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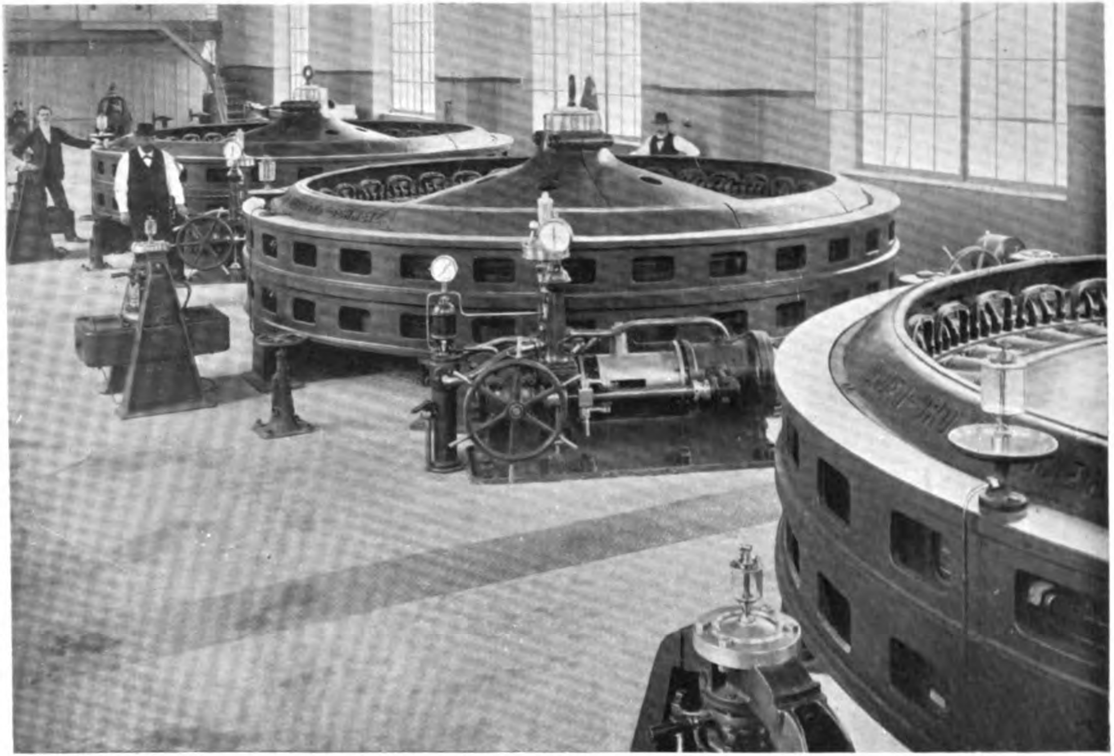

Installation consists of Five Alternators, of Umbrella-type, driven by Water-turbines of 1275 H.P. each. The Alternators, constructed by Messrs. Brown, Boveri & Co., are of the type ATB 48 – 900 – 100

Frontispiece

DYNAMO-ELECTRIC
MACHINERY

A Manual for Students of Electrotechnics

BY

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SEVENTH EDITION

VOLUME II.
ALTERNATING-CURRENT MACHINERY

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PREFACE

In the nine years that have elapsed since the Sixth Edition of this book was published, there has been a vast extension of Alternate-Current Machinery. To present a systematic account of the subject in its actual stage of development has been no small task. The volume is practically new from beginning to end, for out of its 800 pages less than 100 have been taken from the former edition, and these have been revised and re-written.

Chapter II., on Periodic Functions, is entirely new, and leads up to a novel and greatly simplified method of harmonic analysis, which is applied in Chapter IV. to the discussion of the wave-forms of alternating electromotive-forces and currents. In this chapter a practical method of predetermining the wave-form of the electromotive-force of an alternator is described, and the effects on the wave-form of the forms of the polar surfaces and of the distribution of the armature windings are analysed.

Chapter III. is generally descriptive of the various types of Alternators, and of the details of their structure.

Chapter V. is devoted to Magnetic Leakage and the Armature Reactions which affect the working of Alternators. In this chapter much use has been made of the principle of superposition of Magnetic Fields. By adopting a distinction between that part of armature interference which is permanent and that part which is periodic, it has been found possible to simplify the method of predetermining the curve of performance of an alternator on an inductive load, while rendering it more accurate than most of the methods hitherto published, the chief of which are explained.

Chapter VI. deals with winding schemes for Alternators of all types—single-phase, two-phase, and three-phase—and includes the windings, not only of armatures for generators and of stators for induction-motors, but also of rotors for motors. A number of figures, reproduced from photographs, show how the various kinds of windings are carried out in the shops.

One feature of the volume is the prominence given to the methods of design. Chapter VII. is wholly devoted to the design of alternators of the types in current use. Simple but rational formulæ are given for determining the dimensions of the various parts and the numbers of the windings, these formulæ depending upon constants derived from experience of the best makers. The intrinsic value of these formulæ is proved in Chapter IX., where it is shown that they are equally serviceable for the design of high-speed alternators for steam-turbine service and for low-speed machines driven by large reciprocating engines. An Appendix to Chapter VII. describes methods of compounding alternators. In Chapter VIII. analyses are made of fourteen different alternators (not including turbo-alternators), ranging from 37 to 7,500 kilowatts in rating, and from 75 to 770 revolutions per minute. Special attention is directed in Chapter IX. to the recent developments in connexion with steam-turbine driving; and a large amount of hitherto unpublished matter will—thanks to the firms who are chiefly concerned in this development—be found in this section. Chapter X. deals briefly with Synchronous Motors, Motor-Generators, and Converters, while Chapter XI. is concerned with the much-discussed question of the parallel running of Alternators, and the difficulties which have been experienced in the "hunting" of synchronous machinery.

Chapters XII. and XIII. deal with Transformers and their design ; and here, again, the subject is treated from the point of view of the practical designer, with simple formulæ based upon the data afforded by experience.

Chapters XIV., XV. and XVI. are devoted to the Three-phase Induction-Motor, its design, and analyses of examples. The author has adopted a somewhat novel treatment of the semicircle diagram of Heyland, which he deduces direct from the currents in the different parts, and not from the diagrams of magnetic fluxes and leakages, which are puzzling to beginners. He has tabulated for reference the values of the "cardinal quantities"—the power-factor, dispersion-coefficient, overload-factor, etc.—which govern the operation of the induction-motor, and by this plan has greatly simplified the process of motor design. Examples of induction motors of various sizes and styles are analysed.

Chapter XVII., on Single-phase Induction Motors, forms a complement to the preceding, and contains several examples.

Chapter XVIII. concludes the volume with a concise account of the Single-phase Commutator Motor, which, in its various modifications, has lately occupied so largely the attention of electrical engineers.

In the chapters that deal with design, schedules have been inserted to facilitate the various computations, and to systematize the work.

The author is indebted to various assistants, who have helped him at different times in the preparation of this work by collection of data, revision of formulæ, analyses of designs, and writing of digests. During these nine years he had successively the services of Mr. Miles Walker, B.A., Mr. A. C. Eborall, Mr. J. Dennis Coales, B.Sc., Mr. Fielder I. Hiss, and Mr. H. W. Taylor, all of whom have taken part in this work, and to all of whom his acknowledgments are willingly made. To Mr. Taylor, in particular, the author's acknowledgments are due, not only for general assistance in the preparation and revision of the work, but for special care in preparing sundry sections; in particular, the chapters on Armature Reaction, on Parallel Running, and on Single-phase Commutator Motors. The author has also had the assistance of his former student, Mr. Hugh Ledward, in preparing the chapter on the design of Transformers. He is indebted to Mr. Ledward for the useful conception of the flux-factor (p. 610), by which the problems of transformer-design are greatly simplified.

The author is further indebted to several former students for assistance in preparing the drawings for this volume; to Mr. Tchighianoff, for the series of periodic curves in Chapter II.; to Mr. L. E. Bussey, for details of machine construction; and to Mr. Taylor for a number of miscellaneous drawings and diagrams throughout the work.

A treatise such as the present one, dealing with the technicalities of construction and results of actual performance, would be impossible to produce without the co-operation of the engineers and firms who are concerned in the manufacture of machinery such as is here dealt with. The author once more acknowledges, with deep gratitude, his indebtedness to those concerned, who are, indeed, the chief electrical engineers in Europe and America. He has endeavoured, in the text of the work, to acknowledge these obligations, but finds it no easy matter to discharge his duty in this matter of grateful recognition to so many. But he cannot refrain from offering publicly special thanks to two firms: to the Oerlikon Machine Works, which has generously placed him in possession of more than a hundred examples of designs, and to the British Westinghouse Company, which has given him unstinted access to photographs of its machines, and has furnished for this book so many striking pictures in illustration of the modes of winding armatures. The author also desires to particularize in his acknowledgments the following firms and engineers:—The Allgemeine Elektricitäts

Gesellschaft, of Berlin; Messrs. Brown, Boveri and Co., and particularly to Mr. C. E. L. Brown; the Bullock Electric Co., of Cincinnati, and to Mr. B. A. Behrend; the British Thomson-Houston Co., of Rugby, and to Mr. H. S. Meyer; the British Westinghouse Co., and in particular to Mr. Miles Walker; Messrs. Dick, Kerr and Co.; Mr. S. Z. de Ferranti; Mr. V. A. Fynn; Messrs. Ganz and Co., and particularly to Mr. O. T. Bláthy; the General Electric Co., of Schenectady, and particularly to Prof. C. P. Steinmetz; Mr. Alexandre Heyland; Messrs. Johnson and Phillips; Messrs. Kolben and Co., and in particular to Mr. Emil Kolben; the Lahmeyer Co., of Frankfurt, and in particular to Prof. Dr. Joseph Epstein; the Oerlikon Machine Works, and individually to Dr. Hans Behn-Eschenburg; Messrs. Parsons and Co., and specially to the Hon. C. R. Parsons; Mr. Alexandre Rothert; Messrs. Scott and Mountain; Messrs. Witting, Eborall and Co., and in particular to Mr. A. C. Eborall.

S. P. T.

LONDON: 1905.

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LIST OF SYMBOLS EMPLOYED.

For convenience of readers the symbols chiefly employed are here tabulated. Occasionally the same letters do duty in other significations; as, for example, where the three currents of a three-phase system are called A, B and C; or, as on p. 231, where the spans of certain coils are distinguished as *a*, *b*, *c*, etc. Apart from this it has been impossible, owing to the limited number of symbols available, and the recommendation of the Frankfurt Congress to limit the use of Greek letters to *angles* and *coefficients*, to avoid having to employ certain symbols in more than one signification. Thus M is used both for the mean length of one turn of a coil, and for the coefficient of self-induction, but in different parts of the book; and T is used both for the periodic time and for torque. Care has been taken by explanations where necessary in the text that no ambiguity shall arise from this inevitable duplication.

A	area of section, square inches, unless otherwise stated (p. 612).
A	alternator; thus ATB means "alternator, three-phase, B-type," etc. (p. 144, etc.).
A-type	i.e. with internal revolving armature and fixed magnets (pp. 92, 169).
A_1, A_2, A_3, etc.	amplitude of sine-terms in Fourier's series (pp. 48, 61).
AT_A	number of ampere-turns, per pole, of armature (p. 287).
AT_M	number of ampere-turns, per pole, of magnet (p. 287).
A	external area of surface of transformer, in square inches (p. 588).
a	a coefficient for predicting temperature-rise (p. 588).

B_1, B_2, etc.	amplitudes of cosine-terms in Fourier's series (p. 61).
B-type	*i.e.* with revolving magnet wheel and external fixed armature (pp. 93, 173).
B	area of cooling contact of transformer, in square inches (p. 588).
B	(flux-density) lines per square inch.
B_g	lines, per square inch, in the gap (p. 374).
B_t	lines, per square inch, in the teeth (p. 374).
$\mathfrak{B}$	(flux-density in C.G.S. units) lines per square centimetre (p. 542).
b	a coefficient for predicting temperature-rise (p. 588).
b	breadth, or equivalent breadth, of pole, reckoned at armature face, in inches (pp. 201, 384).
b	generally, a breadth, in inches (p. 246).

C	current in amperes, or virtual amperes if alternating (p. 17).
C_1	(in three-phase generators) current in one phase (p. 355).
C_1	(in transformers) current in primary winding (pp. 537, 610).
C_2	(in transformers) current in secondary winding (p. 537).
C_{max}	maximum value of alternating current (p. 542).
C_{virt}	virtual value (*i.e.* root-mean-square) of alternating current (p. 12).
C_M	magnetizing current, in virtual amperes (pp. 289, 542, 685).
C_H	energy-component of current at no-load, in virtual current (pp. 541, 685).
C_O	no-load current, in virtual amperes (pp. 545, 683). On p. 43 means amplitude of current wave.
C_{sk}	short-circuit current, in virtual amperes (p. 683).
C_t	instantaneous value of current, *i.e.* amperes at time t (p. 21).
c	number of circuits in parallel through an armature (p. 434).
c	a constant occurring in the short-circuit test (p. 290).
c	a coefficient in formula for dispersion (p. 682).

D	amplitude of alternating electromotive-force (pp. 4, 41).
D	depth of slot (p. 270).
d	diameter of armature at face, or diameter of bore of stator, in inches (pp. 354, 718).
d_m	diameter of revolving magnet system at face, in inches (p. 384).
d_r	diameter of rotor at face, in inches (p. 700).
d	usual symbol of differentiation (p. 40, etc.).

E	electromotive-force, in volts, or virtual volts.
E_t	instantaneous value of electromotive-force, *i.e.* value of E at time t (p. 195).
E_{max}	maximum value of alternating electromotive-force (p. 195).
E_{virt}	virtual value (*i.e.* root-mean-square) of alternating electromotive-force (p. 195).
E_θ	electromotive-force at time when angle has value θ (p. 4).
E_1	(in three-phase generators) electromotive-force generated in one phase (p. 355).
E_1	electromotive-force induced in primary winding, in virtual volts (p. 535).
E_2	electromotive-force induced in secondary winding, in virtual volts (p. 535).
e_{s_1}	electromotive-force due to primary stray flux (p. 535).
e_{s_2}	electromotive-force due to secondary stray flux (p. 535).

F	mechanical force on a conductor, in pounds' weight (p. 699).
f	frequency, *i.e.* number of cycles per second (p. 3).

g	constant of gravitation, 32 feet per second per second (p. 526).

HP	horse-power, 1 HP = 746 watts.
$\mathcal{H}$	intensity of magnetic field in C.G.S. units, *i.e.* number of gausses or number of lines per square centimetre in air (p. 542).
H	intensity of magnetic field in British units, *i.e.* lines per square inch, in air (p. 650).
h_s	height of pole-shoe at edge (p. 248).
I	moment of inertia, in pound feet units (p. 526).
I	induction motor: thus IS6 (p. 787) means "induction motor, single-phase, 6-pole."
I-type	*i.e.* inductor type of generator, with magnet-winding and armature both stationary (pp. 93, 118).
i	correcting-term for saturation of iron parts (p. 685).
K	capacity, in farads or microfarads (p. 25).
KVA	kilo-volt-amperes (p. 27).
KW	kilowatts (p. 27).
k	Kapp coefficient (p. 196).
k	transformation-ratio (pp. 536, 593).
L	coefficient of self-induction (p. 18).
L_1	coefficient of self-induction of primary circuit, in henries (p. 592).
L_2	coefficient of self-induction of secondary circuit, in henries (p. 592).
l	length generally (pp. 19, 266, 542).
l	length (gross) of core-body, parallel to shaft, in inches (p. 354).
l_i	iron-length, *i.e.* reduced or effective length of core-body, in inches (p. 407).
M	mean length of one turn (p. 368).
M	magnetomotive-force (p. 246). Ampere-turns per pole (p. 249).
M	coefficient of mutual induction (p. 592).
m	symbol for any integer (p. 77).
N	magnetic flux, *i.e.* total number of magnetic lines in one pole or in one magnetic circuit.
N_g	total flux actually crossing the air-gap from the pole (p. 241).
N_m	total flux actually proceeding from any one pole (p. 241).
N_s	stray flux; number of magnetic lines that are dispersed (p. 241).
n	number of revolutions *per second* (p. 4).
n	number indicating order of any harmonic (p. 227).
n_1	number of revolutions per second of rotating flux (p. 658)
n_2	number of revolutions per second of rotor (p. 658).
n_{ph}	number of phases (p. 267).

O-type	*i.e.* with overhung revolving magnet-wheel and fixed armature (p. 94).
P	number of pairs of poles (p. 526).
P_g	the permeance of the magnetic path through air-gap (p. 241).
P_s	the permeance of the path of the stray flux (p. 241).
p	number of poles (p. 6).
p	the pulsation $= 2\pi f$ (p. 20).
Q	torque-factor (p. 700).
Q	quarter-phase (*i.e.* two-phase with the two phases in quadrature); thus AQB (p. 414) means "alternator, two-phase, B-type."
Q_1	specific loading of transformer core, in ampere-turns per inch (p. 619).
q	specific loading, of armature, in ampere-conductors per inch of periphery (p. 347).
RPM	revolutions per minute.
R	resistance generally, expressed in ohms.
R_1	resistance of primary circuit (p. 593), or of one phase (p. 726).
R_2	resistance of secondary circuit (p. 593), or of one phase of secondary (p. 686).
r	resistance of a circuit or conductor (p. 266).
r_1	resistance per inch length of conductor (p. 378).
r_1	in transformer, resistance of primary winding (p. 537).
r_2	in transformer, resistance of secondary winding (p. 537).
S	number of spirals, or turns, in a coil or armature (pp. 39, 534).
S	single-phase, as ASB meaning "alternator, single-phase, B-type" (p. 411).
S_1	number of turns in one phase of armature (p. 269).
S_1	number of turns in primary of transformer (p. 535).
S_2	number of turns in secondary of transformer (p. 535).
S_M	number of magnetizing turns on one pole (pp. 289, 379).
s	area of section of wire, in square inches (pp. 266, 379).
s	slip, in an induction motor (p. 659).
s_1	number of slots per phase per pole (p. 373).
T	period, or periodic time, duration of one cycle, in seconds (p. 41).
T	torque, in pound-feet (p. 700).
t	time, in seconds.
U	revolutions per minute (p. 445).
U	ratio of utilization of magnetism, in induction motors (p. 684).

U-type	*i.e.* umbrella-type of machine, with vertical shaft (p. 98).
u_1	leakage-coefficient of magnetism in stator (p. 683).
u_2	leakage-coefficient of magnetism in rotor (p. 683).

V	voltage, generally.
V	electromotive-force, or difference of potentials between terminals, in volts (p. 349).
V_1	(in three-phase generators) voltage in one phase (p. 349).
V_1	difference of potentials between primary terminals, in volts (p. 537).
V_2	difference of potentials between secondary terminals, in volts (p. 537).
v	peripheral velocity, in feet per minute (p. 356).

W	watts given to the rotor (p. 673).
w	in general, number of watts.
w_c	watts lost in copper of armature by resistance (pp. 368, 404).
w_e	watts lost by eddy-currents (p. 579).
w_f	watts lost by friction, windage, etc. (p. 406).
w_h	watts lost by hysteresis (p. 578).
w_i	watts lost in iron (by hysteresis and eddies) (p. 405).
w_x	watts lost in excitation (p. 406).

X	excitation, *i.e.* number of ampere-turns per pole (p. 291).
X	(in dispersion-formula) means number of slots per pole (p. 681).
X_1	reactance of primary of transformer (in ohms) (p. 632).
X_2	reactance of secondary of transformer (in ohms) (p. 632).
X_{sh}	equivalent reactance (p. 633).
X_p	ampere-turns per pole at full inductive load (p. 363).

Y	(in dispersion-formula) width of slits in slots (p. 681).
Y	flux-factor, in transformers (p. 610).
Y-connected,	*i.e.* the three phases united at a common junction (pp. 39, 294).
y	general symbol for ordinate (p. 51).
y	over-load factor, *i.e.* ratio of maximum load to normal load (p. 695).
y	winding-pitch (see Vol. I., p. 374) (pp. 200, 343, 434).

Z	total number of "conductors" around periphery of armature that are in series in any one circuit (p. 194).
Z_1	total number of conductors in stator (primary) (p. 683).
Z_2	total number of conductors in rotor (secondary) (p. 683).
Z_{sh}	equivalent impedance (p. 632).
z	number of conductors in one slot (p. 217).

GREEK LETTERS.

α	current density, *i.e.* amperes per square inch (p. 369).
α'	amperes per square inch, gross, in the active belt (p. 440).
β	output coefficient = $dl \div$ KVA, in generators, Steinmetz coefficient (p. 354).
β	(in motors) output coefficient = $dl \div$ HP (p. 710).
β'	lines per square inch, gross, in the active belt (p. 440).
γ'	peripheral speed coefficient (p. 440).
δ	length of air-gap from iron to iron, in inches (p. 375).
δ	general symbol for a finite difference (p. 386).
Δ-connected,	*i.e.* the three phases joined up as an equilateral triangle (p. 39).
ϵ	basis of Napierian logarithms (= 2·71828) (p. 42).
ϵ	load-factor, for transformer efficiency (p. 590).
ϵ	edge-thickness of tooth-tops, in inches (p. 681).
η	efficiency, *i.e.* ratio of output to input (pp. 385, 706).
η	hysteretic constant (p. 578).
η_s	efficiency of slot or space-factor, *i.e.* ratio of copper section to area of slot (p. 374).
θ	angle generally, or angle on circle of reference (p. 4).
θ	temperature-rise in degrees Centigrade (p. 625).
λ	ratio of l to d (p. 711).
λ	effective coefficient of self-induction (p. 594).
μ	permeability, *i.e.* ratio of $\mathfrak{B}$ to $\mathfrak{H}$ (p. 542).
ν	coefficient of allowance for magnetic leakage (p. 241).
ξ	output coefficient = $d^2 l \times$ RPM $\div$ KVA, Esson coefficient (p. 354).
π	ratio of circumference of circle to diameter (= 3·1416).
ρ	resistivity of material, in ohms per inch cube (p. 623).
σ	dispersion coefficient (pp. 241, 681, 707).
τ	pole-pitch at armature face = $\pi d \div p$ (p. 355).
ϕ	angle of phase-difference (p. 14), angle of lag (p. 21).
ϕ_2	angle of lag of rotor current (p. 686).
Φ	minimum angle of lag, or angle when cos ϕ is maximum (p. 692).
ψ	pole-breadth ratio (= $b \div \tau$) (p. 354).
ψ	angle of internal lag (p. 267).
Ω	angular velocity of rotating flux (p. 673).
ω	angular velocity of rotor (p. 673).

ERRATA

TO

DYNAMO-ELECTRIC MACHINERY, VOL. I.

(Seventh Edition, 1904.)

Readers are earnestly requested to make these revisions in ink in the volume in the pages where they are needed.—S. P. T.

p. 63, line 12 from bottom—*for* less *read* more.

p. 97, line 9 of second paragraph—*for* performed *read* transformed.

p. 194, line 3—*for* 400 to 600 *read* 254 to 480.

p. 230, line 12, in the equation—
for the sign of multiplication × *read* sign of equality =.

p. 230, line 23, in formula—*for* C *read* C_a.

p. 331, line 11—*for* 250 *read* 950.

p. 343, line 9 from bottom—*for* nC *read* nZ.

p. 355, in Fig. 209—
The spiral connector shown from *a* to 21 should go from *a* to 20.

p. 356, line 21—*for* wire-bound *read* wire-wound.

p. 424, last line—*for* insulation *read* insertion.

p. 503, line 2 from bottom—*for* Plate III. *read* Plate IV.

p. 542, line 9—*for* $a\pi + 2p$ *read* $d\pi \div 2p$.

p. 547, bottom line, in formula—*for* 50,000 *read* 500,000.

p. 590, line 13 from top, and line 3 from bottom—*for* 22·6 *read* 22·5.

p. 647, the sentence which begins on line 5 should be amended to read as follows :—

"This makes the pole-face density 34,300 instead of 32,000; and the tooth-density will be 34,300 + 0·582, about 59,000 at the tops, or about 69,500 having regard to the allowance of insulation between the laminations; making about 92,000 at the roots of the teeth."

p. 648, line 3—*for* 0·35 *read* 0·035.

p. 649, in line 3 of the Table—*for* 9·09 *read* 6·09.

p. 678, line 30—*for* 19,300 *read* 193,000.

p. 775, in the formula at bottom of the page—
for k_2 *read* k^2.

p. 777, line 11 of the text—
for equal *read* unequal.

p. 803, in formula at top of page—

$$for\ \frac{W}{w}\ read\ \frac{w}{W}.$$

p. 835, in equation [IV.], in the numerator—
for r_s *read* r_{sh}
and denominator should read—$r_a + r_m$.

p. 847, line 2 from bottom—*for* 522*c* *read* 522*e*

DYNAMO-ELECTRIC MACHINERY.

VOL. II.

CHAPTER I.

THE PRINCIPLES OF ALTERNATE CURRENTS.

IN alternate-current working the current is rapidly reversed, rising and falling in a succession of pulses or waves. Electricity is in fact oscillating backwards and forwards through the line with great rapidity under the influence of a rapidly-reversing electromotive-force. The adjectives *alternate*, *alternating*, *oscillatory*, *periodic*, *undulatory*, and *harmonic* have all been used to describe such currents. The properties of alternating currents differ in several respects from those of continuous currents. They are affected not only by the resistance of the circuit but also by the magnetic reaction commonly called self-induction or inductance; the inductance of the circuit having a choking effect on the alternating currents, diminishing the amplitude of the waves, retarding their phase and smoothing down their ripples. They are also affected by the capacity or condenser action of the circuit. If a condenser is placed in an electric circuit, it completely blocks the flow for continuous currents; but alternating currents can oscillate into and out of its electrodes as though the condenser allowed them to pass through. The larger the condenser the less does it obstruct the flow. On account of these peculiarities, some preliminary account of alternating currents is needed.

If a coil of suitable form is placed, as in Fig. 1, between the poles of a magnet, and spun around a longitudinal axis, it will have currents generated in it which at each semi-revolution die away and then reverse. In the figure the coil of wire is supposed to be so spun that the upper portion comes towards the observer. In that case, the arrows show the direction of the induced currents delivered to the circuit through the agency of two contact rings (or slip-rings) connected respectively to the ends of the coil. In the position shown, the current will be delivered to the left-hand ring, and returns from the circuit to the right-hand ring; but half a turn later it will be flowing to the right-hand ring and returning from the circuit back to the left-hand ring. Fig. 1 is, in fact, a primitive form of alternator, generating a simple periodically reversed or alternating current, and is, in fact, the kind of alternator known as a "magneto-ringer," used for bell service in telephone sets. The simple revolving coil, by cutting the lines of the magnetic field, sets up periodic electromotive-forces, which change at every half-turn, giving rise to alternate currents. In each whole revolution there will be an electromotive-force which rises to a maximum and then dies away, followed immediately by a reversed electromotive-force, which also grows to a maximum and then dies away. The wave-form depicted in Fig. 2 serves to illustrate this. The heights of the curve above the horizontal line represent

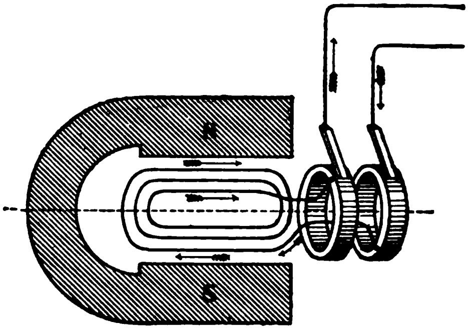

FIG. 1.—SIMPLEST FORM OF ALTERNATOR.

the momentary values of the electromotive-forces ; the depths below, in the second half of the curve, represent the inverse electromotive-forces that succeed them.

Frequency.—Each such complete set of operations is called a *period* or *cycle*, and the number of periods accomplished in a second is called the *frequency* or *periodicity* of the alternations, and is symbolised by the letter *f*. In 2-pole machines *f* is the same as the number of revolutions per second ; but in multipolar machines *f* is greater, in proportion to the number of pairs of poles. Thus, in an 8-pole field with four north poles and four south poles around a centre there will be produced four complete periods in one revolution. If the armature revolves 15 times a second (or 900 times a minute) there will be 60 periods a second, or the frequency will be 60 ~. The symbol ~ must be read as *periods per second*, or *cycles per second*. The standard frequency most in use in Great Britain is 50~, though 25~ is also used for motor driving. Formerly 100~ was in vogue. In the United States 133~ used to be the common frequency, but the later standards are 60~ and 25~. On the Continent, frequencies of 48~, 45~, 42~ and 40~ are found ; 42~ having been preferred because it is the lowest with which an arc-lamp can be worked without the flickering becoming objectionable. Glow-lamps show no flicker down to a frequency of about 20~. Low frequency is preferable for motor-driving : high frequency enables smaller transformers to be used.

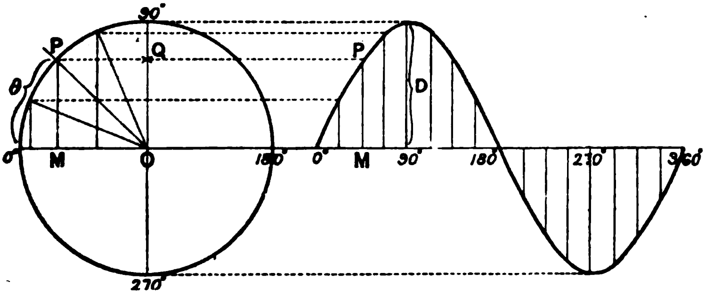

FIG. 2.—SINE CURVE ILLUSTRATING ALTERNATING ELECTROMOTIVE-FORCE.

Angle of Reference. Electrical Degrees.—By revolving a coil in a uniform (bipolar) field, the electromotive-forces set up are proportional to the sine of the angle through which the coil has turned from the position in which it lay across the field. If in this position the flux of magnetic lines through it were N, and the number of spirals in the coil that enclose the N lines be called S, then, as is readily shown, the value of the induced electromotive-force at any time t, when the coil has turned[1] through angle $\theta = 2\,\pi\,n\,t$, will be

$$E_\theta = 2\,\pi\,n\,S\,N\,\sin\theta \div 10^8\,;$$

or, writing D for $2\,\pi\,n\,S\,N\,/\,10^8$, we have

$$E_\theta = D\sin\theta.$$

In actual machines the magnetic fields are not uniform, nor the coils simple loops, so the periodic rise and fall of the electromotive-forces will not necessarily follow a simple sine law. The form of the impressed waves will depend on the shape of the polar faces, and on the form and breadth of the coils. But in most cases we are sufficiently justified in assuming that the impressed electromotive-force follows a sine law, so that the value at any instant may be expressed in the above form, where D is the maximum value or *amplitude* attained by E, and θ an angle of *phase* upon an imaginary circle of reference. As diagrams of lines revolving around a centre are much used in explaining alternate-current actions, the following explanation[2] should be most carefully followed. Consider a point P revolving clockwise round a circle (Fig. 2). If the radius of this circle be taken as unity, P M will be the sine of the angle θ, as measured from 0°. Let the circle be divided into any number of equal angles, and let the sines be drawn similarly for each. Then let these sines be plotted out at

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[1] If n is the number of revolutions per second, $2\,\pi\,n$ will be the total angle (in radians) turned through in one second. Hence, the angle turned through (which we call θ), in any short time t will be equal to t times $2\,\pi\,n$. For example, if $n = 15$, $2\pi n = 94{\cdot}2$ radians per second, and during, say one-eightieth of 1 second, the angle passed over will be $1{\cdot}18$ radians, or about 67°.

[2] Those who are not familiar with the problems of simple harmonic motion should consult some modern treatise of theoretical mechanics on the subject.

equal distances apart along the horizontal line, as in Fig. 2, giving us the *sine-curve.*

Now, the use that we make of this diagram is this. We know that as time goes on, the value of the electromotive-force is changing from instant to instant. To find its value at any particular instant, we *treat time as if it were an ever increasing angle*; we take the number of seconds or the fraction of a second, that has elapsed since a certain instant t (when the electromotive-force was zero), and multiply it by $2 \pi n$, then considering this as an angle expressed in radians, the sine of this angle multiplied by D gives us in volts the electromotive-force for the particular instant. It will therefore be seen that the point P, in revolving uniformly round the circle in Fig. 2, represents the lapse of time. If we consider it revolving at such a speed that it passes through $2 \pi n$ radians in one second, then the perpendicular P M represents (to some scale or other) the electromotive-force at any particular instant. Now taking the horizontal line 0° – 360° to represent time (to some convenient scale), it is evident that after the lapse of the time measured by the distance from 0° to M the electromotive-force has the value M P; and in the same way, at any other instant, the electromotive-force is represented by the perpendicular drawn from that point in the line which represents the instant to the sine-curve shown in the figure. In Fig. 2, one revolution of P around the circle of reference corresponds to one complete alternation or cycle of changes. The value of the electromotive-force (which varies between $+D$ and $-D$ as its maximum values) may be represented at any moment either by the sine P M or by projecting P on to the vertical diameter, giving O Q. As P revolves, the point Q will oscillate along the diameter. We may, therefore, without drawing our sine-curve at all, merely consider a line O P (drawn to some scale to represent D) as revolving round O, and take its projection O Q at any instant as the electromotive-force. Such a diagram is known as a *clock diagram.*

In the case of multipolar machines, the inductive actions that go on in any coil or conductor are repeated at every pair of poles, hence the frequency f will be equal to the number of *revolutions per second* multiplied by the number of *pairs of poles*. Or, if p be the number of poles,

$$f = \frac{p}{2} \times \frac{\text{revolutions per minute}}{60};$$

$$f = \frac{p \times \text{RPM}}{120}.$$

Now in such multipolar machines the electrical cycle of operations repeating $\frac{1}{2}p$ times in each revolution, if we wish to represent the electromotive-force or the current as a sine-function of the periodic time, we see that the cycle of operations repeats itself while the machine moves, not through 360° but through $360° \div \frac{1}{2}p$. Therefore, we refer the periodic change to an imaginary circle of reference, round which a pointer is supposed to revolve once in each cycle, and we refer the sine-function to the degrees around this *circle of reference*, and call them *electrical degrees*. Thus in a 10-pole machine, the cycle of operations goes through a whole 360 electrical degrees, while the machine itself moves over the angular breadth of but two poles, that is 72 actual degrees.

Periodic Currents.—The currents which result from these periodic or alternating electromotive-forces are also periodic and alternating; they increase to a maximum, then die away and reverse in direction, increase, die away, and then reverse back again. If the electromotive-force completes 50 such cycles or reversals in a second, so also will the current.

We may realize the state of things by taking a numerical illustration of an alternating current. For convenience, let the time of one period (itself lasting say one-fiftieth of a second) be considered as divided into a number of equal parts, twenty-four parts will be a convenient number. The current, as we know, rises and falls, reverses, and finally, at the end of the period, reverses back. During the first quarter of the period it is rising to its maximum; during the second quarter it is falling back to zero; during the third quarter it is again rising to a reversed or negative maximum; and during the fourth, it again returns to zero. Supposing that the maximum to which the current attains is 100 amperes. Then its values may be calculated for different times from the corresponding angles by multiplying the sine of the angle by 100. So we represent the current by the following statistical table.

First Quarter.			Second Quarter.			Third Quarter.			Fourth Quarter.		
Stage.	Angle.	Current.	Stage.	Angle.	Current.	Stage.	Angle.	Current.	Stage.	Angle.	Current.
	°			°			°			°	
0	0	0	6	90	+100	12	180	0	18	270	−100
1	15	+ 26	7	105	+ 97	13	195	− 26	19	285	− 97
2	30	+ 50	8	120	+ 87	14	210	− 50	20	300	− 87
3	45	+ 71	9	135	+ 71	15	225	− 71	21	315	− 71
4	60	+ 87	10	150	+ 50	16	240	− 87	22	330	− 50
5	75	+ 97	11	165	+ 26	17	255	− 97	23	345	− 26
6	90	+100	12	180	0	18	270	−100	24	360	0

The + sign indicates the current flowing forward during the first half-period; the − sign indicates its flowing back during the second half-period. These values, if plotted out as a series of equi-distant ordinates give us a sine-curve like Fig. 2. The + values are plotted upwards above the horizontal base-line (which serves as a time-scale as well as a scale of angle), while the − values are plotted downwards.

There is yet another way of representing periodic variations of this kind—namely, by a diagram akin to that used by Zeuner for valve-gears. Let the outer circle (Fig. 3) be as before a circle of reference around which P revolves. Upon each of the vertical radii describe a circle. Then the lengths such as O Q, cut off from the radii, represent the corresponding values of the sine of the angle. If a card with a narrow slit cut radially

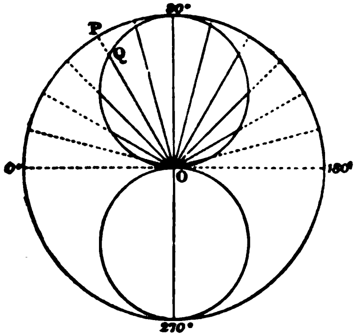

FIG. 3.

in it were made to revolve over this figure, the intersection with the two inner circles would show the varying electromotive-forces in various positions.

The reader who desires to pursue the graphic study of these matters further should consult the excellent treatise of Prof. Fleming,[1] or that of Mr. Blakesley,[2] and sundry papers by Mr. Kapp.[3] Bedell and Crehore[4] devote a whole chapter to the subject. In the case of real machines in which the magnetic fields are not uniform, nor the coils simple loops, the periodic rise and fall of the electromotive-forces will not necessarily follow a simple sine-law. The form of the impressed waves will depend on the shape of the polar faces, and on the form and breadth of the coils. Consider the case of a machine with revolving armature in which the field-magnets consist of a series of fixed poles pointing radially inward. Suppose the pole-faces to be of rectangular outline. If the armature coils are also of simple rectangular form and of the same breadth, the rate of cutting of the magnetic flux will be very nearly a uniform rate that reverses at every pole, and therefore, the form of the electromotive-force curve will be by no means a sine-curve, but will be more nearly like Fig. 4. By appropri-

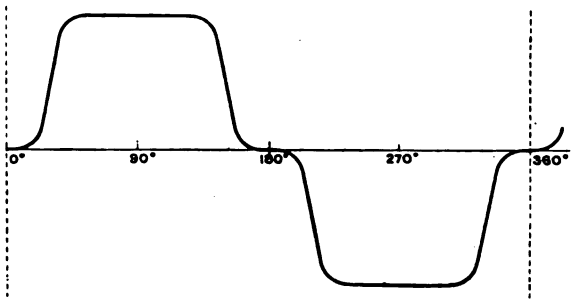

FIG. 4.

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[1] Fleming, *The Alternate Current Transformer*, London, 1889. Also a paper on Polar Diagrams, *Electrician*, xxxv. 43.

[2] Blakesley, *Alternating Currents of Electricity*, London, 1889.

[3] Kapp on "Alternate Current Machinery," *Proc. Inst. Civil Engineers*, 1889, pt. iii.

[4] Bedell and Crehore, *Alternating Currents*, London, 1893.

ately shaping the pole-faces, and spreading the coils, modifications in the wave-form may be produced. These modifications are considered in Chapter IV. But if the design of the machine is such as to give a curve differing widely from a pure wave-form, then the algebraic expression must be also modified; for this case the simple sine law cannot be true for the electromotive-force. In order to test whether in any given dynamo the rise and fall of electromotive-force and of current in the armature coils conforms to the law of sines, experiments are necessary. Joubert, in order to measure the currents of a Siemens alternator, employed an electrometer method, and took off the current at any desired phase by a special commutator, and found an approximate curve of sines.[1] Other methods are described later. As machines are actually built, they give curves which are mostly smooth wave-forms.

In Fig. 5 are given four curves for a half-period. Of these one is a sine-curve, the other three are taken from actual alternators, showing how nearly they agree with a true sine-curve. The one which agrees most nearly is that of the Mordey alternator, which lies just within the sine-curve nearly

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[1] For references as to modern varieties of this method see p. 236. During recent years many experimental methods have been given for determining the shape of the curve followed by the variations of alternating electromotive-forces and currents. The reader should consult the methods pursued by Ryan, *Amer. Inst. Elect. Engineers*, 1888 and 1889; also *Electrician*, xxiv. 263, 1890; Bedell, Miller and Wagner, *Amer. Inst. Elect. Engineers*, x. p. 500; Fleming, *Electrician*, xxxiv. 460, 507, 1895; L. Duncan, *ibid.* 617; Hicks, *ibid.* 698. Fleming's method is applicable to determine the form of the current curve at any part of a circuit. See also a paper by Barr, Burnie and Rodgers, *Electrician*, xxxv. 719, and Duddell, *ibid.*, xxxix. 636, 1897.

Some controversy arose in the columns of the *Electrician* and of the *Electrical World*, in the autumn of 1894, as to whether there was any advantage in alternators giving a sine-curve. Fleming has since found that certain transformers worked with a distinctly higher efficiency when operated by an alternator giving a peaked curve than when operated by one giving a nearly pure sine-curve for the electromotive-force. On the other hand, this form appears to be undesirable for motor-running. As a matter of fact, the form of the current curve depends, not only on the construction of the alternator, but also upon the modifying influences of capacity and self-induction in the circuit. The presence, in the circuit, of transformers with iron cores and of motors will modify the curve; and the modification will specially depend on the degree of saturation to which the iron cores are carried at each cycle. A paper by Barr, Beeton and Taylor, in the *Electrician*, xxxv. 257, 268, is of great importance.

throughout its whole extent. The curve is usually more peaked in machines which have the coils sunk between iron teeth and have much armature-reaction. In the first of the Niagara generators they are, on the contrary, rather flatter-topped and broader than true sine-curves. We are then sufficiently justified in assuming that the impressed electromotive-force generally follows a sine-law. We shall see presently that there are certain quantities, namely, the heating, the power, the torque, and certain mechanical reactions, which instead of having one positive maximum, and one negative maximum in each period, have two maxima of each kind. These may be called double-frequency quantities.

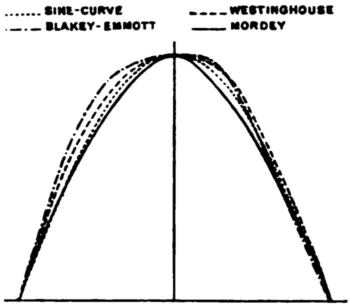

FIG. 5.
CURVES OF ALTERNATORS.

"*Virtual*" *Volts and Amperes.*—Alternate-current voltmeters and alternate-current amperemeters do not measure the arithmetical average values of the volts and of the amperes. They measure what are called *virtual volts* and *virtual amperes.* In a Cardew voltmeter the heating of the wire depends on the square of the current. In an electro-dynamometer the torque depends at every instant on the product of the currents in the fixed and movable parts; therefore, when used as an amperemeter, depends on the square of the current. The attraction (or repulsion) in electrostatic voltmeters is proportional to the square of the volts. The readings which these instruments give us, if first calibrated by using steady currents, are not true means, but are *the square roots of the means of the squares.* Now the mean[1] of the squares of the sine (taken over either one quadrant or a whole circle) is ½; hence the square-root-of-mean-square value of the sine functions is got by multiplying their maximum value by

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[1] See proof, p. 17.

$1 \div \sqrt{2}$, or by 0·707. But[1] the arithmetical mean of the values of the sine is 0·637. Hence an alternating current, if it obey the sine-law, will produce a heating effect greater than that of a steady current of the same average strength, by the ratio of 0·707 to 0·637; *i.e.* about 1·1 times greater. If a Cardew voltmeter is placed on an alternating circuit in which the volts are oscillating between maxima of + 100 and − 100 volts, it will read 70·7 volts, though the arithmetical mean is really only 63·7; and 70·7 steady volts would be required to produce an equal reading.

The matter may be looked at in a different way. If an alternating current is to produce in a given wire the same amount of effect as a continuous current of 100 amperes, since the alternating current goes down to zero twice in each period, it is clear that it must at some point in the period rise to a maximum greater than 100 amperes. How much greater must the maximum be? The answer is that, if it undulates up and down with a pure wave-form, its maximum must be $\sqrt{2}$ (*i.e.* = 1·414) times as great as the virtual mean; or conversely the virtual amperes will be equal to the maximum divided by $\sqrt{2}$. In fact, to produce equal effect the equivalent continuous current will be a kind of mean between the maximum and the zero value of the alternating current; but it must not be the arithmetical mean, nor the geometrical mean, nor the harmonic mean, but the *quadratic* mean; that is, it will be the square-root of the mean of the squares of all the instantaneous values between zero and maximum.

Those not familiar with the subject may be helped by trying for themselves arithmetically to find the quadratic mean value of an alternating current, that has a maximum of 100 amperes, from the table of instantaneous values given on p. 7. Take each number as the successive instants, square each, and find the mean of the

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[1] Or, more strictly,

$$\frac{1}{\theta}\int_0^\theta \sin\theta\, d\theta = \frac{1-\cos\theta}{\theta},$$

whence, if $\theta = \frac{\pi}{2}$, the average is $\frac{2}{\pi}$.

squares. Then take the square root. The successive numbers will run thus:—

First Half Period.

Stage . .	0	1	2	3	4	5	6	7	8	9	10	11	12
Current . .	0	+26	+50	+71	+87	+97	+100	+97	+87	+71	+50	+26	0
Square . .	0	676	2500	5041	7569	9409	10000	9409	7569	5041	2500	676	0

Second Half Period.

Stage . .	13	14	15	16	17	18	19	20	21	22	23	24
Current . .	−26	−50	−71	−87	−97	−100	−97	−87	−71	−50	−26	0
Square . .	676	2500	5041	7569	9409	10000	9409	7569	5041	2500	676	0

The total of the squares amounts to 120,780. Dividing by 24 to get the mean square we have 5032. The square root of this is 70·9. Had the calculation been carried out with precision to several places of decimals instead of assigning whole numbers at the various stages, the value would have been found to be 70·7, which is the exact figure.

The term *virtual*[1] has been used to denote these square-root-of-mean-square values. If an alternate current ampere-meter reads 100 amperes, that means that the current really rises to + 141·4 amperes and then reverses to − 141·4 amperes; but the heating effect and the amount of power delivered, are the same as if the current were 100 continuous amperes, and therefore such a current would be described as 100 *virtual amperes.*

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[1] I adhere to the term *virtual,* as it was in use before the term *efficace* which was recommended in 1889 by the Paris Congress to denote the square-root-of-mean-square value. The corresponding English adjective is *efficacious*; but some engineers mistranslate it with the word *effective.* I adhere to the term *virtual* mainly because the adjective *effective* is required in its usual meaning in kinematics to represent the resolved part of a force which acts obliquely to the line of motion, the effective force being the whole force multiplied by the cosine of the angle at which it acts with respect to the direction of motion. Some authors use the expression "R M S value" (meaning "root mean square") to denote the virtual or quadratic mean value.

It follows from the argument on p. 11 that the virtual electromotive-force of an armature wound for alternate currents will be 1·1 times higher (compare p. 11) than that of the same armature wound as a continuous-current dynamo of the same current-carrying capacity, with the same number of conductors in series; or will be 2·2 times higher if, while the same wire is used, the alternator winding is not re-entrant, but forms a single circuit.

The distinction between virtual and maximum values is important, since certain effects—for example the tendency to pierce insulation with a spark—depend on maximum, not on virtual values. For example, if an electrostatic voltmeter reads 10,000 volts, the maximum value (supposing the law of variation a sine-law) will be 14,142 volts. If the form is more peaked than that of the sine-curve, the maximum will be higher relatively to the virtual value.

We, therefore, have the following relations, provided the volts or amperes follow a pure sine function:—

$$\frac{\text{maximum volts}}{\text{virtual volts}} = 1{\cdot}414;$$

$$\frac{\text{maximum amperes}}{\text{virtual amperes}} = 1{\cdot}414.$$

If the volts or amperes vary in some other way, that is, if their curve be not a pure sine-curve, the ratio of the maximum to the virtual value will not be 1·414 It may be less or more, according to circumstances. See p. 218 for an example worked out of the virtual value or quadratic mean of an irregular curve.

Use of Clock Diagrams.—In these polar diagrams the amperes or the volts that are undergoing periodic cycles of change are represented by the projection on some given line (in this book, the projection on a vertical line is taken) of a line supposed to revolve about a centre. Such diagrams are of so frequent use in the study of alternating currents that a few further points about them are given.

Differences of *phase* are in the clock diagram represented by differences of angular position. For example, if two revolving pointers O V and O C (Fig. 6) are going round at the same rate, but always one a little behind the other, they will not come to their respective maxima at the same instant. Projecting them upon the vertical line we see that at the moment when O V has revolved so far that the angle of position is θ, its projection will have the value O v; while the other pointer, which lags behind by an amount measured by the angle ϕ (= V O C), has for its value as projected, the length O c. When O v gets to its maximum (that is when V arrives at the top), O c will still be behindhand. The values of the two projections are O v = O V . sin θ; and O c = O C . sin ($\theta - \phi$). The angle ϕ is the *difference of phase.*

To add together two different alternating quantities—for instance two electromotive-forces—that have the same period, it is not sufficient simply to add their numerical values. For instance, if there are two coils in series in a circuit in one of which there is being induced an alternating electromotive-force of 40 volts, and in the other an alternating electromotive-force of 30 volts (both having, let us say, the same frequency of 100 periods per second), the total electromotive-force will not be 70 volts unless the two electromotive-forces happen to be exactly "in phase." If there is any difference of phase between them the resultant will be less than 70 because they do not come to their maxima at the same time. To ascertain the value they have when added together we must apply the principle of summation of vectors with which

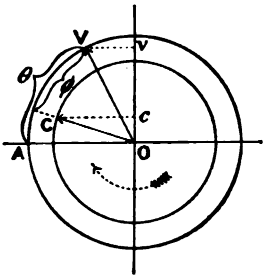


FIG. 6.

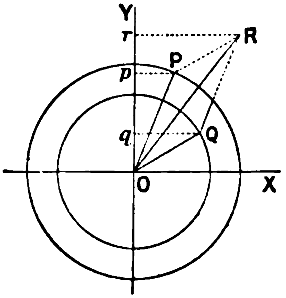


FIG. 7.

every engineer is familiar in the ordinary compounding of forces by constructing a parallelogram, as in Fig. 7.

Let O P and O Q represent two electromotive-forces, of the same period, but with a phase-difference between them of P O Q which we may call angle ϕ. Completing the parallelogram by drawing P R equal and parallel to O Q, we get the resultant O R which represents the relative magnitude and phase of the resultant revolving vector. The projection O r of this line will always be equal to the sum of the projections O p and O q of the two components. Now, by ordinary geometry we have $O R = \sqrt{OP^2 + OQ^2 + 2\ PQ \cos \phi}$. This is obviously a maximum when $\phi =$ zero. For instance, if in the above example O P = 40, O Q = 30, and $\phi = 37°$, it will be found that the resultant O R is 66·6.

If the two components are at right angles to one another, on the diagram one will have its maximum at the instant when the other has its minimum. They are then said to be *in quadrature*, or as some electricians say, in *quarter-phase.* If they are equal in themselves, the resultant will be greater than they in the proportion $\sqrt{2}$ to 1. For example, the resultant of two alternating electromotive-forces of equal period, of 100 (virtual) volts each, that are in quadrature, is 141·4 (virtual) volts.

Products of Periodic Functions.—Suppose we have two periodic functions—say two currents, or a current and an electromotive-force — both varying with the same periodicity, but having different amplitudes and a difference of phase between them. Let one be called $p = O P \cos \theta$; the other $q = O Q \cos \beta$; where O P and O Q are their respective maximum values (as in Fig. 8), and ϕ the angle of phase-difference between them equal to $\beta - \theta$. Now, suppose we want to find the mean value of the product $p q$. This product will itself vary but not as a mere sine function, and therefore is incapable of being represented as a line revolving. It will at

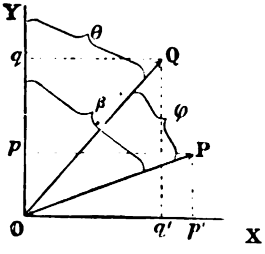


FIG. 8.

certain instants—four times in each cycle—have zero values, for p comes twice to zero, and q comes also twice to zero. It will also have negative values when either p or q is negative. Its mean value will be the mean of all the values of the product during one complete cycle.

At the instant shown the product will be $p\,q = \text{O P} \,.\, \text{O Q} \cos\theta \,.\, \cos\beta$. A quarter-period later the two lines O P and O Q will stand to the axis $-$ O Y in the same relation as they now stand to the axis O X, and the product (being positive) will then be

$$p'\,q' = \text{O P} \,.\, \text{O Q} \sin\theta \,.\, \sin\beta.$$

Taking the mean of these two values, we have

$$\begin{aligned} \frac{p\,q + p'\,q'}{2} &= \tfrac{1}{2}\,\text{O P} \,.\, \text{O Q} \,(\cos\theta \,.\, \cos\beta + \sin\theta \,.\, \sin\beta) \\ &= \tfrac{1}{2}\,\text{O P} \,.\, \text{O Q} \cos(\beta - \theta) \\ &= \tfrac{1}{2}\,\text{O P} \,.\, \text{O Q} \cos\phi. \end{aligned}$$

As an example, let us take the case of the product of 100 alternating volts and 40 alternating amperes, the latter having a difference of phase of 20° from the former. Then the maxima, obtained by multiplying by $\sqrt{2}$, will be 141·4 amperes and 56·56 volts. Then the mean product will have the value, $\frac{1}{2} \times 141{\cdot}4 \times 56{\cdot}56 \times \cos 20°$. Now as $20° = 0{\cdot}94$, hence the mean product will be 3760 watts.

Now this is obviously independent of the actual position of θ or of β; that is to say, for every position the mean of the value between that position and the position at right angles is the same all the way round. Hence this value is the required true mean value of the product.

We shall make use of this theorem later.

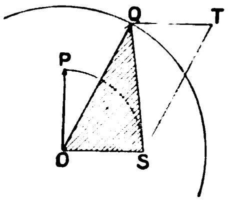


Fig. 9.

A geometrical construction to illustrate the above is given in Fig. 9. Let O P and O Q represent the maximum values of two periodic functions as having phase-difference the angle ϕ or P O Q. Turn either of them (in this case O P) through a right-angle so that it occupies the position O S, then complete parallelogram O Q T S, and draw the triangle O Q S. The area of the parallelogram is equal to O P . O Q cos ϕ, and the area of the triangle is equal to $\frac{1}{2}$ O P . O Q cos ϕ, and therefore represents the mean product.

A further deduction is of use. Suppose p and q to be identical; we shall then obtain the mean value of the square of the periodic function by writing O Q = O P and $\phi = 0$; so that $\cos \phi = 1$. Then we get,

$$\text{mean value of } p^2 = \tfrac{1}{2} (\text{O P})^2.$$

In other words, the mean value of the square of the sine is $\frac{1}{2}$.

If V stands for the virtual value of the volts and C for the virtual value of the amperes, we see that the respective maxima will be $\sqrt{2} \times \text{V}$ and $\sqrt{2} \times \text{C}$. If we replace these for O P and O Q, we get:

$$\text{mean product} = \tfrac{1}{2} \times \sqrt{2}\,\text{V} \times \sqrt{2}\,\text{C} \times \cos \phi\,;$$

which is the same as

$$\text{mean product} = \text{V} \times \text{C} \times \cos \phi,$$

or, the actual *watts* are found by multiplying together the virtual volts and the virtual amperes, and multiplying the products of the cosine of the angle of their phase-difference.

Taking the preceding example, we obtain the same result as before by multiplying:—

$$100 \times 40 \times 0{\cdot}94 = 3760 \text{ watts.}$$

Lag and Lead.—Alternating currents do not always keep in step with the alternating volts impressed upon the circuit. If there is inductance in the circuit the currents will *lag*; if there is capacity in the circuit they will *lead* in phase. Fig. 10 illustrates the lag produced by inductance. The curve marked V represents the alternating volts; that marked C is the current curve. Distances measured from O along the horizontal line represent time. These curves are in fact similar to what would be obtained if curves were plotted from Fig. 6 in the same way as that plotted in Fig. 2, the points V and C being taken instead of the point P. The impulses of current, represented by the blacker line, occur a little *later* than those of the volts. But inductance has another effect of more importance than any retardation of phase; it produces reactions on the electromotive-force, choking the current down. While the current is increasing in strength the reactive effect of inductance tends to prevent it rising, and while the current is falling in strength the reactive effect tends to keep it from falling. To produce a current of 40 amperes in a resistance of 1½ ohms would require—for continuous currents—an E.M.F. of 60 volts. But an alternating voltage of 60 volts will not be enough if there is inductance in the circuit reacting against the voltage. The matter is complicated by the circumstance that the reactive impulses of electromotive-force are also out of step: they are, in fact, exactly a quarter period behind the current.

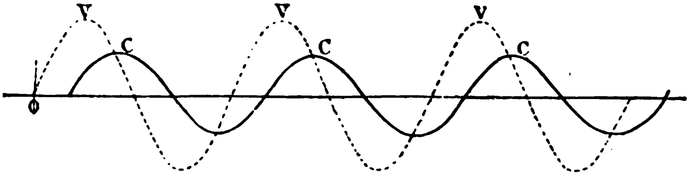


FIG. 10.—CURVE OF CURRENT LAGGING BEHIND CURVE OF VOLTS.

The Reaction of Inductance.—It is known that every current is surrounded with a whirl of magnetic lines all along its length, the number depending on the permeability of the medium, and the distance between the going and returning wires. If the circuit consists of coils whose convolutions lie near one another, the whirls or loops of magnetic lines belonging to one part of the circuit will enclose another part of the circuit; so that whenever the current is growing or dying away these loops of magnetic lines will be cutting across some other part of the circuit. In fact, there will be *self-induction*, and the amount of cutting of magnetic lines that goes on when unit current is turned on or off (and which we may call the co-efficient of self-induction, symbol L) will be proportional to the square of the number of spirals so reacting; or L is proportional to S^2. The presence of an iron core helps the magnetic flux due to each convolution to thread itself around all the other convolutions. If the sectional area, length, and permeability of the magnetic circuit in question are A, l and μ; then $L = 4\pi S^2 \mu \div 10^9 l$; the values being in C.G.S. units. The factor 10^9 is introduced because the unit of induction, the *henry*, is chosen to correspond with the ohm and the other units.

So then whenever, in a circuit having an inductance L, the current is growing, there will be a self-induced electromotive-force reacting and tending to prevent the current growing; and the magnitude of this will be proportional both to L and to the rate of change of current. If an alternate current of C (virtual) amperes is flowing with a frequency of f cycles per second through a circuit of inductance L, the reactive electromotive-force,[1] will be $2\pi f L C$ (virtual) volts. If, for example, $L = 0{\cdot}002$ henry, $f = 50$ periods per second, and $C = 40$ amperes, the reactive electromotive-force will be 25·1 volts. Now, if we wish to drive the 40 (virtual) amperes not only through the resistance of 1½ ohms but against this reaction, we shall require more than 60 volts. But we shall not require 60 + 25·1 volts, since the reaction is out of step with the current. Ohm's law is no longer adequate by itself as a guide. To find out what volts will be needed we must calculate, either by algebra, or by geometry; and for greater simplicity we will have recourse to geometry.

Geometrical Investigation of the Law of Alternate Currents.—Plot out (Fig. 11) the wave form O A b d, to correspond to the volts necessary to drive the current through the resistance, if there were no inductance. The ordinate a A may be taken to scale as 60. This we may call the R C

---

[1] This is calculated as follows. By definition, L, the coefficient of self-induction, or inductance, represents the amount of self-enclosing of magnetic lines by the circuit when the current has unit value; when current has value C the number of lines enclosed is C times L. And, as the self-induced electromotive-force is proportional to the rate of change of this number, we may write $E = L.dC/dt$. Now C is assumed to be a sine function of the time having instantaneous value $C_0 \sin 2\pi f t$; where C_0 is the maximum value of C. Differentiating this with respect to time we get $dC/dt = 2\pi f C_0 \cos 2\pi f t$. The "virtual" values of cosine and sine being equal, we have for E the value $\pi f L C$, but differing in phase from the current by a ¼ period.

curve. Then plot out the curve marked $-p$ L C to represent the volts needed to balance the reaction of the inductance. Here p is written for $2\pi f$. The ordinate at O is 25·1 : and the curve is shifted back one-quarter of the period : for when the current is increasing at its greatest rate, as at O, the self-inductive action is greatest. Then compound these two curves by adding their ordinates, and we get the dotted curve, with its maximum at V. This is the curve of the volts that must be impressed on the circuit in order to produce the current. It will be seen that the current curve attains its maximum a little after the voltage curve. The current lags in phase behind the volts. If O d is the time of one complete period the length $v\,a$ will represent the time that elapses between the maxima of volts and amperes. In Fig. 12 the same facts are represented in a revolving diagram of the same sort as Fig. 6. The line O A represents the working volts R × C, whilst the line A D at right angles to O A represents the self-induced volts p L C. Compounding these

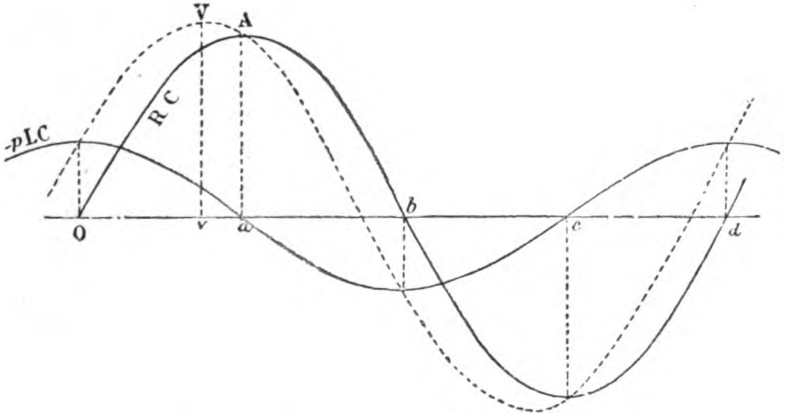


FIG. 11.

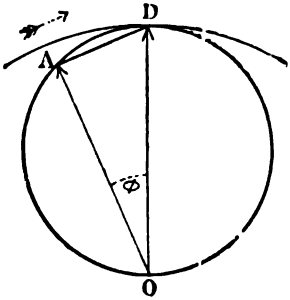


FIG. 12.

as by the triangle of forces, we have as the impressed volts the line O D. The projections of these three lines on a vertical line while the diagram revolves around the centre O give the instantaneous values of the three quantities. The angle A O D, or ϕ, by which the current lags behind the impressed volts, is termed the *angle of lag*. However great the inductance or the frequency, angle ϕ can never be greater than 90°. If O A is 60 and A D is 25·1, O D will be 65 volts. In symbols, the impressed volts will have to be such that $E^2 = (RC)^2 + (pLC)^2$. This gives us the equation:

$$C = \frac{E}{\sqrt{R^2 + p^2 L^2}} \quad . \quad . \quad . \quad . \quad . \quad [I]$$

The denominator which comes in here is commonly called[1] *the impedance*. Comparing this with the law for continuous currents, namely

$$C = \frac{E}{R},$$

we see that the effect of the inductance is to make the circuit act as if its resistance, instead of being R, was increased to $\sqrt{R^2 + p^2 L^2}$. In fact, the alternate current is governed, not by the resistance of the circuit, but by its impedance. The equation tells us the *magnitude* of the current, but not its phase.

In Figs. 13 and 14 the angle of lag is seen to be such that $\tan \phi = pLC / RC$ or $= pL / R$. The current is lagging as if the angle of reference were not θ but $\theta - \phi$, so that the equation for C_t the instantaneous value of C at the moment when $E = D \sin \theta$, is

$$C_t = \frac{D \sin (\theta - \phi)}{\sqrt{R^2 + p^2 L^2}} \quad . \quad . \quad . \quad . \quad . \quad [II]$$

---

[1] The term *impedance* strictly means the ratio of any impressed electromotive-force to the current which it produces in a conductor (see Lodge's *Modern Views*, p. 398), of which the above is only one case. For steady currents the impedance is simply the resistance. For variable currents it may be made up of resistance, of inductance, and (if the circuit has electrostatic capacity), of permittance, in various proportions according to the *form* of the variation. For true periodic currents obeying the sine-law the impedance is the square root of the sum of the squares of resistance and reactance. For currents which vary more suddenly the impedance will depend more on self-induction and less on resistance.

This is Maxwell's law[1] for periodic currents as retarded by inductance. As amperemeters and voltmeters take no account of phase but give virtual values, the simpler form preceding is usually sufficient.

The relation between resistance and impedance is readily got from the triangle in Fig. 14; for clearly the angle ϕ is such that

$$\sin \phi = \frac{p\,L}{\sqrt{R^2 + p^2 L^2}},$$

$$\cos \phi = \frac{R}{\sqrt{R^2 + p^2 L^2}},$$

$$\tan \phi = \frac{p\,L}{R}.$$

If we prefer we may substitute for the impedance in the denominators of the preceding equations its value R / cos ϕ.

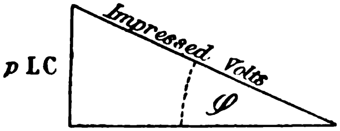


FIG. 13.

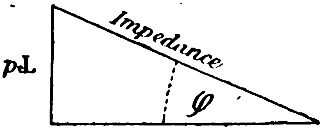


FIG. 14.

The equations established above hold good, whether maximum or virtual values are used. For example, we may write

$$\text{Maximum C} = \frac{\text{maximum E}}{\text{impedance}};$$

or

$$\text{Maximum C} = \frac{\text{maximum E}}{\text{resistance}} \times \cos \phi;$$

and

$$\text{Virtual C} = \frac{\text{virtual E}}{\text{impedance}};$$

or

$$\text{Virtual C} = \frac{\text{virtual E}}{\text{resistance}} \times \cos \phi.$$

---

[1] The analytical proof is given at the end of the present Chapter, p. 42.

The clock diagrams of revolving lines may be drawn either with maximum or virtual values.

Reactance Voltage.—The volts required to drive an alternating current against the reaction of the self-induction, measure the *reactance voltage.* To calculate it, the coefficient of self-induction must be known, for:

$$\text{reactance voltage} = 2\pi f \text{L C} = p \text{L C}.$$

Example.—To find the reactance voltage of a current of 10 amperes, delivered at 50 ~, through a coil having a coefficient of self-induction of 0·15 henry. By the formula, we have:

$$2 \times 3{\cdot}141 \times 50 \times 0{\cdot}15 \times 10 = 471{\cdot}2 \text{ volts.}$$

It must be remembered that this reactance voltage must *not* be added arithmetically to the voltage required to drive the current through the resistance, because it is in quadrature with the ohmic voltage. This is shown in the triangle Fig. 13. They must be compounded vectorially at right angles; or in symbols:

$$\text{E} = \sqrt{(\text{RC})^2 + (p\,\text{LC})^2}.$$

Example.—Taking the same case as before; let the resistance of the coil be 8 ohms. Then the ohmic voltage required to send the 10 amperes through it will be 80 volts; and the whole voltage needed for the coil will be the square root of $(80)^2 + (471{\cdot}2)^2 = 478$ volts.

Another way of regarding the matter is to treat the quantity p L or $2\pi f$ L (which is the inductance part of the impedance) as a sort of resistance. It may be expressed in ohms; but these reactance ohms must not be added to the actual ohms. They must be compounded with them at right angles, as in the triangle in Fig. 14.

Example.—In the same example, the coil has a true resistance of 8 ohms, and its reactance ohms are $2 \times 3{\cdot}141 \times 50 \times 0{\cdot}15 = 47{\cdot}1$ ohms. Hence its whole impedance is the square root of $(8)^2 + (47{\cdot}1)^2 = 47{\cdot}8$ ohms.

Effect of Capacity.—When an electromotive-force is applied to a condenser, the current plays in and out, charging the condenser in alternate directions. As the current runs in at one side and out at the other, the dielectric becomes charged, and tries to discharge itself by setting up an opposing electromotive-force. Its opposing potential rises just as its charge increases. A mechanical analogue is afforded by the bending of a spring, which as it is being bent exerts a back-force proportional to the amount of bending to which it has been subjected. When a periodic force is applied to a spring the elasticity of the spring tends to hasten the return movement. In like manner the electric elasticity of a condenser tends to hasten the return flow of the current.

The effect of capacity introduced into an alternate current circuit is to produce a *lead* in the phase of the current, since the reaction of a condenser, instead of tending to prolong the current, tends to drive it back. The student must clearly distinguish between the case of capacity in series with a circuit and the case of capacity in parallel with a branch of a circuit. What is said here refers to capacity in series, that is to say, the conductor of the circuit is actually cut and the ends joined to a condenser so that no current can flow except into and out of the condenser. If the capacity is in parallel with a branch of a circuit, and we are considering what happens in that branch when there is a *given* alternating pressure at its ends, the capacity in parallel has no effect at all. If we are only given the pressure at some other part of the circuit, then the problem becomes more complex and involves the impedances of the circuit's various branches. Returning then to a simple circuit with a condenser in series, the smaller the capacity of the condenser the more does it react. The reactance due to capacity is therefore written as $-1/pK$,

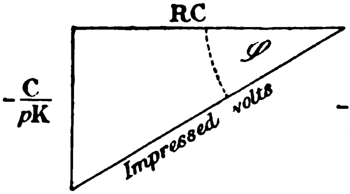


FIG. 15.

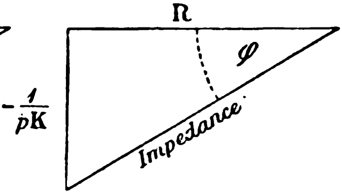


FIG. 16.

being negative and inversely proportional to K (the capacity in *farads*) and to p; and the angle ϕ will be such that $\tan \phi = - 1 / p K R$. The impedance will be $\sqrt{R^2 + 1 / p^2 K^2}$. Figs. 15 and 16 show the construction that is applicable in this case.

If both inductance and capacity are present, $\tan \phi = (pL - 1/p K) \phi R$; the reactance will be $p L - 1/p K$; and the impedance $\sqrt{R^2 + (p L - 1/p K)^2}$. This is illustrated by Fig. 17 in which the triangle for finding ϕ is drawn by setting out pL at right angles to R and then deducting from pL a part equal to $1 / p$K.

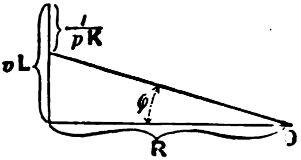


FIG. 17.

The same construction may be applied to a circuit containing several resistances, inductances and capacities in series.

Since capacity and inductance produce opposite effects, they can be used to neutralize one another. They exactly balance if $L = 1 / p^2 K$. In that case the circuit is non-inductive and the currents simply obey Ohm's law.

It will be seen that if in a circuit there is little resistance and much reactance, the current will depend almost exclusively on the reactance. For example, if p ($= 2 \pi f$) were, say, 1000 and L = 10 henries, while R was only 1 ohm, the resistance part of the impedance would be negligible, and the law would become

$$C = \frac{E}{p L}.$$

The current would *lag* by almost exactly 90°.

Self-induction coils with large inductance and small resistance are sometimes used to impede alternate currents, and are called *choking coils*, or impedance coils. This formula is wanted for calculating alternate-current electromagnets; for their apparent resistance is almost entirely due to inductance.

If the current were led into a condenser of small capacity (say $K = \frac{1}{10}$ microfarad, then $1 / p K = 10{,}000$), the current running in and out of the condenser would be governed only by the capacity and frequency, and not by the resistance, and would have the value—

$$C = E p K,$$

and its phase will *lead* by almost exactly 90°.

Resonance.—As pointed out above, the effects of capacity and self-induction tend to oppose one another. They will, in fact, neutralize one another if the frequency is such as to realize the condition :

$$p L = 1/p K,$$

or

$$f = \frac{1}{2 \pi \sqrt{L K}}.$$

If this condition is realised, the circuit acts as if purely non-inductive, and if the resistance of the circuit is small, any electromotive-force, if of this particular frequence, may set up enormous surgings of current. This condition is described as the condition of *resonance.*

A capacity acting laterally across the circuit, as when a condenser is placed across the two mains, has the effect of increasing the flow of current from the dynamo up to the points on the circuit which are connected to it, if the rest of the circuit is non-inductive. But if the rest of the circuit has a preponderating self-induction which would make the current lag, then putting a condenser in parallel with it may actually decrease the current in the mains. These various condenser effects have been considered by various writers. A very clear exposition of them, together with the phenomena observed on the Ferranti mains on the Deptford supply has been given by Fleming.[1]

Mean Power. Power-factor.—The power cannot be calculated by simply multiplying together the *volts* and the *amperes* as with continuous currents ; for when there is any difference of phase the *apparent watts* so calculated are always in excess of the *true watts.* We have seen on p. 16 that the mean product of two periodic functions is equal to half the product of their maximum values multiplied by the cosine of their phase difference : or

---

[1] *Journal Inst. Electr. Engineers*, xx. 362, 1891.

$$\text{Mean power (true } \textit{watts}) = \tfrac{1}{2}\,E_{\max} \times C_{\max} \times \cos\phi,$$

$$= \frac{E_{\max}}{\sqrt{2}} \times \frac{C_{\max}}{\sqrt{2}} \times \cos\phi,$$

$$= E_{\text{virt}} \times C_{\text{virt}} \times \cos\phi.$$

To the product of the volts and amperes (if these have been separately measured), we give the name of *volt amperes.* The *watts* can be calculated from them by multiplying by cos ϕ. Hence we may write:

$$\frac{\text{apparent power}}{\text{true power}} = \frac{\text{volt amperes}}{\text{watts}} = \cos\phi\,;$$

or

$$\text{watts} = \text{volt amperes} \times \cos\phi.$$

Since cos ϕ is the factor by which the apparent power must be multiplied to find the true power, it is termed the *power factor.* Only if the current is in exact phase with the voltage is the power-factor equal to unity. If the current either lags or leads, cos ϕ becomes less than unity. The presence in a circuit of electromagnets, or partially loaded transformers, or induction-motors, causing a lag, reduces the power-factor of that circuit. The following may be taken as usual values of power-factors of circuits:

Nature of Load.	Probable Power-Factor.
Synchronous motors and converters	1·0 to 0·95
Incandescent lighting	0·95
Mixed arc and incandescent lighting	0·85
Induction motors, large and well-loaded	0·80
,, ,, average loaded	0·60 to 0·50
Mixed induction-motors and lighting	0·70

The outputs of alternators are commonly expressed in *kilo-volt-amperes* (abbreviated into KVA), 1 KVA being obviously 1000 volt-amperes. The true power is expressed in kilowatts (KW), the relation between them being again

$$\text{KW} = \text{KVA} \times \cos\phi.$$

One way of dealing with this is to consider the product $E_{\text{virt}} \times \cos\phi$ as the resolved part of the volts that is in phase with the current, and therefore equal to $C_{\text{virt}} \times R$. Hence we may write the mean power (true watts) as C^2_{virt} R. That is to say, if the resistance of the circuit is a plain non-inductive resistance (such as a load of lamps, or a water resistance), the true watts spent in it are found in the usual way by the C^2 R law. There is, however, another way of regarding the matter as follows.

Watt-less Current.—Whenever there is a great phase difference between volts and current (whether a lag due to self-induction or a lead due to capacity), the true watts are, as has already been pointed out, much less than the apparent value that would be obtained by merely multiplying together the virtual amperes and the virtual volts. For, as we have seen, this product must be further multiplied by the cosine of the angle of lag (or lead). Now there are two ways of looking at this matter: the product $E_{\text{virt}} \times C_{\text{virt}} \times \cos \phi$ may be regarded as either the product of the virtual amperes into the resolved part (or effective part) of the virtual volts, or it may be regarded as the product of the virtual volts into the resolved part of the virtual amperes. Just as any force may be resolved into two component forces at right angles to one another, so any alternating current may be resolved into two component alternating currents differing 90° in phase. Or C may be resolved into two parts, C cos ϕ agreeing in phase with the volts, and C sin ϕ in quadrature with the volts. These two resolved parts of the current have received various names. The component in phase with the volts is termed the *energy component*, or *watted component* of the current, or more simply the *working current* or *effective*[1] *current*. The component in quadrature is termed the *watt-less component* or more simply the *watt-less current* or *idle current*. This may be written:

$$\text{energy component} = \text{working current} = C \cos \phi;$$
$$\text{idle component} \quad = \text{watt-less current} = C \sin \phi.$$

In Fig. 18 O E represents the effective part of the impressed electromotive-force O A. Of O E a part O I is found, by dividing by R (p. 21), to represent the current C. Of this current the resolved part O W, in phase with O A, is the working-current, and the part O U,

---

[1] That is, it is the component that can produce effective work. This use of the term is strictly analogous to the use in mechanics, where the resolved component of an oblique force is often termed the effective force. The effective current is equal to the whole current multiplied by the power-factor (i.e. by the cosine of the angle between it and the volts).

which is in quadrature with O A, is the watt-less current. Whenever, for either cause, the angle of lag is great, the watt-less part of the current will be great also. For example, when transformers are left on open circuit, the current in the primary is nearly in quadrature (owing to self-induction) with the impressed volts, and, if it were not for hysteresis or eddy-currents in the iron-cores, would be almost entirely watt-less.

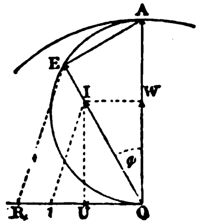


FIG. 18.

For example, if there is a current of 100 virtual amperes lagging 14° behind the impressed volts, this may be resolved into a working-current of $100 \times \cos 14^\circ = 97 \cdot 03$ virtual amperes, and a watt-less current of $100 \times \sin 14^\circ = 24 \cdot 2$ virtual amperes.

An illustration of the compounding of two periodic quantities is afforded by the union of two alternating currents, of similar frequency, but differing in phase. Suppose that there are two circuits, one containing a small resistance but considerable self-induction, and another containing considerable resistance and a small self-induction. Let the resistances be called R_1 and R_2, and the coefficients of self-induction L_1 and L_2, then if they are put in parallel, and a voltage V is applied to them, the respective currents will be

$$C_1 = V \div \sqrt{R_1^2 + p^2 L_1^2};$$

and

$$C_2 = V \div \sqrt{R_2^2 + p^2 L_2^2};$$

these currents will have different lags, respectively such that

$$\tan \phi_1 = pL_1 \div R_1,$$

and

$$\tan \phi_2 = pL_2 \div R_2.$$

But the total current C_3 that they take from the mains will *not* be equal to $C_1 + C_2$, because they differ in phase. In fact, the value will be

$$C_3 = V \div \sqrt{(R_1 + R_2)^2 + p^2(L_1 + L_2)^2};$$

and its lag will be such that

$$\tan \phi_3 = p(L_1 + L_2) \div (R_1 + R_2).$$

This can easily be demonstrated practically by the use of three amperemeters, one in each branch circuit, and the third in the main circuit.

Graphically, the case is realized in Fig. 7, p. 14, where the two currents are represented by O P and O Q, and the resultant current by O R. By the construction, O R will not be equal to O P + O Q, unless the phase-difference (or angle) between them is zero.

Similarly, it is possible to resolve an alternating current into two others that differ from one another by any prescribed amount of phase-difference. In the clock diagram, the one condition is that, as in Fig. 7, the two parts into which the current is split shall be represented to scale by the two sides of a parallelogram, of which the diagonal represents the original current. In this way we may resolve a lagging current into two parts, one in phase with the voltage, the other in quadrature with the voltage. Algebraically, all we have to do is to multiply the current by cos ϕ to find the energy component, and by sin ϕ to find the watt-less component.

Measurement of Alternate-current Power.—The considerations above show that this is a matter for care. If there is no phase-difference between volts and amperes, the apparent watts are the same as the true watts; and in that case amperemeter and voltmeter may be used.[1] But if there is a phase difference a suitable *wattmeter* must be used; the usual form being an electrodynamometer specially constructed so that the high resistance circuit in it shall be non-inductive.

Numerical Example.—Let an impressed electromotive-force of 65 (virtual) volts, alternating with a frequency of 50 periods per second, act upon a circuit having resistance 1·5 ohm, and a coefficient of self-induction of 0·002 henry. Find the lag, the current and the mean power.

To find the lag, we must find the inductance, $2 \pi n$ L, and divide this by the resistance; or

---

[1] Those who are not familiar with this subject should consult the writings of Mr. Blakesley or those of Prof. Fleming. The three-dynamometer method of Blakesley, the three-voltmeter method of Ayrton, and analogous methods, are all of value. Fleming in *Journal Inst. Electr. Engineers*, xxi. 594, 1892, has after much experience given preference to a simple wattmeter method.

$\tan \phi = 2 \pi n L \div R = 2 \times 3·1416 \times 50 \times 0·002 \div 1·5 = 0·419$. Looking in a table of natural tangents, we find that ϕ will be $22° 44'$; whence a table of natural cosines gives us $\cos \phi = 0·9223$. Or, we might calculate $\cos \phi$ directly as $R \div \sqrt{R^2 + 4 \pi^2 n^2 L^2}$. Multiplying $\cos \phi$ into the 65 volts, we get 59·95, say 60, as the effective virtual volts, and dividing by the resistance gives 40 virtual amperes as the current. The mean power is $65 \times 40 \times 0·9223 =$ 2400 watts.

Geometrically this is given in Fig. 19.

Let O A be 65 to any scale, the impressed (virtual) volts. Describe the circle of radius O A, and the semicircle O E A. Draw O B at right angles to O A. On O B set off O R on any convenient scale of resistance, O 1 being taken as 1 ohm. Using same scale, set off O S or R F at right angles, equal to the inductance $2 \pi n L = 0·628$. Join O F. R O F is the angle of lag. Draw E O at right angles to O F, cutting semicircle in E. E O A is also angle of lag, hence E O represents effective virtual volts; and A E the cross-electromotive-force of self-induction $2 \pi n L C$. Join E R and from 1 draw 1 C parallel; C O will represent the current. As O B is O A turned through a right angle, the area of triangle B O C = ½ O A . O C . cos A O C = ½ mean power (see p. 27).

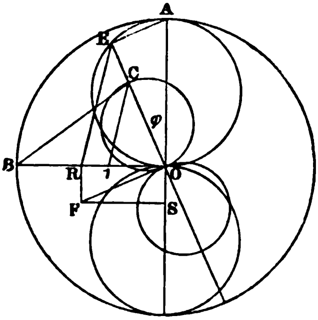
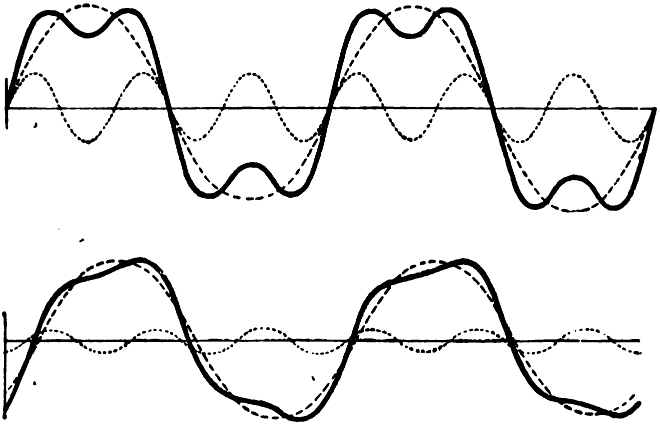


FIG. 19.

There are some reasons why it is desirable that the induction curves of alternators should follow the sine-form (but see p. 9 as to effect of wave-form on transformer efficiency). According to the well-known theorem of Fourier, every complex single-valued periodic function can be analysed down into a series of simple periodic functions differing in amplitude and phase, but all belonging to a harmonic series, having frequencies that are some exact multiple of a single fundamental frequency. Every complex wave-curve may be regarded as built up of sine-curves. For example, the curve shown in Fig. 20 may be looked upon as a compound of the two dotted sine-curves, one of a frequency three times that of the other. Now, if this complex curve represents the impressed electromotive-force of an alternator with curiously-shaped poles, what will be the curve of effective electromotive-force (or of current) when self-induction is present? The amplitude is cut down in proportion nearly to the frequency of the alternation. Hence the component ripple, which has three times the frequency, will be damped out nearly three times as much as the fundamental wave.[1] In Fig. 21 are shown the two waves, as altered by a lag of 41° which cuts down the fundamental to 0·75 and the ripple to 0·35 of their respective amplitudes; the resultant wave being also shown. It is evident that self-induction tends to smooth out the ripples, including all parts of the wave that do not fit to the sine-form. Hence those alternators in which the induction curves of true sine-form are less affected than others by self-induction in the circuit, regulate better, and have a higher plant efficiency.



FIGS. 20 AND 21.

High Frequency Alternations.—Alternations of very high periodicity, going up to as many as 10,000 or 20,000 per second, have been studied by Spottiswoode,[2] and more recently by Tesla,[3] who obtained some very remarkable effects. One of his alternators was of the same type as Mordey's, having numerous polar projections on either side,[4] and another was of the inductor type. With

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[1] See the end of Chapter II., on the analysis of periodic curves.

[2] *Proc. Roy. Soc.*, xxiii. 455.

[3] *American Inst. Electrical Engineers*, May 1891. See *Electrical World*, xvi. 1891, and *The Electrician*, xxvi. 549, 1891.

[4] See *Electrical Engineer* (N.Y.), March 18, 1891.

these excessively high frequencies the currents flow almost exclusively along the surface layers of conductors, instead of flowing through their entire cross-section; even straight rods of copper offering a relatively enormous impedance.

Torque of Alternators.—A very singular result follows the presence of any lag in the current of an alternator. When the amperes flow with the volts, electric energy is being supplied by the machine, and power must be applied to drive it; but when the amperes flow against a counter electromotive-force, then electric energy is leaving the circuit and being turned into mechanical energy, helping to drive the machine. The one is the case of the generator, the other that of the motor. But now consider an alternator with the amperes lagging behind the volts, as indicated by the diagram of Fig. 22. It is clear that in consequence of this lag the amperes are sometimes flowing against the volts instead of with them. In fact, we may divide each complete period such as O X into four parts, during two of which, namely parts II. and IV., the amperes and volts are alike in direction, either both positive, or else both negative; during the other two parts, namely I. and III., the amperes and volts are opposed in direction because the volts have reversed in sign, but the lagging amperes have not yet changed. Now, during the partial periods II. and IV., when there is agreement in sign, the machine is in the condition of being a

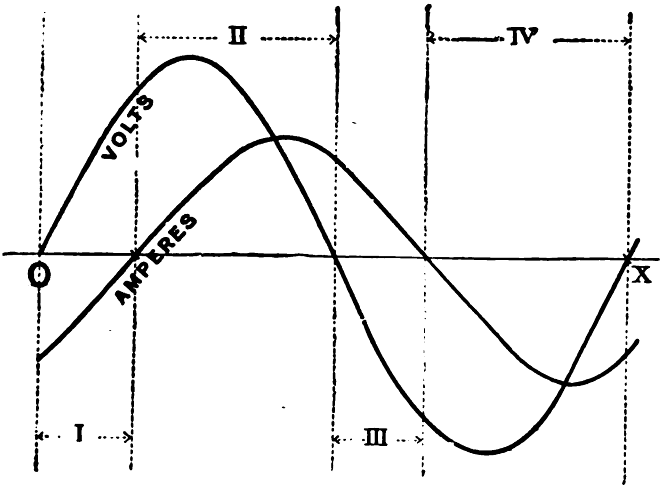

FIG. 22.—EFFECT OF LAG OF CURRENT.

generator, and will require to be driven, the currents in the armature setting up a counter torque. But during the other partial periods I. and III., when there is opposition in sign, the machine is in the condition of being a motor, and will tend to drive itself, the torque helping it on. The conductors are consequently subjected to a raking action, alternately resisting, being driven, and then helping to drive twice in each period. It is clear that if there is little lag there will be little motor action, the partial periods I. and III. being brief; whereas if there is much lag the motor action will increase. If there is a lag of exactly a quarter of a period, the motor and generator actions will be equal. Similarly, if in consequence of capacity the current leads in phase, there will be motor action in partial periods. This subject may be considered in another way. The electromotive-forces change sign just as the conductors are passing (Fig. 23) from one magnetic field to another, where the lines run in an opposite direction. If the currents are in phase with the electromotive-forces, they will always tend to oppose the motion that generates them, and will reverse when the conductor passes into the reversed field as at *a*, *a*. But if the currents lag, the force exerted by the field will help on the motion of those conductors which have passed from one field to the other until such time as the currents have reversed at *b*, *b*.

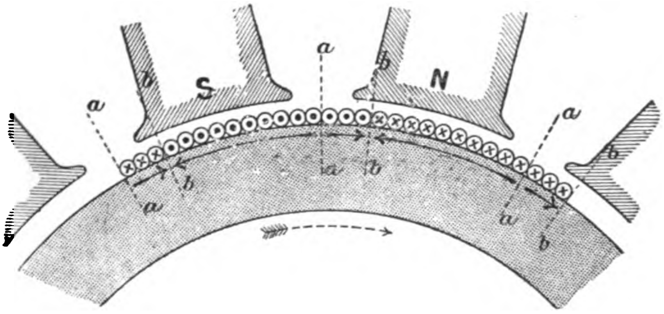

FIG. 23.

It follows that when there is a difference of phase between volts and amperes, the mean power in a cycle is equal to the difference between the power which it gives out during the partial periods II. and IV., and the power which it receives back from the circuit during the partial periods I. and III. If the phase difference is less than 90° the machine acts on the whole as a generator. If it is more than 90° the machine acts as a motor on the whole. If two alternators are coupled in series, one to act as generator, the other as motor, the current will be nearly in phase with the electromotive-force in the one and almost exactly opposed to the electromotive-force in the other. This question is resumed in Chapter X.

POLYPHASE CURRENTS.

During the past fifteen years, systems have arisen in which use is made simultaneously of two or three alternating currents having different phases.

Two-Phase System.—In the two-phase system, two alternating currents of equal period and equal amplitude, but differing in phase by one quarter of a period, are employed. Graphically they may be represented by the two curves marked A and B in Fig. 24. As they differ by an exact quarter of a period, one of them will be at its maximum when the other is at its zero. The values of the A-current may be represented by the equation.

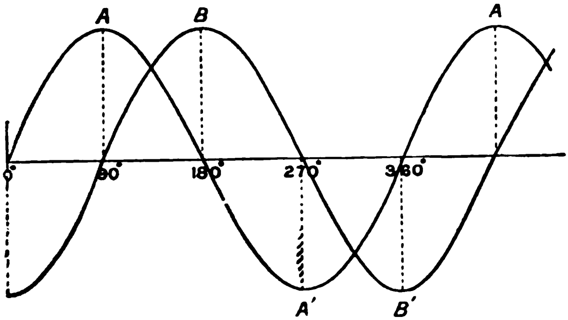

FIG. 24.

$$A_t = A_{\max} \sin \theta,$$

then those of the B-current may be represented by the equation.

$$B_t = B_{\max} \sin (\theta + 90°) ;$$

or, if the angles are reckoned in radians,

$$B_t = B_{max} \sin\left(\theta + \frac{\pi}{2}\right),$$

$$B_t = B_{max} \cos \theta.$$

We may construct, on the plan pursued on p. 7 a statistical table, showing the simultaneous values of the two currents on the supposition that each attains 100 at its maximum. It will be sufficient to give the values for half a period only.

Stage.	Angle.	Value of Current.	
		A-Phase.	B-Phase.
0	0	0	− 100
1	15°	+ 26	− 97
2	30°	+ 50	− 87
3	45°	+ 71	− 71
4	60°	+ 87	− 50
5	75°	+ 97	− 26
6	90°	+100	0
7	105°	+ 97	+ 26
8	120°	+ 87	+ 50
9	135°	+ 71	+ 71
10	150°	+ 50	+ 87
11	165°	+ 26	+ 97
12	180°	0	+ 100

If an alternator is to generate two alternating currents, it must have two independent windings, and these must be so spaced out that when the volts generated in one of the two "phases" are at a maximum, those generated in the other are at zero. In other words, the windings, which must be alike, of an equal number of turns, must be displaced along the armature by an angle corresponding to one quarter of a period, that is, to half the pole-pitch. The windings of the two phases must, of course, be kept separate, hence the armature will have four terminals; or, if it is a revolving armature, it will have four slip-rings. A two-phase system requires four lines for its distribution ; two lines for each phase. It is possible, but not advisable, to reduce the number to 3, by employing one rather thicker line as a common return for each of the phases. If this is done, the voltage between the A-line and the B-line will be equal to $\sqrt{2}$ times the voltage in either phase, and the current in the line used as common return will be $\sqrt{2}$ times as great as the current in either line, assuming the two currents in the two phases to be equal. Another way of using a two-phase system, is to unite the mid-point of the windings of the two "phases" in the generator at a common junction. This is equivalent to making the machine into a four-phase machine with half the voltage in each of the four "phases," which will then be in successive quadrature with each other. Some writers, mostly American, call a two-phase system "quarter-phase," because of the phases being in quadrature with one another.

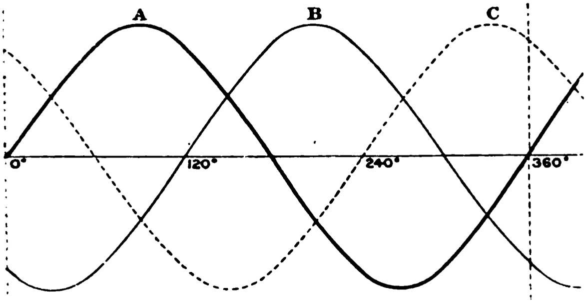

FIG. 25.

Three-Phase System.—In the three-phase system, we have three alternating currents of equal frequency and amplitude, but they differ in phase from one another, by one-third of a period. They are represented by the curves A, B and C in Fig. 25. Inspection of this figure will show that at the instant when any one of the currents is at its maximum, the other two are of exactly half their maximum value, and are flowing in the opposite direction.

The equations for the three phases may be written :—

$$A_t = A_{max} \sin \theta ;$$

$$B_t = B_{max} \sin (\theta + 120^\circ) = B_{max} \sin \left(\theta + \frac{2\pi}{3}\right);$$

$$C_t = C_{max} \sin (\theta + 240^\circ) = C_{max} \sin \left(\theta + \frac{4\pi}{3}\right).$$

As before, we may construct a statistical table of the values of the currents for some concrete case ; that taken is the case of three currents, each having a maximum of 100 amperes. The values at the successive instants are taken, as before, from the figures given on p. 7. It suffices to give the table for half a period.

		Value of the Current.		
Stage.	Angle.	A-phase.	B-phase.	C-phase.
0	0	0	− 87	+ 87
1	15°	+ 26	− 97	+ 71
2	30°	+ 50	− 100	+ 50
3	45°	+ 71	− 97	+ 26
4	60°	+ 87	− 87	0
5	75°	+ 97	− 71	− 26
6	90°	+ 100	− 50	− 50
7	105°	+ 97	− 26	− 71
8	120°	+ 87	0	− 87
9	135°	+ 71	+ 26	− 97
10	150°	+ 50	+ 50	− 100
11	165°	+ 26	+ 71	− 97
12	180°	0	+ 87	− 87

Inspection of this table will show that at any instant the amount of current going out by any one of the three phases is equal to the sum of the currents coming in by the other two phases, or that the algebraic sum of the three currents at any instant is zero.

To generate the three currents will require three independent equal windings on the armature of the generator, and they must be spaced out over its surface so as to be successively $\frac{1}{3}$ and $\frac{2}{3}$ of the period (*i.e.* of the double pole-pitch) apart from one another. The various modes of joining up the coils will be considered in Chapter VII. on Winding Schemes for Alternators. Suffice it to point out that they *might* be used to supply three entirely independent circuits, in which case the armature would appropriately have six terminals or if revolving would be furnished with six slip-rings: two slip-rings for each phase. But this is not the usual method ; it is more usual to group the three sets of windings in one of two different ways—either in Y or in Δ. In the Y-grouping, which is the usual one, one end of each of the three phases is brought to a common junction, which is insulated from external connexions. This leaves three ends, one for each phase, to be brought to three terminals, or, in the case of revolving armatures, to three slip-rings. It is vital to observe that *in this case*, if the voltage generated in each one of the three phases separately is E_1 (virtual) volts, the voltage generated between any two of the terminals will be equal to $\sqrt{3} \times E_1$. Thus if each of the three phases generates 100 volts, and if they are Y-grouped with a common junction, the voltage from the terminal of the A-phase to that of the B-phase will be 173 volts.

ANALYTICAL TREATMENT OF FUNDAMENTAL EQUATIONS OF ALTERNATING CURRENTS.

Beginning with the case of a loop having S_2 turns, placed at such an angle θ (measured from the initial position where it stands right across the field), we see that it no longer encloses the whole number of magnetic lines which are present in the magnetic circuit. When we omit all account of self-induction, we may write

$$\mathbf{N}_1 = S_2 \mathbf{N} \cos \theta \quad . \quad . \quad . \quad . \qquad [I]$$

where $\mathbf{N}_1$ is the amount of flux actually enclosed by the loop in this position.

To get a complete account of the action we must now take into consideration the number of magnetic lines *induced by the circuit on itself.*[1]

If current C flow through a circuit whose coefficient of self-induction or inductance is L, the whole self-induction of the circuit will be equal to L times C; and the product L C will represent the total amount of enclosing of magnetic lines by the convolutions of the circuit.

But we know that if there is a current C in the circuit, we ought to write the equation in full—

$$\mathbf{N}_1 = S_2 \, \mathbf{N} \cos\theta + L\,C \qquad . \qquad . \qquad [II]$$

Now we know that any variation in $\mathbf{N}_1$ will set up induced electromotive-force, and that at any moment the electromotive-force will have the value

$$E = -\frac{d\mathbf{N}_1}{dt}; \qquad . \qquad . \qquad . \quad [III]$$

where we use the negative sign to show that an increase in $\mathbf{N}_1$ will produce an inverse or negative electromotive-force. Any change in $\mathbf{N}_1$, from whatever source arising, will set up electromotive-force. In the absence of armature reactions the only quantities whose variations contribute to the variations of $\mathbf{N}_1$ are θ and C. The angle of position θ varies from 0 to 2π (radians); that is to say, from 0° right round to 360°, and then recurs; and its cosine therefore fluctuates between 1 and -1. The current C varies also from a certain maximum value $+C_{max}$

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[1] Neumann's mathematical investigation of the effect of considering the self-induction of the circuit in relation to a periodic electromotive-force, was published in 1845, but self-inductive phenomena had previously been studied by Henry and by Faraday.

Other mathematical investigations of alternating electric currents have been given by Weber in his *Elektrodynamische Maasbestimmungen*, and by the following:—

Koosen, *Pogg. Ann.*, lxxxvii. 386, 1852.
Le Roux, *Ann. Chim. Phys.* [3], l. 463, 1857.
Clerk Maxwell, *Phil. Trans.*, 1865, p. 473.
F. Kohlrausch, *Pogg. Ann.*, cxlviii. 143, 1873.
Jamin and Richard, *Ann. Chim. Phys.* [4], xvii. 276, 1869.
Joubert, *Ann. de l'École Normale Supérieure*, x. 1881; and *Journal de Physique*, s. ii. t. ii. p. 293, 1883.
Lord Rayleigh, *Phil. Mag.*, May 1886, p. 375.
Hopkinson, Lecture at Instit. Civil Engineers (on Electric Lighting), 1883.
,, *Jour. Soc. Telegr. Engineers*, xiii.
,, *Proc. Roy. Soc.*, Feb. 1887.

Abstracts of the most important of these will be found in Fleming's Book on the *Alternate Current Transformer.*

to an equal negative value $-C_{max}$. We will neglect all the variations of the other quantities, not because these variations would not be instructive—for that would be quite untrue—but because of their lesser practical importance. Then we have

$$E_t = -\frac{dN}{dt} = -\frac{d(S_2 N \cos\theta + LC)}{dt}.$$

Now suppose that while the armature loop has turned through the angle θ, the time occupied—a small fraction of a second—is t. Also take T to represent the time taken for one revolution; so that if there were f revolutions[1] per second, T would be $1/f$ of a second. Then obviously θ will be the $\frac{t}{T}$ part of a whole revolution, and as there are 2π radians in a circle, the angle expressed in radians will be

$$\theta = 2\pi\frac{t}{T} = 2\pi f t = p t;$$

where p is written short for $2\pi f$, and called the *pulsation*.

Asserting this value, and performing the differentiation, we get

$$E_t = 2\pi f S_2 N \times \sin pt - L\frac{dC}{dt}; \quad . \quad . \quad [IV]$$

Consider this equation carefully. It shows us that when the dynamo is on open circuit, so that there is no current, then self-induction would not come in at all. The negative sign also indicates that that part of the electromotive-force which is due to the self-induction opposes the other part. Now write D for the group of symbols $2\pi f S_2 N$. Further, we know that that part of the electromotive-force which is effective in driving the current through the resistance may be calculated by simply applying Ohm's law. So if E_t as found in formula [IV] be the nett or effective electromotive-force at the time t, we may write $E_t = R C_t$; whence

$$R C_t = D \sin\theta - L\frac{dC}{dt}.$$

This is a differential equation of the form

$$a y + b\frac{dy}{dx} = \sin p x.$$

(See Boole's *Differential Equations*, p. 38.)

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[1] For multipolar machines the symbol f will in this case stand for number of cycles per second. In bipolar machines f is literally the number of revolutions.

The solution is

$$C_t = \frac{D \cos \phi . \sin (\theta - \phi)}{R} + c \epsilon^{-\frac{R}{L}t}; \quad . \qquad [V]$$

where ϕ is called the retardation or angle of *lag*, and has the value such that

$$\tan \phi = \frac{2 \pi f L}{R}.$$

In the second term of the expression on the right-hand side of the above equation, the symbol c is a constant of integration, and ϵ is used in its common mathematical sense to represent the number 2·7182, which is the basis of the Napierian (or hyperbolic) logarithms. This second term relates only to the irregularities during the first starting of the current, and dies out as the time t increases in value. The phenomenon of inductive rush, sometimes noticed when current is suddenly switched on or off, is of this nature. In general the exponential term may be omitted.

We have therefore, got our equation for the current at time t as follows :—

$$C_t = \frac{D \cos \phi . \sin (\theta - \phi)}{R}; \quad . \quad . \qquad [VI]$$

which should be compared with the value D sin $\theta \div$ R that the current would have if there were no self-induction. We see by comparing the two expressions that our current still follows a sine-function, but it is the sine-function not of the angle θ, but of the angle $(\theta - \phi)$; that is to say, its waves *lag* behind those of the impressed electromotive-force. Also, the amplitude of the current is reduced, because everything is going on as if the amplitude of the impressed electromotive-force had been altered from D to D cos ϕ. Or, in other words, the effective electromotive-force is equal to the part of the impressed electromotive-force as resolved along the line of the lagging current. If we substitute for cos ϕ its value $R / \sqrt{R^2 + p^2 L^2}$, we reduce the equation to the form

$$C_t = \frac{D \sin (\theta - \phi)}{\sqrt{R^2 + p^2 L^2}}; \quad . \quad . \qquad [VII]$$

which is what we deduced from geometrical considerations.

To establish the equations for the case of a circuit possessing capacity and resistance only, we may proceed very simply to calculate what impressed electromotive-force is needed both to drive the current through the resistance and to charge the condenser. Assume $C = C_0 \sin \theta$. Let the condenser of capacity K (farads) have a charge q at any instant, then its potential will be q / K, and the corresponding electromotive-force needed at that instant to drive the current, will be

$$R\,C + \frac{q}{K} = E.$$

But

$$q = \int C\,dt = -\frac{1}{p} C_0 \cos \theta, \text{ where } \theta = pt = 2\pi f t.$$

Substituting, we get

$$R\,C_0 \sin \theta - \frac{1}{pK} C_0 \cos \theta = E.$$

Now divide both sides by

$$\sqrt{R^2 + \frac{1}{p^2 K^2}}\,;$$

and call

$$\tan \phi = \frac{-1}{R p K}.$$

Then

$$\sin \phi = -\frac{1}{pK} \cdot \frac{1}{\sqrt{R^2 + \frac{1}{p^2 K^2}}},$$

and

$$\cos \phi = \frac{R}{\sqrt{R^2 + \frac{1}{p^2 K^2}}}.$$

$$C_0 (\cos \phi \,.\, \sin \theta - \sin \phi \,.\, \cos \theta) = E \div \sqrt{R + \frac{1}{p^2 K^2}}.$$

$$C_0 \sin (\theta - \phi) = \frac{E}{\sqrt{R^2 + \frac{1}{p^2 K^2}}}.$$

This indicates that the volts will lag in phase behind the current; or, in other words, the current will lead in phase.

Mean Power. The mean power is obtained by integrating the power during one period and dividing by that period, and therefore may be written

$$\frac{1}{T}\int_0^T E\,C\,dt = \frac{1}{T}\int_0^T R\,C^2\,dt = \frac{1}{T}\int_0^T \frac{E^2}{R}\,dt.$$

If we square the expression [VII]found for current, and substitute for the square of the sine its mean value, viz. $\frac{1}{2}$, and then multiply by R, we get as the mean power (in watts)

$$W = \frac{2 \pi^2 n^2 S_2{}^2 N^2 R}{R^2 + 4 \pi^2 n^2 L^2}.$$

This expression, by a well known algebraic rule, will be a maximum for variations of R, when R is such that the two terms in the denominator are equal, or when the resistance equals the inductance. Under these circumstances the highest lag is 45°. But though this is the condition for highest plant efficiency, the regulation is, under these circumstances, bad. Hence it is better to use such a machine for lesser currents than those which would produce so great a lag.

Skin Effect.—When the frequency is high, there is a tendency for the alternate current to distribute itself unequally through the cross-section of the conductor, flowing most strongly in the surface parts. For this reason it has been proposed to use hollow conductors, or flat conductors, rather than solid round wires. But with frequencies not exceeding 100 periods per second, this tendency is negligibly small in copper conductors under one centimetre in diameter. Where the conductor is large, or the frequency high, the effect may be judged by the following instances calculated by Professor J. J. Thomson.[1]

In the case of a copper conductor exposed to an electromotive-force, making 100 periods per second, at 1 centimetre from the surface, the maximum current would be only 0·208 times that at the surface; at a depth of 2 centimetres it would be only 0·043; and at a depth of 4 centimetres less than $\frac{1}{500}$ part of the value at the surface.

If the frequency is a million per second, the current at a depth of 1 millimetre is less than one six-millionth part of its surface value.

The case of an iron conductor is even more remarkable. Taking the permeability at 100 and the frequency at 100 per second, the current at a depth of 1 millimetre is only 0·13 times the surface value; while at a depth of 5 millimetres it is less than one twenty thousandth part of its surface value.

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[1] *Elements of the Mathematical Theory of Electricity and Magnetism* (Cambridge University Press).

CHAPTER II.

ON PERIODIC FUNCTIONS WHICH DO NOT FOLLOW A SIMPLE SINE-LAW.

SINCE the curves of electromotive-force and of current depend upon the distribution of the magnetic field at the poles of the alternator (and on the distribution of the coils of the armature) and as the latter is arbitrary, depending on the shape of the pole-faces, it is evident that in many cases the forms of the curves of electromotive-force and of current will not be those of a simple sine-curve. They may be more peaked (as Fig. 26), or more flattened at the top (as Fig. 27), or irregular in outline (as Fig. 28). They may in some cases (and always if there is distortion due to armature reaction), be unsymmetrical about the peak, as in Fig. 29. The forms of the *current* curves may be affected also by the reaction of apparatus in the circuit. Thus, for example, if the alternator is supplying currents to a transformer or a choking coil the core of which is worked at a high degree of magnetic saturation, the current curve may be distorted to resemble Fig. 30 even though the curve of the electromotive-force is a simple

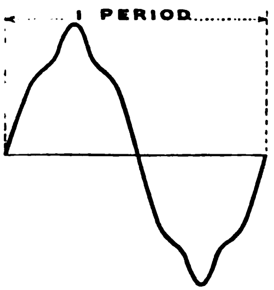

FIG. 26.

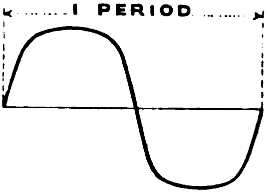

FIG. 27.

smooth sine-curve. Such irregularities demand a careful investigation.

The mathematical method of investigation is due to Fourier, and consists in analysing the compound curve into a set of components each of which follows a simple sine-function. The theory of Fourier is based upon the use of a certain series of terms each of which is a simple sine-function ; the compound periodic function being expressed as the sum of a series of such terms. It is neither necessary nor desirable for present purposes to give a complete account of Fourier's theorem. Those who wish to study the subject more fully are referred to the appropriate mathematical treatises.[1] All that will here be attempted is the most elementary explanatory treatment of its simplest form.

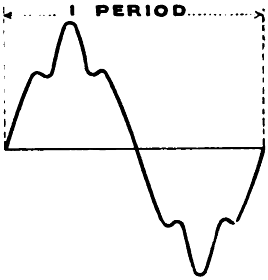

FIG. 28.

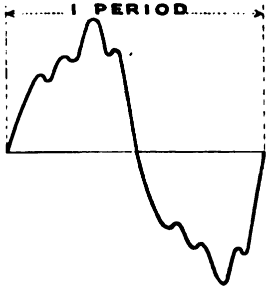

FIG. 29.

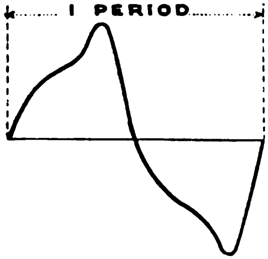

FIG. 30.

Examples of the building up of an elaborate form out of simple elements of definite sizes are common in many very different branches of science and art. Architects build complicated shapes out of simple rectangular bricks. Arithmeticians express varied numbers by building

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[1] For example :—

Perry, *Calculus for Engineers.* London, 1897.

Graham, *An Elementary Treatise on the Calculus for Engineering Students.* London, 1896.

Byerly, *Fourier's Series and Spherical Harmonics.*

together elementary quantities of different sizes. Thus in ordinary arithmetic we deal with "units," "tens," "hundreds" and "thousands"; for instance, the cube of twelve can be built up of elements of four different sizes, one of the "thousand" size, seven of the "hundred" size, two of the "ten" size and eight of the "unit" size; total 1728. And even incommensurable quantities can be approximated to by taking successive quantities of fixed size in a series of ascending smallness. Thus, for example, the number π, the ratio of the circumference of a circle to its diameter, which is more than 3 and less than 4, can be approximated to by taking three units, one tenth, four hundredths, one thousandth, five ten-thousandths, etc.; or in total $3 \cdot 1415 \cdots$. In this last case the successive members belong to a series each term of which is of a magnitude one-tenth of the size of the preceding term. If a builder had to build using pieces of different magnitudes, he could approximate to many curved forms by using appropriate combinations. Suppose a builder had 4 sizes of bricks, which arranged in a descending order of magnitudes are represented by the shapes A, B, C, D, of Fig. 31.

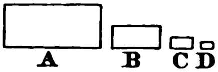

FIG. 31.

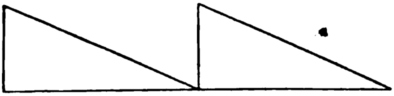
FIG. 32.

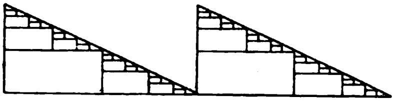
FIG. 33.

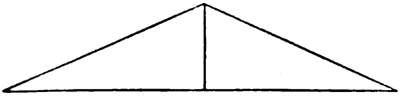
FIG. 34.

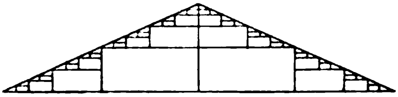
FIG. 35.

Let him be ordered with such bricks to build up the form shown in Fig. 32. He might build it up as in Fig. 33, in which each triangle is built up of 1A, 2B, 4C and 8D shapes. Or he might build the form of Fig. 34, by piling bricks as in Fig. 35.

In the above case the bricks selected were chosen of the relative sizes 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc. Had other smaller sizes been added—$\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, closer approximation might have been obtained. Suppose that instead the series selected had been of the sizes 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, etc., it would have been equally possible by selection to build[1] up appropriate arrangements to imitate given forms.[2]

Now the elements from which Fourier purposed to build up periodic functions are members of a particular series graphically represented by the forms of Fig. 37, or are written algebraically as:—

$$y_1 = A_1 \sin \theta$$
$$y_2 = A_2 \sin 2\theta$$
$$y_3 = A_3 \sin 3\theta$$
$$y_4 = A_4 \sin 4\theta$$
$$y_5 = A_5 \sin 5\theta, \text{ etc.}$$

These smaller curves are known as ***harmonics***, because, in music, they would correspond to the higher harmonics of the fundamental note. The first curve of the series is known as the *fundamental*.

Now each sine wave consists of two parts—a positive and a negative part. Hence in building up curves by superposing

---

[1] Compare: A. Toepler, *Ueber eine Erweiterung der periodischen Reihenentwickelung.* Steiermark. Mittheil. 1872, pp. 64–116.

[2] The following is a help in drawing sine-curves. The *slope* at which a sine-curve crosses the horizontal axis is such that it is directed towards a point above

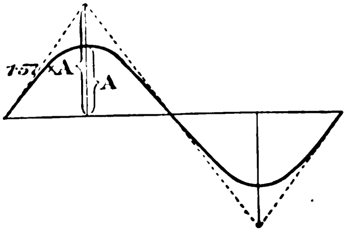

FIG. 36.

the peak situated at a height, having as ordinate $\frac{1}{2}\pi$ times the maximum ordinate of the curve. Or, if A be the amplitude of the curve, then the slope is towards the point whose ordinate is 1·57 times A, as in the accompanying Fig. 36.

such elements we have a possibility that does not occur in building with bricks, namely that we have negative as well as positive elements to reckon with. For example, if we take but two elements—say No. 1 and No. 2—to build with, we shall have the following result in Fig. 38. The resultant curve is formed by adding (algebraically) the ordinates of the No. 2 curve to those of the No. 1 curve. For instance, where the vertical dotted line is drawn, the ordinate of the resultant curve is equal to the sum of the ordinates of the two component curves. The periodic curve obtained by building up No. 2 upon No. 1 begins more steeply than either of its components, and it also ends more steeply. The maximum height of No. 1 being A_1 and that of No. 2 being A_2, the maximum height of the resultant curve will not be $A_1 + A_2$, because the two curves do not come to their respective maxima at the same time. The equation to the curve is of course $y = A_1 \sin \theta + A_2 \sin 2\theta$.

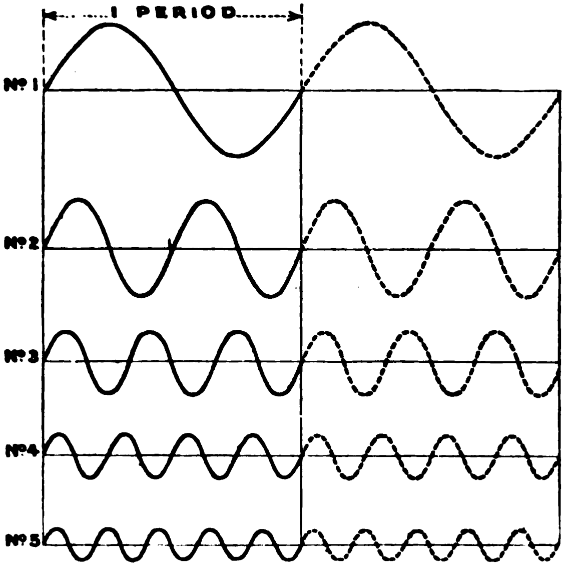

FIG. 37.—HARMONIC SERIES: FIRST FIVE MEMBERS.

In the above example we have supposed that the maximum height A_2 of the second curve was only half as great as that of A_1 of the first curve. But A_2 may have any value either

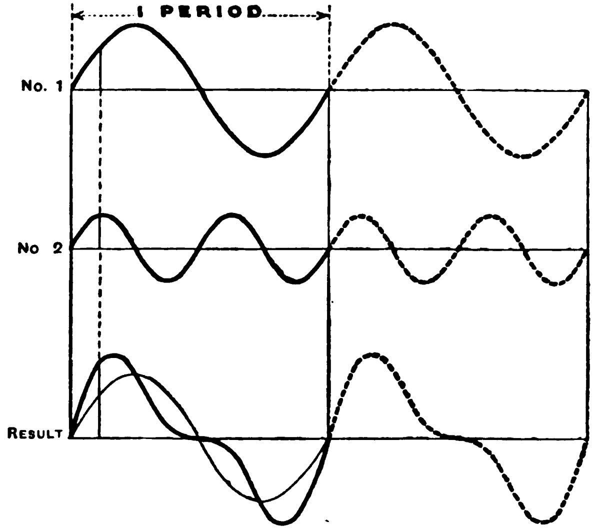

Fig. 38.—Resultant of Fundamental and Second Harmonic.

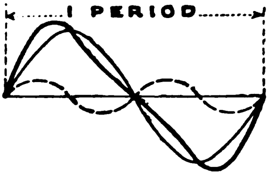

Fig. 39.

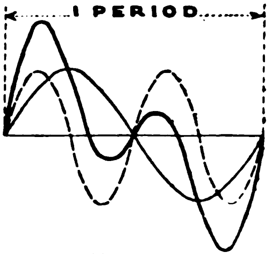

Fig. 40.

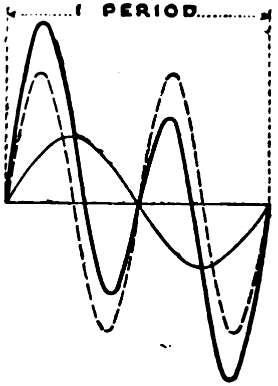

Fig. 41.

greater or less than A_1. In Figs. 39, 40 and 41, are given 3 curves, all conforming to the same equation, but differing amongst themselves in the following respect:—that in Fig. 39 A_2 is only one quarter of A_1; in Fig. 40 A_2 is equal to A_1; and in Fig. 41 A_2 is twice as great as A_1. In each case, the effect of the harmonic is to raise two humps upon the curve in each period.

Now, in general, the origin of the second curve need not coincide with that of the first. Suppose the second wave to start a little later in time—therefore a little further to the right—than the fundamental curve. In that case the wave-form which results is changed. In Fig. 42, which should be compared with Fig. 38 above, the harmonic has been shifted $\frac{1}{8}$ of the whole period of the fundamental to the right; and in Fig. 43 it has been shifted $\frac{1}{4}$ of the whole period of the fundamental. This would be represented in the formula by the addition of a symbol for the lag:

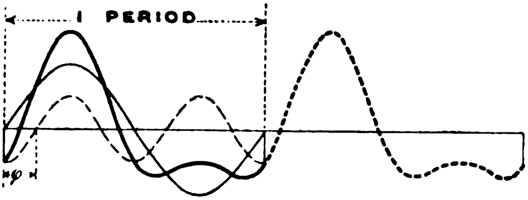

FIG. 42.

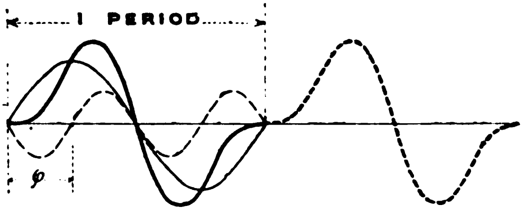

FIG. 43.

$$y = A_1 \sin \theta + A_2 \sin (2\theta - \phi);$$

where ϕ is taken for Fig. 42 as 45°; and in Fig. 43 as 90°.

Let us now see what is the result of another combination, namely that of the fundamental curve with No. 3 of the series; No. 3 being a sine-curve having three times as many waves in the same time as the fundamental No. 1. In this case, if there is no lead or lag, the equation will be

$$y = A_1 \sin \theta + A_3 \sin 3\theta.$$

The form of this curve is shown in Figs. 44, 45 and 46.

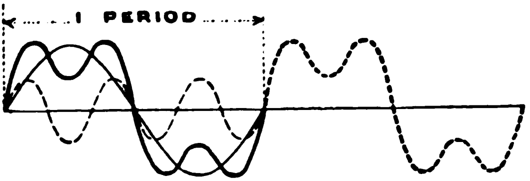

FIG. 44.

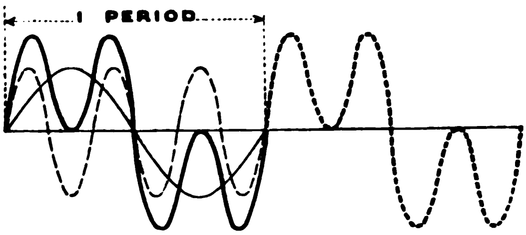

FIG. 45.

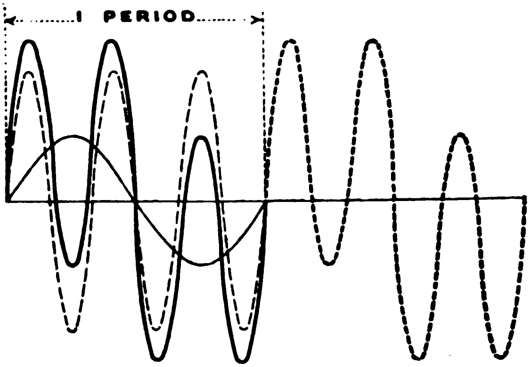

FIG. 46.

In Fig. 44 the amplitude A_3 of the harmonic is taken as only half that of A_1; in Fig. 45 it is equal to A_1 and in Fig. 46 A_3 is twice as great as A_1. In each case the effect of the presence of this third harmonic is to raise three humps upon the curve in each period. If we were to give to this harmonic a lag as in Fig. 47, there results a form which should be compared with Fig. 44 above. There are still three humps upon the curve in each period, but they are now

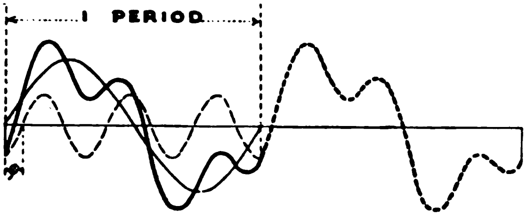

FIG. 47.

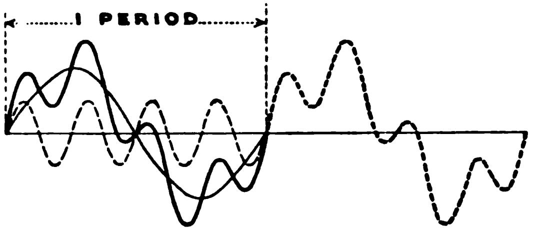

FIG. 48.

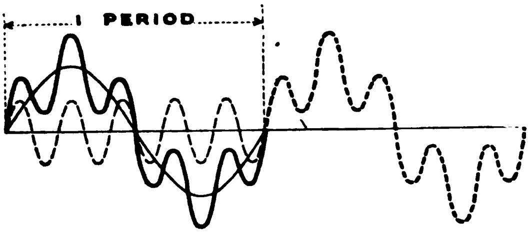

FIG. 49.

distributed a little further to the right. The equation is of course:—

$$y = A_1 \sin \theta + A_3 \sin (3\theta - \phi).$$

By way of comparison let us consider the further cases of the fourth harmonic and fifth harmonic, when superposed on the fundamental. These are depicted in Figs. 48 and 49 respectively.

Next let us consider the effect of taking a harmonic with its amplitude *negative*; the curve starting, therefore, downward instead of upward. In Fig. 50 the second harmonic is taken (negatively); in Fig. 51 the third harmonic; in Fig. 52 the fourth; and in Fig. 53 the fifth.

On comparing these curves with the preceding ones we may come to several conclusions.

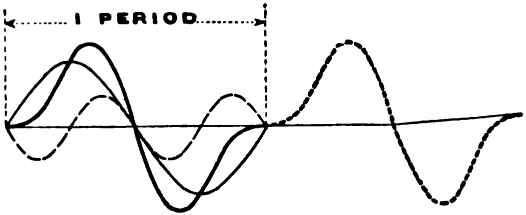

FIG. 50.

1. The number of humps or ripples on the curve (in one period), though not always easily recognised, corresponds to the *order* of the harmonic. Thus the harmonic of the fifth order raises five ripples.

2. *Odd* harmonics (Nos. 3, 5 and 7 of the series) have, when there is no lag, the effect of changing the shape of the curve in a way that is symmetrical in each half-period; as may be seen in Figs. 44, 45, 49, 51 and 53.

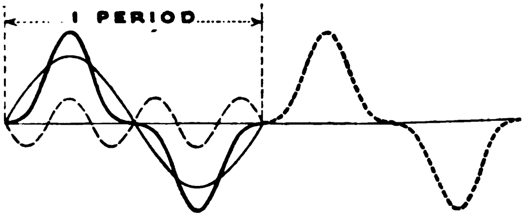

FIG. 51.

3. *Even* harmonics (Nos. 2 and 4 of the series) have, when there is no lag, the effect of changing the shape of the curve in a way that is unsymmetrical. Thus in Figs. 38, 39, 48, and 52, the curve in each half-period is humped up more at one side than the other, giving a kind of saw-tooth effect.

4. The superposition on the fundamental of an odd harmonic (if there is no lag) has the result of altering the peak that occurs in the middle of the half-period of the fundamental sine-curve; the third harmonic lowers the peak (Fig. 44), and the fifth harmonic raises it (Fig. 49); but if the harmonics are taken negatively, the third harmonic raises the peak (Fig. 51), and the fifth harmonic lowers it (Fig. 53). Peaks occur at $\frac{1}{4}$ and hollows at $\frac{3}{4}$ of each complete wave of any simple curve. And at the time—one quarter of a period from the commencement—when the peak occurs in the fundamental, the third harmonic will be in the $\frac{3}{4}$ stage of its period, and the fifth harmonic is the $\frac{5}{4}$ stage of its own period. Similarly the seventh harmonic added gives a depression like the third,

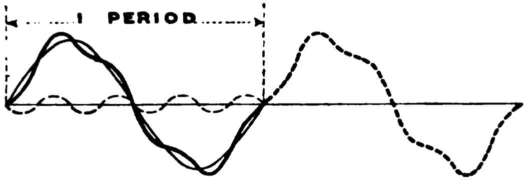

FIG. 52.

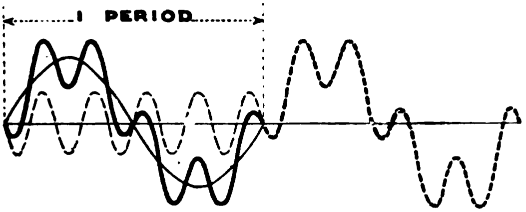

FIG. 53

while the ninth harmonic added gives an elevation like the fifth. If we superpose the three curves Nos. 1, 3 and 5, taking the No. 3 negatively, as in Fig. 54, we shall get a still more pronounced peak; while if we superpose them, taking No. 3 positive and No. 5 negative, we shall get a more pronounced introversion of the summit, as in Fig. 55.

The equation to Figs. 54 and 55 is:

$$y = A_1 \sin \theta + A_3 \sin 3\theta + A_5 \sin 5\theta;$$

but in Fig. 54 A_3 is negative, while in Fig. 55 A_5 is negative.

5. If in any of the curves made by superposing *even* harmonics, we consider the second half-period, we shall see that its shape resembles that of the first half-period, but inverted and also reversed left for right, so that, if we were to take (for example, Fig. 38) and push the second half back underneath the first, as in Fig. 56, the heights above the level are not equal to the depths immediately below them.

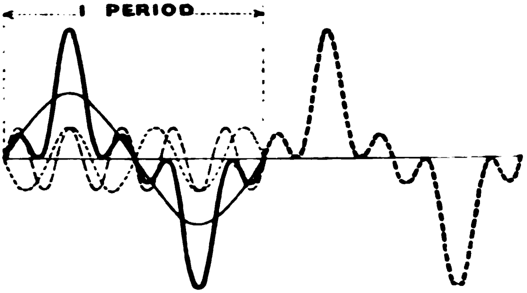

FIG. 54.

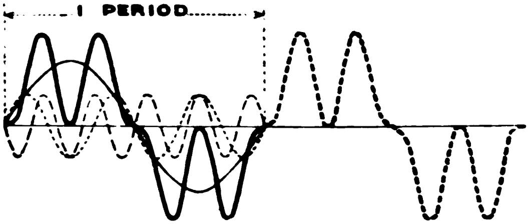

FIG. 55.

6. If, however, we take the curves that are made from *odd* harmonics (even those with lags, and which are also unsymmetrical about the peak) such as Fig. 47, and push the second half underneath the first, as in Fig. 57, we shall find they correspond exactly above and below. There is therefore

a generic difference between curves built up of even harmonics and those built up of odd harmonics.

In the electromotive-force curves of alternators, we have no even harmonics. The second half of the period is the exact counterpart of the first half, for the very simple reason that when one pole is moving past one "side" of any armature coil, the next pole is moving past the other "side" of that coil, so that even if the North poles differed in shape from the South poles, the curves would show no difference. As we shall see, the circumstance that the voltage curves contain odd harmonics only, much simplifies the study of them.

So far we have been considering solely sine-curves, with or without a lag. We have now to touch on a new consideration.

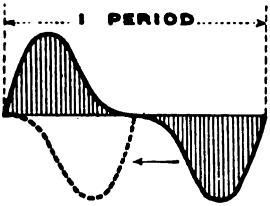

FIG. 56.

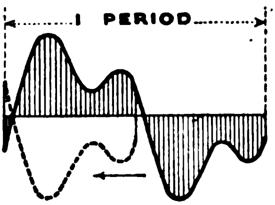

FIG. 57.

It was shown (in Chapter I., p. 30) that it is possible to resolve a sine-curve into two sine-curves that differ from one another by any prescribed difference of phase. The case that most commonly presents itself is the resolution into two that are in quadrature with one another, *i.e.* at right angles in phase; as for example when one resolves a lagging current into two component currents, one an effective current in phase with the voltage, the other an idle or wattless current at right-angles in phase to the voltage.

In this way a lagging curve may be resolved into two components, a sine-curve which does not lag and another sine-curve which lags exactly 90°, or is in other words a cosine-curve. This was done graphically on p. 14, Fig. 7. Algebraically, the problem is as follows:—

To resolve a lagging sine-function of θ into two parts, one in phase with θ, the other at right-angles to θ in phase

$$y = \mathrm{A} \sin (\theta \pm \phi) \quad . \quad . \quad . \quad [1]$$

where ϕ is the angle of lag. Expanding, we have

$$y = \mathrm{A} \sin \theta \cos \phi \pm \mathrm{A} \cos \theta \sin \phi.$$

Now write $\mathrm{A} \cos \phi = \mathrm{M}$; and $\mathrm{A} \sin \phi = \mathrm{N}$; where M and N are the resolved parts of A. Substituting these values, we have at once

$$y = \mathrm{M} \sin \theta \pm \mathrm{N} \cos \theta \quad . \quad . \quad [2]$$

Conversely, if we are given $y = \mathrm{M} \sin \theta + \mathrm{N} \cos \theta$, we may compound these into one single sine term and write it

$$y = \mathrm{A} \sin (\theta + \phi), \quad . \quad . \quad [3]$$

where

$$\phi = \text{arc tan} \frac{\mathrm{N}}{\mathrm{M}}, \text{ and } \mathrm{A} = \sqrt{\mathrm{M}^2 + \mathrm{N}^3} \quad . \quad [4]$$

Example.—It is easy to verify this by examples. Suppose A is = 10, and that there is a phase-difference of 23°. From trigonometrical tables we get $\cos 23° = 0{\cdot}92$, $\sin 23° = 0{\cdot}39$; $\mathrm{M} = 9{\cdot}2$ and $\mathrm{N} = 3{\cdot}9$. Now take any value of θ, say 60°, and work out the corresponding value of y by equation [1] above. $y = 10 \times \sin (60° + 23°) = 10 \times \sin 83° = 9{\cdot}93$. Then calculate y by equation [2], $y = 0{\cdot}2 \times \sin 60° + 3{\cdot}9 \times \cos 60°$. $\operatorname{Sin} 60° = 0{\cdot}866$; $\cos 60° = 0{\cdot}5$; therefore $y = 9{\cdot}2 \times 0{\cdot}866 + 3{\cdot}9 \times 0{\cdot}5$; or $y = 7{\cdot}956 + 1{\cdot}95 = 9{\cdot}926$.

ANALYSIS OF COMPOUND PERIODIC CURVES.

We have been learning how to build up complicated wave-forms by superposition of their elements, and have ascertained many of their principal features. But it must be evident that the same laws that govern the synthesis of a complicated wave-form from its harmonic constituents will also lead to the analysis of the complicated forms into their several constituents. But the process is much less easy than that of building up. If we are merely given a complicated periodic curve and are asked of what elements it can be built up, we find ourselves at once confronted with a number of unknown possibilities. Mere inspection will afford some help, though only of a qualitative kind. For example, we can see that the curve marked A, in Fig. 58, has six ripples in one period, and we therefore know that the sixth harmonic is present: and we could even estimate its relative amplitude. Curve B in Fig. 59 obviously contains no even harmonics, but only odd ones, because the peak is symmetrical about its mid line, but though we might guess (by comparison with Fig. 51 above), that the third harmonic is present with a negative amplitude, we cannot by inspection say what others of the odd harmonics are present. Inspection of curve C, Fig. 60, shows that here we have to deal with a curve in which some at any rate of the components lag, or that, expressing it otherwise, we shall require cosine-terms as well as sine-terms for its expression. Moreover, if we apply the test of shifting the second half of it beneath the first half, we observe that they are not symmetrical above and below, and therefore know that there are some even harmonics present, whether there are odd ones or not. But take now the curves D, E and F, of Figs. 61, 62, and 63. Let the reader study these and try to pronounce of what elements they are constituted.

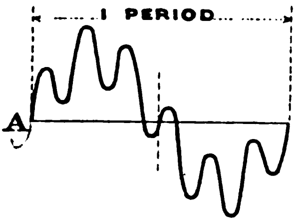

FIG. 58.

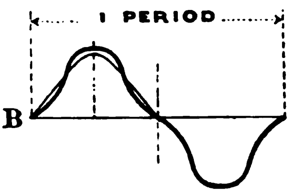

FIG. 59.

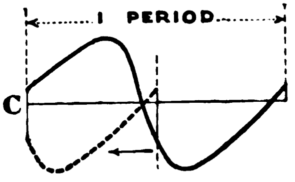

FIG. 60.

Fourier's theorem, briefly stated for the present purpose, is to the effect that *any periodic function,* however complicated, *may* (if it is single-valued) *be considered as built up of a series of harmonic terms, the fundamental one of the series being of the same period as the given function.* In symbols it will be expressed as

$$y = A_1 \sin(\theta + \phi_1) + A_2 \sin(2\theta+\phi_2) + A_3 \sin(3\theta+\phi_3) + \ldots\ldots + A_n \sin(n\theta+\phi_n),$$

where there may be any number of terms, and where in general each term may have a lag as indicated by the letter ϕ. A constant A_0 must be added if the base-line has not been drawn so that the mean ordinate is zero. Or, since

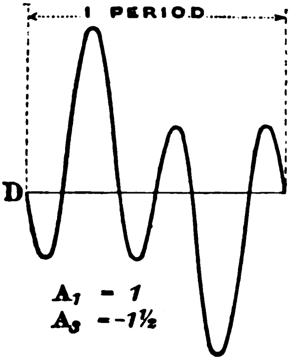

FIG. 61.

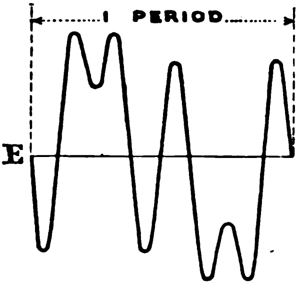

FIG. 62.

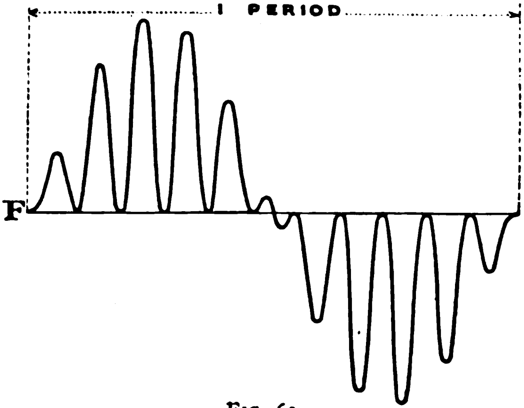

FIG. 63.

any lagging sine-term may be split up into a pair of sine and cosine terms as shown above, the series may be written as:—

$$y = A_1 \sin\theta + B_1 \cos\theta + A_2 \sin 2\theta + B_2 \cos 2\theta + A_3 \sin 3\theta + B_3 \cos 3\theta + \dots\dots + A_n \sin n\theta + B_n \cos n\theta.$$

In the above enunciation it was stipulated that the periodic function should be *single valued*, that is for any value of θ, there should be but one single value of y. In other words, this is a stipulation that the curve shall not, like Fig. 64, turn back upon itself, for then y would not be single-valued. In the cases we have to deal with in electrical applications, the value θ is proportional to *time*, and as time cannot turn back on itself, the functions are necessarily single-valued.

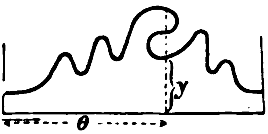

FIG. 64.

The process of finding the appropriate values of the coefficients A_1 A_2 A_3, etc., and B_1 B_2 B_3, etc., that must be put into the Fourier series in order to make it represent any given curve, is called *harmonic analysis*. And, since all sorts of (single-valued) curves can be built up of harmonics by taking higher harmonics to fill up the details, it follows that in some cases many terms of the series must be employed.

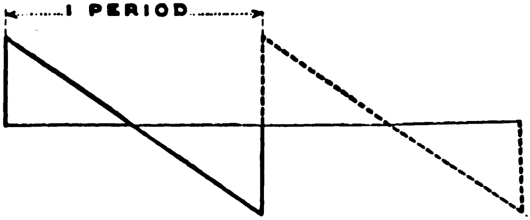

FIG. 65.

Theoretically an infinite number of terms might be required in order to express accurately some given curve of arbitrary form. But in many cases a few terms suffice for an adequate solution; and in many others the coefficients are found to decrease from term to term according to some simple rule. The following examples of curves built up of infinite series will assist.

If $A_1 = 1$, $A_2 = \frac{1}{2}$, $A_3 = \frac{1}{3}$, $A_4 = \frac{1}{4}$, and so on in descending order, and all the B coefficients are zero, we get, with an infinite series, a 'curve' (Fig. 65), simply made up of pieces of straight lines, sloping like a saw-tooth. But if we had taken only as far as eighteen terms the outline would have only filled up to that shown in Fig. 66.

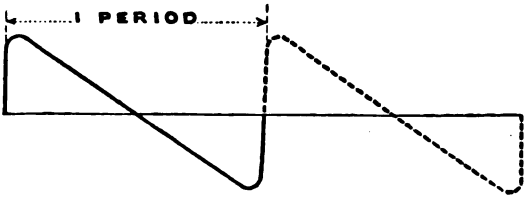

FIG. 66.

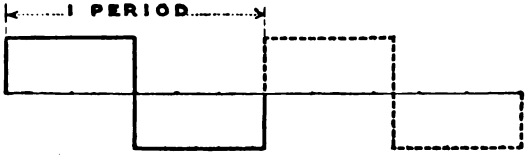

FIG. 67.

Again, if odd coefficients only are taken of the values

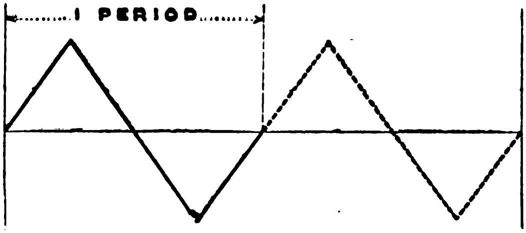

FIG. 68.

$A_1 = 1$, $A_3 = \frac{1}{3}$, $A_5 = \frac{1}{5}$, $A_7 = \frac{1}{7}$, etc.... the form that results is that of Fig. 67. But even if we take 100 terms the corners will not be sharp. The form given in Fig. 68 is that produced by taking the following set of coefficients : $A_1 = 1$,

$A_3 = -\frac{1}{9}$, $A_5 = +\frac{1}{25}$, $A_7 = -\frac{1}{49}$, and so on with alternated + and − values.

Examples.—The reader should try the process of calculating and plotting curves for himself; the following being suitable :—

(1) $A_1 = 1$, $B_1 = 0$, $A_2 = \frac{1}{2}$, $B_2 = 0$, $A_3 = \frac{1}{4}$, $B_3 = 0$.
(2) $A_1 = \frac{1}{2}$, $A_2 = 1$, $A_3 = 2$, $A_4 = 1$, $A_5 = \frac{1}{2}$; all B terms $= 0$.
(3) $B_1 = 1$, $B_3 = \frac{1}{4}$, $B_5 = \frac{1}{16}$, $B_7 = \frac{1}{64}$; all A terms $= 0$.
(4) Odd A terms only, all A's equal, up to A_{15}.
(5) $A_1 = 10{\cdot}2$, $A_3 = 3{\cdot}8$, $A_5 = 2{\cdot}6$.
(6) $A_1 = 1$, $A_2 = 2$, $B_2 = \frac{1}{2}$, $A_9 = \frac{1}{4}$.

Harmonic Analysis.—Now to analyse curves into their harmonic constituents requires one of four possible processes :—

(*a*) *By algebraic reduction of coefficients.*

(*b*) *By integration.*[1] This involves some slight proficiency in the use of integral calculus. The method is described in a note at the end of this Chapter.

(*c*) *By graphic reduction*[2] : as shown in p. 81.

(*d*) *By reduction by grouping.*[3]

(*e*) *By use of the instrument known as a harmonic analyser.*[4]

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[1] See Perry's *Calculus for Engineers*, p. 202.
Graham's *Elementary Treatise on the Calculus*, p. 184.
Perry in *The Electrician*, xxviii., 362, Feb. 5, 1892.
Perry in *Proc. Phys. Soc.*, xiii., 97, 1895.

[2] See Perry in *The Electrician*, xxxv., 285, June 28, 1895.
E. B. Wedmore in *The Electrician*, xxxv., 512, Aug. 16, 1895.
E. B. Wedmore, *Journ. Inst. Elect. Engineers*, xxv., 224, March 1896.
Houston and Kennelly, *Electrical World*, xxxi., 580, May 14, 1898.
F. Loppé in *L'Éclairage Électrique*, xxxi., 449, June 28, 1902.

[3] C. Runge, *Zeitschrift für Mathematik und Physik*, xlviii., 443, 1903.

[4] See Appendix to Thomson and Tait's *Natural Philosophy*, 2nd edition, 1883.
Henrici, in *The Electrician*, xxxiii., 544.
Archibald Sharp, Harmonic Analyser, *Phil. Mag.*, July 1894.
Archibald Sharp, New Method in Harmonic Analysis, *Proc. Phys. Soc.*, xiii. 599, 1895.
G. U. Yule on a Simple Form of Harmonic Analyser, *Proc. Phys. Soc.*, xiii. 403, 1895.
G. Coradi, Der Harmonische Analysator, Zurich, 1894.
A. A. Michelson and S. W. Stretton, *Phil. Mag.*, May 1898.

HARMONIC ANALYSIS BY ALGEBRAIC REDUCTION OF COEFFICIENTS.

The following method of calculating the approximate values of the coefficients is far simpler than any heretofore published, and it suffices amply for the purpose of the electrical engineer. The simplicity of it arises from the following considerations. *First*: as already seen, all even terms are absent, and the Fourier series required consists of odd terms only. *Second*: this being so, the zero line or mean horizontal axis of the curve can always be found midway between the highest and lowest points of the curve, and hence we can always choose the origin at a point where the curve crosses the horizontal axis. *Third*: if the half-period of the curve is symmetrical as to the first and second quarter-periods, that is, is symmetrical right and left of 90°, all cosine-terms will be absent (this is mostly the case for no-load curves of alternators). *Fourth*: if the curve is, like Fig. 56, not symmetrical as to the first and second quarter-periods (as is the case for full-load voltage curves of alternators) then there are cosine-terms present as well as sine-terms. *Fifth*: if sine-terms only, or if cosine-terms only, are present it is easy by mere algebraic considerations to deduce the coefficients from the values of a given set of ordinates by the very *simple process* shown below. *Sixth*: if both sine- and cosine-terms are present, it is always possible to separate them, by the *artifice* of finding sums and differences of the given co-ordinates, and then, having separated them, deduce the A coefficients and the B coefficients separately.

CASE I. *Algebraic Reduction of Coefficients. Sine-terms only.*—We assume that odd sine-terms only are present, and for greater simplicity we will deal with a case in which the higher harmonics beyond that of the fifth order are absent. Suppose, then, that we carry the Fourier series to three terms only, we must write it:

$$y = A_1 \sin \theta + A_3 \sin 3\theta + A_5 \sin 5\theta.$$

The problem before us—harmonic analysis—is to find from the curve, such as that given in Fig. 69, the values of the unknown coefficients A_1, A_3 and A_5. Now to find three unknown quantities we must have at least three equations. We find these as follows. Consider any point on the curve such as P (Fig. 69). Its abscissa is some value of θ, call it θ_1; its ordinate we will call y_1. Because the curve is not a simple sine-curve the height of the ordinate y_1 is not equal to $A_1 \sin \theta_1$, but is made up of that *plus* bits of ordinates belonging to the two other component curves, viz. of a piece equal to $A_3 \sin 3\theta_1$ and another equal to $A_5 \sin 5\theta_1$. In fact

$$y_1 = A_1 \sin \theta_1 + A_3 \sin 3\theta_1 + A_5 \sin 5\theta_1.$$

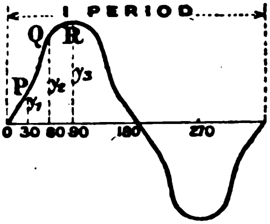


FIG. 69.

In like manner we may write for two other points such as Q and R, the other equations

$$y_2 = A_1 \sin \theta_2 + A_3 \sin 3\theta_2 + A_5 \sin 5\theta_2$$
$$y_3 = A_1 \sin \theta_3 + A_3 \sin 3\theta_3 + A_5 \sin 5\theta_3.$$

We have then three equations in which all the quantities are known except the three coefficients; which can therefore be found, wherever the three points P, Q and R may be put on the curve. But there are some advantages in choosing for P, Q and R, certain particular distances along the curve, so as to give values of $\sin \theta$ that are easily managed. As the curve is symmetrical we may take them all in the first quarter period; and for simplicity take them at equidistant angles, namely 30°, 60° and 90°. Then our three equations become

$$\begin{cases} y_1 = A_1 \sin 30^\circ + A_3 \sin (3 \times 30^\circ) + A_5 \sin (5 \times 30^\circ) \\ y_2 = A_1 \sin 60^\circ + A_3 \sin (3 \times 60^\circ) + A_5 \sin (5 \times 60^\circ) \\ y_3 = A_1 \sin 90^\circ + A_3 \sin (3 \times 90^\circ) + A_5 \sin (5 \times 90^\circ) \end{cases}$$

Now the sines in question have the following values:—

$$\left\{\begin{array}{llllll} \sin 30^\circ = +0{\cdot}5 & ; & \sin 90^\circ = +1 & ; & \sin 150^\circ = +0{\cdot}5 \\ \sin 60^\circ = +0{\cdot}866 & ; & \sin 180^\circ = 0 & ; & \sin 300^\circ = -0{\cdot}866 \\ \sin 90^\circ = +1 & ; & \sin 270^\circ = -1 & ; & \sin 450^\circ = +1 \end{array}\right.$$

Inserting these numerical values, we have :—

$$\left\{\begin{array}{l} y_1 = +0{\cdot}5\ A_1 + 1\ A_3 + 0{\cdot}5\ A_5 \\ y_2 = +0{\cdot}866\ A_1 + 0 - 0{\cdot}866\ A_5 \\ y_3 = +1\ A_1 - 1\ A_3 + 1\ A_5 \end{array}\right.$$

Next further simplify the coefficients by multiplying the first equation through by 2; and multiply likewise the second equation through by 1·154, the reciprocal of 0·866. This gives

$$\left\{\begin{array}{rl} 2y_1 = & +1\ A_1 + 2\ A_3 + 1\ A_5 \\ 1{\cdot}154y_2 = & +1\ A_1 + 0 - 1\ A_5 \\ y_3 = & +1\ A_1 - 1\ A_3 + 1\ A_5 \end{array}\right.$$

Now subtract the third equation from the first, and

$$2y_1 - y_3 = 3\ A_3;$$

whence

$$A_3 = 0{\cdot}667\,y_1 - 0{\cdot}333y_3.$$

Next add the first two equations together, and

$$2y_1 + 1{\cdot}154y_2 = 2\ A_1 + 2\ A_3;$$

$$\therefore\quad A_1 = y_1 + 0{\cdot}577\,y_2 - 0{\cdot}667\,y_1 + 0{\cdot}333y_3$$
$$= 0{\cdot}333y_1 + 0{\cdot}577\,y_2 + 0{\cdot}333y_3$$

Then substituting the above values in equation, we have

$$A_5 = y_3 - 0{\cdot}333y_1 - 0{\cdot}577y_2 - 0{\cdot}333y_3$$
$$+ 0{\cdot}667y_1 - 0{\cdot}333y_3$$
$$= 0{\cdot}333y_1 - 0{\cdot}577y_2 + 0{\cdot}333y_3.$$

Collecting our results, we have

FOR SINES
$$\left\{\begin{array}{l} A_1 = 0{\cdot}333y_1 + 0{\cdot}577y_2 + 0{\cdot}333y_3 \\ A_3 = 0{\cdot}667\,y_1 - 0{\cdot}333y_3 \\ A_5 = 0{\cdot}333y_1 - 0{\cdot}577y_2 + 0{\cdot}333y_3 \end{array}\right.$$

These last three equations enable us readily to get the values of A_1, A_3 and A_5 from the measured heights of the ordinates erected at 30°, 60° and 90° along the given curve.

Example.—In Fig. 70 the heights of the three ordinates are $y_1 = 11$; $y_2 = 12$; $y_3 = 18$. Hence

$$\begin{cases} A_1 = 3\cdot667 + 6\cdot924 + 6 = 16\cdot59 \\ A_3 = 7\cdot333 \qquad\qquad - 6 = \ 1\cdot33 \\ A_5 = 3\cdot667 - 6\cdot924 + 6 = \ 2\cdot74 \end{cases}$$

and the equation to the curve will be

$$y = 16\cdot59 \sin \theta + 1\cdot33 \sin 3\theta + 2\cdot74 \sin 5\theta.$$

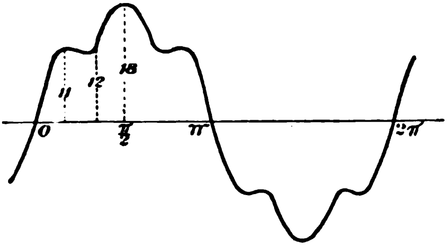


FIG. 70.

If this curve be plotted it will be found to agree very precisely with Fig. 70. The virtual value, or quadratic mean, of the curve will be

$$y_{\text{virt}} = \sqrt{\tfrac{1}{2}\{(16\cdot59)^2 + (1\cdot33)^2 + (2\cdot74)^2\}} = 12\cdot12.$$

It is obvious that the process applied above to the simple case of three terms might be extended to more complicated cases in which more terms are included, so as to obtain a still higher degree of approximation. Thus, if we wish to take into account the seventh harmonic, we must have four equations :—

$$\begin{aligned} y_1 &= A_1 \sin \theta_1 + A_3 \sin 3\,\theta_1 + A_5 \sin 5\,\theta_1 + A_7 \sin 7\,\theta_1 \\ y_2 &= A_1 \sin \theta_2 + A_3 \sin 3\,\theta_2 + A_5 \sin 5\,\theta_2 + A_7 \sin 7\,\theta_2 \\ y_3 &= A_1 \sin \theta_3 + A_3 \sin 3\,\theta_3 + A_5 \sin 5\,\theta_3 + A_7 \sin 7\,\theta_3 \\ y_4 &= A_1 \sin \theta_4 + A_3 \sin 3\,\theta_4 + A_5 \sin 5\,\theta_4 + A_7 \sin 7\,\theta_4 \end{aligned}$$

As before, we may take the angles all in the first quarter period, and as we particularly wish to find the higher harmonics accurately, we will take the angles 18°, 36°, 54° and 72° which have the double advantage of being easily measured by dividing the distance between 0° and 90° into five equal parts and of being so situated as to catch the harmonics at or near a number of their maximum values. Substituting the values of the sines and solving the equations, we have

$$\begin{aligned}
A_1 &= 0\cdot34\,y_1 && + 0\cdot34\,y_3 + 0\cdot65\,y_4 \\
A_3 &= 0\cdot17\,y_1 + 0\cdot525\,y_2 && + 0\cdot17\,y_3 - 0\cdot525\,y_4 \\
A_5 &= 0\cdot45\,y_1 && - 0\cdot55\,y_3 + 0\cdot33\,y_4 \\
A_7 &= 0\cdot38\,y_1 - 0\cdot525\,y_2 && + 0\cdot38\,y_3 - 0\cdot12\,y_4
\end{aligned}$$

The degree to which such approximations will suffice will depend upon the complexity of detail of the given curve. Clearly, if a curve has an outline rippled with eleven ripples in the period it cannot be adequately represented by an analysis which only goes as far as the ninth harmonic.

CASE II. *Algebraic Reduction of Coefficients. Cosine-terms only.*—Since cos 0° = 1, it follows that a curve consisting of cosine-terms only cannot, if the origin is taken at angle 0°, have a zero ordinate there unless the algebraic sum of the coefficients of its terms is zero. For we now have

$$y = B_1 \sin \theta + B_3 \sin 3\theta + B_5 \sin 5\theta, \text{ etc.};$$

and if $\theta = 0$, we shall have

$$y_0 = B_1 + B_3 + B_5 + \text{etc.}$$

Hence, if we have, as laid down above, for the sake of simplicity, chosen our origin at a place where the curve crosses the horizontal axis, it is clear that

$$B_1 + B_2 + B_3 + \text{etc.} = 0.$$

To apply the formulæ to cases where the origin is not taken at this point, we will, however, assume here that y_0 is not of zero value.

If, as before, we are going to work to three terms merely, we shall want three ordinates given. It is (for calculating cosines) no use taking an ordinate at 90°, since cos 90° is = 0, and so are cos (3 × 90°), cos (5 × 90°), etc. So we will take them at the angles of 0°, 30°, and 60°.

Then our three equations will be :—

$$\begin{cases} y_0 = B_1 \cos 0^\circ + B_3 \cos (3 \times 0^\circ) + B_5 \cos (5 \times 0^\circ) \\ y_1 = B_1 \cos 30^\circ + B_3 \cos (3 \times 30^\circ) + B_5 \cos (5 \times 30^\circ) \\ y_3 = B_1 \cos 60^\circ + B_3 \cos (3 \times 60^\circ) + B_5 \cos (5 \times 60^\circ) \end{cases}$$

Inserting the numerical values, we have :—

$$\begin{cases} y_0 = +1 \; B_1 + 1 \, B_3 + 1 \; B_5 \\ y_1 = +0{\cdot}865 \, B_1 \qquad - 0{\cdot}866 \, B_5 \\ y_2 = +0{\cdot}5 \; B_1 - 1 \, B_3 + 0{\cdot}5 \; B_5 \end{cases}$$

and eliminating as before, we get :—

$$\text{For cosines} \begin{cases} B_1 = 0{\cdot}333 \, y_0 + 0{\cdot}577 \, y_1 + 0{\cdot}333 \, y_2 \\ B_3 = 0{\cdot}333 \, y_0 \qquad - 0{\cdot}667 \, y_2 \\ B_5 = 0{\cdot}333 \, y_0 - 0{\cdot}577 \, y_1 + 0{\cdot}333 \, y_2 \end{cases}$$

Example—A curve known to have none but odd cosine-components, since it was symmetrical and had zero at 90°, had at 0°, 30° and 60° the following ordinates: $y_0 = 9{\cdot}3$; $y_1 = 10$; $y_2 = 12{\cdot}1$. The coefficients come out: $B_1 = 12{\cdot}9$; $B_3 = -4{\cdot}96$; $B_5 = 1{\cdot}36$.

CASE III. *When both Sine and Cosine Terms are present.*—As shown above, any harmonic with a lag may be resolved into a non-lagging sine-term and a cosine-term. Hence a curve, such as Fig. 46, might be treated as containing a cosine-component as well as a sine-component of the third order. To apply the simple *process* just explained to cases where both sine and cosine members of the Fourier series are present, we have resort to the following *artifice*, by means of which when we have a number of ordinates given we can dissect them into sine and cosine parts. This artifice depends on the following bases. We know that $\sin \theta = \sin (180^\circ - \theta)$; and that $\cos \theta = -\cos (180^\circ - \theta)$. Suppose then that we take the y_θ for the curve (θ being less than 90°), and take also the ordinate $y_{180-\theta}$ for the supplementary angle, and add them together, any component parts due to presence of cos θ will cancel out; so will any due to cos 3θ or cos 5θ, or, in fact, to any term of the cosine series; while on the other hand, any component parts due to sin θ, sin 3θ, or other odd term of the sine series will be doubled. If, instead, we substract the ordinate $y'_{180-\theta}$ from ordinate y_θ, we have the sine-components cancelling, leaving only cosine-components all doubled. The results in symbols are these:—

$$y_\theta + y_{180-\theta} = 2\{A_1 \sin\theta + A_3 \sin 3\theta + \ldots\ldots A_n \sin n\theta\};$$
$$y_\theta - y_{180-\theta} = 2\{B_1 \cos\theta + B_3 \cos 3\theta + \ldots\ldots B_n \cos n\theta\}.$$

Suppose then we divide a half-period into any number (say 6, as in Fig. 71) of equidistant parts, we shall have a

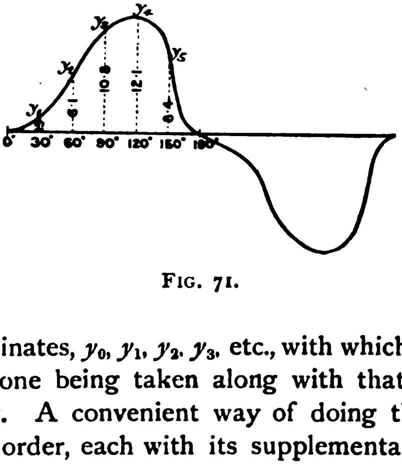


FIG. 71.

number of ordinates, y_0, y_1, y_2, y_3, etc., with which we may deal in pairs, each one being taken along with that belonging to its supplement. A convenient way of doing this is to write them down in order, each with its supplementary one underneath, beginning with y_0, thus:—

	y_0	y_1	y_2	y_3
	y_6	y_5	y_4	y_3
Sum	a_0	a_1	a_2	a_3
Difference	b_0	b_1	b_2	b_3

Then take the sums and the differences of them, as shown the letter a_1 standing for the sum of y_1 and y_5, and the letter b_1 standing for the difference of y_1 and y_5. (The values of a_0 and b_0 will both be zero if the origin has been chosen so that y_0 and y_6 are both zero. Also b_3 will be zero because y_3 is at 90°.)

Now by this simple artifice we have split our ordinates into sine and cosine terms; we may at once use

$$\frac{a_1}{2} \quad \frac{a_2}{2} \quad \text{and} \quad \frac{a_3}{2}$$

as though they were simply 3 ordinates of a curve containing sines only, and put them into the schedule "FOR SINES" on p. 66 in place of y_1, y_2, and y_3, and so compute the A coefficients.

Similarly we may at once use

$$\frac{b_0}{2} \quad \frac{b_1}{2} \quad \text{and} \quad \frac{b_2}{2}$$

inserting them in place of y_0, y_1 and y_2 in the schedule "FOR COSINES" on p. 69, and so compute the B coefficients.

Example of Analysis.—Required the harmonic analysis, to the terms of the fifth order, of the curve, Fig. 71, the ordinates for the angles 0°, 30°, 60°, 90°, 120°, 150°, 180°, being given as 0, 1·5, 6·1, 10·8, 12·1, 8·4, 0.

We begin by arranging the ordinates as directed in two rows.

	0	1·5	6·1	10·8
	0	8·4	12·1	10·8
Sums	0	9·9	18·2	21·6
Differences	0	−6·9	−6·0	0

Dividing by 2 we get the equivalent values to be put into the schedules as follows:—

For sines.	4·95	9·1	10·8
For cosines	0	−3·45	−3·0

For sines

$$\left\{\begin{array}{llll} A^1=0{\cdot}333\times4{\cdot}95 & +0{\cdot}577\times9{\cdot}1 & +0{\cdot}333\times10{\cdot}8= & 10{\cdot}5 \\ A_3=0{\cdot}667\times4{\cdot}95 & & -0{\cdot}333\times10{\cdot}8= & -0{\cdot}3 \\ A_5=0{\cdot}333\times4{\cdot}95 & -0{\cdot}577\times9{\cdot}1 & +0{\cdot}333\times10{\cdot}8= & 0 \end{array}\right.$$

For cosines

$$\left\{\begin{array}{llll} B_1=0{\cdot}333\times0 & +0{\cdot}577\times(-3{\cdot}45) & +0{\cdot}333\times(-3{\cdot}0)= & -3{\cdot}02 \\ B_3=0{\cdot}333\times0 & & -0{\cdot}666\times(-3{\cdot}0)= & 2{\cdot}0 \\ B_5=0{\cdot}333\times0 & -0{\cdot}577\times(-3{\cdot}45) & +0{\cdot}333\times(-3{\cdot}0)= & 1{\cdot}02 \end{array}\right.$$

Hence the equation to the curve is :—

$$y=10{\cdot}5\sin\theta-0{\cdot}3\sin3\theta-3{\cdot}02\cos\theta+2{\cdot}0\cos3\theta+1{\cdot}02\cos5\theta$$

or, combining sine and cosine terms by the method of p. 58, equations [3] and [4],

$$y=10{\cdot}9\sin(\theta-16^\circ)+2{\cdot}02\sin3(\theta-80^\circ)+1{\cdot}02\sin5(\theta+90^c).$$

It is possible to check the results by the following tests. The sum of $B_1 + B_3 + B_5$ should equal y_0, that is, should be zero if the origin is taken where the curve cuts the axis. The sum of $A_1 - A_3 + A_5$ should equal the ordinate at 90°.

HARMONIC ANALYSIS BY INTEGRATION.

Suppose we have a periodic function expressed as

$$y=f(t)$$

where y is a function (single-valued) of the time t, that has elapsed from the origin, when $y = 0$. This, of course, may be exhibited as a curve of some definite form (such as Fig. 72). We will assume the function to be such as to fulfil the conditions mentioned above[1] (p. 64), so we may know that it can

---

[1] These are (1) no constant term A_0; (2) second half of the wave like the first half simply inverted, *but not reversed* right for left.

be expanded in a harmonic series of sine-functions. Then we may write it :—

$$y = A_1 \sin pt + A_2 \sin 2pt + A_3 \sin 3pt + \ldots \ldots + A_n \sin npt;$$

where p is written short for $\frac{2\pi}{T}$; the symbol T standing for the periodic time, or the duration of one whole period.

It is required to analyse the function $f(t)$ so as to obtain the values of the various coefficients A_1, A_2, A_3, etc. of the several terms.

This can be done by integration in a number of successive repetitions of a certain process. This particular process is a mathematical device depending on certain principles which must first be explained.

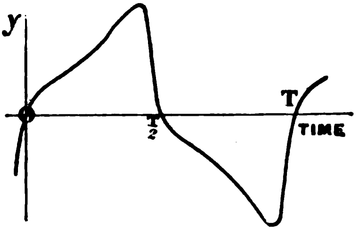


FIG. 72.

Principle I.

$$\int \sin pt.dt = -\frac{1}{p} \cos pt.$$

This is shown in the beginning of every elementary treatise on the Integral Calculus.

Principle II.

$$\int_{t=0}^{t=T} \sin pt \, . \, dt = 0.$$

This may be stated in words: that if a sine-function of the time be integrated from the beginning to the end of one complete period, the value of the integral is zero. This is seen at once on performing the integration by Principle I.

For integrating once for the superior limit, when $t = \mathrm{T}$, the value of the integral is $-\frac{1}{p}\cos p\mathrm{T}$; or (since $p = 2\pi \div \mathrm{T}$) it may be written $-\frac{1}{p}\cos 2\pi$, which (since $\cos 2\pi = 1$) is simply $-\frac{1}{p}$. Then writing the integration a second time for the inferior limit, when $t = 0$, the value of the integral is $-\frac{1}{p}\cos 0$, which (since $\cos 0 = 1$) is also simply $-\frac{1}{p}$. Hence subtracting, so as to obtain the value of the integral between the superior and inferior limits, we have

$$\int_{t=0}^{t=\mathrm{T}} \sin pt \,.\, dt = \left(-\frac{1}{p}\cos p\mathrm{T}\right) - \left(-\frac{1}{p}\cos 0\right)$$

$$= -\frac{1}{p} + \frac{1}{p} = 0.$$

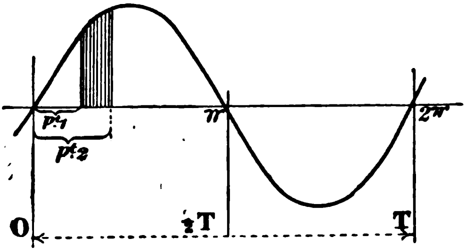


FIG. 73.

These two principles may be illustrated graphically, as follows:—

The integral of a portion of any curve being the area beneath it, if in Fig. 73 we have the ordinate a corresponding to the abscissa pt_1, and the ordinate b corresponding to the abscissa pt_2, so that

$$a = \sin pt_1\,;\; b = \sin pt_2,$$

then the area enclosed under the curve from a to b will be the difference between the values of the areas from 0 up to b and from 0 up to a. Or in symbols

$$\int_{t=t_1}^{t=t_2} \sin pt \,.\, dt = -\frac{1}{p}\cos pt_2 + \frac{1}{p}\cos pt_1.$$

Now suppose we shift the points *a* and *b* so that *a* comes back to the origin when $t = 0$, and that *b* moves right on up to the end of the complete period, so that $t = T$. In that case the area enclosed under one whole period of the curve (from 0 to 2π in length along the axis of abscissæ if reckoned in angle, or from 0 to T if reckoned in time) will consist of two equal areas, positive in the first half-period, negative in the second half-period, and therefore, when added together, equal precisely to zero.

If the area under one whole period of a sine-curve is zero, it will also be zero for any number of whole periods; or

$$\int_{t=0}^{t=T} \sin spt \, . \, dt = 0$$

provided *s* is any whole number.

This will be true wherever the origin may be reckoned from. For example, it will be true for the curve in Fig. 74.

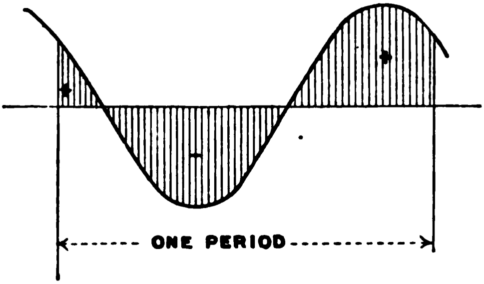


FIG 74.

the sum of the positive and negative areas for one whole period being zero.

Principle III.

$$\int_{t=0}^{t=T} \sin^2 pt \, . \, dt = \tfrac{1}{2} T.$$

In words, this is that the time-integral, over one whole period, of the square of a sine-function is equal to one-half of the periodic time.

This may be verified from the general integral:—

$$\int \sin^2 pt\,.\,dt = \frac{t}{2} - \frac{\sin 2pt}{4p},$$

by simply inserting in this expression $t = \mathrm{T}$ for the upper limit and subtracting from it the similar expression in which the value $t = 0$ has been inserted for the lower limit.

Graphically this principle may be illustrated by Fig. 75 in which a sine-curve is shown, and also a curve in which all the ordinates are squared, their value varying from 0 to + 1, and back, twice in the period. Now the crests and hollows of this curve are identical in outline, since $\sin^2 x = 1 - \sin^2 (90^\circ - x)$. Hence it follows that the total area in one whole period is equal to half the rectangle whose height = 1 and whose length is equal to T. If we had taken the second harmonic, with two sine-waves in the length T, and therefore 4 peaks in the corresponding squares of the sines, the total area would still be = ½ T. Hence also the value of the total area over one whole period T would be the same for any of the higher harmonics of equal amplitude; or, in symbols, it will be true that

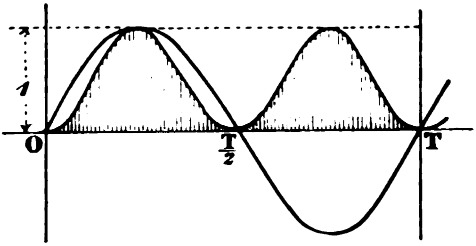


FIG. 75.

$$\int_{t=0}^{T=t} \sin^2 spt\ .\ dt = \tfrac{1}{2}\mathrm{T},$$

if s is any whole number.

Principle IV. If s and r be any whole numbers, then

$$\int_{t=0}^{t=\mathrm{T}} \sin spt\ .\ \sin rpt\ .\ dt = 0.$$

In words, this is that if the ordinates of two sine-curves of *different* periods be multiplied together, and a new curve be plotted with the products as ordinates, the area of this curve will be zero if reckoned out for a period T which is a multiple of each of their individual periods. For example, if $s = 3$ and $r = 4$, there will be three waves of the former and four waves of the latter in one whole period. The proposition, in reality relates to the products of the different harmonics. One might have expected that since the integral (over one period) of $\sin^2 spt$ is $\frac{1}{2}$ T, that the integral of such products as $\sin spt \times \sin rpt$ would not be in every case zero—for instance, when s and r were numbers near to one another, as when say $s = 9$ and $r = 10$. But this is not so, and it is of primary importance to grasp that even if s were say 999, and r were 1000, still the integral of the product would be $= 0$, although the value of the square of the sine-function would be $\frac{1}{2}$ T if both s and r were 999 or if both s and r were 1000.

That the integral of the products *must* be zero is readily seen by remembering that the product

$$\sin spt \, . \, \sin rpt = \tfrac{1}{2} \cos (s - r) pt \; - \; \tfrac{1}{2} \cos (s + r) pt.$$

$$\int_{t=0}^{t=T} \cos mpt \, . \, dt = 0 \, ;$$

where m is any integer. It follows that the corresponding value of each of the two cosine-terms will be zero.

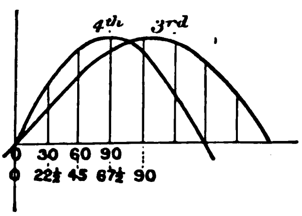


FIG. 76.

A graphical example will perhaps elucidate the point. Consider the product of the third and fourth harmonics, $y = \sin 3\theta$ and $y = \sin 4\theta$. Suppose the whole fundamental period divided into 48 equal intervals: the third harmonic will recur in 16 of these and the fourth in 12 of them. Each interval will correspond in angle to $360° \div 48 = 7\frac{1}{2}°$. Beginning at zero at the end of the first interval, $3\theta = 22\frac{1}{2}°$ and $4\theta = 30°$. Hence, at the end of the first interval, the values of the ordinates will be $\sin 22\frac{1}{2}°$ and $\sin 30°$ respectively, or in numbers $0 \cdot 39$

and 0·50. Similarly at the end of the second interval the respective values of the ordinates will be sin 45° and sin 60°, or 0·707 and 0·866; at the end of the third interval sin $67\frac{1}{2}$° and sin 90°, or, 0·92 and 1·0. (See Fig. 76.) We can then write down at once in two rows the successive values of the sines. For simplicity we will take the nearest figure in the first place of decimals only, and under each pair of ordinates we will set down the products. For convenience in printing the signs + and − are set above the numbers to which they refer.

Interval ..	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Sin 3θ ..	0	+4	+7	+9	+10	+9	+7	+4	0	−4	−7	−9	−10	−9	−7	−4	0	+4	+7	+9	+10	+9	+7	+4	+0
Sin 4θ ..	0	+5	+9	+10	+9	+5	0	−5	−9	−10	−9	−5	0	+5	+9	+10	+9	+5	0	−5	−9	−10	−9	−5	0
Product ..	0	+20	+63	+90	+90	+45	0	−20	0	+40	+63	+45	0	−45	−63	−40	0	+20	0	−45	−90	−90	−63	−20	0

It suffices in this instance to carry out the calculation for a half-period, for, as will be seen, the total of the equidistant ordinates is zero; therefore the total area (the integral over

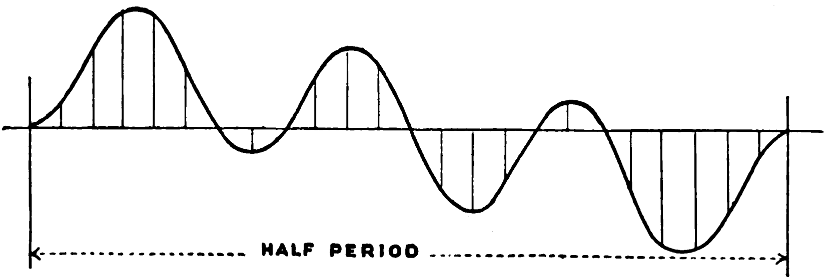


FIG. 77.

the half period) is zero. Fig. 77 is a graph of the products shown in the table.

Having now established the needful principles, let us return to the problem how to find the various coefficients A_1, A_2, A_3, etc., of the several terms of the periodic function

$$\begin{aligned} y &= f(t) \\ &= A_1 \sin pt + A_2 \sin 2pt + A_3 \sin 3pt + \dots \\ &\qquad \dots + A_n \sin npt. \end{aligned}$$

These coefficients we can find, one at a time, by the artifice of multiplying the equation throughout by the sine which occurs in the particular term of which we want to find the coefficient. (Thus to find A_1 we must multiply through by $\sin pt$; or to find A_7 we must multiply through by $\sin 7pt$.) Then, having multiplied through, integrate from 0 to T.

Let us carry out this artifice on the first term, by multiplying by $\sin pt$ and integrating as directed :

$$y \sin pt = A_1 \sin^2 pt + A_2 \sin pt \sin 2pt + A_3 \sin pt \sin 3pt + \dots + A_n \sin pt \sin npt.$$

Now integrate from 0 to T.

$$\begin{aligned} &\int_0^T y \sin pt \,.\, dt \\ &= A_1 \int_0^T \sin^2 pt \,.\, dt + A_2 \int_0^T \sin pt \sin 2pt \,.\, dt + \dots \\ &\qquad \dots + A_n \int_0^T \sin pt \sin npt \,.\, dt. \end{aligned}$$

Now according to Principle III., the integral in the first term on the right hand side is $= \frac{1}{2}$ T; while according to Principle IV., the integrals in all the remaining terms on the right, being integrals of products of different sines, will each be severally zero. So that all will disappear except the first term, thus giving us

$$\int_0^T y \sin pt \,.\, dt = A_1 \times \tfrac{1}{2} T;$$

whence

$$A_1 = \frac{2}{T} \int_0^T y \sin pt \,.\, dt;$$

or A_1 is equal to twice the average value of $y \sin pt$.

In the same way, by operating on other terms, we shall find

$$A_2 = \frac{2}{T}\int_0^T y \sin 2pt \,.\, dt;$$

$$A_3 = \frac{2}{T}\int_0^T y \sin 3pt \,.\, dt;$$

$$A_n = \frac{2}{T}\int_0^T y \sin npt \,.\, dt.$$

The cosine-terms follow similarly.

This being the solution by the calculus of the problem it remains to observe how these processes of multiplying the values of a function by those of some particular sine-function, and then integrating, can be carried out in practice.

Perry's Process.—Professor John Perry gave the first simple arithmetical method in the *Electrician* of Feb. 5, 1892, illustrating it by a case where twenty-four numbers are given as equidistant ordinates of the curve representing the function. [Professor Perry introduces cosine-terms as well as sine-terms, and a constant term A_0 which is simply the mean value of the ordinates of the whole curve. If the ordinates are measured from the mean axis, the term A_0 is eliminated. Cosine-terms are omitted here for simplicity.] The successive twenty-four values of y are then set down on the right-hand column of a schedule. Opposite them are set down the twenty-four values of the sine by which they are to be multiplied. The product

FUNDAMENTAL TERM.

Time.	y	$\sin pt$.	$y \sin pt$.
0	..	0	..
1	..	0·259	..
2	..	0·500	..
3	..	0·707	..
etc.	etc.	etc.	..
		Mean	..
		$A_1 = 2 \times$ mean	..

n^{th} TERM.

Time.	y	$\sin npt$.	$y \sin npt$.
0	..	..	..
1	..	..	..
2	..	..	..
3	..	..	..
etc.	etc.	etc.	etc.
		Mean	..
		$A_n = 2 \times$ mean	..

is then set down to the right and the twenty-four values in this last column are then added up (care being taken as to + and − signs), and divided by 24 to get the mean. Twice the mean is the coefficient sought.

Example.—Find the coefficients A_1, A_3 and A_5 of the Fourier's series for a curve of which the twenty-four equidistant ordinates in one period are: 0, 1·15, 2·7, 5·0, 6·8, 7·7, 8·05, 6·75, 4·7, 3·7, 2·9, 1·8, 0, −1·15, −2·7, −5·0, −6·8, −7·7, −8·05, −6·75, −4·7, −3·7, −2·9, and −1·8.

Answer.—$A_1 = 6{\cdot}95$;
$A_3 = 0{\cdot}91$;
$A_5 = 1{\cdot}14$.

In the *Electrical World and Engineer*, vol. xliii. p. 1023, May 28, 1904, Mr. S. M. Kintner has given an extended schedule for the same process for curves (with odd terms only), for which 36 ordinates in a half-period are given. Sine-values are set down for every 5°, to be multiplied into the successive ordinates for each of the components up to the 17th. This requires no less than 630 separate multiplications by sines of angles to give the coefficients up to A_{17} and B_{17}; a quite disproportionate labour.

Graphic Reduction of Coefficients.—In the *Electrician* for June 28, 1895, Professor Perry described a *graphic* method. In this method, which originated with the late Professor Clifford, resort is had to the artifice of supposing the curve to be wrapped round a circular cylinder whose circumference is the periodic time. The curve is projected upon a diametrical plane passing through $t = 0$. This is carried out by drawing the ordinates on the drawing board not at equal distances but at points (on a horizontal axis), which have been projected from a set of points equidistant around a circle. On planimetering the apparent areas of the curve so projected, the area of projection divided by half the circumference of the cylinder gives A_1. [To get B_1 for cosine-terms, the curve must be projected afresh upon a plane at right angles to the first.] To get the coefficients of the second harmonic, the curve must be wrapped twice round a cylinder of half the diameter, or else its abscissæ must be doubled so that it wraps twice round the same cylinder. This artifice of planimetering the projections virtually multiplies each ordinate by the corresponding sine-value desired in the integration.

A less desirable graphic method, by Mr. A. S. Langsdorf, is described in the *Physical Review*, xii. p. 185, 1901, in which the process of multiplying an ordinate by the sine or cosine of an angle by constructing on it as a diameter a circle, as in Fig. 12, p. 20, where $A D = \sin \phi \times O D$, and $O A = \cos \phi \times O D$.

Wedmore's Method.—The method of graphic reduction worked out by Mr. Basil Wedmore rests on the following principle.

From the circumstance that in any pure sine-curve the positive part in the first half-period is the exact counterpart of that in the second half-period, it follows that if any portion of the curve, equal in length to one whole period, is cut into two equal parts, and the second part is regarded as pushed back so as to be superposed on the first part, and the ordinates are added, the resultant will be zero, as in Fig. 78. Whereas if the curve were shifted through a *whole* period and then two pieces added, the effect would be to double all the ordinates.

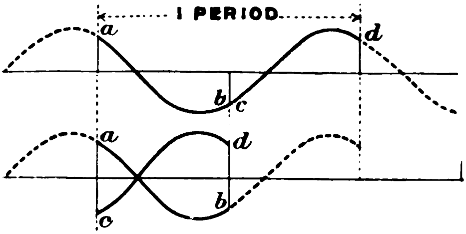


Fig. 78.

Now consider some compound curve, such as Fig. 79, which is known to consist of the fundamental combined with the second harmonic, and cut it in two in the same way and superpose the second half on the first, the result of adding the ordinates will be that so far as the fundamental sine-curve is concerned as a component it will be cancelled out, but the second harmonic component in it will not be cancelled out, since the distance along the axis of abscissæ which is half a period for the fundamental will be a whole period for the second harmonic. Adding the ordinates will then cancel out the fundamental but double the second harmonic. Or if the mean of the two is taken (as in Fig. 79 by the dotted line), the ordinate of which is everywhere equal to half the sum of the two ordinates, this curve will be simply the second harmonic. But by the same reasoning, the shift through a half-period of the fundamental will produce a shift of 1½ periods of the third harmonic, 2½ periods of the fifth harmonic, etc., and therefore also *all* the odd harmonics will at the same time disappear, and the even harmonics (fourth, sixth, etc.), if present in the curve will remain with the second harmonic. Next consider the effect of cutting the whole period into four parts, and superposing the curve on itself after shifting it back one quarter of a period. This will not cause the fundamental to disappear: the resultant so far as the fundamental is concerned will have an amplitude $\sqrt{2}$ times greater than before, but the second harmonic will vanish because this shift of ¼ of the period of the fundamental is the same as ½ period of the second harmonic. With the second harmonic will vanish also the sixth, tenth and fourteenth harmonics, if present, because they are odd multiples of the second. Again, consider the effect of cutting the period into six parts and superposing the curve on itself after shifting it back one-sixth of a period. This will cut out the third harmonic, and also, if they are present, the ninth, fifteenth and other odd multiples of the third. Any harmonic in fact can be made to vanish by thus superposing the curve on itself after shifting it through an exact half-period of that particular harmonic.

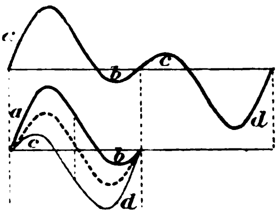


FIG. 79.

Houston and Kennelly's Method.—Another graphic method, due to Houston and Kennelly, which is described in *Electrical*

World and Engineer, xxxi. p. 580, May 14, 1898, has been much used. It consists in dividing up a piece of the given curve into vertical strips, according to the harmonic to be discovered, then ascertaining (either by planimeter or by counting squares), the areas of the odd and even strips, and from these areas calculating the coefficient sought. Take a half-period of the wave: in that half-period there will be n half-waves of the harmonic of the nth order, n being an odd number. Then divide up the half-period into p equally wide strips, p being an odd number, greater than unity, and then reckon out separately the pieces of area enclosed in each strip between the curve and the axis. In making this reckoning, pieces of area that lie above the axis are reckoned positive, while pieces that lie below are reckoned negative. We reckon out first the areas for the odd strips and call its sum S_1, and then for the areas of the even strips likewise, and call their total area S_2. If then we substract S_2 from S_1 we get a remainder which we may call S_p, from which we find the pth coefficient A_p by dividing by the length L of the one whole period of the curve and multiplying by π, so that

$$A_p = \frac{\pi S_p}{L}.$$

Now the reason is this: If we had tried the process, not on a complex curve but on any one harmonic by itself, taking for example five half-waves of the fifth harmonic, and cutting the length up into three strips or into seven strips, we should find the following result. If p has no factor common with n, then the sums of the areas of the odd and even strips are equal, and they cancel on subtraction to zero. This is deducible analytically by expressing the areas as integrals. If, however, we choose $p = n$ then, whether the divisions of the strips intersect at the crossing points of the harmonic curve or not, the areas do not cancel but give a remainder. For example, if there were five half-waves and we divided the total length into five, and began to reckon odd strips and even strips, we should find (as trial will at once show), that since the curve crosses the axis from + to − as many times as there are changes from odd to even strips, our way of subtracting the areas of even strips gives us in the final "difference" of the sums simply the whole area of the row of half-waves irrespective of their sign. So if we were to divide the area of the row by the length of them, namely $\frac{1}{2}$ L, and also by the mean value of the sine, namely, $\frac{2}{\pi}$, we get the maximum ordinate of that harmonic, namely A^n.

Now if there is a remainder only when we make $p = n$ and if the remainder is zero for all other values of p that are prime to n, it follows at once that whatever other harmonics are present in the general form of the curve, if we want to get out the coefficient of the nth harmonic we must divide the half-wave into n strips.

If we begin the divisions into strips at the zero-point of the curve, we shall get out the sine-terms; if we want the cosine-terms we must first divide into n parts, but then begin to reckon the widths of strips from a line drawn half-way on between the dividing points, so that the strips for reckoning cosines are shifted half their own breadth on as compared with the strips for reckoning sines. Thus to find the 5th harmonic coefficients, divide from zero to 180° into 10 parts, the division points being numbered 1 to 9, and carry the one point beyond 180° to an 11th point. Then the odd sine strips will be those from 0-2, 4-6, 8-10, and the even sine strips 2-4, 6-8; while for cosines the odd strips will be 1-3 5-7, 9-11, and the even ones 3-5, 7-9.

Though useful to get out simple coefficients, this process is far less generally useful than either the simple algebraic reduction given on p. 64, or, for more complicated cases where many coefficients are required, the method of reduction next to be described.

Analysis by Groups.—A method of analysis by groups, due to Professor C. Runge (*Zeitschrift für Mathematik und Physik*, xlviii. p. 443, 1903), is of extraordinary power for dealing with higher coefficients. In some respects it resembles the method of algebraic reduction given on p. 69, but carries it much further. Runge divides each period into $4n$ parts, by $4n - 1$ points of division; and then from the method of least squares deduces the following conditions :—

$$4n\ B_0 = \Sigma y_a,$$
$$2n\ A_\lambda = \Sigma y_a \cos a\lambda,$$
$$2n\ B_\lambda = \Sigma y_a \cos a\lambda\ ;$$

where λ represents the order of the harmonic, and a has successive values from 0 to $4n-1$. Then from the rules that

$$y_\lambda - y_{4n-\lambda} = 2\ \Sigma\ A_a \sin a\lambda,$$
$$y_\lambda + y_{4n-\lambda} = 2\ \Sigma\ B_a \sin a\lambda,$$

it is deduced that if the ordinates $y_0, y_1 \ldots y_{4n-1}$ are written in two rows of which the second runs toward the left thus:—

$$\begin{array}{cccccccccc} y_0 & y_1 & y_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\ y_{2n-1} & \\ & y_{4n-1} & y_{4n-2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\ y_{2n+1} & y_{2n} \end{array}$$

and if the differences and sums of those that stand below one another are then taken, the differences will contain sine-terms only and the sums cosine-terms. He then shows that as these sums and differences can be calculated from the doubled coefficients $2A_1\ 2A_2, \ldots 2B_1,\ 2B_2$, etc., so the quantities $2nA$, $2nB$, etc., can be calculated from these sums and differences. The process involves some intricacy in the further argument, sums and differences of a second order being needed to deal with the odd and even orders of coefficients. The scheme of grouping depends on the fact that, in one period of any sine curve, to every angle (save only the angles 0° and 90°) there correspond three others (one in each quadrant), which have the same numerical sine value, + or −, and these four are grouped together for a single multiplication. To separate the odd and even orders of harmonics second sums and differences are required, because when the ordinate at any angle is compared with that of its supplement, though the sines of the fundamental and of the odd harmonics are alike in sign, those of the even harmonics are of opposite sign. But the final result is that by the groupings of numbers in these various stages a schedule is arrived at by which a very complete analysis can be reached with fewer multiplications. Professor Runge draws up a schedule and a worked out example for a case where 36 ordinates in a half-wave are given, in order to determine the Fourier coefficients, both odd and even, up to the 17th harmonic for sines and the 18th for cosines, with only 120 multiplications. The Author has found a great simplification to occur in cases where odd terms only are wanted. Accordingly a shortened form of schedule is here given for the case of a calculation suitable for electrical purposes, using 12 equidistant ordinates in the half-period, and which gives the odd coefficients up to the 11th harmonic, with only *eighteen* operations of multiplication, or *twelve* multiplications if we except the cases where the multiplication is by half or by unity.

The first step is to arrange the 11 ordinates (the 12th being zero) in two rows from left to right and then right to left. They are then added in pairs, giving sums $s_1\ s_2 \ldots s_6$, and subtracted, giving the differences $d_1\ d_2 \ldots d_6$. Some grouping is then done between some of these, giving further numbers for use in finding the third and ninth harmonics. Then the sets of numbers so found are arranged according to schedule in a special table, as shown, each figure so entered having to be multiplied by the sine of the angle opposite which it is entered. These numbers appear in pairs of vertical columns, which are then totalled up, and sums and differences of the two totals in each pair are set down at the bottom. These totals, when each divided through by 6, give the twelve coefficients sought. The calculation takes about half-an-hour.

For rapid work, the still shorter schedule given on p. 90 may be used. It gives the coefficients up to A_6 and B_6 only; but the time taken in making the whole of the calculations is under three minutes.

SCHEDULE FOR THE ANALYSIS OF A PERIODIC CURVE IN WHICH ONLY ODD HARMONICS APPEAR UP TO THE ELEVENTH HARMONIC.

Note.—The two half-periods will be similar, so that if the mean line be taken between the highest and lowest points in the curve, there will be no constant term. For further simplification the origin should be taken where the curve crosses the zero line.

Procedure :—

(1) Divide the half-period into 12 equal parts, and measure the 11 ordinates $y_1\ y_2\ y_3 \ldots y_{11}$; y_0 and y_{12} being each zero.

(2) Then arrange these ordinates as under :—

	y_1	y_2	y_3	y_4	y_5	y_6
	y_{11}	y_{10}	y_9	y_8	y_7	
Adding .	s_1	s_2	s_3	s_4	s_5	s_6
Subtracting .	d_1	d_2	d_3	d_4	d_5	d_6

Note.—s_1 stands for the sum of y_1 and y_{11} ; d_1 for the difference between y_1 and y_{11}. Great care must be taken as to + and − signs throughout.

(3) Group numbers, to obtain values for use with third and ninth harmonics, as follows :—

$$s_1 + s_3 - s_5 = r_1$$
$$s_2 - s_6 \quad = r_2$$
$$d_1 - d_3 - d_5 = e_1$$

(4) Then select from the above numbers and put them in their places in the table below, *multiplying each by the sine set down in the left-hand column before it is entered.*

Angle.	Sine-Terms.						Cosine-Terms.					
	1st and 11th Harmonic.		3rd and 9th Harmonic.		5th and 7th Harmonic.		1st and 11th Harmonic.		3rd and 9th Harmonic.		5th and 7th Harmonic.	
Sin 15° = 0·262	s_1				s_5			d_5				d_1
Sin 30° = 0·500		s_2				s_2	d_4				d_4	
Sin 45° = 0·707	s_3		r_1		$-s_3$			d_3		e_1		$-d_3$
Sin 60° = 0·866		s_4				$-s_4$	d_2				$-d_2$	
Sin 75° = 0·966	s_5				s_1			d_1				d_5
Sin 90° = 1·000		s_6		r_2		s_6	..		$-d_4$		..	
Total first column												
Total second column												
Sum . . .	5 A_1		6 A_3		6 A_5		6 B_1		6 B_3		6 B_5	
Difference . .	6 A_{11}		6 A_9		6 A_7		6 B_{11}		6 B_9		6 B_7	

Result. $y = A_1 \sin \theta + A_3 \sin 3\theta + A_5 \sin 5\theta + A_7 \sin 7\theta + A_9 \sin 9\theta + A_{11} \sin 11\theta + B_1 \cos \theta + B_3 \cos 3\theta + B_5 \cos 5\theta + B_7 \cos 7\theta + B_9 \cos 9\theta + B_{11} \cos 11\theta.$

The following example is the curve shown in Fig. 80.

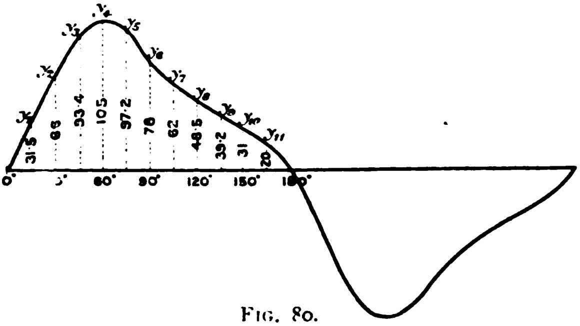


Fig. 80.

The 11 ordinates were 31·5, 66, 93·4, 105, 97·2, 78, 62, 48·5, 39·2, 31 and 20. They are arranged

	31·5	66	93·4	105	97·2	78
	20	31	39·2	48·5	62	
Adding	51·5	97	132·6	153·5	159·2	78
Subtracting	11·5	35	54·2	56·5	35·2	78

Grouping. $51·5 + 132·6 - 159·2 = 24·9$
$97 - 78 = 19$
$11·5 - 54·2 - 35·2 = -77·9$

Entering.—

	Sine-Terms.						Cosine-Terms.					
	1st.	11th	3rd.	9th.	5th.	7th.	1st.	11th.	3rd.	9th.	5th.	7th.
0·262	51·5				159·2			35·2				11·5
0·5		97				97	56·5				56·5	
0·707	132·6		24·9		−132·6			54·2		−77·9		−54·2
0·866		153·5				−153·5	35				−35	
0·966	159·2				51·5			11·5				35·2
1·000		78		19		78	—		−56·5		—	
Multiplying by sines :—												
0·262	13·5				41·7			9·2				3·0
0·5		48·5				48·5	28·2				28·2	
0·707	93·7		17·6		−93·7			38·2		−55·1		−38·2
0·866		132·7				−132·7	30·3				−30·3	
0·966	154·0				49·7			11·1				34·0
1·000		78		19		78	—		−56·5		—	
1st column	261·2		17·6		−2·3		58·5		− 56·5		−2·1	
2nd ,,	259·2		19·0		−6·2		58·5		− 55·0		−1·2	
Sum . .	+ 520·4		+36·6		−8·5		+117·0		−111·5		−3·3	
Difference.	+ 2·0		− 1·4		+3·9		0		− 1·5		−0·9	
Dividing by 6	+ 86·73 + 0·32		+ 6·1 − 0·23		−1·42 +0·65		19·5 0		− 18·58 − 0·25		−0·55 −0·15	

Result. $y = 86·73 \sin\theta + 6·1 \sin 3\theta - 1·42 \sin 5\theta + 0·65 \sin 7\theta - 0·23 \sin 9\theta + 0·32 \sin 11\theta + 19·5 \cos\theta - 18·58 \cos 3\theta - 0·55 \cos 5\theta - 0·15 \cos 7\theta - 0·25 \cos 9\theta.$

The following *Short Schedule* will be found useful in cases where harmonics up to the fifth order only are wanted.

SCHEDULE FOR THE ANALYSIS OF A PERIODIC CURVE IN WHICH ONLY ODD HARMONICS APPEAR UP TO FIFTH ORDER.

(1) Divide the half-period into six equal parts and measure the five ordinates $y_1\, y_2\, y_3\, y_4\, y_5$; y_0 and y_6 being each zero.

(2) Arrange ordinates as under:

$$\begin{array}{lccc} & y_1 & y_2 & y_3 \\ & y_5 & y_4 & \\ \hline \text{Adding . .} & s_1 & s_2 & s_3 \\ \text{Subtracting .} & d_1 & d_2 & d_3 \end{array}$$

(3) Grouping for third harmonic:

$$s_1 - s_3 = r_1$$

(4) Select from above numbers, and put them in their places in the table below, *multiplying each by the sine set down in the left-hand column before it is entered.* Add up the columns as shown, and take sums and differences.

—	Sine-Terms.			Cosine-Terms.		
	1st and 5th Harmonic.		3rd Harmonic.	1st and 5th Harmonic.		3rd Harmonic.
Sin 30° = 0·5 . .	s_1			d_2		
Sin 60° = 0·866 . .		s_2			d_1	
Sin 90° = 1·00 . .	s_3		r_1	—		$- d_2$
First column . . .	..			..		
Second column . .	..			..		
Sum	3 A_1		3 A_3	3 B_1		3 B_3
Difference. . . .	3 A_5			3 B_5		

(5) Divide through by 3, giving the required coefficients.

Result : $y = A_1 \sin\theta + A_3 \sin 3\theta + A_5 \sin 5\theta + B_1 \cos\theta + B_3 \cos 3\theta + B_5 \cos 5\theta$.

(6) Check results as follows:—

$$A_1 - A_3 + A_5 = y_3 ; \quad B_1 + B_3 + B_5 = 0.$$

CHAPTER III.

ALTERNATORS.

THE essential parts of an *alternator*, or generator of alternating currents, are the following :—

(*i.*) A *field-magnet* system to create a magnetic flux or a set of magnetic fluxes. It may be either stationary or rotatory.

(*ii.*) *Exciting coils*, to magnetize the field-magnet cores, these coils being traversed by a continuous current, usually itself supplied from a small independent dynamo, called an *exciter*. The exciting coils must surround the iron-cores of the magnet system ; but they are not necessarily attached to it mechanically. In the exceptional case of "inductor" machines, the coils stand still though the magnet-cores rotate.

(*iii.*) An *armature*, meaning thereby that part of the alternator in which, whether rotatory or stationary, the electromotive-forces are generated. The armature consists necessarily of conductors of copper, in the form of coils or of bars united into a circuit or circuits. These conductors are usually, though not necessarily, wound upon cores of laminated iron. Iron-less armatures are now found only in antiquated types of machine. If the field-magnet system is stationary, the armature must revolve ; or the field-magnet system may revolve and the armature be stationary. Relative motion between them is necessary. Theoretically both might revolve, in opposite directions one to the other ; but this is impracticable.

(*iv.*) *Slip-rings and brushes* are necessary to carry current to and from the revolving windings in all cases, whether the revolving part be field-magnet or armature, save only in the exceptional case of "inductor" machines, in which both the armature and the exciting coils of the magnet-system are stationary. These slip-rings and brushes are as characteristic of an alternator as are the commutator and brushes which are invariably found in continuous-current machines. But in those types where the magnet system revolves, since only a small exciting current has to be conveyed, to the revolving part, two relatively small slip-rings suffice; whereas in those types of alternator where the armature is rotatory, the slip-rings must be relatively large to carry the whole output of current, and may be two, three, four, six or more in number according to the number of "phases" in which the current is to be delivered. If the "phases" are independent of one another, then a pair of slip-rings must be provided for each "phase" in the revolving part, one ring being attached to the beginning, the other to the end of the winding of that phase. Thus a single-phase revolving armature will need two slip-rings, a two-phase armature four. But where the "phases" are interconnected as is usual in three-phase work, the number of slip-rings may be reduced; thus the ordinary three-phase generator with revolving armature has its windings so grouped together that three slip-rings only, not six, are required. The different methods of arranging together these essential parts give rise to a number of well-defined types, capable of simple classification.

CLASSIFICATION OF ALTERNATORS.

Alternators may be broadly divided into two classes as follows:—

I. Single-phase alternators.
II. Polyphase alternators.

Each of these classes may be subdivided with respect to the constructional features into four divisions:—

(*a*) *Outer-pole* type, that is, alternators having stationary external field-magnets and rotating armatures (Type A).

These are mostly small machines, belt-driven. A typical example is afforded by Fig. 81, which represents a 60 kw. single-phase alternator of the Westinghouse Co.

(*b*) *Inner-pole* type, or those with stationary external armatures, and rotating field-magnets (Type B). This type comprises the great majority of modern alternators, the internal revolving magnet-wheel being directly driven from

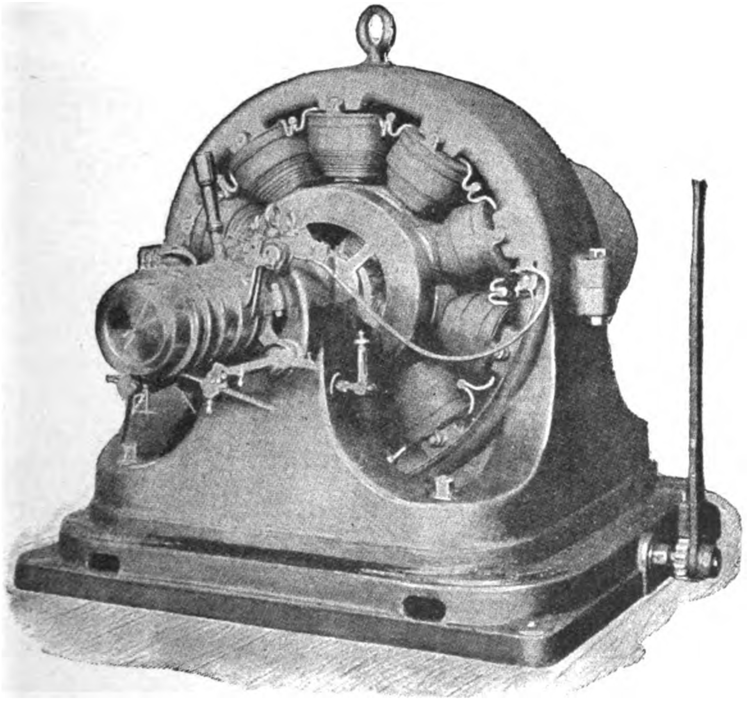


FIG. 81.—ALTERNATOR (TYPE A); OUTER-POLE MAGNET, WITH REVOLVING ARMATURE.

the engine shaft. A recent example is afforded by Fig. 82, which represents a 48-pole, 750 kw. alternator of the General Electric Co., to run at 100 r.p.m.

(*c*) *Inductor type*, or alternators having both field-magnets winding and armature stationary, the amount of magnetic flux from the former through the latter being caused to vary or alternate in direction by the revolution of appropriate pieces of iron, called *inductors* (Type I). This is represented by Fig. 83, an Inductor Alternator of the Oerlikon Co., in which there is no revolving copper, the only moving part being a wheel carrying iron pole-pieces. No slip-rings are required in this type. The small machine mounted on the end of the shaft is the exciter.

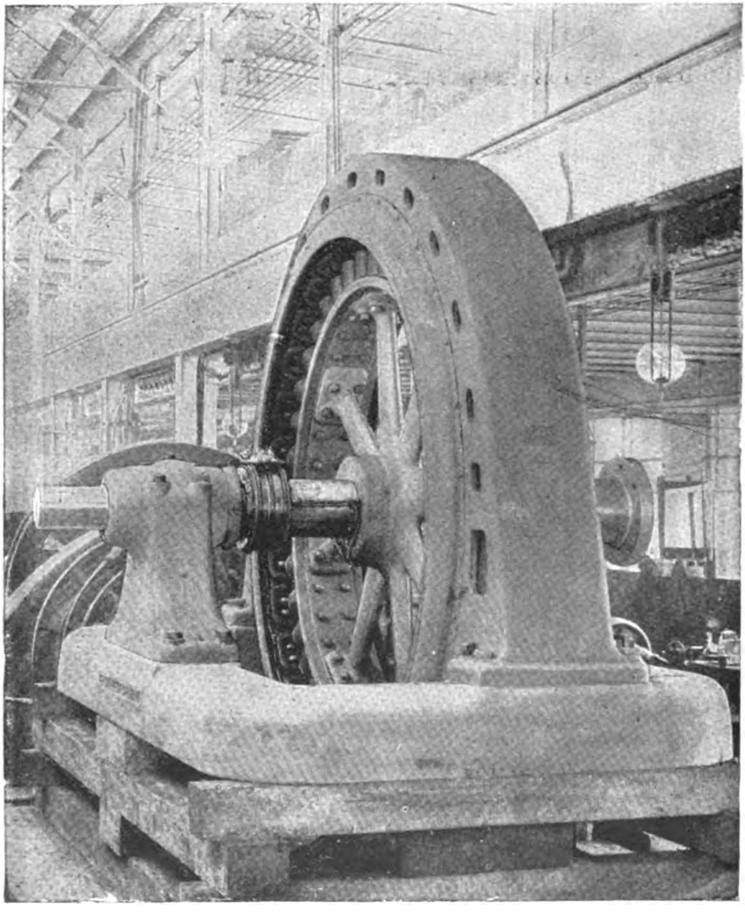


FIG. 82.—ALTERNATOR (TYPE B); INNER-POLE MAGNET-WHEEL, WITH EXTERNAL STATIONARY ARMATURE.

(*d*) *Overhung* type, or alternators, having a fixed internal armature, with an exterior, overhung magnet-wheel with poles pointing inwards (Type O). To this type belongs the machine

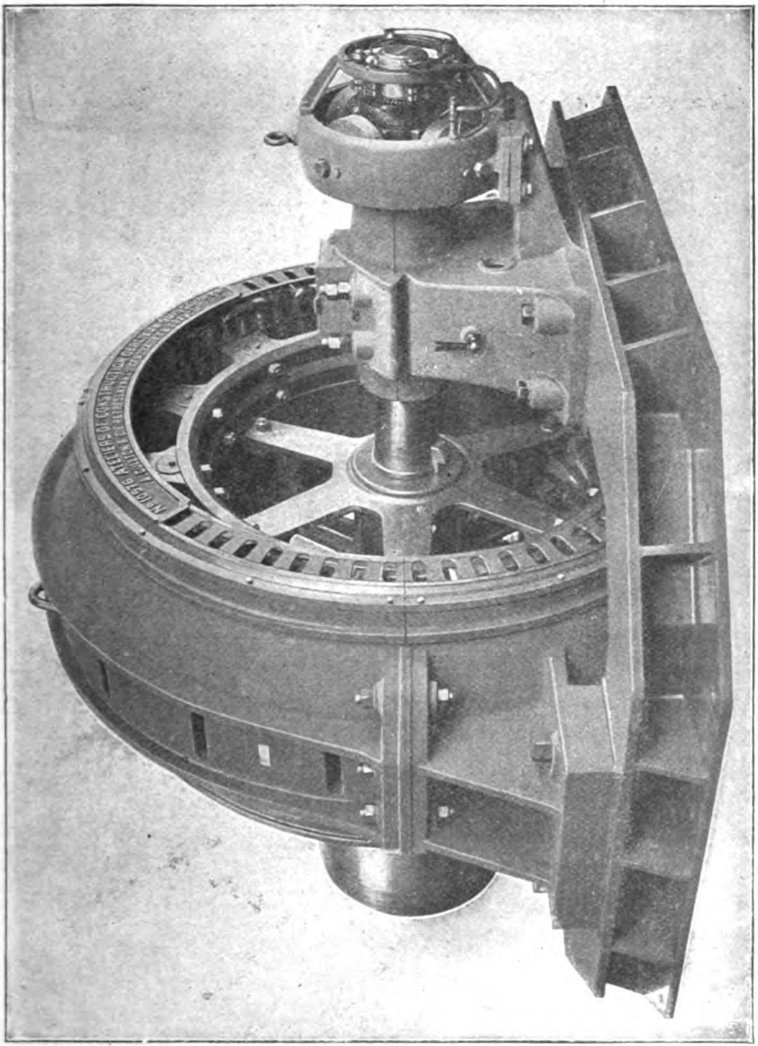


FIG. 83.—INDUCTOR ALTERNATOR, (TYPE I); EXTERNAL STATIONARY ARMATURE (DOUBLE), STATIONARY EXCITING COIL, AND REVOLVING IRON INDUCTOR WHEEL.

by Brown, Boveri and Co. shown in Fig. 84. The stationary armature is mounted on bearings which surround the shaft.

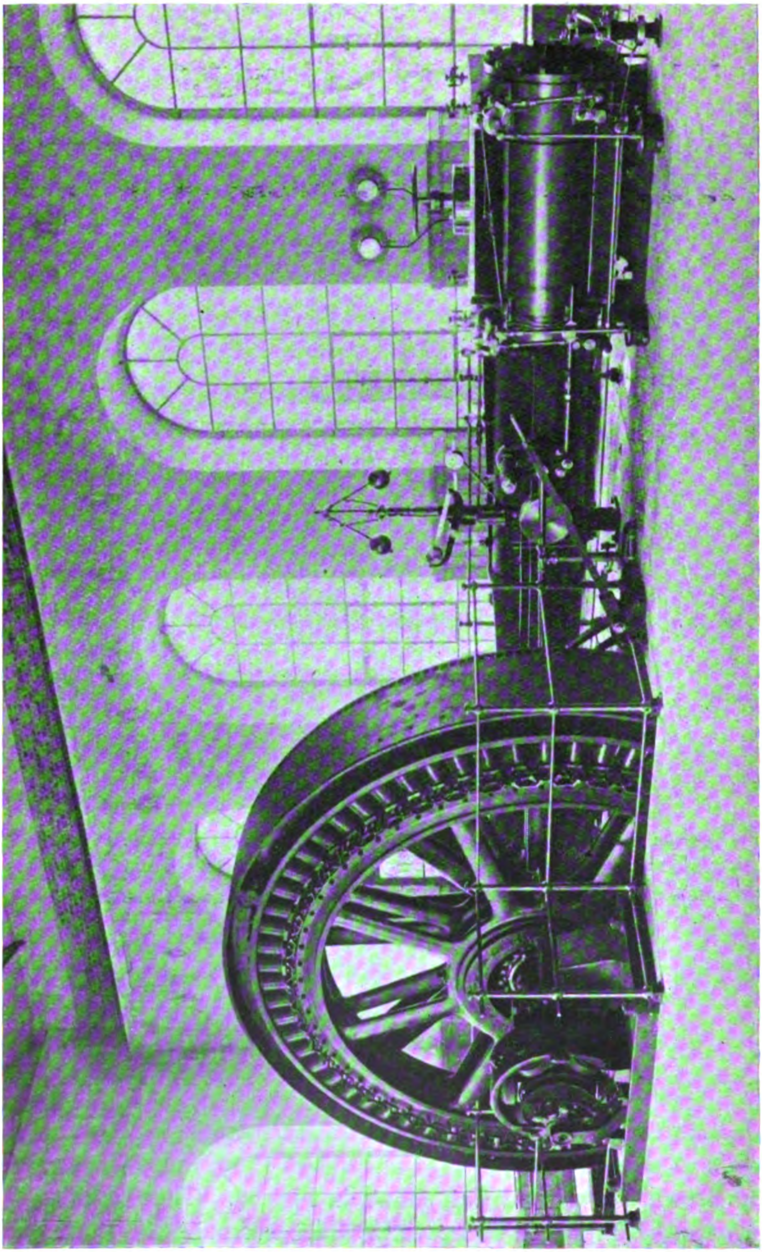


FIG. 84.—ALTERNATOR (TYPE 10); INTERNAL-STATIONARY ARMATURE, AND EXTERNAL OVERHUNG REVOLVING MAGNET-WHEEL

All or any of these types may be made with vertical shafts for the special case of being driven by water turbines, in which case, from the outward resemblance of the revolving part, they

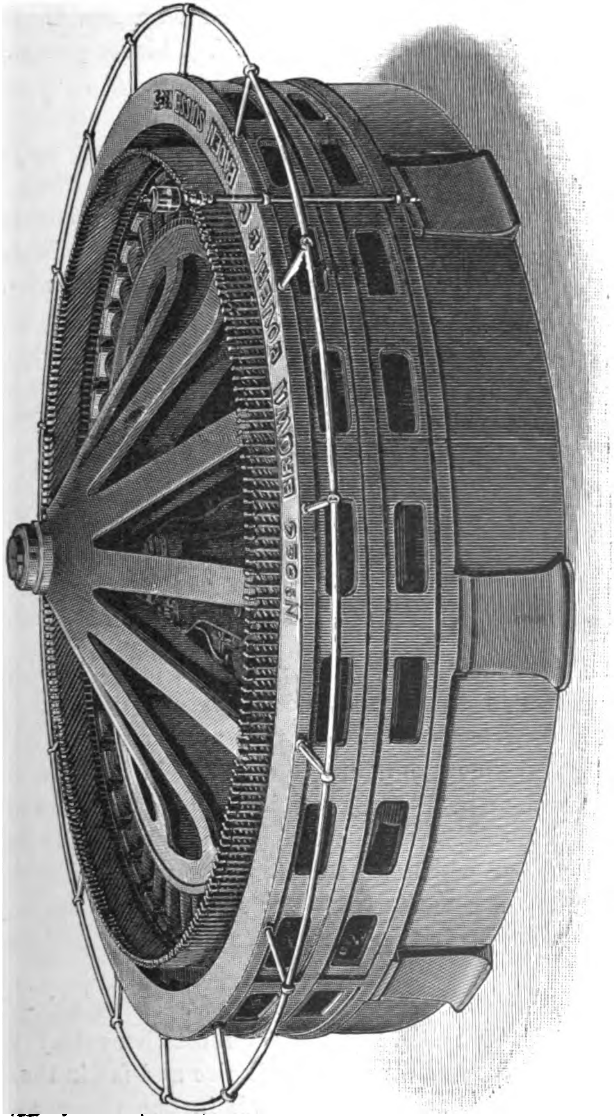
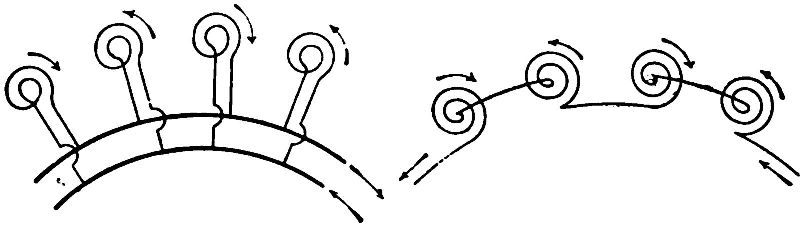


Fig. 85.—Alternator, Umbrella-Type, for Water Turbine Driving.

are sometimes described as being of *umbrella-type.* Fig. 85 depicts one of the alternators constructed for the water-power generating station at Lyons. It is of B-type, having a revolving internal magnet-wheel and an external stationary armature. This type, which originated with Mr. C. E. L. Brown, is illustrated further in Plate IX. (Chèvres generator), and by Plates X. and XI. (Niagara machines).

Several other types are mentioned below.

Coupling Armature Coils.—There are various ways of coupling up the coils of alternators, according to their purpose. If the machine is to be used for high-pressure work, the separate armature coils are invariably connected in series (Fig. 86*b*), while for low and medium pressures the coils may



FIGS. 86*a* AND 86*b*.—DIFFERENT MODES OF COUPLING UP ARMATURE-COILS OF ALTERNATORS.

be parallel connected, or partly in series and partly in parallel. For instance, two or three thinner wires in parallel may be substituted for one thick wire, as they are more readily bent in winding and are more manageable. But in no case where it can be avoided should a set of coils in one half of the armature be put in parallel with a set constituting the other half of the armature, and which are therefore not undergoing induction by the same set of poles. For such arrangements give rise to unequal distribution of current, and consequent increase of heating.

Single-phase and Polyphase Designs.— In single-phase generators, of which Fig. 1 shows the most elementary form, the currents in the armature coils all rise and fall in them at the same instant, that is, they are all in one phase, and, if the armature is revolving, they may be taken off and utilized in the external circuit by means of *two* collector rings and sets of brushes. All single-phase alternators are merely developments of the simple form shown, usually having many armature coils connected up together, and a multipolar inducing field system in order to obtain the desired frequency. Now in order to get the necessary field winding on the magnet limbs, and also to prevent undue magnetic leakage taking place laterally from one limb to the next of opposite sign, it is obvious that the magnet limbs must be spaced out around the armature with a considerable distance between them. Further, there can be no gain in making the armature coils much broader than the magnet pole-pieces, because any additional breadth will not produce more electromotive-force, but will on the contrary increase the resistance and self-induction of the armature. Hence it follows that in any single-phase armature, there must usually be considerable idle space not occupied by the winding. Now let such an armature be taken, and the blank spaces filled up with a second winding—that is, wind a second set of coils in between those of the other set—and connect this winding to a second pair of slip-rings. By replacing the armature so altered in the original field-magnets, we at once have a *two-phase* alternator, that is, we may take from the machine two distinct alternate currents of the same frequency, and (if the two sets of windings are symmetrical and equal in every respect) of the same electromotive-force, but these two currents will differ in phase from one another by 90°, that is their electromotive-force curves will be as shown in Fig. 23, one electromotive-force having its maximum value when the other has its zero value, and so on, due to one set of windings being displaced a quarter period along the armature periphery from the other set. So that merely by putting on this extra wire we are enabled to obtain nearly double the output from the same size of machine.

In Fig. 23 the two phases are marked A and B respectively. If a single-phase generator had been required, only one set of coils could have been employed, such as the A set, the individual coils being connected in series one with another.

