ALTERNATING-CURRENT ELECTRICITY

AND

ITS APPLICATIONS TO INDUSTRY

FIRST COURSE

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PREFACE

THIS book is planned to meet the need for an elementary text on alternating-current electricity, simple enough to be readily understood by beginners, and, at the same time, complete enough to be a proper foundation for engineering practice or for later advanced study.

The authors have tried to present a thorough treatment of the general principles, everywhere closely associating the principles with the machines, the methods and the facts of everyday practice. It is believed that information thus arranged becomes immediately usable, and that, in addition, the student receives the suggestion and incentive for further study on his own initiative. Throughout the text, the steps by which new ideas are introduced have been made small. The increased speed and certainty with which the students advance under this plan has been found, on careful trial, to more than compensate for the longer text made necessary.

In order that a proper background for the study of alternating-current phenomena may be provided, the authors introduced in Chapter I a brief general discussion of the ways in which alternating current and alternating-current machinery are commonly used and the place that they occupy in the larger field of power generation and distribution. It is our belief that the perspective view thus obtained by the student will greatly aid him in all subsequent chapters of the book.

Theoretical demonstrations and discussions are preceded in every instance by practical explanations and common, everyday analogies which must have come within the student's previous knowledge and experience. The fundamental ideas concerning current, e.m.f., phase relations, power and power-factor are brought most prominently to the student's attention, since these ideas are of the first importance in dealing with alternating current; but the authors have purposely postponed the discussion of inductance and capacity until their physical significance and their practical applications have been explained. Armature windings are treated with much greater detail than is usual in textbooks of this kind in the belief that a thorough understanding of that part of the machine in which the e.m.f. is generated is necessary to a clear understanding of the meaning of phase difference, vector summation and polyphase relations. The main object has been to teach, not armature winding, but polyphase alternating currents.

Hydraulic analogies have been used freely for the purpose of giving the student qualitative and physical conceptions rather than a quantitative or mathematical appreciation of the principles under discussion, as the authors have found in their teaching experience that such analogies are more convincing to most students than rigid, abstract demonstrations. For a similar reason, also, many pictures and simple diagrams, each provided with a full and carefully descriptive legend, have been inserted throughout the text. Practical problems have been introduced generously and nearly 1500 will be found distributed throughout and at the ends of the different chapters. These have been carefully planned both to test the student's real knowledge of the subject matter presented in the preceding chapters and to give him valuable engineering data in a form that is convenient and easily remembered.

The text consists of two courses. While there is no break in the continuity between the "First Course" and the "Second Course," each course is, nevertheless, complete in itself and can be comfortably finished in one semester.

The "First Course" includes the general introduction, a description and study of the fundamental principles, and the application of each principle to all of the important departments of electrical engineering practice. The purpose of this course is to acquaint the student with the larger facts of alternating-current phenomena and with the nature of the quantities involved rather than to provide him with an exhaustive study of the appliances.

The "Second Course" explains in greater detail matters relating to the construction and the characteristics of operation of the various common types of alternating-current machines and appliances. Each of these machines and appliances is studied in its relation to all of the principles that have been brought out in the earlier course.

The above arrangement of material, together with the shortening of the steps by which new ideas are introduced, the constant practical applications of principles, the frequent analogies and the elimination of abstract and complicated mathematical demonstrations have been found in the authors' experience to so simplify the subject as to make the study of alternating current entirely practicable much earlier than is usual in technical schools and college courses. This is most important, for alternating current is a subject which requires ample time for thorough assimilation. When alternating-current theory is begun too late, no time remains for subsequent study or for experimental courses dependent upon it.

It is presupposed that the student has studied the equivalent of the first nine chapters of Timbie's "Elements of Electricity," and that he has access to a good laboratory.

In conclusion, the authors wish to express their appreciation and thanks to Mr. Arthur L. Williston, Principal of Wentworth Institute, for inspiration and teaching philosophy, for generous assistance in the preparation of the text, and especially for bringing the first chapter and the introduction into their present form. Also they wish to express to Mr. J. M. Jameson, Vice-President of Girard College, appreciation and thanks for efficient and most valuable editing.

Grateful acknowledgment, too, is extended to Mr. Clifford W. Bates, Sheffield Scientific School, for valuable criticism and suggestions regarding the text and for solutions of the problems.

W. H. T.
H. H. H.

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Alternating-current Electricity
and
Its Applications to Industry
FIRST COURSE

CHAPTER I

MODERN SYSTEMS OF POWER TRANSMISSION

It is the purpose of this chapter to give the reader a bird's-eye view of the particular place occupied by Alternating-current Electricity, and also that occupied by electrical power in general, in the large problem of the generation and distribution of power.

1. Sources of Power. Electricity is not one of the natural sources of power; commercially it is nearly always generated in one way or other from some kind of mechanical power. The chief original sources of power are: first, **coal** which is mined from the earth; second, **oil** and **gas** which flow from driven wells; third, **water** flowing in natural or artificial waterways.

Energy is obtained from fuel, that is, from coal, or oil, or gas, in one of two ways. It may be obtained by burning the fuel under steam boilers, or by internal combustion inside the cylinders of oil engines or gas engines. In both cases heat is generated, and this heat must be transformed into mechanical energy before electricity can be obtained upon a commercial scale.

Heat generated underneath a boiler may be used to make steam under pressure, and this steam passed through the valves of a steam engine or the nozzles of a steam turbine may be made to move a piston or revolve a shaft. Thus heat energy may be transformed into mechanical energy capable of operating the machinery of a mill, or of running an engine, or revolving the armature of an electrical generator. Fig. 1a shows the coal-handling device for a large power plant; Fig. 1b, the belt conveyor that delivers the coal to the hoppers which feed it to the grates beneath the boilers by means of an automatic stoker shown in Fig. 1c.

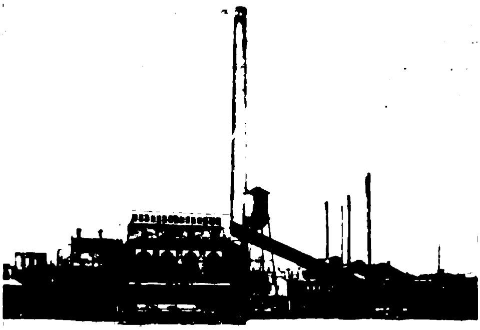


FIG. 1a. Coal-handling device of a modern central power plant. *The Electrical World.*

The steam generated in these boilers drives the four turbines of Fig. 1d, the combined power of which is nearly 20,000 kw. Again, when oil, natural gas or artificial gas manufactured from coal is burned inside the cylinder of a gas engine, heat is generated. The heat expands the gases which result from the combustion, and enables them to push forward the piston of the engine. In this instance, too, heat energy is transformed into mechanical energy and made capable of rotating any shaft to which the engine piston is attached. Fig. 2 shows a 250-kv-a. engine for using natural gas.

In recent years in the United States there have been a large number of very important power installations in which electricity is generated by water-power. The water, in falling from a higher level to a lower, is caused to come

FIG. 1b. A coal conveyor for delivering coal to the hoppers above the boilers. *The Electrical World.*

at a high velocity against the blades of a turbine or water wheel and thus revolve it upon its shaft. The energy in water stored at the higher level may thus be transformed

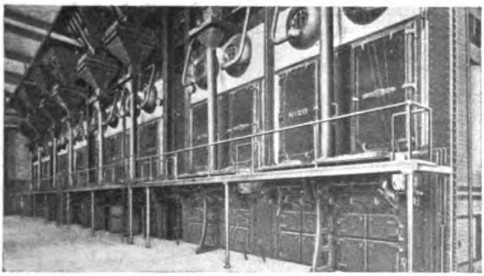


FIG. 1c. A battery of boilers. Note that the coal is fed to the grates by automatic stokers. *Babcock & Wilcox Co.*

into the mechanical energy of a revolving shaft; and this may be used to generate electricity or to operate any kind of machinery. In Fig. 3a is seen the Gatun dam of the Panama Canal Zone; in Fig. 3b, the penstocks which carry the water from Lake Gatun to the 3600-h. p. turbines shown in Fig. 3c, which supply the electric power for the whole Canal Zone.

Steam engines, gas engines and water wheels, when used to convert natural power into mechanical energy, are for convenience sometimes called **prime movers,** when we wish to speak of them collectively under a single term.

2. The Need for Central Power Stations. It very frequently happens that the work that prime movers are intended to perform is scattered over a wide area. In many instances the amount of power that is required for use at any one place is small. To illustrate: In a factory having several buildings scattered over a large tract of land, we find that power is used for incandescent lamps to light each floor of each one of the buildings; for desk fans in offices located in several buildings; and for arc lamps to light the yard. It is also used for operating machine tools in the repair shop in one of the buildings; and for various manufacturing purposes in many other parts of the plant. Each incandescent lamp needs but $\frac{1}{15}$ horse power, each desk fan needs but $\frac{1}{10}$ horse power and each arc lamp needs but about $\frac{2}{3}$ horse power. Each machine tool in the repair shop requires from about $\frac{1}{4}$ horse power to perhaps 3 horse power, and the whole repair shop may require only about 20 or 25 horse power. Large quantities of power are required for manufacturing purposes, but the machines to be driven are located on the several floors of the different buildings.

Under such circumstances it would be neither practicable nor economical to install an engine or a water wheel at each place where power is needed. The only practical plan is to generate in one "**Power Station**" all the power that is needed for the entire plant, and then to distribute this power in some way to the places where it is to be used. In this power station, centrally located if possible, are installed all the prime movers that are necessary. The next problem

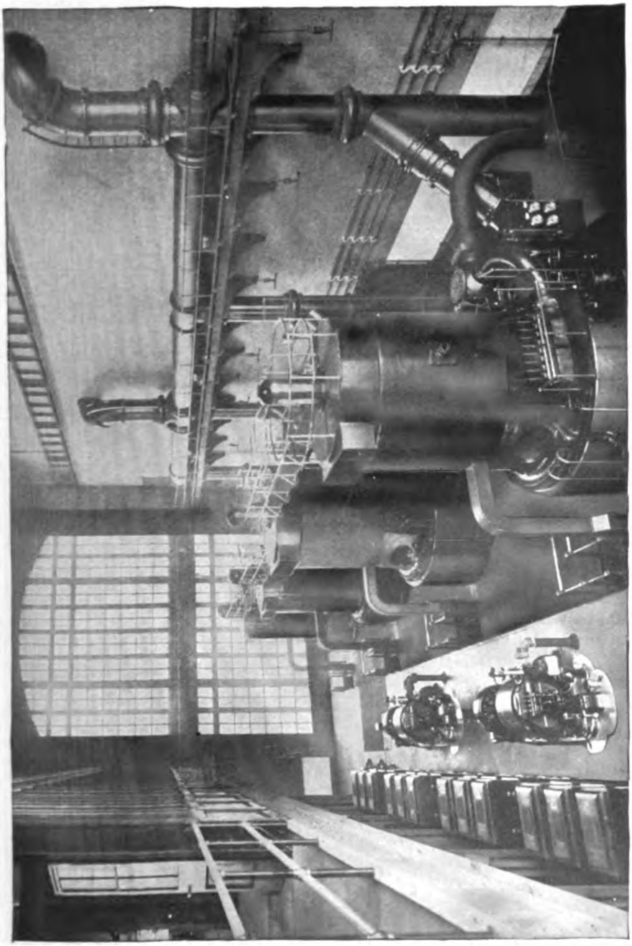


FIG. 1d. A group of steam turbo-generators. Each unit consists of a steam turbine at the bottom and an alternating-current generator at the top. *The General Electric Co.*

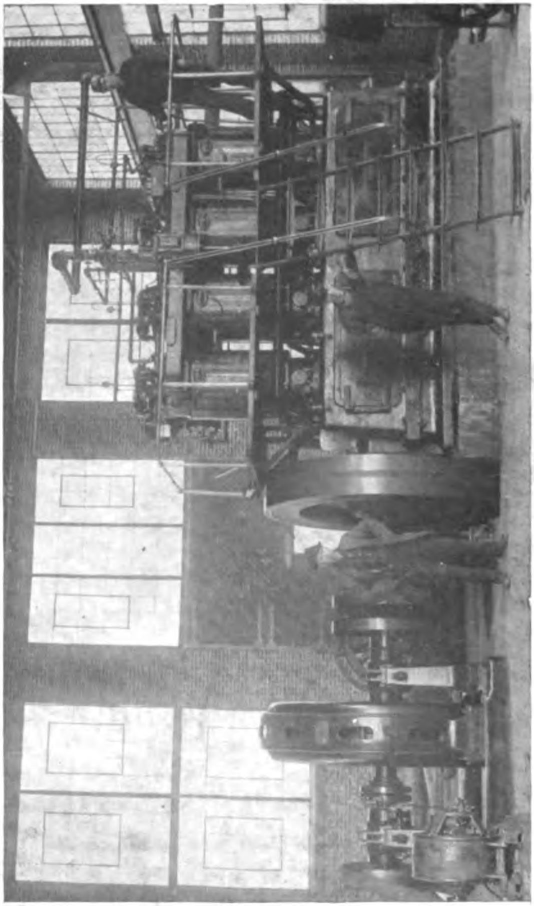


FIG. 2. A gas-engine power plant. Using natural gas, this plant has a capacity of 250 kv-a. *The Bruce-Macbeth Co.*

is to determine the best method of distributing the power that is generated to each separate machine where it is to be used.

3. Selecting a Method for Transmitting Power. There are four common ways of transmitting power to considerable distances: first, mechanical means, by belts or rope-drives and shafting; second, by steam under pressure flowing in mains; third, by compressed air; fourth, by electricity. Which of these four methods of power transmission is best suited to the requirements of a particular plant will depend upon the surrounding conditions, and often requires careful study. Each of these methods has advantages. There are many instances in which each of these four modes of power transmission should be used in preference to all of the others. Where distances are very short, mechanical transmission of power

FIG. 3a. The Gatun dam in the Panama Canal Zone. A head of 75 feet is obtained. *The Electrical World.*

FIG. 3b. The penstocks of the Gatun plant. The water turbines in the process of erection. *The Electrical World.*

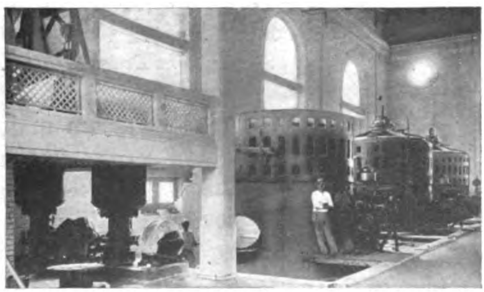


FIG. 3c. The Gatun hydro-electric station with three turbines in place. *The Electrical World.*

by belts and shafting is usually the cheapest and most efficient method. Fig. 4a shows a typical installation of rope transmission, and Fig. 4b is an illustration of the use of belts and shafting for distributing power to the various machines of a textile plant. Where installations are temporary, where the distances to which power must be transmitted are not too great, and where the quantities of

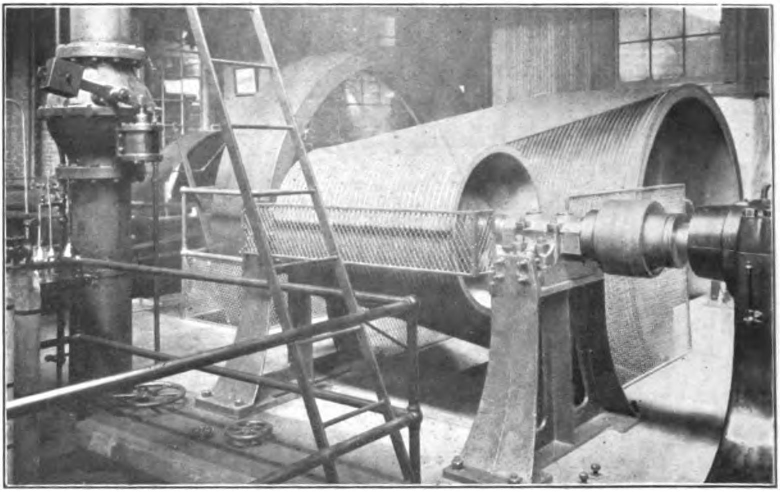


FIG. 4a. Power transmission by means of ropes and pulleys.
The Dodge Mfg. Co.

power required are small, steam under pressure may often be used to advantage. An illustration that might be cited of this type of transmission is the scheme for rock drills, hoisting engines, elevators and pumps needed for a subway or a large building excavation. Compressed air is often used in preference to steam where distances are great, where freezing temperature will be encountered and where the installation is to be more nearly permanent. The rock-drills of Fig. 5a are driven by compressed air, transmitted through a long pipe line. The mine locomotive of Fig. 5b is propelled by compressed air, enough of which can be stored in the three tanks for a trip of several miles. Electricity has many advantages over other types of power transmis-

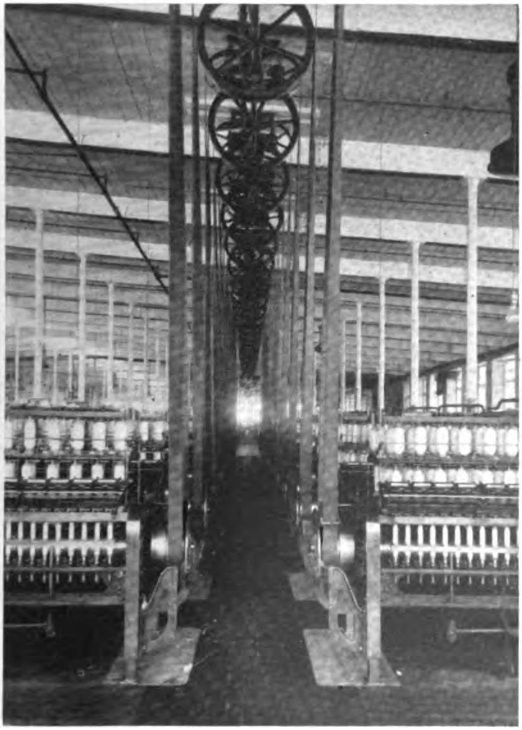


FIG. 4b. Looking down an aisle in a modern textile plant equipped with belts, pulleys and shafting. *The Dodge Idea.*

sion, especially where the distances to which power is to be sent are great. Electricity travels on wires that do not move. These wires may be bent in any direction. They easily pass obstructions and may be supported in a great variety of ways. The cost of such a transmission line is relatively small and, when installed, is subject to but small depreciation and wear. Fig. 6 shows a section of the electric line which transmits 10,000 h. p. from Niagara Falls to Toronto, a distance of 90 miles.

Fig. 5a. Power is conveyed to these rock drills by means of compressed air transmitted through pipe lines. *The Ingersoll-Rand Co.*

Electric power may be started, stopped and controlled by devices that are more accurate and rapid in their operation and more compact and durable in their construction than those which must be used when power is transmitted by other means; and it is suited to a greater variety of uses.

Also, for long-distance transmission, electricity is more economical than any other kind of power. Central power stations which are designed to furnish power to a community for an indefinite variety of purposes have consequently become, in almost every instance, **electric generating stations.**

In these stations the mechanical power of the engines or water wheels drives electric generators. The electricity from these generators is distributed through a **switchboard** over a network of large conductors which are called **feeders or**

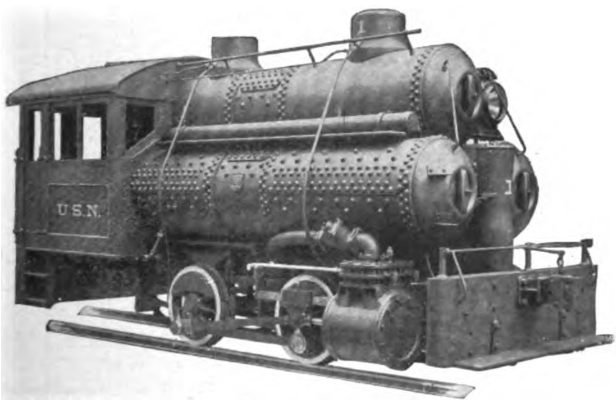


FIG. 5b. A mine locomotive propelled by compressed air. The three storage tanks can be charged in one-half minute with enough compressed air to enable the locomotive to draw a load several miles. *The H. K. Porter Co.*

mains. The current in these mains is often again subdivided at small distributing centers from which it is distributed over small wires to the numerous points where lamps, motors, heating devices or chemical apparatus are to be supplied with power.

4. Selecting the Locations of Central Generating Stations. It is usually best to locate central generating stations at points centrally situated in the areas they are to serve, and so to locate them that the cost of erecting and operating the necessary distributing circuits is a minimum.

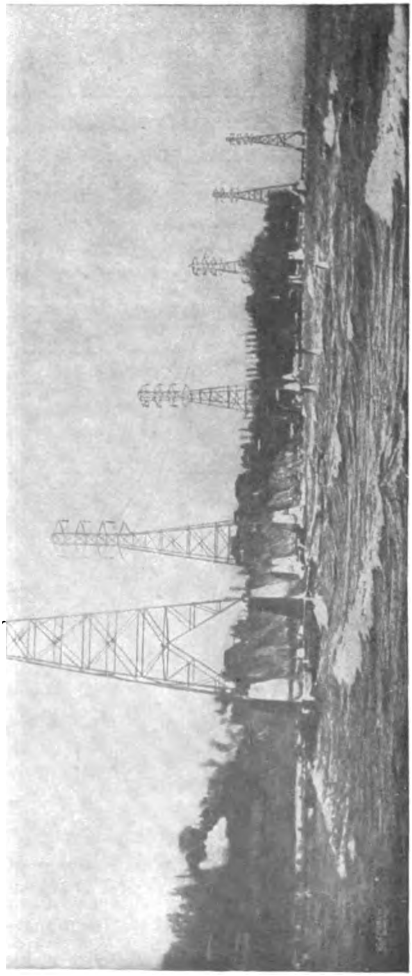


FIG. 6. A section of a modern high-tension transmission line, delivering 10,000 horse power from Niagara Falls to Toronto, a distance of 90 miles. *The General Electric Co.*

Where water is used as the natural source of power, however, the generating station is located at or near the waterfall, as it is cheaper to transmit the electricity to the market where it will be used than it would be to transmit the great quantities of water required for the water wheels.

5. Advantages of Alternating-current Electricity for Central Stations. Experience has shown that large central plants and those having great diversity of service can be operated more efficiently than smaller plants or than plants having little diversity of service; hence there has been continual growth in the economical size for central generating stations. There are now examples of single stations capable of generating 120,000 kw.; and plans are being made for still larger stations. To use the machinery in such plants to the best advantage and operate it at as steady a load as possible, longer and longer transmission lines are being planned.

The desire to transmit electrical power as far as possible with a minimum loss has resulted in the present very general use of alternating-current electricity.

The reason is simple. Electric power depends upon two factors, current and voltage. We may transmit a given amount of power in either of two ways. First, by means of a large current at a low voltage; or, second, by means of a smaller current at a correspondingly higher voltage. The smaller the current used, however, the smaller will be the loss of power in transmission. Hence, for long-distance transmission, in order to secure a small loss, we must use a small current and consequently as high a voltage as is practicable.

In the case of direct current, American engineers have considered it undesirable to use direct current at high voltages. On the other hand, alternating current may be simply and inexpensively stepped up from low voltages, at which it may be generated, to high voltages at which it may be transmitted over wires; and then it may be stepped down again to whatever voltages are desired for use. The instrument used for doing this is called a **transformer.** Such a transformer cannot be used with direct current.

6. Transformer Substations. It is now customary to generate alternating-current electricity in large central stations at voltages as high as 6000 volts or even 12,000 volts, and alternating-current motors are built to operate on voltages as high as 11,000 volts. Beyond these limits alternating current becomes troublesome on account of the difficulty in obtaining suitable insulation between the various parts of the machines.

Where high-voltage transmission is required it is customary to place in central stations **step-up** transformers which increase the voltage from that at which the current is generated to whatever voltage may be desired upon the transmission line. This voltage may be only a few thousand volts or it may be as high as 100,000 or even 150,000 volts, depending upon the quantity of power that is to be transmitted and the distance through which it must be sent. Such transformers are exceedingly compact and their efficiency may be as high as 98 or 99 per cent. They are comparatively inexpensive and have no moving parts to require attendance.

It is not advisable to carry the full voltage of the transmission lines into a town on account of the danger of contact with buildings or trees or with other electrical conductors. Often municipal ordinances forbid it. Therefore, **a transformer substation,** situated on the outskirts of a town, is used to **step down** the voltage from that used on the main transmission line to a voltage that is suitable for the distribution of current to the different consumers. Voltages used on such distributing systems differ very greatly according to circumstances. The voltage may be several thousand volts or it may be below 1000. A pressure of 2300 volts is very frequently used. A portable outdoor transformer substation is shown in Fig. 7a. This station supplied **60 kv-a.**

for operating machinery during the construction work in the Los Angeles Aqueduct.

Near the points where power is consumed small transformers are used for stepping down the voltage of the current in the distributing system to 110 volts, 220 volts, 550 volts,

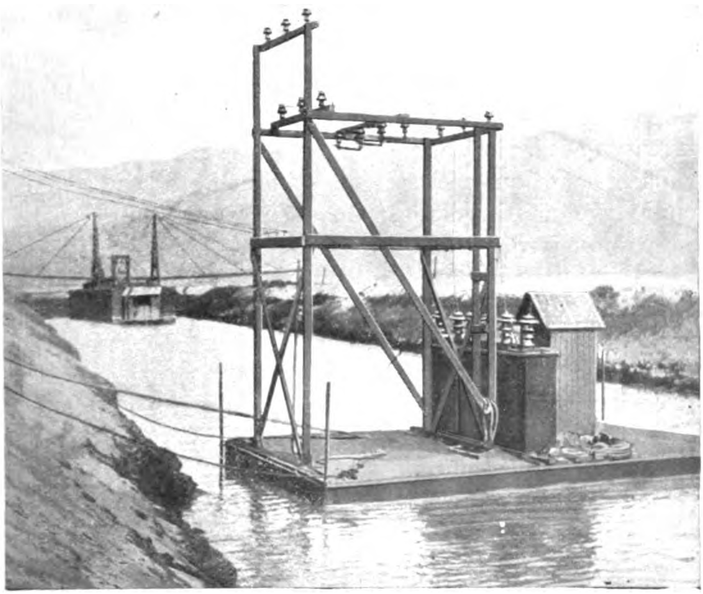


Fig. 7a. A portable transformer substation, for stepping down the pressure from 33,000 volts to 440 volts for operating machinery. Used in the construction work on the Los Angeles Aqueduct. *The Westinghouse Mfg. Co.*

or to whatever other voltage may be required by the lamps or other apparatus.

7. Converter Substations. While a very large percentage, perhaps from 90 to 95 per cent, of the electrical power now generated in central stations in the United States is generated in the form of alternating current, many applications of electrical power require direct current. Therefore, direct current must usually be available from the distributing system, even though alternating current is required for transmission.

Fortunately several devices have been developed for "converting" alternating current into direct current. For the conversion of small amounts of power, various **rectifying** converters are used, of which the **mercury-arc rectifier** is the most familiar example. Such devices are usually connected to the distribution line at each place where direct-current power is to be used: for example, at the garage where direct current is needed to charge storage batteries.

Where large quantities of direct current are needed to supply communities, large manufacturing plants or electric railways, **converter substations** are erected. These substations contain apparatus for converting alternating current at the high voltage used on the transmission line into direct-current electricity at the lower voltage used by the apparatus of the consumer.

This conversion of high-voltage alternating current to low-voltage direct current is done in two stages; the high-tension alternating-current power is transformed first into alternating-current power of lower pressure by means of the step-down transformers already referred to; and then the lower-pressure alternating-current power is converted into direct-current power at a suitable voltage by means of rotating machines, which may be of either of the two following types:

First, the **motor-generator converter** shown in Fig. 7b, which consists of an alternating-current motor operated by the transmission line and mechanically coupled to a direct-current generator. This is the most flexible type of converter and is adaptable to the greatest variety of conditions. It is, however, more expensive than the second type of rotary converter.

Second, the **synchronous converter** (commonly called the

rotary converter). This type of converter, shown in Fig. 7c, does practically all of the work of a motor-generator converter. It has, however, only one armature and only one field structure. It is correspondingly less flexible. It is, however, also less expensive and usually more efficient.

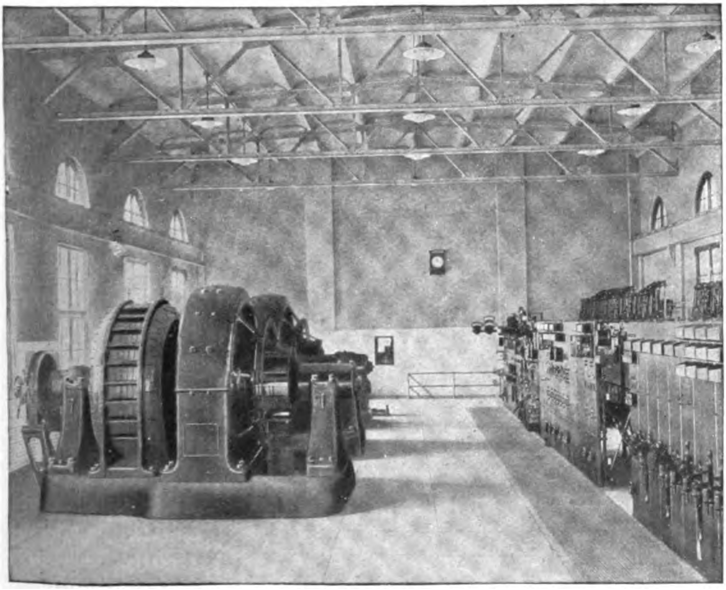


Fig. 7b. A converter substation containing motor-generator converters for the purpose of converting the alternating-current power to direct-current power to be used by locomotives in the Detroit Tunnel.

8. Alternating-current System for Short Transmissions Requiring no Step-up Transformers. An old empirical rule, which has no definite scientific basis but which gives satisfactory results within reasonable limits, states that the proper transmission-line pressure should be about 1000 volts for each mile in length of the line; for instance, 2300 volts may well be used to transmit current within a radius of about two miles from the central station.

In many stations 6600 volts, or 6900 volts, has been adopted as a satisfactory terminal pressure of the generators. When transmitting power not over five or six miles,

Fig. 7c. A converter substation containing synchronous converters. Note that no separate motor is required to drive such a converter. *The General Electric Co.*

therefore, the lines may readily be fed directly from the generators or from the bus-bars on the main switchboard, to which a number of generators in parallel deliver their output.

When lines are operated at a pressure not greater than this, step-up transformers and transformer substations are usually considered unnecessary, and the small distributing transformers, which supply the various services of individual consumers, are attached directly to the transmission line. A typical example of such an alternating-current distributing system is shown in Fig. 8a. This represents the usual three-phase installation using three wires for each circuit.

"A" The main generator A is connected directly to the three-phase transmission line. The alternating-current generator must have its field magnets excited from a separate source of direct current, usually from a small compound-wound direct-current generator driven from an independent source (in this instance by a belt on the pulley X).

"B" A three-wire three-phase transmission line is represented in the figure by the three lines B, B, B. To the transmission line are attached the following service equipment.

"C" A three-phase alternating-current synchronous motor or induction motor wound to operate properly at the full line voltage. The motor load is not shown, but it may be either a mechanical or an electrical machine.

"D" A number of single-phase transformers connected to the three-phase transmission line, stepping down the voltage to a value suitable for small three-phase motors and incandescent lamps.

"E" "A Tub Transformer" or a "Constant-current transformer" connected to a series-arc or series-tungsten lamp circuit such as is usually employed for street lighting. This transformer takes power from one phase of the three-phase line as a variable current at constant voltage. It delivers power to the series-lamp circuit as a constant alternating current at a variable voltage depending upon the resistance of the lamp circuit. If the lamps are of the type that demands direct current, a mercury-arc rectifier may be combined with the tub transformer, thus converting constant alternating current into constant direct current.

"F" A single-phase distributing transformer connected to one phase of the three-phase transmission line. A three-wire constant-voltage lamp circuit is shown connected to the terminals of the low-tension coils. Lighting is always done on a single phase. Various lighting circuits are so connected to the transmission-line wires that the three phases of the transmission line are as nearly equally loaded, or **balanced,** as possible.

"G" A set of three single-phase transformers stepping down the voltage to a value suitable for the low-voltage synchronous converter Y which is being used to convert the three-phase alternating current into direct current for distribution on a three-wire lighting and power system.

9. Alternating-current Systems for Long-distance Transmission where Step-up Transformers are Required. Where the transmission of electric power must be made to distances greater than about five or six miles or at voltages higher than from 6600 to 11,000 volts, the system is usually increased by the addition of step-up or central-station transformers and also transformer substations. A typical long-distance system of this type is illustrated diagrammatically in Fig. 8b. This includes the following principal items of equipment.

"A" A main central-station generator generating three-phase alternating current at a voltage of between 6000 and 11,000 volts.

"B" A set of step-up or station transformers so connected to the main generator as to raise the pressure of the alternating-current generator to the value required on the transmission line.

"C" A three-wire three-phase transmission line C, C, C.

"D" A transformer substation consisting of three single-phase transformers connected to the three-phase high-tension transmission line. These transformers supply current to a set of three three-phase distribution wires E, E, E, at a voltage of perhaps 2300 volts between any two wires or across any one phase.

"F" A variety of apparatus may be connected to the distributing system E, E, E, either through small transformers or directly to the line. The figure shows a small three-phase induction motor operated through transformers at a voltage of 230 volts, a system of incandescent lamps at a voltage of 115 volts, a larger motor operating directly on the distributing line at 2300 volts, and a three-wire alternating-current incandescent lighting system operating at 115–230 volts through a small transformer.

"G" A converter substation consisting of two distinct parts; first, step-down transformers used for reducing the pressure from the high-tension transmission line to a value suitable to drive a synchronous converter; second, a synchronous converter changing alternating-current to direct-current electricity.

In this case the direct-current output of the converter substation is used for operating a suburban railway. It might also be distributed to a town for a great variety of purposes.

SUMMARY OF CHAPTER I

POWER is obtained from coal, oil and water, by the use of prime movers in the form of steam engines, gas engines and water wheels.

CENTRAL POWER STATIONS are established because it is inefficient to place a prime mover at each place where a small amount of power is required.

CENTRAL STATIONS ARE ELECTRICAL because electrical power can be transmitted more cheaply and more conveniently and turned to a greater number of uses than any other form.

THE LOCATION of these power stations is as near the center of the region to be served as possible. Water wheels, however, must be located near the waterfall.

ALTERNATING CURRENT is generated by these central stations because remarkably efficient machinery has been devised for "stepping up" the voltage and getting the great advantage of transmitting at high voltage. The same machine, a transformer, "steps down" the voltage allowing it to be used at a low pressure. Transformers will not operate on direct current.

CONVERTER SUBSTATIONS are placed at points along the transmission line where a large amount of direct current is needed, and synchronous converters or motor-generators are installed which change the alternating current to direct current. For converting a small amount of alternating-current power to direct-current, a mercury-arc rectifier is generally used.

TRANSFORMER SUBSTATIONS are erected wherever it is desirable to step down from the transmission voltage of between 22,000 and 140,000 volts to a city circuit usually of about 2300 volts, for the sake of greater safety to human life. At the immediate points where the power is to be used, small individual transformers change this 2300 volts to the 500, 220 or 110 volts desired.

SHORT TRANSMISSION SYSTEMS for transmitting power six miles or less consist of an alternating-current gen-

erator of from 2000 to 11,000 volts, connected directly to the line. At the receiving end of the line, synchronous motors, induction motors or converters may also be attached directly to the line. By attaching transformers to the line, small motors, incandescent lamps and arc lights may be run at their proper low voltage.

LONG TRANSMISSION SYSTEMS are those which transmit power more than five or six miles. The generator delivers 6600 to 11,000 volts, but this is "stepped up" by station transformers, sometimes as high as 150,000 volts, before it is delivered to the line. Wherever power is to be used, either a transformer substation or a converter substation is erected. The former by means of transformers "steps down" the voltage to about 2300 volts for distribution of alternating-current power over a small area. The latter has a synchronous converter in addition to the transformers and delivers direct-current power to a limited area.

PROBLEMS ON CHAPTER I

Prob. 1–1. In a certain coal mine there is a vein of hard coal 40 ft. wide, 5 ft. deep and 1200 ft. long. This coal averages 13,200 B.t.u. per pound.

(*a*) How many foot-pounds of energy are there in this vein?

(*b*) How many horse power-hours?

Prob. 2–1. A certain factory requires 250 horse power to operate it. How long would the coal in the vein in Problem 1 operate this factory? Assume 25 per cent loss in handling the coal, and that the boiler and engine use only 15 per cent of the energy in the coal. Factory runs 9 hours per day, 6 days a week.

Prob. 3–1. How many kilowatts are required to illuminate a schoolhouse having 50,000 sq. ft. of floor space? The average power required is 1.5 watts per sq. ft.

Prob. 4–1. Assuming that the generator of Problem 3 has an efficiency of 80 per cent, how many tons of soft coal per week of 25 hours are needed to illuminate the school building of Problem 3? The coal averages 13,000 B.t.u. per lb. and 12 per cent of this is turned into mechanical energy by the boiler and steam engine.

Prob. 5-1. If it requires 2.2 lb. of coal per hour to produce one horse power in a good modern steam power plant, how many tons of coal per day of 15 hr. are used by a power plant delivering 25,000 h.p.?

Prob. 6-1. Compute the number of pounds of coal needed per kilowatt-hour on the basis stated in Problem 4.

Prob. 7-1. A certain oil well flows 2000 barrels of oil per day. If this is burned under a boiler, how many horse power will it develop continuously? Assume that 1 lb. of oil contains 17,000 B.t.u. of which 15 per cent is available by this method of using the oil.

Prob. 8-1. One barrel of the oil of Problem 7 is equivalent in heat value to how many pounds of the coal of Problem 1?

Prob. 9-1. How many barrels of oil per day will a locomotive use when developing 1200 h.p. if only 6 per cent of the energy in the oil is available? Assume that the locomotive runs 6 hours per day.

Prob. 10-1. How many tons of coal per day will the locomotive of Problem 9 use under the same conditions?

Prob. 11-1. If it requires 9 barrels of crude oil per day of 10 hr. to run a 250-kw. plant at rated load, what per cent of the energy in the fuel is available by the method used?

Prob. 12-1. The Big Creek reservoir of the Pacific Light and Power Co. is 4.5 miles long, $\frac{1}{2}$ mile wide and has an average depth of 34 ft. The effective height of the reservoir above the water wheel is 1900 ft.

(*a*) How many foot-pounds of energy are stored in this reservoir?

(*b*) How many horse power-hours?

Prob. 13-1. To how many tons of coal averaging 14,000 B.t.u. per pound is the water in the reservoir of Problem 12 equivalent, from the energy standpoint?

Prob. 14-1. The power plants in connection with the reservoir of Problem 12 contain six water wheels of 20,000 h.p. each. How many days would the water in the reservoir alone operate these wheels, assuming that the average load is one-half the capacity of the plants and that the efficiency at this load is 80 per cent?

Prob. 15-1. If the water wheels had only the head of the Keokuk plant, 32 ft., how long would the plant run under the conditions of Problem 14?

Prob. 16-1. How many kilowatts can be delivered by the power plant of Problem 14 operating at full load? Assume an efficiency of 95 per cent for the electric generator.

Prob. 17–1. How many barrels of oil per month would be required to deliver the same power as the water wheels in Problem 14? Assume the oil to be burned under boilers as in Problem 7.

Prob. 18–1. The highest recorded efficiency for water turbines was attained by the four 6000-horse power wheels at New River, Va. Under a head of 49 ft., an efficiency of 93.7 per cent was secured. What flow of water was necessary under these conditions?

Prob. 19–1. What horse power will a water turbine develop when operating under a head of 40 ft. and taking 20,000 cu. ft. of water per minute? Efficiency of the turbine is 80 per cent.

Prob. 20–1. In the Mississippi River hydro-electric development at Keokuk, Iowa, there are 15 turbines each having a normal rating 10,000 h.p. based on a head of 32 ft. Under these conditions they operate at an efficiency of about 88 per cent. What is the flow of water through them?

Prob. 21–1. At full load the generators attached to the turbines in Problem 20 have a guaranteed efficiency of 96.3 per cent. How many kilowatts can each generator deliver under these conditions? Data from General Electric Review.

Prob. 22–1. In the Gatun hydro-electric development shown in Fig. 3a, 3b and 3c, there are three Pelton-Francis turbines, each having a capacity of 3600 horse power when operating under an effective head of 75 ft. The total flow of water through the penstocks is 90,000 cu. ft. per min. What is the efficiency of the turbines under these conditions?

Prob. 23–1. Each generator attached to the turbines in the Gatun plant has a guaranteed efficiency of 95.1 per cent when delivering 2000 kw. What horse power must each turbine develop under these conditions? Data from General Electric Review.

Prob. 24–1. Assuming an efficiency of 83 per cent for the turbines in Prob. 23, how much water per minute must be supplied to each machine at an effective head of 75 feet?

Prob. 25–1. At the average rate of $3.00 per ton, what is the fuel cost of producing one kilowatt-hour, on the basis of Problem 4?

Prob. 26–1. Assume the following conditions in a good gas-producing plant:

The producer delivers 75 per cent of the energy in the coal to gas engine.

The gas engine converts 35 per cent of this energy into mechanical energy of the piston.

The piston delivers 90 per cent of this energy to the shaft.

What is the overall efficiency of the gas-producer plant?

Prob. 27-1. At $2.50 per ton for coal averaging 14,000 B.t.u., what will it cost for fuel per year of 3000 hr. to operate a 100-kw. electric generator with a gas-producer engine? Generator has an efficiency of 90 per cent and producer plant data as in Problem 26?

Prob. 28-1. A 70-h.p. Diesel engine showed on test that it delivered 41.7 per cent of the energy in the oil to the piston. Efficiency of engine from piston to pulley 90 per cent. At 2 cents per gallon (7.6 lb.) for oil, how much will it cost per year of 3000 hours for oil alone? Assume engine to run at 70 horse-power rate for the full time. 1 lb. of oil = 14,500 B.t.u.

Prob. 29-1. In a typical hydro-electric plant there is an effective head of 80 ft. and an average flow of 120,000 cu. ft. per min. of water through the penstocks. The turbines have an efficiency of 89 per cent. The direct-connected generators have an efficiency of 96 per cent. How much power can the generators deliver?

Prob. 30-1. The electric power from the central station of Problem 29 is transmitted over a high voltage system, the efficiencies of the separate parts of which are as follows: Step-up and step-down transformers 98 per cent each; transmission line, 95 per cent; rotary converters, 94 per cent. How much power may be delivered at the converter substation?

Prob. 31-1. A test was made in the shafts and belting of a certain machine shop eight stories high. A jack shaft on each floor is connected by belts to the engine shaft. When the shop was running at full load the sum of the power being delivered by the several jack shafts was 196.7 h.p. The engine was delivering 257.2 h.p.

(*a*) What horse power was lost in the jack shafts and belting?

(*b*) What was the efficiency of the jack shafts and belting?

CHAPTER II

INTRODUCTION

BEFORE beginning a detailed study of alternating-current machines and circuits, and the essential principles that we must understand in order to work intelligently with them, it is desirable to comprehend in a general way the entire system of which each machine and circuit is a part. This will assist us in understanding the physical relationships and the importance of the parts to one another and to the whole system.

Effects follow **causes** so quickly in electrical circuits that any one who expects to operate electrical equipment with safety should fortify himself with a thorough knowledge of all the parts of the system and their interdependence upon one another, just as a surgeon should know the structure of the human body and the functions of all its organs before he assumes the responsibility of performing a surgical operation. The closing or opening of a switch too soon, or too fast, or in the wrong place may result in the interruption of service, the breaking of machinery or the sacrifice of human life. Therefore, the operating electrician must know the workings of the various parts of the system so accurately that he will instantly recognize or anticipate the electrical effects that will follow any action, at whatever point in the system they may appear.

There is nothing especially difficult about understanding alternating-current phenomena, provided the student fully grasps a few simple conceptions regarding the nature of an alternating current and the way in which it differs from a direct current. The most important of these ideas are the following:

Current. Alternating-current electricity differs from direct-current in one important particular only. It is not a uniform continuous flow. In the typical case, the current flows back and forth along the conductors, first in one direction and then in the other, very much as a pendulum swings. The current starts at zero, increases gradually to a maximum in one direction and then gradually decreases again until it becomes zero, when it starts to increase again, but in the opposite direction: increasing to a maximum and decreasing to zero. A graphical illustration of this alternate increase and decrease of current flowing in one direction and then in the other is found in Fig. 37a on page 52. It is most important for the student to get a very clear idea of exactly the way in which the current surges back and forth.

Voltage. The electromotive force or voltage, which causes the flow of current in this case, of course, varies too. It also starts at zero and increases to a maximum and decreases and becomes negative in just the same way that the current does. It is important for the student to remember, however, that the maximum current and the maximum voltage may not occur at exactly the same instant.

As both current and voltage are variables and not constants, we cannot add them together in the same way that we do in the case of direct currents. In the case of two alternating currents it may be that the maximum flow in one direction of one current does not occur at the same instant as the maximum flow in the same direction of the other current. It is conceivable that they might even exactly oppose each other. We must always know accurately what the relation (i.e., the instantaneous magnitude and direction) is at any instant in order that we may add the two currents at that instant. At any particular instant, however, we add alternating currents just as direct currents are added.

Polyphase Currents. If we add together two similar currents that have their maximum flow occurring in the same direction at the same instant, we would get a single alternating current of twice the amperage of either of the original currents. But, if we add the currents in the several wires of a commercial *polyphase* system, we will not get a current equal to the arithmetical sum of amperage in all the wires, because in the wires of such a commercial system the maximum currents do not occur at the same time.

Power. In alternating currents, just as in direct currents, the power at any given instant consists of the product of the current that is flowing at that given instant by voltage at the same instant. But in alternating current both the flow and the pressure are variables and, therefore, we cannot multiply the maximum flow by the maximum voltage. The power in an alternating-current circuit will always be the product of the effective flow of current times some voltage that is between the maximum voltage and the zero voltage.

Resistance. In an alternating current, Ohm's Law applies exactly as it does in a direct current, and the drop of voltage due to resistance over a given distance in a given wire at a given instant may be figured by the usual formula provided we use the instantaneous flow of current that is occurring at the instant. As the current is constantly varying, of course the drop in voltage to the end of the line is varying in a similar way.

Reactance. In studying direct-current electricity we learned that a change in flow of current in one circuit produces a tendency toward a flow of current in any adjacent circuit. This phenomenon is called "Induction." In an alternating-current circuit there is a perpetual change of flow. Consequently, there is a perpetual tendency to cause induction in every adjacent circuit; and furthermore, each wave of current as it surges back and forth in a single circuit produces a similar effect on the impulses that have preceded it in the same circuit. This effect retards the flow of current and produces a decrease in voltage at the other end of a transmission line somewhat similar to the drop due to the resistance of the wire.

Impedance. The effective sum of these hindrances to the flow of an alternating current in a line, due to what we may call its "Ohm's-law" resistance and its "Reactance," is called the "Impedance" of the line.

If the student will try to visualize accurately each of the above seven principles and will get himself into the habit of thinking of the variable quantities of current, voltage and power as they increase and decrease, and will practice dealing with them, adding together and subtracting the currents and the voltages, etc., under all sorts of different conditions, he will find that the difficulty and confusion often found by students who have failed to get a clear idea of these fundamental notions, have entirely disappeared.

CHAPTER II

ALTERNATING CURRENTS. FUNDAMENTAL IDEAS

BEFORE taking up the several principles of alternating-current generation, distribution and use, it is necessary that the student should be familiar with the alternating currents and pressures themselves and with the methods of measuring and computing their value.

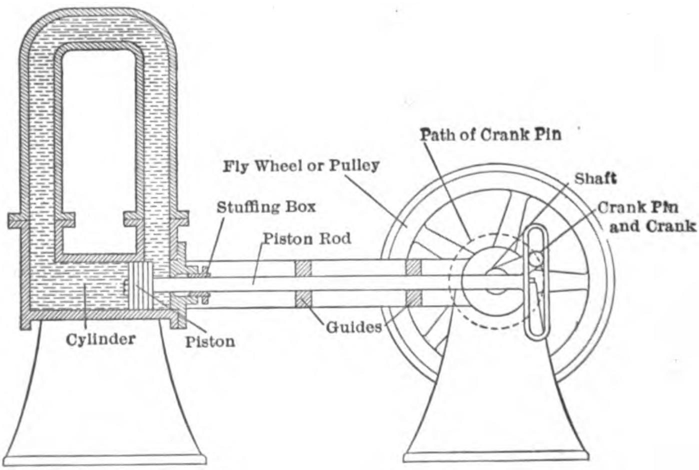


FIG. 9. Engine-driven valveless pump with slotted yoke. The water surges back and forth through the pipe.

10. Cycle. Frequency. An alternating current of electricity differs in no respect from a direct current, except that instead of flowing continuously in one direction, it periodically reverses the direction of its flow. In the best practice at the present time, an alternating current reverses either 50 times a second, or 120 times a second. In a few installations the reversals take place 80 times a second.

If we liken the flow of a direct current to the flow of water in a river, we may liken the flow of an alternating current to the ebb and flow of the tide in a narrow channel. The tide periodically reverses the direction of its flow once about every 6¼ hours or about 4 times a day. A better idea of this ebb and flow of an alternating current can be gained from a study of the engine-driven pump, Fig. 9.

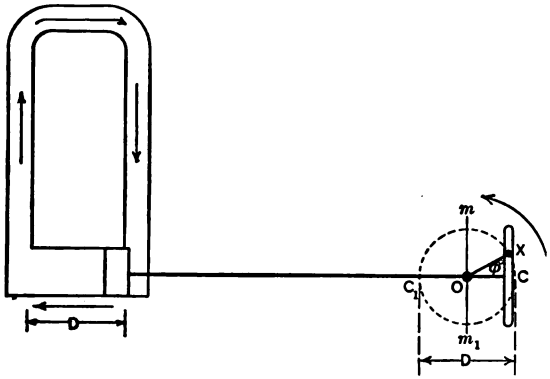


FIG. 10. Skeleton diagram of valveless pump. The direction of the piston motion at any instant depends upon the position of the crank at that instant.

As the valveless piston is moved back and forth in the cylinder, the water which completely fills the system is made to surge back and forth throughout the entire circuit of pipe and cylinder. The number of times the direction of flow changes per minute depends upon the number of revolutions per minute made by the crank shaft driving the piston. Assume the crank shaft to rotate at a uniform speed in the direction marked in Fig. 10 and 11, which are skeleton diagrams of the pump in Fig. 9. It is seen that the direction of flow in the pipe at any given instant depends upon the position of the crank at that instant.

It is also easy to see that as the crank pin passes through the points C and C_1 the piston is at a standstill and no current is flowing. When the crank pin is passing through the points m and m_1, the piston is moving at the maximum rate and the greatest current is flowing around through the pipes. In Fig. 10 the crank pin has just passed the dead or neutral point C, and the piston is traveling to the left, forcing the water around the circuit in a clockwise direction. In Fig. 11 the crank has just passed through the neutral point C_1 and

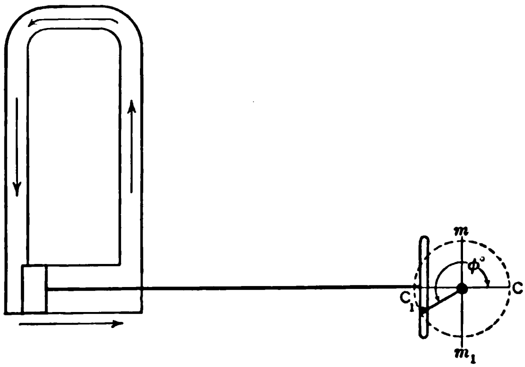


FIG. 11. The crank has moved into a new position, such that the piston motion is reversed.

the piston has started back and is traveling to the right. The direction of the flow of the water has also reversed, and is now counter-clockwise around the pipe system. Note that in each revolution of the crank there are two neutral points (C and C_1) at which the piston is not moving and no current is flowing; and also two maximum points (m and m_1) at which the piston is moving the fastest, and the greatest current is flowing. At all times, however, the **crank shaft** is revolving in the **same direction** and at the **same speed**.

The flow of the water in the above system is a fair picture of what happens in an electric alternating-current system. The current starts flowing in one direction, rises to a maximum value, dies out and stops flowing for an instant. Then it starts flowing in the opposite direction, rising to a maximum value, and dying out until it stops again. This sequence of events is called a **cycle.** Note that in each cycle there are two instants when the current is zero (that is, when it stops flowing) and two instants when the current is flowing at the greatest rate. The number of times the current goes through this cycle each second is called the **frequency.** The usual frequencies of electrical machines in commercial use are 25 cycles per second, and 60 cycles per second. Systems using these frequencies are usually spoken of as 25-cycle and 60-cycle systems.

11. Relation of Current at any Instant to Crank Position. Referring again to the engine-driven pump of Fig. 10, we have seen that the speed and the direction of the **piston** motion at any instant depends upon the position of the crank at that instant. Accordingly, we have said that the amount and direction of **current** in the pipe system at any instant depends upon the position of the crank at that instant.

Let us call *OC* the "zero" position of the crank. It will be found that when a slotted yoke is used, as in this case, the speed of the piston at any instant is proportional to the **sine * of the angle** ϕ, which the crank is making at that instant with the zero position.

Thus, when the crank is at *OM*, it is at an angle of 90° to the zero position *OC*, and the piston is now moving at the fastest speed and the greatest clockwise current is flowing. Let us assume this greatest current to be 100 gal. per minute. When the crank has just reached the position *OX* and is making an angle of only 25° with the zero position, then only a certain fraction of the 100 gal./min. is being forced through the pipe. This fraction will be found to be equal to the sine of the angle 25°, or 0.423. Thus, the current at that instant would be 0.423 of 100 gal./min., or 42.3 gal./min.

* See Appendix A on Trigonometry.

This may be stated as a general rule:

The current at any instant equals the product of maximum current times the sine of the angle which the crank is making with the zero position at that instant. This assumes the crank shaft to rotate at a constant speed.

This rule will be found to be verified when the following problems have been worked out.

If we wish to write this relation in the form of an equation:

Let I_m = maximum current in gal./min.
i = current at any instant in gal./min.
ϕ = angle which crank makes with zero position.

Then

$$i = I_m \sin \phi.$$

Example 1. What is the current in the pipe system in Fig. 10 when the crank is at an angle of 65° with the zero position?

$$\begin{aligned} i &= I_m \sin \phi \\ i &= 100 \sin 65^\circ \\ &= 100 \times 0.906 \\ &= 90.6 \text{ gal./min.} \end{aligned}$$

Prob. 1-2. Assume constant speed for crank shaft.

If the maximum current in pipe system of Fig. 9 is 25 gal./sec., what is the current when the crank is at the 45° position?

Prob. 2-2. (*a*) What is the current in Prob. 1 when the crank is at the 105° position? (*b*) Is it clockwise or counter-clockwise in the pipe system?

Prob. 3-2. (*a*) What current will flow in the system of Fig. 11, if ϕ equals 220°, and the maximum current is 50 gal./sec.? (*b*) Will current be clockwise or counter-clockwise? (Note that the sine of angles between 180° and 360° is negative. See Appendix A.)

Prob. 4-2. (*a*) What current will flow in circuit of Prob. 3 when the crank is at the 300° position? (*b*) Clockwise or counter-clockwise?

Prob. 5-2. If the crank turns at the rate of 120 revolutions per minute, what is the frequency of the alternating current produced through the pipes, in cycles per second?

Prob. 6-2. Consider a pump of such size that 1 inch movement of the piston displaces 3.18 gallons of water, the stroke of the piston being 10 inches. The piston is joined to the crank by a slotted yoke as shown in Fig. 9. The crank rotates uniformly at the rate of 60 rev. per min. By graphical construction and also by trigonometric table find:

(*a*) The distance moved by the piston as the crank moves from a position 29° off dead center, to a position 31° off dead center (average position, 30° off center): also,

(*b*) The piston displacement as the crank moves from 89° to 91° (average position 90°).

(*c*) Calculate the flow in gallons per second through the circuit for each case.

(*d*) Calculate the numerical value of the ratio

$$\left(\frac{\text{av. rate of flow, gal. per second, during movement from } 29° \text{ to } 31°}{\text{av. rate of flow, gal. per second, during movement from } 89° \text{ to } 91°}\right).$$

From a table of sines, find the numerical value of the ratio

$$\left(\frac{\text{sine of } 30°}{\text{sine of } 90°}\right).$$

Draw conclusions.

Prob. 7–2. As in Prob. 6, calculate the numerical value of the ratio $\left(\frac{\text{av. rate of flow, crank moving from } 59° \text{ to } 61°}{\text{av. rate of flow, crank moving from } 89° \text{ to } 91°}\right)$ also, calculate the numerical value of the ratio $\left(\frac{\text{sine of } 60°}{\text{sine of } 90°}\right)$ values being taken from a table of natural sines. Compare these ratios and draw conclusions.

Prob. 8–2. What is the maximum value of current, gal. per second, attained during the cycle, in Prob. 6?

Prob. 9–2. As in Prob. 6, calculate the numerical value of the ratio $\left(\frac{\text{av. rate of flow, crank moving from } 44° \text{ to } 46°}{\text{av. rate of flow, crank moving from } 89° \text{ to } 91°}\right)$.

Also, calculate the numerical value of the ratio $\left(\frac{\text{sine of } 45°}{\text{sine of } 90°}\right)$.

Compare these ratios and draw conclusions.

Prob. 10–2. On the basis of problems 6, 7, 8 and 9, show that the following relation is true:

$$\left(\frac{\text{Rate of flow, gal. per sec., as piston passes through each position}}{\text{Maximum rate of flow, gal. per sec., attained during the cycle}}\right) = \frac{\text{sine } \phi}{\text{sine } 90°} = \frac{\text{sine } \phi}{1},$$

or that **rate of flow as piston passes through $\phi°$ position = maximum rate of flow in cycle × sine $\phi°$.**

Prob. 11–2. What will be the current in Prob. 6 when the crank is at an angle of 70° to the zero position?

12. Relation of Current to Crank Position Expressed by Sine Curve. This relation between the current and the crank position may be represented by plotting a curve between them as in Fig. 12. The different values of the current in gallons per minute are placed on the vertical

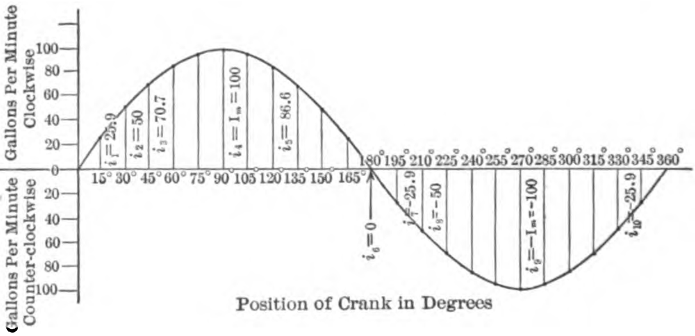


FIG. 12. Relation between current and crank position shown by a curve called "The Sine Curve."

line, reading up for clockwise direction of current and down for counter-clockwise. The crank position in degrees is plotted along the horizontal. Assume the maximum current 100 gallons per minute as before.

When the crank is at zero position the current is zero, thus the curve starts at zero.

When the crank reaches the 15° position, the current equals 25.9 gal./min.

$$\begin{aligned} i_1 &= 100 \sin 15° \\ &= 100 \times 0.259 \\ &= 25.9 \text{ gal./min.} \end{aligned}$$

Thus, at the 15° position on the horizontal axis, draw a line (i_1) upward 25.9 spaces to represent a current of 25.9 gal./min. in a clockwise direction.

When the crank is at the 30° position, the current equals 50 gal./min.

$$\begin{aligned} i_2 &= 100 \sin 30° \\ &= 100 \times 0.500 \\ &= 50 \text{ gal./min.} \end{aligned}$$

Draw a line (i_2) upward 50 spaces at the 30° position on the horizontal axis to represent a 50 gal./min. current clockwise.

Similarly

$$\begin{aligned} i_3 &= I_m \sin 45° \\ &= 100 \times 0.707 \\ &= 70.7 \text{ gal./min.} \end{aligned}$$

Draw i_3 upward 70.7 spaces from the 45° mark on the horizontal. Also

$$\begin{aligned} i_4 &= I_m \sin 90° \\ &= 100 \times 1 \\ &= 100. \end{aligned}$$

Therefore, $$i_4 = 100.$$

Draw i_4 (I_m) upward 100 unit spaces from the 90° mark on the horizontal.

From now on the current grows smaller so that at the 120° position of the crank it has become (i_5) or 86.6 gal./min.

$$\begin{aligned} i_5 &= 100 \sin 120° \\ &= 100 \sin (180° - 120°) \quad \text{(See Appendix A, Art. 9a.)} \\ &= 100 \sin 60° \\ &= 86.6 \text{ gal./min.} \end{aligned}$$

Draw line i_5 upward 86.6 spaces at the 120° point on the horizontal.

As the crank continues to turn, the current grows less and less until it again becomes zero at the 180° position. This is the position OC_1 in Fig. 10.

This fact is also shown by the equation.

Let i_6 = current when the crank is at the 180° position. Then

$$\begin{aligned} i_6 &= I_m \sin 180°, \\ \sin 180° &= \sin (180° - 180°) = \sin 0° = 0, \\ i_6 &= 100 \times 0 \\ &= 0. \end{aligned}$$

The current when the crank is at the 180° position is then 0, and is represented by a dot on the horizontal line at the 180° point.

But as the crank continues beyond the 180° position, the piston begins to move in the reverse direction and when it has reached 195°, a current (i_7) of 25.9 gallons per minute is being sent around in a counter-clockwise direction.

$$\begin{aligned} i_7 &= 100 \sin 195° \\ &= 100 [- \sin (195° - 180°)] \quad \text{(See Appendix A.)} \\ &= 100 (- \sin 15°) \\ &= - 100 \times 0.259 \\ &= - 25.9 \text{ gal./min.} \end{aligned}$$

The minus sign merely means that the current is now flowing in the opposite or counter-clockwise direction.

Thus we draw i_7 downward 25.9 spaces at the 195° position to represent a current of 25.9 gal./min. flowing in the counter-clockwise direction.

When the crank has reached the 210° position, a current i_8 of 50 gal./min. is flowing.

$$\begin{aligned} i_8 &= 100 \sin 210° \\ &= 100 [- \sin (210° - 180°)] \\ &= 100 (- \sin 30°) \\ &= - 50 \text{ gal./min.} \end{aligned}$$

Accordingly we draw i_8 downward 50 spaces at the 210° point to represent a counter-clockwise current of 50 gal./min.

At the 270° position of the crank, the counter-clockwise current has reached its maximum of 100 gal./min. so $i_9 = - I_m$ is drawn at this point on the horizontal to represent this current value.

From the equation we obtain the same result.

$$\begin{aligned} i_9 &= I_m \sin 270°, \\ \sin 270° &= - (\sin 360° - 270°) = - \sin 90° = - 1, \\ i_9 &= 100 \times (- 1) \\ &= - 100 \text{ gal./min.} = - I_m. \end{aligned}$$

As the crank continues toward a completion of its cycle, the value of the counter-clockwise current gradually falls off until at the 360° position of the crank it again becomes zero.

To make clear the computation for this position of the crank, we will calculate the instantaneous current (i_{10}) when the crank is at the 345° position.

$$\begin{aligned} i_{10} &= 100 \sin 345° \\ &= 100 [- \sin (360° - 345°)] \\ &= 100 (- \sin 15°) \\ &= - 25.9 \text{ gal./min.} \end{aligned}$$

Thus the line i_{10} is drawn downward 25.9 spaces at the point 345° on the horizontal to represent a current of 25.9 gal./min. counter-clockwise.

If several intermediate lines are drawn and their ends joined by a smooth curve as in Fig. 12, this line is called a **sine curve,** and gives the clearest possible picture of an alternating current of water or of electricity.

Prob. 12–2. Plot a sine curve for 12 values of the current at different positions of the crank of the system in Prob. 3–2.

Prob. 13–2. Plot a sine curve for 12 values of the current in system of Prob. 6.

Prob. 14–2. Plot two cycles of the sine curve of Prob. 1 using enough points to obtain a smooth curve.

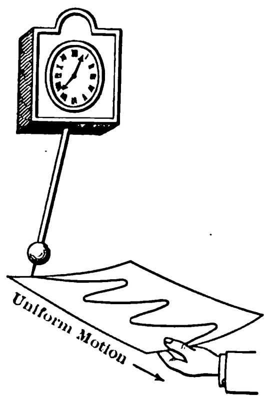


FIG. 13. The pendulum traces a sine curve if smoked paper is moved at uniform speed.

13. The Sine Curve a Standard Wave Form.

The type of motion represented by the sine curve is a very common thing in nature. If we observe accurately any natural object which has a periodic motion, such as a swinging pendulum, a vibrating string, or the rippling surface of a body of water, we find that this form of wave, the **sine curve**, is apparently **Nature's standard.**

Note the curve produced in Fig. 13, by allowing a swinging pendulum to trace its motion on a smoked surface which is moved at a uniform speed at right angles to the swinging.

Fig. 14 shows the curve of the motion of a tuning fork obtained in a similar way, and Fig. 15 represents the cross sections of ripples on the surface of water. These all have the form of the **sine curve.**

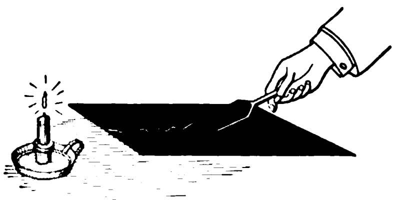


FIG. 14. The curve traced by the tuning fork is a sine curve.

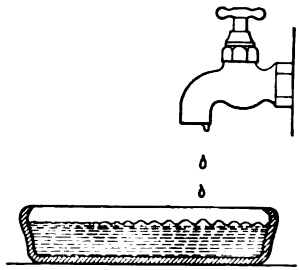


FIG. 15. The waves produced on the water have the form of a sine curve.

Now it is possible to produce alternating currents with an almost endless variety of wave forms, but this natural **sine curve** has been adopted by engineers as the standard, principally for the following reasons:

First. This form produces the least useless disturbance in the circuit and gives the smoothest, most efficient and most useful current wave.

Second. The mathematical computations connected with alternating-current work are much simpler with this form of wave.

It is a common belief among beginners that an alternator naturally delivers an e.m.f. and current whose forms are sine curves, but this is far from the fact. Only by a very careful design of machine and arrangement of windings can this result be accomplished. It requires great effort on the part of the manufacturers to produce a-c. machines which shall deliver an alternating current with this standard wave form. The different means by which this is accomplished are taken up in Chapters VIII and IX.

14. Clock Diagrams, or Vector Diagrams. So far we have learned that the alternating current of water in the enclosed system of Fig. 9 may be represented by a wave form called a sine curve, which presents to the eye a very definite picture of what is happening at each instant in the system. This curve is difficult to draw accurately and is of little use in determining actual values. So if we wish to determine accurately just what current is flowing at any particular instant, we use **the equation of this curve:**

$$i = I_m \sin \phi,$$

where i = gal./min. at any instant,

I_m = maximum gal./min.,

ϕ = angle of crank from zero position.

This equation gives accurate results in every respect, but presents no picture to the eye of what is happening.

Accordingly, a third method of representing alternating currents is in common use, called the **clock diagram** or **vector diagram.** This method enables us to obtain quickly correct numerical values and at the same time have a picture before the eye, of the events taking place in such a circuit as the above in Fig. 9.

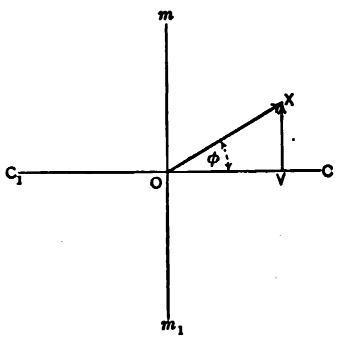


FIG. 16. Clock diagram of current relations in pump with crank at $\phi°$ from the zero position.

We have seen that at any instant the current in Fig. 9 depends upon the position of the crank with regard to the axis C_1C. Accordingly, to find the current at any instant, we draw what amounts to a picture of the crank at that instant, Fig. 16. We draw first a horizontal line CC_1 and a vertical line mm_1, both of indefinite length, just to use as reference lines, very much as the lines CC_1 and mm_1, Fig. 10, are used as reference lines.

Next we draw a line OX making the same angle ϕ with CC_1 that the crank makes with CC_1 in Fig. 10, and assume that this line is drawn to a scale which represents the maximum current in the pipe system.

But we know that at the instant when the crank makes this angle with the horizontal, the current through the pipe is only a certain fraction of the maximum current, and that this fraction is represented by the sine of the angle ϕ. Now, if we draw the line XV at right angles to CC_1 it will be the same fraction of the line OX, that the current at this instant is of the maximum current, since this fraction will in each case be represented by the sine of the angle ϕ. Accordingly, we have at one glance:

First. A line OX, which represents the maximum current.

Second. A line XV, which represents the current at a particular instant.

Third. An angle ϕ, which shows the position of the crank at this instant.

The exact relative value of the instantaneous current to the maximum current is perfectly clear, since

$$\frac{VX}{OX} = \sin \phi.$$

Example 2. The maximum value of the current through the pipe is 48 gal./min. Find the current at the instant when the crank has moved 40° from the horizontal.

Draw OX, Fig. 17, representing 48 gal./min. at an angle of 40° with the horizontal. Then XV represents the instantaneous value of the current.

$$XV = 48 \sin 40^\circ$$
$$= 30.9.$$
$$XV = i = 30.9 \text{ gal./min.}$$

Note that the equation

$$i = I_m \sin \phi$$

is apparent from the construction of the figure.

When the crank reaches the vertical position, or 90° from the zero, the current has reached its maximum. Note in Fig. 18 how the instantaneous value i coincides exactly with the maximum value I_m. The equation also shows this.

$$\begin{aligned} i &= I_m \sin 90°, \\ I_m &= 48, \\ \sin 90° &= 1, \\ i &= 48 \times 1 \\ &= 48 \text{ gal./min.} \end{aligned}$$

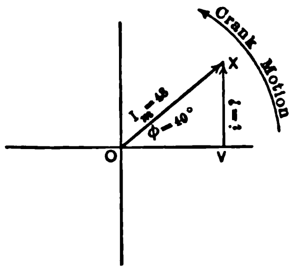


FIG. 17. The line XV represents the value of the current when the crank is at the 40° position.

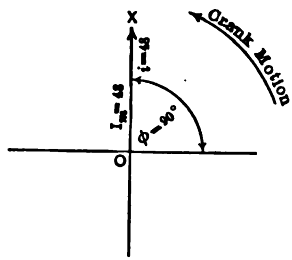


FIG. 18. The line OX represents the current when the crank is 90° from the zero position.

When the crank has passed beyond the maximum position and has reached say 125°, the instantaneous value of the current is easily found by constructing Fig. 19.

$$i = 48 \sin 125°.$$

The figure shows clearly that $\sin 125° = \sin (180° - 125°) = \sin 55°$.

Thus

$$\begin{aligned} i &= 48 \sin 55° \\ &= 48 \times 0.819 \\ &= 39.3 \text{ gal./min.} \end{aligned}$$

Assume that the crank reaches a position 210° from the zero position. The current at that instant can be found from Fig. 20. The line OX or I_m is drawn in the position of the crank, and the line VX, or i, represents the instantaneous current at this instant. Note that its direction is downward, which shows immediately that the value is negative.

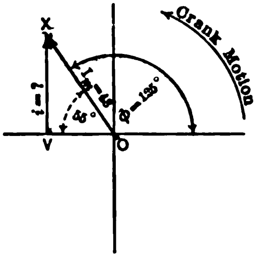


FIG. 19. The line VX represents the value of the current when the crank is 125° from the zero position.

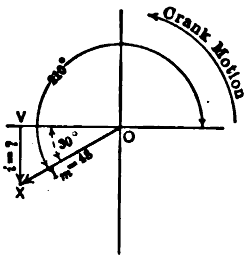


FIG. 20. The line VX represents the value of the current when the crank has reached a position 210° beyond the zero position. Note that XV is negative.

$$i = I_m \sin 210°$$
$$I_m = 48$$
$$\sin 210° = - \sin (210° - 180°)$$
$$= - \sin 30°$$
$$i = 48\,(- 0.500)$$
$$= - 24 \text{ gal./min.}$$

Enough examples have been given to illustrate the ease and certainty with which problems can be solved by this method. The line OX is called a **vector,** hence the name **vector diagram** is given to the figure. Since the diagram often has the appearance of the hands of a clock, it is also called the **clock diagram,** though the rotation is in the opposite direction to the hands of a clock.

In solving all problems in alternating current it is of the utmost importance that all three methods be used for each example, in the following order.

First. Draw a rough sine curve in order to place clearly before the eyes what is happening in the circuit. A sine curve is a panorama photograph or a sort of moving picture of what is happening in the system.

Second. Draw a rough vector diagram, in order to place clearly before the eyes the conditions at any given instant, and to show obviously and definitely the relations existing between the various values at that instant.

A vector diagram is a sort of snapshot of a moving crank.

Third. Paying careful attention to the vector diagram, write the equation for an alternating current which is being considered

$$i = I_m \sin \phi.$$

Example 3. What is the instantaneous value of the water current in system of Fig. 9, when the crank is at the 290° position? Maximum value = 800 gal./min.

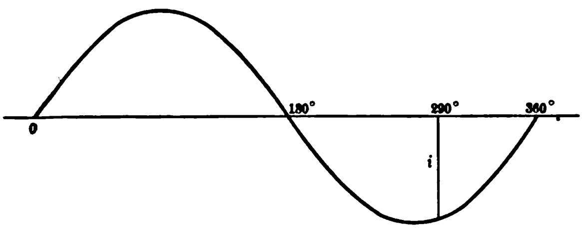


FIG. 21. Rough sine curve showing the direction and relative value of the current when the crank is at the 290° position.

First. Draw sine curve, Fig. 21.

From this we see that the current is negative but is dying out.

Second. Draw vector diagram, Fig. 22.

From this we see that the crank really makes an angle of $-70°$ to the horizontal, and thus the instantaneous current is negative.

Third. From vector diagram write the equation:

$$i = I_m \sin 290°$$

or

$$i = -800 \sin 70°$$
$$= -751 \text{ gal./min.}$$

Solve the following as in above example, using all three methods for each problem.

Prob. 15–2. What current is flowing in a system similar to the one in Fig. 9, when it has completed 65° of its cycle? The maximum current equals 25 gal./min.

Prob. 16–2. An alternating current of water in a system has completed $\frac{1}{8}$ of its cycle and has a value of 15 gal./min. What maximum value of current flows?

Prob. 17–2. What would be the instantaneous current in circuit of Prob. 16 when $\frac{1}{3}$ of the cycle was completed?

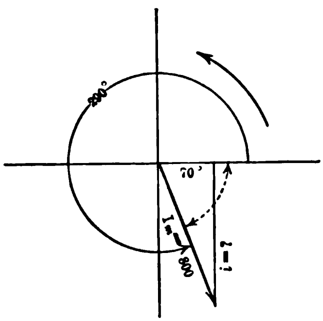


FIG. 22. Vector diagram for case shown in Fig. 21.

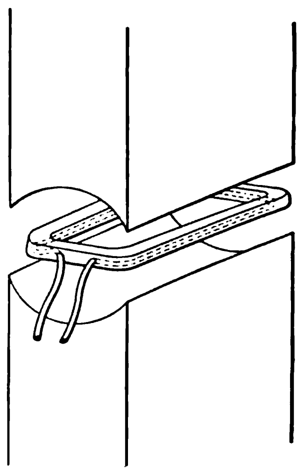


FIG. 23. Simple coil, used to generate an alternating e.m.f. of sine wave form.

Prob. 18–2. What position would crank in Prob. 15 have when the current had a positive or clockwise value of 10 gal./min. and was growing?

Prob. 19–2. What position would crank in Prob. 18 have when the current had an instantaneous value of 10 gal./min. negative or counter-clockwise and was dying?

15. Alternating E.M.F. Whatever we have learned about the alternating current of water in the pipe system of Fig. 9 can be applied directly to an alternating current of electricity. Modern alternators generate an electromotive force or alternating voltage, which goes through a cycle of values exactly similar to the cycle of values which the current of water went through. The same sine wave is used to represent the values at the different stages of the cycle; the same vector diagrams are drawn to represent the relation of any instantaneous value to the maximum value, and the same equation is used to compute the different values.

Before taking up the commercial machines, it is more profitable to study the simplest possible electrical device for generating an alternating electromotive force which has a sine wave form. Such a device is shown in Fig. 23. The field poles are so shaped that the field between them is practically uniform and parallel. When the single coil of wire is revolved in this field, an alternating e.m.f. is set up between the terminals of the coil. This e.m.f. wave has practically the sine form.

In Fig. 9, the crank was assumed to be rotating counter-clockwise at uniform speed. Similarly assume the coil in Fig. 23 to be rotating counter-clockwise at uniform speed.

Just as the position of the engine crank at any instant determined the amount and direction of the water current flowing at that instant, so here the position of the coil at any instant determines the amount and direction of the e.m.f. generated at that instant.

When the crank was horizontal, there was no current flowing through the pipe. Similarly, when the coil is horizontal, as in Fig. 24, it is moving parallel to the lines of force, not cutting them, and thus no e.m.f. is being induced across the terminals. Fig. 25 then represents the vector diagram for this position of the coil, just as it would represent the current in the pipe line at the same instant. The only difference would be the lettering of the lines. Thus E_m here stands for the maximum e.m.f. which is the greatest value that the e.m.f. attains during the cycle. If the diagram represented the water current, this line would be lettered I_m, which stands for the greatest value that the current attains during its cycle. The method of obtaining the amount of this maximum value of the e.m.f. is explained in the following paragraph.

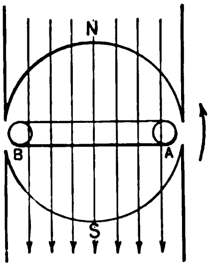


FIG. 24. Coil *AB* is passing through the zero position and is cutting no lines of force.

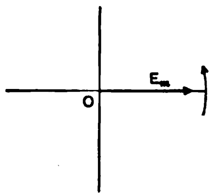


FIG. 25. The vector diagram of the e.m.f. in the coil of Fig. 24. The e.m.f. at this instant is zero.

We have seen that a line drawn from the end of this vector down to the horizontal represents the instantaneous current. Such a line would also represent the instantaneous e.m.f. in this case. But the vector is exactly on the horizontal, so no line can be drawn from the end of it down to the horizontal. Therefore the instantaneous value is zero when the coil is in this position. This we have seen to be the case, since the coil is cutting no lines of force.

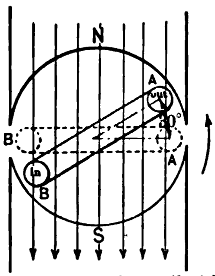


FIG. 26. The coil *AB* is passing through a position 30° from the zero position. An e.m.f. is induced which tends to send a current *out* at *A* and *in* at *B*.

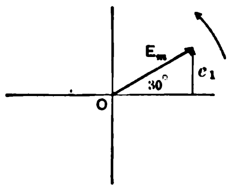


FIG. 27. The vector diagram of the e.m.f. in the coil of Fig. 26. The line e_1 represents the value of the e.m.f. at this instant.

When the coil is passing through the position at an angle of 30° to the horizontal, Fig. 26, there is an e.m.f. induced which tends to cause a current to flow out at A. Test this by "the right-hand rule for induced e.m.f." *

Fig. 27 represents the vector E_m in this position, just as it might represent a crank in this position. The line e_1 represents the voltage that is being induced across the coil when it is passing through this position, just as it might represent the current flowing through a pipe line when the crank was at this position.

But e_1 is merely that fraction of E_m which is represented by the sine of the angle 30°; thus

$$e_1 = E_m \sin 30°.$$

Assume $$E_m = 250 \text{ volts}.$$

Then

$$\begin{aligned} e_1 &= 250 \sin 30° \\ &= 250 \times 0.500 \\ &= 125 \text{ volts}. \end{aligned}$$

That is, if the maximum induced voltage across the coil were 250 volts, there would be only 125 volts induced when the coil was at the 30° position. But when the coil had

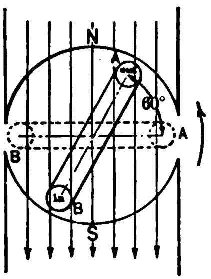


FIG. 28. The coil AB is passing through the 60° position.

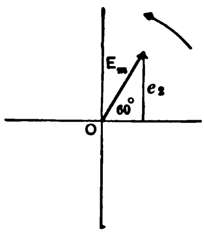


FIG. 29. The vector diagram showing the value of the e.m.f. in the coil in Fig. 28.

reached the 60° position, Fig. 28, it would be cutting lines faster, since it would be moving more nearly straight across them. Thus the induced voltage would be greater at the 60°

* See Timbie's "Elements of Electricity," Art. 105, page 160.

than at the 30° position. To find the value we draw the vector at an angle of 60° to the horizontal, as in Fig. 29. The line e_2 then represents the voltage at this instant. The equation for the value of e_2 is apparent from the diagram.

$$\begin{aligned} e_2 &= 250 \sin 60° \\ &= 250 \times 0.866 \\ &= 217 \text{ volts.} \end{aligned}$$

When the coil has reached the 90° position, Fig. 30, it is moving across the lines of force directly at right angles and is therefore cutting them at the greatest rate. The induced voltage is therefore maximum at this instant.

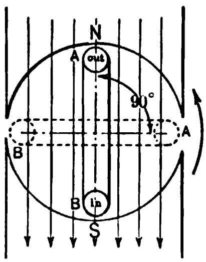


FIG. 30. The coil AB is passing through the 90° position.

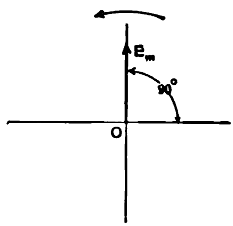


FIG. 31. Vector diagram of e.m.f. in coil at 90° position.

Fig. 31 is the vector diagram for this instant and shows that the instantaneous voltage line must coincide exactly with the maximum voltage line E_m. The equation also shows this fact.

Let e = instantaneous e.m.f. when the angle is 90°.

Then
$$\begin{aligned} e &= E_m \sin 90° \\ \sin 90° &= 1 \\ e &= E_m \times 1 \\ &= E_m. \end{aligned}$$

If E_m is 250 volts, then e, at this instant, equals 250 volts.

Fig. 32 shows the vector diagram of e.m.f. when the coil has reached the 120° position. The vector E_m is drawn at an angle of 120° from the horizontal, and e_3 represents the value of the induced voltage at this instant. By the equation:

$$e_3 = E_m \sin 120°$$
$$\sin 120° = \sin (180° - 120°)$$
$$= \sin 60°.$$

Thus
$$e_3 = E_m \sin 60°$$
$$= 250 \times 0.866 = 217 \text{ volts.}$$

When the coil has reached the 180° position, it has again ceased cutting lines of force, and therefore the induced e.m.f. has again dropped to zero.

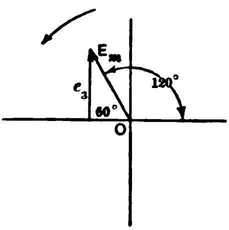


FIG. 32. Vector diagram of the e.m.f. when the coil has reached the 120° position.

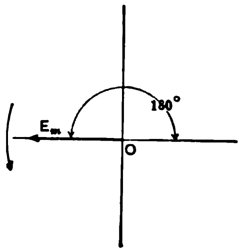


FIG. 33. The vector diagram of the e.m.f. when the coil is at the 180° position.

Fig. 33 is the vector diagram for this instant and the equation obtained from it is

$$e = E_m \sin 180°$$
$$\sin 180° = \sin (180° - 180°)$$
$$= \sin 0°$$
$$= 0.$$

Thus
$$e = 250 \times 0$$
$$= 0.$$

When the coil has passed the 180° position, it begins to cut lines in the opposite direction, as is seen from Fig. 34, where the coil is moving through the 210° position. Thus, the voltage which is induced is in the opposite direction and tends to send a current in at *A* instead of out as before.

Fig. 35 is the vector diagram for the voltage at this instant. Note that the instantaneous voltage line e_1 is below the horizontal and therefore has the negative sign,

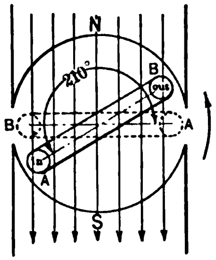


FIG. 34. The coil *AB* is moving through the 210° position and the e.m.f. has been reversed.

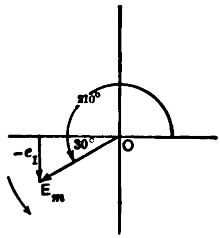


FIG. 35. Vector diagram showing the e.m.f. when the coil is passing the 210° position.

which means merely that the induced e.m.f. has changed direction. The value of e_1 can be found as usual from the equation:

$$
\begin{aligned}
e_1 &= E_m \sin 210^\circ \\
\sin 210^\circ &= -\sin(210^\circ - 180^\circ) \\
&= -\sin 30^\circ \\
&= -0.500 \\
e_1 &= 250 \times (-0.500) \\
&= -125 \text{ volts.}
\end{aligned}
$$

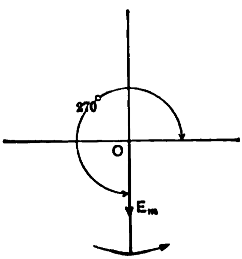


FIG. 36. Vector diagram of the e.m.f. when the coil is passing through the 270° position.

As the coil proceeds, the e.m.f. in this reversed direction becomes larger and larger until it reaches a maximum value at the 270° position as seen from the vector diagram Fig. 36. This maximum value is exactly equal to the maximum value when the coil was at the 90° position. It is merely tending to send a current in the opposite direction and thus has the negative sign.

The induced e.m.f. now begins to decrease as the coil continues to revolve, until it again becomes zero, just as the coil has completed 360° and is starting the cycle over again. During each revolution the induced e.m.f. goes through this cycle of values, having two zero positions (0° and 180°) and two maximum positions (90° and 270°). To represent the instantaneous values at all instants, we plot the sine curve, just as in the case of the alternating current of water in the pipe line from the pump.

Fig. 37 is the sine curve, assuming 250 volts maximum value of the induced e.m.f. Note that the voltage is con-

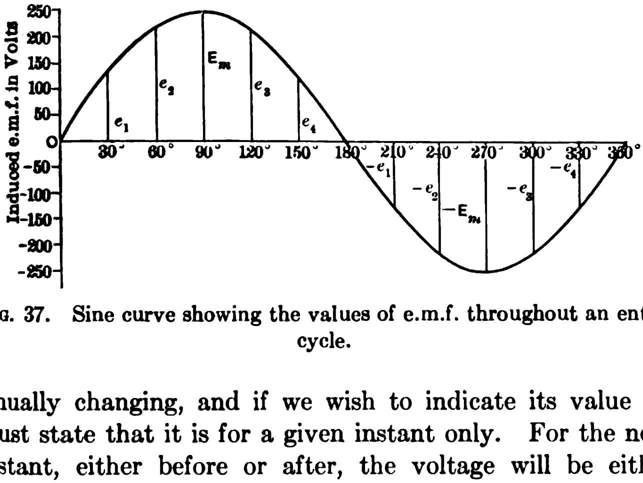


FIG. 37. Sine curve showing the values of e.m.f. throughout an entire cycle.

tinually changing, and if we wish to indicate its value we must state that it is for a given instant only. For the next instant, either before or after, the voltage will be either greater or less. Fig. 37a is an oscillogram from the Proceedings of the A.I.E.E. and shows the curve of the terminal voltage of a 6600-volt generator. Note how closely it follows the form of a sine curve.

Every modern alternator maintains an alternating e.m.f. having approximately this wave form. The closer the approximation, the better the machine, other things being equal.

A machine with a wave form essentially different from this standard sine curve would be of little use in a modern plant. To be sure, commercial alternators are not built of single coils revolving in a bipolar field. Modern types have several pairs of poles and a large number of coils, and in most cases the poles, instead of the armature coils, revolve. The e.m.f. induced in each coil is not necessarily a sine curve, but a number of coils are so arranged that the e.m.f. across the combination has this standard form.*

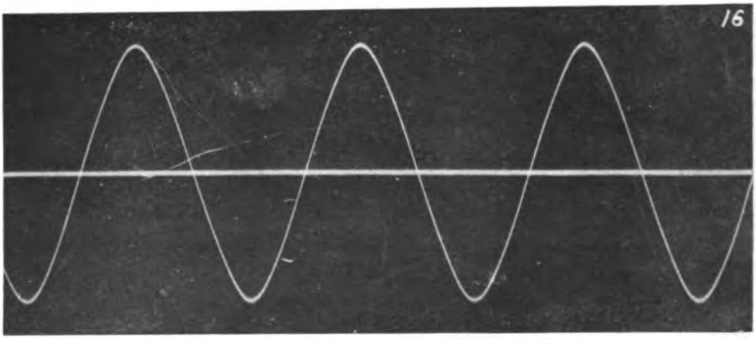


FIG. 37a. Curve 16. — Potential wave no load. 150 kv-a., 60-cycle, three-phase generator, $1\frac{1}{3}$ slots per phase per pole. *Proc. A.I.E.E.*

It is necessary, therefore, for us to think of this cycle of values of an alternating e.m.f. apart from the coil in which it is induced. Accordingly, we divide up each cycle into 360 degrees, regardless of the mechanical position of the coil throughout the cycle. We call these divisions **"electrical degrees"** to distinguish them from the units of angular measure.

An "electrical degree" therefore means a $\frac{1}{360}$th part of the "period" of the alternating e.m.f., or of the **time** required to complete one cycle of values. For this reason, an "electrical degree" is sometimes called also a "time degree," either term distinguishing it from a "space degree," which

* The details of this construction are described and explained in Chapters VIII and IX.

refers to the mechanical position of the coil in space or in relation to the other parts of the machine.

The "period" of the e.m.f. is the time required for a conductor to pass from a position under an N pole to an exactly similar position under the next N pole; it is the time required for a complete cycle of values of the e.m.f., or for the conductor to pass under a pair of poles. The e.m.f. induced in a coil therefore passes through as many cycles, during one revolution of the coil, as there are **pairs of poles.** Thus in a two-pole machine the e.m.f. passes through 360 electrical degrees or one cycle in the same time in which it is passing through 360 mechanical degrees or one revolution. In a four-pole machine the e.m.f. passes through 360 electrical degrees **twice** during the same time in which it is passing once through the 360 mechanical degrees; in an eight-pole, four times, and so on.

Therefore, when an alternating e.m.f. has passed through a quarter of its cycle, we say it has reached the 90° position; when it has passed through half its cycle, 180°, etc. The coil in the meantime would have moved through but 10 mechanical degrees if the machine has 18 poles, or 15 mechanical degrees if it has 12 poles, corresponding in each case to 90 electrical degrees. For the time being, the motion of the coil is immaterial to us. We are interested merely in the varying values of the e.m.f. produced during one cycle. Therefore, in the future, when we plot a sine curve, the horizontal axis will be understood to represent the "electrical or time degrees" through which the quantity is varying; and when we draw a vector diagram, the angle which the vector makes with the horizontal will be understood to be measured in "electrical degrees," since it represents that part of a cycle already passed through by the alternating e.m.f. without regard to the position of any coil.

Thus in Fig. 35 we think of the e.m.f. as having passed through 210° of its cycle and of e_1 representing the value at that instant. To be sure, in the case in hand, the coil in which this voltage was induced had also passed through 210° of its cycle. But this was a special case chosen for the sake of clearness. The position of the coil need not have corresponded to the number of electrical degrees passed through; and would not have, but for the fact that the field was bipolar.

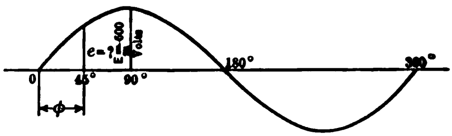


FIG. 38. Rough sine curve showing relative value of the e.m.f. at the 45° position.

Notice how the following examples are solved without any reference to the method of producing the alternating e.m.f.

Example 4. What is the instantaneous value of an alternating e.m.f. when it has passed through 45° of its cycle? The maximum value is 600 volts.

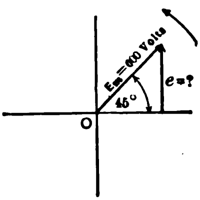


FIG. 39. Vector diagram showing the value of the e.m.f. at the 45° position.

First. Sketch roughly the sine curve as in Fig. 38.

Second. Draw the vector diagram as in Fig. 39.

Third. Write the equation:

$$\begin{aligned} e &= E_m \sin \phi \\ &= 600 \sin 45° \\ &= 600 \times 0.707 \\ &= 424 \text{ volts.} \end{aligned}$$

Solve as above, using sine curve, vector diagram and equation for each problem.

Prob. 20-2. The maximum value of an alternating e.m.f. is 2200 volts. What is the instantaneous value when 65° of the cycle have been completed?

Prob. 21-2. What is the instantaneous value of e.m.f. of Prob. 20 when it has passed through 200° of its cycle?

Prob. 22-2. The instantaneous value of an alternating e.m.f. is 1400 volts at 75°. What is the maximum value?

Prob. 23-2. The value of an alternating e.m.f. is 450 volts 35° after its zero value. What is it at 135°?

Prob. 24-2. Plot the sine curve, to some convenient scale for one complete cycle for Prob. 23.

Prob. 25-2. The maximum value of an alternating e.m.f. is 800 volts. What are the instantaneous values at the following instants: 20°, 80°, 130°, 210°, 300°, 340°?

Prob. 26-2. How much of its cycle has the e.m.f. of Prob. 22 completed, when the value is 300 volts positive and increasing?

Prob. 27-2. What value and what sign will the instantaneous voltage have when an e.m.f. has completed two-thirds of its cycle if E_m is 2300 volts?

Prob. 28-2. At what four instants in one cycle will the e.m.f. of Prob. 27 have the value of 1000 volts?

16. To Find the Maximum Value of an Alternating E.M.F. We learned, when studying the direct-current generator, that the total number of lines cut per second by a conductor determined the average value of the voltage induced in the conductor. Thus

$$\text{Av. } E = \frac{\text{lines cut per sec.}}{10^8}.$$

Now, if the several conductors which make up a coil are bunched or **concentrated,*** there is practically the same voltage induced in each conductor at any given instant. The average voltage in the coil at any instant is, then, merely the voltage in any one conductor multiplied by the number of conductors in the coil.

Of course in an armature where the conductors making up a coil are **distributed*** over the core, all are cutting at different rates at any given instant. The average voltage across such a coil is clearly not the voltage across each conductor multiplied by the number of conductors.

* See Art. 102.

Thus, to find the average voltage induced in an alternator using an armature with **concentrated** windings, we use the above equation, taking into consideration the number of conductors in each coil.

To find the maximum value of the induced e.m.f. of such a generator it is merely necessary to know what part of the maximum this average value is.

Since half of the instantaneous values of an alternating e.m.f. are negative and half are positive, and since the negative values are exactly equal to the positive values, the average of a complete cycle of values must be zero. But the actual average value of the e.m.f. of a generator, as is seen from the above equation, is not zero. It would be just as reasonable to say that the actual average value of the pressure exerted by the piston of the water pump of Fig. 9 is zero, just because the pressure alternates with equal values in opposite directions. The actual average value in both cases is the average of all instantaneous values **regardless of signs,** which indicate direction only.

The average value of an alternating e.m.f. can be found very easily by plotting a number of the instantaneous values at equal intervals throughout the cycle and finding their average. Since the average values for the second, third and fourth quarters of the cycle would be the same as the average value of the first quarter, we need plot the first quarter cycle only.

Prob. 29–2. Let the maximum e.m.f. for convenience be taken as 100 volts. Compute the instantaneous e.m.f. for every 2° during the first quarter of a cycle. Start with 1°, 3°, 5°... to 89°. This will give 45 instantaneous values. The sum of these values divided by 45 will give the average value of the e.m.f. for the first quarter, which equals the average value for the whole cycle as explained above. What fraction is this average value of the maximum value?

The result of the above problem shows that the average value of an alternating e.m.f. is approximately equal to

0.636 of the maximum value. In the form of an equation this may be written,

$$\text{Av. } e = 0.636\, E_m$$

or

$$E_m = 1.57 \text{ Av. } e.$$

Thus to find the maximum e.m.f. which a generator with concentrated armature windings delivers, we have merely to find the average e.m.f. and multiply by 1.57. This is about the only connection in which we ever use the average value of an alternating e.m.f.

Example 5. What is the maximum voltage generated in a coil of a drum armature consisting of 300 series conductors, concentrated winding, which has a speed of 1200 r.p.m.? Each conductor cuts twice through a field of 1,500,000 lines of force during each revolution.

$$\begin{aligned} \text{Av. } e &= \frac{\text{lines cut per sec.}}{10^8} \\ &= \frac{1{,}500{,}000 \times 2 \times 1200 \times 300}{10^8 \times 60} \\ &= 180 \text{ volts.} \\ E_m &= 1.57 \text{ Av. } e \\ &= 1.57 \times 180 \\ &= 283 \text{ volts.} \end{aligned}$$

Prob. 30–2. What is the instantaneous value of the voltage in above example when 40° of the cycle have been completed?

Prob. 31–2. A bipolar a-c. generator with a drum armature, concentrated winding, has a speed of 2400 r.p.m. The field of each pole has 2,500,000 lines of force. Number of series conductors in each path on armature is 600. What is the maximum value of the e.m.f. generated?

Prob. 32–2. What is the instantaneous value of the e.m.f. 60° after the zero value in Prob. 31?

Prob. 33–2. A 4-pole 2-brush a-c. generator has a drum-wound armature with 2000 series conductors in each path, concentrated winding. Speed is 1200 r.p.m. Flux from each pole is 2,400,000 lines. What is average e.m.f.?

Prob. 34–2. What is maximum value of e.m.f. of Prob. 33?

Prob. 35–2. What is the instantaneous value of the e.m.f. of Prob. 33 when 45° of its cycle have been passed through? Would the 45° position of the e.m.f. vector be reached at the instant in which the armature had turned 45 space degrees from its zero or neutral position?

Prob. 36–2. In an alternator, one set of coils containing 200 concentrated conductors is being cut 8 times each second by 10^7 lines. What maximum voltage is induced in the coils?

Prob. 37–2. What is the voltage in coils of Prob. 36 when $1\frac{5}{8}$ of the cycle is completed?

Prob. 38–2. How much of the cycle in Prob. 36 is complete when the instantaneous e.m.f. is 80 volts positive and increasing?

Prob. 39–2. It is desired to generate a maximum alternating e.m.f. of 10,000 volts. The number of concentrated conductors in series in each path of the armature is 500. There are two poles and the machine is run at a speed of 2400 r.p.m. How many lines of force must there be in the field?

Prob. 40–2. A 4-pole 2-brush a-c. generator has 1000 concentrated conductors in series on the armature. Flux from each pole is 6×10^6 lines. It is desired to generate a voltage of 7550 maximum. At what speed must machine run?

Prob. 41–2. What would be the instantaneous voltage when the e.m.f. of machine in Prob. 39 had completed 165° of its cycle?

Prob. 42–2. What would be the average voltage of the generator in Prob. 40 if the speed were reduced one-half and the flux doubled?

Prob. 43–2. (*a*) What is the frequency of the generator in Prob. 31? (*b*) Of the generator in Prob. 39? (*c*) In Prob. 40?

17. Alternating Currents of Electricity. It is natural to expect that when an alternating e.m.f. of sine wave form is impressed upon a circuit, an alternating current of the same wave form will be set up in the circuit; and this is the fact except in a very few special cases which will be taken up later.

The diagrams, Fig. 25–39, which have been drawn to show the relation between instantaneous values and the maximum value of alternating e.m.f., serve just as well to show the relation between instantaneous values and the maximum value of alternating current. We have merely to put i_1, i_2, I_m, i_3, i_4, $-i_1$, etc., in the place of e_1, e_2, etc., and all diagrams show current instead of voltage relations.

The same three methods that we used for e.m.f. values can be used to solve problems in current values.

(1) Thus in Fig. 40, which is identical with Fig. 37 except for the lettering, i_1 is the current at 30° from zero

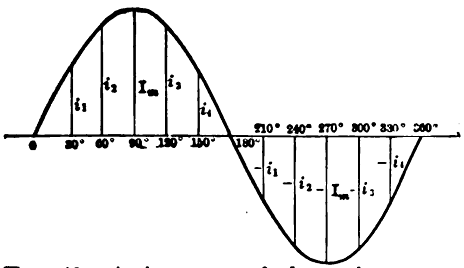


FIG. 40. A sine curve of alternating current showing the values at each instant.

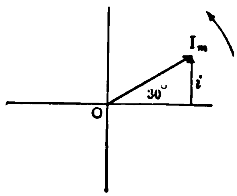


FIG. 41. Vector diagram of the current showing the instantaneous value (i) after 30° have elapsed.

value, i_2 the value of the current at 60°, and I_m at 90°, etc. The current thus follows a curve of the same shape as that of the e.m.f.

(2) The instantaneous values can also be represented by a vector or clock diagram, as in Fig. 41, which is identical with Fig. 27 except for the lettering. I_m represents the maximum value of the current, and i_1 represents the value of the current at the instant 30° after the zero value; i_1 is equal to $I_m \sin 30°$, and can be found from the general equation:

(3) $$i = I_m \sin \phi;$$

when i = instantaneous value of an alternating current
I_m = maximum value
ϕ = phase angle in electrical degrees.

As in the case of the alternating e.m.f. it is best to use all three methods for solving each problem.

Prob. 44-2. What is the instantaneous value of an alternating current at 20° phase? The maximum value is 65 amperes.

Prob. 45-2. What is the maximum value of an alternating current when the value at the 65° instant is 114 amperes?

Prob. 46-2. What are the instantaneous values of current in Prob. 45 at the 180° instant; 200°; 300°?

18. Effective Value of Alternating Current. Just as the average value of an alternating e.m.f. equals 0.636 of the maximum value, so the average value of an alternating current equals 0.636 of its maximum value. But we do not measure an alternating current by its average value, so this fact is of little use. We use instead what is called the **effective value** of the current, for the following reasons.

An alternating current really has no unit of its own, so we measure it in terms of the direct-current unit — the ampere. To have the **alternating-current ampere** exactly equal the **direct-current ampere,** it must produce the same effect as the direct-current ampere. Now an ampere is defined as that steady rate of flow which will deposit a standard amount of silver from a standard solution in one hour. But an alternating current is not a steady current and neither will it deposit any silver from a solution; since whatever it deposits during one-half a cycle it takes off the next half, when it is flowing in the reverse direction.

Accordingly, in order to compare the alternating with the direct current, we must use some other property, which both kinds of current possess. The most natural is the heating effect of each.

Therefore an alternating current is said to be equivalent to a direct current when it produces the same average **heating effect,** under exactly similar conditions. This value is called the **effective value** of an alternating current, and is equal to the amperes of the direct-current equivalent. It is somewhat greater than the average value, being equal to 0.707 of the maximum value.

Why the effective value is greater than the average value, can be explained as follows.

The heating effect of any electric current is proportional at every instant to the square of its value at that instant. Therefore the heating effect of an alternating current during any period of time would depend upon the squares of all the instantaneous values during that length of time. It would not do to get the average of all these instantaneous values and square them in order to find the average heating effect. This average heating effect can be found only by squaring all the instantaneous values and then averaging these.

At first thought it might be supposed that it makes no difference whether we find the average of the values first and then square them, or square the values and then find the average. Suppose we try this on two simple values like 4 and 6.

Squaring them first

$$4 \times 4 = 16$$
$$6 \times 6 = 36$$
$$2\overline{)52}$$

Average of the squares $= 26$

Averaging them first

$$\frac{4 + 6}{2} = 5.$$
$$5 \times 5 = 25.$$

Square of the average $= 25.$

Average of the squares $= 26.$

This shows clearly that the average of the squares of numbers is greater than the square of the average of the numbers.

We have seen that the heating effect depends upon this larger value — the average of the squares. Our problem is then to find some number, which when squared will be equal to the average of the squares of all the instantaneous values. This number is found by squaring all the instantaneous values first, averaging them, and then taking the square root of them. This value then is the number of both direct-current amperes and alternating-current amperes which would produce the same heating effect; the heating effect of each current being proportional merely to the square of this number.

The **effective value** of an alternating current is thus often called the **"square root of mean squares,"** or the R. M. S. (root-mean-square) value because it can be found by squaring a number of the instantaneous values in a cycle, finding the average of these squares, and then extracting the square root. This effective value of an alternating current is the current which the a-c. ammeters indicate and is the value by which alternating currents are measured, unless it is definitely specified otherwise.

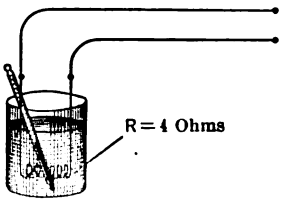


FIG. 42. The alternating current will heat the 4-ohm wire.

That the effective value is equal to 0.707 of the maximum value can be shown by the following problem.

Prob. 47–2. Assume an alternating current, for convenience say of 10 amperes maximum value, and a frequency of 25 cycles. It is desired to determine the direct current to which it is equivalent; that is, its effective value. Assume for convenience this current to run through a 4-ohm wire immersed in water in a calorimeter, Fig. 42.

Construct on fine coördinate paper a sine curve of current i, Fig. 43, having a maximum value of 10 amperes, using at least 10 instantaneous values of current for each loop.

The rate, in calories per second, at which heat is given off by each instantaneous current thus drawn would equal ($0.24\,R$) times the square of this instantaneous current. Therefore, square each value of i, and multiply the result by 0.24×4, or 0.96, and plot curve H and H_1 from these values. Note that the square of a negative (i) is positive; thus H_1 is above the line. The area included in these loops represents and is directly proportional to the heat given off by all the instantaneous currents during one complete cycle. This is seen to be true from the fact that the area of the loops equals the average length of the ordinates multiplied by the abscissæ. The ordinates equal 0.24 i^2R, and the abscissæ are time units. Thus the product of the ordinates times abscissæ equals heat.

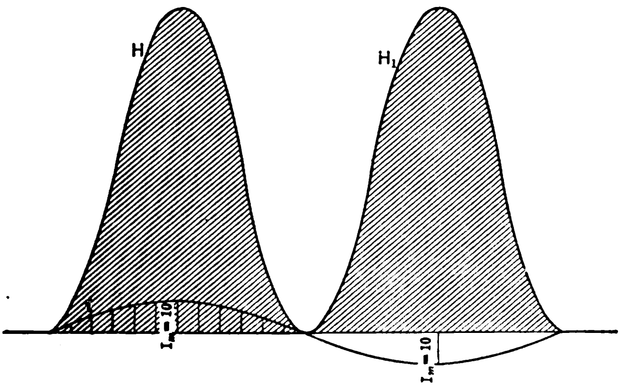


FIG. 43. The area of the loops H and H_1 represent the heat given off in one cycle.

Since the horizontal axis represents the time ($\frac{1}{25}$ second), the area of these two loops must be the total amount of heat which was generated during $\frac{1}{25}$ second. Find the area by noting the number of the smallest squares of the cross-section paper which the curves cover.

Compute from the equation $H = 0.24\, I^2\, Rt$ the value of a direct current which would develop the same amount of heat in this resistance in $\frac{1}{25}$ second. This is the effective value of the above alternating current. State what part of the maximum value, 10 amperes, this effective value is.

Prob. 48-2. The maximum value of an alternating current is 4 amperes. Plot the sine curve of one cycle of this current. Find the instantaneous values for every 2°. Square these instantaneous values and find the average of the squares. Extract the square root to find the effective current. What fraction of the maximum current is this effective current?

From each of the above problems it is found that the effective value of an alternating current is approximately 0.707 of the maximum value. By an equation this may be stated,

$$I = 0.707\, I_m$$

or

$$I_m = 1.41\, I.$$

But

$$1.41 = \sqrt{2}.$$

Accordingly the equation is often written

$$I_m = \sqrt{2}\, I$$

or

$$I = \frac{I_m}{\sqrt{2}},$$

where

I_m = maximum value of the current.
I = effective value of the current.

19. Effective Value of Alternating E.M.F. Since the sine wave is also the standard wave form for an alternating e.m.f., the effective value of the e.m.f. is also 0.707 of the maximum value. This relation is expressed by the equations

$$E = 0.707\, E_m = \frac{E_m}{\sqrt{2}},$$

or

$$E_m = 1.41\, E = \sqrt{2}\, E,$$

where

E_m = maximum value of e.m.f.
E = effective value of e.m.f.

When we speak of an alternating current of so many amperes and an alternating e.m.f. of so many volts, it is always the **effective current** and the **effective voltage** that is understood. The effective values are the standard values of alternating-current quantities, and instruments are graduated to read in terms of effective values.

The effective values of current and voltage may also be used in the vector diagrams instead of the maximum values, since the effective value is always the same definite fractional part of the maximum value of a sine wave. This merely amounts to increasing the scale of the diagram. However, if it is desired to use such diagrams for obtaining instantaneous currents and voltages, it is necessary always to multiply the values taken from the diagrams by $\sqrt{2}$, or 1.41.

Example 5. What is the effective value of an alternating current whose maximum value is 48 amperes?

$$I = 0.707 \times 48 = 33.9 \text{ amperes.}$$

Prob. 49-2. The effective value of an alternating current is 15 amperes. What is the greatest instantaneous value of this current?

Prob. 50-2. What is the average value of the current in Prob. 49?

Prob. 51-2. The effective value of an alternating current is 250 amperes. What is the instantaneous value at the 30° instant?

Prob. 52-2. What is the effective value of an alternating e.m.f. if the instantaneous value at 50° is 500 volts?

Prob. 53-2. What is the effective e.m.f. in Prob. 30?

Prob. 54-2. What is the effective voltage in Prob. 31?

Prob. 55-2. What would a voltmeter read if put across the terminals of the machine in Prob. 33?

Prob. 56-2. If you put a voltmeter across the terminals of the generator of Prob. 36, how much would it indicate?

Prob. 57-2. At what speed must generator in Prob. 39 run to generate 10,000 volts?

Prob. 58-2. At what speed must generator in Prob. 39 run in order to generate 7550 volts?

Prob. 59-2. It requires about 200 amperes alternating current to thaw out in a given time an ordinary ½-inch lead water pipe which has been frozen. If direct current were used, how many amperes would be needed to thaw out the pipe in the same time?

20. Relation Between Effective E.M.F. and Effective Current. Impedance. When studying direct currents we found that a certain ratio always existed between the voltage across the circuit and the current flowing through the circuit. This ratio we called the **resistance** of the circuit. Thus, if it required 20 volts to force 10 amperes direct current through a circuit, it would take 40 volts to force 20 amperes, or 10 volts to force 5 amperes, and we said that the resistance of the circuit was 2 ohms. That is, the ratio of the voltage to the current was 2, or we require two volts pressure to produce each ampere of current.

$$\frac{E}{I} = 2.$$

We express this by the equation

$$\frac{E}{I} = R,$$

where E = direct e.m.f. in volts.
I = direct current in amperes.
R = resistance in ohms.

Similarly there is a certain ratio between the alternating voltage of a given frequency across a circuit and the alternating current which it will force through the circuit.

But this ratio is usually not the same as the ratio would be between the direct voltage and the direct current in the same circuit. Thus 20 volts alternating e.m.f. may not force just 10 amperes alternating current through a line through which 20 volts direct e.m.f. would force 10 amperes direct current.

Accordingly, we call the ratio between the alternating e.m.f. of a given frequency and the current which it forces through a given circuit, the **impedance** of the circuit for that frequency and magnetic condition of the circuit, in order to distinguish it from the resistance. The impedance is measured in ohms, just as is resistance. We write it in the form of an equation

$$\frac{\textbf{Effective e.m.f.}}{\textbf{Effective current}} = \textbf{impedance}\ *$$

or

$$\frac{E}{I} = Z,$$

* Since it does not affect the value of the ratio if maximum values are used for both numerator and denominator, when the current and e.m.f. both have sine wave form the equation may be written

$$\frac{E_m}{I_m} = Z.$$

where E = effective alternating e.m.f. in volts.
I = effective alternating current in amperes.
Z = impedance in ohms.

Thus, if it requires 20 volts alternating e.m.f. to force 10 amperes through a circuit, we would say that the impedance of the circuit was $\frac{20}{10}$, or 2 ohms.

The resistance of the same circuit may be considerably less than 2 ohms. The impedance of a circuit will be the same as the resistance only when there is practically no magnetic or electrostatic effect produced by the current. This is very rarely the case, and the impedance of an electric circuit is an important factor in alternating-current work.

Example 6. What impedance has a circuit through which 110 volts alternating e.m.f. is able to force 5 amperes?

$$Z = \frac{E}{I}$$
$$= \frac{110}{5}$$
$$= 22 \text{ ohms.}$$

Prob. 60-2. The impedance of an electric circuit under certain conditions is 40 ohms. How many amperes can 2100 volts force through this circuit under these conditions?

Prob. 61-2. How many volts would be required to force 18 amperes through the circuit in Prob. 60, assuming all electrostatic and magnetic conditions to remain the same?

Prob. 62-2. What is the maximum value of the current in Prob. 60?

Prob. 63-2. An impedance coil takes 4.35 amperes when across a 110-volt a-c. system. What current will the same coil take when across a 440-volt a-c. system, all other conditions remaining the same?

Prob. 64-2. A transmission line carrying alternating current has a "drop" of 52 volts for 10 miles of line wire when transmitting 45 amperes. What is the impedance of the line per mile for the frequency and the spacing of line wires here used?

Prob. 65-2. An electric circuit has an impedance of 25 ohms when an alternating current of a certain frequency is sent through it. What average value must the voltage across the circuit have when the maximum current is 32 amperes?

SUMMARY OF CHAPTER II

ALTERNATING CURRENT is the term used to distinguish an electric current which flows back and forth in a circuit from a direct current which always flows in the same direction.

A CYCLE is said to be completed when an alternating current has passed through one complete set of values in both directions.

THE FREQUENCY is the number of cycles completed in one second.

THE ACTION OF WATER IN A CLOSED PIPE SYSTEM, operated by a valveless pump driven by an engine with a slotted yoke, gives a fair picture of the action of electricity in a closed circuit. The water current flowing through the system at any given time depends upon the speed of the piston, which in turn depends upon the position of the crank.

The speed of the piston, and thus the amount of the water current, is exactly proportional to the sine of the angle ϕ which the crank is making with the "dead-center" position.

THE EQUATION for the water current flowing at any instant is

$$i = I_m \sin \phi$$

where i = instantaneous current

I_m = maximum current

ϕ = angle which crank is making with "dead-center" position.

THE SINE CURVE is also used to show this relation between current and crank position. This form of wave is nature's standard, as illustrated by pendulum motion, etc., and has been adopted by engineers as the standard wave form which an alternating current should be made to follow.

THE VECTOR DIAGRAM, a snap-shot picture of the current, is also used to show this relation of current to crank position. The vector is a line drawn at the same angle to the horizontal that the crank makes to the "dead-center" position. The length of the vector when drawn to scale represents the maximum current in the pipe, and the vertical projection of the vector then equals the value of the current flowing when the crank is at that given position.

AN ALTERNATING E.M.F. of standard wave form goes through the same cycle as the water current in the above system, and is represented by the same equation, the same sine curve and the same vector diagram.

The equation usually has the form

$$e = E_m \sin \phi$$

where e = the instantaneous voltage

E_m = the maximum voltage

ϕ = the degrees of the cycle which have been completed.

A complete cycle is divided into 360°; thus if a quarter of the cycle has been completed at any instant, 90° are said to have been completed, etc.

FOR A GENERATOR with concentrated armature winding:

$$\text{Av. e.m.f.} = \frac{\text{number of lines cut per sec.}}{10^8},$$

$$E_m = 1.57 \text{ av. e.m.f.}$$

or

$$\text{Av. } e = 0.636\, E_m.$$

AN ALTERNATING CURRENT OF ELECTRICITY is set up by an alternating e.m.f. These currents usually have the same equation, same sine curve and same vector diagrams.

THE EFFECTIVE VALUE OF AN ALTERNATING CURRENT, which is the value always meant unless some other is specified, is that value in amperes of an alternating current which will produce the same heating effect as a direct current of the same number of amperes. The effective value equals the square root of the average of the squares of all the instantaneous values, and is equal to 0.707 or $\frac{1}{\sqrt{2}}$ times the maximum value. The equation is

$$I = 0.707\, I_m$$

$$I = \frac{I_m}{\sqrt{2}}$$

where I = effective current in amperes.
I_m = maximum current in amperes.

Similarly

$$E = 0.707\ E_m,$$

where E = effective e.m.f. in volts.

The effective values are often used in vector diagrams instead of the maximum.

IMPEDANCE is the name given to the result obtained by dividing the effective value of an alternating e.m.f. by the effective value of the alternating current which it can force through a given circuit under given conditions.

In an equation this is expressed

$$Z = \frac{E}{I} = \frac{E_m}{I_m}$$

or

$$E = IZ$$

or

$$I = \frac{E}{Z},$$

where Z is impedance in ohms.

PROBLEMS ON CHAPTER II

66–2. The current in a piping system similar to Fig. 9 is 8 gal./sec. when the crank is at the 70° position. What is the maximum current which is forced through the system at this crankshaft speed?

67–2. At what angles is the crank in Prob. 66 when the current is 6 gal./sec. clockwise?

68–2. At what angles is the crank in Prob. 66 when the current is 6 gal./sec. counter-clockwise?

69–2. The maximum current in a pipe system similar to Fig. 9 is 200 gal./min. At what positions of the crank will the value be 150 gal./min.? State in each case whether the current is clockwise or counter-clockwise.

70–2. The equation for the instantaneous current in a system like Fig. 9 is $i = 150$ gal./hr. $\times \sin 25°$. What is the instantaneous value when 145° of the cycle have been completed?

71–2. Through how many degrees from dead-center has the crank passed when the instantaneous value of the current in Prob. 70 is − 65 gal./hr. and dying?

72–2. Through how many degrees has crank passed when the instantaneous value of the current in Prob. 70 is − 65 gal./hr. and growing?

73–2. The instantaneous value of an alternating water-current is 160 gal./hr. when the crank, Fig. 9, is at the 75° position. What is the instantaneous value when the crank is at the 215° position?

74–2. What is the instantaneous value of current in Prob. 73 when the crank is at the 270° position?

75–2. What is the instantaneous value of the current in Prob. 73 when the crank is at the 90° position?

