fig. 226, show measurements between Sta. 1275 and the face at 15, 28, and 95 feet. The transit notes show the inclination upon which these distances were measured to be + 30° 58′; hence, before they can be platted, they must be reduced to their respective horizontal distances, namely, 13, 24, and 81 feet, the nearest even foot being sufficiently close.

It will be noticed that the course from Sta. 1256 to Sta. 1268 is almost a backsight on the course from Sta. 1255 to Sta. 1256. The foresight from Sta. 1255 to Sta. 1256 is

S 79° 19′ E, while the bearing from Sta. 1256 to Sta. 1268 is N 79° 31′ W. This difference can not be shown on a plan

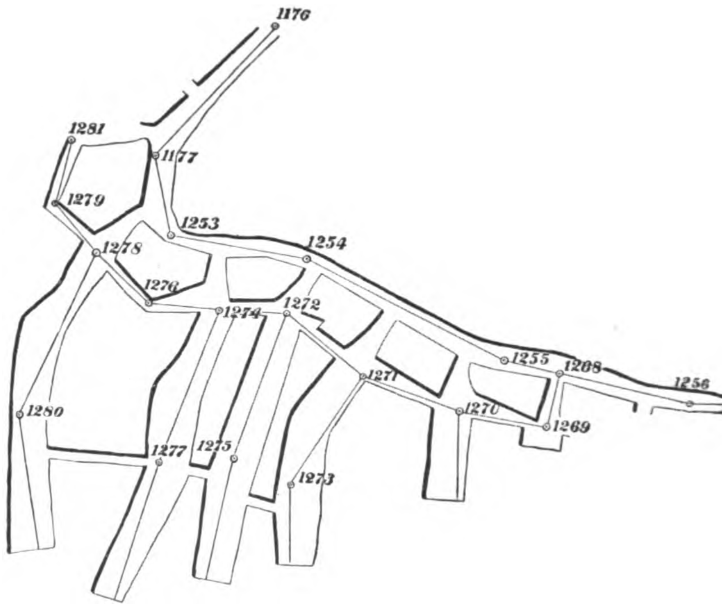


FIG. 227.

drawn to as small a scale as Fig. 227; hence, Stas. 1256, 1268, and 1255 on this plat appear to be in the same straight line.

CURVES.

1194. In addition to the sharp curves in mines, which must be put in with the instrument, particularly where rope haulage is employed, it often happens, when opening a new mine, that the mining engineer is called upon to survey a branch railroad from the main line to the mine, thus necessitating a knowledge of laying out curves.

A line of railroad consists of a series of straight lines and curves. In general, the straight lines, or, more properly, the *tangents*, are first located, and then they are united by curves best fitting the ground lying between the tangents.

There are certain limits of curvature prescribed for all roads, which must not be exceeded. These limits will depend upon conditions to be explained later. Railroad curves are circular, and are divided into *simple*, *compound*, and *reversed* curves.

1195. A **simple curve** has but one radius, as curve *A B*, Fig. 228, whose radius is *A C*. **A compound curve,**

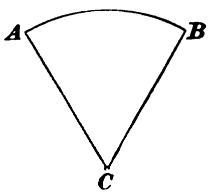


Fig. 228.

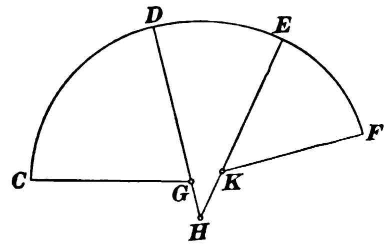


Fig. 229.

shown in Fig. 229, is a continuous curve of two or more arcs of different radii, as *C D E F*, which is composed of the arcs *C D*, *D E*, and *E F*, whose respective radii are *G C*, *H D*, and *K E*. A **reversed curve** (see Fig. 230) is a continuous curve composed of two arcs *L M* and *M N*, of the same or of different radii, described in the opposite directions, and having a common point *M*, called the point of reversal.

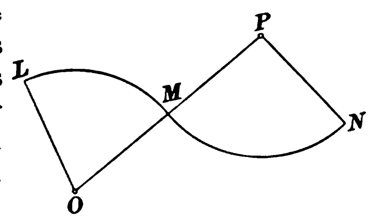


Fig. 230.

Reversed curves, though common in the early days of railroad building in the United States, are now condemned for roads of standard gauge, and are only admitted for narrow-gauge roads, when cheapness of construction is the first requirement.

1196. Geometry of the Circle.—Before attempting to lay out curves, a knowledge of geometry relating to the circle must be mastered. The following propositions are of special importance:

1. A tangent to a circle is perpendicular to the radius

drawn through its tangent point. Thus, *A E*, Fig. 231, is perpendicular to *B O*, and *C E* is perpendicular to *C O*.

2. Two tangents drawn to a circle from any point have equal lengths, and if a chord be drawn joining these points, the angles between the chord and the tangents are equal. Thus, *B E* and *C E* are equal, and the angles *E B C* and *E C B* are equal.

3. An acute angle, subtended by a chord and having its vertex in the circumference of the circle, is equal to half

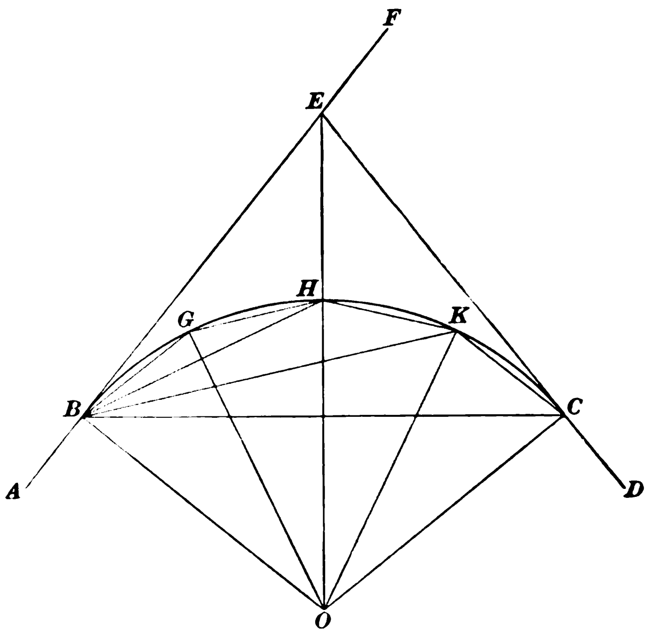


FIG. 231.

the central angle subtended by the same chord; thus, the angle *G B H* is equal to one-half the angle *G O H*.

4. An acute angle, included between a tangent and a chord, and whose vertex is in the circumference, is equal to one-half the central angle subtended by the chord. Thus, the angle *E B G*, being included between the tangent *E B* and the chord *B G*, and having its vertex *B* in the circumference, is equal to half the central angle *B O G*, subtended

by the same chord *B G*. In like manner, the angle *E B H* is equal to half the central angle *B O H*

5. Equal chords subtend equal angles at the center of a circle, and, also, at the circumference, if the angles are inscribed in similar segments. Thus, if the chords *B G*, *G H*, *H K*, and *K C* are equal, then the angles *B O G*, *G O H*, *H O K*, and *K O C* are equal, and the angles *G B H*, *H B K*, and *K B C* are equal.

6. The angle of intersection of two tangents equals the central angle subtended by the chord uniting the tangent points. Thus, the angle *C E F* is equal to the angle *B O C*.

1197. Intersection of Tangents.—Let *A B* and *C D*, Fig. 232, be tangents whose point of intersection and

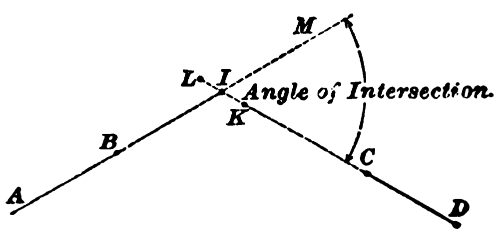


FIG. 232.

the angle which they make with each other are to be determined. First, set up a flag or stake at *B*, and another at *A*, or some other point in the line *A B*. Set up the transit at *C*, backsighting to *D*. Reverse the instrument. Have a flagman hold a rod in the line *C D*, at the same time putting himself in range with the stakes at *A* and *B*. With a little practice, he can nearly determine the intersection *I* of the two lines. Then drive two stakes *K* and *L* firmly in the line *C D*, one on each side of the point *I*, their distance from the point *I* being roughly determined by the size of the angle *A I D*. Carefully center these stakes, driving a tack half its length in each center. Stretch a cord between these tacks. Next, set up the instrument at *B*, backsighting to *A*. Reversing the telescope, set a flag at *I*, which will be the intersection of the line *A B* prolonged with *L D*. Drive a stake flush with the ground at *I* and drive a tack in this stake where the prolongation of *A B* crosses the chord connecting the stakes at *K* and *L*. The point *I* is the intersection of the tangents *A B* and *C D*. The external angle *C I M* formed by the intersecting tangents is called the **angle of intersection.**

1198. Deflection Angles.—When two lines meet in the same plane, they are said to form an angle, and the point of meeting is called the **angular point.** The rate of divergence or deflection of the two lines from their common, or angular, point determines the size of the angle. The unit of angular measurement is the *degree*, equal to $\frac{1}{360}$ part of a circle. Two lines forming an angle of 1 degree with each other will, at a distance of 100 feet from the angular point, deflect or diverge 1.745 feet.

In Fig. 233, the lines *A B* and *A C*, meeting at the point *A*, are supposed to form an angle of 1°, and the angle *B A C* is measured by the arc *B C*, described with the radius *A B*,

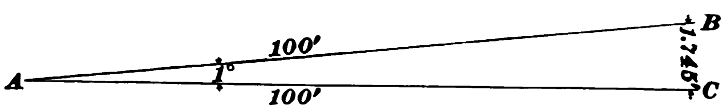


FIG. 233.

which is 100 feet in length. The arc *B C* and the straight line joining the extremities of that arc, that is, the chord *B C*, are assumed to be of equal length.

1199. Degree of Curvature.—The curve from which, as a unit or basis, all other railroad curves are deduced, is called a **one-degree curve.** It is the circumference of a circle whose radius is 5,730 feet, or, more exactly, 5,729.65 feet, in length. Two radii forming an angle of one degree at the center of a one-degree curve will subtend a chord of 100 feet at its circumference. The arc subtended by this chord of 100 feet is assumed to be of the same length as the chord.

In Fig. 234, let *A B* and *A C* be radii 5,729.65 feet in length, forming an angle of 1° at the center *A*; then, the arc *B C* subtended by these radii will be 100 feet in length.

The curve $B\,C$ is called a 1° curve. If, from the point O as a center, with a radius $O\,B$ equal to 2,864.93 feet, we describe an arc $B\,D$, 100 feet in length, the radii $O\,B$ and $O\,D$ will form an angle of 2° at the center O, and the curve $B\,D$

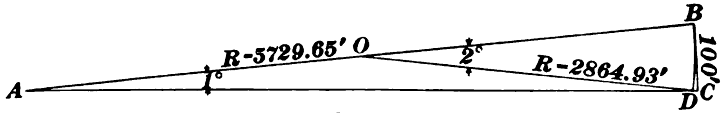


FIG. 234.

is called a 2° curve. A curve whose radius is nearly one-third $A\,B$, or 1,910.08 feet, is a 3° curve, etc.

The **degree of a curve** is determined by the central angle, which is subtended by a chord of 100 feet. Thus, if $B\,O\,G$, Fig. 235, is 10°, and $B\,G$ is 100 feet, $B\,G\,H\,K\,C$ is a 10° curve.

1200. The **deflection angle** of a curve is the angle formed at any point of the curve between a tangent and a chord of 100 feet. The deflection angle is, therefore, **half the degree of the curve.** Thus, if the chord $B\,G$ is 100 feet, the angle $E\,B\,G$ is the deflection angle of the curve $B\,G\,H\,K\,C$, and is half the angle $B\,O\,G$.

If, in Fig. 235, $O\,L$ is drawn perpendicular to $B\,G$, then $\sin B\,O\,L = \frac{B\,L}{O\,B}$. Now, $B\,O\,L = \frac{1}{2}\,B\,O\,G = E\,B\,G =$ the deflection angle for the chord $B\,G$, since $O\,L$, being perpendicular to the chord $B\,G$, bisects the arc $B\,L\,G$. If the deflection angle be denoted by D, and the radius $O\,B$ by R, then, by substituting these values in the given equation, we have

$$\sin D = \frac{B\,L}{R} = \frac{50}{R},$$

from which $$R = \frac{50}{\sin D}. \qquad \textbf{(79.)}$$

1201. For curves of from 1° to 10°, the radius may be found by dividing 5,730 feet (the radius of a 1° curve) by the degree of the curve. The results obtained are sufficiently accurate for all practical purposes. For sharp curves, that is, for those exceeding 10°, formula **79** should be used, especially if the radii are to be used as a basis for further calculation.

For example, the radius of a 4° curve is found by both methods as follows: By first method, $R = 5{,}730$ feet $\div 4 = 1{,}432.5$ feet. By second method, we find the deflection angle D of a 4° curve is 2°. Applying formula **79**, we have

$$R = \frac{50}{.0349} = 1{,}432.67 \text{ feet.}$$

In this case, the error is only 0.17 of a foot, and may be

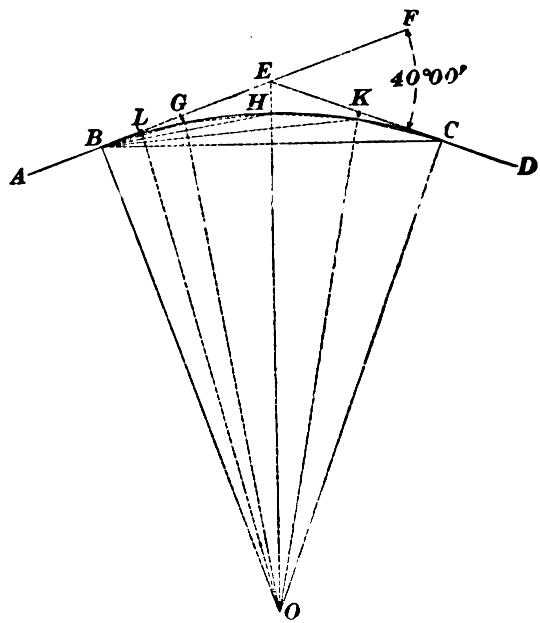


FIG. 235.

ignored in practical work. For a 30° curve we have, by first method,

$$R = \frac{5{,}730}{30} = 191 \text{ feet.}$$

By second method, we have

$$R = \frac{50}{\sin 15^\circ} = \frac{50}{.25882} = 193.18 \text{ feet.}$$

In this case, the error is 2.18 feet, and the error increases as the degree of curve increases.

The *table of radii* given in this book is calculated by formula **79.**

1202. Sub-Chords for Curves of Short Radii.—On curves of short radii, that is, curves of 20° and upwards, center stakes are driven at intervals of 25 feet. In Art. **1199,** it was stated that the standard chord and arc are assumed to be of the same length. This is practically true for curves of large radii, but for curves above 20° the excess of length of arc over the chord constantly increases. If, now, in Fig. 235, the chord BC is 100 feet in length, the arc $BGHKC$ must be *greater* than 100 feet; and if the arcs BG, GH, HK, and KC are equal, that is, each equal to one-quarter the arc BHC, then the equal chords BG, GH, HK, and KC subtending these equal arcs must each be *greater* than one-quarter of BC, which we assumed to be 100 feet. These greater chords must, therefore, be *greater than 25 feet.* Suppose the curve BHC to be a 20° curve, and the chord BC 100 feet; then, the central angle BOC is 20°. As the arc BG is one-quarter of the arc BHC, the central angle BOG is $\frac{20°}{4} = 5°$. The line OL drawn to the middle point of the chord BG is perpendicular to BG, and bisects the angle BOG. The deflection angle $EBG = BOL = GOL$. Let C designate the chord BG; R, the radius OB, and D, the deflection angle $EBG = BOL$. In the right-angled triangle BOL, we have $\sin BOL = \frac{BL}{BO}$. Substituting the above given values, we have $\sin D = \frac{\frac{1}{2}C}{R}$, whence $\frac{1}{2}C = R \sin D$, and $C = 2R \sin D$. The central angle for the chord BG is 5°. The deflection angle D is, therefore, $\frac{5°}{2} = 2° 30'$. $\sin 2° 30' = .04362$. Since the deflection angle $EBC = 10°$ for this case, $R = 50 \div \sin 10° = 287.94$ feet. Hence, chord $C = 2 \times 287.94 \times .04362 = 25.12$ feet.

Accordingly, in measuring the short chords, 25.12 feet is used instead of 25 feet.

1203. Tangent Distances.—When an intersection of tangents has been made and the intersection angle measured, the next question is the degree of curve which is to unite them, which being decided, the next step in order is the location of the points on the tangents where the curve begins and ends. These two points are equally distant from the **point of intersection** of the tangents, which is called the P. I. The point where the curve begins is called the **point of curve,** or the P. C.; the point where the curve terminates is called the **point of tangent,** or the P. T. The distance of the P. C. and P. T. from the P. I. is called the **tangent distance.**

1204. In Fig. 235, let $A B$ and $C D$ be tangents intersecting at the point E and forming an angle $C E F$ with each other. Call the angle of intersection I, the radius $B O$ call R, and the tangent distance $B E$ call T. From Art. **1196,** proposition 6, we have $B O C = C E F$; hence, the angle $B O E = \frac{1}{2} C E F$. From the right triangle $E B O$, we have

$$\tan B O E = \frac{B E}{B O} = \tan \tfrac{1}{2} I = \frac{T}{R},$$

from which $T = R \tan \frac{1}{2} I$. **(80.)**

Now, supposing, in Fig. 235, the intersection angle $I = 40°$, and it is decided to unite the tangents by a 10° curve whose radius (see Table 27) is 573.69; then, since $R = 573.69$, $\frac{1}{2} I = 20°$, and $\tan 20° = .36397$, $T = 573.69 \times .36397 = 208.8$ feet. Measure back from the point E on both tangents the distance 208.8 feet to the points B and C. Drive plugs flush with the ground at both points, and set accurate center points, marked by tacks, in both. Directly opposite each of these plugs drive a stake, called a **guard stake,** because it guards, or, rather, indicates, where the plug is. The stake at B, if the numbering of the stations runs from B towards C, will be marked P. C., and the stake at C will be marked P. T.

1205. To Lay Out a Curve with a Transit.—Having set the tangent points *B* and *C*, Fig. 235, set up the transit at *B*, the P. C. Set the vernier at zero, and sight to *E*, the intersection point. Suppose *B* to be an even, or "full," station, say 18, and that it has been decided to set stakes at each hundred feet. Let the central angle *B O G*, measured by the 100-foot chord *B G*, be 10°; then, the deflection angle *E B G*, having its vertex *B* in the circumference, and being subtended by the chord *B G*, will equal ½ *B O G*, or 5°. Turn an angle of 5° from *B*, which in this case will be to the right, measure a full chain 100 feet from *B*, and line in the flag at *G*; drive a stake at *G*, which will be marked 19. Turn off an additional 5°, making 10° from zero, and at the end of another chain, at *H*, set a stake marked 20. Continue turning deflections of 5° until 20°, or one-half of the intersection angle, is reached. This last deflection, if the work has been correctly done, will bring the fore-chainman to the point of tangency *C*. It is but rarely that the P. C. comes at a full station. When the P. C. comes between full stations, it is called a **sub-station,** and the chord between it and the next full station is called a **sub-chord.** Had the P. C. of the curve come at a sub-station, say 17 + 32, the deflection for the sub-chord of 100 − 32, or 68 feet, the distance to the next station, is found as follows: The deflection for a full station, that is, 100 feet, is $5° = 300'$, and the deflection for 1 foot $= \frac{300'}{100} = 3'$; and for 68 feet the deflection will be $68 \times 3 = 204' = 3°\ 24'$, which is turned off from zero, and a stake set on line, 68 feet from the transit, at station 18. The length of a curve uniting two given tangents, whose intersection angle is determined, is found as follows:

Suppose $I = 32°\ 40'$, and that the tangents are to be united by a 6° curve; 32° 40′, reduced to the decimal form, is 32.666°; as each central angle of 6° will subtend a 100-foot chord, or one chain, there will be as many such chords or chains as 6 is contained times in 32.666, which is 5.444; that is, there will be 5.444 chains in the curve, or 544.4 feet, which is the required length of the curve. The P. C. and P. T. having been set, and the station of the P. C. determined by actual measurement, say 58 + 71, the station of the P. T. is found by adding to 58 + 71, the station of the P. C., the calculated length of the curve, 544.4 feet. 58 + 71 + 544.4 = 64 + 15.4, the station of the P. T.

Another method of calculation is the following: The sum of all the deflection angles is equal to one-half the intersection angle. The intersection angle being 32° 40′, one-half equals 16° 20′, which, reduced to minutes, equals 980′. The deflection for 100 feet is $\frac{6}{2} = 3° = 180'$, and the deflection for 1 foot is $\frac{180'}{100} = 1.8'$; then, 980′, the total deflection, divided by 1.8′, gives 544.4 feet, the required length of the curve.

EXAMPLES FOR PRACTICE.

In the following examples, let I = angle of intersection, T = tangent, and L = length of curve.

1. $I = 16° 13'$; degree of curve = 3°; required, T and L.
   Ans. $\begin{cases} T = 272.13 \text{ ft.} \\ L = 540.55 \text{ ft.} \end{cases}$

2. $I = 59° 20'$; degree of curve = 8° 30′; required, T and L.
   Ans. $\begin{cases} T = 384.32 \text{ ft.} \\ L = 698.04 \text{ ft.} \end{cases}$

3. $I = 21° 35'$; degree of curve = 4° 15′; required, T and L.
   Ans. $\begin{cases} T = 257.03 \text{ ft.} \\ L = 507.84 \text{ ft.} \end{cases}$

4. The degree of a curve is 5° 30′; what is the deflection angle for a chord of 16.2 feet? Ans. 26.7′.

5. The degree of a curve is 7° 15′; what is the deflection angle for a chord of 38.4 feet? Ans. 1° 23½′.

1206. Obstructions in the Line of Curve.—Frequently it happens that the entire curve can not be run in from the P. C. on account of obstructions. This is especially the case in either hilly or wooded country, and the transit has to "move up" to an intermediate point. For example, in Fig. 235, we will suppose that Station H, 200 feet from B, is the last point which can be set from the P. C. at

B. A plug is driven at *H*, flush with the ground, and carefully centered, and a tack driven at the point. The deflection angle *E B H* is 10° to the right. The transit is set up at *H*, an angle of 10° to the left is laid off from zero, and the vernier is clamped. The instrument is then sighted to a flag at *B*, the spindle clamped, and a close sight to the flag taken, the lower tangent screw being used to adjust the sight. The vernier clamp is then loosened, and the vernier set at zero. The line of sight will then be on a tangent to the curve at *H*, and the deflection angles to *K* and *C* can be laid off as before, and the stations *K* and *C* located.

1207. Tangent and Chord Deflections.—Let *A B*, Fig. 236, be a tangent, and *B C E H* a curve commencing at *B*. Produce the tangent *A B* to the point *D*. The line *C D* is a **tangent deflection,** and is the perpendicular distance from the tangent to the curve. If the chord *B C* be produced to the point *G*, making *C G* = *B C* = *C E*, the distance *G E* is a **chord deflection,** and is double the tangent deflection *D C*.

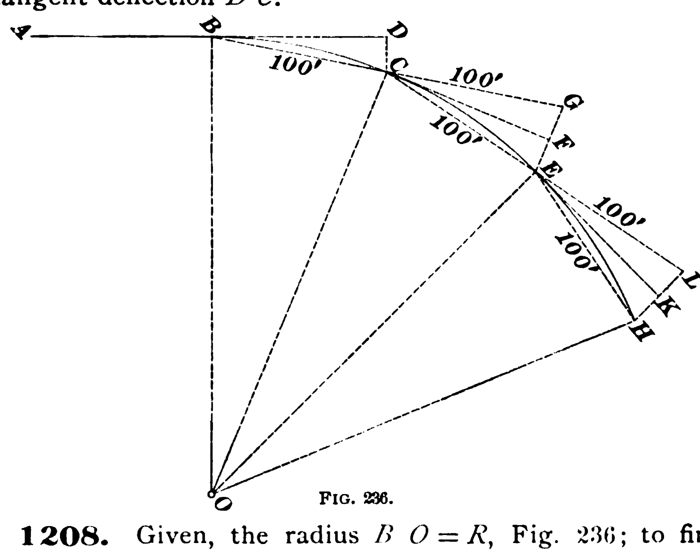


FIG. 236.

1208. Given, the radius *B O* = *R*, Fig. 236; to find the chord deflection *E G*, and the tangent deflection *C D* = *F E*.

TABLE 27.

RADII AND DEFLECTIONS.

Degree.	Radii.	Chord Deflection.	Tangent Deflection.
° ′			
0 5	68754.94	.145	.073
10	34377.48	.291	.145
15	22918.33	.436	.218
20	17188.76	.582	.291
25	13751.02	.727	.364
30	11459.19	.873	.436
35	9822.18	1.018	.509
40	8594.41	1.164	.582
45	7639.49	1.309	.654
50	6875.55	1.454	.727
55	6250.51	1.600	.800
1 0	5729.65	1.745	.873
5	5288.92	1.891	.945
10	4911.15	2.036	1.018
15	4583.75	2.182	1.091
20	4297.28	2.327	1.164
25	4044.51	2.472	1.236
30	3819.83	2.618	1.309
35	3618.80	2.763	1.382
40	3437.87	2.909	1.454
45	3274.17	3.054	1.527
50	3125.36	3.200	1.600
55	2989.48	3.345	1.673
2 0	2864.93	3.490	1.745
5	2750.35	3.636	1.818
10	2644.58	3.781	1.891
15	2546.64	3.927	1.963
20	2455.70	4.072	2.036
25	2371.04	4.218	2.109
30	2292.01	4.363	2.181
35	2218.09	4.508	2.254
40	2148.79	4.654	2.327
45	2083.68	4.799	2.400
50	2022.41	4.945	2.472
55	1964.64	5.090	2.545
3 0	1910.08	5.235	2.618
5	1858.47	5.381	2.690
10	1809.57	5.526	2.763
15	1763.18	5.672	2.836
20	1719.12	5.817	2.908
25	1677.20	5.962	2.981
30	1637.28	6.108	3.054
35	1599.21	6.253	3.127
40	1562.88	6.398	3.199
45	1528.16	6.544	3.272
50	1494.95	6.689	3.345
55	1463.16	6.835	3.417
4 0	1432.69	6.980	3.490
5	1403.46	7.125	3.563
10	1375.40	7.271	3.635
15	1348.45	7.416	3.708
20	1322.53	7.561	3.781
25	1297.58	7.707	3.853
30	1273.57	7.852	3.926
35	1250.42	7.997	3.999
40	1228.11	8.143	4.071
45	1206.57	8.288	4.144
50	1185.78	8.433	4.217
55	1165.70	8.579	4.289
5 0	1146.28	8.724	4.362
5	1127.50	8.869	4.435
10	1109.33	9.014	4.507

Degree.	Radii.	Chord Deflection.	Tangent Deflection.
° ′			
5 15	1091.73	9.160	4.580
20	1074.68	9.305	4.653
25	1058.16	9.450	4.725
30	1042.14	9.596	4.798
35	1026.60	9.741	4.870
40	1011.51	9.886	4.943
45	996.87	10.031	5.016
50	982.64	10.177	5.088
55	968.81	10.322	5.161
6 0	955.37	10.467	5.234
5	942.29	10.612	5.306
10	929.57	10.758	5.379
15	917.19	10.903	5.451
20	905.13	11.048	5.524
25	893.39	11.193	5.597
30	881.95	11.339	5.669
35	870.79	11.484	5.742
40	859.92	11.629	5.814
45	849.32	11.774	5.887
50	838.97	11.919	5.960
55	828.88	12.065	6.032
7 0	819.02	12.210	6.105
5	809.40	12.355	6.177
10	800.00	12.500	6.250
15	790.81	12.645	6.323
20	781.84	12.790	6.395
25	773.07	12.936	6.468
30	764.49	13.081	6.540
35	756.10	13.226	6.613
40	747.89	13.371	6.685
45	739.86	13.516	6.758
50	732.01	13.661	6.831
55	724.31	13.806	6.903
8 0	716.78	13.951	6.976
5	709.40	14.096	7.048
10	702.18	14.241	7.121
15	695.09	14.387	7.193
20	688.16	14.532	7.266
25	681.35	14.677	7.338
30	674.69	14.822	7.411
35	668.15	14.967	7.483
40	661.74	15.112	7.556
45	655.45	15.257	7.628
50	649.27	15.402	7.701
55	643.22	15.547	7.773
9 0	637.27	15.692	7.846
5	631.44	15.837	7.918
10	625.71	15.982	7.991
15	620.09	16.127	8.063
20	614.56	16.272	8.136
25	609.14	16.417	8.208
30	603.80	16.562	8.281
35	598.57	16.707	8.353
40	593.42	16.852	8.426
45	588.36	16.996	8.498
50	583.38	17.141	8.571
55	578.49	17.286	8.643
10 0	573.69	17.431	8.716
10	564.31	17.721	8.860
20	555.23	18.011	9.005
30	546.44	18.300	9.150
40	537.92	18.590	9.295

Degree.	Radii.	Chord Deflection.	Tangent Deflection.
° ′			
10 50	529.67	18.880	9.440
11 0	521.67	19.169	9.585
10	513.91	19.459	9.729
20	506.38	19.748	9.874
30	499.06	20.038	10.019
40	491.96	20.327	10.164
50	485.05	20.616	10.308
12 0	478.34	20.906	10.453
10	471.81	21.195	10.597
20	465.46	21.484	10.742
30	459.28	21.773	10.887
40	453.26	22.063	11.031
50	447.40	22.352	11.176
13 0	441.68	22.641	11.320
10	436.12	22.930	11.465
20	430.69	23.219	11.609
30	425.40	23.507	11.754
40	420.23	23.796	11.898
50	415.19	24.085	12.043
14 0	410.28	24.374	12.187
10	405.47	24.663	12.331
20	400.78	24.951	12.476
30	396.20	25.240	12.620
40	391.72	25.528	12.764
50	387.34	25.817	12.908
15 0	383.06	26.105	13.053
10	378.88	26.394	13.197
20	374.79	26.682	13.341
30	370.78	26.970	13.485
40	366.86	27.258	13.629
50	363.02	27.547	13.773
16 0	359.26	27.835	13.917
10	355.59	28.123	14.061
20	351.98	28.411	14.205
30	348.45	28.699	14.349
40	344.99	28.986	14.493
50	341.60	29.274	14.637
17 0	338.27	29.562	14.781
10	335.01	29.850	14.925
20	331.82	30.137	15.069
30	328.68	30.425	15.212
40	325.60	30.742	15.356
50	322.59	31.000	15.500
18 0	319.62	31.287	15.643
10	316.71	31.574	15.787
20	313.86	31.861	15.931
30	311.06	32.149	16.074
40	308.30	32.436	16.218
50	305.60	32.723	16.361
19 0	302.94	33.010	16.505
10	300.33	33.296	16.648
20	297.77	33.583	16.792
30	295.25	33.870	16.935
40	292.77	34.157	17.078
50	290.33	34.443	17.222
20 0	287.94	34.730	17.365

The triangles $O\ C\ E$ and $C\ E\ G$ are similar, since both are isosceles, and the angle $G\ C\ E =$ angle $C\ O\ E$. Hence, we have $O\ C : C\ E :: C\ E : E\ G$. Denoting the chord $C\ E$ by c, and the chord deflection $E\ G$ by d, we have, from the above proportion, $R : c :: c : d$. Therefore,

$$d = \frac{c^2}{R}. \qquad \textbf{(81.)}$$

To find the tangent deflection, draw $C\ F$ to the middle point of $E\ G$. By Art. **207**, $F\ E =$ the tangent deflection $= D\ C$. Hence, tangent deflection = one-half the chord deflection, or, denoting the tangent deflection by f, we have

$$f = \frac{c^2}{2\ R}. \qquad \textbf{(82.)}$$

EXAMPLES FOR PRACTICE.

1. The degree of curve is 5°, the chord is 67 ft.; what are the tangent and the chord deflections? Ans. { Tan def. = 1.959 ft. Chord def. = 3.918 ft.

2. The degree of curve is 7° 30′, the chord is 23.5 ft.; what are the tangent and the chord deflections? Ans. { Tan def. = 0.359 ft. Chord def. = 0.718 ft.

3. The degree of curve is 6° 15′; the chord is 117 ft.; what are the tangent and the chord deflections? Ans. { Tan def. = 7.465 ft. Chord def. = 14.930 ft.

MAP OF FINAL LOCATION.

1209. In mapping a final location, the measurements should be made from intersection point to intersection point, and the angles platted either by tangents or by latitudes and departures. The points of curve are then located by scaling the tangent distances from the intersection points. The curve centers are best determined by describing intersecting arcs from the tangent points as centers, with radii equal to that of the given curve.

1210. Let it be required to plat by tangents the following location notes:

Station.	Degree of Curve.	Intersection Angle.	Tangent.	Magnetic Bearing.
25 + 50	P. T.			N 1° 00′ E
20 + 10	P. C. 5° L.	27° 00′	275.20 ft.	
14 + 55	P. T.			N 28° 00′ E
10 + 80	P. C. 6° R.	22° 30′	190.03 ft.	
0				N 5° 30′ E

The tangent distances are found by formula **80.**

The first curve is 6° R; the intersection angle I is 22° 30′. The radius of a 6° curve is 955.37 feet. See Table 27. $\frac{1}{2}I = 11° 15'$. Tan 11° 15′ = .19891. 955.37 × .19891 ft. = 190.03 ft. = T, which we place in the column headed tangent, opposite the intersection angle 22° 30′. The second curve is 5° L; the intersection angle is 27° 00′. The radius of a 5° curve is 1,146.28 feet. $\frac{1}{2}I = 13° 30'$. Tan 13° 30′ = .24008. 1,146.28 ft. × .24008 = 275.2 ft. = T, which we place in tangent column opposite the intersection angle 27° 00′.

1211. A plat of these notes is given in Fig. 237. The order of work is the following: First, we select a starting-point A, which we number 0, and through this point we draw a meridian $A\ B$ with its north point at the top of the plat.

The first course has a bearing N 5° 30′ E. From Sta. 0, scale off on the meridian 600 feet, the length of our radius for platting angles. The bearing angle is 5° 30′, and its tangent .09629, which, multiplied by 600, the radius, gives 57.77 feet, the length of the required tangent. Call the extremity of the radius C. At C erect a right perpendicular to $A\ B$, and on it lay off the tangent 57.77 feet, locating the point D. Join A and D. The angle $C\ A\ D = 5° 30'$. The P. C. of the first curve is at Sta. 10 + 80. The tangent distance, as given in the preceding table, is 190.03 feet. Hence, the distance from the starting-point to the first intersection point is the sum of 1,080 and 190.03 feet, which is 1,270.03 feet.

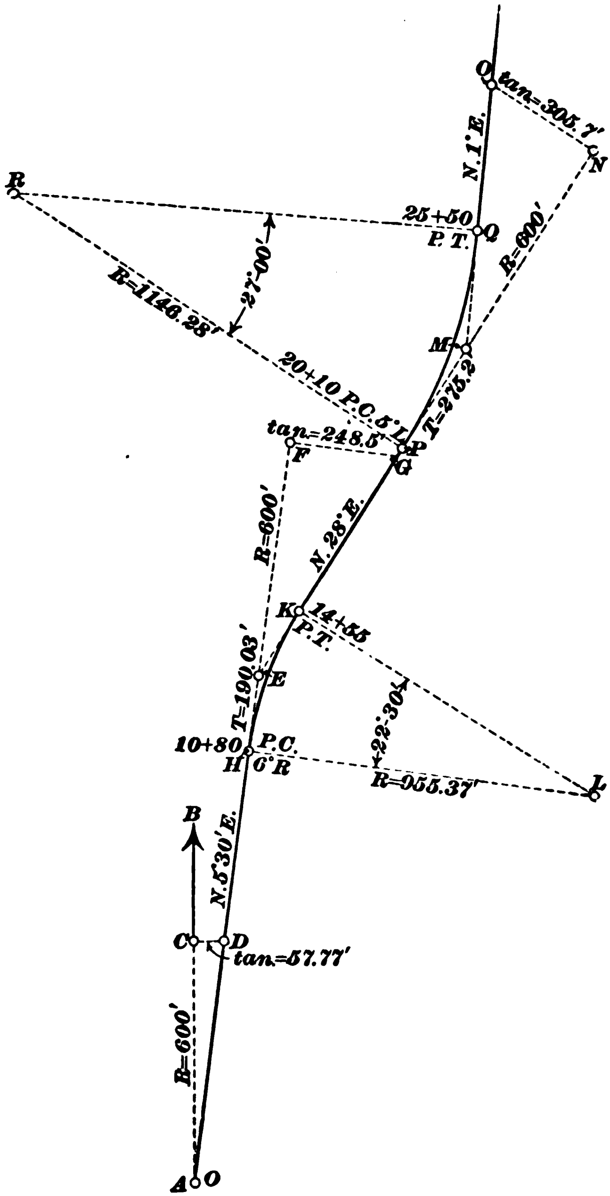

O
tan=305.7'
N
N.1°E.
R
25+50
P. T.
Q
R=600'
27°00'
R=1146.28'
M
20+10 P.C.5°L
T=275.2
tan=248.5'
F
P
G
R=600'
N.28°E.
K
14+55
P.T.
T=190.03'
E
22°30'
10+80
P.C.
H
6°R
R=955.37'
L
B
N.5°30'E.
C
D
tan=57.77'
R=600'
A
O

FIG. 237.

Produce *A D*, making a total distance of 1,270.03 feet to the point of intersection *E*. To this add 600 feet more for the radius by which the next angle is platted. Call the extremity of this radius *F*. The intersection angle of the first curve is 22° 30′. Its tangent is .41421, which, multiplied by 600, the given radius, gives 248.52 feet as the required tangent for platting the angle. At *F* erect a right perpendicular to the radius *E F*, and scale off the tangent *F G* = 248.52 feet, locating the point *G*. Join *E* and *G*. The angle *F E G* is 22° 30′, and the bearing of the tangent *E G* is N 28° E. Next, from the point of intersection *E*, scale off on the lines *E D* and *E G* the tangent distance 190.03 feet, locating the P. C. at *H*, Sta. 10 + 80, and the P. T. at *K*, Sta. 14 + 55. Now, from *H* and *K* as centers, with a radius of 955.37 feet = radius of 6° curve, describe arcs intersecting at the point *L*. Then, from *L* as a center, with the same radius, describe a curve joining the points *H* and *K*. The curve *H K* will be a 6° curve, and will be tangent to the lines *H D* and *K G* at the points of curve *H* and *K*. The next intersection point *M* is in the line *E G* produced. The distance between these intersection points is made up of three parts, viz., the tangent of preceding curve, which we know to be 190.03 feet; the intermediate tangent, that is, the distance from the P. T. of the first curve to the P. C. of the second curve, and the tangent of the next curve following. The P. T. of the first curve is at Sta. 14 + 55; the P. C. of the second curve is at Sta. 20 + 10; the intermediate tangent is, therefore, the difference between 14 + 55 and 20 + 10, which is 555 feet.