Let us go still further, and make the original coils of the single-phase machine somewhat narrower, and then wind on two extra sets, as shown in Fig. 87*a*, where the three sets of coils are marked A, B and C. We have now a ***three-phase*** generator, that is, we may take off from each winding a certain current output at the same voltage and frequency. If the three currents so taken off differ 60 electrical degrees in phase (if the circuits are symmetrical) from one another, their respective electromotive-force curves being as shown in Fig. 87*b*, there would be six collector rings and line wire, required for the three armature windings. But if, as in Fig. 24, p. 37, the three sets of coils are spaced out at 120 electrical degrees apart, that is to say at distances equal

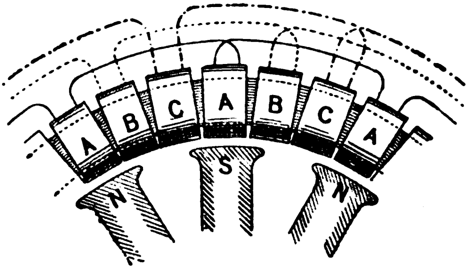


FIG. 87*a*.—A THREE-PHASE GENERATOR.

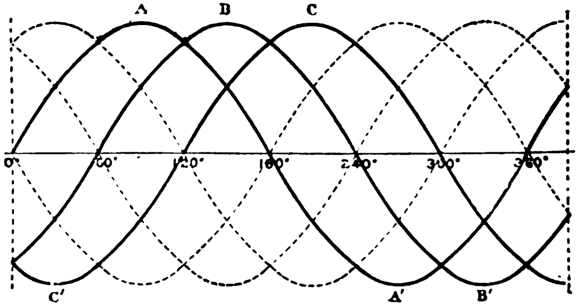


FIG. 87*b*.—THREE-PHASE CURRENTS DIFFERING 60° IN PHASE.

successively to $\frac{1}{3}$ and $\frac{2}{3}$ of the double pole-pitch, the arrangements may be greatly simplified. For then, if one of the ends of each winding are joined together at a common junction, the three other ends may be brought out to three terminals, one for each phase, and then only three lines are required, each winding acting alternately as a return for the other two. This latter plan of using three phases at 120 electrical degrees apart is that which is usually called "three-phase" (Germ. *Drehstrom*). A three-phase alternator with revolving armature has thus essentially three slip-rings and sets of brushes, and three line wires, but a fourth is sometimes employed, to serve as a common return in case of unequal loading. Another way of grouping the three phases in a triangle or "delta" arrangement is described on p. 39.

A three-phase winding utilizes the winding space of an alternate-current armature even better than a two-phase winding, and is, generally speaking, superior to the latter.

From what has been said above, it is clear that if we construct an armature with as many groups of windings as there are poles of the field, we shall have a *single-phase* machine. If we make the coils twice as numerous as the poles of the field we shall get a *two-phase* machine, three times as numerous, a *three-phase* machine, and so on.

Polyphase generators are not only much lighter, and consequently cheaper, than single-phasers of the same type and output, but the reactions due to the armature currents are relatively less than is the case with single-phase machines. Furthermore, polyphase currents offer considerable advantages for driving alternate-current motors, and for transmitting power over long distances, and for these reasons have now come into very general use.

EXCITATION OF FIELD-MAGNETS.

The early forms of alternators were built with permanently magnetized steel field-magnets, but these were soon discarded in favour of electromagnets separately excited. About 1889 began the practice of making them self-exciting by the method of diverting a small current from one or more of the armature coils, which were for this purpose separated from the rest, this current being passed through a commutator which rectified the alternations and made it suitable for magnetizing the field-magnets. But this was not found wholly satisfactory, mainly on account of sparking troubles. Consequently modern alternators are almost invariably excited from separate continuous-current machines, called *exciters*—each alternator may have its own exciter directly coupled to it on a prolongation of the shaft—or the magnets of several alternators may be put in parallel and excited from one or two larger exciters. Authorities differ as to which method is the better in practice, but in each case means must be provided whereby the field excitation can be varied in accordance with the requirements of the load. The advantage of the coupled exciter is that only one machine is put out of service if the exciter breaks down, there being the disadvantage, however, that variations in speed of the engine are magnified in the alternator voltage, because the exciting current depending upon the exciter voltage will also vary with change of speed. The best modern alternators, even in large sizes, have a pressure-drop of from 4 to 15 per cent., the amount depending entirely on the nature of the load. Consequently it may easily happen that the no-load excitation will have to be increased by as much as 50 per cent. in order to maintain the normal pressure at full load. The usual method of varying the field excitation is to interpose a variable resistance in the exciting circuit, or perhaps, if separate exciters are used, in the field circuit of the exciter. Either of these methods necessitates regulation, and this is at present usually done by hand, although a number of systems of automatic regulation have been suggested. These are dealt with at some length at the end of Chapter VII.

CLASSIFICATION OF FIELD-MAGNETS FOR ALTERNATORS.

Multipolar Stationary Magnets.—Practically all alternators are multipolar, because with the customary frequencies of 40 to 50 cycles per second, the speed required for bipolar designs would be mostly impracticable, even with steam turbines. Alternators of type A require a stationary circular yoke from which there project radially inwards a number of pole-cores. Hence the field-magnet of this type resembles generally the

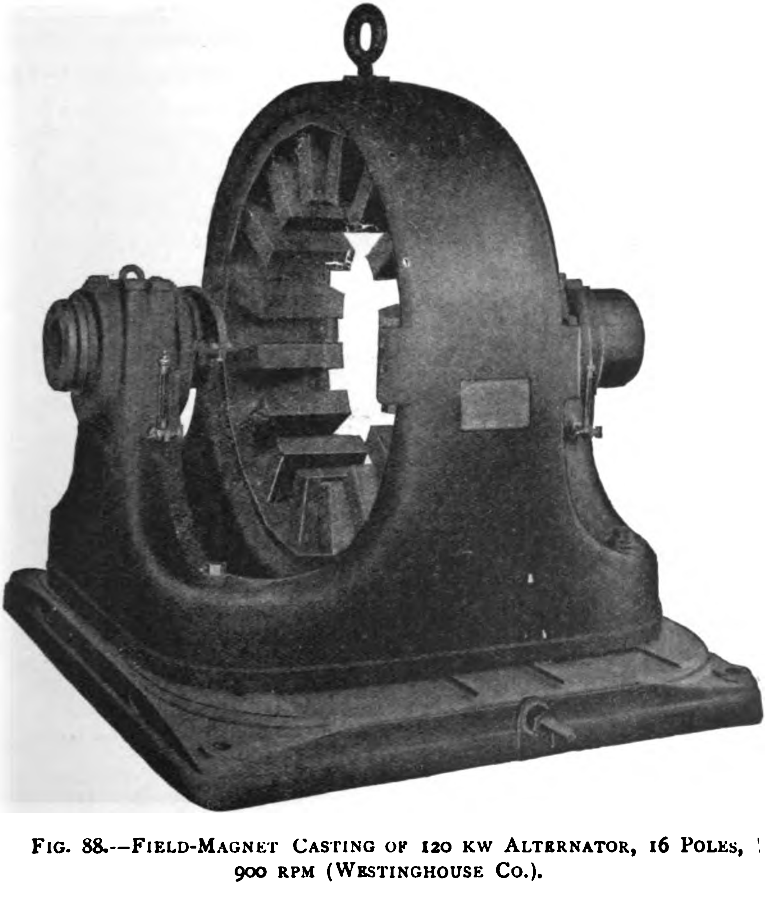


FIG. 88.—FIELD-MAGNET CASTING OF 120 KW ALTERNATOR, 16 POLES, 900 RPM (WESTINGHOUSE CO.).

field-magnet of a modern continuous-current generator; but the poles are usually more numerous and narrower in proportion. Each pole is surrounded by an exciting coil. The yoke is usually of cast-iron or cast-steel. Fig. 88 gives an example of a carcase not yet wound. In this example the pole-cores are cast solid with the yoke. Other makers form them separately and bolt them to the yoke. The poles must, of course, be so excited as to be alternately north and south poles.

Revolving Magnet-Wheels. — For type B, multipolar magnet-wheels are required. Except in small machines, these are almost invariably built up of detachable pole-cores bolted very securely to a foundation-ring. This foundation-ring is a hub or pulley-like structure keyed to the shaft. In large slow-speed machines, the foundation-ring is built like a fly-wheel, with spokes, and indeed, in many cases, actually acts as the fly-wheel for the engine. Fig. 89 shows an 8-pole magnet-wheel for machines of 10 to 100 KW running at 750 RPM. In the particular case shown, the pole-cores consist of laminæ bolted together. Over them are slipped the brass bobbins which receive the exciting coils. These are now almost always made of strip copper, wound edge-wise by a special

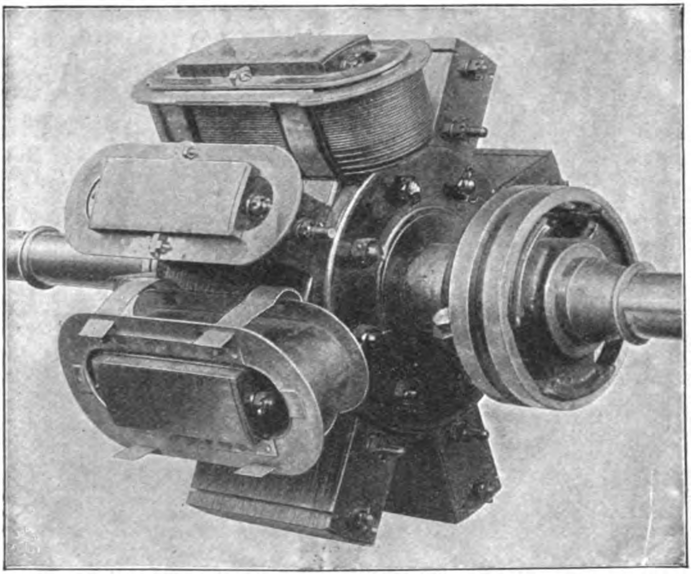


FIG. 89.—MAGNET-WHEEL OF SMALL ALTERNATOR (OERLIKON CO.).

machine. Such a coil is separately shown in Fig. 90. Fig. 89 also shows the two slip-rings by which the exciting current is conveyed to and from the revolving coils.

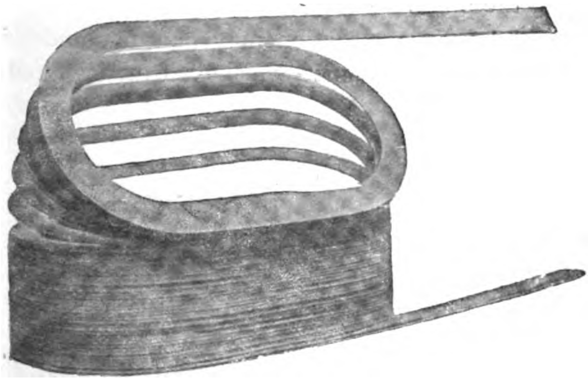


FIG. 90.—EXCITING COIL OF EDGE-WOUND STRIP.

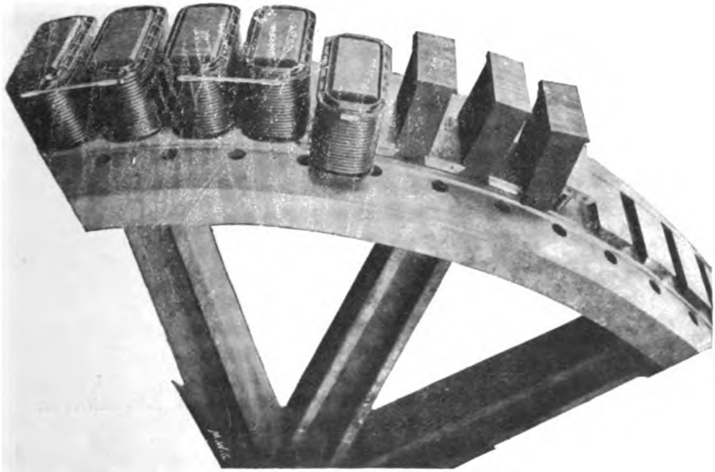


FIG. 91.—SEGMENT OF MAGNET-WHEEL, SHOWING MODE OF FIXING POLE-CORES (OERLIKON CO.).

Fig. 91 depicts a segment of a much larger magnet-wheel designed as a fly-wheel. In this example, the pole-cores slide into dove-tail grooves in the rim of the wheel. Each pole, core is constructed with projecting pole-tips which overhang the bobbin, and secure it from flying off. The cut also shows how the successive coils are coupled up to secure the alternate polarity of the poles.

This mode of construction may be compared with that shown in Fig. 92, a magnet-wheel of American pattern, in which the solid steel pole-cores are secured to the foundation-ring by pairs of screws-bolts. This cut also shows the slip-rings and their connexions going to the coils. It should be compared with Fig. 82, p. 94.

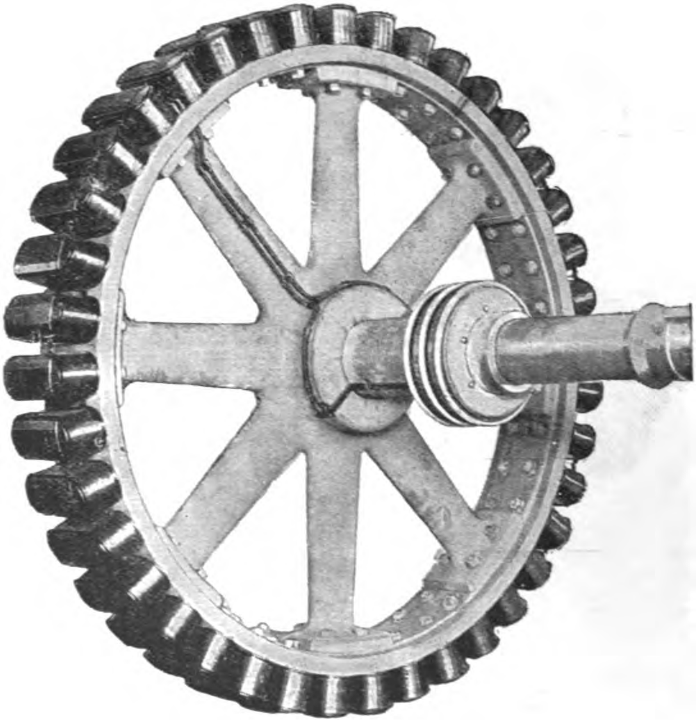


FIG. 92.—REVOLVING MAGNET-WHEEL OF 725 KW ALTERNATOR (GENERAL ELECTRIC CO.).

Other magnet-wheels for machines of B-type are to be found in Figs. 95, 127, and 197, and in Plates V., VI., VII.

and VIII. Fig. 93 shows the magnet-wheel of a 750 KW alternator built by the Union Elektrizitäts-Gesellschaft, ot Berlin, with a very heavy rim mounted upon cheeks of steel plate in lieu of spokes.

It is a controverted point whether it is better for the foundation-ring of the magnet-wheel itself to serve as fly-wheel, or whether a separate fly-wheel should be provided. Certainly some designers first design the magnet-wheel with a rim thick enough to fulfil the magnetic requirement of acting as a yoke for the magnets, which they then thicken up until a sufficient moment of inertia is attained for fly-wheel purposes. The designs of Fig. 338 and of Plates V. and VIII. should be examined as instances. On the other hand, there is given in Fig. 128 a case of a machine in which a separate fly-wheel is provided to which the foundation-ring of the magnet-system is attached.

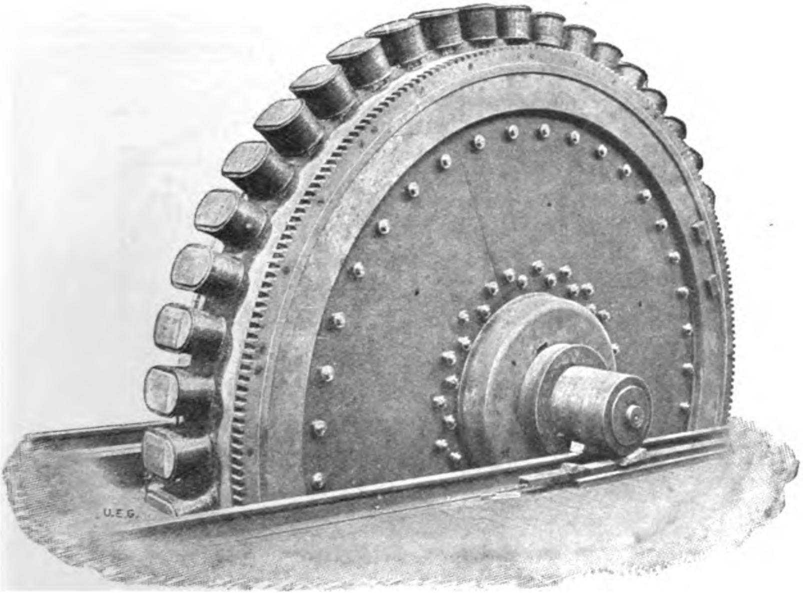


FIG. 93.—MAGNET-WHEEL OF 750 KW ALTERNATOR (UNION CO.).

The type O, with overhung magnet-wheel (see Fig. 84) was created for the express purpose of meeting the need of obtaining a sufficient fly-wheel action. It has the incidental advantage that the centrifugal forces do not tend to drive the exciting coils off the pole-cores, and the magnetic pull on the pole-cores partly helps to balance the centrifugal forces on them. It was this consideration that decided Professor Forbes to adopt this plan of an external yoke with inward-pointing poles in the first alternators built for the Niagara power-station as shown in Fig. 94.

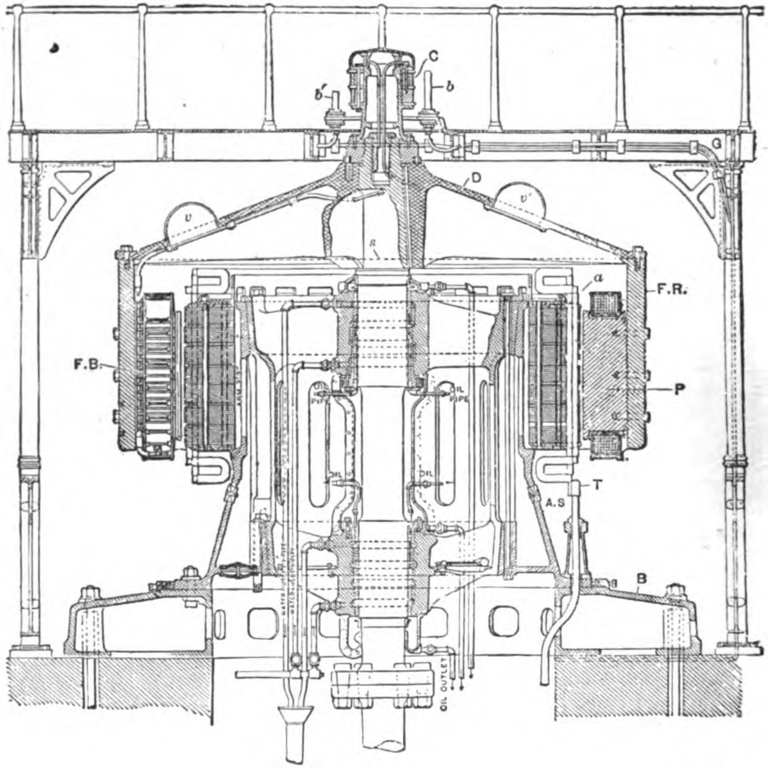


Fig. 94.—Sectional Elevation of Niagara 3750 kw Alternator (1892 Pattern). Scale 1 : 50.

In this figure, F R is the foundation-ring, P the pole-core, F B the foundation-bolt, D the conical frame which suspends the magnet-system from the top of the shaft. The slip-rings to convey the exciting current are at C. The brush-pillars are marked *b*. Ventilating hoods are shown at *v*. B is a bed-plate for the stationary armature, T one of the terminals.

So far every pole-core has been shown with an exciting coil upon it. At one time Mr. Brown proposed to simplify the construction by winding only every alternate pole, putting double the number of coils upon each of the N-poles, leaving the S-poles unwound. This construction, sometimes described as *hemitropic* (*i.e.* having half the poles wound), is shown in Fig. 95, which depicts the revolving magnet-wheel for a machine of the umbrella-type erected about 1895 at Schoenenwerd near Aarau; the outer diameter of the polar face being 106·8 inches, with 80 narrow poles, of which 40 only are wound with magnetizing coils on paper bobbins. Brass guard-frames screwed to the unwound poles secure the coils from flying. This construction, which has the disadvantage of rather greater magnetic dispersion than that in which every pole is wound, is still used for machines of small size. Fig. 96 gives an example. In this case, the unwound poles are of cast-iron, cast solid with the wheel. They are of somewhat larger cross-section than the wound poles, which are cylindrical and of cast-steel. The hemitropic plan of winding reduces the cost, as it halves the number of bobbins, but slightly increases the total weight of copper. Apart from the slight

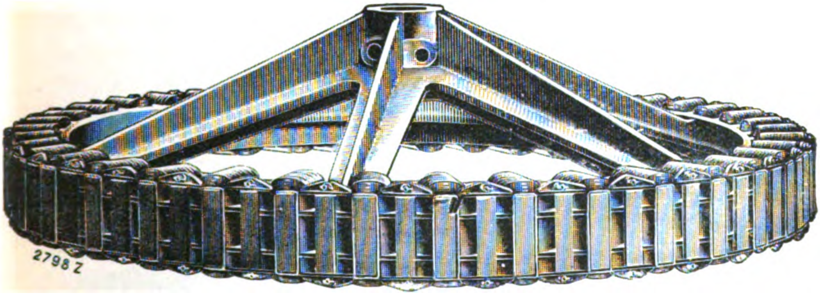


FIG. 95.—MAGNET-WHEEL, WITH ALTERNATE POLES UNWOUND.

disadvantage mentioned above, there is little difference between the effect of winding coils on two poles or of winding the same total number of turns around one pole only; the current *between* the poles is, in either case, flowing in the same general direction in the same number of conductors, hence the magnetomotive-force is the same.

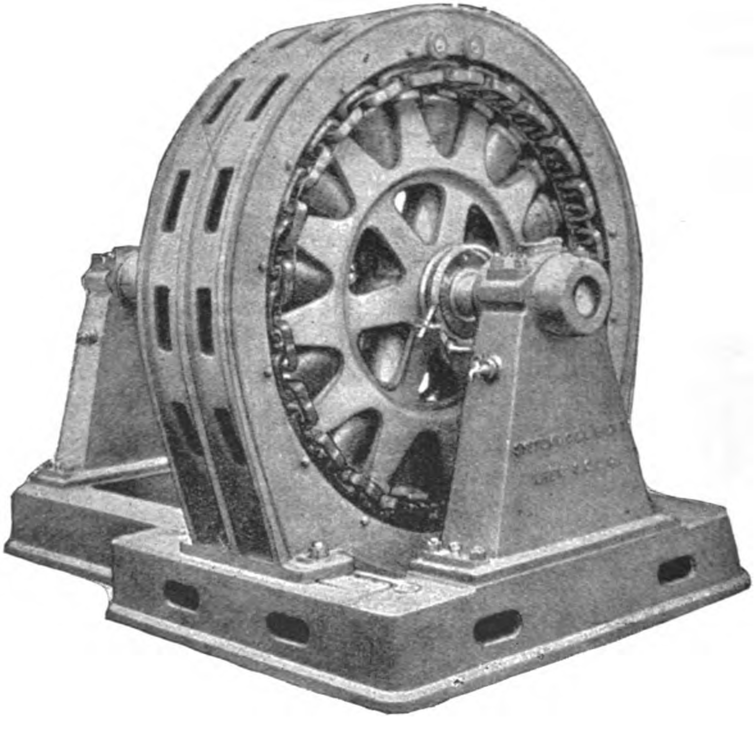


FIG. 96.—TWO-PHASE ALTERNATOR (225 KW), WITH ALTERNATE POLES UNWOUND (BROWN, BOVERI AND CO.).

Field-Magnets suitable for Disk-Armatures.—A form of field-magnet which came early into favour, consisted of two crowns of alternate poles of opposite polarities, thus producing a multipolar field. Fig. 66, page 123 of Vol. I., gives a simple sketch of this type. The poles taken in order round each crown are alternately of N and S polarity; and opposite a

N-pole of one crown faces a S-pole of the other crown. This description will apply to the magnets of the early alternate-current machines of Siemens, and to Ferranti's alternator. The iron plate or frame, shown in Fig. 97 at one side only, serves as the yoke to convey the magnetic flux from one pole-core to the adjacent ones. Since the magnetic lines run in opposite directions between the fixed poles, which are alternately S—N, N—S, as described above, the moving coils will necessarily be traversed by alternating currents; and as the alternate coils of the armature will be traversed by currents in opposite senses, it is needful to connect them up, as shown in Fig. 97, so that they shall not oppose one another's action.

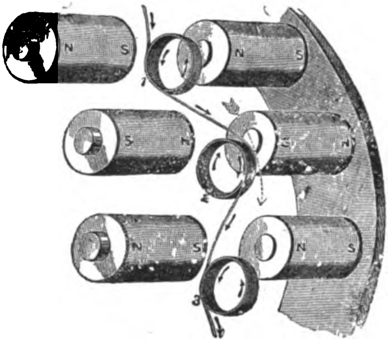


FIG. 97.—ARRANGEMENT OF POLES IN ALTERNATOR.

Siemens' alternators, dating from 1878, realize this design with a thin disk-armature built up of wedge-shaped coils.

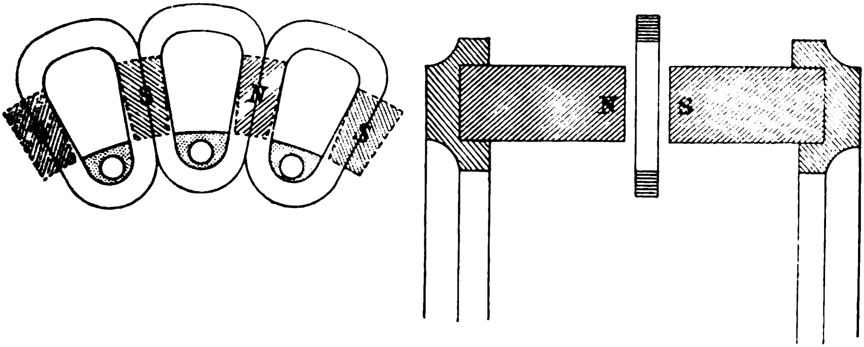


FIG. 98.—FERRANTI ALTERNATOR.

Ferranti's and Mordey's alternators followed the same plan the copper coils being built up into a thin disk, as indicated in Fig. 98 and Plate I.

In machines of this type, it is impracticable to use iron cores for the thin armature; and though in the hands of Ferranti it has given good results, and is still useful for small machines, the type must be regarded as obsolete for larger work.

Homopolar Magnet-Systems.—Thus far it has been assumed that all field-magnets present a system of alternate N and S poles, so that the magnetic lines which penetrate any given part of the armature are continually changing in direction as well as in amount. But this is not necessarily the case: for, seeing that the induction of electromotive-forces depends on the rate at which the magnetic flux interlinked with the armature coils is changing, and not on its actual sign, it is possible to design field-magnet systems in which the flux which enters any given part of the armature never changes its sign, but simply alters in amount. Suppose a N-pole to approach a coil; then during approach the flux it sends through that coil is increasing. As it recedes, the flux diminishes. If, therefore, a series of separate N-poles could be moved past the coils one after the other, an alternating current would still be induced in the coil as the flux increased or decreased in amount. A magnet-system which presents toward any given part of an armature a series of poles of the same kind is described as ***homopolar***, to distinguish it from the ordinary or ***heteropolar*** system which has alternately N and S poles. One feature of a homopolar magnet-system is that there is no need to wind each "pole" separately with an exciting coil, for they may all be arranged to constitute branches of a single magnetic circuit surrounded at its main part by a single exciting bobbin. The substitution of such a magnet in alternators is due to Mr. Mordey.

The characteristic features of the Mordey alternator are the stationary thin disk-armature, and the solid revolving field-magnet. In the latter, though there are two crowns of poles between which the armature lies, all the poles on one side are of one kind, north poles, and all those on the other side are south poles. Hence there is no reversal of the magnetic-field through the armature coils; the number of magnetic lines through any coil simply varying from zero to maximum and back. As a result of this arrangement, there is a great simplification of the means needed to magnetize the field-magnets. One single coil surrounding a central cylinder of iron suffices to magnetize the whole of the poles. There is indeed only one magnetic circuit, branching into separate branches. The construction of the field-magnet is as follows:—

A pulley-shaped iron cylinder, through which the shaft passes, forms the core, and is surrounded by the exciting coil.

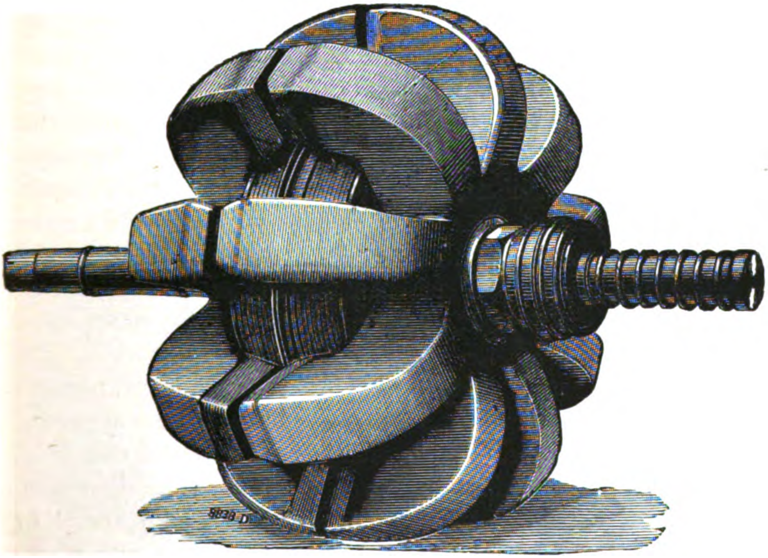


FIG. 99.—HOMOPOLAR FIELD-MAGNET OF MORDEY'S ALTERNATOR.

Against the ends of this core, as shown in Fig. 99, are firmly screwed up the two end castings, each of which is furnished with a number of polar projections varying from 9 in small machines to 60 in large ones, projecting toward one another; the narrow polar gap between them being only just wide enough to admit the armature. The entire field-magnet revolves with the shaft, the exciting coil being supplied with current from a separate machine by means of two contact-rings on the shaft. There is no need for the exciting coil to revolve; but for mechanical reasons it was deemed preferable to wind it actually upon the field-magnet core. The armature coils are of copper ribbon, wound upon narrow wedge-shaped cores of enamelled slate, and insulated with a thin tape between the turns. Each coil is held in a German-silver bracket embedded in ebonite and firmly clamped to the exterior frame. All the metal clampings are outside the magnetic field, and are so arranged that any one coil can be removed in a few minutes without dismounting any part of the machine.

The whole of the armature coils are accessible without removing any part of the machine, because there is a gap between the adjacent poles on either side, rather more than equal to the width of one armature coil. Thus, in any position, half the armature is accessible, and by moving the field-magnet round very slightly, the other half becomes accessible. This facilitates the ordinary cleaning work, while for periodical examination it is easy to withdraw the armature quadrants separately. End play is limited by taking the thrust on a shoulder on the shaft bearing, on the inside end of each bearing. The lubrication is effected by means of a small oil pump.

As the armature is stationary there are no centrifugal forces to be considered, and the coils have to be supported only with a view of resisting the tangential drag of the field. The revolving field-magnet forms an excellent fly-wheel, and as there are no parts liable to fly out, a high speed of driving presents none of the difficulties that arise with many other types of machine. The journals are furnished with a shoulder to limit end play, and the bearing blocks are made adjustable longitudinally, so that the field-magnet may be placed exactly symmetrically with respect to the armature. The electromotive-force is 1 volt per 8½ inches of conductor. The very low resistance of the armature, and almost complete absence of armature reactions, makes the machine almost self-regulating, a point of some importance for parallel running and for operating motors.

In some cases a small continuous-current machine is mounted on the same shaft, as shown in Fig. 100, to excite the field-magnets. In the previous edition of this work a Plate was given depicting one of the 200 KW alternators designed for the Leicester lighting station.

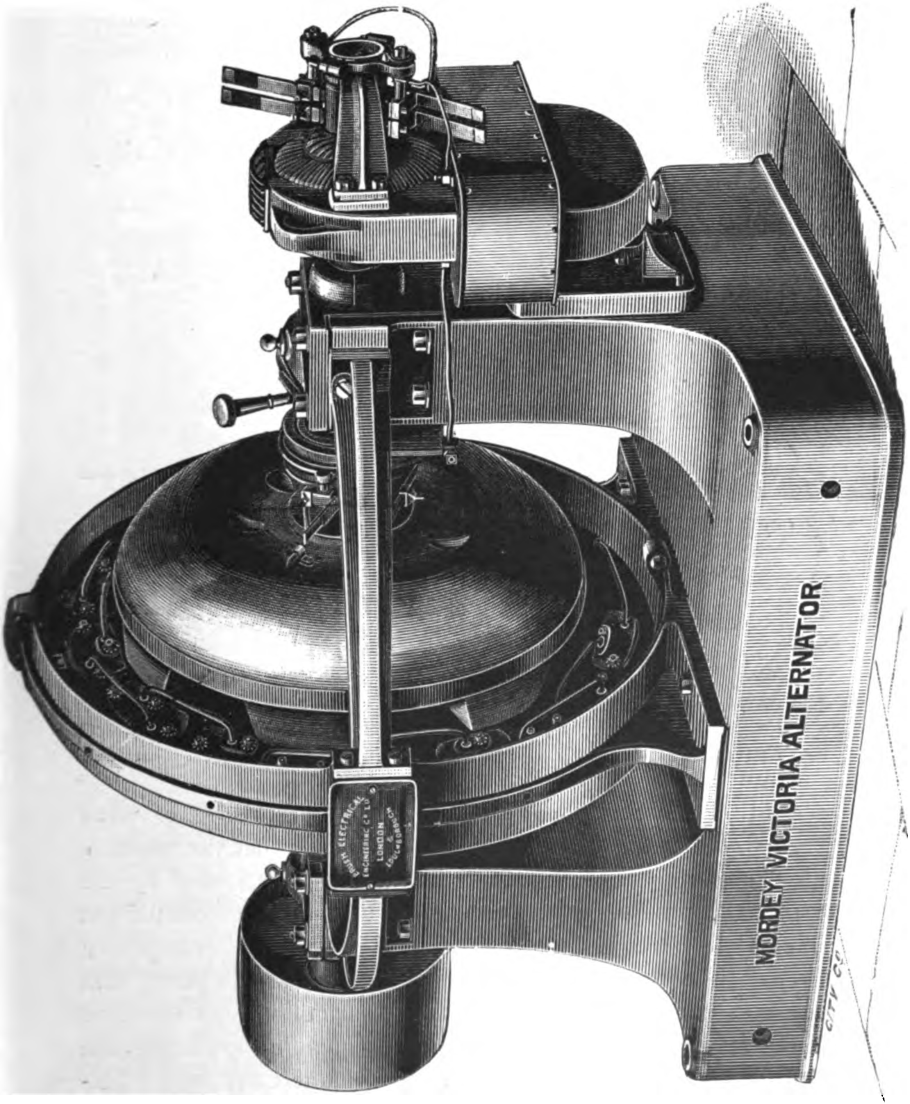


FIG. 100.—MORDEY ALTERNATOR.

Owing to the excellent conditions of ventilation, it comes about that the limit of current-density is not fixed by risk of overheating, but by considerations as to efficiency and self-regulation. The amperage at full load is no less than 3300 amperes per square inch. Loss by hysteresis there is none, owing to absence of any armature core. The eddy-currents in the conductor are trifling: the copper tape needing no further lamination. The coil-holders, moreover, are of German silver, the high specific resistance of which alloy

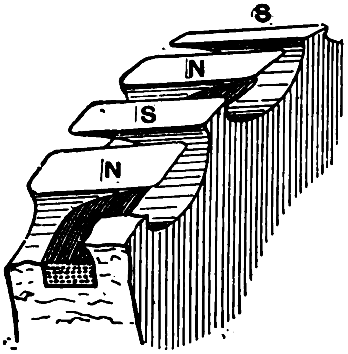


FIG. 101.—CLAW-MAGNET, SHOWING ALTERNATE POLES AND SINGLE-EXCITING COIL.

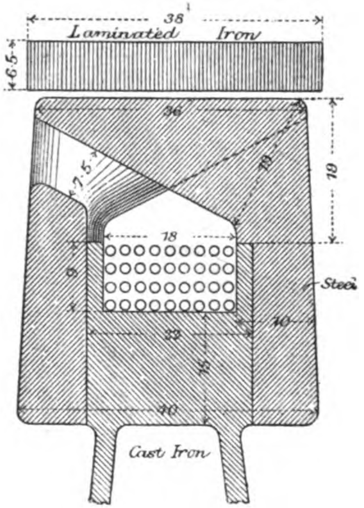


FIG. 102.—FIELD-MAGNET OF BROWN'S ALTERNATOR AT LAUFFEN (1890).

reduces the losses by eddy-currents to $\frac{1}{16}$th or $\frac{1}{20}$th of what they would be if brass were used. A proof that the waste is almost entirely confined to the C^2 R loss is afforded by the fact that a 75 KW machine when driven on open circuit but excited to give its full voltage, only absorbs 3 horse-power, the armature keeping quite cool. It is a curious point that in these machines the losses due to friction, hysteresis and parasitic currents, though moderately great at low loads, are at full load not only proportionally, but actually less. Excellent as these machines are in small sizes and for single-phase work, they have not been found suitable for sizes over 1000 KW, because of the difficulties attendant on the expansion of the thin armature by heating, and the impracticability of arranging in three phases the non-overlapping coils.

Claw Magnets.—A heteropolar form of magnet-system, which, like the Mordey form, has but one exciting bobbin

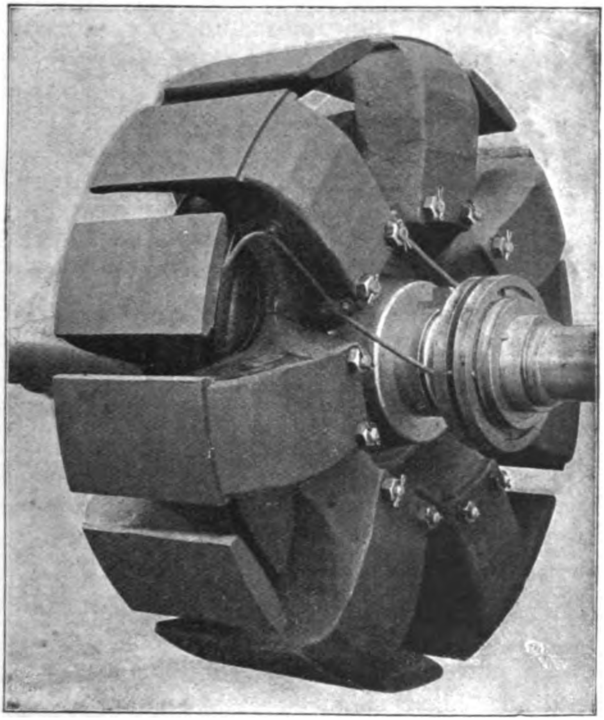


FIG. 103.—CLAW-MAGNET, WITH IMBRICATED POLES (SCHUCKERT AND CO.).

was introduced by Brown in 1890. Imagine the cheeks of a Mordey magnet to be so modified that the polar extensions instead of being opposite to one another and nearly meeting, are elongated into claws, and staggered so that the N-poles at one side project into the spaces between the S-poles on the other side. We thus arrive at the imbricated structure sketched in Fig. 101. Fig. 102 gives the form used by Brown in the historic machine installed at Lauffen on the Neckar. A later example of this construction, which has been adopted by many makers, is shown in Fig. 103.

This form is now obsolete. Though exceedingly mechanical, it has two disadvantages. There is a large tendency to magnetic leakage, which as it increases with the load tends to bad regulation. It is also difficult when the poles are thus magnetically in parallel with one another to ensure their being equally strong owing to blow-holes in the casting.

INDUCTOR ALTERNATORS (Type I).

Following the introduction of the homopolar principle, there was developed a species of machine known as an inductor alternator (Type I), which seemed likely, in the nineties, to supersede all other kinds. In this type of machine none of the copper conductors move, the only moving parts being masses of iron whose motion sets up variations in the magnetic flux. This principle, suggested indeed by several early workers, was revived by the author of this treatise in 1883.[1]

The essential feature of the inductor alternator is that iron only is revolving, and as the design is usually homopolar, the magnetic flux in its armature coils is not alternating, but undulating in character. Thus, with a given maximum flux through each polar mass, the total number of armature turns required to produce a given electromotive-force is just twice that which is required in an alternator having an alternating instead of an intermittent flux through its armature windings.

The apparent disadvantage occasioned by this necessity of doubling the number of turns on the armature is more or less compensated for by the fact that the magnetic flux is not reversed or entirely changed in each period throughout the whole mass of the armature iron. The abrupt changes of flux in the inductor machine are largely confined to the projections on the armature surface between the coils; while with alternators of the inner- and outer-pole types the flux is reversed

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[1] Specification of Patent No. 1639 of 1883, which led up to Mr. Kingdon's form, see *Electrical Review*, xxii. 178, 1888.

throughout the whole mass of the armature iron. This peculiarity makes possible the use of a very high magnetic flux-density in the armature of the inductor alternator without excessive core loss. By using a very high density, a large flux can be used without excessive increase in the amount of magnetic iron required. The use of this large flux makes possible a reduction in the number of armature turns, which compensates more or less for the disadvantage of having only one half of the armature coils in operation at once.

It should be borne in mind, however, that unless these inductor alternators are very carefully designed, especially with regard to magnetic leakage, they become heavy and hence costly, for their output, and their pressure-regulation defective. Generally speaking, their field of application is narrow—on motor loads especially, alternators of the inner- and outer-pole types are as a general rule preferable.

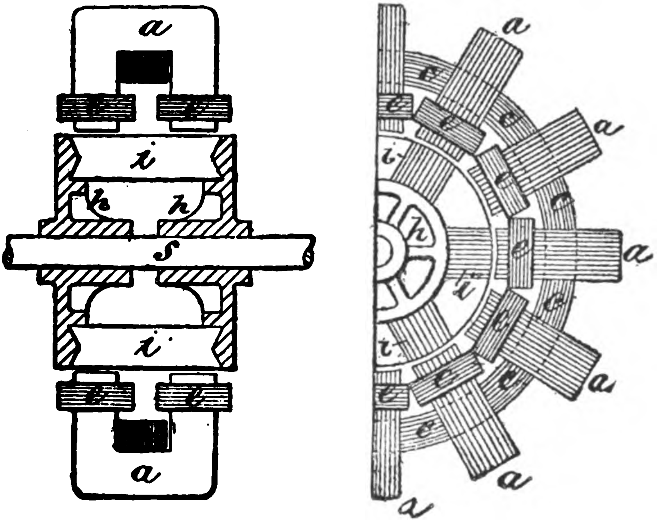

FIG. 104.—MORDEY'S INDUCTOR ALTERNATOR.

The introduction of the homopolar type of inductor machine dates from about 1886 with the designs by Mordey, Fig. 104, kindred designs having been almost simultaneously evolved by Professor Elihu Thomson.

Fig. 104 is a single-phase design, having fixed armature coils *e e* arranged in groups of two upon projecting laminated cores *a a*, between which lies the fixed magnetising coil *c*, surrounding the revolving inner part, a wheel carrying inductor masses of iron *i i*.

Fig. 105 illustrates the application of the same principle to a three-phase generator, and is a design by the Oerlikon Company, being a 4-pole machine. The iron mass which serves as field magnet consists of a steel casting mounted on a shaft, each of its four projecting parts carrying two groups of assembled iron stampings. It is magnetized by means of a stationary coil which surrounds its middle, and thus the eight laminated projections become poles, there being four north poles at one end, and four south poles at the other end of the revolving part. Fig. 109, p. 122, depicts the external appearance of this type of machine. In that figure the small exciter machine is mounted on the end of the shaft. Fig. 106 gives a section through a more modern machine, several of this type having been installed in the Kensington and Notting Hill Station at Shepherd's Bush.

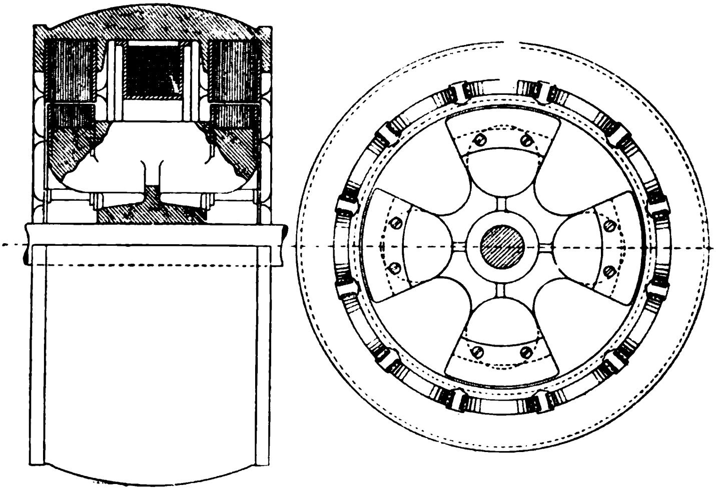

FIG. 105.—HOMOPOLAR INDUCTOR ALTERNATOR (OERLIKON CO.).

The principle of inductor machines will be made more plain by reference to Fig. 107, which is a rough sketch showing how an electromagnet might be made of a cylinder of iron magnetized by a coil laid in a channel around its girth. The projecting spurs or teeth at its two peripheral regions will have polarities as shown. Such an electromagnet might be rotated, yet its magnetizing coil remain stationary.

The armature appropriate to such a revolving magnet would obviously be a double armature, having a set of coils to be acted upon by the N-poles, and another set to be acted upon by the S-poles.

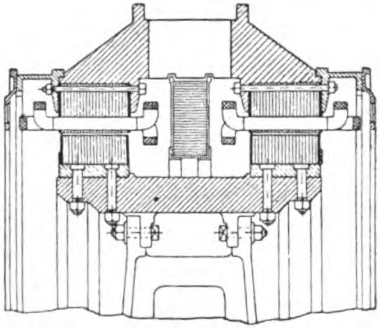

Fig. 106.—Oerlikon Co.'s Inductor Generator (Three-Phase).

Homopolar and Heteropolar Inductor Machines.—A little consideration will show that if the set of N-poles, acting together as a homopolar crown, are situated beside the set of S-poles, as in Fig. 107, the coils of the one armature must not be placed in line with the coils of the other, but must be "staggered," or in other words displaced with respect to one another peripherally through a breadth equal to half the

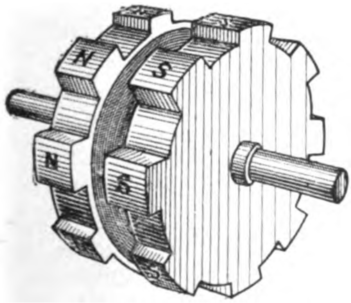

Fig. 107.
Typical Inductor (Homopolar).

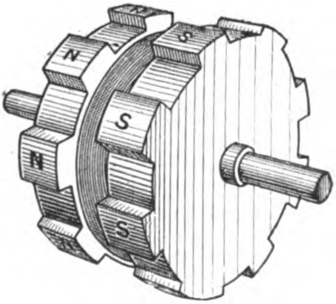

Fig. 108.—Inductor with Staggered Poles (Heteropolar).

distance from one N-pole to the next N-pole. The two armatures thus staggered may then be coupled up in series with one another.

If, however, the projecting polar masses are themselves staggered, as shown in Fig. 108, the coils of the armatures need not be, and in that case a single armature of double breadth may obviously be used, and the rotating inductor then acts as a heteropolar magnet, resembling the claw-magnet, Fig. 103, p. 117, but with short claws. Staggered inductors have been used by Messrs. Brown, Boveri and Company, for example in the generators designed for the Lugano tramway (see Fig. 116), by Mr. Fynn, in England, and also by Messrs. Ganz, of Buda-Pesth. The non-staggered type is preferred by the Oerlikon Company, and by the Allgemeine Elektricitäts-Gesellschaft. Messrs. Stanley and Kelly, of Pittsfield, Massachusetts, were also very early in the field with a two-phase inductor generator of the non-staggered type.

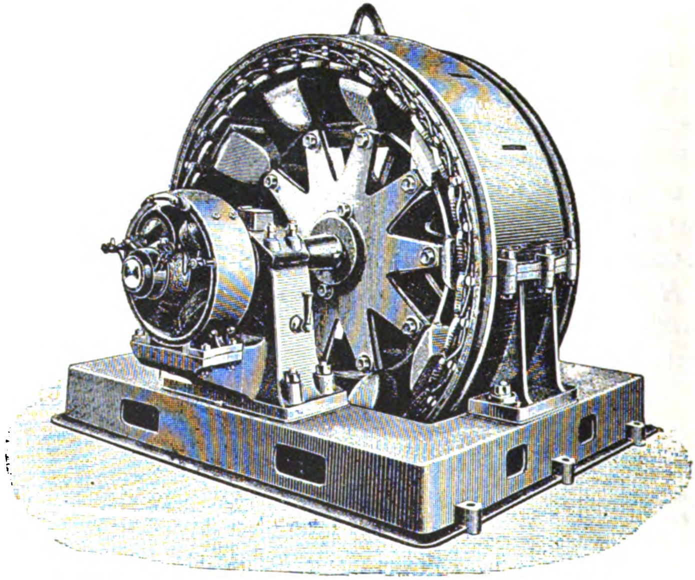
FIG. 109.—OERLIKON CO.'S INDUCTOR GENERATOR (300 H.P.).

Fig. 109 represents a three-phase inductor generator, constructed by the Oerlikon Company, of 210 KW, to run at 250 revolutions per minute. It is designed to give its three currents at a pressure of 3000 volts, the winding being of many turns in each coil of the armature; the coil being non-overlapping, arranged like Fig. 271. It has ten poles on each side, and therefore gives ten cycles pèr revolution or forty-two cycles per second. The small machine mounted on the end of the shaft (Fig. 109), is a small 4-pole exciter. The stationary magnetizing coil of the inductor, which cannot be seen in

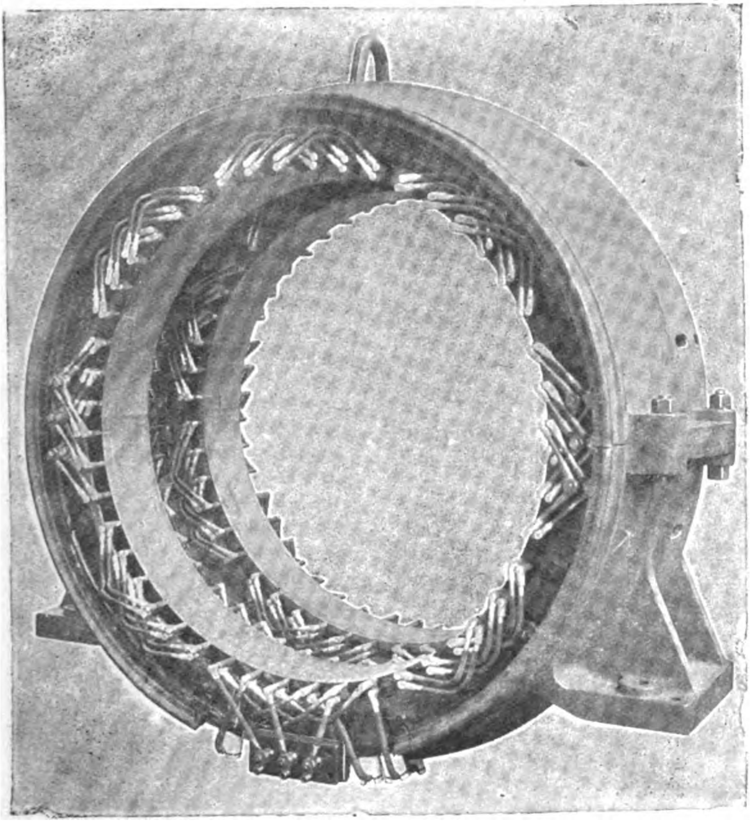

FIG. 110.—ARMATURE OF INDUCTOR GENERATOR (OERLIKON CO.).

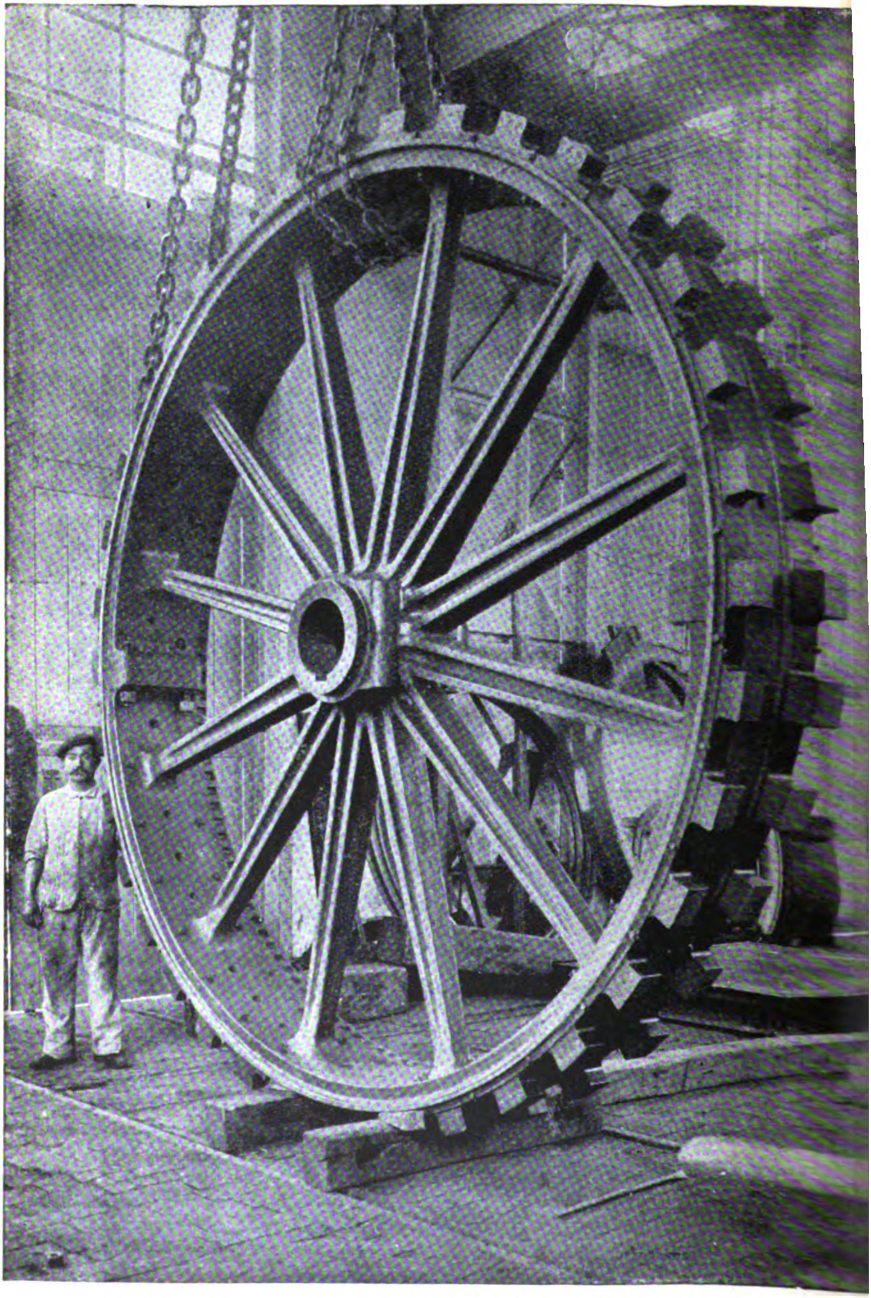

FIG. 111.—INDUCTOR OF SLOW-SPEED GENERATOR OF OERLIKON CO.

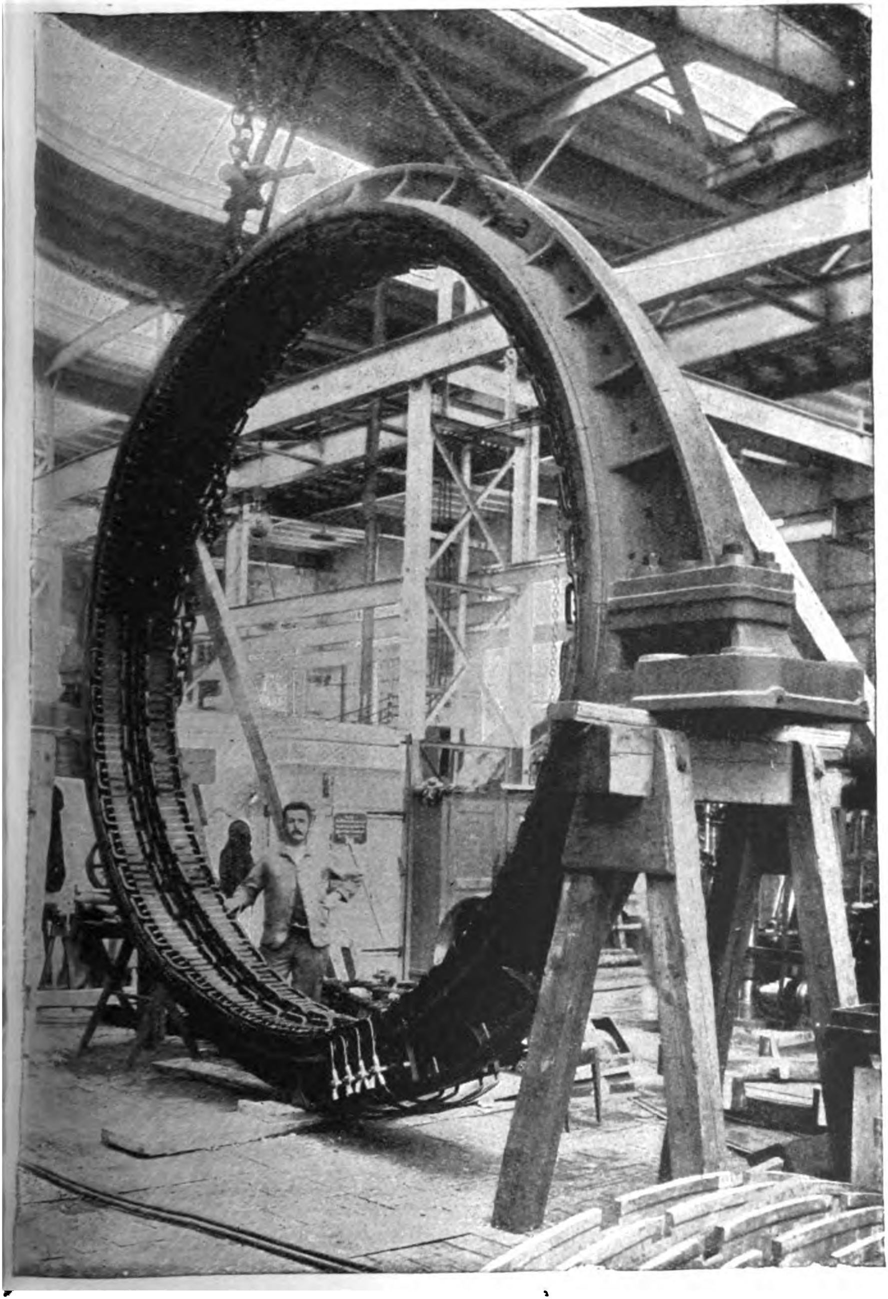

FIG. 112.—ARMATURE OF SLOW-SPEED INDUCTOR GENERATOR OF OERLIKON CO.

the figure, consists of 300 turns of copper wire 5 millimetres in diameter.

Fig. 110 shows a different design, also by the Oerlikon Company, of the armature of an inductor generator intended for a relatively low voltage, and is a wave-wound bar armature having two bars per slot. This cut shows very clearly how when the inductor has its poles in line, the winding of the two armatures must be staggered.

Figs. 111 and 112 depict respectively the inductor wheel, and the stationary double armature, of a very slow speed inductor generator of 300 horse-power constructed at the Oerlikon works. The inductor makes 85 revolutions per minute, and the frequency is forty-six cycles per second. Fig. 112 shows the method of armature winding. The magnetizing coil, which is not shown, would be placed in the annular space between the two armatures, thus surrounding the revolving inductor.

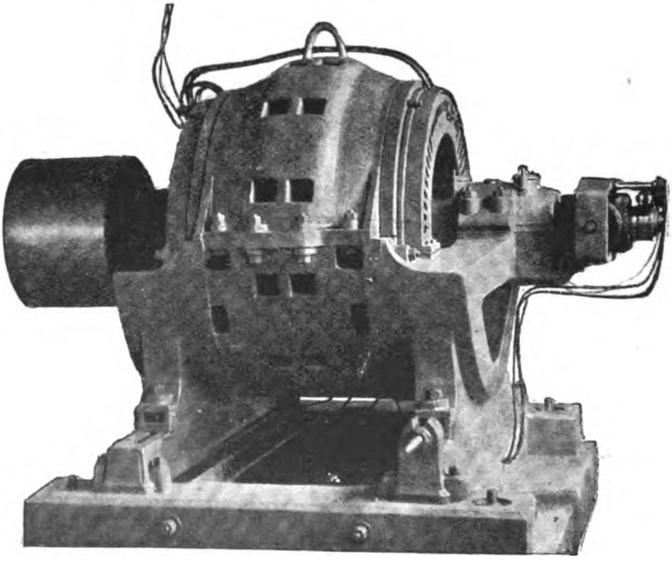

FIG. 113.—INDUCTOR GENERATOR BY KOLBEN AND CO.

As a good example of a small inductor generator, we may take the three-phase machine, Fig. 113, manufactured by Messrs. Kolben and Company, of Prag. This is a 37 kilowatt machine (50 horse-power) when driven at 630 revolutions per minute; the frequency being forty-two cycles per second. If driven at 750 revolutions per minute, it takes 60 horse-power and gives out about 45 kilowatts with a frequency of fifty cycles per second. At full load it yields three currents of 115 amperes each, the full load voltage at the terminals being 190 volts; the power factor being cos ϕ = 0·85. The section given in Fig. 114 shows that the inductor is of cast steel, with laminated polar masses, of external diameter 555 millimetres. The gap is 2·5 millimetres only. The armature core-disks have twenty-four slots each. The inductor has four poles in each of the two sets, so that the number of slots is two per single pole per phase. The surrounding framework which acts as the yoke of the magnetic circuit is of cast-iron. Fig. 114 shows in section how the magnetizing coil lies between the two armatures.

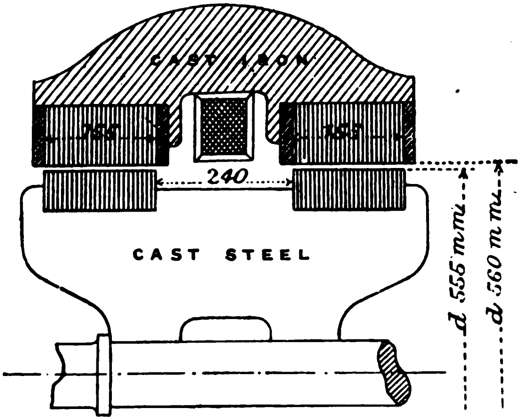

FIG. 114.—KOLBEN INDUCTOR GENERATOR (Section).

Fig. 115 shows the core-disk and the slot, while Fig. 116 gives the winding scheme of this machine. The winding is a Y-grouping, having a common junction at J, and three outer terminals A, B and C. If the three windings are separately tracked out it will be seen that each of them, though grouped in former-wound coils as a "one-slot" winding, goes zig-zagging around each armature from J to the corresponding terminal, the whole constituting a wave winding; each bend of the zig-zag being, however, a complete coil of five turns and each conductor consisting of three wires in parallel, so making apparently fifteen wires through each slot. It will

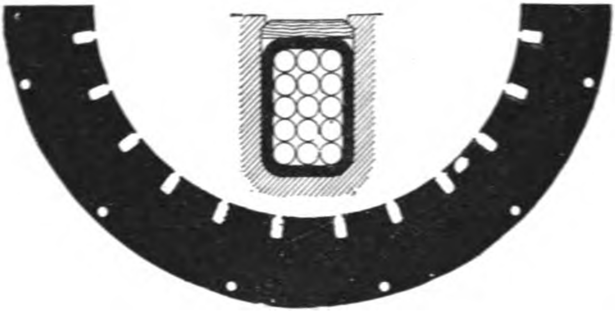

FIG. 115.—KOLBEN GENERATOR. STAMPING OF ARMATURE CORE; AND ARRANGEMENT OF WIRES IN ONE SLOT.

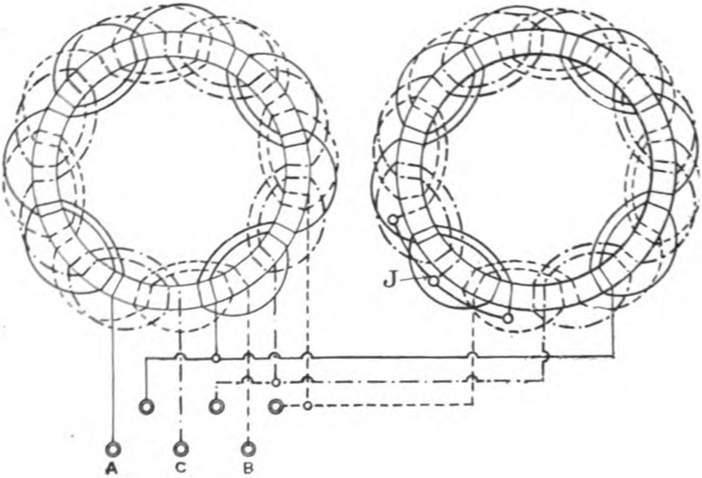

FIG. 116.—KOLBEN GENERATOR. WINDING SCHEME.

also be noticed, by comparing the two halves of the winding scheme, that the winding of one armature is staggered with respect to the other.

The following are the principal data concerning this machine :—

Outer diameter of inductor	555 millimetres.
Inner diameter of core stampings	560 ,,
Effective length of core parallel to shaft	2 × 155 ,,
Thickness of laminations	0·45 ,,
Pole-pitch at armature face	220 ,,
Pitch of winding	220 ,,
Breadth of pole-piece (= pole arc)	172 ,,
Depth of laminations in pole	68 ,,
Pitch of poles on one side	212 ,,
Length of magnetic path in inductor	58 ,,
Section of magnetic path in steel of inductor	1150 sq. ,,
Mean periphery of magnetizing coil	2200 ,,
Internal breadth of winding space	7·5 ,,
Internal depth of winding space	13 ,,
Number of turns of exciting coil	450 ,,
Diameter of wire of exciting coil, bare	4·2 ,,
,, ,, insulated	4·7 ,,
Exciting current at no load (full load volts)	19 amperes.
,, at full load when cos ϕ = 0·85	23 ,,
E.M.F. per circuit from J to terminal, at no load	110 volts.
Number of armature windings in series	40 turns per circuit.
Mean length of armature conductor	1020 millimetres.
Size of armature conductor (three in parallel), bare	4 ,, (diam.).
,, ,, ,, insulated	4·6 ,, ,,
Resistance per circuit at 15° C.	0·0136 ohm.
,, of exciting coil at 15° C.	1·25 ,,
Magnetic flux in armature at no load	3·65 megalines.
Co-efficient of magnetic leakage	1·25
Flux-density in armature, at full load (neck of coil)	16,100 lines per sq. cm.
,, in gap, at full load	13,700 ,,
,, in pole-pieces, at full load	16,300 ,,
,, in body of inductor	15,900 ,,
Short-circuit armature current, when exciting current is 19 amperes	550 amperes.
Exciting current required in order that the short-circuit current shall be equal to full load normal current of 115 amperes	3·9 ,,
Efficiency at full load	88 per cent.

The Allgemeine Elektricitäts Gesellschaft, of Berlin, has made many forms of inductor machine from the designs of

Mr. Dobrowolsky, as shown in Figs. 117 to 122. The first of these, Fig. 117, is constructed on the same lines as the smaller alternators of the Oerlikon Company, Messrs. Kolben, and several German firms. The magnetic circuit passes through an external iron case and two armature core-rings A A built up of stampings with teeth surrounded by coils, and is completed through the yokes J J of the revolving inductor. For three-phase machines the teeth of the fixed armature part are three times as numerous as those of the inductor, as shown in Fig. 118. The 280-kilowatt machines built by the Allgemeine Company for the Strassburg central station are of a different type (Fig. 119), though the principle is the same. In these machines laminated iron inductor masses P revolve between two concentric armatures consisting of toothed core-disks. The excitation is furnished by an annular bobbin bolted up against the side of the cast-steel frame that supports the armatures. Their speed is 150 revolutions per minute. The excitation is 1·4 per cent. of the output, the armature copper loss 2 per cent., and total hysteresis loss is

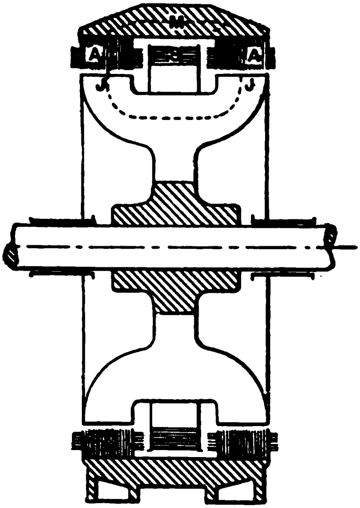

FIG. 117.—SECTION OF DOBROWOLSKY'S INDUCTOR ALTERNATOR.

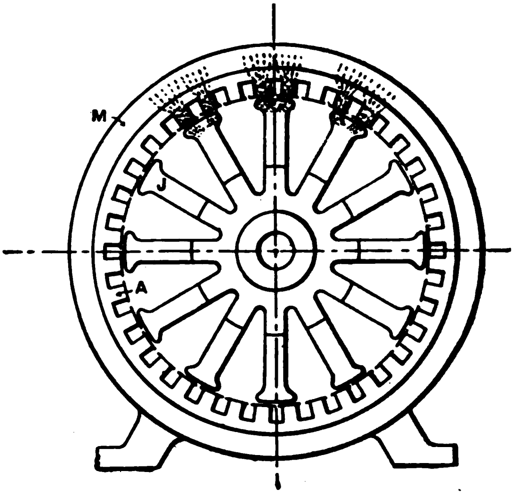

FIG. 118.—END VIEW OF THREE-PHASE INDUCTOR ALTERNATOR.

1·3 per cent. The weight of copper used works out at 4·5 lbs per horse-power, and the iron, excluding shaft and bearings, to about 48 lbs. per horse-power. Two machines of this type have been supplied to operate the three-phase motors in a pumping station in the Memel delta,[1] each of 200 kilowatts, 5000 volts, 50 ~, and 28 amperes per phase. At the full load excitation of 13,600 ampere turns, the short-circuit current per phase was found to be 170 amperes.

Fig. 120 depicts the inductor of a three-phase generator constructed by the Allgemeine Company, of Berlin. This is a 300 kilowatt machine, giving 50 periods at 375 revolutions per minute. The armature, which has a general resemblance to Fig. 120, has forty-eight slots (being six slots per pole, or two slots per pole per phase) in each half.

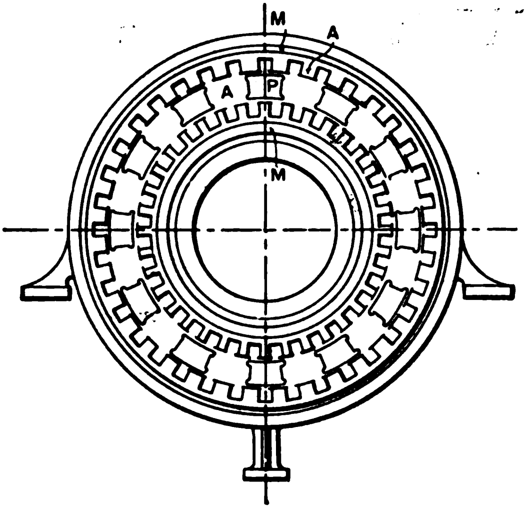

FIG. 119.—THREE-PHASE INDUCTOR ALTERNATOR AT STRASSBURG.

Fig. 121 depicts a section and partial exterior view of one of the large three-phase inductor generators carried out by the Allgemeine Company, from designs which originated at the Oerlikon works, for the great power station at Rheinfelden.

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[1] See *E.T.Z.*, 1897, No. 39.

These machines are of the vertical shaft type for turbine driving. The revolving portion is built up of castings with 110 laminated pole-pieces (arranged in vertical pairs), bolted securely into its recesses. This revolving inductor is 5·74 metres in diameter, the all-over diameter being 6·84 metres. The exciting winding is a single stationary bobbin of 6 metres mean diameter shown in section in Fig. 122. The armature which is exterior to the rotating part is built up also of cast steel with laminated iron faces. These laminated faces are slotted out for the winding, there being six slots per pole.

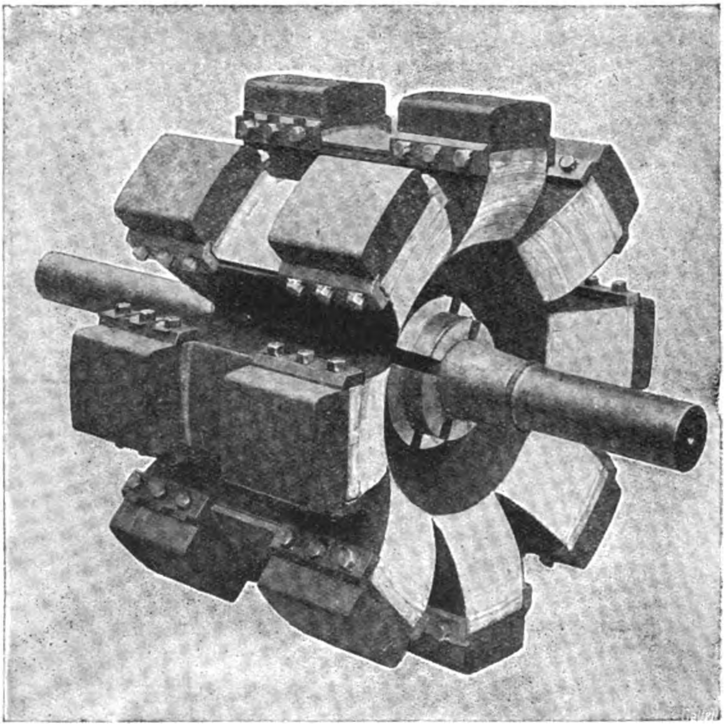


FIG. 120.—INDUCTOR OF ALLGEMEINE CO.

A modern example of inductor alternator built by Messrs. Ganz and Company, of Buda-Pesth, is furnished by a single-

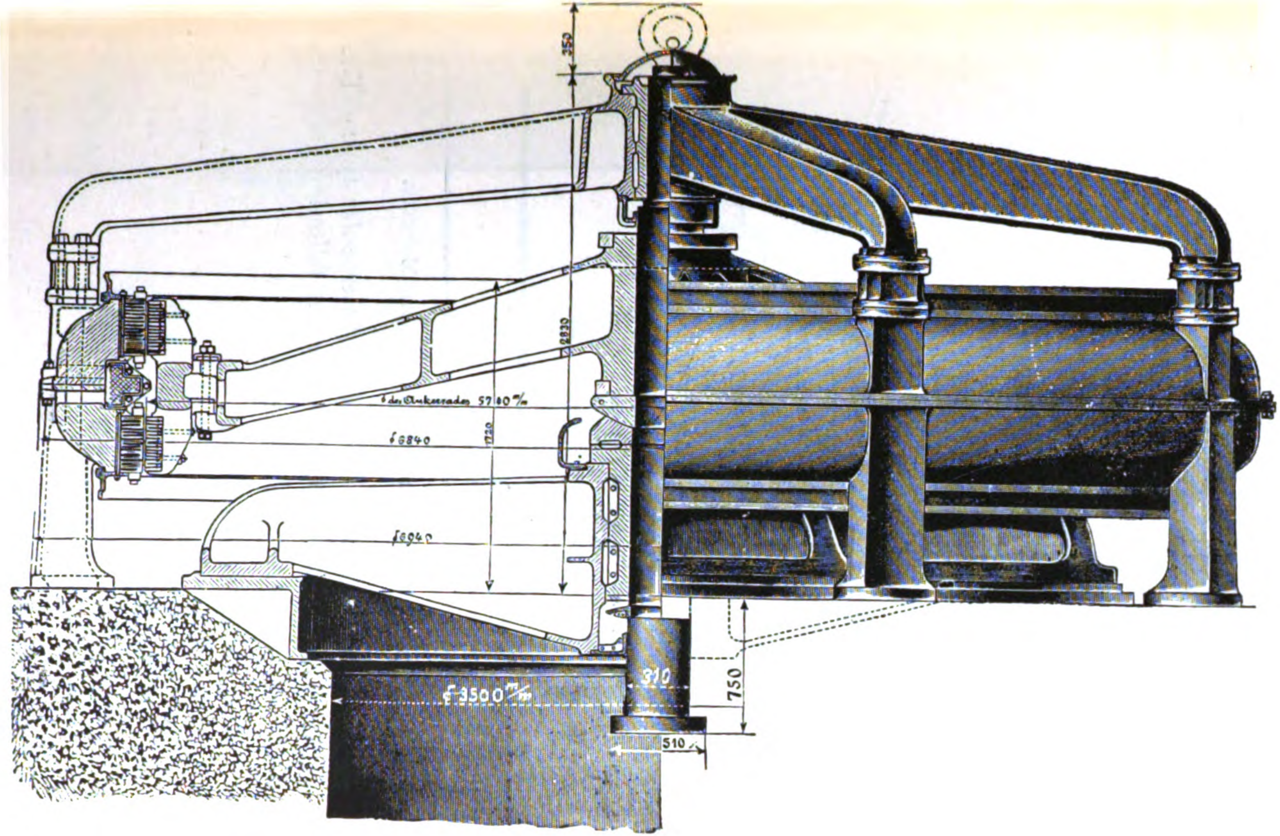


FIG. 121.—VERTICAL SHAFT INDUCTOR GENERATOR, BUILT FOR RHEINFELDEN TRANSMISSION PLANT.

phase machine, AS(I) 60_p—$440_{k.v.a.}$—$100_{r.p.m.}$, supplied by them to the Corporation of Leicester, shown in Fig. 123, and Plate III, which gives a section through the ironwork of armature and inductor. It is of the heteropolar type. The author is indebted to Mr. Blàthy for the following constructional details and data relating to it.

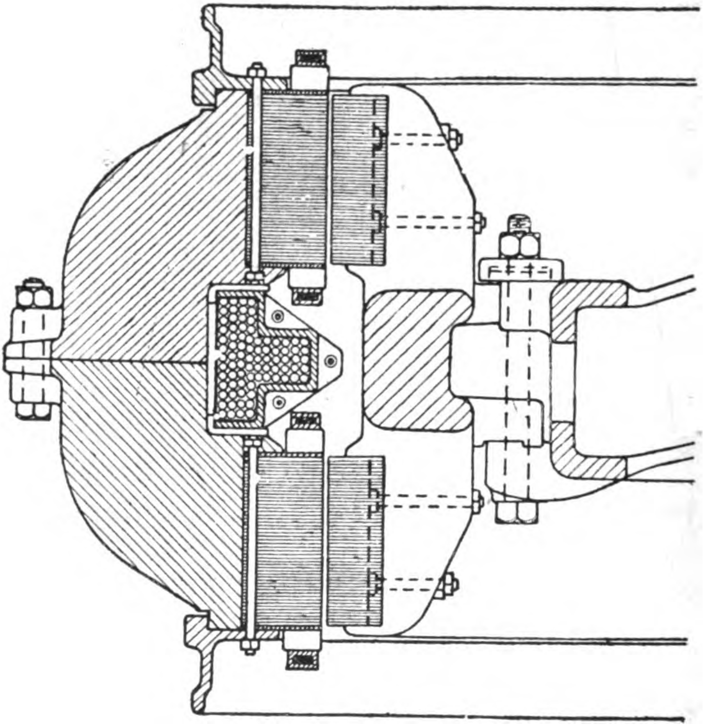
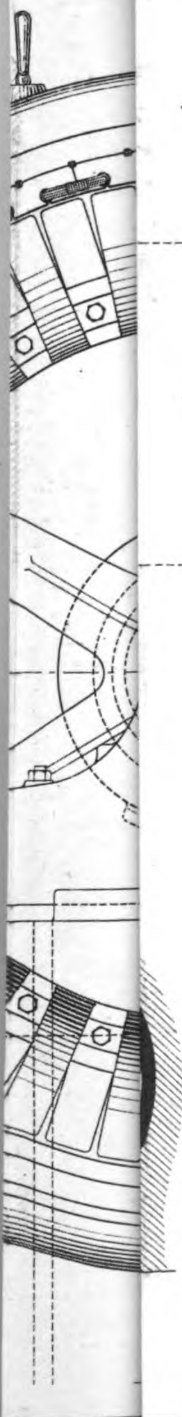
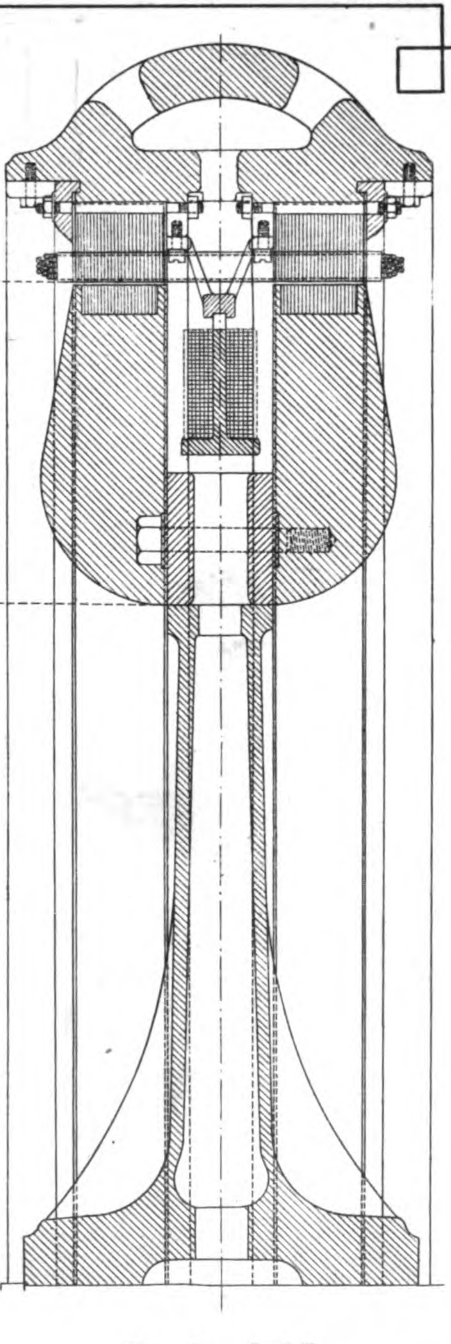


FIG. 122.—DETAILS OF RHEINFELDEN MACHINE.

Kilowatts output for cos $\phi = 0\cdot7$	308
Terminal volts at full-load	2200
Armature current ,, ...	200
Speed in revs. per minute	100
Frequency	50
Number of pairs of inductor poles	30
Number of armature holes	60
,, ,, coils	30
Total number armature conductors	660

Plate III.





Scale 1:15

Thoˢ Kell & Son, Lith.

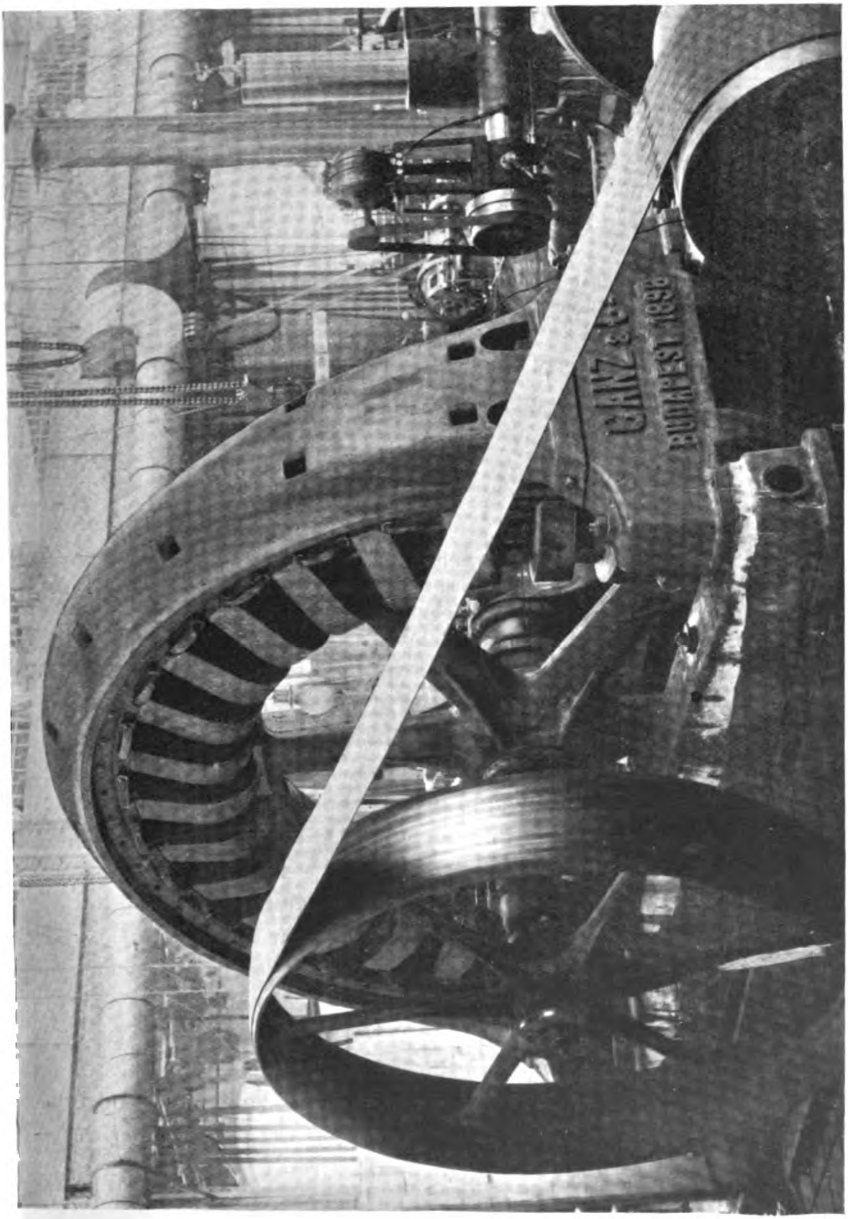


FIG. 123.—GANZ 300 KW INDUCTOR ALTERNATOR, DURING CONSTRUCTION.

Dimensions of armature conductor	19 strands of 1·8 mm. diam.
Resistance armature winding at 20° C.	0·2 ohm.
Number of turns on exciting coil	144
Dimensions of conductor of exciting coil	10 mm. diam.
Resistance at 20° C. of ,,	0·375 ohm
Friction and ventilation loss	8000
Full load C^2 R loss	8000
Total iron losses at full-load	20,000
Excitation losses for 200 amp. cos $\phi = 0·7$	10,850
Final full-load temperature rise, armature	30° C.
,, ,, pole-pieces	10° C.
Length of air gap = clearance	6 mm.
Total weight alternator and exciter complete	53 tons.

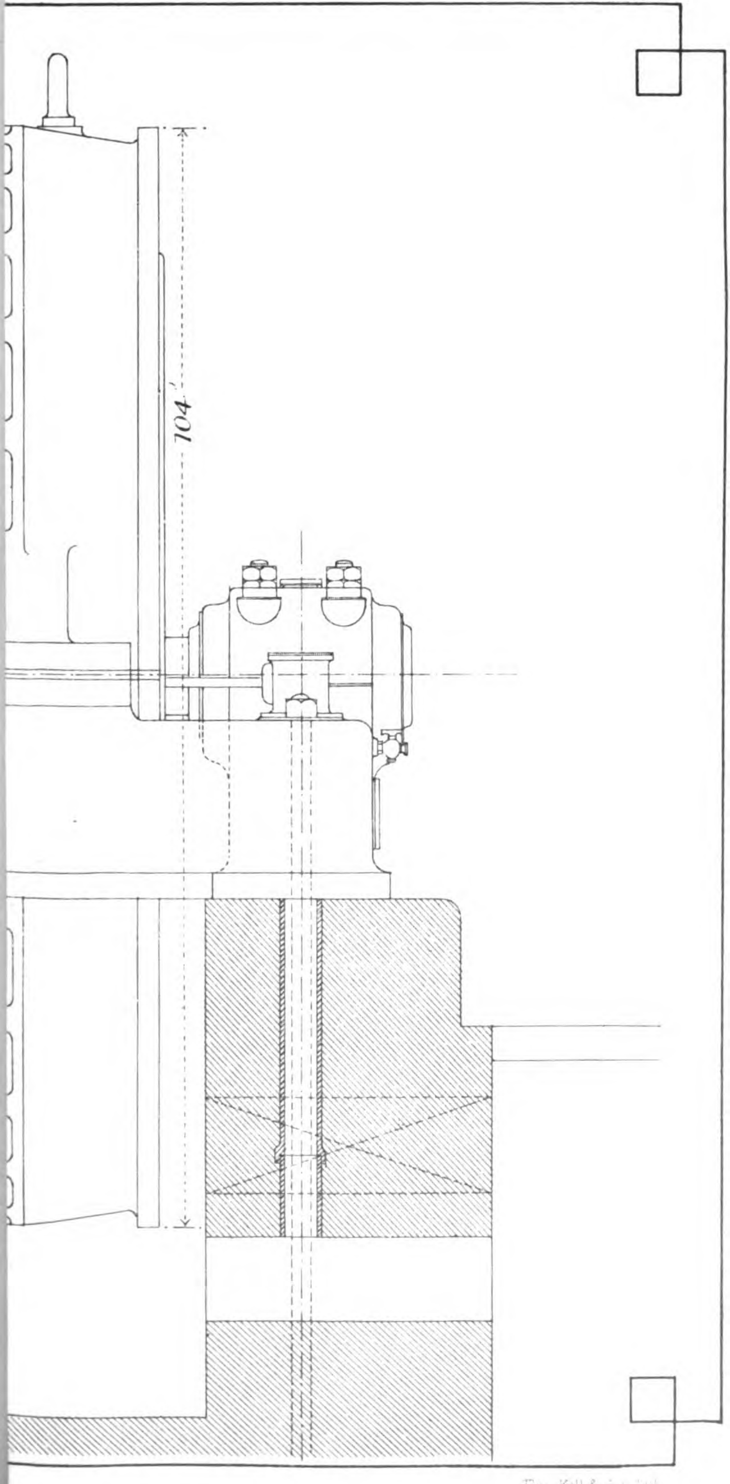
Further analysis of the design shows the following points:—

B_{av} at pole-face = 65,000 lines per sq. in.
q (specific load) = 267 ampere-conductors per inch.
v (surface speed) = 4125 ft. per second.
N (per pole) = 3 megalines.
τ (pole-pitch) = 8·26 inches.
b (pole-breadth) = 6·7 inches.
l (armature length, front to back) = 6·9 inches.
Diameter of micanite insulating tube, external, 2·17 inches.
,, ,, ,, internal, 1·85 inch.

This machine furnishes a striking illustration of a superiority of polyphase apparatus over single-phase, from the designer's point of view. The makers state that the same machine provided with a three-phase armature (weighing practically the same) would give out double the above output for the same heating and an (increased) efficiency of 94 per cent.

Plate IV. shows a single-phase machine, with staggered inductor poles, designed by Mr. Fynn for the Bedford Corporation, Figs. 124 and 125 showing clearly the construction of its armature, inductor and field coil. When driven at 257 revolutions per minute its output is 136 amperes at 2200 volts or 300 kilowatts on a non-inductive load. The armature is built up in two distinct halves, each half being made up in a suitable number of slotted stampings, the slots being rectangular with rounded corners, and partially closed. Between the two halves of the armature is placed the exciting bobbin, made of gun-metal, and securely attached to the cast-iron

Plate IV.



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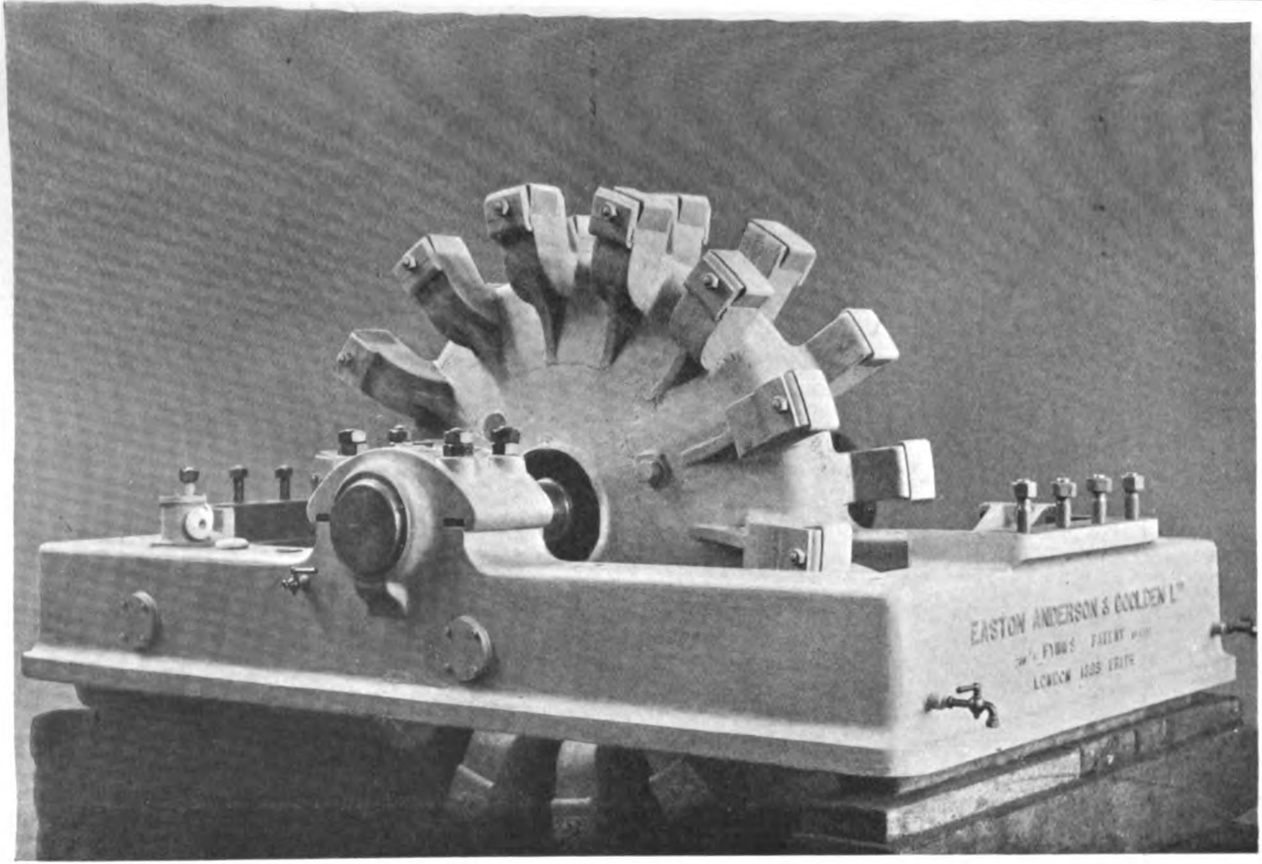

FIG. 124.—INDUCTOR OF FYNN'S SINGLE-PHASE ALTERNATOR.

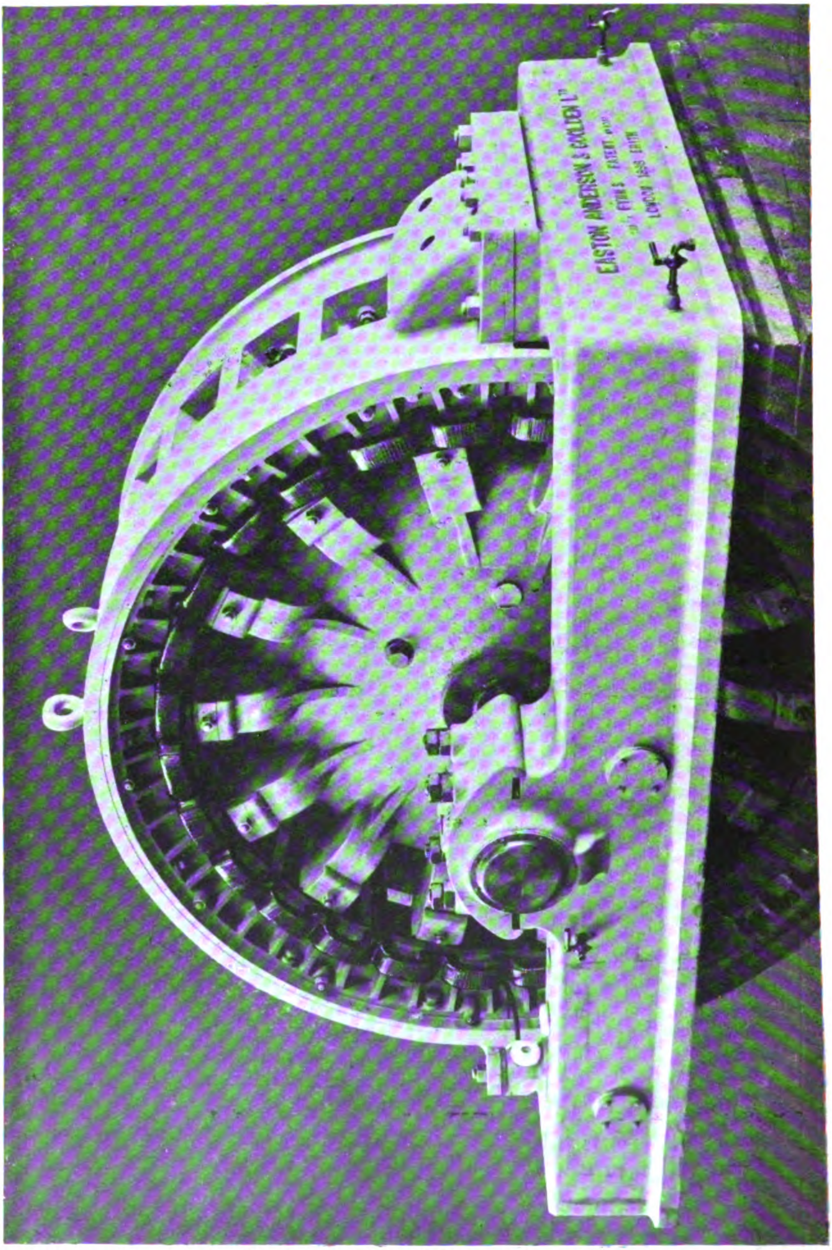

FIG. 125.—FYNN'S SINGLE-PHASE INDUCTOR ALTERNATOR.

frame of the armature by means of radial bolts. The rotating inductor, Fig. 124, consists of a cast-iron structure made in two halves, the poles being provided with laminated pole-pieces at their extremities, which are bolted up as shown. As can be seen from the photograph, they are "staggered" with regard to one another, that is, the N-poles on one side of the inductor are not immediately opposite and facing the S-poles on the other side, but occupy an intermediate position. The manner in which the armature winding is carried out can be clearly seen from Fig. 125. The winding is carried right through the two armatures in paper or micanite tubes, the side of each coil taking up half a slot (Chap. VI., Fig. 241, page 301), so that it becomes an easy matter to rewind a burnt-out or damaged coil. The individual coils are secured by wooden wedge-pieces and connected up in series. The pitch of the poles, that is, half the distance between two adjacent N- or S-poles on one side, is equal to the pitch of the slots, and the breadth of the pole face is made equal to four-fifths of the pole-pitch. Consequently, the Kapp coefficient of the Fynn alternator, is, from Table III., page 202, equal to 2·23. The 300 KVA machine has 28 slots with 20 conductors passing through each slot. The air-gap is $\frac{7}{32}$ inch. At no-load the excitation is 14,800 ampere-turns; increasing to 16,900 on a full non-inductive load, or to 28,100 on a full inductive load. The efficiency on test is 91·9 per cent. Five similar alternators of 125 KVA each are in operation at Torquay lighting station.

It has been already stated that magnetic leakage plays an all-important part in the design of the inductor alternator. If there be much leakage from either armature or field, the machine becomes heavy and defective in regard to pressure regulation. By working with a very small air-gap, not much greater than the necessary clearance, and by careful design of both armature and inductor, magnetic leakage in these machines is reduced to a very small amount, and the whole magnetic field is very stiff and concentrated. But working with such a small air-gap has the following disadvantage. As mentioned on page 119, the armatures of inductor machines must work at higher flux-densities than is usual in alternators of the ordinary types. That is to say, the armature is magnetized, not below the bend of the magnetization curve, but on it. Consequently, if the air-gap is very small, and comparable with the reluctance of the armature iron, the increase of excitation necessary to make up for the drop in pressure at full load becomes large in amount, as a considerable increase of excitation will not cause a corresponding increase in the field (and hence terminal pressure) but will raise it by a small amount only. On the other hand, if the air-gap is large, an increase in the excitation causes a corresponding increase in the terminal pressure, because the air-gap reluctance is constant, the excitation required to overcome it being exactly proportional to the flux itself. If two machines were operated under given conditions, one having a large air-gap and low flux-densities, the other having high flux-densities and a small air-gap, the pressure of the machine of low density could be raised or lowered by small changes of the field excitation, while it would require relatively large increases of excitation to raise the pressure of the machine of high flux-density. If the pressure were maintained constant on both machines, and both were subjected to an inductive load, the case would be the same, and a very large increase of field excitation would be required on the high density machine, while a small increase only would be required on the low density machine. Thus a machine of the inductor type, which must work at high flux-densities on account of economical considerations, and must have a small air-gap if even reasonably good pressure-regulation is required, becomes very inflexible, especially on motor loads. The manner in which this difficulty has been got over is the characteristic feature of the Fynn alternator. Briefly the armature iron is worked at as high a flux-density as possible, determined by the permissible loss, the gap between the inductor poles and armature made very small, and an *artificial air-gap* is introduced into the magnetic circuit at a place where it will do no harm, namely, in the centre of the exciting spool. The inductor casting being made in two halves, by simply screwing out each half to the requisite amount (see Plate IV.) the excitation can be adjusted as desired. This is done once and for all in the test-room before the machine is sent out, the distance between the two halves varying from one-sixteenth to three-eighths of an inch, the amount depending on the size and frequency of the machine. The use of this device for such inductor machines will give extremely good results from the point of view of pressure-regulation.

Messrs. Brown, Boveri and Company have constructed various patterns of inductor alternators, usually for special cases of abnormal speeds. For direct-coupling to high-speed Pelton wheels the firm has used the type shown in Fig. 126. The inductor magnet consists simply of two steel castings bolted together, each casting having four or more arms serving as the poles, the arms of one casting being "staggered' with regard to those of the other. By the action of a central fixed exciting coil, these arms acquire opposite polarities, producing in the stationary external armature (which is also made in two parts) either single or polyphase currents according to the manner of winding. In order to minimize eddy-current losses, the pole tips either have deep cuts made in them, or else consist of iron stampings fixed into the ends of the arms.

For the special types of field-magnet adapted for high-speed steam turbines, the reader is referred to Chapter IX.

MAGNET-WHEEL CONSTRUCTION.

The magnet-wheel consists of a hub, arms, and a rim to which the poles are attached. Whether it itself serves as fly-wheel or not, the principles of fly-wheel construction apply to it. The reader should therefore consult such a work as Unwin's Machine Design, Part II. p. 179, for engineering information.[1] Fly-wheels up to 10 feet in diameter are sometimes cast in one piece, but the hub is apt to split owing to contraction of the casting during cooling, and from the same cause the arms are often under strain. A safer construction,

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[1] For further mention of fly-wheels in connexion with parallel running, see Chapter X.

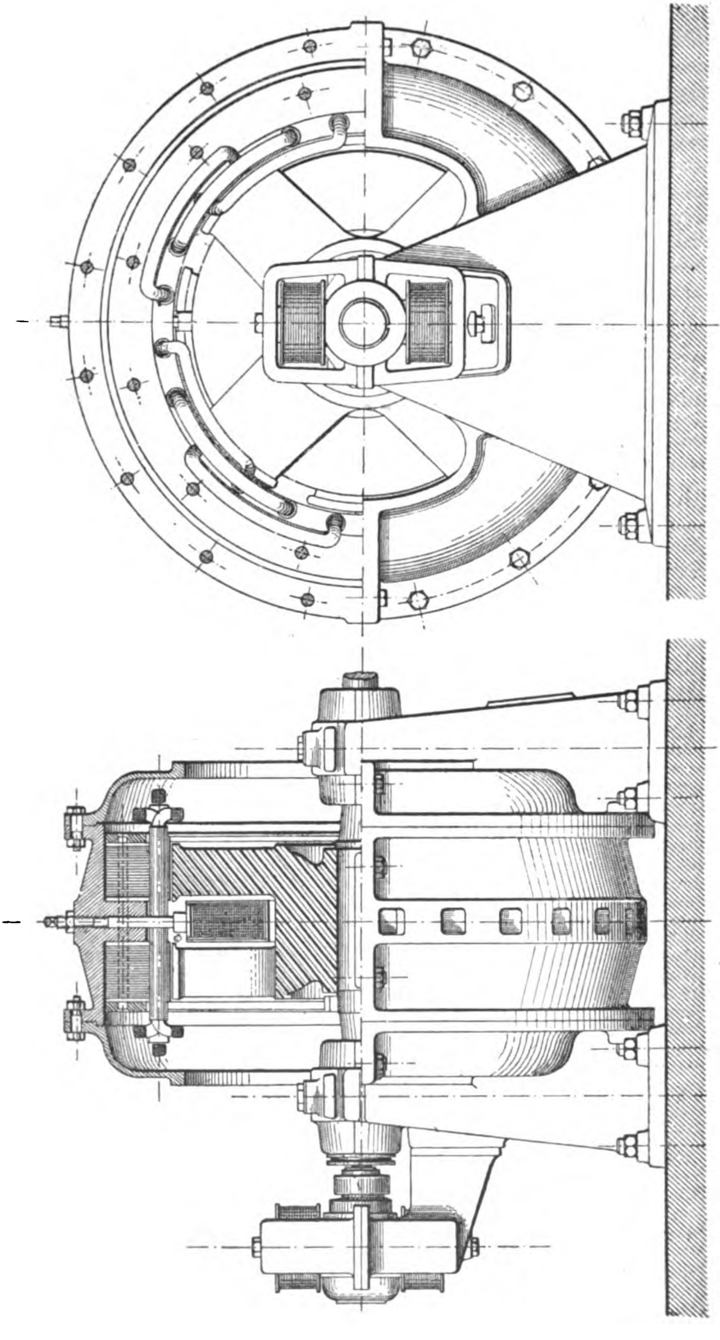

FIG. 126.—BROWN'S THREE-PHASE INDUCTOR GENERATOR, 150 H.P., 600 REVS. 5300 VOLTS.—Scale 1:20.

3

st

s

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s

s

s

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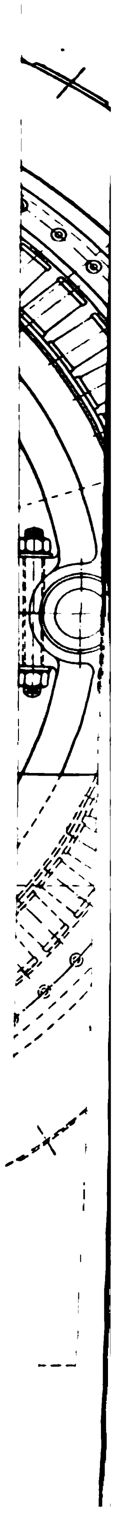
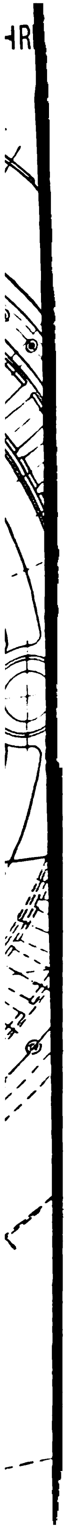
THR





frequent for fly-wheels from 8 to 15 feet in diameter, is to cast them in two halves, of which the rim parts are afterwards joined by dowels and cotters, or by straps or bolts, and the hub-parts by bolts or by rings shrunk on. Still larger wheels are built up in segments similarly held together, or with arms cast separately from the rim and the hub. The various Plates should be examined for details. The stresses due to centrifugal force in a simple rim are for cast-iron expressed by the formula that the stress in pounds per square inch $\doteq 0{\cdot}000027 \times v^2$; v being the peripheral speed in feet per minute. The masses of the pole-cores and bobbins add, however, to the centrifugal forces. Seeing that for each material there is a certain limit of tensile stress up to which it is safe to work, practice dictates that for each type of construction, and for each material there is a certain safe limit of surface speed. Magnet-wheels with cast-iron foundation-rims seldom run at a higher peripheral speed (measured at the pole-faces) than 5000 to 6000 feet per minute; those of cast steel from 6000 to 7000, though there are exceptions. Nor must it be forgotten that the insertion of bolts through the rim to fix the magnet cores or of dowels for jointing the segments, decreases its strength. For machines under 300 KW or so, the foundation wheel is now usually shaped like a simple broad pulley, with a thick rim. For high-speed machines direct-driven, a similar construction is used. Fig. 127 depicts the magnet-wheel of a 290 KVA 10-pole generator, designed for a speed of 600 RPM. A similar construction is used for the 1840 KVA generators running at 200 RPM, constructed by the Oerlikon Company for the Central London Supply Company.

Plate II. (Helios Company) shows a magnet-wheel with double cast spokes; the hub is held on with shrunk-on rings. The rim has deep pulley-like grooves between flanges through which the pole-cores are bolted with bolts parallel to the shaft. Plate V. (Siemens and Halske) depicts a heavy fly-wheel construction cast in two halves, the hub being held together both by bolts and rings. The rim is bolted, dowelled and clamped. The pole-cores are screwed on by screws through the rim.

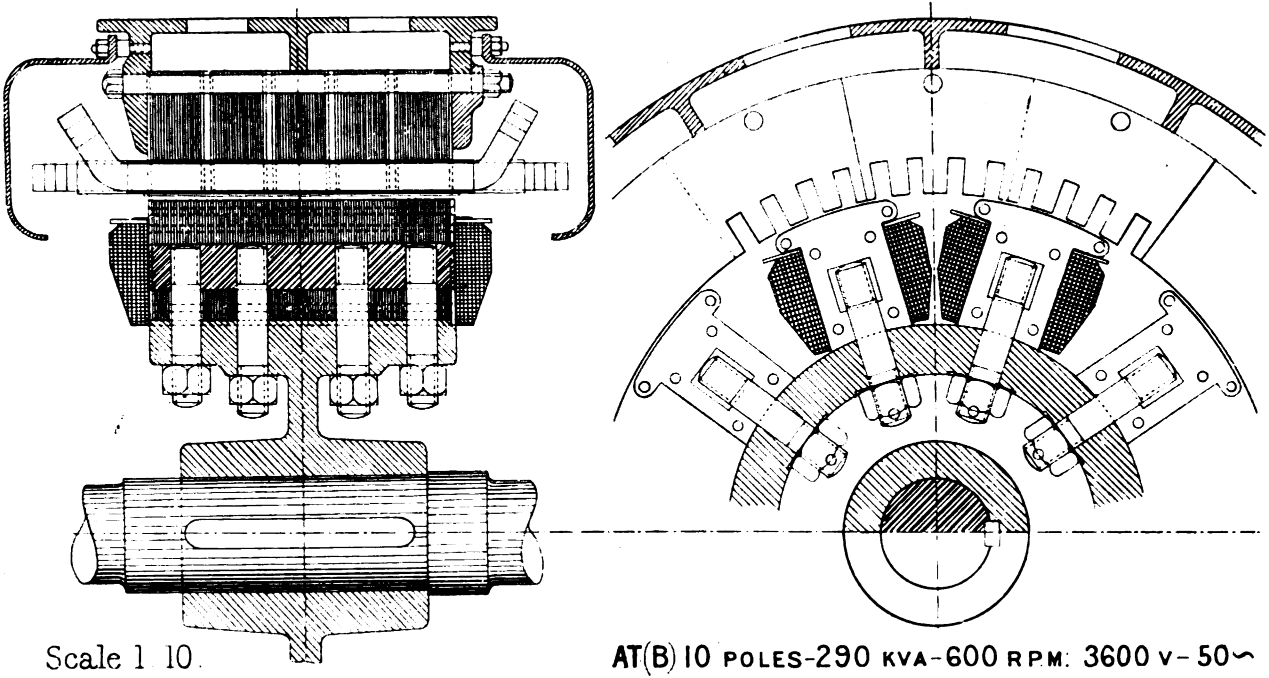


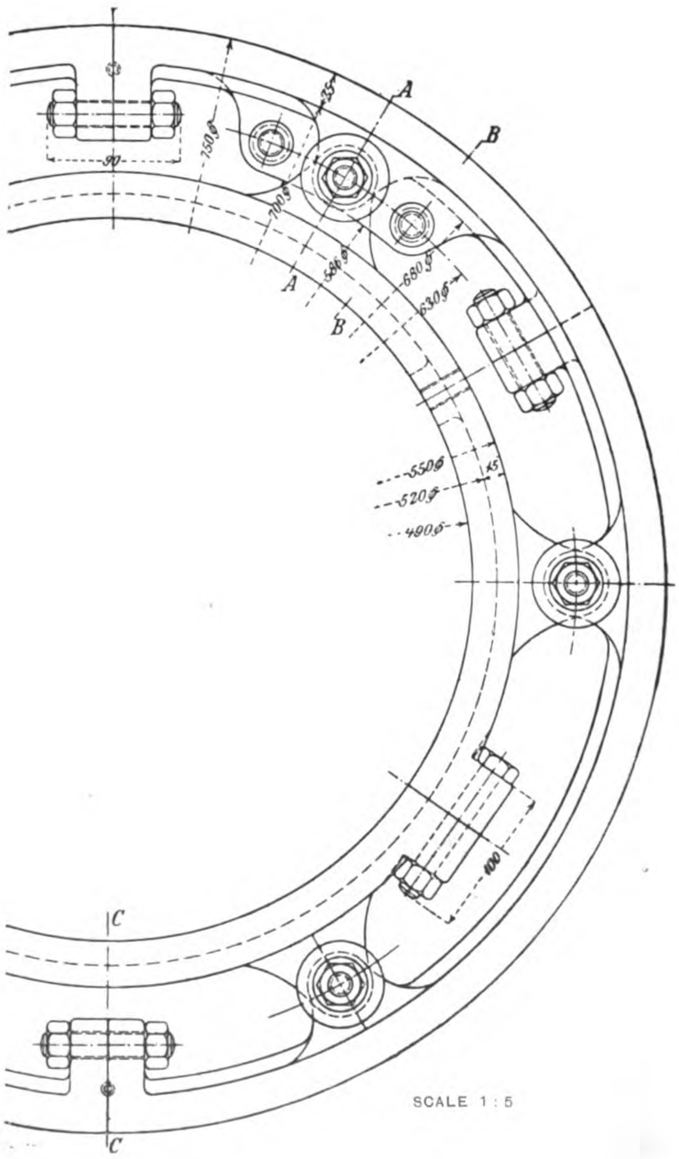
FIG. 127.—HIGH-SPEED THREE-PHASE ALTERNATOR (OERLIKON CO.).

THREE-PHASE ALTERNATOR.

AT (B) $56_P - 900_{KVA} - 107_{RPM} - \frac{1270}{2200}V - 3 \times 236_A - 50 \sim$

CONSTRUCTED BY

INE ELEKTRICITÄTS GESELLSCHAFT (OF BERLIN)



SCALE 1 : 5

Plate VI. (Allgemeine Company) shows a fly-wheel also cast in two halves. The rim is dowelled, cottered and clamped. In this machine the rim is channelled to receive a laminated foundation-ring that is bolted to it and has grooves to hold the dove-tails of the pole-cores.

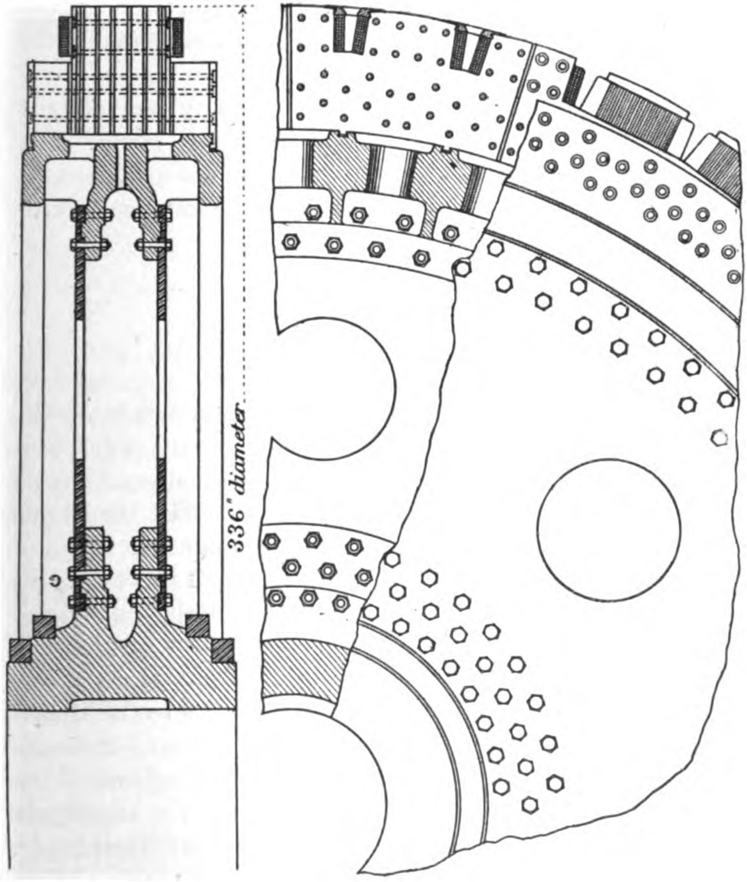


Fig. 128.—Manhattan Alternator at(b) 40—5000—75, by the Westinghouse Co. (Scale 1 : 40.)

Plate VII. (Brown, Boveri and Company) shows a fly-wheel form, with double spokes, between which the screws are inserted which bolt on the pole-cores.

The construction of the magnet-wheels of the 5000 KW generators constructed by the Westinghouse Company for the Manhattan station in New York differs in that the spokes are replaced by continuous webs of steel boiler-plate (a construction used by Gordon in the Paddington alternators of 1882), as shown in Fig. 128.

More recently the Allgemeine Company has designed forms in which the entire rim is built up of iron plates. Herr Lasche, who has discussed[1] these various constructions, has also suggested the replacing of spokes by steel ties like the wire spokes of bicycle wheels.

POLE-CORES AND POLE-TIPS.

Pole-cores are of two types, solid and laminated. Owing to the requirements of speed and frequency, the poles of an alternator, save in abnormal cases, are usually of greater dimensions in the direction parallel to the shaft than in the tangential direction. Those for ordinary frequencies of 40 or 60 cycles per second usually have from 8 to 12 inches as the pole-pitch at the face. The actual span at the pole-face is usually from 5 to 9 inches; and the breadth of the core is generally less than the span, varying from 3½ to 7 or 8 inches. The length from front to back parallel to the shaft is seldom less than 9 inches, and may be as much as 20 inches or more. This being so, it is comparatively seldom that a cylindrical form is adopted for the pole-core, a rectangular section being more suitable. Hence the use of stampings of sheet iron or steel assembled together naturally arises. Even in those cases where a cast cylindrical core is used it is customary to affix a pole-piece of rectangular form. The reader is referred to the following examples in this book :—

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[1] "Der Aufbau und die planmässige Herstellung der Drehstrom Dynamomaschinen." *Zeitschrift des Vereins Deutscher Ingenieurs*, Berlin, 1901.

Solid Pole-Cores.	*Example.*
Rectangular, without shoes	Westinghouse, Fig. 88.
,, with shaped faces	Westinghouse, Plate XIII.
,, with solid shoes	Ferranti, Plate I.
,, with laminated shoes	Ganz, Plate III.
Cylindrical, with solid shoes	Brown, Plate IX.
,, with laminated shoes	Brown, Plate VII.
Laminated Pole-Cores.	
Rectangular, with flat faces	Helios, Plate II.
,, with shaped faces	Niagara, Plates X. and XI.
,, with square projecting tips	Westinghouse, Fig. 120.
,, with shaped projecting tips	Siemens, Plate V.; A.E.G., Plate VI.
,, with skewed edges	Oerlikon, Fig. 197.

Fig. 129 illustrates a number of forms of pole used by different makers.

Of these sketches *f* is a solid form used by Brown. It is more usual to use two screws than one for fixing the cores. Special forms of stampings to serve both as core and yoke are shown in *d*, *e*, and *h*. Form *e* is due to the Oerlikon Company; form *d* to the Westinghouse Company. The method of assembly of the latter is shown again in Fig. 130, where the grooved cast-iron spider is seen, as well as the ventilating ducts between the laminations.

Methods of holding down the pole-cores are shown in Fig. 129 by dovetails *d*, *r*, and *s*. The laminations are usually held together in blocks by thin bolts with riveted or screwed heads. The bolts are usually not insulated, but there are exceptions as in *r* the pole-stampings of the Niagara machines, Plates X. and XI. Sometimes the blocks are attached to the rim by simply screwing the holding-down screws into them as is usually done for solid cores, see Fig. 92, p. 106. Another way is to punch holes through them to admit a steel bar which receives the holding-down screws as in *b*, *c*, or *q*. Another kindred mode with trapezoidal clamping blocks is shown at *s*. A method of keying with key-bolts is shown at *m*. Pole-tips, if solid, are usually screwed on as shown in *k*.

Laminated pole-tips are usually held together with thin bolts, and are either dove-tailed on as in *n* and *o*, or fixed with key-

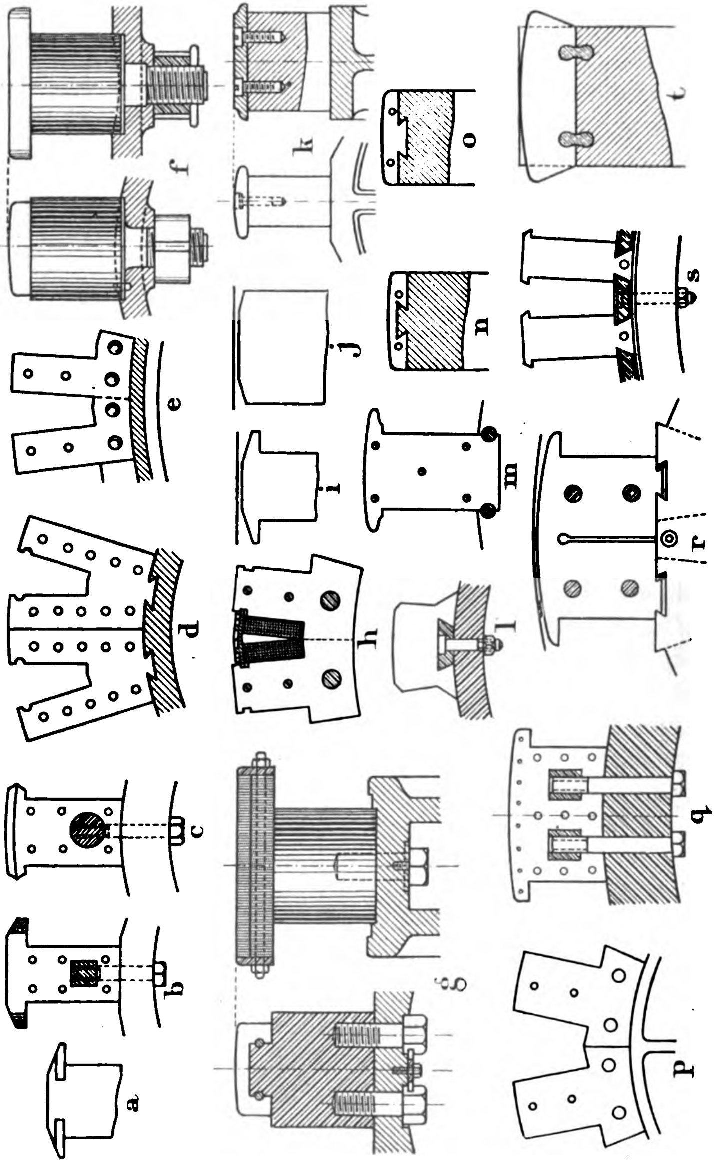


FIG. 129.—FORMS OF POLE-CORES AND POLE-TIPS.

bolts (Brown) as in *g*, or attached by casting tin into 8-shaped perforations as in *t* (Ganz).

As to the forms of pole-tips, it was formerly the practice to shape them (by turning, if solid) so that the gap between them and the armature-face is of uniform width ; or the edges of the pole-faces were slightly rounded, as in *f*, *g*, *k*, *m*, *n*, and *q*. When made of stampings the polar ends were often left simply rectangular, or curved so as to yield a parallel gap, as in *d*, *e*, *h*, *m*, *n*, *p*, and *s*. But makers have gradually become aware of the advantage of securing a certain amount of fringing of the polar field, and this has been attained in several ways, by bevelling the corners as in *a*, *i*, *j*, and *l*, or by use of projecting tips, as in *b*, *k*, *m*, *q*, and *s*. The forms with projecting tips have, indeed, been preferred for mechanically holding-down the exciting bobbins, which otherwise must be independently held by screws or by clamps fitting into

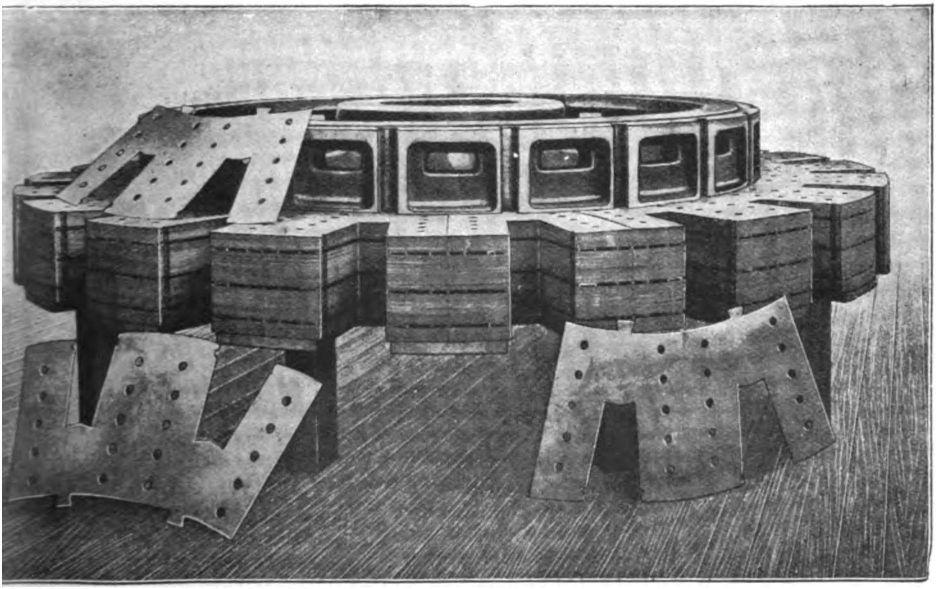


FIG. 130.—ASSEMBLY OF LAMINATED POLE-CORE STAMPINGS ON A CAST-IRON FOUNDATION WHEEL (WESTINGHOUSE CO.).

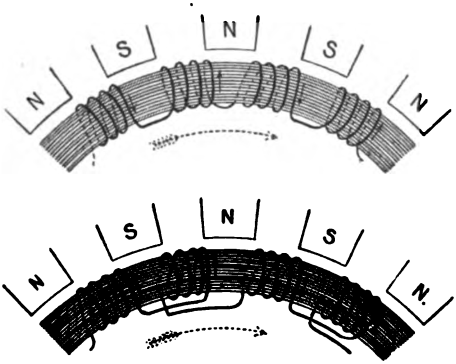
notches as in *d* or *h*. The tendency to saturation of the tips helps to graduate the field at the edges. The Westinghouse Company has used form *j*, where about $\frac{1}{8}$ of the pole-face is bevelled away, and the General Electric Company the form *i*, where about $\frac{1}{4}$ is bevelled away at each side. To secure a still better graduation of fringe the poles of the Niagara (General Electric Co.) machines, *r*, are curved with a radius smaller than that of the polar circle, so that the gap increases regularly toward the tips. Another way of getting a good fringe is that of the Oerlikon Company, *b*, by cutting off varying lengths of the tips so as virtually to make an oblique pole-edge. The importance of this matter is dealt with in Chapter IV. In the form shown in *a* pole-tips of cast-iron are fitted into slots, and are held in by bronze wedges between the neighbouring pole-tips. This construction tends to lessen the distortion caused by armature reaction. In form *c* alternate laminations are made to project beyond those between them so as to produce a higher degree of saturation at the face, a device to increase stability against distortion.

Of the sketches in Fig. 129, *b*, *c*, *e*, *l*, *q*, and *s* are by the Oerlikon Co.; *f* and *g* are characteristic forms with Brown, Boveri and Co. Forms *l* and *t* are used in inductor machines.

ARMATURES.

The armatures of alternators may be of ring, drum, pole, or disk type; but the grouping of the windings is in general different from that which would be adopted for a continuous-current dynamo. The field-magnet being multipolar, a section of the armature winding which is passing an N-pole will have currents induced in it that circulate in an opposite sense to those induced in a section which is at the same moment passing an S-pole. Hence in an alternate-current ring the successive sections must be either wound or connected so as to be alternately right-handed and left-handed. In single-phase alternate-current lap-wound drums the sections do not overlap one another as in continuous-current drum armatures, but in wave-wound drums they overlap, zigzagging between one another as shown in Chapter VI. In two-phase and three-phase armatures it is however usual for the windings of one phase to overlap those of another.

Ring-Armatures.—This type was invented in 1878, almost simultaneously by Gramme[1] and by Wilde,[2] the main difference between them being that, whilst Gramme rotated his field-magnet within a large stationary ring, Wilde rotated his ring armature within an external system of inwardly-pointing field-magnet poles. When ring armatures are used in this type of dynamo, they are not wound in the same manner as for continuous-current armatures. If the successive sections are to be connected up in series then they must be wound as shown in Fig. 131, alternately with right-handed and left-handed windings. If all the sections are coiled right-handedly, then they must be connected as shown in Fig. 132; for the electromotive-force induced in a coil as it passes under an N-pole will circulate around the armature core in an opposite direction to that induced in the neighbouring coil that is passing under an S-pole. For a three-phase ring one may wind each phase with only half as many sections as there are poles, thus making the winding hemitropic.



Figs. 131 and 132.—Ring-Armature Series Windings for Alternators.

If a Gramme ring wound in the ordinary way is connected

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[1] Specification of Patent, 953 of 1878.

[2] Ibid., 1228 of 1878.

down to slip-rings from two points at opposite ends of a diameter, as in Fig. 133, it will yield an alternating current when revolved in a bipolar field. In a multipolar field the ring will need multipolar connexions alternately at points

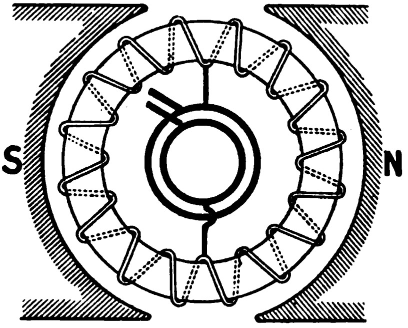

FIG. 133.—SIMPLE BIPOLAR RING ALTERNATOR.

corresponding to the pitch of the poles. In this case, Fig. 134, the various sections of the ring are all in parallel.

A diagram of the Gramme alternator is shown in Fig. 135. The sections of the winding of this machine were four times

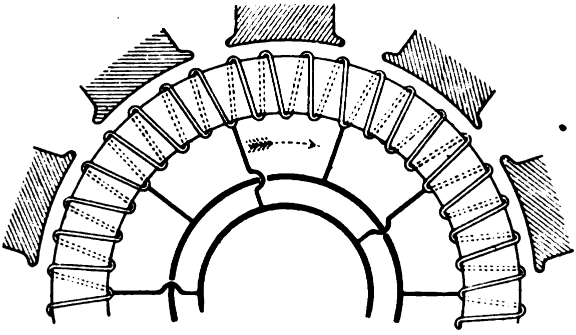

FIG. 134.—PARALLEL RING-WINDING FOR MULTIPOLAR ALTERNATOR.

as numerous as the poles, and might be coupled to feed four separate circuits. It is clear that the revolving poles would come past the four adjacent sections successively, so that the four alternating currents generated would differ *in phase* from one another. Gramme's was in fact a polyphase machine. One form of Gramme alternator, designed for use with Jablochkoff's candles, had four separate circuits differing 45° in phase from each other. Another ring alternator, by De Meritens, with permanent steel magnets, was a favourite about 1879.

In Kapp's early alternator depicted in the fourth edition of this book, the ring lies between a double crown of field-magnet poles. Other ring alternators have been designed by Rankine Kennedy, who used a discoidal ring between alternately-spaced alternate poles within an iron-clad magnet; and by Mordey, who suggested a form with two Pacinotti rings, one laminated, as armature, and one non-laminated as field-magnet.

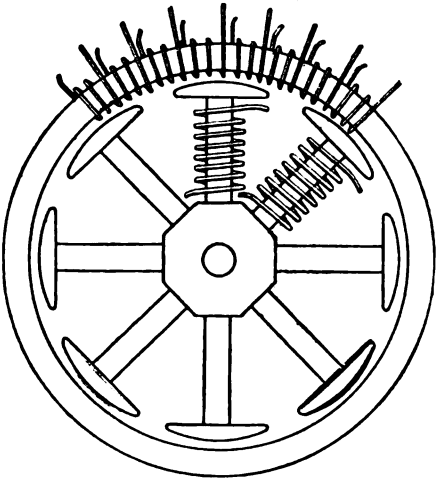

FIG. 135.—GRAMME ALTERNATOR.

Returning to Fig. 133, a little consideration will at once show that such an armature will also serve for the production of polyphase currents. For a two-phase armature, all that has to be done is to connect *four* equidistant points along the winding to four slip-rings. As the ring revolves, the electromotive-forces tend always towards the highest point. Hence two separate alternate currents may be taken from the machine, differing by a quarter period or 90° from one another. Such a simple two-phase alternator was designed as early as 1887 by Bradley.

A *three-phase* alternator might have been made by connecting the ring to three slip-rings at points 120° apart, giving what is known as a three-phase "mesh" winding. Gramme, indeed, wound some of his rings with three independent sets of coils. Such a machine would give three currents in three separate successive phases. If these were grouped as in Fig. 87*a*, p. 100, we might join up the A coils together into one circuit (the coils being wound or connected alternately right-handedly and left-handedly); the B coils being similarly joined up into a second circuit, and the C coils being joined into a third. It is clear that in each set, the electromotive-forces would rise and fall in regular succession, and that the electromotive-force in B would not rise to its maximum until after that in A had passed its maximum and was falling. As the angular distance around the armature from one north pole to the next north pole corresponds to one whole period (p. 2), or to one complete revolution of 360° on the imaginary circle of reference (Fig. 2, p. 3), we see that these three currents will differ from one another by 60 electrical degrees, their electromotive-force curves being as already given in Fig. 87. But the phase-difference can be converted to one of 120 electrical degrees by the simple device of reversing the connexions of the coils of any one phase. For example, if the ends of the B circuit (including all the B coils joined up as shown), are reversed, the effect will be the same as if all the B coils had been shifted on through one pole-pitch, turning 60° into 240°.

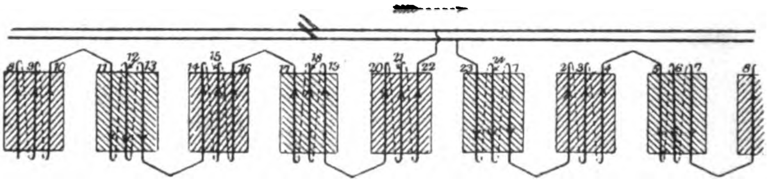

FIG. 136.—ALTERNATE-CURRENT MACHINE: RING-WINDING.

There will then be 120 electrical degrees between the A coils and the C coils, 120 between the C and the B coils, while a third 120 degrees brings the B coils to the A following.

A developed winding diagram of a single-phase ring winding is given in Fig. 136. The two slip-rings are represented by the two parallel lines, the beginning of the winding being connected to one ring, and the end to the other ring, this forming a series winding. It is perfectly possible, but on the whole not advisable (owing to cross currents) to connect up the coils in parallel, or partly in parallel and partly in series, the two free ends being connected to the rings.

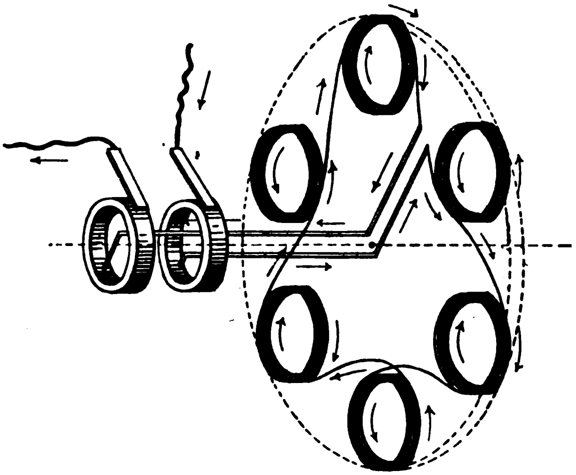

FIG. 137.—DIAGRAM OF SIEMENS DISK-ARMATURE (1878).

Ring-wound machines, having flat rings between two crowns of poles, were at one time designed by Kapp. They are described in earlier editions of this work, and scale drawings were given of a 60 KW machine built at the Oerlikon Works.

Ring-windings for alternators have now gone entirely out of fashion, having been discarded in favour of some of the forms of winding to be presently considered.

Disk-Armatures.—In these machines the armature coils are arranged around the periphery of a thin disk. The field-magnets consist of two crowns of fixed coils, with iron cores arranged so that their free poles are opposite one another. This type was created in 1878 by Herr von Hefner, engineer to Messrs. Siemens and Halske. The connexion of the coils to the slip-rings is shown in Fig. 137.

In the Ferranti alternator, as constructed about 1882, the field-magnet consisted of two crowns of alternate-poles, precisely as in the alternators of Siemens; but the armature consisted of strip copper bent into a wavy star form. There were eight loops in the zigzag (as shown in Fig. 138), and on each side were 16 magnet poles; so that the current flowing radially outward past an N-pole flowed radially inward past an S-pole. The copper strip was wound round on itself (with insulation between) in many layers; the limbs of the star being held in place by insulated bolts passing through star-shaped face-plates. The advantage of the armature of zigzag copper was supposed to lie in its strength and simplicity of construction.

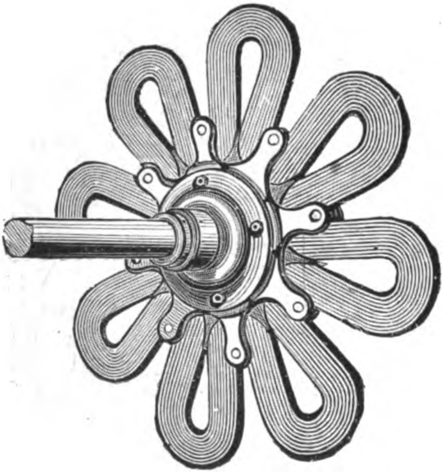

FIG. 138.—FERRANTI ARMATURE (1882).

In the later alternators[1] of Ferranti the zigzag mode of

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[1] See Specification of Patent, No. 3702 of 1883; and for later details, No. 702 of 1887.

winding has been entirely abandoned, and the coils are wound separately and then assembled into a disk. The mode of construction is explained by the figures which follow. Each coil is wound upon a rigid core. The cores are constructed of brass strips spreading fan-wise, with asbestos between, brazed solidly together at one end, and united to a brass piece drilled with an aperture A (Fig. 139). The winding, the inner end of which is soldered to the brass piece, is of ribbon copper slightly corrugated to secure greater rigidity, wound with a tape of thin vulcanized fibre between. The coils are mounted in twos in brass coil-holders, depicted at D, Fig. 141, into which, with interposed layers of mica and fibre, they are secured by bolts which pass through their eyes. The two coils in each holder are separated mechanically and electrically by interposing a piece of fibre of the form shown at H; but the holder constitutes a metallic connection from the eye A of the one to the eye A of the other. Consequently, a current circulating from outside to inside of one coil must circulate from inside to outside of the other. The outside end of each coil is joined to the outside of the nearest coil in the next holder. The holders must of course be insulated, and yet held mechanically and

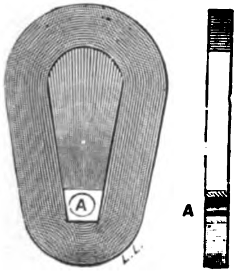


FIG. 139.—SINGLE COIL OF FERRANTI ARMATURE.

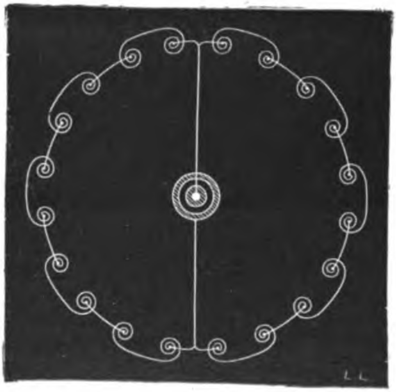


FIG. 140.—CONNEXIONS OF FERRANTI ARMATURE.

firmly. For this purpose they are provided with a tail-piece D′, of circular section, which passes through a porcelain bush E, and is threaded to receive a metal foot which is further secured by a pin passing through D′. The tail-piece, protected by its porcelain bush, passes through the rim of a strong foundation ring, having apertures into which the metal feet are inserted, but which are much wider and longer. The gap between them is then filled up by pouring in a molten compound of sulphur and powdered glass, which secures and

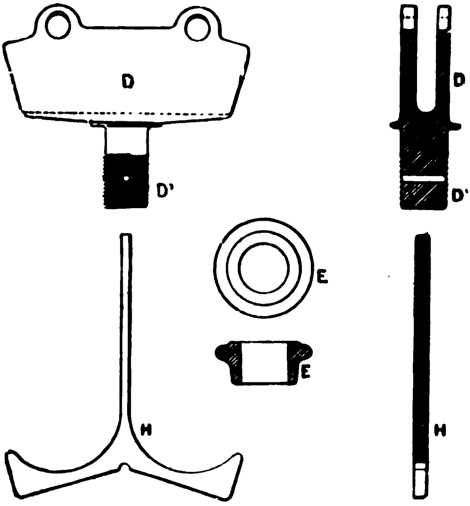


FIG. 141.—DETAILS OF FERRANTI ARMATURE.

insulates them. On the side of each coil-holder projects a small oblique wing, to promote ventilation, as seen in Fig. 142. In all the larger machines the coils are connected up, as shown in Fig. 140, in two series, which are joined together in parallel. This grouping is effected by placing all the coils in one half circumference right-handedly, and in the other left-handedly, and is adopted so as to keep widely apart the coils that differ most in their potentials.

Two copper rods pass inwards from the tail-pieces ot two of the coil-holders at opposite ends of a diameter, and are led to the collecting arrangements which are mounted on the end of the shaft.

Fig. 143 relates to a 225 kilowatt Ferranti alternator, and gives a view of half the armature and half the field-magnet. Here it is seen how the copper connector D^2 passes from the coil-holder D to m_1, a bolt uniting it to the collecting apparatus. The cut also shows how the field-magnet is built in two separate halves, each of which can be racked laterally aside by a lever N and rack M to expose the armature for cleaning or repairs. The speed of this machine is 350 revolutions per minute, and the diameter of the armature 5 feet 6 inches.

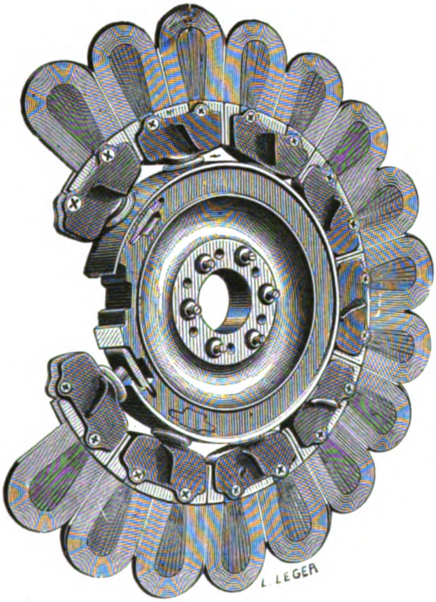


FIG. 142.—FERRANTI ARMATURE, SHOWING MODE OF MOUNTING COIL-HOLDERS.

The low frequency alternators constructed by Mr. Ferranti's firm for the Portsmouth central station and elsewhere, are of entirely different construction, and follow very closely the lines of Brown's Frankfort machine.

In Plate I. is given a drawing of a 1500 KW alternator at Deptford, having crowns of 64 poles each, and running at 156 revolutions per minute. The frequency is therefore 82 cycles per second. It is single-phase, giving 136 amperes at 11,000 volts.

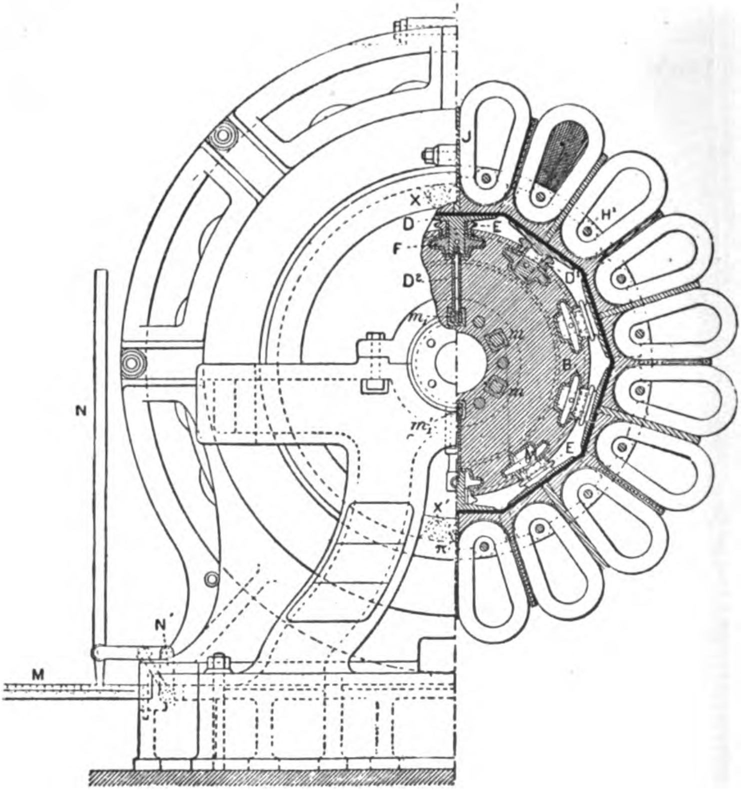
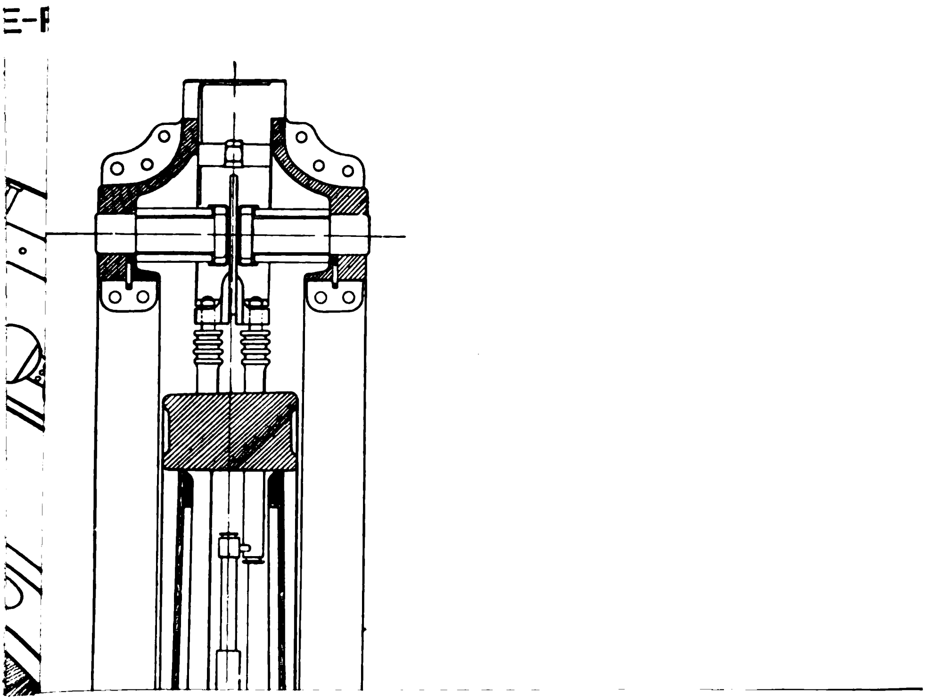


FIG. 143.—FERRANTI ALTERNATOR (225 KILOWATTS). Scale $\frac{1}{20}$.

The Mordey alternator, which has a fixed disk armature, has been described on p. 112 above. In both this machine and that of Ferranti, the employment of two armature circuits in parallel has led to some difficulty in keeping the currents in the two paths equal.

The difficulty is mostly met with in those armatures which have no iron in their cores. Unless the armature coils are

Plate I.



acted on equally and simultaneously all round—a condition very difficult to attain in practice, especially with machines having cast-steel poles—the voltage in the two halves will not be equal, and consequently an internal circulation of current will take place. To avoid this difficulty, Mr. Mordey devised an equalizer consisting of an iron transformer core wound with a single winding whose copper section is great enough to carry comfortably half the current of the machine. The winding of the equalizer is connected between the machine circuits, its middle point constituting one terminal of the machine. If the currents in each winding of the armature are equal, then, as they will flow round the core in opposite directions on their way to the external circuit, there will be no magnetism induced, the two windings neutralizing one another. But any tendency of one portion of the armature to give a larger current than the other portion is resisted by the equalizer which then acts as a transformer, putting an opposing electromotive-force into that circuit which has an excess and adding a corresponding electromotive-force to the circuit of lower voltage.

With alternators having iron cored armatures, the above difficulty is less often met with, because such an armature will in itself act as an equalizer, inequalities of current tending to correct themselves. The true remedy is to have only one armature circuit, or one only for each phase. Or, if for convenience of construction, two or more thin wires are substituted for one thick one, they should always lie side by side in the same channels and should never be coiled around different paths.

Pole-Armatures.—These have been extensively employed for single-phase machines by Messrs. Ganz and Co.[1] on the Continent, and by Messrs. Johnson and Phillips, and Mather and Platt in England. Fig. 144 shows a construction of the former firm, due to M. Zipernowski, in which the field-magnet

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[1] See *Centralblatt für Elektrotechnik*, xii. 544, 1889; also *Electrical Review*, xv. 70, 1884; xvii. 115, 1885; *Electrician*, xxv. 258, 1890; *Electrical World*, xiii. 297, 1889; xvi. 73, 1890; *La Lumière Électrique*, xxxi. 121; and xxxii. 159 and 582, 1889.

cores are made up of U-shaped stampings, and the armature cores of short T-shaped pieces which project through the coils, and are removable singly. The field-magnet being revolving, it is of the B type. At Frankfort, in 1891, a large Ganz alternator was shown by the Helios Co.,[1] of a capacity of 400 kilowatts, giving 200 amperes at 2000 volts at 125 revolutions per minute. The armature consisted of 40 T-shaped punchings like Fig. 144, surrounded with coils each working at 100 volts the whole being coupled up in two series of 20 each. The rotating field-magnet is 299·2 centimetres in diameter, and 38 centimetres wide. The electrical efficiency is given at 95·6, and the nett efficiency at 91·5 per cent. Four very fine examples of the Ganz alternator exist in the central station of Rome,[2] each being of 320 kilowatts capacity, driven direct at 125 revolutions per minute by separate compound engines of 500 horse-power each. They have rotating field-magnets with 40 radiating poles of solid iron, the diameter being over 9 feet. The interior diameter of the armature ring frame is about 9½ feet, the core being built up of sheet iron and paper as described. There are 40 coils, each generating 50 volts, all united in series, and capable of carrying 200 amperes, the wire being 6 millimetres in diameter. The bobbins on which the magnet coils are wound, are made of

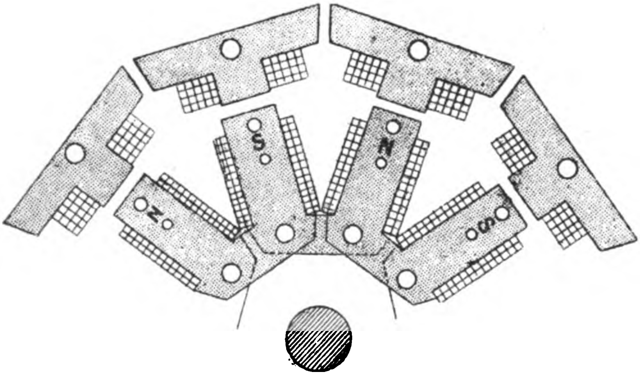

FIG. 144.—GANZ-ZIPERNOWSKI ALTERNATOR.

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[1] See *Electrical Review*, xxix. 503, 1891.

[2] See *Electrician*, xxv. 317, 1890.

split rectangular zinc formers about 15 inches high and 20 inches wide, the windings being more numerous towards the outer end. The armature windings, 30 in each coil, are contained on vulcanized fibre frames 19 inches long, 10 inches wide, and 2 inches deep, and are clamped in place by skeleton bronze frames.

This form was built unchanged till 1897. Since then Messrs. Ganz have adopted both for single-phase and polyphase machines a type described later with a slotted external stator armature, having six, nine or more slots per pole ; and having a magnet-wheel with cylindrical solid steel poles with rectangular pole-pieces. The exciters are mounted on the end of the shaft. Some of these alternators are used for the transmission from Tivoli to Rome. They are three-phase machines rated at 3500 KW, with 24 poles, and run at 210 revolutions per minute. The voltage is 10,000 volts, the diameter of the magnet-wheel 4 metres, and the total weight about 83 tons each. They are direct-driven from water-turbines.

Hopkinson's alternator, which has a pole-armature, is of the A-type. It has fixed external multipolar magnets, with a width of pole-face exceeding three-fourths of the pitch. The armature wires are coiled upon short polar projections of laminated iron having extended faces. The machine is shown in Fig. 145. Its exciter is mounted on a bracket to run on the same shaft.

Owing to the almost complete continuity ot the iron of the magnetic circuit, and the high peripheral speed which the construction of the machine admits of, an exceedingly high efficiency is obtained. The following are particulars of machines of this type constructed by Mather and Platt for the Salford central station.

No. of poles	20
Revolutions per minute	450
Output	40 amperes at 3000 volts.
Resistance of all magnet coils in series . .	7·4 ohms.
Resistance of armature coils in series . .	0·8 ohm.
Exciting current required through magnets at full-load	10 amperes.
Electrical efficiency at full-load . . .	98·4 per cent

In spite of their high self-induction, these machines are suitable for working in parallel, and have so much armature reaction that they can be short-circuited with perfect safety.

To the same type belongs a machine designed by Prof. Kapp for Messrs. Johnson and Phillips. The construction is shown in Fig. 146. The armature core is built up of stampings assembled in removable segmental blocks. Around each block of core in end notches is wound a section of the armature coil. The armature consequently has considerable self-induction. The field-magnet is on the same plan as that

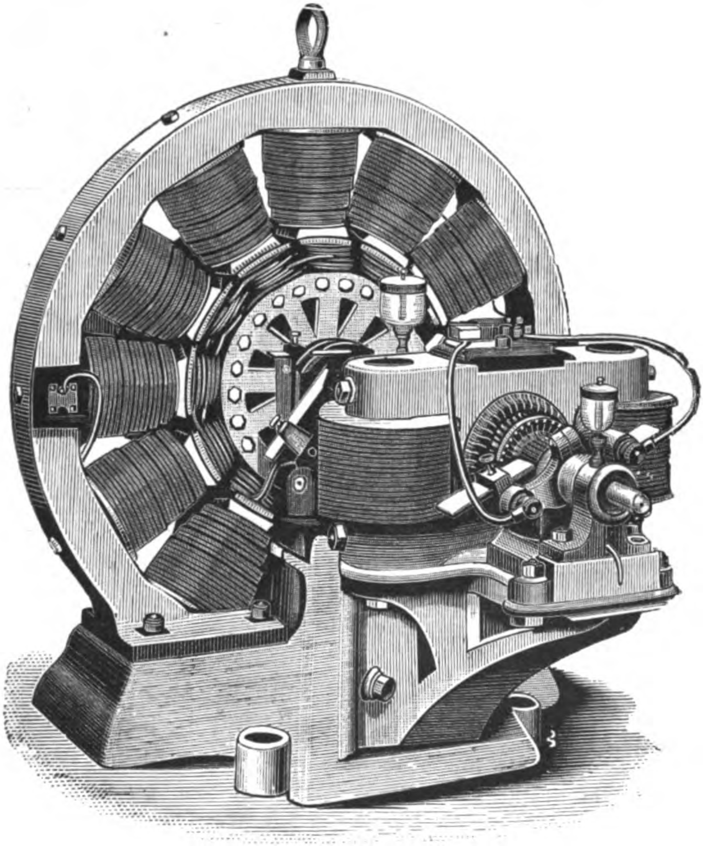

FIG. 145.—HOPKINSON ALTERNATOR (MATHER AND PLATT).

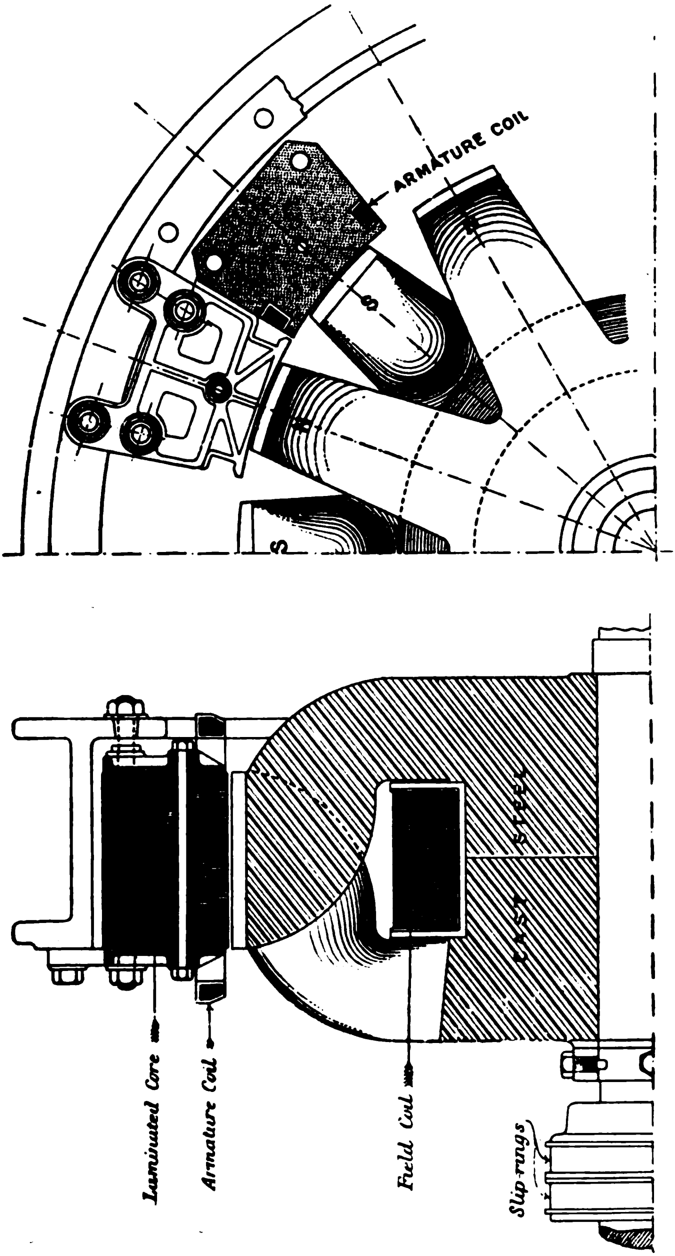

FIG. 146.—KAPP'S ALTERNATOR.

of the Lauffen alternators (see page 117), having but a single coil for excitation.

Plate II. depicts a machine, A S (B) 72 – 660 – 85, of the pole-armature type, by the Helios Company, the pioneer in Germany of the fly-wheel pattern of alternator. The design of this machine, seven of which were built for the central station in St. Petersburg, is clear from the details given in the plate. It gives a normal output of 220 amperes (or 250 at maximum) at a pressure of 3000 volts, running at 85 RPM The exciter is carried on the same shaft. It will be seen that there is one armature coil to each pole, each coil being wound upon a separate block of stampings acting as a single big tooth, and each block mounted with screw-bolts on the external housing in such a way as to be separately removable. The housing is of cast-iron of box-pattern. The diameter of the armature at the face is 235 inches, making the pole-pitch 10·25 inches. The core-length from front to back is 21·7 inches. The gap is 0·4 inch.

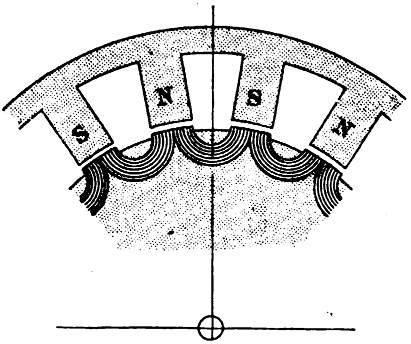

Fig. 147.—Early Form of Westinghouse Alternator.

Drum Armature Alternators.—So far as the active wires are concerned, they may be coupled most effectively as a drum winding. In Fig. 147, which is a diagrammatic picture of the early Westinghouse alternator, the windings lap on the outside of a drum core; the sections being coiled separately on temporary formers are then laid upon the surface of the

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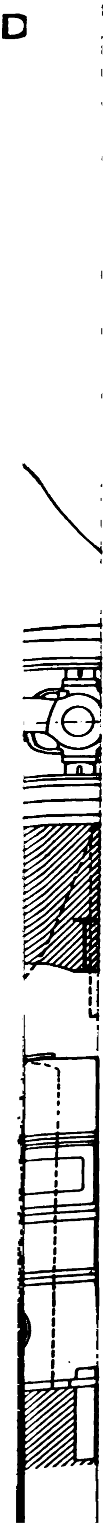

core, with their ends turned down over the end core-disks and firmly secured.

In all recent Westinghouse machines the coils are sunk in slots in the iron of the armature. Large single-phase armatures of the A type are built up of segments, one of which is shown in Fig. 148, composed of thin mild steel stampings bolted together and assembled so as to form a core like the larger cores shown in Fig. 153, p. 171. The joint in the segments being made in the centre of a tooth does not affect the reluctance of the magnetic circuit. The coils, wound on formers, are sprung over the projections of the teeth. Bronze wedges driven in longitudinally make the whole compact.

For two-phase and three-phase drum armatures, the windings are sunk between numerous small teeth in slots. The term *slot* is used to cover not merely open channels but channels that are closed, or partially closed, at the surface, and constituted by holes stamped in the core-disks before assembly. The design of slots, and their relation to the winding-scheme is treated later, see pp. 177 and 295.

FIG. 148.—WESTINGHOUSE CORE SEGMENT.

As is the case with continuous-current drum windings, drum and disk windings may either have their individual coils connected one with another by a *lap-winding*, or by a *wave-winding*. The distinction arises in the following manner. Since the conductors that are passing a north pole generate electromotive-forces in one direction, and those that are passing a south pole generate electromotive-forces in the opposite direction, it is clear that a conductor in one of these groups ought to be connected to one in nearly a corresponding position in the other group, so that the current may flow down one and up the other in agreement with the directions of the electromotive-forces. So after having passed down opposite a north pole face, the conductor may be connected to one that passes up opposite a south pole face, and the winding evidently may be arranged either to lap back, or to zigzag forward.

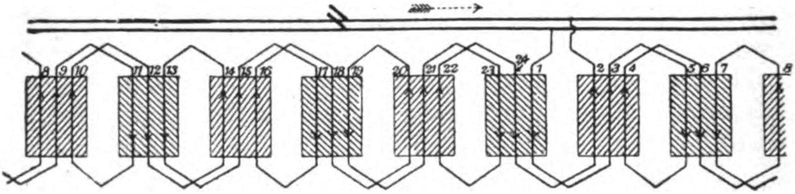

FIG.. 149—ALTERNATE-CURRENT MACHINE: LAP-WINDING.

Wave-windings were independently suggested in 1881, by Lord Kelvin and by Mr. Ferranti. They are excellent for low-voltage machines and for machines that require relatively few conductors per pole. But there are disadvantages in their use for high voltages, owing to the difficulty of maintaining the insulation between each "wave" and the succeeding one.

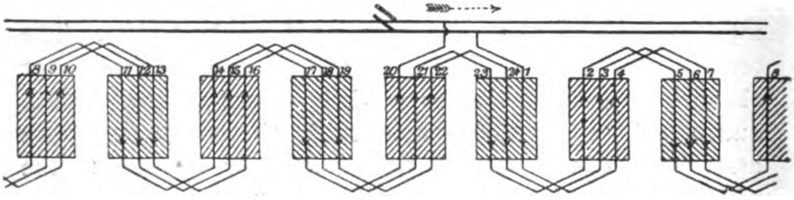

FIG. 150.—ALTERNATE-CURRENT MACHINE: WAVE-WINDING.

As the windings are in series, each lap or group of laps must be connected forward to another, so that the winding becomes progressive. The lap-winding has this feature in common with the wave-winding.

This distinction between lap-windings and wave-windings as applied to alternate-current machines, is illustrated in

Figs. 149 and 150. Fig. 149 represents an 8-pole alternator with lap-winding, each "element" or set of loops extending across the same breadth as the "pitch" or distance from centre to centre of two adjacent poles. Only 24 conductors have been drawn; and it will be noticed that the successive loops are alternately right-handed and left-handed. In Fig. 150 is shown the same alternator with a wave-winding. The electromotive-force of the two machines would be precisely the same; the choice between the two methods of connecting is here purely a question of mechanical convenience in construction, difficulty of insulation, and cost. These two developed diagrams are shown for the same number of active wires.

The drum-type of winding, in its many varieties, is practically the only one now used, ring and disk types having become obsolete.

MODERN ARMATURES (TYPE A).

Except for small sizes, alternators are now seldom built with revolving armatures, except in America. The construction of such machines follows closely that of continuous-current generators, save that in general the poles are more numerous, and that the armatures are furnished with slip-rings, instead of commutators, for collecting the current. The field-magnet poles are usually solid, either cast integrally with the yoke-ring or bolted to it with screws. The armature core consists of toothed segments assembled upon a hub or spider to which the segments are attached, just as in the armatures of continuous-current machines by bolts or dovetails. The coils are either hand-wound or former-wound, and seldom built up of bars, because bar-windings are suitable only for small machines of very low voltage, or for very large machines.

Fig. 151 illustrates a small three-phase alternator constructed by the *Allgemeine Elektricitäts-Gesellschaft*, of Berlin, from the designs of Mr. Dobrowolsky.

It has fixed external radial poles, and a revolving armature with a wave-winding wound in slots. This is the general construction for machines of small output made by this firm, irrespective of the number of phases.

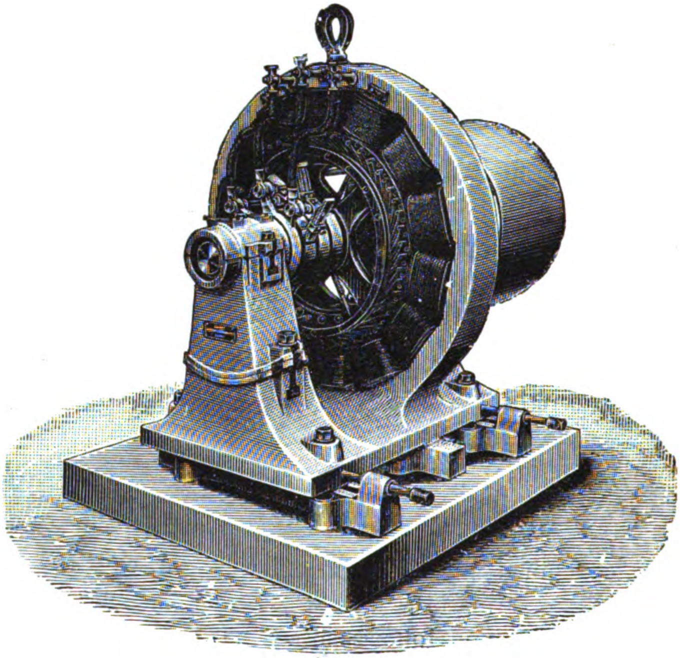

Fig. 151.—Allgemeine Co.'s Three-Phase Alternator (90 KVA).

Figs. 152 and 153 represent a 6-pole three-phase, 160 KVA machine, running at 600 RPM, designed by Mr. Parshall for the Dublin Tramways. The frequency is 50 cycles per second. The armature, which is Y-connected, generates 2500 volts

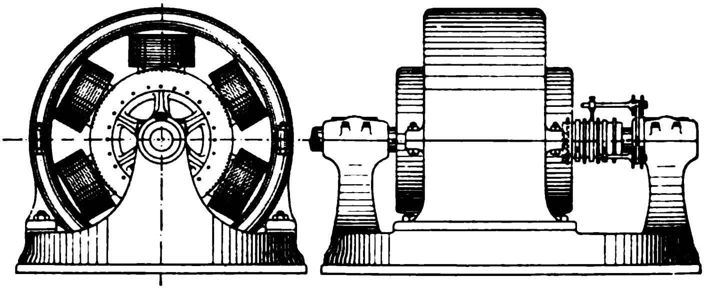

Fig. 152.—AT(A) 6—150—600 (Parshall's Design). (Scale 1 :40.)

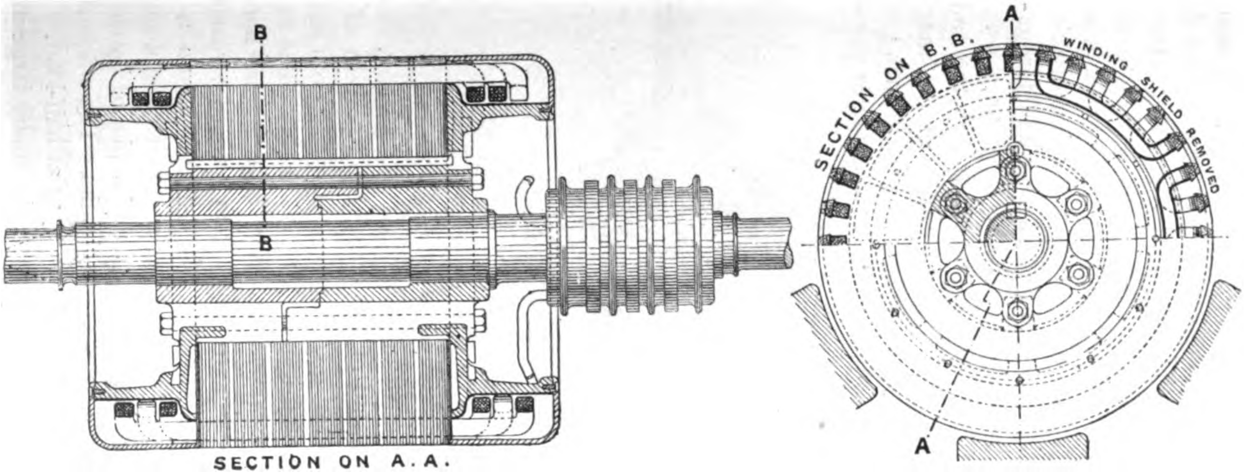

FIG. 153.—ARMATURE OF AT(A) 6—150—600. (Scale 1 : 15.)

between the lines, the pressure in each phase being 1440 volts It has a 2-slot winding (*i.e.* 6 slots per pole), and the end-bends are arranged in three ranges. The armature structure consists of a cast-iron spider in two parts, keyed to the shaft, and drawn together by long bolts. The core-laminations are stamped out as disks from iron 0·014 inch thick. Each disk has a single key-way for alignment. The former-wound coils are secured by wedges in the rectangular slots. Cylindrical projections from the end-plates support the end-bends and coil-connexions. The flux-densities in the machine at full load are: *Teeth*, 96,600 lines per square inch; *Core*, 48,000; *Gap*, 45,000. Current-density 1300 amperes per square inch; temperature-rise at armature surface 30 deg. Centig. The field-magnet frame follows closely the pattern of multipolar continuous-current machines. It is of cast-iron having 6 pole-cores of laminated iron cast into the yoke. The exciting coils are wound on bobbins having a gun-metal flange $\frac{3}{8}$ inch thick, for protecting the coils from armature reaction and to dissipate heat. The yoke is divided across a horizontal diameter. The three slip-rings are built upon a separate hub, and have projecting rings of ebonite placed between them to reduce risk of accidental short-circuits. The efficiency is stated as 92 per cent. at full-load and 89·5 at half-load.

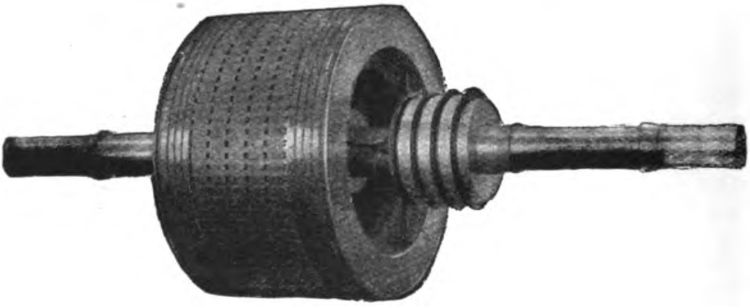

FIG. 154.—THREE-PHASE ARMATURE, COMPLETE, WITH SLIP-RINGS (GENERAL ELECTRIC CO.).

The Westinghouse Co. has built many large machines of this type. Fig. 155 depicts the armature of a two-phase machine of this type. Probably their largest A-type alternators are those at Willesden with 62 poles, and of 1520 KVA output (*see* p. 434).

Fig. 173, p. 189, shows the armature of an 8-pole, 60 KVA three-phase generator by Brown, Boveri and Co. The machine runs at 750 RPM, and the voltage across the terminals of the Y is 2000.

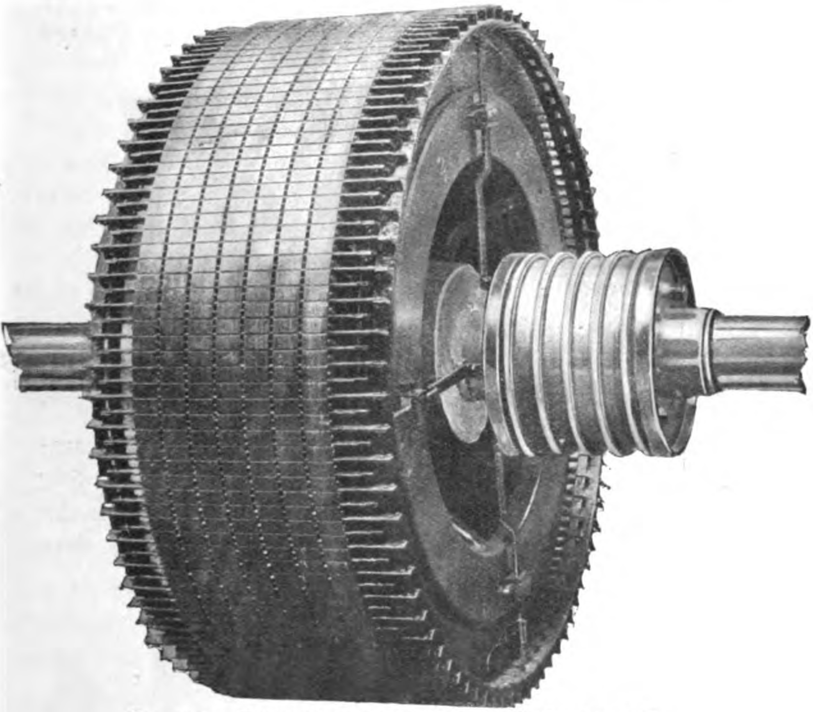

FIG. 155.—REVOLVING ARMATURE AQ(A) 18—500—400.
(BRITISH WESTINGHOUSE CO.)

As an example in mechanical design the machine is excellent.

MODERN ARMATURES (TYPE B).

For stationary external armatures (or *stators*, as they are often called), the almost universal modern construction is a laminated ring core built up of segmental laminations, with slots at its inner periphery to receive the windings, mounted in an external housing to give it stiffness. Windings are treated separately in Chapter VI.

Core-Laminations.—These are seldom stamped out in one piece but are built up of segments which are superposed to break joint. The usual thickness is 15 to 20 mils, and a light insulation is attained by japanning or by pasting on a sheet of manila paper. A large proportion of slow-speed alternators are of a diameter exceeding eight feet, and their assembled armature core is usually from 9 to 16 inches in "length" from front to back. High speed alternators and steam-turbine alternators have armature lengths usually exceeding 16 inches. It is unusual to provide ventilating ducts between the core-laminations in machines that are not more than 10 inches in core-length. The A.E.G. 900 KVA machine, Plate VI., with core-length about 16 inches, has no duct; the Siemens and Halske 850 KVA, Plate V., with core-length 7½ inches, has 1 duct; the Oerlikon 290 KVA, high-speed alternator, Fig. 127, with core-length 15·5 inches, has 4 ducts; the Parsons Turbo-alternator, 3500 KVA, Plate XII., with core-length 30 inches, has 26 ducts.

FIG. 156.—CORE-SEGMENT OF 64-POLE ALTERNATOR (HEYLAND).

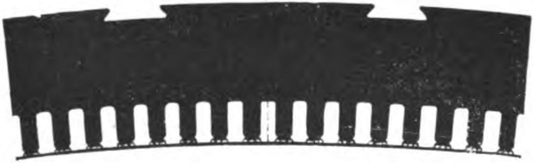

FIG. 157.—CORE-SEGMENT OF 96-POLE ALTERNATOR (KOLBEN AND CO.).

Core-Segments.—These are usually stamped of a size that will subtend at least twice the pole-pitch, and in assembling are arranged so that the segments of one layer break joint with those of the previous layer, so as to procure magnetic continuity in the whole core. To hold them together, and to the housing, it is usual to stamp them with bolt holes at the outer periphery, as far away as possible from the field-poles, so that the bolts need not be insulated. In very large machines bolts are sometimes inserted also nearer to the teeth, and these ought to be insulated. Fig. 156 illustrates a typical kind of core-segment, belonging to a 64-pole 900 KVA alternator designed by Heyland, having 6 slots per pole. The dotted line shows how the face of the stampings is bored out after they are assembled. With this may be compared the segment, Fig. 157, of the 96-pole 3000 KVA alternator of Kolben and Company in the Metropolitan Company's station at Willesden. It has dove-tail attachments and half-closed slots. Fig. 158 is an Oerlikon Company's pattern with dove-tail attachment cut away to afford ventilating space between the core and the housing. It has three slots per pole.

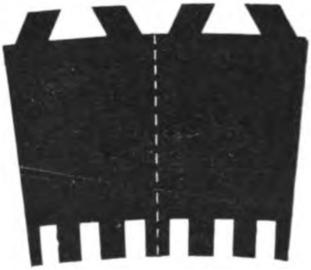
FIG. 158.

Fig. 159 is a core segment of the 46 pole, 900 KW, 2-phase alternator of Brown, Boveri and Company, at Chèvres (see Plate IX.), and has 4 closed slots per pole. Fig. 160 is that of the 28 pole, 1850 KVA, three-phase machines at Paderno (see Plate VII.), also of Brown, Boveri and Company, and has one circular slot per phase per pole. Fig. 115, p. 128, is a cut of the core-segment of the small inductor alternator of Kolben and Company described on p. 127; it has open slots with wedges. Fig. 161 depicts a core-segment of the first type of Niagara generators (umbrella-type with stationary inner armature), built by the Westinghouse Company. Those of the second type supplied in 1896 were very similar. This core-segment should be compared with those of the newest Niagara machines in Plates X. and XI.

Some other forms of core-segment will be found illustrated in the plates and cuts.

It is usual for the core-segments to be insulated lightly with a coat of japan varnish, or now more usually a sheet of thin manila paper pasted on one face. It is a good practice in assembling the core-segments when sufficient have been assembled to make about one inch of thickness, to insert an extra-thick layer of tough varnished paper. In assembling the core it is usual to lay down the housing, or in the case of large machines one half the housing, horizontally upon the floor of the shop, and to assemble the segments in their places in the housing, using wooden rods, or in some cases the actual bolts, passed through the bolt-holes, to keep the successive layers in place. In this assembling the ventilating ducts are preserved in their proper places by the due insertion of ventilating stops or distance-pieces. These are generally made by riveting to a core-segment of extra thickness, ribs of

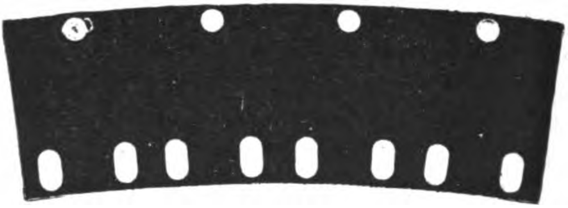

FIG. 159.—CORE-SEGMENT OF TWO-PHASE ALTERNATOR AT CHÈVRES (BROWN, BOVERI AND CO.).

FIG. 160.—CORE-SEGMENT OF THREE-PHASE ALTERNATOR AT PADERNO.

brass about ½ to ¾ inch deep; or sometimes small T-angle iron is used. Plate XVII. shows such riveted ventilating ribs in the left side of the transverse section.

To hold the segments in the housing they are generally bolted between a fixed rim which projects inward from the casting, and a segmental rim at the other face. The tightening bolts usually pass through holes close to the outer periphery of the laminations: the bolts being kept as far as possible from the inner periphery to avoid the generation of parasitic currents in them. Sometimes external dove-tails are provided to the core-segments to hold them in grooves in the housing.

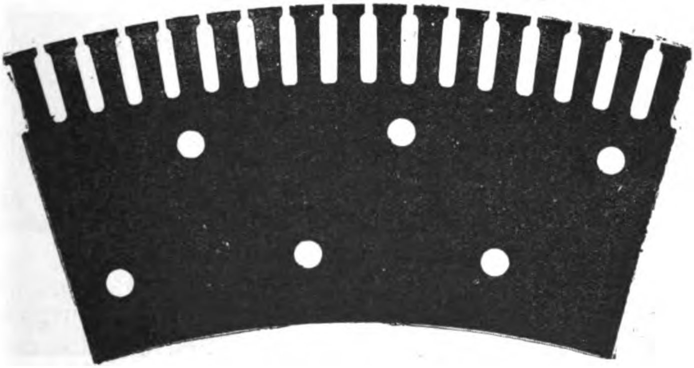

FIG. 161.—CORE-SEGMENT OF NIAGARA ALTERNATORS (1892 PATTERN).

Slots.—The forms used for slots vary from a simple rectangle to a circle. They may be classified as (1) open, (2) closed, (3) half-closed. Open slots are those with straight rectangular sides: they lend themselves to the use of former-wound coils, and are illustrated in Fig. 158 and in Plates V. and XI., also in Fig. 162 *a* and *b*. Entirely closed slots are either circular or of the oblong form with rounded ends shown in Fig. 162 *f*. They can be used either for hand-wound coils, or for bar-windings with bars of appropriate section. Half-closed slots are shown in Fig. 162 *c*, *d* and *e*. They have the advantage over closed slots that the self-induction of the armature winding is reduced. But their use necessitates hand-winding. Recently they have been adapted to former-wound coils in the following way. The coil wound cn the former block is lightly tied together instead of being taped over. A lining is placed in the slots, and then the coil is laid over it and untied, and the wires are then fed in one by one through the slit. This being done the insulation is

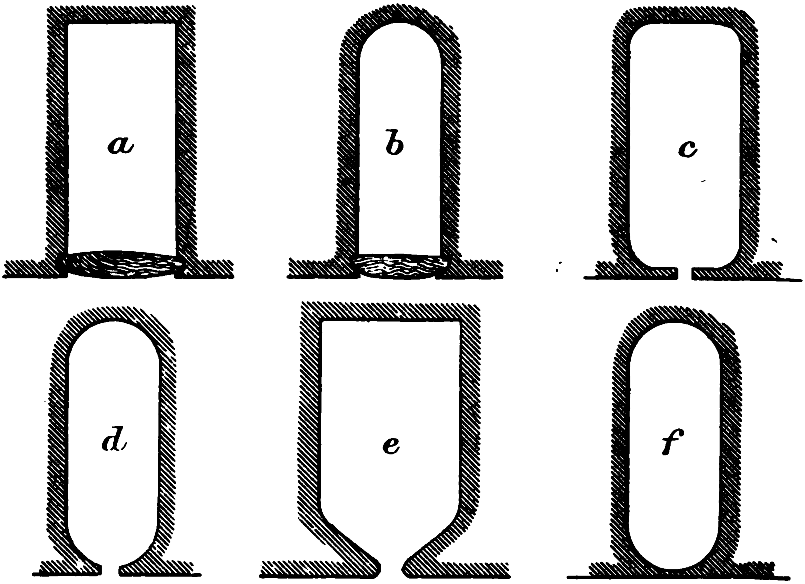

FIG. 162.—TYPICAL FORMS OF SLOTS.

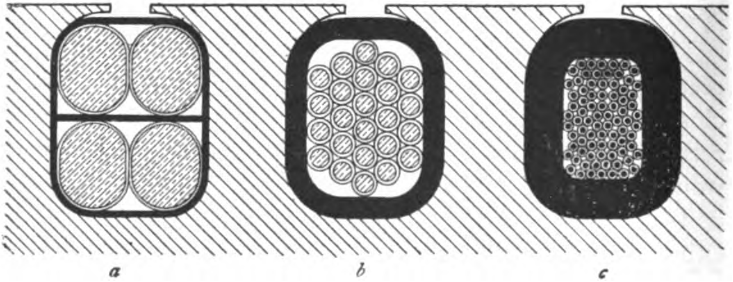

FIG. 163.—SLOT-LININGS FOR DIFFERENT VOLTAGES (KANDO).

folded down, the slit closed by a wooden wedge, and the projecting end-bends of the coil are taped. Slot linings must, of course, be chosen with regard to the voltage of the machine. Tubes of micanite or of press-spahn are usually used for hand-windings. Fig. 163 illustrates in full size three different linings for a slot, *a* being for 500 volts, *b* for 3000 volts, *c* for 10,000 volts. For questions of insulation of slots the reader is referred to the Chapter on *Insulation* in Volume I.

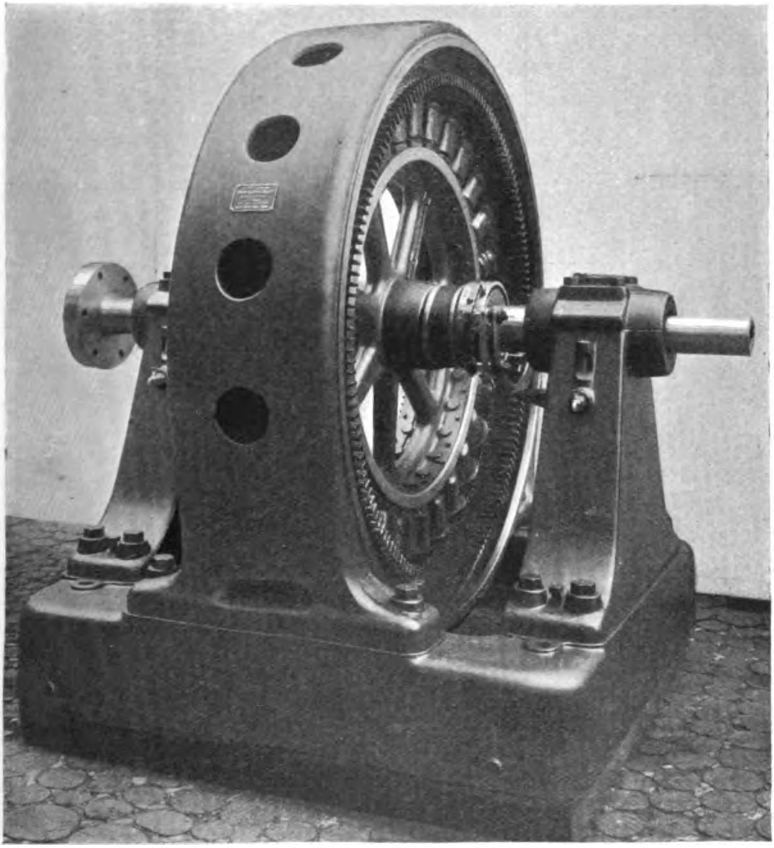

FIG. 164.—75 KVA ALTERNATOR, B-TYPE, THREE-PHASE, 215 RPM (BROWN, BOVERI AND CO.).

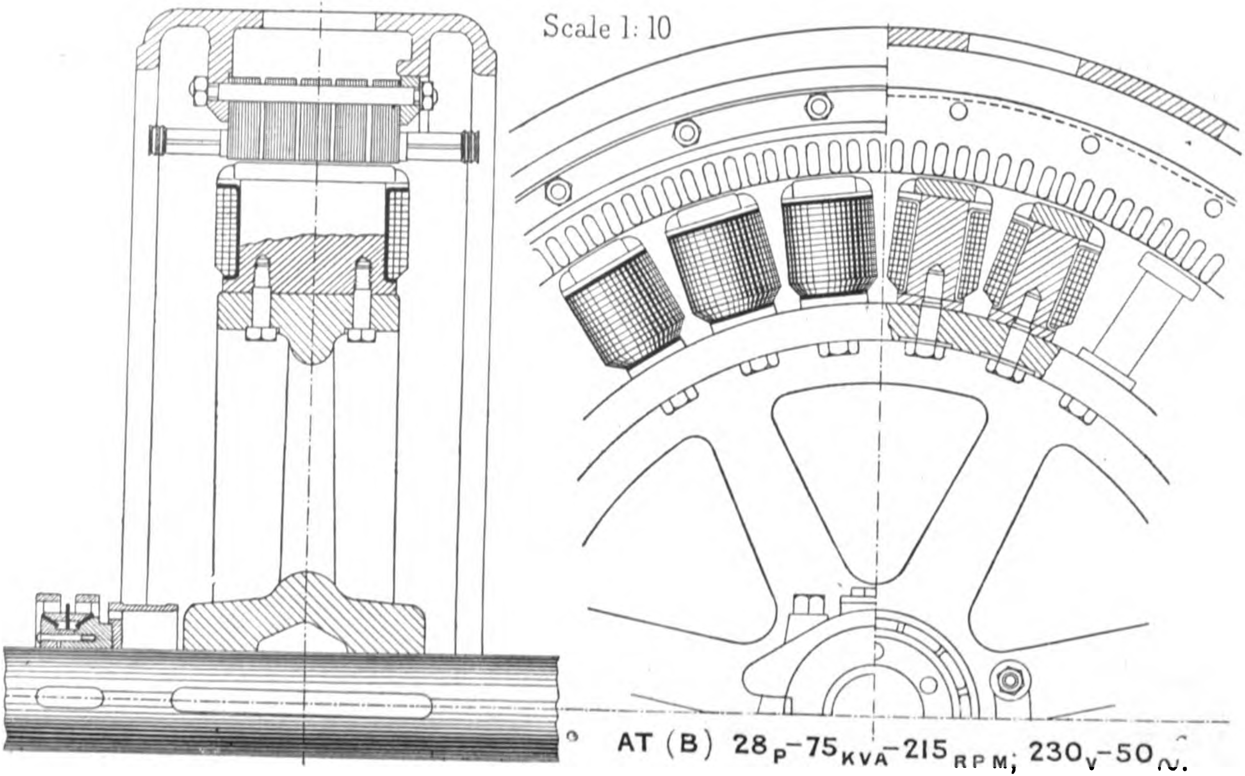

FIG. 165.—SMALL THREE-PHASE GENERATOR BY BROWN, BOVERI AND CO.

Housings.—The housings of alternator armatures are to provide not only a means of clamping together the core-segments, but also a mounting which is sufficiently stiff to prevent the mechanical bendings and distortions which arise owing to the weight of the parts, the attractions of the poles, and the statical torque which is equal and opposite to the torque of the magnet wheel. For small machines, up to 100 KVA, the body of the housing is not infrequently cast in one piece, as a short cylinder with an internal flange against which the core-plates are assembled, and two external feet to be bolted to the bed-plate. A good example is afforded by Fig. 164, which depicts a 75 KVA machine supplied to the St. Petersburg Polytechnicum by Brown, Boveri and Co. A longitudinal section is given in Fig. 165. The armature housings of small alternators, and the stator housings of motors, are frequently built in three pieces—a central flanged cylinder and two flanged convex end plates, which also support the bearings. In some cases the housing is in two parts only which fit together with a telescope joint and are held firmly together and to the core-laminations by bolts. Comparatively seldom the housing of small alternators is divided horizontally.

The housings of large machines require to be designed with particular reference to the provision of an adequately rigid frame-work. Several types may be distinguished : the box-type, the cheek-type, the segmental wheel-type, the tie-rod type, and so forth. The box-type consists of a hollow structure cast in halves or quarters, and bolted together, having internal webs and ribs to stiffen it against bending. Examples of simple box housings are given in Plate VIII. of an 800 KVA machine having 236 inches diameter of bore designed by Heyland ; in Plates II. and V., which are slow engine-driven machines of 660 KVA and 850 KVA capacities respectively, the bore of the former being 235 inches, and of the latter 205 inches ; in Fig. 338, p. 416, of a Witting-Eborall machine of 625 KVA, and with a bore of 177 inches ; and in Fig. 333, p. 397, of a 1500 KVA machine by the Oerlikon Company.

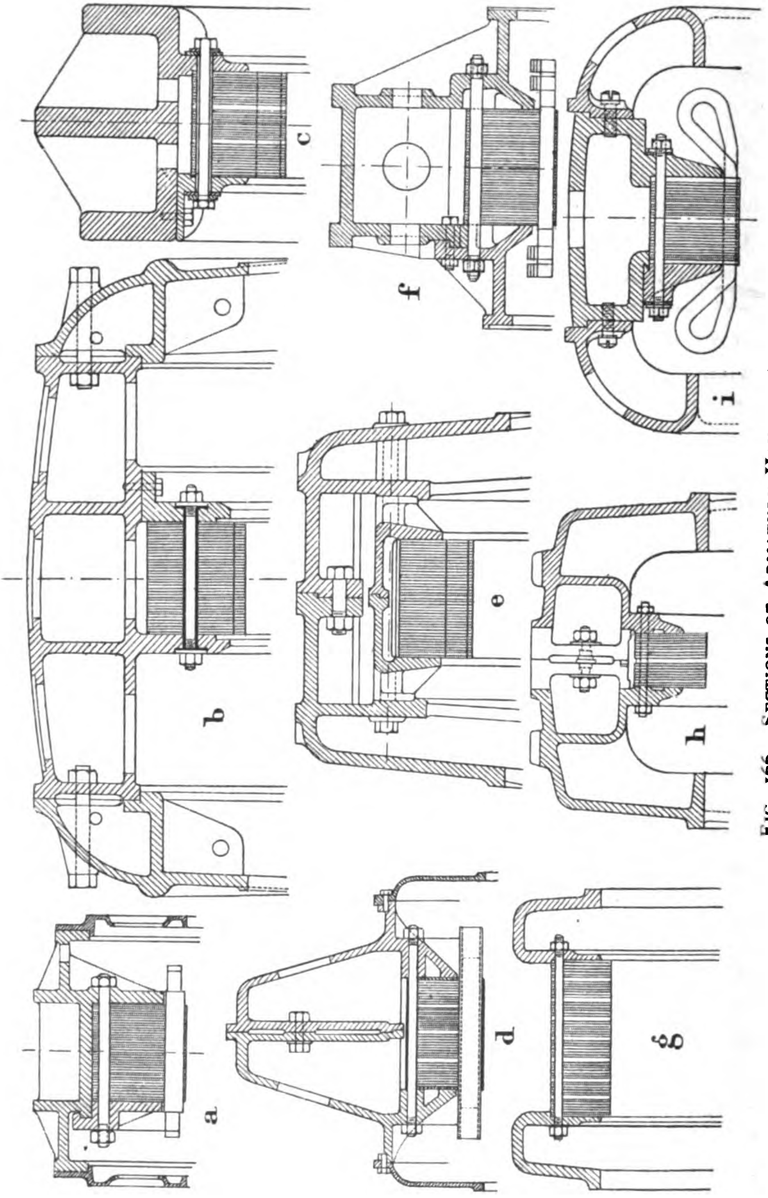

FIG. 166.—SECTIONS OF ARMATURE HOUSINGS.

Fig. 167 shows a more complicated box casting of the Westinghouse Company, used in their Manhattan machines of 5000 KW. and 336 inches bore. In some later machines, only two stiffening ribs were used.

Another example of a box housing is given in *d*, Fig. 166, of a 400 KVA machine by the Karlsruhe Company, the bore being 210 inches. In *c*, we have an example of a Grammont machine of 600 KVA and 150 inches in diameter, in which the top of the box is left out, giving a form much easier for casting. A similar example of a quite small Oerlikon machine of 215 KVA rating is given in *a*, Fig. 166.

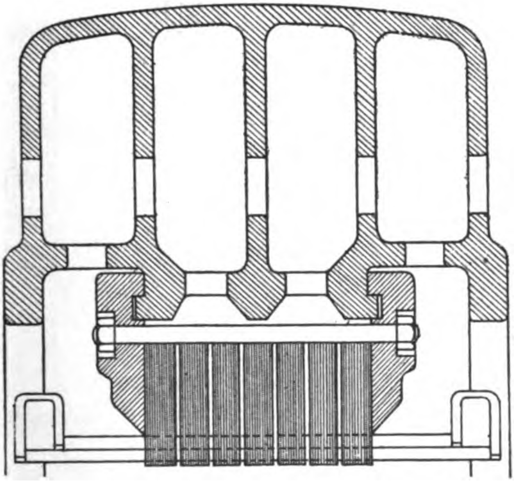

FIG. 167.

An Oerlikon Company's box pattern shown at *f*, is simple to cast. It belongs to a 200 KVA machine, 98 inches diameter. To obviate the great weight of box-castings, the Allgemeine Company has for its machines of moderate size adopted the cheek-type illustrated in *g*, in which all external housing is abandoned except two cheeks bolted laterally to the laminated core-ring. It dispenses with the elevated middle part which was found not to add much to the stiffness. The construction is excellent, particularly for the housings of motors, as it affords good cooling for the laminations.

A larger construction on this same plan, where all the strength of the housing is in massive side cheeks, is shown in Fig. 168, being of a 2000 KVA machine, exhibited at the Paris Exhibition of 1900 by Siemens and Halske.

For large machines, however, a still greater stiffness is required. Brown introduced for the Frankfort machines (Fig. 166, *e*), and later for the larger alternators at Paderno (Plate VII.) the plan of building the armature housing as a hollow segmental wheel with spokes and hollow hubs which turn as on trunnions surrounding the bearings of the shaft. This form holds the core-ring rigid, and at the same time permits it to be barred round for inspection or cleaning. Other types of this construction, but with removable end-arms, are shown in *b*, which is of a 1000 KVA machine by Lahmeyer, exhibited in Paris, with a box diameter of about 230 inches; in *h*, again of a 1000 KVA machine designed by Rothert for the International Company of Liège; and in *i*, which is of a large power motor by the Bullock Company of Cincinnati. For large machines exceeding 100 inches in bore, the tendency of even rigid structures to bend necessitates special considera-

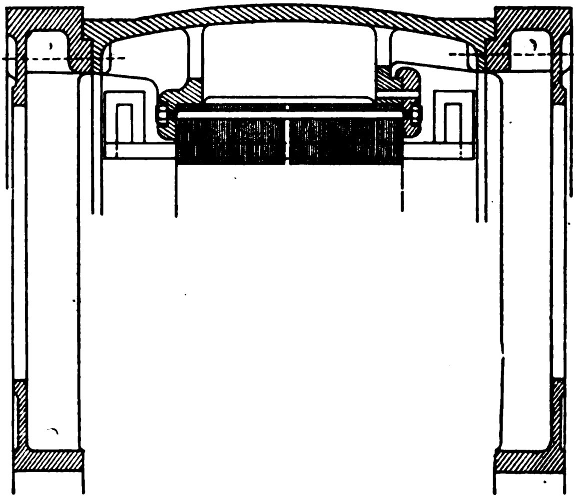

FIG. 168.

tion; and means must be provided to adjust them not only during erection but from time to time. The mechanical considerations as to bending have been studied by Lasche,[1]

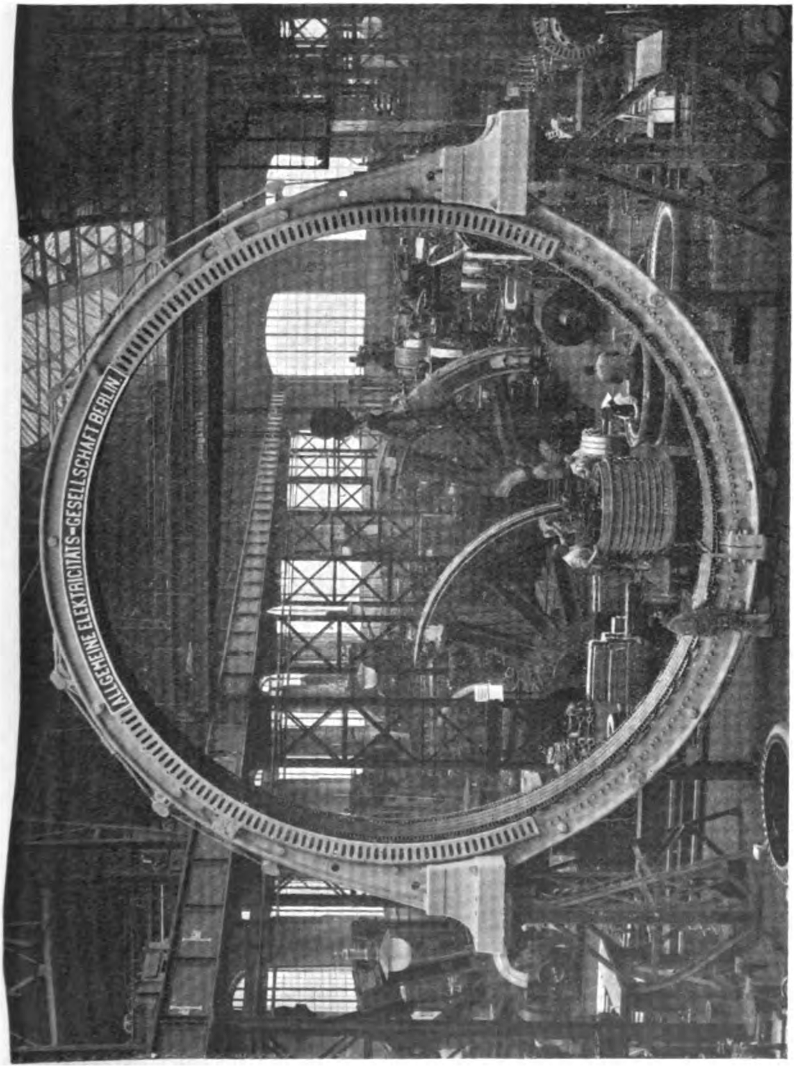

FIG. 169.—STATIONARY ARMATURE OF AT(B) 80—3750—75, CONSTRUCTED BY THE ALLGEMEINE ELEKTRICITÄTS GESELLSCHAFT.

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[1] *Zeitschrift des Vereins Deutscher Ingenieure*, 1901, Nos. 28 and 29.

Schenck,[1] Linsemann,[2] and Niethammer.[3] These stationary armatures are usually furnished with two feet, as in Plates VII. and XVIII., which are from 2 to 3 feet below the level of the centre, and which stand upon levelled bed-blocks upon the foundations, the lower part of the housing descending between the bed-blocks into a pit. Adjusting screws permit of accurate levelling and alignment. This suffices for housings not exceeding 70 or 80 inches in diameter. For those up to 200 inches, it was found expedient to provide as additional support a third foot at the bottom of the pit, as in Plates III., V. and VI. For larger sizes up to 400 inches in diameter two extra feet, or in some cases four extra feet, were added, supporting the lower half of the ring on solid projections in the pit. But the very weight of the upper half causes it to bend; and if it is not perfectly centred around the magnet-wheel, the magnetic pull, being no longer balanced, will tend to further distortion. To prevent the upper half from bending downwards, a pair of horizontal tie-rods were suggested to brace the structure. Then Lasche devised the plan of constructing the entire framework as a system of tie-rods. A fine example is seen in the 3750 KVA alternators at the Manchester Corporation Electricity works (Fig. 169), in which the diameter of the bore is 156 inches. Lasche has designed several

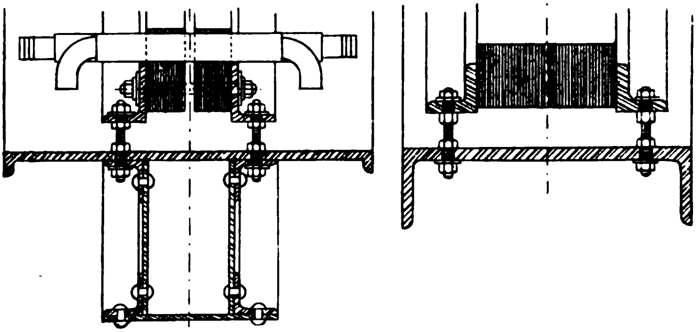

FIG. 170.—HOUSINGS OF WROUGHT IRON.

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[1] *Zeitschrift für Elektrotechnik*, xxii. 167, 1904.

[2] *Elektrotechnische Zeitschrift*, xxiii. 81 and 103, 1902.

[3] *Zeitschrift für Elektrotechnik*, xxii. 367, 1904.

other tie-rod types. The one objection to them is that the tie-rods do not heat equally with the core-ring, but expand unequally.

Another plan of attaining stiffness, without the enormous weight of the castings (sometimes 6 times as great as that of the core-ring), is due to Messrs. Siemens and Halske,[1] who build up the housing of wrought iron or steel rings of T, H or L section, bolted together. Fig. 170 shows two of these patterns.

SLIP-RINGS.

Collecting-rings, as required for revolving armatures, must be designed to meet the following points: (1) number of phases; (2) adequate section for carrying current to be collected; (3) insulation for the voltage prescribed; (4) proper mounting on the shaft; (5) proper connexion with the

FIG. 171.—SPIDER-TYPE MOUNTING OF THREE-PHASE SLIP-RINGS (WESTINGHOUSE CO.).

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[1] British Patent Specifications 8748 and 16837 of 1902, etc.

revolving windings; (6) sufficient surface for brush contacts.

A common mode of construction for low-voltage cases is to mount the copper rings, with suitable insulation between and beneath them, upon a sleeve which is fixed on the shaft, and which is provided with a flange at one end and a clamping-ring at the other. For high voltages more elaborate arrangements are necessary. Fig. 171 illustrates a pattern adopted by the Westinghouse Co. Each slip-ring, which is of cast copper of T-section, is furnished with three internal lugs by means of which it is mounted, with insulating bushes, upon short bolts which project from a light cast-iron spider which is bolted up to the armature spider. In this case the insulation between the rings is air. Fig. 172 shows the corresponding form for a single-phase pair of rings, which also might serve as the collector for bringing the magnetizing current to a revolving magnet-wheel in a machine of the B-type.

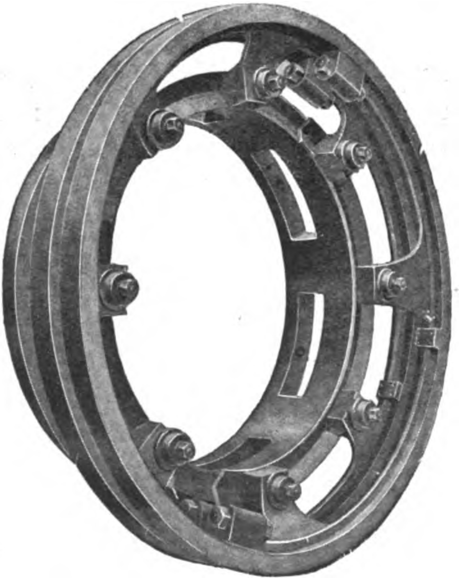

FIG. 172.—SLIP-RINGS FOR SINGLE-PHASE ARMATURE, OR FOR MAGNET-WHEEL (WESTINGHOUSE CO.).

Fig. 173 depicts a recent armature of Brown, Boveri and Co., in which the slip-rings are designed with conical seating

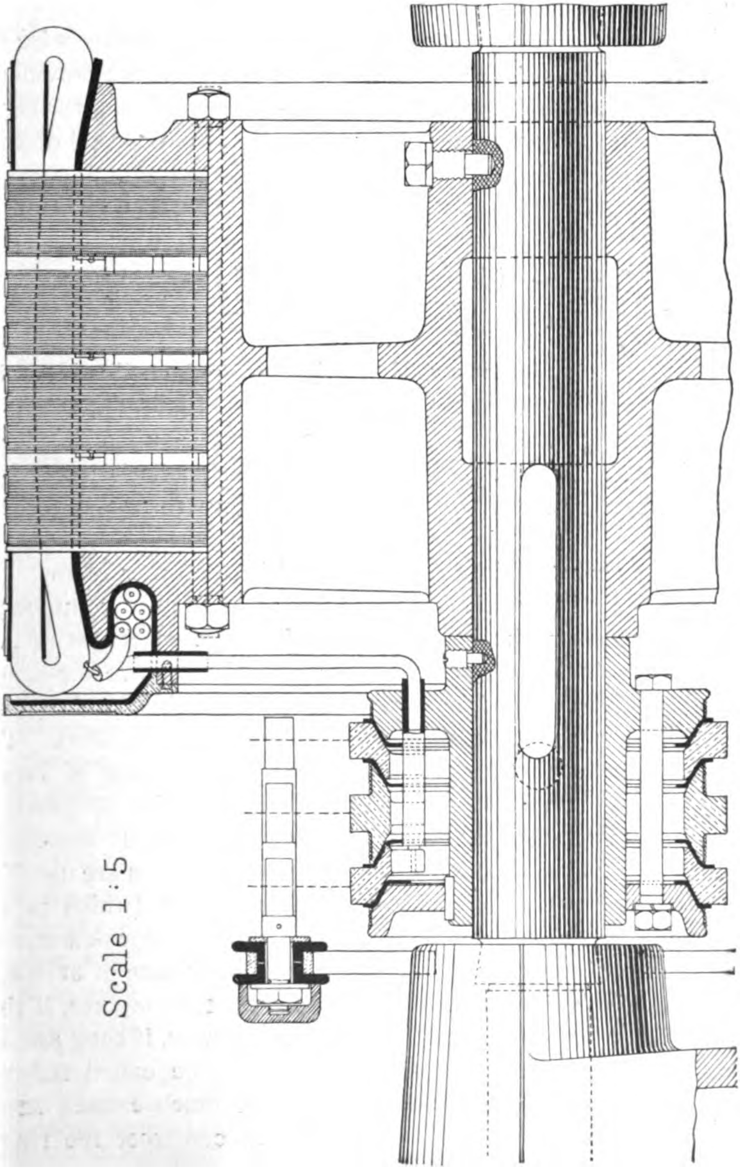

Fig. 173.—Three-Phase Armature (2000 volt), showing Slip-Rings (Brown, Boveri and Co.) at(a) 8—60—750; 2000v; 3 × 17·5a; 50~.

faces, and are pressed together by three bolts between clamping flanges, thus becoming self-centering. Conical washers of micanite between the rings provide insulation. From each ring a lug projects inwardly to which the connecting wire is affixed. Three brush-pillars of different lengths are provided. They are supported, each in a well-insulated bush, through holes in a fixed cast-iron ring mounted on the pedestal of the bearing. In this machine carbon brushes are used.

Some manufacturers provide projecting ridges of insulation between the rings, as shown in Fig. 92, p. 106; Fig. 153, p. 171; and Fig. 155, p. 173. The object of this is to prevent short-circuiting if by chance any metallic object falls upon the rings.

A very simple construction, suitable for the collecting-pair for a revolving magnet-wheel, is shown in Plate VI., wherein the two rings are held together and to their supporting hub by bolts. Connexion to the magnetizing coils is made by conductors sweated to washers that clamp to one or other ring.

Other constructions may be found in the other Plates.

In small machines the brush-pillars are commonly bracketed from the bearings; but in large machines of A-type, and in rotatory converters, where six or more slip-rings are sometimes found, it is more usual to provide special supports rising from the bed-plate to carry the brush-pillars. To lessen the chance of accidents, and in the case of large currents to subdivide the load, it is good practice to provide more than one brush to make contact against each slip-ring. Carbon brushes are often used, but copper brushes are usually preferred, for the higher resistance of carbon (which is of advantage in the case of commutation for continuous-currents) is here a disadvantage. A brush contact surface of at least 1 square inch should be provided for every 100 amperes, if the brushes are of copper, or for every 40 amperes, if they are of carbon. The section of the rings should be calculated so that the mean current-density should not much exceed 1200 amperes per square inch. Some makers construct the rings with an outer copper tyre shrunk on, so as to be replaceable after wear.

In rotatory converters where very large currents have to be dealt with, special construction is necessary, and the bearing surface must be very liberal. The slip-rings on the Westinghouse converter depicted in Plate XV., well illustrate this point.

ALTERNATORS OF EXCEPTIONAL CHARACTER.

We may conclude this descriptive chapter with a brief notice of some exceptional sorts of alternators, namely :—

(i.) *Asynchronous Generators.*
(ii.) *Extra-high Frequency Machines.*
(iii.) *Constant-Power Machines.*

Asynchronous generators are machines in which the rotating magnet-wheel with its definite poles is replaced by a *rotor* having closed circuits : they are in general style identical with asynchronous induction motors having short-circuited rotors (squirrel-cages, etc.), for these machines which, when running as motors, run at a speed slightly below synchronism, act as generators when the speed is raised above that of synchronism. Such machines are not self-exciting ; there must be an alternating or polyphase current already supplied to the mains to which the stationary armature is connected. The possible advantage of such machines[1] is, that in those central stations that are liable to a very sudden increase of load, one or more such machines might be kept turning as a non-loaded motor at a speed just below synchronism until required ; when on merely quickening up the speed of the engine it will begin to generate, thus saving any of the delays that usually occur in the process of synchronizing before switching in.

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[1] See Danielson in *Electrical World*, xxi., 44, 1893 ; *Electrical Review*, xxxii. 169. Also Leblanc, *Éclairage Électrique*, xxi. 328, 1899 ; and Gratzmuller in *L'Industrie Électrique*, Oct. 10, 1902, p. 437. Prof. Clarence Feldmann has published a small monograph *Die Asynchrone Generatoren.*

Extra high frequency machines have several times been proposed for special purposes. In 1902 Tesla proposed for his high-frequency experiments some forms of machines, having very numerous small poles, which would generate currents up to a frequency of 15,000 cycles per second. More recently the Westinghouse Co. has constructed, from the designs of Mr. Lamme, an experimental machine[1] for a frequency of 10,000 cycles per second. This was of the inductor type, having the armature core built of steel ribbon about 2 inches wide and 3 mils thick. It had 200 polar projections with a pole-pitch of only 0·25 inch, and a peripheral speed of 25,000 feet per minute. The armature has 400 slots with one wire per slot, and a bore of about 25 inches, The gap was only 0·03125 inch. The iron losses were small and proportional to the speed. The windage loss was considerably larger and was proportional to the cube of the speed. On constant excitation the voltage fell from 150 volts at no-load to 123 volts with an output of 8 amperes.

Constant-power machines are wanted only for electrochemical or electric heating work, their object being to furnish a nearly constant output irrespective of any variation of either volts or amperes. It is not possible to attain this completely; but it is possible to design a machine such that, as the amperes rise the volts shall fall, and *vice-versâ.* In brief, all that is needed is to exaggerate those armature-reactions which tend to produce a voltage drop. The General Electric Company of Schenectady constructed some such machines on the designs of Mr. Steinmetz for the manufacture of calcium carbide in Willson's electric furnaces. Mr. Steinmetz has furnished the following particulars of one of these machines AS (B) 12—150—600. At 60 volts it yielded 2500 amperes; but when the load was decreased to 1670 amperes the volts rose to 90, the excitation being kept constant. The stationary armature had internal diameter 40 inches, and a core-length of 11 inches, including four ventilating ducts, each

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[1] Lamme, 'Data and Tests on a 10,000 Cycle-per-second Alternator,' *American I.E.E.*, May 1904. See also paper by Duddell before the Physical Society, reported in *Electrician*, liv. 436, Dec. 30, 1904.