76–2. At what positions of the crank would the current in Prob. 73 be − 160 gal./hr.?

77–2. The instantaneous value of an alternating current is 120 gal./hr. when the crank is at the 100° position (Fig. 9). What is the value when the crank has reached the 180° position?

78–2. A 12-pole 60-cycle alternator has 9×10^5 lines per pole. There are 300 conductors concentrated in series on the armature. What maximum e.m.f. does machine generate?

79–2. If machine in Prob. 78 were run as a 25-cycle generator, what maximum e.m.f. would it generate?

80–2. At what speed must generator in Prob. 78 be run to generate a 40-cycle e.m.f.?

81–2. What would be the maximum voltage of generator of Prob. 80?

82–2. A generator produces an e.m.f. which has a maximum value of 1000 volts. What is the value of e.m.f. at an instant 0.001 second after the maximum + value, if the frequency is 60 cycles per second?

83–2. Suppose the sine wave of e.m.f. referred to in Problem 82 had a frequency of 25 cycles per second, what would be the value of e.m.f. 0.001 second after the maximum value + 1000 volts?

84–2. In Problem 82 what would be the e.m.f. 0.01 second after the maximum value + 1000 volts?

85–2. In Problem 83 what would be the e.m.f. 1.25 seconds after the maximum value + 1000 volts?

86–2. What value will the e.m.f. in Problem 82 have at an instant 0.001 second before the maximum value + 1000 volts?

87–2. What are the values of the e.m.f. at instant 0.001 second before and 0.001 second after the zero value, if the maximum instantaneous value is 1000 volts and the frequency 25 cycles per second?

88–2. The value of the e.m.f. of a generator is + 100 volts at an instant 0.001 second after the zero value. What is the maximum value of the sine e.m.f. wave, if the frequency is 60 cycles per second?

89–2. What is the maximum value of the e.m.f. of the generator of Problem 88, if the frequency is 25 cycles per second, other things being equal?

90–2. There is a time interval of 0.01 second between two successive zero values of the e.m.f. of a generator, whose variations are represented by a sine wave. What is the frequency of the e.m.f. in cycles per second, and what is the period in seconds?

91–2. The interval of time between the maximum values of e.m.f. + 1000 and − 1000 volts in a circuit carrying alternating current is 0.004 second. If the e.m.f. is varying according to the sine law, what is the value at an instant 0.001 second after + 1000 volts?

92–2. In the circuit referred to in Problem 91, what is the value and direction of the e.m.f. at an instant 0.001 second after the value + 300 increasing?

93–2. A generator whose e.m.f. alternates according to the sine law is turned very slowly but at uniform speed, with constant field excitation of normal value, and no load. A direct-current voltmeter attached to its terminals deflects alternately above and below the zero mark, reaching the maximum positive value of 125 volts just 44 times in 62 seconds by careful count. If this machine has 8 poles, what must have been its speed in revolutions per minute?

94–2. At what speed in r.p.m. must this generator in Prob. 93 turn in order to give its rated frequency of 60 cycles per second?

95–2. When driven at this speed with the same field excitation as in Problem 93, what will be the maximum value of the e.m.f. produced?

96–2. How many space degrees does the armature of the generator in Prob. 94 move through, during the shortest time required for the e.m.f. to change from + 600 to − 400 volts?

97–2. The e.m.f. of a generator, varying according to the sine law, has a value of + 120 volts at a certain instant, and rises to the maximum value, + 600 volts, 0.0012 second later. What is the frequency, in cycles per second?

98–2. A chronograph, or time-marking device to indicate extremely small intervals of time, is made by drawing a strip of paper between two needle points connected to the terminals of a high-tension 8-pole generator whose e.m.f. varies according to the sine law, and has a maximum value of 1100 volts. The paper is of such kind and thickness that it punctures when the e.m.f. reaches 1000 volts, and there is a high resistance in circuit to prevent an arc from forming after the spark passes. Draw a sketch showing what you think the record would look like if the paper were drawn in a straight line between these needle points at a speed of 1 foot per second, while the generator is being turned at 1500 rev. per minute. Dimension all parts of the sketch.

99–2. Suppose the distance between the middle of adjacent perforations on the strip were 0.15 inch, the speed of the generator having been measured as 1456 r.p.m. at the time the record was taken. What period of time did each inch of length of the strip represent, the perforations being uniformly distributed?

100–2. Sketch the appearance which the record taken as in Problem 98 might have, if the field excitation were adjusted to give a maximum e.m.f. of 2200 volts. From a comparison of the results of these two problems, state how the voltage should be adjusted to get the greatest accuracy in the time record.

101–2. The pole pieces of a certain a-c.-generator are so designed that the e.m.f. induced in each individual conductor on the armature varies according to the sine law, the speed of rotation of the armature being perfectly steady at every instant. The actual speed of the conductor is 4398 feet per minute. The generator has 6 poles. The active length of each conductor is 18 inches and the maximum value of e.m.f. induced in it is 1.8 volts. Calculate the flux density in lines per square inch at the middle of the pole where the flux is most dense, and also at points 10, 20, 30, 40, 50 and 60 mechanical degrees either side of this point.

102–2. Construct a table of possible speeds for 60-cycle generators having various numbers of poles up to 40.

103–2. Construct a similar table for a line of 25-cycle generators.

104–2. A reciprocating pump such as illustrated in Fig. 9 delivers 500 gallons per minute, and makes 30 single strokes per minute.

Calculate:

(*a*) The maximum rate, in gal. per minute, at which water is delivered at any instant during a stroke.

(*b*) The rate of flow as the crank passes dead-centers.

(*c*) The rate of flow as the crank passes successively through positions 22.5°, 45°, 67.5°, 90°, 112.5°, 135°, 157.5° and 180° from dead-center.

(*d*) From these results, the average rate of flow, gal. per minute, for one complete stroke. Compare this average with the value of 500 gal. per minute which was assumed at the beginning of the problem. Explain the difference, if any exists.

105–2. By use of the sine equation for rate of flow, calculate the rate in gal. per minute for the pump in Prob. 104 at successive instants 0.1 second apart, starting from dead-center and continuing until the other dead-center is reached. Calculate the average rate of flow from these values. Compare this result with that obtained in the preceding problem, and explain the difference, if any exists.

106–2. How many conductors must be added to the armature of the generator in Prob. 80 so that it may run as a 40-cycle machine and deliver the maximum e.m.f. of Prob. 78?

107–2. It is desired that the generator in Prob. 80 deliver the same maximum e.m.f. that it does in Prob. 79. If the change is to be produced by changing the field strength, by how many lines must each pole be weakened or strengthened?

108–2. A 60-cycle alternator having 24 poles with 3×10^6 lines per pole delivers an average e.m.f. of 2000. How many conductors are there in series on the armature, the winding being concentrated?

109–2. What is the maximum voltage of generator in Prob. 108?

110–2. At what speed does generator of Prob. 108 run?

111–2. At what speed would generator of Prob. 108 run to deliver a 40-cycle e.m.f.?

112–2. What would be the maximum voltage of the generator in Prob. 111 other things being equal?

113–2. By how many lines must the field of each pole of generator in Prob. 111 be increased to raise the maximum e.m.f. to that of generator in Prob. 108?

114–2. What is the instantaneous positive value of the e.m.f. of generator in Prob. 108, $\frac{1}{200}$ of a second after it has passed through a zero value?

115–2. What is the instantaneous positive value of the e.m.f. of generator in Prob. 111, $\frac{1}{200}$ of a second after it has passed through a zero value?

116–2. What fraction of a second after passing through a maximum value will the instantaneous e.m.f. of generator in Prob. 31 first be 100 volts?

117–2. What fraction of a second after passing through a maximum value will the instantaneous e.m.f. of the generator of Prob. 57 first be 100 volts?

118–2. What e.m.f. does generator in Prob. 40 generate?

119–2. An incandescent lamp takes an alternating current of 0.5 amp. What maximum value does the current through the lamp reach?

120–2. Assume 80 % of the power taken by the lamp in Prob. 119 is given off in heat. How many calories per hour does the lamp radiate, if the resistance is 200 ohms?

121–2. How many calories would the lamp in Prob. 120 radiate per hour if a direct current of the same number of amperes were flowing through it?

122–2. How many calories of heat are generated in one minute in a 40-ohm resistance by the passage of 150 amperes of alternating current?

123–2. How many amperes of direct current would be required to generate the same amount of heat in the resistance of Prob. 122 in the same time?

124–2. The heaters in trolley cars of average size require about 9 amperes direct current. The resistance of a heater averages 61 ohms. How many B.t.u. per hour are used in heating such a car?

125–2. What would be the maximum value of an alternating current which would deliver the same amount of heat through the same resistance to the car of Prob. 124 in an hour?

CHAPTER III

SERIES AND PARALLEL CIRCUITS

VOLTAGE AND CURRENT RELATIONS

21. Alternators in Series: in Same Phase. If two direct-current generators, one delivering 110 volts and the other 90 volts, are joined in series, as in Fig. 44, the combined voltage across the two is exactly equal to the arithmetical sum of the voltages across each, or 200 volts. Of course, if the two generators are so joined that they oppose or "buck"

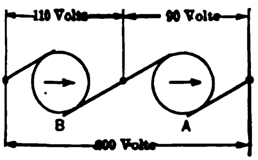


FIG. 44. Two direct-current generators in series. The voltage across the two equals 200 volts.

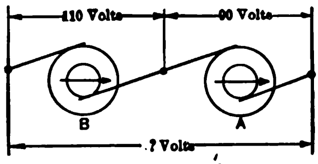


FIG. 45. Two alternating-current generators in series. The voltage across the two may not be 200 volts.

each other, the voltage across the two would be the arithmetical difference between the separate voltages, 110 − 90, or 20 volts.

If two alternating-current generators with sine-wave voltages of the same frequency, but one having an e.m.f. of 110 volts, the other 90 volts (effective), are joined in series as in Fig. 45, the combined voltage across the two is **not necessarily** either the arithmetical sum or the arithmetical difference between the two separate voltages. The total voltage depends upon the phase relations between the voltages of the two generators. Suppose that they are exactly in phase, that is, they both reach the maximum value in the same direction at the same instant; let us see what the resultant voltage is.

The conditions may be illustrated by Fig. 46, which shows two valveless pumps with cylinders of the same length joined together, but *I* has a smaller diameter than *II*. Exactly the same thing is taking place in both pumps at the same instant; that is, one piston is always in the same relative position and traveling in the same direction and at the same rate

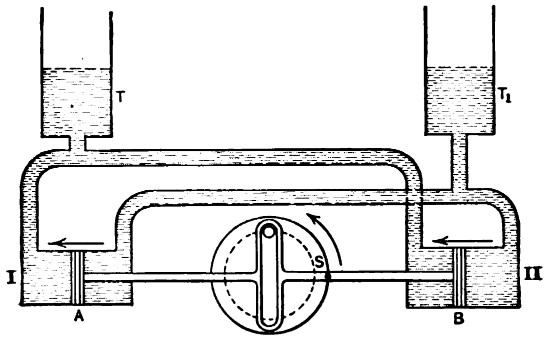


FIG. 46. The pump cylinders *I* and *II* are so joined that the water pumped into *T* equals the sum of the amount pumped by each piston.

as the other. The pumps would then be in phase with each other. At the instant shown, the piston in cylinder *I* is at the central position, *A*, and is tending to fill the stand-pipe *T* and to empty T_1. At this same instant, the piston in cylinder *II* is at a similar position and traveling in the same direction and is also tending to fill stand pipe *T* and empty T_1. Thus they both tend to increase the difference in level of the water between stand-pipes *T* and T_1. The effect produced is the sum of the effects of each. Therefore, the pressure head *h*, which is set up when the pistons in Fig. 47 are at *A* and *B*, respectively, is due to the sum of the volumes of water pumped from cylinder *I* and cylinder *II*.

This is exactly what would take place in the voltage of two generators when joined in series provided they are in phase with each other, as in Fig. 45. The voltage across

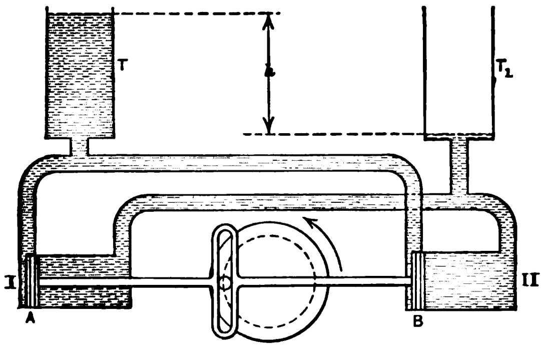


FIG. 47. The pressure head h is due to the sum of the quantities of water pumped by each piston.

the combination would be equal to the arithmetical sum of the voltage across each, 110 + 90, or 200 volts.

Fig. 48 is the sine curve representation of this case. If

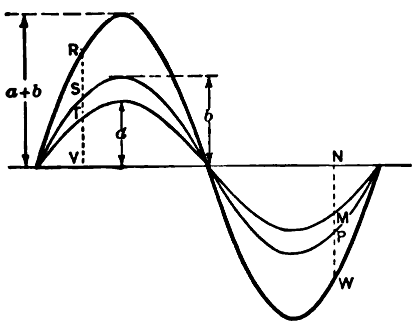


FIG. 48. The heavy curve represents the sum of the other two curves which show the value of the voltage in each generator of Fig. 45 at every instant throughout one cycle.

(a) is the maximum voltage of generator A, and (b) the maximum voltage of generator B, then the maximum voltage across the combination when in phase would be ($a + b$).

The sum of any two instantaneous values taken at the same instant would be the instantaneous value of the voltage across the series combination at that instant. The heavy line, then, is merely the sum of the other two curves. Thus, point R is found by adding the instantaneous values (TV) and (SV) together. Point W is found by adding the values $(-NM)$ and $(-NP)$ together.

Similarly, in the vector diagram, Fig. 49, the vector OA represents the maximum e.m.f. of generator A, and the

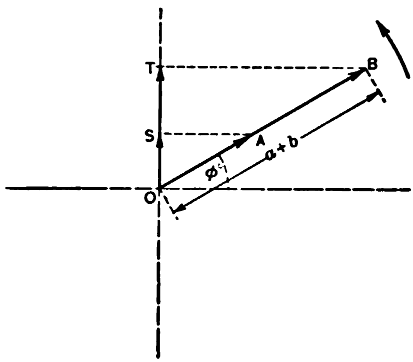


FIG. 49. Topographic vector diagram of e.m.f.'s of the generators in Fig. 45.

vector AB represents the maximum e.m.f. of the generator B. The sum of OA and AB represents the maximum e.m.f. of the two joined in series.

In the same way we can find the instantaneous value across the combination for the instant at which the vectors are drawn, that is, $\phi°$ after the zero position. OS represents the instantaneous value of the e.m.f. OA, and ST represents the instantaneous value of the e.m.f. AB. The sum of OS and ST will then be the instantaneous voltage, at the $\phi°$ position, across the two generators in series.

This way of joining vectors is called the **topographic method.** Note that the tail A of the vector AB is joined to the head A

of the vector OA. Thus the sum of the two vectors is represented by the line OB, which is made up of OA and AB. If OA represents 127 volts and AB, 155 volts, then OB represents 282 volts. Similarly, the sum of the instantaneous values of the e.m.f.'s is shown by the line OT, which is the sum of the lines OS and ST.

Fig. 50 shows the **polar diagram** for the same condition. Note that both vectors start from a common point O. The distance OA represents the vector OA and the distance OB represents the vector OB, although part of it lies along the vector OA. There is, then, no line which represents their sum. But no inconvenience is occasioned by this lack, since we draw the vector diagram mainly to enable us to write correctly the equation from which the voltage across the series combination can be found. It is just as easy to see the equation, $OA + OB = E$, from this diagram as $OA + AB = E$ from the topographic diagram in Fig. 49.

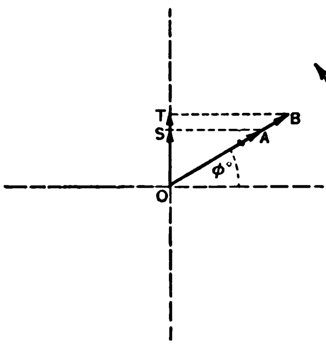


FIG. 50. Polar vector diagram of e.m.f.'s in the generators of Fig. 45.

Similarly, it is just as easy to write the equation for the instantaneous voltage; $OS + OT = e$. As both methods are in common use, they will both be used throughout this text.

It is also customary in many problems to allow the vectors to represent the **effective** instead of the maximum values of e.m.f. or current. This, in reality, merely amounts to changing the scale of the diagram, since the effective value always equals 0.707 of the maximum. This applies to both the topographic and the polar diagram.

Thus OA, in either Fig. 49 or Fig. 50, may be used to represent the 90 volts effective across generator A; and OB, in Fig. 50, and AB in Fig. 49, the 110 volts effective across generator B. From Fig. 50 the effective voltage across the combination is

$$OA + OB = 90 + 110 = 200 \text{ volts,}$$

or, from Fig. 49, the effective voltage across the combination is

$$OA + AB = 90 + 110 = 200 \text{ volts.}$$

Of course, if the effective values are used for the vectors, the lines OS and OT must be divided by 0.707 in order to be numerically equal to the instantaneous values, because an instantaneous value does not equal the effective value times sin ϕ, but the maximum value times sin ϕ.

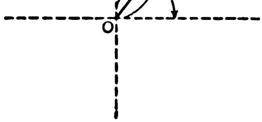


FIG. 51. Polar vector diagram of two e.m.f.'s OA and OB in series.

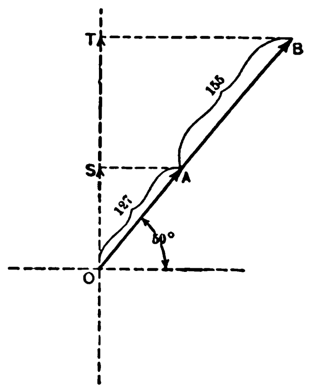


FIG. 52. Topographic vector diagram of two e.m.f.'s OA and AB in series.

Example 1. The maximum e.m.f. of the generator A (Fig. 45) is 127 volts and of B is 155 volts. What is the maximum e.m.f. across the combination?

Draw the polar vector diagram of Fig. 51, or the topographic diagram, Fig. 52. In each OA represents 127 volts. In Fig. 51 OB represents 155 volts; in Fig. 52, AB represents 155 volts, each at an angle of 50° with zero position. From either the equation may be written;

$$E_m \text{ (total)} = 127 + 155 = 282 \text{ volts.}$$

Example 2. What is the instantaneous e.m.f. across the combination of Example 1, when the e.m.f.'s are 50° from the zero values?

Solving Fig. 51,

OS represents the instantaneous value, at the 50° position, of the e.m.f. OA.

OT represents the instantaneous value, at the 50° position, of the e.m.f. OB.

$OS + OT$, the instantaneous value, at the 50° position, of the voltage across the two generators.

$$\begin{aligned} OS &= 127 \sin 50° \\ &= 127 \times 0.766 \\ &= 97.4 \text{ volts;} \\ OT &= 155 \sin 50° \\ &= 155 \times 0.766 \\ &= 119 \text{ volts;} \\ OS + OT &= 97.4 + 119 \\ &= 216 \text{ volts.} \end{aligned}$$

Or solving Fig. 52, OT equals the instantaneous value of the voltage across the two generators at the 50° position.

$$\begin{aligned} OT &= OB \sin 50° \\ &= 282 \times 0.766 \\ &= 216 \text{ volts.} \end{aligned}$$

By both methods the instantaneous value of the voltage across the combination at the 50° position equals 216 volts.

Prob. 1-3. What would be the instantaneous value at the 165° position of the e.m.f. across the generators in Example 1?

Prob. 2-3. At what position would the instantaneous value of the e.m.f. across the combination of Prob. 1 be 250 volts?

Prob. 3-3. The instantaneous value of the e.m.f. at the 40° position across a series combination of two a-c. generators of equal voltage, which are in phase with each other, is 4370 volts. What is the effective e.m.f. of each generator?

Prob. 4-3. Two alternators producing sine wave e.m.f.'s of the same frequency (60 cycles per second) are in series: a voltmeter connected across one of them reads 115 volts, across the other 230 volts, and across the combination 345 volts. What is the phase relation between these e.m.f.'s? Draw a topographic and a polar diagram of vectors to represent them.

Prob. 5-3. What is the maximum instantaneous total e.m.f. of the combination of alternators in Prob. 4? How much time, in seconds, must elapse before and after a maximum value, for the instantaneous voltage to be equal to the effective voltage?

Prob. 6-3. Draw accurately the curve, between volts as ordinates and time in seconds as abscissas, for a 60-cycle alternator generating a sine wave of e.m.f. having an effective value of 115 volts. To the same scale of coördinates, draw another curve representing the e.m.f. of an alternator in series and in phase with the first, this alternator giving a voltmeter reading of 230 volts. Calculate the resultant voltage of the two alternators in series, at instants separated by intervals of $\frac{1}{1440}$ second, and through the points so located draw the curve of resultant voltage. On the same sheet and to the same scales construct a sine curve of e.m.f. having the same maximum ordinate as this curve of resultant voltage, calculating values 15 electrical degrees apart by aid of a table of sines. What do you prove by a comparison of this sine curve with the curve of resultant voltage?

22. Alternators in Series. Opposite Phase. Suppose now that we join two alternating-current generators in series so that they are exactly opposite in phase; that is, one

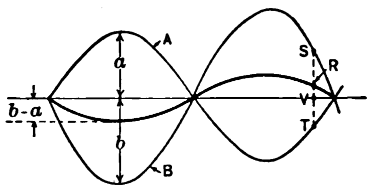


FIG. 53. The heavy line is the difference between the other two curves, which represent the e.m.f.'s of two a-c. generators having a phase difference of 180° with each other.

reaches its maximum positive value just as the other reaches its maximum value in the opposite, or negative, direction through the circuit. Then the resultant voltage is exactly equal to the difference between the two voltages, or 110 − 90

= 20 volts. The generators are then said to differ in phase by 180 electrical degrees. Fig. 53 is the sine curve representation of this case. Note that any instantaneous value on the heavy line curve, which represents the resultant curve of the other two, can be found by subtracting the value on the curve *A* at that instant from the value on the curve *B* at the same instant. Thus the point *R* is found by subtracting the instantaneous value *VT* from the instantaneous value *VS*. Fig. 54 is the polar vector diagram for

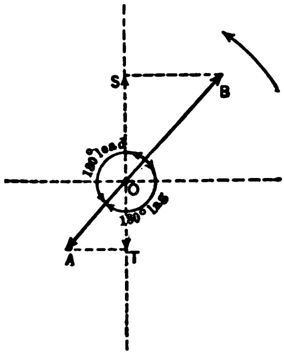


Fig. 54. Polar vector diagram of e.m.f.'s shown in Fig. 53.

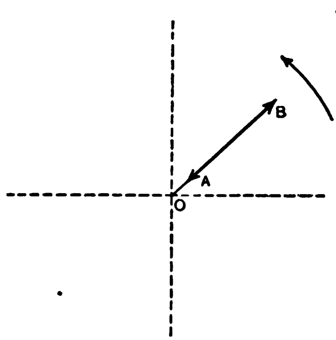


Fig. 55. Topographic vector diagram of e.m.f.'s shown in Fig. 53.

the same case. Note that the vector *OA* representing the maximum e.m.f. of the generator *A* is drawn at an angle of 180° to the vector *OB* which represents the maximum e.m.f. of the generator *B*. This makes them run in exactly opposite directions. The resultant maximum voltage across the two generators would be the difference in length of these vectors drawn to scale.

Fig. 55 is the topographic diagram for this case. *OB* represents the voltage across *B*. From *B* is drawn *BA* in the opposite direction to *OB*, thus cutting down the value of the e.m.f. across the combination to *OA*. Here again there is a line (in this case *OA*) which represents the resulting voltage. In the polar diagram, Fig. 54, for the same case there is no line representing the resulting voltage, but the equation is plain by means of which the value of this result can be computed, $OB - OA = E_m$ (total).

Any instantaneous value equals the difference between the instantaneous values of the two e.m.f.'s at that instant. Thus for the instant at which this vector diagram, Fig. 54, is drawn, the instantaneous value of *OB* is *OS* and of *OA* is *OT*. The instantaneous value of the e.m.f. across the combination at this instant is therefore $OS - OT$.

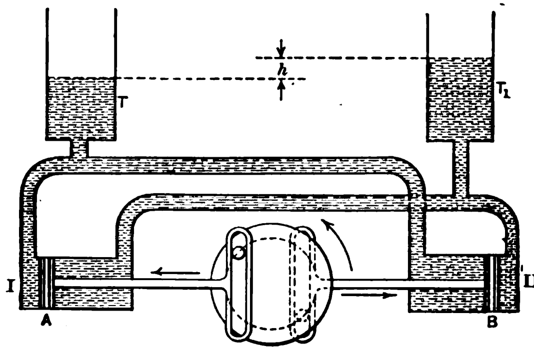


FIG. 56. The pumps are so joined that their actions at every instant oppose each other. The pressure head *h* is due to the difference in the amount of water each pumps.

The action of two alternators in opposite phase joined in series may be illustrated by the action of the two pumps of Fig. 47, but joined as in Fig. 56. Note that the crank-pins, which are on opposite faces of two plates, are always 180° apart, and that the pistons always move in opposite directions with regard to each other. Thus, while piston *B* at this instant is tending to force water into stand-pipe T_1, the piston *A* is tending to draw some away. The pressure head *h* which is set up when the pistons are in the position of Fig. 56 (for instance) is due to the difference of the water pumped from *I* and *II*.

(Draw diagram of connections, rough sine curve, and vector diagram for each problem before making the computations.)

Prob. 7–3. What would be the maximum value of the voltage across the combination such as in Fig. 45 when in phase, if the effective voltage of $A = 220$, and of $B = 250$ volts?

Prob. 8–3. What is the maximum value of the voltage across the combination in Prob. 7, when in opposite phases?

Prob. 9–3. What is the instantaneous value of the voltage across the combination in Prob. 7, at the instant represented by the 40° position of the vectors?

Prob. 10–3. What is the instantaneous value of the voltage across the combination in Prob. 8, when at the 40° position?

Prob. 11–3. What is the voltage across two alternators which are in phase and joined in series, if the maximum value of the e.m.f. of one is 400 volts and of the other, 200 volts?

Prob. 12–3. Two generators are in opposite phases and are joined in series. The instantaneous voltage of one is 100 volts at the 30° phase; of the other, 100 volts at the 60° phase. What is the maximum voltage across the combination?

Prob. 13–3. What is the effective value of the voltage across the combination in Prob. 12?

Prob. 14–3. What would be the effective voltage across the combination in Prob. 12, if the alternators were in phase?

Prob. 15–3. The maximum e.m.f. of three alternators is 153 volts each. What would be the effective voltage across the combination if they were all joined in series and were in phase with one another?

Prob. 16–3. Two alternators giving sine-wave e.m.f.'s of the same frequency, in series and in phase with each other, give a total voltmeter reading of 220 volts across the combination. When the connections of one of the alternators is reversed, the resultant voltage is zero.

(*a*) What is the effective voltage of each alternator?

(*b*) What is the resultant e.m.f. at each of the following instants: 45°, 90°, 135° and 180° **after** the positive maximum instantaneous value?

Prob. 17–3. Two alternators joined in series give a voltmeter reading of 235 volts across the combination. When one of them has its connections reversed, the total voltage becomes 25 volts, as shown by the voltmeter. What is the effective voltage of each alternator? These alternators both produce sine-wave e.m.f.'s of the same frequency.

Prob. 18–3. Three alternators in series, all producing sine-wave e.m.f.'s of the same frequency give a total effective voltage of 110. When the connections of *A* are reversed, the total voltage remains 110. When the connections of *A* are restored as originally, and those of *B* reversed, the total voltage becomes 330. When *B* is restored as originally and *C* is reversed, the total voltage again becomes 110. Draw both polar and topographic vector diagrams to represent the relations between these e.m.f.'s, as originally connected and as reconnected after each change.

23. Alternators in Series: in Quadrature. It is possible to join the two pumps of Fig. 46 and Fig. 56 in such a manner that the two pistons will not be traveling at all instants either in the same direction or in opposite directions.

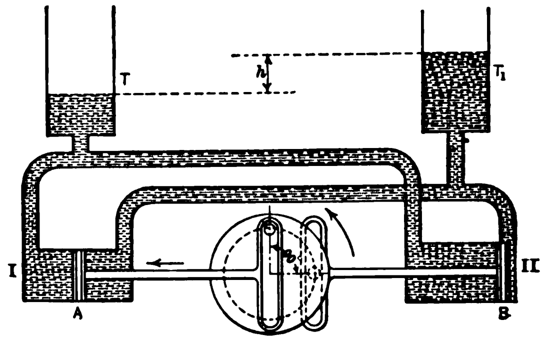


FIG. 57. The two pumps are so joined that they do not continuously aid or hinder each other. The pressure head *h* is due neither to the sum nor the difference of the amounts of water pumped by *I* and *II*.

tions. That is, they may be made to aid each other at some instants and to oppose at other instants. In Fig. 57 the pumps are so joined that the cranks are at an angle of 90° with each other. Thus at the instant when piston *A* is filling *T* and emptying T_1 at the fastest rate, as in Fig. 57, piston *B* is at rest in the zero position. The greatest pressure head (h) which is set up would not at any time equal the sum of the amounts pumped by the two pistons in a complete stroke, as in Fig. 47.

In the same way it is possible to join in series two alternators whose e.m.f.'s are neither in phase nor opposite. For instance, the voltage of A, Fig. 45, may have reached its maximum positive value at an instant when the voltage of B was passing through its zero value and was about to have a small positive value.

The two generators then would be said to have a phase difference of 90°. A might be said to be 90° ahead of B, or B to lag 90° behind A. Fig. 58 shows by means of the sine

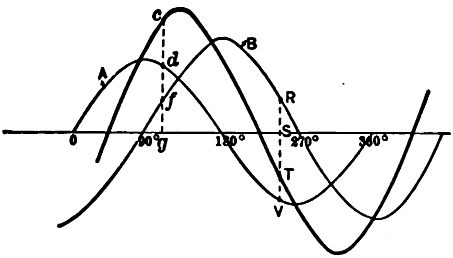


FIG. 58. The sine curves A and B represent two e.m.f.'s having a phase difference of 90°. The heavy line represents the sum of these two curves.

curve, that A reaches its maximum value when B is at zero. B does not reach its maximum until 90° later than A, which has in the meantime dropped to zero and is about to start on its negative values.

The heavy line curve is obtained by adding algebraically the instantaneous values on the curve of each machine. Thus the point (T) on the heavy curve is found by adding ($+RS$) to ($-VS$). This gives ($-TS$), which represents the instantaneous voltage across the combination at that instant. Also (gf) and (gd) are added to give a value (gc) to the instantaneous voltage across the two alternators.

Thus it is possible by means of the voltage curve of each of the two machines to find the curve of the voltage across the combination just as when they were in phase. This resultant curve is a sine curve also.*

This method of finding the resulting voltage, however, is too cumbersome and inexact; accordingly, the vector method is used when numerical results are desired. If the line *B*, Fig. 59, represents the 110 effective volts of generator *B*, then the line *A* will represent the 90 effective volts of generator *A*. Note that *A* is drawn 90° ahead of *B*.

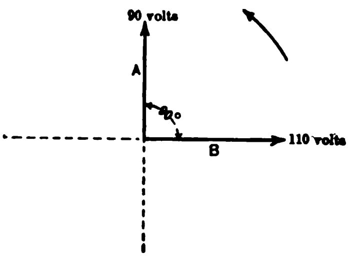


FIG. 59. A polar vector diagram for the case shown in Fig. 58.

We have, then, two electromotive forces acting at an angle of 90° with each other. If we had two mechanical forces or two magnetic forces acting at an angle with each other, we know that the resultant force would be the diagonal of a parallelogram, of which the two forces form the adjacent sides. Thus, in Fig. 60, if *B* represents a force of 110 pounds and *A*, a force of 90 pounds at right angles to it, then *R*, the resultant force, is formed by completing a parallelogram on the forces *A* and *B*, and drawing the diagonal *R* from the point of intersection of the two forces.

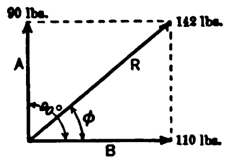


FIG. 60. The line *R* represents the resulting force of the two forces *A* and *B*, which act at right angles to each other.

The value of *R* can be found either by drawing the whole diagram to scale or by using the rule that the square of the hypothenuse of a right triangle equals the sum of the squares of the other two sides. By the latter method,

* See Problem 6, page 84.

$$\begin{aligned} R^2 &= A^2 + B^2 \\ &= 8100 + 12{,}100 \\ &= 20{,}200; \\ R &= \sqrt{20{,}200} \\ &= 142 \text{ lb.} \end{aligned}$$

The angle ϕ, which this resultant force R makes with force B, may be found by measuring it, when the forces are drawn to scale, or by the rule that the tangent of any angle of a right triangle is equal to the ratio of the side opposite the angle to the side adjacent to the angle. By the latter method,

$$\tan \phi = \tfrac{90}{110} = 0.818;$$

0.818 is the tangent of a 39.3-degree angle, hence

$$\phi = 39.3°.$$

Thus the resultant of two forces of 90 lb. and 110 lb., acting at right angles with each other, is equal to a force of 142 lb. acting at an angle of 39.3° to the 110-lb. force.

Since R is neither the arithmetical sum nor the arithmetical difference of A and B, we can not write $R = A + B$. However, in order to express the correct relations, that is, the vector relation, of A and B to R we write the following equation:

$$R = A \oplus B.$$

The sign $\oplus$ means that we add **vectorially** the quantities between which it stands. Thus, the expression $R = A \oplus B$ is read, "R equals the vector sum of A and B," which means that, due allowance being made for the direction in which A and B act, they produce R when combined.

Similarly, we may find the resultant of two electromotive forces A, 90 volts, and B, 110 volts, acting at 90 electrical degrees to each other. We may in the same way write the equation $R = A \oplus B$.

To solve this equation we construct Fig. 61. B and A are drawn to scale at right angles to each other and the parallelogram is completed. Then R, to the same scale, equals 142 volts and acts at an angle of 39.3° to the 110-volt pressure. Or, by applying the trigonometric rule,

$$R^2 = A^2 + B^2,$$
$$R = \sqrt{(90)^2 + (110)^2} = 142 \text{ volts};$$
$$\tan \phi = \tfrac{90}{110} = 0.818,$$
$$\phi = 39.3°.$$

Since the 90-volt vector represents the e.m.f. of the alternator A, Fig. 45, and the 110-volt vector represents the e.m.f. of the alternator B, then the 142-volt vector represents the voltage across the combination. This resultant is found to differ in phase with the e.m.f. of 110 across B, by 39.3°; which means that it reaches its maximum value before the e.m.f. B reaches its maximum value in the same direction, by an amount of time represented by 39.3 electrical degrees, or $\left(\frac{39.3}{360}\right)$ part of one complete period.

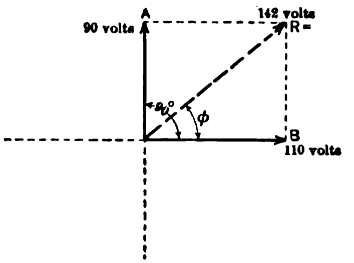


FIG. 61. The line R represents the resulting e.m.f. of the two e.m.f.'s A and B which have a phase difference of 90°.

In the same manner, the resultant of any two alternating electromotive forces acting at right angles, that is, differing in phase by 90°, can be found, together with the phase difference between the resultant and either of the two electromotive forces.

Write the equation and draw the diagram for each of the following problems:

Prob. 19–3. Two alternating electromotive forces with a phase difference of 90° are joined in series. One is 50 volts, the other 75 volts.

(*a*) What is the resulting voltage across the combination?

(*b*) What is the difference in phase between the voltage across the combination and across the 50-volt part?

Prob. 20–3. It is desired to have 220 volts across two generators of the same frequency joined in series. One generator delivers 120 volts. What must be the voltage of the other, if it differs in phase with the first by 90°?

Prob. 21–3. What is the maximum value of the resultant e.m.f. in Prob. 19?

Prob. 22–3. What will be the instantaneous value of the e.m.f. across the 50-volt alternator of Prob. 19, when the e.m.f. across the combination has its maximum value?

Prob. 23–3. What will be the instantaneous value of the e.m.f. across the 90-volt generator of Prob. 19, when the resultant e.m.f. has its maximum value?

Prob. 24–3. What will be the instantaneous value of the voltage across the combination in Prob. 20, when the instantaneous e.m.f. across the 120-volt generator is 40 volts?

Prob. 25–3. When two alternators of the same frequency but with a phase difference of 90° are joined in series, the resulting e.m.f. is 440 volts. If one of the alternators has an e.m.f. of 220 volts, what is the e.m.f. of the other?

Prob. 26–3. What is the phase difference between the e.m.f. across the combination and the e.m.f. across the 220-volt generator of Prob. 25?

Prob. 27–3. What is the instantaneous value of the e.m.f. across each generator of Prob. 25, when the instantaneous value of the resulting e.m.f. is 0?

Prob. 28–3. At what point in its cycle is the phase of the e.m.f. of each machine in Prob. 27?

Prob. 29–3. What is the instantaneous e.m.f. across the combination, when the instantaneous value across the 220-volt generator of Prob. 25 is at a maximum?

Prob. 30–3. (*a*) What is the maximum value of the combined e.m.f.'s in Prob. 25?

(*b*) What is the instantaneous e.m.f. across each generator at the instant when the maximum value of (*a*) occurs?

24. Two-phase Generators. It would be possible to maintain two separate alternators at exactly the same frequency and with a constant phase difference between their e.m.f.'s, but it is simpler and more economical to wind two separate sets of armature coils in the same machine, and locate these windings so that the voltage induced in one set is 90 electrical degrees behind or ahead of the voltage induced in the other set. A machine so wound is called a **two-phase** alternator, or a **quarter-phase** alternator. These two sets of windings are called the **phases** of the machine and are very rarely connected to each other in modern machines. The structural details of this machine are explained on page 473, but its action in delivering two alternating voltages which are at 90 electrical degrees to each

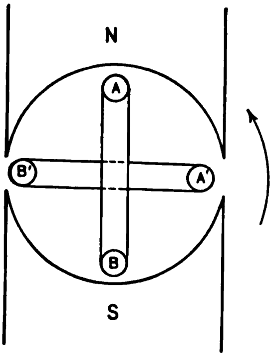


FIG. 62. Simple two-phase alternator. The e.m.f. coil of AB is at a maximum when the e.m.f. of $A'B'$ is zero. Thus there is a phase difference of 90°.

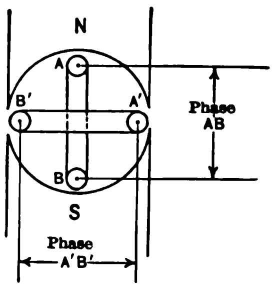


FIG. 63. Phase AB leads phase $A'B'$ by 90°.

other can readily be understood by reference to the following figures. Fig. 24 to 37 (inclusive) show the action of a single-coil, single-phase generator. Now if another coil is added at right angles to the coil of Fig. 24, as in Fig. 62 and 63, the actions will be the same in each coil, with the exception that they will take place 90° later in one coil than in the other. Thus, when the voltage across AB is at a maximum, as in both Fig. 62 and Fig. 63, the voltage across $A'B'$ is zero, and vice versa. The voltages may be represented, then, by two sine curves differing in phase by 90°, as in Fig. 64. E represents the e.m.f. curve of the coil AB; E_1 the e.m.f. curve of the coil $A'B'$. Note that E_1 has a value of zero when E has a maximum value; that is, E has a 90° start on E_1. The vector diagram would be that of Fig. 65, in which E equals the voltage induced in AB, and E_1 the voltage induced in $A'B'$.

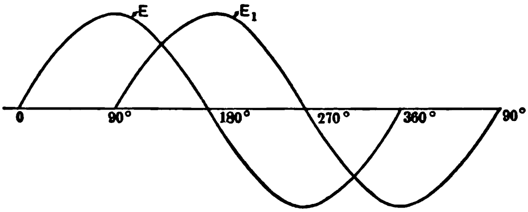


FIG. 64. The sine-wave representation of the e.m.f.'s of Fig. 62 and 63. Curve E leads E_1 by 90°.

Fig. 66 is the conventional representation of such a machine. Note that phase AB may be connected to phase $A'B'$ by joining A' to B. We should then have two e.m.f.'s differing in phase by 90° joined in series. Fig. 67 shows this

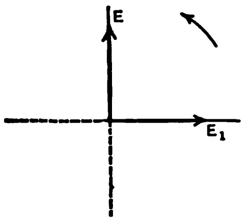


FIG. 65. The polar vector diagram of Fig. 62, 63 and 64. Here also the vector E leads E_1 by 90°.

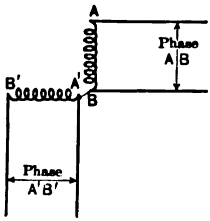


FIG. 66. Convention representation of a two-phase generator.

connection made and a conductor brought out from the juncture of the two phases. This makes the 3-wire 2-phase system, which is unusual.

Prob. 31-3. If the voltage across each phase in Fig. 67 is 110 volts, what is the voltage between the outside wires X and Y?

Prob. 32-3. If 550 volts are desired across the outside wires in Fig. 67, what must be the voltage across each phase?

Prob. 33–3. If the voltage across phase *XN* of Fig. 67 is 120 volts and across phase *YN* is 90 volts, what is the voltage across *XY*?

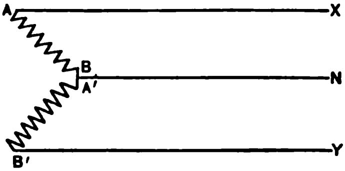


FIG. 67. A 3-wire 2-phase generator. A rather unusual combination.

Prob. 34–3. A two-phase generator, 2300 volts per phase, is connected to a three-wire system for distribution of the power. If the common wire of the two phases becomes connected to ground through the limb of a tree, to what maximum pressure is the insulation between each of the other wires and ground subjected?

Prob. 35–3. Solve Prob. 34–3 on the supposition that it is one of the "outside" wires of the three-wire system (not the common wire) which becomes connected to ground.

Prob. 36–3. If the two-phase generator of Prob. 34–3 is connected to a four-wire distributing circuit, with all wires equally insulated, and one of the wires becomes "grounded," what pressure is brought to bear upon the insulation separating each of the other three wires from ground.

Prob. 37–3. The working voltage for which a certain insulator is designed is 10,000 volts (effective, assuming sine wave form of e.m.f.). What is the largest (effective) voltage at which a single-phase two-wire line may be operated, each wire being supported by these insulators, (*a*) allowing for the possibility of one wire being grounded; (*b*) assuming neither wire to be grounded?

Prob. 38–3. Using the same insulators specified in Prob. 37–3, what is the largest (effective) voltage per phase that may be used on a two-phase four-wire system of distribution?

Prob. 39–3. Solve Prob. 38–3 assuming the two-phase generator to be connected to a three-wire system of distribution.

Prob. 40–3. Show by vector diagram that in reality **three phases** (not "three-phase"), or three e.m.f.'s differing in phase with respect to each other, may be obtained from any three-wire system of distribution connected to a two-phase generator. Specify the voltages and phase relations of these three phases, using a 2300-volt two-phase generator.

25. Three-phase Generators: Open-delta Connection. But coils may be so located on an armature that the e.m.f. across them differs in phase by more or less than 90 electrical degrees. For instance, when three coils are spaced symmetrically or equidistant on an armature, as in Fig. 68, it is clear that the induced voltages A to B, A' to B', and A'' to B'' would be 120° apart, and could, therefore, be represented by the sine curves of Fig. 69a. This would constitute a simple three-phase generator, the type in greatest use at the present time in alternating-current systems. If the three coils are not joined to one another, then the machine supplies power to three separate phases which have no electrical connection. This would require that six wires be used for distribution, and would result in an unnecessary initial expense.

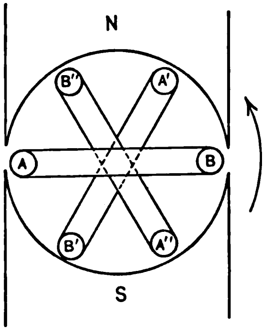


FIG. 68. Simple three-phase alternator. The e.m.f. in each coil has a phase difference of 120° with the e.m.f.'s in the other two coils.

Accordingly, at least two of the phases are joined in series. The other phase may then be left idle, to act as a reserve in case any trouble occurs in either of the two which are in service. One form of such an arrange-

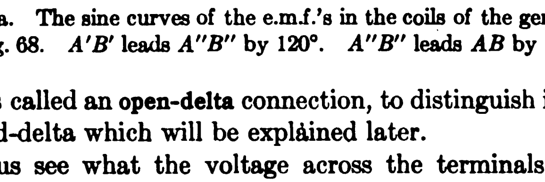


FIG. 69a. The sine curves of the e.m.f.'s in the coils of the generator of Fig. 68. $A'B'$ leads $A''B''$ by 120°. $A''B''$ leads AB by 120°.

ment is called an **open-delta** connection, to distinguish it from a closed-delta which will be explained later.

Let us see what the voltage across the terminals of an open-delta system is. Assume that in Fig. 68 the end B of the phase AB is joined to the end A'' of the phase $A''B''$, leaving the phase $A'B'$ idle. The terminals of the series circuit would be A and B''. The sine curve of this case would then be represented by Fig. 69b. Note that $A''B''$ leads AB by 120° as before. The heavy curve is drawn from

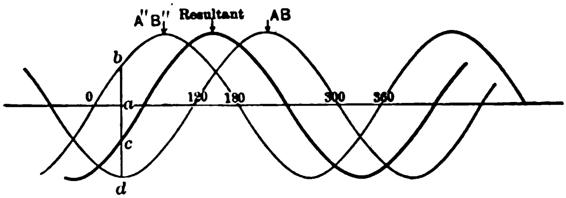


FIG. 69b. The heavy curve represents the resulting e.m.f. where $A''B''$ and AB are joined in series, with $A''B''$ 120° ahead of AB.

the sum of the instantaneous values of the other two curves. Thus, the point (c) is found by adding ($+ ab$) to ($- ad$). Note the peculiar fact that the resultant has no greater maximum value than either of the curves of which it is the sum.

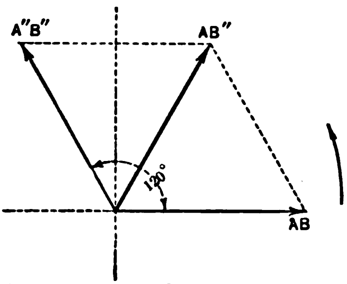


FIG. 70. The polar vector diagram for the sine curves of Fig. 69b. AB'' represents the resulting e.m.f. of joining AB and $A''B''$ in series.

The vector equation for this case would be,

$$AB'' = AB \oplus A''B''.$$

To solve this, construct the vector diagram, Fig. 70. The voltage in the coil $A''B''$ is 120° ahead of the voltage in the coil AB. Thus, if the vector AB represents the effective e.m.f. induced in the coil AB, then the vector $A''B''$, 120° ahead of AB, will represent the effective e.m.f. induced in the coil $A''B''$.

The resultant e.m.f. AB'' across the two phases can then be found as before by completing a parallelogram on these two vectors as sides and drawing the diagonal AB''.

Example 3. In a three-phase generator there is an e.m.f. of 11,000 volts across each phase. What would be the voltage across any two phases joined in series ("open delta")?

This is represented diagrammatically by Fig. 71, in which phase AO is joined to phase OB at O. Phase AB is left idle. Draw the vector diagram, Fig. 72, letting the line OA represent 11,000 volts in one phase, and the line OB, 120° ahead of OA, represent the 11,000 volts in the second phase.

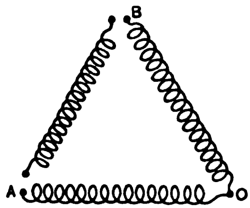


FIG. 71. Conventional representation of the joining of phase OA to OB at O.

Complete the parallelogram and draw the diagonal OC, which will represent the resultant voltage across OA and OB joined in series in this way.

Using the method shown in the Appendix, we can find the value of OC as follows:

$$
\begin{aligned}
OC &= 11{,}000 \oplus 11{,}000,\\
OC^2 &= 11{,}000^2 + 11{,}000^2 + 2 \times 11{,}000 \times 11{,}000 \cos 120^\circ\\
&= 121{,}000{,}000 + 121{,}000{,}000 - 0.5 \times 2 \times 121{,}000{,}000\\
&= 121{,}000{,}000,\\
OC &= 11{,}000 \text{ volts.}
\end{aligned}
$$

It may seem rather strange at first sight that the resultant of two voltages of 11,000 each was only 11,000, but it will be remembered that the sine curve, Fig. 69b, showed the same fact. Also, it is to be recalled that two equal mechanical forces acting at an angle of 120° have a resultant force exactly equal to either one of the forces. Thus, when the voltage across OA is 11,000 volts and the voltage across OB is also 11,000 volts, the voltage across the combination of the two phases joined in series would be 11,000 volts.

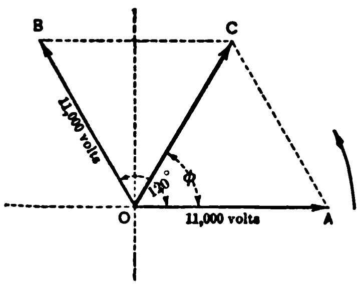


FIG. 72. The vector OC represents the resulting e.m.f. of a series combination of OA and OB which have a phase difference of 120°.

If two circuits are thus joined which have any other difference in phase but 120°, the resulting voltage would not exactly equal the voltage of one of the phases. For instance, suppose that two coils in the armature of an a-c. generator are joined in series in such a way that the e.m.f.'s induced in them are 25° apart. If the voltage across each coil is 80 volts, what is the voltage across the two coils in series?

The general equation is

$$E = 80 \oplus 80,$$

where E is the resulting e.m.f.

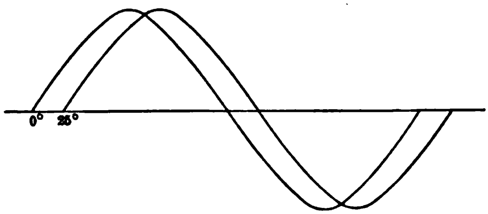


FIG. 73. Rough sine-curves of e.m.f.'s which have a phase difference of 25°.

Sketch roughly the sine curves for this case, as in Fig. 73, in order to get a clear idea of the relations throughout the cycle.

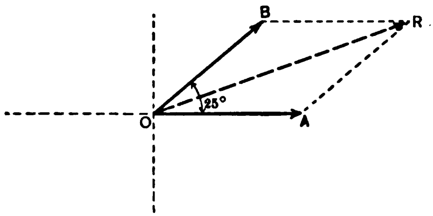


FIG. 74. Polar vector diagram in which the vector *OB* leads *OA* by 25°. *OR* is the resultant of *OB* and *OA*.

Draw *OA* and *OB*, Fig. 74, at 25° to each other. Complete the parallelogram, and draw the diagonal *OR* which represents the resultant voltage across the two coils in series.

$$\begin{aligned} OR^2 &= 80^2 + 80^2 + 2 \times 80 \times 80 \cos 25° \\ &= 6400 + 6400 + 12{,}800 \times 0.906 \\ &= 24{,}400, \\ OR &= \sqrt{24{,}400} = 156 \text{ volts.} \end{aligned}$$

The voltage across the two coils in series must therefore be 156 volts.

Sketch sine curves and vector diagrams for each of the following problems and solve:

Prob. 41–3. Two coils are joined in series. The e.m.f. in one coil is 48 volts, in the other, 70 volts. The phase difference between the e.m.f.'s of coils is 65 electrical degrees. What is the voltage across the combination?

Prob. 42–3. The e.m.f.'s in two coils are 40° apart. The value of one e.m.f. is 86 volts. What must be the value of the other e.m.f. in order that the voltage across the two in series may be 110 volts?

Prob. 43–3. Two coils have e.m.f.'s of 110 and 150 volts respectively. When joined in series the combined voltage is 220 volts. What is the phase difference between the two coils?

Prob. 44–3. If the voltage across the two coils in Prob. 43 when in series had been 260 volts, what must the phase difference have been?

Prob. 45–3. If the voltage across the series combination of the two coils of Prob. 43 had been 40 volts, what would the phase difference have been?

Prob. 46–3. The voltage across each phase of a three-phase generator is 220 volts. What is the voltage across an open-delta combination of the phases?

Prob. 47–3. A choke coil and a resistance are joined in series and an alternating current is sent through the series combination. The drop across the resistance is 65 volts, and across the choke coil 75 volts. The voltage across the combination is 105 volts, What is the phase difference between the drops across the two?

Prob. 48–3. If the current flowing in Prob. 47 is 5 amp., what is the impedance under these conditions of:

(*a*) The choke coil?
(*b*) The resistance?
(*c*) The combination?

26. Reversing Coils or Phases in Transformers and Generators. Upon referring to Fig. 75 it will be seen that the two vectors *OB* and *OA* are at right angles to each other and have as a resultant *OR*. If one vector, say *OA*, be turned around end for end, so that it points to the left as *OA′*, the resultant of *OA′* and *OB* will become *OR′*, instead of *OR*. But note that it has exactly the same numerical value as *OR*, and differs only in its phase relations.

Thus, if the e.m.f.'s of two phases are at 90° to each other, it will make no difference in the numerical value of the resulting voltage which two ends are connected.

But suppose that the e.m.f.'s of two phases are 120° apart. Let us see what the result of reversing the connections of one of the phases will be. In Fig. 76, *OR* is the resultant of *OB* and *OA* when the phases are connected so

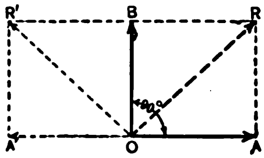


FIG. 75. Vectors *OA* and *OB* are at right angles to each other. Reversing one vector *OA* does not change the value of the resultant *OR*, which becomes *OR'*.

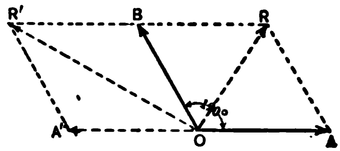


FIG. 76. Vectors *OA* and *OB* are at 120° to each other. Reversing *OA*, changes the value of the resultant *OR* to *OR'*.

that they are 120° apart. But when *OA* is reversed, the resultant becomes *OR'*, which is much greater than *OR*. The cause for this increase in the resultant is evident from an inspection of Fig. 76. Note that when the vector *OA* is reversed and takes the direction *OA'*, it is no longer at an angle of 120° to *OB*, but comes into a position only 60° from *OB*. So that instead of having two vectors at an angle of 120° to each other, we have two vectors at an angle of 60° to each other. The resultant of the latter combination is much greater than of the former, as is clear from Fig. 76.

This effect of reversing a coil is also very evident from the sine curves of the case.

Suppose we reverse phase *A''B''* of Fig. 68 by joining *A''* to *A*, instead of *B''* to *A*. Then all the positive values of the sine wave *A''B''* in Fig. 69b become negative, and vice versa, and $A''B''$ lags 60° behind AB instead of leading it by 120° as before, a change of 60° + 120°, or 180°. This is clear in Fig. 77. The resultant curve, the heavy line, now has much larger maximum values than has either of the components.

From both the vector diagram and the sine curves it can be seen that:

Reversing a coil which generates an e.m.f changes its phase relations to other coils by 180°.

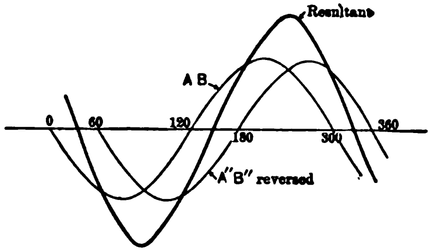


FIG. 77. Curve $A''B''$ is reversed from its position in Fig. 69b. The curve of resulting e.m.f. has much greater maximum values than that of Fig. 69b.

There is usually no convenient way of telling by inspection how two phases of a machine are to be connected in order to produce the desired combined voltage. The method usually employed is to measure the voltage across each set of terminals. Then connect two in series at random. If they do not produce the desired voltage, reverse one of them. This process is called "**phasing-out**" a machine.

Example 4. It is desired to connect two phases of a three-phase generator in open delta. Each phase is found to have a voltage of 220 volts. But when two phases are connected, the voltage across them was found to be 381 volts. Show that by reversing one phase, the correct open-delta voltage of 220 volts can be obtained.

Construct Fig. 78 and determine what the angle ϕ must be to fulfill the conditions in which the resultant OR must be 381, and the vectors OA and OB, 220 each.

By the Cosine Law for parallelograms

$$381^2 = 220^2 + 220^2 + 2 \times 220 \times 220 \cos \phi,$$

$$\cos \phi = \frac{381^2 - 2 \times 220^2}{2 \times 220 \times 220} = 0.500,$$

$$\phi = 60°.$$

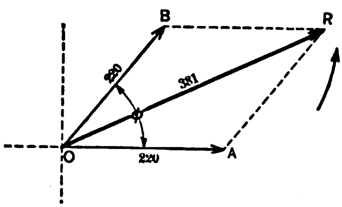


FIG. 78. The coils OA and OB are so joined that the resulting e.m.f. across them is 381 volts.

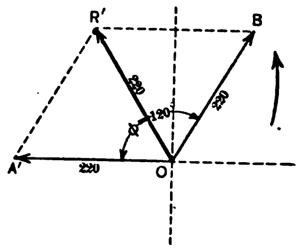


FIG. 79. Coil OA of Fig. 78 is reversed in its connection to coil OB, and the resulting e.m.f. across them becomes 220 volts.

Now if we reverse one phase, say OA, the vector diagram will be as in Fig. 79 and the angle between the phases would be

$$180° - 60° = 120°,$$

$$(OR')^2 = 220^2 + 220^2 + 2 \times 220 \times 220 \cos 120°,$$

$$OR' = \sqrt{220^2 + 220^2 - 220^2} = \sqrt{220^2} = 220.$$

This also checks by solving for the angle by the Cosine Law.

This shows that the true open-delta connection results from reversal of this phase OA, since the reversal gives a resulting voltage of 220 volts.

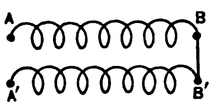


FIG. 80. Two coils joined in series.

Prob. 49-3. When two coils, AB and $A'B'$ are connected as in Fig. 80, the combined e.m.f. across them is 240 volts. The e.m.f. of each coil is 180 volts. Find the difference in phase between the e.m.f.'s of the coils.

Prob. 50-3. If end A' of coil $A'B'$ in Fig. 80 were connected to end B of coil AB, what would the combined e.m.f. be?

Prob. 51–3. If end A' of coil $A'B'$ in Fig. 80 were connected to end A of coil AB, what would the combined e.m.f. be?

Prob. 52–3. It is desired to connect the three-phase generator of Fig. 81 in open delta and run it as a single-phase generator. On testing it out with a voltmeter, the following data are found.

From A to B = 240 volts.
From A to A_1 = 0
From A to A_2 = 0

From A to B_1 = 0
From A to B_2 = 0
From A_1 to B_1 = 240 volts.

From A_1 to A_2 = 0
From A_1 to B_2 = 0
From A_2 to B_2 = 240 volts.

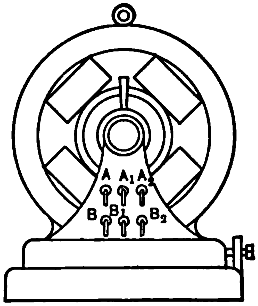


FIG. 81. A three-phase generator with both ends of each phase brought out separately.

(*a*) Name the two terminals of each phase.

(*b*) State what connections you would try in order to produce an open-delta arrangement.

(*c*) Show by vector diagram what voltage there should be across the open-delta arrangement, and what readings you should obtain in order to prove that you have made the proper connection.

Prob. 53–3. If you measured the voltage across the combination of coils as you had connected them for open delta in Prob. 52, and found it to be 416 volts, how would you change the connections?

Prob. 54–3. Suppose that one of the phases which you had intended to use in the open-delta arrangement of Prob. 52 were found to be faulty. Show how you would connect the third phase in its place.

Prob. 55–3. Connect the third phase of the generator described in Prob. 52, in series with the other two phases after the latter have been connected in correct open delta, and calculate the resultant total voltage of the entire series. Then reverse the connections of the third phase, and again calculate the total voltage. Draw vector diagrams to illustrate each condition.

Prob. 56–3. Repeat the work of Prob. 55–3, starting with the incorrect open-delta connection described in Prob. 53–3.

Prob. 57–3. Connect one terminal of the third phase of this machine to the juncture of the other two phases, after the latter have been connected correctly in open delta as described in Prob. 52–3. Calculate the voltage across each of the three pairs of terminals obtainable from the three terminals of the armature thus formed (not including the common juncture). Then reverse the third phase, and again calculate the three voltages. Draw vector diagrams to illustrate.

Prob. 58–3. Repeat the work of Prob. 57–3, starting with the incorrect open-delta connection described in Prob. 53–3.

27. Series Arrangement of More than Two Parts. If more than two e.m.f.'s are joined in series, the resulting voltage may be obtained by finding the resultant of any two, and then combining this resultant with the third voltage,

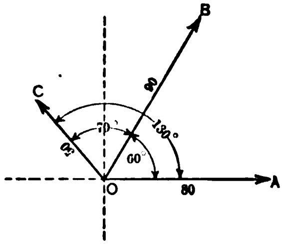


FIG. 82. Polar vector diagram of three e.m.f.'s, OA, OB and OC, joined in series.

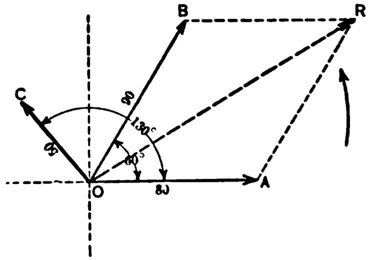


FIG. 83. Polar vector diagram of Fig. 82, showing the resultant OR of the vectors OA and OB.

and so on until all the voltages have been included. For example, assume that a coil with an e.m.f. of 80 volts is put in series with one coil having an e.m.f. of 90 volts leading the first e.m.f. at a phase difference of 60°, and with another coil of 50 volts leading the first e.m.f. at a phase difference of 130°, or leading the second e.m.f. by 70° phase difference.

The general equation is $E = 80 \oplus 90 \oplus 50$.

Polar Diagram.

Fig. 82 shows the polar vector diagram for this arrangement. In Fig. 83, vectors OA and OB have been combined into their resultant OR, which represents the voltage across the series combination of the 80 and 90 volt coils. In Fig. 84 this resultant OR has been combined with the third vector OC, and the resultant OR_1 found, which represents the voltage across the series combination of the three coils.

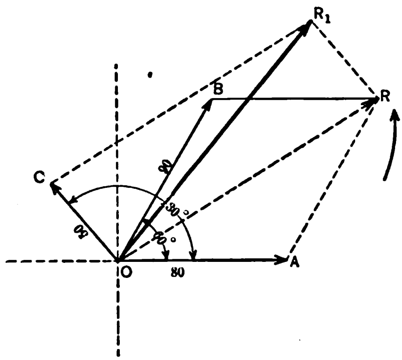


FIG. 84. Showing how OR, the resultant of OA and OB, can be combined with OC to form OR_1, which is thus the resultant of OA, OB and OC.

Topographic Diagram. The topographic method of arriving at this same resultant OR_1 is to lay out OB from the end A of OA at an angle of 60° to OA as in Fig. 85. Note particularly that the 60° angle is on the **outside**, and measured in counter-clockwise direction (leading). Then lay out OC from the B end of OB at an angle of 70° to OB, which makes it at the correct angle, 130°, to OA. Note again that this 70° angle is on the outside, in counter-clockwise direction (leading). Then from O to C is the resultant OR_1.

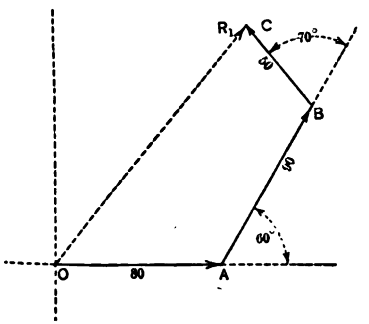


FIG. 85. The topographical vector diagram for three vectors, OA, AB and BC, joined in series.

Note that in laying out this kind of a diagram, the tail of one vector must be attached to the head of the preceding vector. This makes the arrow heads all follow one another, except, of course, that of the resultant vector.

Note also that the phase difference between two vectors so joined is always measured by the outside angle, between the latter vector and the **extension** of the former vector.

Numerical Solution.

The topographical diagram is simpler than the polar to construct and, by means of a few added lines, both are easy to use for the computation of the numerical value of the resultant, and of its phase relations to the other vectors.

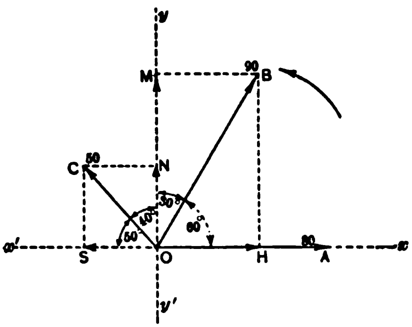


Fig. 86. The three vectors OA, OB and OC are resolved into actions along the horizontal and the vertical axes.

The most common method is by means of the polar diagram shown in Fig. 86. Instead of combining the vectors into a single resultant, each vector is resolved into two components, one on the axis yy' and the other on the axis xx'. Thus OB has a component on the x axis equal to OH, and on the y axis equal to OM. Note that these component actions are all measured from the point O. These show that the vector OB acts **to the right** by the amount represented by the length of OH and acts **up** by the amount represented by the length of OM. In the same way OC acts **to the left** by the amount represented by the line OS, and **up** by the amount represented by the line ON. The vector OA acts to the right only and by the amount represented by the length of OA.

Thus we have a total action, **to the right,** of

$$OH + OA,$$

and **to the left** of

$$OS.$$

The resulting action on the x axis, then, must be

$$(1)\ OH + OA - OS.$$

But $OH = OB \cos 60° = 90 \times 0.500 = 45,$
$OA = 80,$
$OS = OC \cos 50° = 50 \times 0.643 = 32.2.$

Thus $OH + OA - OS = 45 + 80 - 32.2 = 92.8.$

The total action on the x axis is thus 92.8 to the right.

On the y axis the total action is up by an amount equal to

$$OM + ON.$$

But $OM = OB \cos 30° = 90 \times 0.866 = 69.3,$
$ON = OC \cos 40° = 50 \times 0.766 = 39.3.$

Thus $OM + ON = 77.9 + 39.3 = 117.2.$

The total resulting action is therefore

92.8 to the right

and

117.2 up.

Now, for the sake of clearness, lay these values out on another pair of axes, as in Fig. 87, making OP the 92.8 action to the right, and OQ the 117.2 action up.

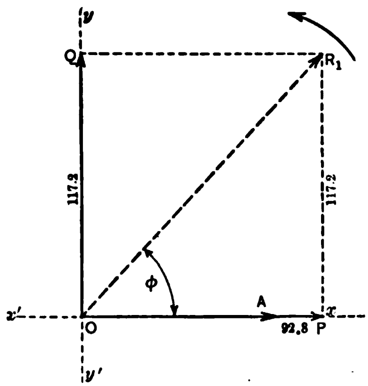


FIG. 87. The actions of the three vectors of Fig. 86 along the horizontal axis are equal to OP; along the vertical axis to OQ. OR is the resultant of the three vectors.

The resultant OR_1 of these two actions will be the resultant of the three vectors, because the two actions OQ and OP are equivalent to the three with which we started.

The value of OR_1 can now be found easily by the equation

$$\begin{aligned} OR_1 &= \sqrt{92.8^2 + 117.2^2} \\ &= \sqrt{8610 + 13,740} \\ &= \sqrt{22,350} \\ &= 149.5 \text{ volts.} \end{aligned}$$

The resultant of the three e.m.f.'s thus has a value of 149.5 volts.

The phase difference between this resultant and the 80-volt phase can be found as follows:

$$\tan \phi = \frac{117.2}{92.8} = 1.262,$$
$$\phi = 51.6°.$$

By means of this method the e.m.f. across a series combination of any number of e.m.f.'s with any phase differences can readily be found.

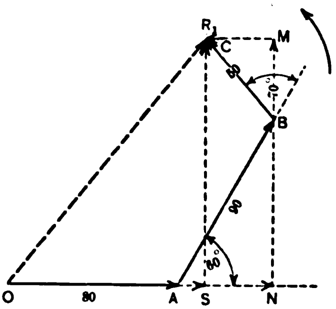


FIG. 88. The three vectors OA, AB and BC of Fig. 85, are here resolved into their vertical and horizontal actions for ease in arriving at the mathematical value of the resultant OR_1.

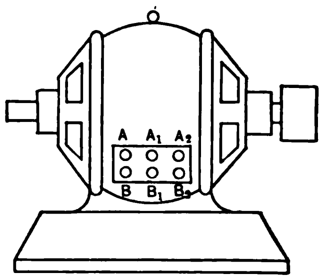


FIG. 89. Special generator with both ends of each coil brought out.

The same method of computing the mathematical values can be applied to the topographical arrangement of vectors.

Note in Fig. 88 that the resultant OR_1 extends **to the right** an amount equal to the line OS. But OS is made up of $OA + AN$ (the amount which B extends to the right) $-$ MC (the amount which C extends to the left). The values of OA, AN and MC are found as shown in solving the polar diagram.

OR_1 extends **up** by the amount of the line SR_1, which equals NB (the amount which B extends up) + BM (the amount which C extends up).

The values NB and BM are found and OR_1 is computed as shown in working out the polar vector diagram. Note that the component actions AN, MC, NB and BM are not measured from the point O, since the vectors themselves do not start from the point O. The component actions of any vector by any method always start where the vector starts.

Draw scheme of electrical connections and rough sketches of both polar and topographic diagrams for each of the following problems. Obtain numerical result by means of the method shown in Fig. 86.

Prob. 59–3. On the armature of a special generator built for the study of phase relations, there are three separate coils, the terminals of which are marked AB, A_1B_1 and A_2B_2 as in Fig. 89. The e.m.f. of each coil is 110 volts.

A_1B_1 differs in phase from AB by 90°.
A_2B_2 leads A_1B_1 by 30°.

(*a*) What e.m.f. is obtained by joining A_1B_1 in series with AB, so that A_1B_1 leads AB by 90°?

(*b*) State the phase relation of the resultant e.m.f. to the e.m.f. of AB, noting whether it leads or lags.

(*c*) Draw rough sketch of sine curves representing all these e.m.f.'s.

Prob. 60–3. (*a*) What resultant e.m.f. is obtained when A_1B_1 is reversed in Prob. 59?

(*b*) State the phase relation of the resultant e.m.f. to the e.m.f. of AB, noting whether it leads or lags.

(*c*) Draw rough sine curves.

Prob. 61–3. (*a*) What is the resulting e.m.f. when AB, A_1B_1, A_2B_2 of Prob. 60 are joined in series so that A_2B_2 leads A_1B_1 by 30° and A_1B_1 leads AB by 90°?

(*b*) State the phase relation of the resulting e.m.f. to the e.m.f. of AB.

(*c*) Draw rough sine curves.

Prob. 62–3. (*a*) What would be the e.m.f. across the series circuit of Prob. 61, if coil A_2B_2 were reversed?

(*b*) State phase relation of resultant e.m.f. to the e.m.f. of AB.

(*c*) Draw rough sine curves.

Prob. 63–3. Assume coil A_1B_1 in Prob. 61 to be reversed and answer (*a*), (*b*) and (*c*), of that problem.

Prob. 64–3. A series circuit consists of two parts AB and BC, in series. A voltmeter placed across the entire circuit, or between A and C, indicates 110 volts, while the voltage across AB is 440 volts and across BC is also 440 volts. What is the phase angle between e.m.f.'s AB and BC?

Prob. 65–3. A series circuit is divided into three parts by points B and C located consecutively between the terminals A and D. The following voltmeter readings are taken under steady conditions of operation: $AB = 100$; $BC = 100$; $CD = 100$; $AC = 173.2$; $BD = 100$; $AD = 200$.

Draw topographic and polar diagrams to show the relations between these e.m.f.'s.

Prob. 66–3. Solve problem 65–3 on the basis of the following voltmeter readings: $AB = 100$; $BC = 100$; $CD = 100$; $AC = 100$ $BD = 100$; $AD = 173.2$.

Prob. 67–3. Solve problem 65–3 on the basis of the following voltmeter readings: $AB = 100$; $BC = 100$; $CD = 100$; $AC = 173.2$; $BD = 173.2$; $AD = 200$.

Prob. 68–3. Solve problem 65–3 on the basis of the following voltmeter readings: $AB = 100$; $BC = 100$; $CD = 100$; $AC = 100$; $BD = 100$; $AD = 0.0$.

28. Closed-delta Connections. We have seen that in a three-phase generator the e.m.f.'s of the three phases are all equal and differ from one another by 120°. When one phase is left idle, and the two remaining phases are so joined in series that the resulting e.m.f. across the two is equal to the voltage across either one, the machine is said to be connected in open delta.

Let us see what the voltage becomes if we join the idle phase in series with the two already in open delta. Assume that by joining A_1 to B, Fig. 81, phases AB and A_1B_1 are joined in open delta. Then in Fig. 90, OR would be the voltage across AB_1, the terminals of the series combination, and would be equal numerically to OB, the e.m.f. of A_1B_1

and to OA, the e.m.f. of AB. Assume that if we now join A_2 to B_1, the phase OC will lead OB by 120°. We then have a series combination the terminals of which would be AB_2. By the polar method of finding the voltage across a series combination of three parts we draw the resultant of two e.m.f.'s OA and OB as in Fig. 90a, and obtain OR as before. This resultant OR must then be combined with the vector OC representing the voltage of A_2B_2. But OR and OC are exactly equal and are seen by the diagram to be exactly opposite. The resulting action, or voltage, is therefore zero, since they exactly neutralize each other. The voltage across AB_2 thus seems to be zero. Testing this by the topographic method we construct Fig. 90b. Draw OA to represent the e.m.f. of AB; from the head end at 120° to OA, draw AB to represent the e.m.f. of A_1B_1; from the head end of AB and at 120° to it draw BC to represent the e.m.f. of A_2B_2. The distance from O to C is then the resultant voltage across the series combination of the three phases. Here again it is seen to be zero, since C falls exactly on O.

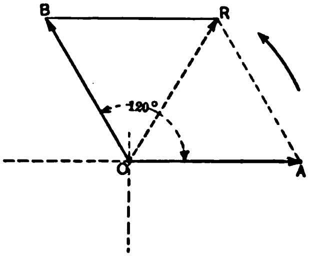


FIG. 90. The vector OR represents the voltage across the two coils AB and A_1B_1, joined in open delta.

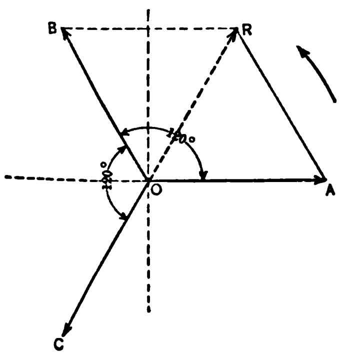


FIG. 90a. OC represents the voltage across the phase A_2B_2. Note that it is equal and opposite to the vector OR, which represents the voltage across the open-delta connection of the other two phases. A combination of OC and OR would result in zero voltage.

The voltage therefore across from A to B_2 is zero. It is perfectly safe then to join these points together. No current would circulate through the coils, since there would be no voltage between the ends joined. This is commonly done and the machine so connected is said to be joined in

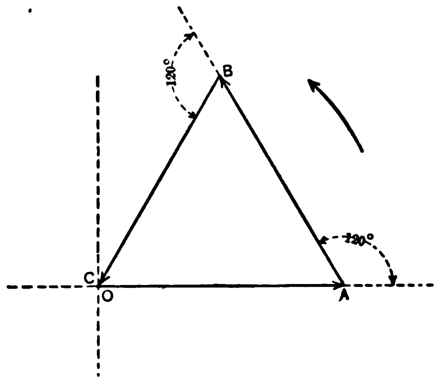


FIG. 90b. A topographic vector diagram for the closed-delta connection, showing that no voltage exists across series combination of the three phases when so joined.

closed delta. The word delta is the name of the Greek letter Δ, which closely resembles the diagrammatic representation of this connection. (See Fig. 90c.)

Usually A_1 is joined to B, A_2 to B_1 and B_2 to A, **inside the**

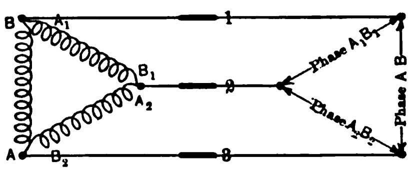


FIG. 90c. Diagram of a delta-connected machine. Merely the terminals 1, 2 and 3 are brought outside of the frame.

frame of the machine and three leads only, one from each juncture, are brought out to a terminal-board similar to that in Fig. 89. We then have three phases for use and but three lead wires, as is seen by Fig. 90c, which shows the connections to the inside of the same machine. Between the leads 1 and 2 is the e.m.f. of phase A_1B_1, between the leads 2 and 3 is the e.m.f. of phase A_2B_2, and between the leads 3 and 1 is the e.m.f. of phase AB.

Prob. 69–3. If the voltage across each path of the armature of the three-phase generator in Fig. 81 is 220 volts, show by the numerical solution of a polar diagram for this case that the voltage across AB_2 is zero when joined in series for a closed-delta connection.

Prob. 70–3. If in attempting to make a closed delta the phase AB (Fig. 81) were connected oppositely, by mistake, what would be the result? Draw a topographic diagram to represent the relations between the e.m.f.'s of the three phases, and calculate thereby the resultant e.m.f. of the series, which would act to produce an internal circulating current around the closed delta when the connections of the phases are completed. Voltage across each phase, 220 volts.

Prob. 71–3. Repeat the solutions of Prob. 70–3, with phase A_1B_1 only, reversed.

Prob. 72–3. Repeat the solutions of Prob. 70–3, with phase A_2B_2 only, reversed.

Prob. 73–3. Repeat the solutions of Prob. 70–3, with phases AB and A_1B_1 both reversed.

Prob. 74–3. Repeat the solutions of Prob. 70–3, with phases A_1B_1 and A_2B_2 both reversed.

Prob. 75–3. Repeat the solutions of Prob. 70–3, with phases AB and A_2B_2 both reversed.

Prob. 76–3. Repeat the solutions of Prob. 70–3, with phases AB, A_1B_1 and A_2B_2 all reversed.

29. Summary of Vector Addition. We have seen that by reversing the connections of a coil producing an e.m.f., we change the phase of the e.m.f. in that coil by 180 electrical degrees. Therefore in representing the e.m.f. of the reversed coil, we use a vector which has been turned 180° from the original position. This brings it into a position exactly opposite the original position. It makes no difference whether we call it a lead of 180° or a lag of 180° with respect to the original position of the vector, since either will bring it exactly opposite the original position.

It is necessary to choose a certain direction through the coil as the positive direction of the e.m.f. Of course the e.m.f. is continually alternating, but we choose one direction as the **positive direction,** just as we choose the upper loop of the e.m.f. sine curve as the positive loop in order to define the relations to each other, of several e.m.f.'s acting in the same circuit.

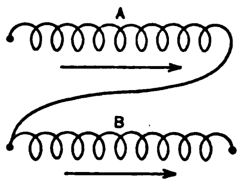


FIG. 91a. Coils *A* and *B* are joined in series. Arrows show the positive directions of e.m.f. in the coils to be in the same direction.

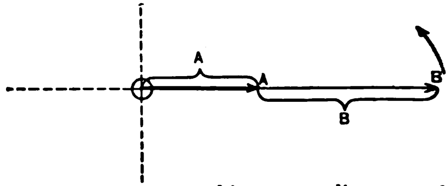


FIG. 91b. Topographic vector diagram of the e.m.f.'s in the coils of Fig. 91a.

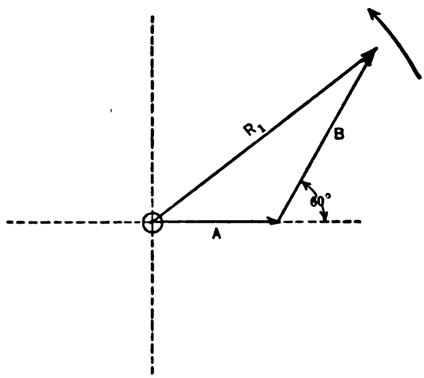


FIG. 91c. Topographic vector diagram of e.m.f.'s in coils of Fig. 91a, if the phase difference is 60°.

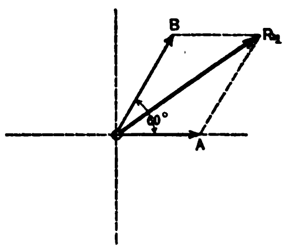


FIG. 91d. Polar diagram corresponding to topographic diagram in Fig. 91c.

Suppose that the e.m.f. in coil *A* (Fig. 91a) reaches its maximum value in the direction chosen as positive (shown by arrow below coil *A*), at exactly the same instant that the e.m.f. in coil *B* reaches its maximum positive value (chosen positive direction shown by arrow below coil *B*). The two e.m.f.'s are therefore **in phase.** Let the (+) end of coil *A* (at head of arrow representing positive direction of e.m.f.) be connected to the (−) end of coil *B*. The positive directions therefore coincide in the series circuit thus formed; the vectors representing these e.m.f.'s are pointed in the same direction, so that in a topographic diagram (Fig. 91b) they lie along the same straight line with head end of one touching tail end of other. The total e.m.f. is equal to the arithmetical sum of the e.m.f.'s *A* and *B*.

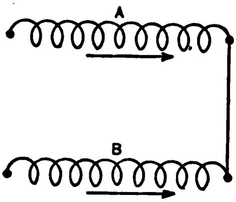


FIG. 92a. The connection of coil *A* to coil *B* has been changed from that of Fig. 91a so that the positive directions of the e.m.f.'s in the two coils oppose each other.

Suppose now that the **e.m.f.'s** remain **unchanged,** while we **reverse** the **connection** of *B* to *A*, as shown in Fig. 92a. Considering the entire series circuit, we see that the e.m.f. in *A* reaches its maximum (+) value at the same instant that the e.m.f. in *B* reaches its own maximum (+) value, but the latter value

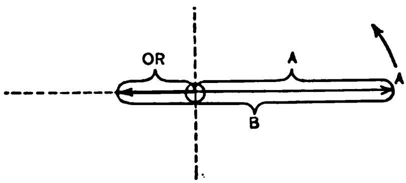


FIG. 92b. The topographic vector diagram for the e.m.f.'s when so joined that they oppose each other as in Fig. 92a.

is opposed to the former on account of the manner of connecting between coils. Plainly, the total e.m.f. is now the arithmetical **difference** between the e.m.f.'s of *A* and *B*, as illustrated by the resultant vector *OR* in the topographic diagram Fig. 92b.

Before we make a summation of vectors in any vector diagram, **it is absolutely necessary that we arrange all vectors so**

that they represent phase relations between the various e.m.f.'s or currents, with respect to the same positive direction through the entire circuit. The angles between vectors on any given plane diagram can represent only one thing: in a force diagram in mechanics, the angles between lines of force represent actual differences of direction in space, whereas in an electrical diagram, where each vector represents an alternating e.m.f., the angles between vectors represent the **time** or **phase** relations between the e.m.f.'s. There are,

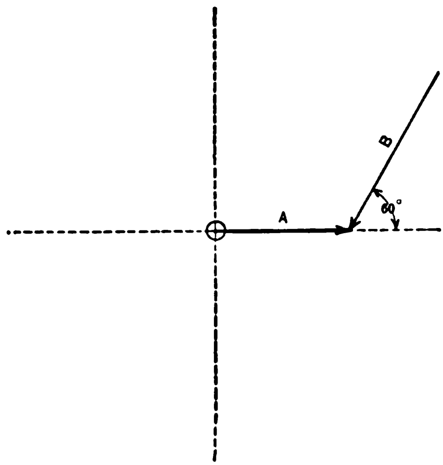


FIG. 92c. The topographic vector diagram of the e.m.f.'s in coils *A* and *B* of Fig. 92a if the e.m.f.'s are at 60° to each other.

however, three ways in which alternating e.m.f.'s differ among themselves, namely, **value or magnitude, time relation** between their maximum values (or phase relation), and **direction** through the electrical circuit. The vector diagram can represent only **two** of these differences: the **length** of each vector represents the effective or the maximum **value** of the e.m.f., and the **angles** between vectors represent difference of phase, or **difference in time** between the instants at which the corresponding e.m.f.'s attain their respective maximum values. As the relative direction of the various e.m.f.'s or currents through their common circuit cannot be represented on the diagram, it is necessary to arrange the vectors so that **differences of direction are eliminated,** before they can be combined to find the total or resultant e.m.f., or current, in the circuit.

This will be clearer if we discuss e.m.f.'s which have some other phase difference than 0° or 180°. Let us say that the e.m.f. in B (Fig. 91a) reaches its maximum value 60° (one-sixth period) **before** the e.m.f. in A reaches its maximum value, in the directions marked by arrows as positive. As the positive directions of both e.m.f.'s in this case point in the same direction through their common circuit, the topographic diagram, Fig. 91c, represents correctly the vector relations, and the vector R_1 represents the resultant or total e.m.f. $A \oplus B$. When we reverse the connections of coil B as in Fig. 92a, we must also reverse the corresponding e.m.f. vector as in Fig. 92c. Now the topographic diagram should be arranged so that the resultant is represented by a line drawn from the tail of the first vector to the head of the last in the series, but this requires that all the arrows point in the same direction; head of one joined to tail of next, and so on. Since Fig. 92c does not allow this, Fig. 92cc is arranged in this way, and is exactly equivalent to Fig. 92c. After this rearrangement of vectors has been made, the resultant is obtained directly, as R_2, by joining the beginning of vector A to the end of vector B, with a straight line; R_2 lags behind A by the angle θ and leads B by the angle ϕ. Notice here, that when B_m occurs 60° earlier than A_m but in the **opposite** direction (as in Fig. 92c), the resultant is exactly

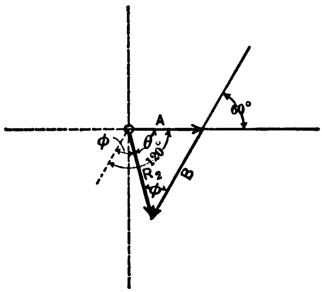


FIG. 92cc. Rearranged topographic diagram of Fig. 92c, putting arrows "tail-to-head."

the same as when B_m occurs 120° later than A_m but in the same direction (as in Fig. 92cc). This must be so, of course, because an e.m.f. reaches its maximum value in one direction just one-half period or 180 electrical degrees before (or after) it passes through its maximum value in the opposite direction. So when B is added in the direction opposite to A and leading A by 60°, it is equivalent to adding B in the same direction as A but leading by (60° + 180°), or (60° − 180°),

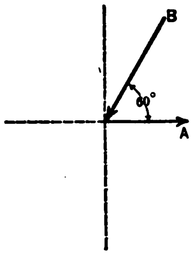


FIG. 92d. Polar diagram corresponding to topographic diagrams 92a, b, c.

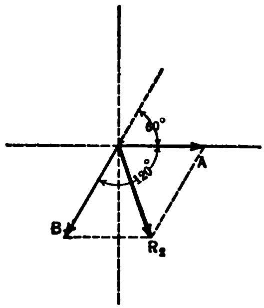


FIG. 92dd. Rearranged polar diagram of Fig. 92d, putting vector B so that it starts from O.

which means leading by 240° or lagging by 120°, as shown in Fig. 92cc. Figures 91d, 92d and 92dd are the polar diagrams corresponding to Fig. 91c, 92c and 92cc, respectively.

Note the following with regard to vector diagrams:

First. The arrows in Fig. 91 and Fig. 92 do not represent any instantaneous direction of e.m.f., but merely the direction which we decide to call the positive direction, in order to draw our vector diagrams conveniently for finding resultants.

Second. Before the parallelogram may be completed in order to find the resultant diagonal in any **polar diagram,** the angles between vectors must be so adjusted that **all vectors point away from the origin or pole.**

Third. Before the resultant vector may be drawn, joining the tail of the first vector to the head of the last of any string of vectors in a **topographic diagram,** the angles between vectors must be adjusted so that **all vectors point in the same direction along the string.**

The following figures show the polar and topographic vector diagrams for phase differences of 60°, 120° and 180°, lead or lag, and connections arranged for either additive or subtractive relation of positive directions in the circuit. These are the relations most commonly met in practice. The diagrams have been arranged to show the differences and the fundamental ideas as clearly as possible. In fact they form a sort of index to the subject of vector diagram interpretation of e.m.f. or current relations in the alternating-current circuit, and if thoroughly mastered they can be used as a sort of reference to check all future vector diagrams.

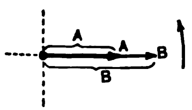
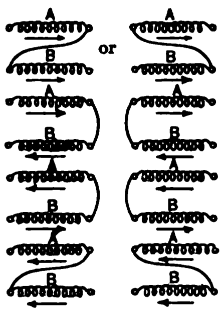


FIG. 93a. Polar diagram of vectors.



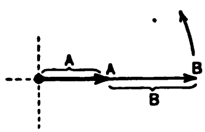


FIG. 93c. Topographic diagram of vectors.

FIG. 93b. Coil connections showing directions in which e.m.f.'s reach maximum values with time differences as represented by the phase angles of the vector diagrams.

A and B are in phase: That is, they both reach their positive maximum values at the same instant and the negative maximum values at the same instant.

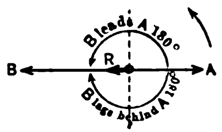
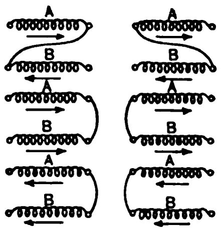


FIG. 93aa. Polar diagram of vectors. R = resultant.



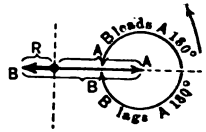


FIG. 93cc. Topograpic diagram of vectors. R = resultant.

FIG. 93bb. Coil connections which produce this result. Note that in all cases the positive direction through one coil is reversed from its corresponding direction in Fig. 93b.

B leads or lags behind A by 180°. That is, $+ B_m$ occurs 180° later or sooner than $+ A_m$, and $- B_m$ occurs 180° later or sooner than $- A_m$, etc., $+ B_m$ therefore occurs at the same instant as $- A_m$ and vice versa.

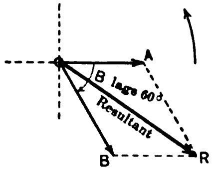


FIG. 94a. Polar diagram of vectors.

FIG. 94b. Two of the many possible connections to produce this effect.

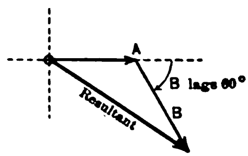


FIG. 94c. Topographic diagram of vectors.

B lags 60° behind A, in the same direction. That is, $+ B_m$ occurs $\frac{1}{6}$ period after $+ A_m$, and $- B_m$ occurs $\frac{1}{6}$ period after $- A_m$.

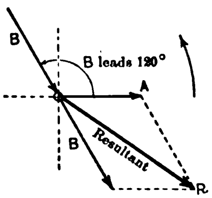


FIG. 94aa. Polar diagram of vectors.

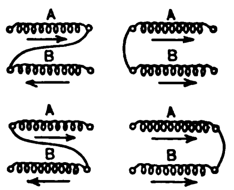


FIG. 94bb. Four of the many possible connections to produce this result.

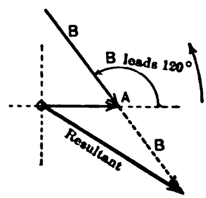


FIG. 94cc. Topographical diagram of vectors.

B leads A by 120°, in the opposite direction. That is, $+ B_m$ occurs $\frac{1}{3}$ period before $- A_m$ and $- B_m$ occurs $\frac{1}{3}$ period before $+ A_m$.

Note that the resulting vector diagrams are identical with Fig. 94a and c, in which B lags 60° behind A. This shows how the reversal of coil connections is equivalent to the reversal of the positive direction through the coil, or to changing the phase of the vector by 180°.

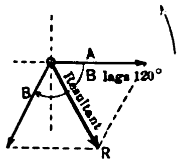


FIG. 95a. Polar diagram of vectors.

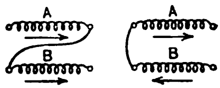


FIG. 95b. Two of the many possible connections to produce this result.

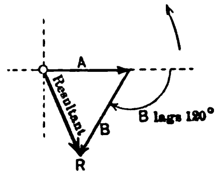


FIG. 95c. Topographic diagram of vectors.

B lags 120° behind A, in the same direction. That is, $+B_m$ occurs 120° later than $+A_m$ and $-B_m$ occurs 120° later than $-A_m$, etc.

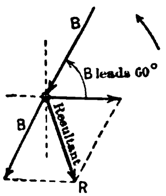


FIG. 95aa. Polar diagram of vectors.

FIG. 95bb. Two of the many possible connections to produce this effect.

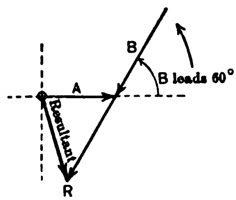


FIG. 95cc. Topographic diagram of vectors.

B leads A by 60°, in the opposite direction. That is, $+B_m$ occurs 60° sooner than $-A_m$ and $-B_m$ occurs 60° sooner than $+A_m$, etc. Note that these diagrams are similar to 95a and c, showing that a lead of 60° in opposed directions is equivalent to a lag of 120° in the same direction.

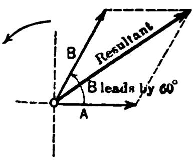


FIG. 96a. Polar vector diagram.

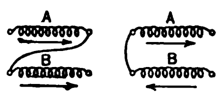


FIG. 96b. Two of the many possible connections to produce this condition.

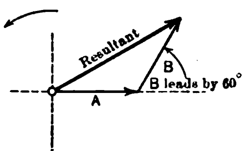


FIG. 96c. Topographic vector diagram.

B leads A by 60°, in the same direction. That is, $+B_m$ occurs 60° before $+A_m$ and $-B_m$ occurs 60° before $-A_m$, etc.

Compare carefully with Fig. 95aa and 95cc to note the meaning of terms used.

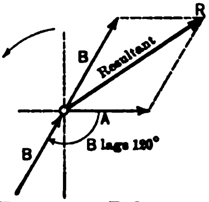


Fig. 96aa. Polar vector diagram.

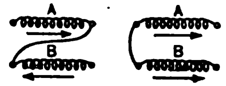


Fig. 96bb. Two of the many possible connections to produce this condition.

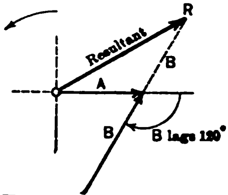


Fig. 96cc. Topographic vector diagram.

B lags behind A by 120°, in the opposite direction. That is, $+B_m$ occurs 120° later than $-A_m$, and $-B_m$ occurs 120° later than $+A_m$, etc.

Compare carefully with Fig. 95a and 95c and note the difference in meaning of the terms, "*lag 120° in the same direction*" and "*lag 120° in the opposite direction.*" Note also the similarity of all Fig. 96 diagrams.

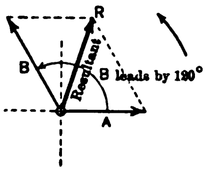


Fig. 97a. Polar vector diagram.

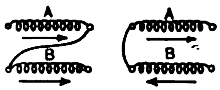


Fig. 97b. Two of the many possible connections to produce this result.

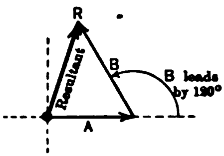


Fig. 97c. Topographic vector diagram.

B leads A by 120°, in the same direction. That is, $+B_m$ occurs 120° before $+A_m$ and $-B_m$ occurs 120° before $-A_m$, etc.

Compare carefully with Fig. 94aa and 94cc to note the difference in meaning of the terms "*leads by 120° in the same direction,*" and "*leads by 120° in the opposite direction.*"

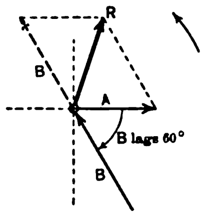


Fig. 97aa. Polar vector diagram.

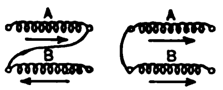


Fig. 97bb. Two of the many possible connections to produce this effect.

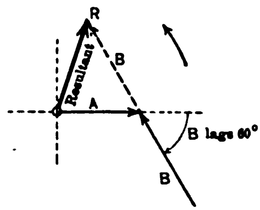


Fig. 97cc. Topographic vector diagram.

B lags behind A by 60°, in the opposite direction. That is, $+B_m$ occurs 60° after $-A_m$ and $-B_m$ occurs 60° after $+A_m$, etc.

Compare carefully with Fig. 94a and 94c to note the difference in meaning between "*lags 60° in the same direction*" and "*lags 60° in the opposite direction.*"

Note also by the similarity of all diagrams in Fig. 97 that "*leads by 120° in the same direction*" is equivalent to "*lags 60° in the opposite direction.*"

Prob. 77–3. The e.m.f. of 110 volts across the coil AB, Fig. 98, leads the e.m.f. of 110 volts across CD by 120°, with respect to positive directions as marked. Construct polar and topographic vector diagrams and compute the e.m.f. between the wires 1 and 3.

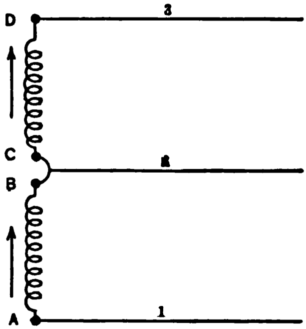


FIG. 98. Positive direction through the coils is the same.

Prob. 78–3. The e.m.f. of 110 volts across the coil RS, Fig. 99, leads the e.m.f. of 110 volts across the coil TV by 120°, but the positive directions are opposed. Construct polar and topographic vector diagrams and compute the e.m.f. between the line wires x and y.

Prob. 79–3. Solve Problem 77 assuming that the e.m.f. of AB leads the e.m.f. of CD by 60°.

Prob. 80–3. Solve Problem 78 assuming that the e.m.f. of RS leads the e.m.f. of TV by 60° with positive directions opposed.

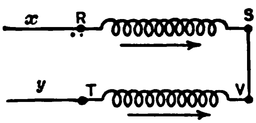


FIG. 99. The voltage across RS leads the voltage across TV by 120° but in the opposite direction.

Prob. 81–3. Assume that the phase difference in Prob. 77 is 15° and recompute the e.m.f. across 1 and 3.

Prob. 82–3. Compute the e.m.f. across x and y, Prob. 78, assuming the phase difference is 15° with positive directions opposed.

Prob. 83–3. Repeat Problems 77 and 78, using a phase difference of 145°.

30. Star or Y Connections. If, by applying the voltmeter test, we found that the correct open-delta connection for the three-phase generator in Fig. 100, was to join A_1 to B, there would be the same voltage from A to B_1 as from A to B when the phases are so joined. The simple diagram of connections would be as in Fig. 100a, which is equivalent to Fig. 97b. The vector diagram would be like Fig. 97c. This is also shown by the topographic vector diagram in Fig. 101, which is self-explanatory.

Suppose now that we reverse the connection to phase A_1B_1 by connecting B_1 to B instead of A_1 to B. The simple connection diagram now becomes Fig. 102 which is like **Fig.** 94bb. The positive directions through the circuit now **oppose** each other. The topographic vector diagram will be like Fig. 94cc, as indicated in Fig. 102a, which is a topographic diagram differing from Fig. 101 only in that the end B_1 instead of the end A_1 of vector A_1B_1 is attached to the end B of vector AB. The magnitude and relative position of these vectors remain unchanged. Fig. 102a shows that the e.m.f.'s AB and A_1B_1 reach maximum values in **opposite** directions through their common series circuit, at instants

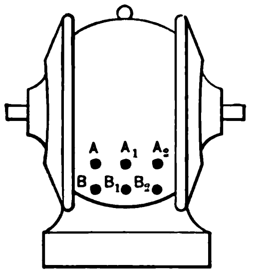


FIG. 100. A three-phase generator with both ends of each phase brought out.

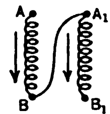


FIG. 100a. The simple diagram of connections for coils AB and A_1B_1 of the generator in Fig. 100.

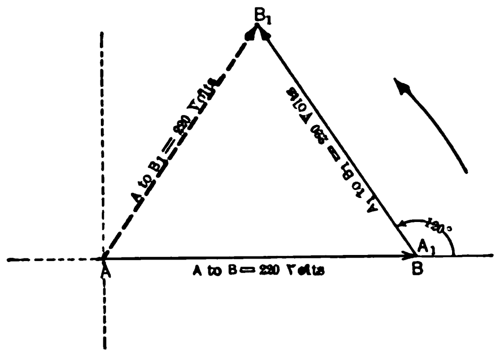


FIG. 101. Topographic vector diagram for the voltage across an open-delta connection.

separated by an interval of time represented by 120 electrical degrees.

Before we can add these vectors and get their resultant, we must redraw the diagram so as to show the differences of phase, or time, between the instants at which they reach their maximum instantaneous values, in the **same** direction through their common circuit. If the e.m.f. (A_1 to B_1) leads the e.m.f. (A to B) by 120°, then the e.m.f. (B_1 to A_1) leads the e.m.f. (A to B) by (120° ± 180°), or lags behind the e.m.f. (A to B) by 60°. But the directions (B_1 to A_1) and (A to B) are identical in the new series connection of coils, hence after reversing the direction of the A_1B_1 vector (or swinging it through 180°), as in Fig. 103, we have the correct arrangement for a topographic diagram; and then a straight line drawn from A to A_1 represents the total e.m.f. of the two phases in series, with B_1 connected to B.

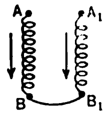


FIG. 102. The simple connection diagram if the coil A_1B_1 of the generator in Fig. 100 is reversed in its connection to coil AB.

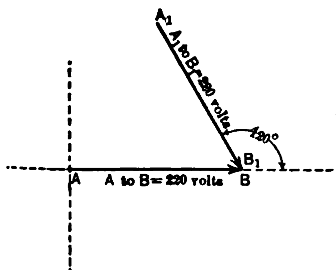


FIG. 102a. Topographic vector diagram of e.m.f.'s in coils AB and A_1B_1 of the generator of Fig. 100, when the connection of A_1B_1 is reversed.

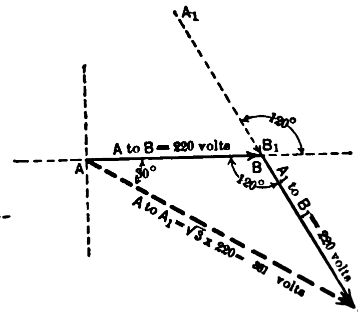


FIG. 103. Topographic solution of the diagram in Fig. 102a. The vector AA_1 represents the e.m.f. across coils AB and A_1B_1 when B_1 is joined to B.

The numerical value of this resultant e.m.f. from A to A_1 (Fig. 103) may be found by the law of sines (see Appendix A).

$$\frac{\text{e.m.f. } A \text{ to } A_1}{\text{e.m.f. } A \text{ to } B} = \frac{\text{sine } 120°}{\text{sine } 30°} = \frac{.866}{.5} = 1.73 = \sqrt{3}$$ (when AB and A_1B_1 are equal),

or $$\text{e.m.f. } (A \text{ to } A_1) = \sqrt{3} \times \text{e.m.f. } (A \text{ to } B),$$

or $$E_{A_1A} = 1.73 \times 220 = 381 \text{ volts.}$$

Thus the voltage across the points A–A_1 of the two phases so joined in series is $\sqrt{3}$, or 1.73, times the voltage across one phase. When one phase of an open-delta is reversed the voltage across the combination always equals $\sqrt{3}$, or 1.73, times the voltage across one phase.

By joining the phase A_2B_2 (also in the reversed direction) to this combination of the other two phases (that is, by joining end B_2 to the juncture of B and B_1), we get the **star** or **Y connection**; so-called because the figure made by the joined phases resembles a star or the letter Y. Thus, when the three phases all have their B ends joined they look like

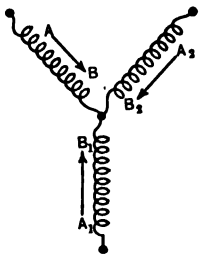


FIG. 104. The conventional diagram for a Y-connected set of coils. Compare with Fig. 105, which is a Δ-connected set.

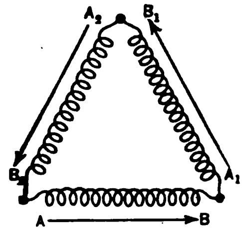


FIG. 105. A Δ-connected set of coils.

Fig. 104. Compare this with their appearance when joined in delta as in Fig. 105. When Y-connected there are three sets of two phases in series, but note that in each set one of the phases is reversed from its position in the delta connection. Thus the voltage across any pair is equal to $\sqrt{3}$ times the voltage across a single phase as shown in the solution of Fig. 103.

Prob. 84–3. What would be the voltage across any two terminals of the generator in Prob. 69 if it were Y-connected?

Prob. 85–3. The voltage across any two terminals of a Y-connected generator is 381 volts. What is the voltage across each component part of any armature path in the machine?

31. Parallel Connections. Suppose that we connect two valveless pumps of exactly the same size in series as in Fig. 106. Note that the head end of one pump is connected to the crank end of the other. Since they are in series the same current is flowing through each as is flowing through the pipe line. Let us assume that the maximum current for each is 100 gal./min.

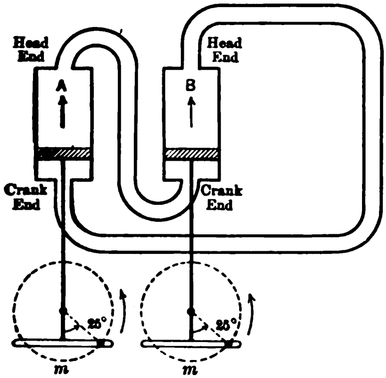


FIG. 106. The two pumps are in series.

Then, at the position at which they happen to be at this instant, 25° from the dead center (m) the current flowing in each would be

$$100 \sin 25° = 42.3 \text{ gal./min.}$$

This would also be the current in the main pipe line. Now suppose we connect the same pumps in parallel as in Fig. 107. Note particularly that now the two head ends are joined, and the two crank ends. **This amounts to turning one** pump around, or reversing it with respect to the path through the pumps.

They no longer pump water through each other in forcing it into the line, but each supplies a separate current to the main pipe line. Thus the current flowing in this pipe line must be the sum of the currents supplied by the two pumps. As the pistons have the same positions as in Fig. 106, they must each be supplying 42.3 gal./min., and the main pipe line must be carrying 2 × 42.3 gal./min., or 84.6 gal./min.

It is to be noted also that in order to supply this current to the line, each pump must exert exactly the same pressure as the other at all times. Thus it must not only have the same maximum pressure as the other, but it must also reach its maximum pressure at the same instant that the other

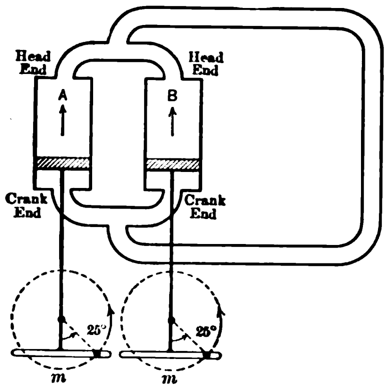


FIG. 107. The two pumps are in parallel. Note that this mode of connection amounts to the reversal of the series connection.

does. In other words, in order to work satisfactorily, the different units of a parallel combination must have the **same pressure,** and be in **phase** and in **synchronism.**

The same thing is true of alternating-current generators. They must not only have equal voltage, but they must also run in phase and in synchronism, that is, reach their maximum, minimum and all other instantaneous values at exactly the same instant.

What happens when two alternators are not synchronized, can easily be understood by referring to Fig. 108, which represents the same two pumps running in parallel, but **out of phase.**

The current being delivered by cylinder A equals

$$100 \sin 90° = 100 \text{ gal./min.}$$

The current being delivered by cylinder *B* equals

$$100 \sin 300° = -100 \sin 60°$$
$$= -86.6 \text{ gal./min.}$$

The statement that cylinder *B* is **delivering** −86.6 gal./min. means that it is **receiving** 86.6 gal./min.

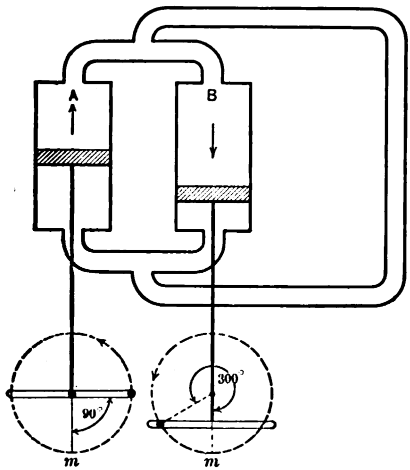


FIG. 108. The two pumps are joined in parallel, but they are not in phase.

Thus, if *A* is delivering 100 gal./min and *B* is receiving 86.6 gal./min., then the pipe line must be getting at this instant only 100 − 86.6, or 13.4 gal./min., although each pump is working as fast as when they were in phase in Fig. 107. In other words, if two pumps were connected in this manner and were out of step, they would be delivering a considerable current of water to each other, and thus the line would receive but little.

This is exactly what happens if two alternators are connected in parallel when they are **out of phase**; they send through each other such large surges of current which do not reach the main line at all, that special protective apparatus is usually installed to prevent the generators from ruining themselves from this cause.

Another way of showing the same thing would be to compute the maximum current for both Fig. 107 and Fig. 108, instead of some other instantaneous current as above.

The topographic vector diagram for Fig. 107, where both are in phase would be as in Fig. 109, where I_A represents the current delivered by pump A, and I_B the current delivered by pump B.

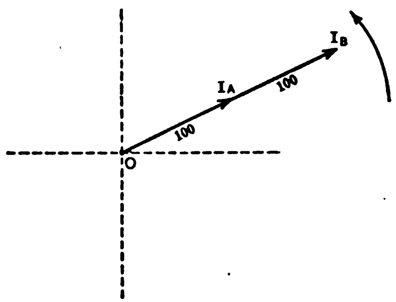


FIG. 109. Topographic vector diagram for current delivered to the line by the pumps in Fig. 107.

The maximum current in the pipe line would be the vector sum, OI_B, of the currents in each cylinder or

$$I_{\text{line}} = I_A \oplus I_B = 100 + 100$$
$$= 200 \text{ gal./min.}$$

Similarly, the vector diagram for Fig. 108, when they are (300° − 90°), or 210°, out of phase would be as in Fig. 110. The current in the pipe line would be the vector sum OI_B of the currents in each cylinder. Note how much smaller OI_B is in this figure than in Fig. 109.

$$I_{\text{line}} = I_A \oplus I_B$$

The value of this line current can be found more easily from the polar diagram constructed in Fig. 111, where OR equals the line current.

$$I_{line}^2 = I_A^2 + I_B^2 + 2 I_A I_B \cos 150°$$
$$= 10{,}000 + 10{,}000 - 20{,}000 \times 0.866$$
$$= 2680,$$
$$I_{line} = \sqrt{2680}$$
$$= 51.8 \text{ gal./min.}$$

The **maximum current** ever flowing through the **main line** would then be but 51.8 gal./min., although through each pump would be flowing a maximum current of 100 gal./min. Compare this with 200 gal./min. in the main line when the pumps were running in phase and in synchronism.

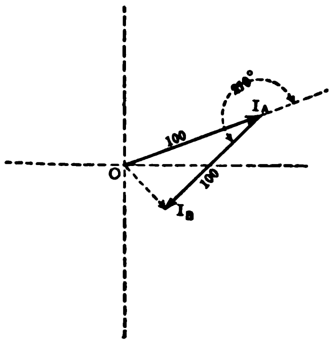


FIG. 110. Topographic vector diagram for the current delivered to the line by the pumps in Fig. 108.

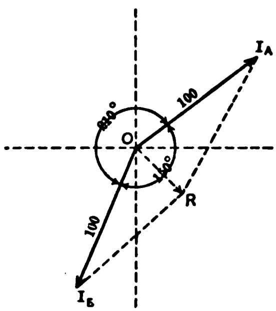


Fig. 111. Polar vector diagram for the current delivered by the pumps in Fig. 108.

Prob. 86–3. The pumps in Fig. 107 are running in phase and in synchronism and each supplies a maximum current of 48 gal./min. What current is flowing in the pipe line, when the pumps are at the 135° position, and in what direction is it flowing, clockwise or counter-clockwise?

Prob. 87–3. If the pumps in Fig. 108 were only 30° out of step, what would be the maximum current in the main pipe line?

Prob. 88–3. What would be the values of the instantaneous currents in each of the pumps in Prob. 87, when the current in the pipe was at the maximum value?

Prob. 89–3. If the pumps in Fig. 108 were 180° out of phase, what would be the maximum current in the pipe line?

32. Parallel Circuits: Voltage and Current Relations. The above discussion has a close analogy in the case of two alternators A and B connected in parallel as in Fig. 112, and delivering current to the common mains. Assume that each generator delivers normally a maximum current of 100 amperes and that the current output of each machine remains the same (100 amperes maximum, or 70.7 amperes effective) under all conditions. Then, if they should get as much as 210° out of step, the resulting current in the line would drop to 36.6 amperes effective if the current of each stays constant (or OI_B, Fig. 110, $\div \sqrt{2}$), as against 141.4 amperes effective (or OI_B, Fig. 109, $\div \sqrt{2}$), when the machines were in phase. As a matter of fact, the impedance of each machine is so small that it requires but very small phase difference to allow one machine to force an enormous surge of current through the other, since it is the e.m.f. of each generator which remains constant and the current increases greatly. Unless protected by automatic devices, the machines may be damaged. Such protection is afforded by an automatic circuit-breaker connected in the leads between the terminals of each generator and the bus-bars; also, by "current-limiting reactance coils."

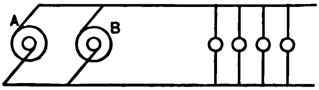


FIG. 112. The alternators A and B are connected in parallel and both supply current to the line if they are in synchronism.

From the foregoing it may be seen that:

First. The voltage across a parallel combination is the **same** as the voltage across each branch.

Second. The current through a parallel combination equals the **vector sum** of the currents through the branches.

The impedance of a parallel circuit will be taken up in the next chapter.

It will be seen that the rules for the current and voltage relations in series and parallel combinations are the same for alternating currents as for direct currents, except that whenever the word **sum** appears in alternating-current work, it must be understood to mean the **vector sum.**

Prob. 90–3. A choke coil carrying 12 amperes is put in parallel with a resistance carrying 6 amperes. The current in the choke coil lags 80° behind the current in the resistance. How much current does the line feeding them carry?

Prob. 91–3. In the parallel circuit of Fig. 113, a-c. ammeter A_3 reads 32 amperes, A_2 reads 21 amperes and A_1 reads 18 amperes. What is the difference in phase between currents through A_1 and A_2?

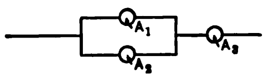


FIG. 113. Ammeters A_1 and A_2 measure the currents in the parallel branches of the circuit. Ammeter A_3 measures the resulting current in the line.

Prob. 92–3. What is the phase difference between A_1 and A_3 in Prob. 91?

Prob. 93–3. If the current through A_1 in Fig. 113 were 90° ahead of the current through A_2 and these ammeters read 20 amperes each, what would A_3 read?

Prob. 94–3. If the current through the a-c. ammeter A_1 were 120° behind the current through A_2, and the two ammeters read 20 amperes each, what would ammeter A_3 read?

Prob. 95–3. A circuit consists of three branches. Through one branch flows an alternating current of 10 amperes. Through the second branch flows an alternating current of 12 amperes, lagging 30° behind the current in the first branch. Through the third branch flows a current of 8 amperes leading the current in the first branch by 50°. How great is the current through the wire feeding the combination?

Prob. 96–3. How large will the instantaneous currents be in the second and third branches of Prob. 95, when the current in the first branch has a value of 13 amp.?

33. Current in the Phases of a Delta-connected Machine. Assume that the three phases AB, BC, and CA of the generator in Fig. 114 are connected in delta as shown and are feeding the line wires 1, 2, 3, to which is attached a three-phase a-c. motor. As we have previously seen, when a generator is delta-connected to the line, there is the same voltage across each pair of line wires as across each phase of the winding. Let us assume this to be 220 volts.

Let us see how the current in each line wire compares with the current flowing in each phase of the generator, which we will assume to be 25 amperes. Consider the current in line wire No. 2. This wire is fed by two currents, from the phases *BC* and *CA* in parallel. Each line wire on any three-phase system may be considered as a return wire for the currents flowing on the other two line wires; the

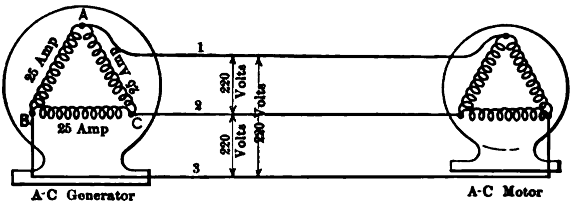


FIG. 114. The phases of the generator are connected in delta and deliver current to the line wires 1, 2 and 3.

current flowing out from the generator along one line wire must be equal in value and of opposite phase to the vector sum of currents flowing out from the generator along all other line wires of the system. In fact, we merely apply to the point *C* where line wire 2 joins the phases *BC* and *CA*, that one of Kirchhoff's laws which states that, wherever any number of conductors join at a point, the sum of currents flowing away from that point must be equal to the sum of currents flowing toward the point. This rule holds algebraically or arithmetically with regard to direct currents or with regard to instantaneous values of alternating currents. It also applies vectorially to effective and maximum values of alternating currents.

