CONTINUOUS AND ALTERNATING CURRENT MACHINERY PROBLEMS

THE WILEY TECHNICAL SERIES

FOR

VOCATIONAL AND INDUSTRIAL SCHOOLS

EDITED BY

J. M. JAMESON

GIRARD COLLEGE

CONTINUOUS AND ALTERNATING CURRENT MACHINERY PROBLEMS

ELEMENTARY PROBLEMS FOR USE IN TECHNICAL SCHOOLS

BY

WILLIAM T. RYAN

Assistant Professor of Electrical Engineering, University of Minnesota.
Member American Institute of Electrical Engineers.

FIRST EDITION

FIRST THOUSAND

PREFACE

THIS little book of problems was prepared to be used in conjunction with Morecroft's elementary text-book on "Continuous and Alternating Current Machinery" as texts for a short course given by the author to civil and mining engineers. It is well understood that the average student is apt to find general principles and mathematical formulæ somewhat vague and uninviting, unless his conception is made somewhat easier by the way of concrete examples and laboratory work. The author believes that Morecroft's text-book, Clewell's Laboratory Manual and this little book of problems make a very effective combination for administering a short elementary course to advanced students in non-electrical engineering courses.

The Problems will also be found suitable for use with any of the available text-books or with a lecture course.

In general the problems are very closely related to actual engineering practice and are so made up as not to be mere mathematical puzzles.

W. T. RYAN.

MINNEAPOLIS, MINN.
January 1, 1915.

CONTENTS

CONTINUOUS AND ALTERNATING CURRENT MACHINERY PROBLEMS

CHAPTER I

ELEMENTARY LAWS OF CONTINUOUS CURRENTS

1. THE armature of a street railway motor has a resistance of 0.0625 ohm at a temperature of 70° F. After operating for several hours the resistance is found to be 0.0775 ohm. What has been the rise of temperature of the armature.

2. The resistance of the filament of a tantalum incandescent lamp at 70° F. is 50 ohms. When operating under normal conditions the temperature is 2025° C., and 115 volts sends a current of 0.39 ampere through the filament. What is the temperature coefficient of the filament?

3. By referring to the areas in circular mils of No. 4 and No. 10 wires as given in the wire table, determine the resistance of 1000 ft. at 20° C. and at 80° C. Then check your results with the wire table. Also determine the resistance by the approximate method and find what the percentage error would be at 20° C.

4. How many circular mils in a wire $\frac{1}{4}$ in. by $\frac{1}{2}$ in.?

5. Ten 530-ohm 25-watt, and five 330-ohm 40-watt Mazda lamps are all in parallel. Resistance of the circuit between the lamps and generator is 0.24 ohm. Find the resistance of the group of lamps and the current taken by them if the voltage at the generator terminals is 116.

6. Approximately 18,000,000 cu.ft. of water per minute falls 200 ft. at Niagara Falls. (*a*) Assuming water turbines available whose efficiency is 86 per cent, what horsepower is available? (*b*) Assuming that all this energy is used up in heating the water, how much warmer is the water below the falls than above?

Note. The distinguished English Scientist, Joule, after whom the practical unit of electrical energy is named, made elaborate experiments to determine exactly what relation existed between the units of heat and work. The most generally accepted experimental results give 778 ft.-lbs. of work as being exactly equivalent to the amount of heat required to raise the temperature of 1 lb. of water 1° F. at or near 39° F., the temperature of its maximum density. This amount of heat is called 1 *British thermal unit* (written B.T.U.). Therefore 1 ft.-lb. = 778 B.T.U. One pound of water falling 1 ft. would represent 1 ft.-lb. of mechanical energy; 550 ft.-lbs. per second represents 1 horse-power.

7. A 60-watt Mazda lamp has a resistance of 240 ohms. If 120 volts are applied to the lamp, determine: (*a*) current taken by lamp; (*b*) the work per hour in Joules; (*c*) the power in watts; (*d*) the power in horse-power; (*e*) the work under (*b*) in foot-pounds; (*f*) the number of B.T.U. developed per minute; (*g*) how many such lamps would a 25-kilowatt generator suffice?

8. *AB* is a resistance whose value is 1 ohm. Two batteries each having an internal resistance of 1 ohm are connected in parallel across *AB*. If the e.m.f. of one of the batteries is 2 volts and of the other 1 volt, how much current will flow through *AB*, and how much of this current will be supplied by the 2-volt battery and how much by the 1-volt battery?

Note. In order to make a general solution of a network of electrical circuits Kirchhoff's rules in addition to Ohm's law are needed. Kirchhoff's first rule: The algebraic sum of all the currents flowing toward a branch point in a network is equal to zero. Kirchhoff's second rule: The algebraic sum of all the electromotive forces acting around a closed circuit of a network

of conductors, is equal to the sum of IR products around the circuit.

At the point A we have,

$$i_1 - i_2 - i_3 = 0. \qquad \cdots \cdots \cdots \quad (1)$$

At the point B we have,

$$i_2 - i_4 - i_5 = 0. \qquad \cdots \cdots \cdots \quad (2)$$

We do not know whether the current, i_4, is flowing up through BD or down as indicated. The assumption is made that it is
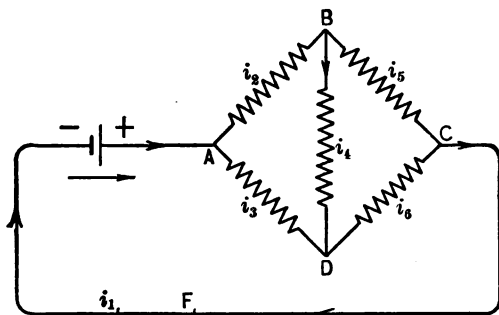
FIG. 1.—Illustrating Kirchhoff's Rules for Solving a Network Problem.

flowing down. If this assumption is wrong, it will be indicated in the final solution by the obtaining of a negative value for i_4.

At the point C we have,

$$i_5 + i_6 - i_1 = 0. \qquad \cdots \cdots \cdots \quad (3)$$

Considering the closed circuit $ABCF$,

$$i_1 r_1 + i_2 r_2 + i_5 r_5 = E. \qquad \cdots \cdots \cdots \quad (4)$$

Considering the closed circuit $ADCF$,

$$E = i_1 r_1 + i_3 r_3 + i_6 r_6. \qquad \cdots \cdots \cdots \quad (5)$$

Considering the closed circuit ABD,

$$0 = i_2 r_2 + i_4 r_4 - i_3 r_3. \qquad \cdots \cdots \cdots \quad (6)$$

If E and r_1, r_2, r_3, r_4, r_5, and r_6 are known, the values of i_1, i_2, i_3, i_4, i_5, and i_6 may be found from the above simultaneous equations.