½ inch wide. There are 12 groups of slots, each group consisting of four slots 1 inch wide, 2·437 inches deep, separated by teeth 0·75 inch wide. The armature turns in series are 16 only. The gap is 0·1875 inch. The 12 magnet-poles are 3·75 inches broad by 11 inches long at the face, the laminated cores being 10 inches long. When working at 60 volts the effective armature ampere-turns per pole are 3330, while the field ampere-turns per pole are 5100. The core-loss on open circuit was found to be 7·45 KW, falling to 2·45 KW when working at 60 volts. The efficiency was 92·8 per cent. at 60 volts, and 92·3 at 90 volts. The temperature rise after ten hours at 60 volts was 39° C. in the armature copper, and in other parts under 24° C.

CHAPTER IV.

ON INDUCED ELECTROMOTIVE-FORCE AND WAVE-FORM OF ALTERNATORS.

ELECTROMOTIVE-FORCE OF ALTERNATORS.

LET N be the total flux of any one pole of an alternator having p poles all of equal strength. Let Z be the number of "conductors" in series in any one of its circuits, and let it make n revolutions per second. The frequency f will be $\frac{1}{2} p \times n$ cycles per second. On the assumption that the form of the distribution of the pole-flux is such that the density of the magnetic field varies from point to point along the periphery as a sine- (or cosine-) function, the flux enclosed by any pair of conductors constituting a simple loop will vary from $+$ N to $-$ N according to the rule

$$N_t = \cos \theta \times N ;$$

where N_t stands for the flux at the instant t (seconds) when the coil has moved through an angle θ (*in electrical degrees*) from the position of enclosing maximum N. Here $\theta = 2 \pi n t \times \frac{1}{2} p = 2 \pi f t.$ The width of the loop is here assumed to be equal to the pole-pitch $\tau = \pi d \div p.$

Hence, differentiating, the rate of cutting the magnetic lines will give as the electromotive-force induced in that one loop, at the instant t,

$$- \frac{dN}{dt} = 2 \pi f N \times \sin \theta \text{ (\textit{lines per second}).}$$

Let Z be the total number of conductors in series in any circuit of the armature. Then as there are $\frac{1}{2}$ Z such loops all in series, all at the same instant passing through similar fields,[1] the total electromotive-force in that circuit, at that instant, will be

$$\pi f Z N \times \sin \theta \text{ (\textit{lines per second})}$$

or

$$E_t = \pi f Z N \times \sin \theta \div 10^8 \text{ \textit{volts}.}$$

The maximum value of this will occur when $\theta = 90°$; or

$$E_{max} = \pi f Z N \div 10^8 \text{ \textit{volts}.}$$

And as we have seen (p. 13) that the *virtual* value of the volts is equal to the maximum value divided by $\sqrt{2}$, or multiplied by $\frac{1}{2}\sqrt{2}$, we have:—

$$E_{virt} = \tfrac{1}{2}\sqrt{2} \times \pi f Z N \div 10^8 \text{ \textit{volts}}$$

or, finally

$$E_{virt} = 2 \cdot 22 \times f \times Z \times N \div 10^8 \text{ \textit{volts}.} \quad . \quad [I]$$

which is often taken as the fundamental formula for alternators. But it was deduced on the assumptions that the distribution of the magnetic flux followed a sine-law, and that the whole of the loops of active conductor in the armature circuit concerned acted simultaneously—in other words, that the winding was not distributed over more than one slot per pole. (This is equivalent also to the assumption that the electromotive-force curve of the alternator is a pure sine-curve.) But the coils are often more or less distributed; they do not always subtend an exact pole-pitch; and the distribution of the flux, which depends on the shaping and breadth of the poles, is often quite different from a sine-distribution. Hence we find in practice that the coefficient which we have deduced as 2·22 is often departed from; and to generalize the expression we will write the formula as:—

$$E_{virt} = k \times f \times Z \times N \div 10^8 \text{ \textit{volts}.} \qquad [Ia]$$

where k is a number which may have different values, usually lying between 2·3 and 1·5 according to the type of construction of the alternator. This number k, the significance of

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[1] This corresponds to the case of single-phase pole-armatures (p. 161), and slotted armatures having only one slot per pole per phase.

which was first pointed out by Prof. Gisbert Kapp, is often called the Kapp coefficient. We saw on p. 11 that the quadratic mean value of a sine-function is greater than the arithmetical mean by 1·1, or more accurately 1·11 times. The number 2·22 deduced above is really made up of two factors 1·11 and 2, the former being the "form-factor," the latter a mere number arising out of the circumstance that each conductor cuts the whole flux N twice in each period.

The Kapp Coefficient.—We have just seen that if the loops of armature winding are equal in span to the pole-pitch and undistributed, and if the field-distribution is sinusoidal, the coefficient $k = 2·22$. But if the windings are spread over a definite breadth on the surface, or if they are distributed in several slots per pole, then the value of k will in general be different from 2·22. In fact there enters in also a "breadth-coefficient" or "winding-factor," which will reduce the value.

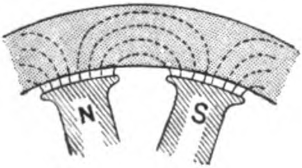

FIG. 174.

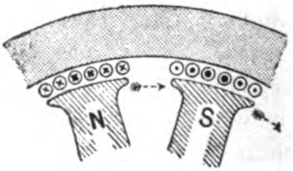

FIG. 175.

Let us consider more closely the effect of breadth of the windings in the coils of the armature. Consider a multipolar revolving field-magnet, such as Fig. 174, in which we will assume that the pole-pieces have been so shaped that the magnetic field in the gap-space between poles and armature cores is distributed in a manner so as to give a regular and smooth wave-form for the curve of electromotive-force induced in any one conductor placed in the gap. We will represent electromotive-forces which act upwards, or towards the reader, by a dot, and those which act downwards, or from the reader, by a cross placed in the section of the conductor. Then it is obvious that there will be induced electromotive-forces acting upwards in those conductors in front of which the S-pole is moving to the right, and downwards in those which the N-pole is passing. But these electromotive-forces will not be equal at the same instant amongst themselves: they will be greatest in those conductors which are most active, that is to say, in those which are passing through the strongest magnetic field. Each conductor will go through an equal cycle of inductive action, but it is clear that they come to their respective maxima one after the other. For convenience we will suppose this maximum to occur in each conductor as the middle of the pole passes it. Now suppose (as is usual in construction) that a number of these conductors are connected up, as in Fig. 176, to form a coil; their electromotive-forces will be added together. If a view is taken, as in Fig. 176, where we are supposed to be looking back at the poles passing from right to left, we shall understand this a little more plainly A moment later the N-pole will come right behind the coil as in Fig. 177. This figure shows that if we wish to use the conductors to the best possible advantage there can be no benefit in having the inner windings of the coil much nearer together than the breadth of the pole-face,[1] since at this instant their electromotive-forces are opposing one another. But the actual electromotive-force generated by a coil of a given number of turns would be greater if they could be all of the same size, so that all should reach their maximum action at the same instant.

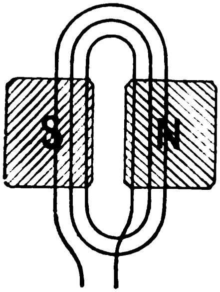

FIG. 176.

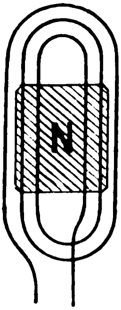

FIG. 177.

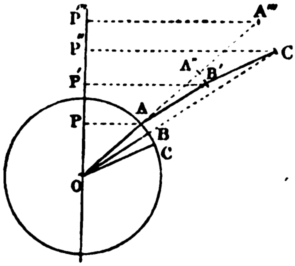

FIG. 178.

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[1] Sometimes, however, this is done for the purpose of obtaining a good wave-form (see p. 233) or for other reasons such as convenience in end-connexions.

This point may be further elucidated by the use of a clock diagram. Suppose the maximum electromotive-force generated in one conductor to be represented by the pointer O A in Fig. 178. Then the projection O A upon the vertical line O P gives the value of the electromotive-force at the instant when the angle A O P corresponds to the phase of the induction that is going on in the period. Let there be two other conductors situated a little further along so that these electromotive-forces would be represented separately by O B and O C. We have to find what the effect will be of joining them all in series. By the rules for compounding vector quantities, we shall find their resultant by drawing from A the line A B′ equal and parallel to O B, and from B′ the line B′ C′ equal and parallel to O C. Then O C′ is the resultant; and its projection O P″ upon the vertical line gives the instantaneous value of the united electromotive-force of the three conductors. Had they all been placed close up to one another at A without any difference of phase between them, the resultant would have been O A‴, and this projected upon the vertical line gives O P‴ as the instantaneous value.

A numerical way of considering the matter may be useful. Suppose each conductor to generate an electromotive-force, the virtual value of which is 1 volt; then if three such conductors are connected up in series their total electromotive-force cannot be three volts, unless they lie so close together that they all receive their maximum values at the same time. Any spreading out of the coils must lower the value of the resultant electromotive-force. The three vectors representing the three electromotive-forces of the three conductors in Fig. 178 form a polygon with three equal sides and a long base. Had we taken a more highly distributed winding, with, say, seven conductors, we should have had a polygon with seven short sides and a long base. In the limiting case we should have a curved arc with a chord for its base. This shows that for a highly distributive winding forming a belt of conductors subtending β electrical degrees, the resultant electromotive-force will be to the sum of the separate electromotive-forces as the chord of the arc they subtend is to that arc.

Based on the above considerations, and on those of breadth and pitch of magnet pole-pieces, Prof. Kapp has calculated different values of k for many different arrangements and types of armature winding. He assumes the flux-density uniform over the pole-face, and from the areas overlapped by the coils in different successive positions arrives at an approximate set of values of the electromotive-forces, of which he then finds the quadratic mean by squaring, adding, and taking the square root. The quadratic mean divided by the arithmetical mean and doubled gives the corresponding value of k. Some of these values relating to the more usual constructions are given in Tables I. and II., p. 200.

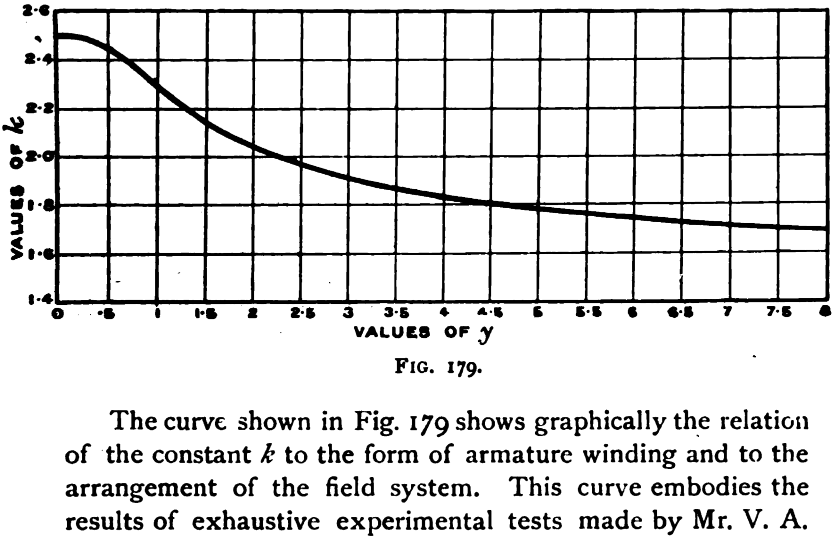

FIG. 179.

The curve shown in Fig. 179 shows graphically the relation of the constant k to the form of armature winding and to the arrangement of the field system. This curve embodies the results of exhaustive experimental tests made by Mr. V. A. Fynn on alternators and induction motors of different designs, and is of the greatest value, insomuch as no assumptions are made as to the distribution of the coils and flux (upon which Prof. Kapp's calculations are based), but the actual value of k for various commercial arrangements has been found by testing the machine. The abscissæ of the curve are simply the mean values of (1) that fraction of the pole-pitch embraced by the armature coil ; and (2) the fraction of the space between the poles taken up by one pole. We write then

$$y = \frac{y_1 + y_2}{2}$$

where

$$y_1 = \frac{\text{Portion of pole pitch embraced by coil per phase}}{\text{Portion of pole pitch not embraced by coil per phase}}$$

and

$$y_2 = \frac{\text{Width of pole at top}}{\text{Pitch minus width of pole at top}}.$$

TABLE I.—VALUES OF THE ELECTROMOTIVE-FORCE COEFFICIENT k FOR SINGLE-PHASE POLE-ARMATURES, AS FIG. 234.

Type of alternator	$\frac{\text{Breadth of tooth}}{\text{Pitch of poles}}$		$\frac{2}{3}$	$\frac{1}{2}$
Inner- and outer-pole types	$\frac{\text{Breadth of pole}}{\text{Pitch of poles}} =$	$\frac{1}{2}$	2·30	2·00
		$\frac{2}{3}$	2·10	1·73

Table I. relates to single-phase *mono-tooth* armatures.

For smooth-core armatures with the windings more or less spread upon the periphery, Table II. may be used.

TABLE II.—VALUES OF THE ELECTROMOTIVE-FORCE COEFFICIENT k FOR SURFACE-WOUND SINGLE-PHASE DRUM ARMATURES, AS FIG. 147.

Type of alternator.	$\frac{\text{Breadth of coil}}{\text{Pitch of poles}}$		0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1
Inner- and outer-pole types	$\frac{\text{Breadth of pole}}{\text{Pole pitch}} =$	$\frac{1}{2}$	2·83	2·50	2·32	2·12	1·64
		$\frac{2}{3}$	2·46	2·23	2·13	1·94	1·49
Inductor type	$\frac{\text{Breadth of pole}}{\text{Pole pitch}} =$	$\frac{4}{5}$	2·23	1·96	1 82	1·66	1·29
		1	2·00	1·76	1·64	1·49	1·16

For slotted armatures having more than one slot per phase per pole, sometimes described as "polytooth" armatures, or as having a multicoil winding, y_2 is the same as before, but

$$y_1 = \frac{\text{Number of wound holes or slots per pole per phase}}{\text{Number of unwound holes or slots per pole per phase}}.$$

In applying this formula to machines of the inductor type, *twice* the true pitch is to be substituted for the pole-pitch in the formulæ above—in other words, the distance from pole to pole on one side (N to N or S to S), is to be taken. An example will make this clear. The inductor machine, drawn out in Plate IV., has poles of breadth 18·5 centimetres, and the pitch of two adjacent poles on one side (= 2 P) is 46 centimetres. The winding of the armature is unicoil, the span of coil (= pitch of slots) being 23 centimetres. Writing τ for pole-pitch, and b for breadth of pole, we have:—

$$2\tau = 46; \; b = 18 \cdot 5; \; b = 23.$$

Consequently

$$y_1 = \frac{23}{(46 - 23)} = 1,$$

$$y_2 = \frac{18 \cdot 5}{27 \cdot 5} = 0 \cdot 672,$$

and therefore

$$y \doteq \frac{1 \cdot 672}{2} = 0 \cdot 836.$$

Referring to the curve, it will be seen that this corresponds to

$$k = 2 \cdot 36.$$

The value of k found from the table is 2·23, the error of the latter figure being, however, on the right side.

Table III. below gives the results according to Kapp.

TABLE III.—VALUES OF THE ELECTROMOTIVE-FORCE COEFFICIENT *k* FOR HOLE AND SLOTTED ARMATURES, AS FIGS. 241 to 249.

Type of Alternator.	Nature of Winding.	Six holes or slots per coil.		Four holes or slots per coil.		Two holes or slots per coil.	As Fig. 252	As Fig. 278
	Number of Phases.	One and two.	One and three.	One and two.	One and three.	One, two and three.	One.	Three.
	$\frac{\text{Breadth of coil}}{\text{Pitch of poles.}}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{4}$	$\frac{1}{6}$	0	1	$\frac{2}{3}$
Inductor type	$\frac{\text{Pole breadth}}{\text{Pole pitch}} = 1$	1·68	1·80	1·72	1·82	2·00	1·16	1·49
	$\frac{\text{Pole breadth}}{\text{Pole pitch}} = \frac{4}{5}$	1·98	2·10	2·02	2·12	2·23	1·29	1·66
Inner- and outer-pole types	$\frac{\text{Pole breadth}}{\text{Pole pitch}} = \frac{2}{3}$	2·16	2·26	2·18	2·3	2·46	1·49	1·94
	$\frac{\text{Pole breadth}}{\text{Pole pitch}} = \frac{1}{2}$	2·36	2·54	2·45	2·59	2·83	1·64	2·12

The formula given above for the electromotive-force induced in one circuit of the windings of an alternate current machine, namely,

$$E_1 = k \times f \times N \times Z_1 \div 10^8,$$

is not directly applicable to the case of inductor machines—that is, in such cases, N does not signify the flux entering the armature from one pole, but half this amount, for the reasons already given (page 118). So that we have for machines A-type or B-type,

$$N = N_1 - N_0.$$

For inductor-type alternators

$$N = \frac{N_1 - N_0}{2},$$

where N_1 is the total flux produced by the magnetizing ampere-turns, and N_0 is the flux lost by dispersion.

With inductor machines, if there is much magnetic dispersion the induced electromotive-force may be still less than that found by calculation as above, because a portion of the leakage flux may actually cut some of the armature conductors, inducing electromotive-forces in the opposite direction to those induced by the main field, and hence reducing the latter by a corresponding amount. This effect, if present, will naturally be greatest at full load, and hence the importance of reducing the dispersion in inductor machines cannot be over-estimated.

For two-phase machines the above formula gives us of course the virtual induced volts per phase, Z_1 being the number of conductors in one phase. Unless the phases are connected in star (page 39), this pressure will also correspond to the voltage on the collector rings or terminals at no load. With three-phase machines the formula also gives the E.M.F. *per phase*, but the line voltage is either equal to this or 1·73 times as large, according as the three circuits in the armature are connected in delta or star fashion (page 39).

In Kapp's calculations the pole-faces are assumed to be approximately flat, and the gap to have parallel faces, so that over the face (save for fringing at the edges), the density of the flux is uniform. In cases where the poles have been shaped away so as to yield a distribution of magnetism at the armature face such as to follow approximately a sine-law, a breadth-coefficient may be calculated in the following manner.

Consider an external armature such as that of the machine illustrated in Plate VIII., and described on page 407, suitable for a three-phase generator, having two slots per phase per pole, that is, having six slots per pole, with a total flux of N lines per pole distributed in an approximately sinusoidal manner. The flux through the coil, shown in Fig. 180, is at its maximum when the pole is directly opposite it. The state of affairs then may be represented by a line B B′, whose radial ordinates represent the flux-density at each point along the stator. The flux-density is zero at the points indicated by O and O′, the distance between which corresponds with a difference of phase of 180°. The points C and C′ which indicate the centre of the holes of the coil in question are at a phase distance of 15°, and the part of the flux intercepted between C and O is useless. The average useful flux-density will therefore be obtained by integrating the sine-function $\text{B}_{\text{max}} \sin \theta$ between the limits of 15° and 165° and dividing by the whole pitch; or

$$\text{average useful B} = \frac{1}{\pi} \int_{\frac{1}{12}\pi}^{\frac{11}{12}\pi} \text{B}_{\text{max}} \sin \theta \, d\theta.$$

This is equal to $\frac{1 \cdot 932}{\pi} \text{B}_{\text{max}}$ instead of $\frac{2}{\pi} \text{B}_{\text{max}}$, the average value over the whole pole area. Thus the electromotive-force is only 0·966 what it would be if the coil had no breadth. The breadth-coefficients corresponding to similar cases with different numbers of slots are given in the table below.

TABLE IV.—BREADTH COEFFICIENTS[1] FOR BURIED CONDUCTORS WHEN THE DISTRIBUTION OF THE FIELD IS SINUSOIDAL.

Number of Slots per pole.	Three-phase.	Two-phase.	Single-phase.
1	..	..	1·000
2	..	1·000	0·707
3	1·000	..	0·666
4	..	0·924	0·653
5	..	..	0·647
6	0·966	0·911	0·643
7	..	..	0·642
8	..	0·906	0·641
9	0·960	..	0·640
10	..	0·903	0·639
12	0·958	0·903	0·638
∞	0·953	0·900	0·637

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[1] See A. Blondel in *Industrie Électrique*, Feb. 25, 1896; *L'Eclairage Électrique*, May 1895; also in his treatise *La Traction Électrique sur Voies Ferrées*, ii. 291. See also the Author's treatise on *Polyphase Currents* (1896), 47, and R. P. Brousson, in *Electrical World*, xxvi. 236, 1895. For an extremely elaborate discussion see Arnold and La Cour *Beitrag zur Vorausberechnung und Untersuchung von Ein- und Mehrphasenstromgeneratoren*. Stuttgart 1901.

To use these breadth-coefficients they should be multiplied by $\frac{\pi}{2} \times \sqrt{2}$ (*i.e.* by 2·22) if the field-distribution follows a sine-curve.

Arnold and La Cour[1] have calculated similar tables of coefficients by a more laborious method than that of Kapp, and find in general somewhat lower values, differing by small amounts up to as much as 8 or 10 per cent. For example, for the case of a three-phase two-slot winding (*i.e.* with six slots per pole), Kapp gives for the case where the pole-span is equal to $\frac{2}{3}$ of the pole pitch the value 2·26, while Arnold finds 2·21. The fringing of the field at the pole-edges (neglected in Kapp's method) tends to cause the coefficient to approximate to 2·22. Grassi,[2] who has adopted a simplified method for taking fringing into account, finds numbers which agree in general with those of Arnold. For a purely distributive winding (three-phase) Grassi finds a coefficient 1·90 while Kapp gives 1·94, if the pole-span is $\frac{2}{3}$ of the pole-pitch. This rises to 2·09 (or according to Kapp 2·12) if the pole-span is narrowed to $\frac{1}{2}$ of the pole-pitch.

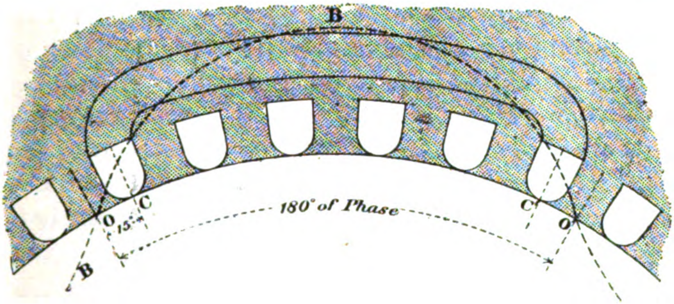

FIG. 180a.—SHOWING METHOD OF CALCULATION OF BREADTH COEFFICIENTS OF BURIED WINDINGS.

Amid so much complication it is not surprising that some designers ignore detail and employ in ordinary calculations simply the number 2·22. If the wave-form is known to be

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[1] Op. cit. See also Arnold *Die Wechselstromtechnik*, vol. iii., 1904.

[2] *Corso di Elettrotechnica*, i. 71.

more peaked (as is usually the case with monotooth armatures), 2·5 to 2·8 may be more nearly true. For three-phase windings and a decently-shaped pole, the following figures are generally useful. One-slot winding (*i.e.* 1 slot per phase per pole) 2·3, two-slot winding 2·2, three-slot winding 2·18. For inductor machines the values run lower, and may be taken about 10 per cent. lower.

ESTIMATION OF DISTRIBUTION OF MAGNETIC FIELD AND WAVE-FORM OF THE ELECTROMOTIVE-FORCE.

The precise form of the wave of the induced electromotive-force in an alternator depends on two conditions: (1) the distribution of the magnetic field, as governed by the shape of the poles and the width of the gap ; (2) upon the arrangement of the armature coils in one, two, three or more slots per pole, and on the shape of these slots; (3) in a secondary degree, upon the distortion of the field by the current in the slots ; and therefore is to this extent affected by the amount and lag of the armature current.

FIG. 181.

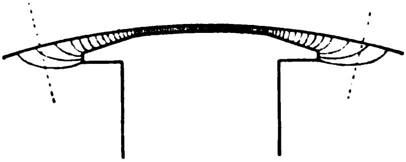

FIG. 182.

There are several ways of arriving at an approximate estimate of the distribution of the magnetic field. Two forms of pole are those depicted in Figs. 181 and 182 ; the former with a gap of uniform width, the latter with the pole face slightly bevelled toward the tips. In the former the field (if undistorted by the presence of wide slots or of armature currents) will be practically uniform all along the gap, but fringing round in the space beyond. In the second case, the field will diminish in density as the gap widens. In the drawings no account has been taken of the effect of the presence of the adjacent

S-poles, which would weaken the fringe of the field and bring it to zero at the neutral line (shown dotted) halfway between the adjacent poles.

If it may be assumed that the whole of the face of a pole is at one definite magnetic potential, and that the whole of the opposing face of the armature is at another definite magnetic potential, then, since the drop of magnetic potential between one face and the other face is alike at all points, it follows that the density of the flux at various points along the gap will be inversely proportional to the reluctance along the respective magnetic paths from iron to iron, and these reluctances will be proportional to the lengths of the several paths. But these paths are curved, at least in the fringe. If, then, we can estimate the lengths of these paths we can deduce the densities at the various points.

There are three ways of making such estimates. A fairly accurate one is to cut out from stout sheet-iron a pair of templates to represent to scale the pole and the opposing armature core ; and, placing them beneath a sheet of cardboard, with a magnetizing coil wrapped round the pole core, find by sprinkling iron filings on the card the exact forms taken by the lines of the field ; then measure the lengths of the curves for a regular series of points equidistant along the armature face. Another method is first to draw the pole and armature face to scale, then set off along the armature face a number of equidistant points, and then, guided by experience and judgment, draw for each point a line, curved or straight as may be judged right, to the pole, to represent the path of the flux. The lengths of such lines are then measured off for use in the calculation. A third method, less accurate, but more rapid, is to draw the outlines to scale and set out a number of equidistant points along the armature face as before, and then mark out another set of equidistant points along the profile of the pole, but, on turning the pole corner, to continue setting out the points not along the side of the pole but along an oblique line arbitrarily chosen at some steep angle—for example, running up to a point on the yoke midway between the roots of two adjacent pole cores. These two sets of points are similarly numbered, and then joined two and two by lines across the gap, the lengths of these last-drawn straight lines being taken as proportional to the several reluctances.

In Fig. 183 the curves in the fringe are quarter-circle arcs, centred around point 3. In Fig. 184 the lines are drawn according to the third method. It is convenient to take 9 points (corresponding to the 90° in one quarter-period) from the zero at the neutral point between poles, to the point opposite the middle of a pole. Two or three points should be taken backward beyond the zero. The length of the connecting lines may be measured off from the drawings in any units —millimetres are very convenient. The numbers should then be tabulated and their reciprocals found. Then these reciprocals should be reduced to percentages in terms of the reciprocal of the minimum length (at the centre of the pole face) which is taken as 100. Then, to allow for the changes due in the fringe of the field to the superposition of the opposing field of the adjacent S-pole, the values at and beyond the zero point should be set out in reverse order, and deducted, leaving the percentage values ranging from 100 at the centre to 0 at the zero point. In the case of inductor alternators, where the successive poles are all of one polarity, the superposed fringe due to the adjacent pole must be added instead of subtracted.

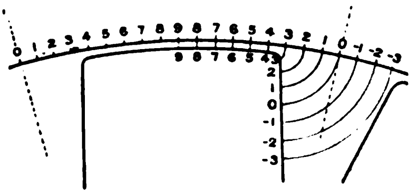

FIG. 183.

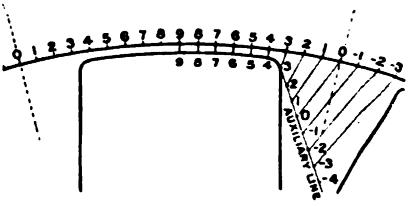

FIG. 184.

It is rather curious that almost equally good results are obtained by any of these methods. But it is useful to remember that, for a given difference of magnetic potential between the two opposing surfaces of iron, the flux will always distribute itself in such a way that the total reluctance of all the paths is a minimum. Hence that way of calculating is best which, when plotted out, gives the largest area of field-curve.

In the following example the data are taken from the 72-pole alternator of the Oerlikon Co. described on p. 397.

Point.	Length (mm.).	Reciprocal.	Percentage of mid-gap.	Deduct superposed fringe.	Resultant percentage density.
9	7	0·143	100	Negligible.	100
8	7	0·143	100	,,	100
7	7	0·143	100	,,	100
6	7	0·143	100	,,	100
5	7	0·143	100	,,	100
4	7	0·143	100	,,	100
3	9	0·111	77·6	− 4·27	73·33
2	33	0·0333	23·3	− 4·94	18·36
1	75	0·0133	9·32	− 5·84	3·48
0	97	0·0103	7·21	− 7·21	0
−1	120	0·00834	5·84	− 9·32	− 3·48
−2	142	0·00705	4·94	−23·3	−18·36
−3	164	0·00610	4·27	−77·6	−73·33

The outline of the pole is given in Fig. 189; the resultant distribution of field for one whole pole-pitch is plotted in Fig. 186; the dotted line gives the percentage uncorrected for the overlap of the neighbouring field. A convenient scale is 100 millimetres for the maximum height, and 90 millimetres from 0 to 9 of the series of points. If squared paper divided in millimetres is not to hand, squared paper divided in twentieths of an inch will answer.

From the distribution of the magnetic field, as thus ascertained, we must now proceed to deduce the form of the electromotive-force curve. On the assumption that the length of the pole-face from front to back, parallel to the shaft, is the same throughout, the number of magnetic lines cut by a conductor in moving through the field,[1] will be pro-

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[1] If it be objected that the conductors do not cut through the field, as they do in the case of smooth cored armatures, but lie in slots, it must be remembered that as the poles sweep past the slots every magnetic line will snap across the slot and cut across the conductors lying in it.

portional to the ordinates of the above curve. If the length be not uniform, then where it is less the length of the corresponding ordinates must be proportionately reduced.

It is convenient to think of the field as stationary, and of the conductors as moving past the poles. Then a conductor in moving from point 0 to point 1 will cut a number of lines proportional to the density of the field, and therefore to the area enclosed beneath the ordinates of the curve between these points. The mean ordinate of this narrow area therefore, gives the mean induced electromotive-force at that part. Or the ordinates of the curve will serve as ordinates for the electromotive-force induced in one conductor if the scale is appropriately changed. This, on the assumption that the slots do not perturb the distribution, would give proportionally the values for the electromotive-forces in a single conductor. But two things must here be remembered. First, that the electromotive-force around any loop of the winding is the sum of the two electromotive-forces in the two "conductors" of which it is made up; secondly, that where any one winding is distributed in more than one slot per pole, the electromotive-forces in the turns lying in the different slots are not precisely in phase with one another. Moreover, even in single-phase machines, if the span of the coil (reckoned from centre to centre of the slots at the pole-face) is, as is some-

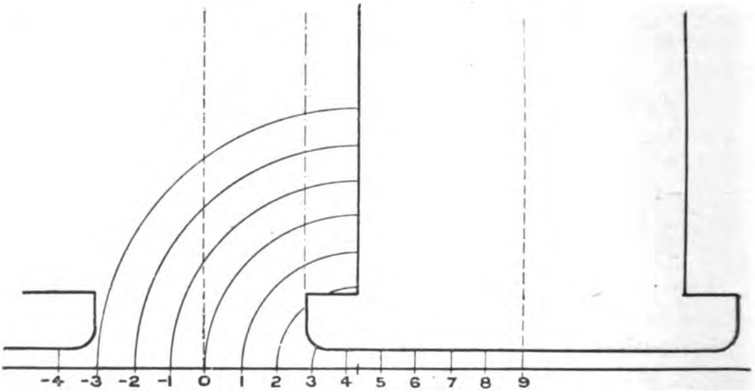

FIG. 185.—GRAPHIC CALCULATION OF DISTRIBUTION OF FIELD.

times the case, less than the pole-pitch, then the net electromotive-force in any one loop will be less than twice that in one "conductor." It is possible to take these things into account in the following alternative ways: (*a*) by reckoning out the separate simultaneous electromotive-forces for the conductors, and superposing them; (*b*) by reckoning out the electromotive-force for one loop on the assumption that it is made up of two conductors situated at a distance apart equal to the pole-pitch, and then multiplying by a "winding-factor" dependent on the number of slots, and finally multiplying by the total number of loops in the winding.

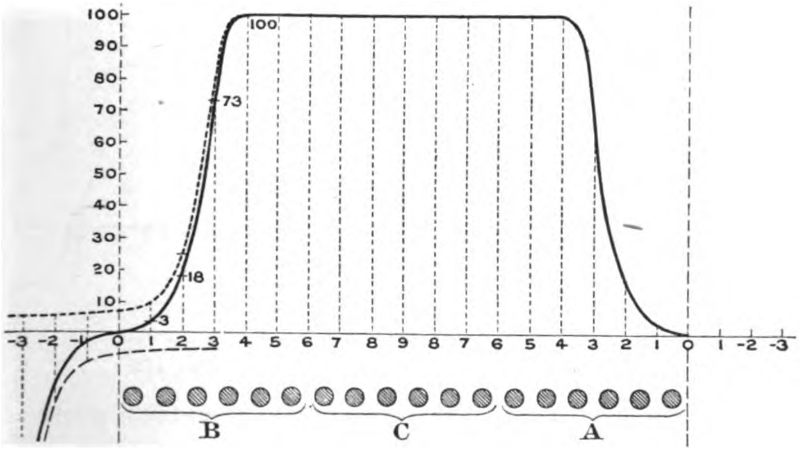

FIG. 186.—DISTRIBUTION OF MAGNETIC FIELD.

Single-phase, One-slot Winding.—Suppose a winding having one slot per pole per phase. In that case each loop of the winding subtends one pole-pitch; so that when one "conductor" at one side of the loop is passing through any particular part of the magnetic field of any pole, the second "conductor," which constitutes the other side of the loop, will at the same moment be passing through the corresponding part of the field of the next pole. Hence, in such a winding, the rise and fall in all the conductors is alike, and the curve of electromotive-force for the whole of the conductors in series will be the same shape as that of one conductor: the scale only will require to be changed.

Single-phase, Two-slot Winding.—If there are two slots per pole per phase, it is clear, from p. 198, that the electromotive-forces cannot all rise and fall simultaneously; and in this case the curve for the electromotive-force in any loop can be found from the curve for one conductor by taking a second similar curve, and shifting it on through the breadth between two adjacent slots, and then adding the ordinates. Similarly the curve for a three-slot or a four-slot winding can be found by superposing three or four curves each properly spaced along, at distances corresponding to the breadths from slot to slot.

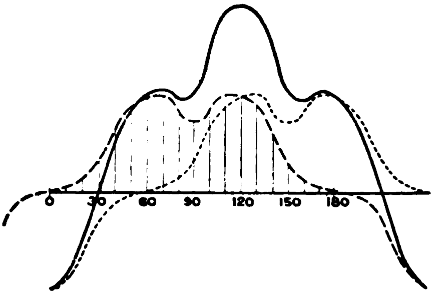

FIG. 187.

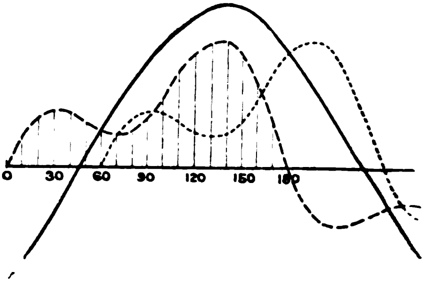

FIG. 188.

Three-phase, Y-grouping.—Suppose the curve of the electromotive-force for one phase of a three-phase armature to be known; and it is required to find the curve for the electromotive-force between the terminals when the phases are combined in Y-grouping with a common junction. The method is to compound two such phase-curves by subtracting their ordinates after shifting one of them 120° along, or, what comes to the same thing, by adding their ordinates after shifting one of them 60° along. The terminal curve may differ very considerably in form from the phase-curve if the latter is not a pure sine-curve, as will readily be seen from the circumstance that, as the result of the shifting along, all the third, ninth, fifteenth, etc. harmonics cancel one another out in the resultant terminal curve, while the fifth, seventh, eleventh, etc., harmonics do not. Any very sharply-peaked phase-curve will result in a double-peaked terminal curve. Figs. 187 and 188 show the result in two cases. The phase-curves are marked E while the terminal voltage curves are marked V. In Fig. 187, the E curve was given by one phase of a three-phase machine of one-slot pattern with square-ended poles, giving a double rounded top. Its terminal curve shows a peak with shoulders. In this case the third and fifth harmonics were initially present; but the third harmonic has vanished from the terminal curve, leaving only fundamental and fifth almost as in Fig. 49, p. 53. In Fig. 188, the E curve contains only fundamental and third harmonic. The latter vanishes, leaving the terminal curve a pure sine-form.

A more general way of treating the subject, capable of enabling either the phase-curve or the terminal curve to be predicted, will now be given. In the example chosen a terminal curve is found.

PREDETERMINING THE FORM OF ELECTROMOTIVE-FORCE CURVES.

Apart from the details of perturbations due to the slots if these are wide, the wave-form of the electromotive-force induced, at no-load, in an armature by a system of conductors distributed in any given manner, can be determined by the following process.

Lay out the conductors so as to show their distribution in front of one pole. Let them be spaced out along the diagram, on the same scale with respect to pole-pitch as the diagram of field distribution. We will use the field-form found in Fig. 186, across the bottom of which are drawn a set of circles to represent conductors, or groups of conductors in slots. The whole width across the three phases A, B and C is equal to one pole-pitch. The case taken is that of a three-phase machine with six slots per phase per pole.[1] The arma-

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[1] For the sake of simplicity it is here represented as if there were but one conductor in each slot. If there are several conductors in each slot, the wave-form will be just the same, but the voltage will be correspondingly higher.

ture is supposed to be Y-connected, so that the three circuits are joined at a common junction, and the wave-form to be determined is the wave-form of the electromotive-force between the terminals of A and B.

As the poles sweep along, the electromotive-force induced in any one conductor is proportional to the ordinate of the field-form immediately above that conductor; and the total electromotive-force in the whole group of conductors in series is equal to the algebraic sum of all the ordinates that stand over all the conductors of that group. To find the wave-form of the electromotive-force we must move the group of conductors from point to point, and at each point take the sum of the ordinates. In doing this for the first time, it will be found convenient to sketch out the conductors upon a slip of paper, and move the paper along from point to point, taking the sum of the ordinates in the successive positions. For the purpose of identifying the successive positions, the slip of paper may be graduated with the numbers 0, 1, 2, 3, 4, etc.; the graduations being spaced equally with the conductors, and a datum mark on the field-form arranged to point to the distinguishing number of the position. The method is illustrated below in tabular form. Vertically across the field are drawn lines the spacing of which also agrees with the spacing of the conductors. On each of these vertical lines may be written the value of the ordinate at that point. For the particular field-form shown in Fig. 186 these ordinates are taken as 0, 3, 18, 73, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 73, 18, 3, 0. In the zero position of the tabular illustration the conductors are so placed that the total electromotive-force between the terminals of A and B is zero, there being as many negative electromotive-forces as positive electromotive-forces. In the position "1" where the zero mark of the field is opposite conductor No. 1, the ordinates to be added are: 100, 100, 100, 73, 18, 3, and 0, −3, −18, −73, −100; and −100; total 100. In the next position No. 2, the sum is 300; in position No. 3 it is 500; in position No. 4 it is 673. The Table shows the calculation in detail for the first seven positions. The whole series obtained in this way is as follows: 0, 100, 300,

+100	+100	+100	+100	+100	+100	+100	4	●	
+100	+100	+100	+100	+100	+100	+100	5	●	
+100	+100	+100	+100	+100	+100	+100	6	●	
+100	+100	+100	+100	+100	+100	+100	1	●	A
+100	+100	+100	+100	+100	+100	+100	2	●	
+100	+100	+100	+100	+100	+100	+ 73	3	●	
+100	+100	+100	+100	+100	+ 73	+ 18	4	●	
+100	+100	+100	+100	+ 73	+ 18	+ 3	5	●	
+100	+100	+100	+ 73	+ 18	+ 3	⊖	6	●	
+100	+100	+ 73	+ 18	+ 3	⊖	− 3	1	●	B
+100	+ 73	+ 18	+ 3	⊖	− 3	− 18	2	●	
+ 73	+ 18	+ 3	⊖	− 3	− 18	− 73	3	●	
+ 18	+ 3	⊖	− 3	− 18	− 73	−100	4	●	
+ 3	⊖	− 3	− 18	− 73	−100	−100	5	●	
⊖	− 3	− 18	− 73	−100	−100	−100	6	●	
− 3	− 18	− 73	−100	−100	−100	−100	1	●	C
− 18	− 73	−100	−100	−100	−100	−100	2	●	
− 73	−100	−100	−100	−100	−100	−100	3	●	
−100	−100	−100	−100	−100	−100	−100	4	●	
−100	−100	−100	−100	−100	−100	−100	5	●	
−100	−100	−100	−100	−100	−100	−100	6	●	
−100	−100	−100	−100	−100	−100	−100	1	●	A
−100	−100	−100	−100	−100	−100	−100	2	●	
−100	−100	−100	−100	−100	−100	− 73	3	●	
−100	−100	−100	−100	−100	− 73	− 18	4	●	
−100	−100	−100	−100	− 73	− 18	− 3	5	●	
−100	−100	−100	− 73	− 18	− 3	0	6	●	
−100	−100	− 73	− 18	− 3	0		1	●	
Position 6 Sum = 894	Position 5 Sum = 791	Position 4 Sum = 673	Position 3 Sum = 500	Position 2 Sum = 300	Position 1 Sum = 100	Position 0 Sum = 0			

500, 673, 791, 894, 994, 1091, 1173, 1173, 1091, 994, 894, 791, 673, 500, 300, 100, and finally 0. The two zero values come at places half of one position to the left of position No. 1, and half of one position to the right of No. 18. After the method is once understood, the successive sums can be obtained without using a moving diagram, for each successive total is obtained from the last by adding the ordinate immediately in advance of the group, and subtracting the ordinate above the last conductor of the group.

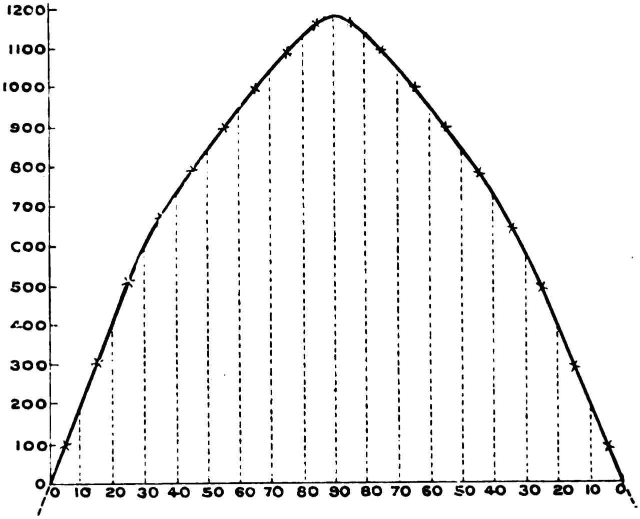

Fig. 189.—Electromotive-Force Wave-Form Deduced from Fig. 186.

In Fig. 189 is plotted the wave-form of the electromotive-force produced by combining the field-form of Fig. 186 with the particular distribution of conductors at the bottom of that figure. The abscissæ represent the successive positions as the poles move forward, and the ordinates give the instantaneous values of the electromotive-force. The scale of the latter, in volts, can be ascertained as follows. Let there be *a* units of length in the maximum ordinate in Fig 186, and let the maximum voltage per conductor (as calculated from the maximum flux-density in the gap and the peripheral speed of the magnet-wheel) be v. Let there be z conductors in each slot. Then any ordinate of the curve multiplied by $\frac{vz}{a}$ gives the instantaneous value of the voltage at the terminals of the group of conductors. If therefore, we multiply any ordinate of the curve of Fig. 189 by $\frac{vz}{a}$ and by the number of groups joined in series (in general the number of groups equals the number of poles) we obtain the instantaneous value of the electromotive-force at the terminals.

In order to find the virtual value of the electromotive-force two methods are possible. We may plot the squares of the ordinates, as ordinates of a new curve, estimate the area of the new curve (either by planimeter or by counting squares on squared paper), divide by the length of base to get the mean of the height, and take the square-root of the quotient. Or we may arithmetically take the square-root of the mean of the squares of the instantaneous values. If the graphic process is adopted regard must be had to the units employed. For instance, suppose that the ordinates of the electromotive-force curve were drawn to such a scale that 1 millimetre represents one volt. The squares of the big ordinates will in general be so high that a new scale must needs be adopted for plotting the squared values. Suppose we take 0·1 millimetre as unit. The area of the curve in square centimetres divided by the base of the curve in centimetres gives the mean height in centimetres; and this height multiplied by 100 gives the mean value of the square of the volts. Taking the square root of this last figure we obtain the quadratic mean, which is the virtual value sought.

If the field-form is a sine-curve, then the electromotive-force curve at no-load will also be a sine-curve.

The *form-factor* is calculated by dividing the quadratic mean by the arithmetical mean. In the example given this works out as follows:—

Numbers.	Squares.
100	10,000
300	90,000
500	250,000
673	452,929
791	625,681
894	799,236
994	988,036
1,091	1,190,281
1,173	1,375,929
1,173	1,375,929
1,091	1,190,281
994	988,036
894	799,236
791	625,681
673	452,929
500	250,000
300	90,000
100	10,000
Total 13,034	Total squares 11,564,184
	Mean square 642,454·6
Arithmetical mean 724·1	Root mean square 801·5

$$\text{Form-factor} = \frac{801\cdot5}{724\cdot1} = 1\cdot105.$$

The circumstance that the form-factor comes out so nearly equal to 1·11 proves that this particular curve is very nearly a true sine-curve.

Predetermination of Wave-Form in Alternators, with very few wide slots per pole.—In the above we have treated as negligible the perturbing effect of the slots. If these are few in number, and wide open at the working face, their effect will be by no means negligible. They will produce irregularities in the curve, because when a slot comes to a position where the flux is changing rapidly, or where the flux-curve is sloping steeply, its presence will tend to shift some of the flux from the denser part, across the slot, to the less dense part on the other side of the slot. Any saturation of the teeth will tend to tone down the effect; in fact in such cases the characteristic shape of the electromotive-force curve will depend less on the distribution of the flux at the pole-edges, and more on the relative sizes of the slots and teeth, but especially on the ratio of pole-span to pole-pitch.

The matter may be treated as follows:—

1. Draw out to scale the profile of the slots over a span of two consecutive poles on squared paper, so that the area subtended at the armature face by the slot opposite the pole-face may easily be determined for any position of the pole, which latter should be drawn out on a separate piece of paper which can be slid along opposite the slots.

2. Construct a Table, as in the calculation of the preceding section (p. 215) in such a way that the first column gives the different successive positions of the pole underneath the slots throughout a half-period, and the other columns give the area of the pole underneath the separate teeth for these different positions.

3. From these areas it is now necessary to estimate the percentage of flux which passes into the teeth at each of the different positions. Here one must make one of two assumptions—(*a*) that there is the same flux per pole for all positions of the pole, whatever area of tooth is above it; or (*b*) that the flux-density in the gap is always the same, and that the total flux from the pole is therefore not constant, but varies in proportion to the area of teeth above it. Neither assumption is correct: for though, as a matter of fact, owing to the great self-induction of the exciting circuit, the total flux through the pole-core (at any given load and excitation) cannot vary greatly, yet, on the other hand, as the magnetomotive-force acting across the gap is (also with given load and excitation) approximately constant, there is a tendency to produce a flux directly proportional to the permeance of the gap. Magnetic dispersion comes in here to adjust the balance of conditions, though it is difficult to lay down any rules. If the iron parts of the alternator, for example either the pole-tips or the pole-cores, are highly saturated, the former assumption will be more nearly true, for in that case any increase in the gap area will alter the total permeance of the magnetic circuit very little. If, on the other hand, the iron parts are far from saturation, the total permeance may be greatly affected by the variance of gap-area. For many machines that do not realize precisely either of the above

conditions the following device enables a closer approximation to be made than either of the preceding assumptions. Take an average of all the areas for the different positions, and then for any position assume that the flux varies according to the square root (or some other fractional power) of the ratio of that area to the mean area.

4. Having thus estimated the flux in the teeth for different positions of the pole, it is an easy matter to find the total flux

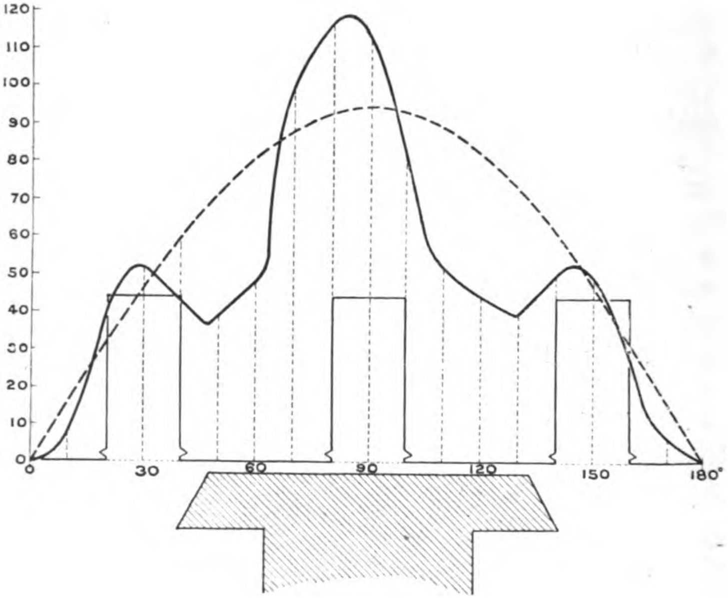

FIG. 190.

linking itself, in any of the positions, with a coil of the armature winding, by taking the difference in the fluxes due to two adjacent poles.

5. The next step is to find the electromotive-force generated at the different positions. This is done approximately by taking the differences between consecutive values of the flux in (4); but a graphic method will probably be found more convenient.

One secondary effect is that as the teeth pass the corners of the poles they tend to produce (independently of fluctuations in the amount of the flux from the pole), small oscillations laterally in the flux; but the conditions are of two kinds. If the pole-width corresponds to an exact number of slot-pitches, then since as the pole when it leaves a tooth at one side simultaneously encounters another at the other side, there will be little fluctuation of total flux but only lateral oscillation; whereas if the pole-width corresponds to an exact number of teeth (*i.e.* to an odd number of half slot-pitches), there will be continual small fluctuations in the flux. In the former case the ripples due to the teeth will be strongest at the highest and lowest points of the electromotive-force curve; in the latter they will be strongest near the zeros of the curve.

An example of an electromotive-force curve in which the effect of the slots is very apparent is afforded by that shown in Fig. 190, which was given by a particular three-phase alternator having a one-slot winding with wide slots. A calculation of the form factor is appended.

Electrical Degrees.	Ordinate.	Ordinate squared.
10°	9	81
20°	41	1,681
30°	51·5	2,652
40°	42	1,765
50°	39	1,521
60°	48	2,304
70°	100	10,000
80°	116	13,456
90°	113	12,769
100°	79	6,241
110°	51·5	2,652
120°	44	1,639
130°	39	1,521
140°	50	2,520
150°	48·5	2,352
160°	25	625
170°	6	36
180°	0	0
Totals	902·5	64,091
Averages	50·1	3,561
Square root of mean square		59·7

$$\text{Form factor} = \frac{59{\cdot}7}{50{\cdot}1} = 1{\cdot}19.$$

The following is the result of a harmonic analysis of this curve :—

$A_1 = 80{\cdot}8$; $A_3 = -8{\cdot}2$; $A_5 = 21{\cdot}9$; $A_7 = -7{\cdot}7$; $A_9 = -5{\cdot}1$;
$A_{11} = -2{\cdot}6$; $A_{13} = -5{\cdot}2$; $A_{15} = -1{\cdot}1$; $A_{17} = -0{\cdot}7$.

$B_1 = 4{\cdot}4$; $B_3 = -5{\cdot}5$; $B_5 = 9{\cdot}1$; $B_7 = -8{\cdot}0$; $B_9 = 1{\cdot}0$;
$B_{11} = -0{\cdot}1$; $B_{13} = -1{\cdot}4$; $B_{15} = 3{\cdot}2$; $B_{17} = -0{\cdot}55$.

Or, uniting the terms and neglecting those above the seventh harmonic, the equation to the curve is :—

$$y = 80{\cdot}9 \sin(\theta + 3^\circ\ 7') - 9{\cdot}9 \sin(3\theta + 33^\circ\ 50') + 23{\cdot}7 \sin(5\theta + 22^\circ\ 35') - 11{\cdot}1 \sin(7\theta + 43^\circ\ 55').$$

It is here evident that the irregularities of outline are due to the influence of the negative third and of the positive fifth harmonics tending to accentuate the peak. Now a glance at Fig. 190, which shows at the bottom the shapes of teeth and of pole-shoe, makes it clear that the effective breadth of pole is about equal to that of two teeth plus the intervening slot. The flux is, therefore, a maximum when a slot is opposite the middle of a pole, and a minimum when a tooth is opposite the middle of a pole. The effective pole-breadth is here about 0·55 of the pole-pitch.

Fig. 191 depicts the voltage (no-load) curve of the same armature when for the pole previously used is substituted one of rectangular form, and of a breadth of about 0·34 of the pole-pitch. The difference between the curves is striking: one would not have imagined that a mere change of pole could effect such an alteration. The curve, instead of presenting three peaks, has two rounded tops with a deep hollow between them. The following is the result of the analysis of this curve :—

$$y = 100{\cdot}9 \sin\theta + 0{\cdot}5 \sin 3\theta - 16{\cdot}1 \sin 5\theta - 3{\cdot}9 \sin 7\theta - 1{\cdot}6 \sin 9\theta + 2{\cdot}9 \sin 11\theta - 0{\cdot}6 \sin 13\theta - 0{\cdot}6 \sin 15\theta + 1{\cdot}7 \sin 17\theta - 1{\cdot}7 \cos\theta - 1{\cdot}3 \cos 3\theta + 0{\cdot}8 \cos 5\theta + 2{\cdot}8 \cos 7\theta + 0{\cdot}8 \cos 9\theta - 0{\cdot}6 \cos 11\theta - 1{\cdot}6 \cos 13\theta + 0{\cdot}1 \cos 15\theta + 2{\cdot}4 \cos 17\theta.$$

The mean ordinate is 61·2; the quadratic mean of the ordinates, 72·0; and the form-factor, 1·18.

It will be seen that the departures from a perfect sine-form are here due mainly to the negative fifth harmonic; and the physical reason for this change in the state of affairs is seen by inspection of the figure. For, owing to the relative widths of teeth, slots and pole-face, and to the fringing at the edges, the flux is now a maximum when one of the large teeth is opposite the middle of the pole, and a minimum when a slot is opposite the middle.

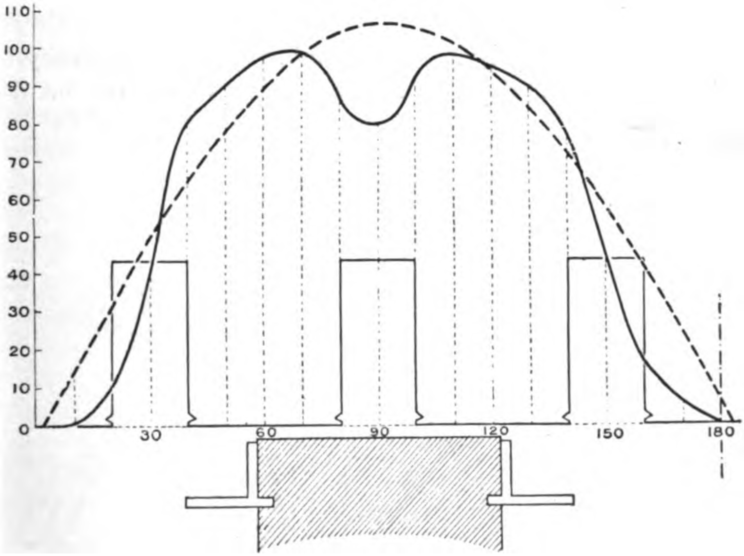

FIG. 191.

Form-factor in Relation to Harmonics.—We have seen above how to estimate the form-factor for a given electromotive-force curve, by actually dividing the quadratic mean by the arithmetical mean value of the ordinate. If the curve has been already analysed down into its constituent harmonics, the form-factor can at once be calculated from the harmonic coefficients as follows:

$$E_{\text{virt}} = \sqrt{\tfrac{1}{2}\,(A_1^2 + A_3^2 + A_5^2 + A_7^2 + \ldots + B_1^2 + B_3^2 + B_5^2 + \text{etc.})}$$

because the mean of the squares of all sine-functions is ½.

Further

$$E_{\text{mean}} = \frac{2}{\pi}\left(A_1 + \tfrac{1}{3}A_3 + \tfrac{1}{5}A_5 + \tfrac{1}{7}A_7 + \text{etc.}\right)$$

since the average value of the sine is $\frac{2}{\pi}$.

Whence at once

$$\text{Form factor} = \frac{\sqrt{\tfrac{1}{2}\left(A_1^2 + A_3^2 + A_5^2 + \text{etc.}\right)}}{\frac{2}{\pi}\left(A_1 + \tfrac{1}{3}A_3 + \tfrac{1}{5}A_5 + \text{etc.}\right)}$$

It is also important to remember that the tendency to pierce insulation depends neither on the virtual nor on the average value of the voltage, but on its maximum value ; and this depends again on the harmonics, according to the relation

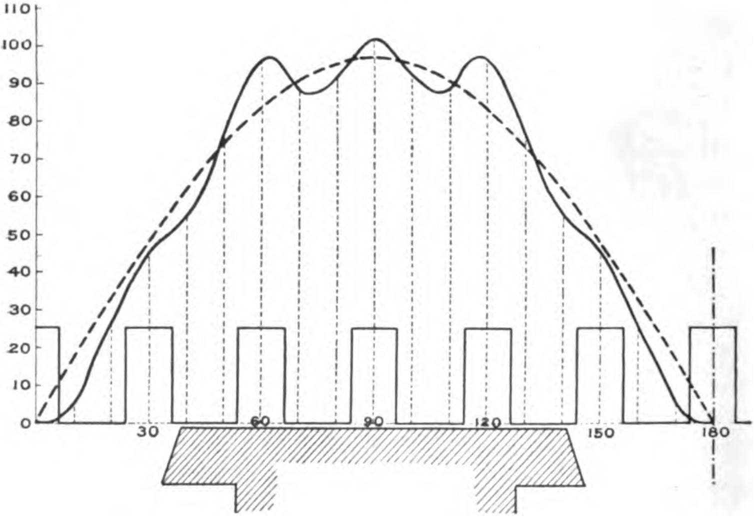

FIG. 192.—NO-LOAD WAVE-FORM OF AT(B) 30—1900—184.

$$E_{\text{max}} = A_1 - A_3 + A_5 - A_7 + \text{etc.} - B_2 + B_4 - B_6 + \text{etc.}$$

Alternator Wave-forms at No-load and Full-load.—The result of observations by one or other of the methods of curve tracing shows that the voltage curves of alternators often depart very far from simple sine-curves, even when there are no armature reactions. The examples afforded by Figs. 190 and 191 above show how perturbing the effect of a few wide slots may be. In that case three wide slots led to the appearance of the third and fifth harmonics. In Fig. 192 a curve is given for a machine with six slots per pole, in which the eleventh harmonic is predominant. This is the phase-curve of the electromotive-force of a three-phase generator; and on analysis yields the equation

$$y = 9{\cdot}66 \sin \theta - 0{\cdot}426 \sin 3\theta - 0{\cdot}56 \sin 5\theta - 0{\cdot}1 \sin 9\theta - 0{\cdot}635 \sin 11\theta + 0{\cdot}2 \sin 13\theta.$$

Professor Rosa has found that a single-phase Westinghouse alternator with seven slots per pole yielded a curve in which

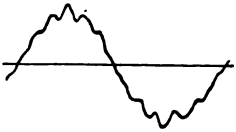

FIG. 193.—NO-LOAD WAVE-FORM: ONE CABLE.

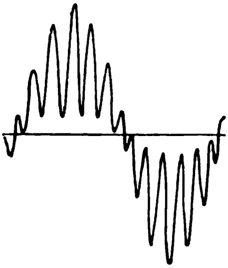

FIG. 194.—NO-LOAD WAVE-FORM : THREE CABLES ON.

the ripples were composed of the thirteenth and fifteenth harmonics, the seventh being absent. The form of a no-load curve may be much modified if the alternator is tested while still connected to mains, even if there be nothing connected across the mains, because in consequence of the capacity of the dielectric and the self-induction of the armature resonance may occur of a frequency corresponding to some one of the harmonics, and thereby exaggerate the ripples.

Figs. 193 and 194 were taken by means of the oscillograph by M. B. Field,[1] from the three-phase alternators of the Glasgow tramways. These have six teeth per pole, which

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[1] *Journ. Inst. Electrical Engineers*, xxxii. 659, 1903.

give rise to ripples chiefly of the eleventh and thirteenth harmonics. In Fig. 194 the thirteenth harmonic has been amplified by the reasonance of the system.

At full-load, the ripples in the electromotive-force curve, are usually less pronounced ; but the effects of loading differ according to the nature of the load. With a non-inductive load (*i.e.* when $\cos \phi = 1$), there is, as will be shown in the chapter on reactions, a cross-magnetizing reaction, which will tend to shift the magnetism of the poles backward, and cause the maximum point of the voltage curves to occur later. This is shown by Fig. 195, which is the full-load curve for the same machine as Fig. 191. If the load is an inductive and wholly lagging load (*i.e.* $\cos \phi = 0$), its reaction will be demagnetizing,

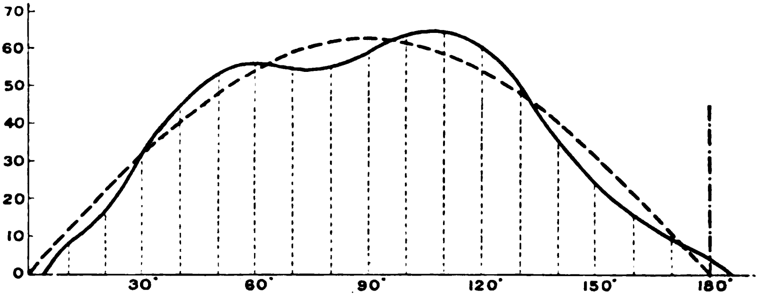

FIG. 195.

and then to keep up the voltage, the field magnets must be more highly magnetized than before. Hence, the teeth will be more highly saturated, magnetic leakage will be greater, and the variations of flux due to the teeth will be stronger than if the load were non-inductive.

Effect of Pole-span upon Harmonics and Form-factor.—It is very instructive to observe the effect upon the electromotive-force curve of taking a pole of any particular breadth. If the pole is not shaped away at its edges, its field will, we know, not be distributed according to a sine-law. There will always be some fringing, but apart from this, the field will (when on no-load) be approximately uniform over the face if the gap is of equal width. A narrow pole will obviously tend to give a narrow field; a broad pole a broad field. But any one of these field distributions may be split up into a series of harmonics over the whole pole-pitch, as follows.

Let the half-span of the pole, reckoned in electrical degrees (in which the pole-pitch is 180°), be called θ. Assuming the field to be uniform over the pole-face, and without fringing, then the form of the flux curve will be rectangular, as shown in Fig. 196.

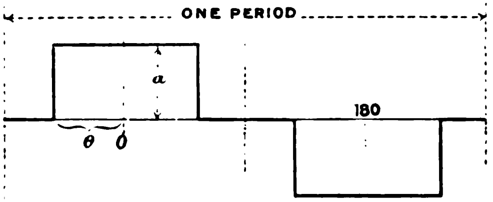

FIG. 196.

The Fourier's expansion of such a periodic curve will be of the form—

$$y = A_1 \cos \theta + A_3 \cos 3\,\theta + A_5 \cos 5\,\theta, \text{ etc.}$$

where $$\theta = 2\,\pi \frac{t}{T},$$

and the general expression for A_n

$$= \frac{4\,a}{T} \int_{+\theta}^{\pi-\theta} \cos n\,\theta \,.\, d\theta\,;$$

$$= \frac{4\,a}{\pi\,n} \sin n\,\theta.$$

In the table below, the amplitudes of the first, third, fifth, seventh, ninth and eleventh harmonics have been calculated for different values of θ, the amplitude, 'a,' of the curve being taken as unity. The value of ψ $\left(\text{the ratio } \frac{2\,\theta_1}{\pi}\right)$ is also given for every value of θ.

TABLE V.—AMPLITUDES OF HARMONICS IN SQUARE-CORNERED FIELDS.

θ_1	ψ	A_1	A_3	A_5	A_7	A_9	A_{11}
deg.							
5	0·055	0·111	−0·110	0·108	−0·104	0·099	−0·095
10	0·111	0·219	−0·213	0·194	−0·171	0·141	−0·109
15	0·166	0·328	−0·303	0·247	−0·176	0·099	−0·030
20	0·222	0·436	−0·367	0·232	−0·117	0	0·079
25	0·278	0·539	−0·410	0·209	−0·015	−0·099	0·112
30	0·333	0·637	−0·424	0·130	0·090	−0·141	0·058
35	0·388	0·732	−0·410	0·022	0·164	−0·099	−0·049
40	0·444	0·871	−0·367	−0·086	0·180	0	−0·114
45	0·50	0·900	−0·303	−0·180	0·104	0·099	−0·082
50	0·555	0·975	−0·213	−0·24	0·031	0·141	0·020
55	0·61	1·042	−0·110	−0·254	−0·076	0·099	0·105
60	0·67	1·105	0	−0·220	−0·158	0	0·101
65	0·72	1·115	0·110	−0·145	−0·181	−0·099	0·010
70	0·78	1·20	0·213	−0·044	−0·139	−0·141	−0·088
75	0·83	1·23	0·303	0·086	−0·047	−0·099	−0·112
80	0·89	1·255	0·367	0·163	0·062	0	−0·040
85	0·94	1·270	0·410	0·230	0·149	0·099	−0·066
90	1·0	1·275	0·424	0·257	0·182	0·141	0·116

From the general expression for A_n, it is seen that the nth harmonic disappears when $\theta_1 = \frac{m\pi}{n}$, where m is any number less than $\frac{n+1}{2}$.

∴ Third harmonic disappears when $\theta_1 = 60°$. The fifth harmonic disappears when $\theta_1 = 36°$ and $72°$. The seventh harmonic disappears when $\theta_1 = 25·7°$, $51·4°$ and $77·2°$. The ninth harmonic disappears when $\theta_1 = 20°$, $40°$, $60°$ and $80°$. The eleventh harmonic disappears when $\theta_1 = 16·2°$, $32·4°$, $48·5°$, $64·7°$ and $81·8°$. Etc., etc.

Now since in a three-phase armature with Y-grouping the third, ninth, fifteenth, etc. harmonics cancel out, it will be seen that the important thing is to avoid having the fifth, seventh and eleventh harmonics. Hence it would appear that if a pole is not shaped away, it is better that the ratio ψ should be about 0·45 or 0·8, than of the amount more frequently found of 0·6 or 0·66. For Δ-grouping, where the third harmonic must be got rid of, the best value of ψ is certainly 0·67.

Effect of Skewing the Poles.—The effects due to the presence of a few wide slots in producing as we have seen abrupt changes of the flux, and superimposing ripples on the curve, are mainly caused by the edges of the slots being parallel to the edges of the pole-faces, so that the movement of a pole-edge past the edge of a slot takes place suddenly. By increasing the number of slots per pole and by using either closed or half-closed slots, these irregularities in the curves are avoided. But with closed and half-closed slots hand-winding must be resorted to. This abruptness could also be altered by making either the slots or the pole-edges slightly askew. This remedy has long been known in the case of continuous-current dynamos, where skewing has been one of the methods for producing a graduated fringe to the field. By shaping the edge of the pole polygonally or by curving it away, any suitable fringe can be devised. Where solid pole-shoes are used they can readily be chamfered, curved or skewed. In the more usual construction of laminations, the face may be chamfered (as in Figs. 129*a*, *b*, *i* and *j*, p. 148) or the assembled blocks of stampings may be bolted askew to the foundation-wheel. It is not practical to build the armature-cores with slots askew. But the Oerlikon Company has lately adopted a mode of procuring a skew-edge to the pole that is of some interest. The pole-core stampings are made in the first instance of T-shape; but the tips are cut away to different amounts at one edge or the other, so as to give the same effect as a skew pole. Fig. 197 depicts a magnet-wheel with poles so constructed. This device had the immediate effect of smoothing out[1] the curve, so that in a machine with six slots per pole, which had a very strongly marked eleventh harmonic ripple, the no-load curve became at once almost a pure sine-curve.

Effect of Armature Coil-span upon Harmonics and Form-factor.—Continuing the questions considered in the previous

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[1] See Paterson, *Elect. Review*, lv. 115, 1904.

section, we will now discuss the effect of different armature coil spans on the wave-form.

The instantaneous electromotive-force which is being generated in any one loop is the sum of those which are being generated by either sides of the loop, and if the coil is so wide that it corresponds to any number of complete periods of any one of the higher harmonics of flux distribution, then that harmonic will not appear in the voltage wave because at any

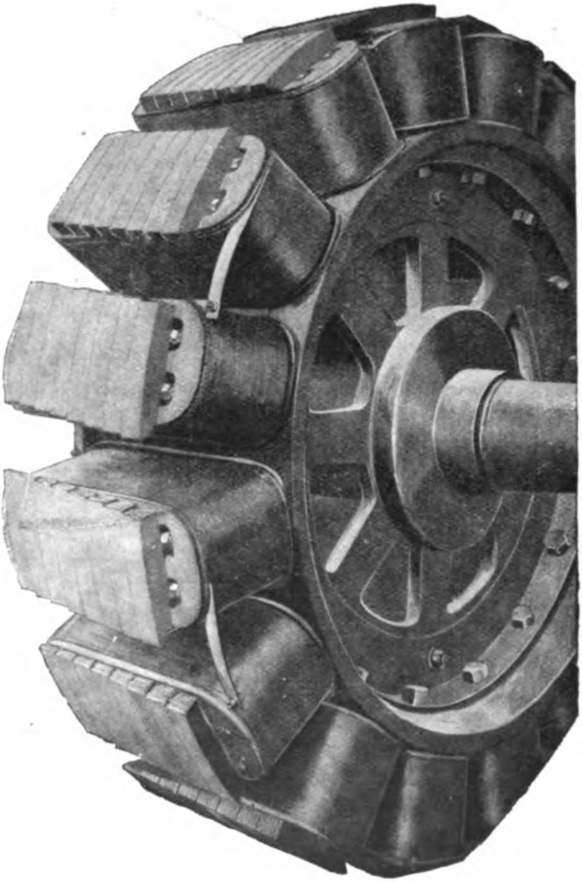

FIG. 197.

instant the nett amount of linking of the flux belonging to that harmonic of flux distribution is nil.

For example, a coil which spans $\frac{2}{3}$ the pole-pitch will not generate the third harmonic, and one which covers $\frac{2}{5}$ or $\frac{4}{5}$ the pole-pitch will not generate the fifth. Now to eliminate both the third and the fifth harmonics, it will not do to connect a coil which does not generate a fifth harmonic in series with one which does not generate a third, because the former will generate a third and the latter a fifth ; it must be done by choosing such positions that both coils generate both these harmonics to the same extent in opposite directions. Again, by suitably placing three coils, the harmonics $3n$, $5n$ and $7n$ may be eliminated.

R. Rüdenberg[1] has discussed this question, and the following are some of his results.

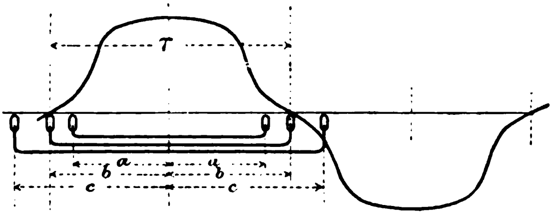

FIG. 198.

Fig. 198 shows a three-slot winding. Denoting by *a*, *b*, and *c* the half-spans of the successive coils, we may describe any coil by the ratio of its half-span to the pole-pitch τ. If, as is usual, the slots are made equidistant, so that $\frac{a}{\tau} = \frac{70}{180}$; $\frac{b}{\tau} = \frac{90}{180}$; $\frac{c}{\tau} = \frac{110}{180}$, then all odd harmonics will be present. They need to be considered up to the thirteenth only. But if, instead, the ratios chosen are $\frac{a}{\tau} = \frac{63}{180}$; $\frac{b}{\tau} = \frac{102}{180}$; $\frac{c}{\tau} = \frac{146}{180}$, then the third, fifth and seventh harmonics will have entirely disappeared from the volt-curve, leaving only the ninth, eleventh and thirteenth with the fundamental. And this will be true whatever the form of the field-curve. The

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[1] *Elektrot. Zeitschr.*, xxv. 252, March 31, 1904.

results may be summed up in the following Table, which gives the coefficients of the harmonics for these two kinds of three-slot winding, also for a special two-slot winding that suppresses the fifth harmonic, and for four kinds of four-slot winding, the last of which, while not absolutely suppressing any harmonic, gives practically none except the ninth and the eleventh.

TABLE VI.—TABLE OF COEFFICIENTS OF HARMONICS (RÜDENBERG).

Style of Winding.	$\frac{a}{\tau}$	$\frac{b}{\tau}$	$\frac{c}{\tau}$	$\frac{d}{\tau}$	A_1	A_3	A_5	A_7	A_9	A_{11}	A_{13}
I. 3-slot, ordinary	$\frac{70}{180}$	$\frac{93}{180}$	$\frac{110}{180}$	..	0·960	−0·667	0·218	0·177	0·333	0·177	0·218
II. 3-slot, special	$\frac{63}{180}$	$\frac{102}{180}$	$\frac{146}{180}$	..	0·810	0	0	0	−0·512	0·137	0·365
III. 2-slot, special	$\frac{78}{180}$	$\frac{42}{180}$	..	..	0·824	0	0	−0·519	0	0·824	−0·519
IV. 4-slot, special	$\frac{29}{180}$	$\frac{55}{180}$	$\frac{89}{180}$	$\frac{115}{180}$	0·801	0	0	0	0	−0·643	0·496
V. 4-slot, equi-distant	$\frac{30}{180}$	$\frac{60}{180}$	$\frac{90}{180}$	$\frac{120}{180}$	0·808	0	−0·058	0·058	0	−0·808	0·808
VI. 4-slot, special	$\frac{30}{180}$	$\frac{69}{180}$	$\frac{97}{180}$	$\frac{128}{180}$	0·804	−0·005	0·019	−0·062	−0·146	−0·156	−0·040

The advantage of proceeding in this way rather than by trusting to shaping the pole-pieces, is that, while the latter process may give a sine-curve at no-load, the armature-reaction at full-load distorts the magnetism, and with ordinary windings also distorts the volt-curve; whereas with these special windings the elimination of harmonics applies equally at full-load because it is independent of the form of the field-curve. The disadvantages are the lower plant-efficiency that results from reduction of the coefficient of the fundamental below unity and the practical difficulty of adopting these special distributions of slots for any but single-phase machines. Such a winding for example as the last in the table constitutes a veritable *sine-filter* which virtually excludes all the harmonics from the volt-curve, whatever the form of the field-curve. But if triplicated at 120° apart to make a three-phase winding, it becomes impossible because the slots of one phase could not be spaced out between those of the overlapping phases. Fig. 199 shows the no-load curves of five of these windings when used in four fields, the curves of which differ exceedingly in form.

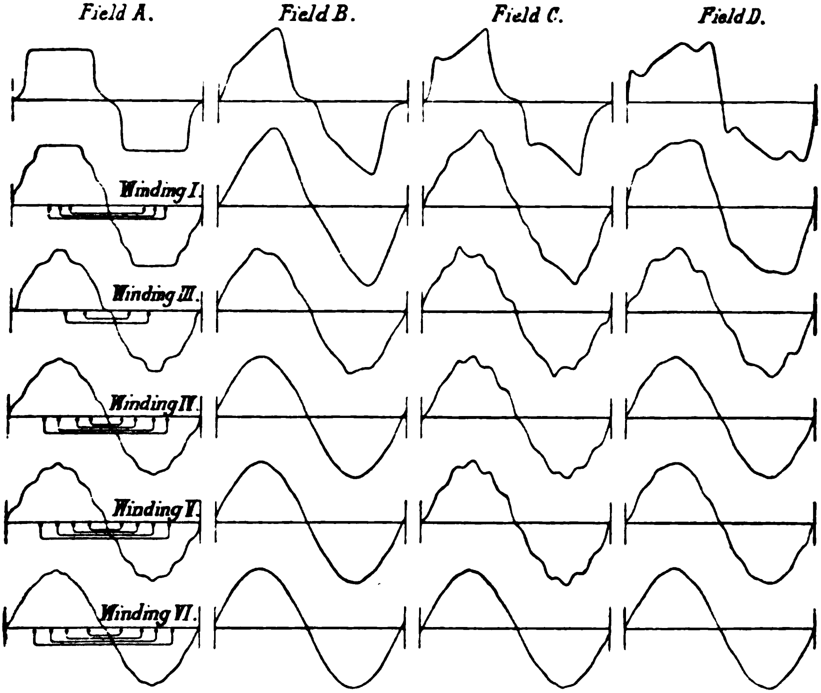

FIG. 199.

It will be noted that in the 4-slot windings the inner coil is very narrow, narrower than any ordinary pole-span. This justifies the use in single-phase machines of such a construction as Fig. 252, p. 309, in spite of the remarks on p. 197.

Either the three-slot special, or the two-slot special of the above table could be used for a two-phase winding. Unequally-spaced slots have been used in some two-phase machines—for example, in the generators at Chèvres (Plate IX.).

ACTUAL ALTERNATOR CURVES. CURVE-TRACING.

The first person to grasp the circumstance that an alternator would induce an electromotive-force the variations of which would follow a definite curve, appears to have been Koosen[1]; but Joubert[2] was the first to propose and carry out the experimental investigation of the form of the curve. Joubert's method consisted in obtaining by means of a contact-maker fixed on the revolving shaft of the alternator, a number of instantaneous voltage readings at a number of successive points on the recurring cycles of electromotive-force. By plotting the successive voltages, a curve of the electromotive-force may be obtained. It does not, however, give the curve for any one cycle, but depends on the assumed similarity of a large number of cycles during the time of observation.

A method of continuous observation dependent on optical means was devised by Frölich,[3] who caused the alternating current to actuate an elastic diaphragm like that of a telephone from which a beam of light was reflected. This has been superseded in recent years by other continuous methods. In fact, the methods of obtaining alternator curves may be classified as under.

I. Point-by-point Methods, with subsequent plotting.
II. Point-by-point Methods, with self-registering appliances.
III. Continuous Methods (Oscillographs), with photographic registration.
IV. Chemical Registration.

Method I. Point-by-point Observation.—The instantaneous contact-maker devised by Joubert consisted of an insulated brass pin fixed in a disk on the revolving armature shaft, which wiped against a small spring or brush once in each revolution; the contact spring being arranged to be moved

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[1] Koosen, *Pogg. Ann.*, lxxxvii. 386, 1852.

[2] Joubert, *Journal de Physique*, 1880, 297; *Comptes Rendus*, xci. 161, 1880.

[3] *Elektrotechnische Zeitschrift*, vii. 210; x. 65 and 345.

around a graduated circle so that the contact should occur at the moment when the armature was in any desired position relatively to the poles. One terminal of the armature coil (or of an exploring coil wound specially upon it) was connected to the revolving pin, the other was joined to one pole of a condenser. The other pole of the condenser was joined to the contact spring. So once in each revolution the condenser was charged up to the voltage in the armature coil while passing through the corresponding position. This voltage is then ascertained by discharging the condenser through a ballistic galvanometer, or by use of a quadrant electrometer, or more recently by use of some commercial form of electrostatic voltmeter. This method has been used by Hopkinson, and with various modifications in the United States by Morton and Thomas (1880), Duncan, Hutchinson and Wilkes (1888), Searing and Hoffman (1889) and Ryan (1890). Ryan modified it into a zero method, and in conjunction with Bedell devised a contact-maker consisting of a minute jet of salt water. Fleming (1895) and Blondel (1902) drove the contact-maker by a separate synchronous motor, thus enabling the observations to be made at any desired place in the circuit. Mershon (1891) described a very simple and handy method in which the voltage so obtained by instantaneous contacts was balanced against that derived from a battery with a potentiometer slide, using a telephone to detect the position of balance. Duncan (1891) devised an arrangement with electrometers of special design.

Differing in many respects from these electrometric or voltmeter methods is the method of Isenbeck (1883), which was used also by the author (see Edition I. of this work), applicable only to small armatures. It consisted in connecting an armature coil, or a small exploring coil wound upon the armature, to a suitable galvanometer, and then suddenly shifting the armature, between stops, through a small definite angle. The galvanometer deflection was proportional to the number of magnetic lines cut, and for a number of successive observations a curve of distribution of the field could be plotted.

Another method, due to Mordey (1884), consists in suddenly switching on or off the current which excites the field-magnets, and observing with a high-resistance ballistic galvanometer the current induced in one armature coil (or in a spiral exploring coil wound on the armature). This is repeated a number of times, the position of the armature being moved on each time through some definite small angle; and so a number of readings are obtained from the successive throws of the galvanometer. These two methods give curves of the field-distribution if the pitch of the exploring coil is equal to that of the poles; or of the no-load voltage if the coils used are the actual armature coils. They cannot be used to ascertain the somewhat distorted curves of electromotive-force or of field at full-load.

Method II. Point-by-point Contacts with Registration.—The point-by-point method was made self-registering by Blondel (1891), Ayrton (1889), Barr, Burnie and Rodgers (1895), Rosa (1897), Callendar (1898), and Hospitalier (1903). The apparatus of Barr, Burnie and Rodgers, and that of Hospitalier ("The Ondograph") includes a gearing which slowly and automatically shifts the contact-maker in position through a cycle of positions, and at the same time actuates a registering voltmeter or wattmeter. In the apparatus of Rosa, a potentiometer slide is worked by hand to produce a balance of electromotive-forces at each position—the movement of the slide being conveyed by pantagraph to a registering point on a slowly revolving drum. The curve so traced consists of a series of minute detached dots. Callendar's Cycle-curve Recorder (*Electrician*, Aug. 26, 1898) records with an ink pen, in a continuous line, the readings of an automatic potentiometer in which the potentials of particular successive points on the cycle of voltage are continuously balanced. By means of worm gearing the contact brushes are slowly revolved concurrently with the motion of the recording sheet. The recording pen is directly attached to the sliding contact of a delicate relay on the potentiometer bridge-wire. This relay is made to actuate a differential pair of motors so as to keep the sliding contact always at the balance point. The drawback of this apparatus is its slow action, about an hour being taken for the inscription of a single curve. Hospitalier's Ondograph (*Electrician*, lii. 298, Dec. 11, 1903) also draws a continuous curve with an ink pen actuated by the pointer of a voltmeter or a wattmeter. The mechanism is propelled by a small synchronous motor, geared down to give a set of successive contacts round a whole cycle in about three minutes. As the recording pen moves about a centre the curves are slightly distorted by the curvilinear movement.

Method III. Oscillographs.—Blondel (*La Lumière Électrique*, xli. 401 and 507, 1891; also *L'Industrie Électrique*, viii. 361, Aug. 28, 1899) and Duddell (*Electrician*, xxxix. 636, Sept. 10, 1897; *Journal I.E.E.*, xxviii. 1, 1899) have independently developed a method of recording photographically the curve of any cycle of voltage, however rapid by an apparatus known as an "Oscillograph." This is a development of the suspended-coil galvanometer, in which the moving part is reduced to a pair of stretched vertical wires lying between the poles of a very powerful electromagnet. The moving part thus has an extremely small moment of inertia, and a natural period of oscillation of the order of $\frac{1}{10000}$ second, so that it can follow the most minute variations during the cycle, or even the fluctuations that occur in non-cyclic phenomena, such as the discharge of a condenser. A minute fragment of silvered mirror attached to the wires reflects a beam of light upon a synchronously revolving mirror, and thence upon a photographic plate; or, for the purpose of eye observation, the moving spot may be reflected upon a screen or upon a sheet of ground glass.

Other continuous methods of kindred nature are those of Frölich mentioned above, and of Elihu Thomson (*La Lumière Électrique*, xxvii. 339, 1888) both of whom used a telephone diaphragm. A galvanometric apparatus using soft iron is described by Blondel in *Comptes Rendus*, clvi. 502 and 748, and in *Electrician*, xxx. 571, 1893; and similar devices have been suggested by Hotchkiss and Millis in the *Physical Review*, iii. 49 and 358; also by McKittrick in the *Transactions of the American Institute of Electrical Engineers*, xiii. 245, 1896. Gérard, in the *Bulletin de l'Académie de Belgique* of 1868, described a similar use of a suspended-coil galvanometer, and Moler (*Physical Review*, i. 214, 1893) has applied an ordinary permanent-magnet voltmeter to record on a smoked cylinder the voltage curves of dynamos. Crehore (*Physical Review*, ii. 122, and iii. 63, 1893) has applied an elegant optical method based upon the rotation of the plane of polarized light; and F. Braun (*Wiedemann's Annalen*, lx. 552) has used the magnetic deflexion of cathode rays in a special vacuum-tube to exhibit cyclical variations of current and voltage.

Method IV. Chemical Registration.—Janet (*Comptes Rendus*, cxviii. 862, 1894) has applied a chemical method of registration to the cyclical curves.

The following is a list of some papers on curve-tracing not discussed in the preceding account.

Townsend, F., *Amer. I.E.E.*, xvi. 75 and 160, 1899.
Mershon, *Electrical World*, xviii. 140, 1891.
Fleming (Cantor Lectures), *Soc. of Arts*, 1896.
Hicks, W. M., *Electrician*, xxxiv. 699, 1895.
Fessenden, R.A., *Electrical World*, xxviii. 688, 1896.
Lutoslawski, M., *Elektrotechnische Zeitschrift*, xvii. 211, 1896.
Abraham, H., *Comptes Rendus*, cxxiv. 758, and cxxv. 92, 1897.
Laws, F. A., *Amer. Acad. Sci.* xxxvi. 321, 1901.
Ayrton, W. E., *Proc. Inst. Civil Engineers*, xcvii. 76, 1889.
Nichols, E. L., *Amer. Ass. Adv. Sci.* xlii. 57, 1893.
Burch, *Electrician*, xxxvii. 532, 1896.
Armagat, *L'Éclairage Électrique*, xii. 346, 1897.
Blondel, *La Lumière Électrique*, xlix. 501, Sept. 16, 1893; *Comptes Rendus*, cxvi. 748, 1893; *Report of the International Congress of Physics*, Paris, 1900; *L'Éclairage Électrique*, xxi. 41 and 161; xxxiii. 115, 1902.
Lyle, T. R., *Philosophical Magazine*, ser. 6, vi. 549, 1903.
Gehrcke, E., *Zeitschrift für Instrumentenkunde*, 1905, p. 34.
Robinson, L. T., *Proc. Am. Inst. Elec. Engs.*, April 1905.

From amongst the investigations mentioned, may be singled out one due to Ayrton, who analysed out the harmonics experimentally by an extremely elegant process of resonating electrically. Using an electro-dynamometer with separated coils, he sent the current to be tested through one coil, whilst through the other he sent an alternating current the frequency of which could be varied between wide limits. The instrument gave no reading except when the second current possessed a frequency accurately coincident with that of one of the harmonics of the current tested. Thus the different harmonics present could be picked out one by one, and individually measured. As the turning moment in the instrument is proportional to the product of the currents in the two coils, it will be seen that sending through one of the coils an alternating current of the same frequency as one of the harmonics is physically the equivalent of the mathematical process of multiplying through by the sine of one frequency, as explained on p. 76.

CHAPTER V.

MAGNETIC LEAKAGE AND ARMATURE REACTIONS IN ALTERNATORS.

IN the design of continuous-current machines it is customary to assume that for each type of machine the *dispersion coefficient*, or coefficient of allowance for magnetic leakage (usually denoted by the Greek letter ν), is a constant. This assumption is not strictly accurate, since owing to armature reaction, the dispersion increases somewhat with the load; nevertheless it introduces no serious trouble into design.

In the case of continuous-current dynamos, the only disadvantage that arises from the presence of the stray field is that a slightly increased sectional area must be given to the pole cores, entailing a slight increase of copper in the exciting coils, with consequent slight increase of prime cost; or else if no increase of iron and copper is allowed, the increased magnetic saturation will involve a slight increase in the exciting current, and therefore, of the watts wasted on excitation.

For alternators, however, the case is far otherwise. The drop of voltage on an inductive load is mainly dependent upon the magnetic leakages, primary and secondary. They increase with the load, and, what is of even more importance they increase with the fall of the power-factor of the circuit on which they may be working. This is one reason why certain types of alternator, though satisfactory on a lighting circuit, have proved themselves very unsatisfactory when applied to a load consisting mainly or largely of motors. Hence a study of the magnetic leakages in alternators becomes of capital importance to a designer of such machines.

FIELD-MAGNET DISPERSION.

Let the magnetic flux, that is to say, the whole number of magnetic lines in the core of one pole of an alternator, be called N ; the number being in modern alternators from 2,500,000 to 50,000,000. Of the total a certain fraction leaks away and is dispersed, so that it does not enter the armature. The useful flux that crosses the gap and enters the armature from one pole may be denoted by N_g. The portion that is dispersed and fails to enter the armature may be called the *stray flux*, and will be denoted by N_s. Then obviously,

$$N_m = N + N_s.$$

The ratio of total flux to useful flux is called the *dispersion coefficient* or *coefficient of allowance for leakage.* It is a number always greater than unity, and in alternators may vary from 1·18 to 1·3 or even to 1·5. It is denoted by the Greek letter ν. Hence by definition

$$\nu = \frac{\text{total flux}}{\text{useful flux}} = \frac{N_m}{N_g}.$$

The ratio which the stray flux bears to the useful flux is called by Rothert *the dispersion.* Its value varies from 0·18 to 0·3 or even in some machines to 0·5. It is here denoted by the Greek letter σ. Hence by definition

$$\sigma = \frac{\text{stray flux}}{\text{useful flux}} = \frac{N_s}{N_g}.$$

Now the value of a flux can be obtained either by dividing the magnetomotive-force by the reluctance of the path, or by multiplying it by the permeance of the path. Thus

$$N_s = M \times P_s;$$

and

$$N = M \times P ;$$

where P_s and P_g are the permeances of the stray path and of the main path respectively.

The main permeance P_g is the reciprocal of the sum of the reluctances of the air-gap, pole-core and yoke, the first of these being by far the greatest. The stray permeance P_s is the sum of the permeances of all the leakage paths from all sides of the pole-shoes and cores.

Obviously between ν and σ exists the simple relation

$$\nu = 1 + \sigma.$$

Hence

$$\nu = 1 + \frac{N_s}{N_g} = 1 + \frac{P_s}{P_g}.$$

The values of P_s and P_g depend upon a number of considerations, such as the shape and size of the pole-cores and pole shoes, their distance apart, the width of the air-gap, the number of exciting ampere-turns, the degree of saturation of the cores, and last but not least on the reaction of the armature.

Neither ν nor σ is a constant; for at full-load, and especially where the armature reaction is great, there is a greater leakage of magnetic lines. This has to be taken into account in designing.

As already remarked, in the design of alternators the leakage must be estimated for each machine, and must be estimated for different loads if the designer would avoid disappointment. Particularly necessary is this in the case of machines of the inductor type, since it is by the dispersion mainly that the voltage drop at full-load is influenced. The designer must know the various causes that contribute to the dispersion, and be able to control his design accordingly. In general, to keep the dispersion small the pole-cores should be short, and of minimum surface, the pole-shoes should not have too wide a span nor be too thick, nor present needless corners, and the axial length of the pole face and of the armature core should not be too great in proportion to the diameter of the working face. In general, to keep the increase of dispersion between no-load and full-load from undue magnitude, it is required that armature reactions shall be relatively small, that the peripheral density of the armature current (ampere-conductors per inch) be not too great, and that the pole-cores be not too highly saturated when excited for no-load.

The field-magnets of modern alternators are of various types (see p. 148). Setting aside for the present the inductor type of machine, there remain the A-type suitable only for high speed machines under 500 kilowatts, and the more usual B-type, with revolving magnets. But of the revolving magnet-wheel there are several varieties ; viz.

(*a*) Alternate radial poles with every pole wound (Fig. 92, p. 106) ;

(*b*) Alternate radial poles with half the poles wound (Fig. 96, p. 110) ;

(*c*) Imbricated poles, with but one exciting coil (Fig. 101, p. 116).

With the last-named the magnetic leakage is large and very variable with the load, and is extremely difficult to predetermine. This form is practically obsolete save for very small machines. The form (*b*), having coils only on the north-poles, and with the south-poles formed of projections cast on the wheel, is suitable only for small machines or high-frequency machines where there is difficulty in getting a sufficient number of poles close together. It has greater leakage than the (*a*) form, and has the defect that the leakage is unequal for the wound and the unwound poles. It also is obsolete save for such special cases. Hence the (*a*) form having all poles wound is the only one that need be considered in detail as to dispersion.

The general nature of the stray field between adjacent poles is shown in Figs. 200 and 201, relating respectively to straight poles and poles provided with shoes. The former are usually built up of laminations, and therefore have a section of rectangular outline : the latter may be of stampings, but may also consist of solid steel cores of circular or oval section to which separate pole-shoes are affixed. As the pole-cores themselves have always parallel sides, the space between them narrows slightly toward the roots. But in large magnet-wheels with 60 or 70 poles the minimum distance between the cores does not differ much from the maximum.

Examples.—I. In the *Oerlikon alternator* with 10 poles, Fig. 127, the pole-cores are 5·5 inches long, and the maximum and minimum distances apart are respectively 5·6 and 2·4 inches.

II. In the Heyland 64-pole machine (p. 407), the pole cores being only 5·3 inches long, the maximum and minimum distances apart are respectively 7·23 and 7·01 inches; a difference of only 3 per cent.

Across this slightly V-shaped space the stray field passes in lines that, save near the outer part, are nearly straight. Quite straight they would not be, even were the sides parallel, because the difference of magnetic potential increases from the roots towards the pole ends. At the roots, where the cores are attached to the yoke, the magnetic potential difference is almost zero,[1] while at the tops of the cores there is a difference of magnetic potential equal to the sum of the ampere-turns on the two cores, tending to drive magnetic lines across. This difference of magnetic potential increases regularly all the way up the cores from root to top: hence the average value may be taken as equal to the ampere-turns on

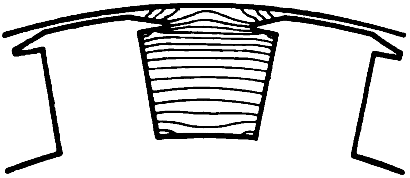

FIG. 200.—STRAY FIELD BETWEEN ADJACENT POLES WITH SHOES.

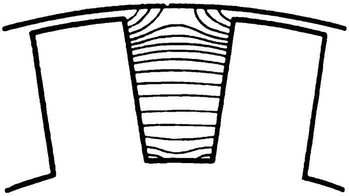

FIG. 201.—STRAY FIELD BETWEEN ADJACENT STRAIGHT POLES.

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[1] It would be exactly zero if there were not a perceptible reluctance offered by the joints and by the metal of the yoke. The reluctance of the joint causes a few of the lines to take paths through the air by a leakage which adds to the useful flux.

one core. The stray field, therefore, will steadily increase in density from the bottom upwards. The process of calculation is given below.

In addition to this stray field between the pole-cores there is also a stray field between the projecting tips or edges of the pole-shoes, see Fig. 200. In some machines the dispersion due to the pole-shoes is greater than that between the flanks of the cores.

There is also a lateral dispersion between adjacent poles as indicated in Fig. 202, where 3 adjacent poles are shown in plan. Owing to the great distance between the poles and the shaft, bearings, and iron housing, etc. no other dispersion of the primary field need be taken into account.

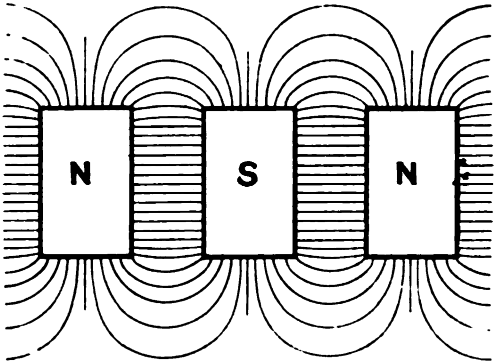

FIG. 202.—LATERAL FIELD BETWEEN ADJACENT POLES.

The whole subject of the calculation of magnetic circuits and of dispersion coefficients is treated in Chapter VI. Vol. I. of the present work; and in the Author's book *The Electromagnet.* Hence a more brief statement of the practical method of calculating the dispersion of alternators will suffice here.

RULES FOR ESTIMATING MAGNETIC LEAKAGE.

For estimating leakages, we may make use of the following rules derived from the theorems[1] of Professor George

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[1] *Journ. Inst. Elec. Eng.*, xv. 551, Nov. 25, 1886.

Forbes, but adapted to our notation. Writing N_s for any stray flux, and M for the magnetomotive-force, in ampere-turns, which produces the leakage, we have—

$$N_s = M \times P_s;$$

where P_s is defined by the general formula—

$$P_s = \text{permeance of stray path}$$

$$= \frac{\text{mean sectional area of stray path}}{\text{mean length of stray path}} \div 0{\cdot}3133.$$

The coefficient 0·3133 signifies here, as elsewhere, the number of ampere-turns required to drive a flux of a density of one line per square inch along a path 1 inch long. It is in act the reluctivity of air in inch units.

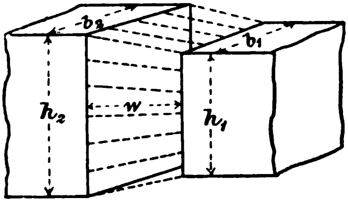

FIG. 203.

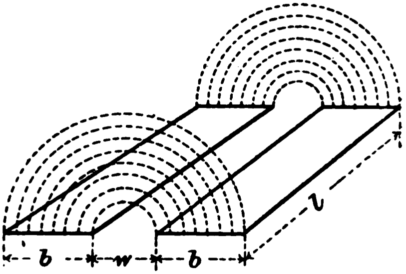

FIG. 204.

RULE I. *Leakage between Parallel Faces.*—On the assumption that the width w between the parallel areas is not relatively large, and that the flux between them follows straight lines, as in Fig. 203, we have—

$$\text{Permeance} = \frac{\frac{1}{2}(b_1h_1 + b_2h_2)}{w} \div 0{\cdot}3133 \quad . \quad [\text{I.}]$$

RULE II. *Leakage between Two Equal Contiguous Areas* (Fig. 204).—On the assumption that the flux follows semicircular paths, we may deduce the following.

From any narrow strip, of breadth dx and length l, the permeance to the corresponding strip is $\frac{l \times dx}{\pi x} \div 0{\cdot}3133$;

where x is the distance of the strip from the centre-line. Hence, integrating for the total breadth b, we get—

$$\text{Permeance} = \int_{x=\frac{1}{2}w}^{x=\frac{1}{2}w+b} \frac{l \,.\, dx}{\pi x} \div 0{\cdot}3133$$

$$= \frac{2{\cdot}3}{0{\cdot}3133} \,.\, \frac{l}{\pi} \,.\, \log \frac{\frac{1}{2}w + b}{\frac{1}{2}w};$$

$$= 2{\cdot}34\, l \,.\, \log\left(1 + \frac{2b}{w}\right). \quad . \quad .[\text{II.}]$$

RULE III. *Leakage between Two Equal Areas at a Greater Distance Apart* (Fig. 205).—On the assumption that the flux follows approximately quadrantal paths joined by straight pieces, we have similarly—

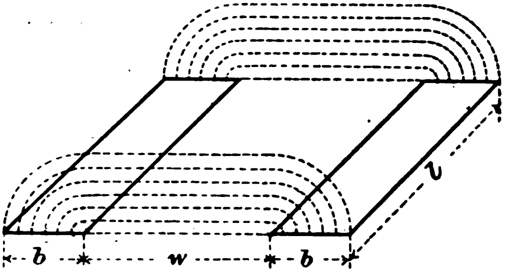

FIG. 205.

$$\text{Permeance} = \int_{x=0}^{x=b} \frac{l \,.\, dx}{\pi x + w} \div 0{\cdot}3133$$

$$= \frac{2{\cdot}3}{0{\cdot}3133} \,.\, \frac{l}{\pi} \,.\, \log \frac{w + \pi b}{w};$$

$$= 2{\cdot}34\, l \,.\, \log\left(1 + \frac{\pi b}{w}\right) \quad . \quad .[\text{III.}]$$

It will be noted that the difference between cases [II.] and [III.] is that it is assumed that when the two surfaces are near together the lines take more highly curved, that is, more reluctant paths. The second term in the brackets in the expressions for the permeance varies correspondingly from $2b \div w$ to $3 \cdot 14 b \div w$.

In cases where these rules cannot be readily adapted, a graphic method may be resorted to. The forms of the leakage paths are sketched out from experience in curves, and the whole stray field is considered as dissected into a number of curved tubes of flux, all in parallel. The permeance of each is then estimated, by the general formula given above, in terms of its mean area and mean path-length, and the sum of all such permeances gives the total permeance. When applying either the special rules or the graphic method to the leakages of field-magnets, it must not be forgotten in using such permeances to deduce the flux, that the magnetomotive-force between adjacent poles is not uniform ; being a maximum between the pole-shoes, and diminishing downwards as we go along the pole-cores to zero at the roots of the poles, which, provided the joints between them and the yoke offer no material reluctance, are at the same magnetic potentials.

APPLICATION TO LEAKAGE OF ALTERNATOR FIELD-MAGNETS.

Consider first the case where the pole-cores are of rectangular form. Fig. 206 indicates the meaning of the symbols used.

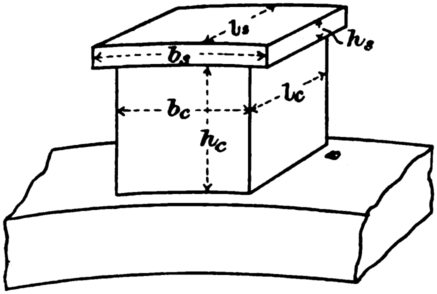

FIG. 206.

We may consider the stray flux as made up of 4 parts :—
N_1 = the flux between the inner faces of the pole-shoe rims ;

N_2 = that between the lateral faces of the pole-shoe rims; N_3 = that between the opposed pole-core faces; N_4 = that between the lateral faces of the pole-cores, following curved paths.

The magnetomotive-force which is effective for producing these stray fluxes is the total number of ampere-turns that are applied to drive the useful flux round one complete magnetic circuit, less the number needed to drive that flux through the two pole-cores and the piece of yoke that joins them. Let us denote this net number of ampere-turns by the symbol M.

Then, applying formula [I.] we have—

$$\begin{aligned} N_1 &= M \times P_1, \\ &= \frac{M}{0{\cdot}3133} \times \frac{l_s \times h_s}{\tau_s - b_s} \times 2. \end{aligned}$$

The factor 2 is to take into account the stray flux from the faces at the two opposite sides of the pole.

Applying rule [II.] for the leakage from half the pole breadth on one pole-shoe to half the pole breadth on the neighbouring pole-shoe, we have, for the total leakage in this respect—

$$\begin{aligned} N_2 &= M \times P_2 \\ &= M \times 2{\cdot}34 \times h_s \times \log\left(1 + \frac{b_s}{\tau_s - b_s}\right) \times 4; \end{aligned}$$

the multiplication by 4 being to take into account the leakage from both halves of the two pole-sides, back and front.

Similarly,

$$\begin{aligned} N_3 &= M \times P_3 \\ &= \frac{M}{0{\cdot}3133} \times \frac{1}{2} \times \frac{l_c \times h_c}{\tau_c - b_c} \times 2; \end{aligned}$$

the division by 2 arising from the mean magnetomotive-force being half that at the polar ends; and the multiplication by 2 as explained for N_1; and

$$\begin{aligned} N_4 &= M \times P_4 \\ &= M \times 2{\cdot}34 \times h_c \times \log\left(1 + \frac{b_c}{\tau_c - b_c}\right) \times 4 \times \tfrac{1}{2}. \end{aligned}$$

Then, since $N_s = N_1 + N_2 + N_3 + N_4$,

$$\text{or } P_s = P_1 + P_2 + P_3 + P_4,$$

we have the dispersion coefficient

$$\nu = \frac{N_a + N_s}{N_a} = 1 + \frac{N_s}{N_a} = 1 + \frac{P_s}{P_a}.$$

This will have different values at full-load and at no-load, for N_a depends on the saturation of iron as well as on air (in the gap), and it is proportional to the total ampere-turns on the magnetic circuit, not on that part only which is taken into account in M.

To calculate the leakage between cylindrical pole-cores, it suffices to substitute for the cylinders cores of octagonal section and equal perimeter. If the cores are not very near together, and nearly parallel (as in cases of machines having over 20 poles), it suffices to substitute square pole-cores of equivalent perimeter.

As an example of the above we will take the case of the 1500 KVA generator described on p. 397, of which sectional views are shown in Fig. 333 and the excitation curve in Fig. 334.

The following is a table of the dimensions (inches) required for the calculation:—

τ_s	Pole pitch at face = 10·29
b_s	Pole-shoe span = 6·69
$\tau_s - b_s$	Width between pole-shoes = 3·6
h_s	Depth of pole-shoe = 0·8
l_s	Length parallel to shaft = 8·5
τ_c	Pitch at centre of core = 9·88
b_c	Breadth of core = 5·11
$\tau_c - b_c$	Width (average) between cores = 4·77
h_c	Length of core radially = 7·6
l_c	Length parallel to shaft = 8·5
l_g	Length of air-gap = 0·275

P_1, the permeance across the inside faces of the pole-shoes,

$$= \frac{8·5 \times 0·8}{3·6 \times 0·3133} \times 2 = 12·1.$$

P_2, the permeance round from the sides of the pole-shoes,

$$= 4 \times 2 \cdot 34 \times 0 \cdot 8 \times \log\left(1 + \frac{6 \cdot 69}{3 \cdot 6}\right)$$
$$= 3 \cdot 4.$$

P_3, the permeance across the inside faces of the pole-cores,

$$= \frac{7 \cdot 6 \times 8 \cdot 5 \times 2}{2 \times 4 \cdot 77 \times 0 \cdot 3133} = 43 \cdot 2.$$

P_4, the permeance round from sides of pole-cores,

$$= 2 \cdot 34 \times 7 \cdot 6 \times \log\left(1 + \frac{5 \cdot 11}{4 \cdot 77}\right) \times 2$$
$$= 2 \cdot 34 \times 7 \cdot 6 \times 0 \cdot 317 \times 2$$
$$= 11 \cdot 3.$$

The total permeance P_s of the stray paths is

$$12 \cdot 1 + 3 \cdot 4 + 43 \cdot 2 + 11 \cdot 3$$
$$= 70.$$

Next to calculate P_g, the permeance of the path for the useful flux.

For a first approximation we may calculate the permeance of the air gaps,[1] and then allow 20 per cent. for the magnetization of the iron parts.

This permeance of two gaps

$$= \frac{1}{0 \cdot 3133} \times \frac{b_s \times l_s}{b_g \times 2}$$
$$= \frac{6 \cdot 69 \times 8 \cdot 5}{0 \cdot 3133 \times 2 \times 0 \cdot 275}$$
$$= 331$$

and allowing 20 per cent. for iron parts, permeance P_g of the whole magnetic circuit

$$= \frac{331}{1 \cdot 20} = 276$$

$$\therefore \quad \nu = 1 + \frac{70}{276} = 1 \cdot 25^{1}$$

---

[1] Both gaps must be taken in in this calculation, because M is the total magnetomotive-force producing leakage flux, i.e. it is the sum of the magnetomotive-forces on two adjacent poles.

This value may now be used to calculate a saturation curve.

When this is done, we may estimate ν to a nearer approximation at the different degrees of excitation by obtaining the values of P_g from the curve obtained as in Fig. 334, p. 398.

At that point on this curve where the iron parts take 20 per cent. of the excitation required for the air-gaps, ν will again come to 1·23, but when the iron is more highly saturated it will be higher.

The following table has been made out from the example already given, columns 2 and 3 being the values calculated for a uniform dispersion coefficient of 1·25. Column 4 gives the permeance of the magnetic circuit on this basis, and the last column gives the new values of the dispersion coefficient for excitations above and below the normal, which is about 6000 ampere-turns per pole.

Volts.	Flux (N_a) (megalines).	Ampere-turns per Magnetic Circuit (Pair of Poles).	Permeance of Magnetic Circuit.	Values of ν.
2000	1·22	3,850	317	1·22
3000	1·82	5,940	307	1·23
4000	2·43	8,040	301	1·23
5000	3·04	10,480	290	1·24
5500	3·34	11,880	281	1·25
6000	3·64	14,340	253	1·27
6500	3·95	17,840	222	1·32
7000	4·25	23,100	184	1·38

Some designers in using the above formulæ take as M the whole-ampere-turns on the magnetic circuit without deducting the ampere turns needed to drive the flux through the joint, the piece of yoke, and the iron of the pole-core. Further some designers omit any calculation for the lateral dispersion (Fig. 202) which undoubtedly adds something to the stray field. The omission of these refinements leaves two slight errors, which, as they are of opposite influence, may generally be neglected. If the coils are not wound uniformly to the same depth, but (as is sometimes preferred) are thickened by additional layers near the poles, the mean path must not be taken as being at a point half-way along the core, but at some higher point half-way along the total number of windings on the core.

Dispersion in Inductor Machines.—In inductor machines, which have two crowns of polar projections, all the poles in one crown being N-poles, and those in the other crown S-poles, there is always a stray field of considerable magnitude. And to calculate the reactions in such machines much care is needed in estimating the dispersion. In these machines it should be remembered that the working flux is the *difference* between the iron-flux and the air-flux. If the iron is much saturated at low loads, then higher excitation gives very little rise in the voltage.

ARMATURE REACTION IN ALTERNATORS.

Every conductor carrying a current creates a magnetic field around itself, whether it is embedded in iron or lies in air. Armature conductors therefore create magnetic fluxes around themselves, and these fluxes will, in part, interfere with the main flux from the poles of the field-magnet, distorting or diminishing it, and will in part form stray fluxes linked around the armature conductors, tending to choke the armature currents. Part of this interference with the main flux will be *permanent*, part will be *periodic*. All effects which are periodic, whether in the form of local stray fluxes, or in the form of armature fluxes which link themselves around the exciting circuit as well as around the armature conductors, act as a true self-induction, and tend to choke the armature current. In any closed path, surrounding any armature conductor or group of armature conductors, the line-integral of the magnetizing forces is simply proportional to the number of amperes so surrounded. If ΣC stands for the whole number of amperes (or, as some writers would say, the whole number of ampere-wires) that are surrounded by any closed path, the magnetomotive-force around the path is, in C.G.S. measure, equal to $\frac{4\pi}{10} \times \Sigma C$. The value of this magnetomotive-force is independent of the actual length of path, and of the question whether the path lies in air or in iron, or partly in both. For example, if in Fig. 207 there are represented 6 conductors, each carrying 40 amperes, lying together in a slot, the total current through that slot is 240 amperes, and the magnetomotive-force around any the three paths indicated by the three thin lines, will have the value 301·6 C.G.S. units. The actual intensities of the fluxes will not be equal round these three paths, but will be proportional to their respective permeances. If the current is alternating, the fluxes will of course also be alternating. If, at any moment, the currents in conductors in neighbouring slots are flowing in the same sense, they tend not only to produce local fluxes around each slot, but also to produce a common flux which surrounds the group. Some of the lines of this flux may pass by the tooth-tips from tooth to tooth, some of them may pass along the air-gap, as shown in Fig. 208. If an iron pole-shoe be near, some of the lines may cross the gap, and pass along the pole-shoe. Where the armature coils project out into the air as end bends at the flanks of the armature core they will be surrounded by stray fields of their own, which may in part traverse the cast-iron end-shields or other neighbouring portions of iron.

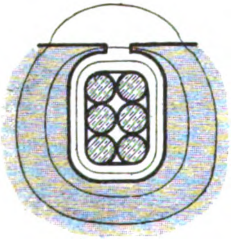

FIG. 207.

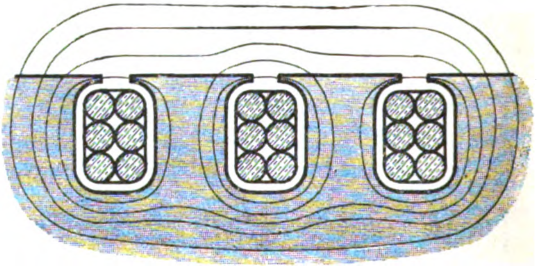

FIG. 208.

The armature conductors of an alternator, whether wound for one phase only, or two, or three, are always grouped in loops or coils which are spaced with symmetry around the periphery so as to correspond to the magnet poles. Because of this grouping, the currents in them will always create in the armature a set of armature poles ; and there will therefore always be a tendency on the part of these currents to produce stray fields from pole to pole of the armature. These armature magnetic fields will under some circumstances pass, as in Fig. 208, along the air-gap from pole to pole, or may cross the gap and pass along the pole-shoes, or under other circumstances, may enter the pole-cores of the field-magnet and interlink themselves with the magnet coils. It is needful to consider these cases more closely.

ARMATURE REACTION IN SINGLE-PHASE ALTERNATORS.

Consider first the case of a single-phase machine, represented in Fig. 209 as having but one slot per pole. Assume that the circuit is non-inductive, and that there is no lag, or $\cos \phi = 1$. In that case the currents will be at their maximum values at the moment when the middles of the moving poles are opposite the slots. The thin lines indicate the flux-paths from the armature magnetomotive-forces, the direction of which is such as to produce conjointly in the armature a set of secondary poles s, n, etc., as shown. The reaction of the armature will, in this case, be to produce a pure *distortion* in the polar fluxes, as in Fig. 210; and this distortion will be pulsatory, ceasing at the moment when the currents are zero, which will occur each time that the poles have moved to a position intermediate between the slots.

Next assume that the circuit is highly inductive, with a lag of 90°, $\cos \phi = 0$. In this case the armature currents attain their maxima when the magnet poles shall have moved on through a ½ pole-pitch, that is a quarter-period later. Fig. 211 represents this state of things. The armature currents now tend to produce the secondary poles n, s, etc. at times when they will *oppose* the polarity of the passing pole, and tend therefore to *demagnetize*, not to distort. The thin lines show magnetomotive-force due to the currents as before. In the single-phase machine this demagnetizing reaction which thus occurs when the current is a lagging one, will be a pulsatory effect, each passing pole suffering a demagnetizing impulse as it passes through the position shown. [It is obvious that in the case of an alternator feeding a condenser, where the currents lead

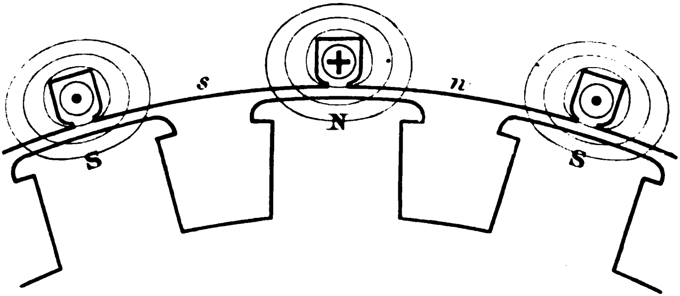

FIG. 209.—ARMATURE CURRENTS. (Case I., no lag, tendency to distort.)

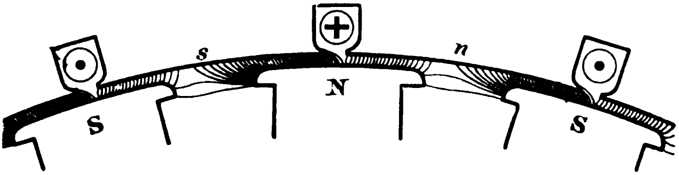

FIG. 210.—RESULTING DISTORTION OF MAGNETIC FLUX.

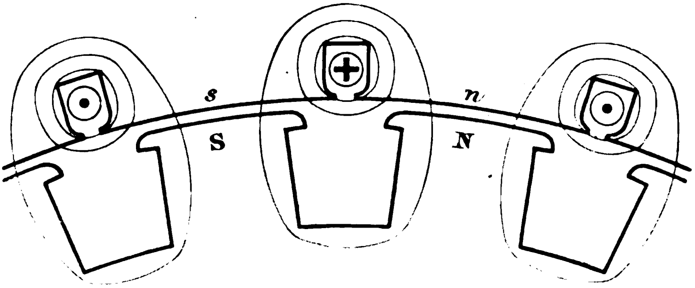

FIG. 211.—ARMATURE CURRENTS. (Case II., lag 90°, tendency to demagnetize.)

instead of lagging, the reaction will be a *magnetizing*, not a demagnetizing one.] If the current has a partial lag, the armature reaction will be partly demagnetizing. Let the angle of lag be ϕ; then *the distorting action is approximately* *proportional*[1] to cos ϕ, *and the demagnetizing reaction to* sin ϕ. The proportionality is inexact, even if the currents follow a pure sine-function of the time, because of the distribution of the field depending on shape of polar surface, the effect of the slots in changing the field distribution, and the complication introduced by the properties of iron.

We see then, in general, that the reaction of a single-phase armature may be regarded as due to a pulsating set of armature-poles, of alternating sign, which appear fixed in space (if the armature is fixed), and which time themselves so that on their emergence they either distort or demagnetize according as whether the current is in phase with the voltage (no lag), or out of phase (lag 90°) with it. In an alternator working as a *generator* the distortion is always a distortion backwards, that is one which will by its magnetic pull tend to oppose the movement of the rotating part. In an alternator working as a synchronous *motor* the distortion is a forward distortion tending by its magnetic pull to help the movement of the rotating part; being in fact the pull which drives the machine magnetically.

ARMATURE REACTION IN THREE-PHASE ALTERNATORS. PROGRESSIVE MAGNETIC FIELD.

If we pass to the case of three-phase generators—and the arguments are applicable to two-phase machines also—we find further considerations to enter. Owing to the very arrangements of polyphase machines there are two, three or more sets of conductors carrying currents that succeed one another in regular succession, as explained on pages 35 to 39. Before a current in any one of the phases dies away, another current in the next phase has grown up in a neighbouring conductor or group of conductors. Hence their magnetic

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[1] Another way of treating the matter is to regard the current as being compounded (see p. 28), of a working component, and a wattless component. The former is proportional to cos ϕ, the latter to sin ϕ. The working component tends to distort only: the wattless component tends to demagnetize only.

fluxes are combined, and shift progressively forward around the armature, following the rotation of the magnet-poles. This progression of the magnetic field of an armature, which here we study as a reaction, is the very foundation of all polyphase motors. The *rotating magnetic field*, or *progressive field*,[1] may be illustrated in the case of three-phase currents by Fig. 212.

In this figure three currents in conductors marked A, B and C to distinguish the three phases, are supposed to be alternating with the same values as those set out on p. 7; each current in turn rising to 100 amperes and falling to − 100 amperes. At the moment when the currrent in the A-phase is zero, that in the B-phase will be − 87 amperes, and that in the C-phase + 87 amperes. But as the three sets of conductors are grouped the return-conductor for the C-phase will lie between the A and B conductors. The curves shown are merely magnetomotive-force curves; the form of the actual lines of the field would depend greatly on the distribution of iron about the conductors. In the stage represented in the top line, the curves surround the B and C conductors. In the stage represented by the second line, the A current has grown to 13 amperes, the B current has changed to 92 amperes and the C current has decreased to 79 amperes. The eye can easily follow the changes of the magnetic curves from stage to stage. Only a quarter-period is shown; and the field during this time progresses through half a pole-pitch.

Superposition of Fields.—We are now in a position to apply the principle of the superposition of magnetic fields to this matter. In the series of sketches in Fig. 213, magnetic curves representing the effect of the armature currents in several different cases are superposed upon the magnetic curves assumed to be due to the field-magnet. The uppermost line shows the primary field due to the exciting coils on the magnet

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[1] See Chapter XIV. on Induction Motors. The reader is also referred to the Author's treatise on *Polyphase Currents*. A rotatory or progressive magnetic field, if thus produced by the interaction of two or more alternating currents, is called on the Continent a *Ferraris field* in honour of the late Galileo Ferraris, who in 1888 described it. A motor having a progressive field (2-phase), operated by commuted battery-currents, was described in 1878 by Mr. Walter Baily.

poles, here represented by 12 lines from each pole. They are shown passing into the armature teeth in two principal positions—(*i*) where the middle of the pole is opposite a tooth, (*ii*) where the middle of a pole is opposite a slot. In the second

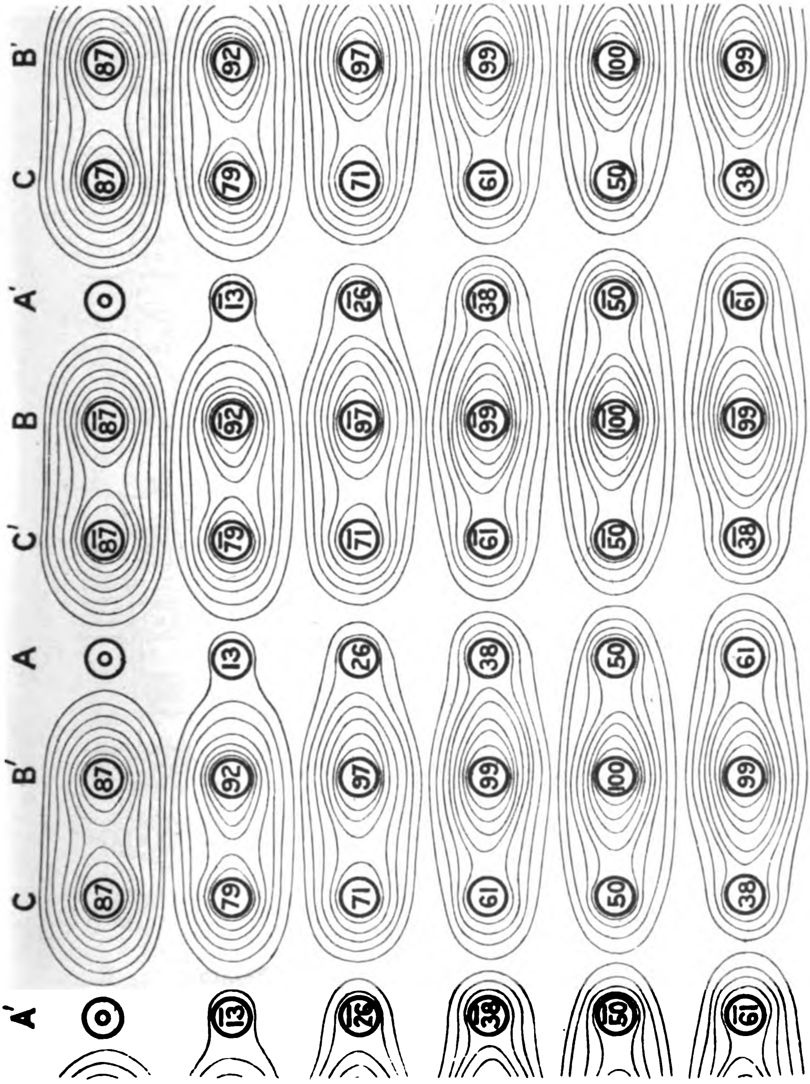

FIG. 212.—PROGRESSION OF MAGNETIC FLUX DUE TO THREE-PHASE CURRENTS.

line is shown the field due to the armature currents on the supposition that there is no lag, and that the magnets are not

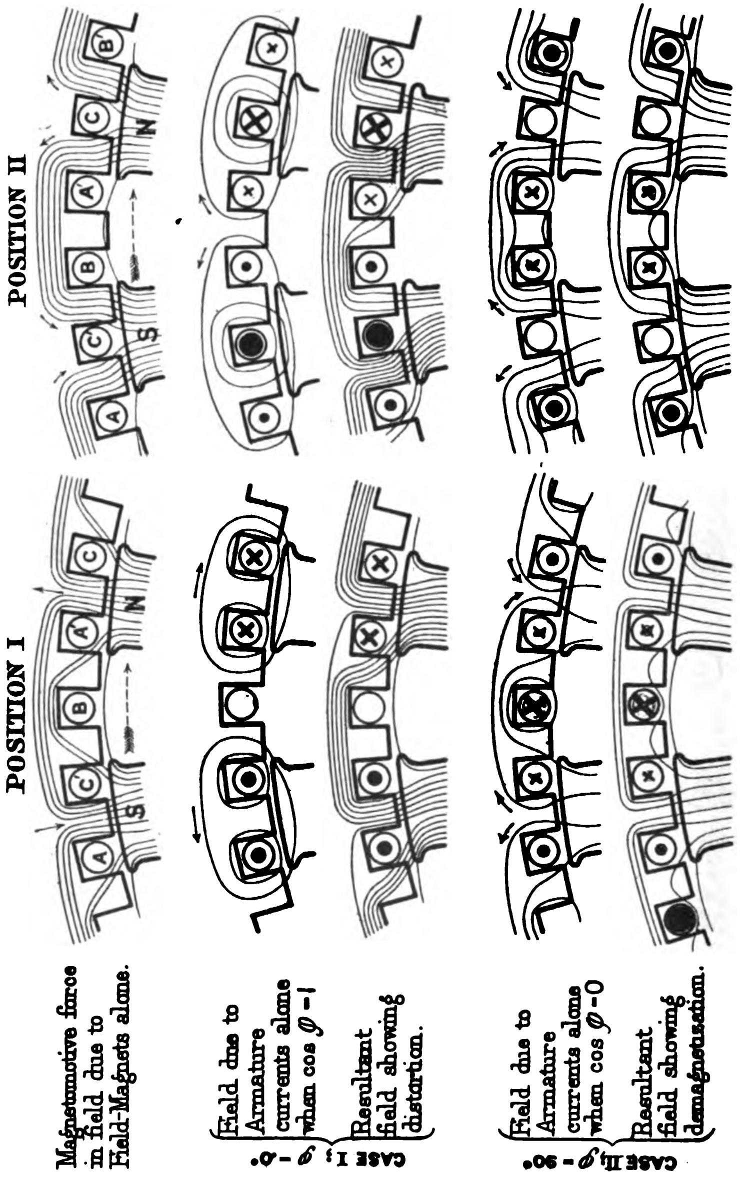

FIG. 213.

excited. If there is no lag the places of strongest current will be opposite the poles. As shown in the right hand figure when the current in one phase (C) is at its maximum those in the other phases (A and B) will be of half strength. In the left hand figure when the current in one phase (B) is at its zero value· those in the other phases will be of equal value, of 87 per cent. of the maximum. In the third line is shown the effect of superposing these fields due to the current upon those due to the magnets as depicted in the first line. Inspection of this resultant field shows how the armature current distorts the field without altering the total number of lines per pole. In the fourth and fifth lines of Fig. 213 are shown the effects of a lagging current. A lag of 90° is assumed; and in that case the maximum current occurs in any conductor one quarter period after the pole has passed, or at a distance of half a pole-pitch behind the middle point of the pole, as in the fourth line. When these armature fields are superposed on those of the magnets in the first line, the resultant fields are those depicted in the fifth line. On inspection it will be seen that in this case there is no distortion; but there is a diminution of the flux from each pole, as the lines due to the armature currents, tending to pass through the pole-cores in the sense opposite to those of the primary magnetism, must be deducted from the total. The twelve lines per pole are correspondingly reduced to eight; and, of these eight, four go astray constituting a leakage field. This illustrates the effect of a lagging current in demagnetizing the field-magnets and in increasing the dispersion.

DISTRIBUTION OF THE PROGRESSIVE ARMATURE FIELD.

Since, then, the regular succession of the armature currents produces a regular progression of the field due to them, it is natural to inquire whether it is not possible, in the case where the three currents are of equal virtual value, to treat the resulting field as a (multipolar) magnetic field in which each pole is of uniform strength (proportional to the virtual value of the currents) revolving at a uniform speed. Now obviously the accuracy of any such correspondence must be affected by the degree to which the distribution of the progressing field is affected by the presence of the slots and tooth-tips; it will also be affected by the distribution of the conductors as to whether they are concentrated, as in a one-slot winding (meaning one slot per phase per pole) or are distributed in a larger number of slots. We may, however, approach the matter by considering how the three magnetomotive-forces superpose themselves.

Consider a diagram of an ordinary 3-phase winding, Fig. 214, having one slot per phase per pole, viewed as if the armature teeth were presented toward the reader. The "teeth" are named *o*, *p*, *q*, *r*, etc. Then it will be clear on inspection that the tooth *p* will be subject to the magnetizing action of the ampere-turns of the coil A, while *q* will be subject to those of the coils A and B acting together, since it is surrounded by the circulation of current in both, and *r* will be subject to the current of coil B alone. Suppose that each coil consists of one turn only, and that the three currents have successive values as set down in the table on p. 7. It will then be seen that the magnetomotive-forces, reckoned in ampere-turns acting around each of the teeth will be as follows:—

—	*p*	*q*	*r*	*s*	*t*	*u*	*v*
At instant 1 . .	+100	+ 50	− 50	−100	− 50	+ 50	+100
„ 2 . .	+ 97	+ 71	− 26	− 97	− 71	+ 26	+ 97
„ 3 .	+ 87	+ 87	0	− 87	− 87	0	+ 87
„ 4 . .	+ 71	+ 97	+ 26	− 71	− 97	− 26	+ 71
„ 5 . .	+ 50	+100	+ 50	− 50	−100	− 50	+ 50
„ 6 . .	+ 26	+ 97	+ 71	− 26	− 97	− 71	+ 26

It is evident that if the permeance over the face of each tooth were uniform and constant, the flux thus made by the armature currents through the face of each tooth would be uniform in distribution, but would vary in amplitude as a sine-function of the time. This field, so distributed, will obviously be progressive, moving forward from tooth to tooth, as the revolving poles of the magnet wheel which create the armature currents move forward. But it presents a certain discrepancy from being purely a progressive field; for since the magnetomotive-force (as considered from point to point along the armature face) changes abruptly from tooth to tooth, the distribution of the field over any pole-pitch at any instant will not follow a sine-curve, but will be irregular. The distribution if plotted as a curve (assuming uniform permeance across the gap) will present square corners. Fig. 215 shows some of the distributions for a three-phase one-slot winding, and Fig. 216 that for a three-phase three-slot winding. These curves relate to magnetomotive-forces, and are only approximately realized in the actual magnetic fields, for the actual fields will be affected by the relative positions of the pole-faces, and by their slope, and also by the width of the slots, the spreading of the field in the gap, and the degree of saturation of the teeth. If the form of the distribution of the field were accurately known in any particular case—for example Fig. 186—it would be possible to resolve the curve of field distribution into harmonic constituents. This indeed has been done by Arnold and La Cour[1], and by Henderson and Nicholson.[2] It can then be shown that if the harmonics of the field-distribution are neglected, and the fundamental sine-distribution is alone regarded, and if the armature currents are also assumed to follow a sine-function of the time, the superposition of the

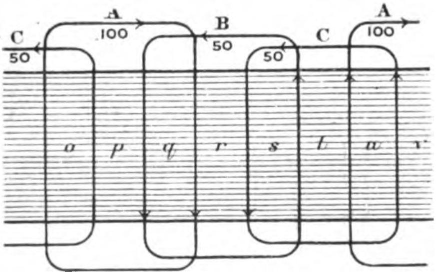

FIG. 214.

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[1] *Die Wechselstromtechnik*, iii. 295.

[2] *Electrician*, liii. 642, Aug. 5, 1904; also *Journ. Inst. Elec. Eng.*, xxxiv. 465, 1905.

field components due to the three phases will yield a rotating field which has a sine-distribution in space, progressing through the double pole-pitch in one period with a uniform velocity, and of unvarying amplitude. Similarly, if the fundamental field-component is neglected and the third harmonic alone is regarded, the resultant due to the superposition of the three phases cancels out. If the fifth harmonic of the field distribution is alone regarded, the resultant due to the superposition of the three phases yields a rotating field having five waves

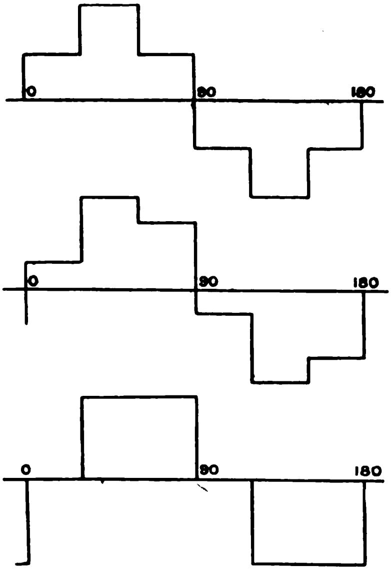

FIG. 215.

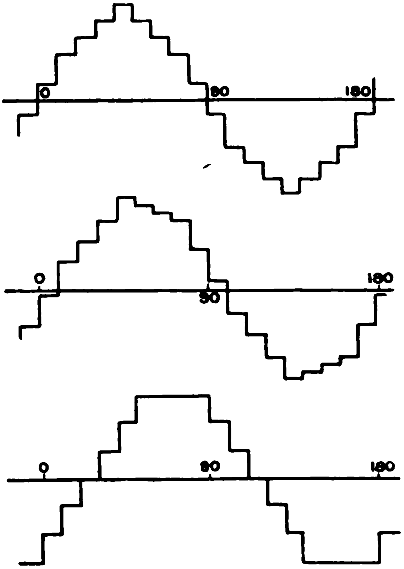

FIG. 216.

in the breadth of the double pole-pitch, progressing in a negative direction with a velocity one fifth of the fundamental field, and of relatively small amplitude. Similarly the seventh components yield a rotating field of seven times as many poles, progressing positively with a velocity one seventh of the fundamental, and of still smaller amplitude. The higher harmonics yield still smaller partial fields: those of the eleventh and seventeenth orders revolving backwards, those of the thirteenth and nineteenth orders revolving forwards, while the ninth, fifteenth and twenty-first orders disappear as the third one. These subsidiary harmonic revolving fields are of small importance: they are partly eliminated by increasing the number of slots over which the winding is distributed ; and the eddy currents which they set up in the pole-faces or in the amortisseurs around the poles, tend to damp them out of existence. The net result is that the interplay of the three currents in the armature windings sets up a system of revolving poles, which may in a sense be regarded as the *images* of the revolving magnet-poles, which images follow the magnet-poles as the image of a candle in a mirror follows the movements of the candle when it is carried past. But these image-poles will not always be exactly opposite to their respective poles. If the circuit is non-inductive the images will be situated exactly mid-way between the poles ; their reaction then being purely a distorting or cross-magnetizing one. If the load is purely inductive, the image-poles will lag so as to come exactly opposite the field-poles, a North pole in the image system being opposite a North pole in the field-magnet system, therefore tending to demagnetize. If the circuit is completed through a condenser, so that the currents lead by 90°, then the images will be shifted *forward* from the middle point, so that again they come opposite the field-poles, a South pole in the image system coming opposite a North pole in the magnet system, therefore tending to increase the magnetization.

PRESSURE REGULATION IN ALTERNATORS.

As in continuous-current machines, the pressure regulation of alternators is defined either as (*a*) the percentage voltage *drop* from the rated value when full-load is suddenly switched on, or (*b*) the percentage *rise* from the rated value when full-load is suddenly switched off. As it is easier to keep within a specification based upon the second method by using designs which otherwise are disadvantageous with respect to the general properties of the machine, it is as well to specify according to the first method.[1]

There are several methods of great or lesser accuracy for

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[1] See a letter by Miles Walker in *Electrician*, lii. 747.

determining, either by direct calculation or by simple experiment on the finished machine, what are the regulating properties of a machine.

The facts that are to be taken into account are more complicated than those involved in the corresponding problem for continuous-current machines. In the case of alternators there are the following matters tending to cause a drop :—

(1) The ohmic resistance of the armature, causing the "copper-drop."

(2) That portion of the armature reaction (*cf.* p. 255) which is *permanent*, and which, therefore, has a demagnetizing effect. It increases as the angle of lag ϕ increases, or as the power-factor cos ϕ diminishes.

(3) The diminution of useful flux in consequence of the cross-magnetizing effect of the armature current, which distorts, and which, therefore, as a secondary effect, increases the saturation. This reaction is approximately proportional to cos ϕ, and is, therefore, greatest when the external circuit is purely non-inductive. It is relatively unimportant.

(4) That portion of the armature reactions which are *periodic* (*cf.* p. 255) and which, therefore, constitutes a self-induction which causes a voltage drop in the ordinary way.

Of these four effects the first and fourth only produce actual voltage drops on the generated electromotive-force, while the second and third act primarily in reducing the flux.

ESTIMATION OF ARMATURE REACTIONS.

(1) *The Copper Drop.*[1]—The resistance per phase is calculated in the ordinary way from the formula

$$r = \frac{9\cdot5 \times l}{10^6 \times s};$$

where l is the length of conductor in any one phase in feet, and s is the cross-section in square inches, and $9\cdot5 \div 10^6$ is

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[1] For full discussion and copper calculations, see Vol. I., Chapter VII. of the present work.

the resistance, at 60° C., of a copper bar 1 foot long and of a cross-section of 1 square inch.

An allowance has to be made on this for skin effect (p. 44) and the equivalent resistance of the eddy-current paths in the pole pieces. It is safe to estimate this addition as from 15 to 20 per cent.

The copper-drop is therefore

$$C_1 \times 1 \cdot 2 \times r$$

and the copper-losses in watts would be

$$n_{ph} \times C_1^2 \times 1 \cdot 2 \times r$$

where n_{ph} is the number of phases.

(2) *Armature Demagnetization.*—As already shown, for any position of a conductor with respect to the pole, a component of it may be taken as directly demagnetizing. In practice, both for want of a more extensive knowledge of the facts and more perhaps in order not to complicate calculations, the demagnetizing component from instant to instant is taken as proportional to sin β, where β is the electrical angular displacement from the mean pole position. (See Fig. 217.)

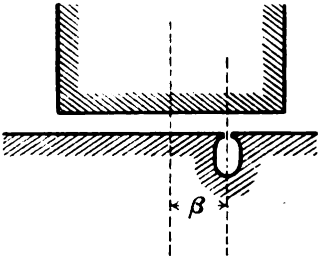

FIG. 217.

Now as to the current carried by the conductor, the electromotive-force generated in it is proportional to cos β, so that for the general case with an internal angle of lag ψ (see below p. 272 and Fig. 219), the current is proportional to

$$\cos(\beta - \psi).$$

Therefore the amount of demagnetization from instant to instant is proportional to

$$\cos(\beta - \psi) \sin \beta$$

which by trigonometrical reduction becomes

$$\cos \beta \sin \beta \cos \psi + \sin^2 \beta \sin \psi.$$

Now in a whole period β varies between 0° and 360°.

$$\therefore \text{ average demagnetization} \propto \frac{\sin \psi}{2}.$$

We may now write the expression for the demagnetizing M.M.F. as

$$1 \cdot 41 \text{ C} \times \left(\frac{Z_1}{2p} \times k_2\right) \times \frac{\sin \psi}{2}$$

where C = virtual value of current being taken from the machine.
Z_1 = conductors on the armature per phase.
p = number of poles.
k_2 = factor of the coils (see p. 204).

The above reasoning applies to the coils of each phase, so that the total armature M.M.F. is obtained by simply multiplying by n_{ph}, the number of phases.

To convert armature M.M.F. into A.T. required on the magnets to compensate it, we must multiply by a dispersion coefficient, which for practical purposes may be taken as ν, the coefficient for the magnets.

The final expression for the ampere-turns per pole on magnets to compensate demagnetization is therefore

$$0 \cdot 71 \times \text{C} \times \frac{Z_1}{2p} \times k_2 \times \sin \psi \times n_{ph} \times \nu.$$

In connection with the assumption that the demagnetizing effect is proportional to sin ψ, we have, in Fig. 218, marked out with respect to a certain pole whose span is two-thirds the pole-pitch (*a*) the positions where maximum current and therefore the position of the permanent back M.M.F. occurs for various power-factors, and (*b*) the presumed corresponding demagnetizing component of the whole M.M.F., *i.e.* sin ψ. It will be seen that even at so low a power-factor as 0·6, the maximum currents still occur within the region of the pole face, and that at a power-factor of 0·8, when the maximum current occurs right under the pole, it is assumed that the demagnetizing component is at least half the value of the total armature magnetomotive-force.

(3) *Armature Inductance.*—There are three ways of dealing with armature inductance.

The first is to express it by means of a coefficient of self-induction L_1 (in each phase), which multiplied by $2\pi f$ gives the armature *reactance* $2\pi f L_1$ (in ohms), and this multiplied by C_1 gives $2\pi f C_1 L_1$, the armature *reactance voltage.* The second is to express it in terms of the stray flux of the armature n_a, and calculate the reactance voltage of the armature (per phase) by the equation

$$l_s = 4{\cdot}44 \times f \times S_1 \times n_a \div 10^8\,;$$

where S_1 is the number of armature-turns in one phase. A third way is to determine the reactance voltage directly by experiment in some actual machines, and from these to deduce practical rules for estimating the reactance voltage.

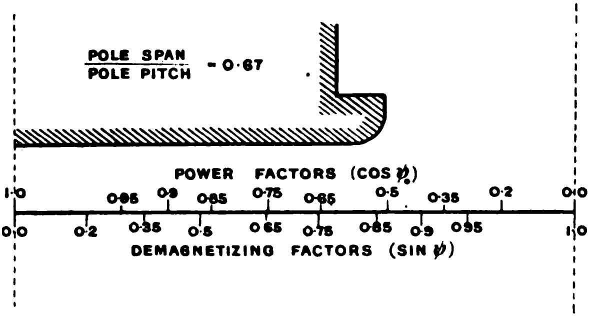

FIG. 218

It is possible to measure the stationary coefficient of self-induction by passing a current of the prescribed frequency through the armature at rest and measuring the voltage requisite for the current used. The ratio gives the (stationary) impedance, which will be found to vary somewhat with the current used, with the degree of magnet excitation, and also with the position of the coils with respect to the magnet-poles. In making the test, therefore, the magnets should be fully excited, the current used should be the full-load value, and several observations should be made with the armature gradually moved over a complete half period, and the average of the results may be taken for use as the impedance of the armature when working.

The resistance r of the armature being known, the reactance can then be calculated by the rule—

$$(\text{impedance})^2 - (\text{resistance})^2 = (\text{reactance})^2.$$

Methods of estimating the stray armature flux have been given by Parshall and Hobart[1] and by Hobart and Punga.[2] If, in any machine of given type, it can be found how many lines of stray flux are created per ampere, per inch length of conductor, then the number so found can be applied for the calculation of the stray flux in other machines. Naturally a conductor embedded in a slot between iron teeth will create around itself more lines per ampere than an equal length of conductor lying free in air. Parshall and Hobart examined nine alternators having slots of varied shape and size, and concluded the number of lines generated by 1 ampere per inch length of the "embedded" conductor varied from 25 in the position of minimum inductance to 40 in the position of maximum inductance, or on the average about 33 lines per ampere per inch. Similarly, they found the free conductor to generate about 3 to 4 lines per ampere per inch. Hobart and Punga, treating the case of conductors lying in the bottom of narrow slots with parallel sides, found the amount of stray flux which thus linked itself around the conductor to increase with the depth to which the conductor was sunk below the opening of the slot. If the depth of a slot be called D, the depth of the top of the conductor from the slot opening be d, and the slot width be w; then the number of lines per ampere per inch length of conductor is given by the empirical formula :—

$$\text{lines per ampere per embedded inch} = 1{\cdot}6\,\frac{\text{D} + d}{w}.$$

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[1] *Engineering*, lxx. 107, 141, 227, 464, 485, 592, 819, 1900.

[2] *Amer. Inst. Elec. Eng.*, Feb. 1904. See also Arnold and La Cour in *Sammlung Elektrotechnischer Vorträge*, Band iii. Heft 1-3, p. 84; also Caufield in *Electrical World*, xliv. 369, 1904.

For the free ends, Hobart and Punga found the number of lines per ampere per inch to be

2·04 for 1 slot per pole.
1·79 for 2 slots per pole.
1·53 for 3 slots per pole.

Adams, in dealing with this subject in the *Harvard Engineering Journal*, Nov. 1902, says that the lines per ampere per inch of embedded length may be taken as varying between 5 and 15, with the open slots and distributed windings used, but that in modern constructions the value is never more than 10. He neglects the end-bends, suggesting that in most cases it is sufficient to allow for them by taking the number for the embedded length slightly higher.

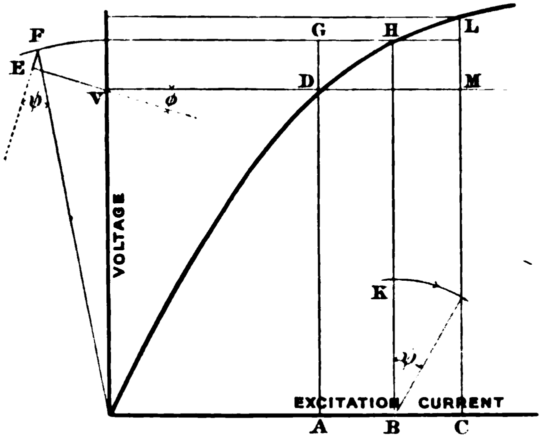

FIG. 219.

Determination of Regulation when Reactions are known.—When the quantities dealt with in the preceding sections have been determined, the pressure regulation for any given load may be determined as shown in Fig. 219. At no-load the excitation O A is necessary to produce the volts A D (= O V). The drop due to resistance is then plotted out as V E, the angle ϕ being the *external* angle of lag, *i.e.* between the current and the *terminal volts*. At right angles to V E, the drop due to resistance, E F, is drawn. Then to have a voltage A D at the terminals, a voltage equal to O F (= A G = B H) must be generated, which requires a no-load excitation equal to O B. Set up B K the total magnetomotive-force of the armature, and tilt it over at angle ψ, so that B C = B K sin ψ, the demagnetizing component. But if B C corresponds to the demagnetizing component, then O C amperes are required at this load to produce the same flux as O B at no-load, and therefore to produce the generated volts B H which give the terminal volts at this load. Therefore the point M is one on a curve connecting terminal volts and excitation on load, and if the load were taken off the terminal voltage would rise in value to C L. The angles connected with the voltage polygon A V E F should be noticed. The angle F A V (= θ) is the phase angle between the generated and terminal voltages, and the angle E F A (= ψ) is the internal angle of lag, *i.e.* the slightly larger lag between the current and the *internal* generated voltage, and is to be used in calculating the armature demagnetization.

To obtain the voltage drop according to the first mode of specification (p. 265) the best method is to complete the characteristic of the machine at this load, by carrying out a similar construction for several other points on the no-load curve, taking each time the same lengths D E and E F set out at the same angles, but altering B C according to the sine of ψ which will be different in each case.

The characteristic of a dynamo machine as first suggested by Hopkinson, and which is invariably used to express the performance of continuous-current generators, consists of plotting the load in amperes against the terminal volts (see Vol. I. Chap. XII.), which gives a curve that shows the regulation of a machine with regard to a fixed voltage for different conditions of load. In the present case, the voltage rise for different loads (at a given constant power-factor) may easily be obtained by the same construction as given above by taking different lengths of E D and E F and then values of B C according to those of E D and ψ in each instance.

To get the voltage drop characteristic a rather laborious process is necessary, consisting of plotting a number of curves as just described, starting with voltages smaller than A D and then interpolating from the series of curves for the curve required.

See further mention about load characteristics on p. 276.

The Short-Circuit Test.—Behn-Eschenburg and many others have made use of this test to determine the regulation of alternators. Even although in some cases the application of the test gives only approximate results, it is useful in affording a ready means of testing large alternators where in the first place it would be practically impossible to provide an artificial loading and in the second the power necessary to operate them at any appreciable load is not available until they are installed in the station and ready for service.

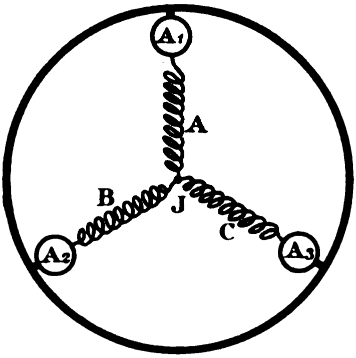

FIG. 220.

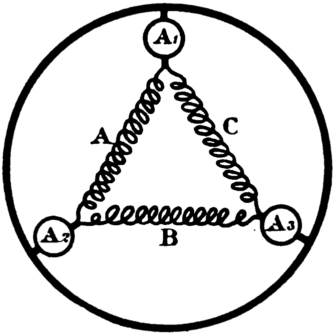

FIG. 221.

To carry out the test the armature of the machine is short-circuited in the manner shown in Fig. 220, for Y connections and in Fig. 221 for Δ connections. The amperemeters which are used in each phase should be practically inductionless and of low resistance, and the average of the three readings should be taken. Before starting the experiment the machine should be run short-circuited with no excitation in order to get rid of permanent magnetism as much as possible. With the speed running normal, a small excitation is put on and the short-circuit currents are read. A series of these readings are taken with the excitation increasing until the short-circuit currents are, say, one and a half the normal full-load value, when the experiment must be stayed for fear of damaging the coils.

The observations are plotted as the *short-circuit* curve and form a straight line which, however, may bend slightly up or slightly down at high currents according to the relative preponderance of saturation effects in the teeth and pole-tips and in magnetic circuit as a whole. If, owing to the heating, it has been impossible to get points on the curve right up to full-load excitation, these must be extrapolated. Fig. 222 shows

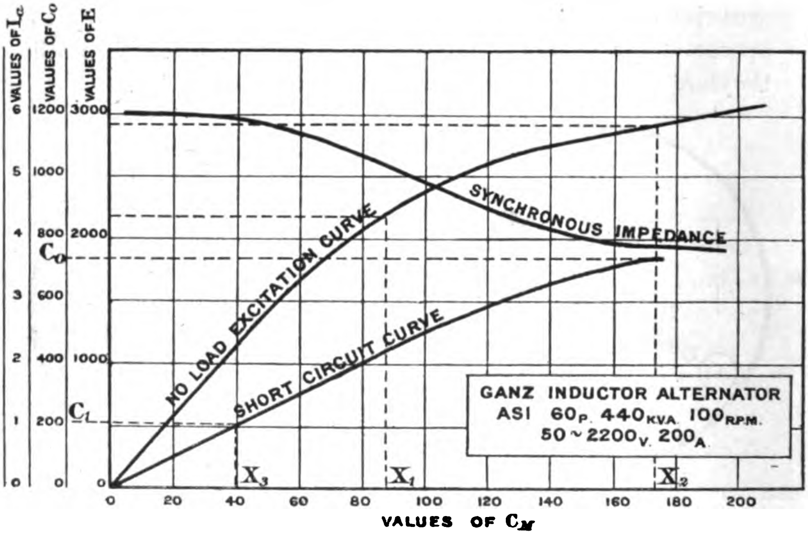

FIG. 222.

the no-load and short-circuit curves as obtained in a test on the Ganz Machine, Plate III. In this Fig. the point X_1 corresponds to the no-load excitation, X_2 to the highest excitation needed by the machine when on full inductive load. To X_2 there corresponds the current C_0 which is the current that would be produced if the machine excited for full-load were to be short-circuited. Another point X_3 is also of importance, for the length O X_3 represents that amount of excitation which will produce a short-circuit current equal to the normal full-load current C_1. *In a good machine* OX_3 *will be less than one* *third of* OX_2. The length OX_3 has a peculiar significance. *It is a measure of the total armature reaction at full-load and with zero power-factor.*

The methods adopted by designers for applying the short-circuit curve to measure the regulating properties of machines[1] may be grouped under three heads :—

(A) Electromotive-force methods.
(B) Magnetomotive-force methods.
(C) Combination methods.

Each of these methods is in the first instance directed to the consideration of the same condition of loading as the short-circuit curve, *i.e.* that of zero power-factor, but each method may be modified so as to apply to all other conditions.

(A) *The Electromotive-Force Method.*—In this method it is assumed that the whole of the reactions in the machine correspond to an impedance, *i.e.* that the full voltage is generated in every case, and that therefore by dividing any value of the E.M.F. by the short-circuit current produced by the same excitation, a value known as the *synchronous impedance* is obtained. It diminishes as the excitation is increased. It must, however, be borne in mind that it is not strictly an impedance, and it needs to be interpreted with some care.

Fig. 222 shows the synchronous impedance curve obtained from the tests on the Ganz I-type machine, Plate III.

By multiplying the values of the synchronous impedance by the full-load current, and taking the resulting voltage away from the corresponding voltage on the no-load characteristic, the *full-load current at zero power-factor* characteristic is obtained, as indicated in Fig. 223. To apply the construction to any other conditions of loading where the current lags by an angle ϕ, the construction shown in Fig. 224 may be used. It depends upon the principle that the angle between a chord and a tangent is equal to the angle in the opposite segment, and

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[1] It should be noted that no method of pressure drop predetermination which is based *entirely* upon the short-circuit curve, can be absolutely reliable, as the conditions which exist during the test in both the field-magnets and the armature are exactly the reverse to those existing on an ordinary loading.

has been drawn for the values when the excitation is O X in Fig. 223, cos ϕ being 0·8. Comparing the two figures, in length X Y = X^1 Y^1 and Y Z = Y^1 Z^1 and in Fig. 224, the angle between X^1 Z^1 and Z^1 Y^1 is $90^\circ + \phi$ so that X^1 Z^1 will give the terminal voltage under these circumstances. To make the construction general we may plot any other load voltage

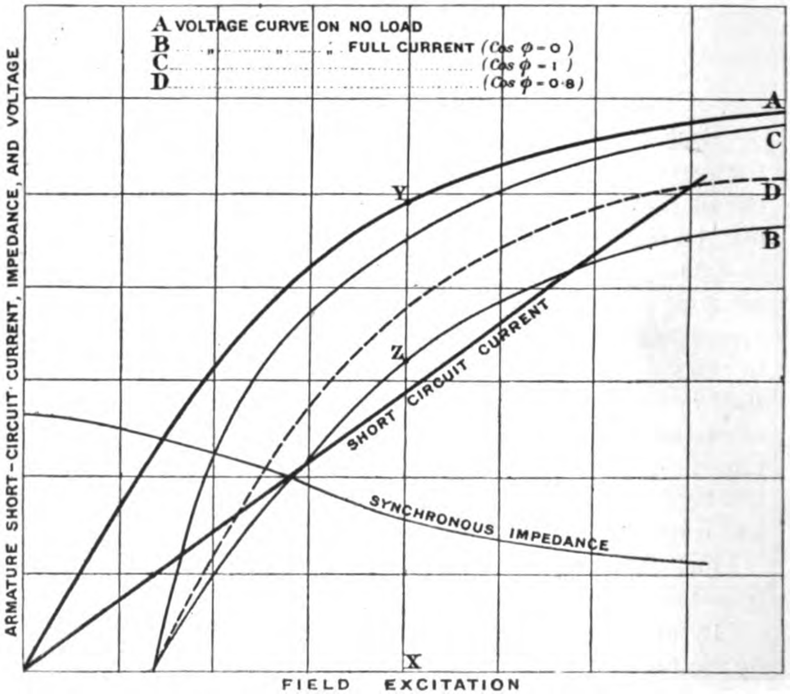

FIG. 223.

along X^1 Y^1 and the centre for the circular arc will be on X^1 B drawn at right angles to X^1 A.

Alternator Characteristics.—Many years ago Esson[1] showed that the voltage characteristic of an alternator working on a non-inductive circuit, and with constant excitation and constant speed, is of a form which droops with an increasing fall downward as the load increases; and under

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[1] *Electrical Review*, xiv. 303, 1884.

weak excitations the terminal voltage drops to zero when the ampere load is increased to a certain finite amount.

By the help of the synchronous impedance idea, we can demonstrate this in a very simple manner.

If E is the generated voltage, r the resistance of the armature, l the synchronous self-induction of the armature, and if R and L apply similarly to the outside circuit, then the current

$$C = \frac{E}{\sqrt{(R + r)^2 + p^2(l + L)^2}}$$

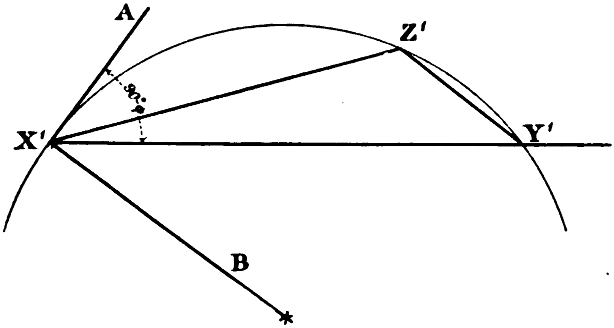


FIG. 224.

And the terminal volts V will be given by

$$C\sqrt{R^2 + p^2 L^2} = \frac{E\sqrt{R^2 + p^2 L^2}}{\sqrt{(R + r)^2 + p^2(l + L)^2}}.$$

For a non-inductive load, L becomes zero, and for a series of loads with a constant power-factor (cos ϕ), R and L have a fixed relationship.

By substituting various values in the expressions, we may plot curves connecting V and C for any power-factor of external load required.

As an example, we have taken the case of the Ganz machine (see Fig. 222) in which $E = 2200$, $r = 0 \cdot 5$, $p\,l = 5 \cdot 0$. Fig. 225 shows the results obtained for power-factors of unity and 0·9.

(B) *The Magnetomotive-Force Method.*—As in method (A) the total reactions were considered as reducible to an equivalent impedance, so in this case, it is supposed that all the reactions may be taken as equivalent to a simple demagnetizing effect, which with full-load current at zero power-factor is represented by the amount O X_0 in Fig. 222. To complete the curve for the full-load current at zero power-factor, the whole curve is shifted, as a whole, to the right as shown in Fig. 226.

For any other power-factor we may reduce the amount of O X_0 in the ratio of sin ϕ to unity and shift the no-load curve to the right by this amount instead. In this last case, however, some writers prefer to treat magnetomotive-forces as vectors. See appendix to this chapter, p. 285.

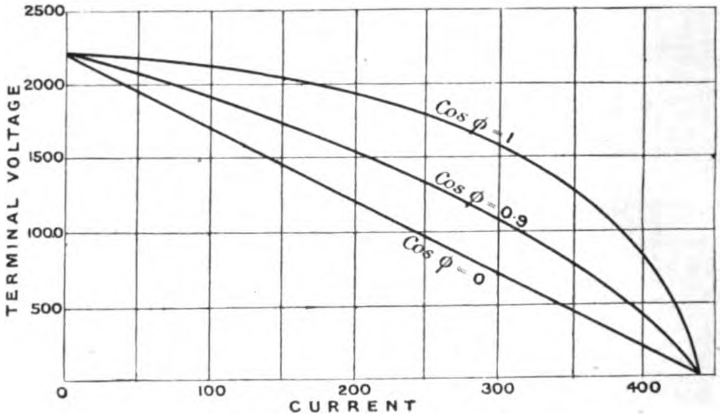


FIG. 225.—CHARACTERISTIC OF ALTERNATOR.

With regard to the accuracy of the above two methods, it will be found that they each give a different result when applied to the same case. Both the methods are unsatisfactory at high power-factors, but at low power-factors it is found that with slow speed or high frequency machines, method (A) gives fairly accurate results, while for high speed machines, especially such as the present types of turbo-alternators and also low frequency machines, the method (B) is the best to use.

(C) *Combination Methods.*—We have seen that method (A) was unsatisfactory because it treated all armature reaction as producing a counter electromotive-force, while method (B) was unsatisfactory because it assumed that all armature reaction could be represented as a demagnetization. The former required the curve to be shifted downwards, the latter required it to be shifted to the right.

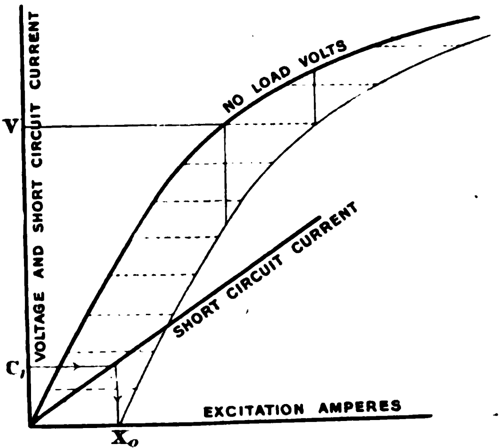


FIG. 226.

It has already been laid down that the drop in alternators is partially due to demagnetization and partially to self-induction. Let us see how these reactions combine in the short-circuit test.

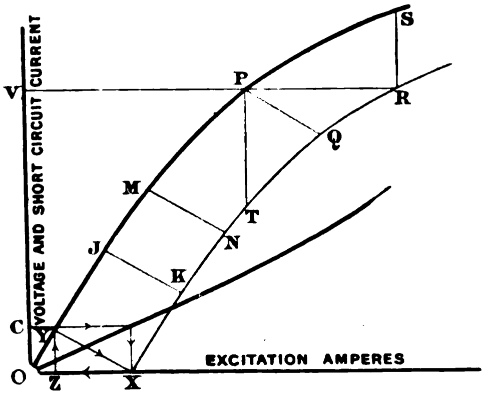


FIG. 227.

The short-circuit current C_{sk} per phase in the armature corresponds, owing to its demagnetizing action, to a certain amount of excitation, less, of course, than O X, Fig. 227, which can be calculated with more or less exactitude from the formula already given, the angle ψ being practically 90°.

Suppose this amount of excitation, which is wiped out, so to speak, to be represented by X Z, then it follows that only flux is being produced which corresponds to a no-load excitation of O Z ; and, therefore, the voltage Z Y, corresponding

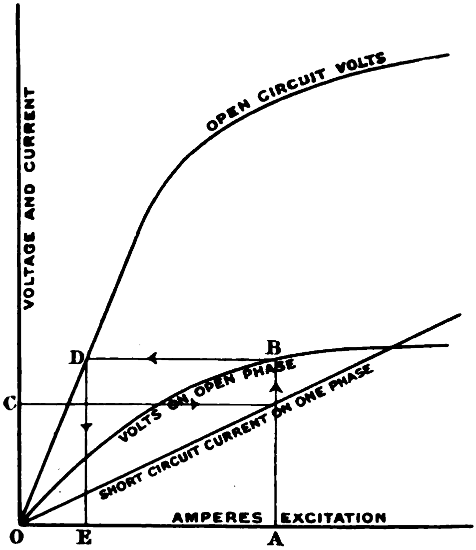


FIG. 228.—PARTIAL SHORT-CIRCUIT TEST.

thereto on the no-load characteristic, is being generated. This voltage is sending current through the armature and the amperemeter only, and is, therefore, the voltage required to overcome the true impedance of the armature under these conditions.

We have, therefore, the triangle X Y Z in which Y Z is the voltage drop due to impedance and Z X is the demagnetizing effect. If now we draw other lines such as J K, M N, P Q, etc., parallel to Z X and of equal length, a new curve is obtained which more nearly approximates to the actual values obtained from the machine, and lies in general within the region between the curves obtained by methods (A) and (B).

To proceed with the foregoing construction, it is necessary to first determine either the self-induction of the armature or the demagnetizing effect. This may be done by calculation by the ways already set forth, but in some circumstances where the data of the machine are not easily accessible, this cannot be done; and in such cases the following test called the *partial short-circuit test* may be made. The test which can be conducted on any three-phase machine is made by short-circuiting one phase or two phases only, and taking readings also of the voltage generated in the third phase. A set of curves is obtained as shown in Fig. 228. At any excitation O A, the short circuit for the one phase is O C, and as the volts A B are generated on the open phase, they are, therefore, also generated in the short-circuited phase. The equivalent excitation to generate these volts (A B = E D) is O E, so that of the total excitation O A, E A is counteracted by the armature current, and therefore E A represents the amperes on the magnet coils which correspond to the current O C on the armature. For any other value of total current in the armature (whether it be distributed between the phases or not), the corresponding current on the magnets is directly proportional. The voltage E D required to send the current against the impedance of the armature in this case has no special significance, because the impedance will be entirely different when all phases are loaded; it is only in connection

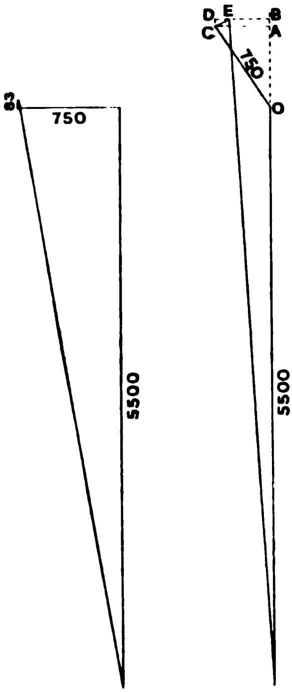


Fig. 229, *a* and *b*.

with the demagnetizing action of the armature currents that deductions may be drawn from this test.

Fig. 227 applies to the determination of the regulation at zero power-factor, but for other power-factors the information derived from the short-circuit tests may be applied in the construction given in Fig. 219, page 271.

EXAMPLE IN THE CALCULATION OF ALTERNATOR REGULATION.

As an example, we will take the St. Étienne machine (see pages 397 to 407 and Figs. 333 to 336). It is specially designed to work on a low power factor, viz. 0·55, and we propose to work out the regulation from the following data.

Slots per pole per phase	1
Conductors per slot	12
Full-load current	160
Turns per field bobbin	35
The no-load excitation curve	Given in Fig. 334.
The short-circuit curve	

(1) Armature magnetomotive-force at full-load

$$= 2{\cdot}12 \times 160 \times 6$$
$$= 2040 \text{ ampere-turns.}$$

This, under short-circuit conditions, corresponds to an exciting current on the magnets of

$$\frac{2040 \times 1 \cdot 1}{33} = 64 \text{ amperes.}$$

The factor 1·1, which allows for dispersion, being taken low, because there is very little magnetism in the machine.

The excitation required for full-load short-circuit current
= 81 amperes.

∴ Excitation actually used in producing flux
= 81 − 64 = 17 amperes.

This excitation generates a voltage of 750 across the phases, and is therefore the voltage drop across the arms of the star due to self-induction, when the armature is carrying the full-load current.

At full-load the drop across the arms of the star due to resistance

$$= 160 \times 0{\cdot}298 \times 1{\cdot}73$$
$$= 82.$$

(2) To calculate the regulation at unity power factor.

The terminal voltage, the resistance drop and the drop due to armature self-induction are related as shown in Fig. 229*a*.

The generated voltage is, therefore,

$$\sqrt{(5582)^2 + (750)^2} = 5630.$$

There is also some demagnetizing at this load, because although the *external* power factor is unity, owing to armature self-induction, the internal power factor (cos ψ) is less than unity. The demagnetizing action is taken as proportional to sin ψ and

$$\sin \psi = \frac{750}{5630} = 0{\cdot}134.$$

At full non-inductive load, therefore, the armature current corresponds to an exciting current of

$$\frac{2040 \times 1{\cdot}25 \times 0{\cdot}134}{3{\cdot}5} = 10 \text{ amperes.}$$

At the part of the no-load volt-curve corresponding to the full voltage, this difference in excitation corresponds to a difference in voltage of 140.

∴ Total voltage rise on non-inductive load

$$= 130 + 140 = 270.$$

This corresponds to a regulation of

$$\frac{270}{5500} = 49 \text{ per cent.}$$

(3) To calculate the regulation at a power factor of 0·55.

The voltage polygon now becomes as shown in Fig. 229*b*

$$\begin{aligned} OA &= 750 \sin\phi = 625 \\ AC &= 750 \cos\phi = 410 \\ AB &= 82 \cos\phi = 45 \\ CD &= 82 \sin\phi = 68 \end{aligned}$$

$$\therefore \quad GB = 5500 + 625 + 45$$
$$= 6170$$

and

$$EB = 410 - 68 = 342.$$

$$\therefore \quad \text{Generated voltage, } GE = \sqrt{(6170)^2 + (342)^2}$$
$$= 6180.$$

In this case

$$\psi = \phi + \psi$$
$$\phi = \cos^{-1} 0 \cdot 55 = 56\tfrac{1}{2}^\circ$$
$$\theta = \sin^{-1} \frac{342}{6180} = \sin^{-1} 0 \cdot 055 = 3^\circ$$
$$\therefore \quad \psi = 59\tfrac{1}{2}^\circ \text{ and } \sin\psi = 0 \cdot 86.$$

∴ The demagnetizing action corresponds to an excitation current of

$$\frac{2040 \times 1 \cdot 25 \times 0 \cdot 86}{35} = 64 \text{ amperes.}$$

This, in region of normal voltage part of the curve, corresponds to a voltage of 600.

∴ Total rise in voltage due to all causes

$$= 630 + 600 = 1230.$$

This corresponds to a regulation of 22·4 per cent.

This is rather higher than the value given by the makers, and is due possibly to the actual self-induction of the machine under conditions of load being less than under short-circuit conditions.

APPENDIX TO CHAPTER V.

In considering the effect of the armature magnetomotive-force, we have in this chapter (see especially p. 272) taken a component proportional to $\sin \psi$, and have then subtracted this component arithmetically from the magnetomotive-force of the magnets. Many writers, however, treat the magnetomotive-forces in this question as *vectors*, not as circuital quantities, but although this may be admissible in considering the coils of an ironless electro-dynamometer, it is not strictly true in alternators having discrete poles separately wound ; for the M.M.F.'s under these circumstances cannot be considered vectors in space, because the direction of the flux produced by their joint action is defined by the shape of the magnetic circuit, and they are not vectors in time, because they are constant quantities. As, however, methods for the determination of pressure regulation based upon the consideration of M.M.F.'s as vectors are used by Rothert and other engineers, a brief *résumé* of the points on which these methods are based will here be given.

In the first place a vector diagram may be constructed to show the relative positions in phase of the primary ampere-turns of the field-magnets $A\,T_M$, the secondary or reacting ampere-turns of the armature $A\,T_A$, and the resultant ampere-turns $A\,T_R$. This is given in Fig. 230, in which O X represents the primary, X R the secondary, and O R the resultant ampere-turns. If the armature current has no lag, R X will be at right angles to O R. If the current lags 90°, then R X will lie along X O, being purely demagnetizing. Now the stray fluxes N_s and n_a are proportional to, and in phase with, the ampere-turns which respectively produce them. Hence we construct the vector diagram Fig. 231, of fluxes and leakages, in which N_s is parallel to $A\,T_M$, and n_a to $A\,T_A$. The resultant flux N_g across the gap will be represented by a line parallel to $A\,T_R$; and by compounding with it the vectors N_s and n_a we obtain the flux in the magnet core N_M, and the flux in the armature N_A. The direction of the vector representing the whole induced electromotive-force will be at right-angles to the resultant flux. It will be noticed that the effect of the presence of n_a is the same as if the armature ampere-turns were increased.

These diagrams are more true for polyphase generators, where the interaction of the several armature currents is the equivalent of a revolving vector of uniform magnitude than for single-phase generators in which the reaction of the armature-currents is of a pulsating character.

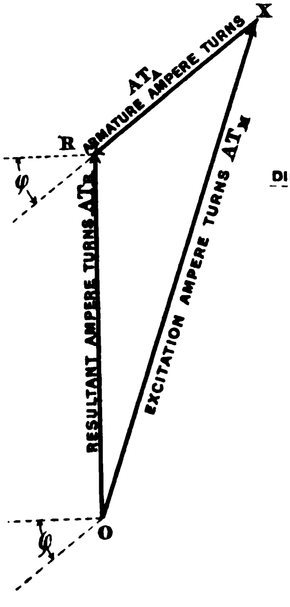


FIG. 230.—VECTOR DIAGRAM OF AMPERE-TURNS.

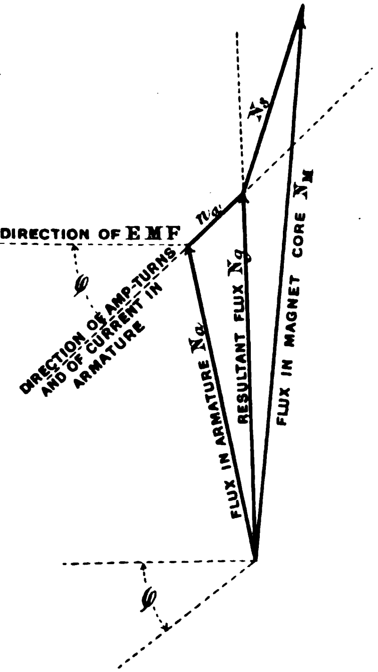


FIG. 231.—VECTOR DIAGRAM OF FLUXES AND LEAKAGES.

It is shown on p. 657, that owing to the overlapping of the coils of the three phases, the effective value of the ampere-turns of the armature per pole may in three-phase machines be reckoned as

$$A\,T_A = 2{\cdot}7 \times C_1 \times S_1 \div p\,;$$

where C_1 is the current (virtual amperes) in one phase, S_1 the number of armature-turns in one phase (*i.e.* equals the total number of conductors on the armature divided by 6), and p the number of poles.

Similarly, in single-phase machines, we have—

$$AT_A = 0{\cdot}9 \times C \times S \div p.$$

Now in this last case, C being the virtual amperes, the operative ampere-turns will vary from $+\sqrt{2}\,C$ to $-\sqrt{2}\,C$, the maximum values occurring, if $\cos\phi = 1$, when the field poles are opposite the slots containing the coils (as in Fig. 209 *ante*), or, if $\cos\phi = 0$, occurring when the poles are midway between the slots containing the coils (as in Fig. 211).

Current on Short Circuit.—The following is a consideration of the short-circuit test according to this method. When the terminals of an alternate current generator having an iron-cored armature are short-circuited at a certain low excitation and speed, the current in the windings will rise to such a value as to make the ampere-turns of the armature approximately equal to the ampere-turns of the field system, or slightly less. Under these conditions the current flowing in the armature will be lagging nearly 90° behind the induced pressure, and will have its maximum value when the coils are immediately opposite the poles. The short-circuit armature ampere-turns would be approximately equal to the excitation ampere-turns were it not for magnetic dispersion intervening.

Let us see why this is. The field excitation induces an electromotive-force in the armature windings which is simply used up in driving the current through the short-circuited coils (1) against ohmic resistance; (2) against the self-induction of the armature, that is, against the inductive electromotive-force produced in the coils by the action of the armature leakage flux. Eddy-currents in the armature cores may also affect the reaction since they may alter both the effective flux from the poles and the self-induction of the armature.

Their effect and that of the ohmic resistance are both very small and almost negligible in comparison with (2), and hence the current in the armature must lag nearly 90° behind the induced electromotive-force. Consequently, as we have already seen, it will exert a strong demagnetizing effect on the main field, and will rise to such a value as to balance the armature field against the latter. This condition of affairs is shown graphically in Fig. 232, in which the line O C_0 represents the phase of the short-circuit current. There will be (1) a small electromotive-force O e_1 in phase with the current representing the pressure required to drive the current against the resistance of the armature; also (2) an electromotive-force O e_2 due to the self-induction of the armature windings and in quadrature with the current O C_0; and finally (3) the resultant of O e_1 and O e_2, given by O e, and which leads the current by

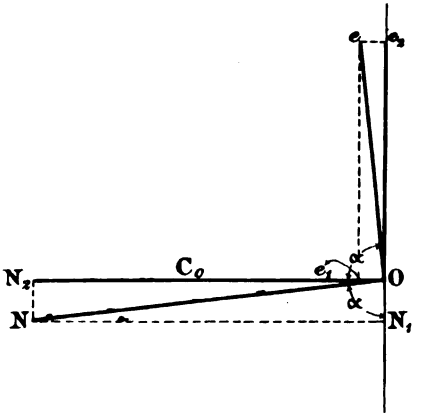


FIG. 232.

the angle a. Briefly we may say that O e_1 represents in magnitude and phase the armature lost volts, O e_2 the inductive drop due to self-induction, or, what is the same thing, the drop due to leakage of the armature magnetic lines, and O e is the electromotive-force necessary to balance these in order to cause the short-circuit current C_0 to flow. In practice O e_1 is, as stated before, very small in comparison to O e_2, thus making the angle a nearly 90°—it is never less than 80° in well designed machines. Hence the short-circuit current is lagging nearly 90° behind the induced pressure, and thus armature reaction has the greatest value it can possibly have for this particular value of armature current C_0.

In order to see how nearly the equality of field and armature ampere-turns holds when the machine is short-circuited, it is convenient to consider the three electromotive-forces O e_1, O e_2, and O e as being produced by the three corresponding magnetic fields to which each is respectively due, namely O N_1, O N_2, and O N in quadrature with them. That is, the main field O N produced by the field excitation in the air-gap, has two components, namely O N_2, which balances the armature reaction, and O N_1, the small field required to produce the total lost volts O e_1. We have further

$$O\,N_2 = O\,N \sin a.$$

As previously stated, in good machines of normal construction, the angle a lies between 80° and 90°, so we may put

$$\sin a = 0 \cdot 98 = \text{constant},$$

and therefore

$$O\,N \propto p \times S_M \times C_M \div \nu$$

and

$$O\,N_2 \propto S_1 \times 1 \cdot 41\, C_0$$

where

S_M is the number of turns per field-pole.
C_M is the exciting current in the field-coils.
S_1 is the number of turns in one phase all round the armature.
C_0 is the virtual value of the short-circuit current.
ν_1 is the leakage coefficient of the field system.

Then we have for a single-phase machine with *short-circuited armature*

$$O\,N_2 = O\,N \times 0 \cdot 98$$

$$1 \cdot 41 \times S_a \times C_0 = p \times S_M \times C_M \times 0 \cdot 98 \div \nu$$

$$C_0 = \frac{p \times S_M \times C_M}{1 \cdot 43 \times S_1 \times \nu_1}.$$

If for the same type of machine for normal sizes and loads the leakage coefficient ν_1 be taken as constant, we may write

$$(1 \cdot 43 \times \nu_1) = \text{constant} = c,$$

and hence obtain as the value of the short-circuit current for single-phase machines

$$C_0 = \frac{p \times S_M \times C_M}{S_1 \times c},$$

where c has values from about 2·7 to 3·5 for three-phase machines, or 1·7 to 2·7 for one-phase and two-phase machines.

With a slight modification, the same formula applies also to the case of three-phase machines, C_0 being the virtual value of the short-circuit current per phase, and S_1 the number of armature turns per phase, when all three phases are short-circuited. If the resultant rotatory field produced by the action of the three currents was of uniform strength, we should have

$$N_2 \propto 1{\cdot}5 \times S_1 \times C_0 \times 1{\cdot}41 \text{ (p. 652).}$$

But this never occurs in practice, even with well designed induction motors, the resultant field being always somewhat greater than 1·5 times the field that would be produced by one phase acting alone, and never more than twice this value. Taking 1·75 as an average figure we have

$$C_0 = \frac{p \times S_M \times C_M}{1{\cdot}43 \times 1{\cdot}75 \times S_1 \times \nu_1}$$

or

$$C_0 = \frac{p \times S_M \times C_M}{S_1 \times c}$$

the constant c having a greater value than before.

It must be noted that the constant "c" is really an empirical number, found by experiment on finished machines of any given type. It would be quite impossible to calculate it with any degree of accuracy, owing to the difficulty of estimating the leakage coefficient and the shape of the current wave. For this reason it has to be deduced for any given type of machine from the result of a simple experiment made on several finished machines as follows.

Predetermination of the Pressure-Drop.—Bearing in mind what has already been said, we will proceed to predetermine the fall of armature pressure for a given design, the value of the load current (C) and power-factor (cos ϕ) being stated.

In this method, the full-load armature ampere-turns (as defined above on p. 286) are calculated, and then the corresponding short-circuit excitation ampere-turns of the field-magnet are calculated by assuming that they will be equal to 1·05 times the number so calculated. For brevity let these short-circuit excitation ampere-turns be called X_0.

Now the ampere-turns necessary to excite to no-load voltage are known. Let them be called X_1. They are given to scale as the length OX_1 on the saturation curve, Fig. 233. But there will be a voltage-drop due to the ohmic resistance of the armature, which, when added to the no-load voltage, increases the excitation to an amount X_2 marked off to scale as OX_2. To this excitation X_2, there has now to be added (vectorially) the excitation needed to balance the inductive drop due to the whole armature reaction, and this is done graphically by setting off from X_2 at an angle equal to 90° + ϕ with OX_2 the line $X_2\ X_0$, the length of which represents to scale the exciting ampere-turns at short circuit which we have called X_0. This gives the resultant OX_0. The length OX_3 on the horizontal axis may then be cut off equal to OX_0. Projecting

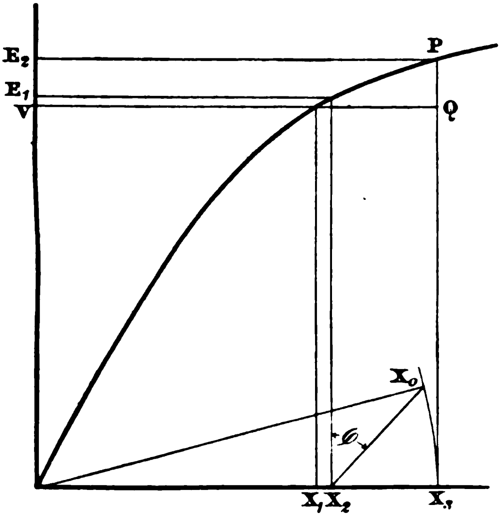


FIG. 233.

upwards from this point to the saturation curve we find the point P, which corresponds to the voltage E_2. The voltage-drop is then the length VE_2, and it ought not to exceed the prescribed amount.

If X_2 and X_0 are known, X_3 can be calculated by the formula:

$$X_3 = \sqrt{X_2^2 + X_0^2 + 2X_2X_0 \cos(\phi + 90^\circ)}.$$

CHAPTER VI.

WINDING SCHEMES FOR ALTERNATORS.

IN general the theory of armature windings for alternators is simple, since in the majority of cases the conductors are an even multiple of the number of poles, and the groupings are usually symmetrical with respect to each pole or with respect to each pair of poles.

A few general principles may be stated at the outset.

In alternators it is the rule that *all the conductors that belong to any one phase are in series with one another*, so that there is but *one circuit* per phase. The exceptions that arise to this rule are: (1) where, for convenience of winding, two or more wires of smaller diameter are laid side by side in parallel instead of one thick wire that might be difficult to bend (for an example see the Oerlikon three-phase 72-pole alternator, 333, p. 398); (2) where the machine is to work at a low voltage, and furnishes very large currents. In such a case there may be 2, 4, or more circuits in parallel in each phase of the winding. As many circuits in parallel as there are poles is the case presented by the winding of the armatures of rotatory converters (Chapter X.). In general, alternators are required to generate high voltages, so that one circuit per phase is appropriate. At one time the single-phase Westinghouse alternators, and the Mordey alternators were wound with the two halves of the armature in parallel. This practice is obsolete. It leads to increased armature losses. For if all the poles of the field-magnet are not equally strong, so that the electromotive-forces in the two halves of the armature are not equal, there will be unequal currents carried by the two halves of the winding, and the heating will be increased.

It will be understood from the preceding paragraph, that in single-phase alternators, as a rule, the winding is never re-entrant, but that the circuit in these machines has a definite end and a definite beginning. The same is true of each of the individual circuits of a polyphase armature. The exceptional cases where re-entrant circuits are used are those of polyphase converters having armature windings so constructed that a commutator can be connected to them exactly as in continuous-current machines. And these are usually designed with a lap-wound drum-winding.

Grouping of Phases. — There are two ways of grouping the separate windings of the various phases. Taking *three-phase* first, there are three independent circuits which therefore have six ends. These *might* be brought out to six separate terminals, or, as in the case of converters to six separate slip-rings. Such a grouping would permit of connexion to three absolutely distinct single-phase circuits. But in three-phase working they are always grouped together in one of the two following ways :—

(*a*) *Star-grouping*, also called Y-*grouping*, in which one end of each of the three circuits is brought to a common junction (usually insulated), and the three other ends are brought out to three terminals (or, in the case of revolving armatures and rotors, to three slip-rings).

(*b*) *Mesh-grouping*, also called Δ-*grouping*, in which the three circuits are joined up together in a triangle to the three corners of which are connected the three terminals (or, in the case of revolving armatures and rotors, to three slip-rings).

In two-phase working, similarly, there are the following possibilities.

(*a*) *Star-Grouping*, wherein the mid-points of each of the two phases being united to a common junction, the four ends are brought out to four terminals (or in the case of revolving armatures or rotors, to four slip-rings). This virtually makes the arrangement a four-phase system.

(*b*) *Mesh-grouping*, in which, the two phases being each divided into two parts, the four are joined up in cyclic order, the end of one to the beginning of the next, as if forming a square, to the four corners of which are joined the four terminals (or, in the case of revolving armatures or rotors, to four slip-rings).

In passing it may be remarked that mesh- or Δ-groupings are now never used in generators, for reasons elsewhere mentioned, but they are still used in the windings of motors of low voltage.

Concentrated and Distributive Windings.—For reasons mentioned on p. 197 concentrated windings yield a higher electromotive-force than distributive windings; but distributive windings give rise to less armature reaction, and lend themselves to better forms of electromotive-force curves. In single-phase armatures the windings *might* be distributed in small slots over the entire face of the armature, as in continuous-current machines, or they *might* be concentrated in a few larger slots—one slot per pole. On the other hand, in three-phase machines the windings *might* be distributed in small slots over belts occupying one-third of the breadth of the pole-pitch (or in hemitropic windings occupying two-thirds of that breadth); or they *might* be concentrated in a few larger slots, namely, three slots per pole, that is, one slot per phase per pole. It is only in the case of distributive windings that there arises any such distinction as was necessary to observe in the case of continuous-current machines between lap-windings and wave-windings. (See Vol. I., p. 360.)

Number of Slots per pole.—Now in order to follow out the connexions when the conductors are distributed in slots, let us consider the question of *numbers of slots per pole* for any one phase.

We may work out the winding scheme on any one of the following suppositions: (i) that in each slot there is one conductor; (ii) that in each slot there is one "side" of a coil; (iii) that one "side" of a coil is subdivided so that it may be distributed in two or more adjacent slots.

It is usual to describe a winding in terms of the number of slots per phase per pole. Thus, if a three-phase machine of 20 poles has 300 slots, therefore having 15 slots per pole, or 5 slots for each of the phases per pole, it will be described as a *five-slot* winding. The voltage, of course, depends on the number of conductors in a slot, but the breadth-coefficient and the wave-form are affected by the number of the slots per pole, and not by the number of conductors within them.

SINGLE-PHASE WINDINGS.

One-slot Windings.—The simplest type of winding, sometimes called the *monotooth* or *unicoil* type, is that in which the periphery of the armature is considered as divided into a series of large teeth, one tooth to each pole, surrounded by the armature coils, one coil (of one or more turns) per pole. Such a winding is presented by Figs. 234 and 235, each of

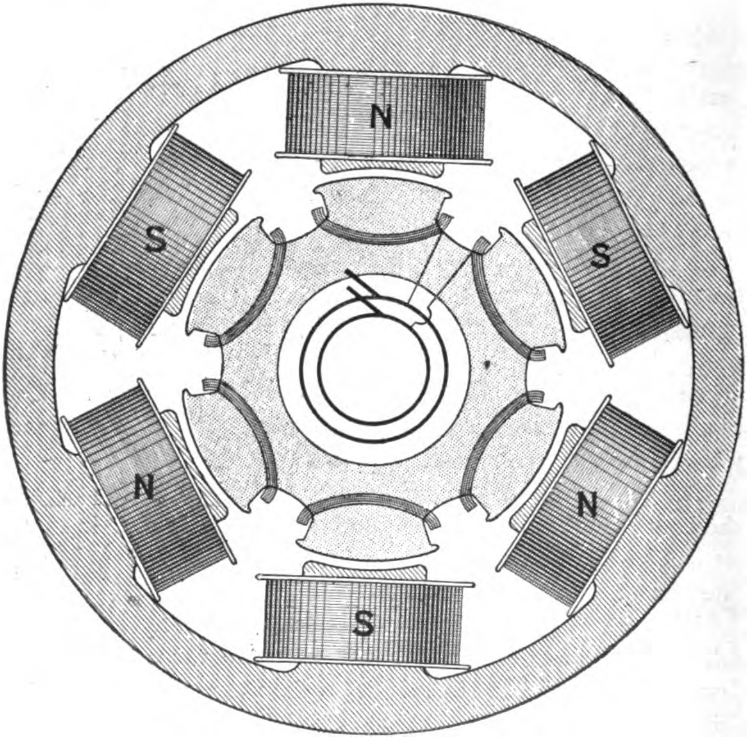


FIG. 234.—6-POLE MONOTOOTH SINGLE-PHASE ALTERNATOR (Type A).

which depicts a monotooth generator, Fig. 234 being of the A type, and Fig. 235 of the B type.

Figs. 236 and 237 depict a Westinghouse monotooth 8-pole armature, and the mode of placing the coils upon the poles.

Suppose that in the machine depicted in Fig. 234 we had the following data :—$p = 6$; $N = 3{,}300{,}000$; $Z = 720$; RPM = 900 ; and $k = 2{\cdot}8$. Then since $f = 45\sim$, we shall have by the formula

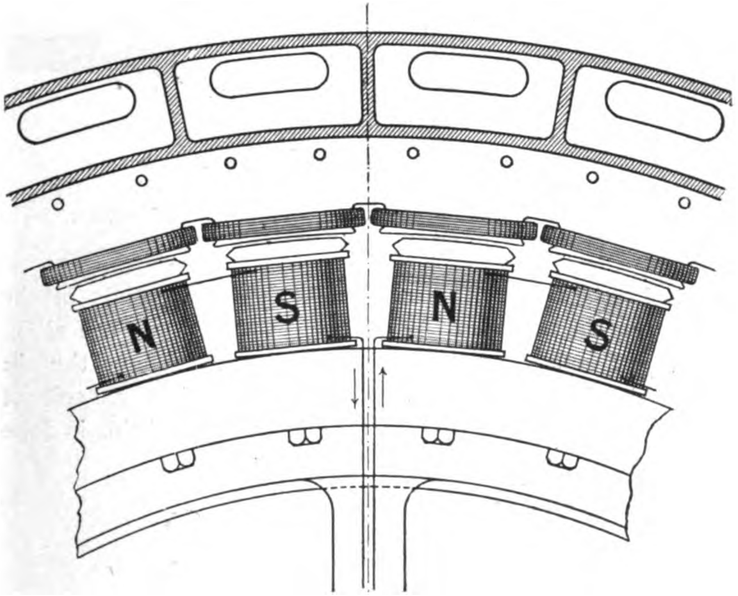


FIG. 235.—MULTIPOLAR MONOTOOTH SINGLE-PHASE ALTERNATOR (Type B).

$$
\begin{aligned}
E &= k.f.Z.N \div 10^8 \\
&= 2{\cdot}8 \times 45 \times 720 \times 3{,}300{,}000 \div 10^8 \\
&= 3000 \text{ volts.}
\end{aligned}
$$

As there are 6 slots, there will be 120 conductors per slot, and the coil wound round each of the six big teeth of the armature will consist of 60 turns. Also the coils must clearly be joined up so that the circulation of current round them must be in alternate senses; for the circulation around a tooth which is passing a north-pole must clearly be in the opposite sense to that around a tooth which at the same instant is passing a south-pole. Now provided this sense of circulation is observed, it matters not how the ends of the active "conductors" are joined up, provided they are all in series. There are 120 conductors through each of the six slots, and it is just as simple to group them in three coils of 120 turns each as in 6 coils of 60 turns each. If this is done we attain Fig. 238, in which only half the teeth are wound. This is known as *hemitropic* winding. Electrically it is exactly the equivalent of the previous case. In single-phase construction it has no particular advantage, but, as we shall

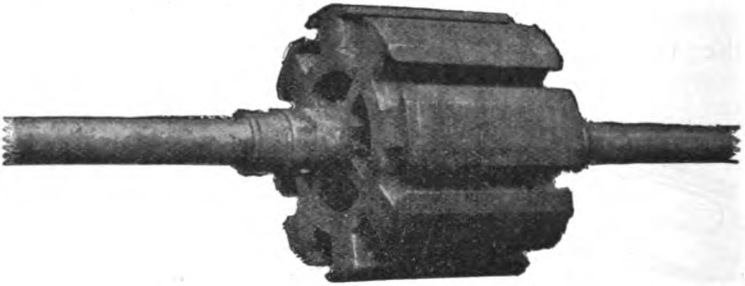


FIG. 236.—ARMATURE OF 60 KVA WESTINGHOUSE 60-CYCLE SINGLE-PHASE ALTERNATOR UNWOUND.

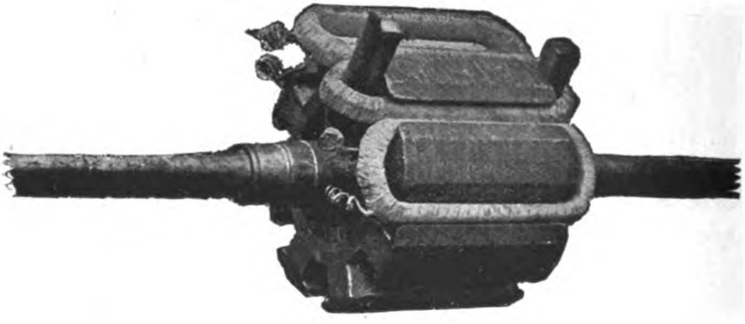


FIG. 237.—ARMATURE OF 60 KVA ALTERNATOR, SHOWING MODE OF PLACING COILS.

see when we come to three-phase windings, it is of distinct value. Fig. 239 is a hemitropic winding on a machine of B-type, and Fig. 240 shows the arrangement in perspective.

To correlate these windings with the more complex kinds which follow, we may compare the developed diagrams Figs. 241 and 242 which correspond respectively to Figs. 234 and 238. The arrows indicate the circulation of current at an instant when the poles, considered as moving to the right and in front of the winding, are in the positions shown.

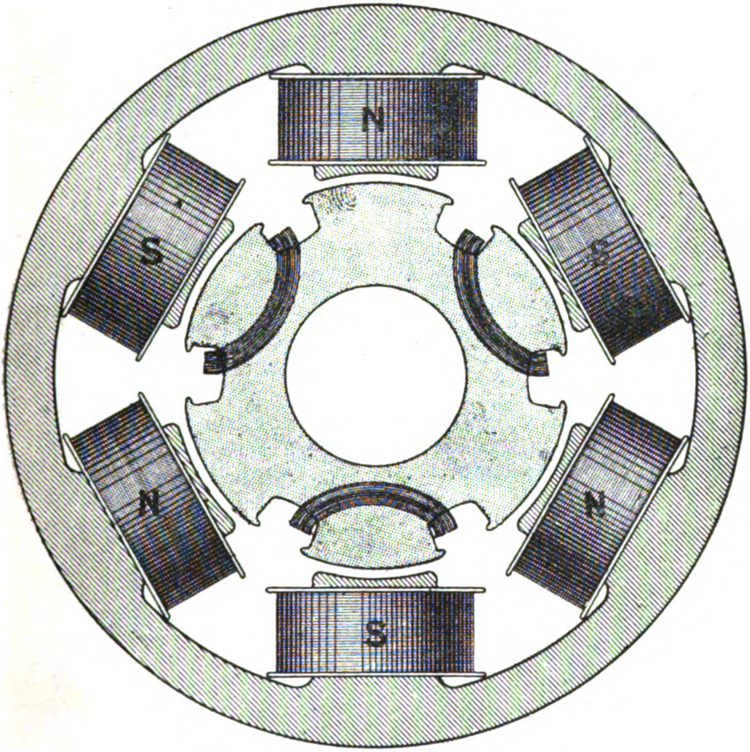


FIG. 238.—6-POLE MONOTOOTH SINGLE-PHASE ALTERNATOR WITH HEMITROPIC ARMATURE (Type A).

In dealing with coils made up of a number of turns it is convenient to regard the number of adjacent conductors which

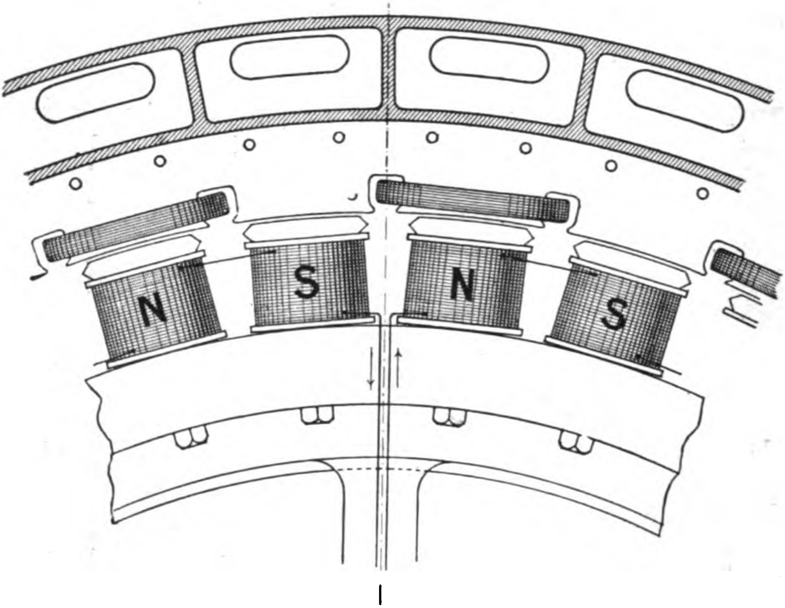


FIG. 239.—MULTIPOLAR MONOTOOTH SINGLE-PHASE ALTERNATOR WITH HEMITROPIC ARMATURE (Type B).

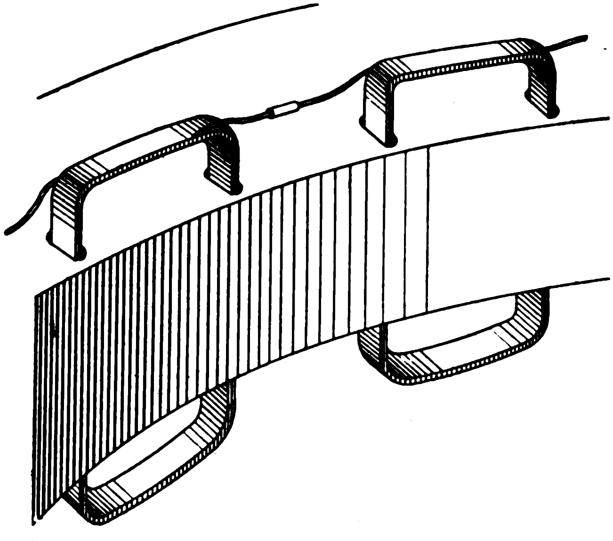


FIG. 240.—HEMITROPIC COIL-WINDING IN PERSPECTIVE.

constitute one of the "sides" of the coil. In Fig. 234 each of the 6 coils has 60 turns, so that there are 60 conductors in

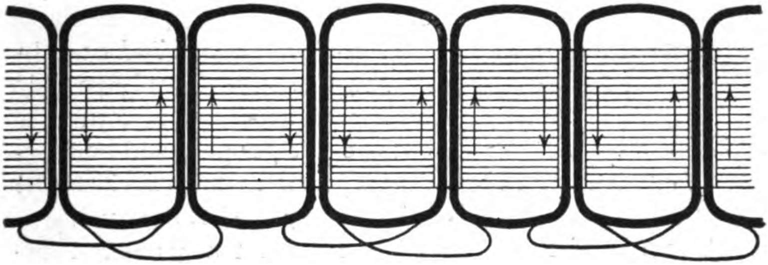


FIG. 241.—DEVELOPED DIAGRAM OF SINGLE-PHASE MONOTOOTH WINDING.

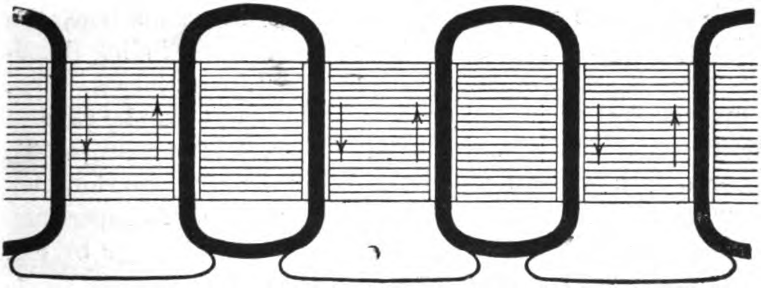


FIG. 242.—DEVELOPED DIAGRAM OF SINGLE-PHASE MONOTOOTH HEMITROPIC WINDING.

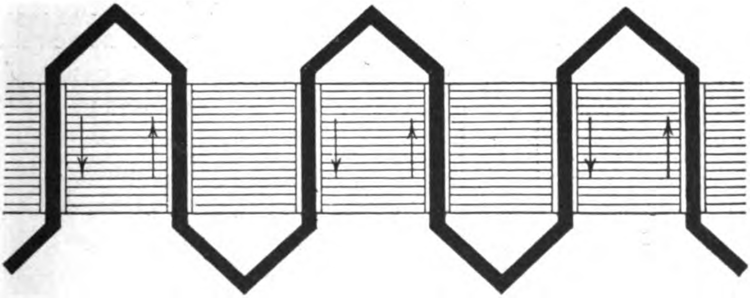


FIG. 243.—ONE SLOT PER POLE, BAR-WINDING.

each "side." Each slot holds two "sides" of 60 wires each. In Fig. 238 each slot holds one "side" of 120 wires. The winding would still be regarded as monotooth, if Fig. 234 had been modified as in Fig. 244 by substituting for each of the six wide slots two smaller slots each admitting only one "side" of a coil.

FIG. 244.
TWO SLOTS PER POLE.

One-Slot Bar-Winding.—In Fig. 243 we have the only possible case for a bar-winding, if there is but one slot per pole, and one conductor per slot. It will obviously be suitable only for the lowest voltages.

There is, of course, nothing to prevent a number of successive "waves" from being set side by side in one slot, save the difficulty of proper insulation. If, in order to attain higher voltages, there must be a number of conductors per pole, then for small and moderate-sized machines, a coil-winding (hand-wound or former-wound) is appropriate. But for very large machines with many poles, the high voltages can be reached with a bar-winding, and the bars are grouped in zigzags. To permit of 2 or more conductors being put into one slot, they must be so grouped that the contiguous bars of copper shall have only a small voltage difference, as in Fig. 245*c* or Fig. 248*b*, and never as in Fig. 245*a* or Fig. 248*a*.

Two-Slot Winding.—If the winding is slightly distributed by putting it into two slots per pole, the result may be still regarded as monotooth, if as in Fig. 244 the two slots are close together. But the winding works out somewhat differently as a matter of construction in different cases. A devoloped diagram for a 12-pole machine having 24 slots is shown in Fig. 245*a*, which depicts a wave-winding that zigzags once round the armature in the slots of odd numbers, and then with a rather longer end-connexion passes from slot 23 to slot 2, and so zigzags again through the even slots.

A simple variant on this is given in Fig. 245*b* for the same machine, in which after the first zigzag through the slots of odd number has been completed, the second zigzag is made by turning backwards from slot 23 to slot 22 by a rather shorter end-connexion, the winding ending at 24 as before.

This variety of retrogressive wave-winding which recurs thus backwards appears to be due to Mr. Parshall.

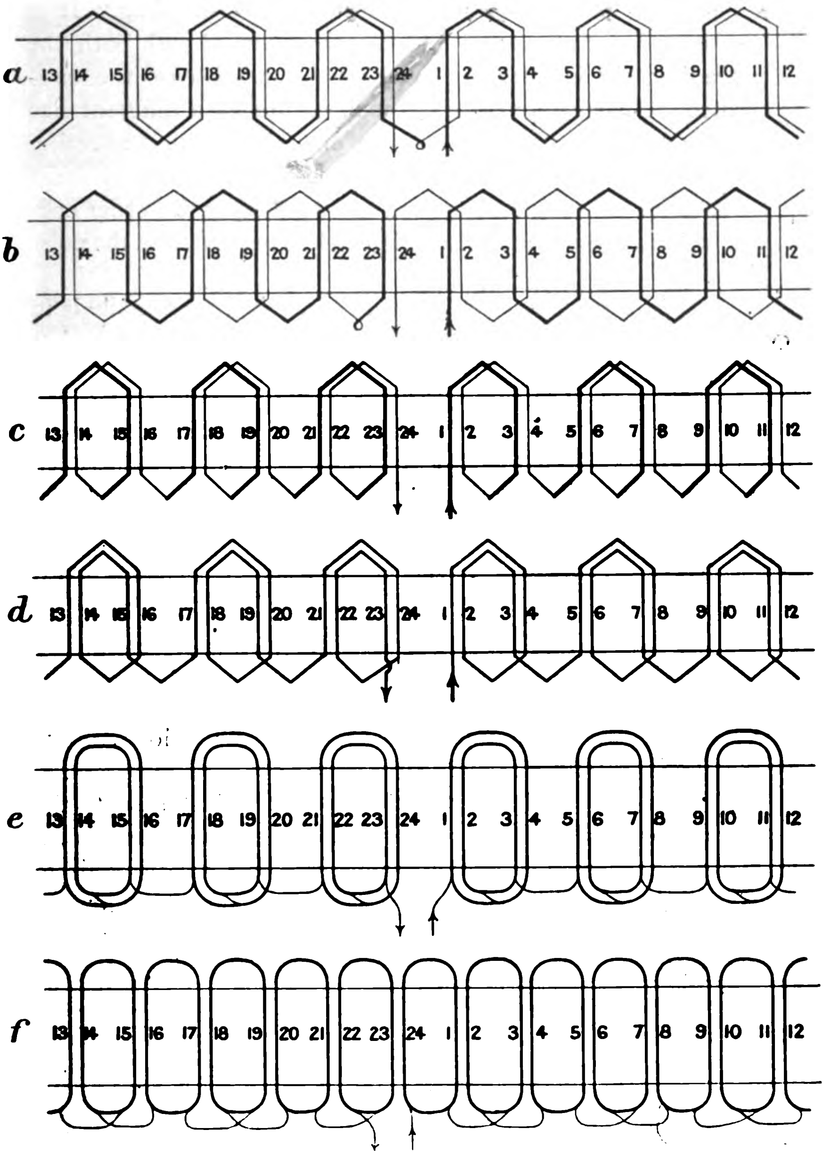


FIG. 245.—SINGLE-PHASE WINDINGS; TWO SLOTS PER POLE.

Another variant makes each alternate bend of the wave to take a second turn around the pole before moving forward. This is in fact a mixture of lap- and wave-winding. It is shown in Fig. 245*c*, and is electrically identical with both the preceding, but is better for insulation. In this case the end-connecting parts at one end of the armature are a different size from those at the other end. At the end where two connecting parts cross, each will span an exact pole-pitch, and they must be so bent as to avoid contact; while those at the other end, and which do not overlap, are slightly less in span than the pole-pitch.

In Fig. 245*d* the only change which has been made is in the form of end-connexions which are now of three sizes, but which avoid any overlap.

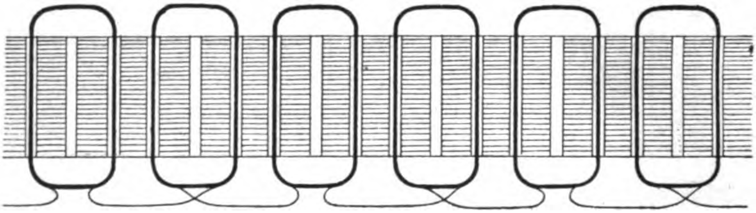


FIG. 246.—SINGLE-PHASE WINDING; THREE SLOTS PER POLE, ONE EMPTY.

All the above windings are suitable only for low-voltage machines with bar-windings.

It is a very obvious transition from the last case to the arrangement suitable for a high-voltage coil-winding shown in Fig. 245*e*, which is a hemitropic winding having coils of two sizes. This is generally preferred to the arrangement of Fig. 245*f*, where all the coils are of one size, and are connected alternately right-handedly and left-handedly.

Another winding, equivalent to the last but one, except that the coils employed will have a slightly lesser breadth, is one in which, though the armature is constructed with three equidistant slots per pole, only two of them are occupied with coils, every third slot being left empty. In this case the winding pitch is two-thirds of the pole-pitch, see Fig. 246

Another plan is to stamp four slots per pole, and fill two only, as in Fig. 247, leaving half the slots empty as if for a second phase. Such arrangements as these are sometimes adopted in order to use stock stampings. See also Fig. 354*d.* Cases in which four slots out of five, and six slots out of ten, are filled, are given in Figs. 301 and 337.

Three Slots per Pole.—Fig. 248*a* shows a thrice-repeated wave-winding intended for three slots per pole. At the end of each zigzag round the series of poles one end-connexion is required of a slightly larger size; these being marked in the figure by loops. The alternative plan of turning back retrogressively at the end of each zigzag is applicable also in this case.

By introducing laps around every alternate pole, as in Fig. 248*b*, a mixed winding is produced in which at one end of the armature there will be overlapping connexions in sets of three, all of a space equal to the pole-pitch, and at the other end there will be two sorts of end-connectors, one sort in sets of two, slightly narrower than the pole-pitch, and another sort still narrower. This is a progressive lap-winding.

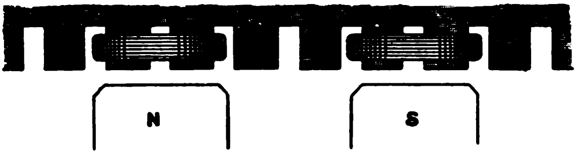


FIG. 247.—SINGLE-PHASE WINDING IN TWO-PHASE STAMPING.

By a further change of end-connexions, Fig. 248*c* is reached.

From this case the transition to a coil-winding distributed over three slots is obvious, the winding being hemitropic (Fig. 248*d*).

Theoretically the winding shown in Fig. 248*e* might be used, being equivalent to the last; but it would have no advantages.

Four Slots per Pole.—It is scarcely necessary to carry the argument further, but it is instructive to see how the arrangements with four slots per pole group themselves, showing two intermediate forms between the pure wave-winding and the progressive lap-winding. The first is suf-

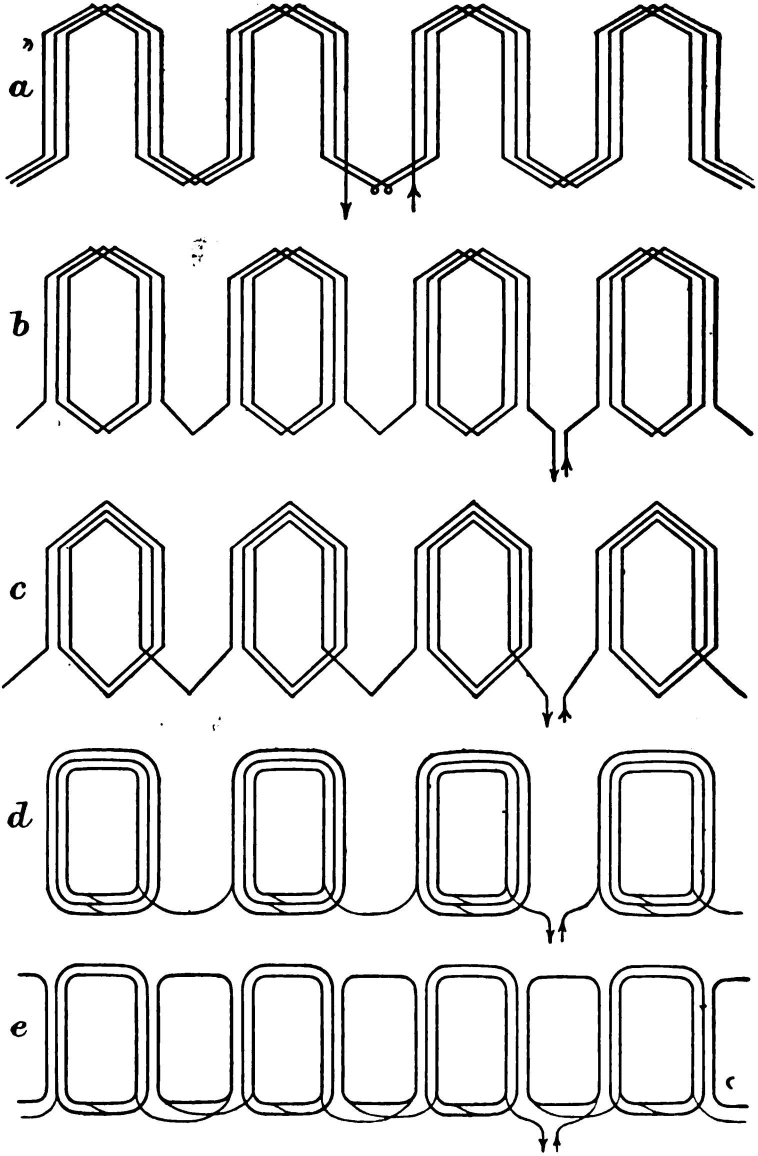


Fig. 248.—Three Slot Single-Phase Windings.

ficiently shown in Fig. 249*a*, while the last is depicted in Fig. 249*b*. The two intermediate ways are shown in Figs. 249*c* and 249*d*. They are all electrically identical.

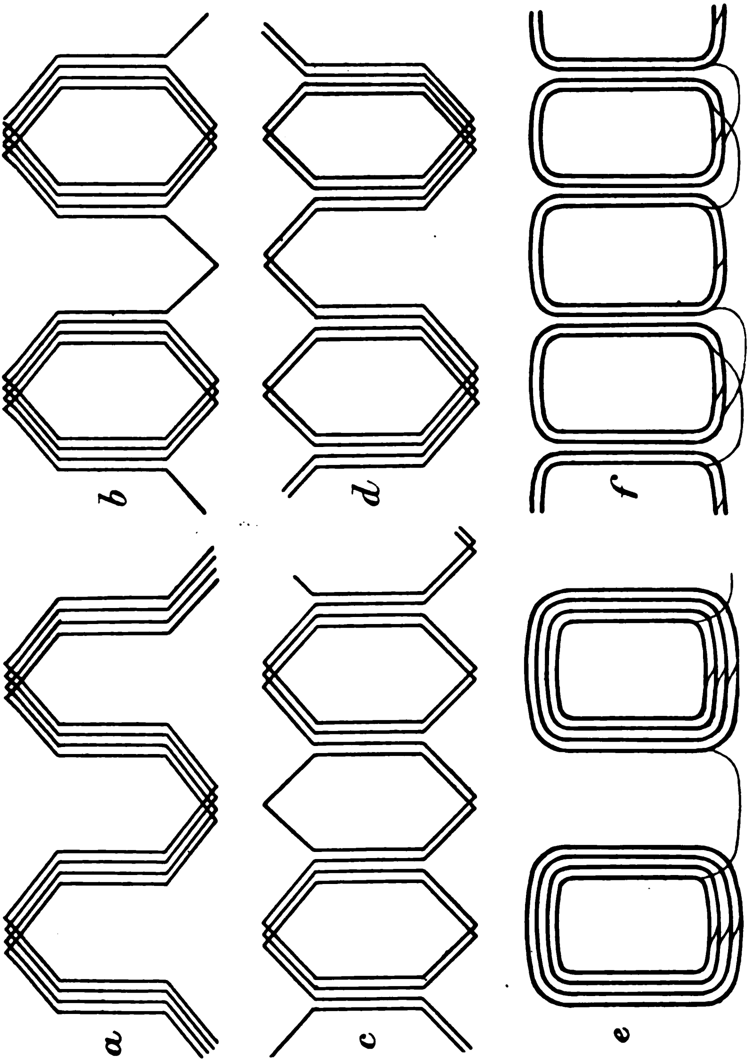


FIG. 249.—FOUR SLOT SINGLE-PHASE WINDINGS.

The corresponding coil-winding diagram is Fig. 249*e*, which is hemitropic. If every pole is wound the diagram will become that shown in Fig. 249*f*, where the connexions of alternate coils must be reversed.