The tangent of the second curve is 275.2 feet. Hence, the distance from the intersection point *E* of the first curve to the intersection point *M* of the second curve is the sum of 190.03, 555, and 275.2 feet, which is 1,020.23 feet. Produce *E G* so as to contain 1,020.23 feet, and 600 additional feet for a radius, the extremity of which call *N*. The intersection angle of the second course is 27° 00′ L; tan 27° = .50953. Radius 600 feet × .50953 = 305.72 feet, the length of the required tangent. Accordingly, at *N* we erect a left perpendicular to the radius *M N*, and on that perpendicular scale off the tangent 305.72 feet, locating the point O. Join M and O. The angle $N M O$ is 27° 00′, equal to the given intersection angle, and the bearing of the tangent $M O$ is N 1° E. From M, on the lines $M K$ and $M O$, scale off the tangent distance 275.20 feet, locating the P. C. at P, Sta. 20 + 10, and the P. T. at Q, Sta. 25 + 50. Then, from P and Q as centers, with radii of 1,146.28 feet, the radius of a 5° curve, describe arcs intersecting at R. From R as a center, with the same radius, describe the curve $P Q$, which is a 5° curve, and is tangent to the lines $M K$ and $M O$ at P and Q. Write the bearings of each tangent in its proper place, being careful that the bearings shall read in the same direction in which the line is being run.

PLATE III.

1212. Plate III contains two maps of railroad location, Figs. 1 and 2, the notes for which are given in Art. **1214.** All the angles are laid off by tangents, and the notes of the *alinement* are given in detail, all of which the student must carefully go over and check.

The student before commencing these drawings should first note that the magnetic meridian (by means of which the direction of the first tangent of each line is determined) is parallel to the right and left border lines of the plate. He must also determine, by measurement from the border lines, the location of the starting-point 0 of each line. Without these precautions, the lines are liable to run off the paper, necessitating a repetition of the work, and involving the erasure of lines, which always soils the paper and mars the appearance of the drawing.

The student will make the drawing to a scale of 300 feet to the inch. If his scale reads only 200 feet to the inch, the distances given to a scale of 300 feet to the inch must be reduced to their equivalent to a scale of 200 feet to the inch. The process of reduction is simple, and may be readily understood from the following: A line which measures 300 feet in length to a scale of 300 feet to the inch will measure but 200 feet to a scale of 200 feet to the inch. Hence, in changing a scale from 300 feet to 200 feet to the inch, the distances and dimensions will scale but $\frac{2}{3}$ of the original distances and dimensions.

EXAMPLE.—A line measures 963 feet to a scale of 300 feet to the inch. What will it measure to a scale of 200 feet to the inch?

SOLUTION.— $\frac{2}{3}$ of $963 = 642$; that is, to a scale of 200 feet to the inch, the line will measure 642 feet. Ans.

1213. The order of platting the notes is as follows: First, draw a meridian as indicated by the arrow. Next, having located the starting-point *A*, Fig. 1, which is numbered 0, draw through that station a parallel meridian *A B*. We find from the notes that the direction of the back tangent *A A'* (which we will consider a part of a line of railroad already constructed) is due north and south, and that Sta. 0 is the P. C. of an 8° R curve with a central or intersection angle of 63° 10'. The tangent distance, we find, by the formula $T = R \tan \frac{1}{2} I$, is 440.7 feet. This distance we scale off on the meridian above the point *A* to a scale of 300 feet to the inch, locating the point *C*, which is the intersection point of the back and forward tangents.

Next, from *C* on the same meridian, we scale off the radius *C D* of 400 feet for laying off the angle of the first curve. The angle of this curve is 63° 10'. The tan of 63° 10' is 1.97681. The radius 400 feet $\times$ $1.97681 = 790.7$ feet, the length of the required tangent. At *D* erect a right perpendicular to *A B*, as the curve is to the right, and upon this perpendicular scale off the calculated tangent 790.7 feet, locating the point *E*. A line joining the points *C* and *E* will give the direction of the forward tangent. On the line *C E*, scale off from *C* the tangent distance, 440.7 feet, locating the point *F*, which is the P. T. of the first curve. From *A* and *F* as centers, with radii of 716.78 feet, the radius of an 8° curve, describe arcs intersecting at *G*. Then, from *G* as a center, with the same radius, describe a curve joining the points *A* and *F*. The curve *A F* is an 8° curve, and is tangent to the lines *A A'* and *F E* at the points *A* and *F*.

We find from the notes that the next curve is 6° R. Its P. C. is at Sta. 13 + 16, and its central angle is 44° 20′. We find its tangent distance is 389.2 feet. We next calculate the distance from the intersection point of the first curve to the intersection point of the second curve. The distance is composed of three parts, viz., the tangent of the preceding curve, viz., 440.7 feet; the intermediate tangent, that is, from the P. T. of the preceding curve at Sta. 7 + 89.6 to the P. C. of the second, or 6°, curve, at Sta. 13 + 16, a distance of 526.4 feet; and the tangent of the 6° curve, viz., 389.2 feet; making a total distance of 1,356.3 feet. Produce the line *C E*, and scale off from *C* on that line a total distance of 1,356.3 feet, locating the point *H*, which is the intersection point of the second, or 6°, curve. Produce *C H* 400 feet to *K* for a radius in laying off the central angle, 44° 20′ R, of the second curve. The tangent of 44° 20′ is .977, which, multiplied by 400, gives 390.8 feet. At *K* erect a right perpendicular to *H K*, and upon it scale off the tangent 390.8 feet, locating the point *L*. The line joining *H* and *L* gives the direction of the forward tangent of the second curve. Next, from the intersection point *H*, scale off on both back and forward tangents the tangent distance 389.2 feet, locating the P. C. of the second curve at *M*, Sta. 13 + 16, and its P. T. at *N*, Sta. 20 + 54.9. Next, from *M* and *N* as centers, with a radius of 955.37 feet, the radius of a 6° curve, describe arcs intersecting at *O*. Then, from *O* as a center, with the same radius, describe a curve joining the points *M* and *N*. The curve *M N* is a 6° curve, and is tangent to the lines *F H* and *H L* at the points of curve *M* and *N*.

1214. The student will draw the tangent distances and the radii and tangents for laying off angles in dotted lines, as they are simply construction lines. The line of survey he will draw in a full, bold line, as shown in the plate. The intersection points and the points of curve and tangent are marked by small circles, the latter being more fully described by their station numbers. Dotted radial lines are drawn

NOTES FOR FIG. 1.

Station.	Deflection.	Total Angle.	Magnetic Course.	Calculated Course.
40	12° 00′			
39	6° 00′			
38 + 00	18° 00′	36° 00′		
37	12° 00′			
36	6° 00′			
35 + 00	P. C. 12° L.			
34				
33 + 04.9	10° 40.3′ P. T.	35° 50′	S 36° 30′ E	S 36° 40′ E
33	10° 30′			
32	7° 00′			
31	3° 30′			
30	7° 14.7′	14° 29.4′		
29	3° 44.7′			
28	0° 14.7′			
27 + 93	P. C. 7° R.			
24				
21				
20 + 54.9	10° 38.8′ P. T.	44° 20′	S 72° 30′ E	S 72° 30′ E
20	9° 00′			
19	6° 00′			
18	3° 00′			
17	11° 31.2′	23° 02.4′		
16	8° 31.2′			
15	5° 31.2′			
14	2° 31.2′			
13 + 16	P. C. 6° R.			
12				
11				
10				
9				
8				
7 + 89.6	15° 35′ P. T.	63° 10′	N 63° 00 E	N 63° 10′ E
7	12° 00′			
6	8° 00′			
5	4° 00′			
4	16° 00′	32° 00′		
3	12° 00′			
2	8° 00′			
1	4° 00′			
0	P. C. 8° R.		North	North

NOTES FOR FIG. 1.

Remarks.	June 28, 1894.
Int. Ang. = 72° 00′ 12° curve, L. R. = 478.34 ft. T. = 347.5 ft. P. C. = 35 + 00 Length curve = 600 ft. P. C. C. = 41 + 00 Def. 100 ft. = 6° 00′ Def. 1 ft. = 3.6′	From intersection to intersection. Tan preceding curve = 264.8 ft. Tan between curves = 195.1 ft. Tan 12° curve = 347.6 ft. Total, = 807.5 ft. tan 72° 00′ = 3.07768 400 ft. × 3.07768 = 1,231.1 ft.
Int. Ang. = 35° 50′ 7° curve, R. R. = 819.02 ft. T. = 264.8 ft. P. C. = 27 + 93 Length curve = 511.9 ft. P. T. = 33 + 04.9 Def. 100 ft. = 3° 30′ Def. 1 ft. = 2.1′	From intersection to intersection. Tan preceding curve = 389.2 ft. Tan between curves = 738.1 ft. Tan 7° curve = 264.8 ft. Total, = 1,392.1 ft. tan 35° 50′ = .72211 400 ft. × .72211 = 288.8 ft.
Int. Ang. = 44° 20′ 6° curve, R. R. = 955.37 ft. T. = 389.2 ft. P. C. = 13 + 16 Length of curve = 738.9 ft. P. T. = 20 + 54.9 Def. 100 ft. = 3° 00′ Def. 1 ft. = 1.8′	From intersection to intersection. Tan preceding curve = 440.7 ft. Tan between curves = 526.4 ft. Tan 6° curve = 389.2 ft. Total, = 1,356.3 ft. tan 44° 20′ = .977 400 ft. × .977 = 390.8 ft.
Int. Ang. = 63° 10′ 8° curve, R. R. = 716.78 ft. T. = 440.7 ft. P. C. = 0 Length of curve = 789.6 ft. P. T. = 7 + 89.6 Def. 100 ft. = 4° 00′ Def. 1 ft. = 2.4′	Radius 1 = 400 ft. tan 63° 10′ = 1.97681 400 ft. × 1.97681 = 790.7 ft.

NOTES FOR FIG. 2.

Station.	Deflection.	Total Angle.	Magnetic Course.	Calculated Course.
13 + 41.7	10° 15′ P. T.	32° 30′	S 79° 00′ E	S 79° 00′ E
13	9° 00′	....	....	....
12	6° 00′	....	....	....
11	3° 00′	....	....	....
10 + 00	6° 00′	12° 00′	....	....
9	3° 00′	....	....	....
8 + 00	P. C. 6° R.	....	....	....
5	....	....	....	....
3	....	....	....	....
0	....	....	S 46° 30′ E	....

NOTES FOR FIG. 1—*Continued.*

Station.	Deflection.	Total Angle.	Magnetic Course.	Calculated Course.
69 + 10.1		End	of Line.	
61 + 65.1	15° 40′ P. T.	31° 20′	N 39° 45′ E	N 39° 40′ E
61	12° 44.1′	....	....	....
60	8° 14.1′	....	....	....
59	3° 44.1′	....	....	....
58 + 17	P. C. 9° R.	....	....	....
55	....	....	....	....
54	....	....	....	....
53	....	....	....	....
52	....	....	....	....
51	....	....	....	....
50 + 00	17° 30′ P. T.	63° 00′	N 8° 15′ E	N 8° 20′ E
49	14° 00′	....	....	....
48	10° 30′	....	....	....
47	7° 00′	....	....	....
46	3° 30′	....	....	....
45	14° 00′	28° 00′	....	....
44	10° 30′	....	....	....
43	7° 00′	....	....	....
42	3° 30′	....	....	....
41 + 00	18° 00′ P. C. C. 7° L.	72° 00′	N 71° 15′ E	N 71° 20′ E

NOTES FOR FIG. 2.

	Remarks. June 28, 1894.
Int. Ang. = 32° 30′ 6° curve, L. R. = 955.37 ft. T. = 278.5 ft. P. C. = 8 + 00 Length curve = 541.7 ft. P. T. = 13 + 41.7 Def. 100 ft. = 3° 00′ Def. 1 ft. = 1.8′	Radius 1 = 400.0 ft. From Sta. 0 to P. C. = 800.0 ft. Tan 6° curve = 278.5 ft. Total from P.C. to P.I. = 1,078.5 ft. tan 32° 30′ = .63707 400 ft. × .63707 = 254.8 ft.

NOTES FOR FIG. 1—*Continued.*

	Remarks. June 28, 1894.
Int. Ang. = 31° 20′ 9° curve, R. R. = 637.27 ft. T. = 178.7 ft. P. C. = 58 + 17 Length curve = 348.1 ft. P. T. = 61 + 65.1 Def. 100 ft. = 4° 30′ Def. 1 ft. = 2.7′	From intersection to intersection. Tan preceding curve = 501.9 ft. Tan between curves = 817.0 ft. Tan 9° curve = 178.7 ft. Total, = 1,497.6 ft. tan 31° 20′ = .60881 400 ft. × .60881 = 243.5 ft.
Int. Ang. = 63° 00′ 7° curve, L. R. = 819.02 ft. T. = 501.9 ft. P. C. C. = 41 + 00 Length curve = 900 ft. P. T. = 50 + 00	From intersection to intersection. Tan preceding curve = 347.6 ft. Tan between curves = 0.0 ft. Tan 7° curve = 501.9 ft. Total, = 849.5 ft. tan 63° = 1.96261 400 ft. × 1.96261 = 785 ft.

NOTES FOR FIG. 2—*Continued.*

Station.	Deflection.	Total Angle.	Magnetic Course.	Calculated Course.
57 + 40		End	of Line.	
47 + 19	8° 46.3′ P. T.	34° 00′	N 11° 15′ E	N 11° 20′ E
47	8° 15′			
46	5° 30′			
45	2° 45′			
44 + 00	8° 13.7′	16° 27.4′		
43	5° 28.7′			
42	2° 43.7′			
41 + 00.8	14° 01.7′ P.C.C. 5° 30′ L.	52° 00′	N 45° 15′ E	N 45° 20′ E
41	14° 00′			
40	10° 30′			
39	7° 00′			
38	3° 30′			
37 + 00	11° 58.2′	23° 56.4′		
36	8° 28.2′			
35	4° 58.2′			
34	1° 28.2′			
33 + 58	P. C. 7° L.			
32				
30 + 36.6	13° 27.8′ P. T.	48° 10′	S 82° 30′ E	S 82° 40′ E
30	12° 00′			
29	8° 00′			
28	4° 00′			
27 + 00	10° 37.2′	21° 14.4′		
26	6° 37.2′			
25	2° 37.2′			
24 + 34.5	P. C. 8° L.			
24				
23				
22 + 14.4	9° 39′ P. T	44° 30′	S 34° 20′ E	S 34° 30′ E
22	9° 00′			
21	4° 30′			
20 + 00	12° 36′	25° 12′		
19	8° 06′			
18	3° 36′			
17 + 20	P. C. 9° L.			
17				
15				

NOTES FOR FIG. 2—*Continued.*

Remarks.	June 28, 1894.
Int. Ang. = 34° 00′	From intersection to intersection.
5° 30′, L. R. = 1,042.14 ft.	Tan preceding curve = 399.5 ft.
T. = 318.6 ft.	Tan between curves = 0.0 ft.
P. C. C. = 41 + 00.8	Tan 5° 30′ curve = 318.6 ft.
Length curve = 618.2 ft.	Total, = 718.1 ft.
P. T. = 47 + 19	tan 34° 00′ = .67451
Def. 100 ft. = 2° 45′	400 ft. × .67451 = 269.8 ft.
Def. 1 ft. = 1.65′	
Int. Ang. = 52° 00′	From intersection to intersection.
7° curve, L. R. = 819.02 ft.	Tan preceding curve = 320.4 ft.
T. = 399.5 ft.	Tan between curves = 321.4 ft.
P. C. = 33 + 58	Tan 7° curve = 399.5 ft.
Length curve = 742.8 ft.	Total, = 1,041.3 ft.
P. C. C. = 41 + 00.8	tan 52° 00′ = 1.27994
Def. 100 ft. = 3° 30′	400 ft. × 1.27994 = 512 ft.
Def. 1 ft. = 2.1′	
Int. Ang. = 48° 10′	From intersection to intersection.
8° curve, L. R. = 716.78 ft.	Tan preceding curve = 260.7 ft.
T. = 320.4 ft.	Tan between curves = 220.1 ft.
P. C. = 24 + 34.5	Tan 8° curve = 320.4 ft.
Length curve = 602.1 ft.	Total, = 801.2 ft.
P. T. = 30 + 36.6	tan 48° 10′ = 1.11713
Def. 100 ft. = 4° 00′	400 ft. × 1.11713 = 446.8 ft
Def. 1 ft. = 2.4′	
Int. Ang. = 44° 30′	From intersection to intersection
9° curve R. R. = 637.27 ft.	Tan preceding curve = 278.5 ft.
T. = 260.7 ft.	Tan between curves = 378.3 ft.
P. C. = 17 + 20	Tan 9° curve = 260.7 ft.
Length curve = 494.4 ft.	Total, = 917.5 ft.
P. T. = 22 + 14.4	tan 44° 30′ = .98270
Def. 100 ft. = 4° 30′	400 ft. × .98270 = 393.1 ft.
Def. 1 ft. = 2.7′	

from the center of each curve to its P. C. and P. T. On one of these radial lines, the length of the radius of the curve is written, and the amount of the central angle is written within the radial lines. The student will need no further directions to enable him to plat the balance of the line and also the notes for Fig. 2, a plat of which is given on this plate.

PRACTICAL METHOD OF LAYING OUT SHARP CURVES IN A MINE.

1215. Curves are frequently so sharp in a mine that it is more convenient to speak of them as curves of so many feet radius, instead of as curves of so many degrees. Thus, a curve of 100-foot radius means one that can be described with a radius of 100 feet. Curves with radii as small as 25 feet are sometimes used in mining work.

Quite unlike surface work, it is necessary to drive a curved entry in a mine before the curve is accurately located. This is no easy task, because the engineer can not be present to direct the workmen as is necessary, perhaps, several times a shift, to get the work properly done; and since the course is continually changing, *sights* can not be put up, as is done for the men who are driving straight headings. These sights consist of two strings suspended, a few yards apart, from the roof near the face of the workings, and in direct line with the desired course of the heading. When the workmen want to know how their place is going, one of them goes back to the sights and directs the other workman to hold his light in line with the strings. This fixes a point at the working face in the proper course, enabling the men to see whether they are too much to the right or left.

1216. Suppose it is required to connect two intersecting headings with a curve of 60-foot radius, around which the cars are to be drawn with a rope, and along which sheave wheels must be accurately set to guide the rope. The first thing to be done is to drive the curved heading, which work must be directed in a practical manner by the mine boss.

Let A and B, Fig. 238 be the two headings intersecting at right angles. The tangent distance $T = R \tan \frac{1}{2} I = 60 \tan 45° = 60$ feet. A distance of 60 feet is measured from c, fixing the point d in the line of the heading B. The point e is also fixed in the same line beyond d at such a distance as will suit the arrangement (Fig. 239) for determining points on the curve. In this case, the distance $e\,d$ is made to equal 10 feet. This completes the engineer's work until the head-

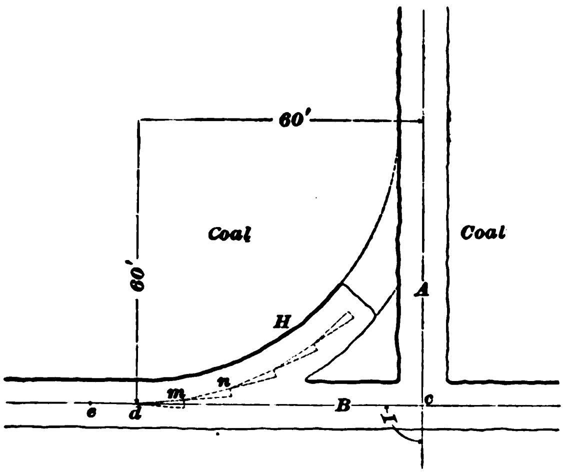


FIG. 238.

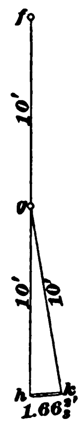


FIG. 239.

ing H is driven, when he comes back and definitely locates the final curve and position of the sheave wheels.

The mine boss now takes charge, and directs the men to work in the proper direction by means of the arrangement shown in Fig. 239, which is provided for him by the engineer in charge. It consists of three inflexible strings, or, better still, fine wires, connected to a small ring g. One of these strings fg is provided with a small loop at its end f, while the other two strings are neatly attached to a piece of wood $k\,h$. The construction can be readily understood by examining the figure. The length of each string fg, $g\,h$, and $g\,k$ is equal to 10 feet. This length is chosen because it is necessary to find points on the curve at least 10 feet apart. The length of the stick $k\,h$ corresponds to a chord deflection of a chord 10 feet long on a curve of 60 feet radius, and is found by substituting the known values in formula **81.** Thus,

$$d = \frac{c^2}{R} = \frac{10^2}{60} = 1.66\tfrac{2}{3} \text{ feet.}$$

The mine boss locates points on the curve near the working face in the following manner: He first places the string *fg* on the line of the heading *B* so that *f* and *g* will fall upon the points *e* and *d*, respectively—both distances *ed* and *fg* being each equal to 10 feet—and fixes the string in this position by means of tacks or pins put through the loop and small ring at the ends of the string *fg*; then he takes hold of the stick *kh* and pulls until both strings attached to it are tight and the center of the stick *hk* is in the same line with the string *fg*. The end *k* of the string *gk* will then lie upon the point *m* of the curve of the heading *H*. To find another point on the curve, fix the string *fg* on the points *d* and *m*, as was done in the previous case on the points *e* and *d*, then tighten up the other two strings and bring *gh* in line with *fg*; the point *k* will again fall upon the curve at the point *n*. In a manner similar to the last step, successive points 10 feet apart may be found near the face as the work progresses, thereby enabling the men to drive the heading on the proper curvature.

It will be understood that only two successive points near the working face need be preserved, which points are made upon ties firmly bedded in slack or dirt, and over which no mules should be allowed to pass. By exercising a little care, the heading *H* may be driven so nearly on the proper curvature with this method that little or no dressing up will be required when the transit is brought in to locate the track and sheave wheels.

HOW TO CARRY A SURVEY INTO A MINE.

1217. The first thing to be done after a careful survey has been made of the surface is to connect the outside and the inside work through the shafts and slopes. It frequently happens that correct surveys of the underground workings and of the surface are made independently, and the two finally connected. It is immaterial how we proceed. We will assume, however, that only a surface survey exists, which we will first connect to the inside work, and then proceed with the inside survey.

1218. In order to connect the outside and the inside survey, it is necessary to determine accurately the magnetic bearing of a line underground, and for this purpose the following methods have been employed:

1. By means of an adit-level or a slope.
2. By means of two shafts.
3. By means of one shaft and two suspended plumb-lines.
4. By means of the transit.
5. By means of one shaft and four suspended plumb-lines.

These methods will now be described in detail.

1219. By Means of an Adit-Level or a Slope.— When the mine is connected with the surface by means of an adit-level or slope, the surface survey is continued into the workings through this opening. In the case of an adit-level or slope of less than 45°, there is no difficulty, beyond the want of absolute rigidity, in setting up the transit, and the danger of moving it in walking about it. The difficulty increases more rapidly than does the pitch. When the slope reaches 60°, there is an impracticability in running a line down a slope, as the line of collimation of the telescope strikes the graduated limb of the instrument. In cases of this kind, the ordinary form of transit becomes useless, and a special form must be procured, provided with a prismatic eyepiece (see Fig. 243), which allows us to take sights upwards. In this case, the transit is set up at the foot of the slope where the longest sight up the same can be secured, and a backsight is taken upon a station of the underground work set up for the occasion. Let A be the station at the foot of the slope where the transit is first set up. Locate B where the transit can be readily set up, and, as far up the slope as we can see (this distance must be at least 100 feet), and in a continuation of $A\,B$, locate C. Set up at B and take foresight to C; locate D under the same conditions that governed the placing of B, and, in a continuation of the line $B\,D$, place E. Set up at D with foresight at E, and locate F and G as before. The survey is carried, by the intermediates B, D, F, etc., to the top, by a series of foresights to C, E, G, etc.

1220. By Means of Two Shafts.—When a slope reaches an angle of 90°, it is called "a shaft" in American practice, though the latter word is applied in foreign work to openings nearly vertical. In the case of two shafts, easily accessible underground one from the other, the sur-

Station.	Distance.	Azimuth Referred to *A*—1 as a Meridian.	Bearing Referred to *A*—1 as a Meridian.	Latitude.		Departure.	
				North.	South.	East.	West.
A—1	17.6	0° 00′	Due North	17.60			
1—2	42.4	357° 33′	N 2° 27′ W	42.36			1.81
2—3	130.0	102° 30′	S 77° 30′ E		28.14	126.92	
3—4	167.5	100° 26′	S 79° 34′ E		30.33	164.73	
4—5	93.0	101° 18′	S 78° 42′ E		18.22	91.20	
5—6	82.8	79° 51′	N 79° 51′ E	14.59		81.50	
6—7	37.4	84° 44′	N 84° 44′ E	3.43		37.24	
7—8	63.2	89° 10′	N 89° 10′ E	.92		63.19	
8—9	61.2	3° 03′	N 3° 03′ E	61.11		3.26	
9—*B*	11.8	318° 14′	N 41° 46′ W	8.80			7.86
				148.81	76.69	568.04	9.67
				76.69		9.67	
				72.12		558.37	

face and the underground survey are connected by means of two plumb-lines, one suspended in each shaft. From any well-defined and convenient stations in the surface survey, lines are run to the points of suspension of the wires in the two shafts. The angles made by these lines with the preceding courses in the surface survey are taken with the transit, and the horizontal distances most carefully measured. In this way, the length and bearing of the line joining the two points of suspension can be calculated by latitudes and departures. A survey is now made underground, joining the two plumb-lines, and, from the data thus obtained, the length and bearing of the line joining the two plumb-lines can be calculated in the same way as the above.

Take as an example: In two shafts two plumb-lines are suspended from points *A* and *B*. We will suppose it has been determined from the surface survey that the length of the line *A B*, joining the two points of suspension, *A* and *B*, is 563 feet, and its azimuth with the magnetic meridian, 118° 36′.

An underground survey joining the plumb-lines is now made, from which the accompanying notes are obtained.

Station *B* is, therefore, 72.12 feet north, and 558.37 feet east of Station *A*.

The direction and distance from *A* to *B* can be calculated,

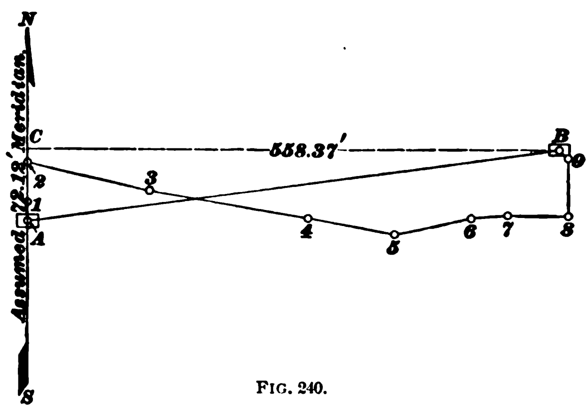


FIG. 240.

and the above notes platted to any convenient scale, as shown in Fig. 240.

In the right-angled triangle *A C B*, *A B* is the hypotenuse, and is calculated thus:

$$A\,B = \sqrt{72.12^2 + 558.37^2} = \sqrt{316,978.35} = 563 \text{ feet.}$$

The bearing of the line *A B*, referred to *A*—1 as a meridian, is found thus:

$$\tan C\,A\,B = \frac{558.37}{72.12} = 7.7422 = \tan 82° \; 38'.$$

Therefore, the bearing from *A* to *B* is N 82° 38′ E, and the distance is 563 feet. This is the bearing when referred to *A*—1 as a *meridian* in the underground survey.

The azimuth of the course joining the points of suspension on the surface is 118° 36′, or its bearing is S 61° 24′ E when referred to the *magnetic meridian*. It is evident that the line *A B* in the underground survey joining the plumb-lines must have this same *magnetic* bearing, S 61° 24′ E, because both *A* and *B* are vertically under the points of suspension.

It is desired to find the true magnetic bearing of the first course, *A*—1, in the underground survey, which was before *assumed* to be the meridian, or north and south, line.

By consulting Fig. 241, it will be plainly seen that the *magnetic bearing* of *A*—1 is N 35° 58′ E, since the azimuth of *A B* referred to the magnetic meridian is 118° 36′, while its azimuth, when referred to the assumed meridian, or course *A*—1, is only 82° 38′. Having now obtained the true magnetic bearing of the first course of the underground survey, no difficulty will be encountered in calculating the magnetic bearings of all the other courses from the azimuths which these courses make with the first course of the underground survey taken as a meridian.

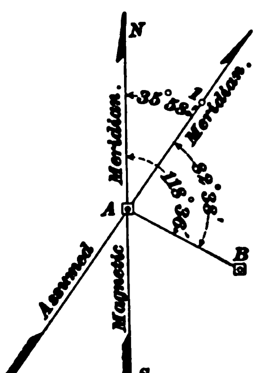


FIG. 241.

1221. By Means of One Shaft and Two Suspended Plumb-Lines.—Two wires are used, which are located as far apart as possible. Two pieces of scantling *c d* and *c f*, Fig. 242, are so spiked across the opposite corners of two compartments of a shaft as to allow the cages to pass up and down without interference. The station *X* is (roughly) located in a line through the corners *x*, *x* and is connected with the outside survey. From this station, locate in the line *X x x* two spads for holding the wires of the plumb-bobs. These are driven up to the head in the scantlings in such a way that the line of sight passes through the center of the holes in their heads. Measure the distances *X a* and *a b*. This completes the work of the survey above ground. The light copper wire is rolled upon a reel, and one end is fastened to a light plumb-bob to keep it free from coils or kinks in descending. It can thus be readily lowered without accident. When at the bottom, the upper end is fastened in the spad, and the heavy "bob" is applied to the lower end and placed in the empty barrel. The cages are then run slowly up and down, with an observer on each, to

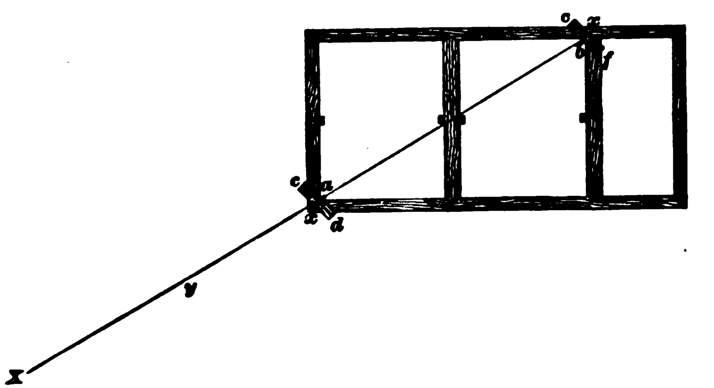


FIG. 242.

see that the wires hang free from top to bottom. By this time the wire will have stretched so that it will be straight, and if there is any slack it is taken up. The barrel is filled with water, and the top boards put upon it. As a last check, measure the distance between the wires below, and see if it agrees with the distance above.

Lining in below a point *y* on the line *a b*, we make a hole in the roof two inches in diameter, and drive in a broad plug. Setting up the transit under *y*, we sight at the wires *a* and *b* alternately. A number of methods for illuminating the wires have been used, several of which are given in text-books. The writer has always found those depending upon a sight of the wire across the flame of a lamp the hardest to obtain, and concludes from experience that the method of illuminating the wires for mine surveying is the best for this also. A large white target is placed behind both wires and illuminated by a large lamp with a reflector behind it. The wire stands out black against it, and can be followed across the target. As there is considerable distance between the wires, and as the transit is comparatively near them, there will be small chance of getting a sight of one when the telescope is focused upon the other, and so the focus has to be set between them. This gives a hazy sight at each, but both are shown against the white background in strong relief.

After the transit head is shifted so that the line of sight coincides approximately with both, focus upon them alternately, and see if the line bisects the swing of each. If so, the work is finished; if not, the shifting of the transit head must follow until the end is attained. It frequently requires two or more hours of steady observation to complete the work, for when it seems as if the proper point is secured, one of the wires will show by its swaying that it has been deflected from the vertical by a peculiar slant of wind, and the result obtained must be checked again. When you are through, there is no absolute certainty that the point you have marked is in the accurate extension of the line *a b* at the surface. Having decided upon the proper place, drive a spad into the plug overhead, hang a plumb-bob to it, and see if it be over the axis of the transit as shown by the screw on the telescope. If not, drive the spad so that the point of the bob does so hang, and the station *y* is said to be in the line *a b*. Measure *y a* and the angles to any station of the underground survey; the line *a b* is connected with the surveys at daylight and below, and the plumb-bobs may be removed.

Thin wires of iron or copper are used to suspend the plummets. They do not contract when wet, they present less surface to the action of air-currents, and can be more precisely read with the transit than larger cords, such as hemp.

The plummets should weigh from five to eight pounds. They should not be attached to the wires at the surface, to avoid any accident which might result from the wires breaking. The wires can be lowered by using smaller weights, and the plummets attached afterwards. The wires should hang perfectly free, and in no case should they come in contact with the sides of the shaft. In order to avoid the vibration of the wires, the plummets are hung in buckets of water, oil, or molasses.

1222. By Means of a Transit.—This method is adapted only to cases where the shaft is not deep, is of an average width, and is not subject to any considerable dropping of water. The instrument is set up at the bottom of the shaft, and is carefully leveled. In order that its telescope may be pointed vertically upwards, a diagonal eyepiece must be employed, which consists of a small right-angled glass prism placed at the eye-end of the telescope, in which the line of sight is reflected from the plane of the hypotenuse. See Fig. 243.

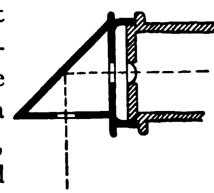


FIG. 243.

The instrument must be in perfect adjustment. After it has been set up at the center of the shaft, the telescope is directed to Station *B* (see Fig. 244), a station in the underground survey. The telescope is then revolved in a vertical plane in the direction of the point *a* at the mouth of the shaft, where a small lamp flame or a white peg is brought exactly in the line of sight. A permanent mark is then placed at *a*. The telescope is then directed to a point *b* on the other side of the mouth of the shaft, in the same manner as before, and a permanent mark placed there.

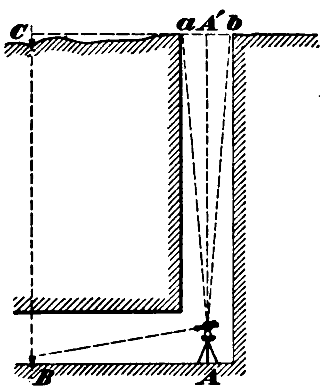


FIG. 244.

This gives two points on the surface in the same vertical plane as the line *A B* underground.

It is evident now that the line on the surface passing through the points *a* and *b*, namely, the line *A′ C*, will have the same direction as the line *A B* underground.

The horizontal distance from *A* to *B* is carefully measured, and exactly the same distance is laid off on the surface from *A′* to *C*. The point *C* will be vertically above *B*.

The bearing of *A′ C* can be correctly determined, which is also the bearing of the underground line *A B*.

1223. By Means of One Shaft and Four Plumb-Lines.—Fig. 245 shows the top set of timbers in a shaft of two hoisting compartments, down which it is desired to

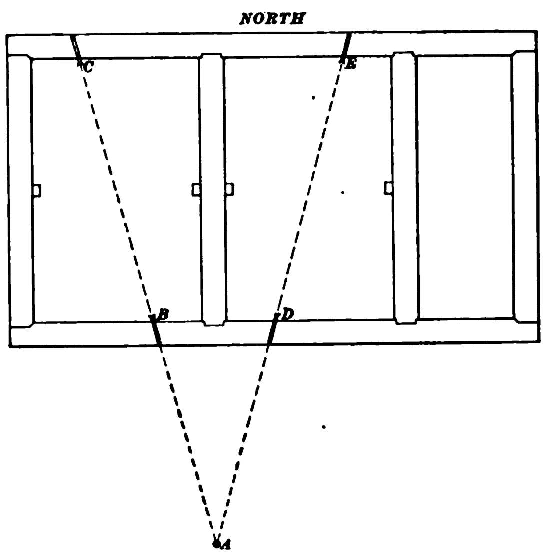


FIG. 245.

carry a known course or meridian on the surface to the entry below. The first thing to be done is to find out which side of the shaft is best adapted for setting up the transit, as the point to be marked in the mines will be vertically under the point on the surface; consequently, the side with the widest opening leading from the foot of the shaft should be selected.

Having carried the meridian to a convenient point near the top of the shaft, and having found that the south side of the shaft is the most accessible, determine, with an ordinary string, the location of the point *A*, from which the hangers for the plumb-lines will be exactly located by means of the transit. Now, mark with chalk on the timbers where the strings cross. These marks, though not accurate, serve as guides in setting the hangers. Make a permanent station at the point *A*, and carry the meridian to it.

The hangers can be made of strap iron, ½ inch thick by 2 inches wide, and at least 16 inches long. In one end of the iron, have a jaw with a fine cut at the apex, or a drill-hole just large enough to contain the wire to be used for plumbing. There should be two or three countersunk holes in the hanger, through which to fasten it to the timbers by means of sheet-iron nails. A top view of the hanger is shown in Fig. 246.

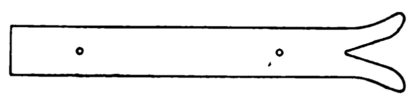


FIG. 246.

In most shafts, there is a space of from 2 to 4 inches wide between the ends of the cage and the sides of the timbers; and in order to hoist and lower the cage to see that the wires are hanging freely, it is best to set the hangers in such a position on the timbers that the wires will hang in the middle of the space.

Fasten the hangers permanently over the chalk-marks previously made on the north side of the shaft, with the jaws pointing towards *A*, and on the south side of the shaft the outer end of the hanger may be fastened temporarily.

Now set the transit over the station at *A*, take the backsight, foresight on the wire-hole of the hanger *C*, and set the wire-hole of the hanger *B* on the same line. Record this course, and foresight on the wire-hole of the hanger *E*, fixing as before the wire-hole of the hanger *D* in the same line. Record this course, and then the meridian to be carried into the workings below is established. Measure carefully and record the distances *A* to *B*, *A* to *C*, and *B* to *C*, the distances *A* to *D*, *A* to *E*, and *D* to *E*, and, finally, the distances *B* to *D* and *C* to *E*; the necessity of taking all these measurements is for the purpose of establishing a point at the bottom of the shaft vertically below *A*, and checking the work in the office.

The transit party can now descend to the bottom of the shaft, taking with them four buckets of oil, the weights or plumb-bobs to be attached to the wire, and all the surveying instruments, leaving a responsible party on the surface to handle the wires. Having arrived at the bottom of the shaft, have the cage hoisted 3 feet above the landing, throw several planks across the timbers on which to set the buckets of oil, signal to the man on top to lower a wire and to fasten it securely, passing it through the wire-hole of the hanger; now, attach the plumb-bob and adjust the wire to such length that, when sustaining the full weight of the plumb-bob, the latter will not touch the bottom of the bucket. Insert the weight in the oil, using care not to leave the full weight on the wire with a jerk, but let the weight down slowly, so that the wire receives the full strain gradually. Set the three remaining wires in a similar way.

After the wires have been hanging a few minutes with the weights attached, the latter may move from one side to the other of the buckets. Watch this carefully, and keep moving the buckets until all the weights hang perfectly free, then leave everything alone until the wires become steady. The cages can now be hoisted and lowered for the purpose of examining the wires to see that they hang free and plumb, care being taken that the cages are not brought too close to the landings, so as not to disturb the hangers at the top, or the buckets at the bottom.

To find a point vertically below *A*, stretch a string along the wires *B*, *C*, being careful not to touch them; stretch another along the wires *D*, *E*; then, with a plumb-line, determine a point on the bottom vertically below the intersection of the strings. Measure the distances *A B*, *A D*, *B D*, and *C E*, and compare them with the corresponding distances at the top of the shaft. If these distances compare favorably, the wires are, in all probability, steady, and the work of determining the desired course with the transit may now be begun.

Set the transit up over the point of intersection just found; backsight on the wires *B*, *C*; foresight on the wires *D*, *E*, and compare the included angle and the distances with the corresponding angle and distances at the surface. If these do not correspond, move the transit in the direction necessary to increase or decrease the angle or distances, as the case may be. Repeat this operation until the exact point vertically below *A* is determined.