EXAMPLE ILLUSTRATING APPLICATION OF KIRCHHOFF'S RULES

A 120-volt dynamo, Fig. 2, and a storage battery are connected in parallel to a number of lamps whose combined resistance is 1 ohm. The voltage of the battery is 116 and the resistance of the battery circuit is 0.10 ohm. The resistance of the dynamo circuit from (B) to (A) is 0.10 ohm. Will the battery be charged
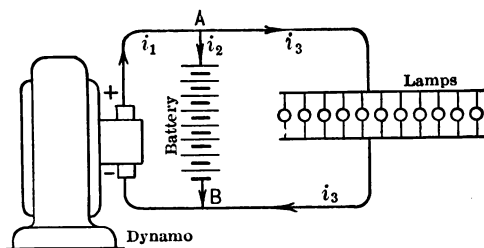
FIG. 2.—Application of Kirchhoff's Rules.

or discharged and what will the current be, in the battery circuit, in the lamp circuit, and in the dynamo circuit?

We will assume the battery is being charged. If it is discharging we will get a negative value for i_2. Considering the point (A), we have,

$$i_1 - i_2 - i_3 = 0. \qquad \ldots \ldots \ldots \quad (1)$$

Considering the dynamo and battery circuit, we have,

$$0.1 i_1 + 0.1 i_2 = 120 - 116 = 4. \qquad \ldots \ldots \quad (2)$$

Considering the dynamo and lamp circuit, we have,

$$0.1 i_1 + i_3 = 120. \qquad \ldots \ldots \ldots \quad (3)$$

Transposing Eq. (2) and multiplying by 10 we get,

$$i_2 = 40 - i_1. \qquad \ldots \ldots \ldots \quad (2a)$$

Transposing Eq. (3) we get,

$$i_3 = 120 - 0.1i_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3u)$$

Substituting the values of i_2 and i_3 from $(2a)$ and $(3a)$ in (1) we get,

$$i_1 - 40 + i_1 - 120 + 0.1i_1 = 0;$$

$$2.1i_1 = 160;$$

$$i_1 = 76.19 \text{ amperes.}$$

The current taken by the lamps is found from Eq. $(3a)$,

$$i_3 = 120 - (0.1)(76.19) = 112.38 \text{ amperes.}$$
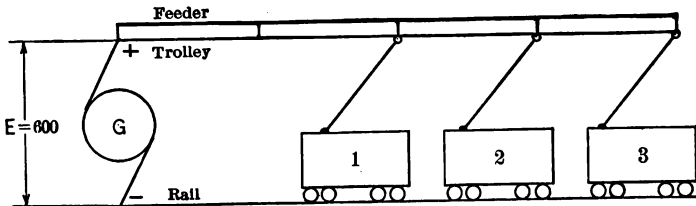
FIG. 3.

The current through the battery circuit is found from Eq. $(2a)$.

$$i_2 = 40 - 76.19 = -36.19 \text{ amperes.}$$

The battery is therefore being discharged instead of charged and supplies 36.19 amperes to the lamps. The remaining 76.19 amperes of the 112.38 amperes taken by the lamps is supplied by the dynamo. The voltage between the points (B) and (A) is $120 - (76.19)(0.1) = 116 - (36.19)(0.1) = 112.38$.

9. In Fig. 3 the first car is 2 miles from the generator and is taking 30 amperes. The second car is 3 miles from the station and is taking 20 amperes. The third car is 4 miles from the station and is taking 20 amperes. The trolley has a resistance of 0.40 ohm per mile. Track resistance = 0.05 ohm per mile. Feeder resistance = 0.20

ohm per mile, and the feeder is connected to the trolley every mile.   Find the voltage across each car.

10. The current through the shunt field of a 12.5-kw. 125-volt generator is 2 amperes.   A rheostat is in series with the shunt field and the voltage across the rheostat is 25 volts, making the e.m.f. across the field 100 volts. (a) What is the resistance of the field, of the rheostat, and of the field and rheostat in series?   (b) How many watts are wasted in heating the rheostat and in heating the field?   (c) What percentage of the rated output of the generator is each loss?

11. At 9 cents per kilowatt hour how much will it cost per week of sixty hours to operate four 25-watt, and eight 40-watt Mazda lamps?

12. What horse-power must an engine deliver to a generator supplying one thousand 40-watt Mazda lamps?   The line loss is 5 per cent, the efficiency of the generator is 90 per cent, and the loss in the belt is 1 per cent.

13. A motor 500 ft. away from the generator requires 40 amperes at 220 volts.   No. 4 wire is used.   Find the e.m.f. at the generator and the per cent of the voltage lost in the line; also find the power lost in the line and the per cent of the power generated thus lost.

14. Required the size wire in problem 13 to limit the loss in the line to 5 per cent of the voltage which is generated.

15. A cast-iron ring 100 ins. long is to carry a flux of 1,000,000 lines.   The area is 20 sq.ins.   What is the permeability at this density?   How many ampere turns will be required?   What is the reluctance of the circuit?

16. A certain magnetic circuit is made up of 160 ins. of sheet iron 20 sq.ins. in cross-section, and 0.50 in. of air 21 sq.ins. in cross-section.   Find the number of ampere turns necessary to set up 500,000 lines of force in the above circuit.

17. Fig. 4 represents a lifting magnet.   The core, B,

is of cast steel, 50 sq.ins. in cross-section. Armature A is of the same material and has a cross-section of 60 sq.ins. There is a space of 0.06 in. between B and A due to dirt and rust. The flux required is 2,000,000 lines. How many ampere turns are needed?

18. What is the reluctance of the magnetic circuit in problem 17?

19. A generator has 120 conductors in series on the armature. The length of the armature is 12 ins. and the diameter is 10 ins. The pole pieces cover three-fourths
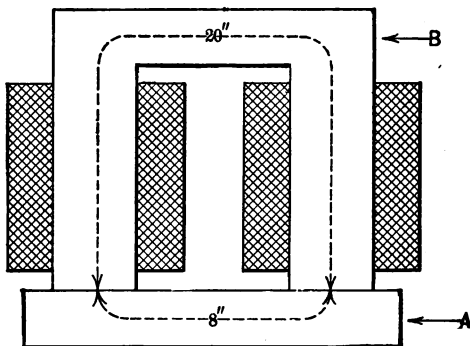
FIG. 4,—Lifting Magnet.

of the armature, and the flux density in the air gap is 60,000 lines per square inch. Required the voltage at 1200 r.p.m.

20. A conductor on the surface of an armature is 20 ins. long. The flux density under a pole is 60,000 lines per square inch; the speed is 60 ft. per second. How many conductors in series will be required to set up 220 volts, assuming the poles to cover 75 per cent of the surface of the armature?