Polyslot Single-Phase Winding.—Comparing Figs. 248*d* and 249*e*, with three and four slots per pole, the question arises how far it becomes advisable to spread the coils of a single-phase generator over any considerable width. Clearly Fig. 249*e* might be altered by adding a still narrower coil inside, and a wider one outside, so that eventually the entire surface of the armature is equally occupied as in Fig. 250. As pointed out on p. 203, there is not much advantage in reducing the interior breadth much below that of the breadth of the pole face. Nor on the other hand is there much advantage in widening the exterior breadth beyond the breadth of the pole-pitch. This undue spreading lowers the value of *k* by reducing the breadth-coefficient, and the use of a larger number of conductors to obtain the same voltage means a

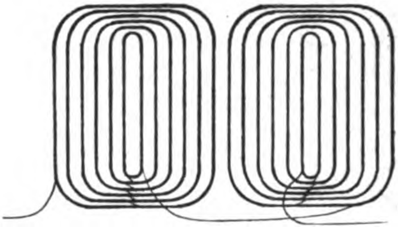


FIG. 250.—POLYSLOT SINGLE-PHASE WINDING.

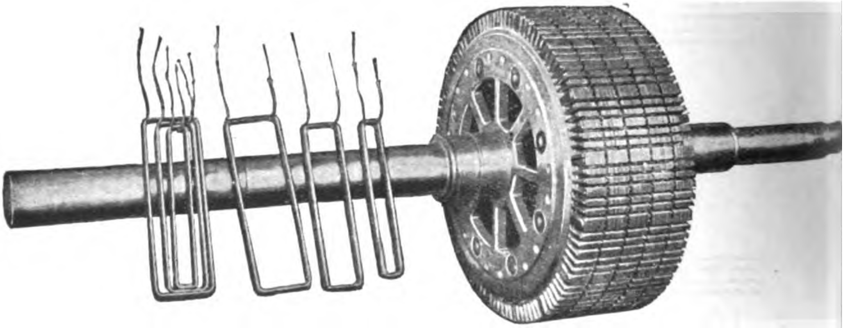


FIG. 251.—CORE AND COILS OF 120 KW SINGLE-PHASE ALTERNATOR.

larger armature self-induction. From this point of view, it would be preferable to concentrate the winding in fewer slots that were closer together. This, however, would accentuate the distorting and demagnetizing reactions of the armature. Between these two disadvantages a compromise must be made, a common practice being to wind in two or three slots per pole, not equally spaced out. An example of this is given in Fig. 251, which is of an A-type 120 KW single-phase machine. Nevertheless, much more distributed windings are sometimes used. In the Johnson and Phillips's

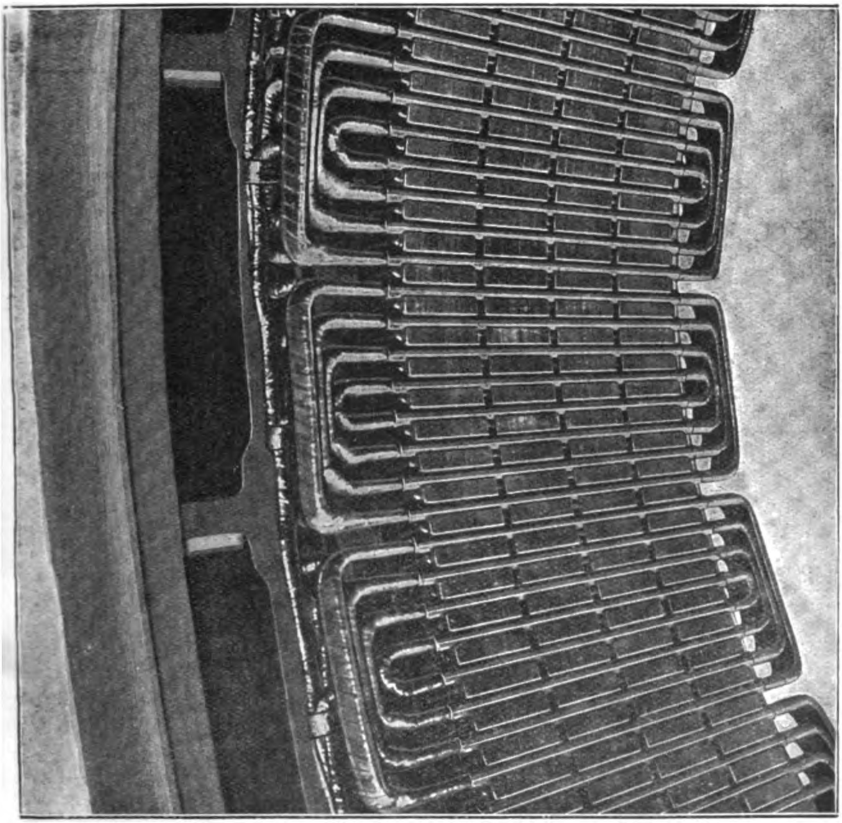


FIG. 252.—PORTION OF SINGLE-PHASE ARMATURE OF THE WESTINGHOUSE CO.

250 KVA machine, p. 411, there are 5 slots per pole, the coil being distributed in 3 of them, and two are left empty. Fig. 252 is from a recent single-phase generator of the Westinghouse Company.

Semi-Symmetrical Windings.—It is possible to produce distributions that are equivalent to 1½ slot per pole, or 2½ slots per pole. This is attained by taking as the unit of spacing not the pair of poles, but a set of 4 poles. Consider the winding shown in Fig. 253*a*. So much of the winding as lies from P to P′, a distance equal to four times the pole-pitch, contains six slots. This might be repeated indefinitely for any machine, the number of whose poles

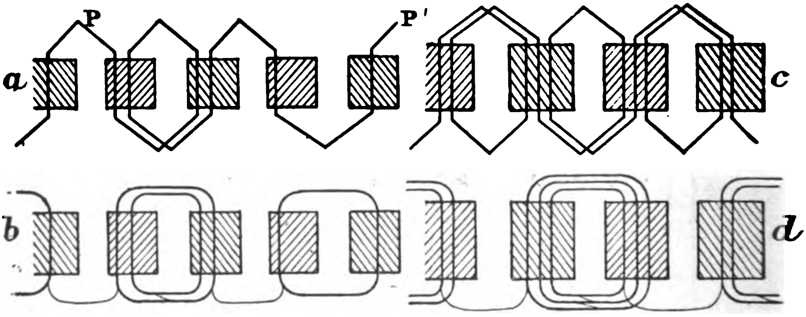


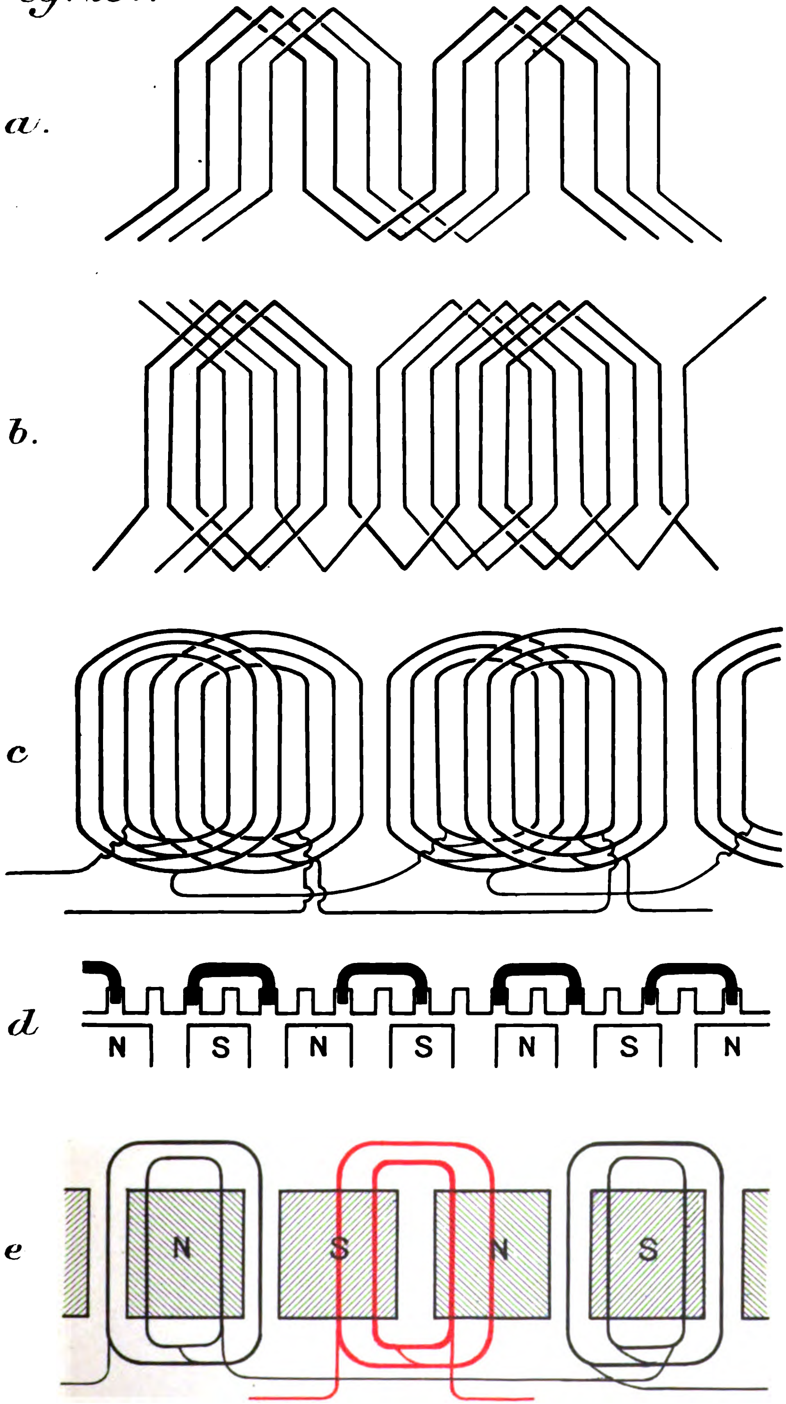
FIG. 253.—SEMI-SYMMETRICAL SINGLE-PHASE WINDING.

is divisible by 4. It could not be used for a machine with 14, or 18, or 22 poles, for example; but could be used for 12, 16, 20, or 24 poles. This is shown as a bar-winding. The equivalent coil-winding (hemitropic) would be that of Fig. 253*b*; there being one slot opposite the first of the 4 poles, two slots opposite the second, two opposite the third, and one opposite the fourth.

To give 2½ slots per pole, the winding (again for 4 poles) would be as in Fig. 253*c*, where there are two slots opposite one of the poles, three opposite the second, again three opposite the third, and two opposite the fourth. The corresponding coil-winding using 2½ slots per pole is shown in Fig. 253*d*.

TWO-PHASE WINDINGS.

Fig. 254.



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TWO-PHASE WINDINGS.

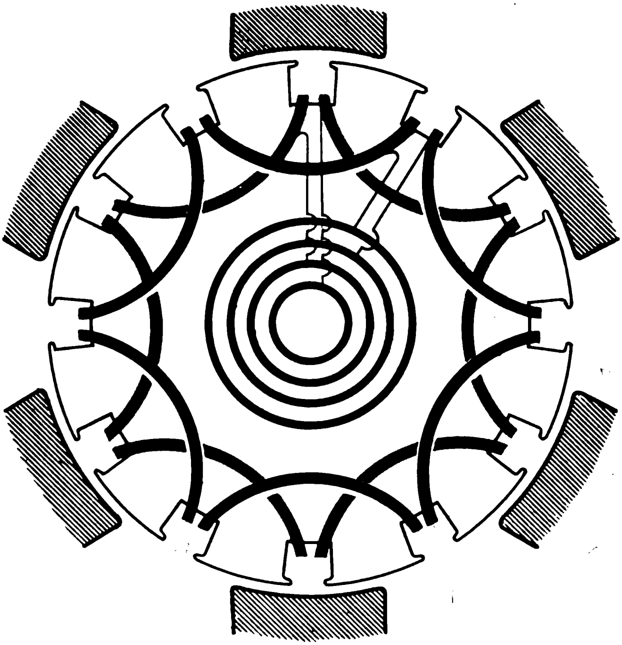

Fig. 255. One Slot per pole per phase.

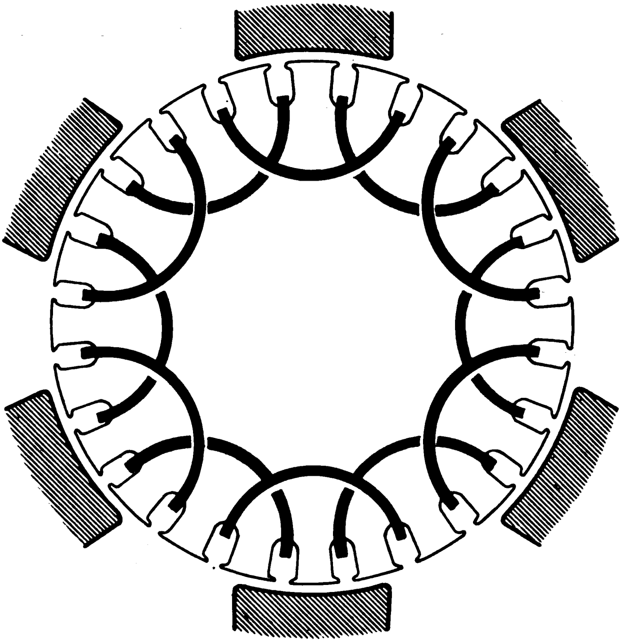

Fig. 256. Two Slots per pole per phase.

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TWO-PHASE WINDINGS.

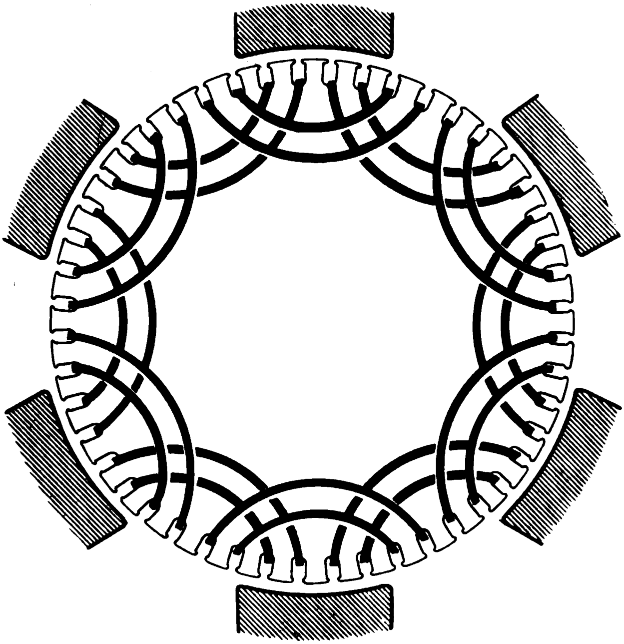

Fig. 257. Four Slots per pole per phase.

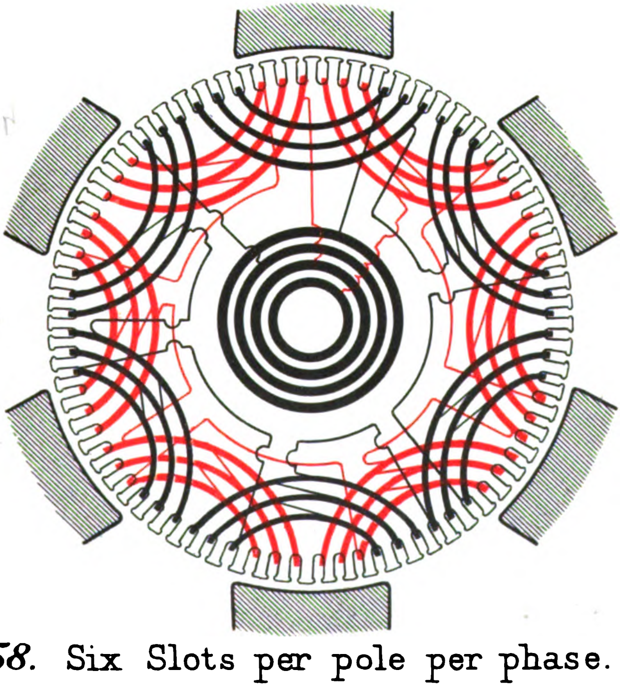

Fig. 258. Six Slots per pole per phase.

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Thos Keil & Son, Ltd

TWO-PHASE WINDINGS.

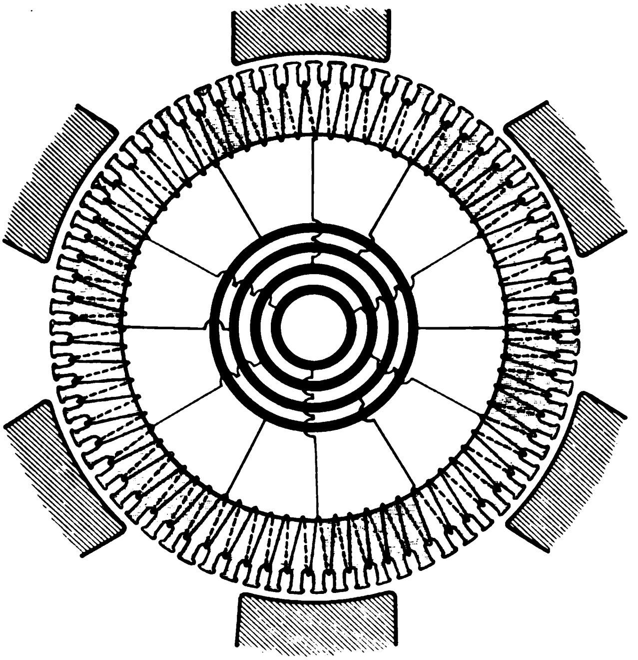


Fig. 259. Ring-Winding, parallel grouping

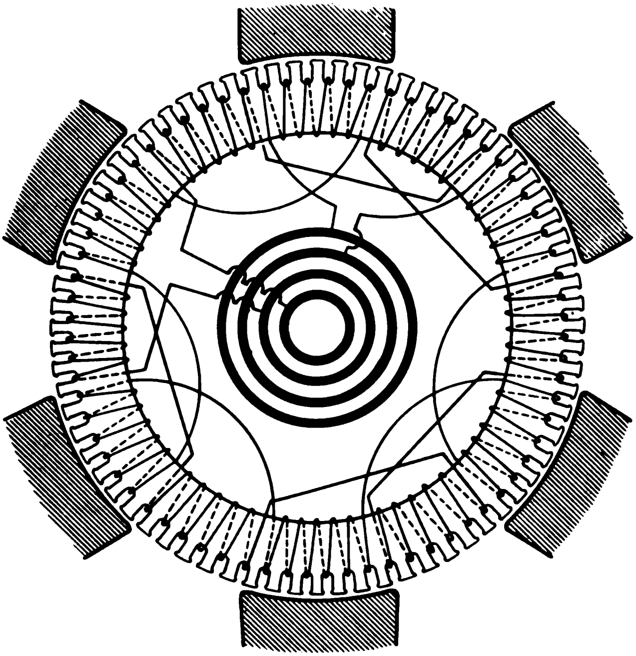


Fig. 260. Ring-Winding series grouping.

E & F. N. Spon Ltd London

[illegible] Kell & Son Ld

To attain 3½ slots per pole the arrangement with respect to the four adjacent poles would need to be 3 slots, 4 slots 4 slots, 3 slots. An example of 2½ slots per pole (per phase) is afforded by the Ganz alternator mentioned on p. 330.

Purely Distributive Windings.—As mentioned on p. 152, any ordinary lap-wound drum-armature or ring-armature can be connected up as a single-phase armature by merely joining to the winding at the appropriate points the connexions to two slip-rings, but this gives a number of circuits in parallel with one another.

TWO-PHASE WINDINGS.

A two-phase winding can be made from *any* of the above single-phase windings, by merely providing a duplicate winding of identical nature, and another set of slots displaced along the periphery of the armature by an amount equal to one half of the pole-pitch. The only exception to the above statement, is the case where the windings have been distributed in belts so broad that there is not room for another belt or set of slots between those already designed for the first of the two phases. For example, by duplicating Fig. 234 we obtain Fig. 255, which will obviously require 12 slots and 4 slip rings, 2 for one phase and 2 for the second phase. Or, again, by duplicating the wave-winding of Fig. 245*a* we obtain Fig. 254*a*, in which there must now be 4 slots per pole, or 2 slots per pole per phase.

Again, taking the mixed lap- and wave-winding of Fig. 248*b*, it can be made into a two-phase winding, as in Fig. 254*b*, by duplicating the system. In the case of bar-windings with curved up end-connectors, like those used in the three-phase armature Fig. 293, two-phase winding presents no new difficulties since the connecting pieces fit in between one another in the same way. With coil-windings the coils must be made of two different shapes, one bent up out of the way of the other. For example, the winding shown in Fig. 254*c* obtained by duplicating Fig. 248*d*, with 3 slots per pole per phase, requires two ranges of coils, one projecting straight out, the other bent up behind them, as in Fig. 262. This winding in two

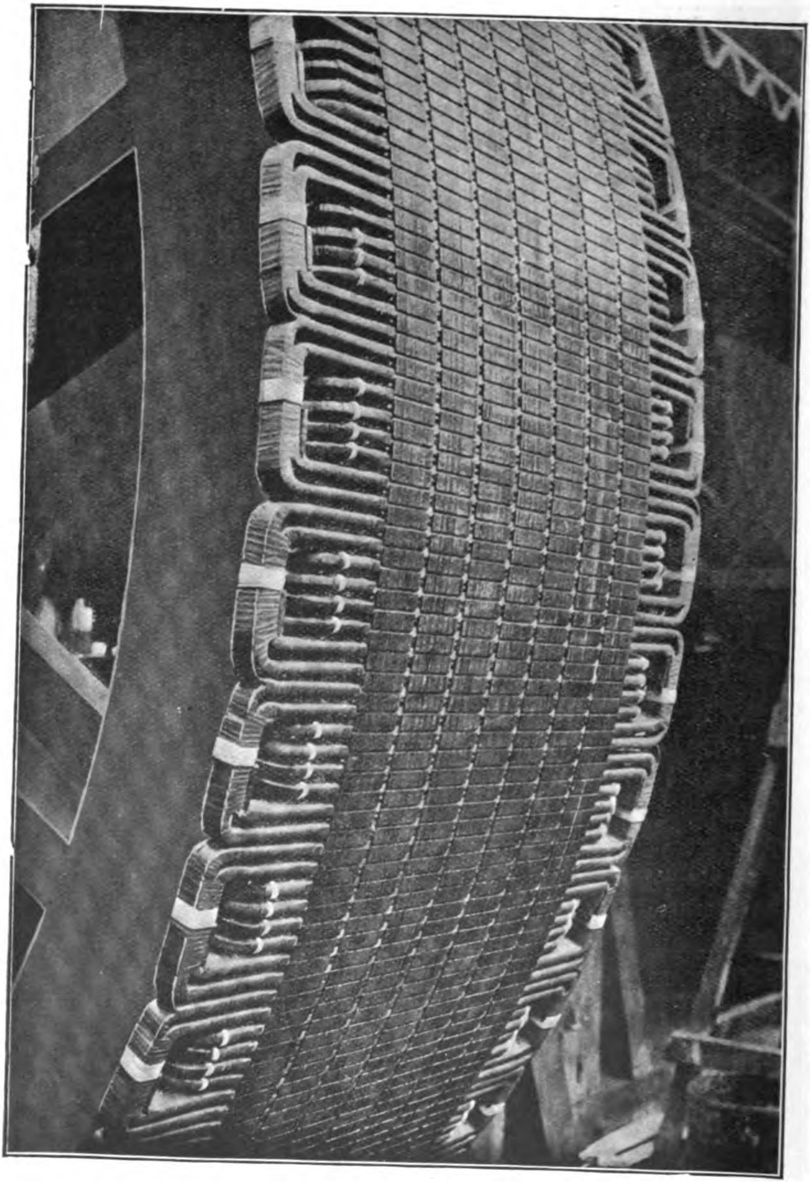


FIG. 261.—FOUR-SLOT, TWO-PHASE WINDING. [AQ(A) 60—600—120; 2200 VOLTS (WESTINGHOUSE CO.).]

ranges is due to Brown. A recent variety, shown in Fig. 261, has the coils which are straight out at one end, turned up at the other. Figs. 255 to 260 depict sundry two-phase windings of rotor armatures. Fig. 255 is a mere duplication of Fig. 234, with the disadvantage of large slots. Fig. 256 is the same winding distributed in twice as many slots. In Fig. 257 the distribution is in 4 slots per pole per phase, and in Fig. 258 is in 6 slots per pole per phase. If in Fig. 255 each one of the six coils of each phase consisted of 120 turns,

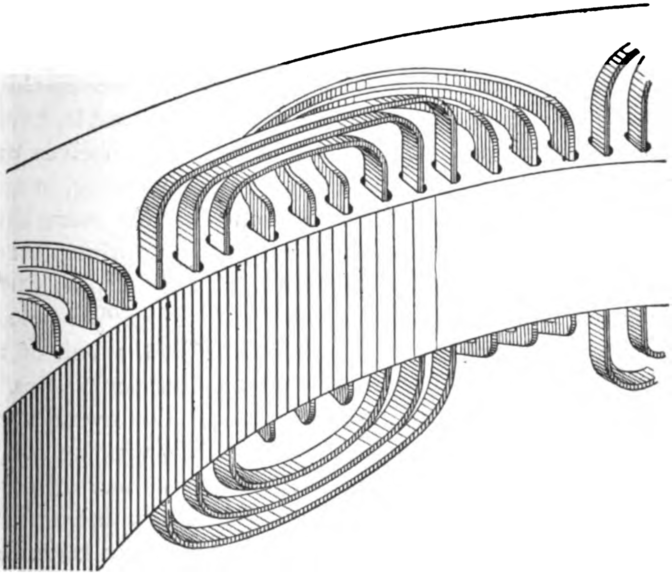


FIG. 262.—TWO-RANGE, THREE-SLOT, TWO-PHASE WINDING.

there would be in each phase 720 conductors in each slot, or, 1440 conductors in all. In Fig. 256 these 1440 conductors would be distributed as 60 conductors in each of 24 slots. In Fig. 257, there would be 48 slots with 30 conductors in each. In Fig. 258, there would be 72 slots with 20 conductors in each. Fig. 259 shows an ordinary ring-winding in 72 slots, and this, when joined down as shown, makes a two-phase winding (used as a mesh-connected 4-phase winding) with three parallel circuits in each phase. By disconnecting the 12 groups of coils and reconnecting as in Fig. 260 to put them into two sizes, a genuine two-phase winding, with two independent phases is obtained. Fig. 155, p. 171, illustrates a revolving two-phase armature, and shows the connexions that go to the slip-rings.

Short-Coil Windings.—To obviate the mechanical difficulty of the overlapping of the coils of the two phases, it has been proposed to use coils of one half the pole-pitch in breadth. Indeed, this method was adopted so far back as 1882, by Gordon, and twenty years before him by Holmes. The narrow coils are disadvantageous in giving a wave-form with a high peak. The method is now rarely used.

Abnormal Two-Phase Windings.—For those machines which have 6, 12, 18, or 24 poles, a winding is possible, having 2 coils for every three poles, the coils being themselves of a breadth equal to the full pole-pitch. In this winding, in each phase as shown in Fig. 254*d* there is one coil for every three poles. This is an extension of the hemitropic principle to a further stage. Fig. 254*e* shows the same special plan with a more distributed winding. We have here 4 slots per pole, of which only two-thirds are filled : or rather as the unit of the winding is a series of 6 poles, we have 24 slots per 6 poles, 16 of the 24 being filled and 8 left empty. The average coil pitch is here 0·75 of the pole-pitch. Such a winding can be applied only to machines with an odd number of pairs of poles. Fig. 263 shows how it maps out for a 6-pole machine.

Distributive Wave-Windings.—If an ordinary re-entrant wave-winding (two-circuit or "series" winding) is taken, and the conductors are numbered in the order of their connexion, then if beginning at any point we go over the winding until one quarter of the conductors in series have been passed over, and mark this point, and then follow out the next quarter of the whole number, and so forth, we shall thus mark out the whole series into four quarters. By connecting these four points to four slip-rings, we attain a mesh-connected 4-phase winding. If the winding is, however, divided at these four points into four independent parts, and reconnected so that the first and third are joined into one circuit (the third must be reversed), and the second and fourth (the fourth being reversed), are joined into another circuit, we obtain two independent circuits, the ends of each of which may be united to two slip-rings as a true two-phase winding. In a two-circuit wave-winding, the number of conductors, is, however, an odd number, being equal to $py \pm 2 = Z$, where y is the mean winding-step (see Vol. I. p. 378); and this number Z will not always divide evenly by 4. Hence, to have equal numbers of conductors in the 4 segments of the winding is not always possible. Thus a 12-pole machine, with a winding-step of 25 will have (as an ordinary singly re-entrant wave-winding) either 298 or 302 conductors. This would give 4 unequal groups of the following numbers 74, 75, 74, 75, or else 75, 76, 75, 76. If odd numbers were thus taken, some of the connecting points would be at the back of the armature if others were at the front. We might take instead 74, 74, 74, 76. or else 74, 76, 76, 76. The slight inequality in the electromotive-forces would be immaterial; or, as a remedy, two of the conductors might be left out altogether to secure equality in numbers. The first Niagara machines (p. 108) were wave-wound with a distributive two-phase winding, having 12 poles and 187 slots, with 2 conductors in each slot. Here $p = 12, y = 31, Z = 374$, making

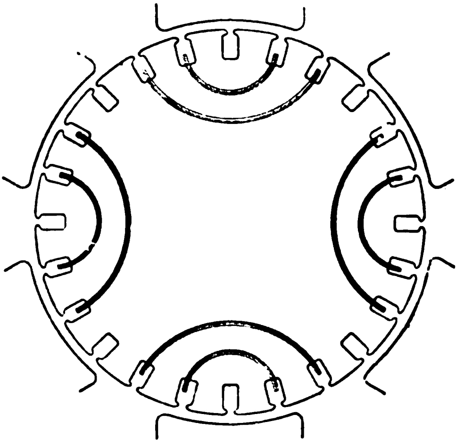


FIG. 263.—TWO-PHASE ARMATURE, A-TYPE (abnormal case).

187 conductors in each phase. For the later Niagara machines, see p. 421.

Two-Phase Machine as Single-Phase.—If the two windings of a two-phase machine are joined up in series, it can be used as a single-phase machine, the voltage being thereby increased in the ratio of $\sqrt{2}$ to 1.

THREE-PHASE WINDINGS.

On the same general principle as that already explained, a three-phase winding can be made from any of the single-phase windings previously described by merely taking three identical single-phase windings and spacing them out successively along the periphery of the armature by amounts equal to one-third and two-thirds, respectively, of the double pole-pitch; the unit, so to speak, in terms of which the spacing must be expressed, being that pitch which corresponds to one whole period. In the above statement it must be understood that each of the three individual windings must itself be so far concentrated into narrow belts that there is room for the two other belts of windings between them. This limits the breadth of the belt of winding in any one phase to one-third of the pole-pitch.

On the above principle the three windings, Figs. 266, 267, and 268, are simply triplicated from the corresponding single-phase windings of Figs. 241, 242, and 243, and require three times as many slots. In the case of Fig. 267 where each phase has as many coils as there are poles, there must necessarily be *three ranges* of end bends. This complication is avoided in Fig. 268 by the adoption of the hemitropic winding; for then, though there are three sets of coils, the end bends can be arranged in *two ranges* only, as appears also from Fig. 269. It will be noted that in any one phase, the phase coloured red for example, the successive coils come alternately in the inner range and in the outer range. Perfect symmetry recurs therefore after every four poles only. As a consequence, if this winding (which is due to Brown) is applied to a machine (whether in a stationary armature or a revolving armature) which has any odd number of pairs of poles, as in a 6-pole or a 10-pole machine, one of the coils must necessarily be askew, going from the inner to the outer range. This is shown in Fig. 272, which exhibits a 10-pole stator with 30 slots. If we number these slots and begin by filling in the black series or A coils they run 1–4, 7–10, 13–16, 19–22, 25–28, and belong alternately to the straight set or outer range, and to the bent-up set or back range. In like manner we fill up the B set, or red coils. If then we fill up the C set or green coils, beginning at 5–8, 11–14, 17–20, 23–26, when we come to 29–2 we see that this coil must come askew. Fig. 264 shows the general appearance of this 2-range winding, with straight and bent-up coils.

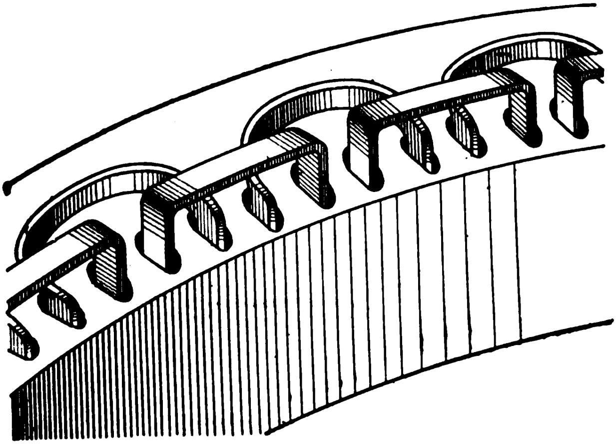


FIG. 264.—TWO-RANGE THREE-PHASE WINDING.

A three-phase winding can also be grouped if preferred in three ranges, as shown in Fig. 273, when the same 10-pole stator is shown again. The coils of each phase are alike; those of the A phase being all in the straight-out range; those in the B phase in a bent-up range; those in the C phase in a bent-down range. This arrangement has the disadvantage that by reason of the bending down of the third range the field-magnet cannot be withdrawn. This is more apparent from the perspective sketch in Fig. 279. If however, the third phase is changed by simply placing its windings a whole pole-pitch further on (as for example by joining 2 to 5 and 8 to 11, etc. in Fig. 273), the arrangement is improved. For, as will appear from the sketch, Fig. 280, the grouping is now such that at distances along equal to the double pole-pitch (where dotted lines are drawn), the stator may be divided. This enables the top half of an armature to be removed by mere disconnection, without unwinding any coil. An example is shown in Fig. 265, which depicts an induction motor, with the upper half of the stator removed.

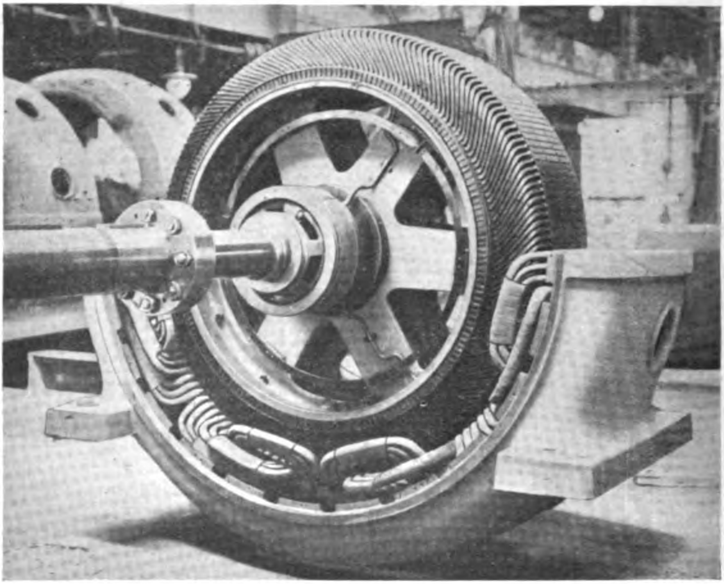
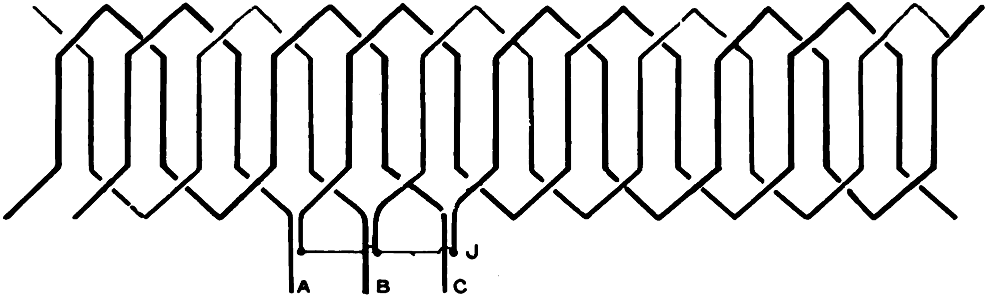


FIG. 265.—THREE-PHASE INDUCTION MOTOR, BY LAHMEYER, WITH THREE-RANGE WINDING.

An alternative method of grouping is the *chain-winding* shown in Fig. 270, in which the coils, made long at one of their sides and short at the other, are bent so that they can lie behind one another. In the case of open slots the coils may be former-wound, and afterwards wedged into their places. This plan of winding is supposed to have some

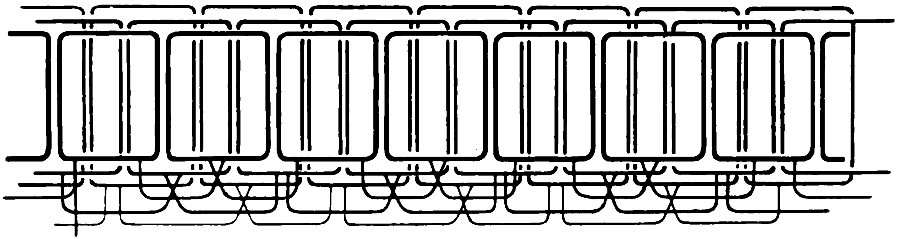
THREE-PHASE WINDINGS

Fig. 266.



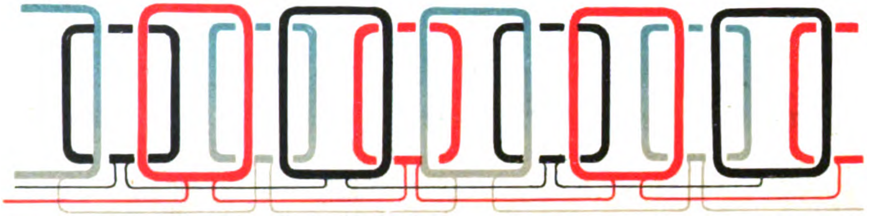
Bar-Winding:
One Slot per
phase per pole.

Fig. 267.



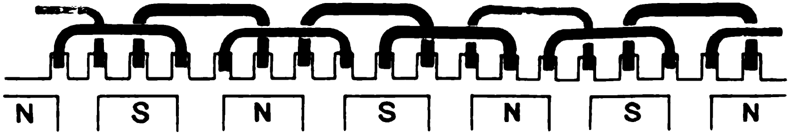
Coil – Winding,
(two sides of coils in 1 slot),
End bends in 3 ranges:
One Slot per
phase per pole.

Fig. 268.



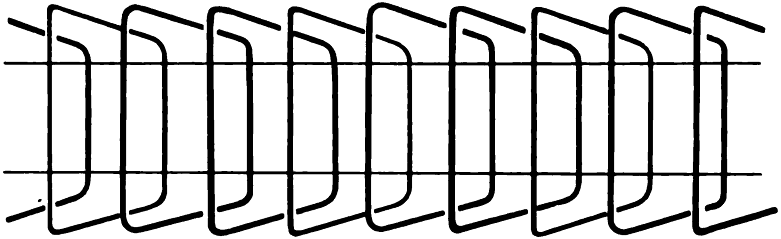
Coil – Winding,
(hemitropic),
End bends in 2 ranges:
One Slot per
phase per pole.

Fig. 269.



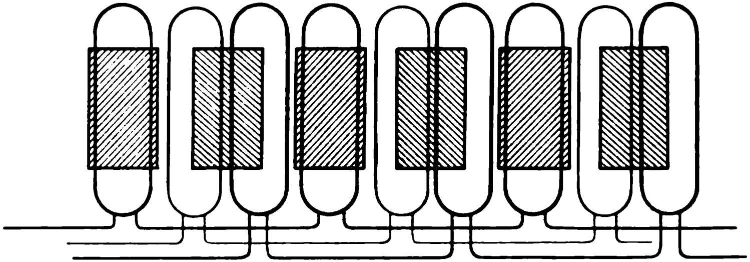
Two Ranges:
One Slot per pole per phase.

Fig. 270.



Coils all similar, with
long and short sides:
One Slot per pole per phase.

Fig. 271.



Short-Coil method of
obviating overlapping.
Coil-breadth = ⅔ pole-pitch.

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THREE-PHASE WINDINGS.

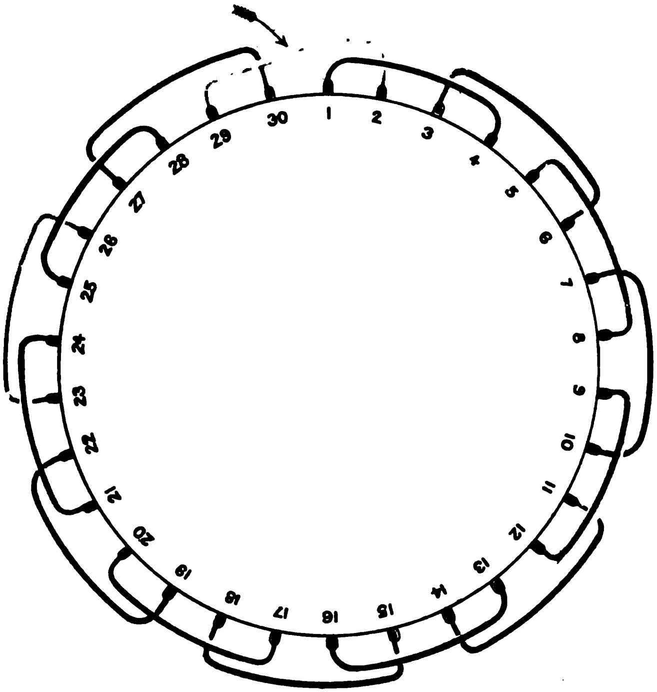


Fig. 272. Ten-pole Stator in Two Ranges.

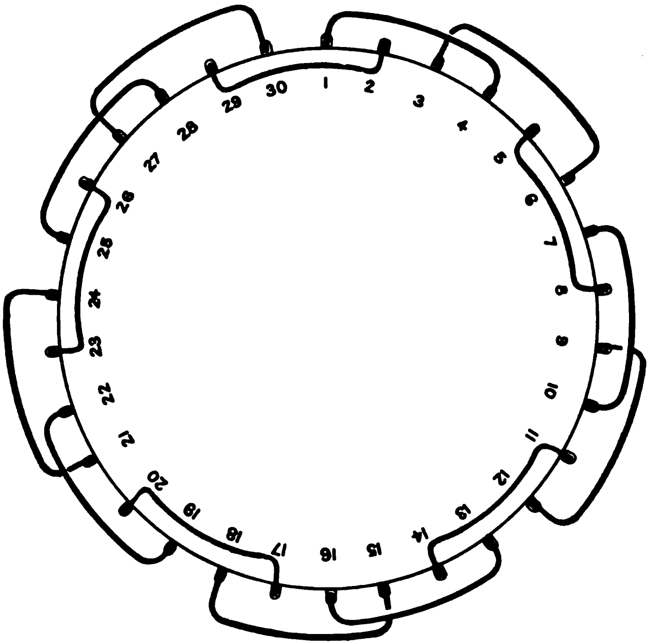
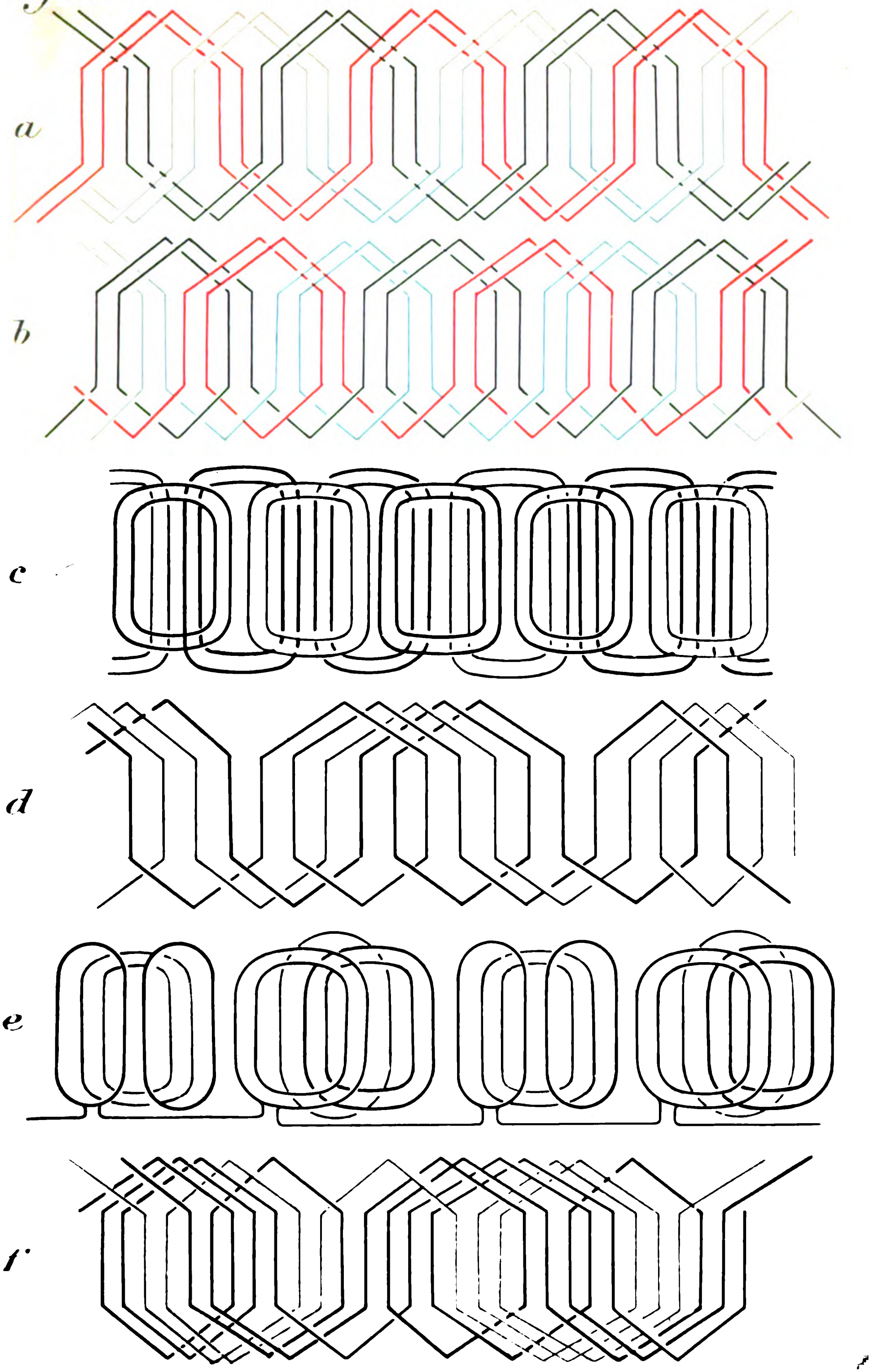


Fig. 273. Ten-pole Stator in Three Ranges.

Fig. 274. THREE-PHASE WINDINGS



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THREE-PHASE WINDINGS.

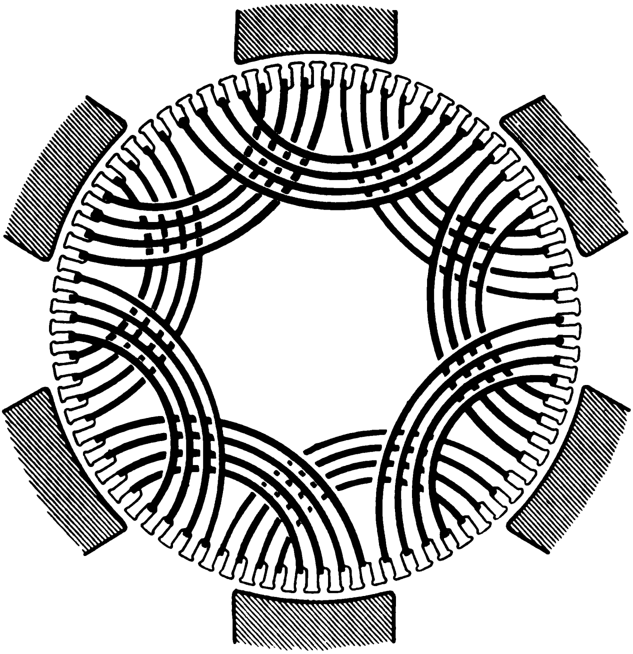

Fig. 275. Four Slots per pole per phase.

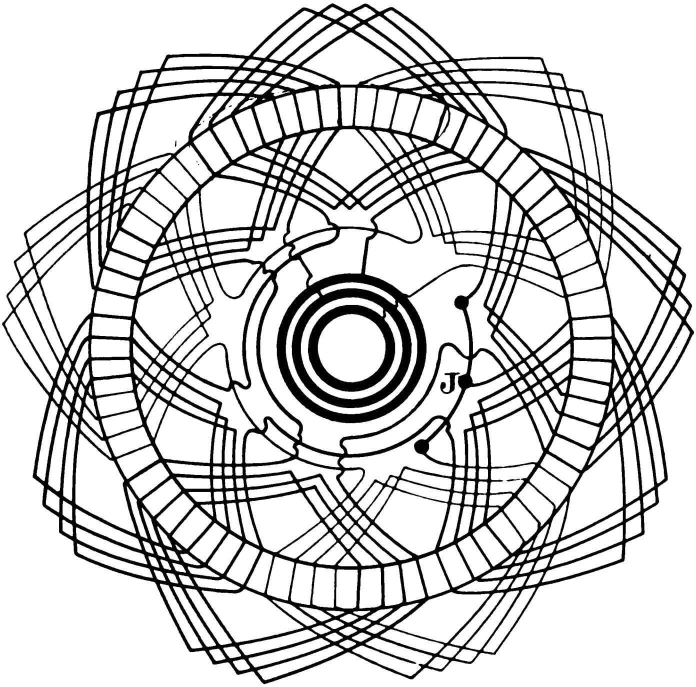

Fig. 276. Lap winding with Y-grouping.

THREE-PHASE WINDINGS.

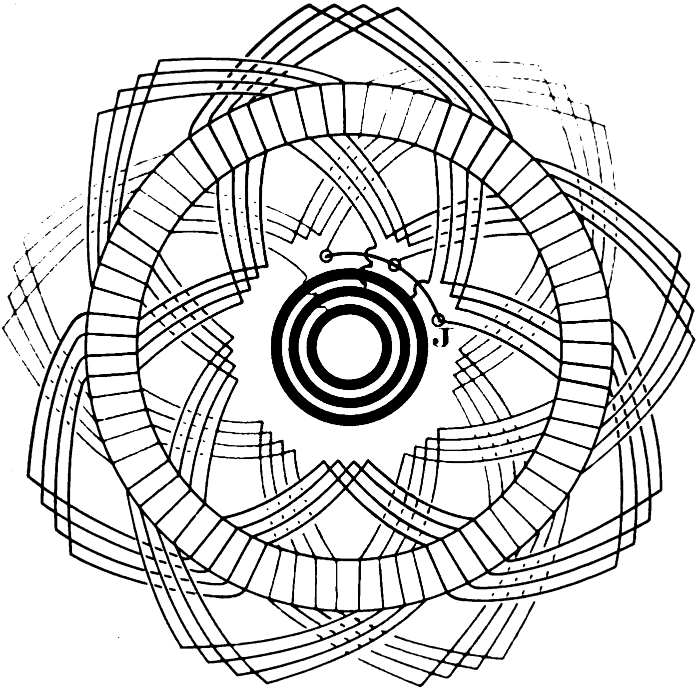

Fig. 277. Wave-winding, with Y-grouping.

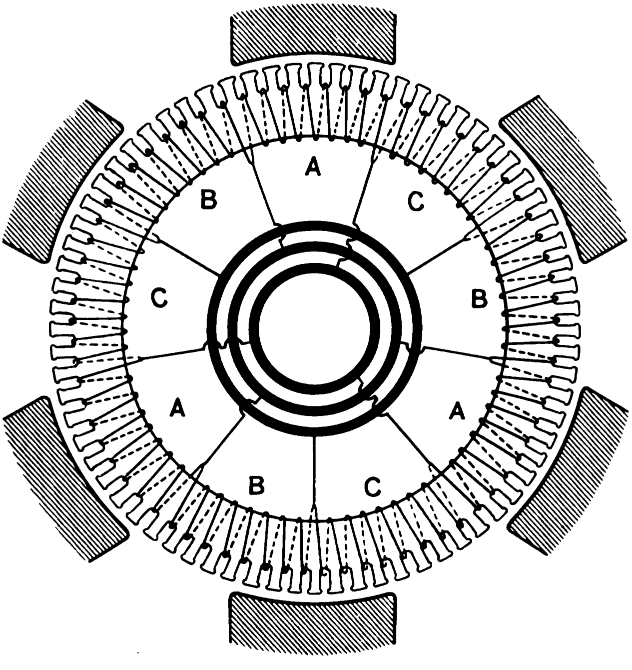

Fig. 278. Re-entrant winding: parallel grouping.

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Thos Kell &

advantage in keeping coils of different phases further out of contact from one another than the 2-range plan.

Another plan, shown in Fig. 281, and suitable for bar-windings, has the bars alternately long and short, with end-connectors made of forked strip.

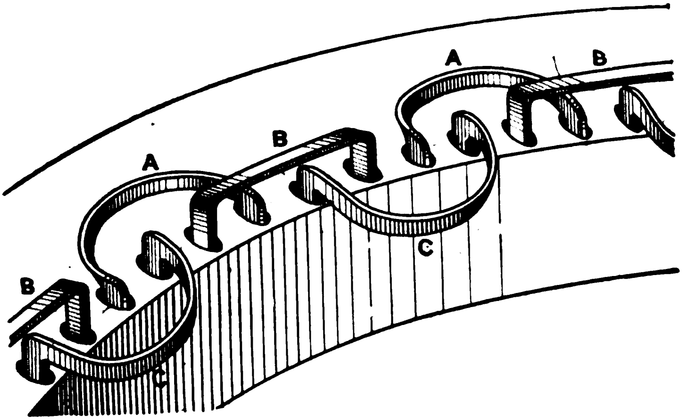


FIG. 279.—INADMISSIBLE ARRANGEMENT IN WHICH THE FIELD-MAGNET CANNOT BE WITHDRAWN.

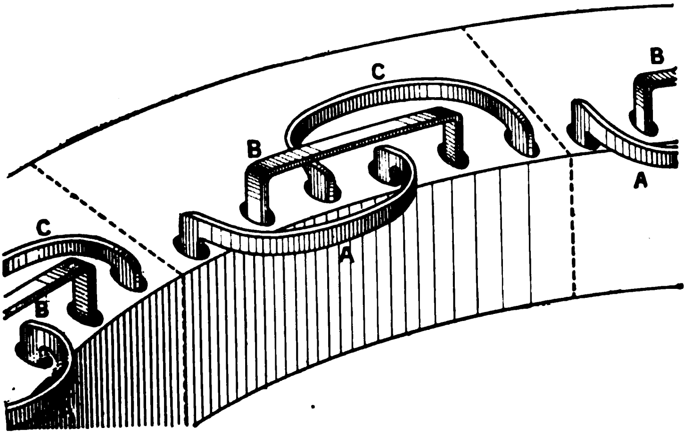


FIG. 280.—ADMISSIBLE ARRANGEMENT IN WHICH THE ARMATURE SEGMENTS CAN BE DIVIDED.

Two Slots per Phase per Pole.—Any of the windings described on pp. 302 to 305 may be triplicated to make a three-phase winding. Fig. 274*a* shows a pure wave-winding, and Fig. 274*b* a mixed wave and lap. The former is preferabie as giving end-connecting pieces of the same size throughout.

For coil-wound armatures the arrangement is obvious, the coils being grouped in two ranges as already described. Fig. 274*c* gives the developed diagram, and Fig. 282 a perspective view of such a winding.

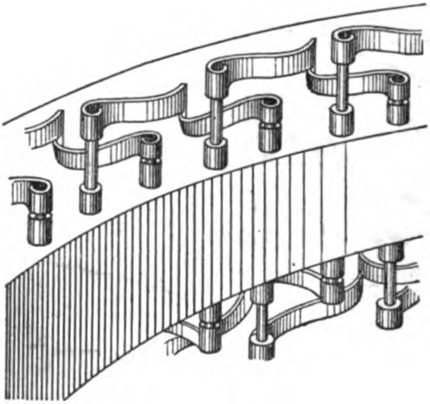


FIG. 281.—SINGLE-RANGE BAR WINDING.

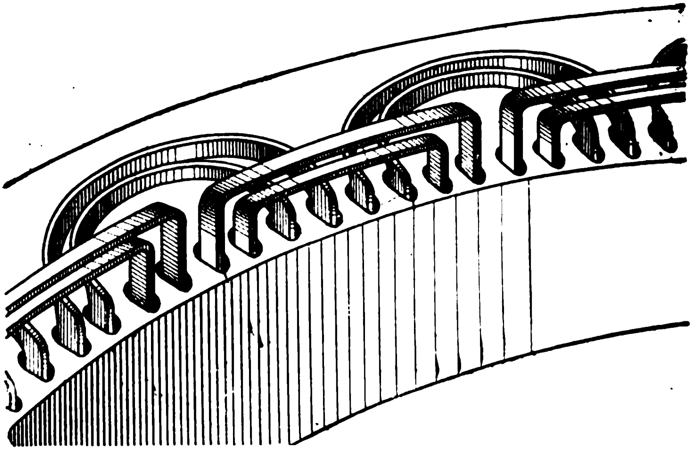


FIG. 282.—TWO-RANGE WINDING: "STRAIGHT OUT" AND "BENT UP."

Fig. 283 also illustrates this type of winding from an 8-pole three-phase stator by Dick, Kerr and Co., while yet another example of a two-slot winding is given in Fig. 284, but in this case with coils after the style shown in Fig. 270.

The detail is shown enlarged in Fig. 285, from which may be seen how a common junction is made for the Y-grouping of the 3 phases, and how the 3 other ends of the 3 circuits are brought out through the frame.

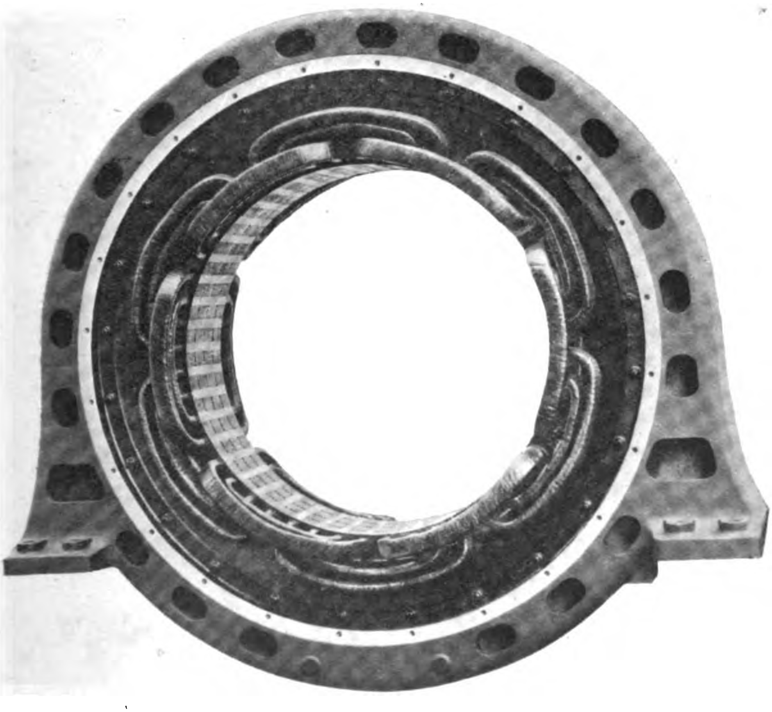


FIG. 283.—STATOR OF 8-POLE THREE-PHASE INDUCTION MOTOR (DICK, KERR & CO.).

Fig. 307, p. 334, depicts a half-armature having a 4½-slot winding with fork-connectors at the ends.

There is no need further to dwell upon cases where 3 and 4 slots per phase per pole are used, for they can be at once obtained from the corresponding single-phase windings on p. 306. Fig. 275 shows a 4-slot coil winding, and Fig. 276

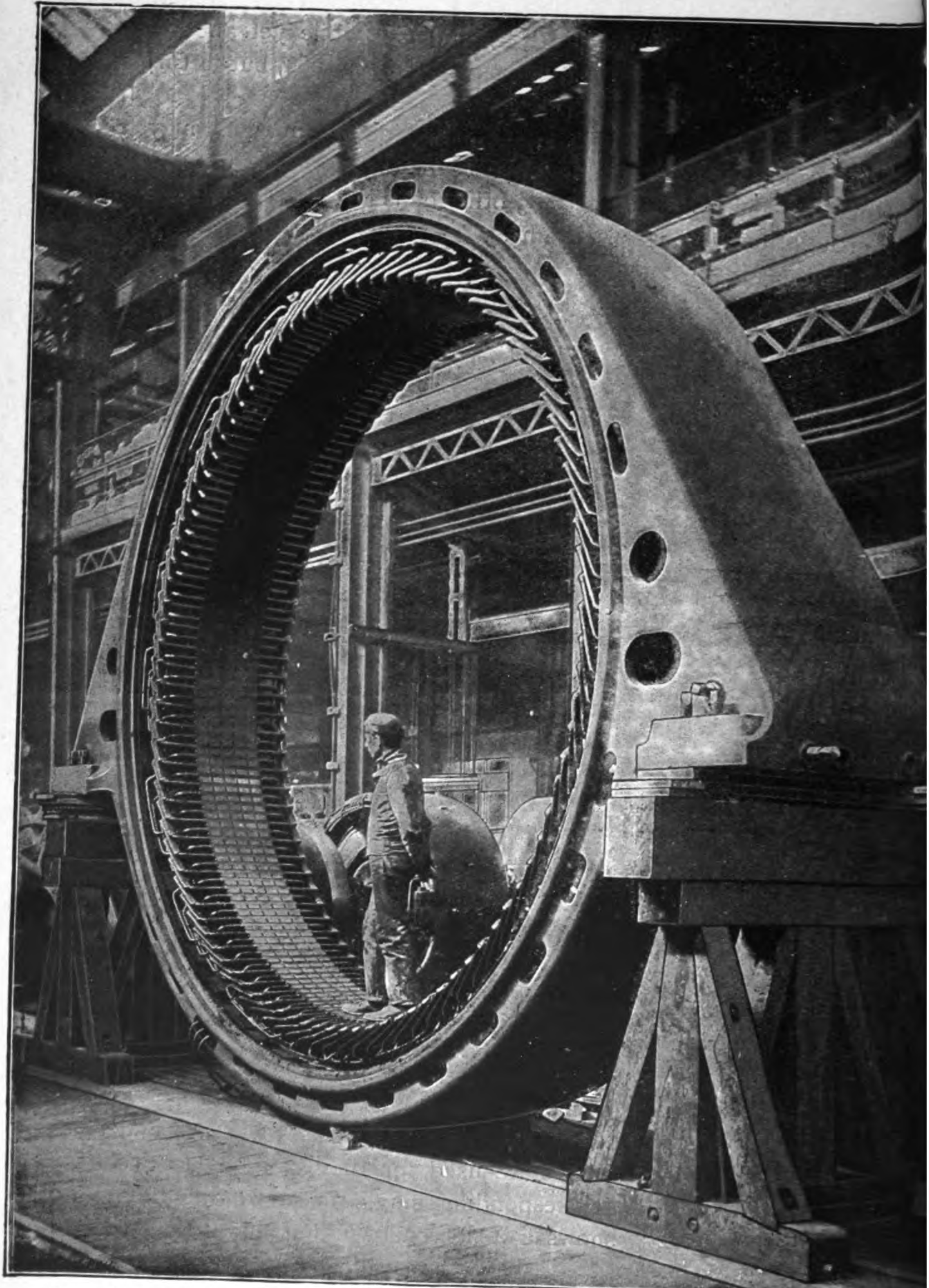


FIG. 284.—AT(B) 32—750—93$\frac{3}{4}$; 6600 V (WESTINGHOUSE CO.), SHOWING CHAIN-WINDING

illustrates a 6-pole rotor winding, with 4 slots per phase per pole, carried out as a lap-winding; the three circuits being united in a Y-grouping, with a common junction and three slip-rings. Fig. 277 gives the same case, but carried out as a pure wave-winding.

Distributive Windings.—The use of a purely distributive winding is illustrated by Fig. 278. Here a re-entrant winding represented as a ring, but which might equally well be a lap-wound drum, is joined to three slip-rings by radial connectors jointed in to the winding at successive points equidistant by ⅔ of the pole-pitch. In the case of a 6-pole machine, there will, therefore, be 9 radial connectors, and the electromotive-force between any two of the slip-rings will be equal to that developed in as much of the winding as lies between any two adjacent connectors. There will be in each phase as many parts in parallel as there are pairs of poles—in this case, 3.

FIG. 285.—DETAIL SHOWING THE COMMON JUNCTION OF THE THREE PHASES (WESTINGHOUSE CO.).

The 3 parts marked A belong to one phase; so also the 3 parts marked B for the second phase; and the 3 parts marked C for the third phase. The grouping is, of course, of the mesh or delta variety.

If each of the 9 segments were disconnected from the others, and then re-united so that all the A segments were joined in series for one phase, all the B segments for a second phase, and all the C segments for a third phase, we should no longer have a re-entrant winding, but the 3 independent phases which then might again be re-grouped either in Y or Δ grouping.

Similarly, a re-entrant wave-winding (two-circuit multipolar series) might be adapted to three-phase work; all that is requisite is to ascertain three points symmetrically (or nearly symmetrically) equidistant along the closed winding; treat the pieces of the winding between these three points as the three distinct phases, re-connecting them, if needs must be, in a Y-grouping.

The Niagara generators offer examples of distributive windings. The earliest (1897) were six machines of 5000 horse-power each, 12-pole, 2-phase, constructed by the Westinghouse Co. There were 187 slots with 2 conductors to each slot. This is 15½ slots per pole or 93½ slots in each phase. The winding was a non-re-entrant wave-winding, the phases being kept separate. The next type (1898) also by the Westinghouse Co., had 322 slots with 1 conductor in each, connected as before in 2 independent, non-re-entrant wave-windings. These machines were all of the O-type, and Fig. 286 shows a bar-winding similar to those actually used. More recently the General Electric Co. (Schenectady) has supplied to Niagara (1903) some other machines of different pattern. Three of 7500-kilowatt size are 12-pole, 3-phase, with a distributive wave-winding. They have 15 slots per pole (*i.e.* 5 per phase per pole) with 2 conductors per slot. Fig. 294 shows the mode of connexion, but for simplicity shows only 2 slots (in place of 5) per pole per phase.

The 5 alternators of 3750-kilowatts are 12-pole, 2-phase, with 13 slots per pole per phase; and while the two phases are kept independent, each consists of two separate wave-windings joined in parallel. The grouping is shown in Fig. 295, where

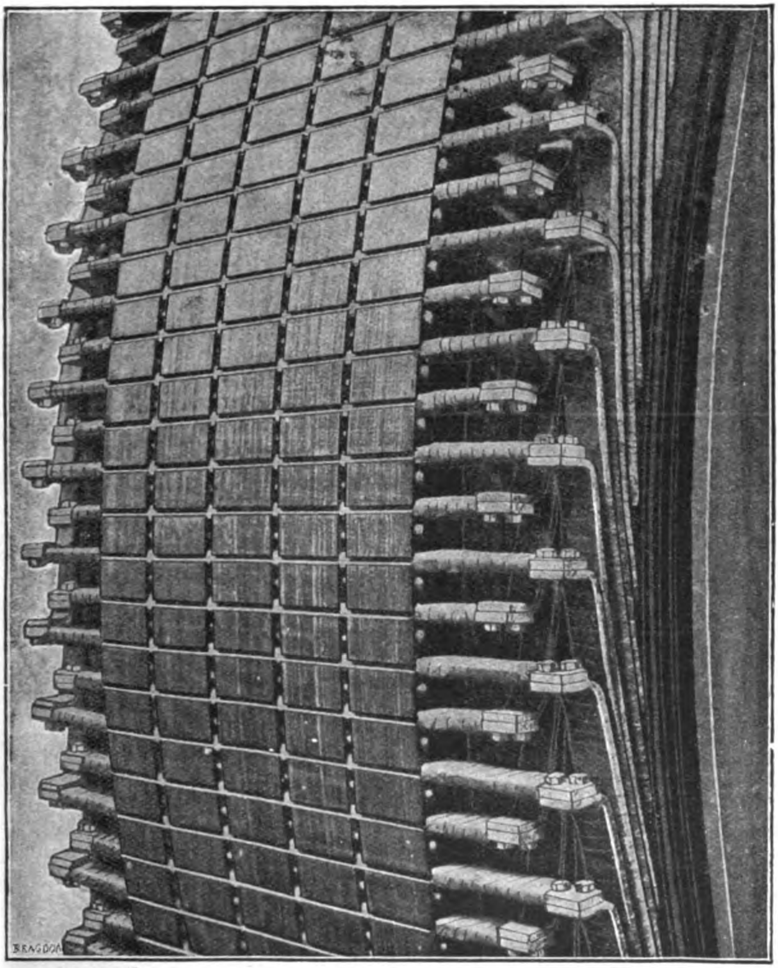


FIG. 286.—FIVE-SLOT THREE-PHASE BAR WINDING (WESTINGHOUSE CO.).

again for greater clearness the drawing is made with 3 slots per pole per phase instead of 13.

SIMILARITY IN COIL SHAPES.

There is one disadvantage in ordinary two-range windings, and that is that two or three separate shapes of coils are required. From the point of view of making, of winding and of supplying spares, it would be cheaper in many cases, if one shape of coil could be made to do for all phases. With such an arrangement also, the resistances and self-inductions of the phases would be more nearly equal. This may be accomplished in three ways, as follows :—

(*a*) By making the *ends* of the coils of different shapes; one end of the coil being straight-out and the other end bent up. Such coils can be former-wound except for the flat piece at the one end, the actual operation of winding the armature consisting in pushing the straight lengths through from one end and soldering up the projecting ends at the other. Half the coils are of course pushed in from one side and the other coils from the other side. The various stages in the construction are well shown in Fig. 287, and a similar armature completed is shown in Fig. 288. Fig. 289 also shows a three-slot winding on this plan in the winding shop during the course of construction.

(*b*) The second method is to make the two *sides* of the coils of different lengths, as shown in Fig. 270, in the style called *chain-winding*. Even with a multiple slot winding the several coils belonging to one phase may be all of the same shape as shown in Fig. 290 of a three-slot winding by Lahmeyer and Co. Fig. 291 is also an illustration of this same idea, being an eight-slot winding by the Westinghouse Co. The coils in this are completely former-wound with quite open slots closed by wedges. Bar windings can, of course, be made always to follow this method of design, as in Figs. 265 and 281. As further examples of such bar windings, Fig. 292 shows one for low voltages, having two conductors per slot, those near the face projecting further from the core than those behind. Forked strips of copper connect from the end of a long conductor to that of a short conductor, at a distance along equal approximately to the pole-pitch. Fig. 293 again shows a type

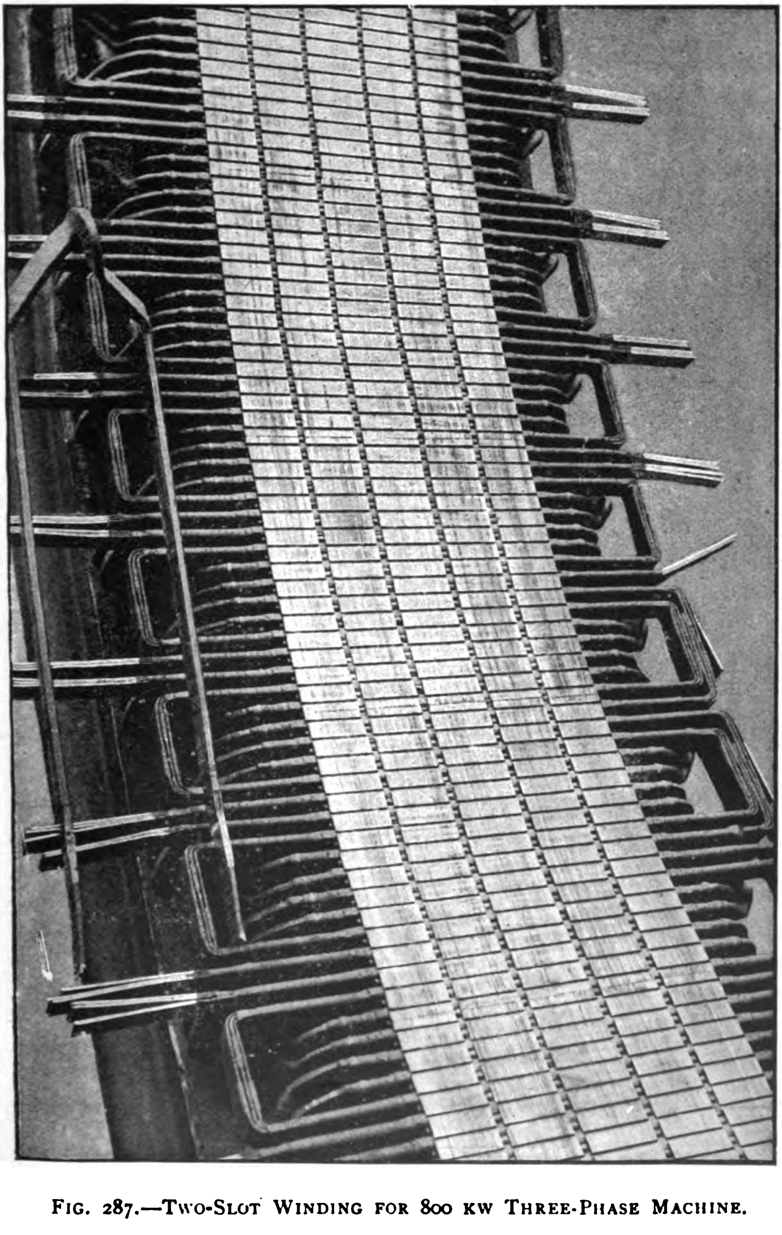


FIG. 287.—TWO-SLOT WINDING FOR 800 KW THREE-PHASE MACHINE.

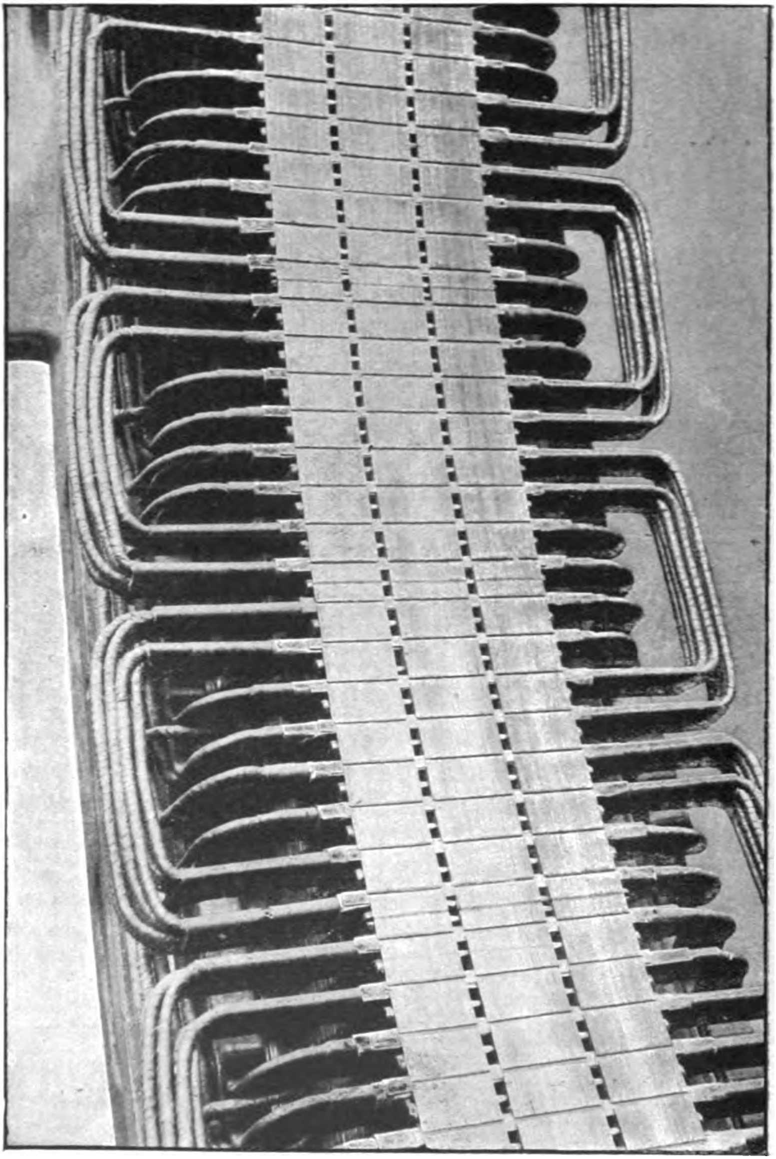


FIG. 288.—TWO-SLOT FORMER-WOUND COIL WINDING.

of bar winding suitable for large power machines. This example is of a Kolben 1500 KW generator for direct coupling to a gas-engine, with a five-slot winding. This is the same

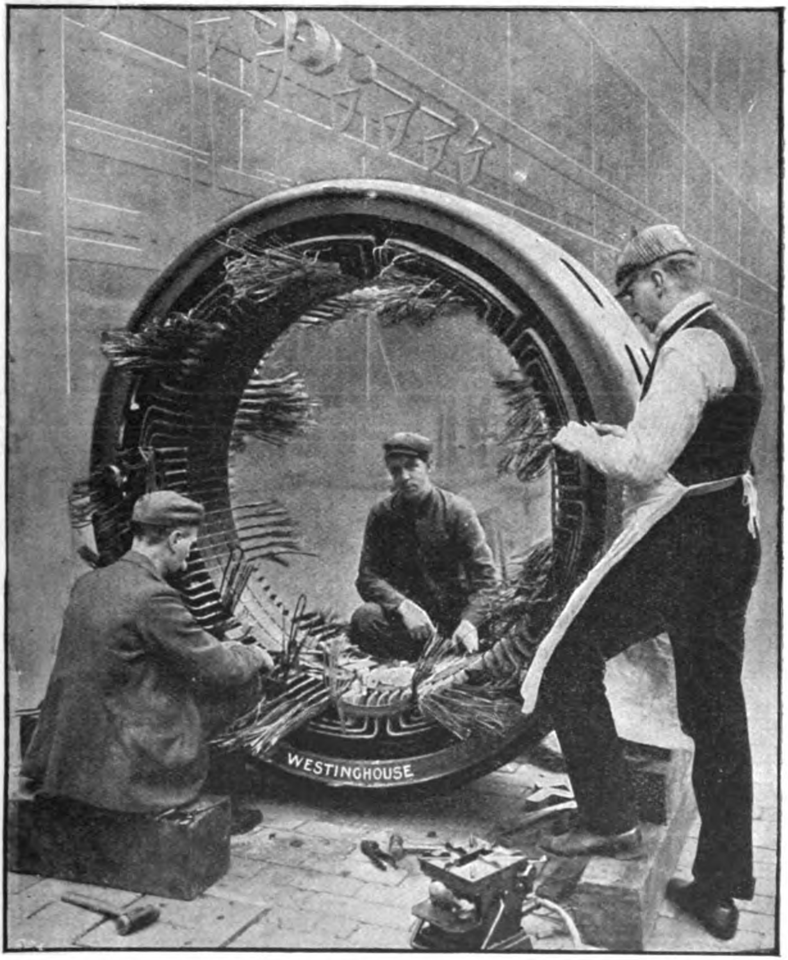


FIG. 289.—THREE-SLOT WINDING WITH FORMER-WOUND COILS, DURING CONSTRUCTION.

type of winding which has been adopted in the later Niagara 3750 KW and 7500 KW machines (see p. 421).

(*c*) The third method is to adopt a short-coil method of winding, similar to that described for two-phase, on p. 314. If the coils are made slightly narrower in span than two-thirds of the pole-pitch, 3 coils (1 for each phase) can be assembled side by side within the breadth of two poles. But the plan is not to be recommended, as the wave-form is too much peaked. The idea is shown in Fig. 271.

Semi-Symmetrical Three-Phase Windings. — By superposing three windings like Fig. 253*a*, with 1½ slot per phase per pole, we get Fig. 274*d*, a winding which can only be used on a machine having its poles a multiple of 4, since the unit pitch of this winding extends over 4 poles.

Fig. 274*e* depicts a coil-winding having also 1½ slot per phase per pole, requiring that the ends of the coils shall be bent into three ranges.

Fig. 290.—Winding of Lahmeyer Armature (Three Phase), 300 KW, 64 poles, 94 RPM, 240 volts.

Fig. 274*f* represents a winding of 2½ slots per pole as used on a 40-pole generator of Ganz and Co., shown at the Paris Exhibition of 1900. The three phases are not quite alike between themselves. In two of the phases the loops are arranged with equidistant spaces, equal to 5 intermediate slots. In the third phase they run 4, 5, 6, 5 ; or might run 4, 6, 6, 4. On the average the spacing comes the same.

Fig. 307 shows a 4½-slot bar-winding on a 3700 KW three-phase armature by the Westinghouse Co.

Creeping Windings.—Another species of winding, known as a "creeping" winding (*Schleichende Wicklung*), is applicable to particular cases. If 3 adjacent coils, each having a pitch of 120 electrical degrees, are set side by side, they will occupy the same breadth as 4 poles, and by repetition will serve for any machine having a multiple of 4 poles, but cannot be used for machines with 6, 10 or 14 poles. Fig. 296 shows this example. In the same way 9 coils, each of 160 electrical degrees, will occupy the same angular breadth as 8 poles; or 9 coils of 200 electrical degrees each will occupy the same angular breadth as 10 poles. Now of these 9 coils any three contiguous ones are nearly in phase, if wound alternately right- and left-handedly. For the 8-pole machine the phase

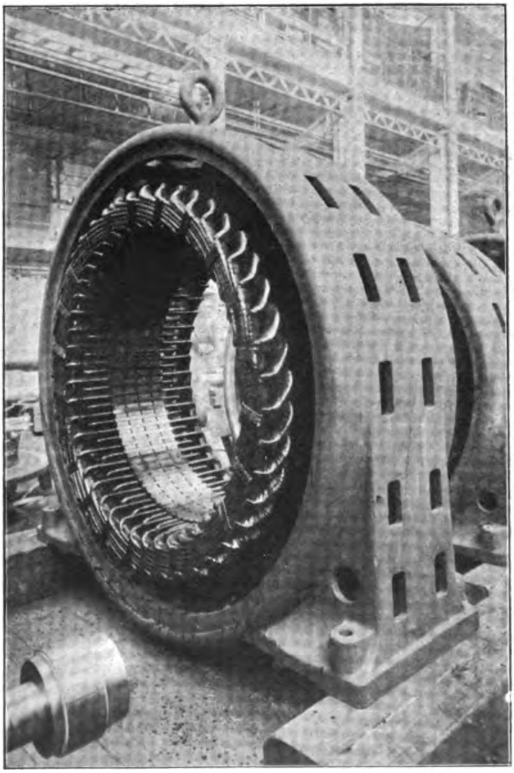


FIG. 291.—EIGHT-SLOT WINDING WITH COMPLETE FORMER WOUND COILS.

difference between adjacent coils is 20° ; for the 10-pole, it is also 20°. The cosine of 20° is 0·9397. Consequently, if 3 adjacent coils are united in series their joint electromotive-force will be equal to 2·879 times that of the middle one of

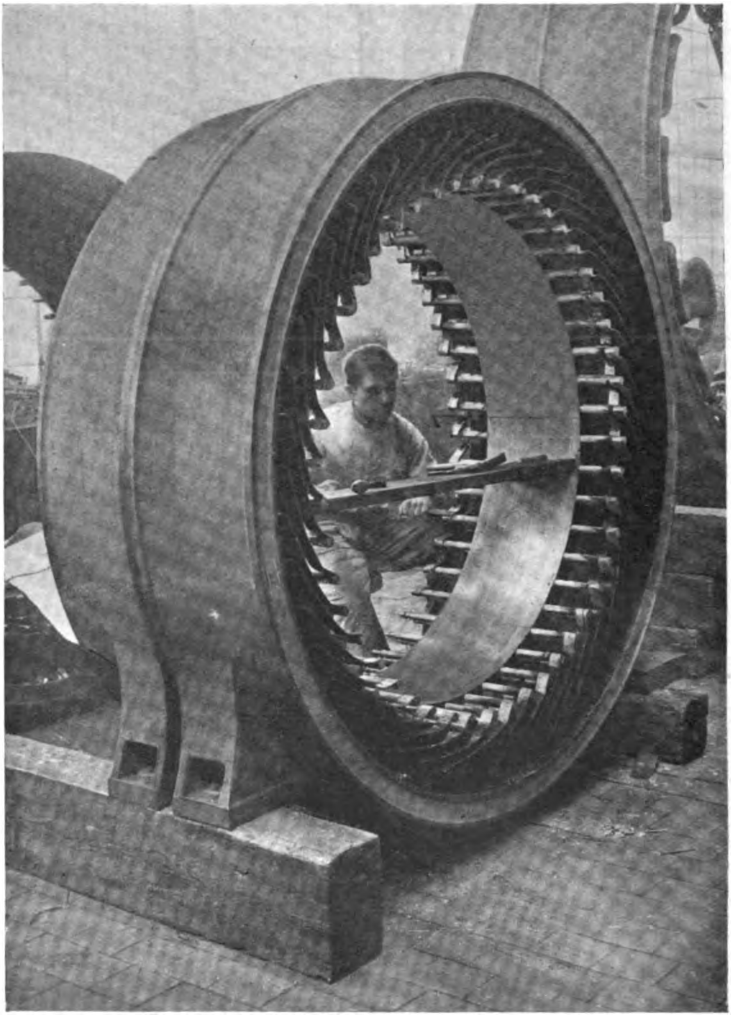


FIG. 292.—ARMATURE OF THREE-PHASE ALTERNATOR, 185 KW FOR 220 VOLTS, DURING WINDING (KOLBEN & CO.).

the three. The 9 coils may therefore be joined up in 3 groups of 3 adjacent coils, for the 3 phases. By repetition, the same

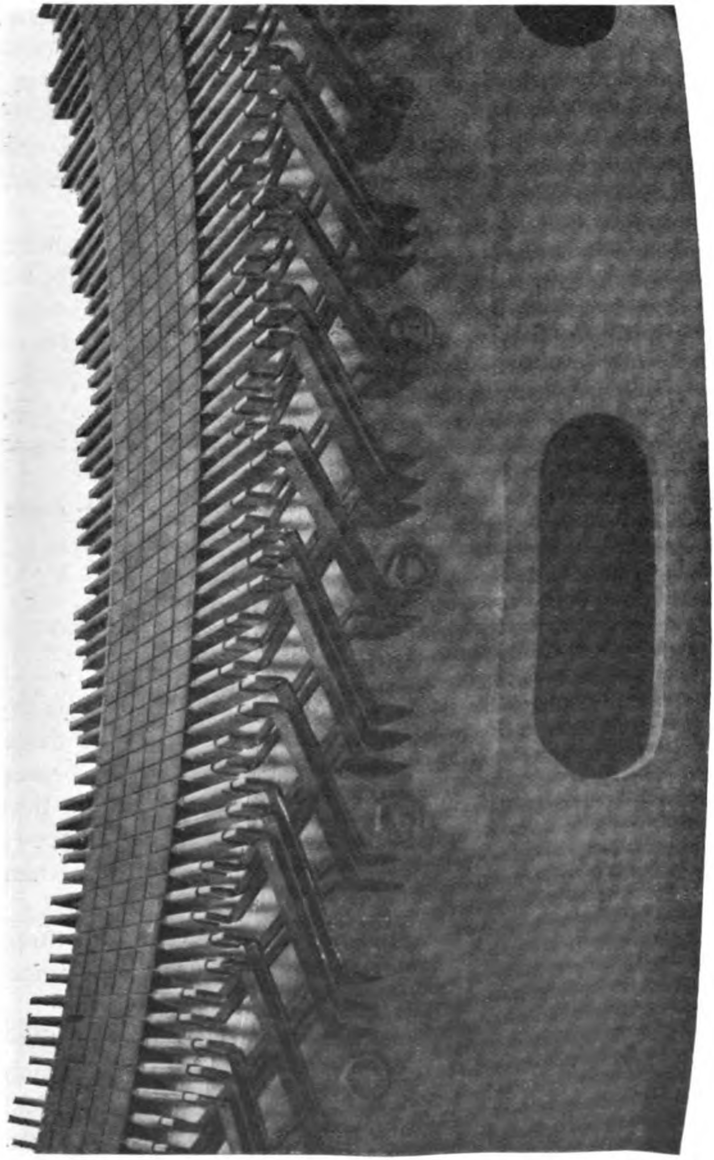


Fig. 293.—Bar Winding on Kolben 1500 kw Three-Phase Generator, 3000 volts, 94 rpm, 64 poles, Coupled directly to Gas Engine.

grouping will suit for any machine having a multiple of 8 or of 10 poles. These two cases are illustrated in Figs. 297 and 298.

Obviously in the same way a series of 15 coils, *i.e.* 3 sets of 5 adjacent coils each, might be constructed of the same angular breadth as 14 poles or as 16 poles. The general formula for such creeping windings would be: $F(3G \pm 1) = p$; where G is the number of adjacent coils of one phase, and F is the number of times that the arrangement is repeated. G must be an odd number, and F may be any whole number, odd or even.

In the Figs. 296, 297 and 298, the coils are represented as occupying 2 slots each. It is obvious that they might be further distributed, and in the case of Figs. 296 and 298, where the coil-breadth exceeds the pole-pitch, it would be advantageous to use 4 or 6 slots per pole. Also the whole might be carried out by wave-windings instead of coil-windings. Fig. 299 gives a creeping winding for 4 poles, with 1½ slots per pole per phase.

Abnormal Three-Phase Windings.—Abnormal three-phase windings are of several kinds. They may be classified as:

1. Unsymmetrical windings.
2. Tripled abnormal single-phase windings.
3. Re-grouped continuous-current windings.

Amongst unsymmetrical three-phase windings may be included any winding in which the three circuits of the three phases are of equal voltage, but which by reason of incorrect spacing on the armature generate electromotive-forces that do not differ by exactly 120° from one another. Fig. 300 is an example, the green and red phases being here less than 120° apart.

Also those cases would be included in which the three phases, though correctly differing in angle of phase-difference, were of unequal voltage.

An extreme case of dissymmetry is afforded by a two-phase winding with the two phases joined up at one end of each to a common terminal. This will generate unsymmetrical electromotive-forces. If there were 100 volts between A and

DISTRIBUTIVE WINDINGS

Fig. 294.

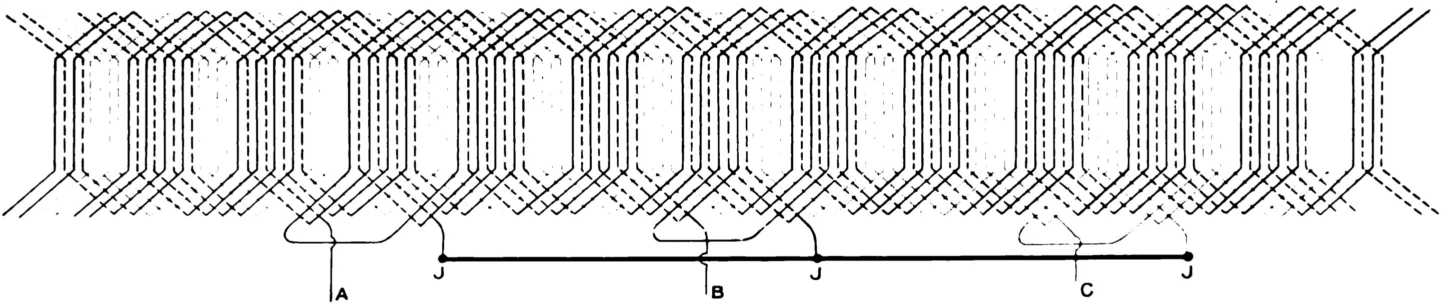
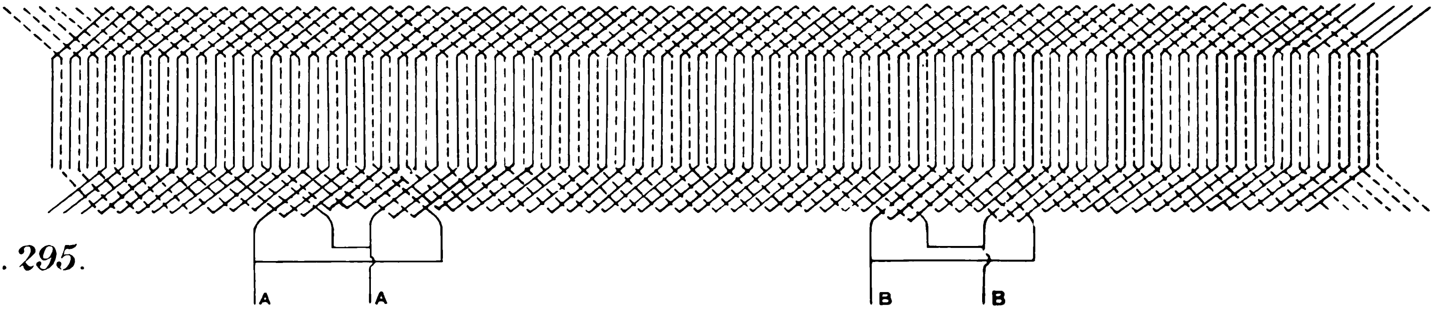


Fig. 295.



DIAGRAMS ILLUSTRATING THE WINDINGS OF THE NEW ALTERNATORS AT NIAGARA

E. & F. N. Spon Ld London

Lith. Kell & Son Ltd.

CREEPING WINDINGS, THREE-PHASE.

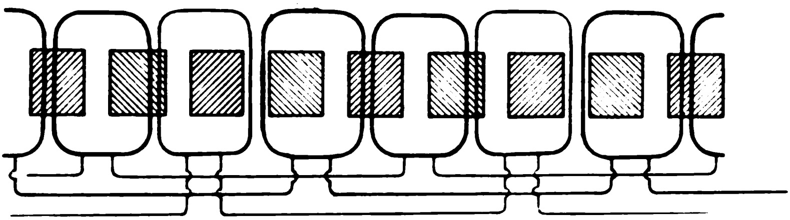


Fig. 296. Three Coils subtending four poles.

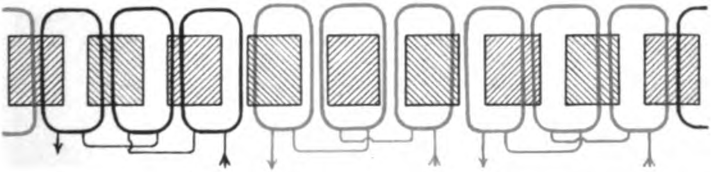


Fig. 297. Nine Coils subtending eight poles.

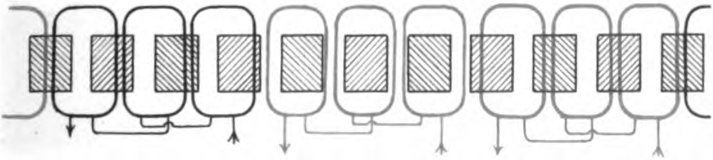


Fig. 298. Nine Coils subtending ten poles.

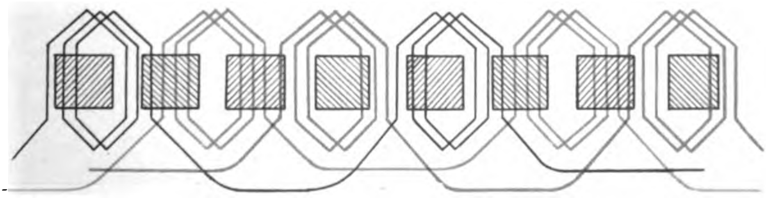
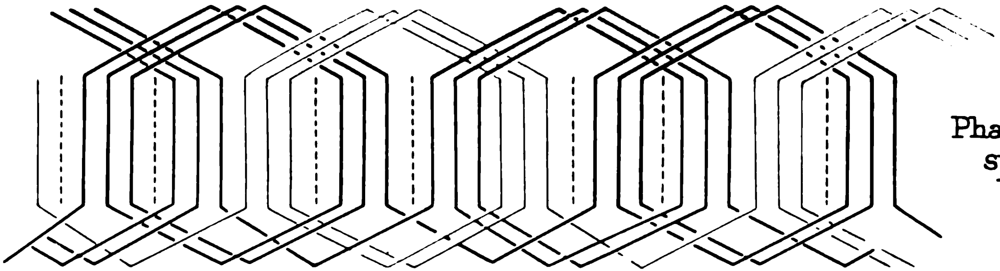


Fig. 299. Creeping winding, 1½ slots per phase per pole.

E. & F. N. Spon Lᵈ London.

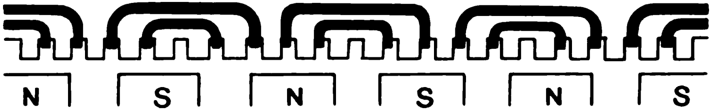
Thoˢ Kell & Son, Lith.

Fig. 300.



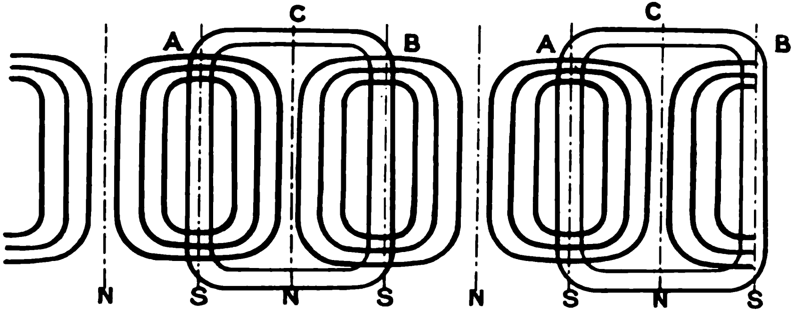
Phases unequally spaced apart.

Fig. 301.



Winding with 15 slots for four poles.

Fig. 302.



Winding for Scott's connexion. (Helios C°)

E & F N. Spon Ld London

Thos Kell & Son Lith.

THREE-PHASE WOUND ROTORS

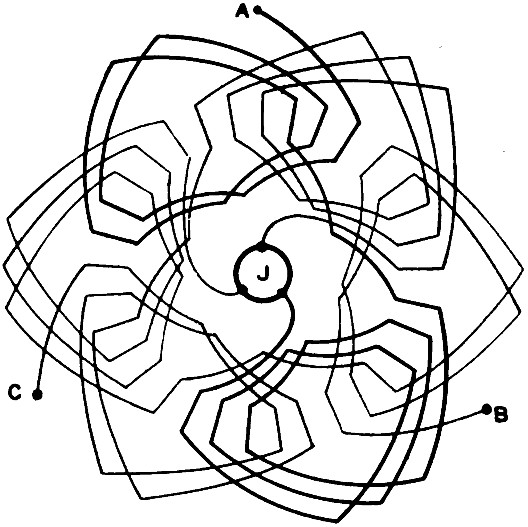


Fig. 303. Three identical circuits each having a prime number of conductors (4-Pole)

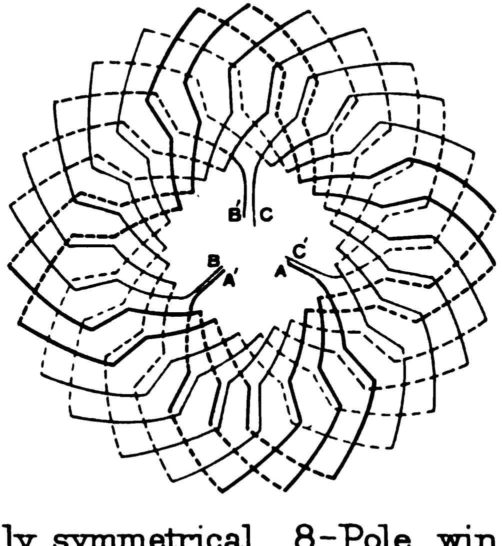


Fig. 304. Fully symmetrical 8-Pole winding made of a reentrant wave winding divided up

E. & F. N. Spon Lᵈ London
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THREE-PHASE ROTOR WINDINGS.

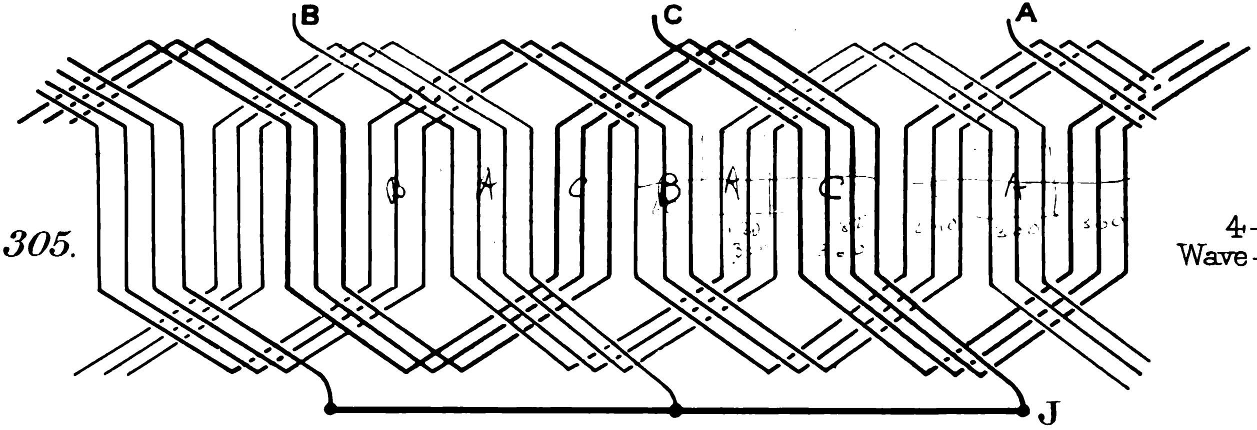

Fig. 305.

4-Pole
Wave-winding.

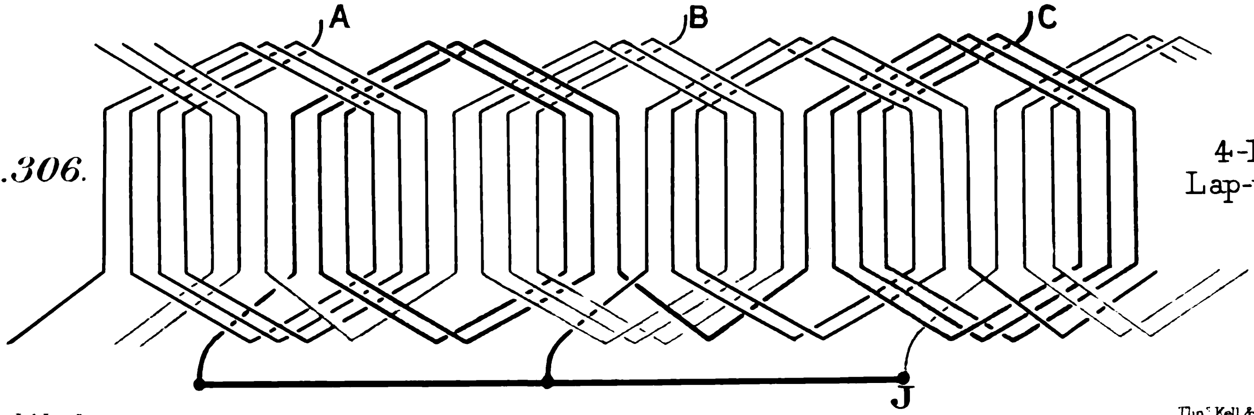

Fig. 306.

4-Pole
Lap-winding.

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Thos Kell & Son, Lith

C (Fig. 308), and 100 volts between B and C, there will be 141 volts between A and B. And while the angle of phase-differ-

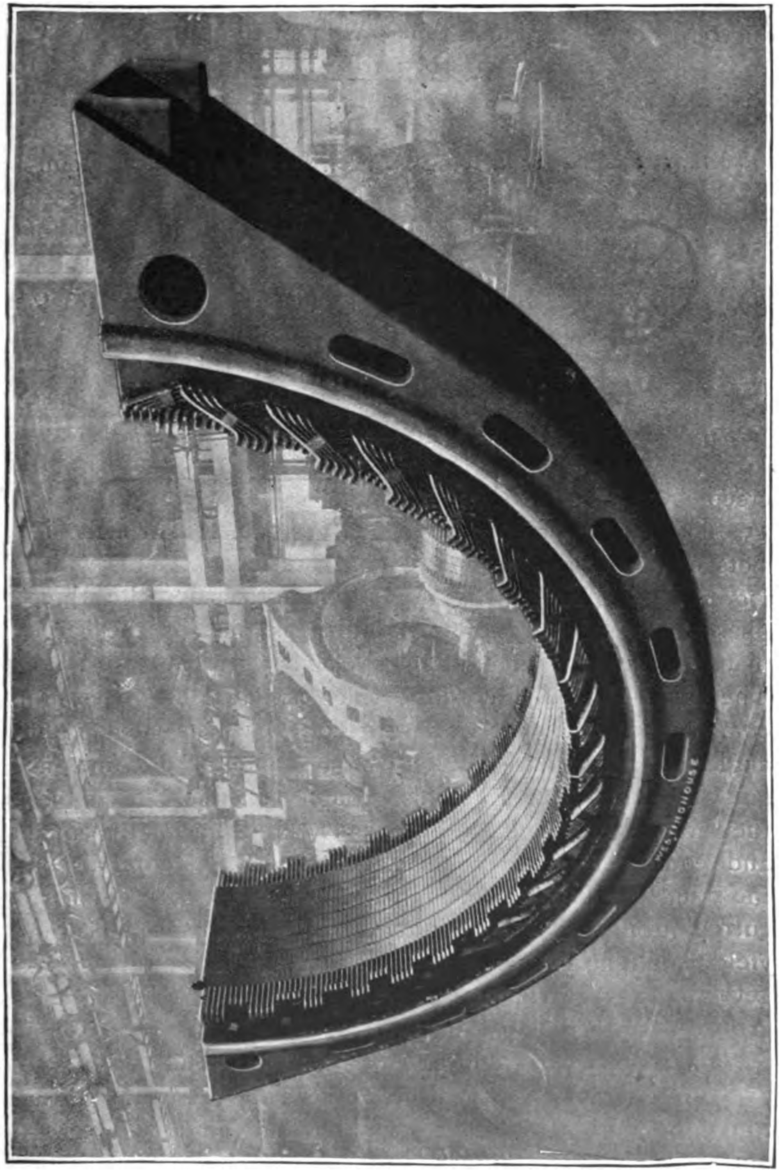


FIG. 307.—HALF-ARMATURE, SHOWING 4½ SLOT WINDING (13½ SLOTS PER POLE), 3730 KW (WESTINGHOUSE CO.).

ence between A and B is 90°, those between B and C and C and A will be 135° each. The so-called monocyclic system is a three-phase system, having one of the three voltages, AB, in Fig. 309, about 1·79 times the value of either of the others, AC and BC, the auxiliary voltage of CD being made a quarter of that of AB.

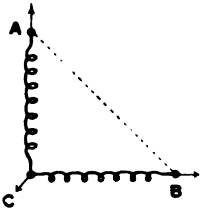


FIG. 308.

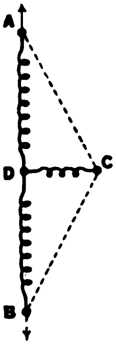


FIG. 309.

By extension of the hemitropic principle to the case where there is but one coil (of a given phase) for every three poles, there was given at p. 310, Fig. 254*e*, an abnormal two-phase winding. The same principle applied to three-phase yields a result identical with Fig. 296, which is a simple creeping winding. A simple variant of this in Fig. 301, with 15 slots for 4 poles or 3¾ slots per pole, of which 12 slots only, or three slots per pole, are filled. This winding, though abnormal, is fully symmetrical for the three phases. Another abnormal winding is that of the large machine of the Helios Company, shown at Paris in 1900. The coils A and B (Fig. 302), which are alike in phase, are combined at their junction with coil C which is at right angles in phase, and gives a voltage about 0·85 times as great as either A or B. This grouping therefore resembles the "monocyclic" system, but yields three practically equal and equally-displaced voltages between the 3 terminals, on the principle of the Scott method of obtaining a three-phase current from a two-phase system. This Helios machine can yield simultaneously a single-phase current from A and B together, and a three-phase current from A, B, and C together, grouped as described.

Six-Phase and Twelve-Phase Windings.—Six-phase and twelve-phase windings are wanted for the operation of rotatory converters. In a six-phase winding the angle of phase-difference between any two adjacent phases is 60°, and the voltage between the terminals of any two adjacent phases is equal to the voltage in any one phase.

A six-phase winding can be made out of a three-phase winding by first disconnecting the three phases from one another, and then uniting together at a common junction the middle point of each of the three phases, as in Fig. 310, thus giving a star-grouping with 6 terminals. If a mesh-grouping is required, each of the three phases must be cut into two equal parts and then they must be reconnected as shown in Fig. 311.

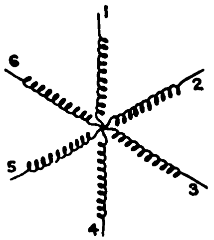


FIG. 310.

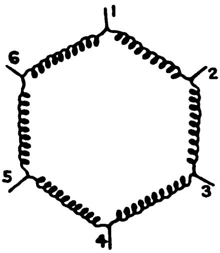


FIG. 311.

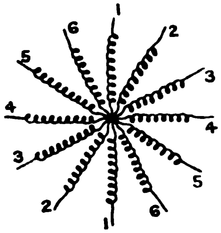


FIG. 312.

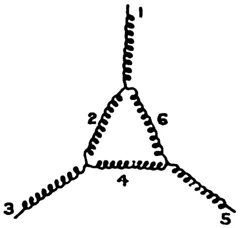


FIG. 313.

In converters, where the armature winding is necessarily one with mesh-grouping, the six phases follow the order of the sections along the closed winding.

The twelve-phase winding is related to the six-phase in the following way: its twelve phases are at 30° to each other, so that they may be regarded as six pairs crossed at the middle-point of each pair, as in Fig. 312. They may be grouped also as a combination of mesh and star, as in Fig. 313. Or, as in converters, they may be grouped in mesh as a 12-point polygon.

If it were desired, a twelve-phase winding can be made out of a creeping winding by having twelve coils of angular breadth 150 electrical degrees each, to subtend the breadth of 10 poles.

Windings for Turbo-Alternators.—Steam-turbines run at such high speeds that the alternators must necessarily have few poles, usually 2 or 4, rarely 6. The following table shows the speeds corresponding to different frequencies. As the poles are so few there must be either numerous slots per pole or else numerous conductors per slot to attain the high voltages required. In generators with revolving armatures, which are rare on account of the difficulty of securing the windings, closed slots, or nearly-closed slots, are imperative. Parsons used orginally a single layer of circular slots.

—	$f = 50\sim$	$f = 42\sim$	$f = 40\sim$	$f = 30\sim$	$f = 25\sim$	$f = 15\sim$
2-pole	3000	2520	2400	1800	1500	900
4-pole	1500	1260	1200	900	750	450
6-pole	1000	840	800	600	500	300

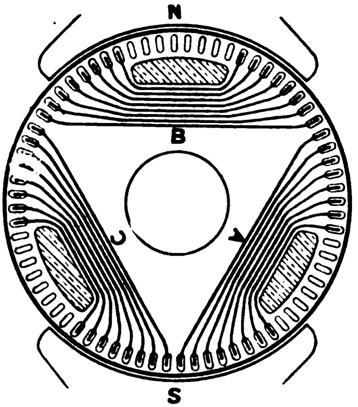


Fig. 314.

Fig. 314 illustrates a 2-pole design by the Oerlikon Co., in which all overlapping of conductors is avoided. It has 72 slots of which only 48 are filled ; there are therefore 8 slots per phase. The projecting claws from the brass end-shield which hold the coils in position are shown shaded. Fig. 315 is a 4-pole design having 48 slots, or 4 slots per phase per pole. This was for a coil-winding having 8 conductors per slot taped together, the end bends forming two ranges.

Fig. 316 is another Oerlikon 2-pole design for a 2-phase armature with 18 slots per pole per phase. The core-disks are spaced out as for 108 slots, but of these, 4 lots of 7 each are not stamped out, and 8 of those stamped are left empty, so that there are 72 slots filled.

Turbo-alternators with stationary armatures present much the same features of winding as other alternators. The Westinghouse 4-pole alternator described on p. 469 has 8 slots per phase per pole. The type of coil winding adopted by this company is shown in Fig. 317, which is of a large synchronous motor running at 750 RPM.

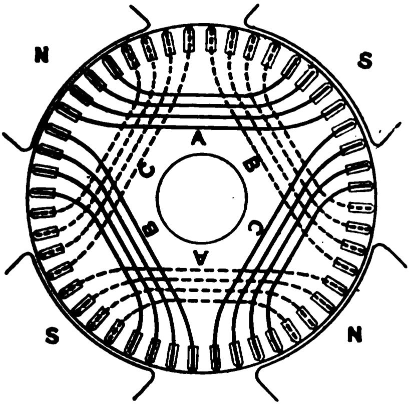


FIG. 315.

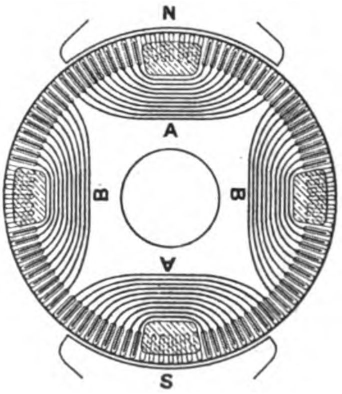


FIG. 316.

A 2-pole turbo-alternator of 200 KVA designed by the Oerlikon Co., has a 7-slot winding in three ranges, as indicated in Fig. 318. The winding accordingly lies in 42 slots, with 1 conductor in each slot. The projecting bends of all three ranges are turned up away from the revolving field-magnet.

Windings suitable for Rotors.—In the special case of rotors for induction motors, it is advisable that the number of slots in the rotor should be prime with respect to those of the stator. This is literally possible for the case of those permanently short-circuited rotors which are provided with squirrel-cage windings (p. 661). But for rotors intended to be used with three slip-rings for the purpose of introducing resistances at starting, the total number of conductors cannot be a prime number unless the three circuits consist of unequal numbers of conductors. This is no great fault ; but in any case it is preferable that they should present perfect symmetry. And if the number of slots cannot be a prime number, but must be

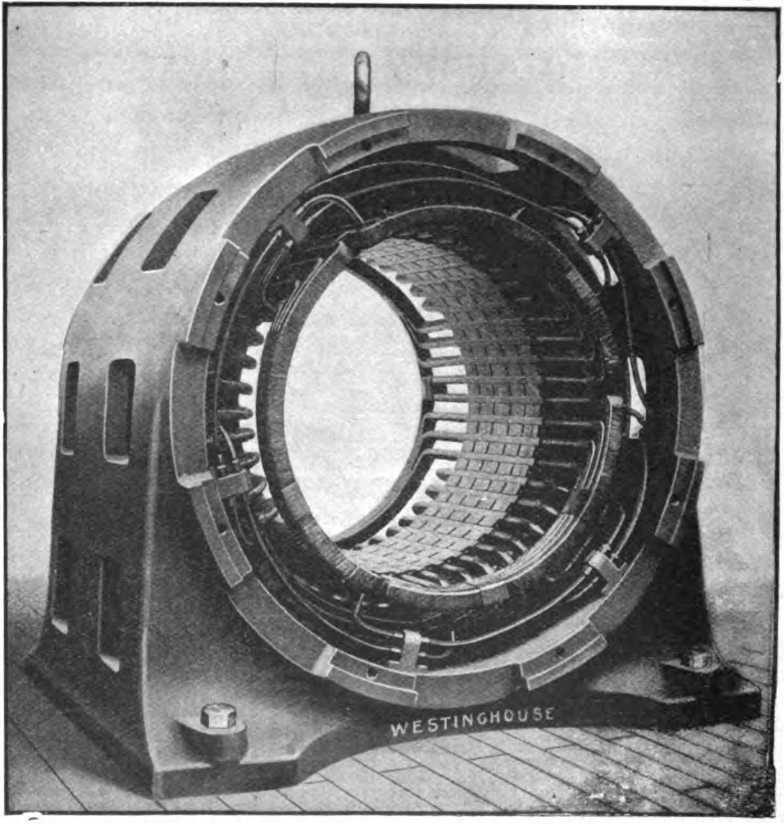


FIG. 317.—STATOR OF 4-POLE SYNCHRONOUS MOTOR, 1000 KW, 750 RPM.

a multiple of 3, it is advantageous that the number of slots in each of the three phases should be a prime number. Further, it is advisable that the number of slots per pole per phase in the rotor should be prime with respect to those of the stator. If the latter is a 3-slot winding, then the former

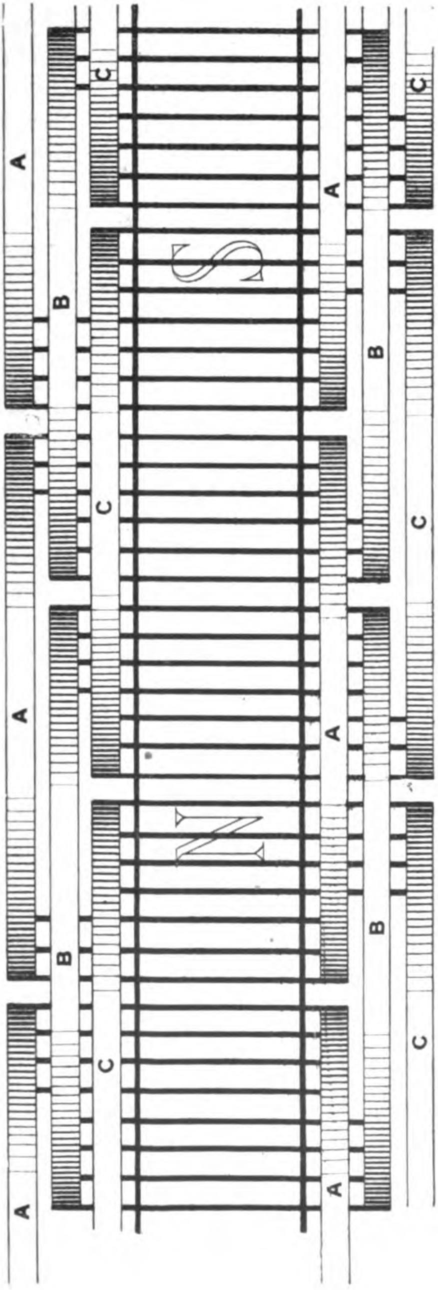


FIG. 318.—SEVEN-SLOT, THREE-RANGE, THREE-PHASE, BAR WINDING OF BIPOLAR TURBO-ALTERNATOR (OERLIKON CO.).

might be either a 4-slot or a 5-slot winding, but must not be a 6-slot winding. To carry out the above principle of incommensurability, for example, on a 4-pole machine which has a 3-slot winding in the stator, and which has therefore 4 × 3 × 3 = 36 stator slots, the suitable number of rotor-slots may be found as follows: Take a 4-slot winding; this would give 4 × 4 = 16 slots for each phase. To make this number prime, add 1 to each phase making 17 slots per phase or 51 slots in total. In some cases one adjusts by subtracting 1 instead of adding. The following table exhibits a number of cases.

TABLE VII.—INCOMMENSURABLE WINDINGS FOR ROTORS.

——	4-Pole.	6-Pole.	8-Pole.	10-Pole.	12-Pole.
Three-slot Winding—					
Slots per phase .	11 or 13	17 or 19	23 or 25	29 or 31	35 or 37
Total number of slots . . .	33 or 39	51 or 57	69 or 75	87 or 93	105 or 111
Four-slot Winding—					
Slots per phase .	15 or 17	23 or 25	31 or 33	39 or 41	47 or 49
Total number of slots . . .	45 or 51	69 or 75	93 or 99	117 or 123	141 or 147
Five-slot Winding—					
Slots per phase .	19 or 21	29 or 31	39 or 41	49 or 51	59 or 61
Total number of slots . . .	57 or 63	87 or 93	117 or 123	147 or 153	177 or 183

This is illustrated in Fig. 303 which shows a rotor-winding of 33 slots, or 11 slots per phase, for a 4-pole machine, mixed lap and wave.

The number of slots in the rotor having thus been settled, it remains to discuss the windings to be inserted. For bar-windings at a low voltage (which are suitable for the larger sizes of machines with many poles) either a lap-winding or a wave-winding may be adopted. If there is but one conductor in each slot, seeing that the number of conductors in one phase will necessarily be an odd number, the common junction will be at one end of the core and the connexions to the slip-rings at the other end. If a two-layer winding is adopted with 2 conductors per slot, there will be as many loops as slots, and the loops may be shaped on suitable formers. If the series in each phase be joined up as a lap-winding, the winding-pitches (reckoned in numbers of slots) must be such that $y_1 - y_2 = 1$, where the mean value of y is approximately equal to the number of slots within one pole-pitch. To make a suitable wave-winding, the total number of conductors in one phase should fulfil the condition of equalling $py \pm a$; where a is the nearest even number to $p \div 3$. Fig. 304 depicts a wave-wound rotor for 8 poles, with 27 slots and 54 conductors, *i.e.* 18 per phase, and is fully symmetrical. It might be joined up as either Δ or Y. For a coil-winding with many turns in each coil, the coils being former-wound to be inserted two-deep in each slot, the rule is the same as for the lap-winding previously mentioned.

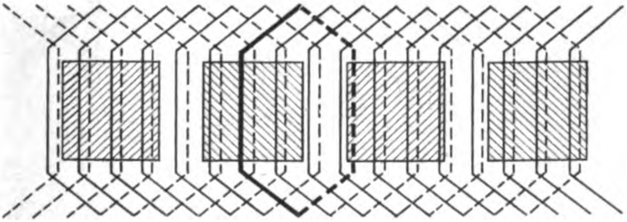


FIG. 319.

Fig. 305 depicts a 4-pole rotor winding (wave) suitable for a bar-wound rotor, a 3-slot winding made incommensurable by adding 1 extra slot to each phase, so that the number of slots is $4 \times 3 \times 3 + 3 = 39$.

Fig. 306 shows the corresponding lap-winding with the same number of slots.

Short-Circuited Rotor Windings.—These are equivalent electrically to squirrel-cage windings, but consist of a number of separated circuits each closed on itself, and surrounding one pole-pitch only or more than one pole-pitch.

The simplest case is where a number—preferably odd—of closed loops each surrounds one pole-pitch. Fig. 319 shows a scheme for a 4-pole rotor, consisting of 17 closed loops. It has 17 slots with 2 conductors per slot. A radial diagram of the same winding is given for comparison in Fig. 320.

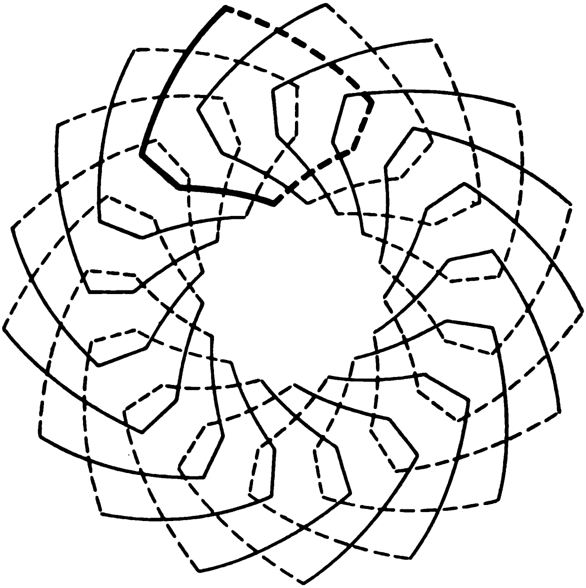


FIG. 320.

If the circuits are closed on a number of loops that come simultaneously opposite 2, 3 or more poles, the effect is the

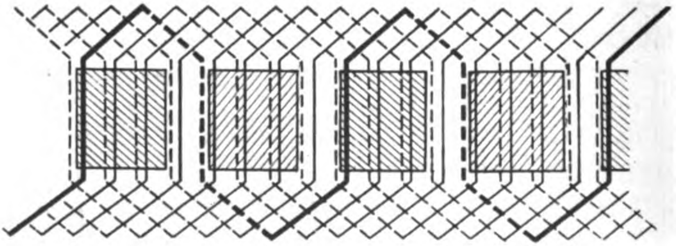


FIG. 321.

same if due regard is had to the joining up of the loops in proper order. This is shown in Fig. 321, which is a wave-winding for 4 poles, with 8 independent waves, each closed on itself. In all such cases it is advisable that the number of closed loops should be a prime number, and that it should be such that it differs considerably from the number of slots per pole on the stator. Thus if the latter is 12, the former should be 7 or 17 rather than 11 or 13.

CHAPTER VII.

DESIGN OF ALTERNATORS.

THE electrical designing of alternators is mostly simple; but there are two matters of importance that do not arise in the designing of continuous-current generators, namely, (1) the shaping of poles and distribution of armature coils, to secure a proper wave-form for the electromotive-force; (2) the limitation of the voltage-drop on an inductive load. Apart from these, the difficulties are mainly mechanical. The present chapter deals with electrical matters only.

TRIAL VALUES AND CONSTANTS.

The following are the results of experience with large modern generators of the type which has found most acceptance; namely, those with stationary armatures and revolving magnet-wheels, with outward-pointing poles of alternate polarity, having exciting coils on each pole (type B), working at 50 ∼ or not at a much higher or lower frequency :—

Flux-density (mean) [1] *at pole face*, 40,000 to 60,000 lines per square inch.

Flux-density in pole-cores (mean), 80,000 to 110,000 lines per square inch.

Current-density in armature copper, 2000 to 2600 amperes per square inch, or even 3200 in small machines.

Current-density in exciting coils, 1200 to 1800 amperes per square inch.

---

[1] This is on the assumption that the field is uniformly spread over the whole face of the equivalent pole, deducting only the ventilating spaces, if any. In some Westinghouse machines a higher density is attained.

Armature amperes per inch periphery,[1] 350 to 600.
Peripheral speed (feet per minute),[2] 4800 to 6000.

The above figures are for normal full-load. In applying the figures as to flux-density in pole-cores, regard must be had to the degree of saturation at the roots of the cores. Owing to the excellent ventilation in alternators, the permissible current-densities are higher than those in continuous-current machines: this is particularly noticeable in the case of the exciting coils, in which, however, no particular importance attaches to the current-density, as the design of this part is not determined by this consideration, but by the watts expended, the amount of cooling surface, and the conditions for cooling. For single-phase alternators the value of the specific loading of the armature (amperes per inch of periphery) is usually lower than is permitted for two-phase and three-phase machines, otherwise the inductive drop may be too great.

To find the Voltage and Current in one Phase.

(1) *Single-phase Machines.*—If the kilowatts KW, voltage V, and probable power-factor cos ϕ (see p. 27) are prescribed, the kilo-volt-amperes KVA are found by dividing KW by cos ϕ, and the current C is then found by multiplying by 1000 and dividing by V.

$$C = \frac{KW \times 1000}{V \times \cos\phi}.$$

Example.—A 450 KW machine, working at 2000 volts, on a circuit in which the power-factor is 0·85 will have as its normal full-load current 264·7 amperes.

(2) *Two-phase Machines.*—If the two phases are *independent*, then each carries half the total current, and each phase must generate the whole voltage $V_{AA} = V_{BB}$ between the

---

[1] This quantity, the "specific load" (symbol q), is reckoned by multiplying the whole number of armature conductors in one phase by the number of virtual amperes at normal full-load in any one phase, multiplying by the number of phases, and then dividing by the number of inches of the periphery. See remarks on p. 355.

[2] See remarks on p. 350.

terminals of its own circuit. Hence the current in either the A-phase or the B-phase will be

$$C_1 = \frac{KW \times 1000}{2 \times V \times \cos\phi}.$$

If the two phases are connected at their middle points, *in star* (making in reality a four-phase system), the voltage V_{AB}, from either of the A terminals to either of the B terminals, will be equal to $\frac{1}{2}\sqrt{2}$ (= 0·707) times either V_{AA} or V_{BB}. The voltage from any terminal to the middle point will, of course, be $\frac{1}{2}$ V_{AA}. Each circuit, as before, carries half the total current. Each phase will, as before, generate the whole voltage $V_{AA} = V_{BB}$. If the voltage between lines V_{AB} is prescribed, the whole voltage in either phase will be $\sqrt{2}$ times as great, and

$$C_1 = \frac{KW \times 1000}{2 \times \sqrt{2} \times V_{AB} \times \cos\phi}.$$

Example.—A 600 KW two-phase machine having 2000 volts in either phase, will have 1414 volts between the A and B circuits, and if $\cos\phi = 0·85$ C_1 will be 176·5 amperes if the phases are either joined in star or are independent.

If the armature windings are connected *in mesh*, the voltage between the lines V_{AB} will be 0·707 of the voltage V_{AA} or V_{BB} across the diagonal of the mesh; and the current C_1 in conductor of the winding will be $\frac{1}{2}\sqrt{2}$ times the current in either the A lines or the B lines, supposed equal between themselves.

Example.—If the previous example had been carried out with a mesh grouping of the windings, C_1 in the windings would be 124·8 amperes only, though the line currents would still be 176·5.

(3) *Three-phase Machines.*—Three-phase alternators, at least those for voltages over 1000 volts, are now always Y-connected. Hence if the voltage prescribed is the voltage *between the lines*, the voltage in one phase is deduced by dividing by $\sqrt{3}$. If the output of the machine in kilowatts

is specified, then the current in one phase is deduced as follows. Multiply the prescribed kilowatts by 1000 to bring to watts, divide by 3 to find the kilowatts per phase, then divide by the voltage in one phase, and finally divide by the power-factor (cos ϕ), if this is stated. If the power-factor of the system is not prescribed, it must be estimated from the nature of the load (see p. 27).

Calling the voltage between lines V, the voltage in one phase V_1, the current in one C_1, we have

$$C_1 = \frac{KW \times 1000}{3 \times V_1 \times \cos \phi};$$

and, as

$$V_1 = V \div \sqrt{3},$$

$$C_1 = \frac{KW \times 1000}{\sqrt{3} \times V \times \cos \phi}.$$

Example.—Find the current in one phase of a three-phase alternator of 600 KW, working at 2100 volts between lines, to be put on a system where the power-factor is cos $\phi = 0 \cdot 8$.

$$(600 \times 1000) \div (1 \cdot 73 \times 2100 \times 0 \cdot 8) = 207 \text{ amperes}.$$

It is however usual, in specifying, to state the amount of current the machine is to deliver at normal full-load. In which case the kilowatts of the machine are deduced by finding the kilo-volt-amperes, and then multiplying by cos ϕ.

$$KVA = 3 \times V_1 \times C_1 \div 1000;$$

or

$$= \sqrt{3} \times V \times C_1 \div 1000;$$

and

$$KW = KVA \times \cos \phi.$$

The necessary horse-power then follows from the efficiency

$$HP = KW \times \frac{1000}{746} \div \eta.$$

At full-load the whole electromotive-force generated in one phase E_1 must be greater than V_1, because of the internal pressure-drop. And as this (see p. 265) is usually considerable in alternators, it is important in designing to allow for it, and to calculate the fluxes, etc., for this higher value.

Peripheral Speeds.—As a high peripheral speed is advantageous in reducing the weight and cost of a machine of given output, and as in large alternators, there is little to fear from centrifugal forces, it is usual to design such machines for high peripheral speeds. In this respect, the design of alternators assimilates itself to the design of fly-wheels, in which for a given type of construction and given materials, the peripheral speed is taken as independent of the diameter. For alternators with magnet-wheels the poles of which are built up upon a strong foundation-rim (itself in some cases serving as fly-wheel) the customary peripheral speeds, measured at the pole-face, range from 4800 to 7500 feet per minute (or say from 24 to 36 metres per second); the lower speeds

TABLE VIII.—PERIPHERAL SPEEDS FOR DIFFERENT DIAMETERS AND SPEEDS OF REVOLUTION.

Revolutions per Minute.	Peripheral Speeds.						
	4000	5000	5500	6000	6500	7000	8000
	Diameters in Inches.						
83	184·2	230·3	253·3	276·3	299·3	322·3	368·4
90	169·9	212·3	233·6	254·8	276·0	297·3	339·7
100	157·8	191·1	210·2	229·3	248·4	267·5	305·7
120	127·4	159·2	175·2	191·1	207·0	223·0	254·8
150	101·9	127·4	140·2	152·9	165·6	178·4	203·9
200	76·4	95·5	105·1	114·6	124·1	133·7	152·8
225	61·2	76 4	84·1	91·7	99·3	107·0	122·5
300	50·9	63·7	70·0	76·4	82·8	89·1	101·9
375	46·0	51·3	66·6	61·5	56·4	71·7	92·0
400	38·2	47·8	52·5	57·3	62·1	67·8	76·4
500	30·6	38·3	42·1	45·9	49·7	53·5	61·2
1000	15·3	19·2	21·0	22·9	24·9	26·7	30·6
1200	..	15·9	17·5	19·1	20·7	22·6	25·5
1500	..	..	14·0	15·3	16·6	17·8	20·4

being chosen for small machines and machines with cast-iron foundation-wheels; the higher for large machines, and those with wheels built up of steel. Special types for steam-turbine engines are designed with peripheral speeds from 12,000 to, 16,000, or even 18,000, feet per minute.

SPEED, FREQUENCY, AND NUMBER OF POLES.

The frequency, or number of cycles per second, which is prescribed for the alternator, limits the speed to certain definite values, according to the number of poles: for the frequency will be equal to the number of pole-pairs in the circumference multiplied by the number of revolutions per second; or, in symbols

$$f = \frac{p}{2} \times \frac{\text{RPM}}{60}.$$

Examples. (1) A 12-pole alternator, running at 450 RPM, will work with a frequency of 45 ~, since $\frac{12}{2} \times \frac{450}{60} = 45$.

(2) A 16-pole alternator, to give a frequency of 50 ~, must run at 375 RPM, and cannot run at any other speed; for $\frac{16}{2} \times \frac{375}{60} = 50$.

(3) An alternator, to run at 360 RPM, and to work at a frequency of 42 ~, must have 14 poles; for $\frac{14}{2} \times \frac{360}{60} = 42$.

Conversely the speed, for a prescribed number of poles and prescribed frequency, is given by the formula

$$\text{RPM} = \frac{f \times 120}{p}.$$

In the following Table IX., the speeds are given for the customary frequencies.

The high speeds for 2-pole and 4-pole machines are practicable only for steam turbines. The low speeds, from 150 downwards, are suitable only for large slow-speed engines and water turbines.

Fig. 322 gives the above relations graphically. It will be found useful for checking the number of poles suitable for any given frequency and speed.

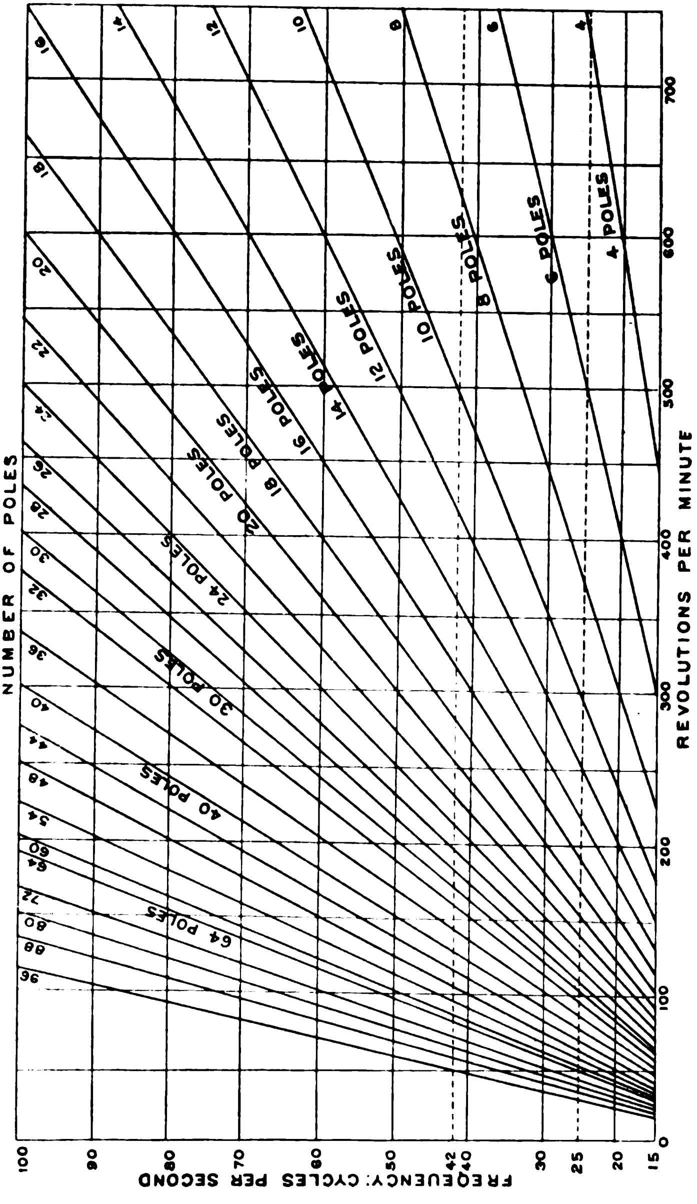


FIG. 322.

TABLE IX.—SPEED, FREQUENCY, AND NUMBER OF POLES.

Number of Poles.	FREQUENCY (in cycles per second).							
	100	60	50	45	42	40	30	25
	Revolutions per Minute.							
2	6000	3600	3000	2700	2520	2400	1800	1500
4	3000	1800	1500	1350	1260	1200	900 .	750
6	2000	1200	1000	900	840	800	600	500
8	1500	900	750	675	630	600	450	375
10	1200	720	600	540	504	480	360	300·2
12	1000	600	500	450	420	400	300	250·5
14	857·1	514·3	428·5	385·7	360	342·8	257·1	214 7
16	750	450	375	337·5	315	300	225	187
18	666·7	400	333·3	300	280	266·7	200	166
20	600	360	300	270	252	240	180	150
24	500	300	250	225	210	200	150	125
28	428·6	257·1	214·3	192·8	180	171·4	128 6	107·1
30	400	240	200	180	168	160	120	100
32	375	225	187·5	168·7	157·5	150	112·5	93·7
36	333 3	200	166·7	150	140	133·3	100	83·3
40	300	180	150	135	126	120	90	75
48	250	150	125	112·5	105	100	75	62·5
50	240	144	120	108	100·8	96	72	60
56	214·3	128·6	107·1	96·4	90	85·7	64·3	..
60	200	120	100	90	84	80	60	..
64	187·5	112·5	93·7	84·3	78·7	75	..	..
70	171·4	102·8	85·7	77·1	72	68·6	..	..
72	166·7	100	83·3	75	70	66·7	..	..
80	150	90	75	67·5	63	60	..	..
84	142·9	85·7	71·4	64·3	60	..	..	..
90	133·3	80	66·6	60	..	..	..	..
96	125	75	62·5	..	..	..	..	..

SIZE IN RELATION TO OUTPUT.

There are several methods of obtaining a trial value for the principal dimensions of an alternator, that is to say, for fixing the diameter *d* and the axial length *l* of the working face of the armature core. Some of the simplest and most handy of these methods are purely empirical: others are based on first principles: the former are available in cases where machines of similar type have already been constructed in other sizes.

Output Coefficients.—Output coefficients are used to express the size of the machine in terms of its normal output and speed. Esson's coefficient, denoted by ξ, deals with the product $d^2 l$; while Steinmetz's coefficient, here denoted by β, deals with the product $d\,l$.

Esson's Rule is

$$d^2 l = \xi \times \frac{\text{KVA}}{\text{RPM}}, \quad . \quad . \quad . \quad [\text{I}]$$

where in practice ξ has values ranging from 27,000 in large alternators to 150,000 or more in small ones, and is somewhat higher for single-phase machines than for two-phase or three-phase machines. For machines of 300 to 1000 kilowatts, it ranges from 54,000 to 33,000. As shown below, its value can be definitely calculated from first principles.

Steinmetz's Rule is

$$d \times l = \beta \times \text{KVA}, \quad . \quad . \quad [\text{II}]$$

where in practice β has values which range between 1·4, for large three-phase generators, and 4 or 4·5 for small single-phase generators; the usual value, for machines of fair size and usual frequencies, being from 2·4 to 3. Its values also may be deduced in each case on the assumption of a given peripheral speed (*see* Equation [X*b*]).

Deduction of Output Coefficient.—In order to deduce the coefficient ξ it is necessary to know from experience the proper values to be assigned to the following quantities:—

k, the Kapp-coefficient (see p. 196).

B_g, the mean flux-density at the pole-face.

q, the specific load as defined on p. 347.

ψ, the ratio of the equivalent pole-span to the pole-pitch.

The specific load q, or number of armature amperes per inch of periphery, is here defined by the formula

$$q = \frac{C_1 Z_1 \times n_{ph}}{\pi \times d};$$

where n_{ph} is the number of phases, C_1 the (virtual) amperes in any one phase (assumed equal in all phases), and Z_1 the whole number of armature conductors in series in any one phase. For trial values, *see* formula [XX], p. 364.

B_g is also defined by the expression

$$B_g = N \div l.\tau.\psi;$$

since $\tau.\psi$ is the equivalent pole-span, and τ the pole-pitch is equal to $\pi d \div p$.

Now we may write in succession four equations:—

$$KVA = E_1 \times C_1 \times n_{ph} \div 1000. \quad . \quad . \quad . \quad [III]$$

$$E_1 = k \times \frac{p}{2} \times \frac{RPM}{60} \times Z_1 \times N \div 10^8 \text{ (see p. 195)} \quad [IV]$$

$$Z_1 = \frac{q \times \pi \times d}{n_{ph} \times C_1}. \quad . \quad . \quad . \quad . \quad . \quad [V]$$

$$N = B_g \times \pi \times d \times \psi \times l \div p. \quad . \quad . \quad . \quad [VI]$$

Inserting the values of [VI] and [V] into equation [IV] and inserting into [III] the expression for E_1 so obtained, and transposing, we get

$$d^2 l = \frac{KVA}{RPM} \times \frac{2}{k \times B_g \times q \times \psi} \times 60 \cdot 8 \times 10^{10}, [VII]$$

which is identical with equation [I] if we put

$$\xi = \frac{2}{k \times B_g \times q \times \psi} \times 60 \cdot 8 \times 10^{10}. \quad . \quad [VIII]$$

Example.—If $k = 2 \cdot 22$; $B_g = 45{,}000$; $q = 430$; and $\psi = 0 \cdot 6$; then $\xi = 47{,}000$.

For three-phase generators of frequencies not outside 40∼

to 60~, the values of ξ are generally accordant with the empirical expression

$$\xi = 27{,}000 \times \frac{\text{KVA} + 300}{\text{KVA}}. \quad . \quad . \quad . \quad [\text{IX}]$$

If now we make the assumption that the peripheral velocity v, expressed in feet per minute, is constant, then since RPM $= 12\,v \div \pi\,d$, inserting this value in [I], and writing

$$\beta = \frac{\xi}{d \times \text{RPM}} = \frac{\xi \times \pi}{12\,v}. \quad . \quad . \quad . \quad [\text{X}a]$$

we get equation [II], and as the rational value of β we deduce

$$\beta = \frac{1}{k \times \text{B}_g \times q \times \psi \times v} \times 31{\cdot}7 \times 10^{10}. \quad [\text{X}b]$$

Example.—If $k = 2{\cdot}22$; $\text{B}_g = 45{,}000$; $q = 430$; $\psi = 0{\cdot}6$; and $v = 5000$; then $\beta = 2{\cdot}46$.

Value of d.—A further relation is derived by equating together two different expressions for the pole-pitch $\tau = \pi\,d \div p = v \div 10 f$, whence

$$d = \frac{p \times v}{10 \times \pi \times f}; \quad . \quad . \quad . \quad [\text{XI}a]$$

or

$$d = \frac{12 \times v}{\pi \times \text{RPM}}. \quad . \quad . \quad . \quad [\text{XI}b]$$

Some Empirical Expressions.—In three-phase machines the value of β may be taken empirically as approximately equal to $13{,}000 \div v$. So we may write

$$d \times l = 13{,}000 \times \text{KVA} \div v. \quad . \quad . \quad . \quad [\text{XII}]$$

Another trial-rule, based on an expression of Esson, in which l is assumed to be given, is

$$d = 220 \sqrt{\frac{\text{KVA}}{l \times \text{RPM}}}. \quad . \quad . \quad [\text{XIII}]$$

The coefficient 220 should be somewhat reduced for very large machines, or increased for very small ones. For single-phase machines it should be increased to 250 or 270.

The simplest of all hand-rules for estimating the diameter of the armature of an alternator, is to assume that the pole-pitch may be taken, in the mean, at $\tau = 10$ inches; which thus gives the periphery (in inches) as equal to 10 times the number of poles, the latter being fixed by the prescribed frequency and the number of revolutions per minute. This gives

$$d = 10p \div \pi, \qquad \text{[XIV}a\text{]}$$

or

$$d = 382 \times f \div \text{RPM} \qquad \text{[XIV}b\text{]}$$

The justification for taking 10 inches as the pole-pitch is afforded by the circumstance that for a peripheral speed of 5000 feet per min. (which = 1000 inches per sec.), if there must be 50 periods per second, a pair of poles must pass 20 inches in one period, making the pole-pitch 10 inches. The pole-pitch will be proportionately greater as the surface-speed is higher, or the frequency lower. At 40 ∼ and a surface-speed of 6000 feet per min., $\tau = 15$ inches. For small machines at 50 ∼, the pole-pitch is sometimes as small as 8 inches, or under, for with small radii a surface-speed of 5000 feet per min. would be too high. In fact, accurately,

$$\tau = \frac{v}{10f} \qquad \text{[XV]}$$

Example.—Find the appropriate dimensions for a 625 KVA three-phase alternator, working at 107 revs. per min.; frequency, 50 ∼; peripheral speed not to exceed 5000 feet per min. Clearly there must be 56 poles. Assume $k = 2 \cdot 22$, $\mathsf{B}_g = 46{,}000$, $q = 430$, $\psi = 0 \cdot 6$. Then by formula [VIII] $\xi = 46{,}500$. Hence by formula [I] $d^2 l = 46{,}500 \times 625 \div 107 = 272{,}000$. Assuming $\tau = 10$, we have $d = 178''$, whence $d^2 = 31{,}700$ and $l = 8 \cdot 6''$. Or, by formula [X*b*] $\beta = 2 \cdot 44$; whence, by formula [II] $dl = 1525$; whence if $d = 178''$, $l = 8 \cdot 6''$. Formula [XI*a*] gives $d = 176''$. Of the remaining empirical formulæ, [XII] gives us $d \times l = 1620$; whence if $d = 178''$, $l = 9 \cdot 1$; whilst formula [XIII] gives $d = 181''$.

Pole-Flux as affecting Size.—Another method of assessing the size of the machine is to fix on a suitable pole-flux. For a machine with a given output, speed, voltage, and frequency, and therefore with a given number of poles, a reduction of the pole-flux will entail an increase of the number of armature windings, and therefore of all reactions tending to bad regulation. On the other hand, an increase of the flux per pole, if attained by enlarging the poles, will not much reduce the reactions, because, unless the whole machine be enlarged also, it will increase the tendency to magnetic dispersion. As the pole-pitch of alternators (at $f = 50 \sim$) is usually about 10 inches, and the pole-span therefore about 6 inches, and as the axial length of the core is seldom less than 7 or more than 16 inches, the gross area of the pole will be from about 42 to 100 square inches. Allowing 10 per cent. for ducts, and assuming a useful flux density of 40,000 to 70,000, it follows that the total flux from a pole will lie between about 1,600,000 and 7,000,000 lines; the lower figure being appropriate for machines under, say, 400 KVA, the higher for machines from 1200 to 2000 KVA. (For turbo-alternators, the values of the flux are usually considerably higher than this scale.) For machines of extra-low frequency the poles are usually larger, because relatively fewer, than in machines of normal frequency. For a 1000 KVA machine at 50 $\sim$ a useful flux of 3,000,000 to 4,500,000 is appropriate, though many existing machines have values outside these limits. As trial values we may take

$$\text{Useful flux per pole} = 1{,}500{,}000 + (\text{KVA} \times 2000).$$

Hence a trial-value for the gross area of the pole may be put empirically at

$$\text{Gross area (sq. in.) of 1 pole} = 33 + \frac{\text{KVA}}{33}. \qquad \text{[XVI]}$$

The following are rational formulæ of service:—

$$\text{N} = \frac{\text{KVA}}{k \times v \times q \times p} \times 1{\cdot}0001 \times 10^{12}, \qquad \text{[XVII]}$$

and

$$\text{N} = \frac{\text{KVA}}{f \times d \times q} \times 14{,}400 \times 10^{6}. \qquad \text{[XVIII]}$$

It follows from these rules that the lower the frequency (other things being equal) the larger the poles. For example, if we take two 500 KVA machines, one for 50 ∼ and the other for 25 ∼, running at equal speeds, the second will have twice as large a pole-flux as the first. A trial-value for gross pole-area can then be calculated by dividing N by 50,000 (assumed for B_g), and this area, divided by the equivalent pole-arc (assumed at 6 inches for a normal type machine at 50 ∼), gives the core-length l.

SETTLING THE SIZE OF FRAME.

If d and l have been provisionally determined, the size of the frame, or housing, of the armature can be fixed; but first the radial depth of the iron behind the armature teeth must be considered. In order to reduce the iron-losses in the core, the flux-density in this part must be kept low. Densities from 40,000 to 60,000 lines per square inch in the nett section of the iron are suitable; and as the insulation between the laminations occupies from 10 to 15 per cent. of the space, it follows that the radial depth must be greater by 10 to 15 per cent. than the half of the equivalent pole-arc. Taking 6 or 7 inches as the normal pole-arc, it is common to find 7 or 8 inches as the ordinary depth of the core behind the slots but some makers give even greater depth. The slots being usually from 2 to 3 inches in depth, this makes the outer diameter of the armature core from 18 to 24 inches greater than d. For small machines and machines of high frequency 16 or even 14 suffices; for large machines and those of low frequency, even 24 inches is sometimes exceeded.

Examples.—(1) A G.E.C. alternator of 300 KW, at 60 ∼, with 16 poles, having $d = 48''$, had slots 2¼ inches deep. The outer diameter of the core was 62″. Here $\tau = 9{\cdot}42''$; pole-arc about $5{\cdot}5''$; $l = 15{\cdot}5''$. The gap was 0·1875″. The diameter at the roots of the teeth was therefore 52·875″; so the core-depth behind the teeth was 4·56″, or about half of τ.

(2) In an 880 KVA alternator, designed by Heyland, with 64 poles, at 50 ∼, having $d = 236''$, $\tau = 11{\cdot}57$, the radial depth of core behind the slots was 6·34″. The external diameter was 252″.

(3) In the new Niagara alternators of the G.E.C., of 7500 KW, for 25 ~, having 12 poles, $d = 150''$, $\tau = 39\cdot2''$, the pôle-arc $27\cdot9''$, the diameter of the core behind the slots is $157\cdot25''$, and the external diameter $180''$. This makes the radial depth of the iron core only $11\cdot625''$, less than half the pole-arc.

The size of the frame being thus settled, and the number of poles being known, this of itself settles the total flux of all the poles that can be used with that frame, and therefore settles the suitable value of N for the particular number of poles in question. It will be noted that in manufacture a frame of a particular size can be made to do duty for various different machines having different numbers of poles. Formula [XVII] may be written in the form

$$p\,\mathrm{N} = \frac{\mathrm{KVA}}{v \times q} \times 455{,}000 \times 10^{6}. \qquad \text{[XIX]}$$

Here p N, the total flux from all the poles, is a measure of the size of the frame. It is evident that, for a given surface speed and a given specific loading of the armature, the size of frame is proportional to the rated output (in KVA) irrespective of the number of poles or of the frequency.

Choice of Frame. Standard Frames.—In selecting a design, manufacturers are often guided by the sizes of frames for which they already have patterns, and which may be adapted to a new machine either by altering the length or by changing the number of poles. Fig. 323 shows a form of diagram convenient for such guidance. It is constructed for a frequency of 50 ~. A different diagram would be needed if machines for any other frequency were to be tabulated. A manufacturer would prepare several, one for each standard frequency. In this diagram are plotted out the kilovolt-amperes of output against the speeds, which, since the frequency is fixed, will necessarily correspond to particular numbers of poles. For example, a 32-pole machine, to work at 50 ~, must necessarily revolve at 186·6 revs. per minute. In the figure, the crosses indicate a particular group of machines taken from the catalogue of a well-known firm. The circles indicate another such group of machines. It will be seen that one of the crosses denotes a 32-pole machine of 1400 KVA, running at 186 6 RPM. Now if this machine were run at the higher speed of 250 RPM, it would make a

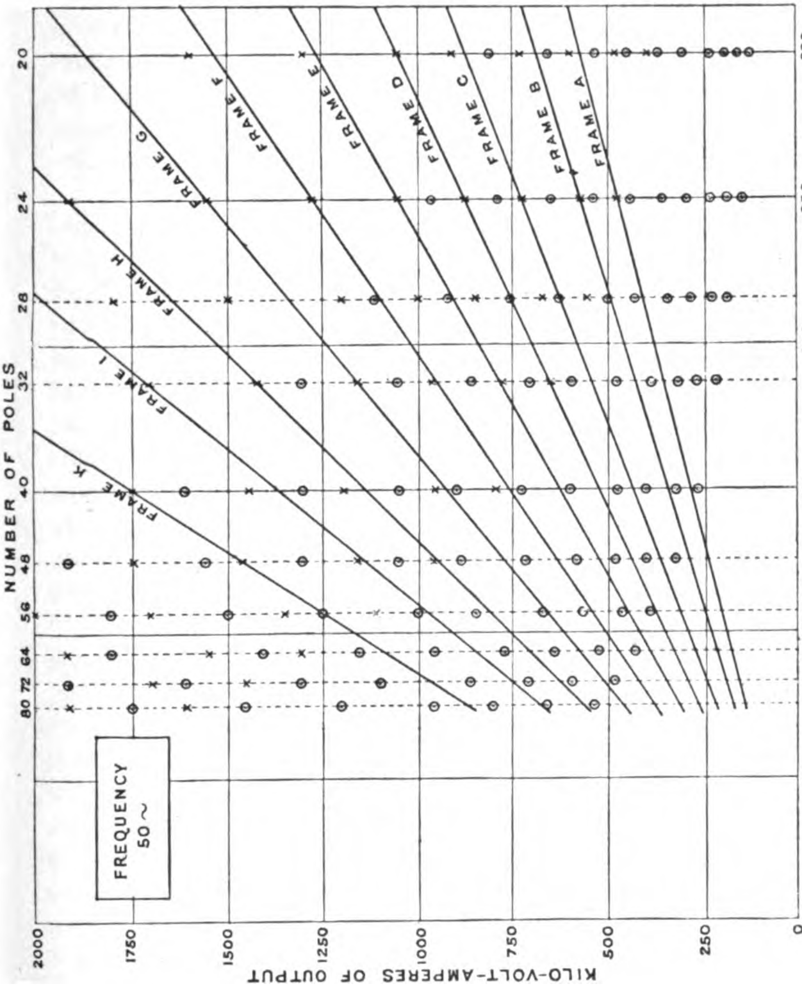


FIG. 323.

1900 KVA machine, provided the number of poles was reduced from 32 to 24. Or the same frame would suit a machine of 950 KVA, with 48 poles, running at 125 RPM. Or, again, if the maker were asked to tender for a 1000 KVA machine at 150 RPM, therefore having 40 poles, he would, on consulting the table, see that he had already a frame suitable for a 925 KVA machine (Frame G), and another (Frame H) suitable for a 1125 KVA machine. If the smaller of these could stand being loaded up to 1000, Frame G would do; if not, he must go to the next larger size. In many cases the smaller frame would admit of being used, or could be adapted by merely increasing the length of core by taking more stampings, and increasing the axial length of the poles correspondingly.

Flux per Pole.—Before proceeding to calculate the armature-winding, it is, however, necessary to consider the value of N from the point of view of the design of the magnet-wheel, and its capability of carrying the needful excitation. The gap is usually so small that the external diameter d' of the magnet-wheel taken at the face of the poles does not differ much from d, the diameter of the armature face; while l', the axial length of the magnet pole face, may be taken as the same as l. In the case of machines with as few as 8 or even 10 poles, the pole-cores, which necessarily radiate at angles to one another, are so oblique to one another that some difficulty may arise as to the winding-space. It is obviously not enough that the pole-core should have a cross-section adequate to carry the desired normal full-load flux N; it is equally necessary that there should be an adequate space between the poles to admit of the requisite number of ampere-turns of excitation. And here it must be remembered that the performance of alternators on inductive loads is mainly controlled by the considerations of armature reaction and magnetic leakage dealt with in Chapter V. That such reactions should be kept within reasonable proportions, the ampere-turns per pole of the magnet should be made from 3 to 4 times as great as the reacting resultant of the ampere-turns per pole of the armature. In three-phase machines, the armature ampere-turns per pole per phase are $C_1 Z_1 \div 2p$, or $q \times \tau \div 6$. The reacting resultant of the three phases together is equal to 2·7 (*i.e.* $1{\cdot}91 \times \sqrt{2}$) times this, *i.e.* is $1{\cdot}35\ C_1 Z_1 \div p$ or $0{\cdot}45\ q\tau$. Hence we ought to have on the magnet pole about 3 to 4 times this number of ampere-turns; or, writing X_p for the number of exciting ampere-turns per pole at full inductive load:—

$$X_p = \{4{\cdot}05 \text{ to } 5{\cdot}4\}\ C_1 Z_1 \div p = \{1{\cdot}35 \text{ to } 1{\cdot}8\}\ q\tau.$$

Examples.—If in the 625 KVA alternator (p. 357) $q = 430$, and $\tau = 10$, X_p ought to be at least 5805. The following are the values of X_p and of the ratio $X_p \div q\,\tau$ in a number of the machines:—

—	X_p	$X_p \div q\tau$
General Electric Co. single phase, 60 KW.	4050	1·93
Oerlikon, three-phase (high speed) 220 KVA	5700	1·265
Oerlikon, single-phase 300 KW	4050	1·9
Oerlikon, AT(B) 20—770—300; 50 ∼ .	7800	1·16
Électricité et Hydraulique, AT(B) 64—760 —94	7850	2·18

The value of this ratio depends upon the closeness with which the machine is required to regulate its voltage Machines having a high ratio will, other things being equal regulate better on an inductive load.

In the case of ordinary alternator magnet poles, the space-factor for the copper is from 0·5 to 0·6 if round copper wires are used, but may rise as high as 0·8 or even 0·92 if copper strip, wound edgewise, is employed. As the current-density is of the order of 1000 amperes per square inch in the copper, it will be of the order of 500 to 600 per square inch of gross available section, rising to 800 to 920 per square inch of gross section if strip winding is employed. Hence every available square inch of section of the space between the poles will accommodate, according to conditions, from 500 to 920 ampere-turns of excitation.

Modern alternators are indeed limited in their output more often by the possible number of ampere-turns that the magnet poles can carry, than by the armature heating or armature reaction. If on examination it is found that the pole cannot carry the required number of ampere-turns of excitation, then its form must be re-designed so as to secure more winding-space, or else the entire machine must be enlarged to give a greater width between poles, therefore enlarging τ and d and v.

Having thus ascertained the capability of the field-magnet as to its limiting values of N and X_p, these should then be reduced some 30 or 35 per cent., so as to allow for the possibility of meeting overloads or full-loads on very low power-factors, and this reduced value of N should be taken as the rated flux at normal full-load.

SETTLING THE NUMBER OF ARMATURE CONDUCTORS.

A trial value for Z_1, the number of conductors in series in any one phase, can be found in several ways. The simplest is to calculate it from q by the formula [V], p. 355, which is

$$Z_1 = \frac{q \times \pi \times d}{n_{ph} \times C_1}.$$

As this evaluation depends on a correct value being assumed for q, it may be well to remember that experience shows that suitable trial values of q for three-phase machines may be conveniently calculated by the empirical formula

$$q = 180\,(1 + \tfrac{2}{3}\log_{10}\text{KVA}). \quad . \qquad [\text{XX}]$$

This gives for a 100 KVA machine the value $q = 450$; for a 1000 KVA machine the value $q = 540$.

If the normal full-load value of N has been provisionally fixed, Z_1 can be calculated from E_1 by formula [IV], which for this purpose is transposed into

$$Z_1 = \frac{120 \times E_1 \times 10^8}{k \times p \times \text{RPM} \times \text{N}}, \quad . \qquad . \;[\text{XXI}a]$$

or

$$Z_1 = \frac{E_1 \times 10^8}{k \times f \times \text{N}}. \quad . \quad . \qquad . \;[\text{XXI}b]$$

In these formulæ k has values varying from 2·22 down to 2·0 according to the wave-form (p. 195); and E_1 the whole voltage to be generated in one phase must be taken with a due allowance for the inevitable drop at full-load. The reactance voltage may be roughly estimated at from 5 to 10 per cent. of the generated voltage and added vectorially to it, with an allowance of $C_1 R_a \times \sin \phi$ for the copper-drop. Fuller treatment is given below.

In any case after a trial-value for Z_1 has been thus arrived at, it must be trimmed to meet the requirements of the numbers of slots and poles. Compare the Chapter on Armature Windings. If the winding is not a purely distributive one (see p. 295), the numbers of slots and of coils, and therefore of conductors, must necessarily stand in some simple numerical relation to the number of poles, otherwise the winding would not be symmetrical. If for a 14-pole three-phase machine, having three slots per pole per phase, the first trial-value for Z_1, calculated by either formula [V] or [XXI*b*], came out at say 833, this number cannot be taken. The number required must be an even multiple of 14 and also of 3, for it must lie symmetrically in 84 slots. So the figure would be adjusted to 840; that is, 10 conductors in each of 84 slots. After so adjusting the value of Z_1, it may be necessary to recalculate N, and if this is seriously altered it may even be needful to go back further in the design and alter *l*, so as to keep the flux-density B in the pole-core at the right figure. If Z_1 has been increased more than 5 per cent., it is advisable to check the increase it has made in *q*; for if this is too high it will be necessary to enlarge the diameter of the armature to keep down the specific loading, and the design must be begun again with a larger value of *d*.

Voltage Drop.—It is usual to specify that the pressure-drop at full-load on a given power-factor shall not exceed a certain percentage of the whole voltage. The usual percentages are:—

$$\text{when } \cos\phi = 1 \begin{cases} 4 \text{ to } 6 \text{ per cent. in large machines.} \\ 5 \text{ to } 8 \quad ,, \quad ,, \text{ in small machines.} \end{cases}$$

$$\text{when } \cos\phi = 0 \cdot 8 \begin{cases} 12 \text{ to } 18 \text{ per cent. in large machines.} \\ 14 \text{ to } 30 \quad ,, \quad ,, \text{ in small machines.} \end{cases}$$

Thus, for example, a design may be prescribed to generate a voltage between lines of 5000 volts (therefore 2886 volts in one phase), and to have not more than 18 per cent. pressure-drop on a power-factor of $\cos \phi = 0 \cdot 8$. Now 18 per cent. of 2886 is 520 volts. The specification requires that when the full-load is one with a power-factor of $\cos \phi = 0 \cdot 8$, that is with a lag of $\phi = 37^{\circ}$, the machine can be excited up to yield $2886 + 520 = 3406$ volts, so as to compensate for the inductive drop of 520 volts. Reference to Fig. 229 will show how by compounding together 2886 and 520 volts at different power-factors we have:—

at $\cos \phi = 1$, 2920 volts ;
at $\cos \phi = 0 \cdot 8$, 3280 volts ;
at $\cos \phi = 0$, 3406 volts.

Now it is always advisable to design a machine to be a little better than will just fulfil the specified conditions ; and as the power-factor depends on the nature of the load, and as the load on a machine may in the course of time be changed, by the substitution of motors for lights, to a highly inductive one, it is advisable to assume that the machine's voltage shall not suffer more than the prescribed drop, even if the power-factor were to fall to $\cos \phi = 0$. This implies that the machine must be so designed, with a margin of possible flux, and of possible excitation, that it can, in case of need, give on full-load 3406 volts instead of 3280. That is to say, while on no-load the flux N_1 must be such as to correspond to 2886 volts, and while on full-load (with a $\cos \phi = 0 \cdot 8$) N_1 must be raised so as to correspond to 3280 volts, it must be capable of being raised to 3406 volts on a full inductive load. If this can be done, then the very worst drop will be only 18 per cent., and the drop at $\cos \phi = 0 \cdot 8$ will be only $3280 - 2886 = 394$ volts instead of 520, or only $13\frac{1}{2}$ per cent.

Example.—Suppose, to realize the preceding case, the design of a 72-pole alternator had shown that a flux of $3 \cdot 52 \times 10^6$ lines per pole was the highest reasonable value of N_1; then, taking $f = 50 \sim$, we have

$$Z_1 = \frac{3406 \times 10^8}{2 \cdot 22 \times 50 \times 3 \cdot 52 \times 10^6} = 867.$$

Now 867 will not do for any machine, being an odd number. If there are to be two slots per phase per pole, there will be 72 × 2 = 144 slots for each phase, and 867 must be adjusted to make it an exact multiple of 144. Now 6 times 144 is 864. So we may take 864 conductors or 432 turns per phase. These will be grouped as 36 double coils per phase, as in Fig. 274*c*, or Fig. 282, or Fig. 288; each slot containing 6 conductors.

The flux N_1 will, of course, be reduced at no-load in proportion from 3406 down to 2886 by lowering N_1 from 3,520,000 to 3,000,000, a reduction of 18 per cent.

Assignment of Losses.—Following the plan adopted for many years by the designers of transformers, we may, if we choose, fix upon a definite figure for the efficiency to be attained by the machine, and apportion out the various inevitable losses. Modern alternators always have a high efficiency from 92 to 95 per cent., not including the power wasted in the bearings. Experience shows that the permissible armature iron-loss may be from two to three times as great as the armature copper-loss. The usual value of the former is from 3 to 4½ per cent. of the rated output, while the copper loss varies from 1¼ to 2½ per cent. The excitation loss may vary from 0·5 per cent. in a machine with a considerable voltage-drop on a non-inductive load, to 2·4 per cent. in a machine in which even on a fully inductive load there is only a small drop. A fair proportion to take, for full-load on a power-factor of unity, would be

Iron-losses, 4 per cent.;
Copper-losses, 2 per cent.;
Excitation-losses, 1 per cent.

For machines that are to work satisfactorily on low power-factors, it is better to reduce the iron-losses, and to increase the copper-losses.

If, in working out the design, due regard is had to the appropriate magnetic and electric densities in the various parts, and to the provision of adequate cooling surface, there need be little fear of the efficiency falling below a reasonable figure. The several losses can be checked during the operation of designing, so as to keep them within the limits above assigned.

Where special high efficiencies are demanded by a customer, the iron loss is reduced by reducing the total flux. This has the effect of increasing the copper-losses, which again are reduced by increasing the sections of the copper, and the cost. In machines of this type, the copper-losses at full-load will, as a rule, be rather greater than the iron-losses.

Selection of Conductor.—The next step is to select a conductor of appropriate size. This is determined by the section requisite to carry the full current. And as the conditions for ventilation are excellent, and as the ohmic pressure-drop is only a fraction of the whole drop at full-load, a high current-density may be allowed to as much as 2600, or even 3000 amperes per square inch (or say from 340 to 380 square mils per ampere) in small machines ; while in a large machine, as for 2000 KVA at 100 RPM, for 5000 volts the density could not exceed 2000 amperes per square inch. The thickness of insulation in high-voltage machines militates against cooling, and keeps down the permissible current-density.

Stranded wires should not be used, and even round wires are objectionable, as the space-factor is unnecessarily low. Square or rectangular wire is much better, as it enables the slots to be smaller. Some makers object to square wires as giving trouble in twisting, and cutting the insulation at the corners. The corners should be rounded in the drawing.

If M the mean length of one turn (in inches) has been estimated, and w_c the permissible number of watts that may be lost in the copper has been assigned, then the appropriate section s (sq. in.) of the conductor may be found from the expression

$$s = \frac{0{\cdot}4}{10^6} \times \frac{C_1^2 \times Z_1 \times M \times n_{ph}}{w_c} \quad . \text{ [XXII]}$$

where n_{ph} stands for the number of phases. This formula is based on the value of the specific resistance of copper as 0·8 microhms per inch cube at 60° C.

When a size has been provisionally adopted, and an estimate made of the mean length per turn (which, since the pole-pitch in ordinary alternators of fly-wheel type for 50 cycles per second, is about 10 inches, cannot differ much from 50 to 65 inches), then, multiplying this by the number of turns per phase, gives the total length per phase, and from this the resistance of one phase is obtained. Then, multiplying by the current in one phase, gives the ohmic drop per phase, which may be checked by observing whether it lies between the permissible limits of 1·25 to 2·5 per cent. of the voltage in one phase.

It should also be noted that the watts lost in ohmic heat, per square inch of the peripheral surface, are related to the current-density in the copper a, by the relation

$$w_c \text{ per square inch} = a \times q \times 0{\cdot}8 \times 10^{-6}.$$

If it is assumed that 1½ watt per square inch may be allowed for copper-loss, then the permissible density will be:—

$$a = \frac{1{,}870{,}000}{q} \quad . \quad . \quad . \quad . \quad [\text{XXIII}]$$

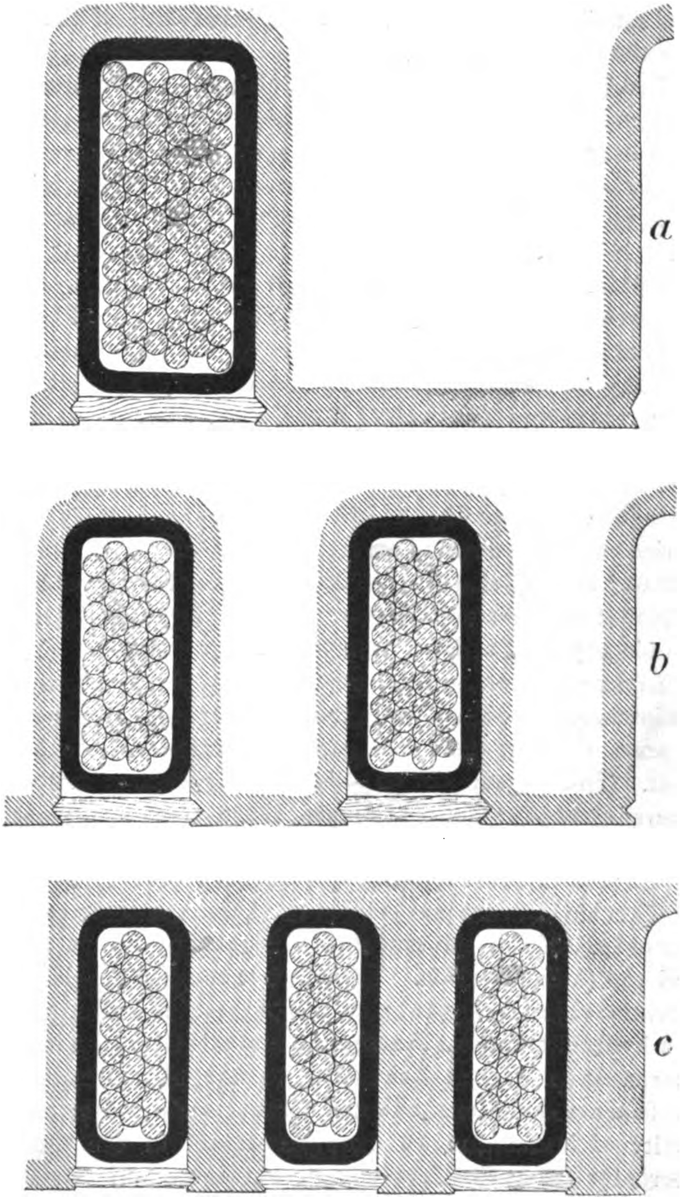
As q seldom exceeds 600, it follows that so far as temperature rise is concerned, a may generally be as high as 3000 amperes per square inch, provided the iron-losses do not exceed the copper-losses.

Before any choice of conductor can be considered final, the question of grouping them, and of surrounding them with the necessary insulation in the slot, must be thoroughly gone into.

SETTLING THE SLOTS.

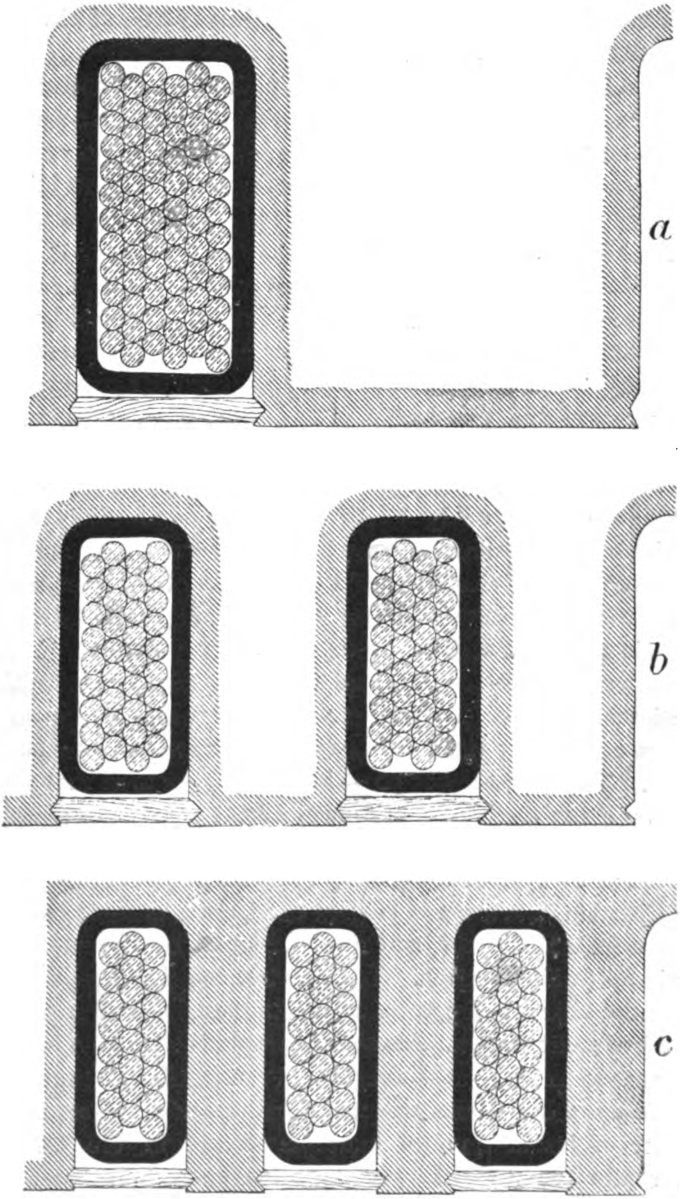
The number of slots per phase per pole is settled by reference mainly to manufacturing conditions and questions of insulation. That the machine may give the best wave-form the number should be 3, 4, or even more; but cheapness of manufacture and simplicity of insulation may require that, for high voltages, the number should be kept down to 3 or 2, or even in extreme cases 1.

That this question of slot insulation is in alternators often a most important consideration is seen from a study of Figs. [illegible] Fig. [illegible] represents a two-slot winding, which was actually employed in a [illegible]000-volt machine. The tooth was [illegible] per cent wider than the slot: and with the use of [illegible] a space-factor of 0·42 was obtained. [illegible] the conductors which now occupy the [illegible] and spacing out the new winding [illegible] as shown in Fig. [illegible]. The space [illegible] and the width of the [illegible] slot has been very much reduced. [illegible] in the space-factor, the necessary [illegible] voltage and the obligation to use [illegible] the total iron width [illegible] namely, in the ratio of [illegible] conductors, and therefore [illegible] means either using a higher [illegible] increasing [illegible] the armature body [illegible] the pole-face [illegible] by the use [illegible] this question of [illegible] the machine. [illegible] where starting [illegible] space-factor has [illegible] [illegible] the slot [illegible] larger total [illegible] in the [illegible] as [illegible] attention [illegible] machine [illegible] economical [illegible]



FIGS. 324 *a*, *b* and *c*.—DIFFERENT ARRANGEMENTS OF CONDUCTORS IN SLOTS, showing effect on space-factor, and on available iron-width of teeth.

That this question of slot insulation is in alternators often a most important consideration is seen from a study of Figs. 324 *a*, *b* and *c*. Fig. 324*b* represents a two-slot winding, which was actually adopted in a 10,000-volt machine. The tooth was about 10 per cent. wider than the slot; and with the use of round wire as shown, a space-factor of 0·42 was obtained. The effect of placing the conductors which now occupy the two slots into three slots, and spacing out the new winding over the same fixed pole-pitch is seen in Fig. 324*c*. The space factor has now been reduced to 0·366, and the width of the tooth with respect to the slot has been very much reduced. In consequence of the reduction of the space-factor, the necessity of using the same pole-pitch and the obligation to use the same number of conductors per pole, the total iron width of the teeth has been greatly reduced, namely, in the ratio of 1 to 0·725. With the same number of conductors, and therefore the same total flux, this means either using a higher density in the teeth or, what is a better procedure, increasing the iron section of the teeth by building the armature body longer. In this machine the flux disturbances in the pole-face due to open slots would hardly have been reduced by the use of an increased number. It is thus seen that this question of the number of slots affects the whole design of the machine. This is more strongly emphasised in Fig. 324*a*, where, starting from the original design in Fig. 324*b*, the whole of the conductors in one pole are placed in one slot. The space-factor has now been brought up to 0·49, a fairly high value for this voltage, and the tooth is now 2·2 times the width of the slot —quite an abnormal figure. At the same time, a larger total iron length across the pole-pitch has been obtained in the ratio of 1·29 to 1. Fig. 324*a* is, of course, not put forward as an alternative design to Fig. 324*b*, but merely to call attention to the important part played by the insulation, an inactive but necessary item, in the production of the most economical design. To adapt Fig. 324*a* for actual use, it would be necessary to make the slot larger still, so that it occupied a more equal proportion of the armature surface, and then to adjust the length of the armature to suit the revisions made in the



FIGS. 324 *a*, *b* and *c*.—DIFFERENT ARRANGEMENTS OF CONDUCTORS IN SLOTS, showing effect on space-factor, and on available iron-width of teeth.

total number of conductors and the iron-width of the teeth over the pole-pitch.

Insulators most generally used in alternator armatures are mica-paper, press-spahn, micanite, and horn-fibre, these often being used in the form of tubes, through which the conductors are threaded singly by hand. For voltages up to 2500, impregnated fabrics, such as empire cloth and oiled paper are used, often, however, in conjunction with sheet press-spahn and horn fibre. An extensive and most valuable series of tests has recently been conducted at the National Physical Laboratory, under Dr. R. T. Glazebrook, by Mr. E. H. Rayner, upon a variety of insulating materials under different conditions of temperature and treatment (*Journ. Inst. Elec. Eng.*, xxxiv. 613, 1905).

The thickness of slot insulation depends of course upon the nature and quality of the insulator, and every maker carries out break-down tests on the special insulator he favours. Hobart,[1] in analysing a set of 27 machines of different makes, finds that an average thickness of slot lining used for 10,000 volts is 4·5 mm., and for 5000 volts about 3·5 mm.

In arranging slots, care must be taken that the breadth of the intervening tooth is not reduced too greatly, otherwise the increase of B in them will cause the iron-losses to be too great. The frequency of 50 ~ is much higher than that which occurs in the teeth of continuous-current generators; but, on the other hand, there is in alternators, generally, ample space for the slots without too greatly saturating the teeth.

Circular slots are now seldom used, but a notable example occurs (p. 425) in the machines designed by Brown for the Paderno (Milan) station. There is but one slot per phase per pole, the object being to have the highest insulation possible in a hydraulic station. Another example is the Ganz inductor alternator (p. 134) at Leicester. Rectangular slots open at the face are sometimes adopted for the same reason of simplicity of insulation: an example occurs in the Oerlikon alternator (p. 401). Low-voltage machines, and bar-wound

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[1] Considerations relating to the design of armature slot insulation in high-voltage alternate-current generators (*Elect. Review*, lvi. 680 and 716, 1905).

machines, having but one or two conductors per slot, are illustrated by the Allgemeine Co.'s 3750 KVA machine at Manchester (p. 185) and by the Niagara machines (pp. 421 to 426).

Semi-closed slots with a narrow slit at the face are preferable to wide open slots; but (save in cases where relatively thin wires are employed that can be fed in, one by one, through the slit) they involve hand-winding. In machines working over 6000 volts, if insulation in the slots does not solidly fill the space between the conductors, trouble may arise from brush-discharges which generate ozone and attack the insulation chemically, so producing a breakdown. A solid insulation filling up all chinks and cavities within the slot is therefore much to be preferred.

Many different examples are given in this book of arrangements of the conductors in the slots. Round wires waste precious space; rectangular wires (with corners slightly rounded to prevent cutting of insulation) are always preferable, from the point of view of economy of space. Yet it is not so important in alternators as in continuous-current generators to economize space in the armature winding, because, as pointed out above, on p. 363, the thing that limits the output from a given size of machine is the pole-flux and the space occupied by the pole excitation. So it will be found that, in spite of the low space-factor involved, round wires are often used for armature coils.

In three-phase generators, having a Δ-grouping of the phases (now seldom employed), a 1-slot winding is to be avoided, as tending to accentuate the third harmonic (see pp. 220 to 223). But in those having a Y-grouping, where the third harmonic is cut out by the method of grouping, this objection to a 3-slot winding is non-existent (see p. 212).

The following rules for settling the breadth and depth of the slots are applicable if the ampere-density in the copper and the space-factor are known:—

Let n_{ph} stand for the number of phases;

s_1 stand for the number of slots per phase per pole;

a stand for the current density in amperes per square inch;

η_s stand for the space-factor, *i.e.* the ratio of the total copper section in the slot to the slot-area.

Then we have

$$\begin{aligned}
&\text{slots per pole} &&= n_{ph} \times s_1\,; \\
&\text{total amperes per pole} &&= q\,\tau\,; \\
&\text{total amperes per slot} &&= q\,\tau \div n_{ph}\,s_1\,; \\
&\text{slot area (square inches)} &&= \frac{q\,\tau}{n_{ph}\,s_1\,\eta_s\,a}. \qquad \text{[XXIV]}
\end{aligned}$$

Now the slot-pitch $= \tau \div n_{ph} s_1$. Hence we have

$$\begin{aligned}
&\text{breadth of tooth} &&= \frac{\tau}{n_{ph}\,s_1} \times \frac{B_g}{B_t}\,; \\
&\text{breadth of slot} &&= \frac{\tau}{n_{ph}\,s_1} \times \frac{B_t - B_g}{B_t}\,; \qquad \text{[XXV]}
\end{aligned}$$

and, dividing (XXIV) by (XXV) we get

$$\text{depth of slot} = \frac{B_t}{B_t - B_g} \times \frac{q}{a\,\eta_s}. \qquad \text{[XXVI]}$$

Example.—Suppose $B_g = 45,000$ and B_t is fixed at 100,000. (It will really come about 10 per cent. higher, because these formulæ neglect the insulation, etc., between the laminations.) Suppose also that, with round wires, and an insulation suitable for a voltage not exceeding 1000 volts, the space-factor is 0·4; that $n_{ph} = 3$; $s_1 = 2$; $a = 1600$; and $q = 500$. Then for a machine in which $\tau = 10$ inches, we have: slot area = 1·30 square inch; breadth of slot = 0·92; depth of slot = 1·419.

Reactive Armature Ampere-Turns.—The equivalent ampere-turns (exercised as a reaction) of the armature current may now be calculated. Calling the armature ampere-turns per pole AT_a, these are given by the formulæ

$AT_a = 2·7 \times C_1 \times$ turns per phase per pole (for three-phase), ... [XXVII]

or

$AT_a = 1·8 \times C_1 \times$ turns per phase per pole (for two-phase), ... [XXVIII] or

$AT_a = 0{\cdot}9 \times C_1 \times$ turns per pole (for single-phase). . . . [XXIX]

It is known that, except for machines with very highly saturated teeth, there will be an unduly large pressure-drop on full inductive load unless the exciting ampere-turns per magnet-pole are about three times as great as the value of AT_a. The numeric $2{\cdot}7$ is arrived at in the following way. The three currents of the three phases are not all at their maximum together. The maximum of any one current is $\sqrt{2}$ times as great as its virtual value. When one is at its maximum the currents in the other two phases in the adjacent slots are at half-value, hence the total effect would be $(\frac{1}{2} + 1 + \frac{1}{2}) \times \sqrt{2}$ times the virtual value if all three were situated in the same slots. When one current is at zero, the two others are each at $0{\cdot}866$ of the maximum. This makes the total effect equal to $(0 + 0{\cdot}866 + 0{\cdot}866) \times \sqrt{2}$ times the virtual value. The mean of these is $1{\cdot}91 \times \sqrt{2} = 2{\cdot}7$. In the short-circuit test, the field-magnet ampere-turns per pole to balance will be about $1{\cdot}05$ times the AT_a as thus calculated.

Choice of Gap-Width.—To decide on the appropriate air-gap δ it may be stated that in large slow-speed machines this seldom exceeds 0·4 inch, and even on machines of 15 feet diameter is sometimes as small as 0·2 inch. The slower the speed and the higher the frequency, the less may the gap be. A common value is 0·3125 (= 5-sixteenths of an inch), or more rarely 0·375 (= 3-eighths of an inch). A larger value is better, for several reasons, for though making the prime cost slightly higher, since it increases the quantity of copper on the field-magnets, it helps to diminish the reactions and the resulting drop on an inductive load. A minimum value may be reckoned from the formula

$$\delta = 5 \times q \times \tau \div B_g. \quad . \quad . \quad [XXX]$$

For example, in an alternator having a specific loading of 420 amperes per inch, a pole-pitch of 10 inches, and a flux-density in the gap of 45,000, the gap ought not to be less than

0·47 inch, unless the pole-cores are very highly saturated, and the magnetic dispersion at full-load low.

In the Westinghouse turbo-alternator (p. 469) the gap (on a 50-inch diameter) is no less than 1¼ inch. This is a very extreme case, but is exceeded by some recent turbo-alternators of the same firm of 5500 KVA, having a gap of about 3 inches. This is mainly a question of closeness of regulation. A machine in which the regulation on full non-inductive load is not to exceed 6 per cent., will require about three times as many ampere-turns per pole on the field-magnet as on the armature. To attain this, *either* the gap must be wide *or* the iron parts must be highly saturated. But saturation of the pole-cores, particularly at the roots of the cores, leads to bad regulation on inductive loads. Hence it is better to use a wide gap, and to have all iron unsaturated except the teeth and the mere pole-tips.

Choice of Radial Depth of Core-Disks.—The depth of the core-disks behind the slots is determined by the permissible amount of iron-losses in the core, and therefore by the flux-densities in the teeth and core. To keep down the losses in the teeth the slots must be kept short; and this is one reason for avoiding the waste of space involved in using stranded conductors or round wires. The usual flux-density permitted in the core beyond the roots of the teeth, is from 25,000 to 40,000 lines per square inch in large machines, or from 40,000 to 60,000 in small machines; and the usual depth of the core is from 4·75 to 8 inches. Densities exceeding 50,000 are admissible only in machines for frequencies below 40 ~. Machines for frequencies of 15 ~ or 20 ~ may have densities from 65,000 to 80,000. The useful flux at full-load being known, the section must be adequate to carry the half of this; hence a trial value may be obtained by taking half the usual flux, dividing by say 40,000 to get the section, and dividing this by the *nett* length of iron from front to back of the core. From the value so obtained the volume of the core should be reckoned out, and the total iron-losses in core and teeth should be computed. These should lie between 2·5 and 4·5 per cent. of the total output. If lower, the core is unnecessarily large; if higher, a larger section must be adopted, involving an increase of depth.

In calculating out the core losses, it is presumed that the flux spreads out equally over the full thickness of the pole-core. In low frequency and high speed machines, such as turbo-alternators in which the length of pole-arc is abnormal, and the number of poles few, it is found that there is a great tendency for the lines to crowd up near the teeth; and in order to keep the losses down, a thickness of core, twice that which is usually considered sufficient, is often used.

Calculation of the Magnetic Circuit.—To obtain a provisional estimate of the ampere-turns of excitation per pole, a provisional length must be assigned to the pole cores. One method of getting a rough trial value is to take 30 times the length of the air-gap as the necessary length of the magnet bobbins, and allow for the flanges and projecting pole-pieces. The calculation will then proceed on the ordinary lines as for continuous-current machines; but care must be taken in several particulars. If the gap be not of uniform length from iron to iron, as when the pole edges are chamfered, the mean length must be taken as the equivalent gap. Also it will not do to guess the value of the dispersion-coefficient; it must be reckoned out as on p. 252 for no-load and for full inductive load, taking into account the increase due to armature reaction. An allowance must be made for the joint between the pole-cores and the yoke, owing to the high flux-density at this part. It suffices to assume an air-film $\frac{1}{100}$ inch thick. The examples given in the next chapter will illustrate how the chief computations are worked out. The calculation must be worked out for a number of different values of the useful flux, and the values so determined plotted out as a saturation curve or no-load characteristic. Rothert recommends that in addition a further calculation should be made to determine a certain hypothetical excitation, viz. the "ampere-turns without reactions." By this expression is meant the ampere-turns that would be needed in the absence of any armature reaction, but using in the calculations the same fluxes and leakages, and the same electromotive-force as is needed at full inductive load to produce the normal voltage at the terminals. This quantity will, in a machine working at high saturations, be some 10 to 15 per cent. higher than the no-load ampere-turns, but only 1 or 2 per cent. higher in one working at low saturation. The ratio between the armature ampere-turns and the "ampere-turns without reaction," gives the *reaction-coefficient*, which is a measure of the probable performance of the machine on an inductive load. If the machines are not highly saturated this reaction-coefficient must not be lower than 0·24, otherwise the inductive drop will be too great. In the above calculation the armature ampere-turns (per pole) should be calculated by the formula [XXVII] already given (p. 374).

If the reaction-coefficient comes out within permissible limits, the magnetic circuit and excitation may be considered so far satisfactory, and the calculation of the magnet winding may be proceeded with.

Calculation of the Magnet-Coil.—In general, the voltage of the exciter is prescribed:—often 100 or 125 volts, rarely 250. For turbo-alternators, it is advisable to keep excitation-voltage down to 50. It may be taken that, in order to provide a sufficient margin of excitation to allow for heating of coils, low power-factor, and the necessity of meeting overloads, some 30 per cent. of the excitation-voltage must be treated as reserve, the available 70 per cent. being taken in calculating the excitation at no-load with the copper coils cold. Let the available volts be called V_x. Then the problem is to find the size of copper conductor to be used on the coils, such that with this voltage applied to the whole of the exciting turns on all the poles, the highest number of exciting ampere-turns per pole required at normal inductive load will result. To solve this problem, we shall first find the resistance per turn, and then from this and the mean length of one turn the resistance per unit of length, which will settle the section. It is well known, from its application to shunt coils, that if a supply voltage is fixed, and the mean length of one turn is fixed, there is only one size of conductor that will give a prescribed number of ampere-turns. (See the discussion of this question in Vol. I., p. 168.) If M be the mean length of one turn, and r_1 the resistance of unit length, then Mr_1 will be the resistance of one turn. If S_m be the number of turns on one coil, then $S_m\,Mr_1$ will be the resistance of that coil; and as there are p coils on the poles, the total resistance of the exciting coils will be $S_m\,Mr_1\,p$. Then the exciting current C_m will be given by Ohm's law as:

$$C_m = \frac{V_x}{S_m\,M\,r_1 p}; \quad . \quad . \qquad [\text{XXXI}]$$

whence we have for the resistance of one turn

$$Mr_1 = \frac{V_x}{p \times C_m\,S_m}; \quad . \qquad [\text{XXXII}]$$

and for the resistance per unit length,

$$r_1 = \frac{V_x}{p \times C_m\,S_m \times M} \quad . \qquad [\text{XXXIII}]$$

For example, if the exciter works at 125 volts, we may take 90 volts as available for V_x. Suppose the alternator to have 36 poles requiring 10,000 ampere-turns of excitation per pole, then $p \times C_m\,S_m = 360{,}000$. Then by rule [XXXII] the resistance of one turn *must* be 90 ÷ 360,000 = 0·00025 ohm. Suppose the mean length of one turn to be 30 inches, then the conductor *must* be of such a section as to have as its resistance per inch 0·00025 ÷ 30 = 0·00000833 ohm. If we take the resistance of an inch cube of copper at 45° C. as 0·0000075 ohm, we get the rule that the necessary section s in square inches will be:—

$$s = \frac{0{\cdot}0000075}{r_1} \quad . \qquad [\text{XXXIV}]$$

Hence, in the present case it will follow that the conductor must have a section of $\frac{75}{833}$ square inch = 0·0896 square inch. It then remains to choose a copper strip of this section, and of suitable breadth and thickness for winding. (Round wire should *not* be used[1] for exciting coils.)

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[1] Mr. Rothert points out that for a section less than 50 square millimetres (0·0775 square inch), square wire is actually preferable to strip so far as space-factor is concerned. He gives the following comparison of the number of con-

Now, having ascertained the right size of conductor, we must next fix the actual number of turns to be used; for, according to shunt principles, if the voltage is fixed, the ampere-turns of a coil are independent of the actual number of turns. If we were to increase the number of turns by 10 per cent., the increased resistance would bring down the current 10 per cent., and so the magnetizing effect would remain the same. On the other hand the waste of energy would be decreased by 10 per cent. Hence we may reduce the heating by piling on more turns of the same conductor. In the case of revolving magnet-wheels, where the ventilation is generally good, and using cotton-covered square wire, with a peripheral speed of 4000 feet per minute, the temperature-rise will not exceed 40° C., if these are allowed 1·17 square inch of coil-surface[1] per watt lost. When the number of poles is few, say 8, 10 or 12, the difficulty that arises is usually that of keeping down the temperature-rise. When the number of poles is great the difficulty is rather that of keeping down the total number of watts lost in excitation, the copper being increased in quantity by adding more turns until the watts lost are sufficiently reduced.

Choice of Length of Pole-Core.—It is of course useless to take

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ductors of equal section (insulated), but of different form, that can be got into a certain given winding area:

Conductors of 120 *sq. mm.* (0·186 sq. in.):

Edgewise strip	44
Square wire	39
Flatwise strip	35
Round wire	33

Conductors of 50 *sq. mm.* (0·0775 sq. in.):

Edgewise strip	87
Square wire	86
Flatwise strip	67
Round wire	66

Edgewise strip has an advantage over square wire in giving better conduction of heat to the surface.

[1] This is modern practice. Formerly a much less surface was permitted, with a higher temperature-rise. In the large alternators in the Paris Exhibition the average allowance was only 0·82 square inch per watt lost. Several machines had over 1·4 square inch. Two had only 0·4 square inch.

a greater length of core than is necessary to accommodate the necessary number of ampere-turns. As was shown on p. 164 of Vol. I., the space-factor, which for round wires falls between 0·4 and 0·6, may, with edgewise copper strip, attain 0·8 to 0·93. With square-drawn wire it varies, according to space occupied by insulation, from 0·65 to 0·75. Hence, at an ampere density in the copper of 1000 amperes per square inch, each square inch of winding space in the transverse section of the coil will carry from 400 to 600 ampere-turns if the wire is round, from 650 to 750 ampere-turns if the wire is square, or from 800 to 920 amperes if edgewise strip is used. The latter is almost universally employed. The permissible winding-depth is seldom less than 1½, and seldom more than 2½ inches: as a working value 1·75 to 2 inches may be assumed. Therefore, with edgewise strip, for every 10,000 ampere-turns, a winding-section of 12·5 to 10·85 square inches is needed, and a clear winding-length of from 5 43 (minimum) to 7·1 (maximum) is needed. If cooling conditions are good, as in revolving magnet-wheels, a current density exceeding 1000 amperes per square inch in the copper may, however, be used; values from 1200 to 1500, and even to 1800 being found. Current density is, however, not so good a guide as the computation of the total watt-loss and the relative cooling surface.

If in designing there turns out not to be room enough, on the pole-cores in the design adopted up to this point, to give the requisite number of ampere-turns without too great a watt-loss, or without too much heating, then the design must be altered and a longer pole-core taken; or, if need be, a larger frame must be assumed for the design.

Example.—In the case of the Oerlikon alternator analysed on p. 397, at full-load 7680 ampere-turns were required per pole. There were 72 poles; the mean of one turn was about 40 inches, and the exciter worked at 100 volts. Taking the available V_x at 70 volts we get by rule [XXXII] above resistance of one turn = 0·000126 ohm, and the resistance per inch length as 0·00000315. Hence by rule [XXXIV] the necessary section will be 0·212 square inch. As a matter of fact, the conductor was strip 1·38 inch broad and 0·157 thick, therefore having a section of 0·216 square inch.

Verification of Dispersion.—The length of the pole-core having been finally settled, drawings should now be made full size of one complete magnetic circuit ; and the various magnetic leakages primary and secondary should be worked over in detail with the greatest care. Unless this is done, no confidence can be felt that when the machine is completed it will give a really satisfactory performance from the point of view of reactions. It will be noted that if rectangular magnet cores that are long in the direction parallel to the shaft are used, the dispersion coefficient will be greater than with cylindrical cores, and may reach the value 1·32 or even 1·35 in machines working at 50 cycles per second.

SCHEDULE FOR DESIGN.

For convenience in design the Schedule given on pp. 383 to 385 has been prepared.

PROCEDURE IN THE DESIGN OF INDUCTOR ALTERNATORS.

It is assumed that the prescribed data for the design are the voltage V, the number of phases n_{ph}, the amperes per phase C_1, the frequency f, and the revolutions per minute RPM. Therefore the number of "poles," p, is fixed, being equal to $f \times 120 \div$ RPM.

Let v, in feet per minute, be the greatest permissible velocity. For large machines with steel foundation rims this may be as much as 8000 feet per minute. (See remarks p. 350). The factor $\frac{1}{5}$ will reduce to inches per second.

Let q be the number of ampere-conductors per inch. This differs in different styles of machines. Where the poles are well apart we can get in more ampere-conductors without making the slots too deep. In high-voltage machines insulation takes up more room and reduces the possible value of q. In practice (see p. 437) q varies from 200 to 650 ampere-conductors per inch ; but the usual values for three-phase machines fall between 300 and 600

SCHEDULE FOR DESIGN OF ALTERNATOR.

Calculated by ______________________

Constructed by ______________________

SPECIFICATION.

TYPE ; No. of Phases = ; Frequency = ; RPM = ; Volts (between lines) = ;
Amperes in 1 Phase C_1 = ; Efficiency η = ; Cos ϕ = .

CONSTANTS: $B_g = 40{,}000$ to $60{,}000$; $B_p = 80{,}000$ to $110{,}000$ lines per sq. in; $v = 4800$ to 6000 feet per minute (for cast iron wheel); $q = 280$ to 600 amp.-cond. per inch.

Values taken: $B_g =$; $q =$; $\psi =$; $v =$; $k =$.

$k \begin{cases} 2{\cdot}16 \text{ for 2 slot winding.} \\ 2{\cdot}14 \text{ for 3 slot winding.} \\ 2{\cdot}12 \text{ for 4 slot winding.} \end{cases}$

$\psi = 0{\cdot}6$ to $0{\cdot}7$.

$E_1 = V \div \sqrt{3} =$; $KVA = 3 \times C_1 \times E_1 =$.

$p = f \times 120 \div RPM =$.

$KW = KVA \times \cos\phi =$; $HP = KW \div 0{\cdot}746 \times \eta$.

Trial values for d and l

(1) $d = \dfrac{12 \times v}{\pi \times RPM} =$

Coefficient $\beta = \dfrac{31{\cdot}7 \times 10^{10}}{k \times B_g \times q \times v \times \psi} =$

(2) $dl = KVA \times \beta =$

Trial Values.

$d =$				
$d. =$				
$l =$				
$v =$				
$\pi d =$				
$\pi d \div p = \tau =$				

Pole breadth $b = \tau \times \psi =$; pole-area $b \times l =$; Trial value for $\mathbf{N} = b \times l \times \mathbf{B}_g =$.

No. slots per phase per pole = .
Total slots per phase . = .
Total number of slots all round = .

Trial values for Z_1.

(1) $Z_1 = q \times \pi \times d \div 3C_1 =$ (maximum)
(2) $Z_1 = \frac{E_1 \times 10^8}{k \times f \times \mathbf{N}} =$ (probable)

Adjust to be exact multiple of Total Slots per phase.
$\therefore Z_1 =$; No. of Conductors per slot = .

Recalculate final value for **N**

Mean length of 1 turn estimated at inches.

$\mathbf{N} = \frac{E_1 \times 10^8}{k \times f \times Z_1} =$; whence $\mathbf{B}_g = \mathbf{N} \div (b \times l) =$.

Section of Conductors (at $a =$ amps. per sq. in.) $= C_1 \div a =$ sq. in.
Nearest standard size is No. Standard Wire Gauge = sq. in.;
Diameter bare = ; diameter covered and varnished = .

Slots.—Slot insulation (at 0·060 per 1000 V) = . Size of slot by .
Slot-pitch = ; slot-width = ; tooth-width = .
[N.B.—Tooth-width should not be less than slot-pitch $\times \mathbf{B}_g \div 100{,}000$.]

Resistance of 1 phase (at 60° C.) $= R_1 = Z_1 \times \frac{\text{mean length 1 turn}}{\text{area of section}} \times \frac{0{\cdot}396}{10^4} =$ ohm.

Ohmic drop $= C_1 R_1 =$ volts; armature copper loss $= 3\, C_1^2 R_1 =$ watts.

Gap.—Should not be less than $\delta = 5 \times q \times \tau \div \mathbf{B}_g =$ in. .
$\therefore d_m = d \mp 2\delta =$ in.

Core-depth behind teeth $= \frac{1}{2}\tau =$.
External diameter armature = .

Excitation.

Probable exciting AT per pole at no-load $= 0{\cdot}5\ \mathbf{B}_g\, \delta =$ amp.-turns.
" " " " full-load $= 0{\cdot}8\ \mathbf{B}_g\, \delta =$ amp.-turns $= C_M S_M$.
Winding area per bobbin at amps. per sq. in. gross = sq. in. . Mean length 1 turn, M = inches.
Excitation loss = winding area × mean length 1 turn × (amps. per sq. in.)$^2 \times 0{\cdot}4 \times 10^6 =$ watts per pole.
Total excitation loss = watts.
Cooling surface per pole = sq. in. Sq. in. per watt = . Exciter voltage $V_X =$ volts.

II. Iron-losses.

Volume iron armature teeth = cub. in. ; B_t in. ditto = lines per sq. in.
,, ,, armature core = cub. in. ; B_c in. ditto = lines per sq. in.
Volume teeth × f × hysteresis waste per cub. in. at frequency 1 ~, from curve = watts.
,, ,, × f^2 × eddy ,, ,, ,, ,, = watts.
Volume core × f × hysteresis ,, ,, ,, ,, = watts.
,, ,, × f^2 × eddy ,, ,, ,, ,, = watts.
Total iron-losses . watts.

Weights and Costs.

Parts.	Material.	Weight.	Cost per lb.	Cost of part.
Armature stampings .				
Armature copper . .				
Insulation of slots . .				
Housing, bolts, etc. .				
Magnet cores . . .				
Magnet copper . . .				
Bobbins, etc. . . .				
Magnet wheel . . .				
Shaft and bearings . .				
Slip-rings, etc. . . .				
Miscellaneous . . .				
	Total weight		Total cost	

Field-Magnet Winding.

Resistance per inch conductor

$$r_1 = \frac{0 \cdot 7 \times V_x}{p \times C_M S_M \times M} = \qquad \text{ohm.}$$

Section of conductor

$$= \frac{0 \cdot 00000078}{r_1} = \qquad \text{sq. in.}$$

Permissible Losses.

Copper	$\% = w_c =$
Iron	$\% = w_i =$
Excitation	$\% = w_x =$
Total	$\% = w_{tot} =$

Efficiency.

$$\eta = \frac{3\, C_1 E_1 \cos \phi}{3\, C_1 E_1 \cos \phi + \text{total losses}} = \quad .$$

Then the following formulæ hold good:—

For Single-Phase Inductor Machines

$$\frac{\mathrm{E}\,.\,\mathrm{C}\times 10^8}{\frac{1}{5}v\times q}=p\,\mathrm{N}.$$

For Three-Phase Inductor Machines

$$\frac{3\,\mathrm{E}_1\,\mathrm{C}_1\times 10^8}{\frac{1}{5}v\times q}=p\,\mathrm{N}.$$

This determines the flux per pole **N**. The diameter d is determined from RPM and v by formula XI*b*, p. 350.

If $\mathbf{B}_g$ the flux density in the gap at the (ideal) pole face is assumed, at, say 50,000 lines per square inch, l is determined.

Next let the exciting ampere-turns on the magnet be called X_0, the following relation holds good for the percentage drop of voltage:—

$$\delta\,v = \text{percentage drop of voltage} = k_3\,.\,\frac{\frac{1}{5}v\times q}{f\times \mathrm{X}_0},$$

where k_3 is a constant determined by practice; k_3 for three-phase inductor machines is $\frac{4}{3}$; or for ordinary alternators with alternating poles, each wound, $=\frac{1}{2}$.

This determines X_0 if $\delta\,v$ is prescribed, for

$$\mathrm{X}_0 = k_3\,\frac{\frac{1}{5}v\,.\,q}{f\,.\,\delta\,v}.$$

X_0 being thus determined, it remains to determine what air-gap will, with this ascertained number of existing ampere-turns per pole, give the requisite value of **N**.

In the usual inductor type, as represented by Fig. 112, the armature is in two parts. In designing such a machine, the first procedure is to design the two armatures, each for half the output. Then the iron inductor part is designed, remembering (*a*) that as there are no reversals of magnetism in the armature the iron will be used with only half as much effect as in ordinary types, (*b*) that the magnetic leakage will be relatively great.

APPENDIX TO CHAPTER VII.

COMPOUNDING OF ALTERNATORS.

Methods for compounding alternators may be divided under the following heads :—

1. By sending rectified or partially rectified currents round the magnet coils of the alternator.

2. By a rheostat in the continuous current excitation circuit of the alternator, its resistance being controlled automatically by the alternating load current.

3. By compounding, in the first place, the continuous current exciter, either

    (*a*) by rectifying current for its magnet winding, or
    (*b*) by supplying alternating current to its armature.

4. By a special designing of the magnet pole so that over certain ranges of load, the armature reactions increase the total flux.

The simplest method of carrying out the first of the above methods is by the use of a rectifying commutator driven by a synchronous motor (as in arc lighting rectifiers). It has been suggested that the recently developed electrolytic rectifiers, or the Cooper-Hewitt mercury vapour rectifier, might be employed for this purpose, thus obviating recourse to revolving machinery with the sparking commutator.

A rather special arrangement for rectifying the alternating current has been used by Baum,[1] and is shown in Fig. 325.

The compensator armature A rotates in synchronism with the generator G. The field of the compensator is provided by the main load on the generator, either directly or indirectly through a transformer. On the armature one end of the winding is connected to a complete slip-ring in the ordinary way, while the other end is connected to an incomplete ring,

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[1] *Trans. Amer. Inst. Elec. Eng.*, May 1902.

broken as shown, the two parts of the broken ring being electrically connected as shown.

If the compensator were excited with continuous current an alternating voltage would be generated in the armature, but the action of the broken slip-ring would be to let pass only certain portions of the current wave, first, of the positive half and then of the negative half. By the use of an alternating current excitation of the same frequency as that of the rotation of the armature, the frequency of the armature voltage is doubled, for while a conductor passes from one pole to the next the polarity of the magnets themselves changes. The action of the broken slip-ring in this case is to allow an intermittent current to pass, which always flows in the same direction, so that when this current is sent round the coils of the main exciter, a permanent increase in its flux takes place.

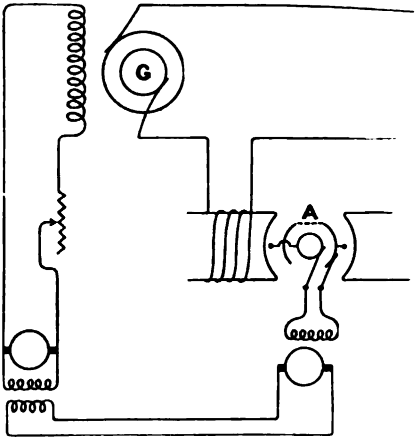

Fig. 325.—Baum's Rectifier.

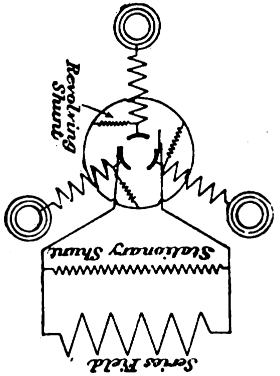

Fig. 326.—Circuits for Compounding a Three-Phase Generator.

To obtain a uniform rectified current from all the phases in a three-phase generator, Mr. E. W. Rice, of the General Electric Co. of Schenectady, adopted the arrangement shown in Fig. 326. The inner ends of the three phases, which would otherwise go to form the common junction, are carried to a three-part commutator which rectifies current from two of the phases at a time and conveys them to a special compound winding on the magnets. To minimize sparking the commutator segments are shunted by resistances, and a further shunt is placed across the field-magnet windings.

For several years Heyland and others have been working on this question of the excitation of alternators by their own

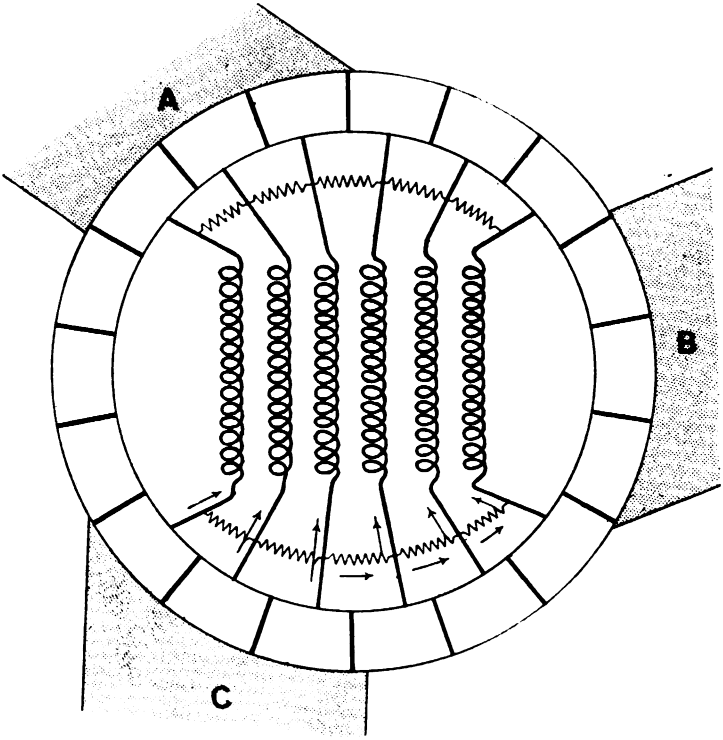

FIG. 327.—HEYLAND'S COMMUTATOR FOR SELF-EXCITING AND SELF-COMPOUNDING ALTERNATORS.

current, and several types of machines have been evolved. Heyland's latest type is shown diagrammatically in Fig. 327, for a bipolar machine. It is in the first place a self-exciting machine corresponding to the *shunt* type of winding, and can be made self-compounding, just in the same way as a continuous-current shunt generator. The magnet coils consist of six parallel windings, each connected to a pair of oppositely situated bars on the commutator.

The diagram shown applies to a three-phase case, and the position of the brushes is such that when on unity power-factor the current in one of the phases is zero, the currents equal in the other phases, while the brush on the first phase covers dead bars of the commutator. As the current grows in the A phase, the commutator revolves until, when the B phase has died away to zero, the brush is covering the dead segments.

In this way the total current flowing in the six parallel circuits is of uniform value. The arrangement may be used for a machine with any number of poles by multiplicating the commutator and cross connecting all bars of equal potential.

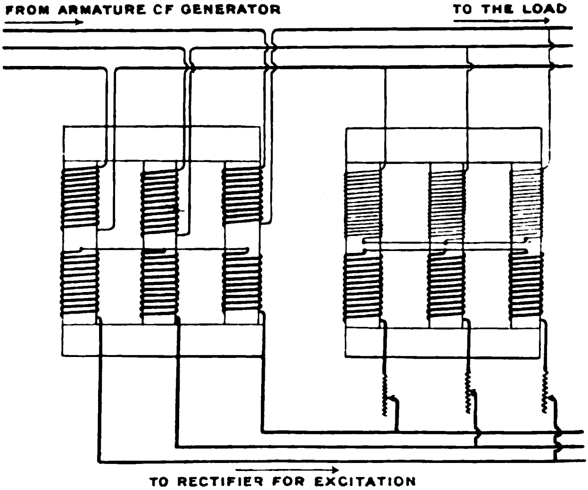

FIG. 328 —CONNEXIONS FOR COMPOUNDING ALTERNATORS.

The resistances placed across adjacent commutator segments help to cut the current down gradually as the brush leaves the set of active segments, thus minimising the tendency to spark.

Compounding for Inductive Loads.—With alternators it is not sufficient merely to compound with the load current going from the machine. The power-factor of that current must also be considered, because on low power-factors the drop for the same load current is greater. In all methods of compounding this is easily arranged for by using both a current transformer and a potential transformer, and compounding the currents obtained from the two secondaries. The connexions of such an arrangement are shown in Fig. 328, the resistances in the secondary of the potential transformers being used for adjusting the amount of compounding or over-compounding. The theory is clearly seen from the vector diagrams in Figs. 329 and 330. O E represents the voltage of the generator. The current in the secondary of the potential transformers will in all circumstances be represented by O A lagging practically 180° behind O E. The load current passing through the primary of the current transformer produces a flux in phase with itself, which flux induces an EMF in the secondary coils at right angles to itself. If the load is non-inductive this EMF and also the current it sends will be represented by O B, and the total current supplied to the compound winding through the rectifier = O P. If, however, the load is partially inductive and lags

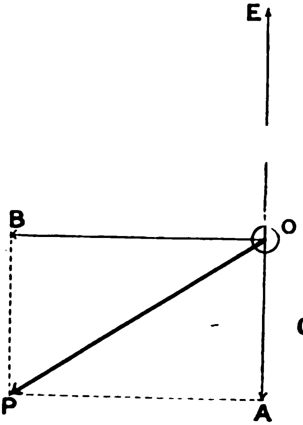

FIG. 329.—COMPOUNDING ON NON-INDUCTIVE LOAD.

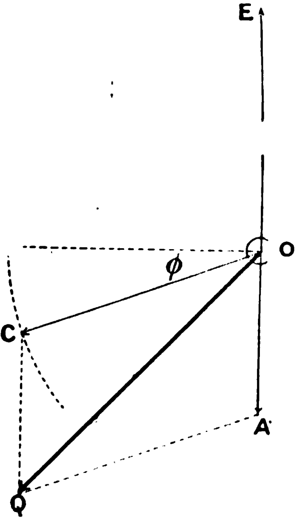

FIG. 330.—COMPOUNDING ON INDUCTIVE LOAD.

by an angle ϕ behind its EMF, the current in the secondary of the current transformer is represented in Fig. 330 by O C, which, being nearer in phase to O A, gives a larger current O Q for compounding.

To regulate by means of an automatically controlled rheostat, arrangements may be used such as have already been described on p. 873 of Vol. I. Such methods are not very satisfactory, because the mechanical parts are liable to get out of order, and also a considerable delay occurs between the putting of the load on the generator and the consequent readjustment of voltage.