Having chosen the positive directions following one another all in the same direction around the delta as in Fig. 115 (this being the only way in which each e.m.f. can lead the preceding one by the same angle, namely 120°), these arrows will show also the positive directions for currents in the delta. Consider the usual case of a "balanced load," in which the currents in the various windings have the same phase difference as the corresponding e.m.f.'s. The current flowing in line wire 2 must be the vector difference of the currents in phases *BC* and *CA* of the generator. This is true regardless of which direction we choose as positive in the line 2. Thus, suppose we choose the positive direction in 2 as **outward** from *C*, as shown in Fig. 115; then, since

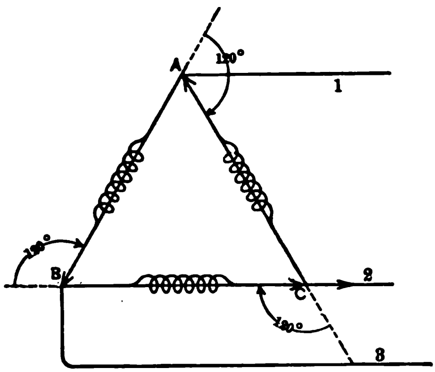


FIG. 115. The arrows show the direction chosen as positive for both e.m.f.'s and currents in a delta-connected machine and leads. Note that the positive direction in *BC* and in the line wire 2 are in the same direction, but that the current in *CA* is in the opposite direction. The arrows do *not* indicate instantaneous values.

the positive directions of the current in *CA* and in line 2 are **away** from *C* while only that in *BC* is **toward** *C*, it must be that the current **out** along 2 is equal to the current in *BC* **minus** the current in *CA*. The subtraction is by vectors, if effective (or maximum) values of current are used. It is by plain arithmetic, if instantaneous values of current are used. In the latter case note that the instantaneous current in *CA* must nearly always be negative or opposite to the marked positive direction, whenever the instantaneous current in *BC* is positive or the same as marked by the arrow on *BC*, on account of the 120° phase difference between *CA* and *BC*. This means that the instantaneous current in line 2 must usually be the numerical sum of the instantaneous currents in *BC* and *CA*, since it is usually the difference between a positive and a negative current.

The positive direction is chosen outward along line 2 in Fig. 115, and the corresponding topographic vector diagram showing the derivation of resultant vector of current in line 2 is shown in Fig. 116. The total current out of terminal *C*

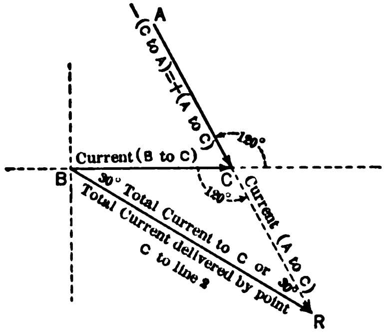


FIG. 116. Topographic vector diagram. The vector *BR* represents the current delivered by the coil *BC* and *AC* to the line No. 2.

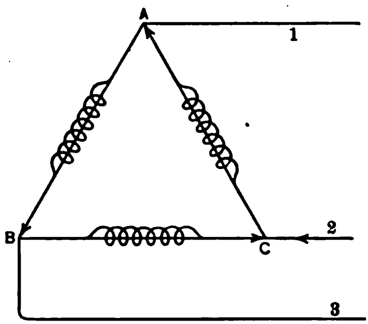


FIG. 117. The result of considering the current in line wire 2 to have its positive direction toward *C*, instead of away from *C* as in Fig. 115. Note that the current in *AC* and line 2 have the same positive direction.

from the generator windings to line 2 is equal to the current (*B* to *C*) minus the current (*C* to *A*); but minus (*C* to *A*) is the same as plus (*A* to *C*), that is, the vector *CA* reversed. After reversing vector *CA*, we add it to the vector *BC* by the topographic diagram of Fig. 116; the resultant *BR* represents the total current delivered by the generator windings out of terminal C to line 2, or $(I_{BC} \ominus I_{CA})$.

The current CA leads the current in BC by 120° but in the reversed direction, if we consider the direction of currents positive as marked in BC and line 2. Fig. 116 thus is like Fig. 94cc, with which it should be compared.

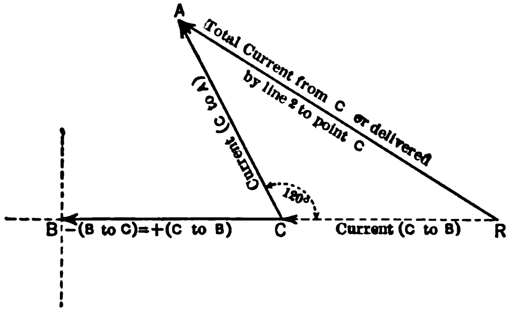


FIG. 118. Topographic vector diagram corresponding to Fig. 117, with the positive direction in line No. 2 reversed. AR represents the current in line wire No. 2.

If the positive direction is chosen **inward** from line 2 toward terminal C, as in Fig. 117, the corresponding vector diagram is given in Fig. 118. The resultant RA represents the vector difference $(I_{CA} \ominus I_{BC})$, which is the total current flowing away from C within the windings of the generator, or **toward** C from line 2. The resultants in Fig. 116 and 118 both have the same value and the same phase relations to the currents in the other parts of the system, therefore it appears to make no difference whatever which direction we choose as positive in a line wire.

From the vector relations in Fig. 116 and 118 we may calculate the value of the current passing between line 2 and terminal C as follows:

$$I_2^2 = (BR)^2, \text{ or } (RA)^2 = (CA)^2 + (BC)^2 + 2\,(CA)\,(BC) \cos 60°$$

(solving as a parallelogram, see Appendix A)

or

$$I_2/(BC) = \text{sine } 120°/\text{sine } 30°$$

(solving by law of sines, Appendix A).

By either method, since CA is numerically equal to BC for a "balanced load" such as is assumed in this case (25 amperes in each phase of the generator winding), we arrive at the result:

$$I_2 = \sqrt{3}\,(BC) = \sqrt{3}\,(CA) = 1.73 \times 25 = 43.3 \text{ amperes},$$

or, in general, **current in each line wire = $\sqrt{3}$ × current in each phase of the delta.** The complete vector diagram, showing relations of value and phase for currents in all parts of the delta and all line wires, is shown by the topographic diagram in Fig. 119 and by the polar diagram in Fig. 120.

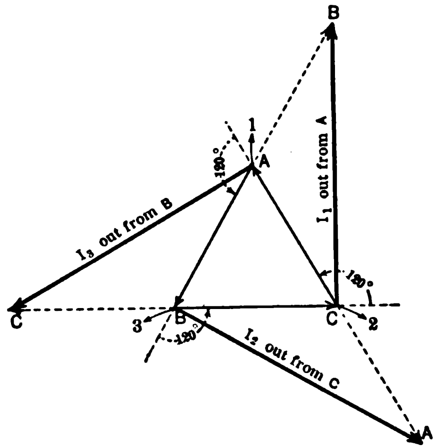


FIG. 119. Topographic vector diagram showing the currents and phase relations of the circuits in the three line wires.

In each case the positive direction of the current in the line is *out* from the machine.

Any machine using more than one phase is said to be a **polyphase** machine. When the currents in all the phases are equal, the phases are said to be **balanced.**

The conditions of a balanced three-phase delta-connected circuit are then:

1. The voltage between any pair of line wires equals the voltage across one phase of the generator.

2. The current in each line wire equals $\sqrt{3}$ times the current in each phase.

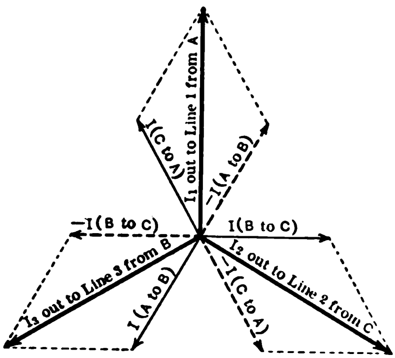


FIG. 120. Polar vector diagram corresponding to the topographic diagrams of Fig. 119.

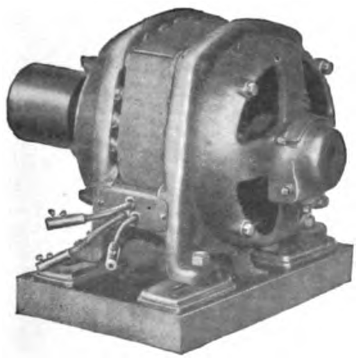


FIG. 121. An induction motor built by the General Electric Co.

Prob. 97-3. Give diagram of Y connections for generator in Fig. 81. Assume each phase to carry 140 amp. and to maintain a voltage of 240 volts across the phase terminals. Compute with aid of vector diagrams:

(*a*) Voltage between each pair of line wires.

(*b*) Current in each line wire.

Prob. 98–3. From data of Prob. 97 make a statement as to a balanced Y connection concerning:

(*a*) Relation of voltage between any pair of line wires to voltage across each phase.

(*b*) Relation between the current in any line wire to the current in each phase.

Prob. 99–3. In the induction motor of Fig. 121, the voltage between leads is 550 volts and each lead carries 15 amp. when the motor is running under full load. If the coils of the motor are delta-connected:

(*a*) What is the voltage across each coil?

(*b*) What current does each coil of the motor carry?

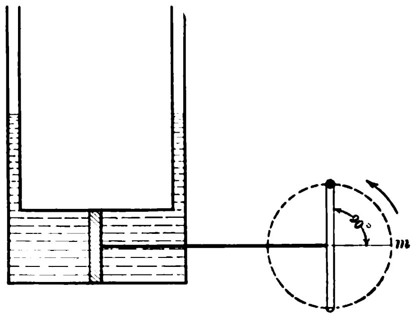


FIG. 122. The current is a maximum when the piston is passing through this position. The pressure, however, is a minimum as the water stands at the same level in each pipe.

Prob. 100–3. Assuming the same voltage between the leads and the same current for each lead wire as in Prob. 99, but with the motor coils Y-connected:

(*a*) What is the voltage across each coil of the motor?

(*b*) What is the current through each coil of the motor?

34. Current and Voltage May Differ in Phase. The current and the pressure are not always in phase with each other, that is, they do not always reach their respective maximum values at the same instant. This may be illustrated by the water analogies in Fig. 122 to 125.

In Fig. 122, the piston has reached the mid-position where the crank is making an angle of 90° with the line of dead centers. Thus it is traveling at its fastest rate, and the amount of current is maximum. However, the pressure set up, that is, the difference in level of the water in the stand pipes, is zero. The vector diagram of this case is shown in Fig. 123, in which the current vector I_m is at the 90° position and has its maximum instantaneous value, while the pressure vector (h) is at the zero position and has a zero instantaneous value.

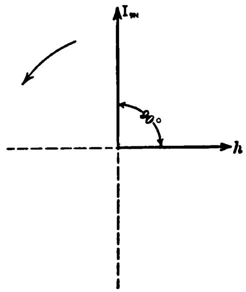


FIG. 123. Polar vector diagram for the condition shown in Fig. 122. The instantaneous value of the current I_m is at a maximum, while the instantaneous value of the pressure h is zero.

In Fig. 124, the piston has reached the extreme left of its stroke, 180° from m, the zero position, and is therefore, at this instant, motionless. Accordingly, the current is zero. But the pressure (h) set up between the two stand pipes is at its greatest possible value. This is represented in Fig. 125 by the vector diagram, in which the

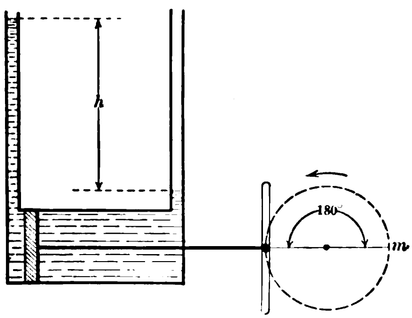


FIG. 124. The current is zero, but the pressure h has reached its maximum.

vector I_m represents the maximum current and (h) the maximum pressure. Note that I_m is at the 180° position and has a zero instantaneous value, while h is at the 90° position and has a maximum instantaneous value.

In each case the current and pressure have as great phase difference as they can ever get; that is, one is zero while the other is at its maximum. They are said to differ in phase by 90°, because in going from zero to maximum value any alternating quantity goes through 90°. An alternating current of electricity, in the same way, can be as much as 90° out of phase with the voltage which produces it. Fig. 126 is the sine curve representation of such a case where the current is lagging 50° behind the e.m.f.

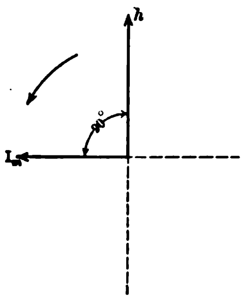


FIG. 125. The polar vector diagram for Fig. 124. The current I_m has become zero, while the pressure h is at a maximum.

Note that the voltage already has a value of e_1 at 50° when the current is zero, showing that the voltage is 50° ahead of the current. When the voltage has its maximum value E_m the current has a value of i_1 only, etc. The

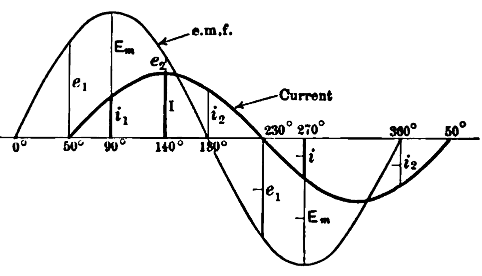


FIG. 126. Sine curve showing a current I lagging 50° behind the electromotive force E.

e.m.f. reaches the maximum at 90°, while the current reaches a maximum when the voltage is at the 140°. Here again the current is 50° late.

The current and voltage of a given part of a circuit may be represented on the same vector diagram, as in Fig. 127, and the vector representing the current need not even be drawn to the same scale as the voltage vector.

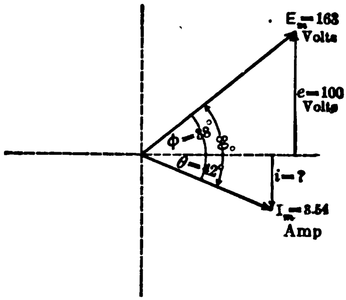


FIG. 127. Polar vector diagram for an e.m.f. of 163 volts, maximum leading a current of 3.54 amp. maximum, by 80 electrical degrees.

Example 5. In a certain choke coil the current lags 80° behind the impressed voltage. The current equals 2.5 amp. and the pressure, 115 volts. What instantaneous value will the current have when the voltage has an instantaneous growing value of 100 volts?

$$I_m = \frac{2.5}{0.707} = 3.54 \text{ amp.}$$

$$E_m = \frac{115}{0.707} = 163 \text{ volts.}$$

Lay out, Fig. 127, $E_m = 163$ to any scale; draw the perpendicular e, the instantaneous value, equal to 100 volts;

$$\sin \phi = \tfrac{100}{163} = 0.614,$$
$$\phi = 38° \text{ approx.}$$

Now draw $I_m = 3.54$ amp. and construct i, the instantaneous value of the current, perpendicular to the x axis.

$$\text{Angle } \theta = 80° - 38° = 42°.$$
$$\begin{aligned} i &= I_m \sin \theta \\ &= 3.54 \sin 42° \\ &= 3.54 \times 0.669 \\ &= 2.37 \text{ amp.} \end{aligned}$$

Since i is below the x axis, it must be negative. Thus the instantaneous value of the current equals -2.37 amp. (in a direction opposite to the pressure) when the pressure has an instantaneous growing value of 100 volts.

Prob. 101-3. In an a-c. circuit the current lags 25° behind the voltage. The maximum value of the current is 45 amperes. What is the instantaneous value of the current when the voltage is passing through 75°?

Prob. 102-3. The current in an a-c. circuit leads the voltage by 50° and has an instantaneous value of 30 amperes when the voltage is zero. What is the maximum value of the current?

Prob. 103-3. (*a*) What is the maximum value of the voltage, Prob. 102, if the instantaneous value is 400 volts, when the current is at its maximum?

(*b*) What is the average value of the voltage?

Prob. 104-3. The maximum value of an alternating voltage is 1600 volts, the maximum value of the current is 70 amperes. If the instantaneous growing positive value of the current is 25 amperes, when the instantaneous value of the voltage is 800 volts, what is the phase difference between current and voltage?

SUMMARY OF CHAPTER III

THE VOLTAGE ACROSS A SERIES CIRCUIT equals the vector sum of the voltages across the parts. In the form of an equation this may be written:

$$E = E_1 \oplus E_2 \oplus E_3 \oplus \ldots .$$

THE CURRENT through a series circuit is the same in all parts.

IN A THREE-WIRE TWO-PHASE SYSTEM, the voltage between the outside wires equals the vector sum of the voltages across the phases. Since each phase is at 90° to the other, the voltage between the "outside wires" equals $\sqrt{2}$ (or 1.41) times the voltage across one phase.

IN AN OPEN-DELTA two phases of a three-phase combination are joined in series, forming between the terminals of the series a third phase. The voltage across this resultant third phase equals the sum of two equal voltages at 120° to each other. This vector sum is exactly equal to the voltage across any phase.

REVERSING A COIL OR A PHASE in the connections of a transformer or a generator, reverses the direction of the vector and causes it to be 180° ahead of or behind its original position.

IN PHASING OUT an a-c. machine, tests of trial connections with a voltmeter will indicate whether or not any coils or connections are reversed.

CLOSED-DELTA CONNECTION. When all three coils of a three-phase generator are connected in series, so that the voltage across any two coils equals the voltage across each phase, there will be no voltage across the terminals of the three in series. It is, therefore, safe to join these terminals, and though it makes a closed ring, no current will circulate. Since the diagrammatic representation of coils so connected resembles the Greek letter Δ (delta), this method is commonly called the DELTA CONNECTION. To distinguish it from the OPEN-DELTA, this is often called a CLOSED-DELTA connection.

STAR OR Y-CONNECTION. When the three phases of a three-phase machine are so joined to a common (neutral) point, that between any two line terminals are two phases only of the machine and these are in series, the phases are said to be star or Y-connected.

The voltage between line terminals is equal to $\sqrt{3}$ times the voltage across each phase.

The current in each line wire is the same as the current in each coil of the armature.

THE VOLTAGE ACROSS A PARALLEL COMBINATION is the same as the voltage across each branch. The current through a parallel combination equals the vector sum of the currents through the branches. In the form of an equation this may be written:

$$I = I_1 \oplus I_2 \oplus I_3 \oplus \ldots.$$

TWO A-C. GENERATORS with same terminal voltage to be run in parallel must be in phase and in step or in synchronism. This is owing to the large local circulating currents which they would exchange, if out of phase, due to the resultant voltage caused by phase difference of e.m.f.'s in the circuit of the two armatures.

THE CURRENT IN EACH LINE WIRE of a balanced 3-phase Δ-connected system is the vector sum of the currents in two PARALLEL coils and is equal to $\sqrt{3}$ times the current in any coil.

The voltage across any pair of the line wires is the voltage across a SERIES connection of two of these coils which differ 120° in phase and exactly equals the voltage across a single coil.

THERE MAY BE AS GREAT A PHASE DIFFERENCE AS 90° between an alternating current and the voltage which causes it to flow. A current which reaches its positive maximum value later than the voltage reaches its positive maximum value is said to lag behind the voltage.

PROBLEMS ON CHAPTER III

Draw rough sine curves, approximate vector diagrams, and write the equations for all problems. Wherever it is possible, the electrical connections should also be drawn.

Prob. 105–3. A choke coil and a field rheostat are joined in series. The current through the combination is 2 amperes. The impedance of the combination is 56 ohms. What is the voltage across the combination?

Prob. 106–3. The voltage across the choke coil in Prob. 105 is very nearly 90° ahead of the voltage across the field resistance; the impedance of the choke coil on this circuit is 20 ohms. What is the voltage across the choke coil?

Prob. 107–3. What is the voltage across the field resistance in Prob. 106.

Prob. 108–3. What is the impedance of the field resistance in Prob. 106?

Prob. 109–3. What will be the instantaneous voltage across the choke coil in Prob. 106, at the instant when the voltage across the field rheostat is 50 volts?

Prob. 110–3. (*a*) What is the difference of phase between the voltage across the rheostat and the voltage across the combination of Prob. 106.

(*b*) Between the voltage across the choke coil and the voltage across the combination?

Prob. 111–3. In a special alternator built for laboratory purposes, both terminals of each of 6 coils are brought out separately as in Fig. 128.

A_5B_6 leads A_5B_5 by 30°.
A_5B_5 leads A_4B_4 by 30°.
A_4B_4 leads A_3B_3 by 30°.
A_5B_3 leads A_2B_2 by 30°.
A_2B_2 leads A_1B_1 by 30°.
E.m.f. of each coil is 100 volts.

Find voltages across all possible series combinations of A_1B_1 and A_2B_2.

Prob. 112–3. Draw rough sine curves for each component and resulting e.m.f. of Prob. 111. What is the phase difference between the resultant and A_1B_1, Fig. 128, in each case?

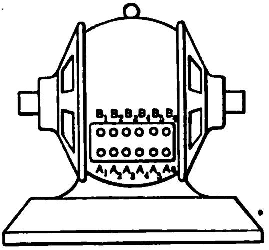


FIG. 128. A special a-c. generator with both terminals of each of the six coils brought out as marked.

Prob. 113–3. Find voltages across all possible series combinations of A_1B_1 and A_3B_3 of generator in Prob. 111. State phase difference between the resultant and A_1B_1.

Prob. 114–3. Find voltages across all possible series combinations of A_1B_1 and A_4B_4 of generator in Prob. 111. State phase difference between resultant and A_1B_1.

Prob. 115–3. Find resultant e.m.f. across the series combination of A_1B_1, A_2B_2, and A_3B_3 of generator in Prob. 111, with A_3B_3 leading A_2B_2, and A_2B_2 leading A_1B_1. State phase difference between resultant and A_1B_1.

Prob. 116–3. Find resultant of series combination of the coils of Prob. 115, if A_4B_4 is added. A_4B_4 leads A_3B_3, which leads A_2B_2, which leads A_1B_1. State the phase difference between the resultant and A_1B_1.

Prob. 117–3. What will be the instantaneous value of the resultant e.m.f. in Prob. 116 when the instantaneous value of the e.m.f. across A_1B_1 is 50 volts.

Prob. 118–3. What will be the instantaneous value of the resultant e.m.f. in Prob. 116 when the instantaneous value of the e.m.f. across A_2B_2 is 50 volts?

Prob. 119–3. What would be the resultant voltage across the series combination of Prob. 116 if coil A_2B_2 were reversed?

Prob. 120–3. What would be the instantaneous value of the voltage across coil A_3B_3 of Prob. 119 when the resultant instantaneous voltage was 25 volts?

Prob. 121–3. If in the series combination of Prob. 115, A_2B_2 is reversed, what will be the voltage across the combination? State phase difference between the resultant voltage and the voltage across A_1B_1.

Prob. 122–3. If in the combination of Prob. 115, both A_2B_2 and A_3B_3 were reversed, what would be the resultant voltage across the combination and what would be the phase difference between it and the voltage across A_1B_1?

Prob. 123–3. What will be the value of the voltage in coil A_1B_1 of Prob. 121 when the resultant voltage has an instantaneous value of 100?

Prob. 124–3. What will be the instantaneous voltages across each part of the series circuit of Prob. 115, when the instantaneous value of the resultant voltage is zero?

Prob. 125–3. In the series circuit of Prob. 115, the coil A_3B_3 is reversed by accident. What voltage will there be across the combination?

Prob. 126–3. What is the voltage across a series combination of two parts, if the voltage across the first part is 85 volts and across the second part is 115 volts? The phase difference between the two parts of the circuit is 40°.

Prob. 127–3. The current in the series circuit of Prob. 126 is 8 amperes and is in phase with the voltage of the first part. What is the phase difference between the current and the voltage across the combination?

Prob. 128–3. Assume the e.m.f. across each phase of Fig. 62 to be 220 volts. If the phases were connected at central point instead of ends as in Fig. 67, what would be the voltage between the points AA'?

Prob. 129–3. When the terminals of the 3-phase generator shown in Fig. 100 are connected, (a) to (a_1) and to (a_2), it is star-connected and the voltage across each pair of terminals is 380 volts. Phases are balanced. Show how you would connect the same leads to make a closed-delta connection and state what the voltage would be across each phase.

Prob. 130–3. (*a*) Show diagram of connections for two ways in which the special generator of Prob. 111–3 can be connected in Δ as a three-phase machine. Use but three coils in each case.

(*b*) Compute voltage across terminals of machine so connected.

Prob. 131–3. Show diagram of connections for two ways in which the generator of Prob. 111–3 can be connected in star as a three-phase generator, using but three coils in each case. Compute the voltage across the terminals of the machine so connected.

Prob. 132–3. (*a*) Using all six coils of generator in Prob. 111–3, show diagram of connections for use as a three-phase Δ-connected generator.

(*b*) Compute voltage across the terminals of the machine so connected.

Prob. 133–3. The current flowing in each lead wire of generator in Prob. 132–3 when Δ-connected is 56 amperes. What current flows in each coil of the machine?

Prob. 134–3. What current would flow in each line wire if the generator of Prob. 133–3 were Y-connected? Assume that the same current flows in each coil whether the generator is Δ- or Y-connected.

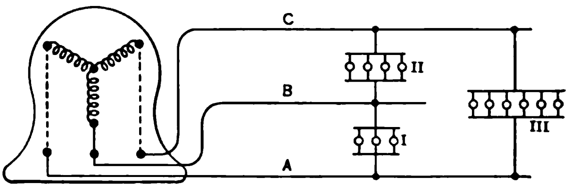


FIG. 129. The current through the lamps is in phase with the voltage across the lamps, but the phases are not balanced.

Prob. 135–3. If each lamp in Fig. 129 takes a current of 5 amperes which is in phase with the voltage across it, what current flows in each line wire? The generator gives correct three-phase e.m.f.'s.

Prob. 136–3. In place of the lamps of Group III of Prob. 135, connect a single-phase induction motor taking a current of 30 amperes which lags 50° behind the voltage across it. Compute the current in each line wire.

Prob. 137–3. In place of the lamps of Group II of Prob. 135, connect a single-phase synchronous motor taking a current of 20

FIG. 130. A star-delta starting switch. *Trumbull-Vanderpoel Co.*

amperes which leads the voltage across it by 35°. Compute the current in each line wire.

Prob. 138–3. In place of the lamps of Group III in Prob. 135-3, connect induction motor of Prob. 136–3. In place of Group II, connect synchronous motor of Prob. 137. Compute current in each line wire.

Prob. 139–3. Fig. 130 shows a star-delta switch, used for connecting the coils of a three-phase induction motor in Y for starting, and then by throwing the switch to put the coils in Δ for running. Draw a diagram of the connections of this switch showing how this may be accomplished. Of course, both ends of each coil of the motor must be available.

Prob. 140–3. (*a*) Show diagram of connections of generator in Prob. 111–3 as a Y-connected three-phase generator using all six coils.

(*b*) Compute terminal voltage across the machine so connected.

CHAPTER IV

POWER AND POWER-FACTOR

35. Current in Phase with Voltage. To find the power (watts) delivered by a direct-current generator we multiply the terminal pressure (volts) by the current delivered (amperes). However, most direct-current generators are supplying power to lighting circuits and machines, which take a varying amount of current during the day. When it is desired to know the average watts delivered by such a machine for a day, one method is to take instantaneous voltmeter and ammeter readings simultaneously at regular short intervals of time for the cycle of one day of 24 hours. By multiplying together these corresponding instantaneous readings of current and pressure, the power in watts taken at these instants is computed. By averaging these instantaneous values of the power determined at regular intervals, the average watts delivered by the generator is found.

In each single circuit of a system carrying alternating current, the power, or the flow of electrical energy per unit time, is varying from instant to instant as the pressure and current change in value and alternate in direction; but we can find the effective power or average rate of energy flow, by a manner exactly similar to that employed for the varying direct current. The power that is flowing between two points in a circuit **at any instant** can be found by multiplying the current **at that instant** by the pressure **at that instant,** between these points. This is true whether the current is direct or alternating, steady or varying.

In the form of an equation, this may be stated

$$p = ie,$$

where

p = power at given instant, in watts,
i = current at given instant, in amperes,
e = pressure at given instant, in volts.

For illustration, let us consider first the easier case of a direct-current generator of which the changes in power are not so rapid.

Fig. 131 shows the curves of volts and amperes delivered by such a machine for the cycle of one day of 24 hours. These data are from an actual case where the generator was used to supply power to a hotel. The corresponding curve of power is plotted in the same diagram, the value of each point on it being the product of the volts and amperes at the same instant, as found from the other two curves.

Thus the value of the power (p) at the instant 6 o'clock P.M. equals the product of the amperes (i) at 6 P.M. by the volts (e) at 6 P.M.

From the curve

$$i = 380,$$
$$e = 220,$$
$$p = 380 \times 220$$
$$= 84{,}000 \text{ watts.}$$

Similarly, the instantaneous value of the power (p_1) delivered at 10 P.M. equals the product of the voltage (e_1) at 10 P.M. by the current (i_1) at 10 P.M.

$$i_1 = 355,$$
$$e_1 = 220,$$
$$p_1 = 355 \times 220$$
$$= 78{,}100 \text{ watts.}$$

The average of all the instantaneous values of the power plotted on this power curve at equal short intervals of time equals nearly 54,000 watts and represents the average power delivered by the generator for a cycle of 1 day of 24 hours.

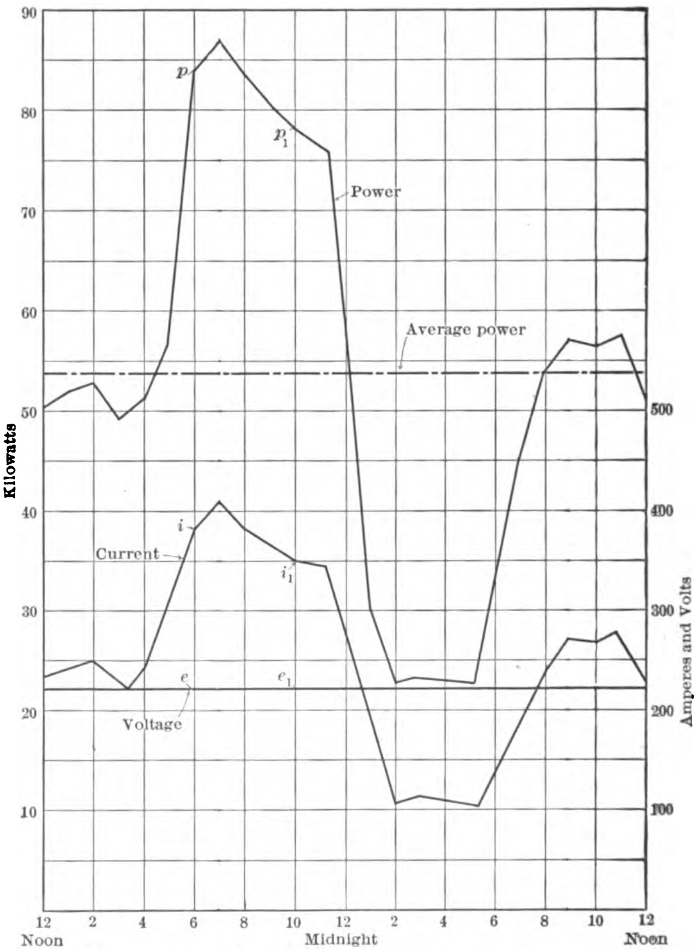


FIG. 131. The curve marked "Power" represents the power taken by a hotel from noon to noon. Each point on this curve is a product of the corresponding values on the voltage and current curves for the same hotel. The average power equals nearly 54 kilowatts.

Similarly, in an alternating-current generator we have a machine which delivers a current varying in value, and at a varying pressure. The fact that a cycle in this case repeats itself many times a second rather than once in 24 hours, and that the amperes and volts vary through great ranges in this exceedingly short interval of time, does not affect in the slightest degree the method of computing the power at any instant which we may choose.

The power which is being delivered **at any instant** is always equal to the pressure at that instant multiplied by the current at that instant, that is,

$$p = ie.$$

So if we draw the curve for current and the corresponding curve for pressure for any length of time, we can find the power at any number of instants by merely multiplying

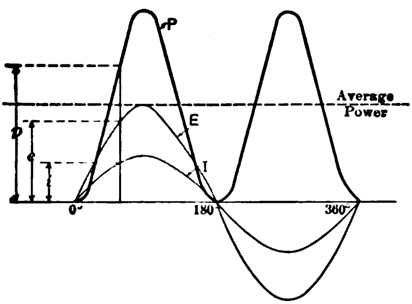


Fig. 132. The curve marked *P* is the product of the curves of voltage *E* and of current *I*, and represents the power delivered by an alternating current and voltage.

simultaneous values of current and pressure. Suppose we consider first a generator in which the current delivered is in phase with the voltage across the terminals, and plot the amperes and the volts for the time of one cycle. The curve *E*, Fig. 132, represents the values of the volts for that space of time, and *I* the values of the amperes.

The corresponding values of the watts would be represented by the curve P, on which every point, as for instance, (p) equals the product of (e), the value of the volts at that instant, times (i) the value of the amperes at the same instant.

The fact that at certain instants the volts and amperes are negative makes no difference where current and voltage are in phase, because the power (volts $\times$ amperes) is always positive, since the volts and amperes become negative at the same instant and the product of two negative quantities is always positive.

The average of all these values of the power in Fig. 132 would be the average power delivered during the cycle, and might be represented by the dash line.

This average power is found to be equal to one-half the greatest instantaneous power. This can easily be verified by solving Prob. 2-4. Also in Fig. 132, note that the power loops above the "average-power" line will just fill the spaces left below this line. The greatest instantaneous value of the power equals the product of the maximum volts times the maximum amperes.

$$P = \tfrac{1}{2} I_m E_m,$$

where

$$P = \text{average power in watts},$$
$$I_m = \text{maximum current in amperes},$$
$$E_m = \text{maximum pressure in volts}.$$

Since we measure the pressure and current in effective values, it is usual to employ these in the formula:

$$I_m = \frac{I}{0.707},$$

$$E_m = \frac{E}{0.707},$$

$$P = \frac{1}{2} \times \frac{I}{0.707} \times \frac{E}{0.707},$$

that is,

$$P = IE.$$

Thus the average value of the power delivered by a single-phase alternating-current generator, when the current and pressure are in phase, equals the product of the effective amperes times the effective volts.

Example 1. What power is a single-phase a-c. generator delivering when it maintains a pressure of 550 volts and delivers a current of 40 amperes in phase with the voltage?

$$\begin{aligned} P &= IE \\ &= 550 \times 40 \\ &= 22{,}000 \text{ watts.} \\ &= 22 \text{ kw.} \end{aligned}$$

Prob. 1-4. Plot the following curves accurately on the largest sheet of coördinate paper available, and to as large a scale as the sheet will admit, putting all curves on same sheeet:

(*a*) One cycle of a sine curve of alternating voltage of 110 volts effective value.

(*b*) One cycle of a sine curve of alternating current of 2 amperes, in phase with the voltage.

(*c*) The curve representing power which this voltage and amperage delivers.

Prob. 2-4. Find the average value of the power in Prob. 1 by means of a planimeter, or by averaging at least 40 values of instantaneous power taken from the curve at equal intervals throughout the cycle. Compare this value of the average power with the values as computed by both of the equations, $P = IE$, and $P = \frac{1}{2} I_m E_m$. Account for any difference.

Prob. 3-4. A circuit has an alternating current of 2.8 amperes flowing through it in phase with the pressure, which is 110 volts. How much power is consumed by the circuit?

Prob. 4-4. What is the maximum power consumed by the circuit in Prob. 3? What is the minimum power consumed by this circuit?

Prob. 5-4. The current taken by a transformer is 14.2 amp. The voltage across the terminals is 2300 volts, and is practically in phase with the current. What power does the transformer take?

Prob. 6-4. In testing a single-phase a-c. generator which was maintaining a pressure of 2200 volts, it was found by means of a wattmeter to be delivering 400 kw. If the current was in phase with the pressure, how many amperes was the generator delivering?

Prob. 7–4. What instantaneous power would the generator of Prob. 6 be delivering when the voltage had passed 40° beyond its zero value?

Prob. 8–4. Suppose the flow of water from the pump illustrated in Fig. 9 to 12 to vary harmonically and to have a maximum value of 100 gal. per minute, and frequency of 50 cycles per minute. Neglecting the inertia reaction of the water, consider that the frictional resistance amounts to 100 lbs. per square inch on the piston at maximum rate of flow and varies in exact proportion to the rate of flow, or water current. Draw to suitable scales the curves representing current, pressure, and power delivered by the piston for one complete cycle. Calculate average power in foot-pounds per minute, and the equivalent horse power and kilowatts. (Delivery of one pound of water against a pressure of one pound per square inch requires the same work as to lift this pound of water through a height of 2.31 feet.)

Prob. 9–4. The velocity of the piston in Fig. 9 to 11 varies harmonically (sine curve between velocity and time), and has an average value of 60 feet per minute. The force exerted on the piston is exactly proportional to the velocity at every instant, the average force throughout each stroke being 10,000 lbs. The piston makes 100 strokes per minute. Draw curves to suitable scales, with velocity, force, and power delivered at the piston as ordinates, and time (in seconds) as abscissas. Calculate the average power (foot-pounds per minute), using one complete cycle.

36. Power with Current at 90° to Voltage. We have seen that there may be a phase difference as great as 90° between the current delivered by an a-c. generator and the voltage between the terminals of the machine.

In determining the average power delivered by the machine when there is a phase difference between the voltage and the current, let us start with the extreme case of a phase difference of 90°. We proceed exactly as we did when the current and voltage were in phase. The current curve and the voltage curve for one cycle are drawn, and the instantaneous power is computed for enough points to determine the power curve. Fig. 133 shows the current curve I lagging 90° behind the voltage curve E. Every point on the power curve P has its ordinate equal to the product of corresponding values of the current and voltage; thus the value $-p$ equals $-i$ (the value of the current at that instant) times $(+e)$ (the value of the voltage at the same instant).

Note the fact that there are two power loops, *B* and *D*, where the power is negative, since all the values of the instantaneous power here are the product of a positive current times a negative voltage or vice versa. Now since the positive values of power represent the power being **delivered by** the generator, the negative values must represent power which is being **returned to** the generator. Therefore, in computing the average power delivered by the generator during one cycle, we must subtract these negative loops (or power returned) from the positive loops (or power delivered) in order to get the real or net power delivered.

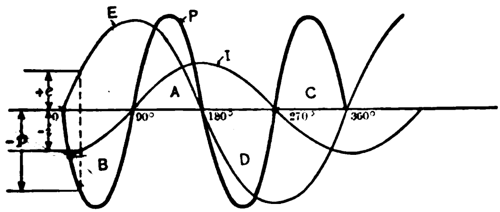


FIG. 133. The heavy line represents the power delivered by an alternating voltage and current when the current lags 90° behind the voltage. Note that the negative power loops *B* and *D* are exactly equal to the positive power loops *A* and *C*.

But in Fig. 133 it will be seen that the negative loops are of the same size as the positive loops. Thus, in this case, where *I* and *E* have 90° phase difference, the circuit gives back to the generator during one quarter-cycle all the power which it received during the preceding quarter-cycle, and the power consumed by the circuit is, therefore, zero.

If the mathematical expression for instantaneous power ($i \times e$) is given a positive sign when electrical energy is flowing out of the dynamo into the external circuit joining its terminals, then a negative sign before the expression for power ($-i \times e$) would mean that the flow of energy at that instant is in the opposite direction, or out of the external circuit into the dynamo. Hence, if **+ie** indicates generator action of the dynamo, then **−ie** indicates motor action. In general, any machine (either mechanical or electrical) which is producing a force, is giving out positive power when the movement is in the same direction as this force, and it is giving out negative power, or is taking in power, when the movement is in a direction opposite to this force. Whether a given portion of circuit (carrying either direct current or alternating current) **receives power** or **delivers power** depends not alone upon the direction of current in that portion nor alone upon the direction of e.m.f. across that portion, but upon the relation between these directions; the numerical value of the power in either case will be at every instant equal to the product of the instantaneous values of volts and amperes for that portion of the circuit.

In the diagram of the pump, Fig. 124, where the pressure is always 90° out of phase with the current, it can be seen that in total no power is delivered by the pump. The water forced up in either standpipe merely uses the pressure thus produced to help force the piston back again, thus returning to the pump all the energy given out by it.

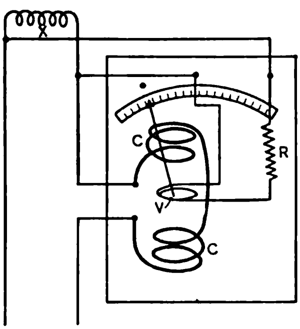


Fig. 134. A wattmeter for measuring power in an electric circuit. In this case the power consumed by the coil x is being measured.

Accordingly, it is reasonable to expect, as the above curves prove, that in the case of an electric current 90° out of phase with the voltage, all the energy is being returned by the circuit as fast as it is received. A wattmeter such as in Fig. 134 placed in such a circuit shows the same result, by reading zero. The **voltage** across the coil x sends a slight current in phase with itself through the movable coil V of the wattmeter. All the **current** through the coil x goes through the stationary field coils C–C of the wattmeter. Now, if the voltage is 90° out of phase with the current, the current through the coil *V* must be 90° out of phase with the current through the field coils *C–C*. So when the current in the movable coil *V* is at its maximum, there is no current in the field coils, thus no moving force; and when the current in the field coils *C–C* is at its maximum there is no current in the movable coil *V*, and again there is no moving force. In the interval (equal to one quarter period) between these two instants there is a slight moving force one way; but during the next quarter period ($\frac{1}{240}$ of a second on a 60-cycle circuit), there is an equal force in the opposite direction as the power is being returned in the opposite direction through the circuit. These time intervals, and pulses of power, are of such short duration that the needle cannot have time to be deflected, and thus reads zero. If a wattmeter could be used which could deflect fast enough in each direction, it would deflect alternately and equally in both directions, showing that as much power was being returned to the generator as was being sent out by it.

The average power consumed in a circuit where the voltage is at 90° to the current is, therefore, always zero.

Prob. 10–4. Following directions in Prob. 1, construct the sine curves for an alternating current of two amperes lagging 90° behind an alternating voltage of 110 volts. Draw power curve on same sheet for this case.

Prob. 11–4. Find the average value of both the positive and negative power, as directed in Prob. 2. Show that no power is being delivered if a generator is delivering this current at this voltage with a phase difference of 90° between them.

Prob. 12–4. In Fig. 46 and 47, the stroke, or distance traveled by each piston between opposite dead-centers, is 1 foot; the area of piston *A* is one sq. ft. and of piston *B*, two sq. ft. The maximum head *h* produced as in Fig. 47 is 10 feet. The crank makes 30 revolutions per minute. Using time in seconds as abscissa, draw curves with **pressure** (*h*, in feet) and **rate of flow** (pounds of water per second out of one standpipe into the other) as ordinates. For instants $\frac{1}{10}$ second apart throughout one cycle calculate, from current and pressure curves or equations representing them, the horse power delivered by or to each piston, and draw the curve of power, representing it as positive from piston to water and negative from water to piston. Neglect all frictional resistances and inertia reaction, and assume that the only force overcome by the pistons is that due to the "head" in the standpipes. Explain fully the meaning of these power curves, and calculate average horse power of each cylinder.

37. Power-factor. Let us now investigate the case when the current and voltage differ by any angle less than 90°. We can proceed exactly as we did when they were in phase or 90° out of phase. Fig. 135 shows the current curve

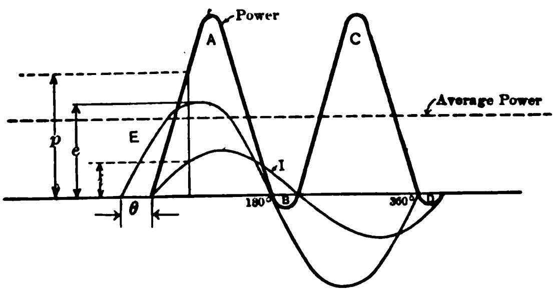


FIG. 135. The current I lags $\theta°$ behind the voltage E. Note that power delivered equals the difference between the positive power loops A and C and the negative power loops B and D.

I lagging θ degrees behind the voltage curve E. Every point on the power curve P is found by multiplying together the corresponding values on the current and voltage curves. Thus, the value (p) at any chosen instant is the product of the values of (e) and (i) at that instant. Note that here again there are two power loops, B and D, which are negative, though here they are smaller than the two positive loops A and C. The average power delivered by the generator will, therefore, be positive, but less than that represented by the two positive loops, since we must subtract from it the amount represented by the negative loops.

This average value, represented by the dash line, must thus be less than the average value of the positive loops, and no longer equal to the product of the effective volts by the effective amperes.

The numerical value of this average power may be arrived at as follows:

When the current and voltage are in phase, the average power equals the effective current times the effective pressure.

When they are out of phase, the current may be thought of as made up of two components, one in phase with the voltage and one at 90° to the voltage.

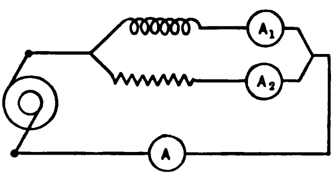


FIG. 136. The current through the ammeter A is the vector sum of the currents through A_1 and A_2.

Of course, there can never be more than one current flowing through one part of a circuit at a time, but we have seen how one current may be the resultant of two or more currents in various phase relations to one another. For instance, there is one current only through the ammeter A in Fig. 136, but it is made up of the currents through ammeters A_1 and A_2. If A_1 reads 40 amp. and A_2 reads 60 amp. and the phase difference between them is 90°, then by the vector diagram, Fig. 137, we see that the current A then can be found as follows:

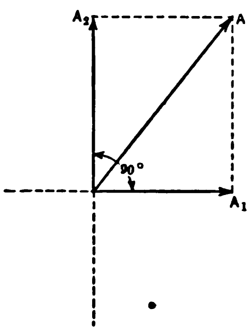


FIG. 137. The vector A represents the current through the ammeter A. It is made up of the sum of vectors A_1 and A_2.

$$A = A_1 \oplus A_2,$$
$$A^2 = A_1^2 + A_2^2,$$
$$A = \sqrt{A_1^2 + A_2^2}$$
$$= \sqrt{1600 + 3600}$$
$$= \sqrt{5200}$$
$$= 72.1 \text{ amp.}$$

Thus a current of 72.1 amp. flows through A. This current is really the resultant of the two currents A_1 and A_2 which have a phase difference of 90°.

Accordingly, if we can compute the resultant of two currents 90° out of phase with each other, we surely can compute these two components when given the resultant and the phase difference; or we may consider any current as the resultant of two components and compute what these components would be.

Therefore, in considering the current I in Fig. 135 as made up of two components one in phase with the voltage E and one at 90° to it, we construct Fig. 138, drawing the vector I to represent the current at an angle of $\theta°$ behind the voltage vector E.

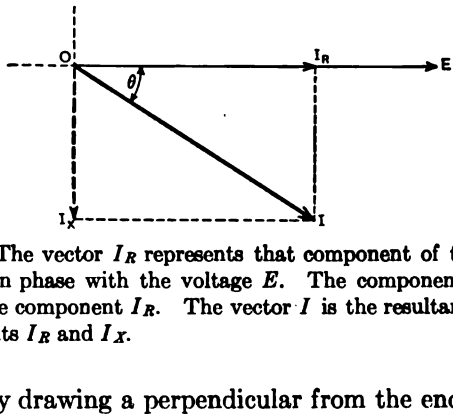


FIG. 138. The vector I_R represents that component of the current I which is in phase with the voltage E. The component I_X lags 90° behind the component I_R. The vector I is the resultant of the two components I_R and I_X.

Then, by drawing a perpendicular from the end of I up to the horizontal axis, we cut off I_R, which will represent that component of I which is in phase with E. By drawing another perpendicular over to the vertical axis, we cut off I_X, which represents that component of I which is 90° behind the voltage E. Thus I_R and I_X represent two alternating currents at 90° to each other, which if combined would result in one current I lagging $\theta°$ behind the voltage E.

By inspecting the figure it can be seen that

$$I_R = I \cos \theta$$

and

$$I_X = I \sin \theta.$$

Thus, if $I = 100$ amperes and $\theta = 20°$,

$$I_R = 100 \cos 20° \\ = 94 \text{ amp.}$$

and

$$I_X = 100 \sin 20° \\ = 34.2 \text{ amp.}$$

Thus a current of 100 amperes could be thought of as being composed of two currents at 90° to each other, one of 94 amperes and the other of 34.2 amperes.

Similarly, in this case of a current I lagging $\theta°$ behind a voltage E, we may think of the current as made up of two currents, one equal to $I \cos \theta$, in phase with the voltage E, and the other $I \sin \theta$ at 90° to the voltage E.

But we have seen that when a current and voltage are at 90° to each other, the power delivered is zero. Thus, the power delivered by the component current $I \sin \theta$ which is at 90° to the voltage is zero. Accordingly, all the power which the current I delivers must come from its component $I \cos \theta$ which is in phase with the voltage. The average power, then, equals the product of the effective volts (E) times that current component ($I \cos \theta$) which is in phase with the voltage.

In the form of an equation this law appears;

$$P = EI \cos \theta,$$

where

P = average or effective power, in **watts**,
E = effective pressure in **volts**,
I = effective current in **amperes**,
θ = angle (electrical degrees) representing phase difference between E and I.

Sometimes it is preferable or more convenient to resolve the voltage E into two components, E_r in phase with I, and E_x 90° out of phase with I, as in Fig. 138a. This makes no real difference in the result, because

$$E_r = E \cos \theta,$$
$$E_x = E \sin \theta,$$
$$E = \sqrt{E_r^2 + E_x^2},$$
$$P = I \times E_r = EI \cos \theta.$$

In either case, the product EI_r or IE_r represents power which flows in only **one direction,** as for instance, out from the generator to the line and receivers, never to return. It is the **effective power,** which one always understands to be meant when the terms power and watts are applied to the circuit without qualification. The product EI_x or IE_x represents power which flows in **both directions** —from the generator out to the line, and from the line back to the

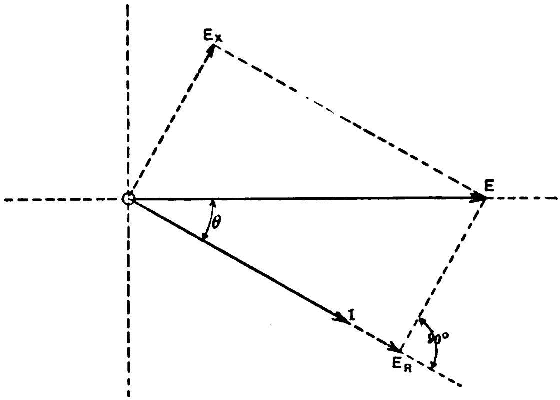


FIG. 138a. The vector E may be resolved into the two components, E_R in phase with the current I, and E_X at 90° to the current.

generator in exactly equal amount. This power is just as real as that which we have called "**effective power**" (which the wattmeter indicates), and it produces very important effects in the circuit. However, as this power merely shuttles back and forth between the generator and the receiving circuit, without being transformed into heat or useful work, it is called **reactive power,** or **reactive volt-amperes.** Obviously, reactive volt-amperes can only be found in circuits which possess an ability to store up energy, receiving it during part of each half-cycle and delivering some of this energy back to the generator during the remainder of each half-cycle. Circuits which possess ability to store energy and deliver power back to the generator periodically are called "inductive" circuits; the energy is stored in the electromagnetic and the electrostatic "fields" induced in the space around the wires when current flows and potential difference exists. These actions will be explained more fully in later chapters. Non-inductive circuits are those in which no energy can be stored in this way.

Example 2. What power is being delivered by a single-phase a-c. generator which maintains a pressure of 220 volts and delivers a current of 50 amp. with a phase difference of 25° between the pressure and the current?

$$\begin{aligned} P &= EI \cos\theta \\ &= 220 \times 50 \times \cos 25° \\ &= 220 \times 50 \times 0.906 \\ &= 9966 \text{ watts.} \\ &= 9.97 \text{ kw.} \end{aligned}$$

Example 3. In the above example, what power would be delivered if the current and voltage were in phase (that is, differed in phase by 0°)?

$$\begin{aligned} P &= EI \cos\theta \\ &= 220 \times 50 \times \cos 0° \\ &= 220 \times 50 \times 1 \\ &= 11{,}000 \text{ watts} \\ &= 11 \text{ kw.} \end{aligned}$$

Example 4. What power would be delivered in the above example if the current and voltage differed in phase by 90°?

$$\begin{aligned} P &= EI \cos\theta \\ &= 220 \times 50 \times \cos 90° \\ &= 220 \times 50 \times 0 \\ &= 0 \text{ watts.} \end{aligned}$$

Prob. 13–4. According to directions in Prob. 1, plot curves for current, voltage, and power, when an alternating pressure of 110 volts delivers a current of 2 amperes which lags 20° behind the pressure.

Prob. 14–4. By methods of Prob. 2, find the average value of the power delivered in Prob. 13. Compare this with the value as computed from the equation $P = EI \cos\theta$.

Prob. 15–4. In the system of pumps specified in Problem 12–4, consider that the force applied by the piston to the water must overcome frictional resistances as well as the pressure head (h) in the storage tanks. The friction reaction, in pounds per square inch at the piston, is proportional to the rate of flow and has a value of 1 lb. per square inch when the rate of flow is one cubic foot of water per second. For simplicity, consider a single pump having a capacity equal to the sum of the two shown, and draw to suitable scale the following curves, using time (seconds) as abscissa for all: (*a*) current, or rate of flow, pounds of water per second; (*b*) head h_s (in feet) due to storage of hydraulic energy, or difference of level in tanks; (*c*) head h_f corresponding to the friction reaction at the piston (2.31 feet head = 1 lb. per square inch); (*d*) total head h_t against which the piston operates; (*e*) power (foot-pounds per second) represented by current and pressure due to receiving tanks; (*f*) power represented by current and back-pressure due to friction; (*g*) total power between liquid and piston. Discuss these curves and their analogies to those of the electric circuit, indicating the correspondences to the quantities I, I_r, I_x, E, E_r, E_x, EI_r, EI_x, IE_r, IE_x, EI, $EI \cos\theta$ in the foregoing discussions.

From the above examples and problems it can be seen that the equation $P = EI \cos\theta$ is a general equation for the average power in an alternating-current circuit, regardless of the phase difference between the voltage and the current.

It is customary to refer to the product of the effective volts by the effective amperes, by the name of **Apparent Power,** and to measure it in **volt-amperes.** The term "$\cos\theta$" is then called the **Power-factor.** Thus, when we want to compute the power we find the apparent power (E times I), and multiply it by the power-factor ($\cos\theta$). When the pressure and the current are in phase, the term ($\cos\theta$) has a numerical value unity (1), and the circuit is said to have **unity power-factor.**

When they differ by about 25°, the term ($\cos\theta$) becomes 0.90 and the circuit is said to have a power-factor of 90%.

The power-factor, then, really indicates what fraction the average power is of the apparent watts, or volt-amperes, (EI). So it is often defined by the equation

$$\textbf{Power-factor} = \frac{\textbf{effective power}}{\textbf{apparent power}} = \frac{\textbf{watts}}{\textbf{volt-amperes}}.$$

Since a wattmeter always reads the effective power, we have only to attach a wattmeter, an ammeter, and a voltmeter to a circuit to find the power-factor, and from it the phase difference between the current and the pressure. **The product of the volts by the amperes gives the apparent power, by which the wattmeter reading is divided to give the power-factor.**

The power-factor has already been shown in a single-phase circuit to be always equal to the cosine of the angle of phase difference between the pressure and the current in the circuit.

Example 5. The ammeter shows that a certain generator is delivering 20 amperes. The voltmeter reads 230 volts. A wattmeter shows that 4 kw. are being delivered.

(*a*) What is the power-factor of the load?

(*b*) What is the phase difference between the current and the voltage?

$$\text{Effective power} = 4000 \text{ watts,}$$
$$\text{Apparent power} = EI$$
$$= 230 \times 20 = 4600 \text{ volt-amperes,}$$
$$\text{Power-factor} = \frac{\text{effective power}}{\text{apparent power}}$$
$$= \frac{4000}{4600}$$
$$= 87\%.$$

But power-factor $= \cos\theta$,

therefore, $0.87 = \cos\theta$,

$\theta = 30°$ approx.

Prob. 16–4. A single-phase induction motor takes a current of 24 amperes which lags 30° behind the impressed voltage of 220 volts. What is the power-factor of the motor at this load and how much power does it take?

Prob. 17–4. A single-phase synchronous motor is taking a current of 30 amp. which leads the impressed voltage of 220 volts by 20°. What is the power-factor of this motor under these conditions and what power does it take?

Prob. 18–4. If the two motors of Prob. 16 and Prob. 17 are put in parallel on the same circuit:

(*a*) What current will flow in the line?

(*b*) What will be the power-factor of the generator?

(*c*) How much power will the generator be delivering, assuming these two motors are alone on a short line? Use two methods to check.

Prob. 19–4. Fig. 139 is from the Proceedings A.I.E.E. and gives the oscillograms of the current and voltage curves of one phase of a 3-phase 500-kv-a. 2300-volt a-c. generator. Assume that the current has a value of 200 amperes and the voltage, 2300 volts. Note that the two curves do not have the same zero line. Draw to the same absciseas the power curve, measuring the instantaneous values of the volts and amperes from the middle of the thick lines.

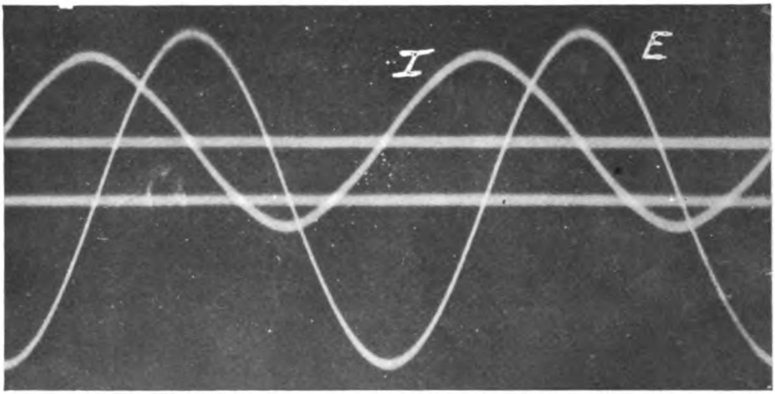


FIG. 139. The current and e.m.f. curves of one phase of a 3-phase generator, taken by an oscillograph. Note that the zero lines of the two curves do not coincide. *Proc. A.I.E.E.*

Prob. 20–4. From the area of the positive and negative loops of the power curve of Prob. 19, compute the average effective power which the generator is delivering to the line.

Prob. 21–4. During a working day of 24 hours, the ammeter and voltmeter connected in circuit with a small single-phase motor driving a drainage pump, indicate steadily 10 amperes and 230 volts respectively. The watt-hour meter reading increases by 44.2 kilowatt-hours during this time. Calculate the power-factor at which the motor operates. Assuming an 80 per cent efficiency for the motor and 60 per cent for the pump, calculate the horse power output of the motor, and the total number of gallons of water raised 20 feet from an excavation.

Prob. 22–4. If under the conditions of Problem 21–4, one-half of the total losses in the motor are constant and the other half are copper losses, which vary as the square of the current, calculate: (*a*) How many amperes the motor would take at the same voltage to deliver the same horse power, if constructed or adjusted to operate at a power-factor of 90 per cent. (*b*) How many kilowatt-hours would be used per day under this condition. (*c*) What the monthly saving would be with energy costing four cents per kilowatt-hour delivered at the motor.

38. Reactive Current, Reactive Pressure, Reactive Volt-amperes. Since no power is taken from the generator when it delivers a current which differs in phase with the voltage by 90°, such a current is said to be **reactive.** It is also sometimes called **wattless,** although this term is not approved. It has been seen that any alternating current at all out of phase with the voltage may be resolved into two component currents. That component part which is out of phase with the voltage by 90° is accordingly called the **reactive component,** and the part which is in phase with the voltage is called the **power component.**

Consider, for instance, Example 5, in which a current of 20 amperes is flowing, 30° out of phase with the voltage. The component in phase with the voltage or the power-component of the current

$$= 20 \cos 30^\circ$$
$$= 20 \times 0.866$$
$$= 17.3 \text{ amperes.}$$

The wattless component, or the one at 90° to the voltage

$$= 20 \times \sin 30^\circ$$
$$= 20 \times 0.500$$
$$= 10 \text{ amperes.}$$

Thus, the current of 20 amperes can be regarded as being made up of a reactive component of 10 amperes at 90° to the voltage, and a power component of 17.3 amperes in phase with the voltage.

Prob. 23-4. How large are the reactive and the power components of the current in the motor of Prob. 16?

Prob. 24-4. How large are the reactive and power components of the motor current in Prob. 17?

Prob. 25-4. Compute the reactive component of the line current in Prob. 18 and compare it with the reactive components of Prob. 16 and 17. Explain reason for change.

As mentioned before, sometimes it is easier to conceive only one current in the circuit, this current being driven by two alternating e.m.f.'s added to each other, the total or resultant of these two imaginary e.m.f.'s being the actual voltage across the circuit which the voltmeter measures. Thus, in Example 5, we could say:

In-phase component or **power component** of line voltage

$$= 230 \cos 30°$$
$$= 199.2 \text{ volts.}$$

Quadrature component or **reactive component** of line voltage

$$= 230 \sin 30°$$
$$= 115 \text{ volts.}$$

In finding the total size or capacity of generators, transformers or distributing lines necessary to supply a number of diverse loads connected together, it is usually convenient to apply a similar analysis to the power in the circuit. Thus, we conceive the total apparent power, or (effective volts × effective amperes), to be made up of two components:

Working volt-amperes, or effective power, or true watts

$$= \text{total volt-amperes} \times \text{power-factor.}$$
$$P_w = EI \cos \theta.$$

Reactive volt-amperes, or circulating power

$$= \text{total apparent power} \times \sin \theta.$$
$$P_0 = EI \sin \theta.$$

It is apparent that:

$$\text{Total } EI = \sqrt{P_w^2 + P_0^2}$$

Since this may be expressed in the form:

$$EI = \sqrt{(EI \cos \theta)^2 + (EI \sin \theta)^2}$$
$$= \sqrt{(EI)^2 (\cos^2 \theta + \sin^2 \theta)},$$
$$EI = EI \sqrt{\cos^2 \theta + \sin^2 \theta} = EI \sqrt{1} = EI.$$

That is, the total apparent power EI in any circuit, or part of a circuit, may be resolved into two components at right angles to each other; one of these is the effective power or watts ($EI \cos \theta$), which is used to deliver energy out of the electrical system (produce torque in motors or heat in conductors), and the other ($EI \sin \theta$) merely oscillates from one form of energy to another within the system (as magnetic or electric fields, mechanical momentum, etc.), without loss.

The advantage of this analysis of the "apparent watts" will be evident from an example:

Example 6. What size generator,* in kv-a. (kilovolt-amperes), is required to supply a system comprising induction motors taking 64 kilowatts at 80 per cent power-factor, incandescent lamps taking 40 kilowatts at 100 per cent power-factor, and idle transformers taking 5 kv-a. at 40 per cent power-factor. For simplicity, assume these values to include the power used to overcome losses and reactions in the corresponding distributing wires.

In solving this problem we may work with amperes as heretofore, or we may work directly with power, which we shall find to be simpler. The loads are probably connected in parallel; we may, therefore, choose a voltage, calculate the amperes supplied to each of the loads (from total kv-a. and volts, or from kw., power-factor and volts), add these currents together vectorially to find the total current to be delivered by the generator, and multiply this resultant current by the voltage to get total volt-amperes output of the generator. It makes no difference to the final answer what voltage we choose, because we first divide and then multiply by it. Thus, assuming 1000 volts pressure:

Current to induction motor load

$$= \frac{\text{total volt-amperes}}{\text{volts}} = \frac{\text{watts} \div \text{power-factor}}{\text{volts}}$$

$$= \frac{(64{,}000 \div 0.80) \text{ volt-amperes}}{1000 \text{ volts}} = 80 \text{ amperes.}$$

Current to incandescent lamp load

$$= \frac{(40{,}000 \div 1.00) \text{ volt-amperes}}{1000 \text{ volts}} = 40 \text{ amperes.}$$

* See Second Course, Chap. 1, on rating of generators.

Current to idle transformers

$$= \frac{5000 \text{ volt-amperes}}{1000 \text{ volts}} = 5 \text{ amperes.}$$

We might plot these vectors in a diagram all to the same scale and combine them graphically, as shown in Fig. 139a, finally arriving at a resultant OR amperes which, multiplied by the chosen

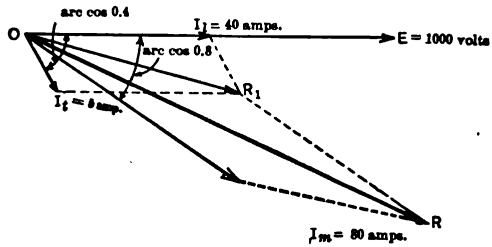


FIG. 139a. A vector diagram in which I_t represents the transformer current; I_m, the motor current; I_l, the lamp current, and OR, the resultant of the three currents.

voltage OE (= 1000 volts), would give us the generator output in volt-amperes. It would be much more accurate to calculate the value of OR by trigonometry, as follows:

Power component of $I_m = I_m \cos\theta_m = 80 \times 0.8 = 64$ amp.
Reactive component of $I_m = I_m \sin\theta_m = 80\sqrt{1.0^2 - 0.8^2}$
$= 80 \times 0.6 = 48$ amperes.

or

Reactive component of $\mathbf{I}_m = \sqrt{\mathbf{I}_m^2 - (\textbf{power component } \mathbf{I}_m)^2}$
$= \sqrt{80^2 - 64^2} = 48$ amps.
Power component of $I_l = I_l \cos\theta_l = 40 \times 1.00 = 40$ amp.
Reactive component of $\mathbf{I}_l = \mathbf{I}_l \sin\theta_l = 40 \times 0.0 = 0$ amp.
Power component of $I_t = I_t \cos\theta_t = 5 \times 0.4 = 2$ amp.
Reactive component of $\mathbf{I}_t = \mathbf{I}_t \sin\theta_t = \sqrt{5^2 - 2^2}$
$= \sqrt{21} = 4.6$ amp.
Power component of $OR = 64 + 40 + 2 = 106$ amperes.
Reactive component of $OR = 48 + 0 + 4.6 = 52.6$ amperes.

Finally,

$$OR = \sqrt{(\text{power comp. of } OR)^2 + (\text{wattless comp. of } OR)^2}$$
$$= \sqrt{(106)^2 + (52.6)^2} = 118.3 \text{ amperes,}$$

and Total "apparent power" output of generator

$$= 118.3 \text{ amps.} \times 1000 \text{ volts}$$
$$= 118{,}300 \text{ volt-amperes}$$
$$= 118.3 \text{ kv-a.}$$

This method is based on the fact that all power components, being in phase with one another, can be added together algebraically, and all reactive components, being also in phase with one another, can also be added together algebraically. The addition of components must be algebraic, with due regard to sign or direction, because some of the connected units may be supplying power to the system as generators at the same time that other units are taking power from the system. Likewise, some loads take lagging reactive components while others take leading reactive components of directly opposite phase. Moreover, the total amperes, or volt-amperes, is equal to the square root of the sum of squared power component and squared reactive component, since the total is the vector sum and the vectors representing these components are at right angles to each other.

In this example we would have:

Power component of motor load in kw.

$$= 64 \text{ (given)}$$
$$= \text{total kv-a.} \times \text{power-factor.}$$

Total kv-a. to motor load $= \dfrac{\text{kw.}}{\text{power-factor}} = \dfrac{64}{0.8} = 80 \text{ kv-a.}$

Reactive kv-a. to motor load $= \sqrt{(\text{total kv-a.})^2 - (\text{kw.})^2}$

$$= \sqrt{80^2 - 64^2} = 48 \text{ kv-a. (lagging).}$$

Power component of lighting load in kw. = 40 (given)

$$= \text{total kv-a.} \times 100 \text{ per cent.}$$

Total kv-a. to lamps $= \dfrac{40 \text{ kw.}}{100\%} = 40 \text{ kv-a.}$

Reactive kv-a. to lamps $= \sqrt{40^2 - 40^2} = 0 \text{ kv-a.}$

Power component of idle transformers in kw.

$$= \text{total kv-a.} \times \text{power-factor}$$
$$= 5 \times 0.4 = 2 \text{ kw.}$$

Total kv-a. to idle transformers = 5 (given).

Reactive kv-a. to idle transformers

$$= \sqrt{5^2 - 2^2} = \sqrt{21} = 4.6 \text{ kv-a. (lagging).}$$

Total effective power output of generator

$$= 64 \text{ kw.} + 40 \text{ kw.} + 2 \text{ kw.} = 106 \text{ kw.}$$

Total reactive power output of generator

$$= 48 \text{ kv-a.} + 0.0 \text{ kv-a.} + 4.6 \text{ kv-a.} = 52.6 \text{ kv-a.}$$

Total apparent output of generator

$$= \sqrt{(\text{total kw.})^2 + (\text{total reactive kv-a.})^2}$$

$$= \sqrt{(106)^2 + (52.6)^2} = 118.3 \text{ kv-a.}$$

Power-factor of total output

$$= \frac{\text{effective power}}{\text{apparent power}} = \frac{106 \text{ kw.}}{118.3 \text{ kv-a.}} = 0.895 = 89.5\%.$$

It is evident that this method is much more direct when dealing with questions of size or power capacity of lines or apparatus, when the pressure or current are not under consideration. In such cases, the vectors in Fig. 139a all represent volt-amperes or kv-a. The length of each vector represents the total kv-a. or apparent power taken by one of the loads. The angle between the vector and the reference axis *OE* (which is the axis of kilowatts) is the angle whose cosine is the power-factor of that load. The projection of the vector on the *OE* axis is the effective power taken by that load. The projection of the vector upon an axis perpendicular to *OE* represents the reactive kv-a. taken by that load. Once this system of representation is thoroughly in mind, it is unnecessary to draw the diagram; the calculations can be made directly as shown just above. **Remember, however, that whenever a load takes a leading current, its vector lies above the OE axis in Fig. 139a and its reactive kv-a. must be called negative and subtracted from the reactive kv-a. of the lagging loads.**

Prob. 26–4. The machines in a certain shop are arranged in three groups, driven by line shafts connected to three similar 50-h.p. alternating-current motors. One of these operates at half load, with an efficiency of 86 per cent and power-factor 75 per cent. The second operates at $\frac{3}{4}$ load, efficiency 88 per cent and power-factor 83 per cent. The third operates at full load, with efficiency of 89 per cent and power-factor 88 per cent. Calculate the kw., reactive kv-a., and total kv-a. supplied to each motor, and to the entire shop. Calculate also the power-factor of the feeder supplying this shop.

Prob. 27–4. If the half-load induction motor of Prob. 26 were replaced by a synchronous motor, which delivers the same power at the same efficiency, but is adjusted to take enough reactive **leading** kv-a. to compensate or neutralize the total reactive lagging kv-a. taken by the other two motors, calculate: (*a*) The reactive kv-a. which must be taken by the synchronous motor; (*b*) the kw. that must be taken by this motor; (*c*) the required size of the synchronous motor, in kv-a.; (*d*) the power-factor at which this motor must operate; (*e*) the power-factor of the feeder under these conditions.

39. Measurement of A-C. Power. Single-Phase. We have seen that the product of amperes by volts gives the apparent power in an a-c. circuit, which is equal to the true power only when the power-factor is unity.

The true power in a single-phase alternating-current circuit is measured by means of a wattmeter attached to the circuit in exactly the same way as to a direct-current circuit.

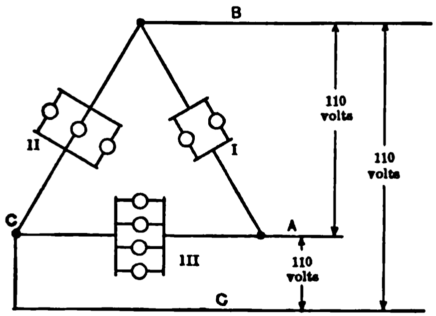


FIG. 140. Groups of lamps attached to a 3-phase system. The power delivered equals the sum of the power consumed by the three groups of lamps.

40. Measurement of Polyphase Power. Balanced Three-phase. In measuring the power delivered by a three-phase generator or taken by any three-phase appliance, whether Δ- or Y-connected, it is merely necessary to measure the power in each coil of the appliance and add the three measurements. The total power must necessarily be the sum of the power in the three phases. Thus, the total power delivered by the three-phase line in Fig. 140 to the lamps must be the sum of the power in Groups 1, 2, and 3. If each lamp is a 100-watt lamp, then:

Phase 1 consumes	2×100	$= 200$	watts.
Phase 2 "	3×100	$= 300$	"
Phase 3 "	4×100	$= 400$	"
Total power consumed by lamps		$= 900$	watts.

Of course, if the phases are balanced and the power is known to be the same in each of the three windings or load circuits, it is only necessary to measure the power in one phase and multiply by three.

However, in most machines which operate on a three-phase system, it is difficult to get at the coils which compose each phase in order to insert a wattmeter in circuit with them. For instance, in Fig. 141, *xy*, *yz*, *zx* represent the phases in the winding of a delta-connected motor. But the connections are made on the inside of the frame of the motor and the leads *A*, *B*, and *C* only are brought out. It might appear that we could measure the power by inserting the current coil of the wattmeter in the lead *A*, putting the voltmeter coil across the leads *AB*, and use the wattmeter reading as the power in the coil *xy*; then doing the same for the other two leads, adding the three readings thus obtained. But this inference would not be correct. To be sure, the voltage used by the wattmeter in the first measurement is the voltage across the coil *xy*, but the current taken by the series coil of the wattmeter would not be the current which flows in the coil *xy*, because the current in the lead *A* is the vector sum of the currents in both the coils *xy* and *zx*.

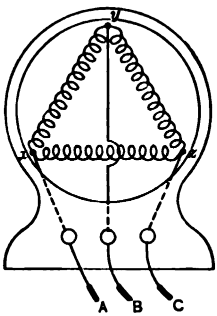


FIG. 141. The coils of this synchronous motor are joined within the frame and three leads *A*, *B* and *C* only are brought out.

The same difficulty would be encountered in sending the current of either of the other leads through the wattmeter. In no case would this current be the current through a single coil of the machine.

But if the phases are exactly balanced, we know that the current in any lead equals $\sqrt{3}$ times the current in any coil of the machine. So if we take the wattmeter reading as first suggested, with the current coil in the lead A and the voltmeter coil across AB, we would know that the current factor of the power reading was $\sqrt{3}$ times the current in the phase xy, therefore, the power indicated should be $\sqrt{3}$ times what it should be for coil xy alone. **However, the current in the lead A is not only $\sqrt{3}$ times as large as the current in the phase xy, but is also out of phase with it.** In Fig. 142, the vector I_A, resultant of I_{xy} and I_{xz} represents the current in the lead A and is seen to be $\sqrt{3}$ times the current in xy **but 30° out of phase with it.**

When the current and the pressure vary according to the sine law, the wattmeter indicates the value of $EI \cos \theta$, where E = effective volts, across the pressure coil of the wattmeter, I = effective amperes through the current coil, and θ = electrical degrees of phase difference between the current and the pressure. If I is the only current and all the current flowing in that part of the circuit across which the pressure E exists, then the wattmeter indicates the true average power in watts delivered to or by that part of the circuit. Thus if a wattmeter is inserted with the current coil in the lead A and with the pressure coil across the phase xy, the wattmeter will read the product of the effective amperes in the lead A times the effective volts across the phase xy, times the cosine of the angle representing phase difference between the current in the lead A and the voltage across the phase xy. But the power consumed by the coil xy does not depend upon the phase angle between the current in the line and the voltage across xy, but it does depend upon the phase angle between the current in xy and the voltage across xy. Since we have assumed the motor to have unity power-factor, the current is in phase with the voltage and this phase angle is 0°. Therefore, the product of the current through xy times the voltage across xy equals the power in xy.

But we have already seen that the wattmeter placed in the line indicating the power, multiplies the product of amperes times the volts by $\cos\theta$. Now, the volts across the line AB are in phase with the current in xy, but are 30° out of phase with the line current as is seen in Fig. 142. Therefore, the line volts must be out of phase with the line current by 30°. Thus this $\cos\theta$ by which the wattmeter automatically multiplies the volt-amperes must be cos 30°. The reading of the wattmeter placed in the line, then, is wrong in two respects for measuring the power in the coil xy.

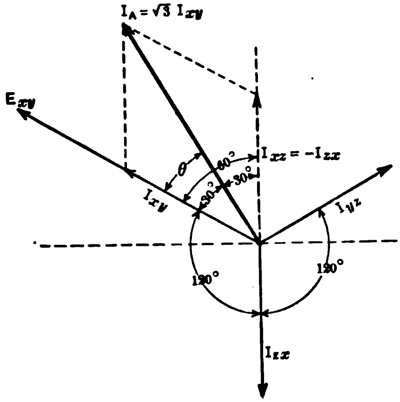


FIG. 142. The vector I_A is the resultant of the currents I_{xy} and I_{xz} and is equal to $\sqrt{3} \times I_{xy}$, but is 30° out of phase with I_{xy}.

First: The current flowing through it is $\sqrt{3}$ times the current in the phase xy, so we must divide the reading by $\sqrt{3}$.

Second: The power-factor of the power so indicated is cos 30°, while the power-factor of the power in xy is 1. So we must also divide the reading by cos 30°.

Accordingly, if we divide the wattmeter reading by both $\sqrt{3}$ and cos 30°, or by ($\sqrt{3} \times \cos 30°$), we will obtain the power in the coil xy. Since we assumed the phases to be balanced, we can now multiply this result by 3 and obtain the power taken by the whole machine. But dividing by

($\sqrt{3} \times \cos 30°$) and then multiplying by 3 is the same as multiplying by

$$\frac{3}{\sqrt{3} \times \cos 30°}, \text{ or } \frac{3}{1.73 \times 0.866}, \text{ or } 2.$$

If the motor were Y-connected and the wattmeter were used as above, the same reasoning would apply, and the reading would have to be multiplied by 2 as in the Δ-connection. In the case of a Y-connection, the current through the wattmeter is the current through one of the coils of the machine, but the voltage across the wattmeter is the resultant voltage across two coils of the machine in series, and is equal to $\sqrt{3}$ times the voltage across one of the coils. This voltage is also 30° out of phase with the current through the coil, which makes the power-factor of the power as measured by the wattmeter cos 30° instead of unity.

Therefore, the rule for finding the power delivered by a balanced three-phase generator, or taken by a balanced three-phase load of unity power-factor, whether connected Δ or Y, is to insert the current coil in one lead and attach the pressure coil between from this lead to one of the others and multiply the reading obtained by 2.

Example 7. What power is consumed by the 3-phase motor of Fig. 141 when running at unity power-factor if the phases are balanced, and a wattmeter indicates 13.2 kw. when the current coil is in lead *B* and the voltage coil between *B* and *C*.

$$\begin{aligned} \text{Power} &= 2 \times \text{wattmeter reading} \\ &= 2 \times 13.2 \\ &= 26.4 \text{ kw.} \end{aligned}$$

It will be seen from an inspection of Fig. 143 that in the case of a balanced three-wire **direct-current** system, the reading of a wattmeter so placed must be multiplied by 2 in order to obtain all the power delivered by the generator or received by the appliances attached to it. Since in a balanced three-wire direct-current system the third wire carries no current, the current coil of the wattmeter must be placed in either of the outside wires. But in a 3-phase system just described the current is the same in each wire, because of the phase-difference between the currents; thus the current coil can be placed in any line wire. Since in a d-c. system the voltage across the outside wires is also greater than the voltage across either side, the voltmeter coil must be placed between an outer and the neutral wire. But in the balanced a-c. system the voltage is the same across all pairs of line wires; thus the voltmeter coil can be placed across any pair, providing the line in which the current coil is inserted is one of the pair.

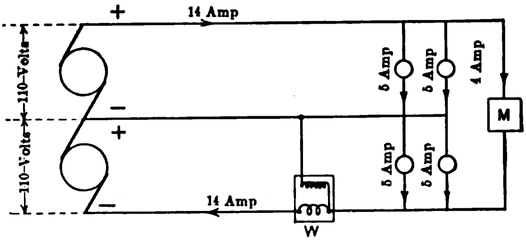


FIG. 143. A balanced 3-wire d-c. system. The reading of the wattmeter must be multiplied by two in order to determine the total power taken by the lamps and the motor *M*.

So, making allowances for the points of difference between an a-c. three-wire three-phase system and a d-c. three-wire system, it is readily seen that when a wattmeter is similarly placed in either, with the load balanced and power-factor unity, the reading must be multiplied by 2 in order to obtain the total power in the system.

41. Two-Wattmeter Method: Unbalanced Three-phase. Keeping in mind the points of similarity and difference between the alternating-current and the direct-current systems just compared, let us investigate the power in the same two systems when unbalanced.

In Fig. 144, which represents an unbalanced 3-wire d-c. system, the power delivered by the generator is **not twice** the reading of wattmeter W_1 (9×110) watts nor is it **twice** the reading of wattmeter W_2 (19×110) watts, but is merely the sum of the two readings $(9 \times 110) + (19 \times 110)$ watts.

Similarly, in any unbalanced 3-wire 3-phase system, there is no single wattmeter reading which can be multiplied by two to obtain the correct total power. But two wattmeters may be inserted, each with its current coil in one line wire

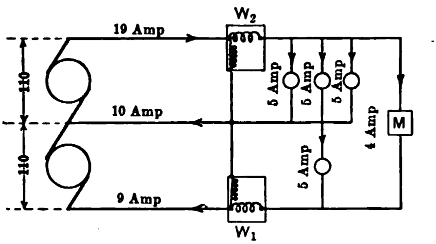


FIG. 144. The power taken by the lamps and motor on this unbalanced 3-wire d-c. line equals the sum of the readings of the two wattmeters.

and its pressure coil between that wire and the third wire, as in Fig. 145, which is similar to Fig. 144 for a d-c. 3-wire system. The sum of the readings of W_1 and W_2 is then the

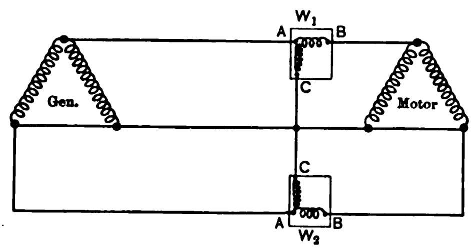


FIG. 145. The power taken by the 3-phase a-c. motor equals the sum of the readings of the two wattmeters just as in the d-c. system shown in Fig. 145.

power taken by the whole system, exactly, as in the case of the unbalanced d-c. system.

There is one other point to be noted concerning this two-wattmeter method of measuring the power in a 3-phase circuit. The current and voltage may be so much out of phase that even on a balanced load when the two wattmeters are connected in exactly similar fashion to their respective lines, one of the wattmeters will indicate in the reverse or negative direction. This wattmeter therefore should have the connections of its pressure coil reversed in order to obtain a reading, and this reading should be subtracted from the reading of the other wattmeter.

The general rule, then, for measuring power in any 3-phase a-c. circuit is to assume the phases unbalanced and to use two wattmeters. Insert the current coil of one meter in any lead, and the current coil of the other in any other lead, being sure always to connect the two coils in the same manner, that is, the similar terminals *A* either both away from the generator or both toward the generator as shown in Fig. 145. Corresponding terminals of both pressure coils should then be connected to similar points on these line wires, and terminal *C* of each instrument should then be connected to the remaining line wire.*

If both instruments now indicate positively, the sum of the readings is the total power in the system. If both indicate negatively, reverse the voltage coils of both and add the readings as before. If one only indicates negatively, reverse the voltage coil of that one and take the difference of the readings.

Great care should be exercised to connect the wattmeters symmetrically with regard to each other. Where the power is sufficiently steady, one wattmeter only may be used, inserting it successively in two lines, and the sum (or difference as explained above) of the readings taken. To facilitate rapid changes of connections, the polyphase board shown in Fig. 146 has been devised.

Note how by means of this device the wattmeter current coil can be quickly inserted in any line and taken out again without interrupting the current through the line. The

* Another reliable method is to consider each wattmeter to be connected positive, when it indicates positive with the third line open.

pressure coil may also be connected between any pair of line wires. This board also secures symmetrical attachment of the meter to all the lines.

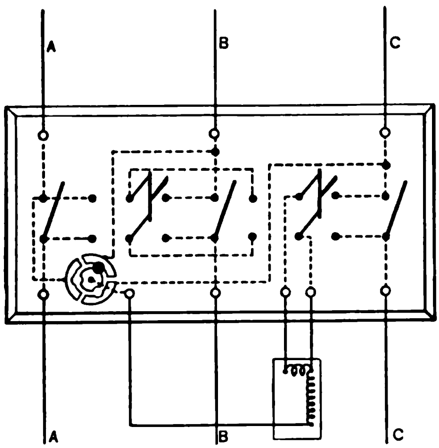


FIG. 146. A polyphase board for connecting one wattmeter quickly and correctly to measure 3-phase power.

Prob. 28-4. Each lamp in Fig. 140 takes 4 amperes. What will a wattmeter indicate when the current coil is inserted in line *A* and the voltage coil is across *AC*?

Prob. 29-4. What will a wattmeter of Prob. 28 indicate if the current coil is inserted in the line *B* and the voltage coil is across *B* and *C*; all connections made symmetrically with wattmeter in Prob. 28.

Prob. 30-4. What should the sum of the two wattmeter indications of Prob. 28 and Prob. 29 be?

Prob. 31-4. Each lamp in Fig. 147 takes a current of 3 amperes in phase with the voltage. The motor is a 5-h.p. induction motor running at full load. It has a power-factor of 80 per cent at full load and an efficiency of 75 per cent. What is the total power supplied to the lamps and the motor?

Prob. 32-4. What current flows in each line wire of Prob. 31 at *A*, *B*, and *C*? (Consider the sequence of phases as counterclockwise in this and following problems.)

Prob. 33–4. What will wattmeter W_1 of Fig. 147 indicate with line loaded as in Prob. 31?

Prob. 34–4. What will wattmeter W_2 of Fig. 147 indicate with line loaded as in Prob. 31?

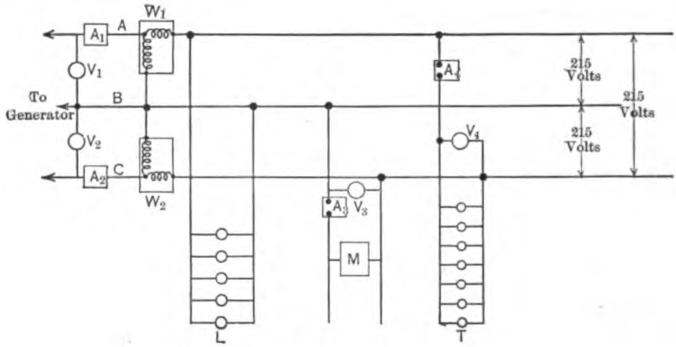


Fig. 147. A three-phase alternating-current circuit unsymmetrically loaded.

Prob. 35–4. What will ammeter A_3 and voltmeter V_3 indicate in Fig. 147 when the line is loaded as in Prob. 31? Assume the line resistance to be negligibly small.

Prob. 36–4. What will ammeter A_4 and voltmeter V_4 indicate in Fig. 147 when the line is loaded as in Prob. 31? Assume the line resistance to be negligibly small.

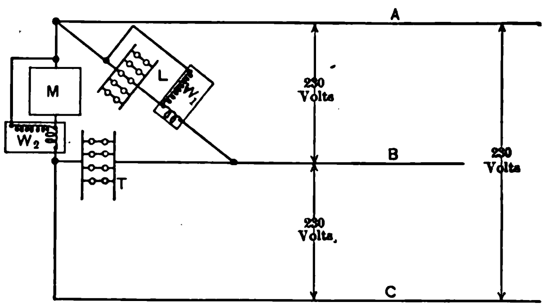


Fig. 148. A three-phase circuit loaded with lamps and an induction motor.

Prob. 37–4. What is the power-factor of the line in Fig. 147 when loaded as in Prob. 31? Assume that the power-factor of a three-phase system means the number by which the total volt-amperes in all phases must be multiplied, to find the total effective watts in all phases.

Prob. 38–4. In Fig. 148 wattmeter W_2 indicates 8.42 kw.; W_1 indicates 0.61 kw. The power-factor of the induction motor M is 72 per cent. The lamps each have the same current and unity power-factor. Find the total power delivered to the lamps and motor.

Prob. 39–4. What are the total volt-amperes and amperes in load M of Prob. 38? What are the watts, volt-amperes, and amperes in each lamp of loads L and T?

Prob. 40–4. What current flows through each of the line wires in Prob. 38?

Prob. 41–4. What is the power-factor of the arrangement in Prob. 38, according to the definition given in Prob. 37?

Prob. 42–4. If the induction motor of Prob. 38 has an efficiency of 80 per cent, at what horse power is it operating?

42. One-Wattmeter Method: Balanced or Slightly Unbalanced Systems. When a three-phase three-wire system, either Δ- or Y-connected, is only a little out of balance, and merely an approximate value of the power is desired, the one-wattmeter method is sometimes used, on account of its greater convenience. Fig. 149 shows the connections. The

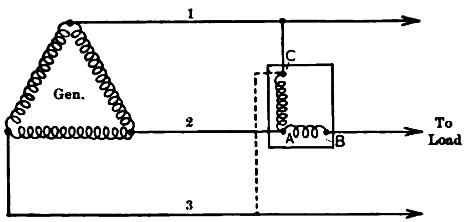


FIG. 149. One-wattmeter method of measuring the power in a 3-phase 3-wire system. Terminal C is attached first to line 1, then to line 3.

current coil is connected in one line wire, say No. 2. The pressure coil is connected across from line 2 to line 1, a reading is taken, and then the terminal of the pressure coil is quickly transferred from line 1 to line 3. If this causes the wattmeter to indicate negatively, the connections of the pressure coil are reversed, and the reading then obtained is given a negative sign. The algebraic sum of the two readings taken in this way is the approximate power being consumed by the load, provided that the phases are nearly balanced. When possible it is generally best to use the two-wattmeter method, which is accurate for a three-wire system under all conditions of balance.

43. Three-Wattmeter Method: Balanced or Unbalanced Systems. If the neutral point of a three-phase system is available, three wattmeters may be used to measure the power delivered. It is necessary merely to connect the current coils of the three wattmeters respectively into the three line wires, placing the pressure coil of each between one end of the current coil and the neutral point. If no neutral is available, **an artificial neutral** may be constructed

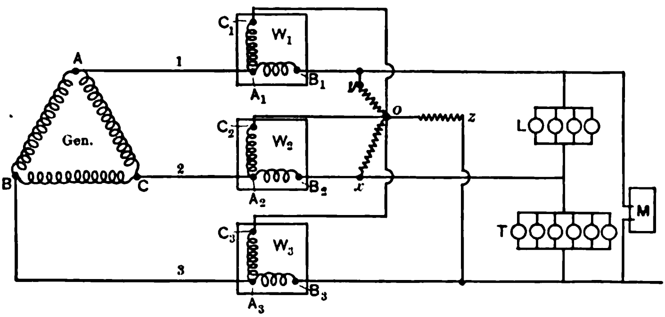


FIG. 150. The three-wattmeter method of measuring power in a 3-wire 3-phase system. The point O is the artificial neutral formed by the three equal star-connected resistances YO, ZO, and XO.

by joining three equal non-inductive resistances in star and connecting the free ends to the system. The resistances must be high enough not to disturb the current in the lines, by drawing an appreciable amount from the line, and yet low enough to be practically negligible when compared with the resistance of the pressure coils of the wattmeters. The three wattmeters as used with their artificial neutral (0), are shown in Fig. 150.

Prob. 43–4. Assume that in Fig. 150 the voltage 1–2 is 200 volts; 3–1, 235 volts, 2–3, 220 volts. What will be the voltage across the pressure coil of each wattmeter?

Prob. 44–4. Assume that each lamp, Fig. 150, takes 10 amperes in phase with the voltage across it, and that the induction motor *M* takes 80 amperes at 80 per cent power-factor.

(*a*) What current flows in each line wire?

(*b*) How much power is taken altogether by the lamps and motor? The line drop is negligibly small.

Prob. 45–4. (*a*) What will each wattmeter in Fig. 150 indicate when the line is loaded as in Prob. 43 and 44?

(*b*) Compare the total wattmeter indication with answer to Prob. 44 (*b*).

44. Computation of Power in Polyphase Circuits. In three-phase circuits we have seen that the phase difference between the line current and the line voltage is not the same as the phase difference between the current and voltage of the coils of a machine. When we state that the power-factor of a machine **which is used as a load** is, say, 0.80, we mean that the power-factor of each phase of the **winding** of that machine is 0.80. Therefore, when cos θ (the cosine of the angle representing phase difference between the volts and the amperes) is used as the power-factor, the angle θ is the angle between the current and the voltage of one and the same coil of the machine. All the coils of a machine generally have the same power-factor; but if there is a slight variation, the average is used in finding the phase difference between the volts and amperes of each coil.

The power-factor of any polyphase system is really and logically the ratio of the total effective power, or total watts, in all parts or phases of the system, to the total apparent power, or total volt-amperes. Thus, when we say that a three-phase alternator is delivering 100 kw. at a power-factor of 80%, we mean that

$$\frac{W_1 + W_2 + W_3}{E_1I_1 + E_2I_2 + E_3I_3} = 0.80,$$

and

$$W_1 + W_2 + W_3 = 100{,}000 \text{ watts} = 100 \text{ kw.},$$

in which W_1, E_1, and I_1 are the effective watts, volts, and amperes in one and the same winding, which forms one of the three phases of the machine.

If the phases were all alike as to voltage ($E_1 = E_2 = E_3$), and amperage ($I_1 = I_2 = I_3$), and wattage ($W_1 = W_2 = W_3$), this would mean that they must also be alike as to power-factor $\left(\frac{W_1}{E_1 I_1} = \frac{W_2}{E_2 I_2} = \frac{W_3}{E_3 I_3}\right)$, and the power-factor of the whole machine would be exactly the same as that of each and every one of its several windings or phases. But if the voltages of the phases are unequal, or the amperages unequal, or the wattages unequal, it is often difficult or impossible to determine the power-factor correctly; and, in fact, a single value of power-factor in such an unbalanced system is practically meaningless, because it is a theoretical and fictitious quantity which does not actually refer to any of the circuits in the system. It simply enables us to determine the size of the machine, approximately. Thus, in the case just given, the alternator would have to be of sufficient size to be rated at least $\frac{100 \text{ kw.}}{0.8} = 125$ kv-a. capacity from all phases together.

The data from which we must calculate power-factor of a polyphase system are, usually, the current in each line wire, the voltage between line wires and the total effective watts as obtained by one of the wattmeter arrangements just described. Inequality of currents or of voltages indicates that the loads or power-factors in the various phases of the generator or motor windings, or load circuits, are unequal. It is a common practice to divide the total effective watts by ($\sqrt{3}$ times the product of average of the amperes in all line wires times average volts between all pairs of line wires). In a three-phase system, according to this method:

$$\text{Power-factor} = \frac{\text{total effective watts}}{\sqrt{3} \times \left(\frac{I_1 + I_2 + I_3}{3} \times \frac{E_1 + E_2 + E_3}{3}\right)}.$$

This method is likely to give a very inaccurate value when the phases are much out of balance; that is, if the total effective watts were divided by the system power-factor calculated by this method under such conditions, the result would **not** be the total volt-amperes in all of the phases. Nevertheless, as it is frequently impossible to apply a wattmeter or voltmeter and ammeter, within the phases of the system (as indicated in Fig. 147), some approximate method, such as that given above, must be used.

In any case it should be clearly understood, however, that the power-factor of a polyphase system indicates (more or less accurately, depending on the closeness of balance) the phase relation of the current in each phase to the e.m.f. in that same phase; it does **not** indicate the phase relation between the current in a line wire and the pressure between two line wires. For instance, in case of the three-phase 100-kw. alternator referred to, the power-factor of which corresponds to a phase difference of 37° (cos 37° = 0.80), we do not mean that the current in the line wires is out of phase with the pressure between line wires by 37°, but we do mean that the current in each phase of the generator winding lags or leads 37° with respect to the voltage across the terminals of that winding; or, that if all the apparatus with which the line is loaded were replaced by one large machine taking the same total power in three balanced phases with the same average current per line wire and same average pressure between line wires, the current in each phase of the winding of this machine would lead or lag 37° with respect to the pressure between terminals of that phase.

Using these facts, we can easily compute the current in the line wires of the above 3-phase generator, for any given voltage between wires.

Example 8. Let us assume that the machine used as a load in the above paragraph is a Δ-connected motor taking 100 kw. with an 0.80 power-factor as in Fig. 151, and that the voltage is 220 volts.

As the three phases are generally balanced, and, as we have seen, the whole load is 125 kv-a., the apparent power taken by each coil would be

$$\frac{125}{3} = 41.7 \text{ kv-a.}$$

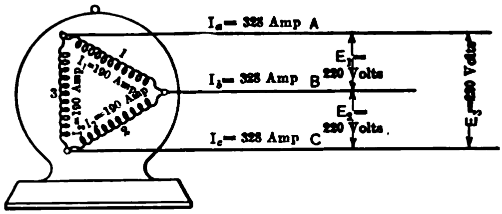


FIG. 151. A 100-h.p. 220-volt induction motor with an 0.80 power-factor.

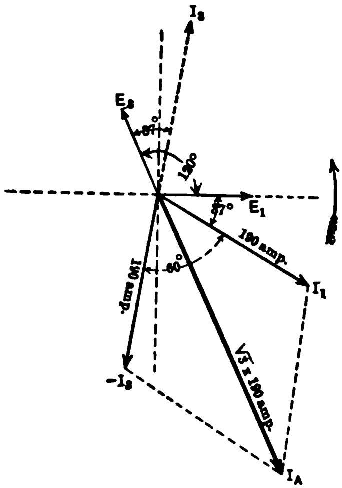


FIG. 152a. The vector I_A represents the current in the line wire A, Fig. 151. It is made up of I_1, the current in coil 1 of the motor and I_3 the current in coil 3 of the motor. Note that the current I_1 lags 37° behind the voltage E_1, and that the current I_3 lags 37° behind the voltage E_3.

The current through each phase must be $\frac{41{,}700}{220} = 190$ amperes. The current in each line wire of a balanced 3-phase system, Δ-connected, equals $\sqrt{3}$ times the current in each phase. Thus the current in the line here equals $\sqrt{3} \times 190 = 328$ amperes. Fig. 151 shows the diagram of connections for this motor and Fig.

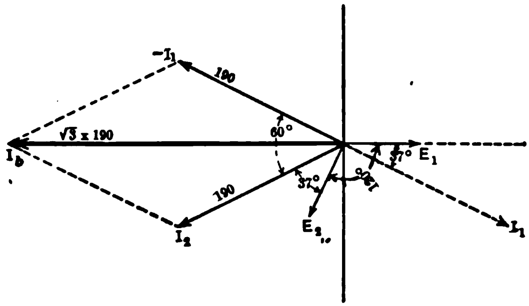


FIG. 152b. The vector I_b is the current in the line B and is made up of I_1 and I_2, the currents in the coils 1 and 2 respectively. Note that I_1 lags 37° behind E_1 and that I_2 lags 37° behind E_2.

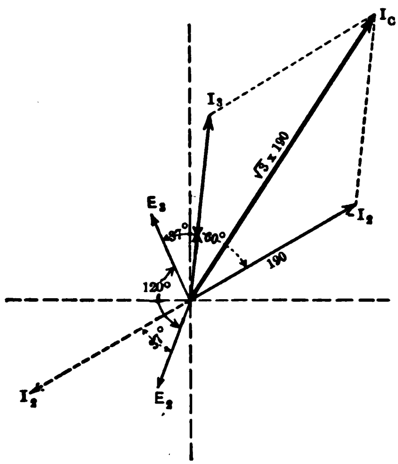


FIG. 152c. I_c is the current in line C and is made up of I_3 and I_2, the currents in coils 3 and 2 respectively. Note that I_2 lags 37° behind E_2 and that I_3 lags 37° behind E_3.

152a is the vector diagram showing the current in the coils and in one line wire. The voltage vectors E_1 and E_3 were not necessary for the construction but were put in to show what the power-factor of 80 per cent meant; that is, that the current I_1 through coil 1 lags 37° behind the voltage E_1 across coil 1, and that the current I_2 through coil 2 lags 37° behind the voltage E_2 of coil 2, etc.

Fig. 152b shows how the currents in coils 1 and 2 combine to produce the current in line *B*. Fig. 152c shows how the currents

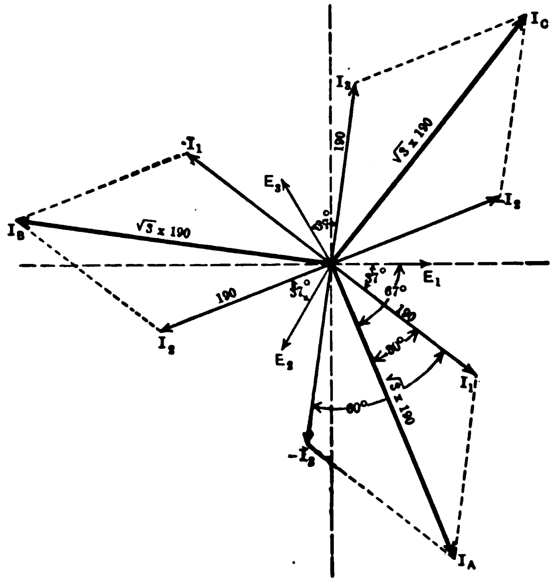


FIG. 152d. Vector diagram showing currents and phase relations of currents and voltages in line and coils of motor. This figure combines Fig. 152a, 152b and 152c.

in the coils 2 and 3 combine to produce the current in line *C*. Fig. 152d is a diagram made by assembling Fig. 152a, 152b, and 152c. It shows the phase relations of the currents and voltages in all the coils and all the line wires.

Note now that to find the amperes in the line wires of a balanced 3-phase Δ-connected system, we divided the total volt-amperes by 3 and then by the volts, and multiplied the result by $\sqrt{3}$.

That is, $$I_{\text{line}} = \frac{\sqrt{3}\text{ apparent power.}}{3\,E_{\text{line}}}$$

But dividing by 3 and multiplying by $\sqrt{3}$ is the same as multiplying by $\frac{\sqrt{3}}{3}$, or $\frac{1}{\sqrt{3}}$.

Thus, $$I_{\text{line}} = \frac{\text{apparent power}}{\sqrt{3}\,E_{\text{line}}}.$$

This is generally written

$$\textbf{Apparent power} = \sqrt{3}\,\textbf{EI},$$

when

E = voltage across any pair of line wires,
I = current through one line wire.

Since effective power = apparent power × power-factor,

$$\textbf{effective power} = \sqrt{3}\,\textbf{EI} \times \textbf{power-factor.}$$

This is the general equation for a balanced, 3-phase system, Δ- or Y-connected.

For if the motor in the example were Y-connected, then the volts across each coil would have been $\frac{220}{\sqrt{3}} = 127$ volts. The apparent power taken by each coil $= \left(\frac{125{,}000}{3}\right) = 41{,}700$ volt-amperes. The current in each coil would have been

$$\frac{41{,}700}{127} = 328 \text{ amperes.}$$

But in a Y-connection each lead carries the same current as each coil. Thus the current in each lead = 328 amperes, as in the case of a Δ-connection.

In this case, to find the current in each line, we divided the apparent power by 3, and then divided this quotient by $\left(\frac{\text{volts}}{\sqrt{3}}\right)$. This is the same as multiplying $\left(\frac{\text{apparent power}}{3}\right)$ by $\left(\frac{\sqrt{3}}{\text{volts}}\right)$, giving as a result $\left(\frac{\text{apparent power}}{\sqrt{3} \times \text{volts}}\right)$.

Thus, $$I_{\text{line}} = \frac{\text{apparent power}}{\sqrt{3}\,E}.$$

This as above is seen to reduce to

$$\text{effective power} = \sqrt{3}\,E_{\text{line}}\,I_{\text{line}} \times \text{power-factor}.$$

Example 9. What power does a balanced 3-phase system with 90 per cent power-factor deliver, when each line wire carries 40 amperes, and the voltage between line wires is 6000 volts.

$$\begin{aligned}\text{Effective power} &= \sqrt{3}\,EI \times (\text{power-factor})\\ &= 1.73 \times 40 \times 6000 \times 0.90\\ &= 374 \text{ kw.}\end{aligned}$$

Prob. 46–4. How much current does each lead of a 3-phase 220-volt induction motor carry, if the motor requires 12 kv-a. to operate it?

Prob. 47–4. How much power does it take to operate a 3-phase 550-volt induction motor at full load if each lead is found to carry 18 amperes, and the power factor is 85 per cent?

Prob. 48–4. A 3-phase 110-volt 5-h.p. induction motor takes 400 watts at no load with a power-factor of 50 per cent. What current does it take from each line wire at no load?

Prob. 49–4. If the power-factor of the induction motor of Prob. 48 rises to 90 per cent at full load, what current does each lead carry at full load, when its efficiency is 85 per cent?

45. Parallel Operation of Motors, etc. When two or more 3-phase appliances are operated in parallel, the total power taken by them all is, of course, the sum of the power taken by each. But in computing the current in the line and the power factor of the combination, it is easiest to consider but one phase of each machine; and since the machines are joined in parallel these phases of the various appliances are in parallel.

Example 10. For example, suppose that the 3-phase induction motor of Example 8, having a power-factor of 80 per cent and taking 100 kw., were operating on the same 220-volt line with a 3-phase synchronous motor taking 100 kw. at 90 per cent power-factor, current leading. Find the line current and power-factor of the load.

In Fig. 153, *M* represents the induction motor and *N* the synchronous motor. We have computed on page 194 by means of a vector diagram that the current in each coil of the induction motor

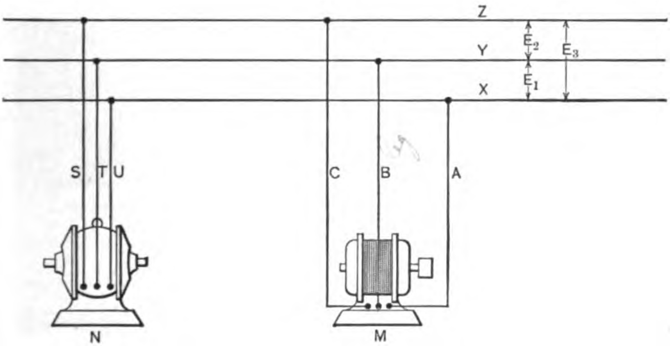


FIG. 153. The 100-kw. synchronous motor *N* with a 0.90 leading power-factor is connected in parallel with the 100-kw. induction motor *M* with a lagging power-factor of 0.80.

M was 190 amperes, and that it lagged 37° behind the voltage across the same coil.

For the synchronous motor *N*,

$$\text{power-factor} = \frac{\text{effective power}}{\text{apparent power}} = \frac{\text{watts}}{\text{volt-amperes}}.$$

$$\text{Thus, total volt-amperes} = \frac{\text{watts}}{\text{power-factor}}$$

$$= \frac{100 \times 1000}{0.90}$$

$$= 111{,}000.$$

In each coil,

$$\text{volt-amperes} = \frac{111{,}000}{3}$$

$$= 37{,}000.$$

Current through each coil,

$$\text{amperes} = \frac{\text{volt-amperes}}{\text{volts}}$$

$$= \frac{37{,}000}{220} = 168.$$

By means of the diagram of connections for the synchronous motor N (Fig. 154) and the vector diagram (Fig. 155), we see that the current which each lead U, S and T must carry equals $\sqrt{3} \times$ 168 amp., or 292 amp. The voltage vectors E_1 and E_2 do not help

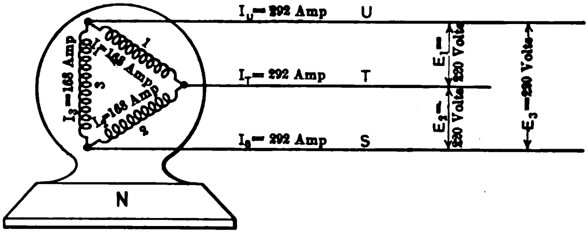


FIG. 154. The internal connections and the current distribution in the synchronous motor N of Fig. 154.

the above solution, but are added to show that the power-factor of 90 per cent means a phase difference of approximately 26° between the current in a coil and the voltage across that coil. Note that in the synchronous motor a power-factor less than unity usually means a **leading** current (see "Second Course"), and that here the current is 26° ahead of the corresponding voltage. The resulting current I_U in the terminal wire U is seen to be 4° behind the voltage E_1 across coil 1.

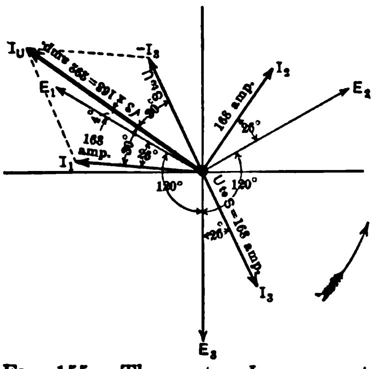


FIG. 155. The vector I_U represents the current in the lead wire U, and is made up of I_1, the current in coil 1 and I_3, the current in coil 3. Note that I_1 leads E_1 by 26° and I_2 leads E_2 by 26° and I_3 leads E_3 by 26°. Note also that I_U lags 4° behind E_1.

From Fig. 152, we see that in the case of the induction motor the terminal wire A is taking 328 amperes from the same line wire X. But this current of 328 amperes lags 67° behind the voltage across coil 1.

The voltage across coil 1 of

each machine is the voltage between the line wires Y and X. Thus, the line x furnishes the synchronous motor N with a current of 292 amp. 4° behind the voltage between X and Y, and it furnishes the induction motor M with a current of 328 amp. 37° behind the voltage between X and Y.

Construct Fig. 156, letting the vector E_1 represent the direction of the voltage between X and Y. I_A represents 328 amperes, the current in the lead wire A of the induction motor, and is drawn 67° behind the voltage vector E_1. I_U represents 292 amperes, the current in the lead wire U of the synchronous motor, and is drawn 4° behind the voltage vector E_1.

The resultant of these, I_X, represents the current which the line wire X carries in order to supply these two motors.

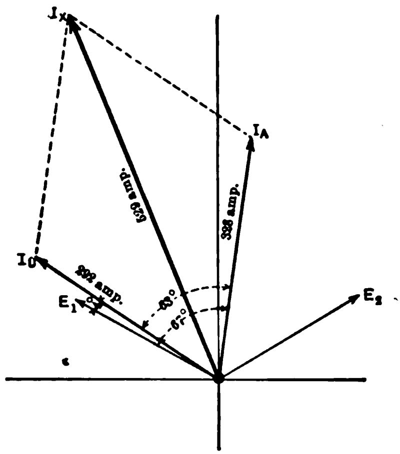


FIG. 156. The vector I_x represents the current in the line wire X of Fig. 153. This line current is a combination of I_A, the current in lead wire A of the induction motor M, and I_U the current in the lead wire U of the synchronous motor N.

$$
\begin{aligned}
I_X &= I_U \oplus I_A. \\
I_X^2 &= I_U^2 + I_A^2 + 2\, I_U I_A \cos 63° \\
&= 292^2 + 328^2 + 2 \times 292 \times 328 \times 0.454 \\
&= 280{,}000, \\
I_X &= \sqrt{280{,}000} \\
&= 529 \text{ amperes.}
\end{aligned}
$$

As the current in all the leads to each motor is the same and at the same phase angle to the corresponding voltage as shown in Fig. 152 and 155, the current in each of the line wires X, Y, and Z would be 529 amperes. To find the power-factor of the combination, or the power-factor of the three-phase system, we have merely to divide the total effective power by the total apparent power.

$$\begin{aligned}\text{Effective power} &= 100 + 100 \\ &= 200 \text{ kw.}\end{aligned}$$

$$\begin{aligned}\text{Apparent power} &= \sqrt{3} \times 529 \times 220 \\ &= 201.6 \text{ kv-a.}\end{aligned}$$

$$\begin{aligned}\text{Power-factor} &= \frac{200}{201.6} \\ &= 99.3 \text{ per cent.}\end{aligned}$$

The same result might be obtained by a longer method consisting of finding the phase difference between the resulting current produced by joining corresponding phases together, and the resulting voltage across the same combination. Thus coil No. 1 of both the induction motor and the synchronous motor are really in parallel across the line wires X and Y, which have the pressure of E_1, or 220 volts, between them.

Draw Fig. 157, letting E_1 represent the voltage across the parallel combination of the coils No. 1 of each machine. Take $I_{1M} = 190$

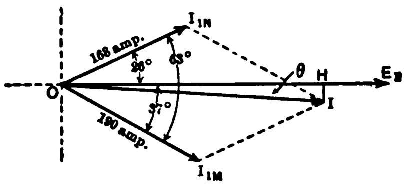


FIG. 157. The vector I represents the current which results when I_{1N}, the current through coil I of the synchronous motor N, is combined with I_{1M}, the current through coil 1 of the induction motor M. It represents the resulting current which must be supplied to three such pairs of coils.

amp., from the vector diagram Fig. 152, to represent the current in the coil 1 of the induction motor M. Note that it lags 37° behind E_1 in Fig. 152, so it must lag 37° behind E_1 in Fig. 157. Take vector $I_{1N} = 168$ amp. from Fig. 155, to represent the current in the coil No. 1 of the synchronous motor. Note that in Fig. 155 it leads E_1 by 26°, so it must lead E_1 by 26° in Fig. 157. Complete the parallelogram and find the resulting current I, as follows:

$$\begin{aligned} I^2 &= I^2_{1N} + I^2_{1M} + 2\,I_{1N} \times I_{1M} \times \cos 63^\circ \\ &= 190^2 + 168^2 + 2 \times 168 \times 190 \times 0.454 \\ &= 93{,}300, \\ I &= \sqrt{93{,}300} \\ &= 305.5 \text{ amperes.} \end{aligned}$$

The angle between this resulting current I through the combination, and the voltage E_1 across the combination, is θ. The corresponding power-factor equals $\cos\theta$. The value $\cos\theta$ can be found as follows:

Draw the line IH perpendicular to E_1. Then $\cos\theta = \dfrac{OH}{OI}$,

$$\begin{aligned} OH &= OI_{1N} \cos 26^\circ + OI_{1M} \cos 37^\circ, \\ OH &= 168 \times 0.9 + 190 \times 0.8 = 303.2, \end{aligned}$$

$$\frac{OH}{OI} = \frac{303.2}{305.5},$$

$$\cos\theta = \frac{303}{305} = 99.3 \text{ per cent.}$$

Since the combination of the No. 1 phases in N and M is exactly similar to the combination of the No. 2 phases in N and M and of the No. 3 phases in N and M, it follows that each phase of the line XYZ must deliver a total current of 305.5 amperes to a total equivalent delta-connected load. Therefore, the current in each of the line wires XYZ must be equal to $(\sqrt{3} \times 305.5)$, or 529 amperes. This checks our result previously obtained. It is common practice, in applying this method and drawing a diagram like Fig. 157, to multiply I_{1N} and I_{1M} each by $\sqrt{3}$ before combining them; then the resultant gives the current in each line wire directly, without applying the $\sqrt{3}$ factor as above. Obviously, it is immaterial to the final value of line current, when we perform this multiplication by $\sqrt{3}$, provided we do it correctly. The quantity $\sqrt{3}\,I_{1N} = 1.73 \times 168 = 292$ amperes is called the **equivalent single-phase amperes** of the motor, because it is the current which a single-phase motor would have to take from a single-phase line to get the same total effective watts at the same voltage between line wires, and the same power-factor.

This latter method makes use of the fact that the power-factor of a line is the cosine of the angle of phase difference between the resultant current in a combination of parallel load circuits, and the voltage across the combination. The first method is based on the simpler fact that the power factor is always the ratio of the effective power to the apparent power.

Still another method may be used by combining directly the true power and the apparent power as vectors and finding their resultant as explained on page 176.

This example also shows how a synchronous motor load may be used to offset an induction motor load and thus better the power-factor of the line. Note that the power-factor of the line is higher when the synchronous motor (taking leading current) is operating in parallel, than when the induction motor (taking lagging current) operates alone.

Note that to supply the 100 kw. to the induction motor alone the line wires each had to carry 328 amperes. But to supply both the induction motor and the synchronous with 200 kw. (twice the load) 529 amperes had to be carried by the line wires. That is, to transmit double the power at the same voltage only about one and one-half as much current was used, because of the improved power factor. Thus comparatively smaller line wires may be used.

Prob. 50-4. At a certain woolen mill a 600-kv-a. synchronous motor is used to improve the power-factor of a 1050-kw. induction motor load which has a power-factor of 64 per cent. At what power-factor does the synchronous motor operate if it raises the total power-factor to 85 per cent when loaded to take 200-kw. effective power? Use 3-phase for convenience in solving at 550 volts.

Prob. 51–4. A 500-kv-a. 3-phase 440-volt synchronous motor is operated at rated kv-a. at 84 per cent leading power-factor in parallel with 3-phase 440-volt induction motors totaling 1000 kv-a. at 80 per cent power-factor. What is the power-factor of the combination?

Prob. 52–4. What will each wattmeter indicate in Fig. 158? (Elec. Journal, 1907).

Prob. 53–4. Find current in each lead in Fig. 159. (Elec. Journal, 1907.)

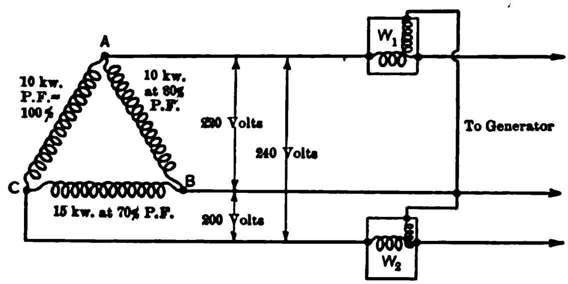


FIG. 158. Two-wattmeter method of measuring a-c. power in three-phase circuits.

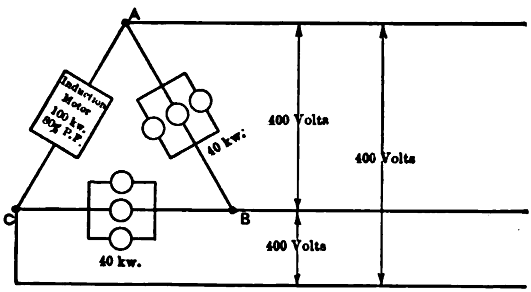


FIG. 159. An unbalanced load on a three-phase three-wire system.

SUMMARY OF CHAPTER IV

THE AVERAGE POWER delivered when the alternating current and pressure are IN PHASE equals the product of the effective volts times the effective amperes. That is,

$$P = EI.$$

WHEN THERE IS A PHASE DIFFERENCE BETWEEN CURRENT AND PRESSURE, of 90°, no power is being delivered. All the energy going from the generator is returned to it during each cycle.