A simple device which is of great advantage is to have three links from an ordinary trace chain placed in the wires on the side towards the transit, and a few feet above the buckets. This not only enables the wires to turn freely, but also enables the transitman to sight through one of the links to the wire beyond, whereby he can place the transit in exact line with the wires more easily than if the links were not there.

STATIONS.

LOCATION OF STATIONS.

1224. As the location of stations is the most important duty of an engineer in surface work, so it takes the first place in underground work, as the accuracy of the work depends on the location of the stations, while its rapidity depends on using the fewest number consistent with completeness. It also stands to reason that the fewer the number of stations, the fewer the chances of error. In underground work, stations should be located under the conditions of permanence, freedom from destroying agencies, and ease of access. Temporary stations for a single sight need not fill all these requirements. Stations are generally established in the roof of the mine, although they are sometimes put in the bottom. In the former case, we must establish a "center" before each set-up of the transit; in this respect, underground work differs from surface work.

The first surveys were made with lamps set on the floor, sighted to, and then set over. Permanence was secured by driving iron nails or tacks in the sills of the track or sets of timber. As acid water soon destroyed these, they were followed by copper tacks or brads, and all were witnessed by notches cut on both sides of the sill, as in outside work, and by a vertical paint mark on the solid wall, with the number of the station. This method is faulty, as the tracks in crooked gangways are seldom placed where one can get the longest sight, and, as they are the traveling-ways, the stations run the chance of being knocked out by passing men or mules, and the whole track on a curved incline is generally sprung by every loaded trip.

As the sights must be as long as careful work will allow, we generally put them in the roof, as that offers the greatest area for a choice, and is not under foot. Any settling of the roof which would affect the accuracy of the station would be equally effective in destroying the accuracy of a station in the floor. We, therefore, choose places that will be least affected by subsequent work, and put the stations in collars, lids, or wedges of props, in the props themselves when they have incline sufficient to allow the transit to be set under them, or in the roof itself. Wherever set, they should not project far from the surface and thus be liable to be brushed away in a low gangway by cars with topping higher than usual, or knocked away by flying fragments from a shot, if near the working faces. Top stations have a mark about them to call attention to their location.

KINDS OF STATIONS.

1225. *First.*—The simplest top station is a shallow conical hole made with the point of the foresightman's hatchet, which is dug into the top rock and rotated; it is called by some a **jigger station.** Corps using these entirely have a **jigger,** consisting of a steel-pointed extension rod with an offset holding a paint-brush. The rod is long enough to allow the point to be driven into the roof at any height, and its rotation marks a circle with the brush, which is also used to mark the number beside the circle. Centers are set under such stations, and sights are given by another tool—also called a jigger. This is an extension rod, beyond the upper end of which projects a piece of sheet iron shaped like an isosceles triangle, with the upper and smaller angle cut off so as to form an end one-quarter of an inch broad, and in this end is cut a U-shaped groove.

The sights are given and the "centers" set by putting the plummet cord in this groove, and placing the end in the "jigger hole" in the roof. The cord must be more than twice the length of the section of the place, as it must be held in the hand, run over the jigger notch, and thence vertically downwards to the plummet, which must come to the bottom when the stations are set. The rod and cord are held in the left hand, and the right is free to steady the "bob," give sight, or set the center. The advantage of this method lies in the rapidity with which the centers are set and the sights given, and the ease with which the highest stations are reached. The disadvantages are the impossibility of making the jigger hole perfectly conical, so that the jigger can be set in the same place on two successive sights, and of making the plummet cord hang exactly in the same place.

1226. *Second.*—Common shingle-nails are driven into collars, or cracks in the roof. The end of the plummet line is noosed and put over the head. This causes an eccentric hanging of the plummet which may cause an error in back and foresight of the width of the nail-head, which will be quite appreciable in a short sight. To do away with this error, a variety of nails (called "spads," "spuds," etc.) are made of iron or copper. Iron will not corrode in dry mines and is much cheaper. The simplest is made by hammering out the head of a horseshoe nail, punching a hole in the flattened head for inserting the cord, and cutting off the point, so as to make the finished spad an inch long. This will bring the head near the surface without having to drill too deep a hole, and will make them unfit for lamp-picks, and therefore not likely to be drawn out for such purposes. Any blacksmith can furnish them for less than one cent each. They are driven broadside to the line of sight, otherwise they would be liable to the same objection as the shingle-nail. To remove all chances of eccentricity, a form is made with a shoulder in which a hole is drilled parallel to the length of the nail. The practice of using staples for stations is antiquated—though given in the last editions of some modern text-books—and should never be used where accuracy is required.

1227. *Third.*—All these varieties of spads are driven into a crack of the roof; but such stations can not be called *permanent*, as the same force that made the crack will tend to open it and let the nail drop. Even if this does not happen, we shall have the water in a wet mine coming in through these cracks, and rotting the nails, or the rock at the sides of the crack, and in a month after the placing of the station, it will be unfit for use.

1228. *Fourth.*—A spad is driven into a wooden plug forced into a hole drilled in the roof. The swelling wood clamps the spad and prevents its coming out as readily as it was put in. The plugs are made of well-dried wood outside, and are carried by the man who sets the stations. The first holes were made by a jumper, and the plugs were two inches square and six inches long. The modern holes are usually made by a twist-drill of as small a diameter as will do the work without bending at the shank. Such drills can be used in slate or clay, but an ordinary drill and hammer must be used in hard rock. The average modern holes are from one-eighth to one-half an inch in diameter, while the plugs are one-half to one inch long as the maximum. The smaller the hole, the quicker the work. All stations should be put in the roof in preference to the under side of the collar, or in any ordinary timbering. The only exception is where the roof is too poor to hold them. In extending a survey, such stations should be checked before they are used, if we wish to swear to the accuracy of our work. The engineer who believes in using collars may find himself in the quandary of the man whose company worked across their line because he started from a collar station. Since its location, the place was working, and the collar was taken down and turned end for end when replaced. *Good side notes, if consulted, would have shown him the change.*

1229. *Fifth.*—A twist-drill $\frac{3}{32}$ of an inch in diameter is used to make a hole in the roof; a piece of cord—or, better, a copper wire—is placed across this, and a hardwood shoe-peg is driven into the hole and binds the cord tight. The plummet is tied to the lower end. A cord will soon rot, and, if in the gangway, is pulled out by the drivers for whip-lashes, while the wire is more permanent; but even this will be pulled out by catching in the topping of a car in a low place.

1230. *Sixth.*—The use of spads is dispensed with, and all the stations put in rock roof where possible. A quarter-inch twist-drill makes a vertical hole one inch deep. Into this, when a sight is to be taken, the foresightman puts a steel clip with serrated edges. This is made by bending a thin piece of steel $\frac{3}{32}$ of an inch wide until its ends nearly touch. When the ends are pressed together, it will go into the hole, and the spring of the sides and the serrated edges hold the clip in the hole, so that it is hard to pull out. The cord passes through a hole in the center of the bend, and is, therefore, in the center of the hole—no matter how the clip is inserted. It is removed by pressing together the ends of the clip. This is the easiest and quickest way of working, as there is no eyehole to be freed from dirt and no knot to be tied and untied. The hanging of the plummet takes a fraction of a second, and the station will remain as long as the roof keeps up. The disadvantages are the drilling of the holes inclined to the vertical by a careless man, and the many roofs that are unfit for piercing with a twist-drill.

MARKING STATIONS.

1231. There should be some regular way of witnessing the stations. In general, a vertical line on the "rib" calls attention to a station in the bottom, near the side marked. A roof station has some geometric figure marked around it, as has been described. If three regular corps are engaged in the same field and meet in the same mines—as the company corps, the corps of the individual operator, and the private corps that is looking after the interest of one or more of the landowners—they must use different signs for stations. The most common are the circle, square, and triangle. If the "circle" corps puts in the station, it has a circle about it. The next corps uses the same station, and puts a square about it and notes "Sta. 472 = Sta. 742 of ( ) Corps." The third corps also uses the station and puts a triangle about the square, and notes "Sta. 617 = Sta. 472 of ( ) Corps, and Sta. 742 of ( ) Corps." If the first corps uses the station again, it notes the numbers given by the other two corps, and these three numbers will aid in identifying it if one or two of the numbers are lost.

DISTINGUISHING STATIONS.

1232. Each station must be lettered or numbered so that it can be readily recognized when the subsequent surveys are made. When set, a station may be at the end of a gangway, while six months later the gangway may be driven hundreds of feet from that place, chambers turned off it, and the place be so utterly unlike its former state that nothing but a fixed mark belonging to that station alone will enable one to recognize it. The methods of distinguishing stations vary widely. In one place, the writer found that each gangway and room had a Sta. 1 at its beginning, and the various stations numbered 1 were designated "Grog Run 1, 2, 3, etc.; Pat James' Gangway 1, 2, etc."; and so on through the map, that showed between fifty and one hundred stations numbered 1, so that a new engineer would have had to learn the mine thoroughly before he could extend a survey. Another way is to use A1, A2, etc., up to A100, and so through the alphabet, *to avoid running up too high in numbers.* A third method is to letter the various sections of the mine A, B, C, etc., the numbers beginning with 1 in each and running up indefinitely.

All of the above have disadvantages, as powder or lamp smoke, mud, mold, or the work of mischievous boys may so obliterate or obscure a mark that it can be recognized only by association with its immediate neighbors, and these may have shared the same fate. You may have only a part of the mine map with you, and because the system of marking strives to get along with as few symbols as possible, you have to go to the office, when there would have been a chance of deciphering the mark if there had been a number of figures to it. The best practice, therefore, is to use large numbers, beginning with Sta. 100 at the mouth of the slope or drift, or the foot of the shaft, and numbering consecutively in each bed. In this way, three figures are used at once, while in old mines the numbers require four digits. The chances of obscuring such a mark are lessened, while the chances of our deciphering it are increased.

CENTERS.

1233. When the station is in the roof, there must be something over which the transit can be set, for the way instruments are constructed it is easier and much more accurate to set over a station than to set under one. The set-up is made over a "center." At first, a cross scratched on the floor, or on a loose piece of slate, a daub of white lead on the same, with a small piece of coal placed under the point of the plummet after it has been steadied, or, finally, a nail driven into a block and afterwards pointed, were used. All of these, except where the mark was on the solid floor—if they were large enough to be stable—were in the way of the observer's feet, while, if small, they were so light as to be readily displaced. It must be noted here that it is not so much the errors that we can foresee and detect that influence the accuracy of the work in our own eyes, but the chances of error from accidents that we can not control and that can not be readily detected. To avoid the above chances, we make the centers as small and as heavy as we can—in other words, we make them of lead. A hole 1¼ inches in diameter and ½ an inch deep is bored in a thick plank; a brad is set in its center with the head down; the hole is filled with melted lead, and the brad is slightly raised to surround the head with lead, and held with pincers in a vertical position till the lead has set. The brad is cut off ¼ of an inch above the lead and pointed. This "center" combines weight and small size, and is generally used.

PAINT.

1234. White lead, or Dutch White, thinned with linseed oil, is ordinarily used. It is carried in a covered tin pail holding a pint. The cover has a hole large enough to admit the brush. The pail generally has to be cleaned out after each day's work, as the brush gathers dirt every time it is used. In case the paint is to be kept for a number of days, it must be covered with water, which can be poured off before using. If the ordinary paint-brush has too long bristles, it can be shortened and kept from wearing by winding with fine wire to the proper length.

THE MINE CORPS.

1235. The method of dividing the work in an underground survey depends on the size of the corps. At least three men are necessary, two others besides the transitman, and more can be used to great advantage.

An ideal corps will be given, and the duties of each man described. The chief of the party should be where he can do the greatest good, and where he can direct the work for his subordinates. The principal point of the survey is the setting of the stations so as to do the work thoroughly with the fewest set-ups, and thus to diminish the chances of error in instrumental work and other measurements.

The chief should locate the stations, and add all the necessary signs to show how the work is to be done. The transitman should not have his attention distracted from his particular work by questions as to procedure. He should work untrammeled. The chief, therefore, should not run the transit. Upon this basis, the ideal mine corps works, and should consist of at least four men from the office and three from the mine. It is divided into two sections. The chief takes the men supplied by the mine—one or more of whom are acquainted with the mining work done since the last survey—and locates the stations.

The transitman follows with the second section to measure angles and distances. When the stations are set, the chief takes his men after the transit party, and gets the side notes, with a check measurement of the distance between stations.

Such a corps goes to the end of the former survey, and identifies the last two stations. The transitman prepares to set up at the last, while the chief, with his party, goes as far as he can see the light from the last station, or to some intermediate point from which one or more sights are to be taken. He then stops, and sends a man along each place where a sight must be taken, as far as their lights can plainly be seen from top to bottom from where he is standing. Over this place he marks a point for a station to be inserted, and generally inserts it himself unless he is pushed with work and must leave it for another to do, when he places a circle about the dot, places the number at the side, and as many arrows as there are new stations, the longer (Fig. 247) pointing to the sight to be last taken and where the transit is to be set up next. Leaving a backsight at the point just set, he sets, successively, stations at the points where the foresightmen have stood in the manner just described till he has covered the new work—the mine boss or some intelligent miner going with him to give him an idea of the "lay of the ground," so that the work can be covered with the fewest number of stations. Sometimes the chief takes the side

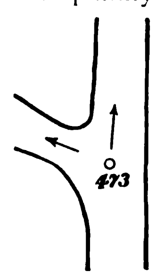


FIG. 247.

notes and measures the distances between stations as fast as they are set. If on a pitch, a circular brass protractor with small plummet is hung at the center of the stretched tape to give the angle at which it is held, the measurement so obtained to serve as a check on the measurements taken by the transit party, which are used as the basis of the work, and the other measurements are solely as checks. In flat work, both measurements should coincide.

In a small corps, and where time is of little importance, the foresightman puts in the stations ahead of the transit, and while he is doing so the transitman takes the side notes. Sometimes the side notes are taken by the same man, while one of the party is taking the transit to the next station and setting it up for the next sight. There are about as many variations from these two methods as there are corps.

The *foresightman* should be intelligent and active, as the amount of work done in a day depends upon his ability to *keep ahead of the transitman.* His duty is to set the center for the next set-up under the station, and also to place the tripod (if three are used in the work), to give the sight, and, in some corps, to carry the front end of the tape and assist in taking the distance. In some corps he also carries the bag with tools for setting stations, so that he generally has a load that makes rapidity of movement difficult, and anything that will diminish the weight carried will tend to quicken the work.

The rapidity with which good work is done varies considerably, but it depends upon the activity of transitman and foresightman. It varies also with the distances between stations. The saving of time should never be sought at the expense of accuracy in the work; it is to be gained by rapidity of moving about, in setting transit, center, etc., and in hanging plummets to give sight. The foresightman and backsightman should be in position to give sight before the transitman is ready, so that he can turn his instrument on one or the other and find them in position.

The *backsightman* has little to do inside, and to compensate for this, he is the one who cleans and oils the tape, gets out new plummet strings, and sees that the tools are ready for the next work, as soon as the corps gets to the office. The *transitman* cleans the transit, unless the corps has subordinates who can be trusted with so delicate an instrument. The blackening from sulphureted hydrogen is rubbed from the silvered surfaces with whiting, and the oil or paint smears are removed with alcohol. Alcohol should be used instead of water for cleaning the instruments, and especially the lenses, which are wiped with jeweler's cotton or soft chamois skin.

INSIDE WORK.

1236. As the beds of anthracite lie at all angles, they give rise to what is known as **flat** and **pitching work,** each of which requires a special method of surveying. *Flat work* is that which is done on beds having so slight a dip that the cars can be drawn to the face of the room. Usually, in this kind of work, there is nothing to prevent a sight being taken from the gangway to the face. The variations in the methods of work in this case depend upon the accuracy with which the work must be performed, as, in some cases, the workings are approaching the boundary-line of the property, and the sides of the rooms must be located accurately. In general, the rooms are driven at right angles to the gangway, unless the dip is too great to haul a car to the face on that line, when they are inclined to the gangway at an acute angle. The width of the rooms in flat work is generally uniform where the roof is good, but where the roof is poor, the entrance is narrowed for a short distance (to better support the gangway) and then widened to the full width, or the whole is driven to the limit narrow, and the side is robbed when the top is drawn, and the whole room caves in. This last must be surveyed before the robbing begins.

The most accurate method of surveying is to run a line along the gangway and put a station at the entrance of each room, whence a sight is taken to the face. This may be varied by putting the stations at alternate rooms and measuring through the cross-cuts to get the thickness of the pillars of the intermediate rooms; or by placing stations at every third room and measuring the thickness of pillars and width of rooms that intervene; or, finally, by running out the gangway with as few sights as possible and paying no attention to the positions of the rooms in setting stations, thence up to the last room to the face, and back through the cross-cuts nearest the face to the former work, where a tie is made. When opportunity offers, sights are made from the face of the rooms to the stations in the gangway for intermediate ties. In case a gangway and an airway have been driven considerably ahead of the rooms, it is always necessary to run lines in each, and tie at the last cross-cut. This must be done in every case where the gangway is approaching the boundary-line, or old workings that have been abandoned and are full of water. In addition to this check, the miners must keep bore-holes twenty feet ahead in the line of the gangway, and every twenty feet they must drill others from the corners of the heading at an angle of 30° with the line of the gangway. In this way, there will be no danger of running into "a house of water," as the Cornish miners call it, if the survey be inaccurate.

1237. *Pitching work* is that which is done on beds inclined so much that a car can not be run to the face. It is generally difficult to see from the gangway to the face, even where the roof is good and the room straight, as a buggy track or chute, or both—when the pitch is slight—fill up the room. Where the pitch is great, or there is a "battery" shutting off the neck of the room, the face can be reached only by several sights. Where the roof is poor, the obstructions are increased, as the rooms are driven narrower, or, if wide, they have center props and stowing in the center. If the coal is full of slate, or if the partings are thick, a great part of the room is taken up with piles of "gob," and with a very poor roof the body of the room that has been worked out is filled with the fallen roof, and the coal is sent out through the triangular manways, where it is almost impossible to take a sight.

Work of this kind is surveyed by lines run in the gangway and back along the faces of the rooms, which are generally clear, even if the bodies of the rooms are filled with the fallen top. Where chance favors, sights are taken to the gangway; but this very seldom happens, as the two lines are as effectually separated as if in different mines. From the stations near the faces, lines are run down the rooms as far as possible to get their direction and to locate the cross-cuts. The very worst case of all is where two beds are separated by a thin parting of rock and the gangway is driven in the lower one alone, the rooms in the upper one being worked by rock chutes into the rooms below, or into the chutes from those rooms. This class of work is hard to ventilate, and hard to survey where the rooms above are ventilated by the air system of the lower beds, but is readily surveyed and mapped where there is an air system for each bed.

MINE SURVEYING AND MAPPING.

(PART 3.)

LEVELING.

INTRODUCTION.

1238. Leveling is the operation of determining the vertical heights of points above some adopted plane of elevation. Wherever convenient, the plane of mean sea-level is taken, although any other plane, whether real or imaginary, may be taken as the basis of operations. Any line in the assumed plane which is taken as the basis of the operations is called the **datum line**, or **datum**.

1239. In mining work, leveling is conducted principally for the following purposes: (1) to determine the difference of level between two or more known points; (2) to determine the elevation of the ground along any desired line, for the purpose of obtaining a profile or a section along that line.

1240. The relative elevations of the various points are to be ascertained without regard to their absolute elevations. But, in order to have a convenient basis of comparison upon which to obtain the relative elevations of the various points, it is convenient to assign an assumed elevation to some one of the points, usually the starting-point.

It is of little importance what elevation is assumed, except that the starting-point should be assumed so far above datum—or, in other words, the position of the datum line should be assumed so far below the starting-point—that no point in the line of levels will be liable to fall below the datum line; for, if any point in the line of levels were to fall below the datum line, its elevation would become minus, which would cause unnecessary inconvenience in keeping the notes. For short lines of levels in ordinary country, it is quite customary to assume the elevation of the starting-point to be one hundred feet above datum. But in rough country, or for an extended line of levels, in order that no point may fall below datum, it will be necessary to assume a greater elevation for the starting-point; an elevation of one thousand feet is not uncommonly assumed.

THE ENGINEER'S LEVEL.

1241. The level is an instrument which, when properly adjusted, will accurately indicate the direction of a level line. For this purpose, instruments of various forms and construction are employed. The instrument commonly used by engineers, however, and which has been found to be the most satisfactory for the general purposes of a leveling-instrument, is what is known as the *engineer's wye level.* It is called a *wye* level, or **Y** level, on account of the **Y**-shaped supports to the telescope, which are well adapted to the accurate adjustment of the instrument, as will be hereafter noticed. As the wye level is the instrument used almost universally in this country for the purpose of leveling, it is important that the student should become familiar with the general principles of its construction and with its adjustments.

An engineer's wye level, embodying the usual features of modern construction, is shown in Fig. 248. It consists essentially of the *telescope c o*, the *spirit-level l l*, the *tripod head h*, and the various appurtenances and devices for attaching the level and telescope to the tripod head, and for adjusting, leveling, and sighting the instrument. In the telescope,

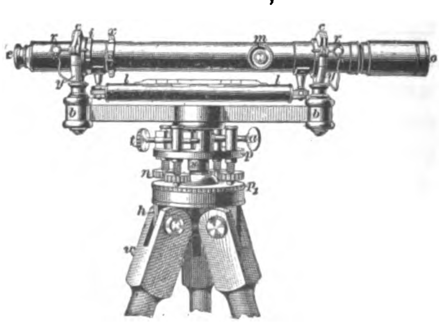


FIG. 248.

e is the *eyepiece*, and *o* is the *object-glass*; *m* is a milled-headed pinion, fitting a rack on the inside of the telescope tube, by means of which the position of the object-glass can be adjusted. The position of the eyepiece *e* is also adjustable. On the inside of the telescope tube at *x* are the *cross-hairs*, or *cross-wires*, at the intersection of which the image of the object observed is focused by the object-glass. The position of the cross-hairs, with reference to the optical axis of the telescope, is adjusted by means of four small capstan-headed screws shown at *x*. The axis of the eyepiece is adjusted, or centered, by means of the grooved-headed screws shown at *i*.

The level is attached to the telescope by adjustable attachments, in order that it may be adjusted parallel to the axis of the telescope. The telescope is supported upon the Y-shaped standards *y*, *y*, in which it is firmly held by the clips *c*, *c*, which are fastened by the small tapering pins *r*, *r*. The standards *y*, *y* are commonly called *wyes*. They are securely attached to the horizontal level-bar *b b*, and one wye is adjustable by means of capstan-headed screws. The bar *b b* is rigidly attached to a steel spindle, commonly called the *center*, fitting very accurately into a socket *s*, which is firmly attached to the upper parallel plate *p*. The lower end of this socket connects, by means of a ball joint, to the lower parallel plate p_1, which is rigidly attached to the tripod head *h*. The plate p_1 is supported upon three wooden legs *w*. The plates *p* and p_1 are called the *parallel plates*, though they are seldom really parallel.

The position of the upper parallel plate, which carries the spindle socket and determines the position of all the upper portion of the instrument, is adjusted by means of the four milled-headed screws *n*, which are commonly called the *leveling-screws*. It is by means of these four screws that the instrument is accurately leveled up when in use. By means of the thumb-screw *a*, the spindle can be clamped in any position in its socket, thus fixing the direction of the telescope, which may then be slowly turned horizontally by means of the milled-headed tangent screw *t*.

The level *l l* consists of a thick glass tube or vial, slightly curved upwards; it is so nearly filled with alcohol as to contain only a small bubble of air, the tube being permanently sealed, and fixed in a metal case having an opening in the top. The interior of the tube, or, at least, the upper portion of it, is ground perfectly cylindrical and to a regular curve. The bubble of air left within the tube will always seek the highest point, and will, by its movements, indicate any change in the position of the tube; when the tube is in a nearly horizontal position, the slightest change in its position will give considerable movement to the air-bubble.

Above the opening in the metal case which encloses the level, and through which the level-tube and the movement of the air-bubble can be seen, is placed a scale containing the same graduations and numbers each way from the center. By means of this scale, the position of the air-bubble, with reference to the center, may be readily seen. The graduations are sometimes made directly on the glass tube. In order that the level may be so adjusted that a line tangent to the curved inner surface of the tube at its center will be parallel to the axis of the telescope, the metal tube in which the level is enclosed can be adjusted both vertically and laterally. In Fig. 248, the capstan-headed screws for the vertical and lateral adjustments of the level-tube are shown at its left-hand and right-hand end, respectively. Both adjustments are made by means of these screws. By means of the left-hand screws, the left-hand end of the bubble-tube can be raised or lowered; and by means of the right-hand screws, the right-hand end of the tube can be moved sideways.

Fig. 249 is a longitudinal section of the telescope *c o* of Fig. 248. As shown at *o*, the object-glass is composed of two lenses, one being a double-convex and one a plano-concave lens. One lens is commonly made of crown glass, and the other of flint glass. This combination of lenses for the object-glass has been found necessary in order to render it achromatic; that is, so that it will not show color by the decomposition of light. The object-glass is not attached to the telescope tube proper, but to a slightly smaller tube *t*, which is accurately fitted into the end of the telescope tube. In order that the image may be focused exactly upon the cross-hairs at *x*, the tube containing the object-glass is made to slide in and out by means of the pinion *p*, working in a rack attached to the object-glass slide, and operated by the milled head *m*. When the telescope is used in the sunshine, the object-glass is protected by a sunshade *s*, which is detachable, and is generally removed when the weather is cloudy. When not in use, the object-glass is covered by a cap which fits over it in much the same manner as the sunshade.

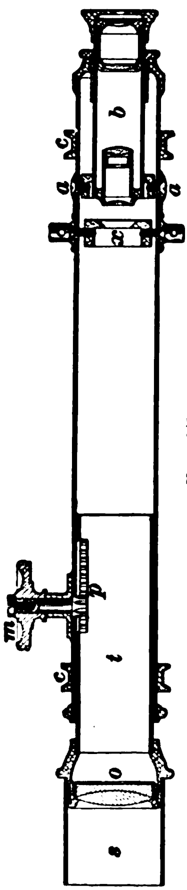


FIG. 249.

The cross-hairs in a telescope are made of very fine platinum wires, or of spider threads. These are attached to a short, substantial diaphragm, or ring, *x*, which is suspended within the telescope tube by four screws. This arrangement is similar to that shown in Fig. 210.

The eyepiece *b*, Fig. 249, is simply a compound microscope, which magnifies the image projected at *x* by the object-glass. It consists of four lenses arranged in a proper manner. Beginning at the eye end, they are known as the *eye*, the *field*, the *amplifying*, and the *object* lens, respectively. In order to accommodate individual peculiarities of vision, such as short-sightedness and long-sightedness, the eyepiece is also made to slide in and out a small amount. The movement of the eyepiece is accomplished in three different ways. In some instruments, it is simply a plain slide, and is pulled out and pushed in with the hand. In other instruments, a rack and pinion movement is employed, similar to that for the object-glass slide. In still other instruments, the eyepiece is carried out or in by a spiral movement, operated by turning a milled ring at the end of the main telescope tube. The spiral movement is generally preferred, although, owing to the fact that it is seldom necessary to move the eyepiece, some prefer the plain slide. The eyepiece is centered for the image by means of the grooved-headed screws shown at *a*, which are the same as those shown at *i*, Fig. 248. This adjustment is omitted by some instrument-makers, who claim that, if the instrument is accurately made, subsequent adjustment of the axis of the eyepiece is wholly unnecessary.

On the outside of the telescope tube, and near its respective ends, are two rings, or collars, *c*, *c*. These collars serve as bearings upon which the telescope rests in the wyes, and are generally composed of bell metal. They must be turned truly circular, and must be of *exactly the same* external diameter. Upon the accuracy of these bearing collars depends, to quite an extent, the accuracy of the instrument; for, unless the collars be perfectly circular and of exactly the same size, the level can not be accurately adjusted by the methods commonly employed. The collars do not rest in the wyes throughout their lower semi-circumference. As the collars are perfectly circular and the wyes are Y-shaped, the collars rest in the wyes upon two small areas of contact, for which very accurate bearings can be obtained.

The center, or spindle, by which the horizontal level-bar is connected to the upper parallel plate, is a solid piece of metal, generally about three inches long, and slightly conical. The metal is preferably steel, and should be turned to very accurately fit the socket, which should be composed of a different metal—generally brass. The arrangement of the parallel plates, and the four milled-headed leveling-screws by which the position of the upper plate is adjusted, will be readily understood from Fig. 248.

ADJUSTMENTS OF THE LEVEL.

1242. First Adjustment.—*To make the line of collimation parallel to the bottoms of the collars.*

Plant the tripod firmly; choose some distant and clearly defined point, the more distant the better, so long as the sight is distinct. Clamp the spindle and remove the pins *r* from the clips *c*, Fig. 248; then, by means of the tangent screw *t*, bring the intersection of the cross-hairs to bear exactly upon the point chosen. Revolve the telescope in the wyes until the level-tube is on top. If the intersection of the cross-hairs is still on the point of sight, it shows that the line of collimation is parallel to the bottoms of the collars. If it is not, move the cross-hairs by means of the capstan-headed screws at *x* over one-half the space of the error, being careful to move them in the opposite direction to that in which it would appear they should go.

1243. Second Adjustment.—*To make the plane of the attached level parallel to the line of collimation.*

Remove the pins *r* and open the clips *c*; turn the telescope until it is over a pair of leveling-screws, and clamp the spindle by means of the thumb-screw *a*. Bring the bubble to the middle of its tube by turning this pair of leveling-screws. Then, revolve the telescope in its supports through an eighth of a revolution, which will make the bubble-tube stand out at an angle with the wyes. If the bubble has moved towards one end of its tube, it shows that a vertical plane passed through the longitudinal axis of the bubble-tube is not parallel to a vertical plane passed through the line of collimation. To correct the error, bring the bubble half way back by means of the check-nuts which regulate the lateral movement of the tube. Repeat the operation until the bubble ceases to run when the partial revolution is made.

To complete the bubble adjustment, level the telescope, take it out of the wyes, and replace it *end for end.* If the bubble remains in the center of the tube, the second adjustment is complete. If it moves towards one end, bring it half way back by means of the check-nuts provided for raising or lowering one end, and bring it the rest of the way, i. e., to the middle of the tube, by means of the leveling-screws. Repeat the operation, one trial being usually insufficient.

1244. Third Adjustment.—*To make the axis of the telescope perpendicular to the spindle, so that the bubble will remain at the center of the tube while the telescope is revolved horizontally.*

Fasten down the clips *c*, unclamp the spindle, and bring the bubble to the center over each pair of leveling-screws in succession. Revolve the telescope 180° horizontally, and correct one-half the movement of the bubble by the lock-nuts at the end of the level-bar, and the remainder by the leveling-screws *n*. Repeat the operation, as several trials may be necessary to make the adjustment exact.

The adjustments of a level in constant use should be tested daily, as any derangement will detract from the accuracy of the work that is done, while it may be so much in error as to render all work absolutely valueless.

LEVELING-RODS.

1245. Leveling-rods are used to determine the points at which a given horizontal line (the line of collimation) intersects lines that are perpendicular to a horizontal plane at the surface of the earth, and to show the distances of such points of intersection from the ground. The rods are divided into feet and hundredths of a foot, and can be read to thousandths of a foot by means of a vernier. Of the various kinds in use, only one will be described.

1246. The **Philadelphia rod,** which is shown in Fig. 250, is cut into two parts so that both ends can be exhibited. It is made of maple or satinwood, in two pieces, sliding one from the other, always in the same direction, so that the same end is always held on the ground, and the graduations start from that end as zero. The graduations are made to tenths, the tenth figures being black, and the feet marked with a large red figure; this rod reads to hundredths.

A target is used to indicate where the line of collimation cuts the rod. The face of the target is divided into quadrants, by a horizontal and a vertical diameter, and the alternate quadrants are painted different colors, as red and white.

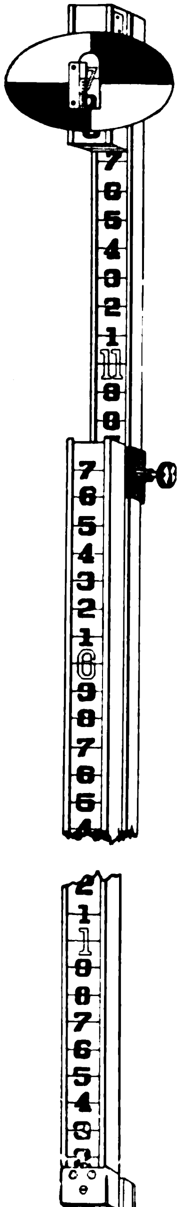


FIG. 250.

Along the right-hand half of the horizontal diameter is a slit about one-eighth inch wide, behind which a lamp is held when the level rod is used in the mines. In sighting the target, this slit looks like a bright line **cut** by the horizontal cross-hair in the telescope.

The opening in the face of the target is a little more than a tenth of a foot long, so that in any position either a tenth or a foot figure can be seen on the face of the rod.

The left edge of this opening is divided into ten equal spaces, each division marking a hundredth of a foot. The divisions start from zero at the horizontal line which separates the colors of the face and read downward. This enables the rod to be read to hundredths of a foot.

For heights *less than* the length of the closed rod, the target is moved along the rod, to which it is slightly attached by springs, and clamped in position by means of a clamp-screw. The reading is then made, the feet and tenths of a foot being read on the rod itself from the ground upward, and the hundredths being read downward on the target.

When a *greater height* than this is required, the target is first clamped with its horizontal line at the highest foot-division of the rod, and the sliding part of the rod carrying the target then moved upward the desired height. In this position, the sliding part is clamped to the other by means of a clamp (see Fig. 250). On the face of this clamp (which is on the opposite side of the rod from the target face) is a scale constructed similar to the one on the target.

The readings are now made on the back of the sliding part *from the top downward*, the feet and tenths of a foot being read directly from the divisions on the sliding part, and the hundredths from the small scale. When the rod is divided into hundredths, the vernier will read to thousandths of a foot.

For mining work, a shorter rod than the one described is necessary, say one that reads to 5 feet when closed and to 9 feet when extended to its full height.

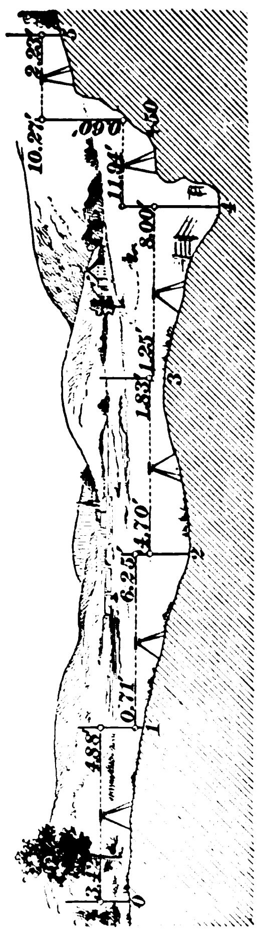


FIG. 251.

LEVELING OPERATIONS.

1247. In Fig. 251 is represented a line of levels prolonged by setting up the level midway between each pair of adjacent points. Points situated at regular distances along surveys are commonly called **stations**, and are numbered consecutively, beginning with zero at the starting-point. The distance of any station from the starting-point of the survey is thus known to be equal to the regular station distance multiplied by the number of the station in question. In the present case, it is assumed that the stations are one hundred feet apart and numbered consecutively, and that the distance to any point between stations is measured from the preceding station. This distance, measured from a regular station to an intermediate point, is called a **plus**. For example, a point midway between Stations 4 and 5, or, in other words, a point 450 feet from the starting-point, would always be designated as *Station* $4 + 50$. Leveling operations are usually referred to some permanent object, either natural or artificial, whose height above the datum is determined and recorded for future reference. Such a permanent object is called a **bench-mark,** and abbreviated B. M.

The point upon which the rod is held for the last sight before the instrument is moved forwards, and upon which it is held for the first sight after the instrument is set up in its new position, is called a **turning-point.** The rod remains

Station.	B. S. +	H. I.	F. S. –	Elevation.
0	3.42			100.00
		103.42		
1	0.71		4.88	98.54
		99.25		
2	4.70		6.25	93.00
		97.70		
3	1.25		1.83	95.87
		97.12		
4	11.94		8.00	89.12
		101.06		
+ 50	10.27		0.60	100.46
		110.73		
5			2.23	108.50
	+ 32.29		– 23.79	
	– 23.79			
	+ 8.50			

at the turning-point while the instrument *turns* or is moved forwards. It will be evident, therefore, that each station in Fig. 251 is a turning-point. As shown in the figure, all the turning-points are at regular stations, except the one between Stations 4 and 5, where the surface of the ground rises so abruptly that it becomes necessary to take a turning-point at a plus.

The first sight taken each time after the instrument is set up, upon a station whose elevation is either known or assumed, is called a **backsight;** all other sights are called **foresights.** A foresight is always taken upon each turning-point, to determine its elevation before the instrument is moved; then, after the instrument is moved forwards, a backsight is taken upon the turning-point, in order to make the line of levels continuous.

1248. The best method of keeping field level notes is what is known as the **method by height of instrument.** The field notes for the operations shown in Fig. 251, as kept by this method, are given in Art. **1247.**

It will be noticed that Station 0 is assumed to have an elevation of 100 feet, and that all the stations are *turning-points*, necessitating that the height of instrument be found between all stations or stations and plusses. These values are recorded in the column headed H. I. (height of instrument) midway between the stations or plusses, on either side of where the instrument was set up. The figures in this column give, for each setting of the instrument, the height of the cross-wires above the datum line. This is obtained each time by adding the backsight to the elevation of the station at which the rod is held when the sight

Station.	Back-sight.	Height of Instru-ment.	Foresight.		Elevation.
			Turning-Point.	Interme-diate.	
0	0.68				100.00
		100.68			
1				2.14	98.54
2				7.68	93.00
3				4.81	95.87
4				11.56	89.12
T. P. + 50	11.86		0.22		100.46
		112.32			
5			3.82		108.50
	+ 12.54		− 4.04		
	− 4.04				
	+ 8.50				

is taken. The elevation of the succeeding station is then obtained by subtracting the foresight on that station from the height of instrument. The column for backsights is, accordingly, marked +, and that for foresights marked −. If we adhere strictly to the significance of the signs given to the backsight and foresight, it will be more nearly correct to state that the backsight is added algebraically to the elevation of the turning-point, for the height of instrument, and the foresight is added algebraically to the height of instrument, for the elevation of the point sighted. The algebraic total of the sum of the backsights and the sum of the foresights, added algebraically to the elevation of the first point, will give the elevation of the last point. It will be noticed that, with this method, the backsight and the foresight on each point and the elevation of the point are all on the same line.

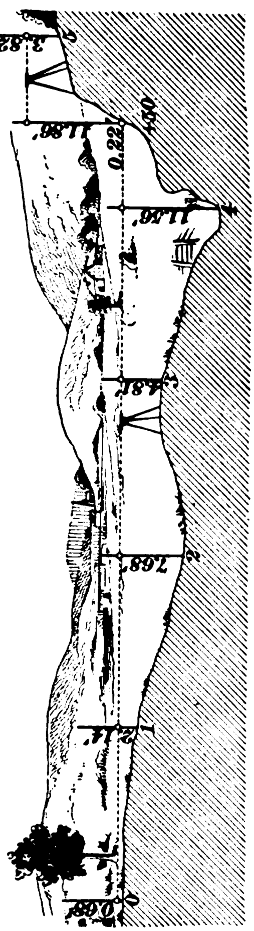


FIG. 252.

The accompanying are the field notes for the operation shown in Fig. 252, as kept by the method of height of instrument.

The form shown here is the same as the preceding, except that, since in this case all the points on which sights are taken are not turning points, a column for the intermediate sights (I. S.) and headed "Intermediate" is introduced for the foresights on intermediate points, in order that the foresights on turning-points may be added for the purpose of checking the operations. All sights on intermediate points are treated as foresights and have negative values Only those sights taken back on turning-points, after moving and setting up the instrument, are considered as backsights.