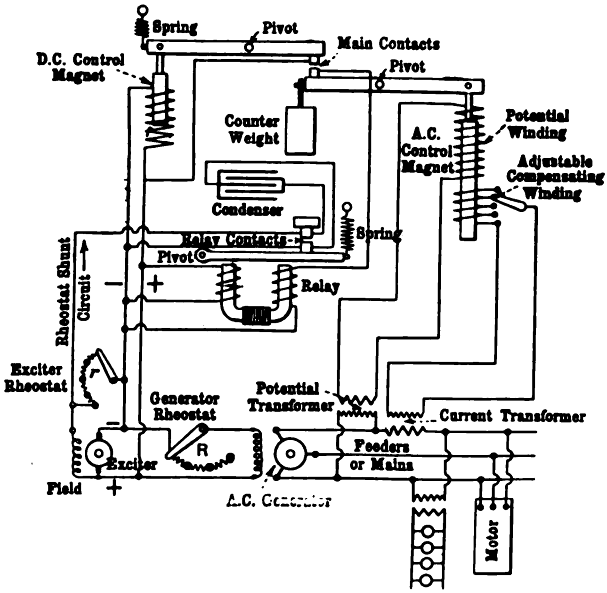

FIG. 331.—TERRIL'S METHOD OF COMPOUNDING ALTERNATORS.

Recently Terril has invented an apparatus which falls under this heading, but which is, however, practically instantaneous in its working. It consists essentially of a vibrating contact which short-circuits a resistance placed in the field circuit of the exciter.

It will be seen from Fig. 331 that the contact is controlled by a differentially wound relay, the current for operating which is taken from the exciter bus-bars and is controlled by the main floating contacts. These latter are controlled on one side by a continuous-current solenoid connected again from the exciter bus-bars, and on the other side by an alternating current solenoid, consisting of a shunt coil across alternate current feeders, and also, if overcompounding is desired, a series coil carrying current proportional to the alternate current load. The function of the continuous-current control solenoid is to maintain a steady, but not necessarily constant, voltage on the exciter bus-bars. The action of the alternate-current control magnet is as follows:—If the A C voltage falls for any reason, the core of the A C solenoid falls, and causes the main contacts to touch, so that the relay is demagnetized and the exciter field resistance becomes shorted, with the result that the generator voltage will be brought up to its normal value. When this is the case, the floating contacts will be separated, and the voltage tends to go down again. In actual operation it is found that the floating contacts are in a continual vibration, the speed of vibration varying according to the load of the machine, so that the exciter field rheostat is shunted during a longer or shorter time, as is required.

With regard to the third method of compounding set forth above, any of the systems already set forth for sending rectified alternating current around the exciting coils of the alternator can also be adopted for sending current round the coils of the exciters. Under the second sub-heading, however, a different method is referred to, which is based upon principles already described on p. 260. We know that the alternating current in an alternating current armature, in the case of a generator, when lagging produces a demagnetizing effect, and when leading a magnetizing effect, and that in the case of a motor the actions are reversed. The application of this principle, therefore, lies in driving the exciter synchronously with the generator, and supplying alternating current proportional to the load to a special winding on the armature wound in such a way that the alternating current, more especially if it is lagging at all, produces a magnetizing effect upon the whole magnet system of the exciter, and thus increases its voltage. It must be remembered, however, that this special winding revolving in the field of the exciter has an electromotive-force generated in it which corresponds to an extra voltage drop which has to be compounded for.

This method has been used by Danielson in Sweden, and Rice in America.

(4) The compensating generator of Miles Walker is an example of this type. It has been shown in pp. 526 to 530 of Volume I., how, owing to the saturation of the iron at the pole face, a cross-magnetizing armature current may diminish the total amount of magnetism in the machine, whereas, if the iron did not become saturated and the magnetism produced was always directly proportional to the current producing it, the effect of a cross-magnetizing action would be to crowd the magnetism up on to one pole side, but not to alter its value. In the arrangement shown in Fig. 332 the chief magnetizing current is sent round the inner coil, and only a small current, if any, is sent round the outside coil, which embraces the whole pole. The result is that at no-load most of the magnetism

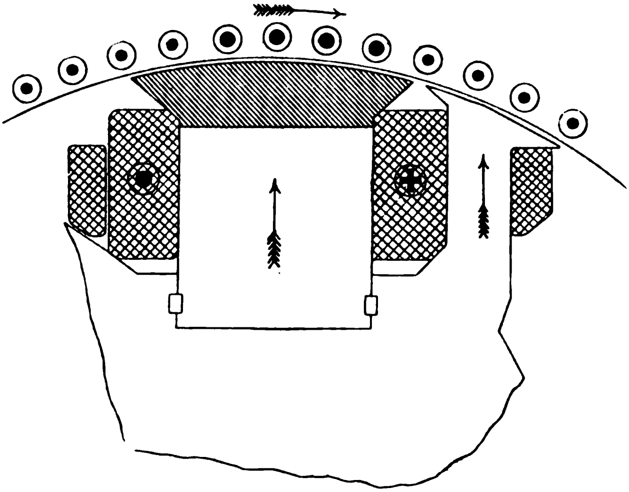

FIG. 332.—WALKER'S METHOD OF COMPOUNDING ALTERNATORS.

flows by the main pole core and saturates it. When a load current in the armature distorts the magnetism in the pole, some is swept into the auxiliary pole on the right and the main pole becoming less saturated is able to carry relatively more magnetism for the diminished M.M.F., and the total flux over the whole pole is increased. The action is therefore the reverse to the one mentioned above. To adjust the machine for various degrees of compensation, the relation between the ampere-turns in the inner and outer coils may be altered. For ordinary power-factors met with, this device answers admirably, but the compensation obviously weakens as the power-factor increases, and a limit is reached when the volts drop as in ordinary machines.

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CHAPTER VIII.

EXAMPLES OF DESIGN OF ALTERNATORS.

THE following examples of alternator design are selected as illustrative of recent practice :—

I. AT(B) 72—1500— 83, by the Oerlikon Machine Works.
II. AT(B) 64— 760— 94, by the Société Électricité et Hydraulique of Charleroi.
III. AS(B) 24— 250—373, by Messrs. Johnson and Phillips.
IV. AS(B) 14— 100—770, by Messrs. Witting, Eborall and Co.
V. AQ(B) 60— 625—100, by Messrs. Witting, Eborall and Co.
VI. AT(B) 48— 270—125, by the Oerlikon Machine Works.
VII. AT(B) 10— 290—600, by the Oerlikon Machine Works.
VIII. AQ(B) 11—3750—250, by the General Electric Co. (Schenectady).
IX. AT(B) 12—7500—250, by the General Electric Co. (Schenectady).
X. AT(B) 28—1640—180, by Messrs. Brown, Boveri and Co.
XI. AQ(B) 96—3500— 75, by Messrs. Kolben and Co.
XII. AQ(B) 46— 900—120, by Messrs. Brown, Boveri and Co.
XIII. AT(B) 28— 75—215, by Messrs. Brown, Boveri and Co.
XIV. AQ(A) 62—1500—116, by the British Westinghouse Co.

It will be noted that Nos. III. and IV. are single-phase machines, Nos. V., VIII., XI., XII. and XIV. are two-phase, and the rest three-phase. No. IV. is a machine of unusually high frequency, viz. 90 cycles per second. Nos. VI. and VII. are selected for contrast, being machines of nearly equal output and same frequency, by the same makers, but one a low-speed, the other a high-speed machine. Nos. IX. and X. are two of the Niagara generators of umbrella pattern, and both of frequency 25~. The ratings vary from 75 KVA (No. XIII.) to 7500 KVA (No. IX.). No steam-turbine alternators are included, as these are specially described in Chapter IX.

Example I.—THREE-PHASE ALTERNATOR of the Oerlikon Machine Works.

$$\text{AT(B) } 72_{\text{P}} - \frac{840 \text{ KW}}{1500 \text{ KVA}} - 83_{\text{RPM}}\,;\ \frac{3200}{5500}{}^{\text{V}} - 3 \times 160_{\text{A}} - 50 \sim\,;$$

$$\cos\phi = 0{\cdot}55.$$

This is a Y-connected machine of simple design, working at St. Étienne (France) on a load with a very low power-factor, which therefore tends to give rise to great armature reaction.

Fig. 333 gives transverse and longitudinal sections showing the general construction of the machine, and the grouping of the windings, together with a full-size drawing of the slot. Fig. 334 shows the no-load and short-circuit curves, and Fig. 335 gives a general view of the machine after erection. These, together with the leading dimensions and data, have been kindly furnished by the Oerlikon Co.

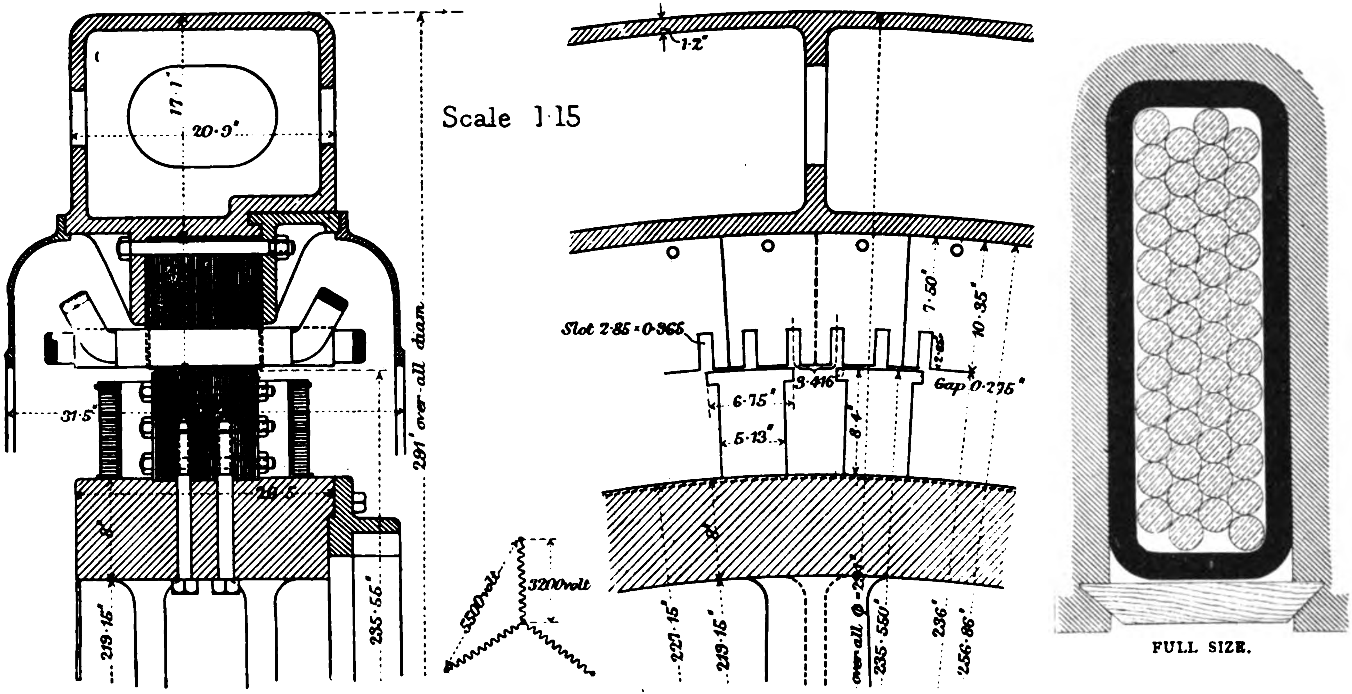
GENERAL SPECIFICATION.

C_1	Full-load current, amperes per phase	160
V	Terminal volts across lines	5500
E_1	Generated volts in one phase	3200
KVA	Apparent kilowatts	1500
KW	True kilowatts	840
	Revolutions per minute	83
f	Frequency (periods per second)	50
	Power-factor ($\cos\phi$)	0·55
p	Number of poles	72
η	Efficiency at full (inductive) load	92½
	,, ½ load	93
	,, ¼ load	91
v	Peripheral speed (ft. per min.)	5115

DIMENSIONS (in inch units).

Armature :—

d	Diam. of bore [6000 mm.]	236
	Outside diameter of core discs	256·86
	Number of slots	216



FULL SIZE.

FIG. 333.—THREE-PHASE ALTERNATOR AT(B) 72—1500—83; CONSTRUCTED AT THE OERLIKON MACHINE WORKS.

	Depth of slot	2·85
	Width ,,	0·965
	Pole-span	6·75
	Slot-pitch	3·42
	Radial depth of iron in core body	7·5
l	Gross length of core body	9
l_i	Nett iron length of core body	8·1
Z	Total number of conductors	2592

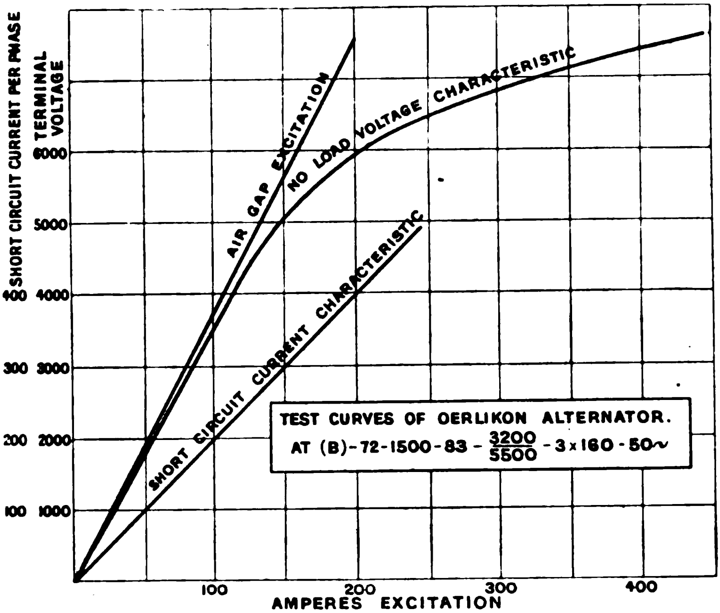


FIG. 334.—SATURATION AND SHORT-CIRCUIT CURVES.

Z_1	"Conductors" per phase	864
	"Conductors" per slot (each of 4 wires in parallel)	12
	Wires per slot	48
	Size of wire, bare	0·160 diam.
	,, insulated	0·185 diam.
	Section of "conductor"	4 × 0·0201 sq. inch
	Style of winding	one slot winding
	Resistance, per phase, of winding at 60° C.	0·298 ohm

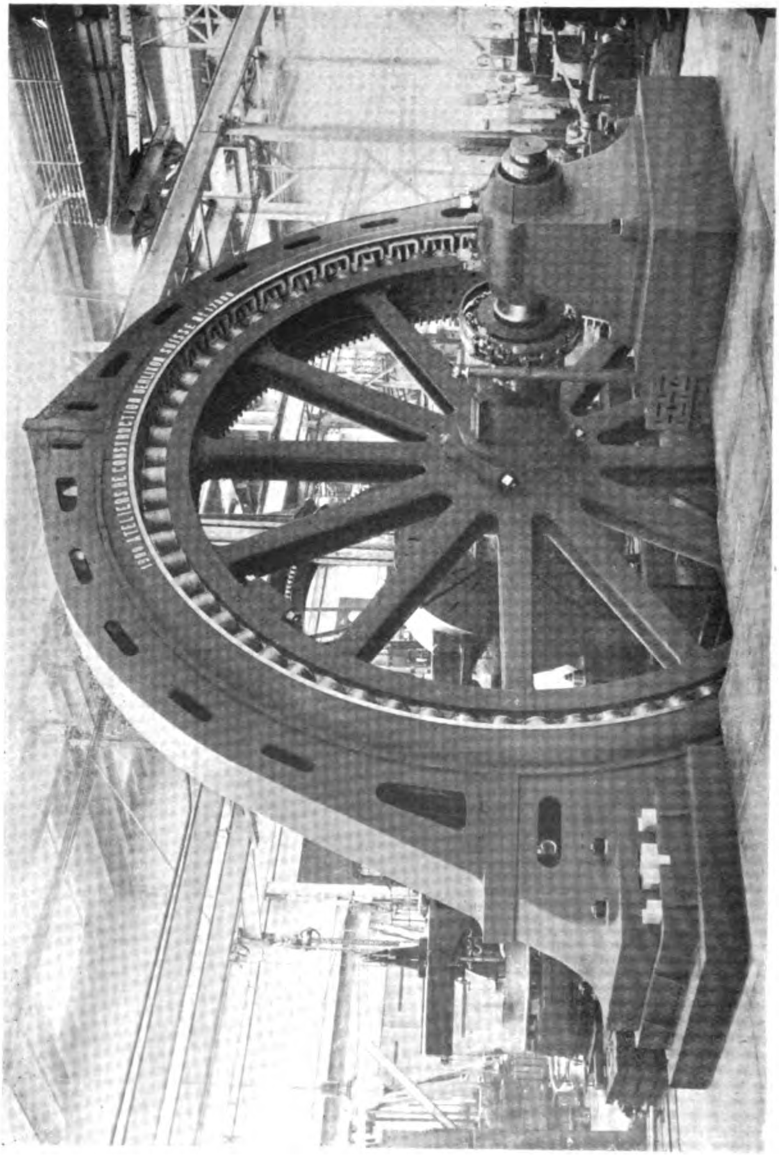


Fig. 335.—Three-Phase Alternator AT(B) 72—1500—83; Constructed by the Oerlikon Machine Works.

Field-Magnets:—

d_m	Diameter at face of magnet-wheel [5986 mm.]	235·5
l_m	Length of pole-face parallel to shaft	8·5
δ	Width of air-gap	0·275
b	Breadth of pole-face	6·75
	Polar angle	3·25°
	Type of winding	edge-wound strip
	Dimensions of conductor	1·38 × 0·157
	Section of conductor	0·2165 sq. inch
S_m	Number of turns per pole	35
M	Mean length of one turn	39·5
	Exciting current at no-load	170 amp.
	,, ,, full inductive load	248 amp.

CALCULATED DATA AND COEFFICIENTS.

τ	Pole-pitch (inches)	10·3
v	Peripheral speed (ft. per min.)	5115
ψ	Ratio of pole-span to pole-pitch	0·655
	corresponding to an electrical angle of	59°
β	Steinmetz coefficient ($d\,l \div$ KVA)	1·42
ξ	Esson coefficient ($d^2 l \times$ RPM + KVA)	27,800
	Slot space-factor	0·35
a	Current density in armature (amps. per sq. inch)	1985
a_m	Current density in field-magnets (amps. per sq. inch), full-load	1150
q	Ampere conductors per inch of periphery	574

In this machine the cores, both of armature and field poles, are constructed of stamped laminations, without ventilating ducts. The armature laminations, clamped between strong end-plates, are supported in a housing of cast iron. The field stampings, bolted together by six bolts from front to back through each pole, are firmly secured to the rim of a substantial wheel by screws which are tapped into holes drilled into the laminated cores. The pole-core stampings have at the face slightly extended projections which hold down the exciting bobbins; the latter, which have semicircular ends, being coiled with copper strip wound edgewise. There are for simplicity of construction only three slots per pole, that is one slot per pole per phase. The projecting end portions of the armature coils are arranged in two ranks. The armature slots are straight, very slightly rounded off at the tips, and are notched just below the tips to admit of the insertion of wedges to secure the coils. The coils are arranged in two ranges, one straight out, the other with end-bends turned outward. Both sets are former-wound. Fig. 335*a* illustrates the way in which the coils of the bent-up set are prepared on former-blocks.

The first step in analysing the design of this machine is to construct the saturation curve or no-load characteristic by a calculation of a magnetic circuit. The electromotive-force in one phase at no-load is 3200 volts ($= 5500$ volts $\div \sqrt{3}$). The winding being a concentrated one we may take the Kapp coefficient at 2·22. The number of turns per phase is 432

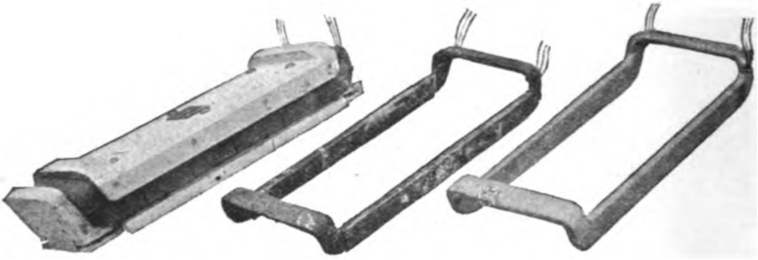


FIG. 335*a*.—FORMER-WOUND COILS OF ALTERNATOR.

(*i.e.* 36 coils of 12 turns each). Hence $Z_1 = 864$. The frequency is 50 ∼, hence we have

$$E_1 = 2{\cdot}22 \times 50 \times 864 \times N \div 10^8 ;$$

or

$$E_1 = 0{\cdot}00009504 \times N.$$

The dispersion-coefficient at no load is 1·20. But with the large lag of current at full inductive load this is increased owing to the largely increased excitation necessary to balance armature reaction; and as the full-load excitation will be about 45 per cent. greater than the no-load excitation, the stray field will be proportionately greater, raising the dispersion from 20 per cent. to 29 per cent., and the dispersion-coefficient from 1·20 to 1·29 (see the calculations pp. 250 to 252). The no-load fluxes may be calculated for a number of values corresponding to different voltages.

E_Δ	E_1	N_a	N_m
3290	1900	2,000,000	2,400,000
4940	2851	3,000,000	3,600,000
5750	3326	3,500,000	4,200,000
6175	3563	3,750,000	4,500,000
6580	3800	4,000,000	4,800,000

It will suffice to give in schedule form the calculations for one of these values. The calculations are made round half of a magnetic circuit, thereby ascertaining the required ampere-turns per pole. The sectional areas and mean magnetic lengths are taken from the drawings. The ampere-turns per inch length of path, in the penultimate column, are taken from curves of magnetization for the appropriate kind of iron (see Vol. I. Plate I.).

		Flux from One Pole (Mega-lines).	Sectional Area (sq. inch).	Flux-Density	Ampere-turns per Inch of Path.	Mean Magnetic Length (inches).	Ampere-turns Needed.
$E_\Delta = 4940.$ $N_a = 3,000,000.$ $\nu = 1{\cdot}23$	Armature body.	1·5	50·6	29,600	3	6·75	20
	Teeth .	3·0	47·5	63,200	9	2·85	25
	Gap . .	3·0	57	52,600	16,450	0·275	4530
	Pole-Core	3·69	39·2	94,200	30	8·4	252
	Yoke . .	1·84	70	26,000	21	8·0	168

Total Ampere-turns needed when $N_a = 3,000,000 = 4995$

It is worthy of note that the pole-core being more highly saturated than the other parts, the number of exciting ampere-turns for it increases disproportionately at the highest excitation.

The other values calculated similarly give the following results for the excitation X_1 at no-load :—

Volts across corners, E_Δ .	3290	4940	5750	6170	6580
Volts per phase, E_1 . . .	1900	2850	3326	3563	3800
Ampere-turns per pole, X_1 .	3340	4995	6550	7170	9100
Amperes excitation per pole, C_M	95·5	142·5	187	216	260

By plotting these values in a curve (Fig. 334) we obtain the working part of the no-load characteristic.

The straight sloping line, also shown, gives the ampere-turns required to drive the flux *through the gap only*, all the additional distances to the right to the no-load curve representing the ampere-turns required for the iron parts.

Calculation of Efficiency.—This might be completed for various loads; but, as an example, that for the full-load must suffice.

1. *Copper-Loss.*—This is—

$$w_c = 3 \times (160)^2 \times 0{\cdot}268 = 20{,}600 \text{ watts.}$$

2. *Iron-Losses.*—The mean diameter of the tooth area is estimated at 238·85 inches, and the tooth takes up 0·725 of the breadth of the slot-pitch. Therefore the volume of iron in the teeth is—

$$(238{\cdot}85 \times \pi \times 2{\cdot}85 \times 8{\cdot}1) \times 0{\cdot}725 = 12{,}700 \text{ cubic inches.}$$

The frequency of magnetization being 50 cycles per second, and the flux density 84,000 lines per square inch, we find on reference to the curves, a loss of 0·012 watt per cubic inch of iron at one cycle per second, and assuming the hysteretic constant at 0·002. Therefore the loss by hysteresis in the teeth at 50 cycles is—

$$0{\cdot}012 \times 50 \times 12700 = 7620 \text{ watts.}$$

Similarly we find the eddy-current loss, assuming core sheets 20 mils thick, to be 0·0001 watt per cubic inch at one cycle per second and at the given flux-density. Therefore the eddy-current loss in the teeth at 50 cycles is—

$$0{\cdot}0001 \times (50)^2 \times 12700 = 3175 \text{ watts}$$

making the total iron-loss in the teeth at full-load—

$$7620 + 3175 = 10{,}795 \text{ watts.}$$

The mean diameter of the core-body being 249·36 inches, we have for the volume of iron in the core-body—

$$249 \cdot 36 \times \pi \times 7 \cdot 5 \times 8 \cdot 1 = 47{,}600 \text{ cubic inches.}$$

The flux-density at full-load is 39,000 lines, which gives a hysteresis loss of—

$$0 \cdot 003 \times 50 \times 47600 = 7140 \text{ watts.}$$

And an eddy-current loss of—

$$0 \cdot 00003 \times (50)^2 \times 47600 = 3570 \text{ watts.}$$

Making the total iron-loss in the core-body—

$$7140 + 3570 = 10{,}710 \text{ watts.}$$

There will be a small additional iron-loss in the tips of the pole-cores, owing to the fluctuations in the distribution of the magnetism due to their movement past the teeth. This is not easy to estimate; for the flux is never reversed in direction, and merely shifts about with a frequency three times that of the periodicity of the system. The loss due to eddy-currents will be greater than that due to hysteresis. It will be confined to the region nearest the gap. As a very rough estimate, let us reckon an eddy-current loss extending to a depth of 0·5 inch, and equal in amount per cubic inch to the eddy-current loss in the teeth. The nett iron area of the pole-face being 4130 square inches, the volume affected in the 72 poles may be reckoned at 2065 cubic inches. Hence, this rough estimate totals up to 516 watts. Adding together the iron-losses in teeth, armature-core, and pole-tips, we get for the total iron-losses the value—

$$w_i = 10795 + 10710 + 516 = 22{,}221 \text{ watts.}$$

3. *Excitation Loss.*—The amperes taken at full inductive load are 248. The mean length of one turn of the winding is

40 inches. The edge strip which is used has a sectional area of 0·216 square inch. There are 35 turns per bobbin, and 72 bobbins in series, and remembering that the resistance of a copper rod, 12 inches long and 1 square inch in cross-section, is 0·00000946 ohm (at 60° Cent.), we have for the total resistance of the winding—

$$\frac{72 \times 35 \times 40 \times 0\cdot00000946}{12 \times 0\cdot216} = 0\cdot386 \text{ ohm.}$$

Therefore the excitation loss is—

$$w_x = (248)^2 \times 0\cdot386 = 22{,}750 \text{ watts.}$$

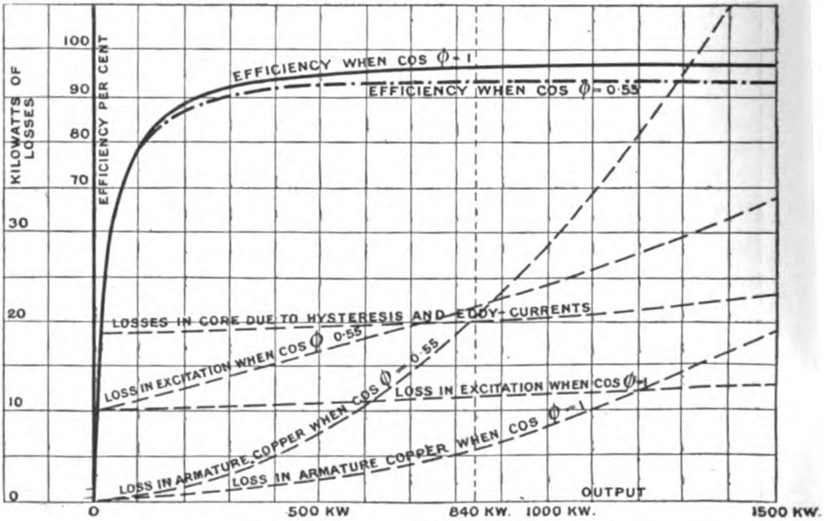


FIG. 336.—TEST-CURVE OF EFFICIENCY AND LOSSES.

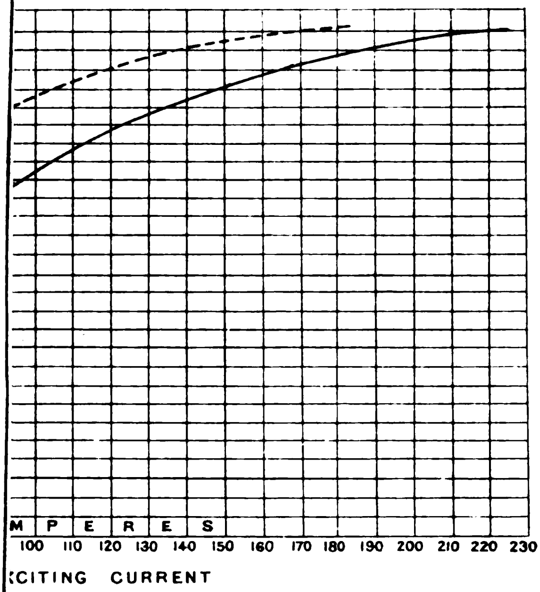
4. *Friction and Windage.*—Assuming this to be 1 per cent. of the true kilowatts output = 8400 watts, we then have for the total losses—

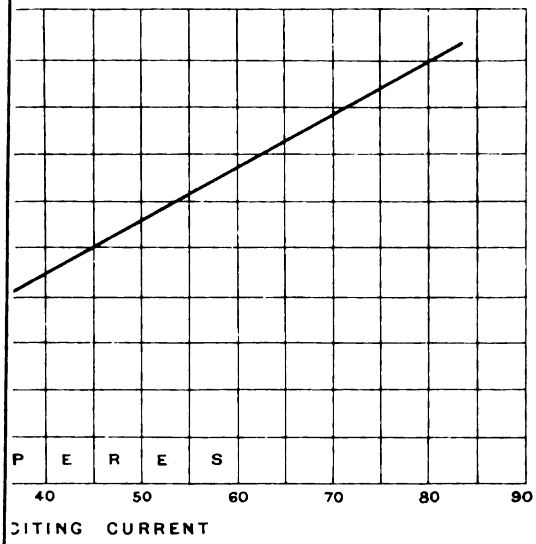
$$w = w_c + w_i + w_x + w_f$$

$$= 20600 + 22221 + 22750 + 8400 = 73971$$

or say 74,000 watts.

Plate VIII.





Thos Kell & Son Lith

Therefore the full-load efficiency is—

$$100 \times \frac{840}{840 + 74} = 91 \cdot 9 \text{ per cent.}$$

As a matter of fact the workshop tests showed a somewhat higher efficiency than that here calculated. The armature copper-loss was under 21 kilowatts at full load; the iron-losses came out at 20, instead of 22; the excitation loss was about 21½ instead of 22·7. Fig. 336 shows the test-curves.

Further references to points concerning this machine will be found on pages 181, 209, 250, 282, 372, and 381.

Example II.—A SLOW-SPEED THREE-PHASE GENERATOR.

This machine was designed by Mr. A. Heyland,[1] for the Société Electricité et Hydraulique, of Charleroi, and was exhibited by them at the Paris Exhibition of 1900. It is now in service at Belgrade.

Sectional drawings and test-curves are given on Plate VIII.

General Specification :—

$$\text{AT(B) } 64\text{—}760\text{—}94\,;\ \frac{1210}{2200}\text{—}3 \times 200\text{—}50\sim.$$

DIMENSIONS (in inch units).

Armature :—

d	Diameter at face	236
l	Length between core-heads. . . .	10
	External diameter	252
	Number of ventilating ducts . . .	2
	Width of each duct	0·4
l_i	Iron length of core	8·75
	Total number of slots	384
	Slots per pole and phase	2
	Dimensions of slot	1·34 × 0·87
	Conductors per slot	3
	Description of armature conductor . .	Two wires in parallel, each of diam. = 0·222
Z_1	Total number of conductors per phase . .	384

---

[1] For a detailed discussion of the design and tests of this machine, see Heyland in *Electrical Review*, xlvii. 195, 931 and 971, 1900. It must be regarded as a design which has been superseded by more economical constructions of the same firm.

Field-Magnets:—

d_m	Diameter over pole-faces	235·3
δ	Length of gap	0·31
b	Span of pole-shoe	5·9
l_m	Axial length of pole-shoe	10·0
	Radial length of pole-shoe and core	5·9
	Axial length of pole-core	7·5
	Width of pole-core (oval)	3·95
	Outside diameter of magnet wheel rim	223·6
	Inside ,, ,, ,,	217·3
	Length of rim parallel to shaft	12·5
S_m	Turns per magnet bobbin	50
	Dimensions of field-magnet conductor	4·18 × 0·03

CALCULATED DATA AND COEFFICIENTS.

τ	Pole-pitch (inches)	11·6
v	Peripheral speed (feet per minute)	5800
ψ	Ratio of pole-span to pole-pitch	0·51
	corresponding to an electrical angle of	46°
	Slot space-factor	0·25
a	Current density in armature (amps. per sq. in.)	2600
a_m	,, ,, field-magnet coils (amps. per sq. inch)	1180
β	Steinmetz coefficient ($dl \div$ KVA)	3·1
ξ	Esson coefficient ($d^2 l \times$ RPM $\div$ KVA)	68800
q	Ampere-conductors per inch of periphery	312

The chief feature about this machine is the small pole-arc to pole-pitch ratio. This causes the dispersion-coefficient to be smaller than is usually the case. It has been taken as 1·15.

The flux per pole is calculated from the usual formula—

$$N = \frac{E \times 10^8}{2 \cdot 22 f Z}$$

$$= \frac{1280 \times 10^8}{2 \cdot 22 \times 50 \times 384}$$

$$= 3\ 0.$$

The following table gives the magnetic calculations of the excitation for no-load.

Part of Magnetic Circuit.	Material.	Flux in Megalines.	Sectional Area in Square Inches.	B.	Ampere-turns per Inch (from a curve.)	Length of Path in Inches.	Ampere-turns Required.
Armature	Wrought Iron.	1·5	55·2	27,200	3	12	36
Teeth .	Wrought Iron.	3·0	27·2	110,500	100	1·42	142
Gap . .	Air .	3·0	51·6	58,200	×0·3133	0·315	5720
Pole-core	Cast Steel.	3·441	29·6	116,500	170	5·9	1005
Yoke .	Cast Steel.	1·72	49	35,000	4	11	44

Total Ampere-turns required 6950

As there are 50 turns per field-magnet bobbin, the exciting current will be $\frac{6950}{50} = 131$ amperes.

This agrees very well with the value shown by the curve on Plate VIII., which was obtained by experiment.

The iron-losses will now be calculated, and to do this, the best method of procedure is to fill in a short table as shown below. Thickness of plates has been taken at 20 mils.

Armature.	Maximum B.	Loss per cub. inch at 50∼.		Total loss per cub. inch.	Volume of iron.	Total loss.
		In hysteresis.	In eddy-currents.			
Teeth .	110,500	0·95	0·05	1·0	4,760	4,760
Core. .	272,00	0·125	0·01	0·135	46,400	6,260

Total iron-loss in machine = 11,020

Copper-losses.

(1) The armature—

From the drawing the mean length of one armature-turn is found to be 67 inches.

There are 192 turns per phase.

Therefore total length per phase $= \frac{67 \times 192}{12} = 1072$ feet.

The section of the armature conductor is

$$2 \times \pi \times \frac{(0 \cdot 222)^2}{4} = 0 \cdot 0773 \text{ sq. inch.}$$

From wire gauge tables a wire, whose section is 0·824 sq. inch, has a resistance per 1000 feet at 60° C. of 0·115 ohm.

.·. Resistance of armature per phase

$$= 0 \cdot 115 \times 1 \cdot 072 \times \frac{824}{773}$$

$$= 0 \cdot 1315 \text{ ohm.}$$

The loss at full-load in the armature per phase is, therefore,

$$(200)^2 \times 0 \cdot 1315 = 5 \cdot 2 \text{ KW.}$$

.·. Total copper-loss in the armature = 15·6 KW.

(2) The field-magnets—

The coils are oval-shaped, and from the drawings the greatest and least diameters of the mean turn are 10 inches and 6½ inches.

This gives the length of the mean turn as 27 inches.

.·. Total length of field-magnet conductor

$$= \frac{27}{12} \times 50 \times 64 = 7200 \text{ feet.}$$

The section of the strip conductor is

$$4 \cdot 18 \times 0 \cdot 0315 = 0 \cdot 132 \text{ sq. inch.}$$

From the B. and S. wire table we find that the resistance at 60° C. of 1000 feet of a wire having this cross-section is 0·0718 ohm.

∴ Resistance of field-magnet coils

$$0{\cdot}0718 \times 7{\cdot}2 = 0{\cdot}516 \text{ ohm.}$$

This resistance is increased to 0·7 by the insertion of the regulating resistance, and the excitation loss will therefore be about

$$(130)^2 \times 0{\cdot}7 = 11{\cdot}9 \text{ KW at no-load,}$$

and will be a little higher (about 13·5 KW) at full-load.

The *total* losses are made up as follows :—

Iron-losses . . .	11·0	kilowatts.
Armature copper-losses .	15·6	„
Field copper-losses . .	13·5	„
Total full-load losses (less windage and friction) . .	= 40·1	„

∴ Total electrical efficiency

$$\frac{720}{720 + 40{\cdot}1} = 94{\cdot}7 \text{ per cent.}$$

Example III.—A HIGH-SPEED SINGLE-PHASE MACHINE, by Johnson and Phillips (Fig. 337).

General Specification :—

ASB, 24—250—375 ; 2200—114—75∼.

DIMENSIONAL DATA (inch units).

Armature :—

d	Inside diameter of stampings . . .	84
	Outside „ „ . . .	96
l	Length between core-heads	23
	Ventilation ducts	none
l_i	Net iron length of core	10·8
	Total number of slots	240
	Slots per pole wound	6
	Dimensions of slot	$1\frac{5}{8} \times \frac{3}{8}$
	Slot lining (micanite tube)	$\frac{1}{16}$
	Arrangement of conductors in slots . .	Four slots with three conductors, and two slots with two conductors.

Z_1	Total number of conductors	384
	Dimension of conductor (bar)	0·30 × 0·165
	Thickness of braiding on conductor	0·020
	Depth of iron behind teeth	$4\frac{5}{8}$
δ	Length of air-gap	$\frac{3}{16}$

Magnet-Wheel:—

d_m	Diameter over face of magnets	$83\frac{3}{8}$
b	Length of pole-arc	$6\frac{3}{4}$
l_m	Length of pole-face parallel to shaft	$11\frac{1}{2}$
	Thickness of pole-shoe (laminated)	$1\frac{15}{32}$
	Width of pole-core	$5\frac{1}{4}$
	Length of pole-core parallel to shaft	12
	Length of pole-core radially	683
	External diameter of fly-wheel	67
	Internal ,, ,,	$58\frac{1}{2}$
	Length parallel to shaft	15
S_m	Number of turns per magnet-coil	120
	Dimensions of magnet-wire	0·25 × 0·15

CALCULATED DATA AND COEFFICIENTS.

τ	Pole-pitch (inches)	11·0
v	Peripheral speed (feet per minute)	8250
β	Steinmetz coefficient (dl + KVA)	4·04
ξ	Esson coefficient ($d^2 l$ × RPM + KVA)	127,300
ψ	Ratio of pole-arc to pole-pitch	0·61
a	Current density in armature-conductors	2300
a_m	,, ,, field-conductors (no-load)	720
q	Ampere-conductors per inch on armature	166
	Form factor of armature-winding	0·88
	Flux per pole (megalines)	3·9
	Average magnetic density on pole-face	50,400
	Average magnetic density in teeth	81,700
	Magnetic density in armature-core	39,200
	Estimated no-load exciting current	27·0
	Part of exciting current required for air-gap	24·1

This machine was designed to fulfil a rather stringent specification with regard to temperature rise and pressure drop, with the result that the rating might be made 50 per cent. higher without the heating being excessive.

On a ten-hour test at an average full-load of 260 KW, the temperature rises were 42° F. on the armature core, and 32° F. on the end connexions. The rise in volts in switching off a full non-inductive current of 114 amperes was 69; this gives a regulation of 3·1 per cent.

It will be noted that there is a peculiarity in the winding of this armature; it has ten slots per pole but four are left empty, and the numbers of conductors in the others are not alike.

The machine is in service in Hong-Kong.

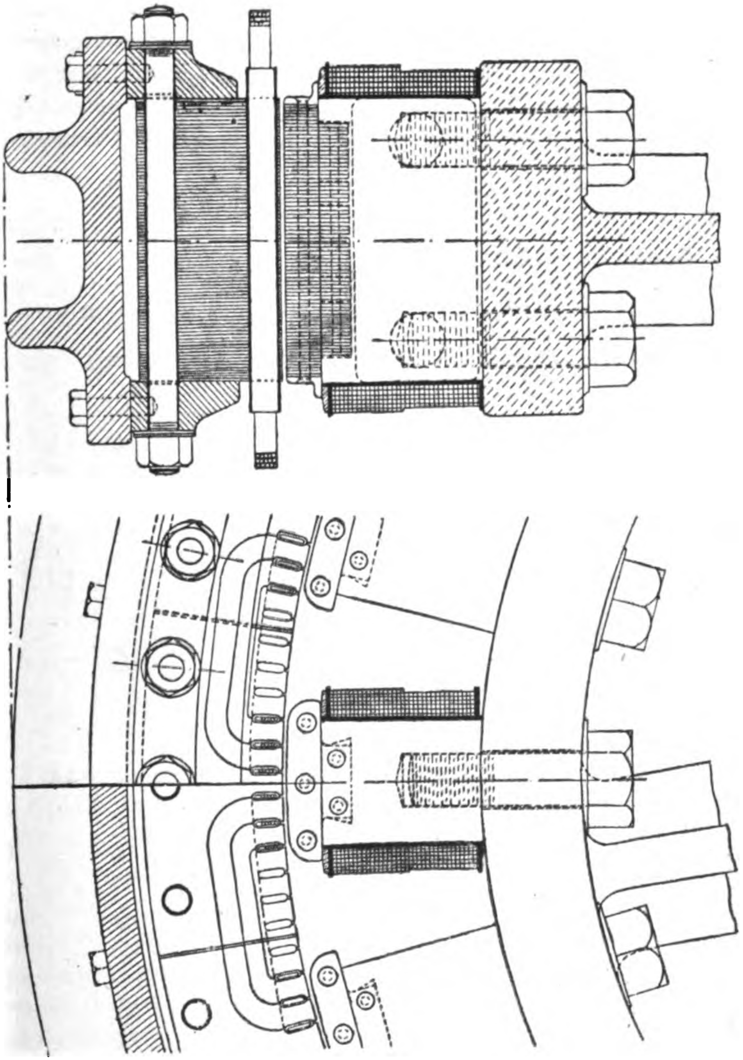


Fig. 337.—Single-Phase Alternator of Johnson and Phillips, AS(B) 24 - 250—375.

Examples IV. and V.—A SINGLE-PHASE GENERATOR, AND A TWO-PHASE GENERATOR OF SIMILAR MECHANICAL DESIGN (Witting, Eborall and Co.).

Although these machines are different in size, rated output, speed, and frequency, their mechanical design is in general quite similar, and therefore drawings of the second one only have been given; Fig. 338 showing the general construction, and Fig. 339 giving details of the windings on both armature and magnet-wheel.

General Specifications.

I. ASB 14—100—770; 2400 volts—41·5 amp.—90∼.

II. AQB 60—625—100; 7500 volts—2 × 41·5 amp.—50∼

DIMENSIONAL DATA (inch units).

		Single Phaser.	Two Phaser.
Armature :—			
d	Diameter at face	36·3	177·5
l	Length between core-heads	12·2	11·4
	Number of ventilating ducts	3	3
	Width of each duct	0·4	0·4
l_i	Nett iron length of core	9·9	9·2
	Slots per pole per phase	6	2
	Slots per pole per phase actually filled	4	2
	Total number of slots	84	240
	Width of slot	0·71	1·0
	Depth of slot	1·18	1·85
	Conductors per slot	12	20
Z_1	Number of conductors per phase	672	2,00
	Diameter of conductor { bare	0·158	0·165
	Diameter of conductor { covered	0·177	0·189
	External diameter of core	49·6	193·25
	Depth of iron behind teeth	5·5	6·0
δ	Length of air-gap	0·2	0·24

		Single Phaser.	Two Phaser.
Magnet-Wheel:—			
d_m	Diameter over face of magnets	35·9	177
b	Length of pole-arc	4·15	5·13
l_m	Length of pole-face parallel to shaft	11·8	10·25
	Thickness of pole-shoe	0·5	0·6
	Width of pole-core	2·52	4·33
	Length of pole-core parallel to shaft	10·65	9·05
	Length of pole-core radially	4·25	7·25
	External diameter of fly-wheel	26·3	161·3
	Estimated magnetic section of fly-wheel	—	159
S_m	Number of turns on magnet-coil	84	142
	Dimensions of magnet-wire { bare	$(0\cdot197)^2$	$(0\cdot213)^2$
	Dimensions of magnet-wire { covered	$(0\cdot228)^2$	$(0\cdot244)^2$

CALCULATED DATA AND COEFFICIENTS.

τ	Pole-pitch (inches)	8·14	9·28
v	Peripheral speed (feet per minute)	7320	4640
β	Steinmetz coefficient ($d\,l$ + KVA)	4·42	3·25
ξ	Esson coefficient ($d^2\,l \times$ RPM + KVA)	123,500	57,500
q	Ampere-conductors per inch on armature	245	362
a	Current density in armature conductors	2120	2020
a_m	Current density in field conductors (no-load)	1055	680
ψ	Ratio of pole-arc to pole-pitch	0·51	0·55
N	Flux per pole (megalines)	1·89	2·66
B_g	Average pole-face density	42,000	50,700
	Average tooth density	89,200	93,700
	Po'e-core density	—	78,000
	Armature-core density	17,300	32,400
	Estimated no-load exciting current	41	31

The chief mechanical features of these machines are a simple box casting for the armature housing, solid steel poles of oval section with a solid steel rectangular pole-shoe and the use of square wire on the magnet-bobbins. Also behind the pole-shoes substantial damping-rings of cast copper have been fitted to facilitate the steady running of the machines. This latter quality is also provided for by the use of a heavy fly-wheel (Fig. 338), which brings the coefficient of cyclic irregularity (see p. 524) down to $\frac{1}{300}$.

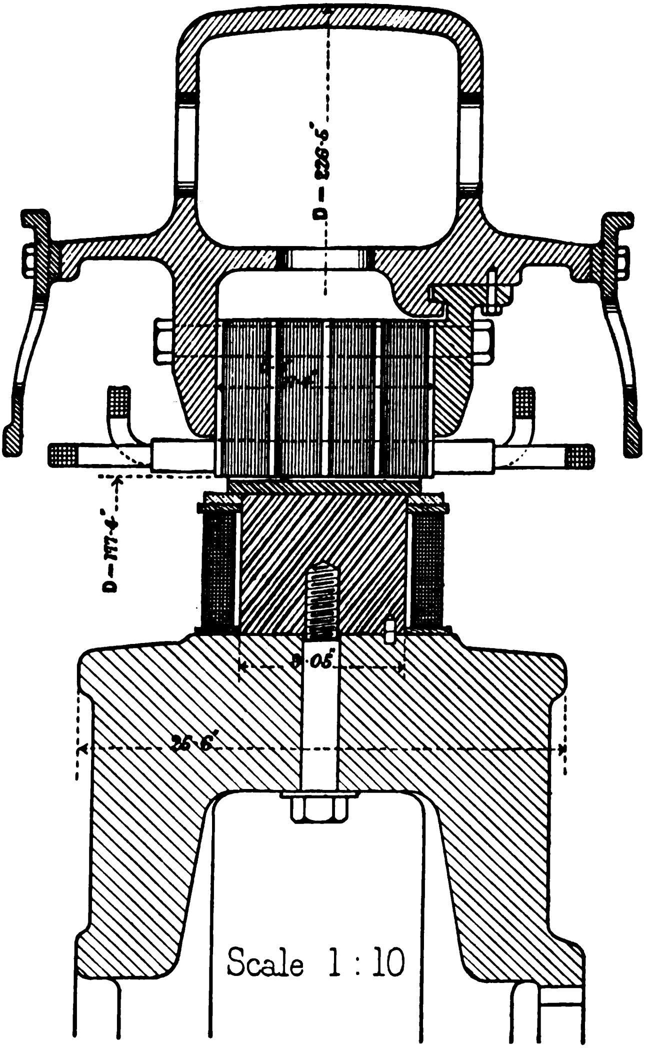


FIG. 338.—SECTION OF TWO-PHASE ALTERNATOR AQB 60—625—100 (WITTING, EBORALL & CO.).

Examples VI. and VII.—THE COMPARATIVE ANALYSIS OF TWO THREE-PHASE GENERATORS OF HIGH AND LOW SPEEDS.

In Fig. 127 on p. 144, sections have been given of a small high-speed machine of the Oerlikon Co. We will now proceed to analyse this machine side by side with another Oerlikon machine, giving about the same output at a similar voltage, but running at a much slower speed.

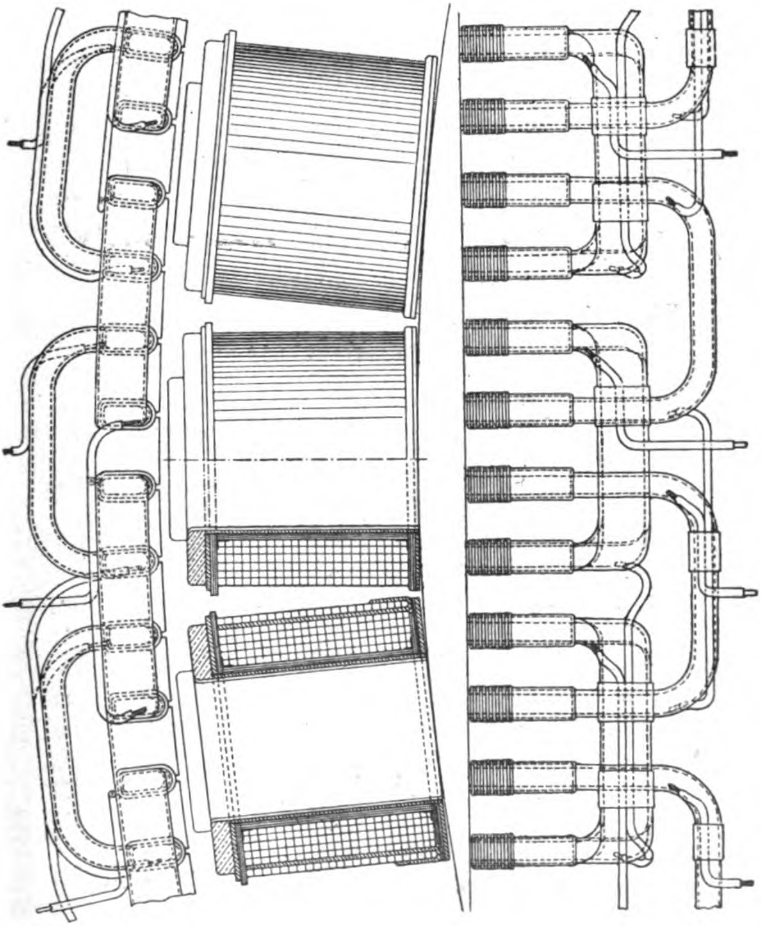


FIG. 339.—ARMATURE-WINDINGS AND POLES OF AQB 60—625—100.

The general specifications of the machines are:—

(*a*) $\text{ATB } 10_{\text{P}}\text{—}290_{\text{KVA}}\text{—}600_{\text{RPM}}\text{; } \frac{2080}{3600}{}_{\text{V}}\text{—}3 \times 46{\cdot}5_{\text{A}}\text{—}50 \sim$

(*b*) $\text{ATB } 48\text{ —}270_{\text{KVA}}\text{—}125_{\text{RPM}}\text{; } \frac{2890}{5000}{}_{\text{V}}\text{—}3 \times 31_{\text{A}}\text{—}50 \sim$

DIMENSIONAL DATA (inch units).

		(*a*)	(*b*)
Armature :—			
d	Diameter at face	35·5	157·7
	External diameter of core-discs	48·5	169·5
l	Length between core-heads	15·8	6·7
	Number of ventilating ducts	4	—
	Width of each duct	¼	—
li	Iron length of core	13·2	6·0
	Total number of slots	60	288
	Slots per pole per phase	2	2
	Dimensions of slot	1·97×0·79	1·58×0·79
	Width of tooth at face	1·07	0·92
	,, ,, at root	1·27	0·95
	Number of conductors per slot	18	17
Z_1	Total number of conductors per phase	360	1636
	Dimensions of bare conductor, 2 wires in parallel	$d = 0{\cdot}118$	$d = 0{\cdot}15$
	,, insulated conductor	$d = 0{\cdot}134$	$d = 0{\cdot}165$
	Depth of core behind slot	4·5	4·4
Field-Magnets :—			
δ	Length of air-gap	0·1	0·1
d_m	Diameter over face of magnet-wheel	35·3	157·5
b	Length of pole-arc	7·1	6·5
l_m	Length of pole parallel to shaft	15·8	6·3
	Width of pole-core	4·3	4·3
	Radial depth from pole-face to yoke	6·7	6·9
	Diameter of conductor on bobbin { bare	0·217	0·335
	Diameter of conductor on bobbin { covered	0·237	0·355
	Number of turns per bobbin	135	68
Yoke :—			
	Outside diameter of rim	21·9	143·7
	Inside diameter of rim	16·4	—
	Width of rim parallel to shaft	15·8	22·5

CALCULATED DATA AND COEFFICIENTS.

		(*a*)	(*b*)
τ	Pole-pitch (inches)	11·15	10·30
ψ	Ratio of pole-arc to pole-pitch	0·62	0·63
v	Peripheral speed (feet per minute)	5570	5150
β	Steinmetz coefficient ($d\,l$ ÷ KVA)	1·94	3·92
ξ	Esson coefficient ($d^2\,l$ × RPM ÷ KVA)	41,200	77,000
q	Specific loading factor (ampere-conductors per inch)	452	307
B_g	Average flux-density in the gap	48,700	43,600
	Current densities (amperes per square inch)—		
a	(1) Armature coils	2120	1750
a_m	(2) Field-magnet coils	405	272
	Slot space factor $\left(\frac{\text{copper section}}{\text{total slot section}}\right)$	0 25	0·24

MAGNETIC DATA.

E_1	No-load voltage per phase	2080	2890
N_i	Armature-flux at no-load (megalines)	5·30	1·70
ψ	Dispersion-coefficient taken as	1·20	1·20
N_m	Flux in magnet system (no-load) (megalines)	6·35	2·04
B_g	Average flux-density in air-gap (no-load)	48,700	43,600
B_t	Flux-density in teeth (no-load)	95,000	62,000
	,, ,, armature core (no-load)	44,700	33,000
	,, ,, field-magnet core (no-load)	104,000	83,500
	,, ,, yoke	77,500	(20,400)
	,, ,, pole-shoe (comb type)	—	107,500
	Total ampere-turns per pole (no-load) estimated at	2050	1600
	Estimated exciting current (no-load)	15	24
	Rating of exciter (volts × amperes)	100 × 60	100 × 103

CALCULATION OF LOSSES.

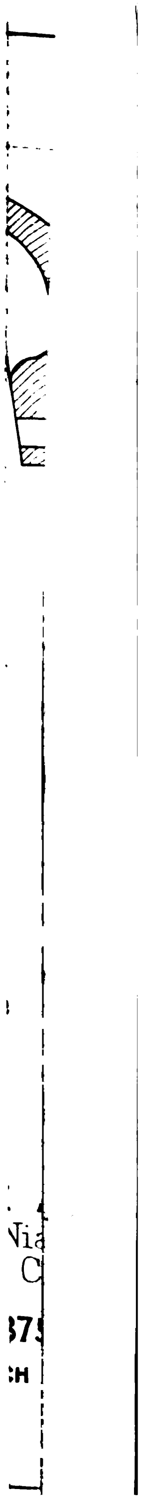
1. *Copper-Losses.* (*a*) *Armature*:—

Mean length of one armature-turn (inches)	79	51
Total number of armature-turns per phase	180	818
Total length of wire in one phase (feet)	1185	3480
Cross section of armature conductor (sq. inch)	0·0218	0·0177
Resistance per phase	0·514	1·15
Ohmic loss in armature (kilowatts)	3·33	3·31

	(*a*)	(*b*)
1. *Copper-Losses.* (*b*) *Field-Magnets*:—		
Mean length of one turn on magnet-coils (inches)	48	32·5
Total number of turns on all bobbins . .	1354	3264
Total length of field-magnet wire (feet) . .	5420	8850
Cross section of field-magnet wire (sq. inch) .	0·037	0·088
Total resistance of field-magnet winding . .	1·38	0·945
Losses in excitation at no-load	1·5	2·4
,, ,, full-load (approx.) . .	(2·5)	(3·5)
2. *Iron-Losses*:—		
Flux-density in armature-core	44,700	33,000
,, ,, teeth	95,000	62,000
Loss per cub. inch in eddies and hysteresis in core	0·25	0 19
,, ,, ,, ,, teeth	1·00	0·47
Volume of core (cub. inches)	8270	13,400
,, teeth (cub. inches)	1820	2540
Total loss in core (kilowatts)	2·07	2·50
,, teeth (kilowatts)	1·82	1·19
Total iron-losses (kilowatts)	3·89	3·69
Armature copper-losses (full-load) . . .	3·33	3·31
Field copper-losses (full-load)	2·5	3·5
Iron-losses (full-load)	3·89	3·69
Total losses (full-load)	9·7	10·5
Full-load electrical efficiency, per cent. . .	96·6	96·3

It is interesting to note that the full-load electrical efficiencies of these machines come out in the end about the same, because the smaller sized machine can be worked harder in both the copper and the iron parts. The reason of this, is that with the higher speed of rotation, ventilation is much more complete, and so a larger amount of heat can be dissipated from each cubic inch of working material.

Attention may be drawn to the pole-faces of the machine Fig. 340, in which every alternate lamina is cut about 1 cm. shorter at the face than the rest, so that the protruding ridges become well saturated. This is known as a *comb*-pole.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camples VIII. and IX.—THE NIAGARA GENERATORS.

the first Niagara station, which was put down in 1895, were installed two different designs of the overhung ella type of machine. These will be found fully described e author's work, *Polyphase Electric Currents* (2nd edition,

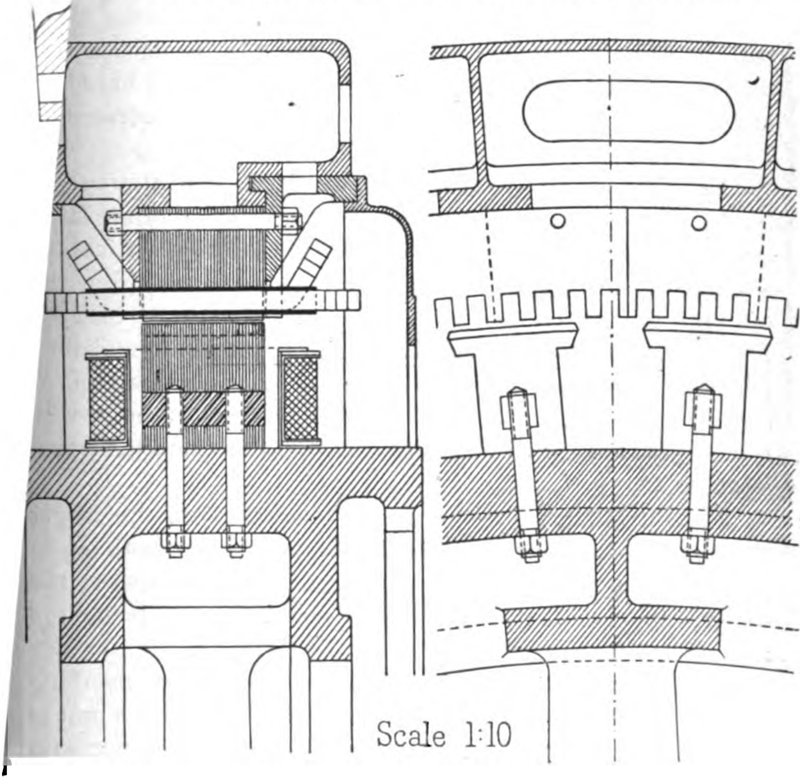


FIG. 340.—THREE-PHASE SLOW-SPEED ALTERNATOR ATB 48—270—125, BY THE OERLIKON MACHINE WORKS.

1900), and a section of the earlier type of machines has already been given in this book on Fig. 94, p. 108.

In 1901, the growth of the system necessitated the laying down of an entirely new and separate station, which, while it works in parallel with the old station, embodies many new features consistent with the advance electrical engineering has made during the few intervening years.

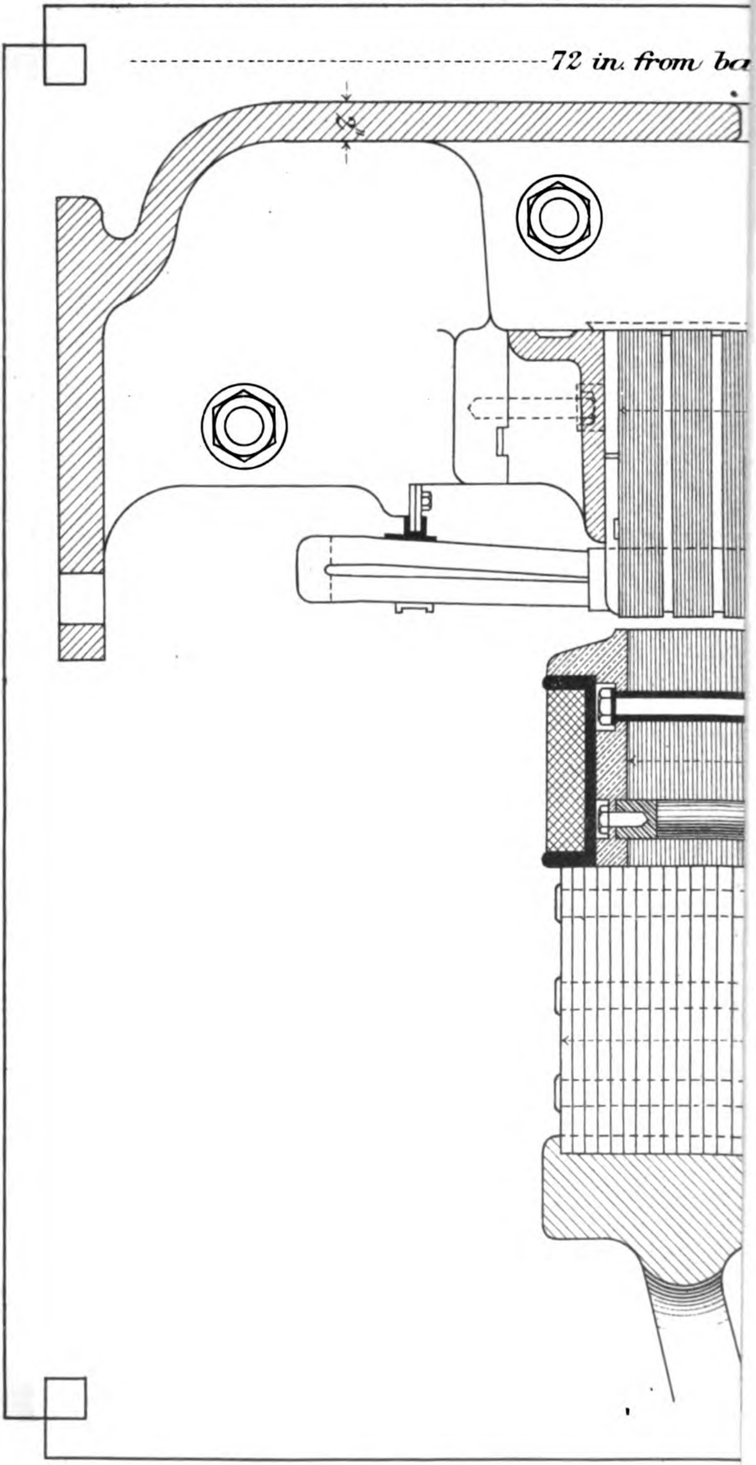
The generator equipment of the new power house was undertaken by the General Electric Co., of Schenectady, and consisted primarily of six 5000 horse-power sets of the same "O" type as had previously been installed by the Westinghouse Co. The remaining five units are, however, of the more usual "B" type with internal revolving field, this type being adopted because of the better accessibility and greater ease of handling and erection. This latter type of generator is illustrated in Plate X.

Anticipating, however, still further increase in the use of Niagara power, the company has now erected another large station on the Canadian side of the falls. The electrical arrangements in this station are somewhat different from the others. The generators have each a rated capacity of 7500 KW, *i.e.* just twice the capacity of the former unit, and they generate three-phase current at 11,000 volts, instead of, as formerly, two-phase at 2200 volts; the two stations will, however, be mutually interconnected by Scott-connected transformers.

The advantages accruing from the adoption of this larger unit are the lower relative cost of the equipment, both as to generator and turbine, as well as to economy in space; the outside dimensions of the old and new machines being 178 and 205 inches respectively.

It should be remembered that the reason for installing what would now be considered so comparatively small a unit as 3750 KW was that in 1895 there was no experience with units anywhere near even that capacity; and that in connection with the question of breakdowns, as the present capacity of the Niagara stations is about 200,000 horse-power, a unit of 10,000 horse-power out of service will cause no inconvenience at all. Plate XI. gives sectional views of this 7500 KW, 10,000 volt generator.

We will now proceed to analyse the design of these two latest types of Niagara machines, side by side.


72 in. from ba
6″

General Specifications :—

(*a*) $\mathrm{AQ(B)}\ 12_{\mathrm{P}}—3750_{\mathrm{KW}}—250_{\mathrm{RPM}}\ ;\ 2200_{\mathrm{V}}—2 \times 850_{\mathrm{A}}—25\sim$

(*b*) $\mathrm{AT(B)}\ 12_{\mathrm{P}}—7500_{\mathrm{KW}}—250_{\mathrm{RPM}}\ ;\ \frac{12000}{6940}{}_{\mathrm{V}}—3 \times 360_{\mathrm{A}}—25\sim$

PRINCIPAL DIMENSIONAL DATA (inch units).

		(*a*)	(*b*)
Armature :—			
d	Diameter at face	132	150
	Outside diameter of core-discs	160	180
l	Length between core-heads	24	48
	Thickness of armature-stampings	0·014	0·014
	Number of ventilation-ducts	8	16
	Width of each duct	$\frac{1}{2}$	$\frac{1}{2}$
li	Iron length of core	18	36
	Total number of slots	312	180
	Slots per pole and phase	13	5
	Dimensions of slots	$2\frac{1}{16} \times 0{\cdot}6$	$3\frac{5}{8} \times 1\frac{3}{8}$
	Width of tooth, at face	0·75	1·37
	,, ,, at root	0·71	1·24
	Conductors per slot	2 (in parallel)	4
Z_1	Total number of conductors per phase	156	240
	Dimensions of bare conductor	$0{\cdot}75 \times 0{\cdot}4$	$1{\cdot}0 \times 0{\cdot}3$
	Depth of core behind slot	$11\frac{15}{16}$	$11\frac{5}{8}$
Gap :—			
δ	Length at centre of pole	$\frac{5}{8}$	$\frac{3}{4}$
	,, pole corners	about $\frac{1}{4}$	about $1\frac{1}{2}$
Field-Magnets :—			
d_m	Diameter over face	$130\frac{3}{4}$	$148\frac{1}{2}$
	Radius of pole-face	42	45
b	Length of pole-arc	22	27·9
l_m	Length of pole parallel to shaft	$23\frac{1}{2}$	47
	Radial depth from pole-face to yoke	$12\frac{5}{8}$	$12\frac{1}{2}$
	Width of pole-core	17	16
Yoke (*constituting fly-wheel*) :—			
	Outside Measurement	$105\frac{1}{2}$	$123\frac{1}{2}$
	Inside diameter	82	$95\frac{1}{2}$
	Thickness, radially	$11\frac{3}{4}$	14
	Length parallel to shaft	28	53

CALCULATED DATA AND COEFFICIENTS.

		(a)	(b)
τ	Pole-pitch (inches)	34·5	39·2
ψ	Ratio of pole-arc to pole-pitch	0·64	0·71
v	Peripheral speed (feet per minute)	8620	9800
β	Steinmetz coefficient	0·88	0·96
ξ	Esson coefficient	29,000	36,000
q	Ampere-conductors per inch	645	500
B_g	Average magnetic density in gap	56,500	44,000
Current-density (amperes per sq. inch) :—			
a	(1) Armature coils	1600	1200
a_m	(2) Field-magnet coils	1550	
	Space-factor (copper section ÷ total section of slot)	0·43	0·24

MAGNETIC DATA.

V_1	No-load voltage per phase	2200	6940
	Armature flux at no-load (megalines)	24·8	52·0
N	Average flux-density in air-gap (no-load)	56,500	44,000
B_g	Flux-density in teeth	123,000	100,000
	,, ,, armature-core	67,000	63,000
	,, ,, magnet pole-core	97,000	84,000
	,, ,, yoke	44,003	42,000
δ	Mean length of air-gap taken as	¾-inch	1⅛-inch
	Total ampere-turns per pole (no-load)	14,700	16,000

Mainly because of the high peripheral speeds they attain, these two machines, which, as will be seen from the plates, are built on the same plan, possess many interesting features in mechanical construction.

The armatures are built up of mild steel stampings, 14 mils in thickness, dove-tailed into a cast-iron outside frame or housing, each stamping spanning round $\frac{1}{12}$ of the complete periphery. Special provision has been made for ventilation, spaces of half an inch being left every three inches and at other intervals of half an inch several laminæ of red paper have been inserted. For convenience of casting, the end-plates at both ends of the core have been made separate, the methods of attachment being clearly shown in the drawings. The windings of both machines are of the bar type and have already been discussed in Chap. VI., p. 324, Figs. 294 and 295, and giving diagrams in which for simplicity the three-phase type is shown for two slots per pole per phase with two conductors per slot instead of a five-slot winding with four conductors per slot and the two-phase type is shown as a three-slot winding instead of one with thirteen slots.

Both windings are, of course, in two layers, with the interesting feature that while the lower conductors are of solid copper bar, the top conductors are of stranded cable pressed to the same shape and dimensions as the copper rod below it in the bottom of the slot. The diminution in actual copper section in the stranded cable is about 23 per cent. The reason for using stranded cable is to diminish the eddy-currents which are set up in large conductors, and it has long been used, having been introduced twenty years ago by Crompton; but it becomes a question as to whether the eddy-current loss obviated by stranding the conductor is not counterbalanced by the extra ohmic loss occasioned by the diminished nett cross-section.

Because of the very large pole-pitch, the end-connexions are lengthy, and so supports from the end-plate have been provided for holding them securely in position.

The revolving field-system consists essentially of three parts, viz.:—

(*a*) A cast steel six-armed spider forced on to a carbon steel shaft; (*b*) a fly-wheel built up on to the spider which, of course, also acts as the yoke of the magnetic circuit; and (*c*) the field-magnet poles built up on to the fly-wheel. By this means a very sound structure is developed, and all risks of flows due to unequal shrinking in a large cast rim, whether cast in one piece with, or separate from, the spider, are avoided. The laminated ring is also cheaper to build up than a large forging.

In the case of the 3750 KW set the shaft diameter at the alternator is 14 inches, and the diameter over the boss of the spider is 24 inches. In the 7500 KW set, these dimensions are 16 inches and 27 inches respectively.

The rim consists of plates, $\frac{1}{8}$-inch in thickness, which are built up to a ridge at the end of the spider arms, and are firmly secured by numerous rivets. The first few plates are slightly longer radially than the others, thus forming a ridge to which the poles may be built up. The pole-cores consist of T-shaped stampings, also $\frac{1}{8}$-inch in thickness, and are secured to the fly-wheel by two keyed dovetail joints, this arrangement allowing the poles to be assembled separately. For, as the overhanging pole-tips provide the necessary good mechanical supports for the magnetizing coils at this high peripheral speed, the coils can only be put on from the rear, and therefore before the pole is attached to the rim.

The pole face is curved away from the armature face by an amount designed to give approximately a sine distribution of the flux in the gap. As the specific current loadings of the armatures are fairly high, fairly wide gaps have been provided in order that the armature leakage and distortion, and therefore the armature self-induction, shall not be too great. Here again, the unequal air-gap is an advantage ; for the flux as a whole will tend to keep in the centre of the pole, *i e.* in the neighbourhood of the smallest gap, and distortion will therefore be lessened.

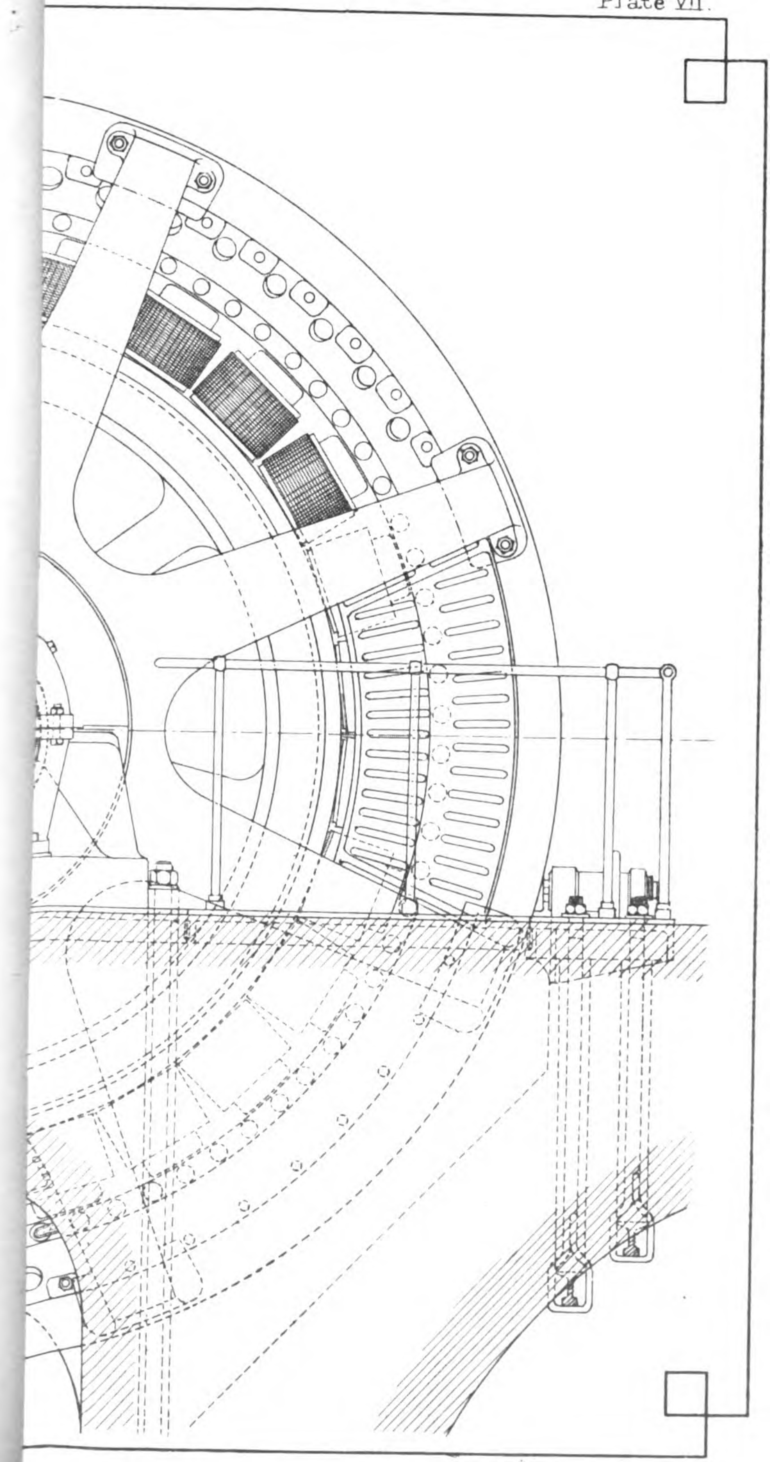
The weights of the magnet-poles and rims are about 20 and 49 tons respectively. The spider arms which support these weights horizontally, are of I sections with outside dimensions of 24 inches by $8\frac{1}{2}$ inches in the case of the small machine, and 37 inches by 11 inches in the larger machine. The average sections of these spokes are 70 square inches and 135 square inches respectively.

Example X.—A THREE-PHASE GENERATOR BY BROWN BOVERI AND CO., six of which are installed at Paderno (Plate VII.).

General Specification.

$$\mathrm{AT(B)}\ 28_{\mathrm{P}}\text{—}1640_{\mathrm{KVA}}\text{—}180_{\mathrm{RPM}}\ ;\ \left\{\frac{13500\text{ to}}{14500_{\mathrm{V}}}\right\}\text{—}3 \times 79_{\mathrm{A}}\text{—}42\sim$$

$$(\cos\phi = 0\cdot 8)$$



Thos Kell & Son Lith

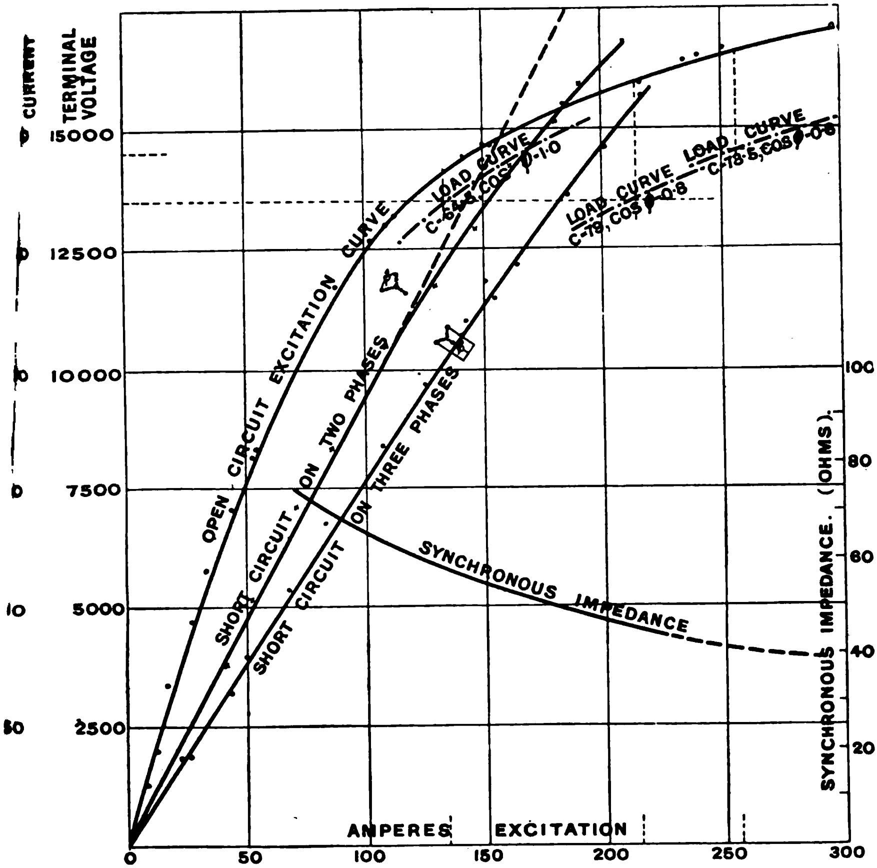


FIG. 341.—SATURATION AND LOAD-CURVES OF ATB 28—1640—180.

DIMENSIONAL DATA (in inch units).

Armature:—

d	Diameter at face	161·7
l	Length between core-heads	21·7
	External diameter	181·5
	Number of ventilating ducts	7
	Width of each duct	0·3
l_i	Iron length of core	17·5
	Total number of slots	84
	Slots per pole and phase	1
	Diameter of slot (circular)	2·56 (6·5 mm.)
	Conductors per slot	30

	Diameter of armature-conductor . . .	$\frac{0·236}{0·276}$
Z_1	Total number of armature-conductors (per phase)	840
δ	Length of air-gap	0·394

Field-Magnets :—

	Diameter over pole-faces	160 9
	Thickness of pole-shoe (laminated) . .	3·0
b	Pole-arc	8·5
l_m	Length of pole-shoe parallel to shaft . .	20·5
	Diameter of pole-core (circular) . . .	11·8
	Radial length of pole-core and shoe . .	14·2
S_m	Turns per field-magnet bobbin . .	73
	Size of field-magnet conductor (strip) . .	1·3 × 0·118
	External diameter of fly-wheel . . .	131·6
	Internal ,, ,, . . .	108
	Width of rim	18·5
	Estimated magnet section of fly-wheel (channel section)	76

CALCULATED DATA AND COEFFICIENTS.

τ	Pole-pitch (inches)	18·1
v	Peripheral speed (feet per minute) . .	7600
ψ	Ratio of pole-span to pole-pitch . . .	0·47
	corresponding to an electrical angle of .	43°
β	Steinmetz coefficient ($dl \div$ KVA) . .	2·14
ξ	Esson coefficient ($d^2l \times$ RPM $\div$ KVA) .	61,000
a	Current density in armature (amperes per sq. inch)	1810
a_m	Current density in magnets (amperes per sq. inch)	780
q	Ampere-conductors per inch of periphery .	392
B_g	Mean flux-density at pole-face . . .	57,300

Example XI.—A LARGE SLOW-SPEED GENERATOR, by Messrs. Kolben and Co., two examples of which were installed by Messrs. Witting, Eborall and Co. for the Metropolitan Electric Supply Co. in their power-house at Willesden, London. Fig. 342 gives a view of one of the sets, and Fig. 343 shows the excitation and short-circuit curves.

General Specification.

AQB 96—3500—75; 11500_V—2 × 152_A—60~

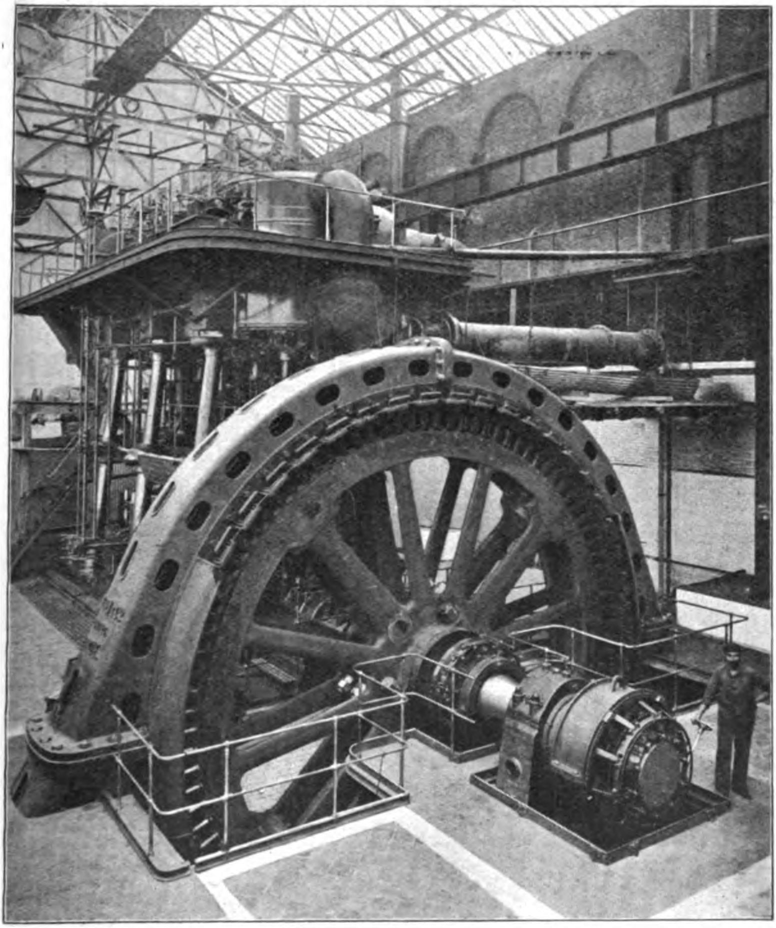


Fig. 342.—View of Slow-Speed Two-Phase Alternator, AQ(B) 96—3500—75, by Kolben & Co., with Sulzer Engine, at Willesden.

Dimensional Data (in inch units).

Armature:—

d	Diameter at face	300·5
l	Length between core-heads	29·2
	External diameter	314·6
	Number of ventilating ducts	11
	Width of each duct	0·4

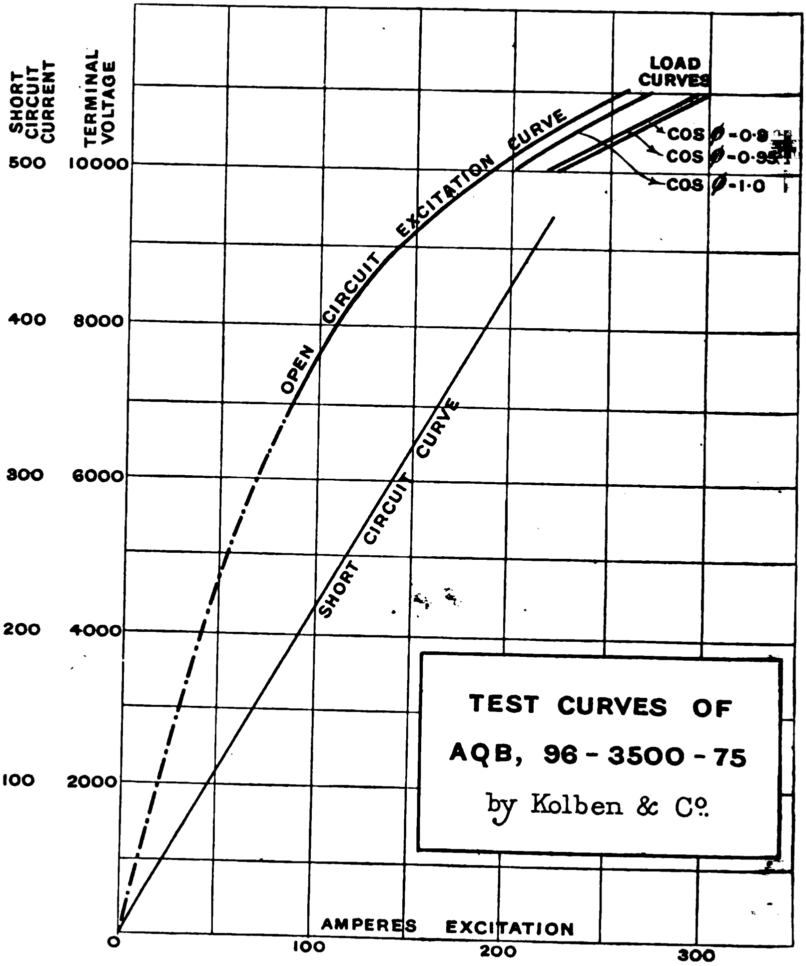
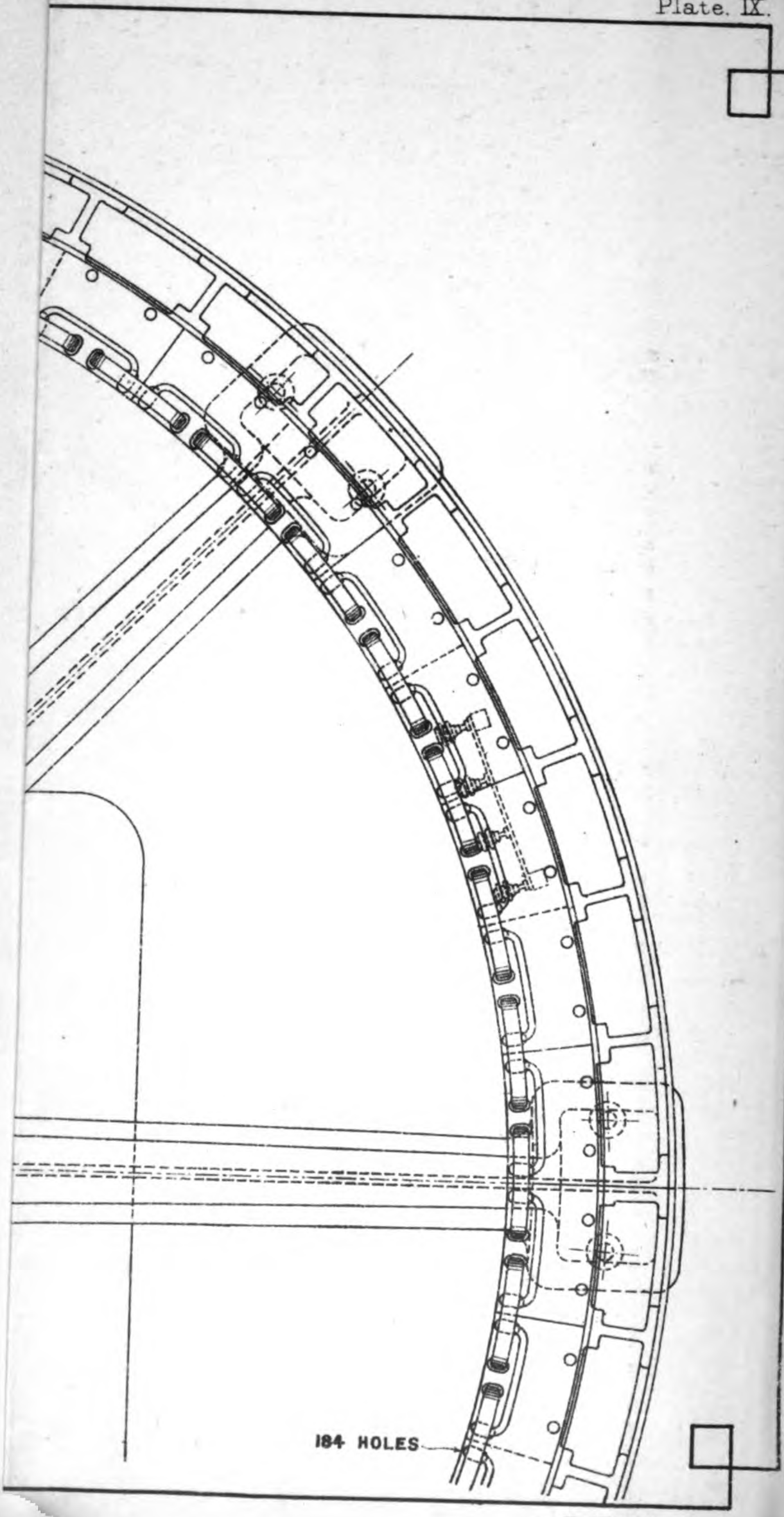


FIG. 343.—TEST-CURVES OF AQ(B), 96—3500—75.

	Total number of slots	576
	Slots per pole and phase	3
	Dimensions of slot	0·73 × 2·23
	Conductors per slot	3
Z_1	Total number of conductors per phase	1440
	Description of armature-conductors	2 bars in parallel 0·315 × 0·165
δ	Length of air-gap	0·43



Thoˢ Kell & Son, Lith

Field-Magnets:—

d_m	Diameter over pole-faces	299·6
l_m	Axial length of pole-shoe	29·2
	,, ,, pole-core	19·5
	Width of pole-core	4·88
b	Span of pole-shoe	6·54
S_m	Turns per field-magnet bobbin	33
	Dimensions of field-magnet conductor	0·22 × 0·985
	Thickness of pole-shoe (laminated)	3·65
	Radial depth of pole-core	9·3

CALCULATED DATA AND COEFFICIENTS.

τ	Pole-pitch (inches)	9·8
v	Peripheral speed (feet per minute)	5920
ψ	Ratio of pole-span to pole-pitch	0·67
	corresponding to an electrical angle of	60°
	Armature slot space-factor	0·32
β	Steinmetz coefficient ($d\,l$ ÷ KVA)	2·5
ξ	Esson coefficient ($d^2 l$ × RPM ÷ KVA)	56,300
q	Ampere-conductors per inch of periphery	463
B_g	Mean magnetic density on pole-face	36,700
a	Current density in armature-conductors at full-load	1470
N	Flux per pole (megalines) in gap	6·3
	Magnetic-density (B) in pole-core	70,000
	,, ,, ,, teeth	62,500
	,, ,, ,, armature-core	24,400
a_m	Current-density in magnet-coils at no-load	880

Example XII.—A LARGE TWO-PHASE WATER-TURBINE DRIVEN UMBRELLA-TYPE GENERATOR, by Brown, Boveri and Co., installed at Chèvres.

General Specification.

AQB 46_P—900_{KW}—120_{RPM} ; 5000_V—2 × 100_A—46∼

DIMENSIONAL DATA (inch units).

Armature:—

d	Inside diameter of stampings	197
	Outside ,, ,,	216
l	Length between core-heads	15·8
	Number of ventilation ducts	5
	Width of each duct	0·2
l_i	Net iron length of core	13·3
	Total number of slots	184
	Slots per pole per phase	2

	Dimensions of slot	1·18 × 2·36
	Conductors per slot	11
	Diameter of armature conductor { bare	0·276
	covered	0·314
Z_1	Total number of conductors per phase	1012
δ	Width of air-gap	0·24
Magnets :—		
d_m	Diameter over face of magnets	198·5
b	Length of pole-arc	7·8
l_m	Length of pole-face parallel to shaft	15·0
	Thickness of pole-shoe (solid)	1·9
	Diameter of pole-core	9·5
	Length of pole-core radially	11·0
	External diameter of fly-wheel rim	172·7
	Length of fly-wheel parallel to shaft	18·1
	Number of turns per magnet-coil	53

CALCULATED COEFFICIENTS AND DATA.

τ	Pole-pitch	13·44
v	Peripheral speed (feet per minute)	9190
ψ	Ratio of pole-arc to pole-pitch	0·58
	corresponding to an angle of	52°
β	Steinmetz coefficient ($dl \div$ KVA)	3·46
ξ	Esson coefficient ($d^2 l \times$ RPM $\div$ KVA)	81,800
a	Current density in armature (amperes per sq. inch)	1670
q	Ampere-conductors per inch of periphery	325
N	Flux per pole	5,400,000
B_g	Pole-face flux density (mean)	43,600

Six machines of this type, having total output equivalent to 7200 horse-power, are installed in the water-turbine station of the Geneva Electricity Supply Co., at Chèvres. The aspect of the machines installed in this power-house closely resembles that given in Fig. 85, p. 97.

Example XIII.—A SMALL HIGH-SPEED THREE-PHASE MACHINE, supplied by Brown, Boveri and Co. to the St. Petersburg Polytechnicum. For sectional sketches and general view of machine, see Figs. 164 and 165, pp. 179 and 180.

General Specification.

ATB 28—75—215 ; $\frac{133}{230}\mathrm{V}$—3 × 189_{A}—50∼

DIMENSIONAL DATA (inch units).

Armature:—

d	Inside diameter of stampings	57·3
	Outside ,, ,,	65·0
l	Length between core-heads	8·9
	Number of ventilation ducts	4
	Width of each duct	0·3
l_i	Net iron length of core	6·9
	Total numbers of slots	168
	Slots per pole and phase	2
	Dimensions of slot	0·51 × 1·69
	Conductors per slot	2
	Description of conductors	2 bars 0·12 × 0·59
Z_1	Total number of conductors per phase	112
	Depth of iron behind teeth	2·2
δ	Width of air-gap	0·2

Magnets:—

d_m	Diameter over face of magnets	56·9
b	Length of pole-arc	3·14
l_m	Length of pole-face parallel to shaft	8·65
	Thickness of pole-shoe (solid)	0·9
	Width of pole-core	2·16
	Length of pole-core parallel to shaft	7·5
	Length of pole-core radially	5·9
	External diameter of fly-wheel rim	43·3
	Internal ,, ,, ,,	39·2
	Length parallel to shaft	9·4
S_m	Number of turns per magnet-coil	370
	Diameter of magnet-coil conductor	0·10 × 0·12
	Exciting current at no-load, 230 volts, 7·2 amperes; at 260 volts, 9 amperes.	
	Short-circuit current at 7·2 amperes excitation, 490 amperes per phase.	

CALCULATED DATA AND COEFFICIENTS.

τ	Pole-pitch (inches)	6·43
v	Peripheral speed (feet per minute)	3380
β	Steinmetz coefficient ($d\,l \div$ KVA)	6·8
ξ	Esson coefficient ($d^2\,l \times$ RPM $\div$ KVA)	84,000
ψ	Ratio of pole-arc to pole-pitch	0·49
	corresponding to an electrical angle of	44°
a	Current-density in armature conductors	1300
q	Ampere-conductors per inch of periphery	210
N	Flux per pole (megalines)	1·07
B_g	Mean pole-face density	39,000

The interest of this machine is that on a small scale it gives all the main features and qualities of the B-type alternator. The peripheral speed and specific loading are lower than usual, with the result that the pole-pitch has gone down from the normal 10 inches to 6·43, and the Steinmetz coefficient is high. The simplicity of construction merits notice.

Example XIV.—A LARGE A-TYPE TWO-PHASE ALTERNATOR, by the British Westinghouse Co.

AQA 62—1500—116; 500_V—2 × 1500_A; 60~

These machines, five of which were supplied to the Willesden station of the Metropolitan Electric Co., are the largest machines with revolving armature in existence. Their field-magnet carcase is illustrated in Fig. 344. In consequence of the low voltage, the winding is unusual. In general appearance it resembles Fig. 155, being a distributed wave-winding, with one conductor per slot. The formula for a wave-winding being $Z = py \pm c$, we have the following values: $Z = 552$, $p = 62$, the winding step $y = 9$, $c = 6$. Hence, there are six paths through the winding, and as Z and c possess as a common factor 3, there will be 3 independent closed windings, each of which comprises 184 conductors. This triplex winding, with symbol O O O (see Vol. I. p. 369), therefore is a special type of combination. Each of the three closed windings will be connected to the four slip-rings at four equidistant points along the winding; and the volts from ring to ring will correspond to the electromotive-force created by 46 conductors in series. In a mesh-connected two-phase winding, the voltage from corner to corner of the mesh is (see p. 15) $\frac{1}{2}\sqrt{2}$, or 0·707 times that across either of the phases; or, if the voltage across each of the two phases is 500, that in the mesh is 353·6. If we increase this to 390 to allow for the drop at full-load, then the flux must be:—

$$N = \frac{390 \times 10^8}{2 \cdot 22 \times 60 \times 46} = 6{,}360{,}000.$$

Taking the pole-face area as 160 square inches, this makes

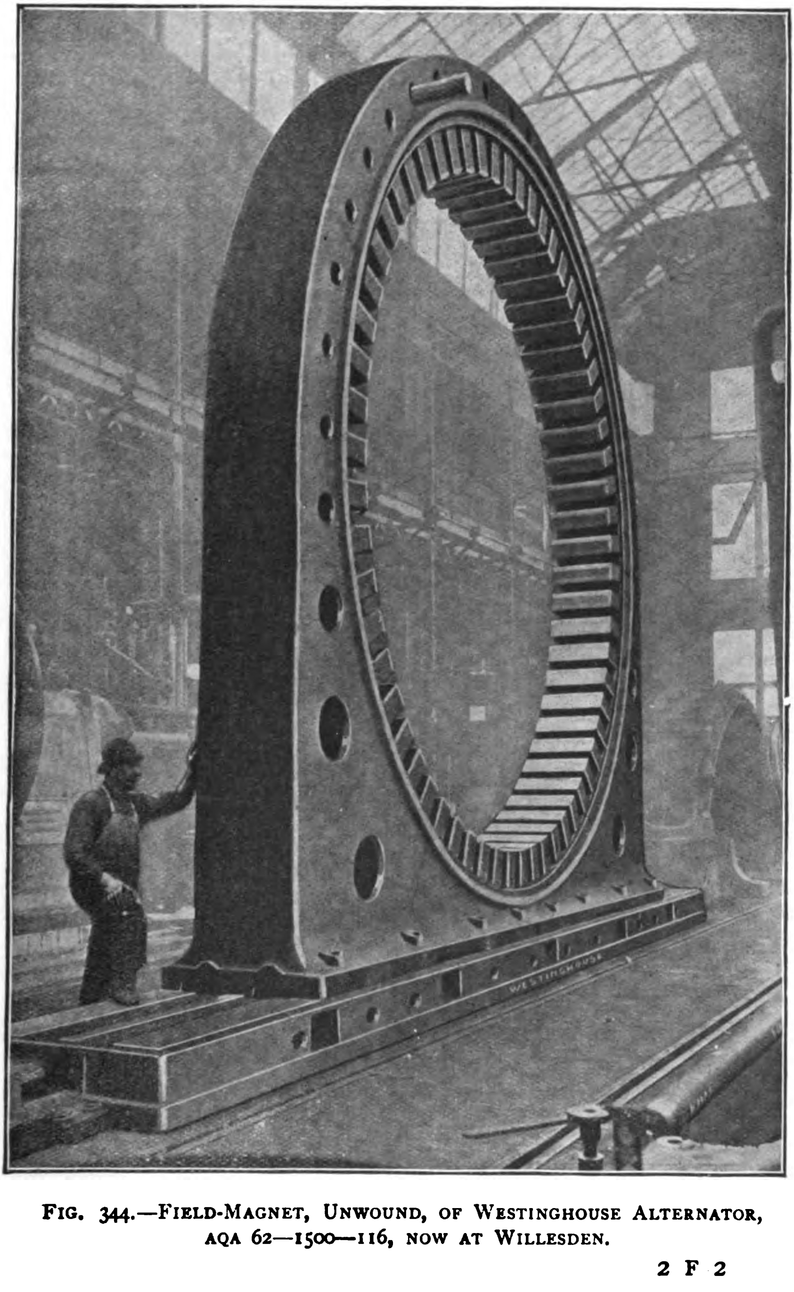


FIG. 344.—FIELD-MAGNET, UNWOUND, OF WESTINGHOUSE ALTERNATOR, AQA 62—1500—116, NOW AT WILLESDEN.

B_g just under 40,000. The current in one conductor will be $\sqrt{2}$ times $\frac{1500}{3}$ = 707 amperes. The principal dimensions of the machine are as follow:—

$d = 200$; $l = 24{\cdot}8$; $\tau = 10{\cdot}01$; $b = 6{\cdot}45$. Slot-pitch = 1·138; slot-width = 0·404, closed to a slit of 0·175 at face; slot-depth 1·18. The gap is 0·43. The pole-core, rectangular, is 24·8 × 6·45 in section; its length is 86·5. The armature-core possesses 8 air-ducts. The conductor, of which there is one in each slot, is 0·316 broad by 0·98 deep, having therefore a section of 0·31 square inches. The ampere-density $a = 2280$. We may deduce the following constants for the machine:—

$B = 39{,}700$; $q = 311$; $\psi = 0{\cdot}645$; $v = 6070$ feet per minute.

Fig. 344, which shows the magnet-frame of this machine while under construction, illustrates the numerous narrow poles which characterize alternators of the A-type.

In addition to the Examples I. to XIV. analysed above, examples of a number of machines of earlier types will be found in Chapter III., pp. 126 to 139; while turbo-alternators are described in Chapter IX., pp. 439 to 479.

In conclusion of these analyses we may collect together the various values of the constants which appear in the 16 machines now analysed. No steam-turbine alternators are included.

Machine XV. is the Ganz inductor described on p. 134, and No. XVI. is the small Kolben inductor described on p. 129.

Readers who desire to follow out other examples of alternator design, will find them in such treatises as Professor Arnold's *Wechselstrom Technik*, vol. iv., and in Professor Gisbert Kapp's work, *Elektromechanische Konstruktionen.* The volume of M. C. F. Guilbert on *The Generators of Electricity at the Paris Exhibition of* 1900, published at Paris in 1902, also contains a vast amount of descriptive and statistical data. There is also an exceedingly valuable and critical article by Herr Alexandre Rothert, entitled *Grands Alterna-*

TABLE X.—DESIGN COEFFICIENTS OF ALTERNATOR EXAMPLES.

—	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.	XIV.	XV.	XVI.
n_{ph}	3	3	1	1	2	3	3	2	3	3	2	2	3	2	1	3
p	72	64	24	14	60	48	10	12	12	28	96	46	28	62	60	8
KVA	1500	760	250	100	625	270	290	3750	7500	1640	3500	900	75	1500	440	37
RPM	83	94	375	770	100	125	600	250	250	180	75	120	215	116	100	630
B_g	58,500	58,200	50,400	42,000	50,700	48,700	43,600	56,500	44,000	57,300	36,700	43,600	39,000	40,000	65,000	44,000
q	594	312	166	245	362	452	307	645	550	392	463	325	210	622	267	400
ψ	0·655	0·51	0·61	0·51	0·55	0·62	0·63	0·64	0·71	0·47	0·67	0·58	0·49	0·645	0·81	0·78
v	5115	5800	8250	7320	4640	5570	5150	8620	9800	7600	9640	6190	3380	6070	4125	3625
β	1·42	3·1	4·04	4·42	3·25	1·94	3·92	0·88	0·96	2·14	2·5	3·46	6·8	3·31	7·06	7·27
ξ	27,800	68,800	127,300	123,500	57,500	41,200	77,000	29,000	36,000	61,000	56,300	81,800	84,000	76,800	111,000	101,000
a	1985	2600	2300	2120	2020	2120	1750	1600	1200	1810	1470	1670	1300	2280	810	1480
$b \div \delta$	24·5	19·0	36·0	20·7	21·4	71·0	65·0	35·2	37·2	21·5	15·2	32·5	15·7	15	35	69
f	50	50	75	90	50	50	50	25	25	42	60	46	50	60	50	42
τ	10·3	11·6	11·0	8·14	9·28	11·15	10·3	34·5	39·2	18·1	9·8	13·44	6·43	10·0	8·26	8·65

teurs, published in *l'Éclairage Électrique* of November 30 and December 6, 1901. In this article Herr Rothert analyses twenty-three different alternators varying in size from a small ATB 32—215—120 by Krizik up to ATB 40—5000—75 by the Westinghouse Co. They represent a great variety of constructional forms, but all, with one exception, are large slow-speed machines of the B-type.

CHAPTER IX.

STEAM-TURBINE ALTERNATORS.

WITH the introduction of steam-turbines there has come about so profound a modification in the design of alternators that a special chapter is devoted to machinery of this type. The dominating characteristic of the type is of course the very high speeds with which these machines run. In recent years, with the introduction of turbines of very great power—up to 10,000 H.P.—the actual speeds have indeed been reduced below those of the small turbines first introduced by Parsons and de Laval, but still they are much higher than those of reciprocating engines or of water-turbines of equal power, and the designs of the alternators differ accordingly. In the fifth edition (1896) of the present work there was indeed described a Parsons's turbo-alternator of 350 KW running at 3000 RPM; but in the construction both of the turbine and of the alternator the type there shown must be regarded as obsolete. The steam turbine itself offers certain advantages over reciprocating steam engines in respect of prime cost, simplicity of construction, lubrication, and economy of space; its relative efficiency in steam-consumption in comparison with the most efficient reciprocating engines is a matter of debate. As regards the turbo-alternator, the real advantage it offers over slower-speed alternators is bound up with the circumstance that in it a much higher surface-velocity is attainable than is practicable in the other types. For, as shown in the discussion in Chapter XVIII. of Volume I., upon the specific utilization of materials, it appears that, considering the cylindrical belt of active material around the armature periphery, consisting of the copper conductors embedded in insulation in the slots, together with the iron teeth between them, the specific utilization of the material of this active belt may be regarded as the product of three factors. These three factors are: (a') the gross current-density per square inch (in a direction parallel tõ the axis) in the active belt; (β') the gross magnetic density per square inch (in a radial direction) in the active belt; (γ') the peripheral velocity (in the tangential direction). If we write the usual symbols, and put s = depth of active belt (*i.e.* length of tooth or depth of slot) in inches, and v = surface-speed in inches per second; then (assuming sine-waves and a power-factor $\cos \phi = 1$) we have

Watts per cubic inch in active belt $= 1 \cdot 1 \times a' \times \beta' \times \gamma'$; where

$$a' = Z\, C_1 \div \pi d s,$$

$$\beta' = N p \div \pi d l,$$

$$\gamma' = v \div 10^8.$$

Now in practice the value of a' seldom exceeds 400 in ordinary alternators, and, owing to the necessity of avoiding too great an armature distortion, cannot well be pushed in turbo-alternators above this figure. The value of β' is in ordinary alternators seldom over 30,000, and cannot well be pushed in turbo-alternators beyond that point. On the other hand while v in ordinary alternators usually lies between 1000 and 1500 inches per second, in turbo-alternators it may attain 2500 or even 3000 inches per second. Hence the specific utilization of material, or watts of output per cubic inch, may in these machines attain nearly double the value it has in the "ordinary" types of alternators. The following table exhibits some of these values for the sake of comparison.

It will be noted how high is the specific output of the large Niagara generator, which is of the high-speed water-turbine type, and approximates closely in its design to the type of machine used with the Curtis steam-turbines.

Since the specific utilization of materials in the active belt is thus about doubled in turbine machines, it might be argued that for a given output the weight and cost would be about halved. But as the total weight of material in a generator is not proportional to the weight of material in the active belt,

TABLE XI.—UTILIZATION COEFFICIENTS OF RECIPROCATING AND TURBINE-DRIVEN ALTERNATORS.

		α'	β'	v	Watts per cubic inch, Active Belt.
Alternators of Older Type :—					
Oerlikon,	ATB 72—1500— 83, p. 397	208	36,000	1023	71·8
Heyland,	ATB 64— 760— 94, p. 407	236	26,000	1160	71·2
Oerlikon,	ATB 10— 290— 600, p. 418	286	27,400	1030	81
Gen. El. Co.,	ATB 12—7500— 250, p. 423	400	31,300	1960	254
Brown,	ATB 28—1640— 180, p. 426	153	25,500	1524	59
Kolben,	AQB 96—3500— 75, p. 428	208	24,600	1180	57·1
Westinghouse,	AQA 62—1500— 116, p. 434	261	25,500	1214	81
Turbo-Alternators :—					
Parsons,	ATB 4—3500—1200, p. 455	300	23,700	2625	188
Westinghouse,	ATB 4—1800—1500, p. 469	271·5	26,000	3020	212
Oerlikon,	ASB 4—1000—1500, p. 465	103	27,800	2700	77
Brown,	ATB 4—1000—1500, p. 460	420	25,500	2620	288
B.T.H. Co.	ATB 6—1000—1880, p. 474	199	30,800	2480	168

and in turbine machines is relatively less, it follows that the total weight (and therefore cost) of turbine machines is correspondingly reduced.

An interesting comparison of size and weight is afforded by the machines furnished by the Westinghouse Co. to the New York Rapid Transit Subway. Some of these are large slow-speed ATB 40—5000—75, direct-driven from reciprocating engines; they are about 40 feet in diameter and weigh about 440 tons each. One such machine is described above on page 146. The others are turbine-driven ATB 4—5000—750; being about 12 feet 6 inches in external diameter and weighing about 105 tons each. A tenfold-speed results in reducing the weight to about ¼, for equal output.

From the engineering point of view there is a further advantage possessed by the steam-turbine in that it requires absolutely no lubrication except at the bearings. All contamination of steam by oil is thus avoided, and super-heating to a high temperature, so objectionable in many types of engine, is thus made practicable, with a resulting gain in economy of fuel.

STEAM TURBINES: TYPES OF CONSTRUCTION.

The essential feature of a steam-turbine consists in a set of nozzles or equivalent devices through which steam at high pressure, usually superheated, passes to impinge upon a series of angularly-placed blades, vanes, or cups, mounted upon a rotating shaft, the whole being enclosed in an outer shell. Usually, arrangements are made by which the steam which has thus impinged upon a set of blades or vanes, is then led through a second set of openings to impinge upon a second set of blades during a further expansion, to be followed again by another series of such operations. The steam, in expanding adiabatically through any such set of nozzles, acquires a great velocity, the potential energy of its heat being converted into kinetic energy of movement which is imported to the vanes, blades or cups upon which it impinges; and thus the potential energy of the steam is converted into work by the rotation of the shaft to which the vanes or blades are mounted. In the most elementary form of turbine, the steam is expanded in a single nozzle and directed against a single crown of vanes; in the more developed forms there are several successive expansions, and several crowns of blades or vanes. There have thus been evolved a number of different varieties of turbine construction.

It is outside the scope of this book to go further into the subject of the steam-turbine, except to point out that in all of them, since the velocity of the steam after adiabatic expansion is very great, there must be as a characteristic feature, a high peripheral velocity, and therefore a high speed of revolution. This last point gives rise to many special problems both mechanical and electrical in connexion with their use. For the benefit of those who wish to pursue further the theory of the steam-turbine and the different types of construction in the market, the following list of recent literature has been inserted.

Parsons Turbine :—
- *Proc. Inst. Civil Eng.*, xcvii. Feb. 1889.
- *Elec.*, xx. 103, 1887.
- *Engineering*, lxxvii. 899, 1904.
- *Journ. Inst. Elec. Engs.*, xxxiii. 587, 794, 1904.
- *Elec. World*, xl. 906, 1902 ; xliii. 356, 925, 1904.
- *Electrician*, lii. 732, 996, 1904.
- *Elec. Rev.*, liv. 605, 1904.

Brush-Parsons Turbine :—
- *Elec. Rev.*, liv. 282, 317, 1904.
- *Electrician*, li. 732, 1904.

Westinghouse-Parsons Turbine :—
- *Electrician*, lii. 942, 1904.
- *Elec. Rev.*, liv. 51, 1904.

De Laval Turbine :—
- *Engineering*, lxxvii. 846, 1904.
- *Electrician*, lii. 777, 1904.

Curtis Turbine :—
- *Engineering*, lxxvii 181, 1904.
- *Elec. World*, xli. 609, 859, 1903 ; xlii. 434, 606, 1903 ; xliii. 356, 1904.
- *Electrician*, li. 1051, 1903 ; lii. 80, 160, 596, 1903 ; liii. 58, 1904 (Governor) ; lv. 162, 1905.
- *Elec. Rev.*, liv. 330, 397, 1048, 1904.

Rateau Turbine :—
- *Engineering*, lxxvii. 863, 1904.
- *Electrician*, liii. 307, 349, 1904.
- *Elec. Rev.*, lii. 133, 1903 ; liv. 1009, 1904.

Zoelly Turbine :—
- *Engineering*, lxxvii. 770, 1904.

Riedler-Stumpf Turbine :—
- *Engineering*, lxxvi. 211, 1904.
- *Electrician*, lii. 902, 1904.
- *Engineer*, xcvi. 587, 611, 1903.
- *Elec. Rev.*, lii. 845, 1903 ; liv. 401, 1904.

Bucholtz Turbine :—
- *Elec. Rev.*, liv. 66, 1904.

Hamilton-Holyworth Turbine :—
- *Elec. Rev. N.Y.*, xlv. 592, 1904.

Crocker Turbine :—
- *Elec. Rev. N.Y.*, xlv. 489, 1904.

Brady Turbine :—
- *Elec. Rev.*, lii. 68, 1903.

PAPERS, ETC., ON GENERAL TURBINE MATTERS.

Set of papers and discussions on several forms of steam turbines.
Proc. Inst. Mech. Engs., 1904, No. 3, June.

Design of Steam-Turbine Generators. Behrend.
Elec. Rev. N.Y., xlv. 375, 1904.
Electrician, liii. 950, 1904.

Several papers on Steam-Turbine Design in
Zeitschrift des Vereines Deutscher Ingenieure, xlviii. May 1904.

Notes on Steam-Turbine Generating Plants.
Electrician, lii. 19, 55, 1902.

Indicator Diagrams from Steam-Turbines. Booth.
Elec. Rev., liv. 500, 1904.

The Steam-Turbine and Superheat.
Elec. Rev., lii. 163, 1903.

The Turbine Problem.
Am. Elec., xvi. 76, 1904.

American Turbo-Dynamos.
Elektrotechnische Zeitschrift, xxv. 1037, 1904.

Gas Turbines.
Proc. Inst. Mech. Engs. Part II., 1904.

Turbines à Vapeur.
Bull. Assoc. Ing. Élec., v. 37, 101, 1905.

Turbo-Dynamos. Niethammer.
Elec. World, xliii. 558, 595 ; xliv. 641, 1904.

Turbine-Driven Homopolar Dynamos (continuous current). Noeggerath.
Trans. Am. Inst. Elec. Engs., xxiv. 1, 1905.
Elec. World, xlv. 251, 1905.

Rotating Field-Magnets for Turbo-Generators, B. A. Behrend.
Electrical World, xlv. 207, 1905.
Electrician, liv. 848, 1905.

Mechanical Construction of Steam Turbines and Turbine Generators. London.
Electrician, liv. 910, 1905.

The following treatises deal solely with steam-turbines:—

The Steam Turbine, by R. M. Neilson.
Steam Turbines, by Dr. A. Stodola.
Bau der Dampfturbinen, by Alfred Musil.
Dampfturbinen, Entwickelung, Systeme, Bau und Verwendung, by Wilhelm Gentsch.
Roues et Turbines à Vapeur, by K. Sosnowski.

GENERAL REMARKS.

Before proceeding to the special types of alternator suited for turbine-work, a few features may be pointed out.

The absence of cranks and reciprocating parts has the result that the difficulties that arise with reciprocating engines from necessity of balancing, and of securing a uniform turning effort—difficulties, which, in many cases, impair the parallel running of even good alternators if driven by reciprocating engines—are reduced to a minimum. On the other hand, the difficulties of design of the magnetic and electrical parts that arise from the presence of centrifugal forces are greatly increased. For centrifugal forces increase with the square of the angular velocity, and are directly as the radius[1] according to the rule:—

$$F = 0{\cdot}0000284\ RU^2;$$

where F is the centrifugal force in pounds weight per pound of peripheral matter, R the radius in inches, and U the revolutions per minute. Thus in Brown's 1000 KW alternator, running at 1500 RPM, with a radius of 16·1 inches, every pound of matter at the periphery tends to fly outwards with a force equal to the weight of 1040 pounds. Hence, strength is a prime consideration in the design of such machines. For securing pole-cores, we must carefully choose, therefore, between the alternatives of a completely solid structure, a dove-tail joint, or a design with massive bolts or screws. The magnetizing coils must be also adequately secured with holding-down devices or confined inside stout rings or caps.

Turbines for outputs below 300 or 400 KW, show little advantage over slower-speed engines, and are therefore unsuitable for small machines.

It will be noted that when comparing together large and

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[1] Another useful approximate rule, given by Mr. W. J. A. London, is that for steel or phosphor bronze or other material of about equal density, the tensile stress in lbs. per square inch at any periphery moving at v feet *per second* is equal to $v^2 \div 10$. Thus in a steel rim running at 18,000 feet per minute, or 300 feet per second, the stress is about 9000 lb. per square inch.

small turbines of any one type, the revolutions per minute go down as the rated power goes up, because the peripheral velocity must be nearly the same whatever the size. Since there is a necessary relation between the speed, the frequency and the number of poles (see p. 351) according to the rule

$$\text{RPM} = \frac{120f}{p}, \qquad \text{or} \qquad p = \frac{120f}{\text{RPM}},$$

it follows that for the standard frequencies such as $f = 50 \sim$ and $f = 25 \sim$ the number of poles must be very few, usually four, occasionally two, seldom six. The following Table XII. exhibits the possible speeds for such machines, with different frequencies:—

TABLE XII.—SPEEDS AND FREQUENCIES.

$f =$		60	50	45	42	40	36	$33\frac{1}{3}$	30	25	$22\frac{1}{2}$	20	15
Number of poles	2	3600	3000	2700	2520	2400	2160	2000	1800	1500	1350	1200	900
	4	1800	1500	1350	1260	1200	1080	1000	900	750	675	600	450
	6	1200	1000	900	840	800	720	667	600	500	450	400	300
	8	900	750	675	630	630	540	500	450	375	$337\frac{1}{2}$	300	225

As the difficulty of turbine-construction is to lower the speed, it follows that the higher frequencies with the smaller numbers of poles are the more favourable. There is, however, some inconvenience in the 2-pole types on account of armature end-connexions and unfavourable magnet form; hence 4-pole types are much to be preferred.

The question of balancing of the revolving parts, though important in all electric machinery, is of cardinal importance in turbine work. As is well known to engineers there is with every running shaft a certain critical speed which is most dangerous, because at that particular speed the vibratory tendencies become cumulative. The critical speed is that particular number of revolutions per second which corresponds with the frequency of the natural vibration of the shaft between its bearings loaded with its rotating part. The ordinary laws of vibration teach us that the natural period of a springing shaft loaded at its middle part with a mass will decrease with the stiffness of the shaft, and increase with the load it carries; or its frequency will increase with the stiffness and decrease with the load. If this natural frequency of the transverse vibrations of the shaft happens to coincide with the impulses given to the shaft by the centrifugal forces due to any want of balance or to any curvature of the shaft, the vibrations become violent, though at speeds even a little above or below this they are quite small. The problem of calculating the forces on a whirling shaft has been studied by Rankine, and by his followers Osborne Reynolds and Dunkerley.[1]

The only way to be sure of freedom from this danger is to design the shaft of such a great stiffness that the critical speed shall lie distinctly above the operative speed. But even if this is arranged, the utmost care must be exercised in accurate balancing of the whirling masses, and in providing shafts designed with minimum spring between their bearings. The length of the shaft between the centres of the bearings must also be kept as short as possible, and the shaft must be thickened at the middle part to prevent springing. In the revolving magnets of Parsons's alternators the ratio of the shaft length between centres to the diameter at the middle rarely exceeds 3 in machines of 1000 KW, and is about 2½ in the 3500 KW machines. The revolving part must be balanced not only statically upon knife edges before it is run, but must also be balanced dynamically by running it at its proper speed upon bearings that will allow it to vibrate if unbalanced. The old plan of marking with a piece of chalk the side of the revolving part which flies out, is rough. A better plan is to provide marks on the end of the shaft and illuminate them by an intermittent light of the same frequency. If any one of the conductors on the revolving part of a turbo-generator can shift its position by even a minute amount by the shrinkage or deterioration its surrounding insulation, trouble may set in even in a machine which at first worked quite satisfactorily. Also, since any flaws in the metal of which the polar masses are

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[1] *Philosophical Transactions*, vol. clxxxv. (A), p. 279, 1894.

made would affect the balancing, the utmost care must be taken to use only homogeneous material.

From the electrical point of view there are also special points to be borne in mind. With these high surface speeds and the low frequencies habitual for power distribution, large pole-pitches are inevitable. The rule (see p. 357) is that

$$\text{pole-pitch (in inches)} = \frac{v}{10f};$$

where v is the peripheral speed in feet per minute. It follows that in turbo-alternators for a frequency of $f = 50 \sim$, the pole-pitch will lie between 25 and 30 inches, while for a frequency of $f = 25 \sim$, it will lie between 50 and 60. These are more than double the values found with the older engine-type of slow-speed alternator; and they involve questions of design of poles.

Again, equality of power in the poles is of importance lest forces tending to unbalance should thereby be introduced. Hence the utmost symmetry must be observed not only in the construction of the poles and in their excitation, but also in their adjustment within the housing that they shall be precisely equidistant from the armature face, otherwise they will exert unequal magnetic pull. From this point of view it is evident that a machine having a wide gap between poles and armature is preferable to one with a narrow clearance, since then any slight decentering will have relatively less effect in setting up inequality of pull.

The largeness of the pole-pitch has an influence on armature construction. It is favourable in so far as it enables a well-distributed winding, with 4, 6, or 8 slots per phase per pole to be used; but it is unfavourable in that it causes the end-bends of the windings to be long, and to project far from the core-body. Now in the regions beyond the ends of the revolving magnet-system there will be, particularly in machines with wide gaps, a considerable stray flux, and the projecting end-bends of the armature will come into the influence of this stray field. This produces no harm in the mere creation of voltage, but it has the result that it subjects the end-bends to mechanical forces which increase with the load, and which, at a time of a momentary short-circuit in the mains, may become simply enormous. Hence the end-bends must be mechanically anchored by stout supports. But these supports must not be of magnetic material, otherwise they will add to the reactance of the armature; neither must they be of any massive metallic structure in which eddy-currents could be induced, otherwise they will become heated greatly. Even the end-shields, which act as protectors over the end-bends, must be designed with the knowledge that they too may have eddy-currents induced in them.

Finally, there is the dominant question of adequate ventilation. Though, as compared with slow-speed types, the iron-losses and copper-losses are relatively small, the radiating surface is still smaller, rendering dissipation of the heat less easy. In no other piece of electric machinery is so large an output concentrated within so small a volume. Five thousand horse-power may be concentrated within 200 cubic feet of a structure of which three-quarters is solid copper, iron or steel. Now, even if an efficiency of 98 per cent. is assumed, the 2 per cent. wasted in heating amounts to a continuous expenditure of 100 horse-power within that mass of metal. This is an amount of heat which, if not removed continuously by radiation or conduction, would raise it in one minute to a red heat. Hence the prime necessity of adequate ventilation in every part. One may either expose the surface as much as possible to give free access to the air, or one may enclose the machine and apply a forced ventilation. Each method has been tried, but preference seems to be given to the latter method, which, though having the advantage of being more silent, demands a certain waste of power in the blowers.

In Chapter III. the design and construction of pole-cores, pole-tips, exciting coils, and housings have been discussed. It remains to see how these structures must be modified to meet the new conditions.

FIELD-MAGNET CONSTRUCTION.

Special constructions for the revolving magnets or rotors of turbo-alternators have been devised in view of the extremely high centrifugal forces, which necessitate special means for securing not only the polar masses, but also the exciting coils. Fig. 345 shows a number of methods of pole construction: Fig. 345*b* shows solid pole-shoes, with a T dovetail, and special clamping-pieces to hold the coils, as used by Parsons; Fig. 345*a* shows two dovetails and coils made of strip wrapped round the core, due to Messrs Siemens and Halske; Fig. 345*d* is the form of dovetail used by Messrs. Dick, Kerr and Co.; Fig. 345*e* is a form devised by the Westinghouse Co., and used by them in ordinary alternators; the wedge-shaped pole-tips are keyed very firmly in place by square feathers, and are removable; Fig. 345*f* is another Siemens and Halske form with screwed pole-piece, the coils being wound on bobbins that are themselves tightened by the insertion of round key-bolts.

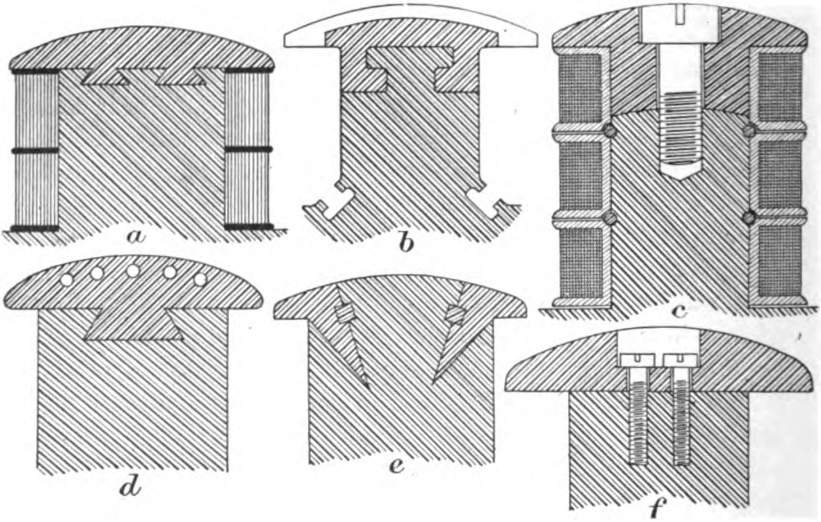

FIG. 345.—FORMS OF POLE-CONSTRUCTION FOR TURBO-ALTERNATOR.

Fig. 346 is a design of the Oerlikon Co., in which the T-shaped dovetail projections of the pole-cores are keyed into the hub; the coils being secured by bronze pieces screwed in between the poles. A very similar plan of keying the pole-cores is used by the General Electric Co. It should be compared with the plan adopted in the Niagara generators, Plates X. and XI., over which it seems to have some advantages.

For rotors having an entire cylindrical periphery, Lamme has patented the construction shown in Fig. 347*a*, in which the conductors are entirely sunk within the iron core, channels being cut not only at the sides but also at the ends, in which to embed the copper conductors. The slots are closed with metal wedges. The method adopted by the Oerlikon Co. is shown in Fig. 347*b*. Constructions very similar to these last have been adopted by Brown, Boveri and Co. and by Messrs. Kolben and Co.

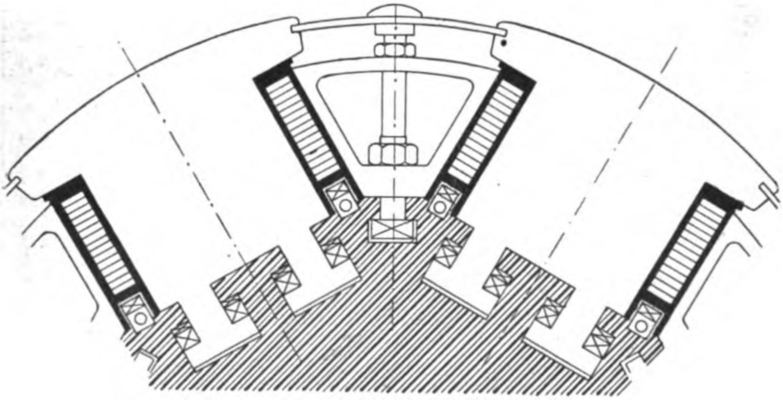

FIG. 346.—MAGNET-CONSTRUCTION OF THE OERLIKON CO.

A type of slotted rotor adopted by the Bullock Co. is shown in Fig. 348. The windings are retained in the slots by wedges of phosphor bronze, and the end-connexions are covered by cylindrical pieces of nickel steel. Inside the end-connexions are located wedge pieces, four of which form an expansible ring, and within these is placed a clamping ring which, when forced into position, causes the wedge pieces which seat against the end-plates to press outwardly against the end-connexions, and to hold them securely in place. This design should be compared with the analogous construction of the Oerlikon Co. in Fig. 356, p. 467.

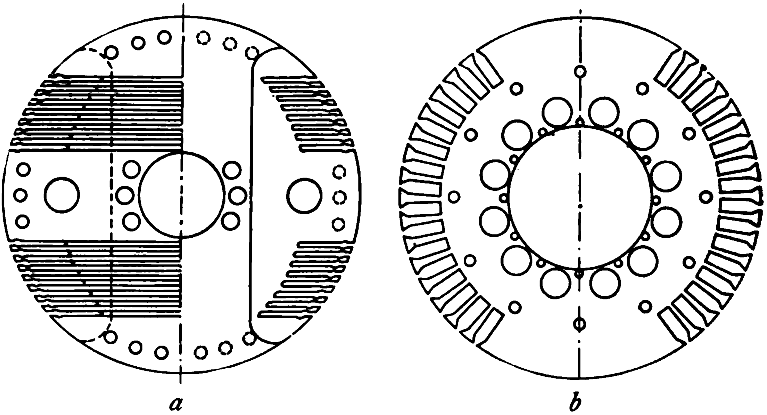

FIG. 347.—METHODS OF CONSTRUCTING SLOTTED ROTORS.

In order to get a perfectly cylindrical construction, which the Allgemeine Elektricitäts-Gesellschaft of Berlin claim minimises the humming when rotating at high speeds, the construction shown in Fig. 349 has been adopted by them. Shaft and spider are in one piece of cast steel, C, and the former wound coils of strip copper are secured in position by the pieces B, B, which are let down into the dove-tailed slots

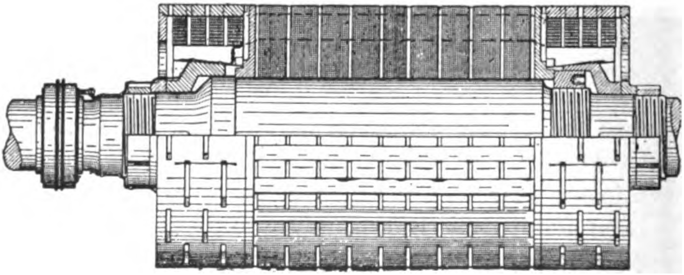

FIG. 348.—ROTOR CONSTRUCTION BY THE BULLOCK CO.

in the spider, and are then finally secured by the wedge D, which in its turn is prevented from flying outwards by the wedge E.

REVOLVING ARMATURE CONSTRUCTION.

For machines of A-type Mr. Parsons adopted, in 1896, a design having (for single-phase) the coils distributed in a number of closed slots below the periphery. One of the earliest Oerlikon plans (for a 2-phase machine) is shown in Fig. 314, p. 338, a 2-pole 20-slot winding, where the coils are threaded through closed slots, and the end-bends are wound under bronze projections, which abut against the unwound parts of the core-disks. For a 3-phase armature the same firm developed the plan shown in Fig. 315, p. 339, which is a 4-slot 4-pole winding.

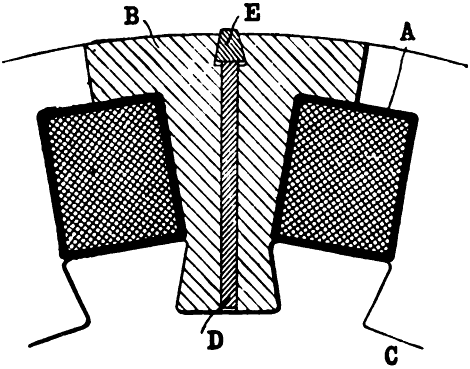

FIG. 349.—ROTOR CONSTRUCTION BY THE ALLGEMEINE CO.

STATOR CONSTRUCTION.

The most special thing about stator-construction is the provision for circulation of air within the housing, as already alluded to. The Allgemeine Co. has used in some smaller turbo-machines the construction adopted for machines of older type, and shown in Fig. 166*g*, p. 182, in which the core-rings are mounted between circular checks without any casing to surround the outer periphery of the machine, thus exposing to the cooling of the air the edges of the core-disks themselves.

A method of holding the end-bends of the armature-coils, as adopted by the General Electric Co. in the construction of their vertical-shaft Curtis turbo-alternators, is shown in Fig. 560, p. 475. The end-bends are threaded by straps of bronze bent to a U-form, which are solidly secured by the bolts that pass through the core-laminations and the core-heads.

PARSONS'S TURBO-ALTERNATORS.

As in the construction of steam-turbines, so also in that of generators to be driven by them, Mr. Parsons must be regarded as the pioneer. His first combined set of turbine and generator was built in 1884, and after use in Gateshead for several years was removed to South Kensington Museum. It ran at 18,000 RPM and developed about 7½ KW in continuous current; the armature being 3 inches in diameter. The first turbo-alternator, built about 1888, had an output of 75 KW. In 1894 a turbo-alternator of 150 KW was installed at Portsmouth, and ran in parallel with other alternators of slow speed. About the same time others, of 350 KW, were supplied to the Metropolitan Electric Supply Co., at Manchester Square and elsewhere. Others of 500 KW were installed in Newcastle, Cambridge, and Scarborough. In 1900, Mr. Parsons constructed, for the town of Elberfeld, two sets of 1000 KW each, of 4-pole A-type running at 1500 RPM. They were found to consume only 19·43 lb. (= 8·81 kilogrammes) of steam per kilowatt-hour at normal load. In 1901 Mr. Parsons supplied to the Neptune Bank station of the Newcastle Electric Supply Co., a turbo-alternator of 1500 KW running at 1200 RPM. This was followed by a 1500 KW 2-phase set for Sheffield Corporation, and by a series of 2000 KW 3-phase sets for the Carville power-house of the Newcastle Electric Supply Co., for which station others of 3500 KW were also made. All this shows a steady progress from small sizes to those of larger size and higher efficiency.

In the fifth (1896) edition of this work the Author gave a

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description and a Plate of a Parsons's single-phase alternator of 360 KW running at 3000 RPM. This machine had 4 stationary external poles and a revolving armature built up of disks having 60 closed slots, in 40 of which the conductors were inserted in one layer.

Plate XII. shows a Parsons's turbo-alternator of 3500 KW, being one of the machines supplied by Messrs. C. A. Parsons and Co. to the Carville power-house of the Newcastle-on-Tyne Electric Supply Co. It is rated for a continuous output of 3500 KW on a power-factor of 0·85, or an overload up to 5000 KW for a period not exceeding 1 hour, or a momentary overload of 5500 KW.

The short specification is as follows:—

$$\text{ATB } 4_{\text{P}} - \frac{4120_{\text{KVA}}}{3500_{\text{KW}}} - 1200_{\text{RPM}}\ ;\ \frac{5000}{2887}{}_{\text{V}} - 3 \times 475_{\text{A}}\ ;\ 40\sim.$$

The magnet system is of the defined pole type, the magnet core being shaped solidly out of a steel forging which is bored at the ends to receive the shafts upon which it is shrunk, and which enter it from each end to about one-quarter of its length. Each of the 4 poles receives a pole-shoe of solid steel, which is slid into it with a T-shaped dovetail joint, the overhanging tips serving to hold down the exciting coils. The pole-shoes are crossed at the faces with a number of grooves (as in the so-called 'comb'-poles, p. 150), so that the face presents a number of ridges which become highly saturated, thus causing the magnetic field to be very *stiff* as it enters the gap. The exciting coils are also held in by bronze clamping-pieces, as shown in the transverse section of Plate XII. The stationary armature has an internal diameter of 43 inches, and its length is 58·8 inches. There are 36 slots in all, or 3 slots per pole per phase; and there are 6 conductors per slot, making 72 conductors per phase. This makes the no-load flux N = 46,600,000 lines per pole, and the mean pole-face density at the armature-face 37,700 lines per square inch; the pole-arc being 21 inches or 0·62 of the pole-pitch. At normal load the specific loading is 762 amperes per inch. In order that the machine shall be well ventilated, 26 ducts are provided in the armature core body. The housing is constructed to allow of very free ventilation, but is open only at the top, so that it acts as a chimney to induce a draught. The peripheral speed at the pole-shoe is 13,200 feet per minute, which creates a centrifugal force equal to 860 pounds weight on every pound of material at the periphery.

Fig. 350 gives a general view of one of the 2000 KW turbine sets at Carville.

BROWN'S TURBO-ALTERNATORS.

Amongst those who early realized the importance of the changes which the adoption of high speeds would bring about in the design of electric machinery was Mr. C. E. L. Brown, of Brown, Boveri and Co. So far back as 1892, when he was designing water-turbine generators for Niagara, he had realized that many details of construction which are quite appropriate for slow speeds become impossible at speeds so high that centrifugal forces outweigh the forces of gravity—in some cases more than a thousand-fold. When there are few poles, perhaps only two or four, of short length radially and long in the axial direction, even the very solid construction of magnet-winding with edge-wise strip copper, is not rigid enough. Exact balancing is at these high speeds an absolute necessity, and if the balance is to remain true after some months of running, every part of the exciting winding must be mechanically held and supported solidly. What efforts have been made to attain this end may be seen by reference to previous paragraphs. Mr. Brown himself patented[1] several forms. The arrangement most generally chosen is one with open slots which admits of the coils, previously shaped on a former, being inserted by their individual turns or groups of turns. Then the coils, so far as they can run in slots, are secured solidly by the insertion in the slots of wedges of some non-magnetic metal such as aluminium or bronze. An insulating wedge is not necessary. At the ends the bends of the coils are confined by bronze caps which also carry ventilating vanes. These create a powerful draught of air.

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[1] English specification, No. 24,632 of 1901.

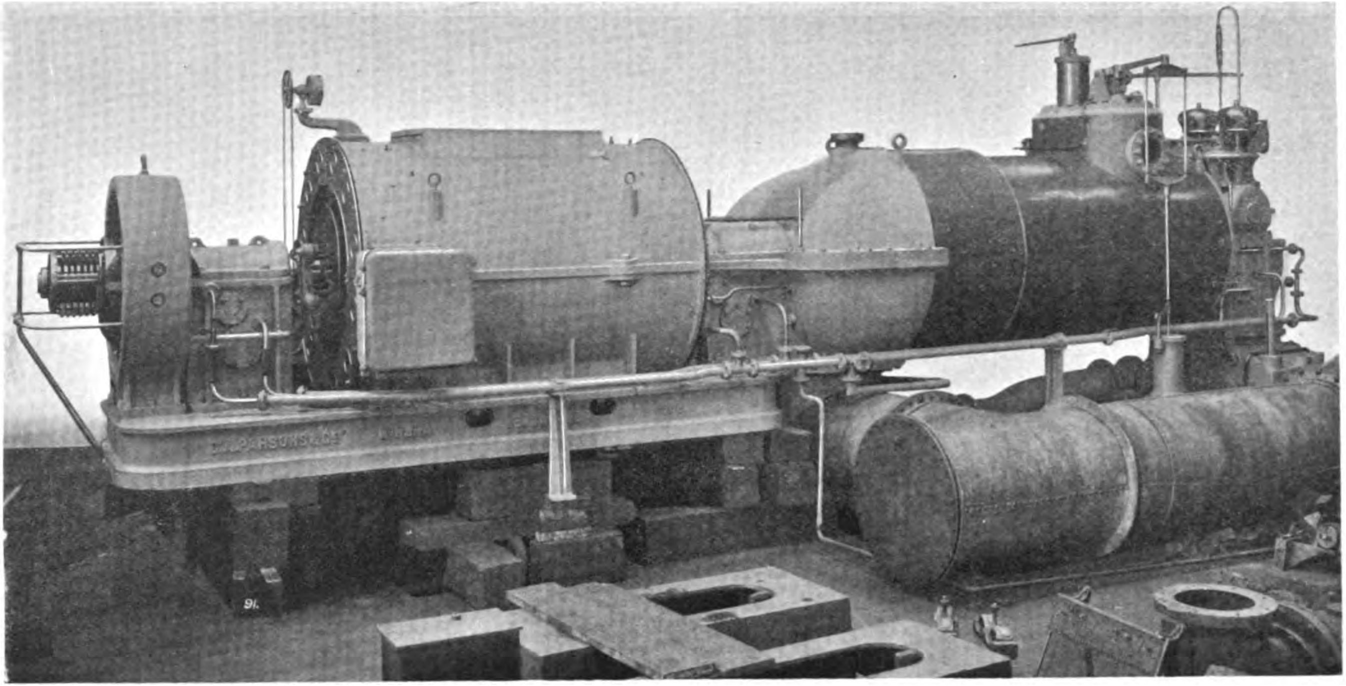


FIG. 350.—PARSONS'S TURBO-ALTERNATOR GENERATING SET AT CARVILLE POWER-HOUSE, NEWCASTLE-ON-TYNE. The steam-turbine is on the right, the alternator in the middle, the exciter on the left.

This type of field-magnet shows, in contrast with the types previously in use, the peculiarity that portions of iron intervene between the individual coils of the same pole, so that the different portions of one and the same pole are not at the same magnetic potential. It is natural that at first sight designers, looking at the matter from the older points of view, would regard this introduction of pieces of iron at intermediate points as an undesirable feature, and would rather substitute non-magnetic material between the turns of the coil. Experiments made on this point showed, however, that such substitution had no advantages, and that this type of field-magnet does not differ in its properties from other types, while as to construction it is by far the cheapest and most rational arrangement. It concentrates the excitation on the centre of the pole, and promotes a sine-distribution of the flux in the gap. It provides an armour-clad design with a rational placing and securing of the coils, and further provides a mechanical and simple solution of the problem how to design the iron parts of the magnet.

Field-magnets so constructed have, by reason of the small number of poles, and particularly in two-pole machines, a small diameter and a relatively great axial length, so that the general form of the magnet is that of a roller having at each end a nave projecting to receive the shaft. Fig. 351 exhibits the various stages of construction. On the extreme right is seen the crude steel forging, shaped like a roller with two long projecting naves already bored to receive the shaft. Next to it is one that has been turned, and has had ventilating ducts cut corresponding to those on the armature. The third one shows the iron completed with longitudinal slots ready to receive the winding. The two axis-ends of special steel have now been inserted, and the slip-rings have also been fitted into place. The field-magnet on the extreme left is finished at its front end; the windings are in place, the wedges have been inserted, the bronze cap that holds down the end-bends, and the ventilating vane-structure attached to it are all visible; but at the further end the windings can be seen as the bronze cap has yet to be added. Fig. 352 shows the complete bipolar

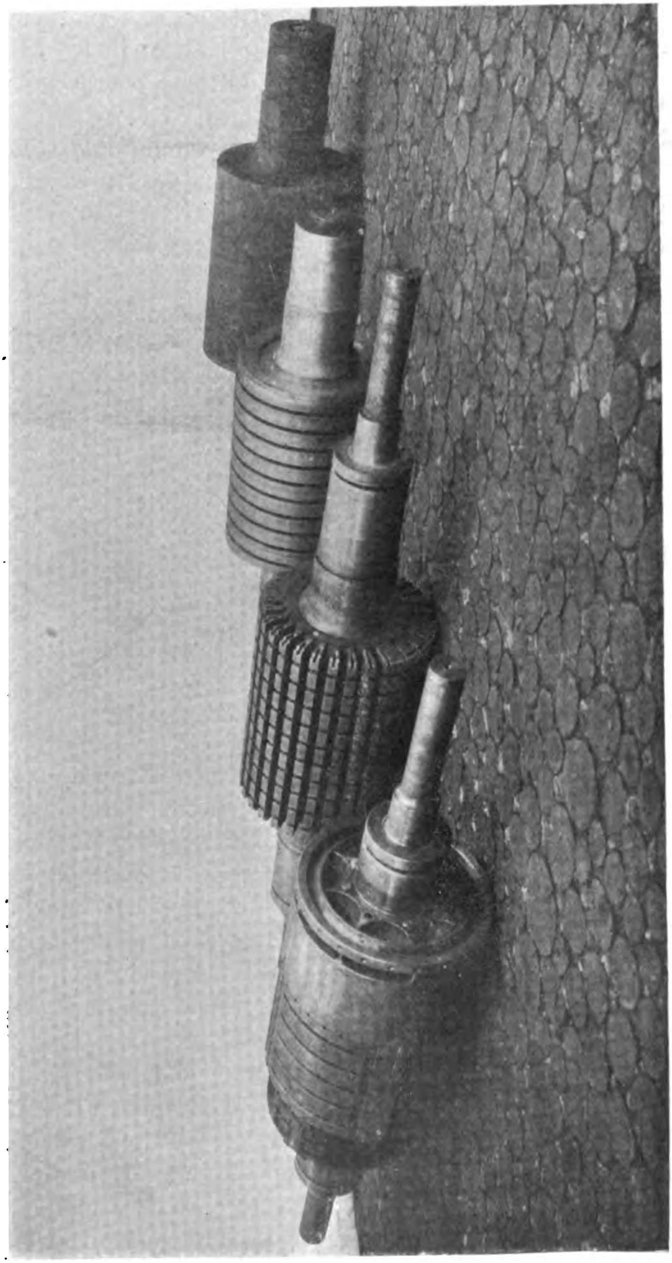


Fig. 351.—Process of Construction of Field-Magnet, Rotating, Bipolar, of Turbo-Alternator of Brown, Boveri & Co.

field-magnet. In this the only electrical parts that can be seen are the two slip-rings and about half an inch length of the connecting wires between them and the projecting naves.

The smallest machines are rated at 100 KW running at 3000 RPM. The largest are 6000 KW running at 750 RPM.

Fig. 353 gives a view in the shops of Messrs. Brown, Boveri and Co., where such machines are under construction. Here two larger 4-pole field-magnets may be seen; one ready to receive its winding, the other wound, but not yet fitted with the bronze caps. It shows very clearly how the coils are arranged in the slots, and makes clear that each pole consists of a wide central portion flanked by two teeth at each side. There is a neutral tooth between the windings of two adjacent poles.

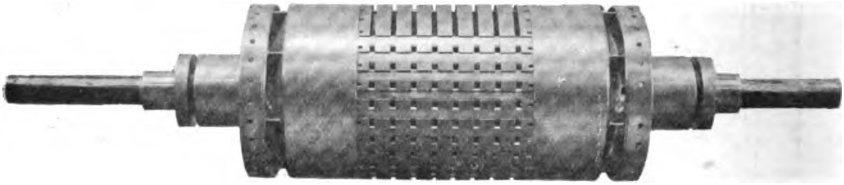


FIG. 352.—ROTATING FIELD-MAGNET OF TURBO-ALTERNATOR, BIPOLAR, COMPLETE WITH TWO BRONZE CAPS (BROWN, BOVERI & CO.).

So solid a construction as that described is eminently adapted for the high speeds of present turbines, and might be used with even higher speeds. The field is very stiff against reactions that tend to distort it. Balancing is simple, and may be done once for all by selecting for the metallic wedges some of lighter or heavier material, or of different thickness as may be required. As the surface does not depart widely from the cylindrical outline, there is less noise than with those types that have discrete projecting poles.

Figs. 354 and 355 represent on a scale of about $\frac{1}{20}$ one of the larger turbo-alternators, of 1000 KW rating, at 1500 RPM, for a frequency of 50 ~. It has four poles of the construction just described. Both the field-magnet and the armature are provided with fourteen ventilating ducts. The armature has a 4-slot winding, carried out in two layers, like an ordinary

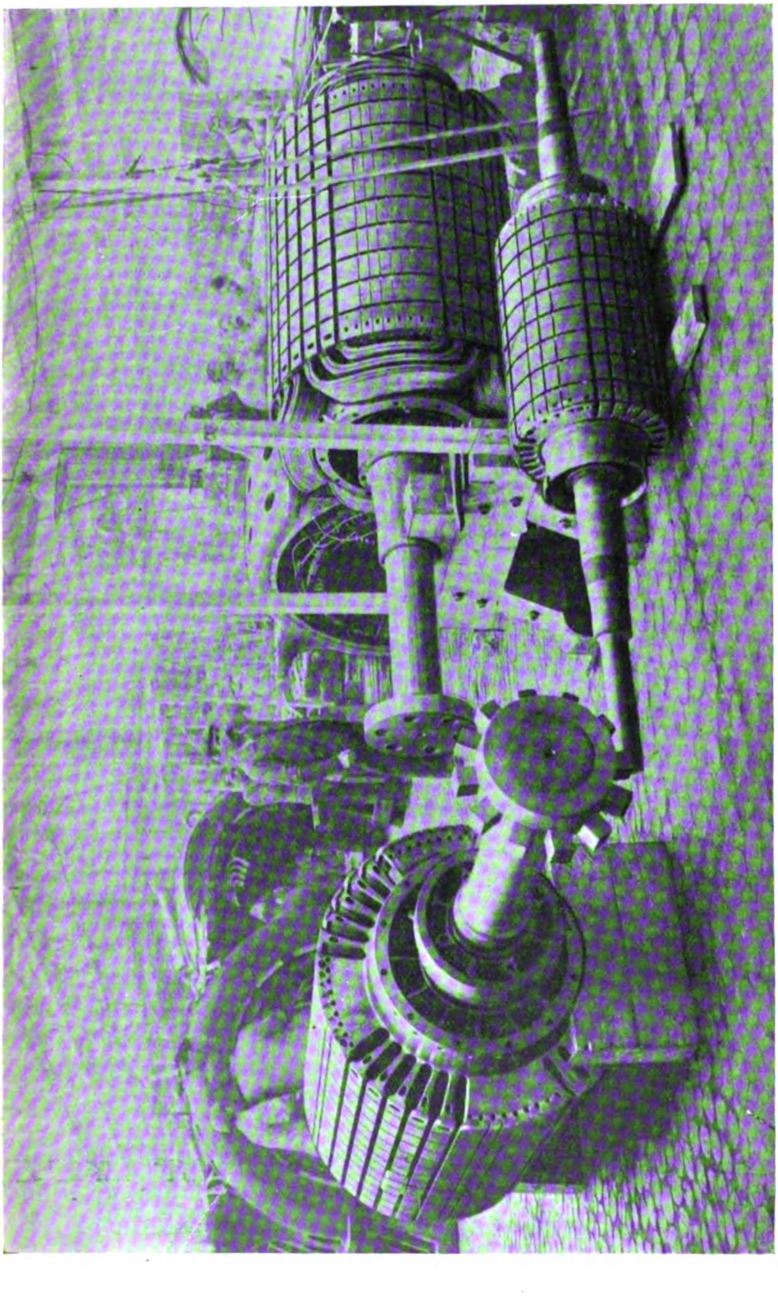


FIG. 353.—TURBO-ALTERNATORS UNDER CONSTRUCTION IN THE SHOPS OF BROWN, BOVERI & CO. Two 4-pole field-magnets are shown, one wound, the other unwound, each for a 3000 KW machine. A smaller bipolar field-magnet, unwound, lies in front.

wave-winding of barrel-type such as Fig. 304. It generates at no-load 1153 volts in each phase, or 2000 volts between lines. The gap is a little over half an inch. The housing is designed

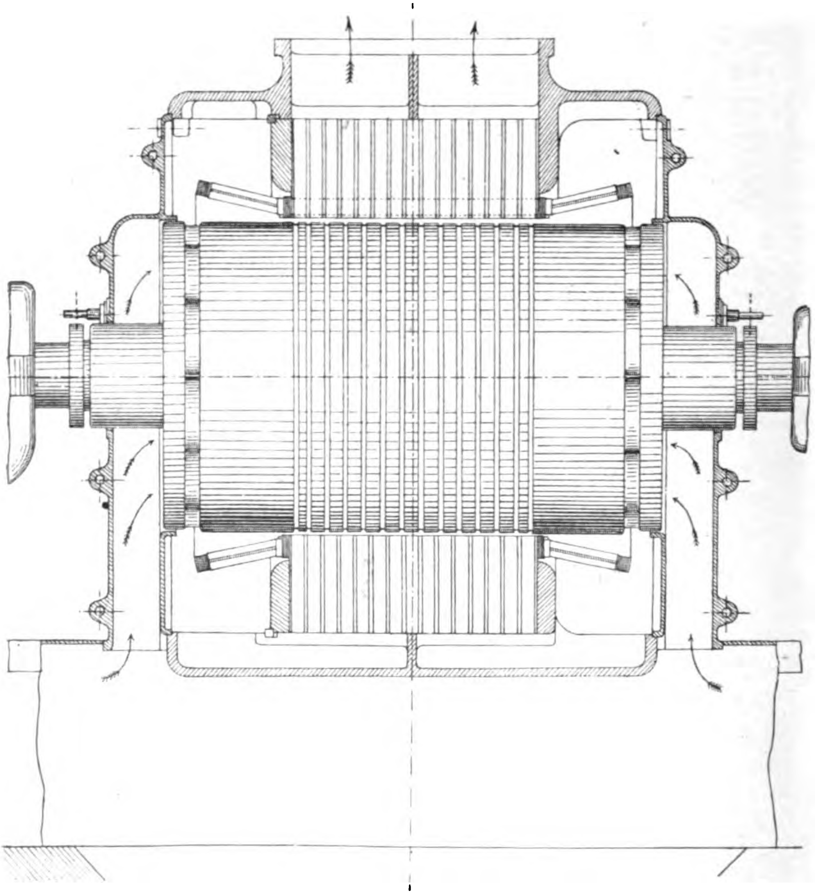


FIG. 354.—BROWN'S TURBO-ALTERNATOR, ATB 4—1000—1500 (in longitudinal section).

with special view to ventilation, for which purpose the ends are separately enclosed, so that cold air may be drawn up from below by the action of the fan-vanes on the ends of the revolving field-magnet, and can emerge at the top, which is formed like a chimney, only after having traversed the armature ducts. This is the first example of a generator furnished with a chimney to promote a draught of air.

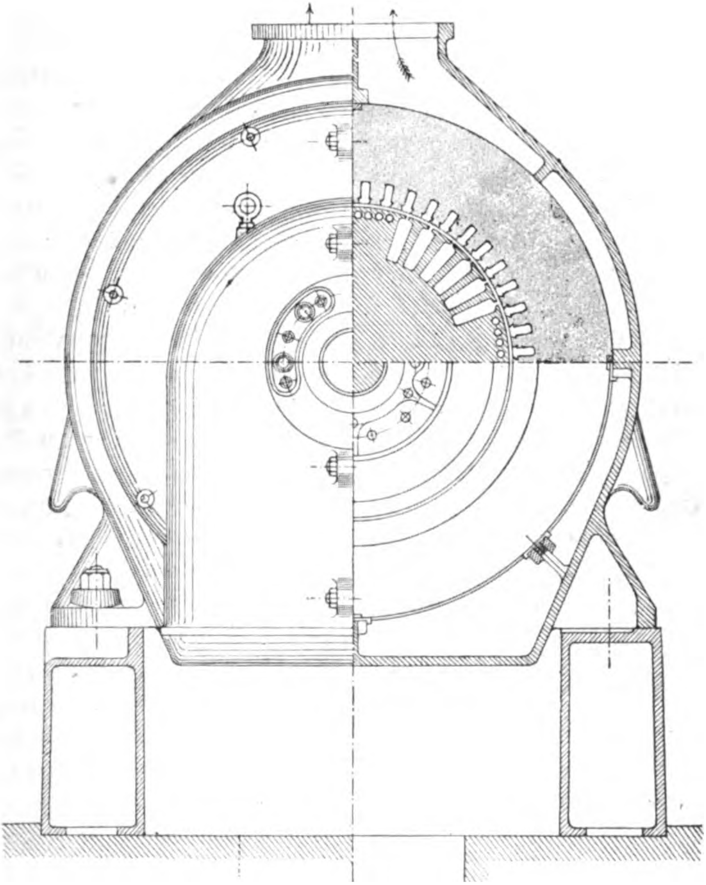


FIG. 355.—BROWN'S TURBO-ALTERNATOR (in transverse section).

Messrs. Brown, Boveri and Co. have also furnished to the station at Frankfort, some turbo-alternators of the pattern ATB 4—2600—1450, working at 3000 volts. The steam-consumption at normal load, using Parsons-Brown turbines constructed at Baden, with condensing plant, was found to be

7·09 kilogrammes (= 15·6 lb.) per kilowatt hour. In a more recent test the steam consumption was found to be reduced to 6·7 kilogrammes (= 14·6 lb.) per kilowatt-hour.

TURBO-ALTERNATORS OF THE OERLIKON CO.

The Oerlikon Co. has broken ground in many directions in turbo-alternator design. The inductor type of construction seemed at first to offer special advantages for turbine work because the rotating part may be made exclusively of iron masses, and early in 1901 a 4-pole 3-phase machine of 1000 KW, running at 1500 RPM, was designed. The inductor had staggered poles built up of iron plates 46 inches in diameter, cut away to form a pair of poles at opposite ends of a diameter. This was followed by a B-type machine of equal output, in which the revolving field-magnet which had an unbroken cylindrical surface 33·2 inches in diameter and 33·5 inches in length, was designed to be excited by a ring-winding carried through 120 slots at the periphery, the winding returning through the interior as in a Gramme armature, but having the direction of its coiling reversed at 4 points so as to constitute a 4-pole magnet. The armature was carried out as an ordinary stator with 3 slots per pole per phase. A little later in the same year an A-type alternator, 2-pole 2-phase, of 400 KW, was designed, running at 2520 RPM, with a frequency of 42~. The armature, 25·5 inches in diameter and 36·5 in length, was made with 80 slots (40 per phase) receiving one conductor each, these slots being arranged in 4 batches with blank spaces between, and spaced as for 108 slots all round. The magnet-system consisted of a cylindrical yoke from which projected inward two salient pole-cores built up of laminæ expanding at the pole so as to subtend about 78° of arc. A fourth design of the same time was of A-type for a 2-pole 3-phase, 200 KW machine at 3000 RPM, for 50~ ; the armature being 19·7 inches in diameter and 26 in length. The core-disks were pierced with 72 holes, only 48 of which were filled ; the winding being carried out as a creeping winding (see Fig. 314, p. 338), with 3 phases non-overlapping, with 8 slots per phase per pole.

The magnet-system closely resembled that of the preceding machine. In the same year there followed another design of A-type, a 4-pole 3-phase machine of 1200 KVA at 1500 RPM, having a 4-slot winding. The armature was 35·5 inches in diameter and 37 in length. The slots were practically closed, and the end-bends in two ranges were secured by bronze blocks and held down by bronze caps. The next type was a 2-pole inductor machine for 2-phase currents, of 400 KW, at 2520 RPM, for $42\sim$. In this machine, in order to secure better dynamical balancing, the polar masses were grouped in a peculiar way; at the middle of the shaft projected one "pole" occupying about 14 inches of its length, while to right and left, at a little distance along, and on the diametrically opposite side of the shaft projected two other "poles" each occupying about 7 inches of length. These projecting poles were of steel plates dovetailed into the solid hub. Although the poles were thus staggered, it was found necessary to construct the armature in three separate parts, as a thick stator standing between two thinner ones. Each stator had 24 half-closed slots, of which 12 only were wound.

A little later came a design of a 2-pole, 200 KW, 3-phase machine, running at 3000 RPM, of B-type. The stator which had a diameter of bore of 18·2 inches, and a length of 24·5 inches, had an ordinary 3-phase 7-slot winding in 42 narrow slots. The rotor was formed with uninterrupted cylindrical outline, built up of core-disks with ventilating holes and assembled with ventilating ducts upon a stout shaft. The periphery was stamped out with slots to receive the exciting coils, the slot-pitch being $\frac{1}{48}$ of the periphery, but only 32 slots were stamped out, in two batches of 16 each, leaving wide portions at two diametrically opposite regions to serve as pole-cores: in other words the winding around each pole was distributed in 8 slots on each side of the unperforated part. The field-magnet wire was a compressed stranded conductor, and the end-bends were solidly clamped down by bronze caps at the ends of the rotor.

These various types served as a basis from which it was possible to work out a definitive construction. For the present the inductor-type, in spite of its inviting possibilities, has been given up because it demands a disproportionate amount of material, and the ventilation is also very difficult. By the device described above of grouping the inductor "poles" for dynamic balancing, and by building up the rotor to a cylindrical outline by dovetailing together on the shaft sectors of iron and brass, the balancing difficulty was overcome, but in order that the exciting coils should be accessible it was necessary to build the stator in three parts, involving complication and expense. Hence the firm has returned to the use of revolving magnets with the exciting coils distributed, as in two of the earlier machines, over a number of slots. In 1901 a 1000 KW machine was built with the ring-winding already described, and in another machine a drum-winding was substituted. As it was found in any completely distributed winding that the conductors lying near the centre of the pole-face contribute little to the total excitation, the slots at this point were, even in the earliest ring-windings, left unwound. In such a case it was useless to stamp out these slots, and accordingly this part of the disk was in subsequent machines left unperforated. This structure is therefore akin to that adopted by Messrs. Brown, Boveri and Co., who apparently arrived at their pattern from other considerations, such as the better securing of the coils that are to surround a pole-core. Whether one starts from the ordinary wound pole and distributes the winding in slots for greater security, or whether one starts from a completely distributed winding and omits the slots in the inoperative part at the middle of the pole, the result comes to much the same thing.

In Fig. 356 is shown a sketch of a single-phase turbo-alternator of the Oerlikon Co., of 1000 KW at 1500 RPM. It has 4 poles, and the rotor, which is 33·5 inches in diameter and 38 inches long, is spaced out as for 32 slots, but only 24 are stamped out to receive the windings, a breadth equal to that of 2 slots plus 3 teeth being left entire at each pole. In the stator, which is 34·3 inches in the bore, and 42½ inches long, there are 48 slots, of which 2 are left unwound at the middle of each pole. All the slots are open, with inserted

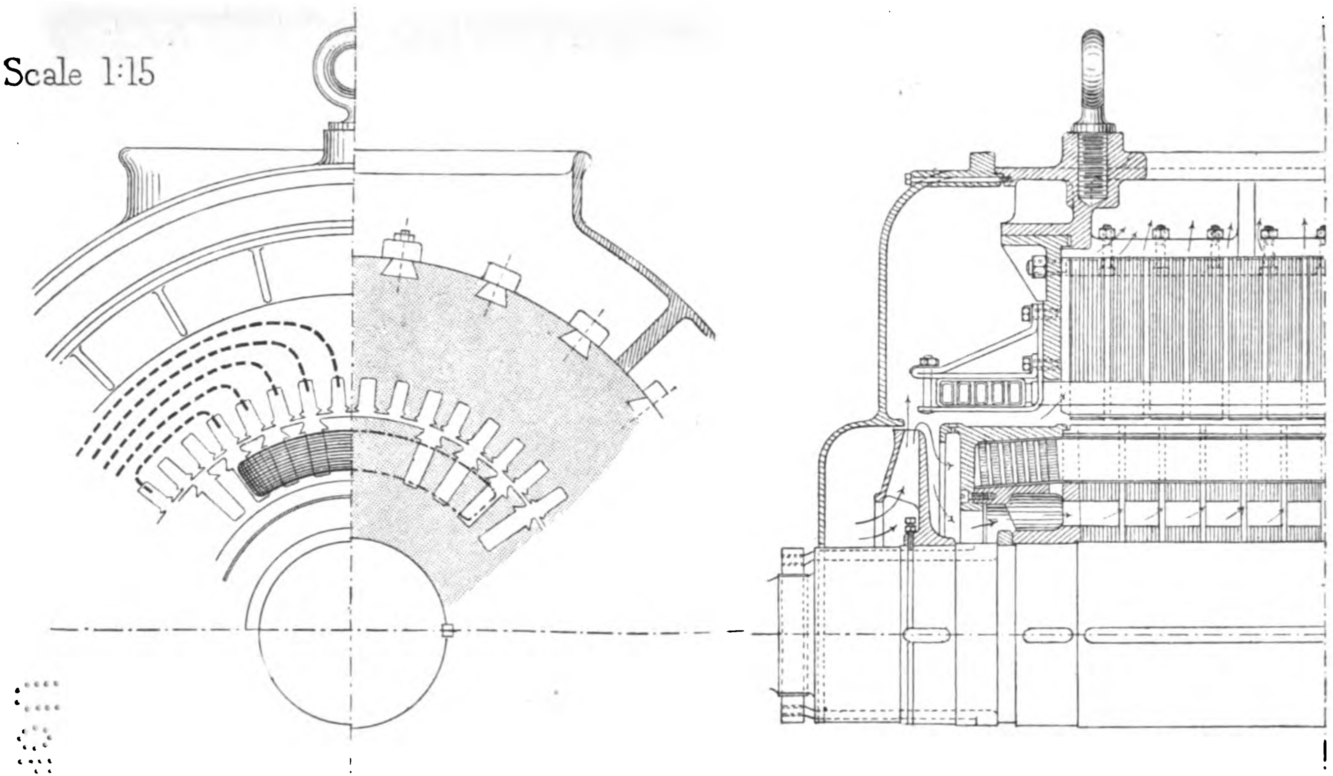


FIG. 356.—SINGLE-PHASE TURBO-ALTERNATOR (ASB 4—1000—1500), OF THE OERLIKON CO.

wedges to confine the windings. The drawing shows how the end-bends of the rotor winding are confined by strong bronze caps. It also shows how the projecting end-bends of the stator winding are held by light but strong metallic supports braced to the end-plates. There is ample provision for circulation of air in this machine, which like those of Brown is provided with a housing with a flue opening at the top ; but the arrangements for circulation are different, the housing being constructed with partitions or concentric baffles which divide the flow of air into a central, an intermediate, and an external region. It is forced into these regions by vanes on the ends of the rotor. In the earlier patterns the housing divided the space into two regions ; and the air forced into the inner region simply found its way through the gap between stator and rotor into the outer region.

The following analysis gives the principal data relating to this machine :—

ANALYSIS OF OERLIKON SINGLE-PHASE TURBO-ALTERNATOR.

ASB 4—1000—1500; 5200_{V}—$192\cdot5_{A}$—$50\sim$.

DIMENSIONAL DATA (inch units).

Armature :—

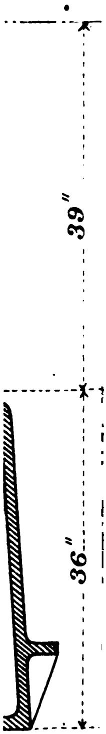
Diameter at face	34·3
Length between core-heads	41·0
Number of ventilating ducts	14
Width of each duct	0·4
Iron length of core	31·8
Total number of slots	48
Number of slots per pole wound	10
Depth of slot	2·76
Width of slot	1·10
Number of conductors per slot	4
Dimensions of conductor	4 wires of diam. = $\frac{0\cdot173}{0\cdot197}$
Total number of conductors	160
Outside diameter of armature-core	59
Depth of iron behind slots	9·6
Length of air-gap	0·4

IRBC

Westi

KVA -1

3000 vc



Field-Magnets (rotor type):—

Diameter over rotor barrel	33·5
Slots per pole for carrying magnetizing coils	6
Depth of slot	4·57
Width of slot	1·38
For spacing of slots, see Drawing.	
Conductors per slot	2 × 22
Dimensions of conductor	0·12 × 0·51
Internal diameter of magnet core	15
Depth of iron behind slots	4·7
Number of ventilating ducts	11
Width of each duct	0·4
Net length of iron in core	30·1
Length between core-heads	37·8

CALCULATED COEFFICIENTS AND DENSITIES.

Pole-pitch (inches)	26·95
Peripheral speed (ft. per min.)	13,470
Steinmetz coefficient	1·41
Esson coefficient	72,000
Winding factor of armature-coils	0·74
Winding factor of field-coils	0·715
Current density in armature copper	2050
Space factor in armature slot	0·11
Flux per pole (megalines)	39·5
Average pole-face density	56,000
Average armature teeth density	80,000
Density in armature-core	65,000
Width of armature tooth at face	1·15
,, ,, at root	1·45
Ampere-conductors per inch on armatuie	286

WESTINGHOUSE TURBO-ALTERNATORS.

In Plate XIII. is illustrated an 1800 KVA turbo-alternator of the British Westinghouse Co.; its short specification being :—

$$\text{ATB } 4_{\text{P}}\text{—}1800_{\text{KVA}}\text{—}1500_{\text{RPM}}\ ;\ \frac{8000}{4630}{}_{\text{V}}\text{—}3\times 139_{\text{A}}\text{—}50\sim.$$

The field-magnet construction is very similar to that of the Parsons type already described; but the entire magnet is cut out of one solid mass of hydraulically fluid-pressed steel To secure partial ventilation, seven ducts about $\frac{3}{4}$ inch wide are cut out from the solid steel and communicate with four circular channels about 3½ inches in diameter, which run the length of the magnet body. The magnetizing coils are laid in narrow channels or grooves ploughed out in the sides of the solid pole-cores, and are keyed in by bronze wedges. The shape of the pole-face should be noted. Starting from the centre of the pole the outline is at first concentric, the gap being constant at 1·25 inch over nearly half the pole-face, then it gradually increases to about 1·7, and then widens more rapidly to 2·25 at the tip. This form gives a flux-distribution over the armature face that closely approximates to a sine-distribution. The diameter over the pole-faces is 47·5, and the axial length 40 inches. This diameter gives a peripheral speed of 18,700 feet per minute, and a centrifugal force at the face of 1530 pounds weight per pound of material.

The armature is 50 inches in diameter and 32 inches long, with 96 slots in all, or 8 slots per pole per phase; each slot holding four conductors, each consisting of two No. 3 SWG wires in parallel, each wire being 0·0499 square inch section. There are in all 30 ventilating ducts in the armature core; and for better ventilation the housing is so designed that the air may circulate in the freest manner.

From the above data it appears that the machine has the following constants:—

$k = 2{\cdot}12$; B at armature face in middle of gap = 32,600;

$\psi = 0{\cdot}634$; q = 340 amperes per inch; N = 32,600,000;

β (Steinmetz coefficient) = 1·17; ξ (Esson coefficient) 87,500.

Armature ampere-turns per pole 6010; field-magnet ampere-turns per pole about 18,000.

One notable feature of this construction is the very wide air-gap, 1¼ inch wide, it being the Westinghouse practice to obtain good regulation on over-loads of low power-factor, by working with field-cores unsaturated and a wide gap; a practice which, as already pointed out, is important in turbine machines. In their larger turbo-alternators the British Westinghouse Co. follow the same rule. In the 3500 KVA alternators of the Metropolitan Railway at Neasden, the gap is 2·25 inches. In the 5500 KVA alternators at the Chelsea power-house of the District Railway, the gap is 3·24 inches.

The fluid-pressed steel used in the construction of the magnet bodies is chosen as having high permeability and great strength, and specially good homogeneity of structure. The steel is manufactured from selected Swedish iron, melted in a Siemens-Martin furnace, cast into a strong steel mould and subjected while still liquid to hydraulic pressure. After removal from the mould the casting is then reheated and forged by a hydraulic press, roughly machined to shape, and then carefully annealed before the final machining.

The 8 alternators supplied to the Chelsea power-house are of 5500 KW, giving 3 × 289 amperes at 11,000 volts between lines. They run at 1000 RPM, the frequency being 33⅓ cycles per second. The specification of these machines requires that the electrical efficiency shall be 97½ per cent. on normal full non-inductive load, 96½ at ¾ load, 95 at half-load, and 90 at ¼ load. The temperature-rise must not exceed 35° C. in any part of the machine at normal load. The voltage-rise with constant excitation and speed when full non-inductive load is thrown off must not exceed 6 per cent. The insulation between armature winding and frame must withstand a puncture test of 30,000 volts for one minute. The armature weighs 50 tons unwound, its windings weigh 3 tons. The revolving field-magnet weighs 37 tons. The magnet-windings are copper-ribbon 2 inches wide, 0·078 thick, held in by phosphor bronze wedges. The end of the housing next the generator is fixed, and to admit of expansion by heat the sides are held in parallel guides, while the other bearing is carried on the housing.

Fig. 357 depicts the newest standard construction of the magnets of a 2-pole, 400 KW machine for 3600 RPM at 60∼.

The method of forming and connecting up the armature conductors of this type is well shown in Fig. 258, which relates to a 4-pole, 1000 KW, 5000 volt machine, running at 1500 RPM, supplied to the De Beers Consolidated mines. It has four slots per pole per phase. The great radial depth of the core-disks behind the slots will be noticed as a feature.

The following is an abstract of the specification of a 750 KVA turbo-alternator AT(B) 4—750—1500, to give 3 × 145 amperes at 3000 volts between lines, at a power-factor of 0·80. The separately excited field-magnet requires 75 amperes at less than 100 volts when delivering normal current at normal voltage, and with power-factor unity. When delivering normal current at normal voltage and with power-factor 0·80, it will take 90 amperes. On power-factor unity the guaranteed voltage drop from no-load to full-load will be not more than 8 per cent. At normal current and voltage, and with power-factor unity the efficiencies will be:—89 per cent. at ¼ load, 91 per cent. at ½ load, 94 per cent. at ¾ load, and 95·5 per cent. at full-load. These figures do not include friction and windage, which together are estimated to absorb about 2 per cent. at full-load.

FIG. 357.—ROTATING FIELD-MAGNET, 2-POLE, 400 KW (BRITISH WESTINGHOUSE CO.).

Temperature rises as measured by thermometer are guaranteed not to exceed the following: Normal current and voltage with a power-factor of unity, after twenty-four hours, not in any part higher than 35° C. With same voltage and power-factor on an overload of 25 per cent. for two hours, a rise not exceeding 45° C. Or with same voltage and power-factor on an overload of 50 per cent. for one hour a rise not exceeding 60° C. If the power-factor be 0·80, the corresponding rises in the three cases will not exceed 40°, 50° and 65° C. respectively.

The field-magnet coils consist of copper strap, wound several layers deep in slots cut in the solid core, held in phase under wedges in top of slots with fibre blocks in addition at corners. After completion the insulation will be subjected to a flash test at 1000 alternating volts.

The armature winding will consist of copper wire in partially closed slots, being former-wound. The winding will be so grouped that connexions having wide difference of potential will not be close together. After completion the insulation will be subjected to a flash test at 6000 alternating volts. The wave form of the voltage in each circuit is substantially a sine-curve, even under an overload of 50 per cent. There will be 8 slots per phase per pole, thus ensuring a smooth wave-form.

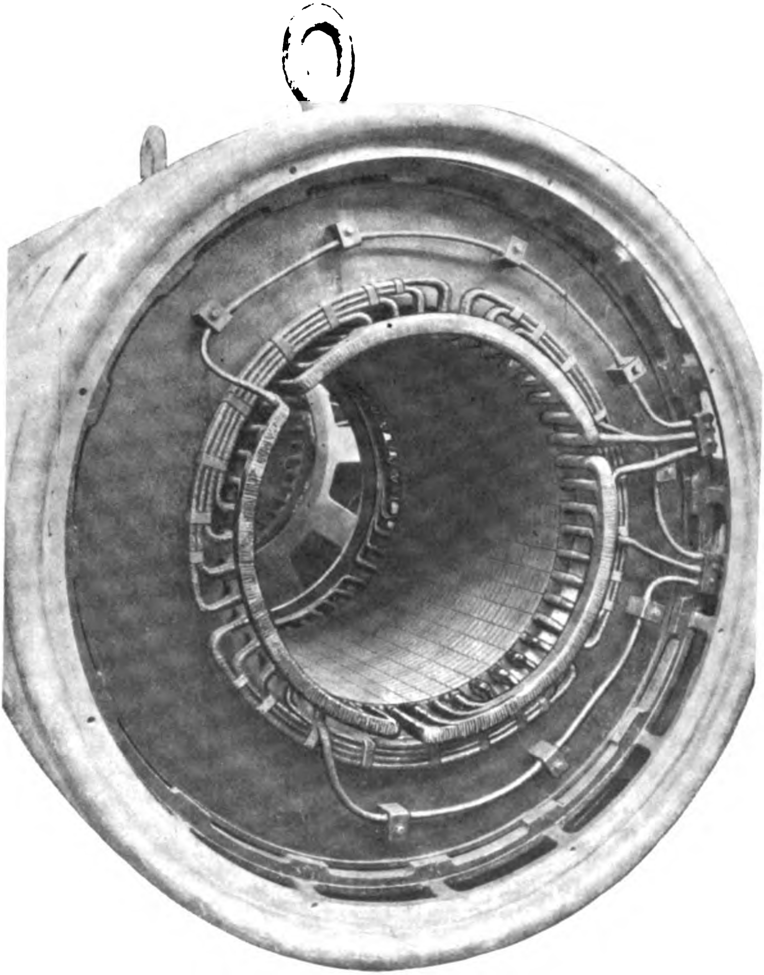


Fig. 358.—Armature of Turbo-Generator, AT(B) 4--1000—1500 (British Westinghouse Co.).

There will be two carbon brushes on each collector ring of the field-magnet.

The approximate weight is 29,000 lb.; the heaviest piece (the armature complete) weighing about 20,000 lb.

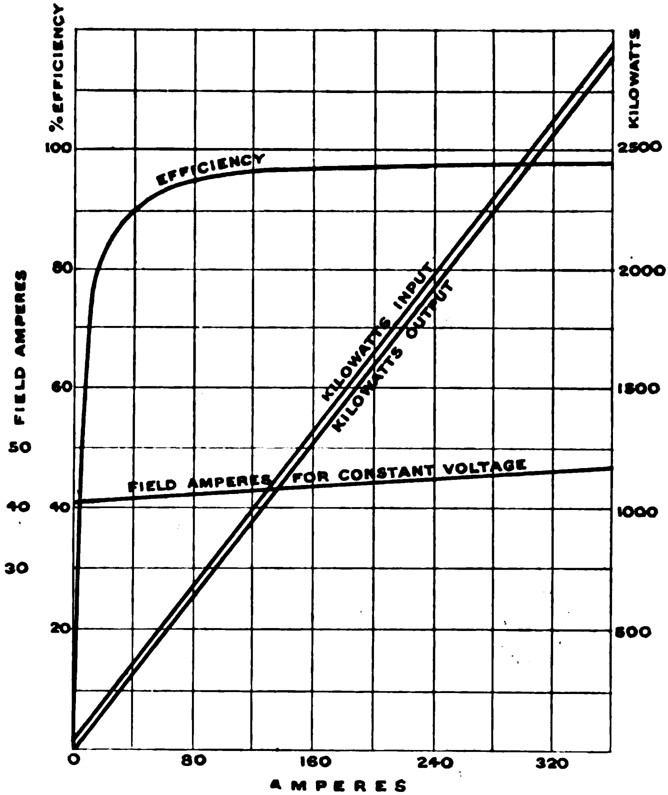


FIG. 359.—TEST-SHEET OF AT(B) 4—1800—1500 (WESTINGHOUSE CO.).

The performance of the 1800 KW alternator described above is plotted in the test-curves given in Fig. 359.

BRITISH THOMSON-HOUSTON CO.'S TURBO-GENERATORS.

The Curtis turbine, with vertical shaft, has been adopted in the United States by the General Electric Co. (Schenectady), and in England by the British Thomson-Houston Co. These companies have developed for service with Curtis turbines a

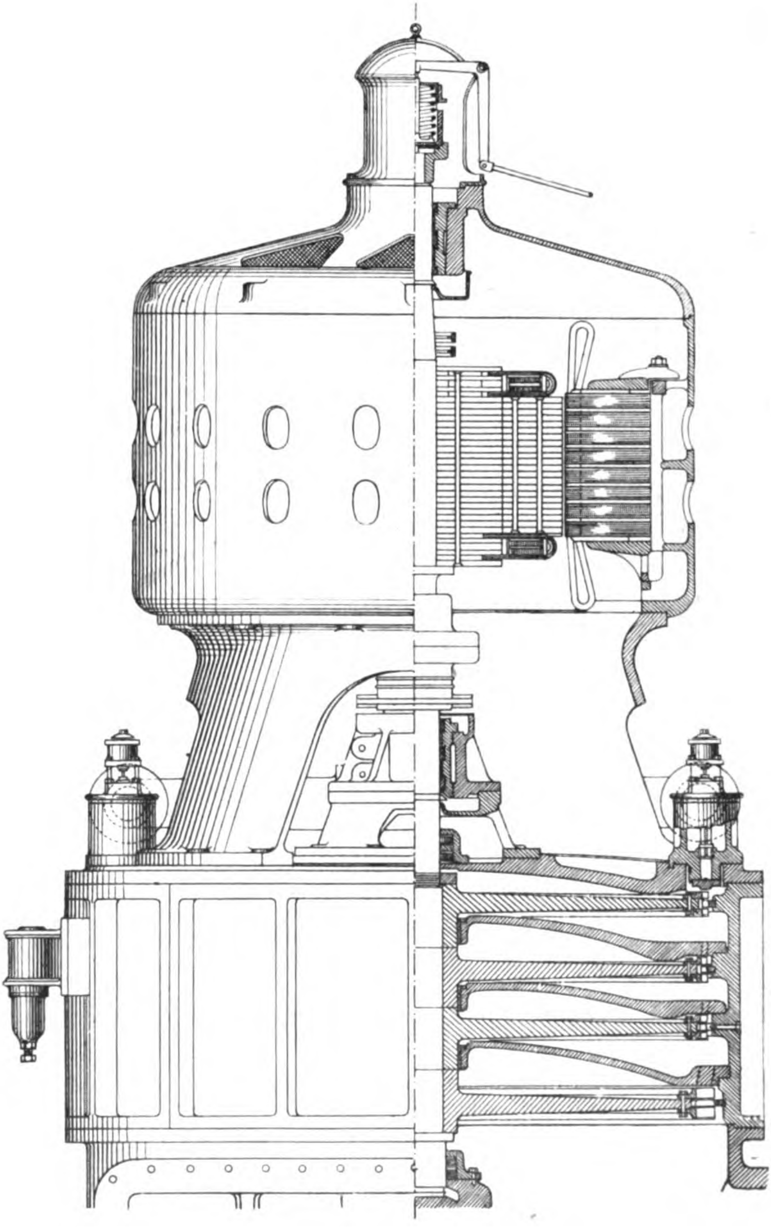


FIG. 360.—TURBO-ALTERNATOR, AT(B) 6—1880—1000, WITH CURTIS TURBINE (BRITISH THOMSON-HOUSTON CO.).

vertical pattern of alternator which has certain points of similarity with the Niagara patterns of machines already described.

Fig. 360 depicts a turbo-generator AT(B) 6—1880—1000; yielding 3 × 99 amperes at 11,000 volts between lines. The diameter of the armature at the face is 48 inches; the core-length between heads 23 inches. There are 54 slots in all, or 9 slots per pole, or 3 slots per pole per phase. Each slot carries 18 conductors. The slots are 3·2 inches deep. The coils are former-wound, all coils being alike in shape, their end-bends lying within one another in the same general mode as those of the winding depicted in Fig. 291, p. 331. The peripheral speed of the moving poles is 12,400 feet per minute.

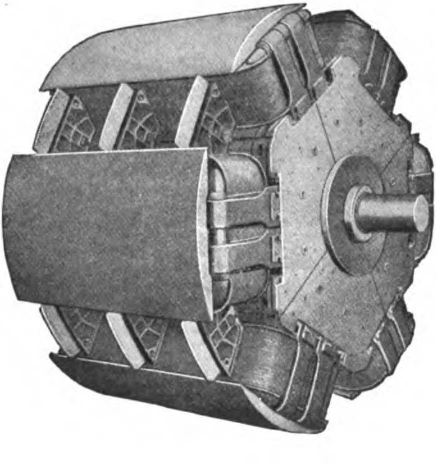


FIG. 361.—SIX-POLE FIELD-MAGNET OF THE THOMSON-HOUSTON CO.

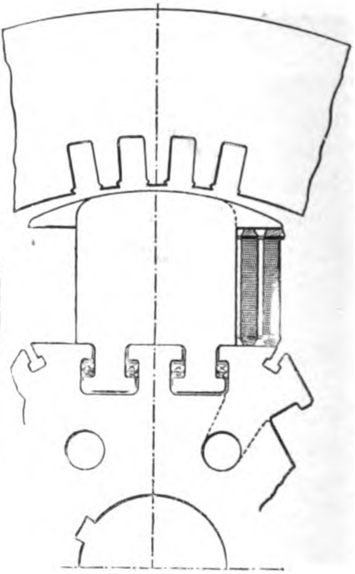


FIG. 362.—DETAILS OF THOMSON-HOUSTON MAGNET CONSTRUCTION.

To eliminate the risk of local flaws due to the unequal cooling in large castings, the Thomson-Houston Co. have in all their designs adopted a laminated construction for the field-magnets. In the earlier type, as shown in Fig. 360, the magnetizing coils are held on by bands of sheet brass, making up the thickness of one lamination, which bend round the coils and then enter the spaces left for them in the rotor body so that they may be riveted up with the rest of the laminations, making one solid piece.

In later constructions the plan shown in Figs. 361 and 362 has been adopted. In this the coils are held in position by wedge-shaped frames, which are secured by dovetails. To minimize reactions the pole-face is eccentric with the shaft, and the pole-tips are made, as shown in Fig. 362, with only half the total thickness of iron.

The following are data of a larger alternator AT(B) 6—5000—500, which gives 3 × 320 amperes, at 9000 volts between lines, at a frequency of 25~. The diameter of armature at the face is 90 inches, the core-length 50 inches, the number of slots 90, or 5 slots per pole per phase, each slot being 2·75 inches deep, and carrying 4 conductors. In this machine the constants are $B_{max} = 34{,}700$; $q = 408$; $v = 17{,}688$ feet per minute.

THE DESIGNING OF TURBO-ALTERNATORS.

The electrical designing of turbo-alternators is far more easy than their mechanical designing: for the difficulties are of much less magnitude, and the process is practically the same as that expounded in Chapter VII., though with altered constants. Assuming that either by shaping or by distributive winding the pole-flux is distributed so as to follow approximately a sine-distribution, then the maximum B_g at the pole-centre will be equal to $\frac{\pi}{2}$ times the average B_g over the whole pole-pitch. The maximum may be taken at 35,000 to 45,000, and the equivalent pole-arc will be $\psi = \frac{2}{\pi} = 0{\cdot}634$ of the pole-pitch. With a well-distributed armature-winding, say 4 slots per pole, k may be taken at 2·13. The specific loading should not (to avoid undue distortion at large overloads) exceed $q = 600$, unless the gap is very wide. In fact, the gap should be about equal to $4 \cdot 8 \times q\tau \div B$, if the iron is quite unsaturated.

Let us take, as an example, the electrical designing of a turbo-alternator, 3-phase, to give, on a power-factor of 0·86, at 6000 volts between lines, a normal current of 3 × 232 amperes, with a frequency of 45∼. The volts in one phase will be $6000 \div \sqrt{3} = 3472$; the KVA will be 3472 × 3 × 232 ÷ 1000 = 2400; the KW will be 2400 × 0·86 = 2125; the horse-power, allowing an efficiency of 97·5 per cent., will be 2125 ÷ (0·746 × 0·975) = 2850. At 45∼ the only possible speeds (see p. 446) are 2700, 1350, and 900 RPM, according as the machine has 2, 4, or 6 poles. We reject the bipolar plan as undesirable, and the 6-pole also, since (as we should find) it will make the total cost greater than the 4-pole plan. We may take $k = 2 \cdot 13$, $q = 480$, and $B = 38{,}000$. If we take as the admissible peripheral speed at the surface of the rotor 14,000 feet per minute or 2800 inches per second, and the equivalent v at the armature-face (for the purpose of calculation) as 15,500, we get the Steinmetz coefficient β by the formula X*b* on p. 356, as:—

$$\beta = \frac{31 \cdot 7 \times 10^{10}}{2 \cdot 13 \times 38000 \times 480 \times 15500 \times 0 \cdot 634} = 0 \cdot 83.$$

Hence

$$dl = \text{KVA} \times \beta = 2400 \times 0 \cdot 83 = 2000;$$

and since $d = 43 \cdot 8$, this makes $l = 45 \cdot 7$; $\tau = 34 \cdot 4$; $b = 21 \cdot 7$.

The gross equivalent area of the pole-face will be

$$21 \cdot 7 \times 45 \cdot 7 = 995 \text{ sq. inches};$$

making

$$N = 995 \times 38{,}000 = 38{,}000{,}000.$$

Then the number of armature-conductors in one phase will be $Z_1 = 3472 \times 10^8 \div (2 \cdot 12 \times 45 \times 38{,}000{,}000) = 96$ provisionally. This number will suit, as with a 4-slot winding there will be 16 slots per phase, or 48 slots all round. Hence there will be 6 conductors per slot. Checking the value of q it is found equal to

$$3 \times C_1 Z_1 \div \pi d = 3 \times 232 \times 96 \div 137 \cdot 5 = 486.$$

The suitable conductor for the armature, at $a = 2000$, will be one of section $s = 0\cdot116$. The slot-pitch is $137\cdot5 \div 48 = 2\cdot85$. To hold 6 conductors, each $0\cdot100$ thick and $1\cdot16$ high, with proper insulation, and a slot-lining of $0\cdot36$ inch, will require a slot about $1\cdot07$ wide, and at least $3\cdot1$ high. The depth of the core-stampings behind the teeth may be $9\cdot5$, making the external diameter of the core-disks $69\cdot75$, or say 70 inches.

The number of armature ampere-turns per pole at normal load will be $\frac{1}{2} \times 486 \times 34\cdot3 = 8370$. The ampere-turns per pole on the field-magnet must be about 3 times this in order to meet reactions, *i.e.* 25,110. If all these are expended on the gap, it follows that the gap δ must be such that $0\cdot3133 \times B \times \delta = 25{,}110$; and as $B = 38{,}000$, it follows that $\delta = 2\cdot1$ inches. This gives $4\cdot2$ on the diameter, so that the rotor diameter would be $43\cdot8 - 4\cdot2 = 39\cdot6$ inches. The centrifugal-force at the face will be (p. 445)

$$0\cdot0000284 \times 19\cdot3 \times 1350 \times 1350 = 990 \text{ pounds}$$

per pound of material at the periphery. The surface speed of the rotor will be 14,000 feet per minute or 233 feet per second, giving a peripheral stress of about 5600 lb. per sq. inch. To give the 25,110 ampere-turns per pole, which at full inductive load may need to be increased to, say 35,000, a winding-area will be needed (at 1500 amperes per sq. inch) of $23\cdot3$ sq. inches. If carried out as in Parsons's method, a winding-length of $6\frac{1}{2}$ to 7 inches will be available, and a winding-depth of about 4 to $4\frac{1}{2}$ inches, which is adequate. If carried out on Brown's plan, three slots will be needed on each side of each pole. The undivided part of the pole takes about $\frac{1}{3}$ of the pole-pitch, the three slots with the teeth between take about another third; hence the slot-pitch is about $3\cdot75$ inches. The three slots at one side must have a total area of at least 27 sq. inches, and may be $5\cdot5$ inches deep and $1\cdot75$ wide, giving a gross area of $9\cdot6$ sq. inches each.

CHAPTER X.

SYNCHRONOUS MOTORS, MOTOR GENERATORS, AND ROTATORY CONVERTERS.

IF two alternate-current machines are joined up in the same circuit as in Fig. 363, they are in parallel when considered as forming part of the lamp circuit, and might be both supplying current to the lamps, but they are in series with one another if we consider the alternator circuit only, for we might cut the lamp circuit out altogether and A_1 might drive A_2 as a motor. Many of the considerations which govern the running of two machines as generator and motor govern the running of two machines in parallel. We shall, therefore, in the present chapter consider the properties of the synchronous machine as a motor and in the next deal with the general questions of the running of all synchronous machinery.

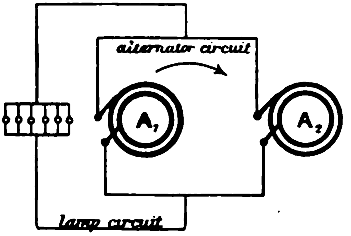


FIG. 363.

The simplest conception of two alternate-current machines in series is that of a closed conductor *a b c d*, Fig. 364, near different points of which two magnets rotate so as to cause it to cut their lines. The part *a b* may be considered as the middle conductor of an alternator coil. The change of position of the field-magnet of an alternator with regard to the centre conductor of one of its coils is represented by the angle θ in Fig. 364. All the phase relations of the magnet's position, the electromotive-force, and the current, can be seen from this figure, and the full theory (so far as at present known) of the synchronous motor can be deduced from it by the aid of a few graphic diagrams.

First of all consider that the magnet A_1 only is rotating. An alternating electromotive-force is thereby induced, and may be represented by the projection upon a vertical line of the line $\overline{O\ E_1}$, drawn to scale to represent the maximum electromotive-force E_1 and which is supposed to rotate clockwise about O in Fig. 365*a*. If the self-induction of the whole circuit is L and the resistance R, we would have from previous considerations (see page 21) the current lagging by the angle ϕ; the vertical projection of R C at any moment representing the electromotive-force which is in phase with it. Observe that a line drawn above the axis of X represents a positive electromotive-force or current, that is, an electromotive-force or current round the circuit in the direction indicated by the big arrow-heads in Fig. 364. The rapidly alternating current flowing along *c d* would tend alternately to turn the magnet A_2 clockwise and counter-clockwise so that it would not start.

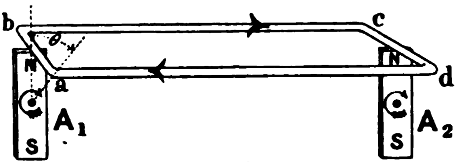


FIG. 364.

If, however, we artificially run A_2 up to the speed of A_1 it will tend to keep on running in synchronism with A_1 and exercise considerable torque. The conditions under which it continues to turn against a mechanical resistance we have now to consider.

In the first place, let us suppose that A_2 has been started in synchronism with A_1 and that at a certain instant the magnets are in the position shown in Fig. 364. The electromotive-force generated in *c d* will be negative in sense and its magnitude will depend on the strength of the magnet A_2. Suppose in the first place that the two magnets A_1 and A_2 are of the same strength, then the electromotive-force which each will induce in the circuit will be represented by the lines $\overline{O\,E_1}$ and $\overline{O\,E_2}$ in Fig. 365*b* ($\overline{O\,E_2}$ at the moment we are considering being negative). The two electromotive-forces being equal and opposite, the resultant electromotive-force in the circuit will be zero, and there will be no current to drive A_2 even against the friction of its bearings.

In the next place, let us suppose that although the magnets are rotating synchronously with one another, they are not each situated in the same position with respect to their alternating current conductors. That is, suppose that A_2 lags (or leads) in phase with respect to A_1. The electromotive-forces may now be represented by $\overline{O\,E_1}$ and $\overline{O\,E_2}$ in Fig. 365*c*, and the resultant electromotive-force will be $\overline{O\,E_3}$. If, as is always the case with alternating-current machinery, there be self-induction in the circuit, the current lags behind the resultant $\overline{O\,E_3}$ as shown in Fig. 365*a*, where the line R C represents the phase of the current. If we let fall the perpendicular $\overline{E_3 D}$ upon the direction of $\overline{E_2\,O}$ we obtain line $\overline{O\,D}$, which divided by the resistance represents the component of the current which is active in producing a torque. As this component is in direct opposition of phase to $\overline{O\,E_2}$, it produces a negative torque, that is, it drives the machine which is producing the electromotive-force $O\,E_2$ as a motor. The power developed by A_2 is proportional to the product $\overline{O\,D}\,.\,\overline{O\,E_2}$.

For every position of A_2 in phase with respect to A_1, the power component of the current has a different value, so that for every load, the motor takes up such a position with respect to the generator that by the applied and back electromotive-forces becoming thus more or less out of phase, a current flows just sufficient to cope with the load. This is the principle of the synchronous motor; and to make the matter more clear, we may make use of the following mechanical analogy.

Two fly-wheels are mounted so that the second is connected to and drawn from the first by means of springs, as in a transmission dynamometer, capable of both elongation and compression. A mark is made across the two fly-wheels to indicate the relative position of one to the other, and the whole arrangement is then brought up to a steady speed of rotation. When a steady motion has been attained, but for a small amount of friction, the marks on the wheels will, in their rotation, be in the same position with regard to one another, as when the wheels were at rest. If now, a brake is applied to the second fly-wheel, it will slip back with respect to the first in order that the spring may become elongated to such an extent as to transmit the force from the first wheel to drive the second wheel against the brake. The action which takes place between a generator and a synchronous motor is exactly similar, the relative positions of the marks on the fly-wheels after the brake is applied corresponding to the angle θ in Fig. 365*a*.

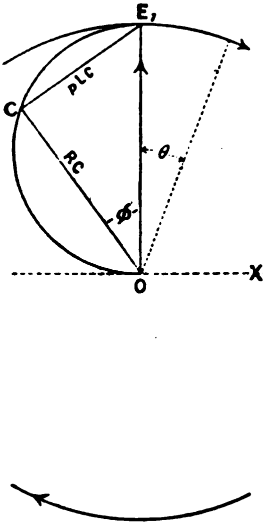


Fig. 365*a*.

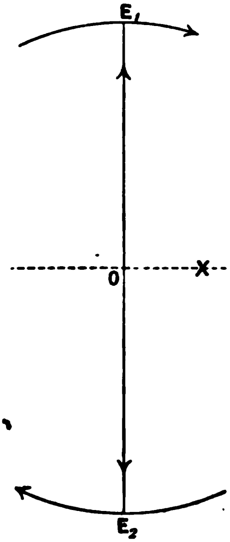


Fig. 365*b*.

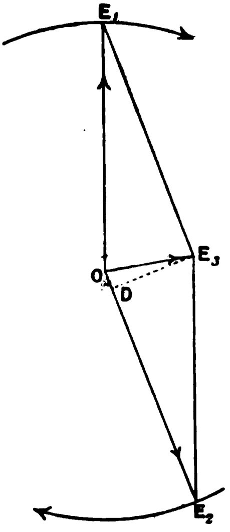


Fig. 365*c*.

So far, we have considered the magnets A_1 and A_2 to be of equal strength, so that the electromotive-forces of the machines have been equal in value. If the magnet A_1 is made stronger than A_2, so that E_1 is greater than E_2, then we may represent the state of affairs in Fig. 366*b*.

In Fig. 366*c*, the diagram has been drawn out again with the conditions reversed, the motor in this case being the more strongly excited. Both Figs. 366*b* and 366*c* have been drawn to represent the same amount of power being developed by the motor; and here it is interesting to note that, whereas in Fig. 365 the phase angle E_1OC between the line voltage and the current is a lagging one, by an alteration of the conditions of excitation as in Fig. 366, the motor is made to act like a condenser and to take a leading current, although the same load is being developed in both cases.

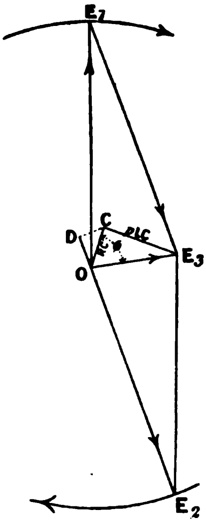


FIG. 366*a*.

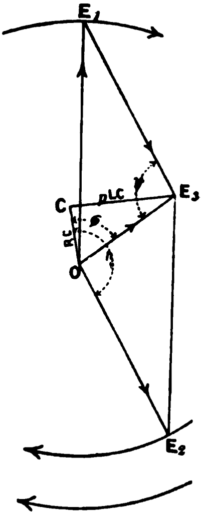


FIG. 366*b*.

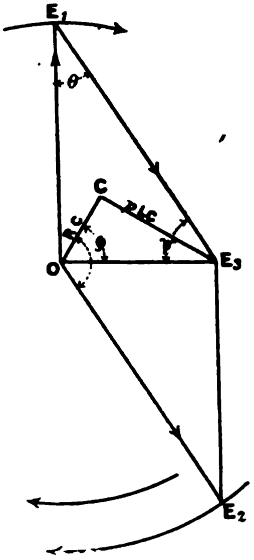


FIG. 366*c*.

Methods of Representing the Performance of Synchronous Motors.—A most usual method for studying the properties of synchronous machines is what is known as the V-curve obtained by plotting the current supplied to the motor under different conditions of excitation, the power developed by the motor remaining constant. It has been noted in the preceding paragraph that while supplying the same power the current may be made to lead or lag by altering the excitation. It is obvious that at some intermediate condition between those shown, the power-factor will be unity, and under this condition the current supplied to the motor will be a minimum; and if the excitation is increased, the current will lead and will increase in value in order to obtain the same power; or if the excitation is diminished, the current will lag and will again increase. The results in fact give a V-shaped curve as shown in Fig. 367. This is an actual curve obtained by Mr. Mordey[1] on a 50 KW machine running unloaded as a motor. Other curves situated above this one may be obtained for various loadings of the motor. In calculating the V-curve, the self-induction and resistance are not the only actions to be taken into account;

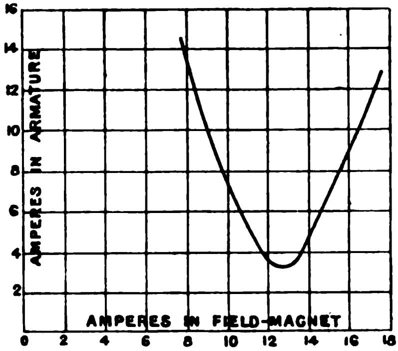


FIG. 367.

for as with alternators the magnetizing action of the armature must be considered. This may be explained as follows:—

It has already been shown on p. 255, that in a generator if the current lags behind the electromotive-force, the field system is partially demagnetized, while if the current leads the field is strengthened. In a synchronous motor the generated electromotive-force is in opposition to the applied voltage, so that a lagging current magnetizes the field system of the motor, and inversely a leading current demagnetizes.

When we consider these points in connexion with what has already been said about the V-curves, we see that as the lagging current magnetizes, the motor will behave as if it were

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[1] On the Testing and Working of Alternators, *Journ. Inst. Elec. Eng.*, Feb. 1893.

excited more than it actually is, and the current will not lag so much. Similarly, with the leading current, taking account of the demagnetizing action gives us a diagram in which the current leads less than would be predicted if only the impedance-drop were to be considered.

The whole theory of the V-curves and the interaction of the currents may be briefly stated as follows:—For any given output of power from the motor, the current may have a minimum value with a power-factor of unity or it may have any higher value than this, the power-factor being correspondingly less than unity, either lagging or leading; and the actual values which the current and power-factor take are such that the resulting field, produced by the mutual action of the under- or over-excited magnets, and the lagging or leading armature current, generates a voltage, which, when added vectorially to the voltage-drops due to resistance and self-induction, give as their resultant the applied or line volts.

The property of an over-excited synchronous motor of causing the current to be in advance of the impressed volts would enable such machines to be used to counteract the tendency of the current to lag when transformers are in circuit, and thus to increase the power-factor of the line. By over-exciting the motors still further, a resultant leading current is produced in the line, and this passing through the generator keeps the voltage to its normal value, and the system becomes self-regulating.[1]

The Power-Curves of Synchronous Motors.—The V-curves deal principally with the variations of current due to variations in excitation. Under working conditions the excitation is kept constant, and the power varies from time to time. We have seen that, in order that a synchronous motor may take a load, its revolving part slips back slightly in its phase with respect to the line voltage. If the motor were to slip back half a period, the electromotive-forces of the line and the motor would be helping one another so that a huge short circuit current would flow, which would be highly inductive and would therefore not develop much power. From this it is

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[1] See paper by Lamme, *Trans. Amer. Inst. Elec. Eng.*, xxi. May 1904.

seen that at a certain position of lag of the motor behind the generator the power developed will be a maximum. As a rule this power is several times the normal rating of the motor. Curves which show the connexion between the output and the displacement of phase (*θ*) are shown in Fig 368, the right hand part of the diagram showing the quantities plotted in rectangular co-ordinates, and the left hand portion being plotted in polar co-ordinates.

The phase displacement (*θ*) here referred to must not be confused with the lag between the line voltage and the current which the motor is taking. As we have already seen, this latter lag may be either a leading or a lagging one; but the displacement angle is always a lagging angle, whatever be the excitation of the motor.[1] The power curves are discussed further on p. 519.

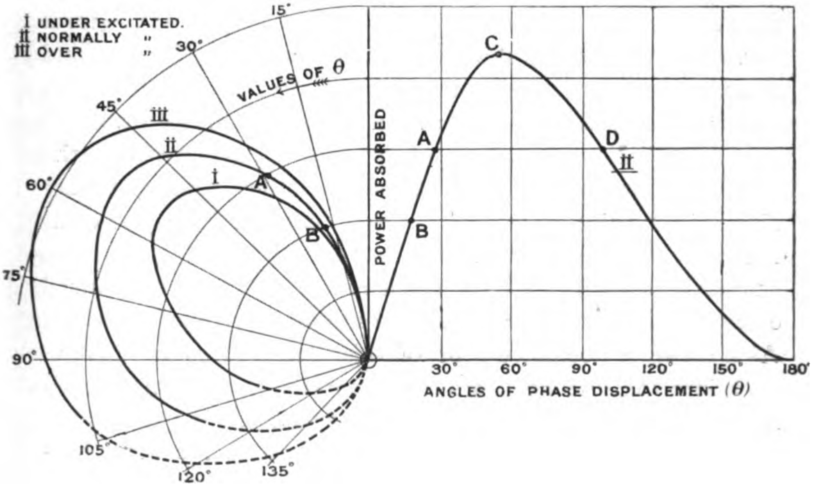

FIG. 368.

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[1] In many problems connected with alternators, the ohmic resistance of the armature is usually neglected. It has been so in Fig. 368, for in the position for no-power the motor is shown to coincide in phase with the applied voltage. From a study of Figs. 365 and 366, pp. 483 and 484, it will be seen that if the resistance be considered, in the case of under-excitation, the motor will be leading on the applied volts by a small angle at no-load, and with over-excitation at no-load the motor will have already started to lag behind the applied volts.

When a synchronous motor is calculated according to the theory given above, it often happens that the results obtained by actual test vary more than would be expected, having regard to the degree of accuracy with which the kindred problem of the regulation of generators can be solved. This is owing to the assumptions of sinusoidal distributions and wave-forms which have been made. Suppose, for instance, that the motor generated a sinusoidal wave. This can oppose that part only of the applied voltage wave which is made up of the fundamental harmonic, with the result that there is simply a short-circuit path through the armature of the motor for the higher harmonics of the applied electromotive-force wave ; and a considerable current, more or less inductive, and unaccounted for in the preceding theory, may flow. The motor itself even may not have a sinusoidal wave-form, and its harmonics may each more or less oppose those of the applied electromotive-force, but on the other hand they may also help the line volts, and a larger current of a particular harmonic will result. It may happen that owing to some characteristic point in the construction of the motor, its induced electromotive-force wave is strong in one particular harmonic. As a consequence a current of this frequency will circulate back into the line and into the generator, and the motor will act as a generator for this particular harmonic ; the power being supplied to it for this purpose through the other harmonics, chiefly the fundamental.

The design of synchronous motors proceeds on exactly the same lines as that of alternators, and the question of voltage regulation in the latter becomes a question of power-factor regulation in the former.

Synchronous motors of small sizes are rarely met with ; and with the larger sizes a high-speed design is generally adopted, this of course giving a more economical machine. As a consequence these motors often approach a design much like that of the turbo-alternator. (See Fig. 127, p. 144 ; and Fig. 317, p. 340.)

MOTOR GENERATOR SETS.

The combination of a motor with a generator may be employed to accomplish any of the following conversions:—

1. Continuous current to continuous current at a different voltage.
2. Alternating current of one or more phases to continuous current (or *vice versâ*).
3. Alternating current of one or more phases at any given frequency into alternating current of—
   (*a*) a different number of phases and same frequency
   or
   (*b*) of the same number of phases and different frequency; or
   (*c*) of a different number of phases and a different frequency.

The motor-generators of the first category have been dealt with in Vol. I. Chapter XXIV.

The conversion of the second sort, from alternating current to continuous, involves the combination of an alternating current motor with a continuous current generator; and this combination may take one of several different forms, viz:—

I. Synchronous motor driving a continuous-current dynamo;
II. Induction motor driving a continuous-current dynamo;
III. Rotatory converter;
IV. Cascade converter.

Nos. I. and II. are simply mechanical combinations of two separate machines coupled together, or built upon a common shaft. Nos. III. and IV. are electrical as well as mechanical combinations, needing particular discussion hereafter. For the present we will deal with the coupled machines of Classes I. and II. commonly described as *Motor-Generator Sets.*

For motor-generator sets synchronous motors are usually preferred to induction motors, because the power-factor of the latter is always low, and there is a 2 or 3 per cent. drop in speed on full-load.

Fig. 369 gives an example of a 250 KW induction motor-generator set installed at Sunderland. The motor takes current at 5500 volts at a frequency of 50 and the continuous current motor generates at 220 volts for the ordinary town supply. The speed of the set is 360 revolutions per minute.

Another example of an induction motor set, this time with a single-phase supply, is also of interest. The set was built by the Oerlikon Co. for use on an electric locomotive in which single-phase current is taken from the line at a voltage of 14,000 and continuous-current is obtained from the converting set to drive the train. The single-phase motor part is described in detail on p. 787, while the continuous-current generator part has already been dealt with in Vol. I. p. 756.

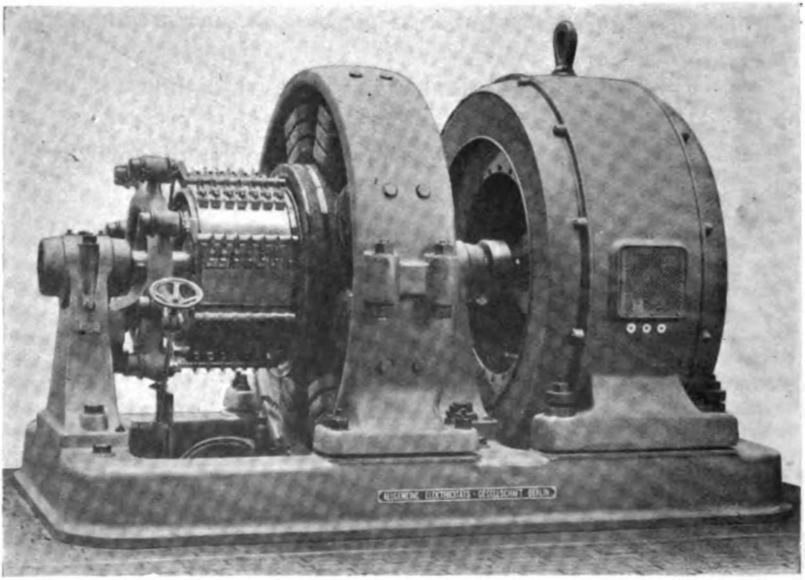

FIG. 369.—MOTOR-GENERATOR SET OF THE ALLGEMEINE ELEKTRICITÄTS GESELLSCHAFT.

An example of a synchronous motor-generator set is given in Plate XIV. It is of 250 KW capacity, by Kolben and Co.,

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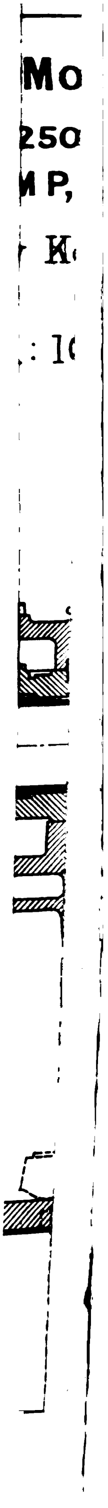
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and is installed at the Stalybridge sub-station on the South Lancashire tramway system.

The principal data of the motor and generator are here given.

A. SYNCHRONOUS MOTOR.

General Specification :—

SM, 12—250—400; 5700v—3 × 25·2A—40~.

Dimensional Data (*inch units*).

Armature :—

Diameter at face		45·7
Length between core-heads		15·8
Number of ventilating ducts		4
Width of each duct		0·4
Total number of slots		72
Slots per pole and phase		2
Number of conductors per slot		32
Total number of conductors per phase		768
Diameter of armature conductors	bare	0·15
	covered	0·17
Dimensions of slot		0·90 × 1·85
Length of gap		0·20

Field-Magnets :—

Diameter over face		45·3
Pole-span		7·5
Length of pole-face parallel to shaft		15·8
Dimensions of pole-core (oval)		14·0 × 4·7
Radial depth of pole shoe and core		7·75
Number of turns on bobbin		800
Diameter of magnet conductor	bare	0·083
	covered	0·106

Calculated Data and Coefficients.

Pole-pitch (inches)	11·9
Peripheral speed (feet per minute)	4760
Ratio of pole-span to pole-pitch	0·63
Steinmetz coefficient	2·89
Esson coefficient	54,000
No-load flux (megalines)	4·83
Average pole-face density	40,800
Current density in armature copper	1,430
Ampere conductors per inch of armature periphery	424
Slot space-factor	0·34

B. CONTINUOUS-CURRENT GENERATOR.

General Specification:—

MP 6—200—400; $\frac{480}{530}{}_{\text{V}}$—$430_{\text{A}}$.

Dimensional Data (inch units).

Armature:—

Diameter at face	33·9
Internal diameter of core-discs	16·6
Length between core-heads	15·0
Total number of slots	132
Dimensions of slot	0·4 × 1·42
Conductors per slot	6
Total number of conductors	792
Dimensions of conductors	0·08 × 0·63
Length of gap	0·157

Field-Magnets:—

Diameter of bore	34·2
Length of pole-arc	13·2
Width of pole parallel to shaft	15
Diameter of pole-core	12
Radial length of pole shoe and core	13·1
Number of shunt turns per pole	1550
Size of shunt conductor	diam. = 0·0815
Number of series turns per pole	9
Size of series conductor	Two parallel bands, each 0·059 × 2·96

Commutator:—

Diameter	26·4
Active length	6·9
Number of segments	396
Width of each segment at face	0·178

Calculated Data and Coefficients.

Pole-pitch (inches)	17·75
Ratio of pole-arc to pole-pitch	0·75
Peripheral speed at armature face (ft. per min.)	3550
Steinmetz coefficient	2·54
Esson coefficient	34,500
No-load flux (megalines) (for 530 volts)	10·0
Average pole-face density	68,000
Current density in armature conductors	1390
,, in shunt ,,	620
,, in series ,,	1200
Ampere-conductors per inch of armature periphery	520
Slot space factor	0·53

ROTATORY CONVERTERS.

These are machines for converting alternating-currents of one, two, or three phases into continuous-currents, or *vice versâ*. They are needed for several different purposes in the electrical industry. Long-distance transmission, with its corollary of the employment of high voltages, has demanded for this service alternating currents. The development of the electric tramway with continuous-current motors has called for methods of feeding at distant points. To feed such a tramway from a long distance, rotatory converters are therefore required. They are also needed for charging accumulators in cases where the public supply is an alternating one. Both these cases might, of course, be met by the use of coupled machines, motor and generator, the motor being either of the synchronous or asynchronous type. But in all such cases the efficiency of the group is necessarily lower than that of either of its component parts. If a motor having, say, a 90 per cent. efficiency is coupled to a dynamo, also of 90 per cent. efficiency, the efficiency of the group cannot possibly exceed 81 per cent. For factory driving with three-phase motors, in cases where the public supply is one with continuous currents, rotatory converters are also wanted.