21. The earth's magnetic field at a certain place dips 60° below the horizontal and its intensity is 0.59 gauss. A horizontal electric light wire stretched due magnetic

east and west carries a current of 2000 amperes flowing from east to west. The wire is 1000 ft. long. Find the force in pounds with which the earth's magnetic field pushes on the wire and specify its direction.

22. There are 600 conductors on the surface of a continuous current armature. Assuming 70 per cent of these to line in a magnetic field of 50,000 lines of force per square inch density. (*a*) How many pounds tangential pull are there on the surface of the armature? The length of each conductor is 15 ins., the diameter of the armature is 12 ins., and the current in each conductor is 20 amperes. (*b*) What torque in foot-pounds is developed? (*c*) If the speed of the armature is 1200 r.p.m. what is the horse-power? (*d*) What would the horse-power be if the speed were 600 r.p.m.?

CHAPTER II

PARTS OF A C.C. DYNAMO-ELECTRIC MACHINE

1. The following data apply to the magnetic circuit of a small modern two-pole machine. The flux required in the armature is 2,400,000 lines. The leakage factor may be assumed to be 1.10.

Part.	Material.	Length in cm.	Area in sq.cm.	Flux.	Flux Density.	Amp. Turns per cm.	Amp. Turns.
Armature.	Laminated iron	20	200	2,400,000			
Teeth....	Laminated iron	1.25 on each side	140	2,400,000			
Air gap..	Air..........	0.25 on each side	240	2,400,000			
Poles....	Laminated iron	15 on each side	180	2,640,000			
Yoke....	Cast steel.....	80	120 each	1,320,000			

Determine: (a) Ampere turns required for each part; (b) total ampere turns required; (c) ampere turns per pole.

Note. This problem will show roughly how the number of ampere turns are obtained. In an actual design there are some other factors, as for example armature reaction, which have to be taken into consideration.

2. Each field coil of a four-pole dynamo requires 12,000 ampere turns for its excitation. If the mean length of a turn is 15 ins. and 110 volts are applied to the four coils, determine resistance per foot of the proper size wire to use. (b) From the wire table find the nearest commercial size.

3. Allowing 1200 circular mils per ampere in order to limit the final temperature rise of the coil to approximately

9

50° C., determine the proper number of turns to use on the field coil of problem 3.

4. 19 volts are applied to each one of the six field coils of a 115-volt motor. The average length of a turn on the field is 28 inches, and the wire is No. 16, B. & S. gage. When the average temperature of the coil is 50° C., what is the magnetizing force in ampere turns? If there are 1000 turns on the coil, what will be the current?

5. (a) About how many amperes would a No. 16 wire safely carry if it were wound into a field coil? (b) If it were wound into a well ventilated armature?

CHAPTER III

THE CONTINUOUS-CURRENT GENERATOR

1. A FOUR-POLE simplex lap-drum-wound armature has 55 coils of 10 turns each. The flux per pole is 2,000,000 lines. Calculate the e.m.f. generated if the speed is 1200 r.p.m.

Note. The formula $e = lHv10^{-8}$ may be reduced to the following more usuable form:

> l = length of wire in series in one path = zl/paths, where
> z = total number of conductors on the surface of the armature.
>
> Average $H = \phi$ poles/$2\pi rl$ where ϕ is the flux per pole, r the
>   radius of the armature, and l its length.

$$v = (2\pi r)(\text{r.p.m.})/60.$$

Therefore,

$$e = \frac{zl\phi \text{ poles } 2\pi r(\text{r.p.m.})}{\text{paths } 2\pi rl60},$$

$$e = \frac{\phi \text{ poles } z(\text{r.p.m.})}{10^8 \text{ paths } 60}.$$

A simplex wave winding has two paths, a duplex wave winding four paths, a simplex lap winding as many paths as there are poles, a duplex lap winding two times as many paths as there are poles, etc.

2. What would the e.m.f. be in problem 1 if the armature were simplex wave wound?

3. A 125-volt, 40-kilowatt, 4-pole generator has 300 conductors. By leaving out two of these conductors this winding may be changed to a two-path wave winding.

What would then be the e.m.f., current capacity, and kilowatt capacity of the machine?

4. The machine of problem 3 is shunt wound and requires 6 amperes for exciting the field. The armature is wound with No. 4 wire. Now many circular mils per ampere are allowed in the armature winding? The field is wound with No. 12 wire. How many circular mils per ampere are allowed in the field? Why are the circular mils per ampere so much smaller in the armature than in the field?

5. A 10-pole simplex lap-wound, 500-kilowatt generator has a terminal e.m.f. of 600 volts on full load. There are 5000 ft. of No. 4 B. & S. copper wire wound on the armature. Assuming 2 volts drop in the brushes, brush contacts and leads, what e.m.f. must be induced in the armature?

6. If there are 1000 conductors on the surface of the armature of problem 5, what must the flux per pole be if the speed is 130 r.p.m.?

7. Show that the amount of wire needed for distribution with given line loss varies inversely as the square of the voltage of transmission.

8. Show that the amount of wire needed with the three-wire system is only three-eighths of that required for the two-wire system if all three wires are made the same size.

9. One thousand 25-watt Mazda lamps are to be supplied with current at an e.m.f. of 110 volts, from a 120-volt generator at a distance of 600 ft. from the lamps. Find the size of copper wire required in the mains.

10. One thousand 25-watt Mazda lamps are to be supplied with current at an e.m.f. of 110 volts from a 240-volt three-wire generator 600 ft. away from the lamps. The system is balanced. Find the size of copper wire required for the outside mains. If the neutral wire is one-half as large as either outside main, what percentage copper wire

is saved by using the three-wire system instead of the two-wire system as in problem 9?

11. The resistance of the armature of a certain generator is 0.15 ohm when delivering 40 amperes and the brush contact drop is 2 volts. The e.m.f. induced in the armature is 126 volts. What is the e.m.f. at the armature terminals?

12. On test a simplex wave-wound shunt generator gives 110 volts and 50 amperes to the external circuit. If the resistance of the armature is 0.15 ohm, the brush contact drop is 2 volts, and the resistance of the field 44 ohms, find the current through each armature path and the B. & S. size wire if 600 circular mils per ampere is used. Also find the total e.m.f. generated by the armature.

CHAPTER IV

THE CONTINUOUS-CURRENT MOTOR

1. A FOUR-POLE simplex lap-wound continuous-current motor has 1,200,000 lines of force per pole, and 55 armature coils with six turns per coil. If the armature current is 40 amperes, what will be the tangential pull in pounds at the rim of a pulley whose diameter is 10 ins.?

Note. The formulae $F = HlI/10$ may be reduced to the following form:

Average $H = \phi$ poles$/2\pi rl$, where $r =$ radius of the armature, l its length, and ϕ the flux per pole;

$I = I_a/$paths, where $I_a =$ total armature current;

$l = Zl$ where $Z =$ number of armature conductors;

$$F = \frac{\phi \text{ poles } I_a Zl}{2\pi rl \text{ paths } 10};$$

$$F = \frac{\phi \text{ poles } I_a Z}{20\pi r \text{ paths }};$$

where

$F =$ tangential pull on the armature wire in dynes.

