A TREATISE

ON

COAL MINING

PREPARED FOR STUDENTS OF

THE INTERNATIONAL CORRESPONDENCE SCHOOLS

SCRANTON, PA.

Volume VI

ANSWERS TO QUESTIONS

Second Edition

SCRANTON

THE COLLIERY ENGINEER CO.

1900

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BURR PRINTING HOUSE,
FRANKFORT AND JACOB STREETS,
NEW YORK.

A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

CONTAINED IN THE

Question Papers Included in Vols. I–III.

The various Keys composing this volume have been given the same section numbers as the Question Papers to which they refer; and the answers and solutions have been numbered to correspond with the questions contained in the Question Papers. In many instances the answer to a question would involve a repetition of statements given in the Instruction Papers; hence, in all such cases, the student has been referred to an article in the Instruction Paper, the reading of which will enable him to answer the question himself.

CONTENTS.

ARITHMETIC.

(PART 1.)

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.

605 = Six hundred five.

28,284 = Twenty-eight thousand, two hundred eighty-four.

9,006,042 = Nine million, six thousand and forty-two.

850,317,002 = Eight hundred fifty million, three hundred seventeen thousand and two.

700,004 = Seven hundred thousand and four.

(6) Seven thousand six hundred = 7,600.

Eighty-one thousand four hundred two = 81,402.

Five million, four thousand and seven = 5,004,007.

One hundred and eight million, ten thousand and one = 108,010,001.

Eighteen million and six = 18,000,006.

Thirty thousand and ten = 30,010.

(7) In adding whole numbers, place the numbers to be added directly under each other so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place 9 units under the units column, and reserve 1 ten for the column

$$\begin{array}{r} 3290 \\ 504 \\ 865403 \\ 2074 \\ 81 \\ 7 \\ \hline 871359 \end{array} \quad \text{Ans.}$$

of tens. $1 + 8 + 7 + 9 = 25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column, and reserve 2 hundreds for the hundreds column. $2 + 4 + 5 + 2 = 13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column, and reserve the 1 thousand for the thousands column. $1 + 2 + 5 + 3 = 11$ thousands, or 1 ten-thousand and 1 thousand. Place the 1 thousand in the column of thousands, and reserve the 1 ten-thousand for the column of ten-thousands. $1 + 6 = 7$ ten-thousands. Place this seven ten-thousands in the ten-thousands column. There is but one figure 8 in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

A simpler (though less scientific) explanation of the same problem is the following: $7 + 1 + 4 + 3 + 4 + 0 = 19$; write the nine and reserve the 1. $1 + 8 + 7 + 0 + 0 + 9 = 25$; write the 5 and reserve the 2. $2 + 0 + 4 + 5 + 2 = 13$; write the 3 and reserve the 1. $1 + 2 + 5 + 3 = 11$; write the 1 and reserve 1. $1 + 6 = 7$; write the 7. Bring down the 8 to its place in the sum.

(8)

$$
\begin{array}{r}
709 \\
8304725 \\
391 \\
100302 \\
300 \\
909 \\
\hline
8407336
\end{array}
\quad \text{Ans.}
$$

(9) (*a*) In subtracting whole numbers, place the subtrahend or smaller number under the minuend or larger number, so that the right-hand figures stand directly under each other. Begin *at the right* to subtract. We can not subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. As 1 *ten* = 10 *units*, we have 10 units + 2 units = 12 units. Then, 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so only 5 tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We can not subtract 3 thousands from 0 thousands, so we take 1 ten-thousand from 5 ten-thousands and add it to the 0 thousands. 1 *ten-thousand* = 10 *thousands*, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten-thousand from 5 ten-thousands and have 4 ten-thousands remaining. Since there are no ten-thousands in the subtrahend, the 4 in the ten-thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

$$\begin{array}{r} 50962 \\ 3338 \\ \hline 47624 \end{array} \text{ Ans.}$$

(*b*)
$$\begin{array}{r} 15339 \\ 10001 \\ \hline 5338 \end{array} \text{ Ans.}$$

(10) (*a*)
$$\begin{array}{r} 70968 \\ 32975 \\ \hline 37993 \end{array} \text{ Ans.}$$

(*b*)
$$\begin{array}{r} 100000 \\ 98735 \\ \hline 1265 \end{array} \text{ Ans.}$$

(11) We have given the minuend or greater number (1,004) and the difference or remainder (49). Placing these in the usual form of subtraction we have $\begin{array}{r} 1004 \\ \text{——} \\ \hline 49 \end{array}$ in which the dash (——) represents the number sought. This number is evidently *less* than 1,004 by the difference 49, hence, 1,004 − 49 = 955, the smaller number. For the sum of the two numbers we then have

$$\begin{array}{rl} 1004 & \textit{larger} \\ 955 & \textit{smaller} \\ \hline 1959 & \textit{sum.} \end{array} \text{ Ans.}$$

Or, this problem may be solved as follows: If the greater of two numbers is 1,004, and the difference between them is 49, then it is evident that the smaller number must be equal to the difference between the greater number (1,004) and the difference (49); or, 1,004 − 49 = 955, the smaller number. Since the greater number equals 1,004 and the smaller number equals 955, their sum equals 1,004 + 955 = 1,959 sum. Ans.

(12) The numbers connected by the plus (+) sign must first be added. Performing these operations we have

$$\begin{array}{r} 5962 \\ 8471 \\ 9023 \\ \hline 23456 \end{array} \textit{ sum.} \qquad \begin{array}{r} 3874 \\ 2039 \\ \hline 5913 \end{array} \textit{ sum.}$$

Subtracting the smaller number (5,913) from the greater (23,456) we have

$$\begin{array}{r} 23456 \\ 5913 \\ \hline 17543 \end{array} \textit{ difference.} \quad \text{Ans.}$$

(13) \$44675 = amount willed to his son.
26380 = amount willed to his daughter.
\$71055 = amount willed to his two children.
\$125000 = amount willed to his wife and two children.
71055 = amount willed to his two children.
\$53945 = amount willed to his wife. Ans.

(14) In the multiplication of whole numbers, place the multiplier under the multiplicand, and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

(*a*)

$$\begin{array}{r} 526387 \\ 7 \\ \hline 3684709 \end{array} \quad \text{Ans.}$$

7 × 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 tens + 4 tens reserved = 60 tens or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7 × 3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve

the 2 thousands. 7×6 thousands $= 42$ thousands; $42 + 2$ thousands reserved $= 44$ thousands or 4 ten-thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten-thousands. 7×2 ten-thousands $= 14$ ten-thousands; $14 + 4$ ten-thousands reserved $= 18$ ten-thousands, or 1 hundred-thousand and 8 ten-thousands. Write the 8 ten-thousands and reserve the 1 hundred-thousand. 7×5 hundred-thousands $= 35$ hundred-thousands; $35 + 1$ hundred-thousand reserved $= 36$ hundred-thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred-thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times $7 = 49$; write the 9 and reserve the 4. 7 times $8 = 56$; $56 + 4$ reserved $= 60$; write the 0 and reserve the 6. 7 times $3 = 21$; $21 + 6$ reserved $= 27$; write the 7 and reserve the 2. $7 \times 6 = 42$; $42 + 2$ reserved $= 44$; write the 4 and reserve 4. $7 \times 2 = 14$; $14 + 4$ reserved $= 18$; write the 8 and reserve the 1. $7 \times 5 = 35$; $35 + 1$ reserved $= 36$; write the 36.

In this case the multiplier is 17 *units*, or 1 *ten* and 7 *units*, so that the product is obtained by adding two partial products, namely, $7 \times 700{,}298$ and $10 \times 700{,}298$. The actual operation is performed as follows:

$$
\begin{array}{rl}
(b) \quad 700298 & \\
17 & \\
\hline
4902086 & \\
700298 & \\
\hline
11905066 & \text{Ans.}
\end{array}
$$

7 times $8 = 56$; write the 6 and reserve the 5. 7 times $9 = 63$; $63 + 5$ reserved $= 68$; write the 8 and reserve the 6. 7 times $2 = 14$; $14 + 6$ reserved $= 20$; write the 0 and reserve the 2. 7 times $0 = 0$; $0 + 2$ reserved $= 2$; write the 2. 7 times $0 = 0$; $0 + 0$ reserved $= 0$; write the 0. 7 times $7 = 49$; $49 + 0$ reserved $= 49$; write the 49.

To multiply by the 1 ten we say 1 times $700298 = 700298$, and write 700298 under the first partial product, as shown, with the right-hand figure 8 under the multiplier 1. Add the two partial products; their sum equals the entire product.

(*c*)
```
    217
    103
    ---
    651
   2170
  -----
  22351
     67
 ------
 156457
 134106
-------
1497517  Ans.
```
Multiply any two of the numbers together and multiply their product by the third number.

(15) If your watch ticks every second, then to find how many times it ticks in one week it is necessary to find the number of seconds in 1 week.

```
    60 seconds = 1 minute.
    60 minutes = 1 hour.
  ----
  3600 seconds = 1 hour.
    24 hours = 1 day.
 -----
 14400
 7200
 -----
 86400 seconds = 1 day.
     7 days = 1 week.
------
604800 seconds in 1 week or the number of times that
  Ans.       your watch ticks in 1 week.
```

(16) If a monthly publication contains 24 pages, a yearly volume will contain 12×24 or 288 pages, since there are 12 months in one year; and eight yearly volumes will contain 8×288, or 2,304 pages.

```
  24
  12
 ---
 288
   8
----
2304  Ans.
```

(17) If an engine and boiler are worth $3,246, and the building is worth 3 times as much, plus $1,200, then the building is worth

```
      $3246
          3
      -----
       9738
plus   1200
     ------
     $10938 = value of building.
```

If the tools are worth twice as much as the building, plus \$1,875, then the tools are worth

$$\begin{array}{rr} & \$10938 \\ & 2 \\ \hline & 21876 \\ \text{plus} & 1875 \\ \hline & \$23751 \end{array} = \text{value of tools.}$$

$$\begin{array}{lr} \text{Value of building} = & \$10938 \\ \text{Value of tools} = & 23751 \\ \hline & \$34689 \end{array} = \text{value of the building and tools. (a) Ans.}$$

$$\begin{array}{lr} \text{Value of engine and boiler} = & \$\ \ 3246 \\ \text{Value of building and tools} = & 34689 \\ \hline & \$37935 \end{array} = \text{value of the whole plant. (b) Ans.}$$

(18) (*a*) $(72 \times 48 \times 28 \times 5) \div (96 \times 15 \times 7 \times 6)$.

Placing the numerator over the denominator the problem becomes

$$\frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = ?$$

The 5 in the *dividend* and 15 in the *divisor* are both *divisible* by 5, since 5 divided by 5 equals 1, and 15 divided by 5 equals 3. *Cross off* the 5 and write the 1 *over* it; also *cross off* the 15 and write the 3 *under* it. Thus,

$$\frac{72 \times 48 \times 28 \times \overset{1}{\cancel{5}}}{96 \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

The 5 and 15 are *not* to be considered any longer, and, in fact, may be erased entirely and the 1 and 3 placed in their stead, and treated as if the 5 and 15 *never* existed. Thus,

$$\frac{72 \times 48 \times 28 \times 1}{96 \times 3 \times 7 \times 6} =$$

72 in the *dividend* and 96 in the *divisor* are *divisible* by 12, since 72 divided by 12 equals 6, and 96 divided by 12 equals 8. *Cross off* the 72 and write the 6 *over* it; also, *cross off* the 96 and write the 8 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times 28 \times 1}{\underset{8}{\cancel{96}} \times 3 \times 7 \times 6} =$$

The 72 and 96 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 6 and 8 placed in their stead, and treated as if the 72 and 96 *never* existed. Thus,

$$\frac{6 \times 48 \times 28 \times 1}{8 \times 3 \times 7 \times 6} =$$

Again, 28 in the *dividend* and 7 in the *divisor* are *divisible* by 7, since 28 divided by 7 equals 4, and 7 divided by 7 equals 1. *Cross off* the 28 and write the 4 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{6 \times 48 \times \overset{4}{\cancel{28}} \times 1}{8 \times 3 \times \underset{1}{\cancel{7}} \times 6} =$$

The 28 and 7 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 4 and 1 placed in their stead, and treated as if the 28 and 7 *never* existed. Thus,

$$\frac{6 \times 48 \times 4 \times 1}{8 \times 3 \times 1 \times 6} =$$

Again, 48 in the *dividend* and 6 in the *divisor* are *divisible* by 6, since 48 divided by 6 equals 8, and 6 divided by 6 equals 1. *Cross off* the 48 and write the 8 *over* it; also, cross off the 6 and write the 1 *under* it. Thus,

$$\frac{6 \times \overset{8}{\cancel{48}} \times 4 \times 1}{8 \times 3 \times 1 \times \underset{1}{\cancel{6}}} =$$

The 48 and 6 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 8 and 1 placed in their stead, and treated as if the 48 and 6 *never* existed. Thus,

$$\frac{6 \times 8 \times 4 \times 1}{8 \times 3 \times 1 \times 1} =$$

Again, 6 in the *dividend* and 3 in the *divisor* are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and write the 2 *over* it; also, cross off the 3 and write the 1 *under* it. Thus,

$$\frac{\overset{2}{\cancel{6}} \times 8 \times 4 \times 1}{8 \times \underset{1}{\cancel{3}} \times 1 \times 1} =$$

The 6 and 3 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 2 and 1 placed in their stead, and treated as if the 6 and 3 *never* existed. Thus,

$$\frac{2 \times 8 \times 4 \times 1}{8 \times 1 \times 1 \times 1} =$$

Canceling the 8 in the dividend and the 8 in the divisor, the result is

$$\frac{2 \times \overset{1}{\cancel{8}} \times 4 \times 1}{\underset{1}{\cancel{8}} \times 1 \times 1 \times 1} = \frac{2 \times 1 \times 4 \times 1}{1 \times 1 \times 1 \times 1}.$$

Since there are *no two remaining numbers* (one in the dividend and one in the divisor) *divisible* by *any number* except 1, without a remainder, it is *impossible* to cancel further.

Multiply all the *uncanceled numbers* in the *dividend* together, and divide their *product* by the *product* of all the *uncanceled* numbers in the divisor. The *result* will be the *quotient*. The *product* of all the *uncanceled numbers* in the *dividend* equals $2 \times 1 \times 4 \times 1 = 8$; the product of all the *uncanceled* numbers in the *divisor* equals $1 \times 1 \times 1 \times 1 = 1$.

Hence, $\frac{2 \times 1 \times 4 \times 1}{1 \times 1 \times 1 \times 1} = \frac{8}{1} = 8$. Ans.

Or,

$$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{72}} \times \overset{\cancel{8}}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{\underset{1}{\cancel{8}}}{\cancel{96}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{6}}} = \frac{8}{1} = 8. \quad \text{Ans.}$$

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

The 50 in the *dividend* and 70 in the *divisor* are both *divisible* by 10, since 50 divided by 10 equals 5, and 70 divided by 10 equals 7. *Cross off* the 50 and write the 5 *over* it; also, *cross off* the 70 and write the 7 *under* it. Thus,

$$\frac{80 \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times 20} =$$

The 50 and 70 are not to be considered any longer, and, in fact, may be erased entirely and the 5 and 7 placed in their stead, and treated as if the 50 and 70 *never* existed. Thus,

$$\frac{80 \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times 20} =$$

Also, 80 in the *dividend* and 20 in the *divisor* are *divisible* by 20, since 80 divided by 20 equals 4, and 20 divided by 20 equals 1. Cross off the 80 and write the 4 *over* it; also, cross off the 20 and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times \underset{1}{\cancel{20}}} =$$

The 80 and 20 are *not* to be considered any longer, and, in fact, may be erased entirely and the 4 and 1 placed in their stead, and treated as if the 80 and 20 *never* existed. Thus,

$$\frac{4 \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times 1} =$$

Again, 16 in the *dividend* and 24 in the *divisor* are *divisible* by 8, since 16 divided by 8 equals 2, and 24 divided by 8 equals 3. *Cross off* the 16 and write the 2 *over* it; also cross off the 24 and write the 3 *under* it. Thus,

$$\frac{4 \times 60 \times 5 \times \overset{2}{\cancel{16}} \times 14}{7 \times 50 \times \underset{3}{\cancel{24}} \times 1} =$$

The 16 and 24 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 3 placed in their stead, and treated as if the 16 and 24 *never* existed. Thus,

$$\frac{4 \times 60 \times 5 \times 2 \times 14}{7 \times 50 \times 3 \times 1} =$$

Again, 60 in the *dividend* and 50 in the *divisor* are *divisible* by 10, since 60 divided by 10 equals 6, and 50 divided by 10 equals 5. *Cross off* the 60 and write the 6 *over* it; also, cross off the 50 and write the 5 *under* it. Thus,

$$\frac{4 \times \overset{6}{\cancel{60}} \times 5 \times 2 \times 14}{7 \times \underset{5}{\cancel{50}} \times 3 \times 1} =$$

The 60 and 50 are not to be considered any longer, and, in fact, may be erased entirely and the 6 and 5 placed in their stead, and treated as if the 60 and 50 *never* existed. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times 14}{7 \times 5 \times 3 \times 1} =$$

The 14 in the *dividend* and 7 in the *divisor* are *divisible* by 7, since 14 divided by 7 equals 2, and 7 divided by 7 equals 1. *Cross off* the 14 and write the 2 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}} \times 5 \times 3 \times 1} =$$

The 14 and 7 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 1 placed in their stead, and treated as if the 14 and 7 *never* existed. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times 2}{1 \times 5 \times 3 \times 1} =$$

The 5 in the *dividend* and 5 in the *divisor* are *divisible* by 5, since 5 divided by 5 equals 1. *Cross off* the 5 of the *dividend* and write the 1 *over* it; also, cross off the 5 of the *divisor* and write the 1 *under* it. Thus,

$$\frac{4\times 6\times \overset{1}{\cancel{5}}\times 2\times 2}{1\times \underset{1}{\cancel{5}}\times 3\times 1}=$$

The 5 in the *dividend* and 5 in the *divisor* are not to be considered any longer, and, in fact, may be erased entirely and 1 and 1 placed in their stead, and treated as if the 5 and 5 *never* existed. Thus,

$$\frac{4\times 6\times 1\times 2\times 2}{1\times 1\times 3\times 1}=$$

The 6 in the *dividend* and 3 in the *divisor* are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and place 2 *over* it; also, cross off the 3 and place 1 *under* it. Thus,

$$\frac{4\times \overset{2}{\cancel{6}}\times 1\times 2\times 2}{1\times 1\times \underset{1}{\cancel{3}}\times 1}=$$

The 6 and 3 are not to be considered any longer, and, in fact, may be erased entirely and 2 and 1 placed in their stead, and treated as if the 6 and 3 *never* existed. Thus,

$$\frac{4\times 2\times 1\times 2\times 2}{1\times 1\times 1\times 1}=\frac{32}{1}=32. \quad \text{Ans.}$$

$$\text{Hence, } \frac{\overset{4}{\cancel{80}}\times \overset{\overset{2}{\cancel{6}}}{\cancel{60}}\times \overset{\overset{1}{\cancel{5}}}{\cancel{50}}\times \overset{2}{\cancel{16}}\times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}}\times \underset{\underset{1}{\cancel{5}}}{\cancel{50}}\times \underset{\underset{1}{\cancel{3}}}{\cancel{24}}\times \underset{1}{\cancel{20}}}=\frac{4\times 2\times 1\times 2\times 2}{1\times 1\times 1\times 1}=\frac{32}{1}=32. \quad \text{Ans.}$$

(**19**) 28 acres of land at \$133 an acre would cost 28 × \$1 3 3 = \$3,724.

```
   2 8
  ----
 1 0 6 4
 2 6 6
 -------
$3 7 2 4
```

If a mechanic earns $1,500 a year and his expenses are $968 per year, then he would save $1500—$968, or $532 per year.

```
 $1500
   968
 $ 532
```

If he saves $532 in 1 year, to save $3,724 it would take as many years as $532 is contained times in $3,724, or 7 years.

```
532 ) 3724 ( 7 years.   Ans.
      3724
```

(20) If the freight train ran 365 miles in one week, and 3 times as far lacking 246 miles the next week, then it ran (3 × 365 miles) — 246 miles, or 849 miles the second week. Thus,

```
              365
                3
             1095
              246
difference    849 miles.   Ans.
```

(21) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in one mile, in 354 miles there are 354 × 5,280 feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, then in going from Philadelphia to Pittsburg, a distance of 1,869,-120 feet, it will make 1,869,120 ÷ 16, or 116,820 revolutions.

```
16)1869120(116820 rev.   Ans.
   16
    26
    16
    109
     96
     131
     128
       32
       32
        0
```

(22) (*a*)

```
576)589824(1024  Ans.
    576
    ----
     1382
     1152
     ----
      2304
      2304
      ----
```

(*b*)

```
43911)369730620(8420  Ans.
      351288
      ------
       184426
       175644
       ------
         87822
         87822
         -----
             0
```

(*c*)

```
505)2527525(5005  Ans.
    2525
    ----
       2525
       2525
       ----
```

(*d*)

```
1234)4961794302(4020903  Ans.
     4936
     ----
       2579
       2468
       ----
        11143
        11106
        -----
           3702
           3702
           ----
```

(23) The harness evidently cost the difference between $444 and the amount which he paid for the horse and wagon.

Since $264 + $153 = $417, the amount paid for the horse and wagon, $444 − $417 = $27, the cost of the harness.

```
$264        $444
 153         417
----        ----
$417        $27  Ans.
```

(24) (*a*)

```
      1024
       576
      ----
      6144
     7168
    5120
    ------
    589824   Ans.
```

(*b*)

```
      5005
       505
     -----
     25025
   250250
   -------
   2527525   Ans.
```

(*c*)

```
      43911
       8420
     ------
     878220
   175644
  351288
  ---------
  369730620   Ans.
```

(25) Since there are 12 months in a year, the number of days the man works is 25 × 12 = 300 days. As he works 10 hours each day, the number of hours that he works in one year is 300 × 10 = 3,000 hours. Hence, he receives for his work 3,000 × 30 = 90,000 cents, or 90,000 ÷ 100 = $900. Ans.

(26) See Art. **71.**

(27) See Art. **77.**

(28) See Art. **73.**

(29) See Art. **73.**

(30) See Art. **75.**

(31) $\frac{13}{8}$ is an improper fraction, since its numerator 13 is greater than its denominator 8.

(32) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(33) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms we divide both numerator and denominator by 4, and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$. Ans.

(34) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6. To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since this denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4 we shall have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form*. Ans.

(35) In order to reduce a mixed number to an improper fraction, we must *multiply the whole number by the denominator of the fraction* and *add the numerator of the fraction to that product.* This *result* is the *numerator of the improper fraction*, of which the *denominator is the denominator of the fractional part of the mixed number.*

$7\frac{7}{8}$ means the same as $7 + \frac{7}{8}$. In 1 there are $\frac{8}{8}$, hence in 7 there are $7 \times \frac{8}{8} = \frac{56}{8}$; $\frac{56}{8}$ plus the $\frac{7}{8}$ of the mixed number $= \frac{56}{8} + \frac{7}{8} = \frac{63}{8}$, which is the required improper fraction.

$$13\frac{5}{16} = \frac{(13 \times 16) + 5}{16} = \frac{213}{16}; \quad 10\frac{3}{4} = \frac{(10 \times 4) + 3}{4} = \frac{43}{4}.$$

(36) The value of a fraction is obtained by dividing the numerator by the denominator.

To obtain the value of the fraction $\frac{13}{2}$ we divide the numerator 13 by the denominator 2. 2 is contained in 13 six times, with 1 remaining. This 1 remaining is written over the denominator 2, thereby making the fraction $\frac{1}{2}$, which is annexed to the whole number 6, and we obtain $6\frac{1}{2}$ as the mixed number. The reason for performing this operation is the following: In 1 there are $\frac{2}{2}$ (two halves), and in $\frac{13}{2}$ (thirteen halves) there are as many units (1) as 2 is contained times in 13, which is 6, and $\frac{1}{2}$ (one-half) unit remaining. Hence, $\frac{13}{12} = 6 + \frac{1}{2} = 6\frac{1}{2}$, the required mixed number. Ans. $\frac{17}{4} = 4\frac{1}{4}$. Ans. $\frac{69}{16} = 4\frac{5}{16}$. Ans. $\frac{16}{8} = 2$. Ans. $\frac{67}{64} = 1\frac{3}{64}$. Ans.

(37) In division of fractions, *invert the divisor* (or, in other words, turn it upside down) *and proceed as in multiplication.*

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \frac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112$. Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \frac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}$. Ans.

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \frac{17 \times 1}{2 \times 9} = \frac{17}{18}$. Ans.

(*d*) $\frac{113}{64} \div \frac{7}{16} = \frac{113}{64} \times \frac{16}{7} = \frac{113 \times 16}{64 \times 7} = \frac{1,808}{448} = \frac{452}{112} = \frac{113}{28}$

```
28)113(4 1/28.  Ans.
   112
   ---
     1
```

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both of the mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \frac{(15 \times 4) + 3}{4} = \frac{60 + 3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \frac{(4 \times 8) + 3}{8} = \frac{32 + 3}{8} = \frac{35}{8}$. The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor and multiply; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \frac{63 \times 8}{4 \times 35} = \frac{504}{140} = \frac{252}{70} = \frac{126}{35} = \frac{18}{5}$.

$$\frac{18}{5}: \quad 5\,)\,18\,(\,3\frac{3}{5} \text{ Ans.} \quad \begin{array}{r} 15 \\ \hline 3 \end{array}$$

(38) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = \frac{1 + 2 + 5}{8} = \frac{8}{8} = 1.$ Ans.

When the *denominators* of the fractions to be added *are alike*, we know that the units are divided into the *same number of parts* (in this case *eighths*); we, therefore, *add the numerators* of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$ or 1 as the sum.

(39) When the *denominators* are *not alike* we know that the units are divided into *unequal parts*, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all these fractions to sixteenths and then add their numerators.

$\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms of the fraction by some number which will make the denominator 16. This number evidently is 4, hence, $\frac{1 \times 4}{4 \times 4} = \frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3 \times 2}{8 \times 2} = \frac{6}{16}$. The fractions now have a common denominator 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16} + \frac{6}{16} + \frac{5}{16} = \frac{4+6+5}{16} = \frac{15}{16}$. Ans.

(40) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5}{8} \times \frac{2}{2} = \frac{10}{16}$. Adding the two fractional parts of the mixed numbers we have $\frac{10}{16} + \frac{7}{16} = \frac{10+7}{16} = \frac{17}{16} = 1\frac{1}{16}$.

The problem now becomes $42 + 31 + 9 + 1\frac{1}{16} = ?$

$$\begin{array}{r} 42 \\ 31 \\ 9 \\ 1\frac{1}{16} \\ \hline 83\frac{1}{16} \end{array}$$ Ans.

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

(41) $29\frac{3}{4} + 50\frac{5}{8} + 41 + 69\frac{3}{16} = ?$. $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}$.

$\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. $\frac{12}{16} + \frac{10}{16} + \frac{3}{16} = \frac{12 + 10 + 3}{16} = \frac{25}{16} = 1\frac{9}{16}$.

The problem now becomes $29 + 50 + 41 + 69 + 1\frac{9}{16} = ?$

29	square inches.
50	square inches.
41	square inches.
69	square inches.
$1\frac{9}{16}$	square inches.
$190\frac{9}{16}$	square inches. Ans.

(42) (*a*) $\dfrac{7}{\frac{3}{16}} = 7 \div \frac{3}{16} = 7 \times \frac{16}{3} = \frac{7 \times 16}{3} = \frac{112}{3} = 37\frac{1}{3}$. Ans.

The line between 7 and $\frac{3}{16}$ means that 7 is to be divided by $\frac{3}{16}$.

(*b*) $\dfrac{\frac{15}{32}}{\frac{5}{8}} = \frac{15}{32} \div \frac{5}{8} = \frac{15}{32} \times \frac{8}{5} = \frac{\overset{3}{\cancel{15}} \times \cancel{8}}{\underset{4}{\cancel{32}} \times \cancel{5}} = \frac{3}{4}$. Ans.

(*c*) $\dfrac{\frac{4+3}{2+6}}{5} = \dfrac{\frac{7}{8}}{5} = \frac{7}{8 \times 5} = \frac{7}{40}$. (See Art. **131.**) Ans.

(43) $\frac{7}{8}$ = value of the fraction, and 28 = the numerator. We find that 4 multiplied by 7 = 28, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32} = \frac{7}{8}$. Hence, 32 is the required denominator. Ans.

(44) (*a*) $\frac{7}{8} - \frac{7}{16} = ?$ When the *denominators* of fractions are *not alike* it is evident that the units are divided into *unequal parts*, therefore, before subtracting, *reduce the* *fractions to fractions having a common denominator.* Then, *subtract the numerators, and place the remainder over the common denominator.*

$$\frac{7 \times 2}{8 \times 2} = \frac{14}{16}. \quad \frac{14}{16} - \frac{7}{16} = \frac{14 - 7}{16} = \frac{7}{16}. \quad \text{Ans.}$$

(*b*) $13 - 7\frac{7}{16} = ?$ This problem may be solved in two ways:

First: $13 = 12\frac{16}{16}$, since $\frac{16}{16} = 1$, and $12\frac{16}{16} = 12 + \frac{16}{16} = 12 + 1 = 13$.

$$\begin{array}{r} 12\frac{16}{16} \\ 7\frac{7}{16} \\ \hline 5\frac{9}{16} \end{array}$$

We can now subtract the whole numbers separately, and the fractions separately, and obtain $12 - 7 = 5$ and $\frac{16}{16} - \frac{7}{16} = \frac{16 - 7}{16} = \frac{9}{16}$. $5 + \frac{9}{16} = 5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$13 = \frac{13}{1} = \frac{13 \times 16}{1 \times 16} = \frac{208}{16}$. $7\frac{7}{16} = \frac{(7 \times 16) + 7}{16} = \frac{112 + 7}{16} = \frac{119}{16}$.

Subtracting, we have $\frac{208}{16} - \frac{119}{16} = \frac{208 - 119}{16} = \frac{89}{16}$ and

$\frac{89}{16} = 16)89(5\frac{9}{16}$ the same result that was obtained by the first method.

$$\begin{array}{r} 16)89(5\frac{9}{16} \\ 80 \\ \hline 9 \\ \hline 16 \end{array}$$

(*c*) $312\frac{9}{16} - 229\frac{5}{32} = ?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this we have $\frac{9}{16} = \frac{9 \times 2}{16 \times 2} = \frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312 - 229 = 83$ and $\frac{18}{32} - \frac{5}{32} = \frac{18 - 5}{32} = \frac{13}{32}$.

$$\begin{array}{r} 312\frac{18}{32} \\ 229\frac{5}{32} \\ \hline 83\frac{13}{32} \end{array} \quad 83 + \frac{13}{32} = 83\frac{13}{32}. \quad \text{Ans.}$$

(45) The man evidently traveled $85\frac{5}{12}+78\frac{9}{15}+125\frac{17}{35}$ miles.

Adding the fractions separately in this case,

$$\frac{5}{12}+\frac{9}{15}+\frac{17}{35}=\frac{5}{12}+\frac{3}{5}+\frac{17}{35}=\frac{175+252+204}{420}=\frac{631}{420}=1\frac{211}{420}.$$

Adding the whole numbers and the mixed number representing the sum of the fractions, the sum is $289\frac{211}{420}$ miles. Ans.

$$\begin{array}{r} 85 \\ 78 \\ 125 \\ 1\frac{211}{420} \\ \hline 289\frac{211}{420} \end{array}$$

To find the least common denominator, we have

$$\begin{array}{r|l} 5 & 12,\ 5,\ 35 \\ \hline 7 & 12,\ 1,\ \ 7 \\ \hline & 12,\ 1,\ \ 1, \end{array} \text{ or } 5\times7\times12=420.$$

(46)

$$\begin{array}{lll} & 573\frac{4}{5}\text{ tons.} & \frac{4}{5}=\frac{32}{40} \\ & 216\frac{5}{8}\text{ tons.} & \frac{5}{8}=\frac{25}{40} \\ \hline \textit{difference} & 357\frac{7}{40}\text{ tons. Ans.} & \frac{7}{40}=\textit{difference.} \end{array}$$

(47) Reducing $9\frac{1}{4}$ to an improper fraction, it becomes $\frac{37}{4}$. Multiplying $\frac{37}{4}$ by $\frac{3}{8}$, $\frac{37}{4}\times\frac{3}{8}=\frac{111}{32}=3\frac{15}{32}$ dollars. Ans.

(48) Referring to Arts. **114** and **116**,

$\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 multiplied by $\frac{7}{8}$ of $\frac{5}{6}$ of 45 =

$$\frac{\cancel{2}\times\cancel{3}\times7\times19\times\cancel{11}\times7\times5\times\overset{\overset{3}{\cancel{15}}}{\cancel{45}}}{\cancel{3}\times4\times\cancel{11}\times\underset{4}{\cancel{20}}\times1\times8\times\underset{\cancel{3}}{\cancel{6}}\times1}=\frac{7\times19\times7\times5\times3}{4\times4\times8}=\frac{13{,}965}{128}=$$

$109\frac{13}{128}$. Ans.

(49) $\frac{3}{4}$ of $16=\frac{3}{\cancel{4}}\times\frac{\overset{4}{\cancel{16}}}{1}=12.\quad 12\div\frac{2}{3}=\frac{\overset{6}{\cancel{12}}}{1}\times\frac{3}{\cancel{2}}=18.$ Ans.

(50) $211\frac{1}{4}\times1\frac{7}{8}=\frac{845}{4}\times\frac{15}{8}$, reducing the mixed numbers to improper fractions. $\frac{845}{4} \times \frac{15}{8} = \frac{12,675}{32}$ cents = amount paid for the lead. The number of pounds sold is evidently $\frac{12,675}{32} \div 2\frac{1}{2} = \frac{\overset{2,535}{\cancel{12,675}}}{\underset{16}{\cancel{32}}} \times \frac{\cancel{2}}{\cancel{5}} = \frac{2,535}{16} = 158\frac{7}{16}$ pounds. The amount remaining is $211\frac{1}{4} - 158\frac{7}{16} = \frac{845}{4} - \frac{2,535}{16} = \frac{3,380}{16} - \frac{2,535}{16} = \frac{845}{16} = 52\frac{13}{16}$ pounds. Ans.

(51)

(tenths. hundredths.)
. 0 8 = *Eight* **hundredths.**

(tenths. hundredths. thousandths.)
. 1 3 1 = *One hundred thirty-one* **thousandths.**

(tenths. hundredths. thousandths. ten-thousandths.)
0 0 0 1 = *One* **ten-thousandth.**

(tenths. hundredths. thousandths. ten-thousandths. hundred-thousandths. millionths.)
. 0 0 0 0 2 7 = *Twenty-seven* **millionths.**

(tenths. hundredths. thousandths. ten-thousandths.)
. 0 1 0 8 = *One hundred eight* **ten-thousandths.**

tenths. hundredths. thousandths. ten-thousandths.

93.0 1 0 1 = Ninety-three, and *one hundred one* **ten-thousandths.**

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the decimal point towards the right, beginning with tenths, to as many places as there are figures, and the *name* of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example above, the simple reading of the figure is *eight*, and the name of its position in the decimal scale is **hundredths,** so that the decimal reading is *eight* **hundredths.** Similarly, the figures in the fourth example are ordinarily read *twenty-seven;* the name of the position of the figure 7 in the decimal scale is **millionths,** giving, therefore, the decimal reading as *twenty-seven* **millionths.**

If there should be a whole number before the decimal point, read it as you would read any whole number, and read the decimal as you would if the whole number were not there; or, read the whole number and then say, "and" so many hundredths, thousandths, or whatever it may be, as "ninety-three, *and* one hundred one ten thousandths."

(52) See Art. **139.**

(53) See Art. **153.**

(54) See Art. **160.**

(55) A fraction is one or more of the equal parts of a unit, and is expressed by a numerator and a denominator, while a decimal fraction is a number of *tenths*, *hundredths*, *thousandths*, etc., of a unit, and is expressed by placing a period (.), called a decimal point, to the left of the figures of the number, and omitting the denominator.

(56) See Art. **165.**

(57) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of the quotient, or .5, since only *one* cipher was annexed to the numerator. Ans.

$$\frac{7}{8}) \; 8)\underline{7.000} \quad .875 \text{ Ans.}$$

$$\frac{5}{32}) \; 32)5.00000(.15625 \text{ Ans.}$$

```
32)5.00000(.15625  Ans.
   32
   ---
   180
   160
   ---
    200
    192
    ---
      80
      64
      ---
      160
      160
      ---
```

Since $.65 = \frac{65}{100}$, then, $\frac{65}{100}$ must equal .65. Or, when the denominator is 10, 100, 1000, etc., point off as many places in the numerator as there are ciphers in the denominator. Doing so, $\frac{65}{100} = .65$. Ans.

$\frac{125}{1000} = .125$. Ans.

(58) (*a*) This example, written in the form of a fraction, means that the numerator (32.5 + .29 + 1.5) is to be divided by the denominator (4.7 + 9). The operation is as follows:

$$\frac{32.5 + .29 + 1.5}{4.7 + 9} = ?$$

```
          32.5
        +   .29
        +  1.5
        --------
   13.7)34.29000(2.5029  Ans.
        274
        ---
 4.7    689
+9.0    685
----    ---
13.7     400
         274
         ----
         1260
         1233
         ----
           27
```

Since there are 5 decimal places in the dividend and 1 in the divisor, there are 5 − 1 or 4 places to be pointed off in the quotient. The fifth figure of the decimal is evidently less than 5.

(*b*) Here again the problem is to divide the numerator, which is $(1.283 \times \overline{8 + 5})$, by the denominator, which is 2.63. The operation is as follows:

$$\frac{1.283 \times \overline{8+5}}{2.63} = ? \quad \overline{8+5} = 13.$$

```
      1.283
    ×    13
      3849
     1283
2.63)16.679000(6.3418  Ans.
     1578
        899                 480
        789                 263
       1100                2170
       1052                2104
         480                 66
```

(*c*) $$\frac{\overline{589 + 27} \times \overline{163 - 8}}{25 + 39} = ?$$

```
  589
+  27
  616

   25
+  39
   64
```

```
          163
       -    8
          155
        × 616
          930
         155
        930
    64)95480.000(1491.875
       64              Ans.
       314
       256
        588
        576
         120
          64
          560
          512
           480
           448
            320
            320
```

There are three decimal places in the quotient, since three ciphers were annexed to the dividend.

(*d*) $$\frac{\overline{40.6+7.1}\times(3.029-1.874)}{6.27+8.53-8.01}=?$$

$$\begin{array}{r} 40.6 \\ +\ \ 7.1 \\ \hline 47.7 \end{array} \qquad \begin{array}{r} 3.029 \\ -\ 1.874 \\ \hline 1.155 \\ \times\ \ 47.7 \\ \hline 8085 \\ 8085\ \ \\ 4620\ \ \ \ \\ \hline 55.093500 \end{array}$$

$$\begin{array}{r} 6.27 \\ +\ 8.53 \\ \hline 14.80 \\ -\ \ 8.01 \\ \hline 6.79 \end{array}$$

$$6.79\,)\,55.093500\,(\,8.1139. \quad \text{Ans.}$$

$$\begin{array}{r} 55.093500 \\ 5432 \\ \hline 773 \\ 679 \\ \hline 945 \\ 679 \\ \hline 2660 \\ 2037 \\ \hline 6230 \\ 6111 \\ \hline 119 \end{array}$$

6 decimal places in the dividend — 2 decimal places in the divisor = 4 decimal places to be pointed off in the quotient.

(59) $$.875=\frac{875}{1{,}000}=\frac{175}{200}=\frac{7}{8}\text{ of a foot.}$$

1 foot = 12 inches.

$$\frac{7}{8}\text{ of 1 foot}=\frac{7}{\cancel{8}_{2}}\times\frac{\cancel{12}^{3}}{1}=\frac{21}{2}=10\frac{1}{2}\text{ inches.}\quad\text{Ans.}$$

(60) 12 inches = 1 foot.

$$\frac{3}{16}\text{ of an inch}=\frac{3}{16}\div 12=\frac{\cancel{3}}{16}\times\frac{1}{\cancel{12}_{4}}=\frac{1}{64}\text{ of a foot.}$$

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, 6 — 0 = 6 places to be pointed off.

```
 1
 -- ) 1.000000(.015625  Ans.
 64     64
       ----
        360
        320
        ----
         400
         384
         ----
          160
          128
          ----
           320
           320
           ---
```

(61) If 1 cubic inch of water weighs .03617 of a pound, the weight of 1,500 cubic inches will be .03617 × 1,500 = 54.255 lb.

```
   .03617 lb.
     1500
  -------
  1808500
  3617
  -------
  54.25500 lb.   Ans.
```

(62) 72.6 feet of fencing at $.50 a foot would cost

```
     72.6 × .50, or $36.30.
      .50
  -------
  $36.300
```

If, by selling a carload of coal at a profit of $1.65 per ton, I make $36.30, then there must be as many tons of coal in the car as 1.65 is contained times in 36.30, or 22 tons.

```
  1.65 ) 36.30 ( 22 tons.   Ans.
         330
         ----
          330
          330
          ---
```

(63)

```
231 ) 17892.00000 ( 77.45454, or 77.4545 to
      1617              four decimal places.   Ans.
      ----
       1722
       1617
       ----
        1050
         924
        ----
         1260
         1155
         ----
          1050
           924
          ----
          1260
          1155
          ----
          1050
```

(64)

$$\frac{\overset{37.13}{\not{7}\not{4}.\not{2}\not{6}} \times \overset{2}{\not{2}\not{4}} \times \overset{.0952}{\not{3}.\not{1}\not{4}\not{1}\not{6}} \times 19 \times 19 \times 350}{\underset{1,000}{\not{3}\not{3},\not{0}\not{0}\not{0}} \times \underset{2}{\not{1}\not{2}} \times \not{4}} =$$

$$\frac{37.13 \times .0952 \times 19 \times 19 \times 350}{1,000} = \frac{446,618.947600}{1,000} =$$

446.619 to three decimal places. Ans.

```
  37.13        19        361          3.534776
  .0952        19        350            126350
 -------      ----     -------     -------------
   7426       171       18050        176738800
 18565        19       1083         10604328
33417        ----     -------      21208656
--------      361      126350      7069552
3.534776                          3534776
                                  --------------
                                  446618.947600
```

(65) See Art. **174.** Applying rule in Art. **175,**

(*a*) $.7928 \times \frac{64}{64} = \frac{50.7392}{64} = \frac{51}{64}$. Ans.

(*b*) $.1416 \times \frac{32}{32} = \frac{4.5312}{32} = \frac{5}{32}$. Ans.

(*c*) $.47915 \times \frac{16}{16} = \frac{7.6664}{16} = \frac{8}{16} = \frac{1}{2}$. Ans.

(66) In subtraction of decimals, *place the decimal points directly under each other*, and proceed as in the subtraction of whole numbers, placing the *decimal point in the remainder directly under the decimal points above.*

```
(a) 709.6300
       .8514
    --------
    708.7786  Ans.
```

In the above example we proceed as follows: We can not subtract 4 ten-thousandths from 0 ten-thousandths, and, as there are no thousandths, we take 1 hundredth from the three hundredths. 1 *hundredth* = 10 *thousandths* = 100 *ten-thousandths.* 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 *thousandths* + 6 *ten-thousandths.* Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths makes it but 2 hundredths now). Since we can not do this, we take 1 tenth from 6 tenths. 1 tenth (= 10 hundredths) + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this can not be done, we take 1 unit from the 9 units. 1 *unit* = 10 *tenths;* 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the tenths place in the remainder. In the minuend we now have 708 units (one unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units; hence, we write 708 in the remainder.

```
(b) 81.963          (c) 18.00          (d) 1.000
     1.700                .18                .001
    ------              -----              -----
    80.263 Ans.         17.82 Ans.          .999 Ans.
```

(*e*) $872.1 - (.8721 + .008) = ?$ In this problem we are to subtract $(.8721 + .008)$ from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

$$\begin{array}{r} .8721 \\ .008 \\ \hline .8801 \end{array} \textit{ sum.}$$

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

$$\begin{array}{r} 872.1000 \\ .8801 \\ \hline 871.2199 \end{array} \text{ Ans.}$$

(*f*) $(5.028 + .0073) - (6.704 - 2.38) = ?$ First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

$$\begin{array}{r} 5.0280 \\ .0073 \\ \hline 5.0353 \end{array} \textit{ sum.}$$

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324.

$$\begin{array}{r} 6.704 \\ 2.380 \\ \hline 4.324 \end{array} \textit{ difference.}$$

The sign between the parentheses indicates that the quantities obtained by performing the above operations, are to be subtracted, namely, that 4.324 is to be subtracted from 5.0353. Performing this operation we obtain .7113 as the final result.

$$\begin{array}{r} 5.0353 \\ 4.324 \\ \hline .7113 \end{array} \text{ Ans.}$$

(67) In subtracting a decimal from a fraction, or subtracting a fraction from a decimal, either reduce the fraction to a decimal before subtracting, or reduce the decimal to a fraction and then subtract.

(*a*) $\frac{7}{8} - .807 = ?$ $\frac{7}{8}$ reduced to a decimal becomes

$$\frac{7}{8}) \, 7.000 \over .875$$

Subtracting .807 from .875 the remainder is .068, as shown.

$$\begin{array}{r} .875 \\ .807 \\ \hline .068 \end{array} \text{ Ans.}$$

(*b*) $.875 - \frac{3}{8} = ?$ Reducing .875 to a fraction we have $875 = \frac{875}{1,000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; hence, $\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2}$. Ans.

Or, by reducing $\frac{3}{8}$ to a decimal, $\frac{3}{8}$) 3.000 / $.375$ and then subtracting, we obtain $.875 - .375 = .5 = \frac{5}{10} = \frac{1}{2}$, the same answer as above.

$$\begin{array}{r} .875 \\ .375 \\ \hline .500 \end{array} \text{ Ans.}$$

(*c*) $\left(\frac{5}{32} + .435\right) - \left(\frac{21}{100} - .07\right) = ?$ We first perform the operations as indicated by the signs between the numbers enclosed by the parentheses. Reduce $\frac{5}{32}$ to a decimal and we obtain $\frac{5}{32} = .15625$ (see example 7).

Adding .15625 and .435,

$$\begin{array}{rr} & .15625 \\ & .435 \\ \hline \textit{sum} & .59125 \end{array}$$

$\frac{21}{100} = .21$; subtracting,

$$\begin{array}{rr} & .21 \\ & .07 \\ \hline \textit{difference} & .14 \end{array}$$

We are now prepared to perform the operation indicated by the minus sign between the parentheses, which is,

$$\begin{array}{rr} & .59125 \\ & .14 \\ \hline \textit{difference} & .45125 \end{array} \text{ Ans.}$$

(*d*) This problem means that 33 millionths and 17 thousandths are to be added. Also, that 53 hundredths and 274 thousandths are to be added, and the smaller of these sums is to be subtracted from the larger sum. Thus, $(.53 + .274) - (.000033 + .017) = ?$

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred thousandths.	millionths.	
.0	0	0	0	3	3	
.0	1	7				
.0	1	7	0	3	3	*sum.*

tenths.	hundredths.	thousandths.	
.5	3		
.2	7	4	
.8	0	4	*sum.*

$$\begin{array}{rrl} & .804 & \textit{larger sum,} \\ & .017033 & \textit{smaller sum,} \\ \hline \textit{difference} & .786967 & \text{Ans.} \end{array}$$

(68) In addition of decimals the *decimal points must be placed directly under each other*, so that *tenths* will come *under tenths*, *hundredths under hundredths*, *thousandths under thousandths*, etc. The addition is then performed as in whole numbers, *the decimal point of the sum being placed directly under the decimal points above.*

```
  .125
  .7
  .089
  .4005
  .9
  .000027
 ---------
 2.214527   Ans.
```

(69)

```
  927.416
    8.274
  372.6
   62.07938
 -----------
 1370.36938  Ans.
```

(70) tenths. hundredths. thousandths. ten-thousandths. hundred-thousandths. millionths.

```
 .017
 .2
 .000047
 --------
 .217047 = Ans.
```

(71) (*a*)

```
   .107
   .013
  -----
    321
   107
 -------
 .001391  Ans.
```

There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are 3 + 3 or 6 decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary six decimal places.

(*b*)

```
    203
    2.03
   -----
    609
  4060
  ------
  412.09
    .203
  -------
  123627
 824180
 --------
 83.65427  Ans.
```

There are two decimal places in the multiplier and none in the multiplicand; hence, there are 2 + 0 or two decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are 3 + 2 or 5 decimal places in the second product.

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parenthesis, and then perform whatever may be required by the sign before the parenthesis.

Multiply together the numbers 2.7 and 31.85.

$$\begin{array}{r} 31.85 \\ 2.7 \\ \hline 22295 \\ 6370 \\ \hline 85.995 \end{array}$$

The parenthesis shows that .316 is to be taken from 3.16.

$$\begin{array}{r} 3.160 \\ .316 \\ \hline 2.844 \end{array}$$

The product obtained by the first operation is now multiplied by the remainder obtained by performing the operation indicated by the signs within the parenthesis.

$$\begin{array}{r} 85.995 \\ 2.844 \\ \hline 343980 \\ 343980 \\ 687960 \\ 171990 \\ \hline 244.569780 \end{array}\ \text{Ans.}$$

(*d*) $(107.8 + 6.541 - 31.96) \times 1.742 = ?$

$$\begin{array}{r} 107.8 \\ +\ \ 6.541 \\ \hline 114.341 \\ -\ \ 31.96 \\ \hline 82.381 \\ \times\ \ 1.742 \\ \hline 164762 \\ 329524 \\ 576667 \\ 82381 \\ \hline 143.507702 \end{array}\ \text{Ans.}$$

(72) (*a*) $\left(\frac{7}{16} - .13\right) \times \overline{.625 + \frac{5}{8}} = ?$

First perform the operation indicated by the parenthesis.

$$\frac{7}{16} = 16\overline{)7.0000}(.4375$$

```
16 ) 7.0000 (.4375
     64
     ---
      60
      48
      ---
      120
      112
      ---
        80
        80
        --
```

We point off four decimal places since we annexed four ciphers.

```
      .4375
      .13
      -----
      .3075
```

Subtracting, we obtain .3075

The vinculum has the same meaning as the parenthesis; hence, we perform the operation indicated by it. We point off three decimal places, since three ciphers were annexed to the 5.

$$\frac{5}{8} = 8\underline{)\,5.000}$$
$$.625$$

Adding the terms included by the vinculum, we obtain

```
  .625
  .625
 -----
 1.250
```

The final operation is to perform the work indicated by the sign between the parenthesis and the vinculum, thus,

```
   .3075
    1.25
  ------
   15375
   6150
  3075
 -------
 .384375  Ans.
```

(*b*) $\left(\frac{19}{32} \times .21\right) - \left(.02 \times \frac{3}{16}\right) = ?$

$.21 = \frac{21}{100}$. $\frac{19}{32} \times \frac{21}{100} = \frac{399}{3200}$. $.02 = \frac{2}{100}$. $\frac{2}{100} \times \frac{3}{16} = \frac{6}{1600} = \frac{3}{800}$.

$\frac{3}{800} = \frac{3 \times 4}{800 \times 4} = \frac{12}{3200}$. $\frac{399}{3200} - \frac{12}{3200} = \frac{399 - 12}{3200} = \frac{387}{3200}$.

Reducing $\frac{387}{3200}$ to a decimal, we obtain

$$
\begin{array}{l}
\frac{387}{3200}\ )\ 387.0000000\ (\ .1209375 \quad \text{Ans.} \\
\qquad\quad 3200 \\
\qquad\quad \overline{\ \ 6700} \\
\qquad\quad\ \ 6400 \\
\qquad\quad\ \ \overline{\ 30000} \\
\qquad\quad\ \ \ 28800 \\
\qquad\quad\ \ \ \overline{\ 12000} \\
\qquad\quad\qquad 9600 \\
\qquad\qquad\ \overline{\ 24000} \\
\qquad\qquad\ \ 22400 \\
\qquad\qquad\ \ \overline{\ 16000} \\
\qquad\qquad\quad 16000 \\
\end{array}
$$

Point off seven decimal places, since seven ciphers were annexed to the dividend.

(*c*) $\left(\frac{13}{4} + .013 - 2.17\right) \times \overline{13\frac{1}{4} - 7\frac{5}{16}} = ?$

$\frac{13}{4} = \frac{13}{4)\,13.00} \atop 3.25$

$$
\begin{array}{r}
4\ )\ 13.00 \\
\hline
3.25
\end{array}
$$

Point off two decimal places, since two ciphers were annexed to the dividend.

$$
\begin{array}{r}
3.25 \\
+\ .013 \\
\hline
3.263 \\
-\ 2.17 \\
\hline
1.093
\end{array}
$$

$\frac{5}{16}$ reduced to a decimal is .3125, since

$$
\begin{array}{l}
\frac{5}{16}\ )\ 5.0000\ (\ .3125 \\
\qquad\ \ 48 \\
\qquad\ \ \overline{\ \ 20} \\
\qquad\ \ \ 16 \\
\qquad\ \ \ \overline{\ \ 40} \\
\qquad\quad\ 32 \\
\qquad\quad\ \overline{\ \ 80} \\
\qquad\qquad 80 \\
\end{array}
$$

Point off four decimal places, since four ciphers were annexed to the dividend.

Then, $7\frac{5}{16} = 7.3125$, and $13\frac{1}{4} = 13.25$, since $\frac{1}{4} = \frac{1}{4)\,1.00}$

$$
\begin{array}{r}
4\ )\ 1.00 \\
\hline
.25
\end{array}
$$

$$\begin{array}{r} 13.25 \\ -\ \underline{7.3125} \\ 5.9375 \end{array} \qquad\qquad \begin{array}{r} 5.9375 \\ \times\ \underline{1.093} \\ 178125 \\ 534375 \\ \underline{593750} \\ 6.4896875 \end{array} \text{ Ans.}$$

(73) (*a*) $.875 \div \frac{1}{2} = .875 \div .5 \left(\text{since } \frac{1}{2} = .5\right) = 1.75$. Ans. Another way of solving this is to reduce .875 to its equivalent common fraction and then divide.

$.875 = \frac{7}{8}$, since $.875 = \frac{875}{1{,}000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; then, $\frac{7}{8} \div \frac{1}{2} = \frac{7}{\not{8}_{4}} \times \frac{\not{2}}{1} = \frac{7}{4} = 1\frac{3}{4}$. Since $\frac{3}{4} =$

$$\begin{array}{r} 4)\,3.00\,(.75 \\ \underline{28} \\ 20 \\ \underline{20} \end{array}$$

$1\frac{3}{4} = 1.75$, the same answer as above.

(*b*) $\frac{7}{8} \div .5 = \frac{7}{8} \div \frac{1}{2} \left(\text{since } .5 = \frac{1}{2}\right) = \frac{7}{\not{8}_{4}} \times \frac{\not{2}}{1} = \frac{7}{4} = 1\frac{3}{4}$, or 1.75. Ans.

This can also be solved by reducing $\frac{7}{8}$ to its equivalent decimal and dividing by .5; $\frac{7}{8} = .875$; $.875 \div .5 = 1.75$. Since there are three decimal places in the dividend and one in the divisor, there are 3 − 1, or 2 decimal places in the quotient.

(*c*) $\dfrac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125} = ?$ We shall solve this problem by first reducing the decimals to their equivalent common fractions.

$.375 = \frac{375}{1{,}000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$. $\frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$, or the value of the numerator of the fraction.

$.125 = \frac{125}{1{,}000} = \frac{25}{200} = \frac{1}{8}$. Reducing $\frac{1}{8}$ to sixteenths, we have $\frac{1 \times 2}{8 \times 2} = \frac{2}{16}$. Then, $\frac{5}{16} - \frac{2}{16} = \frac{3}{16}$, or the value of the denominator of the fraction. The problem is now reduced to

$$\frac{\frac{3}{32}}{\frac{3}{16}} = ? \quad \frac{\frac{3}{32}}{\frac{3}{16}} = \frac{3}{32} \div \frac{3}{16} = \frac{\not{3}}{\underset{2}{\not{3}\not{2}}} \times \frac{\not{1}\not{6}}{\not{3}} = \frac{1}{2} \text{ or } .5. \quad \text{Ans.}$$

(74) $\dfrac{\frac{1.25 \times 20 \times 3}{87 + (11 \times 8)}}{459 + 32} = ?$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

Multiplying the factors of the numerator together, we find their product to be 75.

$$\begin{array}{r} 1.25 \\ \times \quad 20 \\ \hline 25.00 \\ \times \quad 3 \\ \hline 75 \end{array}$$

The fraction $\dfrac{87 + (11 \times 8)}{459 + 32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87 + 88 = 175$.

The numerator is combined as though it were written $87 + (11 \times 8)$, and its result is

$$\begin{array}{r} 11 \\ 8 \text{ times} \\ \hline 88 \\ +87 \\ \hline 175 \end{array}$$

The value of the denominator of this fraction is equal to $459 + 32 = 491$. The problem then becomes

$$\frac{75}{\frac{175}{491}} = \frac{75}{1} \div \frac{175}{491} = \frac{75}{1} \times \frac{491}{175} = \frac{\overset{3}{\not{7}\not{5}} \times 491}{\underset{7}{\not{1}\not{7}\not{5}}} = \frac{1,473}{7} = 210\frac{3}{7}. \quad \text{Ans.}$$

(75) 1 plus $.001 = 1.001$. $.01$ plus $.000001 = .010001$. And $1.001 - .010001 =$

$$\begin{array}{r} 1.001 \\ .010001 \\ \hline .990999 \end{array} \quad \text{Ans.}$$

(76) $.49175 \times 30 = 14.7525$ pounds. Ans.

(77) If the cars at the shaft were the same size as those at the slope, the number of cars from the shaft would then be three times the number from the slope, its output being 3 times that of the slope. But the capacity of the shaft cars is double that of the slope cars; hence, instead of there being 3 times the number of cars coming from the shaft as from the slope, there are only $\frac{3}{2}$ times as many cars. Or $500 \times \frac{3}{2} = 750$ cars. Ans.

(78) $\frac{26}{5.2} = 5$ inches. Ans.

