THE

DIRECT-CURRENT MOTOR

ELECTRO-DYNAMICS

THE DIRECT-CURRENT MOTOR

BY

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PREFACE

THE Electrical Engineer is now confronted with a variety of new problems for whose solution he must look to the full development of the science of Electro-Dynamics. This science in the near future will assume the same relation to the electric motor that the science of Thermo-Dynamics already bears to the steam engine. And since no branch of dynamics has a better claim to be called an exact science than electro-dynamics, it will be able to offer not only convincing but final solutions of these problems. To apply the principles of electro-dynamics to the Direct-Current Motor is the aim of this book.

Writing for electrical engineers particularly, I take for granted a certain acquaintance with the use and design of motors, but as the book is intended to be of service to engineers generally, unexplained technicalities have been avoided as far as possible.

There are now so many excellent text-books on electricity and magnetism, that I offer no apology for omitting discussion of elementary principles here.

I have not considered it necessary to deal with the subject of self-induction, except in connection with the question of sparking. The advanced student will perceive the analogy between the law of acceleration given in Chapter VII. and that for the rise of current in an inductive circuit, and may be tempted to pursue the subject for himself.

The numerical accuracy attempted has been limited to that attainable with an ordinary ten-inch slide rule, on which all the examples have been worked out. Importance is attached to the graphic method of solution, and the diagrams are intended to serve as exercises for the student, who should work out similar problems with different data by the same methods.

I have to thank many friends for assistance, particularly Mr. H. S. Hering, for allowing me to use the results of his tests on electric cars; Mr. L. H. Parker, for providing me with particulars of the construction and performance of the electric locomotives on the Baltimore and Ohio Railroad; Mr. H. P. Curtiss, for placing at my disposal the outcome of his experiments on the Buffalo and Niagara Falls Electric Railway; and the Railway Department of the General Electric Company, for furnishing me with valuable information and data.

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CHAPTER I

THE INDUCTION FACTOR

WHEN a straight conductor carrying a current is placed in a uniform magnetic field in a plane at right angles to the magnetic lines, it experiences a force urging it to move in that plane at right angles to its own length.

If H is the intensity of the field in lines per square centimetre, l the length of the conductor in centimetres, and i the current in the conductor measured in amperes, the force on the conductor is given by

$$f = Hli10^{-1} \text{ dynes} \quad \ldots\ldots\ldots\ldots\ldots(1).$$

The sign of f can be changed by reversing the sign of either H or i; hence the direction of the force can be reversed by altering the sign of the field or of the current. If both are altered at the same time, the direction of the force is unchanged.

Fig. 1 is a section of the armature and pole pieces of a dynamo at right angles to the shaft. A and B are two surface conductors, one on each side of the plane through the centre containing the brushes. The currents in these conductors are in opposite directions, but the lines of force due to the magnets are in the same direction. Hence the forces acting on the conductors constitute a couple. If

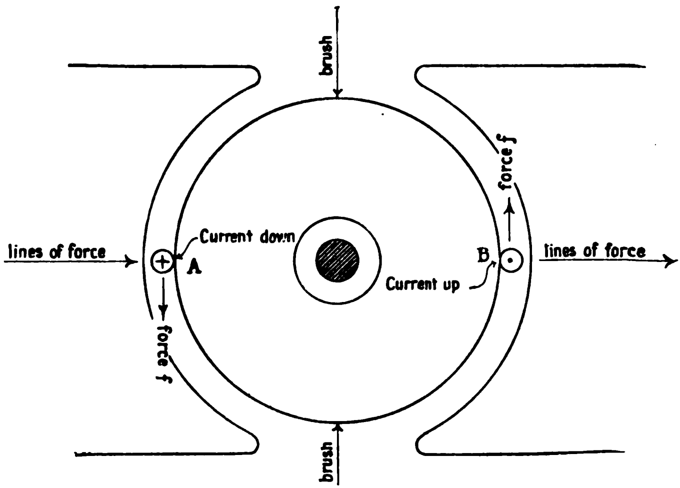


FIG. 1

any number of conductors are arranged evenly round the armature, the action of the field on the current in the conductors on one side of the brush plane, combined with that on the other side, results in a couple tending to turn the armature about its axis.

Let there be A surface conductors, and let their axes lie on a cylindrical surface of radius r and length l centimetres, as shown in Fig. 2. The distance between any two adjacent conductors is $\frac{2\pi r}{A}$ centimetres, and the cylindrical area enclosed by them is $\frac{2\pi rl}{A}$ square centimetres. If dn lines enter the cylindrical surface between them, $\frac{Adn}{2\pi rl}$ will be the intensity of magnetisation at the surface of the

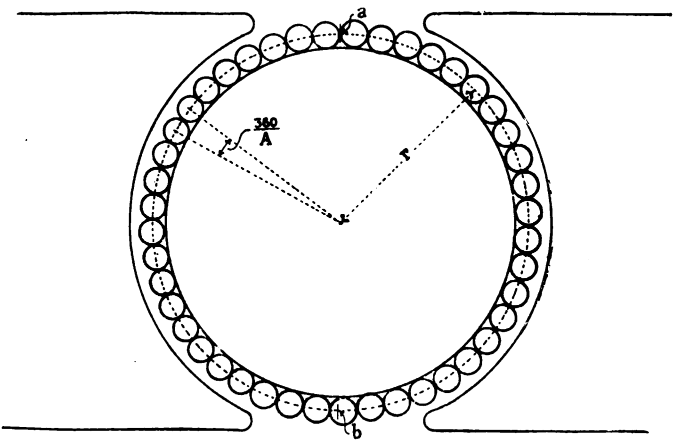


FIG. 2

cylinder, that is, the number of lines per square centimetre crossing the tangent plane to the cylinder at that point.

In Fig. 3, d represents the axis of one of the conductors lying in the cylindrical surface, and we assume that the lines of force enter the cylinder in the direction indicated by the sheaf of arrows, so that the plane at right angles to the lines of force makes an angle of θ degrees with the tangent plane at d, the line edc representing a portion of the cylindrical surface in a plane at right angles to the shaft.

The force f on the conductor is, as we have seen, $Hli10^{-1}$, where H is the number of lines per square centimetre in the plane making θ degrees with the tangent at d. Substitute for H its value in terms of the intensity in the tangent plane, and we get $f=\frac{Aidn}{2\pi r\cos\theta}10^{-1}$. Now, this

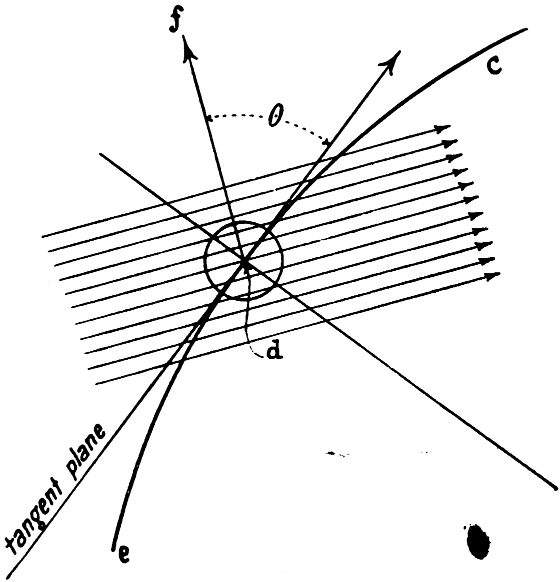


Fig. 3

force acts in a direction making θ degrees with the tangent at d. Hence the resolved part of the force in the direction the tangent is $\frac{Aidn}{2\pi r}10^{-1}$.

The sum of the tangential forces on all the conductors round one side of the armature is thus $\frac{AiN}{2\pi r}10^{-1}$, where N

is the total number of lines of force entering the cylindrical surface from one pole. Hence the couple or torque t, due to the forces on both sides, tending to turn the armature, is $\frac{1}{\pi}AiN10^{-1}$. The whole current, c, passing into or out of the armature will be twice that in each conductor, so we have

$$t=\frac{1}{2\pi}AcN10^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2).$$

We shall find it convenient to express this in inch-pounds. Remembering that a weight of one pound is equal to $4{\cdot}45\times10^{5}$ dynes, we get

$$t=1{\cdot}41AcN10^{-8}\text{ inch-pounds }\ldots\ldots\ldots(3).$$

We have for simplicity supposed that the dynamo has only two poles, and that N lines enter the armature from one pole, and pass from the armature into the other pole. If the dynamo had four poles with N lines per pole and A surface conductors, N and A being the same as in the two-pole machine, and if the current per conductor remained unaltered, the total force on all the conductors would be doubled. But if the four polar divisions of the armature are connected in parallel, a current of c amperes entering the armature will be divided into four instead of into two parts, as with the two-pole machine; hence, if c is the same as before, the current per conductor in the four-pole machine is half what it was in the two-pole machine, so that the force and the torque remain unaltered.

Equation 3 is then a general equation, applicable to machines having any number of poles, provided that N is the number of lines per pole, and that there are as many surface conductors in series between the main terminals of the machine as there are in one polar division of the armature; when this is the case the armature is said to be connected in parallel.

If, however, the armature is connected in series, the conductors lying under one pole being placed in series with those lying under another pole, the force and the torque must be multiplied by a quantity p, expressing the number of polar divisions of the armature thus connected in series. Equation 3 may then be written—

$$t = 1{\cdot}41 pAcN10^{-8} \quad \text{..............(4).}$$

Example 1.—A four-pole dynamo has 440 surface conductors with $16{\cdot}1 \times 10^6$ lines per pole. The armature is series connected, giving $p=2$. The torque for 300 amperes is 60,000 inch-pounds.

Example 2.—A ten-pole dynamo has $28{\cdot}6 \times 10^6$ lines per pole and 1,440 surface conductors; the armature is parallel connected, giving $p=1$. The torque for 800 amperes is 465,000 inch-pounds.

Equation 4 may be written

$$t = 1{\cdot}41 cM \quad \text{....................(5).}$$

M being given by

$$M = pAN10^{-8} \quad \text{.................(6).}$$

where A is the number of surface conductors, p is the number of polar divisions of the armature connected in series, and N is the number of useful lines per pole.

We shall call M **the Induction Factor** of the dynamo.

Example 3.—A two-pole dynamo has 200 surface conductors and $2{\cdot}1 \times 10^6$ lines per pole. The induction factor is $4{\cdot}2$.

Example 4.—A four-pole motor has its armature series connected, with $2{\cdot}8 \times 10^6$ lines per pole and 600 surface conductors. The induction factor is 33·6.

When M is known the torque for any current can at once be obtained from Equation 5.

Example 5.—A motor has an induction factor of 5. The torque on the shaft for 90 amperes in the armature is 635 inch-pounds.

In estimating A we have to count round the entire surface of the armature, and include only those conductors which, being adjacent, are also in series with one another. In many cases one 'conductor,' according to our notation, is, for convenience, made up of two or more insulated wires in parallel, so that in counting we must be careful not to reckon as two conductors two adjacent wires which are connected in parallel with one another.

Example 6.—A railway motor has 60 slots on the armature, and 24 wires per slot, every two of which are in parallel, giving 12 'conductors' per slot, and making $A = 720$.

In a ring-wound armature each turn after leaving the end of the armature away from the commutator passes inside the armature, where it does not cut any lines of force, and so back to the next bar. Hence one turn per commutator bar with a ring armature means as many surface conductors as there are bars; four turns per bar means four times as many surface conductors as there are bars, and so on.

In a drum armature, however, each turn after passing up the armature from the commutator, is brought back on the surface of the armature, and finds its way to the next bar without passing inside the armature; hence each commutator bar provides at least two surface conductors, and the total number of surface conductors is at least twice as many as the number of bars.

Example 7.—A railway motor has 100 bars on the commutator with four turns per bar; the armature is drum wound. A is 800.

Example 8.—A motor has a ring-wound armature, 120 commutator bars, and 8 turns per bar. The number of surface conductors is 960.

Equation 5 shows that we can find the induction factor of a dynamo by observing the current in the armature and the corresponding torque on the shaft.

A lever is bolted to the shaft of the dynamo, and the pull at a measured distance from the centre observed by a spring balance, which should be provided with a metal stirrup to go under the lever. The balance should be raised until the lever is horizontal, and the pull then read.

We shall find that we get a small reading on the scale when there is no current in either magnets or armature. This is due to the friction of the bearings and brushes; we may assume it to be constant throughout the experiment. On exciting the magnets we shall generally find that the pull is slightly increased without having any current in the armature; this will be explained later on, but we may state briefly that a definite torque is required to turn a mass of iron in a magnetic field, the amount of the torque depending on the quality of the iron, the intensity of the field, and the volume of iron turned; if the current exciting the magnets be kept constant, this torque, which we shall call the 'hysteresis torque,' will be constant.

Since the torque in Equation 5 is that due to the action of a current in the armature, we must be careful that we do not credit either the frictional or hysteresis torque to this action ; we must therefore deduct the pull observed with no current in the armature from all subsequent readings, and if the magnet current be kept constant we shall then get the true torque due to the current in the armature.

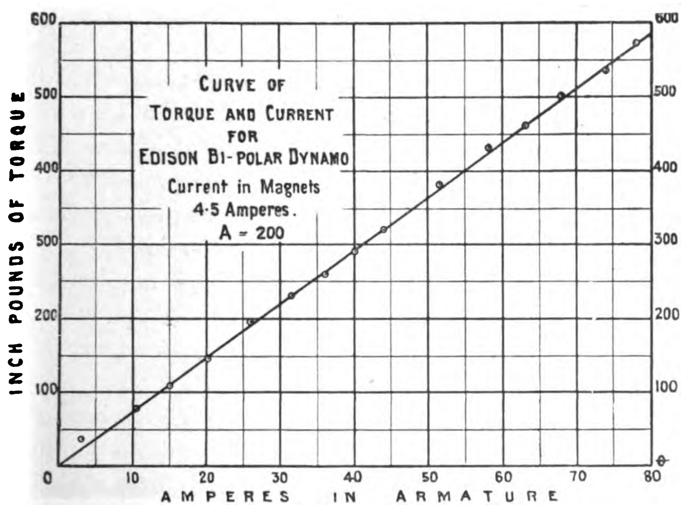


FIG. 4

An ammeter should be placed in the magnet circuit and the current kept constant during the test. The induction factor obtained will be that for this particular current in the magnets; if this current is altered, a different induction factor will be obtained.

The experiment here described, in which **the induction factor** is **found by measuring the torque** when the machine is at rest, shows that the force produced by a current in the armature is independent of the motion of the armature.

In Fig. 4 will be found the results of a torque test made with an Edison 12 K.W. dynamo. This machine has 200 conductors on the armature, and is designed to carry 96 amperes at 125 volts, running at 1,500 revolutions per minute. The conductors are laid on the surface of the iron core. The torque is plotted on a base of amperes in the armature.

The current in the magnets was kept constant at 4·5 amperes. The leverage was 30 inches. A pull of 1·4 pounds due to friction was observed with no current in magnets or armature; this was not increased when the magnets were excited, showing that the hysteresis torque was inappreciable. The induction factor is obtained from Equation 5.

After the experiment a current of 78 amperes was passed through the armature, and a fall of potential of 3·74 volts observed, giving the resistance from brush to brush as 0·048 ohm.