WHEN THE PHASE DIFFERENCE between the current and voltage is $\theta°$, the power delivered equals the effective volts times the effective amperes times the factor, $\cos\theta$. The equation is

$$P = EI \cos\theta.$$

POWER-FACTOR is the name given to the term $\cos\theta$.

APPARENT POWER is the name given to the expression EI, and the units are called VOLT-AMPERES, to distinguish from watts, which is reserved to mean effective power.

$$\text{POWER-FACTOR} = \frac{\text{effective power}}{\text{apparent power}} = \frac{\text{watts}}{\text{volt-amperes}}.$$

The power-factor equals unity when the pressure and current are in phase, and zero when there is a phase difference of 90° between them.

REACTIVE AND POWER COMPONENT. Any alternating current may be considered to be made up of two parts having 90° phase difference. One part, the POWER COMPONENT is in phase with the pressure and equals the current times the cosine of the angle of phase difference between the current and the voltage. The product of this component and the voltage measures the effective power, or watts.

POWER COMPONENT $= I \cos\theta$.

Watts = power component of current × voltage $= E \times I \cos\theta$.

The other component lags or leads 90° from the voltage and is equal to the current times the sine of the angle of phase difference between the current and voltage. This is called the REACTIVE COMPONENT of the current, and no power is delivered by it.

$$\text{REACTIVE COMPONENT} = I \sin \theta.$$

$$\text{Reactive volt-amperes} = \text{reactive component of current} \times \text{voltage} = EI \sin \theta.$$

THE POWER TAKEN BY AN APPLIANCE CARRYING ALTERNATING CURRENT IS MEASURED by means of a wattmeter, one coil of which carries the current which is flowing through the appliance, the other coil receiving the pressure that is across the appliance. The wattmeter reads the product of the volts times the amperes times the power-factor of that part of the circuit to which it is applied.

THE POWER IN ANY POLYPHASE SYSTEM equals the sum of the power in all phases, measured separately.

TO MEASURE THE POWER IN A 3-PHASE 3-WIRE SYSTEM, two wattmeters are generally used. The current-coil of one wattmeter is placed in one line wire, and the current-coil of the other instrument is placed exactly symmetrically with regard to connections, in either of the other two lines. One end of the pressure-coil of each instrument is then connected to the same line as the current-coil, and the other end to the third line wire. If the instruments both indicate negatively, the voltage coil of each is reversed, and the readings added to find the total power in the 3-phase circuit. If one instrument only is negative, the voltage coil of that instrument is reversed and the difference of the readings taken. If both indicate positively, the readings are added.

TO COMPUTE THE POWER IN A BALANCED 3-PHASE CIRCUIT, multiply the product of the effective volts between the line wires by the effective current in each line wire, by $\sqrt{3}$ and by the power-factor.

$$P = \sqrt{3}\, IE \times (\text{power-factor}).$$

The power-factor here is the cosine of the angle of phase difference between the current in each coil of the load and the voltage across that coil.

PROBLEMS ON CHAPTER IV

Prob. 54–4. What power is consumed by a coil the impedance of which is 40 ohms, and power-factor 70 per cent, when an alternating pressure of 220 volts is maintained across it?

Prob. 55–4. What power is consumed by a starting resistance of 40 ohms, non-inductive, when an alternating voltage of 220 volts is maintained across it?

Prob. 56–4. What power would be consumed if the coil and resistance of Prob. 54 and 55 were in parallel across the 220-volt line?

Prob. 57–4. What would be the power-factor of the line, in Prob. 56?

Prob. 58–4. What would be the resultant current in the line in Prob. 57, and what would be the phase difference between the line current and the line voltage?

Prob. 59–4. A generator is supplying two induction motors in parallel which take 7 kv-a. each at 220 volts. Each has a power factor of 80 per cent, lagging current. What is the total kv-a. output of generator? Total watts output? Power-factor of line? What is the current in each line wire if system is single-phase? If the system is two-phase four-wire? If system is three-phase three-wire?

Prob. 60–4. In order to improve the power factor of the above line, one of the induction motors is exchanged for a synchronous motor which carries the same load but takes a leading current. If the power-factor of the synchronous motor is adjusted to 90 per cent, what apparent load in kv-a. must the motor take in order to make the power-factor of the line unity?

Prob. 61–4. (*a*) Compare the apparent power supplied by generator in Prob. 59 with that supplied in Prob. 60? What becomes of the difference?

(*b*) Compare the current supplied by the generators under the conditions of the two problems, assuming a three-phase system.

Prob. 62–4. An induction motor taking a lagging line current of 20 amperes with a power-factor of 75 per cent is connected in parallel with a synchronous motor taking a leading line current of 35 amperes with a power-factor of 85 per cent. What current flows in each wire of the three-phase line which supplies the two motors?

Prob. 63–4. The line voltage in Prob. 62 is 220 volts. What is the phase difference between the current in each line wire and the pressure between this wire and each of the other line wires?

Prob. 64–4. From the line voltage, current per line wire and phase relations found in Prob. 63 compute total power taken by motors.

Prob. 65–4. Check the value of total power taken by motors computed as in Prob. 64, by the sum of values computed for each separately from data in Prob. 62 and 63.

Prob. 66–4. In a two-phase motor having a power-factor of 85 per cent, the two phases are balanced and entirely separate. At full load it takes 15 amperes at 220 volts in each phase. What power does it consume at this load?

Prob. 67–4. If the motor of Prob. 66 has an efficiency of 90 per cent, what is its rating in horse power?

Prob. 68–4. If the motor in Prob. 66 were run on a 3-wire 2-phase system, what current would each line wire carry?

Prob. 69–4. A 15-h.p. 2-phase 4-wire induction motor is guaranteed to have a power-factor of 86 per cent and an efficiency of 88 per cent at full load. What current does each line wire carry, if line voltage is 230 volts?

Prob. 70–4. What current would each line wire carry if the motor in Prob. 69 were used on a 3-wire 2-phase system with the same voltage across each phase in the motor?

Prob. 71–4. (*a*) Show how you would connect the proper instruments to measure the power taken by the motor of Prob. 70.

(*b*) Assuming a power factor of 86 per cent for each coil, what would the total power be?

Prob. 72–4. In Fig. 160 the lamps are Y-connected and a neutral wire brought out from their common juncture to neutral of the generator. Each lamp takes 5 amperes at unity power-factor. What current flows in each line wire?

Prob. 73–4. (*a*) What total power is consumed by the lamps in Prob. 72?

(*b*) What is the power factor of the line if this is the total load?

Prob. 74–4. In the place of Group III of Prob. 72, a single-phase 3-h.p. induction motor with 82 per cent power-factor and 75 per cent efficiency is inserted taking the same current.

(*a*) What power is taken by the motor?

(*b*) What total power is delivered to the system?

Prob. 75–4. What current flows in each line wire of Prob. 74, the e.m.f.'s of the three phases being held 120° apart by the neutral connection?

Prob. 76–4. Calculate the power-factor of the entire polyphase system of Prob. 74, by two methods, as follows:

(*a*) Dividing total effective watts in all phases, by the total apparent watts in all phases.

(*b*) Dividing total effective watts by $\sqrt{3}$ times average line current times average voltage between line wires. Compare these two values of system power-factor.

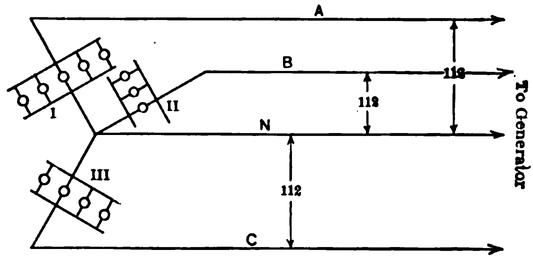


FIG. 160. The three banks of lamps are Y-connected to a 4-wire 3-phase system.

Prob. 77–4. (*a*) How many watts would be indicated by a wattmeter which has its current coil connected in line *A* (Fig. 159) and its pressure coil between *A* and *B*?

(*b*) With current coil in *A* and pressure coil between *A* and *C*?

(*c*) Sum of these readings, or total power according to single-wattmeter method?

Prob. 78–4. (*a*) How many watts would be indicated, with current coil of wattmeter in line *B* (Fig. 159) and pressure coil between *B* and *A*?

(*b*) With current coil in line *C* and pressure coil between *C* and *A*?

(*c*) Sum of these two readings, or total power according to the two-wattmeter method?

Prob. 79–4. (*a*) How many watts would be indicated, with current coil in line *A* (Fig. 160), and pressure coil *A* to *N*?

(*b*) With current coil in *B* and pressure coil *B* to *N*?

(*c*) With current coil in *C* and pressure coil *C* to *N*?

(*d*) Sum of these three readings, or total power according to the three-wattmeter method?

Prob. 80-4. If the current flowing in Prob. 115-3 is 3.5 amp., and is in phase with the voltage of A_1B_1, what power is delivered by the series circuit?

Prob. 81-4. What power is delivered by coil A_1B_1 of Prob. 80-4?

Prob. 82-4. What power is delivered by coil A_2B_2 of Prob. 80-4?

Prob. 83-4. (*a*) What power is delivered by coil A_3B_3 of Prob. 80-4?

(*b*) Compare the sum of the answers to Prob. 81, 82, and 83 with the answer to Prob. 80.

Prob. 84-4. Assume the current in the combination of Prob. 121-3 to be in phase with the voltage across A_2B_2 reversed, what are the power-factors of each coil and of the whole combination?

Prob. 85-4. Assume that a lagging current of 4 amperes flows through the circuit of Prob. 125-3. The average power delivered by the combination is 500 watts. What is the power-factor of the combination?

Prob. 86-4. What is the angle of lag between the current and voltage in each coil of Prob. 85-4, and between the current and the resulting voltage?

Prob. 87-4. How much power is consumed by the first part of the circuit of Prob. 127-3?

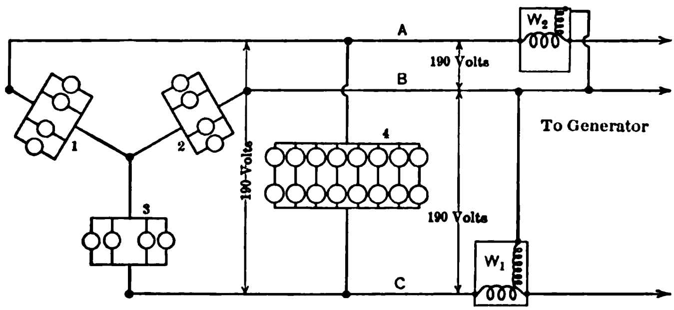


FIG. 161. The three-phase three-wire system in delivering power to four lamp groups. Two wattmeters are being used to measure this power.

Prob. 88-4. How much power is consumed by the second part of the circuit of Prob. 127-3?

Prob. 89-4. How much power is consumed by the circuit of Prob. 127-3?

Prob. 90–4. In Fig. 161 each lamp takes 4 amperes in phase with the voltage across it. What power does the line deliver to the four groups?

Prob. 91–4. What current flows in each line wire of Prob. 90–4?

Prob. 92–4. What would each wattmeter in Fig. 161 indicate, if the line were loaded as in Prob. 90–4?

Prob. 93–4. A single-phase 5-h. p. motor with 85 per cent power factor is represented by *M*, Fig. 162. A three-phase 10-h.p. motor

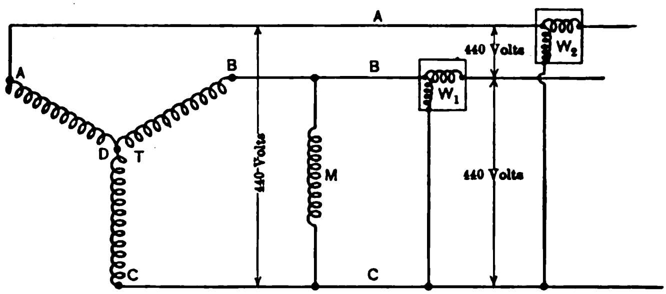


FIG. 162. *M* represents a single-phase induction motor. *T* represents a three-phase induction motor. The two wattmeters measure the power taken by these two motors.

with 80 per cent power-factor is represented by *T*. The single-phase motor has an efficiency of 83 per cent; the 3-phase of 90 per cent. What total power is being supplied to the two motors when both are operating at full load?

Prob. 94–4. What is the power-factor of the entire arrangement in Prob. 93?

Prob. 95–4. What current flows in each line wire of Prob. 93?

Prob. 96–4. What will be the reading of each wattmeter in Fig. 162 when the line is loaded as in Prob. 93?

Prob. 97–4. (*a*) What must be the load, in kv-a., of the synchronous motor in Prob. 60, in order to take the same power as the induction motor which it replaces, and also to make the power factor of the entire system 100 per cent?

(*b*) At what power-factor must this synchronous motor operate?

(*c*) Should it be adjusted to take leading current or lagging current?

CHAPTER V

INDUCTIVE REACTANCE

In studying the relation between a uni-directional electromotive force and the **direct current** which this e.m.f. can force through a given circuit, we learned that for any particular circuit this relation was fixed; that is, the **voltage** across the circuit divided by the **current** in that circuit always gave the same numerical value, which we called the **resistance** of the circuit. We have then defined the resistance of a direct-current circuit as the ratio of the voltage to the current which the voltage can force through it, or

resistance (in **ohms**) = volts per ampere.

When we took up the relation between an alternating e.m.f. of a given frequency and the alternating current which this e.m.f. can force through a given circuit, we saw that there was also a fixed ratio between these, but that this ratio was not necessarily the same as the ratio between a direct e.m.f. and a direct current in the same circuit. Therefore, we called the ratio of volts to amperes in an a-c. circuit the **impedance** of the circuit in order to distinguish it from the ratio of the direct voltage to the direct amperes, or the resistance. Both the resistance and the impedance are measured in **ohms.** It is the purpose of this chapter to explain one of the causes for this difference between the **impedance** of a circuit carrying a-c. power, and the **resistance** of the same circuit carrying direct-current power.

Suppose that we take for our first test circuit, one mile of No. 18 copper wire, which has a resistance of 33.6 ohms per mile. Let us string this up as a half-mile "line and return," with the far ends joined, and the near ends across the terminals of a 110-volt d-c. generator. A d-c. ammeter inserted in the line would read 3.27 amperes, a current which this line could carry comfortably. That is, the resistance which this mile of wire offers to the flow of a direct current equals $\frac{110}{3.27}$, or 33.6 ohms. If we now connect the near ends across the terminals of a 110-volt 60-cycle a-c. generator, and insert an a-c. ammeter in the line, this meter would also read 3.27 amperes. That is, the impedance which this mile of line wire offers to the flow of an alternating current also equals $\frac{110}{3.27}$, or 33.6 ohms. Note that this is exactly equal to the resistance which it offers to a direct current.

For our second test circuit, let us take 345 ft. of No. 12 copper wire with a cotton cover and string it up as "line and return." This wire has a resistance of 0.55 ohm, so that we would not be safe in placing the ends across a 110-volt d-c. generator as before. If we did this, it would allow about 200 amperes to flow, which would burn up the wire. Therefore, let us put only 11 volts across it. An ammeter would now indicate 20 amperes, a safe current for this wire strung up as a line in free air. The resistance is $\frac{11}{20}$, or 0.55 ohm. Similarly, we would not be safe in placing this wire across the terminals of a 110-volt 60-cycle a-c. generator, as approximately 200 amperes would flow. Accordingly we will try 11 volts alternating current, at the same frequency of 60 cycles. An a-c. ammeter would read about 20 amperes, showing that the impedance of the wire so arranged is approximately the same as the resistance; $\frac{11}{20}$, or 0.55 ohm. But now let us wind this same wire on a round wooden core 20 inches long and $1\frac{1}{4}$ inches diameter. There would be 730 turns on this core, which would constitute a weak electromagnet, as in Fig. 163. If we put the coil across the 11 volts direct current, as we did the straight wire, an ammeter would still indicate 20 amperes, showing that, in shaping the wire into a weak electromagnet, we have not changed in the slightest the resistance which it offers to the flow of a direct current. The resistance is still $\frac{11}{20}$ or 0.55 ohm.

But if we put it across the 11 volts alternating at 60

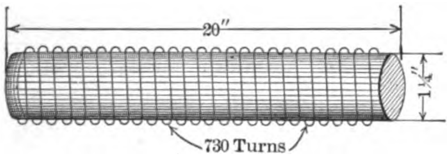


FIG. 163. A weak electromagnet made by winding wire on a wooden core.

cycles, we find that an a-c. ammeter indicates a little less, — about 16.7 amperes. The impedance has become $\frac{11}{16.7}$, or 0.66 ohm. Without changing the wire in any way except to wind it into the form of a weak electromagnet, we have increased the impedance about 20 per cent, while the resistance has not been changed in the slightest degree.

Suspecting that the magnetic feature of the coil may have some influence, let us make as strong a magnet as is convenient, of the same dimensions as the coil with the wooden core. We will, accordingly, wind the 345 ft. around a soft iron ring of 5-in. inside diameter, made of round 1¼-in. stock as per Fig. 164. The length of the iron core would be nearly equal to the length of the wooden core and the 345 ft. of wire would make 730 turns on this ring, as on the wooden core. If we now place this coil across 11 volts direct current we will find that the ammeter still indicates 20 amperes. In winding the wire around an iron ring so as to make a strong electromagnet we have not changed, in the slightest, the resistance which it offers to the flow of direct current. The resistance is still $\frac{11}{20}$ = 0.55 ohm.

Suppose, however, that we try it across 11-volt a-c. 60-cycle mains. An ammeter in the circuit would read about $\frac{1}{100}$ of an ampere only (assuming 1800 as the permeability of the iron core). Apparently the impedance has been increased immensely. In fact, if we put the coil across a 110-volt 60-cycle alternating-current generator, only 0.164 ampere would flow.

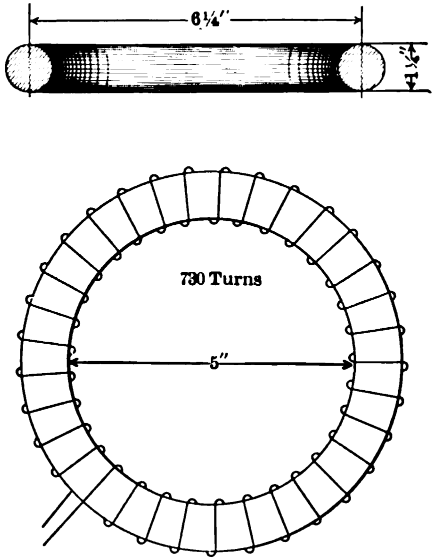


FIG. 164. A strong electromagnet made by winding the wire of Fig. 163 on an iron ring.

In winding the core around the iron ring, therefore, we have increased more than a thousandfold the impedance which it offers to the flow of this alternating current. The impedance has now become $\frac{110}{0.164}$, or 662 ohms. Let us consider the causes of this great increase in the impedance.

46. Inductive Reactance. We started with a wire which, when stretched out approximately straight, offered a fraction of an ohm resistance to the flow of a direct current and the same impedance to the flow of an alternating current. Without changing it any way except merely to wind it about a piece of iron so as to form a strong electromagnet, we raised the impedance to over 660 ohms. This figure is so great that when we compare it with the original, 0.55 ohm, we see that practically all this impedance is due to winding the wire around the iron. In speaking of the impedance of the coil we can thus neglect the original 0.55 ohm due to resistance, and say that practically the whole impedance of the wire is the result of winding it into a coil so that it sets up a strong magnetic field when a current flows through it.

When the impedance of a circuit carrying alternating current is greater than its resistance, we say that this circuit possesses **reactance**; that is, there is some condition present in the circuit which reacts against the voltage and hinders it from forcing through as large an alternating current as we would expect, judging from the resistance to a direct current.

When this reactance in a circuit is largely due to the magnetic field which a current will set up about it, we call the reactance an **inductive reactance** and represent its value in equivalent ohms by the letter X. Now we have seen that the inductive reactance of a short straight line, about which a current produces almost no magnetic field, is practically zero, while the inductive reactance of a coil made of the same piece of wire is quite noticeable, when the coil forms even a weak electromagnet, and very great when a strong electromagnet is produced. There seems to be something about this magnetic field, then, which produces this counter action or, as we have called it, this reactance.

In a circuit carrying direct current we know that sometimes such a strong reaction is set up that the current is cut down below the value which we would expect from Ohm's law. This always happens whenever a motor is running on the circuit. We measure the resistance of the motor with the armature at rest and find it very low, and naturally expect the motor when running to take a current which shall have the value expressed by the fraction $\frac{\text{volts across motor}}{\text{resistance of motor}}$. But we find that the machine takes but a small fraction of this current. On investigation we decide that the cause of this great decrease in current or large increase in apparent resistance is due to the fact that when the armature is revolving the conductors on it cut through the magnetic field and set up a **counter e.m.f.** which opposes the flow of the current. When the armature is standing still, there is no counter e.m.f. and the current would be very great (as indicated by the fraction above) if the same voltage were applied. The faster the armature moves, and the stronger the field, the greater the counter e.m.f. that is produced, and the smaller the current.

If we investigate the reactance of a circuit carrying alternating current, we shall see that the same thing is true. The circuit offers no reactance to an electric current unless the conductors of the circuit cut the magnetic field. When we try to send an alternating current through the coils in Fig. 163 and 164, the current is continually changing in value and direction. The magnetic field is thus continually being built up in one direction, reduced to zero, and built up in the opposite direction. Obviously, in this process, the lines of flux must cut the wires again and again. The stronger the field the greater the e.m.f. thus set up. From an inspection of Fig. 165, we can see that the e.m.f. induced in the wire by this cutting, in each instance, always opposes the action of the current. Thus, if a current is trying to increase, the lines of flux, increasing and spreading, cut the wires in such a direction as to oppose any increase of the current. If the current is trying to decrease, the lines of the dying magnetic field cut the wires in such a direction as to oppose the decrease of the current, or so as to maintain the current.

Consider Fig. 165. Assume that a current is growing in the coil. It enters at A_1 and a field immediately begins to grow out, which spreads to the left and sweeps the wire A_3, from right to left as is shown in Fig. 166. This is equivalent to the wire A_3 cutting to the right across the lines. Thus an e.m.f. is set up which tends to send a current out

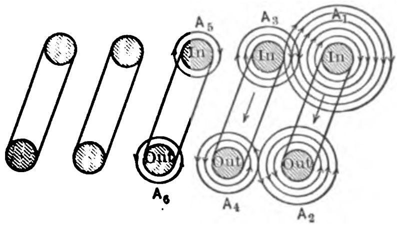


FIG. 165. A growing field around A_1 spreads out and cuts A_3. The growing field around A_3 cuts A_5 and A_1, and sets up an induced e.m.f. From Fig. 166, note that this induced e.m.f. is in the direction opposite to the current.

at A_3 in the opposite direction to the current which is being established in it.* This action takes place in all the wires as the current grows. The spreading field about each wire cuts the other wires in such a direction as to set up a reacting e.m.f. which opposes the growth of the current. A growing current is thus "choked" back in any coil where the field is strong and the turns are numerous and close together.

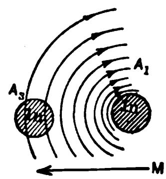


FIG. 166. An enlarged view of the wire ends A_1 and A_3 of Fig. 165. The arrow M shows the direction in which the field about A_1 sweeps across A_3 as it grows. This induces an e.m.f. which tends to send a current *OUT* at A_3 in opposition to the applied e.m.f.

By using the same figures, 165 and 166, and assuming the current to be dying out, we see that the lines in the dying field now sweep across the wires in the opposite direction and set up a reacting e.m.f. which tends to keep the current in these wires from dying out. Thus the action of dying is

* See Timbie's "Elements of Electricity," Art. 104, 105.

also impeded, and it is easy to see that such reactions can greatly hinder an alternating current from flowing through a coil even though the resistance is almost zero. The action of the coil in Fig. 164 in a direct-current circuit is also explained. When a direct current has once reached a steady value it remains constant. Therefore, once a direct steady current is established, the field also remains constant and no cutting of the lines takes place; thus no greater hindrance is offered to the flow of the direct current than the resistance which the wire affords. But we might expect that it would be difficult to start even a direct current flowing, and such is the fact. Fig. 167 shows a curve plotted from data taken to investigate the time required to get a direct current up to its full value, the circuit being highly magnetic. The coil had a resistance of 11 ohms, and was put across 110 volts, direct current. Note that 0.9 second elapsed after the circuit was closed, before the current reached its normal value of 10 amperes. The inductive property of the circuit opposed its growth by setting up a counter e.m.f.

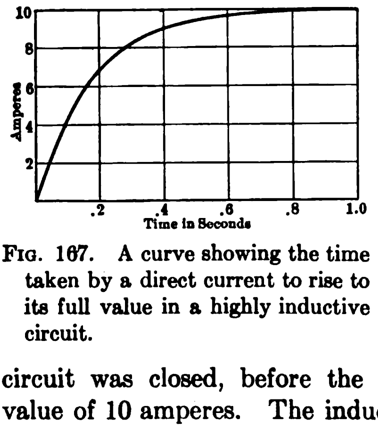


FIG. 167. A curve showing the time taken by a direct current to rise to its full value in a highly inductive circuit.

The opposition to the decrease of a current in an inductive circuit is seen in the flash which takes place when the field switch of a large generator is opened. This arc is so destructive to the copper blades of the switch that special switches are put in a field circuit which reduce the current gradually, not instantaneously. One of these field discharge switches is shown in Fig. 168. When the blades *B* are withdrawn from the clips, it does not disconnect the power from the field which is across the upper clips, because the spring blades *SS* are still held in the clips by their friction. When the blades are withdrawn from clips B, the blade X comes in contact with the clip Y. This connects a resistance across the field terminals because a resistance R is connected across

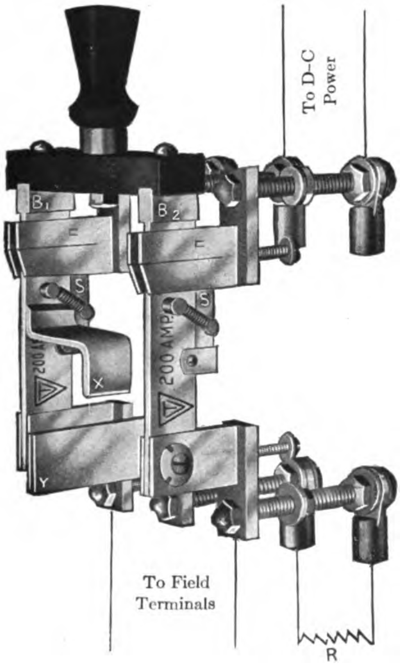


Fig. 168. A Trumbull-Vanderpoel field-discharge switch.

Y and B_2 as shown. As the handle is pulled farther down, the spring blades SS fly out and disconnect the power from the field and the resistance R. But before the spring blades disengage from the upper clips, the blade X makes contact with the lower clip Y. Thus the field is never opened. It is merely connected to a resistance, and the power is then disconnected. The e.m.f. induced in the field coils by the dying magnetic flux produces a current through this resistance; thus the energy stored up in the magnetic field, when the current was compelled to increase against the induced counter e.m.f., now reappears as heat in this resistance instead of an arc at the clips of the switch.

There is a greater damage likely to be done than the burning of the switch contacts when the field current is suddenly stopped. The dying lines of the magnetic field sometimes sweep in such great numbers and so rapidly across the wires

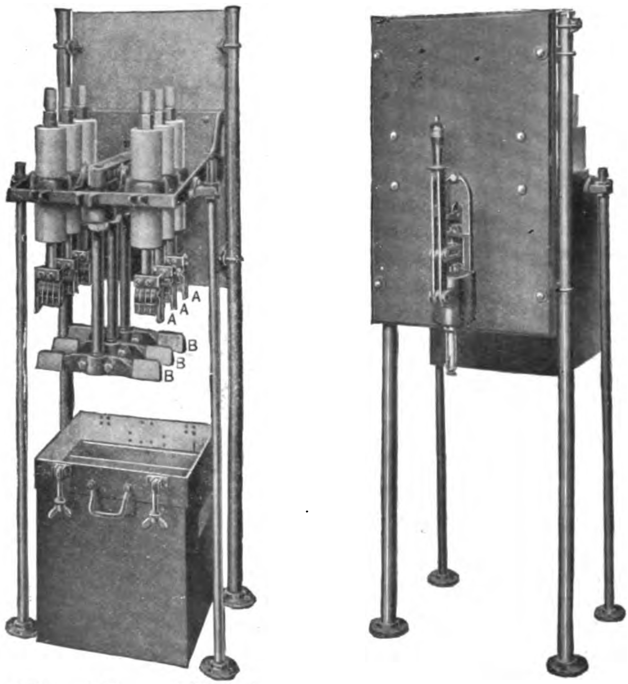


FIG. 169. A General Electric 3-pole oil switch. The oil in the case (shown in position in right-hand view) smothers the arc formed between points A and B as the switch is opened.

in the coils, and so high a voltage is thereby set up, that it punctures the insulation and puts the field coils out of service. The special switch shown above prevents this by introducing a large resistance into the field which allows the field current to die out slowly, and so a smaller e.m.f. is induced. Destructive arcing is also prevented on circuits which have large inductive properties, by means of oil switches such as shown in Fig. 169. This represents a 3-pole switch, the break occurring at the points between *A* and *B*. These points are immersed in oil which is held by the case — here shown lowered in order to make the construction clear. The oil smothers the arc and saves the contact points from being fused or roughened.

In some cases very effective use is made of this counter e.m.f. which is set up, to cut down the currents which

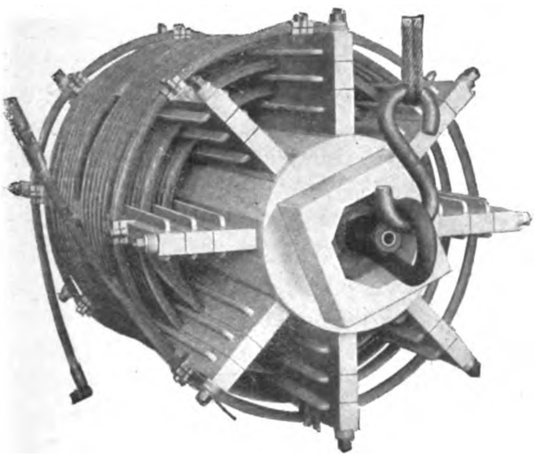


FIG. 170. A General Electric current-limiting reactance coil. The reactance offered by this coil to any sudden growth of current keeps down the amount of current which can flow through the generator to which it is connected, when a short circuit occurs.

would flow in an a-c. generator or transformer if the line wires were suddenly short-circuited. A large coil (Fig. 170) with cement core is placed in series with each lead close to the terminals of the machine. This coil offers but slight counter e.m.f. to the normal current because of the non-magnetic core and the comparatively few turns in the winding. But when a short circuit occurs and the current begins to increase at a tremendous rate, the turns are cut so quickly by the rapidly growing field that enough counter e.m.f. is induced to choke back the current to such an extent as to prevent any destructive stress upon the machine, and to permit the comparatively slow-acting circuit-breaker to disconnect the machine from the circuit.

47. Lenz' Law. Enough has been shown concerning the effects of a strong magnetic field upon the electric circuit within it, to bring out the law which was first stated by Lenz and is called Lenz' law. It states in part that:

While any change is being made in the magnetic field of an electric circuit, an e.m.f. is induced which opposes the change.

Thus we have seen that when a current was growing, and in so doing was setting up a magnetic field, an e.m.f. was induced in the opposite direction, so that it opposed this growth of the current and of the magnetic field. If the growing current had set up no magnetic field there would have been no opposition to its growth. The whole reaction, or opposition, is due to the creation or the destruction of a magnetic field which cuts across the wires composing the circuit and so induces the reacting e.m.f.

Since the magnetic field set up by an alternating current is continually changing, this induced counter e.m.f. is continually acting and thus continually limits the current throughout the system.

48. Inductance. It is often necessary to be able to compute the reactance which such coils as we have described offer to the flow of an alternating current. In taking up this computation it is well to start with the counter e.m.f. of which this reactance is a measure. When such a counter e.m.f. is set up by these changes of current we say that the circuit is **inductive,** or contains **inductance. Inductance may be defined, then, as that magnetic property of a circuit which causes it to oppose any change in the current flowing.**

If there is no current flowing, the inductance opposes the start and growth of one. If a current is already flowing, the inductance of a circuit opposes either any decrease or any increase of this current. **Inductance** in an electrical system is very like inertia in a mechanical system, which opposes any change in the speed of a body. Thus, if we are standing in a rapidly moving car and the motorman suddenly applies the brakes, we feel a strong tendency to go forward in the car and have to brace our feet in order to remain standing. It is the inertia of our bodies which is urging them to keep moving in the same direction and at the same rate while the feet are carried forward by the car. The inertia of our bodies is thus opposing the change of the speed at which they are moving just as the inductance of the electric circuit opposes any change of current flowing. When we try to stop an electric current, the inductance of the circuit tends to keep it going.

Similarly, when a car suddenly starts up, we feel a strong tendency to take a step toward the back of the car. This tendency is again due to the inertia of our bodies, which opposes the change in motion (that is, the speeding-up process), just as the inductance of an electric circuit opposes the start and growth of a current.

In fact it is a universal law which apparently applies to all branches of science, that if we wish to make any changes we must overcome some force which tends to keep things as they are.

The force which tends to keep the current as it is in an electric circuit is the counter e.m.f. which the inductance of the circuit sets up whenever the current changes.

49. Unit of Inductance. The Henry. When a circuit has so much of this inductance that a counter e.m.f. of one volt is set up when the current is changed at the rate of 1 ampere per second, we say that the circuit has 1 **henry** of inductance. To have 2 henrys inductance, a circuit has to have strong enough field per ampere of current flowing, and enough turns of wire around that field, to set up 2 volts e.m.f. when the current is changed at the rate of 1 amp. per sec., or 1 volt may be induced when the current changes at the rate of ½ ampere per second. The volts thus set up are equal to the product of the inductance in henrys times the amperes change per second.

We may write the equation

Induced volts = inductance × rate of change of current,

or,

Inductance (in henrys) = induced volts ÷ rate of change of current (in amperes per second).

The field coils of a 5-kw. generator may have from 10 to 15 henrys inductance, so it is easily seen that the current must be decreased very slowly in order not to produce high enough e.m.f. to puncture the windings. For, assume that the field coils of such a machine carry 2 amperes and that the switch was pulled so suddenly that the current dropped to zero in $\frac{1}{500}$ of a second. A change of 2 amperes in $\frac{1}{500}$ of a second would mean a change at the rate of 1000 amperes per second. With an inductance of 12 henrys in the circuit, the e.m.f. set up would equal

Inductance in henrys × amperes per sec. change

$$= 12 \times 1000 = 12{,}000 \text{ volts.}$$

Note the similarity between the equation for the reacting electromotive **force** due to **inductance,** and the reacting mechanical **force** due to **inertia.**

(1) The mechanical force = mass × rate of change in speed.

(2) The electromotive force = inductance × rate of change in current.

Equation (1) may be written

$$\text{Ave. } f = m \times \frac{v}{t},$$

where

f = value of reacting force due to inertia;

m = mass of the body;

v = speed of the body;

t = time required to change speed from 0 to v, or from v to 0.

Equation (2) may be written

$$\text{Ave. } E = L \times \frac{I_m}{t},$$

where

E = value of induced electromotive force due to inductance;
L = inductance of the circuit;
I_m = maximum value of current, in amperes;
t = time required for current to change from 0 to I_m, or from I_m to 0.

From this equation Ave. $E = L \times \frac{I_m}{t}$, we may find the value of L in henrys for any circuit.

Suppose we consider the inductance of a circuit in which there is an alternating current flowing. The current starts at 0 and rises to its maximum value I_m in t seconds. Therefore the total change in current would be represented by the letter I_m, and the average rate of change by $\frac{I_m}{t}$ (amperes per second).

$$\text{Ave. } E = L\frac{I_m}{t}.$$

But we have seen that this reactive voltage which is set up in a coil is due to the magnetic field sweeping across the wires of the coil. In order to have one volt set up, the total cutting of lines per second must equal 100,000,000, or 10^8.

Thus

Average e.m.f. induced in each turn of coil $= \frac{1}{10^8}$ of $\frac{\phi_m}{t}$, and

(3) $$\text{Ave. } E \text{ (for entire coil)} = \frac{\phi_m N}{10^8\, t},$$

where

ϕ_m = total number of lines set up when the current rises from 0 to I_m amperes;

N = number of turns in the coil, all of which would be cut by all of this growing flux;

t = time required for this flux to cut across the turns of the coil.

We thus have the two equations

(2) $$\text{Ave. } E = L\frac{I_m}{t};$$

(3) $$\text{Ave. } E = \frac{\phi_m N}{10^8 t}.$$

Therefore

(4) $$L\frac{I_m}{t} = \frac{\phi_m N}{10^8 t}.$$

Multiplying each side by t (since t is the same in both equations, being the time required to pass from zero to maximum value of I and ϕ),

(5) $$LI_m = \frac{\phi N}{10^8}.$$

But the general equation for magnetic flux in any circuit is: *

(6) $$\phi = \frac{M}{R},$$

where

(7) $$M = 1.26\,NI$$

and

(8) $$R = \frac{l}{\mu A}.$$

Substituting (7) and (8) in (6) we get

(9) $$\phi_m = \frac{1.26\,NI_m\mu A}{l},$$

in which ϕ_m = maximum total flux in coil attained when $I = I_m$.

Substituting this value of ϕ in Equation (5) we obtain

(10) $$LI_m = \frac{1.26\,N^2 I_m \mu A}{10^8\,l},$$

* See Timbie's "Elements of Electricity," Art. 92 and 94.

and dividing both sides by I_m,

$$L = \frac{1.26\,N^2\mu A}{10^8\,l}, \tag{11}$$

where

L = inductance in henrys;
N = number of turns in the coil;
μ = permeability of the core (assumed for convenience to be constant);
A = cross section of the core in sq. cm.;
l = length of the core in centimeters.

Example 1. What is the inductance of a "choke" coil having 400 turns, if the iron core is 60 centimeters long and 300 sq. cm. section area? Assume $\mu = 1500$.

$$L = \frac{1.26\,N^2\mu A}{10^8\,l};$$
$$N = 400 \text{ turns};$$
$$\mu = 1500;$$
$$A = 300 \text{ sq. cm.};$$
$$l = 60;$$
$$L = \frac{1.26 \times 400 \times 400 \times 1500 \times 300}{10^8 \times 60}$$
$$= 15.1 \text{ henrys.}$$

Example 2. Suppose the current in the above coil to change from 26 to 2 amperes in 3 seconds, what is the average value of the voltage induced?

$$\text{Ave. } E = \text{henrys} \times \left(\frac{\text{change in amperes}}{\text{time to change}}\right) = L \times \frac{I}{t};$$
$$\frac{I}{t} = \frac{26 - 2}{3} = 8 \text{ amp. per sec.};$$
$$\text{Ave. } E = L \times 8$$
$$= 15.1 \times 8$$
$$= 120.8 \text{ volts.}$$

Example 3. If the above change had taken place in 0.004 of a second, what average voltage would have been induced in the coil?

$$\frac{I}{t} = \frac{26 - 2}{0.004} = 6000 \text{ amp. per sec.}$$
$$\text{Ave. } E = L \times 6000$$
$$= 15.1 \times 6000 = 90{,}600 \text{ volts.}$$

Prob. 1-5. A coil of 800 turns is wound on a wrought-iron ring, the mean diameter of which is 15 cm., cross-section area 20 sq. cm. Permeability of iron = 1200. What is the inductance of the coil?

Prob. 2-5. A current of 30 amperes is flowing in coil of Problem 1. If the current is reduced to 18 amperes, the decrease taking place in one-quarter second, what average voltage will be induced?

Prob. 3-5. Compute the inductance of a coil 75 cm. long, 4 cm. in diameter, containing 2000 turns. Air core.

Prob. 4-5. A "choke" coil is formed by winding 200 turns on a ring consisting of annealed sheet iron, permeability 1800, 60 cm. in circumference and 200 sq. cm. cross-section area. What is the inductance of circuit?

Prob. 5-5. It is observed that the field current of an a-c. generator requires 4 seconds to fall from its full value of 40 amperes to zero when the field break-switch is opened. What average voltage is induced by the dying flux, if the inductance of the field winding is 10 henrys?

Prob. 6-5. An electromagnet consisting of 12 coils of 200 turns each, wound in series on an iron core made of two half-rings butted together (mean radius of rings, 25 centimeters and cross section of core, 78.5 square centimeters) had a self-inductance of 180 henrys when carrying a current of 45 amperes. Calculate what the permeability of the iron must have been.

Prob. 7-5. The primary winding of an induction coil, which is 19 inches long and 8 inches in diameter, measured 0.145 ohm resistance and 0.013 henry inductance, while its secondary measured 30,600 ohms and 2000 henrys. Assuming the reluctance of the magnetic circuit to have been the same when both measurements were made, calculate the ratio between the number of turns in primary and secondary windings.

Prob. 8-5. The fields of a certain shunt-wound direct-current dynamo having a rated output of 35 amperes at 100 volts have a resistance of 44 ohms and a self-inductance of 13.6 henrys at a small excitation; the armature of the same machine measures 0.215 ohm resistance and 0.005 henry inductance. (*a*) Calculate what average voltage would be induced in the field winding if it were disconnected from the brushes in such manner as to reduce the current to zero in one-half second. (*b*) Calculate also the maximum rate (amperes per second) at which the field current may be permitted to die away, to prevent the induced voltage from rising above 1000 volts.

50. Interlinkages versus Cutting of Magnetic Lines. There is another very common method for computing the effect of changes in a magnetic field. Instead of considering a number of lines of force as sweeping across the conductors and thus cutting them when the strength of the field is changed, it is customary to take account of the number of lines of force in the magnetic circuit which are linked into each loop of the electric circuit or vice versa. Every electric circuit must consist of at least one complete loop (out from and back to the source of power) and every magnetic line or tube of flux must also form a complete loop. Thus, whenever we see that a magnetic line enters a loop of wire, or a wire enters a magnetic circuit, we know that there must take place an **interlinkage**

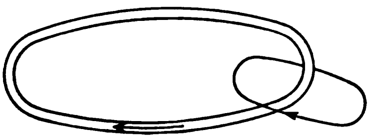


FIG. 171. The line of magnetic force is interlinked with the simple electric circuit. One interlinkage is thus formed.

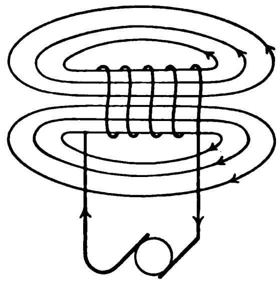


FIG. 172. Six lines of magnetic flux link five turns of the electric circuit. Thus thirty interlinkages are formed.

of the two, similar to the interlinkage of two links in a chain. Since such an interlinkage cannot be made without cutting either one circuit or the other, in order to interlink them, we take no thought of this actual cutting or how it is done, but merely count the number of interlinkages. Thus in Fig. 171 there is one interlinkage, there being one turn in the circuit and one force line. In Fig. 172, 6 lines of force are linked into 5 turns of an electric circuit. Each turn of circuit links with each of 6 lines of flux, or forms 6 interlinkages. The 5 turns must, therefore, form 5×6, or 30 interlinkages. Note that the total cutting is the same as the total interlinkages; for in order to link 6 lines of force into 5 turns of wire there must be 30 cuttings. Thus, if we can produce 100,000,000 interlinkages per second, it is the same as producing a cutting of 100,000,000 lines per second, and one volt e.m.f. is induced. The definition of one henry inductance is, therefore, often given as the inductance of that circuit in which a change of one ampere current produces a change of 10^8 interlinkages between the turns of the circuit and lines of magnetic flux.

The equation for the computation of Inductance is exactly the same as by the method of considering the cuttings. The only difference is that the name "**change of interlinkages**" rather than "**total lines of force cut**" is given to the quantity ϕN. We can then write the equation for induced e.m.f. as follows:

$$\text{Ave. } E = \frac{\text{change of interlinkages, per second}}{10^8}$$

$$= \frac{\phi_m N}{10^8 t}.$$

This equation is identical with equation (3) on page 227. Note also that in the equation for the inductance of a coil

$$L = \frac{1.26\, N^2 \mu A}{10^8 l};$$

since

$$\phi_m = \frac{1.26\, N I_m \mu A}{l},$$

it can be written

$$L = \frac{\phi_m N}{10^8 I_m} = \frac{1}{10^8} \text{ of } \frac{\phi_m N}{I_m},$$

where ϕ_m = total flux when I_m amperes flow.

The inductance of any coil in henrys is thus seen to be merely the total number of interlinkages per ampere flowing in it, divided by 10^8. This forms a very easy method of computing the inductance of the circuits in transformers, alternators and motors.

51. Inductance a Property of the Circuit. When we examine the equation $L = \frac{1.26\, N^2 \mu A}{10^8\, l}$ we see that all the quantities which it contains (N, μ, A, and l) refer to the electric and magnetic circuit. The physical characteristic named **inductance,** then, is a **property of the circuit,** not of the electric current or voltage. It depends entirely upon the shape and size of the circuit and upon the magnetic permeability of the surrounding medium. Another way of stating this might be: Inductance is not a **material thing,** but merely a **term** which expresses the result of a certain **arrangement** of wires, iron, air, etc., in an electric circuit. If the medium consists of iron, which has a high permeability, and the circuit consists of a coil of wire, producing many interlinkages, the inductance of the circuit is great. On the other hand, if the circuit consists of a short straight wire strung in the air, which has low permeability, producing few interlinkages, the inductance of the circuit will be extremely small. The current, voltage, power, etc., have nothing to do with the amount of inductance of the circuit, though these are greatly affected by the inductance in an alternating-current circuit.

52. Inductance of Transmission Lines. While the inductance of a short transmission line is small, it becomes sufficiently large in long lines to produce appreciable and sometimes large effects. The following equation gives the value of the self-inductance of **one mile of one wire** (either outgoing or return) in any system of non-magnetic wires strung in the air.* In polyphase systems the quantity S is to be measured between the two wires strung nearest each other.

$$L_m = .000741 \log_{10} \left(2.568 \frac{S}{d}\right)$$

S = distance between centers of wires, in inches;
d = diameter of wire, in inches.

* From Still's "Overhead Electric Power Transmission."

Prob. 9–5. Compute the inductance of a 100-mile aerial line consisting of two copper wires 20 inches apart, each No. 6 B. & S.

Prob. 10–5. What average voltage would be induced between the terminals of the line in Prob. 9–5, if the current changed from +20 amperes to −20 amperes in $\frac{1}{60}$ of a second.

Prob. 11–5. There were 345 ft. of No. 12 wire, in 730 turns, used to wind the iron ring of Fig. 164. Assuming 1800 as the permeability of the iron, calculate the inductance of the circuit.

Prob. 12–5. When the same wire as in Fig. 164 was wound on the wooden core of Fig. 163, what was the inductance of the circuit?

Prob. 13–5. What would be the inductance of a single-phase transmission, or "line and return," composed of the same wire used on ring in Fig. 164. Assume wires to be strung 30 inches apart.

Prob. 14–5. Calculate the inductance per mile of wire (or per half mile of two-wire single-phase transmission line) for No. 6 B. & S. gauge solid copper wire spaced 10, 20, 30, 40 and 50 inches between centers. Compare these results with those given in the tables in the Appendix.

Prob. 15–5. From the formula given, calculate the inductance per mile of circuit (2 miles of wire), using No. 6 solid wires placed as closely together as possible. The diameter of No. 6 bare is 162 mils and the outside diameter, when rubber-insulated according to National Electric Code for 0 to 600 volts with double braid covering, is 387 mils.

Prob. 16–5. From the formula given, calculate the inductance per mile of wire ($\frac{1}{2}$ mile of single-phase line), for wires spaced 24 inches between centers, using sizes No. 6, 2, 00 and 0000 B. & S. gauge stranded copper cable.

The self-inductance of a line wire is formed by the magnetic flux which surrounds it, or which is linked with it. Fig. 173 represents by the heavy lines the flux around the wire *A* when a current is flowing **in**, and by the light lines the flux around any return wire such as *B*, in which the current would be of the same value, but in the opposite direction, or **out**. Note how all the lines interlinking *A*, which extend beyond *B* are counteracted by the field around *B*. Also the lines around *B* which extend beyond *A* are counteracted by the interlinking lines of *A*. The farther apart the wire *A* and its "return" *B* are, the less interference there is, and therefore, the greater the number of interlinkages per ampere, or the greater the inductance. On inspection of the above formula we see that the farther apart the wires are the greater the inductance of each single line wire.

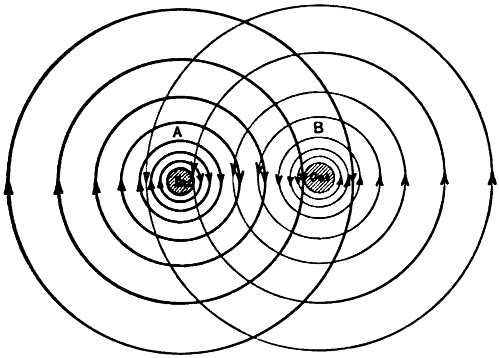


FIG. 173. That part of the magnetic field of the wire *A* which extends beyond the wire *B* weakens the field there, because it opposes the direction of the field around *B*. Also that part of the field of *B* which overlaps *A* weakens the field at that point.

This is in accord with the reason for the usual method of winding a non-inductive coil. The method consists of winding the circuit so that the outgoing and return wires are laid very close to each other. It is done by winding the wire in such a way that the current goes through half the turns of the coil in one direction and then reverses and goes through the other half of the turns, which are laid as closely as possible to the first half. The field around one wire being in the direction opposite to the field around the one lying next to it, is almost completely neutralized.

For the same reason, we never lay in an iron conduit a single wire carrying an alternating current, but always install both of the line wires of a single-phase transmission system in the same conduit. The current in one wire always being in the direction opposite to the current in the other, the magnetic fields around them neutralize each other in spite of the nearness of the iron and the consequent large value for μ. Similarly, all the line wires in any polyphase system must be put into the same iron conduit, whenever iron conduits or protective sheathings are used.

53. Computation of Reactance. It has been shown that some a-c. circuits contain so little resistance that practically the only opposition which the impressed e.m.f. has to overcome when producing a current is the counter e.m.f. set up by the inductance of the circuit. When such a circuit exists, it must be true that this counter e.m.f. set up is practically equal to the impressed e.m.f., since the impressed e.m.f. continues to send greater and greater current through the circuit (there being practically no resistance to limit it), until the counter e.m.f. set up by the resulting magnetic disturbance becomes practically equal to the impressed voltage. Then equilibrium is reached; the current ceases to increase and its effective value remains unchanged.

If E represents the effective value of the impressed voltage in a circuit possessing inductance, but zero resistance, the effective value of the induced counter voltage can thus also be represented by E. But we have seen that the average value of this counter e.m.f. equals the inductance times the rate of change of the current, or in symbols

$$\text{Av. } E = L\frac{I_m}{t},$$

where

L = inductance in henrys;
I_m = maximum value of current, in amperes;
t = time required to change from 0 to I_m amperes, (one-quarter period).

The average rate of change of an alternating current depends upon the maximum value (I_m) which the current attains and the time (t) required to attain it. By reference to the current curve, Fig. 174, we see that the current makes the change from zero to maximum, or vice versa, four times during each cycle. During the part of the curve marked:

(1) it rises from zero to maximum positive value I_m;
(2) it falls from maximum to zero;
(3) it rises from zero to maximum negative value $-I_m$;
(4) it falls from negative maximum value to zero again.

Accordingly, if

f = the number of cycles per second,

then $4f$ = number of changes of current per second between zero and I_m.

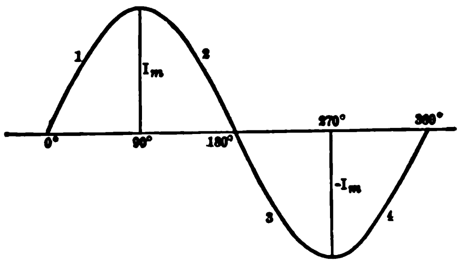


FIG. 174. The sine-wave of current. Note that it changes between the values of O and I_m four times during one cycle.

Therefore, if the current changes $4f$ times each second between the values 0 and I_m amperes, it must change at an average rate of $\frac{I_m}{\frac{1}{4f}}$, or $4fI_m$ amperes per second. That is, if the frequency is 60 cycles per second, and the maximum current 5 amperes, the current makes a 5-ampere change (4×60), or 240 times a second, which is at the same rate as 1200 amperes once every second. Here $4fI_m$ per second $= 4 \times 60 \times 5 = 1200$ amp. per sec.

The average voltage induced by the change of current equals the inductance times this average rate of change.

Thus $$\text{Av. } E = L \times 4fI_m.$$

But if the voltage curve has the sine form *

$$E_m = 1.57 \times \text{av. } E.$$

* For proof that this curve is a sine-wave see Appendix C.

Therefore,

$$\begin{aligned} E_m &= 1.57\,(L \times 4\,fI_m) \\ &= 6.28\,fLI_m \\ &= 2\,\pi fLI_m. \end{aligned}$$

The inductive reactance of a circuit has been defined as the ratio of the volts applied, to the amperes produced, in a circuit where the opposition to the current is due to inductance only (that is, no resistance and no condenser effects). Thus

$$X = \frac{E_{\text{effective}}}{I_{\text{effective}}} = \frac{E_m/\sqrt{2}}{I_m/\sqrt{2}} = \frac{E_m}{I_m},$$

when e.m.f. and current both vary according to sine law. But the applied e.m.f. is exactly equaled by the counter e.m.f. set up in the circuit which equals $2\,\pi fLI_m$; therefore,

$$X = \frac{2\,\pi fLI_m}{I_m},$$

or

$$X = 2\,\pi fL,$$

where

X = inductive reactance, in ohms;
f = frequency, in cycles per sec;
L = inductance, in henrys.

This is the regular equation for inductive reactance. In fact, many prefer to represent the value of inductance by the expression $(2\,\pi fL)$ rather than the symbol X.

Example 4. One of the smaller sizes of reactance coils similar to Fig. 170 has an inductance of 0.13 henry and negligible resistance. What current flows when a 25-cycle generator maintaining 11,000 volts between its terminals is short-circuited through this coil?

$$\begin{aligned} X &= 2\,\pi fL \\ &= 2 \times 3.14 \times 25 \times 0.13 \\ &= 20.4 \text{ ohms.} \\ I &= \frac{E}{X} \\ &= \frac{11{,}000}{20.4} \\ &= 539 \text{ amperes.} \end{aligned}$$

The effect of increasing the frequency, with its accompanying increased rate of change of current, upon the reactance of a circuit, is strikingly illustrated in the use of "choke coils," such as shown in Fig. 175a and b, to protect the generating station or receiving circuit from the effects of line disturbances caused by lightning. (See Chapter IV, Second Course.) The passage of a lightning discharge from cloud to cloud or from clouds to earth causes a quantity of electricity to flow or surge rapidly from one part of the circuit to another, producing very large currents of short duration and setting up rapid oscillations, the frequency of which may run into the millions. Coils *C*, like those shown in Fig. 175a and b, are placed in series between line and generators or between line and load, and the line connected to earth through an air gap *G* near the coil, as shown in Fig. 175c. When the current attempts to rise rapidly through coils *C*, a proportionally high counter e.m.f. is induced in them by their self-inductance, and the surging current finds it easier to jump to earth across the air-gaps *G*, than to pass

FIG. 175a. General Electric choke coil for use on a 6600-volt line.

FIG. 175b. General Electric hour-glass choke coil, 35,000 volts.

through *C* and dissipate its energy destructively upon the machinery. Thus, a coil having an inductance of 0.02 henry may produce a sufficient counter e.m.f. to discharge the line to earth through an air gap which it requires 10,000 volts to break down. If the "wave front" of the current had been of a steepness representing a rise of one ampere per second, 0.02 volt would be induced; but since 10,000 volts are actually induced, the rate of increase of current must have been $\frac{10,000}{0.02}$, or 500,000 amperes per second. The reactance of such a coil at an ordinary commercial frequency of 60 cycles per second would be $2\pi fL = 6.28 \times 60 \times 0.02 = 7.54$ ohms, so that if the normal line current and pressure are 10 amperes and 13,000 volts, respectively, the normal drop of 10×7.54, or 75.4 volts in this reactance does not seriously affect the operation of the line.

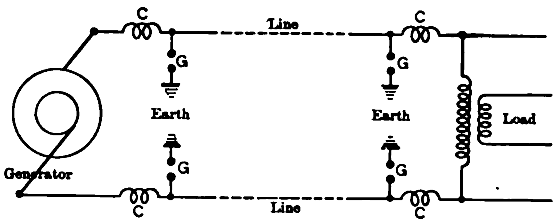


FIG. 175c. The choke coils *C* are placed in the line to choke back any sudden rise in current and cause any excess charge of electricity to discharge across the air-gaps *G* to the ground.

Prob. 17-5. The generator of Example 4 is 3-phase, of 720 kv-a. rated capacity. How many times the normal current through each coil is the short-circuit current?

Prob. 18-5. If the frequency of the generator in Example 4 had been 60 cycles, what current would have flowed through the coil?

Prob. 19-5. How many times the normal current would the short-circuit current through the coil be if the generator of Prob. 18 were rated as a 3-phase 720 kv-a.?

Prob. 20-5. What would be the reactance of the coil in Fig. 163 when on a 60-cycle circuit?

Prob. 21-5. (*a*) What reactance would the coil of Fig. 164 have on a 60-cycle circuit?

(*b*) On a 25-cycle circuit?

Prob. 22-5. What reactance would the wire in the coil of Fig. 164 have, if used as a single-phase "line and return" strung 30 inches apart, and operated at 60 cycles?

Prob. 23-5. What current will 110 volts at 60 cycles force through the coil in Fig. 163? Neglect the resistance.

Prob. 24-5. What current will 110 volts at 60 cycles force through coil in Fig. 164? Neglect the resistance.

Prob. 25-5. If 110 volts, 25 cycles were used across the coil of Fig. 164, what current would be forced through it? Neglect the resistance.

54. Phase Difference between Current and E.M.F. in an Inductive Circuit. Having taken up the method of computing reactance and the equation by which we may find the current in a circuit containing reactance only, we must now consider what phase relation this current bears to the voltage producing it.

We have already seen that the current must lag behind the voltage, because we learned that inductance tends to choke back the current and to hinder it from reaching its maximum value until some time has elapsed after the full or maximum voltage was impressed on a circuit. Fig. 167 showed that in one particular case, when a direct voltage was impressed on such a circuit, the current took 0.9 second to come up to its full value. Similarly, on breaking a circuit, instead of the current ceasing instantly, we have seen that it persists to such an extent that it even flashes across a long air gap. It is thus late in growing to full value and late in dying to zero. Similarly, in an alternating-current circuit, where the voltage is rapidly reversing its direction, the current changes always lag behind the changes of impressed voltage which produce them.

Let us now see how many electrical degrees this lag of current may amount to. We know that the induced counter-voltage in such a circuit depends upon the rate of change of the current (amperes per second), which causes the magnetic field to cut the turns of the coil. When the current is steady, as in a direct-current circuit, there is no counter e.m.f. and the resistance alone keeps the current down. In an alternating-current circuit there are two instants when the current has an unchanging value; one, when it has reached its maximum positive value and the other when it has reached its maximum negative value. In Fig. 176 these points are marked respectively $+I$ and $-I$ on the current curve. At these two instants of steady current, the induced voltage must be zero. Note that the induced voltage, shown by dash line, has zero value at these two instants when the current curve is at its maximum or steady values.

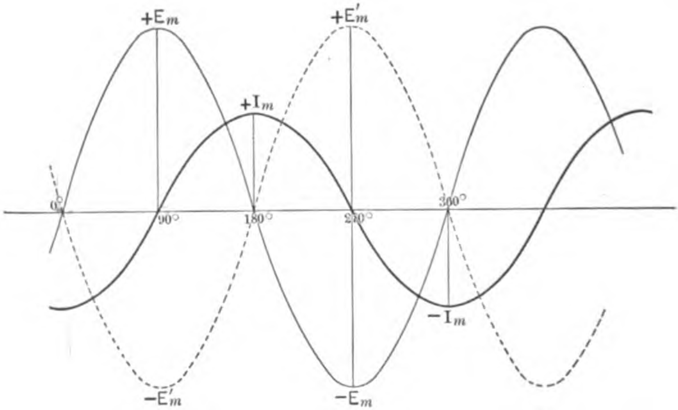


FIG. 176. Curve $+ E_m - E_m$ represents the voltage impressed across a coil possessing inductance only. Curve $- E'_m + E'_m$ represents the counter e.m.f. set up in this coil by the flow of the alternating current $+ I_m - I_m$. Note that the current curve lags 90° behind the impressed voltage curve.

The maximum value of the current, of course, is reached 90 electrical degrees before or after its zero value. Therefore, if the zero value of the induced voltage comes when the current is at its maximum, it must come 90° before or after the zero value of the current. Similarly, the maximum value of the induced voltage must take place at the instant when the current is changing most rapidly. By inspecting again the current curve I of Fig. 176, we see that the curve is steepest and is, therefore, changing in value at the greatest rate just as it is changing from a positive to a negative value or vice versa; that is, as it is passing in either direction through its zero value. The induced voltage, then, has its two maximum values at the two instants when the current is passing through its two zero values. Here again we see that the induced voltage differs in phase with the current by 90°.

Referring to Fig. 176, we see that it is possible to draw two curves, E' (dotted) and E (full line), both of which have the proper **mathematical** phase relation to the current curve to enable them to represent the induced counter voltage; that is, both are 90° out of phase with the curve I. The circuit has negligibly small resistance, therefore, the counter e.m.f. due to self-inductance is the only opposition encountered by the current. Hence the impressed voltage, which produces the current, must be at every instant equal in value and opposite in direction to the induced e.m.f. If curve E' represents induced voltage due to self-induction, curve E must represent the applied voltage, and vice versa. But we have already seen that the **current lags behind the voltage which is applied** to the circuit to produce it; therefore, curve E must represent the applied voltage, since it is the one that leads the current; and curve E' consequently represents the counter e.m.f. of self-induction. If we follow through a cycle of changes, we readily see that E' really opposes the change of I at every instant; while I is in positive direction, E' is in the opposite or negative direction; while I is decreasing in value, E' is increasing in value, and so forth. In most electrical problems we are interested principally in the applied voltage E and the current I which it produces in the circuit, and the counter e.m.f. E', though always present, is seldom represented or considered in our curves and vector diagrams.

Fig. 176 shows the impressed voltage curve E opposite, or at an angle of 180°, to the induced voltage curve E'. Fig. 176a shows the vector E of the impressed voltage drawn in the opposite direction, or at 180° to E', the vector of the induced voltage. It shows also the current vector I 90° out of phase with E' and E, and drawn so that it lags behind E; it therefore leads E by 90°. Here it is very plain that the induced e.m.f. is called a counter e.m.f. because it acts in a direction opposite to the impressed e.m.f. which produces the current. We usually say that the current lags 90° in a circuit containing inductance only, meaning that that is its phase relation to the e.m.f. impressed upon the circuit.

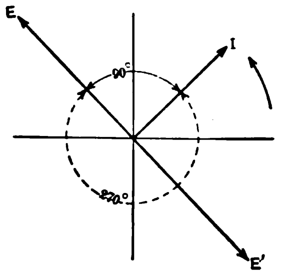


FIG. 176a. The vector diagram for the conditions of Fig. 176. Note that E' is opposite in direction to E and that I lags 90° behind E, just as in Fig. 176.

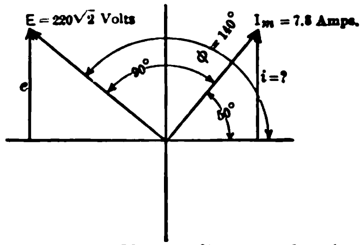


FIG. 177. Vector diagram showing I_m lagging 90° behind E which is at its 140° position.

Example 5. In an alternating-current circuit containing 40 ohms inductive reactance (no resistance or capacity), the effective voltage is 220 volts.

(*a*) What is the effective current?

(*b*) What is the maximum current?

(*c*) What is the current 140° after the voltage passes its zero value?

Solution. Effective current $= \dfrac{\text{effective voltage}}{\text{reactance}}$,

or
$$I = \frac{E}{X_L}.$$

(*a*)
$$I = \frac{220}{40} = 5.5 \text{ amperes} = \text{effective current}$$

and
$$I = 0.707\, I_m,$$
$$I_m = \frac{I}{0.707}$$

(*b*)
$$= \frac{5.5}{0.707} = 7.8 \text{ amperes.}$$

Maximum current = 7.8 amperes.

Draw vector as in Fig. 177, when e.m.f. is at 140° phase; then, by the equation,

$$\begin{aligned} i &= I_m \sin (140°-90°) \\ &= 7.8 \sin (140°-90°) \\ &= 7.8 \sin 50° \\ &= 5.97 \text{ amperes.} \end{aligned}$$

Prob. 26-5. The average voltage in an alternating-current circuit, containing 25 ohms inductive reactance only, is 500 volts. Find:

(*a*) Effective current.

(*b*) Instantaneous value of voltage when current is +15 amperes and increasing.

Prob. 27-5. The maximum value of the current in an alternating-current circuit containing inductance only is 52 amperes. The average value of the voltage is 220 volts. What is the inductive reactance?

Prob. 28-5. In Prob. 27, what will be the instantaneous value of the e.m.f. when the current is + 12 amperes and increasing?

Prob. 29-5. What effective voltage is required to force a current, whose maximum value is 20 amperes, through 18 ohms of inductive reactance?

Prob. 30-5. What instantaneous values may the voltage of Prob. 29 have when the current has a value of 8 amperes?

Prob. 31-5. In an alternating-current circuit containing inductive reactance only, the voltage is 1100 and current 45 amperes. What is the instantaneous value of the current when the voltage is −300 volts and decreasing?

55. Combined Resistance and Inductive Reactance. In the circuits which we have considered thus far, the impedance offered to the flow of an alternating current consisted entirely of reactance, the resistance in each case being too small to be taken into account.

But aside from such apparatus as current-limiting reactance coils and choke coils for use with lightning arresters, the circuits of most electrical devices offer an impedance which is made up of reactance and an appreciable amount of resistance. The impedance of such a circuit is thus a combination of resistance and reactance.

Let us assume a series circuit as in Fig. 178, in which R represents a resistance of 3 ohms, and X, an inductive reactance of 4 ohms. (Note the coil, used to represent an

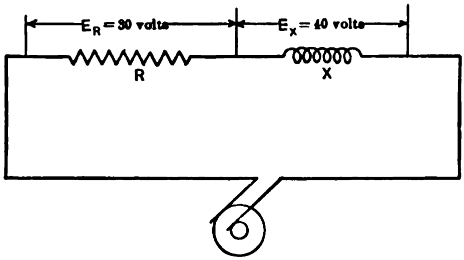


FIG. 178. Diagram of a circuit containing a series combination of reactance X and resistance R.

inductive reactance in order to distinguish it from the zigzag line representing resistance.) Let us assume 10 amperes to flow in the circuit. The voltage across the reactance X

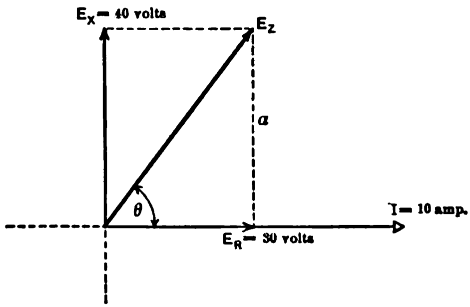


FIG. 179. Vector diagram of the voltage relations in the circuit of Fig. 178. Note that E_x, the voltage across the reactance leads E_r, the voltage across the resistance by 90°.

equals 4×10, or 40 volts; across R the voltage equals 3×10, or 30 volts. But the voltage across an inductive reactance is always 90° ahead of the current. Thus, in Fig. 179, the vector E_X, which represents the 40 volts across the reactance, is drawn 90° ahead of the vector I, which represents the current through the circuit.

The voltage across the resistance is in phase with the current. In any circuit, or any part of a circuit, which produces appreciably no magnetic or electric field-of-force around itself, when current flows, but possesses only resistance which causes all of the power supplied to it to be changed into heat and none of it to be stored up or returned to other parts of the circuit, the current is at every instant exactly proportional to the instantaneous value of e.m.f. across that part of the circuit. Therefore, the current reaches its maximum value at the same instant the e.m.f. reaches its maximum value in the same direction, and the current is zero at the same instant the e.m.f. is zero. That is, the e.m.f. across a non-inductive circuit or part of a circuit, which has resistance only, is exactly in phase with the current through that part of the circuit. Accordingly, the vector E_R, representing the 30 volts across the resistance, is drawn along the current line I to show that it is in phase with the current. The resultant voltage across these two in series would then be E_Z, the diagonal of a parallelogram drawn on E_R and E_X as adjacent sides.

The value of the resultant voltage E_Z can be found from the following equation, since this diagonal is also the hypothenuse of a right-angled triangle.

$$\begin{aligned} E_Z &= \sqrt{E^2{}_X + E^2{}_R} \\ &= \sqrt{40^2 + 30^2} \\ &= \sqrt{2500} \\ &= 50 \text{ volts.} \end{aligned}$$

Knowing the pressure, 50 volts, across the series combination and the current through the series combination, we can now find the impedance.

Impedance (of R and X combined)

$$= \frac{\text{volts (across combination)}}{\text{current (through combination)}},$$

or

$$\begin{aligned} Z &= \frac{E_Z}{I} \\ &= \frac{50}{10} \\ &= 5 \text{ ohms.} \end{aligned}$$

The resulting impedance of a resistance of 3 ohms and an inductive reactance of 4 ohms is thus seen to be but 5 ohms, which is neither the arithmetical sum of nor the difference between the resistance and the reactance.

The impedance of a series combination is not usually found by the above method, although the procedure used is really based on it. Instead of drawing a vector diagram of the voltages, it is customary to draw **a so-called vector diagram of the resistance, reactance and impedance as** in Fig. 180.

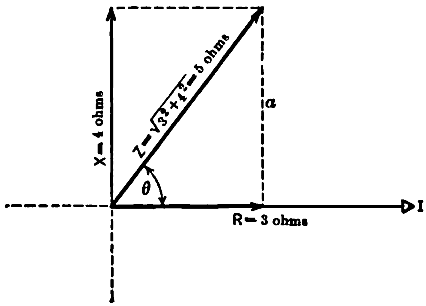


Fig. 180. Vector diagram of the resistance, reactance and impedance of the circuit of Fig. 178.

First, draw a vector *I* representing the current. The resistance *R* is then drawn in phase with the current. The reactance *X* is drawn leading the current by 90°. The impedance *Z* is then the diagonal of a parallelogram of which *X* and *R* are adjacent sides. Of course, strictly speaking, a vector diagram of resistance, reactance and impedance cannot be drawn, because resistance, reactance and impedance have really no direction, such as current and voltage have. It makes no difference which ends of the resistance and of the reactance coils are connected together, the impedance of the combination is the same. We have seen, however, what a difference is made in the resulting e.m.f. and the current by the reversal of the connections between coils carrying alternating e.m.f.'s or currents. By reversing the connections of a coil, the e.m.f. is made to act in the opposite direction, and its vector is turned through 180°. No such effect is produced in the resistance, reactance or impedance of a coil by merely reversing its connection.

The vector diagram of these quantities, however, is usually explained and justified as follows:

The resistance vector R in Fig. 180 is really the voltage vector E_R of Fig. 179, **divided by the current** (30 volts ÷ 10 amp. = 3 ohms); and the reactance vector X is really the voltage vector E_X of Fig. 179 **divided by the current** (40 volts ÷ 10 amp. = 4 ohms). The resultant impedance vector Z of Fig. 180 is the resulting voltage vector E_Z of Fig. 179, **divided by the current** (50 volts ÷ 10 amp. = 5 ohms).

The vector diagram in Fig. 180 of resistance, reactance, and impedance is, then, really a condensed vector diagram of e.m.f.'s which have been divided by a common factor, the current.

From the diagram in Fig. 180, it is seen that

The square of the impedance equals the sum of the squares of the reactance and the resistance.

The equation for impedance is written

$$Z = \sqrt{R^2 + X^2}.$$

Note also from both Fig. 179 and 180 that the angle $\theta°$ is the angle between the resultant or total voltage and the current, since the current must be in phase with E_R. The value of θ can be found as follows:

In Fig. 179, the side (a) of the parallelogram must equal E_X. Observe that

$$\tan\theta = \frac{a}{E_R} = \frac{E_X}{E_R}$$

$$= \tfrac{40}{30}$$

$$= 1.33,$$

$$\theta = 53°.$$

In Fig. 180, the side (a) of the parallelogram equals the side X. Observe that

$$\tan\theta = \frac{a}{R} = \frac{X}{R}$$
$$= \tfrac{4}{3} = 1.33,$$
$$\theta = 53°.$$

Observe that the angle of phase difference between the voltage and current in a series combination can be found from either of the two equations

$$\tan\theta = \frac{E_X}{E_R},$$

or

$$\tan\theta = \frac{X}{R}.$$

The resistance and reactance of a coil are, of course, contained in the same wire, but since the same current is sent through the coil against each, the resistance and reactance of such a coil are treated as being in series. This is in accord with the definition of a series circuit as one in which the same current flows in all parts.

FIG. 181. Part of an electric circuit possessing a resistance of 52 ohms and a reactance of 20 ohms.

In solving problems on impedance, it is always best to draw a diagram of the electrical connections and the vector diagram. Sine curves are also often a great help.

Example 6. What is the impedance of a coil having 52 ohms resistance and 20 ohms inductive reactance, when used in a 60-cycle circuit?

Draw the diagram of electrical connections as in Fig. 181. Construct the vector diagram as in Fig. 182. From vector diagram, write the equation

$$Z = \sqrt{X^2 + R^2}$$
$$= \sqrt{20^2 + 52^2}$$
$$= \sqrt{3104}$$
$$= 55.7 \text{ ohms}.$$

Example 7. What will be the phase difference between the current and the voltage across the coil of Example 6?

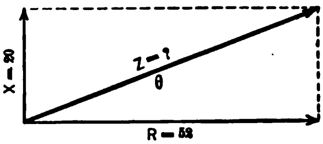


FIG. 182. Vector diagram of the resistance, reactance and impedance of the part of an electric circuit shown in Fig. 181.

The vector of resultant voltage across the coil must lie along the line Z, representing the total impedance in Fig. 182. Similarly, the vector representing the component of voltage used to overcome resistance must lie along the line R and be in phase with the current through the coil. Thus θ must represent the angle of phase difference between the voltage and current, whatever the value of these may be.

$$\tan \theta = \tfrac{20}{52} = 0.385,$$
$$\theta = 21°.$$

Prob. 32–5. What is the impedance of the coil in Fig. 163 when on a 60-cycle circuit?

Prob. 33–5. What current must flow when the coil in Fig. 163 is placed across 11 volts, 60 cycles?

Prob. 34–5. If the coil in Fig. 163 were placed across 11 volts, 120 cycles, what current would flow through it?

Prob. 35–5. What would be the impedance of a "line and return" composed of the wire of coil in Fig. 163, strung 30 inches apart, if a 60-cycle frequency were used?

Prob. 36–5. How much current would flow through a coil of 0.08 henry inductance and 20 ohms resistance when 220 volts at 60 cycles is applied across its terminals?

Prob. 37–5. What would be the phase relation between the current and voltage in the coil of Prob. 36?

56. Power Consumed by Impedance. Since we can find the value of the angle θ between the current and voltage across an impedance, we can also find the power-factor of the appliance. The power-factor, it will be remembered, is the cosine of the angle θ. Thus, in Fig. 179 and 180,

$$\tan \theta = \frac{E_X}{E_R}, \quad \text{or} \quad \frac{X}{R},$$

$$\cos \theta = \frac{E_R}{E_Z}, \quad \text{or} \quad \frac{R}{Z}.$$

But
$$E_Z = \sqrt{E_X^2 + E_R^2}$$
and
$$Z = \sqrt{X^2 + R^2}.$$
Thus
$$\cos\theta = \frac{E_R}{\sqrt{E_X^2 + E_R^2}},$$
or
$$\cos\theta = \frac{R}{\sqrt{R^2 + X^2}}.$$

Example 8. Find the power consumed by a coil having 15 ohms resistance and 20 ohms reactance on a 60-cycle circuit of 110 volts.

Construct connection diagram as in Fig. 183.

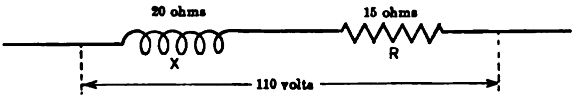


FIG. 183. Diagram of a circuit possessing reactance and resistance.

Draw impedance diagram as per Fig. 184.

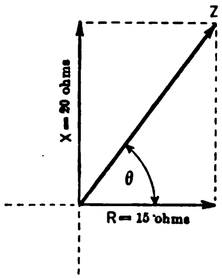


FIG. 184. Vector diagram of the circuit of Fig. 183, possessing reactance and resistance.

$$Z = \sqrt{15^2 + 20^2}$$
$$= 25 \text{ ohms}.$$

The current can then be found by the equation,

$$I = \frac{E}{Z}$$
$$= \tfrac{110}{25}$$
$$= 4.4 \text{ amp.}$$
$$P = IE\cos\theta.$$
$$\cos\theta = \frac{R}{Z}.$$
$$P = 4.4 \times 110 \times \tfrac{15}{25}$$
$$= 290 \text{ watts.}$$

Note that no power is consumed in forcing this current against the reactance of the coil, since that part of the voltage used to overcome the reactance is at 90° to the current. This is further shown by finding the power used in forcing the current through the resistance, and noting that this is all the power consumed by the coil.

$$P = I^2R$$
$$= 4.4^2 \times 15$$
$$= 290 \text{ watts.}$$

Note that the power consumed by the resistance as computed in this way is the total power consumed by the coil as computed by the equation for the coil,

$$P = IE \cos \theta.$$

Prob. 38–5. What power is consumed in a circuit containing 18 ohms impedance? Voltage = 110 volts. Power-factor = 90 per cent.

Prob. 39–5. How many ohms reactance and how many ohms resistance are there in the circuit of Prob. 38–5?

Prob. 40–5. What is the power-factor in a circuit which contains 20 ohms resistance and 14 ohms reactance?

Prob. 41–5. What power is consumed in circuit in Prob. 40, if the voltage is 220 volts?

Prob. 42–5. How many amperes reactive component of current are there in Prob. 41?

Prob. 43–5. A 110-volt 60-cycle circuit contains 20 ohms resistance, only.

(*a*) What current flows?

(*b*) What power is consumed?

Prob. 44–5. How much would the current in Prob. 43 be reduced to, if a coil of negligible resistance and 0.2 henry inductance were placed in series with the 20 ohm resistance?

Prob. 45–5. What power would be consumed by the circuit in Prob. 44?

Prob. 46–5. A coil of 0.015 henry inductance and 4 ohms resistance is connected across 40-cycle 110-volt mains.

(*a*) What is the power-factor?

(*b*) What power is consumed by coil?

57. Impedance in Series. The **resistance of a series combination** equals the arithmetical sum of the resistances of the parts.

The **inductive reactance of a series combination** of inductive reactances equals the sum of the inductive reactances of the separate parts.

This is seen to be true if we consider that the same current would flow through the two when in series and that both resistance drops (or voltages required to overcome resistance) would be in phase with the current and, therefore, in phase with each other, and that both reactance drops lead the current by 90° and are, therefore, in phase with each other and 90° ahead of the resistance drops.

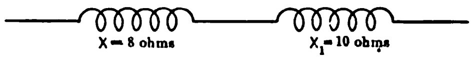


FIG. 185. The two reactances X and X_1 are in series.

In constructing the impedance diagram, therefore, we would draw the two reactances on a line at right angles to the current vector. The resultant of these reactances would be merely the arithmetical sum, inasmuch as they lie along the same line in the same direction. Thus, the resultant of

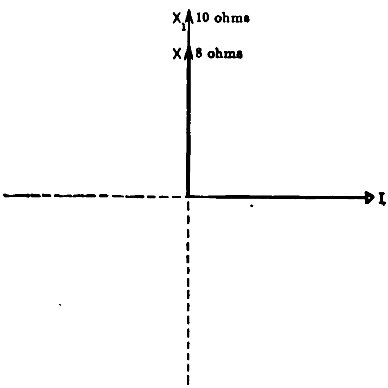


FIG. 186. Vector diagram of the reactances X and X_1 shown in Fig. 185. Note that they both lie along the same line, 90° ahead of the current vector I.

the two reactances 8 ohms and 10 ohms, joined as in Fig. 185, equals the sum of the vectors X and X_1, or 18 ohms, in Fig. 186. Both vectors are drawn at right angles to the current vector I, and leading it.

The **impedance** of a series combination of **resistances** and **inductive reactances** equals the vector sum of the resistances and reactances of the separate parts. This has been shown in the previous pages.

The **impedance** of a series combination of impedances, each composed of resistance and inductive reactance, equals the vector sum of the separate resistances and inductive reactances which combine to make up the impedances. This is self-evident from the previous discussions.

Therefore, in order **to find the impedance of such a combination we must know the reactance and resistance,** or what is equivalent, the power-factor and impedance, **of each component part.** It is impossible to find the impedance of a circuit which contains, say, 8 ohms impedance (Z) in series with 10 ohms impedance (Z_1), unless we know how much resistance or reactance each of the impedances contain, in order to be able to obtain the proper phase relations of one to the other. But if we know that the 8-ohm impedance has a power-factor of 87 per cent and the 10-ohm impedance has a power-factor of 64 per cent, we know that the angle θ, Fig. 187, between Z, the impedance 8 ohms, and R the resistance part of this impedance must equal 30°, since 0.87 must equal cos θ and 0.87 is the cosine of 30°. From the diagram, Fig. 187,

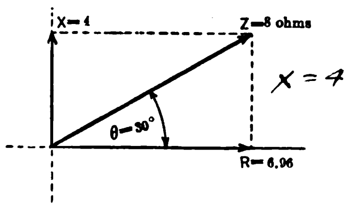


FIG. 187. The impedance Z is made up of R and X. The angle between Z and R must be 30°, because cos 30° = 0.87.

$$\begin{aligned} R &= Z \cos \theta \\ &= 8 \times 0.87 \\ &= 6.96 \text{ ohms}; \\ X &= Z \sin \theta \\ &= 8 \times 0.500 \\ &= 4 \text{ ohms}. \end{aligned}$$

The impedance Z, 8 ohms, is thus made up of 4 ohms reactance and 6.94 ohms resistance. Similarly, by Fig. 188, we see that the 10-ohm impedance Z_1 is made up of R_1 at

50° to Z_1 (since 0.64 = cosine of 50°) and of X_1 at right angles to R_1. From Fig. 188

$$\begin{aligned} R_1 &= Z_1 \cos 50° \\ &= 10 \times 0.64 \\ &= 6.4 \text{ ohms;} \\ X_1 &= Z_1 \sin 50° \\ &= 10 \times 0.77 \\ &= 7.7 \text{ ohms.} \end{aligned}$$

The resistance part of the resultant impedance, then, equals 6.4 + 6.96, or 13.36 ohms, and is represented by the line R_2 in Fig. 189. The reactance part of the resultant equals

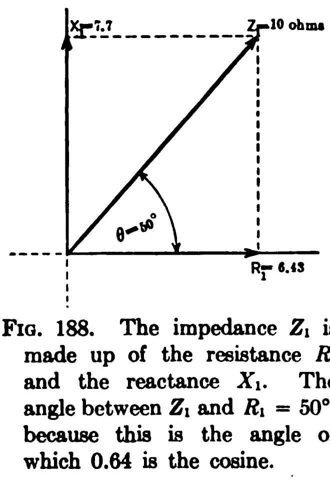


FIG. 188. The impedance Z_1 is made up of the resistance R_1 and the reactance X_1. The angle between Z_1 and R_1 = 50°, because this is the angle of which 0.64 is the cosine.

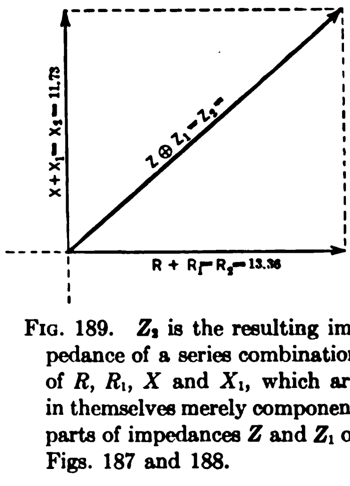


FIG. 189. Z_2 is the resulting impedance of a series combination of R, R_1, X and X_1, which are in themselves merely component parts of impedances Z and Z_1 of Figs. 187 and 188.

4 + 7.7, or 11.7 ohms, and is represented by the line X_2, drawn at 90° to the resistance line R_2. Z_2, the resulting impedance, equals the diagonal of the parallelogram drawn on these two sides.

$$\begin{aligned} Z_2 &= \sqrt{R_2^2 + X_2^2} \\ &= \sqrt{178 + 137} \\ &= 17.7 \text{ ohms.} \end{aligned}$$

The resultant impedance of the two impedances 8 and 10 ohms, respectively, joined in series is therefore 17.7 ohms.

The same result can be obtained by constructing a single diagram as in Fig. 190, drawing Z at 30° to the current

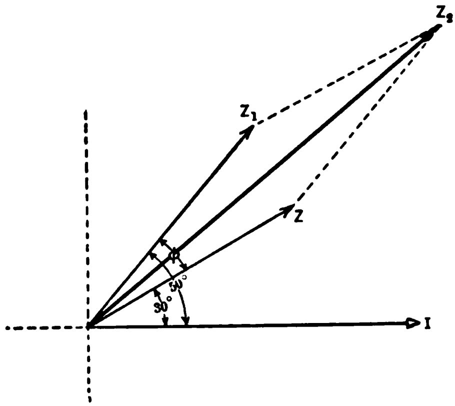


FIG. 190. Z_2 is the resultant impedance of the series combination of impedances Z and Z_1. This figure is equivalent to Fig. 189.

vector, and Z_1 at 50° to the same vector. The angle between Z and $Z_1 = 50° - 30° = 20°$. The value of Z_2, the resultant, can be found from the equation for the diagonal of a parallelogram.

$$\begin{aligned} Z_2 &= \sqrt{Z^2 + Z_1^2 + 2\,ZZ_1 \cos\phi} \\ &= \sqrt{64 + 100 + 2 \times 8 \times 10 \times 0.94} \\ &= 17.7 \text{ ohms.} \end{aligned}$$

Prob. 47-5. In testing the impedance of the different phases of a three-phase automatic starter, it was found that the first phase had an impedance of 20 ohms, the second, 15 ohms, and the third, 18 ohms. If the power-factor of each phase is the same, what would be the impedance of a series combination of the first and second phases?

Prob. 48-5. If the power-factor of the first phase of the starter in Prob. 47 were 0.80 and the power-factor of the third phase were 0.85, what would be the impedance of the series combination of Prob. 47?

Prob. 49–5. If the power-factors of the phases of Prob. 48 were unknown, but the resistance of the first and the third were found to be 16 ohms each, what would be the impedance of a series combination of the first and the third?

58. Impedance of Parallel Combinations. When any number of appliances, containing resistance, reactance or impedance are in parallel:

First: Find the **amperes per volt** through each branch.

Second: Add these currents, paying attention to their **phase relation** to one another. This gives the current through the combination for 1 volt e.m.f. across the combination.

Third: Divide the 1 volt e.m.f. across the combination by the current per volt through the combination. The result is the impedance of the combination, regardless of how many or of what sort the branches are.

Example 9. Find the impedance of the parallel combination of Fig. 191.

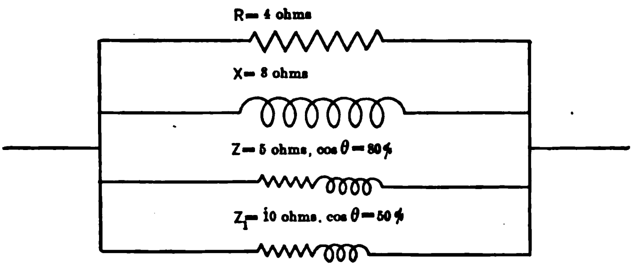


FIG. 191. A parallel combination of impedances, resistances and reactances.

First:

Branch	Amp. per volt	Angle between current and voltage
$R = 4$ ohms	0.25 amp.	In phase
$X = 8$ "	0.125 "	90° lag
$Z = 5$ "	0.20 "	37° " (0.80 = cos 37°)
$Z_1 = 10$ "	0.10 "	60° " (0.50 = cos 60°)

Second: Construct vector diagram, Fig. 192, to find sum of currents per volt through the combination.

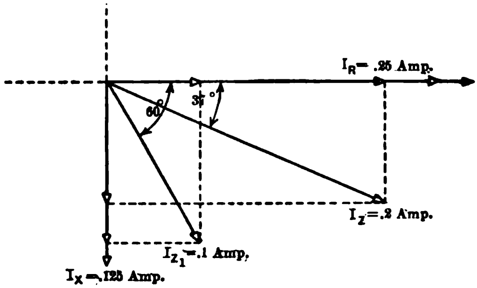


FIG. 192. The vector diagram for the currents which one volt alternating e.m.f. would send through the several parallel paths of Fig. 191.

Component of 0.25 amp. current **in phase** with voltage
$= 0.25 \times \cos 0° = 0.25 \times 1 = 0.25$
Component of 0.125 amp. current **in phase** with voltage
$= 0.125 \times \cos 90° = 0.125 \times 0 = 0.00$
Component of 0.20 amp. current **in phase** with voltage
$= 0.20 \times \cos 37° = 0.20 \times 0.80 = 0.16$
Component of 0.10 amp. current **in phase** with voltage
$= 0.10 \times \cos 60° = 0.10 \times 0.50 = 0.05$
Total current in phase with voltage = 0.46 amp.

Component of 0.25 amp. current **lagging 90°** behind voltage
$= 0.25 \times \sin 0° = 0.25 \times 0 = 0.000$
Component of 0.125 amp. current **lagging 90°** behind voltage
$= 0.125 \times \sin 90° = 0.125 \times 1 = 0.125$
Component of 0.20 amp. current **lagging 90°** behind voltage
$= 0.20 \times \sin 37° = 0.20 \times 0.6 = 0.120$
Component of 0.10 amp. current **lagging 90°** behind voltage
$= 0.10 \times \sin 60° = 0.10 \times 0.866 = 0.087$
Total current lagging 90° behind voltage = 0.332

Construct Fig. 193 drawing vector for 0.332 amp. 90° behind vector for 0.46 amp. and find resultant.

$$I = \sqrt{0.332^2 + 0.46^2}$$
$$= \sqrt{0.3225}$$
$$= 0.568 \text{ amp.}$$

The current through the combination equals 0.57 **ampere per volt.**

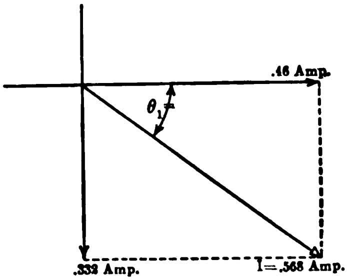


FIG. 193. Vector diagram for the resulting current which 1 volt would force through the parallel combination of Fig. 192.

Third. The impedance of the combination equals the ratio of the voltage across the combination to the current through the combination.

$$Z = \frac{E}{I}$$

$$= \frac{1}{0.57}$$

$$= 1.75 \text{ ohms.}$$

The angle of lag between resulting current and voltage is found as follows:

$$\tan \theta = \frac{0.332}{0.46} = 0.722,$$

$$\theta = 35° \, 50'.$$

The power-factor of the combination can be computed as follows:

$$\cos \theta = \frac{0.46}{0.568} = 0.81,$$

$$\text{Power-factor} = 81 \text{ per cent.}$$

By this method the resulting impedance, power-factor, etc., of all parallel combinations can be found.

Prob. 50–5. An arc lamp containing 11 ohms resistance and 9 ohms reactance at 60 cycles is placed in parallel with a choke coil having 0.03 henry inductance and 6 ohms resistance across a 60-cycle circuit. What is the impedance of the combination?

Prob. 51–5. If the parallel combination of Prob. 50 were placed across a 25-cycle circuit, what would be the impedance of the combination?

Prob. 52–5. What is the power-factor of the combination of Prob. 50?

Prob. 53–5. What is the angle of phase difference between the current and voltage of the combination in Prob. 51?

Prob. 54–5. An induction coil of 20 ohms impedance and 75 per cent power-factor is placed in parallel with a choke coil of 18 ohms impedance and 3 per cent power-factor. What is the impedance and power-factor of the combination?

59. Mutual Inductance. It has been shown that when a current through a circuit changes in value, the accompanying change in the magnetic field causes the wire of the circuit to be cut by its own lines of force. This induces a counter e.m.f. in the circuit which opposes any change in the current. But an e.m.f. is induced not only in the wire of the circuit itself, but also in the wires of every other circuit that may be near enough to be cut by the changing magnetic field. To show this, we will take two coils *A* and *B*, as in Fig. 194. Coil *A* is placed within coil *B*, but has **no electrical connection to it.** Across the terminals of *B* a voltmeter is placed. Now when switch *S* is thrown to power, a current rushes into coil *A* and flows around the coil counterclockwise, as marked. But also, strangely enough, a **momentary** current flows around in coil *B*, through the voltmeter, **only in the opposite direction.** This momentary current in *B* lasts but for an instant, and dies out.

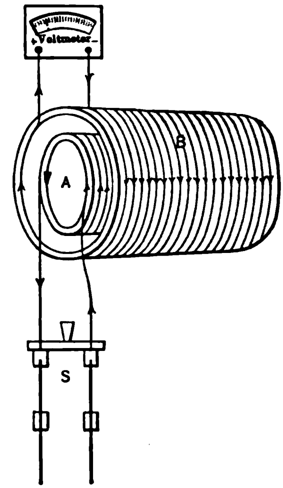


FIG. 194. Coil *B* has no electrical connection to coil *A*, yet when a current is growing in *A*, a reverse current tends to flow in *B*.

If, now, we suddenly open the switch *S*, in order to stop the current in *A*, we notice that another **momentary** current is set up in *B*. This time the **momentary current in B** is in the same direction as the current we are stopping in *A*. This momentary current, like the first one, dies out almost instantly. Thus when we closed the switch and started a current in *A*, we noticed a **momentary** current set up in *B*, opposite in direction to the current we were starting in *A*. On the other hand, while we were stopping the current in *A*, we noticed a momentary current set up in *B*, in the same direction as the one we were stopping. These momentary currents in the coil *B* are due to the e.m.f. induced in the wires of this coil by the changing magnetic field around the wires of coil *A*. Whenever two circuits are so arranged that a change in the current of one produces an induced e.m.f. in the other, the circuits are said to possess the property of **mutual inductance.**

Note that, although in one case the induced current in *B* was in the opposite direction to the current in *A*, and in the other case in the same direction, **in both cases it opposed the change of current taking place** in *A*. It **opposed the setting up** of a current in *A*, when we closed the switch, by setting up a current in the opposite direction. When we opened the switch, it **opposed** the **stopping** of the current in *A*, by setting up one in the same direction as the one we were stopping. In each case as soon as the **change** of current in *A* had ceased, the induced current died out.

Let us also consider the magnetic field set up by these induced currents in *B*. When we sent the current into *A*, we were setting up a field within the coil making the near end a north pole. Note that the induced current in *B* **opposed this setting up** of a north pole by setting up a field with a south pole at this end, tending to neutralize the former. But after the field was once set up, the current in *B* ceased trying to neutralize it and stopped flowing. As soon, however, as we began to destroy the field, by stopping the current in *A*, a current was induced in *B*, which opposed the **dying out of the field,** by setting up a field of its own in the same direction as the one we were destroying. In each case the field of the induced current **opposed the change** that was taking place in the field within the coils.

Here again the induced currents in *B* obey Lenz's law by always opposing the change of the current in the first coil. They do this by tending to set up a magnetic field which shall oppose any change in the magnetic field existing in the first coil.

An induced e.m.f., whether in the circuit in which changes are being made or in adjacent circuits, always has this element of **opposition to change.** This is why inductance is likened to inertia. But it must be remembered that this property lies entirely in the magnetic field of the circuit and not in the current. An electric current offers no opposition to change; in other words, it has no inertia. The entire opposition rests in the inertia of the magnetic field, which acts like a flywheel on the circuit, and opposes any change. It offers this opposition both in the case of mutual and self-induction by cutting the wires of the circuits and setting up a counter e.m.f. We have seen how this takes place in the case of self-induction; we will now study the actions of mutual induction.

Fig. 195 (*a*) and (*b*) represents a cross section of the coils in Fig. 194. When the current is sent into the coil *A*, as

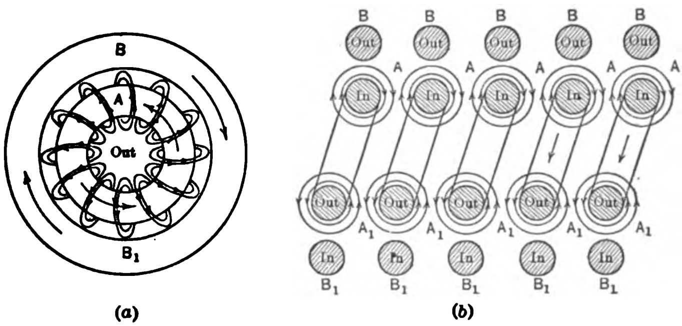


FIG. 195. (*a*) represents an end view of the coils in Fig. 195. (*b*) represents a longitudinal section of the same coils.

marked on the wire ends *A* and A_1, a field grows out around wires as marked. This causes a clockwise field to grow around the top wires of coil *A* and a counter-clockwise field around the bottom wires. This field spreads out in ever-widening rings as the current is increased and cuts across the sides of the coil *B*. This is shown in Fig. 196 (1), (2), (3), where the wire A represents the end of a wire in coil A, and B represents the end of a wire in coil B. It shows that as the current in A grows, the magnetic field caused by it spreads out and cuts across the wires of coil B. The wires on the top of coil B are cut by the lines as they move upward. This is equivalent to the wires of B moving downward and cutting the lines as shown in Fig. 197. By applying the right-hand rule, we see that there would be a voltage induced in B tending to send the current **out** of B. **This is in the opposite direction to the inducing current in A.**

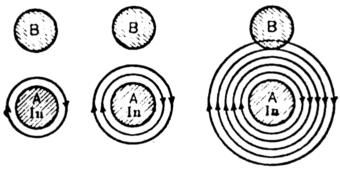


FIG. 196. An enlarged view of three wire ends from the top of Fig. 195*a*. This shows how the growing field around the wires of coil A spread out and cut the wires of coil B.

If we apply this to the lower sides of the coil, Fig. 195, we find that the growing current in coil A is **out** in the wires A_1 at the bottom, and that the wires B_1 of coil B are cut by the growing field in such a way as to induce a voltage which tends to send a current **in** at B_1. Here again we see that the **induced voltage** in coil B **is opposed** in direction **to the increasing current** in coil A. If we look at the end of the coil, Fig. 198, we see that the inducing electric current in A is in a counter-clockwise direction. As long as it is increasing, it is causing an induced current to flow in B in a clockwise direction.

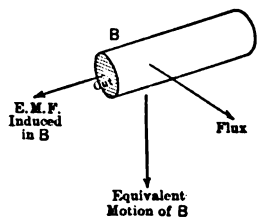


FIG. 197. This shows how the movement up of the growing field across the wires of coil B is equivalent to a downward movement of the wires of B across the field around the wires to A.

The magnetic field in the air core, due to the current in B, is opposed to the building up of the field due to the current in *A*.

The current in *A* is trying to build up a field **out** (Fig. 198), while the current induced in *B* tends to oppose this building up of a field **out** by neutralizing it with a field **in**.

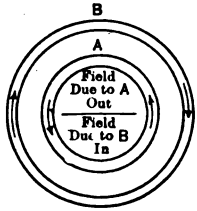


Fig. 198. Note that the field due to the current in *A* is *out* and the field due to the current in *B* is *in*.

60. Induction Coils. There is no electrical connection between coils *A* and *B* of Fig. 194. They are very carefully insulated from each other. Yet all the time that **a current is increasing** in one direction in *A*, it causes a current to build up in *B*. The field of this induced current in *B* **opposes the building up** of the field by the current in *A*. As soon as the current in *A* reaches its normal value, the lines of force around the wires no longer will be spreading out and there will be no lines cutting the sides of *B*, and thus the induced current in *B* will die out.

The induced current thus lasts **only as long as the "primary" current** (the current in *A*) **is growing.** While the current in *A* was growing, it had to overcome both the resistance of the wires composing the coil, and also the opposition of the induced current in *B* to the building up of the field. As soon as the current in *A* reached its normal value, the induced current in *B* died out, and there was left only the resistance of the wires of *A* to be overcome.

In order to produce a persistent voltage across the terminals of coil *B*, therefore, it is necessary to keep the current in *A* continually changing. **Induction coils** are built so that a direct current sent into coil *A* will be interrupted very rapidly by some special device, and the ever-changing current induces a voltage across the coil *B*. Furthermore, an iron core is inserted, which greatly increases the number of magnetic lines, and the coil *B* has many more turns of wire than coil

A, so that the field around each wire of *A* cuts a great many wires of the coil *B*. This causes the voltage induced in *B* to be as many times greater than that across *A*, as is desired. Thus, if coil *B* has ten times as many turns as coil *A*, the voltage induced in *B* will be approximately ten times the voltage across *A*. For if every line of force set up around *A*, for instance, cuts two turns of coil *A* and twenty turns of coil *B*, it is easy to see that ten times as much voltage will be induced in coil *B* as in coil *A*. Of course there are always some lines set up (called **leakage lines**) which do not cut the wires of *B*, so that the ratio of the voltage is always slightly less than the ratio of the turns. Such coils are used for purposes requiring a high voltage, where but little power is necessary, as in electric ignition systems, *X*-ray apparatus, etc.

61. Transformers. The wide use of alternating-current systems is due largely to the possibility of utilizing **transformers,** which depend for their action upon this mutual induction between two coils. Each transformer consists of two coils, one of a few turns of large conductors, called the **low-tension coil,** and the other of many turns of comparatively small conductors, called the **high-tension coil.** The coil by which the power enters a transformer is always called the **Primary**; the coil by which it leaves is called the **Secondary.**

It is well known that it is much more economical to transmit power at high voltages. Accordingly, at a generating station the primary coil is the one of few turns and is connected to the generator. The secondary coil of many turns is connected to the line. The voltage of the line is then approximately as many times higher than the voltage of the generator as the number of turns in the line coil is greater than those in the coil connected to the generator. A transformer so connected is called a "Step-up" transformer. Since it is difficult to use this high voltage for driving motors, etc., it is "stepped down" wherever it is to be used.

This is done by using the same kind of transformer, connecting the high-tension side, as primary, to the line, and

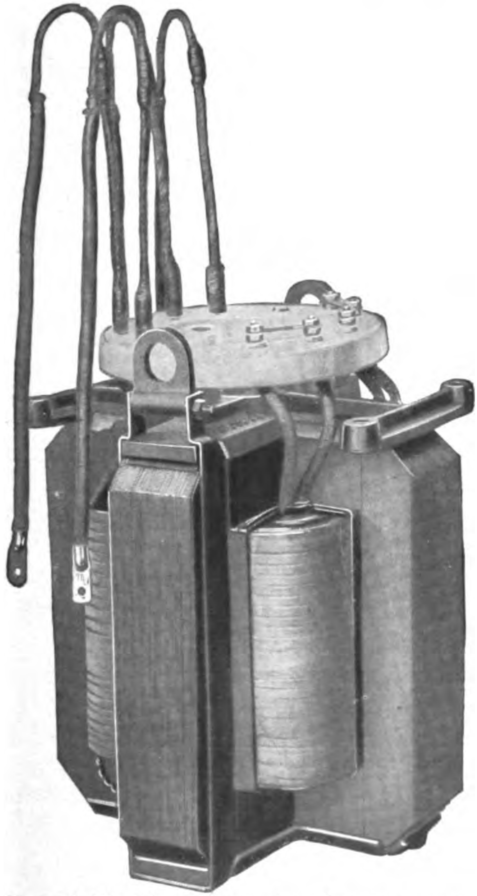


FIG. 199. General Electric 10-kv-a. transformer removed from the case. The high-tension coils are made to operate on 2200 volts; the low-tension on 220 and 110 volts.

connecting the low-tension side, as secondary, to the motor. Thus, by means of two transformers of the same kind, alternating-current power may be generated and used at low voltage, yet transmitted at voltages as high as 150,000 volts (at the present time). The transformers of the present date being remarkably efficient, the loss in transmission is thus reduced to a minimum.

Fig. 199 and 200 show the appearance and construction of a 10-kv-a. transformer for changing from 2200 to 110 or

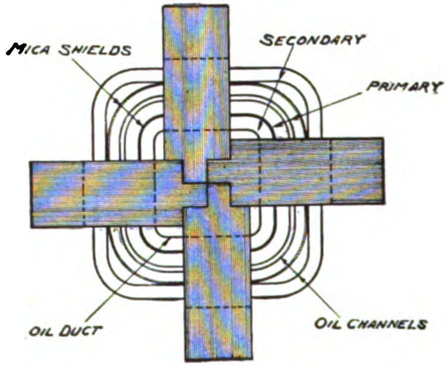


FIG. 200. Horizontal section of the transformer shown in Fig. 199.

220 volts. Note that both the high- and the low-tension coils are wound on the same iron core. The magnetic path is in the form of four rings of laminated iron.

The action of a transformer is exactly similar to the action of the induction coils explained in the previous paragraph except that the current in the primary coil does not have to be periodically broken in order to change the magnetic flux and so induce a voltage in the secondary. The current in the primary is continually alternating so that the flux is also always alternating. This sets up an induced alternating e.m.f. in the secondary. The characteristics and use of transformers of various types are fully described in Chapter XI.

Conventional diagrams for a transformer are shown in Fig. 201 and 202. In Fig. 201 is seen how the method of interlinkages between electric circuit and magnetic circuit facilitates the explanation of the action. On the low-tension side of the transformer there are 3 turns shown; on the high-tension side, 7 turns.

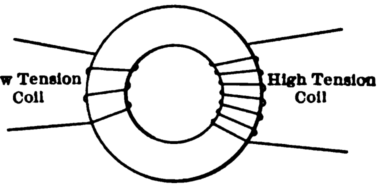


FIG. 201. The conventional diagram of a transformer. Since the high-tension coil is represented with seven turns to three of the low-tension coil the voltage of the high-tension side must be $\frac{7}{3}$ of the low-tension voltage.

If the 3 turns set up ϕ lines, there would be 3 ϕ interlinkages on the low-tension side. Assuming that no lines leak out of the iron core, these ϕ lines would make 7 ϕ interlinkages on the high-tension side. A change of 3 ϕ linkages on the low-tension side thus produces a change of 7 ϕ linkages on the high-tension side. Since the induced voltages are proportional to the linkages changed per second, the voltage on the high-tension side is $\frac{7}{3}$ of that on the low-tension side; or, the ratio of the induced voltages is exactly the same as the ratio of the turns in the coils. The same ratio exists between the e.m.f. impressed on the low-tension side and the induced e.m.f. in the high-tension side, since the impedance and voltage drops in the coils are usually relatively small, and the impressed and induced e.m.f.'s of the low-tension coil are practically equal.

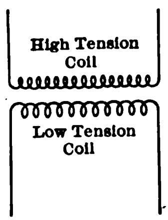


FIG. 202. Another conventional representation of a transformer. Note that the high-tension coils and the low-tension coils have no electrical connection.

The method of transmission by step-up and step-down transformers is shown in Fig. 203. Notice the peculiar circumstance that the alternating-current motors M_1, M_2, M_3 have no electrical connection with the generator G, yet they draw all their power from it. Note also that the transformers T_1, T_2, T_3 are connected in parallel.

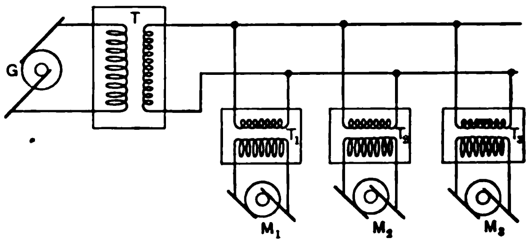


FIG. 203. Conventional diagram showing generator G connected to line through a step-up transformer T. The motors M_1, M_2 and M_3 are connected in parallel to the line through step-down transformers T_1, T_2 and T_3.

SUMMARY OF CHAPTER V

THE RESISTANCE of an electric circuit is the ratio of the direct voltage across it to the direct current which this voltage will force through it.

THE IMPEDANCE of an electric circuit is the ratio of the alternating voltage across it to the alternating current which this voltage will force through it.

WHEN A CIRCUIT OFFERS EXACTLY THE SAME OPPOSITION to the flow of both alternating and direct currents, the impedance of such a circuit is caused by, and is equal to, the resistance only of the circuit.

WHEN A CIRCUIT OFFERS MORE OPPOSITION to the flow of alternating currents than to the flow of direct currents, the impedance is caused, in part at least, by the REACTANCE of the circuit.

IMPEDANCE may be entirely RESISTANCE, entirely REACTANCE, or a combination of the two.

INDUCTIVE REACTANCE (symbol X) differs in one important respect from RESISTANCE.

The opposition which RESISTANCE offers to the flow of a current depends entirely upon the electric CIRCUIT.

The opposition which INDUCTIVE REACTANCE offers to the flow of a current depends not only upon the CIRCUIT, but also upon the number of ALTERNATIONS per second of the current.

(a) THE NUMBER OF DOUBLE ALTERNATIONS per second of the current is called the Frequency (symbol, f).

(b) THE AMOUNT WHICH THE CIRCUIT contributes to the reactance is measured by INDUCTANCE (symbol, L).

THE INDUCTIVE REACTANCE equals 2π times the product of the inductance and the frequency. That is,

$$X = 2\pi f L.$$

THE INDUCTANCE OF A CIRCUIT depends upon the number of magnetic lines (ϕ) which one ampere of current sets up, and also upon the number of times (N) which the circuit loops around these lines.

ONE INTERLINKAGE, or one linkage, is formed when one line of flux is looped once by the circuit. When ϕ lines are looped N times, there are ϕN linkages.

ONE HENRY OF INDUCTANCE is possessed by a circuit when one ampere of current sets up 100,000,000 linkages.

THE NUMBER OF HENRYS INDUCTANCE in a circuit thus equals

$$\frac{\frac{\phi N}{I}}{10^8}$$

or

$$L = \frac{\phi N}{10^8 I}.$$

FOR A LONG COIL THIS BECOMES

$$L = \frac{1.26 N^2 \mu A}{10^8 l},$$

where l is in centimeters and A is in square centimeters.

FOR A MILE OF ONE WIRE OF A TRANSMISSION LINE IN THE AIR

$$L = .000741 \log_{10} \left(2.568 \frac{S}{d}\right).$$

IF THE CURRENT IN A CIRCUIT OF ONE HENRY INDUCTANCE changes in value by one ampere per second, there is a change of 100,000,000 linkages per second. Since 100,000,000 linkages cannot be formed or broken in one second without cutting the circuit 100,000,000 times a second, one volt of e.m.f. must be set up in the circuit so cut. A circuit may therefore be said to have one henry of inductance when a change of current at the rate of one ampere per second induces an e.m.f. of one volt.

INASMUCH AS THIS INDUCED E.M.F. is in such a direction as always to OPPOSE the change of the current, the INDUCTANCE of an electric circuit has been likened to the INERTIA of matter, which causes bodies to OPPOSE any change in motion.

CURRENT-LIMITING REACTANCE COILS make use of the opposition which inductance offers to a change of current. They are large coils of sufficient reactance to oppose the sudden growth (and decay) of large currents. They are placed in the circuits of generators to prevent the growth of large currents in case of short circuits.

LIGHTNING ARRESTERS are reactance coils placed in series with electrical appliances. They allow currents of low commercial frequencies to pass through them unhindered, but offer a large opposition to the rapid surges or high frequencies of current, which are sometimes set up in the line by lightning discharges.

THE CURRENT THROUGH AN APPLIANCE CONTAINING INDUCTIVE REACTANCE ONLY, lags 90° behind the e.m.f. which causes it to flow; thus no power is consumed by maintaining a current against a reactance.

THE CURRENT THROUGH AN IMPEDANCE is considered to be composed of two component currents, at 90° to each other; one in phase with the voltage across the impedance, the other at 90° to the voltage. The component at 90° to the voltage is called the REACTIVE or WATTLESS COMPONENT, since there is no expenditure of power required to maintain it.

THE REACTANCE OF A SERIES COMBINATION OF INDUCTIVE REACTANCES is the arithmetical sum of the separate reactances.

THE REACTANCE OF A PARALLEL COMBINATION OF INDUCTIVE REACTANCES is found by adding the currents through the parallel paths and then dividing the voltage across the combination by this total current.

THE IMPEDANCE OF A SERIES COMBINATION OF RESISTANCES and INDUCTIVE REACTANCES equals the square root of the sum of the squares of the total resistance and the total reactance.

$$Z = \sqrt{(R_1 + R_2 + R_3 + \ldots)^2 + (X_1 + X_2 + X_3 + \ldots)^2}.$$

THE IMPEDANCE OF A PARALLEL COMBINATION OF RESISTANCES AND REACTANCES is found by adding VECTORIALLY the currents through the separate branches of the combination, and dividing the voltage across the combination by this total current.

THE IMPEDANCE OF SERIES OR PARALLEL COMBINATIONS OF IMPEDANCES is found by resolving each impedance into its component resistance and reactance, and proceeding as for series and parallel combinations of these.

MUTUAL INDUCTANCE is the property possessed by any two electric circuits, so constructed that a change in the current through one sets up an induced e.m.f. across the other.

TRANSFORMERS make use of the property of mutual inductance between two coils on the same iron core. One coil, called the low-tension coil, consists of few turns; the other consists of many turns and is called the high-tension coil. An alternating current sent through either coil will induce an alternating e.m.f. in the other. The coil in which this e.m.f. is induced is called the secondary coil; the other is called the primary. The induced voltage across the secondary is in approximately the same ratio to the impressed voltage across the primary, as the turns in the secondary coil are to the turns in the primary coil. When the voltage across the primary is the higher, the transformer is called a "step-down" transformer; when the voltage across the secondary is the higher, a "step-up" transformer.

TRANSFORMERS enable alternating-current power to be generated at low voltage, stepped up to high voltage for long transmission, and then stepped down for use at low voltage. Because of the remarkably high efficiency of transformers, the transmission of large amounts of power over great distances is accomplished with little loss.

PROBLEMS ON CHAPTER V

Prob. 55–5. The Great Western Power Co. transmits power 154 miles over a 3-phase system. The line wires are No. 000 stranded, approximately ½ inch in diameter, spaced 10 ft. apart in a vertical plane on towers. What is the total inductance of each conductor?

Prob. 56–5. The frequency in Prob. 55 is 60 cycles. What is the reactance of each line wire?

Prob. 57–5. The dimensions of the choke coil shown in Fig. 175a are as follows: length, 12 inches; number of turns, 15; diameter, 15 inches. Wire is aluminum, 0.4 inch in diameter. Find the inductance of the coil.

Prob. 58–5. (*a*) What reactance will the coil of Fig. 175a have on a 60-cycle circuit? (*b*) What impedance?

Prob. 59–5. What reactance will the coil of Fig. 175a offer to an oscillating current of 1,000,000 cycles per second set up by a lightning discharge?

The data for Prob. 60 to 75 inclusive are from Ryan's "Design of Electrical Machinery" and apply to the same transformer.

Prob. 60–5. A certain type *H* transformer has 1200 turns in the high-tension windings. How many turns must the low-tension windings have in order to transform 2200 volts to 220 volts?

Prob. 61–5. How many interlinkages must be made per second (average rate) in order to set up the 220 volts (sine wave-form) in the low-tension side of the transformer of Prob. 60?

Prob. 62–5. The transformer of Prob. 61 is to run on 60 cycles. What maximum flux must be set up to produce the proper number of linkages per second?

Prob. 63–5. Allowing 50,000 lines per sq. in. as the flux density in the iron core, what should be the average cross section of the iron path?

Prob. 64–5. Assuming a fair volume for the iron in this transformer as 485 cu. in., what is the average length of the path of the magnetic flux?

Prob. 65–5. What maximum current must be required in the high-tension coils to send the maximum flux through the magnetic circuit of this transformer, with the low-tension side on open circuit? A fair value for permeability of the iron core at this magnetic density would be 2650.

Prob. 66–5. The effective value of the current in the primary coil required to produce the flux through the core, when the secondary side is open, is called the magnetizing current. What is the magnetizing current for this transformer, assuming that the current has a sine wave-form?

Prob. 67–5. This transformer is found to consume 131 watts when the primary high-tension side is connected to the mains with the secondary side open. Practically the whole of this power is consumed in hysteresis loss and in setting up eddy currents in the core. How much current must be used for this purpose in this transformer when it is operating at its proper voltage?

Prob. 68–5. The current used to supply core-loss power and the magnetizing current are in quadrature. What is the total current taken by the high-tension primary side of this transformer when idle on a 2200-volt line?

Prob. 69–5. What is the self-inductance of the primary high-tension coils on this transformer?

Prob. 70–5. On a 60-cycle circuit, what is the reactance of the high-tension primary side of this transformer?

Prob. 71–5. What should be the magnetizing current for this transformer on a 2200-volt line, using the reactance found in Prob. 60? Compare with answer to Prob. 66.

Prob. 72–5. The resistance of the high-tension coils is 5.45 ohms. What is the impedance of the high-tension coils on a 60-cycle circuit?

Prob. 73–5. Using impedance found in Prob. 72, compute current which the high-tension coils would take when connected to a 60-cycle 2200-volt line. Explain why the value differs from that found in Prob. 68.

Prob. 74–5. From the resistance and the current as found in Prob. 68 compute power used in overcoming the resistance of the primary coils when connected to a 2200-volt 60-cycle line.

Prob. 75–5. Recompute the core loss in Prob. 67, correcting the 131 watts taken on open circuit to find the true watts taken for core loss.

Prob. 76–5. Recompute Prob. 68, using the correct core-loss current as found from Prob. 75. What per cent error is made in the value of the no-load primary current by considering that the 131 watts are all core loss?