Two other solutions to the general problem are possible. One is to wind the revolving armature with two sets of windings—one to receive the primary current and revolve as a motor, the other to generate the secondary current. As only one field-magnet and one pair of bearings are required, there is an obvious economy of material, though no great saving in efficiency. Another solution—the more usual one, having the advantage of effecting a considerable increase of efficiency as well as an economy in material—is to wind the armature with but one set of windings, furnished at the end with a commutator, and at the other with appropriate contact rings, the same winding serving both to receive the incoming primary current and to generate the secondary current. It is this specialised type of machine which is called *par excellence* a rotatory converter.[1]

Let us consider a bipolar ring armature, having at its periphery 96 conductors, connected symmetrically down to a 48-part commutator, running at 1200 revolutions per minute, or 20 revolutions per second. There will be two turns of the winding between each commutator bar and the bar next adjacent. That it may run as a 100-volt continuous current machine the magnetic flux through the armature core must be a little over 5 megalines. Fig. 370 will serve to represent diagrammatically this armature when receiving 100 amperes at 100 volts, and running as a motor. The flow of current in the armature winding will, of course, be 50 amperes in each half of the ring, and the input will be 10 kilowatts.

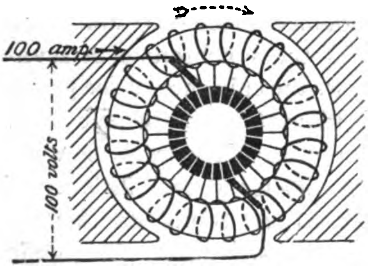

FIG. 370.

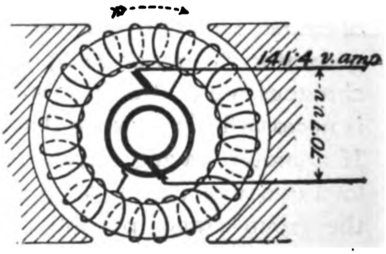

FIG. 371.

Now, suppose a precisely similar armature to be revolving in a precisely similar field, but let its windings be connected at two diametrically opposite points to two slip-rings on the axis, as in Fig. 371. If driven by power it will generate an alternating current. As the maximum voltage between the points that are connected to the slip-rings will be 100 volts, and the virtual volts (as measured by a voltmeter) between the rings will be 70·7 ($= 100 \div \sqrt{2}$), if the power applied in

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[1] The alternating and continuous voltages being both obtained from the same armature, they will be of the same order of magnitude, so that for use with high-pressure transmission the converter requires the use of step-down transformers, whereas, in motor-generator sets, the high voltage may be used direct in the motor armature. But even with the transformers the converter sets are cheaper and more economical than the motor generator equipment.

turning this armature is to be 10 kilowatts, and if the circuit is non-inductive, the output in virtual amperes will be 10,000 ÷ 70·7 = 141·4. If the resistances of each of the armatures is negligibly small, and if there are no frictional or other losses, the power given out by the armature which serves as motor will just suffice to drive the armature which serves as generator. Accordingly, let us suppose them both mounted upon the same shaft, as in Fig. 372, and placed so that each lies in a similar and equal bipolar magnetic field. We have here the well-known combination of a *motor-dynamo*. In every actual case there are, of course, losses (*a*) by friction, hysteresis, and eddy-currents, (*b*) by heating of the resistances in the armatures The former have to be paid for by an increase in the motor current. Suppose them in the present case to amount to 4 per cent. for each armature, then 108 amperes instead of 100 must flow in from the supply circuit. The heat losses manifest themselves electrically by a fall of potential at the terminal of the generator, and by a fall of speed in the motor if the primary voltage is not raised. Let the primary voltage be supposed to be raised the requisite small percentage to keep up the speed and to maintain the secondary voltage at 70·7 on the generator side: the output of the generator will then be 10 kilowatts, the input at the motor 10 kilowatts *plus* the

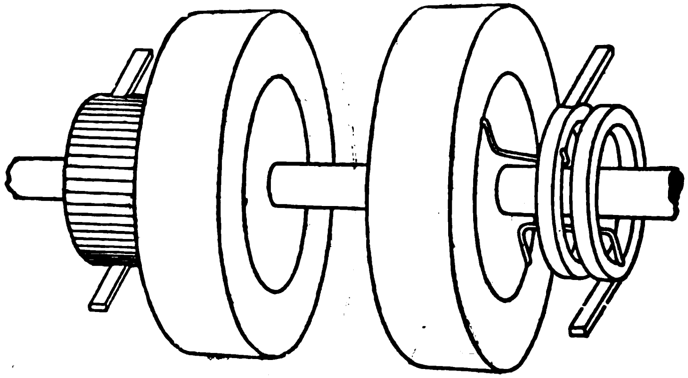

FIG. 372.—MOTOR-DYNAMO.

number of watts required to make up all the various items of lost power.

Though the armatures are of equal resistance, and are respectively receiving and giving out (approximately) equal amounts of electric energy, the armature of the alternate current side (whether used as generator or motor) will heat more than that of the continuous-current side; for it carries $\sqrt{2}$ times as large a current, and the ohmic heat will be proportional to the square of this, or twice that produced in the continuous-current armature by the 100 amperes of working current. In each armature the heat will be developed equally in all the separate coils around the ring.

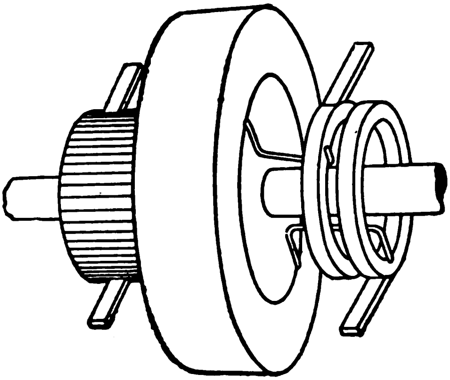

FIG. 373.—CONVERTER.

As the armatures are alike, and as the similarly placed windings in each are passed through identical magnetic fields, there is no reason why one winding should not answer for both purposes. Fig. 373 shows the case in which this change has been made. One armature winding only is used; it is connected at one end to the commutator, at the other to the two slip-rings, and the machine now becomes a simple *rotatory converter*. The total hysteresis and eddy-current losses will obviously be now one-half of their former amount. The total heating due to resistance will be also reduced, because now the single winding has to carry *only the differences of the two currents*, and the ohmic losses will be less. The waste of power, in fact, is approximately halved. How far this economy of losses can be carried depends on the number of phases of the alternating current. But there is another consideration involved—the question of armature reactions and sparkless collection of current. In the combined pair of machines called a *motor-dynamo* the brushes on the continuous-current side must be set, exactly as in any continuous-current machine, with a lead, negative or positive, according as whether it is operating as motor or dynamo. In the *rotatory converter* no lead in either sense need be given to the brushes; for the armature reactions of the motor part being, in general, opposed by those in the dynamo part, they cancel one another to a large extent. This property is common to all those motor-generators in which there is used, whether with one winding or two, a common core in a common field.

The relations between speed and field are peculiar. In the case of those grouped machines, or motor-dynamos (Fig. 372) in which each armature revolves in its own field, the conditions differ from those of the converter (Fig. 373), where there is only one field. If in either case the continuous-current side is the primary (*i.e.* motor) side, the speed of revolution will depend on the field-magnet, the weakening of which will increase the speed. The frequency of the secondary or alternating current will in that case also vary. But the ratio of the primary and secondary voltages will be independent of speed if the fields are alike, or if only one common field is used. The secondary voltage cannot be varied, while the primary voltage is kept constant, unless separate fields and separate windings are employed (see p. 505).

If, on the other hand, the alternating-current side is used as primary, then the machine, whether motor-dynamo or converter, runs as synchronous motor with a fixed speed, and all the considerations concerning the synchronous motor given in the first part of this chapter apply. In this case the voltage ratio remains also nearly constant, even though the excitation of the field is increased or diminished, owing to the peculiar phase relations which take place, as is known, in the currents of synchronous motors when under-excited or over-excited.

Coming to the question how the current in the armature of the converter actually gets through the windings, it will be found that a simple means of answering it is afforded by the principle of the superposition of instantaneous values of currents. Still taking the same concrete case given in Figs. 370 and 371, we may calculate the instantaneous values of the currents in its windings at different epochs in its period of rotation. If we do this for the motor currents and for the generator currents separately, and then superpose them, we shall obtain a number of instantaneous values for the combined armature as converter. This has been done in the series of drawings in Fig. 374. If the continuous-current side acts as motor, taking 100 amperes, there will be 50 amperes flowing in each half of the winding at every instant—*qua motor*—in a direction opposing the electromotive-force in the winding. This is shown in the first diagram in the upper row of Fig. 374. As a single-phase generator the armature is working, as already described, with a virtual voltage of 70·7 volts, and an output (supposing the power factor be unity) of 141·4 virtual amperes. The maximum current will be 141·4 × $\sqrt{2}$ = 200 amperes; and this will be attained at the instant when the two conductors which lead down from the windings to the slip-rings are turned to the position of 90° to the axis joining the middles of the pole-faces. The eight diagrams which follow in the top row of Fig. 374 exhibit the successive values of the current flowing in the armature—*qua generator*—in positions differing successively by 15°. By supposing these eight diagrams successively superposed on the motor diagram which precedes them, we obtain the eight diagrams of the lower row which exhibit the currents actually flowing in the different parts at the particular times when the armature has the position shown.

On examining these diagrams it will be seen that the currents in the armature windings consist of a set of four currents, each in position 0° being 50 amperes. But as the armature turns these change. They increase in the two arcs that are shortening as the points where the slip-ring connectors approach towards the positions occupied by the commutating brushes

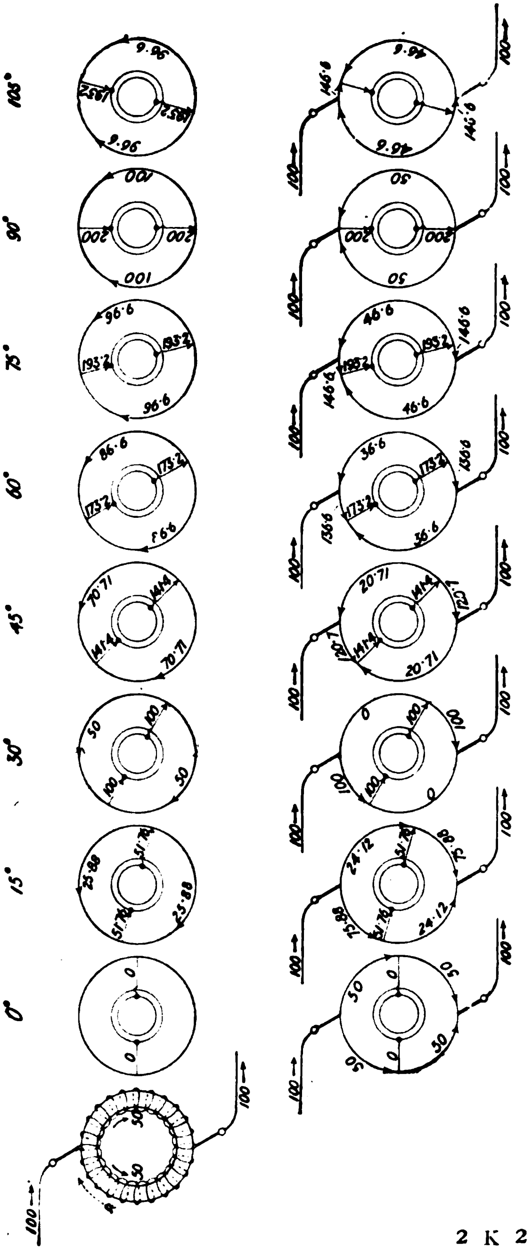

FIG. 374.—DIAGRAM OF CHANGES OF CURRENT IN ARMATURE OF SIMPLE SINGLE-PHASE CONVERTER.

But in the two other arcs that are lengthening, the currents first decrease to zero, then increase again to 50.

Further, the individual parts of the ring winding have very different currents to carry. A coil which is midway along the winding between the two connectors has to carry never more than 50 amperes. Four times in each revolution it carries 50 amperes, and four times it carries none, with intermediate values in intermediate positions. But a coil that is situated next to either of the two connectors has to carry a current which, when it is close to the brush position, rises to a maximum of 150 amperes, and changes abruptly to 50 in passing the brush; rising thus gradually twice to 150, or else rising twice abruptly from 50 to 150, and being zero twice in one revolution. The necessary consequence of this is, that the coils which are close to the slip-ring connectors are much more heated than those which lie midway along the periphery between the connectors. The distribution of the heating is here quite different from that which would obtain in either armature of the motor dynamo machine (Fig. 372). This unequally distributed heating effect is in total somewhat greater than that in the same armature if used purely as a continuous-current motor or generator, but it is less than the total heating effect when this same machine is used to give out the same power as *alternating* current, the ratios of the heating being as 1 : 1·37 : 2 in the three cases. Or, if we consider the output of an armature to be limited by equal heating effects, the several outputs which would give equal total heating would be: as continuous-current generator or motor only, 100 kilowatts; as single-phase generator or motor only, 50 kilowatts; as single-phase converter, 85 kilowatts.

Let us now pass to the consideration of the currents in the armature of a three-phase converter. Fig. 375 gives a series of diagrams to illustrate this case in the same manner as before. The current in any one of the three lines is at its maximum (on a non-inductive circuit, when generator; or when, as motor, the field has normal excitation) when the connector from that line is just passing a commutator brush; and its maximum is $1\frac{1}{3}$ times the continuous current. In

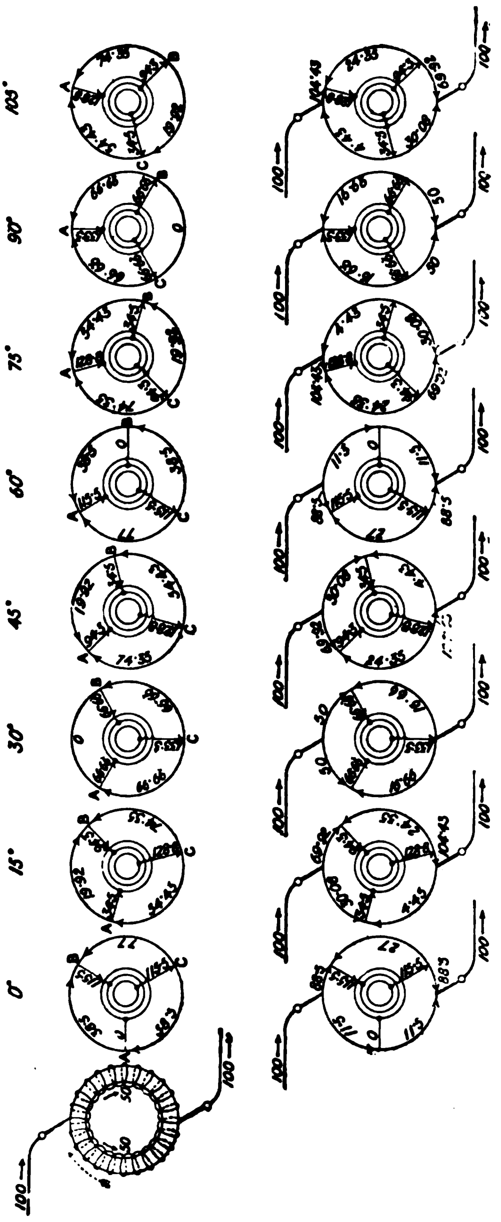

FIG. 375.—DIAGRAM OF CHANGES OF CURRENT IN ARMATURE OF SIMPLE THREE-PHASE CONVERTER.

certain positions of symmetry—for example, in position 30°—the motor and generator actions resulting from the flow are bilaterally similar. In other positions—for example, in position 0°—one side (the left here) is acting wholly as motor, the other side partly as generator and partly as motor. A coil which lies midway between two connector ends will carry a current that has a maximum of 27 in one part of its revolution, and another maximum of 50 in another part of its revolution, there being two each of these maxima in one revolution. A coil situated close to a connector has 133·3 as its maximum twice in each revolution, with an abrupt change to or from 16·66 just as it passes the brush. The inequality of heating of coils is less in a three-phase converter than in a single-phase, but it is greater than in a two-phase converter. The latter case has been examined analytically by Woodbridge and Child.[1] Among other conclusions arrived at, they find that assuming a power factor of unity, if such a converter is driven mechanically so as to be generating *both* a continuous and a two-phase current at the same time, the heating of the armature will be less than would be the case if, with same output, it were used as either a continuous-current generator alone or a two-phase generator alone.

The relation between the respective voltages on the alternating-current side and the continuous-current side of a rotatory converter have long ago[2] been investigated, and expressions for their values in the several instances that may arise have been given for those cases in which it is assumed (*a*) that the alternating currents are simple sine-functions of the time, and (*b*) that the magnetic flux is distributed as a sine-function in space with respect to the periphery of the armature. If we take the continuous currents as being supplied at a constant pressure of 100 volts, the voltmeter readings at the alternate-current side will be as given in the following Table XIII.

Star groupings are out of the question here, as they cannot be applied in armatures of converters. The only two-phase case that is possible is really a four-phase.

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[1] *Electrical World*, xxxi. 12, 216, 1898.

[2] See Ayrton in *Journ. Inst. Elect. Engs.*, xxii., 340, 1893.

TABLE XIII.—VOLTAGE RATIOS OF CONVERTERS.

Number of Slip-Rings.	Angle between Connexions to Rings.	Nature of Service Generated.	Voltage Ratio.	Voltage (virtual volts).
2	180°	single-phase	$\frac{1}{\sqrt{2}}$	70·71
3	120°	three-phase	$\frac{1}{2}\frac{\sqrt{3}}{\sqrt{2}}$	61·23
4	90°	as two-phase	$\frac{1}{\sqrt{2}}$	70·71
4	90°	as four-phase	$\frac{1}{2}$	50·00
6	60°	as three-phase	$\frac{1}{2}\frac{\sqrt{3}}{\sqrt{2}}$	61·23
6	60°	as six-phase	$\frac{1}{2\sqrt{2}}$	35·35

It is easy also to calculate the corresponding relative values of the working currents in the armature and in the line wires.[1] If we assume the above values for the voltages, and proceed to calculate the corresponding currents for an output of 10 kilowatts, we find the values of the currents generated to be as follows, the circuits being supposed non-inductive.

TABLE XIV.—WORKING CURRENTS IN ALTERNATING-CURRENT ARMATURES.

Number of Slip-Rings.	Angle between Connexions to Rings.	Nature of Service.	Armature Current	Line Current.
2	180°	single-phase	70·7	141·4
3	120°	three-phase	54·5	94·3
4	90°	two-phase (4 wires)	50·0	70·7
6	60°	six-phase	47·2	47·2

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[1] A very complete discussion of the voltage relations, with formulæ applicable to the cases of open-coil windings as well as of closed-coil windings, was given by Herr R. M. Friese in the *Elektrotechnische Zeitschrift* of February 15, 1894. Throughout the series of articles he assumes the sine-values of the distribution of the magnetic flux around the periphery of the armature. More recently, Mr. Steinmetz has reconsidered the same problem in the same journal in articles which appeared on March 3 and 10, 1898.

If the current is not in phase with the voltage, the current may of course be obtained by dividing the values given in the table by the power-factor.

Any variation of the flux distribution from a sine-form will alter the conversion ratio. An example of this is afforded by a 4-pole 55-kilowatt converter constructed by the Oerlikon Company. As originally constructed the poles were furnished with pole-pieces, each of about 72° span, therefore having a breadth about 80 per cent. of the pole-pitch. So shaped, the coefficient of conversion was found to be 57·7 per cent., as against the 61·23 per cent. if sine distribution had been present. When supplied at 300 volts on the continuous-current side, its open-circuit three-phase voltage was only 173 volts. The pole-tips were then cut away, so that the arc of pole-span was reduced to a little under 50°; the pole-breadth being now 54 per cent. of the pole-pitch. This had the effect desired of bringing the conversion ratio up to 66·6, the open-circuit three-phase voltage being now 200. The ratio of conversion was constant within 2 per cent. at all loads.

Prof. Gisbert Kapp[1] has examined the question of the influence on the conversion ratio of changes in the distribution of the flux. With the theoretical case of a sine-law distribution he compares analytically other cases, equally unreal in fact, namely, those of a distribution of supposed uniform density over a limited arc of polar span. From his calculations it may be deduced that the conversion ratio will be the same as for the sine-law distribution, if the poles are such that the breadth of their faces is about 70 per cent. of the pole-pitch. If the faces are broader, the voltage ratio for a single-phase machine will be lower than 70·7 per cent., and the ampere ratio higher than 141·4 per cent. If the pole-breadth is reduced to $\frac{2}{3}$ of the pole-pitch, or to $\frac{1}{2}$ the pole-pitch, the respective voltage percentages become as given in the following table.

The problem of changing the voltage conversion ratios outside the range accomplished by a shaping of the pole or an alteration of the number of phase connectors, has engaged

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[1] *Elektrotechnische Zeitschrift*, xix. 621, September 15, and following numbers of September 22 and 29, 1898.

TABLE XV.—VOLTAGE RATIOS AND POLE-ARC RATIOS.

Voltage for	Sine-Law.	Pole Ratio, ⅔.	Pole Ratio, ½.	Relative Heat-waste.
Continuous currents . .	100	100	100	100
Single-phase . . .	70·7	75	82	115
Two-phase . . .	70·7	75	82	115
Three-phase . . .	61·2	65	71	59
Four-phase . . .	50·0	53	58	39
Six-phase . . .	35·35	37	42	27

the attention of several engineers. Mr. Heldt has described[1] an ingenious method of obtaining any desired ratio by the device of including on the alternating-current side additional windings between the slip-rings and the points where these rings are connected up to the armature winding proper. These windings may be wound either forwards or backwards so as to allow a larger voltage on either the alternating or continuous side.

COMPOUNDING OF CONVERTERS.

As already pointed out, the theory of the synchronous motor applies equally well to the rotatory converter; for as the converter is loaded it drops back in angular phase with respect to the generator; and by altering the excitation the current is made to lag or lead accordingly. This is a useful property: for if the continuous-current load is sent round a few series turns on the magnets the current will lead without any adjustment of the shunt rheostat. The result of this is explained by Fig. 376, where O A corre-

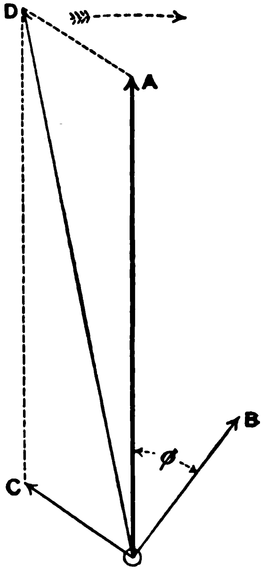

FIG. 376.

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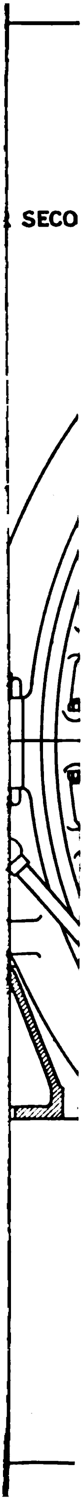
[1] See *Electrical World*, xxviii. 68, July 18, 1896.

sponds to the applied volts and O B represents the current leading. The self-inductive drop OC is 90° behind this current.

The generated voltage and O A must add up vectorially to this voltage O C (leaving out of consideration resistance, for it is small) and the generated back-voltage is therefore represented by O D which is considerably greater than the voltage O A that would be generated, at no-load, when the loading was absent, and when the series-winding was conveying no current. If the alternating voltage is hereby increased, so also is the continuous electromotive-force, and so the voltage in the continuous current lines rises automatically with the load.

Plate XV. gives a good example of a modern converter. This is by the British Westinghouse Co. and is of 1500 KW capacity running at 200 revolutions per minute. The machine was supplied to the Brighton Corporation, and is built for a voltage of 240 to 280 on the continuous-current side with an output of 5350 amperes, which, as there are 30 poles, gives about 350 amperes for each set of brushes. The armature diameter is 108 inches, and the length 12½ inches. This gives a pole-pitch of 11·3 inches and Steinmetz and Esson coefficients of 0·9 and 19,400. This bears out the argument on p. 500, that a converter is always smaller than a continuous or alternating-current machine of the same power. There are 360 slots in the armature, each slot containing four conductors of dimensions ½-inch by $\frac{3}{32}$-inch. The apparent current density in the armature is therefore 3720 amperes per square inch. The air-gap is ¼ inch. This converter is free from hunting troubles provided the engine speed is fairly uniform, and it stands overloads of 100 per cent. without the slightest sparking. This is all the more satisfactory, seeing that the frequency is as high as 50 cycles per second. To obtain such a result every care must be taken in the design in allowing ample dimensions to all active parts. The magnet-frame is of rather unusual construction, consisting of what is really a cast-iron housing into which the T-ended pole-stampings are directly cast. Contrary to usual practice the machine has been built as a three-phase and not as a six-phase converter, as is usually done. The reason of this is that it is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found that for large machines in which the dimensions are amply proportioned the gain in internal losses by the use of

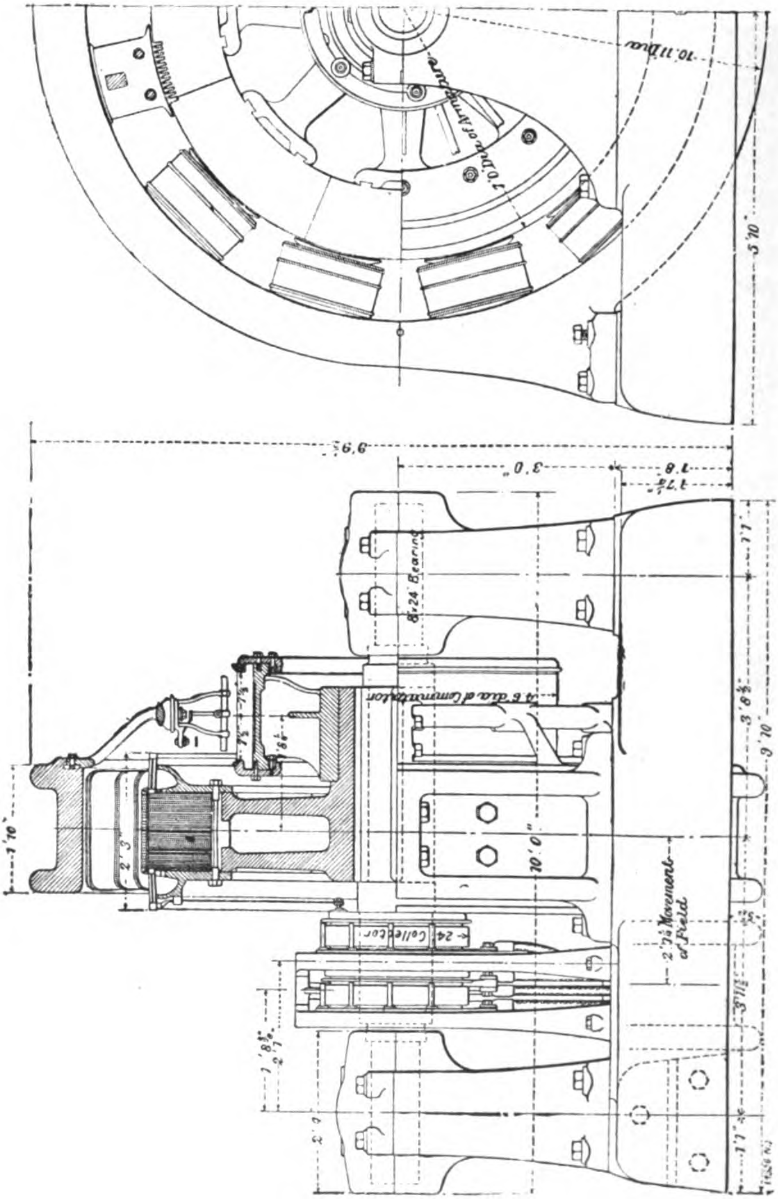

FIG. 377.—THREE-PHASE CONVERTER, DESIGNED BY MR. PARSHALL FOR CENTRAL LONDON RAILWAY.

six-phases is small, and this gain does not compensate for the multiplicity of connexions to the machine both on the armature and the transformers when the number of poles becomes very large. The great thickness of the slip-rings allows for wear.

Fig. 377 gives sections of the three-phase 25-cycle converters supplied to the Central London Railway. They are 12-pole, 900 KW machines running at 250 revolutions per minute. Each weighs about 30 tons, and their guaranteed efficiency is 95 per cent. at full-load, and 93 per cent. at half-load. They take on the alternating-current side 1800 amperes per phase at 310 volts and give out continuous current at 550 volts, this making 275 amperes per set of brushes. With an armature diameter of 84 inches a pole-pitch of 18·8 inches is obtained. The armature length is 13½ inches, which makes the Steinmetz coefficient 1·26 and the Esson coefficient 162,000. The armature has 432 slots, each slot containing two conductors. The poles are laminated and are bolted up to a solid cast-iron yoke. The construction of the slip-ring brush gear should be noted. Pedestals are erected on either side of the machine, and are bolted together over the slip-rings, thus making a very solid arrangement.

For further discussion on the design and construction of rotatory converters and motor-generators see Waters in *Trans. Amer. Inst. Elec. Engs.*, xxiv. July 1905.

CASCADE CONVERTERS.

Recently a novel species of apparatus, having properties intermediate between those of the motor-generator set and the rotatory converter, has been introduced by Professor Arnold and Mr. J. L. La Cour. It is denominated by them a *Cascaden-Umformer*, or cascade converter, and has been put on the English market under the name of "Motor Converter" by Messrs. Bruce, Peebles and Co. Briefly stated, the cascade converter consists of a combination of an induction motor on the same axis as a continuous-current generator, but having the windings of the generator armature joined in cascade, that

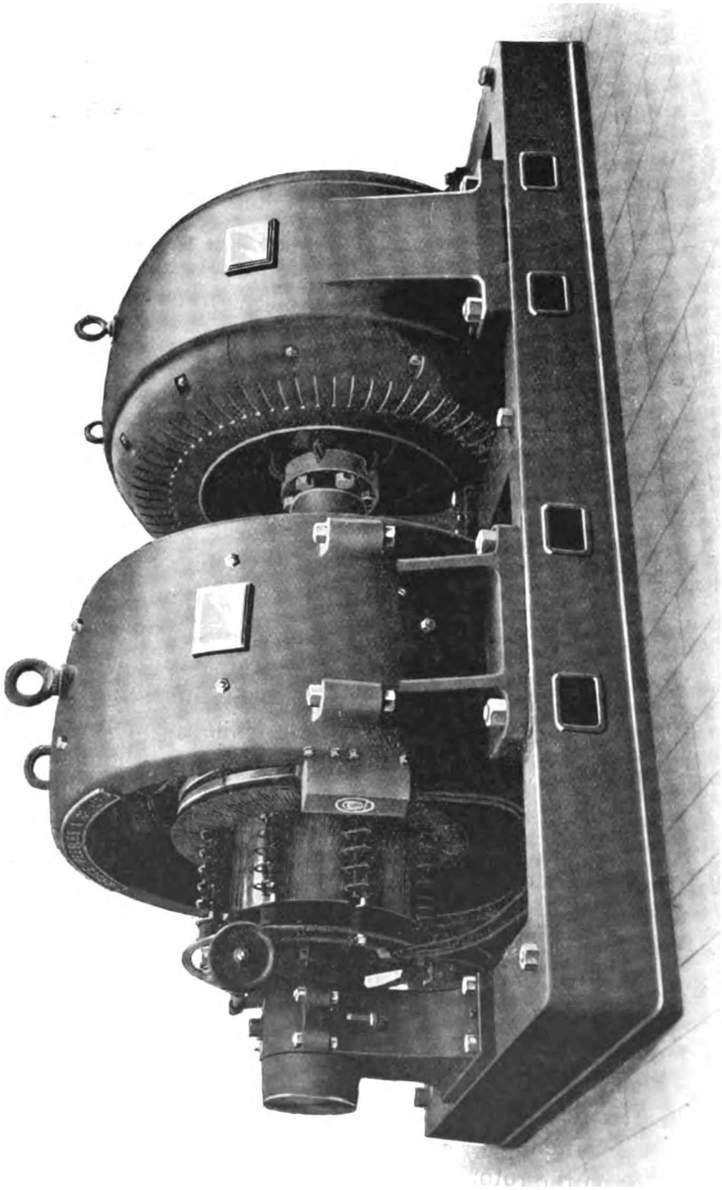

Fig. 378.—Cascade Converter of Arnold and La Cour, Constructed by Bruce, Peebles & Co.

is in series, with those of the rotor of the induction motor. The line supplies 3-phase currents at high voltage direct to the stator of the induction motor and drives it, generating in it currents at a lower voltage depending on the ratio of the windings; but part of the current so generated in the rotor passes at once into the armature of the generator part and is "converted" by the commutator into continuous currents as in a rotatory converter, but is also augmented by currents generated in the windings oi the generator part. Fig. 378 gives a general view of the combined machine. The motor is at the right, the generator on the left; its commutator and the brush gear are seen.

Fig. 379 shows the revolving part of the machine; the rotor of the induction motor is on the right, the armature of the generator on the left. At the end of the continuous current armature, visible in Fig. 379, there are sets of connexions brought down from the windings at equipotential points, and to these are brought, by wires which run through the hollow shaft, the currents from the rotor winding. The rotor and armature run at a speed which (assuming equal numbers of poles in both parts) is *exactly* half the speed of synchronism with the supply circuit. Thus, if the motor is 6-pole on a frequency of 50∼, the speed of revolution of the flux in the stator will be 50 × 60 ÷ 3 = 1000 RPM; while the motor will run at exactly 500 RPM. The rotor windings, therefore, generate alternating currents of half the frequency and supply these to the armature of the generator, and as the connexions are so arranged that these currents tend to set up in the armature a revolving field, rotating at half speed in a sense opposite to that in which the shaft is rotating at half-speed, it follows that by the superposition of this revolving field upon the revolutions of the machine, the magnetic effect is equivalent to a rotation of the armature at whole speed, so that it operates in synchronism, as the armature of a rotatory converter does. Half the electric input into the motor part is, therefore, turned into mechanical energy to drive the shaft, the other half acts inductively on the rotor windings, generating currents therein; so the "motor" part is half motor, half transformer. As to

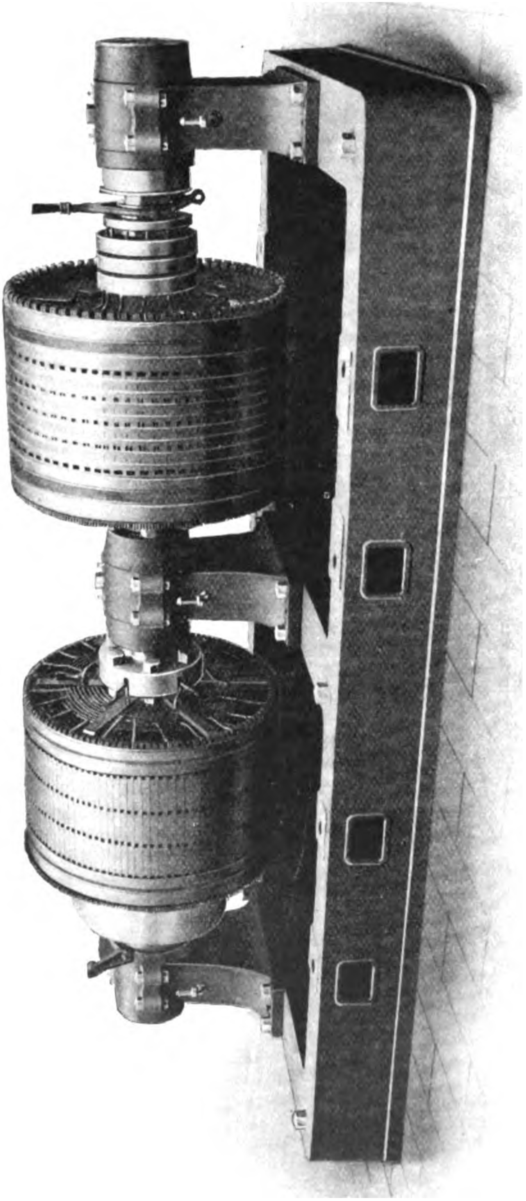

Fig. 379.—Revolving Part of Cascade Converter.

the "generator" part it is half generator, receiving mechanical power by transmission along the shaft to furnish half its output, and it is half converter, turning the currents received from the rotor into continuous-current delivered at the brushes.

The claim is made for this type of combination that it costs less than the motor-generator set, and that it is self-synchronizing and requires no special starting gear. No tests as to the performance of these machines are yet available.

Frequency Changers.—It sometimes occurs that in alternating-current transmission a transformation of frequency is desired. No way of even doubling or halving a frequency by stationary apparatus has been discovered, and even a revolving apparatus for this purpose is unknown, save by coupling together two machines. A motor-generator set in which the numbers of poles on the motor and generator respectively are proportional to the primary and secondary frequencies is the simplest combination. Mention may be made of an enormous set built on this plan by the Bullock Co., of Cincinnati, for the Shawinigan Falls Co., of Montreal, capable of dealing with 8000 HP. Both machines are of the synchronous type, the synchronous motor with ten poles, of 8000 HP., being coupled with a 20-pole generator of 5333 KVA, to change the frequency of 30~ to one of 60~. Both motor and generator are wound for 2300 volts, three-phase. A smaller induction-motor is also built on the same shaft to serve as a starter. The set runs at 300 revs. per minute, and has a nett efficiency of conversion of 92·1 per cent. It works in parallel with five other frequency-changers of 1000 KW each. The generator rated at 5333 KVA on a power-factor of 0·75, is capable of standing an overload of 33⅓ per cent.; and the motor, which is the largest hitherto built, is large enough to drive the generator under all conditions of load-factor. The entire set was designed by Mr. B. A. Behrend.

For other frequency-changing devices, the reader is referred to the author's work on *Polyphase Currents.*

CHAPTER XI.

PARALLEL RUNNING OF ALTERNATORS.

THE parallel running of alternators on the same 'bus-bars presents a problem somewhat different from and more complicated than that of running continuous-current dynamos in parallel, as also for similar reasons does the parallel running of alternate-current synchronous motors.

With continuous-current machines, it suffices simply to see that the voltage has been properly adjusted before the machine is switched in; but with alternators, it is not the virtual values of the voltages alone which must agree, for, inasmuch as the actual voltage obtained from the machine is alternating, and is sometimes positive and sometimes negative, it becomes necessary to see that at any moment the instantaneous values of the voltages of the machines are all equal, so that like currents are flowing from the like sides of all the machines at the same instants. For this condition, three things are necessary, viz.: (1) that the field-current should have been adjusted to produce equality *in voltage* as before; (2) that at any instant the poles and the armatures of all the machines be in the same relative positions—that is, the machines shall be strictly *in phase* with one another; and (3) that the machines continue to be in phase, *i.e.* that they be revolving at the same uniform speed, or in other words, that they be *in synchronism.*

In order to know when these conditions exist, *synchronizers* are employed. A simple form is shown in Fig. 380.

An incandescent lamp is fed from two transformers in series with one another: the primary of one transformer is connected with the 'bus-bars, and that of the other to the alternator to be synchronized. The connexions are so made that when the machines are in synchronism the secondaries of the transformers assist each other in lighting the lamp. When not in synchronism they are in opposition. If the alternator to be thrown in is not going at the right speed it falls into and out of step alternately and the lamp blinks rapidly. The supply of steam is then altered to correct the speed, and the lamp is seen to blink more and more slowly until it takes several seconds between the instant of perfect darkness and the instant of full incandescence. Just at the moment of full incandescence, and when the voltmeter indicates the full pressure, the switches are closed and the lamp forthwith shines without fluctuation, showing that the volts of machine are in step with the volts of 'bus-bars. If the supply of steam is now increased the alternator will take up a portion of the load.

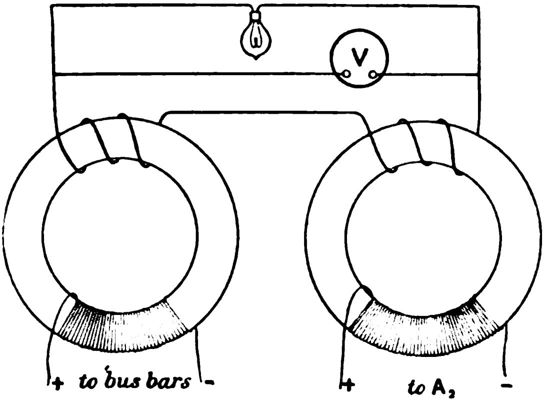

FIG. 380.—A SYNCHRONIZER.

One thing which this simple type of synchronizer does not show is, whether the incoming machine is running too quickly or too slowly, for the light will blink in just the same manner if the frequency of A_2 is 5 per cent. above or below the bus-bar frequency. This is overcome in the case of polyphase currents in the *synchronoscope*, and in the compound arrangement of synchronizers shown in Fig. 381. In the former, the currents from the 'bus-bars are fed into the stator of a small induction motor, and currents from the incoming machine are sent into the rotor, to which a pointer is attached. If the frequencies are the same, there is no relative motion between the stator and rotor fluxes (see p. 658); but if this is not the case, the pointer will revolve one way or the other according to whether the rotor frequency is high or low.

In the arrangement shown in Fig. 381, three lamps are set up in triangular fashion, and the connexions are made to them as indicated. The operation of getting the machines to run into phase with one another is called by the operators "obtaining a phase." When a phase is obtained between the machine *a b c* and the 'bus-bars A B C, the lamp X will be out, while the lamps Y and Z will be at full brilliancy. If, now, *a* is 120° in advance of A, the lamp Z will be out, for *a* and B are then in phase. Thus as the phase changes first one lamp and then another becomes dim in succession, and the suc-

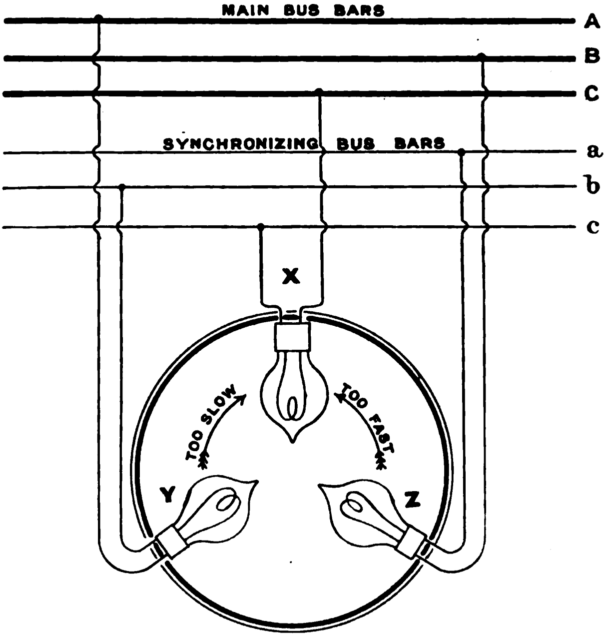

FIG. 381.—THREE-LAMP SYNCHRONIZER.

cession is clockwise or counter-clockwise, according to whether the frequency of the incoming machine *a b c* is too high or too low. In the simple synchronizer in Fig. 380, it is an easy matter to tell when the lamp is most brilliant, but in this method, as a lamp becomes dim before the voltage has reached zero, a voltmeter is used across the lamp X to indicate the exact moment for switching in. A synchronizing voltmeter, consisting of two voltmeters, one on a phase of each set of 'bus-bars, is of course also inserted to enable the excitation to be adjusted properly.

STARTING OF SYNCHRONOUS MACHINERY.

In the case of synchronous motors and rotatory converters, arrangements must be made for starting them up, for they can exert a torque at synchronous speed only. Converters, and synchronous motor generator sets, may, of course, be started as continuous-current motors from the continuous-current mains, or, if these are not already alive, from a small battery which is sometimes installed for this purpose, or better still from a small induction motor-generator set just capable of providing the starting current for one of the synchronous sets; the method of procedure in all these cases being obvious. Failing ability to start up with continuous current, the alternating current must be used, there being two methods in general used by which this is done. The first, which is not to be recommended, is to open the field-magnet circuit in several places and then to apply the alternating voltage to the slip-rings and let the machine start up as an induction motor by eddy currents in the pole pieces and in the copper damping shoes and bridge pieces between the poles (see p. 529); the second is to have a small induction motor mounted on the end of the shaft, the rotor being of the slip-ring type, so that a fine adjustment of speed for the purpose of paralleling may be obtained by the insertion of a resistance. With regard to the first of these methods, those machines which are the worst from economical and regulation points of view will start up easily without taking excessive currents from the lines and without thereby producing an abnormal drop in voltage or disturbing the running of other synchronous machinery on the system. Further, large electromotive-forces are generated in the field coils which produce great stress on their insulation.

When a converter is started up in this way, it is necessary to ensure that it is finally excited to give the right polarity. To do this a voltmeter of the permanent magnet type is connected across the brushes on the continuous-current side, and when the machine is nearly up to speed the pointer swings slowly from one side to the other and the field-switch is closed when the needle is on that side which indicates the correct polarity of the machine.[1]

The starting up of a generator is carried out somewhat as follows: The engine driver stands at the stop-valve and controls the engine by hand regulation, the governor playing no part in the operation. In order that the supply of steam may be regulated with a nicety at the valve, the main valve is usually provided with a by-pass valve, which is only about $\frac{1}{10}$ the area of the big valve. In order to indicate to the driver the speed of the engine with regard to synchronism a voltmeter and a synchronoscope such as have already been described (p. 514) are mounted in a prominent position. When the driver sees the switchman connect the machine to the 'bus-bars, he then opens the main stop-valve, thus allowing the engine an uninterrupted supply of steam so that it becomes capable of taking up a load. Now the switchman, when synchronizing, had adjusted the machine voltage to the 'bus-bar voltage so that no unnecessary rush of current took place at the moment of switching in, but now in order that the alternator may respond to the load given to it by the engine, he must forthwith increase the excitation still further, in order to allow for the voltage drop on load.

We have seen how synchronous machines are brought to the conditions for parallel running. As they only continue in this condition while they are rotating quite uniformly and equably, it becomes necessary to inquire what are the criteria

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[1] For further discussion on the starting, etc., of rotatory converters and motor-generator sets, see Eborall in *Journ. Inst. Elec. Eng.*, xxx. 702, and Pearce in *Electrician*, liii. 518, 552, and 597, and discussions.

that a stable regime shall still continue when the machinery is running under commercial conditions, *i.e.* with loads which are constantly varying, and with a primary source of power such as a reciprocating engine whose fuel is measured out to it by a governor at more or less frequent intervals. In the remaining part of this chapter these questions will be discussed.

HUNTING IN SYNCHRONOUS MACHINERY.

To introduce the subject we will revert to our mechanical analogy (p. 482) of the two fly-wheels connected by a spring susceptible of both tension and compression. Consider what will happen when we put a brake on the second fly-wheel. The springs begin to elongate, and, if they are fairly flexible, they will elongate more than the necessary amount, because when that position is reached the second fly-wheel is rotating slightly slower[1] than the first, and before the spring stops its stretching the two wheels must be rotating together again. When this occurs, as we have seen, the springs have elongated too much, and so are exerting more force on the second wheel than is necessary to drive it against the brake. Consequently they accelerate the second wheel, and again contract the springs. It will thus be seen that the velocity of the second wheel oscillates about the synchronous speed when a load is put on and also when the load is taken off. Owing to the friction, chiefly in the spring itself, the oscillations gradually die out, and the second wheel takes up a steady speed, but is shifted in its phase behind the first.

In alternating current circuits a precisely similar action takes place between the generators and the motors, or even between the generators themselves, and the phenomenon is known as *hunting*.

Our example just given of the two fly-wheels corresponds to a varying load on a synchronous motor; but there is another way in which hunting will be produced. Suppose the speed

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[1] The difference in angular speed can be found by taking the angle by which the second machine has fallen behind the first, and dividing it by the time taken to fall back.

of the first fly-wheel corresponding to the supposed constant frequency of the alternate current generator increases. The second fly-wheel will follow the first in this increase in speed, but because of the flexible connexion between the two, there will be a lag between the two machines, and oscillations or hunting between them will result.

The hunting of motors may be further explained by reference to the load diagram given in Fig. 368, p. 487. Suppose that there is a load O A upon the motor. It will then be lagging behind the no-load position by an angle θ_1. Now suppose the load on the motor is suddenly altered to a value represented by the length of the line O B. The motor while it stays in the position of O A, only exerts the power represented by the length of O A, so that when the greater load is put on it begins to slow up, *i.e.* the angle θ_1 automatically increases, simply because there is more load on the motor than it can, for the moment, deal with. As the angle θ_1 increases, the power given out by the motor also increases until it comes up to the value of O B, when the motor is lagging behind the applied electromotive-force by an angle θ_2. But the motor will not remain at this point, because it has been retarded to get there, and is, therefore, moving at a somewhat lower speed than synchronism ; and so it overshoots to the point B and is then accelerated in speed because the power supplied is more than that given out by the motor. This will bring the motor back to synchronism, but at the same time it will be at more than its proper angle of lag, θ_2, and so will accelerate still further until it again attains the position corresponding to θ_2, when it will be running at more than synchronous speed. This process of swinging backwards and forwards about the position of θ_2 will continue, but, owing to various frictional effects, mechanical and electrical, with decreasing amplitude until the motor finally settles down in the position θ_2, rotating at the synchronous speed.

It is readily seen that swinging will also take place for exactly similar reasons when the load is decreased to O A from O B.

HUNTING IN GENERATORS.

In order that an engine may deal with a varying load and still keep a fairly constant speed it is provided with a governor, which controls the supply of steam either by reducing the pressure by throttling it, or by adjusting the amount let into the cylinder at each stroke, the former method being preferred for high speed engines, and the latter for slow speed engines.

The usual type of governor, of which there are, of course, numerous variations, is shown in Fig. 382. To the spindle A, which in high speed engines may be the shaft of the engine, two arms B B, carrying heavy balls, are suspended from joints at C. When the shaft A revolves, the arms, owing to the centrifugal action, tend to fly outwards. The links D D, which connect the arm B to a collar E, which is loose on the shaft A, enable the in-and-out movements of the balls to be communicated to the lever F, which directly or indirectly controls the supply of steam.

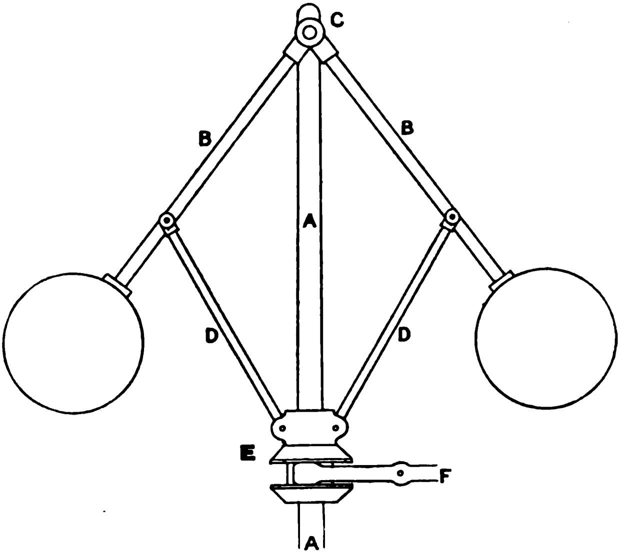

FIG. 382.

Governors may be divided into two classes, the isochronous and the non-isochronous. In the former, which are quite unsuitable for driving alternators in parallel, the mechanism is so arranged that the engine is running at the same speed whatever be the position of the governor balls. If the speed does increase, the balls begin to rise and settle down in such a new position as will enable the engine to deal with its new load at the same speed. It should be noted as an essential point in all sensitive governors, and especially in those of this type, that if the motion of the balls is not checked or damped to a certain extent by means of a dash pot, the moment a variation in speed occurs, the balls suffer a large displacement, more than is necessary for the new load, and so a periodic oscillation of the governor is set up, which is known as *hunting of the governor*, and which causes electromechanical hunting to follow in the alternating-current system.

In the second class of governor there is a certain speed for each position of the balls. The difference of speed between no-load and full-load position is not great, and, according to the sensitiveness of the governor, varies as a rule between 2 and 5 per cent. of the no-load speed.

This tendency to hunt is provided against by the use of a dash pot, connected with the governor rod, which consists essentially of a piston, making a very loose fit in a cylinder which is filled with oil or grease, so that when the speed varies and the governor comes into action its motion is damped, but not entirely checked, by the fluid resistance of the oil.

We may plot a curve, as in Fig. 383, connecting the speed of an engine, as controlled by its governor, and the load. For a set of engines driving alternators in parallel, all their characteristic curves should be alike, because on these alone depends how nearly equally the load is divided among the several units. Suppose two sets, with engine characteristics such as are shown by the lines A and B in Fig. 383, to be put in parallel, and that a certain load is put on. Now the engines must be working at the same speed, so that they will settle down to such a speed that the sum of the powers given out by the two engines equals the given total power.

The load will not be equally distributed, and it is obvious that the more nearly horizontal the engine characteristics are, then the more unequally will the load be divided between the sets. If the engine characteristics are as shown in Fig. 383, drooping more at high loads than when only slightly loaded, then the load when full will be more equally shared between them; and it is a matter of common observation that alternators run better in parallel when well loaded than when only lightly loaded, especially when the governors are so set that the engines are equally loaded at, say, half-load, as with B and C, Fig. 383. The extreme difficulty of running well on low loads in this case is clearly seen.

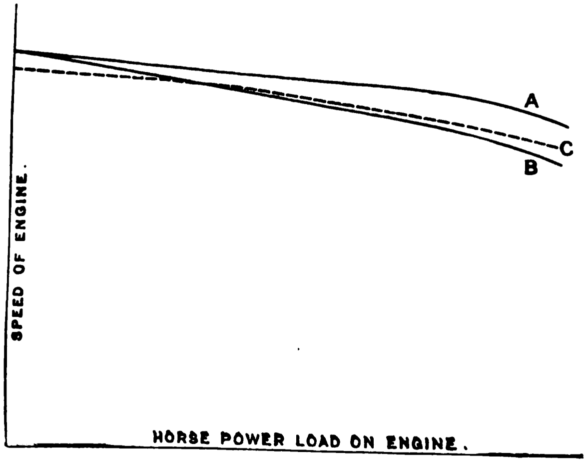

FIG. 383.

Let us consider what happens when the load of the station changes. Supposes it increases :—

1. The engines begin to slow up.

2. When this change of speed takes place the governor alters its position and admits more steam. Note that the governor did not act when the load went up, but after the speed had begun to drop. The speed will, for the time being, go down further than that given by the engine characteristic, and its lowest value will be determined by the moment of inertia of the fly-wheel and the amount by which the governor is damped. The greater the former the more energy

is there stored in the engine, and the more easily will it resist a sudden change in speed. On the other hand, it is preferable to have the engine damped, because, as the speed has fallen below the proper amount, the governor, if freely acting, will let in a lot of steam and so send the speed up higher than is required. In cases where the governor is not damped, the action may become cumulative and very unstable running may result. In some systems the load undergoes rapid variations, and in these cases particularly a governor is required, which, while governing for any permanent change, does not respond appreciably to any momentary fluctuations.

In order that two sets may share a load satisfactorily in any desired proportion there are two things to be adjusted, namely, the governor, the adjustment of which means altering the engine characteristic, and the field excitation of the alternator. With regard to the engines, some engineers control the sensitiveness of the governor from the switchboard by means of a small reversing motor which alters the position of the dead weight on the governor. With regard to the alternators, the distribution of current in their armatures depends upon their respective impedances and the extent to which they are excited. The field rheostat is, therefore, the means of equalizing the current between them.

If one of the alternators be excited more than is necessary, it cannot give out more power than the engine can give to it, and it will then slip back in phase, and the greater current which it will send will have a larger wattless component. The switchman may, therefore, set his governor by comparing his wattmeters, and can then adjust his field rheostat so as to get a minimum amount of current from his several amperemeters. This process may be carried out whether the alternators are similar or not, for in the latter case, where the impedance consists of different proportions of reactance and resistance, the machines will have some phase difference between themselves.[1]

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[1] For an exhaustive examination of the action of governors in connection with the driving of alternators, see a paper by de Marchena in *Bull. Soc. Int. Elec.*, ii. 754, 1902. Abstracted in *Sc. Abs.*, vi. 193*b*.

There are other variations of engine speed which sometimes cause trouble, namely, those small changes which occur during every complete revolution of the engine, due to uneven turning effort upon the crank. They are measured by what is known as the *coefficient of cyclic irregularity*, which is obtained by dividing the greatest variation in speed in one revolution by the mean speed, and is of the order of from $\frac{1}{100}$ to $\frac{1}{300}$. In turbines cyclic irregularity is absent entirely, while it is most prominent in gas engines, for in these the energy is put forth at less frequent intervals than in steam engines. The coefficient may, of course, be made as small as desirable by the use of sufficiently large fly-wheels.

SURGING IN ALTERNATE CURRENT MACHINERY.

The term *surging* is applied to the current fluctuations between the various machines when hunting occurs. It is the electrical consequence of the phenomenon, hunting being of course the mechanical action.

As explained at the commencement of the previous chapter, the questions of running alternators in parallel, and of driving a synchronous motor are identical. Consider at any moment when the machines are swinging, and their electromotive-forces are represented by O A and O B, Fig. 384, differing in phase by an angle θ. Because of this phase-difference, there will be a resultant electromotive-force O R which will be effective in sending a short-circuit current between the two machines in question. This current is represented by OC. Taking the components of this current on the voltage lines OA and OB, we find that on the former it has a negative sense, and on the latter it has a positive sense. This means that the current will be generated in the machine OA, and will be a motor current in the machine OB, and will therefore tend to bring the machines into step again.

Fig. 385 shows the opposite case, where the second machine has swung backward. The lettering is the same as before, and in this case it is seen that the first machine is acting as a generator and the second as a motor, so that the tendency is again to bring the machines into step.

For small displacements of phase, the out-of-balance voltage, O R, is proportional to the angular displacement θ ; and, when this is so, the power-component of the current is proportional to θ, and is therefore equal to E θ.

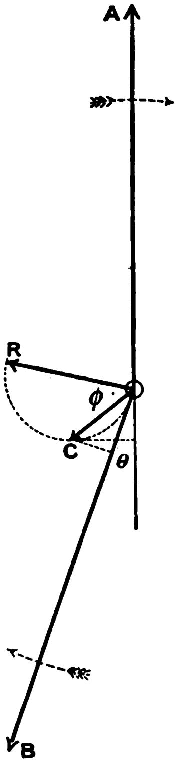

FIG. 384.

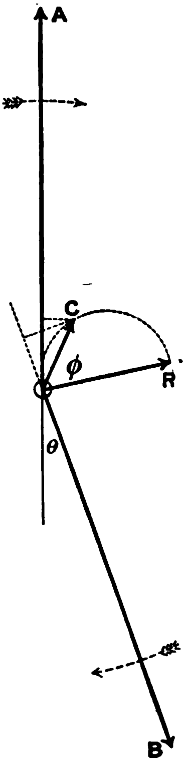

FIG. 385.

The short-circuit current sent by this is given by $\frac{E\theta}{\sqrt{R^2+p^2 L^2}}$, where R and L are the resistance and self-induction respectively of the complete system in which hunting is taking place.

The component of this current on the voltage line is

$$\frac{\mathrm{E}\,\theta \sin \phi}{\sqrt{\mathrm{R}^2 + p^2\,\mathrm{L}^2}} = \frac{\mathrm{E}\,\theta\, p\,\mathrm{L}}{\mathrm{R}^2 + p^2\,\mathrm{L}^2} = \frac{\mathrm{E}\,\theta}{p\mathrm{L}},$$

since in the practical case R is small compared with p L.

The power in watts developed in one machine and given to the other, is given by

$$\frac{\mathrm{E}^2\,\theta}{p\,\mathrm{L}},$$

and therefore, the torque in ft.-lbs. in action to bring the system back into phase

$$= \frac{\mathrm{E}^2\theta}{p\,\mathrm{L}\,n} \times \frac{550}{2\pi \times 746}$$

$$= \frac{\mathrm{E}^2\,\theta}{p\,\mathrm{L}\,n} \times 0\cdot0117,$$

where n = revolutions of machine per second. This torque is resisted by that due to the acceleration of the machine, which may be written

$$\frac{\mathrm{I}}{g\,\mathrm{P}} \cdot \frac{d^2\,\theta}{d\,t^2},$$

where I = moment of inertia of the machine in ft.-lbs. units and P the number of pairs of poles, and $g = 32\cdot2$.

We have then the following equation

$$\frac{\mathrm{I}}{g\,\mathrm{P}} \cdot \frac{d^2\,\theta}{d\,t^2} + \frac{\mathrm{E}^2\,\theta}{p\,\mathrm{L}\,n} \times 0\;0117 = 0.$$

From this we have a periodic motion, whose period in seconds is given by

$$\mathrm{T} = 2\,\pi \sqrt{\frac{p\,\mathrm{L}\,n\,\mathrm{I}}{\mathrm{E}^2 g\,\mathrm{P} \times 0\cdot0117}} = 25\cdot6\,\frac{n}{\mathrm{E}}\sqrt{\mathrm{L}\,\mathrm{I}},$$

putting $p = 2\,\pi\,\mathrm{P}\,n$.

It is to be noticed that the relation between the reactance and the resistance has an important bearing upon the restitutive or controlling forces. If the circuit were absolutely non-inductive, there would be no controlling force, for the components of the current on the electromotive-forces would be equal both in magnitude and direction, and equal power is developed by each machine to overcome resistance simply. For all other relative values of resistance and self-induction, there is more or less controlling force according to whether the ratio of resistance to reactance is large or small. Between the values 0 and θ of ϕ, the second machine still acts as a generator as well as the first, but when this point is reached the forward swinging machine acts alone as generator, both for overcoming resistance and controlling the backward swinging machine.

In the expression for the controlling force on p. 526, we have struck out R, because it is invariably small compared with p L, but it will be readily seen that if these quantities were to be oppositely disposed in magnitude, the controlling force would become exceedingly small and would be given by

$$\frac{E\,(p\,L)\,\theta}{R^2}.$$

The above explanation of phase swinging is essentially the same as that put forward by the late Dr. Hopkinson in his lecture on "Electric Lighting," before the Institution of Civil Engineers in 1883. On this occasion Dr. Hopkinson showed theoretically that if two alternators acting as generators are coupled electrically in series, they are in an unstable state, for if anything happens to alter slightly the exact agreement in phase, the mutual electrical action will tend to increase this difference in phase instead of counteracting it.

From this, it follows that when generators are connected in parallel and a slight disagreement of phase occurs, the mutual action will be to bring the machines back into step. The theory as here given has been directed rather towards this latter aspect of the problem, because it is the one which has nowadays to be faced. The parallel running of alternators had been experimentally observed by Wilde some years previously.

In considering the equation for the swinging of alternators we have up to now only dealt with the terms which involve the displacement (θ) of the machine during the swing and the acceleration $\left(\frac{d^2\theta}{dt^2}\right)$ during the swing. We have yet to consider the term which involves the velocity $\left(\frac{d\theta}{dt}\right)$ and which determines the nature of the swing, whether when once started by external causes, the oscillations will die out or will continue with gradually increasing amplitudes until they accumulate with such violence that it ultimately becomes impossible to keep the machines in parallel on the load.

In the vibrations of a spring, if the moving parts are immersed in a liquid the vibrations are quickly damped out, while if periodic small forces are applied to the weights in the direction in which they are at that instant travelling, the amplitudes of the vibrations will, on the other hand, increase very rapidly.

In synchronous machinery, the latter tendency is present only in special cases and then only to a small extent,[1] while, on the other hand, the damping tendency is always present and can always be increased by the use of special devices, consisting essentially of masses of low-resistance material such as copper or brass in the neighbourhood of the pole, in which eddy-currents are set up by any shifting of the flux with respect to the pole. In the *amortisseur* of Hutin and Leblanc, copper bars are embedded in slots in the pole-faces and form a complete squirrel cage around the magnet-wheel as in an induction motor (see p. 661).

The field created by the armature currents, as shown on p. 257 and in Fig. 212, rotates with the magnet system, so that in steady running no current is generated in the amortisseur bars except a small amount due to the slight fluctuations in value of the flux from point to point. When the machine hunts, a relative motion occurs between the armature flux and the amortisseur bars, so that large currents are generated in

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[1] For a detailed discussion on this point, see Bertram Hopkinson in *Proc. Roy. Soc.*, lxxii. 235, 1903, or *Electrician*, li. 656, 704, 886, 1903.

them, and these, reacting on the armature-field, tend to check the speed variation between the two parts.

Some makers are content with surrounding each of the poles with a heavy copper ring, this forming in many cases part of the framework of the magnet-spool. The rings around adjacent poles are sometimes inter-connected.

Fig. 386 shows an arrangement of this description adopted by the Westinghouse Co., this particular device also having rods embedded in the pole. In some of their rotatory converters the Westinghouse Co. cover the trailing half of the pole-face with a complete sheet of copper. Another form of damper consists of a massive copper rectangle fitted between adjoining pole-tips. In the *Journ. Inst. Elec. Engineers*, xxx. 737, Eborall gives a drawing of dampers of this type used on a 725 horse-power synchronous motor. Fig. 387 shows a similar arrangement which is practically solid metal. Dampers of this description are wedged tightly behind the pole-tips so that they also form a clamp to hold down the coils.

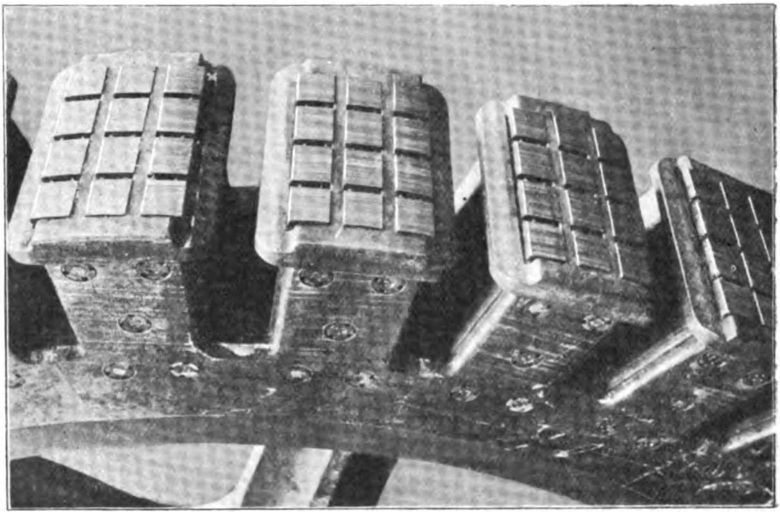

FIG. 386.—COPPER DAMPERS SURROUNDING FIELD-MAGNET POLES (BRITISH WESTINGHOUSE CO.).

Much has been written on this subject of the hunting of alternators running in parallel, and volumes of mathematics have been published about it. For those who would pursue the subject further, we append a list of the more important recent papers dealing with it.

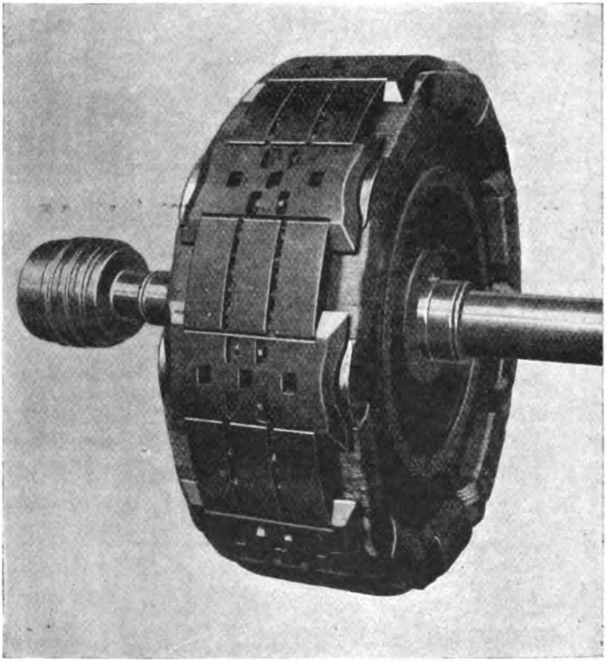

FIG. 387.—MAGNET-WHEEL, 8-POLE, WITH COPPER DAMPING WEDGES BETWEEN THE POLES (WESTINGHOUSE).

Papers dealing more especially with the engine problem :—

Benischke, *Elek. Zeits.*, xx. 870, 1899.
Sykes, *Elec. World*, xxxv. 177, 1900.
Emmet, *Elec. World*, xxxv. 95, 1900 ; *Trans. Am. Inst. Elec. Engs.*, xix. 329, 1902.
Steinmetz, *Trans. Am. Inst. Elec. Engs.*, xix. 325, 1902.
Dettmar (Gas Engines), *Elek. Zeits.*, xx. 728, 1899.
Kruesi, *Elec. World*, xxxviii. 591, 1901.
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CHAPTER XII.

TRANSFORMERS.

WHENEVER electric energy is to be transmitted to a distance, considerations of economy dictate that high voltages[1] shall be employed. On the other hand, considerations respecting safety to person as well as those respecting the pressures suitable for lamps, dictate that the voltage at which the energy should be supplied to the consumer should be comparatively low, or from 100 to 250 volts at the most. Tramway motors are worked at 500 to 600 volts; but the current is often generated at 6000 to 10,000 volts. Hence devices are required which shall receive the currents at high pressure from the feeders or main lines, and shall transform the energy so as to give out larger currents at lower pressures. Such devices are called *transformers*.

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[1] This may be briefly explained thus. The energy supplied per second is the product of two factors, the current and the pressure at which that current is supplied, or in our notation,

$$V\,C = \text{electric energy per second (in } \textit{watts}\text{)}.$$

The magnitudes of the two factors may vary, but the value of the power supplied depends only on the product of the two; for example, the energy furnished per second by a current of 10 amperes supplied at a pressure of 2000 volts is exactly the same in amount as that furnished per second by a current of 400 amperes supplied at a pressure of 50 volts; in each case the product is 20,000 watts. Now the loss of energy that occurs in transmission through a well-insulated wire depends also on two factors, the current and the resistance of the wire, and in a given wire is proportional to the square of the current. In the above example the current of 400 amperes, if transmitted through the same wire as the 10-ampere current, would, because it is forty times as great, waste sixteen hundred times as much energy in heating the wire. Or, to put it the other way round, for the same loss of energy one may use, to carry the 10-ampere current at 2000 volts, a wire having only $\frac{1}{1600}$ of the sectional area of the wire used for the 400-ampere current at 50 volts. The cost of copper conductors for the distributing lines is therefore very greatly economised by employing high pressures, and using step-down transformers to reduce the pressure to that needed for the lamps.

For transforming continuous currents a revolving apparatus is required consisting, in principle, of a motor (driven by the incoming or primary current) driving a generator, which induces a secondary current at the desired (low) pressure (see Vol. I., Chapter XXIV.).

For converting alternating currents into continuous currents revolving apparatus is also needed. Such combinations, known as *motor-generators* and *rotatory converters*, are especially considered in Chapter X.

For transforming alternating currents into other alternating currents at a different voltage all that is needed is a stationary apparatus consisting of a suitable core of laminated iron with primary and secondary coils wound upon it—in fact, an induction coil. These *alternate-current transformers* form the subject of the present chapter.

GENERAL NOTIONS ABOUT ALTERNATE-CURRENT TRANSFORMERS.

The simplest and earliest form of transformer was the iron ring of Faraday, Fig. 388, upon which he wound two coils, a primary and a secondary. In elementary treatises on electricity it is explained how an electromotive-force is induced in the secondary whenever the primary current is increasing or diminishing, because the magnetic lines made in the iron core by the primary current thread through the secondary coil and act inductively. The same thing occurs in the form shown in Fig. 389, where the two coils are wound one outside the other upon a straight core of iron wires.

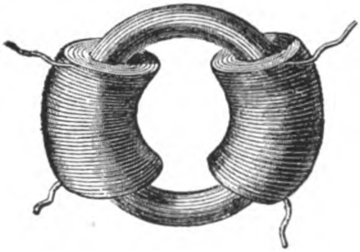

FIG. 388.—FARADAY'S RING, WITH PRIMARY AND SECONDARY COILS.

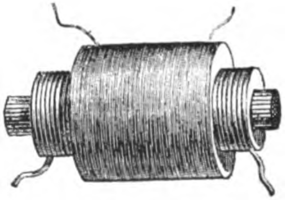

FIG. 389.—SIMPLE INDUCTION COIL, WITH STRAIGHT CORE.

An alternating transformer may be regarded as a species of dynamo, in which neither armature nor field-magnet revolves, but instead the magnetism of the iron circuit is made to vary through rapidly repeated cycles of alternation, by separately exciting it with an alternating current. The primary coil of the transformer corresponds to the field-magnet coil of the dynamo; the secondary of the transformer to the armature coil of the dynamo.

If an alternating current having a frequency of f periods per second be sent into either of the coils, there will be set up in the other coil an alternating electromotive-force having the same frequency, because the iron core is undergoing an alternating magnetization also of f cycles per second. The effect on the second circuit is the same as if the magnetized iron core were being plunged into and removed from the second coil f times per second.

Our first step shall then be to calculate the electromotive-force induced in a coil of any given number of turns by an alternating magnetic flux in the core within it. Let S be the number of spirals or turns in the coil, and N the maximum value of the flux in the core. Suppose that the changes of the flux follow a sine law, we may then write for the value of the flux at time t after the maximum has occurred,

$$\mathrm{N}_t = \mathrm{N} \cos 2\,\pi f t.$$

But the electromotive-force in any one turn is proportional to the rate at which N is changing, or to $d\,\mathrm{N}/dt$. Further, we must multiply by S, and divide by 10^8 to bring to volts. Performing the differentiation, we get

$$\mathrm{E}_t = 2\,\pi f\,\mathrm{S}\,\mathrm{N} \sin 2\,\pi f t \div 10^8.$$

The virtual value of this electromotive-force is obtained by substituting for $\sin 2\,\pi f t$ its square-root-of-mean-square value (see p. 10), namely $\frac{1}{\sqrt{2}}$, giving us

$$\mathrm{E} = 4{\cdot}44 f\,\mathrm{S}\,\mathrm{N} \div 10^8 \quad . \quad . \quad . \quad . \quad [1]$$

This formula is fundamental in transformer calculations.

Now consider a simple magnetic circuit, having wound on it a primary coil of S_1 turns, and a secondary coil of S_2 turns. We may conceive it like Fig. 390; but to avoid complications at first, we will suppose that there is no magnetic leakage, that is to say, all the magnetic lines created by the current in the primary coil link through the secondary coil. The impressed electromotive-force applied to the terminals of the primary coil sets up a primary current which produces an alternating magnetic flux, and this alternating flux in turn induces electromotive-forces, not only in the secondary coil but also a back-electromotive-force[1] in the primary. These two induced electromotive-forces will be strictly proportional[2] to the respective numbers of turns, and absolutely in phase with one another. We may write them

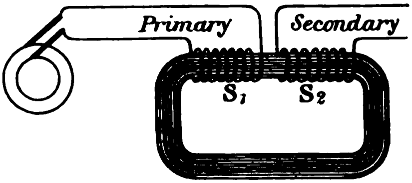

FIG. 390.—ELEMENTARY TRANSFORMER.

$$E_1 = 4{\cdot}44 f S_1 N \div 10^8 \quad . \quad . \quad . \quad . \quad . \qquad [2]$$

$$E_2 = 4{\cdot}44 f S_2 N \div 10^8 \quad . \quad . \quad . \quad . \quad . \qquad [2a]$$

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[1] Whenever an electric current does any useful work other than mere heating, it does so by flowing against an opposing electromotive-force. In driving motors, in charging accumulators, and in operating transformers, the greater this back-electromotive-force that is overcome by a given current, the greater is the amount of power thus utilized by the current. In the case of power being spent in heating a resistance the drop of potential occasioned by driving a given current through the resistance acts similarly as a back-electromotive-force, and is a measure of the amount of power expended in driving this current through the resistance.

[2] Similarly if there is a stray-flux of N_s magnetic lines, being a magnetic leakage that will give rise to self-inductive reactions in either of the two coils, the values of these reacting electromotive-forces will be respectively

$$e_{s_1} = 4{\cdot}44 f S_1 {}_{s_1} + 10^8,$$

$$e_{s_2} = 4{\cdot}44 f S_2 N_{s_2} \div 10^8.$$

These formulæ are used on p. 539 below.

we have, therefore,

$$\frac{E_1}{E_2} = \frac{S_1}{S_2}.$$

This ratio is called the *ratio of transformation*, and is in this chapter denoted by *k*.

Two main cases now arise for consideration: (i.) when the secondary circuit is open; (ii.) when the secondary circuit is closed on a load of lamps or other resistance.

If the secondary circuit is open, though electromotive-force may be induced in it there will be no secondary current, and therefore no reaction of any kind due to this coil. It might as well be absent. The only reaction will be that of the primary coil on itself. As in a motor running light, so in the transformer at no-load, the back-electromotive-force will be almost equal to the impressed electromotive-force. The latter must be slightly greater, for there must be enough volts unbalanced to drive the requisite small magnetizing current through the internal resistance of the primary coils; as there are hysteresis and eddy-current losses they also must be provided for by a small additional primary current. But, save for these, the whole action of the primary, when the secondary is open, is that of a *choking coil*, and the induced electromotive-force E_1 will be almost in quadrature with the primary current. (Compare Fig 431, p. 596.)

Now pass to the case where the secondary is closed upon a load of lamps or other resistance. We will suppose this resistance to be for the present a simple non-inductive resistance. There will be a secondary current in phase with the induced electromotive-force E_2, therefore in phase also with E_1, therefore also in almost exact opposition of phase to the primary current. When the primary is rising to its maximum, the secondary will also be rising to its maximum, but flowing the opposite way round. While the primary is magnetizing the secondary is demagnetizing; and it is clear that the magnetic flux, on which the counter-electromotive-force in the primary depends, could not be as great as before unless more current flowed from the primary source. In fact, more current will of itself flow in the primary because of the demagnetizing effect of the secondary current. The effect of the presence of the current in the second circuit is then to *unchoke* the primary. The primary coil now acts not merely as a choking coil to dam back the primary current, but as a *working* coil, inducing current in the secondary by flowing sufficiently strongly to keep up the alternating magnetic flux in spite of the demagnetizing action of the secondary current. If only half the lamps are on, then the primary will act partly as a choking coil and partly as a working coil. If the primary impressed volts are kept constant, the secondary volts at the terminals of the lamp circuit will be nearly constant also; and the apparatus is in fact self-regulating, more current flowing into the primary of itself when more lamps are turned on in the secondary circuit.

The elementary theory of this simple case of a transformer without leakage, working on a non-inductive load of lamps, is quite easy. Adopting the same notation as used for motors and dynamos, let V_1 represent the volts of supply as measured at the primary terminals, and V_2 the volts at the secondary terminals. Let r_1 be the internal resistance of the primary and r_2 that of the secondary. Call the ratio of transformation $k = S_1 / S_2 = E_1/E_2$. Since (apart from small hysteresis losses, here neglected) the work done *by* the fluctuating magnetism of the core is equal to the work done *on* it, we may further write $E_1 C_1 = E_2 C_2$; whence it follows that $C_1 = C_2 / k$. The volts lost in the primary are $r_1 C_1$ those in the secondary $r_2 C_2$. Hence we may write

$$V_1 = E_1 + r_1 C_1,$$
$$V_2 = E_2 - r_2 C_2.$$

Writing the first of these as:

$$E_1 = V_1 - r_1 C_1 = V_1 - r_1 C_2/k,$$

and inserting E_1/k for E_2 in the second equation, and substituting, we get

$$V_2 = \frac{V_1}{k} - \left(\frac{r_1}{k^2} + r_2\right) C_2 ;$$

which shows that everything goes on in the secondary as though the primary had been removed, and we had substituted for V_1 a fraction of it proportional to the number of windings, and at the same time had added to the internal resistance an amount equal to the internal resistance of the primary reduced in proportion to the square of the number of windings.

Example.—In a Mordey $1\frac{1}{2}$ kilowatt transformer, $S_1 = 300$; $S_2 = 12$; $r_1 = 10$ ohms; $r_2 = 0{\cdot}014$ ohm; $V_1 = 1000$; find e when $C_2 = 36$ amperes. Here $k = 25$, so that on open circuit the secondary volts would be exactly $\frac{1}{25}$ of the primary volts, or 40 volts. But working out by the formula for the output of 36 amperes the terminal volts V_2 drop to $38{\cdot}92$.

Magnetic Leakage.—We will now pass on to consider briefly the practical case of a transformer having magnetic leakage. Returning to Fig. 390, it is sufficiently obvious that even with the secondary circuit unloaded, the magnetic lines produced by the primary magnetizing current cannot all pass through the secondary winding. With such an elementary arrangement it is clear that there will always be a certain amount of stray field, not linking through the secondary, and this amount will naturally be dependent on the demagnetizing action of the secondary coils, and hence increase with the current in the latter. It will be readily seen by a little consideration that the stray or leakage field in a transformer produces self-induction in its windings, because any such field (varying as it does with the primary current) must induce an electromotive-force in the windings affected by it, and as field and electromotive-force are always in quadrature, as follows from the considerations discussed on p. 534, that the electromotive-forces produced by the leakage fields will be 90° behind the primary and secondary currents respectively, and hence the result is exactly the same as if additional self-induction had been actually added to the primary and secondary windings. So the effects produced by a transformer having a given amount of leakage with a particular secondary current could be reproduced by a transformer having no leakage, but with choking coils of definite windings introduced into its primary and secondary circuits.

Magnetic leakage in a transformer has two effects on its performance—firstly it increases the drop of pressure in the secondary, and secondly it tends to diminish what is known as the "power-factor" of the transformer, of which more will be said hereafter. The first effect—that of increasing the secondary drop, is of great practical importance, especially in transformers designed for motor work. Consider for a moment the case of a transformer having a certain amount of leakage working on a non-inductive load. If leakage had been absent, the secondary drop would merely be due to the ohmic resistance of the two windings, and would therefore be

$$\delta V_2 = C_2 r_2 + \left(\frac{C_1 r_1}{k}\right),$$

the drop in the primary, $C_1 r_1$, being expressed in terms of the secondary drop by simply dividing by the ratio of transformation. But as leakage is present, the actual drop will be greater. Let N_{s_1} and N_{s_2} be the values of the leakage fields in the primary and secondary windings respectively. Then

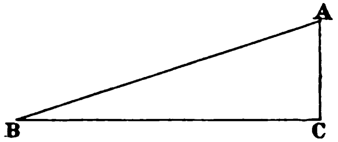

FIG. 391.

$$e_{s_1} = 4 \cdot 44 f S_1 N_{s_1} \div 10^8 = C_1 X_1$$
$$e_{s_2} = 4 \cdot 44 f S_2 N_{s_2} \div 10^8 = C_2 X_2$$

where e_{s_1} and e_{s_2} are respectively the values of the reactance voltages produced by the leakage fields N_{s_1} and N_{s_2}, and X_1 and X_2 the respective reactances. So then we can construct the diagram, Fig. 391, in which we make

Length of side A C = $C_2 r_2 + (C_1 r_1 \div k)$
" " B C = $C_2 X_2 + (C_1 X_1 \div k)$.

The resultant of these two, namely A B, gives a quantity upon which the pressure regulation of the transformer entirely depends. It is proportional to the secondary current, and always bears the same relation to it, no matter what the load. But its phase relation to the secondary pressure depends entirely on the nature of the load, that is, whether the current is leading, lagging, or in phase, and wholly determines the magnitude of the secondary drop, the latter having a maximum value for a given current when the secondary current is lagging behind the terminal pressure by an angle equal to $(90 - a)$ where a is the angle A B C. This point will be gone into more fully later on (see p. 601).

The magnetomotive-forces tending to produce leakage increase from no-load to full-load; hence the leakage is a maximum at full-load, and is practically the same whether that load be inductive or non-inductive. But the voltage-drop is greater on an inductive-load, because with an increase in lag the drop due to leakage comes more nearly into phase with that due to copper resistance.

In order to minimize the leakage effects in a transformer, three points must be attended to. These are: (1) the magnetic reluctance of the iron core must be kept as low as possible—that is, the core must be compact, and free from bad joints; (2) the depth of winding in both coils must be kept down as much as possible; (3) the primary and secondary windings should be sub-divided, and interplaced, sections of the secondary winding being sandwiched in between sections of the primary winding, or else arranged in coaxial cylinders surrounding one another. This last point is most important, and is invariably carried out in modern transformers of good design.

It will be noted that magnetic leakage has no effect on the efficiency of a transformer, unless the leakage field is allowed to produce eddy-currents in the iron case, which very rarely occurs in practice. A transformer having leakage will merely require a little more copper in its windings, and therefore cost a little more, while at the same time its pressure regulation will be impaired, as shown above.

Kapp has indicated a method of estimating the amount of magnetic dispersion by calculation, from a short-circuit test, of the self-induced voltage. Short-circuiting the secondary through an amperemeter, he supplies the primary at a reduced alternating voltage which is gradually raised until the secondary current is equal to the normal full-load. Assuming the secondary circuit to be of negligible resistance, he concludes that the half of the applied voltage balances the primary self-induction. Grassi has, however, shown[1] that the secondary resistance is in many cases not negligible in comparison with the reactance, and that Kapp's method therefore over-estimates the effect.

Power-Factor.—It will now be well to consider in detail the case of a commercial transformer with its primary connected to a pair of mains, but without load on the secondary. As this is the case of a lighting transformer for the greater part of the twenty-four hours, it is clear that such a transformer should not only absorb a minimum amount of power from the mains, but should take as small a *current* as possible from them, in order to minimize heat-losses in mains and generators. Now the current taken by a transformer on no-load has to do two things—firstly, it must magnetize the core to the right degree (so that the secondary pressure is always available for use when required), and secondly, it must balance the "core losses." These core losses are due to hysteresis and eddy-currents in the laminated iron core of the transformer, and they act in exactly the same way as a very small load on the secondary would do; that is, a certain number of watts are taken from the mains by the primary merely for the purpose of supplying the losses in the iron. The wasted power-current of the transformer is thus very easily found. Let W be the reading on a wattmeter placed in the primary circuit, V_1 the volts on the mains, and C_H the value of the required power current. Then we have

$$C \times V_1 = w, \quad \text{or} \quad C_H = \frac{w}{V_1}.$$

This of course assumes that the copper loss is negligible, which is quite permissible at no-load. The magnitude of C_H

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[1] *Atti dell' Associazione Elettrotechnica Italiana*, iv., pt. 3, 1900.

therefore determines the watts absorbed at no-load, and hence in order to keep these small, not only must the core be thoroughly laminated, so as to minimize the eddy-current losses, but the iron itself must be as soft and well annealed as possible in order to keep down the hysteresis loss.

The other component of the no-load current, which is known as the "magnetizing current," can be estimated by abstract magnetic principles. Let C_M be its virtual value, $\mathfrak{B}$ the maximum value of the flux-density in the transformer core, l the mean length of the magnetic path in the core in cms., and S_1 the number of primary turns. Then we have [1]

$$\mathfrak{B} = \frac{1{\cdot}257 \times S_1 \times (C_M \times 1{\cdot}41) \times \mu}{l},$$

or

$$C_M = \frac{\mathfrak{B} \times l}{S_1 \times \mu \times 1{\cdot}76} = \frac{\mathfrak{B}\, l}{S_1\, \mu} \times 0{\cdot}568.$$

It is convenient to retain $\mathfrak{B}$ as the *maximum* value of the flux-density, on account of the iron-losses being a direct function of the *maximum*, and not of the *virtual* value. In British units the above formula becomes

$$C_M = \frac{B \times l''}{S_1 \times \mu \times 4{\cdot}47}.$$

Magnetizing Current.—The modern way to estimate the magnetizing current is to resort to *curves* of magnetic data (see Vol. I. Chapter IV.). The maximum flux N in the core is calculated from the formula $E_1 = 4{\cdot}44 \times f \times S_1 \times N \div 10^8$. The maximum flux-density B is obtained from N by dividing it by the sectional area of iron in the core. Reference to an ordinary saturation curve for the iron under consideration will then give the corresponding value of ampere-turns per inch. Multiplying by l, the number of inches of length of the mean magnetic path around the core, will give the maximum

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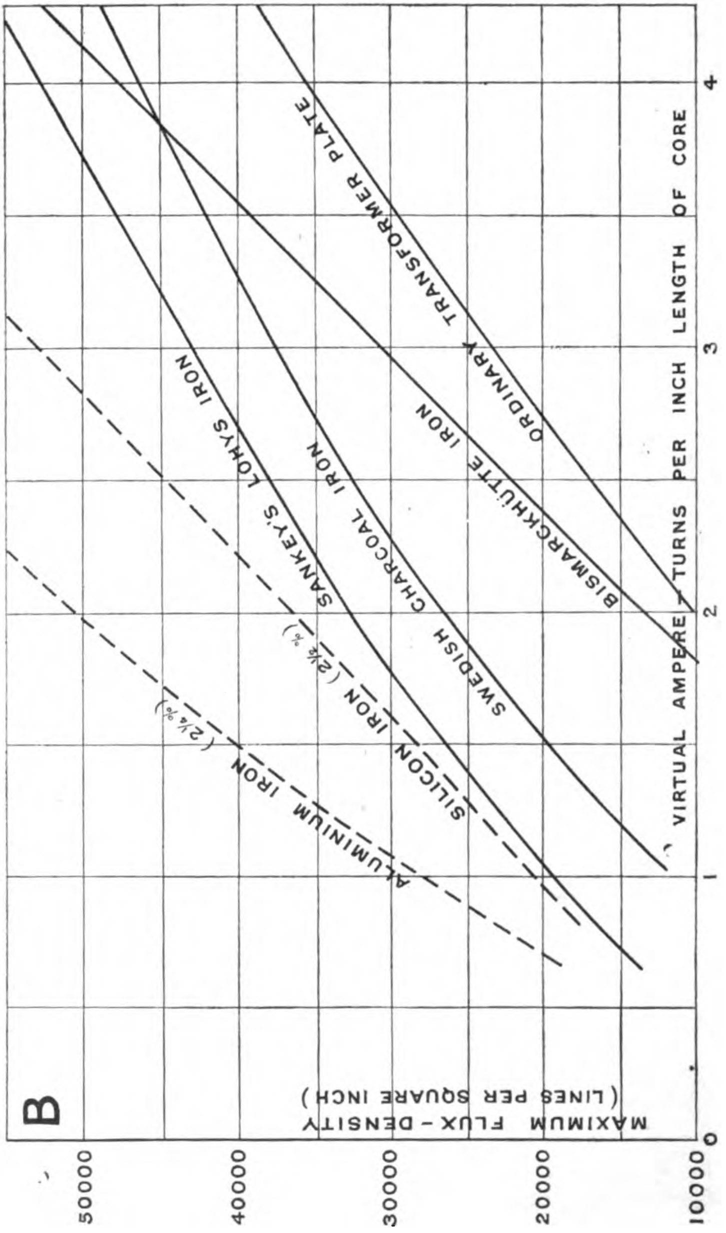
[1] Note that the *maximum* value of the magnetizing current will be $C_M \times \sqrt{2}$; also the maximum value of the magnetomotive-force will be $\frac{4\pi}{10} S_1 (C_M \times \sqrt{2})$, and this will be equal to $\mathfrak{H}\, l = \mathfrak{B}\, l / \mu$, where $\mathfrak{B}$ is the *maximum* corresponding value of the flux-density in the core.


B
MAXIMUM FLUX-DENSITY (LINES PER SQUARE INCH)
50000
40000
30000
20000
10000
0
1
2
3
4
VIRTUAL AMPERE-TURNS PER INCH LENGTH OF CORE
TRANSFORMER PLATE
ORDINARY
BISMARCKHÜTTE IRON
SWEDISH CHARCOAL IRON
SANKEY'S LOHYS IRON
SILICON IRON (2½ %)
ALUMINIUM IRON (2¼ %)

number of ampere-turns required. Dividing by 1·41 will give the virtual number of ampere-turns; and finally dividing by S_1 will give the number of (virtual) amperes of the magnetizing current C_M. In Fig. 392 are given curves of several kinds of transformer iron ***plotted specially in terms of virtual ampere-turns per inch.*** Values taken off from curves thus plotted merely require to be multiplied by l and divided by S_1 to give C_M.

The curve for the iron from the Bismarckhütte, for flux-densities as generally used in transformers, very nearly fits to the empirical equation:

$$\text{Virtual ampere-turns per inch} = 1\cdot25 + \frac{B_{max}}{17{,}000}.$$

The curve for ordinary English transformer-iron, for these densities, is more nearly expressed by the rule:

$$\text{Virtual ampere-turns per inch} = 1\cdot1 + \frac{B_{max}}{12{,}500}.$$

Now this current C_M is a wattless current, because it must obviously be in phase with the magnetic flux in the core, and this latter at no-load is practically in quadrature with the supply pressure—it is exactly in quadrature with the back electromotive-force E_1 of the primary. But, while not affecting the amount of power taken from the mains, it nevertheless affects the no-load current, as may be seen at once from Fig. 393. Here O C_H represents the power current to scale, and in phase with the supply pressure O V_1. O C_M is the magnetizing current as found above, drawn to scale at right angles to the supply pressure. The resultant O C_0 of these two currents gives the actual no-load current, or the current that would be read on an amperemeter placed in the primary circuit. This current lags behind the supply volts by the angle ϕ. We have then $C_0 \times V_1$ as the

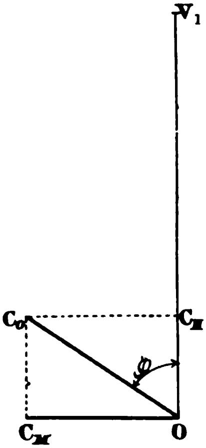


FIG. 393.

apparent power taken by the primary, while $C_H \times V_1$ gives the true power, and is in fact the wattmeter reading, as stated above. The ratio of the latter to the former, or of C_H to C_0, gives what is known as the *no-load power-factor* of the transformer, and the magnitude of this quantity is a measure of its quality. In practice it varies at from about 65 per cent. to 75 per cent. It will be noticed that the no-load power-factor is simply the cosine of the angle by which the no-load primary current lags behind the supply volts, because the ratio of C_H to C_0 is the cosine of the angle ϕ.

A low value of the no-load power-factor denotes that a large wattless current is being taken from the mains, wasting power in heat in mains and generators, which in turn implies that a large proportion of the station-plant has to be kept running all day merely to supply the large no-load currents required by the transformers. In order, therefore, to prevent waste from this cause, and to bring the power-factor as nearly equal to 100 per cent. as possible, the iron employed in the transformer cores must have a high permeability, and must therefore be worked at a low flux-density. Further, the iron circuit itself must be short, free from bad joints, and, as far as possible, from magnetic leakage.

If C_M and C_H are known, the value of the no-load current C_0 is easily found. Evidently, as follows from Fig. 393—

$$C_0 = \sqrt{C_M^2 + C_H^2}.$$

CONSTRUCTION OF TRANSFORMERS.

The principal points underlying the construction of transformers have already been discussed. In regard to the iron circuit, we have seen that it must be as compact as possible, and free from gaps of any kind ; that it must be of great enough section to carry the necessary flux at a moderate flux-density ; and further, that it must be built up of thin soft iron plates, preferably lightly insulated from one another ; the iron employed having as little hysteresis as possible. With regard to the copper circuits, the mean length of turn must be as small as possible in order to minimise $C^2 R$ losses and leakage drop; the primary and secondary windings must be wound in sections and properly distributed, and, what is of great importance, the cooling surface afforded by the transformer must be amply large enough to get rid of the heat produced in the iron and copper parts. With regard to the mechanical features of the design, the coils should, for manufacturing reasons, be of such form that they can be wound in a lathe, and preferably of circular shape, as giving the best arrangement of material, and they should be readily accessible for repair if necessary.

Return now to Fig. 388, which depicts Faraday's ring-transformer. Its iron core was not laminated; and the placing of the two coils was such that there was a great tendency to magnetic leakage across the ring from top to bottom. Two obvious improvements are (1) to make the core of wire or washers of the same mean diameter as the ring; (2) to wind the primary and secondary coils in sections, sandwiched between one another, as in Fig. 394.

FIG. 394.—RING TRANSFORMER, WITH SANDWICHED COILS.

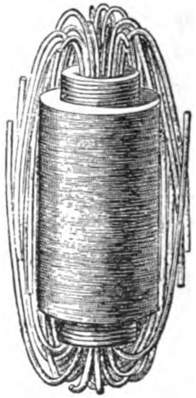


FIG. 395.—VARLEY'S CLOSED-CIRCUIT TRANSFORMER.

Now turn to Fig. 389, p. 533, which depicts the cylindrical type of induction coil, also used by Faraday, further developed by Callan, Masson, and Ritchie, and perfected for spark purposes by Ruhmkorff. For the purpose of an alternating current transformer, however, it has a bad magnetic circuit; for the magnetic lines will have to find their return paths through the surrounding air: it will take a relatively large magnetizing current, and there will be some leakage, though not quite as much as if the two coils had been wound separately on the two ends of the core instead of over one another. Fig. 395 depicts a form due to Varley, which is an obvious improvement, the magnetic circuit being much better closed. The Pyke and Salomons transformer was like this, but had the coils sandwiched along the core. The Ferranti transformer, Figs. 404 and 413, also resembles this form, but has its core of ribbons of sheet iron. If we imagine the two coils made quite short and set side by side on the core, the elongated form of Fig. 395 might be reduced to the squat shape of Fig. 396, which is a form introduced by Zipernowsky. The primary and secondary coils are first laid upon one another, and the iron core is then wound through and over them by a shuttle, so that the whole of the copper is enclosed within the iron. In the sketch (Fig. 396), the front portion of the iron winding is represented as removed to show the interior. Prof. Kapp has proposed the name of "shell-transformers" for this type of apparatus as distinguished from those with a mere straight or a non-expanded internal core, which he calls "core-transformers." But the two types run into one another. All shell-transformers have a core, and all core-transformers, if they have closed magnetic circuits at all, have some portion of iron returning outside the windings; so it is only a question of detail how far this return portion is spread out as a shell. It is certain that excellent transformers are made in accordance with both extremes of type.

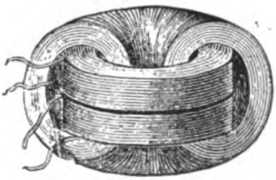
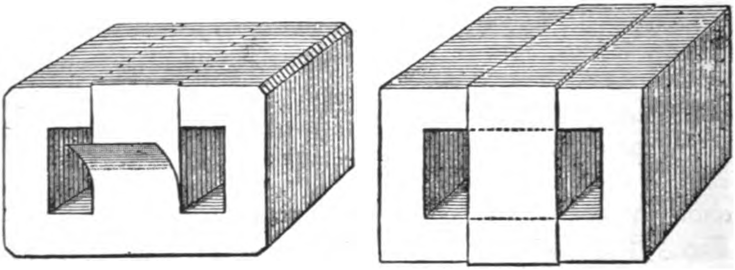


FIG. 396.—ZIPERNOWSKY'S SHELL-TRANSFORMER.

Shell-Type of Transformer.—Modern transformers, without exception, have cores built up of thin sheet stampings. These stampings vary in thickness from 13 to 20 mils, and are always lightly insulated before being built up either with japan varnish or thin tissue or other paper. Paper is generally preferred for transformers, in which case the section of iron is from 85 per cent. to 90 per cent. of the gross section of core. Varnish is often used for choking coils and small arc lamp transformers, and occupies 5 per cent. to 7 per cent. of the gross section. The forms shown in Figs. 397 and 398 are typical of a class in which the stampings when assembled constitute a long central core and an external shell, with two long apertures to receive the coils. Different firms build up the stampings differently, and wind the coils in different ways.



FIGS. 397 and 398.—CORE-PLATES OF TRANSFORMERS (WESTINGHOUSE AND MORDEY).

To avoid waste of material Mordey introduced the method shown in Fig. 398, where the cross-pieces that form the core are simply the rectangular portions stamped out of the external plates that form the shell. If the external dimensions of the shell-plate are 6 by 4 inches, the core-plates will be 4 by 2 inches, and each of the windows will be 2 by 1 inches. These pieces are interlaced as shown, being built up, however, around the coils (not shown in Fig. 398), which are previously wound upon a light rectangular former A, Fig. 399, made of hard wood steeped in ozokerit.

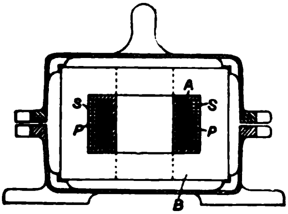


FIG. 399.—MORDEY'S TRANSFORMER (Transverse Section).

Fig. 400 shows in diagram four different ways of disposing the primary and secondary windings in the space available in the apertures. Apart from an allowance for the small extra amount of primary current for magnetizing, the quantities of copper needed for primary and secondary are equal (for minimum heat-waste and drop); for if the secondary wire has only $\frac{1}{k}$ as many turns as the primary it will have to carry *k* times as much current, and therefore require a section *k* times as great. It is usual to make the primary of a round wire well insulated, and the secondary of insulated copper ribbon or rectangular strip. And as the insulation of the fine primary wire takes up a relatively greater space, the total space left for the primary is greater than that for the secondary. Owing to the conditions of imperfect ventilation, a high amperage

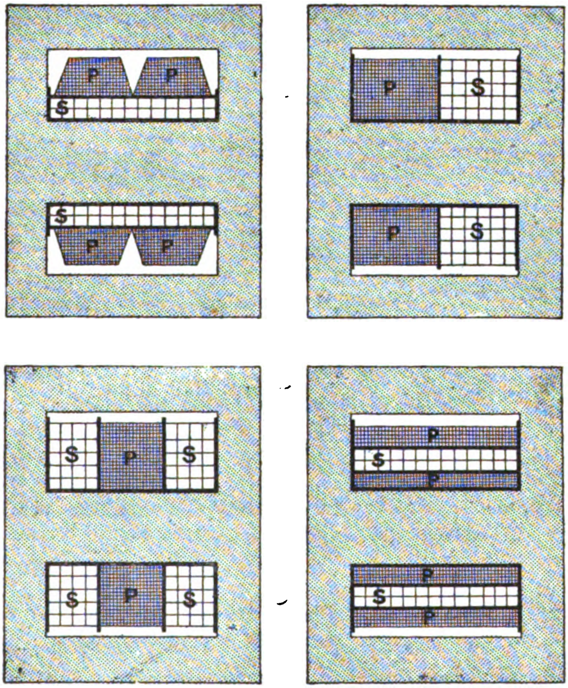


FIG. 400.—VARIOUS MODES OF DISPOSING TRANSFORMER WINDINGS.

cannot be used ; a current density of 600 amperes per square inch being considered rather high.

In Figs. 401 and 402 are depicted, without showing the jointing of the cores, two types of construction still in vogue. The first of these is the long shell type just discussed ; with its exceedingly compact magnetic circuit and its elongated coils built on a special frame. The second represents a type used by the Oerlikon Co. (compare Fig. 403) from the original designs of Mr. C. E. L. Brown, having a long core over which the coils, wound in cylindrical shapes on bobbins, can be slipped. The shell-yokes are then added, being furnished with faced joints. This feature of placing the windings cylindrically over one another upon a long core is found excellent for avoiding leakage and inductive drop, and it therefore gives good regulation. As will be noticed, an approximation to cylindrical form is procured by use of graduated sizes of core-plates. The fine-wire high-voltage winding is divided into two parts for the purpose of keeping far apart the portions which differ greatly in potential ; and the winding is coned at its ends so as to obviate the use of bobbin cheeks ; insulation in oil or air being better without them than with them.

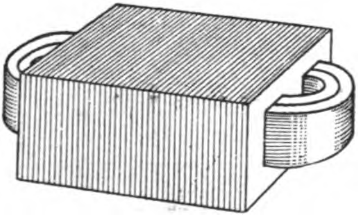


FIG. 401.—SHELL-TYPE TRANSFORMER, with elongated coils.

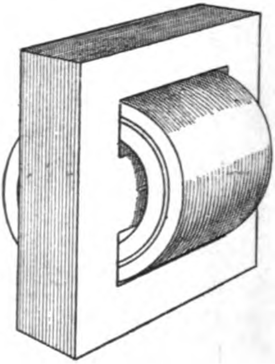


FIG. 402.—SHELL-TYPE TRANSFORMER, with coaxial cylindrical coils.

Brown's later designs, built by his firm, are somewhat

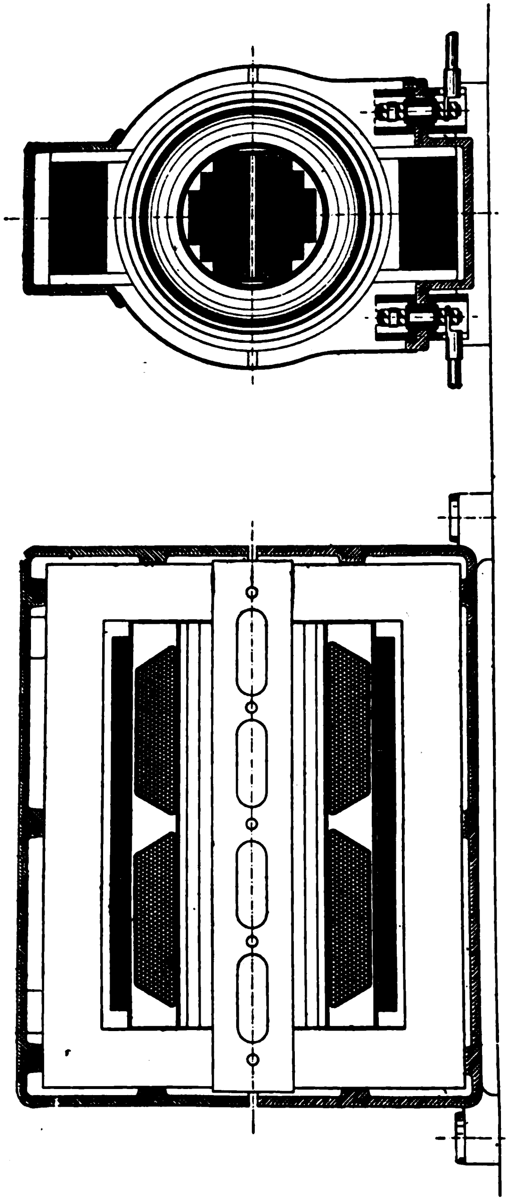


FIG. 403.—OERLIKON CO.'S 20-KILOWATT TRANSFORMER.

different. For outputs up to about 15 kilowatts, his transformers have a similar internal core, built up with plates of graduated sizes, over which, on a seamless paper cylinder, is slipped the secondary winding, generally of strip copper, and over this again the primary winding in two coned coils; but the part forming the yoke is not in two portions, as in Fig. 403, but in one of the same shape, and double iron section, fitted by faced joints.

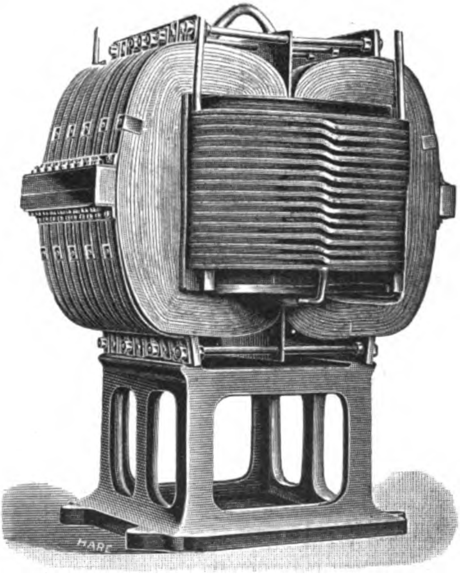


FIG. 404.—FERRANTI'S TRANSFORMER.

Ferranti's shell transformer (Fig. 404) for extra high pressure work, has a core made of a large number of thin strips of iron, which pass vertically up through the middle of the copper coils, and are bent round below and above on each side and interlapped so as to complete the magnetic circuit. The coils are made of copper strip, very carefully insulated, and compacted together in sections by insulating material. There are three coils thus built up, the innermost being a portion of the primary, outside this the secondary, and outside this again the rest of the primary. Sheets of ebonite are interposed in the spaces between these coils, so as to prevent sparking across from the high-pressure coils. There is also room for air-ventilation in the vertical spaces where these sheets of ebonite are wrapped round between the three piles of coils.

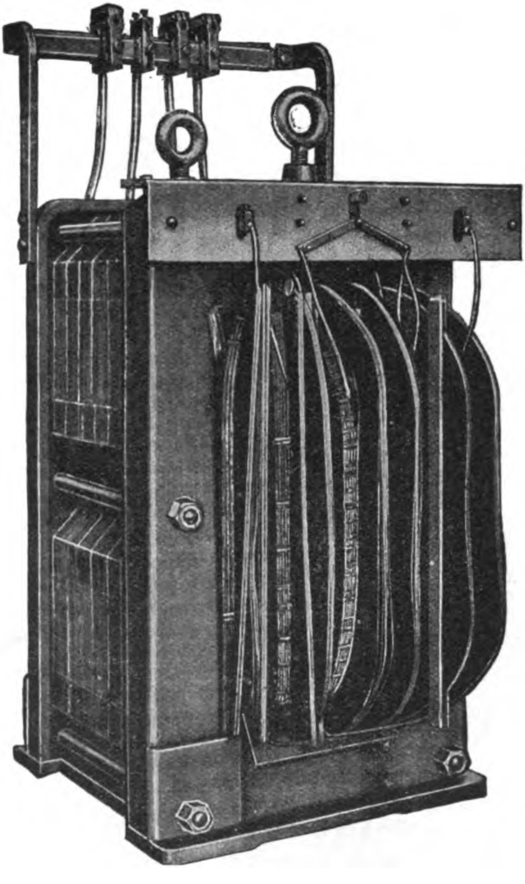


FIG. 405.—WESTINGHOUSE TRANSFORMER, 375 KW, 15,000 VOLTS.

Fig. 405 shows a modern transformer of the shell-type by the Westinghouse Company. The primary and secondary coils are both made up into flat pancake sections, the primaries being sandwiched between the secondaries. To promote

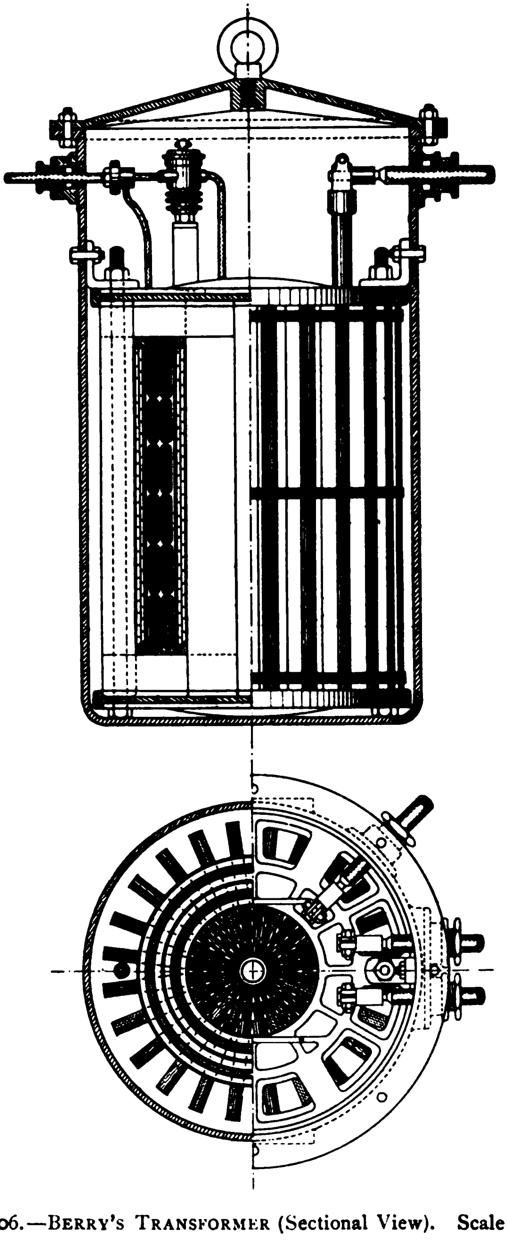


FIG. 406.—BERRY'S TRANSFORMER (Sectional View). Scale 1 : 12.

cooling the projecting parts of the coils are bent out away from one another. The whole transformer is immersed in oil in a corrugated iron case, and is self-cooling. The high-voltage terminals are mounted on a marble slab. The efficiency is about 98 per cent. at full-load, and 96·1 at ¼-load. The voltage drop is 3·3 per cent. on a power-factor of 0·9. The

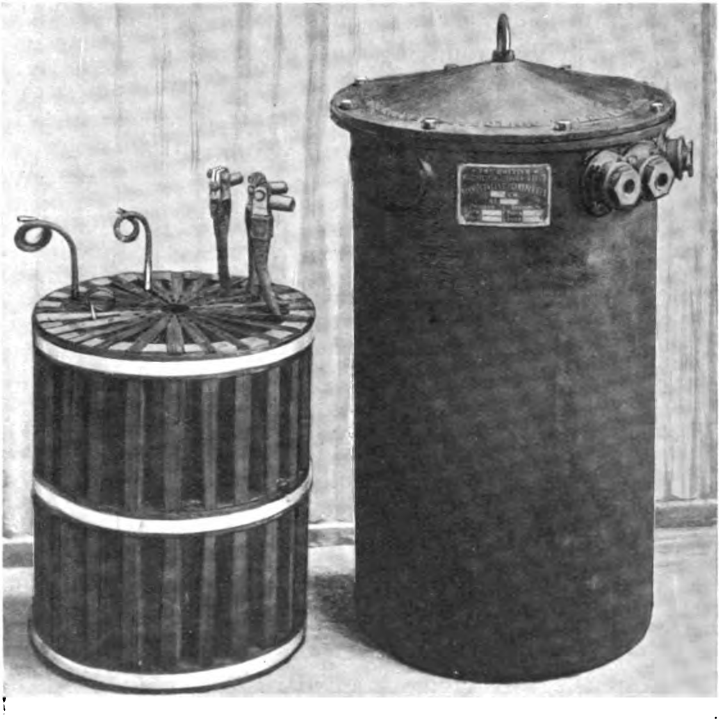


FIG. 407.—BERRY'S TRANSFORMER.

375 KW transformer of this type, insulated for 10,000 volts, weighs 3000 lb. The case weighs 2800 lb. and contains 3000 lb. of oil. Its base is 65 × 52 inches, and its height 65 inches.

The newest transformer of shell-type is that of Berry, which differs in important features from any of the preceding forms. The coils, which are built as concentric cylinders, surround a central core of laminated plates, the magnetic circuit being completed by other laminations built up to enclose the whole as it were in a cage. By judiciously arranging the sizes of the stampings, so as to obtain minimum length of copper circuit with maximum cooling surface for the iron, Mr. Berry has produced a transformer which for compactness and efficiency surpasses all other shell-type transformers with natural cooling. The secondary winding is in two parts, one immediately surrounding the inner core, the other outside the primary and contiguous to the outer laminations. An air-space is left on each side of the primary winding, which, for better insulation, is wound in sections that are slipped on and stand one above the other. The stampings are interleaved at the joints to reduce the reluctance. Copper strip is used for winding the secondaries, and square copper wire for the primaries. Fig. 406 gives a sectional view of a 30 KW transformer, and Fig. 407 shows a general view.

Core-Type of Transformer. — The form represented in Fig. 408 is that adopted by Messrs. Johnson and Phillips, originally from the designs of Kapp, and may be described as an improved Faraday ring. The cores are built up of varnished plates of graduated sizes, so that the section of each limb is approximately octagonal. The cores are served with tape and coated with shellac varnish. The stampings are in rectangular strips imbricated at the joints, and secured by insulated bolts. Sleeves of insulating material receive the coils, which being cylindrical are slipped over one another on the longer limbs of the core. Afterwards when placed concentrically on the core, sheet ebonite is interposed between them; the fine-wire primary lying outside the secondary. A cast-iron watertight case encloses the whole. The transformer illustrated in Plate XXV. is of this pattern.

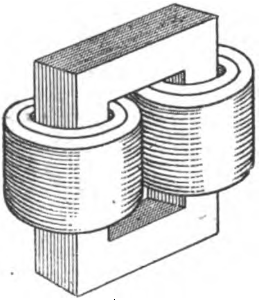
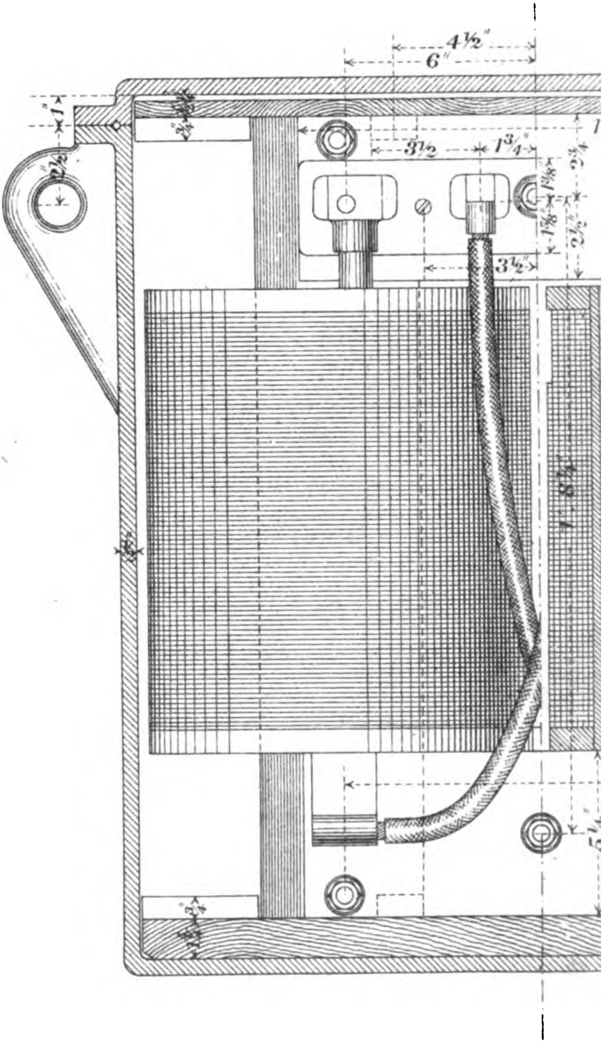


FIG. 408.—CORE-TYPE TRANSFORMER.


4½"
6"
1"
¾"
2½"
3½
1¾"
1⅜"
2¾
1⅞"
2½"
3½"
1' 8¼
¾"
5¼

In the core-type transformers of Brown, Boveri and Co., for the larger outputs, the windings are distributed over the

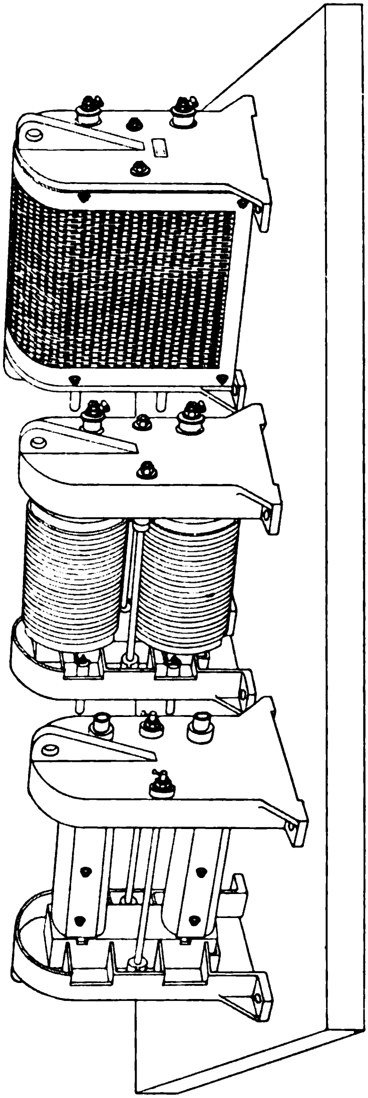


Fig. 409.—Core-Type Transformer of Brown, Boveri and Co.

two cores, there being part of the secondary and primary on each, as this gives a better distribution of material and better pressure regulation. One of these transformers is shown in Fig. 409 in three stages of construction. The iron parts forming the yokes are not bolted together, but are forced between the jaws of the cast-iron frame, and the ends faced up in this position. The high-pressure coils are wound with paper-covered wire, as the firm's experience conclusively shows that this is the best insulating material for all high-tension work.

Fig. 410 illustrates the so-called "Hedgehog" transformer of Swinburne,[1] having as core a bundle of iron wires which, after receiving the copper coils, are spread out at their ends so as to reduce the magnetic reluctance, which is in any case great, the magnetic circuit being an open one. It was supposed to be more efficient, as the weight of iron is so small, reducing the eddy-current and hysteresis losses. But owing to its incomplete magnetic circuit it requires a very large magnetizing current, and therefore at low loads wastes a disproportionate amount of energy in the primary mains. It is now generally agreed that closed-circuit forms are preferable: they have the further advantage of an entire absence of waste from eddy-currents in the copper conductors, however massive.

FIG. 410.—SWINBURNE'S HEDGEHOG TRANSFORMER.

Modern Methods of Construction.—In recent years makers have gradually fallen into line with one another; so that while both core-type and shell-type persist, general methods both of electrical and mechanical construction have been adopted.

All single-phase core-type transformers are practically built up in one of the two ways shown in Figs. 411 and 412. In the former the two limbs (set vertical for better ventilation) are built up of two blocks of stampings, and the two yokes

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[1] *Journal Inst. Electr. Engineers*, xx. 183, 1891.

are likewise built up of stampings, and the four blocks are put together with butt joints.

In the latter way the stampings are interleaved between one another, forming imbricated joints. The second method

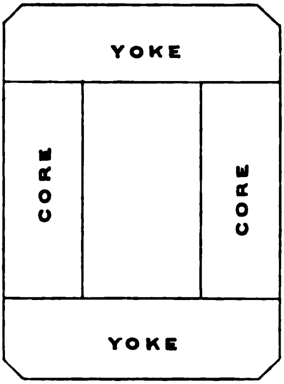


FIG. 411.—CORE LAMINATIONS WITH BUTT JOINTS.

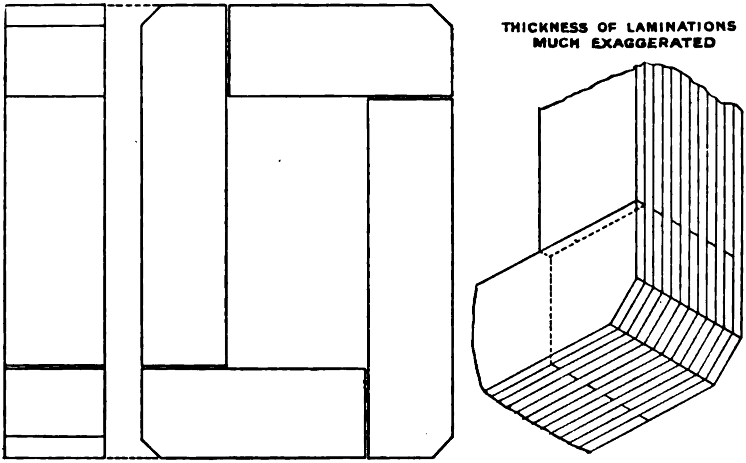


FIG. 412.—CORE LAMINATIONS WITH IMBRICATED JOINTS.

is magnetically superior, as the interleaving reduces the reluctance of the joints to a negligible quantity, and therefore reduces the magnetizing current at no-load. Butt joints are cheaper, though if used, a thin sheet of paper (2 mils thick), should be interposed at each joint to prevent the circulation of eddy-currents. This, however, increases the no-load current. It is a question whether this disadvantage outweighs the advantage of greater ease of erection and repairs.

In both forms some arrangement is necessary for clamping the yokes, and pressing them on to the wound limbs. For good constructions, see the examples given later. Bolts passing through the iron must of course be insulated. To obviate this necessity, some makers trust to wooden dowels for holding the cores in place; or, in transformers of small size, they even use a careful binding with tape which serves at the same time as insulation for the secondary (low-voltage) winding. If the laminations are not held tightly together, and the yokes not firmly clamped, the mechanical vibrations of the cores will be so great as to cause the transformer to hum loudly.

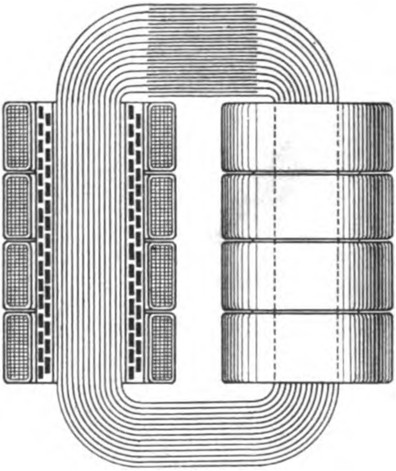


Fig. 413.—Ferranti Core-Type Transformer.

Messrs. Ferranti build core-type transformers on a somewhat different plan from the preceding. They take long strips of iron and bend them to a U-shape. On the legs of the U the previously-wound coils are slipped. When they are in position the projecting upper ends of the strips are bent over and interleaved with one another. The joint so formed is then pressed tightly down. There is thus only one joint in the magnetic circuit. This construction is represented diagrammatically in Fig. 413.

In the preceding paragraphs it has been assumed that the cores are built up to a rectangular section. Up to about 50 KW, indeed, the square section is the best form: but for transformers rated for larger outputs a less simple form, built up of strips of different widths so as to assimilate toward a cruciform or an octagonal form of section (as in Figs. 415 and 440, and Plate XXIII.) may be employed with advantage.

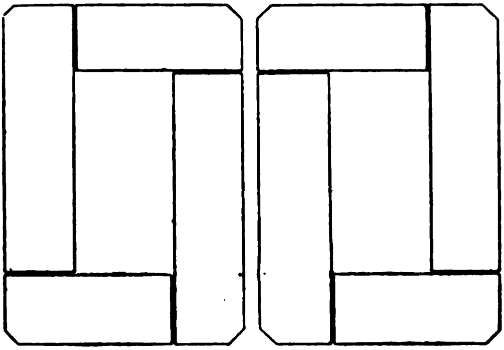


FIG. 414.—ARRANGEMENT OF STAMPINGS OF SHELL-TRANSFORMER.

The yokes are nearly always designed with a rectangular form of cross-section, the dimensions being such that the thickness of the pile of stampings is the same as in the limbs, and the width of them the same as that of the limb at its widest part. Rarely the laminations which form the yokes are bent out in parcels, fan-wise, away from each other to gain cooling surface. An example of this is seen in the Kolben transformer, Fig. 445.

Transformers of the shell-type are nowadays also built up of two sizes of stampings. Formerly, as in Fig. 397, laminations stamped each in one piece, to give minimum waste of material, were used. But the modern plan is to arrange pieces of two sizes only, as in Fig. 414. Successive layers are arranged to overlap at the joints, to diminish magnetic leakage. The gap between the two halves, which may be about ¼ inch wide, is useful in promoting cooling.

To assemble laminations they are threaded upon wooden pins or dowels, distance-pieces to form air-ducts being inserted at short intervals. The air-ducts may be 0·4 or 0·3 inch wide, and at 2 to 3 inches apart. For oil cooling the ducts may be at 3 to 4 inches apart from one another. Such an arrangement is shown in Fig. 415, which is a design for the core of a 250 KW transformer by the Lahmeyer Co. A suitable cast-iron clamp is fixed at the top and another at the bottom, for clamping the laminations together. The top clamp is usually designed to support the porcelain beds for the terminals, or to receive eye-bolts for lifting, as in Fig. 444.

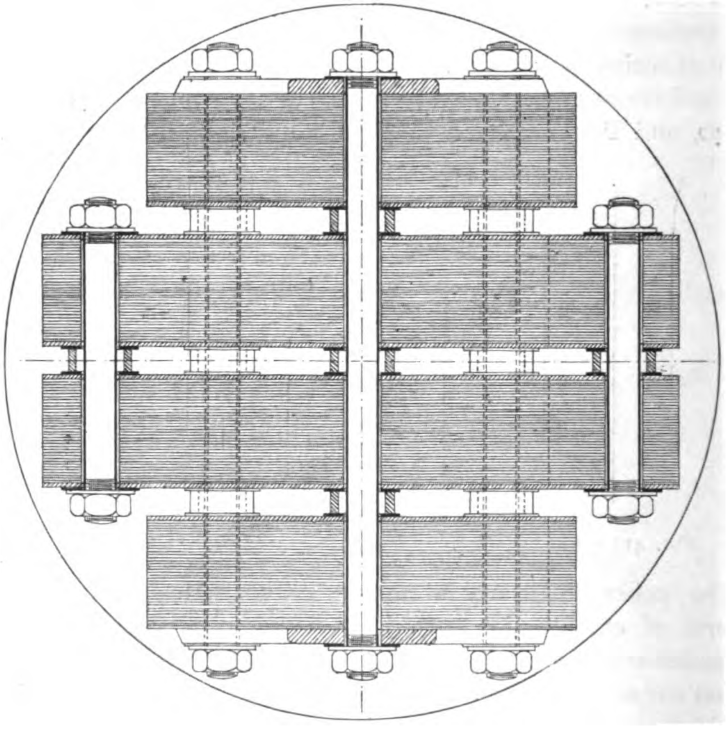


FIG. 415.—SECTION OF LIMB OF TRANSFORMER (LAHMEYER CO.).

Transformer Windings.—There is nothing more important in a transformer than the insulation, particularly the insulation of the high-voltage winding : for the worst fault that a transformer can have is to break down owing to defective insulation. Hence there should be no stinting of either quality or quantity of the insulation.

The core-type of transformer can be wound in three different ways, viz. (*i.*) where the windings are arranged as two concentric cylinders (Fig. 416*a*) ; (*ii.*) where the windings are sandwiched in successive thicknesses one between the other (Fig. 416*b*) ; (*iii.*) where the windings are sandwiched concentrically (Fig. 416*c*). The last-named arrangement does not appear to have any advantages compensating for the increased cost, and the complication. The second arrangement may in certain cases have less magnetic leakage than the first, but is less suitable for any case where the voltage is high, because in it insulation is more difficult. The first may be considered the standard method of winding transformers of core-type.

In this method the secondary, which for step-down transformers is the low-voltage winding, is always wound next to the core, with the primary outside. It is good practice to insert a sheet of insulation—hard paper preparation, or else micanite—between the two windings. Many makers, however, rely on the insulation of an air-gap between the two.

Regarding the arrangement of the coils there is another consideration, namely the voltage-drop. If there is to be no inductive voltage-drop there must be no magnetic dispersion, and this can never be absolutely attained because the two windings cannot occupy the same space at the same time. Failing this impossible condition, the next best thing is that the geometrical mean distance of each part of the primary from each part of the secondary (in any given transverse section of the winding) should be a minimum. This is the object of sandwiching, which, however, when carried too far makes adequate insulation impossible. The art of design is here, as in so many other cases, the art of compromise between conflicting conditions.

As to windings suitable for the shell type, several different dispositions have already been suggested in Fig. 400. Of these the best from the insulation point of view is the first,

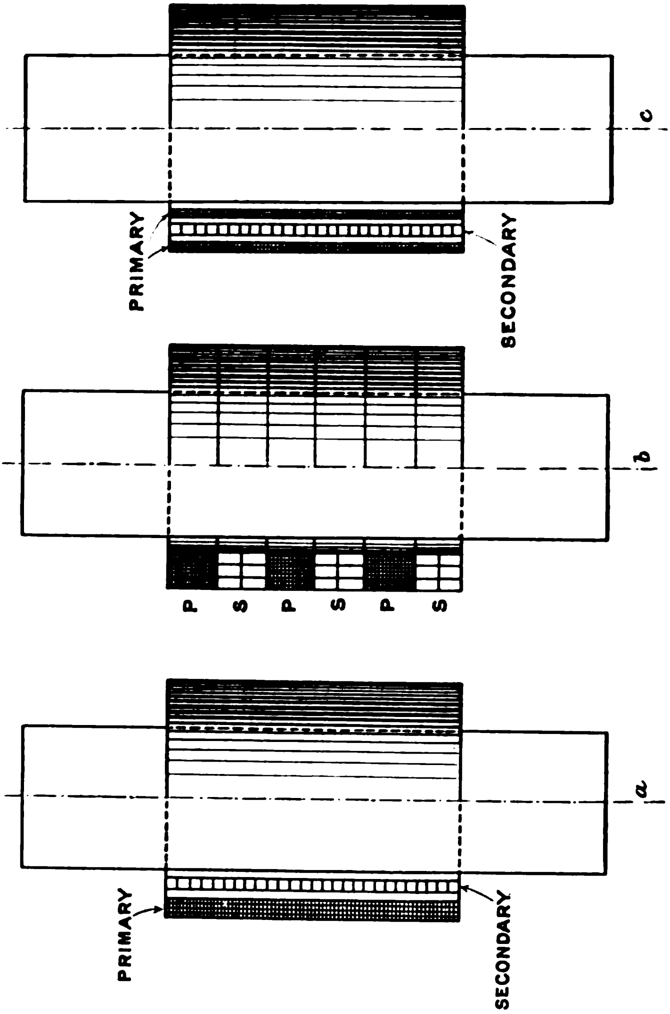


FIG. 416.—MODES OF ARRANGING WINDINGS.

while the fourth will probably have the least magnetic dispersion.

For almost every purpose, the simple arrangement of concentric cylindrical coils, whether for core-type as in Fig. 416*a*, or for shell-type as in Fig. 400*a*, will be found satisfactory. But in order to minimize the risk of breakdown, the high-voltage coil should itself be made up as a number of separate smaller coils, from 4 to 10 in number, piled upon one another, and each separately insulated so as to withstand the full voltage. And, in addition, between all the layers of each such smaller coil there should be a thickness of insulation adequate to guard against short-circuiting of any layer.

As insulation for the outside of the coils we may use unbleached linen tape, painted over with several coats of the best insulating varnish, or mica-cloth and tape combined. For lower voltages some of the stock insulating fabrics such as "Empire cloth" will suffice. Between each layer and the next should be inserted one or two layers of soft Japanese paper, or red rope paper, or mica paper, or even a brushing of varnish.

Each manufacturer has his own preferences, and experience to draw upon; and practice varies a good deal. The following figures may be taken as a fair guide to customary practice with regard to insulation of the outsides of coils:—

VOLTAGE.	THICKNESS OF INSULATION.	
	Mica-cloth and tape, or varnished tape.	Empire cloth and similar materials.
	mils.	mils.
2,000	35	35
5,000	70	40
7,500	100	—
10,000	150	—
15,000	150	—
20,000	150	—
30,000	150	—

For voltages above 10,000 a ring of micanite should be placed between the coils, of a thickness of 50 mils or more, up to 100 mils for 30,000 volts. In the case of extra-high pressure transformers, a layer of mica or micanite should be inserted between the layers inside the coil at suitable intervals.

A point of great importance is the provision of a sufficiently wide *air-gap* between the primary and secondary windings. It is good practice to insert a cylinder or sheet of micanite between the two coils. In a 30,000-volt transformer this may be as much as 350 mils thick. Such a cylinder serves the same function as a guard-plate between the two windings, and is far preferable thereto.

An arrangement of the secondary winding used by the Oerlikon Machine Works is very interesting, and mechanically good. They make their windings of bare copper, or bare aluminium strip, insulated by a sufficient air-gap between the several turns. The stiffness of the material makes this construction quite satisfactory. The secondary winding may also be made of ordinary copper wire covered as for continuous-current dynamo windings, viz. taped with a half-lap taping, dry. It may be wound directly on the core, and afterwards served with insulating varnish.

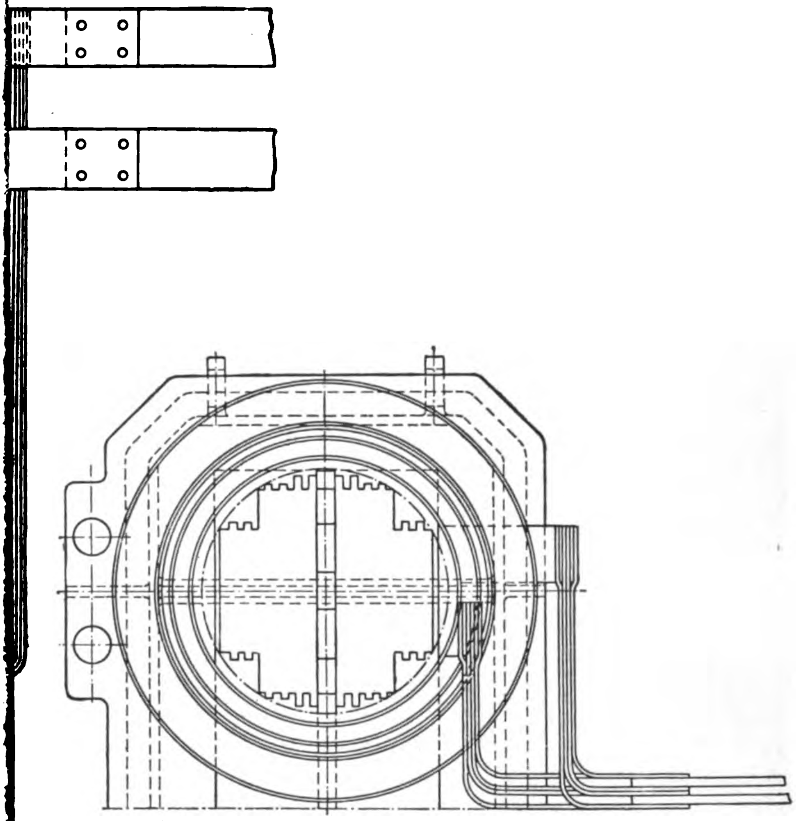
Shell-type transformers are often built with sandwiched coils. The winding is made up of a handy size of copper strip. If necessary in the case of the low-pressure winding several such strips may be wound in parallel. In addition to the insulation between the strips it is advisable to insert corner-pieces of leatheroid or white fibre. To gain cooling surface the coils of shell-transformers may be shaped to spread out at the ends, as is done in the Westinghouse transformers (see Fig. 405, p. 553).

It should be noted that in all cases the coils must be firmly fixed; for otherwise they may (as the result of magnetic leakage) tend to vibrate up and down the limbs, to the great risk of spoiling the insulation.

Attention should be given to the entrance and exit of the high-voltage leads. Cases might be cited of transformers being built with the windings very carefully, even extravagantly, insulated, while the leads to the coils were of ordinary low-pressure flexible conductor resting in places against the iron core clamps!

Plate XVIII.

T.



POLYPHASE TRANSFORMERS.

For the special forms of transformer used for two-phase and three-phase currents, and for transforming two-phase to three-phase currents, or *vice versâ*, the reader is referred to the author's treatise on *Polyphase Electric Currents.* Suffice it here to point out that for three-phase work it is possible to use either three separate single-phase transformers, one for each phase, or a three-phase transformer with three cores combined with yokes common to the three phases. Such three-phase transformers are in this country seldom built above 150 KW normal rating. Above this output their advantages with regard to better balancing are outweighed by their increased cost. In certain cases it may, of course, be advisable to use three-phase transformers of much larger rating and size. In this respect Continental practice differs from English and American practice, various Continental firms making it a rule in all cases to use one three-phase transformer in preference to three single-phase transformers.

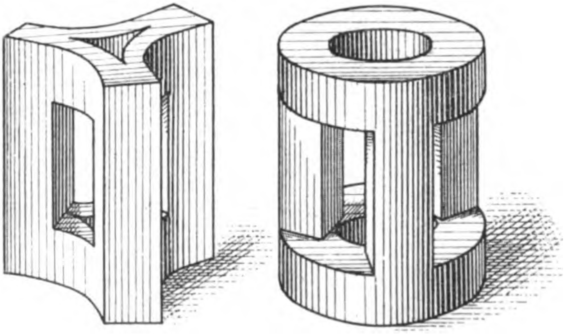


FIG. 417.—THREE-PHASE TRANSFORMER: TRIGONAL PATTERNS.

Three-phase transformers are seldom built of the shell-type, generally of the core-type, with three cores, which in most cases stand upright. Three arrangements are possible. In the first arrangement the three cores are grouped triangularly, with three-cornered yokes, or their equivalent, at the top and at the bottom. This construction, in two varieties, is illustrated by the skeleton diagram of Fig. 417. It gives perfect symmetry for the three phases; but there is some trouble in the mechanical arrangement of the yokes. If these are stamped out triangularly and inserted horizontally between the three cores, it is necessary to interpose a layer of insulation, other-

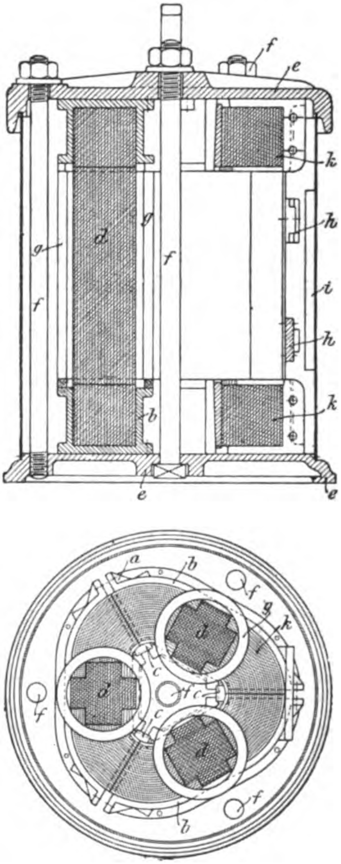


FIG. 418.—THREE-PHASE TRANSFORMER WITH NATURAL COOLING (KUMMER AND CO.).

wise there would be deleterious parasitic currents formed in the stampings. Fig. 418 illustrates a trigonal transformer by Kummer and Co., of Dresden, in which the yokes *k* are formed of bundles of bent strips assembled vertically, and meet the limbs *d* with butt joints above and below, so avoiding the necessity for an interposed layer of insulation. They are clamped horizontally by cast-iron pieces *g*, and held together vertically by the four screw-bolts *f*. Fig. 419 shows the same transformer with the protecting case removed. A 70-KW transformer of this make showed on test an efficiency of 96 per cent. at full-load, and 90 per cent. at half-load.

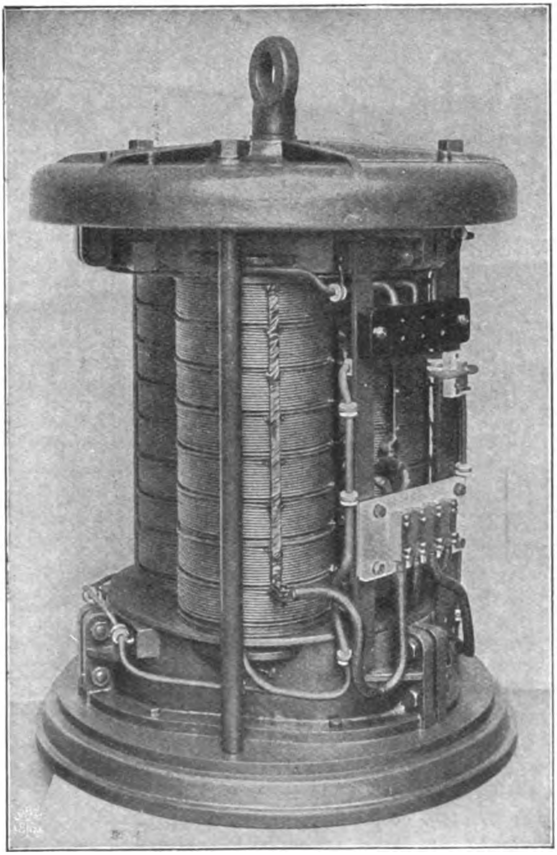


FIG. 419.—THREE-PHASE TRANSFORMER, with case removed.

The three-phase transformer of Siemens and Halske, Figs. 420 and 421, differs in having the yokes of horizontal triangular stampings fitted in between the ends of the core-limbs. In the pattern shown, provision is made for cooling by natural draught through openings at the top and bottom. This firm insulates its core-stampings with thin paper pasted on one face of each; and in the larger sizes clamps the stampings together by insulated bolts. The high-voltage coils are outside the low-voltage coils. Vertical screw-bolts clamp the yokes against the limbs. The magnetic dispersion

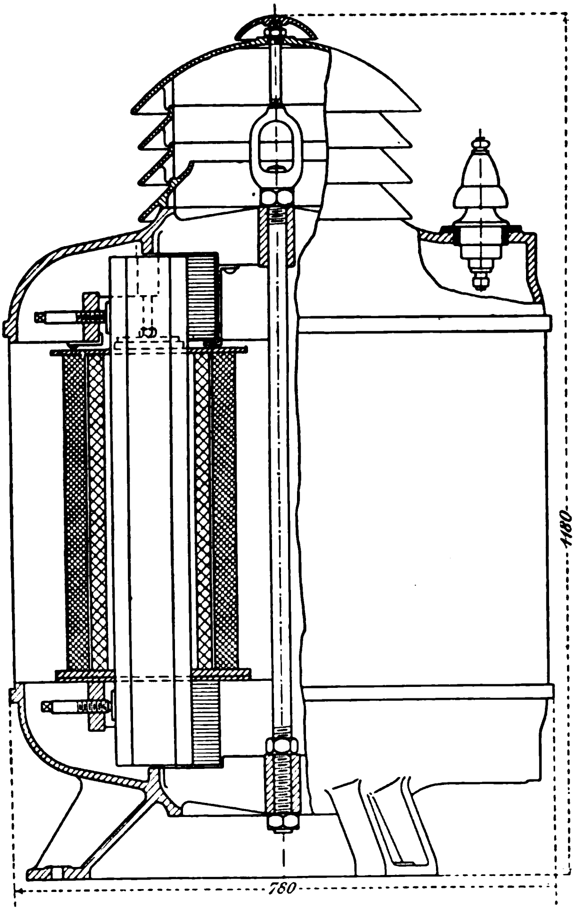


FIG. 420.—THREE-PHASE TRANSFORMER, 20 KW (SIEMENS AND HALSKE). Scale 1 : 10.

is low, and consequently the regulation is good, even on inductive loads. The full-load efficiency varies from 98 per cent. in the 150-KW size to 92 per cent. in the 1-KW size.

In the second arrangement, Fig. 422, the three cores stand side by side, in line, with straight yokes across the top and bottom. The mechanical construction is simpler, but it is magnetically and electrically not quite symmetrical; as the magnetic circuit for the middle core is slightly better than that for either of the end cores, since for it the nett length of the return-parts through the iron is slightly shorter. Hence it is found that the magnetizing current at no-load is slightly smaller for that phase which is wound on the middle core than that of either of the other phases. This is, however, of small importance, and makers generally prefer the straight-core pattern to the trigonal one. The straight-core pattern is usually built up of four sets of stampings, arranged as in Fig. 422, breaking joints in successive layers, or else with butt-joints, as for example the Kolben transformer, Figs. 444, 445, and 446.

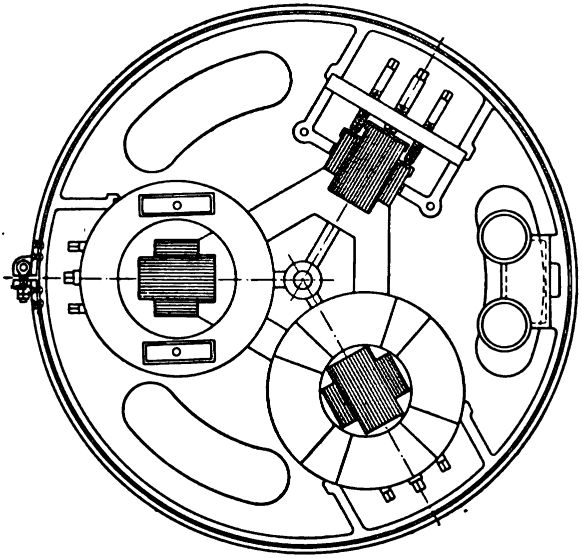


FIG. 421.—TRANSVERSE SECTION OF 20 KW THREE-PHASE TRANSFORMER.

The Allgemeine Company has adopted for air-cooled three-phase transformers a straight-yoke pattern, with the three cores horizontal, and with coils sandwiched. Plate XXIV. illustrates this arrangement for a 100-KW at 50 cycles per second. The short yokes are butt-jointed between the ends of the cores. The transformers of this pattern at Strassburg show only a 3 per cent. voltage-drop on a full inductive load.

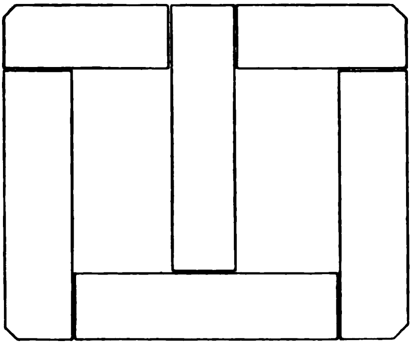


FIG. 422.—THREE-PHASE TRANSFORMER: STRAIGHT-YOKE PATTERN.

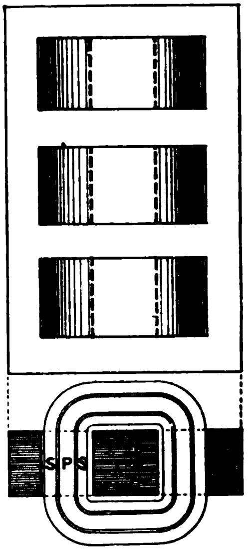


FIG. 423.—THREE-PHASE TRANSFORMER: SHELL TYPE.

In the third arrangement of three-phase transformers the three wound cores stand perpendicularly above one another, while the yokes are disposed as shown in Fig. 423. This pattern therefore belongs to the shell type. It may also be built up of cylindrical form; and indeed the Berry three-phase transformers are so arranged.

IRON FOR TRANSFORMERS.

All the general facts about the magnetization of iron have been set forth in Volume I., Chapter IV., together with information about permeability, hysteresis, and eddy-currents. Some additional curves

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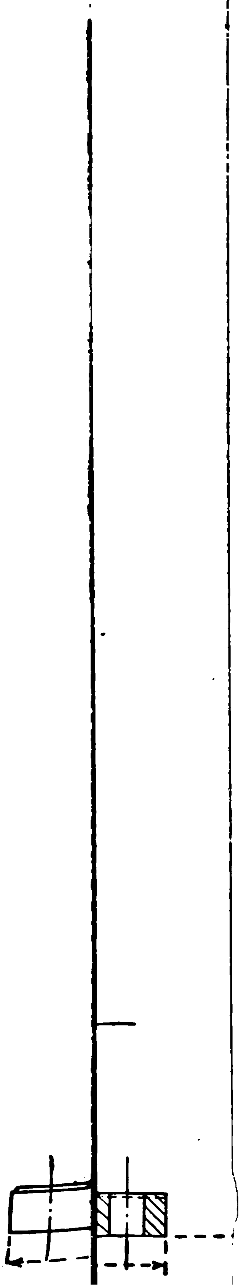

have been given in Fig. 392, page 543. Here we will add a few points in special relation to transformers. Considering the important part played by the iron in a transformer, and the influence which its quality has on the performance of the latter, its selection in order that the best results may be attained becomes a matter of considerable importance to the transformer builder. Even the best commercial iron plate contains impurities in the shape of carbon, silicon, phosphorus, and manganese, and the amount of these impurities greatly influences the hysteresis losses. As an example of this, the case may be cited of two samples of iron plate tested by Mr. Kolben, one being of the best English transformer iron, and the other an ordinary grade of Continental iron. His results are embodied in the tabulation given below:—

Number of Sample.	Total watts lost per lb. of plate at $50 \sim$ and $\mathfrak{B} = 5000$.	Chemical Analysis.		
		Carbon per cent.	Silicon per cent.	Manganese per cent.
I.	0·48	0·036	0·021	0·042
II.	0·93	0·050	0·032	0·065

It will be noticed that the purer English iron (sample No. I.) had only about half the iron loss of the Continental sample. The thickness of plate was 13 and 20 mils respectively, but this would make hardly any difference, as only about 10 per cent. of the total loss is due to eddy-currents.

No very precise information is yet forthcoming as to the specific influence of different elements in combination with iron. It would seem that the presence of silicon increases the permeability slightly, while reducing the hysteresis. Barrett, Brown and Hadfield,[1] working with steels prepared by Hadfield at the Hecla Steel Works, Sheffield, found two alloys having a higher permeability than the purest Swedish charcoal iron. In a field of 4 C.G.S. units the Swedish iron showed a flux density of $\mathfrak{B} = 11,150$, so that $\mu = 2790$. In the same field a silicon steel containing $2\frac{1}{2}$ per cent. of silicon showed $\mathfrak{B} = 12,300$, and $\mu = 3075$; while an aluminium steel with $2\frac{1}{4}$ per cent. of aluminium showed $\mathfrak{B} = 13,800$, and $\mu = 3450$. Messrs. Sankey and Sons, of Bilston, adopting a special process of annealing, have produced an iron known as "Lohys" iron, which in the same field gives $\mathfrak{B} = 10,000$, and $\mu = 2500$; the speciality of this iron being its low hysteresis.

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[1] *Journ. Inst. Elect. Engineers*, xxxi. 674, Feb. 1902.

As tested by Ewing, its hysteresis waste at $\mathfrak{B} = 4000$ and at 100 $\sim$ is only 0·32 watts per lb., while the Swedish charcoal-iron under the same test wasted 0·38 watts per lb. The corresponding figures for Hadfield's alloys were: silicon-iron 0·26 watts per lb., aluminium-iron 0·23 watts per lb.

Ageing of Iron.—There is yet another fact to be considered as regards the iron used for alternate-current apparatus, viz. that the iron loss in many samples increases with time. The rapid reversals of magnetism in transformer cores, for instance, taken in conjunction with the unavoidable heating to which they are subjected, cause some sort of molecular change in the iron, this change causing the hysteresis loss to increase greatly. Attention was first called to this phenomenon in 1894 by Partridge[1] and Mordey,[2] and it has been carefully investigated by the latter and by other observers. It was commonly found that the core losses double themselves in from nine to twelve months; and this is of course a very serious matter. The very best and purest irons seem to deteriorate more than the common brands, the amount of deterioration depending on the temperature to which they are subjected in normal use, and on the amount of pressure employed in clamping up the plates. Mordey showed that it was due to protracted heating. Dr. G. Stern in the years 1897 to 1900 examined many specimens of iron. He found samples by three different German firms to show, when kept at temperatures from 60° to 100° C., an increase in hysteresis waste, varying from 27·7 to 93 per cent. He also found specimens in which after exposure during fifteen months to a temperature of 45° C. the waste had risen 15 per cent., and was still rising; while in another specimen kept at 20° C. for seventeen months, the waste had risen 12 per cent. Two samples by English makers showed no increase. He found that specimens having a low hysteresis coefficient were more liable to deteriorate by ageing than others with a higher coefficient; and that there was more ageing with a low flux-density than with a higher one. Also that annealing the laminations after they were stamped or cut out was of no use. Mordey had so far back as 1890 proposed as a test for iron-losses that

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[1] Partridge, *Electrician*, xxxiv. 160, Dec. 7, 1894.

[2] Mordey, *ib.* 219, Dec. 21, 1894; also *Proc. Roy. Soc.* lvii. 224, Dec. 1894. See also *Electrician*, liii. 780, 1904. The following are the more important contributions to the investigation of this question:—

Blathy, *Electrician*, xxxiv. 191, Dec. 14, 1894: Ewing, *ibid.* 297, Jan. 11, 1895; Roget, *ibid.* xli. 182, and xlii. 530, also *Proc. Roy. Soc.* lxiii. 258, 1898; Ford, *Proc. Amer. Inst. Elec. Engineers*, xvii. 227, April 1900; Stern, *Elektrotechnische Zeitschrift*, xxiv. 407, May 28, 1903; Charpy, *Bulletin de la Société Internationale des Électriciens*, 835, 1902.

"the loss in iron sheets or stampings 0·014 inch thick made up in the form of a transformer and tested by a wattmeter should be 0·38 watt per pound at a magnetization of $\mathfrak{B} = 2500$, and at 100 cycles per second." By this test at that date the loss in the best soft Swedish charcoal iron varied from 0·3 to 0·6 watt. Since that date British manufacturers have greatly improved the quality of their products, and the figure 0·3 can now be always guaranteed for this test. At the same time the difficulty about ageing has entirely disappeared, for makers can now guarantee a quality free from this defect. Apparently steel made by the Bessemer process is liable to age whilst that made by the Martin process does not. As, however, there are irons still in the market which have higher losses, and which deteriorate with age, it is necessary for transformer manufacturers to test the samples furnished to them.

Iron-Testing.—It follows from what has been said above, that all iron plate used for alternate-current work should be tested before being employed for the construction of transformers, induction motors, alternator armatures, etc., in order that a high standard of quality may be maintained, and the delivery of a faulty batch of plates detected. The best commercial way of testing such plates is to make up a sample to a known weight, and then to actually measure the watts absorbed by it when subjected to a given magnetization at a given frequency. A simple apparatus designed by Prof. Kapp for this purpose, and which has been found to give very satisfactory results in practice, is shown in Fig. 424. It consists of two magnetizing coils of known turns and joined in series encircling respectively the bundle of sample plates A, and a yoke B. This latter consists of a number of iron plates of the shape indicated, built up to a suitable thickness and bolted together. The sample is built up of the specimen plates, which are merely rectangular stampings bound up with tape. The iron section of the sample is made equal to the iron section of the yoke, and the weights of both yoke and sample are known. Both are insulated with varnish or thin paper according to the standard practice of the particular works. The sample is prevented from shifting about by means of wooden clamps. To calibrate the apparatus, a number of plates made from the same iron as the yoke are built up as a sample. An alternate current of known voltage and frequency is then sent round the coils, and the reading on a wattmeter placed in circuit noted. This reading gives the watts lost for sample and yoke, and as they are made of the same quality plate these watts can be allotted to sample and yoke in proportion to their weight (see page 479). If now, another sample is made up and substituted for the last, the wattmeter reading gives the sum of the losses in yoke and sample. As the former was found in the first test, we also know the latter. The losses are found for various flux-densities by simply varying the pressure on the terminals of the magnetizing coils, and calculating out the corresponding flux-densities from the formula already given:

$$E = 4\cdot 44 \times f \times S \times (B \times A) \div 10^8.$$

In this formula A is the iron area of the sample or yoke and E the applied volts less the lost volts due to the ohmic resistance of the magnetizing coils. It will be noticed that the only effect of the joints in the magnetic circuit of the apparatus is slightly to increase the current—they will not influence the wattmeter reading.

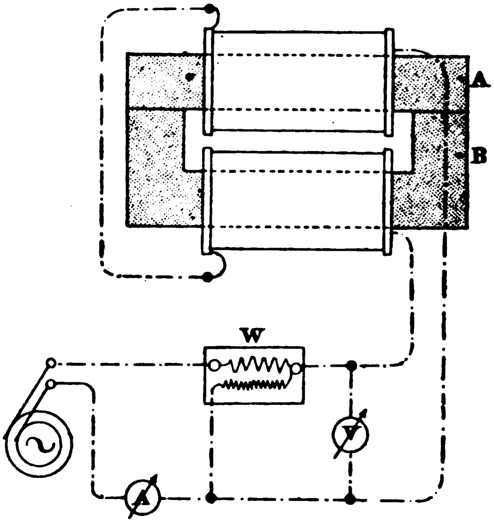

FIG. 424.—APPARATUS FOR TESTING IRON.

A modification of the apparatus has been made by Professor J. Epstein, of Frankfort, in which the whole of the iron employed is of the sample to be tested. This is shown in Fig. 425. It has been adopted by the German Electrotechnic Society for their standardization of iron. The magnetic circuit consists of 4 cores, each being 500 mm. long, 30 mm. wide, of strips of the customary thickness (from 0·3 to 0·5 mm.) at least 2½ kilogrammes of iron being taken. Each strip is coated on one face with tissue paper to prevent contact. The cores are then built up on a wooden board to form a square, each core being surrounded by a magnetizing coil, all four coils being joined in series. The coils are wound on press-spahn tubes, having an aperture 38 mm. square, and 435 mm. long. Each coil is of 150 turns of a wire of 14 sq. mm. section. The corners of the square are clamped together tightly with wooden clamps. An alternate current of frequency 50 ~ is sent through the coil, and its strength adjusted so that a maximum flux-density of $\mathfrak{B} = 4000$ (*i.e.* $\mathbf{B} = 25{,}800$) is reached. It has been agreed to denote by the term "figure of loss" (Verlustziffer) the number of watts lost per kilogramme. The standard temperature is taken as 30° C.; and the density of iron as 7·77. Under these circumstances ordinary good transformer iron shows a figure of loss of 3 to 3·6 watts per kilogramme, or 1·36 to 1·64 watt per lb. For a similar method see G. F. C. Searle, in *Journ. Inst. Electr. Engineers*, xxxiv. 55, 1904.

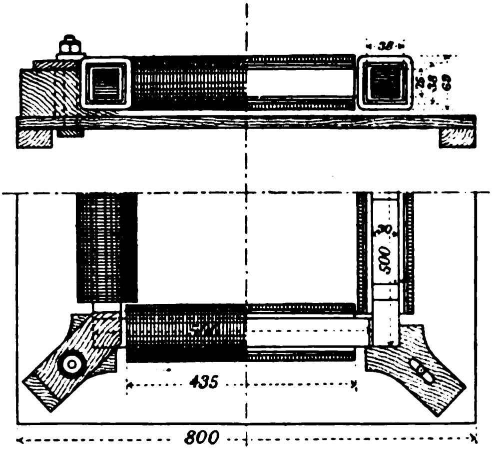

FIG. 425.—EPSTEIN'S APPARATUS FOR TESTING IRON-LOSSES.

Eddy-current Losses.—It will be noted that both these methods of test measure the total iron-loss, that due to eddy-currents being included along with that due to hysteresis. For commercial purposes, it is seldom necessary to separate the two, but it should be remembered that while the hysteresis-waste is proportional to $\mathbf{B}^{1\cdot6}$ and to f, the eddy-waste is proportional to $\mathbf{B}^2$ and to f^2. Further, the eddy-waste increases as the square of the thickness of the laminations, while the hysteresis-waste is independent of the thickness or indeed may be slightly greater for thinner sheets owing to the slightly harder nature of the surfaces of the sheet. If the frequency is reduced from

50~ to 25~, the eddy-waste is reduced to relatively half the importance compared with the hysteresis-waste; but still it is by no means negligible, and is indeed of the same order of importance even when the thinnest sheets are used. It is difficult to get sheets less than 13 mils thick, and a reduction to 10 mils would make little difference. An important investigation by Mordey and Hansard[1] demonstrates the importance of the eddy-waste, and shows that the simple laws generally assumed to be followed by hysteresis and eddies are often widely departed from, so that the only satisfactory way of pre-determining the iron-losses in a transformer is to make wattmeter tests of samples under conditions as nearly as possible like those in actual working.

CALCULATION OF IRON-LOSSES.

In Volume I. methods of calculation were shown, but for present purposes the following recapitulation will suffice.

Hysteresis Constant.—Steinmetz found from numerous experiments that the number of ergs lost per cubic centimetre per cycle by hysteresis alone may be expressed by the empirical formula:

$$\text{ergs per cycle} = \eta \times \mathfrak{B}^{1\cdot6};$$

where η is a numeric depending on the quality of the iron used, and $\mathfrak{B}$ is the *maximum* (C.G.S.) to which the flux-density is carried in the cycle. The customary value of η for transformer iron is 0·0013, but it is not a constant, for it depends in some way on the frequency. It should be determined for a specimen of the iron proposed, and the determination should be made with a current of the same frequency as that to be used with the transformer. In no case ought iron to be accepted for which the hysteretic constant exceeds 0·0016; the makers will now guarantee to supply a transformer iron (actually a very soft mild steel) for which η does not exceed 0·0012. The value of the index of $\mathfrak{B}$ is also not constant, but varies between 1·4 and 1·8; but 1·6 is the most usual value.

From the above formula we may deduce practical formulæ for the hysteresis loss as follows. Here f is the frequency and w_h the watts lost:—

$$w_h = f \times \eta \times \mathfrak{B}^{1\cdot6} \times 10^{-4} \times \text{volume of iron};$$

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[1] *Electrician*, liii. 790, Sept. 2, 1904.

the volume being given in *cubic decimetres.* For British engineers the formula becomes

$$w_h = 1 \cdot 31 \times f \times \eta \times \mathrm{B}^{1 \cdot 6} \times 10^{-6} \times \text{weight of iron};$$

where **B** is the *maximum* value of the flux-density in *lines per square inch*, and the weight is given in *pounds.*

Eddy-current Losses.—These are proportional to the square of the flux-density, to the square of the frequency, and to the square of the thickness of the sheet. Denoting the thickness, expressed in mils, by t, the number of watts lost by eddies w_e, may be written

$$w_e = 0 \cdot 0064 \left(t \times \frac{f}{100} \times \frac{\mathrm{B}}{1000} \right)^2 \times \text{weight of iron.}$$

To calculate out the iron-losses afresh for each case from the formulæ would be too laborious. To shorten calculations we may make use of curves deduced from tests made, on the plan described above, for the particular brands of iron to be used, showing the total iron-losses per pound as a function of the flux-density at a given frequency. Fig. 426 shows the results of some tests made in this way, and which may be useful in designing. The curve A A is for special English transformer plate 13 mils in thickness; curve B B ordinary transformer or armature plate 20 mils thick; while curve C C shows the permeability corresponding to different flux-densities, and which is practically the same for each sample. The curves shown apply only to a frequency of 50 ∼ ; for other frequencies it is sufficient to take the losses as being proportional to the frequency, as they are nearly entirely due to hysteresis.

Another example is afforded by the curves given in Fig. 427. Here one curve represents the watts per pound lost by hysteresis at a frequency of 50 cycles per second, for different values of **B**. The other represents the corresponding number of watts lost by eddies, at the same frequency, the thickness of the iron sheet being assumed at the usual value of 14 mils.

From these curves, the known maximum value of **B**, and the weight of the core, we can at once calculate the two iron-losses of the transformer, if the frequency is of the standard value of 50 cycles. If the frequency be of any other value, say 40 or 25 cycles, then it must be remembered that the hysteresis loss will be proportional to the frequency simply, while the eddy-current loss will be proportional to the square of the frequency.

It has been found independently by Steinmetz, by Fleming, and by Wedding that the efficiency of a given transformer depends to some extent upon the form of the electromotive-force impressed by the generator, a peaked form giving a higher efficiency, a flat-topped square-shouldered form giving a lower efficiency than a pure sine-curve. The reason depends on the fact that the hysteresis losses increase disproportionately with the higher flux-densities. For, since the value of the volts at any instant depends on the rate of change in the magnetic flux, a square-shouldered

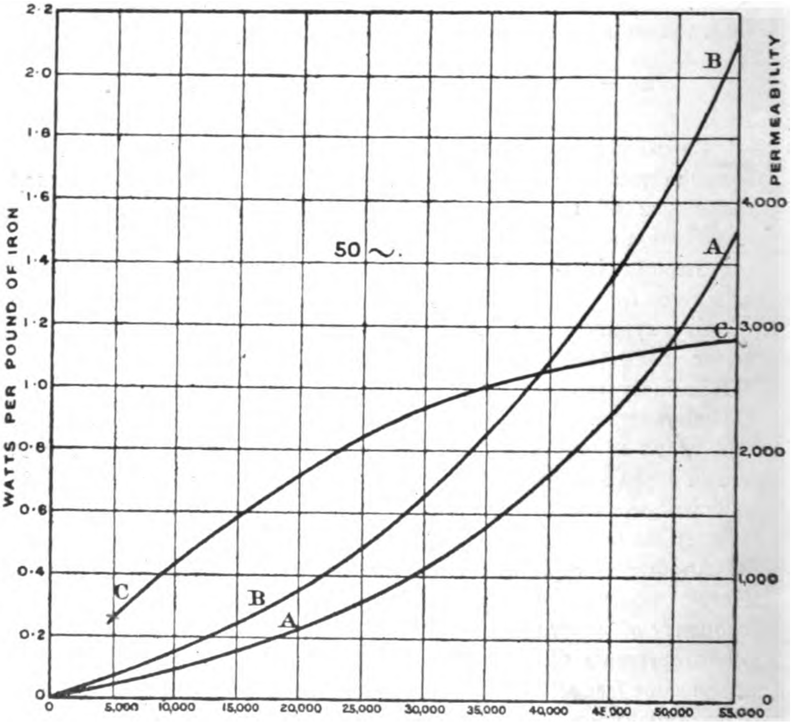

FIG. 426.—TRANSFORMER IRON LOSSES.

volt curve will imply a high-peaked curve of flux-density, and *vice versâ.* Dr. Roessler, in an investigation[1] on this subject, found that at no-load the primary winding when the volts followed a sine-law absorbed 1·5 times as much energy as when a peaked wave was used. He pointed out that one objection to the peaked wave was that it

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[1] *Elek. Zeit.*, August 1, 1895; *Engineer*, August 9, 1895.

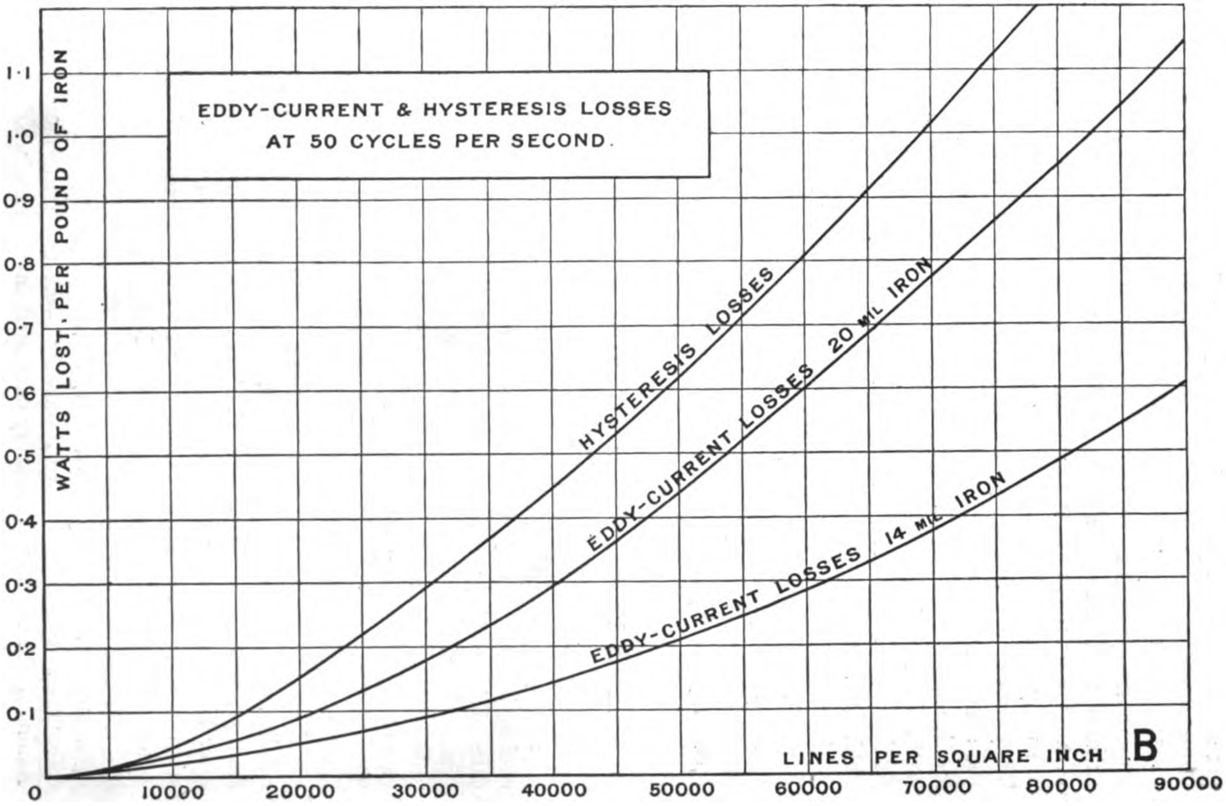
EDDY-CURRENT & HYSTERESIS LOSSES
AT 50 CYCLES PER SECOND.
WATTS LOST. PER POUND OF IRON
1·1
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
10000
20000
30000
40000
50000
60000
70000
80000
90000
LINES PER SQUARE INCH
B
HYSTERESIS LOSSES
EDDY-CURRENT LOSSES 20 MIL IRON
EDDY-CURRENT LOSSES 14 MIL IRON

FIG. 427.

put a greater stress upon the insulation than a sine-wave of the same virtual value.

Messrs. Beeton, Taylor and Barr have also published[1] some important experimental results, which clearly show that if the virtual value of the electromotive-force applied to the transformer is always of the same value, the total iron-losses will vary by an amount which is simply dependent upon the *area* of the electromotive-force wave. They proved experimentally that the hysteresis loss depends only on the area of the electromotive-force wave, and that the eddy-current loss is constant, for a given virtual applied electromotive-force. For further information on this subject the reader should also consult writings of Bedell and Crehore, Kapp, Fleming, Weekes, and Feldmann.

Whatever the wave-form of the applied primary voltage, that of the secondary voltage will copy it in every detail, but the form of the current curve, particularly at low loads, may depart widely from that of the voltage curves. It is governed by the reactances in the secondary circuit, and by that of the magnetization of the core.

HEATING, VENTILATION, AND COOLING OF TRANSFORMERS.

Owing to the fact that a transformer is a stationary piece of apparatus, not receiving ventilation from moving parts, its efficient cooling becomes a very strong feature of the design, especially in the case of large high-pressure transformers. The effective cooling is rendered more difficult owing to the fact that transformers are invariably enclosed in more or less air-tight cases, except in very dry situations, where a perforated metal covering may be permitted. The final degree to which the temperature rises after some hours' continuous working, depends on the total losses in iron and copper, on the total radiating surface, and on the facilities afforded for cooling. In regard to these latter, practice differs considerably amongst different makers. Mr. C. E. L. Brown, for instance, soon after his experience with the Lauffen-Frankfurt transmission, introduced the use of oil as a filling for transformer cases, and this practice is now becoming very general. Not only will a transformer immersed in an oil-filled case have from 20 per cent. to 30 per cent. smaller temperature rise than if it were in the same case without oil, but it is also very securely protected from moisture. Again, if the oil is mechanically circulated in the case, which has

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[1] *Proc. Inst. Elec. Eng.*, July 1896.

been done in America in the case of large transformers, the temperature rise is still less. Another firm has employed in their largest sizes hollow secondary conductors, through which cold water circulates continuously. In the case of the large 1000 kilowatt transformers at Niagara, built by the General Electric Co., an air-blast is employed, and that firm has now a standard line of air-blast transformers. The blast is produced by means of a small motor running from the secondary side, and which is coupled to a blower, cool air being thus circulated through the case. The amount of power consumed by the blower motor averages from one-eighth to one-tenth per cent. of the transformer output—in the case cited above it is only one-fifteenth per cent. In all transformers of any size, the provision of suitable ventilating ducts in the core is absolutely essential, otherwise there may be a portion of the winding at a much higher temperature than the surface temperature. In the case of the Brown, Kapp, etc., transformers, the octagonal arrangement of the core plates provides flues inside the coils up which oil or air may circulate, and which materially assists the cooling. It must be remembered that not only is the cool running of a transformer essential from the insulation point of view, but the cooler it is the better will be its pressure regulation.

With regard to the number of watts per square inch total radiating surface allowed in actual practice, no hard-and-fast rule can be given, on account of the great variety of causes affecting this quantity. An outside figure is perhaps 0·2 watt per square inch for transformers in case without oil, and 0·3 watt with oil, but for any given design, working constants must be got from practice. The curves given in Fig. 428, which are the results of some careful tests made at the Technical College, Finsbury, may perhaps serve as a guide to the probable heating of transformers, more especially to those with the coils outside the iron. They show the rise in temperature corresponding to different values of watts lost per square inch for transformers in air, in case without oil, and in case with oil. Similar curves have been published by Professor Kapp.[1]

One of the most important considerations with respect to transformers is the means adopted for getting rid of the heat which is inevitably developed in the apparatus by the waste energy. Heat can be removed by three processes: convexion, conduction, and radiation. While radiation and convexion are the principal means by which the heat escapes from the case of an entirely enclosed transformer, it is

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[1] *Transformers*, by G. Kapp, 1896.

to conduction and convexion rather than to radiation that we must look for conveying the heat from the core and conductors to the case.

When a transformer is run continuously at full load, the temperature gradually rises until at the end of some hours it becomes constant. The difference between this constant temperature and that of the surrounding atmosphere is called "the temperature-rise at full-load." Its amount constitutes a most important feature in the commercial value of the transformer. There are several reasons why a high temperature-rise is undesirable. Among these the most weighty are :—

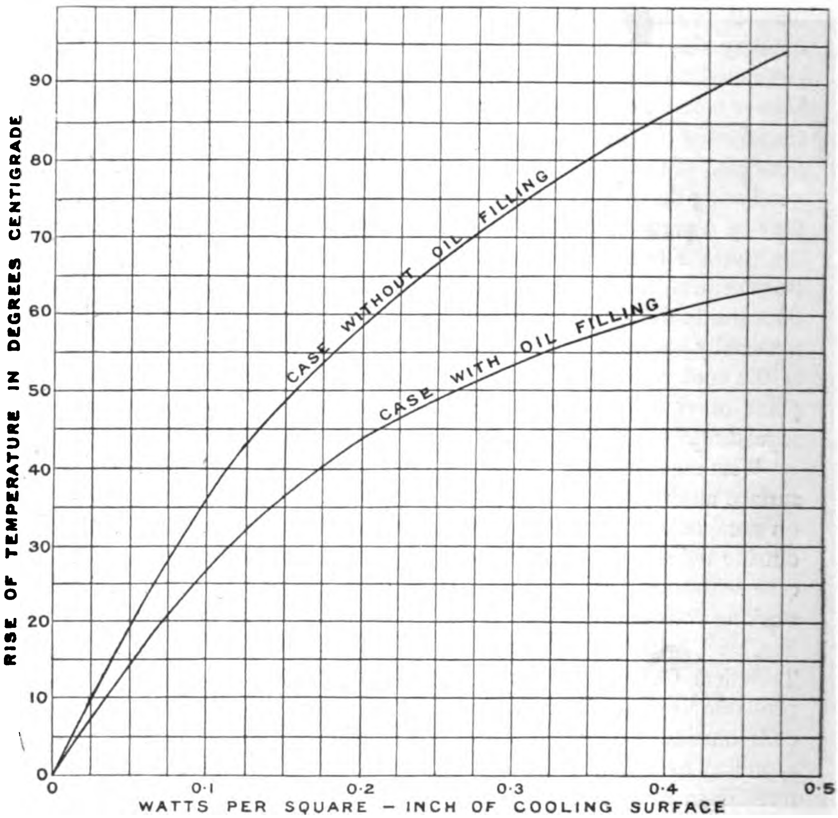

FIG. 428.—CURVES OF TEMPERATURE-RISE.

(*i.*) The rapid deterioration (due to carbonization and disintegration) of the insulation which sets in very rapidly with high temperatures.

(*ii.*) The hysteresis losses in the iron, which increase steadily if the temperature is high, (See p. 574.)

(*iii.*) The danger of fire with a high temperature.

Ordinary insulating materials are liable to char on being overheated; but they degenerate at temperatures far below that at which any such action occurs, in fact their temperature ought not to be raised above the boiling point of water, 100° C. Now if the temperature of the surrounding air be taken as about 20° C. (or in summer 25° C.), and if, to provide a margin of safety, 85° C. be set as the extreme limit, it is clear that the rise of temperature permissible above that of the surrounding air must be fixed at 60 degrees or so. The customary stipulation for transformers is that the temperature-rise in any part shall not exceed 60° C. It is true that for the Valtellina Railway, Messrs. Ganz and Co. have built transformers with a special non-inflammable insulation that will admit, without danger, of an overload of 500 per cent., but this is entirely exceptional.

For transformers of small sizes (up to 10 KVA), the heat produced may be got rid of by means of the natural convexion-current of warm air that rises from the transformer, combined with the conduction of heat from the base into the floor. But with larger transformers some other way must be devised of limiting the temperature-rise; because, while the weight increases as the cube of the linear dimensions, the area of cooling surface only increases as the square.

The transformer may be placed in a large tank filled with oil. The oil carries away the heat to the sides of the case, whence it is emitted to the air outside. The oil also acts as an insulator. The kind of oil used is of little importance, so it be free from water and from acid. The Westinghouse Company used a clear thin oil like signal oil, while other companies use any good machine oil freed from water. Castor oil is good but expensive. A step further in this method of cooling is to cool the oil by means of water circulating in pipes through the oil and round the tank. The rise of temperature in the oil is about 0·6 to 0·7 as great as that of the transformer itself.

The Westinghouse Company give the following data respecting the amount of oil required for their transformers. The higher figures relate to oil transformers not cooled by water.

In the case of water-cooled transformers, the supply of water seems to vary greatly in different sizes and makes of transformer, so that it is impossible to give figures.

Kilowatts of Rated Output.	Weight of Oil, in Pounds.
¼ to ½	9
1 to 2	15
3	22
4 to 5	31
7½ to 10	53 to 200
15	128 to 410
20 to 25	185 to 420
30 to 37½	245 to 600
50	375 to 700
100	900 to 1300
500	3000 to 4000

Another much used way of cooling a transformer is by *forced draught.* A current of air at a low pressure is forced through the air-ducts and case of the transformer. A special ventilating fan may be fixed to each transformer, or, where transformers are banked together, an air-chamber may be built underneath the floor, from which the air passes upwards through holes and so through the ventilating ducts in the transformers, one large fan serving for moving the air in the chamber. The power needed to drive the fan involves a slight drop in the efficiency, which, however, will not exceed ½ per cent. The disadvantage of this method is the slight danger of fire. A short-circuit in a transformer after it has been working some time will almost certainly burn out its insulation, and the fire may spread to the others in the station. With the oil-cooled transformer the insulation often rots under the action of the oil, so that a repair practically means re-winding the whole transformer.

To illustrate the cooling effect of the oil upon a transformer some curves are given in Fig. 429 for a 500 KVA Westinghouse oil-filled, water-cooled transformer. The top curve gives the secondary copper temperature, the second, the primary copper temperature. The third curve is that of the oil, and the last, the temperature of the outlet water.

For their air-cooled (forced draught) transformers, the Oerlikon Machine Works allow in the core a loss of 0·07 watt per sq. cm. of core surface, which corresponds to a rise of 60° C. For the secondary winding a rise of 60° C. is also allowed, and the permissible loss is

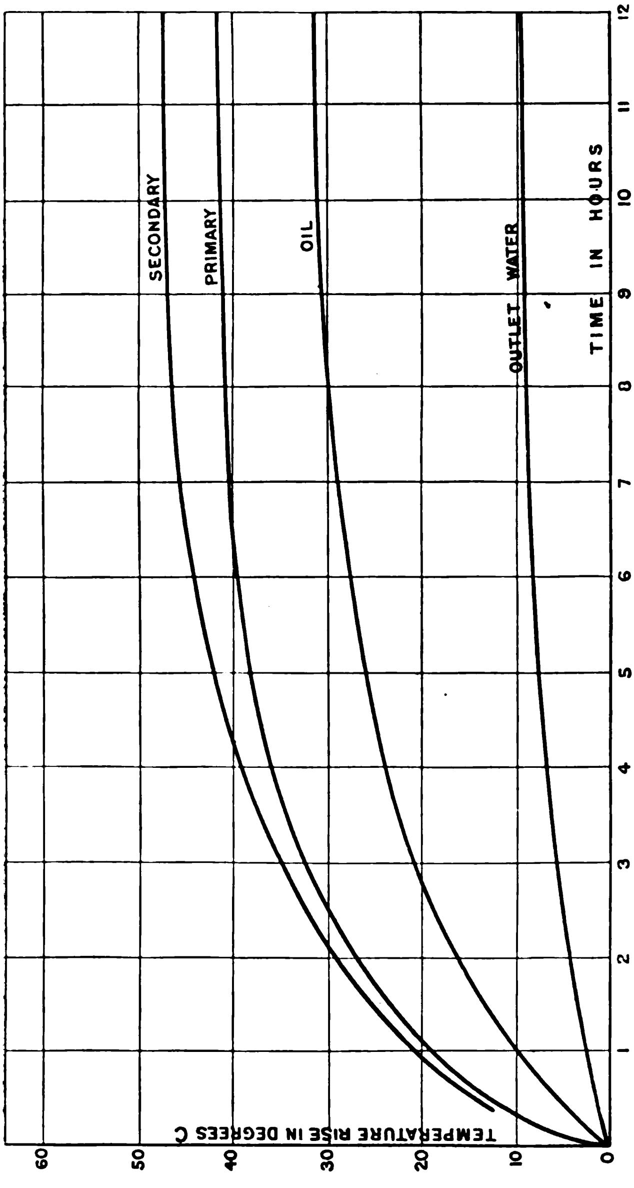

FIG. 429.—HEATING OF OIL-FILLED WATER-COOLED TRANSFORMER, 5000–2000 VOLTS.

stated to be 0·06 watt per sq. cm. of coil surface; while for the primary, with a permissible rise of 40° C., the permissible loss is fixed at 0·03 watt per sq. cm. of coil surface. The volume of air required per minute may be calculated, in cubic feet, by the formula:

cubic feet per minute = total losses (watts) × constant;

where the value of the constant varies from 0·0032 to 0·004.

A suitable formula for finding the temperature-rise is as follows:

Let A be the total external surface, in square inches, of the transformer directly in contact with the air or oil, including the outer surface of both primary and secondary coils.

Let B be the area, in square inches, in contact with the flow.

Then the temperature-rise will be

$$t = \frac{1000 \times \text{watts lost}}{a \cdot A + b \cdot B};$$

where the coefficients a and b have the following values:—

(*i.*) For transformers cooled by natural draught $a = 3$ to 3·6

(*ii.*) For transformers cooled in oil . . . $a = 3·5$ to 4·5

(*iii.*) For transformers cooled in oil with water cooling or means of circulation . $a = 4·2$ to 6·0

(*iv.*) For transformers cooled by air-blast . . $a = 4·9$ to 6

The value of b varies between 4·5 and 6.

The better the cooling facilities, such as the possibility for free circulation of oil, the pressure of numerous air-ducts, etc. the higher will be the value of the coefficient a.

EFFICIENCY OF TRANSFORMERS.

By definition the efficiency of a transformer is

$$\text{efficiency} = \frac{\text{input} - \text{losses}}{\text{input}}.$$

The losses consist of two parts, viz. constant iron-losses and the varying copper-losses.

The efficiency may therefore be written as

$$\eta = \frac{\mathrm{E\,C} - \mathrm{C}^2 r - w_i}{\mathrm{E\,C}}$$

$$\eta = 1 - \frac{\mathrm{C}\,r}{\mathrm{E}} - \frac{w_t}{\mathrm{E\,C}}.$$

The term $\frac{C\,r}{E}$ we may call the loss in efficiency due to the copper and $\frac{w_i}{E\,C}$ the loss in efficiency due to iron. These are expressed in Fig. 430, the first by a straight line through the origin and the second by a rectangular hyperbola. These two curves cross in a point X, and before this point is reached the iron-losses are more important in determining the efficiency loss, but when this load is passed the copper-losses chiefly determine the shape of the curve. If the efficiency curve is to be very square-shouldered, *i.e.* if the efficiency is to be relatively high at low loads, then the iron-losses must be kept low. The point X also gives the load of maximum efficiency. The above remarks apply, of course, to many other classes of electrical machinery in which the iron-losses are constant, but they have been given here because the question of transformer efficiency is one of great importance to the central station engineer,

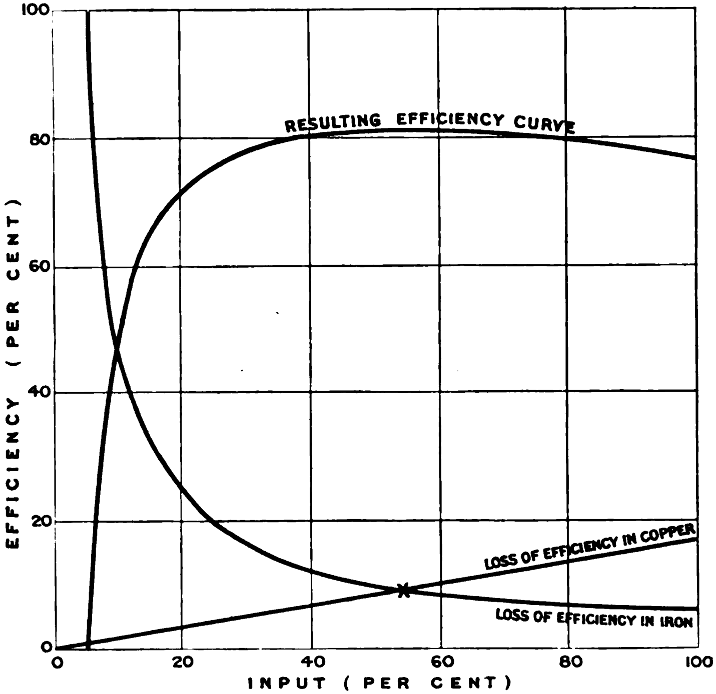

FIG. 430.

who must have his transformers on day and night although they are working at next to no-load for the greater part of the time.

The All-Day Efficiency of a Transformer.—In central stations the ratio of the average output to the maximum output is called the *load factor.* It varies between 10 to 15 per cent. for a lighting station to about 25 per cent. for one with a good day motor load. To estimate the transformer copper-losses during a complete day, it will be sufficiently accurate for our purpose to take them as those produced by the average load, in which case it will be expressed as energy in watt-hours by the expression

$$24\,\epsilon^2\,C^2\,r$$

where ϵ = the load-factor, and C the normal maximum load.

The iron-loss, which continues all the time, will be given by the expression

$$24\,w_i$$

so that the all-day efficiency will be given by the expression

$$\frac{24\,\epsilon\,E\,C - 24\,\epsilon^2\,C^2\,r - 24\,w_i}{24\,\epsilon\,E\,C}$$

$$= 1 - \frac{\epsilon\,C\,r}{E} - \frac{w_i}{\epsilon\,E\,C}.$$

The loss in efficiency is now

$$\frac{\epsilon\,C\,r}{E} + \frac{w_i}{\epsilon\,E\,C},$$

in which it is to be remembered C is the maximum load on the transformer.

To make the all-day efficiency a maximum the two terms in the above expression must be equal, so that

$$\frac{\epsilon\,C\,r}{E} = \frac{w_i}{\epsilon\,E\,C}$$

i.e.

$$\frac{w_i}{C^2\,r} = \epsilon^2$$

or when

$$\frac{\text{iron-losses}}{\text{maximum load copper-losses}} = \epsilon^2.$$

In most cases this of course is not a practical solution, as the question of pressure-drop and prime cost must be considered. The losses in a transformer have to be paid for in any case, and it becomes a question of proportioning the expense of the energy wasted in heating and the interest on the outlay on a more expensive but again more efficient apparatus.

It is usual, therefore, to specify for transformers that they shall have not only a certain full-load efficiency, but that the efficiency at ⅙-load shall attain a certain value. A good modern transformer rated at 100 KW ought to have an efficiency of 98 per cent. at full-load and of 95 at ⅙-load. This attainment of high efficiency at low loads can only be realised by keeping iron-losses at a minimum, by use of the very finest iron, and by reduction of the primary copper-loss. One way of raising the efficiency is to use square copper-wire instead of round for the primary, as this reduces the waste space.

MATHEMATICAL THEORY OF ALTERNATE-CURRENT TRANSFORMERS

There are two ways of treating the theory of transformers. In the first, which lends itself the more easily to simple treatment, and has been used on pp. 533 to 544, the fundamental consideration is the alternating magnetic flux in the core, which induces electromotive-forces in the two windings, and is itself due to the resultant of the two sets of ampere-turns in the coils. This method has been elaborated by Hopkinson.[1] In the second the calculations are effected by introducing the notion of coefficients of mutual and self-induction into the differential equations for the two circuits. The latter method, due to Maxwell,[2] consists in finding the electromotive-force induced in the second circuit by the variations of current impressed upon the first circuit.

First let us consider the coefficients of mutual and self-induction. In order to calculate the mutual action of the two circuits, we want to know the amount of cutting of magnetic lines by the *secondary* coils that takes place when unit current is made to flow, or is stopped in the *primary* coils. Let M be used as a symbol for this quantity. It will be proportional to the number of turns in the secondary coil, because each turn encircles the iron core and cuts

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[1] *Proc. Roy. Soc.*, February 1887.

[2] *Philosophical Transactions*, clv. pt. i. 459, 1865. In this paper Maxwell shows that the effect of the second circuit is to add to the apparent resistance and diminish the apparent self-induction of the first circuit. The student will find the equations more fully treated by Mascart and Joubert, *Électricité et Magnétisme*, i. 593 and ii. 834; also by Hopkinson, *Journal Soc. Teleg. Engineers*, xiii. 511, 1884; Ferraris, *Mem. Acad. Sci.* (Turin), xxxvii. 1885; and by Vaschy, *Annales Télégraphiques*, 1885–6, or *Théorie des Machines Magnéto et Dynamo-Électriques*, 31. A summary of Maxwell's work is given in Fleming's book.

the magnetic lines; it will also be proportional to the number of turns in the primary coil, because, *cæteris paribus*, the magnetism evoked in the iron core is proportional to the ampere-turns that excite it; it will also be proportional at every stage to the permeability of the iron core. We may, in fact, calculate M by the magnetic principles laid down previously. Suppose the iron core to form a closed circuit of length l, section A, permeability μ; and that S_1 and S_2 are the respective numbers of turns in primary and secondary. Then, if the primary current is unity (in absolute C.G.S. units), the magnetomotive-force due to it will be $4 \pi S_1$, and the reluctance will be $l / A \mu$. Dividing the former by the latter, we shall have an expression for the number of lines in the core; this multiplied by S_2 gives the amount of cutting of lines by the secondary circuit; or in symbols

$$M = 4 \pi S_1 S_2 A \mu / l.$$

The name given to this quantity is the *coefficient of mutual induction.* If the current in the primary has the value C_1 (absolute C.G.S. units), then the amount of cutting by the secondary on turning this current on or off will be M C_1. And if the rate of increase or decrease of the primary current at any instant is known, this multiplied by M will give the electromotive-force impressed at that instant on the secondary circuit.

Considerations precisely analogous to those above will show that there will be a *coefficient of self-induction*, which we will call L_1, which represents the amount of cutting, by the primary coil, of the magnetic lines created in the coil when the primary coil carries unit current; and, as before, the value of this coefficient will be

$$L_1 = 4 \pi S_1^2 A \mu / l.$$

As S_1 is itself usually large, L_1 will be enormous. Further, there will be a coefficient of self-induction L_2 in the secondary circuit, such that

$$L_2 = 4 \pi S_2^2 A \mu / l.$$

In a well-built transformer it is clear that

$$M = \sqrt{L_1 L_2}.$$

If, however, all the magnetic lines due to one circuit are not enclosed by the other, M will have a less value than is indicated by the above relation. (See a paper by Dr. Bedell read at the Chicago Congress, 1893.)

The ratio between the two electromotive-forces and the two sets of windings,

$$\frac{S_1}{S_2} = k,$$

we call the *coefficient of transformation.*

If it is assumed that there are equal weights of copper used in the primary and secondary coils, then the following relations will hold good:—

—	Primary.	Second ry.	Ratio.
Windings	S_1	S_2	k
Resistance	r_1	r_2	k^2
Self-induction	L_1	L_2	k^2
Reactance	$p\,L_1$	$p\,L_2$	k^2
Electromotive-force	E_1	E_2	k
Current	C_1	C_2	k^{-1}
Heat-waste	$C_1^2 r_1$	$C_2^2 r_2$	1

Also $$M = \frac{L_1}{k} = k\,L_2.$$

Maxwell's Theory.—At any given instant the impressed electromotive-force in the primary circuit must be sufficient not only to drive the current C_1 through the resistance R_1 of that circuit, but must also be adequate to counterbalance the reactions arising from mutual and self-induction. These at that instant will have the respective values $M\frac{d\,C_2}{d\,t}$ and $L\frac{d\,C_1}{d\,t}$.

Accordingly we write as the differential equation of the first circuit—

$$E_1 - M\frac{d\,C_2}{d\,t} - L_1\frac{d\,C_1}{d\,t} - R_1 C_1 = 0; \qquad (1)$$

where E_1 is the impressed electromotive-force of the generator which is supposed to fulfil the condition $E_1 = D \sin 2\pi n t$ (see p. 4). If the supposition is admitted that a constant (alternating) potential can be maintained at the terminals of the primary coil (by proper compounding of the alternator, or otherwise), then the letters E, L, and R_1, may be taken to apply to that part of the primary circuit only which lies between the terminals of the primary coil. From this differential equation we have to deduce a value for $M\frac{d\,C_2}{d\,t}$. For brevity we will write p for $2\pi n$: and $-p^2 C$ for $\frac{d^2 C}{dt^2}$, because C is also assumed to be a sine-function. Then differentiating equation (1) we get

$$\frac{dE_1}{dt} + Mp^2 C_2 + L_1 p^2 C_1 - R_1 \frac{dC_1}{dt} = 0. \qquad (2)$$

Now multiply this by R_1 to get equation (3), and multiply equation (1) by $L_1 p^2$ to get equation (4), and add (3) and (4) to get (5).

$$R_1 \frac{dE_1}{dt} + M p^2 R_1 C_2 + L_1 p^2 R_1 C_1 - R_1^2 \frac{dC_1}{dt} = 0. \qquad (3)$$

$$L_1 p^2 E_1 - L_1 p^2 M \frac{dC_2}{dt} - L_1^2 p^2 \frac{dC_1}{dt} - L^1 p^2 R_1 C_1 = 0. \qquad (4)$$

$$\left(R_1^2 + L_1^2 p^2\right) \frac{dC_1}{dt} = R_1 \frac{dE_1}{dt} + L_1 p^2 E_1 + M p^2 \left(R_1 C_2 - L_1 \frac{dC_2}{dt}\right). \qquad (5)$$

Now multiply every term by $\frac{M}{R_1^2 + L_1^2 p^2}$, and write the following abbreviations:—

$$\frac{Mp}{\sqrt{R_1^2 + L_1^2 p^2}} = \frac{1}{k},$$

$$R_1/k^2 = \rho,$$

$$L_1/k^2 = \lambda.$$

Then

$$-\frac{1}{kM}\left(\frac{R_1}{p} \cdot \frac{dE_1}{dt} + L_1 E_1\right) = E_2 = \frac{1}{k} E_1 \sin(pt - \phi),$$

where ϕ relates to the phase of the electromotive-force; and we may write equation (5) as

$$M \frac{dC_1}{dt} = \rho C_2 - \lambda \frac{dC_2}{dt} - E_2. \qquad (6)$$

Now the differential equation for the second circuit is—

$$M \frac{dC_1}{dt} + L_2 \frac{dC_2}{dt} + R_2 C_2 = 0; \qquad (7)$$

there being in this circuit no other electromotive-forces than those due to mutual and self-induction. Inserting in (7) the value obtained in (6), we get as the final equation—

$$(R_2 + \rho) C_2 + (L_2 - \lambda) \frac{dC_2}{dt} - E_2 = 0. \qquad (8)$$

Examination of the quantity k shows us that if R_1 be small enough or p large enough, it becomes equal to $\frac{L_1^2}{M}$; or is the same thing as the ratio of the windings for which we have used the same symbol. Then returning to interpret equation (8) we see that it shows us that the whole effect is equivalent to that which would happen if, the primary circuit being absent, there were introduced into the secondary circuit an electromotive-force equal to E_1 divided by k, and at the same time the resistance were increased by a quantity equal to R_1/k^2, and the self-induction were diminished by a quantity equal to L_1/k^2. If there are equal weights of copper in the two windings, $L_2 = L_1/k^2$, and $R_2 = R_1/k^2$; and the effect when the transformer is fully at work is to make ρ equal to the internal resistance of the secondary, and λ equal to L_2, so that the internal resistance is virtually doubled and the self-induction nearly wiped out.

Professor Perry has contributed several important papers [1] on the theory of transformers, in which he has treated leakage and multiple secondaries mathematically.

The theory of the transformer in terms of complex quantities has been given [2] by Steinmetz. For other examples of mathematical treatment the reader is referred to the works of Fleming, of Bedell and Crehore, of Kapp, and of Feldmann.

Phase Relations in Transformers.—Keeping in mind always as the chief consideration in the operation of a transformer, the alternating magnetic flux in the core, we must next study the relations to it of the other quantities. It may be remarked that in a system of supply at constant pressure, this flux scarcely varies between no-load and full-load. As in a compounded dynamo, so in a regulated transformer, to keep the volts at the terminals of the lamp-circuit constant needs at full-load an increase of but 2 or 3 per cent. in the magnetic flux to compensate for the drop. To simplify matters we will suppose, however, that a drop is allowed to occur, but that the flux always alternates around the same cycle. Also, for simplification, suppose the ratio of transformation to be = 1, so that ampere-turns in each coil may be plotted to same scale as amperes. For any other ratio it will at any time be easy to substitute any given value of the ratio k. Then $E_1 = E_2$, and both are at right-angles to the line O N, Fig. 431, which on the clock diagram represents the time when the flux is at its maximum in either direction. Consider first the case of *no-load*; then the only current will be that in the primary, and if there were neither hysteresis nor eddy-currents in the core it would

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[1] *Phil. Mag.*, August 1891; and *Proc. Roy. Soc.*, li. 455, May 1892.

[2] *Alternating-current Phenomena*, p. 193.

be an entirely wattless current, in quadrature with the primary impressed volts and in phase with flux. Let the value of this *magnetizing current*, C_M, be represented by the line O C_M. But as hysteresis and eddy-currents put a small load upon the transformer there will necessarily be a small component of current C_H in phase with the volts.

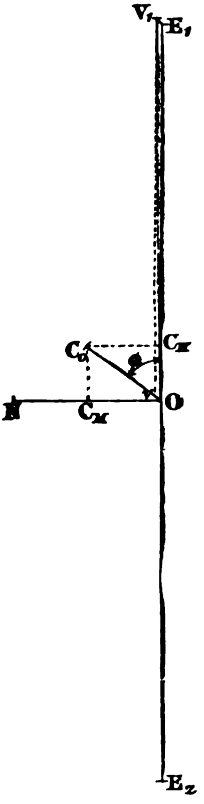

FIG 431.

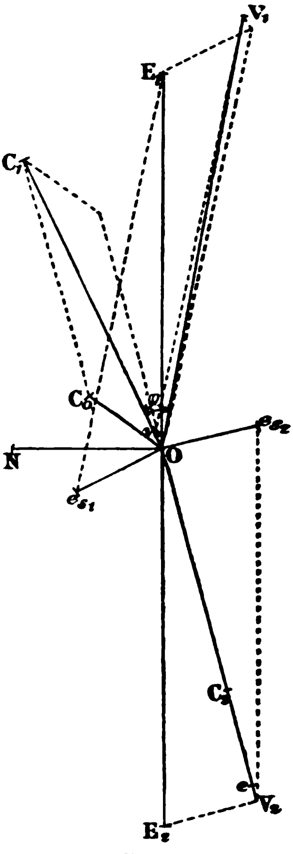

FIG. 432.

This may be represented by the line O C_H. The actual no-load current will be the resultant of O C_M and O C_H namely O C_0. The *power-factor* at no-load will be the ratio of the true watts to the apparent watts, or is $C_H \div C_0$. To furnish the small electromotive-force O v requisite to drive the current C_0 through internal resistance of the primary, the impressed primary volts must have a magnitude and phase, such that O V_1 shall be the resultant of O E_1 and O v. But as the no-load current is, say, only 3 per cent. of the full current, and as the primary lost volts at full-load will not be more than 2 per cent., O v will not be more than about $\frac{1}{1500}$ of O E_1, and the difference of phase between O V_1 and O E_1 will be insignificant.

At *full-load* the phase relations are somewhat different, and they differ according as to whether the load on the secondary circuit is a plain resistance, or whether it is inductive, causing a lag of the secondary current behind the secondary terminal pressure. Let the former case be first considered, namely that of a commercial transformer loaded on, say, a bank of lamps. Taking the flux line as a line of reference as in the last case, we have (Fig. 432) O C_0 as the no-load current found by the last construction, and O E_1 and O E_2 at right angles to O N as the back-electromotive-force of the primary and the induced electromotive-force of the secondary respectively. Now as the transformer is loaded, there will be a certain amount of magnetic leakage present, the actual amount depending on the design, and, as we have seen before, the effect of this is to introduce two electromotive-forces in quadrature with the primary and secondary currents respectively. If the line O e_{s_2} represents the one in quadrature with the secondary current, we have O V_2 as the resultant of it and of the induced secondary pressure. Subtracting a small amount e V_2 to represent the ohmic drop in the secondary due to the load, we get O e as the terminal secondary volts available for doing work. As the lead is non-inductive, we mark off on this line a piece O C_2 representing the secondary current to scale, so that the power given out by the secondary will be O e × O C_2. Producing O C_2 to an equal length upwards, and compounding it with the no-load current vector O C_0, we obtain the line O C_1 as representing the primary current in magnitude and direction.

Now the vector representing leakage in the primary we know to be in quadrature with this, and so set off Oe_{s_1} to represent it to scale. The volts lost in driving the primary current through the windings must obviously be in phase with it, namely O v. Compounding these two with O E_1 we get the line O V_1 as representing the primary impressed pressure in position and magnitude, for it must clearly counterbalance the back-electromotive-force of the primary, annul the leakage effect, and make good the lost volts.

On comparing this figure with the last, it will be seen at once that the angle by which the primary current lags behind the impressed pressure is much smaller—in other words the power-factor is much nearer unity than when the transformer was unloaded. The diagram also shows that the smaller the no-load current, the more nearly would C_1 and C_2 be in complete opposition of phase, and further, in order to keep the secondary induced pressure the same value as when the transformer was unloaded, we have to increase the primary pressure V_1—in other words, the ratio of transformation has increased.

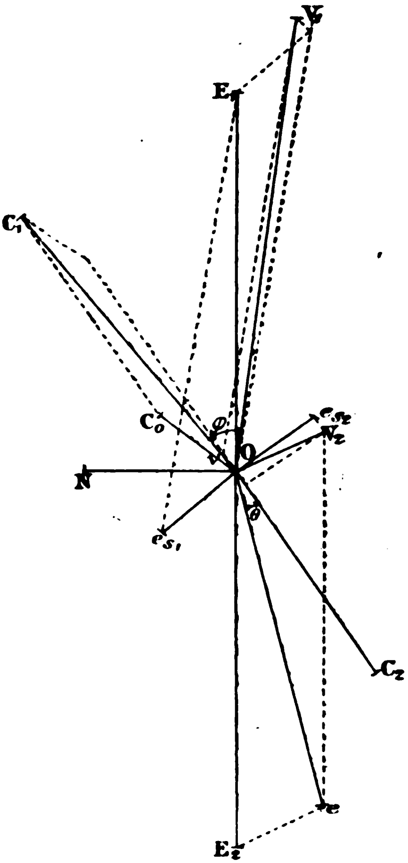

FIG. 433.

Fig. 433 shows the diagram for the same transformer giving out the same current under the same terminal pressure to an inductive load. The secondary current O C_2 is lagging behind the terminal volts by an angle θ. The lettering employed is the same as in the previous diagrams, and the construction is made in a similar manner. In order to get the secondary terminal pressure we have to remember that in this case the lost volts are in phase with the current, but not with the terminal electromotive-force. Hence O e is found by first combining the lost volts with e_{s_2}, giving O V_2, and then combining OV_2 with O E_2, which gives the terminal pressure O e. The power given out by the secondary is of course O C_2 × O e × cos θ, or less than before, but as will be noticed, the primary current has increased somewhat, due to the diminished power-factor. An inspection of the diagram will also show that the ratio of transformation has still further increased, that is, the secondary pressure drops more than in the last case, although the secondary current has the same value. This is due to the lag in the secondary circuit, causing the current to come more completely into opposition of phase with regard to the primary current, as can be seen from the diagram. This in turn causes the two inductive electromotive-forces due to leakage, namely e_{s_1} and e_{s_2} to come more nearly into line, and also to act at a more favourable angle for producing a pressure drop—that is, a lagging secondary current allows magnetic leakage to come into play to a greater extent than when the current is in phase with the terminal pressure. The converse holds in the case of a *leading* secondary current.

It follows, then, that the drop in pressure of a transformer will be a maximum when the primary and secondary currents are in exact opposition of phase, and this condition can only occur in a transformer having no iron losses. But there is one case in practice in which it holds very nearly, namely, when a transformer is short-circuited at a reduced primary voltage, and this case we will now proceed to investigate. Let us take the same transformer as in the other cases, short-circuit its secondary through an amperemeter, and then supply to the primary side sufficient pressure to drive the full-load current through the secondary windings. The diagram representing this state of things is shown in Fig. 434. We have then no secondary terminal pressure, a greatly diminished primary impressed pressure, and very trifling iron losses on account of the small flux-density in the core. But the lost volts in primary and secondary are the same, and so are the leakage volts, on account of the secondary current having the same value as before. It will be seen from the diagram that the secondary current O C_2 is lagging behind the secondary induced pressure by a large angle, and this would be 90° if the secondary winding had no resistance, making O e nothing. It will also be noticed that O E_1, the primary impressed volts, is practically equal to $e_{s_1} + e_{s_2}$, so that a very simple experiment will give us the magnitude of these quantities for any transformer in which the copper-losses are very small, as is invariably the case in practice. Moreover, owing to the small iron-losses causing O C_0 to be so small, the primary and secondary currents are practically in exact opposition of phase, as also are e_{s_1} and e_{s_2}, and E_2 and V_1.

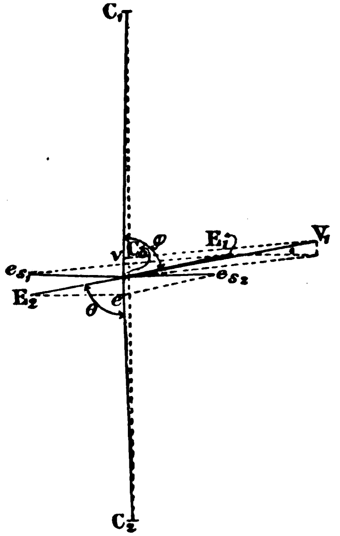

FIG. 434.

Owing to the fact that the primary impressed pressure O V_1 is simply needed to drive the normal currents through the windings against their resistance and the reactive effects of magnetic leakage, it follows that O V_1 is simply the *maximum* drop the transformer can have with the current O C_2 when loaded in the ordinary way, because the relation of the primary and secondary currents is such as to cause magnetic leakage to come into play to the greatest possible extent. We see, for instance, that if this transformer was taken, and actually loaded up on a circuit, having sufficient self-induction in it to cause the full-load secondary current to lag behind the secondary induced pressure by the angle θ, the primary pressure would have to exceed the secondary pressure by an amount O V_1—that is, the drop would be equal to O V_1, and any variation in the angle of lag in the secondary circuit would cause the drop to diminish. If any other transformer were taken, not having an equal transformation ratio, but ratio k_{1c} the maximum drop that transformer could have with the full-load current would be simply O $V_1 \div k$.

Short-Circuit Test.—It will now readily be seen that we can very easily determine the maximum drop of pressure which a transformer will have with a given current, without actually putting it on the load, and this, especially in the case of large transformers, is a great commercial consideration. All we have to do is to short-circuit its secondary through an amperemeter, and then gradually supply pressure to the primary until the amperemeter indicates the rated full-load current. This value of impressed pressure divided by the transformer's ratio of transformation will give the *maximum drop* it can have for full-load current.

Example.—A certain 50-kilowatt Brown transformer was designed to transform from 2000 to 105 volts. On making the test as above described, it was found that 56 volts had to be applied to the primary in order that 475 amperes might flow through the short-circuited secondary. Hence maximum drop at 475 amperes

$$= 56 \times \frac{105}{2000} = 2 \cdot 9 \text{ volts} = 2\tfrac{3}{4} \text{ per cent.}$$

The primary pressure required to drive a given current through the short-circuited secondary of a transformer is often known as the

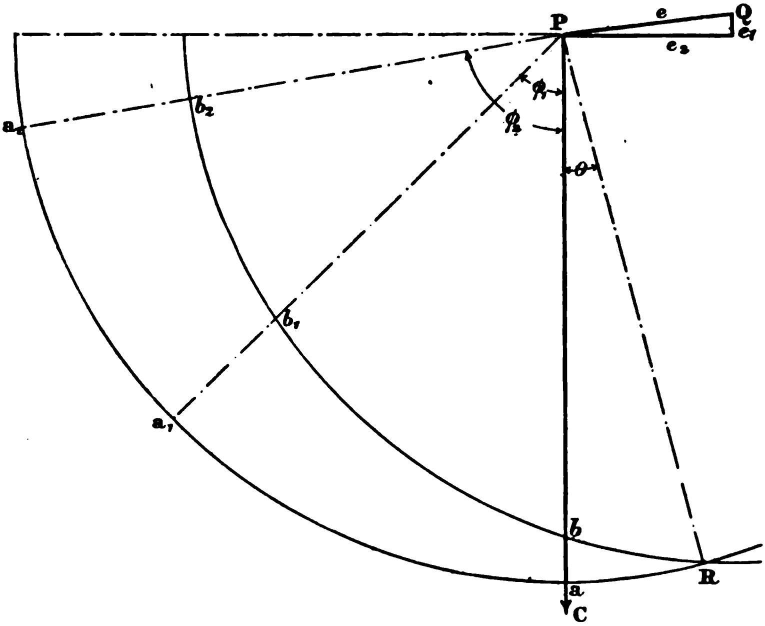

FIG. 435.—KAPP'S DIAGRAM OF TRANSFORMER REGULATION.

"impedance volts" for that current, and is the side P Q of the triangle shown in Fig. 435. It has been stated before that the phase relation of the impedance volts to the secondary pressure wholly determines the drop of the latter. On short-circuit, for instance, the two are in line, and in direct opposition, and hence the drop is a maximum, the lag of the secondary current behind the induced pressure being $(90° - a = \theta)$. (Fig. 434.) Now the effect of altering the angle of lag in the secondary circuit is simply that of causing the impedance volts to work against the secondary pressure at an angle, and therefore will produce a smaller drop. As the angle θ is still further decreased, the secondary drop will lessen, and then (as the current begins to lead instead of lag) fall to nothing, finally becoming a negative drop, that is, a rise, as the angle of lead becomes greater and greater. We see that by making the secondary current lead enough, we can actually cause a rise of secondary pressure with increasing load. This is, of course, due to the reversed effect of the leakage lines—a leading secondary current causing the inductive electromotive-forces produced by them to act with the secondary pressure, and not against it. The maximum *rise* in pressure is attained when the current in the secondary is leading the pressure by an amount equal to $(90^\circ + \alpha)$. When this happens, the impedance volts are again in line with the secondary pressure, but in this case assisting it, producing a corresponding rise instead of drop of the terminal pressure of the transformer.

The case of the short-circuited transformer given above, has been extended by Prof. Kapp[1] to determine its pressure-regulation for any definite conditions of lead or lag in the secondary circuit, and by making use of his very useful diagram, we are enabled to determine what the voltage drop will be when the secondary is loaded on a circuit of any given power-factor, without the necessity of actually loading the transformer up, thus effecting a great saving of time, trouble and expense, especially in the case of large transformers.

Equivalent Resistance.—We have seen that each circuit acts as though there were transformed into it a certain proportion of the resistance and reactance of the other circuit. In fact, we may write :—

$$\text{Equivalent resistance of primary} = R_1 + R_2 \frac{S_1^2}{S_2^2};$$

$$\text{Equivalent resistance of secondary} = R_2 + R_1 \frac{S_2^2}{S_1^2}.$$

Similarly, we may write :—

$$\text{Equivalent reactance of primary} = X_1 + X_2 \frac{S_1^2}{S_2^2};$$

$$\text{Equivalent reactance of secondary} = X_2 + X_1 \frac{S_2^2}{S_1^2}.$$

The equivalent impedance of either circuit is obtained by squaring its equivalent resistance and its equivalent reactance, adding these squares, and taking the square root.