In Fig. 5 let ab represent the length πr, where r is the mean distance of the axes of the conductors from the centre of the shaft; ab is thus equal to one half of the armature circumference measured round the circle of radius r.

Set off along ab points 1, 2, 3, 4 . . . to represent the positions of the conductors. The distance from point to point will be $\frac{1}{A}$ of the complete circumference, assuming for the present that the conductors are laid side by side. At each point draw a vertical line, to represent by its length the number of lines of force per square centimetre entering the armature across the cylinder of radius r, on the surface of which the axes of the conductors are supposed to be lying, as shown in Fig. 2. The ends of these vertical ordinates will form a curve, the shape of which will depend upon the variation of the intensity of magnetisation from point to point round the armature. The curve will generally be quite irregular and follow no definite law; methods of obtaining it experimentally will be given later on. We shall call such a curve the

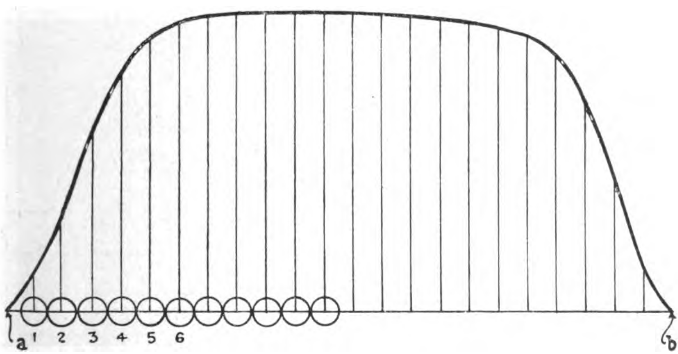


FIG. 5

magnetisation curve, and the points a, b, the neutral points; these points will be a distance πr apart if the field is symmetrical.

The area of each element of the curve being the product of the intensity at any point into the distance over which the intensity may be assumed to be constant, will represent dn, the number of lines of force entering the armature between two adjacent conductors, and the whole area of the curve will represent N, the whole number of lines of force entering the armature from one pole.

In Fig. 6 magnetisation curves are given for a high tension arc light dynamo. This machine has a ring armature with 80 sections and 47 turns per section, making $A = 3{,}760$. The areas of the curves for 3, 6, and 9 amperes in the magnets are as 100, 159, and 178. The corresponding values of N as obtained by a method to be presently described are $1{\cdot}05 \times 10^6$, $1{\cdot}68 \times 10^6$, and $1{\cdot}89 \times 10^6$.

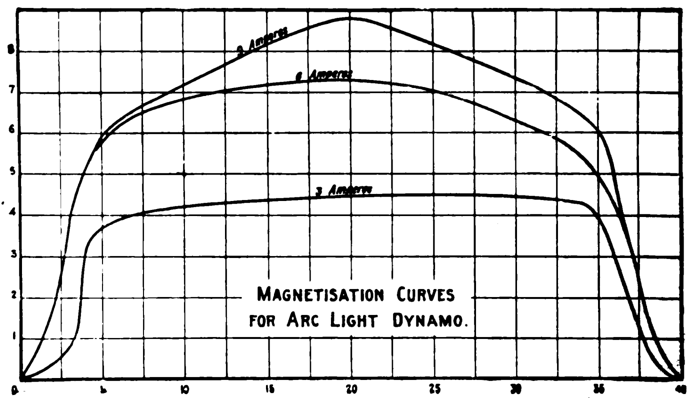


FIG. 6

Let us suppose that the armature is turned through an angle equal $\frac{360}{A}$ degrees. Each conductor will then sweep through the space that lies between it and the next conductor. The number of lines of force occupying this space is represented by the area of the strip of the curve between the two conductors; hence each conductor will cut a number of lines of force represented by the area of the strip next to it. Let dn be the area of any such strip, and dt the time occupied in turning the armature through

$\frac{1}{A}$ of a revolution, then the rate of cutting lines of force is $\frac{dn}{dt}$; there will therefore be an induced tension at the ends of each conductor equal to $\frac{dn}{dt}$ c.g.s units of tension, or $\frac{dn}{dt}10^{-8}$ volts, so that if each conductor were separate from all the other conductors, we should observe a reading of $\frac{dn}{dt}10^{-8}$ volts on a voltmeter connected to the ends of any one conductor, dn depending upon the position of the conductor.

In practice, if the brushes are placed at the neutral points ab, each conductor is in series with all the other conductors round one side of the armature between the brushes, so that the effect is as if we had one long conductor cutting lines at different rates along its length; the total number of lines cut by this imaginary long conductor in dt seconds is represented by the whole area of the curve; hence the whole tension induced is $\frac{N}{dt}10^{-8}$ volts.

If the armature is rotating uniformly at n revolutions per second, $dt = \frac{1}{nA}$; hence the tension induced in all the conductors in series between the neutral points, and observed on a voltmeter connected by brushes to these points, will be given by the equation

$$e = AnN10^{-8} \text{ volts} \quad \ldots\ldots\ldots\ldots\ldots\ldots(7).$$

If the dynamo has more than two poles, and if the conductors between two adjacent neutral points are placed in series with these between two other neutral points, p

representing as before the number of polar divisions of the armature thus connected in series, we may write:

$$e = pAnN10^{-8} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8).$$

We have assumed that the conductors are spaced evenly round the armature. A similar proof would hold good if the conductors lay two or more deep, with equal spaces between those in each layer.

If in Equation 8 we insert M in place of $pAN10^{-8}$, we get

$$e = Mn \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9).$$

From this we see that the induced tension in a dynamo is given by multiplying the induction factor by the number of revolutions per second. The induction factor may thus be defined as the induced volts divided by the number of revolutions per second.

We must not however suppose that because M is thus defined it depends on the motion of the armature. Equation 6 shows that M depends only upon p, A, and N, and does not in any way involve the speed.

We have here a **second method for finding** the value of **the induction factor**—namely, to drive the dynamo as a generator, and observe the induced tension at the terminals of the machine, and the revolutions per second.

In making this test we must be careful to insure that the tension recorded is the true induced tension, expressed by Equation 7. The voltmeter will only indicate this tension provided that the brushes are placed precisely at the neutral points, for in this position only will all the conductors between two brushes be generating a tension of like sign. Any forward or backward lead will place two sets of conductors with opposing tensions between two brushes, and consequently make the reading of the voltmeter less than the true induced tension.

If the speed be plotted along a horizontal axis and the readings of the voltmeter along a vertical axis, then if M is constant for any given current in the magnets, the locus of the observed points should be a straight line, and this should pass through the ~~axis~~ origin if there be no residual magnetisation.

Fig. 7 gives the results of an experiment made on the arc light dynamo, previously described. In this experiment the current was kept constant, first at two and then at four amperes, and the speed varied, the tension corresponding to each speed being observed and plotted in the figure. There was no current in the armature. The points found lie on straight lines passing through the origin, showing that the induction factor is independent of the speed.

If a current is passing in the armature during a test, the tension observed at the brushes will be less than the induced tension by an amount CR volts, where C is the current and R the resistance of the armature from brush to brush. The product CR is called the 'heat drop.' Hence to find M when a current is passing, we must measure C; knowing R, we can add the product CR to the measured volts at the brushes.

Example 9.—We wish to know the torque on the shaft of a direct coupled generator due to a current of 300 amperes passing in the armature. The speed of the dynamo as indicated by a tachometer is 400 r.p.m., while the voltmeter reads 120 volts on open circuit. Dividing the volts by the revolutions per second, we find the induction factor to be 18; the torque thrown on the

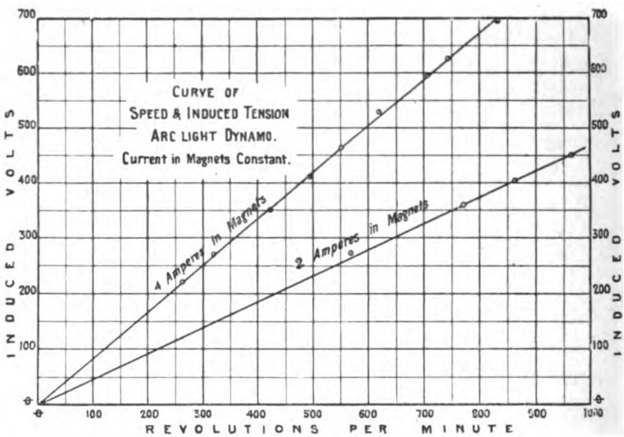

CURVE OF
SPEED & INDUCED TENSION
ARC LIGHT DYNAMO.
Current in Magnets Constant.
4 Amperes in Magnets
2 Amperes in Magnets
INDUCED VOLTS
REVOLUTIONS PER MINUTE

FIG. 7

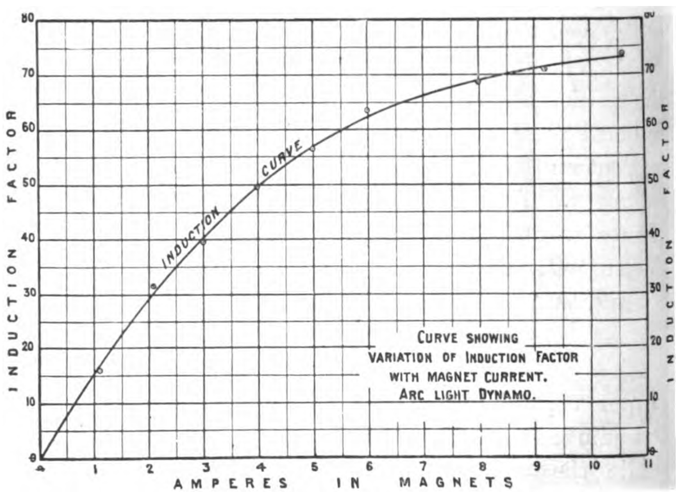

INDUCTION CURVE
CURVE SHOWING
VARIATION OF INDUCTION FACTOR
WITH MAGNET CURRENT.
ARC LIGHT DYNAMO.
INDUCTION FACTOR
AMPERES IN MAGNETS

FIG. 8

shaft when a current of 300 amperes is passed through the armature is then 7,610 inch-pounds.

Example 10.—A generator is running at 440 r.p.m. The tension on the brushes is 550 volts when the machine is delivering 400 amperes. The internal resistance of the armature is 0·0375 ohm. What is the pull on the belt due to the current in the armature if the pulley is 40 inches diameter? The heat drop is 15 volts, so that the true induced tension is 565, and M is 77. The pull due to the armature current is equal to the torque divided by the radius of the pulley, or 2,166 pounds. If we had neglected the heat drop, the pull would have appeared to be 56 pounds less than this.

The induction factor can be varied by changing the current in the magnets. We may obtain values of M for different values of the magnetising current by either of the methods previously described, the torque or the speed method. If the values of M are plotted on a base of current in the magnets, we get a curve that we shall call **the 'induction curve.'**

Fig. 8 gives the induction curve of the arc light dynamo already described, obtained by running the armature at a measured speed and observing the induced tension on open circuit for different currents in the magnets. The curve plotted in the figure is for descending values of the current.

The curve commonly called the 'Characteristic' may be obtained from the induction curve by multiplying the vertical ordinates by any given number of revolutions per second, thus giving a curve of volts for the given speed on a base of magnet current. The induction curve is quite independent of speed, since it represents the magnetic condition of the dynamo for different magnet currents, and in no way involves the idea of motion.

When a current is made to pass through the armature of a dynamo, the magnets being excited, the field due to the magnets is shifted and the distribution of magnetisa-

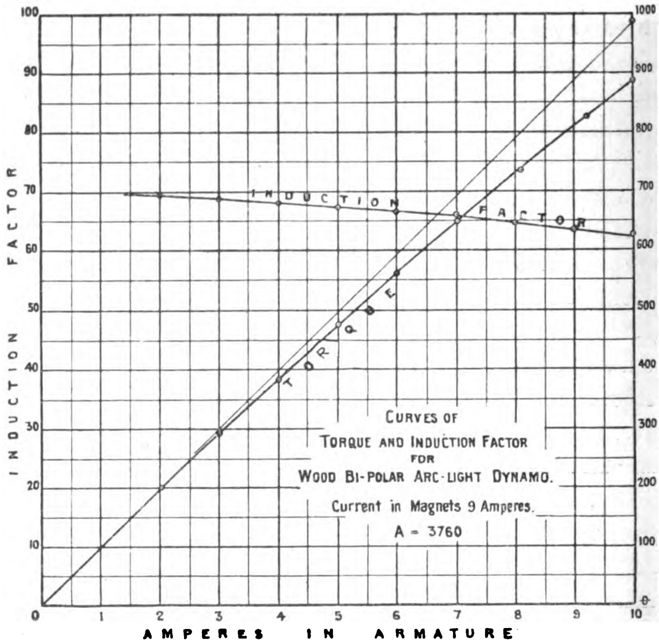


FIG. 9

tion changed, the result often being an alteration in the area of the magnetisation curve and a diminution in the value of *M*. The amount of this diminution depends very largely upon the form of the pole pieces.

In some machines where the value of the induction factor is practically constant for all currents in the armature, while in others M is perceptibly diminished as the armature current is increased.

Experiments were made on the arc light dynamo, previously described, to ascertain the amount of the reduction of M due to this cause. The current in the magnets was kept constant at 9 amperes, and the armature current increased from nothing to 10 amperes. The induction factor was observed by the torque method. The results are plotted in Fig. 9. M decreases from 70 to 63.

The experiment made on the Edison 12 K.W. dynamo and plotted in Fig. 4 shows no change in the value of M due to this cause. This matter will be discussed in greater detail in a later chapter under the name of Armature Reaction.

CHAPTER II

CONDITIONS OF UNIFORM MOTION

WHEN a conducting circuit of R ohms resistance is connected to a line having a tension of E volts, the rate of communication of energy to the circuit is measured by the product of the tension of the line into the current flowing; if this current is c amperes, energy is supplied to the circuit at the rate of cE watts.

The energy thus supplied is expended in two ways, in heating the circuit and in doing work. The rate of expenditure of energy in the form of heat we know to be equal to the product of the resistance into the square of the current, or c^2R watts. We now have to find an expression for the rate of expenditure of energy in doing work.

Let us suppose that part of the conducting circuit is placed near some magnetic field, so that when a current is flowing in the circuit the action of the current on the magnetic field produces a tendency to move the circuit. We will further suppose that the circuit is contrived in such a way that continuous motion is possible.

The action of the current on the field may be represented by the moment of a force of F pounds acting at a distance of r inches from the axis of rotation. The work done in one revolution is then $2\pi rF$ inch-pounds. If the conducting system makes n revolutions per second the work done per second is $2\pi rFn$ inch-pounds. This is called the rate of working, being the work done in unit time. We have then an expression for the rate of doing work stated in terms of inch-pounds per second. Divide this by 12 and we get foot-pounds per second, divide again by 550 and we get the rate of working expressed in horse-power, multiply by 746 we get finally the same quantity expressed in watts. If we now substitute for Fr the letter t, representing the torque in inch-pounds, we get

$$\text{the rate of doing work} = w = 0{\cdot}71tn \text{ watt} \quad \ldots(10).$$

This equation has been derived from purely mechanical considerations, and does not necessarily involve anything electrical.