Prob. 77–5. (Data taken from Electric Journal, April, 1914.) Tests on a current-limiting reactance similar to that shown in Fig. 170, which was designed to carry 200 amperes continuously, showed that it had a self-inductance of about 0.0032 henry. One of these reactances is placed in series with each line wire connected to the terminals of a three-phase 15,000-kv-a. 6600-volt 60-cycle 360-r.p.m. water-wheel type generator. Calculate the value of this reactance in ohms, the drop across each reactance, when rated output of the generator is being delivered in volts and in per cent of the terminal voltage of each phase, assuming the phases to be Y-connected and the resistance negligible in comparison with the reactance. How many kilovolt-amperes are being carried by each reactance?

Prob. 78–5. If the three line wires of Prob. 77 were to be short-circuited together while the terminal voltage and speed were maintained constant, what voltage drop would there be across each reactance, and what kilovolt-amperes carried by it? What would be the short-circuit current per line wire or per phase, in per cent of the normal current?

Prob. 79–5. The reactance of each phase of the winding of the generator in Prob. 78–5 is 0.69 ohm when operating at rated frequency. The total reactance of each phase including the external current-limiting coil is 1.90 ohms. Calculate the final value of current flowing in each phase of the generator winding when all phases are short-circuited:

(*a*) Between the windings and the reactance.

(*b*) Between the reactances and the line wires. The generator continues to operate with full normal field flux and speed. Express each of these currents in percentage of rated full-load current.

Prob. 80–5. Calculate the maximum instantaneous value of voltage across the terminal of the current-limiting reactance in Prob. 77–5, and the value of current and of power in the reactance coil at this instant.

Prob. 81–5. In Prob. 78–5, what will be the values of current and of e.m.f. at the instant when the power in the reactance is zero?

Prob. 82–5. What will be the values of e.m.f. and power in one of the reactances of Prob. 77–5 at the instant when the current through it is zero?

Prob. 83–5. A certain induction coil takes 10 amperes when connected across a 230-volt 60-cycle line. When the frequency is reduced 50 per cent the current is increased 50 per cent. What are the values of resistance and inductance of this coil?

Prob. 84–5. A 15,000-kv-a. 6600-volt 60-cycle 3-phase Y-connected alternator has a reactance which is stated to be 15 per cent; that is, when operating at rated frequency and load, the e.m.f. required to overcome the reactance of each phase of the winding is 15 per cent of the terminal e.m.f. of that phase. Calculate the reactance drop in volts, and the reactance in ohms.

Prob. 85–5. If the total power loss due to resistance in the three phases of the armature winding of Prob. 84 at rated full load is 2 per cent of the power output at unity power-factor, calculate the resistance of each phase, in ohms. From this and the result of Prob. 84–5, calculate the impedance of each phase.

Prob. 86–5. A coil having inductance and resistance takes 110 watts and 2 amperes when connected across 110-volt 60-cycle mains. What is the resistance, reactance, and inductance of this coil? What is the power-factor and angle of phase difference between pressure and current?

Prob. 87–5. If the wire in the coil of Prob. 86–5 were unwound and rewound non-inductively, what current and watts would be taken from the same mains? What would be the phase difference between pressure and current?

Prob. 88–5. When all three wires of a three-phase transmission line are short-circuited together at the far end, the instruments on this line at the station switchboard indicate as follows: Volts between wires, 11,000; amperes in each wire, 100; total watts supplied to line, 1350 kilowatts. Calculate the resistance and reactance of each line wire, in ohms.

Prob. 89–5. Calculate what the current and wattage in the coil of Prob. 46–5 would be, if the frequency were increased to 60 cycles at the same voltage.

Prob. 90–5. How much of the power (kv-a.) in Prob. 46–5 is "give-and-take," or reactive? How many total kilovolt-amperes are taken by the coil from the mains? At each of the four instants when the power taken from the mains is zero, how much power previously stored in the magnetic field around the coil is being returned and dissipated as heat? What are the values of current and line voltage at each instant?

Prob. 91–5. What is the maximum value of negative power, or power returned by the coil of Prob. 90, to the mains, and what are the values of current and e.m.f. at this instant? How much power is being given up by the coil at this instant, and how much of it is being transformed into heat by the resistance of the coil?

Prob. 92–5. One unit alone takes 10 kw. at 60 per cent power-factor when connected across a certain line. Another unit alone takes 10 kw. at unity power-factor when connected across the same line. How many kilowatts will these two units together take when connected in series across the same line, and what will be the power-factor of the entire circuit? What fraction of the total voltage will each unit take?

Prob. 93–5. Two impedances are in series across 220-volt 60-cycle mains. There is the same voltage across each, but the resistance of one of them is 15 ohms while that of the other is 7 ohms. The total power consumed by the entire circuit is 550 watts. Calculate the inductance of each of these impedances, the power-factor of each, and the power consumed in each.

Prob. 94–5. Verify the statement that when several units, all having the same resistance and the same reactance, are connected in series, the total voltage across the entire circuit is equal to the arithmetical sum of the voltages across the units, or that they divide the total voltage equally among them.

Prob. 95–5. Verify the statement that when several circuits each having resistance and inductance, and all of the same power-factor, are connected together either in series or in parallel, the power-factor of the entire circuit is the same as that of each of its parts.

Prob. 96–5. One coil alone takes 10 amperes at 60 per cent power-factor from 110-volt 60-cycle mains. Another coil, alone, takes 10 amperes at 80 per cent power-factor from the same mains. What is the impedance, resistance, reactance, and inductance of each coil, and what will be the current, power, and power-factor if they are connected in series across the same mains?

Prob. 97–5. In Prob. 96–5, what would be the voltage across each coil when they are in series?

Prob. 98–5. What would be the answers to Prob. 96–5 and 97–5 if the frequency were doubled?

Prob. 99–5. What would be the answers to Prob. 96–5 and 97–5 if the frequency were reduced to zero, or if direct current were used?

Prob. 100–5. Verify the statement that when several units, all having the same resistance and the same reactance, are connected in parallel, the total current in the main circuit is equal to the arithmetical sum of the currents through the units, or that the units divide the total current equally among them.

Prob. 101–5. If the two units specified in Prob. 96–5 are connected in parallel to the same mains, what will be the total amperes, watts, and power-factor of the combination?

Prob. 102–5. What would be the resistance and reactance of a single coil equivalent to the parallel combination of Prob. 101–5?

Prob. 103–5. Calculate the value and direction of current in each coil of Prob. 101–5, at the instant when the current in the mains is zero. Calculate the e.m.f. applied at this instant, and the value and direction of power in each coil and in the line.

Prob. 104–5. Four units all having the same resistance and inductance take altogether 4 kw. at 80 per cent power-factor when connected in parallel across 220-volt 60-cycle mains. What would be the current, power, and power-factor if they were connected in series across the same mains?

CHAPTER VI

ALTERNATING-CURRENT GENERATORS: SINGLE PHASE

An electromotive force is generated in a conductor whenever magnetic lines of force are cut by the conductor. It is immaterial whether the conductor moves and cuts the lines

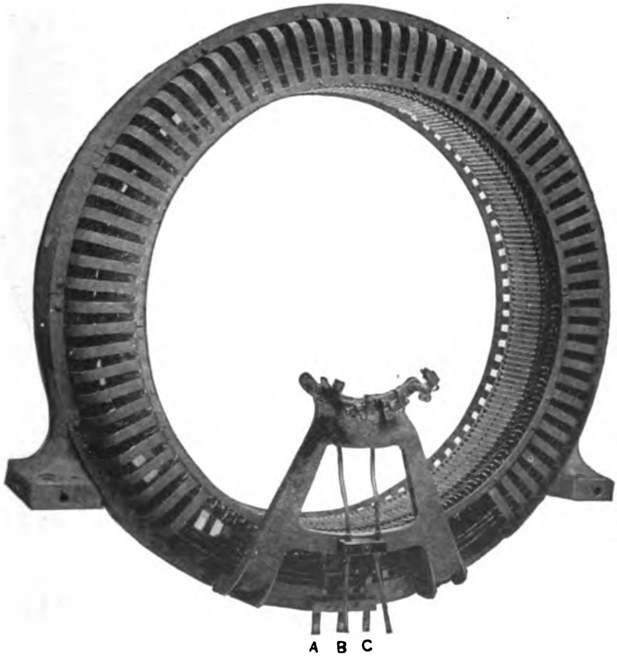


Fig. 204. The stationary armature of a 200-kv-a. alternator. *Westinghouse Electric & Mfg. Co.*

or whether the lines move so as to be cut by the conductor. In direct-current generators, it is always the conductors which move. In other words, direct-current generators have

rotating armatures. In alternating-current generators, especially of the larger capacities, it is usually the field which rotates, although rotating armatures may be used. Whether it is the field or the armature, that part of the machine which

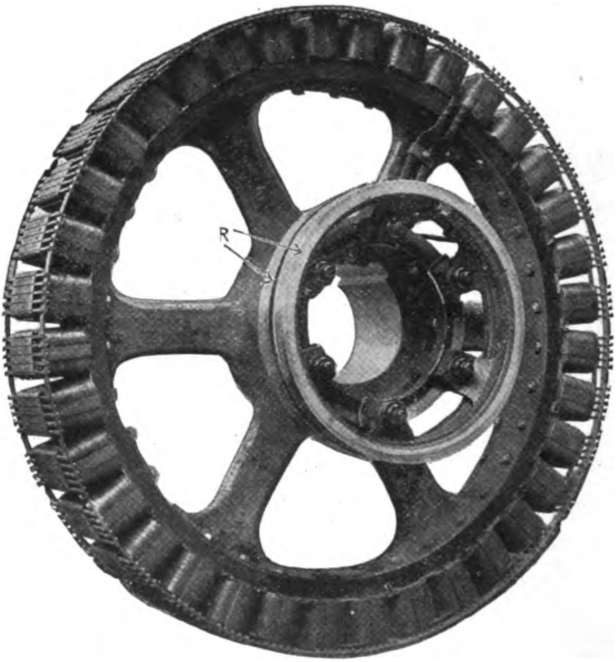


Fig. 205. The rotating field for the armature of Fig. 204. Direct current is supplied to the rings for the field coils.

rotates is always called the **rotor,** and the stationary part, the **stator.**

62. Rotating Field Type. The e.m.f. is always generated in the conductors, or **inductors,** as they are called in this case, which are wound on the armature, whether it be the rotor or the stator. The greatest advantage which the stationary armature has over the rotating armature is that the power is generated in conductors which do not move and therefore it can be delivered to the external circuit without the aid of any sliding contacts. Also, the insulation necessary for high voltages is easier to construct and maintain. Fig. 204 is a picture of this type of armature as constructed by the Westinghouse Mfg. Co. for a 200-kv-a. generator. All the power output is taken from the three stationary leads, *A*, *B* and *C*.

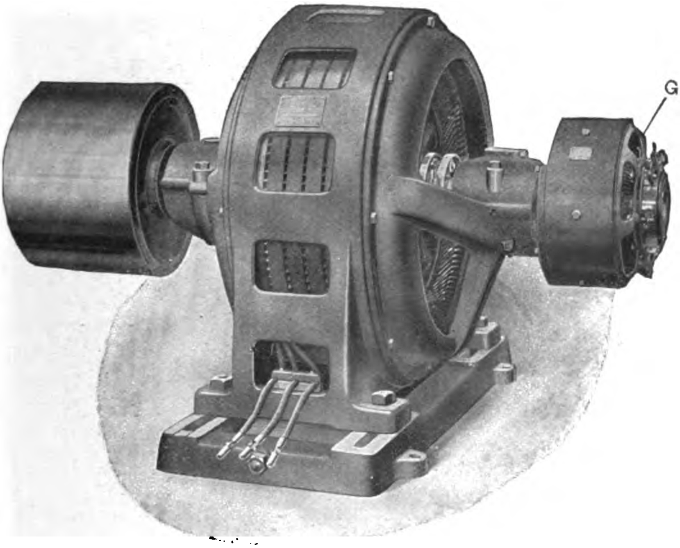


FIG. 205a. A small alternator. The direct current for exciting the fields is taken from the small d-c. generator *G*, which is coupled to the shaft.

The rotating field for this machine is shown in Fig. 205. It consists merely of a ring of pole pieces, **alternate north and south** of course, each being wound with its own field coil. The magnetic flux from these poles sweeps across the windings of the stationary armature and induces the e.m.f. in them. The current supplied to the field coils must be direct current, since the polarity of each pole never changes during the operation of the machine. This **exciting current,** as it is called, is supplied usually from some outside source and delivered by brushes, which make contact with the collector rings *R* in the diagram. In Fig. 205a the exciting current is being supplied to the field of an alternating-current generator by the

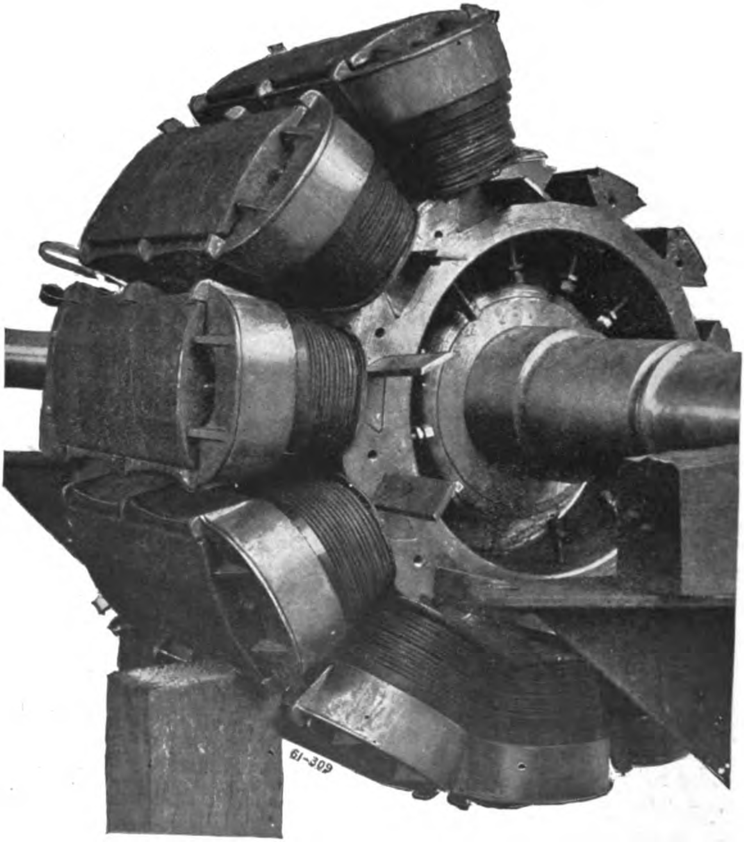


FIG. 206. Revolving field of a 300-kv-a. Wagner alternator. Notice the shape of the faces of the laminated poles, the form of the field coils and the method of fastening the pole cores to the rotor frame.

small direct-current generator coupled to it, called an **exciter.** Fig. 206 shows more clearly the construction of a revolving field. Note that the poles, and consequently the magnetic fields from them, are distinct and sharply defined. Such fields are thus said to be of the **definite pole** type, to distinguish them from a type of revolving field used in high-speed generators the poles of which are not so clearly defined. Fig. 207 shows, in course of construction, the rotating field

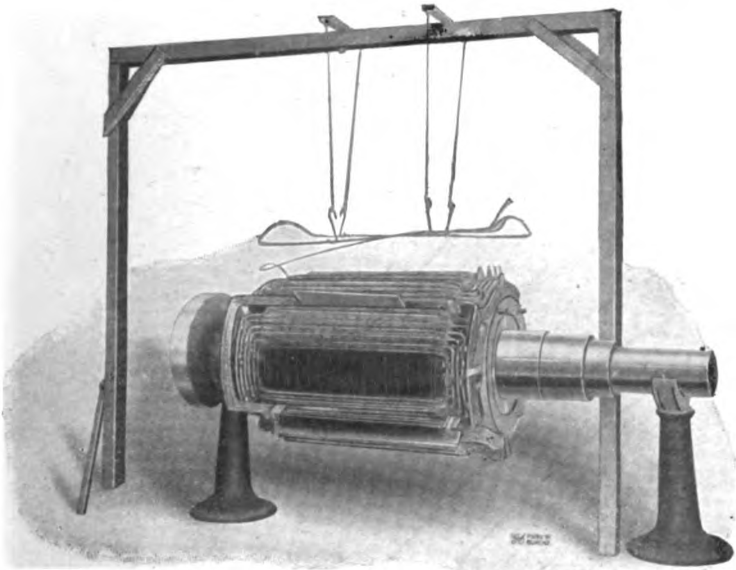


FIG. 207. The revolving field of a Turbo-generator, with the coils in process of assembly. This shows the cylindrical construction of the field core employed for the high-speed rotors. The field coils are laid in slots in the field core, which is built of punched laminations similar to a rotating armature with the windings partly distributed. *General Electric Co.*

of a high-speed turbo-generator, built by the General Electric Co., in which the poles are more or less indistinct, but may be discerned by tracing out the windings. A little inspection of the figure shows that there are four poles formed by the field coils, which are distributed over the surface much in the manner of armature windings.

63. Revolving Armature Type. A 25-kv-a. alternator of the revolving armature type made by the General Electric Co. is shown in Fig. 208. Notice the small commutator *C*

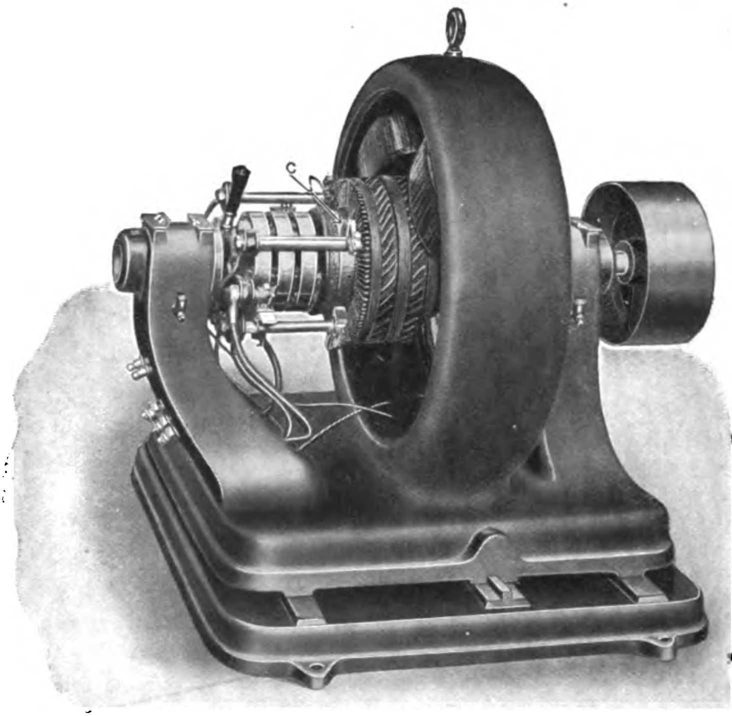


FIG. 208. An alternator with a revolving armature. Three-phase, 4-pole, 1800 r.p.m., 60 cycles, 25-kw., self-excited. A small separate winding is connected to a commutator and furnishes direct current to the field coils. *General Electric Co.*

to the right of the collecting rings. This is connected to a small winding on the armature which is separate from the main alternating-current winding, and which furnishes direct current at from 40 to 80 volts pressure, to supply the field coils. A self-excited generator such as this, with exciting e.m.f. and main alternating e.m.f. both generated by the same field, has to be of rather special design, because any change of field flux, due to the magnetic reaction of armature currents, changes the excitation voltage and thereby produces a still further change of alternating voltage. Hence, to get a good regulation for the alternating e.m.f. requires a "stiff" field, or one little affected by changes of the armature current.

Whatever the type or form, in which the **field** is constructed, the flux under all conditions remains fairly fixed in direction around each polar region. The number of the poles is two or some multiple of two. All poles of the same machine are constructed alike, and produce fields which are identical in distribution and value, but alternately north and south around the frame.

In whatever form or type the **armature** is constructed, the **inductors** are always connected together symmetrically with respect to the field poles. They are always connected so as to **add** together their useful effects and produce the largest possible electrical output at the terminals of the machine, using the smallest possible amount of material.

64. Value of Electromotive Force in Armature Inductors. We have learned that one volt is induced in an inductor when it cuts 100,000,000 lines of flux in one second. However, it is not necessary for the inductor to move for a whole second and cut 10^8 lines in order to generate one volt. It may move for one-half second and cut 50,000,000 lines. This would generate an e.m.f. whose average value during that one-half second of time was one volt, because it would be cutting lines **at the rate of** 100,000,000 per second. Similarly, it may move but $\frac{1}{1000}$ of a second and cut 100,000 lines. During this time, it would also generate an e.m.f. whose average value was one volt, for here again it would be cutting **at the rate of 10^8 lines per second.**

We may consider a period of time so short that the average value during that time is practically an instantaneous value.

Thus the induced e.m.f. in an inductor at any instant equals $\frac{1}{10^8}$ of the number of lines which would be cut in one second, if the inductor continued to cut at the same rate for a full second. Of course a conductor actually never cuts at uniform rate for anywhere near a whole second. In spite of the fact that all rotors are turned at practically constant speed, the armature conductors are constantly changing their direction of motion relative to the flux, and are cutting through a field in which the lines are not uniformly distributed. The e.m.f. in each conductor, then, is not steady from instant to instant, but may be different for two successive instants, no matter how short a time elapses between these instants, at which you choose to examine or calculate it. For this reason, it is often necessary to compute the e.m.f. at given instants. When so calculated, it is understood that at instants just before and just after the one chosen, the e.m.f. may have a different value. The symbol for instantaneous e.m.f. is here always taken to be (e), to distinguish it from average, effective and maximum values.

If we know the relative speed at any instant of the field and the cutting inductor, the intensity of the field which is being cut, and the cutting length of the inductor, we can compute the e.m.f. which is being generated at just that instant. We merely have to write an equation, as follows, which really gives $\frac{1}{10^8}$ of the number of lines which would be cut if the cutting continued at the same rate for one second.

$$e = \frac{BlV}{10^8},$$

where

e = instantaneous e.m.f. in volts.

B = lines per square inch (or per square centimeter) in the field at the point where the inductor is at the given instant: the area to be measured in the plane in which the inductor moves.

l = cutting length of inductor in inches (or centimeters); only so much of the inductor is to be measured as is actually cutting.

V = velocity of inductor relative to field, in inches per second (or centimeters per second).

Care must be used to measure units of length either all in inches or all in centimeters. Thus, if B is measured in lines per square inch, l must be in inches, and V in inches per second.

Example 1. The following data would apply to the generator of Fig. 204.

Diameter of frame at depth of armature winding	= 8 ft.
Cutting length of conductor	= 6 in.
Speed	= 200 r.p.m.

What would be the instantaneous e.m.f. induced in a conductor as that part of a pole face was passing it where flux density became 6300 lines per square centimeter?

$$e = \frac{BlV}{10^8}.$$

$$B = 6300 \times 6.45 = 40{,}600 \text{ lines per square inch.}$$

$$l = 6 \text{ inches.}$$

$$V = \frac{8 \times 12 \times 3.14 \times 200}{60} = 1000 \text{ inches per second.}$$

$$e = \frac{40{,}600 \times 6 \times 1000}{10^8}$$

$$= 2.45 \text{ volts.}$$

Prob. 1–6. At an instant later than that of Example 1, the portion of the pole face nearer the trailing tip was passing the same inductor and the flux at this point was 6500 lines per square centimeter. What e.m.f. was induced in the inductor at this instant?

Prob. 2–6. Assume that the greatest flux density of the lines coming from the poles of the generator in Example 1 was 7000 lines per square centimeter. At what speed would the rotor have to turn in order to induce a maximum voltage of 1.28 volts in a single inductor?

Prob. 3–6. If the greatest allowable peripheral speed of the rotor of Problem 2 is a mile per minute, what is the greatest instantaneous e.m.f. which can safely be set up in a single inductor? Assume that the outside diameter of rotor is 95 inches.

65. All Dynamos Generate Alternating Electromotive Forces.* A direct-current generator is one which delivers a unidirectional e.m.f. and current; that is, one terminal is constantly of positive polarity, the other of negative polarity. An alternating-current generator (commonly called an "alternator") is one which delivers a current that periodically reverses its direction through the circuit; that is, each terminal is alternately positive and negative in polarity. If the alternations were slow enough, an ordinary (permanent magnet type) direct-current voltmeter attached to the terminals of the alternator would show the nature of the e.m.f. by deflecting first to right and then to left of the zero mark. Notwithstanding the very great difference in the nature of the e.m.f. and current delivered from the terminals, and the tremendous differences thereby produced in the actions which go on in the external circuit, there is really a strong internal resemblance between these two types of dynamos. This resemblance, if pointed out, should make it easier for one who has studied the direct-current generator to understand also the alternating-current generator.

A typical direct-current generator is shown, diagrammatically, in Fig. 209. Only a part of the winding is given, to avoid confusion of lines in the diagram, and the simplest style of winding in ordinary use has been chosen. The fact that each of the collecting brushes carries current steadily in one direction, as indicated by the polarity definitely marked upon it (positive or negative), is due solely to the action of the commutator. As the rotation of the armature sweeps each conductor on its surface through the magnetic flux in the air gap between the faces of the poles and the iron core of the armature, the e.m.f. induced in it alternates as it passes north and south poles alternately. Thus the current produced in each armature conductor by this e.m.f. is really

* The only exception to this statement is the "homopolar generator" (sometimes called the unipolar generator), which is rarely built on account of great difficulties in construction.

an alternating current. The "right-hand rule," concerning the relative direction of flux, motion and induced e.m.f., states that if we extend the thumb, forefinger and middle finger of the right hand at right angles to one another, pointing the **thumb** in the direction of motion of the **inductor,** and the **forefinger** in the direction of the **flux,** the **middle finger** will point in the direction of the induced e.m.f. We find, as

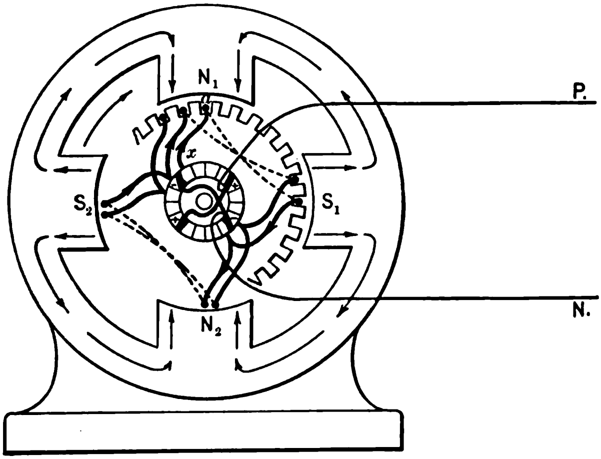


FIG. 209. A simple direct-current generator. Note that the e.m.f. induced in conductor *a* is *in* (away from the reader) as it passes an *N*-pole while rotating in a clockwise direction.

we apply this rule to some one inductor, such as *a*, through a complete revolution, that it generates an e.m.f. **into** the paper or **away** from the reader while passing in clockwise direction under pole N_1, **out** or **toward** the reader while passing clockwise under S_1, **in** under N_2, and **out** under S_2. If the speed is steady, and the magnetic flux is distributed similarly under all the poles, which we may safely assume because of their uniformity and symmetry of construction, we see that the e.m.f. goes through exactly the same set of values, and in the same order, in moving from N_1 past S_1 to N_2, that it goes through in moving from N_2 past S_2 to N_1. This complete set of values which is repeated again and again, we have seen, is called a "cycle," and the time (in seconds) required to complete one cycle is called the **period** of the alternating e.m.f. or current. Evidently the period of the e.m.f. induced in any single inductor is the time required for it to pass a **pair of poles**, and the number of cycles per second (called the **frequency** of the e.m.f.) is equal to the number of pairs of poles passed per second. The **frequency,** then, is **equal to the number of revolutions per second times the number of pairs of poles.**

Prob. 4–6. The speeds and numbers of poles employed by the Crocker-Wheeler Company for a line of direct-current lighting and power generators of from 200 to 1000 kw. capacity are as follows:

From 175 to 225 r.p.m., 8 poles.
From 100 to 200 r.p.m., 10 poles.
From 100 to 135 r.p.m., 12 poles.
From 80 to 100 r.p.m., 16 poles.
From 80 to 100 r.p.m., 22 poles, or more for the largest sizes.

Calculate the range of frequencies of alternating e.m.f. generated in the armature coils of this line of direct-current generators. Does there seem to be any uniformity of practice in regard to the frequency?

Prob. 5–6. Sometimes in large systems the power is generated and transmitted at low frequency, but must be transformed to a higher frequency before it may be applied to certain uses. To do this, two alternators having different numbers of poles are coupled together, one of them running as a motor in step with the alternations of the e.m.f. in the line, the other delivering alternating current at the higher frequency as a generator. Such a combination is called a **frequency-changer.** What combinations of numbers of poles may be used on these two machines, in order to take current from the line at 25 cycles frequency and deliver current at 60 cycles?

Prob. 6–6. If we increase the flux per pole by 20 per cent without altering the frequency, in an alternator which before the change was developing 220 volts between terminals, what voltage do we get?

Prob. 7–6. At what speed must the field revolve in the Westinghouse alternating-current generator of Fig. 204 and 205 in order to deliver

(*a*) A 60-cycle e.m.f.?
(*b*) A 25-cycle e.m.f.?

66. The Commutator is a Reversing Switch. The foregoing paragraphs are true for either a direct-current or an alternating-current generator. These two types differ mainly in the manner of getting the e.m.f. and current from the inductors to the external circuit. Usually the method of connecting the inductors together into the circuits, or the "winding" of the armature, is different for the alternating-current and direct-current generators; but this difference is not an essential one, and very often does not exist at all.* But they must differ essentially in the manner of getting the e.m.f. and the current from the armature winding to the external circuit. The **direct-current dynamo requires a commutator,** connected in series between the armature winding and the external circuit, to commutate or rectify the induced e.m.f. and current, or, to reverse the alternate pulses so as to deliver them constantly in the same direction to the external circuit. The commutator acts like a multipolar reversing switch, arranged on the shaft between the armature winding and the terminals of the direct-current armature. It is connected in such a way that each inductor, or each group of inductors, has its connections to the armature terminals **reversed** just at the instant when its induced e.m.f. reverses — that is, at the moment it passes out from the field or influence of one of the poles into the field of the next adjacent pole. If an inductor, or group of inductors, delivering alternating e.m.f. were connected to the left side of the reversing switch in Fig. 210, and this switch were thrown first to the right and then to the left **in synchronism** (i.e., in equal time) with the alternations of e.m.f. — that is, so that the connection was reversed just at the same instant when the e.m.f reversed, there would be produced in the circuit at the right of the switch a unidirectional e.m.f., as indicated. If the switch were not reversed exactly in synchronism with the current, flowing in the alternating-current circuit, or not exactly at the instant when the current reached its zero value once in

* See Chapters VIII and IX.

each half-cycle, destructive sparking or arcing would be produced at the switch points. If used to rectify a 60-cycle alternating current, the switch would have to be reversed 120 times in every second and every time at exactly the instant when the current is zero. This seems too rapid to be prac-

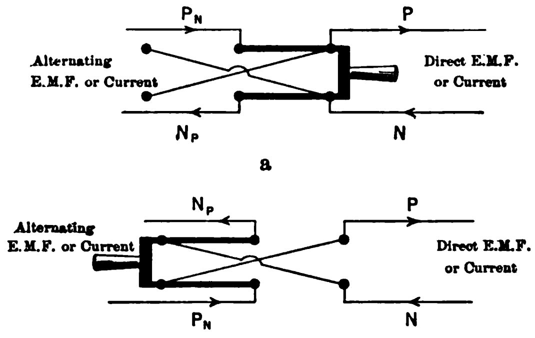


FIG. 210. If the switch were thrown first one way then the other in time with the alternations of the alternating current, a direct current would be delivered to the line on the right. A commutator does exactly this for the alternating e.m.f. of the armature.

ticable, but it is exactly what is done in the so-called "vibrating-type rectifier," which is a device for changing small amounts of power from alternating into direct current for charging automobile- and ignition-batteries. The switch is of very light construction, the contacts are close together, the adjustment must be very accurate, and auxiliary automatic devices are necessary to operate the switch successfully, but the principle is as shown in Fig. 210.* In a direct-current generator the commutator performs the office of this reversing switch. It receives an alternating e.m.f. from the windings of the armature and delivers a direct e.m.f. to the external circuit.

67. Collecting-Rings for Alternators. If we desire to get **alternating current** in the external circuit, no such reversing

* See Second Course, Chapter IX, on Conversion of Alternating Current.

switch is necessary — the series of inductors composing the armature winding can be connected directly to the external circuit — and the terminals of the winding become directly the terminals of the armature. If the armature revolves, as in very small alternators, a collecting-ring or sliding contact is necessary between each stationary line wire and the corresponding terminal of the moving armature winding. If the armature is stationary, as in most alternators, no such

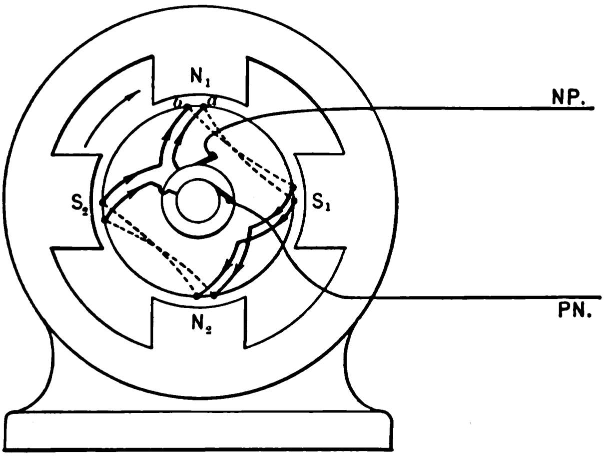


FIG. 211. The armature of Fig. 209 arranged to deliver alternating e.m.f. to the circuit. The windings are connected in series and the two terminals brought out to a pair of collecting-rings.

device is required in the circuit of the armature, but two sliding contacts as *R* in Fig. 205 are necessary to carry direct-current into the rotating field coils for producing flux. To illustrate the essential structural differences between direct-current and alternating-current generators, the same armature which was used with a commutator in Fig. 209 to deliver direct current to the external circuit is shown, rearranged so as to deliver alternating current to the external circuit, in Fig. 211. Notice that instead of a commutator consisting of numerous segments of hard forged copper, insulated from each other and from the frame of the machine, two simple rings are used. This makes the alternating-current generator simpler and less troublesome, and usually cheaper for a given power capacity, than the direct-current generator — other things, such as speed and grade of materials used, being equal. These diagrams show the machine as seen from the end where the current-collecting device (brushes, with collecting-rings or commutator) is located. Wires used to connect the inductors at the further end of the armature are shown as dotted lines, while connections at the near end are shown as full lines. One collecting-ring is shown inside the other simply to make the diagram clear; they are in fact all of the same diameter and mounted side by side along the shaft.

68. Open-circuit and Closed-circuit Windings. Now suppose that the winding of the direct-current dynamo of Fig. 209 were continued as started, using a proper relation between the number of slots, poles, commutator bars and **pitch** of the winding (or distance between adjacent inductors in the series, or between sides of a coil). We should find that the winding would finally end exactly where it started, the beginning and the end would be soldered together, and the armature winding would form a **closed circuit** (independent of any external circuit) by re-entering into itself. The commutator bars are merely tapped off at equidistant points along the winding. However, if the corresponding alternating-current winding were to be continued as shown in Fig. 211, the completed winding would be an **open circuit** (its two ends terminating at the collecting-rings) until closed by making a connection externally between the brushes. All direct-current dynamos necessarily have closed windings.* Alternating-current machines may

* The only exceptions are some of the old forms of generators used to produce direct current for arc lighting circuits, the Brush arc-dynamo, for instance. These are no longer being installed.

have either closed or open windings. The latter are most usual, but the former are used in cases where the machine must deliver very large currents at low voltage, in which case the winding consists of several similar paths connected in parallel to the terminals of the armature, thus forming one or more closed circuits or meshes within the armature winding. Closed windings are also used where the same machine must handle either direct current or alternating current, or both, as in synchronous converters, double-current generators, or Dobrowolsky three-wire direct-current generators, which will be found treated elsewhere.

In a later chapter a rather full explanation of the subject of armature windings for alternating-current machines is given, in a way intended to be useful not only in tracing out the internal connections of alternating-current generators and motors so that they may be assembled, rearranged or repaired intelligently, but useful also in helping the worker to understand what alternating e.m.f.'s and currents are really like, what phase difference means and does, and how to use vector diagrams. At this point, we shall consider only the simplest and clearest complete diagram which can be used to represent the actions going on in an alternator, using it as a basis for explanations of such actions.

69. Magnetic Path: Leakage. Fig. 212 represents diagrammatically a two-pole armature having 24 inductors upon its outer surface, spaced equidistant from each other. The thin full lines represent some of the lines of magnetic flux, passing out of the *N*-pole into the air gap, thence into and through the steel core of the armature, through the air gap under the adjacent *S*-pole, and into this *S*-pole. The magnetic circuit is always completed from *S*-pole to *N*-pole through some form of iron or steel yoke, not shown here. The actual amount and distribution of the magnetic flux is not intended to be shown in this sketch; it depends upon a great many conditions, to be explained later, as becomes necessary in order to understand the operation of the machine. As the armature rotates and carries them around, each of the conductors on the armature has an e.m.f. induced in it while it is cutting through lines of flux. The only lines that can be cut are those entering or leaving the steel core of the armature. The cylindrical armature core is made hollow, as shown, to economize material and to improve the circula-

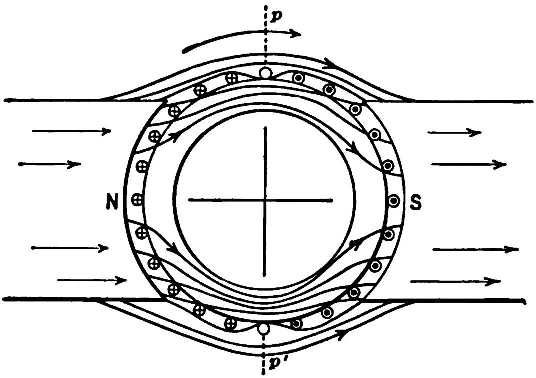


FIG. 212. Magnetic path through the armature of a simple two-pole generator.

tion of air for carrying off the heat due to the losses, so as to keep the temperature down to a value which will not injure the insulation around each armature conductor. The cross-section of this core is made no larger than necessary to keep the density of magnetic flux in it down to a point where the hysteresis and eddy-current losses will be reasonable. The flux finds it easier to pass through iron than through air, so there are practically no lines of flux within the hollow core — practically all of those that enter the armature core prefer to pass through it, as shown. A few lines will always pass from one pole to the other through the air without entering the armature, because this **leakage** path (for flux which is useless because the armature conductors cannot cut it) always exists in parallel with the path of the **useful** flux (into and through the armature). The proportion of leakage flux to useful flux is kept as low as possible in order to improve the voltage regulation and efficiency of the dynamo, and increase the amount of load it can carry. This proportion depends upon the construction of the machine and upon the amperes and power-factor of the current delivered by the armature.

Prob. 8–6. Draw a sketch showing the magnetic circuits and distribution of flux in a four-pole dynamo. Mark the polarity of all the poles and the direction of each line of flux. Make the sketch large enough to be clear, and do not crowd the lines of flux.

Prob. 9–6. As in Problem 8–6, draw the magnetic circuits of a six-pole dynamo.

Prob. 10–6. Draw sketches showing what you think would happen to the magnetic flux in a six-pole dynamo if the bearings should wear so as to allow the armature to drop a little.

Prob. 11–6. Draw sketches showing how you think the flux distribution of the same dynamo would be altered if pole-shoes were put on the pole-cores, making the pole-faces cover say 80 per cent of the circumference of the armature, instead of 60 per cent.

70. Grouping of Armature Inductors. Drum and Ring Windings. While the armature of Fig. 212 is rotating in a clockwise direction, each armature conductor is generating an e.m.f. in a direction away from the reader (represented by a **cross** within the circle) as it cuts the flux under or near an *N*-pole, and an e.m.f. toward the reader (represented by a **dot** within the circle) as it cuts the flux in the field of an *S*-pole. These symbols will be adhered to; to remember them, the reader may consider that an arrow within the inductor is pointed in the direction of the e.m.f., and that he sees either the feather end ⊕ when the arrow and current are going away from him or the pointed end of the arrow ⊙ when the current is flowing toward him. Notice that the direction of e.m.f. in the various inductors naturally divides them into groups, the number of groups being equal to the number of poles, two in this case. The **neutral points** (p) are the points which mark the boundaries between the fields of adjacent poles — the points where a line of flux from an *N*-pole just touches the armature and emerges again immediately to go to the nearest *S*-pole. The e.m.f. induced in any inductor reverses direction as it passes a neutral point — being in one direction while it is moving on one side, in the opposite direction while it continues its motion on the other side, and zero just at the moment it passes the neutral point. The field is comparatively sparse or weak in the region between the pole tips, where the air path is long and the magnetic reluctance consequently high. Hence the e.m.f. throughout this space is almost zero, and the neutral point is said to be in the midst of a **neutral zone.** When the flux is distributed symmetrically with respect to the centers of the poles, as it is at zero load, the neutral points are situated midway between the adjacent tips of adjacent *N*- and *S*-poles, as illustrated in Fig. 212 at p and p'.

To get the greatest total useful effect from all the inductors upon the armature, we must group them together in such a way that they add their effects together, and do not oppose one another. If we desire particularly to get high voltages, we connect them together so that their induced e.m.f.'s are in **additive series.** If we would rather get large current capacity at moderate voltage, we group part of them in series and similar groups **in parallel.** The connections between inductors may be made in either of two ways, called **drum winding** and **ring winding** respectively. The drum winding is illustrated by Fig. 209 and 211. The end-connectors between the inductors are like chords across the ends of the armature, curving somewhat so as to pack together properly, require the minimum amount of wire, avoid the shaft and permit removal of the rotor. Every armature conductor which is wound lengthwise of the shaft is located on the outside surface of the armature core, and is a useful inductor contributing to the e.m.f. of the machine.

The ring winding produces effects exactly similar to those of the drum winding, and although long ago discarded on account of inherent defects, the diagram of it is much more convenient to use in explaining the action of electrical machinery. A sufficient excuse for using this diagram instead of that for drum-wound armature is that it saves time. In Fig. 213, adjacent inductors, those numbered 1 and 2, for instance, have been placed in electrical series so that their e.m.f.'s are added together and not opposed, by means of a small radial end-connector x on the front end of the armature

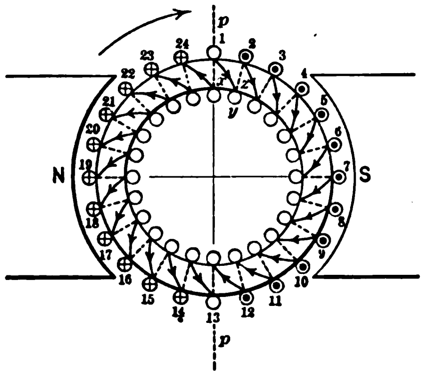


FIG. 213. A ring-wound armature. Note that only the numbered conductors cut lines of force. The others (x, y and z) merely serve to connect the numbered conductors to one another.

ture core, a conductor y on the inside of the core and another end-connector z at the further end of the armature. Such method of connection is repeated successively between all adjacent inductors. This armature winding is called the **ring winding** and is seen to be of the **closed** or **re-entrant type.** None of the end-connections like x and z, nor any of the inside conductors like y, contribute anything to the total e.m.f. of the series of inductors, because they cut no lines of flux. It is plain that the armature circuit contains more inactive material in the form of connections like x, y, z, than of active material in the form of inductors like those numbered. This proportion is also usually higher in a ring winding than in a drum winding for the same armature, resulting in a more expensive or less efficient dynamo for the same volts and amperes capacity. Moreover, the ring winding must be wound in place upon the armature laboriously and by hand, instead of being done separately by machines in formed coils which are subsequently laid upon the armature and soldered together into a complete circuit, as in the case of the drum winding. Finally, the inductance of the ring winding is greater than that of the equivalent drum winding, which would make an alternator have a poorer voltage regulation.* For these reasons, the drum winding is used universally for all types of dynamos, direct current or alternating current. However, the equivalent ring winding, on account of its simplicity, will be used for illustration instead of the drum winding wherever possible.

In a winding like that of Fig. 213, or any equivalent re-entrant drum winding, a current could flow locally within the closed mesh in the armature, even without any external circuit or load connected, if there were any resultant e.m.f. acting around this mesh. Any such local circulating current would heat up the armature and reduce the amount of useful current and power, or load, that it could deliver to an external circuit. However, the resultant e.m.f. around the closed mesh is at every instant equal to zero in such a case as this, where all of the inductors are in series.† This is because just as many lines of flux must **leave** the armature to return to *S*-poles of the field magnets, as **enter** the armature from the *N*-poles, however many these poles may be. Since all of the inductors are attached to the armature at uniform distances apart, and move at the same speed through a symmetrically distributed flux, the total number of **entering** lines cut per second by the inductors must at every instant be equal to the total number of **emerging** lines cut per second.

* See Second Course, Chapter I.

† There may be a circulating current under certain conditions in a parallel-wound or two-circuit armature; see pages 407 and 470.

Thus the e.m.f.'s induced in the closed circuit by these two actions are **opposite** in direction and **equal in value,** and the resultant is zero volts at all moments. Therefore the fact that this is a closed winding does not make its electrical action differ essentially from that of an open winding.

Prob. 12–6. Make a neat sketch similar to Fig. 213, but for a four-pole ring winding with 24 inductors equally spaced.

Prob. 13–6. In Fig. 214, the flux is assumed to be distributed uniformly under the face of each pole, and no flux entering the armature beyond the edges of the poles. Assuming the flux density to

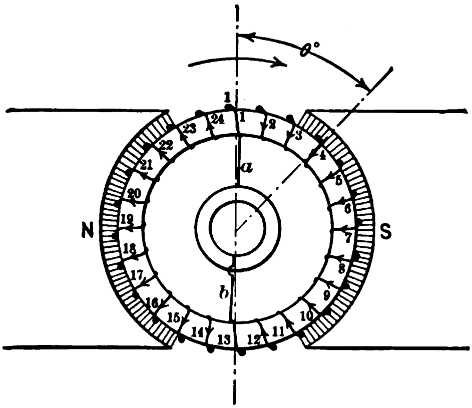


FIG. 214. A simple ring-wound armature.

be 50,000 lines per square inch at every point in the air gap between the armature and the pole faces, diameter of armature 6 inches, frequency 60 cycles, poles covering 58.3 per cent of the circumference of armature, length of pole-face parallel to shaft 8 inches, calculate the e.m.f. in each numbered inductor and the total e.m.f. in the closed mesh of the armature winding, at the instant shown. Calculate the total useful flux per pole.

Prob. 14–6. Repeat the calculations of Problem 13–6 and find the e.m.f. in each numbered inductor and the total e.m.f. of the ring, at an instant one-quarter period later.

Prob. 15–6. Suppose that the density of the useful flux in Fig. 214 varies from place to place around the entire circumference of the armature, in such a manner that the density (in lines entering armature per square inch of its surface) at any point θ degrees removed from the neutral point is represented by the equation $B = 45{,}810 \times \sin\theta°$. Conductors 1 and 13 lie on the neutral points. Calculate:

(*a*) The flux density, lines per square inch, at the position occupied by each numbered inductor in Fig. 214.

(*b*) The e.m.f. induced in each inductor at the moment the armature is passing through the position illustrated in Fig. 214.

(*c*) The total e.m.f. acting at this instant in the closed ring winding.

Prob. 16–6. Repeat the calculations of Problem 15–6, and find the e.m.f. in each numbered inductor and the total e.m.f. in the closed ring, at an instant one-quarter period later.

71. Tapping Points on Closed-circuit Windings. To get e.m.f. and current from the rotating armature to the stationary line wires, we must select points in the winding to which we may attach the sliding-contact connections, or collecting-rings. For a single-phase circuit having two wires, we need two rings. Unless these rings are attached to points on the winding having certain positions relative to each other, two difficulties will be encountered. First, the e.m.f. will be reduced by having some of the inductors oppose others in the same series or circuit; and, second, the current will be unequally distributed in the armature conductors, resulting in overheating the windings and in reduced output.

The proper location of taps depends upon the number of poles on the alternator.

This will be made clear by a number of examples. Fig. 214 shows the same facts as Fig. 213, but in a more conventional way. Only the front end-connections are shown in Fig. 214, and the direction in which the induced e.m.f. of the inductors acts through the circuit is shown by arrows marked upon these end-connections. In a two-pole single-phase armature such as this, each of the collecting-rings should be tapped to a single point of the windings. These points should be chosen so that, when one of them is under the middle of the *N*-pole, the other is at the same instant under the middle of the *S*-pole. This brings the taps diametrically opposite each other, as shown at *a* and *b* in Fig

214. The armature winding is thus divided into two parallel paths between the rings, each path being a series combination consisting of one-half the total number of inductors. The total e.m.f. between rings at each instant is equal to the sum of the e.m.f.'s induced in all of the inductors of one path, at that instant, this sum being the same for each path. Whatever current may be delivered to an external circuit connected between the rings will be divided equally between these two parallel paths, because their e.m.f.'s are equal and they are so constructed as to have equal resistances and impedances. To show how the total e.m.f. between rings alternates, consider Fig. 215 to 219, these being views of the same machine at suc-

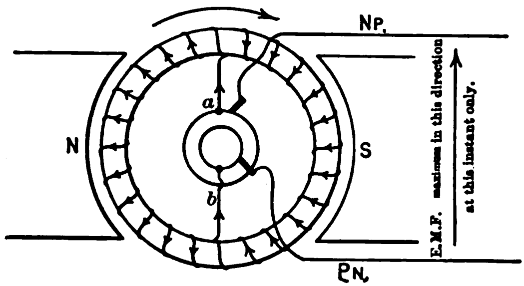


FIG. 215. The voltage between the rings *a* and *b* is a maximum at this instant.

cessive instants one-quarter revolution, or one-quarter period, apart. The e.m.f. between rings has its maximum value in Fig. 215, because all of the series inductors in either path have e.m.f.'s in the same direction at this instant. One-quarter revolution later, as in Fig. 216, the e.m.f. between *a* and *b* has become zero, because each path has half of its series inductors inducing an e.m.f. equal and opposite to that in the other half of the same series. After another quarter revolution, the conditions are as in Fig. 217, and the e.m.f. is again at its maximum value, but in the reverse direction. After another quarter period, the e.m.f. is again zero as in Fig. 218; and when a whole revolution has been

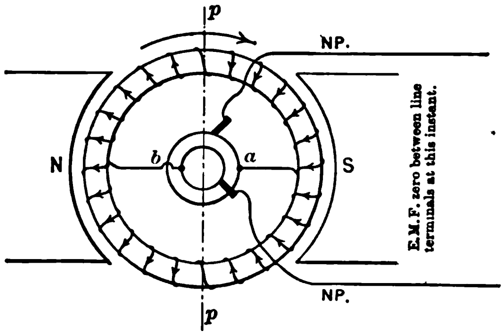


FIG. 216. The voltage between the rings a and b is now zero. The armature has turned through one-quarter of a revolution.

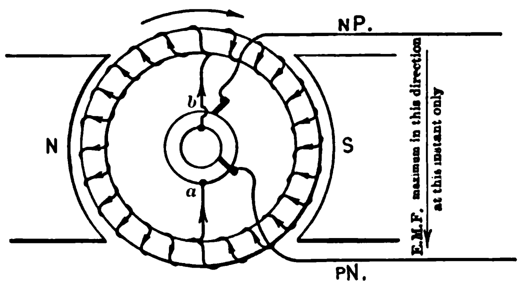


FIG. 217. The voltage across a and b is again a maximum although in the direction opposite that of Fig. 215.

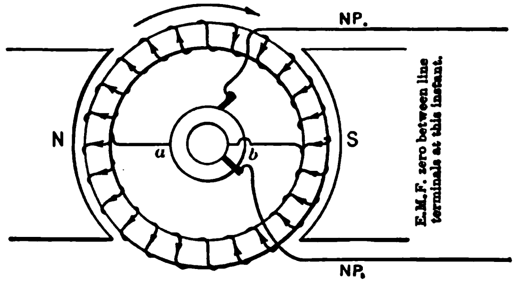


FIG. 218. The armature has turned through another quarter revolution and the voltage between a and b has again become zero as in Fig. 216.

completed, the cycle is complete (Fig. 219), and the e.m.f. is at its maximum value again as in Fig. 215. The voltage between rings will depend upon the height and form of the wave of e.m.f. induced in each inductor, upon the number and spacing of the inductors, and upon the resistance and reactance of the armature circuit and the armature current.

Prob. 17-6. Assuming a uniform flux distribution without fringing, under conditions as in Problem 13-6, calculate the total e.m.f.

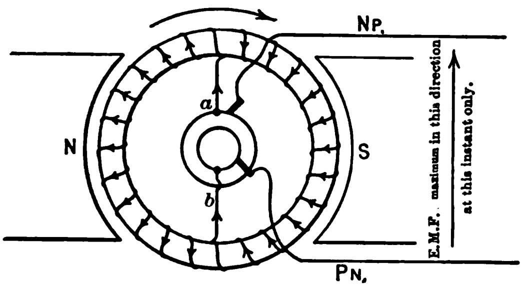


FIG. 219. The armature has now made a complete revolution and is in the same position as in Fig. 215. The voltage between the rings *a* and *b* is again a maximum, and in the same direction as in Fig. 215.

between rings at the instant that the armature passes through the position illustrated in Fig. 214.

Prob. 18-6. The armature is delivering a current of 100 amperes at the instant it is moving through the position illustrated in Fig. 215. How many amperes are flowing in each one of the inductors at this instant?

Prob. 19-6. (*a*) If the total induced e.m.f. between rings is a sine wave of 100 volts effective value, and the current in the armature is also a sine wave, with an effective value of 100 amperes, but lagging practically one-quarter period or 90 electrical degrees behind the induced e.m.f., what is the direction and value of the current in each inductor at the instant the armature is passing through the position shown in Fig. 216, when the e.m.f. is zero?

(*b*) Which inductors are opposing the motion of the armature (carrying current in same direction as induced e.m.f., and acting as generators), and which inductors are aiding the motion of the armature (carrying current in direction opposite to induced e.m.f., and acting as motors)?

(c) Measuring the generator or motor action in watts, or product of instantaneous amperes and instantaneous induced volts, calculate the net total generator action and motor action of the entire winding at the instant pictured in Fig. 216.

Prob. 20–6. (a) Repeat the calculations of Problem 19–6, for the same effective values of voltage and current, and the same phase lag of current behind e.m.f. (zero power-factor), but consider the armature at the instant pictured in Fig. 215, when the e.m.f. is maximum.

(b) From comparison of the results of Problems 19–6 and 20–6, and similar ones which may be solved for a number of equidistant armature positions, what can you say concerning the amount of torque and power required to drive a generator on a purely inductive load with zero power-factor? The armature resistance is neglected here.

Prob. 21–6. Assuming the useful flux to be distributed around the armature so that the intensity of the field through which the inductors move at successive instants or positions varies harmonically, and that B conforms to the equation given in Problem 15–6, calculate the total instantaneous e.m.f. between the two collecting-rings in Fig. 215.

Prob. 22–6. Calculate the total e.m.f. between rings as in Problem 17–6, for successive positions of the armature $\frac{1}{24}$ revolution apart, or successive instants 15 electrical- or time-degrees apart, and covering one complete period or cycle. Draw a curve between time as abscissas (frequency 60 cycles) and instantaneous volts between rings (zero load) as ordinates. Calculate the effective value of this e.m.f. curve.

Prob. 23–6. Solve Problem 22–6 for the machine described in Problems 21–6 and 15–6. Compare the wave-form of resultant e.m.f. between rings with that of the e.m.f. in a single inductor.

Prob. 24–6. Draw on tracing paper or cloth a sine curve of e.m.f. ($e = E_m \sin \theta$) to the same scale of abscissas used in Problem 23–6, using the same maximum value of e.m.f. as found to exist in the curve of e.m.f. between rings in Problem 23–6. Lay this sine curve over the curve in the preceding problem, and compare the forms of the two curves. Does this prove to you that the sum or resultant of any number of sine curves of any height or amplitude, and any phase relation, but all of the same frequency, is also a sine curve?

If the taps to the two rings had been made at some other points on the bipolar winding, as shown in Fig. 220, it may be shown by the same process of reasoning that the e.m.f. between rings or taps is alternating, but has a smaller maximum value and also a smaller effective value. The sum of e.m.f.'s in the path *apb* is at every instant equal to the sum of e.m.f.'s in the path *ap'b*. To demonstrate this, extend the line of *a* straight through the center to *a'*, and extend *b* to *b'*. The sum of e.m.f.'s between *a* and *b'* is always equal to the sum of e.m.f.'s from *b* to *a'* at the same instant. Since these

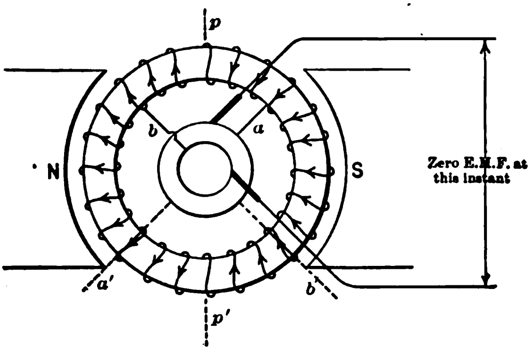


FIG. 220. Note that the armature winding is tapped at two points 90° apart, instead of 180°.

two sums are in opposite directions in the same series path (*ap'b*), they neutralize each other exactly, leaving as the resultant total e.m.f. of path *ap'b* only the inductors in the series *b'p'a'*. This total e.m.f. is seen to be the same in value and toward the same ring as the total e.m.f. in the other path *apb*. The position of the armature in Fig. 220 is such that both of these totals have zero value. Hence if the total e.m.f.'s in the two parallel paths are equal, any current which may be delivered to the external circuit through the rings, will be divided **unequally** between these paths, the path *apb* carrying a much larger share than the path *ab'p'a'b*, because its impedance is less, it having fewer turns. If the impedances were of the same ratio as the resistances, the former would carry three times as much current as the latter. In Fig. 221 the e.m.f. between rings has its maximum value, which is seen to be less than the maximum in Fig. 215 for the same armature.

The result of a comparison between this case and that where windings were tapped at diametrically opposite points shows that for a two-pole machine the greatest alternating e.m.f. is obtained when the armature windings are tapped

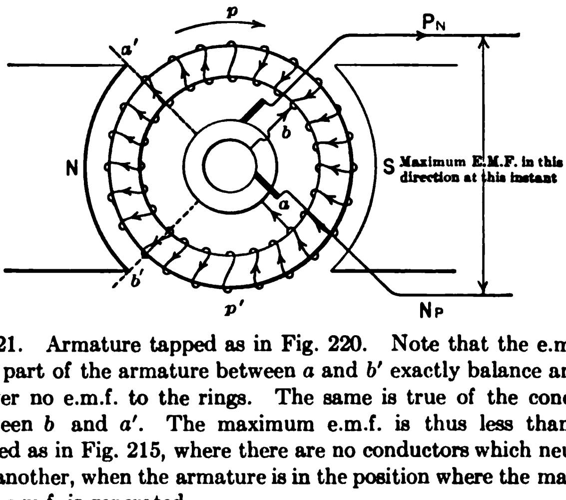


FIG. 221. Armature tapped as in Fig. 220. Note that the e.m.f.'s in that part of the armature between a and b' exactly balance and thus deliver no e.m.f. to the rings. The same is true of the conductors between b and a'. The maximum e.m.f. is thus less than when tapped as in Fig. 215, where there are no conductors which neutralize one another, when the armature is in the position where the maximum total e.m.f. is generated.

at points diametrically opposite each other. When tapped at any other points the e.m.f. is less and the current distribution in the windings will not be uniform.

Prob. 25–6. Under the conditions stated in Problem 13–6, calculate the form of the e.m.f. wave produced between rings tapped as in Fig. 220. After drawing the e.m.f. wave to scale, calculate the effective value of the e.m.f. Compare these results with those of Problem 22–6.

Prob. 26–6. Under the conditions stated in Problem 15–6, calculate the form of the e.m.f. wave produced between rings tapped as in Fig. 220. After drawing the e.m.f. wave to scale, calculate the effective value of the e.m.f. Compare with results of Problem 23–6.

Prob. 27–6. (*a*) At rated full load a certain two-pole single-phase ring-wound armature like that in Fig. 215 delivers from its rings a current of sine wave-form and 100 amperes effective value; what should be the effective value of the amperes flowing in each path, or in each inductor?

(*b*) If the two rings are tapped as in Fig. 220, is the effective value of current the same for all inductors in both paths? Compare the values of the current in each conductor of a single path.

(*c*) Assuming the impedance of either path to be proportional to the length of conductor in the path, calculate the ratio of the effective amperes in one path to the effective amperes in the other path.

(*d*) In what ratio are the watts used up in heating each path in Fig. 220 greater than they would be when delivering the same amperes output, if the two rings were properly tapped as in Fig. 215?

Prob. 28–6. In Problem 27–6, by what percentage are the total watts lost in the whole armature greater or less when tapped as in Fig. 220 than when tapped properly as in Fig. 215, assuming the same amperes output in both cases and the same winding?

Prob. 29–6. By what percentage must the amperes output be changed from rated full-load amperes, if this machine is tapped as in Fig. 220 instead of properly as in Fig. 215, in order that the total rate of heat development in the whole armature winding shall not be above that permitted at rated full load?

Prob. 30–6. The percentage reduction in load capacity due to incorrect tapping of the collecting-rings is really determined more by the local heating of particular inductors or paths than by the average heating of the entire armature. By what percentage must the amperes output from the rings be reduced below rated full-load amperes output, when tapped as in Fig. 220, in order to prevent any single inductor from developing heat faster than it would develop at rated full load when the same winding is tapped properly as in Fig. 215?

72. Armature Taps for a Synchronous Converter. The method of obtaining a direct current from this same armature by applying a commutator is illustrated in Fig. 222. The segments of the commutator are thoroughly insulated from one another and from the frame; they are tapped to equidistant points of the armature winding, these taps virtually dividing the winding into coils. In fact, the few ring-

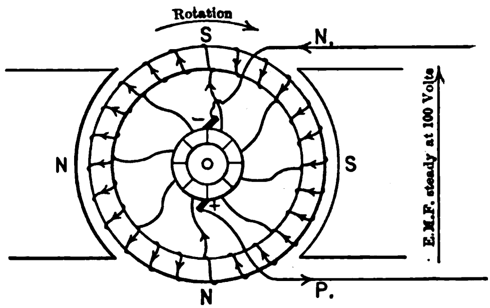


FIG. 222. The armature of Fig. 213–221, tapped and connected to commutator so as to deliver direct current.

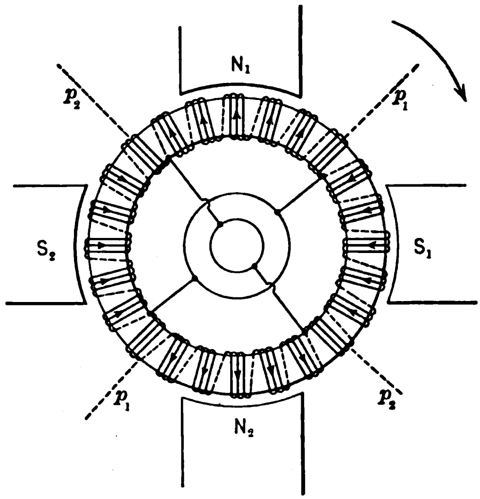


FIG. 223. A ring-wound armature in which the winding consists of equally-spaced coils of equal number of turns each.

wound machines which are manufactured, have their wire wound on in equidistant coils of equal number of turns as in Fig. 223, the adjacent ends of adjacent coils being soldered

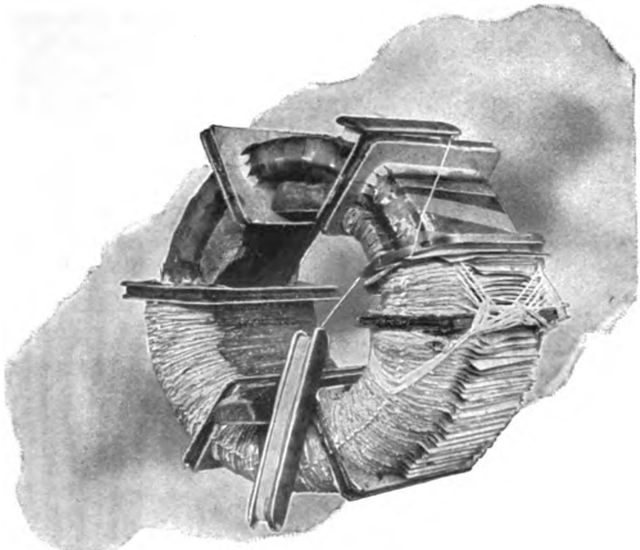


FIG. 224. A ring-wound armature in the process of construction. The coils are wound on the core by hand. *Fort Wayne Works.*

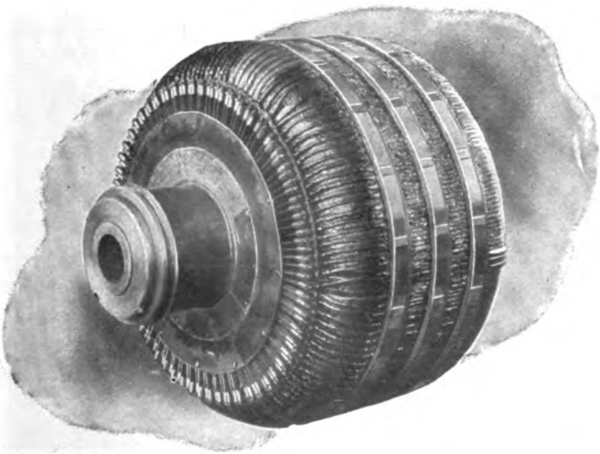


FIG. 225. The finished ring-wound armature of Fig. 224. Note that adjacent coils are soldered together, and leads brought from the juncture points to the commutator segments.

together. For a direct-current machine, the junction points of coils are tapped to the commutator bars. See Fig. 224 and 225.

The same winding may deliver both direct current and alternating current at the same time; it is then called a double-current generator. Fig. 226 shows the connections for this purpose to be merely a combination of those in Fig. 222 and 215. Instead of tapping separately from the windings to the commutator bars and to the collecting-rings, it is

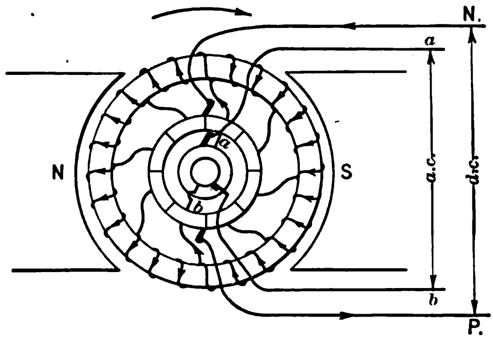


FIG. 226. The armature of Fig. 213–222 tapped and furnished with both commutator and collecting-rings. It can thus deliver both direct and alternating current at the same time. This is also the method of tapping a synchronous converter.

possible to tap from the rings to the proper commutator bars. Thus, single-phase alternating current may be had from any direct-current generator, simply by mounting upon the shaft two insulated rings and connecting them to commutator bars separated by a proper distance. A synchronous converter or a Dobrowolsky three-wire direct-current generator could be improvised in this way.

Prob. 31–6. When the speed and field excitation of a two-ring double-current generator like that in Fig. 226 are such as to produce 220 volts direct current at the terminals N and P, the e.m.f. between the rings a and b is about 160 volts effective. If the total load of 10 kw. were equally divided between the alternating-current and the direct-current circuits, how many amperes must be delivered to each circuit? The alternating-current circuit is non-inductive.

Prob. 32–6. Suppose the dynamo shown in Fig. 226 were operating as an "inverted" converter (that is, converting direct e.m.f. to alternating e.m.f.), taking direct current from 220-volt mains and delivering alternating current at 160 volts. When delivering 10 kw. to the single-phase alternating-current circuit at unity power-factor, it operates at an efficiency of 88 per cent. What is the alternating-current amperes output and the direct-current amperes input?

Prob. 33–6. If the converter of Problem 32–6 were delivering its 10 kw. to an inductive load having a power-factor of 0.8, what would be the amperes and kilovolt-amperes on the alternating-current side, and the amperes and kilowatts on the direct-current side? The efficiency in this case is 86 per cent.

Prob. 34–6. By inspection of Fig. 226, what relation do you observe between the e.m.f. at the direct-current brushes and the maximum instantaneous value of the alternating e.m.f. between the collecting-rings? Draw a sketch of this machine as it appears at the instant when the alternating e.m.f. between rings has zero value.

Prob. 35–6. If the e.m.f. between rings in Fig. 226 were exactly of sine wave-form, what would be its effective value when the e.m.f. between direct-current terminals is 230 volts?

73. Multipolar Alternators. When we consider a **four-pole** alternator with a closed winding, we find that the method of tapping must differ from that which is proper for a two-pole alternator. Fig. 227 illustrates the same ring-wound armature used in the preceding figures, but without any collecting-rings attached, and placed in a four-pole field. It will be noticed that the same inductors are now naturally divided into **four** groups or zones, giving four neutral points p between groups, instead of two. The inductors in each cutting zone are adjacent to each other and all of them have e.m.f. in the same direction. The conventional sketch, with direction of the e.m.f.'s marked on the end-connections between inductors, is shown in Fig. 228. The total resultant e.m.f. around the closed ring is zero, notwithstanding the

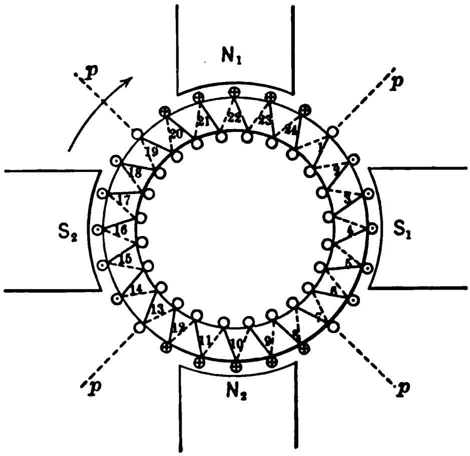


FIG. 227. The armature of the preceding figures placed in a four-pole field. Note that there are now four neutral zones.

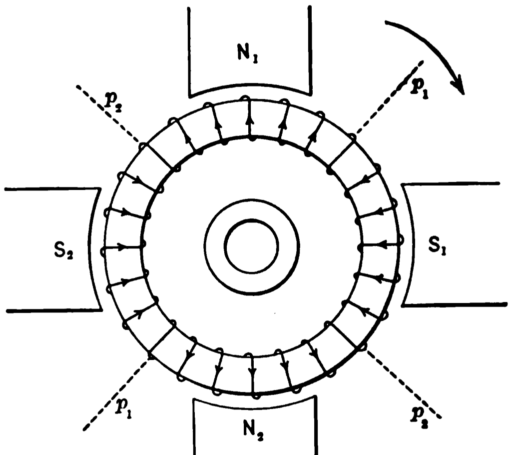


FIG. 228. The direction of the induced e.m.f. in each inductor is here shown in the conventional way for the conditions of Fig. 227.

increased number of poles and the new grouping of inductors. Fig. 229 and 230 show two positions of a single-phase or two-ring armature, tapped at two diametrically opposite points, as was shown in Fig. 215 to be proper for a two-pole field, but here used with a four-pole field. In both of these positions, as in any position which the student may choose to draw, the e.m.f. between the rings is zero. That is, a closed armature winding tapped properly for two poles will not deliver anything when used with a four-pole field.

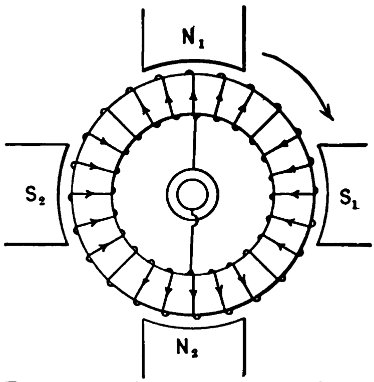


FIG. 229. The armature tapped for a two-pole field and placed in a four-pole field. Note that the e.m.f. across the rings is zero.

In order that alternating-current armatures be interchangeable they must be not only magnetically and mechanically similar, but they must be wound for the same number of poles.

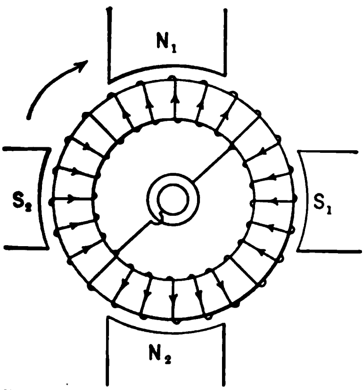


FIG. 230. The armature of Fig. 229 turned 90 electrical degrees. Still there is no e.m.f. induced across the rings.

When the **closed winding** is used with a **four-pole armature, each ring** should be tapped to **two points** which are electrically similar — that is, so located that there is no voltage between them, or so that one of these points of the winding is under the middle of one N-pole at the same instant the other point is at the middle of the other N-pole. Leads from these two tapping points then go to the same ring. Two corresponding taps are taken from points directly under *S*-poles and brought to the other ring. The taps for one ring should be located in such a way with respect to the taps of the other ring that all of the parallel paths through the armature between the two rings shall be exactly similar — that is, an equal number of series inductors in each path, giving equal total e.m.f.'s and equal resistances. In this way only will the total armature amperes be divided equally between the

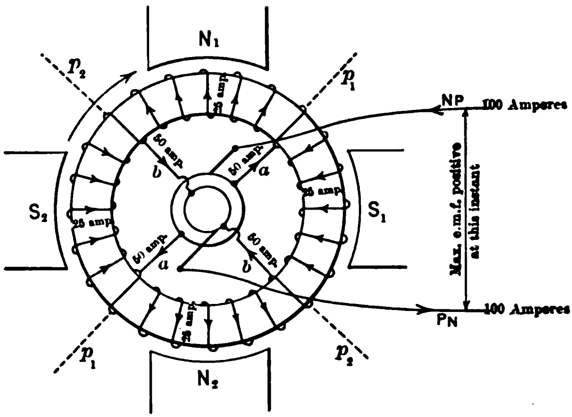


FIG. 231. The armature of the preceding figures properly tapped for a four-pole field. It is now at the position of maximum voltage between the rings.

paths, so that every inductor carries equal current and is equally heated, and the total volts and amperes are the greatest possible to be obtained from the winding. Thus, Fig. 231 is a single-phase ring winding tapped properly for four poles, pictured in the position where the instantaneous e.m.f. has its maximum value, in what we may choose to call the **positive** direction for the various circuits. Fig. 232 shows the same winding at an instant one-quarter period later (note that this corresponds to one-eighth revolution in a four-pole alternator), when the e.m.f. between rings is zero. Fig. 233 shows the winding again at an instant one-half period later than Fig. 231, or when the e.m.f. between

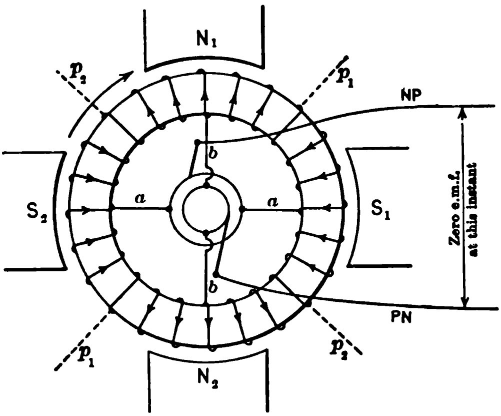


FIG. 232. The armature 90 electrical degrees ahead of its position in Fig. 231. The e.m.f. here is zero.

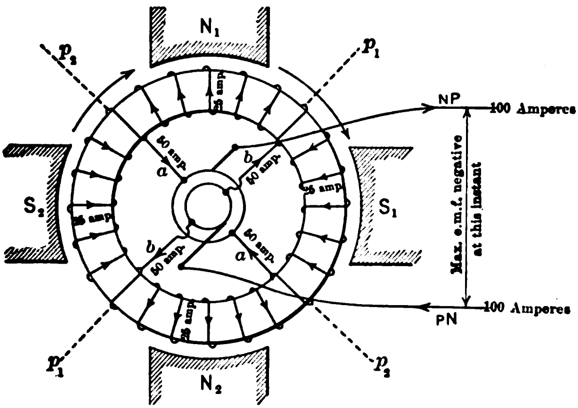


FIG. 233. The armature 90 electrical degrees ahead of its position in Fig. 232 and 180 electrical degrees ahead of its position in Fig. 231. The e.m.f. between rings is again a maximum but in the opposite direction.

rings has its maximum value in the opposite or negative direction. If the rings are connected to a load, and the entire circuit is non-inductive, then the current in every inductor will be in phase with the total induced e.m.f., and the arrows in Fig. 231 will represent the direction of currents as well as e.m.f.'s in every path or part of the circuit. These pictures show the manner in which a current whose instantaneous value is 100 amperes in the external circuit would divide under these conditions. If taps *b* had not been located exactly midway between taps *a*, the result would have been that two of the four parallel paths through the armature would have been shorter and of lower impedance than the other two, and the current and heating in the short paths would have been excessive, for the same (rated full load) amperes output from the collecting-rings. The volts also between rings would have been reduced, and both of these defects would cut down the power which could be delivered from the generator without injuring it.

From the foregoing, a general rule might be deduced, to tap any multipolar winding for two rings.

Holding the rotor stationary in any position, attach one of the two rings to all points on the winding situated directly under the middle of North poles, and attach the other ring to all points situated in corresponding positions, directly under the middle of all South poles.

Thus, in an eight-pole two-ring closed winding, each ring would be tapped to four equidistant points of the winding, the taps from one ring being midway between the taps from the other ring.

Winding schemes for single-phase alternators are explained in Chapter IX.

Prob. 36–6. (*a*) Draw a single-phase ring-wound armature in a six-pole field.

(*b*) If the armature has altogether 24 inductors, how many inductors are there in series in each of the parallel paths?

(*c*) If this machine is rated to deliver 120 amperes at 120 volts, what is the total effective volts generated in each path, and the amperes delivered by each path?

Prob. 37–6. The armature of a two-pole single-phase alternator has been burned out, and to resume operation again as quickly as possible, someone proposes to insert another spare two-ring armature that happens to be available of the same dimensions and rated voltage and amperes capacity. This armature, however, was wound and tapped for a four-pole field.

(*a*) Draw sketches to show the new armature in positions of maximum and zero total instantaneous e.m.f. in the two-pole field, and explain whether or not it will operate, and why.

(*b*) If you decide that it will not operate, how would you change it to make it operate?

Prob. 38–6. One of the taps between the winding and one ring of a single-phase four-pole alternator with a closed winding, having been insecurely attached originally, becomes disconnected.

(*a*) By what percentage is the current in each path thereby made greater or less than it should be, when the alternator is delivering its rated full-load current to the external circuit?

(*b*) On account of this mishap, what is the greatest percentage increase, and the greatest percentage decrease, in the watts heat loss in any armature conductor, at rated full-load amperes output?

Note. To obtain percentage values, as in the above problem, the exact value of total volts or amperes delivered by the armature, or of resistance or impedance per inductor in the armature winding, are immaterial because they cancel out in the computation. Any voltage or any current or any resistance per conductor may be assumed, provided only that these values are adhered to throughout the calculations. The effects of armature reaction are neglected in this problem.

Prob. 39–6. What is the greatest percentage of rated full-load current output that could be delivered from the alternator in Problem 38–6 without heating the entire armature at a total rate exceeding that permitted at rated full load?

Prob. 40–6. If no single inductor may be permitted, without danger, to develop heat at a wattage rate greater than it does at rated full load with the machine in good working order, by what percentage must the full-load output be reduced on account of the breaking of the tap in Problem 38–6?

SUMMARY OF CHAPTER VI

AN ELECTROMOTIVE FORCE is generated whenever there is relative motion between a conductor and a magnetic field so that one cuts the other.

IN A DIRECT-CURRENT GENERATOR it is always the conductors (forming the armature) which move.

IN AN ALTERNATOR either the conductors or the field may move, although usually it is the field. THUS REVOLVING-FIELD alternators are the more common, the field coils being excited by a direct current from an outside source.

THE INSTANTANEOUS E.M.F. in any conductor may be found from the following equation:

$$e = \frac{BlV}{10^8},$$

where

e = e.m.f. at any instant.
B = density of flux in which the conductor lies at that instant.
l = cutting length of the conductor.
V = relative speed of field and conductor at given instant.

B, l and V must all have same LENGTH units.

THE ARMATURE WINDING of a closed-circuit alternator is exactly the same as that of a closed-circuit direct-current generator, and an alternating e.m.f. is induced in both.

THE COMMUTATOR serves as a reversing switch for the direct-current generator and rectifies the voltage delivered to the terminals.

THE COLLECTING-RINGS are used in a single-phase alternating-current generator, in the place of the commutator. Taps are brought to these rings in such a manner that the greatest possible e.m.f. exists across them.

ONE RING IS ATTACHED to all points situated directly under the middle of all North poles; the other is attached to all points situated under the middle of all South poles at the same instant. Any other method of tapping the armature winding and attaching it to the collecting-rings results in a lower e.m.f. and unbalanced currents in the armature windings; hence a smaller output.

THE OUTPUT OF A GENERATOR is limited by the maximum heating of any single armature conductor.

BOTH COMMUTATOR AND COLLECTING-RINGS may be attached to the same armature winding. Thus the same machine may supply both alternating and direct currents at the same time, as a double-current generator. Or it may receive alternating-current power at the collecting-rings to drive it as a motor, and deliver direct current from the commutator as a generator, or vice versa. In this case it is called a synchronous converter.

PROBLEMS ON CHAPTER VI

Prob. 41–6. Draw a curve to rectangular coördinates, to illustrate the distribution of useful flux around the armature in Problem 15–6. As abscissas use the position of some point on the moving armature, measured in inches of its path moved through after passing the neutral point; the corresponding ordinate will be the field intensity of the useful flux entering the armature at this point, or B as obtained from the equation. Plot entering flux upward from the zero of B, and emerging flux downward, and make the curve cover an entire revolution of the armature. Calculate also the total useful flux per pole of this machine, as determined from the area of this curve, or its length and average height. Compare this value with the total flux per pole in Problem 13–6.

Prob. 42–6. The armature of Fig. 215 produces through an external short-circuit between the rings a sine wave of current whose effective value is 100 amperes. If the inductive reactance of the armature is five times as great as its resistance, redraw Fig. 215, 216, 217 and 218, showing the direction and value of the current in each inductor for these positions of the armature when operating under these conditions.

Prob. 43–6. The armature of Fig. 215 has a total induced e.m.f. of 230 volts between rings and delivers current to a circuit whose total resistance, including the whole armature, is 4 ohms and total inductive reactance 3 ohms. Calculate:

(*a*) The effective amperes delivered from the rings.

(*b*) The effective amperes delivered by each path.

(*c*) The instantaneous amperes in each of the inductors on the armature in the position pictured in Fig. 215.

Prob. 44–6. If we decrease the flux per pole by 30 per cent without altering the frequency, in an alternator which before the change was delivering 220 volts at the terminals, what voltage do we get? Zero load.

Prob. 45–6. If we reduce the frequency of a 60-cycle alternator to 25 cycles without altering the field excitation, by what percentage is its terminal voltage (at zero load) increased or diminished?

Prob. 46–6. If we increase the frequency of a 25-cycle alternator to 60 cycles and desire to keep the same terminal voltage, by what percentage must we increase or diminish the total flux per pole?

Prob. 47–6. Solve Problem 27–6 on the supposition that the builder, in attempting to tap the rings properly as in Fig. 215, or to inductors 1 and 13, made a slight mistake of one inductor, and got the rings tapped to inductors 1 and 12.

Prob. 48–6. Solve Problem 28–6 for the conditions stated in Problem 47–6.

Prob. 49–6. Solve Problem 29–6 for the conditions stated in Problem 47–6.

Prob. 50–6. Solve Problem 30–6 for the conditions stated in Problem 47–6.

Prob. 51–6. Would there be any objection to using a ring-wound armature tapped for two poles, in a four-pole field, provided it were mechanically suited to the new location?

Prob. 52–6. If the armature of a two-polé alternator had burned out and a spare armature of exactly the same dimensions and rated voltage for a similar four-pole alternator were available, could it be used or not? Explain.

Prob. 53–6. A four-pole single-phase ring-wound alternator has the same number of uniformly-spaced inductors as a certain other single-phase ring-wound alternator having two poles. How must the amount of flux per pole compare in these two machines, in order that they may both give the same e.m.f. between rings, at same speed at zero load?

Prob. 54–6. If all dimensions of the armature are the same in both of the alternators of Problem 53–6, how do their armature resistances compare (as measured between rings)? Allowing the same total watts I^2R loss for the entire armature in both cases, what is the ratio between the permissible amperes output from the rings in these two machines?

Prob. 55–6. If the two alternators of Problem 53–6 were operated with the same flux per pole, how would their terminal e.m.f.'s compare? How would the kilowatts capacity at unity power-factor compare, taking into account also the results of Problem 54–6?

Prob. 56–6. (*a*) If the two alternators of Problem 53–6 were required to operate at the same frequency, how would their speeds compare?

(*b*) Having the same number of inductors per pole, but with these relative speeds, how should the total flux per pole compare when producing the same terminal e.m.f. at zero load?

CHAPTER VII

ALTERNATING-CURRENT GENERATORS: POLYPHASE

MASTERY of the **principles** set forth in this chapter will enable the student to understand the actions and relations in any network or system of interconnected alternating-current circuits. It should be understood, however, that most of the demonstrations given refer to a distributed ring winding only. The ring winding enables us to understand quite easily what is meant by a **polyphase** alternator. Usually polyphase alternator windings are of the open-circuit drum type, as will appear in Chapter IX, but the ring winding enables us to work with the polyphase idea more directly. Moreover, the polyphase ring winding illustrates very closely a polyphase synchronous converter winding.

74. Two-phase Alternator: Meaning of Lead and Lag. Consider the same two-pole ring-wound armature pictured in Fig. 215, having altogether 24 inductors, or 24 coils, each containing as many inductors as may be necessary to deliver the required e.m.f. at the rings. Suppose four collecting-rings to be mounted upon the shaft instead of two, and let one pair of these rings r_1 be tapped to diametrically opposite points of the winding, as a_1 and b_1, Fig. 234. The other pair of rings r_2 will be tapped also at diametrically opposite points, as a_2 and b_2; but the points a_2 to b_2 will be chosen midway between the points a_1 to b_1. From Fig. 234 it will be seen that the e.m.f. a_1 to b_1 has its **maximum** instantaneous value **at the same moment** that the e.m.f. a_2 to b_2 has its **zero** instantaneous value. From Fig. 235, representing conditions at an instant one-quarter period later, it is seen that the e.m.f. a_1 to b_1 has decreased to its zero value while the e.m.f. a_2 to b_2 has increased to its maximum value. If the variations of e.m.f.

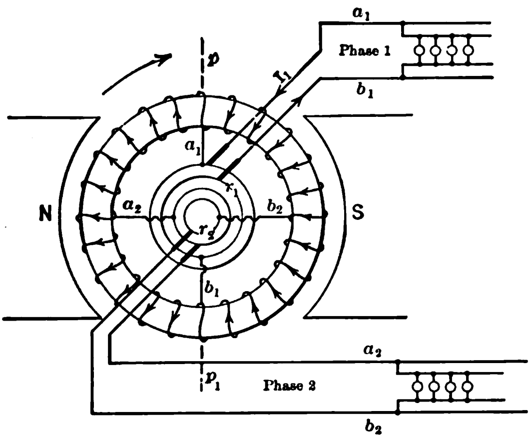


FIG. 234. The armature is tapped at four points. The voltage across the points a_1–b_1 is at a maximum at this instant, while the voltage across a_2–b_2 is zero. a_1–b_1 and a_2–b_2 thus have a phase difference of 90°.

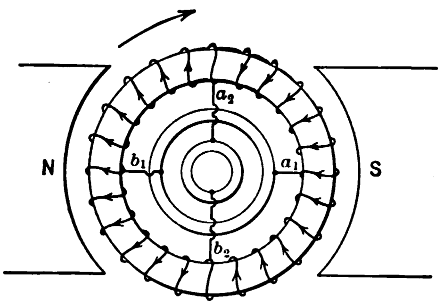


FIG. 235. The armature of Fig. 234 has turned through 90 electrical degrees and now the voltage across a_1–b_1 is zero while the voltage across a_2–b_2 is maximum.

across both r_1 and r_2 were observed for one or more complete cycles, it would be seen that the form of the e.m.f. waves and the maximum and effective values of e.m.f. across these two pairs of rings are equal, but that there is a phase difference of one-quarter period, or 90 electrical degrees, between them. If the direction of e.m.f. from the a ring to the b ring is chosen as positive in **both** pairs (a_1 to b_1, and a_2 to b_2), or if the e.m.f. from b toward a be chosen as positive in **both** pairs (b_1 to a_1, and b_2 to a_2), then in either case the e.m.f. across r_1 would be said to **lead** the e.m.f. across r_2 by 90 degrees (for direction of rotation as shown); because the voltage across a_1 to b_1 would reach its maximum value in the direction chosen as positive, just one-quarter period **before** the voltage across a_2 to b_2 would reach its maximum value in the direction chosen as positive. But if the positive direction across r_1 is chosen a_1 toward b_1 while the positive direction across r_2 is chosen b_2 toward a_2, then the e.m.f. across r_1 would be said to **lag** behind the e.m.f. across r_2 by 90 degrees; because the former would then reach its maximum value in the direction chosen as positive, just one-quarter period **after** the latter would reach its maximum value in the direction chosen as positive. It is thus seen that lead and lag are descriptive terms having a purely relative and not an absolute meaning, **when applied to separate circuits.** The same phase difference, say of 120 degrees, between the currents in circuit A and in circuit B, may be described accurately either as I_A leading I_B by 120 degrees, or as I_A lagging behind I_B by 60 degrees, depending upon which direction through each of the two circuits is chosen to be called the positive direction. When we are dealing with e.m.f.'s or currents, or current and e.m.f., **in one and the same circuit,** however, the terms lead and lag have very definite meanings not dependent upon the actual direction chosen as positive, because the chosen positive direction refers to everything that goes on in the same circuit. Thus, when one e.m.f. reaches its maximum positive value one-quarter period before another e.m.f. acting in the same circuit reaches its maximum positive value, the first e.m.f. without doubt **leads** the second e.m.f., no matter which direction through the circuit is chosen as positive. The first thing to be done, therefore, in solving problems with any combination of different circuits in which the e.m.f.'s or currents are not in phase, is to choose, **for each circuit**, one direction which is to be called positive. Only then may vector diagrams be drawn to have any definite and certain meaning.

75. Closed-circuit Two-phase Systems must have Four Line Wires. The two-wire or single-phase circuit connected across rings r_1 in Fig. 234, we shall call Phase 1; the other single-phase circuit connected across rings r_2, we call Phase 2; these two circuits **considered together** are called **a two-phase (four-wire) circuit.** It has been explained in Chapter III how in some two-phase circuits the number of wires is reduced to three by combining two of the four wires into one. To do this in Fig. 234 would be equivalent to connecting a short-circuit externally between tap a_1 and either b_2 or a_2, or between tap b_1 and either b_2 or a_2. In any case, a section of the armature winding, generating a very considerable effective e.m.f., would be short-circuited and burned out. It is, for this reason, impossible to distribute two phases from a closed type armature winding by means of three wires. This limitation does not affect most alternators, because they usually have open windings, but it does prevent a synchronous converter from being connected to any three-wire two-phase system, except through transformers (see *Second Course*, Chapter IX).

76. Four-phase Alternators. The closed winding tapped at four electrically equidistant points to four collecting-rings, as in Fig. 234, may be considered also as a **four-phase** alternator. According to the ordinary definition, a two-phase circuit is any combination of wires from which two similar e.m.f.'s of equal value and 90 electrical degrees phase difference may be obtained. Such circuits usually have four distributing wires, but may sometimes have only three. **No other phase relation,** even between similar e.m.f.'s of equal value, would permit the combination to be called a **two-phase** circuit, in accordance with usual custom. The two-phase generator is often called a "quarter-phase" generator, because the e.m.f.'s in the two single-phase circuits which may be connected to it have one-quarter period, or 90 electrical degrees, phase difference.

In the ordinary quarter-phase alternator with open winding, the connection between the two phases is merely mechanical and magnetic, and not usually electrical unless they are connected somewhere to a three-wire distributing system. That is, if you were to test out the four terminals of the quarter-phase armature winding with a voltmeter, you would be able to find only two combinations of two wires which would deflect a voltmeter. These would be separate pairs with no electrical connection between them and each pair would be one of the phases. But in Fig. 235 you may obtain four additional combinations or pairs of terminals which would deflect a voltmeter, namely a_1 to a_2, a_2 to b_1, b_1 to b_2, and b_2 to a_1. A separate two-wire or single-phase circuit could be connected to each of these pairs of terminals, and current could be taken from all at once, or from any one pair or combination of pairs. If positive directions in these various circuits be chosen as a_1 to a_2, a_2 to b_1, b_1 to b_2, and b_2 to a_1 (that is, in uniform or continuous rotation around the closed mesh or ring), then the e.m.f. a_1 to a_2 leads the e.m.f. a_2 to b_1 by 90 electrical degrees or one-quarter period; a_2 to b_1 leads b_1 to b_2 by 90 degrees; b_1 to b_2 leads b_2 to a_1 by 90 degrees; and b_2 to a_1 leads a_1 to a_2 by 90 degrees. Although the four circuits are all alike as far as the voltmeter readings show, each differs in phase from the next adjacent by the same amount, or 90 degrees, hence they are said to form a four-phase circuit when considered all together.

The four terminals of a two-phase machine with closed winding therefore form a four-phase as well as a two-phase circuit. The difference between the two-phase and the four-phase from the same machine is very real as far as concerns the value of the voltage, because a voltmeter connected across any single phase (as a_1a_2) of the four-phase combination would show a very different reading from that obtained when it is connected across either single phase (as a_1b_1) of the two-phase combination. In fact, it may be seen from the way the groups or circuits which generate these e.m.f.'s in the armature winding are made up in Fig. 235, that the e.m.f. (a_1b_1) of **one** of the **two-phase circuits** is at every instant equal to the resultant of the e.m.f.'s of **two** (a_1a_2 and a_2b_1) of the **four-phase circuits.** This is equivalent to saying that the effective voltage (a_1 to b_1) is equal to the vector sum of the effective voltages (a_1 to a_2) and (a_2 to b_1) when the positive directions are chosen so as to coincide.

77. Relation between Two-phase and Four-phase E.M.F.'s. To get the numerical relation between two-phase and four-phase voltages in Fig. 235, we begin by deciding what shall be considered the positive direction of e.m.f. in every part of the armature winding. As before, let us assume that a current is flowing in the positive direction in the winding when it passes from one inductor towards the next one counted in a counter-clockwise direction around the closed circuit of the armature. That is, a_1 to a_2, a_2 to b_1, b_1 to b_2, b_2 to a_1. An e.m.f. is positive when it tends to produce a positive current. This is equivalent to saying that the e.m.f. in any coil or division of the armature winding is **positive** while that coil is **rotating in a clockwise direction under a North pole,** with the armature wound as in Fig. 235. In a **ring winding,** any one coil attains the maximum instantaneous value of e.m.f. at the moment the middle of the coil passes through the point where the useful flux is most dense — in this case, under the middle of a pole. Now, with clockwise rotation, as in Fig. 235, the middle of coil a_2b_1 passes under the middle of N-pole just one-quarter revolution (corresponding to one-quarter period, or 90 degrees in a two-pole machine) after the middle of coil a_1a_2 passes under the middle of the same N-pole. The middle of coil b_1b_2 passes the middle of the N-pole just one-quarter period after middle of coil a_2b_1 passes middle of same N-pole and so on, as the armature continues to turn. This is equivalent again to saying that the e.m.f. a_2 to b_1 lags 90 degrees behind e.m.f. a_1 to a_2; e.m.f. b_1 to b_2 lags 90 degrees behind e.m.f. a_2 to b_1; e.m.f. b_2 to a_1 lags 90 degrees behind e.m.f. b_1 to b_2; and e.m.f. a_1 to a_2 lags 90 degrees behind e.m.f. b_2 to a_1.

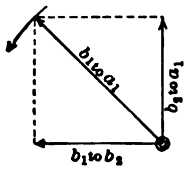


FIG. 236. Note in Fig. 234 that the voltage b_1–b_2 leads the voltage b_2–a_1 by 90°. The resultant b_1–a_1 is, therefore, the vector sum of these two since they are in series.

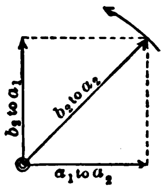


FIG. 237. The voltage across b_2a_1 leads the voltage across a_1a_2 by 90°. The resultant voltage across the two is b_2a_2.

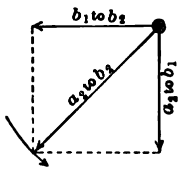


FIG. 238. The voltage across a_2b_1 leads the voltage across b_1b_2 by 90°. The resultant voltage of the two in series is a_2b_2.

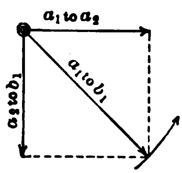


FIG. 239. The voltage across a_1a_2 of Fig. 234 leads the voltage across a_2b_1 by 90°. The resultant voltage of these two in series is a_1b_1.

78. Vector Diagram for E.M.F. Relations in a Four-phase Alternator. The vector diagrams to show these relations are as given in Fig. 236, in which the same length of vector is chosen to represent the effective value of voltage across any one of the four phases. The vectors are all considered to be rotating counter clockwise as usual, at the rate of one complete revolution per cycle. Fig. 236 shows the relations between (b_1 to b_2) and (b_2 to a_1) and their resultant (since they are in series), which is (b_1 to a_1); Fig. 237 shows the relations between (b_2 to a_1) and (a_1 to a_2) and their resultant in series, which is (b_2 to a_2); Fig. 238 shows the relations between (b_1 to b_2) and (a_2 to b_1) and their resultant in series, which is (a_2 to b_2); Fig. 239 shows the relations between (a_2 to b_1) and (a_1 to a_2) and their resultant in series, which is (a_1 to b_1). Fig. 240 combines all of these preceding vector diagrams, and shows the relationship, both as to phase and value, between all the various e.m.f.'s which it is possible to measure on this four-ring alternator. All the e.m.f.'s are drawn to the same scale. In this assembly diagram it is easily seen that the e.m.f.'s (b_1 to b_2) and (a_1 to a_2) are of equal effective value but opposite phase; and e.m.f.'s (b_2 to a_1) and (a_2 to b_1) are of equal value but opposite phase. Also, we see that e.m.f. (b_2 to a_2) is equal and opposite to e.m.f. (a_2 to b_2), which is, of course, true because these two e.m.f.'s are taken between exactly the same points, the difference being only in the direction which is chosen to be positive. That is, the e.m.f. (a_1 through a_2 to b_1) is of the same value and phase as the e.m.f. (a_1 through b_2 to b_1); and of the same value but opposite phase to the e.m.f. (b_1 through b_2 to a_1).

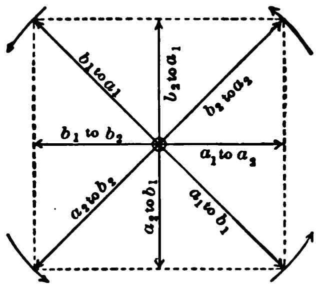


FIG. 240. The vector diagram combining the vector diagrams of Fig. 236–239. This diagram shows the voltage across the different parts of the armature in their true values and phase relations to one another.

Reversing the direction called positive in any circuit changes all the phase relations between it and any other circuit by just 180 degrees, or reverses the vector representing the e.m.f.'s and currents of that phase in any diagram, just as though we had reversed the connections while keeping the positive direction fixed.

Notice also, in Fig. 240, that the effective e.m.f. per phase of the two-phase circuit is represented by the diagonal of a square and is equal to $\sqrt{2}$ (or 1.414) times the effective e.m.f.

per phase of the four-phase circuit. This diagram would represent conditions just as truly for a four-ring machine of any number of poles, if it were tapped symmetrically. Drawing a vector diagram to represent these relations, of course, presumes that the e.m.f.'s in all phases are sine waves, which is practically true in most machines, but may not be strictly so. Any scale of voltages may be used, depending upon the particular values of speed, useful flux per pole, and number of inductors per pole, used in the particular machine to which the diagram refers.

79. The Currents in the Coils of a Two-phase Generator. Balanced Load. By similar vector diagrams we may find the relation between the amperes in each line wire and the amperes in each armature coil. The amperes delivered from rings r_1 are drawn equally from the two parallel paths $a_1a_2b_1$ and $a_1b_2b_1$. The amperes delivered from rings r_2 are drawn equally from the two parallel paths $a_2b_1b_2$ and $a_2a_1b_2$. The e.m.f.'s in one of these phases (or pair of paths), through the winding, are 90 degrees out of phase with the e.m.f.'s in the other phase (or pair of paths). If the currents in the two phases have identical phase relation to the corresponding e.m.f.'s which produce them, or have equal power-factors, then the current in paths $a_2b_1b_2$ and $a_2a_1b_2$ lags behind the e.m.f. across rings r_2, by the same amount that the current in paths $a_1a_2b_1$ and $a_1b_2b_1$ lags behind the e.m.f. across rings r_1. That is, the phase difference between the currents in path $a_1a_2b_1$ or path $a_1b_2b_1$, and the currents in path $a_2b_1b_2$ or path $a_2a_1b_2$, is 90 degrees, because the phase difference between the e.m.f. across rings r_1 and the e.m.f. across rings r_2 is 90 degrees, and each current lags the same amount behind its e.m.f. Now any single coil in the armature is **common to two paths**; for instance, the coil a_1a_2 is common to path $a_1a_2b_1$ and path $a_2a_1b_2$. And since the currents in these two paths are 90 degrees out of phase, it follows that the current in coil a_1a_2 is actually the vector resultant or sum of two currents differing in phase by 90 degrees, each current being equal in value to one-half of the amperes delivered to one of the phases, or to one-half of the current delivered from one of the rings.

Thus, to take a concrete example, suppose the alternator illustrated in Fig. 240a, which is merely Fig. 234 repeated for convenience, is operating two-phase, and delivering 100 amperes from rings r_1 to Phase 1 and 100 amperes from rings r_2 to Phase 2, both phases having equal terminal pressure of say 230 volts, and equal power-factors. The 100 amperes from taps a_1 and b_1 to Phase 1 are made up of 50 amperes from

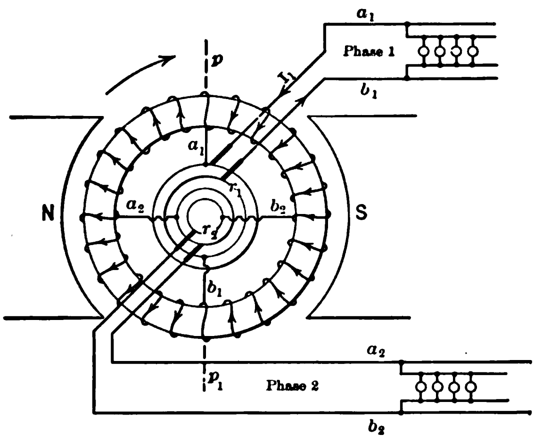


FIG. 240a. The coil a_1a_2 is in one path which carries half the current of phase 1 and is also in one path which carries half the current of phase 2. The coil a_1a_2 thus carries two component currents which have a phase difference of 90° with each other.

path $a_1a_2b_1$ and 50 amperes from path $a_1b_2b_1$, these two paths being in parallel with respect to rings r_1, and having equal e.m.f.'s and equal impedances. Similarly, the 100 amperes from taps a_2 and b_2 to Phase 2 is made up of 50 amperes from path $a_2b_1b_2$ and 50 amperes from path $a_2a_1b_2$. Hence the total current flowing in coil a_1a_2 **which is part of both paths** is made up of two currents, each having the same value of 50 amperes but a phase difference of 90 degrees between them; the value of this total current is therefore $\sqrt{(50)^2 + (50)^2} =$

70.7 amperes, as indicated in Fig. 241. Note that 70.7 = ½ of 100 times $\sqrt{2}$. If I = current in main line, and I_A = current in each coil of the armature winding, then

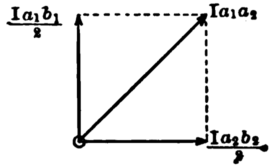


FIG. 241. The current in the section a_1a_2 is made up of half the current delivered to phase 1, Ia_1b_1, and half the current delivered to phase 2, Ia_2b_2. These two parts have a phase difference of 90°.

$$I_A = \frac{\sqrt{2}\,I}{2} = \frac{I}{\sqrt{2}} = 0.707\,I.$$

80. Currents in Each Coil for Unbalanced Loads. If the two phases had been **unbalanced but of equal power-factor,** the internal conditions within certain ones of the armature coils would have been as shown in Fig. 242, where it is assumed, for example, that Phase 1 of Fig. 240a delivers 160 amperes while Phase 2 delivers 40 amperes. The current in any coil or conductor of the armature is here seen to be equal to $\sqrt{80^2 + 20^2} = 82.5$ amperes. If the voltages of the two phases were to remain equal after the load is unbalanced, it is evident that the total volt-amperes and total watts output from the generator to both phases would be the same when one phase was delivering 160 amperes and the other 40 amperes as when they were both delivering 100 amperes. Yet the current in certain armature conductors and coils is increased in the ratio 82.5/70.7 by reason of the unbalancing of the load. Since the resistance of every conductor or coil is practically constant, the rate at which heat is developed in these conductors is increased in the ratio

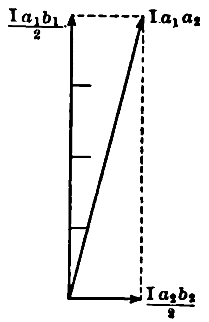


FIG. 242. The vector diagram for the current in coil a_1a_2 if phase 1 were carrying four times the current of phase 2. Ia_1b_1 is the current in phase 1 and Ia_2b_2 the current in phase 2.

$$\frac{R \times 82.5^2}{R \times 70.7^2} = \frac{6800}{5000} = \frac{1.36}{1.00}.$$

The interpretation of this result is, that if the total load of a two-phase alternator with closed winding is maintained constant but the phases are unbalanced in the ratio 4:1, the rate of heat development in some of the armature conductors is increased 36 per cent beyond what it should be with balanced load. This would burn out some parts of the armature winding if the machine were operated at a total load equal to its rated full load. The calculation would be altered slightly when applied to any specific actual machine, because of the fact that the voltages of the phases are not equal on unbalanced load. If this unbalancing were carried to the limit, so that rated full load for the whole generator were carried on one phase only, and no current whatever on the other phase, the two-phase alternator would in reality be a single-phase machine. In this particular case, 200 amperes would be drawn from a_1b_1, and nothing from a_2b_2, and the current in all conductors in each path $a_1a_2b_1$ and $a_1b_2b_1$, and therefore in the whole armature, would be 100 amperes. The rate of heating in every armature conductor would therefore be increased above what it should be, in the ratio $100^2/70.7^2 = 2.0/1.0$. The whole armature would be heated twice as rapidly as it should be if the same load were divided equally between the two phases instead of being all concentrated on one phase.

If it is desired to operate this two-phase alternator as a single-phase alternator, the total output must be reduced below the rated full-load for two phases, but it may be larger than half of the rated two-phase load. We have seen that if the rated two-phase load is 100 amperes on each phase, each conductor is allowed to carry 70.7 amperes without overheating. Operating only one phase with 70.7 amperes in each of the two parallel paths (into which the winding is divided by the taps of this phase) makes the largest allowable amperes output on this single phase equal to 2×70.7 amperes, or 141.4 amperes. The voltage would probably be fixed at the same value as before by the requirements of the distributing system. Even if it were not, it could not be increased much on account of the danger of overheating the field windings and armature core. Hence the total kilowatt or kilovolt-ampere output would be proportional to the total amperes output or would be less than output under rated conditions in the ratio 141.4/(100 + 100), or 70.7 per cent of rated (two-phase) full load.

81. Complete Current and Voltage Distribution for Unbalanced Load. It is both instructive and useful to see what happens in the **entire** armature when the load on the phases becomes unbalanced — that is, different either as to amperes or as to power-factor. We cannot do this from the partial diagrams shown in Fig. 241 and 242; these tell the truth, only as it relates to a **part** of the winding. To get the entire truth, we must draw the vector diagram so as to include the entire armature, and not merely a single coil. In Fig. 243, we have repeated the complete e.m.f. diagram of Fig. 240, as a groundwork upon which to place the current vectors. Suppose that we now consider the complete armature of Fig. 240a to be operating two-phase with balanced loads, the equal currents in the two phases being each in phase with the induced e.m.f. which produces it. We lay out a vector I_1 (the current in Phase 1) in phase with $E_{a_1b_1}$, and a vector I_2 of equal length in the direction of $E_{a_2b_2}$, to represent the current in Phase 2. Now one-half of I_1 and one-half of I_2 combine vectorially to produce the current $I_{a_2b_1}$ which flows in the armature conductors between taps a_2 and b_1. We combine $\frac{I_1}{2}$ and $\frac{I_2}{2}$ directly, as they were drawn, because the positive direction for Phase 1 (from a_1 through a_2 to b_1) coincides, within the coil a_2b_1, with the positive direction for Phase 2 (from a_2 through b_1 to b_2). On account of the symmetry of the diagram we can see, merely by inspection, that this resultant current in coil a_2b_1 is equal to $\sqrt{(I_1/2)^2 + (I_2/2)^2} = \frac{1}{2}$ of $\sqrt{I_1^2 + I_2^2}$, which agrees with our previous calculation (see Fig. 241). Now consider coil a_1a_2 in

Fig. 240a, to which these vector diagrams all refer. The current $I_{a_1a_2}$ consists of two components; one whose numerical value is $\frac{I_1}{2}$ flowing from a_1 toward a_2, and belonging to the branch $a_1a_2b_1$ of Phase 1; the other has a numerical value of $\frac{I_2}{2}$ amperes but flows from a_2 toward a_1, belonging to the branch $a_2a_1b_2$ of Phase 2. Note particularly that the positive

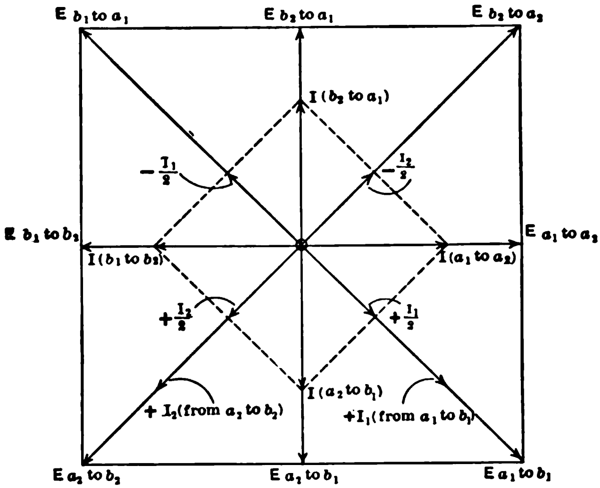


FIG. 243. The outside square and diagonals represent the voltage relations of Fig. 240a, and are merely a copy of Fig. 240 on an enlarged scale. The inside square and diagonals represent the currents in various parts of the armature.

directions of the two component currents through this coil are opposite each other. Moreover, these two currents differ in time-phase by one-quarter period, or 90 electrical degrees. With regard to coil a_1a_2 the problem is then to find the resultant of two currents, of $\frac{I_2}{2}$ and $\frac{I_1}{2}$ amperes, the positive directions of which are opposite, and with $\frac{I_2}{2}$ reaching its maximum value at a moment 90 degrees after $\frac{I_1}{2}$ reaches its maximum value (in the opposite direction). Fig. 244 is a simple representation of the currents $\frac{I_1}{2}$ and $\frac{I_2}{2}$, showing how they flow in opposite directions through that part of the winding between a_1 and a_2. But any periodic quantity alternating as a sine wave will reach its maximum value in either direction just one-half period, or **180 degrees, before and after** its maximum value in the **opposite** direction.

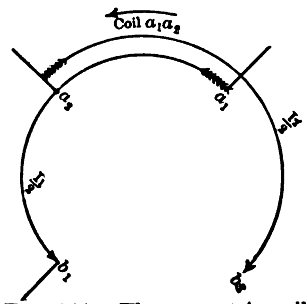


FIG. 244. The current in coil a_1a_2 is made up of two currents $\frac{I_1}{2}$ and $\frac{I_2}{2}$, the positive directions of which are opposite.

Now, we have seen that $\frac{I_1}{2}$ reaches its maximum value in the opposite direction to what is positive for $\frac{I_2}{2}$, just 90 degrees ahead of $\frac{I_2}{2}$. But $\frac{I_1}{2}$ must reach its maximum value in the **same** direction as $\frac{I_2}{2}$, either 180 degrees sooner or 180 degrees later than this. If $\frac{I_1}{2}$ reaches its maximum value in the **same direction** as $\frac{I_2}{2}$ 180 degrees **sooner,** then it must reach it 90° + 180°, or 270° ahead of the maximum positive value of $\frac{I_2}{2}$. We would then say that $\frac{I_2}{2}$ lags 270 degrees behind $\frac{I_1}{2}$.

But if $\frac{I_1}{2}$ reaches its maximum value in the **same direction** as $\frac{I_2}{2}$, 180 degrees **later** than it reached its maximum value in the opposite direction, then it must reach it 90° − 180°, or

90° behind the maximum positive value of $\frac{I_2}{2}$. We would then say that $\frac{I_2}{2}$ leads $\frac{I_1}{2}$ by 90 degrees.

But a lead of 90 degrees is exactly the same as a lag of 270 degrees, as is seen by referring to Fig. 245. This diagram is really a piece of the larger Fig. 243, containing only the cur-

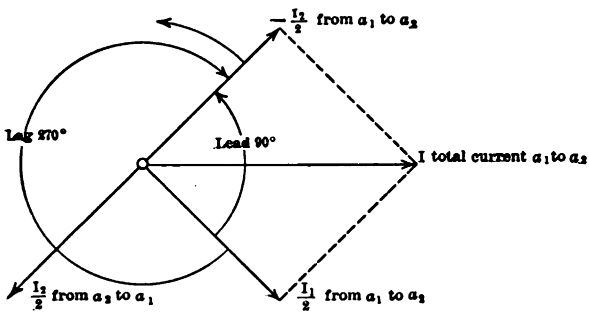


FIG. 245. The vector $\frac{I_2}{2}$ may be said to lead the vector $\frac{I_1}{2}$ by 90°, or to lag 270° behind it.

rent vectors about which we are speaking, in the same relation they have in the larger figure. The vector of $\frac{I_2}{2}$ had to be reversed before it could be added to the vector of $\frac{I_1}{2}$, because it is presumed by agreement, that the angles between vectors representing various alternating quantities in the same circuit are equal to the phase difference with respect to the same positive direction for all quantities. We desire to find the resultant vector representing current from a_1 to a_2 rather than from a_2 to a_1, for the reason that we desire the positive direction of current in each coil to coincide with the direction which we chose to consider positive for the e.m.f. in the same coil. This was counter clockwise in the closed ring, or from a_1 to a_2, from a_2 to b_1, from b_1 to b_2 and from b_2 to a_1. This enables us to read easily from the diagram the phase angle between current and e.m.f. in each coil.

In a similar manner, it is seen from Fig. 243 that $I_{b_1b_2}$ is the vector sum of $\frac{I_2}{2}$ direct and $\frac{I_1}{2}$ reversed, and $I_{b_2a_1}$ is the vector sum of $\frac{I_2}{2}$ reversed and $\frac{I_1}{2}$ reversed. The minus sign before vectors shows that they are reversed. Fig. 243 proves

(1) That the current in each coil of the armature is in phase with the total induced e.m.f. in that same coil, when the current output to each external circuit is in phase with the e.m.f. induced in that circuit.

(2) That the current in all coils or parts of the armature winding is the same when the external load is balanced between the phases.

(3) That in a two-phase ring-wound armature with closed winding, having load of equal power-factor on the two phases, the current in each armature conductor is equal to $\frac{1}{2}\sqrt{I_1^2 + I_2^2}$.

So much could have been stated on the basis of the incomplete diagrams in Fig. 241 and 242. But we could not, without the complete diagram, such as Fig. 246 (or of equivalent mathematical equations), understand what is happening in all the armature coils when the power-factors of the several phases become different. Suppose this same ring-wound two-phase generator is supplying rated full-load current of 100 amperes to each phase, but that the current in Phase 1 lags 60 degrees behind the induced e.m.f. of Phase 1 (that is, the power-factor is 0.5 for the entire circuit of Phase 1, including the armature), and that the current in Phase 2 is in phase with the induced e.m.f. in Phase 2. In Fig. 246 the complete vector diagram of e.m.f.'s deduced in Fig. 240 is first repeated. To this is added a vector I_1 whose length represents 100 amperes to scale, and which has a phase lag of 60 degrees behind the e.m.f. of Phase 1 ($E_{a_1b_1}$) which produces it; also, a vector I_2 whose length represents 100 amperes in Phase 2, and which is in phase with the e.m.f. of

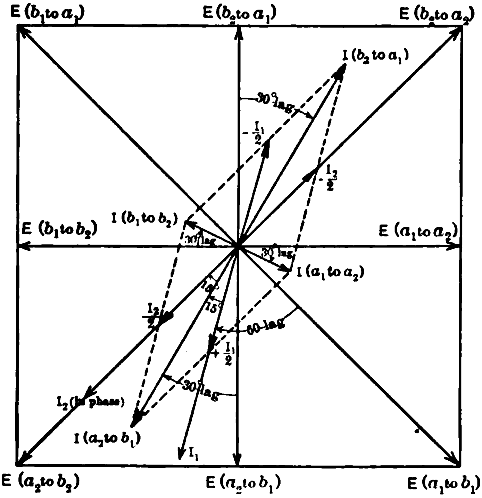


FIG. 246. The large square represents the voltage across the coils and machine terminals. The shorter vectors represent the currents in the various coils when the power-factors of the phases are unlike.

Phase 2 ($E_{a_2b_2}$). Proceeding as with Fig. 243, we find that:

$I_{a_1a_2}$ equals vector sum of $\frac{I_1}{2}$ direct and $\frac{I_2}{2}$ reversed.