In order to get the torque in dyne centimeters we must multiply by the radius, r, therefore,

$$T = \frac{\phi \text{ poles } I_a Z}{20\pi \text{ paths }},$$

where

$T =$ torque in dyne cms.

One foot-pound is equal to $(1.356)(10^7)$ dyne-cms., therefore,

14

$$T = \frac{\phi \text{ poles } I_a Z}{20\pi \text{ paths } (1.356)(10^7)};$$

$$T = \frac{0.118 I_a Z \phi \text{ poles}}{10^8 \text{ paths}};$$

where

T = torque in foot-pounds = tangential pull in pounds at 1-ft. radius.

The formulae for horse-power is,

$$\text{H.P.} = \frac{2\pi n T}{33000},$$

where

T = torque in foot-pounds;
n = speed in r.p.m.

2. What horse-power will be produced in the motor of problem 1, (a) if the speed is 1200 r.p.m., and the armature current 50 amperes? (b) If the speed is 600 r.p.m. and the armature current is 50 amperes?

3. (a) A shunt motor has an armature resistance of 0.10 ohm, and 2 volts brush contact drop. It is connected to 230-volt mains and the armature current is 100 amperes. What is the counter e.m.f.? (b) A shunt motor is developing a counter e.m.f. of 213 volts. If the armature resistance is 0.10 ohm, the brush drop 2 volts, and it is connected to 220-volt mains, how much current will flow through the armature?

4. (a) How much current would flow through the armature of problem 3 if it were connected directly to the mains and were not generating any counter e.m.f.? (b) If a starting rheostat having total resistance of 2 ohms is used, how much current will flow through the armature at the moment of starting?

5. The field winding of a shunt motor has a resistance of 110 ohms and the e.m.f. applied to it is 220 volts. (a) What is the amount of power expended in field excitation? (b) The full-load intake of the motor is 50 amperes at 220

volts, and the efficiency 90 per cent; express the power consumed in the field in terms of the full-load output of the motor.

6. From the following data determine the armature resistance of a motor:

$$\begin{aligned} \text{Impressed voltage} &= 230 \text{ volts;} \\ \text{Counter e.m.f.} &= 215 \text{ volts:} \\ \text{Armature current} &= 30 \text{ amperes;} \\ \text{Brush contact drop} &= 1.9 \text{ volts.} \end{aligned}$$

7. A 220-volt shunt motor has an armature resistance of 0.40 ohm and a brush contact drop of 2 volts. The field resistance is 220 ohms. The full-load speed is 1000 r.p.m. and the motor is taking 41 amperes. (a) At what speed must this machine run as a generator in order to deliver 40 amperes at 220 volts? (b) What is the counter e.m.f. when running at full load as a motor? (c) What e.m.f. is induced in the armature when run as a generator delivering 40 amperes at 220 volts.

8. The efficiency of the motor of problem 7 as a motor when running at full load is 85 per cent. (a) What H.P. is delivered by the machine? (b) How much useful torque is being produced. (c) Assuming 6 per cent of the torque actually developed as being used up in overcoming friction and windage, what is the flux per pole which actually enters the armature? There are four poles and the armature is simplex lap-wound and has 55 coils of six turns each.

9. A shunt motor connected to 110-volt supply mains takes 3 amperes through its armature and runs at a speed of 1000 r.p.m. The resistance of the armature including brushes and brush contacts is 0.2 ohm. The resistance of the field circuit is doubled and the speed is observed to rise to 1500 r.p.m. and the armature current to 5 amperes. In what ratio has the armature flux, ϕ, been changed by doubling the field resistance?

10. (*a*) Find the speed of a railway motor in r.p.m. corresponding to a car speed of 20 miles per hour, the gear ratio between the motor and axle being 14 : 68, and the diameter of the car wheels 33 ins. (*b*) Assuming that 15 per cent of the armature torque is used up in motor gear, and axle friction and windage, find the armature torque in footpounds corresponding to a tractive effort of 2000 lbs. delivered by the motor to the car. (*c*) What is the horsepower of the motor?

CHAPTER V

EFFICIENCY OF DYNAMO-ELECTRIC MACHINES

1. A CERTAIN motor has a normal current density of 30 amperes per square inch of brush contact. (*a*) What is the voltage drop for both brushes together (on basis of 0.8 volt+0.2×amperes per square centimeter)? (*b*) What will it be for 50 per cent overload? (*c*) For one-half load?

2. What is the brush contact loss in watts in problem 1, when, (*a*) the current density is 10 amperes per square inch; (*b*) when it is 30 amperes? (*c*) when it is 40 amperes? The normal full-load current of the motor is 61 amperes, and the field current is 1 ampere.

3. The following data apply to a 10-horse-power, 220-volt motor:

> Shunt field resistance (hot) = 220 ohms;
> Armature resistance (hot) = 0.45 ohm;
> Area of brush contacts,
>   (all+brush contacts)   = 8 sq.cm.;
> Rated current   = 41 amperes;
> Rated voltage   = 220 volts;
> Current density full load = 5 amps. per sq.cm.;
> Stray power (no load)   = 490 watts.

(*a*) Plot curves between current input (up to 60 amperes) and (1) brush contact loss in watts; (2) shunt field loss; (3) stray power loss (may be assumed to be constant at all loads); (4) armature copper loss; (5) total loss. (*b*) Plot a curve between efficiency and current input.

4. (*a*) Determine what the current input is for the motor of problem 3 when the output (input-losses) is 10

horse-power. (*b*) The name-plate rating of the above machine is 41 amperes, 220 volts and 10 horse-power. What efficiency would this indicate?

5. Calculate the efficiency of the motor of problem 3 when used as a generator and delivering 40 amperes at 220 volts to the external circuit.

6. The motor of a motor generator set takes 100 amperes at 230 volts from supply mains, and delivers 89 per cent of its power intake to the generator which it drives. The generator delivers 90 per cent of its power intake to a receiving circuit at 115 volts. Find the current supplied by the generator.

CHAPTER VI

ELEMENTARY PRINCIPLES OF ALTERNATING CURRENT

1. (*a*) Rotate a vector and taking its projections plot a sine wave of current, $i = I_m \sin \alpha$, where I_m is the maximum value of the current. (*b*) Divide a half cycle into narrow strips and obtain the average ordinate in terms of I_m.