We have already seen that when a current of c amperes flows in a circuit acted upon by a magnetic system, the torque is given by $t = 1{\cdot}41cM$, where M is the induction factor. If we insert this in place of t in Equation 10 we see that the rate of working is given by

$$w = cMn = ce \text{ watts} \quad \ldots\ldots\ldots\ldots\ldots(11).$$

where e is the tension induced by the motion. We thus arrive at the very important result that **the rate of working is** expressed by **the product of the current and the induced volts.**

This equation was derived by simply considering the forces acting on the conducting system, and obtaining an expression for the work done per second in terms of the current in the circuit and the induction factor. If the motion is in the direction of the force produced by the current, work is being done by the current; if the motion is in the contrary direction, work is being done on instead of by the moving system. In both cases the rate of working is given by the product of the current and the induced tension.

Example 11.—A dynamo with an induction factor of 4 runs at 1,500 revolutions per minute, and has a current of 70 amperes passing in the armature. The induced tension is 100 volts, so that the rate of working is 7,000 watts. Nothing is here stated as to whether the dynamo is acting as a motor and doing work, or acting as a generator and having work done on it; it is sufficient to know that a current of 70 amperes is passing, and that the induced tension is 100 volts, in order to ascertain the rate of working, positive or negative.

We have now obtained expressions for the rate of expenditure of energy in our circuit in the two possible ways, as heat and as work. The Principle of the Conservation of Energy tells us that the energy supplied to the circuit, per second, must be equal to the energy spent in heating together with that spent in doing work. We are thus able to write down the equation of energy for the circuit thus:

$$cE = ce + c^2R \quad \text{.................(12).}$$

This is the energy equation when the line is the source of energy, the current is then doing work and the dynamo is acting as a motor.

If no work is being done, the induced tension is nothing since there is no motion, and consequently $c = \frac{E}{R}$.

When work is being done $c=\frac{E-e}{R}$, it thus appears that when work is being done the current is less than that given by Ohm's law. It follows from this that the tension induced by the motion tends to reduce the current and acts in the contrary direction to that of the line. But the current from the line is producing the motion, hence **the induced tension tends to stop the motion.** This is a proposition of great importance, since it gives us a means of determining the direction of the induced tension under all circumstances. We see that it is simply one way of expressing the energy equation for the circuit as laid down by the Principle of the Conservation of Energy.

This truth may be established in another way. Take the case of a generator where the current flows in the direction of the induced tension. The current in the armature produces a torque, the direction of which depends upon that of the current, and of the sign of the field. If the torque due to the current assisted the motion we should have arrived at a condition of perpetual motion; the Principle of the Conservation of Energy therefore demands that the torque due to the current should oppose the motion, hence the induced current must flow in the armature in such a direction as to oppose the motion.

We here assume that the circuit is closed, and the current passing in the armature in the direction of the induced tension, but as it is evident that the phenomenon of induction will be present, even if the circuit be broken, and no current be permitted to pass, we are led to this general law: Any displacement of the relative positions of a closed circuit and of a magnetic system induces a tension tending to send a current in a direction opposing the motion.

In the case with which we shall be concerned chiefly, when the dynamo is used as a motor, the current in the armature does not flow in the direction of the induced tension, so that it is important to remember that a tension is induced in virtue of the motion of the conductors in the magnetic field, even though the current that actually flows is not in the direction of this induced tension.

We have stated that energy is communicated to a motor in two ways—as **heat and** as **work**. We must now inquire more precisely into the distinction here assumed. We know that energy in any form can be expressed as the product of a force into a distance. Thus, if a body weighing ten pounds is raised through a distance of five feet, the energy communicated to the body is fifty foot-pounds. If the body is raised through this distance in one second, the rate of communication of energy is fifty foot-pounds per second, or 68 watts. This would be the rate of communication of energy to an incandescent lamp taking 0·68 ampere at 100 volts, but in this case the energy would be communicated in the form of heat. There is this important difference: in the case of the weight the energy is expended in overcoming a force acting in a definite direction, whereas in the lamp there is no such force to be overcome.

When energy is spent in overcoming a resisting force we say that work is being done. Thus we say that we are doing work when we are raising the weight, because we are overcoming a resisting force. With the lamp, however, we cannot say that we are overcoming any resisting force; it is true that we are communicating energy, but we speak of the energy thus communicated as being in the form of heat.

Professor Clerk Maxwell has stated that 'the only difference between these two kinds of communication of energy is that the motions and displacements which are concerned in the communication of heat are those of molecules, and are so numerous, so small individually, and so irregular in their distribution that they quite escape all our methods of observation, whereas when the motions and displacements are those of visible bodies consisting of great numbers of molecules moving together, the communication of energy is called work. Hence we have only to suppose our senses sharpened to such a degree that we could trace the motions of molecules as easily as we now trace those of large bodies, and the distinction between work and heat would vanish, for the communication of heat would be seen to be a communication of energy of the same kind as that which we call work.'[1]

It is important to remember that doing work implies overcoming a resisting force which can be measured experimentally. We know that we must be overcoming some force when we pass a current through a rheostat, since the dimensions of rate of communication of energy are always the same, viz. a force multiplied by a distance and divided by a time; but in the case of heat we cannot measure the force experimentally, and therefore we cannot say that we are doing work.

This becomes clearer if we express the rate of communicating energy in terms of t and n, using the equation already found:

$$w = 0{\cdot}71tn \text{ watt} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10).$$

[1] *Scientific Papers*, vol. ii. p. 669.

This equation is true for all forms of communication of energy, as heat or as work. When we can measure t we are doing work, but when the energy is all going in the form of heat, t cannot be measured. In the case of the 68-watt lamp, all that we know is that the product of a torque of t inch-pounds into a speed of n revolutions per second is 96; if we knew t, we could deduce n, and *vice versâ*.

The molecular movements constituting heat cannot therefore assist or resist motion; if we could marshal all the molecules, assuming that we had command of sufficiently delicate mechanism, and oblige them to move in the same direction, we could then make them resist or assist motion; but if we could do this, we could also apply a force to them and stop their motion completely, thus taking all the heat out of a body and making it perfectly cold. Our physical inability to accomplish this result is stated in what is called the Second Law of Thermodynamics. (See Professor Clerk Maxwell's 'Theory of Heat.')

In considering the energy supplied to an electric motor we require to know precisely how much of the energy is spent in overcoming resistance to motion. It is not sufficient to know that the energy used in overcoming friction, for instance, is eventually dissipated in heat, we must know whether friction offers a resistance to motion. Similarly, we must not consider the energy used in heating the armature as spent in overcoming resistance to motion.

In the two cases here selected it is not difficult to see which should be classed as heat and which as work. But cases may arise when it is not easy to effect the right classification. For instance, we know that when a mass of iron is rotated in a magnetic field a certain expenditure of energy is necessary on account of what has been called hysteresis. This energy is, no doubt, eventually dissipated in the form of heat, but this fact does not warrant us in classing it under heat in the performance of an electric motor. We must know whether the effect of hysteresis is to oppose the motion.

Now, although the motions concerned in the expenditure of energy by hysteresis can be described as being 'those of molecules,' and as being 'so small individually that they quite escape all our methods of observation,' yet we know that energy spent in hysteresis is spent in overcoming a definite resistance to motion, a resistance that can be measured. Hence we conclude that hysteresis must be classed as an expenditure of energy in the form of work.

This conclusion may at first sight appear to be a contradiction of Clerk Maxwell's statement that 'when the motions and displacements are those of visible bodies consisting of great numbers of molecules moving together, the communication of energy is called work,' for we could hardly describe the phenomena of hysteresis in such terms.

Clerk Maxwell, however, foresaw the possibility of 'our senses being sharpened to such a degree that we could trace the motions of the molecules as easily as we now trace those of larger bodies.' The researches of Professor Ewing and others have enabled us to do this in the case of hysteresis, so that we can actually trace the motions of the molecules accompanying this phenomena as easily as we can trace those of larger bodies, and measure the resistance to motion that is offered by the combined action of all the molecules in the iron.

The forces acting on the shaft of a dynamo may be divided into two classes, those which assist and those which resist the motion.

Thus, when a dynamo is acting as a generator and driven by a steam engine, the force of the engine assists the motion, while the action of the current in the armature, if any, and the friction resist the motion.

If the dynamo is acting as a motor, the forces of the current in the armature acting on the magnetic field assist the motion, while the friction and the useful working forces resist the motion.

We shall use the term **load** to denote the forces in a motor tending to resist the motion, and we shall express these in terms of inch-pounds of torque.

A motor may be said to be 'free to move' when the load is finite. If the shaft is blocked in any way the load is infinite and the motor is not free to move. It does not follow that the shaft will turn if the motor be free to move, since the torque produced by the action of the current in the armature may not be sufficient to cause motion.

When the magnets of a dynamo are excited and the armature caused to rotate, either as motor or generator, we may divide the torques acting on the shaft as follows:

1. The torque on the pulley; this assists the motion in a generator and resists the motion in a motor.

2. The action of the current in the armature, given by $t = 1{\cdot}41 cM$; this assists the motion in a motor and resists the motion in a generator.

3. The action of eddy currents in the armature and pole-pieces; these oppose the motion in all cases.

4. Friction of the bearings and brushes, and wind resistance; these oppose the motion in all cases.

5. Hysteresis, arising from the fact of a definite torque being needed to turn a mass of iron in a magnetic field; this always opposes the motion.

If the torque assisting the motion exactly balances that resisting the motion, the motion will be either nothing or uniform; uniform motion implies a balance between the assisting and resisting torques, assuming that motion has been established. For if the assisting torque is greater than the resisting torque, there will be a surplus available for acceleration.

Take the case of a locomotive pulling a train; if the horizontal effort of the locomotive at the draw-bar is greater than the frictional resistance of the train it will accelerate; when the draw-bar pull is equal to the resistance the motion will be uniform. If, before there is any motion, the draw-bar pull is only just equal to the resistance the train cannot start. So in a motor, to effect a start the assisting torque must be greater than the resisting torque, leaving a surplus which we shall call the 'accelerating torque,' the motor will then speed up until the assisting and resisting torques balance, and uniform motion is the result.

If a motor with an armature of R ohms resistance be connected to a line having a tension of E volts, the greatest possible current is $\frac{E}{R}$ amperes. If the induction factor of the motor be M, the torque produced by this current in the armature, as given by Equation 5, is $1{\cdot}41\frac{EM}{R}$ inch-pounds. If the load is greater than this amount the armature cannot move; if it is less than this the difference is available for acceleration, and the motor will speed up until the torque due to the current is equal to that due to the load. The process of speeding up will be discussed in a subsequent chapter. We shall for the present confine our attention to the **conditions of uniform motion.**

When the motion is uniform, *i.e.* when the speed is neither increasing nor decreasing, the torque due to the current must exactly balance that due to the load, for if it were greater or less than this there would be a torque available either for acceleration or retardation. It follows that for uniform motion **the current is determined by the load,** and is given by Equation 5, where t is the torque due to the load, *i.e.* the sum of all the torques resisting the motion. The current is thus independent of the tension of the line and of the speed.

As an illustration we may take the case of a railway motor driving a loaded car. Suppose that the tension of the line falls from 500 to 200 volts. Since the current depends on the load, which is unaltered by the decrease of the tension, the motor will take the same current at 200 volts as it did at 500 volts.

In Fig. 10, let DO be drawn to represent the maximum possible speed $\frac{E}{M}$ in revolutions per second, and DB the maximum current $\frac{E}{R}$ amperes. Join BO, then the intercept of any vertical ordinate between DB and BO represents both the speed and the induced tension for any current, since, if M is constant, the induced tension is proportional to the speed. When the load on the motor is such that for steady motion the current given by Equation 5 is DK amperes, the speed is given by KF and the induced volts also by KF. Since DO is equal to E, the tension of the

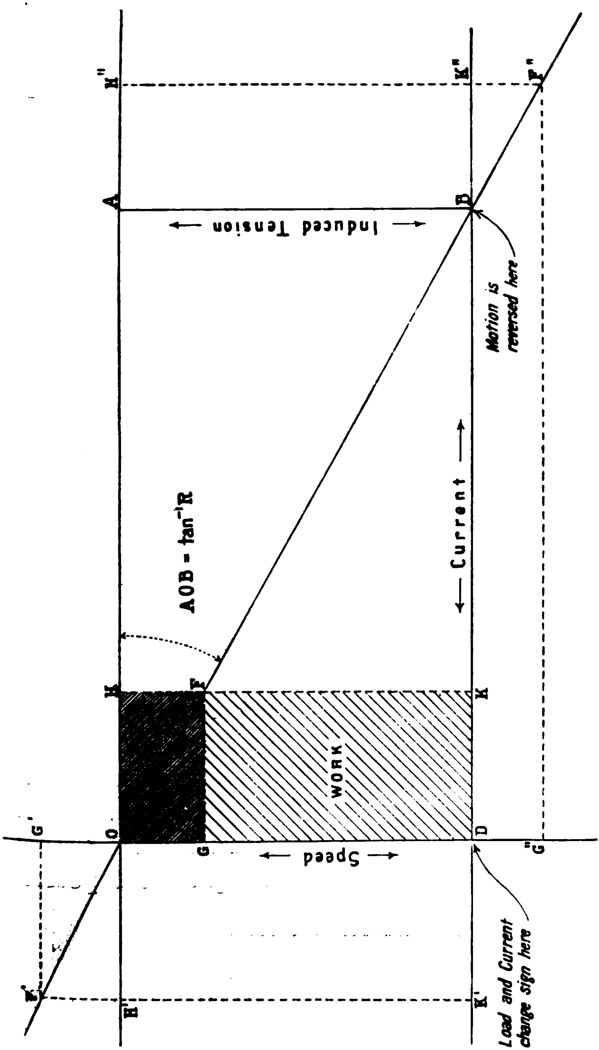

Induced Tension
Motion is reversed here
AOB = tan⁻¹R
Current
WORK
Speed
Load and Current change sign here
A
B
K
K''
F
F''
O
G
G'
G''
D
H'
K'
F'

FIG. 10

line, and $DB=\frac{E}{R}$, it follows that the tangent of the angle *AOB* is equal to the resistance of the motor. The same diagram could be constructed by setting off $AOB=\tan^{-1}R$, and *OD* equal to the tension of the line. *DB* would then be the maximum current.

The diagram shows us that when the load is so great as to require a current *DB* to balance it, the whole of the energy from the line is expended in heating the resistance, the line watts being represented by the area *OABD*. As the load is reduced the motor begins to turn, the line watts is diminished, but some of the energy from the line is now used in doing work, the ratio of the areas *DKFG*, *DKHO*, being the ratio of the mechanical watts to the line watts. This ratio represents the proportion of the energy from the line that is used in overcoming resisting torque.

At the point *B* the speed is nothing, the mechanical watts is nothing, and the torque has its maximum value. On the other hand, at the point *D* the speed is a maximum and the torque is nothing. Hence the mechanical watts increases from nothing to some maximum and then decreases to nothing again, the speed increasing as the torque decreases.

The mechanical watts is a maximum when $DK=KB$ (not necessarily when *DKFG* is a square), *i.e.* when the current is half *DB*, the area of *DKFG* is then seen to be one fourth of *DBAO*. Hence the **greatest possible value of** w **the mechanical watts** is $\frac{1}{4}\frac{E^2}{R}$.