$I_{a_2b_1}$ equals vector sum of $\frac{I_1}{2}$ direct and $\frac{I_2}{2}$ direct.

$I_{b_1b_2}$ equals vector sum of $\frac{I_1}{2}$ reversed and $\frac{I_2}{2}$ direct.

$I_{b_2a_1}$ equals vector sum of $\frac{I_1}{2}$ reversed and $\frac{I_2}{2}$ reversed.

After we have performed these reversals and summations, we find, by interpreting the vectors according to the values and scales chosen in this particular case, that:

(1) The currents $I_{(a_2 \text{ to } b_1)}$ in coil a_2b_1 and $I_{(b_2 \text{ to } a_1)}$ in coil b_2a_1 are each about 96.6 amperes, whereas they cannot exceed 70.7 amperes without danger of burning them out, according to the rating of the machine. That is, half of the conductors on the armature are being heated above normal rate in the ratio

$$\frac{R \times (96.6)^2}{R \times (70.7)^2} = \frac{1.860}{1.000},$$

or 86.0 per cent faster than they should be heated. This mathematical figure would be modified somewhat in practice by consideration of the other two coils, which are not developing heat up to the permitted rate, and are therefore tending to absorb some heat from these overheated coils and thus keep the local temperature rise from being as great as it would appear to be from these figures.

(2) The current in coils a_1a_2 and b_1b_2 is about 25.9 amperes, whereas it is permitted by the rating to be 70.7 amperes without danger. That is, the other half of the armature conductors are being heated below normal rate in the ratio

$$\frac{25.9^2}{70.7^2} = \frac{0.1340}{1.0000},$$

or 13.4 per cent as fast as they could be heated.

(3) If the temperature rise within the armature (which tends to injure the insulation and therefore limits the permissible load) were proportional to the average rate or total rate of heat development over the entire armature, this manner of loading would be perfectly safe; because the average rate of heating for the two halves would be

$$\left(\frac{1.860 + 0.134}{2} = \frac{0.997}{1.000}\right)$$

That is, the armature would be heated just about as fast as permissible and would therefore run at the highest temperature permissible, though no higher.

(4) But the danger in overloading a dynamo appears in each individual wire and coil, on account of the rise of temperature that occurs within the insulation when the heat attempts to pass through it and get to the body of the armature and to the air currents which are supplied for ventilation. The insulation, usually having low thermal conductivity, dams up the heat, so to speak, as it tries to pass through and escape from the electrical conductors, which are completely encased by the insulation and in which the heat is generated when current flows. Nevertheless it would not be correct to consider the temperature rise (and danger) of the insulation around any particular conductor as being dependent only upon the rate of heat generation in that conductor; because if a nearby conductor or part of the machine happened to be developing heat at less than the normal or permissible rate and is therefore at less than normal temperature, the heat from the first conductor would naturally be dissipated faster than usual toward the cooler regions, and it would reach a temperature correspondingly less than it might be supposed to reach if considered without regard to its surrounding conductors or parts. If we were to consider each coil by itself, in this case, we find that coils a_2b_1 and b_2a_1 would have their insulation quickly ruined, resulting in breakdown and stoppage of the generator, unless the current in these coils were reduced to normal value of 70.7 amperes corresponding to the rated full load of the alternator; that is, a reduction to $\frac{70.7}{96.6}$, or $\frac{73.2}{100.0}$ part of its present value. This means, that the current output of each phase would have to be reduced to 73.2 per cent

of its present value of 100 amperes, or to 73.2 amperes, which at constant voltage means a proportional reduction in the kilowatts output of the whole generator.

(5) The total power output of the entire machine is now equal to

Power generated in Phase 1

$$= E_1I_1 \cos\theta_1 = E_1 \times 100 \times 0.5 = 50.0\,E_1 \text{ watts}$$

Power generated in Phase 2

$$= E_2I_2 \cos\theta_2 = E_2 \times 100 \times 1.0 = 100.0\,E_2 \text{ watts}$$

Total power generated by the machine = $(50\,E_1 + 100\,E_2)$

Assuming $E_1 = E_2 = 100$ volts, total power generated = 15,000 watts. In order to make sure that the temperature of every part of the insulation does not exceed the amount permitted at the rated full load of the machine, the output at constant voltage would have to be reduced to $0.732 \times 15{,}000 = 11{,}000$ watts. The maximum steady output from this machine, with unity power-factor on both phases, is $(100\times100\times1.0) + (100 \times 100 \times 1.0) = 20{,}000$ watts. Therefore, the net effect of a 50 per cent reduction in power-factor of only one phase is to reduce the load capacity of the **entire machine** by almost 50 per cent.

These calculations have been followed through in some detail because they illustrate several important points:

(1) How much general knowledge concerning the results of operation under various conditions may be obtained for any machine, even without much specific knowledge concerning its construction other than that given by the nameplate, provided only that a man knows vector diagrams thoroughly, with some arithmetic and trigonometry. The facts herein deduced could be applied to any two-phase machine with closed winding. To be sure, closed windings and two-phase are not universally used, but they offer the clearest introduction to the more difficult calculations which appear in the "Second

Course," Chapter IX, in connection with synchronous converters. The methods of reasoning would be the same in any case. Such facts concerning the internal currents in an interconnected system or a closed winding of several phases could not easily be learned by experiment, and, if not calculated, might be discovered only after burning out a machine.

(2) How to solve problems relating to values and phase relations of currents and e.m.f.'s in a complicated network. The results even have an approximate relation to the performance of a synchronous converter (which, though not ring-wound, has a closed multi-circuit armature winding), particularly when carrying a mechanical load. They illustrate in a concrete fashion, principles which may be applied to other machinery and circuits.

If anyone were inclined to doubt the accuracy of a diagram like Fig. 246, the following check could be applied:

The sum of the watts generated in all the coils on the armature ought to be equal to the total watts in the two-phase circuits.

Fig. 246 is drawn to the scales $\begin{cases} E_{a_1b_1} = E_{a_2b_2} = 100 \text{ volts.} \\ I_1 = I_2 = 100 \text{ amperes.} \end{cases}$

Measuring the other vectors to the same scale, we have:

$$E_{a_1a_2} = E_{a_2b_1} = E_{b_1b_2} = E_{b_2a_1} = 100/\sqrt{2} = 70.7 \text{ volts.}$$
$$I_{a_1a_2} = I_{b_1b_2} = 25.9 \text{ amperes.}$$
$$I_{a_2b_1} = I_{b_2a_1} = 96.6 \text{ amperes.}$$

$I_{a_1a_2}$ reaches its maximum value in the direction from a_1 to a_2 just 30 degrees after $E_{a_1a_2}$ reaches its maximum value in the same direction, or $I_{a_1a_2}$ lags 30 degrees behind $E_{a_1a_2}$. Also, $I_{a_2b_1}$ lags 30 degrees behind $E_{a_2b_1}$; $I_{b_1b_2}$ lags 30 degrees behind $E_{b_1b_2}$; and $I_{b_2a_1}$ lags 30 degrees behind $E_{b_2a_1}$. Hence we may write the following:

Power developed in coil $a_1a_2 = 70.7 \times 25.9 \times \cos 30° =$ 1,587 watts
Power developed in coil $a_2b_1 = 70.7 \times 96.6 \times 0.866 =$ 5,913 watts
Power developed in coil $b_1b_2 = 70.7 \times 25.9 \times 0.866 =$ 1,587 watts
Power developed in coil $b_2a_1 = 70.7 \times 96.6 \times 0.866 =$ 5,913 watts
Total power developed in all coils = 15,000 watts

Power output in Phase 1

$= E_{a_1b_1} \times I_{a_1b_1} \times \cos\theta_1 = 100 \times 100 \times 0.5 = 5{,}000$ watts

Power output in Phase 2

$= E_{a_2b_2} \times I_{a_2b_2} \times \cos\theta_2 = 100 \times 100 \times 1.0 = 10{,}000$ watts

Total power output in the two phases $= 15{,}000$ watts

Prob. 1-7. A 220-volt two-phase two-pole alternator with a ring winding delivers at rated full load, 50 amperes to each phase of the load.

(*a*) Draw a sketch showing the windings and taps, the rings and the loads.

(*b*) If the load is all of unity power-factor, what is the total watts output of the generator?

(*c*) If the load is inductive with an 80 per cent power-factor in both phases, what is the kw. output of the alternator?

(*d*) What is the total kv-a. rating of this alternator?

Prob. 2-7. (*a*) What other e.m.f. could be obtained from the rings of the alternator of Problem 1-7 while delivering rated full load?

(*b*) Between which rings, on your sketch of the winding?

(*c*) What is the phase angle between this e.m.f. and each of the two-phase e.m.f.'s?

Prob. 3-7. Under the conditions stated in Problem 1-7, how many amperes flow in each armature conductor?

Prob. 4-7. If the alternator of Problem 1-7 were loaded as a four-phase generator, what is the largest value of amperes that could be used in each of the four phases without making the amperes in each conductor of the armature any greater than when loaded as rated (see Problem 1-7)? The four phases are balanced (same amperes in each) and non-inductive.

Prob. 5-7. What changes of connections in Fig. 234 would make the armature suitable as a single-phase generator in a four-pole field?

Prob. 6-7. If used thus as a single-phase armature in a four-pole field, how many amperes could the generator of Problem 5-7 deliver without allowing any armature conductor to be heated at any greater wattage rate than was permitted under rated full-load conditions (see Problems 1 and 3)?

Prob. 7-7. When the generator of Fig. 234 is operating as a two-phase generator at rated frequency and voltage, the phases, carrying non-inductive loads, become unbalanced so that one of them carries 120 amperes instead of 100 as rated. What is the greatest current that may be taken on the other phase without permitting any conductor on the armature to carry more current or develop heat faster, than is permitted at rated full-load?

Prob. 8-7. If the machine of Problem 7-7 were able to maintain practically full rated terminal volts on both phases without excessive losses in either field coils or armature core, would the permissible kilowatts output on this unbalanced load be greater or less than rated full-load kilowatts, and by what percentage?

Prob. 9-7. If only one of the two phases of the generator in Problem 7-7 were loaded and the output on the other phase were zero, how many amperes could be delivered by the loaded phase without developing any greater I^2R loss in the **whole** armature winding than is permitted under rated full-load conditions?

82. Three-phase Alternator: Delta-Connected. Fig. 247 represents a two-pole alternator with ring winding, tapped in a manner suitable for connection to a "three-phase circuit." A three-phase circuit is a combination of wires, or coils, so related to each other that three distinct alternating e.m.f.'s of the same frequency may be obtained between various pairs or combinations of terminals or wires. These e.m.f.'s have **equal effective values** and a phase difference of one-third period, or **120 degrees, between any two of the e.m.f.'s.** It has already been shown (see Art. 40 and Chapter IV) that three such distinct phases may be transmitted over four or three line wires instead of six and that three is the usual number. It will be shown in this chapter that the use of three phases rather than one, two or four phases, when all systems are on a similar basis, permits the greatest amount of power to be handled with a given amount and cost of material in the generators. In the "Second Course," Chapter IV, the same truth is demonstrated for transmission lines. We desire, therefore, to connect the coils of the simple ring winding shown in Fig. 213, to **three collecting-rings,** in such a way that the three e.m.f.'s obtainable between these rings shall compose a correct three-phase, and so that the currents may be distributed as nearly uniformly as possible in the armature coils when the phases are balanced. The method of tapping the rings to the windings, shown in Fig. 247, is suitable only for two poles; for multipolar machines, methods will be shown in the chapter on Synchronous Converters.

The arrow on each end-connection in Fig. 247 indicates the direction of e.m.f. induced in any inductor at the instant it passes through this position in the magnetic field. Each arrow thus belongs to its particular position and is fixed, so to speak, and refers to each inductor successively as the rotation of the armature brings that inductor into that position. At the instant the armature occupies the position shown in Fig. 247, the total e.m.f. in coil *bc* (sum of instantaneous

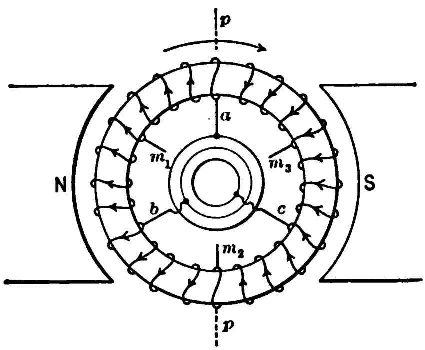


FIG. 247. The armature of the previous figures tapped for three-phase power.

values of all inductors in series in this coil) is zero; while coils *ab* and *ca* at this instant have total e.m.f.'s which are equal to each other, but in their positive directions are opposite with reference to the closed mesh or ring. The effective e.m.f. in all three coils is the same, because they all have the same number of series inductors, and rotate together at the same speed in the same flux. The instantaneous e.m.f. in coil *ab* reaches its maximum value in the direction from *a* to *b*, at the moment when the middle (m_1) of this coil passes a point under the middle of the *N*-pole. The e.m.f. in coil *bc* reaches its maximum value from *b* to *c*, at the instant its middle (m_2) passes the same point in the magnetic field, which occurs just one-third period, or 120 degrees, after the maximum from *a* to *b*; and the e.m.f. in coil *ca* reaches its maximum value from *c* to *a*, at the instant m_3 passes the same point in the field, which is two-thirds period, or 240 degrees, after the maximum e.m.f. *a* to *b*, or 120 degrees before the maximum e.m.f. *a* to *b*, or 120 degrees after the maximum e.m.f. *b* to *c*. Therefore the coils *ab*, *bc* and *ca* differ in phase from one another, successively, by 120 degrees, when the positive directions are chosen from *a* to *b*, from *b* to *c* and from *c* to *a* (or all vice versa) — that is, in the same direction around the closed mesh or ring of the winding. Fig. 248 is a

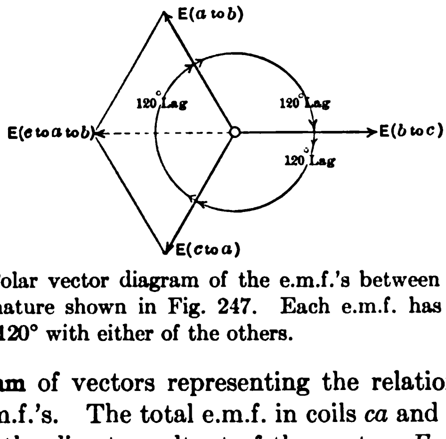


FIG. 248. Polar vector diagram of the e.m.f.'s between the tappings of the armature shown in Fig. 247. Each e.m.f. has a phase difference of 120° with either of the others.

polar diagram of vectors representing the relations of these induced e.m.f.'s. The total e.m.f. in coils *ca* and *ab* together is equal to the direct resultant of the vectors E_{ca} and E_{ab} as they stand, since the relative directions of the vectors have been drawn to represent phase relations of e.m.f.'s whose positive directions are the same with respect to the series circuit. It is seen from the diagram that this resultant $E_{c \text{ to } a \text{ to } b}$ is equal in value, and exactly opposite in phase, to the e.m.f. induced in coil *bc*, which is represented by the vector $E_{b \text{ to } c}$. If vectors $E_{c \text{ to } a \text{ to } b}$ and $E_{b \text{ to } c}$ were combined, the resultant would be zero. This agrees with our previous conclusion that the total e.m.f. in the closed winding is zero, regardless of how it is tapped. This is true of instantaneous values as well as of effective values of e.m.f. Fig. 249 is a topographic diagram showing the same vector relations. Here the vectors are drawn end to end, and the resultant of any number of e.m.f.'s in series is represented vectorially by a straight line drawn from the beginning of the first vector of the series to the end of the last vector of the series. Here again it is seen that the resultant e.m.f. of coils *c–a* and *a–b* in series is the vector $E_{c\ to\ a\ to\ b}$, which is equal and opposite to the vector $E_{b\ to\ c}$. Furthermore, when all three vectors have the same length and same 120 degrees angular difference in the same direction, as shown here, the end of vector $E_{a\ to\ b}$ reaches back exactly to the beginning of vector $E_{b\ to\ c}$, indicating that the total e.m.f. around the mesh is zero, because a line between these two ends of the series would have zero length.

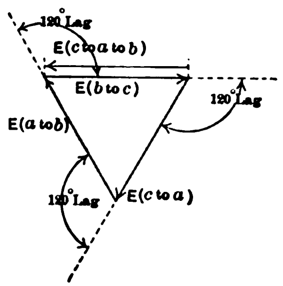


FIG. 249. Topographic vector diagram of the voltage relations in Fig. 247.

As the armature coils are connected together and tapped to the rings in Fig. 247, the winding represents what is called the **mesh connection** or the **delta connection** of coils. The synchronous converter has a mesh or delta connection of necessity, because it must operate with direct current as well as alternating current, and the former requires a rotating armature with a closed winding. However, this is not the most usual method of connecting together the different phases in the windings of ordinary **alternators** with drum-wound armatures. It is most usual to make a **star** or "Y" connection between the coils; this is an open-circuit winding, and gives the greatest terminal voltage that can be obtained in a three-phase winding for a given maximum voltage per coil, and is particularly desirable in generators for high-tension work.

83. Star Connection. In Fig. 250, we have cut open the closed ring winding at three equidistant points, *a*, *b*, *c*, forming three separate groups or phases out of the armature winding. As before explained, the e.m.f. in Coil 2 will lag 120 degrees behind the e.m.f. in Coil 1, and the e.m.f. of Coil 3 will lag 120 degrees behind the e.m.f. of Coil 2, or 240 degrees behind that of Coil 1, the positive directions having been chosen either all clockwise or all counter clockwise around the ring. Starting with one end of Coil 1 at the cut *b*, we

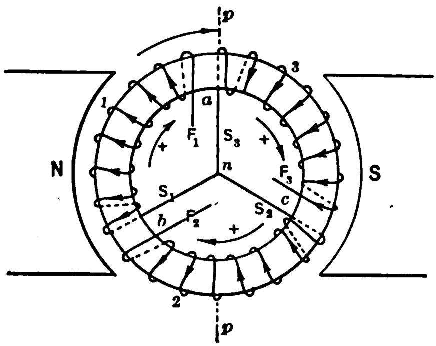


FIG. 250. The same armature Y connected. Note that the winding had to be cut at *a*, *b*, and *c* before a Y connection could be made.

have labeled the ends of each coil S (Start) and F (Finish), as we follow the winding progressively around the ring; that is, S_1, S_2 and S_3 are similar ends of the three phases of the winding, and F_1, F_2 and F_3 are the similar other ends of the three phases, respectively. If now we connect S_1, S_2 and S_3 together, as shown at n (results would be exactly the same if we were to connect F_1, F_2 and F_3 together instead), we find that we have between the three remaining terminals a correct three-phase e.m.f. This e.m.f., however, is about 73 per cent greater than the e.m.f. across any group of the armature winding. The e.m.f. between any two of the three terminals of the mesh-connected winding of Fig. 247 was exactly the same as the e.m.f. across **one** of the three groups of the armature winding, as could be seen by simple inspection, since the terminals of the armature, or rings, were tapped directly to the ends or junctions of the armature groups. Hence we have this important result; that if we connect an armature winding according to the star or Y method, we get a three-phase terminal voltage just 73 per cent greater than if we connect the same armature winding according to the mesh or delta method. The vector diagrams accompanying Fig. 250 illustrate how this comes about. Fig. 251 shows how the e.m.f. between terminals F_2 and F_1 is made up of the e.m.f.'s $E_{S_1 \text{ to } F_1}$ and $E_{S_2 \text{ to } F_2}$ which are in series between these terminals. Note that starting at F_2 and tracing the winding through to F_1, we have to go opposite to the positive direction in Coil 2. The chosen positive directions of these two e.m.f.'s are thus opposed to each other in this series, and their **phase difference is 120 degrees only with reference to these positive directions.** We have seen that the vector sum of two e.m.f.'s having a phase difference of 120 degrees in **opposite** directions is in reality equal to the vector sum of two similar e.m.f.'s having a phase difference of 60 degrees in the **same** direction through the circuit of the series. Fig. 251, 252 and 253 show separately

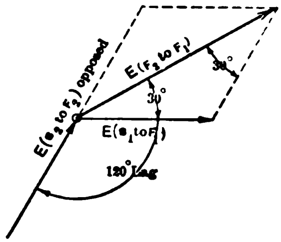


FIG. 251. Polar vector diagram for one phase of the Y-connected armature of Fig. 250. Note that the vector $E_{S_2 \text{ to } F_2}$ has the opposite positive direction to the vector $E_{S_1 \text{ to } F_1}$.

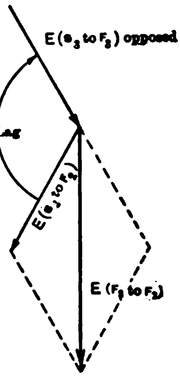


FIG. 252. Vector diagram for the voltage across F_1–F_2. Note that it is made up of the voltage across S_2–F_2 reversed and S_1–F_1.

the summations of $E_{S_1F_1}$ and $E_{S_2F_2}$ to find $E_{F_2F_1}$; of $E_{S_2F_2}$ and $E_{S_3F_3}$ to find $E_{F_3F_2}$; and of $E_{S_3F_3}$ and $E_{S_1F_1}$ to find $E_{F_1F_3}$.

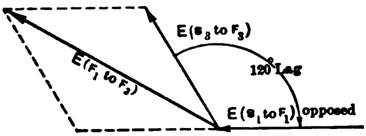


FIG. 253. The vector diagram for the voltage across the phase F_1–F_3. Note that it is made up of the voltage across S_1–F_1 reversed and S_3–F_3.

Fig. 254 is an assembly showing all these e.m.f.'s together. It is seen, by simple trigonometry, that

$$E_{F_2F_1} = E_{F_3F_2} = E_{F_1F_3} = \sqrt{3}\, E_{SF},$$

and the phase differences are 120 degrees each to each, provided only that $E_{S_1F_1} = E_{S_2F_2} = E_{S_3F_3}$, and that the phase

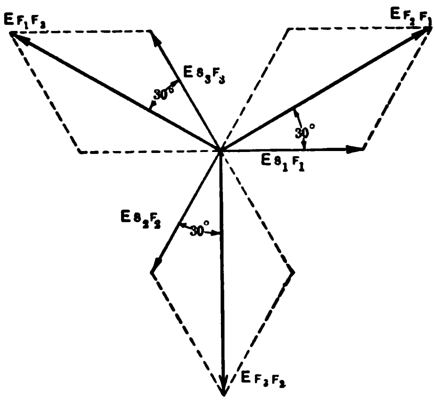


FIG. 254. The vector diagram of Fig. 251, 252 and 253 combined to show the phase relations of the voltages across the three phases and between line wires.

differences of these e.m.f.'s are 120 degrees each to each, as is usual.

Prob. 10–7. Prove that the ratio of current in each line wire to current in each armature coil, in a delta-connected three-phase armature with balanced load, is the same as the ratio of volts between terminals to volts in each armature section, in a star-connected three-phase armature. See Fig. 254.

Prob. 11–7. A delta-connected three-phase armature is rated to deliver 100 amperes from each terminal, with 230 volts between each pair of terminals, at 60 cycles frequency. What would be its rated full-load amperes per line wire and volts between terminals, if the phases or groups in the armature were connected star instead of delta?

Prob. 12–7. What is the maximum steady load, in kilowatts, which can be obtained from machine in Problem 11–7:

(*a*) Connected delta.

(*b*) Connected star?

(*c*) In general, what difference will be produced in the rating of a generator as to amperes (per terminal or per line wire), volts (between any two terminals or line wires) and kv-a. (or kw. at unity power-factor), if the internal connections of the phases in the armature are changed from Y to delta?

Prob. 13–7. A three-phase delta-connected armature is rated 100 kv-a. (or 100 kw. at unity power-factor), 2300 volts:

(*a*) What is the current in each part of the armature at rated full-load output, unity power-factor?

(*b*) What would be the amperes, volts and kv-a. rating of this machine, if the armature groups are connected in star?

Prob. 14–7. A three-phase delta-connected alternator is rated 200 kv-a. at 6600 volts:

(*a*) If this armature were to be used to deliver its full rated load (kilovolt-amperes) from a single phase or pair of terminals (presuming the voltage could be kept constant), what would be the amperes in each section of the armature winding? Draw a sketch to illustrate your solution.

(*b*) By what percentage would the rate of heat development in each armature coil exceed that permitted under rated conditions?

Prob. 15–7. In the generator specified in Problem 14–7, what are the greatest amperes and kilovolt-amperes that can be delivered at rated voltage from any one pair of terminals, without developing heat in any armature conductor at a wattage rate exceeding that permitted at full load?

Prob. 16–7. If the thermal conductivity from one part of the armature of Problem 15–7 to another were so good that the temperature of each part were determined by the average or total watts dissipated as heat in the entire armature, calculate what maximum amperes and kilovolt-amperes could be delivered from a single phase of this machine.

Prob. 17–7. (*a*) Draw a vector diagram for the case in Problem 16–7 of a three-phase delta-connected armature delivering load at unity power-factor from a single phase only, and calculate the watts of power developed in each of the three sections of the closed armature circuit.

(*b*) Does the sum of power developed in all of the armature groups check up with the power between the terminals of the single loaded phase?

84. What Determines the Number of Phases to Use. It has already been shown that the power in any single phase varies from instant to instant in a manner which depends upon the power-factor (see Art. 37). In every cycle there are two instants at which the power is zero, and two instants at which it has the same maximum value. If the current is in phase with the e.m.f. which produces it, the flow of power is unidirectional, always out from the generator to the line or consuming device, and the "**power-factor**" of the circuit is **unity.** But if the current either leads or lags with respect to the e.m.f. which produces it, there are negative pulses of power interspersed between positive pulses, the circuit returning, during one part of each cycle, a portion of the power delivered to it by the generator during the other part of the cycle. The **power-factor** is in consequence **less than unity.**

In any correct two-phase, three-phase or four-phase circuit (as defined in the preceding articles), the instantaneous power in each one of the phases varies just as in any single-phase circuit operating with an equal power-factor. But when the polyphase circuit is balanced, it can be shown, either mathematically or graphically, that the **total power** in all of the phases is at every instant the same; in other words, the total flow of power is steady, some circuits supplying enough while the other circuits are either giving none or taking back some, to make the total power constant at all instants.

This important fact really led to the development of polyphase systems. The fluctuation of power does not make single-phase systems objectionable for supplying power for heating purposes, or for chemical purposes. In fact, single-phase power is entirely suitable for lighting, provided the frequency is not less than about 40 cycles per second, because the fluctuations of power are so rapid that the heating and lighting effects do not have a chance to die out between pulses of power, so to speak. But great trouble and expense have been involved in getting **motors** to work satisfactorily on single-phase alternating current. A motor must deliver power steadily against a uniform resisting torque. Hence if it is supplied with a flow of power which pulsates and has zero, or even negative, values at certain regularly recurring instants, there must be certain parts of each cycle during which the motor is giving out more power than it is receiving, and other parts when it is receiving much more than it gives out. The average value of the variable power taken in must be in excess of the steady power given out, by an amount equal to the sum of all losses within the machine. The motor must therefore possess features which will enable it to store energy during one part of a cycle to be given out during another part. To furnish this storage capacity requires a larger, heavier motor with a greater amount of copper and iron in it. The expense is further increased by the fact that the single-phase motor will not start itself unless furnished with some auxiliary devices, usually of the nature of special windings on both rotor and stator, with commutator and brushes, all of which add to the complication, chances of trouble and cost of manufacture and maintenance. A polyphase alternating-current motor, on the other hand, does not need this capacity to store energy from one part of the cycle to another, because the total electrical power input is steady like the mechanical power output. Moreover, the polyphase motor is simpler and more rugged and durable than the single-phase motor, costs less to manufacture and maintain, and will start more quickly, or with a bigger load, and with less disturbance to the distributing system, than the single-phase motor. The advantages of the polyphase motor practically forced the development of polyphase systems and generators. Of the various polyphase systems available, the three-phase system, using three wires, has been adopted in practice as standard, because of its simplicity.

The two-phase system usually requires four line wires for transmission. It has been shown (Art. 75) that two of these wires may be combined into one, making three wires (provided the generator does not have a closed or re-entrant winding). But two disadvantages are thereby introduced: the common wire carries a current which is the vector sum of currents in the two phases, and it should therefore be larger than either of the other two wires, which is an inconvenience; and the resultant voltage between the two smaller wires is about 41 per cent higher than the voltage of either of the two phases, which means that correspondingly better insulation is required on the smaller wires.

The four-phase system absolutely requires four wires for transmission in order to connect four loads to four distinct and different pairs or phases. As to uniformity of power flow, the four-phase possesses no advantage over two-phase. In fact there is only one real phase difference between the four phases, because two of the e.m.f.'s reach their maximum values at exactly the same instant, and the other two reach their maximum values also at exactly the same instant, there being one-quarter period between these instants.

The three-phase system requires only three line wires to be perfectly symmetrical, so that the voltage between any two line wires is the same. It will be shown later that this system requires the least material and costs the least in the distributing system for a given pressure than any of the others mentioned. Here we shall demonstrate the same fact regarding only the generators for these several systems.

85. Electromotive Force Relations: Single-phase, Two-phase, Three-phase. In all of the diagrams used thus far, the armature has had altogether 24 inductors upon its outside surface, equal distances apart. As it is bipolar, each whole revolution completes one cycle of e.m.f. in every inductor. As we follow the winding around the ring, the e.m.f. in each successive inductor lags behind the e.m.f. in the preceding inductor by one-twenty-fourth of one period, or 15 electrical degrees. When the ring is tapped for single phase as in Fig. 215, the armature is divided into two parallel paths, with half of the total number of inductors arranged in series in each path. The total e.m.f. between terminals is therefore equal to the vector sum of 12 e.m.f.'s, each differing from the next by 15 degrees, all in the same direction through the series.

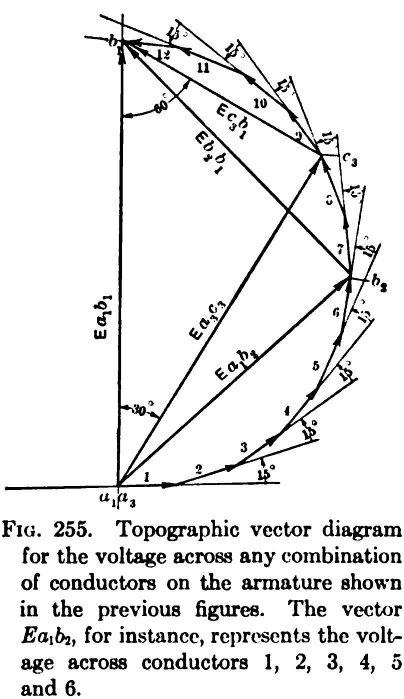


FIG. 255. Topographic vector diagram for the voltage across any combination of conductors on the armature shown in the previous figures. The vector Ea_1b_2, for instance, represents the voltage across conductors 1, 2, 3, 4, 5 and 6.

Adding the e.m.f. of each inductor to that of the succeeding one in the series, as a string of vectors, according to the topographic method of vector addition, we get a diagram like Fig. 255. The great convenience of this type of diagram is due to the fact that a chord drawn between any two points of the diagram represents correctly both the value and phase relations of the resultant or sum of all e.m.f.'s whose vectors are included in the series between the ends of this chord. Thus, the total e.m.f. of the six equal e.m.f.'s in inductors No. 1, 2, 3, 4, 5 and 6, between a_1 and b_2, is represented by the chord a_1b_2 in Fig. 255. The **length** of vector a_1b_2 represents the effective voltage of the group from a_1 to b_2 to the same scale of volts used in laying out the component voltages, and the **angle** between vector a_1b_2 and any other vector of the diagram represents the phase relation between the e.m.f.'s of the corresponding parts of the armature circuit. It may be proved by the rules of geometry, or by accurate drawing of the diagram, that the vector a_1b_1, representing the sum of the e.m.f.'s of twelve inductors 15 degrees apart in series, is the diameter of a semicircle drawn through the ends of all the component vectors. Also, if we choose any point between a_1 and b_1, such as b_2, the e.m.f.'s a_1b_2 and b_2b_1 are 90 degrees out of phase with each other, and their resultant is always equal to a_1b_1, that is, $E_{a_1b_1} = \sqrt{E_{a_1b_2}^2 + E_{b_2b_1}^2}$. If, as in this case, a_1b_2 and b_2b_1 are chosen equal to each other, each being the resultant of the same number (six) of similar component e.m.f.'s, each is the e.m.f. of one phase of a four-phase machine. Therefore E (single-phase) $= \sqrt{2} \times E$ (four-phase) which agrees with our previous work.

If now we choose a point c_3 separated from a_1 by eight inductors, we find that the following relations exist between the vector $E_{a_3c_3}$, representing the e.m.f. between a_1 and c_3, and the vector $E_{c_3b_1}$, representing the e.m.f. in the remaining four coils of the half-winding, between c_3 and b_1:

$E_{a_3c_3}$ lags behind $E_{c_3b_1}$ by 90 degrees, or one-quarter period.
$E_{a_3c_3}$ lags behind $E_{a_1b_1}$ by 30 degrees.
$E_{c_3b_1}$ leads $E_{a_1b_1}$ by 60 degrees.
$E_{a_1b_1} = \sqrt{E_{a_3c_3}^2 + E_{c_3b_1}^2}$.
$E_{a_3c_3} = E_{a_1b_1} \times (\sqrt{3}/2) = 0.866\, E_{a_1b_1}$.
$E_{c_3b_1} = E_{a_1b_1} \times (\frac{1}{2}) = 0.5\, E_{a_1b_1}$.

But a_3c_3 is one of the three-phase voltages. From these facts we may deduce the following rules for closed windings:

(1) If the coils of any given armature winding are grouped to form a correct three-phase, the voltage across each

phase of the winding will be equal to ($\sqrt{3}/2$, or 0.866) times the voltage that would be obtained by dividing it into two equal parallel groups for single-phase.

(2) If this winding is grouped to form six equidistant phases of equal e.m.f.'s (equivalent to tapping the winding of Fig. 256 at inductors 4 and 20, thus forming six equal and equidistant groups a_1–4, 4–c_3, c_3–b_1, b_1–b_3, b_3–20, and 20–a_1), the e.m.f. in each of the six phases is equal to one-half the voltage that would be obtained by dividing the whole number of coils into two equal parallel groups for single phase ($E_{a_1b_1}$).

86. Relative Kv-a. Capacity of Single-phase, Quarter-phase and Three-phase Generators. We shall calculate now the relative power capacity of the same armature with windings arranged in these several ways. Although the actual numbers used make no difference in the relative results, we shall select some in order to make the example realistic. Suppose, for simplicity, that we consider a re-entrant ring winding which is capable of delivering 100 amperes at 100 volts from the terminals a_1b_1 only, in Fig. 256. The maximum output which may be permitted without overheating any part of the machine is therefore 100 amperes × 100 volts = 10,000 volt-amperes = 10 kilovolt-amperes = 10 kilowatts if the power-factor is unity. The greatest allowable current in any single coil or inductor is ½ of 100 amperes = 50 amperes.

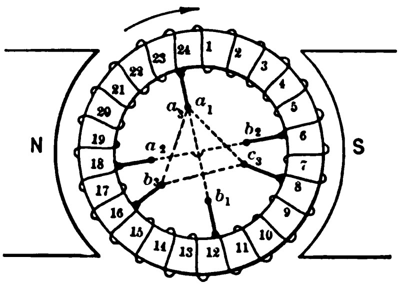


FIG. 256. The armature of the previous figures tapped for single-phase, two-phase, three-phase and four-phase.

Now consider this same winding tapped for two-phase, as at a_1 and b_1, a_2 and b_2, in Fig. 256. It has been shown (Art.

79 and Fig. 243) that with such arrangement of winding, the current in each armature conductor is $\left(\frac{\sqrt{2}}{2}\right)$ times the current delivered from either a_1b_1 or a_2b_2, on balanced load of any power-factor. Hence the greatest allowable current output from either phase (two parallel paths in each phase) of the two-phase is $\frac{2 \times 50}{\sqrt{2}}$ amperes. If the flux from each pole, and the speed, remain constant, the e.m.f. induced in each coil is not altered by the change in the connections or manner of loading; hence the e.m.f. across a_2b_2 is the same as that across a_1b_1, which remains the same as when operating single-phase.* Hence the greatest permissible total output when operating two-phase will be

$$\left(2 \times \frac{50}{\sqrt{2}} \times 100\right) + \left(2 \times \frac{50}{\sqrt{2}} \times 100\right) = \frac{20{,}000}{1.414} \text{ volt-amperes}$$
$$= 14.14 \text{ kv-a.}$$

Now consider the same winding tapped for three-phase, as at a_3, b_3, c_3, in Fig. 256. It was shown by means of Fig. 255 that the e.m.f. between any two of these points, as a_3c_3, is equal to 0.866 times the e.m.f. between a_1 and b_1, which was the single-phase e.m.f. Hence

Power output from a_3c_3 = power from c_3b_3 = power from b_3a_3 = (50 amperes) $\times$ (0.866 $\times$ 100) volts.

Greatest total three-phase output = $3 \times 50 \times 0.866 \times 100$ = 13,000 volt-amperes = 13.0 kv-a.

To check this:

The actual amperes output from each terminal or to each line wire, delta-connected as shown, is $I_l = \sqrt{3} \times 50$ amperes = 86.6 amperes. The volts between any two line wires is $E_l = 0.866 \times 100$ volts = 86.6 volts. The total volt-amperes given out to a three-phase line equals $\sqrt{3}\, E_l I_l$

* In these calculations the effects of armature reaction in altering the flux and induced e.m.f. are ignored, as being of too complicated character to be considered in such simple calculation. See Chapter I, Second Course.

(to get watts we would multiply volt-amperes by power-factor), which reduces to (1.732 × 86.6 volts × 86.6 amperes), or 13,000 volt-amperes. If this same winding had been star-connected instead of delta-connected, the results would have been as follows:

Greatest allowable current output from each terminal $= I_l = 50$ amperes.

Greatest allowable e.m.f. between any two terminals $= E_l = \sqrt{3} \times 86.6$ volts $= 150$ volts.

Total volt-amperes delivered to three-phase line $= \sqrt{3}\, E_l I_l = 1.732 \times 150 \times 50 = 13{,}000$ volt-amperes.

It therefore makes no difference in the **power rating** of a three-phase machine whether it be connected star or delta; such change affects only the rated volts and amperes, but not their total product.

87. Best Tapping for Three-phase Generator. It appears from the above, that although the single-phase, two-phase and three-phase machines all contain the same amount of copper and iron, and have the same speed and cost, it is permissible to take 41.4 per cent more power out of the two-phase, and 30 per cent more power out of the three-phase, than out of the single-phase machine, operating under the same conditions as to power-factor, permissible watts loss and temperature rise. This makes the two-phase alternator appear to be the best machine. **This is not true in practice,** because the three-phase winding is usually arranged to better advantage than as shown at $a_3b_3c_3$ in Fig. 256. Windings are in fact usually arranged so that each phase has one group of coils or inductors for each pole — or so that each phase is distributed symmetrically with regard to every single pole, and not with regard to every pair of poles. To illustrate, in Fig. 257, representing a three-phase delta winding, each phase consists of one large group (a_3b_3, or b_3c_3, or c_3a_3) for the pair of poles. But in Fig. 258 the same number of inductors or coils are arranged so that each phase consists of one pair of equal groups per pair of poles, **or one group per pole.** Instead of cutting the winding at only three equidistant points to form phases as in the former case, we have cut the total number of coils which lie under a pair of poles into six groups, making three phases each with **one group per pole per phase.** Each group contains a number of coils equal to $\frac{1}{3}$ of the total number of coils per pole of the whole machine.

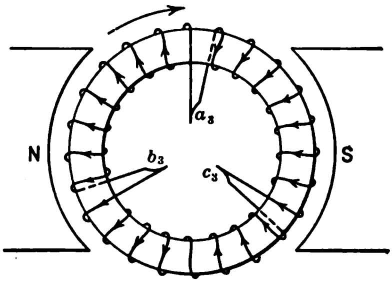


Fig. 257. A simple diagram of the delta arrangement of the windings in Fig. 256.

In Fig. 258 it is seen that groups 1, 2, 3, 4, 5 and 6 have the same number of inductors or coils in each, and therefore generate equal effective voltages. Each group in Fig. 258 has $\frac{1}{2}$ as many inductors as each group in Fig. 257. In Fig. 255 it is seen that the resultant voltage across the eight coils from a_3 to c_3 is equal to only $\sqrt{3}$, or 1.732, times the resultant voltage across four coils from c_3 to b_1, yet the number of similar adjacent coils from a_3 to c_3 is $\frac{8}{4}$, or twice the number of coils from c_3 to b_1.

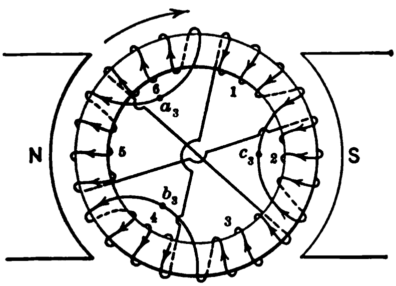


Fig. 258. The armature windings of Fig. 257, cut up into six coils which are rejoined into three phases. Note that coils 6 and 3 produce phase a_3b_3, coils 4 and 1 form phase b_3c_3, and coils 2 and 5 form phase c_3a_3.

The voltage across four adjacent coils is therefore $\frac{1}{1.73}$, or 0.578, of the voltage across eight adjacent coils, or somewhat more than half as much. The reason for this is plain from Fig. 255, where it is seen that the more coils there are in series, the greater is the phase difference between each added one and the first and thus the less it adds to the voltage across the series. The scheme of cutting the ring into groups of three coils each as in Fig. 258 and 259 merely offers an opportunity of rejoining the coils, so that only those coils will be in series in which the e.m.f.'s are nearest in phase.

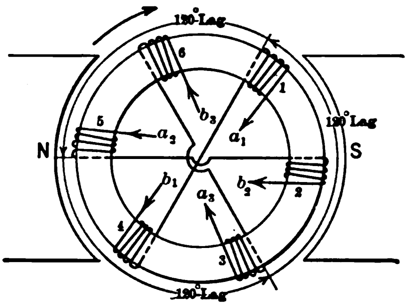


FIG. 259. The six coils of Fig. 258, arranged for three separate phases a_1b_1, a_2b_2 and a_3b_3.

It is seen, from Fig. 258 and 259, that the e.m.f.'s in Groups 1 and 4 reach their maximum instantaneous values at the same moment, but in opposite directions around the ring, because while one of them is passing under *N* the other is passing similarly under *S*. Now if coils 1 and 4 can be connected in series so that their maximum instantaneous e.m.f.'s add together in the same direction through this series, the total effective e.m.f. of these two groups of four coils each will be (2×0.578), or 1.156 times the effective e.m.f. of a single group of eight adjacent coils in Fig. 255. Note that this is done in Fig. 258 and 259 by means of one simple connection which is explained in the following page. There are three such pairs of groups in Fig. 258 — that is, 1 and 4, 2 and 5, 3 and 6, and they may be combined into a three-phase mesh or star, because there is a suitable phase relation between groups as shown by Fig. 260 and 261. The current capacity of each group is the same in both methods of joining them, but inasmuch as the voltage per phase is 1.156 times as great in Fig. 258 as in Fig. 257, it follows that the power capacity of each of the three phases is 1.156 times as great as before. This makes the greatest allowable load, on the same basis as before, equal to 13.0 kv-a. × 1.156 = 15.0 kv-a.

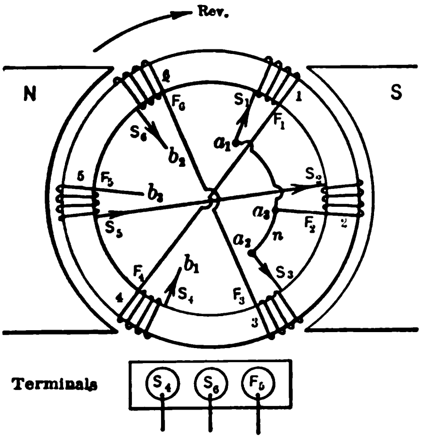


FIG. 260a. The coils of Fig. 259 joined in star.

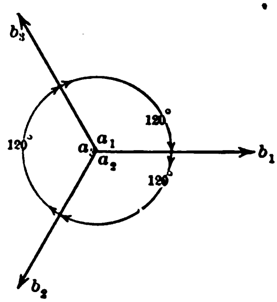


FIG. 260b. Polar vector diagram of the voltages across the coils joined in Y as in Fig. 260a. Note the three have the true three-phase relation of phase difference.

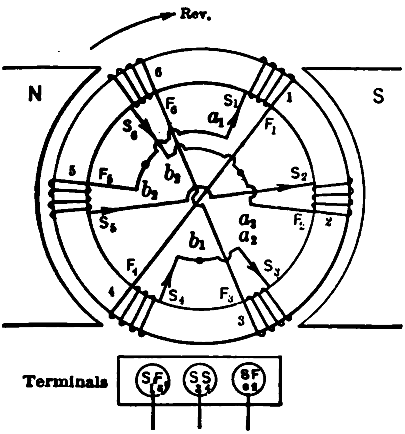


FIG. 261a. The coils of Fig. 259 joined in delta.

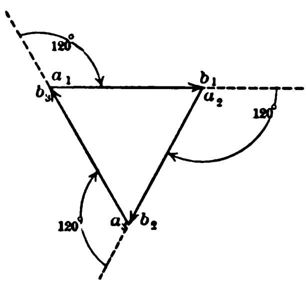


FIG. 261b. The topographic vector diagram for the voltages across the delta connection of Fig. 261a.

It is significant that we have obtained this increase from 13 to 15 kv-a. in the capacity of the same machine, amount-ing to 15.6 per cent increase, merely by the application of a little knowledge of vector addition, which shows us the best way to arrange the coils. Evidently it does not pay to add more coils in series when the phase lag of the added e.m.f.'s is large. For this reason the windings of some machines are not distributed uniformly over the entire armature, but are concentrated in groups corresponding to the poles, so that only a fraction of the periphery of the armature is occupied. This is particularly noticeable in single-phase windings. Notice now, that as a result of subdividing the three-phase winding so as to have a group of coils under each pole for each phase, we have changed the relative power capacity of the three machines, for the same size and approximately the same cost, as follows:

Single-phase: two-phase: three-phase = 10.0: 14.14: 15.0.

These relations are general, and applicable to any given armature winding, regardless of the style of winding, the total number of coils and inductors, or of the number of poles, provided only that all the coils are exactly similar and that the total number of coils per pole is divisible by the number of phases desired. Thus, if any given machine had a total number of equidistant similar coils, which is exactly divisible by either three times the number of poles or two times the number of poles, the machine may be changed from three-phase to two-phase, or vice versa, merely by altering the end-connections between the coils. This would not change the frequency for a given speed (or the speed obtained with a given frequency, if the machine happens to be a motor), but it would change the voltage at which it is proper to operate the machine. The exact form and manner of making or altering the end-connections between coils for any given winding would depend upon the style of winding, as hereafter explained (see Chapter IV). In any case, if the connections were made correctly, the terminal voltage produced for a given voltage per coil, for a given speed and flux per pole, and a given number of coils per pole per phase, would be indicated by the vector diagrams as given, regardless of the style of winding.

It is worth noting here, how the three phases in Fig. 258 are connected together, because it is a fertile source of error. Fig. 259 has been specially drawn so as to show the coils in each group squeezed together or concentrated, to make the picture clearer. A little study will show that on account of the number of poles and the position of the groups relative to each other, the e.m.f. in Groups 5 and 2 reaches its maximum instantaneous value in the direction from a_2 to b_2, just one-third period, or 120 degrees, after the e.m.f. in Groups 1 and 4 reaches its maximum instantaneous value in the direction from a_1 to b_1, or, that $E_{a_2 \text{ to } b_2}$ lags 120 degrees behind $E_{a_1 \text{ to } b_1}$; similarly, that $E_{a_3 \text{ to } b_3}$ lags 120 degrees behind $E_{a_2 \text{ to } b_2}$, or 240 degrees behind $E_{a_1 \text{ to } b_1}$; and that $E_{a_1 \text{ to } b_1}$ lags 120 degrees behind $E_{a_3 \text{ to } b_3}$. If we desire a correct three-phase star or Y connection, we must connect the three points a_1, a_2 and a_3 together (this is called the neutral point), and take our three-phase loads from b_1–b_2, b_2–b_3 and b_3–b_1. Fig. 260 indicates the equivalent of this connection, and at the same time shows the vector diagram according to which the e.m.f.'s are combined to get the terminal e.m.f.'s of the several phases, or terminal voltage of the machine. Notice that in Fig. 260a we have connected Groups 2 and 5 together in a sense opposite to that in which Groups 1–4 and Groups 3–6 are connected, instead of having all pairs connected in the same sense as in Fig. 259. Then we have connected the "Starting" ends of Groups 1–4 and 3–6 to the "Finishing" end of Groups 2–5, instead of connecting the Starting ends of all groups together as described above for Fig. 259. One of these changes exactly compensates the other, and the purpose is to shorten the connections between each group and neutral. If we desire a correct three-phase mesh or delta connection for Fig. 259, we should join b_1 to a_2, b_2 to a_3 and b_3 to a_1, and connect the three-phase loads to these junction points, which are the terminals of the winding. Fig. 261 illustrates this connection, and the vector diagram representing it. Taking the positive direction to be from a toward b in all phases of the winding, the phases are successively 120 degrees apart; and the resultant of three such phases in series so that their positive directions are the same throughout the series (as connected in the delta) is seen to be zero by the closure of the diagram in Fig. 261.

88. Open and Closed Windings. Relative Capacity for Different Methods of Connection. The same coils may be connected together to form either a **closed winding,** as shown in Fig. 262 for a single-phase alternator, or an **open winding,** as shown in Fig. 263. For the same flux and speed, the same machine with open winding, connected as in Fig. 263, will safely deliver only half as much current, but at twice as great

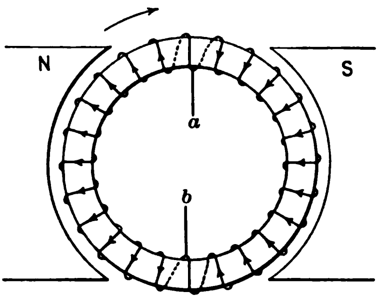


Fig. 262. Single-phase closed winding. 100 amp., 100 volts, 10 kv-a. capacity.

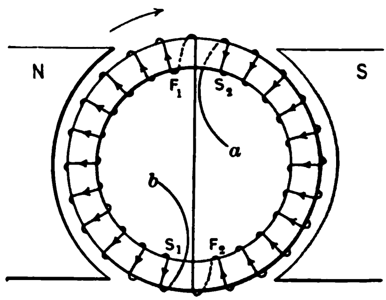


Fig. 263. Single-phase open winding. 50 amp., 200 volts, 10 kv-a. capacity.

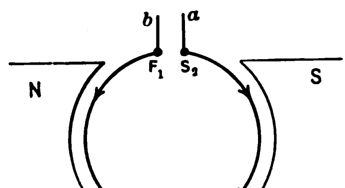


Fig. 264. Incorrect tapping and connection for single-phase winding. 50 amp., zero volts, zero capacity.

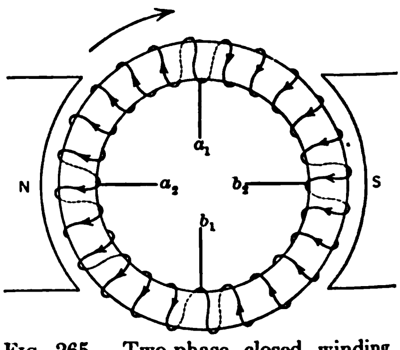


Fig. 265. Two-phase closed winding. 70.7 amp., 100 volts for each phase, a_1b_1 and a_2b_2. Total capacity = $2 \times 70.7 \times 100 = 14.14$ kv-a.

voltage, as it would with coils arranged in the closed winding shown in Fig. 262. The total kv-a. capacity would be the same in either case. Care must be exercised to connect the parts of the open winding so that the total e.m.f. is as great as possible, or so that the coils connected in series to form each phase are not in opposition. Failure to observe this may result as indicated diagrammatically in Fig. 264, which illustrates the effect of connecting F_2 to S_1, leaving S_2 and F_1 as the terminals of the single-phase winding. Fig. 265, 266 and 267 illustrate the corresponding arrangements for a two-phase winding, Fig. 265 representing the closed winding, Fig. 266 the open winding for same coils and Fig. 267 the effect of connecting the halves of each phase so as to oppose instead of help each other.

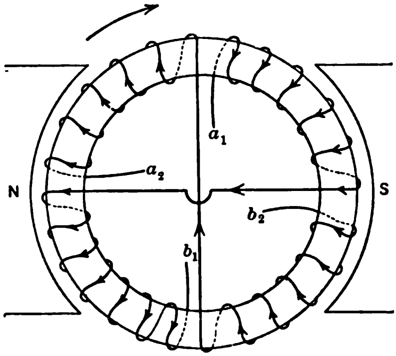


Fig. 266. Two-phase open winding. 50 amp., 141.4 volts in each phase, a_1b_1 and a_2b_2. Total capacity = $2 \times 50 \times 141.4 = 14.14$ kv-a.

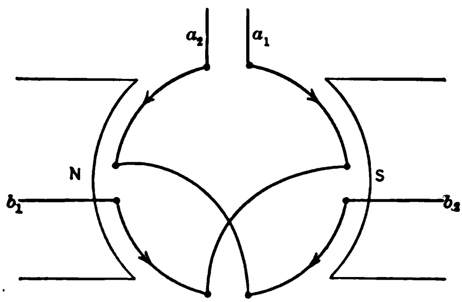


Fig. 267. Incorrect connections for two-phase winding. 50 amp., zero volts for each phase. Total capacity = zero kv-a.

Example 1. Fig. 268 represents for a two-pole three-phase alternator, the connections of a ring winding equivalent to the drum winding most frequently used in alternating-current machines, namely a Y connection with **one group of coils per phase per pole.** All groups in each phase are arranged so as to be in phase with each other, thus delivering at the terminals the greatest e.m.f. possible to obtain from the machine with the given coils, speed and flux. At rated frequency or speed, and normal flux or current per field coil, this machine is capable of delivering 200 kv-a. at a voltage of 2300 between any two terminals, with balanced load. This tells us indirectly the following important facts concerning the machine:

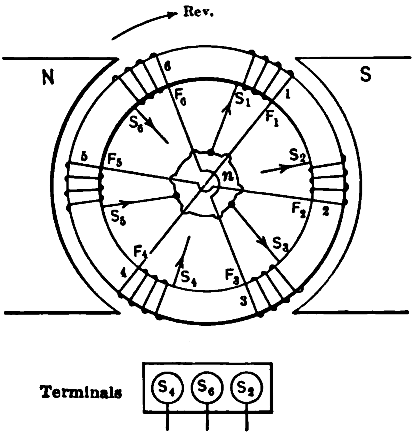


FIG. 268. Best three-phase star-connection for this armature. At a rating of 200 kv-a., 2300 volts for the generator, each coil can carry 50.2 amp.

(*a*) The **greatest current** that any coil of the winding can be allowed to carry steadily without overheating it and ruining the insulation. Thus, if I_c represents the rated full-load current per coil, then, using effective values throughout,

$$200{,}000 \text{ volt-amperes} = \sqrt{3} \times 2300 \text{ volts} \times I_c \text{ amperes.}$$

$$I_c = \frac{200{,}000}{1.732 \times 2300} = 50.2 \text{ amperes.}$$

(*b*) The **greatest e.m.f.** that can be delivered by any one of the six groups of coils into which the winding is divided (as 1 and 4, or 3 and 6, or 5 and 2) while carrying a current of 50.2 amperes, and while the machine is turning at rated speed, without requiring a flux which will cause excessive hysteresis and eddy-current losses in the armature core, or a field current which will overheat the field coils. Thus, if E_g be the volts across each group of coils, as across S_1–F_1 or S_2–F_2 or S_6–F_6, at full rated load, then

$$2300 = \sqrt{3} \times (2\,E_g),$$

as is evident from the corresponding vector diagram, Fig. 269, hence $$E_g = 2300/2\sqrt{3} = 664 \text{ volts.}$$

Check: The sum of the volt-amperes developed in all three phases of the winding ought to be equal to the total volt-amperes output from the terminals of the machine. In this case, the volts per phase of the winding $= 2 \times 664 = 1328$, and the amperes $= 50.2$. Hence, total volt-amperes of three phases of winding $= 3 \times 1328 \times 50.2 = 200{,}000$. It would not be correct, **in general,** to say that the total volt-amperes in all the coils or groups of the winding (as $1 + 2 + 3 + 4 + 5 + 6$) should be equal to the total volt-amperes output from the terminals, because it is not always true that the e.m.f.'s of all coils or groups in the same phase of the winding (as S_1–S_4) are in phase with each other, although the current would

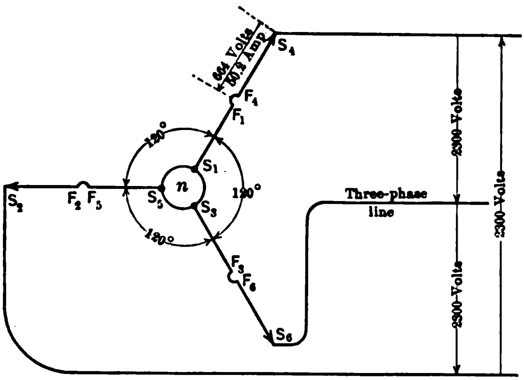


FIG. 269. Polar vector diagram for armature in Fig. 268. Note that the voltage across each coil, at the rating of Fig. 268, must be 664 volts. Note also that the voltages across any two coils in series as S_1F_1 and F_4S_4 are in phase.

be the same. Hence the power-factor might be different for different coils, and, therefore, their volt-amperes could not be added together arithmetically, but must be added together vectorially. **In the case just shown,** we could have checked just as well by simply adding arithmetically the volt-amperes in all six groups, but only because the Groups 1 and 4, which compose Phase 1 of the winding, are in phase with each other, and so on. This is not so in some cases which follow. The phase angle between current and e.m.f. in each of the three phases of the winding is always equal to the angle whose cosine is the power-factor of the external circuit, whenever the load is balanced. This equality does not extend any further than the whole phases, and thus is not necessarily true for each coil or group of coils in a phase.

Prob. 18–7. Explain by means of vector diagrams what results would be obtained by making the following connections in Fig. 259.

(*a*) a_1 to a_2, b_2 to b_3, a_3 to b_1, junction points being armature terminals.

(*b*) b_1 to b_2, a_2 to a_3, b_3 to a_1, junction points being armature terminals.

(*c*) b_1 to b_3, a_3 to b_2, a_2 to a_1, junction points being armature terminals.

(*d*) a_1, b_2 and a_3 together, others as terminals of the armature.

(*e*) b_1, b_2 and b_3 together, others as terminals of the armature.

Prob. 19–7. Specify which coil terminals should be connected together and which would be the terminals of the armature, for a star connection in Fig. 259, and show the simplest change by which it can be converted into a delta connection.

Prob. 20–7. If you were given a machine which was delta-connected, and required to reconnect it in Y, show how you would tag the terminals of each armature group, and which terminals you would join together after separating the original groups. Illustrate by sketches of the winding.

Prob. 21–7. The generator shown in Fig. 258 gives a voltage of 2200 between any two terminals when connected correctly as a three-phase delta. Show by sketch how you would connect it up in Y so as to make Groups 1 and 2 form one phase of the winding, Groups 3 and 4 another phase, Groups 5 and 6 another phase. What would be the terminal e.m.f. when so connected, and what percentage would be sacrificed in the maximum permissible total power output of the alternator?

NOTE. In the diagrams referred to in the following problems, the arrows on each coil indicate the direction of the instantaneous e.m.f. corresponding to the position shown, not the conventional positive directions; they are put there to aid the student to see whether the coils are connected correctly in series or in parallel.

Prob. 22–7. What will be the rated full-load values of amperes per terminal, volts between terminals, and total kv-a. capacity, of the same winding illustrated in Fig. 268 and specified in Exam. 1, but when connected as in Fig. 270? (Fig. 271 is the corresponding vector diagram.)

Prob. 23–7. Solve Problem 22–7 on the basis of connections shown in Fig. 272. (Fig. 273 is the corresponding vector diagram.)

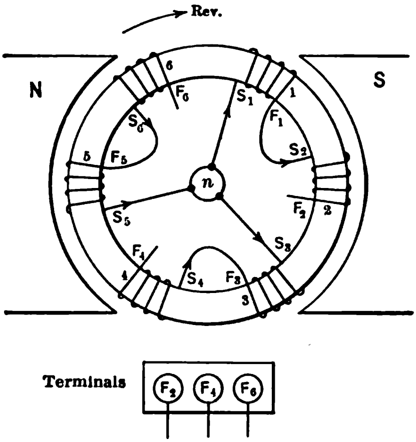


FIG. 270. The armature of Fig. 268 reconnected in star. Arrowheads denote instantaneous direction of e.m.f.

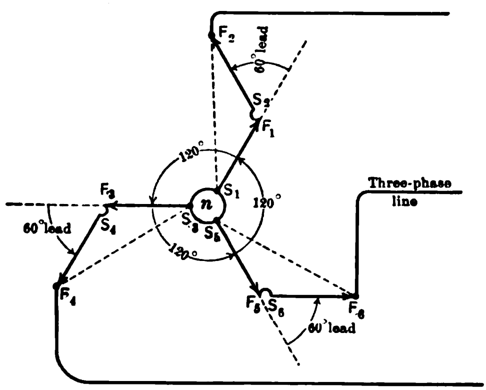


FIG. 271. The vector diagram for the armature as connected in Fig. 270. Note that the voltages in the coils S_1F_1 and S_2F_2 are not in phase, as the voltages in S_1F_1 and F_4S_4 of Fig. 269 were.

Prob. 24-7. Solve Problem 22-7 on the basis of connections shown in Fig. 274. (Fig. 275 is the corresponding vector diagram.)

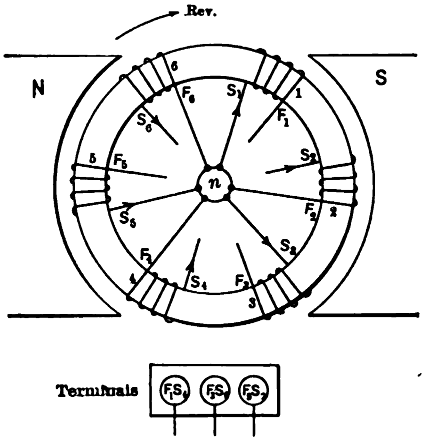


FIG. 272. Another star connection for the armature of Fig. 268.

Prob. 25-7. Solve Problem 22-7 on the basis of connections shown in Fig. 276. (Fig. 277 is the corresponding vector diagram.)

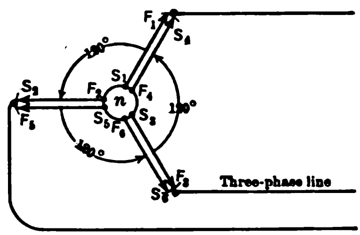


FIG. 273. Polar vector diagram for the armature connected as in Fig. 272.

Prob. 26-7. Solve Problem 22-7 on the basis of connections shown in Fig. 278. (Fig. 279 is the corresponding vector diagram.)

Prob. 27-7. From the solutions of the five preceding problems, explain which methods of connection should be used to achieve each of the following results:

(*a*) Obtain the greatest possible three-phase line current output from the given coils.

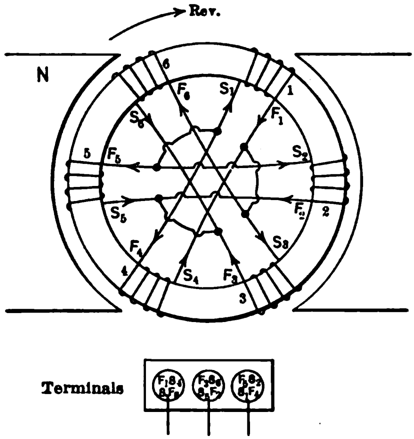


FIG. 274. Another delta connection for the armature of Fig. 268.

(*b*) Obtain the greatest possible three-phase terminal voltage from the given coils.

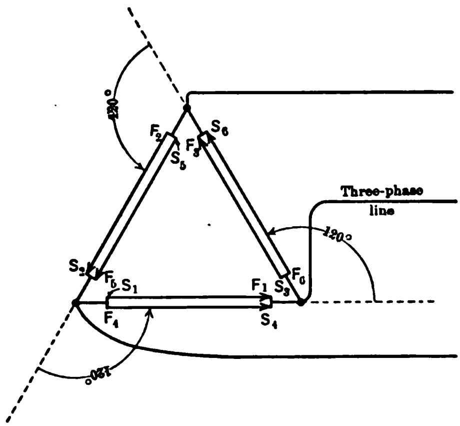


FIG. 275. The topographic vector diagram for the armature connected as in Fig. 274.

(*c*) Obtain the greatest possible three-phase total load, in kv-a. from the given coils.

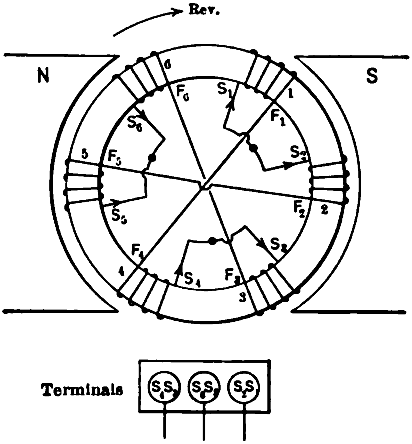


FIG. 276. Another delta connection for the armature of Fig. 268.

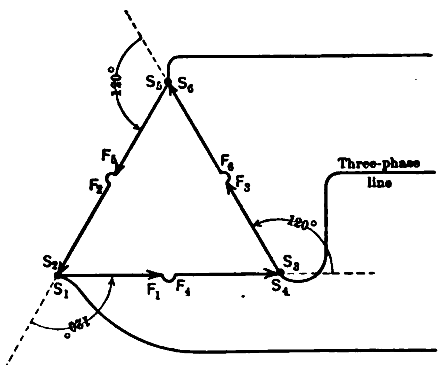


FIG. 277. The topographic vector diagram for the voltage of the armature connected as in Fig. 276.

Prob. 28-7. Using one group of coils per phase per pole, redraw Fig. 268 and 276 as they would appear for

(*a*) A four-pole machine.

(*b*) A six-pole machine, the coils for different pairs of poles to be in series.

Prob. 29-7. Solve Problem 28-7 for Fig. 270 and 278.

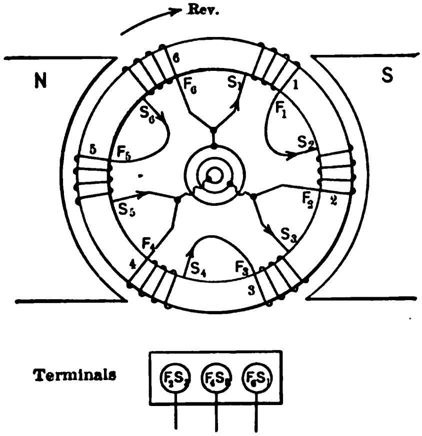


FIG. 278. The armature of Fig. 268 connected in delta in still another way.

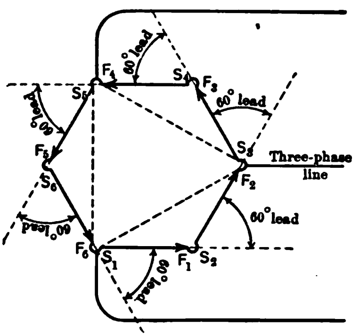


FIG. 279. The topographic vector diagram of the voltage in the armature connected as in Fig. 278.

SUMMARY OF CHAPTER VII

A TWO-PHASE GENERATOR is one which delivers two e.m.f.'s having equal value but a phase difference of 90 electrical degrees.

A THREE-WIRE SYSTEM FOR DISTRIBUTING TWO-PHASE E.M.F.'S can be connected to an open armature winding, but not to a closed winding without short-circuiting a part of the latter.

TWO-PHASE DISTRIBUTING SYSTEMS usually employ four wires. Four wires MUST be used with closed armature windings.

A FOUR-PHASE SYSTEM may be taken from a closed circuit winding tapped for two phases. In this case the e.m.f. of each phase of the two phases equals $\sqrt{2}$, or 1.41, times the e.m.f. of each of the four phases.

THE CURRENT IN EACH COIL OF A TWO-PHASE GENERATOR WITH CLOSED WINDING, on balanced load, equals 0.707 of the current in each line wire.

UNBALANCING THE LOAD, by making either the currents or the power-factors of the phases unequal, causes unequal currents and unequal heating in the various parts of the armature winding. To avoid burning the insulation on any coils, the output of the armature must be reduced below the rated full-load value.

A THREE-PHASE GENERATOR is one which delivers three e.m.f.'s having equal value but a phase difference of 120 electrical degrees each to each.

THREE-PHASE SYSTEMS are most commonly used because

(1) The single-phase motor is difficult to start and costly to build.

(2) The two-phase system requires AT LEAST three wires for transmission, one of which carries 1.41 as much current as either of the other two. Generally four wires are used, which makes the cost of copper greater than in the three-phase system.

(3) Three-phase generators are most economical. The same winding tapped for three phases delivers 1.50 times as much power as when tapped for single phase, and 1.06 times as much as when tapped for two phases.

THE THREE-PHASE GENERATOR is usually star-connected, though it may be delta-connected. The kilovolt-ampere capacity for a given machine is the same for both connections. The voltage between the terminals of a star-connected machine is $\sqrt{3}$, or 1.73, times the voltage between the terminals of the same machine delta-connected.

PROBLEMS ON CHAPTER VII

Prob. 30–7. Draw a sketch of a two-phase armature with a continuous ring winding tapped for a six-pole field.

Prob. 31–7. How many amperes flow in each armature conductor of the machine of Problem 30–7 when it is supplying a balanced two-phase load of 45 kilowatts at 90 per cent power-factor and 230 volts?

Prob. 32–7. What result would be obtained by using a two-phase armature wound and tapped for two poles, in a four-pole field?

Prob. 33–7. What result would be obtained by using a two-phase armature wound and tapped for four poles, in a two-pole field?

Prob. 34–7. Draw the internal and external connections of a four-phase ring-wound armature (continuous winding) tapped for an eight-pole field.

Prob. 35–7. A certain two-pole ring-wound arc-lighting generator has a rating of 10 amperes at 500 volts, direct current. Show the connections of the external direct-current circuit to the windings through the brushes and commutator. Calculate the amperes flowing through each armature conductor at rated full load.

Prob. 36–7. Show how this same winding of the generator in Problem 35–7 would be connected to deliver single-phase alternating current, instead of direct current, to the external circuit. Calculate how many amperes (effective value) harmonic alternating current could be delivered to the single-phase circuit without overheating any conductor on the armature.

Prob. 37–7. Operating the machine of Problem 36–7 as an alternating-current generator with the same speed and field flux as when delivering direct current, calculate the voltmeter reading at the alternating-current terminals. The alternating e.m.f. has very nearly a sine wave-form. Calculate the greatest kilowatt load that could be taken from the generator single-phase.

Prob. 38–7. If the current in one phase of the two-phase ring-wound alternator of Problem 1–7 lags 30 degrees behind the induced e.m.f. in that phase, and the current in the other phase lags 60 degrees behind the induced e.m.f. which produces it, each current having the rated full-load value of 50 amperes, what is the current in each armature conductor or each section of the winding? Assume that there is no distortion of phase relations between the induced e.m.f.'s due to the armature reaction. By what percentage is the heating in each coil above or below normal or rated value?

Prob. 39–7. Can a given ring-wound alternator deliver any more kilowatts or kilovolt-amperes when tapped and loaded two-phase than when tapped and loaded single-phase? Allow the same current in each armature conductor in both cases, and the same power-factor. In case of the two-phase, assume phases balanced. How does the e.m.f. on each of the two phases compare with that on the single-phase?

Prob. 40–7. Draw a diagram like Fig. 243 for a two-phase ring-wound two-pole alternator delivering half of its rated full-load current of 100 amperes to one phase, and one and one-half times rated full-load current (or 150 amperes) to the other phase, both loads being non-inductive. Assume the induced voltages to be equal in both phases, say 250 volts. Calculate the current in each armature coil, phase angle between current and e.m.f. in each coil, and power developed in each coil. Calcualte also the total kilowatts generated in the two phases, and thus check your solution. By what percentage is the rate of heating of each armature coil greater or less than it should be at normal full load?

Prob. 41–7. In Fig. 234, which coils in the armature would be overheated, and by what percentage, if both phases carried full-load current, but phase a_2b_2 had a power-factor of 0.50, lagging, while phase a_1b_1 had a unity power-factor? Solve by a diagram similar to Fig. 246.

Prob. 42–7. What is the greatest permissible current output from each phase of a two-phase two-pole ring-wound alternator, rated 230 volts, 100 amperes per phase, both phases having a power-factor of 86.6 per cent, and with one phase delivering 20 per cent more current than the other phase, without allowing any conductor on the armature to be heated faster than under rated full-load conditions? What per cent of its rated total kilowatts load is the generator carrying in this case?

Prob. 43–7. If the alternator specified in Problem 14–7 were reconnected in star or Y for three-wire system, what would be the rated voltage and current output? If full-load current were taken from one pair of terminals, or one phase only, at rated voltage, at what percentage of its rated full-load kv-a. would this generator be working? The average watts I^2R loss in the whole armature would then be what percentage of the normal full-load value? Load is non-inductive.

Prob. 44–7. What are the greatest amperes and kv-a. that could be taken from a single pair of terminals of the alternator of Problem 43–7 when star-connected?

(*a*) On the supposition that the watts I^2R heating in no single armature conductor shall be greater than at rated full load.

(*b*) On the supposition that the total or average watts I^2R heating in the entire armature shall not be greater than at rated full load.

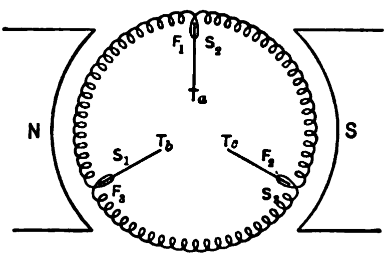


FIG. 280. Diagram of a delta-connected armature.

Prob. 45–7. Draw a complete vector diagram for a three-phase star-connected alternator, rated 50 kv-a., 2200 volts, loaded with full-load current from one pair of terminals only, (*a*) at unity power-factor, and (*b*) at 86.6 per cent power-factor, lagging. In each case, calculate the power developed in each section or phase of the armature winding, and find whether the sum of these checks with the total power developed at the single-phase terminals. Is any section of the armature winding being heated at an excessive rate? If so, which one, and by what percentage above normal rate? Explain the meaning of your results exactly, by means of a sketch of the winding.

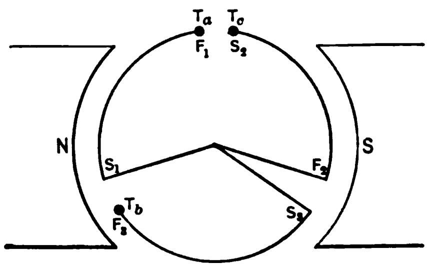


FIG. 281. Diagram of the way in which armature of Fig. 280 was reconnected for a star arrangement.

Prob. 46–7. In changing the phases of the armature winding shown in Fig. 280 from the delta connection to the Y connection, the latter connection is made as represented diagrammatically in Fig. 281. The machine was rated 50 kv-a., 2200 volts when delta-connected. What will be the voltage across each pair of terminals in Fig. 281. How would you remedy the difficulty?

Prob. 47-7. A Y-connected three-phase alternator armature is to be altered to the delta connection. The armature coils *A*, *B*, *C* were originally connected as in Fig. 282a. Fig. 282 b, c, d and e represent four possible couplings of the coils for mesh connection. Which of these connections are correct and which incorrect? Why? Illustrate by vector diagrams what results would be obtained in each case.

Prob. 48-7. In reconnecting the generator of Problem 21-7 after making repairs, the Y connections are made correctly to give greatest output, except that coils 1 and 4 are connected opposing instead of helping each other. Show such connections by a sketch,

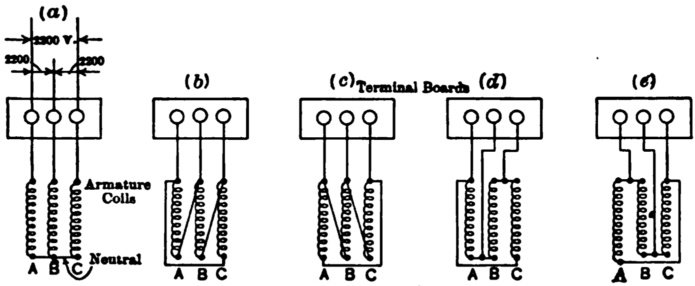


FIG. 282. In changing the star connection of (a) to a delta connection, which arrangements, (b), (c), (d), or (e), are correct?

and explain by aid of a vector diagram what effects will be produced on the terminal voltages of the several phases.

Prob. 49-7. If the correct delta connections between phases were made on the machine in Problem 21-7 to give greatest output, except that coils 1 and 4 happened to be joined together opposing instead of aiding each other, what would be the results? Illustrate the connections and results by a sketch and a vector diagram.

Prob. 50-7. A certain six-pole two-phase generator, with open windings, rated 230 volts, 30 kv-a., has altogether 72 identical equidistant coils in its armature winding. What is the phase difference between adjacent coils? How many coils per pole per phase? What is the e.m.f. across each coil at rated full load, assuming sine wave of e.m.f. in each coil? How would you rearrange the connections between the coils so as to get a three-phase winding? How many coils per pole per phase would there then be? Illustrate by vector diagrams and sketches.

Prob. 51–7. After the alternator of Problem 50–7 has been reconnected for three-phase power, what would be its rated amperes output per line wire, rated volts between line wires, and rated total kv-a. capacity:

(*a*) If the three phases were Y-connected?

(*b*) If the three phases were delta-connected?

The speed is fixed, by the driving engine and the frequency required by the apparatus connected to the distributing system. The flux per pole cannot be increased without excessive losses and heating in the armature iron and in the field copper, and it cannot be decreased without losing some power capacity and making the armature reaction greater and the voltage regulation poorer. The volts per coil are therefore just the same as before the change of connections. Assume each coil to generate a sine wave of e.m.f. Illustrate your solution by vector diagrams.

Prob. 52–7. What is the least total number of similar coils, uniformly distributed around the armature, that an alternating-current generator or motor with six poles may have, without making it impossible to change the machine from two-phase to three-phase by simply rearranging the connections between coils? How many coils per pole? If the machine had a three-phase rating of 7.5 kv-a., 230 volts, and was Y-connected, what would be its two-phase rating, in amperes and volts per phase, and total kv-a.? Illustrate solution by sketches of internal connections, and vector diagrams.

Prob. 53–7. Draw a four-pole alternator with 24 similar and equidistant armature coils, connected together for two phases with open winding. Note how this is done in Fig. 266 for a two-pole machine. If each coil produces a sine wave of e.m.f. of 50 volts effective value and can carry 20 amperes without overheating, what is the e.m.f. between the terminals of each phase, and the total kv-a. capacity when so connected?

Prob. 54–7. Draw a sketch of connections for the alternator of Problem 53–7 as a three-phase machine, delta-connected. Calculate the amperes output per terminal, volts between terminals, and total kv-a. capacity.

Prob. 55–7. (*a*) How would you reconnect the 24 similar and equidistant armature coils of a 60-cycle 1800-r.p.m. two-phase alternator so as to make it deliver three-phase power? (*b*) How would you do it if the machine has 36 coils?

Prob. 56–7. What is the greatest voltage that it would be practicable to get, three-phase, by reconnecting a single-phase alternator rated 60 cycles, 1800 r.p.m., 7.5 kv-a., 220 volts. As a single-phase machine the armature has an open winding consisting of 36 similar and equidistant coils, connected all in series.

Prob. 57–7. A 60-cycle 900-r.p.m. three-phase alternator has 96 armature coils, all similar and equidistant. It is rated 100 kv-a., 2300 volts, and is Y-connected. How many coils per pole per phase? What is the phase difference between the e.m.f.'s of adjacent coils? If each coil delivers an e.m.f. of approximately sine wave-form, what is its effective value in volts? What is the current which each coil can carry without overheating, at rated full load?

Prob. 58–7. With the same speed and field excitation, what volts, amperes and kv-a. would the alternator of Problem 57–7 deliver if reconnected for single-phase, open winding? Draw a vector diagram and a sketch to illustrate.

Prob. 59–7. Solve Problem 28–7 for Fig. 272 and 274.

Prob. 60–7. How would the three delta windings shown in Fig. 274, 276, 278 compare with each other as to the weight of copper required to deliver any specified voltage and current at the terminals, with the same total I^2R loss in the entire armature winding, and the same frequency, number of poles and useful flux per pole in each case? Choose any convenient values of E, I and total I^2R; if the chosen values are adhered to throughout the calculation, they will not affect the comparative results.

Prob. 61–7. Solve Problem 60–7 for the three star windings shown in Fig. 268, 270 and 272.

Prob. 62–7. The statement is made that the best delta arrangement of winding requires 73 per cent more inductors (and correspondingly greater active length of conductor in the winding), each of 58 per cent greater size, than the best star arrangement of the same coils, to deliver the same amperes and volts at the three-phase terminals, with the same total I^2R loss in the winding. Prove whether or not this statement is true.

Prob. 63–7. Windings are sometimes designed so that when one type of connection is used between groups of coils, the machine will deliver 11,000 volts, three-phase, and when the connections between groups are changed to another type, the same machine operating with practically the same flux and speed will deliver 6600 volts, three-phase. Illustrate by sketches how this may be done.

Prob. 64–7. How many kv-a. could be taken continuously by a single-phase external circuit connected between two of the three terminals of the alternator in Fig. 268, rated as in Example 1? Assume that the terminal voltage can be kept up to the rated three-phase value without requiring excessive field current; and that the current output is limited by the heating of any single coil. This is generally true in practice, especially in large generators, it being possible to burn out a single coil or group of coils completely without seriously damaging the adjacent coils.

Prob. 65–7. How many kv-a. can be taken single-phase from the machine specified in the example of Paragraph 88, when the windings are arranged as in Fig. 270, and on the assumptions specified in Problem 64–7?

Prob. 66–7. Solve Problem 65–7 for the same windings arranged as in Fig. 272.

Prob. 67–7. Solve Problem 65–7 for the same windings arranged as in Fig. 274.

Prob. 68–7. Solve Problem 65–7 for the same windings arranged as in Fig. 276.

Prob. 69–7. Solve Problem 65–7 for the same windings arranged as in Fig. 278.

Prob. 70–7. If the alternator specified in the example of Paragraph 88 were delivering a balanced load of 100 kv-a. at 80 per cent power-factor to three-phase induction motors, how much single-phase lamp load at unity power-factor could it carry in addition continuously across one pair of terminals, on the assumptions specified in Problem 64–7.

Prob. 71–7. If the alternator of Problem 70–7 were connected as in Fig. 274 and were delivering a balanced load of 150 kv-a. at 87 per cent power-factor, lagging, to a transmission line, how much single-phase load at 65 per cent power-factor, lagging, could it carry in addition across one pair of terminals, on the assumptions specified in Problem 64–7?

CHAPTER VIII

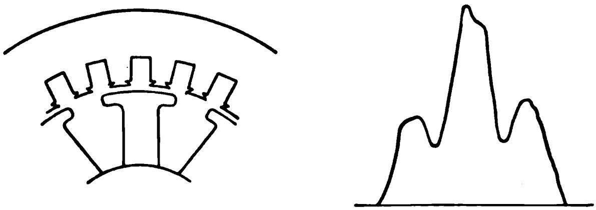
THE GENERATION OF PROPER E.M.F. WAVE-FORM

THE great importance of proper e.m.f. wave-form was not fully appreciated before large power plants and long-distance and high-voltage transmission lines became necessary. It is difficult, and often impossible, to operate alternating-current machines interconnected on the same system unless they all generate very nearly the same wave-form of e.m.f. A very slight departure from a sine wave-form of e.m.f. causes to flow between the parts of the system heavy currents which are reactive in the sense that they deliver no power except the heat loss that they produce in the conductors. Moreover, the effects of an alternating e.m.f. upon a circuit cannot be calculated accurately or conveniently unless it varies according to the simple sine law. Therefore, the first and most necessary consideration about an alternator is that it shall generate very nearly a sine wave of e.m.f.; and as considerable variations of wave-form may be produced by differences in construction and method of operation, or even by such a usual fact as wearing of the bearings, it is desirable for us to study this subject briefly.

The **wave-form of e.m.f.** depends upon the **way in which the magnetic flux is distributed** around the poles, and the **way in which the inductors in each path are distributed** over the surface of the armature. The **actual** form of the wave representing the variation of e.m.f. and current in any circuit from instant to instant is easily measured by means of several devices, and in fact may be automatically drawn or traced out on a screen or on a photographic film, by the particular instrument known as the "oscillograph." Fig. 285 to 288 are copies of oscillographic records taken from operating machinery and presented in Proceedings A.I.E.E., Feb., 1913, by W. J. Foster.

89. Actual Wave-forms, by Test, Showing Effects of Shaping the Poles. The effect upon e.m.f. wave-form due to changing the shape of pole faces is illustrated in curves of Fig. 285, 286 and 287 taken on a 60-cycle 150-kw. 6600-volt three-phase belt-driven generator.

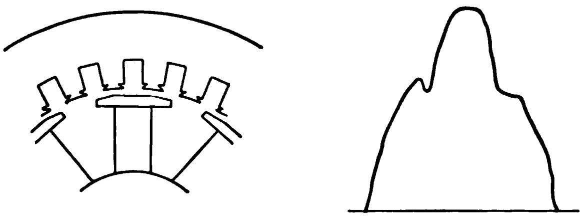
The only changes made between these several records were on the pole faces, the air gap being slightly increased and the pole tips beveled off between records 285 and 286, and the curvature of the



(a) (b)

FIG. 285. The unbeveled pole tips in (a) produce the ragged curve of e.m.f. shown in (b).

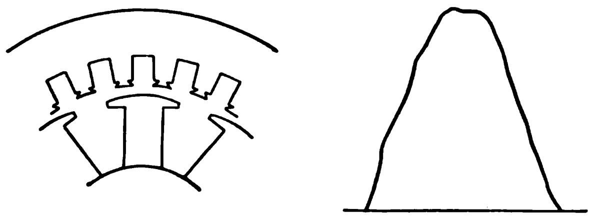
pole faces being made smoother between records 286 and 287. The effects upon wave shape are seen to be tremendous — abnormal, in fact — due to the relatively large ratio between width of slot and length of air gap which existed in this particular alternator. All three curves were taken at zero load.



(a) (b)

FIG. 286. The pole tips of Fig. 285 have been beveled to the shape shown in (a). The resulting smoothing of the e.m.f. curve is shown in (b).

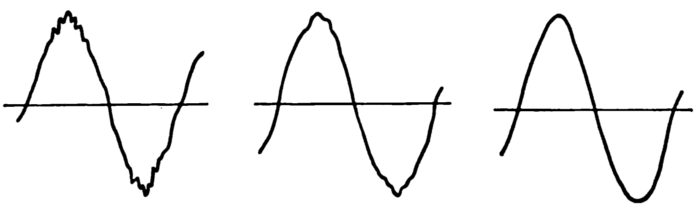
The same general effects are illustrated by Fig. 288a and 288b, taken from an 18-kv-a. 25-cycle 750-r.p.m. three-phase 110-volt alternator, operating without load. In Fig. 288a the radius of curvature of the faces of the revolving poles was $8\frac{1}{4}$ inches, whereas in Fig. 288b this radius was cut down to $7\frac{3}{4}$ inches, this being the only change. This not only increased the difference between the air gap at the center and at the tips of the pole, causing the flux to concentrate rather more at the center of the poles and tend to stay there against the redistributing or distorting effect of the armature currents, but there was also an increase in the actual average length



(a) (b)

FIG. 287. The curvature of the beveled pole tips of Fig. 286 has been smoothed to (a) and the curve of e.m.f. becomes much smoother as is shown in (b).

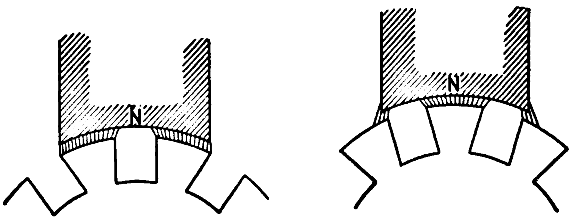
of the air gap and in the ratio of air gap to slot width. The smoothing-out of the no-load e.m.f. curve is due principally to the latter cause. The reason will be seen from inspection of Fig. 289a and 289b, which show how the effective sectional area of the magnetic path for the flux through the air gap and teeth into the steel core



(a) (b) (c)

FIG. 288. The e.m.f. curve (a) was taken and then the radius of curvature of the pole face was made one-half inch less, and the e.m.f. curve (b) resulted. At full load the curve (b) becomes curve (c).

of the armature varies from instant to instant as the armature rotates, and the number of teeth under the pole face changes. The change is usually small and is purposely exaggerated in the sketch to make it clear; nevertheless the reluctance of the path of the flux through the air gap and teeth is actually caused to change periodically at a high frequency. This produces a corresponding periodic pulsation in the strength of the useful flux and, therefore, in the induced e.m.f., the **frequency** of the fluctuation depending upon the speed of the machine and the number of teeth, and the **amount** of fluctuation depending upon the ratio of change in the magnetic reluctance. This fluctuation of induced e.m.f. appears as ripples in the e.m.f. wave, fine ones if the number of teeth or slots per pole is large, and long ones if this number is small. If we make the air gap longer while keeping the number of slots per pole unaltered, it is seen that the magnetic reluctance of air gap and teeth is greater, hence, the same actual fluctuation produces a relatively less effect upon the useful flux and induced e.m.f., and the ripples are smoothed out. Fig. 288c was taken from the same machine carrying its rated full load at unity power-factor, the pole face being same as in



(a) (b)

Fig. 289. By comparing (a) and (b) it will be seen that the path of the flux from pole to armature is ever changing as the armature teeth and slots pass the pole face. This causes the ripples in the e.m.f. curve of Fig. 288 (a).

Fig. 288b. Practically all traces of the original ripples have been smoothed out. This effect of load upon wave-form of e.m.f. is quite complex and not always the same. In this case the wave is smoothed, but sometimes the distortion of flux due to loading is such that the wave is made more irregular than at zero load. The inductive reactance of the armature, as well as the saturation of the teeth, affect the result.

90. Calculation of Wave-form. Flux Radial and Uniform from Each Pole. Let us represent graphically the variations of e.m.f. induced in a **single inductor** on the two-pole 60-cycle 3600-r.p.m. alternating-current generator shown in Fig. 214, on the basis of a uniform flux density of 60,000 lines per square inch in the air gap, an armature diameter of 6 inches, and an active length of 12 inches for each armature inductor. Assume the flux to pass radially into the armature core, without any fringing at the pole tips. Following any one chosen inductor, say that labeled **1**, through successive marked positions $\frac{1}{24}$ cycle apart, we see that

From position 1 to position 3, e.m.f. = 0.0 volt.

From position 3 to position 11, e.m.f. is constant at a value of

$$\frac{60{,}000 \times (3.14 \times 6'' \times 3600) \times 12''}{10^8 \times 60} \text{ volts} = +\,8.15 \text{ volts},$$

in a direction out of the paper toward the reader.

From position 11 to position 15, e.m.f. = 0.0 volts.

From position 15 to position 23, e.m.f. = − 8.15 volts, **away from the reader.**

From position 23 to position 1, e.m.f. = 0.0 volt, completing one cycle.

Plotting these e.m.f.'s as ordinates with respect to positions as abscissas, we get the heavy line in Fig. 290, which is the form of

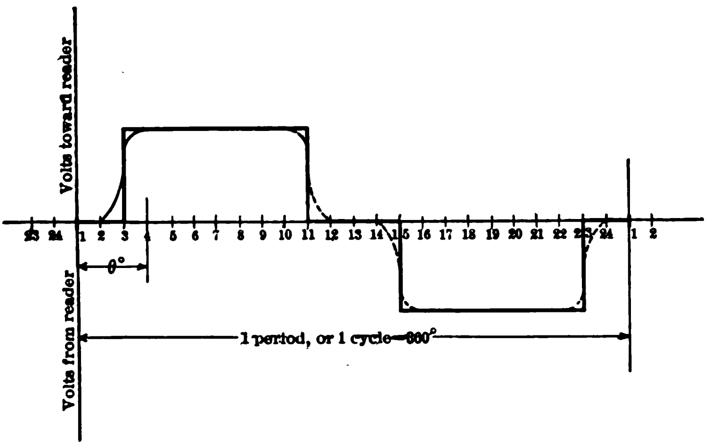


FIG. 290. The wave-form produced in concentrated armature windings with flux uniformly distributed over pole face and passing radially into the armature.

"e.m.f. wave" induced in each single inductor on this armature. If ten inductors or ten turns of winding were bound closely together into a bundle or coil, forming what would be called a concentrated winding, the total e.m.f. for the entire bundle or coil-side would be $10 \times 8.15 = 81.5$ volts instead of 8.15 volts, but the form of the e.m.f. wave would be exactly the same.

91. Effects of Fringing of Flux and of Armature Reaction. In the actual machine, the flux would always spread out or stray somewhat near the pole tips, so that the e.m.f. would not begin and end abruptly, but gradually, as indicated approximately by the dotted lines in Fig. 290. Moreover, when the circuit is completed externally and the generator delivers load, the magnetic effect of current flowing through the armature wires would not only change the value of the flux, but also distort it somewhat, shifting it away from the leading pole tip and making the density correspondingly greater near the trailing tip.* The e.m.f. induced as the inductor passes through any point is equal to the rate of cutting flux at that point.

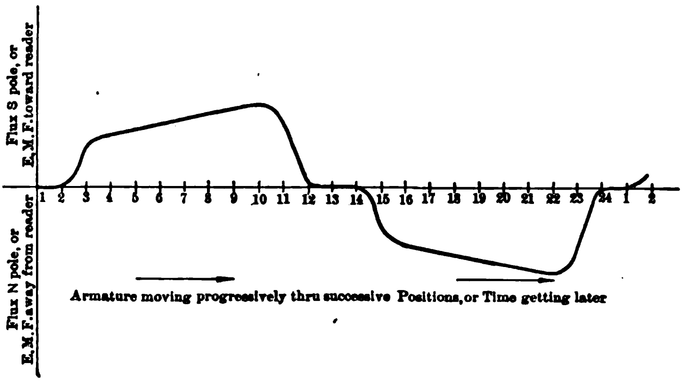


FIG. 291. The current flowing through the armature conductors may cause the flux to be crowded into the "trailing" pole tips as this curve shows. The e.m.f. curve of Fig. 290 would then assume this same shape.

As the speed is constant, the form of the **e.m.f. curve** will be exactly the same as that of a **flux curve,** which shows flux density (as ordinate) at each position (as abscissa). The effect of loading the generator then, will be to change the form of the curve in Fig. 290 (representing either distribution of flux or values of e.m.f. induced, according to the scale of ordinates used) to some other form like Fig. 291. The amount of change depends upon the extent to which armature reaction affects the useful flux, which again depends upon the relation of the amperes and turns in the armature winding to the ampere-turns in the field winding, distribution of armature winding,

* The leading pole tip is the tip which the conductor first comes under when it reaches the pole.

and power-factor of load. Note that the e.m.f. gradually increases as the inductor approaches the trailing tip of the pole where the flux is most dense.

92. Effects of Shaping the Pole Tips. In an effort to reduce the amount of this flux distortion due to magnetic reaction of the loaded armature, and to make the generated e.m.f. approach more closely to a sine wave, the face of the pole is often given a radius of curvature differing from that of the armature, or the tips of the poles are beveled somewhat, as illustrated in Fig. 293 and 292, respectively. The flux resists being crowded to the trailing pole tips of the generator, because this would require more of it to pass through a longer air gap, and flux always tends to choose the path whose magnetic

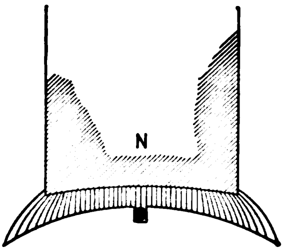


FIG. 292. A pole face with a curvature less than that of the armature, causing higher flux density under the middle of the pole, and less distortion of flux and e.m.f. due to load.

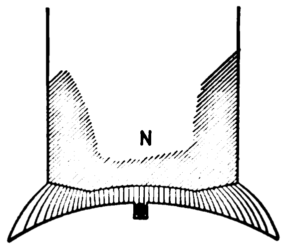


FIG. 293. A beveled pole face showing lines crowded toward center of pole. It is more difficult to crowd these lines into one tip because of the greater reluctance at the tips.

reluctance is least, just as current tends to prefer a circuit whose resistance or impedance is least. The effect of such cutting of pole tips upon flux distribution and the form of e.m.f. wave generated is represented by Fig. 294. In some concentrated windings for alternators, the form of e.m.f. wave is controlled entirely by shaping the pole pieces; but in most modern machines the series inductors in each phase or circuit of the armature winding are distributed in at least two or three slots under each pole, and the effects of this distribution of winding, or widening of the armature coils into bands of conductors, upon the form of e.m.f. wave, is very marked, and is most commonly employed to produce a close approximation to a sine wave-form.

93. Actual Wave-forms Showing Effects of Distributing the Windings. The effect, upon the wave-form of induced e.m.f., produced by distributing the winding in different numbers of slots per pole, is illustrated in Fig. 295a, b and c, these being reproductions of oscillographic records taken at no-load from a 3000-kv-a. three-

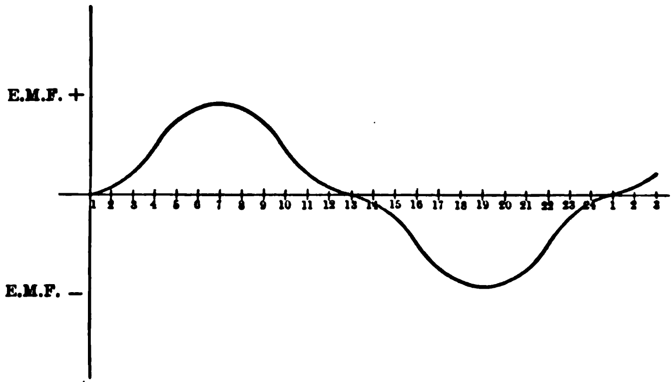
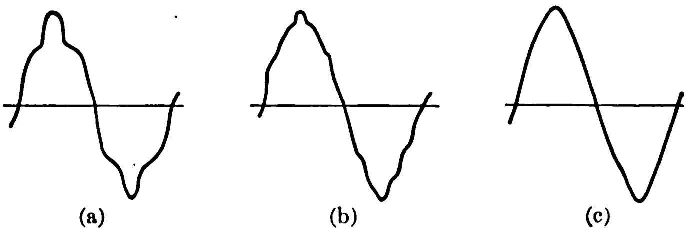


FIG. 294. The effect upon the e.m.f. of beveling the pole tips. The e.m.f. is much greater when the conductor is passing the center of the pole than when passing the tips.

phase star-connected 6600-volt alternator. Fig. 295a shows the e.m.f. wave when the winding (of each phase) is concentrated in one slot per pole per phase. In Fig. 295b it is distributed in two slots per pole per phase, and the wave is seen to be generally smoother.



(a) (b) (c)

FIG. 295. The e.m.f. curve with the windings concentrated in one slot is shown in (a), in two slots in (b) and in three slots in (c).

In Fig. 295c the winding is still further distributed in three slots per pole per phase, and the e.m.f. curve is quite smooth. These curves are not all to the same scale.

94. Exploring-coil Method of Finding Wave-forms of E.M.F. in a Single Coil. The distribution of flux under the poles of the machine is usually explored by measuring the wave of e.m.f. induced in an "exploring coil" or separate concentrated winding wound in one of the slots under each pole, alongside the regular winding. The form of the curve between **time** and **e.m.f.** induced in such a coil is exactly the same as the curve between **corresponding position** and **flux density** in that part of the air gap through which the coil is passing (if the speed of rotation is exactly constant, as usual). Fig. 296 is an oscillographic record of the wave of e.m.f. induced in an exploring coil on a 1400 kv-a. 50-cycle three-phase 10,000-volt alternator, the coil being wound so as to span the full pole pitch of 12 teeth, or so that its sides were exactly under the centers of adjacent poles at the same instant. Notice first the saw-tooth fluctuations of high frequency in the e.m.f. wave due to the pulsations of flux caused by the teeth passing under the pole faces. These could be reduced in amplitude or smoothed out by making the air gap greater or the number of slots per pole greater. Notice also the generally flat-top appearance of the wave similar to Fig. 290 for a concentrated winding; also, the comparatively wide neutral

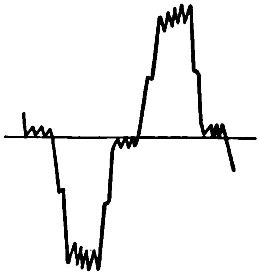
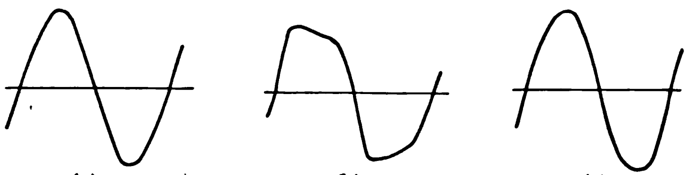


FIG. 296. The e.m.f. curve, taken by means of an exploring coil, of a 10,000-volt generator.



(a) (b) (c)

FIG. 297. The curve (b) shows the effect of a load with a leading power-factor upon the generator voltage. The curve (c) shows how this curve was affected by the use of a compensating coil wound on the field structure.

zones where the inductors cut no flux and the e.m.f. is practically zero.

95. Actual Wave-forms Showing Effects of Loading an Alternator. Fig. 297a and 297b illustrate the effect of load upon the terminal e.m.f. of an alternator, and Fig. 297c shows how this effect is neutralized by use of a compensating coil wound on the field structure. The machine on which these curves were obtained was a 50-kv-a. 60-cycle single-phase alternator, having a cylindrical revolving field or rotor. The load was a condenser, such as an underground cable unloaded, taking an almost wholly reactive charging current from the generator (see Second Course, Chapter V).

In general, the same three-phase winding will give an e.m.f. wave more nearly approaching a sine curve, between the terminals, when the phases are Y-connected than when delta-connected. Fig. 298a is a copy of the oscillograph record of terminal e.m.f. at zero load on a Y-connected three-phase generator, rated 2100 kv-a., 60 cycles, while Fig. 298b is for the same machine delta-connected.

96. Calculation of Wave-form, Showing Effects of Distributing the Windings. The e.m.f. between two terminals of an alter-

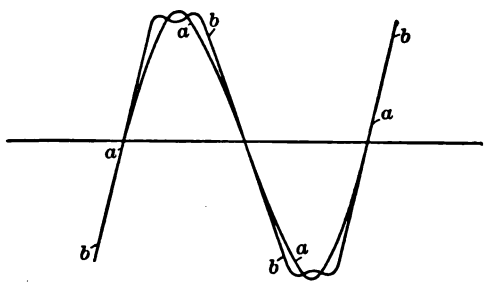


FIG. 298. *a* is the e.m.f. curve of a Y-connected alternator. *b* is the e.m.f. curve of the same generator Δ-connected. Note the fact that the Y-connection gives the much smoother curve.

nating-current generator at any given instant is equal to the sum of all the e.m.f.'s which are being induced at that instant in the individual inductors that are in series on the armature between these terminals. This sum is an **algebraic sum**, having regard to the **directions** around the circuit of the component instantaneous e.m.f.'s in the various inductors. At all instants, except two in each cycle (corresponding to the maximum values of e.m.f. illustrated in Fig. 215 and 217), some of the inductors in each of the parallel paths between collecting-rings will have e.m.f.'s opposed to those in the other inductors of the same path. In fact, the zero values of e.m.f. between rings are due to cancellation of opposite and equal e.m.f.'s in series, and not to inactivity of the inductors.

To illustrate the effect of distribution of winding upon wave-form of e.m.f. let us find the curve of e.m.f. produced between the collecting-rings in Fig. 211, under the following simple conditions:

(*a*) When there is only one turn per coil or per pair of slots, or one inductor per slot.

(*b*) When the flux distribution is such as to induce an e.m.f. wave of the rectangular form illustrated by Fig. 290 (heavy line), in each inductor.

(*c*) When the total number of equidistant slots on the armature is equal to 48, of which only 2 per pole, or 8 altogether, are in use for this winding.

Curve 299a represents the e.m.f. induced in the inductor in one slot; curve 299b, the e.m.f. induced in the conductor in the adjacent

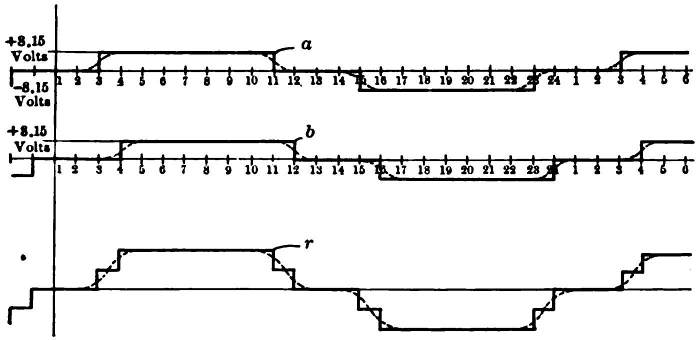


FIG. 299.

a = Curve of e.m.f. induced in one slot.
b = Curve of e.m.f. induced in adjacent slot.
r = Curve of resultant e.m.f., two adjacent inductors in series.

slot under the same pole, which is in additive series with the former, as may be seen by inspection of the connections in Fig. 211. The e.m.f. in conductor *b* attains its maximum value later than the e.m.f. in conductor *a*, but persists longer before it decreases again to zero; in fact, there is a "difference of phase" between e.m.f.'s *a* and *b*, this difference being the time necessary to pass from the middle of one slot to the middle of the adjacent slot. Since there are altogether 48 slots under 2 pairs of poles, or 24 slots per cycle, the slot pitch, or distance between adjacent slots, is $\frac{1}{24}$ of 180 mechanical degrees, represented by $\frac{1}{24}$ of 360 electrical degrees in a four-pole machine, or 15 electrical degrees or time degrees. Curve *b* is shown displaced by this amount with respect to curve *a*. The ordinate of curve r is at all points, positions or instants, equal to the (algebraic) sum of the ordinates of curves a and b at the same instant. The winding consists of two inductors under each pole located with respect to each other and to the pole as a and b are.* Hence for the four-pole machine here considered, the form of the curve of **total e.m.f. between rings** will be as developed in curve r, Fig. 299. Each ordinate, however, will be four times as great, since there would be a pair of conductors in a corresponding position under each pole and

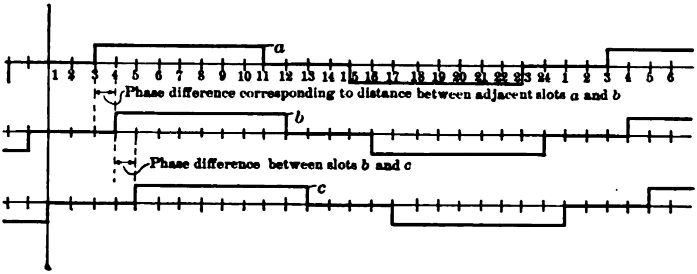


FIG. 300. Curves of e.m.f. in each conductor of a generator using three adjacent slots per pole out of a total of twelve slots per pole.

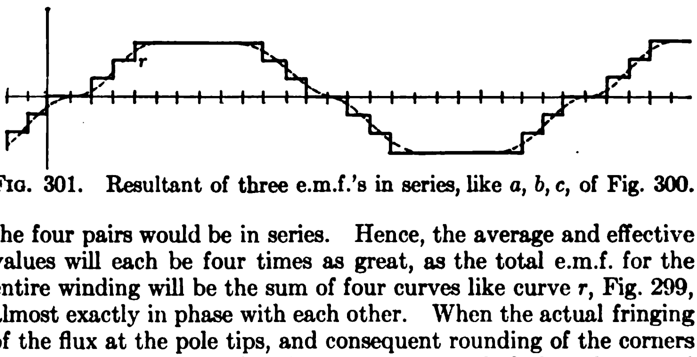


FIG. 301. Resultant of three e.m.f.'s in series, like a, b, c, of Fig. 300.

the four pairs would be in series. Hence, the average and effective values will each be four times as great, as the total e.m.f. for the entire winding will be the sum of four curves like curve r, Fig. 299, almost exactly in phase with each other. When the actual fringing of the flux at the pole tips, and consequent rounding of the corners of e.m.f. curves, are taken into account, we find that the actual resultant e.m.f. curve is about as shown by the dotted lines in Fig. 299.

* The *band* of conductors cannot always be located in *exactly* similar manner with respect to all poles when using this particular (wave) winding, on account of the necessity for the winding to progress around the armature, but the difference is small.

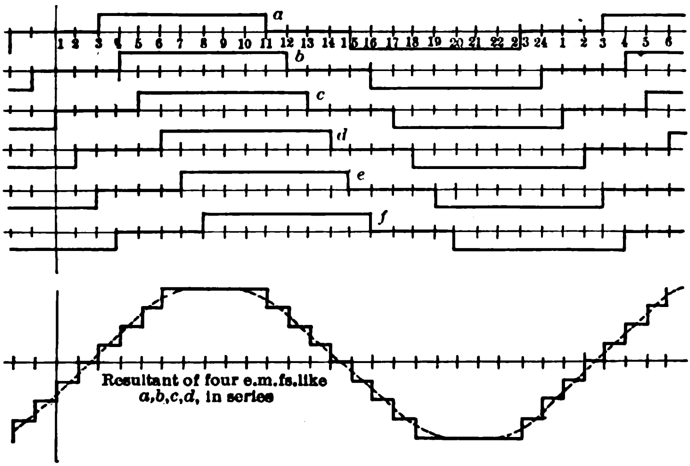


Fig. 302. The e.m.f. developed in each conductor of a generator using four slots a, b, c and d per pole out of a total of twelve slots per pole. The lower curve is the resultant voltage across the four conductors in series.

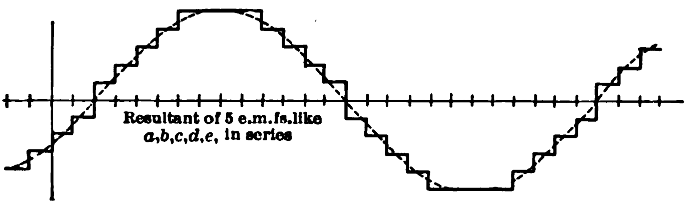


Fig. 303. Note that the curve becomes more peaked as we add more conductors per pole.

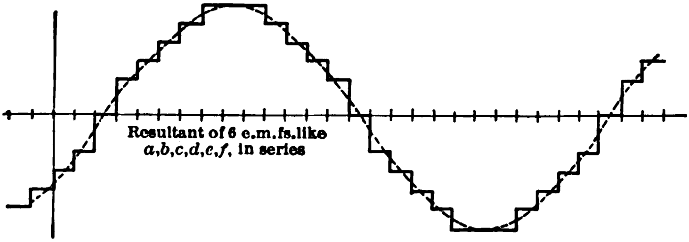


Fig. 304. The curve becomes still more peaked with six conductors in series.

Fig. 300 and 301 illustrate the composition of the curve of resultant e.m.f. in a winding occupying three slots per pole out of 24 slots per pair of poles, or 12 slots per pole, flux density uniform over the entire pole face or air gap. The heavy line is developed on the assumption of no fringing, and the dotted line illustrates the result obtained when fringing occurs as in the actual machine. The scale of ordinates for the curve will depend upon the flux per pole, frequency or speed, and number of series conductors per slot.

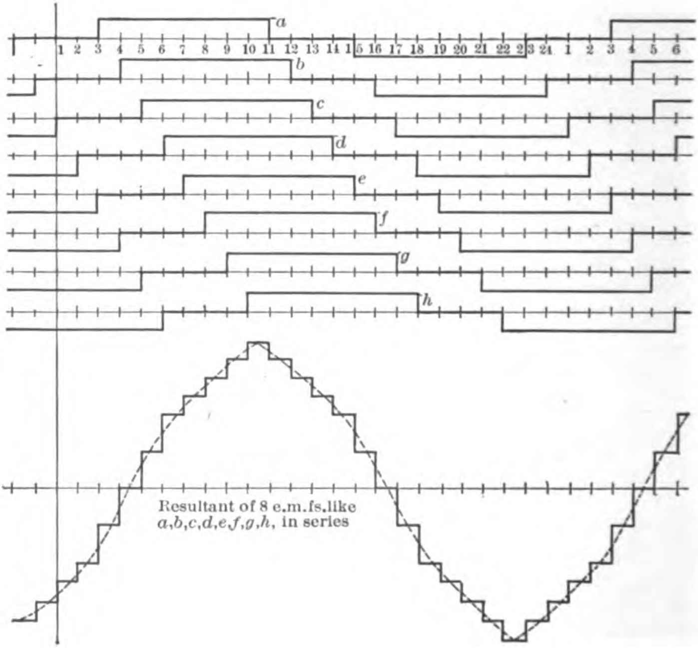


FIG. 305. The e.m.f. curve is now very peaked.

Fig. 302 is developed for the same armature with a winding occupying 4 slots per pole, Fig. 303 for 5 slots per pole, Fig. 304 for 6 slots per pole, Fig. 305 for 8 slots per pole, and Fig. 306 for 12 slots per pole. Notice that, as we start from a winding **concentrated** in one slot per pole, and widen the winding into a band consisting of 2, 3, 4, 5, 6, 8 and 12 slots per pole (all inductors being connected in series with each other), the curve of resultant e.m.f. changes from a low flat-topped wave (for concentrated winding, as in Fig. 290 or 299) to a peaked wave (for **distributed** winding as in Fig. 305). Then, as the winding becomes still more extensively distributed, the

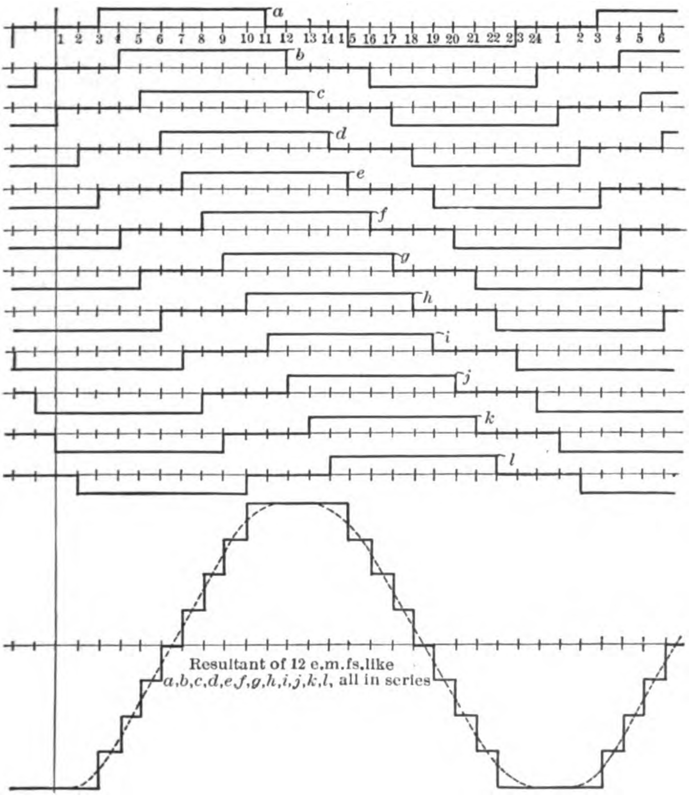


FIG. 306. As too many slots per pole are filled with conductors in series, the e.m.f. curve again becomes flat-topped. Note that the resultant e.m.f. is very little higher than that of Fig. 305 although there are 50 per cent more inductors and copper.

wave becomes flat-topped again and higher or greater in amplitude (though not nearly in proportion to the increase in the number of inductors), as shown in Fig. 306. For usual values of "polar embrace," a close approximation to the sine wave-form for the resultant e.m.f. is obtained by distributing the winding over about one-third of the total number of slots, as is done in a three-phase armature whose winding fills all of the slots.

Prob. 1–8. An alternator, such as illustrated in Fig. 214, operates at 60 cycles frequency with a peripheral speed of 5000 feet per minute. The active length of each inductor is 10 inches. The poles cover $\frac{2}{3}$ of the circumference of the armature, and there are 24 inductors. The flux is uniform in the air gap and has an average density of 50,000 lines per square inch; assume no fringing at the pole tips. Draw to scale the e.m.f. wave induced in a single inductor. Calculate the average and effective values of this e.m.f.; also, the **form-factor,** or ratio of effective to average e.m.f., and the **amplitude-factor,** or ratio of maximum to effective e.m.f.

Prob. 2–8. Again in this same alternator the flux fringes out similarly at both pole tips, so that the density increases uniformly from zero value at position 2 to the maximum (specified in Problem 1–8) value at position 4, remains at that value from 4 to 10, then decreases uniformly to zero at position 12. The flux distribution under the other pole is, of course, exactly similar. Draw a curve between flux density as ordinates and positions as abscissas. Draw also a curve between instantaneous volts induced in a single inductor as ordinates, and time in seconds as abscissas. Compare these curves.

Prob. 3–8. From the curve in Problem 2–8, calculate the average and effective values of e.m.f. in one inductor, and the form-factor and amplitude-factor. By comparing results with Problem 1–8, draw some conclusions concerning the effect of rounding the corners of the e.m.f. curve. Calculate the total flux per pole in Problem 2–8, and the total flux per pole in Problem 1–8, and find whether the effective e.m.f. per inductor has been changed in greater or less ratio than the change in flux per pole. Calculate the average flux density and find whether the average e.m.f. per inductor has been changed in the same ratio as the average flux density.

Prob. 4–8. When the generator of Problem 2–8 delivers a certain amount of current at a certain power-factor, the flux from each pole is both changed in amount and rearranged over the pole face, so that the density now rises uniformly from 20 per cent below the zero-load average density (50,000 lines per square inch), at position 4, to 10 per cent above zero-load average density, at position 10, fringing out with a uniformly decreasing density from position 4 to position 2, and from 10 to 12, being zero from 24 to 2 and from 12 to 14. Draw the curve of flux distribution and the curve of e.m.f. in a single inductor, as described in Problem 2–8. Calculate average and effective values of e.m.f., and form-factor. Calculate average flux density and total flux per pole. Compare results with Problems 3–8 and 1–8 and find whether the average e.m.f. and the effective e.m.f. have been altered in greater or less ratio than the total flux per pole.