(79) $133.68 \times 1,728 = 230,999.04$ cu. in.

$\frac{230,999.04}{231} = 1,000$ gallons, nearly. Ans.

(80)

Cage,	1,850 pounds.
Car,	967 pounds.
Coal,	1,235 pounds.
Total load,	4,052 pounds.
Friction, $\frac{4,052}{4}$ =	1,013 pounds.
Rope, $.88 \times 250$ =	220 pounds.
Total strain,	5,285 pounds. Ans.

(81) $800 \times 1\frac{1}{2} = 1,200$ gallons pumped before water is turned on.

800 gallons flowing in per hr.
$(5 + 3) \times 60 = 480$ gallons flowing out per hr.
Difference $= 320$ gallons, net quantity flowing in per hr

$\frac{2,000 - 1,200}{320} = 2\frac{1}{2}$ hours. $1\frac{1}{2} + 2\frac{1}{2} = 4$ hours. Ans.

(82) $900 \times \frac{2}{3} = 600$ tons, to railroad. Ans.

$900 \times \frac{1}{4} = 225$ tons, to private trade. Ans.

$600 + 225 = 825$ tons.

$900 - 825 = 75$ tons, to team sales. Ans.

(83) $720 \times \frac{2}{3} = 480$ tons, lump.
$720 \times \frac{1}{3} = 240$ tons, screenings.
$240 \times \frac{2}{3} = 160$ tons, nut.
$240 \times \frac{1}{3} = 80$ tons, steam coal.

$480 \times \$1.50 = \$720$
$160 \times 1.00 = 160$
$80 \times .25 = 20$

Total sales $= \$900$ Ans.

(84) $\$7,230 \div 328 = \$22.04$ per foot. Ans.

(85) $62.5 \times 1.27 = 79.375$ pounds. Ans.

(86) $2,000 \div 93.5 = 21.39$ cu. ft. Ans.

(87) $.49175 \div .03617 = 13.5955$ cu. in. Ans.

(88) $1 - \frac{5}{8} = \frac{3}{8}$
$270 = \frac{3}{8}$ of the whole.
$\frac{270}{3} = 90 = \frac{1}{8}$ of the whole.
$90 \times 5 = 450 = \frac{5}{8}$ of the whole, or the negroes employed. Ans. 450 negroes.

(89) Pounds of coal mined $= 5,000 \times 344 = 1,720,000$ lb.
Each car contains $2,000 \times 20 = 40,000$ lb.
Number of cars $= 1,720,000 \div 40,000 = 43$. Ans.

(90) Lump, $25 \times 20 = 500$ tons.
Lump, $20 \times 16 = 320$ tons.

Total,	$820 \times 1.40 =$	\$1,148
Nut,	$15 \times 16 = 240 \times .90 =$	216
Steam,	$12 \times 16 = 192 \times .25 =$	48
Total receipts,		\$1,412 Ans.

(91) Props, $1,000 \times 4 = 4,000$ feet.
Props, $1,200 \times 5 = 6,000$ feet.

Total,	$10,000 \times .01 =$	\$100.00
Caps,	$\frac{2,640 \times 7.50}{1,000} =$	19.80
Total cost,		\$119.80 Ans.

ARITHMETIC.

(PART 2.)

(92) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. Hence, 25% of 8,428 lb. $= .25 \times 8{,}428$ lb. $= 2{,}107$ lb. Ans.

(93) Here $100 is the base and $1\% = .01$ is the rate. Then, $.01 \times \$100 = \$1$. Ans.

(94) $\frac{1}{2}\%$ means one-half of one per cent. Since 1% is .01, $\frac{1}{2}\%$ is .005, for, $2\overline{).010}$ giving $.005$. And $.005 \times \$35{,}000 = \$175$. Ans.

(95) Here 50 is the base, 2 is the percentage, and it is required to find the rate. Applying rule, Art. **193,**

$$\text{rate} = \text{percentage} \div \text{base};$$
$$\text{rate} = 2 \div 50 = .04 \text{ or } 4\%. \quad \text{Ans.}$$

(96) By Art. **193,** rate = percentage ÷ base.* As percentage = 10 and base = 10, we have rate = 10 ÷ 10 = 1 = 100%. Hence, 10 is 100% of 10. Ans.

(97) (*a*) Rate = percentage ÷ by base. Art. **193.** As percentage = $176.54 and base = $2,522, we have

$$\text{rate} = 176.54 \div 2{,}522 = .07 = 7\%. \quad \text{Ans.}$$

$$2522\overline{)176.54}$$
$$.07$$

* Remember that an expression of this form means that the first term is to be *divided by* the second term. Thus, as above, it means percentage *divided by* base.

(*b*) Base = percentage ÷ rate. Art. **192.**
As percentage = 16.96 and rate = 8% = .08, we have

$$\text{base} = 16.96 \div .08 = 212. \quad \text{Ans.}$$

```
.08 ) 16.96
      212
```

(*c*) Amount is the sum of the base and percentage; hence, the percentage = amount minus the base.

Amount = 216.7025 and base = 213.5; hence, percentage = 216.7025 − 213.5 = 3.2025.

Rate = percentage ÷ base. Art. **193.**

Therefore, rate = 3.2025 ÷ 213.5 = .015 = $1\frac{1}{2}$%. Ans.

```
213.5 ) 3.2025 ( .015 = 1½%
        2135
        10675
        10675
```

(*d*) The difference is the remainder found by subtracting the percentage from the base; hence, base − the difference = the percentage. Base = 207 and difference = 201.825, hence percentage = 207 − 201.825 = 5.175.

Rate = percentage ÷ base. Art. **193.**

$$\text{Therefore, rate} = 5.175 \div 207 = .025 = .02\frac{1}{2} = 2\frac{1}{2}\%. \quad \text{Ans.}$$

```
207 ) 5.175 ( .025
      414
      1035
      1035
```

(98) In this problem $5,500 is the amount, since it equals what he paid for the farm + what he gained; 15% is the rate, and the cost (to be found) is the base. Applying rule, Art. **197,**

base = amount ÷ (1 + rate); hence,
base = $5,500 ÷ (1 + .15) = $4,782.61. Ans.

```
1.15)5500.0000(4782.61
     460
     ---
      900
      805
      ---
       950
       920
       ---
        300
        230
        ---
         700
         690
         ---
          100
          115
          ---
```

The example can also be solved as follows: 100% = cost; if he gained 15%, then 100 + 15 = 115% = \$5,500, the selling price.

If 115% = \$5,500, 1% = $\frac{1}{115}$ of \$5,500 = \$47.8261, and 100%, or the cost, = 100 × \$47.8261 = \$4,782.61. Ans.

(99)

24 % of \$950	= .24 × 950 =	\$228
$12\frac{1}{2}$% of \$950	= .125 × 950 =	118.75
17 % of \$950	= .17 × 950 =	161.50
$53\frac{1}{2}$% of \$950	=	\$508.25

The total amount of his yearly expenses, then, is \$508.25, hence his savings are \$950 − \$508.25 = \$441.75. Ans.

Or, as above, 24% + $12\frac{1}{2}$% + 17% = $53\frac{1}{2}$%, the total percentage of expenditures; hence, 100% − $53\frac{1}{2}$% = $46\frac{1}{2}$% = per cent. saved. And \$950 × .465 = \$441.75 = his yearly savings. Ans.

(100) The percentage is 961.38, and the rate is $.37\frac{1}{2}$. By Art. **192,**

Base = percentage ÷ rate

= 961.38 ÷ .375 = 2,563.68, the number. Ans.

Another method of solving is the following:

If $37\frac{1}{2}\%$ of a number is 961.38, then $.37\frac{1}{2}$ times the number = 961.38 and the number = $961.38 \div .37\frac{1}{2}$, which, as above = 2,563.68. Ans.

```
.375)961.38000(2563.68
     750
     ----
     2113
     1875
     ----
      2388
      2250
      ----
       1380
       1125
       ----
        2550
        2250
        ----
         3000
         3000
         ----
```

(101) Here $1,125 is 30% of some number; hence, $1,125 = the percentage, 30% = the rate, and the required number is the base. Applying rule, Art. **192,**

Base = percentage ÷ rate = $1,125 ÷ .30 = $3,750.

Since $3,750 is $\frac{3}{4}$ of the property, one of the fourths is $\frac{1}{3}$ of $3,750 = $1,250, and $\frac{4}{4}$ or the entire property, is 4 × $1,250 = $5,000. Ans.

(102) Here $4,810 is the difference and 35% the rate. By Art. **198,**

Base = difference ÷ (1 − rate)
= $4,810 ÷ (1 − .35) = $4,810 ÷ .65 = $7,400. Ans.

```
.65)4810.00(7400
    455
    ---
     260          1.00
     260           .35
     ---           ---
      00           .65
```

Solution can also be effected as follows: 100% = the sum diminished by 35%, then (1 − .35) = .65, which is $4,810.

If 65% = \$4,810, 1% = $\frac{1}{65}$ of 4,810 = \$74, and 100% = 100 × \$74 = \$7,400. Ans.

(103) In this example the sales on Monday amounted to \$197.55, which was $12\frac{1}{2}$% of the sales for the entire week; i. e., we have given the percentage, \$197.55, and the rate, $12\frac{1}{2}$%, and the required number (or the amount of sales for the week) equals the base. By Art. **192,**

Base = percentage ÷ rate = \$197.55 ÷ .125;

or,

```
.125)197.5500(1580.4  Ans.
     125
     ---
      725
      625
      ----
      1005
      1000
      -----
        500
        500
        ---
```

Therefore, base = \$1,580.40, which also equals the sales for the week.

(104) 16.5 miles = $12\frac{1}{2}$% of the entire length of the road. We wish to find the *entire* length.

16.5 miles is the percentage, $12\frac{1}{2}$% is the rate, and the entire length will be the base. By Art. **192,**

Base = percentage ÷ rate = 16.5 ÷ $.12\frac{1}{2}$.

```
.125)16.500(132 miles.  Ans.
     125
     ---
      400
      375
      ---
       250
       250
       ---
```

(105) Here we have given the difference, or \$35, and the rate, or 60%, to find the base. We use the rule in Art. **198**,

$$\text{Base} = \text{difference} \div (1 - \text{rate})$$

$$= \$35 \div (1 - .60) = \$35 \div .40 = \$87.50. \quad \text{Ans.}$$

```
.40)35.000(87.5
     320
     ---
      300
      280
      ---
       200
       200
       ---
```

Or, 100% = whole debt; 100% − 60% = 40% = \$35.

If 40% = \$35, then $1\% = \frac{1}{40}$ of $\$35 = \frac{35}{40}$, and 100% = $\frac{35}{40} \times 100 = \$87.50$. Ans.

(106) 28 rd. 4 yd. 2 ft. 10 in. to inches.

```
  × 5½
  ----
   154
  +  4
  ----
   158 yards
  ×  3
  ----
   474
  +  2
  ----
   476 feet
  × 12
  ----
  5712
  + 10
  ----
  5722 inches. Ans.
```

Since there are $5\frac{1}{2}$ yards in one rod, in 28 rods there are $28 \times 5\frac{1}{2}$ or 154 yards; 154 yards plus 4 yards = 158 yards. There are 3 feet in one yard; therefore, in 158 yards there are 3×158 or 474 feet; 474 feet + 2 feet = 476 feet. There are 12 inches in one foot, and in 476 feet there are 12×476 or 5,712 inches; 5,712 inches + 10 inches = 5,722 inches. Ans.

(107)

```
12 ) 5722 inches.
 3 ) 476 + 10 inches.
5½ ) 158 + 2 feet.
      28 + 4 yards.
```

Ans. = 28 rd. 4 yd. 2 ft. 10 in.

EXPLANATION.—There are 12 inches in 1 foot; hence, in 5,722 inches there are as many feet as 12 is contained times in 5,722 inches, or 476 ft. and 10 inches remaining. Write these 10 inches as a remainder. There are 3 feet in 1 yard; hence, in 476 feet there are as many yards as 3 is contained times in 476 feet, or 158 yards and 2 feet remaining. There are $5\frac{1}{2}$ yards in one rod; hence, in 158 yards there are 28 rods and 4 yards remaining. Then, in 5,722 inches there are 28 rd. 4 yd. 2 ft. 10 in.

(108)

$$\begin{array}{rl} 5 & \text{weeks 3.5 days.} \\ \times\ 7 & \\ \hline 35 & \text{days in 5 weeks.} \\ +\ 3.5 & \\ \hline 38.5 & \text{days.} \end{array}$$

Then, we find how many seconds there are in 38.5 days.

$$\begin{array}{rl} 38.5 & \text{days} \\ \times\ 24 & \text{hours in one day.} \\ \hline 1540 & \\ 770 & \\ \hline 924.0 & \text{hours in 38.5 days.} \\ \times\ 60 & \text{minutes in one hour.} \\ \hline 55440 & \text{minutes in 38.5 days.} \\ \times\ 60 & \text{seconds in one minute.} \\ \hline 3326400 & \text{seconds in 38.5 days. Ans.} \end{array}$$

(109) Since there are 24 gr. in 1 pwt., in 13,750 gr. there are as many pennyweights as 24 is contained times in 13,750, or 572 pwt. and 22 gr. remaining. Since there are 20 pwt. in 1 oz., in 572 pwt. there are as many ounces as 20 is contained times in 572, or 28 oz. and 12 pwt. remaining.

Since there are 12 oz. in 1 lb. (Troy), in 28 oz. there are as many pounds as 12 is contained times in 28, or 2 lb. and 4 oz. remaining. We now have the pounds and ounces required by the problem; therefore, in 13,750 gr. there are 2 lb. 4 oz. 12 pwt. 22 gr.

```
24 ) 13750 gr.
     20 ) 572 pwt. + 22 gr.
          12 ) 28 oz. + 12 pwt.
                2 lb. + 4 oz.
```

Ans. = 2 lb. 4 oz. 12 pwt. 22 gr.

(110)

```
100 ) 4763254 li.
     80 ) 47632 + 54 li.
              595 + 32 ch.
```

Ans. = 595 mi. 32 ch. 54 li.

EXPLANATION.—There are 100 li. in one chain; hence, in 4,763,254 li. there are as many chains as 100 is contained times in 4,763,254 li., or 47,632 ch. and 54 li. remaining. Write the 54 li. as a remainder. There are 80 ch. in one mile; hence, in 47,632 ch. there are as many miles as 80 is contained times in 47,632 ch., or 595 miles and 32 ch. remaining. Then, in 4,763,254 li. there are 595 mi. 32 ch. 54 li.

(111)

```
1728 ) 764325 cu. in.
        27 ) 442 + 549 cu. in.
              16 cu. yd. + 10 cu. ft.
```

Ans. = 16 cu. yd. 10 cu. ft. 549 cu. in.

EXPLANATION.—There are 1,728 cu. in. in one cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cu. ft. and 549 cu. in. remaining. Write the 549 cu. in. as a remainder. There are 27 cu. ft. in one cubic yard; hence, in 442 cu. ft. there are as many cubic yards as 27 is contained times in 442 cu. ft., or 16 cu. yd. and 10 cu. ft. remaining. Then, in 764,325 cu. in. there are 16 cu. yd. 10 cu. ft. 549 cu. in.

(112) We must arrange the different terms in columns, taking care to have like denominations in the same column.

	rd.	yd.	ft.	in.	
	2	2	2	3	
		4	1	9	
			2	7	
	3	$2\frac{1}{2}$	0	7	
or	3	2	2	1	Ans.

EXPLANATION.—We begin to add at the right-hand column. $7 + 9 + 3 = 19$ in.; as 12 in. make one foot, 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column, and reserve the 1 ft. to add to the next column.

1 (reserved) + 2 + 1 + 2 = 6 ft. Since 3 ft. make 1 yard, 6 ft. = 2 yd. and 0 ft. remaining. Place the cipher in the column of feet and reserve the 2 yd. for the next column.

2 (reserved) + 4 + 2 = 8 yd. Since $5\frac{1}{2}$ yd. = 1 rod, 8 yd. = 1 rd. and $2\frac{1}{2}$ yd. Place $2\frac{1}{2}$ yd. in the yards column, and reserve 1 rd. for the next column; 1 (reserved) + 2 = 3 rd.

Ans. =	3 rd.	$2\frac{1}{2}$ yd.	0 ft.	7 in.	
or,	3 rd.	2 yd.	1 ft.	13 in.	
or,	3 rd.	2 yd.	2 ft.	1 in.	Ans.

(113) We write the compound numbers so that the units of the same denomination shall stand in the same column. Beginning to add with the lowest denomination, we find that the sum of the gills is $1 + 2 + 3 = 6$. Since there are 4 gi. in 1 pint, in 6 gi. there are as many pints as 4 is contained times in 6, or 1 pt. and 2 gi. We place 2 gi. under the gills column and reserve the 1 pt. for the pints column; the sum of the pints is 1 (reserved) + 5 + 1 + 1 = 8. Since there are 2 pt. in 1 quart, in 8 pt. there are as many quarts as 2 is contained times in 8, or 4 qt. and 0 pt. We place the cipher under the column of pints and reserve the 4 for the quarts column. The sum of the quarts is 4 (reserved) + 8 + 3 = 15. Since there are 4 qt. in 1 gallon, in 15 qt. there are as many gallons as 4 is contained times in 15, or 3 gal. and 3 qt. remaining. We now place the 3 under the quarts column and reserve the 3 gal. for the gallons column. The sum of the gallons column is 3 (reserved) + 4 + 6 + 3 = 16 gal. Since we can not reduce 16 gal. to any higher denomination, we have 16 gal. 3 qt. 0 pt. and 2 gi. for the answer.

gal.	qt.	pt.	gi.
3	3	1	3
6	0	1	2
4	0	0	1
	8	5	0
16 gal.	3 qt.	0 pt.	2 gi.

(114) Reduce the grains, pennyweights, and ounces to higher denominations.

24) 240 gr.	20) 125 pwt.	12) 50 oz.
10 pwt.	6 oz. 5 pwt.	4 lb. 2 oz.

Then, 3 lb. + 4 lb. 2 oz. + 6 oz. 5 pwt. + 10 pwt. =

lb.	oz.	pwt.	
3			
4	2		
	6	5	
		10	
7 lb.	8 oz.	15 pwt.	Ans.

(115) Since "seconds" is the lowest denomination in this problem, we find their sum first, which is 11 + 29 + 25 + 30 + 12, or 107 seconds. Since there are 60 seconds in 1 minute, in 107″ there are as many minutes as 60 is contained times in 107, or 1 minute and 47 seconds remaining. We place the 47 under the seconds column and reserve the 1 for the minutes column. The sum of the minutes is 1 (reserved) + 17 + 26 + 19 + 16, or 79. Since there are 60 minutes in 1 degree, in 79 minutes there are as many degrees as 60 is contained times in 79, or 1 degree and 19 minutes remaining. We place the 19 under the minutes column and reserve the 1 degree for the degrees column. The sum of the degrees is 1 (reserved) + 10 + 20 + 13 + 11, or 55 degrees. Since we can not reduce 55 degrees to any higher denominations, we have 55° 19′ 47″ for the answer.

deg.	min.	sec.
11	16	12
13	19	30
20	0	25
0	26	29
10	17	11
55°	19′	47″

(116) Since "inches" is the lowest denomination in this problem, we find their sum first, which is 11 + 8 + 6, or 25 inches. Since there are 12 inches in 1 foot, in 25 inches there are as many feet as 12 is contained times in 25, or 2 feet and 1 inch remaining. Place the 1 inch under the inches column, and reserve the 2 feet to add to the column of feet. The sum of the feet is 2 feet (reserved) + 2 + 1 = 5 feet. Since there are 3 feet in 1 yard, in 5 feet there are as many yards as 3 is contained times in 5 feet, or 1 yard and 2 feet remaining. Place the 2 feet under the column of feet, and reserve the 1 yard to add to the column of yards. The sum of the yards is 1 yard (reserved) + 4 + 5 = 10 yards. Since there are $5\frac{1}{2}$ yards in 1 rod, in 10 yards there are as many rods as $5\frac{1}{2}$ is contained times in 10, or 1 rod and $4\frac{1}{2}$ yards remaining. Place the $4\frac{1}{2}$ yards under the column of yards, and reserve the 1 rod for the column of rods. The sum of the rods is 1 (reserved) + 304 + 215 + 130 = 650 rods. Place 650 rods under the column of rods. Therefore, the sum is 650 rd. $4\frac{1}{2}$ yd. 2 ft. 1 in. Or, since $\frac{1}{2}$ a yard = 1 ft. 6 in., and since there are 320 rods in 1 mile, the sum may be expressed as 2 mi. 10 rd. 5 yd. 0 ft. 7 in. Ans.

	mi.	rd.	yd.	ft.	in.	
		130	5	1	6	
		215	0	2	8	
		304	4	0	11	
		650	$4\frac{1}{2}$	2	1	
or,	2	10	5	0	7	Ans.

(117) Since "square links" is the lowest denomination in this problem, we find their sum first, which is 21 + 23 + 16 + 18 + 23 + 21, or 122 square links. Place 122 square links under the column of square links. The sum of the square rods is 2 + 3 + 2 + 2 + 2 + 3, or 14 square rods. Place 14 square rods under the column of square rods. The sum of the square chains is 323 square chains. Since there are 10 square chains in 1 acre, in 323 square chains there are as many acres as 10 is

A.	sq. ch.	sq. rd.	sq. li.
21	67	3	21
28	78	2	23
47	6	2	18
56	59	2	16
25	38	3	23
46	75	2	21
255	3	14	122

contained times in 323 square chains, or 32 acres and 3 square chains remaining. Place 3 square chains under the column of square chains, and reserve the 32 acres to add to the column of acres. The sum of the acres is 32 acres (reserved) + 46 + 25 + 56 + 47 + 28 + 21, or 255 acres. Place 255 acres under the column of acres. Therefore, the sum is 255 A. 3 sq. ch. 14 sq. rd. 122 sq. li. Ans.

(118) Before we can subtract 300 ft. from 20 rd. 2 yd. 2 ft. and 9 in., we must reduce the 300 ft. to higher denominations.

Since there are 3 feet in 1 yard, in 300 feet there are as many yards as 3 is contained times in 300, or 100 yards. There are $5\frac{1}{2}$ yards in 1 rod, hence in 100 yards there are as many rods as $5\frac{1}{2}$ or $\frac{11}{2}$ is contained times in $100 = 18\frac{2}{11}$ rods.

$$100 \div \frac{11}{2} = 100 \times \frac{2}{11} = \frac{100 \times 2}{11} = \frac{200}{11}$$

```
11 ) 200 ( 18 2/11 rd.
     11
     --
      90
      88
      --
       2
```

Since there are $5\frac{1}{2}$ or $\frac{11}{2}$ yards in 1 rod, in $\frac{2}{11}$ rods there are $\frac{2}{11} \times \frac{11}{2}$, or one yard, so we find that 300 feet equals 18 rods and 1 yard. The problem now is as follows: From 20 rd. 2 yd. 2 ft. and 9 in. take 18 rd. and 1 yd.

We place the smaller number under the larger one, so that units of the same denomination fall in the same column. Beginning with the lowest denomination, we see that 0 inches from 9 inches leaves 9 inches. Going to the next higher denomination, we see that 0 feet from 2 feet leaves 2 feet. Subtracting 1 yard from 2

rd.	yd.	ft.	in.
20	2	2	9
18	1	0	0
2	1	2	9

yards, we have 1 yard remaining, and 18 rods from 20 rods leaves 2 rods. Therefore, the difference is 2 rd. 1 yd. 2 ft. 9 in. Ans.

(119)

A.	sq. rd.	sq. yd.	
114	80	25	
75	70	30	
39	9	$25\frac{1}{4}$	Ans.

EXPLANATION.—Place the subtrahend under the minuend so that like denominations are under each other. Then begin at the right with the lowest denomination. We can not subtract 30 from 25, so we take one square rod $(= 30\frac{1}{4}$ square yards) from 80 square rods, leaving 79 square rods; adding $30\frac{1}{4}$ square yards to 25 square yards, we have $55\frac{1}{4}$ square yards; subtracting 30 from $55\frac{1}{4}$ square yards leaves $25\frac{1}{4}$ square yards; we now subtract 70 square rods from 79 square rods, which leaves 9 square rods; next, we subtract 75 acres from 114 acres, which leaves 39 acres, which we place under the column of acres.

(120) If 10 gal. 2 qt. and 1 pt. of molasses are sold from a hogshead at one time, and 26 gal. 3 qt. are sold at another time, then the total amount of molasses sold equals 10 gal. 2 qt. 1 pt. plus 26 gal. 3 qt.

Since the pint is the lowest denomination, we add the pints first, which equal 0 + 1, or 1 pint. We can not reduce 1 pint to any higher denomination, so we place it under the pint column. The number of quarts is 3 + 2, or 5. Since there are 4 quarts in 1 gallon, in 5 quarts there are as many gallons as 4 is contained times in 5, or 1 gallon and 1 quart remaining. We place the 1 quart under the quart column, and reserve the 1 gallon to add to the column of

gal.	qt.	pt.
10	2	1
26	3	0
37 gal.	1 qt.	1 pt.

gallons. The number of gallons equals 1 (reserved) + 26 + 10, or 37 gallons.

If 37 gal. 1 qt. and 1 pt. are sold from a hogshead of molasses (63 gal.), there remains the difference between 63 gal. and 37 gal. 1 qt. 1 pt., or 25 gal. 2 qt. and 1 pt.

63 gal. is the same as 62 gal. 3 qt. 2 pt., since 1 gal. equals 4 qt. and 1 qt. = 2 pt.

Beginning with the lowest denomination, 1 pt. from the 2 pt. 1 pint from 2 pints leaves 1 pint. One quart from 3 quarts leaves 2 quarts, and 37 gallons from 62 gallons leaves 25 gallons. Therefore, there are 25 gal. 2 qt. and 1 pt. of molasses remaining in the hogshead. Ans.

gal.	qt.	pt.
62	3	2
37	1	1
25	2	1

(121) If a person were born June 19, 1850, in order to find how old he would be on Aug. 3, 1892, subtract the earlier date from the later date.

On August 3, 7 mo. and 3 da. have elapsed from the beginning of the year, and on June 19, 5 mo. and 19 da.

Beginning with the lowest denomination, we find that 19 days can not be taken from 3 days, so we take 1 month from 7 months. The 1 month which we took equals 30 days, for in all cases 30 days are allowed to a month. Adding 30 days to the 3 days, we have 33 days; subtracting 19 days from 33 days, we have 14 days remaining. Since we borrowed 1 month from the months column, we have 7 − 1, or 6 months remaining; subtracting 5 months from 6 months, we have 1 month remaining. 1850 from 1892 leaves 42 years. Therefore, he would be 42 years 1 month and 14 days old. Ans.

yr.	mo.	da.
1892	7	3
1850	5	19
42	1	14

(122) If a note given Aug. 5, 1890, were paid June 3, 1892, in order to find the length of time it was due, subtract the earlier date from the later date.

Beginning with the lowest denomination, we find that 5 can not be subtracted from 3, so we take a unit from the next

yr.	mo.	da.
1892	5	3
1890	7	5
1	9	28

higher denomination, which is months. The 1 month which we take equals 30 days. Adding the 30 days to the 3 days, we have 33 days. 5 days from 33 days leaves 28 days. Since we took 1 month from the months column, only 4 months remain. 7 months cannot be taken from 4 months, so we take 1 year from the years column, which equals 12 months. 12 months + 4 months = 16 months. 7 months from 16 months = 9 months. Since we took 1 year from the years column, we have 1892 − 1, or 1891 remaining. 1890 from 1891 leaves 1 year. Hence, the note ran 1 year 9 months and 28 days. Ans.

(123) Write the number of the year, month, day, hour, and minute of the earlier date under the year, month, day, hour, and minute of the later date, and subtract.

22 minutes before 8 o'clock is the same as 38 minutes after 7 o'clock. 7 o'clock P. M. is 19 hours from the beginning of the day, as there are 12 hours in the morning and 7 in the afternoon. December is 11 months from the beginning of the year.

10 o'clock A. M. is 10 hours from the beginning of the day. July is 6 months from the beginning of the year. The minuend would be the later date, or 1,888 years, 11 months, 11 days, 19 hours, and 38 minutes.

The subtrahend would be the earlier date, or 1,883 years, 6 months, 3 days, 10 hours, and 16 minutes.

Subtracting, we have

yr.	mo.	da.	hr.	min.
1888	11	11	19	38
1883	6	3	10	16
5	5	8	9	22

or, 5 yr. 5 mo. 8 da. 9 hr. and 22 min. Ans.

16 minutes subtracted from 38 minutes leaves 22 minutes; 10 hours from 19 hours leaves 9 hours; 3 days from 11 days leaves 8 days; 6 months subtracted from 11 months leaves 5 months; 1,883 from 1,888 leaves 5 years.

(124) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

17 ft.	3 in.
	51
879 ft.	9 in.

and begin at the right to multiply. $51 \times 3 = 153$ in. As there are 12 inches in 1 foot, in 153 in. there are as many feet as 12 is contained times in 153, or 12 feet and 9 inches remaining. Place the 9 inches under the inches, and reserve the 12 feet. 51×17 ft. $= 867$ ft. 867 ft. $+ 12$ ft. (reserved) $= 879$ ft.

879 feet can be reduced to higher denominations by dividing by 3 feet to find the number of yards, and by $5\frac{1}{2}$ yards to find the number of rods.

3) 8 7 9 ft. 9 in.
5.5) 2 9 3 yd.
5 3 rd. $1\frac{1}{2}$ yd.

Then, answer $=$ 53 rd. $1\frac{1}{2}$ yd. 0 ft. 9 in.; or 53 rd. 1 yd. 2 ft. 3 in

(125)

	qt.	pt.	gi.	
	3	1	3	
			4.7	
	1 8.2 qt.	0	.1	
or,	1 8 qt.	0 pt.	1.7 gi.	
or, 4 gal.	2 qt.	0 pt.	1.7 gi.	Ans.

Place the multiplier under the lowest denomination of the multiplicand, and proceed to multiply. 4.7×3 gi. $= 14.1$ gi. As 4 gi. $= 1$ pt., there are as many pints in 14.1 gi. as 4 is contained times in $14.1 = 3.5$ pt. and .1 gi. over. Place .1 under gills and carry the 3.5 pt. forward. 4.7×1 pt. $= 4.7$ pt.; $4.7 + 3.5$ pt. $= 8.2$ pt. As 2 pt. $= 1$ qt., there are as many quarts in 8.2 pt. as 2 is contained times in $8.2 = 4.1$ qt. and no pints over. Place a cipher under the pints, and carry the 4.1 qt. to the next product. 4.7×3 qt. $= 14.1$; $14.1 + 4.1 = 18.2$ qt. The answer now is 18.2 qt. 0 pt. .1

gi. Reducing the fractional part of a quart, we have 18 qt. 0 pt. 1.7 gi. (.2 qt. $= .2 \times 8 = 1.6$ gi.; $1.6 + .1$ gi. $= 1.7$ gi.). Then, we can reduce 18 qt. to gallons ($18 \div 4 = 4$ gal. and 2 qt.) $=$ 4 gal. 2 qt. 1.7 gi. Ans.

The answer may be obtained in another and much easier way by reducing all to gills, multiplying by 4.7, and then changing back to quarts and pints. Thus,

$$\begin{array}{rl} & 3 \text{ qt.} \\ \times & 2 \text{ pt.} \\ \hline & 6 \text{ pt.} \\ + & 1 \text{ pt.} \\ \hline & 7 \text{ pt.} \\ \times & 4 \text{ gi.} \\ \hline & 28 \text{ gi.} \\ + & 3 \text{ gi.} \\ \hline & 31 \text{ gi.} \end{array}$$

3 qt. 1 pt. 3 gi. $=$ 31 gi.
31 gi. $\times 4.7 = 145.7$ gi.

$$\begin{array}{r|l} 4 & 145.7 \text{ gi.} \\ \hline 2 & 36 \text{ pt.} + 1.7 \text{ gi.} \\ \hline & 18 \text{ qt.} + 0 \text{ pt.} \end{array}$$

Ans. $=$ 18 qt. 1.7 gi.;
or, 4 gal. 2 qt. 1.7 gi.

(126) (3 lb. 10 oz. 13 pwt. 12 gr.) $\times 1.5 = ?$

$$\begin{array}{rl} & 3 \text{ lb. } 10 \text{ oz. } 13 \text{ pwt. } 12 \text{ gr.} \\ \times & 12 \\ \hline & 36 \text{ oz.} \\ + & 10 \\ \hline & 46 \text{ oz.} \\ \times & 20 \\ \hline & 920 \text{ pwt.} \\ + & 13 \\ \hline & 933 \text{ pwt.} \\ \times & 24 \\ \hline & 22392 \text{ gr.} \\ + & 12 \\ \hline & 22404 \text{ gr.} \end{array}$$

22,404 gr. $\times 1.5 = 33,606$ gr.

$$\begin{array}{r|l} 24 & 33606 \text{ gr.} \\ \hline 20 & 1400 \text{ pwt.} + 6 \text{ gr.} \\ \hline 12 & 70 \text{ oz.} + 0 \text{ pwt.} \\ \hline & 5 \text{ lb.} + 10 \text{ oz.} \end{array}$$

Since there are 24 gr. in 1 pwt., in 33,606 gr. there are as many pwt. as 24 is contained times in 33,606, or 1,400 pwt. and 6 gr. remaining. This gives us the number of grains in the answer. We now reduce 1,400 pwt. to higher denominations. Since there are 20 pwt. in 1 oz., in 1,400 pwt. there are as many ounces as 20 is contained times in 1,400, or 70 oz. and 0 pwt. remaining; therefore, there are 0 pwt. in the answer. We reduce 70 oz. to higher denominations. Since there are 12 oz. in 1 lb., in 70 oz. there are as many pounds as 12 is contained times in 70, or 5 lb. and 10 oz. remaining. We can not reduce 5 lb. to any higher denominations. Therefore, our answer is 5 lb. 10 oz. 6 gr.

Another but more complicated way of working this problem is as follows:

	lb.	oz.	pwt.	gr.	
	3	10	13	12	
				1.5	
	4.5	15	19.5	18	
or,	4	21	19	30	
or,	5	10	0	6	Ans.

To get rid of the decimal in the pounds, reduce .5 of a pound to ounces. Since 1 lb. = 12 oz., .5 of a pound equals .5 lb. × 12 = 6 oz. 6 oz. + 15 oz. = 21 oz. We now have 4 lb. 21 oz. 19.5 pwt. and 18 gr., but we still have a decimal in the column of pwt., so we reduce .5 pwt. to grains to get rid of it. Since 1 pwt. = 24 gr., .5 pwt. = .5 pwt. × 24 = 12 gr. 12 gr. + 18 gr. = 30 gr. We now have 4 lb. 21 oz. 19 pwt. and 30 gr. Since there are 24 gr. in 1 pwt., in 30 gr. there is 1 pwt. and 6 gr. remaining. Place 6 gr. under the column of grains and add 1 pwt. to the pwt. column. Adding 1 pwt., we have 19 + 1 = 20 pwt. Since there are 20 pwt. in 1 oz., we have 1 oz. and 0 pwt. remaining. Write the 0 pwt. under the pwt. column, and reserve the 1 oz. to the oz. column. 21 oz. + 1 oz. = 22 oz. Since there are 12 oz. in 1 lb., in 22 oz. there is 1 lb. and 10 oz. remaining. Write the 10 oz. under the ounce column, and reserve the 1 lb. to add to the lb. column. 4 lb. + 1 lb. (reserved) = 5 lb. Hence, the answer equals 5 lb. 10 oz. 6 gr.

(127) If each barrel of apples contains 2 bu. 3 pk. and 6 qt., then 9 bbl. will contain 9 × (2 bu. 3 pk. 6 qt.).

We write the multiplier under the lowest denomination of the multiplicand, which is quarts in this problem. 9 times 6 qt. equals 54 qt. There are 8 qt. in 1 pk., and in 54 qt. there are as many pecks as 8 is contained times in 54, or 6 pk. and 6 qt. We write the 6 qt. under the column of quarts, and reserve the 6 pk. to add to the product of the pecks. 9 times 3 pk. equals 27 pk.; 27 pk. plus the 6 pk. reserved equals 33 pk. Since there are 4 pk. in 1 bu., in 33 pk. there are as many bushels as 4 is contained times in 33, or 8 bu. and 1 pk. remaining. We write the 1 pk. under the column of pecks, and reserve the 8 bu. for the product of the bushels. 9 times 2 bu. plus the 8 bu. reserved equals 26 bu. Therefore, we find that 9 bbl. contain 26 bu. 1 pk. 6 qt. of apples. Ans.

	bu.	pk.	qt.
	2	3	6
			9
	18	27	54
or,	26	1	6

(128) (7 T. 15 cwt. 10.5 lb.) × 1.7 = ? When the multiplier is a decimal, instead of multiplying the denominate numbers as in the case when the multiplier is a whole number, it is much easier to reduce the denominate numbers to the lowest denomination given; then, multiply that result by the decimal, and, lastly, reduce the product to higher denominations. Although the correct answer can be obtained by working examples involving decimals in the manner as in the last example, it is much more complicated than this method.

$$
\begin{array}{rl}
 & 7\text{ T. } 15\text{ cwt. } 10.5\text{ lb.} \\
\times & \underline{20} \\
 & 140\ \text{cwt.} \\
 & \underline{15} \\
 & 155\ \text{cwt.} \\
\times & \underline{100} \\
 & 15500\ \text{lb.} \\
 & \underline{10.5} \\
 & 15510.5\ \text{lb.}
\end{array}
$$

15,510.5 lb. × 1.7 = 26,367.85 lb.

There are 100 lb. in 1 cwt., and in 26,367.85 lb. there are as many cwt. as 100 is contained times in 26,367.85, which equals 263 cwt. and 67.85 lb. remaining. Since we have the number of pounds for our answer, we reduce 263 cwt. to higher denominations.

```
100 ) 2 6 3 6 7.8 5 lb.
 20 ) 2 6 3 cwt. + 67.85 lb.
        1 3 T. + 3 cwt.
```

There are 20 cwt. in 1 ton, and in 263 cwt. there are as many tons as 20 is contained times in 263, or 13 tons and 3 cwt. remaining. Since we cannot reduce 13 tons any higher, our answer is 13 T. 3 cwt. 67.85 lb. Or, since .85 lb. = .85 lb. × 16 = 13.6 oz., the answer may be written 13 T. 3 cwt. 67 lb. 13.6 oz.

(129)

7)	358 A.	57 sq. rd.	6 sq. yd.	2 sq. ft.	
	51 A.	31 sq. rd.	0 sq. yd.	8 sq. ft.	Ans.

We begin with the highest denomination, and divide each term in succession by 7.

7 is contained in 358 A. 51 times and 1 A. remaining. We write the 51 A. under the 358 A. and reduce the remaining 1 A. to square rods = 160 sq. rd.; 160 sq. rd. + the 57 sq. rd. in the dividend = 217 sq. rd. 7 is contained in 217 sq. rd. 31 times and 0 sq. rd. remaining. 7 is not contained in 6 sq. yd., so we write 0 under the sq. yd. and reduce 6 sq. yd. to square feet. 9 sq. ft. × 6 = 54 sq. ft. 54 sq. ft. + 2 sq. ft. in the dividend = 56 sq. ft. 7 is contained in 56 sq. ft. 8 times. We write 8 under the 2 sq. ft. in the dividend.

(130)

12)	282 bu.	3 pk.	1 qt.	1 pt.	
	23 bu.	2 pk.	2 qt.	$\frac{1}{4}$ pt.	Ans.

12 is contained in 282 bu. 23 times and 6 bu. remaining. We write 23 bu. under the 282 bu. in the dividend, and reduce the remaining 6 bu. to pecks = 24 pk. + the 3 pk. in the dividend = 27 pk. 12 is contained in 27 pk. 2 times and 3 pk. remaining. We write 2 pk. under the 3 pk. in the dividend, and reduce the remaining 3 pk. to quarts. 3 pk. = 24 qt.; 24 qt. + the 1 qt. in the dividend = 25 qt. 12 is contained in 25 qt. 2 times and 1 qt. remaining. We write

2 qt. under the 1 qt. in the dividend, and reduce 1 qt. to pints = 2 pt. + the 1 pt. in the dividend = 3 pt. $3 \div 12 = \frac{3}{12}$ or $\frac{1}{4}$ pt.

(131) We must first reduce 23 miles to feet before we can divide by 30 feet. 1 mi. contains 5,280 ft.; hence, 23 mi. contain 5,280 × 23 = 121,440 ft.

121,440 ft. ÷ 30 ft. = 4,048 rails for 1 side of the track.

The number of rails for 2 sides of the track = 2 × 4,048, or 8,096 rails. Ans.

(132) In this case where both dividend and divisor are compound, reduce each to the lowest denomination mentioned in either and then divide as in simple numbers.

$$\begin{array}{rl} 1 \text{ bu. } 1 \text{ pk. } 7 \text{ qt.} & \\ \times 4 & \\ \hline 4 \text{ pk.} & \\ +1 & \\ \hline 5 \text{ pk.} & \\ \times 8 & \\ \hline 40 \text{ qt.} & \\ +7 & \\ \hline 47 \text{ qt.} & \end{array} \qquad \begin{array}{rl} 356 \text{ bu. } 3 \text{ pk. } 5 \text{ qt.} & \\ \times 4 & \\ \hline 1424 \text{ pk.} & \\ +3 & \\ \hline 1427 \text{ pk.} & \\ \times 8 & \\ \hline 11416 \text{ qt.} & \\ +5 & \\ \hline 11421 \text{ qt.} & \end{array}$$

$$\begin{array}{r} 47\,)\,11421\,(\,243 \\ 94 \\ \hline 202 \\ 188 \\ \hline 141 \\ 141 \\ \hline \end{array}$$

11,421 qt. ÷ 47 qt. = 243 boxes. Ans.

(133) We must first reduce 16 square miles to acres. In 1 sq. mi. there are 640 A., and in 16 sq. mi. there are 16 × 640 A. = 10,240 A.

$$\begin{array}{l} 62\,)\,10240 \text{ A.} \\ \hline 165 \text{ A. } 25 \text{ sq. rd. } 24 \text{ sq. yd. } 3 \text{ sq. ft. } 80+ \text{ sq. in. Ans.} \end{array}$$

62 is contained in 10,240 A. 165 times and 10 A. remaining. We write 165 A. under the 10,240 A. in the dividend and reduce 10 A. to sq. rd. In 1 A. there are 160 sq. rd., and in 10 A. there are $10 \times 160 = 1,600$ sq. rd. 62 is contained in 1,600 sq. rd. 25 times and 50 sq. rd. remaining. We write 25 sq. rd. in the quotient and reduce 50 sq. rd. to sq. yd. In 1 sq. rd. there are $30\frac{1}{4}$ sq. yd., and in 50 sq. rd. there are 50 times $30\frac{1}{4}$ sq. yd. $= 1,512\frac{1}{2}$ sq. yd. 62 is contained in $1,512\frac{1}{2}$ sq. yd. 24 times and $24\frac{1}{2}$ sq. yd. remaining. In 1 sq. yd. there are 9 sq. ft., and in $24\frac{1}{2}$ sq. yd. there are $24\frac{1}{2} \times 9 = 220\frac{1}{2}$ sq. ft. 62 is contained in $220\frac{1}{2}$ sq. ft. 3 times and $34\frac{1}{2}$ sq. ft. remaining. We write 3 sq. ft. in the quotient and reduce $34\frac{1}{2}$ sq. ft. to sq. in. In 1 sq. ft. there are 144 sq. in., and in $34\frac{1}{2}$ sq. ft. there are $34\frac{1}{2} \times 144 = 4,968$ sq. in. 62 is contained in 4,968 sq. in. 80 times and 8 sq. in. remaining.

We write 80 sq. in. in the quotient.

It should be borne in mind that it is only for the purpose of illustrating the method that this problem is carried out to square inches. It is not customary to reduce any lower than square rods in calculating the area of a farm.

(134) To square a number, we must multiply the number by itself once, that is, use the number twice as a factor. Thus, the second power of 108 is $108 \times 108 = 11,664$. Ans.

```
  108
  108
 ----
  864
108
-----
11664
```

(135)

```
          181.25
          181.25
          ------
          90625
         36250
        18125
      145000
      18125
     -----------
      32851.5625
          181.25
     -----------
     16425 78125
     65703 1250
    32851 5625
   2628125000
   328515625
   -------------
   5954345.703125
```

The third power of 181.25 equals the number obtained by using 181.25 as a factor three times. Thus, the third power of 181.25 is 181.25 × 181.25 × 181.25 = 5,954,345.703125. Ans.

Since there are 2 decimal places in the multiplier, and 2 in the multiplicand, there are 2 + 2 = 4 decimal places in the first product.

Since there are 4 decimal places in the multiplicand, and 2 in the multiplier, there are 4 + 2 = 6 decimal places in the final product.

(136)

```
             27.61
             27.61
             -----
             2761
           16566
          19327
          5522
          --------
          762.3121
             27.61
          --------
          7623121
        45738726
       53361847
      15246242
      -----------
      21047.437081
             27.61
      -------------
      21047437081
    126284622486
   147332059567
   42094874162
   -----------------
   581119.73780641
```

The fourth power of 27.61 is the number obtained by using 27.61 as a factor four times. Thus, the fourth power of 27.61 is 27.61 × 27.61 × 27.61 × 27.61 = 581,119.73780641. Ans.

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are 2 + 2 = 4 decimal places in the first product.

Since there are 4 decimal places in the multiplicand and 2 in the multiplier, there are 4 + 2 = 6 decimal places in the second product.

Since there are 6 decimal places in the multiplicand and 2 in the multiplier, there are 6 + 2 = 8 decimal places in the final product.

(**137**) (*a*) $106^2 = 106 \times 106 = 11,236$. Ans.

$$\begin{array}{r} 106 \\ 106 \\ \hline 636 \\ 1060 \\ \hline 11236 \end{array}$$

(*b*) $\left(182\frac{1}{8}\right)^2 = 182\frac{1}{8} \times 182\frac{1}{8} = 33,169.515625$. Ans.

$\frac{1}{8} = 8\overline{)1.000}$, $.125$

$$\begin{array}{r} 182.125 \\ 182.125 \\ \hline 910625 \\ 364250 \\ 182125 \\ 364250 \\ 1457000 \\ 182125 \\ \hline 33169.515625 \end{array}$$

Since there are 3 decimal places in the multiplier and 3 in the multiplicand, there are $3 + 3 = 6$ decimal places in the product.

(*c*) $.005^2 = .005 \times .005 = .000025$. Ans.

$$\begin{array}{r} .005 \\ .005 \\ \hline .000025 \end{array}$$ Ans.

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3 + 3 = 6$ decimal places in the product.

(*d*) $.0063^2 = .0063 \times .0063 = .00003969$. Ans.

$$\begin{array}{r} .0063 \\ .0063 \\ \hline 189 \\ 378 \\ \hline .00003969 \end{array}$$ Ans.

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4 + 4 = 8$ decimal places in the product.

(*e*) $10.06^2 = 10.06 \times 10.06 = 101.2036$. Ans.

$$\begin{array}{r} 10.06 \\ 10.06 \\ \hline 6036 \\ 100600 \\ \hline 101.2036 \end{array}$$

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, there are $2 + 2 = 4$ decimal places in the product.

(138) (*a*) $753^3 = 753 \times 753 \times 753 = 426{,}957{,}777$. Ans.

$$\begin{array}{r} 753 \\ \underline{753} \\ 2259 \\ 3765 \\ \underline{5271} \\ 567009 \\ \underline{753} \\ 1701027 \\ 2835045 \\ \underline{3969063} \\ 426957777 \end{array}$$

(*b*) $987.4^3 = 987.4 \times 987.4 \times 987.4 = 962{,}674{,}279.624$. Ans.

$$\begin{array}{r} 987.4 \\ \underline{987.4} \\ 39496 \\ 69118 \\ 78992 \\ \underline{88866} \\ 974958.76 \\ \underline{987.4} \\ 389983504 \\ 682471132 \\ 779967008 \\ \underline{877462884} \\ 962674279.624 \end{array}$$

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, there are $1 + 1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the multiplicand and one in the multiplier, there are $2 + 1 = 3$ decimal places in the final product.

(*c*) $.005^3 = .005 \times .005 \times .005 = .000000125$. Ans.

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3 + 3 = 6$ decimal places in the first product; but, as there are only 2 figures in the product, we prefix four ciphers to make the six decimal places.

$$\begin{array}{r} .005 \\ \underline{.005} \\ .000025 \\ \underline{.005} \\ .000000125 \end{array}$$

Since there are six decimal places in the multiplicand and 3 in the multiplier, there are $6 + 3 = 9$ decimal places in the final product. In this case we prefix six ciphers to form the nine decimal places.

(*d*) $.4044^3 = .4044 \times .4044 \times .4044 = .066135317184$. Ans.

```
        .4044
        .4044
        -----
        16176
       16176
     161760
    ---------
    .16353936
        .4044
    ---------
     65415744
    65415744
  654157440
-------------
.066135317184
```

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4 + 4 = 8$ decimal places in the first product.

Since there are 8 decimal places in the second multiplicand and 4 in the multiplier, there are $8 + 4 = 12$ decimal places in the final product; but, as there are only 11 figures in the product, we prefix 1 cipher to make 12 decimal places.

(139) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. Ans.

(140) $3^4 = 3 \times 3 \times 3 \times 3 = 81$. Ans.

(141) (*a*) $67.85^2 = 67.85 \times 67.85 = 4{,}603.6225$. Ans.

```
     67.85
     67.85
    ------
     33925
    54280
   47495
  40710
  ---------
  4603.6225  Ans.
```

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the product.

(*b*) $967{,}845^2 = 967{,}845 \times 967{,}845 = 936{,}723{,}944{,}025$. Ans.

```
          967845
          967845
          -------
         4839225
        3871380
       7742760
      6774915
     5807070
    8710605
    ------------
    936723944025
```

(*c*) A fraction may be raised to any power by raising both numerator and denominator to the required term.

$$\text{Thus, } \left(\frac{3}{8}\right)^2 = \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3}{8 \times 8} = \frac{9}{64}. \quad \text{Ans.}$$

(*d*) $\left(\frac{1}{4}\right)^2 = \frac{1}{4} \times \frac{1}{4} = \frac{1 \times 1}{4 \times 4} = \frac{1}{16}.$ Ans.

(142) (*a*) $5^{10} = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 =$ 9,765,625. Ans.

(*b*) $9^5 = 9 \times 9 \times 9 \times 9 \times 9 = 59{,}049.$ Ans.

$$\begin{array}{r}
5 \\
5 \\ \hline
25 \\
5 \\ \hline
125 \\
5 \\ \hline
625 \\
5 \\ \hline
3125 \\
5 \\ \hline
15625 \\
5 \\ \hline
78125 \\
5 \\ \hline
390625 \\
5 \\ \hline
1953125 \\
5 \\ \hline
9765625
\end{array}
\qquad\qquad
\begin{array}{r}
9 \\
9 \\ \hline
81 \\
9 \\ \hline
729 \\
9 \\ \hline
6561 \\
9 \\ \hline
59049
\end{array}$$

(143) (*a*) $1.2^4 = 1.2 \times 1.2 \times 1.2 \times 1.2 = 2.0736.$ Ans.

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off $1 + 1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off $2 + 1 = 3$ decimal places in the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off $3 + 1 = 4$ decimal places in the final product.

```
     1.2
     1.2
     ---
     2 4
   1 2
   -----
   1.4 4
     1.2
  ------
   2 8 8
  1 4 4
  ------
  1.7 2 8
      1.2
 --------
  3 4 5 6
 1 7 2 8
 --------
 2.0 7 3 6
```

(*b*) $11^6 = 11 \times 11 \times 11 \times 11 \times 11 \times 11 = 1{,}771{,}561$. Ans.

```
     11
     11
    ---
    121
     11
   ----
   1331
     11
  -----
  14641
     11
 ------
 161051
     11
-------
1771561
```

(*c*) $1^7 = 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$. Ans.

(*d*) $.01^4 = .01 \times .01 \times .01 \times .01 = .00000001$. Ans.

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, we must point off $2 + 2 = 4$ decimal places in the first product; but, as there is only 1 figure in the product, we prefix 3 ciphers to make the 4 necessary decimal places.

$$\begin{array}{r} .01 \\ .01 \\ \hline .0001 \\ .01 \\ \hline .000001 \\ .01 \\ \hline .00000001 \end{array}$$

Since there are 4 decimal places in the second multiplicand and 2 in the multiplier, we must point off $4 + 2 = 6$ decimal places in the second product. It is necessary to prefix 5 ciphers to make 6 decimal places.

Since there are 6 decimal places in the third multiplicand and 2 in the multiplier, we must point off $6 + 2 = 8$ decimal places in the product. It is necessary to prefix 7 ciphers to make 8 decimal places in the final product.

(*e*) $.1^5 = .1 \times .1 \times .1 \times .1 \times .1 = .00001$. Ans.

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off $1 + 1 = 2$ decimal places in the first product. It is necessary to prefix 1 cipher to the product.

$$\begin{array}{r} .1 \\ .1 \\ \hline .01 \\ .1 \\ \hline .001 \\ .1 \\ \hline .0001 \\ .1 \\ \hline .00001 \end{array}$$

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off $2 + 1 = 3$ decimal places in the second product. It is necessary to prefix 2 ciphers to the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off $3 + 1 = 4$ decimal places in the third product. It is necessary to prefix 3 ciphers to this product.

Since there are 4 decimal places in the fourth multiplicand and 1 in the multiplier, we must point off $4 + 1$ or 5 decimal places in the final product. It is necessary to prefix 4 ciphers to this product.

(144) (*a*) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637$. Ans.

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, we must point off $4 + 4 = 8$ decimal places in the product; but, as there are only 5 figures in the product, we prefix three ciphers to form the eight necessary decimal places in the first product.

$$\begin{array}{r}
.0133 \\
.0133 \\
\hline
399 \\
399 \\
133 \\
\hline
.00017689 \\
.0133 \\
\hline
53067 \\
53067 \\
17689 \\
\hline
.000002352637
\end{array}$$

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we must point off $8 + 4 = 12$ decimal places in the product; but, as there are only 7 figures in the product, we prefix 5 ciphers to make the 12 necessary decimal places in the final product.

(*b*) $301.011^3 = 301.011 \times 301.011 \times 301.011 =$ 27,273,890.942264331. Ans.

$$\begin{array}{r}
301.011 \\
301.011 \\
\hline
301011 \\
301011 \\
3010110 \\
9030330 \\
\hline
90607.622121 \\
301.011 \\
\hline
90607622121 \\
90607622121 \\
906076221210 \\
271822866363 0 \\
\hline
27273890.942264331
\end{array}$$

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, we must point off $3 + 3 = 6$ decimal places in the first product.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, we must point off $6 + 3 = 9$ decimal places in the final product.

(*c*) $\left(\frac{1}{8}\right)^3 = \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} = \frac{1 \times 1 \times 1}{8 \times 8 \times 8} = \frac{1}{512}$. Ans.

(*d*) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer, and multiply the denominators together for the denominator of the answer.

$$\left(3\frac{3}{4}\right)^3 = \frac{15}{4} \times \frac{15}{4} \times \frac{15}{4} = \frac{15 \times 15 \times 15}{4 \times 4 \times 4} = \frac{3,375}{64} = 52.734+.$$

Ans.

$$3\frac{3}{4} = \frac{3 \times 4 + 3}{4} = \frac{12 + 3}{4} = \frac{15}{4}.$$

15	64)3375.000(52.734+
15	320
75	175
15	128
225	470
15	448
1125	220
225	192
3375	280
	256
	24

Since *three* ciphers were annexed to the dividend, *three* decimal places must be pointed off in the quotient. It is easy to see that the next figure will be a 3; hence, write the sign +, as shown.

(**145**) Evolution is the reverse of involution. In involution we find the *power* of a number by multiplying the number by itself one or more times, while in evolution we find the *number* or *root* which was multiplied by itself one or more times to make the power.

(146) (*a*)

```
                 1
                 1
                 20
                  8
                 28
                  8
                 360
                   6
                 366
                   6
                 3720
                    7
                 3727
                    7
                 3734
√3'4 8'6 7'8 4.4 0'1 0 = 1867.29 +    Ans.
 1
 248
 224
  2467
  2196
   27184
   26089
3734 ) 1 0 9 5.0 0 0 (.293 or .29 +
        7 4 6 8
        3 4 8 2 0
        3 3 6 0 6
          1 2 1 4 0
```

EXPLANATION.—Applying the short method described in Art. **272**, we extract the root by the regular method to four figures, since there are six figures in the answer, and $6 \div 2 + 1 = 4$. The last remainder is 1095, and the last trial divisor (with the cipher omitted) is 3734. Dividing 1095 by 3734, as shown, the quotient is .293 +, or .29 + using two figures. Annexing to the root, gives 1,867.29 +. Ans.

```
(b)  (a)  3        √9'00'00'99.40'09'00 = 3000.0165 + Ans.
          3  (b)   9
          ―        ―――――――――――――――
     (d)  60 (c)   0000994009
           0            600001
          ―――         ――――――――
          600          39400800
            0          36000156
          ――――         ――――――――
          6000          3400644
             0
          ―――――
          60000
              0
          ――――――
          600000
               1
          ――――――
          600001
               1
          ―――――――
          6000020
                6
          ―――――――
          6000026
```

EXPLANATION.—Beginning at the decimal point we point off the whole number into periods of *two* figures each, proceeding from *right* to *left;* also, point off the decimal into periods of *two* figures each, proceeding from *left* to *right.* The largest number whose square is contained in the first period, 9, is 3; hence, 3 is the first figure of the root. Place 3 at the left, as shown at (*a*), and multiply it by the first figure in the root, or 3. The result is 9. Write 9 under the first period, 9, as at (*b*), subtract, and there is no remainder. Bring down the next period, which is 00, as shown at (*c*). Add the root already found to the 3 at (*a*), obtaining 6, and annex a cipher to this 6, thus making it 60, which is the *trial divisor*, as shown at (*d*). Divide the dividend (*c*) by the trial divisor, and obtain 0 as the next figure in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 60, and annex a cipher, thereby making the next trial divisor 600. Bring down the next period, 00, annex it to the dividend already obtained, and divide it by the trial divisor. 600 is contained in 0000, 0 times, so we place another cipher in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 600, and annex a cipher, thereby making the next trial divisor 6,000. Bring down the next period, 99. The trial divisor 6,000 is contained in 000099, 0 times, so we place 0 as the next figure in the root, as shown, and also add it to the trial divisor 6,000, and annex a cipher, thereby making the next trial divisor 60,000. Bring down the next period, 40, and annex it to the dividend already obtained to form the new dividend, 00009940, and divide it by the trial divisor 60,000. 60,000 is contained in 00009940, 0 times, so we place another cipher in the root, as shown, and also add it to the trial divisor 60,000, and annex one cipher, thereby making the next trial divisor 600,000. Bring down the next period, 09, and annex it to the dividend already obtained to form the new dividend, 0000994009, and divide it by the trial divisor 600,000. 600,000 is contained in 0000994009 once, so we place 1 as the next figure in the root, and also add it to the trial divisor 600,000, thereby making the complete divisor 600,001. Multiply the complete divisor, 600,001, by 1, the sixth figure in the root, and subtract the result obtained from the dividend. The remainder is 394,008, to which we annex the next period, 00, to form the next new dividend, or 39,400,800. Add the sixth figure of the root, or 1, to the divisor 600,001, and annex a cipher, thus obtaining 6,000,020 as the next trial divisor. Dividing 39,400,800 by 6,000,020, we find 6 to be the next figure of the root. Adding this last figure, 6, to the trial divisor, we obtain 6,000,026 for our next complete divisor, which, multiplied by the last figure of the root, or 6, gives 36,000,156, which write under 39,400,800 and subtract. Since there is a remainder, it is clearly evident that the given power is not a perfect square, so we place + after the root. Since the next figure is 5, the answer is 3,000.017 −.