For any value of w less than $\frac{1}{4}\frac{E^2}{R}$ there are two possible values, one with t large and n small, the other with

n large and t small, both having an equal area in the diagram.

We have now to inquire what determines the speed when the load is fixed. If we put Mn for e in Equation 12, n being the revolutions per second, we get

$$n=\frac{E-cR}{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots(13).$$

from which we see that when E, R and M are fixed, the speed depends only upon c; but c depends upon the load, hence **the speed depends on the load**.

If cR is less than E, n is positive. If the load is nothing cR is nothing and $n=\frac{E}{M}$, so that if we had no load and no frictional or other resistance to oppose the motion, the armature would run at a speed given by dividing the tension of the line by the induction factor; this is therefore the maximum possible speed of a motor. If $cR=E$, n is nothing. This is simply a different way of stating the fact that if the current is a maximum, the speed is nothing.

We can also express the speed thus:

$$n=\frac{E}{M}-\frac{tR}{1\cdot41M^2} \quad \ldots\ldots\ldots\ldots\ldots(14).$$

Example 12.—Let $E=100$, $M=8$, and $R=2$. The maximum current is 50 amperes; motion is only possible if the load is less than 564 inch-pounds. Suppose that the load is 300 inch-pounds, the current required to balance the load is 26·6 amperes and the heat drop is 53·2 volts; the speed is thus 351 r.p.m. The maximum possible speed for no load is 750 r.p.m.

From Equation 13 we see that when cR is negative and greater than E, n is negative, *i.e.* the direction of rotation is reversed, and consequently the induced tension is of the same sign as the tension of the line. Suppose that the load is such that, to balance it, a current DK'' is required. For uniform motion we must have $H''\ K''$ the tension, of the line, plus $K''\ F''$ the induced tension, equal to $H''\ F''$, the heat drop. We thus have a greater current flowing than that given by $c=\frac{E}{R}$.

It is important to notice that the direction of rotation is reversed only when the current is greater than $\frac{E}{R}$; this shows that we can reverse the motion of a dynamo by putting on a load greater than that given by $1{\cdot}41\frac{EM}{R}$; the dynamo then ceases to be a motor and becomes a generator.

The conditions here described can be represented experimentally as follows: A motor is taken and connected up to a line of constant tension through an ordinary starting rheostat, and an adjustable rheostat is placed in series with the armature, so that the resistance can be varied and the motor brought to a standstill. On the shaft is placed a pulley with a cord attached, which is wound round the pulley and hangs down on one side, loaded with weights in a pan. The weights and the rheostat are adjusted until the armature is at rest, and on turning it with the hand, as ready to move one way as the other, the current through the armature being observed with an ammeter.

The following experiments may be made:

1. Increase the load by adding weights and observe

how the current has to be increased by reducing R in order to obtain a balance.

2. Decrease the load and observe the diminished current required for a balance.

3. Keep the load constant, and diminish the resistance; note (*a*) the temporary increase of current; (*b*) the direction of rotation, say clockwise; (*c*) the tendency of the current to resume its previous value.

4. Keep the load constant, and increase the resistance; note (*a*) the temporary decrease of current; (*b*) the direction of rotation, now counter-clockwise; (*c*) the tendency of the current to resume its previous value.

5. Cut the cord and notice how the armature speeds up to its maximum speed. Take the speed and the tension at the brushes. Deduce M, calculate the torques for the currents previously observed, and see how they correspond with the torques obtained by measuring the pulley diameter, and multiplying by the weights used.

These conditions might be realised in the case of a lift, the unbalanced weight of a car taking the place of the weights in the experiment. A certain current is required to hold the car in position. If the resistance is decreased or the tension of the line increased, the car rises. If the resistance is increased or the tension decreased, the car descends, the direction of rotation being reversed, and the dynamo acting as a generator but without reversal of current, the induced tension helping that of the line.

Suppose, on the other hand, that the tension and the resistance were constant, and the load gradually increased. The speed would diminish until, for a certain load, the motor would come to rest, the conditions assumed being such that the current at this point would not injure the motor. If the load is still further increased, the car will descend, and the motion will cause the current to be increased. If the load is suddenly taken off, as may happen should the car be jammed on its way down, the speed of the motor will increase to its maximum value. This is a source of danger, as the ropes would be paid off the rope drum and hang slack over the car. A slight shock might then cause the car to fall, unless special safety arrangements are provided.

To return to the diagram. If the load were nothing, the current also would be nothing, and the motor would run at the maximum speed *DO*. In practice, it would be impossible to reduce the load on a motor to nothing, since there is always some load due to friction of bearings and brushes. We might, however, reduce the load absolutely to nothing, by applying a small force in the contrary direction, sufficient to balance the torque due to the friction. If this were done, the current in the motor armature would be nothing, and the speed would be given by $\frac{E}{M}$.

Suppose now that we were to apply more force than was necessary to overcome the frictional resistance, the sign of the load would then be changed, and instead of resisting the motion, the load would be assisting the motion. But we have seen that the torque due to the load must be equal and opposite to that due to the current in the armature, hence **if the load changes sign, the current must also change sign**.

This conclusion is of great practical importance; it shows us that the current and the load change sign together. If the load is nothing the current is nothing, if the load resists the motion the current flows from the line, if the load assists the motion the current flows into the line.

We can thus change the sign of the current by reversing the load, so that a dynamo will act as a generator or as a motor according to the sign of the load, and the change from the one to the other is through the value of zero load.

During the process of a reversal of the load the sign of the induced tension is unaltered, hence a dynamo does not reverse its direction of rotation when changing from generator to motor. This is seen also from Equation 13, which shows that whatever be the sign of c, n can only be negative when c is negative, and cR greater than E.

The direction of rotation may of course be reversed by changing either the sign of the terminal tension or the sign of the magnetic field. If both are changed simultaneously, the direction of rotation is unchanged. The usual method of reversing is to bring the motor to rest, reverse the sign of the terminal tension, and then start up in the ordinary way.

The change in the sign of the current consequent upon a change in the sign of the load is clearly shown in the figure. For any current, OH', delivered into the line, the energy per second expended in heat is represented by the area $OG'F'H'$; the energy given to the line, being the product of the current and the tension of the line, is represented by the area $ODK'H'$; the whole energy expended is thus represented by the area $DK'F'G'$, and this is equal to the input of energy in the form of work done. This we see to be true, since $K'F'$ represents the speed and hence the induced tension; the area $DK'F'G'$ is therefore the product of the current and the induced tension, and therefore equal to the work done per second. The induced tension is now greater than that of the line, the difference, $H'F'$, being what is called the 'effective' tension.

Example 13.—Suppose that we have a shunt-wound dynamo directly connected to a steam engine running at 462 r.p.m., and giving 100 volts at the brushes on open circuit. The induction factor is then 13. Let the dynamo be acting as a generator, and charging accumulators. Assume the speed and induction factor to be constant. Suppose, also, that the torque required to turn the engine and armature is 400 inch-pounds. When the dynamo is delivering 100 amperes into the accumulators, the torque due to the current is 1,300 inch-pounds and resists the motion. The friction also opposes the motion, so that we have 1,700 inch-pounds of torque opposing the motion. Suppose, now, that the steam valve is suddenly closed. If there is no automatic cut-out, the tension of the cells immediately reverses the current in the armature, but the direction of rotation remains the same, since the current in the magnets is not reversed. The load on the dynamo is now only that due to friction; hence the current taken from the cells will be 22 amperes. The speed is given by Equation 13. If $R=0{\cdot}015$, we see that the effect of shutting off the steam will simply be to reduce the speed of the dynamo by 1·5 r.p.m., and to reverse the current from 100 amperes in one direction to 22 amperes in the other.

We are now dealing with a motor of constant induction factor and resistance, whose terminals are connected to a line of constant tension, the only variable being the load, which may change in sign and amount. We must be careful not to confuse the results obtained by altering the sign of the load with those obtained by changing the value of the induction factor or the tension of the line; these cases will be considered later.

If *DK* represents the current taken by a motor when running on a line of tension *E*, the area *OHKD* represents the product cE, and thus expresses graphically the energy per second supplied to the motor from the line. Since $HOF = \tan^{-1} R$, the area *OHFG* is the product c^2R, or the energy per second expended in heat. Further, since *HK* is the tension of the line, and *HF* equal to the product cR, or the heat drop, it follows that *FK* is the induced tension. The area *GFKD* is then the energy per second expended in doing work, and this is shown graphically to be equal to the difference between the energy per second supplied to the motor from the line and the energy per second spent in heating the resistance.

This diagram shows how we may find graphically the tension of the line required to do work at a given rate. When the torque and the induction factor are given, we can obtain the current. Let this be *DK*. If the speed is given, knowing *M*, we can obtain the induced tension. Let this be *KF*. The area *GFKD* is thus equal to the given rate of working. The energy expended per second in heat depends on the resistance, since *c* is now fixed. Draw *GFO* equal to $\tan^{-1} R$; *OD* will then be the required tension of the line.

We have supposed the load to vary from the maximum $1{\cdot}41 \dfrac{EM}{R}$ to nothing, and have shown that if the load did thus vary, the current would diminish from DB to nothing, and the speed would increase from nothing to DO. We here assume that the normal resistance of the motor is increased by inserting a starting rheostat in the circuit, so that the current drawn from the line when the motor is at rest is not greater than the maximum safe current. If this rheostat were left in the circuit, the motor would attain a speed given by Equation 13, where R is the resistance of the motor and the starting rheostat. In practice, when a sufficient speed has been attained, and the current reduced, the starting rheostat is taken out, until finally the resistance of the motor only is left in the circuit. Our diagram will now serve to show the variations of the speed with the current, but only a small portion of the speed curve will be used, and it will be inclined to the horizontal at an angle whose tangent is equal to the resistance of the motor only.

The general equation of energy involves three terms: 1. The term cE, expressing the energy per second from the line if c is negative, and into the line if c is positive, the former being the case of the motor, and the latter that of the generator. 2. The term ce, expressing the product of the current and the induced tension, and hence the work done. 3. The term c^2R, giving the rate of expenditure of energy in the form of heat.

We have (α) the case of the motor when c is negative; current is passing from the line, and the induced tension is opposed to that of the line. (β) c positive; the case of the generator when c is positive; the direction of rotation and the induced tension are the same as in the motor, but the sign of the current is now changed, since the current is passing into the line. (γ) c negative, but with the sign of e changed;

current is now passing from the line; but the direction of rotation and therefore the induced tension are reversed. This is the case when the load is greater than $1{\cdot}41\frac{EM}{R}$, thus causing the dynamo to turn backwards; work is then being done on the dynamo, as in the case of the generator, but all the energy, including that from the line, goes in heating the resistance.

The three cases can be stated thus:

(a) $cE = c^2R + ce$ . . . motor.
(β) $ce = cE + c^2R$ . . . generator.
(γ) $c^2R = cE + ce$ . . . generator.

The rate of supply and expenditure of energy in the three cases here enumerated is shown graphically in Fig. 11. The distance *ak* is taken to represent the tension of the line, and *ab* represents the current for no motion. Since the line watts is the product of the tension of the line and the current, the former being constant, distances measured vertically from *ab* may represent the line watts for different currents; thus at *a* the line watts is nothing, while at *b* it is *bd*; and for all currents the line watts for any current is given by the ordinate intercepted by the straight line *ad* continued both ways beyond *a* and *d*.

The heat watts for any current will be given by the ordinate of a parabola passing through the points *a*, *g*, and *d*, where *ng* is one-half *nh*. This curve can also be continued indefinitely beyond the points *a* and *d*.

The mechanical watts for any current is the difference between the line watts and the heat watts for points lying between *a* and *b*; for points to the left of *a* the mechanical watts is the sum of the line watts and the heat watts; for

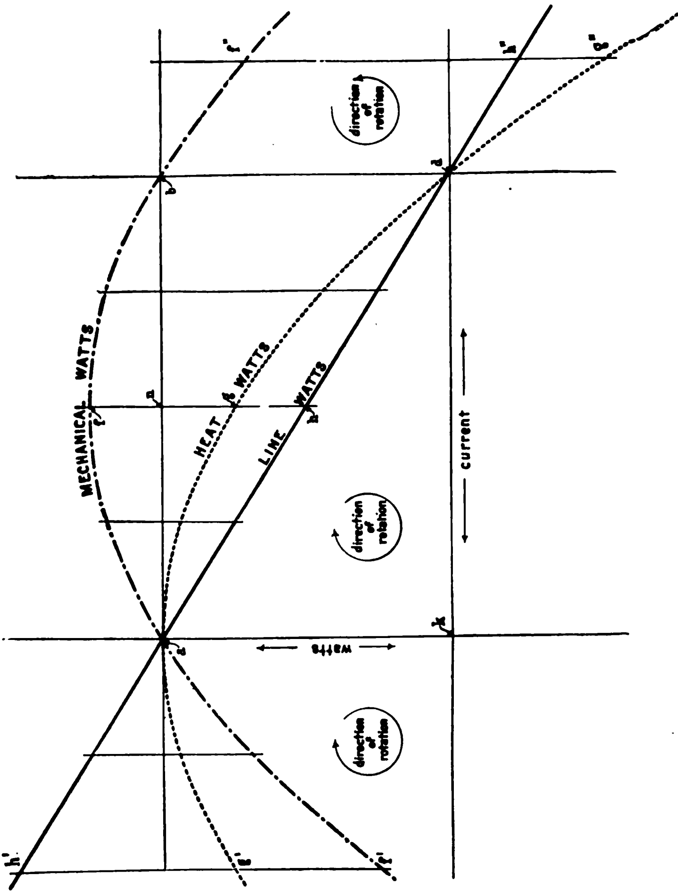

MECHANICAL WATTS
HEAT WATTS
LINE WATTS
current
watts
direction of rotation
direction of rotation
direction of rotation

FIG. 11

points to the right of d, the heat watts is the sum of the line watts and the mechanical watts.

This diagram illustrates how the difference between the line watts and the heat watts increases from nothing at a, to a maximum at n, and decreases again to nothing at b, showing that when the dynamo is acting as a motor the rate of doing work is a maximum when the current is equal to $\frac{E}{2R}$.

Fleming's rules for determining the direction of the force on a conductor and of the induced tension will be found useful. Compare Fig. 12. Let the first finger, the middle finger, and the thumb of the right hand be held at right angles to one another to represent three rectangular axes; then if the first finger point in the direction of the lines of force, and the motion be in the direction of the thumb, the middle finger gives the direction of the induced tension, *i.e.* the direction in which the induced current would flow if it were free to do so. We have already seen that this does not give the direction of the current in all cases, since in a motor the current does not flow in the direction of the induced tension.