Prob. 5–8. Calculate the average and effective values of the e.m.f. curve (*a*) in Fig. 299 and of curve (*r*) (heavy lines) using any scale of volts that may be convenient, and ordinates not more than 15 electrical degrees apart. Is the average value of the resultant equal to the arithmetical sum of the average values of the components? Is the effective value of the resultant equal to the arithmetical sum of effective values of the components? Is it equal to the vector sum of the components, if each component is represented by a vector (length of vector proportional to effective value of component e.m.f., angle between vectors equal to phase difference between components)? Calculate the form-factor of one component and of the resultant e.m.f., and compare these values.

97. Irregular Wave-forms of E.M.F. Undesirable Effects. If the wave-form of e.m.f. produced by an alternator departs much from the standard form (sine wave), the following undesirable effects result:

(1) **Parallel Operation.** Cross-currents.

When it operates in parallel with another alternator of dissimilar wave-form, currents are exchanged locally between the armatures, which deliver no power to the external circuit, but nevertheless heat up the armatures and reduce their capacity for delivering real power output (see Art. 134). If the difference is very great, the machines may not be able to operate together at all.

(2) **Peaked Waves.** Better insulation necessary.

For a wave-form more peaked than the standard sine wave, the "amplitude factor," or ratio (maximum value to mean effective value), is greater than for the sine wave; and for a wave which is more nearly flat-topped than the sine wave, this factor is less. Insulation may break down at any instant during the cycle — usually, of course, at or near the maximum or peak value of e.m.f. For a given mean effective value of voltage, that alternator will have the greatest stress upon its insulation whose wave-form is most peaked, other things being equal. Hence, to give the same factor of safety from danger of break-down and short-circuit at any given operating voltage, the machine with the more peaked wave-form or higher amplitude-factor would require thicker and more expensive insulation; or, for the same insulation and effective voltage, the danger of break-down would be greater with a peaked than with a flat-topped e.m.f. wave.

(3) **Ripples. Effect of Capacity and Inductance.**

When a generator whose wave-form is peaked or flat, or has ripples in it, is connected to a circuit having condenser effect (see Second Course, Chapter V) — as a long transmission line operated at high voltage or an underground cable operated even at moderate voltage, the wave-form of the current will be very much further from the sine form than that

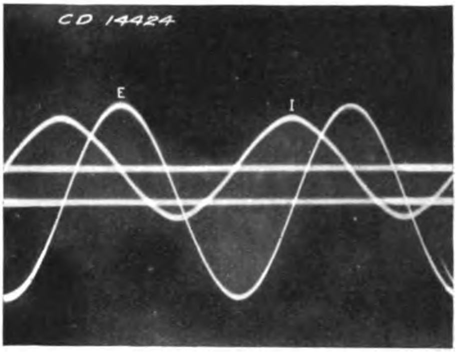


Fig. 307. The sine wave E produces a sine wave of current I even in a circuit containing either inductance or capacity. Oscillograms furnished by C. M. Davis.

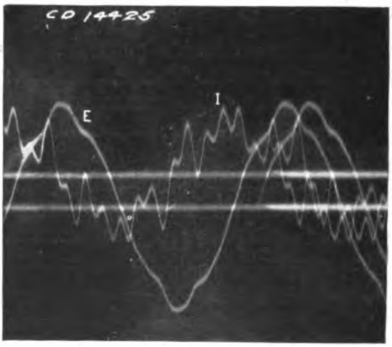


Fig. 308. The non-sine wave E produces a very irregular current curve I when applied to a circuit of large capacity. Oscillogram furnished by C. M. Davis.

of the e.m.f. which produces it. The shorter or higher-frequency ripples in the e.m.f. wave produce the greater irregularities in the current wave. In Fig. 307, the curve I shows the wave-form of current produced in a load having condenser characteristics (an underground cable), when the e.m.f. E of true sine wave-form is impressed upon it. Note that the current I has also a sine wave-form. Fig. 308 shows the current I which a non-sine wave-form E produced when impressed upon the same circuit. The wave E is not nearly so far from sine form as many which may be observed in practice, yet these slight irregularities of e.m.f. are seen to cause large effects on the current in a circuit having electrostatic capacity. Fig. 308a is an oscillogram showing the irregular current curve produced by an e.m.f. which has not quite a sine wave-form. When this **same generator** is connected to an inductive circuit taking a lagging current and

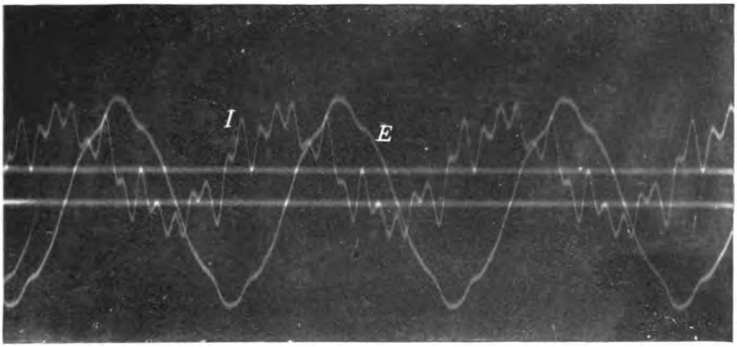


FIG. 308a. The wave-form E has a slight ripple in it which produces a very irregular current curve I when applied to an unloaded transmission line. Oscillogram furnished by C. M. Davis.

having an electromagnetic instead of an electrostatic field around it, the current wave takes the form of I in Fig. 309. Notice that the ripples in the wave of terminal e.m.f. which were quite pronounced when the current was leading, are all smoothed out when the current lags. Since nothing has been altered except phase relation (the shape of pole pieces, distribution of winding, etc., remaining unchanged), the effect of the current wave-form upon the wave-form of terminal e.m.f. must be due to armature reaction or to internal voltage-drop caused by reactance and resistance. These curves were taken from a three-phase 150-kv-a. 48-pole 2300-volt generator. See Trans. A.I.E.E., Feb., 1913, C. M. Davis. In general, inductive circuits smooth out irregularities in the e.m.f. and current waves, and long transmission lines or cables, lightly loaded, exaggerate them.

(4) Flat Waves. Hysteresis Losses Greater.

The wave-form of the e.m.f. has also an effect upon the energy losses in the iron core of any apparatus, such as motors, generators or transformers, connected to the circuit. If we impress upon exactly similar circuits, two e.m.f.'s of equal effective values, but one having a flat-top wave and the other a peak-topped wave, the e.m.f. with the flat-top wave will set up the greater alternating magnetic flux around the coils. As

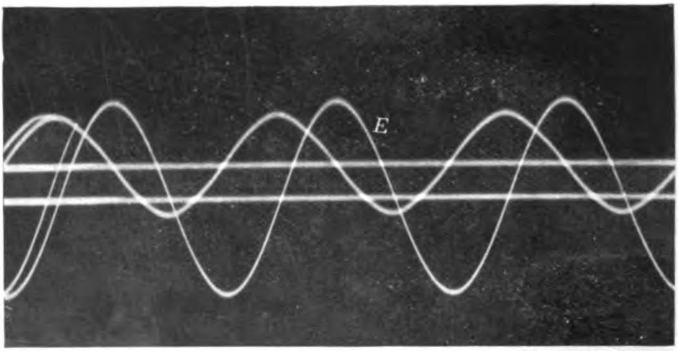


FIG. 309. An inductive load will often smooth out the wrinkles in a wave and make it appear as the voltage curve *E*. Oscillogram furnished by C. M. Davis.

the coils are identical, that which has the greater flux will also have the greater flux density. The hysteresis loss, per pound of iron, at a fixed frequency, varies as about the 1.6 power of the maximum value of flux density attained in the core throughout the cycle. The exact value of the loss and rate of variation with flux density depends upon the quality of the iron, as shown by numerous experimental researches.* If the maximum flux density in the core is doubled (volume, temperature and number of turns being constant), the hysteresis loss will be increased to about 3.03 times its former value. If B_m be increased to three times, hysteresis loss is increased to about 5.8 times; if B_m is increased to four

* See American Handbook for Electrical Engineers, pages 902–909.

times, hysteresis loss is increased to about 9.19 times its original value, and so on, these ratios being subject to slight change for different grades of iron. The eddy-current loss per pound of iron core, at fixed frequency, for a fixed number of turns in the coil and constant effective value of e.m.f. applied, is independent of the form of the e.m.f. wave.

The variation of total core losses with wave-form at constant effective voltage across a given coil will, therefore, be equal to the variation of the hysteresis loss only. As in ordinary practice the eddy-current loss is reduced by use of thinner laminations until it does not exceed 25 per cent of the hysteresis loss, the ratio of change in the entire core loss will be not far different from the ratio of change of the hysteresis loss. Good practice as defined by the Standardization Rules of the A.I.E.E. requires that the actual e.m.f. wave shall not deviate at any instant from the equivalent sine wave (whose square-root-mean-square value is the same) by an amount greater than 10 per cent of the maximum value of this equivalent sine wave. See Art. 205 to 206 of Standardization Rules. It has been shown that, within the limits of this permissible variation, the core losses may vary as much as 7.8 per cent below, and 5.8 per cent above, that which would be obtained with the true sine wave. Since the core losses remain approximately constant at all loads, as long as the apparatus is connected to the source of power and in readiness to serve its purpose, any variation in these core losses is of great economic importance. In lighting transformers, for instance, which are always excited and ready to serve, the core losses go on every hour in the day and every day in the year, whereas the copper losses are of appreciable value only while the transformer is carrying a fair percentage of its rated load, which is usually only a few hours per day. Consequently, a change of 10 per cent in the core losses, which is seen to be easily possible by change of the wave-form, means almost a 10 per cent change in the annual energy losses in the transformer if the average load, or load-factor, is low. This would make an appreciable dollars-and-cents difference in the annual cost of operation, particularly in the case of a system operating large numbers of small transformers.

(5) Measuring Instruments. Power-factor Meter.

The standard types of voltmeters, ammeters, and wattmeters are not appreciably affected by the wave-form. This holds true, no matter whether the actuating principle is magnetic attraction between coils containing no iron or heating. Each type gives an indication depending only upon the square-root-mean-square or effective value. The indications of soft-iron instruments, or induction instruments depend to some extent upon the wave-form. The variation is usually small, however, in good modern meters. The indications of a **power-factor meter,** however, vary considerably with changes in the wave-form of the e.m.f. or current. If either or both of these waves depart from sine form, there may be considerable actual power delivered or consumed even with zero phase difference between the waves of current and e.m.f. The power-factor meter, whose indications are proportional to the cosine of this **phase difference** between the E and I waves which have sine wave-forms, will, therefore, indicate correctly only when the wave-forms are exactly like those with which the calibration of the instrument was made.

Modern refinements in alternating-current apparatus and engineering practice make the subject of wave-shape increasingly important. It is no longer permissible to disregard the question of wave-shape if only the various apparatus on the system will merely **operate** together — it must operate **properly** — as economically, as smoothly, and as usefully as possible. As the determination of wave-shape requires rather unusual and expensive instruments, variations, that are unsuspected, frequently cause much difficulty in electric circuits.

Prob. 6-8. In a certain circuit the wave-shape of the e.m.f. is semicircular. On account of certain peculiar reactions in the circuit, this e.m.f. produces a current wave of the same frequency whose shape is triangular. As these two curves are perfectly symmetrical, the phase difference between them may be easily determined, because it is the difference in time between the instants at which they reach their respective maximum values in the same direction through the circuit. The maximum value of the e.m.f. is 10,000 volts, and of the current 10 amperes. The current lags 60 degrees behind the e.m.f. Calculate the effective value of the e.m.f. and the effective value of the current. Multiply together the simultaneous instantaneous values of e.m.f. and current at equal intervals of time (interval not more than 10 electrical degrees) for one complete cycle of e.m.f. From the curve of instantaneous power so determined, calculate the average power. Find the ratio of average power to the product of effective volts and effective amperes — this is, the "power-factor" of the circuit, according to the definition of this term.

Prob. 7–8. The power in the circuit of Prob. 6 is indicated by a power-factor meter, which was calibrated with e.m.f. and current having sine wave-shapes. Assume that the deflection of the pointer of this instrument is proportional to the phase angle even if the waves have not the sine wave-form and that the power-factor, marked on the scale at each position of the pointer, is the power-factor of a circuit having the same phase angle between e.m.f. and current, both of which have sine wave-shapes. Would the power-factor meter register the power-factor of this particular circuit correctly? If not, by what percentage of the correct value would its readings be too high or too low?

Prob. 8–8. If the phase angle between current and e.m.f. in Problem 6–8 were zero, by what percentage would the power-factor meter read too high or too low?

Prob. 9–8. Repeat Problems 6–8 and 7–8, using a flat-topped rectangular e.m.f. wave of the same maximum value, and the same current wave as before, placing them 60 degrees out of phase.

Prob. 10–8. Repeat Problem 9–8, placing the e.m.f. and current in phase with each other.

Prob. 11–8. Choosing any convenient scale of volts and amperes, calculate the mean effective volts and amperes for the curves in Fig. 308. Multiply instantaneous volts and amperes together at instants 10 electrical degrees apart for one complete cycle of e.m.f., and calculate the correct average power from these values of instantaneous power. (If the figure were large enough to scale from the results would be more accurate if intervals were taken to be 5 degrees.) Calculate ratio of correct average power, to product of effective volts and effective amperes; this is the correct power-factor, according to definition. If the power-factor meter on this circuit had been calibrated with e.m.f. and current having sine wave-shape, what power-factor would it indicate? What correction (add or subtract, and how much) would have to be applied to its reading in order to get correct results with these particular irregular wave shapes?

SUMMARY OF CHAPTER VIII

THE WAVE-FORM OF THE TERMINAL E.M.F. is important as it affects the accuracy of our calculations for the generator and for the alternating-current circuit. These calculations are usually founded on the assumption that both the current and the e.m.f. have a sine wave-form. The form of these waves will depend on the form of the induced e.m.f. wave, and upon the form of the voltage reactions, or " drops " within the machine, due to resistance and reactance.

THE FORM OF THE VOLTAGE REACTIONS depends upon the wave-form of current delivered, which in turn depends upon the e.m.f. wave and the character of the receiving circuit.

THE FORM OF THE INDUCED E.M.F. DEPENDS UPON:

(1) The distribution of the flux over the pole face or around the armature.

(2) The distribution of the winding on the armature, number of slots per pole, etc.

(3) The shape and length of the air gap and the relative magnetic strength (ampere-turns) of field magnets and armature winding.

(4) The power-factor of the load or the phase difference between current and induced e.m.f.

(5) The manner of connecting together the phases of a polyphase machine.

ARMATURE WINDINGS MAY BE EITHER CONCENTRATED OR DISTRIBUTED. The wave-form generated by a concentrated winding at no load is usually somewhat flat-topped. When the winding is distributed, the wave approaches a peaked form, although it is possible to distribute it so extensively as to revert to a flat-topped form. In the concentrated winding, using only one slot for each phase under each pole, the desired sine wave-form is obtained by shaping the pole faces so as to distribute the flux suitably. Most windings used in practice, however, are distributed in two or three slots for each phase under each pole. This distribution of the winding causes a marked approach to sine wave-form entirely apart from the effect of shaping the pole faces.

RIPPLES IN THE E.M.F. WAVE may be smoothed down by increasing the length of the air gap, increasing the number of slots per pole, beveling the pole tips and, in a polyphase alternator, by connecting the phases together in star instead of in mesh.

THE EFFECT OF LOAD upon the wave-form of terminal e.m.f. may be to make it either more or less like a sine wave, depending upon numerous conditions of operation and structural features. A compensating winding may be used on the poles to neutralize the distorting effect.

NON-SINE WAVE-FORMS OF E.M.F. have the following undesirable effects:

(1) Machines with unlike wave-forms operating in parallel exchange local currents which reduce their output.

(2) Machines with too-peaked curves require better insulation than those with sine curves.

(3) Ripples are greatly increased when the load takes a leading current, and are smoothed out by a lagging current.

(4) The hysteresis loss is increased by the use of flat-topped waves. Since the core-loss of transformers is about three-quarters hysteresis loss, the wave-form of e.m.f. used may greatly affect the "all-day" efficiency and the cost of operating these machines.

(5) The indications of most modern meters are very nearly independent of the wave-form. Power-factor meters, however, are made to indicate the cosine of the angle of phase difference between the current and the voltage only when both have sine wave-forms. The power-factor is really the ratio of effective power to apparent power, and this is NOT equal to the cosine of the angle of phase difference, UNLESS both the current and e.m.f. are of the sine wave-form. Thus power-factor meters do not read correctly on circuits which do not employ standard wave-forms.

PROBLEMS ON CHAPTER VIII

Prob. 12–8. Solve Problem 5–8 for Fig. 302 and one of the component e.m.f.'s and compare this solution with that of Problem 5–8. What would the average and effective values of e.m.f. have been for a winding consisting of the same total number of inductors moving at the same speed in the same field, but concentrated into one small bundle (or slot) under each pole, instead of divided equally among four equidistant bundles or slots 15 degrees apart, under each pole? Is either the average or effective total e.m.f. of the winding increased in proportion to the number of inductors, using the winding distributed as in Fig. 302? Answer the last question for a concentrated winding of equal total number of inductors.

Prob. 13–8. Solve Problem 12–8 for the winding whose resultant e.m.f. is as shown in Fig. 305. To what actions do you ascribe the decreasing effectiveness of inductors added in series when they are placed so that they broaden the band or make the winding cover a larger fraction of the pole pitch?

Prob. 14–8. Solve Problem 12–8 for the winding whose resultant e.m.f. is as shown in Fig. 306. This is called a **thoroughly-distributed** winding, because the inductors are distributed uniformly over the entire face of the armature. Show by diagram how you would connect the inductors from 12 to 24 into the series. If connected properly, how would they alter the e.m.f. curve, Fig. 306? If connected improperly, how would they alter the curve of Fig. 306?

Prob. 15–8. The ratio of the total effective e.m.f. of the entire winding as it is actually distributed upon the surface of the armature, to the total effective e.m.f. that would be obtained if the same total number of inductors had been concentrated into equal groups, one group or slot under each pole, is sometimes called the "phase constant" or "distribution constant" of the winding. Calculate the phase constants for Fig. 306, 305, 304, 303, 302, 301 and 299r, and compare them. What would the phase constant indicate to you concerning good policy in arranging the winding on an armature?

Prob. 16–8. Draw the curve of e.m.f. between collecting-rings (at zero load) for a single-phase alternator having six poles and 72 equidistant slots, the winding being partly distributed — that is, occupying only four slots under each pole. The useful flux per pole is 1,000,000 lines, uniformly distributed over the pole face and entering the armature radially at all points, with no fringing at edges of the pole. The poles embrace $\frac{7}{12}$ of the circumference of the armature. Frequency 60 cycles per second. Three conductors per slot. Calculate the effective value of e.m.f. between rings.

Prob. 17–8. If the armature of machine in Problem 16–8 were wound so as to have one conductor in every slot, all conductors being connected so as to add together in a single series circuit, what would be the curve of e.m.f. between terminals at zero load, and what would the voltmeter read across the terminals?

Prob. 18–8. By what percentage would the flux per pole have to be changed with the winding as in Problem 16–8 in order to give a voltmeter reading as in Problem 17–8?

Prob. 19–8. What would be the total effective voltage of a concentrated winding for the machine of Problem 16–8, consisting of four conductors per slot? What e.m.f. for eight conductors per slot? What is the phase constant for the winding in Problem 16–8 and in Problem 17–8?

Prob. 20–8. If the armature of Problem 16–8 were wound up for three phases, using four slots per pole per phase, and the phases were delta-connected, what would be the voltmeter reading between any two terminals? Draw to scale the curve of instantaneous e.m.f. obtained between terminals of the armature.

Prob. 21–8. Repeat Problem 20–8, but with the three phases of the winding Y-connected between terminals.

Prob. 22–8. Draw a sine wave of current, having the same phase lag as the actual current curve I, with respect to the actual e.m.f. curve E, in Fig. 308. Let its maximum value be called I_m. Calculate the curve of instantaneous power for a complete cycle, each ordinate being expressed in terms of a number times the unknown value I_m. Make calculations for instants not more than ten electrical degrees apart. Find the average power for one cycle, in terms of I_m. Find what value of I_m will make the average power with sine wave of current, the same as the average power with the actual wave of current (see results of Problem 11–8). The same e.m.f. wave E is used in both cases, of course. The heat losses in the transmission line wires are proportional to the mean of the squared values of instantaneous current. By what percentage in this case are the line losses due to I^2R different with the irregular current wave which actually flows than they would have been if the current wave had been of sine form, while delivering the same actual power with the same angle of lag and the same e.m.f. curve?

Prob. 23–8. Replace the irregular E curve in Fig. 308 with a sine curve in exactly the same position, and having the same effective value as the actual curve E. Then proceed as in Problem 22–8, and calculate the percentage change in I^2R losses on the transmission line, due to changing both e.m.f. and current waves to sine form, while delivering the same power with the same phase lag.

Prob. 24-8. Assuming as a convenient scale that the maximum value on the voltage curve of Fig. 308 is 1000 volts, calculate from Fig. 308 the following:

(*a*) Effective value of the e.m.f. curve *E*.

(*b*) Maximum value of an equivalent sine wave having the same effective value.

(*c*) Percentage by which the maximum value of the actual curve is greater or less than the maximum value of the equivalent sine curve.

This figure (*c*) represents the sacrifice or advantage as the case may be, in the amount of stress on the insulation of the circuit and in the probability of its failure, due to the deviation of the actual e.m.f. wave from the standard sine form. If the breakdown stress is three times the effective value of the equivalent sine wave, what will be the factor of safety of the insulation using the distorted e.m.f. wave in Fig. 308?

Prob. 25-8. A Type *H* transformer manufactured by the General Electric Co., operating on non-inductive load of full rated value, 3 kv-a., with frequency and voltage at value stated on the name plate, has an efficiency of 96.9 per cent, according to the manufacturers' advertisements, the iron losses being 35 watts and I^2R losses 62 watts. This rating, efficiency and losses are for continuous operation with sine wave of e.m.f. All of the energy which this transformer can deliver can be sold at five cents per kilowatt-hour for ten hours per day every day in the year, such being the character of the load and the demand or market for energy the load being non-inductive. The transformer must remain connected to the power supply in readiness to serve the customers at all times. The line is supplied from a generator giving a flat-topped wave instead of a sine wave, the distortion of wave-form being sufficient to increase the iron losses by 10 per cent with the same effective value of e.m.f. The total watts lost in the transformer cannot be allowed to be greater than at rated full load on sine wave of e.m.f., without ultimately injuring the transformer and shortening its useful life.

(*a*) By how much must the I^2R loss be reduced to compensate for the increase in iron loss?

(*b*) By what percentage must the current output be reduced on this account? Assume any convenient value of voltage, say 100 volts.

(*c*) What is the corresponding reduction in kilowatts output of the transformer?

(*d*) By how many dollars per year is the income of the central station from this transformer reduced below what it might be if the iron losses had not been increased?

(*e*) By how many dollars per year is the cost of losses in this transformer increased, assuming that the value of the energy lost is the same as that of the energy sold?

(*f*) What is the total annual loss in dollars, to the central station, due to this change of wave-form of the e.m.f., the customer's bill being based on reading of meter on output side of the transformer?

CHAPTER IX

ARMATURE WINDINGS OF ALTERNATING-CURRENT MACHINES

IN this chapter will be considered the windings of armatures only. The field windings will not be discussed because these carry direct current and have no proper place here. The armature of a generator has been defined as that part of the generator which **delivers** power to the external circuit. Accordingly, the armature of a motor is that part of a motor which **would deliver** power to the circuit if the motor were run as a generator. We have seen that the armature of both alternating- and direct-current machines always carries alternating current. The armature is always the rotor of a direct-current machine, but may be either the stator or the rotor of an alternating-current machine. It is usually the stator.

The typical **alternating-current windings** for motors and generators are essentially alike. The form of the slots in the steel core of the armature, into which the windings are placed, differs usually with the type of machine, but the same form and arrangement of coils may be used in every case. The subject of windings may be found discussed in great detail in specialized books, which should be consulted if it is desired to learn much concerning the relative merits of these windings and the proper places to apply each type, or if it is desired to know (as well as may be, without really doing it) how actually to manufacture the windings and assemble them on the machine. Here we shall emphasize only the electrical relations and principles involved. Explanations will be made in most cases with particular reference to **alternating-current generators** in order to make clearer the electromotive-force relations involved. It is usually more convenient both to make and to understand a diagram representing a winding, if it refers to a rotating armature. Hence the general use of such diagrams throughout this book does not mean that rotating armatures are common. In practice, all armatures of machines larger than 25 kilowatts, other than synchronous converters, are of the stationary type. Whether the armature is the stator or the rotor, the form of the windings may be exactly the same; — the coils on a moving armature being placed in slots on the outside surface of a rotating cylinder, while the coils of a stationary armature are placed in slots on the inside cylindrical surface of the stator.

98. Structure of the Winding. Windings are studied altogether by diagrams in which the armature circuits are represented by lines arranged in various conventional ways. Before committing ourselves to the study of such diagrams, it is well to gain a concrete idea of the physical appearance of the windings. Mechanically, the winding usually consists of a number of coils which are connected together to form a series or group. The coils may be all alike, or there may be a number of different shapes and sizes of coils in the same machine. Each coil may consist of a single turn, each side forming one of the inductors; or it may consist of several turns of round copper wire or strip of rectangular section. Again, each turn may consist of several wires or strips connected in parallel, in preference to a heavy, stiff bar of the same conductivity. The various turns of any coil are insulated from each other by micanite, or cotton tape impregnated with insulating paint, sufficiently well to stand any e.m.f. that can occur between successive turns. The whole coil also is thoroughly insulated by being wrapped with empire cloth, with perhaps cotton tape around the outside to bind it together or prevent mechanical injury to the insulation. The thickness and character of the insulation used depends upon the voltage of the machine.

Each slot in the armature core is usually lined with some strong form of sheet insulation before the side of a coil is put into it. The slot may contain one side of one coil, or one side of each of two different coils, depending upon the style of winding. When these have been properly placed,

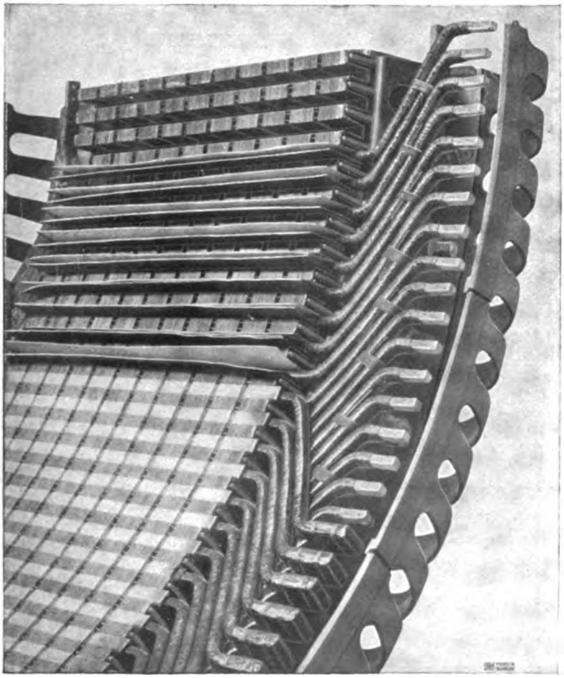


FIG. 310. Bar-wound armature (stator) in the process of construction. Each bar forms one side of a coil. In this case there are two bars per slot. *General Electric Co.*

a wedge of hard wood or fiber is driven into the slot over the top of the coil to make the construction rigid and to protect the coils from injury.

99. Appearance of the Windings. Fig. 310 shows a bar-wound armature (stator) in course of construction.

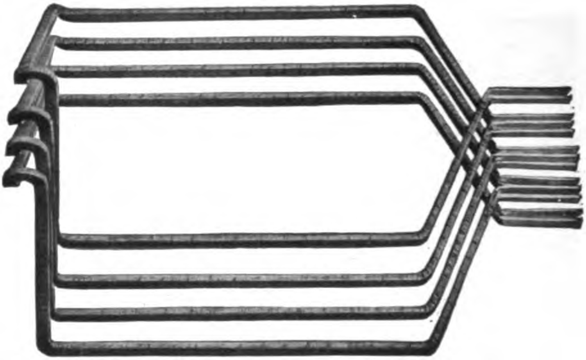


FIG. 311. Appearance of coils like those in Fig. 310, when removed from the slots. *Westinghouse Mfg. Co.*

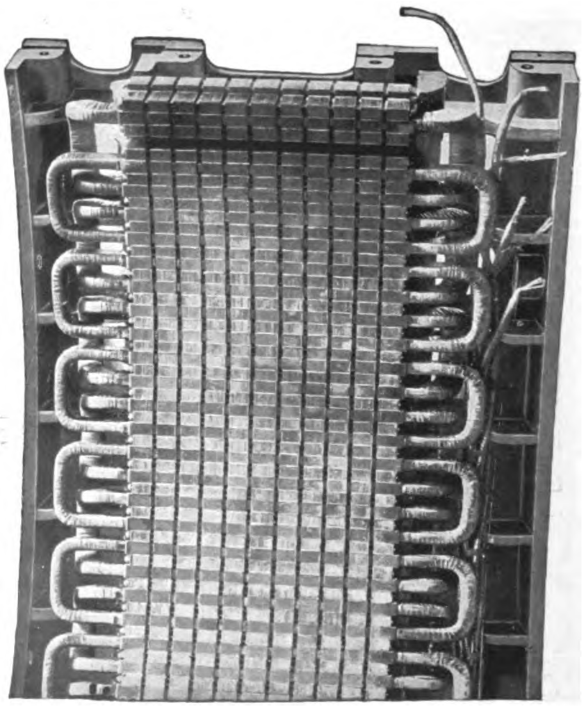


FIG. 312. Three-phase chain-winding, one slot per pole per phase. Each coil consists of several turns as shown in Fig. 313. *General Electric Co.*

Each coil consists of one turn, the left side or inductor lying at the bottom of one slot and the right side or inductor lying at the top of another slot, making what is called a "two-layer winding" (see Art. 109). Though not for the same generator, Fig. 311 shows what such coils look like when taken out of the slots; each of the four coils in this picture consists of two copper bars in parallel, one above the other. Fig. 312 shows a coil winding of the particular type commonly known as a chain-winding, on account of its apparent likeness to the form of a chain. Each coil of this winding consists of several turns, and the coils when removed from the slots look something like Fig. 313. There are seen to be two different forms of coil in Fig. 312 and Fig. 313. Either of these windings could be connected up for single phase or three phase by following the suggestions already given in Chapter VIII and further ones to be given in this chapter. However, **Fig. 312 is a typical three-phase winding,** with one slot per pole per phase. Fig. 314 is a chain-winding for a three-phase 500-kw. vertical shaft alternator driven by a

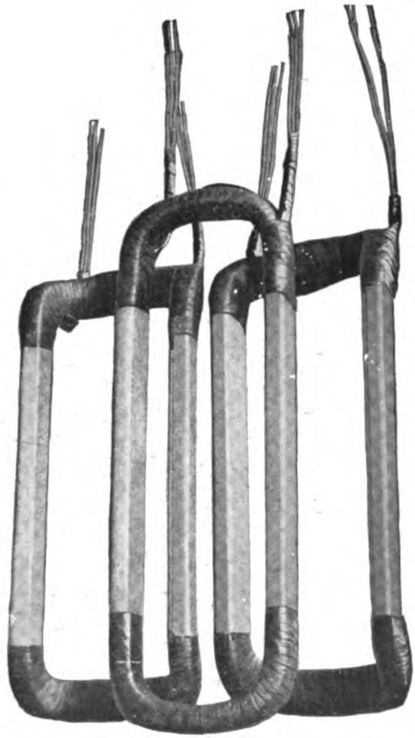


FIG. 313. Appearance of the coils in the chain-winding of Fig. 312. Note that the coils consist of two different forms. *Allis-Chalmers Co.*

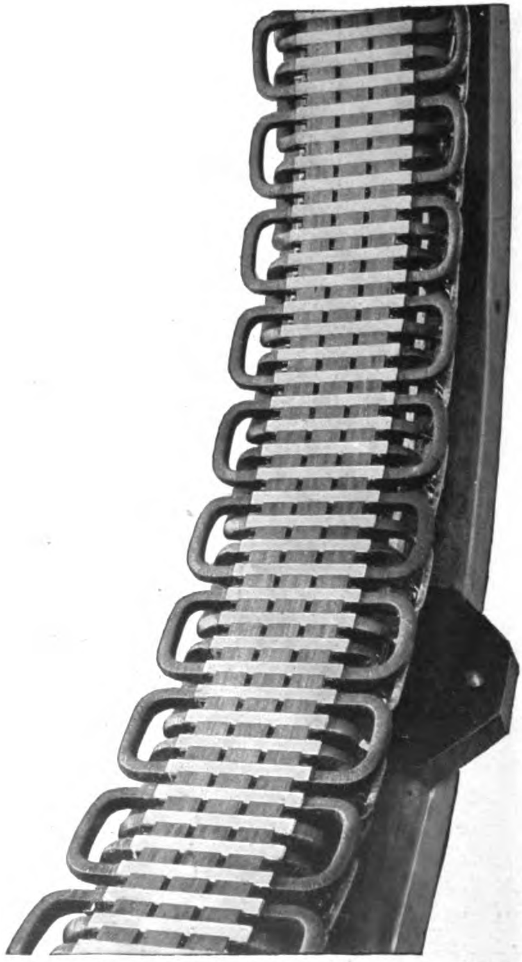


FIG. 314. A portion of the chain-wound armature of a three-phase 500-kv-a. vertical-shaft alternator; water-wheel type. One slot per pole per phase, single-layer winding. *Allis-Chalmers Co.*

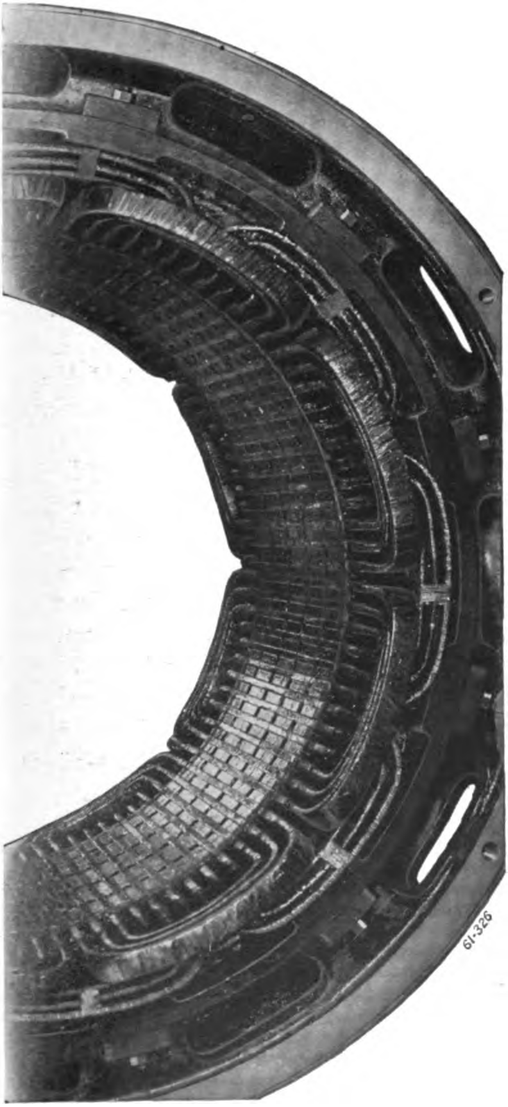


FIG. 315. A three-phase armature winding having three slots per phase per pole. *Wagner Electric Co.*

water wheel. It has one slot per pole per phase and is a single-layer winding. Fig. 315 shows a three-phase stator with three slots per phase per pole; Fig. 316 shows the same kind of a winding in course of construction on a different machine. Fig. 317 illustrates a two-phase 200-kw. stator with three slots per phase per pole.

100. Pitch. Element of Winding. A few terms should be explained which have general application and are frequently used in describing windings. The **pole-pitch** is the distance between centers of adjacent poles (*N* and *S*). Sometimes it is expressed in inches measured on the periphery of the armature, but in relation to windings it is usually expressed in terms of slots. That is, if a machine has, altogether, 72 equidistant slots and 6 poles, the pole-pitch is 12 slots. When a coil spans exactly the distance between centers of adjacent poles, it is said to be a **full-pitch** coil or winding; but if it spans less than this, so that the two sides of a coil are not exactly under the centers of adjacent poles at the same instant, it is said to have a **fractional pitch.** If the coil is less than full pitch, it is said to be a **short-pitch** coil or winding. Such coils have a very pronounced effect in smoothing out the wave of induced e.m.f., making it approach more closely to sine form if properly designed. An **element of winding** is the term used for the inductor or bundle of inductors composing one side of a single coil; a slot may contain one or two winding elements, depending on whether it is a single-layer or a two-layer winding.

The **e.m.f.** which can be induced in a **single inductor** (or conductor which cuts flux on the surface of the armature) **depends upon** the **effective length** of the inductor (that part of its total length which is active or cuts flux), the **linear speed at which the flux moves** with relation to the armature and the **flux density in the air gap** where the cutting of flux occurs. Using practicable values of armature length, peripheral speed and air-gap density for flux, the effective value of e.m.f. which can be induced in a single inductor is

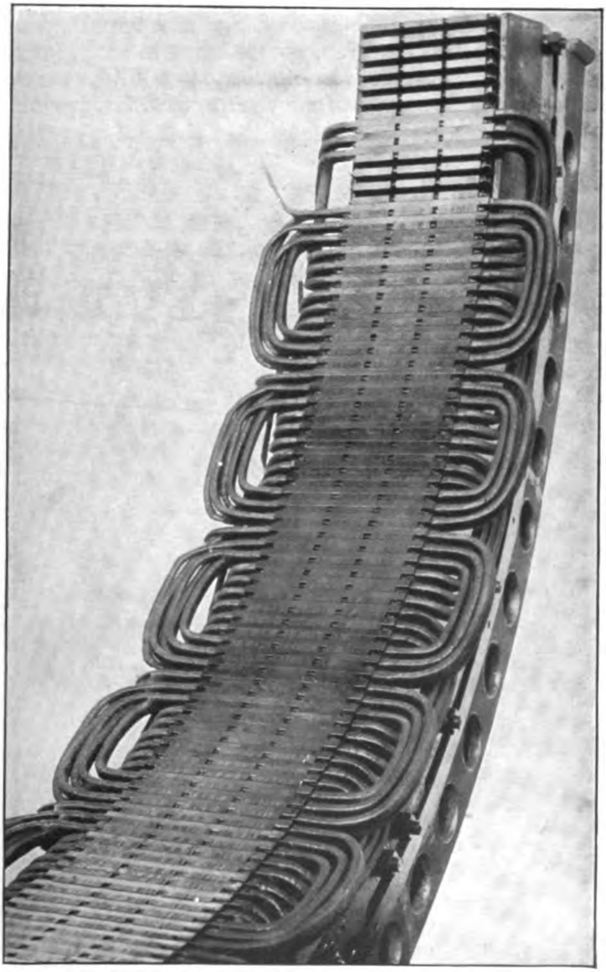


FIG. 316. A stator in the process of construction, having three slots per phase per pole. *Allis-Chalmers Co.*

not large, from 5 to 9 volts per foot of active length of a single inductor being the usual value.

Prob. 1–9. Usual practice in the design of alternating-current generators for low and moderate speeds employs an **average** flux density in the air gap equal to from 40,000 to 60,000 lines per square inch, and a peripheral speed of from 4800 to 6000 feet per minute.

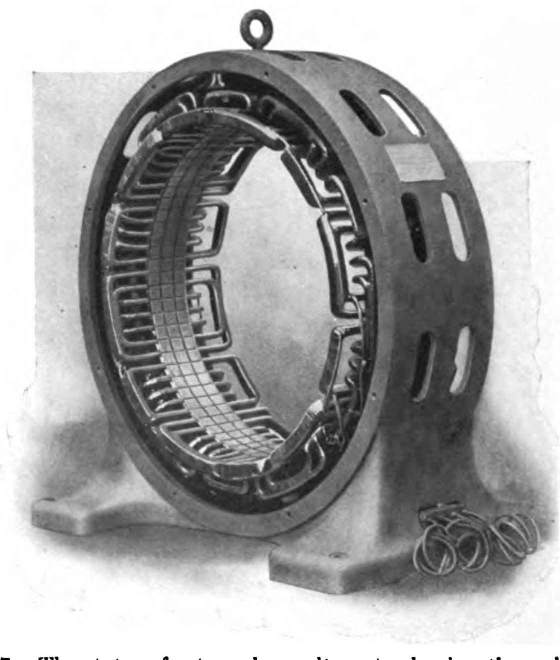


FIG. 317. The stator of a two-phase alternator having three slots per phase per pole. *Westinghouse Electric & Mfg. Co.*

Using these values, calculate the lower and upper limiting values for the **average** e.m.f. induced in an armature inductor having one foot useful length. Assuming the flux distribution to be such that this induced e.m.f. varies according to the sine law, calculate the limiting value of the **effective** e.m.f. per foot useful length of inductor.

Prob. 2–9. If good proportions in the design of a certain generator require the useful length of each inductor to be not over two feet, how many inductors must be connected in series (assuming the e.m.f. in all of them to be in phase with each other) to give an effective e.m.f. of 6900 volts between the terminals of the winding? Use average speed and flux density of Prob. 1.

101. Simplest Armature Connections. It is necessary to multiply the number of inductors in the armature circuit, not only to produce the voltages suited for commercial circuits and apparatus, and for the economical handling of

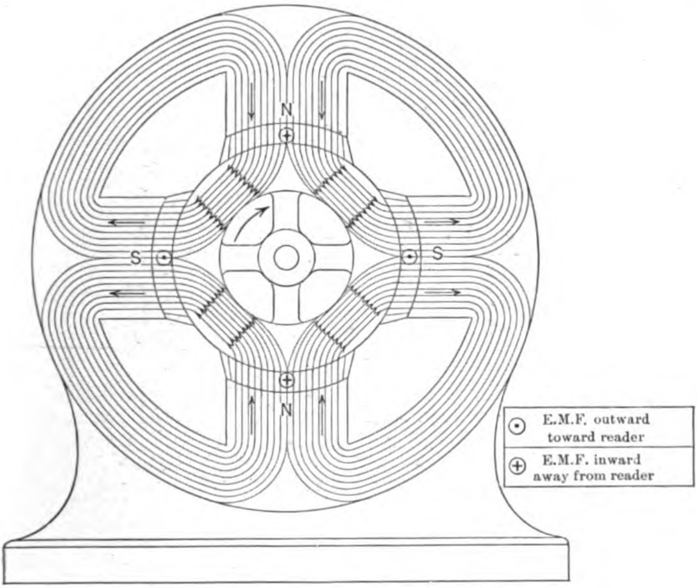


FIG. 318. The simplest possible single-phase armature winding; — one inductor per pole.

electric power in quantity, but also in order to supply two or more different phases to the external circuits connected to the generator, as has been found desirable in the operation of many types of electrical apparatus, particularly motors. (See Chapter IV, Second Course.) We shall explain here a few typical methods in regular use for arranging a multitude of inductors upon the surface of the armature, and for connecting them together properly into one or more circuits or phases. This subject of windings is of great importance to the designer and manufacturer, but concerns the operator or repair-man only to the extent that he must be able intelligently to trace out the winding of any given machine, and restore or rearrange the connections when necessary.

In Fig. 318 and 318a, observe the simplest possible case — **a single-phase concentrated** winding with **one inductor per pole.**

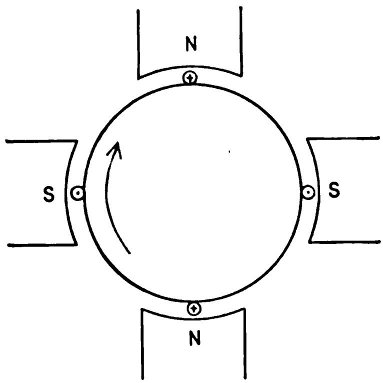


FIG. 318a. The conventional representation of the generator shown in Fig. 318.

Four-pole construction has been chosen, merely for simplicity. The principles involved could be applied and explained exactly as well for a two-, six- or eight-pole field, or for any number of poles, but the sketches would not be so easy to trace through. With the clockwise direction of rotation as indicated, any inductor will have induced in it, according to Lenz's law, an e.m.f. **away** from the reader (indicated by the mark ⊕), while passing under a North pole, and an e.m.f. **toward** the reader (indicated by the mark ⊙), while passing under a South pole. All of the inductors are spaced so that they occupy exactly corresponding positions under the poles at any given instant. The flux is distributed in exactly similar manner under all poles, because the mechanical construction is symmetrical. Hence the e.m.f.'s in all inductors reach their maximum values at the same instant and their zero values at the same instant — that is, they are all in the same phase, or in phase with each other. In Fig. 319, the same inductors are shown connected together in series into a single circuit, in such a way that the various e.m.f.'s do not oppose and cancel each other, but add together. The wave-form of the e.m.f. for the whole circuit will be exactly the same as that for any single one of the component inductors, as they are all identical

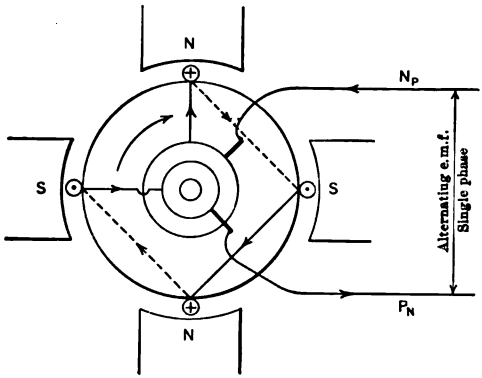


FIG. 319. The four inductors are so connected that they form a single-phase winding, the e.m.f. of which is the sum of the e.m.f.'s in all the inductors.

and exactly in phase with each other. But the maximum e.m.f. of the winding would be equal to the (arithmetical) sum of the maximum values of all the (four) inductors which are in series. Likewise, the effective e.m.f. of the entire winding will be equal to the (arithmetical) sum of the effective values of the component e.m.f.'s, and the average total e.m.f. would be equal to the arithmetical sum of the average values of the component e.m.f.'s. The connections between the ends of the inductors, by which they are placed electrically in additive series, are shown as chords of the circle. End-connectors at the rear or pulley end of the armature are shown by dotted lines. End-connections at the near end, where the collecting rings are, are shown as full lines. Arrows and signs represent direction of e.m.f. in the various parts of the circuit, at this particular instant only, pictured by the sketch.

101a. Advantages of Slotted Core. In the very early days of dynamo construction, it was the practice to make the armature core smooth, and simply to lay the inductors on the outside surface, as shown in Fig. 319, holding them there by friction only, by means of bands of binding wire wound tightly around the armature, separated from the conductors by bands of strong insulation. The smooth-core armature was soon discarded, however, on account of two great defects:

(1) The large magnetic forces exerted on the wires when the machine carried heavy loads, and particularly when a short-circuit occurred, often twisted the winding around on the smooth core, ruining the insulation and necessitating rewinding.

(2) When, as usual, a large number or large size of inductors must be employed, the air distance between armature core and pole faces required for the conductors, insulation, and mechanical clearance, altogether, becomes so large that a very great number of ampere-turns are needed on the field magnets to produce the required amount of flux against such a large reluctance. This increases the cost and lowers the efficiency of the machine.

Therefore, to transmit the magnetic forces more safely to the shaft, to reduce the amount of copper wire needed and the power lost in the field, and to improve the voltage regulation and efficiency of the generator, the inductors are sunk below the cylindrical surface of the armature into **slots** which are lined with insulation. The slots are usually, in large machines, rectangular in shape, narrow, deep radially, and open at the top to permit easy insertion of the winding, which is prepared separately in the form of coils. These coils are laid in the slots and connected together properly to form the complete winding. Note that the winding of Fig. 320 is exactly like that of Fig. 319, excepting that it has been sunk into slots, which have been cut in the armature core and lined with some sheet form of insulation, such as fiber, leatheroid, presspahn or paper.

Prob. 3-9. Draw, to scale, the curve of total e.m.f. induced between the collecting rings during one complete revolution of the armature shown in Fig. 319 and 320, on the following assumptions: Poles cover two-thirds of the periphery of the armature; useful

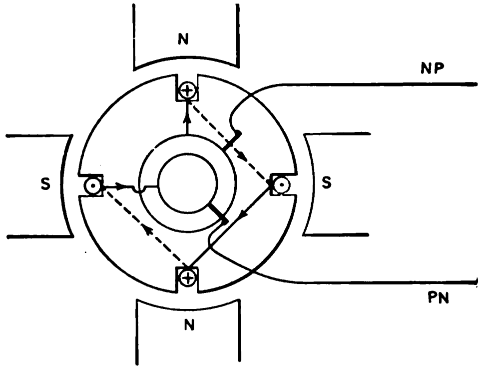


FIG. 320. Simple armature winding of Fig. 319, with the coils sunk into slots.

length of magnetic field parallel to shaft or axis of rotation, 10 inches; diameter of armature, 10 inches; peripheral speed, 5000 feet per minute; density of flux under pole face, 50,000 lines per square inch; flux uniformly distributed under pole face, density falling off uniformly between pole tips and neutral points midway between poles. In drawing the e.m.f. curve, consider the direction of e.m.f. as positive when it is from PN toward NP (thus, $PN \rightarrow NP$) in external circuit.

Prob. 4-9. (*a*) Calculate the average value and effective value of the e.m.f. wave, and the form factor of the generator in Prob. 3.

(*b*) How many conductors must be grouped in each slot, and connected in additive series by suitable winding, to give a voltmeter reading as close as possible to 230 volts between collecting rings at zero load?

(*c*) By what percentage must the flux per pole be increased or diminished at same speed, using this number of conductors, to give exactly 230 volts reading?

Prob. 5–9. What would be the frequency of the e.m.f. of the generator in Problem 4–9?

102. Concentrated Single-phase Windings. Definitions. Conventional Diagrams. When it becomes necessary to increase the total voltage generated by using more than one conductor in each slot, we have to choose between several methods for connecting them together in series. Fig. 321 and 322 both represent **concentrated single-phase** windings, being in this respect similar to Fig. 320 and 319. But a comparison of Fig. 322 with 321 will show a marked difference in the manner of connecting the conductors together, notwithstanding the fact that the value and wave-form of the total e.m.f. induced between the collecting rings is the same in both cases, as all the conductors have an e.m.f. in phase with each other and are connected in additive series. (Follow the arrows, to show the direction in which the e.m.f.'s act in the circuit.)

In discussing armature windings, three types of diagrams are used, each having both advantages and disadvantages. Fig. 321a, 321b and 321c, all representing the same winding, illustrate these three types of diagrams. Fig. 321a is a diagrammatic view of that end of the armature adjacent to the collecting rings. Fig. 321b is a circular or radial-type diagram of the winding only, not showing either armature core or slots. The radial lines represent conductors on the armature, and the arrows on them represent direction of e.m.f. induced at the **instant** they are moving **through the position shown in the diagram.** A loop around any group of conductors denotes that they are all in the same slot. All lines other than radial ones, in the armature circuit, represent connections between conductors or coils, at either end of the armature. Fig. 321c is a diagram of the view of the winding which would be obtained if the armature core were to be cut through at xy in Fig. 321a and the end y uncurled counter clockwise so that the armature core and collecting rings become straightened out as if on a flat sheet or plane. This is called a **developed view** of the winding, and is the one

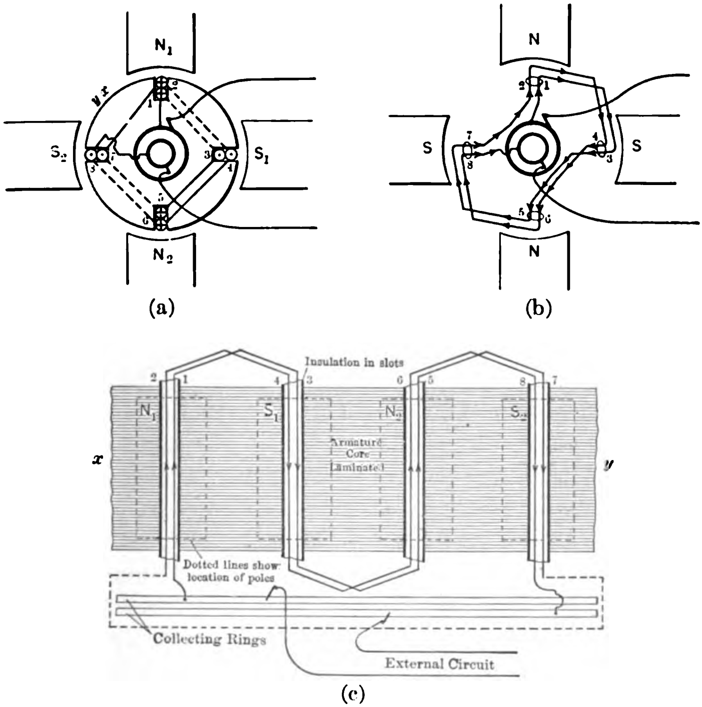


FIG. 321. Concentrated single-phase wave winding. (a) Diagrammatic view from collecting-rings end. (b) Circular or radial diagram showing windings only. (c) Developed view of armature surface.

perhaps most usually met in the literature of this subject. It is the clearest, to an experienced person, and also the easiest to draw; but it is a little difficult for the beginner to see the connection between the severed ends at x and y, especially when the winding has many poles and many conductors.

In either 321b or 321c, the conductors must be shown side by side, although in fact they may be disposed **either** side by side or one above the other, in the slots. When arranged in two layers and numbered, the conductors underneath receive odd numbers, and the top conductors in the slot have even numbers. It is important to adopt and adhere to such a convention, when the pitch of the armature-coils is expressed (as it frequently is in practice) in terms of inductors rather than in terms of slots. The student may adopt that diagram which affords the clearest view of any given winding, as will be done in this brief discussion.

Figure 321 represents a method of connecting conductors together, which is called a **wave winding.** The reason for the name can be most clearly seen from Fig. 321c, which shows how the circuit progresses around the armature continuously (in a clockwise direction, Fig. 321a or 321b), with a sort of **wavy** undulation, backward and forward, parallel to the shaft. Figure 322 represents the same number of conductors arranged in additive series in the same number of slots, but after the manner known as a **lap winding.** The reason for this title will be seen most clearly from Fig. 322c, which shows how the circuit **laps** back upon itself in a **loop** or coil, under one pair of poles, before progressing forward by an end-connection to a similar loop or coil under the next pair of poles. The distinction between lap and wave winding will be made clearer from subsequent consideration of an armature having two or more slots per pole per phase, than from a "unislot" armature having only one slot per pole per phase, as in Fig. 321 and 322. Thus, Fig. 323 and 324 both represent single-phase **wave windings** for a four-pole armature, there being altogether eight inductors in series — so far exactly like the winding in Fig. 321. The arrangement of Fig. 323 and 324, however, spreads the inductors apart into two slots per pole per phase, and is called **a distributed** winding, whereas that of Fig. 321 has the inductors bunched together in one slot per pole per phase, and

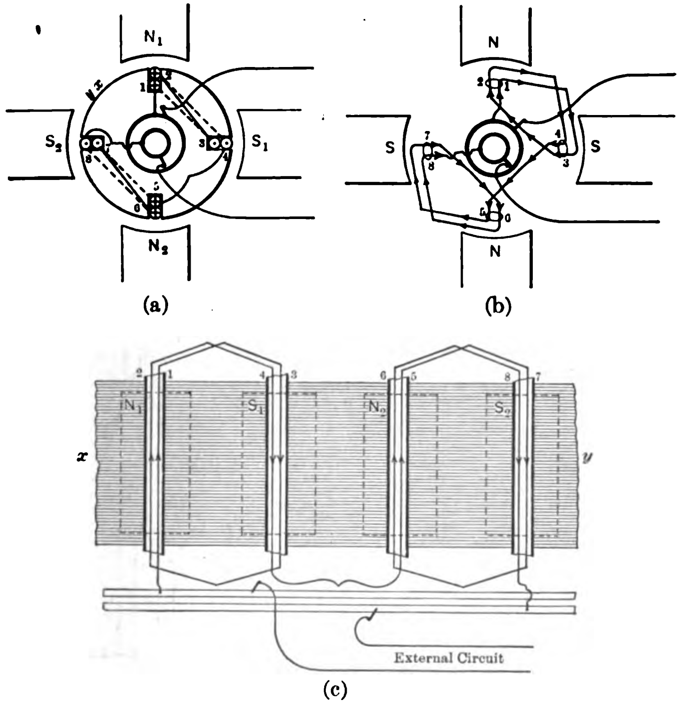


FIG. 322. Concentrated single-phase lap winding, equivalent to the wave winding in Fig. 321.

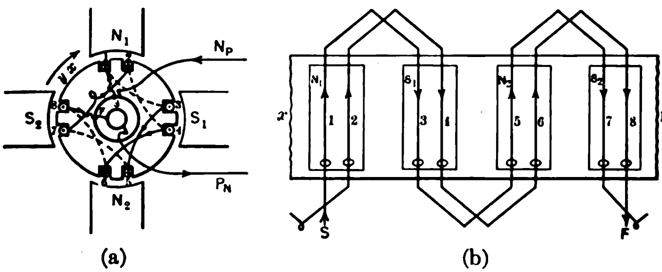


FIG. 323. Distributed single-phase wave winding. Progressive type.

is called a **concentrated** winding. The reader is asked to study closely and compare the windings shown in Fig. 323 and 324, which are all electrically equivalent, and to learn thereby the distinction implied by the adjectives **progressive** and **retrogressive** as applied to the wave winding. A curl in the diagram of any end-connection denotes that it has to be a little longer or shorter than the other end-connections

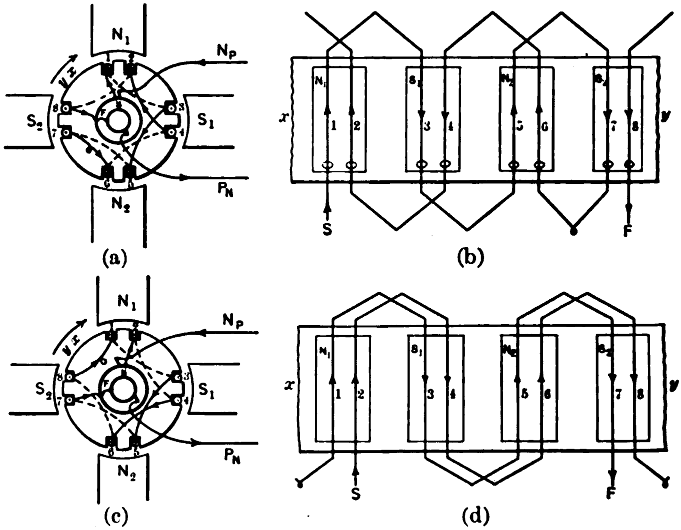


FIG. 324. Distributed single-phase wave winding, showing two different forms of retrogressive type.

in the same winding. Fig. 323a requires a special end-connection a little longer than the rest, for each complete passage of the circuit around the armature; whereas Fig. 324a and 324c require a shorter special end-connection.

Fig. 325 shows a single-phase distributed **lap winding**, for the same four-pole armature, two slots per pole per phase, one inductor per slot. The (curled) end-connector between pairs of poles makes it appear somewhat similar to a wave winding, but this similarity disappears as the number of slots per pole per phase increases.

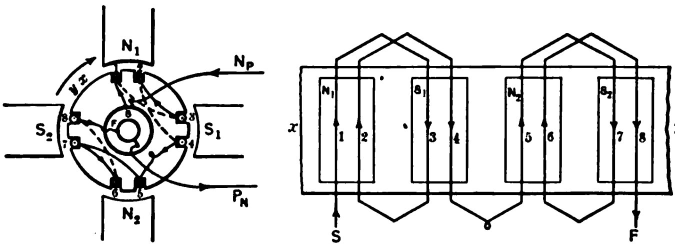
Prob. 6–9. An 8-pole single-phase alternator having a concentrated winding requires altogether 30 conductors in each slot to produce the required e.m.f. of 2300 volts at the standard frequency of 60 cycles per second:

(*a*) Draw a diagram to represent these conductors arranged in coils and connected as a lap winding.

(*b*) Calculate the speed of the alternator, in r.p.m.

(*c*) Calculate the greatest permissible diameter of the armature, if the peripheral speed is not to exceed 5000 ft. per minute under normal conditions.

Prob. 7–9. (*a*) Calculate the required flux per pole of the generator in Problem 6–9 to generate this terminal voltage (2300) at



(a) (b)

FIG. 325. Lap winding to correspond to the wave winding of Fig. 324.

zero load, at this speed with this winding. Assume the e.m.f. in each inductor approximates closely to a sine wave.

(*b*) The average flux density in the air gap under the pole face being 60,000 lines per square inch, and the poles covering 60 per cent of the periphery of the armature, calculate the effective length of the armature.

Prob. 8–9. Draw a progressive wave winding for a single-phase six-pole armature having 24 equidistant slots, there being one inductor in every slot.

Prob. 9–9. Draw a retrogressive wave winding for a single-phase eight-pole armature having 24 equidistant slots, there being one inductor in every slot.

Prob. 10–9. Draw a lap winding for a single-phase six-pole armature having 24 equidistant slots, there being one side of each of two coils in every slot and two turns to every coil. How many inductors per slot are there? How many inductors altogether, in series?

103. Whole-coiled Winding. Another fundamental distinction is illustrated by a comparison of Fig. 326 and 327, both of which represent a four-pole single-phase alternator, with one slot per pole per phase, and a total of eight induc-

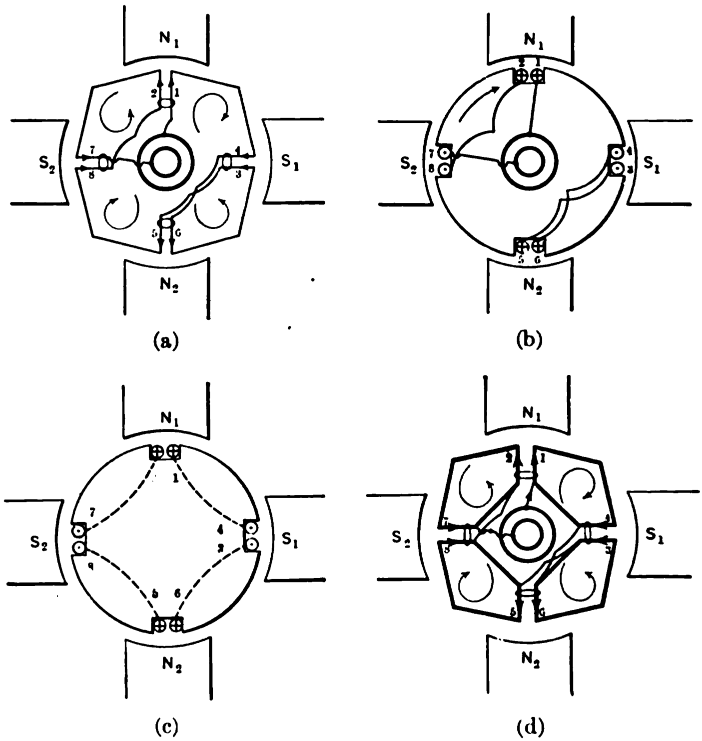


FIG. 326. Whole-coiled winding. (b) Ring-end connections. (c) Pulley-end connections (viewed from ring end). (d) Note that there are as many coils (in heavy lines) as poles.

tors or more, the windings being thus of the concentrated type. In Fig. 326a the eight conductors are grouped into coils so that there is one complete turn or coil for each pole. This winding is called a **whole-coiled winding.** The fact that there is one whole coil per pole is not very apparent in Fig.

326a, which represents a winding for only two inductors or one turn per pole per phase. But it is clearly shown in Fig. 326d, which represents a winding for any number of turns per pole per phase, or any even number of inductors per pole per

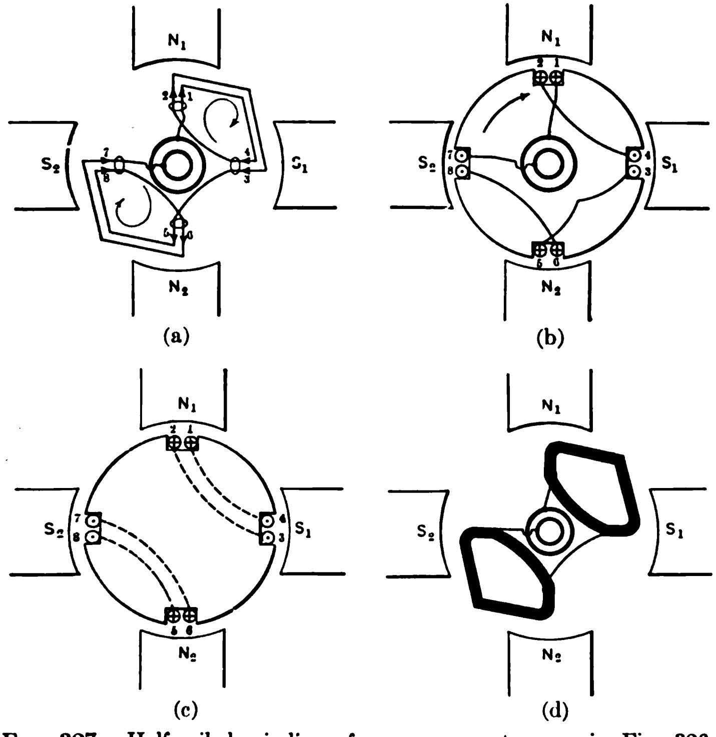
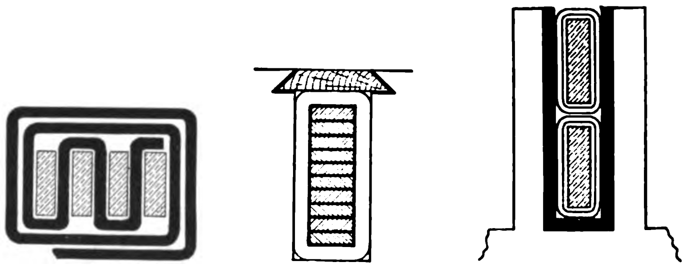


FIG. 327. Half-coiled windings for same armature as in Fig. 326. Note in (d) that there are half as many coils as poles.

phase, since there are usually two coil-sides in each slot. Fig. 326c, showing the back-connections (viewed through the armature from the ring end), should also justify the "whole-coiled" designation. Fig. 326b shows merely the connections made at the front, or ring end, of the armature. The copper inductors may be either round wires or bands having rectangular cross section. Small motors usually employ wire coils, but large motors and generators, in which the amperes to be carried in each armature circuit are considerable, usually employ the copper strips. In very large armatures the strip is enlarged into a heavy copper bar, there being usually in such cases only one turn per coil or one inductor per slot. This is called a "bar winding," and is illustrated in Fig. 310 and 311. Fig. 328a shows a cross section of one side of a strip coil, with the strip wound flatwise, while Fig. 328b shows a slot containing one coil-side, the coil consisting of ten turns of strip wound edge-



(a) (b) (c)

Fig. 328. Method of arranging the coils in the armature slots. (a) The strips are wound flatwise. (b) The strips are wound edgewise. (c) A two-layer winding.

wise. Fig. 328c shows a slot containing one side of each of two coils, composing what is called a **two-layer winding,** consisting, in this particular case, of coils of strip or bar having one turn each.

104. Half-coiled Winding. Now compare Fig. 327 with Fig. 326. Notice that in Fig. 327 the same number of inductors in the same slots on the same armature are now connected in additive series into a single-phase winding, in such a way that they, with their end-connections, form coils, the number of coils being only **half** the number of poles. This is called a **half-coiled winding.** Each coil has twice as many turns as a coil of the corresponding whole-coil winding and each slot contains one coil-side instead of two. In Fig. 327c, the two turns in series under poles N_1 and S_1 would in fact be taped together to form a coil, as would any number of inductors in the same slots. Fig. 327d and 326d show clearly that with a **half-coiled** winding it is possible to split the armature into sections without cutting through any coils — merely breaking a soldered connection between coils; whereas in the **whole-coiled** winding it is impossible to split the armature without either removing some coils from the slots, or cutting and ruining some coils. It is often desirable to build large generators or motors so that the machines may be shipped in sections, each of which is complete in itself, and ready to be bolted together to make a finished machine; or so that the upper half of the (stationary) armature may be lifted away from the bed-plate and bearings, so as to expose or remove the rotor or field structure, for cleaning or repairs. The half-coiled winding is readily adapted to this form of construction.

Prob. 11–9. Draw a diagram for a whole-coiled concentrated single-phase winding for a six-pole alternator, using four inductors per slot.

Prob. 12–9. Draw a half-coiled concentrated single-phase winding for a six-pole alternator, using 24 inductors altogether upon the armature.

Prob. 13–9. What is the least total number of inductors out of which a whole-coiled winding may be made for an 8-pole single-phase alternator? In general, to what rule must the number of inductors per pole per phase conform in order to permit a whole-coiled winding? Is there any corresponding rule for a half-coiled winding?

Prob. 14–9. Sketch an 8-pole single-phase alternator with a concentrated winding arranged so that the upper half of the stationary armature may be lifted off by its ring-bolt at the top, exposing the rotating field in its bearings, with least possible disturbance to the windings.

Prob. 15–9. Draw a concentrated winding for a six-pole armature having altogether 36 inductors. (*a*) Whole-coiled. (*b*) Half-coiled.

105. Distributed Windings. The concentrated or uni-slot winding has the advantage that it gives the greatest e.m.f. possible to obtain from a given number of inductors, the generated e.m.f.'s of a single circuit being all in phase with each other. Or we might say, for a given e.m.f., speed and field, the concentrated winding requires the least number of inductors. The chief disadvantage is the difficulty in producing a sine wave-form of e.m.f., since control of the wave-form in this case can be had only by shaping the pole face. Moreover, concentrating the winding increases the number of turns per coil; and as the inductance of a coil increases as the square of the number of turns, the reactance of a winding is much larger when it is concentrated than when it is distributed, and this results in poorer voltage regulation. It is more usual to distribute the series inductors in any phase or circuit of the armature winding, among two, three or more slots under each pole. This tends to produce a smoother e.m.f. wave, but on account of their phase differences the various e.m.f.'s in any series circuit in the armature are added less effectively. This will be clearer from a calculation based on Fig. 329a, which represents a single-phase winding, **thoroughly distributed** — that is, it occupies **every one** of a number of slots spaced **equidistant all around the armature.** In this particular case, there are 4 poles and 24 slots.

In the alternator, to which the winding diagram Fig. 329b applies, suppose that there is one inductor in each slot, and that each inductor produces an e.m.f. of sine wave-form, having an effective value of 10 volts. Inductors 1 + 2 + 3 + 4 + 5 + 6 form together a coil ("spiral" winding) so connected that the e.m.f.'s are added together vectorially to give the total e.m.f. between the terminals of the coil. Similarly, inductors 7 to 13 add together to form a coil, as do inductors 13 to 19, and 19 to 24; and finally, all four of these coils are connected together in a series, so that the coil e.m.f.'s add together vectorially. Of course, **any** two e.m.f.'s in series produce a total which is in the **broad** sense a vector sum; but in the **usual** sense, two e.m.f.'s are connected in **additive** series when the effective voltage across the entire series is greater than across either part of the series; and they are in **subtractive series** when the total effective e.m.f. is **less** than that across either part of the series. Two e.m.f.'s may be changed from additive to subtractive series by reversing

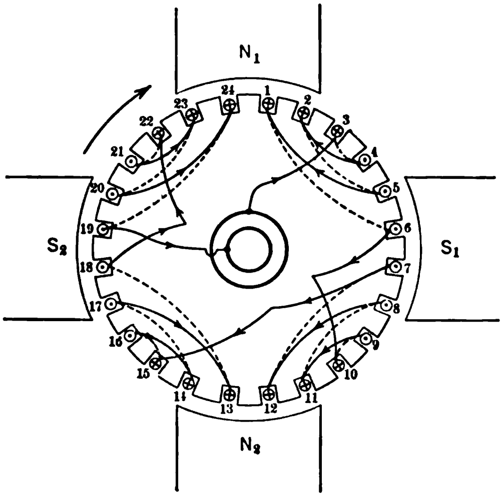


FIG. 329a. A thoroughly-distributed single-phase winding. All the 24 slots are occupied.

the connections to one of them. In Fig. 329b, the arrows represent instantaneous directions of e.m.f. in the various conductors as they move through the positions shown. The e.m.f.'s in 1, 2, 3, 4, 5 and 6 are all in **additive series** because the maximum instantaneous value, so produced on the entire coil, is greater than could be produced by any other interconnection of the same series of inductors. Let us find the effective voltage across this coil.

The e.m.f. in any conductor reaches its maximum value in the direction we shall choose to call positive, as it passes under the middle of pole N_1. Consider conductors No. 5 and No. 6. For clockwise rotation, conductor No. 5 attains its positive maximum e.m.f. when inductor No. 6 has moved $\frac{1}{24}$ revolution beyond this position. The positive maximum e.m.f. in No. 5, therefore, would occur at an instant $\frac{1}{24}$ of 2 periods (there being two cycles of e.m.f. per revolution in a 4-pole machine), or $\frac{1}{12}$ of one cycle of

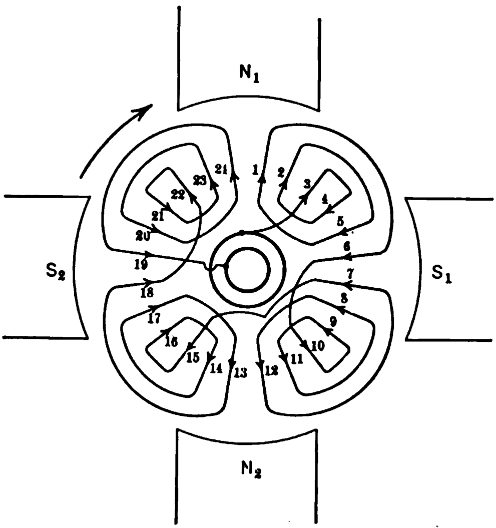


Fig. 329b. Another diagram to show the winding of the armature in Fig. 329a.

360°, or 30 electrical degrees, after the e.m.f. in No. 6 has passed its positive maximum. In other words, for the particular number of poles and slots used in Fig. 329, or in general for a **slot pitch** of 30 electrical degrees between centers of adjacent slots, the e.m.f.'s induced in such slots differ in phase successively by 30 electrical degrees. Similarly, the e.m.f. in No. 4 is 30° behind No. 5; No. 3 is 30° behind No. 4; No. 2 is 30° behind No. 3; No. 1 is 30° behind No. 2. Fig. 330 is a (topographic) vector diagram showing the relation between the e.m.f.'s E_6, E_5, E_4, E_3, E_2 and E_1, on the basis that they are numerically equal to each other, and just 30 electrical degrees apart. Having laid out this diagram, it is merely a matter of elementary trigonometry to show that the resultant e.m.f. (vector *OR*) of all six inductors in additive series is equal to the numerical sum of their effective e.m.f., times the factor 0.644, or equal to the effective value of e.m.f.'s in any **one** inductor multiplied by (6×0.644), or 3.864. If there had been only three adjacent inductors 30 electrical degrees apart, in additive series, the total effective e.m.f. would have been equal to that of one inductor, multiplied by the factor (3×0.911), or 2.732. The numbers 0.966, 0.911, 0.836, given in Fig. 330, are the ratios between the actual e.m.f. of the series and the numerical sum of the component e.m.f.'s, on the basis (as already noted) that the e.m.f.'s composing the series are all of **sine wave-form, equal in value, and 30 degrees apart, successively** and all in additive series.

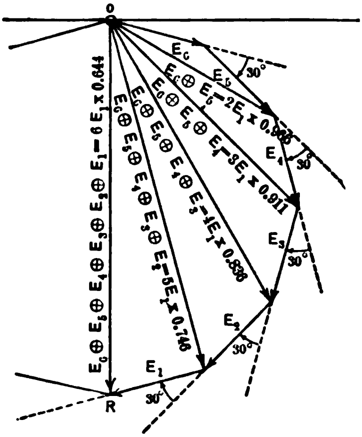


FIG. 330. Topographic vector diagram of voltage across one group of inductors in winding of Fig. 329.

Further, **the resultant e.m.f.'s** in the four whole groups or spirals of Fig. 329b reach their maximum values all at the same instant and in the same direction through the circuit. Fig. 331 indicates this fact and shows that the total e.m.f. of the entire winding of Fig. 329, occupying 24 slots, is equal to only 15.5 times the e.m.f. induced in a single slot.

106. Comparison of Distributed to Concentrated Winding. The vector diagrams of Fig. 330 and 331 show that the total e.m.f. induced in the series of inductors (1 + 2 + 3

+ 4 + 5 + 6) is equal to $0.644 \times 6 \times 10 = 38.64$ volts, R.M.S. (root-mean-square, or effective value), when there is one inductor in each slot and ten volts induced in each inductor. The total e.m.f. induced between the rings of Fig. 329 is $4 \times 38.64 = 154.6$ volts effective, since there are 4 groups like this one, all in additive series and in phase.

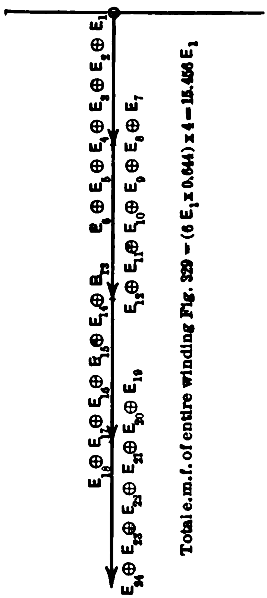


FIG. 331. Topographic vector diagrams of voltages across the four series groups in winding of Fig. 329.

If this winding were concentrated, using only one slot per pole, with six inductors in each slot, the total e.m.f. of an additive series of the same six inductors would be $6 \times 10 = 60$ volts. In other words, to induce any given value of total effective e.m.f. between the terminals of a winding so distributed would require just $\frac{60}{38.64} = 1.56$ times as many inductors, or about 56 per cent more copper in the armature winding, than would be required if the winding were concentrated into one slot per pole. (This does not consider the amount of copper used in the ends of coils and end-connections between coils.) If the cross section of the copper were the same in both cases, the resistance of the winding and the I^2R loss in the armature would be correspondingly greater for the distributed winding than for a concentrated winding of the same kilovolt-ampere and voltage rating (same amperes per circuit). This would make the efficiency of the machine having the distributed winding less. In order to get the efficiency of the distributed winding to equal that of the concentrated winding, using the same slots, inductors, field and speed, the cross section of the copper must be increased also by 56 per cent, to keep its total resistance the same as for the concentrated winding. The combined effect of increasing the section and the length is to increase the weight and cost of copper very greatly. But the **distributed winding** gives a **smoother e.m.f.** wave, approaching more closely to the sine form, eliminating the ripples and peaks shown in Fig. 286a, which are usually found in the e.m.f. from a concentrated winding. The **armature reaction** also is less than for a concentrated winding; that is, the changes in value and distribution of the magnetic flux entering the armature, due to changes of load, are reduced. This gives a better voltage regulation and a more constant form of e.m.f. wave with varying load, for the distributed winding.

Prob. 16–9. (*a*) Calculate the e.m.f. induced in the whole winding of Fig. 329, there being ten conductors in each slot, and an e.m.f. of sine wave-form and 5 volts effective value in each conductor.
(*b*) Draw the diagram for this winding, after the manner of Fig. 329b.

Prob. 17–9. If the winding in Fig. 329 were distributed in four of the six equidistant slots per pole, by what percentage would the total e.m.f. be less than that which would be induced in an equal total number of conductors arranged as a concentrated winding in one slot per pole? Solve on basis of Fig. 330.

Prob. 18–9. Show how the vector diagrams corresponding to Fig. 330 and 331 would appear for a thoroughly-distributed single-phase winding for a six-pole armature with 24 slots. In this case, what e.m.f. would each of the vectors in this diagram represent?

Prob. 19–9. Draw the vector diagrams corresponding to Fig. 330 and 331 for a thoroughly-distributed single-phase winding for six-pole armature with 36 slots. If the terminal e.m.f. of the entire winding is 2300 volts, how many volts per slot?

107. How a Desired E.M.F. is Built Up. This same armature (Fig. 329) could be made to give **any** required voltage between the collecting rings, by simply increasing the number of conductors in each slot, and connecting them together in additive series in the form of coils, as illustrated in Fig. 332. If it be required to produce 300 volts between the collecting rings, we must put $\frac{300}{154.6}$ = approximately 2 conductors in each slot, each conductor generating 10 volts,

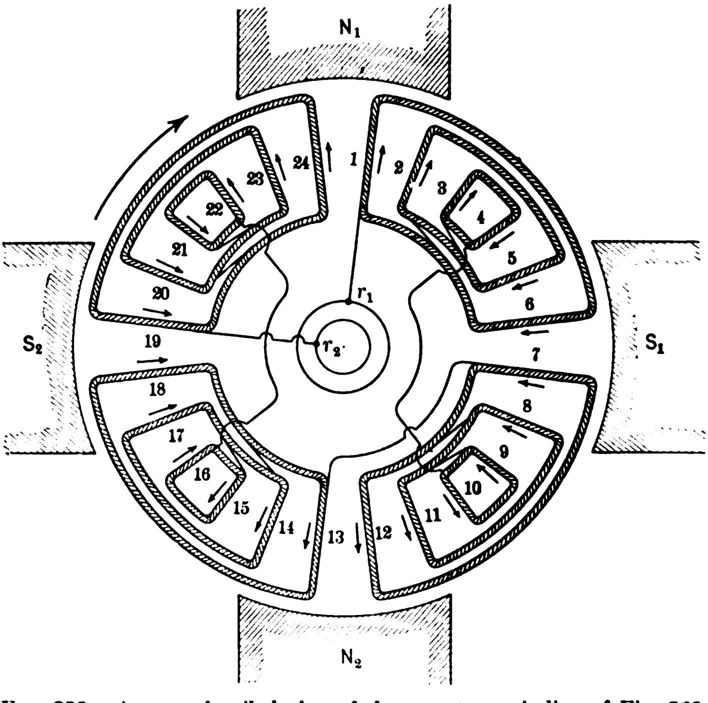


Fig. 332. A more detailed plan of the armature winding of Fig. 329, showing the form of the coils. Note that three forms of coil are used. The end-connections between coils are not exactly those of Fig. 329, but are equivalent to them.

effective, connecting them all together as shown in detail by the winding diagram of Fig. 333. The exact voltage of 300 would then be obtained by reducing the flux slightly, in fact to $\frac{300}{2 \times 153.6}$, or 97.7 per cent of its former amount.

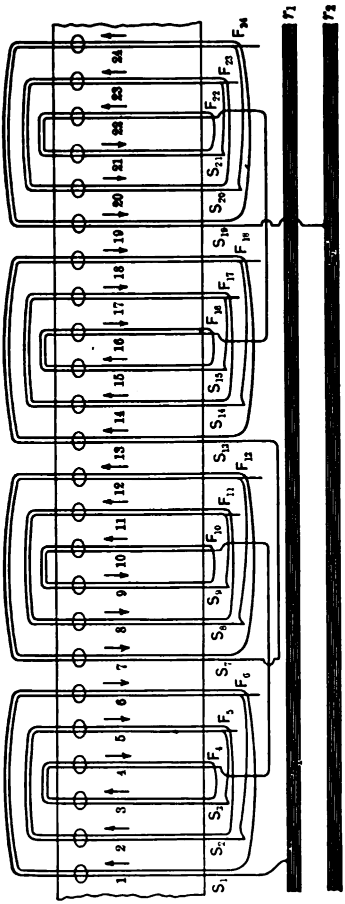


FIG. 333. A developed plan of the armature winding shown in Fig. 332.

Notice that three kinds of coils are required, that they are all wound up similarly, but are connected differently under the different poles. If S_1, S_2, S_3, etc., represent the starting end of any coil, and F_6, F_5, F_4, etc., represent the finishing end of the same coil (respectively), the coils are all wound up right-handedly or clockwise in this figure. To connect the three coils of any one group together in additive series, the finishing end of one coil is connected to the starting end of the next coil of the same group, or the starting end of one coil to the finishing end of the next coil. The first group, for instance, starts at S_1, and ends at F_4, while the second group of coils starts at S_7 and ends at F_{10}, the third group starts at S_{13} and ends at F_{16}, the fourth group starts at S_{19} and ends at F_{22}. Observing the armature as in Fig. 332, when all groups are in the position where the maximum total instantaneous value of e.m.f. is generated in them, we see that the e.m.f. acts clockwise around one group of coils (as 1–6, 2–5 and 3–4), while it acts counter clockwise around the next adjacent group of coils (as 7–12, 8–11 and 9–10). Consequently, in connecting the groups together, the end-connections between groups must be made from finishing end of one group to finishing end of the next group, from starting end of this group to starting end of the next adjacent group, and so on around the armature. This is necessary in order that the e.m.f. in all of the conductors composing the entire winding may act together in a way to make the e.m.f. of the whole winding as great as possible, at the instant when the maximum cyclic value is reached. The details of sequence in connecting the ends of adjacent coils together, as shown in Fig. 333, are ordinarily understood but not shown. In other words, coils are treated and connected up together, just as single turns, provided they are all wound up in a similar sense, as used and as illustrated in Fig. 333. When thus composed of coils, the spiral winding has an appearance resembling a chain and is sometimes called a **chain winding** (see Fig. 314, 312, 315). Vector diagrams similar to Fig. 330 are sufficient to calculate the terminal e.m.f. of an **entire winding**, whether thoroughly or only partly distributed, provided the spacing of the slots and number of poles is known, and the e.m.f. induced in each slot is of sine wave-form. This diagram is not sufficient, however, to compare the relative contribution of each **individual coil** of the chain winding, toward the total e.m.f. or the total power output. Fig. 333a is a polar diagram of vectors corresponding to Fig. 330. It demonstrates that the slots (or winding elements, whichever you choose to con-

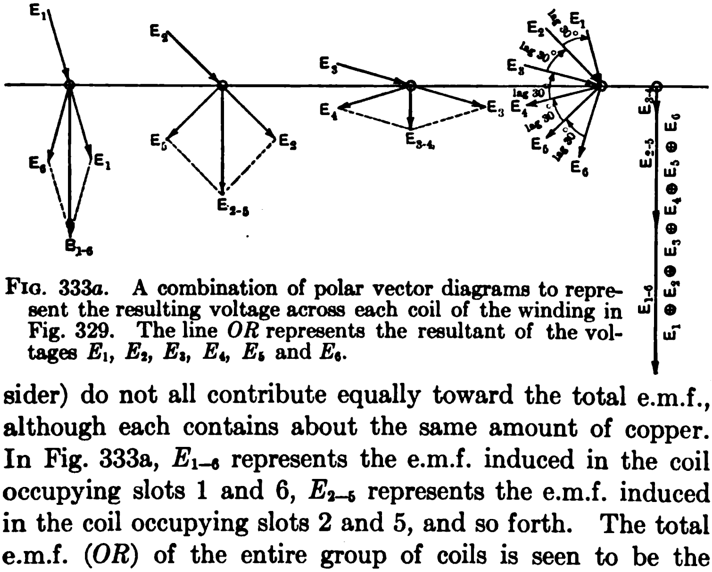


FIG. 333*a*. A combination of polar vector diagrams to represent the resulting voltage across each coil of the winding in Fig. 329. The line *OR* represents the resultant of the voltages E_1, E_2, E_3, E_4, E_5 and E_6.

sider) do not all contribute equally toward the total e.m.f., although each contains about the same amount of copper. In Fig. 333a, E_{1-6} represents the e.m.f. induced in the coil occupying slots 1 and 6, E_{2-5} represents the e.m.f. induced in the coil occupying slots 2 and 5, and so forth. The total e.m.f. (*OR*) of the entire group of coils is seen to be the same as in Fig. 330. If we desire to economize by cutting out the less useful of the elements in Fig. 332, so as to make the winding only partly distributed, where should we begin — which coils are least worth keeping?

108. Economy of Copper. Partly-distributed Windings. Fig. 334 represents a thoroughly-distributed single-phase winding for four poles. Just for variety, the total number of slots has been taken as 32 in this case. Using thus 8 slots per pole, the phase difference between adjacent slots or elements of winding is $\frac{1}{8}$ of 180°, or $22\frac{1}{2}$ electrical degrees. If in Fig. 335 vectors E_1, E_2, E_3, etc., represent to scale the equal e.m.f.'s induced in each winding element of Fig. 334 (or all the inductors composing one side of a coil), then Fig. 336 represents how the eight elements composing four coils (which together form just $\frac{1}{4}$ of the entire winding, the other quarters being exact repetitions of this one) all add

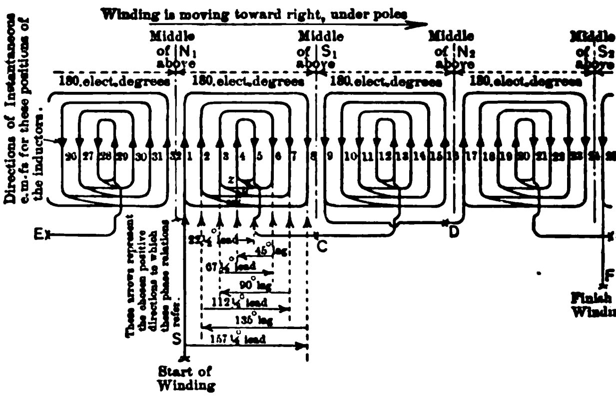


FIG. 334. Developed plan of a thoroughly-distributed single-phase armature winding for four poles. There are 32 slots, or 8 slots per pole. The slots are therefore $\frac{1}{8}$ of 180 electrical degrees apart. Note that therefore the e.m.f.'s of adjacent elements have a phase difference of $22\frac{1}{2}$°.

together to produce the total e.m.f. E (S to C) for the entire group. Fig. 336 is similar to Fig. 330, the slight differences being merely in lengths of vectors and the phase angle between adjacent slots. But the question here is, **exactly how** is this total e.m.f. E (S to C) of one group built up? What is the e.m.f. in each of the four coils composing the group?

Number the winding elements successively 1, 2, 3, etc., as shown. Choosing the **positive direction** of e.m.f. in every element to be **upward** in Fig. 334, the e.m.f.'s in adjacent elements successively have phase difference of 22½°, as illustrated by Fig. 335. The arrows in Fig. 334 indicate the e.m.f. at the particular **instant** the winding moves through the position shown (if they be considered to belong to and move with the inductors); or they indicate the direction of e.m.f. induced in any inductor as it moves through that particular **location** in the field (if the arrows be considered as fixed in position and direction). It is seen that in order to get the greatest maximum instantaneous e.m.f. for the entire winding, we have connected elements 1 and 8 together into a coil in such way that their fixed **positive** directions are in opposite direction through the series, which brings their **instantaneous** e.m.f.'s usually in the same direction so as to add together, because the two sides of the coil are at all instants moving under opposite poles. So it is also with elements 2 and 7, which form another coil (within the largest coil 1–8); and so also with coils 3–6 and 4–5. But for a considerable fraction of the time both sides of each small coil, as 4–5, are generating instantaneous e.m.f.'s directly opposite to each other because the narrow span of the coil compels both sides to be moving under the same pole; and at all such instants the total e.m.f. of the coil is equal to the arithmetical difference of the e.m.f.'s in the elements, instead of the arithmetical sum. Hence the e.m.f. of this coil is less than that of the others, although it has the same number of inductors and moves at the same speed in the same field. Another

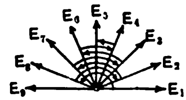


FIG. 335. Polar vector diagram of the e.m.f.'s induced in the inductors of four coils. There are four of these four-coil groups in the whole winding.

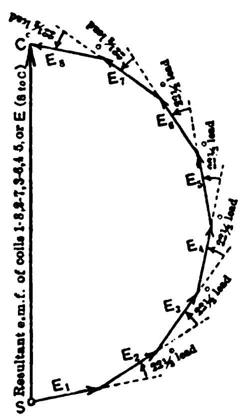


FIG. 336. Topographic vector diagram of the e.m.f.'s in one four-coil group.

way to look at it, is that coil 4–5 encloses a smaller total quantity of flux than any other. Hence as it moves from place to place in the field where the flux density is not uniform, the rate of variation of flux enclosed by the coil, or the number of lines of force appearing or disappearing within it per second as it moves from a weak field into a dense field or vice versa, is less than in the other coils, and the e.m.f. induced in it is therefore also correspondingly less. If the flux were uniformly distributed around the armature, the coil 4–5 most of the time would generate no e.m.f. at all, because the instantaneous e.m.f.'s in its two sides would be not only opposite in direction, but equal in value also. The same remarks apply also to the other narrow coils, in proportion to their narrowness. This has a great effect upon the resultant e.m.f. of the coils and of the entire winding. To illustrate this vectorially, we draw Fig. 336a, which is intended to show exactly how the e.m.f.'s in the various elements of the group are added together vectorially in their order of succession as we pass through the armature circuit in Fig. 334. Starting from *S*, Fig. 334, we pass through element No. 1 in the positive direction. Then by the back end-connection we go ahead in the direction of rotation of the armature (leading angle) a distance equal to 7 slots and represented by $7 \times 22\frac{1}{2}° = 157\frac{1}{2}°$. Then we continue the series by passing through element No. 8 in the negative direction (the **positive** direction upon which our phase relations are founded being **upward** through every inductor). Then we go backward (lag) 6 slots, or 135°, to get to element No. 2 and pass through element No. 2 in its positive direction; then go forward (lead) 5 slots, or $112\frac{1}{2}°$, to element No. 7, and through element No. 7 in its negative direction; then backward (lag) 4 slots, or 90°, to element No. 3, and through element No. 3 positively; then forward (lead) 3 slots, or $67\frac{1}{2}°$, to element No. 6, and through element No. 6 negatively; then backward (lag) 2 slots, or 45°, to element No. 4, and through element No. 4 positively; then forward (lead) 1 slot, or $22\frac{1}{2}°$, to element No. 5, and through element No. 5 negatively. This brings us to the end-connector marked C, which is the end of this group of coils, there being a similar group under every other pole, all groups being connected together in series to compose the entire winding.

The vector relations between the e.m.f.'s in these consecutive elements of the series are represented by the topographic diagram in Fig. 336a. Starting at S, we lay out a vector, parallel and equal to E_1 of Fig. 335, representing the total e.m.f. of all inductors composing element No. 1, or that side of coil 1–8 which lies in slot No. 1. At the end of 1,

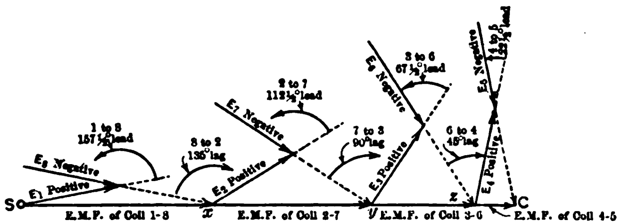


FIG. 336a. A topographic vector diagram of the e.m.f.'s in the windings of Fig. 334. Note that the e.m.f. of coil 4–5 is small because of the great phase difference between the e.m.f.'s in the inductors 4 and 5 composing its sides, or because the e.m.f.'s of its sides are in opposition most of the time.

we lay out an angle in a leading (counter clockwise) direction, and draw a vector parallel and equal to E_8, of Fig. 335, **but pointed in the opposite direction,** because as we follow the series we go through element No. 8 in a direction opposite to its positive direction. The total e.m.f. in elements 1 and 8, or in coil 1–8, is therefore equal to the resultant of 1 and -8, which is Sx, as indicated on Fig. 336a. From x we lay out a vector parallel to E_2, representing the e.m.f. induced in No. 2, the next element which we pass through in its positive direction. Then from the end of E_2, Fig. 336a, we lay out another vector parallel and equal to E_7 of Fig. 335, but in the opposite direction, representing negative E_7. And so we proceed through the series, until we finally come to the end of the group, which is represented by the point C, Fig. 336a. When this has been done, we find that the vector resultants Sx (representing total e.m.f. of coil 1–8), xy (representing total e.m.f. of coil 2–7), yz (representing total e.m.f. of coil 3–6) and zC (representing total e.m.f. of coil 4–5) are **all in phase** with each other (or along the same straight line), but are **not equal** to each other. This is exactly in accord with our previous conclusions. But we now have the exact angular relations and a diagram which enables us to calculate the e.m.f. of any single coil. Thus, it appears that the e.m.f. of coil 4–5 is equal to 2 cos $78\frac{3}{4}°$ (or 0.39) times the e.m.f. of one element, whereas the e.m.f. of coil 1–8 is equal to 2 cos $11\frac{1}{4}°$ (or 1.96) times the e.m.f. of one element, or five times as much e.m.f. for the same amount of copper.

Consider a concrete example. Each coil of this winding has, let us say, 10 turns. Each pole furnishes 1,000,000 lines of useful flux. The frequency is 60 cycles, and the flux distribution is such that the generated e.m.f. is of sine wave-form, as must be assured in order to represent the e.m.f.'s by vectors. What will be the voltage generated in the smallest coil 4–5, in this case?

Each inductor passes under 60 pairs of poles in one second, or 120 poles. 120 poles × 1,000,000 lines per pole = 120,000,000 lines cut per second by each conductor, average. Average e.m.f. in each inductor is therefore 1.2 volts. The form factor, or ratio $\left(\frac{\text{effective value}}{\text{average value}}\right)$, of a sine wave, is 1.11. Hence the effective e.m.f. in each inductor is 1.11 × 1.2 = 1.332 volts. Each coil-side, or element of winding, consists of 10 inductors, all close together in the same slot so that their e.m.f.'s are in phase and therefore add together arithmetically. Hence the total e.m.f. in each element or slot is 13.32 volts effective. The resultant e.m.f. in the small coil 4–5 (represented in Fig. 336a by the vector zC) is therefore equal to 2 × cos $78\frac{3}{4}°$ times 13.32 volts, or 5.2 volts. If the e.m.f.'s of the other coils were worked out on the same basis, their sum total (represented by SC) for the whole group would be found to have a value about 68.3 volts. All groups are in phase with each other, and as there is one group for each pole, the e.m.f. generated in this entire winding is 68.3 × 4 = 273 volts.

It appears, however, that the four smallest coils (like 4–5) contribute only $\frac{5.2}{68.3}$, or 7.6 per cent, of the total e.m.f., although they contain almost $\frac{1}{4}$, or 25 per cent, of the total amount of copper on the winding (exactly $\frac{1}{4}$ if we consider only the active length of the conductors). If we take into account the ends of the coils and end-connections between coils the case is not so bad against the small coil. As the same current passes through all coils, the I^2R losses are in the same ratio as the resistances. And as the size of wire is the same throughout, the resistances are in proportion to the lengths of conductor in the coils. Hence the coils 4–5 in this case are responsible for nearly 25 per cent of the total armature I^2R loss. And since the resultant e.m.f.'s of all coils are in phase with each other, the phase relation between the current and the e.m.f. of each coil is the same throughout the winding, hence the volt-amperes and watts contributed by each coil toward the total output of the generator are in the same numerical ratio as their resultant e.m.f.'s. Hence the coils 4–5 all together contribute only 7.6 per cent of the total kilovolt-ampere or kilowatt output. Altogether the case looks pretty bad for these small coils; and it is largely for this reason that the winding is grouped into two or three phases instead of a single phase, resulting in a more advantageous connection of these elements into the armature circuit.

108a. Lap Winding. The same armature shown in Fig. 329 may be wound with a lap winding, as shown in Fig. 337a. In distinguishing between these two types, notice that the coils of the lap winding are naturally grouped so that there is one group of coils for each pair of poles, with only a single conductor as an end-connection between the groups. Fig. 337b is a developed diagram of the same type of winding for the same armature, but with two inductors per slot, or two turns per coil, the winding being pictured at the instant when the e.m.f. between terminals has its maximum value.

Fig. 337c is the same as Fig. 337b except it shows the winding at the instant, one-half period later, when the e.m.f. between the terminals is zero. Notice in the latter case that all inductors have e.m.f.'s greater than zero, because none are on the neutral points midway between poles. Note also that the zero e.m.f. between rings is due to cancellation of equal opposite values of e.m.f. in various parts of the circuit.

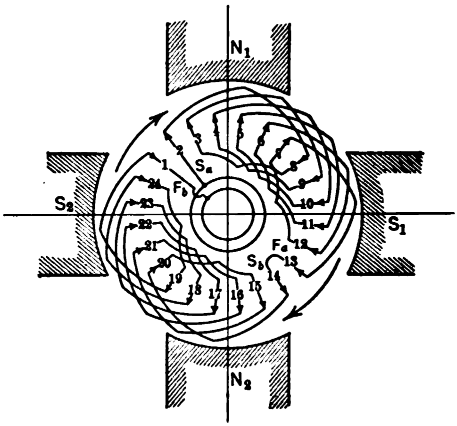


FIG. 337a. A lap winding for the armature of Fig. 329. Note that there is but one group of coils for each pair of poles instead of for each pole.

Prob. 20–9. Calculate how many turns there must be in each of the coils in Fig. 332 to give an e.m.f. of 2300 volts, effective, between rings at zero load, there being an e.m.f. of sine wave-form and 10 volts effective value in each conductor.

Prob. 21–9. In Fig. 332, what must be the total active length, in inches, of all the inductors in a single coil, in order that the total e.m.f. induced in the entire winding may be 6900 volts effective. The e.m.f. in each conductor is of sine wave-form, average density of useful flux under each pole 50,000 lines per square inch, peripheral speed of rotor 5000 feet per minute, frequency 60 cycles per second. If the active length of a single conductor in any coil-side is 10 inches, how many conductors must there be in each slot, and how many turns in each of the coils?

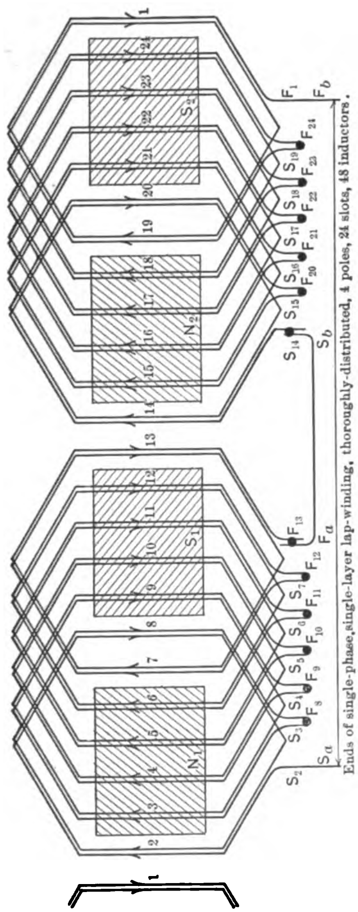


Fig. 337b. A developed diagram for the lap winding of Fig. 337a, using two inductors per slot. The e.m.f. between the terminals is at this instant a maximum.

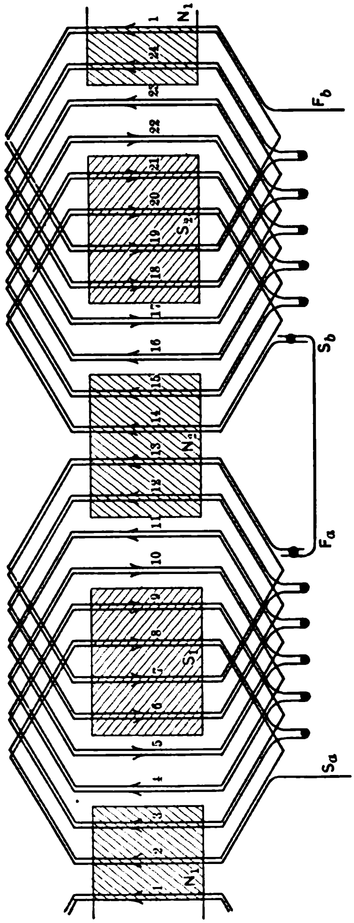


FIG. 337c. The lap winding of Fig. 337a at the instant of zero e.m.f. between the terminals. Note that the terminal e.m.f. is zero because the e.m.f.'s in the several coils oppose one another.

Prob. 22-9. Draw a wave winding thoroughly distributed for the armature shown in Fig. 329a, to deliver 230 volts between the collecting rings at zero load, with a frequency of 60 cycles when the flux per pole is 5,500,000 lines, so distributed that a sine wave of e.m.f. is induced in each conductor. Show clearly the coils and their connections and calculate how many conductors per coil and how many turns per coil are required. Would the solution be more feasible if the armature were to have one slot more or less, or an odd number of slots?

Prob. 23-9. Draw a wave winding for the armature shown in Fig. 329a, occupying two adjacent slots per pole, with one conductor per slot. Calculate the effective e.m.f. of this entire winding from the data given in Fig. 330, for a frequency of 60 cycles, and a total flux of 5,000,000 lines from each pole, there being a sine wave of e.m.f. in each inductor. Compare this e.m.f. with that which would be produced by a spiral winding with the same number of inductors in the same slots.

Prob. 24-9. Solve Problem 23-9 on the supposition that only four adjacent slots out of the six per pole are filled by this winding.

Prob. 25-9. Solve Prob. 23-9 for a wave winding filling all the six slots per pole.

Prob. 26-9. If the single-phase generator with thoroughly-distributed spiral winding, 4 poles, 8 slots per pole, as illustrated in Fig. 334, can deliver up to 100 kv-a. continuously at 2300 volts without overheating any part of the winding, what is the voltage between the ends of each of the four groups of coils? What is the voltage between the ends of coil 1-8? of coil 2-7? of coil 3-6? of coil 4-5?

Prob. 27-9. If the slots 4-5 were left vacant in Fig. 334, by what percentage would the number of inductors in each of the remaining slots have to be increased in order to deliver the same terminal voltage at the same frequency with the same flux?

109. Two-layer Windings. So far we have considered mostly single-layer windings. In such there is usually only one side of one coil in each slot. In the whole-coiled windings, to be sure, there are usually in each slot one side of each of two coils, but these two coil-sides, or winding elements, are placed side by side, and not one above the other. When a slot contains two coil-sides placed one above the other, we have what is known as a two-layer winding. Fig. 338 illus-

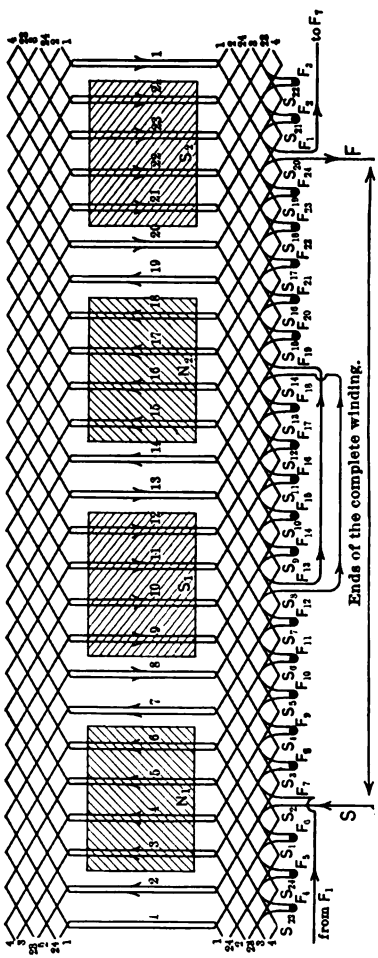


FIG. 338. Single-phase, two-layer lap-winding, thoroughly distributed, 4 poles, 24 slots, 48 T conductors, where T = number of turns per coil. Showing winding in position at which total e.m.f. has maximum instantaneous value.

trates such a winding for the same armature that we have been considering in the previous figures, — namely, four poles, thoroughly distributed, single phase, two coil-sides per slot, 24 slots. Fig. 338 shows the complete winding, from one collecting ring (S) to the other (F). Fig. 339

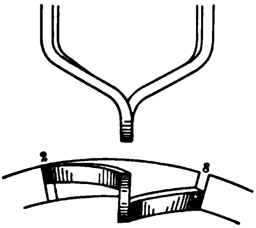


FIG. 339. The form of a single coil in the two-layer winding shown in Fig. 338.

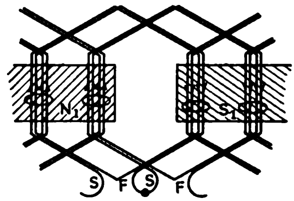


FIG. 339a. Details of the coil connections of a two-layer winding.

illustrates the form of a single coil. Note that it lies in the upper half of slot 2 and in the lower half of slot 8. This coil may consist of any number of turns. The turns in the coil may be wound side by side, as in Fig. 328a, or one above the other as in Fig. 328b, or the coil may consist of a single turn of copper bar, disposed in the slots as indicated in Fig. 328c. Fig. 340 shows a possible slot section for this winding. The principal advantage of the two-layer windings is that the turns or coils are all exactly alike, so that the number of forms required to wind the coils on during manufacture, and the number of coils necessary to keep in stock for repairs, is much less than for other types; — the spiral or chain winding, for instance. One limitation to its use is, that the number of inductors in each slot must be a multiple of two, since there are two coil-sides in each slot, consisting of an equal number of inductors. Fig. 353 is a photograph of a two-layer winding.

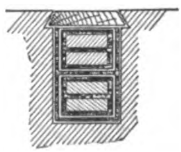


FIG. 340. Possible cross-section of the two-layer winding of Fig. 338.

110. Armature Circuits with More than One Path. All the drum windings so far shown, have been **single-circuit windings**; that is, there has been only one path for the current from one collecting ring to the other, all of the in-

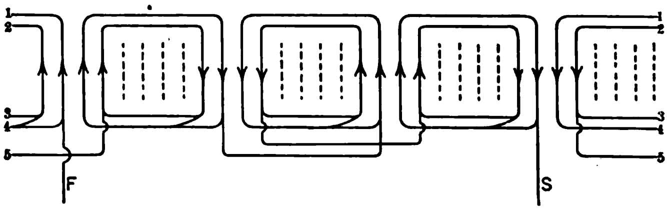


FIG. 341. Two-phase, four-pole spiral- or chain-coil winding. Four slots per pole per phase. Only one phase is shown; the other phase is laid in the slots represented by the dotted lines.

ductors in each phase of the winding being connected together in series. Often it is desirable to use two or more circuits in parallel between two rings, forming one phase of the winding, in order either to increase the current-carrying capacity of the armature or to enable the manu-

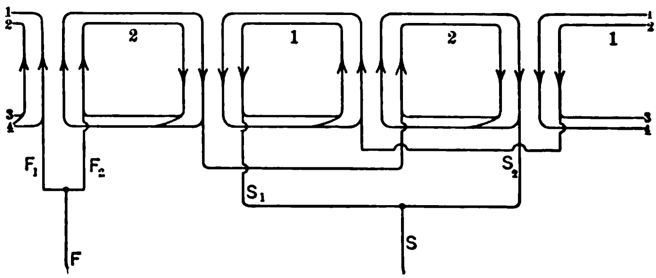


FIG. 342. Same winding as in Fig. 341, but connected in parallel instead of in series in order to carry twice the current at half the voltage.

facturer to use the same punchings and coils for machines of several voltages, simply by rearranging the end-connectors between the coils. Figures 341 to 344 illustrate how the same coils may be connected together all in series as one large group, giving a single circuit between the collecting rings or terminals S–F, or in two parallel groups capable of handling together twice the amperes at half the voltage between the terminals S–F. Fig. 341 and 342 show a special

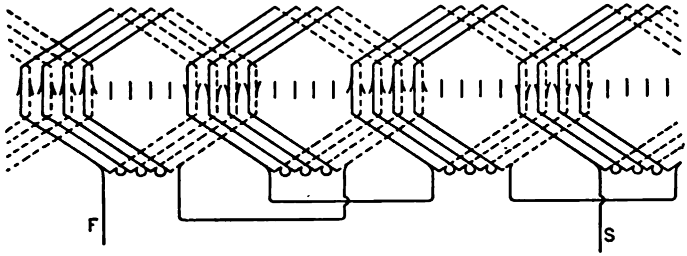


FIG. 343. A double-layer winding for the armature of Fig. 341, connected as a single circuit.

winding, frequently called a **chain-winding** (on account of its likeness to a chain of links), for a two-phase 4-pole armature with four slots per pole per phase. See Fig. 317 for a photograph of this type of winding. These figures

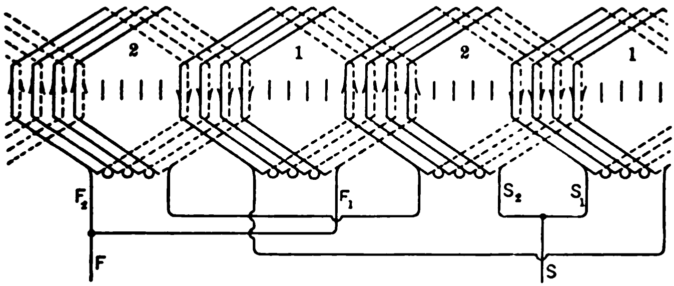


FIG. 344. The two-layer winding of Fig. 343, connected for two-circuit operation, capable of delivering twice as much current at half the voltage.

actually show only the connections for one phase; the slots which hold the coils for the other phase are shown dotted. The connections of the second phase would be exactly similar to those of the first, but displaced by half of one pole pitch. If the winding of Fig. 341 were good for 100 kv-a, 2000 volts, 50 amperes, then with end-connectors rearranged as in Fig. 342, the same machine would deliver 100 amperes at 1000 volts when driven at the same speed with the same excitation. But the power capacity is still 100 kv-a., and the current in each conductor, the flux per pole and the speed are unchanged. Fig. 343 and 344 show a two-layer winding for the same armature, arranged for a single-circuit and a two-circuit connection respectively.

111. Improper End-connections: Local Currents. In making the two-circuit arrangement of winding in alternators, care must be exercised to avoid either inequality in number of inductors, or inequality of e.m.f. in the two circuits which are connected in parallel, or phase difference between the total e.m.f.'s in the two circuits. For example, Fig. 345 shows an improper arrangement of a two-circuit winding for one phase of a three-phase, six-pole armature with 36 slots altogether, or two slots per pole per phase. Fig. 345 is a radial diagram of a wave winding for this machine in which inductors 1, 7, 13, 19, 25 and 31 compose one of the two parallel circuits, and inductors 2, 8, 14, 20, 26 and 32 compose the other circuit. By the construction shown here, the e.m.f.'s in all of the inductors composing either group are in phase with each other. Consequently, the total e.m.f. of group S_1–F_1 is equal to six times the e.m.f. induced in inductor No. 1, and in phase with it; and the total e.m.f. of group S_2–F_2 is equal to six times the e.m.f. induced in inductor No. 2, and in phase with it. The e.m.f.'s in inductors No. 1 and No. 2 are equal to each other in value, but have a phase difference corresponding to one slot pitch. This phase difference amounts to $\frac{1}{36}$ of 3 cycles, or $\frac{1}{12}$ of 360°, or 30 electrical degrees in this 6-pole 36-slot machine. If the e.m.f. induced in each inductor is 10 volts effective, Fig. 346 shows that there is a resultant e.m.f. OR equal to 31.0 volts acting in the closed circuit formed by the two parallel paths S_1F_1 and S_2F_2. This e.m.f. will produce in the windings, at zero load (or at any load) a local current equal to 31 volts divided by the total impedance of the winding. This local current would heat up the armature as long as the machine generates e.m.f., regardless of the

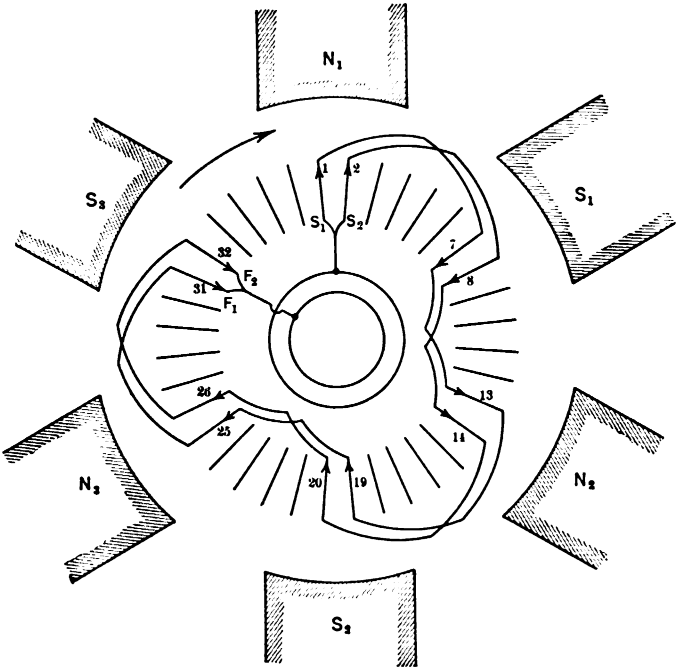


FIG. 345. A radial diagram showing an apparently correct method of making a two-circuit connection. The method is incorrect, however, as the e.m.f.'s of the two parallel paths are not in phase and thus a local current will circulate.

load, and this useless heating reduces the capacity of the machine to deliver useful output. It appears, therefore, that the **two paths** S_1F_1 and S_2F_2 **cannot be connected in parallel unless they are wound so that their e.m.f.'s. are in phase with each other.** That is, the two sets of inductors must be located either in the same slots, as in Fig. 344, or in slots located similarly with respect to each pole, as shown in Fig. 342. When this condition is fulfilled, the vectors S_1F_1 and S_2F_2 in Fig. 346 will be exactly opposite in phase and the resultant e.m.f. *OR* acting in the closed mesh will become zero for all values of total induced e.m.f.

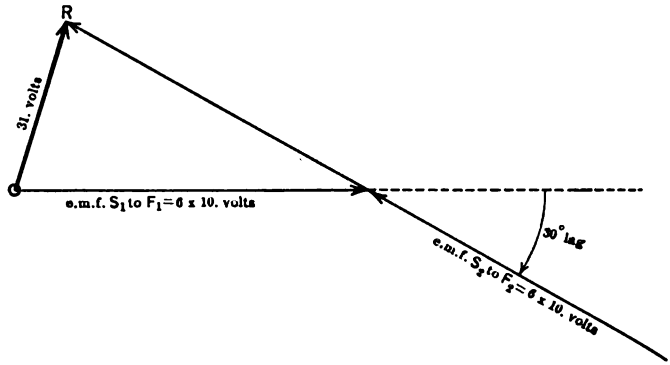


FIG. 346. The e.m.f. of 31 volts represented by the vector *OR* is the resulting voltage around the closed circuit of the armature connected as in Fig. 345.