In this problem there are *seven* periods—four in the whole number and three in the decimal—hence, there will be *seven* figures in the root, *four* figures constituting the whole number, and three figures the decimal of the root. Hence, $\sqrt{9,000,099.4009} = 3,000.017 -$.

(*c*)

3	$\sqrt{.00'12'25} = .035.$ Ans.
3	00
60	12
5	9
65	325
	325

Pointing off periods, we find that the first period is composed of ciphers; hence, the first figure of the root will be a cipher. No further explanation is necessary, since this problem is solved in a manner exactly similar to the problem solved in Art. **264.** Since there are *three* decimal periods in the power, there will be three decimal figures in the root.

(147) (*a*)

1	$\sqrt{1'07'95.21} = 103.9$ Ans.
1	1
20	0795
0	609
200	18621
3	18621
203	
3	
2060	
9	
2069	

(*b*)

2	$\sqrt{7'30'08.04} = 270.2$ Ans.
2	4
40	330
7	329
47	10804
7	10804
5400	
2	
5402	

(*c*)

9
9
180
4
184
4
1880
8
1888
8
1896

$\sqrt{90.00'00'00} = 9.487 -$ Ans.

81
900
736
16400
15104

1896) 1296.00 (.68 + or .7 −
11376
15840
15168

Having found the first three figures, we find the fourth by division, as shown.

(*d*) $\sqrt{.09} = .3$. Ans.

(148) (*a*)

6
6
12
6
180
8
188
8
196
8
2040
9
2049
9
2058
9
2067

36
72
10800
1504
12304
1568
1387200
18441
1405641
18522
1424163

$\sqrt[3]{.327'680'000} = .6894 +$ Ans.

216
111680
98432
13248000
12650769

1424163) 597231.00 (.41+ or .4+
5696652
2756580
1424163

Here we find the first three figures in the regular way, and the fourth figure by the short method. See Art. **284.**

Explanation.—(1) When extracting the *cube* root we divide the power into periods of three figures each. Always begin at the decimal point, and proceed to the *left* in pointing off the whole number, and to the *right* in pointing off the decimal. In this power $\sqrt[3]{.32768}$, a cipher must be annexed to 68 to complete the second decimal period. Cipher periods may now be annexed until the root has as many figures as desired.

(2) We find by trial that the largest number whose cube is contained in the first period, 327, is 6. Write 6 as the first figure of the root, also at the extreme left at the head of column (1). Multiply the 6 in column (1) by the first figure of the root, 6, and write the product 36 at the head of column (2). Multiply the number in column (2) by the first figure of the root, 6, and write the product 216 under the figures in the first period. Subtract and bring down the next period 680; annex it to the remainder 111, thereby obtaining 111,680 for a new dividend. Add the first figure of the root, 6, to the number in column (1), obtaining 12, which we call the *first correction;* multiply the first correction 12 by the first figure of the root, and we obtain 72 as the product, which, added to 36 of column (2), gives 108. Annexing two ciphers to 108, we have 10,800 for the trial divisor. Dividing the dividend by the trial divisor, we see that it is contained about 8 times, so we write 8 as the second figure of the root. Add the first figure of the root to the first correction, and we obtain 18 as the *second correction.* To this annex *one* cipher, and add the second figure of the root, and we obtain 188. This, multiplied by the second figure of the root, 8, equals 1,504, which, added to the trial divisor 10,800, forms the *complete divisor* 12,304. Multiplying the complete divisor 12,304 by 8, the second figure of the root, the result is 98,432. Write 98,432 under the dividend 111,680; subtract, and there is a remainder of 13,248. To this remainder annex the next period 000, thereby obtaining 13,248,000 for the next new dividend.

(3) Adding the second figure of the root, 8, to the number in column (1), 188, we have 196 for the *first new*

correction. This, multiplied by the second figure of the root, 8, gives 1,568. Adding this product to the last complete divisor, and annexing two ciphers, gives 1,387,200 for the next trial divisor. Adding the second figure of the root, 8, to the first new correction, 196, we obtain 204 for the *new second correction.* Dividing the dividend by the trial divisor 1,387,200, we see that it is contained about 9 times. Write 9 as the third figure of the root. Annex *one* cipher to the *new second correction*, and to this add the third figure of the root, 9, thereby obtaining 2,049. This, multiplied by 9, the third figure of the root, equals 18,441, which, added to the trial divisor, 1,387,200, forms the complete divisor 1,405,641. Multiplying the complete divisor by the third figure of the root, 9, and subtracting, we have a remainder of 597,231. We then find the fourth figure by division, as shown.

(*b*)

4	16	$\sqrt[3]{74'088} = 42$ Ans.
4	32	64
8	4800	10088
4	244	10088
120	5044	
2		
122		

(*c*)

4	16	$\sqrt[3]{92'416} = 45.212 -$ Ans.
4	32	64
8	4800	28416
4	625	27125
120	5425	1291000
5	650	1220408
125	607500	612912)70592.000(.115
5	2704	612912
130	610204	930080
5	2708	612912
1350	612912	3171680
2		3064560
1352		107120
2		
1354		

(*d*)

7	49	$\sqrt[3]{.373'248} = .72$ Ans.
7	98	343
14	14700	30248
7	424	30248
210	15124	
2		
212		

(149)

1	1	$\sqrt[3]{2.000'000'000} = 1.259921 +$ Ans.
1	2	1
2	300	1000
1	64	728
30	364	272000
2	68	225125
32	43200	46875000
2	1825	42491979
34	45025	4755243)4383021.000(.9217 or .922−
2	1850	42797187
360	4687500	10330230
5	33831	9510486
365	4721331	8197440
5	33912	4755243
370	4755243	34421970
5		
3750		
9		
3759		
9		
3768		

This example shows what a great saving of figures is effected by using the short method. The figures obtained by the division are 9217, thus making the last figures of the answer 922, according to Art. **272.** This is not correct in this case; the true answer to eight decimal places being 1.25992104 +; hence, the first three figures found by division should be used in this case. The reason for the apparent failure of the method in this case to give the seventh figure of the root correctly is because the fifth figure (the first obtained by division) is 9. Whenever the first figure obtained by division is 8 or 9, it is better to carry the root process one place further, before applying Art. **272,** if it is desired to obtain absolutely correct results.

(150) (*a*)

1	1	$\sqrt[3]{1'758.416'743} = 12.07$ Ans.
1	2	1
2	300	758
1	64	728
30	364	30416743
2	68	30416743
32	4320000	
2	25249	
34	4345249	
2		
3600		
7		
3607		

(*b*)

1	1	$\sqrt[3]{1'191'016} = 106$ Ans.
1	2	1
2	30000	191016
1	1836	191016
300	31836	
6		
306		

(*c*) $\sqrt[3]{\frac{4}{32}} = \sqrt[3]{\frac{1}{8}} = \frac{\sqrt[3]{1}}{\sqrt[3]{8}} = \frac{1}{2}$. Ans.

(*d*) $\sqrt[3]{\frac{27}{512}} = \frac{\sqrt[3]{27}}{\sqrt[3]{512}} = \frac{3}{8}$. Ans.

(151)

1
1
2
1
30
4
34
4
38
4
420
4
424
4
428
4
4320
2
4322
2
4324

1
2
300
136
436
152
58800
1696
60496
1712
6220800
8644
6229444
8648
6238092

$\sqrt[3]{3.000'000'000} = 1.442250 -$ Ans.

1
2000
1744
256000
241984
14016000
12458888
6238092) 1557112.000 (.2496 or .250 –
12476184
30949360
24952368
59969920
56142828
3827092

(152) (*a*)

1
1
20
1
21
1
220
1
221

$\sqrt{1'23.21} = 11.1$ Ans.

1
23
21
221
221

(*b*)

1
1
200
7
207
7
2140
2
2142

$\sqrt{1'14.92'10} = 10.72 +$ Ans.

1
1492
1449
4310
4284
26

(*c*)

7
7
140
0
1400
9
1409

$\sqrt{50'26'81} = 709$ Ans.

49
12681
12681

(*d*)

2
2
400
3
403

$\sqrt{.00'04'12'09} = .0203$ Ans.

00
04
4
1209
1209

(153) (*a*)

1	1	$\sqrt[3]{.006'500'000} = .18663-$ Ans.
1	2	1
2	300	5500
1	304	4832
30	604	668000
8	368	602856
38	97200	103788) 65144.00 (.627 or .63 −
8	3276	622728
46	100476	287120
8	3312	207576
540	103788	79544
6		
546		
6		
552		

(*b*)

2	4	$\sqrt[3]{.021'000'000} = .2759-$ Ans.
2	8	8
4	1200	13000
2	469	11683
60	1669	1317000
7	518	1113875
67	218700	226875) 203125.0 (.89 or .9 −
7	4075	1815000
74	222775	216250
7	4100	
810	226875	
5		
815		
5		
820		

(*c*)

2	4	$\sqrt[3]{8'036'054'027} = 2{,}003$ Ans.
2	8	8
4	12000000	036054027
2	18009	36054027
6000	12018009	
3		
6003		

(*d*)

1	1	$\sqrt[3]{.000'004'096} = .016$ Ans.
1	2	000
2	300	004
1	216	1
30	516	3096
6		3096
36		

(*e*)

2	4	$\sqrt[3]{17.000'000} = 2.5713-$ Ans.
2	8	8
4	1200	9000
2	325	7625
60	1525	1375000
5	350	1349593
65	187500	198147)25407.00(.128 or .13—
5	5299	198147
70	192799	559230
5	5348	396294
750	198147	162936
7		
757		
7		
764		

(154) (*a*) In this example the index is 4, and equals 2×2. The root indicated is the fourth root, hence the square root must be extracted twice. Thus, $\sqrt[4]{\ } = \sqrt{\ }$ of the $\sqrt{\ }$ and $\sqrt[4]{6561} = \sqrt{\sqrt{6561}} = \sqrt{81} = 9$. Ans.

8	$\sqrt{65'61} = 81$	$\sqrt{81} = 9$ Ans.
8	64	
160	161	
1	161	
161		

(*b*) In this example the index is 6, and 6 equals 3×2 or 2×3. The root indicated is the sixth root; hence, extract both the square and cube root, it making no particular difference as to which root is extracted first. Thus,

$$\sqrt[6]{\ } = \sqrt[3]{\ } \text{ of the } \sqrt{\ }, \text{ or } \sqrt{\ } \text{ of the } \sqrt[3]{\ }.$$

Hence, $\sqrt[6]{117,649} = \sqrt[3]{\sqrt{117,649}} = \sqrt[3]{343} = 7$. Ans.

3	$\sqrt{11'76'49} = 343$	$\sqrt[3]{343} = 7$ Ans.
3	9	
60	276	
4	256	
64	2049	
4	2049	
680		
3		
683		

(*c*) $\sqrt[6]{.000064} = \sqrt[3]{\sqrt{.000064}} = .2$. Ans.

$\sqrt{.000064} = .008$. $\sqrt[3]{.008} = .2$. Hence, $\sqrt[6]{.000064} = .2$. Ans.

(*d*) $\sqrt[3]{\frac{3}{8}} = ?$ $\frac{3}{8} = .375$, since $8\overline{)3.000}$
.375

7	49	$\sqrt[3]{.375'000'000} = .72112+$ Ans.
7	98	343
14	14700	32000
7	424	30248
210	15124	1752000
2	428	1557361
212	1555200	1559523) 194639.00 (.124 or .12 +
2	2161	1559523
214	1557361	3868670
2	2162	3119046
2160	1559523	749624
1		
2161		
1		
2162		

Hence, $\sqrt[3]{\frac{3}{8}} = .72112+$. Ans.

(**155**) (*a*) $\sqrt{\frac{1225}{5476}} = \frac{\sqrt{1225}}{\sqrt{5476}}$.

3	$\sqrt{12'25} = 35$
3	9
60	325
5	325
65	

Hence, $\sqrt{\frac{1225}{5476}} = \frac{35}{74}$. Ans.

7	$\sqrt{54'76} = 74$
7	49
140	576
4	576
144	

(*b*)

5	$\sqrt{.33'64} = .58$ Ans.
5	25
100	864
8	864
108	

(*c*)

3	$\sqrt{.10'00'00'00} = .31623-$ Ans.
3	9
60	100
1	61
61	3900
1	3756
620	632) 144.00 (.227 or .23 −
6	1264
626	1760
6	1264
632	496

(*d*) $25.0\frac{3}{4} = 25.075.$

5	$\sqrt{25.07'50'00'00'00} = 5.00749+$ Ans.
5	25
10000	075000
7	70049
10007	495100
7	400576
100140	9452400
4	9013401
100144	438999
4	
1001480	
9	
1001489	

(*e*) $.000\frac{4}{9} = .0004444444+.$

2	$\sqrt{.00'04'44'44'44} = .02108+$ Ans.
2	00
40	04
1	4
41	44
1	41
4200	34444
8	33664
4208	780

(156) (*a*) $\sqrt[4]{2} = \sqrt{\sqrt{2}}.$

1	$\sqrt{2.00'00'00'00} = 1.41421356+$
1	1
20	100
4	96
24	400
4	281
280	11900
1	11296
281	60400
1	56564
2820	28284) 3836.0000 (.13562 or .1356+
4	28284
2824	100760
4	84852
28280	159080
2	141420
28282	176600
2	169704
28284	6896

1	$\sqrt{1.41'42'13'56} = 1.1892+$ Ans.
1	1
20	41
1	21
21	2042
1	1824
220	21813
8	21321
228	49256
8	47564
2360	1692
9	
2369	
9	
23780	
2	
23782	

It is required in this problem to extract the fourth root of 2 to four decimal places; hence, we must extract the square root twice, since $\sqrt[4]{} = \sqrt{}$ of the $\sqrt{}$. In the first operation we carry the root to 8 decimal places, in order to carry the root in the second operation to 4 decimal places.

(*b*) $\sqrt[6]{6} = \sqrt[3]{\sqrt{6}}.$

2	$\sqrt{6.00'00'00'00'00'00} = 2.4494897428+$
2	4
40	200
4	176
44	2400
4	1936
480	46400
4	44001
484	239900
4	195936
4880	4396400
9	3919104
4889	489896) 477296.00000 (.974280 or .97428 +
9	4409064
48980	3638960
4	3429272
48984	2096880
4	1959584
489880	1372960
8	979792
489888	3931680
8	3919168
489896	12512

It is required in this problem to find the sixth root of 6; hence it is necessary to extract both the square and cube roots in succession, since the index, 6, equals 2×3 or 3×2. It makes no particular difference as to which root we extract first, but it will be more convenient to extract the square root first. The result has been carried to 10 decimal places; since the answer requires but 5 decimal places, the remaining decimals will not affect the cube root in the fifth decimal place, as the student can see for himself if he will continue the operation.

1	1	$\sqrt[3]{2.449'489'742'800} = 1.34801 -$ Ans.
1	2	1
2	300	1449
1	99	1197
30	399	252489
3	108	209104
33	50700	43385742
3	1576	43352192
36	52276	5451312) 33550.000 (.006 or .01 −
3	1592	32707872
390	5386800	842128
4	32224	
394	5419024	
4	32288	
398	5451312	
4		
4020		
8		
4028		
8		
4036		

(157) (*a*)

1	$\sqrt{3.14'16} = 1.7725 -$ Ans.
1	1
20	214
7	189
27	2516
7	2429
340	354) 87.00 (.245 + or .25 −
7	708
347	1620
7	1416
354	204

(*b*)

8
8
160
8
168
8
1760
6
1766
6
1772

$\sqrt{.78'54'00} = .8862 +$ Ans.
64
1454
1344
11000
10596
1772) 404.0 (.22 or .2 +
3544
496

(158) (*a*)

1
1
2
1
30
4
34
4
38
4
420
6
426
6
432
6
4380
4
4384
4
4388

1
2
300
136
436
152
58800
2556
61356
2592
6394800
17536
6412336
17552
6429888

$\sqrt[3]{3.141'600'000} = 1.4646 -$
Ans.
1
2141
1744
397600
368136
29464000
25649344
6429888) 3814656.0 (.59 or .6 –
32149440
5998120

(*b*)

8	64	$\sqrt[3]{.523'600'000} = .80599+$ or $.8060-$ Ans.
8	128.	512
16	1920000	11600000
8	12025	9660125
2400	1932025	1944075)1939875.00(.99
5	12050	17496675
2405	1944075	1902075
5		
2410		

(159) $11.7 : 13 :: 20 : x.$ The product of the means equals the product of the extremes.

$11.7\,x = 13 \times 20$

$11.7\,x = 260$

$$x = \frac{260}{11.7}$$

11.7) 260.000 (22.22 + Ans.

```
 234
 ---
  260
  234
  ---
   260
   234
   ---
    260
    234
    ---
     26
```

(160) (*a*) $20 + 7 : 10 + 8 :: 3 : x.$

$27 : 18 :: 3 : x$

$27\,x = 18 \times 3$

$27\,x = 54$

$x = \frac{54}{27} = 2.$ Ans.

(*b*) $12^2 : 100^2 :: 4 : x.$

$144 : 10{,}000 :: 4 : x$

$144\,x = 10{,}000 \times 4$

$144\,x = 40{,}000$

$$x = \frac{40,000}{144}$$

```
144 ) 40000.0 ( 277.7 +   Ans.
      288
      1120
      1008
       1120
       1008
        1120
        1008
         112
```

(161) (*a*) $\frac{4}{x} = \frac{7}{21}$, is equivalent to $4 : x :: 7 : 21$. The product of the means equals the product of the extremes. Hence,

$$7x = 4 \times 21$$
$$7x = 84$$
$$x = \frac{84}{7} \text{ or } 12. \quad \text{Ans.}$$

(*b*) In like manner,

$\frac{x}{24} = \frac{8}{16}$ is equivalent to $x : 24 :: 8 : 16$.

$$16x = 24 \times 8$$
$$16x = 192$$
$$x = \frac{192}{16} = 12. \quad \text{Ans.}$$

(*c*) $\frac{2}{10} = \frac{x}{100}$ is equivalent to $2 : 10 :: x : 100$.

$$10x = 2 \times 100$$
$$10x = 200$$
$$x = \frac{200}{10} = 20. \quad \text{Ans.}$$

(*d*) $\frac{15}{45} = \frac{60}{x}$ is equivalent to

$15 : 45 :: 60 : x$.

$$15x = 45 \times 60$$
$$15x = 2,700$$
$$x = \frac{2,700}{15} = 180. \quad \text{Ans.}$$

(*e*) $\frac{10}{150} = \frac{x}{600}$ is equivalent to

$10 : 150 :: x : 600$.

$$150x = 10 \times 600$$
$$150x = 6,000$$
$$x = \frac{6,000}{150} = 40. \quad \text{Ans.}$$

(162) $x : 5 :: 27 : 12.5.$

$$\begin{array}{r} 5 \\ \hline 12.5\,)\,135.0\,(\,10\tfrac{4}{5}\text{ Ans.} \\ 125 \\ \hline \end{array} \qquad \frac{100}{125} = \frac{4}{5}$$

(163) $45 : 60 :: x : 24$

$$60x = 45 \times 24$$
$$60x = 1{,}080$$
$$x = \frac{1{,}080}{60} = 18. \text{ Ans}$$

(164) $x : 35 :: 4 : 7.$

$$7x = 35 \times 4$$
$$7x = 140$$
$$x = \frac{140}{7} = 20. \quad \text{Ans.}$$

(165) $9 : x :: 6 : 24.$

$$6x = 9 \times 24$$
$$6x = 216$$
$$x = \frac{216}{6} = 36. \quad \text{Ans.}$$

(166)

$\sqrt[3]{1{,}000} : \sqrt[3]{1{,}331} :: 27 : x.$

$10 : 11 :: 27 : x.$

$$10x = 297.$$
$$x = \frac{297}{10} = 29.7. \quad \text{Ans.}$$

$\sqrt[3]{1{,}000} = 10.$

$\sqrt[3]{1{,}331} = 11.$

$$\begin{array}{r} 1 \\ 1 \\ \hline 2 \\ 1 \\ \hline 30 \\ 1 \\ \hline 31 \end{array} \qquad \begin{array}{r} 1 \\ 2 \\ \hline 300 \\ 31 \\ \hline 331 \end{array} \qquad \begin{array}{r} 1'331\,(\,11 \\ 1 \\ \hline 331 \\ 331 \\ \hline \end{array}$$

(167) $64 : 81 = 21^2 : x^2.$

Extracting the square root of each term of any proportion does not change its value, so we find that $\sqrt{64} : \sqrt{81} = \sqrt{21^2} : \sqrt{x^2}$ is the same as

$$8 : 9 = 21 : x$$
$$8x = 189$$
$$x = 23.625. \quad \text{Ans.}$$

(168) $7 + 8 : 7 = 30 : x$ is equivalent to

$$15 : 7 = 30 : x.$$
$$15x = 7 \times 30$$
$$15x = 210$$
$$x = \frac{210}{15} = 14. \quad \text{Ans.}$$

(169) 2 ft. 5 in. = 29 in.; 2 ft. 7 in. = 31 in. Stating as a direct proportion, 29 : 31 = 2,480 : x. Now, it is easy to see that x will be greater than 2,480. But x should be less than 2,480, since, when a man lengthens his steps, the number of steps required for the same distance is less; hence, the proportion is an inverse one, and

$$29 : 31 = x : 2{,}480,$$
or, $$31x = 71{,}920;$$
whence, $$x = 71{,}920 \div 31 = 2{,}320 \text{ steps. Ans.}$$

(170) This is evidently a direct proportion. 1 hr. 36 min. = 96 min.; 15 hr. = 900 min. Hence,

$$96 : 900 = 12 : x,$$
or, $$96x = 10{,}800;$$
whence, $$x = 10{,}800 \div 96 = 112.5 \text{ mi. Ans.}$$

(171) This is also a direct proportion; hence,

$$27.63 : 29.4 = .76 : x,$$
or, $$27.63x = 29.4 \times .76 = 22.344;$$
whence, $$x = 22.344 \div 27.63 = .808 + \text{lb. Ans.}$$

(172) 2 gal. 3 qt. 1 pt. = 23 pt.; 5 gal. 3 qt. = 46 pt. Hence,

$$23 : 46 = 5 : x,$$
or, $$23x = 46 \times 5 = 230;$$
whence, $$x = 230 \div 23 = 10 \text{ days. Ans.}$$

(173) First cause, 5 men and 8 hours; second cause, x men, 10 hours. The effect is the amount of work, which is the same in each case.

$$\begin{array}{c|c} \not{5} & x \\ \not{8} & \not{1}\not{0} \\ 4 & \not{2} \end{array} = \text{work} \;\Big|\; \text{work}$$

$$x = 4 \text{ men. Ans.}$$

(174) Taking the times as the causes,

$$\begin{array}{r|l} \not{2}\not{0} & \not{2}\not{5} \\ & \not{5} \\ & \\ \not{4}\not{0} & x \\ 2 & \end{array} = \begin{array}{r|l} & 14 \\ & \not{7}\not{0} \\ \not{5}\not{4}\not{0} & \not{6}\not{3}\not{0} \\ \not{2}\not{7} & \\ 3 & \end{array}$$

hence, $3x = 2 \times 14 = 28$, or $x = 9\frac{1}{3}$ hr.

(175) $10{,}000^2 : 20{,}000^2 = 8 : x$;

or $1^2 : 2^2 = 8 : x$;

or $1 : 4 = 8 : x$;

hence, $x = \dfrac{4 \times 8}{1} = 32$ pounds. Ans.

(176) $\begin{matrix} 10 \\ 320 \end{matrix} : \begin{matrix} 8 \\ 600 \end{matrix} = 400 : x$;

hence, $x = \dfrac{8 \times 600 \times 400}{10 \times 320} = 600$ hoists. Ans.

(177) $20{,}000^3 : 30{,}000^3 = 16 : x$;

or $2^3 : 3^3 = 16 : x$;

or $8 : 27 = 16 : x$;

hence, $x = \dfrac{27 \times 16}{8} = 54$ horsepower. Ans.

FORMULAS.

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(178) Substituting for D, x, B, and i their values,

$$C = \frac{D - x}{B + i} = \frac{120 - 12}{10 + 3.5} = \frac{108}{13.5} = 8. \quad \text{Ans.}$$

A line between two numbers signifies that the one above the line, or numerator, is to be divided by the one below the line, or denominator.

(179) Substituting for A, h, D, and x their values,

$$\frac{A\,h + D}{2\,x + 6} = \frac{(5 \times 200) + 120}{(2 \times 12) + 6} = \frac{1{,}000 + 120}{24 + 6} = \frac{1{,}120}{30} = 37\tfrac{1}{3}.$$

$$37\tfrac{1}{3} + D = 37\tfrac{1}{3} + 120 = 157\tfrac{1}{3}. \quad \text{Ans.}$$

When there is no sign between the letters, multiplication is understood.

(180) Substituting for B, h, A, x, and i their values

$$r = \frac{3.246 \times B \times h}{\dfrac{A\,x + h}{A\,i - B}} = \frac{3.246 \times 10 \times 200}{\dfrac{(5 \times 12) + 200}{(5 \times 3.5) - 10}} = \frac{6{,}492}{\dfrac{260}{7.5}} =$$

$$6{,}492 \div \frac{260}{7.5} = 6{,}492 \times \frac{7.5}{260} = 187.269 +. \quad \text{Ans.}$$

(181) Substituting for A, D, i, and B their values,

$$v = \sqrt{\frac{A\,D}{i\,B + 1.5}} = \sqrt{\frac{5 \times 120}{(3.5 \times 10) + 1.5}} = \sqrt{\frac{600}{36.5}} =$$

$$\sqrt{16.4383} = 4.05 +. \quad \text{Ans.}$$

The square root sign extends over both numerator and denominator, thus indicating that the square root of the entire fraction is to be extracted.

(182) Substituting for B, x, h, and A their values,

$$u = \sqrt[3]{\frac{B\,x}{.00018\,h\,(A^2 - x)}} = \sqrt[3]{\frac{10 \times 12}{.00018 \times 200 \times (5^2 - 12)}} =$$

$$\sqrt[3]{\frac{120}{.036 \times (25 - 12)}} = \sqrt[3]{\frac{120}{.036 \times 13}} = \sqrt[3]{\frac{120}{.468}} =$$

$$\sqrt[3]{256.41 +} = 6.35 +. \quad \text{Ans.}$$

(183) Substituting for h, D, and A their values,

$$f = \frac{10\,(h - D)^2}{\sqrt[3]{D + A}} = \frac{10\,(200 - 120)^2}{\sqrt[3]{120 + 5}} = \frac{10 \times 80^2}{\sqrt[3]{125}} = \frac{64{,}000}{5} = 12{,}800. \quad \text{Ans.}$$

(184) Substituting for B, A, and D their values,

$$g = \frac{(B - A)^2 - \sqrt[3]{D + A}}{A^3 - (1 + D)} = \frac{(10 - 5)^2 - \sqrt[3]{120 + 5}}{5^3 - (1 + 120)} =$$

$$\frac{5^2 - \sqrt[3]{125}}{125 - 121} = \frac{25 - 5}{4} = \frac{20}{4} = 5. \quad \text{Ans.}$$

(185) Substituting for A, B, and h their values,

$$k = \sqrt{\frac{A\,B^2}{\sqrt[3]{A\,h}}} = \sqrt{\frac{5 \times 10^2}{\sqrt[3]{5 \times 200}}} = \sqrt{\frac{5 \times 100}{\sqrt[3]{1{,}000}}} = \sqrt{\frac{500}{10}} =$$

$$\sqrt{50} = 7.071 +. \quad \text{Ans.}$$

(186) Substituting for A, h, D, x, and B their values,

$$T = \sqrt{\frac{A^2\left[490 + \frac{(h\,x)^2}{D^2}\right]}{h + \frac{x}{D}(A^2 - B)^2}} = \sqrt{\frac{5^2\left[490 + \frac{(200 \times 12)^2}{120^2}\right]}{200 + \frac{12}{120}(5^2 - 10)^2}} =$$

$$\sqrt{\frac{25\,(490 + 400)}{200 + (\frac{1}{10} \times 225)}} = \sqrt{\frac{25 \times 890}{200 + 22.5}} = \sqrt{\frac{22{,}250}{222.5}} =$$

$$\sqrt{100} = 10. \quad \text{Ans.}$$

GEOMETRY AND TRIGONOMETRY.

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(273) When one straight line meets another straight line at a point between the ends, the sum of the two adjacent angles equals two right angles. Therefore, since one of the angles equals $\frac{4}{5}$ of a right angle, then, the other angle equals $\frac{10}{5}$, or two right angles, minus $\frac{4}{5}$. We have, then, $\frac{10}{5} - \frac{4}{5} = \frac{6}{5}$, or $1\frac{1}{5}$ right angles.

(274) The size of one angle is $\frac{1}{6}$ of two right angles, or $\frac{1}{3}$ of a right angle.

(275) The pitch being 4, the number of teeth in the wheel equals 4×12, or 48. The angle formed by drawing lines from the center to the middle points of two adjacent teeth equals $\frac{1}{48}$ of 4 right angles, or $\frac{1}{12}$ of a right angle.

(276) It is an isosceles triangle, since the sides opposite the equal angles are equal.

(277) An equilateral heptagon has seven equal sides; therefore, the length of the perimeter equals 7×3, or 21 inches.

(278) A regular decagon has 10 equal sides; therefore, the length of one side equals $\frac{40}{10}$, or 4 inches.

(279) The sum of all the interior angles of any polygon equals two right angles, multiplied by the number of sides in the polygon, less two. As a regular dodecagon has 12 equal sides, the sum of the interior angles equals two right angles × 10 (= 12 − 2), or 20 right angles. Since there are 12 equal angles, the size of any one of them equals 20 ÷ 12, or $1\frac{2}{3}$ right angles.

(280) Equilateral triangle.

(281) No, since the sum of the two smaller sides is not greater than the third side.

(282) No, since the sum of the three smaller sides is not greater than the fourth side.

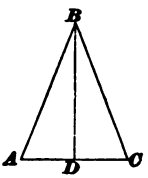

FIG. 1.

(283) Since the two angles A and C, Fig. 1, are equal, the triangle is isosceles, and a line drawn from the vertex B will bisect the line AC, the length of which is 7 inches; therefore,

$$AD = DC = \frac{7}{2} = 3\frac{1}{2} \text{ in. Ans.}$$

(284) The length of the line $= \sqrt{12^2 - 9^2} + \sqrt{15^2 - 9^2}$, or 19.94 inches.

(285) The sum of the three angles is equal to $\frac{8}{4}$, or 2 right angles; therefore, since the sum of two of them equals $\frac{5}{4}$ of a right angle, the third angle must equal $\frac{8}{4} - \frac{5}{4}$, or $\frac{3}{4}$ of a right angle.

(286) One of the angles of an equiangular octagon is equal to $\frac{1}{8}$ of 12 right angles, or $1\frac{1}{2}$ right angles, since the sum of the interior angles of the equiangular octagon equals 12 right angles.

(287) The sum of the acute angles of a right-angled triangle equals one right angle; therefore, if one of them equals $\frac{5}{8}$ of a right angle, the other equals $\frac{8}{8} - \frac{5}{8}$, or $\frac{3}{8}$ of a right angle.

(288) (See Art. **734.**)

(289) In Fig. 2, $AB = 4$ inches, and $AO = 6$ inches. We first find the length of DO. $DO = \sqrt{\overline{OA}^2 - \overline{DA}^2}$; but $\overline{OA}^2 = 6^2$, or 36, and $\overline{DA}^2 = \left(\frac{4}{2}\right)^2$ or 4; therefore, $DO = \sqrt{36 - 4}$, or 5.657.

$DC = CO - DO$, or $DC = 6 - 5.657$, or .343 inch. In the right-angled triangle ADC, we have AC, which is the chord of one-half the arc ACB, equals $\sqrt{2^2 + .343^2}$, or 2.03 inches.

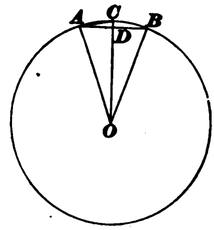

FIG. 2.

(290) The method of solving this is similar to the last problem.

$$DO = \sqrt{9 - 4}, \text{ or } 2.236. \quad DC = 3 - 2.236 = .764.$$

$$AC = \sqrt{2^2 + .764^2}, \text{ or } 2.14 \text{ inches.}$$

(291) Let HK of Fig. 3 be the section; then, $BI = 2$ inches, and $HK = 6$ inches, to find AB. HI (= 3 inches) being a mean proportional between the segments AI and IB, we have

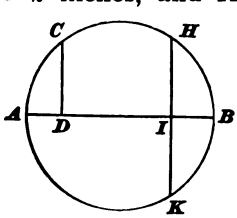

FIG. 3.

$$BI : HI :: HI : IA,$$

$$\text{or} \quad 2 : 3 :: 3 : IA.$$

$$\text{Therefore,} \quad IA = 4\frac{1}{2}.$$

$AB = AI + IB$; therefore, $AB = 4\frac{1}{2} + 2$, or $6\frac{1}{2}$ inches.

(292) Given $OC = 5\frac{3}{4}$ inches, and $OA = \frac{17}{2}$, or $8\frac{1}{2}$ inches, to find AB (see Fig. 4). CA, which is one-half the chord AB, equals

$$\sqrt{\overline{OA}^2 - \overline{OC}^2};$$

$$\text{therefore,} \quad CA = \sqrt{(8\tfrac{1}{2})^2 - (5\tfrac{3}{4})^2},$$

or 6.26 inches. Now, $AB = 2 \times CA$; therefore, $AB = 2 \times 6.26$, or 12.52 inches.

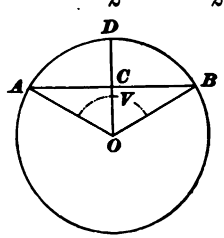

FIG. 4.

(293) The arc intercepted equals $\frac{3}{4}$ of 4, or 3 quadrants. As the inscribed angle is measured by one-half the intercepted arc, we have $\frac{3}{2} = 1\frac{1}{2}$ quadrants as the size of the angle.

(294) Four right angles $\div \frac{2}{7} = 4 \times \frac{7}{2}$, or 14 equal sectors.

(295) Since 24 inches equals the perimeter, we have $\frac{24}{8}$, or 3 inches, as the length of each side or chord.

Then, $2 \times \sqrt{\left(\frac{3}{2}\right)^2 + 3.62^2} = 7.84$ inches diameter.

(296) Given, $AC = \frac{AB}{2} = \frac{10.5}{2}$, or 5.25 inches. AO and $AP = 13$ inches. (See Fig. 5.)

The required distance between the arcs DD' is equal to $OA + AP - OP$. In the right-angled triangle ACO, we have

$$OC = \sqrt{\overline{AO}^2 - \overline{AC}^2},$$

or $OC = \sqrt{169 - 27.5625} = 11.9$ inches.

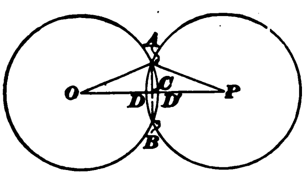

FIG. 5.

Likewise, $CP = \sqrt{\overline{AP}^2 - \overline{AC}^2} = 11.9$. $OP = OC + CP = 11.9 + 11.9 = 23.8$ inches. $OA + AP = 13 + 13 = 26$ inches. $26 - 23.8 = 2.2$ inches. Ans.

(297) Given $AP = 13$ inches, $OA = 8$ inches, and $AC = 5.25$ inches. Fig. 6.

$OC = \sqrt{\overline{AO}^2 - \overline{AC}^2} = \sqrt{8^2 - 5.25^2} =$ 6.03 inches.

$CP = \sqrt{\overline{AP}^2 - \overline{AC}^2} = 11.9$ inches.

$OP = OC + CP = 6.03 + 11.9 =$ 17.93 inches.

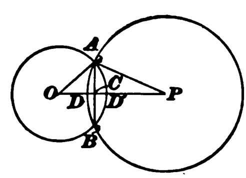

FIG. 6.

$DD' = OA + AP - OP = 8 + 13 - 17.93 = 3.07$ inches. Ans.

(298) $AB = 14$ inches, and $AE = 3\frac{1}{4}$ inches, Fig. 7. $CE = ED$ is a mean proportional between the segments

AE and EB. Then,

$$AE : CE :: CE : EB,$$

$$\text{or}\quad 3\frac{1}{4} : CE :: CE : 10\frac{3}{4},$$

$$\text{or}\quad \overline{CE}^2 = 3\frac{1}{4} \times 10\frac{3}{4} = 34.9375.$$

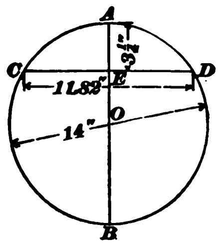

FIG. 7.

Extracting the square root, we have

$$CE = 5.91.$$

$2 \times CE = CD = 2 \times 5.91$, or 11.82 inches. Ans.

(299) In 19° 19′ 19″ there are 69,559 seconds, and in 360°, or a circle, there are 1,296,000 seconds. Therefore, 69,559 seconds equal $\frac{69,559}{1,296,000}$, or .053672 part of a circle. Ans.

(300) In an angle measuring 19° 19′ 19″ there are 69,559 seconds, and in a quadrant, which is $\frac{1}{4}$ of 360°, or 90°, there are 324,000 seconds. Therefore, 69,559 seconds equal $\frac{69,559}{324,000}$, or .214688 part of a quadrant. Ans.

(301) Given, $OB = OA = \frac{23}{2}$, or $11\frac{1}{2}$ inches, and angle $AOB = \frac{1}{10}$ of 360°, or 36°. (See Fig. 8.) In the right-angled triangle COB, we have

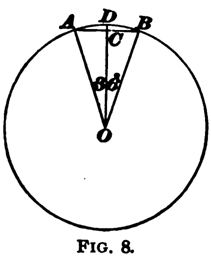

FIG. 8.

$$\sin COB = \frac{CB}{OB}, \text{ or } CB = OB \times \sin COB.$$

Substituting the values of OB and $\sin COB$, we have

$$CB = 11\frac{1}{2} \times \sin 18°,$$

$$\text{or}\quad CB = 11\frac{1}{2} \times .30902 = 3.55.$$

Since $AB = 2\,CB$, $AB = 2 \times 3.55 = 7.1$ inches.

The perimeter then equals $10 \times 7.1 = 71$ inches, nearly. Ans.

(302)

$$\begin{array}{rrrl} 90° = 89° & 59' & 60'' & \\ 35° & 24' & 25.8'' & \\ \hline 54° & 35' & 34.2'' & \text{Ans.} \end{array}$$

(303) The side $BC = \sqrt{\overline{AB}^2 - \overline{AC}^2}$, or $BC = \sqrt{\overline{17.69}^2 - \overline{9.75}^2} = \sqrt{217.8736} = 14$ ft. 9 in. To find the angle BAC, we have $\cos BAC = \frac{AC}{AB}$, or $\cos BAC = \frac{9.75}{17.69} =$ .55115.

.55115 equals the cos of 56° 33′ 15″.

Angle $ABC = 90° -$ angle BAC, or $90° - 56° 33' 15'' =$ 33° 26′ 45″.

(304)

$$\begin{array}{rrr} 159° & 27' & 34.6'' \\ 25° & 16' & 8.7'' \\ 3° & 48' & 53\ \ '' \\ \hline 188° & 32' & 36.3'' \end{array}$$

(305) Sin 17° 28′ = .30015.

Sin 17° 27′ = .29987.

.30015 − .29987 = .00028, the difference for 1′.

$.00028 \times \frac{37}{60} = .00017$, difference for 37″.

.29987 + .00017 = .30004 = sin 17° 27′ 37″.

Cos 17° 27′ = .95398.

Cos 17° 28′ = .95389.

.95398 − .95389 = .00009, difference for 1′.

$.00009 \times \frac{37}{60} = .00006$, difference for 37″.

.95398 − .00006 = .95392 = cos 17° 27′ 37″.

Tan 17° 28′ = .31466.

Tan 17° 27′ = .31434.

.31466 − .31434 = .00032, difference for 1′.

$.00032 \times \frac{37}{60} = .00020$, difference for 37″.

.31434 + .0002 = .31454 = tan 17° 27′ 37″.

$$\left.\begin{array}{l} \text{Sin } 17° 27' 37'' = .30004 \\ \text{Cos } 17° 27' 37'' = .95392 \\ \text{Tan } 17° 27' 37'' = .31454 \end{array}\right\} \text{Ans.}$$

(306) From the vertex B, draw BD perpendicular to AC, forming the right-angled triangles ADB and BDC. In the right-angled triangle ADB, AB is known, and also the angle A. Hence, $BD = 26.583 \times \sin 36° 20' 43'' = 26.583 \times .59265 = 15.754$ feet. $AD = 26.583 \times \cos 36° 20' 43'' = 26.583 \times .80546 = 21.411$. $AC - AD = 40 - 21.411 = 18.589$ feet $= DC$. In the right-angled triangle BDC, the two sides BD and DC are known; hence, $\tan C = \frac{BD}{DC} = \frac{15.754}{18.589} = .84750$, and angle $C = 40° 16' 53''$. Ans.

$BC = \frac{BD}{\sin C} = \frac{15.754}{\sin 40° 16' 53''} = \frac{15.754}{.64654} = 24.37$, or 24 ft. 4.4 in. Ans.

Angle $B = 180° - (36° 20' 43'' + 40° 16' 53'') = 180° - 76° 37' 36'' = 103° 22' 24''$. Ans.

(307) This problem is solved exactly like problem No. 305.

Sin of $63° 4' 51.8'' = .89165$.
Cos of $63° 4' 51.8'' = .45274$.
Tan of $63° 4' 51.8'' = 1.96949$.

(308) $.27038 = \sin 15° 41' 12.9''$.
$.27038 = \cos 74° 18' 47.1''$.
$2.27038 = \tan 66° 13' 43.2''$.

(309) The angle formed by drawing radii to the extremities of one of the sides equals $\frac{360°}{11}$, or $32° 43' 38.2''$. Ans. The length of one side of the undecagon equals $\frac{4 \text{ ft. } 3 \text{ in.}}{11}$, or 4.6364 inches. The radius of the circle equals $\frac{\frac{1}{2} \text{ of } 4.6364}{\sin \text{ of } \frac{1}{2}(32° 43' 38.2'')} = \frac{2.3182}{.28173} = 8.23$ inches. Ans.

(310) If one of the angles is twice the given one, then it must be $2 \times (47° 13' 29'')$, or $94° 26' 58''$. Since there are two right angles, or 180°, in the three angles of a triangle, the third angle must be $180 - (47° 13' 29'' + 94° 26' 58'')$, or $38° 19' 33''$.

(311) If one of the angles is one-half as large as the given angle, then it must be $\frac{1}{2}$ of 75° 48′ 17″, or 37° 54′ 8.5″. The third angle equals 180° − (75° 48′ 17″ + 37° 54′ 8.5″), or 66° 17′ 34.5″.

(312) From the vertex *B*, draw *B D* perpendicular to *A C*, forming the two right-angled triangles *A D B* and *B D C*. In the right-angled triangle *A D B*, *A B* is known, and also the angle *A*. Hence, $B\ D = \sin\ A \times A\ B = \sin\ 54° \ 54'\ 54'' \times 16\frac{5}{12} = .81830 \times 16\frac{5}{12} = 13.434$ feet.

Sine of angle $C = \frac{B D}{B C} = \frac{13.434}{13.542} = .99202$, and, hence, angle $C = 82°\ 45'\ 30''$. Ans.

Angle $B = 180° - (54°\ 54'\ 54'' + 82°\ 45'\ 30'') = 180 - 137°\ 40'\ 24'' = 42°\ 19'\ 36''$. Ans.

$A D = A B \times \cos A = 16\frac{5}{12} \times \cos 54°\ 54'\ 54'' = 16\frac{5}{12} \times .57479 = 9.43613$ ft.

$C D = B C \times \cos C = B C \times \cos 82°\ 45'\ 30'' = 13\frac{13}{24} \times .12605 = 1.70692$ ft.

$A C = A D + C D = 9.43613 + 1.70692 = 11.143 = 11$ ft. $1\frac{3}{4}$ in. Ans.

(313) If one-third of a certain angle equals 14° 47′ 10″, then the angle must be 3 × 14° 47′ 10″, or 44° 21′ 30″. $2\frac{1}{2} \times$ 44° 21′ 30″, or 110° 53′ 45″, equals one of the other two angles. The third angle equals 180° − (110° 53′ 45″ + 44° 21′ 30″), or 24° 44′ 45″.

(314) Given, *B C* = 437 feet and *A C* = 792 feet, to find the hypotenuse *A B* and the angles *A* and *B*.

$A B = \sqrt{\overline{A C}^2 + \overline{B C}^2} = \sqrt{792^2 + 437^2} = \sqrt{818,233} = 904$ ft $6\frac{3}{4}$ in. Ans.

$\text{Tan } A = \frac{437}{792} = .55176$; therefore, $A = 28° 53' 18''$. Ans.

Angle $B = 90° - 28° 53' 18''$, or $61° 6' 42''$. Ans.

(315) In Fig. 9, angle $A\,O\,B = \frac{1}{8}$ of 360°, or 45°. Angle $m\,O\,B = \frac{1}{2}$ of 45°, or $22\frac{1}{2}^{\circ}$. Side $A\,B = \frac{1}{8}$ of 56 feet, or 7 feet. Now, in the triangle $m\,O\,B$, we have the angle $m\,O\,B = 22\frac{1}{2}^{\circ}$, and $m\,B = \frac{7}{2}$, or $3\frac{1}{2}$ feet, given, to find $O\,B$ and the angle $m\,B\,O$.

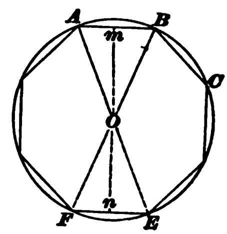

FIG. 9.

$$\text{Sin } m\,O\,B = \frac{m\,B}{O\,B}, \text{ or } O\,B = \frac{m\,B}{\sin m\,O\,B}.$$

Substituting their values, $O\,B = \frac{3.5}{\sin 22\frac{1}{2}^{\circ}} = \frac{3.5}{.38268} = 9.146$ feet.

$B\,F$, the diameter of the circle, equals $2 \times B\,O$; therefore, $B\,F = 2 \times 9.146 = 18.292$ feet $= 18$ feet $3\frac{1}{2}$ inches.

$$\text{Angle } B\ O\ m = 22° 30'.$$
$$B\ O\ m + O\ B\ m = 90°.$$
Therefore, $O\ B\ m = 90° - B\ O\ m = 90° - 22° 30'.$
$$= 67° 30'.$$
$$A\ B\ C = 2\ O\ B\ m = 2\ (67° 30') = 135°.$$
Ans.

By Art. **703,** the sum of the interior angles of an octagon is $2\ (8 - 2) = 12$ right angles. Since the octagon is regular, the interior angles are equal, and since there are eight of them, each one is $\frac{12}{8} = 1\frac{1}{2}$ right angles. $1\frac{1}{2} \times 90° = 135°$.

(316) Lay off with a protractor the angle $A\ O\ C$ equal to $67° 8' 49''$, Fig. 10. Tangent to the circle at A, draw the line $A\ T$. Through the point C, draw the line $O\ C$, and continue it until it intersects the line $A\ T$ at T. From C

draw the lines $C\,D$ and $C\,B$ perpendicular, respectively, to the radii $O\,E$ and $O\,A$. $C\,B$ is the sine, $C\,D$ the cosine, and $A\,T$ the tangent.

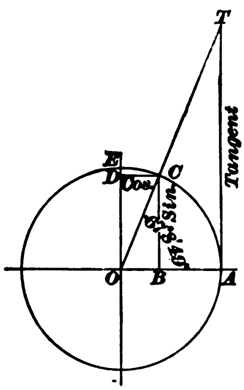

FIG. 10.

(317) Suppose that in Fig. 10, the line $A\,T$ has been drawn equal to 3 times the radius $O\,A$. From T draw $T\,O$; then, the tangent of $T\,O\,A = \frac{T\,A}{O\,A} = 3$. Where $T\,O$ cuts the circle at C, draw $C\,D$ and $C\,B$ perpendicular, respectively, to $O\,E$ and $O\,A$. $C\,D$ is the cosine and $C\,B$ the sine. The angle corresponding to tan 3 is found by the table to equal 71° 33′ 54″; therefore, sin 71° 33′ 54″ = .94868 and cos 71° 33′ 54″ = .31623.

(318) The angle whose cos is .39278 = 66° 52′ 20″.
Sin of 66° 52′ 20″ = .91963.
Tan of 66° 52′ 20″ = 2.34132.

For a circle with a diameter $4\frac{3}{4}$ times as large, the values of the above cos, sin, and tan will be

$$\left.\begin{array}{l} 4\frac{3}{4} \times .39278 = 1.86570 \text{ cos.} \\ 4\frac{3}{4} \times .91963 = 4.36824 \text{ sin.} \\ 4\frac{3}{4} \times 2.34132 = 11.12127 \text{ tan.} \end{array}\right\} \text{Ans.}$$

(319) See Fig. 11. Angle B = 180° − (29° 21′ + 76° 44′ 18″) = 180° − 106° 5′ 18″ = 73° 54′ 42″.

From C, draw $C\,D$ perpendicular to $A\,B$.

$A\,D = A\,C \cos A = 31.833 \times \cos 29° 21' = 31.833 \times .87164 = 27.747$ ft. $C\,D = A\,C \sin A = 31.833 \times \sin 29° 21' = 31.833 \times .49014 = 15.603$.

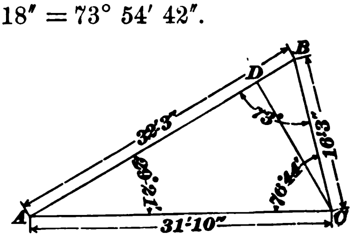

FIG. 11.

$$BC = \frac{CD}{\sin B} = \frac{15.603}{\sin 73° 54' 42''} = 16.24 \text{ feet} = 16 \text{ ft. } 3 \text{ in.}$$

$$BD = \frac{DC}{\tan B} = \frac{15.603}{\tan 73° 54' 42''} = 4.5 \text{ feet.}$$

$$AB = AD + DB = 27.747 + 4.5 = 32.247 = 32 \text{ ft. } 3 \text{ in.}$$

$$\text{Ans.} \begin{cases} AB = 32 \text{ ft. } 3 \text{ in.} \\ BC = 16 \text{ ft. } 3 \text{ in.} \\ B = 73° 54' 42''. \end{cases}$$

(320) In Fig. 8, problem 301, AB is the side of a regular decagon; then, the angle $COB = \frac{1}{20}$ of 360°, or 18°. To find the side CB, we have $CB = OB \times \sin 18°$, or $CB = 9.75 \times .30902 = 3.013$ inches. Since $AB = 2 \times CB$, $AB = 2 \times 3.013$, or 6.026 inches, which multiplied by 10, the number of sides, equals 60.26 inches. Ans.

(321) Perimeter of circle equals $2 \times 9.75 \times 3.1416$, or 61.26 inches. $61.26 - 60.26 = 1$ inch, the difference in their perimeters. Ans.

In order to find the area of the decagon, we must first find the length of the perpendicular CO (see Fig. 8 in answer to question 301); $CO = OB \times \cos 18°$, or $CO = 9.75 \times .95106 = 9.273$. Area of triangle $AOB = \frac{1}{2} \times 9.273 \times 6.026$, or 27.939, which multiplied by 10, the number of triangles in the decagon, equals 279.39 square inches. Area of the circle $= 3.1416 \times 9.75 \times 9.75$, or 298.65 square inches.

$298.65 - 279.39 = 19.26$ square inches difference. Ans.

(322) The diameter of the circle equals $\sqrt{\frac{89.42}{.7854}} = \sqrt{113.8528}$, or 10.67 inches. Ans.

The circumference equals 10.67×3.1416, or 33.52 inches. Ans.

In a regular hexagon inscribed in a circle, each side is equal to the radius of the circle; therefore, $\frac{10.67}{2} = 5.335$ inches is the length of a side. Ans.

(323) Angle $m\,O\,B = \frac{1}{16}$ of 360°, or $22\frac{1}{2}^\circ$. $m\,O = \frac{1}{2}$ of $m\,n = \frac{1}{2}$ of 2, or 1 inch. (See Fig. 12).

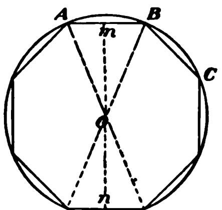

FIG. 12.

Side $m\,B = O\,m \times \tan 22\frac{1}{2}^\circ$, or $m\,B = 1 \times .41421 = .41421$.

$A\,B = 2\,m\,B$; therefore, $A\,B = .82842$ inch.

Area of $A\ O\ B = \frac{1}{2} \times .82842 \times 1 = .41421$ square inch, which, multiplied by 8, the number of equal triangles, equals 3.31368 square inches.

Wt. of bar equals $3.31368 \times 10 \times 12 \times .282$, or 112 pounds 2 ounces. Ans.

(324) $16 \times 16 \times 16 \times \frac{1}{6} \times 3.1416 = 2{,}144.66$ cu. in. equals the volume of a sphere 16 inches in diameter.

$12 \times 12 \times 12 \times \frac{1}{6} \times 3.1416 = 904.78$ cu. in. equals the volume of a sphere 12 inches in diameter.

The difference of the two volumes equals the volume of the spherical shell, and this multiplied by the weight per cubic inch equals the weight of the shell. Hence, we have $(2{,}144.66 - 904.78) \times .261 = 323.61$ lb. Ans.

(325) The circumference of the circle equals $\frac{5\frac{13}{32} \times 360}{27}$, or 72.0833 inches. The diameter, therefore, equals $\frac{72.0833}{3.1416}$, or 22.95 inches.

(326) The number of square inches in a figure 7 inches square equals 7×7, or 49 square inches. $49 - 7 = 42$ square inches difference in the two figures.

$\sqrt{7} = 2.64$ inches is the length of side of square containing 7 square inches. The length of one side of the other square equals 7 inches.

(327) (*a*) $17\frac{1}{64}$ inches = 17.016 inches.

Area of circle = $17.016^2 \times .7854 = 227.41$ sq. in. Ans.
Circumference = $17.016 \times 3.1416 = 53.457$ inches.

$$16° \; 7' \; 21'' = 16.1225°.$$

(*b*) Length of the arc $= \frac{16.1225 \times 53.457}{360} = 2.394$ inches. Ans.

(328) Area = $12 \times 8 \times .7854 = 75.4$ sq. in. Ans.
Perimeter = $(12 \times 1.82) + (8 \times 1.315) = 32.36$ in. Ans.

(329) Area of base $= \frac{1}{4} \times 3.1416 \times 7 \times 7 = 38.484$ sq. in.

The slant height of the cone equals $\sqrt{11^2 + 3\frac{1}{2}^2}$, or 11.5434 in.

Circumference of base = $7 \times 3.1416 = 21.9912$.

Convex area of cone $= 21.9912 \times \frac{11.5434}{2} = 126.927$.

Total area = $126.927 + 38.484 = 165.41$ square inches. Ans.

(330) Volume of sphere equals $10 \times 10 \times 10 \times \frac{1}{6} \times 3.1416 = 523.6$ cu. in.

Area of base of cone $= \frac{1}{4} \times 3.1416 \times 10 \times 10 = 78.54$ sq. in.

$\frac{3 \times 523.6}{78.54} = 20$ inches, the altitude of the cone. Ans.

(331) Volume of sphere $= \frac{1}{6} \times 3.1416 \times 12 \times 12 \times 12 = 904.7808$ cu. in.

Area of base of cylinder $= \frac{1}{4} \times 3.1416 \times 12 \times 12 = 113.0976$ sq. in.

Height of cylinder $= \frac{904.7808}{113.0976} = 8$ inches. Ans.

(332) (*a*) Area of the triangle equals $\frac{1}{2} A\,C \times B\,D$, or $\frac{1}{2} \times 9\frac{1}{2} \times 12 = 57$ square inches. Ans.

(*b*) See Fig. 13. Angle $BAC = 79° 22'$; angle $ABD = 90° - 79° 22' = 10° 38'$. Side $AB = BD \div \sin 79° 22' = 12 \div .98283 = 12.209$ inches.

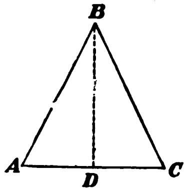

Fig. 13

Side $AD = BD \times \tan 10° 38' = 12 \times .18775 = 2.253$ inches.

Side $DC = AC - AD = 9.5 - 2.253 = 7.247$ inches.

Side $BC = \sqrt{\overline{DB}^2 + \overline{DC}^2} = \sqrt{12^2 + 7.247^2} = \sqrt{196.519} = 14.018$ inches.

Perimeter of triangle equals $AB + BC + AC = 12.209 + 14.018 + 9.5 = 35.73$ inches. Ans.