The necessary complement of the preceding rule is the following: Let the first finger, the middle finger, and the thumb of the left hand be held at right angles to one another; then if the direction of the first finger represent that of the lines of force, and the direction of the second finger represent that of the current, the thumb will point in the direction of the force produced by the action of the current on the magnetic field. It does not follow that this is the direction of the motion, since when the dynamo acts as a generator the direction of motion is opposed to

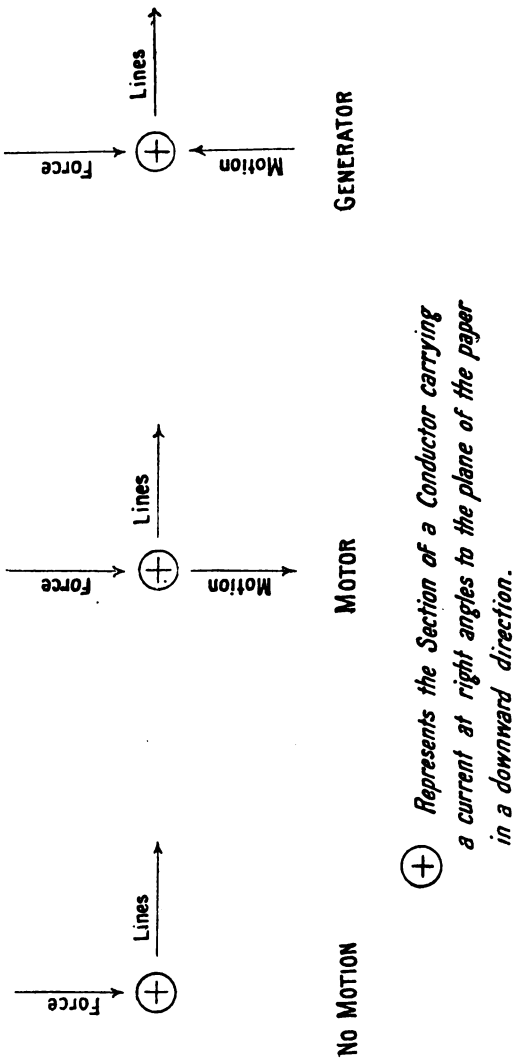

Force
Lines
Motion
GENERATOR
Force
Lines
Motion
MOTOR
Force
Lines
NO MOTION
Represents the Section of a Conductor carrying a current at right angles to the plane of the paper in a downward direction.

FIG. 12

the direction of the force between the wire and the field.

The rule giving the direction of the induced tension is as true for motors as it is for generators, and that giving the direction of the force is as true for generators as it is for motors.

CHAPTER III

EQUATIONS FOR THE INDUCTION FACTOR

WE shall now show how to find the induction factor of a motor that has to fulfil certain specified conditions.

The problem may be stated in the most general way as follows: Given the tension E of the line, the resistance R of the motor, n the revolutions per second of the motor, and w the rate of doing work expressed in watts, **to find the induction factor.**

If c is the current passing in the motor when fulfilling the required conditions, we know that $M=\frac{E-cR}{n}$. Now $w=cMn$, hence $M=\frac{E}{n}-\frac{Rw}{Mn^2}$, and we get the quadratic

$$M=\frac{E}{2n}\left\{1\pm\sqrt{\left(1-\frac{4Rw}{E^2}\right)}\right\}\text{............}(15).$$

There are two possible values for M, the one a high value obtained by taking the positive sign, the other a low value given by taking the negative sign. Of these two we shall select the former as giving the smaller current. Note that w is the total rate of working, and includes all torque losses.

When the rate of working is given in horse-power the equation for the induction factor may be written thus:

$$M=\frac{E}{2n}\left\{1+\sqrt{\left(1-2984\frac{R\,HP}{E^2}\right)}\right. \text{.........(16).}$$

Here HP is the horse-power on the motor shaft, and n the revolutions per second.

Example 14.—A motor has to do work at the rate of 7 kilowatts, the tension of the line is 125 volts, the resistance of the motor is 0·06 ohm, the speed has to be 1,200 revolutions per minute. The expression $\frac{4Rw}{E^2}$ we find to be 0·107, and the term under the root is 0·893. Hence the value of the induction factor is 6·07. If a motor with this induction factor and the given resistance be connected to a line having a tension of 125 volts, it will run at 1,200 revolutions per minute when working at the rate of 7 kilowatts. By taking the negative sign inside the bracket, we see that a motor with an induction factor of 0·172 would also fulfil these conditions, so that we need some further information to guide us as to which value of M to adopt.

We know that the current is given by $w=cMn$; from this equation we find that with an induction factor of 6·07 the current would be 58 amperes, while with an induction factor of 0·172 it would be 2,030 amperes. The latter is practically impossible, though we must not forget that even when carrying this current the motor would be running at the given speed, if its resistance remained unaltered.

We shall in all cases take the positive sign, so as to get the greatest possible value of M and the least possible value of C.

In Equation 15, n is the speed of the motor in revolutions per second. If the shaft of the motor is geared or belted to a second shaft, so that the speed of the former is v times that of the latter, we can write :—

$$M=\frac{E}{2vn}\left\{1\pm\sqrt{\left(1-\frac{4Rw}{E^2}\right)}\right\} \quad \text{.........(17).}$$

where n is now the speed of the second shaft, which we shall call the **main shaft**. We shall call v the **velocity ratio.**

We should notice here that the expression $\frac{4Rw}{E^2}$ is unaltered by the introduction of gearing, since it is immaterial whether the work is being done directly by the motor or through the medium of gearing. In practice some of the work would be represented by overcoming the friction of the gearing, some by the friction of the shaft of the motor; the larger part of the work, however, would be done on the main shaft; the term w includes all of these.

Example 15.—Work has to be done at the rate of 7 kilowatts on a main shaft rotating at 1,200 r.p.m.; the shaft is to be turned by a motor geared to the main shaft with a velocity ratio 1·3; the resistance of the motor is 0·06 ohm, and the tension of the line 125 volts; to find the induction factor of the motor. We see at once that M will be less than that in Example 14, in the ratio of 1·3 to 1, or $M=4{\cdot}67$; the speed of the motor will be 1,560 r.p.m.; the current will be 58 amperes, the same as in Example 14.

When the motor shaft is geared to a main shaft we may express the rate of working in terms of the speed and torque of the motor shaft or of the main shaft; if no work is being done in overcoming friction in the gearing or in the bearings of the motor shaft, the work done on the one shaft is equal to the resistance overcome on the other, and since the motor shaft runs at v times the speed of the main shaft, it follows that the torque on the former is to that on the latter as 1 is to v.

Hence, when a motor of induction factor M is driving a main shaft, on which the torque is t inch-pounds, the torque on the motor shaft is $\frac{t}{v}$, where v is the velocity ratio, and the current is $\frac{t}{1{\cdot}41Mv}$. This is the current in the motor required to overcome the torque t on the main shaft. In practice we shall require more current than this, since there is always some friction to be overcome in the gearing and in the motor itself.

The main shaft may be connected to the motor shaft either by belting or by toothed gearing. In the former case v will be the ratio of the diameter of the pulley on the main shaft to that on the motor shaft. In the latter case v will be the ratio of the number of teeth in the toothed wheel on the main shaft, which we shall call the **gear wheel,** to the number of teeth in the toothed wheel on the motor shaft, which we shall call the **pinion.**

The following table gives the numbers of teeth in gear wheel and pinion with the corresponding velocity ratios often used in railway motors.

Number of Teeth in Gear Wheel.	Number of Teeth in Pinion.	Velocity Ratio.
62	22	2·82
67	17	3·94
69	15	4·60
67	14	4·78

In many cases it is convenient to be able to express M in terms of the torque on the shaft to which the motor is geared. If t is the torque in inch-pounds on the main shaft, which rotates at n revolutions per second, the induction factor of the motor is given by

$$M = \frac{E}{2vn}\left\{1 + \sqrt{\left(1 - 2{\cdot}84\frac{Rtn}{E^2}\right)}\right\} \quad \ldots\ldots(18).$$

Example 16.—A motor having a resistance of 0·04 ohm is geared to a main shaft with a velocity ratio of 4. The main shaft has to rotate at 5 revolutions per second. The tension of the line is 150 volts. Find the induction factor of the motor, when the whole resistance to be overcome is 8,000 inch-pounds of torque on the main shaft. The term under the root is 0·798, so that the value of M is 7·1. The motor rotates at 1,200 r.p.m. The current is 200 amperes. We may check the results thus. Since the motor rotates at 20 revolutions per second the induced tension is 142 volts; the heat drop is 8 volts, and the sum of these two makes up the tension of the line. Suppose now that we require, over and above the 8,000 inch-pounds of torque on the main shaft, 400 inch-pounds of torque to turn the motor shaft in its bearings when the magnets are excited, and 600 inch-pounds to move the gearing, the torque in both cases being measured at the motor shaft.

The term t in Equation 18 represents the torque on the main shaft, hence in order to allow for the resistance to motion of the motor shaft and the gearing, we must increase the 8,000 inch-pounds on the main shaft by 4,000 inch-pounds. We might, of course, reduce all resistances to torque measured at the motor shaft; we should then have to multiply n by v. With an induction factor of 7·1, the current required to overcome the whole resistance would now be 300 amperes; the speed of the motor would then be 1,170 revolutions per minute. To get the specified speed we must calculate the value of M again; we find it to be 6·90, and the current to be 308 amperes.

If, in any of the equations for the induction factor, the expression under the square root should be negative, a solution of the equation is impossible. This simply states in another way what we have already found, namely, that for any motor there is a certain maximum rate of working that cannot be exceeded. This maximum we found to be one fourth of the maximum possible watts from the line, or $\frac{1}{4}\frac{E^2}{R}$. There are three ways of putting the limiting condition.

$$w \text{ must not exceed } \frac{1}{4}\frac{E^2}{R}$$

$$HP \text{ must not exceed } \frac{1}{2984}\frac{E^2}{R}$$

$$t\,n \text{ must not exceed } \frac{1}{2 \cdot 84}\frac{E^2}{R}$$

For a given tension, the limit of w, HP or tn depends upon the resistance of the motor.

Example 17.—A motor is to work on a line having a tension of 500 volts. If the resistance is 3·34 ohms, the maximnm rate of working is 25 horse-power. If the resistance is reduced to 1·67 ohms, the maximum possible horse-power is double.

We must not infer that a motor would be designed to work at its maximum possible output, for as we shall see later on, the efficiency would then be very low; we here simply state what are the conditions under which a solution of the equations for the induction factor is possible.

For a given resistance the maximum output varies as the square of the tension of the line. Thus if the tension is halved, the maximum possible horse-power is reduced to one-fourth of its former value.

If the main shaft is provided with a driving wheel of diameter d inches, the whole resistance to be overcome can be represented by an equivalent force of T pounds at the circumference of the driving wheel, and we may then substitute $\frac{Td}{2}$ for t, the torque on the main shaft.

First, suppose that the two shafts are fixed in space, as in Fig. 13, where A is the motor shaft, and B is the main shaft. If n is the speed of the main shaft, we may write

$$s = 15{\cdot}71\, nd \ldots\ldots\ldots\ldots\ldots\ldots (19).$$

where s is the velocity in feet per minute of a point on the rim of the driving wheel. Equation 18 may then be written in the form:

$$M = \frac{7{\cdot}85Ed}{vs}\left\{1 + \sqrt{\left(1 - \frac{1}{11{\cdot}07}\,\frac{RTs}{E^2}\right)}\right\} \ldots (20).$$

This equation is applicable to all kinds of hoisting machinery.

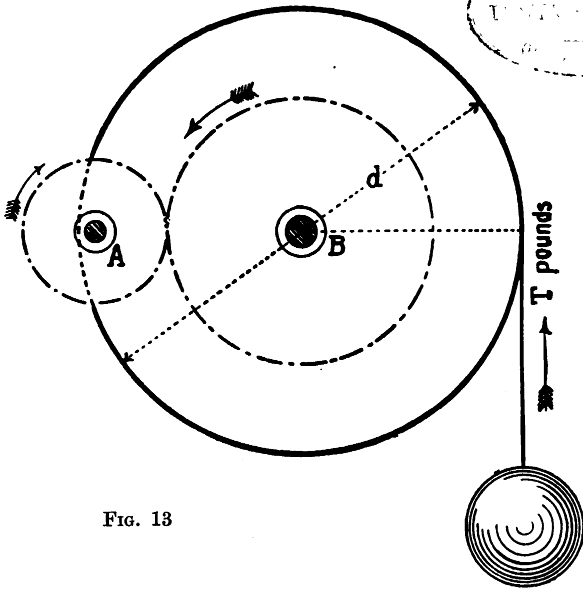


FIG. 13

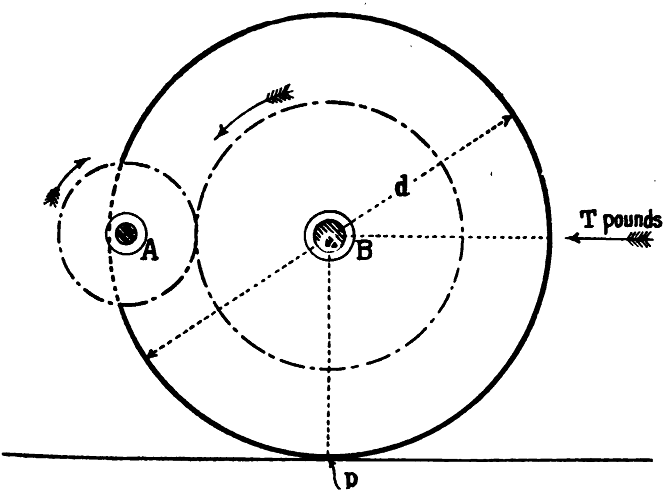


FIG. 14

Example 18.—A motor is to be designed to work a lift under the following conditions. The tension of the line is 125 volts. The diameter of the rope drum is 36 inches. Worm gearing is used with a velocity ratio of 60. An unbalanced weight of 1,200 pounds has to be raised at speed of 180 feet per minute. The resistance of the motor is to be 0·06 ohm. A torque of 90 inch-pounds is required on the motor shaft to overcome friction. Find the induction factor of the motor. The frictional torque may be expressed by an equivalent pull at the rim of the rope drum. Using equation $T = \frac{2vt}{d}$ we find that the friction may be represented by a pull of 300 pounds, so that the value of T in the equation for M is 1,500 pounds. We find the induction factor to be 6·37. The current, given by $c = \frac{Td}{2{\cdot}82\ Mv}$ is 50 amperes.

If the two shafts are free to move in a straight line, while the driving wheel rolls without slipping on a rail parallel to AB, as shown in Fig. 14, the torque produced by the motor tends to drive B in a direction parallel to the rail. For the effort exerted by the motor can, as before, be represented by a force of T pounds at the rim of the driving wheel. Since p, the point of contact between the wheel and the rail, is at rest at any instant, the moment of the force T acting at p, taken about the centre of the wheel, is equal to the moment of a force F acting at the centre of the wheel taken about the point p; it follows that F is equal to T, in other words, the effort exerted at the centre of the driving wheel, in a direction parallel to AB, is equal to the force T exerted at the rim of the wheel.

We can then write down the value of the induction factor of a motor which, when geared to a main shaft with velocity ratio v, will give a tractive effort of T pounds at the centre of a driving wheel of diameter d inches, E being the tension of the line, and R the resistance of the motor; we have

$$M = \frac{Ed}{11 \cdot 2vS}\left\{1 + \sqrt{\left(1 - \frac{7 \cdot 96RTS}{E^2}\right)}\right\} \quad \ldots\ldots(21).$$

where S is now the velocity of the centre of the driving wheel, expressed in miles per hour.