In modern transformers the equivalent reactance is usually from 0·8 to 1·5 times as great as the equivalent resistance.

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[1] See *Elektrotechnische Zeitschrift*, p. 260, 1895.

SPECIAL FORMS OF TRANSFORMERS.

Under this heading may be included transformers for constant current, auto-transformers, and boosters.

Constant-current Transformers.—The former have a limited application for use in series arc and glow-lamp systems, and generally depend upon magnetic leakage for their constant-current regulation. At the normal load, for instance, they deliver current with a large drop of pressure; any further increase of load simply brings about a much greater drop, thus tending to keep the current in the circuit at a constant value. In order to bring this about, the transformer windings and iron circuit would be so arranged as to cause a large leakage drop—in some designs a magnetic shunt is employed directly to promote leakage. Such transformers necessarily have a lower efficiency and power-factor than those designed for minimum leakage, and they are also large and costly for their output. There are two main types. In the first a magnetic shunt is provided in the transformer directly to promote magnetic leakage. The same result would of course be obtained if a large choking coil were placed in the circuit, for when the resistance is small compared with the reactance, it may be varied considerably without much altering the value of the current. In the second type of constant current transformer, one part, either the coils or the core, is movable, so that as more lamps are switched on, less leakage takes place, owing to the coils being attracted nearer together, or the magnetic circuit becoming more compact.

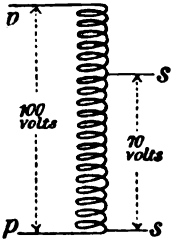

FIG. 436.
AUTO-TRANSFORMER.

Auto-transformers.—The auto-transformer (or "one coil" transformer) merely consists of a suitable iron circuit provided with a single winding, and connected across the mains. To some point in it, at a greater or less distance from one end, according to the voltage required, a branch wire is attached and current is drawn off between this branch and one end. In Fig. 436 the ends *pp* are attached to the primary mains, while *ss* act as the secondary terminals, giving out a lower voltage, and acting as a pressure-*reducer*. It will be seen that a greater current can be drawn off in this way than is actually supplied by the mains, as the portion of coil that is common to the circuits acts as the secondary of a transformer. If the connexions were made the other way, so that the lesser number of coils were connected to the mains, the voltage at the outer terminals would be raised; the arrangement then serving as an *augmentator* of pressure. The current carried by either section will be inversely proportional to the number of turns in that section, and if the auto-transformer is made up in two coils, these will both occupy about the same space, as the section of wire in each part will be proportional to the number of turns in the other. Although considerably less labour and material are required than if there were two separate coils, the arrangement

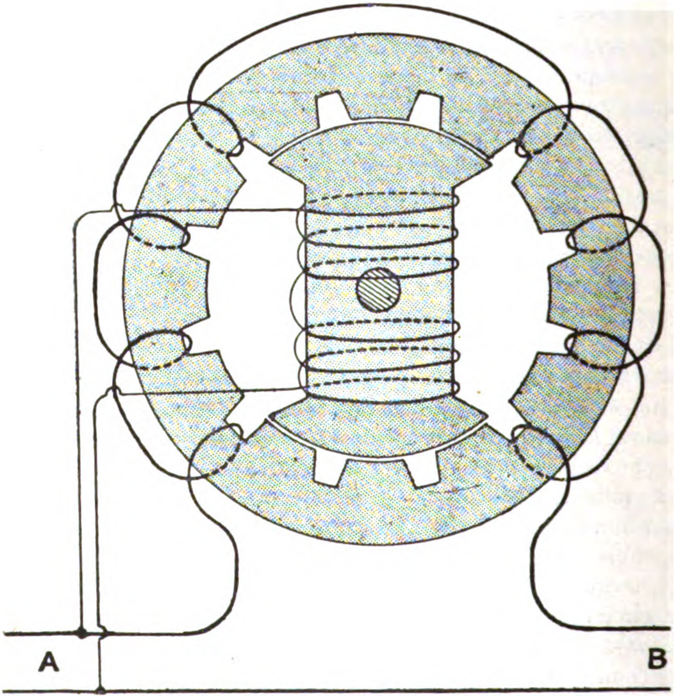

FIG. 437.—BOOSTING TRANSFORMER WITH VARIABLE VOLTAGE.

is, on the whole, not so good, as the pressure regulation is impaired on account of increased magnetic leakage. For this reason auto-transformers are only used for comparatively small work, such as for supplying arc lamps, starting induction motors, and in similar cases where the first requirement is cheapness.

For distribution by the three-wire system, the secondaries of the substation transformers are often wound to 200 or 220 volts, with an extra terminal halfway along the winding for connection to the neutral wire. Such an arrangement has the advantage of helping to equalise the pressure between each of the two outers and the neutral in the event of the system being unbalanced.

Boosting Transformers.—Owing to the variable length of the feeders in alternate-current systems of supply, it generally happens that arrangements have to be made in the station by which the pressure on the different feeders can be independently regulated in order to keep it constant at the feeding points. One way of doing this is by means of "boosting" transformers. These are simply transformers of small size connected into each feeder, the secondary winding being in series with it and the 'bus-bars, and more or less of the primary winding being directly connected to the 'bus-bars through a multiple contact regulating switch. It is evident that if the primary winding is completely cut out, the feeder has practically the same pressure as the 'bus-bars, but on the other hand, if the whole or part of the primary winding is also connected to the 'bus-bars, the feeder-pressure will be either reduced or augmented, by an amount depending on the number of primary windings in use. The drop of pressure in the feeder can thus be made up in each case by means of the transformer, the 'bus-bars being maintained at the constant pressure required by the shortest feeder.

The above method of boosting up was suggested by Kapp some years ago, and subsequent improvements have been to abolish the switch (naturally a source of weakness), and to vary the arrangement of the magnetic circuit rather than alter the windings. Such a device[1] is shown in Fig. 437, which shows diagrammatically the arrangement of booster supplied to Bolton by the Manchester Edison and Swan Company. The position of the movable primary circuit shown in the diagram corresponds to the position of maximum effect. If the primary be moved through 90° there will naturally be no effect on the secondary, as half the coils would generate an electromotive-force in one direction, and the other half an electromotive-force in the opposite direction, the two thus neutralizing one another. Between these two positions any desired additional pressure can be supplied to the feeder in series with the secondary, and this is done quite smoothly and gradually, thus having no perceptible effect on the lights.

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[1] E. W. Cowan and A. Still, the Regulation of Pressure, etc., in Alternating Current Systems, *Electrician*, xxxvi. 762, 1896. See also A. Russel, on Boosting with Alternating Currents, in *Electrician*, xxxviii. 73, 1896.

CHAPTER XIII.

THE DESIGN OF TRANSFORMERS.

AN application of the principles and formulæ given on previous pages, combined with the observance of the ordinary rules of design, will enable anyone to readily design a transformer which will be more or less satisfactory in its working. But to do this in the best possible manner, that is to obtain the best result for the smallest expenditure in material and labour, requires not only considerable experience of transformer designing in general, but special experience of the type selected. Due regard, too, must always be paid to the conditions under which the transformer will usually work, as these conditions modify the design very considerably. We know that there are two sources of loss in any transformer—the core-loss, due to hysteresis and eddies in the iron core, and which is practically independent of the load, and the copper-losses, due to the resistance of the transformer windings, which vary with the load. Now, theoretically, the best design for a given output would be the one in which these two came out equal, as then the total losses become a minimum. But when the working condition of the transformer, and the cost of electrical energy, are taken into account, it is often found better to depart somewhat from the theoretically best conditions in the design.

Let us first consider the case of a transformer used wholly for lighting work. For the greater part of the day it will be idle or lightly loaded on the mains, with the core-losses continuously going on, and the copper-losses will probably only become of any importance for the few hours of full-load. This, then, would appear to be a case in which the copper-losses could be made greater than the iron-losses with advantage, due regard of course being given to the question of pressure regulation, and so we find in practice, that lighting transformers generally have greater total losses in the copper than in the iron, unless the cost of energy is very small, as in some water-power stations. In this case the initial cost of the transformer would probably be the first consideration. The case of a transformer used for operating induction motors is quite different. Here the transformer is always more or less loaded—perhaps even working at a constant load—and hence we find that the best results are obtained when the iron and copper losses are made equal. An exception to this occurs perhaps when the transformer has to run several small motors—the case of an electrically-driven crane might be cited—in which case it will not only be on the average more or less lightly loaded, but the load will always have a low power-factor, which tends, as we know, greatly to increase the pressure-drop. Here the transformer would probably be designed to have the total copper-losses somewhat less than the iron-losses, in order to keep the secondary pressure-drop within reasonable limits, for good pressure regulation is quite as important in motor work as for lighting.

In addition to the above, it should be remembered that the copper-losses will be a minimum when they are equally divided between primary and secondary windings, and this proportion should always be adhered to as far as possible. An exception may sometimes be advantageously made for transformers with special cooling arrangements, such as forced draught, etc.

The final rise of temperature of any transformer after continuous running at full-load should never exceed 60° C., especially for those boxed up in cases without oil filling.

Procedure in Design.—With regard to the actual method of designing a transformer to have a given output at a given voltage and frequency, there are several modes of procedure that may be employed. A very general way is as follows. The required efficiency and voltage-drop being given, first settle the type of core and the methods of winding and cooling to be employed. Next, from consideration of the normal working conditions, divide up the permissible total loss into two portions as iron and copper losses. From previous experience of the type, the dimensions and weight of the core will be approximately known. By dividing the permissible iron-loss by the weight of the core we know how many watts may be wasted per pound of iron. Knowing the quality of the stampings to be employed for the core, a reference to such a test curve as shown in Fig. 426, A A or B B, gives us at once the permissible maximum flux-density **B** in the core. This multiplied by the iron area of the core gives the total magnetic flux **N**. From this value of **N** and the known conditions of supply, the primary turns S_1 are easily found from the equation already given. The number of secondary turns S_2 follows immediately from this and the given transformation ratio.

The copper areas are determined with the reference to the copper-losses, allowable pressure-drop, the depth of winding, and the permissible heating. The drawing can now be set out, and it will then be seen if there is sufficient winding space on the cores to take the bobbins and the previously determined windings. Should this not be the case, the dimensions of the core must be suitably altered. We must now calculate from the drawing : —

(1) *The Cooling Surface.*—A reference to such curves as shown in Fig. 428, and from the watts lost per square inch of cooling surface, will give the probable temperature rise. (2) *The total secondary drop of pressure at full-load*—depending on the conductors used, their total length, and also greatly on the arrangement of the primary and secondary windings. (3) *The no-load Current.* One or other of these is sure to come out too high, and the whole design must be correspondingly modified in the directions already indicated, until the design fulfils the specified conditions. When this result has been attained, calculate out the weight and cost of active material per kilowatt. This gives some idea of the total cost, and the next thing to do is to work out two or more other complete designs in which the flux-density has different values up and down, which will give a different allotment of iron and copper losses, and different temperature rise, pressure drop, and no-load current. These designs must also be altered until they fulfil the terms of the original specification, and then the weight and cost per kilowatt worked out as before. A comparison of the several designs will at once show which one is the best to select for the purpose, for, if they all fulfil the specified conditions, the cheapest to manufacture will be adopted.

Alternative Procedures.—In the procedure advocated by Weekes, the iron and copper losses are first fixed. Then the dimensions of the iron circuit are independently got out from experience, and the weight calculated. From this the iron-loss per lb. is calculated, and thence by use of a curve the corresponding value of B is deduced. This multiplied by the cross-section of iron gives N, and then S_1 and S_2 follow from the voltages. This plan may lead to quite unsuitable values of B, and in such cases involves re-calculation by trial and error till a reasonable value has been reached.

Fleming starts by assuming the value of B, the ampere-density in the copper, and the copper-loss. From the current, the current-density, and the specific resistance of copper the cross-sections, lengths and resistances of the windings are found, and from the voltage, the length of winding, and B the area of it deduced. This procedure may lead to unsuitable weights of iron, and impracticable iron-losses, also involving recalculation several times.

Adams fixes B from experience, and then decides on S_2 and A from experience. He gives curves from experience, to determine for a transformer of given output the number of secondary turns which will make the cost of active material per kilowatt a minimum. Adopting this the section of iron follows. Then the permissible copper-losses are fixed; and the mean length of one turn being estimated, the necessary sectional areas of the windings are found, and disposed so as to make the iron circuit as short as possible. The iron-losses are then estimated from the weight of iron and the assumed value of B. The comparison of the losses with the prescribed efficiency and the cooling surface will then probably show that recalculation is necessary.

R. Bauch constructs a number of formulæ based on the use of two coefficients, one of them being the nett area of core divided by the square of the mean length of the primary winding (a coefficient varying from 0·027 to 0·033), and the other an expression for the watts lost per unit cube of the iron divided by B^2 and by f^2, thus assuming that both hysteresis and eddy-current losses vary alike. This coefficient varies between $4{\cdot}2 \times 10^{-14}$ and 5×10^{-14}.

Other modes of procedure are based on the values deduced from experience of the ratio of the ampere-turns at full-load to the mean magnetic length of the iron circuit; but this ratio is not by any means constant even in one type of transformer.

Procedure Recommended.—The following method will be found to be simple, as the *flux-factor* on which it is based has values which though differing in different types are found to be fairly constant for a series of transformers of any one given type.

It cannot be too strongly stated that the design of transformers, as of dynamos and motors, is an art, quite as much as a science, and that proficiency in it is to be attained only by practice. Nevertheless scientific rules and formulæ save much time at the initial stages; in the later stages judgment and experience are needed to arrive at the finished design which best fulfils the conflicting conditions of the problem.

TRANSFORMER DESIGN.

To determine the appropriate dimensions and windings of a transformer to fulfil any prescribed specification, we may avail ourselves of formulæ which are readily deduced from the considerations already laid down, while experience enables us to simplify the calculations by introduction of factors the values of which are derived from practice.

The Dimension Formulæ.—We deal first with simple-phase transformers. From p. 535 above we have the rule

$$E_1 = 4{\cdot}44 \times f \times S_1 \times N_1 \div 10^8 ; \quad . \quad [1]$$

where E_1 is the primary induced volts, which are (save for a slight drop) equal to the primary volts of supply at the terminals ; f is the frequency ; S_1 the number of primary windings ; and N_1 the maximum value of the flux in the core. The constant 4·44 is made up of the numeric 4 and the form-factor of the voltage, which for sine-waves is equal to 1·11 (*see* p. 195).

If we multiply both sides of equation [1] by C_1, the primary full-load current,[1] we obtain the expression :—

$$C_1 E_1 = 4{\cdot}44 \times f \times C_1 S_1 \times N_1 \div 10^8 ;$$

and, remembering that $KVA = C_1 E_1 \div 1000$, we may arrange this as follows :—

$$C_1 S_1 \times N_1 = \frac{KVA \times 10^{11}}{4{\cdot}44 \times f} \quad . \quad . \quad [2]$$

Next consider the ratio borne by the maximum flux N_1 to the full-load ampere-turns $C_1 S_1$. Let us call this ratio the *flux-factor*, and denote it by the symbol Y, so that

$$\frac{N_1}{C_1 S_1} = Y.$$

Then, if we know the value of Y for any type of transformer (air-cooled, oil-cooled, etc.) we can at once determine the separate values of N_1 and of $C_1 S_1$. [For the sake of clearness it may be well to remember that C_1, the full-load current being many times larger than C_m, the magnetizing-current, the full-load ampere-turns, $C_1 S_1$, will be many times greater than the magnetizing ampere-turns.]

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[1] When these formulæ are to be applied to polyphase transformers, E_1 will be the primary voltage generated in one phase, and C_1 will be the primary full-load current in one phase. Similarly KVA will be the apparent kilowatts in one phase.

The most suitable value of the flux-factor Y is dependent upon many things, the method of cooling, the purposes which the transformer is to subserve, the nature of the load, the permissible voltage-drop, and the maximum voltage. In the absence of specific information on these points, such as might be obtained from tests on completed transformers of the prescribed type, the following values may be taken :—

TABLE XVI.—FLUX-FACTOR FOR FREQUENCIES OF 40~ TO 60~.

Method of Cooling.	Voltage. Virtual volts.	Flux-Factor, Y
Core-Type.		
Natural draught . . .	0 to 6,000	55 to 70
	6,000 and above	70 to 75
Oil-cooled	0 to 6,000	75 to 90
	6,000 to 10,000	85 to 100
	10,000 and above	100 to 150
Air-blast or Water-cooled .	0 to 10,000	110 to 160
	above 10,000	160 to 240
Shell-Type.		
Natural draught . . .		500 to 700
Air-blast and Water-cooled .		600 to 1000

For higher frequencies the value of Y will be somewhat smaller, and for lower frequencies somewhat larger than those given in the table.

This is in accordance with the experience that for a given output and voltage, low-frequency transformers must be larger and heavier than high-frequency transformers.

Having, then, decided on a suitable value of Y, we can insert it in the following formulæ, which are deduced from equation [2] and the definition of Y.

$$C_1 S_1 = \sqrt{\frac{\text{KVA} \times 10^{11}}{4\cdot44 \times f \times Y}}\,; \quad . \quad [3]$$

$$N_1 = \sqrt{\frac{\text{KVA} \times Y \times 10^{11}}{4\cdot44 \times f}} \quad . \quad [4]$$

Thus we have by the simplest rules found the values of the maximum flux, and of the full-load ampere-turns, and may proceed to the design of the core and of the windings.

Example.—To design a transformer to the following specification : core-type, oil-cooled, single-phase; 100 KVA; 50 cycles; transforming down from 5000 to 500 volts.

For such a transformer a suitable value of Y would be 90. By equation [3], therefore,

$$C_1 S_1 = \sqrt{\frac{100 \times 10^{11}}{4{\cdot}44 \times 50 \times 90}} = 22{,}350.$$

And by equation [4],

$$N_1 = \sqrt{\frac{100 \times 90 \times 10^{11}}{4{\cdot}44 \times 50}} = 2{,}050{,}000 \text{ lines.}$$

It should be borne in mind when fixing the value of Y, that, as a rule, the greater the value of Y the cheaper the transformer will be, but the lower will be the all-day efficiency.

The Core and the Windings.—It is presumed here that the general form of the coil has been decided upon by experience. We will first of all consider fully the single-phase core-type with two cores wound, and afterwards show where the treatment of the other forms differs from this form.

The single-phase core-type transformer is diagrammatically represented in Fig. 438.

We have already found the value of the flux N_1. In order to find the necessary iron section we must now assume a suitable value for the flux-density B, and then the necessary iron-area A in square inches is found from the relation

$$A = N_1 \div B \quad . \quad . \quad . \quad [5]$$

Suitable and customary values for B may be taken from the curve, Fig. 439, which gives the result of experience with transformers of modern make. These curves do not, however, give the limits for B. With very well cooled transformers (cooled by water-circulation), in which the value of Y is proportionately high, B may approach 70,000 lines per square inch, as in some of the large transformers built by the Oerlikon Machine Works; but this is exceptional. High flux-densities can naturally be used with low-frequency transformers owing to the relatively smaller losses by hysteresis and eddy-currents.

Having fixed B, and calculated the corresponding value

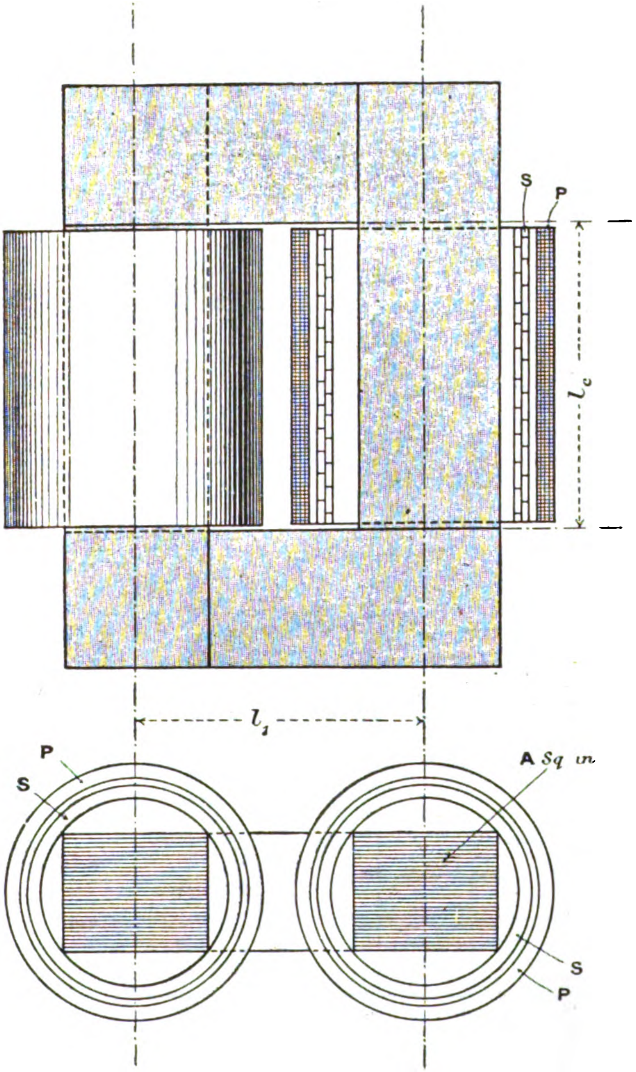


FIG. 438.—CORE-TYPE TRANSFORMER.

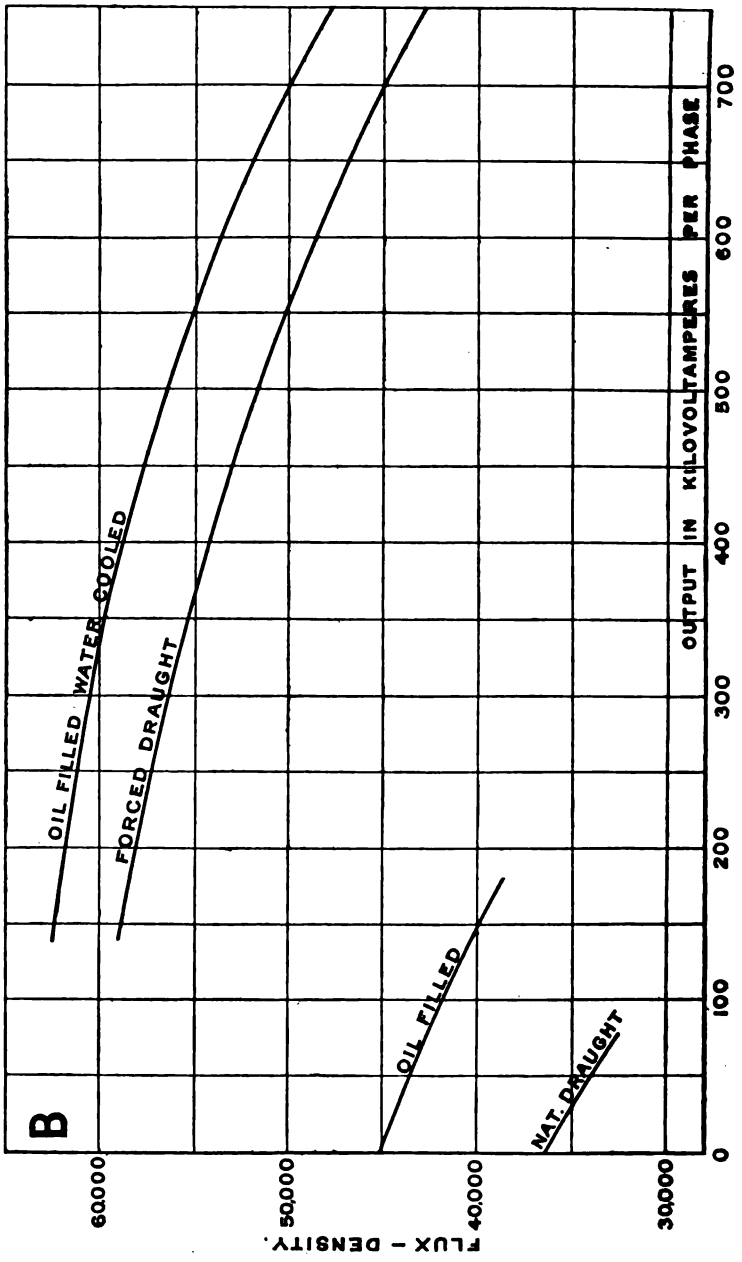


Fig. 439.—Flux-Densities in Relation to Rated Output.

of A, we next proceed to build up this area out of laminations, bearing in mind the extra cost which will be entailed for dies, and for additional complications in erecting, if our design involves several different sizes of core-stampings. For sizes up to about 50 KVA the plain rectangular (therefore square, for reasons of economy), form of section of core will be adopted. For transformers rated above that output, the core may be built up of laminations of two widths, as shown in Fig. 440*a*. The proportions given in the figure should be approximately adhered to as suitable to give the smallest circumscribing circle in relation to the area of iron enclosed. The thickness of the principal packet of stampings should be about three-fifths of their breadth, and each of the two other packets should be in breadth three-fifths as broad as the principal packet, and their thickness one-third of their breadth. The allowance for insulation between the laminations (japan varnish or paper) should lie between 5 and 7 per cent. In fact, if A is the nett iron section needed, the breadth of the principal bundle must be $1 \cdot 13 \times \sqrt{A}$, and its thickness $0 \cdot 678 \times \sqrt{A}$. The two other bundles should be of breadth $0 \cdot 678 \times \sqrt{A}$ and thickness $0 \cdot 226 \times \sqrt{A}$. The diameter D of the circumscribing circle will be $1 \cdot 316 \times \sqrt{A}$.

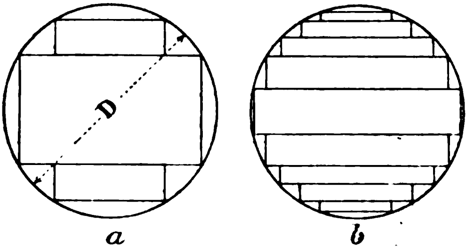


FIG. 440.

For transformers of very high outputs it must pay to build up the core of three or four different sizes of stampings as indicated in Fig. 440*b* ; but the gain is not very great.

The question of the introduction of one or more air-ducts depends upon the method of cooling employed. For iron cores of above 36 square inches area, a single half-inch duct will be of advantage. Some constructors put in one duct only whatever the size of the transformer. The Oerlikon transformers have one duct 20 millimetres wide in each core of whatever size. Compare the Lahmeyer construction shown in Fig. 415.

A convenient rule relating to sectional areas of cores is that, for a given secondary voltage at a given frequency, and with a given flux-density, the area of section of core will vary inversely as the number of secondary turns, as may be seen by the formula

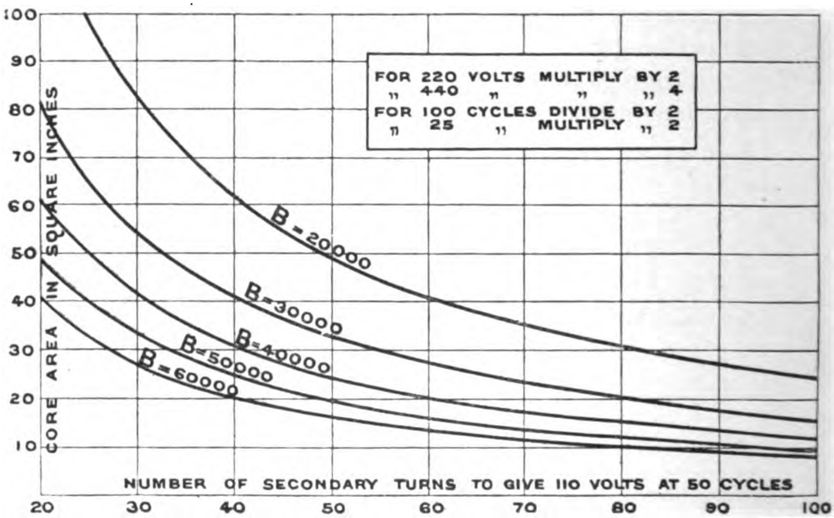


FIG. 441.

$$A = \frac{E \times 10^8}{4{\cdot}44 \times f \times S_2 \times B}.$$

The curves given in Fig. 441 may be useful, as showing the number of secondary turns required with any particular section of core, or *vice-versâ*, for the case where the secondary pressure is 110 volts, and the frequency 50 ~. If B is given, and either S_2 or A is fixed, then the other will vary directly as E_2 and inversely as f; so that the values in the curve can readily be re-calculated for any other voltage or frequency.

We now proceed to the design of the windings.

The object in winding design is to attain the minimum mean length of one turn, and thus cheapen the transformer as much as possible. In order to do this it is the practice of some makers to insulate the core and to wind the secondary direct on the core. The type of winding having been decided upon, we can proceed to its calculation.

From formula [3] we have :—

$$C_2 S_2 = C_1 S_1 = \sqrt{\frac{KVA \times 10^{11}}{4{\cdot}44 \times f \times Y}}.$$

Hence we can find immediately (knowing C_1 and C_2) the values of

S_2 = the number of turns in the secondary,

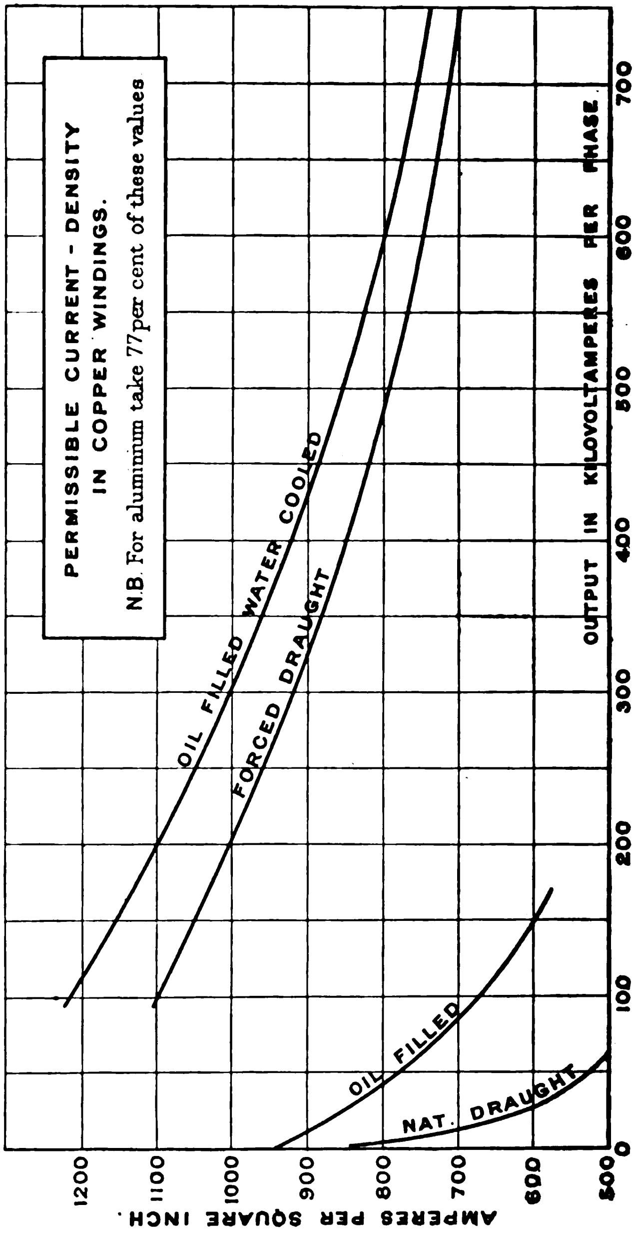
S_1 = the number of turns in the primary.

The secondary winding is usually built up of copper strip, or in some cases of aluminium strip. The area A_2 is determined from the expression :

$$A_2 = \frac{C_2}{a} \text{ square inches ;}$$

where a is the permissible current density in amperes per square inch. Suitable values of a may be taken from the curves in Fig. 442. These values are for copper. For aluminium the current-density would have about three-quarters (or more precisely 77 per cent.) of these values. Having found the necessary sectional area, we then choose a strip of suitable breadth and thickness to give this area. It is rarely that a number of strips are used in parallel in place of one thick strip, because of the liability—even though the magnetic leakage may be small—to unequal flux-density in the parallel paths which causes unequal loading with current and consequent increase in the copper-losses, and in the resistance-drop.

The calculation of the requisite size of the primary copper-



PERMISSIBLE CURRENT - DENSITY IN COPPER WINDINGS.

N.B. For aluminium take 77per cent of these values

FIG. 442—CURRENT-DENSITY IN COPPER.

winding proceeds in exactly the same way. The current-density for the primary should be the same as that in the secondary winding if the habitual load on the transformer is a fairly full one. But for those transformers which are destined for all-day work, where the load for many hours may be a small one, it is better to make the current-density in the primary somewhat lower than that in the secondary, so as to reduce the all-day copper-losses.

The question of the length of the wound portion of core depends principally upon the permissible winding depth. For a minimum length of magnetic circuit, the core with its yokes should be of square form. This, however, would in most cases necessitate too great a winding depth. A good method of procedure is to assume the winding length l_c from the following formula:—

$$l_c = \text{const.} \times \sqrt{A}.$$

Suitable values of the constant are:—

Natural-cooled, very small sizes	4
Natural-cooled, moderate sizes	5 to 7
Oil-cooled	4 to 6
Water-cooled	2·5 to 4
Air-blast	6 to 8

With the value of l_c obtained, the depths of winding necessary are then computed. If they are found to exceed the following limits, the winding length should be increased, and the depths recalculated.

For the natural-cooled type, the total depth of both primary and secondary windings (excluding the insulation between them) should not exceed 2 inches. In the case of large oil-cooled and water-cooled transformers, this limit may be somewhat exceeded, but 3 inches should be regarded as the maximum permissible.

Another way of settling the core-length is to work to a certain number of ampere-turns (primary, full-load) per inch length of core. This factor we may term the *specific loading* of the core, and denote it by the symbol Q_1. It is defined by the equation:—

$$Q_1 = C_1S_1 \div l_m,$$

where l_m is the number of inches of length of mean magnetic path around the magnetic circuit. The value of Q_1 varies in practice from 250 to 400. The smaller values are found in smaller sizes of transformers. The larger the value of Q_1, the larger, other things being equal, will be the inductive voltage-drop.

Having thus dimensioned the windings fully, we can now finish the core. The length necessary between the yokes is found above (l_c). The axial distance of cores (l_1), now that we know the depth of the windings, is immediately obtained. The yokes should have the same section as the cores, since (as proved[1] by La Cour), for maximum economy the flux-density in the yoke should be the same as that in the core.

We have now determined all the electrical dimensions of the transformers, and all that remains is to design the frame, case, etc.

The treatment of the three-phase transformer (core-type) differs from the above only in that the number of ampere-turns C_1S_1 in any one phase will be placed on one leg instead of being distributed on two cores as in the single-phase transformer. If the transformer, too, is built with the three legs in line, the yoke must be of the same area of the core, if it is wished to have the same flux-density in each. If the transformer is built with the cores at the apices of an equilateral triangle, the yokes need only have $\frac{1}{\sqrt{3}} = 0 \cdot 58$ of the area of the core.

Example.—We now have to design the core and windings for the transformer previously specified. We have already found (page 612) that

$$C_1 S_1 = 22350,$$

and

$$N_1 = 2 \cdot 05 \times 10^6.$$

Now turning to the curves (Fig. 439) we see that a suitable value of B will be 42,000 lines per square inch. Hence the necessary iron area is 49 square inches. For this output we will build up the core

---

[1] *Die Wechselstrom Technik*, i. 355.

of two series of stampings with one ½ inch air duct. A section of the dimensions, shown in Fig. 443 (allowing 8 per cent. for insulation), gives us a nett area of 49·4 square inches corresponding to a flux-density of 41,600 lines per square inch.

Now we turn to the winding. The secondary current C_2 = 100,000 ÷ 500 = 200 amperes. A suitable current-density for the secondary winding will be 670, the necessary area is therefore $\frac{200}{670} = 0 \cdot 3$ square inch. A copper strip will be suitable, and for flexibility made of four strips laid together. Hence we may select a strip 0·6 inch wide and 0·125 inch thick. So $4 \times 0 \cdot 6 \times 0 \cdot 125 = 0 \cdot 3$ square inch.

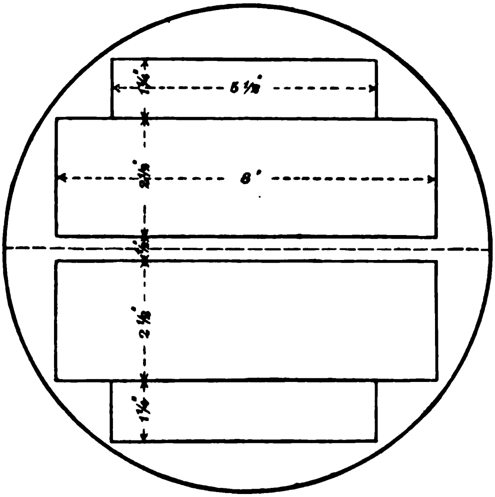


FIG. 443.

The number of secondary turns required is 22,350 ÷ 200 = 112. We will arrange the secondary turns on the two legs in series. The four copper strips will be taped together with two tapings (half-lap) to the size 0·62 × 0·52 inch. The former diameter (*i.e.* the internal diameter of the coil) will be 10·25 inches. The external diameter of the secondary winding is 11·29 inches, and the over-all length of the coil 34·7 inches. We, therefore, make the length of the core $(l_c) = 36$ inches $= 5 \cdot 15 \sqrt{A}$.

We can now proceed to determine the primary winding. Between the two coils we will insert a cylinder of micanite 0·1 inch thick as insulation to prevent a contact between the primary and secondary windings. The primary current $C_1 = 100{,}000 \div 5000 = 20$ amperes. Taking the same current-density as for the secondary we see that we require a conductor of 0·03 inch area. We will use two wires of No. 8 B and S gauge in parallel with one another, their combined area being 0·026 square inch.

We will connect the primary windings of each core in series. Each leg will therefore have 1120 turns of wire, two turns in parallel. We will sub-divide this winding into eight coils. Each coil will consist of five layers of 28 turns each. Between each layer and the next are placed two thicknesses of special Japanese paper 0·004 inch thick. Each entire coil is separately insulated, being served with four tapings of tape, each taping being well brushed over with three coatings of insulating lac. The external dimensions of the coil are reckoned out as follows:—axial length $(28 \times 0{\cdot}143) + (2 \times 0{\cdot}07) = 4{\cdot}14$ inches; depth $(5 \times 0\ 143) + (4 \times 0{\cdot}004) + (2 \times 0{\cdot}07) = 0{\cdot}871$ inch; the internal diameter of the coil being 11 8 inches.

It will be sufficient here simply to indicate the different treatment required for the shell type of transformer. Here, as can be seen from Fig. 400, the whole of the ampere-turns pass through the space between the four stampings. In order to have a minimum weight of iron the sectional form of this "window" should be nearly square. The values of B for this type will be much the same as for the core type, or a little lower. The nett area of section of the core will be equal to

$$2b \times l \times 0{\cdot}92 = N \div B.$$

The value of b, the breadth of the strips (Fig. 414), may be taken from the empirical formula

$$l = \text{const.} \times b;$$

where the value of the constant will be:

For the natural cooled type, from 4 to 6;
For the oil-cooled type, from 3 to 5;
For the air-blast type, from 5 to 7

the lower values applying to the smaller sizes.

The windings are always sandwiched one between the other, and often spread out at the ends in order to get better cooling surface. The necessary length of the stampings will be found from the area necessary for the windings. The width of the opening should not exceed the value of $2b$.

The total over-all length of the core will be greater than the value of l obtained by formula. Air-ducts must be inserted at intervals of from 3 to 5 inches apart, except in the case of small oil-cooled forms.

CALCULATION OF LOSSES AND PRESSURE-DROP.

I. *The Iron-Losses.*—The method of estimating these has already been explained above on p. 578, and curves given, from which the values can be taken off.

II. *The Copper-Losses.*—The copper-losses, as is well known, are given by the expression :

$$W_c = C^2 R.$$

The primary copper-losses are therefore

$$W_{c_1} = C_1^2 R_1,$$

and the secondary copper-losses

$$W_{c_2} = C_2^2 R_2 ;$$

R_1 and R_2 being the resistances of the primary and secondary respectively.

Now the resistance

$$R = \frac{L}{A} \times \rho \text{ ohms.}$$

If L be in inches and A in square inches, then ρ the specific resistance will be the resistance of an inch cube of the material, and this varies with the temperature. Now at a temperature of 0° C. the resistance per inch cube is :

For *copper* $\rho_0 = 0{\cdot}00000063$ ohm
For *aluminium* $\rho_0 = 0{\cdot}00000105$ ohm.

The change in the value of ρ with temperature is given by the formula :

$$\rho_t = \rho_0 (1 + at),$$

where t is the temperature in Centigrade degrees, and a the temperature-coefficient.

For *copper* $a = 0{\cdot}0039$
For *aluminium* $a = 0{\cdot}004.$

As an average value we may assume that $t = 60^\circ$ C.; hence we may assume that ρ_t will have the values:—

For *copper* $\rho_t = 0{\cdot}00000078$
For *aluminium* $\rho_t = 0{\cdot}00000135$.

From these data we may now calculate the ohmic resistances of the primary and secondary windings, or rather by them we may calculate these resistances as they would be if we were dealing with continuous currents. But with alternating currents the effective resistances will be from 5 to 25 per cent. greater (quite apart from any apparent increase due to self-induction), owing to the alternating current not dividing itself equally through the area of the copper (skin effect), to eddy-currents in the windings, and to losses due to vibration of the windings, and other obscure influences. To cover this increase in the ohmic losses we may allow an increase of, say, 15 per cent.; that is to say, we will take the effective ohmic resistances as being 15 per cent. higher than those calculated from the specific resistance. There are too many obscure factors to permit of a more accurate calculation. The effective resistances may, however, be easily determined for a finished transformer by the short-circuit test. But for another transformer of the same type, the observed value would quite probably differ from that of the one tested by quite as great an amount as the value calculated differed from the value observed.

Example.—We will proceed to calculate the iron and copper losses for the 100 KVA transformer we are designing.

The volume (nett) of iron in the cores and yokes is 5700 cubic inches; hence, taking the specific weight of iron at 0·28 lb. per cubic inch, the total weight of the iron laminations is 1600 lb. The flux-density in the core is 42,000 at its maximum. Hence by a reference to the curves we see that the hysteresis loss will be 0·46 watt per lb., and the eddy-current loss 0·18 watt per lb.; or together, 0·64 watt per lb. Multiplying by 1600 makes the total iron-losses 1024 watts.

The area of section of the primary winding is 0·026 square inch. The number of turns is 1120. The mean length of 1 turn is 39·8 inches, so that the total length $L_1 = 44{,}500$ inches. Hence the resistance $R_1 = 44{,}500 \times 0{\cdot}00000078 \div 0{\cdot}026 = 1{\cdot}33$ ohm; but assuming for skin-effect and eddy-currents, etc., a probable increase of 20 per cent. we must put $R_1 = 1{\cdot}6$ ohm.

Similarly we have $A_2 = 0{\cdot}3$ square inch. The number of turns is 112, and the mean length of 1 turn 33·8, making $L_2 = 3790$ inches. Hence $R_2 = 3790 \times 0{\cdot}00000078 \div 0{\cdot}3 = 0{\cdot}0095$; or allowing an increase of 20 per cent, as before, the effective resistance becomes $R_2 = 0{\cdot}0114$.

The primary copper-loss will be $(20)^2 \times 1{\cdot}6 = 640$ watts. The secondary copper-loss will be $(200)^2 \times 0{\cdot}114 = 456$ watts. The total full-load losses will be $1024 + 640 + 456 = 2120$ watts.

The next point to consider is the cooling: the formulæ for which were given on p. 588.

Example.—For our 100 KVA transformer, the value of A is 9400 sq. inches; that of B 200 sq. inches. For the coefficient a we will take the value 4·2; for b the mean value 5·5. The temperature-rise θ will therefore be:

$$\theta = \frac{1000 \times 2120}{(4{\cdot}2 \times 9400) + (5{\cdot}5 \times 200)} = \frac{2{,}120{,}000}{40{,}600} = 52{\cdot}2^\circ \text{ C.}$$

So that this design fulfils the prescribed condition as to rise in temperature at full-load.

The ohmic drop at full-load is calculated most simply by first deducing the equivalent total resistance of the transformer, by adding to the effective primary resistance R_1, the effective secondary resistance R_2, multiplied by the square of the winding-ratio (see p. 602). Or, calling the equivalent total resistance R′, we have:

$$R' = R_1 + R_2 \frac{S_1^2}{S_2^2}.$$

Then the voltage-drop δV_1 on a full non-inductive load, that is the drop due to resistance alone, will be:

$$\delta V_1 = C_1 R' \text{ volts.}$$

Example.—The winding ratio is $\frac{1120}{112} = 10$. Hence the square of this = 100 and since $R_1 = 1{\cdot}06$, and $R_2 = 0{\cdot}0114$, we have: $R' = 1{\cdot}6 + (0{\cdot}0114 \times 100) = 1{\cdot}6 + 1{\cdot}14 = 2{\cdot}74$ ohms. The voltage-drop on a non-inductive load is, therefore, $20 \times 2{\cdot}74 = 55{\cdot}8$ volts, which is 1·12 per cent. of the primary voltage.

No-load Current.—We may estimate the no-load current. The length of the mean magnetic path is 98 inches. The maximum flux-density is 41,600. Referring to the saturation-curve of the iron used (see Fig. 392) we find that to attain this there is needed an excitation of 3 virtual ampere-turns per inch. Hence the whole number of exciting ampere-turns is 294. As there are 1120 primary turns, the magnetizing current C_M will, therefore, be $298 \div 1120 = 0 \cdot 263$ ampere. The energy component can be found by dividing the total iron-losses which amounted to 1024 watts, by the primary voltage 5000 volts, giving $C_M = 0 \cdot 205$ ampere. Then the no-load current will be $C_0 = \sqrt{(0 \cdot 263)^2 + (0 \cdot 205)^2} = 0 \cdot 334$ ampere. The no-load copper-loss is negligible, being only $0 \cdot 334 \times 0 \cdot 334 \times 1 \cdot 6 = 1 \cdot 78$ watt.

Short-Circuit Impedance and Inductive Drop.—The equivalent resistance R' we found to be 2·74 ohms. If we take the equivalent reactance (p. 602) as 1·5 times the equivalent resistance, we shall have $X_s = 4 \cdot 11$ ohms, and the short-circuit impedance would be 4·9 ohms. The impedance voltage at a full-load of 20 amperes would then be $4 \cdot 9 \times 20 = 98$ volts. If we assume a power-factor of 0·8 on a full-load, we shall have $98 \times 0 \cdot 8 = 78 \cdot 4$ volts as the primary voltage-drop on a circuit of that power-factor. This is but 1·6 per cent. of the primary voltage.

Efficiency.—The output at full-load is 100 × 1000 watts, and the total losses are estimated at 2120 watts, hence the full-load efficiency will be

$$\frac{100{,}000}{100{,}000 + 2120} = 0 \cdot 98 \text{ ; } \textit{i.e.} \text{ 98 per cent.}$$

At full-load the iron-losses are 1024 watts and the copper-losses are 1096 watts, so that the full-load efficiency is practically the same as the maximum efficiency.

At one-quarter of full-load the copper-losses will be only 88·5 watts, so that the efficiency will be

$$\frac{25 \times 1000}{(25 \times 1000) + 1093 \cdot 5} = 0 \cdot 93 \text{ ; } \textit{i.e.} \text{ 93 per cent.}$$

This low efficiency at low load is due to the comparatively large value of the iron-losses. The transformer is therefore suitable for a power load rather than a lighting load.

In determining the all-day efficiency, we will assume that the transformer will be connected to the mains for 14 hours out of the 24. During 9 of these hours, the transformer is fully loaded and during 3 hours it is running at about quarter-load ; during the other

2 hours it is practically idle. Such conditions as there obtain in the case of a factory, which is partly working overtime.

The output of power in watt-hours will be

$$(100{,}000 \times 9) + (25{,}000 \times 3) = 975{,}000 \text{ watt-hours.}$$

The total iron-losses will be

$$14 \times 1025 \text{ watt-hours} = 14{,}300 \text{ watt-hours.}$$

The total copper-losses will be

$$(1096 \times 9) + (68 \cdot 5 \times 3) + 0 = 10{,}080 \text{ watt-hours.}$$

The all-day efficiency in such a case will therefore be

$$\frac{975{,}000}{975{,}000 + 14{,}300 + 10{,}080} = 0 \cdot 975 \text{ ; } \textit{i.e.}\ 97\tfrac{1}{2} \text{ per cent.,}$$

which is a fairly high all-day efficiency.

Weights.—The following may be taken for calculations of weights:

—	Specific Gravity.	Weight of 1 cubic inch.
		lb.
Iron	7·77	0·28
Copper	8·91	0·3215
Aluminium	2·66	0·097

Example.—In the example before us the nett volume of iron is 5700 cubic inches. Hence the weight will be 5700 × 0 28 = 1600 lb. The primary copper has section 0·026 and length 44,500 inches, so its volume is 1160 cubic inches, and weight 1160 × 0·3215 = 373 lb. The secondary copper has 0·3 section and 3790 inches length, so its volume is 1263, and weight 1263 × 0·3215 = 407 lb. Total weight of copper 780 lb. Taking copper at 10*d.* per pound and iron at 4*d.* per pound we have:—

	d.	*s.*	*d.*
780 lb. of copper at 10*d.* =	7800 =	650	0
1600 lb. of iron at 4*d.* =	6400 =	533	4
	14200 =	1183	4

Cost of active materials, not including insulation, per KW $\Big\} = \frac{1183}{100} =$ 11*s.* 10*d.*

It will be convenient to give in schedule form the results of this computation.

SCHEDULE FOR DESIGN OF TRANSFORMER.

General Specification.—Type: oil-cooled, core-type. Output, 100 kilowatts; number of phases, 1; frequency, 50 ∼; primary volts, $V_1 = 5000$; secondary volts, $V_2 = 500$; efficiency, $\eta = 0 \cdot 98$.

Constants Assumed.—Flux-factor, $Y = 90$; flux-density in core, $\mathsf{B}_{max} = 41{,}600$; insulation-factor in core $= 0 \cdot 92$; core-length coefficient, $l_c \div \sqrt{A} = 5 \cdot 13$.

Magnetic Dimensions (inch units).

Kilowatts output per phase, KW . . .	100	
Kilowatts input (primary) per phase, $KW \div \eta = KW_1$.	102	
Primary ampere-turns (see page 610) $C_1 S_1 = \sqrt{\frac{KW_1 \times 10^{11}}{4 \cdot 44 \cdot f \cdot Y}}$	22,350	
Flux $\mathsf{N} = Y \times C_1 S_1$	$2 \cdot 05 \times 10^6$	lines
Nett area of section of iron in core $= \mathsf{N} \div \mathsf{B}_{max} = A$.	49·4	sq. inches
Gross area of section of iron in core $= A \div 0 \cdot 92$ (see sketch).	53·7	,,
Dimensions of yoke	$8 \times 6\frac{3}{4}$	
Nett area of iron in yoke	49·6	
Flux-density in yoke, B_y	41,200	per sq. in.
Length of core, $l_c = 5 \cdot 13 \times \sqrt{A}$ . . .	36	
Ampere-turns excitation needed per inch of core-length.	3	virt. amp.
Arrangement of cores (see sketch)		
Distance between centres of cores . . .	14	inches
Weight of cores	1000	lb.
Weight of yokes	610	,,
Total length of magnetic path in cores, l_4 .	72	inches
,, ,, ,, ,, in yokes, l_3 .	26	,,
Total length of mean magnetic path . .	98	,,
Ampere-turns needed per inch in core, CS_c .	3	
,, ,, ,, ,, in yoke, CS_y .	3	
Total ampere-turns for cores, $l_4 \times CS_c$ . .	216	
,, ,, for yokes, $l_3 \times CS_y$ . .	71	
Total ampere-turns for cores and yokes, CS_m .	294	
No-load magnetizing current, $C_m = CS_m \div S_1$.	0·263	amp.

	Primary.		Secondary.	
Windings :—				
Current, per phase .	$C_1 = KW_1 \div E_1$	20	$C_2 = KW_2 \div E_2$	200
Turns, per phase . .	$C_1 S_1 \div C_1$	1120	$C_2 S_2 \div C_2$	112
Current - density (per sq. in.)	a_1	770	a_2	670
Area of conductor (sq. in.)	A_1	2 × 0·013	A_2	0·3
Dimensions of conductor, bare	..	2 No. 8 B & S	..	4 (0·6×0·125)
Dimensions of conductor, insulated	..	0·143	..	0·62 × 0·52
Arrangement of winding (see sketch)	..	..	..	..
Mean length one turn of winding	l_1	39·8	l_2	33·8
Material of winding .	..	Copper	..	Copper
Weight of winding (lb)	..	373	..	407

Iron-Losses :—

Iron-losses in core per lb.	0·64	watt per lb.
,, in yoke ,, ...	0·63	,, ,,
Total iron-losses in core	384	watts
,, ,, in yoke	640	,,
Total iron-losses in transformer	1024	,,

	Primary.		Secondary.	
Copper-losses :—				
Length of winding (inches)	$L_1 = l_1 \times S_1$	44,500	$L_2 = l_2 \times S_2$	3790
Coefficient of resistance (warm)	ρ	0·00000078	ρ	0·00000078
Resistance of winding (ohm)	$\rho \times L_1 \div A_1$	1·33	$\rho \times L_2 \div A_2$	0·0095
Percentage added for eddy-currents . .	..	20%	..	20%
Effective resistance (ohm)	R_1	1·6	R_2	0·0114

Total effective resistance, $R_t = R_1 + (S_1 \div S_2)^2 R_2$.	2·74	ohms
Total copper-losses $C_1^2 R_t$ at 1¼ load . . .	1710	watts
,, ,, ,, at full-load . . .	1096	,,
,, ,, ,, at ¾ load . . .	615	,,
,, ,, ,, at ½ load . . .	274	,,
,, ,, ,, at ¼ load . . .	68·5	,,

Heating of Transformer :—

Total losses at full-load	2121	watts.
Area of surface directly in contact with air or oil *A*.	9400	sq. inch.
Ditto with flow *B*	200	,,
Coefficient *a*	4·2	..
Coefficient *b*	5·5	..
Temperature-rise $\theta = \dfrac{1000 \times \text{total losses}}{a A + b B}$	52	deg. Centig.
Volume of air required per minute to cool		cub. ft. per min.

Efficiency and Voltage-Drop :—

Non-inductive voltage-drop, $C_1 R_t$	55·8	volts
Percentage ,,	1·12	per cent.
Total impedance Z	4·1	ohms
Impedance ratio $Z_t \div R_t$	1·5	
Total reactance $X_t = \sqrt{(Z_t^2 - R_t^2)}$	3·05	ohms
Probable short-circuit current	1020	amps.
Voltage-drop inductive load at cos $\phi = 0$	1·6	per cent.
Efficiency at 1¼ load	98	,,
,, full-load	98	,,
,, ¾ load	97·9	,,
,, ½ load	97·5	,,
,, ¼ load	93	,,
All-day efficiency under probable working conditions.	97·5	,,

Weights and Costs per Kilowatt :—

Total weight of iron, 1600 lb. at 4*d*. per lb.	= 26*l*. 13*s*. 4*d*.
Total weight of copper, 780 lb. at 10*d*. per lb.	= 32*l*. 10*s*. 0*d*.
Total cost of active material	= 59*l*. 3*s*. 4*d*.
Weight of active material per kilowatt	23·8 lb
Cost of active material per kilowatt	11*s*. 10*d*.

The Testing and Working of Transformers.—The suitability of a transformer for its work depends upon several points, some of which have already been noted. They may all be ascertained by tests on the finished motor. These tests are of four kinds: the no-load test; the short-circuit test; the heating test; the insulation test.

The No-load Test.—The current, which the transformer takes at no-load, as appears on p. 545, is made up of two parts: (*i.*) an energy component (in phase with the terminal volts), namely that current which supplies the energy necessary to make up for the no-load losses, which component we may call C_H; and (*ii.*) a wattless component (in quadrature with the terminal volts), which is the true magnetizing current serving to excite the alternating magnetic flux, and which we called C_M. Then the no-load current C_0 is such that

$$C_0 = \sqrt{C_H^2 + C_M^2};$$

and the no-load power-factor is

$$\cos \phi_0 = C_H \div C_M.$$

The test is to find the values of C_0, C_H and C_M.

Three instruments are required: A suitable amperemeter, a suitable voltmeter, and a suitable wattmeter. The secondary of the transformer being open, the terminals of the primary are connected to an alternating supply of the correct frequency and voltage. The voltage V_1 is read off on the voltmeter, the current taken C_0 is read off on the amperemeter, and the no-load loss W_0 is read off on the wattmeter. For accurate work the resistance of the primary coil R_1 should also be known. The energy-component C_H is calculated thus:—

$$C_H = W_0 \div V_1;$$

whence

$$C_M = \sqrt{C_0^2 - C_H^2}$$

and $\cos \phi_0$ at once follows.

The no-load copper-loss is $C_0^2 R_1$, and if this is deducted from W_0 the remainder represents the no-load iron-losses.

To separate the hysteresis and eddy-current losses a more elaborate test is needed. A series of readings may be taken with the wattmeter either while the voltage is varied, and the frequency kept constant, or while the frequency is varied, the voltage kept constant. Then as hysteresis varies directly as the frequency and as $B^{1 \cdot 6}$, and eddy-current losses as B^2 and f^2, it is readily seen that from either series of readings simultaneous equations may be formed, from which the hysteresis and eddy-current constants are quickly obtained.

The Short-circuit Test.—For this test the secondary coil is short-circuited, and to the primary terminals there is applied a greatly reduced voltage of the correct frequency. This reduced voltage may be obtained by a preliminary transformation, or by the introduction of resistances, but in any case some resistances should be introduced to afford a means of regulation. We then regulate the resistance or voltage so that the current flowing in the primary circuit is equal to the full-load current C_1, and having attained this we take the voltmeter reading E_{sh} and the wattmeter reading W_{sh}.

From these readings we deduce (1) the equivalent resistance R_{sh} of the short-circuited transformer,

$$R_{sh} = W_{sh} \div C_1^2,$$

and the equivalent impedance,

$$Z_{sh} = E_{sh} \div C_1.$$

Now we know (p. 602) that the equivalent resistance is

$$R_{sh} = R_1 + R_2 \cdot \frac{S_1^2}{S_2^2};$$

which is therefore determined. And, further, the equivalent impedance is made up of the equivalent resistance and the equivalent reactance compounded at right-angles. The equivalent reactance (see p. 602) is known to be

$$X_{sh} = X_1 + X_2 \cdot \frac{S_1^2}{S_2^2}.$$

As then

$$Z_{sh} = \sqrt{R_{sh}^2 + X_{sh}^2},$$

it follows that

$$X_{sh} = \sqrt{Z_{sh}^2 - R_{sh}^2}.$$

From this we calculate the probable voltage-drop at full-load on an inductive circuit with any given power-factor as shown in Kapp's diagram, Fig. 435.

The Heating Test.—Every transformer ought to be tested as to its rise of temperature. For small transformers this presents no difficulty. For large transformers there arises the difficulty of working them on an adequate load. In such cases the best procedure is that of Sumpner, who connects up two similar transformers to circulate power at full-load from one to the other as in the Hopkinson dynamo test, using an external source to make good the losses. Obviously it cannot be applied to any single transformer. For such a case one method of procedure is to measure the iron-losses at no-load with a wattmeter. Also similarly measure the copper-losses when the full-load current is being sent through the short-circuited transformer. Then increase the voltage so as to waste in the copper a total number of watts equal to the known total losses, and proceed to watch the temperature-rise.

It is usual in temperature tests to insert thermometers in the air-ducts between the core-plates, and also in contact (under a pad of cotton wool), with the copper coils. Mercury thermometers should not be used, as they will heat with eddy-currents in the mercury: thermometers of alcohol or glycerine must be used. The temperature-rise in the copper is more accurately ascertained by measuring the increase of resistance of the copper.

A transformer intended for work on a load-factor of 100 per cent. should be tested on a run of twenty-four hours; one for a load-factor of 75 per cent. on a run of eighteen hours, and so forth.

Insulation Test.—A common practice is to test the insulation of the transformer at double the normal high voltage. Thus a transformer intended to work at 4000 volts would be tested at 8000. A small transformer furnishes the testing voltage which is applied as a flashing test for a few minutes, the test being made (*a*) from primary to secondary, and (*b*) from primary to the iron core. The test ought to be made while the transformer is still hot, after a run.

Regulation Test.—A test should also be made of the regulation of the transformer to ascertain how great is the voltage-drop (*a*) on a non-inductive load, (*b*) on an inductive load, while the primary voltage is maintained constant. The two tests should be made not at full-load only, but at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and $1\frac{1}{4}$ loads also, and a curve plotted from the tests.

Analysis of a finished transformer may prove instructive. For this purpose we will take the small transformer shown in Plate XVI., designed some years ago by Mr. A. C. Eborall to a specification of which the following is a summary:—

Output	20 horse-power on a motor load of 80 per cent. power-factor
Frequency	50 cycles
Ratio of transformation	2000 to 120 volts at full-load
Efficiency	Not to be less than 96 per cent. at full-load
Maximum drop of pressure . . .	Not to exceed 6 per cent. at full-load
Temperature-rise . .	Not to exceed 50° C. at full-load.

From considerations of first cost an oil-cooled transformer was necessary, and the type of core and arrangement of winding adopted, namely, two vertical limbs with concentric bobbins, was selected on account of its giving a good distribution of material and small pressure-drop. The mode of bringing out and connecting up the ends of the thick secondary conductors has been omitted from the plate for the sake of clearness. The transformer case is also not shown, and is merely a cast-iron box provided with a watertight cover, and many external ribs for the purpose of assisting the cooling. The connections are brought out through insulated stuffing-boxes at each end of the case.

General Arrangement.—The core is of the shape shown in Fig. 440*a*, built up of stampings. In building it up, the L-shaped stampings are pushed through the previously-wound bobbins from alternate ends, the joints in one layer being covered up by those in the next, so that a practically jointless iron circuit is the result. Thick end plates are used for the sides of the core, and also where the size of plate changes, and the stampings after being assembled are bolted together by means of paper-insulated bolts running right through the core, and provided with a nut at each end to facilitate the building up of the plates. The latter are insulated with thin paper on one

side of each stamping, and which occupies 15 per cent. of the gross area of the core.

The primary and secondary windings are lathe wound, the former on moulded micanite, and the latter on paper, bobbins. Each of the two primary bobbins is wound in two sections, these being separated by a split micanite ring as shown. Micanite bobbins are excellent for this kind of work, as they possess high insulation, and are mechanically nearly as strong as those made of brass. The paper bobbins used for the secondary windings are made of compressed paper, and are provided with paper flanges, which are not really essential when copper strip is used for the secondary conductor, as in this case. As the two cores stand vertically in the case, the left hand side shown in the plate being really the lower end of the transformer, the bobbins are supported partly by the lower iron yoke, and partly by means of four small gunmetal brackets, as shown on the drawing. The spaces inside each bobbin (due to the fact that the cores are of cruciform section) materially assist the cooling, by providing vertical flues up which the oil can circulate.

The primary winding is of round copper insulated with paper to 15 mils (30 mils total); and the secondary of copper strip, cotton insulated to 25 mils (50 mils total), and wound sidewise.

Calculations, &c.—From the drawing, and the workshop data, we find the following:—

Gross area of core	26·25 sq. in.
Mean length of iron path	65 inches.
Total number of turns, primary . . .	960
„ „ „ secondary . .	60
Number of layers per bobbin, primary . .	6
„ „ „ secondary .	2
„ turns per layer, primary . .	80
„ „ „ secondary . .	15
Dimensions of conductor, primary .	d = 0·144 in.
„ „ secondary . .	(0·8 × 0·25) in.
Mean length of one turn, primary . .	32·0 inches
„ „ „ secondary . .	25·5 „
Depth of winding per bobbin, primary .	1·1 „
„ „ „ secondary .	0·65 „
Total cooling surface	3300 sq. in.

From these data we get:—

Area of iron . . . . . . = $26{\cdot}25 \times 0{\cdot}85$
= $22{\cdot}3$ sq. in.
Weight of iron . . . . . = $22{\cdot}3 \times 65 \times 0{\cdot}28$
= 406 lb.
Section of primary conductor . . = $(0{\cdot}144)^2 \times 0{\cdot}785$
= $0{\cdot}0163$ sq. in.
Section of secondary „ . . = $0{\cdot}8 \times 0{\cdot}25$
= $0{\cdot}2$ sq. in.
Total length of primary conductor . = $32 \times 960 \div 12$
= 2560 feet
„ „ secondary „ . = $25{\cdot}5 \times 60 \div 12$
= $127{\cdot}5$ feet
Total weight of primary conductor . = $2560 \times 0{\cdot}0163 \times 3{\cdot}85$
= 161 lb.
„ „ secondary „ . = $127{\cdot}5 \times 0{\cdot}2 \times 3{\cdot}85$
= 99 lb.

The secondary full-load current will be

$$C_2 = \frac{20 \times 746}{120 \times 0{\cdot}8} = 156 \text{ amperes,}$$

and the primary current (assuming an efficiency of 96 per cent.) will be approximately

$$C_1 = 156 \times \frac{60}{960} \times \frac{100}{96} = 10{\cdot}15.$$

As the magnetizing current has to be allowed for, we will say $C_1 = 10{\cdot}5$ amperes, a value well on the safe side.

Using the formulæ given on page 623 we find the resistance of the primary winding for a temperature-rise of about 50° C. to be

$$R_2 = \left[0{\cdot}000008 \times \left\{1 + (0{\cdot}004 \times 50)\right\} \times \frac{2560}{0{\cdot}0163}\right]$$

$$R_1 = 1{\cdot}51 \text{ ohm.}$$

Similarly we have for the secondary

$$R = \frac{0{\cdot}000008 \times 1{\cdot}2 \times 128}{0{\cdot}2},$$

$$R_2 = 0{\cdot}00615 \text{ ohm.}$$

Hence the full-load $C^2 R$ losses at 65° C. are

$$\begin{aligned} \text{Primary} &= (10{\cdot}5)^2 \times 1{\cdot}51 = 166{\cdot}5 \\ \text{Secondary} &= (156)^2 \times 0{\cdot}00615 = 149{\cdot}5 \end{aligned}$$

$$\text{Total copper-loss} = \underline{316} \text{ watts.}$$

We will now proceed to calculate the flux-density in the core, and the core-losses.

We have at no-load

$$E_1 = 4{\cdot}44 \times n \times S_1 \times N \div 10^8,$$
$$2000 = 4{\cdot}44 \times 50 \times 960 \times B \times 22{\cdot}3 \div 10^8.$$
$$\therefore \quad B = 42{,}000 \text{ lines per sq. inch.}$$

A reference to the curve AA for best transformer iron given in Fig. 426 shows us that the total loss per pound of core when worked at this density and 50 periods per second is 0·8 watt. Hence the total iron-loss is

$$0{\cdot}8 \times 406 = 324{\cdot}8 \text{ watts, say } 325.$$

The total watts wasted at full-load by this transformer when hot are therefore $(316 + 325) = 641$ watts. The full-load efficiency is hence

$$\frac{20 \times 746 \times 100}{(20 \times 746) + 641} = 95{\cdot}6 \text{ per cent.}$$

The next point to be considered in the design is the voltage-drop from no-load to full-load. We have for the secondary electromotive-force at no-load

$$E_2 = \frac{2000 \times 60}{960} = 125 \text{ volts.}$$

Now the ohmic drop in the primary at full-load is $10{\cdot}5 \times 1{\cdot}51 = 15{\cdot}85$ volts. That in the secondary is $156 \times 0{\cdot}00615 = 0{\cdot}96$ volt. By dividing the primary drop of 15·85 volts by the ratio of transformation, we get its effect on the secondary. So that we may say that the drop of secondary pressure caused by the ohmic resistance of the two windings is

$$\left(0{\cdot}96 + \frac{15{\cdot}85 \times 60}{960}\right) = 1{\cdot}95 \text{ volt.}$$

To obtain the maximum drop with full-load current we must take the resultant of this and the total inductive drop due to leakage, as already explained. Now experience shows that for this type of transformer and arrangement of windings, the inductive drop due to leakage is very approximately three times the ohmic drop.[1] Taking therefore the inductive drop as being 6 volts, we at once find the maximum drop for 156 amperes from Fig. 435, the length of the side P Q being 6·31 volts, or 5 per cent. of the secondary voltage at no-load—a value well within the specification. If we work out the full-load drop for different secondary power-factors by means of Mr. Kapp's construction previously referred to, we find :

Power-factor of secondary circuit	100 %	80 %	60 %	40 %	20 %	10 %
Secondary electromotive-force at 156 amperes . . .	122·84	119·84	119	118·7	118·76	118·85

Showing that the drop of secondary pressure under the normal conditions as specified, would be 5·16 volts, or 4·12 per cent. of the secondary pressure at no-load.

The current taken by the primary of the transformer when the secondary circuit is open is easily found. We have

$$C_H = \frac{325}{2000} = 0 \cdot 162 \text{ ampere.}$$

Referring to the iron curves, Fig. 392, p. 443, we see that an ordinary sample at a maximum flux-density of 42,000 requires about

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[1] Prof. Gisbert Kapp has (*Elektrotechnische Zeitschrift*, April 14th, 1898) given a formula by means of which the inductive drop may be very approximately calculated from the drawing of the transformer. Let e_s be the inductive drop expressed as a percentage of the no-load secondary pressure, x the mean of the primary and secondary ampere turns at full-load expressed in units of 10^3, N the total flux in the core in units of 10^6, b the distance from copper to copper between primary and secondary windings, a the mean depth of primary and secondary windings, l the length of bobbin, and m the mean perimeter of the space between primary and secondary coils, all lengths being taken in centimetres. Then for this type of transformer and method of winding

$$e_s = k \times \frac{x}{N}\left(\frac{b}{2}+\frac{a}{3}\right) \times \frac{m}{l} \text{ per cent.,}$$

where k is a constant varying from 0·17 to 0·3. In the above transformer we have $x = 9 \cdot 7$, $N = 9\ 36$, $b = 0 \cdot 58$, $a = 2 \cdot 225$, $m = 70 \cdot 6$, $l = 37$. Assuming $k = 0 \cdot 2$, this gives $e_s = 4 \cdot 05$ per cent. of the secondary volts at no-load, and hence gives a value for the inductive drop of 5·06 volts, a value less than that taken above.

3·5 ampere turns per inch of path; and hence the magnetizing current is

$$\frac{3·5 \times 65}{960} = 0·237 \text{ ampere.}$$

Hence we find the no-load current as

$$C_0 = \sqrt{(0·162)^2 + (0·237)^2} = 0·29 \text{ ampere,}$$

or about $2\frac{3}{4}$ per cent. of the full-load primary current.

The total cooling surface is about 3300 sq. inches, and the total loss 641 watts. Hence the watt loss per square inch of cooling surface is 0·194, which gives a temperature-rise (Fig. 428) of about 43° C.

Weights and Cost per Kilowatt.—These work out as follows:

260 lb. copper, at $10\frac{1}{2}$*d.* per lb. . . .	227·5 shillings
406 „ iron „ 2*d.* „ . . .	67·7 „
666 lb. active material costing . . .	295·2 shillings

Therefore we have

$$\text{Weight of active material per kilowatt} = \frac{666}{0·746 \times 20} = 45 \text{ lb.}$$

$$\text{Cost of active material per kilowatt} = \frac{295·2}{0·746 \times 20} = 19·7 \text{ shillings.}$$

If this same transformer were used for running glow lamps, where the power-factor of the secondary circuit would be 100 per cent., we should obtain a cheaper apparatus with higher efficiency and better regulation for the same heating. The output, for instance, would be now 25 horse-power, the efficiency 96·7 per cent., and the secondary pressure drop 2·16 per cent. The weight and cost of active material per kilowatt come out as 36 lb. and 16·76 shillings respectively.

With regard to the amount of winding space allowed for the above windings, it must be remembered that a certain amount of space is always lost in winding, and also that the space occupied by one extra turn has to be allowed for in the length direction, as this amount is lost in changing over from one layer to the next. Thus in this transformer we find that the dimensions of the winding spaces are (14 × 0·65) inches for the secondary, and (14·3 × 1·1) inches for the primary, which affords a plentiful allowance for winding clearance.

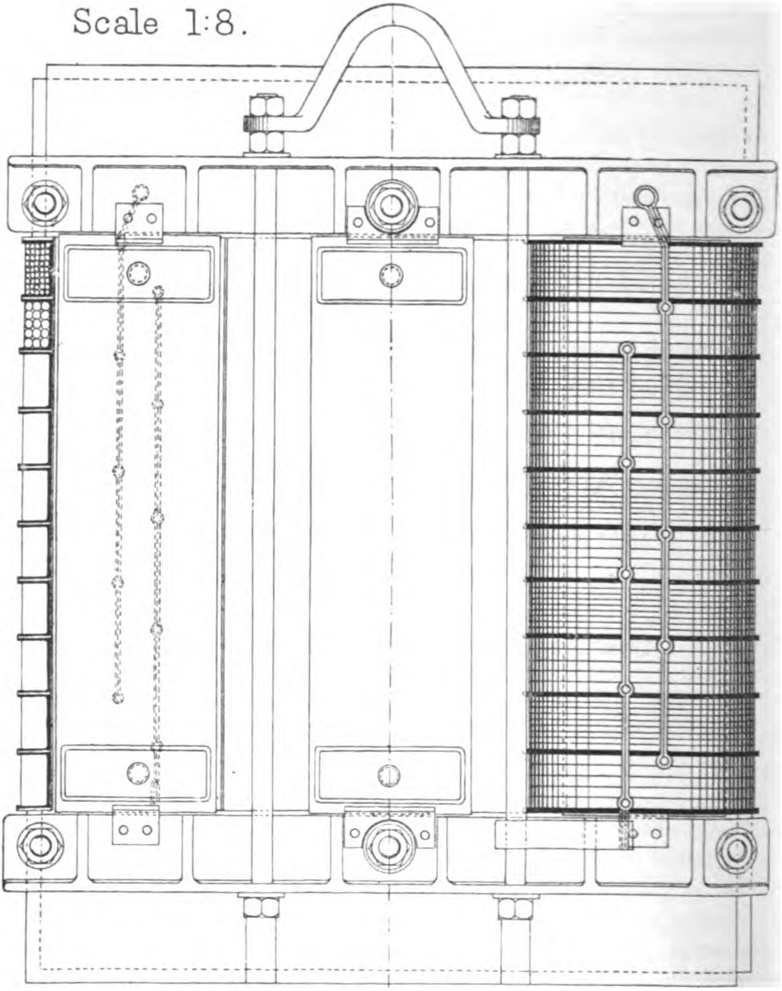

Fig. 444.—Three-Phase Transformer, 60 kw (Kolben and Co.).

A somewhat unusual type of transformer, though of standard pattern, is that of Kolben and Co. depicted in Figs. 444, 445 and 446. It is a low-voltage three-phase transformer with cores of square cross-section and coils wound on square formers. It is rated at 60 KW, and transforms from 330 to 110 volts, with a frequency of 42 cycles per second.

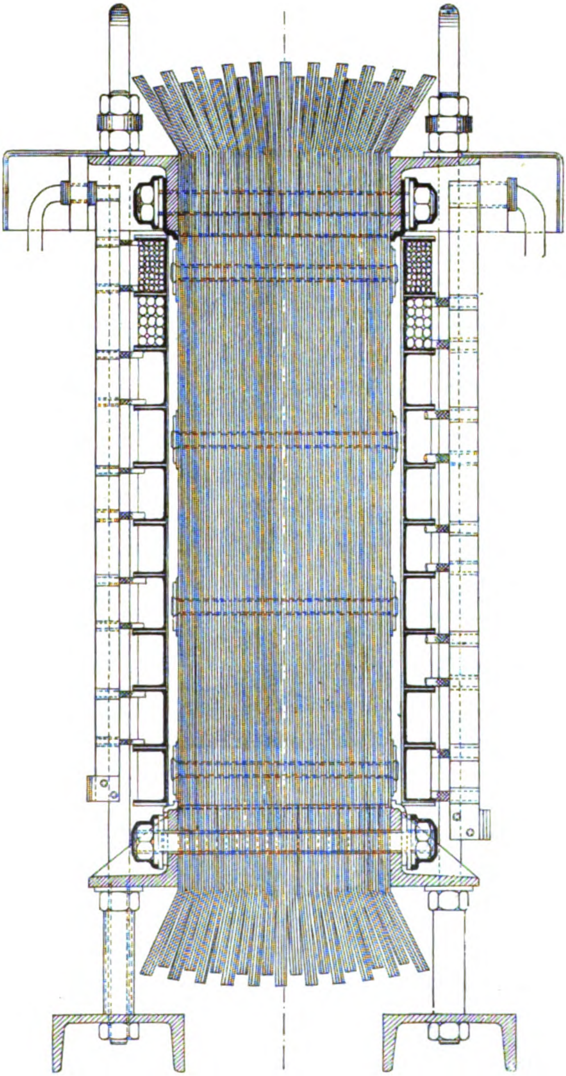

FIG. 445.—THREE-PHASE TRANSFORMER, 60 KW (KOLBEN AND CO.).

Type of transformer	Three-phase, core
Full-load kilo-volt-amperes	60
Primary voltage	330
Secondary voltage	110
Primary amperes	105
Secondary amperes	315
Frequency	42

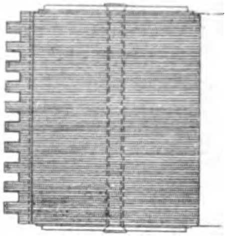

FIG. 446.—DETAIL OF CORE-SECTION.

DIMENSIONS (inch units).

Core :—

Length	24·6
Outside width	6·8
Outside breadth	8·85
Estimated total iron section of core	54

Yoke :—

Extreme length	29·8
Breadth	8·85
Depth	6·8
Estimated total iron section of yoke	54

Windings :—

Number of turns per bobbin, primary	44
,, ,, ,, secondary	15

Number and arrangement of bobbins—
Primary, five per phase, all in parallel.
Secondary ,, ,, ,,

Dimensions of conductor—

Primary, diameter	bare	0·197
	insulated	0·22
Secondary ,,	bare	0·335
	insulated	0·372

Arrangement of conductors on bobbin—
Primary, 4 layers of 9 wires, 1 layer of 8 wires
Secondary, 3 layers of five conductors

Estimated mean length of 1 turn—

Primary	24 inches
Secondary	24 inches

CALCULATED DATA AND COEFFICIENTS.

Maximum flux in core (megalines)	2·27
Maximum flux-density in core (lines per sq. in.)	42,000
,, ,, in yoke	42,000
Magnetizing ampere-turns required for each core and part of yoke	110
No-load magnetizing current per phase	2·45
Full-load ampere-turns on primary	4680
Value of flux-factor $\left(Y = \frac{N_1}{C_1 S_1}\right)$	485
Ampere-turns per inch length of core	190
Core coefficient $\left(\frac{l_c}{\sqrt{A_c}}\right)$	3·34
Current density (amps. per sq. inch)—	
Primary	690
Secondary	713

LOSSES AND EFFICIENCY.

Iron-loss by hysteresis (watts)	655
,, by eddy-currents (watts)	645
Resistance of primary windings (at 15° C.)	0·00725
Loss in primary (half-load)	71
,, ,, (full-load)	285
Resistance of secondary windings (at 15° C.)	0·0009
Loss in secondary (half-load)	84
,, ,, (full-load)	320
Total losses at half-load	1450
,, at full-load	1905
Efficiency at half-load, per cent.	95·5
,, at full-load ,,	97

Plate XVIII. gives a drawing of a 200 KW transformer of the Oerlikon Co. with air-blast cooling. The secondary winding is made of bare aluminium strip insulated with asbestos separators and supports. It carries 4000 amperes at 50 volts. Some of these transformers have the secondary wound edge-wise, while others have the conductors superposed with several windings in parallel.

CHAPTER XIV.

INDUCTION MOTORS.

AN induction motor is one in which the part corresponding to the field-magnet (usually the rotating part) is entirely disconnected from the supply mains, and receives its currents not by conduction but by induction. It is in one sense therefore a species of transformer. The armature is usually stationary and is constructed precisely like the armature of an alternator : it is usually denominated as the *stator.* The revolving part, which corresponds to the field-magnet, is denominated the *rotor.* In small machines, and in some large ones, the rotor consists of a mere cylinder of laminated iron, having a system of copper bars or closed circuits embedded in slots at the periphery ; such simple forms being called *squirrel-cage rotors.* In large machines, and in some small ones, the rotor is constructed of toothed core-disks having regular windings laid in the slots, and connected up as a symmetrical Y-connected three-phase winding, the ends of the three circuits being brought out to three slip-rings on the shaft, in order that, at starting, resistances may be inserted in the rotor circuits : rotors so constructed are known as *wound rotors.*

The induction motor, the most primitive form of which with a disk-armature was described by Baily in 1879, was developed as an immediate result of the discovery, by Ferraris (1885) and by Tesla (1888), of the rotating magnetic field produced by combining together the operation of two, three, or more alternating currents differing from one another in phase.

For a full account of the development of the induction motor the reader is referred to the author's treatise, *Polyphase Electric Currents.*

PRODUCTION OF A ROTATING MAGNETIC FIELD.

The theory of the production of the rotatory magnetic field is extremely simple.

It is appropriate to consider how currents which are different in their phases can be combined to produce resultant magnetic fields.

We may take it that when a simple alternating current is carried in a coil around a core, the magnetism along the core will be an alternating magnetism. If the core is merely air, we shall have an alternating field. If the core is of iron, the flux of magnetic lines through it will be an alternating flux ; that is to say, a flux which begins, increases to a maximum then dies away, reverses in direction, increases to a reversed maximum, and dies away, to begin the cycle over again. The frequency of this alternating flux will be the same as that of the current. If the iron is properly laminated, and there are no secondary circuits to perturb by reactions, the rise and fall of the magnetic flux will be nearly in phase with that of the surrounding current. Any eddy-currents in the core and any secondary currents induced by the core in neighbouring conductors, will inevitably have the effect of retarding the phase of the alternating magnetic flux, and of causing it to lag. Such reactions by induced currents in closed secondary circuits play a vitally important part, as we shall now show in the induction motor.

It is self-evident that (in the absence of secondary reactions) a magnetizing force which alternates along a fixed direction will produce an *alternating* magnetic flux ; whereas a magnetizing force which is constant in value, but is continuously changing in direction—rotating in space—will tend to produce a *rotating* magnetic flux. Whether this resulting rotating magnetic flux will have a constant value or a uniform speed of rotation, will depend not only upon the uniformity of the impressed rotatory magnetizing force, and on the influence of secondary currents, but will also depend on the shape of the magnetic masses, as to whether they also are magnetically symmetrical around the axis of rotation of the magnetizing forces.

For the present, to gain simplicity in grasping the subject, we will consider the problem of the combinations of magnetizing forces to produce a resultant magnetizing force.

Let us consider two magnetizing forces such as would be produced by two short coils of large diameter placed at right angles to each other as shown in Fig. 447. It will be seen that their magnetizing forces which are at right angles to each other, will combine to produce a resultant magnetizing force whose direction and amount depends not only on the direction of the plane of the coils but also upon the value for the time being of the current in each coil. We will suppose that the coils remain at right angles to one another and that the currents in them have the same frequency.

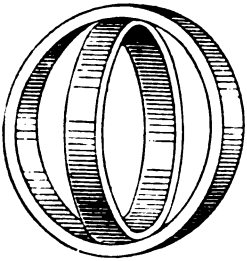

FIG. 447.

If the direction and intensity of a magnetic field may be represented by the direction and length of a line, then we may apply the ordinary parallelogram rule for the compounding of vectors, and find the resultant of two magnetic fields that differ in direction and magnitude by compounding the vectors that represent them, and drawing the diagonal. Let the magnetizing force, at the centre, due to one of the coils be represented by the line A and that of the other by the line B at right angles to A. First let A be in phase with B and equal to it. Then Fig. 448 shows the direction and amount of the resultant R. When component A has the small value O A_1 and component B the small value O B_1, the resultant will be O R_1. When A grows to O A_2 and B to O B_2 the resultant will be O R_2; and it is evident that if the magnitudes of O A and O B increase and decrease together, the resultant O R will also vary in the same phase, but will remain fixed in its own direction. In brief, two alternating vectors of equal period and in identical phase have as their resultant another alternating vector of equal period, of identical phase, and of fixed direction.

If, however, as in Fig. 449, the two components go through their periodic changes with a difference between their phases, not increasing and decreasing together, the resultant will no longer have a fixed direction. Let the phase-difference between A and B be equal to 90° so that when A is at its maximum B is zero, and *vice versâ*. Then it is evident that the resultant will change, as in the figure, from OR_1 to OR_2, OR_3, etc. The exact shape of the curve $R_1\,R_2\,R_3$ will depend upon the law of the current-variation in the coils. If A and B have equal maxima and follow a sine law, the curve (as proved below) will be a circle; that is to say, R will be constant in amount and its direction will rotate about O.

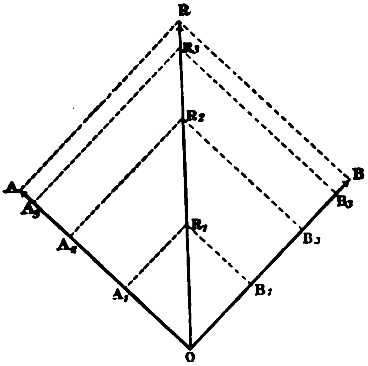

FIG. 448.

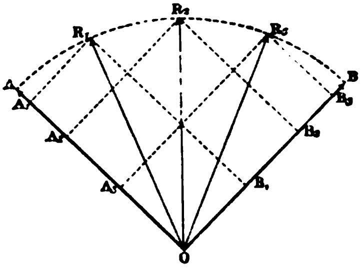

FIG. 449.

In 1883 Marcel Deprez communicated[1] to the *Académie des Sciences* an important theorem on the production of a true rotatory magnetic field, by the combination of two alternating magnetic fields having as their difference of phase a quarter period.

It is well known that a uniform circular motion can be decomposed into two rectilinear harmonic motions at right-angles to one another, the two having equal amplitude, equal period, and a phase difference of one-quarter period. Let P be a point uniformly revolving around centre O. The projections of the radius O P upon the two axes (Fig. 450) are

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[1] *Comptes Rendus*, 1883, ii. 1193.

O M and O N. If the radius O P be called r we have O N = $r \sin \theta$, and O M = $r \cos \theta = r \sin (\theta + 90°)$. While P revolves the point N will oscillate up and down the line Y Y′; the amplitude of its motion being equal to the radius of the circle. Also the point M will oscillate along the line X′ X with equal amplitude and in equal time; but O N will be at its maximum when O M has zero value, and *vice versa.* It follows kinematically that a uniform circular motion may be produced out of two straight line motions, by combining them at right angles, provided they are harmonic, of equal period, of equal amplitude, and differing by an exact quarter period.

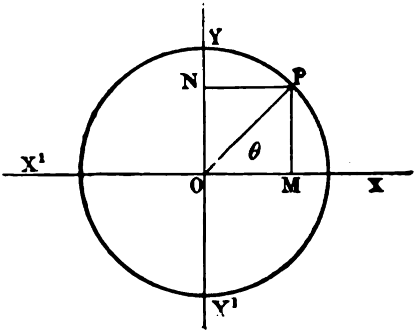

FIG. 450.—DEPREZ'S THEOREM.

Geometrically, this is as follows: We are given two harmonically varying movements of equal amplitude set in direction at right angles to one another. Each follows a sine-function of the time, but they differ by 90° in phase. We may write them as:

$$y = r \sin \theta$$
$$x = r \sin (\theta + 90°) = r \cos \theta.$$

In Fig. 450, y is represented by O N and x by O M. As x and y are at right angles, the resultant of them will be obtained by squaring them, adding and taking the square-root.

$$\sqrt{(x^2 + y^2)} = \sqrt{(r^2 \sin^2 \theta + r^2 \cos^2 \theta)};$$
$$= r \sqrt{(\sin^2 \theta + \cos^2 \theta)}.$$

But

$$\sin^2\theta + \cos^2\theta = 1,$$

whence

$$\sqrt{(x^2 + y^2)} = r.$$

The resultant then will be of constant value, equal in magnitude to the maximum of either component, and will rotate around the centre at a uniform speed, making one revolution in each cycle. It becomes a *rotating vector.*

Mechanically this motion is equivalent to that of two pistons having equal travel, working by two connecting rods upon the same crank pin, but placed at right angles to one another (Fig. 451). If motion of rotation is given to the shaft it will be decomposed into two rectilinear motions; the apparatus then acting as a two-throw pump. If on the other hand the cylinders are made to produce two rectilinear motions one ahead of the other by a quarter period in time, the apparatus will combine these motions in a true circular motion and becomes equivalent to a two-crank engine with parallel cylinders.

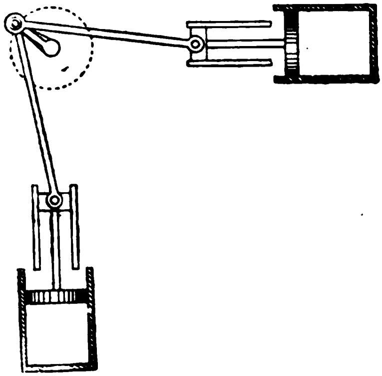

FIG. 451.

Deprez saw that a similar combination can be magnetically effected. If an alternating current is led round a coil so as to produce an alternating or oscillating magnetic field along the line O X, and a second alternating current is led round a second coil so as to produce a second alternating magnetic field along the line O Y, then the result will be a ***rotatory*** magnetic field, provided these two magnetic fields are of equal period and amplitude, and differ exactly a quarter in phase.

If we regard the above as representing two harmonically varying fields, H_A due to the current in the A-phase, and H_B due to the current in the B-phase, we shall have

$$H_A = H_{max} \sin 2\pi f t,$$

$$H_B = H_{max} \cos 2\pi f t,$$

and their resultant will be :—

$$\surd(H_A^2 + H_B^2) = H_{max} ;$$

and will be a field rotating with frequency *f*. If the component fields are of equal period, but not of exactly equal amplitude, the result will be equivalent to an *elliptically*-rotating magnetic field; that is to say, one in which the strength and direction of the field is represented by the successive values of the radius vector drawn to an ellipse from its centre, and sweeping over equal areas in equal times. An elliptically-rotatory field will also be produced if the two component magnetic fields, though equal in period and amplitude, do not differ by exactly a quarter period. For a perfect rotatory field, corresponding to uniform circular motion, the two components must vary precisely as the sine and the cosine[1] of an angle respectively.

In the above we have used two harmonic components at right-angles, differing by a quarter period, or, in brief, a two-phase arrangement. But this is not by any means the only combination that will produce a rotatory magnetic field. The mechanical analogues of the three-crank engine, and of the three-throw pump, at once suggest other solutions. In the former instance three cylinders are used, with three pistons which operate in successive phases differing by one-third of a period from one another. If the three cylinders are set (as in a Brotherhood's engine) at 120° to each other, their connecting-rods may actuate a single crank, Fig. 452. If the three

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[1] See also Ferraris, "Rotazioni elettrodynamiche," *Turin Acad.*, March 1888.

cylinders are set parallel side by side, then there must be three cranks spaced out in angular positions 120° from one another. If the angular positions of the cranks were not exactly 120° apart, the phase-differences of the motions would not be exactly one-third of the period. The phase-difference of the motion must correspond to the peripheral spacing of the mechanism that combines them. It is a kinematic principle that in combining harmonic motions to produce rotation, the space-phase of angle in the combining mechanism must be the supplement of the angle which represents the time-phase of motion, otherwise the resulting motion will not be a *uniform* rotation.

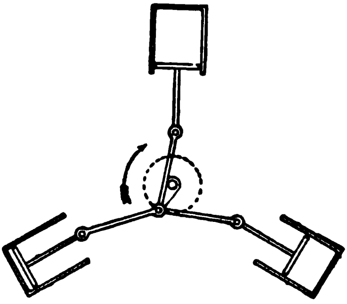


FIG. 452.

The three-phase system of currents (or *drehstrom*), p. 37, for producing a rotatory magnetic field, is the electrical analogue of the three-crank mechanism. In dealing with such combinations of magnetic fields, we may proceed analytically. We have three coils, or three pairs of coils, each producing a component magnetic field which alternates along a fixed direction, and we want to find the resultant field when they are combined together. When the coils are placed at an angle to each other we have to take into account not only the strength of each component field determined by the phase of the current, but also the direction of it. This is most easily done by splitting up the field produced by each circuit into components along two axes X and Y. For instance, taking the coils on a ring such as in Fig. 453, the coils *b* and *e* will together produce a horizontal flux in the direction of O *b* along the axis of X in Fig. 454, which will change in value following the law H sin θ. The coils *d* and *a* will produce a field in the direction of O *d* which will follow the law H sin (θ – 120°). Similarly the coils *f* and *c* will produce a flux in direction O *f* following the law H sin (θ – 240°).

Adding together the component of these along the axis of X, we get

$$\text{H} \sin \theta - \text{H} \sin (\theta - 120^\circ) \cos 60^\circ - \text{H} \sin (\theta - 240^\circ) \cos 60^\circ$$
$$= \frac{3}{2} \text{H} \sin \theta.$$

And, taking the components along the axis of Y,

$$\text{H} \sin (\theta - 240^\circ) \cos 30^\circ - \text{H} \sin (\theta - 120^\circ) \cos 30^\circ = \frac{3}{2} \text{H} \cos \theta.$$

If we draw a line O R representing $\frac{3}{2}$ H to scale, making the angle θ with the axis of Y, we shall see that as θ increases with time and R revolves round O, the projection of O R upon the axis of X is $\frac{3}{2}$ H sin θ, and on the axis of Y it is $\frac{3}{2}$ H cos θ. Therefore, O R represents the direction of the resultant of the field at any moment. The field *at the centre* has, therefore, a constant value equal to one-and-a-half times the maximum field produced by one pair of coils, and rotates at a constant angular velocity. Except at the centre the field is non-uniform. Such a field is called by Ryan a *pivotal* field, to distinguish it from a *progressive*, or peripheral field, both being varieties of rotating fields.

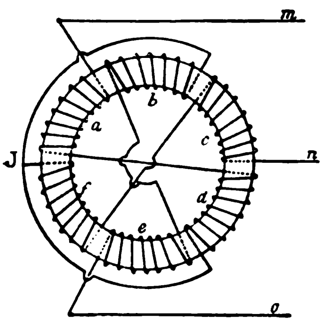


FIG. 453.

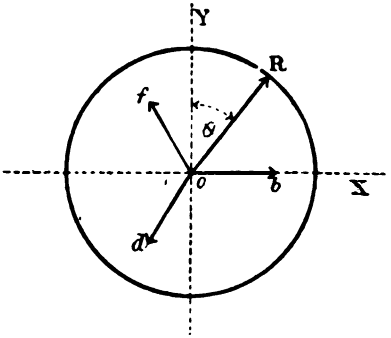


FIG. 454.

Generally we may say that, with a symmetrical grouping of coils around a centre, if the number of phases is called n_{ph}, the ratio of the resultant field *at the centre* to the field produced by one phase is $\frac{1}{2}$ n_{ph}.

We see, then, what are the time and space relations in simple 2-phase and in simple 3-phase working. To produce a uniformly rotating magnetic field, we may have as components either two equal fields differing by a quarter period in time, and set at 90° (i.e. a quarter circumference) to one another in space; or, instead, three equal fields differing by a third of a period from one another in time, and set mutually at 120° (i.e. a third of a circumference) to one another in space. Obviously other cases might arise, for it is clear that by putting the two cylinders in Fig. 451 at some other angle than a right angle, the uniform circular motion of the crank will produce at the pistons two simple harmonic motions of equal period, which did not differ by a quarter period. As a simple kinematic principle we have, therefore, that a uniform circular motion may be compounded out of two equal simple harmonic motions that do not differ by a quarter period, provided the space-phase of their angular positions is equal to the supplement of the time-phase of their motions.

Returning to the case of 2-phase and 3-phase combinations properly so-called, it may be remarked that even though the amplitudes of the components be equal and their phase relations properly adjusted, the result cannot be a uniform circular motion unless the individual components are truly harmonic —that is to say, follow exact sine-functions. Now we know that in many cases the form of the curves of electromotive-forces, currents and magnetomotive-forces in the actual alternating current systems in use differs considerably from that of a simple sine curve. It is easy to see in general what will be the effect of any departure from the simple sine form. Taking a 2-phase combination, if the component curves are of the peaked type, such as Fig. 26, p. 45, the resultant curve will have the general form of Fig. 455, while if the component curves are of the broad-topped round-backed variety, the resultant will have the general form of Fig. 456. If one or both the curves present a rippled outline, owing to the presence of a sub-component of higher order of frequency, the contour of the resultant curve will also be rippled.

In the paragraphs immediately preceding, the combination of component vectors has purposely been discussed from the rather abstract or kinematic point of view. The rotatory phenomena in induction motors are both more concrete and more complex. In them the impressed magnetic field seldom has a simple uniform circular rotation. They are mostly multipolar; they have projecting poles, teeth and other discontinuities of structure, all of which must have a tendency to cause the magnetic field to rotate more or less by jumps, and with variations in its magnitude from point to point. This is, however, of minor importance, for, as we shall see, the necessary tendency of the induced effects in the conducting revolving masses is to react against all departures from simple and uniform rotation. Further, in the ideal case, what is sought is not a uniformly rotating magnetic field, but such a combination of rotating magnetic field with a set of induced currents that the conductors carrying the latter (or the iron core in which they are embedded) shall be urged around its axis with a sufficient and sufficiently uniform torque. The torque at different points of the revolution is not uniform in steam engines,

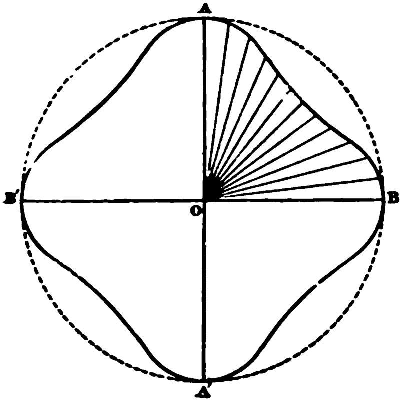


Fig. 455.

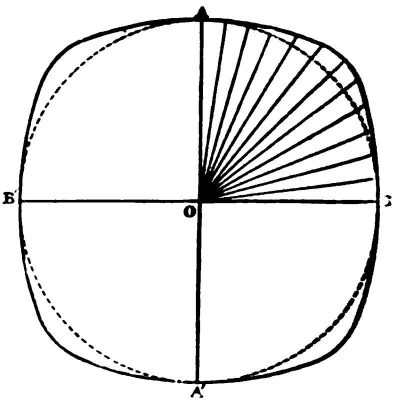


Fig. 456.

even in those provided with two or three cranks. But even in the worst polyphase motor the torque is much more uniform than in the best reciprocating engine. No polyphase motor, no single-phase motor even, needs any fly-wheel to regularize the irregularity of the turning effort.

Lastly, it may be well to remind the student that the principle of vector combination (such as in the well-known parallelogram of vectors) is only applicable to magnetomotive-forces, magnetic fluxes, and electric currents when we are considering these quantities *as vectors*, that is to say, when their actual direction in space is being taken into account, and, therefore, obviously cannot be employed in dealing with quantities of a *circuital* nature such as the total magnetomotive-force, or as the total magnetic flux in a circuit, or in combining currents flowing from several wires into a common wire. There the quantities in question have merely a *scalar* value, their directions varying throughout their circuit. If we are considering the magnetic force at a point, we have something with a perfectly definite direction, and may, therefore, combine it with another magnetic force at the point. Similarly, when we are considering magnetic fields whose directions at a particular instant are uniform over the space we are considering—as in the case of the magnetic fields in the theorem of Marcel Deprez on p. 648, and the theorem which follows it, as to the resultant field in a particular three-phase motor—the principles of vector combination are applicable. But in a multipolar motor, where the flux is along curved paths, as shown in Fig. 457, the flux as a whole cannot be considered as a vector, and it is for this reason that in Chapter V., in the discussion of the progression of the magnetic field, the diagrams have been drawn to show how the circuital magnetomotive-forces progress along the periphery of the rotor.

The student must clearly distinguish between the application of the polygon of vectors in the case where *vector* quantities are being added, and the application of the same geometrical construction when the quantities that are being added are *scalar* quantities that happen to follow a sine function of the time. In the latter case the quantities have their phase relations represented by the inclination of lines to one another; the legitimacy of the process depending solely upon the peculiar properties of the sine-function.

Multipolar Progressive Fields.—Up to this point we have treated the rotatory field as though it were a simple bi-polar arrangement producing a field of a certain value at a centre around which it turned as on a pivot. But in reality the field is produced in the narrow gap between stator and rotor, and is almost always multipolar, and progresses along the periphery. The flux is distributed in the gap in a manner so as to be generally sinusoidal, though the discontinuity of the iron due to slots may produce irregularities in detail. The number of poles is, of course, determined by the stator.

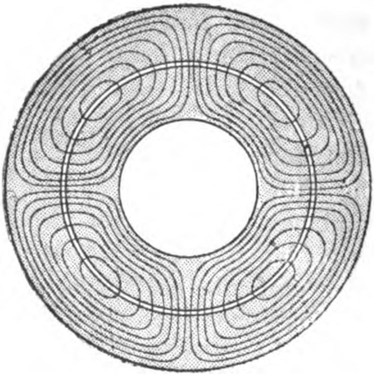


FIG. 457.—MAGNETIC FLUX IN A 4-POLE STATOR.

The main duty of the stator is to produce a magnetic field, whose lines shall spring from its inner cylindrical surface in a radial direction, and after traversing part of the rotor shall re-enter the stator radially, as shown in Fig. 457, producing north and south poles which move rapidly over the face of the stator and sweep the conductors of the rotor as a simple rotating field would do. The number of these pairs of poles will depend upon the frequency for which the motor is designed, and the speed at which it is to be run; the number of pairs of poles being equal to the frequency divided by the number of revolutions per second. The stator windings must, of course, be spaced out in accordance with the principles laid down in Chapter VI. The currents in these windings will then conspire to generate a set of poles which regularly shift around the face precisely as the induced poles in an armature shift around, as discussed on pp. 257 to 265. Figs. 212 and 216 may be taken as completely illustrative of the progression of a stator field, or rather of the combined magnetomotive-forces which create the progressive field. We have next to consider the conjoint effect of the several phases.

Now in 3-phase stators the magnetomotive-force at the mid-point of the pole is less than three times the ampere-turns of one phase, because the three currents are not all at their maximum at the same time. But it is greater than 1·5 times the maximum ampere-turns of one pole as deduced on p. 652 for the central field, because the 1·5 rule assumed the three sine-functions to be vectorially compounded at 120° apart, whereas as arranged on the periphery of the stator they are more nearly in one straight line, and the overlap of the B and C phases over the A-phase, though each may comprise 120 electrical degrees of the double pole-pitch, does not give a vectorial combination at 120° in space. When the A current is at its maximum the B and C currents are at values of half-maximum (the sum then being $1 + \frac{1}{2} + \frac{1}{2} = 2$); and when the A current is at its zero the B and C currents are at 0·866 of the maximum (the sum then being $0 + 0{\cdot}866 + 0{\cdot}866 = 1{\cdot}73$). Its *mean* value can then be found by taking the value

$$\frac{6}{\pi}\int_0^{\frac{\pi}{6}} 2 \cos \theta \,.\, d\theta$$

because it recurs every 60°, and is therefore the mean over 30°. It therefore equals $\frac{6}{\pi} \times 2 \times 0{\cdot}5 = 1{\cdot}91$; and as this is the *mean*, the maximum will be $1{\cdot}91 \times 1{\cdot}41 = 2{\cdot}7$ times the virtual value of the ampere-turns due to any one phase.

But, further, it must be noted that the magnetizing effect in the motor as a whole is dependent on the rotor currents as well as on the stator currents.

It must here be borne in mind that an induction motor is essentially a species of transformer; the currents supplied to the primary or stator windings inducing secondary currents in the windings of the rotor. This process follows transformer principles strictly. At any moment the ampere-turns in the secondary (rotor) are nearly equal, and nearly opposite in phase to the primary (stator) ampere-turns. They would be strictly equal and opposite were it not for two considerations, the magnetizing component of current in the primary, and the presence of magnetic dispersion. These will be considered presently. In the meantime, we have to bear in mind that the actual field in the gap and in the body of the magnetic circuits through the gap, is the resultant of the two opposing sets of magnetizing forces; and that as the result of the interplay of the three currents in the three phases there is generated a progressive multipolar "field," the lines of which travel around at a high speed although the iron stator itself stands still.

SLIP.

The stator then produces a revolving multipolar flux, and the revolutions of this invisible magnetic field set the pace for the rotor. The rotor, swept by the magnetic lines as they go by, becomes the seat of induced currents, the mechanical reaction of which drives it in the same direction. But the speed of the rotor never actually attains equality with that of the invisible revolving field; even at no-load, when it runs at its highest speed, it falls slightly short of synchronism. At full-load it runs a little slower—in practice from 2 to 3 per cent. (or in small machines more than 3 per cent.) slower—than the revolving field. This slight drop in speed below synchronism is denominated *the slip*. It is an essential feature of all induction motors, which therefore are sometimes called *asynchronous motors*, and is in fact that which produces the driving forces.

Let

n_1 = the revolutions per second of the flux,

and

n_2 = the revolutions per second of the rotor.

Then n_2 is *always less than* n_1. The difference of the speeds $n_1 - n_2$ is *the slip* in revolutions per second. It is often convenient to express the slip as by the ratio it bears to the primary speed; and the fraction

$$\frac{n_1 - n_2}{n_1} = s \quad . \quad . \quad . \qquad [I]$$

is also known as the *slippage*, or the *percentage slip*.

The necessity for a slip is readily understood from induction principles, and its discovery is due to Ferraris. It had been known from the early experiments of Arago, Babbage and Herschel, and Baily, that a mass of copper (or any conducting metal) if placed in a revolving magnetic field, will be urged to revolve in the direction of the revolving field. The early experimenters used copper disks; Ferraris used a copper cylinder.

Consider the most elementary case—that of a cylinder of copper situated in a rotatory field, as in Ferraris's early motor. The effect is equivalent to that produced by a pair of magnet poles placed at opposite sides of the cylinder, and revolving around it. Suppose the north pole to be in front of the cylinder (Fig. 458) and to be moving past it from right to left (or clockwise as viewed from above). The inductive action will be the same as if the pole stood still while the cylinder revolved from left to right. This will set up electromotive-forces in the part which is passing under the pole in a direction shown by the arrows, upward; and there will be set up, as the result, a pair of eddy-currents as indicated in the sketch. Now the mechanical force which a conductor carrying a current experiences when in a magnetic field, is always in a direction at right angles both to the lines of the magnetic field and

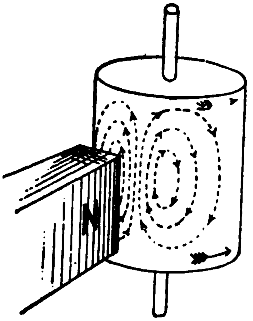


FIG. 458.—EDDY-CURRENTS INDUCED IN A COPPER CYLINDER.

to the line of flow of the current. That portion of the copper which carries the upward current across the field will be urged laterally to the left, whilst those parts in which the current is flowing horizontally will simply be urged up or down, and will contribute nothing to the torque. On the other hand, the parts of the copper in which the currents are flowing downwards will—if they lie in the same magnetic field—experience forces tending to turn in the other sense. To obtain the best mechanical effect the currents induced in the rotor ought to be led through paths situated in a strong field of the proper polarity. Clearly, then, a better result will ensue if the downward returning currents are led into some path where they will return across a field of opposite polarity from that across which they flowed up. Then they will doubly tend to produce rotation.

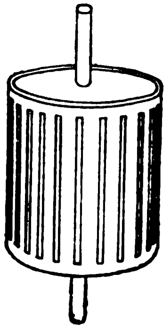


FIG. 459.—ELEMENTARY SQUIRREL-CAGE ROTOR.

As a first stage to this, it will obviously be an improvement to make in the copper cylinder a number of parallel slits, which extend nearly to the ends of the cylinder as in Fig. 459, or to build it up of a number of parallel bars all joined together by a ring at each end. Dobrowolsky, who appears to have been the first to introduce the latter construction under the name of *Schluss-anker*, seems to have thought that the insulation of these bars from the iron core was of little importance. He regarded the bars as merely veins of copper lying buried in a solid mass of iron.

A solid cylinder of iron will of course serve as a rotor, as it is magnetically excellent; but the high specific resistance of iron prevents the flow of induced currents from taking place sufficiently copiously; and a solid cylinder of iron is improved by surrounding it with a mantle of copper, or by a squirrel-cage of copper bars (like Fig. 459), or by imbedding rods of copper (short-circuited together at their ends with rings) in holes just beneath its surface. But since all eddy-currents that circle round as those sketched in Fig. 458

are less advantageous in their mechanical effect than currents confined to proper paths; and as they, whether mechanically advantageous or not, consume power and spend it in heating effects, it is better to adopt a still more careful method of construction—namely to build up the iron core of thin disks or rings of soft sheet iron, to insulate them (lightly) from one another, and to insulate them (fully) from contact with the copper bars which constitute the conducting circuit. So we arrive at the form, Fig. 460, of rotor which has been so generally employed for small motors, and even for quite large ones, of the *squirrel-cage* of copper rods imbedded in a laminated and insulated iron core, and provided with a short-circuiting ring of copper (or in some cases of German silver) at each end.

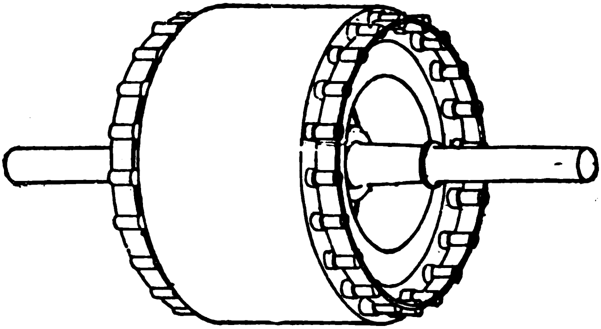


FIG. 460.—SHORT-CIRCUITED ROTOR OR "SQUIRREL-CAGE" ROTOR.

But this simple form was not arrived at without experiment. Brown and other pioneers experimented with many forms of construction, which are detailed in the author's book on *Polyphase Currents.* Experience has decided in favour of rotors built up of laminations, and having the conductors imbedded in slots with closed or nearly-closed tops, in which the windings are placed. These windings may be of three different types:—

(*a*) *Squirrel-cage,* consisting of bars short-circuited together by end-rings;

(*b*) *Three phase windings*, with slip-rings for insertion of starting resistances ;

(*c*) *Multiphase short-circuited windings*, consisting of a number of individual closed loops, each of which spans approximately across a pole-pitch.

All these are separately considered later.

MOTOR CONSTRUCTION.

It will be convenient here to describe a few typical forms of induction motors, and parts of motors. Fig. 461 shows an Oerlikon three-phase motor of 8 horse-power taken apart to show the structure. The stator is a 3-phase 4-pole design, with hemitropic winding, like Fig. 268, but it has 36 half-closed slots, so that there are three slots per pole per phase. The stator housing is a simple cylindrical casting with end-flanges to which are bolted the end-plates that carry the bearings. The rotor is a four-pole wave-winding with 48 bars, each lying in a closed slot, joined together by evolute connectors, and resembling Fig. 277, and is a 4-slot winding.

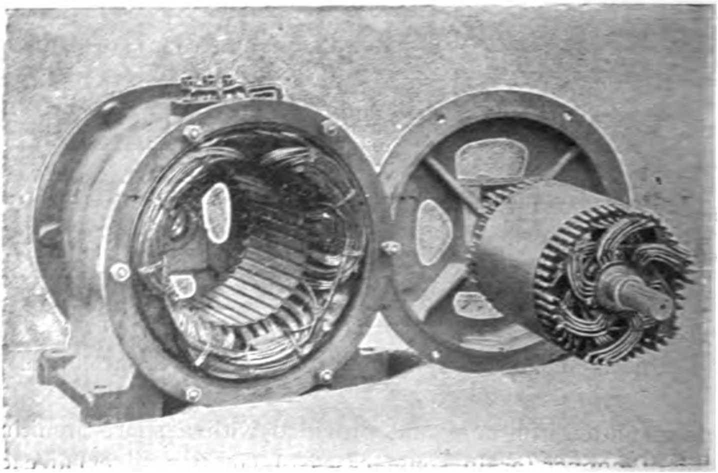


FIG. 461.—FOUR-POLE MOTOR (OERLIKON CO.).

Each of the three circuits is simply closed on itself, so that this rotor acts as a squirrel-cage.

Fig. 462 illustrates a small 5 horse-power motor of the

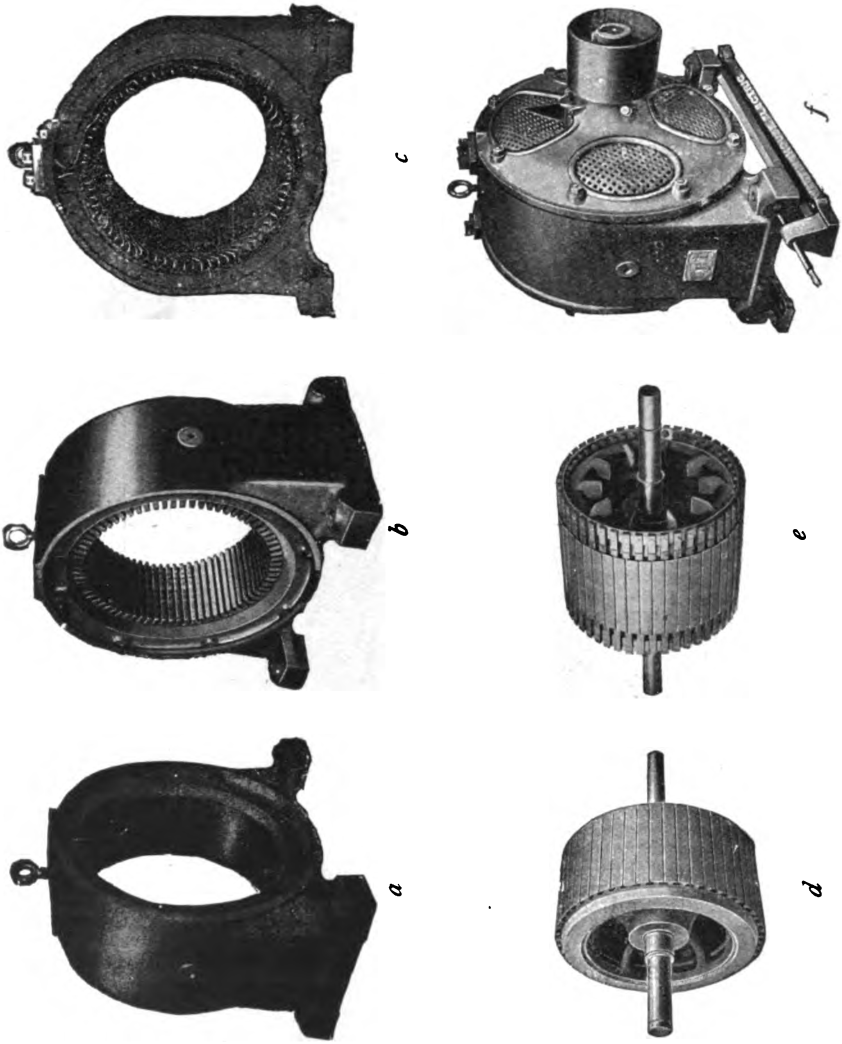


Fig. 462.—Small Induction Motor (British Westinghouse Co.).

British Westinghouse Co. It is designed to run at 1120 RPM on a 60-cycle supply. It is a 6-pole machine, and the synchronous speed would be 1200 RPM, hence there is a 6·6 per cent. slip. It stands about 26 inches high. Fig. 462*a* is the stator housing cast in one piece. Fig. 462*b* shows the housing with the stator core-body inserted, with open slots to receive the windings, which are shown in their place in Fig. 462*c*. Fig. 462*d* shows the rotor core-body assembled on its shaft. Fig. 462*e* is the completed rotor with squirrel-cage construction, and ventilating vanes at the ends. Fig 462*f* depicts the completed motor with its protecting end-shields.

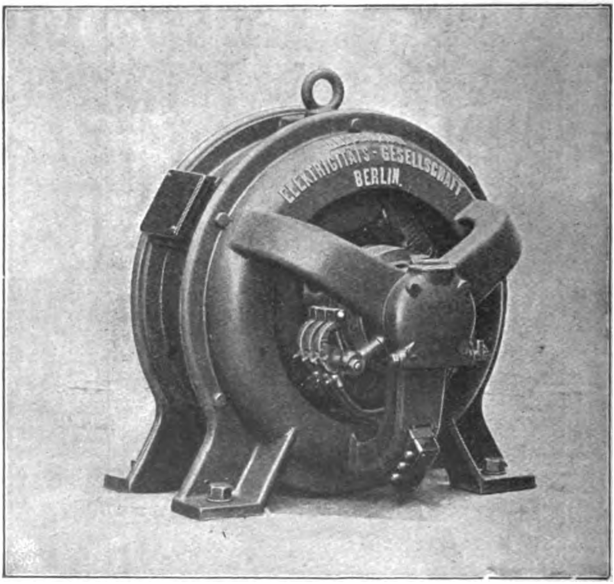
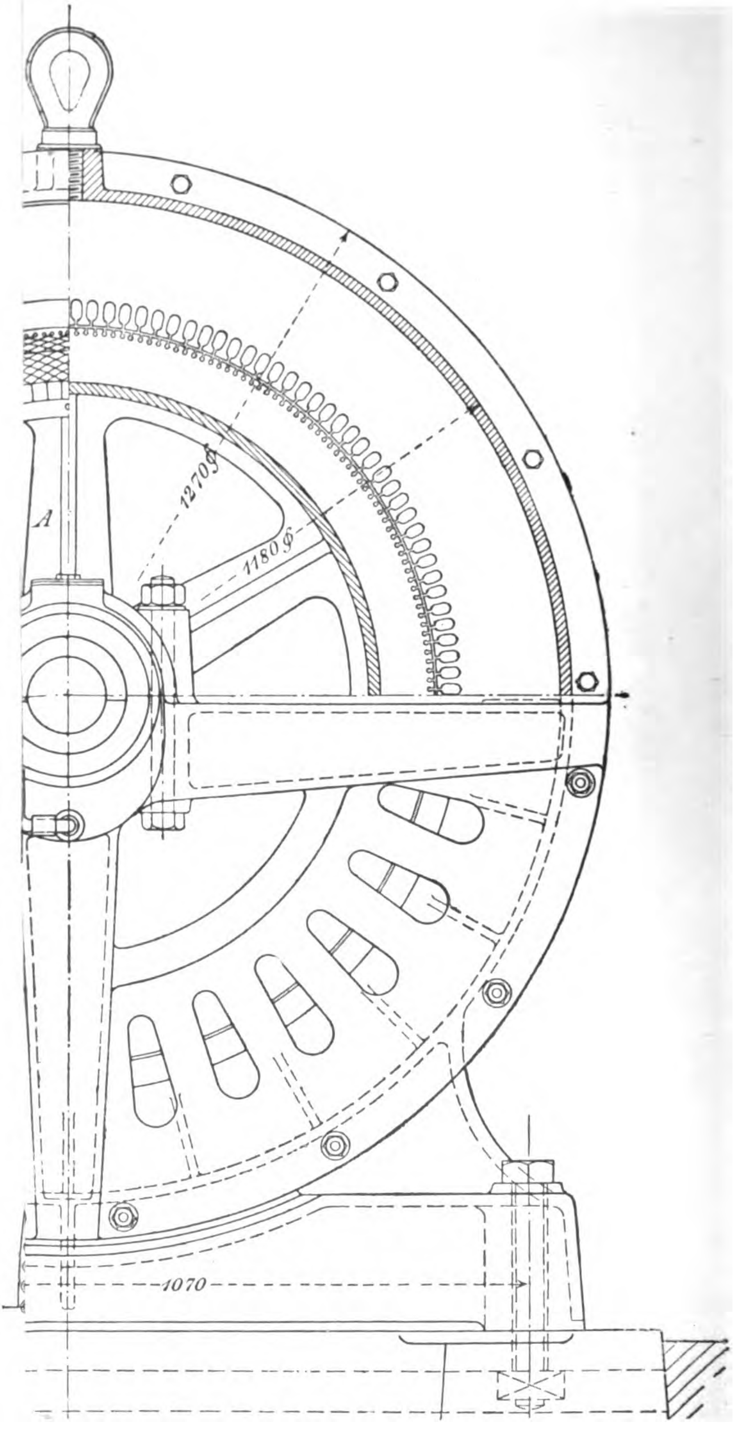


Fig. 463.—5 H.P. Motor of Allgemeine Co.

Fig. 463 shows a construction of the Allgemeine Elektricitäts Gesellschaft for small motors, as carried out for a 5 horse-power motor with wound rotor and slip-rings. The overload capacity is 2½ times the normal load. The tests show that at normal load cos ϕ was about 0·91, and the efficiency 84 per cent.; the slip being about 1½ per cent. Fig. 464 depicts another three-phase motor of the Allgemeine Co., also of the slip-ring type, and of longer proportions axially. A larger

Plate XXI.



motor of the Allgemeine Co., depicted in Plate XXI., is described later.

Stator Housings.—Simple cylindrical housings cast in one piece, with end plates faced to fit, and secured by bolts to flanges on the main carcase, as shown in Fig. 465, are the usual pattern for small machines. The cylindrical body is sometimes provided with apertures for better ventilation of the core; and the core itself, except in small sizes, is constructed with a ventilating duct. To clamp the core-disks, the end-plate must be secured, either by bolts passing, as in Fig. 466, through the periphery of the core-laminations, or else by a steel clamping-ring sprung into a groove in the housing, as in Fig 465. A special metallic clamping with a ring sprung in, is shown in Plate XXI. For larger machines a box-casting of greater stiffness is necessary, and the construction, such as Fig. 467, then resembles that usual in the armature housings

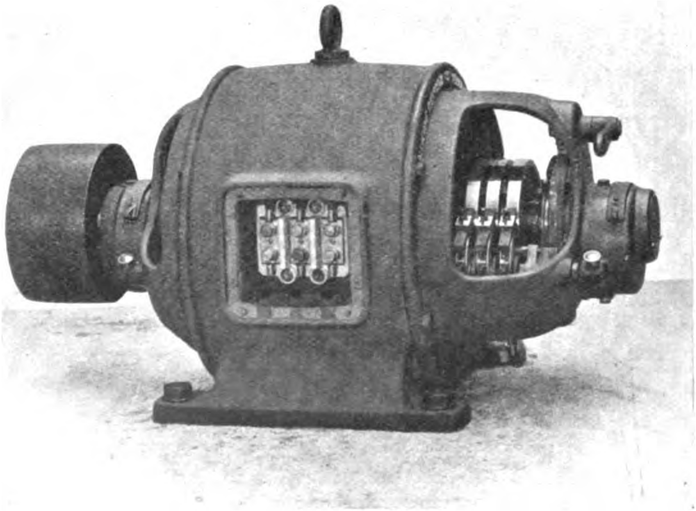


FIG. 464.—THREE-PHASE MOTOR, 30 H.P., 960 RPM, 50 CYCLES PER SECOND, ALLGEMEINE CO.

of large alternators. Plate XXIII. depicts an Oerlikon machine of 570 horse-power, in which the housing is constructed like that of the Frankfurt and the Paderno alternators, p. 184,

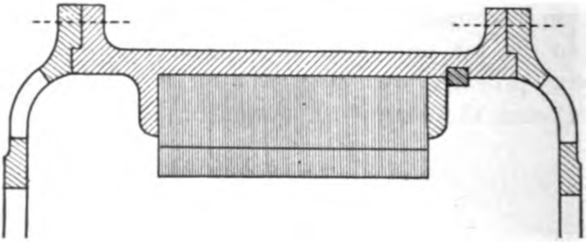


FIG. 465.

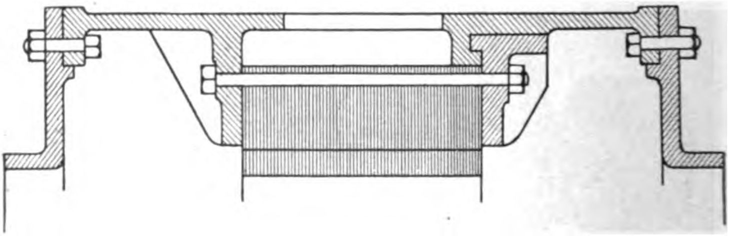


FIG. 466.

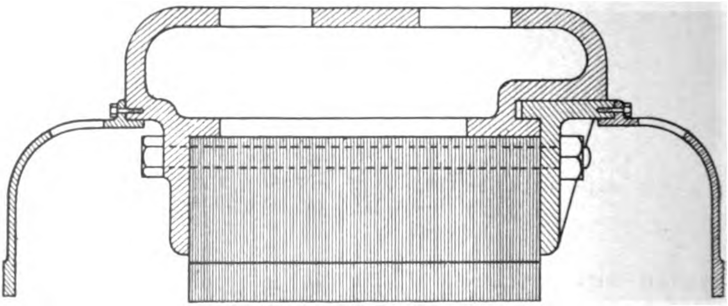


FIG. 467.

with hollow trunnion bearings, so that it can be turned round for inspection or cleaning.

Rotor Construction.—The core-bodies of rotors are always built up of laminated sheet-iron or sheet-steel, and in those of smallest size, the core-disks are mounted directly upon the shaft, as in Fig. 468, with stout end-plates to clamp them together. For squirrel-cage motors these end-plates may be made of cast copper or gun-metal, and serve both as clamping-plates and as end-rings to short-circuit the conductors. For machines of medium size, the core-rings are usually mounted on a hub of cast iron or gun-metal keyed to the shaft, and admitting of ventilation; the laminations being clamped together between end-plates as are those of dynamo armatures. For very large rotors, segmental core-laminations must be used, and these must be assembled on a spider which provides means for holding them in against centrifugal forces. In fact, their construction is practically identical with that of alternator armatures of the A-type.

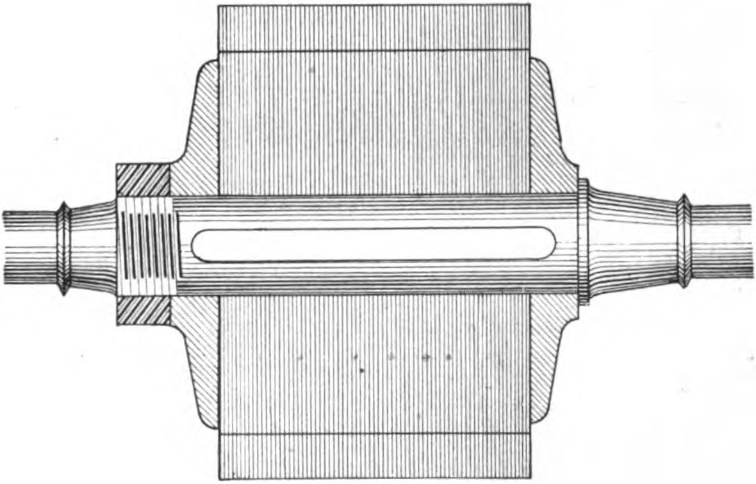


FIG. 468.—SIMPLE ROTOR BODY.

In small squirrel-cage rotors, the conductors are usually cylindrical, carried in circular slots slit at the face. As the resistance need not be very low, and high current densities are admissible, and no great insulation is needed, the rotor slots generally appear to be very small, relatively to the stator slots. The end-rings are by some constructors made of massive cast copper; others purposely make them as a thin hoop in order to increase the resistance, or even make the hoop of German silver. Messrs. Kolben use a form of short-circuited rotor, in which each conductor is at its ends joined down by a radial copper strip, or riser, to a small solid metal cylinder mounted on the shaft, there being one such block at each end.

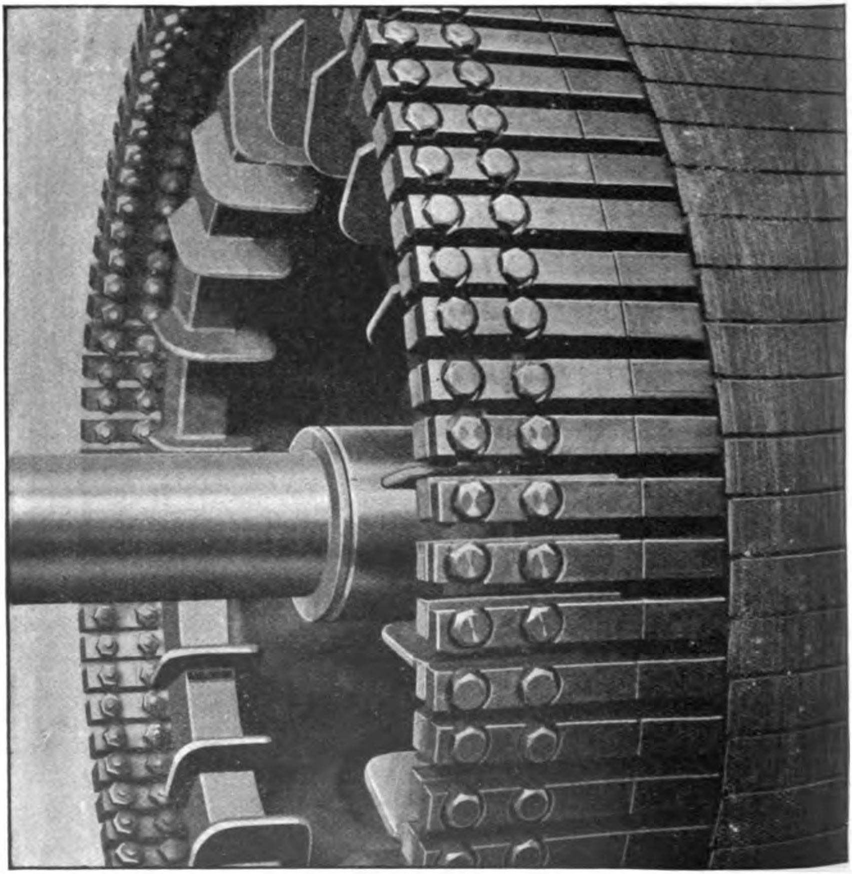
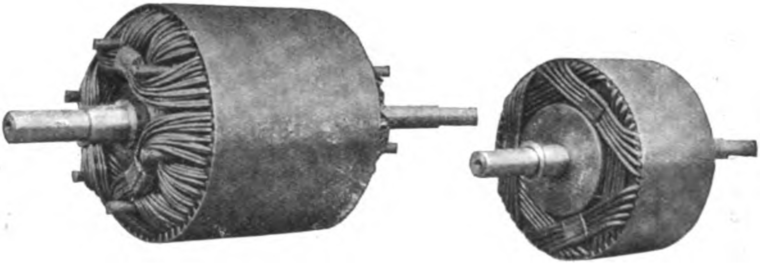


FIG. 469.—CONSTRUCTION OF SQUIRREL-CAGE ROTOR OF A 50 H.P. MOTOR (BRITISH WESTINGHOUSE CO.).

Fig. 469 shows the construction with rotor bars of square section adopted by the Westinghouse Company.

Care must be taken in squirrel-cage rotors, that the joints are well made, otherwise the distribution of current will be irregular ; for, as all the conductors are of low resistance, the distribution will be largely determined by the differences of resistance at the joints.

Short-circuit Wound Rotors.—Figs. 470*a* and 470*b* show two small rotors of the Oerlikon Co. having a winding that is short-circuited, but which is not exactly the equivalent of a squirrel cage. Each of them is for a 4-pole machine, and so far as slots are concerned each is a three-phase pattern with 5 slots per pole per phase. But the larger one has more wires per slot, and is provided with pegs to hold down the bends. These windings need only the slightest insulation—a coat of lacquer suffices ; and each coil when wound is closed on itself. It is claimed for this form that, having so very few soldered joints, there is less liability to breakdown. On the other hand, as the currents at any instant must be equal in a whole belt of wires, the distribution is not quite so favourable as in a true squirrel-cage, where in each slot the current can proportion itself to the field, and reach a nearer approach to a sine-distribution. Also with this kind of wound rotor, the total number of slots (or even of slots per pole) cannot be made incommensurable with those of the stator ; so that it is less advantageous for starting.



FIGS. 470*a* and 470*b*.—SHORT-CIRCUITED ROTORS, WITH WIRE WINDING (OERLIKON CO.).

A form of short-circuit rotor which is not subject to this criticism, and which has some advantages over the ordinary squirrel-cage, is that depicted in Figs. 319 and 320, p. 343, in which a number of self-closed loops, each embracing one pole-pitch, are placed two deep in a prime number of slots.

Wound Rotors with Slip-Rings.—So far as windings are concerned these are discussed in the chapter on windings on pages 236 to 343. For wound rotors, especially those wound to work at rotor voltages approaching 500 volts, the slots are relatively larger; and in modern forms they are nearly always designed of a shape to admit of the conductors being arranged for a barrel-winding, that is, with an upper and a lower layer in each slot. This is not however necessarily so, and some good makers still employ as a rotor winding a regular 3-slot, 4-slot, or 5-slot winding (such as Figs. 275, 276, 305, or 306) with end-bends grouped in two ranges. To secure end-bends from moving under centrifugal forces, they are often wound

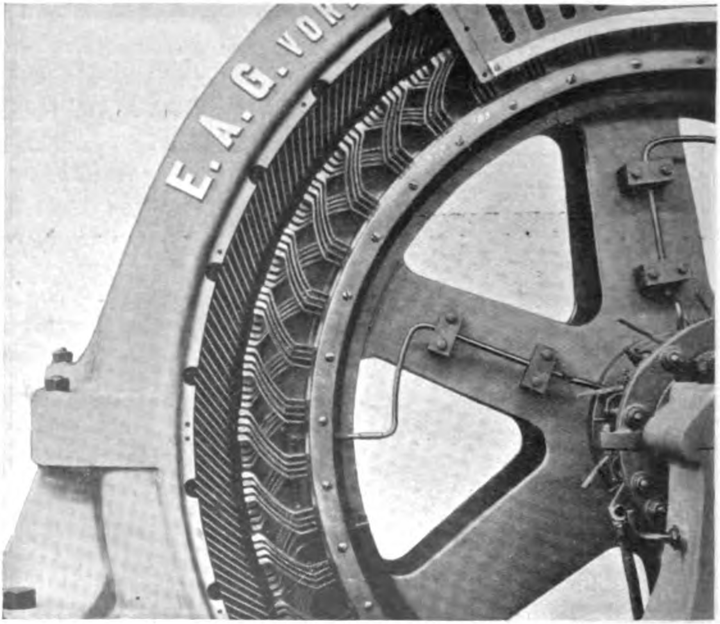


FIG. 471.—PORTION OF 3-PHASE INDUCTION MOTOR, 200 H.P. AT 200 RPM (LAHMEYER CO.).

under pins that project from the rotor cheeks, as is shown in Fig. 470, p. 669, and Fig. 499, p. 742.

Fig. 471 depicts a 200 H.P. 3-phase motor by the Lahmeyer Co., of Frankfurt, running at 200 RPM on a 50-cycle supply, and having therefore 30 poles. A portion of the end shield has been removed to display the windings. The stator has a wave-wound, 3-slot winding resembling Fig. 290, p. 330, but with two layers of conductors. The rotor has a 4-slot winding in a single layer, on the plan of Fig. 305, with evolute fork connexions. Fig. 471 shows very clearly two of the three connecting wires that join the winding down to the slip-rings. Another good example of the connexion of the rotor windings to the slip-rings is afforded by Fig. 265, p. 318, the rotor of this machine being a distributed 2-layer winding similar to the stator winding of Fig. 471, but with 10 slots per phase per pole.

ELEMENTARY THEORY.

Slip and Induced Electromotive-force.—The relation of the slip to the driving forces may be stated simply. Suppose a motor to be without load, and devoid even of friction at its bearings, the smallest driving forces will then suffice to keep it at full speed. If it ran actually at full speed, that is in absolute synchronism with the revolving fluxes, there would be no relative motion between the fluxes and the rotor conductors, there would be no cutting of magnetic lines, therefore no induced electromotive-forces, no currents in the rotor, and no driving forces. If a small load, such as friction, be present it will run a little slower, or in other words there will be a small slip; the fluxes will then cut across the conductors and induce electromotive-forces. The effect will be the same as if the flux stood still and the rotor were to revolve backwards at a speed equal to the difference between n_1 and n_2. As the slip is small, the induced currents will be small, and these currents will produce a small driving force—enough in fact to keep the rotor going at nearly full speed on that small load. If the load is further increased, the rotor will run a little slower, and with this increased slip the induced currents and the driving forces will further increase. Assuming, as is true in well-designed motors within the range of their normal load, that the fluxes are of constant value, and that the lag of the rotor currents is also small, then the driving-forces will be proportional to the slip: or in other words as the load is increased the slip automatically increases, so that the rotor gives driving forces proportional to the load. Suppose a motor to be designed with 6 poles, for a circuit where the frequency is 40 cycles per second. The synchronous speed will then be 800 revolutions per minute; for there are 2400 cycles per minute, and each revolution corresponds to 3 cycles. Now, if friction necessitated a slip of 0·5 per cent., the highest speed on no-load would be 796 RPM, and if to obtain full-load torque a 3 per cent. slip was needed, the full-load speed would drop to 776 RPM. The mechanical characteristic of the motor would be, as in Fig. 472, a straight line which slopes gently away from the ideal or synchronous speed.

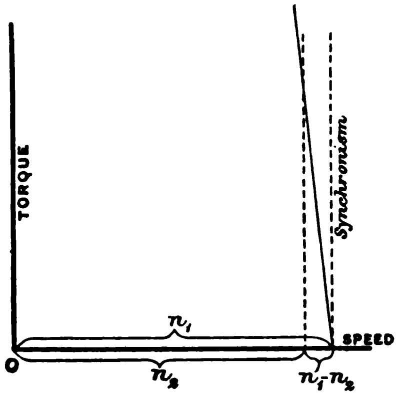


FIG. 472.

Slip and Frequency.—The frequency in the stator being f_1 cycles per second, that in the rotor will be the same when the motor is standing still. But when running it will be reduced in the proportion of the slip; so that—

$$f_2 = f_1 \times \frac{n_1 - n_2}{n_1} = f_1 \times s \quad . \quad . \quad [II]$$

At top speed, when the rotor is revolving almost as fast as the field, the magnetism will shift round in the rotor from point to point quite slowly. In the example cited above the rotor frequency at full-load would be only 1·2~; and at no-load only 0·2 ~.

On account of the extremely low frequency in the rotor when running, the iron-losses in its core and teeth are practically negligible.

Slip, Torque, and Rotor Efficiency.—Let Ω stand for the angular speed of the rotatory magnetic flux, and ω for the angular speed of the rotor. Then $\Omega - \omega$ is the difference of the angular speeds, and

$$s = \frac{\Omega - \omega}{\Omega} = \frac{n_1 - n_2}{n_1}.$$

Let W be the watts imparted by the revolving field to the rotor, and *w* the power actually used in turning the rotor. Then W − *w* = the power lost in the rotor in heating its conductors.

Now W is proportional to Ω and to the torque between stator and rotor, and *w* is proportional to ω and to the same torque; whence we have as the ratio of output to input in the rotor

$$\frac{w}{W} = \frac{\omega}{\Omega} \quad . \quad . \quad [III]$$

From this we see that the efficiency *of the rotor* is the same as the ratio of the speeds. If there is a 3 per cent. slip, *i. e.* if the rotor speed is 97 per cent. of the speed of the rotating flux, then the rotor part will have a 97 per cent. efficiency. The motor will of course have other losses in the stator copper and in the iron.

Further, since the rotor frequency f_2 is low when the motor is running on load, the rotor current will be very nearly in phase with the field; and as the field is nearly constant, the torque will be proportional to the currents in the rotor. But these are (as will be evident from induction principles) proportional to the angular slip $\Omega - \omega$. Hence also it follows that T will be proportional to $\Omega - \omega$, and may be written:

$$T = b(\Omega - \omega), \quad . \quad . \quad [IV]$$

where b is a constant depending on the construction of the motor, strength of field, resistances, and the like.

Then we have:

$$\text{Useful watts, } w = T\omega = b\,\omega\,(\Omega - \omega).$$
$$\text{Watts given to rotor } W = T\Omega = b\,\Omega\,(\Omega - \omega).$$
$$\text{Watts wasted in rotor } W - w = \quad b\,(\Omega - \omega)^2.$$

Hence we may at once apply the now well-known diagram of motor efficiencies by drawing a square A B C D (Fig. 473), having its side A B numerically equal to Ω, and cutting off a piece B F equal to ω. The area A F H D represents the total watts supplied, the area A F G K, or G L C H, the watts utilised, and the square K G H D the watts wasted in heating the conductors of the rotor. The efficiency will approach unity as F moves up towards A; and, as with continuous-current motors, if it were not for the weakening of the field by armature reaction, the output would be a maximum when $\omega = \frac{1}{2}\Omega$, the efficiency being then only 50 per cent. When, however, the motor is running at much below its proper speed, magnetic leakage and other causes play such an important part that the equation above no longer holds. Fig. 473 is, however, applicable to cases of normal running, and illustrates the likeness between these motors and continuous-current motors.

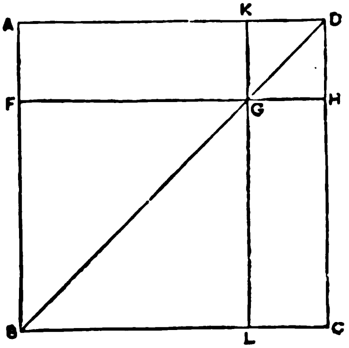


FIG. 473.

Induction Principles.—When we regard the induction motor as a transformer, we shall see that as it consists of a set of primary circuits, which are supplied from the mains, and of a set of secondary circuits, both of which sets surround magnetic paths that are common to both, it follows that, with certain reservations, we may apply the principles of ordinary transformers. The magnetic flux which is common to the two windings is an alternating flux, the frequency of which depends on the supply. It will (see p. 535) set up electromotive-forces in both windings; a back electromotive-force in the primary or stator winding; and in the secondary, or rotor winding, another electromotive-force, namely that which drives the rotor current. Let Z_1 be the number of stator conductors in series in any one phase of the stator, and Z_2 be the number of conductors in series in any one phase of the rotor (supposed to be a wound rotor, or if a squirrel-cage we must take $Z_2 = 2$, that is as acting as one turn only) and let the flux (per pole) that is common to stator and rotor be denoted as **N**. Then we shall have for the two electromotive-forces induced in the primary and secondary the following expressions:—

$$\left.\begin{aligned} E_1 &= k \times f_1 \times Z_1 \times \mathbf{N} \div 10^8 \\ E_2 &= k \times f_2 \times Z_2 \times \mathbf{N} \div 10^8 \end{aligned}\right\} \quad . \quad [V]$$

Here k is the Kapp coefficient (see p. 196) and is of value about 2·14 for a three-slot winding, or about 2·12 for a four-slot or five-slot winding. We may compare these expressions with those given on p. 535 for stationary transformers. Assuming that k is practically the same for both windings, we have:

$$\frac{E_2}{E_1} = \frac{f_2 \times Z_2}{f_1 \times Z_1}; \quad . \quad . \quad . \quad [VI]$$

or, inserting the value of f_2 from [II], we get

$$\frac{E_2}{E_1} = \frac{n_1 - n_2}{n_1} \cdot \frac{Z_2}{Z_1} = s \times \frac{Z_2}{Z_1} \quad . \quad . \quad [VII]$$

Now the slip s is equal to 1 at standstill when the motor is starting, and diminishes to, say, 0·03 at full-load, and to about 0·005 at no-load. The ratio $Z_2 \div Z_1$ is the transformation-ratio (compare p. 536) when the motor is at rest.

Hence we see that the motor, so far as it acts as a transformer, acts as though its secondary had a variable number of windings, diminishing as the speed goes up, because of the slip going down. The primary induced electromotive-force E_1, being, when the motor is running, a back electromotive-force, it is necessarily a little smaller than the voltage of supply in any one phase.

MAGNETIC LEAKAGE.

In the preceding, N was taken as the common flux per pole. But, as in every transformer, so here, there will be some magnetic leakages. The actual flux through a pole of the stator will be slightly greater, perhaps by some 1½ per cent., than that which finds its way across the gap into the rotor, and the rotor currents also set up other magnetic leakages of about the same amount. *The question of magnetic leakage is of immense importance in induction motors,* and demands most careful study. The rotor ampere-turns produce a back magnetomotive-force which (1) tends to diminish the flux coming from the stator, and (2) sets up a stray flux around the rotor conductors. This may be illustrated by consider-

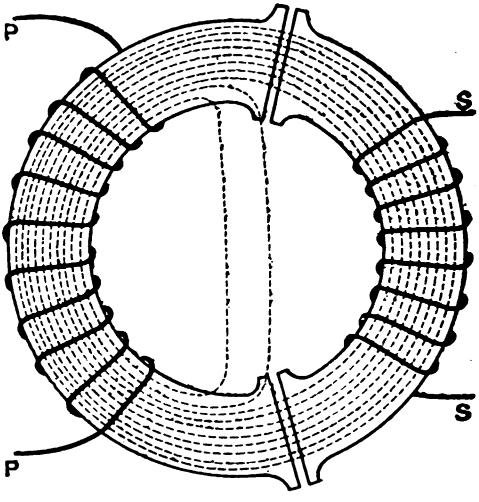


FIG. 474.

ing a simple hypothetical case. Let an iron ring divided by 2 air gaps be surrounded by 2 circuits, a primary PP and a secondary SS. Suppose at first, that, as in Fig. 474, there is current only in the primary, creating a certain flux represented by 10 lines, drawn dotted in the diagram, of which in consequence of dispersion only 8 succeed in traversing the gaps and linking themselves with the secondary winding, while 2 form a stray flux. Now if both primary and secondary currents are present simultaneously the effect will be represented by Fig. 475. The two magnetomotive-forces oppose one another and the only effect is the production of a greater amount of leakage, for when current is allowed to flow through the secondary, because of the constant voltage applied to the primary which must always be choked back, more current will flow in to keep up the primary flux undiminished. The total stray field is now 4. The proportions are absurd; but if we had taken Fig. 474 as having 1000 lines of which 2 went astray and 998 entered the rotor, and Fig. 475 with a total leakage of 4 lines, we should have been able to obtain a diagram more nearly representing the actual case. It will be noted that the inevitable presence of a gap between stator and

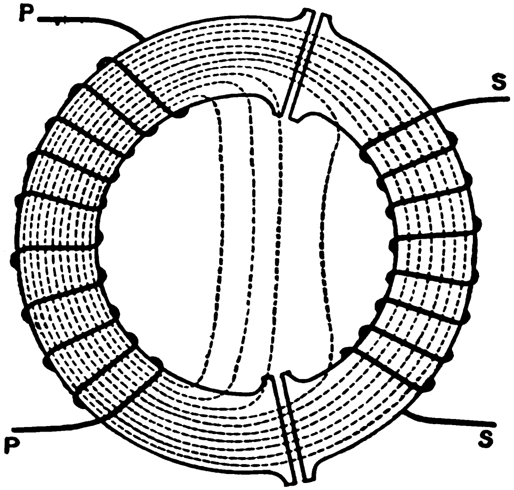


FIG. 475.

rotor tends to promote dispersion, and that if the teeth have projecting tops these will also promote dispersion. A further source of magnetic dispersion in the motor is of course the projecting end-bends of the coils, because these set up local stray fields around themselves. Fig. 476 is an attempt to complete the picture by adding in each winding projecting coils which set up self-closed fluxes. If both circuits carried alternating currents in *exact* opposition of phase, the stray field, with the exception of that at S would all be accounted a primary leakage, since it does not interlink itself with the secondary.

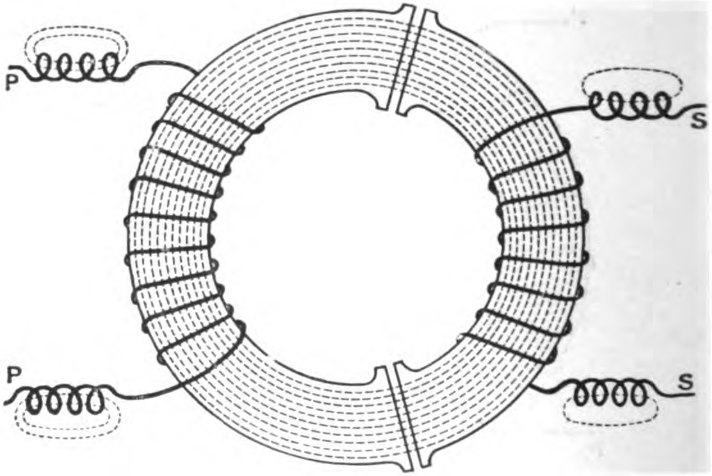


FIG. 476.

If the relative phases were in any other stage than in exact opposition, there would be some stages where some of the stray flux at the gaps would interlink itself with the secondary.

There are three chief sources of magnetic dispersion in induction motors; they are known as: (1) zigzag dispersion, (2) peripheral dispersion, (3) flank dispersion. The zigzag dispersion is illustrated by Figs. 477 and 478. Since stator and rotor must have a different number of teeth per pole, it follows that if at some part of the periphery the teeth are nearly opposite, as in Fig. 477, at some other part they will alternate in position, as in Fig. 478. And, therefore, of neces-

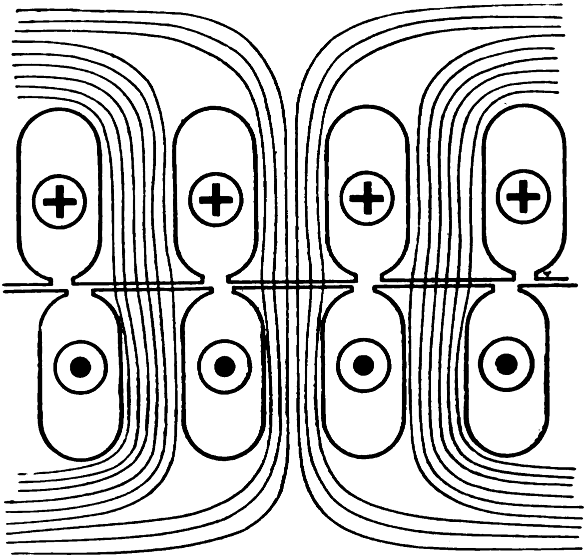


FIG. 477.

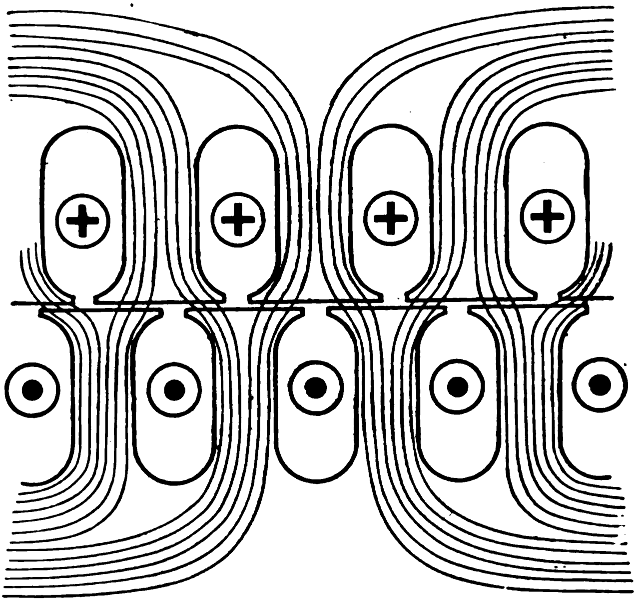


FIG. 478.

sity there must always be, as in the latter case, some of the lines which are not interlinked with the rotor conductors (those represented in the middle slot of the rotor), and such lines are accounted for the moment as stray lines. The peripheral dispersion is that which finds its way along the gap, or along the tops of the teeth from pole to pole, as indicated in Fig. 479, where the thick black line represents diagrammatically the main flux, and the thin line the additional stray flux along the periphery, which interlinks itself with the primary conductors only. The flank dispersion is that which occurs laterally

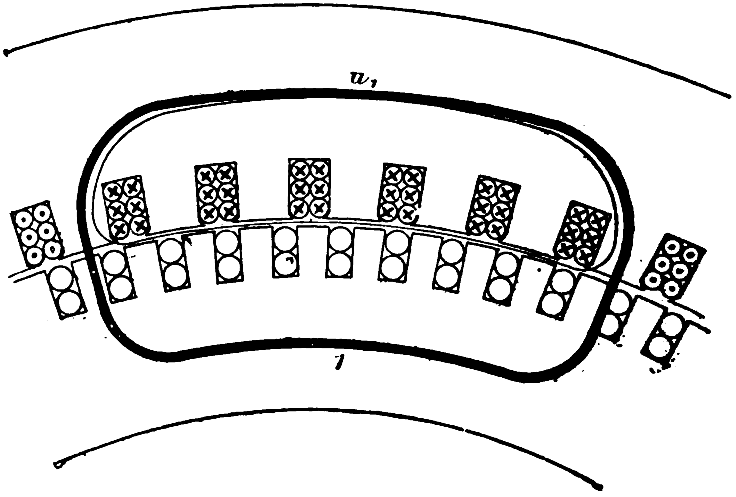


FIG. 479.

at the flanks of the motor, owing to the stray field issuing from the polar parts of both stator and rotor, Fig. 480, and to the self-closed linkages round the end-bends of the windings. Behn-Eschenburg, to whom we owe a most exhaustive investigation[1] of the dispersion in inductive motors, has assigned, from an elaborate comparison of many motors built by the Oerlikon Co., a formula to take into account these three sources of magnetic leakage, which, for present purposes, may be written as follows:—

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[1] *Journ. Inst. Elec. Eng.*, xxxiii. 239, 1904.

$$\sigma = \frac{3}{X^2} + \frac{10\,\delta\,\epsilon}{X\,Y\,\tau} + \frac{5\,\delta}{l_i}, \quad . \quad . \text{[VIII]}$$

where σ is the dispersion-coefficient,[1] X the mean number of slots per pole in stator and rotor taken together, Y the width of the slits in the slots, δ the width of the motor's air-gap, ϵ the edge-thickness of the overhanging tooth-tops, τ the pole-pitch, and l_i the nett iron length of the core. The first term relates to the zigzag dispersion, the second to the peripheral dispersion, the third to the flank dispersion. In the case of wholly-closed slots with thin iron bridges at the face, as the bridges become fairly saturated it suffices to take the second term at a value about four times greater than if air-slits were substituted. For the definition of σ see pp. 241 and 242.

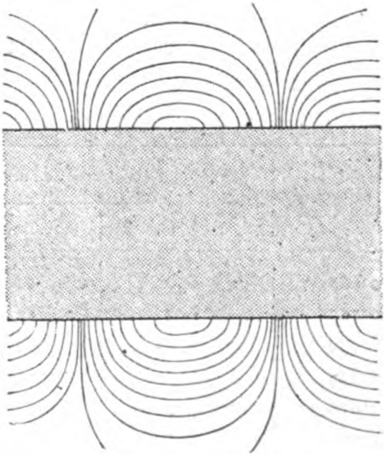

FIG. 480.—MAGNETIC LEAKAGE AT FLANKS OF POLES.

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[1] This dispersion-coefficient depends partly on the leakages in the stator and partly on those in the rotor. If M be the coefficient of mutual induction between primary and secondary, and L_1 and L_2 be their respective coefficients of self-induction, the definition of σ is given as

$$\sigma = \frac{L_1 L_2 - M^2}{L_1 L_2}.$$

As these induction coefficients are not readily ascertained from the design, it is more practical to work with the empirical formulæ obtained by Behn-Eschenburg from actual practice.

An elaborate discussion of the predetermination of the dispersion and of the magnetizing current, by Dr. G. Benischke, will be found in the *Elektrotechnische Zeitschrift*, xxv. 834, Sept. 22, 1904.

A rougher rule for finding a value for the dispersion coefficient σ has been suggested by Behrend[1] in the form :—

$$\sigma = c \times \frac{\delta}{\tau}, \quad . \quad . \quad . \quad [IX]$$

where c is a number varying from about 6 to 16. According to Hobart[2] the values of c depend on the ratio of the pole-pitch to the effective length of the core, being smaller for long core-bodies, and also smaller for open slots than for closed. He gives the following table :—

TABLE XVII.—HOBART'S VALUES OF CONSTANT.

Ratio of Pole-Pitch to Effective Length of Core.	Values of c in Behrend's Formula.	
	Open Slots.	Quite Closed Slots.
0·667	6·0	12·5
0·715	6·2	12·6
0·770	6·4	12·7
0·834	6·7	12·9
0·91	7·1	13·1
1·0	7·5	13·4
1·11	8·2	13·8
1·25	8·8	14·3
1·43	9·5	14·8
1·67	10·3	15·4
2·0	11·3	16·1
2·5	12·2	16·8
3·33	13·8	17·7

Experimentally, the magnetic leakages may be explored in two ways. Assuming the rotor to be of the wound variety, let a voltmeter be connected across two of the rotor slip-rings while the normal voltage is applied to the stator windings.

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[1] The Induction-Motor, by B. A. Behrend, 1902, p. 36.

[2] *Elektrotechnische Zeitschrift*, xxiv. 934, Nov. 12, 1903.

The latter voltage is proportional to the total primary flux (which we may call N_1) and to Z_1, while the former is proportional to that part of the primary flux which interlinks with the rotor windings, viz. N, and to Z_2. Hence the ratio

$$\frac{N}{N_1} = \frac{V_1}{V_2} \cdot \frac{Z_2}{Z_1} = u_1, \quad . \quad . \quad [X]$$

will be a little greater than unity. Following Hopkinson's way of treating leakage coefficients, we may write this as u_1. It will have a value somewhere about 1·023 in ordinary motors. Similarly we may connect a voltmeter to the primary terminals and apply a voltage to the rotor, and so obtain a measure of the secondary leakage, which will be expressed by a similar coefficient u_2. During such tests the rotor should be turned by hand to different positions, and the means of the readings taken. These leakage factors, u_1 and u_2, will be very nearly equal; they are practically constant over the working range of the motor. The dispersion coefficient of the motor as a whole will be obtained by multiplying them together and subtracting unity; or

$$\sigma = u_1 u_2 - 1 \quad . \quad . \quad . \quad . \quad [XI]$$

A simpler approximate way of finding σ is to measure with an amperemeter in the primary circuit the no-load current C_0, and also the short-circuit stator current C_{sk} when the rotor is held fixed. As C_{sk} is usually very large, and might damage the motor, it is better to apply a reduced voltage to the stator during this test, and multiply the observed short-circuit current by the ratio of the voltages, to obtain the value that would have been observed had full primary voltage been applied; the error due to difference of magnetic saturation in the two cases is not serious. Then, subject to small corrections for the ohmic resistance and the no-load iron-losses, the value of σ is given by the rule:—

$$\sigma = \frac{C_0}{C_{sk}} \quad . \quad . \quad . \quad . \quad . \quad [XII]$$

In well-designed modern motors σ is always small, seldom exceeding 0·04, except in small or old machines, more usually 0·035, and may even be reduced below 0·025 in large motors. For many purposes it is convenient to consider the reciprocal of σ, which we here term the *utilization constant*, and denote by U. As σ denotes the fraction of the flux which is dispersed, so U denotes the ratio in which the flux utilized is greater than that which strays. So we have :—

$$U = \frac{C_{sh}}{C_0} = \frac{1}{\sigma} = \frac{1}{u_1 u_2 - 1} \quad . \quad . \quad . \quad [XIII]$$

Now if a motor is well designed, C_0, the no-load current, will be small, because the gap is narrow, and the other reluctances are low; and C_{sh} ought to be very great, because the lower the magnetic leakage, the smaller will all self-inductive actions be, and the greater will be the rush of current in the short-circuit test. Hence it follows that U ought to be large. It is seldom lower than 25, except in small or old machines, more usually 28 to 30, and may exceed 40 in large motors of modern design.

For further considerations on the no-load and short-circuit currents, see J. L. La Cour, *Leerlauf und Kurzschluss-Versuch*, p. 55 (Braunschweig, 1904).

MAGNETIZING CURRENT.

If the flux-density in all the iron-parts is low, then the only reluctance in the magnetic paths that need be taken into account is the air-gap. If δ be the width of the gap between stator face and rotor face, and B_{max} the value of B in the gap at the middle of the pole-face, then by ordinary rules (see Vol. I., p. 92) the number of ampere-turns per pole needful to produce a flux of this density will be :—

$$\text{magnetizing AT per pole} = B_{max} \times \delta \times 0{\cdot}3133.$$

But the average value of the flux-density over the whole pole-pitch will be equal to $B_{max} \times \frac{2}{\pi}$; so that the expression becomes :

$$= B_{av} \times \delta \times 0{\cdot}492.$$

If the saturation in the teeth exceeds 60,000 lines per square inch, it will be needful to make some allowance for their contribution to the reluctance of the path. The simplest way to do this is to calculate what addition to the gap-width would give an equivalent reluctance (usually 10 to 15 per cent. is abundant), and this may be inserted in the formula by taking the effective δ as equal to $\delta\ (1 + i)$ where i is this small correction for the iron parts.

Now it was shown above, on p. 657, that the conjoint effect of the 3-phases in producing magnetomotive-forces was such that it equalled 2·7 times the virtual value of the ampere-turns due to any one phase. Now if C_m is the magnetizing current, seeing that the number of turns per pole in any one phase of the stator is $\frac{1}{2}\ Z_1 \div p$, it follows that the total effect of all three phases in producing magnetizing forces will be :—

$$\text{magnetizing AT per pole} = 1 \cdot 35 \times C_m \times Z_1 \div p.$$

Equating this to the former expression, we get

$$1 \cdot 35 \times C_m \times Z_1 \div p = B_{av} \times \delta\ (1 + i) \times 0 \cdot 492,$$

whence

$$C_m = 0 \cdot 364 \times B_{av} \times \delta(1 + i) \times p \div Z_1 \quad . \text{[XIV]}$$

or

$$C_m = 0 \cdot 23 \times B_{max} \times \delta\ (1 + i) \times p \div Z_1 \quad \text{[XIV}a\text{]}$$

which is the formula used for calculating magnetizing currents.

NO-LOAD CURRENT.

The actual no-load current is a little greater than the true magnetizing current, because (as also in transformers, p. 544) there is a small component needed in phase with the voltage, to provide the energy wasted in the stator iron-losses, and for friction at the bearings. If the no-load loss be called w_i, then this small energy component C_h will be equal to $w_i \div 3\ E_1$. If the vertical line OE, Fig. 481, be taken as the relative phase of the voltage, then the true magnetizing current

C_m will lag 90° behind it, and the small energy-component will be in phase with OE, and the actual no-load current will be compounded of these. It w consequently lag somewhat less th 90°, and its magnitude will be

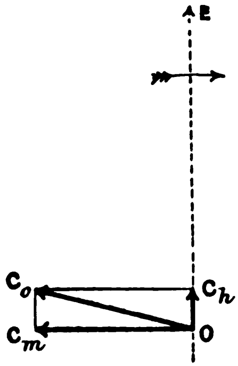

FIG. 481.

$$C_o = \sqrt{C_m^2 + C_h^2}. \quad . \qquad . \quad [XV]$$

LAG OF ROTOR CURRENT.

The lag of the rotor current behind the electromotive-force induced in the rotor windings will depend on the impedance of these circuits, that is upon their resistance and the reactance of the rotor leakage-field. If we write R_2 for the resistance of any one phase of the rotor circuit, and L_2 for its coefficient of self-induction, then the reactance will be $2\pi f_2 L_2$, and the impedance will be $\sqrt{R_2^2 + 4\pi^2 f_2^2 L_2^2}$.

Then the rotor current will be

$$C_2 = \frac{E_2}{\sqrt{R_2^2 + 4\pi^2 f_2^2 L_2^2}};$$

and the lag ϕ_2 of the rotor current will be such that

$$\tan \phi_2 = \frac{2\pi f_2 L_2}{R_2} = \frac{2\pi f_1 s L_2}{R_2}$$

and

$$\sin \phi_2 = \frac{2\pi f_2 L_2}{\sqrt{R_2^2 + 4\pi^2 f_2^2 L_2^2}}.$$

It will be noted that as the speed goes up, the slip steadily diminishes, and f_2 goes down with it; so that at all speeds on load from no-load to full-load, the reactance term will be small relating to R_2, and the lag angle ϕ_2 will not be large. On the other hand, at standstill, as when starting, f_2 is the full frequency f_1, and the reactance term becomes enormous, so that R_2 is relatively negligible and the lag angle ϕ_2 will swing round to practically 90°.

Now $E_2 = E_1 \times \frac{Z_2}{Z_1} \times \frac{f_2}{f_1}$, as shown on p. 675.

Hence the standstill value of the rotor current when $f_2 = f_1$ may be written :—

$$C'_2 = \frac{E_2}{2\,\pi f_1\, L_2} = \frac{E_1}{2\,\pi f_1\, L_2} \times \frac{Z_2}{Z_1}. \qquad . \text{ [XVI]}$$

When running at any load, the rotor current will have the value

$$C_2 = \frac{E_1}{\sqrt{R_2^2 + 4\,\pi^2 f_2^2\, L_2}} \times \frac{Z_2}{Z_1} \times \frac{f_2}{f_1}.$$

Multiplying numerator and denominator by $2\,\pi\, L_2$, we get

$$C_2 = \frac{E_1}{\sqrt{R_2^2 + 4\,\pi^2 f_2^2\, L_2}} \times \frac{Z_2}{Z_1} \times \frac{2\,\pi f_2\, L_2}{2\,\pi f_1\, L_2}.$$

Rearranging

$$C_2 = \frac{E_1}{2\,\pi f_1\, L_2} \times \frac{Z_2}{Z_1} \times \frac{2\,\pi f_2\, L_2}{\sqrt{R_2^2 + 4\,\pi^2 f_2^2\, L_2}}$$

and this is

$$C'_2 = C'_2 \text{ (at standstill)} \times \sin \phi_2. \qquad . \qquad \text{[XVII]}$$

From this we may deduce a most important consequence. In Fig. 482 let the line O E_2 represent the relative phase of the voltage induced in any phase of the rotor. Then we know that the rotor current C_2, represented by the vector O Q, will lag behind it by an angle ϕ_2. At top speeds, when slip is small and the load light, O E_2 will be small, C_2 will be small, and ϕ_2 will be small. But if the load is increased, the slip will be greater, O E_2 will be greater, C_2 will be greater, *and it will swing round further to the right*, because the lag ϕ_2 will be greater. Now if the point Q swings round to the right as O Q increases, it follows that it must move upon some curve ; and it is easy to show that this curve *must be a semicircle.*

Let O Q represent the secondary current lagging at its proper angle ϕ_2. Draw Q S at right angles to O Q until it meets at S, the horizontal line drawn through O. On O S draw the semicircle through C_2. It is clear that the angle O S Q will be equal to ϕ_2, and sin ϕ_2 will equal O Q ÷ O S; whence O Q = O S × sin ϕ_2. Now O S represents the standstill current in the rotor, that is to say the current which would occur in the rotor if, when standing still, the motor were suddenly switched on to the mains at full voltage. The standstill current will, owing to the high slip, lag at 90° and be limited only by the self-induction.

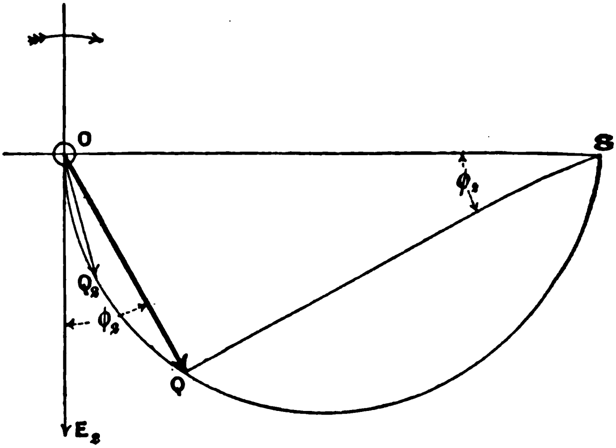

FIG. 482.—LAG OF ROTOR CURRENT.

STATOR AND ROTOR CURRENTS.

In accordance with transformer principles, the actual magnetization of the iron parts that act as core, depends upon the resultant ampere-turns which surround it. The secondary ampere-turns are nearly equal, and nearly opposite in phase, to the primary ampere-turns. They would be precisely equal and opposite at all stages were it not for the magnetizing component of the primary, which, being itself a current lagging

90°, causes the primary to lag behind the primary electromotive-force by an angle ϕ_1 which is rather greater than the angle ϕ_2 by which the secondary current lags behind E_2.

In Fig. 483 O Q is a vector representing the rotor ampere-turns, O P represents the stator ampere-turns. They are at such an angle apart that their resultant O M, in the parallelogram O P M Q, represents the magnetizing ampere-turns. The same lines which represent the ampere-turns may serve to represent currents. O P will be the stator current, O M the magnetizing current; and O Q will represent the rotor current reduced in the proportion of the relative numbers of windings, that is will equal $C_2 \times Z_2 \div Z_1$. The primary current O P may then be regarded as made up of a vector M P equal and opposite to O Q, compounded with O M.

Now in the previous section we have seen that the point Q will move round, as the load alters, upon a semicircle that passes through O. As P M is equal to O Q, it follows that P *will also move round on a semicircle* drawn from M, above the horizontal line.

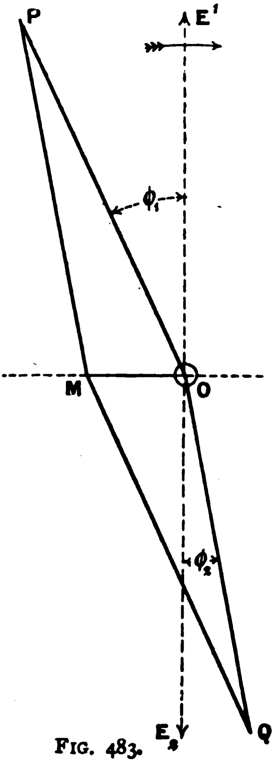

Fig. 483.

THE HEYLAND SEMICIRCLE DIAGRAM.

Amongst those who have investigated the theory of the induction motor, Blondel,[1] Kapp,[2] Behrend,[3] and Heyland,[4]

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[1] *Éclairage Électrique*, May 1895; *Industrie Électrique*, Feb. 25, 1896.

[2] *Elektromechanische Konstruktionen*, 1898, 32.

[3] *Elektrot. Zeitschr.*, Feb. 1896, 63; and *The Induction Motor*, 1902.

[4] *Elektrot. Zeitschr.*, 1894; *Electrician*, xxxvi. (1896), 505, 578, 651, 719 and 753. See also Heyland's pamphlet, 'Eine Methode zu experimentellen Untersuchungen an Induktionsmotoren.' Stuttgart, 1900.

have given graphic constructions; but it is chiefly to Heyland that we owe the elegant diagram that most fully exhibits the properties of the motor in action. The principle of the Heyland diagram will be readily understood.

Let O E as before be the datum line for the induced voltages due to the revolving flux. The magnetizing current is represented by the horizontal line O M. O P is the stator current, and M P is the equivalent rotor current (reduced in the ratio of the primary and secondary windings). Then as just shown, the point P will move around on a semicircle, the base of which, M D, is horizontal. As shown in Fig. 482,

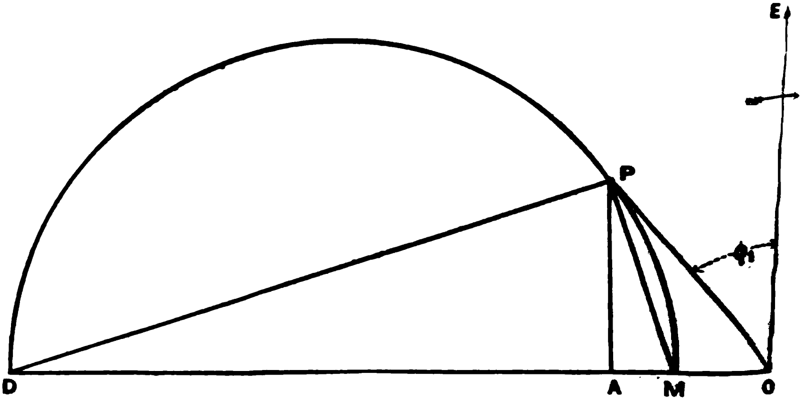

FIG. 484.—THE HEYLAND SEMICIRCLE.

the base of the semicircle (O S in that figure) represents the rotor current at standstill. Hence D M in the present diagram represents the rotor current at standstill (also on the assumption that the scale is changed in the ratio of the primary and secondary windings). As the load alters the point P shifts around the semicircle. At no-load, when P M is zero, the point P moves down indefinitely near to M. As the load increases P moves up the semicircle, and at full-load it will be about in the position shown in Fig. 484. If the motor is over-loaded both P O and P M increase, and P moves over the top to the left. In the diagram, Fig. 484, the line P M is really the component of the stator current that is equal and opposite to the rotor current, P O the actual stator current being compounded of P M and O M the magnetizing current. Hence, at standstill, the short-circuit current in the stator is represented by O D. It follows that the utilization-coefficient U, which is the ratio of C_{sk} to C_0 will be the ratio between the lengths of O D and O M ; and reciprocally σ is the ratio of O M to O D.

Now in a well designed motor in which there is little magnetic leakage, C_0 is small and C_{sk} is large. It follows that in a well designed motor O will be relatively very near M, and the primary current line O P will for a given load be more nearly vertical.

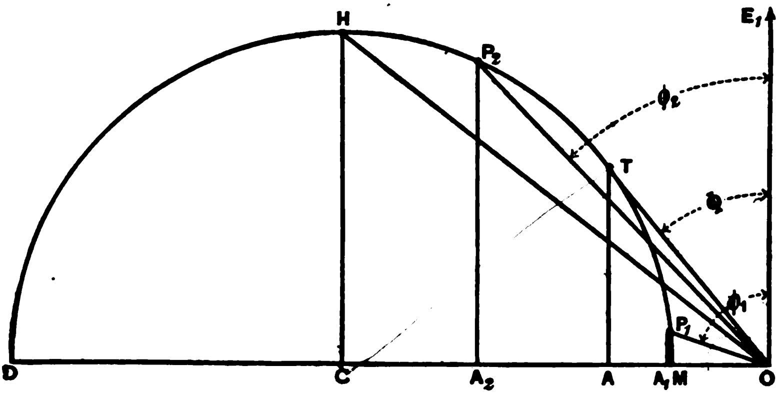

FIG. 485.

Power-Factor.—The angle P O E, or ϕ_1, is the angle of lag of the stator current. If from P we drop the perpendicular P A, the angle O P A is also equal to ϕ_1. The power-factor is the cosine of ϕ_1. Now it will be noticed that, as P moves round the semicircle from M, through P_1 to P_2 (Fig. 485), the angle ϕ changes, and there is one particular load which will make this angle a minimum, namely when P moves to the position T where O P becomes a tangent to the circle. For loads less than this, or greater than this, the lag is greater. If ϕ is at its minimum, cos ϕ is at its maximum. This load may be called the ideal best load for a motor, as it is that load which interferes least with the pressure-regulation of the supply system. It is therefore desirable that motors should work at about this point. A motor intended to work always at a constant load should be designed so that its normal full-load should coincide with its ideal load. A motor intended to work on a variable load should be so designed that its full-load point should come somewhat above the point of ideal load—say at about ¾ load. This angle of minimum lag or maximum power-factor is so important that it deserves to be denoted by a special symbol, for which purpose we take the Greek capital Φ. Then, since the tangent to a circle is a geometric mean between the two secants drawn from the same point, it follows that O T is a mean-proportional between O M and O D; or the stator current at ideal best load will be a mean-proportional between the no-load current and the short-circuit current. Or, if C_1, the normal load, is assumed to occur at the value that makes cos Φ a maximum, we have

$$C_m = C_1 \div \sqrt{U} = C_1 \times \sqrt{\sigma},$$

and

$$C_{sh} = C_1 \times \sqrt{U} = C_1 \div \sqrt{\sigma}.$$

Further the angle Φ is related to U, since

$$\cos \Phi = \frac{C\,T}{C\,O} = \frac{\frac{1}{2}(U - 1)}{U - \frac{1}{2}(U - 1)} = \frac{U - 1}{U + 1} = \frac{1 - \sigma}{1 + \sigma}.$$

It immediately follows that

$$U = \frac{1 + \cos \Phi}{1 - \cos \Phi}, \text{ and that } \sigma = \frac{1 - \cos \Phi}{1 + \cos \Phi}.$$

Input.—The amount of electric power put into the motor in any one phase, is $E_1 C_1 \cos \phi_1$. Now if P O represents the stator current C_1, to any scale, the perpendicular P A will represent $C_1 \cos \phi_1$; and therefore, assuming E_1 constant P A will be proportional to the input. A slight correction is needed as will be shown for the stator losses.

In passing, it may be noted that the stator current O P may be considered as resolvable into two components—O A, which is the wattless component, and A P the energy component, the torque being proportional to the latter in so far as it is not wasted on supplying iron-losses and copper-losses in the stator. The greatest possible torque will occur when P has moved to the top point H.

Corrected Diagram.—While the above simple semicircle diagram suffices for most purposes, a further approximation to complete accuracy is desirable; and the next step is to show how the various losses may be taken into account. In the first place the iron-losses in the stator, which are of the order of 2 to 3 per cent. of the input, are practically constant. If therefore in Fig. 486 we cut off at the foot of P A a piece A U to represent these losses, we may draw the horizontal line U N through U, meeting the no-load current line O N at N. To represent the copper-losses is less easy, because these losses are not constant. Each will at full-load be about 3 per cent. of the input; but in squirrel-cage motors and crane motors the rotor copper-loss is often more than 3 per cent. But they go by the square of the currents, and the voltage drops they represent are not constant, but are proportional to the respective currents. Heyland has used the following device to represent them. P being in the position for normal load, join P D. Then from P D cut off at W a fractional piece P W, to scale as representing the stator copper-drop of voltage, as a fraction of the whole stator voltage. Now this piece P W is itself proportional to the load, and therefore approximately proportional to P M, so that the angle P M W is practically constant. Now in the narrow triangle M P W the angle M P W is a right-angle (because P moves along the semicircle). Hence the external angle M W D, which is greater than a right-angle, will be constant, and as P moves round its semicircle, W will also move round the arc of another circle drawn through M, W and D, and by similar triangles the height of W X is reduced from the height P U in the same proportion as the length W D has been reduced from P D. Hence the height of the perpendicular line W X will represent that amount of power which is actually imparted from the stator to the rotor, since it is equal to the input P A less the portions trimmed off at the top and the bottom to represent the copper and iron losse the stator.

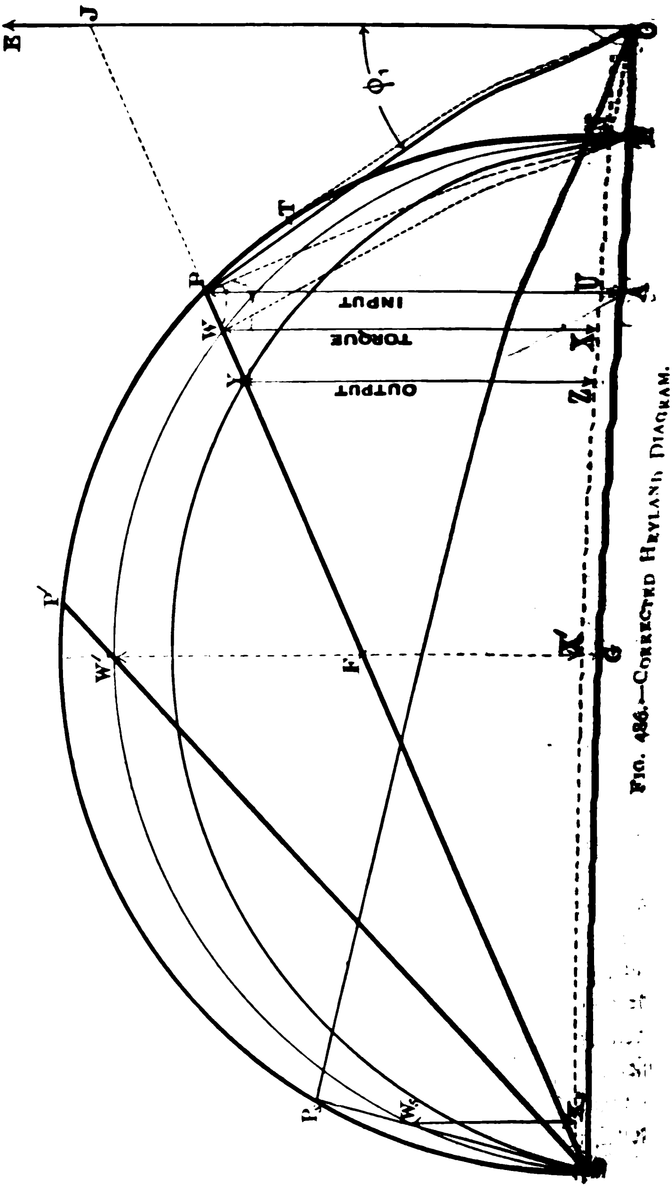

FIG. 486.—CORRECTED HEYLAND DIAGRAM.

In a similar way, by deducting another fractional part, W Y along the line W D to represent the fraction of voltage-drop in the copper of the rotor, we obtain a point Y which will move along the arc of a third circle, and such that the vertical length Y Z represents the output of power of the rotor, for it is equal to the input P A less the portions trimmed off to represent the stator and rotor losses.

The efficiency will be the ratio of output to input; or,

$$\eta = YZ \div PA.$$

Torque and Overload Torque.—We have seen that the vertical line W X represents by its length the amount of power actually given by the stator to the rotor. And as the speed is constant (to within 3 per cent. or so) at all loads in the working range of the motor, it follows that this line W X also represents the torque of the rotor. Now the point W moves around its own circle; hence it follows that the motor will exert a maximum torque when P has moved round to such a point that W is at the highest point of its circle. Now, if at normal full-load P is high up, it is obvious there will be very little margin for overload. It was formerly one of the complaints made against induction motors that they were not able to cope with momentary overloads, but would pull up dead when overloaded even with as little as a 50 per cent. increase over the normal full-load effort. But this complaint, however justified in early types, is not reasonable to-day. For if a motor is well designed, O M will be relatively very small. The position of T (the tangent position of P O, when cos ϕ is a maximum) will then be low down on the right, and the normal full-load position of P will be either at T or a little above it, and there will be a good margin for overload. If we assume that the stator copper and iron losses are small, so that W is very close to P, and U close to A, so that at its maximum W X is practically equal to the radius of the semicircle, it follows that the ratio of the torque at ideal load to the maximum torque will be equal to sin Φ. Let us write the symbol y for the ratio of maximum torque to the torque at the ideal load; it is a measure of the *capacity for overload.*

Then if the motor is so designed that normal full-load occurs at maximum power-factor, we have the relation—

$$y = \frac{1}{\sin \Phi} = \frac{1 + \sigma}{2\sqrt{\sigma}}.$$

For instance, if the motor is required to have an overload capacity of 2 (that is, must be able to exert a torque twice its normal full-load torque), it must not produce at full-load a lag greater than that whose sine is ½, *i. e.* 30 degrees.

A correction is needed to express accurately the angle ϕ if the stator dispersion is not very small, for then it sets up a self-inductive reaction. This may be seen by reference to Fig. 487. Let the vertical line O E_1 represent that part of the applied primary voltage which balances the induced back-electromotive-force in the stator windings. The primary current lags behind this by the angle E_1 O P. Draw O L_1 at right-angles to O P of a length to represent the stator reactance voltage $2\pi f_1 C_1 L_1$. Let O r represent the ohmic drop, due to C_1 and in phase with it. Then O q, the resultant of O L_1 and O r, is equal to the additional voltage which must be supplied. Let O q be compounded with O E_1 giving O V_1 which represents to scale in amount and relative phase, the voltage of the supply (for 1 phase). It is a little greater than O E_1; and the true angle of lag is the angle V_1 O P, and the true power-factor is the cosine of this angle.

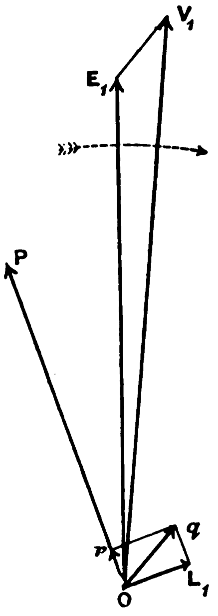

FIG. 487.

Standstill Torque.—If the rotor stands still there is obviously no output of power, that is to say, Y Z is zero. Now this occurs if P moves so far around the semicircle that the line P D becomes a tangent to the innermost of the three circles, and the standstill torque will then be represented by

$W_s X_s$. Now if both rotor and stator resistances are small, the tangent line $D P_s$ will be nearly perpendicular, and $W_s X_s$ will be small. It would be increased if the rotor resistance were increased, but not if the stator resistance were increased. In fact, as we shall see, it is an advantage at starting to have resistance in the rotor windings.

Slip.—There are several ways of representing the slip in the Heyland diagram. The simplest is as follows: As slip is proportional to tan ϕ_2 (see p. 686), and as ϕ_2 is equal to the angle P D A, it is clear that lengths proportional to the slip may be read off on any vertical line such as O J by the height from the base to the point where the line D P crosses it. If the value of any one point, say that corresponding to full-load, is known, the others can be divided out proportionally.

Summary of Results.—The foregoing results are as follows: The actions which go on in an induction motor are represented in the Heyland diagram, Fig. 486, thus:—

Magnetizing current	O M
No-load current	O N
Short-circuit current	O D
Stator current	O P
Rotor current (equivalent)	P M
Input	P A
Output	Y Z
Efficiency	Y Z ÷ P A
Torque	W X
Standstill torque	$W_s X_s$
Maximum torque	W′ X′
Slip	O J

For a different graphical treatment, with a circle-diagram, together with interesting researches on the effect of the form of the field curve, see a pamphlet by O. S. Bragstad, entitled *Beitrag zur Theorie und Untersuchung von mehrphasigen Asynchronmotoren*, Stuttgart, 1902.

Physical Cause of Torque.—A current flowing through a conductor does not produce any driving force upon it unless it lies in a magnetic field, or unless it lies in a slot between teeth through which there is a flux. In the former case the propelling drag comes on the conductor itself: in the latter it acts on the teeth. The physical action in the rotor may be considered by the aid of the following figures.

Let there be a mass of iron such as the upper part of Fig. 488, having a slot in it, and let it be placed in close juxtaposition to a second piece of iron such as is represented in the lower part, through which passes another slot or hole. Suppose that for any reason there is a magnetic flux passing downwards from the one piece of iron to the other. The general paths of the magnetic lines are indicated by the ten representative lines of the figure. Five of these will pass the slots on the right, and five on the left. Next consider the same pieces of iron, without this vertical magnetic field; but let currents flow through copper conductors in the slots. The cross and dot marks, Fig. 489, respectively indicate currents going towards and from the plane of the diagram. These currents would produce magnetic fluxes around themselves in directions indicated by the surrounding lines, which may be taken as representing the magnetism due to the currents in a single stator conductor, and to a single rotor conductor respectively. These lines (two representative ones being taken in each case) show pure leakages, since those due to the stator current do not cross into the rotor or embrace the rotor conductor; and those due to the rotor do not embrace any stator conductor. Such a field contributes nothing to the driving forces, as none of the lines pass from stator to rotor. Lastly, suppose the fluxes of Fig. 488 to be superposed upon those of Fig. 489; the resultant flux being represented in Fig. 490. There are still ten lines from top to bottom; but they are no longer symmetrically spaced. Seven now pass to the right of the upper slot, and of these seven four cross along the gap obliquely and form part of the seven on the left of the lower slot. In accordance with the principle that there is a tension along the lines of a field, there will now be tangential components of force tending to drag the rotor toward the right, with an equal reaction tending to urge the stator toward the left. Any magnetic lines which merely surround any conductor, or any two or more conductors belonging only to one of the two parts, contribute nothing to the driving forces, and are so much magnetic leakage, tending to choke the respective currents. Now in a motor, whether on load or at starting, the currents in the rotor at any part of the periphery are nearly equal, and are practically opposite in phase, to those in the neighbouring part of the stator, and are therefore in the case represented in Fig. 489. They cannot *drive*, unless there is present a field (like Fig. 488) due to circulation of current at some other part of the periphery. In fact, as in the diagram Fig. 483, p. 689, the magnetizing component of current is nearly at right angles in phase to both stator and rotor currents, if those do not lag much. Now the actual direction of the fluxes is (electrically) at right angles to the circulation of the currents that produce them. In Fig. 483, p. 689, the vertical line O E represents the direction of the magnetic flux and of the electromotive-force it generates; for the conductors of the rotor, swept by the rotating flux, experience the maximum induction of electromotive-force when the densest field is sweeping past. Hence for maximum torque effects, the current in the conductor should not lag, but be a maximum at a moment when the maximum field is sweeping past it. If C_{max} is the strength of a current, in a conductor of length l inches, and B_{max} is the maximum density of the field, then the mechanical force (in pounds' weight) it experiences, if there is no difference of phase, is

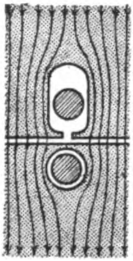

FIG. 488.

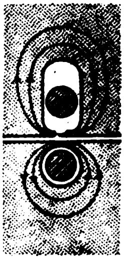

FIG. 489.

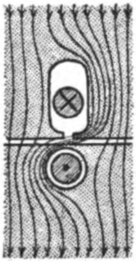

FIG. 490.

$$F_{max} = B_{max} \times C_{max} \times l \div 11{,}303{,}000,$$

and hence

$$F_{av} = B_{av} \times C_{virt} \times l \div 8{,}016{,}000.$$

The joint effect of three sets of conductors, of Z in series in each phase, spaced out as a three-phase winding in, say, two slots per phase per pole, will be not quite three times as great, because there will be a breadth-coefficient about 0·954. Further, since the two other phases act as independent windings just in the same manner as the one already considered, the result is that the three phases together exert just three times the force of one phase. Hence, still on the supposition of unity power-factor in the rotor, the total tangential mean force of the rotor will be

$$F_{total} = 3 \times B_{av} \times C_2 \times Z_2 \times l \div 8{,}400{,}000.$$

The torque in pound-feet will be given by multiplying this by the radius of the rotor periphery in feet, making the torque

$$T = d_r \times B_{av} \times C_2 Z_2 l \div 67{,}200{,}000 \quad . \quad [\text{XVIII}]$$

where d_r is the rotor diameter in inches.

Now if there is a lag ϕ_2 of the current, the expression must be multipled by cos ϕ_2. But

$$\cos \phi_2 = R_2 \div \sqrt{R_2^2 + 4\pi^2 f_1^2 s^2 L_2^2},$$

where s is the slip (see p. 673); and

$$C_2 = k f_1 s Z_2 N \div \sqrt{R_2^2 + 4\pi^2 f_1^2 s^2 L_2^2} \times 10^8.$$

If then we write, for brevity, as the torque-factor

$$Q = d_r \times B_{av} \times k \times Z_2^2 \times N \times l \div (67{\cdot}2 \times 10^{14}),$$

we shall have

$$T = Q \times \frac{R_2 \times f_1 s}{R_2^2 + 4\pi^2 f_1^2 s^2 L_2^2} \quad . \quad . [\text{XIX}]$$

This gives the value of the torque in relation to the slip s, and may be plotted out as a curve such as in Fig. 491, in which, if the speed is reckoned from left to right, the slip will be reckoned backward from the point of full speed. We observe that at standstill there is a certain torque, which, as the motor gets up speed, rises to a maximum, and then, as the motor nears its top speed, falls regularly to zero, at no-load. The working range is the part from the full-load point F to the full-speed point.

At standstill, when $s = \frac{n_1 - n_2}{n_1} = 1$, and for low speeds at starting, the second term in the denominator of the expression [XIX] will be large compared with the first term, and, if we neglect the first term, then the equation becomes

$$\text{T at starting} = Q \frac{R_2}{4 \pi^2 f_1 s L_2^2}, \quad . \quad . \quad . \text{[XX]}$$

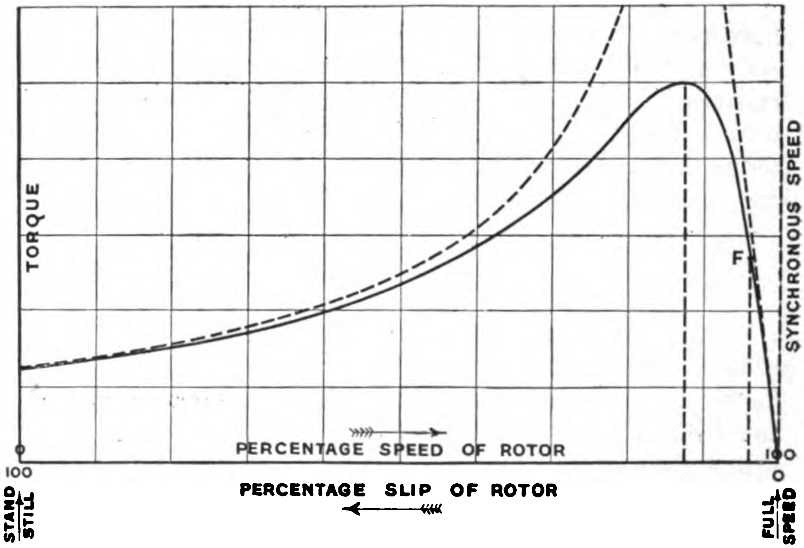

FIG. 491.

from which we see that *the initial torque is proportional directly to the rotor resistance* R_2. It also increases at first as the speed gets up, being *inversely* proportional to the slip.

At top speed, however, and in general throughout the working range, when the slip has fallen to 0·13 or less, and s^2 is therefore 0·0009 or less, the second term becomes negligible compared with R_2, and we then have

$$\text{T on load} = Q \frac{f_1 s}{R}. \quad . \quad . \quad . \text{[XXI]}$$

from which it appears that at speeds near synchronism the torque is *directly* proportional to the slip, and inversely proportional to the *resistance*. It may also be inferred that for a given motor, working on a given voltage and frequency, the slip that will be needful to yield a given torque is directly proportional to the rotor resistance.

Equation [XXI], if plotted, gives the sloping straight line drawn dotted in Fig. 491. Equation [XX] is the equation to a hyperbola, also shown dotted in Fig. 491. The actual curve of performance is asymptotic to both these lines.

It will be noted that the curve of torque rises to a certain maximum at a speed below full-load speed. Theoretically (as can be deduced by differentiating the expression for T and equating to zero) this maximum (on a given resistance) should occur with such a value of the slip as would make the two terms of the denominator equal to one another, or when consequently

$$s = \frac{R_2^2}{4\pi^2 f_1^2 L_2^2}.$$

From considerations named above, it will be seen that while resistance in the rotor is favourable at starting in increasing the torque, it is unfavourable when the motor is running, and reduces the amount of torque that the motor will give with a given permissible slip. Hence the objection sometimes raised to the squirrel-cage form of rotor, that unless it has so much resistance as to make its efficiency low when on load, its starting torque will be small.

Effect of Inserting Resistance at Starting.—As the starting torque is directly proportional to the rotor resistance, motors can be designed to start with any desired torque, up to the maximum (which is usually about twice the full-load torque), by the insertion of resistance. For this purpose the rotor must be of the wound type, and be provided with slip-rings whereby its three circuits can be connected to three non-inductive resistances that are themselves arranged either in Y or in Δ grouping. They should be arranged also so that they can be cut out in stages by a suitable switch, as the motor speeds up. The effect of adding a resistance is graphically depicted in Fig. 492, where the curves are given for several different resistances. Increase of resistance raises the initial torque, but does not raise the maximum torque. And it causes the maximum torque to occur at a lower speed, or in other words increases the slip.

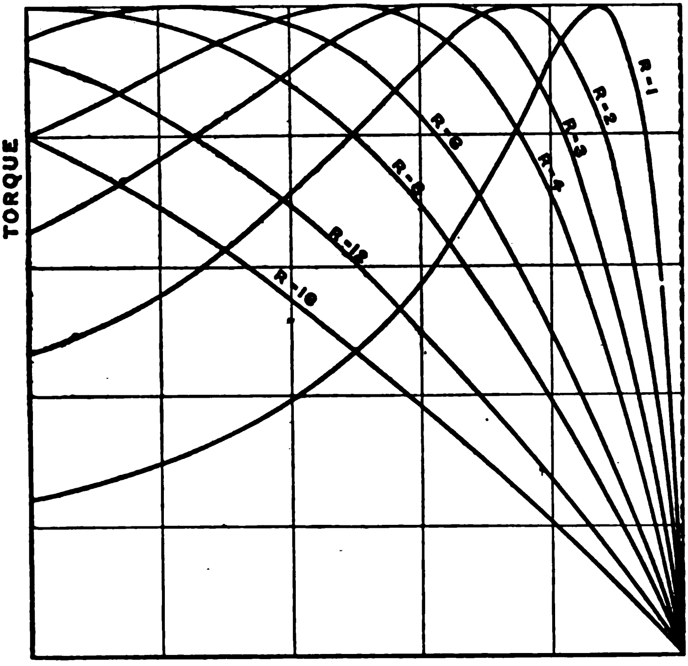

FIG. 492.

The amount of resistance necessary to cause the motor to start with a torque equal to full-load torque, and take a current no greater than full-load current, may be found by trial experimentally. Or it may be deduced as follows. Let R_0 be the total resistance to which the resistance of the rotor circuit is to be increased. Then, if C_2 is a current equal to full-load current in the rotor,

$$C_2 \sqrt{R_0^2 + 4\pi^2 f_1^2 L_2^2} = k f_1 Z_2 N \div 10^8 ;$$

and at full-load, with frequency $f_2 = s \times f_1$

$$C_2 \sqrt{R_2^2 + 4\pi^2 f_2^2 L_2^2} = k f_2 Z_2 N \div 10^8 ;$$

whence

$$\frac{R_0}{R_2} = \frac{f_1}{f_2} = \frac{1}{s}.$$

The resistance so inserted must not be inductive. An inductive resistance would indeed check the rush of current at starting, but it would lessen the initial torque instead of increasing it, as it would increase that which causes the initial torque to be feeble, namely the retardation in phase of the current. Wires or strips of rheostene, eureka metal, or other suitable alloy of high resistance are usually employed; or in workshops, where a rougher and cheaper appliance is admissible, three liquid resistances are often used, and are joined up in the manner indicated in Fig. 493. A type of three-phase water rheostat made by Siemens and Halske is shown in Fig. 494. The bobweights balance the rheostat plates, and dashpots are provided to prevent the switch from being closed too suddenly.

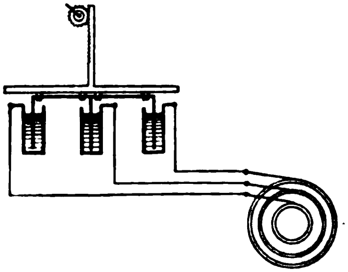
FIG. 493.—DOBROWOLSKY'S STARTING RESISTANCE.

Numerous devices have been suggested for the automatic insertion of resistance at starting, and for the short-circuiting of it when the speed rises, by centrifugal gear. Some of these are described in the *Electrical World*, xlii. 297, 1903. Zani has proposed amongst other expedients the use of three choking coils of low resistance put permanently in parallel with the three rotor resistances. At starting, owing to the high rotor-frequency, they practically do not conduct, so that the rotor resistance is high; but as synchronism is approached, the rotor-frequency falls so low that the choking-coils practically short-circuit the rotor resistances.

Determination of Efficiency of a Three-Phase Motor.—In the following convenient way of determining the efficiency, measurement must be made of the following quantities:—Voltage between lines, V; resistance of one phase of stator, R_1; speed at no-load, n_0; speed at full-load, n_2; no-load current in one phase, C_0; full-load current in one phase, C_1; no-load watts in one phase, w_0; full-load watts in one phase, w_1.

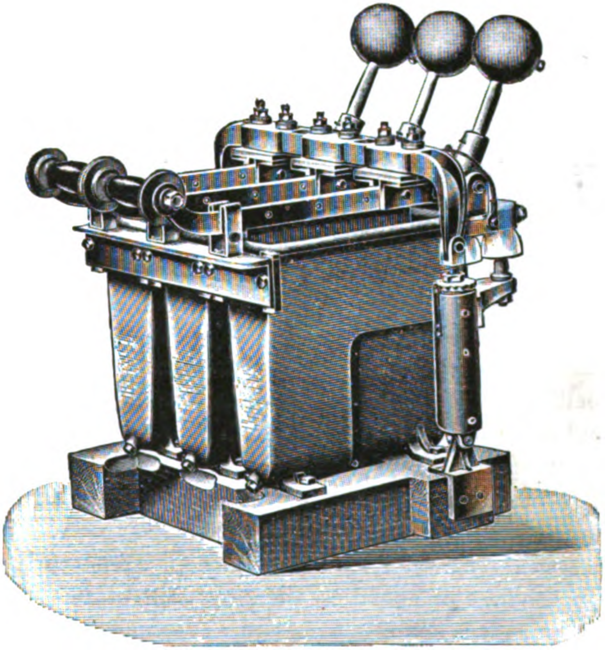

FIG. 494.—THREE-PHASE WATER RHEOSTAT. (SIEMENS AND HALSKE.)

Now the rotor copper-losses are proportional to the slip, and may be written—

$$w_{rot} = 3\, w_1 \left\{ \frac{n_0 - n_2}{n_0} \right\}.$$

The stator copper-losses at full-load, over and above any losses at no-load, may be written:—

$$w_{stat} = 3\, R_1 \left\{ C_1^2 - C_0^2 \right\}.$$

The efficiency, being the ratio of input minus losses to input, will be

$$\eta = \frac{w_1 - w_0 - w_{rot} - w_{stat}}{w_1};$$

and the full-load power-factor

$$\cos \phi = \frac{\sqrt{3}\, w_1}{V C_1}.$$

A method for separation of the losses in induction motors has been given by Angermann in the *Elektrotechnische Zeitschrift*, xxvi. 295, March 1905.

Induction Motors of Changeable Speed.—The variations of speed that can be produced by inserting resistances in the rotor are uneconomical. The only way to change the speed (the frequency of supply being fixed) is to alter the number of poles ; and to do this it is obvious that the stator must either have more than one winding, or the windings must be so designed that by a suitable switching apparatus the effective number of poles can be varied. The Oerlikon Co. has made extensive use of this in its *Drehstromstufenmotoren*, or multiple-speed induction motors. It has recently published an account[1] of a 500 H.P. motor which can run (on a 50-cycle circuit) at speeds of 367, 294, 183 and 147 RPM, and of a 60 H.P. motor which can run at 1450, 950, 750 and 460 RPM, the windings giving 2, 3, 4 and 6 poles respectively.

Tandem Grouping of Induction Motors.—Another mode of obtaining change of speed is the grouping of two motors in tandem, or, as it is sometimes called, *in cascade*, upon one shaft. The currents generated in the rotor of the first motor are led into the stator of the second. This plan, suggested independently in 1897 by Görges and by Steinmetz, has much the same result as the series-parallel arrangement of continuous-current motors. When the motors are thus put in series they tend to run at half the synchronous speed that they will tend to if used singly or in parallel. Dr. Max Breslauer has devoted a special research to this subject in his pamphlet, *Das Kreisdiagramm des Drehstrommotors*, Stuttgart, 1903.

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[1] See articles by Dr. Behn-Eschenburg in the *Schweiz. Bauzeitung*, xi. No. 16, 1902 ; also in *Elektrische Bahnen und Betriebe*, Part 4 of 1905, where test-curves are given. See further the Author's work, *Polyphase Currents*.

CHAPTER XV.

DESIGN OF INDUCTION MOTORS.

The Cardinal Relations of Induction Motors.—A helpful preliminary to the design of induction motors is to tabulate the relations between the different quantities that govern its performance. These are: its no-load or magnetizing current C_m; its short-circuit current C_{sh}; its minimum angle of phase-difference Φ; its dispersion coefficient σ; and its capacity for overload (*i.e.* the ratio of its maximum torque to the torque at ideal full-load), y. All these quantities are related to one another, as we have seen.

Calling by the symbol U the ratio $C_{sh} \div C_m$, we then have the following cardinal relations:—

$$U = (y + \sqrt{y^2 - 1})^2 = \frac{1 + \cos \Phi}{1 - \cos \Phi} = \frac{1}{\sigma};$$

$$\cos \Phi = \frac{\sqrt{y^2 - 1}}{y} = \frac{U - 1}{U + 1} = \frac{1 - \sigma}{1 + \sigma};$$

$$y = \frac{U + 1}{2\sqrt{U}} = \sqrt{\frac{1}{1 - \cos^2 \Phi}} = \frac{1}{\sin \Phi} = \frac{1 + \sigma}{2\sqrt{\sigma}};$$

$$\sigma = (y - \sqrt{y^2 - 1})^2 = \frac{1}{U} = \frac{1 - \cos \Phi}{1 + \cos \Phi}.$$

Hence, if any one of these four cardinal quantities, U, Φ, y, or σ is given, all the others can at once be determined. It is, for example, quite frequent to find it specified that the motor shall not pull up if subjected to a torque twice as great as the torque at normal full-load. If normal full-load be taken as the ideal load that makes the lag a minimum, then we see at once, by putting $y = 2$, that Φ must not exceed 30°, that U must not be less than 13·93, and σ must not be greater than 0·0718. For convenience, the numerical table following has been prepared.

TABLE XVIII.—CARDINAL RELATIONS OF INDUCTION MOTORS.

$\cos\Phi$	Φ	U	$\sqrt{U}$	$\sigma = \frac{1}{U}$	$\sqrt{\sigma}$	y	$x = \frac{\sqrt{U}}{2}$
1	0	∞	∞	0	0	∞	∞
0·966	15°	58	7·6	0·0172	0·131	3·86	3·80
0·96	16° 15′	49	7·0	0·0204	0·143	3·57	3·50
0·954	17° 15′	43·5	6·59	0·0230	0·152	3·37	3·29
0·95	18° 11′	39	6·24	0·0256	0·160	3·20	3·12
0·94	19° 57′	32·4	5·69	0·0309	0·176	2·93	2·84
0·93	21° 34′	27·6	5·25	0·0362	0·190	2·72	2·63
0·92	23° 4′	24	4·90	0·0417	0·204	2·55	2·45
0·91	24° 30′	21·2	4·60	0·0472	0·217	2·41	2·30
0·90	25° 50′	19	4·36	0·0526	0·229	2·29	2·18
0·88	28° 22′	15·65	3·96	0·0639	0·253	2·10	1·98
0·866	30°	13·93	3·73	0·0718	0·267	2·00	1·865
0·85	31° 48′	12·3	3·51	0·0813	0·285	1·90	1·751
0·83	33° 54′	10·75	3·28	0·0930	0·305	1·79	1·64
0·80	36° 52′	9·0	3·0	0·1111	0·333	1·67	1·5

There is a further relation, namely, the ratio of the current required to give maximum torque to the current at ideal full-load. This ratio x is equal to $\frac{1}{2}\sqrt{U}$.

The primary current at the particular load which gives minimum angle of lag Φ, and therefore which gives the maximum power-factor, is not necessarily coincident with the normal full-load current If it is so coincident then C_1 at normal full-load $= C_m \times \sqrt{U} = C_{sh} \div \sqrt{U}$.

The effort from no-load to full-load increases faster than the primary current.

For small motors of 1 H.P. or less, cos Φ is seldom more than 0·80; for motors from 5 to 10 H.P. seldom more than 0·86; for 10 to 25 H.P. seldom more than 0·90. For larger motors it may reach from 0·90 to 0·92, but this figure is not readily surpassed.

Output Formulæ.—As in the cases of continuous-current generators (Vol. I., p. 541), of continuous-current motors (Vol. I., p. 851), and of alternators (see p. 354 of this volume), so with induction motors, it is possible to state a rational relation between the dimensions of the machine and its output.

$$\text{KVA} = 3 \times \text{C}_1 \times \text{E}_1 \div 1000;$$

$$3\ \text{C}_1 = q\pi d \div \text{Z}_1;$$

$$\text{E}_1 = k \times \frac{p}{2} \cdot \frac{\text{RPM}}{60} \times \text{Z}_1 \times \text{N} \div 10^8;$$

$$\text{N} = \text{B} \times l \times \pi d \div p.$$

Inserting these and eliminating, we get:—

$$\text{KVA} = \text{RPM} \times d^2 l \times k.q.\,\text{B} \div (121 \cdot 6 \times 10^{10});$$

whence

$$d^2 l = \frac{\text{KVA}}{\text{RPM}} \times \xi; \quad . \quad . \quad . \quad . \quad [1]$$

where ξ, the *output coefficient*, is written for

$$\xi = \frac{121 \cdot 6 \times 10^{10}}{k \times \text{B} \times q}. \quad . \quad . \quad . \quad . \quad [2]$$

The B here used is the *average* flux-density taken over the pole-area regarded as an unbroken surface, and q, the specific loading, is $3\ \text{C}_1\text{Z}_1 \div \pi d$. Formula [1] is sometimes referred to as the d^2l rule. In three-phase motors the value of the output coefficient varies between 24,000 and 90,000, the smaller values referring to the larger motors.

Example.—If we assign to the various quantities the following mean values; $k = 2 \cdot 2$; $\text{B} = 25{,}000$ lines per square inch; $q = 400$; we get as a mean value for the output coefficient, $\xi = 55{,}000$.

A trial-value for d can always be got, if a limiting value has been assigned to the surface-speed v, by the relation:

$$d = \frac{12 \times v}{\pi \times \text{RPM}}; \quad . \quad . \quad [3]$$

where d is in inches and v in feet per minute.

Another variety of output formula is known as the dl formula (similar to that explained on p. 356 for generators). It is derived as follows. The peripheral velocity v, if expressed in feet per minute, is related as we have seen to the angular speed by the relation RPM = $12\,v \div \pi d$. Now we have:

$$d^2l = \frac{\text{KVA}}{\text{RPM}} \times \xi,$$

and since $\text{KVA} = \text{HP} \times \frac{746}{1000 \times \eta \times \cos\phi}$, it follows that

$$d^2l = \frac{\text{HP} \times 0{\cdot}746 \times \pi \times d}{12 \times v \times \eta \times \cos\phi} \times \xi;$$

whence

$$dl = \text{HP} \times \frac{0{\cdot}196 \times \xi}{v \times \eta \times \cos\phi}.$$

Now as

$$\xi = \frac{121{\cdot}6 \times 10^{10}}{k \times \text{B} \times q},$$

we may write:

$$dl = \text{HP} \times \beta, \quad . \quad . \quad . \quad . \quad . \qquad [4]$$

where

$$\beta = \frac{23{\cdot}7 \times 10^{10}}{k \times \text{B} \times q \times v \times \eta \times \cos\phi}. \qquad [5]$$

If we assign to the various quantities the following[1] average values, $k = 2{\cdot}2$; $\text{B} = 25{,}000$ (av.); $q = 400$; $v = 3000$; $\eta = 0{\cdot}9$; $\cos\phi = 0{\cdot}9$; then we find $\beta = 4{\cdot}5$. In fact, β varies from values of about 2 in large motors to about 7 (or even more) in those of 1½ to 1 HP. It is small in large well-ventilated machines of high peripheral speed.

The coefficient β may, for the ordinary frequency of 50 ~ be empirically taken as approximately equal to from 13,000 to 18,000 divided by v; but for lower frequencies it will be rather greater.

Another way of arriving at a dl formula is to estimate the number of watts lost in the rotor (from 2 to 4 per cent. of the full-load watts), and then to allow from ¾ to 1 square inch of

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[1] Strictly the value of v used in the formula should be the full-load value—about 3 per cent. lower than that of synchronism.

rotor core-surface for each watt so lost. This makes dl equal to about $\frac{1}{\pi}$ times the watts thus lost. Putting the rotor watt loss as 3 per cent., each horse-power of output would necessitate from 15 to 23 square inches of surface; or the value of dl would be from 4 to 7 times the HP, thus justifying the rule.

Or, as a convenient empirical form, if η be the efficiency at full-load, as the surface must be greater the greater the inefficiency, we may write:

$$dl = \text{HP} \times 55\,(1 - \eta). \qquad . \qquad [6]$$

If we would have the polar faces as nearly square as may be[1], so that $l = \tau = \pi d \div p$, we then have from [4] and [5],

$$d^2 = \text{HP} \times \frac{7{\cdot}61 \times 10^{10} \times p}{k \times \text{B} \times q \times v \times \eta \times \cos\phi};$$

whence

$$d = 2{\cdot}82 \times 10^5 \times \sqrt{\frac{\text{HP} \times p}{k \times \text{B} \times q \times v \times \eta \times \cos\phi}}. \qquad [7]$$

Having found d thus, then,

$$l = \tau = \frac{d\pi}{p}. \qquad . \qquad . \qquad [8]$$

If the ratio $l \div d = \lambda$ has already been fixed upon in any proportion, then a value for the diameter can be calculated by a rule due to Heubach:

$$d = 5{\cdot}5 \times \sqrt[3\frac{1}{2}]{\frac{\text{HP} \times 746}{\text{RPM} \times \lambda}}. \qquad . \qquad [9]$$

DESIGN OF THREE-PHASE MOTORS.

Design proceeds on the assumption that certain specified conditions are to be fulfilled by the motor. These are: that when installed upon mains supplied at a given voltage,

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[1] The reason sometimes given for this proportion is that it makes the mean length of the coils a minimum; but as the coils are spread over several slots, the minimum is not accurately attained. There is some advantage in having l rather longer than τ.

with a given frequency of alternation, it shall be capable of working at the prescribed amount of horse-power, while running at the prescribed speed; that it shall not overheat on a continuous run at full-load; that it shall not have a lower efficiency than that prescribed; that the drop of speed from no-load to full-load shall not exceed a certain small percentage. It is usual, also, further to prescribe one of the following conditions: (*a*) that it shall be able to produce momentarily a maximum torque, usually stated at from 1½ to 2 times the torque at normal full-load (this provision being to ensure that the motor shall not suddenly pull up dead if accidentally overloaded); (*b*) that at normal full-load it shall not produce a greater dephasement of current (or a lower power-factor) than a certain prescribed amount; (*c*) that the no-load current shall not exceed a certain prescribed fraction of the normal full-load current. It is of no use to prescribe more than one of these latter conditions (*a*, *b*, and *c*), because, as shown in the preceding section on *Cardinal Relations*, they are dependent on one another, and all depend on the dispersion-coefficient of the machine. But the fulfilment of these conditions, in whatever way prescribed, affects the design in detail.

In the following section there is developed a logical method of procedure in design, a procedure which, like that in all other branches of the subject, is based upon the use of data calculated from examples of machines that are known to be successful, and from which the various constants and coefficients that are used have been found.

Different designers follow different orders of procedure; and some designers follow orders that are different in different cases. To an experienced designer it does not matter much at what point in a design he begins: he has to fulfil a certain specification; he ends by achieving his design. The order of the intermediate steps is not of very great importance, as he knows his way about, and can weigh the significance of each step taken. But to the beginner, and to the designer who, though familiar with the design of other kinds of electric machinery, has had no practice with induction-motors, there is some advantage in following a definite and logical order. Such an order of procedure is accordingly here given. It will enable the designer to see from the beginning what he is about: and it will be such as to permit him from stage to stage to check his results.

Order of Procedure.—It is assumed that the specified matters are:—HP, RPM, V (or E_1), f (cycles per sec.), η, and either cos Φ (power-factor) or else y (factor of overload). The first point in procedure then will be from cos Φ or y to calculate (see Table on p. 708) the cardinal relations Φ, cos Φ, U, $\sqrt{U}$, σ, $\sqrt{\sigma}$, y and x. Then from the frequency and the speed find the number of poles. From HP, η, and cos Φ and E_1 find the KVA, and the full-load current C_1. From C_1 and $\sqrt{U}$ or $\sqrt{\sigma}$, the values of the magnetizing current and the short-circuit current can then be found, and the Heyland diagram to which the motor must conform can then be drawn. All these things had best be settled before a single dimension has been fixed. The next thing is to find trial values for d and l by one of several possible processes; and τ the pole-pitch follows, and the gap-width δ can be fixed. Then a trial-value can be found for either Z_1 (in the stator) or N (flux in one pole), and the value of the former adjusted to meet the required number of stator slots, and the section of the stator conductor can be fixed. Similarly Z_2 for the rotor, and the size of rotor conductor can next be fixed. Next the various losses in copper and iron can be assigned, the outer dimensions of the core-disks fixed, the efficiency and slip calculated, and the weights and costs computed.