112. Wear on Bearings Causes Local Currents. Fig. 347 and 348 illustrate another source of trouble in parallel windings. It shows one phase of a two-phase whole-coiled chain winding for a six-pole 24-slot armature. Fig. 347 illustrates a revolving-armature type of machine in which the six armature coils are divided up into two groups, each group consisting of three adjacent coils in additive series. Group S_2F_2 consists of winding elements 2, 5, 6, 9, 10 and 13. Group S_1F_1 consists of winding elements 1, 22, 21, 18, 17 and 14. As long as the armature or rotor continues to be perfectly aligned, so that the air gap is exactly the same under all poles and the flux entering the armature from all poles equal, the e.m.f.'s induced in the two parallel paths will be equal in value as well as exactly opposite in phase. But when the wear of the bearings allows the armature to drop and the air gap to become shorter for the lower poles and longer for the upper poles, the flux for the lower poles becoming thereby greater than the flux for the upper poles, there is a part of each half-revolution of the armature during which the e.m.f. is greater in one path than in the other path. Thus there is a resultant e.m.f. and circulating cur-

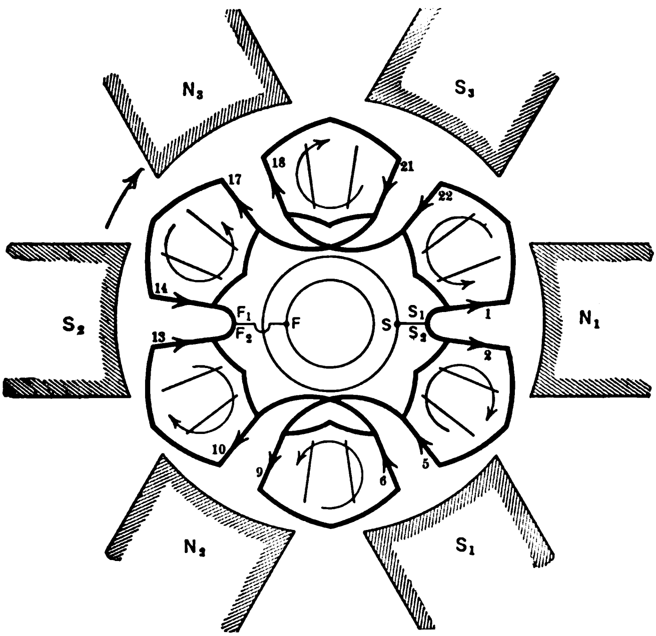


FIG. 347. The revolving armature has worn down the bearings so that it is nearer the lower poles N_2 and S_1 than N_3 and S_3. The resulting greater flux will cause a greater voltage across the lower armature path than across the upper.

rent in the closed mesh between the paths, at zero load or at any load.

Fig. 348 illustrates a similar condition for a revolving-field type of machine where the rotor has sagged due to wear of bearings, so that every pole has a larger flux during its half-revolution below the horizontal than during its upper half-revolution. Fig. 348 corresponds to the same instant of time as Fig. 347, the e.m.f. between the terminals *S* and *F* being at its maximum in both cases. If the coils were divided between the two parallel groups in such a way that one group was composed mostly of lower ones and the other circuit mostly of upper ones (the extreme case being as shown in Fig. 348), the total e.m.f. in one group

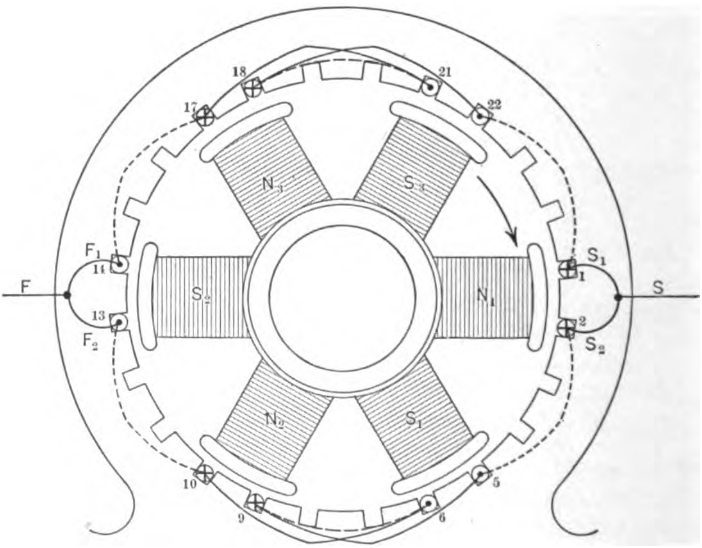


FIG. 348. The revolving field has worn the bearings down so that the same effect is produced in the armature windings as in Fig. 347.

would have a larger value than that in the other group. Thus there would be a resultant e.m.f. acting around the closed mesh, producing a circulating current which would heat the armature and reduce its capacity to deliver power. This defect could be remedied in the revolving-field machine by dividing the winding of the stator, or armature, **vertically,** so that neither of the parallel circuits would have a preponderance of the strong poles. Pulsations of the flux would induce an e.m.f. in the field winding, tending to neutralize the changes of flux, so that the condition relieves itself in some measure.

Prob. 28–9. Rearrange the connections of Fig. 334 so as to make it a two-circuit winding. If the armature had a rating of 200 kv-a. at 2300 volts before the change, what is its rating now? How many amperes can it deliver in each case?

Prob. 29–9. Rearrange the connections of Fig. 338 so as to make it a two-circuit winding. Answer questions of Problem 28–9 on same data.

Prob. 30–9. (*a*) When the winding shown in Fig. 341 is delivering 2300 volts from the terminals *S–F* at zero load, what is the e.m.f. generated in a single small coil of the winding?

(*b*) What is the e.m.f. generated in a single large coil of the winding?

(*c*) What is the e.m.f. of a pair of coils, or one of the four groups in Fig. 341?

Prob. 31–9. Answer questions of Problem 30–9 for the winding shown in Fig. 342.

Prob. 32–9. (*a*) When the winding shown in Fig. 343 is delivering 2300 volts at its terminals *S–F* at zero load, what is the e.m.f. generated in a single coil of the winding?

(*b*) What is the e.m.f. of one of the four groups of coils in Fig. 343?

Prob. 33–9. (*a*) Redraw the diagram of Fig. 341 so as to fill all of the slots on the armature with a thoroughly-distributed single-phase winding.

(*b*) By what percentage would the useful amount of copper be increased in so doing?

(*c*) By what percentage would the terminal voltage be increased, for the same frequency and field excitation?

(*d*) By what percentage would the number of turns per coil, and amount of copper, in the existing winding have to be increased to get the same increase in voltage without using the additional slots?

Prob. 34–9. (*a*) Redraw the diagram of Fig. 343 so as to fill all of the slots on the armature with thoroughly-distributed single-phase winding.

(*b*) By what percentage would the amount of copper be increased by so doing?

(*c*) By what percentage would the terminal voltage be increased, for the same frequency and field excitation?

Prob. 35–9. (*a*) By what percentage would the resistance of a single-phase winding between terminals be greater when thoroughly distributed, as in Problem 34–9, than when semi-distributed, as in Fig. 343, using the same size copper in the coils? By what percentage would the I^2R loss be increased if the current delivered remained the same as before?

(*b*) By what percentage must the amperes output be reduced to keep the same total I^2R loss as in the semi-distributed single-phase winding shown in Fig. 343?

(*c*) By what percentage would the kilovolt-amperes be increased or diminished, taking account of both the change in voltage and the necessary change in current capacity?

113. Polyphase Windings for Drum Armatures. Having studied, until they are thoroughly understood, the explanations of two-phase and three-phase ring windings (both closed and open types) which were given in Chapters VI and VII, we should now be able to produce a polyphase drum winding for any given armature to which they are applicable. Each of the phases of the polyphase winding has its connections between coils arranged in exactly the manner which has just been described for a single-phase drum winding. In the three-phase windings, the star and delta connections between the three phases (each phase taken as a whole) are made in a manner exactly similar to that described for a ring winding. For a two-phase winding the total number of slots should, of course, be exactly divisible by two, in order that each phase may have the same number of winding elements with the same phase relations between them. Also the number of slots, or of coils, in each phase is usually, but not necessarily, exactly divisible by the number of poles, in order that each phase may have a group of similar elements, similarly connected, under each pole. Likewise, in order to permit a three-phase winding to be made up, the total number of equidistant slots in the armature, or the total number of coils or winding elements which are to be placed in these slots, should be exactly divisible by the number of phases, and usually also by the number of poles. Machines having a fractional number of **slots per phase per pole** are quite frequently used, however, and produce in general a better shape of e.m.f. wave; but the total number of **slots per phase** must be a whole number. Let us consider a few examples.

A two-phase drum winding for a four-pole machine is shown in the simplest possible form, as a radial diagram, in Fig. 349. It is a concentrated wave winding, having one slot per pole per phase, full pitch. Each phase has one element of winding, or one slot, under each pole, and the elements of phase B are distant from the corresponding elements of phase A by exactly 90 electrical degrees. The whole winding is thus divided into two exactly similar halves, electrically distinct from each other, just like two single phases, their positions being relatively fixed so that S_A passes under middle N_1 just one-quarter period after (or before, depending on direction of rotation) S_B passes the same point. They are thus tied together in phase relation through the magnetic field and the mechanical distribution of the winding. The rings of phase A would be connected to S_A and F_A, and the rings of phase B to S_B and F_B.

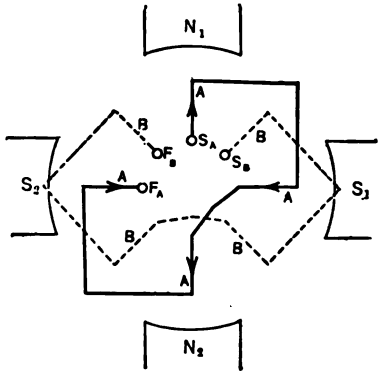


FIG. 349. A radial diagram of a two-phase winding for a four-pole machine.

A three-phase drum winding for a four-pole machine is shown in Fig. 350 to 351. The winding is the simplest practicable one to make, occupying only one slot per pole per phase. We first make sure that the total number of slots is divisible by 3 (phases) and 4 (poles). Then, holding the rotor stationary, we mark one element (or slot) under the middle of each pole as belonging to phase A, and connect them properly together in series so that their e.m.f.'s add together when they are in the position where the maximum instantaneous e.m.f. for the whole group is induced (as shown in Fig. 350a). We mark one end of this series S_A in Fig. 350b, and the other end F_A. With the rotor still fixed in the same position and starting from S_A, we proceed to count the slots or coils in one direction around the armature until we have passed two-thirds of those which lie between the middle of adjacent poles. Since the distance between the middle of adjacent poles is 180 electrical degrees, we have now passed over 120°. Put S_B label on this slot, element or coil, and locate the other three B elements or coils with respect to each other, just exactly as the A elements are related to each other. Connect the B group in additive series just exactly as the A group was connected, and mark F_B on the finishing end of the B series. Now from S_B continue to count slots or coils around in the same direction until you have passed as far ahead of S_B as S_B is ahead of S_A. This will be 120° from S_B or 240° from S_A. Mark this slot or coil S_C, and locate the other C slots or coils in similar

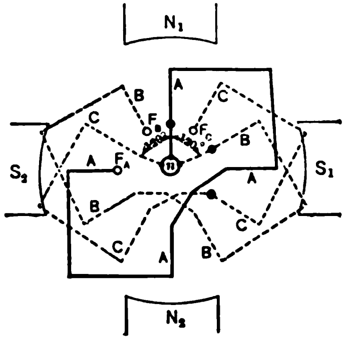


FIG. 350a. A radial diagram of a three-phase winding for a four-pole machine, star-connected.

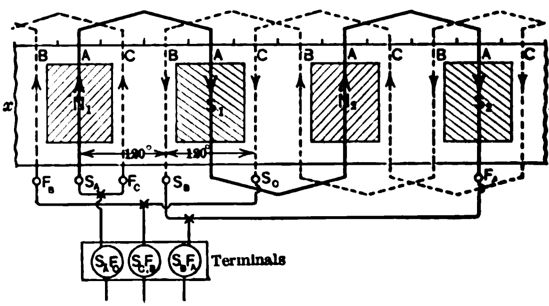


FIG. 350b. Developed diagram of the three-phase armature winding of Fig. 350a, delta-connected.

positions with respect to all other poles. Connect the C slots together in additive series just as the A slots were connected, and put the F_C label on the finishing end of the series. If the e.m.f. in phase A is at its maximum value from S_A to F_A at the position shown in Fig. 350a, and 350b, it is apparent that the e.m.f. S_A to

F_A must pass through one-third cycle or 120° before the e.m.f. from S_B to F_B reaches its maximum value (or is brought into the same position in the magnetic field by a counter-clockwise rotation of the rotor). Also, the e.m.f. from S_A to F_A must pass through $\frac{2}{3}$ cycle or 240°, before the e.m.f. from S_C to F_C reaches its maximum value. Since these three e.m.f.'s reach their maximum value in a direction **away from S** just 120° apart, consecutively, we must **connect the S ends together to neutral,** to get a star-connection. The **terminals** of the three-phase armature thus connected in **star** are **F_A, F_B, F_C,** as shown in Fig. 350a. The connections shown in Fig. 351 differ slightly in the method of connecting the C elements or coils together, but the **result** is exactly the same as in Fig. 350a.

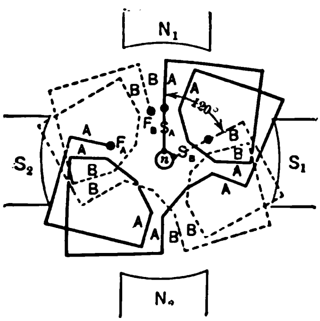


FIG. 350c. Three-phase winding having two slots per pole per phase. Two phases only are shown here.

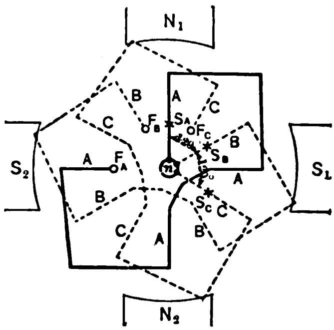


FIG. 351. Another diagram of a three-phase four-pole winding, star-connected; — equivalent to Fig. 350a.

Fig. 350b is a developed view of this same winding (three-phase, four-pole, one slot per pole per phase) **connected in delta or mesh.** Notice that if only we mark clearly that end of each phase which is separated by 120° from the **similar** end of the preceding phase, the connection becomes very simple, because we merely connect the finishing end of one phase to the starting end of the next phase 120° ahead, and so on. Thus we connect F_A to S_B, F_B to S_C and F_C to S_A. These junction points are then the terminals of the delta winding. The equal e.m.f.'s (S_A to F_A), (S_B to F_B), (S_C to F_C) are 120° apart successively in the same direction through the closed mesh or series; and it has been shown that this relation gives a resultant e.m.f. of zero volts around the mesh (see Art. 28).

If the winding is to occupy **two slots per pole per phase,** requiring

$3 \times 4 \times 2 = 24$ slots altogether, the connections would be as shown in Fig. 350c. In order to avoid confusion of lines, only Phases *A* and *B* have been drawn in and a lap winding with one inductor per slot has been chosen. Suppose we are given this same four-pole armature, with its 24 slots, completely filled by a two-layer lap winding, as in Fig. 352a. There are as many coils as slots, and all coils are exactly alike. Each slot contains two

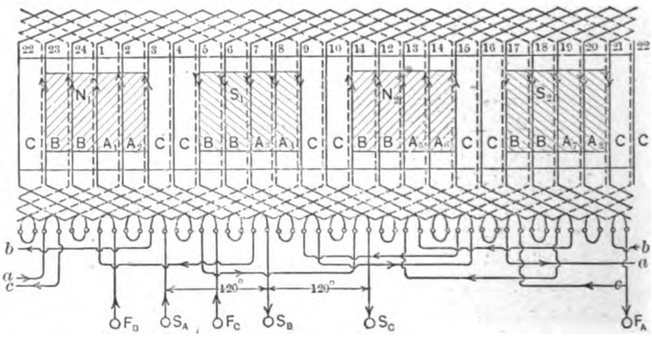


Fig. 352a. A rotor having 24 slots, wound as a three-phase four-pole armature. Two-layer winding. All coils alike.

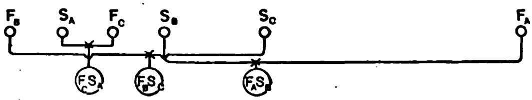


Fig. 352b. The armature of Fig. 352a delta-connected.

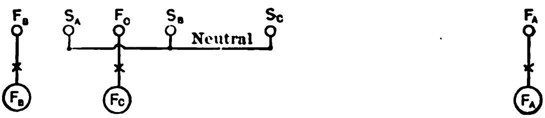


Fig. 352c. The armature of Fig. 352a, star-connected.

coil-sides or winding elements, — the one at the bottom is the right-hand side of a coil lying to the left of the slot, and the one at the top is the left-hand side of a coil lying to the right of the slot. The elements at the bottom of the slot are shown in dotted lines. Such an arrangement of coils used on the stator of a machine is illustrated by Fig. 353 and is typical of all lap-wound direct-current machines, and synchronous converters; but it may be used for any sort of an alternating-current winding, closed or open. If we label the similar end of every coil as its starting end *S*, and the other end *F* (as it is wound up on the form, for instance), then we would get a closed winding by simply soldering the *F* of one coil to the *S* of the one lying in the next slot, and so on all around the armature until the last *F* is soldered to the first *S*. This closed winding could be tapped at equidistant points, depending on the number of poles, just exactly as the

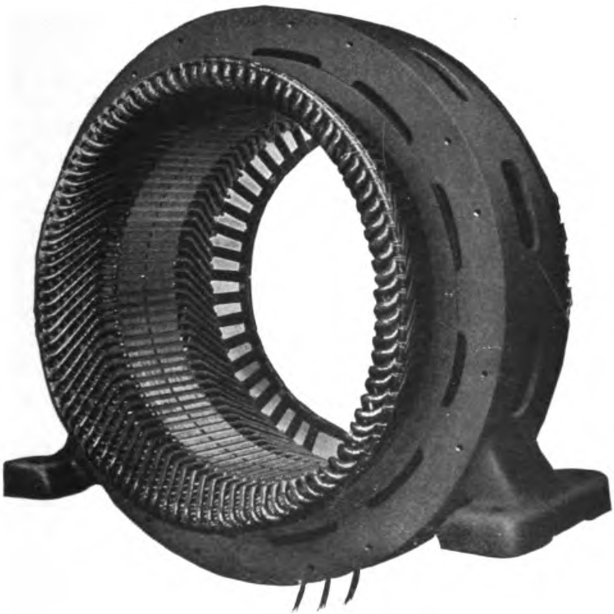


FIG. 353. A stator wound like the rotor in Fig. 352a. It has a full pitch winding, with a coil pitch of eight. *Westinghouse Electric & Mfg. Co.*

ring winding was tapped in Chapters VI and VII, and the same rules deduced there would then apply here. Or this closed winding could be opened up at two or more points and the parts be connected in series as an open winding for any number of phases as illustrated in Art. 88 and Fig. 262 to 267. The main difference between it and a ring winding is that the span or pitch of the coil in this lap winding fixes the number of poles within narrow limits, whereas any ring winding could be tapped for any number of poles. We recognize Fig. 352 as being properly a four-pole armature, because each coil spans one-quarter of the circumference, and to get the greatest e.m.f. in a coil, we know that its opposite sides should both come as nearly as possible simultaneously under the middle of adjacent poles. We have 24 coils altogether, for 3 phases and 4 poles, which allows two coils per pole per phase. These two will, of course, be adjacent coils, in order that their e.m.f.'s shall be as nearly as possible in phase with each other, so as to get the greatest possible resultant e.m.f. from the series.

With rotor fixed in Fig. 352a we choose a coil located exactly under the middle of N_1 and S_1, and label it A_1. Its starting end we mark S_A. The finish end of coil A_1 we connect to the starting end of coil A_2 which is adjacent to coil A_1. We now locate coils A_3 and A_4, also belonging properly to Phase A, because they are located with relation to pole S_1 and N_2 just exactly as coils A_1 and A_2 are located with relation to pole N_1 and S_1. Similarly, we locate A_5 and A_6 under poles N_2 and S_2, and A_7 and A_8 under poles S_2 and N_1. We then group A_3 and A_4 in additive series by soldering the finish of A_3 to the start of A_4. Similarly, we connect A_5 and A_6 together, and A_7 and A_8 together. Now, since the e.m.f. is clockwise around coils A_1 and A_2, counter clockwise around coils A_3 and A_4, clockwise around A_5 and A_6, and counter clockwise around A_7 and A_8, we see that in order to get these groups of coils together into additive series we must connect A_1, A_2, A_5 and A_6 together similarly, but A_3, A_4, A_7 and A_8 oppositely. If the student will carefully trace through the series of coils composing Phase A, starting at S_A and going right through to F_A, he will see that the instantaneous e.m.f.'s are all in the same direction at about the time when the e.m.f.'s induced in the coils of Phase A are greatest (which is about the position shown in the figure).

Phase B has been started at one end, S_B (similar to the end S_A), of a coil located 120 electrical degrees from coil A_1, and from this point through to F_B the connections and arrangement of coils are an exact duplicate of Phase A, except as to actual position in the magnetic field. Likewise Phase C is a duplicate of Phase A_1, but Sc is located 120° further along in the same direction from S_B, or 240° from S_A. This gives us the six terminals of the three phases all properly labeled. In Fig. 352b are shown the proper connections between these six terminals to give a three-phase delta. In Fig. 352c are shown the connections between the same six terminals to give a three-phase star. The only way to get a thorough understanding of these windings and connections is to trace through the lines and circuits on the diagram slowly and carefully, and then, by aid of the explanations given, to draw out some diagrams completely. Fig. 353 is a photograph of a two-layer winding for a Westinghouse generator.

The **chain winding** is the one most commonly used for alternating-current generators and motors, in addition to the two-layer

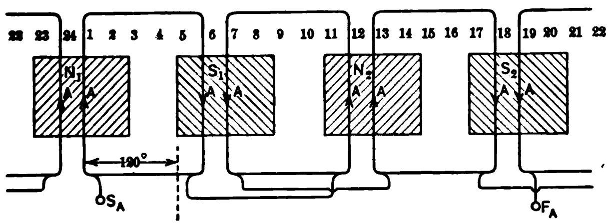


FIG. 354. Phase *A* of a three-phase chain winding for the drum armature of Fig. 352. Two slots per pole per phase.

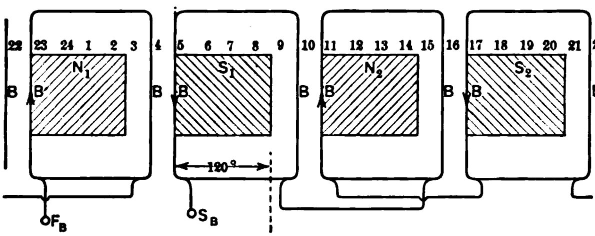


FIG. 355. Phase *B* of the three-phase chain winding.

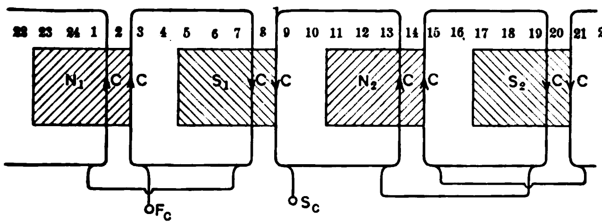


FIG. 356. Phase *C* of the three-phase chain winding.

windings. Fig. 354 represents a three-phase four-pole chain winding, using two slots per pole per phase on the same 24-slot armature which we have been using throughout for illustration. In order not to confuse the diagram, Phases *A*, *B* and *C* have been drawn out separately, in Fig. 354, 355 and 356. Notice that they are exactly alike, except as to relative position on the armature. Phase

B is 120° from Phase *A*, and Phase *C* is 120° further in the same direction, from Phase *B*, or 240° from Phase *A*, the positive direction of e.m.f. in each phase being from the *S* end to the *F* end. When the three phases are assembled altogether as in Fig. 357, it is seen that there must be a different shape or length of coil for

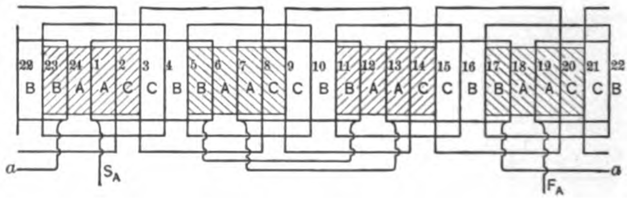


FIG. 357. The three-phase chain winding, the three phases of which are shown separately in Fig. 354, 355 and 356. The end-connections of phase *A* only are shown. Three forms of coils are necessary.

each phase in order that the ends of the coils shall not interfere with each other. This is expressed usually by saying that the end-bends of the coils are in three **ranges.** This is due to the fact that the winding is a single-layer winding (the coils of a two-layer winding are all exactly alike), and also because we have made all coils in each phase the same shape. In Fig. 357 the end-connec-

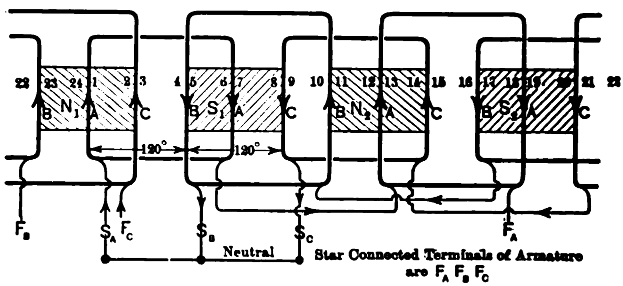


FIG. 358. Three-phase chain winding using one slot per pole per phase. Two coil forms only are needed. See Fig. 312 and 314.

tions between coils are shown only for Phase *A*, to avoid confusion. The system of connections would be exactly the same as in other figures which are complete.

A three-phase chain winding using one slot per pole per phase, in which the coils are made in only **two forms,** is shown in Fig. 358. Photographs of this sort of winding may be seen in Fig.

312 and 314; it is very convenient and common because of the economy of winding-forms and stock required. It is obtained by using the two different shapes for alternate coils in the same phase. Fig. 358 shows the complete connections for each phase and also

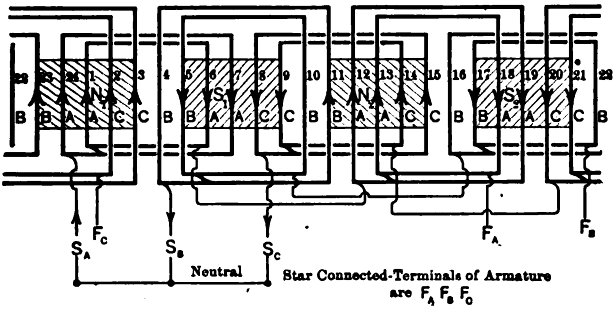


FIG. 359. Three-phase chain winding using two slots per pole per phase. See Fig. 315 and 317.

between the phases for a three-phase star. The student should easily be able to rearrange the end-connections between phase-terminals S_A and F_A, S_B and F_B, S_C and F_C to get a three-phase delta. Fig. 359 shows a three-phase chain winding using two

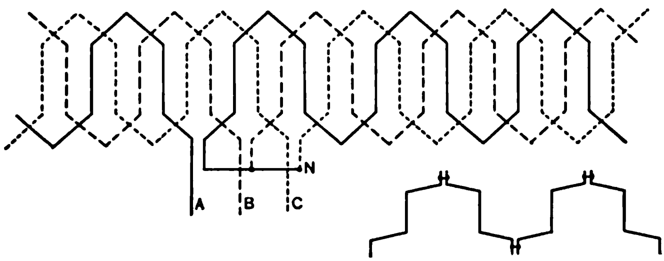


FIG. 360. Three-phase bar winding, (wave) using one slot per pole per phase; star-connected.

slots per pole per phase, connected in star. Windings similar to this, except that they are for three slots per pole per phase, are photographed for Fig. 315 and 317.

A few other typical forms are merely illustrated in the following figures: Fig. 360 is a three-phase bar winding (wave) using one slot per pole per phase. It may be extended as shown for any number of pairs of poles and is drawn star-connected. Fig. 361 is known as a "skew-coil" winding; although there is only one winding element or coil-side in each slot, all coils are of the same shape for all phases, and conflicts of the coil-ends are avoided by making one side of each coil longer than the other side. Fig. 362 illustrates what is called a "short-coil winding" for a three-phase

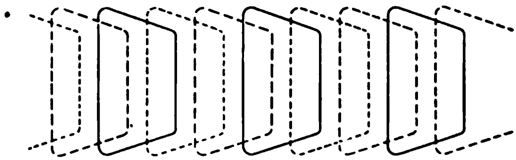


FIG. 361. Skew-coil winding. All coils are alike, each having one side shorter than the other.

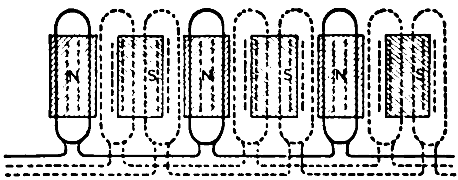


FIG. 362. Short-coil winding. Each coil is slightly less than two-thirds of the pole pitch to avoid overlapping of coils. The e.m.f.'s of the two sides of a coil do not add to such good advantage as in other types.

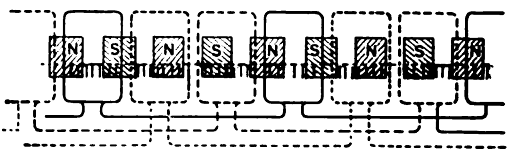


FIG. 363. A "creeping winding." The coils have a fractional pitch. Three coils cover four poles in this case. The small dash lines represent slots left vacant for the sake of clearness.

machine using two slots per pole per phase. By making the breadth of each coil only $\frac{2}{3}$ of the pole pitch, overlapping of coils is altogether avoided, and all coils in the entire winding are exactly alike. The series e.m.f.'s composing each phase are not added to as good advantage as in other types of winding using coils nearer full-pitch, and, therefore, more copper would be needed for the same capacity. The wave-form is also likely to be more peaked.

Fig. 363 shows a "creeping winding" in which the coils are fractional pitch and the series of coils in each phase is arranged so as to gain or lose one or more poles as you trace it around the armature. In Fig. 363 three adjacent coils each spanning 240°, together cover 720° or four poles. Fig. 364 shows a single-phase whole-coiled winding for 8 poles, using 3 slots per pole, for an armature having altogether 64 slots. A photograph of a two-phase armature having a winding like this for a two-phase machine is shown in Fig. 317.

Prob. 36–9. Draw a developed diagram of connections for the full-pitch winding whose photograph appears in Fig. 310, assuming the alternator to deliver e.m.f. at 60 cycles when driven at 900 r.p.m. Connect the winding in three-phase star.

Prob. 37–9. How many poles should there be on a revolving field to be used with the stator shown in Fig. 317? Draw out a

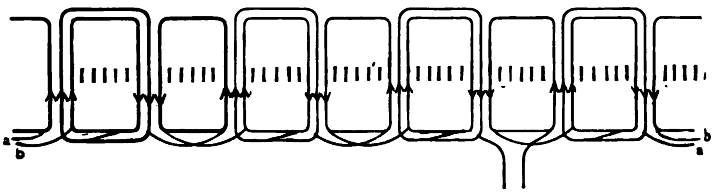


FIG. 364. Single-phase whole-coiled winding for eight poles, using three slots per pole. See Fig. 317 for two-phase winding of this type.

developed winding diagram for the two-phase winding on this machine, showing the connections of the coils to each other and to the terminals of the machine.

Prob. 38–9. How many poles should there be on the rotor to be used with the stator shown in Fig. 353? Draw out a developed winding diagram for this machine as a two-phase stator.

Prob. 39–9. The 500-kw. Allis-Chalmers alternator, whose winding is shown in part in Fig. 314, delivers 11,000 volts between terminals of the star-connected three-phase winding, at 60 cycles per second, when driven by a water wheel at 200 r.p.m. By aid of the explanations given concerning Fig. 358, which is a diagram of this same style of winding for a smaller number of poles, draw a vector diagram showing how the e.m.f. in the various coils of one phase are combined, and calculate:

(*a*) The e.m.f. which must be generated in each complete phase of the winding — Fig. 314.

(*b*) The e.m.f. which must be generated in a single coil of this winding.

Prob. 40-9. (*a*) How many poles should be on the rotor used with the stator shown in Fig. 315?

(*b*) Draw a developed diagram of connections for the winding of this machine after the manner of Fig. 359, showing complete connections between all coils in each phase and between the delta-connected phases.

(*c*) If the terminal voltage of this machine is 2300 volts, what is the e.m.f. across each of the three coils of different pitch used?

SUMMARY OF CHAPTER IX

The conductors which compose the armature winding of electrical machines are formed into COILS and placed in the slots of the armature core.

THE POLE-PITCH of an armature is the number of slots or of inductors on the armature, between the center of one pole and the center of the next pole.

IN A FULL-PITCH armature coil, if one side of the coil lies under the center of a pole, the other side of the coil will lie under the center of the next pole.

IN A FRACTIONAL-PITCH coil both sides will not lie under pole centers at the same instant. Such a coil generates less e.m.f. than a full-pitch coil under similar conditions, but may generate a more nearly perfect wave-form of e.m.f.

AN ELEMENT OF WINDING consists of all the conductors forming one side of a single coil.

Windings are usually placed in slots on the armature in order:

(1) To avoid being shifted by the large torque put upon them at heavy load or by short circuits.

(2) To decrease the air gap between armature core and pole faces.

A BAR WINDING has only one turn in each coil, usually of heavy copper having rectangular cross-section.

A CONCENTRATED WINDING is one in which each of the phases occupies only one slot per pole. In such windings, the wave-form of generated e.m.f. depends principally upon the shape of the pole faces, but also somewhat upon the pitch or span of the coil. The effective voltage of the winding may be increased by using more than one turn per coil.

A WAVE WINDING is one which leads us continuously in one direction around the armature as we follow along one of the circuits of the winding. A development of the winding on a flat surface resembles a wave (see Fig. 321c). A wave winding may be either PROGRESSIVE or RETROGRESSIVE, but this affects only the end-connections between coils, and not the value of the e.m.f. generated.

A LAP WINDING is one which leads us alternately clockwise and counter clockwise around the armature, as we follow continuously along one of the circuits of the winding. That is, it laps back upon itself as we pass from one coil to the next, as shown in Fig. 322c.

When there is the same number of coils formed as there are poles, the winding is said to be a WHOLE-COILED WINDING. When only half as many coils as poles are formed, it is said to be a HALF-COILED WINDING. A half-coiled winding allows the armature to be " split " for repairs, etc., without cutting through any of the coils. Also, the reactance of the armature may be less than with a whole-coiled winding.

In order to produce a smoother wave of e.m.f. the inductors forming each armature path are often not grouped together in a few slots, but are spread out into many slots and distributed over the armature face. This arrangement is called a DISTRIBUTED WINDING.

In a concentrated winding the e.m.f.'s of the several conductors forming a single armature path are all practically equal to one another at any given instant. The e.m.f.'s in the conductors of a distributed winding differ materially among themselves at any given instant, since the conductors occupy different relative positions in the magnetic field at that instant.

In order to obtain the instantaneous voltage across any armature path, it is necessary to add algebraically the e.m.f.'s induced in the separate conductors. It is usually inefficient to use many inductors of which the e.m.f. is more than 60 or 70 degrees out of phase with the e.m.f. across the armature path.

In order to cover economically as much as possible of the armature surface with active copper, two-phase or three-phase windings are used. By thus dividing the inductors into two or three circuits, only such inductors need be joined in series as have e.m.f.'s more nearly in phase with the e.m.f. across the circuit.

When TWO-LAYER windings are used, one side of a coil occupies the lower half of one slot, while the other side occupies the upper half of another slot. One form will serve for all coils used in such a winding.

In PARALLEL or MULTI-CIRCUIT WINDINGS where two or more paths through the armature coils are joined in parallel, the greatest care must be taken to see that:

(1) The e.m.f.'s across all paths are equal.

(2) The e.m.f.'s across all paths are exactly in phase with one another.

(3) The clearance between all poles and the armature are equal.

(4) The paths are so interlaced that the minimum unbalance will result if the clearance does change.

If any of these conditions are not fulfilled, local currents will circulate through the armature paths, and reduce the permissible output of the machine.

Any closed winding can be tapped for either single-, two- or three-phase, or any number of phases. In any case, the phases are mesh-connected, and the winding is of the parallel or multi-circuit type.

By properly proportioning the number of armature slots and coils to the number of poles, and by the proper connection of the armature windings, either single-, two- or three-phase connection can be made by OPEN-WINDINGS. The open-winding can be connected either in star or mesh.

The more common forms for the coils used are:

(1) Double-layer coils (all alike for a given armature).

(2) Chain (two or more forms for each armature).

(3) Skew-coil (all alike).

(4) Short-coil (all alike, but wasteful of copper).

(5) Creeping-coil (all alike).

PROBLEMS ON CHAPTER IX

Assume in all of the problems that the e.m.f. generated in each inductor is of approximately sine wave-form.

Prob. 41–9. Each inductor in the winding shown in Fig. 333, generates say 10 volts. Draw a vector diagram to show what voltage is induced between the terminals of the coil occupying slots 3 to 4 — that is, the volts induced between S_3 and F_4.

Prob. 42–9. What is the phase difference for the winding shown in Fig. 333, in electrical or time degrees, between:

(*a*) The e.m.f.'s in the various conductors composing one coil-side or winding element in any slot?

(*b*) The e.m.f.'s of the two winding elements composing coil 1 and 6?

(*c*) The e.m.f.'s of the winding elements composing coil 2–5?

(*d*) The e.m.f.'s of the winding elements composing coil 3–4?

Prob. 43–9. On same assumptions as in Problem 41, calculate the total voltage induced in the coil S_2F_5, occupying slots 2 and 5.

Prob. 44–9. On same assumptions as in Prob. 41, calculate the total voltage induced in the coil S_1F_6, occupying slots 1 and 6.

Prob. 45–9. (*a*) In the same winding (Fig. 333), what is the phase relation (direction from S toward F being considered positive in every case) between the terminal e.m.f. at zero load of coil 3–4 and coil 2–5?

(*b*) What is the phase relation between coil 2–5 and coil 1–6?

(*c*) Between coil 1–6 and coil 7–12?

(*d*) Between group 1–2–3–4–5–6 and group 7–8–9–10–11–12?

(*e*) Between group 1–2–3–4–5–6 and coil 7–12?

(*f*) Between group 1–2–3–4–5–6 and coil 3–4?

(*g*) Between the entire winding and coil 3–4?

Prob. 46–9. The alternator whose winding is shown in Fig. 333, when operating at rated frequency of 60 cycles, with normal field excitation, gives 2300 volts between its terminals, at zero load. In replacing the coil which occupied slots 9 and 10, which has been burned out, the operator gets its terminals reversed, — that is, F_4 connected to S_9 instead of to F_{10}, and F_{11} connected to

F_{10} instead of to S_9. What e.m.f. is now obtained at the terminals of the generator, when operated with same frequency and field strength as before?

Prob. 47–9. Same as Problem 46–9, but with coil 8–11 reversed instead of coil 9–10.

Prob. 48–9. Same as 46–9, but with coil 7–12 reversed instead of coil 9–10.

Prob. 49–9. In making the end-connections in Fig. 333, between coils, by what percentage would the total zero-load voltage between the terminals of the armature be increased or diminished, for any given speed and field excitation, if by mistake F_4 had been connected to S_7, and F_{10} had been connected to S_{13}?

Note. In the above problem assume any convenient voltage per coil, say 1 or 10 or 100 volts; the actual value used will be immaterial in determining percentage change, as it cancels out in the calculation.

Prob. 50–9. In Fig. 333 what is the maximum value of e.m.f. tending to break down the insulation between the end-connector F_4 to F_{10}, and the coil-connections S_9 to F_{11}, S_8 to F_{12} and S_7 to S_{13}, which it crosses, when the effective voltage between the terminals of the entire winding is 2300 volts effective?

Prob. 51–9. If the smallest of the three coils in each group, Fig. 333, were left out of circuit, and the connections were completed otherwise as in Fig. 333, by what percentage would the terminal voltage at zero load be reduced, for the same speed and field current?

Prob. 52–9. Draw vector diagrams of both topographic and polar types to show how an e.m.f. of 20 volts effective is built up between the terminals F_6 and S_1 in Fig. 333. What is the effective voltage induced in a single inductor?

Prob. 53–9. Draw vector diagrams as in Problem 52 to show the composition and value of the effective e.m.f. between the coil terminals S_1 and S_2.

Prob. 54–9. Draw vector diagrams in Problem 52 to show the composition and value of the effective e.m.f. between the coil terminals S_1 and S_3.

Prob. 55–9. Draw vector diagrams as in Problem 52 to show the composition and value of the effective e.m.f. between the coil terminals S_3 and S_9.

Prob. 56–9. Draw vector diagrams as in Problem 52 to show the composition and value of the effective e.m.f. between the coil terminals S_3 and S_8.

Prob. 57-9. Draw vector diagrams as in Problem 52 to show the composition and value of the effective e.m.f. between the coil terminals S_3 and S_7.

Prob. 58-9. When operated at rated frequency with its maximum field current, a single-phase armature wound as in Fig. 333 gives 3200 volts between terminals at zero load. The coils occupying slots 1, 2, 3, 4, 5 and 6 had to be removed for repairs. If, when they are being restored, the connections of coil 1-6 are reversed, what will be the terminal e.m.f. of the machine under the same conditions as before? Draw a vector diagram to illustrate your solution.

Prob. 59-9. Solve Problem 58 for coil 2-5 alone reversed, all others correct. See Prob. 46-9.

Prob. 60-9. Solve Problem 58 for coil 3-4 reversed, others correct. See Prob. 46-9.

Prob. 61-9. Solve Problem 58 for coils 1-6 and 2-5 both reversed, coil 3-4 correct.

Prob. 62-9. Solve Problem 58 for coils 2-5 and 3-4 both reversed, coil 1-6 correct.

Prob. 63-9. Solve Problem 58 for coils 1-6 and 3-4 both reversed, coil 2-5 correct.

Prob. 64-9. Solve Problem 58 for coils 1-6, 2-5 and 3-4 all reversed.

Note. In the following problems assume e.m.f. of sine wave-form to be induced in each conductor of the winding. Note that the armature in Fig. 337 is of exactly the same construction as that in Fig. 329b, *i.e.*, single-phase, four poles, thoroughly distributed in 24 equidistant slots. Note also that the winding of Fig. 337b is similar to that of Fig. 333 in all respects except the manner of grouping the conductors into coils and joining them together in series. To find the real electrical distinctions between the spiral or chain winding and the lap winding, therefore, compare the solutions of the following problems based on Fig. 337 with those obtained from similar problems on the chain winding based on Fig. 333.

Prob. 65-9. (*a*) What is the phase difference in electrical degrees, for the winding shown in Fig. 337, between the e.m.f. of the winding elements 2 and 8, composing the coil S_7-F_8?

(*b*) Between the winding elements in coil S_3-F_9?

(*c*) Between the winding elements composing coil S_7-F_{13}?

(*d*) Compare results with those of Problem 42-9.

Prob. 66-9. Each conductor in the winding shown in Fig. 337b generates say 10 volts:

(*a*) Draw a vector diagram to show what voltage is induced between the terminals, S_2–F_8, of the coil which occupies slots 2 and 8.

(*b*) What voltage is induced within coil 3–9?

(*c*) What voltage is induced within any other coil of this winding?

(*d*) Compare these results with those of Problems 41–9, 43–9 and 44–9.

Prob. 67–9. From Fig. 337b, what is the phase relation between the terminal e.m.f.'s at zero load (direction from S toward F being considered positive in every case):

(*a*) Of coil 2–8 and coil 3–9?

(*b*) Of coil 3–9 and coil 4–10?

(*c*) Of coil 4–10 and coil 5–11?

Prob. 68–9. From Fig. 337, what is the phase difference of the e.m.f. at no-load:

(*a*) Between any coil and the next adjacent coil?

(*b*) Between coil 2–8 and coil 4–10?

(*c*) Between coil 2–8 and coil 5–11?

(*d*) Between coil 2–8 and coil 6–12?

(*e*) Between coil 2–8 and coil 7–13? Compare results with those of Prob. 45–9.

Prob. 69–9. (*a*) What is the phase relation in Fig. 337 between the terminal e.m.f.'s at zero load, of coil 2–8 and coil 14–20?

(*b*) Between group S_a–F_a, and group S_b–F_b?

(*c*) Between the entire winding and group S_a–F_a?

(*d*) Compare results with corresponding ones of Problem 45–9.

Prob. 70–9. (*a*) Draw vector diagrams of both topographic and polar types, to show the composition and value of the total e.m.f. between the coil terminals F_{13} and S_7 in Fig. 337b:

(*b*) Between F_{13} and S_6.

(*c*) Between F_{13} and S_5.

(*d*) Between F_{13} and S_4.

(*e*) Between F_{13} and S_3.

(*f*) Between F_{13} and S_2. The e.m.f. in each inductor is 10 volts.

(*g*) Compare results with corresponding values in Problems 52–9 to 57–9.

Prob. 71–9. At rated frequency and normal field excitation, the winding shown in Fig. 337b produces 2300 volts between its terminals at zero load:

(*a*) What is the e.m.f. between the terminals of any single coil?

(*b*) What e.m.f. is there between the terminals of a group of coils as, between S_a and F_a?

Prob. 72–9. What would be the terminal e.m.f. of the entire winding with same field excitation and speed, if in repairing a burn-out in the armature of Fig. 337b, the coil 2–8 had its connections reversed, *i.e.*, S_2 connected to S_3 instead of F_8 to S_3? Give vector diagram showing how the total voltage is made up.

Prob. 73–9. Same as Prob. 72–9, but for coil 3–9 reversed, instead of coil 2–8.

Prob. 74–9. Same as Prob. 72–9, but for coil 4–10 reversed, instead of coil 2–8.

Prob. 75–9. Same as Problem 72–9, but for coil 5–11 reversed, instead of coil 2–8.

Prob. 76–9. Same as Problem 72, but for coil 6–12 reversed, instead of coil 2–8.

Prob. 77–9. (*a*) Same as Problem 72, but for coil 7–13 reversed, instead of coil 2–8.

(*b*) Compare results of Problems 72 to 77 with those of Problems 46–9 to 48–9.

Prob. 78–9. If, in connecting up the two groups of coils in Fig. 337b, F_a had been connected to F_b instead of to S_b, by what percentage would the terminal e.m.f. of the entire winding be increased or diminished at any given frequency and field excitation? Compare results with those of Problem 49–9.

Prob. 79–9. Draw a progressive wave winding for the armature illustrated in Fig. 337a, four-pole, single-phase, 24 slots, one inductor per slot.

Prob. 80–9. Draw a retrogressive wave winding for the armature illustrated in Fig. 337a.

Prob. 81–9. Draw a lap winding for an 8-pole armature with 24 slots, 2 inductors per slot, single-phase, thoroughly distributed.

Prob. 82–9. Draw a vector diagram showing the composition and value of the terminal e.m.f. of the whole of the winding developed in Problem 81–9. (The e.m.f. in each conductor is 10 volts.)

Prob. 83–9. (*a*) Draw a winding diagram for a wave winding for an 8-pole, 24-slot armature, one conductor per slot, 10 volts per inductor.

(*b*) Show by means of a vector diagram how the terminal e.m.f. of the entire winding is made up, and calculate its effective voltage.

Prob. 84–9. Draw a "snapshot" picture of the machine illustrated in Fig. 329, at the instant that its e.m.f. has the value 1380 volts, the outer collecting ring being of positive polarity. The effective value of e.m.f. as indicated by a voltmeter across the rings is 2300 volts.

Prob. 85–9. (*a*) Draw a spiral winding (like Fig. 332) for a 6-pole armature having altogether 54 equidistant slots, the winding occupying only two-thirds of the total number of slots.

(*b*) Calculate the number of conductors per slot and turns per coil required to produce a total voltage of 2300 at 60 cycles, with 5,000,000 useful lines per pole, distributed so as to produce a sine wave of e.m.f. in every conductor.

Prob. 86–9. By vector diagrams similar to Fig. 330 and 331, find the total voltage induced in a partly-distributed single-phase winding for a 60-cycle, 900-r.p.m. alternator having 48 equidistant slots, the winding occupying two-thirds of the total number of slots; sine wave of e.m.f. having 20 volts effective value is induced in each of the 192 inductors composing the entire winding.

Prob. 87–9. Solve Problem 27–9, leaving slots 3, 4, 5 and 6 vacant in Fig. 334.

Prob. 88–9. (*a*) Solve Problem 27–9, leaving slots 2, 3, 4, 5, 6 and 7 vacant.

(*b*) How many slots per pole would this leave?

(*c*) Show how the winding would then look.

(*d*) Would it be a concentrated winding?

Prob. 89–9. Draw a diagram of a concentrated winding for the machine shown in Fig. 334 for:

(*a*) Whole coils.

(*b*) Half coils.

(*c*) Calculate by what percentage the number of conductors per slot, and also by what percentage the total number of conductors in the whole machine, would have to be greater or less than in the winding shown in Fig. 334, in order to generate the same total voltage, with the same frequency and flux.

Prob. 90–9. (*a*) Considering the active length of the conductor and neglecting the ends of the coils, by what percentage is the ohmic resistance of this equivalent concentrated winding of Problem 89–9 greater or less than that of the thoroughly-distributed winding shown in Fig. 334 for the same armature? The slots are the same size in each case.

(*b*) When generating the same total kv-a. at the same total voltage, by what percentage is the total I^2R loss in the equivalent concentrated winding greater or less than in the thoroughly-distributed winding?

Prob. 91–9. (*a*) Allowing the same total I^2R loss in the concentrated as in the equivalent thoroughly-distributed winding, at the same rated full load, by what percentage should the circular-mil area of the conductors in the concentrated be greater or less than in the distributed winding?

(*b*) By what percentage would the quantity of copper be greater or less?

Prob. 92–9. (*a*) If, in Fig. 334 the phase difference between the total induced e.m.f. of the entire winding and the current delivered is 30°, what is the power-factor of each coil in the winding?

(*b*) What fraction of the total kv-a. output of the entire winding does each of the coils generate?

(*c*) What fraction of the total IR drop in the whole winding is the IR drop of each coil, neglecting the end-connections?

Prob. 93–9. (*a*) Grade the coils in the winding of Fig. 334, according to the amount of power they contribute to the total power generated by the machine, under the conditions of Prob. 92–9.

(*b*) If it were decided to use a winding only partly distributed, using say only four out of the eight slots per pole, which coils would you drop out first?

(*c*) Show the winding as you would rearrange it.

(*d*) What percentage of the former armature I^2R loss would be saved?

Prob. 94–9. Repeat all diagrams of Fig. 334, 335 and 336, for a 6-pole single-phase winding thoroughly distributed in 6 slots per pole per phase.

Prob. 95–9. Repeat all diagrams of Fig. 334, 335, 336 for a 720-r.p.m. 60-cycle alternator, having altogether 120 slots, the single-phase circuit occupying only 80 of these.

Prob. 96–9. Draw a diagram corresponding to Fig. 337b, for winding the armature of the alternator specified in Prob. 94–9.

Prob. 97–9. (*a*) Arrange the winding in Fig. 334 so that it occupies only 4 slots under each pole, each slot containing one inductor generating an e.m.f. of sine wave-form, 30 volts effective value. Note that there are 8 slots per pole on the armature.

(*b*) Calculate the vector sum of the e.m.f.'s generated in the group of slots under each pole by use of a diagram, such as Fig. 330, and from this calculate the total e.m.f. between rings as in Fig. 331.

(*c*) Check this result by calculating the total e.m.f. of each coil by means of a diagram, such as in Fig. 336a, and adding the e.m.f.'s of all of the coils together successively to get the total e.m.f. between rings. These two methods will give same result if applied accurately.

Note. The armature shown in Fig. 338, though different in arrangement of winding, is identical in inductors, if $T = 1$, with that shown in Fig. 333; *i.e.*, it is four-pole, single-phase, thoroughly distributed, 24 slots, 2 inductors per slot.

Prob. 98–9. (*a*) When 10 volts are induced in each inductor, what is the e.m.f. between the terminals of any single coil in Fig. 338, say the one occupying upper 2 and lower 8?

(*b*) Upper 3 and lower 9, etc.? Compare results with those of Problems 41–9 to 44–9.

Prob. 99–9. (*a*) What is the total number of coils used in Fig. 338?

(*b*) How many used in Fig. 337b?

(*c*) How many used in Fig. 333?

(*d*) What is the maximum e.m.f. acting upon the insulation between the upper and lower inductors in slot 2, Fig. 338, when the terminal e.m.f. of the entire winding is 2300 volts and the load zero?

Prob. 100–9. What e.m.f. is being induced between the terminals of each coil, as for instance the one in upper 2 and lower 8 in Fig. 338, when the entire winding delivers 2300 volts at zero load?

Prob. 101–9. What is the e.m.f. between the connectors F_{13}–F_{19} and S_8–S_{14} where they cross at the end of the armature, Fig. 338? Terminal e.m.f. of the entire winding is 2300 volts.

Prob. 102–9. (*a*) Draw a winding diagram for a two-layer winding to generate 2300 volts between terminals, single-phase, thoroughly distributed, 24 slots, 6 poles. Assume 6 volts induced in each inductor.

(*b*) Show how a section of the slot might appear.

Prob. 103–9. Suppose that in making the end connections while assembling the winding in Fig. 338, F_{13} had been connected to S_8 and F_{19} to S_{14}. By what percentage would the total voltage of the entire winding be greater or less than it should be, at normal excitation and frequency, and zero load?

Prob. 104–9. Suppose that, in replacing the coil S_2–F_8 of Fig. 338, after making some repairs, its connections were reversed. By what percentage would the total e.m.f. of the winding at normal frequency and excitation and zero load, be increased or diminished?

Prob. 105–9. (*a*) Draw a thoroughly-distributed spiral single-phase winding for four poles on the 24-slot armature which we have been using for illustration, arranged for single circuit, as in Fig. 341.

(*b*) Can you wind it also for single-phase two-circuit? How?

(*c*) If not, why not?

(*d*) Could you do it if the winding were not thoroughly distributed?

(*e*) How?

Prob. 106–9. (*a*) Draw a spiral winding for this 24-slot armature of Prob. 105, thoroughly distributed, single phase, single-circuit, for 6 poles.

(*b*) Repeat for a two-circuit winding.

Prob. 107–9. Repeat Problem 105–9, using a two-layer winding.

Prob. 108–9. (*a*) Draw a vector diagram showing how the terminal e.m.f. of a stator like Fig. 353, but having only 8 poles for 96 slots, or pole pitch of 12 with coil span of 8, is built up when connected three-phase-delta.

(*b*) If the machine is rated 100 kv-a., 440 volts, at 60 cycles, what is its speed and what is the voltage between the ends of each coil in the winding, assuming a sine wave of e.m.f. to be generated?

Prob. 109–9. (*a*) Draw a vector diagram showing how the terminal e.m.f. of the stator in Fig. 317 is built up.

(*b*) The machine is rated 200 kv-a., two-phase, 440 volts. At what speed must it be driven to give a frequency of 60, and what is the voltage across each of the two spans of coil used?

Prob. 110–9. (*a*) Draw a duplicate winding in Fig. 364 in such a way that the two windings together compose a two-phase machine, with three slots per pole per phase.

(*b*) What percentage of the total number of slots is filled?

(*c*) Draw a vector diagram showing how a terminal voltage of 2300 per phase would be built up, and calculate from this diagram the voltage across each type of coil used in the winding.

Prob. 111–9. (*a*) Draw a half-coiled chain winding, single-phase, 8-pole, using 3 slots per pole out of a total of eight slots for a 64-slot armature, to be equivalent to the winding shown in Fig. 364.

Prob. 112–9. (*a*) Draw a complete winding diagram like Fig. 360, and the corresponding vector diagram, for a wave-wound armature with single-layer bar winding, three-phase-delta, 10 poles, one slot per pole per phase.

(*b*) Calculate the e.m.f. that must be induced in each conductor to give a terminal voltage of 2300 to the whole winding.

Prob. 113–9. Draw a skew-coil winding like Fig. 361 for a 600-r.p.m. three-phase machine to give 60 cycles. Draw in the poles and the connections between coils to form phases; show the phases connected in star.

Prob. 114–9. If the speed and flux are such in the machine referred to in Problem 113–9 that each coil-side generates a sine wave of e.m.f. whose effective value is 100 volts, calculate the e.m.f. between terminals of the machine at zero load.

Prob. 115–9. (*a*) If the flux per pole in Fig. 362 is of such strength and distribution that each coil-side develops a 60-cycle e.m.f. of sine wave-form and effective value 100 volts, what will be the total e.m.f. per phase and the terminal e.m.f. of such a short-coil winding for a three-phase star-connected armature to operate at 900 r.p.m.?

(*b*) Draw a complete developed diagram of this winding showing coil connections.

(*c*) Draw a vector diagram to explain how your terminal voltage was obtained.

Prob. 116–9. (*a*) Draw a vector diagram to accompany Fig. 363, showing the value and the composition of the terminal e.m.f. of a 12-pole, star-connected three-phase machine, if the e.m.f. induced in each coil-side is of the sine wave-form and 100 volts effective value. Note that there are 27 slots under 4 poles, and that each coil spans 8 slots with 1 slot between adjacent coils.

(*b*) Draw the complete winding diagram for this condition.

(*c*) If it were desired to use 4 coils in each phase, *i.e.*, 3 groups together in each phase for the 12 poles, how would the winding appear and how many volts would be induced in each of the various coils used?

Prob. 117–9. Can you connect the coils in Fig. 361 so as to make a two-phase generator with the same number of poles? If not, why not?

Prob. 118–9. Can you connect the machine shown in Fig. 353 as a two-phase generator with the same number of poles?

(*b*) If so, show the complete winding diagram.

(*c*) Using the same rotor operating at the same speed as in Problem 108–9, calculate the terminal voltage across each phase, the rated amperes output per phase, total rated kv-a. and frequency, as a two-phase generator, with the same maximum allowable temperature rise in each armature coil.

Prob. 119–9. (*a*) Try to connect the machine shown in Fig. 317 as a three-phase generator in star with the same number of poles.

(*b*) Draw the complete diagram of connections between coils and between phases, and show by vector diagram how you figure your results.

Prob. 120-9. A two-phase induction motor receives 75 amperes at 220 volts in each phase, the two phases being separate windings. The power-factor of each phase is 0.80. Compute and plot the sum of the instantaneous power received by both phases at instants 10 electrical degrees apart throughout one complete cycle.

Prob. 121-9. A three-phase induction motor, delta-connected, receives 50 amperes in each lead wire at 220 volts between lead wires. The power-factor is 0.80. Compute and plot the sum of the instantaneous power received by the three phases at instants 10 electrical degrees apart throughout one complete cycle.

Prob. 122-9. (*a*) Compute and plot at instants 10 degrees apart throughout a complete cycle the sum of the instantaneous power received by the three windings of a three-phase star-connected synchronous motor. Current in each lead wire, 100 amperes; voltage between lead wires, 2300 volts; power-factor, 0.90 leading.

(*b*) What conclusion do you draw from Prob. 120, 121, and 122 concerning the advantage of using polyphase power for driving motors?

APPENDIX A

SIMPLE TRIGONOMETRIC FUNCTIONS

In a triangle certain relations exist among the sides and angles which can be learned easily, and which will greatly shorten the work of computing alternating-current values.

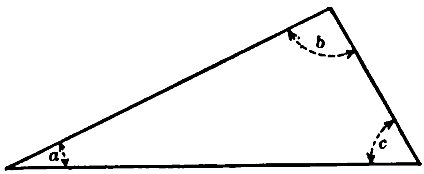


Fig. 1a. The sum of the angles *a*, *b* and *c* must equal 180° whatever the shape of the triangle.

1a. The Angles of a Triangle. — **In any triangle, the sum of all the angles always equals 180°.** Thus in Fig. 1a, $\angle a + \angle b + \angle c = 180°$. So if $\angle a = 25°$ and $\angle b = 95°$, then $\angle c$ must be large enough to make up the 180°.

$$\angle a + \angle b = 25° + 95° = 120°,$$
$$\angle c = 180° - 120° = 60°,$$

or

$$\angle a + \angle b + \angle c = 25° + 95° + 60° = 180°.$$

Problem 1a. In a right triangle, one of the acute angles is 30°. What is the other?

Problem 2a. A triangle has one angle equal to 27°, another equal to 46°. What must the third angle be?

2a. The Right Triangle. The Relations of the Sides to One Another. Consider the relation of the lengths of the sides to one another in the right triangle in Fig. 2a. The longest side *H* is called the **hypothenuse**, and is always **opposite** the right angle. The other two sides are called legs. It has been found that:

The square of the length of this hypothenuse is exactly equal to the sum of the squares of the lengths of the legs.

In the form of an equation, this is written

$$H^2 = A^2 + B^2.$$

From this we get

$$H = \sqrt{A^2 + B^2}.$$

Thus if $A = 3$, $B = 4$,

then $A^2 = 9$, $B^2 = 16$,

and $H^2 = 9 + 16 = 25$,

or $H = \sqrt{25} = 5.$

Accordingly, if we know the length of two legs of a right triangle, or of one leg and the hypothenuse, we can always find the length of the third side.

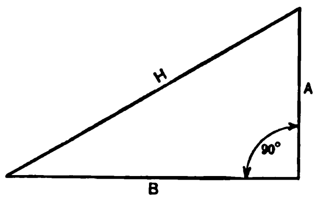


FIG. 2a. In a right triangle the square of the hypothenuse, H, equals the sum of the squares of the legs, A and B. Thus $H^2 = A^2 + B^2$.

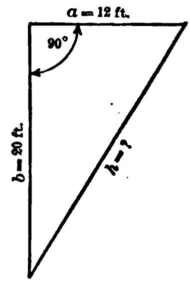


FIG. 3a. In the right triangle $h^2 = b^2 + a^2$.

Example 1a. One leg of a right triangle is 12 ft., the other is 20 ft. How long is the hypothenuse?

Construct Fig. 3a.

$$h^2 = a^2 + b^2,$$
$$h = \sqrt{a^2 + b^2} = \sqrt{144 + 400} = \sqrt{544} = 23.3 \text{ ft.}$$

Example 2a. One leg of a right triangle is a line representing 120 pounds, the hypothenuse represents 200 pounds. How many pounds does the other represent?

Construct Fig. 4a.

$$h^2 = a^2 + b^2,$$
$$a^2 = h^2 - b^2,$$
$$a = \sqrt{40{,}000 - 14{,}400}$$
$$= \sqrt{25{,}600}$$
$$= 160 \text{ pounds.}$$

Problem 3a. In a right triangle the two legs are 40 and 90 respectively. How much is the hypothenuse?

Problem 4a. One leg of a right triangle represents a pressure of 110 volts. The hypothenuse represents 175 volts. How many volts must the other leg represent?

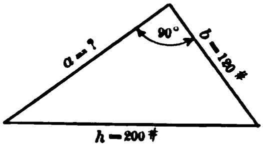


FIG. 4a. In a right triangle $h^2 = a^2 + b^2$ or $a^2 = h^2 - b^2$.

Problem 5a. The side opposite the right angle of a right triangle measures 4.17 inches. One leg measures 3.07 inches. What must be the length of the other leg?

Problem 6a. How long a ladder is necessary to reach a window 15 ft. from the ground, if the foot of it cannot be placed nearer the building than 8 ft. on level ground?

Problem 7a. A rectangular lot measures 100 yd. by 80 yd. How many yards of travel are saved by going "across lots" in getting from one corner to the opposite corner?

3a. Relations Among the Sides and Angles in a Right Triangle. Sine. Consider the right triangle in Fig. 5a, having one angle of 30°. In such a triangle the length of the side opposite the 30° angle, that is, the side **a**, will always be exactly one-half of the length of the hypothenuse **h**. Thus if the hypothenuse is 10 inches long, **a** will be ½ of 10, or 5 inches long. If **h** represents a force of 90 pounds, then **a** will represent a force of ½ of 90, or 45 pounds.

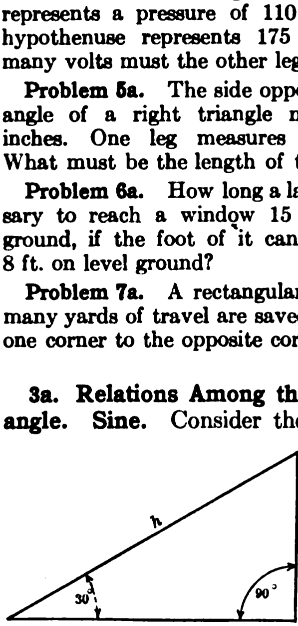


FIG. 5a. In a right triangle having a 30° angle, the side opposite the 30° angle is always one-half as long as the hypothenuse.

Suppose that we have a right triangle, Fig. 6a, with a 53° angle in the place of the 30° of Fig. 5a. Then the length of the leg **a** (opposite the 53° angle) will be $\frac{8}{10}$ of the length of the hypothenuse **h**. Thus if **h** is 10 in. long, **a** will be $\frac{8}{10}$ of 10, or 8 inches. If **h** is 40 inches long, **a** will be $\frac{8}{10}$ of 40, or 32 inches long.

THIS FRACTION which the side opposite an angle is of the hypothenuse has been given the name of SINE of the angle. Thus the sine of 30° is 0.5, which means that in any right triangle containing a 30° angle, the side opposite this 30° angle is 0.5 of the hypothenuse. It is usually written

$$\sin 30° = 0.500.$$

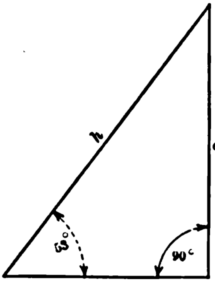


FIG. 6a. In a right triangle having a 53° angle the side opposite this 53° is always $\frac{8}{10}$ of the hypothenuse.

A table has been made of the sines of all angles between 0° and 90°, and can be found on page 521, columns 2 and 6.

By means of this table, if one of the angles of a right triangle is known, together with the length of the hypothenuse, then the length of the two legs can be found. Or if any angle and one leg is known, the hypothenuse and the other leg can be found.

Example 4a. If the hypothenuse of the right triangle of Fig. 7a is 14 inches long, what are the lengths of the sides **a** and **b**? The angle between **h** and **a** = 25°. From the definition of sine, we know that

$$\sin 25° = \frac{\text{side opposite } 25°}{\text{hypothenuse}}.$$

That is,

$$\sin 25° = \frac{b}{h}.$$

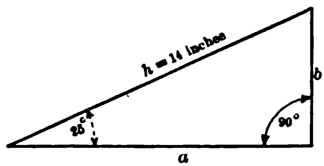


FIG. 7a. The fraction $\frac{b}{h}$ has a certain value called the sine of 25°.

From the table, $\sin 25° = 0.432.$

Therefore
$$\frac{b}{h} = 0.423.$$
$$b = 0.423\,h$$
$$= 0.423 \times 14$$
$$= 5.92 \text{ inches.}$$

Since the sum of the angles of the triangle must equal 180°, the angle opposite **a** must equal 180° − 25° − 90° = 65°.

$$\text{Sin } 65° = \frac{\text{side opposite } 65°}{\text{hypothenuse}},$$

or

$$\frac{a}{h} = \sin 65°.$$

From the table,

$$\sin 65° = 0.906.$$

Therefore

$$\frac{a}{h} = 0.906,$$

$$a = 0.906\,h$$
$$= 0.906 \times 14$$
$$= 12.7 \text{ inches.}$$

This checks with the rule that the square of the hypothenuse equals the sum of the squares of the other two sides, because

$$12.7^2 + 5.92^2 = 14^2.$$

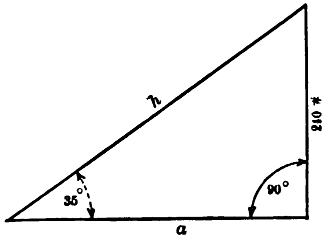


FIG. 8a. The fraction $\frac{240}{h} = \sin 35°$. The fraction $\frac{a}{h} = \sin (90° - 35°)$.

Example 5a. One of the angles of a right triangle is 35° and the side opposite it represents a force of 240 pounds. How large must be the forces which are represented by the hypothenuse and the other side? Construct Fig. 8a.

By definition of sine,

$$\sin 35° = \frac{\text{side opposite } 35°}{\text{hypothenuse}},$$

$$\sin 35° = \frac{240}{h}.$$

From table,

$$\sin 35° = 0.574.$$

Thus

$$0.574 = \frac{240}{h},$$

or

$$0.574\,h = 240,$$

$$h = \frac{240}{0.574}$$
$$= 418 \text{ pounds.}$$

The angle opposite leg a equals

$$180° - 90° - 35° = 55°,$$

$$\sin 55° = \frac{\text{side opposite } 55°}{\text{hypothenuse}}$$
$$= \frac{a}{h}$$
$$= \frac{a}{418}.$$

From table,

$$\sin 55° = 0.819.$$

Thus

$$\frac{a}{418} = 0.819,$$
$$a = 0.819 \times 418$$
$$= 343 \text{ pounds.}$$

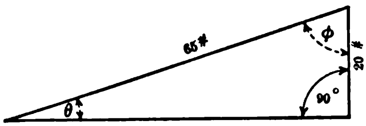


FIG. 9a. The fraction $\frac{20}{65} = \sin\theta$.

By means of this relation which exists between the side opposite an angle and the hypothenuse, the number of degrees in the angles of a right triangle may be found.

Example 6a. Determine the angles in a right triangle, having 20 pounds for one leg and 65 pounds for hypothenuse.

Construct Fig. 9a.

By definition,

$$\sin\theta = \frac{20}{65} = 0.308.$$

From table,

$$0.309 \text{ is sine of } 18°.$$

Therefore

$$\text{angle } \theta = 18° \text{ approx.,}$$
$$\text{angle } \phi = 180° - 90° - 18°$$
$$= 72° \text{ approx.}$$

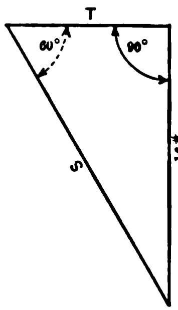


FIG. 10a. In the right triangle find the value of the sides S and T.

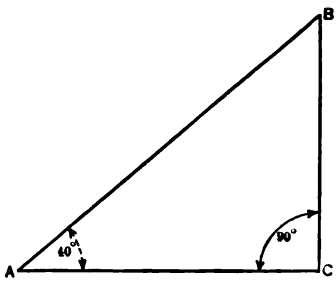


FIG. 11a. A right triangle.

Problem 8a. Find the value of S and T in Fig. 10a.

Problem 9a. A 40-ft. ladder leaning against a house makes an angle of 72° with the ground. To what height does it reach on the side of the house, and how far is the foot of the ladder from the house?

Problem 10a. In Fig. 11a, AB is 48 inches long. How long are BC and AC?

Problem 11a. A 25-ft. ladder leaning against a house makes an angle of 62° with the ground. To what height does it reach on the side of the house, and how far is the foot of the ladder from the house?

Problem 12a. Find the value of Z in Fig. 182.

Problem 13a. One leg of a right triangle represents an e.m.f. of 240 volts. The hypothenuse represents 300 volts. What is the angle opposite the 240 leg?

Problem 14a. How many volts does the other leg of the right triangle in Problem 13a represent and what is the value of the angle opposite it?

Problem 15a. A pole resting on the ground is inclined at an angle of 20° from the vertical. From the top of the pole to the ground, measured on a vertical line, is 15 ft. How long is the pole?

Problem 16a. What must be the value of the angle ϕ in Fig. 12a, in order to make $MX = 110$? $OM = 200$?

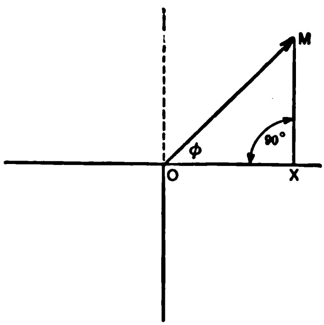


FIG. 12a. The triangle OMX is a right triangle.

Problem 17a. What is the value of the line OX in Prob. 16a?

Problem 18a. If angle ϕ remains the same as in Prob. 16a, how long must OM be in order to make $OX = 84$?

Problem 19a. Find the value of E_2 in Fig. 179.

Problem 20a. Find the value of Z in Fig. 184.

Problem 21a. What is the value of (i) in Fig. 17?

Problem 22a. In Fig. 19, what is the value of (i)?

Problem 23a. Find (i) in Fig. 20.

Problem 24a. What is the value of (i) in Fig. 22?

Problem 25a. What is the value of (e) in Fig. 39?

Problem 26a. In Fig. 35, E_m equals 220 volts. What is the value of $(-e_1)$?

Problem 27a. Find the values of CS and BH in Fig. 86.

Problem 28a. AB, Fig. 88, equals 840 volts. What is the value of BN?

Problem 29a. In Fig. 138, $OI = 62$ amperes, $\theta = 22°$. What value has I_RI?

Problem 30a. In Fig. 138, $OI = 62$ amperes, $\theta = 22°$. What value has I_XI?

4a. Cosine. We have seen that in a right triangle, as in Fig. 13a, the leg **a** is always a definite fraction of the hypothenuse. This fraction is called the sine of the angle which is opposite the side.

Thus $$\frac{a}{h} = \sin\theta.$$

It is also true that **b**, the side adjacent to the angle θ, is a definite fraction of the hypothenuse. This fraction is called the **COSINE. The cosine of an angle then may be defined as that fraction which the side adjacent to the angle is of the hypothenuse.**

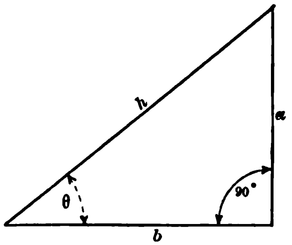


FIG. 13a. The fraction $\frac{b}{h}$ is called the cosine of the angle θ.

Thus $$\cos\theta = \frac{b}{h}.$$

If $$\theta = 60°, \text{ and } h = 40,$$
then b must equal ½ of 40, or 20, because cos 60° = 0.500. On page 521 is a table of cosines.

Example 7a. In Fig. 13a, $\theta = 40°$, $h = 12$ ft. What length does **b** represent? By definition,

$$\cos 40° = \frac{\text{side adjacent } 40°}{\text{hypothenuse}},$$

$$\cos 40° = \frac{b}{h}$$

$$= \frac{b}{12}.$$

From table,

$$\cos 40° = 0.766.$$

Then

$$\frac{b}{12} = 0.766,$$

$$b = 12 \times 0.766$$

$$= 9.19 \text{ ft.}$$

Problem 31a. Solve Prob. 14a by the cosine table.

Problem 32a. One angle of a right triangle is 35°. The side adjacent to this angle represents 900 amperes. How many amperes does the hypothenuse represent?

Problem 33a. Solve Prob. 18a by the cosine table.

Problem 34a. If in Fig. 13a **a** = 75, **h** = 120, what value has the angle between them?

Problem 35a. How long must the hypothenuse of a right triangle be when one of the angles is 20° and the side adjacent to this angle is 2.16 inches long?

Problem 36a. In Fig. 60, what is the value of ϕ?

5a. Tangent. Consider Fig. 14a. Not only does a definite relation exist between **a** and **h**, and between **b** and **h**, but also between **a** and **b**.

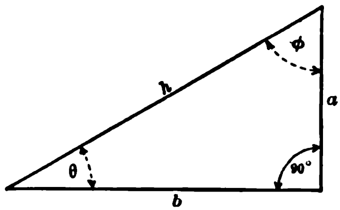


FIG. 14a.

The relation of **a** to **h** is called the sine of the angle θ.

The relation of **b** to **h** is called the cosine of the angle θ.

The relation of a to b is called the tangent of the angle θ.

The fraction $\frac{a}{h} = \sin\theta$.

" " $\frac{b}{h} = \cos\theta$.

" " $\frac{a}{b} = \tan\theta$.

As it is generally written

$$\sin\theta = \frac{a}{h} = \frac{\text{side opposite }\theta}{\text{hypothenuse}},$$

$$\cos\theta = \frac{b}{h} = \frac{\text{side adjacent }\theta}{\text{hypothenuse}},$$

$$\tan\theta = \frac{a}{b} = \frac{\text{side opposite }\theta}{\text{side adjacent }\theta}.$$

Example 8a. If in Fig. 14a the angle θ is 25° and **b** is 9 inches long, how long is **a**?

By definition,

$$\tan 25^\circ = \frac{\text{side opposite }25^\circ}{\text{side adjacent }25^\circ} = \frac{a}{b} = \frac{a}{9}.$$

From table on page 521,

$$\tan 25^\circ = 0.466.$$

Thus

$$\frac{a}{9} = 0.466,$$

$$a = 9 \times 0.466 = 4.19.$$

Problem 37a. In Fig. 14a, if $a = 45$ pounds and $b = 65$ pounds, what value has angle θ? (Use tangent table.)

Problem 38a. Solve Prob. 17a by means of tangent table.

Problem 39a. One leg of a right triangle represents 110 volts. The angle opposite this leg is 28°. How many volts does the other leg represent?

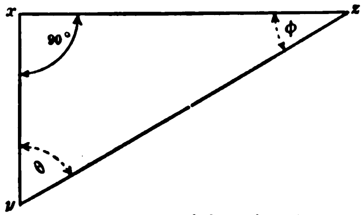


FIG. 15a. A right triangle.

Problem 40a. In Fig. 10a, find the value of T by means of the tangent table.

Problem 41a. In Fig. 15a, $XZ = 500$, $YX = 300$. Find the value of the angle θ.

Problem 42a. Find ZY in Fig. 15a, and angle ϕ.

Problem 43a. One of the legs of a right triangle is $\frac{3}{8}$ of the hypothenuse. What is the angle between this leg and the hypothenuse?

Problem 44a. What fraction of the hypothenuse in triangle of Prob. 43a is the other leg?

6a. Solution of Oblique Triangles and Parallelograms. Triangles which are not right triangles are called **oblique triangles.** They can be solved either by dividing them up into right triangles, or adding enough to them to make them right triangles. The right triangles thus formed can then be solved as above. Parallelograms are solved in the same way.

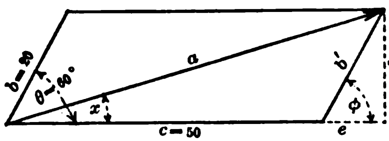


FIG. 16a. By adding the lines d and e to the parallelogram, a right triangle is made by the sides a, d and $(c + e)$.

Example 9a. In alternating-current work it is often necessary to find the diagonal a of a parallelogram, Fig. 16a, of which the sides b and c and the angle between are known.

Draw d perpendicular to e which is an extension of c. It can be proved that b′ always equals b and that angle ϕ always equals angle θ.

$$\text{Angle } \theta = 60°. \quad \text{Therefore angle } \phi = 60°.$$

$$b = 20. \quad \text{Therefore } b' = 20.$$

$$\frac{d}{b'} = \sin\phi = \sin 60° = 0.866,$$

$$\begin{aligned} d &= 0.866\, b' \\ &= 0.866 \times 20 \\ &= 17.3. \end{aligned}$$

$$\frac{e}{b'} = \cos\phi = \cos 60° = 0.500,$$
$$e = 20 \times 0.500$$
$$= 10.$$
$$\text{Tan } x = \frac{d}{c+e} = \frac{17.3}{50+10}$$
$$= \frac{17.3}{60} = 0.288.$$

In table,

$$0.287 = \tan 16°.$$

Therefore $\quad$ angle $x = 16°$ approx.,

but $\quad \dfrac{d}{a} = \sin x = \sin 16°,$

$$\frac{17.3}{a} = 0.276,$$
$$17.3 = 0.276\,a,$$
$$a = \frac{17.3}{0.276},$$
$$= 62.6.$$

Thus **a** = 62.6 and acts at an angle of 16° to *C*.

7a. The Diagonal of a Parallelogram. The above method requires so many construction lines and so much computation that it is customary, when the length of the diagonal is required, to use the following rule which is very easy to remember.

The square of the diagonal of a parallelogram equals the sum of the squares of the two sides plus twice the product of the two sides times the cosine of the angle between the two sides.

The sides referred to are always those two which come together at the point where the diagonal starts.

Thus in Fig. 16a, the square of the diagonal, $\mathbf{a}^2$, equals the sum of the squares of the two sides ($\mathbf{b}^2 + \mathbf{c}^2$) plus twice the product of these two sides times the cosine of the angle between them ($2\,\mathbf{bc}$ cos 60°).

Note that the sides **b** and **c** come together at the point where the diagonal **a** starts, and that it is the angle 60° lying between **b** and **c**, which is used in the equation.

We use this equation to solve for **a** as follows:

$$a^2 = b^2 + c^2 + 2\,bc \cos 60°$$
$$= 20^2 + 50^2 + 2 \times 20 \times 50 \times 0.500$$
$$= 400 + 2500 + 1000$$
$$= 3900,$$
$$a = \sqrt{3900}$$
$$= 62.4.$$

The diagonal is thus found much more quickly and accurately by this rule than by completing right triangles and solving them.

Problem 45a. One side of a parallelogram represents a force of 900 pounds; the adjacent side, a force of 650 pounds. The angle between the two sides is 40°. What force does the diagonal drawn from their point of intersection represent?

Problem 46a. In Fig. 190, find the value of Z_2 when $Z = 8$ ohms and $Z_1 = 10$ ohms.

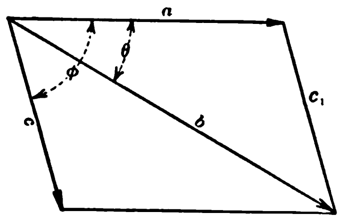


FIG. 17a. Find the value of c when a, b and ϕ are known.

Problem 47a. The diagonal of a parallelogram represents a voltage of 360 volts. One side represents 100 volts. What does the other side represent if the angle between the two sides is 35°? The diagonal is drawn from the point of intersection of the two sides mentioned.

Problem 48a. Find the value of c in Fig. 17a, if $a = 10$, $b = 15$ and $\phi = 60°$.

Problem 49a. What value must b have in Fig. 16a in order that a may have the value 75, c and θ having the original values?

Problem 50a. Two forces, one of 240 pounds, the other of 160 pounds, make an angle of 50° to each other. Construct a parallelogram and find the length of the diagonal which represents the resultant of the two forces.

Problem 51a. In the parallelogram of Fig. 17a, $a = 90$, $b = 125$, $\phi = 75°$. Find the side c.

Problem 52a. What is the value of OR in Fig. 83?

Problem 53a. Find the value of OR in Fig. 84.

Problem 54a. Find the angle which OR of Fig. 84 makes with OA.

Problem 55a. Find the value for OR_1 of Fig. 84, using results obtained in Prob. 53a and 54a.

Problem 56a. In Fig. 91c, $A = 400$ volts, $B = 600$ volts. What is the value of R_1?

Problem 57a. Find the value of I in Fig. 157.

8a. The Oblique Triangle. Relation between Sides and Angles. The Cosine Law. It will be noted from Fig. 17a that the parallelogram is divided into two triangles by the diagonal b. The sides of one triangle are a, b and c. The angle θ is between the sides a and b.

Considering these lines as a part of a parallelogram, we have seen that the following equation holds:

$$b^2 = a^2 + c^2 + 2\,ac \cos \theta.$$

If, now, we consider these lines as sides of a triangle, we find that the following very similar equation can be used:

$$c_1^2 = a^2 + b^2 - 2\,ab \cos \theta.$$

In order to distinguish this equation from that for the parallelogram, it is merely necessary to note that the sign of the term $2\,ab \cos \theta$ has been changed to a minus, and that the angle θ is always the angle opposite that side which stands alone on the other side of the equation. Thus the value c_1^2 stands alone on one side of the equation and therefore the angle θ is used.

The most common use of this rule is for finding the angles of a triangle of which the three sides but no angles are known.

Suppose, in the triangle abc_1, of Fig. 17a, that $a = 80$, $b = 120$ and $c_1 = 60$. The value of the angle θ can be found as follows:

$$\begin{aligned}
60^2 &= 80^2 + 120^2 - 2 \times 80 \times 120 \cos \theta.\\
19{,}200 \cos \theta &= 20{,}800 - 3600\\
\cos \theta &= \frac{17{,}200}{19{,}200}\\
&= 0.896\\
\theta &= 26^\circ\ 20'.
\end{aligned}$$

The angle opposite the side a can be found in the same way.

Let x = angle opposite side a.

Then

$$\begin{aligned}
a^2 &= b^2 + c_1^2 - 2\,bc \cos x\\
80^2 &= 120^2 + 60^2 - 2 \times 120 \times 60 \cos x\\
\cos x &= \frac{120^2 + 60^2 - 80^2}{2 \times 120 \times 60}\\
&= .806\\
x &= 36^\circ\ 20'.
\end{aligned}$$

$$\begin{aligned}
\text{The remaining angle} &= 180 - \theta - x\\
&= 180 - 26^\circ\ 20' - 36^\circ\ 20'\\
&= 117^\circ\ 20'.
\end{aligned}$$

In any triangle any two sides have the same relation to each other as the sines of the angles opposite the sides.

This rule is called the **Law of Sines** and enables us to solve quickly diagrams in which we know two angles and one side of a triangle, as in Fig. 18a, where two angles of 60° and 50° are given and one side, **a** = 24.

The other angle must equal 70°, in order to have the sum of the angles add to 180°.

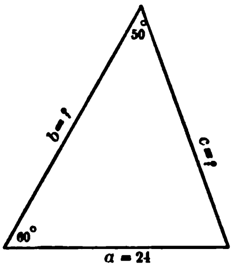


FIG. 18a. The fraction $\frac{a}{c}$ equals the fraction $\frac{\sin 50°}{\sin 60°}$.

Using the Law of Sines, we have

$$\frac{c}{a} = \frac{\text{sine of angle opposite } c}{\text{sine of angle opposite } a}$$

$$= \frac{\sin 60°}{\sin 50°}$$

$$= \frac{0.866}{0.766}.$$

Thus

$$\frac{c}{a} = 1.13.$$

That is,

$$c = 1.13\,a$$
$$= 1.13 \times 24$$
$$= 27.1.$$

Similarly

$$\frac{b}{a} = \frac{\text{sine of angle opposite } b}{\text{sine of angle opposite } a}$$

$$= \frac{\sin 70°}{\sin 50°}$$

$$= \frac{0.940}{0.866}.$$

Thus

$$\frac{b}{a} = 1.23$$

$$b = 1.23\,a$$
$$= 1.23 \times 24$$
$$= 29.5.$$

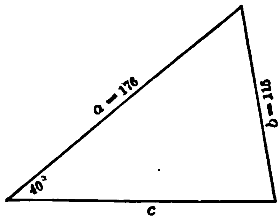


FIG. 19a. An oblique triangle.

Problem 58a. In Fig. 10a, find the value of S and T by the law of sines.

Problem 59a. Find the value of the side c and the other two angles in Fig. 19a.

Problem 60a. In Fig. 20a find the length of the sides **a** and **b**.

Problem 61a. Find the value of the angle between R_1 and A in Fig. 91c. $A = 42$, $B = 61$.

Problem 62a. What is the value of the angle between OR and OA in Fig. 78?

Problem 63a. Compute the value of the angle between I_t and OR_1 in Fig. 139a. $OR_1 = 42.2$.

9a. The Sine of Angles Greater than 90°. In alternating-current computation, we often use such expressions as sin 120°, sin 330°, etc., yet it will be noted that the trigonometric tables do not give the sine, cosine, or tangent for angles greater than 90°.

The reason is that for angles beyond 90° the tables would merely have to repeat the values already given. Thus sin 91° is the same as sin 89°, sin 92° = sin 88°, etc.

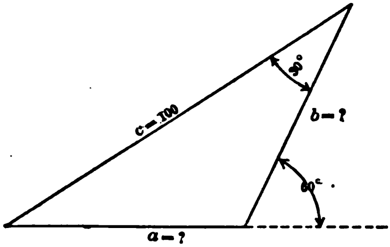


FIG. 20a. To find the value of the sides a and b in an oblique triangle.

When looking up a function of an angle greater than 90°, it is necessary, then, to know what angle in the tables has the same value for this function as the one with which we are dealing. This can best be determined as follows:

Suppose it is desired to find the sine of the angle 120°.

Instead of constructing a triangle, erect the axes xx' and yy', Fig. 21a. Draw OR at an angle of 120° to OX. Drop a perpendicular RV from R to xx'. Then $\frac{RV}{RO}$ is the sine of 120°.

But we see that $\frac{RV}{RO}$ must also be the sine of the angle θ. Since $\theta + 120° = 2$ right angles $= 180°$, then $\theta = 180° - 120° = 60°$.

Thus

$$\sin 120° = \sin (180° - 120°) = \sin 60° = 0.866.$$

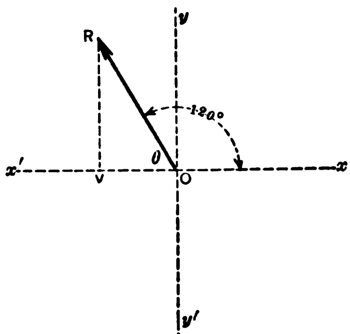


FIG. 21a. The fraction $\frac{RV}{RO} = \sin 120° = \sin \theta° = \sin (180° - 120°) = \sin 60°$.

In the same way the sine of all angles between 90° and 180° can be found. Thus:

$$\sin 92° = \sin (180° - 92°) = \sin 88°,$$
$$\sin 150° = \sin (180° - 150°) = \sin 30°,$$
$$\sin 165° = \sin (180° - 165°) = \sin 15°,$$

etc.

The rule is

The sine of any angle between 90° and 180° equals the sine of an angle whose value is 180° minus the given angle.

It is best not to memorize any rule, but in each case to draw a rough diagram and note the above relation.

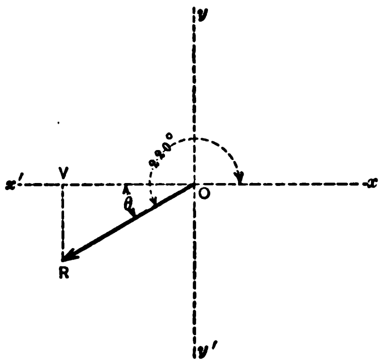


FIG. 22a. The fraction $\frac{VR}{OR} = \sin 220° = -\sin \theta° = -\sin (220-180°) = -\sin 40°$.

Suppose it is required to find the sine of an angle between 180° and 270°, say 220°. Construct the axes xx' and yy' of Fig. 22a, and draw OR at an angle of 220° to Ox. Note that OR is always drawn at the given angle to Ox. Draw RV perpendicular to xx'.

Now $$\frac{VR}{OR} = \sin 220°.$$

But, also, $$\frac{VR}{OR} = \sin \theta,$$

$$\theta = 220° - 180° = 40°.$$

Therefore
$$\sin 220° = \sin (220° - 180°)$$
$$= \sin 40°,$$
$$\sin 40° = 0.643.$$

But note that in this case VR is below the line xx'. Whenever this occurs, the line VR is said to be negative. Thus the fraction $\frac{VR}{OR}$ must have a minus sign before it and

$$\sin 220° = -\sin 40°$$
$$= -0.643.$$

The sine of any angle between 180° and 270° must have the minus sign, and equals the sine of an angle whose value is 180° less than the given angle.

Thus $\sin 185° = -\sin(185° - 180°) = -\sin 5°,$
$\sin 260° = -\sin(240° - 180°) = -\sin 60°.$

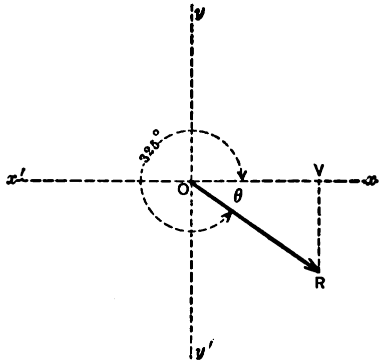


FIG. 23a. The fraction $\frac{VR}{OR} = \sin 325° = -\sin\theta° = -\sin(360° - 325°) = -\sin 35°.$

Similarly, the sine of any angle between 270° and 360° can be found by drawing the axes xx' and yy' of Fig. 23a and putting in the line OR at the given angle, say 325°, to the line Ox.

$$\frac{VR}{OR} = \sin 325°.$$

But
$$\frac{VR}{OR} = \sin\theta,$$
$$\theta = 360° - 325° = 35°.$$

Therefore $\sin 325° = \sin 35°$. But VR is below the line and therefore is negative, thus $\frac{VR}{OR}$, or $\sin 35°$, must be negative.

Thus
$$\sin 325° = -\sin(360° - 325°)$$
$$= -\sin 35°$$
$$= -0.574.$$

Similarly $\sin 300° = -\sin(360° - 300°) = -\sin 60,$
$\sin 350° = -\sin(360° - 350°) = -\sin 10°,$
etc.

The sine of any angle can be found by the above method. Care must always be taken to note whether the line RV is plus or minus; that is, whether it is above or below the line xx'.

10a. Cosine of Angles Greater than 90°. Suppose it is required to find the cosine of any angle between 90° and 180°, say 120°. Draw Fig. 21a again.

$$\frac{OV}{OR} = \cos 120°.$$

But, also,
$$\frac{OV}{OR} = \cos \theta°.$$

θ has been found to be equal to $180° - 120° = 60°$.

Thus
$$\cos 120° = (\cos 180° - 120°) = \cos 60° = 0.500.$$

But when the line OV extends to the left of the axis yy' it has the minus sign. Thus the fraction $\frac{OV}{OR}$ must be negative.

Therefore
$$\begin{aligned}\cos 120° &= -\cos (180° - 120°)\\ &= -\cos 60°\\ &= -0.500.\end{aligned}$$

Similarly
$$\begin{aligned}\cos 92° &= -\cos (180° - 92°) = -\cos 88°,\\ \cos 175° &= -\cos (180° - 175°) = -\cos 5°.\end{aligned}$$

In Fig. 22a it will be noted that the line OV is still to the left of yy', so that the cosine of any angle between 270° and 180° is also negative. But in Fig. 23a the line OV extends to the right of yy', and thus the cosine of any angle between 270° and 360° is positive.

Note that in finding the cosine of any angle greater than 90°, we use the same figure which is drawn for finding the sine and the same rules apply. The only difference is in the sign of the function.

11a. Tangent of Angles Greater than 90°. To find the tangent of any angle between 90° and 180°, say 120°, construct Fig. 21a. Note that OV is minus.

$$\tan 120° = \frac{VR}{-VO}.$$

But, also, $\frac{VR}{-VO} = -\tan \theta = -\tan (180° - 120°) = -\tan 60°.$

Thus
$$\tan 120° = -\tan 60°.$$

Similarly
$$\begin{aligned}\tan 98° &= -\tan (180° - 98°) = -\tan 82°,\\ \tan 170° &= -\tan (180° - 170°) = -\tan 10°,\end{aligned}$$
etc.

An interesting fact must be noticed in the case of the tangent of angles between 180° and 270°. By Fig. 22a it can be seen that $\tan 220° = \frac{VR}{OV}$, and that both VR and OV are negative.

This makes the fraction positive, since it is the quotient of a negative by a negative. The tangent of all angles between 180° and 270° must be positive.

Thus
$$\tan 220° = \tan (220° - 180°) = \tan 40°,$$
$$\tan 265° = \tan (265° - 180°) = \tan 85°,$$
etc.

By Fig. 23a it will be seen that in the fraction $\frac{VR}{OV}$, the tangent of any angle between 270° and 360°, OV has become positive while VR remains negative. Thus the fraction $\frac{OR}{OV}$ is negative and the tangent of all these angles must be negative.

Thus
$$\tan 325° = -\tan (360° - 325°) = -\tan 35°,$$
$$\tan 275° = -\tan (360° - 275°) = -\tan 85°.$$

To find the sine, cosine, or tangent of any angle greater than 90°, proceed as above, each time drawing the axes and the line OR, and noting whether either or both of the lines OV or RV are negative. Look up in the tables the function of that angle which the figure shows to have an equal function.

For angles greater than 360°, subtract 360° before proceeding.

Problem 64a. Find the cosine, sine, and tangent of 155°.

Problem 65a. Reduce the following to functions of acute angle.

(a) Sin 164°.
(b) Cos 195°.
(c) Tan 173°.

Problem 66a. What is the value of tan 310°?

Problem 67a. Find the value of sin 815°.

Problem 68a. Of what angle greater than 180° is 0.492 the sine?

Problem 69a. What are the angles less than 360° the cosine of which is 0.500?

Problem 70a. Find the value of e in the equation $e = 400 \sin 260°$.

Problem 71a. Find the value of E in the equation
$$-42 = E \cos 130°.$$

Problem 72a. Find the value of θ, greater than 90°, which fits the equation
$$96 = 400 \cos \theta.$$

Problem 73a. What is the value of the angle θ in Fig. 17a, if the values of Problem 51a are applied to the figure?

Problem 74a. Find the value of e in Fig. 177.

Summary of the Trigonometry Needed to Solve Alternating-current Problems.

(1) In any triangle the sum of all the angles equals 180°.

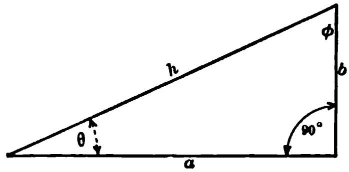


FIG. 24a. A right triangle.

(2) In a right triangle, Fig. 24a.

(a) $h^2 = a^2 + b^2$.

(b) $\text{Sin}\,\theta = \frac{b}{h}, \quad \cos\theta = \frac{a}{h}, \quad \tan\theta = \frac{b}{a}.$

(c) $\text{Sin}\,\phi = \frac{a}{h}, \quad \cos\phi = \frac{b}{h}, \quad \tan\phi = \frac{a}{b}.$

(3) The sides of any triangle hold the same relation to one another as the sines of the angles opposite them.

(4) In any parallelogram, the square of the diagonal equals the sum of the squares of the two sides, which meet where the diagonal starts, plus twice the product of these two sides times the cosine of the angle between them.

$$a^2 = b^2 + c^2 + 2\,bc\cos\theta,$$

a = diagonal.

$\left.\begin{matrix} b \\ \& \\ c \end{matrix}\right\}$ = sides which meet where diagonal a starts.

θ = angle between b and c.

(5) Oblique triangles and parallelograms can be solved, without the use of rules (3) and (4), by dividing them into right triangles and using rules (1), (2) and (3).

(6) The sine of any angle is equal to the sine of some angle less than 90°. This angle can be found by drawing diagrams like Fig. 21a, 22a and 23a. Some of the sines may be negative.

(7) The statements in (6) are true of the cosine and tangent of any angle.

Problem 75a. If the two forces in Problem 45a are at an angle of 150°, what would the resultant be?

Problem 76a. In the parallelogram of Fig. 25a,

$$a = 800,\ b = 600,\ \phi = 132°.$$

Find:

(*a*) The line *c*.
(*b*) The angle θ.

Problem 77a. In the parallelogram of Fig. 17a,

$$a = 900,$$
$$c = 400,$$
$$\phi = 70°.$$

Find the value of *b*.

Problem 78a. Find the value of θ in Prob. 77a.

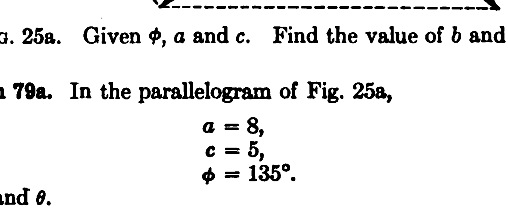


FIG. 25a. Given ϕ, *a* and *c*. Find the value of *b* and θ.

Problem 79a. In the parallelogram of Fig. 25a,

$$a = 8,$$
$$c = 5,$$
$$\phi = 135°.$$

Find *b* and θ.

Problem 80a. One side of an oblique triangle is 12 inches long, the adjacent side is 7 inches long. The angle between sides is 30°. How long is the third side?

Problem 81a. If the angle between the two sides given in Prob. 80a were 120°, how long would the third side be?

Problem 82a. A triangle is composed of sides, 18, 29, and 37 inches long. What are the angles of this triangle?

Problem 83a. Explain how you would determine, by means of a steel tape only, whether the corner of a certain lot is an exact right angle.

Problem 84a. How long must a guy wire be, if attached to a 30-ft. pole at a point 15 ft. from the ground in order to attach to an anchor in level ground at a distance of 15 ft. from the foot of the pole.

Problem 85a. A guy wire attached to one pole at a point 20 ft. from the ground is attached to the next pole at a point 6 ft. from the ground. The poles are 100 feet apart and set in level ground. How long is the guy wire, neglecting sag?

Problem 86a. How long are the sides of an A-shaped transmission line tower, if the top or apex is 30 feet above ground, and the feet are 10 feet apart at the ground? What are the angles at the apex, and between each side and ground?

Problem 87a. What is the length of the longest straight piece of material or bar which can be put on a flat-car whose dimensions are 40 ft. × 8 ft.

Problem 83a. What is the horizontal distance between two vertical transmission line towers which are 500 ft. apart as measured along the wire, which slopes to an angle of 20° to the horizontal and has no sag?

APPENDIX B

TABLE I

NATURAL SINES, COSINES, TANGENTS AND COTANGENTS

0° to 3°

	sin	cos	tan	cot	
0	.000000	1.00000	.00000	∞	90°
5′	1454	1.00000	145	687.55	55′
10′	2909	1.00000	291	343.77	50′
15′	4363	0.99999	436	229.18	45′
20′	5818	998	582	171.89	40′
25′	7272	997	727	137.51	35′
30′	.008727	996	.00873	114.59	30′
35′	.010181	995	.01018	98.218	25′
40′	1635	993	164	85.940	20′
45′	3090	991	309	76.390	15′
50′	4544	989	455	68.750	10′
55′	5998	987	600	62.499	5′
1°	.017452	.99985	.01746	57.290	89°
5′	.01891	982	.01891	52.882	55′
10′	.02036	979	.02036	49.104	50′
15′	181	976	182	45.829	45′
20′	327	973	328	42.964	40′
25′	472	969	473	40.436	35′
30′	618	966	619	38.188	30′
35′	763	962	764	36.178	25′
40′	.02908	958	.02910	34.368	20′
45′	.03054	953	.03055	32.730	15′
50′	199	949	201	31.242	10′
55′	345	944	346	29.882	5′
2°	.03490	.99939	.03492	28.636	88°
5′	635	934	638	27.490	55′
10′	781	929	783	26.432	50′
15′	.03926	923	.03929	25.452	45′
20′	.04071	917	.04075	24.542	40′
25′	217	911	220	23.695	35′
30′	362	905	366	22.904	30′
35′	507	898	512	22.164	25′
40′	653	892	658	21.470	20′
45′	798	885	803	20.819	15′
50′	.04943	878	949	20.206	10′
55′	.05088	870	.05095	19.627	5′
3°	.05234	.99863	.05241	19.081	87°
10′	524	847	533	18.075	50′
20′	.05814	831	.05824	17.169	40′
30′	.06105	813	.06116	16.350	30′
40′	395	795	408	15.605	20′
50′	685	776	700	14.924	10′
	cos	sin	cot	tan	

87° to 90°

4° to 10°

	sin	cos	tan	cot	
4°	.06976	.99756	.06993	14.301	86°
10′	.07266	736	.07285	13.727	50′
20′	556	714	578	13.197	40′
30′	.07846	692	.07870	12.706	30′
40′	.08136	668	.08163	12.251	20′
50′	426	644	456	11.826	10′
5°	.08716	.99619	.08749	11.430	85°
10′	.09005	.99594	.09042	11.059	50′
20′	295	567	335	10.712	40′
30′	.0585	540	629	.385	30′
40′	.9874	511	.09923	10.078	20′
50′	.10164	482	.10216	9.7882	10′
6°	.10453	.99452	.10510	9.5144	84°
10′	.10742	421	.10805	.2553	50′
20′	.11031	390	.11099	9.0098	40′
30′	320	357	394	8.7769	30′
40′	609	324	688	.5555	20′
50′	.11898	290	.11983	.3450	10′
7°	.12187	.99255	.12278	8.1443	83°
10′	476	219	574	7.9530	50′
20′	.12764	182	.12869	.7704	40′
30′	.13053	144	.13165	.5958	30′
40′	341	106	461	.4287	20′
50′	629	067	.13758	.2687	10′
8°	.13917	.99027	.14054	7.1154	82°
10′	.14205	.98986	351	6.9682	50′
20′	493	944	648	.8269	40′
30′	.14781	902	.14945	.6912	30′
40′	.15069	858	.15243	.5606	20′
50′	356	814	540	.4348	10′
9°	.15643	.98769	.15838	6.3138	81°
10′	.15931	723	.16137	.1970	50′
20′	.16218	676	435	6.0844	40′
30′	505	629	.16734	5.9758	30′
40′	.16792	580	.17033	.8708	20′
50′	.17078	531	333	.7694	10′
10°	.17365	.98481	.17633	5.6713	80°
10′	651	430	.17933	.5764	50′
20′	.17937	378	.18233	.4845	40′
30′	.18224	325	534	.3955	30′
40′	509	272	.18835	.3093	20′
50′	.18795	218	.19136	.2257	10′
	cos	sin	cot	tan	

80° to 86°

11° to 20° | 21° to 30°

	sin	cos	tan	cot			sin	cos	tan	cot	
11°	.19081	.98163	.19438	5.1446	79°	21°	.35837	.93358	.38386	2.6051	69°
10′	366	107	.19740	5.0658	50′	10′	.36108	253	.38721	.5826	50′
20′	652	.98050	.20042	4.9894	40′	20′	379	148	.39055	.5605	40′
30′	.19937	.97992	345	.9152	30′	30′	650	.93042	391	.5386	30′
40′	.20222	934	648	.8430	20′	40′	.36921	.92935	.39727	.5172	20′
50′	507	875	.20952	.7729	10′	50′	.37191	827	.40065	.4960	10′
12°	.20791	.97815	.21256	4.7046	78°	22°	.37461	.92718	.40403	2.4751	68°
10′	.21076	754	560	.6382	50′	10′	730	609	.40741	.4545	50′
20′	360	692	.21864	.5736	40′	20′	.37999	499	.41081	.4342	40′
30′	644	630	.22169	.5107	30′	30′	.38268	388	421	.4142	30′
40′	.21928	566	475	.4494	20′	40′	537	276	.41763	.3945	20′
50′	.22212	502	.22781	.3897	10′	50′	.38805	164	.42105	.3750	10′
13°	.22495	.97437	.23087	4.3315	77°	23°	.39073	.92050	.42447	2.3559	67°
10′	.22778	371	393	.2747	50′	10′	341	.91936	.42791	.3369	50′
20′	.23062	304	.23700	.2193	40′	20′	608	822	.43136	.3183	40′
30′	345	237	.24008	.1653	30′	30′	.39875	706	481	.2998	30′
40′	627	169	316	.1126	20′	40′	.40141	590	.43828	.2817	20′
50′	.23910	100	624	.0611	10′	50′	408	472	.44175	.2637	10′
14°	.24192	.97030	.24933	4.0108	76°	24°	.40674	.91355	.44523	2.2460	66°
10′	474	.96959	.25242	3.9617	50′	10′	.40939	236	.44872	.2286	50′
20′	.24756	887	552	.9136	40′	20′	.41204	.91116	.45222	.2113	40′
30′	.25038	815	.25862	.8667	30′	30′	469	.90996	573	.1943	30′
40′	320	742	.26172	.8208	20′	40′	734	875	.45924	.1775	20′
50′	601	667	483	.7760	10′	50′	.41998	753	.46277	.1609	10′
15°	.25882	.96593	.26795	3.7321	75°	25°	.42262	.90631	.46631	2.1445	65°
10′	.26163	517	.27107	.6891	50′	10′	525	507	.46985	.1283	50′
20′	443	440	419	.6470	40′	20′	.42788	383	.47341	.1123	40′
30′	.26724	363	.27732	.6059	30′	30′	.43051	259	.47698	2.0965	30′
40′	.27004	285	.28046	.5656	20′	40′	313	133	.48055	809	20′
50′	284	206	360	.5261	10′	50′	575	.90007	414	655	10′
16°	.27564	.96126	.28675	3.4874	74°	26°	.43837	.89879	.48773	2.0503	64°
10′	.27843	.96046	.28990	.4475	50′	10′	.44098	752	.49134	353	50′
20′	.28123	.95964	.29305	.4124	40′	20′	359	623	495	204	40′
30′	402	882	621	.3759	30′	30′	620	493	.49858	2.0057	30′
40′	680	799	.29938	.3402	20′	40′	.44880	363	.50222	1.9912	20′
50′	.28959	715	.30255	.3052	10′	50′	.45140	232	587	768	10′
17°	.29237	.95630	.30573	3.2709	73°	27°	.45399	.89101	.50953	1.9626	63°
10′	515	545	.30891	.2371	50′	10′	658	.88968	.51319	486	50′
20′	.29793	459	.31210	.2041	40′	20′	.45917	835	.51688	347	40′
30′	.30071	372	530	.1716	30′	30′	.46175	701	.52057	210	30′
40′	348	284	.31850	.1397	20′	40′	433	566	427	.9074	20′
50′	625	195	.32171	.1084	10′	50′	690	431	.52798	1.8940	10′
18°	.30902	.95106	.32492	3.0777	72°	28°	.46947	.88295	.53171	1.8807	62°
10′	.31178	.95015	.32814	.0475	50′	10′	.47204	158	545	676	50′
20′	454	.94924	.33136	3.0178	40′	20′	460	.88020	.53920	546	40′
30′	.31730	832	460	2.9887	30′	30′	716	.87882	.54296	418	30′
40′	.32006	740	.33783	.9600	20′	40′	.47971	743	.54673	291	20′
50′	282	646	.34108	.9319	10′	50′	.48226	603	.55051	165	10′
19°	.32557	.94552	.34433	2.9042	71°	29°	.48481	.87462	.55431	1.8040	61°
10′	.32832	457	.34758	.8770	50′	10′	735	321	.55812	1.7917	50′
20′	.33106	361	.35085	.8502	40′	20′	.48989	178	.56194	796	40′
30′	381	264	412	.8239	30′	30′	.49242	.87036	577	675	30′
40′	655	167	.35740	.7980	20′	40′	495	.86892	.56962	556	20′
50′	.33929	.94068	.36068	.7725	10′	50′	.49748	748	.57348	437	10′
20°	.34202	.93969	.36397	2.7475	70°	30°	.50000	.86603	.57735	1.7321	60°
10′	475	869	.36727	.7228	50′	10′	252	457	.58124	205	50′
20′	.34748	769	.37057	.6985	40′	20′	503	310	513	.7090	40′
30′	.35021	667	388	.6746	30′	30′	.57054	163	.58905	1.6977	30′
40′	293	565	.37720	.6511	20′	40′	.51004	.86015	.59297	864	20′
50′	565	462	.38053	.6279	10′	50′	254	.85866	.59691	753	10′
	cos	sin	cot	tan			cos	sin	cot	tan	

70° to 79° | 60° to 69°

31° to 37°

	sin	cos	tan	cot	
31°	.51504	.85717	.60086	1 6643	59°
10′	.51763	567	483	534	50′
20′	.52002	416	.60881	426	40′
30′	250	264	.61280	319	30′
40′	498	.85112	.61681	212	20′
50′	745	.84959	.62083	107	10′
32°	.52992	.84805	.62487	1.6003	58°
10′	.53238	650	.62892	1.5900	50′
20′	484	495	.63299	798	40′
30′	730	339	.63707	697	30′
40′	.53975	182	.64117	597	20′
50′	.54220	.84025	528	497	10′
33°	.54464	.83867	.64941	1.5399	57°
10′	708	708	.65355	301	50′
20′	.54951	549	.65771	204	40′
30′	.55194	389	.66189	108	30′
40′	436	228	.66608	.5013	20′
50′	678	.83066	.67028	1.4919	10′
34°	.55919	.82904	.67451	1.4826	56°
10′	.56160	741	.67875	733	50′
20′	401	577	.68301	641	40′
30′	641	413	.68728	550	30′
40′	.56880	248	.69157	460	20′
50′	.57119	.82082	.69588	370	10′
35°	.57358	.81915	.70021	1.4281	55°
10′	596	748	455	193	50′
20′	.57833	580	.70891	106	40′
30′	.58070	412	.71329	.4019	30′
40′	307	242	.71769	1.3934	20′
50′	543	.81072	.72211	848	10′
36°	.58779	.80902	.72654	1.3764	54°
10′	.59014	730	.73100	680	50′
20′	248	558	547	597	40′
30′	482	386	.73996	514	30′
40′	716	212	.74447	432	20′
50′	.59949	.80038	.74900	351	10′
37°	.60182	.79864	.75355	1.3270	53°
10′	414	688	.75812	190	50′
20′	645	512	.76272	111	40′
30′	.60876	335	.76733	.3032	30′
40′	.61107	.79158	.77196	1.2954	20′
50′	337	.78980	.77661	876	10′
	cos	sin	cot	tan	

53° to 59°

38° to 45°

	sin	cos	tan	cot	
38°	.61566	.78801	.78129	1.2799	52°
10′	.61795	622	.78598	723	50′
20′	.62024	442	.79070	647	40′
30′	251	261	.79544	572	30′
40′	479	.78079	.80020	497	20′
50′	706	.77897	498	423	10′
39°	.62932	.77715	.80978	1.2349	51°
10′	.63158	531	.81461	276	50′
20′	383	347	.81946	203	40′
30′	608	.77162	.82434	131	30′
40′	.63832	.76977	.82923	.2059	20′
50′	.64056	791	.83415	1.1988	10′
40°	.64279	.76604	.83910	1.1918	50°
10′	501	417	.84407	847	50′
20′	723	229	.84906	778	40′
30′	.64945	.76041	.85408	708	30′
40′	.65166	.75851	.85912	640	20′
50′	386	661	.86419	571	10′
41°	.65606	.75471	.86929	1.1504	49°
10′	.65825	280	.87441	436	50′
20′	.66044	.75088	.87955	369	40′
30′	262	.74896	.88473	303	30′
40′	480	703	.88992	237	20′
50′	697	509	.89515	171	10′
42°	.66913	.74314	.90040	1.1106	48°
10′	.67129	.74120	.90569	1.1041	50′
20′	344	.73924	.91099	1.0977	40′
30′	559	728	.91633	913	30′
40′	773	531	.92170	850	20′
50′	.67987	333	.92709	786	10′
43°	.68200	.73135	.93252	1.0724	47°
10′	412	.72937	.93707	661	50′
20′	624	737	.94345	599	40′
30′	.68835	537	.94896	538	30′
40′	.69046	337	.95451	477	20′
50′	256	.72136	.96008	416	10′
44°	.69466	.71934	.96569	1.0355	46°
10′	675	732	.97133	295	50′
20′	.69883	529	.97700	235	40′
30′	.70091	325	.98270	176	30′
40′	298	.71121	.98843	117	20′
50′	505	.70916	.99420	058	10′
45°	.70711	.70711	1.00000	1.0000	45°
	cos	sin	cot	tan	

45° to 52°

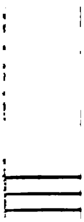
Table II

PROPERTIES OF ANNEALED COPPER WIRE

B. & S. gage.	Area in circ. mils. d^2	Diameter in mils. d		Number of strands in cable.	Resistance per mile at 20° C. or 68°F. (approx.).	Weight per mile in pounds (approx.).	
		Solid.	Stranded.			Solid.	Stranded.
14	4,107	64.05	73	7	13.3	65.6	70
12	6,530	80.81	92	7	8.40	104	108
10	10,380	101.9	116	7	5.27	166	172
8	16,510	128.5	146	7	3.31	264	269
6	26,250	162.0	184	7	2.08	420	428
5	33,100	181.9	206	7	1.65	530	544
4	41,740	204.3	232	7	1.31	667	682
3	52,630	229.4	260	7	1.04	841	866
2	66,370	257.6	292	7	0.824	1062	1,087
1	83,690	289.3	333	19	0.656	1337	1,368
0	105,540	324.9	373	19	0.518	1687	1,730
00	133,080	364.8	418	19	0.412	2127	2,190
000	167,810	409.6	470	19	0.328	2682	2,740
0000	211,600	460.0	528	19	0.259	3381	3,470
250,000		500.0	575	37	0.217	....	4,090
300,000		547.7	630	37	0.185	....	4,980
350,000		591.6	681	37	0.159	....	5,740
400,000		632.5	728	37	0.137	....	6,570
450,000		670.8	772	37	0.122	....	7,470
500,000		707.1	814	37	0.111	....	8,210
550,000		741.6	855	63	0.100	....	9,020
600,000		774.6	893	63	0.0898	....	9,850
650,000		806.2	929	63	0.0845	....	10,660
700,000		836.7	964	63	0.0793	....	11,480
750,000		866.0	998	61	0.0739	....	12,320
800,000		894.4	1031	61	0.0687	....	13,130
900,000		948.7	1093	61	0.0633	....	14,850
1,000,000		1000	1152	61	0.0528	....	16,420
1,250,000		1118	1289	91	0.0438	....	20,300
1,500,000		1225	1412	91	0.0304	....	24,600
1,750,000		1323	1526	127	0.0311	....	28,700
2,000,000		1414	1631	127	0.0275	....	32,800

Table III

1 cu. ft. of water weighs 62.5 lb.
1 horse power = 33,000 ft. lb. per minute.
1 kilowatt = 1.34 horse power.
1 B.t.u. = 780 ft. lb.
1 cu. in. of copper weighs 0.3195 lb.
1 cu. in. of aluminum weighs 0.0963 lb.
1 cu. ft. of $\begin{Bmatrix}\text{solid}\\\text{broken}\end{Bmatrix}$ anthracite coal weighs approx. $\begin{cases}100\text{ lb.}\\60\text{ lb.}\end{cases}$
1 cu. ft. of $\begin{Bmatrix}\text{solid}\\\text{broken}\end{Bmatrix}$ bituminous coal weighs approx. $\begin{cases}84\text{ lb.}\\52\text{ lb.}\end{cases}$
1 barrel of crude oil = 41 gal. = 310 lb.
1 ton = 2000 lb.



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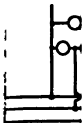

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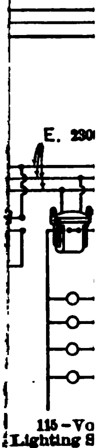

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