(333) The diagonal divides the trapezium into two triangles; the sum of the areas of these two triangles equals the area of the trapezium, which is, therefore,

$$\frac{11 \times 7}{2} + \frac{11 \times 4\frac{1}{4}}{2} = 61\frac{7}{8} \text{ square inches. Ans.}$$

(334) Referring to Fig. 17, problem 350, we have OA or $OB = \frac{10}{2}$ or 5 inches, and $AB = 6\frac{3}{4}$ inches.

$\sin COB = \frac{CB}{OB} = \frac{6\frac{3}{4} \div 2}{5} = .675$; therefore, angle $COB = 42° 27' 14.3''$.

Angle $AOB = (42° 27' 14.3'') \times 2 = 84° 54' 28.6''$. Ans.
$CO = OB \times \cos COB = 5 \times .73782 = 3.6891$.

Area of sector $= 10^2 \times .7854 \times \frac{84° 54' 28.6''}{360°} = 18.524$ sq. in.

Area of triangle $= \frac{6.75 \times 3.6891}{2} = 12.450$ sq. in.

$18.524 - 12.450 = 6.074$ sq. in., the area of the segment. Ans.

(335) Convex area $= \frac{\text{perimeter of base} \times \text{slant height}}{2} = \frac{63 \times 17}{2} = 535.5$ square inches. Ans.

(336) See Fig. 14. Area of lower base $= 18^2 \times .7854 = 254.469$ sq. in.

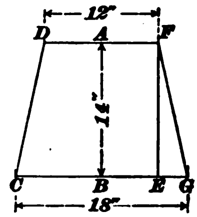

FIG. 14.

Area of upper base $= 12^2 \times .7854 = 113.0976$ sq. in.

$GE = BG - AF = 9 - 6$, or 3 inches.

Slant height $FG = \sqrt{\overline{GE}^2 + \overline{EF}^2} = \sqrt{3^2 + 14^2} = 14.32$ inches.

Convex area $=$

$$\frac{\text{circumference of upper base} + \text{circumference of lower base}}{2} \times$$

slant height, or convex area $= \frac{37.6992 + 56.5488}{2} \times 14.32 = 674.8156$ sq. in.

Total area $= 674.8156 + 254.469 + 113.0976 = 1{,}042.3$ sq. in. Ans.

Volume $=$ (area of upper base $+$ area of lower base $+$ $\sqrt{\text{area of upper base} \times \text{area of lower base}}) \times \frac{1}{3}$ of the altitude $=$ $(113.0976 + 254.4696 + \sqrt{113.0976 \times 254.4696})\frac{14}{3} = 2{,}506.84$ cubic inches. Ans.

(337) Area of surface of sphere 27 inches in diameter $= 27^2 \times 3.1416 = 2{,}290.2$ sq. in. Ans.

(338) Volume of each ball $= \frac{10}{.261} = 38.3142$ cu. in.

$$\text{Diameter of ball} = \sqrt[3]{\frac{38.3142}{.5236}} = 4.18 \text{ inches. Ans.}$$

(339) Area of end $= 19^2 \times .7854 = 283.5294$ sq. in. Volume $= 283.5294 \times 24 = 6{,}804.7056$ cubic inches $= 3.938$ cubic feet. Ans.

(340) Given, $BI = 2$ inches and $HI = IK = \frac{14}{2} = 7$ inches to find the radius.

$BI : HI :: HI : AI$, or $2 : 7 :: 7 : AI$;

therefore, $AI = \frac{49}{2} = 24\frac{1}{2}$ inches.

$$AB = AI + BI = 24\frac{1}{2} + 2 = 26\frac{1}{2} \text{ inches.}$$

$$\text{Radius} = \frac{AB}{2} = \frac{26\frac{1}{2}}{2} = 13\frac{1}{4} \text{ inches. Ans.}$$

(341) (*a*) Area of piston $= 19^2 \times .7854 = 283.529$ sq. in., or 1.9689 square feet.

Length of stroke plus the clearance $= 1.14 \times 2$ ft. (24 in. = 2 ft.) = 2.28 ft.

$1.9689 \times 2.28 = 4.489$ cubic feet, or the volume of steam in the small cylinder. Ans.

(*b*) Area of piston $= 31^2 \times .7854 = 754.7694$ sq. in., or 5.2414 square feet.

Length of stroke plus the clearance $= 1.08 \times 2 = 2.16$ ft.

$5.2414 \times 2.16 = 11.321$ cubic feet, or the volume of steam in the large cylinder. Ans.

$$(c)\quad \text{Ratio} = \frac{11.321}{4.489}, \text{ or } 2.522:1. \quad \text{Ans.}$$

(342) (*a*) Area of cross-section of pipe $= 8^2 \times .7854 =$ 50.2656 sq. in.

$$\text{Volume of pipe} = \frac{50.2656 \times 7}{144} = 2.443 \text{ cu. ft.} \quad \text{Ans.}$$

(*b*) Ratio of volume of pipe to volume of small cylinder $= \frac{2.443}{4.489}$, or 0.544:1. Ans.

(343) (*a*) In Fig. 15, given, $OB = \frac{16}{2}$, or 8 inches, and $OA = \frac{13}{2}$, or $6\frac{1}{2}$ inches, to find the volume, area and weight:

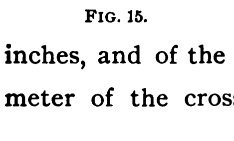

FIG. 15.

Radius of center circle equals $\frac{8 + 6.5}{2}$ or $7\frac{1}{4}$ inches. Length of center line $=$ $2 \times 3.1416 \times 7\frac{1}{4} = 45.5532$ inches.

The radius of the inner circle is $6\frac{1}{2}$ inches, and of the outer circle 8 inches; therefore, the diameter of the cross-section on the line AB is $1\frac{1}{2}$ inches. Then, the area of the ring is $1\frac{1}{2} \times 3.1416 \times 45.553 = 214.665$ square inches. Ans.

Diameter of cross-section of ring = $1\frac{1}{2}$ inches.

Area of cross-section of ring $= \left(1\frac{1}{2}\right)^2 \times .7854 = 1.76715$ sq. in. Ans.

Volume of ring $= 1.76715 \times 45.553 = 80.499$ cu. in. Ans.

(*b*) Weight of ring $= 80.499 \times .261 = 21$ lb. Ans.

(344) The problem may be solved like the one in Art. **790.** A quicker method of solution is by means of the principle given in Art. **826.**

(345) The convex area $= 4 \times 5\frac{1}{4} \times 18 = 378$ sq. in. Ans.

Area of the bases $= 5\frac{1}{4} \times 5\frac{1}{4} \times 2 = 55.125$ sq. in.

Total area $= 378 + 55.125 = 433.125$ sq. in. Ans.

Volume $= \left(5\frac{1}{4}\right)^2 \times 18 = 496.125$ cu. in. Ans.

(346) In Fig. 16, $OC = \frac{AC}{\tan 30°}$. $\left(\frac{1}{6}\right.$ of 360° = 60°, and since $AOC = \frac{1}{2}$ of AOB, $AOC = 30°\left.\right)$.

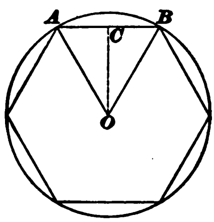

FIG. 16.

$OC = \frac{6}{.57735} = 10.392.$

Area of $AOB = \frac{12 \times 10.392}{2} = 62.352$ square feet.

Since there are 6 equal triangles in a hexagon, then the area of the base $= 6 \times 62.352$, or 374.112 square feet.

Perimeter $= 6 \times 12$, or 72 feet.

Convex area $= \frac{72 \times 37}{2} = 1,332$ sq. ft. Ans.

Total area $= 1,332 + 374.112 = 1,706.112$ sq. ft. Ans.

(347) Area of the base = 374.112 square feet, and altitude = 37 feet. Since the volume equals the area of the base multiplied by $\frac{1}{3}$ of the altitude, we have

$$\text{Volume} = 374.112 \times \frac{37}{3} = 4,614 \text{ cubic feet.}$$

(348) Area of room $= 15 \times 18$ or 270 square feet.

One yard of carpet 27 inches wide will cover $3 \times 2\frac{1}{4}$ (27 inches $= 2\frac{1}{4}$ ft.) $= 6\frac{3}{4}$ sq. ft. To cover 270 sq. ft., it will take $\frac{270}{6\frac{3}{4}}$, or 40 yards. Ans.

(349)

$$\begin{aligned} \text{Area of ceiling} &= 16 \times 20 = 320 \text{ square feet.} \\ \text{Area of end walls} &= 2(16 \times 11) = 352 \text{ square feet.} \\ \text{Area of side walls} &= 2(20 \times 11) = \underline{440} \text{ square feet.} \\ \text{Total area} &= 1{,}112 \text{ square feet.} \end{aligned}$$

From the above number of square feet, the following deductions are to be made:

$$\begin{aligned} \text{Windows} &= 4(7 \times 4) = 112 \text{ square feet.} \\ \text{Doors} &= 3(9 \times 4) = 108 \text{ square feet.} \end{aligned}$$

Baseboard less the width of the three doors

$$\text{equals } (72' - 12') \times \frac{6}{12} = 30 \text{ square feet.}$$

Total No. of feet to be deducted $= 250$ square feet.

Number of square feet to be plastered, then, equals 1,112 $- 250$, or 862 square feet, or $95\frac{7}{9}$ square yards. Ans.

(350) Given $A\,B = 6\frac{7}{8}$ inches, and $O\,B = O\,A = \frac{10}{2}$ or 5 inches, Fig. 17, to find the area of the sector.

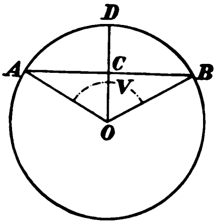

FIG. 17.

Area of circle $= 10^2 \times .7854 = 78.54$ square inches.

$\text{Sin } A\,O\,C = \frac{A\,C}{O\,A} = \frac{6\frac{7}{8} \div 2}{5} = .6875$; therefore, $A\,O\,C = 43° \; 26'$.

$A\,O\,B = 2 \times A\,O\,C = 2 \times 43° \; 26' = 86° \; 52' = 86.8666°$.

$\frac{86.8666}{360} \times 78.54 = 18.95$ square inches. Ans.

(351) Area of parallelogram equals

$7 \times 10\frac{3}{4}$ (129 inches $= 10\frac{3}{4}$ ft.) $= 75\frac{1}{4}$ sq. ft. Ans.

(352) (*a*) See Art. **778.**

Area of the trapezoid $= \frac{15\frac{7}{12} + 21\frac{11}{12}}{2} \times 7\frac{2}{3} = 143.75$ sq. ft. Ans.

(*b*) In the equilateral triangle ABC, Fig. 18, the area, 143.75 square feet, is given to find a side. Since the triangle is equilateral all the angles are equal to $\frac{1}{3}$ of 180° or 60°. In the triangle $ABD = ADC$, we have $AD = AB \times \sin 60°$. The area of any triangle is equal to one-half the product of the base by the altitude, therefore, $\frac{BC \times AD}{2} = 143.75$. $BC = AB$ and $AD = AB \times \sin 60°$; then, the above becomes

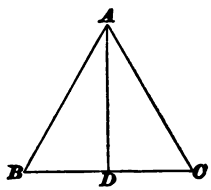

FIG. 18.

$$\frac{AB \times AB \sin 60°}{2} = 143.75,$$

or $$\frac{AB^2 \times .86603}{2} = 143.75,$$

or $$AB^2 = \frac{2 \times 143.75}{.86603}.$$

Therefore, $AB = \sqrt{\frac{287.50}{.86603}} = 18$ ft. 2.64 in. Ans.

(353) (*a*) Side of square having an equivalent area $= \sqrt{143.75} = 11.99$ feet. Ans.

(*b*) Diameter of circle having an equivalent area $= \sqrt{\frac{143.75}{.7854}} = \sqrt{183.0277} = 13\frac{1}{2}$ feet. Ans.

(*c*) Perimeter of square $= 4 \times 11.99 = 47.96$ ft.

Circumference or perimeter of circle $= 13\frac{1}{2} \times 3.1416 = 42.41$ ft.

Difference of perimeter $= 5.55$ ft. $=$ 5 feet 6.6 inches. Ans.

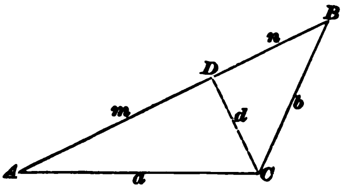

FIG. 19.

(354) In the triangle ABC, Fig. 19,

$AB = 24$ feet,
$BC = 11.25$ feet, and
$AC = 18$ feet.

$m + n : a + b :: a - b : m - n$,
or $24 : 29.25 :: 6.75 : m - n$.

$$m - n = \frac{29.25 \times 6.75}{24} = 8.226562.$$

Solving for m and n (see Art. **761**),

$$m = \frac{(m+n) + (m-n)}{2} = \frac{24 + 8.226562}{2} = 16.113281 \text{ ft.};$$

$$n = \frac{(m+n) - (m-n)}{2} = \frac{24 - 8.226562}{2} = 7.886719 \text{ ft.}$$

In the triangle ADC, side $AC = 18$ feet, side $AD = 16.113281$; hence, according to Rule **3**, Art. **754**, $\cos A = \frac{16.113281}{18} = .89518$, or angle $A = 26° 28' 5''$. In the triangle BDC, side $BD = 7.886719$, and side $BC = 11.25$ ft. Hence, $\cos B = \frac{7.886719}{11.25} = .70104$, or angle $B = 45° 29' 23''$.
Angle $C = 180° - (45° 29' 23'' + 26° 28' 5'') = 108° 2' 32''$.

Ans. $\begin{cases} A = 26° 28' 5''. \\ B = 45° 29' 23''. \\ C = 108° 2' 32''. \end{cases}$

GASES MET WITH IN MINES.

(QUESTIONS 355–444.)

(355) See Art. **828.**

(356) See Art. **834.**

(357) See Art. **841.**

(358) See Art. **849.**

(359) Carbonic acid gas, because it contains more matter per unit of volume, and is more compact and heavier than marsh-gas. See Art. **830** and Table 19 (Art. **865**).

(360) See Art. **831.**

(361) Applying formula **2,** we have

$$\text{Sp. Gr.} = \frac{175}{62.5} = 2.8. \quad \text{Ans.}$$

(362) See Arts. **835** and **836.**

(363) It drives or tends to drive the molecules apart. See Art. **843.**

(364) See Arts. **846, 847,** and **848.**

(365) See Art. **860.**

(366) See Art. **864.**

(367) See Art. **881.**

(368) The amount of matter in a body, regardless of the space it occupies, is called mass, while the space which the body occupies, regardless of the amount of matter, is called the volume. See Art. **829.**

(369) Applying formula **4,** we have $62.5 \times 1.5 = 93.75$ lb., the weight of 1 cu. ft. of anthracite coal. Hence, the weight of 1 cu. yd. or 27 cu. ft. $= 27 \times 93.75$ lb. $= 2,531.25$ lb. Ans.

(370) See Art. **836.**

(371) See Art. **838.**

(372) See Art. **840.**

(373) A compound substance is a substance formed of molecules which are unlike in their nature. See Arts. **836** and **842.**

(374) See Art. **844.**

(375) See Art. **847.**

(376) See Art. **850.**

(377) See Art. **851.**

(378) Applying formula **8,**

$$v_1 = \frac{6 \times 8}{80} = \frac{3}{5} = .6 \text{ cu. ft.} \quad \text{Ans.}$$

(379) See Art. **853.**

(380) See Art. **859.**

(381) See Art. **860.**

(382) See Art. **866.**

(383) Applying formula **21,**

$$W = \frac{1.3253 \times 300 \times 30 \times 1.5291}{459 + 70} = 34.48 \text{ lb.} \quad \text{Ans.}$$

(384) See Art. **877.**

(385) See Art. **887.**

(386) See Art. **897.**

(387) See Arts. **902, 903,** and **904.**

(388) See Art. **871.**

(389) See Art. **861.**

(390) See Art. **898.**

(391) See Art. **895.**

(392) See Art. **899.**

(393) Applying formula **12,**

$$20 : W_1 :: 12 : .0763, \text{ or } W_1 = \frac{20 \times .0763}{12} = .1272 \text{ lb. } \textbf{Ans.}$$

(394) See Art. **889.**

(395) Applying formula **7,**

$$p_1 = \frac{3 \times 36}{4} = 27 \text{ lb. per square inch. Ans.}$$

(396) See Art. **870.**

(397) See Arts. **882** and **883.**

(398) See Art. **876.**

(399) See Art. **848.**

(400) See Art. **845.**

(401) See Art. **874.**

(402) One pound of bituminous coal, when burned, furnishes 14,400 B. T. U. (see Table 18); hence, 2,000 pounds will furnish 2,000 × 14,400 = 28,800,000 B. T. U. Ans.

(403) See Arts. **878** and **879.**

(404) See Art. **873.**

(405) See Art. **856.**

(406) See Art. **839.**

(407) See Art. **841.**

(408) The carbon in the coal = .88 × 300 lb. = 264 lb., and since the molecular weight of carbonic acid gas (CO_2) is 12 + 32 = 44, the carbon in the gas must be $\frac{12}{44}$ of the weight of the gas. Therefore, if 264 lb. of carbon be used to produce carbonic acid gas, 264 lb. will represent $\frac{12}{44}$ of the resulting product. Hence, $\frac{44}{44}$, or the whole of the gas formed, $= \frac{264 \times 44}{12} = 968$ lb. Ans.

See Art. **838** and Table 17.

(409) Dissociation is the disunion of the elements forming a compound. See Art. **837.**

(410) See Art. **834.**

(411) Applying formula **1,** we have

$$\text{Sp. Gr.} = \frac{10}{10 - 8.6} = 7.14. \quad \text{Ans.}$$

(412) To gases only. See Art. **841.**

(413) One molecule of CH_4 yields one molecule of CO_2, and since they are both compound gases, a molecule of each occupies the same volume. Hence, 1,200 cu. ft. of CH_4 will yield 1,200 cu. ft. of CO_2. Ans. See Art. **841.**

(414) Gases, liquids, and solids. See Art. **833.**

(415) See Art. **863.**

(416) See Art. **869.**

(417) See Art. **900.**

(418) See Art. **907.**

(419) See Art. **922.**

(420) See Art. **916.**

(421) Applying formula **20,**

$$V = \frac{15 \times 20 + 9 \times 15}{21} = 20.71 \text{ cu. ft.} \quad \text{Ans.}$$

(422) See Art. **918.**

(423) See Art. **922.**

(424) See Art. **862.**

(425) See Art. **923.**

(426) See Art. **919.**

(427) See Arts. **906** and **915.**

(428) See Art. **890.**

(429) Applying formula **14,** we have

$$p_1 = 14.7 \left(\frac{459 + 212}{459 + 50}\right) = 19.38 \text{ lb. per square inch.} \quad \text{Ans.}$$

(430) See Art. **901.**

(431) See Arts. **907** and **910.**

(432) See Art. **913.**

(433) See Art. **915.**

(434) See Art. **891.**

(435) Applying formula **15,** we have

$$P = \frac{.37052 \times 6.5 \times (459 + 84)}{76} = 17.21 \text{ lb. per square inch.}$$

Ans.

(436) See Art. **911.**

(437) See Arts. **912** and **914.**

(438) See Art. **892.**

(439) See Arts. **893** and **894.**

(440) See Art. **900.**

(441) See Art. **896.**

(442) Weight $= .0766 \times .559 \times 100 = 4.28$ lb. Ans. See Art. **832.**

(443) Because the square root of the density of carbonic acid gas is greater than that of marsh-gas. See Art. **865.**

(444) The specific gravity of marsh-gas is .559. Using formula **21,**

$$W = \frac{1.3253 \times 650 \times 29.5 \times .559}{459 + 60} = 26.75 \text{ lb. Ans.}$$

MINE VENTILATION.

(PART 1.)

(445) See Arts. **925, 927,** and **932.**

(446) (*a*) Using formula **27,**

$$t = \frac{v}{g} = \frac{1,876}{32.16} = 58.333 \text{ sec.},$$

the time the ball would require to reach the highest point. Hence, $58.333 \times 2 = 116.66$ seconds, or 1.944 min. Ans.

(*b*) By using formula **28,**

$$h = \frac{v^2}{2g} = \frac{1,876^2}{2 \times 32.16} = 54,716.66 \text{ ft.},$$

the height the ball will rise. Hence, $54,716.66 \times 2 = 109,433.3$ ft., the total distance over which the ball will pass. Ans.

(447) See Art. **937.**

(448) See Art. **936.**

(449) Using formula **43,**

$q = a\,v = 7 \times 7 \times 300 = 14,700$ cu. ft. per min. Ans.

(450) Since the water-gauge is equivalent to a certain pressure, law **(3)**, Art. **980,** may be used. Hence, substituting W and W_1 for p and p_1, respectively,

$$W : W_1 :: q^2 : q_1^2, \text{ or } 2 : 8 :: 15,000^2 : p_1;$$

whence, $p_1 = 30,000$ cu. ft. Ans.

(451) Law **(15)** evidently applies to this case. Calling the original quantity 1,

$$H : H_1 :: q^3 : q_1^3; \text{ or } 2 : H_1 :: 1^3 : 2^3;$$

whence, $H_1 = 16$ horsepower. Ans.

(452) Since $p = 5.2\,W$, $p = 5.2 \times 1.5 = 7.8$ lb. per sq. ft. Applying formula **36**,

$$P = pa = 7.8 \times 6 \times 7 = 327.6 \text{ lb.} \quad \text{Ans.}$$

(453) The perimeter of the 6 ft. × 6 ft. airway is $6 \times 4 = 24$ ft.; of the 8 ft. × $4\frac{1}{2}$ ft., $8 \times 2 + 4\frac{1}{2} \times 2 = 25$ ft. Since both airways have the same length, the 6 ft. × 6 ft. airway has less rubbing surface than the 8 ft. × $4\frac{1}{2}$ ft., its perimeter being less. Hence, the 6 ft. × 6 ft. airway will pass the greater quantity of air.

(454) Applying the method illustrated in Art. **992**,

$\sqrt{\frac{a_1^3}{s_1}} = \sqrt{\frac{36^3}{36,000}} = 1.1384$, since $a_1 = 6 \times 6 = 36$, and $s_1 = 4 \times 6 \times 1,500 = 36,000$.

$\sqrt{\frac{a_2^3}{s_2}} = \sqrt{\frac{42^3}{46,800}} = 1.2582$, since $a_2 = 6 \times 7 = 42$, and $s_2 = (2 \times 6 + 2 \times 7) \times 1,800 = 46,800$.

$\sqrt{\frac{a_3^3}{s_3}} = \sqrt{\frac{30^3}{29,700}} = .9535$, since $a_3 = 6 \times 5 = 30$, and $s_3 = (2 \times 6 + 2 \times 5) \times 1,350 = 29,700$.

$\sqrt{\frac{a_4^3}{s_4}} = \sqrt{\frac{25^3}{30,000}} = .7217$, since $a_4 = 5 \times 5 = 25$, and $s_4 = 4 \times 5 \times 1,500 = 30,000$.

sum = 4.0718

$$\left.\begin{aligned} q_1 &= \frac{1.1384}{4.0718} \times 45,000 = 12,582 \text{ cu. ft. per min. for (1).} \\ q_2 &= \frac{1.2582}{4.0718} \times 45,000 = 13,905 \text{ cu. ft. per min. for (2).} \\ q_3 &= \frac{.9535}{4.0718} \times 45,000 = 10,539 \text{ cu. ft. per min. for (3).} \\ q_4 &= \frac{.7217}{4.0718} \times 45,000 = 7,974 \text{ cu. ft. per min. for (4).} \end{aligned}\right\} \text{Ans.}$$

sum = 45,000

(455) Apply law **(22)**. Since $p = 5.2\,W$, $p = 5.2 \times 1 = 5.2$ lb. per square foot. Then, $p : p_1 :: d_1^5 : d^5$, or $5.2 : p_1 :: 5^5 : 6^5$; whence, $p_1 = 12.94$ lb. per sq. ft. Ans.

(456) Since quantity and velocity are directly proportional, we may substitute v for q in law **(15)**, obtaining $u : u_1 :: v^3 : v_1^3$, or, calling the power originally required 1,

$1 : u_1 :: 4^3 : 8^3$; whence, $u_1 = 8$; i. e., the ratio of increase will be 8 : 1. Ans.

(457) Applying law (**5**), and calling the original pressure and length each 1,

$$p : p_1 :: l : l_1, \text{ or } 1 : p_1 :: 1 : 2;$$

whence, $p_1 = 2$, and the ratio is 2 : 1. Ans.

(458) Applying law (**3**),

$$p : p_1 :: q^2 : q_1^2, \text{ or } 1 : p_1 :: 1^2 : 2^2;$$

whence, $p_1 = 4$, and the ratio is 4 : 1. Ans.

(459) Applying law (**15**),

$$u : u_1 :: q^3 : q_1^3, \text{ or } 1 : u_1 :: 1^3 : 2^3;$$

whence, $u_1 = 8$, and the ratio is 8 : 1. Ans.

(460) Since the volumes are proportional to the absolute temperatures, we may write $v : v_1 :: T : T_1$, T being $459 + 30 = 489$, and T_1 being $459 + 70 = 529$. Hence, $10{,}000 : v_1 :: 489 : 529$, or $v_1 = 10{,}818$ cu. ft. per min. Ans.

(461) Since $p = 5.2\ W$, $p = 5.2 \times 2 = 10.4$ lb. per sq. ft. Applying formula **48**,

$$H = \frac{pq}{33{,}000} = \frac{10.4 \times 120{,}000}{33{,}000} = 37.82 \text{ horsepower, nearly.}$$

Ans.

(462) Substituting W and W_1 for p and p_1 in law (**3**),

$$W : W_1 :: q^2 : q_1^2, \text{ or } 3 : W_1 :: 20{,}000^2 : 30{,}000^2;$$

whence, $W_1 = 6\frac{3}{4}$ in. Ans.

(463) 5 ft. per sec. $= 5 \times 60 = 300$ ft. per min. Applying formula **45**;

$$a = \frac{q}{v} = \frac{8{,}000}{300} = 26\tfrac{2}{3} \text{ sq. ft.} = \text{area of No. 1 split. Ans.}$$

$$a = \frac{10{,}000}{300} = 33\tfrac{1}{3} \text{ sq. ft.} = \text{area of No. 2 split. Ans.}$$

$$a = \frac{12{,}000}{300} = 40 \text{ sq. ft.} = \text{area of No. 3 split. Ans.}$$

$$a = \frac{14{,}000}{300} = 46\tfrac{2}{3} \text{ sq. ft.} = \text{area of No. 4 split. Ans.}$$

$$a = \frac{16{,}000}{300} = 53\tfrac{1}{3} \text{ sq. ft.} = \text{area of No. 5 split. Ans.}$$

(**464**) Applying formula **50**,

$$p = \frac{33{,}000\,H}{q} = \frac{33{,}000 \times 40}{112{,}000} = 11.79 \text{ lb. per sq. ft., nearly.}$$

Hence, $W = \frac{11.79}{5.2} = 2.27$ in., nearly. Ans.

(**465**) Applying the method described in Art. **992**,

$$\sqrt{\frac{a_1^3}{s_1}} = \sqrt{\frac{48^3}{192{,}000}} = .75894$$

$$\sqrt{\frac{a_2^3}{s_2}} = \sqrt{\frac{48^3}{280{,}000}} = .62846$$

$$sum = 1.38740$$

Then, $q_1 = \frac{.75894}{1.3874} \times 10{,}000 = 5{,}470$ cu. ft. per min. in A.

$q_2 = \frac{.62846}{1.3874} \times 10{,}000 = 4{,}530$ cu. ft. per min. in B.

Ans.

(**466**) (*a*) The easiest way to work this example is to calculate the ventilating pressure for each split; if all are equal, no regulators will be required, but if some, or all, are different, regulators must be introduced into those splits having the lesser values. The pressure may be calculated by using formula **44** to find the velocity, and then using formula **38**; but an easier way is to use the following formula, which is obtained by transposing terms in formula **q**, Art. **979**:

$$p = \frac{k\,s\,q^2}{a^3}.$$

Applying this formula, we have

$$p = \frac{k\,s\,q^2}{a^3} = \frac{.0000000217 \times 280{,}000 \times 5{,}000^2}{48^3} = 1.374 \text{ lb. per sq. ft. for } A.$$

$$p = \frac{k\,s\,q^2}{a^3} = \frac{.0000000217 \times 150{,}000 \times 10{,}000^2}{50^3} = 2.604 \text{ lb. per sq. ft. for } B.$$

$$p = \frac{ksq^2}{a^3} = \frac{.0000000217 \times 360{,}000 \times 20{,}000^2}{72^3} = 8.372 \text{ lb. per}$$
sq. ft. for C.

$$p = \frac{ksq^2}{a^3} = \frac{.0000000217 \times 160{,}000 \times 15{,}000^2}{48^3} = 7.064 \text{ lb. per}$$
sq. ft. for D.

Hence, to distribute the air as required by the example, regulators must be placed at A, B, and D. Ans.

(b) After placing the regulators, the pressure will be 8.372 lb. per sq. ft. in all the splits. Therefore, applying formula **48**,

$$H = \frac{pq}{33{,}000} = \frac{8.372 \times 50{,}000}{33{,}000} = 12.685 \text{ horsepower. Ans.}$$

(467) Using formula **21**,

$$W = \frac{1.3253\,VBD}{T},\quad W = \frac{1.3253 \times 30 \times 1 \times 1}{459 + 62} = .076313 \text{ lb.}$$

Now, applying formula **34**,

$$M = \frac{5.2\,G}{W} = \frac{5.2 \times .4}{.076313} = 27.256 \text{ ft. Ans.}$$

(468) In the last example, the weight of a cubic foot of air at 62° F. and 30 inches barometer was found to be .076313 lb.

Hence, applying formula **32**,

$$v = \sqrt{\frac{2gF}{w}} = \sqrt{\frac{2 \times 32.16 \times 2.08}{.076313}} = 41.873 \text{ ft. per sec.} =$$
2,512.38 ft. per min. Ans.

(469) None of the laws will apply to this case, but the example may be worked as follows: Denoting the quantity passed with 15 horsepower by 1, we have for the quantity passed with 36 horsepower [applying law (**15**)],

$$H : H_1 :: q^3 : q_1^3, \text{ or } 15 : 36 :: 1^3 : q_1^3;$$

whence, $q_1^3 = 2.4$, and $q_1 = \sqrt[3]{2.4}$.

Now, applying law (**3**) and substituting W and W_1 for p and p_1, respectively,

$$W : W_1 :: q^2 : q_1^2, \text{ or } .6 : W_1 :: 1^2 : \sqrt[3]{2.4^2};$$

whence, $W_1 = 1.076$ in., nearly. Ans.

(**470**) The rubbing surfaces are $(2 \times 6 + 2 \times 8) \times 8{,}000 = 224{,}000$ sq. ft., and $(2 \times 6 + 2 \times 8) \times 10{,}000 = 280{,}000$ sq. ft. Hence, applying law (**10**), Art. **980**,

$$q : q_1 :: \sqrt{s_1} : \sqrt{s}, \text{ or } 10{,}000 : q_1 :: \sqrt{280{,}000} : \sqrt{224{,}000};$$

whence, $q_1 = 8{,}945$ cu. ft. per min., nearly. Ans.

(**471**) See Arts. **975** and **976.** Since the airways have similar sections,

$$10 : x :: \sqrt[5]{5{,}000} : \sqrt[5]{8{,}000}, \text{ or } x = 10.99 \text{ ft.};$$

also, $10 : 10.99 :: 8 : x$, or $x = 8.792$ ft.

Hence, the required section is 8.792 ft. × 10.99 ft., say 8.8 ft. × 11 ft. Ans.

(**472**) Since the airway is square and $a = 64$ sq. ft., the length of a side $= d = \sqrt{64} = 8$ ft. Representing the pressure by 1, the units of power required would be $u = pq = 1 \times 15{,}000 = 15{,}000$ ft.-lb. Since the power is to remain the same, the pressure for the new airway must be less (the length remaining the same), since the quantity is greater. Hence, $u = p_1 q_1 = p_1 \times 20{,}000 = 15{,}000$, or $p_1 = \frac{15{,}000}{20{,}000} = .75$; i. e., the new pressure is .75 of the original pressure. By using formula **55,** $q = \sqrt{\frac{p d^5}{4 k l}}$, whence, $q^2 = \frac{p d^5}{4 k l}$; also, $q_1 = \sqrt{\frac{p_1 d_1^5}{4 k l}}$, whence, $q_1^2 = \frac{p_1 d_1^5}{4 k l}$. Dividing the first by the second, $\frac{q^2}{q_1^2} = \frac{p d^5}{p_1 d_1^5}$, the denominators canceling out, being equal, or $q^2 : q_1^2 :: p d^5 : p_1 d_1^5$.

Substituting the values of q, q_1, p, d, and p_1,

$$15{,}000^2 : 20{,}000^2 :: 1 \times 8^5 : .75 d_1^5;$$

whence, $$d_1 = \sqrt[5]{\frac{20{,}000^2 \times 8^5}{.75 \times 15{,}000^2}} = 9.507 \text{ ft.}$$

Hence, the area $= 9.507^2 = 90.38$ sq. ft. Ans.

(473) Applying formula **56**,

$$A = \frac{.0004\,q}{\sqrt{W}} = \frac{.0004 \times 8,000}{\sqrt{\frac{1}{4}}} = 6.4 \text{ sq. ft.} \quad \text{Ans.}$$

(474) (*a*) Applying the method described in Art. **992**,

$$\sqrt{\frac{a_1^3}{s_1}} = \sqrt{\frac{30^3}{19,800}} = 1.1678$$

$$\sqrt{\frac{a_2^3}{s_2}} = \sqrt{\frac{36^3}{19,800}} = 1.5350$$

$$\sqrt{\frac{a_3^3}{s_3}} = \sqrt{\frac{24^3}{16,800}} = .9071$$

$$\sqrt{\frac{a_4^3}{s_4}} = \sqrt{\frac{20^3}{12,960}} = .7857$$

$$sum = 4.3956$$

$$q_1 = \frac{1.1678}{4.3956} \times 60,000 = 15,942 \text{ cu. ft. for 1st split.}$$

$$q_2 = \frac{1.5350}{4.3956} \times 60,000 = 20,952 \text{ cu. ft. for 2d split.}$$

$$q_3 = \frac{.9071}{4.3956} \times 60,000 = 12,381 \text{ cu. ft. for 3d split.}$$

$$q_4 = \frac{.7857}{4.3956} \times 60,000 = 10,725 \text{ cu. ft. for 4th split.}$$

Ans.

$$sum = 60,000$$

(*b*) Velocity in main split = 60,000 ÷ 80 = 750 ft. per min., since sectional area = 8 × 10 = 80 sq. ft. Applying formula **38** to find the pressure,

$$p = \frac{k s v^2}{a} = \frac{.0000000217 \times 54,000 \times 750^2}{80} =$$

8.24 lb. per sq. ft., nearly.

To find the pressure necessary to force the air through the splits, consider split No. 1.

$$\text{Velocity} = \frac{15,942}{30} = 531.4 \text{ ft. per min.}$$

Applying formula **38**,

$$p_1 = \frac{k s_1 v_1^2}{a_1} = \frac{.0000000217 \times 19,800 \times 531.4^2}{30} =$$

4.04 lb. per sq. ft., nearly.

Total pressure = 8.24 + 4.04 = 12.28 lb. per sq. ft.

Hence, water-gauge $= \frac{12.28}{5.2} = 2.36$ in., nearly. Ans.

(475) According to law **(15)**,

$u : u_1 :: q^3 : q_1^3$, or $1 : u_1 :: 1^3 : 2^3$; whence, $u_1 = 8$.

That is, the power must be increased to 8 times its original amount in order to double the quantity. Ans.

(476) It is evident, from the conditions of the example, that one airway is 5 times the length of the other. Calling the length of the short airway 1, and the quantity passing through it 1; and the length of the long airway 5, and the quantity passing through it q_1, we have, applying law **(20)**, Art. **980**,

$l : l_1 :: q_1^2 : q^2$, or $1 : 5 :: q_1^2 : 1$; whence, $q_1 = \sqrt{\frac{1}{5}} = .4472$.

Since $q = 1$, $q + q_1 = 1 + .4472 = 1.4472$. Hence,

$q = \frac{1}{1.4472} \times 100,000 = 69,099$ cu. ft. per min. in short airway.

$q_1 = \frac{.4472}{1.4472} \times 100,000 = 30,901$ cu. ft. per min. in long airway.

} Ans.

(477) (*a*) Applying formula **35**,

$$M = \frac{D(t - t_1)}{459 + t} = \frac{300\,(130 - 50)}{459 + 130} = 40.75 \text{ ft.} \quad \text{Ans.}$$

(*b*) Applying formula **35**,

$$M = \frac{300\,(150 - 50)}{459 + 150} = 49.26 \text{ ft.}$$

The weight of a cubic foot of air at 50° F. and 30 inches barometer is

$$W = \frac{1.3253 \times 30}{459 + 50} = .07811 \text{ lb.}$$

Hence, the pressure per square foot = .07811 × 49.26 = 3.85 lb. Ans.

(478) (*a*) Applying the method illustrated in Art. **992,**

$$\sqrt{\frac{a_1^3}{s_1}} = \sqrt{\frac{36^3}{96{,}000}} = .69714$$

$$\sqrt{\frac{a_2^3}{s_2}} = \sqrt{\frac{30^3}{66{,}000}} = .63960$$

$$\sqrt{\frac{a_3^3}{s_3}} = \sqrt{\frac{25^3}{80{,}000}} = .44194$$

$$sum = 1.77868$$

$q_1 = \frac{.69714}{1.77868} \times 50{,}000 = 19{,}597$ cu. ft. per min. for 1st split.

$q_2 = \frac{.63960}{1.77868} \times 50{,}000 = 17{,}980$ cu. ft. per min. for 2d split. Ans.

$q_3 = \frac{.44194}{1.77868} \times 50{,}000 = 12{,}423$ cu. ft. per min. for 3d split.

$$sum = 50{,}000$$

(*b*) Applying formula used in solving example 466,

$$p = \frac{k s q^2}{a^3} = \frac{.0000000217 \times 400{,}000 \times 50{,}000^2}{100^3} =$$

21.7 lb. per sq. ft.

Therefore, $H = \frac{pq}{33{,}000} = \frac{21.7 \times 50{,}000}{33{,}000} =$

32.88 horsepower for first case. Ans.

$$p_1 = \frac{k s_1 q_1^2}{a_1^3} = \frac{.0000000217 \times 96{,}000 \times 19{,}597^2}{36^3} =$$

17.15 lb. per sq. ft.

Therefore, $H = \frac{pq}{33{,}000} = \frac{17.15 \times 50{,}000}{33{,}000} =$

25.98 horsepower for second case. Ans.

(479) Applying formula **41,**

$$l = \frac{s}{o} = \frac{54{,}000}{2 \times 9 + 2 \times 6} = 1{,}800 \text{ ft.} \quad \text{Ans.}$$

(480) 7 ft. 3 in. = $7\frac{1}{4}$ ft.; 11 ft. 9 in. = $11\frac{3}{4}$ ft. Hence, applying formula **43**,

$q = av = 7\frac{1}{4} \times 11\frac{3}{4} \times 434 = 36,971$ cu. ft. per min., nearly. Ans.

(481) Applying law (**3**),

$p : p_1 :: q^2 : q_1^2$, or $1 : p_1 :: 120,000^2 : 180,000^2$; whence, $p_1 = 2\frac{1}{4}$.

Therefore, the original pressure must be increased $2\frac{1}{4}$ times. Ans.

(482) Applying law (**3**),

$$p : p_1 :: q^2 : q_1^2, \text{ or } 19.2 : p_1 :: 160,000^2 : 120,000^2;$$

whence, $p_1 = 10.8$ lb. per sq. ft. Ans.

(483) For a water-gauge of .9 in., $p = 5.2 \times .9 = 4.68$ lb. per sq. ft. Applying formula **48**,

$$H = \frac{pq}{33,000} = \frac{4.68 \times 70,000}{33,000} = 9.927 \text{ horsepower. Ans.}$$

(484) Applying formula **35**,

$$M = \frac{D(t - t_1)}{459 + t} = \frac{300\,(120 - 45)}{459 + 120} = 38.86 \text{ ft. Ans.}$$

(485) For a water-gauge of .6 in., $p = 5.2 \times .6 = 3.12$ lb. per sq. ft. Applying law (**3**),

$$p : p_1 :: q^2 : q_1^2, \text{ or } 3.12 : p_1 :: 12,000^2 : 24,000^2;$$

whence, $p_1 = 12.48$ lb. per sq. ft. Ans.

(486) Quantity passing in first case is $6 \times 8 \times 300 =$ 14,400 cu. ft. per min. Applying law (**3**),

$$p : p_1 :: q^2 : q_1^2, \text{ or } 4 : p_1 :: 14,400^2 : 24,000^2;$$

whence, $p_1 = 11\frac{1}{9}$ lb. per sq. ft. Ans.

(487) Applying formula **48**,

$$H = \frac{pq}{33,000} = \frac{2.5 \times 20,000}{33,000} = 1\tfrac{17}{33} \text{ horsepower.}$$

Applying law (**15**),

$$H : H_1 :: q^3 : q_1^3, \text{ or } 1\tfrac{17}{33} : H_1 :: 20,000^3 : 25,000^3;$$

whence, $H_1 = 2.959$. Ans.

(488) (*a*) Applying formula **45,**

$$a = \frac{q}{v} = \frac{30,000}{500} = 60 \text{ sq. ft. Ans.}$$

(*b*) If the current be divided equally, $30,000 \div 2 = 15,000$ cu. ft. per min. must pass in each split, and area of either split $= \frac{15,000}{500} = 30$ sq. ft. Ans.

(*c*) Perimeter of large airway $= \sqrt{60} \times 4 = 31$ ft., nearly.

Sum of perimeters of two small airways $= \sqrt{30} \times 4 \times 2 =$ 44 ft., nearly.

Since the lengths of all the airways are equal, it is evident that the two small airways together have more rubbing surface than the large one; hence, they offer more resistance and require greater power in the proportion of 44 : 31, or 1.4194 : 1. Ans.

(489) The easiest method of solving this example is as follows:

By formula **r,** Art. **979,** $s = \frac{u}{k v^3}$ for the first airway, and $s_1 = \frac{u}{k v_1^3}$ for the second airway, since u, the power, is the same for both airways. By transposing the terms v^3 and v_1^3 in their respective equations, $s v^3 = \frac{u}{k}$ and $s_1 v_1^3 = \frac{u}{k}$. Since $s v^3$ and $s_1 v_1^3$ equal the same thing, i. e., $\frac{u}{k}$, they are equal to each other; in other words, $s v^3 = s_1 v_1^3$. For, if $5 \times 6 = 30$ and $2 \times 15 = 30$, it is clearly evident that $5 \times 6 = 2 \times 15$. Since the lengths of the airways are the same, the rubbing surfaces are proportional to the perimeters, and o and o_1 may be substituted for s and s_1. Hence, $o v^3 = o_1 v_1^3$. Now, $o = 2 \times 6 + 2 \times 10 = 32$ ft.; $o_1 = 2 \times 5 + 2 \times 6 = 22$ ft., and $v = \frac{24,000}{6 \times 10} = 400$ ft. per min. Therefore, $o v^3 = o_1 v_1^3$, or $32 \times 400^3 = 22 \times v_1^3$; whence,

$$v_1 = \sqrt[3]{\frac{32 \times 400^3}{22}} = 453.21 \text{ ft. per min.} =$$

velocity in small airway.

Applying formula **43,**

$q_1 = a_1 v_1 = 5 \times 6 \times 453.21 = 13,596$ cu. ft. per min., nearly. Ans.

(490) (*a*) The rubbing surface $= 8 \times 3.1416 \times 1,800 =$ 45,239 sq. ft. Applying the formula,

$$p = \frac{k s q^2}{a^3},$$

$$p = \frac{.0000000217 \times 45,239 \times 40,000^2}{(.7854 \times 8^2)^3} =$$

12.36 lb. per sq. ft., nearly. Ans.

(*b*) Applying formula **48,**

$$H = \frac{p q}{33,000} = \frac{12.36 \times 40,000}{33,000} = 14.982 \text{ horsepower. Ans.}$$

(491) The velocity before putting in regulator $= \frac{35,000}{70} = 500$ ft. per min.

The velocity after putting in regulator $= \frac{21,000}{70} = 300$ ft. per min.

Then, as in Art. **998,** $p : p_1 :: v^2 : v_1^2$, or (substituting W and W_1 for p and p_1) $.75 : W_1 :: 500^2 : 300^2$; whence, $W_1 = .27$ in. Therefore, $W - W_1 = .75 - .27 = .48$ in.

Applying formula **56,**

$$A = \frac{.0004 q}{\sqrt{W}} = \frac{.0004 \times 21,000}{\sqrt{.48}} = 12.12 \text{ sq. ft., nearly. Ans.}$$

(492) (*a*) See Art. **941.**
(*b*) See Art. **943.**
(*c*) See Art. **985.**

(493) (*a*) See Art. **997.**
(*b*) See Art. **993.**
(*c*) See Arts. **985** to **987** and Art. **999.**

(494) In Art. **995,** it is shown that $p : p_1 :: s v^2 : s_1 v_1^2$; substituting in this proportion the values given, and replacing p and p_1 by W and W_1,

$.7 : W_1 :: 1 \times 8^2 : 3 \times 10^2$; whence, $W_1 = 3.28$ in. Ans.

(495) (*a*) The rubbing surfaces of the splits are $(2 \times 7 + 2 \times 6) \times 2,000 \times 3 = 156,000$ sq. ft., and $(2 \times 7 + 2 \times 6) \times 5,000 \times 3 = 390,000$ sq. ft. Apply formula **q,** Art. **979,** to the short split.

Since $p = 5.2 \times 2.5 = 13$ lb. per sq. ft., $q = a\sqrt{\frac{pa}{ks}} = \sqrt{\frac{pa^3}{ks}} = \sqrt{\frac{13 \times 42^3}{.0000000217 \times 156{,}000}} = 16{,}868$ cu. ft. per min. Ans.

Applying the same formula to the long split,

$$q = \sqrt{\frac{pa^3}{ks}} = \sqrt{\frac{13 \times 42^3}{.0000000217 \times 390{,}000}} =$$
10,668 cu. ft. per min. Ans.

(*b*) The total quantity $= 16{,}868 + 10{,}668 = 27{,}536$ cu. ft. per min. Applying formula **48**,

$$H = \frac{pq}{33{,}000} = \frac{13 \times 27{,}536}{33{,}000} = 10.85 \text{ horsepower. Ans.}$$

(*c*) As in example 491, $W : W_1 :: v^2 : v_1^2$. But $v = \frac{16{,}868}{42} = 401.6$ ft. per min., nearly; hence, $v_1 = \frac{401.6}{2} = 200.8$ ft. per min. Therefore, $2.5 : W_1 :: 401.6^2 : 200.8^2$; whence, $W_1 = .625$ in. Applying formula **56**, we have, since $16{,}868 \div 2 = 8{,}434$, and $2.5 - .625 = 1.875$,

$$A = \frac{.0004q}{\sqrt{W}} = \frac{.0004 \times 8{,}434}{\sqrt{1.875}} = 2.46 \text{ sq. ft., nearly. Ans.}$$

(496) According to formula **53**, $u = ksv^3$. Since $v = \frac{q}{a}$, $v^3 = \frac{q^3}{a^3}$ and $u = ks\frac{q^3}{a^3}$, or $\frac{sq^3}{a^3} = \frac{u}{k}$. Now, as the power is the same for both airways, we have $\frac{s_1 q_1^3}{a_1^3} = \frac{u}{k}$. Hence, $\frac{sq^3}{a^3} = \frac{s_1 q_1^3}{a_1^3}$. Assuming that both airways have the same length, we can substitute o and o_1 for s and s_1. Therefore, $\frac{oq^3}{a^3} = \frac{o_1 q_1^3}{a_1^3}$. Substituting the values given,

$$\frac{3.1416 \times 18 \times 50{,}000^3}{(.7854 \times 18^2)^3} = \frac{3.1416 \times 6 \times q_1^3}{(.7854 \times 6^2)^3};$$

whence, $q_1 = 8{,}012.5$ cu. ft. per min. Ans.

(497) (*a*) See Art. **973.**
(*b*) See Art. **958.**
(*c*) See Art. **968.**

(498) (*a*) The solution is similar to that given in Art. **1000,** $2^5 = 32$; hence, the first figure is 2.

(3) $35 - 32 = 3$. Annexing five ciphers gives 300,000.

(4) $2^4 \times 5$ with four ciphers annexed $= 800{,}000$; 2^3 with four ciphers annexed $= 80{,}000$; the sum $= 880{,}000$.

(5) Since 300,000 will not contain 880,000, the second figure of the root is 0, and the first two figures are 20.

(6) $20^5 = 3{,}200{,}000$; $3{,}500{,}000 - 3{,}200{,}000 = 300{,}000$.

(7) $20^4 \times 5 = 800{,}000$; $300{,}000 \div 800{,}000 = .37$, or .4.

(8) 20^3 with a cipher annexed $= 80{,}000$; $80{,}000 \times .4 = 32{,}000$, and $800{,}000 + 32{,}000 = 832{,}000$.

(9) $300{,}000 \div 832{,}000 = .36$, the fourth and fifth figures. Hence, $\sqrt[5]{35} = 2.036$. Ans.

(*b*) (1) Pointing off into periods gives 642′68937.

(2) $3^5 = 243$; $4^5 = 1{,}024$; hence, the first figure of the root is 3.

(3) $642 - 243 = 399$; annexing the second period gives 39,968,937.

(4) $3^4 \times 5$ with four ciphers annexed $= 4{,}050{,}000$; 3^3 with four ciphers annexed $= 270{,}000$; the sum $= 4{,}050{,}000 + 270{,}000 = 4{,}320{,}000$.

(5) $39{,}968{,}937 \div 4{,}320{,}000 = 9 +$. But 9 is evidently much too large; hence, try 6, making the first two figures of the root 36.

(6) $36^5 = 60{,}466{,}176$; $64{,}268{,}937 - 60{,}466{,}176 = 3{,}802{,}761$.

(7) $36^4 \times 5 = 8{,}398{,}080$; $3{,}802{,}761 \div 8{,}398{,}080 = .452$, or .4.

(8) 36^3 with a cipher annexed $= 466{,}560$; $466{,}560 \times .4 = 186{,}624$, and $8{,}398{,}080 + 186{,}624 = 8{,}584{,}704$.

(9) $3{,}802{,}761 \div 8{,}584{,}704 = .443$, say .44.

Hence, $\sqrt[5]{64{,}268{,}937} = 36.44$. Ans.

MINE VENTILATION.

(PART 2.)

(499) See Art. **1002.**

(500) See Art. **1004.**

(501) See Art. **1005.**

(502) See Art. **1006.**

(503) Using formula **57,**

$$W = \frac{1.3253 \times 30.25}{459 + 350} = .0495.$$

$$.0495 \times 500 \times 3 = 74.25 \text{ lb. Ans.}$$

(504) Using formula **57,**

$$W = \frac{1.3253 \times 29.3}{459 + 32} = .0791 \text{ lb., nearly. Ans.}$$

(505) Using formula **43,** $q = a v = 8 \times 10 \times 800 =$ 64,000 cu. ft. of air at 32° F. Then, using formula **58,** and substituting for T, t, and v their numerical values as given, we have

$$459 + 60 = \frac{V}{64,000} \times (459 + 32),$$

and $$519 = \frac{491}{64,000} V = .007672\, V,$$

and $$V = \frac{519}{.007672} = 67,650 \text{ cu. ft., nearly. Ans.}$$

(506) (*a*) Using formula **57,**

$$W = \frac{1.3253 \times 29.8}{459 + 0} = .086043 \text{ lb.,}$$

average weight of 1 cu. ft. downcast air;

$$W = \frac{1.3253 \times 29.8}{459 + 300} = .052034 \text{ lb.,}$$

average weight of 1 cu. ft. upcast air.

$.086043 \times 540 = 46.46322$ lb. per sq. ft., pressure in downcast column.

$.052034 \times 540 = 28.09836$ lb. per sq. ft., pressure in upcast column.

18.36486 lb. per sq. ft., difference. Ans.

(*b*) Using formula **60,** in which we substitute for .077, the average weight of 1 cu. ft. of the downcast air, as obtained in (*a*), we have

$$p = \frac{300 - 0}{459 + 300} \times .086043 \times 540 = 18.365 \text{ lb. Ans.}$$

(507) Using formula **59,**

$$M = \frac{360 - 60}{459 + 360} \times 200 \times 3 = 219.78 \text{ ft. Ans.}$$

(508) See Art. **1009.** The isolation of the ribs by side drifts. The isolation of the roof by an air-space between the two arches. The grate area must be proportioned to its work, or vary inversely as the square root of the depth of the shaft. The sectional area over and around the furnace must be proportional to the quantity of air required.

(509) Using formula **61,**

$$s = \frac{34}{\sqrt{250}} = 2.15 \text{ sq. ft. per horsepower.}$$

Using formula **48,** the horsepower is

$$H = \frac{50{,}000 \times 2 \times 5.2}{33{,}000} = 15.76 \text{ H. P., nearly.}$$

$$2.15 \times 15.76 = 33.884 \text{ sq. ft. Ans.}$$

(510) (*a*) See Art. **1011.** Its object is to isolate the return air of a gaseous mine from the flaming gases and sparks of the furnace.

(*b*) 150 feet.

(511) See Arts. **1012** and **1013.**

(512) (*a*) Any mechanical device for producing an air-current.

(*b*) The centrifugal fan and the steam-jet are familiar examples.

(513) The most prominent types of centrifugal ventilators now in use are represented by the Waddle, Schiele, Guibal, and Capell fans. See Art. **1044.**

(514) (*a*) By blowing and by exhausting.

(*b*) The efficiencies of each method are practically the same. See Art. **1043.**

(515) Blades curve backwards from the direction of their motion, and are so tapered that the breadths of the blades at different distances from the center vary inversely as their distances from the center. See Art. **1045.**

(516) The Schiele fan consists of a central disk provided with duplicate sets of blades upon its two sides. Air enters upon each side. The fan is surrounded by a spiral casing, which conducts the air to an evase chimney. See Art. **1046.**

(517) It provides a uniformly increasing sectional area about the fan, which gives a uniform velocity to the air-current all around the circumference. See Art. **1046.**

(518) To reduce the velocity of discharge and loss of energy. See Art. **1042.**

(519) See Art. **1047.**

(520) See Art. **1048.**

(521) (1) It is safer and (2) it has a uniform efficiency in deep and shallow mines alike.

(522) The *furnace* rarefies the air of one shaft by heat, thus causing a difference in pressure, which is the ventilating pressure; the *fan*, by exhaustion or compression, creates the difference in pressure between the intake and discharge openings of a mine. See Arts. **1021** and **1022.**

(523) Using formula **64,**

$$v = 18\sqrt{8.41} = 52.2 \text{ ft. per sec.} \quad \text{Ans.}$$

(524) The velocity of the air entering the fan should not exceed 18 feet per second. See Art. **1030.**

(525) See Art. **1030.**

$$\frac{175,000}{2} = 87,500 \text{ cu. ft. on each side.}$$

Using formula **66,**

$$d = .0343\sqrt{87,500} = 10.146 \text{ ft. Ans.}$$

(526) (*a*) It is the surface of the imaginary cylinder whose diameter is the diameter of the port of entry of the fan, and whose length is the breadth of the fan-blades. See Art. **1031.**

(*b*) The diameter of the port of entry and the width of the blades.

(527) See Art. **1030.**

$$\frac{250,000}{2} = 125,000 \text{ cu. ft. on each side.}$$

Using formula **66,** $d = .0343\sqrt{125,000} = 12.127$ ft. Now, using formula **67,** second case, $b = \frac{1}{2}d = \frac{1}{2} \times 12.127 = 6.06$ ft. Ans.

(528) See Art. **1031.**

$$\sqrt{\frac{153.9384}{.7854}} = 14.0 \text{ ft., diameter of port of entry.}$$

Using formula **67,**

$$b = \frac{1}{4}d = \frac{1}{4} \times 14 = 3.5 \text{ ft. Ans.}$$

(529) See Art. **1032.**

(530) See Art. **1050.**

(531) See Art. **1051.**

(532) That the velocity is not too low. See Art. **1052.**

(533) (*a*) Too high a velocity of the air-current renders a safety-lamp unsafe, as the flame may be blown through the gauze of the lamp.

(*b*) 450 feet per minute. See Art. **1053.**

(534) (*a*) Doors, stoppings, brattices, curtains, regulators, and overcasts, or bridges.

(*b*) See Art. **1054.**

(535) The water-gauge or manometer, and the anemometer. The water-gauge is used to measure the difference of pressure between the intake and return airways. The manometer is used for the same purpose. The anemometer is used to determine the velocity of the current. See Art. **1057.**

(536) See Arts. **1058, 1059,** and **1060.**

(537) See Arts. **1061, 1062,** and **1063.**

(538) Density of air depends mainly upon two factors, barometric pressure measured by the barometer, and temperature measured by the thermometer. See Art. **1064.**

(539) (*a*) Freezing-point of water = 0° C. and 32° F.; boiling-point of water = 100° C. and 212° F.

(*b*) 212° − 32° = 180° F. = 100° C. See Arts. **1066** and **1067.**

(540) Using formula **76,**

(*a*) $F = \frac{9}{5} \times 350 + 32 = 630 + 32 = 662°$ F. Ans.

(*b*) $F = \frac{9}{5}(-10) + 32 = -18 + 32 = 14°$ F. Ans.

(*c*) $F = \frac{9}{5}(-25) + 32 = -45 + 32 = -13°$ F. Ans.

(541) Using formula **77,**

(*a*) $C = \frac{5}{9}(365 - 32) = \frac{5}{9}(333) = 185°$ C. Ans.

(*b*) $C = \frac{5}{9}(5 - 32) = \frac{5}{9}(-27) = -15°$ C. Ans.

(*c*) $C = \frac{5}{9}(-49 - 32) = \frac{5}{9}(-81) = -45°$ C. Ans.