2. Construct the curve of square ordinates, divide into
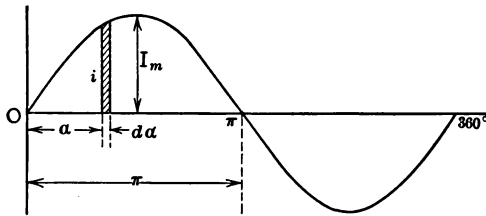
Fig. 5.—Current Wave Form.

narrow strips and obtain by measurement the average value of the squares of the instantaneous values; then take the square root of this average and express it in terms of I_m.

Note. One method of proving that $I_{av} = 0.636 I_m$ is as follows: In Fig. 5, $i = I_m \sin \alpha$ and

$$I_{av} = \text{average value of } i = \frac{\int_0^\pi I_m \sin \alpha \, d\alpha}{\pi};$$

$$I_{av} = \frac{\left[I_m \cos \alpha \right]_0^\pi}{\pi} = \frac{2 I_m}{\pi} = 0.636 \ I_m.$$

20

The fact that $I_v = 0.707 I_m$ may be shown as follows:

$$I_v = \sqrt{i^2} = \sqrt{\frac{\int_0^\pi I_m{}^2 \sin^2 \alpha d\alpha}{\pi}} = \sqrt{\frac{I_m{}^2 \left[\frac{\alpha}{2} - \frac{1}{4} \sin^2 \alpha\right]_0^\pi}{\pi}};$$

$$I_v = \sqrt{\frac{I_m{}^2 \pi}{2\pi}} = \sqrt{\tfrac{1}{2} I_m{}^2} = 707 I_m.$$

By a similar process it may be proved that $W = EI \cos \phi$, where W is the power in watts, E the current in amperes (virtual value), E the e.m.f. in volts (virtual value), and ϕ the angle by which E and I differ in phase.

3. (a) Prove that $W = EI$ where E and I are virtual values and are in phase. (b) Prove that $W = EI \cos \phi$ where ϕ is the angle by which E and I differ in phase.

4. An alternating current varies sinusoidally from 0 to 200 amperes as time elapses. (a) What will an a.c. ammeter read if placed in the circuit? (b) The maximum value of the current in an alternating current circuit is 50 amperes and the maximum value of the e.m.f. 160 volts. The current and e.m.f. are 30 degrees out of phase. What is the power in the circuit?

5. (a) What is the virtual value of an alternating e.m.f. when the instantaneous value at 60° is 300 volts? (b) What is the average e.m.f ?

6. The average voltage in an alternating-current circuit containing 10 ohms resistance (no capacity and no inductance) is 110 volts. What is the average current? (b) What is the virtual current? (c) What is the power?

Note. In all problems to follow, the terms "amperes" and "volts" and the symbols E and I will mean virtual values unless otherwise specified.

7. In a given circuit the resistance is 10 ohms, the inductance 0.03 henry and the frequency 60 cycles per second. (a) What current will flow if the e.m.f. is 125

volts? (b) What is the phase displacement? (c) What is the power? (d) What current would a continuous-current e.m.f. of 125 volts send through the circuit?

8. The exciting current of a 220-volt, 10 horse-power continuous current motor is 1 ampere. The resistance of the field is 220 ohms, and the inductance is 90 henrys. Find what this current would be if a 60-cycle per second e.m.f. of 220 volts were applied to the field circuit.

9. In a given circuit the resistance is 1 ohm and a 220-volt, 60-cycle per second e.m.f. causes 110 amperes to flow in the circuit. Find the inductive reactance, resistance, and impedance.

10. A condenser whose capacity is 100 micro-farads is connected to a 60-cycle per second 220-volt circuit. (a) Assuming the resistance of the lead wires as equal to zero, what charging current will flow? (b) What will the power be?

11. (a) A 60-cycle per second 110,000-volt, 100-mile transmission line has a capacity of 0.50 micro-farad. What will the charging current be? (b) A 60-cycle per second 10,000-volt, 10-mile transmission line has a capacity of 0.05 micro-farad. (b) What will the charging current be? (c) What will the charging kilovolt amperes be in each case?

12. A circuit having a resistance of 10 ohms, a capacity of 0.0003 farad and an inductance of 0.03 henry has a 60-cycle per second, 230-volt e.m.f. applied to it. Determine (a) current; (b) phase displacement; (c) power.

13. Two circuits having resistances of 5 and 10 ohms respectively, and inductances of 0.02 and 0.03 henry respectively are in series and have a 60-cycle per second, 230-volt e.m.f. applied to them. Determine (a) current in amperes; (b) power in each circuit and in the two circuits; (c) e.m.f. in volts across each circuit.

14. Given two circuits in series. The resistance of the first circuit is 10 ohms, the inductive reactance 10 ohms and the capacity reactance 5 ohms. The resistance of the

second circuit is 5 ohms, the inductive reactance is 5 ohms and the capacity reactance 10 ohms. Determine the impedance of the two circuits in series. If the e.m.f. applied to the above circuit is 500 volts, determine, (a) current; (b) phase displacement; (c) power.

15. The two circuits of problem 14 are connected in parallel. Determine: (a) Current in each circuit if the applied e.m.f. is 500 volts. (b) Total current. (c) Power in each circuit and the total power. (d) Total impedance in ohms.

16. The two circuits of problem 14 are connected in parallel and are connected in series with a third circuit whose resistance is 5 ohms, inductive reactance 5 ohms, and capacity reactance 0 ohms. Required the impedance of the combination and the current and power due to an e.m.f. of 600 volts.

17. A 10-horse-power, 200-volt, single-phase induction motor, power factor 80 per cent, guaranteed efficiency 86 per cent, is connected to a generator by a transmission line having a resistance of 0.5 ohm and an inductive reactance of 0.5 ohm. Required the generator voltage.

Note. Problems like the above are solved as follows:

$$I = \frac{10 \times 746}{(200)(.80)(.86)} = 54 \text{ amperes;}$$

$$IR \text{ of line} = (54)(0.5) = 27 \text{ volts;}$$

$$IX \text{ of line} = (54)(0.5) = 27 \text{ volts;}$$

Equivalent IR of motor $= (200)(0.80) = 160$ volts;

Equivalent IX of motor $= \sqrt{(200)^2 - (160)^2} = 120$ volts;

$$E_0 = \sqrt{(160+27)^2 + (120+27)^2} = 235.7 \text{ volts;}$$

$$\phi_0 = \tan^{-1} \frac{147}{187}.$$

If the above motor were two phase, the problem would be solved on the basis of transmitting one-half of the power over two of the four wires connecting the motor to the generator. If it were three phase, it would be solved on the basis of transmitting one-half of the power over two of the three wires connecting the motor with the generator. It is therefore evident that a three-phase line requires only 75 per cent as much copper as a single-phase or a two-phase line.
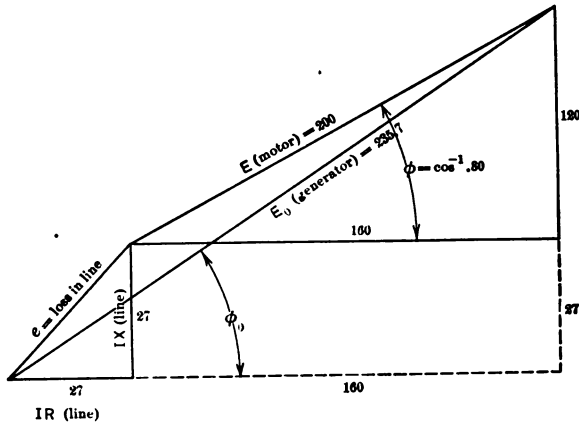
Fig. 6.—Illustrating Method of Solving Problem 17.