We have assumed that the whole resistance to motion can be represented by a force of T pounds acting at the rim of the driving wheel. This resistance may for convenience be divided into frictional and gravitational resistance. The frictional resistance may be further divided into friction of the motor and its equipment, and friction of the shafts and axles driven by the motor. The force exerted in overcoming the latter is sometimes called the 'useful effort,' though strictly the effort is not more useful than that exerted in overcoming the friction of the gearing.

In Equation 21, T includes all resistances to motion. In the case of a locomotive the frictional resistance of the axles of the carriages is represented by a force exerted at the draw bar: we may also require a force at the draw bar to overcome a gravitational force due to an incline. Besides these there will be the frictional resistance of the motors and their equipments. In a motor car there is strictly no draw bar pull, but there will be its equivalent if the car is ascending an incline.

When a motor is described as capable of exerting a tractive force of x pounds 'available for useful effort,' this does not express the whole force the motor exerts, since it neglects the effort exerted in overcoming the frictional resistances of the motor equipment. Thus, if a motor is rated at 1,000 pounds 'available for useful effort,' this would not be the T of our equations; we must increase the given amount by the equivalent force required to overcome the resistances of the motor equipment. The ratio of the whole force exerted to that 'available for useful effort' is the measure of the mechanical efficiency of the equipment.

Example 19.—A locomotive is to be designed to run at sixteen miles an hour while exerting a force of 3,600 pounds at the draw bar. Two motors are to be used, connected in parallel and geared to the shafts of the driving wheels with velocity ratio of 4·78. The driving wheels are to be 33 inches in diameter. The tension of the line is 500 volts, and the resistance of the motors 0·2 ohm each. A force of 600 pounds has to be provided for, over and above the draw bar pull, to overcome the frictional and other resistances to motion in the locomotive itself. To find the induction factor of the motors. As there are two motors the load may be divided equally between them, giving 2,100 pounds as the value of T per motor. Inserting the given values of E, d, v, R, and S, in Equation 21 we find the induction factor to be 36·3. This result may be checked by finding the value of the induced tension. The motor will rotate at thirteen revolutions per second when the train is running at sixteen miles an hour, hence the induced tension will be 472 volts. The current per motor is 141 amperes, and the heat drop is 28 volts, the two together making up the tension of the line.

The gear wheel and the shaft B may be fixed in space, and the motor and its shaft A free to turn about B as a centre. This arrangement is found in swing bridges, turrets, and crane stages, where the gear wheel is placed in a horizontal plane, the turret or stage rotating about B as a centre, and the shaft A attached to the rotating stage so that when the motor turns the stage also turns. Equation 18 is applicable here; the term n previously used for the speed of rotation of the driving wheel now becomes the speed of rotation of A round B, while t is still the resistance to motion, measured in inch-pounds of torque at the centre of B. In this arrangement t generally consists entirely of friction. The time occupied in getting up speed is so large a part of the whole time of working in cases of this kind that we must defer further consideration of the problem until we have considered the question of acceleration.

Equation 20 may be written in the form:

$$\frac{Mv}{d} = 7{\cdot}85\frac{E}{s}\left\{1 + \sqrt{\left(1 - \frac{RTs}{11{\cdot}07E^2}\right)}\right\} \quad \ldots(22).$$

From which we see that when the speed and the tractive effort are fixed the ratio $\frac{Mv}{d}$ is fixed. By making $\frac{v}{d}$ large we can make M small; in other words, if we are at liberty to adjust the values of v and d, we can reduce the weight of the motor. To get M as small as possible we must have $\frac{v}{d}$ as large as possible, or v large and d small. In most cases of hoisting machinery we are at liberty to select values for v and d that will give us the best value of M. Taking d as small as is practically convenient, we can get any required value of v by the use of worm gearing if necessary. We are not here limited for space, and can make the gear wheel on the main shaft as large as we please.

For instance, in Example 18 the ratio $\frac{Mv}{d}$ has to be 10·6. By the use of worm gearing we can make $v=60$, and we get $M=6·37$, with a rope drum 36 inches in diameter. If we reduce the diameter of the rope drum to 24 inches the induction factor may be reduced to 4·25 without altering any of the conditions. There is, however, a practical limit to the reduction of M; namely, the speed of the motor. In the example before us this would be 1,150 revolutions per minute, for $v=60$ and $d=36$. If the diameter is reduced to 24 inches the speed of the motor is increased to 1,725; this will involve an increase in the velocity of the line of contact of the toothed wheels, and experience shows that the tendency of the gears to cut increases with the speed, giving us a practical limit to the speed of the motor.

In railway work single reduction gearing is now universally used, and since there is a practical limit to the reduction of the number of teeth in the pinion, it follows that to increase the ratio $\frac{v}{d}$ we must either increase the diameter of the gear wheel or decrease the diameter of the driving wheel. We are here limited by the necessary clearance between the gear wheel and the ground, for the diameter of the gear wheel is limited by that of the driving wheel.

In street railway practice the diameter of the driving wheel now almost universally adopted is 33 inches. With 14 teeth in the pinion and 67 in the gear wheel, we find in a well-designed equipment a clearance of $4\frac{5}{8}$ of an inch between the casing of the gear wheel and the level of the rail. If we take this to be the least permissible clearance, $\frac{v}{d}$ is limited to 0·145.

CHAPTER IV

SHUNT-WOUND MOTORS

In this chapter we shall assume that the **magnets are excited by a shunt winding** entirely independent of the armature circuit. We have already seen that the speed is given by the equation

$$n = \frac{E}{M} - 0{\cdot}71\,\frac{Rt}{M^2} \quad \text{..............(23).}$$

When R, M and E are constant, the speed is less than the maximum by an amount depending on the load.

Example 20.—A motor with an induction factor of 5 and internal resistance 0·2 ohm is connected to a line having a tension of 120 volts. With no load the speed is 1,440 revolutions per minute. With a torque of 850 inch-pounds the speed is less than the maximum by 290 revolutions.

The term R in the equation for the speed includes all resistance in the circuit, not only the internal resistance of the motor, but also any resistance that may be in series with it, as, for instance, a rheostat used for the purpose of starting or regulating, E being measured outside such resistance.

Example 21.—If $E = 120$, $M = 5$, $R = 0{\cdot}2$, the maximum possible speed is 1,440 r.p.m. If the load is 845 inch-pounds, the speed will be 1,150 r.p.m. If the resistance is increased by 0·8 ohm, the speed will fall to 288 revolutions per minute.

If the resistance be reduced suddenly, the current that will flow through the motor will depend upon the speed at which it was running before the resistance was altered.

Example 22.—If the resistance of 0·8 ohm in Example 21 is taken out suddenly while the motor is running at 288 r.p.m., the induced tension being then 24 volts, the current at the moment of cutting out the resistance will be 480 amperes. To find the speed at which the motor must run before it would be safe to take the extra resistance entirely out, so that the current shall not exceed, say, 150 amperes, we must use the Equation 13, c being here 150 amperes, and R being 0·2 ohm. We find the required speed to be 1,008 r.p.m.; the resistance must therefore be gradually reduced as the motor speeds up until it runs at this speed, when the rheostat may be cut out altogether. In this way we can determine the resistance of each step of a rheostat that can be used with a given motor, so that the current shall not exceed a certain fixed amount.

A rheostat is sometimes placed in series with a motor to obtain a variation in the speed. When this is done, care must be taken that it is not placed between the main terminals and the magnets if the motor is shunt wound, as in that case we should of course reduce the value of the induction factor.

If R_1 is the whole resistance in the circuit of a motor when running at n revolutions on a line having a tension of E volts, the current being constant and equal to c amperes, the whole resistance R_2 required in the circuit in order to reduce the speed to $\frac{1}{x}$ of n will be given by

$$R_2 = \frac{E}{c}\left(1 - \frac{1}{x}\right) + \frac{R_1}{x} \quad \text{...........(24).}$$

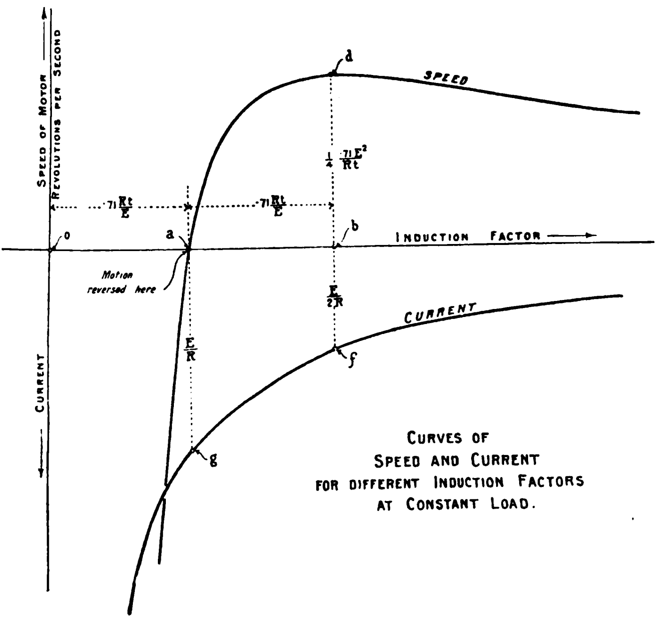
Example 23.—A motor of resistance 0·3 ohm is running on a line of 120 volts tension, the current being 40 amperes. What must be the resistance of a rheostat in the circuit in order that the speed may be halved? From the equation we find at once that 1·65 ohms is required, of which 0·3 ohm is in the motor itself, so that the resistance of the rheostat must be 1·35 ohms. The result may be checked by showing that in the first case the induced tension was 108 volts and in the second 54 volts. The current remains unaltered.

When the load on the motor is constant, **the speed** may be **altered by changing the induction factor,** the tension of the line and the resistance remaining the same.

From Equation 23 we see that when $M = \cdot 71\frac{Rt}{E}$ the speed is nothing. This represents the condition when the greatest current that can flow from the line is only just sufficient to balance the torque; the current is then equal to $\frac{E}{R}$.

If we differentiate Equation (23) with respect to M, and equate to nothing, we shall find that the speed is a maximum when $M = 2\frac{\cdot 71Rt}{E}$, the speed then being $\frac{1}{4}\frac{\cdot 71E^2}{Rt}$ and the current $\frac{E}{2R}$.

In Fig. 15 values of M are plotted parallel to the horizontal axis and values of n and c parallel to the vertical axis. The speed is nothing at a. From a to b


SPEED OF MOTOR
REVOLUTIONS PER SECOND
SPEED
d
$\frac{1}{4}\,\frac{.71E^2}{Rt}$
$.71\frac{Rt}{E}$
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o
a
b
INDUCTION FACTOR
Motion reversed here
$\frac{E}{2R}$
CURRENT
$\frac{E}{R}$
f
g
CURRENT
CURVES OF
SPEED AND CURRENT
FOR DIFFERENT INDUCTION FACTORS
AT CONSTANT LOAD.

FIG. 15

the speed increases with the induction factor; $ob = 2\frac{\cdot 71Rt}{E}$; at b the speed is a maximum and $bd = \frac{1}{4}\frac{\cdot 71E^2}{Rt}$. If the induction factor is increased beyond ob, the speed will diminish until, when M is infinite, the speed is nothing. The current curve is also plotted in the figure; $ag = \frac{E}{R}$ and $fb = \frac{E}{2R}$.

From considerations of efficiency motors are usually worked on the speed curve to the right of d, where an increase of M involves a decrease of the speed. But it is easy to arrange an experiment in which the strength of the magnets can be varied and the motor brought to rest by increasing M. Care must be taken that the current when the speed is nothing shall not be great enough to injure the motor.

The experiment should be arranged so that the load, when unbalanced, tends to turn the armature in the reverse direction. If the induction factor is reduced below the value oa, the direction of rotation will be changed and the speed will increase in the reverse direction as the value of M is reduced. When M is nothing, the speed will be infinite, the current will also be infinite.

In practice the load on the motor is generally incapable of reversing the motion, so that if the induction factor be reduced to nothing, the speed would tend first to increase, and then to decrease to nothing, the current meanwhile increasing to its final value $\frac{E}{R}$; the safety fuses would of course prevent so large a current from passing.

The values of the speeds for different induction factors have been plotted in Fig. 19 on a base of current in the armature, by a method that will be described later on.

If the induction factor remains constant, the speed will diminish as the load increases. If we wish the speed to remain constant, we must arrange that the induction factor shall vary so that the effect due to the increased load is counterbalanced.

Motors are usually worked under conditions when a decrease of M involves an increase of the speed; when this is so, we must arrange that M shall decrease with an increase of the load. This we can do by winding round the magnets a circuit that shall carry the whole or part of the main current, so that as this increases it shall produce a magnetisation of contrary sign to that of the shunt winding. The motor is then said to be **compound wound**, and the series winding, as it is called, acts in opposition to the shunt winding.

Example 24.—A motor with an internal resistance of 0·13 ohm has to run at constant speed of 1,500 r.p.m., on a line of 125 volts tension. The maximum current is 150 amperes. At no load the induction factor has to be 5, while at full load it has to be 4·28. If we have the induction curve, and know the number of turns in the shunt winding, we can at once see how many turns carrying the maximum current are required to reduce M to its proper value at full load. If the induction curve is straight between the maximum and minimum values of M, the speed will be constant at all loads. If, as is usually the case, it is concave to the current axis, M will be too high for medium loads, and the speed will consequently be too low.

In many cases the reduction of M due to armature reaction may of itself compensate for the falling off of speed due to the heat drop, without the use of series winding. Hence a motor in which M decreases, as the armature current increases, will give better speed regulation than one in which M remains constant at all loads.

Example 25.—If the high-tension dynamo of Fig. 9 has an armature resistance of 8 ohms, it will run at 772 r.p.m, at no load on a 900 volt circuit, and at 784 r.p.m., with a load of 9 amperes.

Equation 23 gives us the change in the speed of a motor when the tension of the line is varied, the induction factor and the resistance remaining constant. If the load is constant and incapable of reversing the motion, the speed will be nothing until $E = \cdot 71 \frac{Rt}{M}$. As E increases, n will increase, the relation between the two being represented by a straight line such as $a\,b$ in Fig. 16, where $oa = \cdot 71 \frac{Rt}{M}$. Since the load is constant, the current is also constant.

Variations in the speed consequent upon changes in the value of E may be investigated experimentally by connecting the motor through a rheostat to a line of constant tension. The resistance should be adjusted so that the motor runs at a certain measured speed; the current and the tension at the terminals of the motor should be noted. The resistance should then be altered, the load being adjusted so that the current remains constant. Since the tension at the terminals of the rheostat is equal to the product of the current and the resistance, and therefore varies with the resistance, the tension at the motor terminals will be altered by altering the resistance; we shall thus be able to vary the tension at the motor terminals, and observe the corresponding speeds.