1249. Fig. 253 is a sectional view of a plane in a mine showing the various positions of the level and rod by which the notes following were obtained. This form of notes is convenient where the distances between the points are not required, and the object is to determine the difference in elevation between two points.

The level is set up at the foot of the plane and the rod

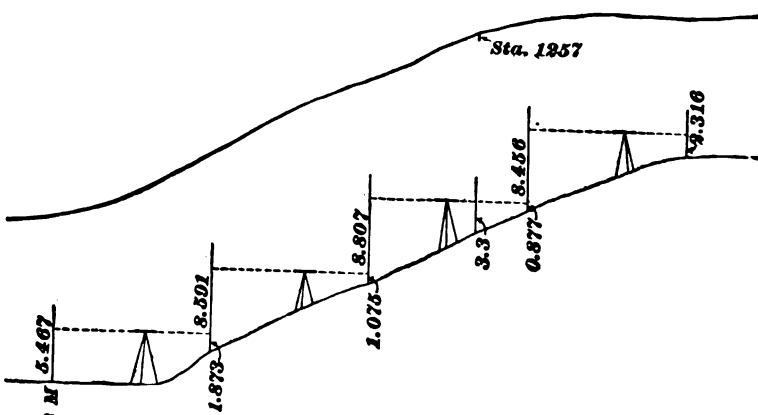


FIG. 253.

held on a *bench-mark* previously established, whose elevation is known to be 887.463 feet (above sea-level). A target-reading of the rod is taken and found to be 5.467 feet. Being the first sight of the survey, it is, according to the preceding definitions, a *backsight*, and is recorded in the B. S. column opposite the name of the station (B. M.) in the left-hand column. At the same time a description of the B. M. is entered opposite to it in the "Remarks" column. The rod-reading (5.467 feet) is the height of the line of collimation (line of sight) of the instrument above the B. M.; hence, it must be added to the elevation of the latter to give the *height of instrument* (H. I.). Accordingly, 887.463 + 5.467

FORM OF LEVEL NOTES.

Station.	B. S.	H. I.	F. S.	I. S.	Elevation.	Remarks.
B. M.	5.467	892.930			887.463	B. M. is in No. 3 tunnel, to right of Sta. 1256.
T. P.	8.591	899.648	1.873		891.057	
T. P.	8.807	907.380	1.075		898.573	
St. 1257.				3.3	904.000	Rail under Sta. 1257.
T. P.	8.456	914.959	0.877		906.503	
Rail.			2.316		912.643	Rail at top of plane.
	31.321		6.141			

= 892.930, which value is written in the column headed H. I. and opposite B. M. in the left-hand column.

It being necessary to run the line of levels up the plane, a point must be selected on which a rod-reading can be taken, both before and after the instrument is moved forwards. A point used for this purpose is a *turning-point;* hence, the letters T. P. are entered below B. M. in the column headed "Station."

The rodman, having selected a T. P., holds the rod on it and moves the target as directed by the levelman, the rod-reading thus found being 1.873 feet. According to the definitions, this is a foresight, and is entered in the column headed F. S., and opposite the T. P. in the left-hand column. The distance, 1.873 feet, shows that the point on which the rod was held is 1.873 feet below the line of collimation, or H. I.; hence, the elevation of the T. P. is 892.930 − 1.873 = 891.057 ft. Place this value in the column headed "Elevation," and opposite the T. P. in the left-hand column.

The level is now removed from its first position and set up on the plane, at such a height, however, as not to look over the top of the rod. When this has been done, the rod (extended to its full length) is sighted to as it is held on the same object as before, and then, when the target is properly lowered, it shows a height of 8.591 feet. As this is the first sight after the setting up of the level, it must be a backsight, according to the definitions. It is entered in the notes in the column headed B. S., and opposite the T. P. in the left-hand column. The reading is the height of the line of collimation above the T. P.; hence, it must be added to the elevation of the T. P. to get the H. I., 891.057 + 8.591 = 899.648, which is written opposite T. P. in the H. I. column. The rodman now goes up the plane and selects a firm point to serve as another T. P., the object being selected at such a height that, when the rod is held vertically upon it, the horizontal cross-hair *cuts* the rod. The target is moved down along the rod until it is properly *bisected* by the horizontal cross-hair. The foresight-reading thus obtained is 1.075 feet, which is entered in the F. S. column, opposite T. P., written in the left-hand column. As in the case of the previous T. P., H. I. minus the F. S. gives the elevation of the T. P. as 899.648 − 1.075 = 898.573 feet, which must be entered opposite T. P. in the "Elevation" column.

The level is now taken up the plane and a backsight taken on the rod held on the last T. P. The rod-reading is 8.807 feet, which is entered in the B. S. column, opposite T. P. The H. I. is found as before, namely, the rod-reading is added to the elevation of the T. P. This gives 898.573 + 8.807 = 907.380 feet, which is entered in the H. I. column opposite T. P.

The rodman now goes up the plane and holds the rod on the rail under Sta. 1257. This is an intermediate sight, and is read to the nearest tenth of a foot, the target on the rod not being brought into service for the purpose. The reading, 3.3 feet, is entered in the I. S. column, and opposite Sta. 1257, written in the left-hand column. The entry in the "Remarks" column shows that the rod was held on the rail, and not, as is often done, bottom side against the station in the roof.

The rodman next moves farther up the plane and selects another turning-point. Here the operations are precisely the same as described for the two preceding turning-points.

Finally, in its position near the top of the plane, the level is sighted to the rod held on the rail at the top of the plane or knuckle. The reading obtained (2.316) is the last one of the survey; hence, it is a foresight, and is entered in the F. S. column, opposite the word "Rail," written in the left-hand column. The word "Rail" is further explained in the "Remarks" column.

The elevation of the rail at the top of the plane is found to be 912.643. The B. M. at the start has an elevation of 887.463, which gives a difference of 912.643 – 887.463 = 25.180 feet.

1250. The accuracy of the calculations is checked as follows: Add the B. S. readings; the sum is 31.321 feet. Next, add the F. S. readings; the sum is 6.141 feet. Subtract the lesser from the greater, and the difference should be the same as the difference in elevation between the first and the last point of the survey. Thus, 31.321 – 6.141 = 25.180 feet, the same as the difference between the B. M. and rail at top of plane. It will be noticed that this would not be the case if the foresight column also contained the intermediate sights; hence, the advantage in having a separate column for the latter.

The accuracy of the leveling operations themselves can be checked only by repeating the work, preferably by running down the plane and using different turning-points from those employed when running up.

PROFILES.

1251. A **profile** represents a vertical section of a line of survey, and shows in an exaggerated manner all abrupt changes in elevation, by using a small scale for horizontal distances as compared with the scale used for vertical distances.

For railroad work, a horizontal scale of 400 ft. = 1 in. and a vertical scale of 20 ft. = 1 in. are commonly used; while for mining work, a horizontal scale of 50 ft. = 1 in. or 100 ft. = 1 in. and a vertical scale of 5 ft. = 1 in. are quite generally employed.

1252. The principal use of a profile is to enable the engineer to establish a **grade line,** or a line showing the relative proportion of excavation and fill in the complete work.

PLATE IV.

1253. This plate represents a plan and profile of a portion of a straight entry 10 feet wide in a mine where it is desired to make a uniform grade between Sta. 134 and a point 25 feet beyond Sta. 186, as given by the transit survey, and a minimum height of 6.5 feet for the entry. The plate is ruled like the ordinary ready-made profile paper which is purchased by the engineer.

The level notes in Art. **1254** are taken between Sta. 134 and a point 25 feet beyond Sta. 186.

The student should notice that in these notes the horizontal distances between stations are recorded, and that Sta. 0 is taken as the B. M., which in this case is assumed to have an elevation of 30 feet above the *datum line.*

1254. For convenience and accuracy in platting, the stations are designated as in transit surveying—in 100-foot lengths. Thus, Sta. 3 and Sta. 3 + 70 mean that these stations are 300 feet and 370 feet from Sta. 0, respectively. In profile work it is not necessary to find the elevation of a T. P. unless the T. P. be taken on the line of survey and at an abrupt change in the elevation of that line, in which case T. P. enclosed in parenthesis should be placed alongside the station number in the column of stations. Owing to falls of the roof in an entry, the curvature or line of the roof is not the same as that of the bottom, necessitating that measurements be taken on the roof as well as on the bottom of the entry. Measurements taken from the roof are intermediate sights, and are recorded in the I. S. column with a minus sign in front of them. This method enables the

LEVEL NOTES FOR A PROFILE.

Station.	B. S. +	H. I.	F. S. –	I. S.	Elevation.	Remarks.
(B. M.) 0	1.378	31.378			30.00	B. M. is di-
0				– 5.1	36.48	rectly under
0 + 25				– 3.7	35.08	Sta. 134, in
0 + 50				3.3	28.08	Mine No. 6.
0 + 75				– 5.3	36.68	
1				5.9	25.48	
1 + 25				– 6.1	37.48	
T. P.	2.634	28.976	5.036			
1 + 55				3.0	25.98	
1 + 55				– 4.0	32.98	
(T. P.) 2	1.754	24.254	6.476	1.8	22.50	
2 + 25				– 5.2	29.45	
(T. P.) 2 + 50	1.827	21.827	4.254	1.8	20.00	
3				5.8	16.03	
3 + 15				– 1.1	22.93	
3 + 50				6.3	15.53	
3 + 70				– .6	22.43	
4				– 3.2	25.03	
4 + 20				2.9	18.93	
4 + 60				2.8	19.03	
5				– 4.3	26.13	
5				1.7	20.13	
6				1.3	20.53	
6 + 25				– 5.2	27.03	
(T. P.) 6 + 50	1.432	20.932	2.327	1.4	19.53	End, 25 ft.
7				– 2.9	23.83	beyond Sta.
7 + 25			5.400	5.4	15.53	186.

engineer to calculate the elevation of a point on the roof in the same manner as he determines the elevation of a point on the bottom; namely, by subtracting, algebraically, the intermediate sight from the height of the instrument. Thus, the elevation of Sta. 0 + 75 = 31.378 − (− 5.3) = 36.678 feet.

1255. It is a good idea to have a plan of the work upon the same sheet of paper that the profile is on, for the purpose of comparison. The plan should be made to the horizontal scale of the profile, and should be placed directly over it, so that each station on the plan can be projected from the corresponding station on the profile.

DRAWING THE PROFILE.

1256. First assume the datum line near the bottom of the sheet, and, starting near the left-hand end of this line, lay off to the horizontal scale the distances given in the "Stations" column. Then, from the *o* point on the datum line, lay off vertically and to the vertical scale the elevation of Sta. 0, which is 30 feet. This fixes the point *a*, which is, in this case, the B. M. The figures 30 and 0 at the point *a* represent the distances the point is from the datum line and Sta. 0, respectively. All other points are similarly marked. It will be seen, according to the notes, that a minus sign is recorded in the I. S. column opposite Sta. 0, showing that a measurement was taken from the roof at that station. Hence, we lay off from the *o* point on the datum line the distance 36.48 feet, fixing the point *z* in the roof at Sta. 134 of the transit survey. At the point 0 + 25 on the datum line erect a perpendicular, as there is no vertical line on the paper passing through this point; and upon this line lay off the distance 35.08 feet, fixing the point *y* in the roof. Join *z y*. From the point 0 + 50 on the datum line, lay off vertically the distance 28.08 feet, fixing the point *b* in the bottom. Join *a b*. Again, at the point 0 + 75 erect a perpendicular, and lay off upon it the distance 36.68 feet, fixing the point *x* in the roof. Join *y x*. At the point 1 lay off the distance 25.48 feet, fixing the point *c*. Join *b c*. At the point 1 + 25 erect a perpendicular and lay off upon it the distance 37.48 feet, fixing the point *w*. Join *x w*.

We now come to a T. P. in the notes, whose distance from Sta. 0 is not recorded, because it was not taken at any abrupt change of elevation along the entry, being used simply to determine the height of the instrument when moved forwards. Passing to point 1 + 55, erect a perpendicular and lay off upon it the distances 25.98 feet and 32.98 feet, fixing the points *d* and *v*, respectively. Join *c d* and *w v*. Sta. 2 was used as a T. P., and, therefore, the letters T. P., enclosed in parenthesis, are placed alongside of it. At the point 2 lay off the distance 22.5 feet, fixing the point *e*. Join *d e*. At the point 2 + 25 erect a perpendicular, and lay off upon it the distance 29.45 feet, fixing the point *u*. Join *v u*. At the point 2 + 50 lay off the distance 20 feet, fixing the point *f*. Join *e f*.

The student will proceed in a similar manner, laying off vertically from the remaining points on the datum line the corresponding elevations, until the point *n* is reached. By joining *a n*, the grade line is established, and by drawing another line 6.5 feet above and parallel to the grade line, the top, or roof, line is fixed. It now becomes evident where "filling," "ripping down the top," and "lifting bottom" are necessary. The auxiliary, or construction, lines should be drawn with a lead pencil and finally erased. They are shown dotted on the profile, to aid the student.

METHOD OF CALCULATING CUBICAL CONTENTS OF FILLS, CUTS, ETC.

1257. In Fig. 254 is shown a profile of an entry in which the grade and the roof line are established. From the figure it is evident that the cubical contents of the fill is equal to the area of the polygon *a b c d e*, multiplied by the width of the entry, which we will assume to be 10 feet.

In order to find the area of the polygon *a b c d e*, it is divided into triangles and trapezoids, by drawing vertical lines, if necessary, from the points *b*, *c*, *d* to the grade line. The area of each is found separately, and finally all are added together. The lengths of the lines *b b'*, *c c'*, and *d d'* are found by measuring them upon the map. The distances the point *e* is from the lines *d d'* and *f f'* are also found by measurement. Whenever it is necessary to measure a distance between two points, it is a good plan to connect the points with a straight line, if they are not already connected by a line on the map, and place the resulting measurement upon it.

The area of $abb' = 1.9 \times \frac{100}{2} = 95$ sq. ft.; the area of $bcc'b' = \frac{1.9 + 4.3}{2} \times 55 = 170.5$ sq. ft.; the area of $cdd'c' = \frac{4.3 + 3.8}{2} \times 45 = 182.25$ sq. ft.; the area of $dd'e = 3.8 \times \frac{74}{2} = 140.6$ sq. ft. Hence, the area of the polygon $abcde =$

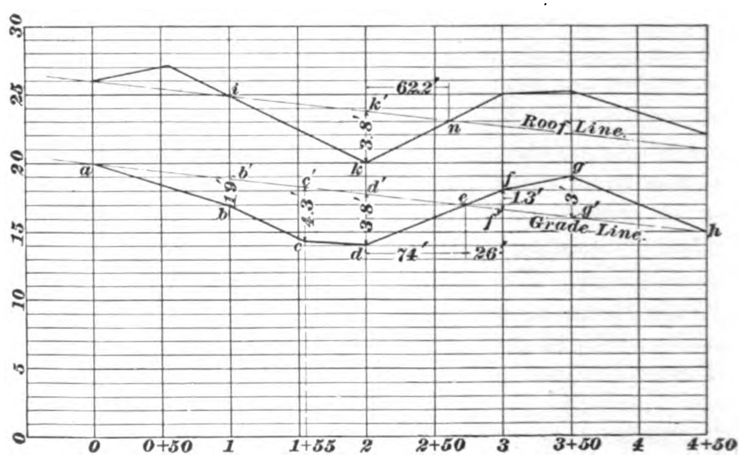


FIG. 254.

$95 + 170.5 + 182.25 + 140.6 = 588.35$ sq. ft., and the cubical contents of the fill $= 588.35 \times 10 = 5{,}883.5$ cu. ft. In a similar manner, we divide the polygon *e f g h* into the triangles *e f f'* and *g g' h* and the trapezoid *f g g' f'*. The area of $eff' = 1.3 \times \frac{26}{2} = 16.9$ sq. ft.; the area of $fgg'f' = \frac{1.3 + 3}{2} \times 50 = 107.5$ sq. ft.; the area of $gg'h = 3 \times \frac{100}{2} = 150$ sq. ft. Hence, the area of the polygon $efgh = 16.9 + 107.5 + 150 = 274.4$ sq. ft., and the cubical contents of the cut $= 274.4 \times 10 = 2{,}744$ cu. ft. The area of $ikk' =$

$3.8 \times \frac{100}{2} = 190$ sq. ft.; the area of $k k' n = 3.8 \times \frac{62.2}{2} = 118.18$ sq. ft. Hence, the area of the polygon $i k n = 190 + 118.18 = 308.18$ sq. ft., and the cubical contents of the cutting $= 308.18 \times 10 = 3{,}081.8$ cu. ft.

TOPOGRAPHICAL SURVEYING.

CONTOURS.

1258. The transit survey gives merely the general figure and the area of the ground surveyed, and the relative positions of certain conspicuous features with reference to one another. It frequently is necessary to make a more minute and careful examination of the ground, and by means of certain observations to determine accurately the accidents of form and the irregularities of the surface. In placing upon paper the results of such observations as the above, we are enabled to distinguish the hills from the valleys, the rivers and brooks from the plains, etc. This method of surveying is called **topographical surveying.**

In addition to the horizontal measurements which are determined in the progress of an ordinary transit survey, it is necessary and essential to have the vertical heights; these are obtained by leveling.

There are several methods of representing irregularities of the surface, such as hills, valleys, etc. The principal method, however, is one in which a system of curved lines lying within horizontal planes is used to represent the irregularities of the surface; these curved lines are called **contours.**

It will be necessary to have a clear conception of the meaning of contours, to appreciate how the irregularities of the surface are represented by them. Suppose the accompanying diagram (Fig. 255) to represent a hill having a series of *equidistant horizontal planes* passed through it. The contours will be the horizontal projections of the intersections of these planes with the hill. They will have the same appearance on the map as they would to an observer directly above the hill in a balloon. Suppose the observer to be in the balloon, and the hill to be surrounded at its base by a body of water. The line which this water makes with the base of the hill will be the first contour line. Conceive the water to rise up around the hill five feet at a time. The

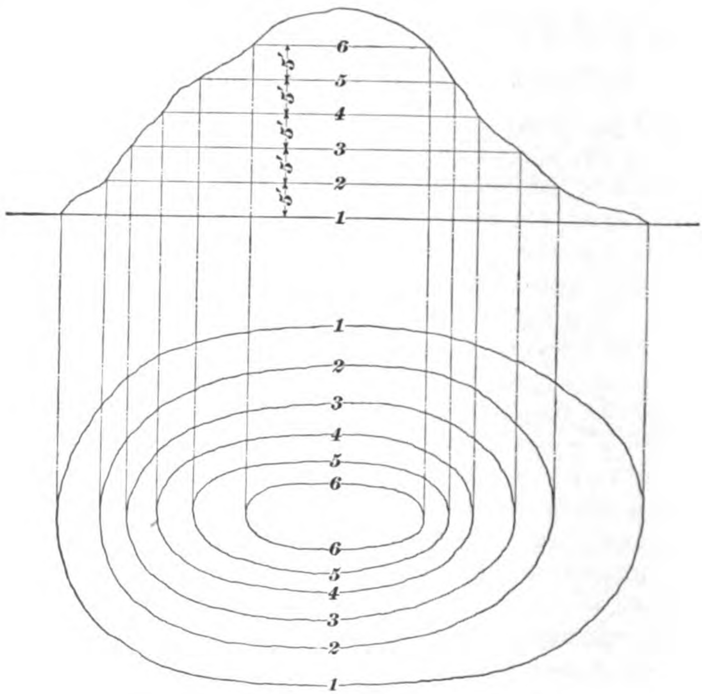


FIG. 255.

successive contours would be the successive water-lines around the hill.

It is obvious from Fig. 255 that these contours will be nearer together or farther apart as the ascent of the hill is steep or gentle.

If any piece of ground be intersected by such a system of equidistant planes, and the contours thus determined be accurately shown on paper, the map will furnish a very correct idea of the irregularities and striking characteristics of the ground.

The horizontal plane on which the contours are projected, and to which they are referred, is called the **plane of reference.** This plane may be assumed in any position, and the distances of the contours above or below it are noted on them.

It is best to assume the plane of reference as lower than any point to be represented, so that all of the contours will lie above it.

The vertical distance which the horizontal planes are apart depends entirely upon the nature of the ground surveyed. In mountainous districts the contours are taken 100 feet apart; in the U. S. Coast Survey they are 20 feet apart, and for ordinary engineering purposes they are 5 feet apart, or even less.

TO DETERMINE AND REPRESENT THE CONTOUR OF A SINGLE HILL.

1259. Fig. 256 is a picture of a hill shown in elevation, or as it would appear to one approaching it on the surface of the earth. *A* is the summit of the hill, and such observations with transit and level are required as are necessary to represent it on paper in contour.

To proceed, drive a stake at *A*, and directly over the nail driven in its center set up the transit. From *A* measure any line, as *A B*, down the hill, using the telescope to arrange all points in the same straight line. At different points of the line, as *a*, *b*, *c*, *d*, let stakes be driven, and let the *horizontal* distances (for *horizontal* and not pitch distances are required) be measured by a chain, utmost care being used to hold the chain in a truly horizontal position. Of course, if the instrument is in perfect adjustment, and has a vertical circular arc attachment, it can be used to get the angle of depression of the lines from the horizontal, and the horizontal distances can be calculated from the pitch distances. The desirability of either method will depend upon the particular case in question.

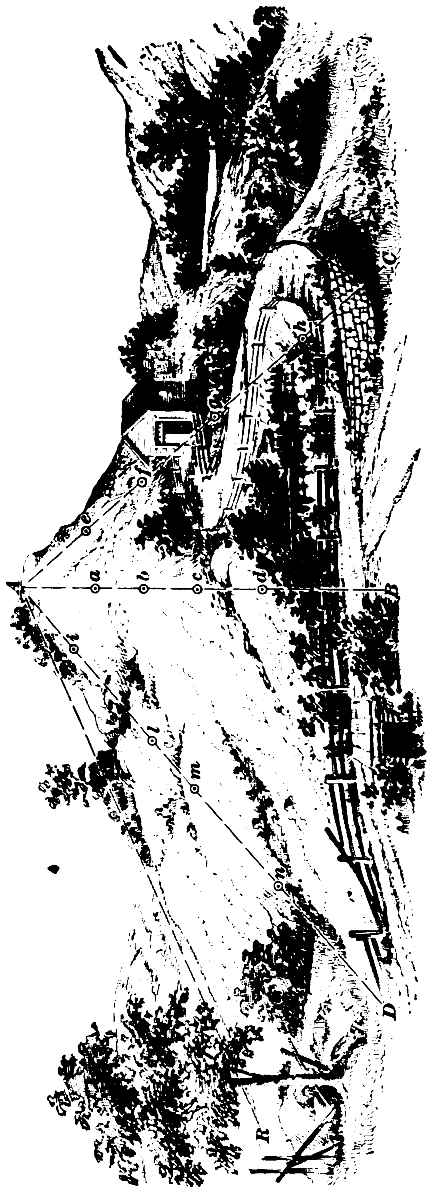


FIG. 256.

The degree of the irregularities of the surface, and the accuracy with which the work is to be shown, must govern the distances between the stakes. Their differences of level ought not to exceed twice the horizontal distances between the horizontal planes of section.

Having arranged the stakes on this line, and having measured the horizontal distances between them, let any number of lines, such as *A C*, *A D*, *A R*, etc., be run down on all sides of the hill in exactly the same manner as in the case of *A B*, and let similar stakes be placed on each. With the transit, find the angular distance apart of all these lines, viz., angle $B A D = 30°$, $B A C = 25°$, etc.

Assume a datum line, preferably below the lowest point on any line, and proceed on each line separately to find the heights of the various stakes on it with reference to the assumed datum. This is done with the level and rod, the same principles and precautions being observed as in running section levels.

The data now obtained are sufficient, not only to determine the intersections of the horizontal planes with the surface of the hill—in other words, the contours—but also for drawing such contours on paper.

PLATTING THE WORK.

1260. Refer to Fig. 257. Draw on the surface of the paper the line $A B$, and from it lay off the angles $B A D =$

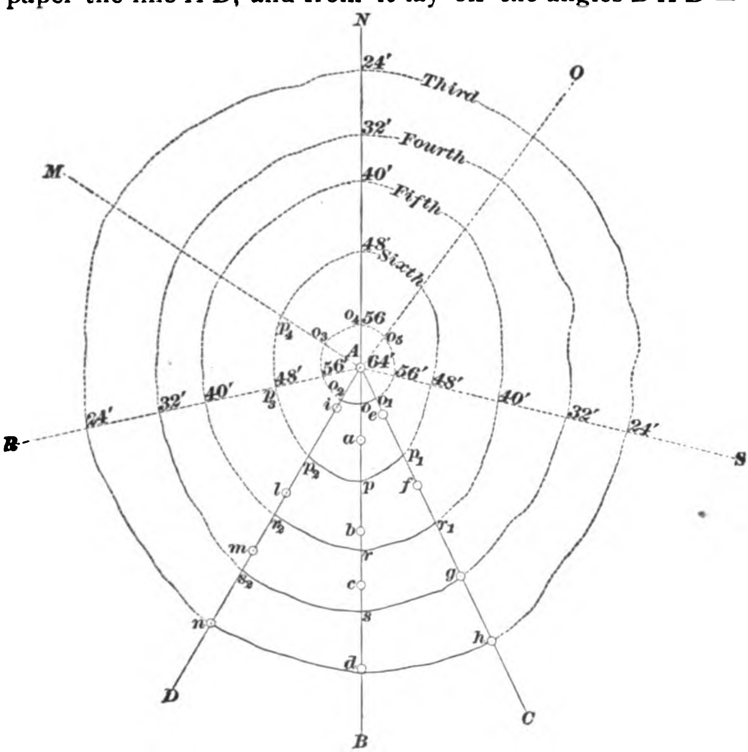


FIG. 257.

30° and $B A C = 25°$, and in this way lay off around the point A, the summit of the hill, every angle made by every line with *A B*. In this way, every line, as *A R*, *A S*, etc., will be represented. It must be remembered now that Fig. 257 is a horizontal projection of Fig. 256; that is, it represents the view of the hill an observer would have if he were vertically above *A* in a balloon.

In this example, it will be necessary, in order to illustrate the method of platting, to take only the three lines *A B*, *A C*, and *A D*, and assume certain data, and then proceed to draw the contours between them.

LINE *A B*.

Horizontal Distances.	*Heights above Datum.*
	A = 64 feet.
A a = 40 feet.	*a* = 52 feet.
a b = 50 feet.	*b* = 44 feet.
b c = 30 feet.	*c* = 35 feet.
c d = 46 feet.	*d* = 24 feet.

LINE *A C*.

	A = 64 feet.
A e = 28 feet.	*e* = 53 feet.
e f = 45 feet.	*f* = 44 feet.
f g = 55 feet.	*g* = 32 feet.
g h = 38 feet.	*h* = 18 feet.

LINE *A D*.

	A = 64 feet.
A i = 25 feet.	*i* = 55 feet.
i l = 55 feet.	*l* = 42 feet.
l m = 38 feet.	*m* = 35 feet.
m n = 48 feet.	*n* = 21 feet.

Assume the contours to be placed vertically 8 feet apart. In this case, the summit *A* will be in the eighth contour, counting from the datum. *A*, being the actual summit, or highest point, will be represented as a point. The difference in elevation between *A* and *a* is 12 feet, and as the contours are to be only 8 feet apart, the seventh contour plane must intersect the line *A B* between *A* and *a*, the first stake. If the descent between *A* and *a* is assumed as uniform (and in so doing no appreciable error is introduced for such short distances), the following proportion is applied:

$$\left\{\begin{array}{c}\text{The difference of}\\ \text{level between}\\ A \text{ and } a\end{array}\right\} : \left\{\begin{array}{c}\text{The hor.}\\ \text{distance}\\ A\,a\end{array}\right\} = \left\{\begin{array}{c}8 \text{ ft.}\end{array}\right\} : \left\{\begin{array}{c}\text{The hor. dist. from}\\ A \text{ to the seventh}\\ \text{contour line.}\end{array}\right\}$$

$$12 : 40 = 8 : A\,o.$$

Whence, horizontal distance $A\,o = \frac{40 \times 8}{12} = 26.66 +$ feet.

This distance is now laid off on the line $A\,B$ equal to $A\,o$. In like manner, the points at which this contour intersects the other lines $A\,C$, $A\,D$, etc., are determined, and the distances from A to those points o_1, o_2, o_3, etc., are laid off to scale on the lines $A\,C$, $A\,D$, $A\,M$, etc. The curved line passing through the points thus determined represents the seventh contour above the plane of reference. The other contours, sixth, fifth, fourth, etc., are determined and drawn in exactly the same way.

The points on the lines $A\,B$, $A\,C$, $A\,D$, etc., through which the sixth contour is to pass, are found as follows: This contour must be 8 feet vertically below the seventh contour. The point o on the line $A\,B$ is 4 feet vertically above a, and the sixth contour must cross $A\,B$ 4 feet vertically below a. The point of crossing on the line $A\,B$ is found by the following proportion:

$$\left\{\begin{array}{c}\text{The diff.}\\ \text{of level of}\\ a \text{ and } b\end{array}\right\} : \left\{\begin{array}{c}\text{The}\\ \text{hor. dist.}\\ a\,b\end{array}\right\} = \left\{\begin{array}{c}4 \text{ feet}\end{array}\right\} : \left\{\begin{array}{l}\text{The hor. dist. from}\\ a \text{ to the sixth con-}\\ \text{tour line.}\end{array}\right\}$$

Whence, $8 : 50\ 4 : = a\,p = 25$ feet,

and so may the distances $b\,r$, $c\,s$, etc., be found.

The curved line passing through the points thus found represents the sixth contour. And similarly any number of contours can be drawn. The plane of reference, or datum level, is 24 feet below the third contour shown in the figure.

1261. A profile, Fig. 258, along any line, as $A\,B$, can be platted in accordance with rules already given. Thus, using a scale of 20 feet to the inch for heights above datum, and 100 feet to the inch for horizontal measurements, the

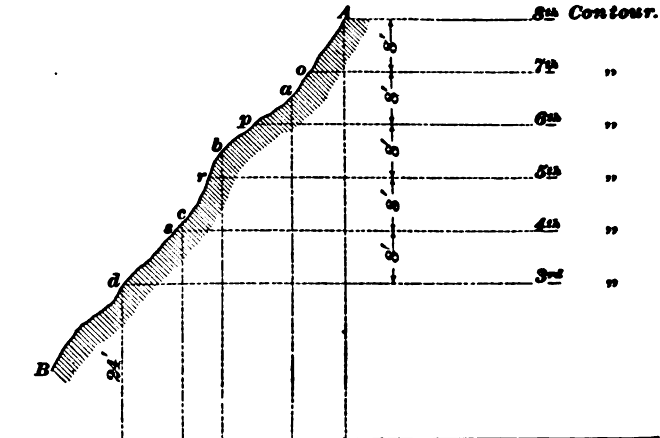


FIG. 258.

irregularities are exaggerated and appear to better advantage to the student or engineer.

HOW TO DETERMINE AND REPRESENT ON PAPER THE CONTOURS OF ANY IRREGULAR PIECE OF GROUND.

1262. With a transit or level, range a line of stakes *A*, *B*, *C*, *D*, *E*, etc., Fig. 259, along one side or through the middle of the ground to be surveyed, placing the stakes at convenient distances apart, say 100 feet. These stakes *A*, *B*, *C*, *D*, *E*, etc., should be marked with red chalk or by any other characteristic mark. The transit or level is now used to range in a line of stakes from each of these points, as from *A*, the line a_1, a_2, a_3, a_4, etc., and similarly from *B*,

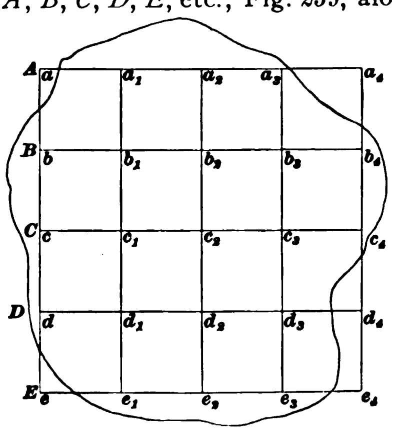


FIG. 259.

C, *D*, *E*. The ground is now divided into squares of 100 feet.

FIELD NOTES.

Benches.	+ Sights.	Height of Inst.	− Sights.	Stations.	Heights.
B. M.	4.136	12.142			8.006
			12.100	c_2	0.000
			1.900	d_2	10.200
c_3	11.906	22.441	1.607	c_3	10.535
			6.000	c_1	16.400
			6.900	c_3	15.500
			6.800	d_3	15.600
			1.900	d_1	20.500
			3.700	c_3	18.700
			6.800	b_4	15.600
			1.700	c_4	20.700
b_3	11.914	33.448	0.907	b_3	21.534
			8.000	e	25.400
			4.000	b_2	29.400
			4.100	d	29.300
			1.900	c_1	31.500
			5.000	a_3	28.400
			9.900	a_4	23.500
			1.700	d_4	31.700
			1.100	c_4	32.300
			0.100	a_2	33.300
C	11.813	45.225	0.036	c	33.412
			4.800	b_1	40.400
			2.600	b	42.600
a_1	8.925	52.669	1.481	a_1	43.744
			3.200	a	49.500

Proceed thus: Set up the level, and take a reading on some bench-mark whose height above some plane of reference has been previously determined. Then take readings on the prominent stakes immediately around the instrument. Go beyond these stakes, set up the instrument in a new position, and sight to one of these known stakes as a turning-point, reading the rod placed upon it to thousandths, and get the heights of all other stakes now in sight as before. Then go beyond these again, and so on, until the heights of all the stakes have been determined.

In the above example, c_3 is the lowest point. The instrument is set up in any convenient place, and a reading is taken upon a bench-mark whose height above some datum has been previously determined to be 8.006 feet. This reading is 4.136 feet, and is recorded in the notes as a plus sight. The height of the instrument is then 8.006 + 4.136 = 12.142 feet.

Readings of the rod are now taken from this position upon the stakes at c_2, d_2, c_3, reading c_3 to thousandths, because it is to be used as a turning-point. The level is now removed to a new position, and the rod read upon c_3 to thousandths. This reading is 11.906 feet, and is recorded as a plus sight. The height of instrument is, therefore, 10.535 + 11.906 = 22.441 feet. From this position read as many stakes as possible, and then use b_3 as a turning-point, and so on. The notes give all the necessary data for constructing the contours.

1263. In order to plat the work, lay off on paper, preferably engineers' cross-section paper, the lines of stakes to a scale of say 100 feet to the inch. Then, each 100-foot square on the ground will be represented by a square inch on paper. At each stake, carefully mark out on the plat its height above the plane of reference.

It is required now to construct the contours, supposing them 5 feet apart, having obtained the heights of all the stakes above the datum, etc.

Taking, for example, the contour whose height is 45 feet, it is plainly seen it must fall between *A* and *B*, Fig. 260, whose heights are respectively 49.5 feet and 42.6 feet. Its distance from *A* will be found by proportion, as shown before in regard to the hill.

{The diff. in height of *A* and *B*} : {The diff. in height of *A* and the contour} = {The dist. from *A* to *B*} : {The dist. from *A* to the contour.}

Whence, (49.5 – 42.6) : (49.5 – 45) = 100 ft. : 65.22 ft. = 1 in. : .6522 in., the scale being 100 feet to the inch. Then, since each space in the plat is 10 feet, or 0.1 inch, we mark

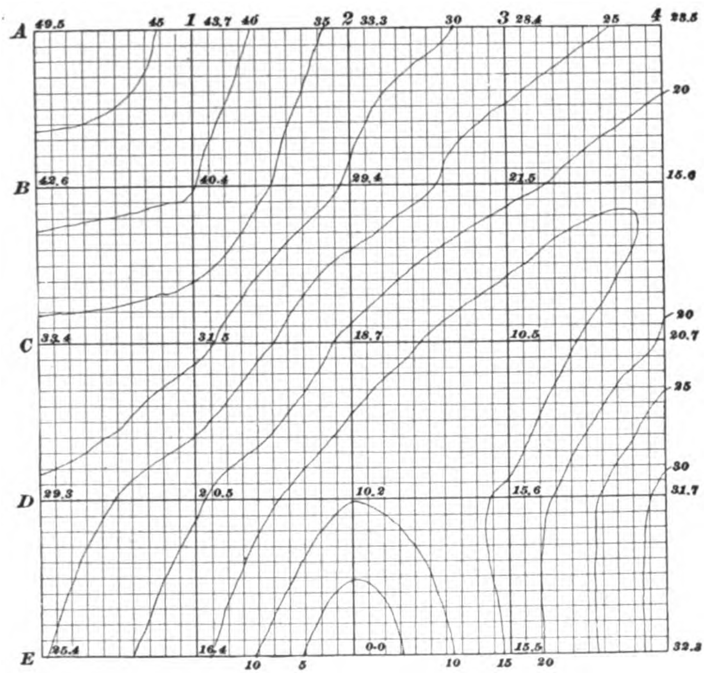


FIG. 260.

off between *A* and *B* 6.5 of the small divisions. Between *A* and *1*, (49.5 – 43.7) : (49.5 – 45) = 1 : .7758 inch, nearly 7.8 divisions. And so we may find where each contour intersects the sides of the rectangles. The curved lines passing through the points of intersection thus determined are the contours.

GENERAL PLAN OF A BITUMINOUS MINE.

PLATE V.

1264. This plate, which consists of a general plan of the inside workings and the surface arrangements of a bituminous mine, is given for the double purpose of affording the student practice in the principles already learned, and enabling him to get a clear idea of how the mining engineer completes his survey for presentation to the company or to any of its officials requiring information regarding any point, such as the stage of development of the mine, the methods of working the mine, the exact location of certain entries or rooms with reference to the property or crop lines, etc.

1265. It requires a great many notes to make a map of a mine and of the surface arrangements; but, in order to obviate this difficulty, and at the same time accomplish the above purposes, great regularity is assumed for the inside workings, so that after the student has platted a portion of the work he can duplicate it in such a way as will give the complete map the same appearance as if every part had its own special notes. Further, it is assumed that the mine has been operated for some time and that all the workings have been systematically laid out with the transit. That portion of the mine which has been worked out before this survey is shown on the map by cross-lines.

1266. Before making the survey, the engineer determines a true meridian in some convenient place, as described in Part 1, Arts. **1118** to **1120,** inclusive. If, however, it is not the proper time to determine a true meridian and the work must be done immediately, any line may be laid out and taken as the base line of the survey until such time as the true meridian is fixed, when the entire survey can be made to conform to it as explained before.

In general, the surface survey should be re-run by the engineer, so as to fix definitely the property lines, and to be sure all is right. This he does by obtaining a copy of the original survey as recorded on the deed. The surface survey is based on the same meridian as the mine survey.

1267. The crop line is run for the purpose of showing, when placed on the map, the probable area underlaid with coal, the relative position of the workings with respect to the main body of coal, and, in a measure, the topography of the property. When an extensive plant is to be erected, a number of *contour* lines should be run and placed on a map, to enable the engineer to locate the buildings to the best advantage.

DRAWING THE PLATE.

1268. Two methods of platting the courses will be used in drawing this plate, so as to familiarize the student with both and enable him to judge which method will be the more suitable for any work which he may be called upon to do. Frequently, however, it is advantageous to use both while making a map. Thus, all the main courses, such as those giving the direction and location of the entries or gangways, should be platted by total latitude and total departure, while the less important courses, as those fixing the position of the rooms, can be platted by means of the protractor. Owing to the ease and rapidity with which the work can be done with a protractor, many engineers use it entirely. The student will plat the notes for the mine and the surface arrangements with the protractor. All work should be first drawn with a hard, sharp lead-pencil and finally inked in with a drawing pen.