18. A 15-horse-power, 220-volt, single-phase motor, power factor 90 per cent, efficiency 90 per cent is connected to a generator by a transmission line having a resistance of 0.5 ohm. Required the generator voltage.

19. A 20-horse-power, 220-volt, two-phase motor, power factor 90 per cent, efficiency 90 per cent, is connected to a two-phase generator by a transmission line, the resistance of each wire being 0.6 ohm and the inductive reactance 0.3 ohm. Required the generator voltage.

20. Required the generator voltage if the motor in problem 19 were three-phase, and if three of the wires specified in problem 19 were used.

21. Six hundred 25-watt, 110-volt Mazda lamps are

connected to a generator through a transmission line having a resistance of 0.05 ohm and 0.03 ohm inductive resistance.   Required the generator voltage.

Note.   The power factor of incandescent lamps may be taken as unity.

22. What frequency will give resonance in a circuit having 1 ohm resistance, 10 micro-farads capacity, and 0.350 henry   inductance.

23. An induction motor taking a lagging current of 50 amperes, power factor 80 per cent, is connected in parallel with an over-excited synchronous motor which is taking a leading current of 60 amperes, power factor 90 per cent. Required current taken by the two motors and the resultant power factor.

CHAPTER VII

THE ALTERNATING-CURRENT GENERATOR

1. DETERMINE the frequency in cycles per second of a current given by a ten-pole alternator running at 720 r.p.m. (*b*) A generator is to be direct-connected to a Corliss engine running at 120 r.p.m. If the frequency is to be 60 cycles per second, how many poles must the alternator have?

2. (*a*) A single-phase generator has a safe current-carrying capacity of 100 amperes and is rated at 250 volts. What would the rating be in kilovolt amperes? (*b*) A two-phase alternator has a safe current-carrying capacity of 100 amperes per phase, and is rated at 250 volts. Required the rating in kilovolt amperes. (*c*) A three-phase alternator has a safe current-carrying capacity of 100 amperes per line and is rated at 250 volts. Required the rating in kilovolt amperes.

3. A 200-kilovolt ampere alternator delivers 100 kilowatts at 80 per cent power factor to a number of induction motors. How many 40-watt Mazda lamps at unity factor will the generator carry in addition. The total lines losses are to be 10 per cent.

4. A certain alternator is observed to require 10 amperes exciting current when delivering normal full-load current of 80 amperes at the rated full load e.m.f. of 125 volts and unity power factor. The armature impedance is found to be 0.35 ohm and the resistance 0.15 ohm. Required the total induced e.m.f. of the machine when the exciting current is 10 amperes.

5. A three-phase generator has equal loads of 50 kilo-

26

watts, 100 per cent power factor delta-connected across each phase. Voltage across each phase is 125. (*a*) What is the current in each line wire? (*b*) If the alternator were Y-connected what would the current be in each winding? (*c*) What would it be if the alternator were delta-connected?

6. A 125-volt, 100-kilovolt ampere, delta-connected alternator is reconnected in Y. What will be the voltage? What will the rating be in kilovolt amperes?

CHAPTER VIII

THE TRANSFORMER

1. A CERTAIN 10-kva. transformer requires 0.088 ampere when the high-tension side is connected to 2200-volt mains, and the secondary is open circuited. The hysteresis and eddy-current losses are 110 watts. Required the magnetizing current.

Note. The no-load primary current, I_0, is the resultant of the iron loss component, I_w, and the magnetizing component, I_m.

$$I_c = \sqrt{I_w{}^2 + I_m{}^2}.$$

I_w is equal to the iron losses in watts divided by the primary voltage. I_w is in phase with the primary voltage and I_m is in quadrature.

2. What would a watt meter read if it were placed in the primary circuit specified in problem 1? What is the angle of phase displacement between the impressed primary voltage and I_0?

3. There are 1200 turns on the primary of the above transformer, and 120 turns on the secondary. (a) What will the secondary voltage be with no load on the secondary? (b) A sufficient number of incandescent lamps, unity power factor, are connected to the secondary of the above transformer to cause 40 amperes to flow in the secondary. Required the current and its phase displacement in the primary.

Note. Since the ratio of transformation is 10 to 1, 40 amperes in the secondary will cause an increase of 4 amperes in the primary, and since the power factor is unity it will be in phase with

28

the primary voltage, therefore in phase with I_w and in quadrature with I_m.

4. The area of the magnetic circuit of a certain 15-kva. transformer is 12 sq.ins. The mean length of the magnetic circuit is 36 ins. The maximum flux density is 50,000 lines per square inch. A fair value for the permeability at this density would be 2650. There are 1000 turns on the primary. Required the magnetizing current.

Note. The formula, $\phi = 0.4\pi N I \mu A / l$ applies, but it must be remembered that I is the maximum value of the current if ϕ is taken as the maximum value of the flux.

Substituting for B, A/l and $I\sqrt{2}$ for I_m we get

$$I_m = \frac{0.5627Bl}{N\mu},$$

where N is the number of primary turns, l is expressed in centimeters and B in lines per square centimeter, I_m is the virtual value of the magnetizing current in amperes.

5. A certain transformer has a ratio of 10 to 1, and the windings are so designed that the current densities in the primary and in the secondary are the same. (a) What is the ratio of primary to secondary resistance, if both windings have the same mean length of turn? (b) If the current density in the secondary were made 10 per cent greater than the primary and if the mean length of a primary turn were 10 per cent greater, what would the ratio be?

6. The magnetic core of a 30-kva. transformer has a net cross-sectional area of 20 sq.ins. and an average length of 60 ins. The high-tension winding is designed for an impressed voltage of 6600 at 60 cycles. The maximum flux density in the core is 60,000 lines per square inch. At this density the permeability is 2300 and the iron loss 0.35 watts per cubic inch. How many turns are required on the high-tension winding? What is the magnetizing current, the no-load current and the power factor?

7. A 10-kva. transformer has a core loss of 102 watts and 149 watts total copper loss at full load; the primary and secondary copper losses are equal. Determine the efficiency at $\frac{4}{4}$, $\frac{5}{4}$, $\frac{4}{4}$, $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$ load.