(542) See Art. **1068.**

(543) See Art. **1069.**

(544) Dip workings, because, as a rule, the *intake* air is cooler than the return air from the workings, and it is natural for the heavier, cool air to flow to the dip, and the lighter air to the rise. See Art. **1071.**

(545) (*a*) Positive air columns are those whose weight acts in the direction in which the current moves.

(*b*) Negative air columns are those whose weight is opposed to the direction of the current. See Art. **1070.**

(546) By ascensional ventilation we understand such a method of ventilation that the general course of the current will be towards the rise.

(547) The algebraic sum of the weights of the positive and negative air columns in any mine is always equal to the weight of the *motive* column, or the pressure per square foot producing the flow of air. See Art. **1072.**

(548) The main feature in such a case is the manner of splitting the air at each pair of cross-entries, when these entries are sufficiently developed to warrant the expense. See Art. **1074.**

(549) The velocity of the divided current, which must not fall below 3 or 4 feet per second in non-gaseous mines and 5 or 6 feet per second where gas is given off. See Art. **1074.**

(550) In non-gaseous mines the haulage roads should be made the return airways for two reasons. See Art. **1075.** In gaseous mines, haulage is done upon the intake airway in order to lessen, as far as possible, liability to explosion. See Art. **1077.**

(551) That the ventilation shall be ascensional.

(552) (1) The establishment of the air-current. (2) The direction of the current should not be altered, except upon most urgent demand. (3) Repairs of stoppings, doors, and brattices must be made rapidly, as no great advance can be made ahead of the air. See Art. **1081.**

(553) See Art. **1083.**

(554) Care must be taken to begin sealing off a fire at its side next to the return air, and work towards the intake, as then there is less opportunity for the entrapping of pure air, which would give rise to an explosion under certain conditions.

MINE SURVEYING AND MAPPING.

(PART 1.)

(555) See Art. **1087.**

(556) See Arts. **1091** and **1093.**

(557) See Art. **1098.**

(558) As 11.55 feet. See Art. **1093.**

(559) The distance could be measured by the ordinary method of measuring up or down hill, as explained in Art. **1098,** or by measuring the actual length of the slope and multiplying this length by the cosine of the angle of dip or rise. See Art. **1100.**

(560) See Art. **1099.**

(561) See Arts. **1101** to **1103.**

(562) See Art. **1101.**

(563) See Arts. **1104** and **1105.**

(564) The order of the letters on the face of the compass is N-W-S-E taken in a clockwise direction, the E and W being transposed to facilitate the taking of the bearings. See Art. **1107.**

(565) It is graduated to half-degrees. See Art. **1106.**

(566) See Arts. **1109** and **1110.**

(567) See Art. **1111.**

(568) See Arts. **1112** and **1113.**

(569) The zero of the vernier has moved towards the right more than one whole degree division, and lies between

1° 00′ and 1° 30′. Reading towards the left from the zero of the vernier, we reach the 15′ mark without having found a coinciding line. We therefore begin at the extreme right of the vernier and read towards its zero. The twelfth mark from the right end, or the twenty-seventh from the zero, according to the order followed, and which is shown by the top row of figures, is the coinciding line. Consequently, the reading is 1° 00′+ 27′= 1° 27′. See Art. **1114.**

(570) Observe, the zero of the vernier has been moved to the left a little more than two whole degree divisions on the limb. Reading the vernier in the direction opposite to its motion, we observe that the first line on the vernier which exactly coincides with a line on the limb is the 01′ line. The reading is, therefore, 2° 01′. See Art. **1115.**

(571) The zero of the vernier has moved to the left over three and one-half whole degree divisions, and lies between 3° 30′ and 4°. Reading the vernier in the direction opposite to its motion, we observe that 15′ is the first line on the vernier which exactly coincides with a line on the limb. The reading is, therefore, 3° 30′+ 15′ = 3° 45′. See Art. **1115.**

(572) The zero of the vernier has moved to the right not quite two whole degree divisions, and lies between 1° 30′ and 2° 00′. Reading towards the left from the zero of the vernier, we reach the 15′ mark without having found a coinciding line. We therefore begin at the extreme right of the vernier and read towards its zero. The fourth mark from the right end, or the nineteenth mark from the zero in the order followed (shown by the top row of figures), is found to coincide; consequently, the reading is 1° 30′+ 19′= 1° 49′. See Art. **1114.**

(573) See Arts. **1116** and **1117.**

(574) See Art. **1118.**

(575) For putting up points by which rooms, or chambers, can be driven very approximately on the proper course. See Arts. **1101** and **1122.**

(576) See Art. **1121.**

(577) Since the butt entry runs N 30° E, and the rooms run N 20° W, the center line of the entry and that of a room make an angle of 30° + 20° = 50°. Hence, applying formula **78,**

$$D = \frac{54}{\sin 50°} = \frac{54}{.76604} = 70.49 \text{ ft. Ans.}$$

(578) See Art. **1131.**

(579) The total latitude of a course is the distance its end (not beginning) is north or south of some station, usually the first, to which all the courses of the survey are referred. The total departure of the course is the distance its end is east or west of the same station. See Arts. **1135** to **1137.**

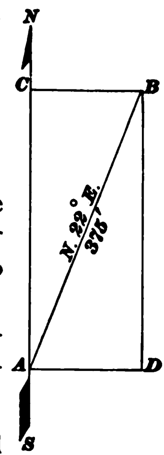

FIG. 20.

(580) Let NS, Fig. 20, be a meridian, and AB be the course; then, its latitude $= DB =$ $375 \times \cos 22° = 347.7$ feet, and its departure $= CB = 375 \times \sin 22° = 140.5$ ft. Ans. See Art. **1132.**

(581)

Station.	Bearing.	Distance.	Cosine.	Sine.	Latitude. North.	Latitude. South.	Departure. East.	Departure. West.
1–2	S 46° 30′ E	207.6	.68835	.72537		142.90	150.59	
2–3	S 74° 30′ E	309.5	.26724	.96363		82.71	298.24	
3–4	N 33° 15′ E	188.0	.83629	.54829	157.22		103.08	
4–5	N 56° 00′ W	276.0	.55919	.82904	154.34			228.82
5–6	Due West	213.5	.00000	1.00000				213.50
6-1	S 51° 54′ W	139.3	.61704	.78676		85.95		109.59
					311.56	311.56	551.91	551.91
					311.56		551.91	

(582)

Station.	Total. Latitude. North.	South.	Departure. East.	West.
1–2		142.90	150.59	
2–3		225.61	448.83	
3–4		68.39	551.91	
4–5	85.95		323.09	
5–6	85.95		109.59	
6–1				

(583) Station 3 has a total south latitude of 225.61 feet and a total east departure of 448.83 feet. Hence, its bearing will equal the angle whose tangent is $\frac{448.83}{225.61} = 1.98941$. By looking in a table of natural tangents, it will be found that 1.98941 corresponds very nearly to the tangent of 63° 19′. Therefore, the bearing from 1–3 is S 63° 19′ E. Ans.

(584) Care must be exercised in calculating the total latitudes and the total departures where more than one station has been located from the same station. See Art. **1138.**

(585)

Station.	Bearing.	Distance.	Cosine.	Sine.	Latitude. North.	South.	Departure. East.	West.
1–2	Due E	130	.00000	1.00000			130.00	
2–3	N 8° E	137	.99027	.13917	135.67		19.07	
3–4	N 81° W	186	.15643	.98769	29.10			183.71
4–5	Due S	54	1.00000	.00000		54.00		
5–6	S 36° W	125	.80902	.58779		101.13		73.47
6–7	S 45° E	89	.70711	.70711		62.93	62.93	
7–1	N 40° 18′ E	69.86	.76267	.64679	53.28		45.18	
					218.05	**218.06**	**257.18**	**257.18**

(586)

Station.	Bearing.	Distance.	Cosine.	Sine.	Latitude. North.	Latitude. South.	Departure. East.	Departure. West.
1–2	N 37° 13′ E	413.6	.79635	.60483	329.37		250.16	
2–3	N 10° 56′ E	246.7	.98185	.18967	242.22		46.79	
3–4	S 17° 23′ E	253.0	.95433	.29876		241.45	75.59	
4–5	S 43° 37′ E	216.0	.72397	.68983		156.38	149.00	
5–6	S 33° 43′ W	789.0	.83179	.55509		656.28		437.97
					571.59	1054.11	521.54	437.97
						571.59	437.97	
						482.52	83.57	

Station 6 is, therefore, 482.52′ south, and 83.57′ east of Station 1.

Hence, $\frac{83.57}{482.52} = \text{tangent } A = .17319$. See Fig. 21.

The angle A is, therefore, equal to $9° \ 49'+$, and the bearing from 1–6 is S 9° 49′ E.

The distance from 1–6 $= \sqrt{482.52^2 + 83.57^2} =$ 489.7 ft. Ans.

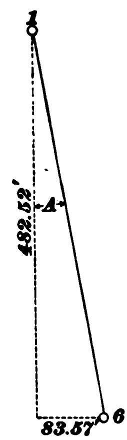

FIG. 21.

(587) This disagreement in the readings shows that the magnetic variation has changed as much as the difference between the two readings, or 30′. The variation has been from west to east; therefore, turn the vernier 30′ to the right, and the reading will be the same.

(588) They facilitate the calculating of latitudes and departures. See Art. **1133.**

(589) See Art. **1140.**

(590) The actual distance between the two points on the surface $= 4.378 \times 200 = 875.6$ ft. Ans. See Art. **1141.**

(591) The distance between them on the map $= \frac{537.8}{150} =$ 3.585 in. Ans. See Art. **1141.**

(592) (*a*) See Art. **1146.**
(*b*) See Art. **1152.**
(*c*) See Art. **1148.**

(593) See note, Art. **1146.**

(594) Platting by bearing and platting by latitude and departure; the latter method is the better, particularly where the total latitudes and the total departures are employed. See Arts. **1155, 1156,** and **1162.**

(595) The closing line of a survey is the straight line between two points connected by a survey. Its bearing and length are determined as explained in Arts. **1164** and **1165.**

(596) See Art. **1156.**

(597) See Art. **1166.**

(598) To find the area of the survey, we must calculate first the area enclosed by the outside lines *M*, *N*, *O*, *P*, Fig. 22,

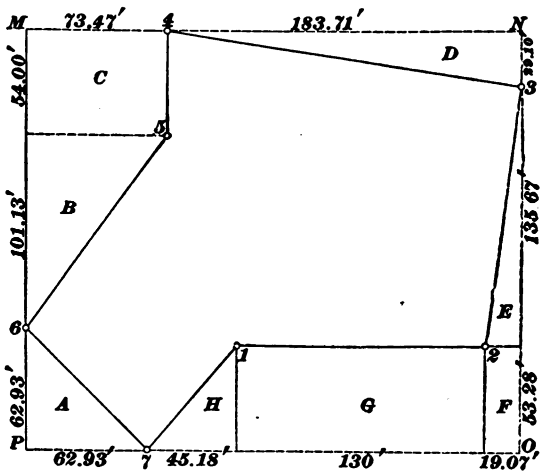

FIG. 22.

and from it subtract the combined areas of *A*, *B*, *C*, *D*, *E*, *F*, *G*, and *H*. This remainder will be the area of the survey.

Square Feet.

Area of rectangle $MNOP = 218.05 \times 257.18 =$ 56,078.10

Area of triangle $A = \frac{62.93 \times 62.93}{2} =$ 1,980.09

Area of triangle $B = \frac{101.13 \times 73.47}{2} =$ 3,715.01

Area of rectangle $C = 73.47 \times 54 =$ 3,967.38

Area of triangle $D = \frac{183.71 \times 29.10}{2} =$ 2,672.98

Area of triangle $E = \frac{135.67 \times 19.07}{2} =$ 1,293.61

Area of rectangle $F = 53.28 \times 19.07 =$ 1,016.05

Area of rectangle $G = 130 \times 53.28 =$ 6,926.40

Area of triangle $H = \frac{53.28 \times 45.18}{2} =$ 1,203.60

Area of $A + B + C + D + E + F + G + H =$ 22,775.12

The area of the survey is, therefore, $56{,}078.10 - 22{,}775.12 =$ 33,302.98 sq. ft. Ans.

(599) In order to calculate the area of the survey

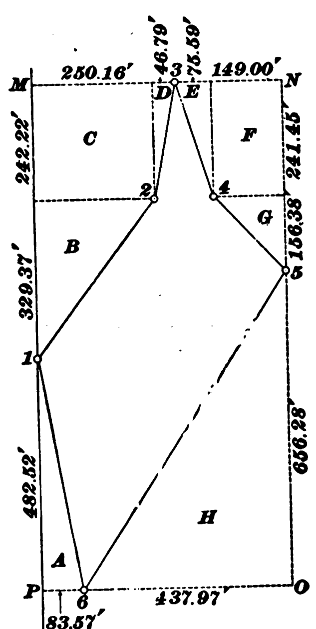

FIG. 23.

shown in Fig. 23, it will be necessary first to determine the

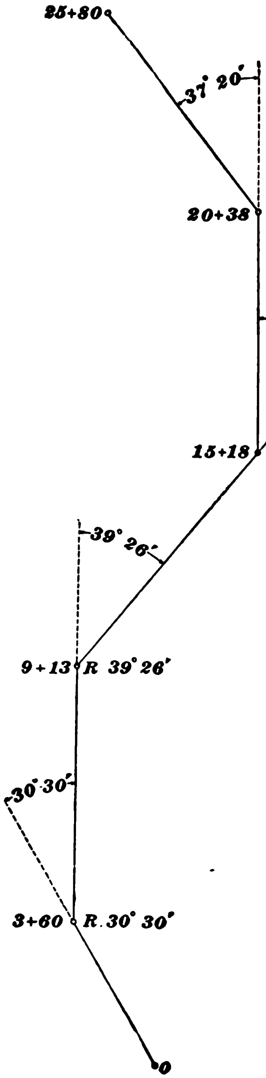

FIG. 24.

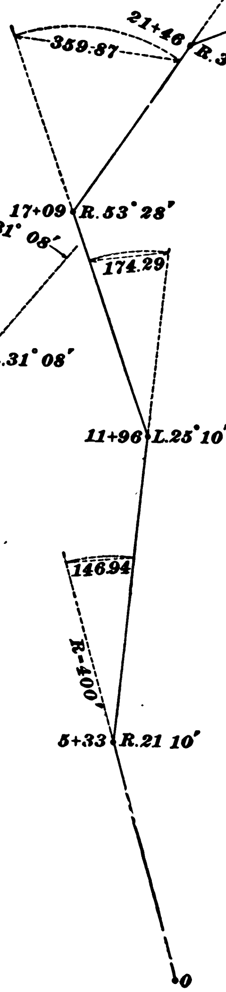

FIG. 25.

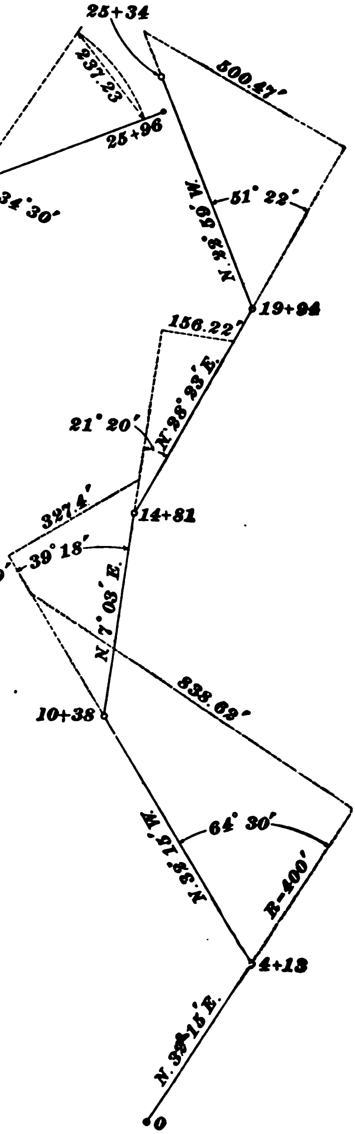

FIG. 26.

area enclosed by the outside lines MN, NO, OP, and PM, and then subtract from it the combined areas of A, B, C, D, E, F, G, and H.

Square Feet.

The area of $MNOP = 1{,}054.11 \times 521.54 = 549{,}760.53$

$$\text{The area of } A = \frac{482.52 \times 83.57}{2} = 20{,}162.10$$

$$\text{The area of } B = \frac{329.37 \times 250.16}{2} = 41{,}197.60$$

$$\text{The area of } C = 250.16 \times 242.22 = 60{,}593.76$$

$$\text{The area of } D = \frac{242.22 \times 46.79}{2} = 5{,}666.74$$

$$\text{The area of } E = \frac{241.45 \times 75.59}{2} = 9{,}125.60$$

$$\text{The area of } F = 241.45 \times 149.00 = 35{,}976.05$$

$$\text{The area of } G = \frac{156.38 \times 149.00}{2} = 11{,}650.31$$

$$\text{The area of } H = \frac{437.97 \times 656.28}{2} = 143{,}715.48$$

Total area of $A + B + C + D + E + F + G + H = 328{,}087.64$

The area of the survey is, therefore, 549,760.53 − 328,087.64 = 221,672.89 sq. ft. Ans.

(600) Fig. 24 shows the required plat. For details of platting a line by means of the protractor, see Arts. **1143** to **1145.**

(601) Fig. 25 shows the required plat. For method of platting a broken line by means of chords, see Art. **1147.**

(602) Fig. 26 shows the required line platted by tangents, as explained in Art. **1152.**

MINE SURVEYING AND MAPPING.

(PART 2.)

(603) See Art. **1167.**

(604) The advantages of the transit over the vernier compass are mainly due to the telescope on the transit, and its vertical arc, by means of which vertical angles can be measured. See Art. **1167.**

(605) A direct vernier. See Art. **1168.**

(606) There are four adjustments of the transit, which should be made whenever the instrument is to be used. The first adjustment is made to remove any lack of level that may be in the limb; the second is to bring the intersection of the cross-wires into the optical axis of the telescope; the third is to correct any derangement which the standards may have suffered; and the fourth adjustment is to make the line of sight of the telescope a level line when the bubble in the attached bubble-tube is in the center of the tube. These adjustments are made as explained in Arts. **1171** to **1174.**

(607) The reading is $70° \ 30' + 21'$, or $70° \ 51'$. For an explanation of the method of taking this reading and those of the following three questions, see Arts. **1176** to **1179.**

(608) The reading is $263° + 13' = 263° \ 13'$.

(609) The readings are $54° + 23' = 54° \ 23'$, and $305° + 37' = 305° \ 37'$.

(610) The readings are $60° \ 30' + 13' = 60° \ 43'$, and $119° + 17' = 119° \ 17'$.

(611) (*a*) The horizontal limb of the instrument is the horizontal plate carrying the small level tubes.

(*b*) The axis of the instrument is the vertical line passing through the center of the instrument perpendicular to the line of collimation.

(*c*) The line of collimation is the optical axis of the telescope.

(*d*) The standards are the supports for the horizontal axes of the telescope with its attached level, and they rest on and are made fast to the horizontal limb. See Arts. **1168** and **1169.**

(612) A horizontal angle measured from any point to two objects is the angle included between two vertical planes passing through the point and the objects. It may also be defined as the horizontal projection of the angle formed by drawing lines from one point to two other points. The method of measuring horizontal angles is explained in Art. **1180.**

(613) See Art. **1181.**

(614) (*a*) N E. (*b*) E. (*c*) S E. (*d*) S. (*e*) S W. (*f*) W. (*g*) N W. (*h*) N. See Art. **1182.**

(615) See Art. **1183.**

Stations.	Azimuths with *A B*.	Bearings with *A B*.
A	0°	Due North
B	35° 30′	N 35° 30′ E
C	110° 30′	S 69° 30′ E
D	270° 00′	Due West
E	330° 45′	N 29° 15′ W

(616) If the given first course *A B* is not really a north and south line, its magnetic bearing must be obtained by the compass, and then the magnetic bearings of the succeeding courses can be calculated by adding the magnetic bearing of *A B* to, or subtracting it from (as the case may be), the azimuths of the given courses, according as the magnetic bearing of *A B* is N E or N W of the meridian or north and south line. See Art. **1183.**

(617) See Art. **1181.**

(618) See Art. **1200.**

(619) See Art. **1209.**

(620) See Art. **1190.**

(621) The magnetic readings are taken merely to check the azimuth readings. See Art. **1191.**

(622) (*a*) A one-degree curve is one on which a 100-foot chord will subtend a central angle of one degree.

(*b*) A five-degree curve is one on which a 100-foot chord will subtend a central angle of five degrees. See Art. **1199.**

(623) See Arts. **1218** to **1223.**

(624) See Arts. **1225** to **1230.**

(625) There is not only danger of the timber being more or less displaced by the overlying weight, but it is possible that the timbermen may, in repairing the cross-bar or collar, so displace the station that the displacement would not be noticed. In this case, all work run from such a station would be wrong. It is better to have a station destroyed entirely than to have it displaced in such a way that it would not be noticed. See Art. **1228.**

(626) See Arts. **1184** and **1185.**

(627) They are technically called tangents. See Art. **1194.**

(628) The degree of a curve is determined by the central angle subtended by a chord of 100 feet. See Art. **1199.**

(629) See Art. **1207.**

(630) A right-angled glass prism placed at the end of the telescope. See Art. **1222.**

(631) See Art. **1205.**

(632) $24° \ 30' = 24.5°$. Hence, there will be as many hundred-foot lengths in the curve as 5 is contained in 24.5°, or $\frac{24.5°}{5} = 4.9$ lengths = 490 ft. Ans. See Art. **1205.**

(633) See Art. **1201.**

(634) Applying formula **80,**

$$T = R \tan \tfrac{1}{2} I = 819.02 \times \tan \frac{36°}{2} = 819.02 \times .32492 = 266.12 \text{ ft. Ans.}$$

The value of R for a 7° curve is found in the table of Radii and Deflections.

(635) See Art. **1188.**

(636) See Art. **1195.**

(637) See Arts. **1186** and **1189.**

(638) See Arts. **1215** and **1216.**

(639) See Art. **1192.**

(640) Applying formula **81,**

$$d = \frac{c^2}{R} = \frac{9^2}{45} = \frac{81}{45} = 1.8 \text{ ft. Ans.}$$

(641) The deflection angle for 100 feet on a 6° 30′ curve is $\frac{6° \ 30'}{2} = 3° \ 15' = 195'$; hence, the deflection angle for 1 foot on the same curve is $\frac{195'}{100} = 1.95'$, and for 48 feet it is $1.95' \times 48 = 93.6' = 1° \ 33' \ 36''$. Ans. See Art. **1205.**

(642) They are usually designated as curves of so many feet radius. See Art. **1215.**

(643) See Art. **1203.**

(644) See Art. **1205.**

(645) Since the deflection angle for a 10° curve is 5°, 5 stations 100 feet apart could be located with the transit at the P. C. of the curve, because $\frac{25}{5} = 5$. To locate other stations, the transit should be moved up to the last station, and the vernier plate firmly clamped at a reading of 360° − 25° = 335°. A backsight should then be taken to the P. C. station, being careful to keep the vernier plate clamped, and the line of sight of the telescope made to cut the P. C. station by means of the lower tangent screw. The vernier clamp is loosened and the instrument made to read 0°, at which reading the line of sight of the telescope will be tangent to the curve. From this point the operation is the same as starting from the P. C. of the curve. See Art. **1206.**

(646) A 9° curve has a radius of 637.27 ft. See Table 27. Hence, applying formula **81,**

$$d = \frac{c^2}{R} = \frac{120^2}{637.27} = \frac{14,400}{637.27} = 22.6 \text{ ft., nearly. Ans.}$$

Also, applying formula **82,**

$$f = \frac{c^2}{2R} = \frac{120^2}{2 \times 637.27} = \frac{14,400}{1,374.54} = 11.3 \text{ ft., nearly. Ans.}$$

(647) See Art. **1231.**

(648) See Art. **1237.**

(649) It is best to use large numbers for stations in the mine; for if a station is partly obliterated, it is more easily deciphered when marked with a large number than when marked with a small number. See Art. **1232.**

(650) See Art. **1233.**

(651) See Art. **1235.**

(652) Fig. 29 shows the platted survey and a vertical section through Sta. 1 and the face of the tunnel.

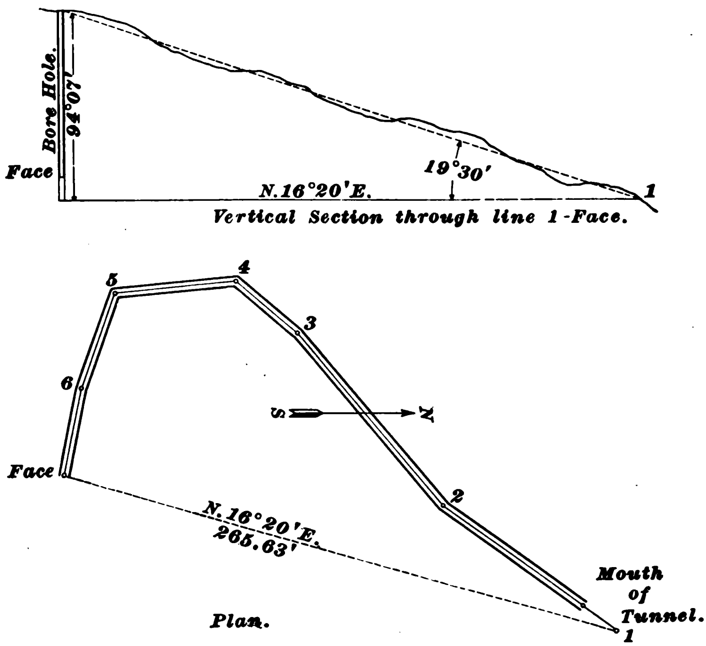

FIG. 29.

Station.	Bearing.	Dis-tance.	Cosine.	Sine.	Latitude.		Departure.	
					North.	South.	East.	West.
1–2	S 36° 50′ W	99.1	.80038	.59949		79.32		59.41
2–3	S 49° 47′ W	104.2	.64568	.76361		67.28		79.57
3–4	S 40° 00′ W	37.1	.76604	.64279		28.42		23.85
4–5	S 4° 55′ E	56.5	.99632	.08571		56.29	4.84	
5–6	S 71° 15′ E	46.0	.32144	.94693		14.79	43.56	
6–Face	S 77° 30′ E	40.7	.21644	.97630		8.81	39.74	
					000.00	254.91	88.14	162.83
						000.00		88.14
						254.91		74.69

(*a*) The face, therefore, is 254.91 feet south, and 74.69 feet west of Sta. 1. Hence, its bearing from Sta. 1 is found thus:

$$\frac{74.69}{254.91} = \text{tangent of bearing} = .29300 = \tan 16^\circ\ 20'.$$

Hence, the bearing is S 16° 20′ W from Sta. 1 to face of tunnel, or N 16° 20′ E from face of tunnel to Sta. 1.

The distance from Sta. 1 to face $= \sqrt{254.91^2 + 74.69^2} =$ 265.63 feet. Ans.

Station.	Distance.	In the Tunnel.		On the Surface.	
		Vertical Angle.	Vertical Height.	Vertical Angle.	Vertical Height.
1–2	99.1	+ 1° 18′	+ 2.25	+ 10° 35′	+ 18.52
2–3	104.2	+ 0° 31′	+ 0.94	+ 15° 43′	+ 29.32
3–4	37.1	+ 0° 45′	+ 0.49	+ 14° 27′	+ 9.56
4–5	56.5	− 0° 34′	− 0.56	+ 16° 17′	+ 16.50
5–6	46.0	+ 3° 37′	+ 2.91	+ 12° 21′	+ 10.07
6–Face	40.7	+ 3° 30′	+ 2.49	+ 13° 56′	+ 10.10
		Total,	+ 8.52		+ 94.07
					8.52

(*b*) Depth of face below surface = 85.55 ft. Ans.

(*c*) The vertical angle from Sta. 1 to a point on the surface vertically over the breast is found thus:

$$\frac{94.07}{265.63} = 0.35414 = \tan 19^\circ\ 30';$$

hence, 19° 30′ is the vertical angle. Ans.

MINE SURVEYING AND MAPPING.

(PART 3.)

(655) See Art. **1238.**

(656) See Art. **1239.**

(657) The best method of keeping notes in leveling is the method known as "height of instrument." For explanation of this method, see Art. **1248.**

(658) See Art. **1258.**

(659) When only the elevation between two points is required, the operation of leveling is very simple. The level is set up near one of the points and a reading taken on the rod placed upon the point of starting, which is assumed to have a certain height above the datum line. The height of the instrument is then determined, and a sight taken to the rod placed as far in the direction in which it is decided to run the line of levels between the two points as it will be possible to take a reading upon the rod. The level is then moved in the proper direction as far past this point as it will be possible to get a reading upon the rod still held upon the point last located. The height of the instrument is again determined, and a sight taken to the rod moved beyond the level as before. This process is continued until a reading is obtained on the rod placed upon the last point. The last reading taken from the last height of instrument will be the elevation of the last point above the datum line; then the difference in the elevation of the two points will simply be the difference of their heights above the datum line. Throughout the entire operation readings are taken on turning points only. See Art. **1249.**

(660) See Arts. **1251** and **1252.**

(661) See Art. **1241.**

(662) See Art. **1247.**

(663) See Art. **1240.**

(664) The slight curvature in the bubble-tube. The less the curvature the more sensitive the level. See Art. **1241.**

(665) This is done for uniformity, or so that by adding algebraically the backsight to the elevation of the turning point, the height of instrument is determined, or by adding algebraically the foresight to the height of instrument, the elevation of the last point is determined. See Art. **1248.**

(666) See Art. **1238.**

(667) See Arts. **1242** to **1244.**

(668) See Art. **1257.**

(669) The object of a topographical survey is to determine accurately the irregularities of the surface for the purpose of making a map on which such irregularities will be plainly shown. See Art. **1258.**

(670) See Art. **1258.**

(671) Level notes are checked by adding algebraically the foresights to the backsights. This sum should equal the difference in elevation of the first and the last station. See Art. **1248.**

(672) See Art. **1254.**

(673) See Art. **1259.**

(674) The leveling operations can only be checked by repeating the work. See Art. **1250.**

(675) See Art. **1246.**

(676) See Art. **1241.**

(677) The stations which are at regular distances apart are numbered consecutively, beginning at 0 and running up. A station between two regular stations is marked with a plus sign between the number of the regular station immediately preceding it and the number of feet beyond the same station. Thus, a station between Stations

3 and 4, and 35 feet beyond Station 3, would be marked as Station 3 + 35. See Art. **1247.**

(678) A general map of the mine, the property on which it is located, and the surface arrangements enables one to see at a glance the relative positions of the entries and rooms of the mine to the property lines, buildings, or bodies of water on the surface. See Art. **1264.**

(679) Buildings or other objects are located either by taking rights and lefts from some established line, or by sighting to them with the transit, and measuring the distances over the lines of sight. See Art. **1268.**

(680) It enables the engineer to determine the probable area underlaid with coal, as well as to determine the relative positions of the entries, room, etc., with the outcrop. See Art. **1267.**

(681) See Art. **1266.**

(682) See Art. **1267.**

(683) See Art. **1251.**

(684) See Art. **1266.**

(685) Referring to Fig. 30, $AM = AG \times \sin 43° =$

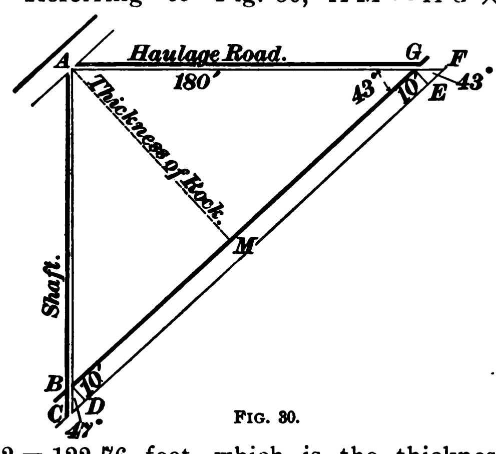

FIG. 30.

$180 \times .682 = 122.76$ feet, which is the thickness of rock between the seams measured at right angles to the pitch. Ans.

(686) (*a*) Referring to Fig. 30, it will be seen that $AB + BC =$ the depth of the shaft.

$$AB = 180 \times \tan 43° = 180 \times .93252 = 167.85 \text{ ft.}$$

$$BC = \frac{10}{\sin 47°} = \frac{10}{.73135} = 13.67 \text{ ft.}$$

Hence, the depth of the shaft $= 167.85 + 13.67 = 181.52$ ft. Ans.

(*b*) The distance on the pitch from the foot of the shaft to the level of the haulage-road in the underlying seam is equal to CF.

$$CF = \sqrt{\overline{AC}^2 + \overline{AF}^2} = \sqrt{(AC)^2 + (AG + GF)^2}.$$

$$GF = \frac{10}{\sin 43°} = \frac{10}{.682} = 14.66 \text{ ft.}$$

Therefore, $AF = AG + GF = 180 + 14.66 = 194.66$ ft.; hence, $CF = \sqrt{181.52^2 + 194.66^2} = 266.16$ ft. Ans.

(687) In the right-angled triangle ABC, Fig. 31, we have given the angle $CAB = 30°$. To find the length of AB, we have

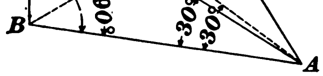

FIG. 31.

$$\tan CAB = \frac{CB}{AB}, \text{ or } AB = \frac{CB}{\tan CAB}.$$

Assuming CB to equal 1 foot, and substituting,

$$AB = \frac{1}{\tan 30°} = \frac{1}{.57735} = 1.732.$$

In the right-angled triangle ABE, we know the length of AB, and also the horizontal angle $BAE = 30°$. To find the length of AE, we have

$$\cos BAE = \frac{AB}{AE}, \text{ or } AE = \frac{AB}{\cos BAE}.$$

Substituting, we have

$$AE = \frac{1.732}{\cos 30°} = \frac{1.732}{.866} = 2.$$

In the triangle AED, $AE = 2$ feet, and $DE = CB =$ 1 foot. Hence, to find the angle DAE, which is the pitch of the rooms, we have

$$\tan DAE = \frac{DE}{AE} = \frac{1}{2} = .5,$$

which corresponds to 26° 34′, nearly. Ans.

(688) The dimensions of a claim, as allowed by the United States Mineral Laws, are 1,500 feet in the direction, or on the strike, of the vein, and 300 feet on each side of the middle of the vein at the surface. See Art. **1269.**

(689) The general principles in surveying metalliferous mines are the same as in surveying coal mines. The differences are only in the details. See Art. **1274.**

(690) Four drawings are necessary to represent the workings of a metalliferous mine: (1) the surface plan; (2) the working plan; (3) a longitudinal section; (4) a transverse section. See Art. **1274.**

(691) See Art. **1274.**

(692) See Art. **1274.**

(693) See Art. **1274.**

(694) See Art. **1274.**

(695) When the lode is very nearly flat, the longitudinal section is made along the lode. See Art. **1274.**

(696) The datum line is assumed to be 91.397 feet below the first bench-mark.

The level must be in perfect adjustment. It is then set up in some convenient place, and the reading of the rod is taken on the bench-mark. It is equal to 4.576 feet, and is recorded in the notes in the plus or backsight column opposite B. M. As the B. M. is assumed to be 91.397 feet above the datum line, the height of the instrument (or line of collimation) above this datum will be $91.397 + 4.576 =$ 95.973 feet.

The unit of measurement in the column of distances is 100 feet. Readings are taken at intermediate points (as at

340 and 670 feet in this example) where there are any abrupt changes in the inclination of the surface.

Station 1 is a turning point, T. P. The reading of the rod, held vertically on it, is 3.726 feet. This reading is recorded as a minus sight, and the surface height is the difference between 95.973 and 3.726 = 92.247 feet.

The levelman now goes forward as before, sets up his instrument in a convenient place, levels it and takes a backsight upon the rod, the target reading being taken to thousandths. The height of instrument will be the elevation of the T. P. plus this reading, which is on that account recorded, as in the notes, as a *plus* sight.

The height of instrument is, therefore, 92.247 + 5.420 = 97.667 feet.

The rodman now goes forward with the rod, and sets it upon Station 2, while the levelman sights to it without changing his former position. This reading is recorded as a minus sight, and by subtracting it from the height of instrument at the last turning point, we obtain the surface height; thus, 97.667 − 4.5 = 93.167 feet.

Station 3 is a turning point, T. P. The reading of the rod on it is 3.170 feet, whence the surface height = 97.667 − 3.170 = 94.497 feet. The levelman goes forward, backsights upon the rod, and has the target set so that the reading can be taken to thousandths. The height of the instrument is, therefore, 94.497 + 4.910 = 99.407 feet.

The rodman goes forward with the rod, sets it upon Station 3 + 40, and the levelman sights to it from his former position. The reading 4.9 feet is recorded as a minus sight, and the surface height is found to equal 99.407 − 4.9 = 94.507 feet.

Station 4 is a turning point, T. P. The reading of the rod upon it is 6.386 feet, whence the surface height = 99.407 − 6.386 = 93.021 feet.

The levelman goes forward, backsights upon the rod, and has the target set so that the reading can be taken to thousandths. The height of the instrument is, therefore, 93.021 + 3.380 = 96.401 feet.

(697)

Distances.	B. S. +	Height of Instru-ment.	F. S. — T. P.	F. S. — Interme-diate Sights.	Elevations.
B. M.	4.576	95.973			91.397
100 T. P.	5.420	97.667	3.726		92.247
200				4.5	93.167
300 T. P.	4.910	99.407	3.170		94.497
340				4.9	94.507
400 T. P.	3.380	96.401	6.386		93.021
500				4.6	91.801
600 T. P.	2.760	93.761	5.400		91.001
670				3.1	90.661
700				3.8	89.961
800 B. M.			6.925		86.836

Total, + 21.046 — 25.607
+ 21.046

Difference = — 4.561
+ 91.397

+ 86.836

} Proof of the correctness.

(698) Fig. 32 is a profile made from the notes in example 696, after calculating the elevation of each station.

(699) See Arts. **1262** and **1263.**

(700) The best method of platting the notes for a mine map is to plat the main passages by means of total latitude and total departure, and the rooms or chambers by means of the protractor. This method possesses both the advantage of accuracy and of rapidity. See Art. **1268.**

(701) See Art. **1251.**

(702) Its proper location is determined by a careful examination of the notes. See Art. **1268.**

(703) See Art. **1255.**

(704) Begin traversing the transit notes by ruling eleven columns, and head them as shown in Art. **1137.**

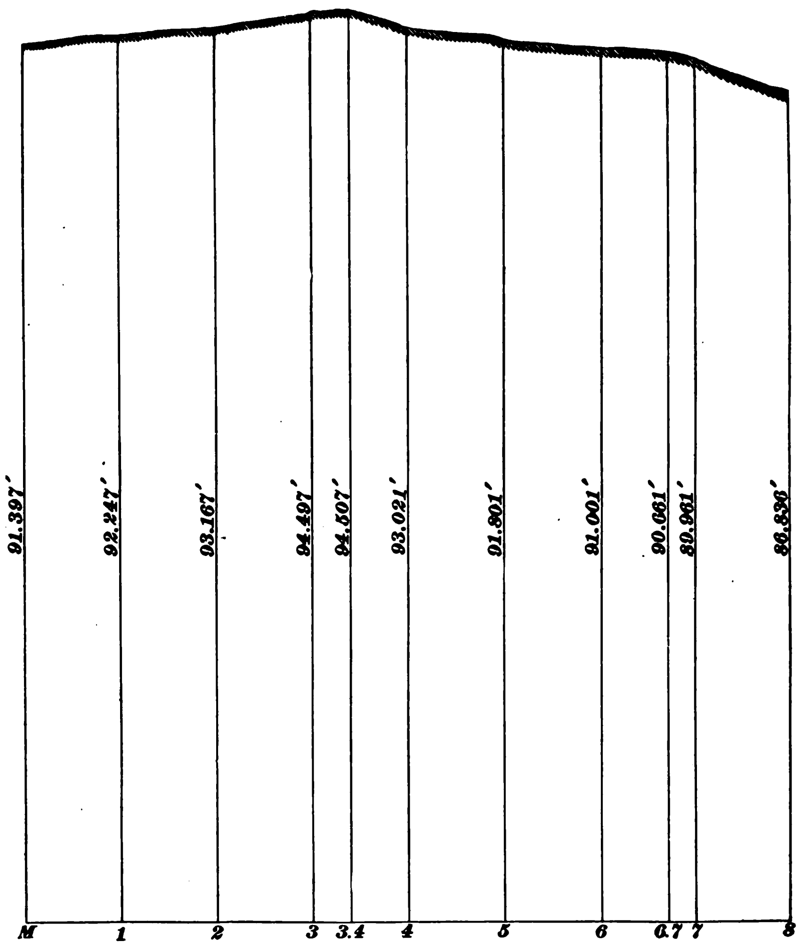

FIG. 32.

Fill in the stations and courses as given in the question, and then reduce all districts measured on an inclination to horizontals by multiplying their measured length by the cosine of the angle of inclination; then, fill in the column of distances.

Next, multiply each horizontal distance by the cosine and the sine of its course (bearing), the products being the latitude and the departure, respectively. Place these in their proper columns.

Calculate the *total* latitudes and the *total* departures with reference to Sta. 302 as the initial station.

Begin the platting by drawing two lines at right angles, their intersection being the position of Sta. 302. Measure on the meridian line a distance north equal to the total latitude of Sta. 303, which is 17.12 feet, and mark the point. Measure due east from this point a distance of 241.74 feet, and mark the point, which, if no error has been made, is the proper location of Sta. 303. To check the correctness of this location, measure the distance from the point to Sta. 302, and if it scales 242.35 feet, as nearly as can be judged, the work is correct.

Plat and check each of the other stations in precisely the same manner, joining consecutive stations by straight lines, thus forming a skeleton upon which the side notes are built.

Before platting these side notes reduce the distances on the tape, where the side measurements were taken, to horizontal lengths. For this purpose a traverse table is very convenient, as it is sufficiently exact to take the vertical angle to the nearest even quarter degree and the distance to the nearest foot. For example, the sight from Sta. 321 to face was on an inclination of 16° 40′. The nearest quarter degree is 16° 45′, and a traverse table gives the horizontal distance for the side-note distances of 48, 56, 66, 144, 152, 160, and 171 feet, as 46, 54, 63, 138, 146, 153, and 164 feet, respectively.

Plat the gangway side notes first, beginning at Sta. 302. On the line joining 302 with 303, mark off to scale points 51, 91, 122, 141, etc., feet from Sta. 302. At these points measure to scale right and left the distances given in the notes. Where "chutes" are indicated, sketch them in at once, and from their corners draw lines (free-hand) through the other points located to the next corner of a chute. Locate all side-note distances at *right angles* to the line of sight. When all the side notes (gangway and chambers) have been platted, the work is complete, as shown in Fig. 33.

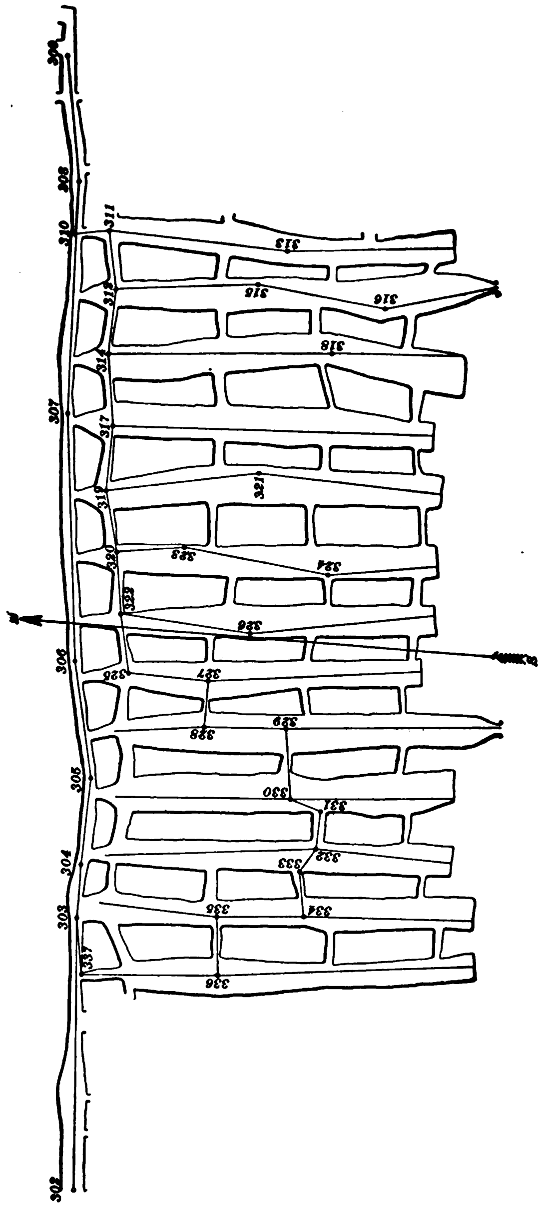
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FIG. 33.

TRAVERSED SURVEY.

Stations.	Bearings.	Distances.	N.	S.	E.	W.	Total.			
							N.	S.	E.	W.
302							0.0	0.0	0.0	0.0
302–303	N 85° 57′ E	242.35	17.12		241.74		17.12		241.74	
303–304	N 89° 33′ E	47.47	.37		47.47		17.49		289.21	
304–305	S 87° 45′ E	77.06		3.03	77.00		14.46		366.21	
305–306	N 77° 51′ E	104.69	22.03		102.34		36.49		468.55	
306–307	N 84° 02′ E	220.76	22.95		219.56		59.44		688.11	
307–308	N 88° 21′ E	206.18	5.94		206.10		65.38		894.21	
308–309	N 80° 36′ E	113.44	18.53		111.92		83.91		1,006.13	
308–310	S 89° 35′ W	46.87		.34		46.87	65.04		847.34	
310–311	S 8° 55′ E	30.16		29.80	4.67		35.24		852.01	
311–312	S 78° 14′ W	52.78		10.76		51.67	24.48		800.34	
311–313	S 2° 12′ W	161.26		161.14		6.19		125.90	845.82	
312–314	N 86° 48′ W	57.40	3.20			57.31	27.68		743.03	
312–315	S 6° 33′ E	127.05		126.22	14.49			101.74	814.83	
315–316	S 6° 38′ W	115.22		114.45		13.31		216.19	801.52	
314–317	S 81° 47′ W	63.86		9.13		63.20	18.55		679.83	
314–318	S 4° 30′ E	200.46		199.84	15.73			172.16	758.76	
317–319	N 88° 27′ W	58.08	1.57			58.06	20.12		621.77	
319–320	S 75° 39′ W	55.52		13.76		53.79	6.36		567.98	

TRAVERSED SURVEY—*Continued.*

Stations.	Bearings.	Distances.	N.	S.	E.	W.	Total.			
							N.	S.	E.	W.
319–321	S 10° 51′ E	137.76		135.30	25.93			115.18	647.70	
320–322	S 82° 05′ W	54.78		7.54		54.26		1.18	513.72	
320–323	S 8° 14′ E	60.24		59.62	8.63			53.26	576.61	
323–324	S 6° 12′ W	130.84		130.07		14.13		183.33	562.48	
322–325	S 78° 07′ W	52.67		10.85		51.54		12.03	462.18	
322–326	S 4° 08′ W	117.42		117.11		8.46		118.29	505.26	
325–327	S 1° 24′ W	72.08		72.06		1.76		84.09	460.42	
327–328	N 88° 43′ W	40.97	.92			40.96		83.17	419.46	
328–329	S 3° 21′ E	73.16		73.03	4.28			156.20	423.74	
329–330	S 81° 42′ W	62.61		9.04		61.95		165.24	361.79	
330–331	S 17° 50′ W	28.38		27.02		8.69		192.26	353.10	
331–332	N 87° 20′ W	33.52	1.56			33.48		190.70	319.62	
332–333	N 58° 15′ W	24.52	12.90			20.85		177.80	298.77	
333–334	S 79° 31′ W	40.10		7.30		39.43		185.10	259.34	
334–335	N 4° 53′ W	77.79	77.51			6.62		107.59	252.72	
335–336	S 84° 15′ W	51.80		5.19		51.54		112.78	201.18	
336–337	N 4° 17′ W	122.52	122.18			9.15	9.40		192.03	
337–303	N 80° 34′ E	50.50	8.28		49.82		17.68		241.85	

ECONOMIC GEOLOGY OF COAL.

(705) See Art. **1279.**

(706) See Art. **1297.**

(707) See Art. **1302.**

(708) (*a*) and (*b*) See Art. **1332.**

(709) Deposits of coal in the Sub-Carboniferous period are called false coal measures. (Art. **1340.**)

(710) (*a*) and (*b*) See Art. **1341.**

(711) See Art. **1351.**

(712) See Arts. **1315** and **1352.**

(713) See Art. **1325.**

(714) See Art. **1308.**

(715) (*a*) See Art. **1338.** (*b*) No; there were no materials for the formation of coal during the Silurian Age on the American continent.

(716) See Art. **1346.**

(717) See Arts. **1345** and **1347.**

(718) No. 21, Fig. 375, is a trilobite.

(719) No rule can be given for determining the displacement of a fault. (Art. **1320.**)

(720) (*a*) and (*b*) See Art. **1305.**

(721) (*a*) and (*b*) See Art. **1341.**

(722) See Art. **1350.**

(723) The Acadian epoch. (See Art. **1336.**)

(724) Pennine fault. (See Art. **1324.**)

(725) See Art. **1311.**

(726) The Silurian. (Art. **1339.**)

(727) Using formula **83**, Art. **1287**,

$$T = 50.68 + \frac{900 - 19.68}{67.2} = 63.78^\circ. \quad \text{Ans.}$$

(728) (*a*) See Art. **1330.** (*b*) Dawn of animal life; old life; middle life; recent life; era of mind. (See Art. **1331.**)

(729) (*a*) and (*b*) See Art. **1349.**

(730) The Corniferous. (See Geological Chart for North America.)

(731) (*a*) The mountain limestone belongs to the Lower Carboniferous epoch.

(*b*) The millstone grit belongs to the coal measures. (See Geological Chart.)

(732) (*a*) and (*b*) See Arts. **1340** and **1355.**

(733) See Arts. **1296** and **1307.**

(734) See Art. **1309** and glossary, Art. **1382.**

(735) No. (See Art. **1300.**)

(736) We should follow the life system rather than the rock system. (See Art. **1330.**)

(737) See Art. **1350.**

(738) No. (See Art. **1365.**)

(739) See Art. **1339.**

(740) No.

(741) The Silurian. (Art. **1336.**)

(742) (*a*) and (*b*) See Art. **1288.**

(743) See Art. **1304.**

(744) See Art. **1340.**

(745) See Art. **1316.**

(746) See Art. **1343.**

(747) Thickness $b\,c = a\,b \times \sin 70° = 1{,}000 \times .93969 =$ 939.69 ft. Ans.

(748) See Art. **1311.**

(749) See Arts. **1344** and **1345.**

(750) (*a*) See Art. **1339.** (*b*) No.

(751) Fossils of fishes. (Art. **1339.**)

(752) See Art. **1359.**

(753) No. (Art. **1339.**)

(754) See Art. **1325.**

(755) See Art. **1320.**

(756) See Art. **1296.**

(757) Yes. (See Art. **1307.**)

(758) 2,000. (See Art. **1353.**)

(759) Pennsylvania, Virginia, Kentucky, and Indiana. (See Geological Chart.)

(760) (*a*) and (*b*) See Arts. **1281, 1282,** and **1283.**

(761) Anticlinal axis. (Art. **1300.**)

(762) See Art. **1303.**

(763) See Art. **1312.**

(764) (*a*) and (*b*) See Art. **1327.**

(765) See Art. **1353.**

(766) See Art. **1341.**

(767) See Art. **1307.**

(768) No. The dip may be inclined to either side of the line of strike. (See Art. **1299.**)

(769) See Art. **1291.**

(770) See Art. **1316.**

(771) See Art. **1364.**

(772) See Arts. **1308** and **1310.**

(773) (*a*) and (*b*) See Art. **1300.**

(774) The rocks of the Cretaceous period are less frequently metamorphic than the older rocks. (See Art. **1363.**)

PROSPECTING FOR COAL AND LOCATION OF OPENINGS.

(775) See Art. **1383.**

(776) See Art. **1390.**

(777) See Arts. **1424, 1433,** and **1434.**

(778) See Art. **1447.**

(779) See Art. **1434.**

(780) The presence of coal is determined by boring. (See Art. **1397.**)

(781) (*a*) From No. 3 to No. 2 rise = 275 ft. − 260 ft. = 15 ft. 15 ft. in 540 ft. or 1 in $\frac{540}{15}$ = 36 ft. From No. 2 rise to No. 4 will be $\frac{960}{36} = 26\frac{2}{3}$ ft.; 260 ft. − $26\frac{2}{3}$ ft. = $233\frac{1}{3}$ ft. Ans.

(*b*) From No. 2 to No. 1 rise = 260 ft. − 180 ft. = 80 ft. 80 ft. in 1,500 ft., or 1 in $\frac{1,500}{80}$ = 18.75 ft. From No. 2 vein rises towards No. 1, 1 in 18.75, and it will rise to level of No. 4, or $26\frac{2}{3}$ ft., in going $18.75 \times 26\frac{2}{3} = 500$ ft. Ans.

(*c*) From No. 2 vein will fall in the opposite direction at same rate, 1 ft. in 18.75 ft., and it will fall to the level of No. 3, or 15 ft., in going $18.75 \times 15 = 281.25$ ft. Ans.

(*a'*) True dip is at right angles to the line of strike. We first find the length of the line $x\,y$, in which the vein falls 260 ft. − $233\frac{1}{3}$ ft. = $26\frac{2}{3}$ ft. Thus, $\tan a = \frac{500}{960} = .52083$; hence, angle $a = 27° \; 30'$; then, $x\,y = 960 \times \sin a = 960 \times$

sin 27° 30′ = 960 × .46175 = 443.28 ft. And true dip = $\frac{443.28}{26\frac{2}{3}}$ = 16.63, or dip = 1 ft. in 16.63 ft. Ans.

(*e*) See Fig. 34.

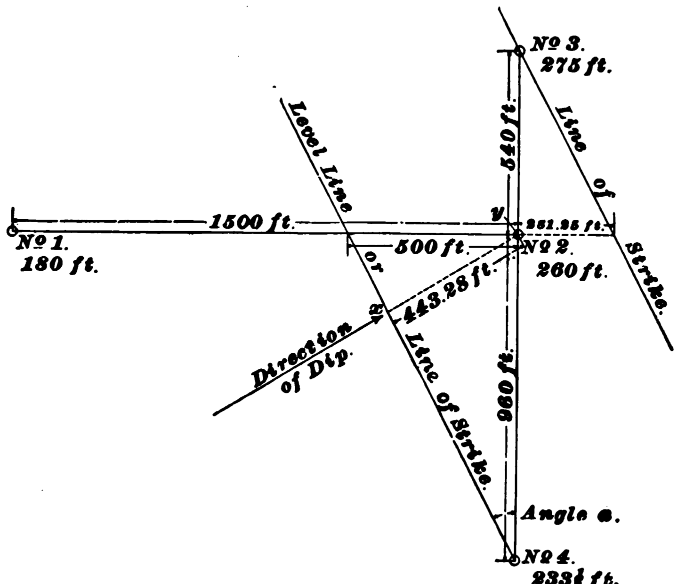

FIG. 34.

(782) See Art. **1386.**

(783) See Art. **1441.**

(784) See Art. **1437.**

(785) See Art. **1405.**

(786) See Art. **1394.**

(787) Specific gravity = $\frac{\text{Weight in air}}{\text{Difference}} = \frac{530}{530 - 105} = \frac{530}{425}$ = 1.247. Ans.

(788) Depth of bore hole = 550 × tan 46° 20′ = 550 × 1.04766 = 576.2 ft. Ans.

(789) See Art. **1384.**

(790) See Arts. **1398** and **1399.**

(791) See Art. **1437.**

(792) See Art. **1392.**

(793) See Arts. **1424** and **1434.**

(794) See Arts. **1449** and **1452.**

(795) See Art. **1436.**

(796) See Art. **1396.**

(797) 30° and 120° to 150°. (See Art. **1410.**)

(798) See Art. **1418.**

(799) See Art. **1442.**

(800) See Art. **1398.**

(801) See Arts. **1385** and **1389.**

(802) See Art. **1396.**

(803) See Art. **1421.**

(804) See Arts. **1408** and **1412.**

(805) See Art. **1393.**

(806) See Art. **1432.**

(807) (1) One acre = 43,560 sq. ft.

$43,560 \times 4\frac{1}{2} = 196,020$ cu. ft.

Weight of coal $= 196,020 \times 62.355 \times 14 = 17,111,958$ lb.

$17,111,958 \div 2,000 = 8,556$ short tons. Ans.

(2) Tons per inch per acre is 141, since specific gravity is 1.4; $4\frac{1}{2}$ ft. = 54 in.

$141 \times 54 = 7,614$ tons. Ans.

(808) See Art. **1388.**

(809) See Art. **1396.**

(810) See Art. **1448.**

(811) See Art. **1417.**

(812) See Arts. **1451** and **1452.**

SHAFTS, SLOPES AND DRIFTS.

(QUESTIONS 813–888.)

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(813) See Art. **1453.**

(814) The shaft should be placed on the lowest boundary of the field, or at such a distance from it as will provide for a sump on the low side of the shaft bottom, and the workings driven to the rise. This will ensure good drainage for the working places, and the most advantageous grade for haulage purposes. (See Art. **1456.**)

(815) See Art. **1454.**

(816) See Art. **1457.**

(817) See Arts. **1462** and **1463.**

(818) See Art. **1470.**

(819) It is carried down to the bed rock, or "hard pan." (See Art. **1464.**)

(820) See Art. **1464.**

(821) See Art. **1478.**

(822) See Art. **1474.**

(823) See Art. **1485.**

(824) See Art. **1486.**

(825) See Art. **1489.**

(826) Substituting values in formula **87,** Art. **1487,** we have

$$N = \frac{(16^2 - 15^2) \times .7854 \times 65}{.25 \times .3333 \times .6666} = 28,486.458.$$

Less 10% = 25,638 bricks. Ans.