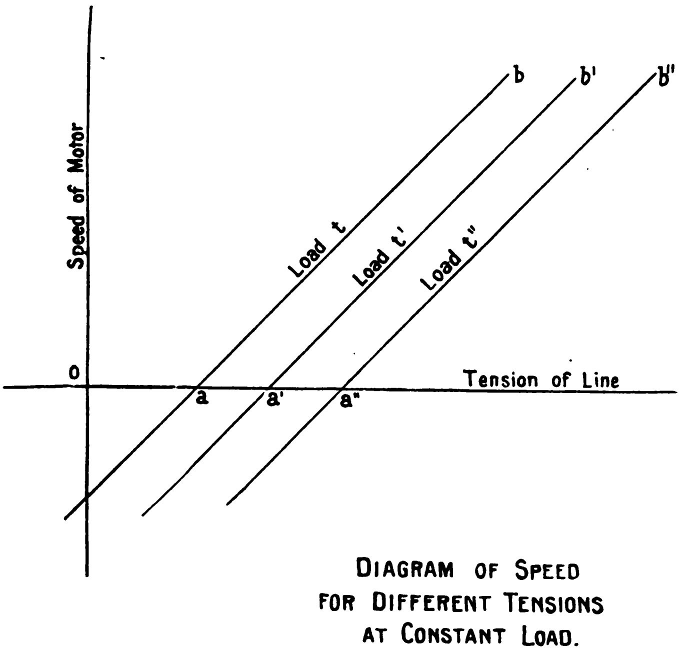


FIG. 16

If the torque is increased but kept constant during the changes in the tension, we shall obtain a relation between E and n given by $a'b'$ in the figure, $a'b'$ being parallel to ab, the intercept oa being equal to $\cdot 71 \frac{Rt'}{M}$, where t' is the new torque. Thus if the loads are kept constant during the changes of E, the speed curves are parallel to one another. If the induction factor is altered as well as the load, the speed curves will be differently inclined to the axis of speed (see Fig. 19).

We shall now consider the conditions of motion when **the load** is **a function of the speed.** Suppose that the resistance to motion increases with the speed, and that it can be represented by a curve, where vertical ordinates represent the speed in revolutions per second, and whose horizontal ordinates represent the torque resisting the motion.

For uniform speed the torque due to the current assisting the motion must be equal to the torque resisting the motion. Hence there will be a certain speed at which the motor must run when it is carrying a certain current. For, if M is constant, a given current means a given assisting torque; and there is only one speed at which the motor can run for that torque. If the speed is greater than that given by the curve, the resisting torque will be greater than the assisting torque due to the current, and the motor will be retarded, and if the speed is less than that given by the curve, the resisting torque will be less than the assisting torque, and the motor will accelerate.

If we place such a motor in series with a circuit in which the current varies, the speed of the motor will depend upon the current. The motor would then act as a **current meter,** and the total revolutions at the end of any time would depend upon the current that had been passing during that time. If the curve of torque and speed is straight and passes through the origin, the speed is proportional to the current. In practice it is not possible to make the curve straight nor to make it pass through the origin.

If t_f is the torque required to overcome the initial friction, an initial current $c_f = \cdot 71 \frac{t_f}{M}$ will be required to start the meter. If n_1 and n_2 are the speeds for currents c_1 and c_2, we shall have, assuming the torque curve to be straight,

$$\frac{n_1}{n_2} = \frac{c_1 - c_f}{c_2 - c_f} \quad \text{..................(25).}$$

For example, suppose that the curve is straight and inclined to the axis of current or torque at an inclination of 1·98, *i.e.* the ratio of any vertical ordinate, in terms of revolutions per minute, to the corresponding horizontal ordinate, measured up to the point where the curve cuts the current axis, in amperes, is 1·98. Suppose, also, that the starting current is 0·2 ampere. When the current is 3 amperes the speed is 5·55 r.p.m.; if the starting current were nothing it would be 5·95 r.p.m., the meter is therefore running 6·7 per cent. too slow. When the current is 15 amperes the speed is 29·3 r.p.m., it should be 29·7 r.p.m., the meter is now running 1·3 per cent. too slow. We see then that the accuracy increases with the total current.

Curves for three types of current meter are given in Fig. 17. The curves in the figure give (1) the speed, and (2) the ratio of the speed to the total current, both on a base of total current. In the Hookham meter the starting current is 0·2 ampere, and the maximum current 25 amperes. In the Perry meter the starting current is 0·25 ampere, the maximum current 20 amperes. In the

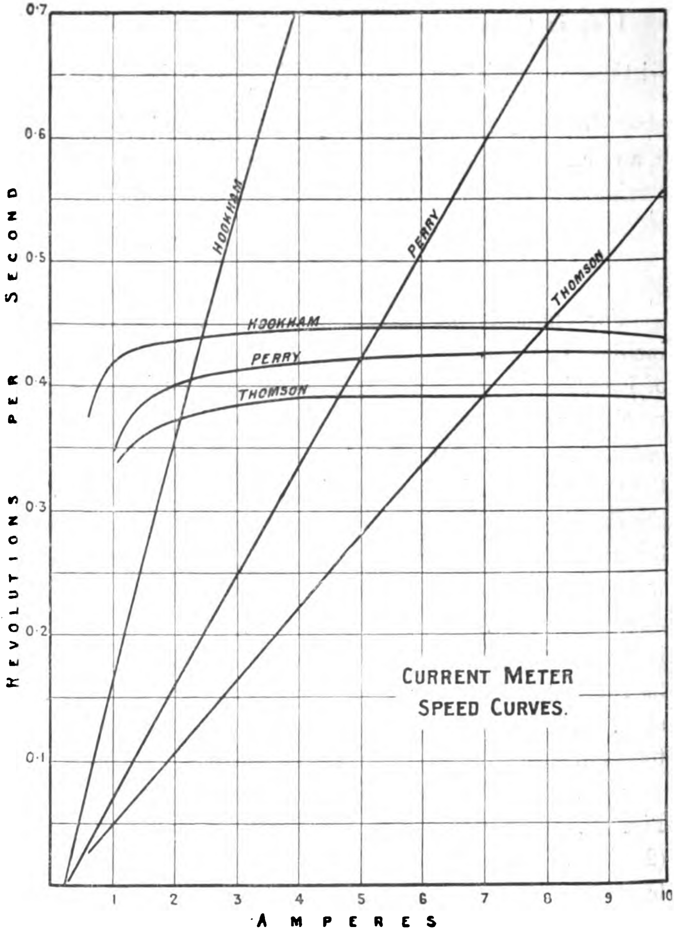

0·7
0·6
0·5
0·4
0·3
0·2
0·1
Revolutions per Second
HOOKHAM
PERRY
THOMSON
HOOKHAM
PERRY
THOMSON
Current Meter
Speed Curves.
1
2
3
4
5
6
7
8
9
10
Amperes

Fig. 17

Thomson meter the starting current is 0·14 ampere, the maximum current 10 amperes. The curves are obtained from data given by Mr. G. W. D. Ricks, in a paper read before the British Association, August 1897; see the 'Electrician' of August 27, 1897.

If the torque curve is not straight, a further error will be introduced, tending to make the meter run slower or faster according as the curve is concave or convex to the current axis. In all these meters the curves are concave to the current axis, and for small speeds the error due to the starting torque is the greater of the two. For high speeds, when this error is very small, the error due to the bend of the curve becomes apparent, and the accuracy curve turns down again.

Hitherto we have supposed M to be constant. If M varies, the speed will depend on the product of M and c, so that if c is fixed and M increased, the speed will increase with M, or if both c and M increase, the speed will increase with their product.

If the magnetising coil is wound as a shunt across the main line terminals of the circuit in which the current is flowing, the current in the shunt will be proportional to the tension, and if the magnetic circuit contains no iron the induction curve will be a straight line, *i.e.* the value of M will be proportional to the tension, hence the speed will be proportional to the watts in the circuit. This is the principle of the Thomson **watt-meter**. For a given current the speed increases directly in proportion to the tension.

We shall now consider the case when **two dynamos are mechanically coupled** so that they rotate at the same speed, and have their main terminals connected in parallel to the same line. The magnets should be shunt wound and separately excited, and provided with rheostats so that the induction factor of each dynamo can be adjusted to any desired value and kept constant. The connections will be made so that the dynamos tend to turn in the same direction when both are acting as motors.

We have further to suppose that by means of a pulley fitted on to one of the dynamo shafts and connected by a belt to a third dynamo, or by simply coupling a third dynamo direct on to the shaft line, we can produce a load of any required amount or of any required sign; in other words, we suppose that we can vary the mechanical torque on the shaft, making it either positive so that the dynamos have to act as motors, or negative so that they act as generators. Any arrangement will act provided the motion can be assisted or retarded at will.

The two dynamos may be distinguished by the letters *A* and *B*. Suppose that dynamo *A* has an induction factor of 6 and a resistance of 1·2 ohms, and that dynamo *B* has an induction factor of 4 and a resistance of 1·09 ohms. Let the tension of the line be 120 volts.

Set off *ab* in Fig. 18 equal to 100 amperes, the maximum current in *A*. Set off *ad* equal to 110 amperes, the maximum current in *B*. Let *af* represent 1,200 r.p.m., the speed of *A* when its induced tension is equal to the tension of the line. Let *ag* represent 1,800 r.p.m., the speed of *B* when its induced tension is equal to the tension of the line. Join *fb* and *gd*. These will be the speed curves of the two dynamos on a base of current in the armature.

Since the dynamos are mechanically coupled, we can at once find from the diagram the current that each is

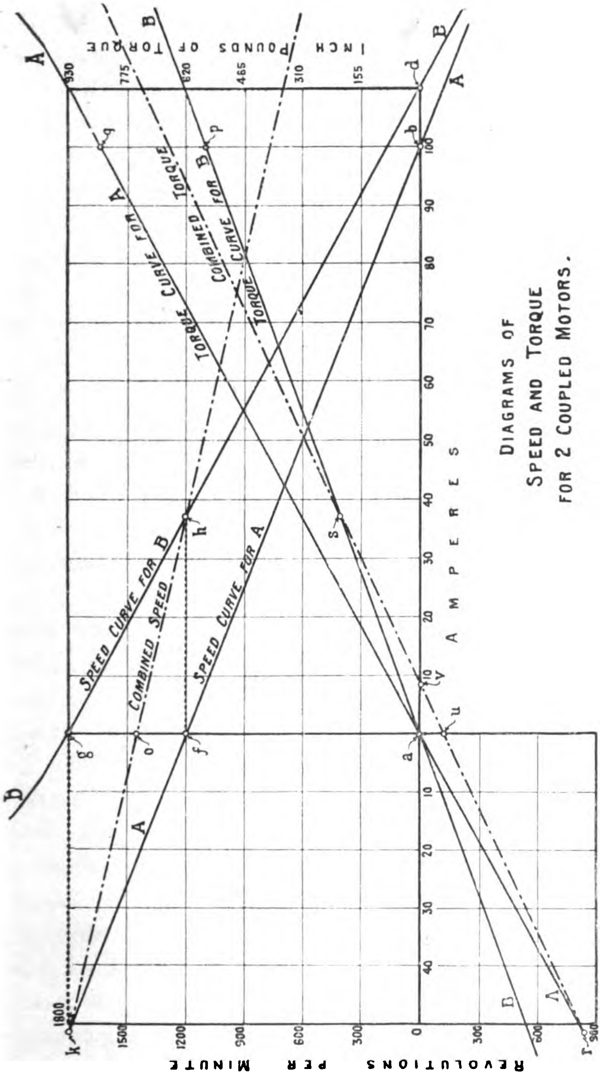

DIAGRAMS OF
SPEED AND TORQUE
FOR 2 COUPLED MOTORS.
SPEED CURVE FOR B
COMBINED SPEED
SPEED CURVE FOR A
TORQUE CURVE FOR A
COMBINED TORQUE
COMBINED TORQUE CURVE FOR B
INCH POUNDS OF TORQUE
AMPERES
REVOLUTIONS PER MINUTE
930
775
620
465
310
155
1800
1500
1200
900
600
300
0
300
600
900
10
20
30
40
50
60
70
80
90
100

FIG. 18

taking for any given speed. When the speed is nothing, the currents are 100 and 120 amperes. When the speed is 1,200 r.p.m. the current in *A* is nothing, and that in *B* is 36·7 amperes. When the speed is 1,800 r.p.m. the current in *B* is nothing, and *A* is now generating a current of 50 amperes. If the speed is above 1,800 both dynamos are acting as generators, if it is below 1,200 both are acting as motors, between these speeds *A* is acting as a generator and *B* as a motor.

If horizontal lines are drawn through *g* and *f* to cut the two speed curves in *k* and *h*, and if a line is drawn through the points *h* and *k*, this line is **the combined speed curve**, and will give the speed for any current from the line. The horizontal ordinates of this curve represent the current from or into the line. Where the combined speed curve cuts the axis of speed, the current from the line is nothing, and the current from *A* is the same as that into *B*.

Now let *bq* represent the torque in *A* for a current of *ab* amperes, and let *bp* represent the torque in *B* for the same current; these distances will be related to one another in the ratio of the two induction factors. Join *aq* and *ap*, and produce in both directions. These lines will then be the torque curves for the two dynamos, on a base of amperes.

From *k* draw a vertical line to cut the torque curve of *A* in the point *r*. Then since at the speed *aq* there is no current in *B*, the ordinate of the torque curve of *A* at *r* represents the combined torque of the two dynamos, and hence *r* is a point on the curve of combined torque, the horizontal ordinates of which would give the current from or into the line. A second point on this curve may be found by drawing a vertical line from *h* to cut the torque curve of B in s. Here there is no current in A, and B is doing all the work, hence s is a point on the combined torque curve. Join rs and produce it. This line will give the combined torque for any current, from or into the line.

The curve of combined torque cuts the speed axis at a point u, below the origin, indicating that when the current from the line is nothing, a small negative torque has to be supplied to assist the motor and maintain the current in the two dynamos. Since the current in each dynamo is here the same, we can write $\frac{E-M_a n}{R_a}=\frac{E-M_b n}{R_b}$, where R_a, R_b are the two resistances, and M_a, M_b the two induction factors, E the tension of the line, and n the speed in revolutions per second. Solving this equation, we have for the speed when the current from the line is nothing

$$n=E\frac{R_a+R_b}{R_aM_b+R_bM_a} \qquad \text{..............(26).}$$

Inserting the given values for R and M, we find that the speed av in this case is 1,452 revolutions per minute.

If $R_a=R_b$, $n=\frac{E}{\frac{1}{2}(M_a+M_b)}$. Hence two coupled motors with equal resistances and unequal induction factors behave as a single motor with an induction factor equal to the mean of the two induction factors.

Knowing the speed, av, we can find the induced tension in each dynamo. Using the equation $e = Mn$, we see that the induced tension of A is 145·2 volts, and of B is 96·8 volts. The induced tension of A, now acting as a generator, is considerably greater than that of the line, which is 120 volts, while the induced tension of B, now acting as a motor, is less than that of the line; both, however, are of the same sign, since both oppose the tension of the line. A, however, sends as much current into the line as B takes from the line, with the result that no current passes either from or into the line, but a current circulates in the two dynamos.

When there is **no current passing into or out of the line**, the current in each motor may be written $c=\frac{E-M_a n}{R_a}$. Using the value of n in Equation 26, we may express the value of each current thus:

$$c=E\frac{M_b-M_a}{R_aM_b+R_bM_a} \quad \text{............(27).}$$

Now the torque at this point is the difference between the torques on the two motors, of which one assists and one resists the motion, and may therefore be written

$$t=1{\cdot}41\ c\ (M_a-M_b) \quad \text{..............(28).}$$

Substitute for c, and we have

$$t=-1{\cdot}41\ E\frac{(M_a-M_b)^2}{R_aM_b+R_bM_a} \quad \text{...... . ...(29).}$$

In our example this torque is 60 inch-pounds; this will be the torque at which no current is passing from the line, and when the current generated by A is the same as that passing through B.