8. Determine the all-day efficiency for the transformer of problem 7 if it is operated two hours at full load, one hour at $\frac{3}{4}$ load, five hours at $\frac{1}{4}$ load and sixteen hours at no load.

9. A 40-kva. transformer has 200 watts hysteresis loss and 40 watts eddy current loss. If the magnetic density in this transformer is 60,000 lines per square inch what would the effect on hysteresis, eddy current and total core loss be of increasing the density to 90,000 lines per square inch. Of decreasing it to 40,000?

10. How would the losses in problem 9 be effected if there were no change in density, but if the frequency were to become 50 cycles per second instead of 60?

11. A 10 kva., 2200—220-volt transformer has the following constants:

> Resistance of primary $= 5.45$ ohms.
> Resistance of secondary $= 0.0545$ ohm.
> Core loss (at 2200 volts) $= 130$ watts.
> Magnetizing current $= 0.10$ ampere.

Required primary current, primary copper losses and the efficiency when the secondary current is (a) 50 amperes, (b) 40 amperes, (c) 30 amperes, (d) 20 amperes, (e) 10 amperes.

12. The load on the secondary of the transformer in problem 11 consists of incandescent lamps operating at unity power factor, the leakage has a negligible effect on the regulation, and it is sufficiently accurate for commercial purposes to subtract the primary IR drop from the impressed voltage, divide by the ratio of transformation, and then subtract from the resultant the secondary IR drop, in order to get the secondary voltage. On this basis what

would the secondary voltage be when secondary current is 45 amperes? The ratio of transformation is 10 to 1.

13. The effect of magnetic leakage in the transformer of problem 12 is equivalent to an inductive reactance of 10 ohms placed in series with the primary winding and outside of the transformer when the secondary current is 45 amperes. What will the secondary voltage be with a load of 45 amperes at 100 per cent power factor on the secondary?

CHAPTER IX

ALTERNATING-CURRENT MOTORS

1. A SYNCHRONOUS motor running on a 2200-volt line is so excited that the armature current and impressed e.m.f. are in phase. The current in the armature is 30 amperes. The resistance of the armature is 0.6 ohms, and the inductive reactance $2\pi f L_a I_a$, is 3 ohms. Required the c.e.m.f. of the motor.

Fig. 7 shows the e.m.f. diagram of an alternator and synchronous motor in which the assumption is made that the coefficient of self-induction of the armature has a constant value for all conditions of load and excitation. The e.m.f., E_g, generated in the armature forms the hypothenuse of a right-angled triangle one side of which is equal to the total inductive pressure drop both internal and external, while the other side is equal to the sum of the external IR drop plus the $I_a R_a$ drop in the armature. E is the terminal e.m.f. and ABC is the angle between the terminal voltage and the armature current. The angle OBC is the angle between the e.m.f. generated in the armature and the armature current. In the motor diagram E_g is the e.m.f. induced in the armature, is the c.e.m.f. of the synchronous motor. E is the voltage applied to the motor. In the diagram the current lags behind E by the angle OBC. It is at once evident that as E_g is increased by increasing the exciting current, the point B will swing to the right, if E and OA remain constant. E, being the impressed e.m.f., does not change and OA varies only a small amount due to the fact that L_a varies somewhat as E_g is changed. The error made therefore by assuming L_a to remain the same is very small and the vector diagram is made extremely simple. In the problems given in this chapter L_a is to be assumed as remaining constant. It is at once evident from the diagram that if E_g is made large enough, the current will lead the impressed voltage E by an angle which may be made quite large by considerably over-exciting the motor.

32

2. (a) Required the c.e.m.f. of the motor in problem 1 if the current is 30 amperes and leads the impressed e.m.f. by 15 degrees.   (b) If it lags behind the impressed e.m.f. 15 degrees.

3. Required the phase displacement between the cur-
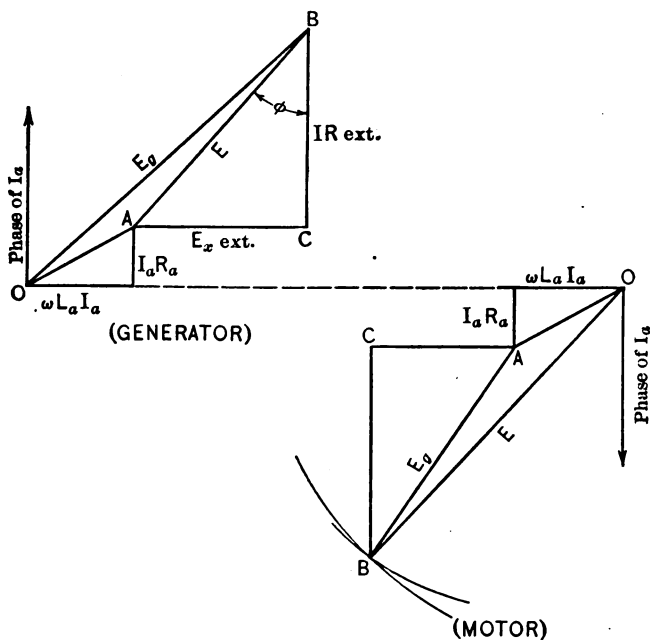
FIG. 7.—E.M.F. Vector Diagrams A.C. Generator and Synchronous Motor.

rent and impressed e.m.f. in the motor of problem 1, when the armature current is 40 amperes and the c.e.m.f. is equal to the impressed e.m.f.

4. The motor of problem 1 is connected in series with a choke coil having 5 ohms inductive reactance and 0.20 ohm resistance.   The e.m.f. impressed on the combination is 2300 volts.   The current in the armature is 30 amperes

and leads the impressed e.m.f. by 30°.   Required the
e.m.f. at the motor terminals and the c.e.m.f. of the motor.

5. An 8-pole induction motor is connected to 60-cycle
per second, 220-volt mains and when fully loaded runs at
850 r.p.m.   What is the slip in r.p.m.?   Assuming zero
slip when the motor is unloaded, what would the r.p.m. be?

6. In problem 5 what is the frequency of the current in
the rotor when the speed is 850 r.p.m.?

7. Two 10-horse-power, three-phase induction motors,
power factor 80 per cent, efficiency 85 per cent, are opera-
ting in parallel with a 25-horse-power, three-phase syn-
chronous motor excited so as to have a leading power
factor of 90 per cent, efficiency is 90 per cent.   The above
motors are connected to 220-volt mains.   How much
current does each motor take, and what is the total current?

8. Five hundred, 40-watt Mazda lamps, unity power
factor and 20 horse-power in single-phase induction motors,
average efficiency 80 per cent, average power factor 80
per cent, and a single-phase, 25-horse-power synchronous
motor, efficiency 90 per cent, are operating in parallel on
125-volt mains.   Required the power factor of the syn-
chronous motor in order to make the power factor of the
system unity.