Other orders of procedure are as follows:—Behn-Eschenburg finds in the following order, σ, C_1, C_m, C_{sh}; then assuming v, q, and the width of gap from experience he finds d, τ, $\mathbf{B}_{max}$, Z_1, **N**, and from $\mathbf{B}_{max}$ and **N** finds l. Lastly from the dimensions he checks by controlling formulæ the values found for q, **N** and σ.

Behrend proceeds in a somewhat similar order, assuming values for v, and finding successively p, d, τ, δ, σ, cos ϕ, C_1, C_{sh}, C_m, **B** (assumed), Z_1 which is then adjusted, **B**, **N** (from E) and finally l.

Boy de la Tour finds the various losses, then C_1, C_m, assumes the leakage factors, the primary resistance, deduces the starting current, fixes δ, assumes ratio of $l \div d$, finds product $d \times Z_1$, takes trial-values of d so that $d \times Z_1$ will fit to a suitable number of slots, fixes Z_1, deduces d, and so lastly finds l.

Heubach first assumes a suitable ratio of $l \div d$ to give square poles, then finds d by a special output formula from HP and RPM, so getting also l and τ. He fixes on the type of winding, and settles δ by Kapp's rule (see p. 720), and from pole-surface and assumed B finds a trial-value of N_1 and from E_1 and frequency deduces Z_1, which is then adjusted. C_1 is found from the output, and the copper sections of stator and rotor are found. Then by a set of special formulæ he calculates σ from the number and shapes of slots. The calculations of C_m, of the resistances, the iron-losses, and the efficiency follow.

H. M. Hobart has read before the Congress at St. Louis, in September 1904, a paper on a method of design in which he assumes from practice an output-coefficient, equivalent to taking the Esson coefficient at values from 67,000 for 10 HP to 33,500 for motors up to 1000 HP. He finds the cost to be proportional to the diameter of the rotor and to $l + 0{\cdot}7\ \tau$. From these he works out a number of examples, assuming that the pole-pitch is not less than 7·1 inches, the peripheral speed not more than 8000 feet per minute, and finds that high frequencies, though they lower the cost, increase the dispersion.

Constants Suitable for Design.—It will be seen in the preceding review that in all cases design proceeds on experience as to suitable values for sundry constants to be used in finding trial-values in a new design. Experienced designers have their own views; but for those who cannot draw upon a store of experience of their own, the following formulæ will be found to give reasonable values.

Flux density $\mathsf{B}_{av} = 40{,}000 \left(\frac{\text{H.P.} + 50}{\text{H.P.} + 100}\right)$ lines per sq. inch;

Specific load $q = 650 \left(\frac{\text{H.P.} + 25}{\text{H.P.} + 50}\right)$ ampere-conductors per inch;

Peripheral speed $v = 5000 \left(\frac{\text{H.P.} + 25}{\text{H.P.} + 50}\right)$ feet per minute;

Efficiency $\eta = 0{\cdot}94 \left(\frac{\text{H.P.}+4}{\text{H.P.}+5}\right)$;

Power-factor $\cos\Phi = 0{\cdot}92 \left(\frac{\text{H.P.}+4}{\text{H.P.}+5}\right)$;

Current-density a = 1500 to 2000 amperes per sq. inch at full-load in the stator. In wound rotors the same or less. In squirrel-cage motors 3000 if of copper, or 2000 if of brass.

Experienced designers may use values differing considerably from these. For instance, for a 10 HP machine the formula for q gives a specific loading of only 380 amperes per inch periphery, whereas many designers would adopt a higher specific loading for a machine of this output, possibly as great as 420 or 450. The formula has been set at a low value purposely to assist those who might find some difficulty in dealing with the more highly loaded machine.

Procedure in Design.—Given HP, RPM, f (cycles per sec.), V (between lines), or E_1 (voltage in one phase), η, and $\cos\Phi$ (power-factor at full-load, assumed to be the maximum power-factor), or y (overload factor).

Example.—To design a 16 H.P. three-phase motor running at a speed not exceeding 840 revs. per min., to be used on a supply system of frequency of 42 ~, at a voltage of 190 volts between lines; the efficiency at full-load being 0·9 (*i.e.* 90 per cent.); the motor to be capable of giving a maximum torque equal to twice the normal full-load torque.

1. *Find the Cardinal Relations.*—By the formula on p. 707, or the table on p. 708, we have to find the values of $\cos\Phi$, Φ, U, $\sqrt{U}$, σ, $\sqrt{\sigma}$, y and x.

Example.—In this example, the overload factor y is given as 2. Referring to the table on p. 708, we see that $\cos\Phi = 0{\cdot}866$; $\Phi = 30^\circ$; $U = 13{\cdot}9$; $\sqrt{U} = 3{\cdot}73$; $\sigma = 0{\cdot}0718$; $\sqrt{\sigma} = 0{\cdot}267$; $x = 1{\cdot}865$.

2. *Find the Number of Poles.*—The formula holds good from p. 351 : —

$$p = \frac{120 \times f}{\text{RPM}} . \quad . \quad . \qquad [\text{I}]$$

If this does not come out an exact number the nearest *even* number must be taken. From this number of poles the RPM must then be recalculated. This value will be the synchronous speed of no-load ; the speed at full-load being of course 2 to 3 per cent. lower, according to the slip. The following are the values of the synchronous speeds for the corresponding frequencies.

TABLE XIX.—SPEEDS AND FREQUENCIES FOR INDUCTION MOTORS.

—	Revolutions per Minute (no-load).					
	$f = 50$	$f = 48$	$f = 42$	$f = 40$	$f = 30$	$f = 25$
2-poles	3000	2880	2520	2400	1800	1500
4-poles	1500	1440	1260	1200	900	750
6-poles	1000	960	840	800	600	500
8-poles	750	720	630	600	450	375
10-poles	600	576	504	480	360	300

Example.—By formula [I] $p = 120 \times 42 \div 840 = 6$. This needs no trimming. The 6-pole machine will run at 840 RPM on no-load, dropping to about 814 at full-load, if the slip is 3 per cent. It may be remarked in passing, that for a 6-pole machine, if the pole-faces are about square, the ratio of l/d, which is $= \pi \div p$, will be about 0·52.

(3) *Find the Voltage in one Phase* E_1.—The voltage from line to line V is equal to $\sqrt{3}$ times the voltage in one phase E_1 ; hence

$$E_1 = V \div \sqrt{3} = V \times 0{\cdot}577, \quad . \qquad [\text{II}]$$

and therefore, at no-load this is the amount of electromotive-force which the coils of one phase must generate. At full-load this back electromotive-force will drop slightly, but as the drop due to ohmic resistance in the stator coils is exceedingly small, we may, for the purpose of design, neglect it, or simply reduce the calculated value by 1 per cent., or less.

Example.—The prescribed line voltage V is 190 volts. Hence $E_1 = 190 \div 1{\cdot}732 = 110$ volts.

4. *Find the Kilovoltamperes and Full-Load Current.*—The formulæ are :—

$$\text{KVA} = \frac{\text{HP} \times 746}{1000 \times \eta \times \cos\phi} \quad . \quad . \quad [\text{III}]$$

$$C_1 = \frac{\text{KVA} \times 1000}{3\,E_1} \quad . \quad . \quad . \quad [\text{IV}]$$

These formulæ are obvious ; they follow from the definitions and from the elementary properties of three-phase currents.

Example.—Our 16 H.P. machine will be rated at

$$\frac{16 \times 746}{1000 \times 0{\cdot}9 \times 0{\cdot}866} = 15{\cdot}3 \text{ KVA};$$

and the full-load current will be

$$\frac{15{,}300}{3 \times 110} = 46{\cdot}4 \text{ amperes in each phase.}$$

This may be otherwise calculated thus: 16 HP multiplied by 746 gives 11,936 watts. As the prescribed efficiency is 0·9, 11,936 ÷ 0·9 gives 13,262 primary watts. Dividing by the power-factor 0·866 shows that there will be 15,313 apparent watts or 15,313 voltamperes. This will be 5104 voltamperes per phase: and dividing by the voltage in one phase 110 gives 46·4 amperes.

5. *Find the No-load (Magnetizing) Current and the Short-circuit Current.*—We are here taking the magnetizing current C_m as being the no-load current ; neglecting for the present the small watted component that will be occasioned by the iron-losses in the stator.

The formulæ are :—

$$C_m = \frac{C_1}{\sqrt{U}} = C_1 \times \sqrt{\sigma} \quad . \quad . \quad [\text{V}]$$

$$C_{sh} = C_1 \times \sqrt{U} = \frac{C_1}{\sqrt{\sigma}}. \quad . \quad . \quad [\text{VI}]$$

These equations are explained above on p. 692.

Example.—Here $\sqrt{\mathrm{U}} = 3{\cdot}73$; $\sqrt{\sigma} = 0{\cdot}267$. Hence we have:—

$$C_m = \frac{46{\cdot}5}{3{\cdot}73} = 12{\cdot}4 \text{ amperes}; \quad C_{sh} = 46{\cdot}5 \times 3{\cdot}73 = 173 \text{ amperes.}$$

This does *not* mean that however badly we design the rest of our motor C_m will come out only 12·4 amperes. What it means is that, carrying out the rest of the design so that C_m is not greater than 12·4 amperes, we shall get a motor that will fulfil the prescribed specification, and have the required overload-capacity.

6. *Draw the Heyland Semicircle Diagram.*—Everything needed for this is now at hand.

7. *Decide on suitable Values for* B_{av}, B_{max}, q, *and* v.—Suitable trial-values for B_{av}, q, and v can be assigned by aid of the rules given for these constants on p. 714 above. B_{max}, meaning the flux-density in the gap at the phase where it is a maximum, namely at the middle of the pole-face, is got from B_{av} by simply multiplying it by $\frac{\pi}{2}$.

Example.—Inserting 16 as HP into the formulæ on p. 714 we get as suitable values for this machine the numbers: $\mathrm{B}_{av} = 22{,}800$; $\mathrm{B}_{max} = 35{,}000$; $q = 400$ ampere-conductors per inch; $v = 3100$ feet per minute.

8. *Find Values for d and l.*—There are numerous ways of ascertaining suitable values for *d*. The simplest way of all is to take the peripheral speed v (feet per minute) from experience, and apply the rule:—

$$d = \frac{v \times 12}{\mathrm{RPM} \times \pi} \quad . \quad . \quad [\mathrm{VII}]$$

But this affords no clue to the proper value of l; and having been settled by reference *solely* to surface-speed, it will not do to assume that l may be made equal to the pole-pitch. In fact, the rule needs to be supplemented by finding either $d^2 l$ or $d\,l$ by one of the output formulæ given above. These are

$$(1) \quad d^2 l = \frac{\mathrm{HP}}{\mathrm{RPM}} \times \frac{90{\cdot}8 \times 10^{10}}{k \times \mathrm{B} \times q \times \eta \times \cos\phi}. \quad [\mathrm{VIII}]$$

$$(2) \quad d\,l = \text{HP} \times \frac{23 \cdot 7 \times 10^{10}}{k \times \text{B} \times q \times v \times \eta \times \cos\phi}. \quad [\text{IX}]$$

$$(3) \quad d\,l = \text{HP} \times 55\,(1 - \eta) \quad . \quad . \quad . \quad [\text{X}]$$

Of course, it must be borne in mind that, though the use of the above rules gives a value for d that is suitable for the case to which the calculation is applied, that is no proof that a different value of d might not be used by a designer of experience. The manufacturing firms have standard sizes of core-disks, and naturally prefer to use such wherever possible.

Another rough way of settling the size of core-disk is to assume that the pole-pitch τ will be equal to $300 \div f$ inches (this assumes a surface-speed of 3000 feet per minute), and then multiply this by p to find the periphery. So that the trial-value of d would be given by the formula :—

$$d = \frac{300 \times p}{\pi \times f} \quad . \quad . \quad . \quad [\text{XI}]$$

Example.—In the example before us the different rules work out as follows :—

By rule [VII], taking v by rule at foot of p. 714—

$$d = \frac{3100 \times 12}{840 \times \pi} = 14 \cdot 2 \text{ inches.}$$

By Heubach's rule, p. 711—

$$d = 5 \cdot 5 \times \sqrt[3\frac{1}{2}]{\frac{16 \times 746}{840 \times 0 \cdot 52}} = 14 \cdot 1 \text{ inches.}$$

By rule [XI]—

$$d = \frac{300 \times 6}{3 \cdot 14 \times 42} = 13 \cdot 4.$$

The output rules give :—

By rule [VIII]—$d^2l = 1100.$
By rule [IX]— $d\,l = 78.$
By rule [X]— $d\,l = 88 \cdot 1.$

We therefore take a few trial-values of d, and ascertain the corresponding values of l, thus :—

	d	13	13·5	14	14·25	14·5	15
	d^2	169	182	196	203	210	225
l	if $d^2 l$ = 1100	6·5	6·05	5·6	5·4	5·25	4·9
	if $d\,l$ = 88	6·8	6·5	6·3	6·2	6·05	5·9
	τ	6·8	7·05	7·3	7·45	7·57	7·85

Any of these dimensions would make a working motor, but in order to have room for 9 slots per pole we will provisionally adopt $d = 14 \cdot 5$, $l = 6 \cdot 05$ as the dimensions.

9. *Settle Suitable Numbers of Slots per Pole in Stator and Rotor.*—Except in the very smallest motors, the minimum number of slots per pole in the stator should be 9, *i.e.* three per phase. The rotor winding, if not actually incommensurable, should then have either 12 or else 15 slots per pole. In doubtful cases, where the pole-pitch seems ill-adapted for the number of slots mentioned (for example a pole-pitch of 5 inches seems too small for 9 slots, but would be ample for 6 slots) final decision should be left until the value of Z_1 has been ascertained, in order that the suitability for packing the wires in the slots may be considered. For selection of rotor windings such that the three phases are alike, but that the number of slots per phase shall be incommensurable with the slots per phase of the stator, see p. 342.

Example.—The pole-pitch is $\pi\, d \div p = 3 \cdot 14 \times 14 \cdot 5 \div 6 = 7 \cdot 57$. If there are 9 slots per pole (for a 3-slot winding), the slot-pitch will be 0·84. This makes 54 stator slots, or 18 slots per phase. If the rotor had a 4-slot winding, with 12 slots per pole, the total number would be 72. Adding 3 makes 75, or 25 per phase, which is incommensurable with the 18 in the stator as required. Stator slots 54; rotor slots 75. The slots will be nearly closed, with a slit of about $\frac{1}{10}$ inch at the face. With a 3-slot winding k will be about 2·14.

10. *Fix the Air-Gap.*—Mechanical considerations alone limit the air-gap length δ, which from the magnetic point of view ought to be as small as possible. The rule given by Kapp

$$\delta = 0 \cdot 008 + \frac{d}{1000} \quad . \quad . \quad [\text{XII}]$$

is a good one. Another rule is to take $\frac{1}{500}$ of diameter.

Hobart gives the rule that

$$\delta = \frac{1}{24{,}000}\sqrt{d \times l \times v},$$

but takes 1 millimetre where this formula would yield smaller values.

Example.—Applying the rule, we get $\delta = 0{\cdot}008 + 0{\cdot}016 = 0{\cdot}024$. Or if we take $\frac{1}{500}$ diameter, we have $\delta = 0{\cdot}032$. The mean is 0·028. Now on a rotor 14 inches in diameter a clearance of $\frac{1}{64}$ inch would be quite feasible mechanically. Hence it is quite safe to take 0·028. This makes the rotor diameter 14·444.

11. *Fix the Number of Stator Conductors.*—There are three different modes of getting at a trial-value of Z_1 using the following formulæ:—

$$Z_1 = \frac{q \cdot \pi \cdot d}{3\,C_1} \quad . \quad . \quad [\text{XIII}a]$$

$$Z_1 = \frac{B \cdot p \cdot \delta}{C_m} \times 0{\cdot}365 \quad . \quad [\text{XIII}b]$$

$$Z_1 = \frac{E_1 \times 10^8}{k \cdot f \cdot \tau \cdot l \cdot B} \quad . \quad [\text{XIII}c]$$

These three values will generally differ slightly from one another. The first deduced from q gives a maximum number for Z_1 that must not be exceeded without risk of raising the specific loading. The second derived from C_m gives a minimum value for Z_1, below which it is not advisable to go, otherwise the permissible magnetizing current at no-load will be exceeded. Hence, if the values found by [XIII*a*] and [XIII*b*] do not agree, the value selected should fall between them. The decision as to the *precise* number must obviously rest on the choice of a suitable number of wires per slot; the trial-value of Z_1 being adjusted so as to become an exact multiple of the slots per phase and of the wires per slot. If, in any case, it is found that the value of Z_1 given by the first of the three formulæ is lower than that given by the second, it is clear that d has been taken too small. To remedy this a higher value of d must be taken, and the computations from that point begun afresh.

Example.—Applying the above rules, we get for the three values of Z_1: by rule [XIII*a*], which gives the maximum, 140; by rule [XIII*b*], which gives the minimum, 113; by the rule [XIII*c*], 135. Now these conductors have to lie within 18 slots. But $6 \times 18 = 108$; $7 \times 18 = 126$; $8 \times 18 = 144$. The choice clearly falls on 126, with 7 wires per slot.

12. *Fix the Flux per Pole.*—This follows at once by the formula:

$$N = \frac{E_1 \times 10^8}{k.f.Z_1}. \quad . \quad . \quad [XIV]$$

Example.—This works out: $110 \times 10^8 \div (2 \cdot 1 \times 42 \times 126) = 992{,}000$ or practically $N = 1{,}000{,}000$. This may be checked by comparison with pole-area. For the area is $7 \cdot 57 \times 6 \cdot 05$ in gross $= 45 \cdot 7$ square inches, which at a density of 22,800 lines per square inch makes $N = 1{,}030{,}000$.

13. *Assign the Various Losses.*—The total number of watts which may be lost is calculated thus

$$\frac{HP \times 746}{\eta} = \text{watts of input};$$

$$\frac{HP \times 746}{\eta}(1 - \eta) = \text{watts lost.}$$

Friction may be estimated as consuming about 1 per cent. The rotor-resistance, to which the slip is proportional, will account for a loss of power from 1½ in the largest motors down to 2½, 3, or 3½ per cent. in small motors. (In crane motors from 7 to 12 per cent.) The iron-loss in the rotor is negligible. The remainder is divided between the stator copper-loss $3C_1^2R_1$, and the stator iron-losses. Maximum efficiency is attained when the no-load losses (friction and iron) are equal to the load-losses. If the maximum efficiency is to be reached at say 80 per cent. of the full-load, the full-load copper-losses may be made half the total.

The following table shows the apportionment of the losses in 3 modern motors. They are given in percentages of the input.

TABLE XX.—CUSTOMARY PERCENTAGES OF LOSSES.

—	2 H.P.		14 H.P.		200 H.P.	
	Per cent.	Watts	Per Cent.	Watts.	Per cent.	Watts.
Stator copper-loss .	3·87	83	3·0	375	1·37	1990
Stator iron-losses .	5·90	127	5·0	625	1·10	1720
Rotor copper-loss .	4·70	99	3·0	375	1·08	1690
Rotor iron-loss . .	..	..	..	..	0·05	90
Friction, etc. . .	1·03	22	1·0	125	1·00	1560
	15·50	331	12·0	1500	4·6	7050

Example.—In our machine the watts input are: 16 × 746 ÷ 0·9 = 13,260 watts. The efficiency being 0·9, the watts lost will be 1326. Allowing 1 per cent. or 132·6 for friction, and 3 per cent. or 397·8 for slip, there remain 796·4, of which provisionally 400 may be assigned to iron-losses, and 396·4 to copper.

14. *Fix the Size and Arrangement of Stator Conductors.*—A preliminary judgment as to the requisite size of stator conductor may be drawn by assuming as a suitable current density 1800 to 2200 amperes per square inch, or say 2000 as a mean value. Dividing the full-load current by this figure will give a trial-value for the necessary cross-section; and reference to a set of wire-gauge tables will at once give the nearest corresponding size. An estimate must also be made of the allowance for insulation, and the wires must be grouped so as to lie conveniently within the slot. The dimensions of this can then be estimated and the width of the tooth deduced. This should not be so small as to force up the magnetic saturation unduly, otherwise there will be too great a loss from hysteresis. A safe magnetic density in the teeth is from 75,000 to 85,000 lines per square inch.

A more precise predetermination of the necessary copper section can be made if the mean length of one turn of the stator winding has been carefully estimated. If the permissible watt loss in the stator copper, at full-load be called W_s, and the mean length of one turn, in inches, be called M, then the necessary area of section of the wire, in square inches, will be :—

$$s = \frac{1 \cdot 2}{10^6} \times \frac{C_1^2 \times Z_1 \times M}{W_s} \quad . \quad . \quad [XV]$$

Example.—As $C_1 = 46 \cdot 5$, at 1700 amperes per square inch the necessary cross-section will be 0·0274. The nearest size of wire is No. 6 S.W.G., which has 0·02895, and has a diameter of 0·192 inch bare. If the seven wires are arranged in two rows of four and three each, the width of the double row allowing for insulation, and for a short lining of 25 mils thickness, will necessitate a slot about 0·51 inch wide. As the slot-pitch (at the face) is 0·84 inch, the minimum width of tooth will be 0·33 inch, and this (if the iron were solid) would run up the saturation to 22,800 × 0·84 ÷ 0·33 = 58,000. But as the insulation between the laminations reduces the effective iron length to about 85 per cent. of the measured length, the saturation will run up to about 58,000 ÷ 0·85 = 69,000. The slot will be about 1·05 inches in extreme length.

By the formula [XV], if M is 33 in., and $C_1 = 46 \cdot 5$, and the permissible copper-loss is 390 watts,

$$s = \frac{1 \cdot 2 \times 46 \cdot 5 \times 46 \cdot 5 \times 126 \times 33}{10^6 \times 390} = 0 \cdot 029 \text{ sq. inch};$$

from which the wire-tables at once give the gauge as No. 6 S.W.G.

15. *Fix the Number of Rotor Conductors.*—This is determined by the voltage desired in the rotor. For wound rotors with slip-rings, the usual practice is to arrange the windings so that the voltage between slip-rings is from 50 to 100 volts at starting. It is, however, better to use some higher voltage[1], say 200, 250, or even 500 volts for large motors, because with the higher voltage the currents to be handled are smaller ; and though the watts to be absorbed in the starting resistances are in any given motor the same independently of the voltage, the switch contacts may be lighter if the currents are small. Then by ordinary transformer principles the ratio between the number of rotor conductors per phase Z_2, and stator conductors per phase Z_2 will

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[1] See L. J. Pumphrey in *Elect. Rev.*, lvii. p. 46, 1905.

be as (say) 50 volts is to the primary voltage between lines; or as a trial-value, if V_2 is the slip-ring voltage

$$Z_2 = Z_1 \times V_2 \div V_1 . \quad . \quad . \quad [\text{XVI}]$$

But the number so obtained must be adjusted to make a suitable rotor-winding on the principles laid down in p. 339, and may be varied within wide limits since there is no importance in generating an exact voltage. If this leads to an abnormally great rotor current, say over 200 amperes per phase, it is advisable to increase the voltage by taking Z_2 higher.

Example.—V_1 is 190 volts, and $Z_1 = 126$. If V_2 be taken as 50 volts, the trial-value for Z_2 will be $126 \times 50 \div 190 = 48$. Now we have seen that a suitable (incommensurable) number of rotor slots per phase is 25. If we put two conductors in a slot, Z_2 becomes 50 instead of 48. So settle $Z_2 = 50$. The total number of rotor slots is 75 and of rotor conductors 150.

16. *Fix the Dimensions of the Rotor Conductors.*—If Z_2 were equal to Z_1, then they would at full-load carry equal currents (neglecting the question of magnetizing component of C_1). For present purposes it suffices to assume that C_2, the current in one phase of the rotor, will vary inversely as the number Z_2. Hence we have:—

$$C_2 = C_1 \times Z_1 \div Z_2 . \quad . \quad . \quad [\text{XVII}]$$

From this the appropriate size of conductor can be determined by a rule similar to [XVI], allowing again from 1500 to 2200 amperes per square inch as a suitable density.

Example.—By the formula $C_2 = 46{\cdot}5 \times 126 \div 50 = 128$ amperes, $128 \div 1600 = 0{\cdot}080$ sq. in. The rotor slot-pitch being $14{\cdot}5 \times \pi \div 75 = 0{\cdot}6$ inch, an appropriate conductor would be a rectangular wire 0·25 inch wide by 0·33 deep. This allows a slightly larger section, viz. 0·082 sq. in. The two wires would lie one below the other, in a slot about 0·325 inch wide and 0·98 deep.

17. *Estimate Copper Resistances.*—Before accepting these windings, particularly the stator winding, as final, the resist-

ances ought to be estimated, to afford a check upon the watts that will be lost respectively in them.

Taking the resistance of a rod of copper 1 foot long and 1 sq. inch in section, at 60° C. as 9·5 microhms, we have the resistance of one phase of the stator winding R_1 as

$$R_1 = \frac{Z_1}{2} \times M \times \frac{0 \cdot 8}{s \times 10^6}, \quad . \quad [XVIII]$$

where M is the mean length, in inches, of one turn of the stator winding, and s the cross-section. To estimate M accurately is not easy on account of the bends of the windings. It must be remembered too, that, in all cases of frequencies as high even as 50, there is an additional resistance due to the skin-effect.

Example.—The estimated mean length of one turn in the stator is 35 inches; that of the rotor about 33. Hence $R_1 = \frac{126 \times 35 \times 0 \cdot 8}{2 \times 0 \cdot 02895 \times 10^6} =$ 0·06 ohm. Hence the ohmic loss in one phase will be 46·5 × 46·5 × 0·06 = 130 watts, or in all 3 phases together 390 watts. The permissible amount was 396·4. In the rotor the similar calculation gives: $R_2 = \frac{50 \times 33 \times 0 \cdot 8}{2 \times 0 \cdot 082 \times 10^6} = 0 \cdot 0080$. The rotor current being 128 amperes, the loss will be 3 × 128 × 128 × 0·0080 = 394 watts. The amount allowed was 397·8.

18. *Fix External Diameter of Stator and Internal Diameter of Rotor.*—The density of the flux beyond the roots of the teeth should not much exceed 20,000 in the stator, or 30,000 in the rotor. Now as the radial depth must be sufficient to carry half the flux of one pole, and as the mean density of the flux in the gap is usually from 20,000 to 25,000, it follows that the appropriate radial depths in stator and rotor will be respectively equal to a little more than, and a little less than half the pole-pitch at the face. The ultimate decision in the case of the stator rests on the permissible iron-losses.

Example.—The pole-pitch at the face is 7·57, the half of which is 3·78. And as the gap-density is 22,800, trial-values for the two radial depths will be $3 \cdot 78 \times \frac{22,800}{20,000} = 4 \cdot 3$, and 3·78 ×

$\frac{22{,}800}{30{,}000} = 2{\cdot}9$ inches. The diameter of the stator face is 14·5, adding 4·3 inches at each side makes the external diameter 23·1 inches. The diameter of the rotor at the face is 14·444, deducting twice 2·9 makes the internal diameter 8·644 inches. To check the stator dimensions it may be remembered that the permissible iron-losses are 400 watts. Now the net volume of the stator teeth excluding insulation may be estimated at 255 cubic inches, and the flux-density in them is 69,000. The volume of the iron behind the stator teeth is about 1430 cubic inches, with a flux-density of 20,000. Reference to curves of losses in iron (see pp. 578 to 581 *ante*, or Vol. I. pp. 103 and 105) enables us to estimate these. We have:—

$$\left\{\begin{array}{lll} \text{in teeth, by hysteresis,} & 0{\cdot}012 & \text{watt per cycle, per cubic inch.} \\ \text{,, by eddies,} & 0{\cdot}00017 & \text{,, ,,} \end{array}\right.$$

$$\left\{\begin{array}{lll} \text{in core, by hysteresis,} & 0{\cdot}003 & \text{watt per cycle, per cubic inch.} \\ \text{,, by eddies,} & 0{\cdot}00003 & \text{,, ,,} \end{array}\right.$$

Hence the losses are

	watts.
in teeth $(0{\cdot}012 \times 42 \times 225) + (0{\cdot}00017 \times 42 \times 42 \times 225) =$	221
in core $(0{\cdot}003 \times 42 \times 1430) + (0{\cdot}00003 \times 42 \times 42 \times 1430) =$	181
total iron-losses =	402

The permissible iron-loss was set down as 400 watts.

In preliminary design, and with the usual flux-densities, it suffices as a rough estimate to put down the friction and iron-losses together as amounting to about 0·4 watt per cubic inch of iron.

19. *Estimate the Efficiency at Full-Load.*—By formula

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{HP} \times 746}{(\text{HP} \times 746) + \text{the losses}} \quad . \quad [\text{XIX}]$$

Example.—By the foregoing the losses are estimated as follows: Stator copper 390, rotor copper 394, iron 402, friction, etc., 132·6, total losses 1312·6 watts. Hence we have

$$\eta = \frac{16 \times 746}{(16 \times 746) + 1312{\cdot}6} = 0{\cdot}9, \textit{ i.e.} \text{ 90 per cent.}$$

20. *Controlling Calculations.*—It remains to check some of

the points in the design, viz. the *dispersion coefficient* σ, the *specific loading* q, the *pole-flux* N, the *slip*, and the *no-load current.*

To check the dispersion coefficient we will use Behn-Eschenburg's formula :—

$$\sigma = \frac{3}{X^2} + \frac{10 \times \delta \times \epsilon}{X \times Y \times \tau} + \frac{5\,\delta}{l_i} \quad . \quad . \quad [XX]$$

where the ϵ is the thickness of the edges of teeth where they overhang ; X the mean number of slots per pole in stator and rotor together; Y the width of the slits at the tops of the slots, and l_i the nett length of iron in the core.

$$q = \frac{3\,C_1\,Z_1}{\pi\,d} \quad . \quad . \quad . \quad . \quad [XXI]$$

$$N = \frac{HP \times 3{\cdot}56 \times 10^{11}}{p \times q \times v \times \eta \times \cos\phi} \quad . \; [XXII]$$

$$\text{Slip} = \frac{3\,C_2^2\,R_2}{3\,C_2^2\,R_2 + (HP \times 746)} \quad [XXIII]$$

Magnetizing Current

$$C_m = 0{\cdot}365 \times B \times p \times \delta \div Z_1 \quad [XXIV]$$

$$\text{No-load current} = \sqrt{C_m^2 + \left(\frac{W_h}{3\,E_1}\right)^2} \quad . \quad . \; [XXV]$$

Example.—For calculating σ we must insert the following values: Stator slots per pole are 9, rotor slots per pole 11½, hence X = 10·25 ϵ = 0·070; Y = 0·1; l_i = 5·4. Hence

$$\sigma = \frac{3}{105} + \frac{10 \times 0{\cdot}028 \times 0{\cdot}07}{10{\cdot}25 \times 0{\cdot}1 \times 7{\cdot}57} + \frac{5 \times 0{\cdot}028}{5{\cdot}4}$$
$$= 0{\cdot}0285 + 0{\cdot}0025 + 0{\cdot}0258 = 0{\cdot}0568.$$

This makes $\cos\phi = 0{\cdot}89$; and $y = 2{\cdot}19$. So the motor more than fulfils the specification.

The specific loading is $3 \times 46{\cdot}5 \times 126 \div (\pi \times 14{\cdot}5) = 388$ amperes per inch.

The flux from one pole is $\frac{16 \times 3{\cdot}56 \times 10^{11}}{6 \times 388 \times 3100 \times 0{\cdot}9 \times 0{\cdot}88}$, which is practically one million.

The slip is $394 \div (394 + 11936) = 0{\cdot}0324 = 3\frac{1}{4}$ per cent. Hence the speed at full-load will be 813·3 revolutions per minute.

The magnetizing current $= 0{\cdot}365 \times 22800 \times 6 \times 0{\cdot}028 \div 126 = 11{\cdot}6$ amperes.

The no-load current will be $\sqrt{\left\{(11{\cdot}6 \times 11{\cdot}6) + \left(\frac{402}{3 \times 110}\right)^2\right\}}$ $= \sqrt{135{\cdot}5} = 11{\cdot}65$ amperes.

21. *Calculate the Auxiliary Starting-Resistance.*—The rule is that the resistance (in each phase) must be made up to the value

$$R_0 = R_2 \times \frac{1}{1 - \text{slip}} . \qquad . \quad [\text{XXVI}]$$

Example.—$R_2 = 0\ 008$, and the slip is 0·0324. Hence R_0 must be made up to $0{\cdot}008 \div 0{\cdot}9676 = 0{\cdot}0083$.

22. In the foregoing it has been assumed that the maximum power-factor is required to occur at normal full-load. But cases may arise where it is desired that the maximum power-factor shall occur at over-load or at some specified underload. For example, if the motor (or a number of motors) will probably be used on the average at only $\frac{3}{4}$ load, then, in order that the amount of wattless current in the mains may be reduced to a minimum, it may be well to design the motor so that maximum power-factor shall occur at $\frac{3}{4}$ load. To meet such requirements in the design, increase or decrease the magnetizing current by altering the number of conductors in the slot, while the total number of ampere-conductors will remain the same. Then increase or decrease the length of the core correspondingly so as to make the flux per pole fit to the new number of wires per slot and to the prescribed voltage. An increase in the number of conductors with the resulting reduction in the full-load current will be followed by the selection of a smaller wire ; and it may be that such a modification in the design may lead to a change in the size of slots or even of core-disks. It may be remembered that if a standard type of stator winding has been assumed, for example, one with nine slots per pole, and if the magnetic densities have also been

fixed from experience in advance, the number of ampere-conductors per slot (of given style) is thereby fixed irrespective of the length of core, and the short-circuit current (and, therefore the diameter of the Heyland semicircle) is also approximately fixed; so that increasing the number of conductors per slot, and consequently decreasing the magnetizing current, will cause the maximum load-factor to occur with a smaller primary current, that is with a lesser current than that of normal full-load.

If, on the other hand, the motor is so designed that the maximum power-factor occurs only at an over-load, then throughout the normal range of working up to full-load the amount of wattless current will be greater than need be; but the motor will have a greater capacity for over-load. For instance, if the 16 H.P. motor, used as example above, were to be modified so that its maximum cos Φ of 0·866 should occur at an over-load of 40 per cent., the number of wires per slot in the stator must be reduced from 7 to 5, and the core length l increased from 6·05 to 8·45. Then the power-factor at normal full-load would drop to about 0·83, while the over-load factor above normal load would be increased from 2 to 2·8.

SCHEDULE FOR
DESIGN OF THREE-PHASE MOTOR

Calculated by ______

Constructed by ______

SPECIFICATION: HP; RPM; V; $f =$ cycles; $\eta =$ efficiency; $\cos\Phi =$ ______, at max.

No. of Poles $= \frac{120 f}{\text{RPM}} =$. *Cardinal relations* $\sigma = \frac{1}{U} = \frac{1 - \cos\Phi}{1 + \cos\Phi} = (y - \sqrt{y^2 - 1})^2 =$. $\begin{Bmatrix}\text{Overload factor } y = \\ 1 + \sin\Phi = \end{Bmatrix}$.

$\cos\Phi =$; $y =$; $\sigma =$; $U =$; $\sqrt{U} =$; $\Phi =$.

TRIAL VALUES $B_{av} = 40{,}000\left(\frac{HP + 50}{HP + 100}\right)$; $q = 650\left(\frac{HP + 25}{HP + 50}\right)$; $v = 5000\left(\frac{HP + 25}{HP + 50}\right)$; $k = \begin{Bmatrix}2{\cdot}13 \text{ for 3-slot winding} \\ 2{\cdot}12 \text{ for 4-slot winding}\end{Bmatrix}$

taken as $B_{av} =$ $\frac{\text{lines}}{\text{sq. in.}}$; $q =$ $\frac{\text{amp. wires}}{\text{inch}}$; $v =$ $\frac{\text{ft.}}{\text{min.}}$; $B_{max} =$ $\frac{\text{lines}}{\text{sq. in.}}$

$E_1 = V \div \sqrt{3} =$; $KVA = \frac{HP \times 746}{1000 \times \eta \times \cos\Phi} =$; $C_1 = \frac{KVA \times 1000}{3\,E_1} =$; Slip-ring voltage $V_2 =$.

$C_m = C_1 \times \sqrt{\sigma} =$; $C_{sh} = C_1 \div \sqrt{\sigma} =$;

Trial Value of $d = \frac{v \times 12}{\text{RPM} \times \pi} =$ inches.

$d\,l = HP \times \beta$; where $\beta = \frac{23{\cdot}7 \times 10^{10}}{k \,.\, B_{av} \,.\, q \,.\, v \,.\, \eta \,.\cos\Phi} =$.

(Rough Check) $d\,l = HP \times \beta$ (where β varies from 3 to 7).

Values Fixed $d =$; $l =$; $\tau =$.

Gap $\delta = 0{\cdot}008 + \frac{d}{1000} =$; d' of rotor = .

Trial Values.

d					
$d\,l$					
l					
πd					
$\frac{\pi d}{p} = \tau$					
v					

From τ fix slot-pitch not less than $0{\cdot}5$ nor more than $0{\cdot}9$ inch.

$$\left\{\begin{array}{ll} Z_1 = \dfrac{q.\pi.d}{3C_1} = & \text{(maximum).} \\ Z_1 = \dfrac{\mathbf{B}.p.\delta \times 0{\cdot}365}{C_m} = & \text{(minimum).} \\ Z_1 = \dfrac{E_1 \times 10^8}{k.f.\tau.l.\mathbf{B}} = & \text{(probable).} \end{array}\right\} \xrightarrow{\qquad}$$

$$\mathbf{N} = \frac{E_1 \times 10^8}{k.f.Z_1} = \qquad \text{lines.}$$

Depth of core behind teeth $\left\{\begin{array}{l}\text{in Stator} = 0{\cdot}5\,\tau = \quad .\\ \text{in Rotor} = 0{\cdot}4\,\tau = \quad .\end{array}\right.$

External Diameter Stator = .

Internal Diameter Rotor = .

$\left\{\begin{array}{ll}\text{Volume iron Stator teeth} = & \text{cub. in.}\\ \text{Volume iron Stator core} = & \text{cub. in.}\end{array}\right.$

Watts lost.	Friction etc. (1% of Input), w_f	=	
	Rotor copper (3% of Input), w_r	=	
	Stator copper w_s	=	
	Stator iron (hyst + eddies) w_i	=	
	Rotor iron (negligible)		
	Total Losses ..	..	

$$\text{Efficiency } \eta = \frac{\text{HP} \times 746}{(\text{HP} \times 746) + \text{total losses}} = \qquad .$$

Watts Input = HP × 746 ÷ η = .

Watts lost = HP × 746 × $(1-\eta) \div \eta$ = .

STATOR. $\left\{\begin{array}{l}\text{No. of slots per phase per pole} = \quad .\\ \text{No. of slots per pole} = \quad .\\ \text{Total No. of slots per phase} = \quad .\\ \text{Total No. of slots all round} = \quad .\end{array}\right.$

Adjust to exact multiple of slots per phase. Z_1 = .

ROTOR. $\left\{\begin{array}{l}\text{No. of slots per phase per pole} = \quad .\\ \text{No. of slots per pole} = \quad .\\ \text{No. of slots per phase} = \quad .\\ \text{Total No. slots all round} = \quad .\end{array}\right.$

Rotor Conductors. Trial value, $Z_1 \times V_2 \div V_1$ = .
Adjust to be an incommensurable × 2 = $= Z_2$.

Rotor Current $C_2 = C_1 \times Z_1 \div Z_2$ = amp.

Resistance in 1 phase, at 60° C.

$$\text{Stator, } R_1 = Z_1 \times \frac{\text{mean length of turn}}{\text{area of section}} \times \frac{0{\cdot}396}{10^6} = \qquad \text{ohm.}$$

$$\text{Rotor, } R_2 = Z_2 \times \frac{\text{mean length of turn}}{\text{area of section}} \times \frac{0{\cdot}396}{10^6} = \qquad \text{ohm.}$$

Copper Losses $\left\{\begin{array}{ll} w_s = 3\,C_1^2\,R_1 \text{ of Stator} = & \text{watts.}\\ w_r = 3\,C_2^2\,R_2 \text{ of Rotor} = & \text{watts.}\end{array}\right.$

$\left\{\begin{array}{l}\text{Slot insulation thickness Stator} = \quad .\\ \text{Slot insulation thickness Rotor} = \quad .\end{array}\right.$

$\left\{\begin{array}{l}\text{Slot dimensions Stator} = \quad \times \quad .\\ \text{Slot dimensions Rotor} = \quad \times \quad .\end{array}\right.$

$\left\{\begin{array}{l}\text{Tooth width Stator} = \quad .\\ \text{Tooth width Rotor} = \quad .\end{array}\right.$

$\left\{\begin{array}{lll}\text{Section Stator Conductor at} & \dfrac{\text{amps}}{\text{sq. in.}} = & \text{sq. in.}\\ \text{Nearest wire S.W.G. No.} & \text{has} & \text{sq. in.}\\ \text{Section Rotor Conductor at} & \dfrac{\text{amps}}{\text{sq. in.}} = & \text{sq. in.}\\ \text{Suitable dimensions} & \text{by} & \text{inches.}\end{array}\right.$

IRON LOSSES.

Vol. Stator teeth $\times f \times$ hysteresis waste per cub. in. at frequency 1 ~, from curve = watts wasted hysteresis teeth =
" " " $\times f^2 \times$ eddy " " " " " = " " eddies " =
Vol. Stator core $\times f \times$ hysteresis " " " " " = " " hysteresis core =
" " " $\times f^2 \times$ eddy " " " " " = " " eddies " =

Total iron losses ... =

Controlling Formulæ.

$$\sigma = \frac{3}{X^2} + \frac{10 . \delta . \epsilon}{X . Y . \tau} + \frac{5 \delta}{l_i} = \qquad ; \text{ where } \begin{cases} X = \text{mean No. of slots (Stator and Rotor) per pole.} \\ Y = \text{width of slits of slots.} \\ \epsilon = \text{edge thickness of teeth.} \\ l_i = \text{iron length of core.} \end{cases}$$

Starting Rheostat, resist. p. phase = ohms.
Total radiating surface = sq. in.
Probable temperature rise = deg. Centig.

$q = 3 C_1 Z_1 \div \pi d =$.

$$N = \frac{HP \times 3 \cdot 56 \times 10^8}{p . q . v . \eta . \cos \Phi} = \qquad .$$

$$\text{Slip} = \frac{w_r}{w_r + (HP \times 746)} = \qquad .$$

$C_m = 0 \cdot 365 \times B \times p \times \delta \div Z_1 =$ amp.

$C_h = w_i \div 3 E_1 =$ amp.

No-load Current $\sqrt{C_m^2 + C_h^2} =$ amp.

WEIGHTS AND COSTS.	Material.	Weight.	Cost per lb.	Cost of part.
Stator Stampings .				
Rotor Stampings .				
Stator Copper . .				
Rotor Copper . .				
Shaft and pulley .				
Spider				
Housing, etc. . .				
Brush Fittings . .				
Miscellaneous . .				
	Total weight		Total cost	

CHAPTER XVI.

EXAMPLES OF INDUCTION MOTORS.

THE following examples will serve to illustrate the principal types of induction motors, ranging from 8 H.P. at 1470 RPM to 570 H.P. at 75 RPM :—

I. IM 8— 25— 750, by Messrs. Scott and Mountain.
II. IM 4— 8—1450, by the Oerlikon Machine Works.
III. IM 6— 10—1000 (two-phase), by Messrs. Johnson and Phillips.
IV. IM 6—335— 980, by the Oerlikon Machine Works.
V. IM 36—570— 75, by the Oerlikon Machine Works.
VI. IM 24—200— 100, by the Oerlikon Machine Works.
VII. IM 44—500— 163, by the Bullock Co.
VIII. IM 6—150— 750, by Messrs. Brown, Boveri and Co.
IX. IM 8—150— 550, by the Oerlikon Machine Works.
X. IM 6—120— 760, by the Oerlikon Machine Works.
XI. IM 6—150— 900, by Messrs. Ganz and Co.

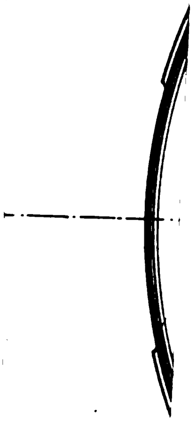
Example I.—ANALYSIS OF A 25 H.P. THREE-PHASE MOTOR, constructed by Messrs. Scott and Mountain.

$$\text{IM } 8_{P}—25_{HP}—750_{RPM}\,;\ 250_{V}—3 \times 53_{A}\,;\ 50\sim.$$

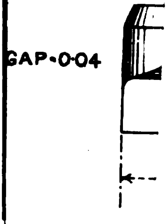
This motor, drawings of which are given in Plate XX., and of which Fig. 495 shows a general view, may be taken as a good example of the ordinary type of induction motor with slip-rings, suitable for fairly low output. Designed as a 25 H.P. motor to run at no-load with a synchronous speed of 750 RPM on a supply system at 50 cycles per second, it has 8 poles ; and at full-load it runs at 720 RPM, with a slip of 4 per cent. and an efficiency of 90 per cent., the power-factor at full-load being 0·91.

TAIN MOT

$\frac{0}{5}$V, 53×3A, 50-



. SIZE)



The following dimensional data are given :—

Stator :—

Diameter at face	17 inches
External diameter of core-disks	24 ,,
Length between core-heads.	6 ,,
Total number of slots.	72

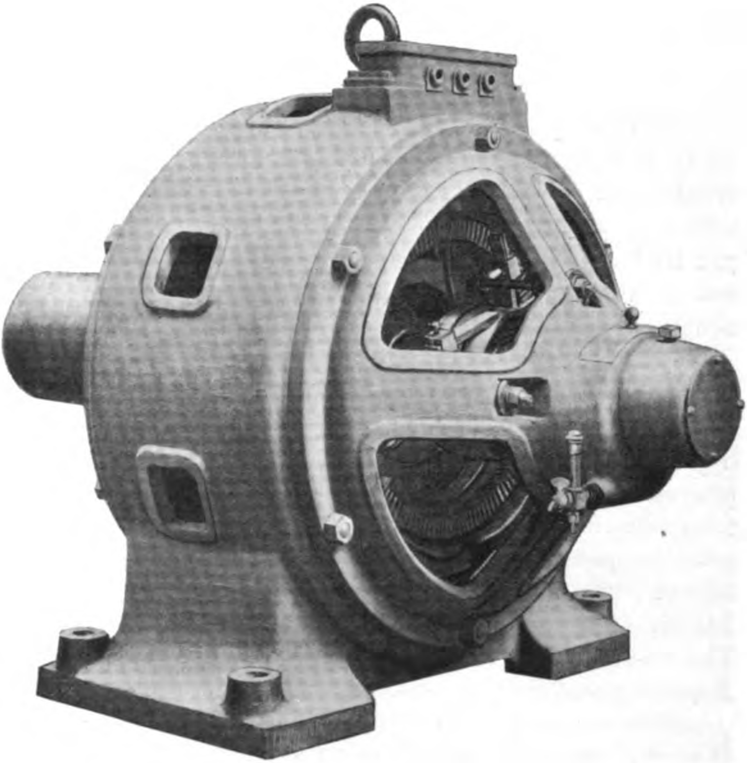


FIG. 495.—THREE-PHASE MOTOR OF 25 H.P. (SCOTT AND MOUNTAIN.)

Slots per pole per phase	3
Dimensions of slots	1·15 × 0·51 inch
Conductors per slot	9
Total number of conductors per phase . . .	216
Dimensions of conductor, two wires in parallel, of bare diam.	0·120 inch
Radial depth of iron behind teeth. . . .	2·225 inches

Rotor :—

Diameter at face	16·92 inches
Length of gap	0·04 inch
Internal diameter of core-disks	11 inches
Length between core-heads	6 inches
Total number of slots	120
Slots per pole per phase	5
Dimensions of slot	1·1 × 0·22 inch
Conductors per slot	2
Total number of conductors per phase	80
Dimensions of conductor	0·45 × 0·135 inch
Radial depth of iron below teeth	1·9 inch

The details of the slots are given in Plate XX.

In an 8-pole machine supplied at 50 cycles per second, the synchronous speed will be $50 \times 60 \div 4 = 750$ RPM. Then with a 4 per cent. slip the speed will drop to $0{\cdot}96 \times 750 = 720$ RPM. The peripheral speeds at these revolutions work out to 3320 and 3190 feet per minute respectively; these being quite usual values for motors of this size.

Next to calculate the consumption of current per phase at full-load we have: The output of 25 H.P. corresponds to $25 \times 0{\cdot}746 = 18{\cdot}65$ KW output. Dividing by the efficiency 0·90, we get the input as 20·72 KW; and dividing by the power-factor 0·91, we get the input in KVA as 22·8. The total volt-amperes being 22,800, there will be 7600 volt-amperes per phase. The voltage between lines being 250, that for one phase will be $250 \div \sqrt{3} = 144$. Dividing 7600 by 144 gives 53 amperes as the full-load current in one phase. The no-load current in one phase was found to be 11 amperes. Assuming that the power-factor at full-load was the maximum power-factor (see p. 707), this makes the utilization coefficient $U = 23$, since $\sqrt{23} = 4{\cdot}8 = 53 \div 11$. Then σ will be 0·0434, and $\cos \Phi = 0{\cdot}915$. The short-circuit current should, therefore, be 253 amperes, and the capacity for overload 2·49 times the normal torque. Taking the Kapp coefficient as 2·15, we find that the flux per pole will be

$$N = \frac{144 \times 10^8}{2{\cdot}15 \times 216 \times 50} = 624{,}000.$$

The pole-pitch is $\pi \times 17 \div 8 = 6{\cdot}66$, and the core-length is

6 inches, making the pole-area 40 sq. inches. Hence the average flux-density over the pole-face will be $B_{av} = 15{,}600$. From which, if no allowance is made for reluctance of the iron parts, the magnetizing current can be calculated by formula [XXIV] on p. 728, as $0·365 \times 15{,}600 \times 8 \times 0·04 \div 216 = 8·4$ amperes. The specific loading is $3 \times 216 \times 53 \div (17\,\pi) = 645$ amperes per inch of periphery. The output coefficients (see p. 709) are $\xi = 17 \times 17 \times 6 \times 735 \div 22·8 = 55{,}400$, and $\beta = 17 \times 6 \div 25 = 4·08$.

The current densities are as follows. In the stator 53 amperes are carried by 2 wires in parallel, each being of diameter 0·120 inch, therefore having section of 0·0113 sq. inch. Hence, the density is $53 \div 0·0226 = 2350$ amperes per sq. inch. In the rotor the current will be a little less than $53 \times 216 \div 80$, say 140 amperes. The conductor section is $0·45 \times 0·135 = 0·061$ sq. inch. Hence the rotor ampere-density will be $140 \div 0·061 = 2300$ amperes per sq. inch.

We found the mean value of B in the gap to be 15,600; hence the maximum B at the middle of the pole-face will be $15{,}600 \times \pi \div 2 = 24{,}600$. The stator slot-pitch is 0·75, and the tooth width about 0·24. The nett iron-length of the core is about 0·9 of the gross length. Hence the maximum density in the stator teeth will be $24{,}600 \times 0·75 \div (0·24 \times 0·9) = 86{,}000$. The rotor slot-pitch is 0·443 and the tooth-width 0·223, hence, similarly, the maximum tooth-density will be about 55,000. In the stator core-body the flux-density will be about 25,000, and that in the rotor core-body about 32,000.

The iron-losses may be estimated as follows:—

Volume of stator teeth	115 cub. in.
Flux-density in stator teeth at maximum .	86,000 lines per sq. in.
Hysteresis waste per cubic inch at 1 cycle per sec., as deduced from curves (Vol. I. pp. 103 and 105)	0·020 watt
Eddy-current waste per cubic inch at 1 cycle per sec.	0·0002 watt
Volume of stator core-body, effective . .	2100 cub. in.
Flux-density in core-body at maximum .	25,000 lines per sq. in.
Hysteresis waste per cubic inch at 1 cycle per sec.	0·003 watt
Eddy-current waste per cubic inch at 1 cycle per sec.	0·00002 watt

Hence the losses will be :—

Stator teeth hysteresis 115 × 50 × 0·020	. . =	115
Stator teeth eddies 115 × 50 × 50 × 0·0002	. =	57
Stator core hysteresis 2100 × 50 × 0·003	. . =	315
Stator core eddies 2100 × 50 × 50 × 0·00002	. =	105
Total iron-losses	. . . . . =	592 watts

From the drawings the mean length of one turn on the stator is taken as 34 inches, and on the rotor as 30 inches. This makes the total length of stator copper in one phase to

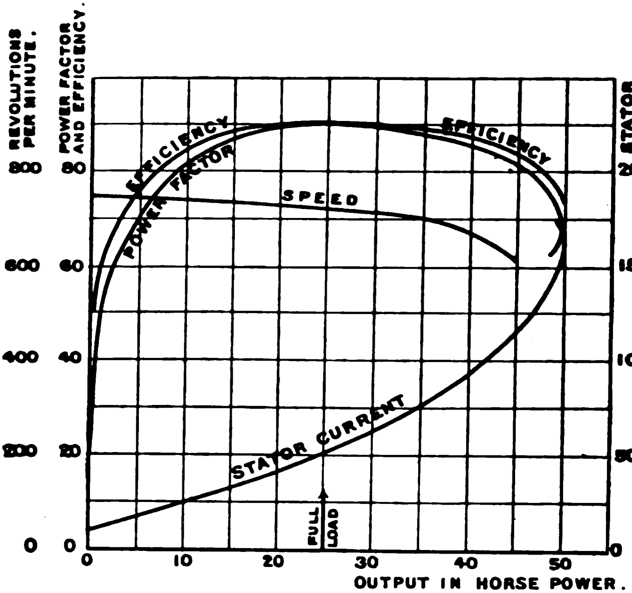


FIG. 496.—TEST CURVES OF 25 H.P. INDUCTION MOTOR. (SCOTT AND MOUNTAIN.)

be 34 × 108 ÷ 12 = 306 feet, and on the rotor 30 × 40 ÷ 12 = 100 feet. The section of the stator conductor is 0·0226 sq. inch, that of the rotor is 0·061. Taking 9·5 microhms as the resistance at 60° C. of 1 foot length of copper bar 1 sq. inch in cross-section, we have as the resistance of the stator winding 306 × 9·5 ÷ (0·0226 × 10^6) = 0·129 ohm ; similarly, that of the rotor 0·0155 ohm.

Fig. 496 gives the test-curves of this motor, which show that at normal full-load the efficiency and the power-factor both reach 90 per cent., while the overload which the motor will endure is just double the normal.

The stator windings are carried out with hand-wound coils, threaded through micanite tubes, with a 3-slot winding in two ranges. The rotor windings are carried out as a two-layer barrel-winding, with bars that are connected at the ends with copper clips sweated on.

The drawings show that the housing is excellently ventilated. The assembled stator core-disks are clamped between an end-flange, which is cast solid with the housing, and an end-ring that is secured in its place by a key-ring sprung into a groove in the inner face of the housing. The rotor core-rings are secured to the hub in a similar way.

Example II.—ANALYSIS OF A SMALL THREE-PHASE MOTOR, constructed by the Oerlikon Machine Works.

$$\text{IM } 4_{\text{P}}\text{—}8_{\text{HP}}\text{—}1450_{\text{RPM}}\text{ ; } 500_{\text{V}}\text{—}3 \times 9{\cdot}5_{\text{A}}\text{ ; } 50\sim.$$

The synchronous speed of this motor would be 1500, but it drops to 1450 RPM on a load of 8 H.P. The tests show that when run at 6 H.P. it has an efficiency of 0·82, and a power-factor of 0·86. At 8 H.P. the efficiency is slightly lower, the power-factor slightly higher. The diameter of the stator bore d is 8·7 inches, the core length l is 4·35. The output coefficient $\beta = dl \div$ H.P. = 4·7. The pole-pitch is 6·8, and the peripheral speed 3430 feet per minute.

The stator (see Figs. 497 and 498) has 72 slots, that is 24 slots per phase per pole. The slots are 1·06 by 0·248, and contain 18 conductors each, making 1296 conductors in all, or 432 conductors per phase. The stator conductor is 0·0785 inch diameter bare, or 0·0985 covered. Its section is 0·00484, and as the full-load current is 9·5 amperes, the current density is just under 2000 amperes per square inch. The stator slot lining is 24 mils thick.

The rotor diameter is 8·66, making the gap 0·020 (*i.e.* 0·5 millimetre). The rotor has 60 slots, that is 15 per pole,

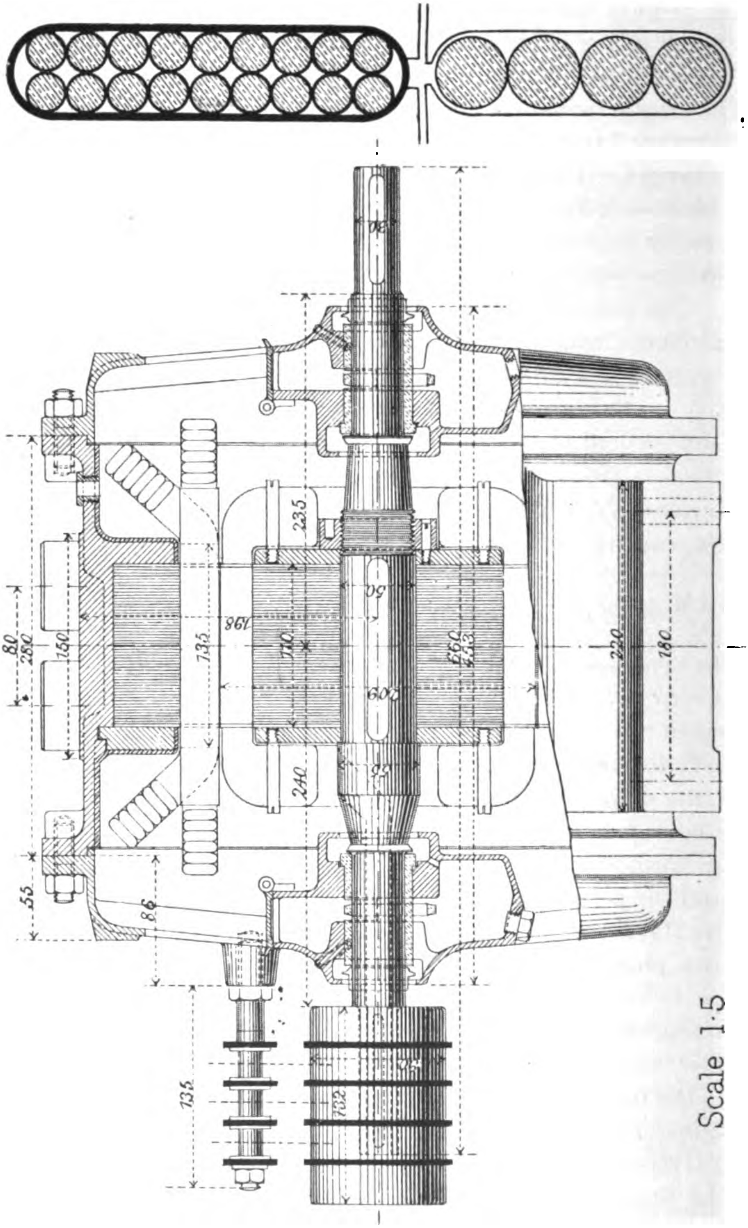


FIG. 497.—THREE-PHASE MOTOR, 8 H.P. AT 1450 RPM (OERLIKON CO.). Slots twice full size.

or 20 per phase, or 5 per pole per phase, with 4 conductors per slot. These are of a round wire 0·149 inch in diameter bare, or 0·165 covered. It is wound under pegs as shown in the drawing, and in appearance resembles Fig. 470 *a*, p. 669. Three slip-rings on the shaft are joined to the three windings.

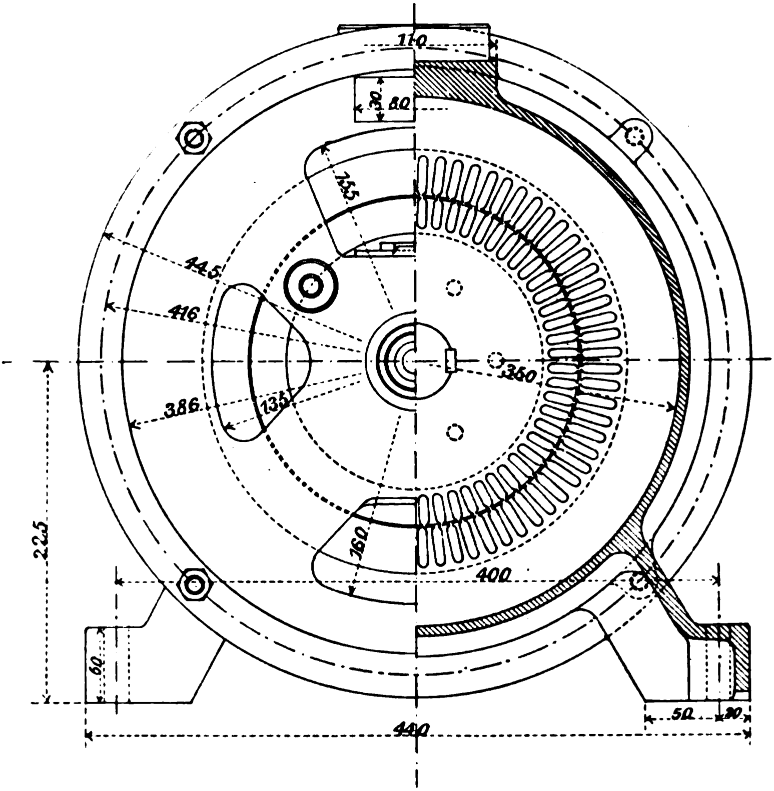


FIG. 498.—THREE-PHASE MOTOR, 8 H.P. AT 1450 RPM (transverse section).

The average density of the flux over the whole pole arc is about 21,000 lines per square inch. This makes the flux per pole about 635,000 lines. The specific loading is about 450 ampere-conductors per inch.

Example III.—ANALYSIS OF A TWO-PHASE INDUCTION MOTOR OF 10 H.P., constructed by Messrs. Johnson and Phillips.

This motor, of which a drawing is given in Fig. 499, has the specification :—

$$\text{IM } 6_P\text{—}10_{HP}\text{—}1000_{RPM}\text{ ; } 200_V\text{—}2 \times 17{\cdot}5_A\text{ ; } 50\sim.$$

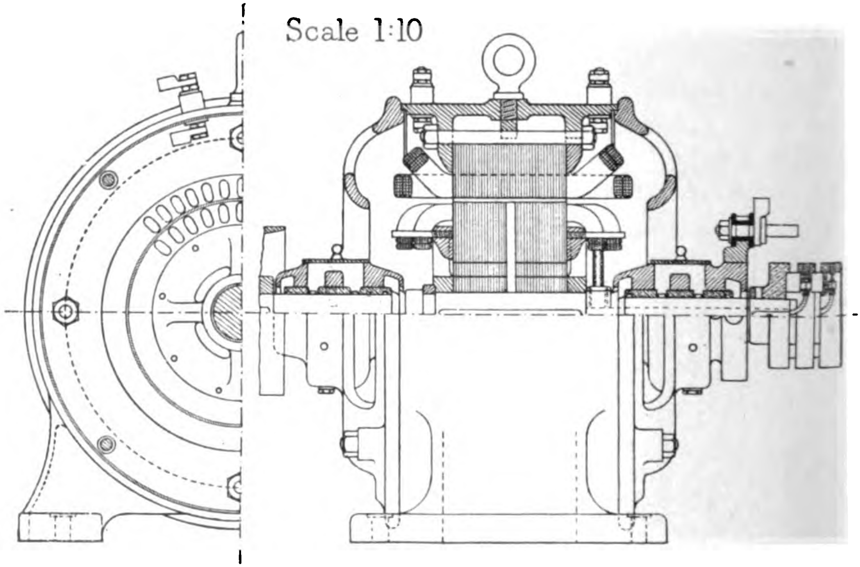


FIG. 499.—TWO-PHASE INDUCTION MOTOR, 10 H.P. AT 1000 RPM, BY JOHNSON AND PHILLIPS.

The supply is by four wires, with 200 volts between the lines. This may be regarded as either a two-phase supply at $282{\cdot}8$ ($= 200 \times \sqrt{2}$) volts per phase, or as a 4-phase supply at ($= 200 \div \sqrt{2}$) volts per phase. The diameter of the stator bore is 12 inches, the core-length 6 inches. Hence, the output coefficient $\beta = 7{\cdot}2$. The stator has 48 slots, that is 8 slots per pole, with 10 conductors per slot. The rotor has 36 slots, and is wound as a star-connected three-phase winding, there being 5 conductors per slot. The stator conductor is 0·144 inch in diameter bare, at 0·164 covered. The rotor conductor is made up of two wires in parallel, each being 0·150 in diameter bare, and 0·170 covered. There are, therefore, 240 conductors per phase in the stator, and 60 conductors per phase in the rotor.

The flux, calculated from the voltage per phase, the frequency, and the conductors per phase, is 1,100,000 lines; and as the pole area is 34 square inches the average B is 32,400 lines per square inch. The current in one primary conductor is 17·5 amperes, and there are 480 conductors around the periphery of 37·6 inches; hence the specific loading q = 224 amperes per inch. The peripheral speed is 3140 feet per minute. The current densities are 1100 amperes per square inch in the stator, and 1300 in the rotor. The gap is 0·0235 inch, and the no-load current 7·5 amperes.

According to the tests the efficiency at full-load is 86 per cent., at half-load 81. The power-factor at full-load is 0·86. The slip at full-load is 3 per cent.

Example IV.—ANALYSIS OF DESIGN OF A HIGH-SPEED THREE-PHASE MOTOR, constructed by the Oerlikon Machine Works.

$$\text{IM } 6_{\text{P}}\text{—}335_{\text{HP}}\text{—}980_{\text{RPM}};\ 2000_{\text{V}}\text{—}3 \times 82{\cdot}5_{\text{A}}\text{—}50\sim.$$

This is a special type of three-phase induction motor, with ventilated cores, having a high efficiency of 0·94 and a power-factor of the unusually high value of 0·93 (at its maximum).

The following are the principal data:—

Stator.—Diameter at face 29·745; the external diameter of the core-disks 45·15; making the stator-depth 7·702. The over-all diameter of the housing is 55·1 inches. The gross-length of the stator-core is 13·8 inches, with two air-ducts about ½ inch wide. There are 72 half open slots, each 0·75 inch wide by 1·7 inch extreme length. Each slot contains 10 wires grouped 2 in parallel so as to act as 5 conductors, the wire being in diameter 166 mils bare, 180 mils covered.

Rotor.—The diameter at face is 29·625 inches; internal diameter 12·75 inches; length of core 13·8 with two ducts, each about 0·8 wide. There are 89 slots, closed all but a narrow slit at the face, each about 0·472 inch wide, by 1·46 deep. In each slot are 2 conductors, each made up of 3 strips of copper 0·434 by 0·107 inch. The cores are mounted upon a cast-iron spider on a shaft 6·3 inches thick. The gap is 0·040.

Windings.—The stator-coils are hand-wound, the bends being in two ranges, both of which are bent up at about 45°, but one projecting out further than the other one before it bends. The rotor winding is a 2-layer barrel drum-winding, with connectors taken off at 3 points, and carried down through the shaft (which is tubular at one end) to the slip-rings.

Fig. 500 gives sketch-sections of this machine.

We will now analyse the design and deduce the constants.

At 50 periods per second a 6-pole machine will make $50 \div 3$ revolutions per second or 1000 RPM if up to synchronism. The motor is designed to give a full-load speed of 980 RPM, that is to say, there will be a 2 per cent. slip.

At the synchronous speed of 1000 RPM the surface-speed of the rotor will be 7760 feet per minute. With a 2 per cent. slip, the surface-speed will be 7600 (an unusually high surface-speed).

The output is 335 H.P., which, expressed in KW, is $335 \times 0·746 = 250$; and the KW input will be $250 \div 0·94 = 266$; while the KVA are found by dividing by $\cos \phi$ $(=0·93)$, viz. $= 286$. The volts between lines are 2000, therefore in one phase are equal to $2000 \div \sqrt{3} = 1154$. The volt-amperes in one phase being $286 \times 1000 \div 3 = 95{,}333$, the amperes in one phase will be $96{,}333 \div 1154 = 82·5 = C_1$.

As $\cos \Phi$ is stated to have a value of 0·93, it follows that $\sigma = \frac{1 - \cos \Phi}{1 + \cos \Phi} = 0·0362$; whence, $\sqrt{\sigma} = 0·19$; $U = 27·5$; and $\sqrt{U} = 5·25$. Hence $C_m = 82·5 \times 0\ 19 = 15·7$ amperes; and $C_{sh} = 432$ amperes. It is, therefore, possible at once to draw the Heyland semicircle. The over-load factor $y = 2·7$. The stator has a 4-slot winding, hence, we may

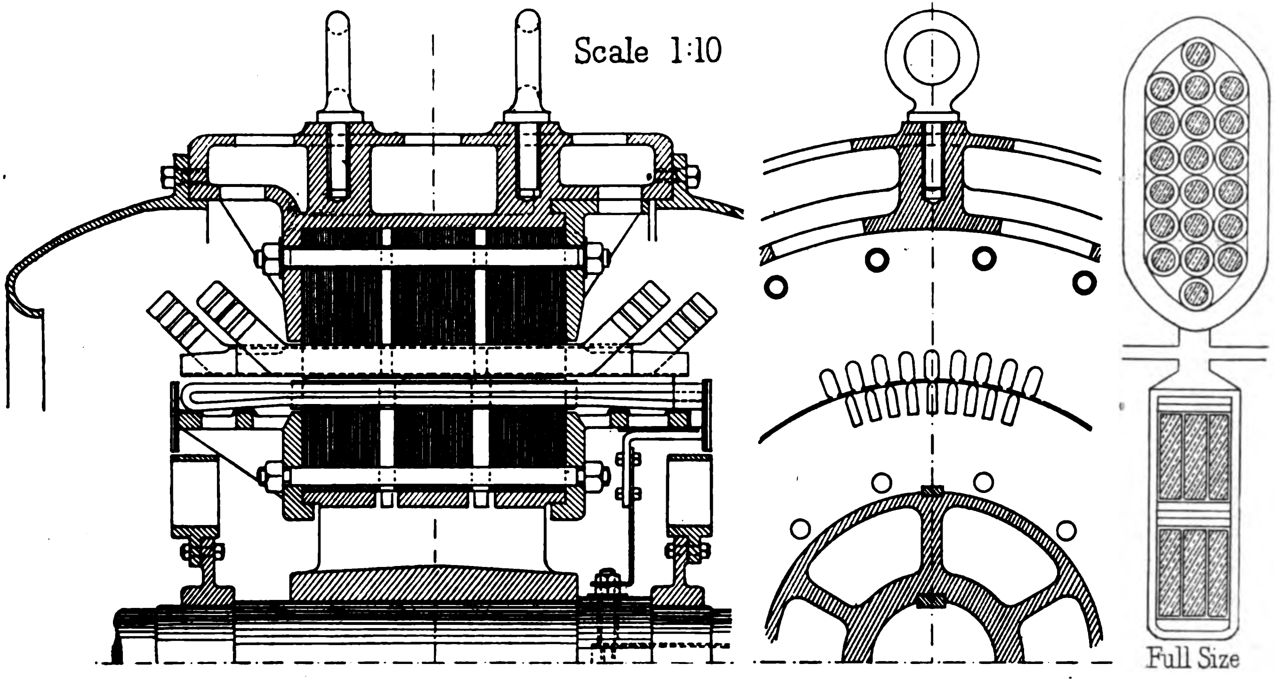


FIG. 500.—HIGH-SPEED THREE-PHASE MOTOR, 335 H.P. AT 980 RPM (OERLIKON CO.).

take k as $= 2{\cdot}12$. There are 24 slots to each phase, each containing 10 conductors, so $Z_1 = 240$. Hence, by equation [XIV] p. 722, $N = \frac{1154 \times 10^8}{2{\cdot}12 \times 50 \times 240} \times 4{,}550{,}000$ lines. The periphery of the stator-face being $29{\cdot}745 \times \pi = 93{\cdot}5$, the pole-pitch is 15·55 inches. The core-length, less the ducts, being 12·8 inches, the gross area of pole-face will be $12{\cdot}8 \times 15{\cdot}55 = 200$ sq. inches; whence it follows that the mean flux-density in the gap B_{av} is $4{,}550{,}000 \div 200 = 22{,}750$ lines per sq. inch.

There being 240 conductors per phase, each carrying at full-load 82·5 amperes, and as the periphery is 93·5 inches, it follows that q the specific loading is $3 \times 240 \times 82{\cdot}5 \div 93{\cdot}5 = 636$ ampere-conductors per inch.

Output Coefficients.—In this machine $d^2 l$ amounts to 12,200, while KVA ÷ RPM = 0·292, making the output coefficient ξ (p. 709) = 41,700. From $dl = 411$ we get the coefficient β (p. 710) as 1·225, an unusually low figure.

Current Densities.—In the stator 82·5 amperes flow in conductor, consisting of 2 wires of 0·0241 sq. inch section each, making the density 1700 amperes per sq. inch. In the rotor the current is about 317 amperes, with a cross-section of about 0·139 inch, making the density about 2260 amperes per sq. inch.

Magnetic Densities.—The average B in the gap is 22,750. Hence the maximum B is about 35,500. The stator slot-pitch is about 1·35 inch, and the slot is 0·75 inch wide, making the tooth-breadth 0·6 inch. As the core-length is 13·8 inches, the nett iron length about 11·5 inches and the leakage coefficient about 1·018, we have as tooth-density,

$$B_t = 35{,}500 \times \frac{1{\cdot}35}{0{\cdot}6} \times \frac{13{\cdot}8}{11{\cdot}5} \times 1{\cdot}018 = 97{,}000.$$

In the rotor, similarly, the slot-pitch being 1·0 inch, and the slot-width 0·472 inch, the tooth-breadth is 0·528 inch. So taking the nett iron length as 11·2 inches, and the leakage coefficient 1·018, the tooth-density becomes

$$35{,}500 \times \frac{1{\cdot}0}{0{\cdot}528} \times \frac{13{\cdot}8}{11{\cdot}2} \times 1{\cdot}018 = 84{,}000.$$

Magnetizing Current.—Neglecting the reluctance of the iron parts, the magnetizing current should be, by formula [XXIV] p. 728, $C_m = 0·365 \times 22{,}750 \times 8 \times 0·060 \div 240 = 15·6$ amperes. This is consistent with the claim that $\cos \phi = 0·93$.

Copper-Losses.—The mean length per turn of the stator-winding would appear from the drawings to be about 90 inches, that of the rotor about 76. Hence, at 60° C., the resistance of one phase would be :—

$$\text{Stator: } 240 \times \frac{90}{0·0481} \times \frac{0·396}{10^6} = 0·17.$$

$$\text{Rotor: } 60 \times \frac{76}{0·139} \times \frac{0·396}{10^6} = 0·015.$$

The voltage drops are as follows :—

Stator: $82·5 \times 0·17 = 14$ volts.
Rotor: $317 \times 0·015 = 4·7$ volts.

The watts lost are :—

Stator: $3 \times 82·5 \times 82·5 \times 0·17 = 3500.$
Rotor: $3 \times 317 \times 317 \times 0·015 = 4500.$

Iron-Losses.—The net volume of iron in the stator teeth is about 1700 cubic inches ; that of the stator core body, so far as it acts magnetically, is about 7000. Reference to tables of losses or to curves (see p. 581, or Vol. I. p. 103) shows that in the former, with a maximum B of 97,000, the waste of power per cubic inch per cycle per second by hysteresis is 0·020 watt, and by eddies (assuming a thickness of 0·025 inch for the stampings) is 0·00025 watt. Hence the loss in the teeth will be, by hysteresis, $1700 \times 50 \times 0·020 = 1700$ watts ; and by eddies $1700 \times 50 \times 50 \times 0·00025 = 1350$ watts, or in total 3050 watts. As to the core-body the maximum density will be about 56,000. At this density, the waste per cycle per second per cubic inch, by hysteresis, is about 0·008 watt, and by eddies is 0·00009 watt. Hence the loss in the core will be, by hysteresis, $7000 \times 50 \times 0·008 = 2800$; and by eddies, $7000 \times 50 \times 50 \times 0·00009 = 1580$; or in total 4380 watts. Adding losses in teeth and core together we get 7430 watts as the total iron-loss.

Efficiency.—Assuming that friction, etc. absorb 1 per cent. of the output, the losses therefore appear to be: friction, etc. 2500, iron-losses 7430, copper stator losses 3500, copper rotor losses 4500, total 17,930 watts. The output (335 H.P.) being 250,000 watts, the efficiency will be 250,000 ÷ 267,930 = 0·933 or 93·3 per cent. The efficiency claimed is 94 per cent.; which may be, if the iron-loss estimated above has been over-estimated.

Slip.—The rotor copper waste being 4500 watts, the corresponding slip will be 4500 ÷ 267,930 = 0·017 or 1·7 per cent.; and the full-load speed will be 1000 × (1 − 0·017) = 983 RPM.

Coefficient of Dispersion.—This may be checked by Behn Eschenburg's formula, p. 728. The following are the details for insertion: $X = 13{\cdot}5$; $Y = 0{\cdot}25$; $\delta = 0{\cdot}04$; $\epsilon = 0{\cdot}1$; $\tau = 15{\cdot}55$; $l_i = 11{\cdot}5$; whence we have

$$\sigma = \frac{3}{181} + \frac{10 \times 0{\cdot}04 \times 0{\cdot}1}{13{\cdot}5 \times 0{\cdot}25 \times 15{\cdot}55} + \frac{5 \times 0{\cdot}04}{11{\cdot}5}$$

$$= 0{\cdot}0165 + 0{\cdot}0008 + 0{\cdot}0174 = 0{\cdot}0347.$$

This would make the maximum power-factor 0·932.

LARGE INDUCTION MOTORS FOR SLOW-SPEED PUMPS.

One of the special problems in connection with induction motors is their design for use with large slow-speed pumps generally for the purpose of draining mines.

The very slow speed at which these motors must necessarily run, is not a condition which conduces to a good performance with respect to power-factor and efficiency.

A slow speed, even with low frequencies, necessitates many poles; and so to get the proper pole-pitch, the diameter must be large, and questions of floor space available and cost in most cases put a limit on the diameter, which becomes, therefore, a good deal smaller than is desirable for good design.

Further, for mechanical reasons, the gap must be longer than for smaller machines of the same output, and this also produces a large amount of leakage, and therefore also a low power-factor. (See p. 691.)

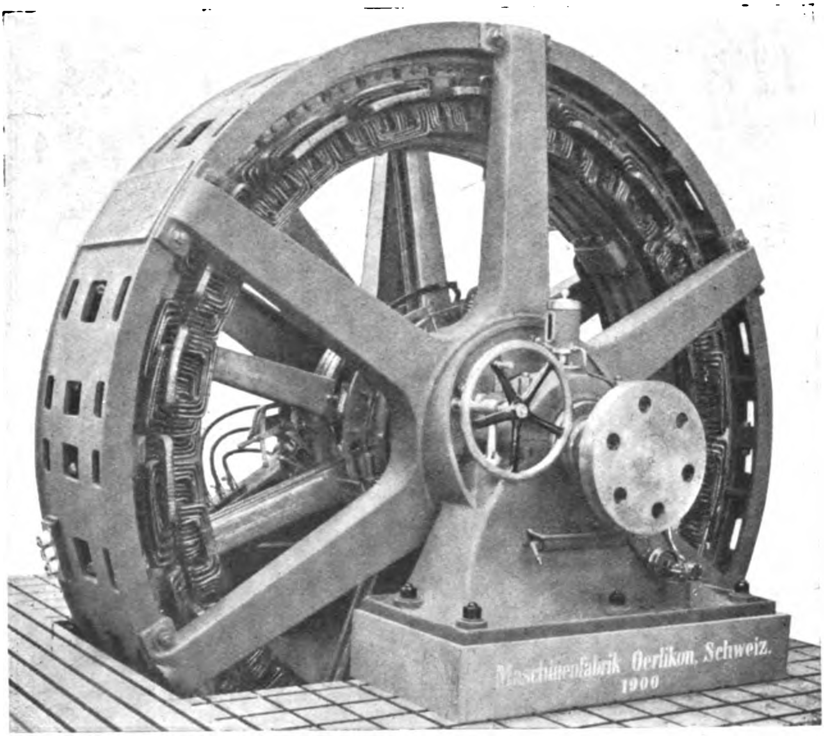


FIG. 501.—SLOW-SPEED THREE-PHASE INDUCTION MOTOR OF 570 H.P. AT 75 RPM (OERLIKON CO.).

In some installations where the pumping motors form the greater part of the load, current is generated at a frequency to suit the motors, rather than at one which is desirable for lighting purposes. In two cases considered below, the frequencies adopted were 20 and 22·5 cycles per second.

Plate XXIII. gives sectional views of a 570 H.P. three-phase motor installed by the Oerlikon Co., of Zürich, for draining the mine of Hösch Steel Works at Dortmund. It runs at 75 RPM at full speed, and has a wound rotor with slip-rings, so that it may start up with a torque 20 per cent. above normal. A general view is given in Fig. 501.

In the second column of the table, data are given concerning a similar but smaller motor of 200 H.P., working at the lower frequency of 20 cycles per second, installed by the same company at Monterrad. The third motor analysed in the table was built by the Bullock Co., of Cincinnati, for a large American water-supply works.

Examples V., VI., VII.—COMPARATIVE ANALYSIS OF SLOW-SPEED INDUCTION MOTORS.

General Specifications.—

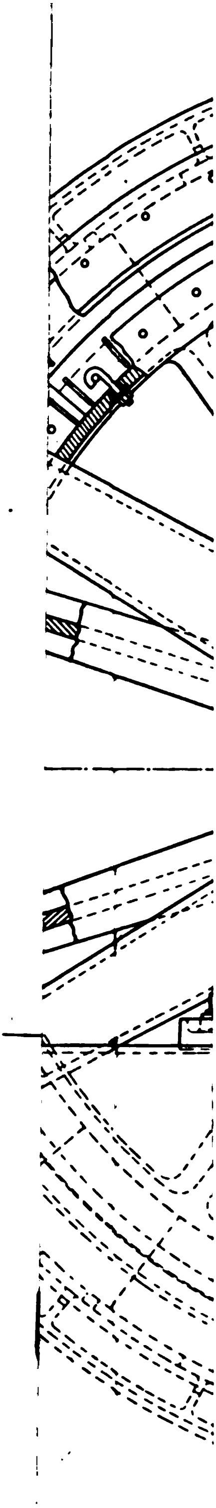
(*a*) IM 36_{P}—570_{HP}—75_{RPM} ; 1900_{V}—$22\frac{1}{2}$ ~ ; Δ-connected.

(*b*) IM 24_{P}—200_{HP}—100_{RPM} ; $\frac{980}{570}$V—20 ~ ; Δ-connected.

(*c*) IM 44_{P}—500_{HP}—163_{RPM} ; $\frac{2000}{1160}$V—60~ ; Y-connected.

DIMENSIONAL DATA (inch units).

	(*a*)	(*b*)	(*c*)
Stator :—			
Diameter at face	130	70·8	156
External diameter . . .	143·8	84·3	182
Length between core-heads . .	14·6	15	8¼
No. of ventilating ducts . .	1	—	1
Width of each duct . . .	0·6	—	½
Iron length of core . . .	12·6	13·5	7·1
Total number of slots . . .	324	216	528
Slots per pole and phase . .	3	3	4
Depth of slot	1·97	1·89	1·70
Width of slot	0·63	0·55	0·45
Number of conductors per slot .	8	5	4
Total number of conductors per phase.	864	360	704
Dimensions of conductor . .	3 wires of dia. =0·150	3 wires of dia. =0·173	2 strips 0·12×0·32
Depth of iron behind slot . .	4·9	4·9	6·3



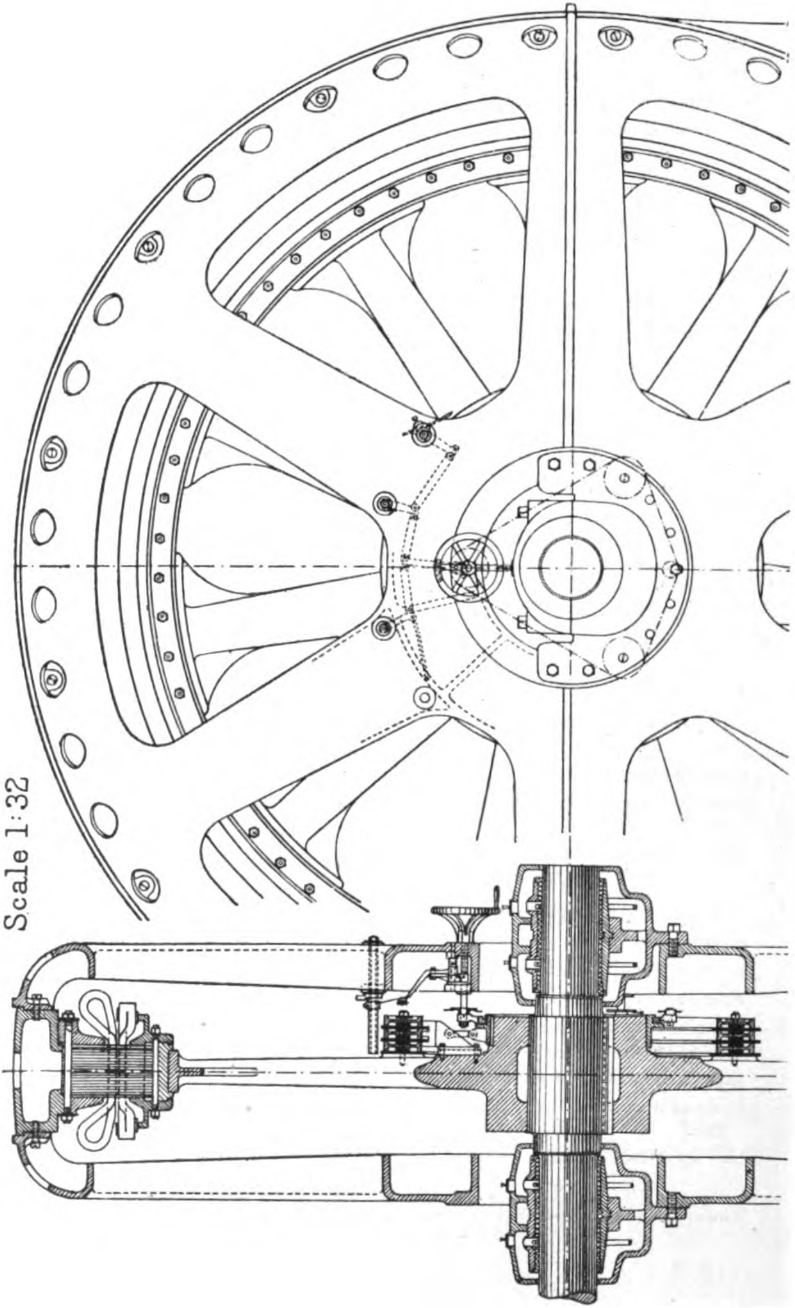


FIG. 502.—SLOW-SPEED THREE-PHASE INDUCTION MOTOR OF 500 H.P. AT 163 RPM (BULLOCK CO.).

Rotor :—	(*a*)	(*b*)	(*c*)
Diameter at face	129·85	70·68	$155\frac{7}{8}$
Internal diameter . . .	119	61·4	$144\frac{1}{4}$
Length between core-heads . .	14·6	15	$8\frac{1}{4}$
No. of ventilating ducts . .	1	—	1
Width of each duct . . .	0·6	—	$\frac{1}{2}$
Iron length of core	12·6	13·5	7·1
Total number of slots . . .	432	281	660
Slots per pole and phase . .	4	4	5
Depth of slot	1·28	1·26	$1\frac{1}{2}$
Width of slot	0·512	0·394	$\frac{3}{8}$
Number of conductors per slot .	1	1	1
Total number of conductors per phase.	144	96	220
Dimensions of conductors . .	19 wires of dia. = 0·142	12 wires of dia. = 0·173	4 strips 1·2 × 0·065
Depth of iron behind slot . .	4·2	3·4	4·3
Length of air-gap . . .	0·071	0·059	$\frac{1}{16}$

CALCULATED DATA AND COEFFICIENTS.

Pole-pitch (inches) . . .	11·34	9·26	11·15
Peripheral speed (feet per minute) .	2550	1850	6650
Square inches on rotor per horse-power.	9·9	16·6	8·1
Esson's coefficient . . .	32,500	40,800	72,000
Full-load stator current (per phase in motor).	98·5	109	118
Ratio of transformation . .	1 : 3	4 : 15	5 : 16
Full-load rotor current (estimated at).	650	345	318
Current-density (stator) . .	1850	1550	1540
,, ,, (rotor). . .	2170	1230	1020
Flux per pole in megalines . .	4·4	3·54	1·42
Average flux-density in gap . .	28,000	26,200	17,900
Average flux-density in stator teeth	60,500	62,000	34,700
Flux-density in stator core . .	38,000	27,000	15,800
Estimated magnetizing ampere-turns required per pole.	802	540	390
Estimated exciting current per phase	33	39	23
Ratio of diam. of rotor to length of gap.	1830	1200	2500
Slot space-factor, stator . .	0·36	0·35	0·40
,, ,, rotor. . .	0·50	0·57	0·55
Ampere-conductors per inch (stator)	624	530	568

ESTIMATION OF LOSSES.

	(*a*)	(*b*)	(*c*)
Copper-Losses (1) *Stator* :—			
Mean length of one turn (inches) .	76	79	66½
Number of turns per phase . .	432	180	352
Total length of conductor per phase (feet)	2730	1185	1950
Section of conductor (sq. in.) .	0·053	0·0705	0·0768
Resistance per phase at 60° C. .	0·488	0·246	0·24
Full-load stator copper-loss (KW).	13·8	8·7	10
Copper-Losses (2) *Rotor* :—			
Mean length of one turn (inches) .	73	75	54
Number of turns per phase . .	72	48	110
Total length of conductor per phase (feet)	438	300	445
Section of conductor (sq. in.) .	0·30	0·28	0·312
Resistance per phase at 60° C. .	0·0138	0·010	0·0153
Actual full-load rotor current per phase	516	356	346
Full-load rotor copper-loss (KW) .	11·1	3·8	5·5
Iron-Losses, Stator :—			
Flux-density in teeth (maximum) .	90,000	93,000	52,000
,, in core . . .	38,000	27,000	15,800
Loss per lb. in teeth (watts) . .	1·24	0·84	1·42
,, ,, in core (watts) . .	0·25	0·10	0·20
Weight of teeth (lbs.) . . .	1430	750	810
,, of core (lbs.) . . .	6200	4920	6600
Total loss in teeth (KW) . .	1·29	0·63	1·15
,, in core (KW) . .	1·55	0·49	1·32
Total iron-loss in stator (KW) .	2·80	1·12	2·47

POWER FACTOR AND DISPERSION COEFFICIENTS.

Ratio of air-gap breadth to pole-pitch	160	157	178
Ratio of pole-pitch to effective length of core	0·9	0·69	1·57
Value of U, (*a*) by Hobart and Behrend's method	12·25	12·6	19·8
Value of U, (*b*) by Behn-Eschenburg's method	16·1	18·3	19·2
Actual value of U . . .	10·8	(15·7)	17
Actual maximum value of cos ϕ .	0·82	(0·88)	0·89
Overload factor	1·79	2·1	2·14

Remarks.—(*a*) Owing to the motor having to be trans poted in pieces through the narrow passages of the mine to the pump-chamber, the following points were stipulated : (1) that none of the component parts of the motor should exceed 6 feet 6 inches in any direction, and (2) that no such part should weigh more than 5 tons.

To fulfil these conditions the stator had to be made in two halves ; and in order to have little connecting up to do after the machine had left the works, the three-phase coil-windings on both the stator and rotor are carried out in three ranges.

In all these slow-speed motors special consideration has to be given to the stiffness of the motor frame, because any slight eccentricity in the mounting of the revolving part produces enormous strains. The housing is usually supported by rigid end-shields constructed with spokes, mounted on trunnion bearings (see Brown's Paderno Alternator, Plate VII.).

The motor is started up with full-load torque by the insertion of resistance in the rotor, the usual arrangements for short-circuiting and lifting the brushes when full speed is attained being fitted. Because of the considerable fluctuations in shaft-effort required even for three-throw pumps, the rotor was designed with a considerable moment of inertia, and for the same reason the full-load current density in the rotor conductors is carried to a much higher figure than is usually the case, a larger full-load percentage slip being thereby obtained, although, of course, the copper-loss in the rotor is increased.

A short-circuit test was made on this machine, and the current was found to be 110 amperes with the terminal voltage reduced to 390, and again 200 amperes at 600 volts.

(*b*) The important consideration in connection with this installation was the danger from fire-damp in the mine. In order that no risks are run, the motor is provided with a permanently short-circuited rotor, and the motor is started up with the generator, which is first excited to full-load value and is then slowly brought up to speed, the governor being regulated in order to allow the engine to start up with a heavy load. All switches and protective devices for the motor are placed in the generating station above ground. In order to take up the irregular working effort of the pump, the motor has been connected by a flexible coupling to its pump.

(*c*) Fig. 503 gives the test-curves of the Bullock Co.'s motor, which was designed by Mr. B. A. Behrend. It will be noted that at full-load of 500 H.P. the total losses are almost exactly double of the core and friction losses.

Fig. 504 shows a 130 H.P. motor constructed by Lahmeyer and Co., of Frankfort, for mine-pumping. It runs at 75 RPM on a supply of frequency 15~, at 750 volts. The stator has 288 slots (12 per pole), wound with a wire of 0·164 inch diameter. The rotor has an eleven-phase short-circuited bar-winding in 264 slots (11 per pole) of conductors 0·72 inch diameter.

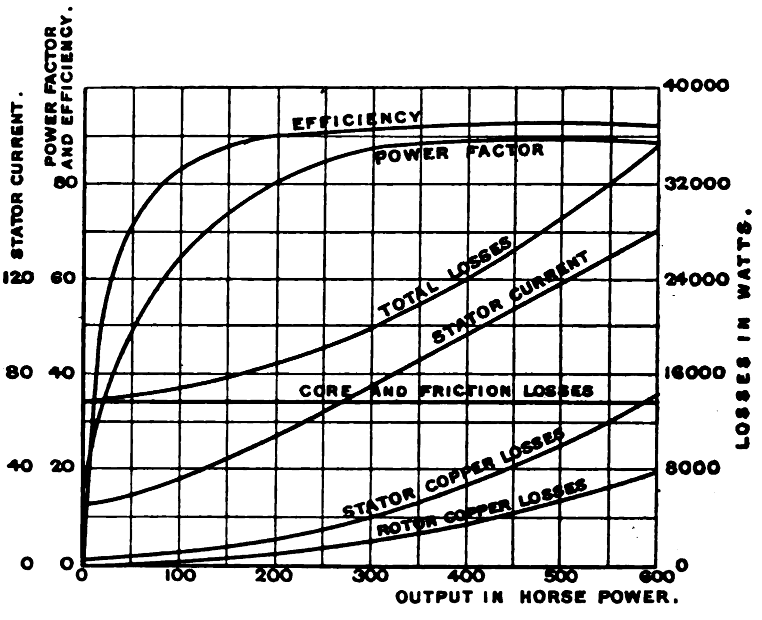


FIG. 503.—TEST-CURVES OF BULLOCK CO.'S INDUCTION MOTOR.

Messrs. Witting, Eborall and Co., have described some similar pump motors of 150 H.P., running at 68 RPM, on a 21 cycle supply. This required a rotor diameter of 114 inches with a clearance of only 0·07 inch. This feature called for special stiffness of the stator structure, the rotor bearings being carried by special end-shields consisting of spider-

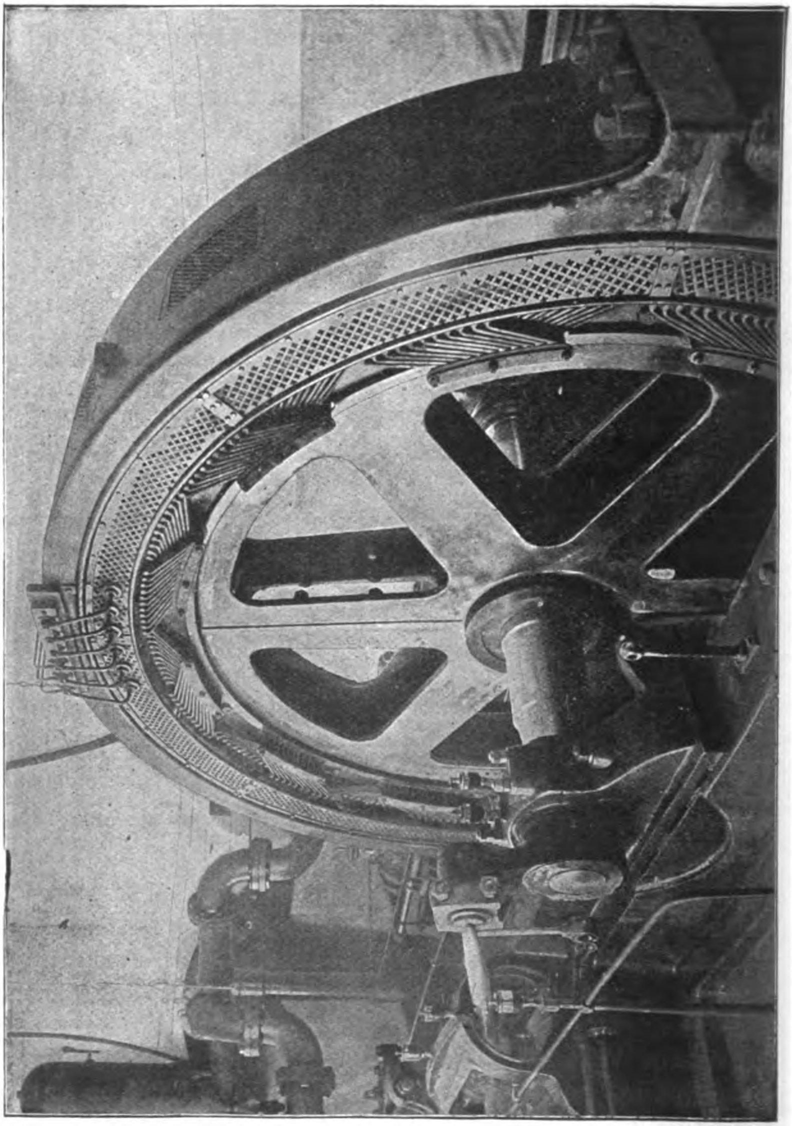


FIG. 450.—INDUCTION MOTOR FOR PUMPING: 130 H.P. AT 75 R.P.M.

shaped castings bolted to the stator frame around its whole periphery. The overload capacity was about 270 H.P., and the efficiency over 85 per cent. for all loads from 35 to 225 H.P.

Examples VIII., IX., X.—COMPARATIVE ANALYSES OF THREE THREE-PHASE MOTORS FOR TRACTION.

General Specifications.—

(*a*) IM 6_{P}—150_{HP}—760_{RPM} ; 500_{V}—38~.

Constructed by Brown, Boveri and Co. for Jungfrau Railway (see Plate XXII.)

(*b*) IM 8_{P}—150_{HP}—570_{RPM} ; 500_{V}—38~.

Constructed by Oerlikon Co. for Jungfrau Railway goods traffic.

(*c*) IM 6_{P}—120_{HP}—760_{RPM} ; 500_{V}—38~.

Constructed by Oerlikon Co. for Jungfrau Railway passenger traffic.

DIMENSIONAL DATA (inch units).

	(*a*)	(*b*)	(*c*)
Stator :—			
Diameter at face	28·8	25·6	23·65
External diameter . . .	40·5	36·25	34·3
Length between core-heads . .	11·04	16·5	9·45
No. of ventilating ducts . .	—	1	—
Width of each duct . . .	—	0·8	—
Iron length of core . . .	9·9	14·1	8·5
Total number of slots . . .	108	72	108
Slots per pole and phase . .	6	3	6
Depth of slot	1·89	1·32	1·54
Width of slot	0·47	0·53	0·34
Number of conductors per slot .	5	3	3
Total conductors per phase . .	180	72	144
Number of wires in parallel per conductor.	1	3	2
Diameter of wire. . bare .	0·256	0·190	0·197
Diameter of wire. . covered .	0·296	0·212	0·220
Iron length behind slot. . .	4·0	3·1	3·8
Length of gap	0·059	0·043	0·047

Rotor:—	(*a*)	(*b*)	(*c*)
Diameter at face	28·7	25·5	23·56
Internal diameter . . .	18·1	7·9	15·0
Length between core-heads . .	11·04	16·5	9·45
Number of ventilating ducts . .	—	1	—
Width of each duct . . .	—	0·8	—
Iron length of core	9·9	14·1	8·5
Total number of slots . . .	126	120	162
Slots per pole and phase . .	7	5	9
Width of slot	0·32	0·32	0·24
Depth of slot	1·89	1·26	0·925
Number of conductors per slot .	2	1	1
Total conductors per phase . .	84	40	54
Dimensions of conductor . .	0·16 × 0·71	17 wires dia.=0·118	5 wires dia.=0·158
Depth of iron behind slot . .	3·4	3·2	3·35

Calculated Data and Coefficients.

	(*a*)	(*b*)	(*c*)
Pole-pitch (inches) . . .	15·1	10·1	11·85
Peripheral speed (feet per minute) .	5730	3840	4500
Sq. in. of rotor surface per H.P. .	6·65	8·9	5·6
Full-load stator current (per phase)	160	160	125
Ratio of transformation . .	15 : 14	9 : 5	2 : 1
Current-density (stator) . .	1560	1880	2050
,, ,, (rotor) . .	1260	1550	2500
Magnetic data for Y-connections (running conditions) :			
Flux per pole in megalines .	2·30	2·87	2·49
Average flux-density in air-gap	15,400	20,200	22,500
Ditto in stator teeth . .	27,500	29,000	46,900
Flux-density in stator core .	29,000	32,600	35,000
Estimated exciting current per phase (full-load).	45	46	34
Ratio of diam. of rotor of length of gap.	490	595	504
Ampere-conductors per inch (full-load).	477	430	727
Magnetic data for Δ-connections (starting conditions) :			
Flux per pole in megalines .	3·98	4·95	4·30
Average density in air-gap .	26,600	34,800	38,500
Average density in stator teeth	47,700	50,000	80,000
Density in stator core . .	50,200	56,400	60,500

These motors were built at different times for service on the mountain railway up the Jungfrau. The object with which the data, kindly furnished by the respective makers, are set down here, is not for the purpose of comparative criticism, as this would be manifestly unfair to the makers. It is solely for the illustration of constructive principles, all the motors having done excellent service. The conditions for the design of such motors differ from those of motors for factory work or for pumping. Being geared, they may run at fairly high speeds. It will be noted that in all of them the gaps exceed one millimetre, on rotors of about 2 feet diameter. The advantage of employing induction motors on steeply inclined railways, rather than the continuous-current series motors usually employed for ordinary tramway lines is that by their automatic braking action during descent the speed cannot much exceed the speed of synchronism.

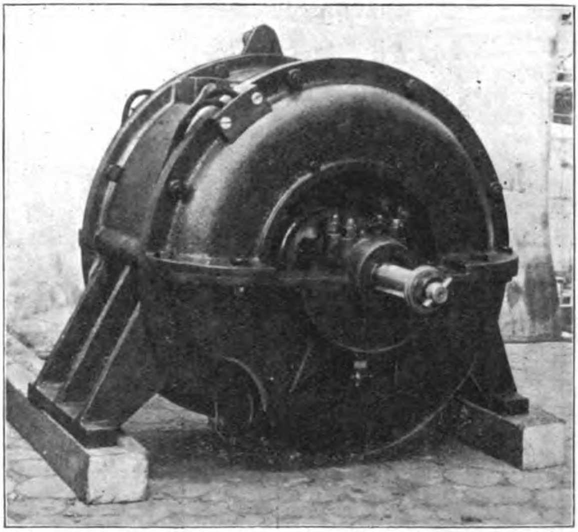


FIG. 505.—JUNGFRAU RAILWAY MOTOR, 125–150 H.P.
(BROWN, BOVERI AND CO.).

The motors of Brown, Boveri and Co. weigh about 2·6 tons each. There are two on each of the locomotives equipped by them, the locomotive complete weighing about 13 tons. Plate XXII. gives a view of this motor, the general appearance of which is seen in Fig. 505.

The passenger-traffic locomotives equipped by the Oerlikon Co. have each two motors of 120 H.P., weighing 2·1 tons each. After three hours at full-load the temperature-rise is stated not to exceed 25° C. in any part. The no-load current is 25 amperes, and the no-load loss 4200 watts. The short-circuit current is reckoned at 800 amperes. At normal load the efficiency is stated at 0·92, the power-factor at 0·90, the slip at 1½ per cent., while the copper-losses are 2½ per

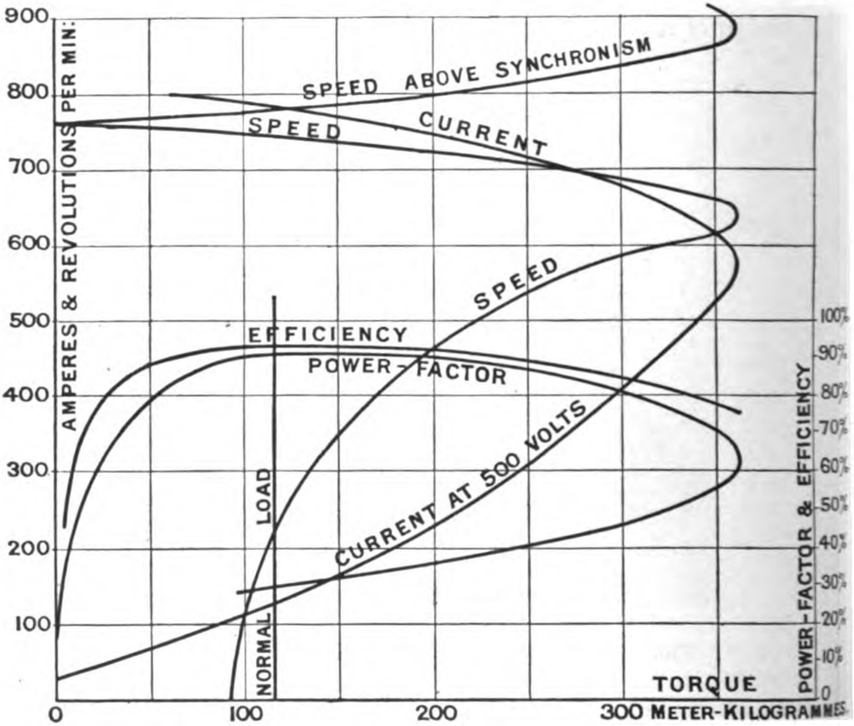


FIG. 506.—TEST CURVES OF JUNGFRAU RAILWAY MOTOR, 120 H.P. (OERLIKON CO.).

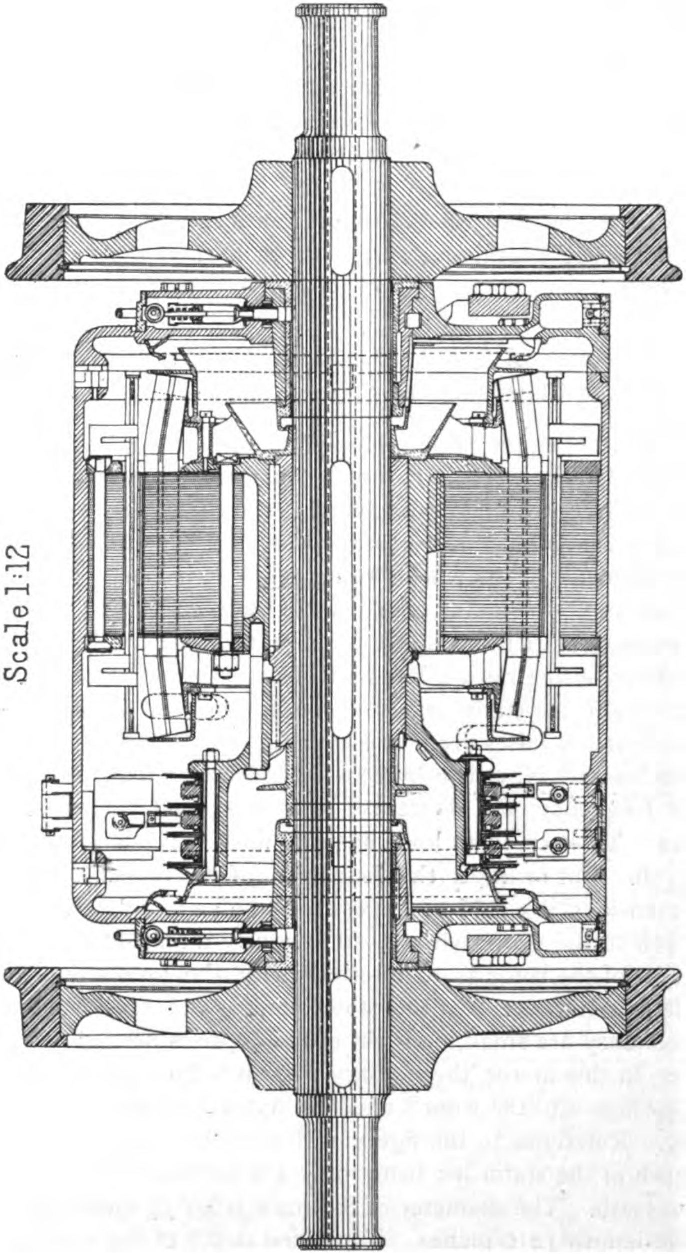


Fig. 507.—Three-Phase Railway Motor of 150 H.P. (Ganz and Co.). (Scale 1 : 15.)

cent. The resistance of one phase in the stator is 0·04 ohm, that in the rotor 0·01 ohm.

In 1904 a new locomotive of Brown, Boveri and Co. was added to the equipment of the line. This is slightly heavier than the older type, and differs in the detail of having the rotor provided, not only with the usual slip-rings for connexion to the starting resistances, but also with a commutator to introduce continuous current from a small battery during descent.

Example XI.—A DIRECT-DRIVING THREE-PHASE MOTOR, of Ganz and Co.

Another example of a three-phase motor for railway service is afforded by a 150 H.P. motor constructed by Ganz and Co. for the Lecco-Collico railway, and shown in Fig. 507. The rotor is mounted direct on the shaft, and runs at 900 RPM, the diameter of the wheels being about 49 inches. The stator is mounted on sleeves on the same shaft, so that it is always concentric notwithstanding the rise and fall of the shaft in running on the rails. The motor is wound for six poles, with 72 slots in the stator and 90 in the rotor. One peculiarity of construction arises from the necessity of keeping down the iron-losses. Now the iron-losses in the primary depend on the frequency, the degree of saturation, and the volume of iron. To keep them low the iron must have a great radial depth. But owing to the limitation of the possible external dimensions, it would be impossible to give an adequate radial depth to the stator without so seriously diminishing the diameter of the rotor as to interfere with the production of an adequate torque. On the other hand, as the iron-losses in the secondary are small, no great radial depth is needed. Therefore, in this motor, the primary current is introduced by three slip-rings into the rotor; and the stator becomes the secondary. Reference to the figure will show how small the radial depth of the stator is; being only 4·8 inches, or 3·375 behind the teeth. The diameter of the rotor is 30·75 inches, and the core-length 12·6 inches. The radial depth of the rotor is 9·9

TABLE XXI.—DESIGN COEFFICIENTS OF INDUCTION MOTOR EXAMPLES.

—	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
n_{ph}	3	3	2	3	3	3	3	3	3	3
p	8	4	6	6	36	24	44	6	8	6
H.P.	25	8	10	335	570	200	500	150	150	120
RPM	750	1,450	1,000	980	75	100	163	750	550	550
B_g	15,600	21,000	32,400	22,750	28,000	26,200	17,900	15,400	34,800	38,500
q	645	450	224	635	624	530	568	477	430	727
v	3,320	3,430	3,140	7,760	2,550	1,850	6,650	5,730	3,840	4,500
β	4·08	4·7	7·2	1·225	3·16	5·3	2·58	2·12	2·84	1·78
ξ	55,400	59,000	86,400	41,700	32,500	40,800	72,000	37,000	32,600	26,200
a_s	2,350	2,000	1,100	1,700	1,850	1,550	1,540	1,560	1,880	2,050
a	2,300	3,000	1,300	2,260	2,170	1,230	1,020	1,260	1,550	2,500
f	50	50	50	50	22½	20	60	38	38	38
τ	6·66	6·8	6·3	15·5	11·34	9·26	11·15	15·1	10·1	11·85
U	23	17·7	15·7	27·5	10·8	15·7	17·0	12·6	12·0	13·6
cos Φ	0·91	0·88	0·88	0·93	0·82	0·88	0·89	0·85	0·85	0·86
y	2·5	2·22	2·08	2·7	1·79	2·1	2·14	1·92	1·88	2·0
δ	0·04	0·020	0·0235	0·040	0·071	0·059	0·062	0·059	0·043	0·047
$d+\delta$	425	435	510	743	1,830	1,325	2,520	490	595	504

inches, or 6·3 behind the teeth. The over-all diameter of the motor is 41·5 inches. The primary voltage is 1850 volts. The 3 slip-rings of copper are separated by mica disks, and 8 carbon brushes make contact on each ring. Both the windings are carried out as wave-windings; those of the rotor being of barrel type, while those of the stator are a mixed type with both barrel windings and connecting forks. The motor can exert during acceleration a maximum power of 750 H.P. Starting resistances of kruppin are introduced into each of the three phases of the secondary in 29 sections; the first 4 sections being for use in starting, the remaining 25 for speed-regulation during running.

The elaborate precautions to prevent the oil from the bearings reaching the electrical parts should be noticed.

In concluding this chapter of examples of induction motors, the preceding Table XXI. displays the various design coefficients in convenient form for comparison.

CHAPTER XVII.

SINGLE-PHASE INDUCTION MOTORS.

MOTORS for use with a single-phase—that is to say, an ordinary—alternate current have the obvious advantage that they need only two lines instead of three or four wherewith to supply them.

In construction these resemble very closely induction motors of the ordinary two-phase type, but as an alternating current in only one phase is supplied, that current is passed through a winding adapted to impress a simple alternating magnetomotive-force. When the motor is in operation the rotor currents react in such a way that the resultant flux is a rotating one which produces a torque almost as effectively as the rotating flux of a polyphase motor. We have now to consider exactly how this comes about, and in what respects these motors differ from polyphase motors.

It should be noted in the first place that the rotating field and torque only come into existence after the motor has been started, so that special starting devices are requisite. The most usual method of starting is to supply to an auxiliary winding on the motor a current differing in phase from the current supplied to the main winding. The difference of phase is produced by one of the various *phase-splitting* devices to which we shall refer at the end of this chapter.

The consideration of these motors has been very much simplified by a method of treating the subject suggested by the late Professor Galileo Ferraris.[1] He regarded the simple alternating field impressed by the stator current as capable of

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[1] Galileo Ferraris, "A Method for the Treatment of Rotating or Alternating Vectors, with an Application to Alternate-current Motors." *The Electrician*, xxxiii. 110, 129, 152, 184, 1894, translated from *Mem. Reale. Accad. d. Sci. Torino*, serie ii. tomo xliv. Dec. 3rd, 1893.

being resolved into two rotating fields which rotate in opposite directions.

If we represent by the vector b_1 which rotates clockwise uniformly about O, Fig. 508, the magnitude and direction of a rotating magnetic field, and by b_2 the magnitude and direction of another field of the same strength rotating in the opposite sense with the same frequency n_1, it will be seen that the direction of the resultant field is always along the line B, and the magnitude of the resultant field will alternate between the values $+2b$ and $-2b$ following a sine-function of the time, so that we may write $B = 2b \sin 2\pi n_1 t$.

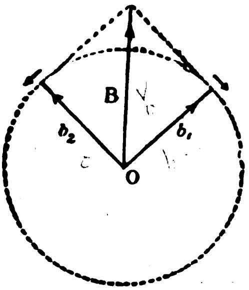

FIG. 508.

Conversely, if we have an alternating field following the law $B_0 \sin 2\pi n_1 t$, as in a monophase motor, we may resolve it into two oppositely rotating fields of the same frequency n_1, and consider the effect of each field separately upon the rotor.

If the rotor turns clockwise with a frequency n_2, the frequency of rotation of the clockwise field with respect to the rotor will be $n_1 - n_2$, and the frequency of rotation of the counter-clockwise field with respect to the rotor will be $n_1 + n_2$.

The way in which the torque changes with slip is now clearly seen by an application of the slip diagram of the polyphase motor already given in Fig. 491, namely, by drawing one diagram for the forward rotating field and another, reversed, for the backward rotating field. In Fig. 509, the ordinates of the curve O P Q U represent the torque for different slips due to the forward rotating field, the synchronous speed being n, and the speed of the rotor m. The ordinates of the curve W_1 Q W represent the torque due to the backward rotating field. Subtracting these from the ordinates of the first curve we obtain T P_1 Q_1 as the true torque curve. This curve is not drawn to scale. The backward torque falls to one-eightieth part of the forward torque at full-load. It will be seen that the resultant torque falls to zero at a speed a little below the synchronous speed; but this occurs when the slip is such a small fraction of 1 per cent. that it hardly serves in practice to differentiate the monophase motor from the polyphase motor. The point Q where the torques due to the oppositely rotating fields are equal and opposite is of course the standstill position. It will also be noticed how the maximum torque is reduced in the single-phase motor.

In the theory of Ferraris the effect of the two impressed fields in producing a torque is considered with regard to the self-induction and resistance of the rotor conductors.

We will consider in the first place a bipolar motor, in which the current supplied to the stator follows a simple sine-function of the time, and we will suppose that the winding is

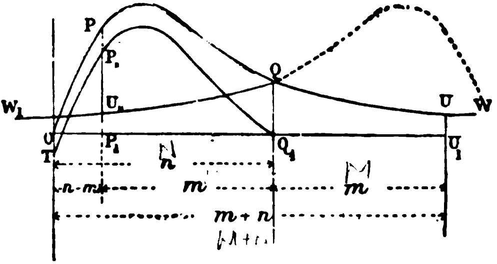

FIG. 509.

so distributed on the motor that the flux-density in the air gap is distributed as a sine-function round the periphery.

The motor is supposed to be running at full-load. We have to show why it runs, how the currents and magnetic field are distributed, and how the torque can be calculated.

In leading up to the Heyland construction for the single-phase induction motor, we must first see how the currents and fluxes are distributed in the motor. For this purpose we are supposed to be given the value of the stator current, the constants of the motor, and the amount of the slip.

Now it is clear that it does not matter for the purposes of the theory whether we regard the field impressed by the stator currents as split up into two oppositely rotating components, or whether we regard the current itself as so split up. We will take the latter convention for reasons which will presently appear. The current diagram obtained is at once convertible into a magnetic diagram by turning it through a right angle and altering the scale.

Let the vertical line in Fig. 510 represent the stator current, a simple alternating current passing always through the same coils of the stator so that the path of its magnetic effect remains constant. A magnetic field produced by a current of this kind we will describe as a *simple alternating field*,[1] as distinguished from a *rotating field*, whose path is always changing in direction. The whole period of one complete alternation is here divided into sixteen intervals for the purpose of plotting the diagram. When the stator current is at its positive maximum, it is represented by the vertical line 5 1, and as it sinks harmonically to zero, the end of the line is represented after successive intervals of time by the points 2, 3, 4 and 5 in the same vertical line. As the current rises to a negative maximum, the successive values are given by the points 6, 7, 8 and 9. Now a simple alternating current of this kind may be regarded as being composed of two currents of half the maximum value rotating in opposite directions. Let the lines 5 P and 5 p be two vectors, each equal to one-half the vertical line 5 1. The vector 5 P is supposed to be rotating clockwise as indicated by the arrow, and 5 p is rotating counter-clockwise. They are depicted at the instant 2 when their resultant is 5 2.

Each of these rotating currents will act upon the rotor in exactly the same way as the rotating current of a three-phase motor, so that the diagrams already described are at once applicable. We have only to consider how the scale of the diagrams is to be fixed. We will suppose that the rotor is revolving clockwise with a frequency n_2, and that it is going only 2·5 per cent. slower than the vector 5 P whose frequency is n_1. The frequency of the revolving vector 5 p with respect to the rotor will be $n_1 + n_2$, or 197·5 per cent.

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[1] The word *oscillating* is becoming specialised in its application to those currents and fields whose oscillations are being damped out, as in "electric oscillations." But for this, we should have spoken of an oscillating field.

We can now draw a diagram like Fig. 484 or Fig. 486, for each of the rotating vectors, just as if each constituted the

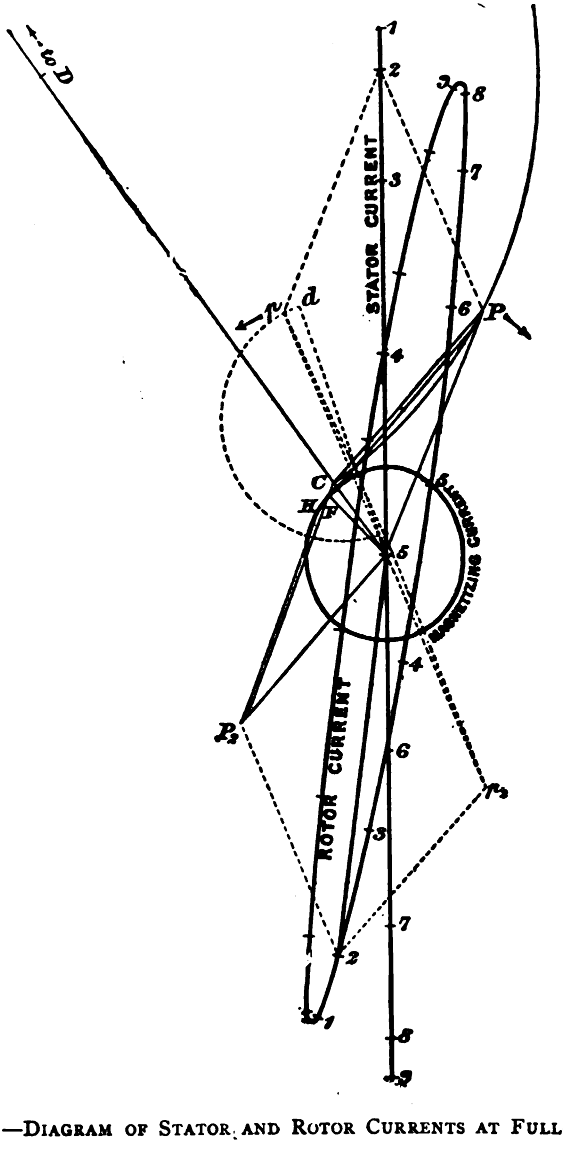

FIG. 510.—DIAGRAM OF STATOR AND ROTOR CURRENTS AT FULL-LOAD.

stator current of an independent polyphase motor, the slip in one case being $n_1 - n_2$, and in the other $n_1 + n_2$. Observe that in these diagrams it is prescribed that the stator current shall be the same for each. The diagram, in which the slip is only 2·5 per cent., is the same as part of Fig. 484, the point O of which corresponds to the point 5 of Fig. 510. The core-loss current O C_H and several other lines of Fig. 484 are omitted to avoid complication, and the parallelograms C 5 and H 5 have been completed as in Fig. 483, in order to bring the line 5 P_2, which represents the rotor current, into its true position. For a given motor the position of the point P in the semicircle C P D depends only on the slip, so that in Fig. 485 we can find the position of P for any slip $n_1 + n_2$ on that semicircle. It is obvious that when the slip $n_1 + n_2$ is 80 times as great as the slip $n_1 - n_2$, the position for the other motor of the point p on its semicircle must be very near to d. Having found the point p we have the true shape of the diagram of the motor for that particular slip, and it only remains so to proportion the semicircles for the two motors, that the stator current 5 p shall be represented to the same scale as the equal stator current 5 P. This has been done in Fig. 510, in which the line 5 d represents the line O D on the former diagrams, and it is divided so that the diameter of the dotted semicircle bears to the whole length of 5 d the ratio of U to U + 1. The point p is correctly placed for a particular motor when the slip is 197·5 per cent. It will be seen that the rotor current 5 p_2 is almost opposite in phase to 5 p; and the magnetizing current, the resultant of these two, is exceedingly small. The extra lines drawn through F parallel to C P and H P_2 respectively relate to matters concerning magnetic leakages that do not here concern us.

Combining 5 P_2 and 5 p_2 for the particular instant 2, shown in the figure, we get 5 2, forming a radius vector of the long ellipse. The radius vector of this ellipse at any instant 1, 2, 3, or 4 gives the value and position of the rotor current at that instant. The ellipse is plotted by combining the vectors 5 P_2 and 5 p_2 at sixteen successive positions, as these vectors make one revolution in opposite directions.

We see from this that the distribution of current in the rotor of a monophase motor is very different from that in the rotor of a polyphase motor. It follows from p. 657 that in a polyphase rotor the current consists of a band whose maximum remains almost constant, and which rotates uniformly, so that it might be fairly represented by a radius vector of constant length. In the monophase rotor the radius vector varies between very wide limits, so that it represents a current which is almost of a simple alternating character with only a small rotating component. The direction of this alternating current is fixed relatively to the stator, notwithstanding the motion of the rotor, and it very nearly coincides with the direction of oscillation of the stator current. Thus we see that in both polyphase and monophase motors the stator current is more or less perfectly *reflected* in the rotor. If the reluctance of the magnetic path and the resistance of the rotor conductors could be reduced, the reflexion would become more and more perfect, until in the limit the vector radii representing the rotor currents would be exactly the same as those representing the stator currents with the sign changed. The fact that the magnetic path has a certain reluctance, calls for a finite magnetizing current to bring into existence the magnetic-flux which is to provide the back electromotive-force. It is mainly due to this magnetizing current that the image of the stator current in the rotor is distorted and diminished in intensity. The magnetizing current in the clockwise rotating diagram is represented by the vector 5 F, which is the resultant of the stator and rotor currents 5 P and 5 P_2, after the deduction of the effects due to the magnetic leakage. The line 5 F also represents to the flux scale the value of the clockwise rotating flux, the direction of which lies at right angles to this line, that is to say, almost parallel to 5 P_2. The counter-clockwise rotating magnetizing current is exceedingly small. It is represented by the resultant of 5 p and 5 p_2. It produces a flux of exceedingly small value, which is just sufficient to generate the current 5 p_2 in the rotor conductors by rotating at the great speed $n_1 + n_2$. Thus the resultant magnetizing current will be represented by the radius vector of the little ellipse in the centre of the figure, which differs very little from the circle described by the point **F**.

The resultant field is necessarily almost a simple rotating field, and is represented by a radius vector which describes an ellipse of very small eccentricity, whose major axis is at right angles to the major axis of the magnetizing current ellipse.

Thus it comes about that the simple alternating current in the stator combined with the currents on the revolving rotor produce an almost simple revolving field. The actual existence of this field is evidenced by the fact that coils wound on the stator in positions differing from the positions of the main stator coils, have an electromotive-force generated in them differing in phase from the voltage of supply, and the voltage per conductor is very little short of the voltage per conductor in the main coils.

It will be seen that this revolving field, acting in conjunction with the current 5 P_2, with which it remains almost in phase, will produce a very considerable torque, which can be calculated in the way explained on pages 699 and 700. The current 5 p_2 also reacts upon the field and produces a torque which alternates in sign as this current goes into and out of phase with the field. Its effect in one revolution is almost equal to zero, there being only a small backward torque due to the very small backward rotating component. The value of this can be calculated in precisely the same way. The effective working torque is the resultant of these, and is therefore not uniform as in polyphase motors, but is pulsatory like the torque due to a single-acting steam engine. The pulsations, however, are so frequent that they are not noticed in practice.

Fig. 510 can be extended by the addition of lines representing electromotive-forces, so as to obtain the phase relations and values of the back electromotive-forces due to the rotation of the field.

It will be noticed that this figure as now constructed represents the distribution and phase relations of currents and magnetic fluxes at one particular load. If the load is changed the amount of the slip changes, and with it the rotor and stator currents. It will be seen from Fig. 510 that the $C^2 R$ losses of a monophase motor are twice as great as those of a polyphase motor of the same size, because the backward rotating current of the rotor is as great or greater than the forward rotating current, and it is producing a small negative torque. The power-factor is not so high as in a polyphase motor, and it will be seen (p. 582) that we are apt to have more losses, from the fact that the field is not distributed as a simple sine-function.

There are, however, two matters to which we must call attention in the application of the Heyland diagram to a monophase motor. The first is that the magnetizing current will not be the same as for a polyphase motor. The total rotating band of current represented by 5 F in Fig. 510 is, as we have seen, almost the same as for a polyphase motor, but it cannot be regarded as a simple component of the stator current as in Figs. 484 and 486. It is in fact the resultant of the stator and rotor currents, and at some instants exists only in the rotor. Nevertheless the magnetism (if we trace it to its ultimate source) has to be supplied by the stator. What then is it that can be called the magnetizing current in the stator? The fact that the magnetism revolves while the stator current simply alternates, tells us that there cannot be a true magnetizing current in the stator; but if we allow the motor to run at synchronous speed, and measure the current supplied under these conditions, we get what is sometimes called the magnetizing current. It is more properly called the no-load current. From the construction of Fig. 510, we see that this no-load current or stator current at synchronous speed is exactly twice 5 C; for when 5 P becomes so small as to coincide with 5 C there is an equal current 5 *p* rotating in the opposite direction, giving a resultant in the stator equal to twice 5 C. And it is found by experiment that the no-load current of a monophase motor is just double the magnetizing current required for a polyphase motor of the same size. Of course, in comparing two motors in this respect account must be taken of the number of conductors in the stator of each, and the difference of phase where several phases are combined (see pages 14 and 37).

Secondly, it is to be noted that as the slip increases, the dotted semicircle upon which p lies becomes greater and greater, and the backward rotating field, instead of being small and insignificant, becomes so great as to seriously detract from the torque yielded by the motor. In effect the point of maximum torque occurs earlier, and at starting there is no torque at all. As the backward rotating field increases the forward field decreases, so that the electromotive-force in the stator conductors shall remain constant. And the forward rotating current at starting reaches only one-half the value it would reach in a polyphase motor. At the same time there is a backwardly rotating current of the same value.

All these facts—the double no-load current, the smaller power-factor, the smaller maximum torque, and the lower corresponding values of rotor and stator currents—are summed up in a very simple modification due to Heyland.

By the aid of a construction like that of Fig. 483, we may see exactly how the currents are distributed in stator and rotor at synchronous speed. In the first place we know that the stator is being supplied with a simple alternating current. Divide this into oppositely rotating halves. The clockwise rotating half must be equal to 5 C (Fig. 510), because there cannot be any clockwise rotating current in the rotor when it is running synchronously; C P being absent, the stator current 5 P coincides with 5 C, and this is the current required to create the forwardly rotating flux. The opppositely rotating stator current must be of the same value, but its magnetic effect is almost wiped out by a rotor current of nearly equal and opposite value, the difference between them being sufficient to create the small flux which at the speed $2 n_1$ generates the rotor current. If we take the instance at which the stator no-load current is at its maximum, the state of affairs can be represented as in Fig. 511.

O C_M represents the forward rotating magnetizing current, C_M C_0 the backward current, and the latter is almost counteracted by the rotor current O C′. The total no-load current is thus O C_0. The electromotive-force of supply is, of course, 90° in advance of the line O C_0, and may be represented by the line O $\mathcal{E}$.

If we represent the stator current when the motor is loaded by a line such as O P, it is found by experiment that the locus of the point P, as the load is gradually increased, is almost a semicircle. Heyland has given the following construction for approximately determining this locus. The line O C_M, Fig. 512, is set off to scale to represent the magnetizing current calculated as for a polyphase motor (see p. 685). The line is then produced toward the left to D so that C_M D = U times O C_M (p. 684).

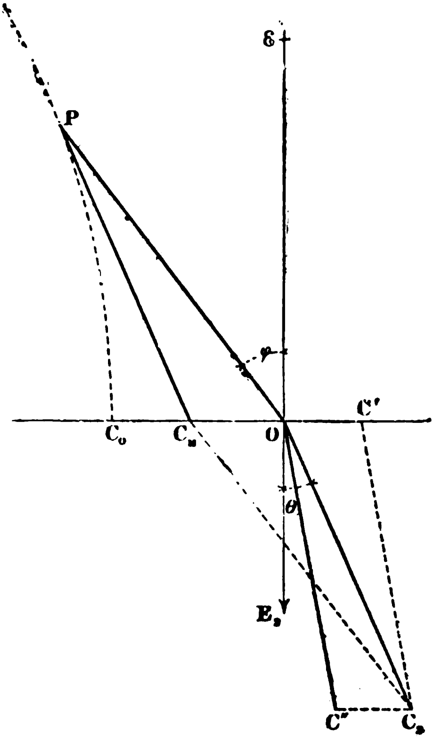

FIG. 511.

If a semicircle like the dotted one in the figure were drawn on C_M D this would be the locus of P for a polyphase motor; but for a monophase motor we proceed differently. We cut off C_0 C_M equal to C_M O and describe the semicircle upon D C_0. Then this semicircle is sufficiently near the locus of P for all practical calculations. P O (to the current scale) gives the stator current and P C_M the rotor current divided by u^2, and the transformation ratio (see p. 675). The two oppositely turning rotor currents can be found by joining P C_0 and drawing C_M R parallel to it. P O is then bisected in R. At the instant that the stator current is at its maximum, the line C_M R, gives the phase relatively to O $\mathcal{E}$ of the forward rotating rotor current. If to this we add O C_M we get O R, the forward rotating stator current. At the instant when the stator current is at its maximum the backward rotating stator current is in phase with this, and is therefore represented by R P. It will be seen that ϕ, the angle of lag of the current behind the electromotive-force O $\mathcal{E}$, is greater in a monophase motor than in a polyphase motor. This is found to be so in practice. The line P A represents the torque of the motor, but to a scale different from the scale of torque for a polyphase motor. The easiest way of finding the torque scale is to find the power supplied to the motor for a particular position of P, deduct the C^2 R losses, and divide the remaining power by the angular speed. This gives the torque which the perpendicular P A represents.

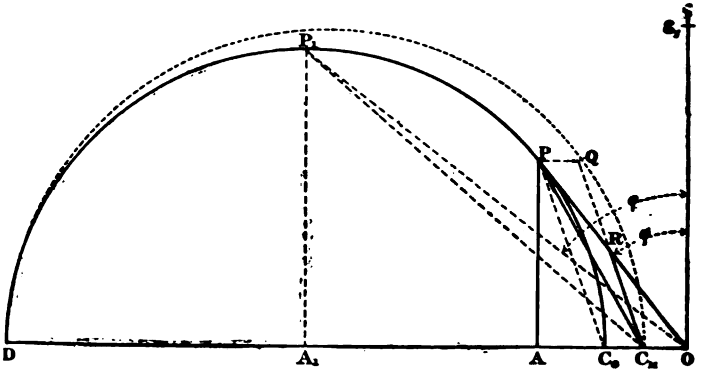

FIG. 512.

Another construction for arriving at the phase relations and values of the currents and electromotive-forces in a monophase motor has been devised by Mr. M. B. Field.[1]

It was found above (see p. 700) that a field rotating with a speed s relatively to the rotor produced a torque

$$T = Q \frac{r s}{r^2 + 4\pi^2 L^2 s^2}.$$

The torque produced by the two oppositely rotating fields will be

$$T = Qr \left[\frac{n_1 - n_2}{r^2 + 4\pi^2 L^2 (n_1 - n_2)^2} - \frac{n_1 + n_2}{r^2 + 4\pi^2 L^2 (n_1 + n_2)^2} \right]$$

which expression[2] is that given by Ferraris, and expresses in an algebraic formula the same facts as the geometric diagram given in Fig. 509, p. 767.

It is not necessary to consider the torque exerted by reason of the fact that the currents due to one rotating field flow in conductors that are immersed in the *oppositely* rotating field, because the frequency of these currents differs by $2n_2$ from the frequency of the opposite field, and consequently this torque will be a rapidly reversing one, causing a mere vibration.

The Starting of Single-Phase Induction Motors.—As we have already seen, the single-phase induction motor is not self-starting; but when once started (in either direction) by artificial means, a torque will be exerted, and the motor will then run up to near synchronism.

The means invariably employed for starting consist in using some form of "phase-splitting" device—that is, the stator is wound with two sets of windings, a "working" winding and a "starting" winding, the two windings being displaced from one another by 90°, just as are the two phases of a two-phase

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[1] On Oscillatory and Rotatory Magnetic Fields and the Theory of the Single-Phase Motor, *Elec. Rev.*, xliv. 194, 271, 439, 491, 579, 1899.

[2] Compare Hutin and Leblanc, *La Lumière Électrique*, xl. 418, 1891.

motor. At starting, one winding is put on the mains directly, while the other is connected to them through a choking coil or capacity. The result is that the currents in the two windings have a phase displacement from one another, the amount of which depends upon the nature of the starting device and arrangement of the windings, being always considerably less than 90°. Consequently a more or less irregularly rotating field is produced, which acts upon the rotor in exactly the same way as the field of a two-phase motor would, only not so effectively, causing the rotor to start, and run up to speed. As with all rotatory field motors, the starting torque is very greatly increased by the insertion of resistance in the rotor circuit to keep the rotor current more in phase with the forwardly-rotating field. The resistance which gives best effect in a single-phase motor has been investigated[1] by Maffiotti and Pescetto, who found that in the motors examined by them the best resistance was expressed by the equation $r = 0{\cdot}4 \times 2 \pi n \mathrm{L}$. Compare p. 704.

The starting winding will take up about one-third of the stator periphery. The windings are always embedded in holes or slots, just as for polyphase motors, and in order to minimize magnetic leakage, are invariably drum-wound. Thus in practice each coil of the working winding would be wound quite straight, its shape being a simple rectangle, while the coils of the starting winding would be bent up and over the others. As the starting winding is only in use for short periods, the current density in it may be very high, compensating in this manner for the smaller winding space available.

Rotors for single-phase motors are built in exactly the same manner as those for polyphasers. They may be of either the wound or the squirrel-cage type, but the latter are used only for the smallest of motors ; and if the frequency is above 60 cycles per second, wound rotors are certainly preferable. The reason for this is that a very imperfectly rotating field is produced by the phase-splitting device, so that it is impossible to start a single-phase motor of any size with a reasonable

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[1] G. B. Maffiotti and Col. F. Pescetto, 'Circa il motore asincrono monofase del Brown.' *Revista d'Artiglieria e Genio*, iv. 1897.

current consumption, unless a non-inductive resistance is temporarily used in the rotor windings.

Figs. 513 to 516 show four different ways of splitting the phase for starting purposes. The first is only effective for small motors. At starting, the two windings are put in series on the mains, the starting winding being shunted by a non-inductive resistance. The current will therefore lag more in one winding than in the other, the phase-difference being sufficient to cause an irregular rotatory field. The starting coils and resistance are cut out, and the working coils put direct on the line, as soon as the motor has nearly attained full speed. The arrangements shown in Figs. 514, 515, and 516 are identified with the firm of Brown, Boveri and Co. Fig. 514 is the same as Fig. 513, except that a condenser is substituted for the non-inductive resistance. The lagging current in the starting coils acts in just the same way as in the other case,

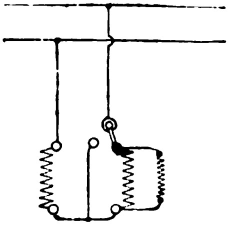

FIG. 513.

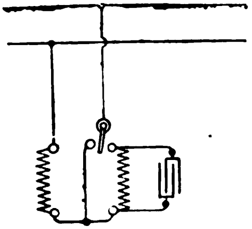

FIG. 514.

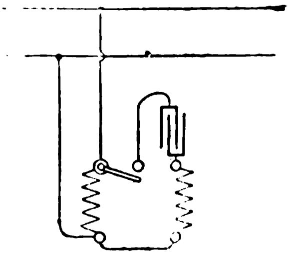

FIG. 515.

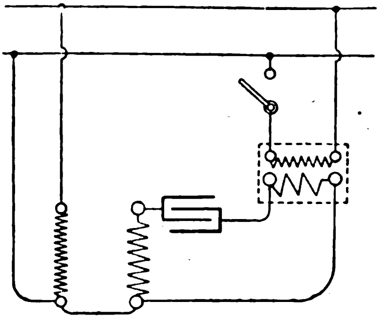

FIG. 516.

causing the rotor to quickly run up. The arrangement is very effective for starting motors up to about 5 H.P., after which that shown in Fig. 515 is better when used with a non-inductive resistance in the rotor. The rotor resistance is inserted and the line current is switched directly on the working winding, the starting coils, in series with a condenser, being put in parallel with it. As the rotor runs up, the rotor resistance is gradually reduced, the condenser and starting coils being cut out at about two-thirds of the normal speed. Fig. 516 shows the arrangements used for starting the 70 H.P. high voltage monophase motors at Frankfort, put in by Messrs. Brown. The working winding is switched directly on the 2800-volt mains, but the starting winding and the condenser are connected to the mains through a small transformer, which reduces the pressure to a lower value more suited to the condenser. At about two-thirds of normal speed the primary of the transformer would be switched out, and the rotor resistance short-circuited. Some other methods of phase-splitting are described in the *Electrical Review*, xlix. 299.

The condensers used with the Brown single-phase motors are all of the liquid type, and hence of very simple construction. Each condenser consists of a number of thin iron plates separated by thin fibre strips, and arranged in an enamelled iron vessel containing soda solution.

In the specification of British patent No. 24,098 of 1892, Brown describes a number of methods of starting single-phase motors, including the methods using auxiliary windings with self-induction and capacity, and also including some methods in which a commutator is used at starting.

The difference of phase in the currents in the two windings of the stator may be caused by the windings having themselves unequal coefficients of self-induction, or by putting resistance or capacity in series with one, and inductance in series with the other; or any combination of these may be employed as described below.

Fig. 517 is a drawing of switch arrangements made by the Oerlikon Machine Company for the purpose of starting these motors. It is shown with the "chopper" of the switch in the

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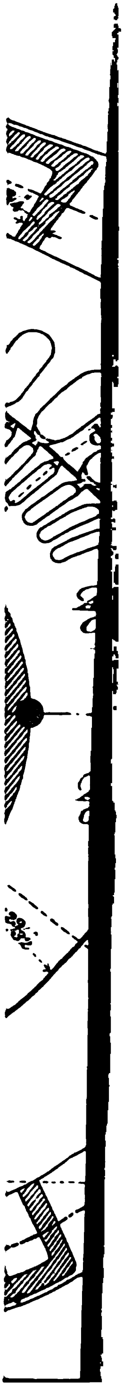

full-speed position. When in the starting position the blades of the chopper connect together the pieces *d*, *e* and *f* and also *g* with *h*, and the connexions are such that in this position the main coils and the starting coils, the latter with a non-inductive resistance in series, are connected across a reduced voltage through a small transformer. The switch is thrown over at about two-thirds of full speed, when the starting phase is cut out entirely.

An example of the winding of a single-phase stator is afforded by Fig. 518, which depicts that of an induction-motor designed by Mr. V. A. Fynn. The diameter of the bore is

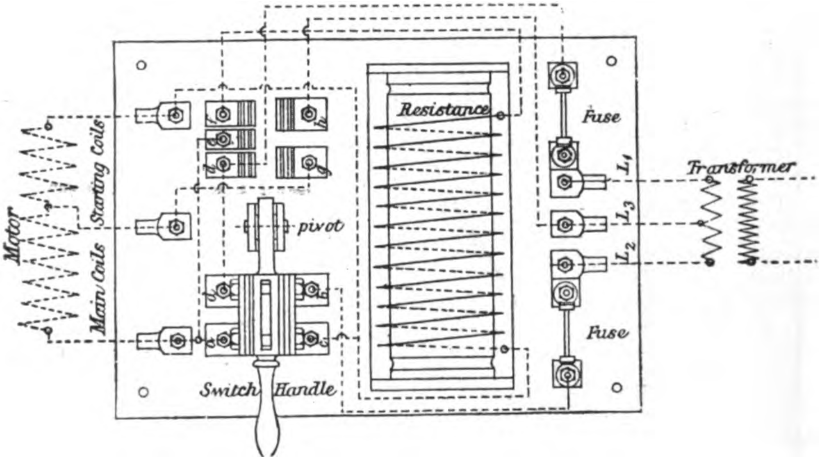

FIG. 517.—STARTING GEAR FOR SINGLE-PHASE MOTOR.

26 inches and the core-length is 12 inches. This is a 14-pole machine, giving 35 H.P. at 788 RPM; being designed for a frequency of 92 cycles per second, and a supply at 1900 volts. It will be seen that the principal stator winding consists of 14 coils, each spanning two-thirds of the pole-pitch. They are hand-wound with 70 wires per slot in micanite tubes placed in half-closed slots, about 3 inches deep. The other winding, which constitutes the auxiliary starting phase, consists of 7 coils, of breadth equal to the pole-pitch, being therefore a hemitropic winding, and the end-bends are turned up behind those

of the principal coils. The rotor of this machine has 84 slots, each of which receives two conductors of strip copper, which are connected up in a wave-winding and their ends brought out to three slip-rings on the shaft for the purpose of inserting a starting-resistance. Plate XXIV. gives a scale-drawing of this machine.

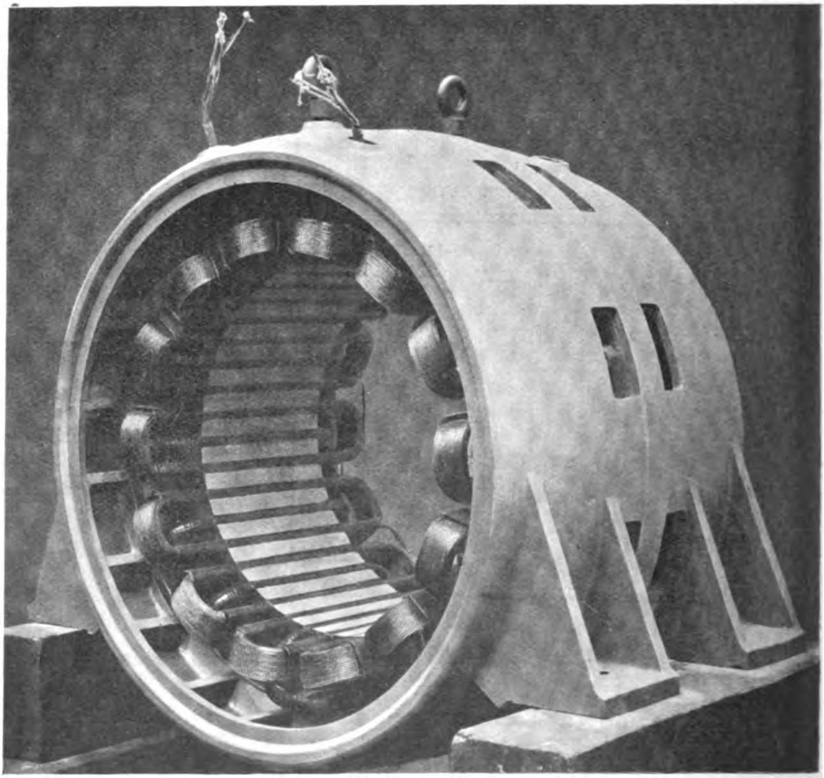

FIG. 518.—STATOR OF FYNN'S 35 H.P. SINGLE-PHASE INDUCTION MOTOR, showing two sets of coils.

Fig. 519 shows the external appearance of a Kolben single-phase motor of 6 horse-power for running on a 110-volt, 50-periods per second circuit. It will be seen that the motor is fitted with a fast and loose pulley placed inside the bracket bearing.

A starting device used by the General Electric Company, of Schenectady, shown in Fig. 520, consists[1] of a resistance coil in series with a choking coil placed across the mains. From the middle point where the two coils join, an auxiliary connexion is then made, as for a third wire, to a suitable point on the stator winding. The motor starts as an unsymmetrical three-phase machine, taking some power wastefully at first through this third terminal, but as speed increases running as a single-phaser and pumping back a little power from its third terminal.

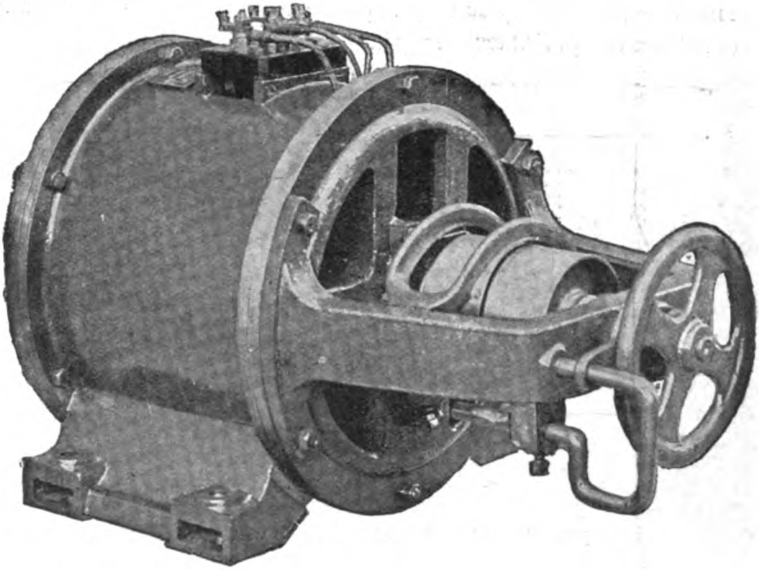

FIG. 519.—SINGLE-PHASE INDUCTION MOTOR (KOLBEN AND CO.).

Fig. 521 is a diagram of connexions supplied by the British Thomson-Houston Co., showing another way of running an ordinary three-phase motor on a simple alternating circuit. Two terminals of the motor are connected direct to the mains, and the third terminal is carried to one or other of the mains through an auto-transformer, the high potential terminals of which are connected to a dry condenser. In a recent test made on this method it was found to be advantageous to keep the transformer and condenser always in circuit, as the

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[1] See E. J. Berg, in *American Electrician*, ix. 204, 1897.

increased power factor and efficiency when running light under these conditions more than compensated for the small loss in the transformer.

Attempts have been made from time to time to make single-phase motors self-starting without the addition of any condenser or separate starting gear. Heyland has devised a very successful motor of this type, the principle of which will be seen from Fig. 522. The chief winding is distributed in the smaller slots, the leakage self-induction being kept as small as possible by having separate teeth distributed over a considerable peripheral length. The starting winding consists of coils closed on themselves and wound through the square holes shown in the figure, the leakage self-induction being aggravated by the bunching together of the conductors and the leaving of a filament of iron to bridge the slot. Currents are induced in this closed winding which lag almost 90°, and produce a cross-field, much as another phase winding would. The starting torque is greater than the full-load torque, and the amount of current at starting is only 1·7 times the current at full-load. Further description of this motor will be found in the articles referred to below.[1]

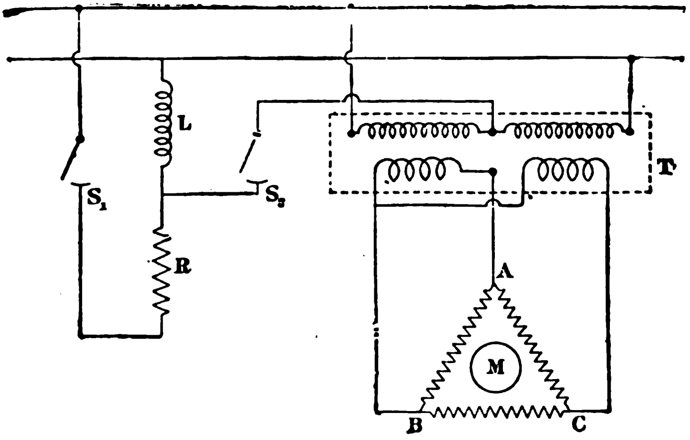

FIG. 520.—GENERAL ELECTRIC CO.'S STARTING DEVICE.

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[1] *Electrical Engineer*, Sept. 3, 1897, p. 306; and *Journ. Inst. Elec. Eng.*, xxix. 816, 1900.

A very simple method of starting single-phase motors is employed by Arno.[1] The motor is wound with one set of windings only, there being no attempt to split the phase. Into the circuit of the rotor is introduced during starting a resistance sufficient to cause the starting current not to exceed double the full-load current. The rotor is then given by hand a small initial velocity. It at once runs in the direction in which it is started, gaining torque as its speed rises ; the added resistance being gradually cut out as the motor comes up to its speed.

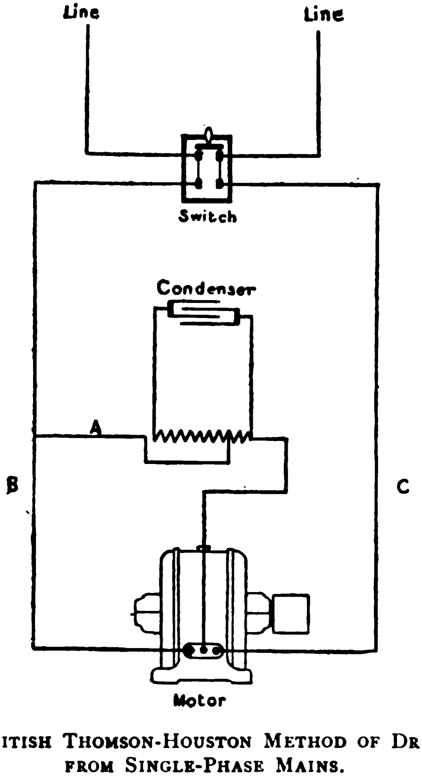

FIG. 521.—BRITISH THOMSON-HOUSTON METHOD OF DRIVING MOTOR FROM SINGLE-PHASE MAINS.

Notable among single-phase motors are those made by an English firm, the Langdon-Davies Motor Company. Mr.

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[1] *Bulletin de la Société Internationale des Électriciens*, Dec. 1, 1897.

Langdon-Davies has made very exhaustive researches upon the distribution of the magnetic field in motors wound with coils in various positions and with currents in various phases, for the purpose of ascertaining the method of winding best suited to give a large starting torque, a uniform field, and a high efficiency. His experiments have resulted in placing upon the market a most successful self-starting motor, which is now widely used on single-phase circuits. An early account of these motors will be found in the *Electrician* for June 19, 1896, and also in *Engineering* of the same date. More recently further improvements have been introduced, including special methods of winding the rotor to increase the starting torque.

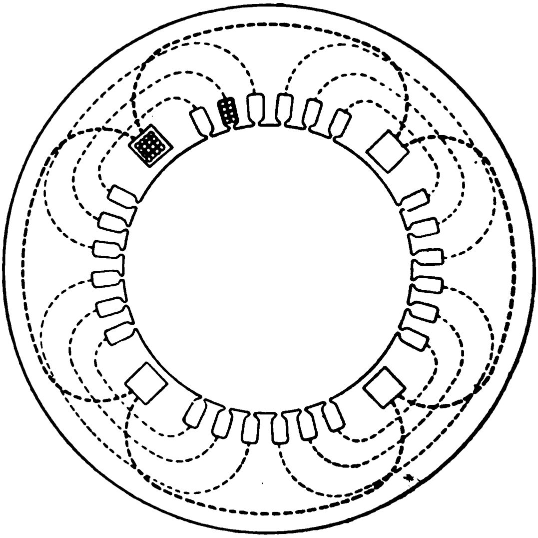

FIG. 522.—STATOR OF HEYLAND'S SELF-STARTING MONOPHASE MOTOR.

Some constructors obtain a starting torque by providing the rotor with a commutator. The action of commutator motors is however so distinct, that the treatment of it is reserved to another chapter.

Example of a Single-Phase Induction-Motor.—The example selected is a high-speed motor, belonging to a motor-generator set for use on an electric locomotive, constructed by the Oerlikon Machine Works. The generator part of this set is described in Vol I., p. 757, Fig. 481. The general specification is :—

$$\mathrm{IS}\,6_{\mathrm{P}} - 520_{\mathrm{HP}} - 1000_{\mathrm{RPM}}\,;\; 14{,}000_{\mathrm{V}} - 31_{\mathrm{A}}\,;\; 50 \sim.$$

This motor is built for extreme compactness. Figs. 523 and 524 show that the stator has two separate windings in slots of different sizes, and that the rotor is of the squirrel-cage type.

The principal dimensions are as follows :—

INCH MEASURES.

Stator :—

Diameter of bore	31·75
Gross iron length of core	11·9
Nett ,, ,,	10
External diameter of core-disks	53·25
Large slots, 24 in number, or 4 per pole, size	1·03 × 3·7
Number of wires per slot, 100, of diam., bare	0·127
,, ,, ,, ,, covered	0·151
Slot insulation	0·020
Small slots, 60 in number, or 10 per pole, size	0·4 × 2·18
Number of conductors per slot, 2, size, bare	0·87 × 0·285
,, ,, ,, ,, covered	0·96 × 0·375
Slot insulation	0·020

The smaller slots receive a low-voltage winding, and serve as a starting-phase, current of about 520 amperes being supplied to them at 700 volts. The larger slots are for the high-voltage winding which receive current direct from the overhead lines at 14,000 volts. Both sets of slots are half-closed, and the coils are hand-wound ; the end-bends of the high-voltage winding are bent up at the side next to the generator for greater compactness. The air-gap is 2 millimetres.

Rotor :—

External diameter	31·67
Gross iron length of core	11·9
Internal diameter	16·3
Slots, 196 in number, of size	0·257 × 1·410
Conductors, 196 in number, of size	0·237 × 1·390

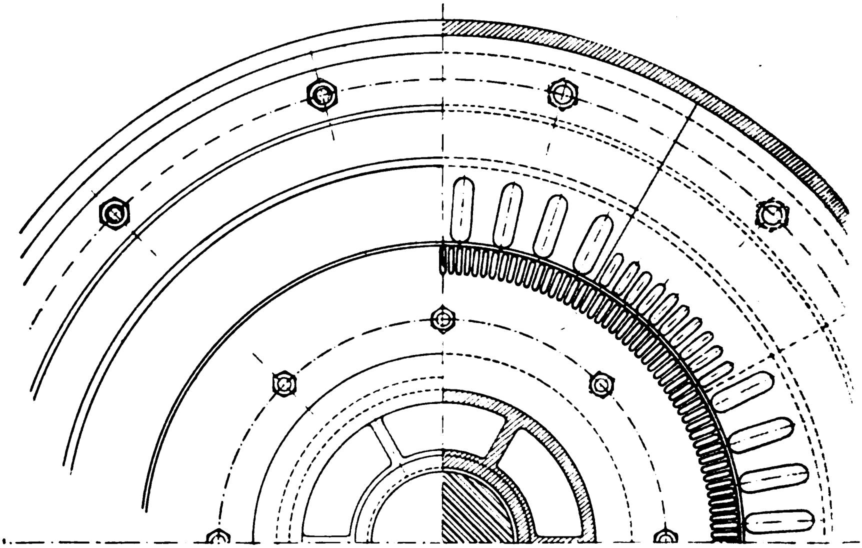

FIG. 523.—HIGH-SPEED SINGLE-PHASE INDUCTION MOTOR, 520 H.P. OF OERLIKON CO. (Scale 1 : 10.)

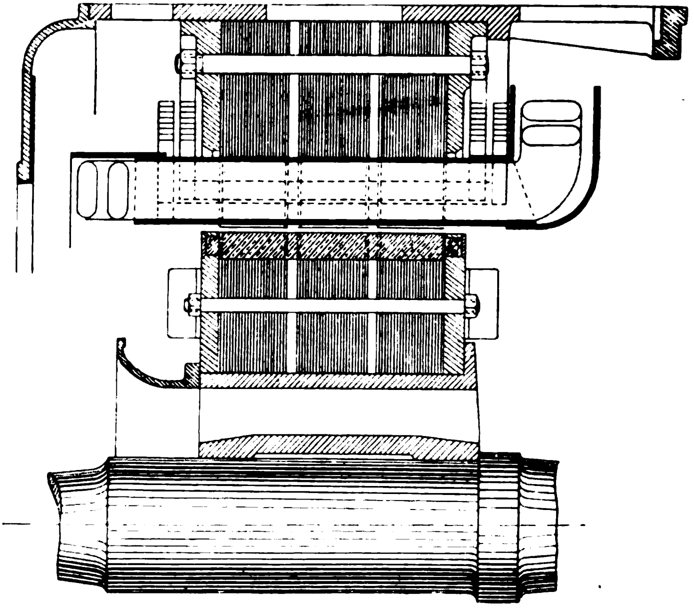

FIG. 524.—HIGH-SPEED SINGLE-PHASE INDUCTION MOTOR, 520 H.P. (Longitudinal section, scale 1 : 10.)

The rotor core is mounted on a cast-iron spider with ample ventilation.

The following articles may be consulted for a more detailed discussion of the subject.

A. Heyland, *Electrician*, xxxvi. 753, 1896.
Arno, *Electrician*, xl. 254.
Wilson, Prof. E., *Inst. Elec. Eng.*, xxviii. 321, March 1899; and *Electrician*, xlii. 729, 801, 824, 861.
Leblanc, *Bull. Soc. Inter. Électriciens*, xv. 488.
C. P. Steinmetz, *Amer. Inst. Elec. Eng.*, xv. 35, 1898.
M. B. Field, *Elec. Review*, xliv. 194, 271, 439, 492, 579.
A. C. Eborall, *Inst. Elect. Eng.*, xxix. 816, 1900.
W. S. Franklin, *Amer. Inst. Elect. Eng.*, xxi. April 1904; and Discussion.
A. Boyd, *Electrician*, liii. 912, 1904.

CHAPTER XVIII.

ALTERNATING-CURRENT COMMUTATOR MOTORS.

THE question of the "electric equipment" of railways has of late directed considerable attention to the single-phase commutator motor because of the following advantages it possesses for this purpose over other types.

(1) It enables power to be transmitted to the train at a much higher voltage than the highest at which it is practicable to use continuous currents, thus effecting economies in the matter of line-losses as well as in the cost of converting substations; (2) it provides the high torque, necessary for rapid acceleration at starting, which is unobtainable by the single-phase induction motor, but which is obtained with the help of the commutator; and (3) it thereby supersedes the three-phase system in which a high starting torque is obtained only at the expense of the unavoidable complications for collecting current from two or three insulated conductors.

The idea of the alternating commutator motor is not new.[1] It was much discussed some fifteen years ago, but owing to the high frequencies of supply then in vogue, it was abandoned for the time being in favour of the more efficient continuous-current motor and the more simple commutatorless three-phase induction motor.

In this chapter the elementary theory of the various types of commutator motors will be discussed, and attention will be directed to those points in their design which determine good efficiency and sparkless commutation.

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[1] Alex. Siemens in 1884 pointed out the possibility of using the principle of the continuous-current series motor for alternating currents (*Journ. Inst. Elec. Engs.*, xviii. 527, 1884), and Elihu Thomson introduced his repulsion motor in 1887 (*Elec.*, xxv. 35).

THEORY OF A CLOSED COIL ARMATURE ROTATING IN AN ALTERNATING MAGNETIC FIELD.

Consider the typical bipolar-field shown in Fig. 525.

1. *When the armature is stationary.*—By transformer action the alternating flux passing in and out of the armature coils from one pole to the other induces an alternating electromotive-force in the coils between A and B; and the current, which will flow by connecting A and B electrically together, will interact with the coils on the magnets, and the ordinary laws of transformers will apply. Between any other two oppositely placed points as E F, this induced or transformer voltage will be less, since the electromotive-force between B and E is opposed by that between A and E. Further, there will be no transformer electromotive-force across the axis C D, because the opposing voltages between D B and B C will be equal.

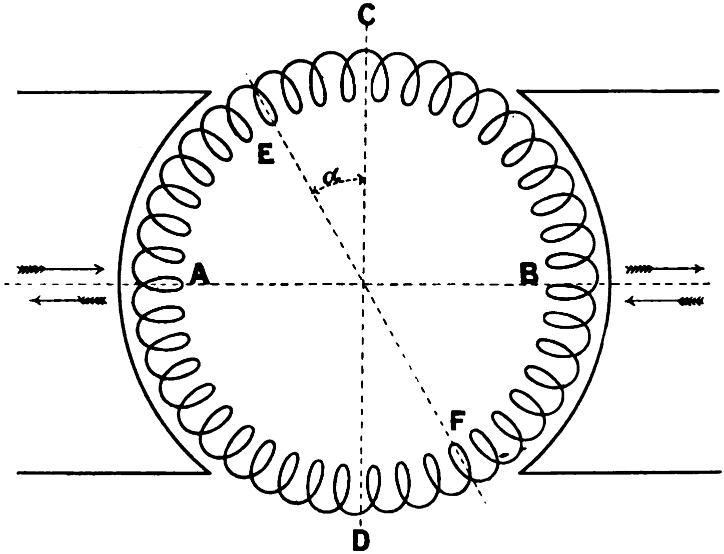

FIG. 525.

The actual value of this induced electromotive-force may be calculated according to the usual transformer rules, p. 535, and the factor of correction for any position of the brushes other than A B is taken as proportional to the sine of the angle of brush displacement a. This applies only when a sine-distribution of the flux can be assumed, and with the square-cornered fields (p. 226), obtained in the ordinary continuous-current motors, this assumption may be inaccurate.

The wave-form of the induced electromotive-force will be the same as that at the terminals of the magnet-coils; for the same flux which induces it, induces also in the primary an electromotive-force which exactly opposes the applied wave-form.

2. *When the armature is revolving*, the induced electromotive-force which could be tapped off the windings when the armature was stationary, is now obtained by collecting with brushes placed at the points A B on the commutator. The induced electromotive-force will be the same in every respect as before, for with respect to the brushes there is always a set of coils, similarly placed on the armature, in which the electromotive-forces are being induced.

There is, however, a second electromotive-force produced when the armature is rotating, and this is due to conductors *cutting* the flux, and therefore corresponds to the electromotive-force generated in any continuous-current machine. It must, therefore, not be confused with the transformer electromotive-force considered above, which was due to *interlinkage* of the coils with the alternating field; and in order to distinguish between them in the present Chapter, that due to *cutting* simply will be designated the *generated* electromotive-force and that due to *interlinkage* or transformer action, the *induced* electromotive-force.

The generated electromotive-force will be an alternating one of the same frequency as the supply, and will be in value directly proportional to the speed of the armature, and its wave-form will be the same as that of the variations of flux in the magnet-system (see Chapter IV.).

Referring again to Fig. 525, it is to be noticed that whereas the value of the induced electromotive-force across

C D is zero, the generated electromotive-force across this axis is a maximum, and is on the other hand zero across the axis A B. For any other axis, the value is usually taken as proportional to the cosine of the angle of brush displacement a, on the same assumption of a sine-distribution of the flux as before. In other respects, the value of the generated electromotive-force is calculated just as that in a continuous-current machine.

To sum up this introductory point: the armature, whether ring-wound or drum-wound, rotating in an alternating magnetic field, acts partly as a dynamo armature, partly as the secondary of a transformer. It is the seat, therefore, of two different electromotive-forces: (1) the *generated* electromotive-force, proportional to speed and to cos θ; the other (2) the *induced* electromotive-force, independent of the speed, proportional to sin θ. Both depend on the frequency of the supply, and both are proportional, *cæteris paribus*, to the number of windings on the armature. It will readily be seen that the same argument might be extended to multipolar designs, with appropriate arrangements of armature windings, commutator and brushes.

Chief Types.—The chief types[1] of alternating-current commutator motor are:—

1. The series motor (Fig. 526).
2. The repulsion motor (Fig. 529).
3. The shunt motor (Fig. 530).
4. The compensated motors, including—
   - (*a*) Any of the above types constructed with compensating windings (Figs. 536 and 537).
   - (*b*) Heyland's compensated induction motor (Fig. 539).
   - (*c*) Motors constructed upon the Latour-Winter-Eichberg plan (Figs. 540 and 541).

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[1] Compare with the classification adopted by Atkinson in *Proc. Inst. Civil Engineers*, cxxxiii. 113, 1898.

THE SERIES MOTOR.

The series motor is perhaps the simplest of the above types. The arrangement of its connexions, shown in Fig. 526, is exactly similar to that of the continuous-current series motor. That the torque is produced in the same way is readily understood when it is remembered that the direction of rotation of the ordinary series tramway-motor is independent of the direction in which the voltage is applied. At any moment the torque will be proportional to the product of the current and the flux which it is at that moment producing in the magnet-system; and the average torque will be, *cæteris paribus*, the product of the average current and the average flux it produces; so that if the iron parts are unsaturated, as they must be if the iron-losses are not to be too high, ***the torque will be proportional simply to the square of the current***, there being no question of power-factor entering into the consideration.

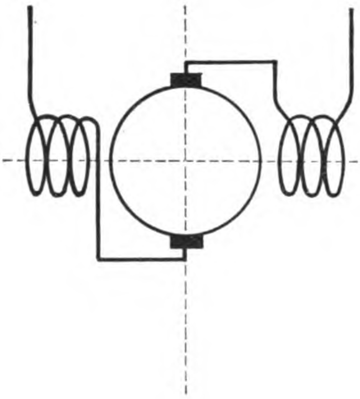

FIG. 526.

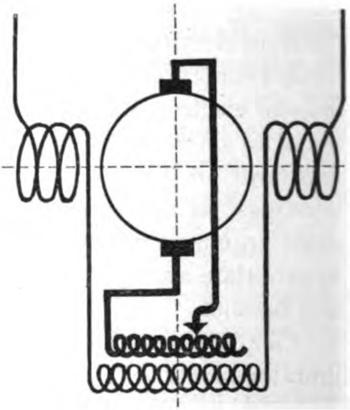

Fig. 527.

At any condition of loading when we have the current lagging behind the applied electromotive-force by an angle ϕ, we may construct a diagram as that in Fig. 528, in which A B represents the total applied voltage and B C the direction of the current. Now the applied voltage A B is made up of three parts, viz. the ohmic drop in the motor which is in phase with the current, the reactance drop which is at right-angles to the current, and the back generated electromotive-force of the armature, which is also in phase with the current. These three quantities are represented in the diagram by D E, A D, and E B, the angle A D B being a right-angle. For any other condition of load, when the current has taken up another angle of lag, the angle at the point corresponding to D will again be a right-angle; and as such lines as D E and E B representing the ohmic drop and the armature generated voltage must always be in the same straight line, it follows that the point D will move on a semicircle, and that the line A D will represent the value of the current for angle of lag ϕ. But this is not the most convenient method of representing the current. As the resistance and the reactance of the motor are constant, the length of the lines A D and D E representing the voltage drops due to these causes will also be constant, and therefore the angle D A E will be constant, and so by simple geometry the angle A E B is constant for all values of ϕ. We may, therefore, draw a circular arc through E and for any position of E, as A E may be taken to represent the current and E B is that part of the applied voltage which is balanced by the voltage generated in the armature of motor, the speed will be given by the ratio $\frac{E\,B}{A\,E}$. In order to repre-

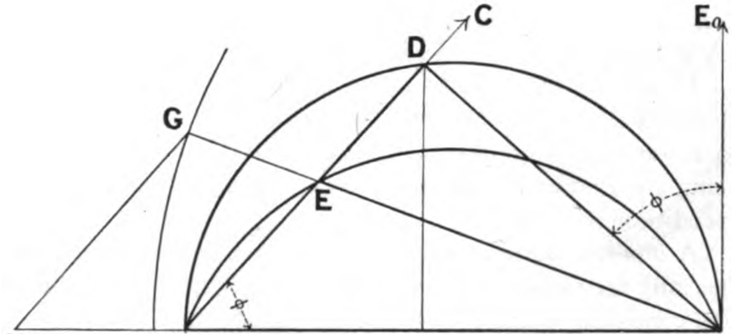

FIG. 528.

sent the speed by one line, we have now to draw an arc of any radius as A H, produce A E to meet it in G, and through G to draw G K parallel to E B. G K then represents the speed. For the triangles A E B and A G K are similar, and therefore

$$\frac{\text{E B}}{\text{A E}} = \frac{\text{G K}}{\text{A G}} = \text{G K} \times \text{constant.}$$

The torque is given by $(\text{A D})^2$. To represent this by one line we drop a perpendicular D F on A B and then A F gives the torque for this condition of load. For the triangles A D B and D F A are similar. Therefore

$$\frac{\text{A F}}{\text{A D}} = \frac{\text{A D}}{\text{A B}} \text{ or A F} = \frac{(\text{A D})^2}{\text{A B}} = (\text{A D})^2 \times \text{constant.}$$

If desirable, we may in our diagram erect a perpendicular A E_0 at A and represent the angle of lag of the current by the angle between A E_0 and A D.

It will be seen from the above constructions that the characteristics of the alternate-current series motor are similar to those of the continuous-current series motor, the torque being a maximum at starting and decreasing to zero when the speed becomes infinite.

A modified form of alternate-current series motor adapted specially for railway work is shown in Fig. 527 ; the modification consisting in the use of a series transformer, by means of which, while the full pressure of the line may be applied to the stator, only a low pressure current is conveyed to the armature. By having regulating and reversing switches on the secondary side of the transformer the motor may be reversed and started up without the use of resistances in the primary side.

It is important to notice that the characteristics of the series motor are preserved in this motor ; for as it has been shown on pp. 595 and 602, the currents, resistances and reactances in the secondary circuit of a transformer can always be expressed in terms of those of the primary.

THE REPULSION MOTOR.

By a further development of the series motor we obtain the repulsion motor, Fig. 529.

In this the field coils only are supplied with current, and in order that a torque is produced the brushes are set at an angle to the field axis. Therefore we see that in this motor the induced voltage in the armature coils is used as well as the generated voltage, and it is in this that the repulsion motor resembles the series motor shown in Fig. 527, for the series transformer instead of being a separate piece of apparatus is formed by the coils of the motor itself.

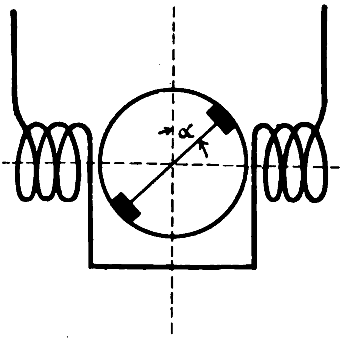

FIG. 529.—REPULSION MOTOR.

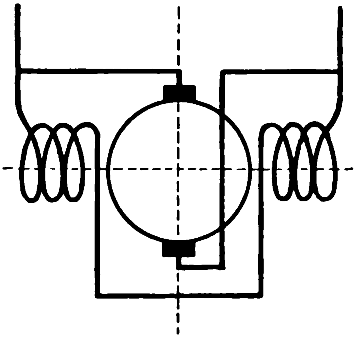

FIG. 530.—SHUNT MOTOR.

In Fig. 529, suppose the brush axis is displaced at an angle *a*, then at the start an electromotive-force is induced between these points, proportional to sin *a*. We see from this that altering the position of the brushes affords a means of regulating the current at starting.

Although the repulsion motor starts with a maximum torque, unlike the series, this is reduced to zero at a definite speed. This is because of the *generated* electromotive-force which comes into play when the armature revolves, being at right angles in time with the *induced* electromotive-force, and the current flowing as a result of these two electromotive-forces gradually becomes out of phase with the latter, and at a certain speed, depending upon the brush angle a, the torque disappears entirely.

The following diagram, Fig. 531, very much like that for the series motor, but modified to take account of the transformer action, has been given by Osnos.[1]

O E representing the applied electromotive-force, O M represents the magnetizing current obtained when the brushes are lifted from the commutator. Then, for any given brush position the short circuit standstill current is taken, and this, O P_s, is plotted out at the proper angle. A semicircle is then drawn through M P_s with its centre on M D, and a circular arc is also drawn to pass through D P_s O. This circular arc is the locus of the stator current, and with the help of the supplementary semicircles all the particulars of the performance of the rotor may be obtained graphically. Where the arc touches the first semicircle at P_0, we have the no-load current, *i.e.* the value of the current when the motor has attained the maximum speed. For any other point P we have the speed represented by $\frac{\text{P B}}{\text{M B}}$ and the torque by O P × A P.

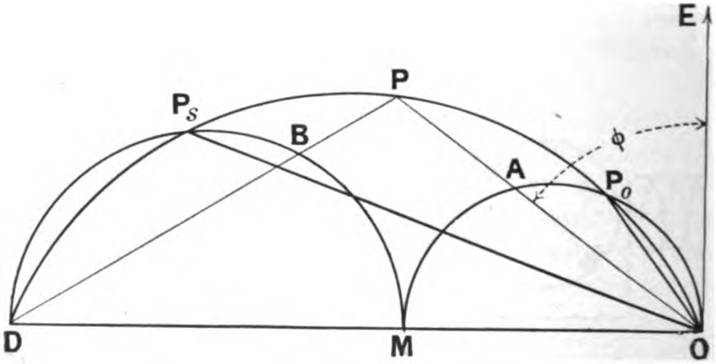

FIG. 531.

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[1] *Elektrotechnische Zeitschrift*, xxiv. 905, 1903. For an exhaustive treatment of the repulsion motor with general diagram for all angles of brushes, see C. F. Creedy in *Elec. Rev.*, liv. 203, 322 and 368, 1904.

Advantage has been taken of the high initial starting torque of repulsion motors to start single-phase induction motors. The wound rotor of an induction motor is easily provided with a commutator, and when synchronism has been obtained it may be short-circuited as a three-phase armature and the brushes lifted off the commutator.

The Wagner Co., of America, have for some years adopted this plan, the commutator being automatically short-circuited at a certain predetermined speed by a centrifugal device, at the same time the brushes being lifted. Figs. 532 and 533 show sections of a Wagner Co.'s motor.

One disadvantage in this arrangement is that at the moment when the machine is changed from a repulsion motor to an induction motor a rush of current is likely to occur. This objection is partly overcome in the Schuler motor (D.R.P. 140,925), in which the rotor windings are during the process of starting up connected through three slip-rings to a three-legged resistance which is gradually cut out as synchronous speed is attained.

The Fynn motor (Patent Spec. 22712 of 1902) goes even further than this. It has already been pointed out that even in starting up a repulsion motor the current may be excessive, and either resistance must be introduced across the otherwise short-circuited brushes or the applied voltage to the stator must be reduced. In the Fynn motor only a portion of the rotor windings (Fig. 534) are used to form a repulsion motor, the rotor current being thus reduced, but all the windings are connected up to the three-phase starter as in the Ferranti-Schuler machine, and are used when the motor has attained synchronous speed.

THE SHUNT MOTOR.

The shunt motor (Fig. 530) as it stands is not of much practical importance because of the heavy wattless current it must of necessity take at all loads. We will consider briefly how this is so, by reference to the vector diagrams. In Fig. 535 the triangle O Q R applies to the shunt circuit, the applied

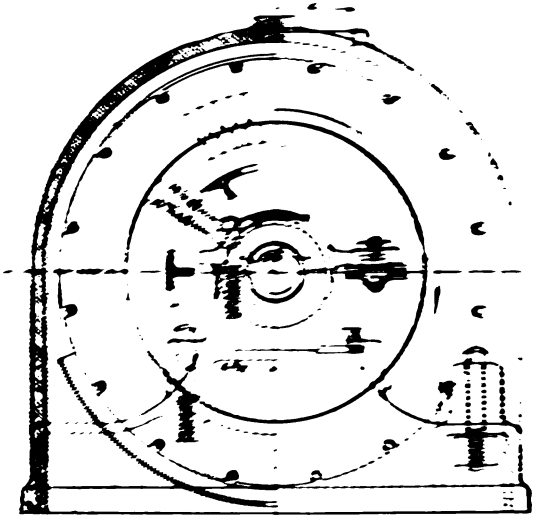


Fig. 532.—Wagner Motor (Cross Section).

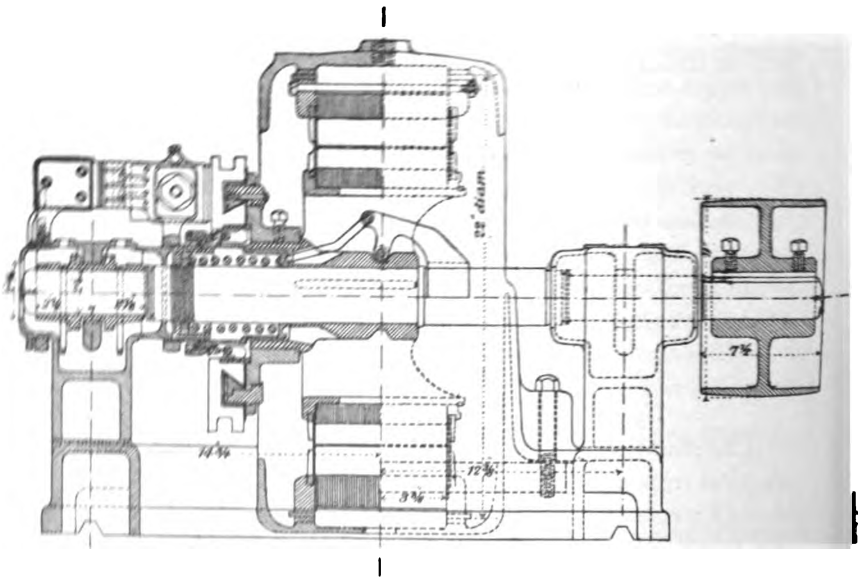


Fig. 533.—Wagner Motor (Longitudinal Section).

voltage O Q being split up into Q R and O R representing the ohmic and reactance drops respectively in the shunt windings. The flux is in phase with Q R, so that the armature generated voltage is also in phase with this line, and at a certain speed may be represented by the length Q S which leaves O S as the unbalanced voltage to provide the armature current to drive the motor. According to the resistance and self-induction of the armature, O S is split up into O T and T S, O T representing the reactance drop and S T the ohmic drop. The angle marked ϕ_2 gives the lag of the current behind the applied voltage.

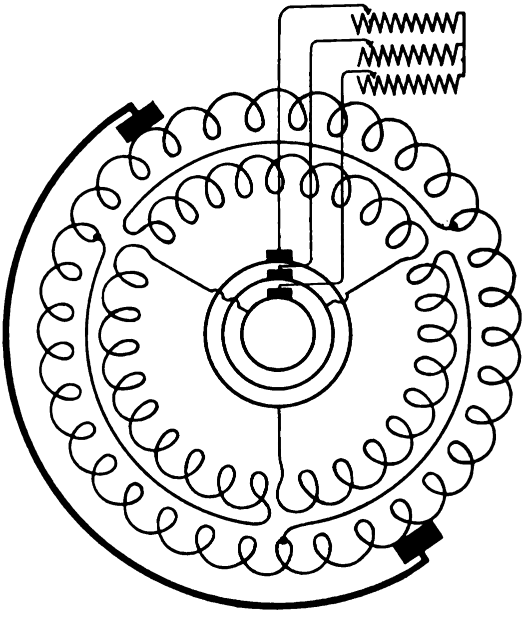


FIG. 534.—ROTOR WINDINGS OF FYNN MOTOR.

The torque of the motor will then be

$$RQ \cdot TS \cdot \cos(\phi_1 - \phi_2).$$

It will be noted that, inasmuch as the direction of E S in Fig. 535 is fixed, there will be no speed at which the current is zero, because there is no speed at which S corresponds with O.

When the speed is so high that S is nearly vertically over E, the armature current will begin to lead on the applied volts, *i. e.* T S will be nearly parallel to O E. This condition is reached at a lower speed if the shunt current is more nearly in phase with the applied voltage.

From the above deductions it is seen that the shunt motor has inherently many properties which render it unsuitable for practical use. For a discussion on the compensated shunt motor, in which it is claimed many of these properties are eliminated, see Bethenod in *L'Éclairage Électrique*, xlii. 321, and xliii 287, 1905.

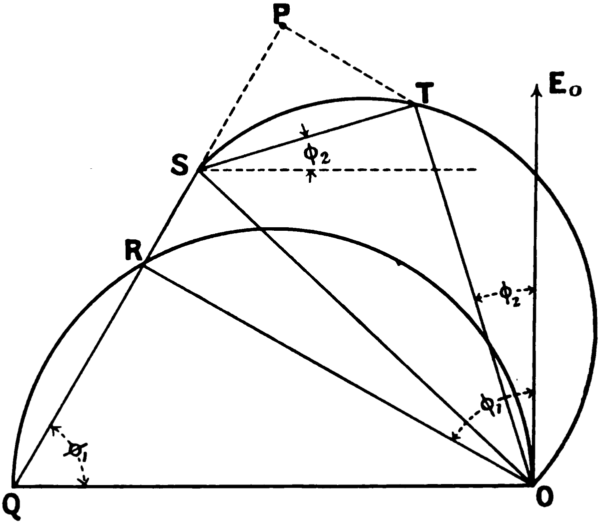
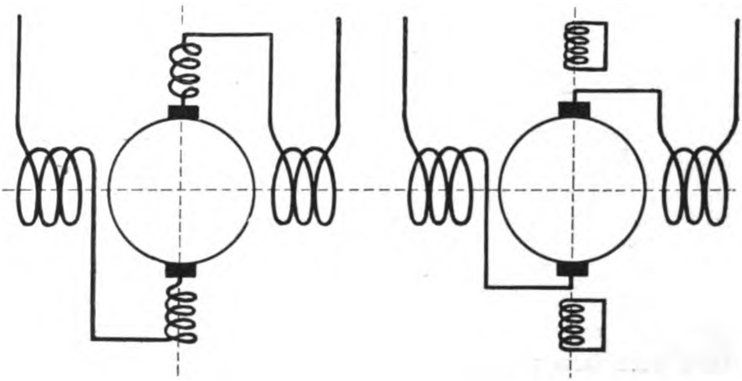


FIG. 535.

COMPENSATED MOTORS.

In the simple-series motor, Fig. 526, there will be a distortion of the flux as in the continuous-current motors. As the distorting magnetomotive-force is in phase with that of the magnets, the distortion of the flux will be a fixed effect.

If the poles are definite as in continuous-current machines, this distortion may not seriously affect the running of the motor, but with a magnetizing system like that universally adopted in induction motors the flux will be shifted as a whole in the direction of the distortion, which will produce the same effect as if in the former case the brushes had been shifted forward; whereas for good commutation they should have been shifted backward. As in continuous-current machines, this distortion is undesirable since it is not conducive to sparkless working, and also reduces to a more or less extent the torque exerted by the motor. As remedies the devices which have already been described in Chapter X., Vol. I., may all be used, but notice will only be taken here of those which have been actually adopted for the alternate-current case. The simplest remedy is that first adopted by Eichemeyer of compensating coils, Fig. 536, which are wound at right angles (electrically) to the main magnetizing coils, and are therefore placed on the stationary part in a position exactly to oppose the current on the revolving part; and so no distortion results. In the almost equivalent arrangement in Fig. 537 the compensating current is induced by transformer action. It will not be so effectual because there must be *some* distortion, although only a small amount, to produce current in the compensating coils; and as these coils must have self-induction, the current in them will not be exactly in phase with the armature distorting current. Fig. 538 shows the arrangement of this method adopted by Lamme, of the Westinghouse Co. When undistorted, the



FIGS. 536 AND 537.—COMPENSATED SERIES MOTORS.

flux will distribute itself equally on either side of the centre line through the pole; but when distorted some must link the centre slot, which may contain a solid bar of copper, or, as is here shown, two rods above and below which are short-circuited at the ends. The Oerlikon Co., in their single-phase railway motors, use separately wound polar projections as were designed by Sayers for continuous-current. See Vol. I., p. 262.

Another method of lessening distortion, but not entirely eliminating it, is to have great magnetic reluctance in the path of the distorting armature flux, and this again is carried out by various makers in ways exactly similar to those which have been adopted in continuous-current machinery (*loc. cit.*). Besides the device just mentioned, Lamme cuts slots in the pole face which serve the double purpose of making more saturated what iron is left, and of giving a longer path round the holes for the distortion fluxes.[1]

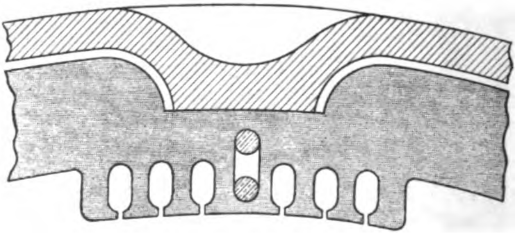


FIG. 538.—LAMME'S ANTIDISTORTION POLE.

Dr. Finzi,[2] of Milan, also produces cross-reluctance by special shapes of poles, giving graduated air-gaps, and with an air-slit right down the centre.

A method of compensation applied more especially to three-phase induction motors, but really applicable to all kinds of commutator motors, is that due to Heyland, shown in Fig. 539. It consists essentially of providing the rotor of the ordinary induction motor with a commutator, and arranging the windings and the angle of the brushes so that the flux is produced in the rotor instead of in the stator. As will be

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[1] Lamme's Brit. Spec. 26746 of 1902 was reprinted in *Elec.*, lii. 522, 1904.

[2] Brit. Spec. 17185 of 1903.

seen from the diagram, which is of the three-phase case, the current is introduced into the rotor at right angles in space to the direction of the coils in the stator, so that a magnetizing current sent into the rotor in phase with the applied voltage, will generate an electromotive-force in the stator coils at right-angles in space; and, as the fluxes are rotating, it will be also at right-angles in phase. In this way the motor has a power-factor of unity over a wide range of load. By adjusting the current sent into the rotor, the motor can be made to take a

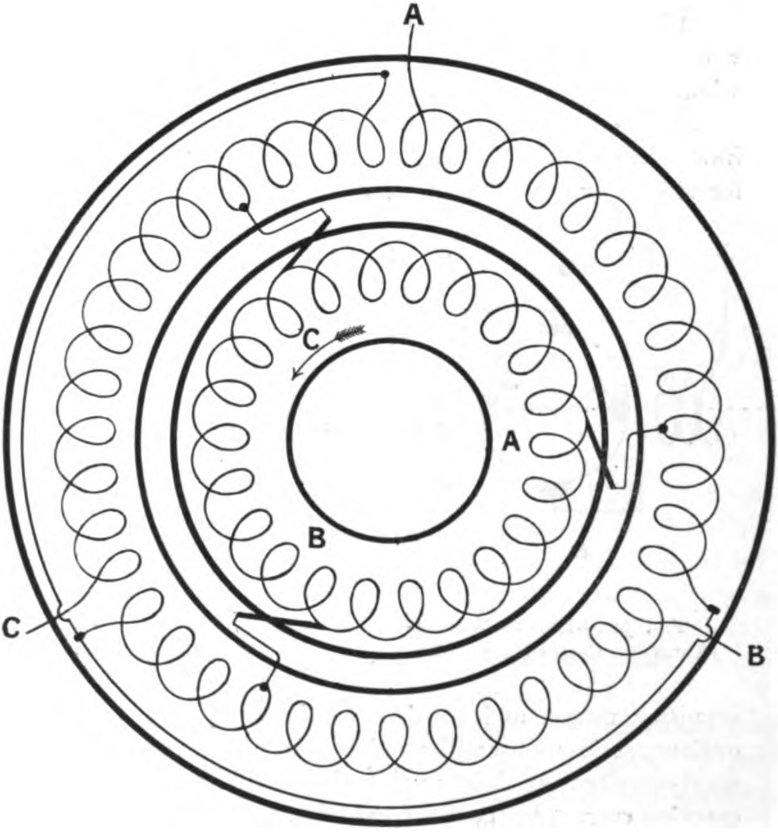

FIG. 539.—STATOR AND ROTOR WINDINGS OF HEYLAND'S COMPENSATED INDUCTION MOTOR.

leading current. The rotor excitation may be carried out with a special winding or with the usual short-circuited windings. It has even been found possible to use a squirrel-cage rotor, just pressing the brushes on the short-circuiting ring; but this method is not to be recommended.

It is to be noted that the Heyland arrangement does not correspond exactly to a series motor, because the rotor winding is in parallel with a portion of the stator winding.

A type of motor somewhat analogous to the above, is that independently used by Latour, and by Winter and Eichberg for single-phase working. It has been described as a series-repulsion motor, and consists, as shown in Fig. 540, of an ordinary series motor with another pair of brushes, permanently short-circuited at right angles electrically, so that the coils carrying current to them are exactly opposed by the windings on the stator.

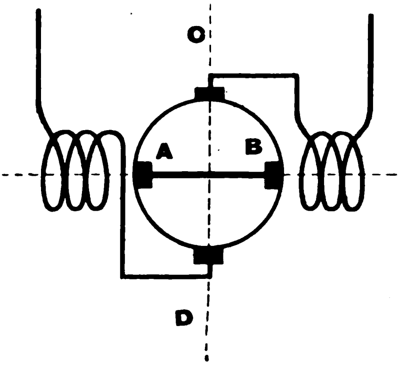

FIG. 540.—LATOUR-WINTER-EICHBERG COMPENSATED MOTOR.

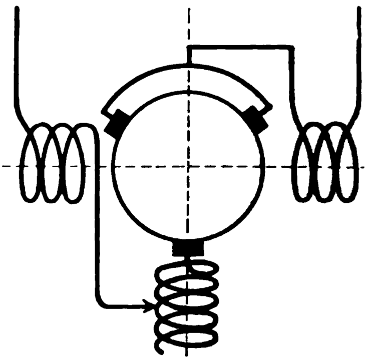

FIG. 541.—BRUCE-PEEBLES COMPENSATED MOTOR.

The action may be explained as follows. With an undistorted flux coming from the field-windings, only an *induced* electromotive-force is produced across the brushes A B. When, however, the current is flowing through the armature across the brush axis C D, a distortion takes place, and then a *generated* electromotive-force is produced across AB. The current which flows in consequence of this tends to check the distortion, which produces and also reacts with the stator windings to improve the power-factor.

A further example of this type of motor recently brought out by Bruce, Peebles and Co. is shown in Fig. 541.

Relative advantages of defined poles and continuous stators with distributed windings.—In all continuous-current designs, except those of the abnormal steam turbine type, the defined pole magnet system is invariably used. In the case of induction motors, where the field revolves, the continuous iron type of magnets has, of course, been adopted, and it has also been suggested by some that the commutator motors for alternating currents should be made with this type of construction.

When the stator windings are placed in slots on the stator surface, not only the leakage self-induction of the stator windings is reduced, but that of the rotor also, for the two sets of windings being thus placed closer together, their reactionary fluxes act *mutually* on one another, and thereby reduce the self-inductive effect which would lower the power-factor.

If the stator windings are concentrated in one coil on a defined magnet-pole, unless special compensating windings are provided the rotor currents set up distortions in the pole-pieces. These may either be laminated to reduce the eddy-current losses, or solid, in which case, although the eddies will occur, they will minimise the distortions causing them.

With the defined poles, any distortions which are set up have hardly any effect on the primary coils.

An economy in exciting copper results from having the windings concentrated in one coil, for the form-factor for the coil is unity (see p. 204), and a smaller current is, therefore, required to produce the same amount of flux. Again, the pole-face area is much better utilized by the flux in the case of the concentrated coil; for there is a uniform flux distribution in this case, whereas with the distributed coils the flux distribution shows a definite peak.

COMMUTATION IN ALTERNATE-CURRENT MOTORS.

The general principles of commutation have already been given in Vol. I., Chapter X., and we will now discuss them in their special application to the alternate-current case.

Commutation is rendered difficult in alternate-current machines, because—

(1) Of the shorter gap, which is necessary to diminish the magnetic leakage that would otherwise lower the power-factor;

(2) As a rule very high flux-densities may not be used in alternate-current motors, in view of keeping down the iron-losses;

(3) In motors where the currents in the field and the armature are not in phase with one another, *i.e.* when the maximum armature current does not occur at the same moment as the maximum field, times must occur when current must be commuted in no field at all;

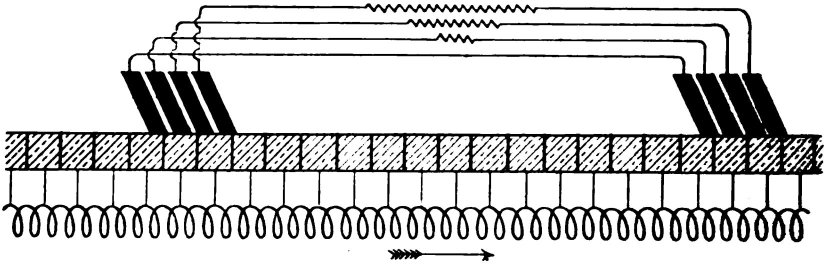

FIG. 542.—ARNOLD'S REPULSION MOTOR BRUSHES.

(4) As already explained (p. 791), electromotive-forces are always *induced* in the armature coils by transformer action, and, when a brush bridges over two segments of the commutator, the coil undergoing commutation is like a short-circuited transformer, and a large current circulates in it. This induced current further tends to decrease the torque of the motor.

Inasmuch as the difficulties experienced in commuting alternating currents are of the same kind as those which occur with continuous currents, the remedies are similar.

Laminated brushes similar to Mordey's (Vol. I., p. 500) have been suggested, and Fig. 542 shows special arrangements patented by Arnold for repulsion-motors. There are four independent pairs of brushes with different resistances in the cross connexions, so that the whole brush (represented as one set of small brushes) does not offer such good paths for local short circuits; moreover the main current may be cut off automatically as the coils pass under brushes short-circuited through more and more resistance.

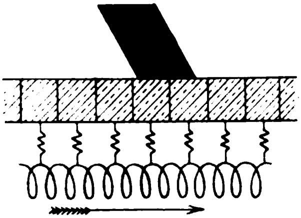

FIG. 543.

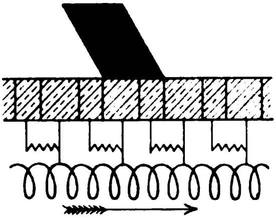

FIG. 544.

Various other devices are shown in Figs. 543 to 545. In Fig. 543 the insulation between the bars is of considerable thickness, so that the brush does not short-circuit two bars for any length of time; in fact, by making the bar,. the insulation, and the brush, all of the same width, no short-circuiting takes place. The two brushes are connected by a resistance, and the current is tapped off from the centre, or preferably nearer the hinder brush.

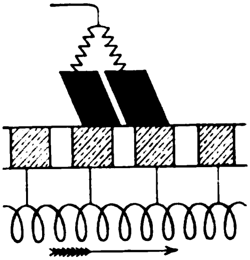

FIG. 545.

In the discussion on the effect of resistance on commutation in Vol. I., p. 248, it was shown that a resistance in the coil undergoing commutation was not necessarily a certain remedy, and it was pointed out that a variable resistance, such as is given, more or less, by the brush itself, as a commutator bar leaves it, was more desirable. The use of a high resistance connexion, as shown in Fig. 544, first suggested by Kelly in 1891, is, however, warrantable in alternate-current motors because of the transformer action, absent in continuous-current machines; but an arrangement such as that of Fig. 545, where a resistance is automatically inserted in the coil as it finishes commutation, would appear to be more satisfactory.

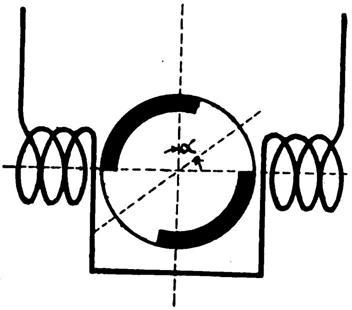

FIG. 546.—CRAMP'S REPULSION MOTOR.

Other arrangements with two sets of windings on the armature and double commutators, so that no coil is ever short-circuited by brushes at the same time, have been proposed. Cramp has suggested that in the repulsion motor (Fig. 528), instead of having two sets of brushes stretching over two or three bars, large brushes or sets of brushes should be arranged to stretch over a large arc of the commutator as in Fig. 546, and the *local* short-circuit currents become the currents which provide the torque.

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Osnos, M., 'Das Kreisdiagram des Repulsionmotors,' *Elek. Zeits.*, xxiv. 903, 1903. 'Die Einphasen Wechselstromkommutatormotoren (a review of all commutator motors to date),' *Elek. Zeits.*, xxv. 1 and 25, 1904.

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Schnetzler, K., 'Ein Neuer Repulsion Motor und seine Vorausberechnung,' *Elek. Zeits.* xxvi. 72 and 91, 1905.

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APPENDIX

THE STANDARDIZATION OF VOLTAGES, FREQUENCIES, AND RATINGS IN ENGLAND, UNITED STATES, AND GERMANY.

WHEN every manufacturer has his own stock sizes of machines, and when every consulting engineer designs distribution systems at voltages and frequencies which differ from those of others, confusion arises, and there is entailed a non-economical increase in the cost of production. To remedy this, engineers in the different countries have, through the medium of their respective Engineering Institutions, appointed Committees to draw up standard sets of rules for the *voltages*, *frequencies*, *sizes*, *etc.* of electrical machinery. In England, the work has been undertaken by the *Engineering Standards Committee*, a body appointed conjointly by the Institutions of Civil, Mechanical, and Electrical Engineers, the Institute of Naval Architects, and the Iron and Steel Institute. In the United States, rules have been drawn up by the American Institute of Electrical Engineers; and in Germany by the Verband Deutscher Elektrotechniker.

Below are given the recommendations of the British Standards Committee; and also, as notes on these, the American and German rules are mentioned.

I. PRESSURES AND FREQUENCIES.

The following are the Resolutions with reference to British Standard Pressures and Frequencies :—

1. RESOLVED that the Standard Low Pressures for direct and alternating-current work, measured at the terminals of the consumer, be :—

   110, 220, 440, 500, volts.

   Though not included in the above standard pressures, 380 volts shall be considered as the recognised pressure to be maintained between the principal conductors in a three-phase system with neutral wire, the pressure then being 220 volts between the three conductors and the neutral.

2. RESOLVED that the Standard High Pressures for alternating-current work, measured at the terminals of the generator, be :—

   2200, 3300, 6600, 11,000, volts.

3. RESOLVED that the Standard Primary Pressures for alternating-current transformer work, measured at the primary terminals of the transformer, be :—

   2000, 3000, 6000, 10,000, volts.

4. RESOLVED that the Standard Secondary Pressures for alternating-current transformer work, measured at the secondary terminals of the transformer, be :—

   115, 230, 460, 525, volts at no-load.

5. RESOLVED that the Standard Direct Current Pressure for tramway work, measured at the terminals of the motor, be :—

   500 volts.

6. RESOLVED that the Standard Frequency for alternating-current work be :—

   50 periods per second.

But where the circumstances of the case demand a lower frequency, a standard of 25 periods per second shall be adopted.

N.B.—The above Standard Pressures are subject to a permissible variation of 10 per cent. on either side.

In America, for low voltage continuous-current generators, the average terminal voltages recommended are 125, 250, and 550 to 600; while for both continuous and alternating current, low-pressure distribution voltages of 110 and 220 may be considered as standard. The voltage for continuous-current power plant is 500 volts.

It is not surprising to find that in America, where long-distance transmissions at extra-high pressures are common, the transmission voltages are much higher than those in England, the standard values at the receiving end being 6000, 10,000, 15,000, 20,000, 30,000, 40,000 and 60,000 volts, with an allowance of a no-load excess of 10 per cent. to provide for the drop in transmission. A uniform alternatimg voltage of 2200 is also recommended for distribution purposes, which allows of voltage ratios in the distributing transformers of $\frac{1}{10}$ or $\frac{1}{20}$.

The American Institute recommends three standard frequencies of 25 ~, 60 ~, and 120 ~, as these are already in extensive use.

The German regulations for voltages are briefly given in the following tables; their standard frequencies are 25 ~ and 50 ~.

Continuous-Current.		Alternating-Current (1, 2, or 3 Phases).	
For Motors.	For Generators.	For Motors or Primary of Transformers.	For Generators or Secondary of Transformers.
volts.	volts.	volts.	volts.
110	115	110	115
220	230	220	230
440	470	500	525
500	550	1000	1050
		2000	2100
		3000	3150
		5000	5250

II. RATING OF GENERATORS AND MOTORS.

(Except for traction motors.)

1. Two ratings shall be recognized by the British Engineering Standards Committee—

(A) Continuous Working.
(B) Intermittent Working.

(A) The output of generators and motors for continuous working shall be defined as the output at which they can work continuously for six hours and conform to the prescribed tests.

(B) The output of motors for intermittent working shall be defined as the output at which they can work for one hour and conform to the prescribed tests.

N.B.—The duration of test for machines above 250 KW is still under consideration.

2. Every generator and motor shall carry, in a conspicuous position, a name plate giving the output and other particulars enumerated below.

In the absence of any statement to the contrary, the output given shall always be understood to mean the output for continuous working under rating (A).

Name plates for machines under class (B) shall bear the word " Intermittent."

3. The output and full-load speed marked on the name plate shall be those taken when the machine is at its normal working temperature, as determined at the close of the test run referred to above.

4. All generators shall have their outputs stated in kilowatts (KW).

All motors shall have their outputs stated in B.H.P.

5. The following information shall be given on the name plates :—

Generators	*Direct Current.*	KW.	Volts.	Amperes.	RPM.
	Alternating Current.	KW.	Volts.	Amperes.	Power-Factor.
	Full-load Excitation Volts and Amperes.			Frequency.	RPM.

Motors	D.C. (Continuous working)	BHP	Volts.	RPM
	D.C. (Intermittent working)	BHP (Intermittent)	Volts.	RPM
	A.C. (Continuous working)	BHP	Volts. Frequency	RPM Power Factor.
	A.C. (Intermittent working)	BHP (Intermittent)	Volts. Frequency	RPM Power Factor.

The above applies to combined machines, such as motor generators, boosters, rotary converters, which shall have name plates giving information applying both to input and output.

It will be noted that in these regulations so far no standard test conditions were laid down, as further investigation was necessary as to the safe limit of temperature at which electrical machinery can be allowed to work for lengthened periods of time. This matter has therefore been taken in hand by a sub-committee, who have undertaken tests at the National Physical Laboratory under Dr. Glazebrook. The points to be investigated were the following:—

1. The *maximum* temperature to which the insulating materials at present used in the manufacture of electrical apparatus could be exposed for lengthened periods of time without electrical or mechanical deterioration.
2. Permissible temperature-rise deduced from these experiments.
3. The relation between the mean temperature of any coil obtained by measurement of rise in resistance and the *maximum* temperature at the hottest portion of the coil.

Pending the further report on these matters much information will be found of these tests in the paper by Mr. Rayner in the *Journal of the Institution of Electrical Engineers*, xxxiv. 613, 1905.

The American rules stipulate that all ratings, whether of generators or motors, shall be stated in kilowatts, and in the case of alternate-current machines the apparent power shall be expressed in kilowatts (kilovolts-amperes). With regard to tests to decide the rating by the heating which takes place under load, elaborate rules have been drawn up, of which the following are the more important.

Rule 33 says that the following maximum values of temperature-rise (above a standard test-room temperature of 25° C.) should not be exceeded; viz.:—

In dynamos, alternators, and converters:—Field-magnets and armatures, 50° C. by resistance; commutators, collecting-rings, and brushes, 55° C. by thermometer; electric circuits 50° C. by resistance; squirrel-cage and short-circuit rotors, 55° C. by resistance.

In transformers for continuous service:—electric circuits, 50° C. by resistance; other parts, 40° C. by thermometer.

In transformers for intermittent work:—when tested as near as possible to normal manner of working, their electric circuits must not show a rise of more than 50° C. by resistance.

For tramway motors and railway motors a special rating is provided, namely that, working continuously at their rated output for one hour, at 500 volts, the temperature-rise shall not exceed 75° C.

Rules 89 to 92 deal with questions of the overloads which different apparatus shall stand without self-destruction by heating, sparking, or mechanical breakdown, and with a further increase in temperature of 15° C. above that of full-load conditions.

In rule 92 the following overload capacities are recommended :—

In dynamos and alternators, 25 per cent. for two hours.

In all motors, save those used for intermittent service, 25 per cent. for two hours, and 50 per cent. for one minute.

In synchronous converters, 50 per cent. for one half-hour.

In transformers, 25 per cent. for two hours.

In exciters for alternators and converters, 10 per cent. more overload than is required for the excitation of the machine at its guaranteed overload, and for the same period of time.

In the corresponding German regulations apparatus is divided into three sections, according to the following conditions of working :—

(*a*) Intermittent working, as in the case of motors for cranes, lifts, tramways, etc., in which the periods of work and rest alternate with each other, each lasting but a few minutes.
(*b*) Working for shorter periods than are necessary for attainment of the final temperature, and with periods of rest long enough for the temperature of the machine to drop approximately to that of the surrounding air.
(*c*) Continuous working long enough to reach final temperature.

In testing for temperature-rise, apparatus which comes under class (*a*) shall be tested for one hour; that under class (*b*) for such time as shall be specified on the name-plate; that under class (*c*) for ten hours, if uniform temperature is not previously attained.

The temperature-rises, as measured by thermometer, are specified according to the nature of the insulation, and are :—

For class (*a*)	with cotton insulation	70° C.
	with paper insulation	80° C.
	with mica or asbestos insulation	100° C.
For classes (*b*) and (*c*)	with cotton insulation	50° C.
	with paper insulation	60° C.
	with mica or asbestos insulation	80° C.

A higher rise by 10 per cent. is allowed for stationary windings.

As regards overloads, the following conditions are briefly specified :—Generators, motors, and converters, 25 per cent. for half an hour; or for these, and for transformers, 40 per cent. for three minutes.

III. LIST NUMBERS FOR SIZES OF MACHINES.

DIRECT-CURRENT GENERATORS.

6. The List Nos. represent the KW at which the machine can work when running continuously as a generator.

List Nos. and speeds of direct current generators (up to 100 KW :—

List No.	Standard Motor Carcase.	RPM.	List No.	Standard Motor Carcase.	RPM.
6	7½	1075	32	40	750
8	10	1000	40	50	675
12	15	900	60	75	625
16	20	850	80	100	575
24	30	800	100	..	500

British Standard generators of 100 KW and above, whether for direct or alternating-current work, shall conform to the following list of sizes and speeds recommended for generators to be directly coupled to steam or gas engines.

KW.	Revolutions per Minute.		
	Slow.	Medium.	High.
100	..	250	500
150	..	250	428
200	..	250	375
250	..	250	375
300	94	214	375
400	94	214	375
500	83	214	300
750	83	188	250
1000	83	188	250

N.B.—The "Slow" speeds in the above table are tentative.

ALTERNATING-CURRENT GENERATORS.

7. British Standard alternators of any type, in addition to the requirements laid down in previous clauses, in so far as the latter apply, shall conform to the following regulations :—

(*a*) They shall give an E.M.F. curve which, under all working conditions, shall be as nearly as possible a sine wave.

(*b*) For exciting the field-magnets the standard pressures shall be :—

65, 110, or 220 volts.

(*c*) The term "Alternator" shall not include "Exciter." The latter, when necessary, shall be separately specified and subject to the regulations for standard direct current generators.

(*d*) The regulation of an alternator shall be defined as the difference between the rated full-load pressure and the no-load pressure with the same speed and excitation. This difference expressed as a percentage of the rated full-load pressure, shall be termed the percentage "pressure-rise" of the alternator.

(*e*) They shall not have a greater percentage pressure rise than 6 per cent. (6%) on a non-inductive load, and 20 per cent. (20%) on an inductive load, the latter being here considered as one having a power-factor of 0·8.

This pressure-rise may be tested on a non-inductive or inductive load, according to the requirements of the specification.

The figures in (*b*) and (*e*) shall not apply to compounded alternators.

MOTORS.

8. All motors for the purposes of tests shall be rated under the following classes:—

(i) Open.
(ii) Protected.
(iii) Ventilated.
(iv) Totally enclosed.

(i) and (iv) require no definition.

(ii) A *Protected* motor is defined as a motor in which the armature, field coils, and other live parts are protected mechanically from accidental or careless contact, so as not to materially interfere with ventilation.

(iii) A *Ventilated* motor is defined as a motor in which, while ventilation is provided for, access to the armature, field coils, and other live parts is only to be obtained by opening a door in, or removing a portion of, the enclosing case.

N.B.—An alternating-current motor, Class (iii), in which the slip-rings are outside the protection, shall be considered as coming under Class (ii).

9. The List Nos. represent the B.H.P. at which the machine can work when running continuously as a motor, at the standard pressure of 220 volts, up to and including two (2) B.H.P., and above that size, at the standard pressure of 440 volts.

10. The following are the List Nos. of British standard sizes of motors:—

LIST NOS. (direct-current)—

¼, ½, 1, 2, 3, 5, 7½, 10, 15, 20, 30, 40, 50, 75, 100.

LIST NOS. (single-phase) 50 ~

1, 2, 3, 5, 7½, 7½A, 10, 10A, 15, 20, 25.

LIST NOS. (two- and three-phase) 50 ~

1, 2, 3, 5, 7½, 7½A, 10, 10A, 15, 20, 25, 30, 40, 50, 50A, 75, 100.

11. List Nos. and speeds of motors (up to B.H.P.)—

DIRECT-CURRENT MOTORS.

List No.	RPM at Full Load.	List No.	RPM at Full Load.	List No.	RPM at Full Load.
¼	1600	5	1000	30	750
½	1400	7½	1000	40	700
1	1400	10	900	50	650
2	1100	15	850	75	600
3	1100	20	800	100	550

ALTERNATING-CURRENT INDUCTION MOTORS.

Single phase, 50 ~

List No.	RPM at No-Load.	List No.	RPM at No-Load.	List No.	RPM at No-Load.
1	1500	7½	1500	15	1000
2	1500	7½A	1000	20	1000
3	1500	10	1500	25	700
5	1500	10A	1000		

Two- and three-phase, 50 ~

List No.	RPM at No-Load.	List No.	RPM at No-Load.	List No.	RPM at No-Load.
1	1500	10	1500	40	750
2	1500	10A	1000	50	750
3	1500	15	1000	50A	600
5	1500	20	1000	75	600
7½	1500	25	750	100	500
7½A	1000	30	750		

The figures referring to alternating-current motors give the no-load or synchronous speeds; allowance should, therefore, be made for a reduction in speed at full-load of from about 7½ per cent. in the smallest motors to 2½ per cent. in the largest motors.

In America and in Germany no attempts appear to have been made to secure adoption of a similar standard list of sizes.

A large section is devoted to the question of regulation. It is defined on the same pressure-rise basis as the above, but no standard limits are prescribed.

No standard voltages are prescribed for exciters, nor for rotors of three-phase motors; and no classification or listing of motors as in the British regulations has been prescribed.

On the other hand, the American regulations go deeply into questions of efficiency and insulation, and also provide standards for comparison of luminous intensities.

In the German regulations, no standard sizes of machines are proposed. With regard to regulation, the pressure-rise is to be specified, and is to be given for the normal armature current with an inductionless load, and also for a current one-third of the normal but on an inductive load such that the power-factor is not greater than 0·3. No standard regulations are, however, prescribed to limit the pressure-rise.

On the matter of insulation, the German rules are fairly detailed, and the following insulation tests are given. Apparatus for pressures up to 5000 volts to be tested at double the working pressure, but in no case at less than 100 volts. Apparatus for pressures between 5000 and 10,000 volts to be tested at 5000 volts in excess; and apparatus for pressures above 10,000 at a 50 per cent. excess pressure.

On the question of efficiency, the German rules are very concise. They put forward a series of six methods of testing for efficiency, with full instructions in each case.

The full text of the several regulations may be found as follows:—

British Regulations:—

Reports of Engineering Standards Committee, 28 Victoria Street, Westminster:

No. 7. Standard Tables of Copper Conductors and Thicknesses of Di-electric.

No. 17. Interim Report on Electric Machinery.

No. 19. Report on Temperature Experiments on Field coils.

No. 22. Report on Effect of Temperature on Insulating Materials.

See also *Electrician* liii. 679, 1904, for reprint of No. 17.

American Regulations:—

Transactions of American Institute of Electrical Engineers, xix. May, 1902.

German Regulations:—

Electrotechnische Zeitschrift xxiii. 755, 1902. Summary in *Electrician* xlvii. 339, 1901.

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