The mine should be platted first. By looking over the notes of the surface arrangements, it will be seen how Sta. 100, which is at the drift mouth, is connected to the meridian, or base line. Draw the meridian line in the proper relative position on the paper, and take Sta. 0 at such a point on this line that when the work is completed it will have the proper position on the map. In this case, the student can be guided by the map which he is required to duplicate. In ordinary work, the only guide to determine the location of the point at which the platting should begin is a careful examination of the notes.

At Sta. 0 draw a line making an angle of 97° 38′ with the base line, remembering that the azimuth is always measured in a clockwise direction, and determine a point on this line 538 feet from Sta. 0. From this point, draw a line making an angle of 73° 51′ with the base line.* This may be done

TRANSIT NOTES.

Stations.	Azimuths.	Bearings.	Distances.	Remarks.
100–101	25° 08′	N 25° 06′ E	208	Sta. 100 is at the drift mouth.
101–102	25° 08′	N 25° 07′ E	220	
102–103	25° 08′	N 25° 05′ E	312	
103–104	25° 08′	N 25° 06′ E	180	
104–105	346° 50′	N 13° 08′ W	53	
104–106	25° 08′	N 25° 08′ E	140	
105–107	25° 08′	N 25° 07′ E	40	
105–108	205° 08′	S 25° 08′ W	190	
108–109	205° 08′	S 25° 06′ W	216	
109–110	205° 08′	S 25° 03′ W	180	
110–111	205° 08′	S 25° 05′ W	144	
111–drift mouth	205° 08′	S 25° 07′ W	200	
106–112	115° 08′	S 64° 52′ E	30	
112–113	25° 08′	N 25° 06′ E	32	
112–face	205° 08′	S 25° 08′ W	308	
112–114	115° 08′	S 64° 50′ E	42	

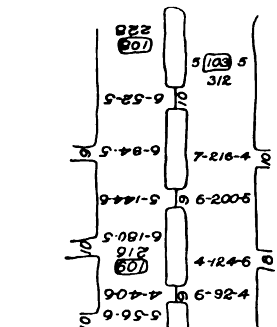
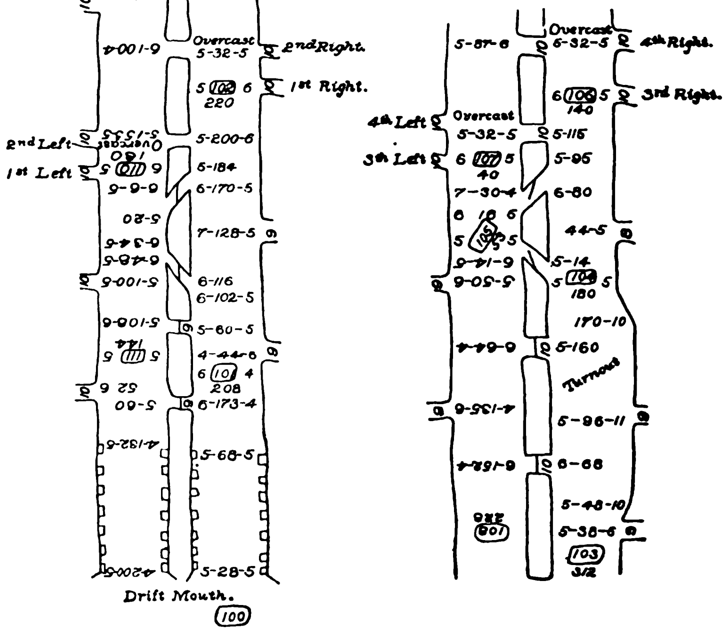
by laying off the angle at Sta. 0 and transferring the course to the required point by means of the parallel ruler, or the line may be laid off directly from its point of beginning by means of the **T** square and protractor. Lay off on this line a distance of 172 feet, fixing Sta. 100.

* For the sake of brevity, let it be understood hereafter, when a line is drawn making a certain angle, that the angle meant is the one formed by the line and the base line.

Now, noticing the transit notes of the mine survey, it will be observed that Stas. 100, 101, 102, 103, and 104 are on the same straight line, which has an azimuth course of 25° 08′. Hence, by drawing a line from Sta. 100 making an angle of 25° 08′, and laying off along this line a distance of 208 feet, a distance beyond this of 220 feet, a distance beyond this of 312 feet, and a distance beyond this again of 180 feet, Stas. 101, 102, 103, and 104, respectively, are located.

At Sta. 104 draw a line making an angle of 346° 50′, and lay off on this line a distance of 53 feet, locating Sta. 105. Also draw another line from this station making an angle of 25° 08′, and lay off a distance of 140 feet, locating Sta. 106. At Sta. 105 draw a line making an angle of 25° 08′, and lay off the distance 40 feet, locating Sta. 107; also, draw a line from this station making an angle of 205° 08′, and lay off on this line a distance of 190 feet, a distance beyond this of 216 feet, a distance beyond this of 180 feet, a distance beyond this of 144 feet, and a distance beyond this again of 200 feet, locating Stas. 108, 109, 110, and 111, and the drift mouth, respectively. At Sta. 106 draw a line making an angle of 115° 08′, and lay off a distance of 30 feet, locating Sta. 112. At Sta. 112 draw a line making an angle of 25° 08′, and lay off a distance of 32 feet, locating Sta. 113. This is about as far as the transit notes are given, but the student will draw a line from Sta. 112, making an angle of 115° 08′, and lay off distances 42 feet apart, locating all the stations between Stas. 112 and 132. See side notes for butt entries. At each of these stations draw a line making an angle of 205° 08′, and lay off on each line the corresponding distance shown at the face of each room in the side notes. This will fix the end, or working face, of each of the rooms in the third right heading. Similarly, draw a line from Sta. 113 making an angle of 115° 08′, and lay off distances 42 feet apart, locating all stations between Stas. 113 and 151. See side notes for butt entries. At each of these stations draw a line making an angle of 25° 08′, and lay off on each of these lines the corresponding distance given at the face of the working places in the fourth right heading.

It will now be convenient for our purpose to plat the side notes. This, however, is not done in practice until all the transit notes have been platted. Starting at Sta. 100, it will be observed in the side notes for the main entries that rights and lefts were taken at a distance of 28 feet from Sta. 100. Lay off from Sta. 100, 28 feet, and at this point measure 5 feet to the right and 5 feet to the left, fixing a point on each side at the beginning of the drift. At a distance of 68 feet from Sta. 100 fix a point 5 feet to the right and another point 5 feet to the left. Join the two right-hand points and the two left-hand points with a free-hand line drawn as straight as possible. No attempt

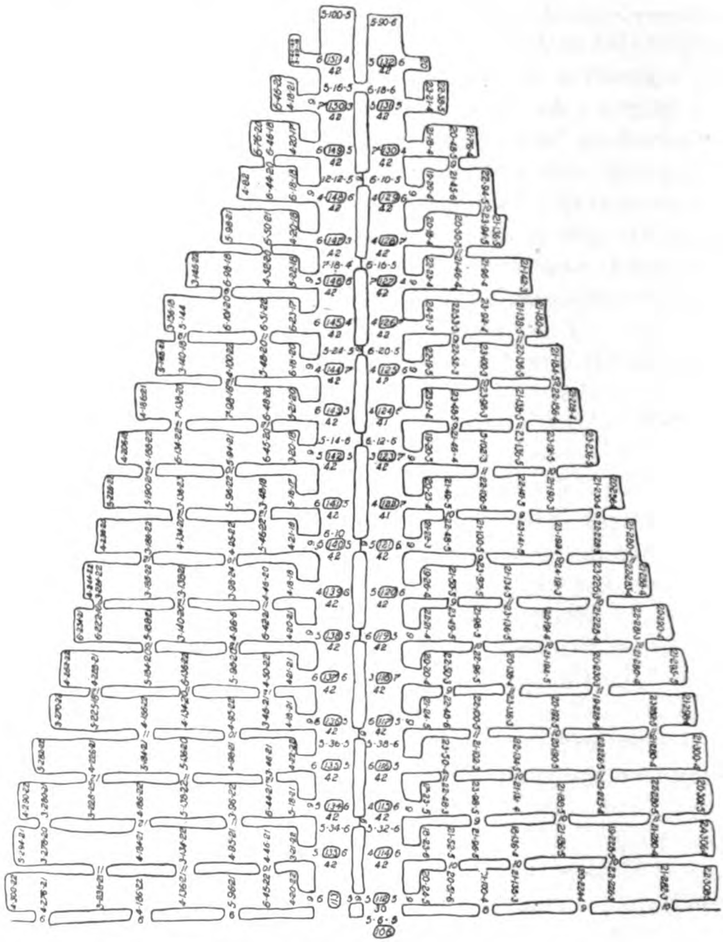



SIDE NOTES FOR MAIN ENTRIES.

should be made to show the timbers or slight irregularities of the sides.

At a distance of 173 feet from Sta. 100 there is a cross-cut, or "breakthrough," which is 9 feet wide. At this point a distance of 4 feet to the right determines a point at the side of the heading, and a distance of 6 feet to the left determines a point at the middle of the cross-cut, and by laying off 4½ feet on either side of this point the inside and the outside corner of the cross-cut are located. The point to the right is joined to the one just back of it and on the right, and the outside corner of the cross-cut is joined to the point just back of it and on the left. At Sta. 101, which is 208 feet from Sta. 100, and where rights and lefts were taken, determine a point 4 feet to the right and join it to the point on the right just back of it, and one 6 feet to the left, which is joined to the inside corner of the cross-cut to the left. In a similar manner, the student will follow up the lines run by the transit, and fix points as given by the side notes. Each point should be joined to the one immediately back of it as soon as it is determined, so as to avoid confusion and mistakes.

After platting all the side notes given, the student will proceed to duplicate the notes for third and fourth right on the left of the main headings, by considering Sta. 113 placed directly to the left of Sta. 107 and 30 feet from it, just as Sta. 112 is directly to the right of Sta. 106 and 30 feet from it. This will make Sta. 112 correspond to Sta. 153, as shown on the map. When this is finished, the student will consider Sta. 102 as Sta. 106, and reproduce the work beyond Sta. 106 just as it has been produced beyond Sta. 102, with the exception that while platting the headings and rooms to the right and left he will consider Stas. 121 and 140 to correspond to Stas. 199 and 200, respectively. This will diminish the number of working places at this part of the mine. When this is completed, the student will continue the main headings for 536 feet from Stas. 193 and 194, making cross-cuts 9 feet wide and at intervals of 90 feet. The pillars on the right and left of that portion of the main headings which run through the worked-out territory are each 18 feet wide. A margin of 60 feet must be left between the worked-out territory and the crop or property



SIDE NOTES FOR BUTT ENTRIES.

lines. The inside boundary of the worked-out territory to the right is at right angles to the main headings and 290 feet beyond Sta. 102. The inside boundary of that on the left is also at right angles to the main headings and 290 feet beyond Sta. 110. Cross-line the worked-out portion, as shown on the map. In practice, the worked-out portions of a mine are usually shown on the map by a special color.

The next step is to plat the surface survey by means of total latitude and departure. To do this, it is convenient to divide the map into 5-inch squares (10-inch squares would be used when the scale is 100′ = 1″) by drawing vertical lines 5 inches apart and horizontal lines also 5 inches apart on the paper. The lines should be so drawn that a vertical and a horizontal line will pass through that station to which the total latitudes and departures refer.

By examining the notes for the surface arrangements, it will be seen that, by laying off from Sta. 0 a distance of 540 feet on a line making an angle of 201° 48′ with the base line, Sta. *A* of the surface survey will be located. From this point the survey is platted as described in Part 1, Arts. **1162** and **1163**, and according to the notes here given.

To plat the crop line, lay off a distance of 748 feet from *B* towards *C* on the line *B C* of the surface survey, locating Sta. 1 of the crop line. At Sta. 1 draw a line making an angle of 111° 55′ with the original base line, and lay off the distance 178 feet, locating Sta. 2. Proceed in a similar manner until Sta. 12 is reached. Sta. 13 is located by laying off 498 feet from *C* towards *D* on the line *C D* of the surface survey. At Sta. 13 draw a line making an angle of 58° 13′ with the original base line, and lay off 298 feet, locating Sta. 14. In like manner the student will continue the work to Sta. 17, where the crop line ends.

It remains to plat the notes for the surface arrangements. By examining these notes, it will be readily seen that the buildings or other objects are located either by taking rights and lefts from some established line, or by sighting to them with the transit from some established station and measuring the distances over the lines of sight to them. The road to the mine and the car-tracks to the lumber-yard and the rock-dump are located by running a line with the transit along the middle of them, establishing stations at the prominent curves only.

TRAVERSED NOTES OF THE SURFACE SURVEY.

Stations.	Bearings.	Horizontal Distances.	Latitude.		Departure.		Total Latitude.		Total Departure.	
			North.	South.	East.	West.	North.	South.	East.	West.
A	N 14° 30′ W	174.50	168.94			43.69	168.94			43.69
B	N 10° 22′ E	1,156.35	1,137.48		208.09		1,306.42		164.40	
C	N 12° 06′ W	928.25	907.62			194.58	2,214.04			30.18
D	N 23° 22′ E	644.15	591.32		255.48		2,805.36		225.30	
E	N 77° 33′ E	898.32	193.67		877.19		2,999.03		1,102.49	
F	S 83° 23′ E	1,176.42		135.56	1,168.59		2,863.47		2,271.08	
G	Due South	580.36		580.36			2,283.11			
H	S 13° 30′ W	570.10		554.35		133.09	1,728.76		2,137.99	
I	S 13° 04′ E	558.22		543.77	126.20		1,184.99		2,264.19	
J	S 3° 00′ W	530.20		529.47		27.75	655.52		2,236.44	
K	S 19° 36′ W	694.00		653.79		232.80	1.73		2,003.64	
L	S 89° 58′ W	2,003.64		1.73		2,003.64				

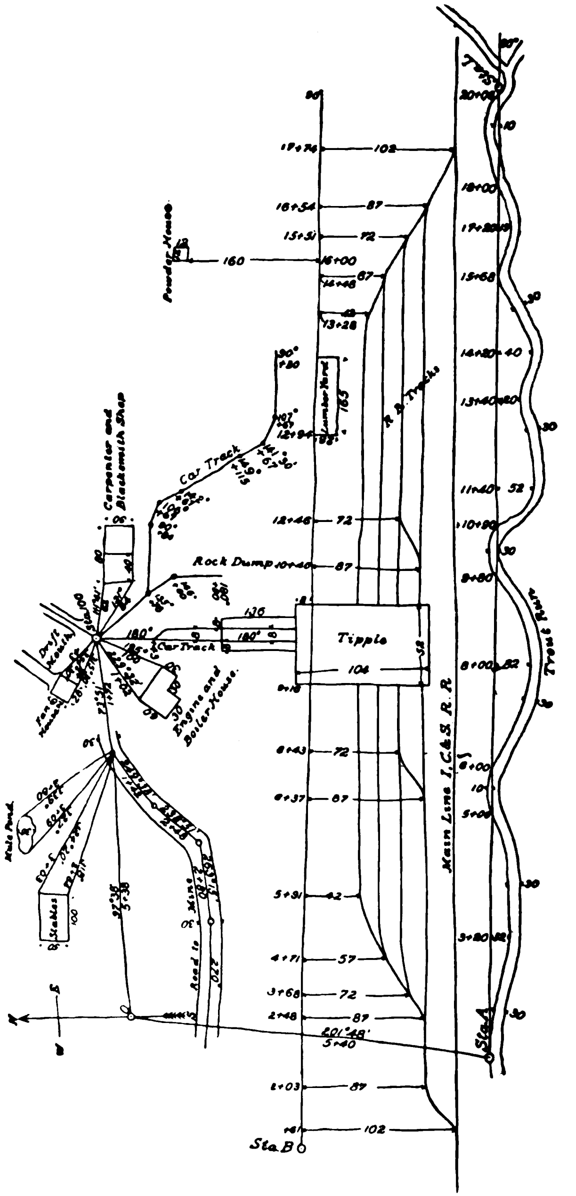
The railroad-tracks are located by taking distances to the right from a line passing through Sta. B of the surface survey and making an angle of 90° with the original base line, and the creek is located by taking rights and lefts from a line drawn from Sta. A to Sta. L.

From what has been given, it will not be necessary to explain in detail every step required to plat these notes. The

NOTES FOR CROP LINE.

Stations.	Azimuths.	Distances.	Remarks.
1	111° 55′	178	Sta. 1 is on the
2	122° 53′	186	line *BC* of sur-
3	90° 18′	192	face survey and
4	131° 53′	164	748 feet from *B*.
5	109° 51′	166	
6	127° 40′	84	
7	93° 58′	182	
8	110° 11′	314	
9	98° 55′	364	
10	65° 15′	142	
11–12	80° 50′	250	End.
13	58° 13′	298	Sta. 13 is on
14	38° 42′	392	line *CD* of sur-
15	334° 08′	268	face survey and
16–17	355° 37′	357	498 feet from *C*.

student will first start at Sta. 100 and locate all buildings and tracks which are to be located from this station, as indicated by the notes. Then he will locate the railroad-tracks, etc., from the east and west line passing through Sta. B, and finally locate the creek by taking rights and lefts from a line passing through Stas. A and L, all as indicated by the notes. The direction and length of any course is either placed at its end or written along it, so as to read in the same direction as the course was run.



NOTES FOR SURFACE ARRANGEMENTS.

METALLIFEROUS MINE SURVEYING.

1269. The methods of locating mining claims are usually crude, except where an official survey is made under the direction of the U. S. Surveyor-General. The dimensions of a claim, as allowed in the U. S. Mineral Laws, are 1,500 feet in the direction, or on the strike, of the vein, and 300 feet on each side of the middle of the vein at the surface.

Generally, in the prospector's location of his claim, the line of strike is not accurately determined, and, consequently, both the direction and the dimensions of the claim are only approximate. However, provided no trespass is committed, he is allowed to *swing* his claim around into its proper position after the strike has been definitely determined. The law provides that the end lines shall be parallel, which makes the claim essentially a parallelogram. The side lines of the claim may be straight lines extending between the ends of the opposite end lines, or they may be broken lines to include the vein, if it should be curved, so as to pass outside the straight side lines. In any case, only 1,500 feet of the vein can be taken, measured along the center line of the claim. The above are the essential features which govern the shape and extent of a mining claim.

1270. The law requires that 10 feet of the vein shall be exposed before the location of the claim is undertaken by the prospector. After the discoverer has sunk his shaft 10 feet, he then has his claim surveyed and the survey recorded. Before the survey is made, he decides how much of 1,500 feet he will extend his claim on either side of his discovery shaft. Various local conditions govern him in this, such as trespass, the nature of the lode, etc. The survey is begun by ranging off a center line along the strike of the lode, and then measuring from the discovery shaft in each direction as much of the 1,500 feet as has been decided upon. Through the ends of this center line the end lines are ranged out, usually at right angles to it, and half the width of the claim, or 150 feet, is measured on each side of the center line. The location must then be distinctly marked on the ground, so that its boundaries can readily be traced. Permanent monuments must be placed at the four corners, and also on the side lines midway between the corners.

1271. If the claim is on surveyed land, it may be tied to some section-corner of the government survey; if it is on unsurveyed land, the tie must be made with some prominent natural object, such as a mountain peak, or with some permanent locating monument established especially for the purpose.

On each claim located, and until a patent has been issued therefor, not less than one hundred dollars' worth of labor shall be performed, or one hundred dollars' worth of improvements made, during each year. This original location may be made by any surveyor, or by the miner himself, but the survey on which the patent, or possessory title, from the government is issued is made by a U. S. Deputy Mineral Surveyor.

1272. A patent for any land claimed and located for valuable deposits may be obtained in the following manner: Any person, having claimed and located a piece of land for such purposes, who has complied with the law, may file in the proper land office an application for a patent, under oath, showing such compliance, and a plat and field notes of the claim made by or under the direction of the U. S. Surveyor-General, showing accurately the boundaries of the claim, which shall be distinctly marked by monuments on the ground, and shall post a copy of such plat, together with a notice of such application for patent, in a conspicuous place on the land embraced in such plat previous to the filing of the application for the patent; and shall file an affidavit of at least two persons that such notice has been duly posted; and shall file a copy of the notice in such land office; and shall thereupon be entitled to a patent for the land in the following manner: The Register of the land office, upon the filing of such application plat, field notes. notices, and affidavits, shall publish for the period of sixty days a notice that such application has been made, in a newspaper to be by him designated as published nearest to such claim, and he shall also post such notice in his office for the same period.

1273. The claimant at the time of filing the application, or at any time thereafter within the sixty days of publication, shall file with the Register a certificate of the U. S. Surveyor-General that five hundred dollars' ($500) worth of labor has been expended, or of improvements made upon the claim by himself or grantors; that the plat is correct, with such further description, by such further reference to natural objects or permanent monuments, as shall identify the claim, and furnish an accurate description to be incorporated in the patent.

At the expiration of the sixty days of publication, the claimant shall file his affidavit showing that the plat and notice have been posted in a conspicuous place on the claim during such period of publication.

If no adverse claims have been filed with the Register and the Receiver of the proper land office at the expiration of the sixty days of publication, it shall be assumed that the applicant is entitled to a patent upon the payment, to the proper officer, of five dollars per acre, and that no adverse claim exists; and, thereafter, no objection from third parties to the issuance of a patent shall be heard, except it be shown that the applicant has failed to comply with the terms of these rules.

The Surveyor-General of the United States may appoint in each land district containing mineral lands as many competent surveyors as shall apply for appointments to survey mining claims.

1274. The general principles in surveying metalliferous mines are the same as in surveying coal-mines. The differences are only in the details. Four drawings are necessary to represent the workings of a metalliferous mine: (1) the surface plan; (2) the working plan; (3) a longitudinal section; (4) a transverse section.

The surface plan gives a general representation of the whole concession or tract to be worked. It should be made on a scale of 100 feet to the inch, and on it the boundary of the property of every landowner should be distinctly marked and all the lodes indicated. The working plan gives a general view of the underground workings as they would be

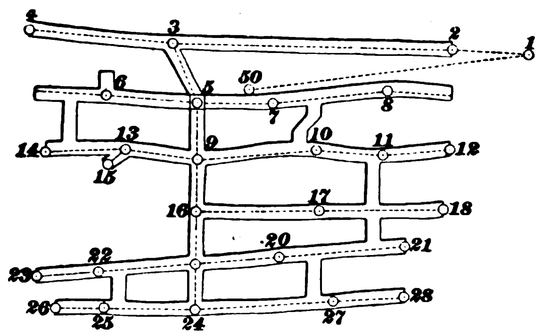


FIG. 261.

seen from above if the ground were transparent. This plan should be drawn on a large scale; 20, 30, 40, and 50 feet to the inch are scales often used for the purpose.

The longitudinal section is drawn on the supposition that a section of the ground is cut away, and that a side view of the mine is exposed. All the vertical shafts, the stopes, the grade of the levels, and the surface line with elevations of the mine buildings will be correctly shown. The levels, diagonal shafts, and winzes will have a false appearance. The levels will appear perfectly straight, however crooked their

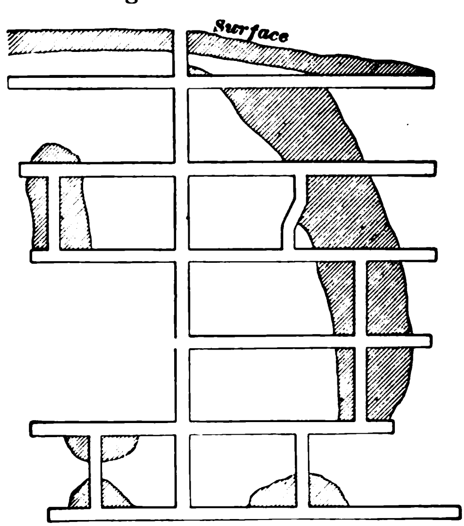


FIG. 262.

course may be, the diagonal shafts and winzes will appear perpendicular, and the cross-cuts will be represented as open doorways.

The transverse section is of great value, as it shows the dip of the lodes, or veins. In the transverse section, the view is taken at one end of the workings, at right angles to the longitudinal section. Thus, the inclinations of the shafts and winzes sunk on the lode are shown. The levels driven on the lode will be represented as open doorways; the cross-cuts will be correctly shown; and all variations in the dip of the lode from the surface to the bottom of the mine will be shown.

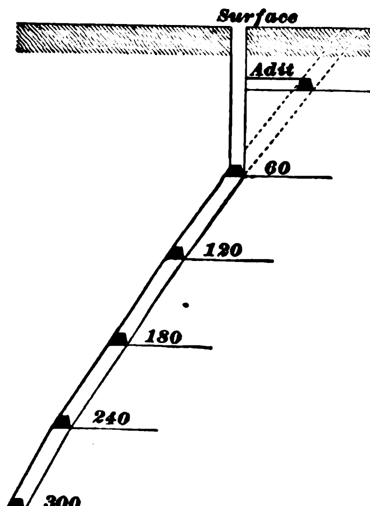


FIG. 263.

What is meant by the above descriptions of the several sections will be clearer if the student imagines the mining claim, surface, and workings to be enclosed in a glass box, whose sides are the vertical planes drawn through the side and end lines of the claim. The working plan is the view which would be obtained by looking down through the top of the box. The longitudinal section is the view which would be obtained by looking through the side of the box *parallel* to the strike of the lode. The transverse section is the view which would be obtained by looking through the end of the box which is *perpendicular* to the strike of the lode and *parallel* to its dip.

When the lode is very flat, the longitudinal section is made along the lode. In this way, a true idea is given of the ground worked, but an erroneous one with regard to depth. This method of projecting the section is necessary to enable the ground stoped away to be shown, as, when the lode is so very flat, the back of one level in a vertical section would touch the floor of the next. As a rule, lodes are so nearly vertical that a perpendicular plane may be taken for the section.

The horizontal and the vertical angles and the distances are measured exactly the same as in the survey of a coal-mine, and the same rules for platting apply for the ground plan. The longitudinal and the transverse sections, Figs. 262 and 263, respectively, are constructed in the same manner as the cross-sections previously described. The shaft is on a pitch, causing the gangways on the different levels to appear on the plan, Fig. 261.

1275. The workings of a metalliferous mine are shown in Fig. 261. The mine has an adit-level, and, below that, 60, 120, 180, 240, and 300 foot levels. The adit is north of the shaft. The engine shaft contains the pumps which lift the water from the sump, or lowest point of the shaft, to the adit-level, which comes out to the surface on the adjacent hillside. This shaft was sunk vertically to intersect the lode at the 60-foot level, a cross-cut being driven to the adit. Then, instead of continuing vertically, necessitating the driving of cross-cuts to the lode, the shaft follows the latter. The shaded portions shown in the longitudinal section, Fig. 262, represent the projection of the ore masses removed by stoping. In practice, such portions are not shaded, but are tinted, purple for tin, green for copper, blue for lead, yellow for gold, etc. The draftsman can, of course, adopt any colors to represent the various minerals.

Between the 60 and the 120 foot level the winze and rise did not meet, owing to an error of the surveyor.

Fig. 263 is a section through the shaft and at right angles to the direction of the levels.

SURVEY OF A METALLIFEROUS MINE.

1276. The notes for the survey from which the plan and sections shown in Figs. 261, 262, and 263 were constructed are as follows:

Began at Station 1.

1–50	S 80° W	+ 11° 03′	205.0

Station 50 is notch and nail in root of stump.

1–2	S 89° 15′ W	0° 00′	52.5

Station 2 at mouth of adit.

2–3	S 89° 15′ W	0° 00′	198.0

Station 3 at cross-cut, ribs of adit 5 ft. right, 3 ft. left.

2 + 50 = ribs 5 ft. right, 3 ft. left. Adit 6 ft. high.

2 + 80 = first side stoping, left. Stoping through to level below.

2 + 100 = ribs 5 ft. right, 3 ft. left.

2 + 143 = second side stoping, left. Stoping through to level below.

2 + 150 = ribs 5 ft. right, 3 ft. left.

2 + 150 } = stoping, right. Stoping at 2 + 150 begins at
2 + 172 } top of adit and runs diagonally to a point 12 ft. above adit at 2 + 172.

2 + 193 = first side of cross-cut.

3–4	N 87° W	0° 00′	102.0

Station 4 is face of adit; ribs 5 ft. right, 3 ft. left.

3 + 3 = west side of cross-cut.

3 + 50 = ribs 3 ft. right, 4 ft. left.

3–5	S 24° 45′ E	0° 00′	45.0

3 + 41 = north side of shaft. Shaft is 8 ft. square.

Station 5 is center of shaft, 60 ft. to 60-foot level.

Then on 60-foot level began at center of shaft, and called it Station 5.

5–6	N 88° W	0° 00′	64.0

At 6, ribs 4 ft. right, 4 ft. left. Level 6 ft. high.

At 6, center of rise, 8 ft. wide.

6 to face. N 88° W 0° 00′ 49.0, ribs 4 ft. right, 4 ft. left.

6 + 23 = center of winze, 8 ft. wide.

6 + 3 } = stoping right and left. Stoping up 16 ft. at
6 + 40 } center.

5–7	N 89° 30′ E	0° 00′	52.5

At 7, ribs 2 ft. right, 6 ft. left.

7–8	N 81° 30′ E	0° 00′	82.0

At 8, ribs 2 ft. right, 8 ft. left.

7 + 30 = center of winze 8 ft. wide.

$\left.\begin{array}{l}7+22\\7+75\end{array}\right\}$ = stoping, left, through to adit.

$7+\left.\begin{array}{l}34\\80\end{array}\right\}$ = stoping, right, through to level below.

8–face, S 87° 20′ E	0° 00′	45.0, ribs 2 ft. right, 6 ft. left.	
5–9	Due South	− 56° 19′	72.1

Station 9 on 120-foot level. Shaft from 60-foot level to bottom is 10 ft. wide.

9–10	N 83° 45′ E	0° 00′	83.0

At 10, ribs 3 ft. right, 5 ft. left.

9 + 50 = ribs 2 ft. right, 6 ft. left.

9 + 72 = center of rise, 8 ft. wide.

10–11	Due East	0° 00′	47.0

At Station 11, ribs 4 ft. right, 4 ft. left.

10 + 9 = first side stoping, left, cut into rise 22 ft. above level.

10 + 11 = first side stoping, right, through to level below.

10 + 25 = ribs 6 ft. right, 2 ft. left.

10 + 40 = center of winze, 8 ft. wide.

11–12	N 83° E	0° 00′	48.0

Station 12 is face, ribs 4 ft. right, 4 ft. left.

11 + 19 = inside of stoping, left, through to level above.

11 + 20 = inside of stoping, right, through to level below.

9–13	N 84° 10′ W	0° 00′	52.5

At Station 13, ribs 5 ft. right, 5 ft. left.

13–14	S 87° 15′ W	0° 00′	55.0

Station 14 is face, ribs 5 ft. right, 3 ft. left.

13 + 7 ft. = point of pillar of hole on left side.

$\left.\begin{array}{l}13+3\\13+50\end{array}\right\}$ = stoping, right, through to level above.

13 + 35 = center of rise, 8 ft. wide.

13–15	S 43° W	0° 00′	15.0

Station 15 is face, ribs 4 ft. right, 4 ft. left.

13 + 6 ft. = point of pillar on right.

9–16	S 0° 30′ E	− 58° 44′	70.5

Station 16 on 180-foot level.

16–17	N 87° 10′ E	0° 00′	87.0

At Station 17, ribs 4 ft. right, 4 ft. left.

17–18 N 87° 10′ E 0° 00′ 87.0

Station 18 is face, ribs 4 ft. right, 4 ft. left.

17 + 32 = center of rise and center of winze, each 8 ft. wide.

17 + 20 }
17 + 70 } = stoping, right and left, through to levels above and below.

16–19 S 1° 15′ W – 57° 19′ 71.3

Station 19 is on 240-foot level.

19–20 N 83° E 0° 00′ 61.0

At Station 20, ribs 2 ft. right, 6 ft. left.

20–21 N 83° 15′ E 0° 00′ 87.0

Station 21 is face, ribs 4 ft. right, 4 ft. left.

20 + 21 = winze, 8 ft. wide.

20 + 62 = rise, 8 ft. wide.

20 + 49 }
20 + 82 } = stoping, left, through to level above.

19–22 S 83° W 0° 00′ 65.5

At Station 22, ribs 5 ft. right, 3 ft. left.

19 + 24 = first side stoping, left, cuts into winze 15 ft. below level.

19 + 54 = winze, 8 ft. wide.

22–23 S 83° 45′ W 0° 00′ 44.0

Station 23 is face, ribs 4 ft. right, 4 ft. left.

22 + 6 = inside stoping, left, cuts into winze 15 ft. below level.

19–24 S 2° 45′ E – 61° 56′ 68.0

Station 24 is on 300-foot level.

24–25 S 88° 35′ W 0° 00′ 64.0

At Station 25, ribs 5 ft. right, 3 ft. left.

24 + 27 = first side stoping, right, cuts into rise 22 ft. above level.

24 + 55 = center of rise, 8 ft. wide.

25–26 S 88° 30′ W 0° 00′ 34.0

Station 26 is face, ribs 5 ft. right, 3 ft. left.

25 + 11 = inside stoping, right, cuts into rise 22 ft. above level.

24–27 N 84° 30′ E 0° 00′ 98.5

At Station 27, ribs 4 ft. right, 4 ft. left.

24 + 44 = first side stoping, left, cuts into rise 22 ft above level.

24 + 81 = center of rise, 8 ft. wide.

27–28	N 86° 15′ E	0° 00′	93.0

Station 28 is face, ribs 4 ft. right, 4 ft. left.

27 + 12 = inside stoping, left, cuts into rise 22 ft. above level.

On the longitudinal section, the ore masses removed by stoping are determined by measurements made along the levels when the stoping extends from level to level; but when it does not extend through, the length of the stoping is measured along the level, and its height or depth is measured from the side of the level.

PRACTICAL PROBLEMS.

1277. The following problems are given to illustrate such practical work as the mining engineer is often required to do. The student should thoroughly review the principles of plane trigonometry which relate to the solution of triangles, before he begins to study these problems.

1. The main entry from the bottom of the shaft runs due north 3,600 feet. A cross-entry is started due east at a distance of 200 feet from the face and driven 2,465 feet. (*a*) What length of roadway started 250 feet from the shaft will be required to connect with the face of the cross-entry? (*b*) What will be the bearing of this connecting roadway?

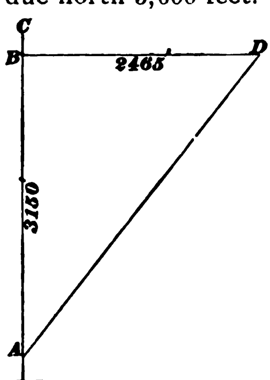


FIG. 264.

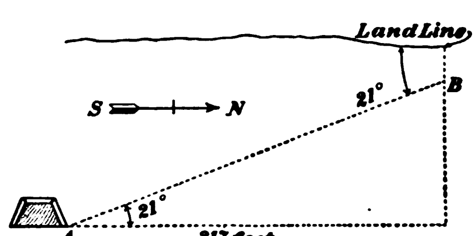


FIG. 265.

(*a*) In Fig. 264, the distance from the shaft S to the face C of the main entry is 3,600 feet. The distance $A\ S$ is 250 feet and the distance $B\ C$ is 200 feet; therefore, $A\ B = S\ C - (A\ S + B\ C) = 3{,}600 - (250 + 200) = 3{,}600 - 450 = 3{,}150$ feet. The length $B\ D$ is 2,465 feet. From the right-angled triangle $A\ B\ D$,

$$A\ D = \sqrt{(A\ B)^2 + (B\ D)^2} = \sqrt{3{,}150^2 + 2{,}465^2} = \sqrt{15{,}998{,}725} = 3{,}999.8 \text{ ft.} \quad \text{Ans.}$$

(*b*) From the right-angled triangle $A\ B\ D$, $\tan B\ A\ D = \frac{B\ D}{A\ B} = \frac{2{,}465}{3{,}150} = .78254$, which corresponds to an angle of 38° 03′ (nearly). The bearing of the road from A to D is, therefore, N 38° 03′ E. Ans.

2. If the distance measured on the map from the haulage road to the land line is 317 feet due north, and the seam dips 21° due south, how far can a breast, or room, be driven up the pitch before it reaches the land line?

The room is to be driven up a pitch of 21° from the haulage road to the land line, and the horizontal distance from the haulage road to the land line is 317 feet; therefore, by consulting Fig. 265, it can be seen that the line $A\ B$ represents the distance up the pitch, and since it is the hypotenuse of a right-angled triangle, it is calculated thus:

$$A\ B = \frac{317}{\cos 21^\circ} = \frac{317}{.93358} = 339.55 \text{ feet.} \quad \text{Ans.}$$

3. A seam of coal 7 feet thick crops at the north line of a tract of land; the seam dips 9° south, and the surface has a regular fall of 5° in the same direction; how far south of the north line must one go to sink a shaft, so that it will cut the seam at a point 1,100 feet on the pitch from the crop, and what will be the depth of the shaft from the surface to the bottom slate?

In Fig. 266, $A\ C = 1{,}100$ ft., the distance from the outcrop A to the point C in the seam to which the shaft is to be sunk. $A\ E$ is the horizontal distance measured south from the outcrop.

$B\ D = B\ C + C\ D =$ depth of shaft to bottom slate. $A\ E$ can be calculated from the right-angled triangle $A\ E\ C$, thus:

$A\,E = 1{,}100 \times \cos 9° = 1{,}100 \times .98769 = 1{,}086.46$ ft.

To calculate the depth of the shaft $B\,D$:

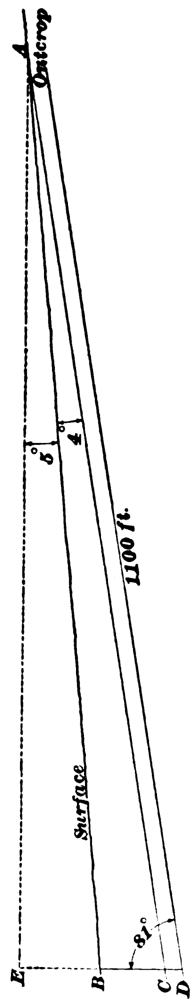
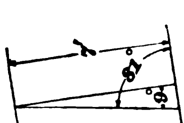


FIG. 266.



$$B\,D = B\,C + C\,D.$$

$B\,C$ must first be calculated, which can easily be done by finding $E\,C$ and $E\,B$ and subtracting them.

$E\,C = 1{,}100 \times \sin 9° =$

$1{,}100 \times .15643 = 172.07$ ft.

$E\,B = A\,E \times \tan 5° =$

$1{,}086.46 \times .087489 = \underline{\;95.05\;}$ ft.

Difference $B\,C = E\,C - E\,B = \;\;77.02$ ft.

Now, calculate $C\,D$. The thickness of the seam is 7 feet, but $C\,D$ does not exactly represent the thickness.

$$C\,D = \frac{7}{\sin 81°} = \frac{7}{.98769} = 7.09 \text{ ft.}$$

Therefore, $B\,D = B\,C + C\,D = 77.02 + 7.09 = 84.11$ ft. Ans.

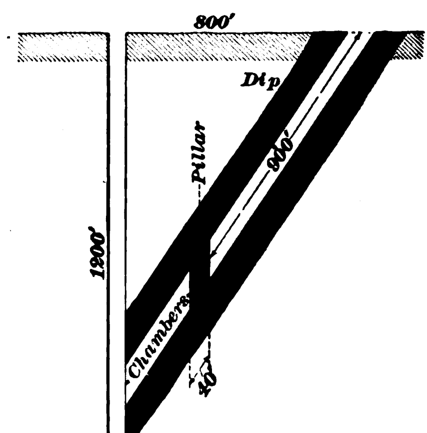


FIG. 267.

4. A colliery has been worked down three lifts, on a steep north dip. The slope is down 300 yards. North from the mouth of the slope a horizontal distance of 800 feet, a shaft has been sunk, cutting the seam

at a depth of 1,200 feet. (*a*) How far from the shaft level can the chambers be driven, to leave a chain pillar 40 feet thick (measured on the dip) between them and the slope gangway, and (*b*) what is the angle of dip?