CHAPTER X

POLYPHASE POWER

1. Two watt-meters are connected into a balanced three-phase system to measure the power. (a) If one of the meters reads two-thirds of the power what is the power factor? (b) If one of the meters reads all the power, what is the power factor? (c) If both meters read exactly the same, what is the power factor?

2. Three circuits each having a resistance of 1 ohm and a reactance of 0.5 ohm are connected in delta across a three-phase, 220-volt circuit. Determine (a) line current, (b) current in each circuit, (c) total power supplied.

3. The three circuits described in problem 2 are connected in Y across a three-phase, 220-volt circuit. Determine (a) line current, (b) current in each circuit, (c) total power supplied.

4. Three circuits each having zero reactance and resistance of 10, 20 and 30 ohms respectively are connected in delta across a three-phase, 440-volt circuit. (a) What is the current in each circuit? (b) What is the current in each line wire? (c) What is the total power supplied?

5. The circuits referred to in problem 4 are connected in Y across a three-phase, 440-volt circuit. (a) What is the current in each circuit? (b) What is the total power supplied?

6. Three unequal lamp loads are connected in delta at the end of a three-phase transmission line having a resistance of 0.1 ohm and a negligible reactance. If the voltage at the generator is 115 and there are fifty 25-watt lamps in the first circuit, one hundred 40-watt lamps

in the second circuit and two hundred 40-watt lamps in the third circuit, what will be the line voltages at the lamps?

7. (a) If the three lamp-loads of problem 6 were connected in Y and the generator voltage were 200, what would be the voltage applied to each set of lamps? (b) Why is it not practicable to Y connect incandescent lamps to a three-phase line?

8. Two alternating-current generators operating in parallel deliver power to a balanced three-phase load. The output of each generator is measured by the 2-watt meter method. The terminal voltage is 2200. The watt-meter readings are as follows:

Generator No. 1. $W_1 = 200$ kw. $W_2 = 250$ kw.
Generator No. 2. $W_1 = 210$ kw. $W_2 = 240$ kw.

What is the total power supplied?.

DIMENSIONS AND RESISTANCES OF PURE COPPER WIRE

Gauge No.	Diameter (Inches) — Bare	Single Cotton Covered	Double Cotton Covered	Area — Circular mils (d_a) 1 mil = .001 Inch	Weight and Length, sp. gr. = 8.89 — Lbs. per 1000 Feet	Feet per Lb.	Ohms per 1000 Feet — At 20° C.	At 50° C.	At 80° C.	Feet per Ohm — At 20° C.	Ohms per Pound — At 20° C.
0000	.460			211600.00	640.5	1.561	.04893	.05467	.06058	20440	0.00007639
000	4.0			167805.00	508.0	1.969	.06170	.06893	.07640	16210	0.0001215
00	.365			133079.40	402.8	2.482	.07780	.08692	.09633	12850	0.0001931
0	3.5			105592.50	319.5	3.130	.09811	.1096	.1215	10190	0.0003071
1	.289		.303	83694.20	253.3	3.947	.1237	.1382	.1532	8083	0.0004883
2	2.8			66373.00	200.9	4.977	.1560	.1743	.1932	6410	0.0007765
3	229			52634.00	159.3	6.276	.1967	.2198	.2435	5084	0.001235
4	204			41742.00	126.4	7.914	.2480	.2771	.3071	4031	0.001963
5	182			33102.00	100.2	9.980	.3128	.3495	.3873	3197	0.003122
6	162			26250.50	79.46	12.58	.3944	.4406	.4883	2535	0.004963
7	144		.156	20816.00	63.02	15.87	.4973	.5556	.6158	2011	0.007892
8	128		.140	16509.00	49.98	20.01	.6271	.7007	.7765	1595	0.01255
9	114		.126	13504.00	39.63	25.23	.7908	.8835	.9791	1265	0.01995
10	102	.108	.112	10381.00	31.43	31.82	.9972	1.114	1.235	1003.	0.03173
11	0907	.097	.101	8234.00	24.93	40.12	1.257	1.405	1.557	795.3	0.05045
12	0808	.087	.091	6529.90	19.77	50.59	1.536	1.771	1.963	630.7	0.08022
13	0720	.078	.082	5178.40	15.68	63.79	1.999	2.234	2.476	500.1	0.1276
14	0641	.071	.075	4106.80	12.43	80.44	2.521	2.817	3.122	396.6	0.2028
15	0571	.063	.067	3256.7	9.85	101.4	3.179	3.552	3.936	314.5	0.3225
16	0508	.055	.059	2582.9	7.81	127.9	4.009	4.479	4.964	249.4	0.5128
17	0453	.049	.053	2048.2	6.20	161.3	5.055	5.648	6.259	197.8	0.8153
18	0403	.044	.048	1624.3	4.91	203.4	6.374	7.122	7.892	156.9	1.296
19	03-9	.039	.043	1252.4	3.89	256.5	8.038	8.980	9.952	124.4	2.061
20	0320	.036	.040	1021.5	3.09	323.4	10.14	11.32	12.55	98.66	3.278
21	0285	.032	.036	810.10	2.45	407.8	12.78	14.28	15.83	78.24	5.212
22	0253	.029	.033	642.70	1.94	514.2	16.12	18.01	19.96	62.05	8.287
23	0226	.027	.031	509.45	1.54	648.4	20.32	22.71	25.16	49.21	13.18
24	0201	.024	.028	404.01	1.22	817.6	25.63	28.63	31.73	39.02	20.95
25	0179	.022	.026	320.40	.97	1031.	32.31	36.10	40.01	30.95	33.32
26	0159	.020	.024	254.01	.77	1300.	40.75	45.52	50.45	24.54	52.97
27	0142	.018	.022	201.50	.61	1639	51.38	57.40	63.62	19.46	84.23
28	0126	.017	.021	159.79	.48	2067	64.79	72.39	80.22	15.43	133.9
29	0118	.015	.019	126.72	.38	2607	81.70	91.28	101.2	12.24	213.0
30	0100	.014	.018	100.5	.30	3287.	103.0	115.1	127.6	9.707	338.6
31	00893	.0125		79.71	.24	4145.	129.9	145.1	160.8	7.698	538.4
32	00795	.0115		63.20	.19	5227.	163.8	183.0	202.8	6.105	856.2
33	00708	.0105		50.13	.15	6591.	206.6	230.8	255.8	4.841	1361.
34	00531	.0098		39.74	.12	8311.	260.5	291.0	322.5	3.839	2165.
35	00562	.0086		31.52	.10	10480.	329.4	366.9	406.7	3.045	3441.
36	00500	.0080	.011	25.00	.08	13210.	414.2	462.7	512.9	2.414	5473.