(827) See Art. **1453.**

(828) Steam. (See Art. **1498.**)

(829) See Art. **1472.**

(830) See Art. **1457.**

(831) The water should be shut off by building a cofferdam, as shown at *K*, Fig. 425. (See Art. **1466.**)

(832) Buntons are timbers placed horizontally across the shaft to carry the cage guides, column pipes, etc.; also to strengthen the timbering. They are set into the rock and keyed firmly. (See Art. **1467.**)

(833) The size of the shaft will depend upon what it is used for; i. e., it may be used for hoisting, ventilating (return and intake airways), and pumping, which require a shaft of greater dimensions than when an air-shaft is sunk near by. (See Arts. **1458** and **1459.**)

(834) The circular form should be used, shutting off the water by means of iron tubbing. (See Arts. **1457** and **1476.**)

(835) See Art. **1482.**

(836) See Art. **1475.**

(837) See Art. **1488.**

(838) A wedging curb is a permanent curb which must be used with any form of walling that is required to be water tight. (See Art. **1490.**)

(839) See Art. **1457.**

(840) See Art. **1461.**

(841) See Art. **1454.**

(842) See Art. **1456.**

(843) Substituting the values in formula **86, Art. 1480,** we have

$$t = \left\{ \frac{1}{\sqrt{1 - \frac{.434 \times 2 \times 375}{90{,}000 \times \frac{1}{8}}}} - 1 \right\} 6 \times 15 + \tfrac{1}{8} = 1.46 \text{ inches.}$$

Ans.

(844) See Art. **1468.**

(845) See Art. **1473.**

(846) See Art. **1483.**

(847) See Art. **1491.**

(848) By the use of cement, by freezing, and by pumping off the water. (See Arts. **1494, 1523,** and **1518.**)

(849) See Art. **1498.**

(850) They are the holes bored in the middle of the shaft bottom for the purpose of blasting it out, before the lateral holes are fired to trim up the shaft to the proper dimensions. (See Art. **1499.**)

(851) See Art. **1503.**

(852) See Art. **1513.**

(853) A number of detonators are used in the same hole. (See Arts. **1505** and **1506.**)

(854) See Art. **1500.**

(855) See Art. **1496.**

(856) See Art. **1497.**

(857) Dynamite is exploded by means of a shock produced by an exploder, or detonator, while powder is exploded by a spark or flame. (See Arts. **1501** and **1502.**)

(858) By incandescence in the bridge *E D* (Fig. 446). (See Art. **1505.**)

(859) See Art. **1506.**

(860) See Art. **1502.**

(861) Shots are said to be fired (1) in series, when they are connected as shown in Fig. 447; (2) in parallel, when connected as shown in Fig. 448; (3) in multiple series, when connected as shown in Fig. 449. (See Art. **1508.**)

(862) The method shown in Fig. 452 should be used. (See Art. **1516.**)

(863) See Arts. **1519, 1520,** and **1521.**

(864) See Arts. **1523** to **1527.**

(865) The method shown in Fig. 453 should be employed. (See Art. **1517.**)

(866) See Art. **1528.**

(867) See Art. **1521.**

(868) See Art. **1539.**

(869) Great difficulty is found in keeping the holes exactly vertical. (See Art. **1540.**)

(870) The Kind-Chaudron method should be used. (See Arts. **1531** to **1538.**)

(871) The water in the shaft will prevent the inflow of quicksand or other soft material, and assist in lowering the tubbing, and no pumping will be required while sinking. (See Arts. **1537** and **1538.**)

(872) See Art. **1541.**

(873) See Art. **1542.**

(874) The widening of a shaft should begin at the top, and the process requires that the mine be idle while the work is being done. (See Art. **1543.**)

(875) Wooden guides and iron guides (**T**-iron, round-iron), and wire-rope guides. (See Arts. **1545** to **1551.**)

(876) By turnbuckles and weights hung upon their lower ends. (See Arts. **1548** and **1551.**)

(877) See Art. **1552.**

(878) The point at which the face of the slope will have a vertical height greater than the timbers. (See Art. **1553.**)

(879) Where the top is very soft and falls in as quickly as the excavation takes place. (See Art. **1553.**)

(880) See Art. **1555.**

(881) They are set so that their tops lean a few degrees up the pitch from the perpendicular to the slope. (See Art. **1561.**)

(882) See Arts. **1561** and **1563.**

(883) See Art. **1560.**

(884) See Art. **1564.**

(885) The face is arranged in two lifts, the top one being carried forwards in advance of the lower one. (See Art. **1567.**)

(886) See Art. **1569.**

(887) The vertical distance in feet of the outcrop above the level of the parting $= 600 \tan 30' = 600 \times .00873 = 5.238$ feet. The vertical distance in feet that the drift mouth will be below the level of the parting $= 600 \times \frac{1.5}{100} = 9$ feet. Therefore, $5.238 + 9 = 14.238$ feet. Ans.

(888) By substituting the values in formula **85,** Art. **1459,** we have

$$L = \frac{1,200 \times 2,000 \times 600}{36,000 \times 8 \times 50 \times 4 \times 3} + 1 = 9 \text{ feet } 4 \text{ inches.}$$

Total length of shaft = width of hoisting compartments + width of pumpway + width of buntons $= 2\,(5 + 2) + 6 + 2 \times \frac{10}{12} = 21$ ft. 8 in.

The shaft is 21 ft. 8 in. × 9 ft. 4 in. in the clear. Ans.

METHODS OF WORKING COAL MINES.

(PART 1.)

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(889) See Art. **1575.**

(890) The practice in the locality in which the shaft or slope is to be sunk. (See Art. **1576.**)

(891) $\frac{625}{4} + 125 = 281\frac{1}{4}$ feet, the radius of shaft pillar. (See Art. **1576.**)

(892) Because the plane of fracture is nearly perpendicular to the slope, while it is nearly parallel to the line of the shaft. (See Art. **1578.**)

(893) Because the overlying strata increase as the slope advances downwards. (See Art. **1579.**)

(894) A hard top and a soft bottom. (See Art. **1580.**)

(895) Because the pillars do not increase in strength (width) as the slope is driven downwards. (See Art. **1579.**)

(896) The pillars in the upper seam should be formed vertically over those in the lower seam. (See Art. **1616.**)

(897) Shafts should be sunk so that the tracks on the cages are parallel with the strike of the seam, and slopes should be sunk on the full dip. (See Art. **1582.**)

(898) See Arts. **1583, 1584,** and **1585.**

(899) They should not be less than 100 feet wide. (See Art. **1581.**)

(900) From 1 to 2 per cent. grades. (See Art. **1585.**)

(901) They should be formed so that their longer sides will be parallel to the line of dip. (See Art. **1593.**)

(902) See Art. **1595.**

(903) See Art. **1596.**

(904) See Art. **1601.**

(905) See Art. **1602.**

(906) See Art. **1592.**

(907) See Art. **1594.**

(908) They are turned off both butt entries when the seam is comparatively flat, or when the seam is inclined and the butt headings run to the rise or dip; but when the seam is inclined and the productive headings run along the strike, the rooms are turned off on one side only. (See Art. **1606.**)

(909) The former; because when the rooms are turned off both of the butt headings, one group of rooms is driven to meet another group coming from the next pair of butt entries, necessitating only one-half the amount of productive or butt entry-driving. (See Art. **1606.**)

(910) By the panel system. (See Art. **1612.**)

(911) See Art. **1650.**

(912) It is arranged in sections, each of which is made level by the refuse of the seam. (See Art. **1625.**)

(913) From 15° to 30°. (See Art. **1626.**)

(914) See Art. **1630,** and Figs. 499 and 500.

(915) See Art. **1629,** and Fig. 498.

(916) See Art. **1631.**

(917) See Arts. **1635** and **1639.**

(918) The danger is that, in the case of a collapse of a manway, the entire ventilation of all the breasts of the section in which the collapse occurred is destroyed. (See Art. **1638.**)

(919) By leaving a pillar at the mouth of the breast to protect the airway, as shown at *A*, Fig. 503.

(920) The air is conducted around the breast in which the accident occurred through the airway and the small passages leading from the manway chutes to the airway. These passages are shown in Fig. 504 as *c* and *d*, respectively.

(921) They are batteries placed in chutes to prevent the air-current from taking a short cut from the gangway to the breast airways. (See Art. **1637.**)

(922) It is that system of mining by which the coal from the upper seam is run through rock chutes to the seam below. (See Art. **1643.** For the undetermined points, see Art. **1645.**)

(923) See Art. **1646.**

(924) It is a breast driven over the gangway for the purpose of getting a large portion of the gangway pillar, which would otherwise be lost. (See Art. **1641.**)

(925) See Art. **1642.**

(926) One bore hole should be drilled straight ahead, and flank bore holes should be drilled on each side. (See Art. **1653.**)

(927) The thickness and character of the parting between them. (See Art. **1649.**)

(928) See Art. **1621.**

(929) It should be opened up in a manner similar to that shown in Fig. 504, Art. **1639.**

(930) They should be driven parallel to the face cleats, so that the rooms turned perpendicularly off them will be driven on the face cleats. (See Art. **1613.**)

(931) It should be set so that it leans from 2° to 6° up the pitch from the perpendicular to the seam. Theoretically, the post should be set perpendicular to the seam, or strata; because, when the weight of the overlying strata which is to be supported is resolved into two components, one along the roof and the other perpendicular to it, the post must support that component of the weight which is perpendicular to the seam, while the roof itself holds in equilibrium the other. The post is inclined slightly up the pitch, in order that it will tighten rather than fall out in case the roof slides down the pitch. (See Arts. **1654, 1655,** and **1656.**)

(932) See Art. **1657.**

(933) By rounding their bottoms. (See Art. **1658.**)

(934) See Art. **1658.**

(935) By placing the "sights" near one side of the heading. (See Art. **1661.**)

(936) It gives, in general, a very crooked or circuitous road, which is very hard on the cars and makes mechanical haulage difficult. (See Art. **1661.**)

(937) See Art. **1660.**

(938) The pitch and thickness of the seam. (See Art. **1662.**)

(939) By lifting the bottom on the rise side. (See Fig. 515, Art. **1664.**)

(940) See Fig. 518, Art. **1667.**

(941) See Figs. 523, 524, and 525, Art. **1672.**

(942) See Art. **1675.**

(943) From 25 to 45 pounds per yard. (See Art. **1676.**)

(944) Wooden rails are laid outside the **T** rails to obtain a greater friction when the wheel is spragged or the brake is applied. (See Art. **1676.**)

(945) The spikes have not the proper relative position in the ties. (See Art. **1676.**)

(946) One that gives an unbroken main road, such as that shown in Fig. 527, Art. **1678.**

(947) They are used at turnouts or landings, or where two tracks come together, one of which is for the loaded cars and the other for the empties. (See Arts. **1583, 1588,** and **1693.**)

(948) There should be a difference in the relative heights of the lead and follower rails. (See Art. **1680.**)

(949) See Fig. 532.

(950) See Art. **1687.**

(951) See Art. **1694.**

(952) By means of a slope carriage, or gunboat. (See Art. **1695.**)

(953) If both sides of the mine do not produce the same amount of coal, caging becomes proportionately more difficult. (See Art. **1696.**)

(954) The size of the mine cars. (See Art. **1697.**)

(955) One bore hole should be drilled straight ahead, and flank bore holes should be drilled on the high side only. (See Art. **1653.**)

(956) $(15 \times 4) + (15 \times 5) = 135$ ft. Ans. (See Art. **1651.**)

(957) $[(6 \times 3) + (6 \times 5)] \times 2 = 96$ ft. Ans. (See Art. **1651.**)

(958) See Art. **1704.**

(959) As water exerts a pressure of 0.434 lb. per sq. in. per foot of depth, the pressure per sq. in. under a 200-foot head of water equals $200 \times .434 = 86.8$ lb. Then by formula **88** the thickness of the dam is

$$T = 360 \times \left(1 - \sqrt{1 - \frac{20 \times 86.8}{8{,}000}}\right) =$$

$$41.44 \text{ in.} = 3.45 \text{ ft.} = 3 \text{ ft. } 6 \text{ in., nearly. Ans.}$$

By doubling the calculated thickness to ensure safety we have 3 ft. 6 in. $\times 2 = 7$ ft. (See Art. **1710.**)

(960) Using formula **88,**

$$T = 84 \times \left(1 - \sqrt{1 - \frac{20 \times 108.5}{2,500}}\right) =$$

53.48 in. = 4 ft. 6 in., nearly. Ans.

4 ft. 6 in. × 2 = 9 feet. (See Art. **1710.**)

(961) Applying formula **89,**

$$T = 96 \times \left(1 - \sqrt[3]{1 - \frac{15 \times 130.2}{2,500}}\right) =$$

38.15 in. = 3 ft. 2 in., nearly. Ans.

3 ft. 2 in. × 2 = 6 ft. 4 in. (See Art. **1710.**)

(962) It prevents leakage, and transmits the pressure from one concentric arch to the other. (See Art. **1708.**)

(963) The best Portland cement. (See Art. **1709.**)

METHODS OF WORKING COAL MINES.

(PART 2.)

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(964) See Art. **1711.**

(965) See Art. **1713.**

(966) If longwall advancing be used, the method should be similar to that of Fig. 555 or 559; but if longwall retreating be used, the method shown in Fig. 580 should be used. The combined method (Fig. 582) may also be used.

(967) See Art. **1717.**

(968) By first bearing in a short distance at one side of his room or allotted portion of the working face and continuing the shallow mining across it, by which time the weighing action will have softened the coal just beyond the mining and make it comparatively easy to repeat the operation and deepen the mining. (See Art. **1735.**)

(969) See Art. **1730.**

(970) Because it is otherwise a difficult matter to maintain a continuous line of working face, which is essential to the best working of longwall. (See Art. **1750.**)

(971) See Art. **1769.**

(972) Where the roof is brittle sufficient breadth can not be maintained to take the car along the face, making it necessary to approach the face by a greater number of roads. (See Art. **1712.**)

(973) See Art. **1755.**

(974) Shafts are protected either by pillars left at the bottom of the shaft or by carefully stowing up the space made vacant by taking out all the coal. (See Art. **1714.**)

(975) The face should advance parallel to the principal or face cleats, i. e., the "end on" plan should be adopted. (See Art. **1725.**)

(976) See Arts. **1735** and **1736.**

(977) There is danger of throwing excessive weight upon the face and destroying the timber. (See Art. **1730.**)

(978) It is so arranged that each miner will have a long rise side and a short dip side with reference to the road leading to the face. This saves him the work of shoveling coal up hill. (See Art. **1763.**)

(979) To take advantage of gravity. (See Art. **1727.**)

(980) Where the roof is bad and roads 10 or 12 yards apart approach the face, the coal is shoveled to the road-heads; but where the seam is low, a buggy is used to convey the coal along the face to the road-heads. (See Arts. **1712** and **1755.**)

(981) See Art. **1716.**

(982) See Art. **1752.**

(983) See Art. **1743.**

(984) See Art. **1712.**

(985) See Art. **1765.**

(986) The inclination of the seam and the condition of the packwalls. (See Art. **1766.**)

(987) To properly regulate the weight upon the working face. (See Art. **1729.**)

(988) See Art. **1721.**

(989) By being able to make a deep holing or mining with a very low front. (See Art. **1736.**)

(990) The steps are made between the pairs of roads. (See Art. **1745.**)

(991) The seam is divided into layers or lifts which are worked independently, or so that one lift is a short distance ahead of the other. (See Art. **1771.**)

(992) At the road-head. (See Art. **1759.**)

(993) If the rise be too steep for the mule to enter the room with the car, the coal is conveyed to the level either by chutes or incline planes. (See Art. **1745.**)

(994) By taking a proper width of coal out of both the high and low side of the level. (See Art. **1754.**)

(995) It should advance perpendicularly to the face cleats. (See Art. **1723.**)

(996) The weighting action of the roof. (See Art. **1751.**)

(997) From the roof, the floor, the seam itself, and from the surface. (See Art. **1711.**)

(998) They impede ventilation, increase the cost of production, and cause the coal to be crushed more than when the continuous face is employed. (See Art. **1716.**)

(999) See Art. **1731.**

(1000) So as to get the line of fracture, which takes place along the rib side, below the level, in order to secure good roof in the level and support to the lower edge of the loosened mass, whereby it will be prevented from slipping down hill and destroying the packwalls. (See Art. **1752** and Fig. 561.)

(1001) The pressure of the gas in the coal greatly assists in the work of extraction or in loosening the coal at the working face. (See Art. **1738.**)

(1002) The principal difficulties are found in the tendency of the roof to gravitate away from the working face, and in keeping the packwalls in good condition. (See Art. **1729.**)

(1003) See Art. **1740.**

(1004) See Art. **1792.**

(1005) The cleats may have any direction with reference to the line of dip. (See Art. **1728.**)

(1006) See Art. **1742.**

(1007) Upon the height of the seam and amount of stowage. (See Art. **1746.**)

(1008) A pair of entries (slopes) should be driven in the lower seam from the outcrop to the boundary, if possible, and directly on the line of maximum dip. Then a cross-heading should be driven connecting the three seams, which are finally opened out by driving headings or levels to the right and left in each seam from the cross-heading and not more than 600 feet away from it. These headings are connected at their far ends, and sometimes in the middle, for the purpose of ventilation. On the rise side of each level longwall faces should be started, giving in all 6 places. The faces should be worked in the ascending order and lead each other about 30 feet. The cross-heading or slant should be moved forwards about every 60 feet. (See Art. **1777** and Fig. 581.)

(1009) By systematic and efficient support of the roof near the face by means of props, nogs, chocks, etc. (See Art. **1734.**)

(1010) When the coal is situated at a shallow depth, and it is cheaper to sink extra shafts than to maintain permanent haulways through the gob. (See Art. **1746.**)

(1011) By the escape of the gas in advance of the working face. (See Art. **1738.**)

(1012) The dip and the tendency of the roof to gravitate away from the working face is lessened. (See Art. **1733.**)

(1013) See Art. **1784.**

(1014) See Arts. **1794** to **1797.**

(1015) Special attention must be given to the first packwalls, because more settling takes place at that time than in the ordinary working. (See Art. **1749.**)

(1016) See Art. **1734.**

(1017) By taking down top and lifting bottom. (See Art. **1746.**)

(1018) By leaving a portion of the solid coal on the roof over them. (See Art. **1750.**)

(1019) See Art. **1785.**

(1020) By the escape of the gas through the adjoining strata and by excessive weight upon the working face. (See Arts. **1738** and **1798.**)

(1021) See Art. **1728.**

(1022) "Blind pits" are used to lower coal from one seam to another. (See Art. **1756.**)

(1023) They may have any relative position, but in new coal fields they should be reasonably close together so as to speedily secure a permanent return airway. (See Art. **1783.**)

(1024) The overlying weight is distributed over a greater portion of the bottom. (See Art. **1788.**)

(1025) See Art. **1739.**

(1026) When the top has settled completely. (See Art. **1746.**)

(1027) When the mining or holing is done at the top or near the middle of the seam. (See Art. **1756.**)

(1028) By the use of dynamite. (See Art. **1795.**)

(1029) A constant output is secured. (See Art. **1783.**)

(1030) By driving main headings to the boundary and commencing the necessary narrow work there and bringing it back in advance of the working face. (See Art. **1775** and Fig. 580.)

(1031) See Art. **1797.**

(1032) Longwall retreating. (See Art. **1782.**)

(1033)

Let $r =$ radius, in feet, of pillar to support shaft only.

Then $r + 96 =$ radius, in feet, of pillar to support shaft and buildings.

Hence, $r + 96 = \frac{135 \times 3}{2} = 202.5$, or $r = 202.5 - 96 = 106.5$.

But depth of shaft $D = 4\,r = 426$ feet. Ans. (See Art. **1576.**)

MECHANICS.

(PART 1.)

(1034) See Arts. **1800** and **1801.**

(1035) Reducing 14 minutes to seconds, $14 \times 60 =$ 840 seconds.

$$840 \times 40 = 33{,}600 \text{ ft.} = 6\tfrac{4}{11} \text{ miles. Ans.}$$

(1036) See Art. **1839.**

(1037) Using formula **94**,

$$P a = W b, \text{ or } P \times F c = W \times F b.$$

Hence, $W \times 3\tfrac{1}{2} = 85 \times 21;$

$$W = \frac{85 \times 21}{3\frac{1}{2}} = 510 \text{ lb. Ans.}$$

(1038) Applying formula **99**,

$$N = \frac{80 \times 28}{21} = 106\tfrac{2}{3} \text{ rev. per min. Ans.}$$

(1039) (*a*) Applying formula **102**,

$$D = \frac{1\frac{1}{2} \times 50}{3.1416} = 23.87 \text{ in. Ans.}$$

(*b*) See Art. **1872.** Addendum $= .3$ of the pitch. $1.5 \text{ in.} \times .3 = .45 \text{ in.}$ $.45 \text{ in.} \times 2 = .9 \text{ in.} =$ difference between the diameter of the pitch circle and the outside diameter. Hence, outside diameter $= 23.87 + .9 = 24.77$ in. Ans.

(1040) Apply formula **107.**

$$r = \frac{45 \times 212}{180} = 53 \text{ R. P. M. Ans.}$$

(1041) Apply formula **110.**
Pitch = $\frac{1}{13}$ in.; therefore,

$$W = \frac{6.2832 \times 24 \times 11}{\frac{1}{13}} = 21{,}563.94 \text{ lb.} \quad \text{Ans.}$$

(1042) The pull on the support equals the centrifugal force of the ball. Hence, applying formula **112,**

$$F = .00034 \times 5 \times \tfrac{33}{12} \times 350^2 = 555\tfrac{1}{3} \text{ lb.} \quad \text{Ans.}$$

(1043) Apply formula **113.**

$$K = \frac{2 \times 600^2}{64.32} = 11{,}194 \text{ ft.-lb.} \quad \text{Ans.}$$

(1044) 7 ft. = 84 in. Arc of contact = $\frac{84}{63 \times 3.1416} \times 360° = 153°$. $800 + 3(180 - 153) = 881$. Applying formula **115,**

$$W = \frac{881 \times 150}{3{,}000} = 44.05 \text{ in.}$$

Using formula **117,**

$$W_1 = 44.05 \times \tfrac{2}{3} = 29.37 \text{ in., or say } 29.5 \text{ in.} \quad \text{Ans.}$$

(1045) See Arts. **1803** to **1823.**

(1046) There are 1,760 yd. in 1 mile. If a man can run 100 yd. in 12 seconds, in 1 second he can run $\frac{100}{12}$ yd.; that is, his velocity is $\frac{100}{12}$ yd. per sec. Applying formula **92,**

$$t = 1{,}760 \div \tfrac{100}{12} = 1{,}760 \times \tfrac{12}{100} = 211.2 \text{ sec.} = 3 \text{ min. } 31.2 \text{ sec.} \quad \text{Ans.}$$

(1047) See Art. **1840.**

(1048) See Arts. **1860** and **1861.**

(1049) 13 ft. = 156 in. Applying formula **99,**

$$N = \frac{91 \times 108}{156} = 63 \text{ rev. per min., the speed of the engine.} \quad \text{Ans.}$$

(1050) Applying formula **102,**

$$D = \frac{2\frac{1}{2} \times 192}{3.1416} = 152.79 \text{ in.} \quad \text{Ans.}$$

(1051) Apply formula **108.**

$$R = \frac{81 \times 80}{18} = 360 \text{ rev. per min. Ans.}$$

(1052) Pitch = $\frac{1}{8}$ in. Using formula **110,**

$$W = \frac{6.2832 \times 60 \times 26}{\frac{1}{8}} = 78,414.336 = \text{the theoretical pressure.}$$

Since the efficiency is but 40%, the actual pressure is 78,414.336 × .40 = 31,365.7 lb. Ans.

(1053) See Art. **1899.**

(1054) First determine the speed of the center of gravity of the section in feet per second. This point revolves in a circle whose diameter is 6 ft. $1\frac{5}{8}$ in. × 2 = 12 ft. $3\frac{1}{4}$ in. = 12.2917 ft. Distance traveled in one revolution = 12.2917 × 3.1416 = 38.6156 ft. Distance traveled in one second = $\frac{38.6156 \times 150}{60}$ = 96.539 ft. Hence, applying formula **113,**

$$K = \frac{13,000 \times 96.539^2}{64.32} = 1,883,661.7 \text{ ft.-lb. Ans.}$$

(1055) Arc of contact = $\frac{18}{14 \times 3.1416} \times 360° = 147°$. 800 + 3 (180 − 147) = 899. Applying formula **115,**

$$W = \frac{899 \times 2.5}{2,000} = 1.12 \text{ in., say 1 in. Ans.}$$

(1056) (*a*) See Arts. **1810** and **1835.**
(*b*) and (*c*) See Art. **1809.**

(1057) $\frac{48}{12}$ × 3.1416 = 12.5664 ft. = circumference of pulley. $\frac{3,000}{12.5664}$ = 238.73 revolutions in 1 minute, or 60 seconds. To make 100 revolutions will require $\frac{100}{238.73}$ × 60 = 25.13 sec., nearly. Ans.

(1058) 4 ft. 6 in. = 54 in. 54 × 2 × $\frac{3}{4}$ × .261 = 21.141 lb. = weight of lever. Considering the weight of the lever to be concentrated at its center of gravity, we have three weights of 47, 21.141, and 71 lb., with the smaller weight, $\frac{54}{2} = 27$ in., from the other two. To find the center of gravity of the two large weights, apply formula **93.** $l_1 = \frac{47 \times 54}{71 + 47} = 21.508$ in. = the distance bc in Fig. 35. Consider both weights to be concentrated at b; that is, imagine both

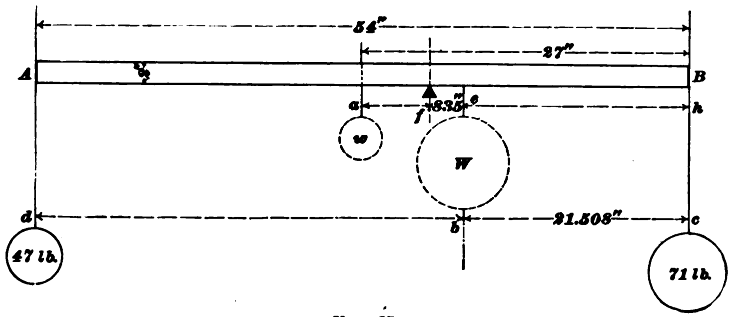

FIG. 35.

weights removed and to be replaced by the dotted weight W, equal to $71 + 47 = 118$ lb. The dotted circle w represents the weight of the bar. The distance $ae = 27 - 21.508 = 5.492$ in. Distance of balancing point f from e is found by means of formula **93** to be $\frac{21.141 \times 5.492}{118 + 21.141} = .835$ in. Finally, $fh =$

$$21.508 + .835 = 22.343 \text{ in.} = \text{short arm.} \quad \text{Ans.}$$
$$54 - 22.343 = 31.657 \text{ in.} = \text{long arm.} \quad \text{Ans.}$$

(1059) See Art. **1862.**

(1060) Apply formula **99,** after reducing the 2 ft. to inches.

$$N = \frac{32 \times 63}{24} = 84 \text{ per min.} \quad \text{Ans.}$$

(1061) Apply formula **103.**

$$T = \frac{11.48 \times 3.1416}{1\frac{1}{8}} = 32 \text{ teeth.} \quad \text{Ans.}$$

(1062) Apply formula **108** to find the number of revolutions of the driving gear. $R = \frac{75 \times 88}{44} = 150$ rev. per min. of the driving gear and also of the 8-inch pulley. Using formula **96** to find the diameter of pulley on the shaft,

$$D = \frac{8 \times 150}{200} = 6 \text{ in.} \quad \text{Ans.}$$

(1063) (*b*) Using formula **110,**

$$W = \frac{6.2832 \times 25 \times 15}{\frac{1}{4}} = 9{,}424.8 \text{ lb.} = \text{theoretical pressure. Ans.}$$

(*a*) $9{,}424.8 - 5{,}000 = 4{,}424.8$ lb. Ans.

(1064) See Art. **1899.** $\frac{51}{62.5} = .816$, the specific gravity. Ans.

(1065) 660 ft. per min. $= \frac{660}{60} = 11$ ft. per sec.

Applying formula **113,**

$$K = \frac{325 \times 11^2}{64.32} = 611.4 \text{ ft.-lb., nearly.} \quad \text{Ans.}$$

(1066) Applying formula **109,**

$$\text{H. P.} = .01 \times 1^2 \times 1{,}200 = 12. \quad \text{Ans.}$$

(1067) See Arts. **1824** and **1826.**

(1068) See Art. **1830.**

(1069) In Fig. 36, ABC represents the triangle. The center of gravity is found as explained in Art. **1845.** The distance of the center of gravity from the side $AC = 1\frac{3}{32}$ in. Ans.

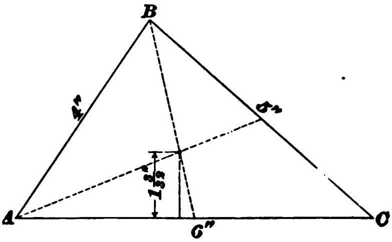

FIG. 36.

(1070) Apply formula **96.**

$$D = \frac{36 \times 60}{40} = 54 \text{ in.} \quad \text{Ans.}$$

(1071) (*a*) Applying formula **98**,

$$n = \frac{12 \times 80}{8} = 120 \text{ R. P. M.} \quad \text{Ans.}$$

(*b*) Applying formula **98**,

$$n = \frac{120 \times 20}{6} = 400 \text{ R. P. M.} \quad \text{Ans.}$$

(*c*) Applying formula **98** again,

$$n = \frac{400 \times 6}{4} = 600 \text{ R. P. M.} \quad \text{Ans.}$$

(1072) Applying formula **103**,

$$T = \frac{3.1416 \times 34.15}{1\frac{3}{8}} = 78 \text{ teeth.} \quad \text{Ans.}$$

(1073) Since there are two parts to the rope, the pulleys will raise a load of $225 \times 2 = 450$ lb. Ans.

(1074) 5 ft. 6 in. = 66 in.

(*a*) $66 \div 6 = 11$ = velocity ratio. Ans.

(*b*) $11 \times 5 = 55$ lb. Ans.

(1075) See Art. **1896.** $55 \times .65 = 35.75$ lb. Ans.

(1076) Referring to the table of Weights per Cubic Foot, a cubic foot of platinum weighs 1,343.8 lb. Therefore, 1 cu. in. weighs $\frac{1,343.8}{1,728}$ lb., and 10 lb. will contain $10 \div \frac{1,343.8}{1,728} = 10 \times \frac{1,728}{1,343.8} = 12.86$ cu. in., nearly. Ans.

(1077) See Art. **1907.**

(1078) Speed of a point on the pitch circle in feet per minute $= \frac{30}{12} \times 3.1416 \times 100 = 785.4$ ft. per min. Apply formula **109.**

$$\text{H. P.} = .01 \times 785.4 \times 1.57^2 = 19.36. \quad \text{Ans.}$$

(1079) See Art. **1826.**

(1080) See Art. **1831.**

(1081) Volume of sphere $= .5236 \times 5^3 = 65.45$ cu. in. 1 cu. in. of cast iron weighs .261 lb.; hence, weight of ball $= 65.45 \times .261 = 17.08$ lb. Weight of a cu. in. of steel is .284 lb.; hence, weight of handle $= (\frac{7}{8})^2 \times .7854 \times 40 \times .284 = 6.83$ lb. Distance of center of gravity of rod from center of ball $= \frac{40}{2} + \frac{5}{2} = 22\frac{1}{2}$ in. Apply formula **93.** Distance of center of gravity of both ball and rod from center of ball $= \dfrac{6.83 \times 22.5}{17.08 + 6.83} = 6.427$ in. Ans.

(1082) Applying formula **96,**

$$D = \frac{180 \times 30}{240} = 22\tfrac{1}{2} \text{ in. Ans.}$$

(1083) Applying formula **100,**

$$P = \frac{6{,}000 \times 6 \times 5 \times 8 \times 3}{18 \times 12 \times 15 \times 12} = 111\tfrac{1}{9} \text{ lb.}$$

Since there is a loss of 20%, $111\frac{1}{9}$ represents 80% of the total force. Hence, the force actually required $= 111\frac{1}{9} \div .80 = 138\frac{8}{9}$ lb. Ans.

(1084) Apply formula **104.**

$$P = \frac{3.1416 \times 24.16}{38} = 1.9974 \text{ in. Ans.}$$

(1085) See Art. **1858.** Since there are eight parts of the rope, the force required $= 1{,}890 \div 8 = 236\frac{1}{4}$ lb. Ans.

(1086) (*a*) Velocity ratio $= \dfrac{1{,}000}{50} = 20$. Ans.

(*b*) Efficiency $= \frac{50}{95} = .5263 = 52.63\%$. Ans.

(1087) Volume $= (\frac{1}{2})^2 \times .7854 \times 10 = 1.963$ cu. in. One cu. in. of lead weighs .411 lb. (see table of Weights per Cubic Foot); consequently, $1.963 \times .411 = .807$ lb. $= 12.91$ oz. Ans.

(1088) Using formula **114,**

$$B = 3\tfrac{1}{4} \times \frac{11 + 7}{2} + 2 \times 38 = 105\tfrac{1}{4} \text{ ft.} = 105 \text{ ft. } 3 \text{ in. Ans.}$$

(1089) (*a*) $18 \times 60 \times 60 = 64{,}800$ miles per hour. Ans.
(*b*) $64{,}800 \times 24 = 1{,}555{,}200$ miles per day. Ans.

(1090) See Art. **1825.**

(1091) See Art. **1832.**

(1092) Length of power arm = 4 ft. − 4 in. = 48 in. − 4 in. = 44 in. According to formula **94,** $P \times 44 = 1{,}500 \times 4 = 6{,}000$; hence, $P = \frac{6{,}000}{44} = 136\frac{4}{11}$ lb. Ans.

(1093) Length of power arm = 4 ft. = 48 in. Hence, as in the preceding question, $P = \frac{6{,}000}{48} = 125$ lb. Ans.

(1094) Apply formula **97.**

$d = \frac{10 \times 88}{110} = 8$ ft., the diameter of the pulley. Ans.

(1095) See Arts. **1869** and **1870.**

(1096) Apply formula **104.**

$$P = \frac{3.1416 \times 36.56}{42} = 2.7347 \text{ in.} \quad \text{Ans.}$$

(1097) See Art. **1885.** $4{,}000 \times 45 = 400 \times$ the force. Hence, force $= \frac{4{,}000 \times 45}{400} = 450$ lb. Ans.

(1098) See Arts. **1889** and **1895.**

(1099) One foot of the wire will weigh $(\frac{1}{16})^2 \times .7854 \times 12 \times .303 = .011155$ lb. (See table of Weights per Cubic Foot.) Consequently, 10 lb. will contain $\frac{10}{.011155} = 896$ ft., nearly. Ans.

(1100) 14 ft. = 168 in. Applying formula **114,**

$$B = 3\tfrac{1}{4} \times \frac{18 + 14}{2} + 2 \times 168 = 388 \text{ in.} = 32 \text{ ft. } 4 \text{ in.} \quad \text{Ans.}$$

(1101) 30 miles per hour $= 5{,}280 \times 30 = 158{,}400$ ft. per hour $= \frac{158{,}400}{60} = 2{,}640$ ft. per min. $= \frac{2{,}640}{60} = 44$ ft. per sec. Ans.

(1102) $\frac{28}{12} \times 1{,}500 = 3{,}500$ ft. in 6 min. $= \frac{3{,}500}{360} = 9\frac{13}{18}$ ft. per sec., since 6 min. $= 6 \times 60 = 360$ sec. Ans.

(1103) See Art. **1833.**

(1104) Apply formula **97.**

$$d = \frac{40 \times 120}{160} = 30 \text{ in.} \quad \text{Ans.}$$

(1105) See Arts. **1871** and **1872.**

(1106) See Arts. **1876** and **1877.**

(1107) The weight which comes on the block and tackle is the same as the force required to pull the body up the plane, or is equal to $\frac{50{,}000 \times 125}{1{,}200} = 5{,}208\frac{1}{3}$ lb. Since there are 12 parts to the rope, the force required to be exerted on the free end is $5{,}208 \div 12 = 434$ lb. Ans.

(1108) See Art. **1898.**

(1109) See Arts. **1901** to **1905.**

(1110) 19 ft. 3 in. = 231 in. Applying formula **114,**

$B = 3\frac{1}{4} \times \frac{20 + 8}{2} + 2 \times 231 = 507\frac{1}{2}$ in. = 42 ft. $3\frac{1}{2}$ in. Ans.

(1111) (*a*) 15 miles per hour $= \frac{15 \times 5{,}280}{60 \times 60} = 22$ ft. per sec. Since the bodies are moving in opposite directions, they are moving *away* from each other, and their distance apart is increasing at the constant rate of $11 + 22 = 33$ ft. per sec. In 8 min. the distance between them will be $\frac{33 \times 8 \times 60}{5{,}280} = 3$ miles. Ans.

(*b*) $825 \div 33 = 25$ sec. Ans.

(1112) 2 min. 10 sec. = 130 sec. 2 miles = 10,560 ft Applying formula **90,**

$$v = \frac{10{,}560}{130} = 81.23 \text{ ft. per sec.} \quad \text{Ans.}$$

(1113) See Art. **1838.**

(1114) Applying formula **95,** letting P represent the required force,

$$P \times 30 \times 20 \times 10 \times 15 = 1{,}250 \times 6 \times 5 \times 4 \times 7,$$

or $$P = \frac{1{,}250 \times 6 \times 5 \times 4 \times 7}{30 \times 20 \times 10 \times 15} = 11\tfrac{2}{3} \text{ lb. Ans.}$$

(1115) Applying formula **98,**

$$n = \frac{20 \times 150}{16} = 187\tfrac{1}{2} \text{ rev. per min. Ans.}$$

(1116) See Art. **1872.**

(1117) Apply formula **105.**

$$T = \frac{60 \times 40}{100} = 24, \text{ the number of teeth. Ans.}$$

(1118) See Art. **1885.** $\frac{750 \times 50}{80} = 468\tfrac{3}{4}$ lb. Ans.

(1119) Substituting in formula **112,**

$$F = .00034 \times 128 \times \frac{8\frac{3}{4}}{12} \times 180^2 = 1{,}028.16 \text{ lb. Ans.}$$

(1120) One cubic foot of water weighs 62.5 lb.; hence, 20 cu. ft. weigh $62.5 \times 20 = 1{,}250$ lb. The work done $=$ $1{,}250 \times 50 = 62{,}500$ ft.-lb. Ans.

(1121) Arc of contact $= \frac{21}{15 \times 3.1416} \times 360° = 160°$. $800 + 3\,(180 - 160) = 860$. Applying formula **116,**

$$H = \frac{5 \times 1{,}960}{860} = 11.4 \text{ H. P. Ans.}$$

(1122) $18{,}000 + 10{,}000 = 28{,}000$ lb. = the load which the screw must overcome.

Using formula **111,**

$$P = \frac{\frac{1}{3} \times 28{,}000}{6.2832 \times 15} = 99 \text{ lb., nearly. Ans.}$$

(1123) $\frac{9 \times 3.1416 \times 100 \times 60}{5,280} = 32.13$ miles per hour = the velocity. Applying formula **91**,

$$s = 32.13 \times 1\frac{1}{4} = 40.16\frac{1}{4} \text{ miles. Ans.}$$

(1124) If the ball fitted the gun loosely and the gun was held horizontally, the ball would roll out and fall to the floor, since, according to the first law of motion, every body tends to preserve its velocity unless acted upon by some force. The ball has a velocity due to the train of 100 ft. per sec. When the gun is fired, the force applied to the ball apparently gives it a velocity of 100 ft. per sec. in the opposite direction, but it really stops the ball and brings it to rest relatively to a point on the earth. The gun and car keep up their motion and draw away from the ball, which is stationary with respect to a point on the earth, and the ball falls to the ground.

(1125) $30 \times 14\frac{1}{2} \times 2 = 870$. $870 \div 5 = 174$ lb. Ans.

(1126) Substituting in formula **98**,

$n = \frac{42 \times 108}{36} = 126$ revolutions per minute of the countershaft. Ans.

(1127) See Art. **1872.**

(1128) Apply formula **106.**

$$t = \frac{34 \times 360}{170} = 72 \text{ teeth. Ans.}$$

(1129) Answer from your own observation.

(1130) The number of foot-pounds of work done in 1 minute is

$$10^2 \times .7854 \times 41.38 \times \tfrac{16}{12} \times 450 = 1,949,991 \text{ ft.-lb.}$$

Dividing by 33,000 to obtain the horsepower,

$$\frac{1,949,991}{33,000} = 59.091 \text{ horsepower, nearly. Ans.}$$

(1131) Since the width of a double belt is but $\frac{2}{3}$ of that of a single belt to transmit the same horsepower, a single belt doing the same work as the 20-inch double belt in this example must be $20 \div \frac{2}{3} = 20 \times \frac{3}{2} = 30$ inches wide.

$$\text{Arc of contact} = \frac{5.75}{4 \times 3.1416} \times 360° = 165°.$$

$$800 + 3\,(180 - 165) = 845.$$

Applying formula **116,**

$$H = \frac{30 \times 2,800}{845} = 99.4 \text{ H. P. Ans.}$$

MECHANICS.

(PART 2.)

(1132) That force which will produce the same final effect upon a body as all the other forces acting separately or together.

(1133) This example is solved by the parallelogram of forces, as in Art. **1917.** Measuring the diagonal, the total pressure on the shaft is found to be $7\frac{1}{4}$ tons, nearly. Ans.

(1134) See Arts. **1932** and **1933.**

(1135) Applying formula **122,**

$$W = 12{,}000 \times (\tfrac{3}{8})^2 = 1{,}687.5 \text{ lb. Ans.}$$

(1136) Apply formula **125,** and use 1,000 instead of 600, as the rope is of steel.

$$W = 1{,}000 \times (5\tfrac{1}{4})^2 = 27{,}562.5 \text{ lb. Ans.}$$

(1137) Applying formula **132,**

$$\text{force} = 6^2 \times .7854 \times 60{,}000 = 1{,}696{,}464 \text{ lb. Ans.}$$

(1138) (*a*) If a 5-inch line = 20 lb., a 1-inch line = 4 lb. $1 \div 4 = \frac{1}{4}$ inch = 1 lb. Ans.

(*b*) $6\frac{1}{4} \div 4 = 1.5625$ inches = $6\frac{1}{4}$ lb. Ans.

(1139) See Art. **1964.**

(1140) The method of obtaining the resultant is shown in Fig. 37. The forces are laid off to scale to form a polygon, and the closing line gives the direction and magnitude of the resultant. See Art. **1918.**

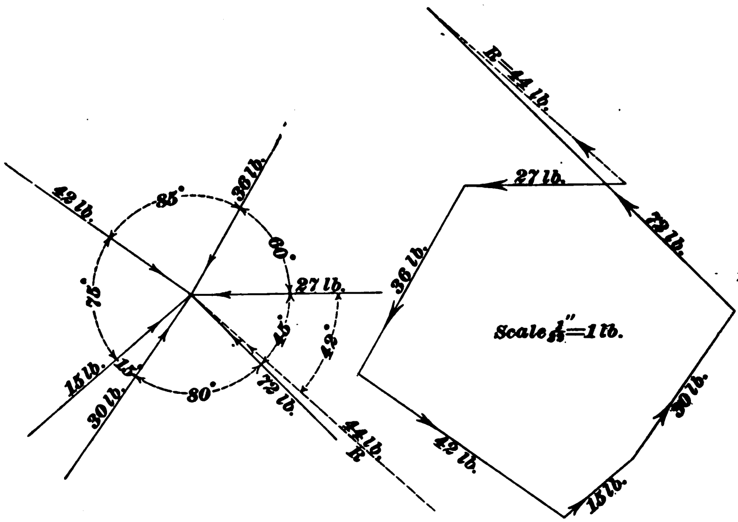

FIG. 37.

(1141) Apply formula **120.**

$$S = \frac{12,400}{3.5} = 3,543 \text{ lb. per sq. in., nearly. Ans.}$$

(1142) Apply formula **123.**

$$W = 100 \times 4^2 = 1,600 \text{ lb. Ans.}$$

(1143) Area of cross-section $= 8^2 \times .7854 = 50.2656$ sq. in. 10 ft. $= 120$ in. $= L$. Crushing strength $= 3.5$ tons per sq. in. (see Table 33). $a = 187.5$ (see Table 36). Substituting these values in formula **127,**

$$W = \frac{3.5 \times 50.2656}{\frac{120^2}{187.5 \times 8^2} + 1} = 80 \text{ tons, very nearly.}$$

Hence, $80 \div 6 = 13\frac{1}{3}$ tons = safe load. Ans.

(1144) Those forces by which the given force may be replaced, and which will produce the same effect on a body as the given force.

(1145) Apply formula **119.**

$$A = \frac{12,000}{5,000} = 2.4 \text{ sq. in., the area of the bolt.}$$

$$\text{Diameter} = \sqrt{\frac{2.4}{.7854}} = 1.74 \text{ in.}+. \quad \text{Ans.}$$

(1146) Using formula **125,** with a constant of 1,000 for steel wire,

$$W = 1,000 \times 4.75^2 = 22,562.5 \text{ lb.} \quad \text{Ans.}$$

(1147) First calculate the load it will sustain in the middle by means of formula **130.**

$$\text{Load in middle} = \frac{4 \times 10^2 \times 8 \times 30}{28} = 3,428\tfrac{4}{7} \text{ lb.}$$

$$\text{Uniform load} = 3,428\tfrac{4}{7} \times 2 = 6,857\tfrac{1}{7} \text{ lb.} \quad \text{Ans.}$$

(1148) Apply formula **133.** From Table 40, the proper constant is 70.

$$\text{Horsepower} = \frac{10^3 \times 200}{70} = 2,857\tfrac{1}{7}. \quad \text{Ans.}$$

(1149) Southeast in the direction of the diagonal of a square. See Fig. 38.

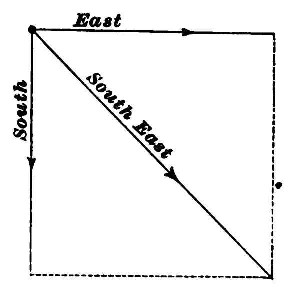

FIG. 38.

(1150) Total pressure on the head $= 19^2 \times .7854 \times 180 = 51,035$ lb.

Tension in each stud $= \frac{51,035}{14} = 3,645$ lb. Applying formula **119,**

$$A = \frac{3,645}{5,000} = .729 \text{ sq. in., the area of the bolt.} \quad \text{Ans.}$$

(1151) Apply formula **124.**

$$\text{Circumference} = .1\sqrt{4,200} = 6.48 \text{ in., say } 6\tfrac{1}{2} \text{ in.} \quad \text{Ans.}$$

(1152) Substitute in formula **127.** For this case, $C = 18$, $S = 6 \times 2\frac{1}{2} = 15$ sq. in., $L = 10 \times 12 = 120$ in., $a = 1,500$, and $d = 2\frac{1}{2}$ in. Consequently,

$$W = \frac{18 \times 15}{\frac{120^2}{1,500 \times 2.5^2} + 1} = 106.467 \text{ tons.}$$

$$\frac{106.467}{6} = 17.7445 \text{ tons} = 35,489 \text{ lb. Ans.}$$

(1153) $2\frac{7}{16}$-inch shafting. See Art. **1964.**

(1154) See Fig. 39.

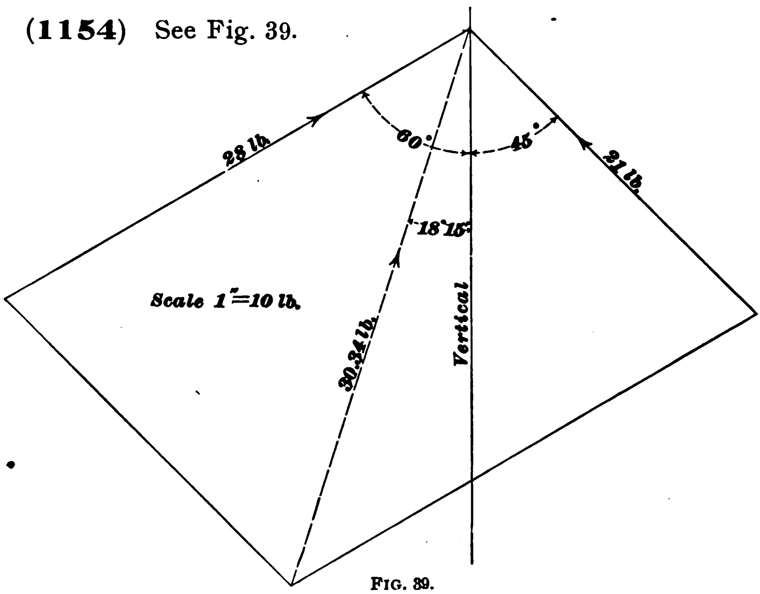

FIG. 39.

(1155) See Fig. 40. By trigonometry, $bc = 87 \times \sin 23° = 87 \times .39073 = 33.994$ lb. $ac = 87 \times \cos 23° = 87 \times .92050 = 80.084$ lb.

(1156) Apply formula **121.**

$W = 18,000 \times .5^2 = 4,500$ lb., the load. Ans.

(1157) Applying formula **126,**

$C = .0408\sqrt{14,000} = 4.83$ in., the circumference, nearly. Ans.

(1158) Applying formula **131,**

$$\text{Load} = \frac{4 \times 2^3 \times .6 \times 150}{6} = 480 \text{ lb.} \quad \text{Ans.}$$

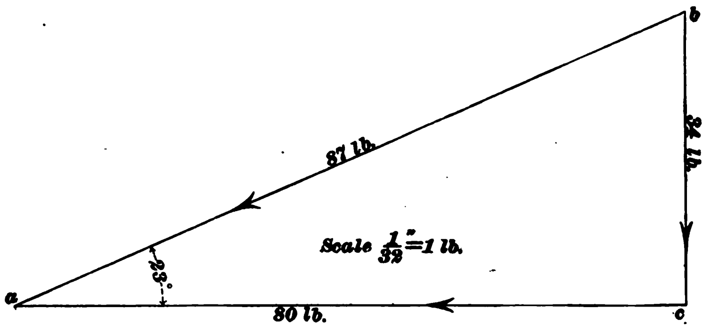

FIG. 40.

(1159) Apply formula **135.** The constant for cast iron is 90 (see Table 40).

$$\text{Diameter} = \sqrt[3]{\frac{90 \times 1{,}000}{80}} = 10.4 \text{ in.} \quad \text{Ans.}$$

(1160) See Fig. 41.

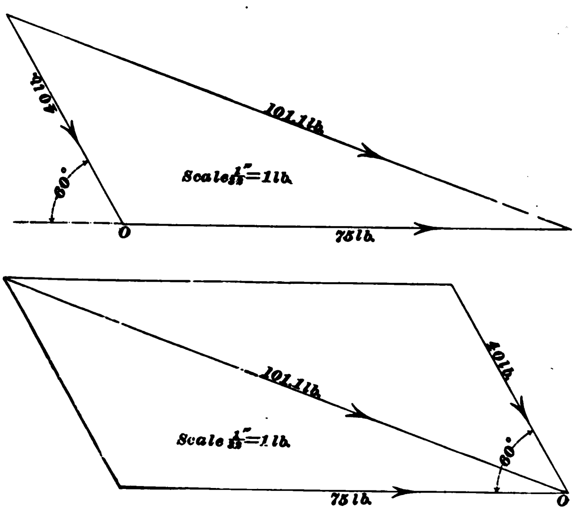

FIG. 41.

(1161) See Fig. 42.

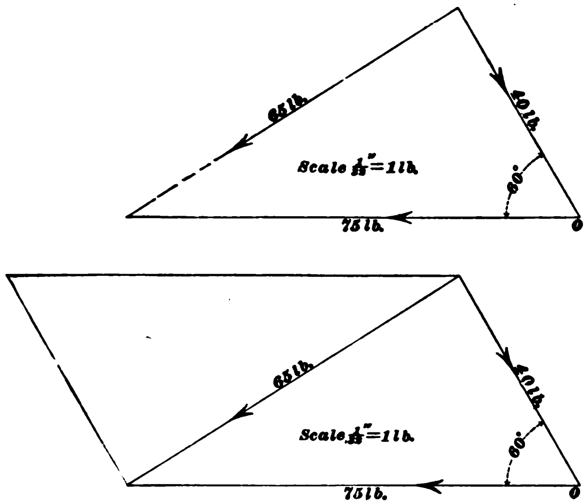

FIG. 42.

(1162) 46 − 27 = 19 lb., acting in the direction of the force of 46 lb. Ans.

(1163) Area of cross-section $= 1\frac{3}{4} \times 3 = 5.25$ sq. in. Applying formula **118,**

$W = 5.25 \times 6{,}000 = 31{,}500$ lb., the safe load. Ans.

(1164) Apply formula **124.**

Circumference $= .1\sqrt{W} = .1\sqrt{2{,}400} = 4.9$ in. Ans.

(1165) See Fig. 43.

(1166) The graphical construction is clearly shown in Fig. 44.

(1167) See Arts. **1926** to **1928.**

(1168) Apply formula **121.**

$W = 18{,}000 \times (\frac{13}{16})^2 = 11{,}883$ lb., the greatest safe load. Ans.

(1169) Apply formula **126,** and use .0316 instead of .0408, since the rope is of steel.

$C = .0316\sqrt{8{,}000} = 2.83$ in. Ans.

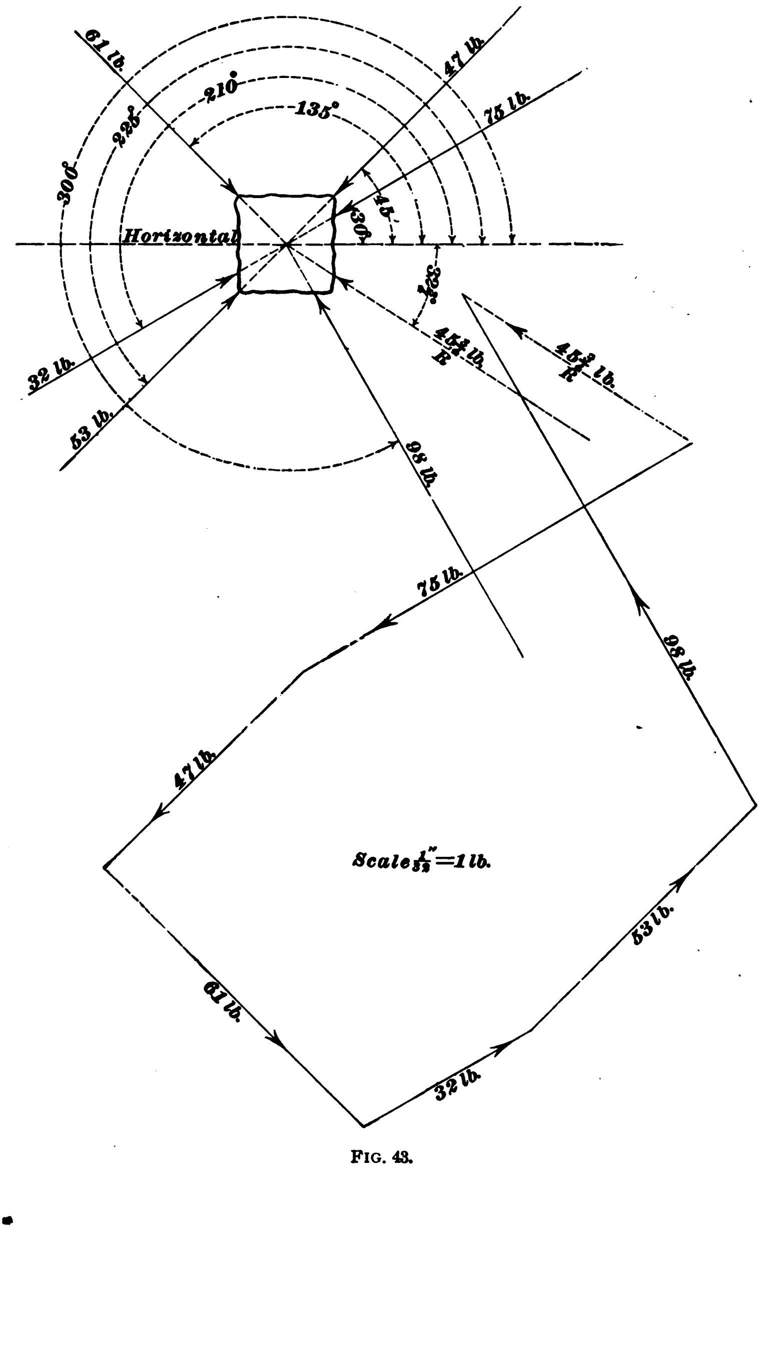
61 lb.
47 lb.
75 lb.
210°
135°
225°
300°
45°
30°
Horizontal
32 lb.
53 lb.
98 lb.
75 lb.
98 lb.
47 lb.
Scale 1/32" = 1 lb.
53 lb.
61 lb.
32 lb.

FIG. 43.

(1170) Apply formula **130,** and multiply the result by 2.

$$W = \frac{4 \times 6^2 \times 2 \times 160}{20} \times 2 = 4,608 \text{ lb., the load. Ans.}$$

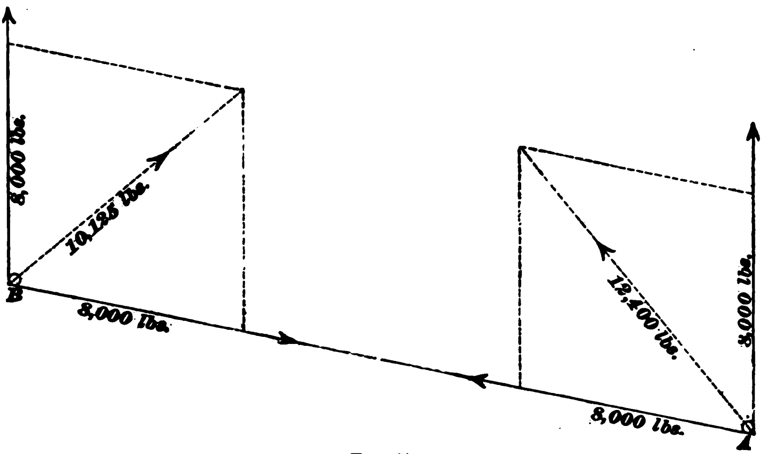

FIG. 44.