(*a*) By consulting Fig. 267, it will be seen that there is a right-angled triangle whose base, 800 feet, is equal to the horizontal distance from the mouth of the slope to the shaft; its altitude is equal to 1,200 feet, the depth of the shaft, and its hypotenuse is equal to 900 feet, the length of the slope, plus 40 feet, the length of the pillar, plus the unknown length of the chambers. This last length can be calculated thus:

The hypotenuse $= \sqrt{1,200^2 + 800^2} = 1,442.2$ ft.

Hence, the length of the chambers $= 1,442.2 - (900 + 40) = 502.2$ ft. $= 167.4$ yd. Ans.

(*b*) The angle of dip is calculated thus:

$$\text{Tangent of dip} = \frac{1,200}{800} = 1.5 = \text{tangent } 56° \ 19', \text{ nearly.}$$

Therefore, the angle of dip $= 56° \ 19'$. Ans.

5. Find the depth of a shaft which is to be sunk at a point 350 feet horizontally from the outcrop of a vein, to intersect a cross-cut of 125 feet. The dip of the vein is 60°, and the difference in level between the outcrop and the mouth of shaft is 50 feet.

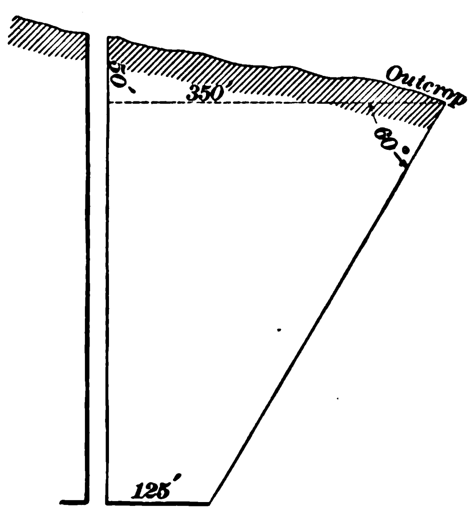


FIG. 268.

If, Fig. 268, a line is drawn from the outcrop downwards parallel to the shaft, and the cross-cut then produced to the left until it intersects this line, a right-angled triangle will be formed whose altitude, plus 50 feet, will equal the depth of the shaft, since the difference in elevation of mouth of shaft and outcrop is 50 feet. The base of the triangle will equal 350 — 125 = 225 feet, which is apparent from the drawing; and the angle opposite will equal 30°, the complement of the 60° angle of dip; hence, $\frac{225}{\tan 30^\circ} = \frac{225}{.57735} =$ 389.7 ft. The depth of shaft = 389.7 + 50 = 439.7 feet. Ans.

6. If the face of a haulage road driven due east is at the land line, which has a course of S 46° 15′ E, and a breast, or room, is started at right angles to the haulage road 60 feet

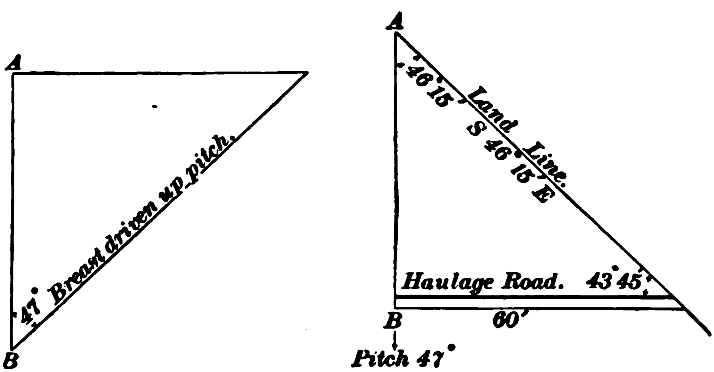


FIG. 269.

back from face of gangway, and the pitch of the seam is 47° south, at what distance from the haulage road will the breast, or room, reach the land line?

A plan and section are given, Fig. 269, to better represent the meaning of this problem. From the plan on the right we see that $A\ B = 60 \times \tan 43^\circ\ 45' = 60 \times .95729 = 57.44$ feet, which is the horizontal distance from the haulage road to the land line. Therefore, if a room be driven up a pitch of 47°, at right angles to the haulage road, its length will be $\frac{57.44}{\cos 47^\circ} = \frac{57.44}{.682} = 84.2$ ft. Ans.

7. It is intended to sink a shaft at the end of a level driven from Smith's shaft; the accompanying notes show the

Station.	Bearing.	Distance.
1–2	N 2° 00′ W	70 feet.
2–3	N 82° 45′ E	40 feet.
3–4	N 81° 30′ E	30 feet.
4–5	East	80 feet.
5–6	S 78° 00′ E	50 feet.
6–7	S 83° 15′ E	40 feet.
7–8	S 75° 00′ E	20 feet.
8–9	N 85° 00′ E	30 feet.
9–10	N 77° 45′ E	20 feet.
10–11	N 9° 00′ W	40 feet.

survey from the center of the shaft to the eastern end of the level. Calculate the distance and bearing from Smith's shaft to the point at the surface where the new shaft is to be sunk.

Traverse the notes as follows:

Station.	Bearing.	Distance.	Cosine.	Sine.	Latitude.		Departure.	
					North.	South.	East.	West.
1–2	N 2° 00′ W	70 ft.	.99939	.03490	69.96			2.44
2–3	N 82° 45′ E	40 ft.	.12620	.99200	5.05		39.68	
3–4	N 81° 30′ E	30 ft.	.14781	.98902	4.43		29.67	
4–5	East	80 ft.	.00000	1.00000			80.00	
5–6	S 78° 00′ E	50 ft.	.20791	.97815		10.40	48.91	
6–7	S 83° 15′ E	40 ft.	.11754	.99307		4.70	39.72	
7–8	S 75° 00′ E	20 ft.	.25882	.96593		5.18	19.32	
8–9	N 85° 00′ E	30 ft.	.08716	.99619	2.61		29.89	
9–10	N 77° 45′ E	20 ft.	.21218	.97723	4.24		19.54	
10–11	N 9° 00′ W	40 ft.	.98769	.15643	39.51			6.26
					125.80	20.28	306.73	8.70
					20.28		8.70	
					105.52		298.03	

Sta. 11 is, therefore, 105.52 feet north and 298.03 feet east of Sta. 1.

The tangent of the bearing from 1 to 11 $= \frac{298.03}{105.52} =$ 2.8244; hence, the bearing from 1–11 = N 70° 30′ E, and

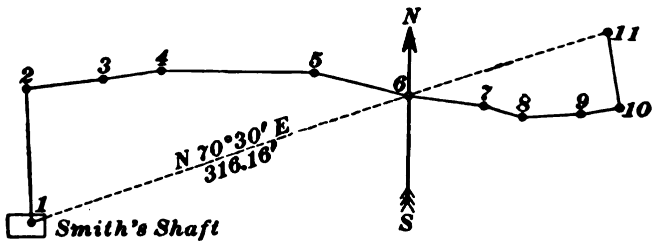


FIG. 270.

the distance $= \sqrt{298.03^2 + 105.52^2} = 316.16$ ft., nearly. See Fig. 270.

8. Having given the accompanying notes from which the plat shown in Fig. 271 is constructed, (*a*) find the length and bearing of the course 6–1; (*b*) find the area enclosed in the survey.

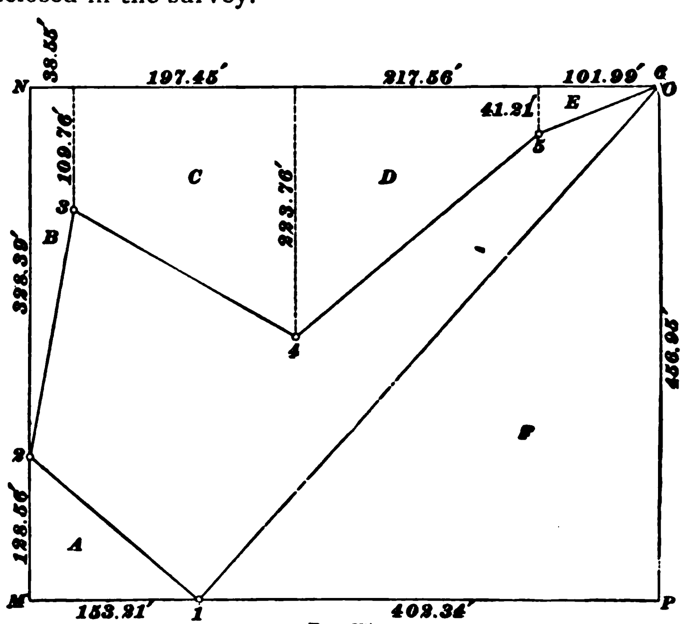


FIG. 271.

Station.	Bearing.	Distance.
1–2	N 50° W	200 feet.
2–3	N 10° E	222 feet.
3–4	S 60° E	228 feet.
4–5	N 50° E	284 feet.
5–6	N 68° E	110 feet.
6–1		

(*a*) Traverse the notes as follows:

Station.	Bearing.	Distance.	Cosine.	Sine.	Latitude.		Departure.	
					North.	South.	East.	West.
1–2	N 50° W	200 ft.	.64279	.76604	128.56			153.21
2–3	N 10° E	222 ft.	.98481	.17365	218.63		38.55	
3–4	S 60° E	228 ft.	.50000	.86603		114.00	197.45	
4–5	N 50° E	284 ft.	.64279	.76604	182.55		217.56	
5–6	N 68° E	110 ft.	.37461	.92718	41.21		101.99	
6–1	To be calculated.							
					570.95	114.00	555.55	153.21
					114.00		153.21	
					456.95		402.34	

Sta. 6 is, therefore, 456.95 feet north and 402.34 feet east of Sta. 1.

The tangent of the bearing from

$$1 \text{ to } 6 = \frac{402.34}{456.95} = .88049;$$

hence, the bearing from 1 to 6 = N 41° 22′ E, very nearly, and the distance $= \sqrt{456.95^2 + 402.34^2} = 608.84$ ft. Ans.

(*b*) To find the area enclosed by the survey:

Area of enclosing rectangle $MNOP =$

$$456.95 \times 555.55 = \underline{253{,}858.57} \text{ sq. ft}$$

Area of triangle $A =$

$$\frac{128.56 \times 153.21}{2} = 9{,}848.34 \text{ sq ft.}$$

Area of trapezoid $B =$

$$\frac{109.76 + 328.39}{2} \times 38.55 = 8{,}445.34 \text{ sq. ft.}$$

Area of trapezoid $C =$

$$\frac{109.76 + 223.76}{2} \times 197.45 = 32{,}926.76 \text{ sq. ft.}$$

Area of trapezoid $D =$

$$\frac{41.21 + 223.76}{2} \times 217.56 = 28{,}823.44 \text{ sq. ft.}$$

Area of triangle $E =$

$$\frac{101.99 \times 41.21}{2} = 2{,}101.50 \text{ sq. ft.}$$

Area of triangle $F =$

$$\frac{402.34 \times 456.95}{2} = 91{,}924.63 \text{ sq. ft.}$$

$$\overline{174{,}070.01} \text{ sq. ft.}$$

Total area of survey = 253,858.57 − 174,070.01 = 79,788.56 square feet. Ans.

9. Having decided to abandon a drift at a mine and sink a slope, in order to obtain better haulage, drainage, and outside arrangements, the survey following was made.

The slope will be driven due south from Sta. 1 on a pitch of 30°, and the work will commence both at the surface and in the mine. Determine (*a*) the horizontal distance from Sta. 4 to a point in the monkey drift where a horizontal heading driven due north will intersect the line of the slope at the point from which the work in the mine will commence; (*b*) the length of the heading leading from the monkey drift to the foot of the slope; and (*c*) the length of the slope.

(*a*) After traversing the notes, Sta. 4 is found to be 709.56 feet east of Sta. 1, or the line of the slope. Hence,

$$\frac{709.56}{\text{os}\,(90^\circ - 89^\circ\,15')} = 709.6$$ feet, the horizontal distance ich must be measured off along the monkey drift from Sta. determine the starting-point of the horizontal heading.

Stations.	Azimuths.	Bearings.	Vertical Angle.	Horizontal Distances.	Latitude. North.	Latitude. South.	Departure. East.	Departure. West.
1–2	91° 30′	S 88° 30′ E	− 17° 15′	540		14.15	539.84	
2–3	140° 00′	S 40° E	− 23° 07′	258		197.63	165.84	
3–4	179° 30′	S 0° 30′ E	− 5° 13′	444		444.00	3.88	
4– Along monkey drift.	270° 45′	N 89° 15′ W	+ 4° 02′					
						655.78	709.56	

(*b*) The height of Sta. 1 above Sta. 2 = 540 tan 17° 15′ = 167.68 feet; the height of Sta. 2 above Sta. 3 = 258 tan 23° 07′ = 110.14 feet; the height of Sta. 3 above Sta. 4 = 444 tan 5° 13′ = 40.54 feet. The distance of Sta. 4 below the horizontal heading = 709.6 tan 4° 02′ = 50.03 feet. Hence, 167.68 + 110.14 + 40.54 − 50.03 = 268.37 feet = the distance the horizontal heading is below Sta. 1. The distance the starting-point of the horizontal heading is north of Sta. 4 = 709.6 sin 0° 45′ = 9.29 feet, and since Sta. 4 is, according to the notes, 655.78 feet south of Sta. 1, 655.78 − 9.29 = 646.49 feet is the distance the starting-point of the horizontal heading is south of Sta. 1. The slope will strike the level of the heading at a distance south of Sta. 1 $= \frac{268.37}{\tan 30°} = 464.83$ feet. Hence, 646.49 − 464.83 = 181.66 feet = the length of the heading.

(*c*) The length of the slope $= \frac{268.37}{\sin 30°} = 536.74$ feet.

A SERIES

OF

QUESTIONS AND EXAMPLES

RELATING TO THE SUBJECTS
TREATED OF IN THIS VOLUME.

It will be noticed that the various Question Papers that follow have been given the same section numbers as the Instruction Papers to which they refer. No attempt should be made to answer any of the questions or to solve any of the examples until the Instruction Paper having the same section number as the Question Paper in which the questions or examples occur has been carefully studied.

ARITHMETIC.

(PART 1.)

(1) What is arithmetic?

(2) What is a number?

(3) What is the difference between a concrete number and an abstract number?

(4) Define notation and numeration.

(5) Write each of the following numbers in words:

(*a*) 980; (*b*) 605; (*c*) 28,284; (*d*) 9,006,042; (*e*) 850,317,002; (*f*) 700,004.

(6) Represent in figures the following expressions:

(*a*) Seven thousand, six hundred. (*b*) Eighty-one thousand, four hundred, two. (*c*) Five million, four thousand, seven. (*d*) One hundred eight million, ten thousand, one. (*e*) Eighteen million, six. (*f*) Thirty thousand, ten.

(7) What is the sum of 3,290 + 504 + 865,403 + 2,074 + 81 + 7? Ans. 871,359.

(8) 709 + 8,304,725 + 391 + 100,302 + 300 + 909 = what? Ans. 8,407,336.

(9) Find the difference between the following:

(*a*) 50,962 and 3,338; (*b*) 10,001 and 15,339.

Ans. { (*a*) 47,624. (*b*) 5,338.

(10) (*a*) 70,968 − 32,975 = ? (*b*) 100,000 − 98,735 = ?

Ans. { (*a*) 37,993. (*b*) 1,265.

(11) The greater of two numbers is 1,004 and their difference is 49; what is their sum? Ans. 1,959.

(12) From 5,962 + 8,471 + 9,023 take 3,874 + 2,039. Ans. 17,543.

(13) A man willed $125,000 to his wife and two children; to his son he gave $44,675, to his daughter $26,380, and to his wife the remainder. What was his wife's share? Ans. $53,945.

(14) Find the products of the following:
(*a*) 526,387 × 7; (*b*) 700,298 × 17; (*c*) 217 × 103 × 67.

Ans. { (*a*) 3,684,709. (*b*) 11,905,066. (*c*) 1,497,517.

(15) If your watch ticks once every second, how many times will it tick in one week? Ans. 604,800 times.

(16) If a monthly publication contains 24 pages in each issue, how many pages will there be in eight yearly volumes? Ans. 2,304.

(17) An engine and boiler in a manufactory are worth $3,246. The building is worth three times as much, plus $1,200, and the tools are worth twice as much as the building, plus $1,875. (*a*) What is the value of the building and tools? (*b*) What is the value of the whole plant?

Ans. { (*a*) $34,689. (*b*) 37,935.

(18) Solve the following by cancelation:

(*a*) $\frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = ?$ (*b*) $\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$

Ans. { (*a*) 8. (*b*) 32.

(19) If a mechanic earns $1,500 a year for his labor, and his expenses are $968 per year, in what time can he save enough to buy 28 acres of land, at $133 an acre? Ans. 7 years.

(20) A freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles, the next week; how far did it run the second week? Ans. 849 miles.

(21) If the driving wheel of a locomotive is 16 ft. in circumference, how many revolutions will it make in going from Philadelphia to Pittsburg, the distance of which is 354 miles, there being 5,280 feet in one mile? Ans. 116,820 rev.

(22) What is the quotient of
(*a*) 589,824 ÷ 576? (*b*) 369,730,620 ÷ 43,911? (*c*) 2,527,525 ÷ 505? (*d*) 4,961,794,302 ÷ 1,234?

Ans. { (*a*) 1,024.
(*b*) 8,420.
(*c*) 5,005.
(*d*) 4,020,903.

(23) A man paid $444 for a horse, wagon, and harness. If the horse cost $264 and the wagon $153, how much did the harness cost? Ans. $27.

(24) What is the product of
(*a*) 1,024 × 576? (*b*) 5,005 × 505? (*c*) 43,911 × 8,420?

Ans. { (*a*) 589,824.
(*b*) 2,527,525.
(*c*) 369,730,620.

(25) If a man receives 30 cents an hour for his wages, how much will he earn in a year, working 10 hours a day and averaging 25 days per month? Ans. $900.

(26) What is a fraction?

(27) What are the terms of a fraction?

(28) What does the denominator show?

(29) What does the numerator show?

(30) How do you find the value of a fraction?

(31) Is $\frac{13}{8}$ a proper or an improper fraction, and why?

(32) Write three mixed numbers.

(33) Reduce the following fractions to their lowest terms: $\frac{4}{8}$, $\frac{4}{16}$, $\frac{8}{32}$, $\frac{32}{64}$. Ans. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$.

(34) Reduce 6 to an improper fraction whose denominator is 4. Ans. $\frac{24}{4}$.

(35) Reduce $7\frac{7}{8}$, $13\frac{5}{16}$, and $10\frac{3}{4}$ to improper fractions. Ans. $\frac{63}{8}$, $\frac{213}{16}$, $\frac{43}{4}$.

(36) What is the value of each of the following: $\frac{13}{2}$, $\frac{17}{4}$, $\frac{69}{16}$, $\frac{16}{8}$, $\frac{67}{64}$? Ans. $6\frac{1}{2}$, $4\frac{1}{4}$, $4\frac{5}{16}$, 2, $1\frac{3}{64}$.

(37) Solve the following:

(*a*) $35 \div \frac{5}{16}$; (*b*) $\frac{9}{16} \div 3$; (*c*) $\frac{17}{2} \div 9$; (*d*) $\frac{113}{64} \div \frac{7}{16}$; (*e*) $15\frac{3}{4} \div 4\frac{3}{8}$.

Ans. (*a*) 112. (*b*) $\frac{3}{16}$. (*c*) $\frac{17}{18}$. (*d*) $4\frac{1}{28}$. (*e*) $3\frac{3}{5}$.

(38) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = ?$ Ans. 1.

(39) $\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ Ans. $\frac{15}{16}$.

(40) $42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Ans. $83\frac{1}{16}$.

(41) An iron plate is divided into four sections; the first contains $29\frac{3}{4}$ square inches; the second, $50\frac{5}{8}$ square inches; the third, 41 square inches, and the fourth, $69\frac{3}{16}$ square inches. How many square inches are in the plate? Ans. $190\frac{9}{16}$ sq. in.

(42) Find the value of each of the following:

(*a*) $\dfrac{7}{\frac{3}{16}}$; (*b*) $\dfrac{\frac{15}{32}}{\frac{5}{8}}$; (*c*) $\dfrac{\frac{4+3}{2+6}}{5}$

Ans. (*a*) $37\frac{1}{3}$. (*b*) $\frac{3}{4}$. (*c*) $\frac{7}{40}$.

(43) The numerator of a fraction is 28, and the value of the fraction $\frac{7}{8}$; what is the denominator? Ans. 32.

(44) What is the difference between (*a*) $\frac{7}{8}$ and $\frac{7}{16}$? (*b*) 13 and $7\frac{7}{16}$? (*c*) $312\frac{9}{16}$ and $229\frac{5}{32}$?

Ans. (*a*) $\frac{7}{16}$. (*b*) $5\frac{9}{16}$. (*c*) $83\frac{13}{32}$.

(45) If a man travels $85\frac{5}{12}$ miles in one day, $78\frac{9}{15}$ miles in another day, and $125\frac{17}{35}$ miles in another day, how far did he travel in the three days? Ans. $289\frac{211}{420}$ miles.

(46) From $573\frac{4}{5}$ tons take $216\frac{5}{8}$ tons. Ans. $357\frac{7}{40}$.

(47) At $\frac{3}{8}$ of a dollar a yard, what will be the cost of $9\frac{1}{4}$ yards of cloth? Ans. $3\frac{15}{32}$ dollars.

(48) Multiply $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 by $\frac{7}{8}$ of $\frac{5}{6}$ of 45.
Ans. $109\frac{13}{128}$.

(49) How many times are $\frac{2}{3}$ contained in $\frac{3}{4}$ of 16?
Ans. 18 times.

(50) Bought $211\frac{1}{4}$ pounds of old lead for $1\frac{7}{8}$ cents per pound. Sold a part of it for $2\frac{1}{2}$ cents per pound, receiving for it the same amount as I paid for the whole. How many pounds did I have left? Ans. $52\frac{13}{16}$ pounds.

(51) Write out in words the following numbers: .08, .131, .0001, .000027, .0108, and 93.0101.

(52) How do you place decimals for addition and subtraction?

(53) Give a rule for multiplication of decimals.

(54) Give a rule for division of decimals.

(55) State the difference between a fraction and a decimal.

(56) State how to reduce a fraction to a decimal.

(57) Reduce the following fractions to equivalent decimals: $\frac{1}{2}$, $\frac{7}{8}$, $\frac{5}{32}$, $\frac{65}{100}$, and $\frac{125}{1000}$.

Ans. $\begin{cases} .5. \\ .875. \\ .15625 \\ .65. \\ .125. \end{cases}$

(58) Solve the following:

(*a*) $\dfrac{32.5 + .29 + 1.5}{4.7 + 9}$; (*b*) $\dfrac{1.283 \times \overline{8 + 5}}{2.63}$;

(*c*) $\dfrac{\overline{589 + 27} \times \overline{163 - 8}}{25 + 39}$; (*d*) $\dfrac{\overline{40.6 + 7.1} \times (3.029 - 1.874)}{6.27 + 8.53 - 8.01}$.

Ans. (*a*) 2.5029. (*b*) 6.3418. (*c*) 1,491.875. (*d*) 8.1139.

(59) How many inches in .875 of a foot? Ans. $10\frac{1}{2}$ in.

(60) What decimal part of a foot is $\frac{3}{16}$ of an inch?
Ans. .015625.

(61) A cubic inch of water weighs .03617 of a pound. What is the weight of a body of water whose volume is 1,500 cubic inches? Ans. 54.255 lb.

(62) If by selling a carload of coal for $82.50, at a profit of $1.65 per ton, I make enough to pay for 72.6 ft. of fencing at $.50 a foot, how many tons of coal were in the car?
Ans. 22 tons.

(63) Divide 17,892 by 231, and carry the result to four decimal places. Ans. 77.4545 +.

(64) Find the value of the following expression when the result is carried to three decimal places:

$$\frac{74.26 \times 24 \times 3.1416 \times 19 \times 19 \times 350}{33{,}000 \times 12 \times 4} = ?$$ Ans. 446.619 –.

(65) Express (*a*) .7928 in 64ths; (*b*) .1416 in 32ds; (*c*) .47915 in 16ths.

Ans. (*a*) $\frac{51}{64}$. (*b*) $\frac{5}{32}$. (*c*) $\frac{8}{16}$.

(66) Work out the following examples:

(*a*) $709.63 - .8514$; (*b*) $81.963 - 1.7$; (*c*) $18 - .18$; (*d*) $1 - .001$; (*e*) $872.1 - (.8721 + .008)$; (*f*) $(5.028 + .0073) - (6.704 - 2.38)$.

Ans. (*a*) 708.7786. (*b*) 80.263. (*c*) 17.82. (*d*) .999. (*e*) 871.2199. (*f*) .7113.

(67) Work out the following:

(*a*) $\frac{7}{8} - .807$; (*b*) $.875 - \frac{3}{8}$; (*c*) $(\frac{5}{32} + .435) - (\frac{21}{100} - .07)$; (*d*) What is the difference between the sum of 33-millionths and 17-thousandths, and the sum of 53-hundredths and 274-thousandths?

Ans. (*a*) .068. (*b*) .5. (*c*) .45125. (*d*) .786967.

(68) What is the sum of .125, .7, .089, .4005, .9, and .000027? Ans. 2.214527.

(69) $927.416 + 8.274 + 372.6 + 62.07938 = ?$

Ans. 1,370.36938.

(70) Add 17-thousandths, 2-tenths, and 47-millionths.

Ans. .217047.

(71) Find the products of the following expressions:

(*a*) $.013 \times .107$; (*b*) $203 \times 2.03 \times .203$; (*c*) $2.7 \times 31.85 \times (3.16 - .316)$; (*d*) $(107.8 + 6.541 - 31.96) \times 1.742$.

Ans. (*a*) .001391. (*b*) 83.65427. (*c*) 244.56978. (*d*) 143.507702.

(72) Solve the following:

(*a*) $(\frac{7}{16} - .13) \times \overline{.625 + \frac{5}{8}}$; (*b*) $(\frac{19}{32} \times .21) - (.02 \times \frac{3}{16})$; (*c*) $(\frac{13}{4} + .013 - 2.17) \times \overline{13\frac{1}{4} - 7\frac{5}{16}}$.

Ans. (*a*) .384375. (*b*) .1209375. (*c*) 6.4896875.

(73) Solve the following:

(a) $.875 \div \frac{1}{2}$; (b) $\frac{7}{8} \div .5$; (c) $\dfrac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125}$.

Ans. { (a) 1.75. (b) 1.75. (c) .5.

(74) Find the value of the following expression:

$$\cfrac{1.25 \times 20 \times 3}{\cfrac{87 + (11 \times 8)}{459 + 32}}.$$

Ans. $210\frac{3}{7}$.

(75) From 1 plus .001 take .01 plus .000001.

Ans. .990999.

(76) A cubic inch of mercury weighs .49175 pound; what is the weight of 30 cubic inches of the same?

Ans. 14.7525 pounds.

(77) The output of a certain shaft is 3 times that of a slope; the mine cars at the shaft have twice the capacity of those at the slope. If the slope sends out 500 cars per day, what number of cars are hoisted from the shaft?

Ans. 750 cars.

(78) If one inch of water gauge corresponds to a pressure of 5.2 pounds per square foot, how many inches of water gauge will be produced when the pressure is increased to 26 pounds per square foot? Ans. 5 inches.

(79) There are 231 cubic inches in one U. S. gallon. How many gallons will be contained in a tank whose capacity is 133.68 cubic feet, there being 1,728 cubic inches in one cubic foot? Ans. 1,000 gal., nearly.

(80) The weight upon a certain hoisting rope is as follows: Cage, 1,850 pounds; mine car, 967 pounds; coal, 1,235 pounds; the friction produces a strain equal to one-fourth of this total load. The weight of the rope is .88 pound per foot, and the shaft is 250 feet deep. Find the total strain upon the upper end of the rope, when cage is at the bottom of the shaft. Ans. 5,285 pounds.

(81) A tank delivers 5 gallons of water per minute to a mill and 3 gallons per minute to a factory. It is filled by a

pump which discharges 800 gallons per hour. The capacity of the tank is 2,000 gallons. If the pump works one hour and a half before the water is turned on to mill and factory, how long will it take to fill the tank, supposing it to have been empty at the start? Ans. 4 hours.

(82) The entire output of a coal mine is 900 tons per day. Of this, two-thirds is furnished to a railroad, one-fourth to private trade, and the balance supplies team sales. How many tons go to the railroad, how many to the private trade, and how many to team sales?

Ans. { 600 tons, railroad.
225 tons, private trade.
75 tons, team sales.

(83) The output of a mine is 720 tons per day. Two-thirds of the output is lump coal, and one-third is screenings. Of the screenings, two-thirds is nut coal which is sold at $1.00 per ton, and one-third is steam coal which is sold at 25 cents per ton. The lump coal is sold at $1.50 per ton. What is the total income per day from coal sales?

Ans. $900.

(84) The cost of sinking a shaft 328 feet deep was $7,230. What was the average cost per foot? Ans. $22.04.

(85) If soft coal weighs 1.27 times as heavy as water, and the weight of a cubic foot of water is 62.5 pounds, what is the weight of a cubic foot of this coal? Ans. 79.375 pounds.

(86) If a cubic foot of hard coal weighs 93.5 pounds, how many cubic feet are in a ton of this coal, or 2,000 pounds? Ans. 21.39 cu. ft.

(87) If a cubic inch of water weighs .03617 pound, and a cubic inch of mercury .49175 pound, how many cubic inches of water will balance the weight of one cubic inch of mercury? Ans. 13.5955 cu. in.

(88) Five-eighths of all the men employed in a certain shaft are negroes, the remainder, numbering 270, being white men; find the number of the negroes.

Ans. 450 negroes.

(89) The average daily output of each miner, in a certain

slope, is 5,000 pounds of coal. If there are 344 miners at work, what will be the number of 20-ton cars loaded in one day at this mine? Ans. 43 cars.

(90) The output of a certain mine for one day is as follows: 25 cars lump coal, 20 tons each; 20 cars lump, 16 tons each; 15 cars nut, 16 tons, and 12 cars steam coal, 16 tons each. The lump coal sells at $1.40 per ton; the nut, 90 cents per ton, and the steam coal, 25 cents per ton. What are the day's receipts at this mine? Ans. $1,412.

(91) (*a*) What is the total cost of 1,000 4-foot props, and 1,200 5-foot props, at one cent per running foot? (*b*) What is the cost of 2,640 caps at $7.50 per thousand? (*c*) What is the total cost? Ans. (*c*) $119.80.

ARITHMETIC.

(PART 2.)

(92) What is 25 per cent. of 8,428 lb.? Ans. 2,107 lb.

(93) What is 1 per cent. of $100? Ans. $1.

(94) What is $\frac{1}{2}$ per cent. of $35,000? Ans. $175.

(95) What per cent. of 50 is 2? Ans. 4%.

(96) What per cent. of 10 is 10? Ans. 100%.

(97) Solve the following:

(*a*) Base = $2,522 and percentage = $176.54. What is the rate? (*b*) Percentage = 16.96 and rate = 8 per cent. What is the base? (*c*) Amount = 216.7025 and base = 213.5. What is the rate? (*d*) Difference = 201.825 and base = 207. What is the rate?

Ans. (*a*) 7%. (*b*) 212. (*c*) $1\frac{1}{2}$%. (*d*) $2\frac{1}{2}$%.

(98) A farmer gained 15% on his farm by selling it for $5,500. What did it cost him? Ans. $4,782.61.

(99) A man receives a salary of $950. He pays 24% of it for board, $12\frac{1}{2}$% of it for clothing, and 17% of it for other expenses. How much does he save in a year?

Ans. $441.75.

(100) If $37\frac{1}{2}$ per cent. of a number is 961.38, what is the number? Ans. 2,563.68.

(101) A man owns $\frac{3}{4}$ of a property. 30% of his share is worth $1,125. What is the whole property worth?

Ans. $5,000.

(102) What sum diminished by 35% of itself equals $4,810? Ans. $7,400.

(103) A merchant's sales amounted to $197.55 on Monday, and this sum was 12½% of his sales for the week. How much were his sales for the week? Ans. $1,580.40.

(104) The distance between two stations on a certain railroad is 16.5 miles, which is 12½% of the entire length of the road. What is the length of the road? Ans. 132 miles.

(105) After paying 60% of my debts, I find that I still owe $35. What was my whole indebtedness? Ans. $87.50.

(106) Reduce 28 rd. 4 yd. 2 ft. 10 in. to inches.
Ans. 5,722 in.

(107) Reduce 5,722 in. to higher denominations.
Ans. 28 rd. 4 yd. 2 ft. 10 in.

(108) How many seconds in 5 weeks and 3.5 days?
Ans. 3,326,400 sec.

(109) How many pounds, ounces, pennyweights, and grains are contained in 13,750 gr.?
Ans. 2 lb. 4 oz. 12 pwt. 22 gr.

(110) Reduce 4,763,254 links to miles.
Ans. 595 mi. 32 ch. 54 li.

(111) Reduce 764,325 cu. in. to cu. yd.
Ans. 16 cu. yd. 10 cu. ft. 549 cu. in.

(112) What is the sum of 2 rd. 2 yd. 2 ft. 3 in.; 4 yd. 1 ft. 9 in.; 2 ft. 7 in.? Ans. 3 rd. 2 yd. 2 ft. 1 in.

(113) What is the sum of 3 gal. 3 qt. 1 pt. 3 gi.; 6 gal. 1 pt. 2 gi.; 4 gal. 1 gi.; 8 qt. 5 pt.? Ans. 16 gal. 3 qt. 2 gi.

(114) What is the sum of 240 gr. 125 pwt. 50 oz. and 3 lb.?
Ans. 7 lb. 8 oz. 15 pwt.

(115) What is the sum of 11° 16′ 12″; 13° 19′ 30″; 20° 25″; 26′ 29″; 10° 17′ 11″? Ans. 55° 19′ 47″.

(116) What is the sum of 130 rd. 5 yd. 1 ft. 6 in.; 215 rd. 2 ft. 8 in.; 304 rd. 4 yd. 11 in.? Ans. 2 mi. 10 rd. 5 yd. 7 in.

(117) What is the sum of 21 A. 67 sq. ch. 3 sq. rd. 21 sq. li.; 28 A. 78 sq. ch. 2 sq. rd. 23 sq. li.; 47 A. 6 sq. ch. 2 sq.

rd. 18 sq. li.; 56 A. 59 sq. ch. 2 sq. rd. 16 sq. li.; 25 A. 38 sq. ch. 3 sq. rd. 23 sq. li.; 46 A. 75 sq. ch. 2 sq. rd. 21 sq. li.?

Ans. 255 A. 3 sq. ch. 14 sq. rd. 122 sq. li.

(118) From 20 rd. 2 yd. 2 ft. 9 in. take 300 feet.

Ans. 2 rd. 1 yd. 2 ft. 9 in.

(119) From a farm containing 114 A. 80 sq. rd. 25 sq. yd., 75 A. 70 sq. rd. 30 sq. yd. are sold. How much remains?

Ans. 39 A. 9 sq. rd. 25½ sq. yd.

(120) From a hogshead of molasses, 10 gal. 2 qt. 1 pt. are sold at one time, and 26 gal. 3 qt. at another time. How much remains? Ans. 25 gal. 2 qt. 1 pt.

(121) If a person were born June 19, 1850, how old would he be August 3, 1892? Ans. 42 yr. 1 mo. 14 da.

(122) A note was given August 5, 1890, and was paid June 3, 1892. What length of time did it run?

Ans. 1 yr. 9 mo. 28 da.

(123) What length of time elapsed from 16 min. past 10 o'clock A. M., July 4, 1883, to 22 min. before 8 o'clock P. M., Dec. 12, 1888? Ans. 5 yr. 5 mo. 8 da. 9 hr. 22 min.

(124) If 1 iron rail is 17 ft. 3 in. long, how long would 51 rails be, if placed end to end? Ans. 53 rd. 1½ yd. 9 in.

(125) Multiply 3 qt. 1 pt. 3 gi. by 4.7.

Ans. 4 gal. 2 qt. 1.7 gi.

(126) Multiply 3 lb. 10 oz. 13 pwt. 12 gr. by 1.5.

Ans. 5 lb. 10 oz. 6 gr.

(127) How many bushels of apples are contained in 9 bbl. if each barrel contains 2 bu. 3 pk. 6 qt.?

Ans. 26 bu. 1 pk. 6 qt.

(128) Multiply 7 T. 15 cwt. 10.5 lb. by 1.7.

Ans. 13 T. 3 cwt. 67.85 lb.

(129) Divide 358 A. 57 sq. rd. 6 sq. yd. 2 sq. ft. by 7.

Ans. 51 A. 31 sq. rd. 8 sq. ft.

(130) Divide 282 bu. 3 pk. 1 qt. 1 pt. by 12.

Ans. 23 bu. 2 pk. 2 qt. ¼ pt.

(131) How many iron rails, each 30 ft. long, are required to lay a railroad-track 23 miles long? Ans. 8,096 rails.

(132) How many boxes, each holding 1 bu. 1 pk. and 7 qt., can be filled from 356 bu. 3 pk. and 5 qt. of cranberries?

Ans. 243 boxes.

(133) If 16 square miles are equally divided into 62 farms, how much land will each contain?

Ans. 165 A. 25 sq. rd. 24 sq. yd. 3 sq. ft. 80 + sq. in.

(134) What is the square of 108? Ans. 11,664.

(135) What is the cube of 181.25? Ans. 5,954,345.703125.

(136) What is the fourth power of 27.61?

Ans. 581,119.73780641.

(137) Solve the following: (*a*) 106^2; (*b*) $(182\frac{1}{8})^2$; (*c*) $.005^2$; (*d*) $.0063^2$; (*e*) 10.06^2.

Ans. (*a*) 11,236. (*b*) 33,169.515625. (*c*) .000025. (*d*) .00003969. (*e*) 101.2036.

(138) Solve the following: (*a*) 753^3; (*b*) 987.4^3; (*c*) $.005^3$; (*d*) $.4044^3$.

Ans. (*a*) 426,957,777. (*b*) 962,674,279,624. (*c*) .000000125. (*d*) .066135317184.

(139) What is the fifth power of 2? Ans. 32.

(140) What is the fourth power of 3? Ans. 81.

(141) What are the values of: (*a*) 67.85^2; (*b*) $967,845^2$? (*c*) $(\frac{3}{8})^2$? (*d*) $(\frac{1}{4})^2$?

Ans. (*a*) 4,603.6225. (*b*) 936,723,944,025. (*c*) $\frac{9}{64}$. (*d*) $\frac{1}{16}$.

(142) What is (*a*) the tenth power of 5? (*b*) the fifth power of 9?

Ans. (*a*) 9,765,625. (*b*) 59,049.

(143) Solve the following: (*a*) 1.2^4; (*b*) 11^6; (*c*) 1^7; (*d*) $.01^4$; (*e*) $.1^5$.

Ans. (*a*) 2.0736. (*b*) 1,771,561. (*c*) 1. (*d*) .00000001. (*e*) .00001.

(144) Find the values of the following: (*a*) $.0133^3$; (*b*) 301.011^3; (*c*) $(\frac{1}{8})^3$; (*d*) $(3\frac{3}{4})^3$.

Ans. { (*a*) .000002352637. (*b*) 27,273,890.942264331. (*c*) $\frac{1}{512}$. (*d*) $52\frac{47}{64}$, or 52.734375.

(145) In what respect does evolution differ from involution?

Note.—In the answers to the following examples, a minus sign after a number indicates that the last digit is not quite as large as the number printed. Thus, 12.497 — indicates that the number really is 12.496 +, and that the 6 has been made a 7 because the next succeeding figure was 5 or greater. For example, had it been desired to use but three decimal places in example 137 (*b*), the answer would have been written 33,169.516 —.