(1171) Apply formula **134.**

$$R = \frac{85 \times 80}{4^2} = 106\tfrac{1}{4} \text{ revolutions per minute. Ans.}$$

(1172) See Arts. **1929** to **1931.**

(1173) Apply formula **122.**

$$W = 12,000 \times (\tfrac{5}{8})^2 = 4,687.5 \text{ lb. Ans.}$$

(1174) Apply formula **125.**

$$W = 600 \times 6^2 = 21,600 \text{ lb. Ans.}$$

(1175) 4 ft. = 48 in. Area to be sheared $= 48 \times \frac{1}{2} =$ 24 sq. in. Applying formula **132,**

$$W = 24 \times 40,000 = 960,000 \text{ lb., the force required. Ans.}$$

(1176) Applying formula **134,**

$$\frac{70 \times 200}{7^3} = 40.8 \text{ revolutions per minute, nearly. Ans.}$$

(1177) See Art. **1963.** Area to be punched $= 1 \times 3.1416 \times \frac{7}{16} = 1.37445$ sq. in. Applying formula **132,**

$$\text{force} = 1.37445 \times 40{,}000 = 54{,}978 \text{ lb. Ans.}$$

(1178) Apply formula **133.**

$$H = \frac{(1\frac{7}{8})^3 \times 180}{95} = 12.49. \quad \text{Ans.}$$

(1179) Total pressure against the head $= 44^2 \times .7854 \times 100 = 152{,}053.44$ lb. Applying formula **119,**

$$\text{area of studs} = \frac{152{,}053.44}{5{,}000} = 30.41 \text{ sq. in., nearly.}$$

$$30.41 \div 1.057 = 29 \text{ studs. Ans.}$$

(1180) Apply formula **125.**

$$\text{Load} = 600 \times 4^2 = 9{,}600 \text{ lb. Ans.}$$

(1181) Apply formula **128.**

$$\text{Load} = \frac{2.5^2 \times 1.5 \times 100}{4\frac{8}{12}} = 201 \text{ lb., nearly. Ans.}$$

(1182) Apply formula **133.**

$$\text{Horsepower} = \frac{(2\frac{7}{16})^3 \times 120}{85} = 20.445. \quad \text{Ans.}$$

(1183) Area of cross-section $= (1\frac{1}{2})^2 \times .7854 = 1.7671$ sq. in. Apply formula **118.**

$$\text{Safe steady load} = 12{,}000 \times 1.7671 = 21{,}205.2 \text{ lb. Ans.}$$

(1184) Apply formula **123.**

$$W = 100 \times 6^2 = 3{,}600 \text{ lb., the safe load. Ans.}$$

(1185) Substituting the values of $C = 40$, $S = 14^2 \times .7854 - 11.5^2 \times .7854 = 50.0693$, $L = 20 \times 12 = 240$, $a = 562.5$, and $d = 14$ in formula **127,** we have

$$W = \frac{40 \times 50.0693}{\frac{240^2}{562.5 \times 14^2} + 1} = \frac{2{,}002.772}{1.5225} = 1{,}315.45 \text{ tons.}$$

$$\frac{1{,}315.45}{6} = 219.24 \text{ tons. Ans.}$$

(1186) See Art. **1963.** Area punched $= 1\frac{1}{2} \times 3.1416 \times \frac{3}{4} = 3.5343$ sq. in. Force $= 3.5343 \times 60,000 = 212,058$ lb. Ans.

(1187) See Fig. 45.

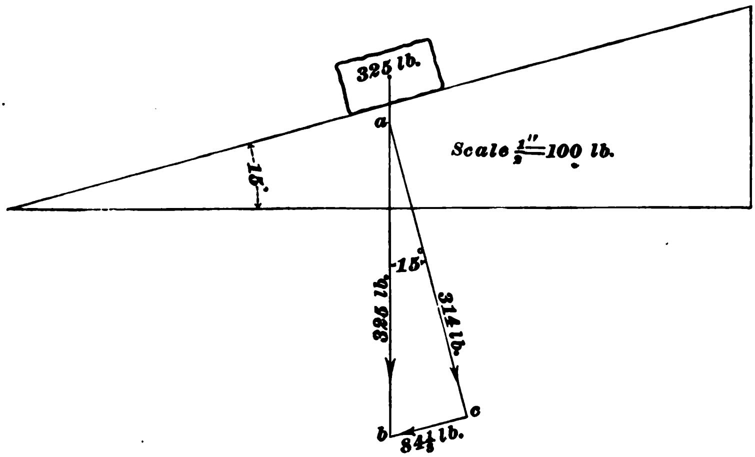

FIG. 45.

(*a*) $ac = 325 \times \cos 15° = 325 \times .96593 = 313.93$ lb. Ans.

(*b*) $bc = 325 \times \sin 15° = 325 \times .25882 = 84.12$ ib. Ans.

STEAM AND STEAM-BOILERS.

(1188) See Arts. **1970** to **1977.**

(1189) See Arts. **1972** to **1974.**

(1190) Less. See Art. **1982.**

(1191) See Arts. **1975** to **1977.**

(1192) (*a*) See Art. **1979.**
(*b*) and (*c*) See Art. **1978.**
(*d*) See Art. **1982.**

(1193) See Arts. **1984** to **1986.**

(1194) (*a*) and (*b*) See Arts. **1980** and **1981.**
(*c*) 1 B. T. U. = 778 ft.-lb.
$30\frac{1}{2}$ B. T. U. = $30\frac{1}{2} \times 778 = 23,729$ ft.-lb. Ans

(1195) 35 H. P. = $35 \times 33,000$ ft.-lb. per min. = $35 \times 33,000 \times 60$ ft.-lb. per hour = $\frac{35 \times 33,000 \times 60}{778}$ B. T. U. per hour = 89,074.5 B. T. U. per hour.

But this is the heat actually used, or 20% of the whole. Hence, the heat required is $89,074.5 \div .20 = 445,372.5$ B.T.U. per hour. Ans.

(1196) One horsepower $= 33,000 \times 60$ ft.-lb. per hour $= \frac{33,000 \times 60}{778}$ B. T. U. per hour.

Each pound of coal gives 14,000 B. T. U., of which 8%, or $14,000 \times .08 = 1,120$ B. T. U., is utilized. Hence, the coal required per hour per H. P. is

$$\frac{33,000 \times 60}{778} \div 1,120 = 2.27 \text{ lb.} \quad \text{Ans.}$$

(1197) The specific heat of sulphur is .2026. (See Table 41.) By formula **136,** $U = c\,W(t_1 - t) = .2026 \times 22\frac{1}{2} \times (68 - 44) = 109.4$ B. T. U. Ans.

(1198) (*a*) See Art. **1984.**

(*b*) To raise the ice from 17° to 32° requires for each pound $.504 \times (32 - 17) = 7.56$ B. T. U. To melt it requires 144 B. T. U. Hence, 1 lb. requires $144 + 7.56 = 151.56$ B. T. U. 11 lb. requires $11 \times 151.56 = 1{,}667.16$ B. T. U. Ans.

(1199) By formula **136,** $U = c\,W(t_1 - t) = .4805 \times 6 \times (342 - 310) = 92.256$ B. T. U. Ans.

(1200) Using formula **137,**

$$T = \frac{w\,c\,t + w_1\,c_1\,t_1 + w_2\,c_2\,t_2 + \ldots\ldots}{w\,c + w_1\,c_1 + w_2\,c_2 + \ldots\ldots} =$$

$$\frac{18 \times .0951 \times 305 + 13 \times .1138 \times 278 + 32 \times 1 \times 56}{18 \times .0951 + 13 \times .1138 + 32 \times 1} = 77.45°. \text{ Ans.}$$

(1201) (*a*) 966 B. T. U. Ans.

(*b*) To raise a pound of water from 63° to 212° requires $212 - 63 = 149$ B. T. U. To change it into steam requires 966.069 more B. T. U. $966 + 149 = 1{,}115$ B. T. U. for 1 lb. Hence, $8 \times 1{,}115 = 8{,}920$ B. T. U. are required. Ans.

(1202) To change 1 lb. of ice from 23° to 32° requires $(32 - 23) \times .504 = 4.536$ B. T. U. To melt the ice requires 144 B. T. U. To change the water at 32° to water at 212° requires 180 B. T. U. per pound. To change the water at 212° into steam at 212° requires 966 B. T. U. per pound. $4.536 + 144 + 180 + 966 = 1{,}294.536$ B. T. U. per pound. For 2.2 pounds, $1{,}294.536 \times 2.2 = 2{,}847.98$ B. T. U., as required. Ans.

(1203) See Arts. **1991** to **1993.**

(1204) See Art. **2024** and Arts. **2027** to **2030.**

(1205) In the return-tubular boiler the one or two large flues are replaced by a large number of small tubes. In other respects, the boilers are quite similar in principle.

(1206) See Art. **2024.**

(1207) See Arts. **2017, 2023, 2025,** and **2028.**

(1208) See Art. **2024.**

(1209) See Art. **2023.**

(1210) See Art. **2011.**

(1211) See Art. **2011.**

(1212) (*a*) See Art. **2004.**

(*b*) The temperature at which combustion takes place is always the same for the same substance. The nitrogen reduces the temperature of the furnace, since a portion of the heat given off by combustion is required to heat the nitrogen.

(1213) No. See definition of combustion, Art. **2003.**

(1214) See Art. **2007.**

(1215) See Art. **2008.**

(1216) The number of heat units required to convert a pound of water at 32° into steam at 400° may be found by means of formula **140.**

$$H = 1{,}081.4 + .305 \times 400 = 1{,}203.4 \text{ B. T. U. Ans.}$$

(1217) In order to use formula **140,** the temperature must be known. This can be found when the pressure is known, by means of formula **138.** Applying the formula, $t = 14\sqrt{175} + 199 = 384.2°$, the temperature of saturated steam having a pressure of 175 pounds per square inch. Now, using formula **140,**

$$H = 1{,}081.4 + .305 \times 384.2 = 1{,}198.6 \text{ B. T. U. Ans.}$$

(1218) Since the expansion follows Mariotte's law, the final pressure may be found by the formula $p_1 = \frac{p\,v}{v_1}$. Substituting, $p_1 = \frac{60 \times 5}{5 \times 2.5} = 24$ lb. per sq. in. above vacuum. $24 - 14.7 = 9.3$ lb. per sq. in. above atmosphere. Ans.

(1219) From Table 42, column 5, the total heat of combustion of one pound of coal is found to be 14,133 B. T. U.

$11 \times 13 \times 5 = 715$ pounds of coal burned in 5 hours.

$14,133 \times 715 = 10,105,095$ B. T. U. generated by the combustion of the coal. Ans.

(1220) According to Table 42, the amount of air required for the complete combustion under a blast draft is found to be 14 pounds. Hence, the amount of air required for combustion of the coal in Question 1219 is

$$715 \times 14 = 10,010 \text{ pounds. Ans.}$$

(1221) The number of pounds of water having a temperature of 62° which can be converted into steam having a temperature of 212° is found, from Table 42, column 6, to be 12.67 pounds. Hence, the total quantity of water which could be evaporated under the above conditions by the combustion of 715 pounds of coal is

$$12.67 \times 715 = 9,059.05 \text{ pounds. Ans.}$$

(1222) Since the pressure is 3,600 pounds per *square foot* above a vacuum, and there are 144 square inches in a square foot, the pressure above a vacuum is $\frac{3,600}{144} = 25$ pounds per *square inch*. Consequently, the pressure per square inch above the atmosphere is $25 - 14.7 = 10.3$ pounds. Ans.

(1223) See Art. **2001**.

(1224) According to formula **138**, the required temperature is

$$t = 199 + 14 \times \sqrt{152} = 371.62° \text{ F. Ans.}$$

(1225) Applying formula **139**, we have for the required pressure $p = \left(\frac{232 - 199}{14}\right)^2 = 5.56$ pounds per square inch gauge-pressure. Ans.

(1226) 132 tons equal $132 \times 2,000 = 264,000$ pounds. $264,000 \times 296 = 78,144,000$ = foot-pounds of work necessary to raise the coal to the top of the shaft. Since 1 B. T. U. = 778 foot-pounds, the heat supplied is

$$\frac{78,144,000}{778} = 100,442.15 \text{ B. T. U. \quad Ans.}$$

(1227) $277,160 \times 778 = 215,630,480$ foot-pounds of work done by the engine in two hours.

Hence, $\frac{215,630,480}{2} = 107,815,240$ ft.-lb. done in one hour. Ans.

(1228) The strength of any construction is always that of its weakest part. In the present example the diameter and thickness of the steam and water drums only are given, the thickness of the flues, mud-drum, and boiler-shell, and the diameter of the boiler-shell being omitted. Such being the case, we must confine ourselves to the strength of the steam and water drums, assuming that the other parts of the boiler have been made correspondingly strong. The pressure which the steam-drum will safely sustain is found by formula **141** to be $\frac{16,608 \times \frac{5}{16}}{24} = 216.25$ pounds per square inch, and the pressure which the water-drum will safely sustain is found by the same formula to be $\frac{16,608 \times \frac{5}{16}}{20} = 259.5$ pounds per square inch. Since the safe pressure upon the steam-drum is less than that which can be sustained by the water-drum, the pressure on the boiler must not exceed the safe pressure which can be sustained by the steam-drum; that is, 216.25 pounds per square inch. Ans.

(1229) From Table 43, it is seen that from 14 to 18 square feet of water-heating surface are required to produce one horsepower with a return-tubular boiler. Using 16 square feet as a mean, we obtain

$$\frac{1,620}{16} = 101\tfrac{1}{4} \text{ H. P. \quad Ans.}$$

(1230) In the same manner as in example 1229, it is found that about 11 square feet of heating-surface are required to produce 1 horsepower with a water-tube boiler. Hence,

$$\text{H. P.} = \frac{3,025}{11} = 275 \text{ horsepower. Ans.}$$

(1231) Applying formula **142,** the height of the chimney is found to be

$$h = \left[\frac{348}{3.33 \times 12 - 2\sqrt{12}}\right]^2 = \left[\frac{348}{3.33 \times 12 - (2 \times 3.464)}\right]^2 =$$

111 ft. Ans.

(1232) The dome and the dry-pipe. See Arts. **2022** and **2023.**

(1233) See Art. **2019.**

(1234) Blow-off pipes are provided to remove the collected sediment. The boiler is also provided with manholes or handholes for cleaning purposes.

(1235) See Art. **2013.**

(1236) To avoid overheating and burning out the upper plates of the furnace. So long as the water is in contact with the plates which are next to the fire, they can not be overheated or burned.

(1237) See Art. **2023.**

(1238) See Art. **2017.**

(1239) See Art. **2018.**

(1240) Answer from the result of your own observations.

(1241) The steam-pipe conveys the steam after it is generated from the boiler to the place where it is used The feed-water pipe is the one through which the water is introduced to the boiler. A blow-off pipe is one attached to the lower part of the boiler or to a mud-drum. It is used to empty the boiler of the whole or a part of its contents.

(1242) See Art. **2012.**

(1243) The arm of the safety-valve is a lever in which the power is the total steam-pressure on the valve, $6 \times 81 = 486$ pounds. The power arm is 2 inches, and the weight is 54 pounds. Calling the weight arm b, we have, from formula **94**,

$$Pa = Wb, \text{ or } 486 \times 2 = 54 \times b.$$

Hence, $$b = \frac{486 \times 2}{54} = 18 \text{ in. Ans.}$$

(1244) According to formula **141**, $p = \frac{10,224 \times \frac{3}{8}}{30} =$ 127.8 pounds per square inch, the greatest pressure under which it would be safe to operate a boiler of these dimensions.

(1245) See Art. **2033.**

(1246) (*a*) See Art. **2035.**

(*b*) The top and sides of the furnace, and the tubes.

(1247) Using formula **141**, $p = \frac{13,152 \times \frac{5}{16}}{45} = 91\frac{1}{3}$ lb. per sq. in., the safe working pressure. Therefore it would be unsafe to use 110 lb. pressure.

(1248) (*b*) According to Table 43, a vertical boiler has from 15 to 20 square feet of heating-surface per horsepower. Assuming 18 sq. ft. per H. P., the heating-surface will be $35 \times 18 = 630$ square feet. Ans.

(*a*) Since the heating-surface is 25 to 30 times the grate area, the latter must lie between $\frac{630}{25} = 25.2$ sq. ft. and $\frac{630}{30} =$ 21 sq. ft.; say about 23 sq. ft. Ans.

(*c*) One horsepower is equivalent to an evaporation of 30 pounds of water per hour, the feed being at 100° and the steam-pressure at 70 pounds. The evaporation per hour is, therefore, $35 \times 30 = 1,050$ lb. Ans.

STEAM-ENGINES.

(1249) The stationary parts of a plain slide-valve engine are the steam-cylinder, steam-chest, supply-pipe, exhaust-pipe, guide-bars, shaft-bearings, and the bed or frame of the engine.

(1250) The expansion curve of steam on an indicator-card represents the decrease of pressure of the steam after cut-off, corresponding to the increase of volume.

(1251) It passes its central position during the interval between the point of release of the steam from the head end of the cylinder, and the point of compression of the steam in the crank end of the cylinder, during the forward stroke of the piston, and conversely for the backward stroke.

(1252) Plain slide-valves usually cut off between one-half and full stroke.

(1253) The points of cut-off and release are marked, as shown in Figs. 46, 47, 48, and 49. The perpendicular distances from these points to the atmospheric line are measured. Multiplying the lengths of these perpendiculars by 45, the scale of the spring, we obtain

$$\left.\begin{array}{l} 1.3750 \text{ in.} \times 45 = 61.8750 \text{ lb. for cut-off} \\ .5625 \text{ in.} \times 45 = 25.3125 \text{ lb. for release} \end{array}\right\} \text{Fig. 46.}$$

$$\left.\begin{array}{l} 1.3750 \text{ in.} \times 45 = 61.8750 \text{ lb. for cut-off} \\ .6800 \text{ in.} \times 45 = 30.6000 \text{ lb. for release} \end{array}\right\} \text{Fig. 47.}$$

$$\left.\begin{array}{l} 1.3800 \text{ in.} \times 45 = 62.1000 \text{ lb. for cut-off} \\ .1200 \text{ in.} \times 45 = 5.4000 \text{ lb. for release} \end{array}\right\} \text{Fig. 48.}$$

$$\left.\begin{array}{l} 1.3700 \text{ in.} \times 45 = 61.650 \text{ lb. for cut-off} \\ .1200 \text{ in.} \times 45 = 5.4000 \text{ lb. for release} \end{array}\right\} \text{Fig. 49.}$$

To find the back-pressure, in each case, find the perpendicular distance between the lowest point of diagram and

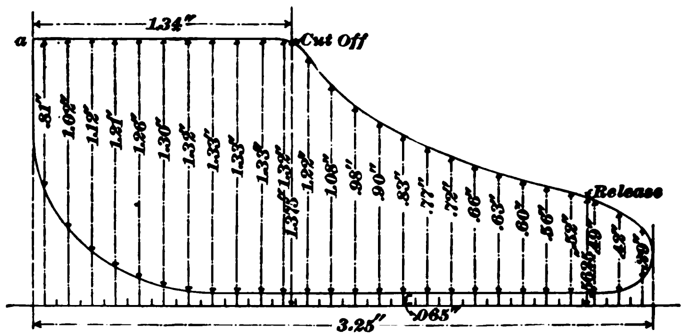

FIG. 46.

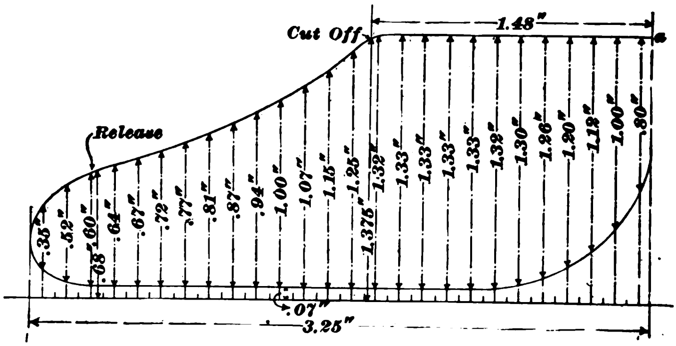

FIG. 47.

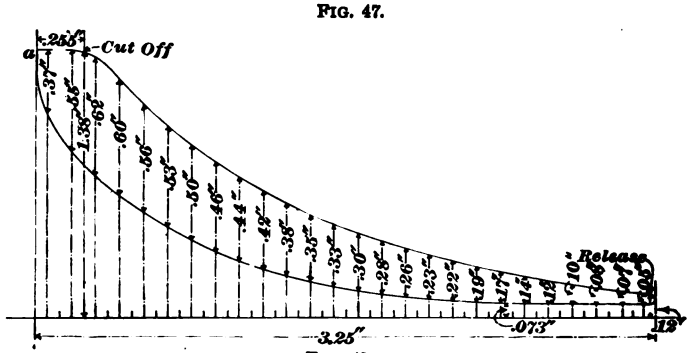

FIG. 48.

the atmospheric line; multiply by 45, the scale of the spring, and the products will be as follows:

.065 in. × 45 = 2.925 lb. back-pressure for Fig. 46.

.070 in. × 45 = 3.15 lb. back-pressure for Fig. 47.

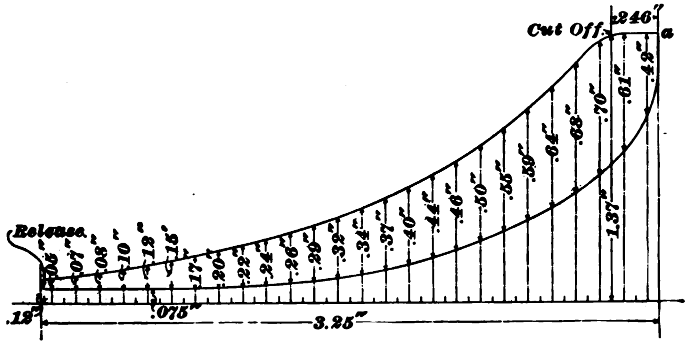

FIG. 49.

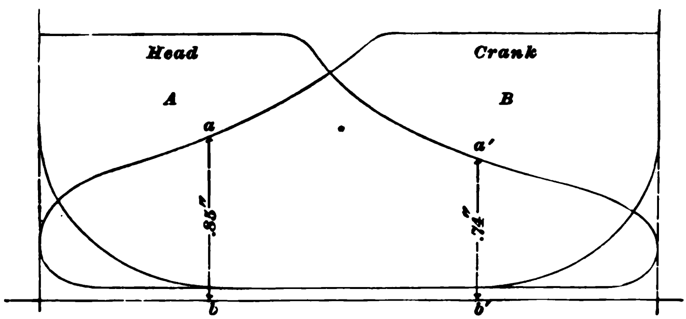

FIG. 50.

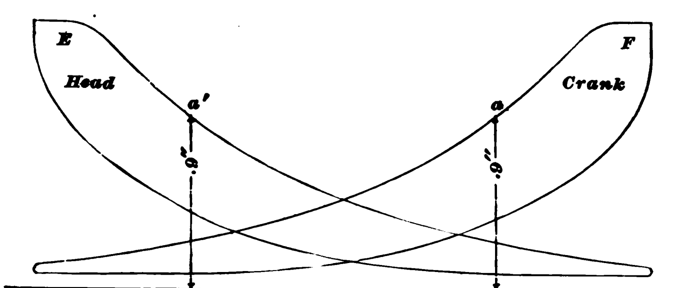

FIG. 51.

.073 in. × 45 = 3.285 lb. back-pressure for Fig. 48.

.075 in. × 45 = 3.375 lb. back-pressure for Fig. 49.

To determine the steam-pressure in the cylinder at the point of compression, we must combine diagrams *A* and *B* and *E* and *F*. These diagrams are combined by placing *B* upon *A* and *F* upon *E*, the atmospheric and extreme right and left hand lines coinciding. The height *a b* of the diagram *B*, in Fig. 50, represents the pressure of the steam in the crank end of the cylinder at the point of compression of the diagram *A*. This is as it should be, since the compression curve is drawn by the pencil of the indicator when the piston is making its return stroke. In a similar manner, the pressure of the steam in the head end of the cylinder at the point of compression in the crank end is the height *a' b'* of *A*. In Fig. 51 the height *ab* represents the pressure at compression for *E*, and *a' b'* the same for *F*. These results tabulated are as follows:

$$.85 \text{ in.} \times 45 = 38.25 \text{ lb. for } A.$$
$$.74 \text{ in.} \times 45 = 33.30 \text{ lb. for } B.$$
$$.90 \text{ in.} \times 45 = 40.50 \text{ lb. for } E.$$
$$.90 \text{ in.} \times 45 = 40.50 \text{ lb. for } F.$$

(1254) Project the extreme right and left hand points of the indicator-diagrams upon the atmospheric line; divide the distance between them into any number of equal spaces—26 in this case—and through the centers of these spaces draw lines across the diagram perpendicular to the atmospheric line. Now measure the length in inches of each of these perpendicular lines (the lengths are given in all the figures), and take their sum; divide this sum by the number of the equal spaces into which the atmospheric line is divided, and multiply the quotient by the scale of the spring.

Sum of the perpendiculars of the diagram of Fig. 46 = 24.02 in.; then,

$$\frac{24.02}{26} \times 45 = 41.573 \text{ lb., M. E. P.}$$

Sum of the perpendiculars of the diagram, Fig. 47, = 26 in.; then,

$$\tfrac{26}{26} \times 45 = 45 \text{ lb., M. E. P.}$$

The average M. E. P. for both diagrams is

$$\frac{41.573 + 45}{2} = 43.29 \text{ lb. per sq. in.} \quad \text{Ans}$$

(1255) Sum of the perpendiculars of the diagram, Fig. 48, = 8.32 in.; then,

$$\frac{8.32}{26} \times 45 = 14.40 \text{ lb., M. E. P.}$$

Sum of the perpendiculars of the diagram, Fig. 49, = 8.97; then,

$$\frac{8.97}{26} \times 45 = 15.525 \text{ lb., M. E. P.}$$

The average M. E. P. for the two diagrams is

$$\frac{14.40 + 15.525}{2} = 14.96 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(1256) Area of 15-inch piston $= 15^2 \times .7854 = 176.715$ square inches.

Using formula **143,**

$$\text{I. H. P.} = \frac{43.29 \times 2 \times 176.715 \times 175}{33{,}000} = 81.14 \text{ I. H. P.} \quad \text{Ans.}$$

(1257) Proceeding as in example 1256,

$$\text{I. H. P.} = \frac{14.96 \times 2 \times 176.715 \times 175}{33{,}000} = 28.04 \text{ I. H. P.} \quad \text{Ans.}$$

(1258) The actual horsepower is $81.14 - 28.04 = 53.1$ H. P. Ans.

Applying formula **146,** the efficiency is

$$\frac{53.1}{81.14} = .654 = 65.4 \text{ per cent.} \quad \text{Ans.}$$

(1259) The force of gravity and the centrifugal force.

(1260) See Art. **2098.**

(1261) The piston, piston-rod, cross-head, connecting-rod, crank, crank-shaft, eccentric, eccentric-rod, slide-valve, and fly-wheel.

(1262) In order that the energy stored in them may be utilized in carrying the crank over its dead-center position, and also to cause the engine to run at a more uniform speed.

(1263) Compression is taking place. See Figs. 50 and 51.

(1264) Any portion added to the length of a valve more than is absolutely necessary, in order to cover the outside edges of the steam-ports when the valve is in its central position, is called the outside lap of the valve. It is added to enable the valve to cut off the live steam before the piston reaches the end of its stroke.

(1265) Apply rule, Art. **2059.** Cut-off in the diagram, Fig. 807, takes place at a point 1.34 inches from *a*. See Fig. 46.

Therefore, cut-off equals $\frac{1.34}{3.25}$, or 41% of stroke.

Cut-off in the diagram, Fig. 808, takes place at a point 1.48 inches from *a*. See Fig. 47.

Therefore, cut-off equals $\frac{1.48}{3.25}$, or 46% of stroke, nearly.

Cut-off in the diagram, Fig. 809, takes place at a point .255 inch from *a*. See Fig. 48.

Therefore, cut-off equals $\frac{.255}{3.25}$, or 7.8% of stroke.

Cut-off in the diagram, Fig. 810, takes place at a point .246 inch from *a*. See Fig. 49.

Therefore, cut-off equals $\frac{.246}{3.25}$, or 7.6% of stroke.

In each case the length of the diagram is 3.25 inches.

(1266) The indicated horsepower of this engine will be about one-half greater than the actual horsepower, or $\frac{65}{2} + 65 = 97.5$ horsepower. See example, Art. **2077.**

A fair piston speed is 500 feet per minute.

Assume the cut-off to be taken at $\frac{2}{3}$ and the boiler-pressure to be 70 pounds per square inch. Applying formula **144**, the M. E. P. $= .9\,[.937\,(70 + 14.7) - 17] = 56.13$ pounds per square inch. Letting $d =$ diameter of cylinder,

$$\text{I. H. P.} = \frac{d^2 \times .7854 \times 56.13 \times 500}{33{,}000} = 97.5, \text{ or}$$

$$d = \sqrt{\frac{97.5 \times 33{,}000}{.7854 \times 56.13 \times 500}} = 12.08 \text{ inches, or say } 12 \text{ inches.}$$

Taking the ratio of stroke to diameter of cylinder as $1\frac{1}{2}$, we have stroke $= 12 \times 1\frac{1}{2} = 18$ inches. The number of revolutions of the crank is

$$\frac{500 \times 6}{18} = 166\tfrac{2}{3} \text{ revolutions per minute.}$$

(1267) A combination of two single-cylinder engines of exactly the same description and dimensions, which have their cranks rigidly connected to a common crank-shaft and take the steam at the same pressure, is called a *duplex* engine.

Compound engines are those having two cylinders, of which the working lengths are usually the same, but the diameter of one, the high-pressure cylinder, is less than that of the other, the low-pressure cylinder, and the steam, instead of entering both cylinders at boiler-pressure, enters first the high-pressure cylinder, and is exhausted from there into the low-pressure cylinder

(1268) One in which the cylinder is in a vertical or upright position.

(1269) The stroke of an engine is the distance passed over by the piston when moving from one end of the cylinder to the other end, and is equal to the *throw of the crank*, or to the diameter of the circle described by the center of the crank-pin.

(1270) An eccentric is a disk, or wheel, so arranged upon a shaft that the center of the wheel and that of the shaft do not coincide. It is equivalent to a crank having the same throw, and is used to give motion to the slide-valve.

(1271) It is the period during which the steam remaining in the cylinder after the exhaust-valve has closed is compressed as the piston continues the return stroke. It begins at the instant that the valve closes the port to the exhaust-steam.

(1272) It shortens the period of release and lengthens both the period of expansion and compression.

(1273) It permits an earlier cut-off, together with a greater range and more perfect steam distribution.

(1274) Using formula **143**,

$$\text{I. H. P.} = \frac{62.4 \times .7854 \times 18^2 \times \frac{24}{12} \times 2 \times 175}{33{,}000} =$$

$$336.825 \text{ I. H. P.} \quad \text{Ans.}$$

(1275) By setting the cranks at right angles, both engines can not be on a dead-center at the same time.

(1276) See Arts. **2097** and **2098.**

(1277) By the *bore* of a cylinder is meant its diameter.

(1278) Steam is called live steam when it leaves the boiler and before doing any work in the cylinder. The energy stored in the live steam is potential energy.

(1279) The fly-wheel supplies the force necessary to overcome the retarding effect of compression.

(1280) (*a*) The dead-center positions occur when the piston reaches the end of its stroke, and the centers of the cross-head pin, crank-pin, and crank-shaft are in the same straight line.

(*b*) Twice.

(1281) A steam-engine indicator is an instrument which draws a diagram showing the pressure of the steam in the cylinder at every point of the stroke. See Fig. 679 for method of fastening to cylinder.

(1282) See Art. **2097.**

(1283) See Art. **2039.**

(1284) It is the resistance against being pushed into the condenser or the atmosphere which the exhaust-steam exerts on the piston.

(1285) By period of release is meant that period during which the steam is escaping into the atmosphere or condenser. The point of compression marks the end of release.

(1286) See Art. **2049.**

(1287) Two. One spring is to resist any upward motion of the indicator-piston, and the other is to carry the drum back to its first position when the pull on the cord is discontinued.

(1288) The back-pressure line would then fall below its present position a distance represented by a pressure of $\frac{5}{6} \times 14.7 = 12\frac{1}{4}$ pounds $= \frac{12\frac{1}{4}}{45} = .27$ inch, nearly. Then, for the same mean effective pressure, the cut-off would be earlier.

(1289) See Ans. **2091.**

(1290) See Art. **2039.**

(1291) See Art. **2042.**

(1292) Release is taking place.

(1293) The varying pressures of the steam while being compressed.

(1294) See Art. **2050.**

(1295) See Art. **2055.**

(1296) See Art. **2055.**

(1297) See Art. **2078.**

(1298) See Art. **2039.**

(1299) See Art. **2050.**

(1300) At the end. See Art. **2045.**

(1301) See Arts. **2048** and **2050.**

(1302) See Art. **2055.**

(1303) Using formula **144** and the constants in Table 44, the M. E. P. for $\frac{3}{10}$ cut-off is

$$.9\,[.708\,(75 + 14.7) - 17] = 41.86 \text{ lb. per sq. in. Ans.}$$

For cut-off at $\frac{1}{2}$ stroke,

$$\text{M. E. P.} = .9[.847(75+14.7) - 17] = 53.16 \text{ lb. per sq. in. Ans.}$$

(1304) See Art. **2080.**

(1305) See Art. **2092.**

(1306) See Art. **2039.**

(1307) See Art. **2044.**

(1308) Closed. See Art. **2045** and Fig. 670 (*a*).

(1309) See Art. **2045.**

(1310) Using formula **145,**

$$S = \frac{lR}{6} \text{ or } l = \frac{6S}{R} = \frac{350 \times 6}{175} = 12 \text{ inches. Ans.}$$

(1311) See Art. **2080.**

(1312) See Arts. **2094** and **2098.**

(1313) See Art. **2039.**

(1314) See Art. **2043.**

(1315) See Art. **2052.**

(1316) Applying the rule, Art. **2056,**

$$\text{length} = \frac{96 \times 3}{12} = 24 \text{ inches. Ans.}$$

(1317) Applying formula **145,**

$$S = \frac{lR}{6} = \frac{48 \times 50}{6} = 400 \text{ feet per minute. Ans.}$$

(1318) See Art. **2085.**

(1319) See Arts. **2096** and **2097.**

(1320) See Art. **2044.**

(1321) See Art. **2044.**

(1322) Formula **143** gives

$$\text{I. H. P.} = \frac{PLAN}{33{,}000} = \frac{43.4 \times \frac{18}{12} \times 22^2 \times .7854 \times 2 \times 200}{33{,}000} =$$

300 H. P., nearly. Ans.

(1323) See Art. **2061.**

(1324) Applying formula **145,**

$$S = \frac{lR}{6}, \text{ or } R = \frac{6S}{l};$$

therefore,

$$R = \frac{6 \times 750}{60} = 75 \text{ rev. per min.} \quad \text{Ans.}$$

(1325) See Art. **2081.**

(1326) See Art. **2095.**

AIR AND AIR COMPRESSION.

(1327) See Art. **2101.**

(1328) (*a*) 4 ft. = 48 in. A cubic inch of mercury weighs .49 lb.; hence, the pressure exerted by 48 inches of mercury = $48 \times .49 = 23.52$ lb. per sq. in. Ans.

(*b*) Since the pressure of 1 atmosphere is 14.7 lb. per sq. in., a pressure of 23.52 lb. per sq. in. is equivalent to $23.52 \div 14.7 = 1.6$ atmospheres. Ans.

(1329) A pressure of 1 atmosphere will support a column of water 34 ft. high. Since the column of water is 15 ft. high, the height of the confined air is $34 - 15 = 19$ ft., or, in other words, the tension of the confined air in pounds per square inch is equal to the weight of a column of water 1 in. square and 19 ft. high. The pressure exerted by a column of water 1 ft. high and having a cross-section of 1 sq. in. = $12 \times .03617 = .434$ lb. Hence the tension of the confined air = $.434 \times 19 = 8.246$ lb. per sq. in. Ans.

(1330) See Arts. **2117** and **2118.**

(1331) See Art. **2118.**

(1332) There would be no loss, because the air would have no opportunity to lose heat by radiation in the pipes. The heat stored in the air during compression would be available for useful work.

(1333) See Arts. **2121** and **2126.**

(1334) See Art. **2131.**

(1335) See Art. **2136.**

(1336) See Art. **2145.**

(1337) (*a*) See Art. **2113.** (*b*) More work is required to compress air adiabatically.

(1338) Applying formula **157** and substituting,

$$p_1 = p\left(\frac{459 + t_1}{459 + t}\right) = 40 \times \left(\frac{459 + 55}{459 + 120}\right) =$$

$$40 \times \frac{514}{579} = \frac{20,560}{579} = 35.51 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(1339) (*a*) $14.7 + 9 = 23.7$ lb. per sq. in., the tension. Since the area of the piston remains constant, the volume at any point of the stroke will be proportional to the distance passed over by the piston; hence, we may substitute the latter for the former in formula **151.**

$v_1 = \frac{p v}{p_1} = \frac{14.7 \times 80}{23.7} = 49.62$ in., distance between piston and end of stroke. The distance passed over by piston $= 80 - 49.62 = 30.38$ in. Ans.

(*b*) Area of piston $= 80^2 \times .7854$ sq. in.

Volume of air at point of discharge $= 80^2 \times .7854 \times 49.62 =$ 249,417.91 cu. in.

$$249,417.91 \div 1,728 = 144.34 \text{ cu. ft.} \quad \text{Ans.}$$

(1340) Since the required horsepower is 25 and the loss is 35%, the horsepower of the engine must be $25 \div (100\% - 35\%) = 25 \div .65 = 38.46$ H. P.

To calculate the M. E. P., formula **144** may be used.

$$\text{M. E. P.} = .9\,[.904\,(92 + 14.7) - 17] = 71.5 \text{ lb. per sq. in.}$$

To find the diameter of the steam-cylinder, substitute in formula **148.**

$$D = 79.6\sqrt[3]{\frac{38.46}{1\frac{3}{8} \times 71.5 \times 340}} = 8.34; \text{ or say } 8\tfrac{3}{8} \text{ in.}$$

Length of stroke $= 8\frac{3}{8} \times 1\frac{3}{8} = 11\frac{1}{2}$ in. Consequently, the steam-cylinder should be $8\frac{3}{8} \times 11\frac{1}{2}$ in. Ans.

(1341) The steam-cylinder will show the greater I. H. P. The difference represents the horsepower required to overcome the friction of the moving parts of the compressor.

(1342) See Art. **2149.**

(1343) (*a*) Volume of cylinder $= \frac{20^2 \times .7854 \times 32}{1{,}728} =$ 5.8178 cu. ft.

$$32 - 26 = 6 \text{ in., length of stroke unfinished.}$$

The volume at discharge is $\frac{6}{32}$ of volume at beginning of stroke, or $\frac{6}{32} \times 5.8178 = 1.0908$ cu. ft. Ans.

(*b*) To calculate the weight, substitute in formula **161,** taking the values of P, V, and T at beginning of stroke.

$$W = \frac{PV}{.37052\,T} = \frac{14.7 \times 5.8178}{.37052 \times (459 + 76)} = .43143 \text{ lb. Ans.}$$

(1344) Since the area of the cylinder remains constant, any variation in the volume will be proportional to the distance between the piston and end of stroke; hence, we may substitute the latter for the volume in formula **150.**

$$p_1 = \frac{p\,v}{v_1} = \frac{14.7 \times 32}{6} = 78.4 \text{ lb. per sq. in. Ans.}$$

(1345) Using formula **150,**

$$p_1 = \frac{p\,v}{v_1} = \frac{(3 \times 14.7) \times 1}{2.5} = 17.64 \text{ lb. Ans.}$$

(1346) Applying formula **159** and substituting,

$$V = \frac{.37052\,W\,T}{P} = \frac{.37052 \times 7.14 \times (459 + 75)}{1.5 \times 14.7} =$$

64.068 cu. ft. Ans.

(1347) $p = 3\frac{1}{2}$ atmospheres $= 3\frac{1}{2} \times 14.7 = 51.45$ lb. per sq. in.

Applying formula **154** and substituting,

$$p : W :: p_1 : W_1.$$

$$51.45 : 13 :: p_1 : 2.$$

$$p_1 = \frac{2 \times 51.45}{13} = 7.915 \text{ lb. per sq. in. Ans.}$$

(1348) Volume at beginning of stroke =

$$\frac{48^2 \times .7854 \times 60}{1,728} = 62.832 \text{ cu. ft.}$$

Substituting in formula **161** to obtain the weight of the air,

$$W = \frac{PV}{.37052\,T} = \frac{14.7 \times 62.832}{.37052 \times (459 + 60)} = 4.8031 \text{ lb.}$$

Volume at time of discharge =

$$\frac{48^2 \times .7854 \times (60 - 50)}{1,728} = 10.472 \text{ cu. ft.}$$

To calculate the tension, substitute in formula **158**, taking the values of T and V at the time of discharge and the value of W as 4.8031.

$$P = \frac{.37052\,W\,T}{V} = \frac{.37052 \times 4.8031 \times (459 + 130)}{10.472} =$$

100.096 lb. per sq. in. Ans.

(1349) Applying formula **159** and substituting,

$$V = \frac{.37052\,W\,T}{P} = \frac{.37052 \times 1 \times (459 + 127)}{27} = 8.042 \text{ cu. ft.}$$

Ans.

(1350) A pressure of 4,000 lb. per sq. ft. is equivalent to $\frac{4,000}{144}$, or 27.777 lb. per sq. in. Using formula **159**,

$$V = \frac{.37052\,W\,T}{P} = \frac{.37052 \times .5 \times 559}{27.777} = 3.728 \text{ cu. ft. Ans.}$$

(1351) Applying formula **156** and substituting,

$$v_1 = v\left(\frac{459 + t_1}{459 + t}\right) = 4 \times \left(\frac{459 + 115}{459 + 40}\right) = 4.6012 \text{ cu. ft. Ans.}$$

(1352) See Art. **2147.**

(1353) Since the ordinary temperature is given in each case, we add 459° to obtain the corresponding absolute temperatures.

459° + 32° = 491°; 459° + 212° = 671°; 459° + 62° = 521°; 459° + 0° = 459°; 459° − 40° = 419°.

(1354) $P = 10$ atmospheres $= 10 \times 14.7 = 147$ lb. per sq. in. Applying formula **160** and substituting,

$$T = \frac{PV}{.37052\,W} = \frac{147 \times 4}{.37052 \times 3.5} = 453.417^\circ.$$

$453.417^\circ - 459^\circ = -5.583^\circ$, or 5.583° below zero. Ans.

(1355) See Art. **2134.**

(1356) Applying formula **150** and substituting,

$$p_1 = \frac{p\,v}{v_1} = \frac{130 \times 11.798}{75} = 20.45 \text{ lb. per sq. in. Ans.}$$

(1357) Applying formula **160,** $T = \dfrac{PV}{.37052\,W}$.

Substituting, $T = \dfrac{18 \times 14}{.37052 \times 1.2} = 566.77^\circ$. $566.77^\circ - 459^\circ = 107.77^\circ$. Ans.

(1358) Applying formula **156** and substituting,

$$v_1 = v\left(\frac{459 + t_1}{459 + t}\right) = 21 \times \left(\frac{459 + 420}{459 + 60}\right) = 35.57 \text{ cu. ft. Ans.}$$

(1359) To obtain absolute pressure, 1 atmosphere must be added to the gauge-pressure. $6 + 1 = 7$ atmospheres. Substituting in formula **161,**

$$W = \frac{PV}{.37052\,T} = \frac{14.7 \times 7 \times 12}{.37052 \times (459 + 90)} = 6.07033 \text{ lb.,}$$

weight of 12 cubic feet. $6.07033 \div 12 = .50586$ lb., weight per cubic foot. Ans.

(1360) .5 lb. = 8 oz. 1 lb. 6 oz. = 22 oz.

Applying formula **154** and substituting,

$$p : W :: p_1 : W_1.$$
$$14.7 : 8 :: p_1 : 22.$$

$$p_1 = \frac{14.7 \times 22}{8} = 40.425 \text{ lb. per sq. in. Ans.}$$

(1361) Apply formula **156.** $v_1 = v\left(\frac{459 + t_1}{459 + t}\right)$.
Substituting,

$$v_1 = 4,516\left(\frac{459 + 80}{459 + 260}\right) = 4,516 \times \frac{539}{719} = 3,385.42 \text{ cu. in.}$$

3,385.42 ÷ 1,728 = 1.96 cu. ft. Ans.

(1362) $P = 1\frac{1}{4} \times 14.7$ lb. per sq. in.
Applying formula **161** and substituting,

$$W = \frac{PV}{.37052T} = \frac{1\frac{1}{4} \times 14.7 \times 55}{.37052 \times (459 + 88)} = \frac{1,010.625}{202.67444} = 4.986 \text{ lb.}$$ Ans.

(1363) Since the temperature and volume in both vessels are the same, the pressure of the mixture will be equal to the sum of the pressures.

2 atmospheres = 2 × 14.7 = 29.4 lb. per sq. in.

29.4 + 40 = 69.4 lb. per sq. in. Ans. See Art. **2167.**

(1364) We would understand that the mercury had fallen 7 inches, and that there was enough air in the condenser to produce a pressure of $\frac{30 - 23}{30} \times 14.7$, or $\frac{7}{30} \times 14.7 = 3.43$ lb. per sq. in. Ans. See Art. **2155.**

(1365) 144 × 14.7 = 2,116.8 lb. per sq. ft. Ans.

(1366) If the weight of 3 cu. ft. under a pressure of 30 lb. per sq. in. is .27 lb., the weight per cu. ft. $= \frac{.27}{3} = .09$ lb.
Applying formula **154** and substituting,

$$p : W :: p_1 : W_1, \text{ or } 30 : .09 :: 65 : W_1.$$

$$W_1 = \frac{.09 \times 65}{30} = .195 \text{ lb.}$$ Ans.

(1367) To find the absolute temperature, we substitute in formula **160,** the values of P, V, and W given in Question 1366.

$$T = \frac{PV}{.37052\ W} = \frac{30 \times 3}{.37052 \times .27} = 899.64°.$$

Ordinary temperature = 899.64° − 459° = 440.64°. Ans.

(1368) Since the pressures and volumes are unequal, we apply formula **162** in order to obtain the tension of the mixture.

$$P = \frac{p\,v + p_1\,v_1}{V}.$$

Substituting,

$$P = \frac{14.7 \times 12 + 3 \times 14.7 \times 8}{20} = \frac{176.4 + 352.8}{20} =$$

26.46 lb. per sq. in. Ans.

(1369) Applying formula **163** and substituting,

$$V = \frac{p\,v + p_1\,v_1}{P} = \frac{14.7 \times 12 + 3 \times 14.7 \times 8}{24} = 22.05 \text{ cu. ft.}$$
Ans.

(1370) See Art. **2155.**

(1371) Since a cubic inch of mercury weighs .49 lb., $\frac{1}{40}$ of a cubic inch weighs $\frac{1}{40} \times .49 = \frac{.49}{40}$ lb. Consequently, a height of $\frac{1}{40}$ in. of mercury is equivalent to a pressure of $\frac{.49}{40}$ lb. per sq. in. 1 sq. ft. = 144 sq. in. The equivalent pressure upon a sq. ft. $= \frac{.49}{40} \times 144 = 1.764$ lb. Ans.

(1372) (*a*) See Art. **2155.**

(*b*) The height of the mercury in the tube shows the number of inches of vacuum.

Since the mercury column is $4\frac{1}{2}$ inches shorter than the barometer column, the gauge will show $30 - 4\frac{1}{2} = 25\frac{1}{2}$ inches vacuum. Ans.

(1373) Since a column of mercury 30 in. high will support a column of water 34 ft., 1 in. of mercury will support a column of water of $\frac{1}{30} \times 34$, or $\frac{34}{30}$ ft. in height.

Hence, 27 inches of mercury will support $27 \times \frac{34}{30} = 30.6$ ft. of water. Ans.

(1374) See Art. **2142.**

(1375) See Art. **2140.** Each rock-drill requires a receiver volume of 10 cubic feet; therefore, to supply 8 rock-drills, the volume of the receiver should be $8 \times 10 = 80$ cu. ft. Ans.

(1376) See Art. **2119.** (1) There is a loss due to useless heating of the air during the compression; this is reduced by the water-jacket or by water injection. (2) The loss due to the friction of the engine and compressor can only be reduced by careful workmanship and design in the building of the compressor. (3) There is a slight loss due to leakage and friction of air in pipes. The loss due to friction becomes large when the pipe is very long and of small diameter; therefore, this loss is reduced to a minimum by using large pipes for conveying the air.

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(1377) (*b*) To obtain the discharge in cubic feet per second, apply formula **180.**

$$Q_a = .41\, b \sqrt{2g}\,[\sqrt{h^3} - \sqrt{h_1^3}] =$$

$$.41 \times \tfrac{30}{12} \times \sqrt{2 \times 32.16}\,[\sqrt{(5\tfrac{1}{2})^3} - \sqrt{(3\tfrac{1}{2})^3}] = 52.21 \text{ cu. ft. per sec.}$$

Ans.

(*a*) Area of weir $= b\,d = 2.5 \times 2 = 5$ sq. ft.

Apply formula **179.**

$$v_m = \frac{Q_a}{b\,d} = \frac{52.21}{5} = 10.44 \text{ ft. per sec. Ans.}$$

(*c*) To get the discharge in gallons per hour, multiply (*b*) by 60×60 (seconds in an hour) and by 7.48 (gallons in a cubic foot). Thus, $52.21 \times 60 \times 60 \times 7.48 = 1{,}405{,}910.9$ gal. per hour. Ans.

(1378) First find the coefficient of friction by using formula **182** and Table 45.

$$v_m = 2.315\sqrt{\frac{h\,d}{f\,l}} = 2.315\sqrt{\frac{76 \times 7.5}{.025 \times 12{,}000}} = 3.19 \text{ ft. per sec.}$$

From the table, $f = .0243$ for $v_m = 3$, and $.023$ for $v_m = 4$; the difference is $.0013 =$ difference for a difference of velocity of 1 ft. per sec. Then, $.0013 \times .19$ ft. per sec. $= .000247$, or say .0002, using but four decimal places = difference for a difference of velocity of .19 ft. per sec. Therefore, $f = .0243 - .0002 = .0241$, or say .024.

Use formula **186.** Substitute in it the value of f here found, and multiply by 60 to get the discharge per minute.

$$Q = .09445\, d^2 \sqrt{\frac{h\,d}{f\,l + .125\,d}} \times 60 =$$

$$09445 \times 7.5^2 \sqrt{\frac{76 \times 7.5}{.024 \times 12{,}000 + .125 \times 7.5}} \times 60 = 447.7 \text{ gal. per min.}$$ Ans.

(1379) (*a*) Use formula **181.**

$$v_m = 2.315 \sqrt{\frac{h\,d}{f\,l + .125\,d}} \times 60 =$$

$$2.315 \sqrt{\frac{76 \times 7.5}{.024 \times 12{,}000 + .125 \times 7.5}} \times 60 = 195.08 \text{ ft. per min.}$$ Ans.

(*b*) 447.7 gal. per min. ÷ 60 = 7.46⅛ gal. per sec. = 1 cu. ft. per sec., nearly. Ans.

(1380) Use formula **167.**

$$v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 10} = 25.36 \text{ ft. per sec.}$$ Ans.

(1381) Use formulas **185** and **183.**

$$v_m = \frac{24.51\,Q}{d^2} = \frac{24.51 \times 42{,}000}{6.5^2 \times 60 \times 60} = 6.768 \text{ ft. per sec.}$$

From Table 45, $f = .021$ for $v_m = 6.768$; hence,

$$h = \frac{f\,l\,v_m^2}{5.36\,d} + .0233\,v_m^2 =$$

$$\frac{.021 \times 1{,}500 \times 6.768^2}{5.36 \times 6.5} + .0233 \times 6.768^2 = 42.48 \text{ ft.}$$ Ans.

(1382) Area of top or bottom of cylinder equals $20^2 \times .7854 = 314.16$ sq. in. Area of cross-section of pipe $= (\frac{3}{8})^2 \times .7854 = .1104$ sq. in. 25 lb. 10 oz. = 25.625 lb. 25.625 ÷ .1104 = 232.11 lb. pressure per sq. in. on top or bottom exerted by the weight and piston.

Pressure due to a head of 10 ft. = .434 × 10 = 4.34 lb. per sq. in.

Pressure due to a head of 13 ft. = .434 × 13 = 5.642 lb. per sq. in.

(Since a column of water 1 ft. high exerts a pressure of .434 lb. per sq. in. See Art. **2289.**)

(*a*) Pressure on bottom = pressure due to weight + pressure due to head of 13 ft. = 232.11 + 5.64 = 237.75 lb. per sq. in. Ans.

(*b*) Pressure on the top = pressure due to weight + pressure due to head of 10 feet = 232.11 + 4.34 = 236.45 lb. per sq. in. Ans.

(*c*) Total pressure, or equivalent weight on the bottom, = 237.752 × 314.16 = 74,692.17 lb. Ans.

(1383) $.434 \times 1\frac{1}{2} = .651$ lb., pressure due to the head of water in the cylinder above the orifice.

236.45, pressure on top per sq. in. + .651 = 237.1, total pressure per sq. in. Area of orifice = $1^2 \times .7854 = .7854$ sq. in.

$$.7854 \times 237.1 = 186.22 \text{ lb. Ans.}$$

(1384) First find the coefficient of friction by formula **182** and Table 45.

$$v_m = 2.315\sqrt{\frac{h\,d}{.025\,l}} = 2.315\sqrt{\frac{120 \times 4}{.025 \times 4{,}000}} = 5.072 \text{ ft. per sec.},$$

or say 5 ft. per sec.

From the table, $f = .023$ for $v_m = 4$ and .0214 for $v_m = 6$.

$$\frac{.023 - .0214}{6 - 4} = .0008. \quad .023 - .0008 = .0222 = f \text{ for } v_m = 5.$$

Use formula **182,** because the pipe is longer than 10,000 times its diameter.

$$\text{Hence, } v_m = 2.315\sqrt{\frac{120 \times 4}{.0222 \times 4{,}000}} = 5.38 \text{ ft. per sec. Ans.}$$

(1385) Use formulas **182** and **181.**

$$v_m = 2.315\sqrt{\frac{120 \times 4}{.025 \times 2{,}000}} = 7.17 \text{ ft. per sec.}$$

From the table, $f = .0214$ for $v_m = 6$ and .0205 for $v_m = 8$.

$$\frac{.0214 - .0205}{8 - 6} = .00045$$ decrease for an increase of velocity of 1 ft. per sec. $7.17 - 6 = 1.17$. $.00045 \times 1.17 = .0005265$, total decrease. $f = .0214 - .0005265 = .0208735$, or .0209, using four figures.

Hence, the velocity of discharge =

$$v_m = 2.315\sqrt{\frac{120 \times 4}{.0209 \times 2{,}000 + .125 \times 4}} = 7.79 \text{ ft. per sec.}$$ Ans.

(1386) (*a*) $f = .0205$ for $v_m = 8$. Therefore, using formula **183**,

$$h = \frac{.0205 \times 5{,}280 \times 8^2}{5.36 \times 10} + .0233 \times 8^2 = 130.73 \text{ ft.} \quad \text{Ans.}$$

(*b*) Using formula **184**, $Q = .0408\, d^2 v_m = .0408 \times 10^2 \times 8 = 32.64$ gal. per sec. $32.64 \times 60 \times 60 = 117{,}504$ gallons per hour. Ans.

(1387) A column of water 1 inch square and 2.304 ft. high weighs 1 lb.; hence, to produce a pressure of 30 lb. per sq. in., it will require a column of water $2.304 \times 30 = 69.12$ ft. high = head. Using formula **172**,

$$v = .98\sqrt{2gh} = .98\sqrt{2 \times 32.16 \times 69.12} = 65.34 \text{ ft. per sec.} \quad \text{Ans.}$$

(1388) (*a*) 36 in. = 3 ft. A column of water 1 inch square and 1 ft. long weighs .43403 lb. $.43403 \times 43 = 18.6633$ lb. per sq. in. on the bottom of the cylinder. $.43403 \times 40 = 17.3612$ lb. per sq. in. on the top of the cylinder. Area of base of cylinder $= 20^2 \times .7854 = 314.16$. $314.16 \times 18.6633 = 5{,}863.26$ lb., the total pressure on the bottom. Ans.

(*b*) $314.16 \times 17.3612 = 5{,}454.19$ lb., total pressure on top. Ans.

(1389) Use formula **168.** $h = \frac{33^2}{64.32} = 16.931$ ft. per sec. Ans.

(1390) (*a*) Use formula **178** and multiply by $7.48 \times 60 \times 60$ to reduce cubic feet per second to gallons per hour.

$$Q_a = .41 \times \tfrac{21}{12} \times \sqrt{2 \times 32.16 \times (\tfrac{15}{12})^3} \times 7.48 \times 60 \times 60 = 216{,}551 \text{ gal. per hr.} \quad \text{Ans.}$$

(*b*) By formula **179**,

$$v_m = \frac{Q_a}{bd} = \frac{.615 \times \frac{2}{3} \times \frac{21}{12} \times \sqrt{2 \times 32.16 \times (\frac{15}{12})^3}}{\frac{21}{12} \times \frac{15}{12}} = 3.676 \text{ ft. per sec.} \quad \text{Ans.}$$

(1391) $f = .0193$ for $v_m = 12$. Therefore, using formula **183**,

$$h = \frac{f l v_m^2}{5.36 d} + .0233 v_m^2 = \frac{.0193 \times 6{,}000 \times 12^2}{5.36 \times 3} + (.0233 \times 12^2) =$$
1,040.37 ft. Ans.

(1392) $\frac{5^2 \times .7854}{144}$ = area of pipe in sq. ft. Using formula **165**,

$$Q = A v = \frac{5^2 \times .7854}{144} \times 7.2 = \text{discharge in cu. ft. per sec.}$$

$\frac{5^2 \times .7854}{144} \times 7.2 \times 7.48 \times 60 \times 60 \times 24 = 634{,}478$ gal. discharged in 1 day. Ans.