(146) Find the square root of the following: (*a*) 3,486,-784.401; (*b*) 9,000,099.4009; (*c*) .001225.

Ans. { (*a*) 1,867.29 +. (*b*) 3,000.017 —. (*c*) .035.

(147) Extract the square root of (*a*) 10,795.21; (*b*) 73,008.04; (*c*) 90; (*d*) .09.

Ans. { (*a*) 103.9. (*b*) 270.2. (*c*) 9.487 —. (*d*) .3.

(148) Extract the cube root of (*a*) .32768; (*b*) 74,088; (*c*) 92,416; (*d*) .373248.

Ans. { (*a*) .6894 +. (*b*) 42. (*c*) 45.212 —. (*d*) .72.

(149) Extract the cube root of 2 to six decimal places.

Ans. 1.259921 +.

(150) Extract the cube root of (*a*) 1,758.416743; (*b*) 1,191,016; (*c*) $\frac{4}{32}$; (*d*) $\frac{27}{512}$.

Ans. { (*a*) 12.07. (*b*) 106. (*c*) $\frac{1}{2}$. (*d*) $\frac{3}{8}$.

(151) Extract the cube root of 3 to six decimal places.

Ans. 1.442250 —.

(152) Solve the following: (*a*) $\sqrt{123.21}$; (*b*) $\sqrt{114.921}$; (*c*) $\sqrt{502,681}$; (*d*) $\sqrt{.00041209}$.

Ans. { (*a*) 11.1. (*b*) 10.72 +. (*c*) 709. (*d*) .0203.

(153) Solve the following: (*a*) $\sqrt[3]{.0065}$; (*b*) $\sqrt[3]{.021}$; (*c*) $\sqrt[3]{8,036,054,027}$; (*d*) $\sqrt[3]{.000004096}$; (*e*) $\sqrt[3]{17}$.

Ans. { (*a*) .18663 −. (*b*) .2759 −. (*c*) 2,003. (*d*) .016. (*e*) 2.5713 −.

(154) Solve the following: (*a*) $\sqrt[4]{6,561}$; (*b*) $\sqrt[6]{117,649}$; (*c*) $\sqrt[6]{.000064}$; (*d*) $\sqrt[3]{\frac{3}{8}}$.

Ans. { (*a*) 9. (*b*) 7. (*c*) .2. (*d*) .72112 +.

(155) Extract the square root of (*a*) $\frac{1225}{5476}$; (*b*) .3364; (*c*) .1; (*d*) $25.0\frac{3}{4}$; (*e*) $.000\frac{4}{9}$.

Ans. { (*a*) $\frac{35}{74}$. (*b*) .58. (*c*) .31623 −. (*d*) 5.00749 +. (*e*) .02108 +.

(156) (*a*) Extract the fourth root of 2 to four decimal places; (*b*) extract the sixth root of 6.

Ans. { (*a*) 1.1892 +. (*b*) 1.34801 −.

(157) Extract the square root of (*a*) 3.1416 and (*b*) .7854 to four decimal places.

Ans. { (*a*) 1.7725 −. (*b*) .8862 +.

(158) Extract the cube root of (*a*) 3.1416 and (*b*) .5236 to four decimal places.

Ans. { (*a*) 1.4646 −. (*b*) .8060 −.

Find the value of x in the following:

(159) $11.7 : 13 :: 20 : x$. Ans. 22.22 +.

(160) (*a*) $20 + 7 : 10 + 8 :: 3 : x$; (*b*) $12^2 : 100^2 :: 4 : x$.

Ans. $\begin{cases} (a)\ 2. \\ (b)\ 277.7+. \end{cases}$

(161) (*a*) $\frac{4}{x} = \frac{7}{21}$; (*b*) $\frac{x}{24} = \frac{8}{16}$; (*c*) $\frac{2}{10} = \frac{x}{100}$; (*d*) $\frac{15}{45} = \frac{60}{x}$; (*e*) $\frac{10}{150} = \frac{x}{600}$.

Ans. $\begin{cases} (a)\ x = 12. \\ (b)\ x = 12. \\ (c)\ x = 20. \\ (d)\ x = 180. \\ (e)\ x = 40. \end{cases}$

(162) $x : 5 :: 27 : 12.5$. Ans. $10\frac{4}{5}$.

(163) $45 : 60 :: x : 24$. Ans. 18.

(164) $x : 35 :: 4 : 7$. Ans. 20.

(165) $9 : x :: 6 : 24$. Ans. 36.

(166) $\sqrt[3]{1,000} : \sqrt[3]{1,331} = 27 : x$. Ans. 29.7.

(167) $64 : 81 = 21^2 : x^2$. Ans. 23.625.

(168) $7 + 8 : 7 = 30 : x$. Ans. 14.

(169) A man whose steps measure 2 ft. 5 in. takes 2,480 steps in walking a certain distance. How many steps of 2 ft. 7 in. will be required for the same distance?

Ans. 2,320 steps.

(170) If a horse travels 12 mi. in 1 hr. 36 min., how far will he travel at the same rate in 15 hr.? Ans. 112.5 mi.

(171) If a column of mercury 27.63 in. high weighs .76 of a pound, what will be the weight of a column of mercury having the same diameter, 29.4 inches high? Ans. .808 + lb.

(172) If 2 gal. 3 qt. 1 pt. of water will last a man 5 da., how long will 5 gal. 3 qt. last him, if he drinks at the same rate? Ans. 10 da.

(173) If 5 men by working 8 hours a day can do a certain amount of work, how many men by working 10 hours a a day can do the same work? Ans. 4 men.

(174) If a man travel 540 miles in 20 days of 10 hours each, how many hours a day must he travel to cover 630 miles in 25 days? Ans. $9\frac{1}{3}$ hr.

(175) In the ventilation of a mine, the ventilating pressure varies as the square of the quantity of air passing, for any particular airway. If a pressure of 8 pounds per square foot will pass 10,000 cubic feet of air per minute in a certain airway, what pressure will be required to double this amount of air? Ans. 32 lb. per sq. ft.

(176) If 400 hoists are made in 10 hours from a shaft 600 feet deep, how many hoists can be made in 8 hours from a shaft 320 feet deep, the speed of hoisting being the same in each case? Ans. 600 hoists.

(177) In a ventilating current the power required to circulate the current, in any airway, varies as the cube of the quantity of air in circulation. If, then, it requires 16 horsepower to circulate 20,000 cubic feet of air, in a certain airway, per minute, what power will be required to circulate 30,000 cubic feet per minute, in the same airway?

Ans. 54 horsepower.

FORMULAS.

$A = 5$	$h = 200$
$B = 10$	$x = 12$
$i = 3.5$	$D = 120$

Work out the solutions to the following formulas, using the above values for the letters:

(178) $C = \frac{D - x}{B + i}$. Ans. $C = 8$.

(179) $Q = \frac{A h + D}{2 x + 6} + D$. Ans. $Q = 157\frac{1}{3}$.

(180) $r = \frac{3.246 B h}{\frac{A x + h}{A i - B}}$. Ans. $r = 187.269 +$.

(181) $v = \sqrt{\frac{A D}{i B + 1.5}}$. Ans. $v = 4.05 +$.

(182) $u = \sqrt[3]{\frac{B x}{.00018 h (A^2 - x)}}$. Ans. $u = 6.35 +$.

(183) $f = \frac{10 (h - D)^2}{\sqrt[3]{D + A}}$. Ans. $f = 12{,}800$.

(184) $g = \frac{(B - A)^2 - \sqrt[3]{D + A}}{A^2 - (1 + D)}$. Ans. $g = 5$.

(185) $k = \sqrt{\frac{A B^2}{\sqrt[3]{A h}}}$. Ans. $k = 7.071 +$.

(186) $T = \sqrt{\frac{A^2 \left[490 + \frac{(h x)^2}{D^2} \right]}{h + \frac{x}{D} (A^2 - B)^2}}$. Ans. $T = 10$.

GEOMETRY AND TRIGONOMETRY.

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(273) If one of the angles formed by one straight line meeting another straight line equals $\frac{1}{2}$ of a right angle, what is the other angle equal to? Ans. $1\frac{1}{2}$ right angles.

(274) If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, what is the size of one angle? Ans. $\frac{1}{3}$ of a right angle.

(275) The diametrical pitch of a gear wheel is the number of teeth in the wheel divided by the diameter of the wheel. If the pitch is 4, and the diameter of the gear is 12 inches, what is the size of an angle formed by drawing lines from the center to the middle points of two adjacent teeth? Ans. $\frac{1}{12}$ of a right angle.

(276) If a triangle has two equal angles, what kind of a triangle is it?

(277) In an equilateral heptagon one of the sides equals 3 inches; what is the length of the perimeter? Ans. 21 in.

(278) The perimeter of a regular decagon is 40 inches; what is the length of a side? Ans. 4 in.

(279) What is one angle of a regular dodecagon equal to? Ans. $1\frac{2}{3}$ right angles.

(280) A triangle has three equal angles; what is it called?

(281) Can a triangle be formed with three lines whose lengths are 12 inches, 7 inches, and 4 inches? Give reasons for your opinion.

(282) Can a quadrilateral be formed with lines whose lengths are 20 inches, 9 inches, 4 inches, and 7 inches? Give reasons.

(283) A certain triangle has two equal angles. If, from the vertex of the other angle, a perpendicular is drawn to the side opposite, which is 7 inches long, what are the lengths of the two parts of the side thus divided by the perpendicular?

(284) The shortest distance from a given point to a given line is 9 inches; the distances from this point to the two extremities of the line are 12 inches and 15 inches; what is the length of the line? Ans. 19.94 in.

(285) The sum of two angles of a right-angled triangle is $\frac{5}{4}$ of a right angle; what is the other angle equal to?
Ans. $\frac{3}{4}$ of a right angle.

(286) What is one of the angles of an equiangular octagon equal to? Ans. $1\frac{1}{2}$ right angles.

(287) In a right-angled triangle one acute angle equals $\frac{5}{8}$ of a right angle; what is the other angle equal to?
Ans. $\frac{3}{8}$ of a right angle.

(288) Given, three points, A, B, and C, and the distance from A to B equal to $1\frac{1}{2}$ inches, from B to C $1\frac{1}{2}$ inches, and from C to A 2 inches; pass a circle through these three points.

(289) The chord of an arc in a circle whose radius is 6 inches is 4 inches long; what is the length of the chord of half the arc? Ans. 2.03 in.

(290) If the diameter of the circle in the last problem had been 6 inches, what would have been the length of the chord of half the arc? Ans. 2.14 in.

(291) The diameter of a plane section of a sphere is 6 inches, and its height is 2 inches; what is the diameter of the sphere? Ans. $6\frac{1}{2}$ in.

(292) The length of a perpendicular from the center of a circle to a chord is $5\frac{3}{4}$ inches; if the diameter of the circle is 17 inches, what is the length of the chord?
Ans. 12.52 in.

(293) The sides of an inscribed angle intercept three-fourths of the circumference; how many quadrants are there in the angle? Ans. $1\frac{1}{2}$ quadrants.

(294) How many equal sectors are there in a circle, if each sector measures $\frac{2}{7}$ of a right angle? Ans. 14 sectors.

(295) If the perimeter of a regular inscribed octagon is 24 inches, and the length of the perpendicular from the center to one of the sides is 3.62 inches, what is the diameter of the circle in which the octagon is inscribed?

Ans. 7.84 in.

(296) Two equal circles intersect so that the common chord of the two arcs of intersection measures $10\frac{1}{2}$ inches. If the circles are struck with a 13-inch radius, what is the greatest distance between the two intersecting arcs?

Ans. 2.2 in.

(297) In the last example, if the radius of one circle is 13 inches, and of the other 8 inches, what is the greatest distance between the arcs? Ans. 3.07 in.

(298) If the height of a plane section of a sphere is $3\frac{1}{2}$ inches, and the diameter of the sphere is 14 inches, what is the diameter of the flat surface of the section?

Ans. 11.82 in.

(299) What part of a circle is an arc of 19° 19′ 19″? Express it decimally. Ans. .053672 of a circle.

(300) What part of a quadrant would an angle of 19° 19′ 19″ be? Express it decimally. Ans. .214688 of a quadrant.

(301) A regular decagon is inscribed in a circle whose diameter is 23 inches; what is the perimeter of the decagon?

Ans. 71 in., nearly.

(302) What is the difference between 90° and 35° 24′ 25.8″?

(303) In a right-angled triangle *A B C*, the hypotenuse *A B* = 17.69 feet, and the side *A C* = 9 ft. 9 in.; find the other three parts.

Ans. { 56° 33′ 12.5″.
33° 26′ 47.5″.
14 ft. 9 in.

(304) Add 159° 27′ 34.6″, 25° 16′ 8.7″, and 3° 48′ 53″.

(305) Find the sine, cosine, and tangent of 17° 27′ 37″.

(306) In a triangle ABC, $AB = 26$ feet 7 inches, $AC = 40$ feet, and the included angle $A = 36° 20' 43''$; find the remaining parts.

Ans. $\begin{cases} C = 40° 16' 52''. \\ B = 103° 22' 25''. \\ BC = 24 \text{ ft. } 4.4 \text{ in.} \end{cases}$

(307) Find the sine, cosine and tangent of 63° 4′ 51.8″.

(308) Sine = .27038, cosine = .27038, and tangent = 2.27038; find the corresponding angles.

(309) A polygon of eleven sides is called an undecagon. If a regular undecagon whose perimeter is 4 feet 3 inches be inscribed in a circle, what is the size of an angle formed by drawing radii to the extremities of one of the sides? Also, what is the radius of the circle?

Ans. $\begin{cases} 32° 43' 38.2''. \\ \text{Radius} = 8.23 \text{ in.} \end{cases}$

(310) One angle of a triangle is 47° 13′ 29″; what are the other two angles if one of them is twice the given angle?

Ans. $\begin{cases} 38° 19' 33''. \\ 94° 26' 58''. \end{cases}$

(311) One angle of a triangle is 75° 48′ 17″; what are the other two angles if one of them is half as large as the given angle?

Ans. $\begin{cases} 37° 54' 8.5''. \\ 66° 17' 34.5''. \end{cases}$

(312) In a triangle ABC, the side $AB = 16$ feet 5 inches; the side $BC = 13$ feet $6\frac{1}{2}$ inches, and the angle $A = 54° 54' 54''$; find the remaining parts.

Ans. $\begin{cases} B = 42° 19' 36''. \\ C = 82° 45' 30''. \\ AC = 11 \text{ ft. } 1\frac{3}{4} \text{ in.} \end{cases}$

(313) If one-third of an angle of a certain triangle = 14° 47′ 10″, what are the angles, one of the other two being two and one-half times the given angle?

Ans. $\begin{cases} 24° 44' 45''. \\ 44° 21' 30.'' \\ 110° 53' 45''. \end{cases}$

(314) In a right-angled triangle *A B C*, the two sides are 437 feet and 792 feet in length; find the hypotenuse and the two acute angles.

Ans. $\begin{cases} 28° 53' 19''. \\ 61° 6' 41''. \\ 904 \text{ ft. } 6\frac{3}{4} \text{ in.} \end{cases}$

(315) Find by trigonometry and prove by geometry that the angle between two adjacent sides of a regular octagon inscribed in a circle is 135°. If the perimeter of the octagon is 56 feet, what is the diameter of the circle?

Diameter = 18 feet $3\frac{1}{4}$ inches.

(316) Draw a diagram showing the sine, cosine, and tangent of 67° 8′ 49″.

(317) Given, the tangent of a certain angle = 3. (*a*) Draw a diagram showing an angle having this tangent, and mark its sine and cosine. (*b*) Give their values from the tables.

(318) If the cosine of an angle is .39278, what are the actual lengths of the cosine, tangent, and sine of the same angle in a circle whose diameter is $4\frac{3}{4}$ times as large?

(319) In a triangle *A B C*, the angle $A = 29° 21'$; angle $C = 76° 44' 18''$, and the side $A C = 31$ feet 10 inches; find the other three parts.

Ans. $\begin{cases} B C = 16 \text{ ft. } 3 \text{ in.} \\ A B = 32 \text{ ft. } 3 \text{ in.} \\ B = 73° 54' 42''. \end{cases}$

(320) A regular decagon is inscribed in a circle whose radius is $9\frac{3}{4}$ inches; what is the perimeter of the decagon?

Ans. 60.26 inches.

(321) In the above question, what is the difference between the perimeter of the decagon and the circle; also, what is the difference of their areas?

Ans. $\begin{cases} 1 \text{ in.} \\ 19.26 \text{ sq. in.} \end{cases}$

(322) The area of a circle is 89.42 square inches; what is its diameter and circumference? What is the length of a side of the largest regular hexagon that could be inscribed in it?

Length of side = 5.335 in.

(323) The distance between two parallel sides of a wrought iron octagon bar is 2 inches; what is the weight of

a bar 10 feet long, a cubic inch of wrought iron weighing 0.282 pound? Ans. 112 lb. 2 oz.

(324) The outside and inside diameters of a cast iron spherical shell are 16 inches and 12 inches; what is its weight, a cubic inch of cast iron weighing 0.261 pound?

Ans. 323.61 lb.

(325) The length of an arc of a circle is $5\frac{1}{3}\frac{1}{2}$ inches by measurement. If the number of degrees in the arc is 27, what is the diameter of the circle? Ans. 22.95 in.

(326) What is the difference between a plane figure containing 7 square inches and one 7 inches square? If both figures are perfect squares, what are the lengths of the sides?

(327) (*a*) What is the area of a circle whose diameter is $17\frac{1}{64}$ inches? (*b*) What is the length of an arc of 16° 7′ 21″ in the above circle? Ans. Length of the arc = 2.394 in.

(328) What is the area of an ellipse whose axes are 12 inches and 8 inches? What is its perimeter?

(329) What is the entire surface of a cone whose base is 7 inches in diameter, and whose altitude is 11 inches?

Ans. 165.41 sq. in.

(330) What is the height of a cone having the same volume and diameter as a 10-inch sphere? Ans. 20 in.

(331) What is the height of a cylinder having the same volume and diameter as a 12-inch sphere? Ans. 8 in.

(332) (*a*) What is the area of a triangle whose base is $9\frac{1}{2}$ inches, and whose altitude is 12 inches? (*b*) If the angle which one side forms with the base is 79° 22′, what is the perimeter of the triangle? Ans. Perimeter = 35.73 in.

(333) The diagonal of a trapezium is 11 inches; the lengths of the perpendiculars from the opposite vertexes upon this diagonal are $4\frac{1}{4}$ inches and 7 inches; what is the area of the trapezium?

(334) The length of a chord of a segment in a circle

whose diameter is 10 inches is $6\frac{3}{4}$ inches; what is the area of the segment and the number of degrees in its arc?

Ans. { 84° 54′ 28.6″.
6.074 sq. in.

(335) What is the convex area of a pyramid whose slant height is 17 inches, the perimeter of its base being 63 inches?

(336) What is the volume and entire area of a frustum of a cone whose upper base is 12 inches and lower base is 18 inches in diameter, and whose altitude is 14 inches?

Ans. { 2,506.997 cu. in.
1,042.38 sq. in.

(337) What is the area of the surface of a sphere 27 inches in diameter? Ans. 2,290.2 sq. in.

(338) Wishing to make some dumb-bells to weigh 20 pounds each exclusive of the handle, the balls to be equal spheres, what must be the diameter of the balls, a cubic inch of cast iron weighing 0.261 pound? Ans. 4.18 in.

(339) What is the volume of an engine cylinder, in cubic feet, whose diameter is 19 inches, and whose stroke is 24 inches? Ans. 3.938 cu. ft.

(340) The chord of the arc of a segment is 14 inches long, and the height of the segment is 2 inches; what is the radius? Ans. $13\frac{1}{4}$ in.

(341) The cylinders of a compound engine are 19 and 31 inches in diameter, and the stroke is 24 inches; if the clearance at each end in the small cylinder is 14% of the **stroke**, and in the large cylinder 8% of the **stroke**, (*a*) what is the total volume in cubic feet of the steam in the small cylinder during one stroke? (*b*) In the large cylinder? (*c*) What is the ratio between the two?

Ans. { 4.489 cu. ft.
11.321 cu. ft.
Ratio = 2.522 : 1.

(342) In the above example the pipe which connects the small or high-pressure cylinder to the large or low-pressure cylinder is 8 inches in diameter and 7 feet long. (*a*) What

is its volume in cubic feet? (*b*) What is the ratio of its volume to that of the high-pressure cylinder?

Ans. { 2.443 cu. ft.
Ratio = 0.544 : 1.

(343) (*a*) What is the volume and area of a cylindrical ring whose outside diameter is 16 inches and inside diameter 13 inches? (*b*) If made of cast iron, what is its weight? Ans. Weight = 21 lb.

(344) If all the dimensions in Fig. 91, Art. **790**, be doubled, what will be its area? Ans. 453.92 sq. in.

(345) The altitude of a parallelopipedon is 18 inches; its base is a square, one edge measuring $5\frac{1}{4}$ inches; what is its convex area, entire area, and volume?

Ans. { 378 sq. in.
433.125 sq. in.
496.125 cu. in.

(346) What is the convex area and entire area of a hexagonal pyramid, the slant height being 37 feet, and one edge of the base measuring 12 feet?

Ans. { 1,332 sq. ft.
1,706.112 sq. ft.

(347) If the altitude of the pyramid in the last problem had been 37 feet, what would have been its volume?

Ans. 4,614 cu. ft.

(348) How many yards of Brussels carpeting, 27 inches wide, will it take to cover a room 15 feet by 18 feet?

Ans. 40 yd.

(349) How many square yards of plaster will it take to cover the sides and ceiling of a room 16 × 20 feet, and 11 feet high, with four windows, each 7 × 4 feet, and three doors, each 9 × 4 feet over all, the baseboard coming 6 inches above the floor? Ans. $95\frac{1}{9}$ sq. yd.

(350) What is the area of a sector if the chord of the arc is $6\frac{1}{8}$ inches long, and the diameter of the circle is 10 inches?

Ans. 18.95 sq. in.

(351) What is the area in square feet of a parallelogram whose base is 129 inches long, if the shortest distance between the base and side opposite is 7 feet?

(352) The parallel sides of a trapezoid are 15 feet 7 inches, and 21 feet 11 inches long; the altitude is 7 feet 8 inches. (*a*) What is the area of the trapezoid? (*b*) What is the length of a side of an equilateral triangle having the same area?

Ans. { 143.75 sq. ft.
18 ft. 2.64 in.

(353) (*a*) What would be the length of a side of a square having the same area as the trapezoid in the last problem? (*b*) The diameter of a circle? (*c*) How much shorter is the circumference of the circle than the perimeter of the square?

Ans. { 11.99 ft.
13½ ft.
5 ft. 6.6 in.

(354) In a triangle $A\,B\,C$, $A\,B = 24$ feet, $B\,C = 11$ feet 8 inches, and $A\,C = 18$ feet; required, the three angles.

Ans. { $A = 26°\ 28'\ 5''$.
$B = 45°\ 29'\ 23''$.
$C = 108°\ 2'\ 32''$.

GASES MET WITH IN MINES.

(355) What is chemistry ?

(356) What is the difference between an atom and a molecule ?

(357) By what means can we determine the volume of gases resulting from any given chemical reaction ?

(358) What is meant by the density of air, and at what height above sea-level is it one-half as great as at the sea-level ?

(359) Which is the more dense, marsh-gas or carbonic acid gas, and why ?

(360) What is specific gravity, and what are the standards or units of measure used (*a*) for solids and liquids ? (*b*) for gases ?

(361) If a cubic foot of slate weighs 172 pounds, what is its specific gravity ? Ans. 2.8.

(362) What is the difference between an element and a compound ?

(363) What effect has the force of repulsion on matter ?

(364) Describe the principle of the barometer.

(365) What is firedamp, and what are its properties ?

(366) What is meant by diffusion of gases ?

(367) Name the principal gases formed when a charge of powder is exploded.

(368) What is the difference between mass and volume ?

(369) If the specific gravity of anthracite coal is 1.5, what will a cubic yard of it weigh ? Ans. 2,531.25 lb.

(370) Is brine, or salt water, a chemical compound or a mechanical mixture?

(371) What is atomic weight, and how does it differ from the ordinary meaning of weight?

(372) What is a chemical equation?

(373) What is a compound substance?

(374) Define the British Thermal Unit.

(375) What is a vacuum?

(376) Why is it difficult to pull two flat wet pieces of glass apart?

(377) What is meant by tension of gases, and what effect has compression on confined gas, the temperature remaining the same?

(378) If 8 cubic feet of air have a tension of 6 pounds per square inch, what will be the volume when the tension is 80 pounds per square inch, the temperature remaining the same? Ans. .6 cu. ft.

(379) What effect has temperature on the volume, when the pressure remains the same?

(380) Of what is marsh-gas a product, and what are its properties?

(381) How is firedamp detected in the mine?

(382) Explain what is meant by "occluded gas."

(383) What is the weight of 300 cubic feet of carbonic acid gas at a temperature of 70° F., barometer 30 inches?

Ans. 34.48 lb.

(384) What are the two principal classes of explosives?

(385) What causes *blown-out* and *windy* shots?

(386) What conditions influence and determine the character of an explosion of gas in a coal-mine?

(387) What is a safety-lamp? State the principle of its action.

(388) Upon what does the force of a gas explosion depend?

(389) What produces white damp, what is its effect on human life, and how may it be detected?

(390) What are the most common causes of the ignition of gas in mines?

(391) What are the Sprengel explosives, and what peculiar good feature do they possess?

(392) What causes tend to lessen the heat of an explosion?

(393) If 20 cubic feet of air at a temperature of 60° F., and a pressure of one atmosphere, are compressed to 12 cubic feet (the temperature still remaining at 60° F.), what will the compressed air weigh per cubic foot? Ans. .1272 lb.

(394) Name the three general classes of detonating explosives.

(395) If a volume of 3 cubic feet of air is under a pressure of 36 pounds per square inch, what will the pressure be when the volume is increased to 4 cubic feet?

Ans. 27 lb. per sq. in.

(396) What causes sudden outbursts of gas?

(397) Which is the better blasting powder: that composed of potassium nitrate, sulphur, and carbon, or that composed of sodium nitrate, sulphur, and carbon?

(398) What is spontaneous combustion?

(399) How many kinds of barometers are there?

(400) Give the composition of the atmosphere about us.

(401) What is oxidation?

(402) How many B. T. U. are produced by burning 2,000 pounds of bituminous coal? Ans. 28,800,000 B. T. U.

(403) What is the difference between deflagration and detonation?

(404) What is the result of combustion?

(405) Explain the result of mixing two liquids which do not act chemically upon each other, and which have different densities; also, explain the result of mixing two gases which do not act chemically upon each other, and which have different densities.

(406) Give the symbols of (*a*) water; (*b*) marsh-gas; (*c*) carbonic acid gas; (*d*) carbonic oxide gas. State the advantage of adopting such a system of symbols.

(407) What is atomic volume?

(408) What weight of carbonic acid gas will be produced by burning 300 pounds of coal containing 88% of carbon?

Ans. 968 lb.

(409) What is dissociation?

(410) Explain (*a*) the force that binds atoms together; (*b*) the force that binds molecules together.

(411) A piece of iron weighs 10 pounds when weighed in air, and 8.6 pounds when weighed in water; what is its specific gravity? Ans. 7.14.

(412) To what do the first and second laws of volume apply?

(413) If a volume of 1,200 cubic feet of marsh-gas is mixed with pure air in such a proportion that when exploded all the carbon in the marsh-gas combines with the oxygen in the air, what volume of carbonic acid gas will result, the carbonic acid gas and the marsh-gas being subjected to the same pressure and temperature? Ans. 1,200 cu. ft.

(414) What are the three forms of matter?

(415) What are the properties of sulphureted hydrogen gas, and how is the presence of the gas detected?

(416) What causes *feeders* or *blowers* of gas?

(417) In what ways do various kinds of coal-dust influence the character of an explosion?

(418) Describe the Davy lamp.

(419) What is the most practical manner of detecting firedamp in mines?

(420) How is the percentage of gas in the atmosphere measured by a Pieler lamp?

(421) What must be the volume of a vessel to hold two gases whose volumes are 15 cubic feet and 9 cubic feet, and whose tensions are 20 pounds and 15 pounds, respectively,

in order that the pressure of the mixture may be 21 pounds per square inch, the temperature remaining the same throughout? Ans. 20.71 cu. ft.

(422) What effect has a small amount of carbonic acid gas upon the flame-cap of a safety-lamp?

(423) Explain the manner in which a test is made for firedamp with a safety-lamp.

(424) What is black damp, what produces it, what is its effect on human life, and how can it be detected?

(425) Describe the Shaw gas-testing machine.

(426) What oils are generally used in safety-lamps?

(427) State the requirements of a good safety-lamp (*a*) for general mining use; (*b*) for testing purposes

(428) What is nitroglycerine, and what ruptive pressure is exerted by it when exploded?

(429) If a quantity of air confined in a cylinder and at a temperature of 50° F. is heated until it has a temperature of 212° F., what will be the resulting tension if the original tension was 14.7 pounds per square inch?

Ans. 19.38 lb. per square inch.

(430) State the conditions under which mine explosions are most frequently produced.

(431) In what respect does the Clanny lamp differ from the Davy lamp?

(432) Why is it that the Mueseler lamp gives a better light than either the Marsaut lamp or Clanny lamp?

(433) Describe the Ashworth-Hepplewhite-Gray lamp.

(434) What is dynamite, and what proportion of nitroglycerine is contained in each grade?

(435) If a volume of 76 cubic feet of confined air weighs 6.5 pounds and has a temperature of 84° F., what pressure per square inch does it exert?

Ans. 17.21 lb. per square inch.

(436) In what respects is the Evan Thomas lamp better than the Clanny lamp?

(437) What peculiar feature is possessed by the Marsaut lamp, and how may its illuminating power be improved?

(438) What is guncotton, and how is it exploded?

(439) What are (*a*) tonite? (*b*) potentite? (*c*) gelatine-dynamite?

(440) When gas is exploded, what effects are caused by coal-dust suspended in the air?

(441) Upon what does the strength of an explosive depend, and which is the best type for use in working non-gaseous coal-mines?

(442) If the specific gravity of marsh-gas at a temperature of 60° F., barometer 30 inches, is 0.559, what will 100 cubic feet of it weigh? Ans. 4.28 lb.

(443) Why does black damp diffuse in air slower than firedamp?

(444) What is the weight of 650 cubic feet of marsh-gas at a temperature of 60° F., the barometer being at 29.5 inches? Ans. 26.75 lb.

MINE VENTILATION.

(PART 1.)

(445) Define (*a*) gravitation; (*b*) the mass of a body; (*c*) acceleration.

(446) If a cannon-ball is shot vertically upwards with an initial velocity of 1,876 feet per second, (*a*) what time will elapse before it reaches the ground again? (*b*) What distance will it travel?

Ans. { (*a*) 1.944 min., nearly.
(*b*) 109,433.3 ft.

(447) What are the requirements that should be considered in fixing the quantity of air for any particular mine?

(448) State the quantity of air per man, per minute, required by law (*a*) in the anthracite region; (*b*) in the bituminous regions of the Union.

(449) When the velocity of the current is 300 feet per minute, what quantity of air is passing through a 7 ft. × 7 ft. airway? Ans. 14,700 cu. ft. per min.

(450) If a water-gauge of 2 inches passes 15,000 cubic feet of air per minute, what quantity per minute will a water-gauge of 8 inches pass in the same airway?

Ans. 30,000 cu. ft.

(451) If 2 horsepower pass 14,000 cubic feet of air per minute, to what must the power be increased to double the quantity? Ans. 16 H. P.

(452) What is the total ventilating pressure in an airway 6 ft. × 7 ft., the water-gauge being 1.5 inches?

Ans. 327.6 lb.

(453) If you have two airways under the same pressure, one 6 ft. × 6 ft. × 5,000 ft., and the other 8 ft. × $4\frac{1}{2}$ ft. × 5,000 ft., which will pass the greater quantity of air, and why?

(454) In a certain mine the total quantity of air passing down the downcast shaft is 45,000 cubic feet per minute. At the foot of the downcast it is divided into four splits as follows: Split (1) 6 ft. × 6 ft., 1,500 feet long; Split (2) 6 ft. × 7 ft., 1,800 feet long; Split (3) 6 ft. × 5 ft., 1,350 feet long; Split (4) 5 ft. × 5 ft., 1,500 feet long. Calculate the amount of air passing in each split when no regulators are used.

Ans. { 12,582 cu. ft. per min. for (1).
13,905 cu. ft. per min. for (2).
10,539 cu. ft. per min. for (3).
7,974 cu. ft. per min. for (4).

(455) If the current in an underground road 6 feet square is maintained by a pressure represented by 1 inch of water-gauge, what pressure per square foot will be required to pass the same quantity of air along a road 5 feet square, the two roads being of the same length? Ans. 12.94 lb. per sq. ft.

(456) If the velocity of an air-current is 4 feet per second, and it is required to increase it to 8 feet per second, what will be the ratio of increase in the power?

(457) If the airways of a mine were increased to double their length, other conditions remaining the same, in what proportion would you have to increase the ventilating pressure?

(458) In order to obtain double the quantity of air, in what proportion must the ventilating pressure be increased?

(459) In question 458, in what proportion would the power have to be increased to obtain the same result?

(460) The anemometer shows a current of 10,000 cubic feet of air per minute to be passing down the intake of a mine where the temperature is 30° F. Supposing no increase of the flow from the gases of the mine, what quantity of air will be passing per minute up the return where the temperature has risen to 70° F.? Ans. 10,818 cu. ft. per min.

(461) The quantity of air produced by a fan is 120,000 cubic feet per min. and the water-gauge is 2 inches. What is the horsepower absorbed in the mine? Ans. 37.82 H. P.

(462) If a water-gauge of 3 inches passes 20,000 cubic feet of air per minute in a certain mine, what water-gauge will be required to pass 30,000 cubic feet per minute through the same mine under similar conditions? Ans. $6\frac{3}{4}$ in.

(463) A current of 60,000 cubic feet of air per minute is circulated in a certain mine in five splits as follows: Split No. 1, 8,000 cu. ft.; Split No. 2, 10,000 cu. ft.; Split No. 3, 12,000 cu. ft.; Split No. 4, 14,000 cu. ft.; Split No. 5, 16,000 cu. ft. Calculate the sectional area for each split, in order that the air may travel at a uniform velocity of 5 feet per second in all the splits.

Ans. { No. 1, $26\frac{2}{3}$ sq. ft.
No. 2, $33\frac{1}{3}$ sq. ft.
No. 3, 40 sq. ft.
No. 4, $46\frac{2}{3}$ sq. ft.
No. 5, $53\frac{1}{3}$ sq. ft.

(464) The quantity of air passing per minute in a mine is 112,000 cubic feet, the effective power of the furnace is 40 horsepower. Required, the height of the water-gauge in inches. Ans. 2.27 in.

(465) A current of 10,000 cubic feet of air per minute is passing into a mine having two splits as follows: Split *A*, 4 ft. × 12 ft., 6,000 feet long; Split *B*, 6 ft. × 8 ft., 10,000 feet long. Find the amount of air passing through each split.

Ans. { *A*, 5,470 cu. ft. per min.
B, 4,530 cu. ft. per min.

(466) Suppose 50,000 cubic feet of air per minute to be passing in four splits, as follows:

Split *A*, 6 ft. × 8 ft., 10,000 ft. long, and 5,000 cu. ft. per min.

Split *B*, 5 ft. × 10 ft., 5,000 ft. long, and 10,000 cu. ft. per min.

Split *C*, 6 ft. × 12 ft., 10,000 ft. long, and 20,000 cu. ft. per min.

Split *D*, 4 ft. × 12 ft., 5,000 ft. long, and 15,000 cu. ft. per min.

(*a*) In which splits would regulators have to be introduced to accomplish such division of air, and why? (*b*) What is the entire horsepower required for the circulation in the splits? Ans. (*b*) 12.685 H. P.

(467) What is the height of the motive column if the water-gauge reads .4 inch? The temperature in the downcast shaft is 62° F., and the barometer is 30 inches.

Ans. 27.256 ft.

(468) If the air in a mine has a temperature of 62° F., and is kept in motion by a ventilating pressure of 2.08 pounds per square foot, what will be the theoretical velocity of the air, the barometer being 30 inches?

Ans. 2,512.38 ft. per min.

(469) If the water-gauge reads .6 inch when 15 horsepower is used, what will it read when the power is increased to 36 horsepower? Ans. 1.076 in.

(470) If a ventilating current of 10,000 cubic feet of air per minute passes through an airway 6 ft. × 8 ft. and 8,000 feet long, what current will pass when the length of the airway is increased to 10,000 feet?

Ans. 8,945 cu. ft. per min.

(471) 24,000 cubic feet of air per minute are passed through an airway 8 ft. × 10 ft. and 5,000 feet long. What should be the dimensions of a similar airway 8,000 feet long to pass the same quantity? Ans. 8.8 ft. × 11 ft.

(472) A square airway has a sectional area of 64 square feet, and has passing through it 15,000 cubic feet of air per minute. What must be the sectional area of a similar airway to pass 20,000 cubic feet per minute, the power remaining the same? Ans. 90.38 sq. ft.

(473) An airway is passing 20,000 cubic feet of air per minute, and it is desired to reduce this quantity to 8,000 cubic feet by means of a regulator. If the difference of pressure on the two sides of the regulator is equivalent to $\frac{1}{4}$ inch of water-gauge, what must be the area of the opening? Ans. 6.4 sq. ft.

(474) An airway 8 ft. × 10 ft. passes 60,000 cubic feet of air per minute to a point 1,500 feet distant from the downcast shaft, where it splits into four airways of the following dimensions: 1st, 6 ft. × 5 ft., 900 feet long; 2d, 6 ft. × 6 ft., 825 feet long; 3d, 6 ft. × 4 ft., 840 feet long; and 4th, 5 ft. × 4 ft., 720 feet long. (*a*) What is the quantity passing through each split, and (*b*) what should be the reading of the water-gauge for the mine, neglecting the shafts?

Ans. (*a*) 1st. 15,942 cu. ft. per min.
2d. 20,952 cu. ft. per min.
3d. 12,381 cu. ft. per min.
4th. 10,725 cu. ft. per min.
(*b*) 2.36 in.

(475) How much (i. e., in what proportion) must the ventilating power be increased to double the quantity of air?

(476) In a certain mine the entire circulation of 100,000 cubic feet of air per minute is divided into two splits having equal cross-sections. If the resistances of the splits are to each other as 5 is to 1, what quantity of air will pass through each?

Ans. 69,099 cu. ft. per min. in short airway.
30,901 cu. ft. per min. in long airway.

(477) The depth of a furnace shaft is 300 feet; the temperature of the upcast column of air is 130° F., and that of the downcast column, 50° F. (*a*) What is the length of the motive column? (*b*) What would be the pressure producing ventilation should the temperature of the upcast be raised to 150° F., while that of the downcast remained at 50° F.? Assume the barometer reading to be 30 inches.

Ans. (*a*) 40.13 ft.
(*b*) 3.85 lb. per sq. ft.

(478) Suppose that 50,000 cubic feet of air per minute pass through an airway 10 ft. × 10 ft. and 10,000 feet long, and that a change is made by dividing the current into three splits of the following dimensions: 1st, 6 ft. × 6 ft. and 4,000 feet long; 2d, 5 ft. × 6 ft. and 3,000 feet long; 3d, 5 ft. × 5 ft. and 4,000 feet long. (*a*) What quantity will pass through each of the splits that are now substituted for

the original airway, assuming that the total quantity re mains the same? (*b*) What horsepower is required in each case?

Ans. (*a*) 1st. 19,597 cu. ft. per min. 2d. 17,980 cu. ft. per min. 3d. 12,423 cu. ft. per min. (*b*) 1st case, 32.88 H. P. 2d case, 25.98 H. P.

(479) If an airway 9 ft. × 6 ft. has a total rubbing surface of 54,000 square feet, what is its length?

Ans. 1,800 ft.

(480) What quantity of air is passing through an airway 7.25 ft. × 11.75 ft., when the velocity is 434 feet per minute?

Ans. 36,971 cu. ft. per min.

(481) In what proportion must the ventilating pressure be increased to change the volume of air from 120,000 cubic feet to 180,000 cubic feet per minute? Ans. 2¼ times.

(482) If a pressure of 19.2 pounds per square foot passes 160,000 cubic feet per minute, what pressure is required to pass 120,000 cubic feet per minute through the same airway? Ans. 10.8 lb. per sq. ft.

(483) What horsepower is required to pass 70,000 cubic feet of air per minute when the water-gauge reading is .9 inch? Ans. 9.927 H. P.

(484) The upcast shaft is 300 feet deep, and the temperature of the ascending air is 120° F. The temperature of the downcast shaft being 45° F., what is the height of the motive column? Ans. 38.86 ft.

(485) If .6 inch of water-gauge passes 12,000 cubic feet of air per minute, what pressure per square foot is required to pass 24,000 cubic feet? Ans. 12.48 lb. per sq. ft.

(486) If a pressure of 4 pounds per square foot produces a velocity of 300 feet per minute in a 6 ft. × 8 ft. airway, what pressure is required to pass 24,000 cubic feet per minute through the same airway? Ans. 11⅑ lb. per sq. ft.

(487) If 20,000 cubic feet of air are passed per minute

by a pressure of $2\frac{1}{2}$ pounds per square foot, what must be the horsepower to pass 25,000 cubic feet per minute?

Ans. 2.959 H. P.

(488) (*a*) What is the area of a square airway through which a current of 30,000 cubic feet of air per minute is passing at a velocity of 500 feet per minute? (*b*) What should be the area of each of two square airways to divide the current into two equal splits, the length of the airways and the velocity of the air being the same in both cases? (*c*) Which of these arrangements requires the greater power, and in what proportion?

Ans. (*a*) 60 sq. ft.
(*b*) 30 sq. ft.
(*c*) The two small airways, in the ratio of 1.4194 : 1.

(489) If 24,000 cubic feet of air per minute pass through a 6 ft. × 10 ft. airway, what quantity will pass through a 5 ft. × 6 ft. airway of the same length, the power remaining the same? Ans. 13,596 cu. ft. per min.

(490) If 40,000 cubic feet of air per minute are passing in a circular airway 8 feet in diameter and 1,800 feet long, what is (*a*) the pressure per square foot? (*b*) the horsepower required?

Ans. (*a*) 12.36 lb. per sq. ft.
(*b*) 14.982 H. P.

(491) A 7 ft. × 10 ft. airway is passing 35,000 cubic feet of air per minute, and it is desired to reduce this quantity to 21,000 cubic feet per minute by means of a regulator. The water-gauge reading $\frac{3}{4}$ inch, what must be the area of the opening in the regulator? Ans. 12.12 sq. ft.

(492) What do you understand by the terms (*a*) motive column? (*b*) ventilating pressure? (*c*) split?

(493) (*a*) What is the chief use of the regulator? (*b*) To what is it equivalent? (*c*) What are the effects produced by splitting the air, and what advantages are obtained by splitting?

(494) The water-gauge at a mine is .7 inch. If the length of the airway is increased to three times its original

length, and the velocity is increased from 8 to 10 feet per second, what will the water-gauge read? Ans. 3.28 in.

(495) There are two splits in a mine, one 2,000 yards long and the other 5,000 yards long; each has a section 7 ft. × 6 ft. (*a*) With a water-gauge of $2\frac{1}{2}$ inches for underground friction, what quantity will pass through each split? (*b*) What horsepower is required for the two splits? (*c*) What area of regulator opening will be required to reduce the quantity in the short split one-half?

Ans. (*a*) 1st. 16,868 cu. ft. per min.; 2d. 10,668 cu. ft. per min.
(*b*) 10.85 H. P.
(*c*) 2.46 sq. ft.

(496) If 50,000 cubic feet of air pass in a circular airway 18 feet in diameter, what quantity will pass in an airway 6 feet in diameter, the power remaining the same?

Ans. 8,012.5 cu. ft. per min.

(497) (*a*) What are similar figures? (*b*) Define the coefficient of friction as used in mine ventilation. (*c*) Define power and work.

(498) Extract the fifth root (*a*) of 35, and (*b*) of 64,268,-937, by means of the rule given in Art. **1000.**

Ans. (*a*) 2.036.
(*b*) 36.44.

MINE VENTILATION.

(PART 2.)

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(499) By what two methods is air set in motion so as to produce a current?

(500) Mention the different means at hand for producing ventilation in mines.

(501) (*a*) What conditions are necessary to produce natural ventilation in a mine? (*b*) How does this form of ventilation differ from all others?

(502) Explain the use of the furnace. Where is it always placed, in the ventilation of a mine?

(503) The depth of a certain upcast shaft is 500 yards; the mean barometric pressure is 30.25 inches, and the mean temperature of the air in the shaft is 350° F. What is the weight of a column of air in this shaft, having a base of 1 square foot? Ans. 74.25 lb.

(504) What is the weight of a cubic foot of air when the barometric reading is 29.3 inches and the temperature is 32° F.? Ans. .0791 lb., nearly.

(505) If the anemometer records a velocity of 800 feet per minute in the intake airway of a mine, where the sectional area measures 8′ × 10′ and the thermometer shows a temperature of 32° F., what should be the volume of air passing in this same airway per minute, at a point where the temperature has risen to 60° F.? Ans. 67,650 cu. ft., nearly.

(506) The depths of the downcast and upcast shafts are each 540 feet. Their respective average temperatures are 0° F. and 300° F. Calculate the pressure per square foot

that produces ventilation in this mine when the average barometric pressure is 29.8 inches. Use two methods.

Ans. 13.365 lb.

(507) What is the height of motive column which produces ventilation, when the depths of the upcast and downcast shafts are each 200 yards, and their respective average temperatures are 60° F. and 360° F.? Find the motive column in the downcast shaft. Ans. 219.78 ft.

(508) Mention some of the essential points in the construction of a mine furnace.

(509) What will be the area of fire-grate of a furnace which is to supply 50,000 cubic feet of air per minute against a 2-inch water-gauge when the depth of the furnace shaft is 250 feet? Ans. 33.884 sq. ft.

(510) (*a*) Explain the object of a dumb drift. (*b*) What is the minimum height above the furnace of the point where a dumb drift may enter a shaft with safety?

(511) Describe the method of ventilating by means of a waterfall.

(512) (*a*) What do you understand by a mechanical ventilator? (*b*) Mention some examples of such ventilators.

(513) What are the most prominent types of centrifugal ventilators now in use?

(514) (*a*) In what two ways do centrifugal fans act? (*b*) What can you say of the relative efficiencies of these two modes of action?

(515) Describe the general characteristics of the Waddle fan.

(516) What are some of the chief points in the construction of the Schiele fan?

(517) What is the effect of the spiral casing surrounding the circumference of a ventilating fan?

(518) What is the purpose of the évasée chimney?

(519) Describe the general form of the Guibal fan.

(520) What can you say of the Capell fan?

(521) In what two respects is the fan a better means of ventilation than the furnace?

(522) Plainly stated, what is the difference between the action of the furnace and the action of the fan?

(523) What is the velocity of air blowing through an orifice into a vacuum under a pressure of 8.41 pounds per square foot? Ans. 52.2 ft. per sec.

(524) What is the velocity at which air should enter the fan to produce the best results?

(525) A fan is designed to produce a current of 175,000 cubic feet of air per minute. What should be the diameter of its central orifice if it receives its air upon each side?
Ans. 10.146 ft.

(526) (*a*) What do you understand by the throat of a fan? (*b*) What determines the area of the throat of a fan?

(527) What should be the width of blade in a fan designed to throw 250,000 cubic feet of air per minute? The fan receives its air upon each side. Ans. 6.06 ft.

(528) The area of the port of entry of a fan which receives its air upon one side is 153.9384 square feet. What should be the breadth of its blades? Ans. 3.5 ft.

(529) What do you understand by manometric efficiency as relating to fans?

(530) Name three essential elements to the efficient ventilation of a mine.

(531) How is the quantity of air that is necessary for the ventilation of a mine determined, and what amount is customary in non-gaseous mines? What amount is usually specified for a gaseous mine?

(532) In order to secure thorough ventilation in a mine, what is necessary with respect to the *velocity* of the air-current?

(533) (*a*) What danger arises in gaseous mines from too high a velocity? (*b*) What is the maximum velocity allowed under the Anthracite Mine Law of Pennsylvania?

(534) What means are necessary in order to properly conduct an air-current to the working face? Mention, also, the essential points necessary to be observed with respect to each of these means.

(535) What instruments are used for measuring the resistance of airways, and what does each measure, respectively?

(536) Describe the water-gauge and the manner of using it for determining the ventilating pressure in a mine.

(537) Describe the anemometer and the manner of using it, stating also what precautions are necessary in order to obtain an average velocity for the entire area of the airway.

(538) Upon what does the density of air mainly depend, and what instruments are used in determining the density (the weight of a cubic foot) of air?

(539) (*a*) Name the freezing and boiling points of the Centigrade and Fahrenheit scales, respectively. (*b*) How many degrees of the Fahrenheit scale correspond to 100° of the Centigrade?

(540) Convert (*a*) 350° C. into the corresponding Fahrenheit reading; (*b*) – 10° C.; (*c*) – 25° C.

Ans. { (*a*) 662° F.
(*b*) 14° F.
(*c*) – 13° F.

(541) What readings of the Centigrade scale correspond to the following: (*a*) 365° F.; (*b*) 5° F.; (*c*) – 49° F.?

Ans. { (*a*) 185° C.
(*b*) – 15° C.
(*c*) – 45° C.

(542) To what is the pressure *per square foot* of sectional area in an airway, due to any air column, always equal?

(543) Give examples of vertical and inclined air columns in the practical ventilation of mines, and state how the pressure per square foot at the base of the inclined column is calculated.

(544) Which class of workings is more easily ventilated, *rise* or *dip* workings, and why?

(545) What is meant by (*a*) positive air columns? (*b*) negative air columns?

(546) What do you understand by ascensional ventilation?

(547) To what is the algebraic sum of the weights of all the columns, positive and negative, equal?

(548) What is the main feature in the ventilation of a flat, non-gaseous seam?

(549) What is the practical limit to the splitting of air-currents?

(550) How does the presence or absence of gas affect the arrangement of the haulage roads with respect to the ventilating current?

(551) What is the main point to be considered in the ventilation of inclined seams?

(552) Mention some important points in connection with the entrance of a mine after an explosion.

(553) Name the different methods of treating mine fires.

(554) What precaution is necessary to be taken in building stoppings for the isolation of a mine fire? Explain the reason why such precaution is necessary.

MINE SURVEYING AND MAPPING.

(PART 1.)

(555) Define mine surveying.

(556) Explain the construction of the steel tape and of the chain.

(557) Describe the method of measuring up hill, and that of measuring down hill. Which is the more accurate, and why?

(558) How would a distance of 3 yd. 2 ft. $6\frac{3}{8}$ in. be recorded in mine surveying?

(559) What two methods could be used in measuring the horizontal distance between a point at the top of a slope having a uniform grade, and a point at the foot of the slope? Which method is the better in this case?

(560) What points must be borne in mind while measuring distances?

(561) Describe a surveyor's compass fully.

(562) What objection is there to using a compass for mine surveys?

(563) What adjustments are necessary for a compass, and how are they made?

(564) Give the order of the letters on the face of a compass, and the reason for such arrangement.

(565) State how the face of a compass is graduated.

(566) What is a vernier? Explain its principle.

(567) (*a*) What is a retrograde vernier? (*b*) What is a direct vernier?

(568) Of what advantage is a vernier on a compass?

(569) What is the reading of the vernier shown in Fig. 272 ?

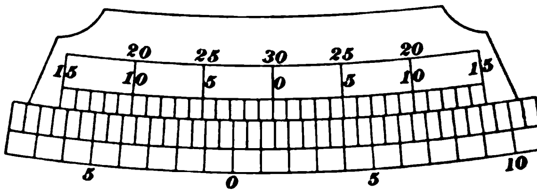

FIG. 272.

(570) What is the reading of the vernier shown in Fig. 273 ?

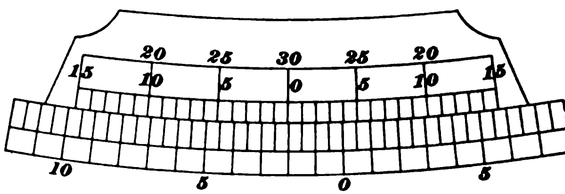

FIG. 273.

(571) What is the reading of the vernier shown in Fig. 274 ?

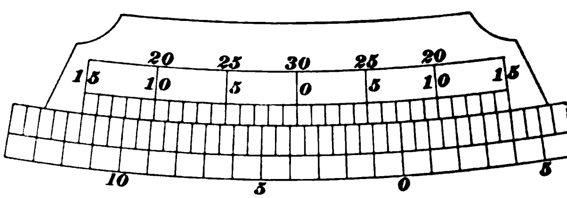

FIG. 274.

(572) What is the reading of the vernier shown in Fig. 275 ?

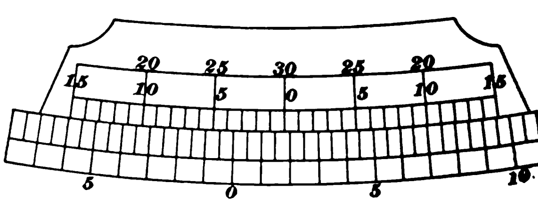

FIG. 275.

(573) Define (*a*) true azimuth; (*b*) true bearing; (*c*) magnetic bearing; (*d*) assumed meridian.

(574) Explain how a true meridian is determined.

(575) For what is the compass principally used in the mine?

(576) Explain how the station at which local attraction acts most strongly is discovered.

(577) The butt entry runs N 30° E, and the rooms turned off from it run N 20° W; what is the distance measured along the entry between the center lines of the rooms, if the perpendicular distance between these lines is 54 feet? Ans. 70.49 ft.

(578) Define (*a*) latitude; (*b*) departure; (*c*) northing; (*d*) southing; (*e*) easting; (*f*) westing.

(579) What is (*a*) total latitude? (*b*) total departure?

(580) Calculate and show by sketch the latitude and the departure of a course whose bearing is N 22° E, and whose length is 375 feet.

(581) Calculate the latitudes and the departures for the following courses, and show the method employed:

Stations.	Bearings.	Distances.
1–2	S 46° 30′ E	207.6 feet.
2–3	S 74° 30′ E	309.5 feet.
3–4	N 33° 15′ E	188.0 feet.
4–5	N 56° W	276.0 feet.
5–6	Due West	213.5 feet.
6–1	S 51° 54′ W	139.3 feet.

(582) Calculate the total latitudes and the total departures for the notes given in example 581, by taking Sta. 1 as the basis of calculations.

(583) From the total latitudes and the total departures

found in example 582, determine the bearing from Sta. 1 to Sta. 3. Ans. S 63° 19′ E.

(584) What special precaution must be taken in calculating total latitudes and total departures?

(585) Traverse and close the following survey:

Stations.	Bearings.	Distances.
1–2	Due East	130 feet
2–3	N 8° E	137 feet
3–4	N 81° W	186 feet
4–5	Due South	54 feet
5–6	S 36° W	125 feet
6–7	S 45° E	89 feet
7–1	N 40° 18′ E	69.86 feet

(586) The horizontal angles and distances of a survey are as follows:

1–2	N 37° 13′ E	413.6 feet.
2–3	N 10° 56′ E	246.7 feet.
3–4	S 17° 23′ E	253.0 feet.
4–5	S 43° 37′ E	216.0 feet.
5–6	S 33° 43′ W	789.0 feet.

How far north or south and east or west of Sta. 1 is Sta. 6? Calculate the bearing and the distance from Sta. 1 to Sta. 6.

(587) If the notes of an old survey give the bearing of a certain course as N 43° 15′ E, and the compass now gives a bearing of N 42° 45′ E for the same course, how should the compass be adjusted in order to accurately re-run the old lines?

(588) Of what advantage are traverse tables?

(589) Explain what is meant by platting to scale.

(590) By careful measurement, the distance between two points on a map drawn to a scale of 200 ft. = 1 in. is found

to be 4.378 inches. What is the actual distance between these points on the surface ?

(591) Two stations are 537.8 feet apart. What will be the distance between them on a map drawn to a scale of 150 ft. = 1 in. ?

(592) Explain the method of laying off angles (*a*) by chords; (*b*) by tangents; and (*c*) by bearings.

(593) Give method of calculating chords for laying off angles.

(594) What are the different methods of platting a survey ? Which is the best, and why ?

(595) What is meant by the closing line of a survey? Explain how its length and bearing are determined.

(596) What is meant by traversing a survey ?

(597) Describe a method of calculating the area of a survey.

(598) Find the area of the survey given in example 585.

(599) Find the area of the survey given in example 586, after calculating the closing line.

(600) Plat the following line by means of a protractor, using a scale of 200 ft. = 1 in., and a radius of 400 feet:

Stations.	Angles.
25 + 80	End of Line
20 + 38	L 37° 20′
15 + 18	L 31° 08′
9 + 13	R 39° 26′
3 + 60	R 30° 30′
0	

(601) Plat the following line by means of chords, using a scale of 200 ft. = 1 in., and a radius of 400 feet:

Stations.	Angles.
25 + 96	End of Line
21 + 46	R 34° 30′
17 + 09	R 53° 28′
11 + 96	L 25° 10′
5 + 33	R 21° 10′
0	

(602) Plat the following line by means of tangents, using a scale of 200 ft. = 1 in., and a radius of 400 feet:

Stations.	Bearings.
25 + 34	End of Line
19 + 94	N 22° 59′ W
14 + 81	N 28° 23′ E
10 + 38	N 7° 03′ E
4 + 13	N 32° 15′ W
0	N 32° 15′ E

MINE SURVEYING AND MAPPING.

(PART 2.)

(603) What is a transit?

(604) State the advantage which the transit has over the vernier compass.

(605) What kind of a vernier is used on the transit?

(606) How many adjustments of the transit are necessary? Explain each in detail.

(607) If the zero mark on the vernier is between 70° 30′ and 71°, and the first line on the vernier that exactly coincides with a line on the limb, or plate, of the transit is marked 21, what is the reading?

(608) If the zero mark on the vernier is between 263° and 263° 30′, and the first vernier line that coincides with a line on the limb, or plate, is 13, what is the reading?

(609) What are the two readings shown in Fig. 276?

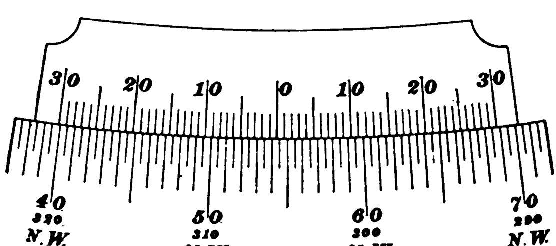

FIG. 276.

(610) What are the two readings shown in Fig. 277?

(611) What is (*a*) the horizontal limb of the transit? (*b*) the axis of the instrument? (*c*) the line of collimation? (*d*) What are the standards?

(612) Define horizontal angle, and describe fully the method of measuring such an angle; i. e., explain how to set up the instrument over the point; the position of the vernier to start with; what clamps are used, how they are used, when they are used, and for what they are used. Explain how the objects are sighted, and how perfect bisection with the cross-hairs is effected.

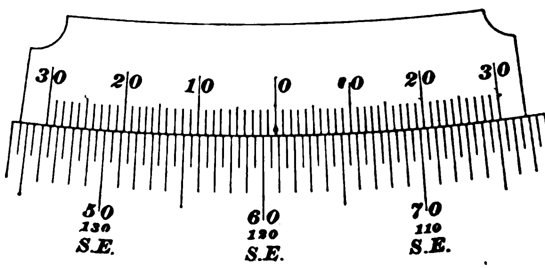

FIG. 277.

(613) Explain the method of finding with the transit the azimuths of several successive courses with a given first course.

(614) What is the direction of a course whose azimuth lies (*a*) between 0° and 90°; (*b*) exactly 90°; (*c*) between 90° and 180°; (*d*) exactly 180°; (*e*) between 180° and 270°; (*f*) exactly 270°; (*g*) between 270° and 360°; (*h*) exactly 360°, or 0°?

(615) Calculate the bearings of the following courses referred to *A B* as a meridian:

Stations.	Azimuths with *A B*.
A	0° 00′
B	35° 30′
C	110° 30′
D	270° 00′
E	330° 45′

(616) How may the magnetic bearings of several suc-

cessive courses be found from their azimuths when only the magnetic bearing of the given first course is known?

(617) What is meant by the meridian of the survey?

(618) Define the deflection angle of a curve.

(619) In making a map of final location, how should the measurements be made, the angles platted, and the curve centers determined?

(620) Explain the method of measuring to the end of a pillar which is rounded off.

(621) Why are the magnetic readings taken in addition to the azimuths?

(622) What is (*a*) a 1° curve? (*b*) a 5° curve?

(623) Give several methods of carrying a survey into a mine, and explain fully the one in which one shaft and four plumb-lines are used.

(624) Give the different kinds of stations used in mine surveying.

(625) What danger is there in putting a station on a timber used to support the roof?

(626) Explain two methods of prolonging a line intercepted by an obstacle.

(627) What are the straight portions of a railroad technically called?

(628) What determines the degree of a curve?

(629) Explain what is meant by the tangent deflection and the chord deflection of a curve.

(630) What device is necessary to take a sight vertically upwards with the transit?

(631) Show by sketch the method of laying out curves with the transit.

(632) The intersection angle of a 5° curve is 24° 30′. What is the length of the curve? Ans. 490 ft.

(633) How are the radii of curves of from 1° to 10° determined in practice?

(634) What is the tangent distance for a 7° curve, if the intersection angle I is 36° ? Ans. 266.12 ft.

(635) Give a form for keeping transit notes.

(636) What is (*a*) a simple curve ? (*b*) a compound curve ? (*c*) a reverse curve ?

(637) What are side notes ? Give a sketch to illustrate them.

(638) Explain fully the method of directing the miners who are driving a curved entry, so that they will keep the entry on the proper curvature.

(639) With regard to vertical angles, how are the rise angles distinguished from the dip angles in the notes ?

(640) What is the chord deflection for a chord of 9 feet on a curve having a radius of 45 feet ? Ans. 1.8 ft.

(641) The degree of a curve is 6° 30′; what is the deflection angle for a chord of 48 feet ? Ans. 1° 33′ 36″.

(642) How are sharp curves designated in mine work ?

(643) Define (*a*) the P. I., (*b*) the P. C., and (*c*) the P. T. of a curve.

(644) Define (*a*) sub-station; (*b*) sub-chord.

(645) If, in laying out a 10° curve with the transit, an obstacle prevents turning off a deflection angle greater than 25°, explain fully how points farther on will be located upon the curve.

(646) What are the chord and the tangent deflection for a chord of 120 feet on a 9° curve ?

Ans. { Chord def. = 22.6 ft.
Tangent def. = 11.3 ft. }

(647) How are the stations marked in a mine where several corps are doing work ?

(648) State the disadvantages in surveying pitching work.

(649) How should stations be numbered in a mine ?

(650) What are centers, and how are they used ?

(651) Give a detailed description of a mine corps.

(652) Given the following field notes:

Stations.	Vertical Angles. In Tunnel.	Vertical Angles. On Surface.	Bearings.	Horizontal Distances.	Remarks.
1–			S 36° 50′ W	19.1 feet	To mouth of tunnel
1–2	+ 1° 18′	+ 10° 35′	S 36° 50′ W	99.1 feet	
2–3	+ 0° 31′	+ 15° 43′	S 49° 47′ W	104.2 feet	
3–4	+ 0° 45′	+ 14° 27′	S 40° 00′ W	37.1 feet	
4–5	– 0° 34′	+ 16° 17′	S 4° 55′ E	56.5 feet	
5–6	+ 3° 37′	+ 12° 21′	S 71° 15′ E	46.0 feet	
6–face	+ 3° 30′	+ 13° 56′	S 77° 30′ E	40.7 feet	

(*a*) Calculate the horizontal distance and direction of a point on the surface over the face of tunnel to Sta. 1. (*b*) Find the depth of the bore-hole from this point to the face of the tunnel. (*c*) Calculate the vertical angle from Sta. 1 to this point on the surface over the face.

Ans. { (*a*) 265.63 ft., N 16° 20′ W.
(*b*) 85.55 ft.
(*c*) 19° 30′.

MINE SURVEYING AND MAPPING.

(PART 3.)

(655) Define leveling.

(656) For what purposes is leveling used in mining work?

(657) State and explain the best method of keeping notes in leveling.

(658) Define contours, and state what determines their distance apart.

(659) Explain the necessary steps in determining with the level the difference in elevation of two points.

(660) What is a profile, and what is its principal use?

(661) Describe the engineer's level.

(662) Define (*a*) turning-point; (*b*) backsight; (*c*) foresight; and (*d*) a plus.

(663) What should be considered in assuming the elevation of a bench-mark or the starting-point in leveling operations?

(664) What determines the sensitiveness of a level?

(665) Why is the backsight column marked + and the foresight column marked − in the level notes?

(666) What is a datum line?

(667) Give the different adjustments of the level and explain each fully.

(668) Describe a method of calculating the cubical contents of a cut or fill in an entry.

(669) What is the object of making a topographical survey?

(670) Define plane of reference.

(671) How are level notes checked?

(672) How are the readings taken on the roof in the mine designated in the notes, and why so marked?

(673) Describe fully the method of determining the contours of a single hill.

(674) By what means can the operations of leveling be checked?

(675) Describe the Philadelphia rod; state the use of both verniers.

(676) Explain the construction of the telescope of a level.

(677) How are the stations numbered in leveling operations?

(678) Of what advantage is a general map showing the workings of the mine, the property lines, and the surface arrangements?

(679) Explain how the notes for locating on the map the buildings, railroads, etc., of the surface arrangements are taken.

(680) What advantage is there in placing the crop line on the mine map?

(681) What is done in case a mine must be surveyed immediately, and it is not the proper time to determine a true meridian; i. e., the North star is not at its northern elongation?

(682) In what case is it necessary to determine a considerable number of contour lines while making a survey for a mining plant?

(683) State the principle by which a profile is made to exaggerate the irregularities of the surface.

(684) Why should the engineer re-run the property lines before making a general map?

(685) If a level tunnel *A G*, Fig. 280, driven through the rock at right angles to the strike of a seam, is 60 yards long to the top slate of an underlying seam, and both seams

have an inclination of 43°, what is the thickness of the rock between the seams, measured at right angles to the pitch? Ans. 122.76 ft.

(686) If, in Fig. 280, a perpendicular shaft *A C* is sunk from the haulage road in the overlying seam to the bottom

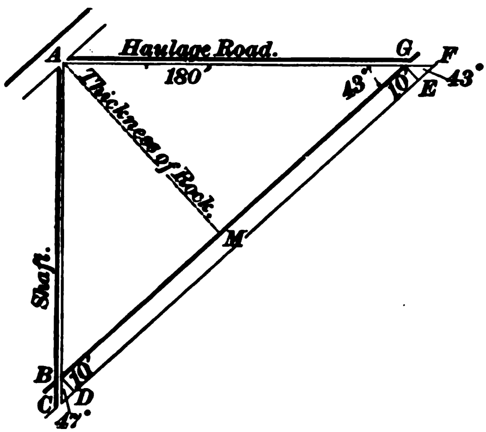

FIG. 280.

slate of the underlying seam, which is 10 feet thick, (*a*) how deep will the shaft be? (*b*) What will be the distance, on the pitch, from the foot *C* of the shaft to the level *F* of the haulage road in the underlying seam? Ans. { (*a*) 181.52 ft. (*b*) 266.16 ft.

(687) If the pitch of a seam, as shown in Fig. 281, is 30°, the line of strike due east and west, and the rooms are driven N 30° E, what will be the degree of pitch on which the rooms are driven? Ans. 26° 34', nearly.

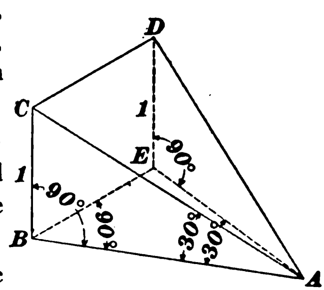

FIG. 281.

(688) What are the essential features which govern the shape and extent of a mining claim?

(689) What is the difference between a survey made in a coal-mine and one made in a metalliferous mine?

(690) How many drawings are necessary to correctly represent the workings of a metalliferous mine ?

(691) What does the surface plan show ?

(692) What does the working plan show ?

(693) What does a longitudinal section show ?

(694) What does a transverse section show ?

(695) How is the longitudinal section made when the lode is flat ?

(696) Describe fully (to the end of point 4) the various steps in running a section level of which the following are the notes:

Stations.	B. S. +	Ht. of Inst.	F. S. −	Elevations.	Remarks.
B. M.	4.576			91.397	
1	5.420		3.726		
2			4.500		
3	4.910		3.170		
3 + 40			4.900		
4	3.380		6.386		
5			4.600		
6	2.760		5.400		
6 + 70			3.100		
7			3.800		
8			6.925		

(697) Fill up the notes given in example 696, showing the height of instrument, and the elevation of every station. Indicate which stations are T. P's, and insert a column for intermediate sights. The stations are 100 feet apart, and here and there are intermediate stations, as at 340 feet. Give the proof to show the correctness of your work.

(698) Draw a profile from the notes given in example 696, using a scale of 10 feet to the inch for heights, and a scale of 100 feet to the inch for horizontal distances.

(699) Explain fully the method of determining and representing on paper the contours of any irregular piece of ground.

(700) What is the best method of platting the notes for a mine map?

(701) What are the different scales used for profile work?

(702) How is the location of the point from which the platting is started usually determined?

(703) In making a profile, what advantage is there in placing a plan of the work and the profile on the same paper? State what should be their proper relative positions, and the scale to which the plan should be drawn.

(704) Traverse the following transit notes, getting the latitude and the departure of each course, and the total latitude and the total departure of each course with reference to Sta. 302. Then, from the traversed notes and the side notes given in Figs. 282, 283, and 284, make a map of that portion of the mine from which the notes were taken. Be careful to reduce the pitch distances in the transit and the side notes to the required horizontal distances before beginning to traverse.

TRANSIT NOTES.

Stations.	Bearings.	Vertical Angles.	Distances.
302–303	N 85° 57′ E	....	242.35
303–304	N 89° 33′ E	....	47.47
304–305	S 87° 45′ E	....	77.06
305–306	N 77° 51′ E	....	104.69
306–307	N 84° 02′ E	....	220.76
307–308	N 88° 21′ E	....	206.18
308–309	N 80° 36′ E	....	113.44
308–310	S 89° 35′ W	....	46.87
310–311	S 8° 55′ E	+ 17° 45′	31.67
311–312	S 78° 14′ W	+ 2° 33′	52.83
311–313	S 2° 12′ W	+ 16° 04′	167.82
313–face	S 6° 20′ E	+ 16° 15′	156.00
312–314	N 86° 48′ W	− 4° 45′	57.60
312–315	S 6° 33′ E	+ 15° 58′	132.15
315–316	S 6° 38′ W	+ 16° 12′	119.98
316–*x*	S 14° 55′ E	+ 16° 30′	106.00
314–317	S 81° 47′ W	− 0° 23′	63.86
314–318	S 4° 30′ E	+ 17° 23′	210.05
318–face	S 3° 20′ E	+ 17° 30′	124.00
317–319	N 88° 27′ W	− 4° 33′	58.26
317–face	S 2° 27′ E	+ 16° 45′	300.00
319–320	S 75° 39′ W	+ 0° 53′	55.53
319–321	S 10° 51′ E	+ 16° 35′	143.74
321–face	S 2° 47′ W	+ 16° 40′	171.00
320–322	S 82° 05′ W	....	54.78
320–323	S 8° 14′ E	+ 16° 54′	62.96
323–324	S 6° 12′ W	+ 17° 07′	136.90
324–face	S 8° 40′ E	+ 17° 00′	102.00
322–325	S 78° 07′ W	+ 2° 38′	52.72
322–326	S 4° 08′ W	+ 17° 14′	122.94
326–face	S 9° 51′ E	+ 17° 15′	176.00
325–327	S 1° 24′ W	+ 17° 18′	75.49
327–328	N 88° 43′ W	− 1° 37′	40.98
327–face	S 6° 48′ E	+ 20° 04′	202.00
328–*x*	N 7° 03′ W	− 19° 45′	86.00
328–329	S 3° 21′ E	+ 19° 48′	77.75
329–*x*	S 4° 30′ E	+ 19° 50′	204.00
329–330	S 81° 42′ W	+ 1° 30′	62.63
330–*x*	N 3° 48′ W	− 20° 03′	166.00
330–face	S 4° 40′ E	+ 20° 10′	156.00
330–331	S 17° 50′ W	+ 15° 28′	29.45
331–332	N 87° 20′ W	− 2° 28′	33.55
332–*x*	N 6° 22′ W	− 20° 22′	196.00
332–face	S 1° 35′ W	+ 20° 28′	130.00
332–333	N 58° 15′ W	− 15° 05′	25.39
333–334	S 79° 31′ W	+ 1° 51′	40.12
334–*x*	S 2° 53′ E	+ 20° 30′	162.00
334–335	N 4° 53′ W	− 21° 14′	83.45
335–*x*	N 2° 05′ E	− 20° 15′	108.00
335–336	S 84° 15′ W	− 3° 05′	51.88
336–face	S 6° 30′ E	+ 23° 26′	250.00
336–337	N 4° 17′ W	− 23° 23′	133.48
337–303	N 80° 34′ E	....	50.50
303–302	S 85° 57′ W	....	....

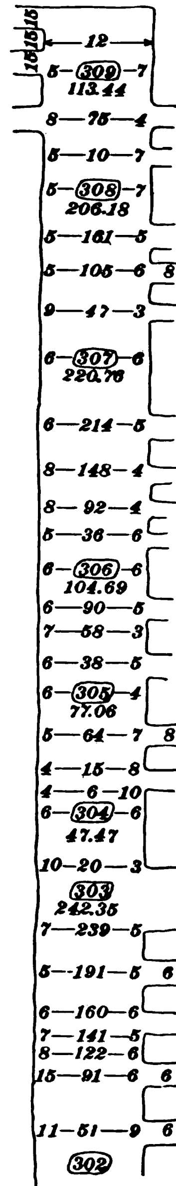

FIG. 282.

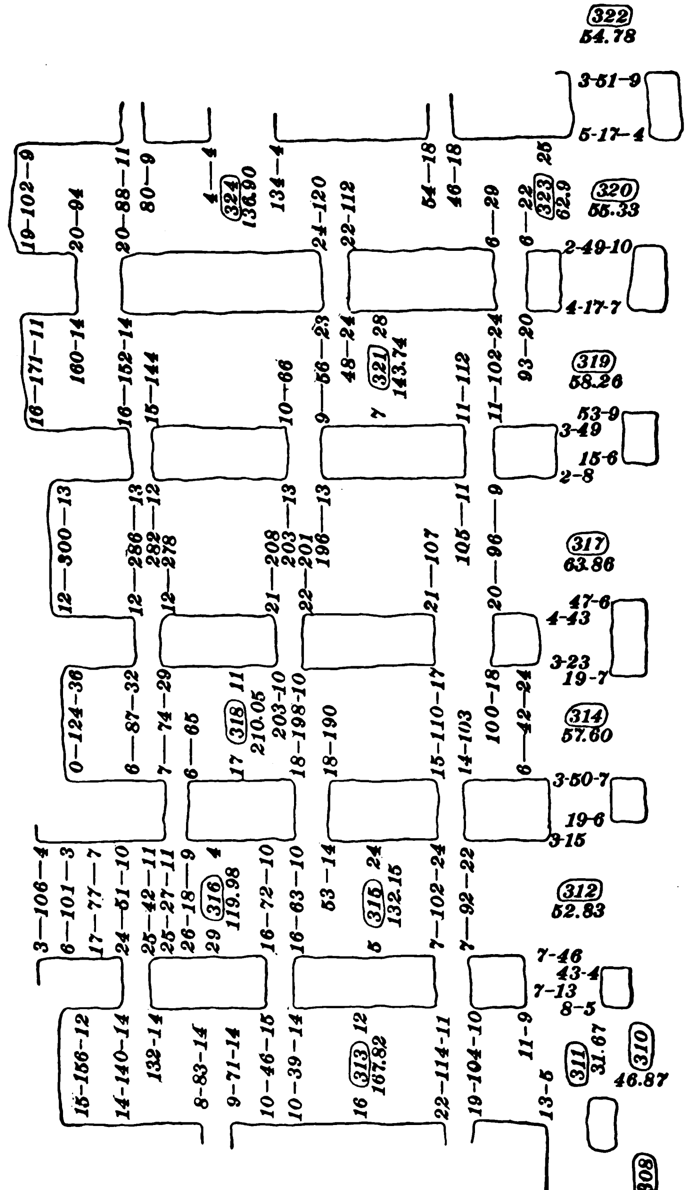
322
54.78
3-51-9
5-17-4
320
55.33
2-49-10
4-17-7
319
58.26
53-9
3-49
15-6
2-8
317
63.86
47-6
4-43
3-23
19-7
314
57.60
3-50-7
19-6
3-15
312
52.83
7-46
43-4
7-13
8-5
46.87

FIG. 283.

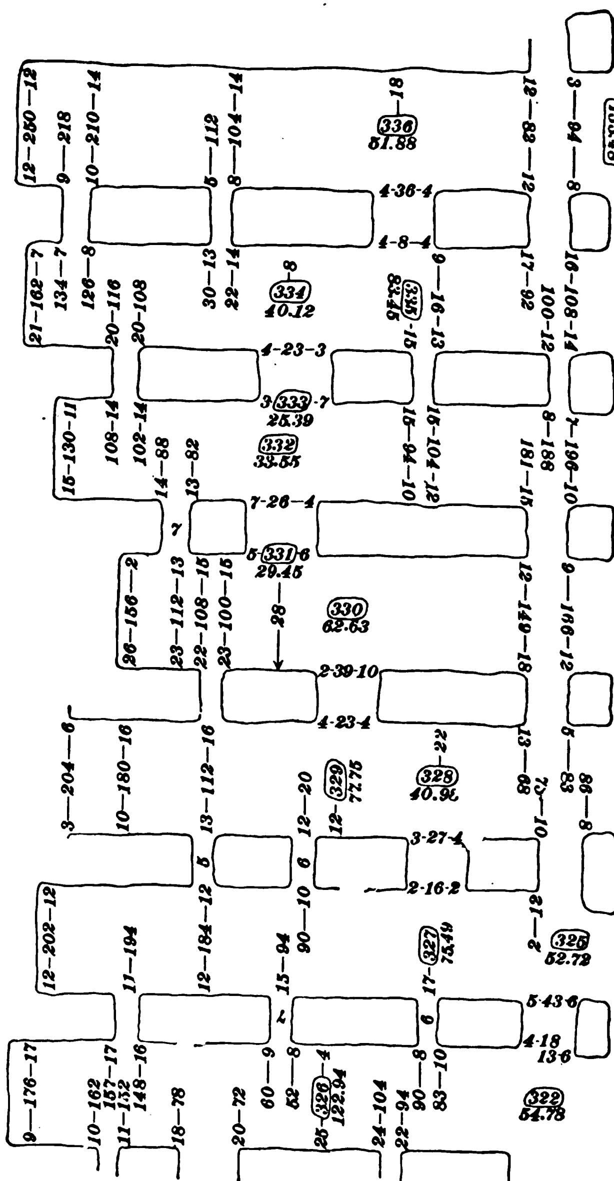
337
133.48
336
51.88
334
40.12
333
25.39
332
33.55
331
29.45
330
62.63
329
72.75
328
327
75.49
325
52.72
326
122.94
322
54.78

FIG. 284.