(1393) 38,000 gal. per hour $= \frac{38{,}000}{60 \times 60}$ gal. per sec. $= Q$.
Using formula **185**,

$$v_m = \frac{24.51\, Q}{d^2} = \frac{24.51 \times 38{,}000}{5.5^2 \times 60 \times 60} = 8.5526 \text{ ft. per sec.} \quad \text{Ans.}$$

(1394) Use formula **178.**

(*a*) $Q_a = .41 \times b\sqrt{2 g d^3} = .41 \times \frac{27}{12} \times \sqrt{2 \times 32.16 \times (\frac{36}{12})^3} =$
38.44 ft. per sec. Ans.

(*b*) $Q = \frac{Q_a}{.615} = \frac{38.44}{.615} = 62.5$ cu. ft. per sec. **Ans.**

(1395) Use formulas **167** and **169.**

(*a*) $v = \sqrt{2 g h} = \sqrt{2 \times 32.16 \times 45} = 53.8$ ft. per sec. Ans.

(*b*) $2.304 \times 10 = 23$ ft., nearly = height of a column of water which will give a pressure of 10 lb. per sq. in. $45 + 23 = 68$ ft.

$$v = \sqrt{2 g (h_1 + h)} = \sqrt{2 \times 32.16 \times 68} = 66.153 \text{ ft. per sec.}$$
Ans.

(1396) Use formula **184.**

$Q = .0408\, d^2 v_m = .0408 \times 6^2 \times 7.5 = 11.016$ gal. per sec. Ans.

(1397) Head = 41 ÷ .434 = 94.47 ft.

Using formula **172**,

$$v = .98\sqrt{2gh} = .98\sqrt{2 \times 32.16 \times 94.47} = 76.39 \text{ ft. per sec.}$$

Ans.

(1398) Divide by 60 × 60 to get the discharge in gal. per sec., and by 7.48 to obtain the discharge in cu. ft. per sec.

$$\text{Area in sq. ft.} = \frac{4^2 \times .7854}{144} = .087267.$$

$$v_m = \frac{Q}{A} = \frac{12{,}000}{60 \times 60 \times 7.48 \times .087267} = 5.106 \text{ ft. per sec. Ans.}$$

(1399) (*a*) Area of pump-piston $= (\frac{1}{2})^2 \times .7854 = .19635$ sq. in.

Area of plunger $= 10^2 \times .7854 = 78.54$ sq. in.

Pressure per sq. in. exerted by piston $= \frac{100}{.19635}$ pounds.

Hence, according to Pascal's law, the pressure on the plunger is $\frac{100}{.19635} \times 78.54 = 40{,}000$ lb. Ans.

(*b*) According to the principle given in Art. **2181**,

$D \times 1\frac{1}{2}$ inches $= W \times$ distance moved by plunger, or $100 \times 1\frac{1}{2} = 40{,}000 \times$ required distance; hence, the required distance $= \frac{100 \times 1\frac{1}{2}}{40{,}000} = .00375$ in. Ans.

(1400) (*a*) Use formula **180**, and multiply by 7.48 and 60 to reduce the discharge from cu. ft. per sec. to gal. per min.

$$Q_a = .41\, b\sqrt{2g}\left[\sqrt{h^3} - \sqrt{h_1^3}\right] \times 60 \times 7.48 =$$

$$.41 \times \tfrac{14}{12} \times \sqrt{64.32}\left[\sqrt{(9 + \tfrac{20}{12})^3} - \sqrt{9^3}\right] \times 60 \times 7.48 =$$

13,491.22 gallons per minute. Ans.

(*b*) In the second case,

$$Q_a = .41 \times \tfrac{20}{12} \times \sqrt{64.32}\left[\sqrt{(9 + \tfrac{14}{12})^3} - \sqrt{9^3}\right] \times 60 \times 7.48 =$$

13,322.47 gallons per minute. Ans.

(1401) Area of weir = 14 × 20 ÷ 144 sq. ft. Use formula **166,** and divide by 60 × 7.48 to reduce gal. per min. to cu. ft. per sec.

(*a*) $v_m = \frac{Q}{A} = \frac{13{,}491.22 \times 144}{60 \times 7.48 \times 14 \times 20} = 15.46$ ft. per sec. Ans.

(*b*) $\frac{13{,}322.47 \times 144}{60 \times 7.48 \times 14 \times 20} = 15.264$ ft. per sec. Ans.

(1402) In Art. **2197** it is stated that the theoretical mean velocity is $\frac{2}{3}\sqrt{2gh}$. Hence,

$$v_m = \tfrac{2}{3}\sqrt{2 \times 32.16 \times 3} = 9.26 \text{ ft. per sec.} \quad \text{Ans.}$$

(1403) (*a*) 4 ft. 9 in. = 4.75 ft. 19 − 4.75 = 14.25. Using formula **170,**

$$R = \sqrt{4hy} = \sqrt{4 \times 4.75 \times 14.25} = 16.454 \text{ ft.} \quad \text{Ans.}$$

(*b*) 19 − 4.75 = 14.25 ft. Ans.

(*c*) 19 ÷ 2 = 9.5. Greatest range = $\sqrt{4 \times 9.5^2} = 19$ ft. Ans.

(1404) Use formulas **182** and **186.**

$$v_m = 2.315\sqrt{\frac{hd}{fl}} = 2.315\sqrt{\frac{25 \times 5}{.025 \times 1{,}300}} = 4.54 \text{ ft. per sec.}$$

From the table, $f = .023$ for $v_m = 4$, and .0214 for $v_m = 6$.

$\frac{.023 - .0214}{2} \times .54 = .000432$. $f = .023 - .000432 = .022568$.

$$Q = 60 \times 60 \times .09445 \times 5^2 \times \sqrt{\frac{25 \times 5}{.022568 \times 1{,}300 + .125 \times 5}} =$$

17,368.95 gal. per hr. Ans.

(1405) Obtain the values by approximating to those given in Table 45. Thus, for $v_m = 2$, $f = .0265$, and for $v_m = 3$, $f = .0243$.

Difference = .0022. .0022 × .37 = .000814, or say .0008. .0265 − .0008 = .0257 = f for $v_m = 2.37$. Ans.

.0243 − .023 = .0013. .0013 × .19 = .000247, or say .0002. .0243 − .0002 = .0241 = f for $v_m = 3.19$. Ans.

$.023 - .0214 = .0016.$ $\frac{.0016}{2} \times 1.8 = .00144$, or say .0014. $.023 - .0014 = .0216 = f$ for $v_m = 5.8$. Ans.

$.0214 - .0205 = .0009.$ $\frac{.0009}{2} \times 1.4 = .00063$, or say .0006. $.0214 - .0006 = .0208 = f$ for $v_m = 7.4$.

$.0205 - .0193 = .0012.$ $\frac{.0012}{4} \times 1.83 = .000549$, or say .0005. $.0205 - .0005 = .02 = f$ for $v_m = 9.83$. Ans.

$.0205 - .0193 = .0012.$ $\frac{.0012}{4} \times 3.5 = .00105$, or say .0011. $.0205 - .0011 = .0194 = f$ for $v_m = 11.5$. Ans.

(1406) The specific gravity of sea-water is 1.026 (see table of Specific Gravity); hence, the weight of a cubic foot of sea-water $= 1.026 \times 62.5 = 64.1$ lb.

Total area of cube $= \frac{10.5^2 \times 6}{144}$ sq. ft. 1 mile = 5,280 ft.

Hence, total pressure on cube $= \frac{10.5^2 \times 6}{144} \times 5{,}280 \times 3.5 \times 64.1 = 5{,}441{,}609.25$ lb. Ans.

(1407) (*b*) The pressure per square inch equals the weight of a volume of water 1 in. square and 12 in. high; that is, it equals

$$1 \times 1 \times .03617 \times 12 = .434 \text{ lb., nearly. Ans.}$$

(*a*) Total pressure on the bottom = area of bottom in square inches multiplied by the pressure per square inch $= 8^2 \times .7854 \times .434 = 21.82$ lb. Ans.

(1408) 8,000 gal. per hr. $= \frac{8{,}000}{60}$, or $133\frac{1}{3}$ gal. per min.

Plunger speed per min. $= 7 \times 10 = 70$ ft. Applying formula **190** and substituting,

$$d = 5.535\sqrt{\frac{G}{S}} = 5.535\sqrt{\frac{133\frac{1}{3}}{70}} = 7\tfrac{5}{8} \text{ in. Ans.}$$

(1409) See Art. **2266.**

(1410) The height to which a pump will lift water is directly proportional to the atmospheric pressure; that is, proportional to the length of the mercury column.

Letting x represent the height to which the pump will lift the water on top of the mountain, we have the proportion, 30 : 22 :: 25.5 : x, or $30x = 22 \times 25.5$; whence, $x =$ 18.7 ft. Ans.

(1411) Area of dam $= 40 \times 12 = 480$ sq. ft.

$\frac{1}{2} \times 12 = 6$ ft., depth of center of gravity below the level of the liquid.

The total pressure on the dam $= 40 \times 12 \times 6 \times 62\frac{1}{2} =$ 180,000 lb. Ans.

(1412) (*a*) Apply formula **190.**

$$d = 5.535\sqrt{\tfrac{750}{100}} = 15 \text{ inches. Ans.}$$

(*b*) Use formula **195.**

$$d_1 = .35\sqrt{G} = .35\sqrt{750} = 10 \text{ inches. Ans.}$$

(*c*) Use formula **196.**

$$d_2 = .25\sqrt{G} = .25\sqrt{750} = 7 \text{ inches. Ans.}$$

(1413) First obtain the coefficient of friction from formula **182** and Table 45.

$$v_m = 2.315\sqrt{\frac{h\,d}{f\,l}} = 2.315\sqrt{\frac{40 \times 6}{.025 \times 840}} = 7.82 \text{ ft. per sec.}$$

From the table, $f = .0214$ for $v_m = 6$ and .0205 for $v_m = 8$.

$\frac{.0214 - .0205}{2} = .00045$ decrease for an increase of velocity of 1 ft. per sec. $7.82 - 6 = 1.82$.

$.00045 \times 1.82 = .0008$, nearly, total decrease. $f = .0214 - .0008 = .0206$.

To obtain the discharge in gal. per sec., substitute this value of f in formula **186,** and multiply by 60×60 to get the discharge in gal. per hr.

$$Q = .09445\,d^2\sqrt{\frac{h\,d}{f\,l + .125\,d}} \times 60 \times 60 =$$

$$.09445 \times 6^2 \times \sqrt{\frac{40 \times 6}{.0206 \times 840 + .125 \times 6}} \times 60 \times 60 =$$

44,553.6 gal. per hr. Ans.

(1414) If the area of the tube is $\frac{1}{2}$ sq. in., and that of the cylinder 80 sq. in., a force of 80 lb. on the small piston will raise a weight of $\frac{80}{\frac{1}{2}} \times 80 = 12{,}800$ lb. on the large piston. Since the length between the hand and the fulcrum is $7\frac{1}{2}$ times the distance between the piston-rod and the fulcrum, a force of 80 lb. on the end of the lever will raise a weight of $7\frac{1}{2} \times 12{,}800 = 96{,}000$ lb. Ans.

(1415) (*a*) Using formula **190**,

$$d = 5.535\sqrt{\frac{200}{2 \times 150}} = 4\tfrac{1}{2} \text{ in.} \quad \text{Ans.}$$

(*b*) Use formula **195**.

$$d_1 = .35\sqrt{G} = .35\sqrt{200} = 5 \text{ in.} \quad \text{Ans.}$$

(*c*) Use formula **196**.

$$d_2 = .25\sqrt{G} = .25\sqrt{200} = 3\tfrac{1}{2} \text{ in.} \quad \text{Ans.}$$

(*d*) Applying formula **192** and substituting,

$$H = .00038\,G\,h = .00038 \times 200 \times 250 = 19 \text{ H. P.} \quad \text{Ans.}$$

(1416) (*a*) Since the pressure exerted by a column of water 1 foot high = .434 lb. per sq. in., the pressure exerted by a column of water 210 ft. high $= 210 \times .434 = 91.14$ lb. per sq. in. Ans.

(*b*) Applying formula **167** and substituting,

$$v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 210} = 116.22 \text{ ft. per sec.} \quad \text{Ans.}$$

(1417) To calculate the diameter of the steam-cylinder, we apply formula **194**. But we must first obtain the value of H, or the horsepower, by formula **192**. $H = .00038\,Gh$. Substituting, $H = .00038 \times \frac{27{,}000}{60} \times 240 = 41.04$ H. P. for both sides of the pump. $\frac{41.04}{2} = 20.52$ H. P. for each side. Substituting in formula **194**,

$$D = 205\sqrt{\frac{20.52}{90 \times 85}} = 10\tfrac{5}{8} \text{ inches.} \quad \text{Ans.}$$

Apply formula **190.**

27,000 gal. per hr. $= \frac{27,000}{60}$, or 450 gal. per min. for both sides. $\frac{450}{2} = 225$ gal. for one side $= G$.

$$d = 5.535\sqrt{\frac{G}{S}} = 5.535\sqrt{\frac{225}{90}} = 8\tfrac{3}{4} \text{ inches. Ans.}$$

(1418) (*a*) A column of water 1 foot high and having a cross-section of 1 sq. in. weighs .434 lb. Hence, the pressure per sq. in. at the bottom of the stand-pipe $= .434 \times 70 = 30.38$ lb. per sq. in. Ans.

(*b*) At a distance of 30 ft. from the top of the water the pressure is $.434 \times 30 = 13.02$ lb. per sq. in. Ans.

(1419) See Art. **2216.**

(1420) (*a*) Apply formula **191.**

$G = .03264\, d^2 S = .03264 \times 14^2 \times 100 = 639.744$ gal. per min. due to one side of pump. $639.744 \times 2 = 1,279.488$ gal., total discharge per minute. $1,279.488 \times 60 = 76,769.28$ gal. per hour. Ans.

(*b*) To obtain the height to which water can be raised, we apply formula **193;** but, before we can substitute in this formula, we must obtain the horsepower by applying the formula $H = \frac{PLAN}{33,000}$. Remembering that $L \times N =$ piston speed, we have

$$H = \frac{45 \times 22^2 \times .7854 \times 100}{33,000} = 51.8364 \text{ H. P.}$$

Substituting in formula **193,**

$$h = \frac{H}{.00038\, G} = \frac{51.8364}{.00038 \times 639.744} = 213.22 \text{ feet. Ans.}$$

(1421) $307 \times .434 = 133.238$ lb. per sq. in. Ans.

(1422) The time of making the stroke depends simply on the acceleration of the pit-work, which in turn depends solely on the difference between the weight of the pit-work and water column minus the frictional resistances. Now, if this difference is too great, the stroke will be made too quickly for safety and convenience, and, to obviate this, the weight of the descending pit-work must be made less or the weight of the ascending water column greater. This is accomplished by balancing the pit-work, as explained in Arts. **2247** to **2249.**

(1423) First find the value of f from Table 45.

$$f = .0243 \text{ for } v_m = 3 \text{ and } .023 \text{ for } v_m = 4.$$

Difference $= .0013$. $3.3 - 3 = .3$. Then, $.0013 \times .3 = .00039$, total decrease. $f = .0243 - .00039 = .02391$.

Substituting in formula **183,**

$$h = \frac{f l v_m^2}{5.36\, d} + .0233\, v_m^2 =$$

$$\frac{.02391 \times 2{,}000 \times (3.3)^2}{5.36 \times 2.5} + .0233 \times (3.3)^2 = 39.12 \text{ ft.} \quad \text{Ans.}$$

(1424) (*b*) 80,000 gal. per hr. $= \frac{80{,}000}{60} = 1{,}333\frac{1}{3}$ gal. per min.

To obtain the actual horsepower, apply formula **192.**

$H = .00038\, G\, h = .00038 \times 1{,}333\frac{1}{3} \times 420 = 212.8$ H. P. Ans.

(*a*) The theoretical horsepower $= \frac{2}{3} \times 212.8 = 141.87$ H. P. Ans.

(1425) Applying formula **187** and substituting,

$$D = \frac{835.5\, G\, h}{W} = \frac{835.5 \times 30{,}000 \times 290}{600} = 12{,}114{,}750 \text{ ft.-lb.}$$

Ans.

(1426) We first calculate the value of f from formula **182** and the table.

$$v_m = 2.315 \sqrt{\frac{h\, d}{f\, l}} = 2.315 \sqrt{\frac{220 \times 6}{.025 \times 6{,}500}} = 6.597 \text{ ft. per sec.}$$

From the table, $f = .0214$ for $v_m = 6$ and $.0205$ for $v_m = 8$.

$\frac{.0214 - .0205}{2} = .00045 =$ decrease for an increase of velocity of 1 ft. per sec. $.00045 \times .597$ ft. per sec. $=$ .000268, total decrease. $.0214 - .000268 = .02113 = f$.

Substituting in formula **182**,

$$v_m = 2.315\sqrt{\frac{220 \times 6}{.02113 \times 6,500}} = 7.17 \text{ ft. per sec.} \quad \text{Ans.}$$

(1427) (*a*) See Art. **2290.**

Head $= 45 \times 2.304 = 103.68$ ft. Ans.

(*b*) $2.304 \times 86 = 198.144$ ft. Ans.

(*c*) $2.304 \times 108 = 248.832$ ft. Ans.

(1428) (*b*) Applying formula **191** and substituting, $G = .03264 d^2 S = .03264 \times 15^2 \times 100 = 734.4$ gal. per min. Ans.

(*a*) To calculate the diameter of the steam-cylinder, we first obtain the horsepower from formula **192,** then substitute in formula **194.**

$$H = .00038\,G\,h = .00038 \times 734.4 \times 310 = 86 \text{ H. P.}$$

$$D = 205\sqrt{\frac{H}{PS}} = 205\sqrt{\frac{86}{50 \times 100}} = 27 \text{ in.. Ans.}$$

(1429) Applying formula **167** and substituting,

$$v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 13.7} = 29.685 \text{ ft. per sec.} \quad \text{Ans.}$$

(1430) According to Pascal's law, the pressure per square inch on each piston is the same. In order that the weights shall balance, they must be proportional to the area of the piston. Hence, we have the proportion

$$5 : 73 :: 22 : x.$$

$$x = \frac{22 \times 73}{5}, \text{ or } 321.2 \text{ lb.} \quad \text{Ans.}$$

(1431) As the lips are applied to the tube and the breath drawn in, the air in the tube above the surface of the water is drawn into the mouth, and a partial vacuum in the tube is the result of the operation. Now, as there is very little pressure on the water in the tube, and as the water outside the tube is exposed to the pressure of the atmosphere, 14.7 lb. per sq. in., the water must be forced up the tube by the greater pressure of the atmosphere. The action known as suction is, therefore, only a manifestation of atmospheric pressure.

(1432) (*a*) Applying formula **191** and substituting,
$G = .03264\,d^2 S = .03264 \times 11^2 \times 100 = 394.944$ gal. per min.

$394.944 \times 60 = 23{,}696.64$ gal. per hour. Ans.

(*c*) Use formula **192.**
$H = .00038\,G h = .00038 \times 394.944 \times 300 = 45.024$ H. P. Ans.

(*b*) Applying formula **194,**

$$D = 205\sqrt{\frac{H}{PS}} = 205\sqrt{\frac{45.024}{50 \times 100}} = 19\tfrac{1}{2} \text{ inches. Ans.}$$

(1433) Because the water helps to fill up the pores in the flat surface and the glass, thus creating a partial vacuum between the surfaces.

(1434) First finding the value of f by formula **182** and Table 45, we have

$$v_m = 2.315\sqrt{\frac{h d}{f l}} = 2.315\sqrt{\frac{15 \times 3.5}{.025 \times 88}} =$$
11.3 ft. per sec., nearly.

From the table, $f = .0205$ for $v_m = 8$ and $.0193$ for $v_m = 12$.

$\frac{.0205 - .0193}{4} = .0003$, decrease for an increase of 1 ft. per sec.

$.0003 \times 3.3 = .00099$, total decrease.

$f = .0205 - .00099 = .01951$.

To obtain the discharge in gal. per sec., substitute this value of f in formula **186.**

$$Q = .09445\,d^2\sqrt{\frac{h d}{f l + .125 d}} =$$

$$.09445 \times (3.5)^2\sqrt{\frac{15 \times 3.5}{.01951 \times 88 + .125 \times 3.5}} = 5.711 \text{ gal. per sec.}$$

$5.711 \times 60 = 342.66$ gal. per min. Ans.

(1435) (*a*) To obtain the diameter of the steam-cylinder, we calculate the horsepower from formula **192,** then substitute in formula **194.**

$$H = .00038\,G\,h = .00038 \times 300 \times 225 = 25.65 \text{ H. P.}$$

$$D = 205\sqrt{\frac{H}{PS}} = 205\sqrt{\frac{25.65}{110 \times 50}} = 14 \text{ in.} \quad \text{Ans.}$$

(*b*) Applying formula **190** and substituting,

$$d = 5.535\sqrt{\frac{G}{S}} = 5.535\sqrt{\frac{300}{110}} = 9\tfrac{1}{8} \text{ in.} \quad \text{Ans.}$$

(*c*) Assume the number of strokes per minute to be 110; then, stroke $= \frac{110}{110} = 1$ ft. $= 12$ in. Ans.

(*d*) Use formula **195.**

$$d_1 = .35\sqrt{G} = .35\sqrt{300} = 6 \text{ in.} \quad \text{Ans.}$$

(*e*) Apply formula **196.**

$$d_2 = .25\sqrt{G} = .25\sqrt{300} = 4\tfrac{1}{2} \text{ in.} \quad \text{Ans.}$$

(1436) (*a*) To obtain the theoretical discharge, apply formula **167.** $v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 15.75} = 31.83$ ft. per sec., since 15 ft. 9 in. $= 15.75$ ft.

$31.83 \times 60 = 1{,}909.8$ ft. per min.

Area of orifice $= 11.2$ sq. in. $= \frac{11.2}{144}$ sq. ft.

To calculate the theoretical quantity in cu. ft. per min., substitute in formula $Q = A\,v$.

$$Q = \frac{11.2}{144} \times 1{,}909.8 = 148.54 \text{ cu. ft. per min.} \quad \text{Ans.}$$

(*b*) Applying formula **174** and substituting,

$$Q_a = .615 \times \frac{11.2}{144}\sqrt{2 \times 32.16 \times 15.75} = 1.5224 \text{ cu. ft. per sec.}$$

$$1.5224 \times 60 = 91.344 \text{ cu. ft. per min.} \quad \text{Ans.}$$

(1437) Because if any air be left in the siphon, it will exert a pressure on the water in the arms of the siphon that will exactly balance the atmospheric pressure on the surface of the water outside, which tends to force the water up the arms.

Therefore, the water in each arm is in equilibrium, and no motion can take place. As soon, however, as the air is expelled, either by filling the siphon with water or by pumping the air out, the water is no longer in equilibrium, and will begin to flow.

(1438) Piston speed per minute $= 9 \times 5 = 45$ ft.

(*a*) Applying formula **191**,

$G = .03264 d^2 S = .03264 \times 19^2 \times 45 = 530.24$ gal. per min. Ans.

(*b*) $530.24 \times 60 = 31{,}814.4$ gal. per hr. Ans.

(1439) Applying formula **187** and substituting,

$$D = \frac{835.5 G h}{W} = \frac{835.5 \times 80{,}000 \times 340}{400} = 56{,}814{,}000 \text{ ft.-lb. Ans.}$$

(1440) See Art. **2290.**

(*a*) $2.304 \times 80 = 184.32$ ft. Ans.

(*b*) $2.304 \times 30.5 = 70.272$ ft. Ans.

(*c*) $2.304 \times 108 = 248.832$ ft. Ans.

(*d*) $2.304 \times 215 = 495.36$ ft. Ans.

(1441) Applying formula **191** and substituting,

$G = .03264 d^2 S = .03264 \times 14^2 \times 100 = 639.744$ gal. per min.

$639.744 \times 60 = 38{,}384.64$ gal. per hr., the delivery for one side.

Total delivery $= 38{,}384.64 \times 2 = 76{,}769.28$ gal. per hr. Ans.

(1442) $f = .0205$ for $v_m = 8$.

Substituting in formula **183**,

$$h = \frac{f l v_m^2}{5.36 d} + .0233 v_m^2 = \frac{.0205 \times 5{,}000 \times 8^2}{5.36 \times 4} + .0233 \times 8^2 = \frac{6{,}560}{21.44} + 1.49 = 307.46 \text{ ft. Ans.}$$

(1443) Since the area of the orifice is greater than $\frac{1}{20}$ of the area of the cross-section of the vessel, we use formula **171.**

$$v = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}} \cdot \sqrt{\frac{2 \times 32.16 \times 12}{1 - \frac{(11 \times 11)^2}{(36^2 \times .7854)^2}}} = \sqrt{\frac{771.84}{1 - .0141}} =$$

27.979 ft. per sec. Ans.

(1444) Applying formula **187** and substituting,

$$D = \frac{835.5\,G\,h}{W} = \frac{835.5 \times 4{,}000{,}000 \times 125}{7{,}460} = 55{,}998{,}660 \text{ ft.-lb.}$$

Ans.

(1445) Force available to accelerate the moving mass = $20 - (12 + 3) = 5$ tons $= F$. Weight to be accelerated $= 20 + 12 = 32$ tons $= W$. By formula **188,** acceleration $= f = \frac{gF}{W} = \frac{32.16 \times 5}{32} = 5.025$ ft. per sec.

By formula **189,** $t = \sqrt{\frac{2s}{f}} = \sqrt{\frac{2 \times 10}{5.025}} = 1.995$ sec. = time occupied in passing over 10 feet. This is at the rate of $\frac{10}{1.995} \times 60 = 300$ ft. per min.

Since the speed must not exceed 200 ft. per minute, the pit-work must be counterbalanced. Suppose a counterweight of 2 tons be tried, assuming that the frictional resistances are not increased.

Then, force $= F = 20 - (12 + 3 + 2) = 3$ tons.

Weight $= 20 + 12 + 2 = 34$ tons.

$$f = \frac{gF}{W} = \frac{32.16 \times 3}{34} = 2.84 \text{ ft. per sec.}$$

$$t = \sqrt{\frac{2s}{f}} = \sqrt{\frac{2 \times 10}{2.84}} = 2.65 \text{ sec., nearly.}$$

$$\frac{10}{2.65} \times 60 = 226.42 \text{ ft. per sec.}$$

This speed is also too great, so we will try a counterbalance of $2\frac{1}{2}$ tons.

$$\text{Force} = F = 20 - (12 + 3 + 2\tfrac{1}{2}) = 2\tfrac{1}{2} \text{ tons.}$$

$$\text{Weight} = W = 20 + 12 + 2\tfrac{1}{2} = 34\tfrac{1}{2} \text{ tons.}$$

$$f = \frac{gF}{W} = \frac{32.16 \times 2\frac{1}{2}}{34\frac{1}{2}} = 2.33 \text{ ft. per sec.}$$

$$t = \sqrt{\frac{2s}{f}}\sqrt{\frac{2 \times 10}{2.33}} = 2.93 \text{ sec., nearly.}$$

$$\frac{10}{2.93} \times 60 = 204.78 \text{ ft. per min.}$$

This is near enough for practical purposes, but if a counterweight of 2.6 tons be tried, it will reduce the acceleration so that the speed of the pit-work is almost exactly 200 ft. per min.

(1446) By the difference of cylinder volumes. The steam is admitted into the high-pressure cylinder and exhausted into the low-pressure cylinder.

(1447) See Art. **2269.**

(1448) Apply formulas **195** and **196.**

(a) $d_1 = .35\sqrt{G} = .35\sqrt{\frac{70,000}{60}} = 12$ in. Ans.

(b) $d_2 = .25\sqrt{G} = .25\sqrt{\frac{70,000}{60}} = 8\frac{1}{2}$ in. Ans.

(1449) 100,000 gal. per hr. $= \frac{100,000}{60}$ gal. per min.

Applying formula **192,**

$$H = .00038\,Gh = .00038 \times \frac{100,000}{60} \times 480 = 304 \text{ H. P. Ans.}$$

(1450) Apply formula **184.**

$$Q = .0408\,d^2 v_m = .0408 \times 7^2 \times .721 = 14.414232 \text{ gal. per sec.}$$

$14.414232 \times 60 \times 60 = 51,891.24$ gal. per hr. Ans.

(1451) See Art. **2260.**

(1452) See Art. **2271.**

(1453) See Arts. **2225, 2226, 2259, 2271,** and **2280.**

(1454) 200 ft. per min.; 400 ft. per min.; 100 ft. per min.

(1455) Applying formula **191** and substituting,

$G = .03264\, d^2\, S = .03264 \times 15^2 \times 95 =$ number of gal. per min.

$.03264 \times 15^2 \times 95 \times 60 = 41,860.8$ gal. per hr. Ans.

(1456) See Arts. **2250** to **2254.**

(1457) Applying formula **172** and substituting,

$v = .98 \sqrt{2gh} = .98 \sqrt{2 \times 32.16 \times 69.12} = 65.34$ ft. per sec. Ans.

(1458) (*a*) Area of piston $= (\frac{7}{8})^2 \times .7854 = .6013$ sq. in.

Pressure per sq. in. exerted by piston $= \frac{50}{.6013} = 83.15$ lb.

A column of water 1 foot high and of 1 sq. in. cross-section weighs .434 pound, and therefore exerts a pressure of .434 pound per sq. in. The height of a column of water to exert a pressure of 83.15 lb. per sq. in. must be $\frac{83.15}{.434} = 191.6$ feet. Consequently, the water will rise 191.6 feet.

The diameter of the hole in the squirt-gun has nothing to do with the height of the water, since the pressure per square inch will remain the same, no matter what may be the diameter.

(*b*) Using formula **170,**

$R = \sqrt{4hy} = \sqrt{4 \times 10 \times 191.6} = 87.54$ ft. Ans.

MINE HAULAGE.

(1459) See Art. **2298.**

(1460) See Art. **2299.**

(1461) See Art. **2300.**

(1462) See Art. **2301.**

(1463) See Art. **2303.**

(1464) See Art. **2304.**

(1465) See Art. **2305.**

(1466) See Arts. **2305** and **2311.**

(1467) See Art. **2306.**

(1468) See Art. **2307.**

(1469) See Art. **2308.**

(1470) See Art. **2309.**

(1471) See Art. **2310.**

(1472) See Art. **2311.**

(1473) See Art. **2312.**

(1474) See Art. **2312.**

(1475) See Art. **2312.**

(1476) See Art. **2313.**

(1477) See Art. **2314.**

(1478) As the hold or grip increases directly as the square of the number of coils, the proportion of grip the latter will have compared with the former is as $2^2 : 4^2$, or as 4 : 16; that is, the rope turned four times around the drum will have 4 times the grip or hold that the rope coiled twice around the drum has. See Art. **2315.**

(1479) It would equal two complete coils on one wheel, and the grip or haulage power would be four times that of a single coil around one wheel. See Art. **2315.**

(1480) See Arts. **2317** and **2318.**

(1481) See Art. **2319.**

(1482) See Art. **2320.**

(1483) See Art. **2321.**

(1484) See Art. **2322.**

(1485) 50 lb. See Art. **2324.**

(1486) See Art. **2324.**

(1487) See Art. **2325.**

(1488) Applying formula **197,** we have

$$.12 \times 3{,}500 + \frac{3{,}500}{40} = 507.5 \text{ lb. Ans.}$$

(1489) See Art. **2327.** By adding the weight of the empty car to that of the loaded car and dividing the sum by the coefficient of friction, or 40.

(1490) Applying formula **197,** we have $F_1 = .10 \times 4{,}200 + \frac{4{,}200}{40} = 525$ lb., the force required to move the rope. Applying formula **199,** we have $F = .10 \times (4{,}000 - 1{,}800) - \frac{4{,}000 + 1{,}800}{40} = 75$ lb., the available gravity force due to one pair of cars. Therefore, the number of cars that must run in a train is $525 \div 75 = 7$ cars. Ans.

(1491) See Art. **2328.**

(1492) The rope weighs $250 \times 1.5 = 375$ lb. Applying formula **197,** we have $F_1 = .25 \times 375 + \frac{375}{40} = 103.125$ lb., the power necessary to raise the rope. Applying formula **199**, and assuming that the loaded car is at the bottom of the jig and the balance car at the bottom of the plane, we have $F = .25\,(4{,}000 - 2{,}900) - \frac{4{,}000 + 2{,}900}{40} = 102.5$ lb., the available gravity force at the descent of the full car. Now, as it requires 103.125 lb. to move the rope, and there is only 102.5 of gravity force available, it is plain that this jig will not operate.

(1493) See Art. **2328.**

(1494) See Art. **2329.**

(1495) See Art. **2329.**

$$\frac{(2 \times 1{,}800) + 1{,}200}{40} + .25\,(2 \times 1{,}800 + 1{,}200) = 1{,}320 \text{ lb.}$$

Ans.

(1496) See Arts. **2330** and **2331.**

(1497) See Arts. **2332** and **2333.**

(1498) See Art. **2337.**

(1499) See Art. **2339.**

(1500) See Art. **2340.**

(1501) See Art. **2342.**

(1502) See Art. **2343.**

(1503) See Art. **2343.**

(1504) See Art. **2344.**

(1505) See Art. **2345.**

(1506) See Art. **2346.**

(1507) See Art. **2347.**

(1508) Applying formula **200,**

$$T = \frac{(18 \times 4{,}000) + (5{,}000 \times .88)}{40} + .05\,(18 \times 4{,}000 + 5{,}000 \times .88) =$$

5,730 lb. Ans.

(1509) The velocity of the train is $\frac{10 \times 5,280}{60} = 880$ ft. per min. $880 \times 5,730 = 5,042,400$ = foot-pounds of work per minute required of the engine. $5,042,400 \div 33,000 = 152.8$ H. P. Ans.

(1510) See Art. **2349.**

(1511) See Art. **2350.**

(1512) See Art. **2351.**

(1513) The weight of the rope $= 3,000 \times 2 \times .88 = 5,280$ lb. The weight of 25 empty cars weighing 1,500 lb. each $= 1,500 \times 25 = 37,500$ lb.; therefore, the resistance due to friction $= \frac{5,280 + 37,500}{40} = 1,069.5$ lb. The resistance due to gravity $= 37,500 \times .05 = 1,875$ lb. Then, $1,069.5 + 1,875 = 2,944.5$ lb., the tension on the rope. 12 miles per hour $= \frac{5,280 \times 12}{60} = 1,056$ ft. per minute.

$$\frac{1,056 \times 2,944.5}{33,000} = 94.2 \text{ H. P.} \quad \text{Ans.}$$

Or, by formula **201**, the tension can be found as follows:

$$T = \frac{37,500 + 5,280}{40} + .05 \times 37,500 = 2,944.5 \text{ lb.}$$

The H. P. can be found by formula **202**, as follows:

$$H = \frac{2,944.5 \times 1,056}{33,000} = 94.2 \text{ H. P.} \quad \text{Ans.}$$

(1514) See Art. **2354.**

(1515) See Art. **2355.**

(1516) See Art. **2356.**

(1517) See Arts. **2357** and **2358.**

(1518) See Art. **2358.**

(1519) See Art. **2359.**

(1520) See Art. **2360.**

(1521) See Art. **2365.**

(1522) As the roads are level, there is no tension due to grade, and formula **201** becomes simply $T = \frac{W + w}{40} = \frac{90{,}000 + 9{,}100}{40} = 2{,}477.5$ lb., the tension in the main rope. Ans.

To find the tension in the tail-rope, the weight of the train of empty cars is found.

Then, $T_1 = \frac{30{,}000 + 9{,}100}{40} = 977.5$ lb. Ans.

As the conditions of the problem require the maximum tension on the rope, we take that on the main rope, or 2,477.5 lb., and applying formula **202,**

$$H = \frac{2{,}477.5 \times \frac{(10 \times 5{,}280)}{60}}{33{,}000} = \frac{2{,}180{,}200}{33{,}000} = 66.1 \text{ H. P.} \quad \text{Ans.}$$

(1523) Applying formula **201,** we have

$$T = \frac{90{,}000 + 12{,}480}{40} + .03 \times 90{,}000 = 5{,}262 \text{ lb.} \quad \text{Ans.}$$

(1524) Applying formula **203,** we have

$$P_1 = \frac{90\,(6{,}000 + 60)}{6{,}000} = 90.9 \text{ H. P.}$$

(1525) As the gravity force due to the pitch of the incline reduces the tension on the main rope, it must be treated negatively. Then, formula **201** becomes

$T = \frac{W + w}{40} - a \times W = \frac{100{,}000 + 7{,}040}{40} - .04 \times 100{,}000 = -1{,}324$ lb., or the negative tension on the main rope. Ans.

This means that there is not only no tension on the main rope, but an excess of gravity force equal to 1,324 lb. The gravity force in the case of hauling the train of empty cars is positive, and can be found by use of formula **201.**

$$T = \frac{40{,}000 + 7{,}040}{40} + .04 \times 40{,}000 = 2{,}776 \text{ lb.,}$$

the tension of the tail-rope. Ans.

No horsepower is exerted on the main rope, because, as shown previously, the tension is negative. By using formula **202,** the horsepower exerted over the tail-rope is

$$H = \frac{2,776 \times \dfrac{(5,280 \times 11)}{60}}{33,000} = 81.4 \text{ H. P. Ans.}$$

(1526) By the use of formula **204,** we have $T = \frac{5,000 \times 20 + 4,000\,(3.65 + .6)}{40} - .04 \times (100,000 - 12,200) = (2,925 - 3,512) = -587$ lb., the negative tension. For the tension in the tail-rope, formula **205** is used. $T_1 = \frac{40,000 + 4,000\,(3.65 + 6)}{40} + .04\,(40,000 - 12,200) = 1,425 + 1,112 = 2,537$ lb., the tension on the tail-rope. Now, as the tension on the main rope is negative, there is no power applied to it; on the tail-rope, however, in which there is a tension of 2,537 lb. with the trains running 11 miles per hour, we have $\text{H.P.} = \frac{2,537 \times \dfrac{(5,280 \times 11)}{60}}{33,000} = 74.4$ H.P. Ans.

(1527) See Art. **2372.**

(1528) See Art. **2372.**

(1529) See Art. **2373.**

(1530) $6,000 + 4,800 + 2,500 + 7,000 + 3,000 = 23,300$; $\frac{23,300}{5} = 4,660$, the mean length. $\frac{5,280 \times 12 \times 10}{4,660 \times 3} = 45.3$, practically 46 trains. Ans.

(1531) $\frac{2,500}{46 \times 2.5} = 21.7$, say 22 cars. Ans.

(1532) Allowing $\frac{1}{3}$ of the time for stoppage, the rope travels for $\frac{2}{3}$ of $10 = 6\frac{2}{3}$ hours, and hauls coal for but $\frac{1}{2}$ this time, or $3\frac{1}{3}$ hours. Hence, the distance the rope travels while hauling coal is $5,280 \times 11 \times 3\frac{1}{3} = 193,600$ feet, and since the mean length of the haulage roads, which is found by dividing their total length by 4, is

$$\frac{4,250 + 3,012 + 756 + 514}{4} = 2,133 \text{ feet},$$

the number of loaded trains is

$$\frac{193,600}{2,133} = 90.76, \text{ or } 91. \quad \text{Ans.}$$

The number of cars in each train is

$$\frac{2,500}{91 \times 2.5} = 11. \quad \text{Ans.}$$

The weight of the rope is equal to its weight per foot multiplied by twice the maximum haul, or $4,250 \times 2 \times 1.5 = 12,750$ pounds, and the weight of a loaded car is $2,000 + 2.5 \times 2,000 = 7,000$ pounds.

Substituting in formula **201**,

$$T = \frac{7,000 \times 11 + 12,750}{40} + .03 \times 77,000 = 4,553.75 \text{ pounds.}$$

The speed of the train is equal to

$$\frac{5,280 \times 11}{60} = 968 \text{ feet per minute.}$$

Applying formula **202**,

$$H = \frac{4,553.75 \times 968}{33,000} = 133.6 \text{ horsepower, nearly.} \quad \text{Ans.}$$

(1533) See Art. **2374.**

(1534) See Art. **2375.**

(1535) See Art. **2375.**

(1536) See Art. **2376.**

(1537) See Art. **2377.**

(1538) See Art. **2377.**

(1539) See Art. **2377.**

(1540) See Art. **2378.**

(1541) See Art. **2379.**

(1542) See Art. **2379.**

(1543) See Art. **2380.**

(1544) See Arts. **2384** and **2385.**

(1545) See Arts. **2384** and **2386.**

(1546) See Art. **2382.**

(1547) See Art. **2382.**

(1548) See Art. **2387.**

(1549) See Art. **2387.**

(1550) See Arts. **2387** and **2388.**

(1551) See Art. **2388.**

(1552) See Arts. **2390** to **2392.**

(1553) See Art. **2395.**

(1554) See Art. **2395.**

(1555) See Arts. **2397** and **2398.**

(1556) See Art. **2401.**

(1557) By formula **207,** the number of cars on the rope is $\frac{2,500 \times 5,230}{2 \times 5,280 \times 8 \times 1.5} = 103.18$, say 103. Ans.

And, by formula **208,** the distance the cars are apart is $\frac{5,230}{103.18} = 50.68$ ft. Ans.

(1558) See Art. **2406.**

(1559) See Art. **2407.**

(1560) See Arts. **2408** and **2409.**

(1561) See Art. **2411.**

(1562) By formula **207,** the number of loaded cars on the rope at one time is $\frac{976 \times 4,720}{2.5 \times 5,280 \times 10 \times 1.25} = 27.919$. The weight of the loaded cars on one side of the rope will then be $4,000 \times 27.919 = 111,676$ pounds. Taking the weight of an empty car at 1,200 pounds, the weight of the empty cars on the rope will be $1,200 \times 27.919 = 33,502.8$. The weight of the rope is $4,720 \times 2 \times 3 = 28,320$ pounds. Then, substituting in formula **210**, $T = \frac{(111,676 + 33,502.8 + 28,320)}{40} =$ 4,337.47 pounds, the tension on the rope. Ans.

A velocity of $2\frac{1}{2}$ miles an hour is equal to $\frac{2.5 \times 5,280}{60} =$ 220 feet per minute. Using formula **202**, the horsepower is

$$H = \frac{4,337.47 \times 220}{33,000} = 28.92 \text{ H. P.} \quad \text{Ans.}$$

(1563) As the two sides of the rope balance each other and the cars balance each other, only the weight of the coal is subject to the gravity of the grade. Substituting in formula **210**, we have

$$T = \frac{(111,676+33,502.8+28,320)}{40} + .025(111,676-33,502.8) =$$

6,291.8 pounds tension on the rope. Ans.

The velocity is $\frac{2.5 \times 5,280}{60} = 220$ feet per minute. The horsepower is, therefore, $\frac{6,291.8 \times 220}{33,000} = 41.9$ horsepower. Ans.

(1564) Substituting in formula **210**, we have

$$T = \frac{(111,676+33,502.8+28,320)}{40} - .025(111,676-33,502.8) =$$

2,383.14 lb., the tension on the rope. Ans.

The horsepower required is $\frac{2,383.14 \times 220}{33,000} = 15.89$ horsepower. Ans.

(1565) See Art. **2414.**

(1566) See Arts. **2414** and **2415.**

(1567) See Art. **2418.**

(1568) See Art. **2420.**

(1569) See Art. **2422.**

(1570) See Arts. **2428** to **2430.**

(1571) See Art. **2431.**

(1572) See Arts. **2432** and **2433.**

(1573) See Art. **2435.**

(1574) See Art. **2437.**

(1575) See Art. **2439.**

(1576) See Art. **2440.**

(1577) See Art. **2443.**

(1578) See Art. **2444.**

(1579) See Art. **2446.**

(1580) See Art. **2446.**

(1581) See Art. **2449.**

(1582) See Art. **2450.**

(1583) See Art. **2450.**

(1584) See Art. **2316.**

HOISTING AND HOISTING APPLIANCES.

(1585) See Art. **2452.**

(1586) Electric motors and steam or compressed-air engines.

(1587) See Art. **2463.**

(1588) See Arts. **2454** and **2455.**

(1589) See Arts. **2458** and **2459.**

(1590) See Art. **2460.**

(1591) See Art. **2462.**

(1592) See Art. **2464.**

(1593) See Art. **2464.**

(1594) See Arts. **2465** to **2469.**

(1595) See Art. **2465.**

(1596) See Art. **2467.**

(1597) See Arts. **2466** and **2467.**

(1598) See Art. **2468.**

(1599) See Art. **2469.**

(1600) See Art. **2470.**

(1601) See Art. **2470.**

(1602) See Art. **2471.**

(1603) The minimum diameter of drum is 60 times the diameter of the rope.

(1604) (*a*), (*b*), and (*c*) See Art. **2471.**
(*d*) See Art. **2473.**

(1605) (*a*) Assume the weight of the rope to be 2,000 lb. Then, the load on the rope is

Material........................	3,000 lb.
Car	1,800 lb.
Cage...........................	2,400 lb.
Rope...........................	2,000 lb.
Total	9,200 lb.

Using a factor of safety of 10, the breaking load is 92,000 lb. = 46 tons. Referring to Table 46, a 1-inch plow-steel rope with 19 wires to the strand has a breaking load of 47 tons; its weight is 1.58 lb. per ft. In this case, the weight is 1,200 × 1.58 = 1,896 lb., which is quite close to the assumed weight. Therefore, a 1-inch rope should be used. Ans.

(*b*) The smallest allowable drum has a diameter 60 times that of the rope, or 60 in. = 5 ft. Ans.

(1606) Using two cages, the gross load is

Material........................	3,000 lb.
2 cars	3,600 lb.
2 cages.........................	4,800 lb.
Rope...........................	1,896 lb.
Total	13,296 lb.

The net load is

Material........................	3,000 lb.
Rope...........................	1,896 lb.
Total	4,896 lb.

Actual load = net load + 10% of gross load = 4,896 + .10 × 13,296 = 6,225.6 lb. Ans.

(1607) See Art. **2474.**

(1608) The working diameter of the drum is 60 + 1 = 61 in. = $\frac{61}{12}$ ft.

$$6{,}225.6 \times \tfrac{61}{12} \times 3.1416 = 99{,}421.6 \text{ ft.-lb.} \quad \text{Ans.}$$

(1609) Using formula **211**,

$$D = 1.97\sqrt[3]{\frac{99,421.6}{48.76 \times 1.5}} = 21.82, \text{ say } 22 \text{ in. Ans.}$$

Stroke $= 22 \times 1.5 = 33$ in. Ans.

(1610) Area of piston $= 12^2 \times .7854 = 113.1$ sq. in. The piston travels per revolution $\frac{24}{12} \times 2 = 4$ ft. Total pressure on piston $= 113.1 \times 40$. Work = total pressure × distance traveled by piston $= 113.1 \times 40 \times 4 = 18,096$ ft.-lb. Ans.

(1611) See Arts. **2471** and **2472**.

(1612) (*a*) Using formula **211**,

$$D = 1.97\sqrt[3]{\frac{36,000}{40 \times 2.5}} = 14 \text{ in. Ans.}$$

Stroke $= 14 \times 2.5 = 35$ in. Ans.

(*b*) Area of piston $= 14^2 \times .7854 = 153.938$ sq. in.
Length of crank $= \frac{35}{2} = 17\frac{1}{2}$ in.
Turning moment = total pressure on piston × length of crank $= 153.938 \times 40 \times 17\frac{1}{2} = 107,757$ in.-lb. Ans.

(1613) See Art. **2476**.

(1614) It may be smaller. See Art. **2477**.

(1615) Larger. See Art. **2478**.

(1616) See Art. **2480**.

(1617) See Art. **2482**.

(1618) See Art. **2484**.

(1619) Least diameter of drum $= 1\frac{1}{2} \times 60 = 90$ in.
Effective diameter $= 90 + 1\frac{1}{2} = 91\frac{1}{2}$ in. $= 7\frac{5}{8}$ ft.
Circumference $= 7\frac{5}{8} \times 3.1416 = 24$ ft., nearly.

Number of turns $= \frac{1,800}{24} = 75$.

Adding 5 turns for friction and for possible overwinding, the number of turns is 80.
Width for each turn $= 1\frac{1}{2} + \frac{1}{4} = 1\frac{3}{4}$ in.

$$80 \times 1\frac{3}{4} = 140 \text{ in.} = 11 \text{ ft. } 8 \text{ in. Ans.}$$

(1620) See Art. **2488.**

(1621) See Arts. **2488** and **2491.**

(1622) See Arts. **2485** and **2526.**

(1623) Assume, first, that the rope weighs 2,000 lb. Then, the load on the rope is

Material	4,000 lb.
Car	3,000 lb.
Cage	3,200 lb.
Rope	2,000 lb.
Total	12,200 lb.

Using a factor of safety of 10, the breaking load is 61 tons, which, from Table 46, requires a $1\frac{3}{8}$-inch rope, the weight of which is 3 lb. per foot. The weight of rope is $3 \times 800 = 2,400$ lb., which adds 400 lb. to the previous total weight, making it 12,600 lb., or 6.3 tons. The breaking load is, therefore, 63 tons, and the $1\frac{3}{8}$-inch rope is correct. The minimum diameter is $1\frac{3}{8} \times 60 = 82\frac{1}{2}$ in. $= 7$ ft., nearly. Ans.

Using formula **213,**

$$D = \frac{7\,(4,000 + 2 \times 6,200 + 2 \times 2,400)}{4,000 + 2 \times 6,200} = 9.05 \text{ ft., say 9 ft.}$$ Ans.

(1624) See Art. **2492.**

(1625) See Arts. **2498** to **2503.**

(1626) See Art. **2505.**

(1627) See Arts. **2507** and **2510.**

(1628) See Art. **2511.**

(1629) See Art. **2512.**

(1630) See Art. **2516.**

(1631) See Art. **2514.**

(1632) See Art. **2516.**

(1633) See Art. **2516.**

(1634) See Arts. **2518** and **2520.**

(1635) See Art. **2497.**

(1636) See Art. **2529.**

(1637) See Arts. **2530** and **2531.**

(1638) See Art. **2532.**

(1639) See Art. **2535.**

(1640) See Art. **2535.**

(1641) See Art. **2537.**

(1642) See Arts. **2538** and **2539.**

(1643) See Art. **2540.**

(1644) See Art. **2543.**

(1645) See Art. **2546.**

(1646) See Art. **2547.**

(1647) See Art. **2549.**

(1648) See Art. **2549.**

(1649) See Art. **2559.**

(1650) See Art. **2560.**

(1651) See Arts. **2563** and **2564.**

SURFACE ARRANGEMENTS OF BITUMINOUS MINES.

(1652) By limiting the size of the opening through which the cars are hauled Small seams necessitate the use of low cars, and a bad roof necessitates narrow headings, and, consequently, comparatively narrow cars. See Art. **2568.**

(1653) See Art. **2569** and Figs. 947 and 948.

(1654) See Arts. **2572** to **2575** and Fig. 947.

(1655) See Arts. **2576** to **2579** and Fig. 948.

(1656) See Art. **2611.**

(1657) See Art. **2583.**

(1658) From 1 to 1.3 times the vertical height of the center of the sheaves above the center of the drum, or from 60 to 78 feet. See Art. **2586.**

(1659) See Art. **2589.**

(1660) See Art. **2589.**

(1661) See Art. **2596.**

(1662) See Art. **2597.**

(1663) See Art. **2600.**

(1664) See Art. **2601.**

(1665) See Figs. 947 and 948 and Art. **2602.**

(1666) See Art. **2606.**

(1667) See Art. **2607.**

(1668) See Art. **2610.**

(1669) See Arts. **2613** and **2620.**

(1670) See Art. **2614.**

(1671) See Art. **2616.**

(1672) See Arts. **2618** and **2619.**

(1673) See Art. **2622.**

(1674) $\frac{8}{2} = 4$ trips for each car per day. $4 \times 2 = 8$ tons for each car per day. Hence, $\frac{1,500}{8} = 187.5$, or 188 cars. Ans.

(1675) See Figs. 963 and 964 and accompanying description.

(1676) See Art. **2625.**

(1677) See Art. **2630.**

(1678) See Arts. **2630** and **2640.**

(1679) See Figs. 965 and 966 and accompanying descriptions.

(1680) See Art. **2631.**

(1681) See Art. **2631.**

(1682) See Art. **2634.**

(1683) See Art. **2634.**

(1684) See Art. **2634.**

(1685) See Art. **2638.**

(1686) See Arts. **2639** and **2640.**

(1687) See Art. **2644.**

(1688) See Art. **2647.**

(1689) See Art. **2649.**

(1690) See Art. **2650.**

(1691) See Art. **2654.**

(1692) See Art. **2654.**

(1693) See Art. **2660.**

(1694) See Art. **2657.**

(1695) See Art. **2655.**

(1696) See Art. **2655.**

(1697) See Art. **2669.**

(1698) See Art. **2670.**

(1699) See Art. **2663.**

(1700) See Art. **2664.**

(1701) See Art. **2664.**

(1702) See Art. **2664.**

(1703) See Art. **2665.**

(1704) See Art. **2665.**

(1705) See Art. **2665.**

(1706) See Art. **2666.**

(1707) See Art. **2667.**

(1708) See Art. **2669.**

(1709) See Art. **2672.**

		Tons.
(1710)	Output of lump coal $= 1{,}500 \times .70 =$	1,050
	Output of nut coal $= 1{,}500 \times .15 =$	225
	Output of pea coal $= 1{,}500 \times .08 =$	120
	Output of slack coal $= 1{,}500 \times .07 =$	105

The lengths of sidings are:

$$\left.\begin{array}{ll} \text{Lump,} & \dfrac{1{,}050 \times 34}{30} = 1{,}190 \text{ ft.} \\ \text{Nut,} & \dfrac{225 \times 34}{30} = 255 \text{ ft.} \\ \text{Pea,} & \dfrac{120 \times 34}{30} = 136 \text{ ft.} \\ \text{Slack,} & \dfrac{105 \times 34}{30} = 119 \text{ ft.} \end{array}\right\} \text{Ans.}$$

See Art. **2673.**

(1711) See Art. **2674.**

(1712) See Arts. **2675** and **2676.**

(1713) See Art. **2675.**

(1714) See Art. **2676.**

(1715) 8 hr. $= 8 \times 60 = 480$ min.
$480 \div 20 = 24$ trips.
$1{,}500 \div 24 = 62$, the number of tons hauled per trip.
$62 \div 2 = 31$, number of cars per smallest trip.
$31 \times 8 = 248$ ft., smallest length of siding.
248 ft. $\times 2 = 496$ ft., proper length. Ans. See Art. **2677.**

(1716) See Art. **2684.**

(1717) See Arts. **2695** to **2698.**

SURFACE ARRANGEMENTS
OF
ANTHRACITE MINES.

(1718) A drift is driven in the coal seam, while a tunnel is driven across the measures. See Arts. **2720** and **2721.**

(1719) See Art. **2747.**

(1720) See Art. **2797.**

(1721) See Art. **2862.**

(1722) See Art. **2802.**

(1723) See Arts. **2738** and **2739.**

(1724) (*a*) Since the inclination of the dump chute is $3\frac{1}{2}$ inches per foot, or $\frac{7}{24}$, the tower end is $\frac{7}{24} \times 200 = 58\frac{1}{3}$ feet higher than the breaker end. Therefore, the tower end of the chute is $90 + 58\frac{1}{3} = 148\frac{1}{3}$ feet above the wall or the breaker, or $148\frac{1}{3} - 12 = 136.33$ feet above the wall of tower. Ans.

(*b*) The length of the chute is the length of the hypotenuse of a right-angled triangle of which the base is 200 feet and the altitude is the rise, $58\frac{1}{3}$ feet. The length is, therefore, $\sqrt{200^2 + 58\frac{1}{3}^2} = 208.33$ ft. Ans.

(1725) See Arts. **2731** and **2904.**

(1726) See Art. **2724.**

(1727) See Arts. **2778** to **2792.**

(1728) See Art. **2858.**

(1729) See Arts. **2793** and **2794.**

(1730) See Art. **2826.**

(1731) See Art. **2868.**

(1732) See Art. **2740.**

(1733) See Art. **2893.**

(1734) See Art. **2743.**

(1735) See Art. **2806.**

(1736) See Art. **2875.**

(1737) See Art. **2731.**

(1738) See Art. **2877.**

(1739) See Art. **2803.**

(1740) See Art. **2729.**

(1741) At right angles. See Art. **2895.**

(1742) See Art. **2745.**

(1743) See Art. **2803.**

(1744) See Art. **2867.**

(1745) See Art. **2840.**

(1746) See Art. **2749.**

(1747) See Art. **2800.**

(1748) See Art. **2823.**

(1749) See Arts. **2866** to **2874.**

(1750) See Art. **2750.**

(1751) See Art. **2799.**

(1752) See Art. **2835.**

(1753) See Art. **2731.**

(1754) See Art. **2842.**

(1755) See Art. **2770.**

(1756) See Art. **2896.**

(1757) See Art. **2735.**

(1758) See Art. **2758.**

(1759) See Art. **2833.**

(1760) See Arts. **2829** and **2924.**

(1761) See Art. **2830.**

(1762) See Art. **2856.**

(1763) See Art. **2771.**

(1764) See Art. **2889.**

(1765) See Art. **2854.**

(1766) See Art. **2925.**

(1767) See Art. **2817.**

(1768) See Art. **2764.**

(1769) See Art. **2822.**

(1770) See Art. **2828.**

(1771) See Art. **2924.**

(1772) See Art. **2763.**

(1773) The Guibal. See Art. **2766.**

(1774) See Art. **2829.**

(1775) See Art. **2767.**

(1776) See Art. **2866.**

(1777) The description is given in Art. **2783**

(1778) See Art. **2723.**

(1779) See Art. **2866.**

(1780) See Art. **2851.**

(1781) See Art. **2762.**

(1782) See Art. **2782.**

(1783) See Art. **2883.**

(1784) See Art. **2773.**

(1785) See Art. **2849.**

(1786) See Art. **2830.**

(1787) See Art. **2884.**

(1788) See Arts. **2778** and **2792.**

(1789) See Art. **2846.**

(1790) See Art. **2857.**

(1791) See Arts. **2831** and **2896.**