The current may also be found by dividing the difference between the two tensions, namely, 48·4, by the sum of the two resistances, 2·29, giving us a current of 21·2 amperes. This may be checked by comparison with the diagram.

The expenditure of energy due to this current passing in the two dynamos can be calculated, since we know the resistances. We find that the C^2R loss in A is 538 watts, and in B is 489 watts, making a total of 1,027 watts. This loss has to be made up by the mechanical input represented by the torque, and given by the intercept au of the curve of combined torque on the speed axis.

If in the equation for the torque $t = 1{\cdot}41\ cM$ we put for M its equivalent $\frac{e}{n}$, we can write this equation thus, $t = 1{\cdot}41\frac{w}{n}$, where w is the mechanical watt output or input according to sign. Inserting the values of w and n as found above, we find the corresponding torque to be 60 inch-pounds.

The required torque may be determined in yet another way. The torque in A, due to 21·2 amperes when the induction factor is 6, is 179·5 inch-pounds; this torque resists the motion. The torque in B, due to the same current, when the induction factor is 4, is 119·5 inch-pounds; this torque assists the motion. For uniform motion the assisting and resisting torques must be equal, so that we require a torque of 60 inch-pounds to assist the motion, and this must be supplied from some external source.

The assumption has been made that the only torques to be considered are those due to the currents passing in the two dynamos, and that if the difference between the assisting and retarding torques be supplied from outside we shall have uniform motion. In practice we cannot run two dynamos in this manner without experiencing a further resisting torque, due to the friction in the bearings, hysteresis, &c. If, then, the torque supplied to such an arrangement proves by measurement to be greater than that required to make up the difference between the assisting torque of the current in B and the resisting torque of that in A, the amount by which it is greater represents the torque lost in friction, &c., in the combination. This will be considered further when we come to discuss the application of these principles to the testing of dynamos.

When the resistances and induction factors of two dynamos are given, and also the tension of the line, we see that there is a definite speed at which they must run in order that the current generated by the one shall be equal to that received by the other; so that our arrangements for the experiment must admit of the machines being run at the required speed. When this is done the current from the line will be nothing, and will remain nothing as long as the speed remains constant. If the speed be increased some current will pass into the line, while if the speed be decreased current will be received from the line.

Let us now suppose that the torque supplied from the external source be removed altogether, and that the torque lost in friction is nothing. We shall find that a current will pass from the line in amount equal to the intercept ov, where the current axis is cut by the curve of combined torque. At the same time the speed will fall off, the new speed being found by the vertical ordinate of the combined speed curve drawn through the point v.

Since no torque is supplied from outside, the assisting torque of B must equal the resisting torque of A, hence the currents in the two dynamos must be related to one another in the inverse ratio of their induction factors.

Further, the whole expenditure of energy in the form of heat must be supplied by the current from the line. So that we have the equation

$$E(c_b - c_a) = c_a^2 R_a + c_b^2 R_b \quad \ldots\ldots\ldots\ldots (30).$$

Where c_a and c_b are the currents in the two motors. Substituting the value of c_b in this equation we have

$$c_a = EM_b \frac{M_a - M_b}{M_a^2 R_b + M_b^2 R_a} \quad \ldots\ldots\ldots\ldots (31).$$

While the difference between c_a and c_b, *i.e.* the current from the line, is given by

$$c = E \frac{(M_a - M_b)^2}{M_b^2 R_a + M^{a2} R_b} \quad \ldots\ldots\ldots\ldots\ldots (32).$$

Inserting the given values of E, M, and R in these equations we find that the current from the line in this case is 8·2 amperes, the current in A is 16·5 amperes, and that in B is $\frac{M_a}{M_b} \times 16{\cdot}5$ or 24·7 amperes.

The reduced speed may be found by considering the fact that the induced tension of A, minus the heat-drop, must be equal to the tension of the line, and thus equal to the induced tension of B plus its heat-drop. This may be stated as follows:

$$nM_a - c_a R_a = nM_b + c_b R_b \quad \ldots\ldots\ldots\ldots (33).$$

From this equation we deduce the value of n in terms of the current in A:

$$n = c_a \frac{R_a M_b + R_b M_a}{M_b(M_a - M_b)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (34).$$

Inserting the value of c_a as found above we get

$$n = E\frac{R_a M_b + R_b M_a}{M_a^2 R_b + M_b^2 R_a} \quad \text{..............(35).}$$

Using the given data, the new speed appears to be 1,398 r.p.m., so that when the external torque is removed, the speed is reduced from 1,452 to 1,398 revolutions per minute.

These results may be checked by equating (1) the torques in the two dynamos, (2) the watts from the line and the sum of the heat watts in the two dynamos, (3) the effective tension and the product of the resistance and the current in each dynamo.

When two dynamos coupled in parallel are both intended to act as motors, the speed of the combination will be determined by the load or torque on their common shaft, or by the total torque when they are connected by some mechanical arrangement, as for instance when two motors are driving one railway car.

For a given total torque each motor will take its own share of the load; the amount of the torque taken by each motor can be ascertained from the diagram. Suppose, for example, that the total torque is 273 inch-pounds. Set this off on the curve of combined torque and draw a vertical line to cut the combined speed curve; this gives the speed. Then draw a horizontal line to cut the torque curves of *A* and *B*, the intersections give the current passing through each dynamo; in this case *A* is taking 5 amperes and *B* is taking 41 amperes. From these intersections draw vertical lines to cut the torque curves of *A* and *B*, this will give the torques of the two motors, 42 inch-pounds and 231 inch-pounds respectively, the sum making up the total torque. The dynamo *A*, having the larger induction factor, will take the smaller current for all loads until the currents in the two dynamos are equal. The torque at this point is negative, and is given by Equation 29.

If the combined torque increases negatively, *i.e.* still further assists the motion, A will carry more current than B. The torque at which B ceases to act as a motor is given by

$$t = -1{\cdot}41 \frac{E}{R_a} \cdot \frac{M_a}{M_b}(M_b - M_a) \quad \text{...........(36).}$$

In our example this is 423 inch-pounds.

For heavy loads the dynamo with the larger induction factor will do more work than the other, since the torque is greater though the speeds are the same; as the total load diminishes, the distribution of the load between the two motors becomes more uniform. There is a certain **load at which the two dynamos** have equal torques, and consequently **are working at the same rate.** The torque at this point can be readily found.

Since the currents in the two machines bear to one another the inverse ratio of their induction factors, we can write:

$$c_a = \frac{E - M_a n}{R_a} = \frac{M_b}{M_a} \cdot \frac{E - M_b n}{R_b} \quad \text{.........(37).}$$

From which we deduce the speed,

$$n = E \frac{R_b M_a - R_a M_b}{R_b M_a^2 - R_a M_b^2} \quad \text{..............(38).}$$

The currents in the two machines may be written respectively:

$$c_a = E \frac{M_a M_b - M_b^2}{R_b M_a^2 - R_a M_b^2} \text{ and } c_b = E \frac{M_a^2 - M_a M_b}{R_b M_a^2 - R_a M_b^2} \text{...(39).}$$

Taking now the sum of the two torques, we find that the total torque becomes

$$t = 2{\cdot}82 E \frac{M_a^2 M_b - M_b^2 M_a}{R_b M_a^2 - R_a M_b^2} \quad \text{...........(40).}$$

In the example illustrated in the diagram the speed at this point is 626 r.p.m., c_a is 48 amperes, c_b is 72 amperes, the total current is 120 amperes, and the combined torque 812 inch-pounds.

When the total torque is less than that given by Equation 40 the dynamo with the larger induction factor will do less work than the other, until at a certain load it will do no work at all, and all the work will be done by the motor with the smaller induction factor.

The torque for which **one motor** is **doing all the work** may be found by considering the fact that at this load the induced tension of the other motor is equal to that of the line. Hence we can write:

$$\frac{E}{M_a} = \frac{E}{M_b} - {\cdot}71 \frac{tR_b}{M_b^2} \quad \text{..............(41).}$$

From which we find the required torque to be

$$t = 1{\cdot}41 \frac{EM_b}{R_b M_a}(M_a - M_b) \quad \text{...........(42).}$$

In our example this torque is 207 inch-pounds.

It has been assumed that when one of two coupled motors is generating a current, the other is in danger of being burnt out at heavy loads, on the supposition that it would have to do all the useful work and also drive the second motor as a generator. In practice it will be found that it is only at light loads that one of the two motors will act as a generator, the effect of an increase in the total load being to equalise the torques and consequently the currents in the two motors.

When two dynamos are coupled mechanically so as to run at the same speed, and are connected electrically in parallel on the same line, we have seen that it is possible to run them under such conditions, that there is no current passing either into or out of the line. In this condition the internal losses of the dynamos are made up by the mechanical energy put into the combination.

If we know the resistances of the two machines, and measure the current passing between them, then the torque input is given by the consideration that the mechanical equivalent of the heat loss is equal to $\cdot 71tn$, or $c^2\ (R_a + R_b)$. From this we can write down the torque required to make up the heat losses at any speed. If now the torque input is measured and proves to be greater than that indicated by the above expression, we know that the difference represents the torque in the two dynamos lost in friction, hysteresis, &c. We have thus a method of finding out what these losses amount to.

Take the case already considered and illustrated in Fig. 18. Since the resistances of the two dynamos are 1·2 and 1·09, and their induction factors 6 and 4, we have to run the combination at 1,452 r.p.m. to make the current from the line nothing; the current in each dynamo is then 21·2 amperes. Using these values we found that the torque required to make up the heat loss was 60 inch-pounds. This is the torque input on the assumption that the only losses to be made up are the heat losses; if, however, there are frictional and other torque losses as well, the torque input must be greater than 60 inch-pounds; suppose that we found it to be 80 inch-pounds, then we should know that the frictional losses in the combination were equal to 20 inch-pounds.

Since the current from the line is nothing we might disconnect the line altogether without making any change in the conditions. We have here **Hopkinson's method of testing dynamos.** See 'The Electrician,' March 12, 1886, and the 'Proceedings of the Institution of Civil Engineers,' Vol. LXXXIII. p. 235. This method consists in mechanically coupling two similar dynamos, their common shaft being provided with a pulley. The two machines are driven by a belt, the power transmitted being measured by a dynamometer.

Suppose that a test of a generator has to be made, the conditions being that it has to run at a given speed and give out a certain current at a certain terminal tension. First we must arrange to drive the combination at the given speed and to keep it constant throughout the test. Then we must adjust the induction factor of the motor so that when the combination is running at the specified speed, we shall have the proper current passing in the two dynamos. When this is the case, the belt and the motor together will be driving the generator, and the power transmitted by the belt will represent the heat and torque losses in the two machines. Knowing the terminal tension, the resistance of the generator, and the speed n at which it has to run, we know the induction factor, call this M_a. Then the difference between the two induced tensions is the effective tension and equal to the product of the current into the sum of the two resistances, so that M_b, the induction factor of the motor, is given by

$$M_b = M_a - \frac{c(R_a + R_b)}{n} \quad \text{...........(43).}$$

If then the combination be driven at the proper speed, the generator will give the specified current at the specified tension.

Example 26.—A generator has to be tested under the following conditions: The speed is to be 780 r.p.m., the current 350 amperes, with a tension of 110 volts on the brushes. The resistance of the armature is 0·01 ohm. The induced tension of the generator must be greater than the tension at the brushes by the amount of the heat-drop, which is 3·5 volts at 350 amperes. Hence we have 113·5 volts as the induced tension, giving 8·73 as the induction factor of the generator. Inserting this value in Equation 43, we find that the induction factor of the motor must be 8·19. From Equation 29 we see that the torque required to make up the heat loss is 266 inch-pounds. Suppose that the dynamometer reading showed a torque input of 716 inch-pounds, it would follow that there were frictional losses to be made up, over and above the heat losses, amounting to 450 inch-pounds of torque. On the assumption that the two machines are in all respects similar, we may then say that the frictional and other torque loss in the generator is 225 inch-pounds.

Since the two induction factors are not equal it is not strictly accurate to divide the torque losses equally between the two dynamos. We see that in this case the induction factors are respectively 8·73 and 8·19, showing that there is a difference in the magnetic condition of the two machines. The difference between the induction factors for a given speed and current in the generator increases with the resistance in the two dynamos.

This method is of course equally applicable to the testing of a motor which has to comply with specified conditions.

Example 27.—A motor has to be tested while running at 900 r.p.m. on a line of 125 volts tension, the torque on the motor shaft being 3,150 inch-pounds and the resistance of the motor 0·015 ohm. A second dynamo is provided similar in all respects and having the same resistance as the motor to be tested. Equation 18 gives us the value of the induction factor of a motor running on a line of given tension, with a specified torque, and at a given speed. From this equation we find the value of the induction factor of the motor to be 8·06 ; it will then be taking 278 amperes. Using Equation 43 we find the induction factor of the generator to be 8·616. The torque required to make up the heat loss will be 218 inch-pounds. If the dynamometer indicates 600 inch-pounds, there is a torque loss in the motor of 191 inch-pounds.

Returning now to the original arrangement where two dynamos are connected in parallel on a line of constant tension. We found that when the resultant torque on the shaft was nothing, there was a current passing from the line representing the energy required to make up the heat losses in the two machines. The current then passing in the motor is greater than that in the generator by the amount of the current from the line. If there is no other loss than that due to heat, the current from the line will be given by Equation 32. If, however, there are losses due to friction, hysteresis, &c., the line current will be greater than this, and the difference will represent these losses. We have here a **modification of Hopkinson's method** of testing, namely one in which the losses are measured electrically.

Example 28.—A four-pole lighting generator is to be tested under the following conditions: The speed is to be 75 r.p.m., the current 800 amperes, with a tension of 250 volts at the brushes. The resistance of the armature is 0·00585 ohm. The heat-drop at full load is 4·7 volts. Hence the induced tension of the generator must be 254·7 volts, and the induction factor is therefore 203·73. The generator will then be giving out 800 amperes, and there will consequently be a torque of 229,700 inch-pounds on the coupling between the two dynamos. We have now to find the induction factor of the motor; assuming there are no torque losses, it has to deliver 229,000 inch-pounds of torque to the generator at 75 r.p.m., the tension of the line being 250 volts. Inserting these values in Equation 43, and assuming the resistance to be the same as for the generator, we find the induction factor of the motor to be 196. To give the required torque the motor will then draw 831·1 amperes, 800 of which will be supplied from the generator and 31·1 from the line.

The results may be checked thus:

Watts supplied from the line, $31·1 \times 250$.	7,775	
Watts lost in heating the generator, $800^2 \times ·00585$.		3,744
Watts lost in heating the motor, $831·1^2 \times ·00585$.		4,040
Total watts lost in heat . . .		7,784

The difference is due to the limits of the accuracy of the slide rule.

Suppose, now, that the current from the line as actually measured is 91·6 amperes; we must deduct 33·6 amperes for the heat losses, as the current in the motor is now 891·6 amperes, leaving 58 amperes for the torque losses in the two machines, or if divided equally between them, a torque loss of 7,240 watts in the generator. If expressed as torque, we find that it amounts to 5,800 inch-pounds. (Compare the results of tests of a 200 k.w. generator at the Nice Central Lighting Station, described in the 'Industrie Electrique' for October 10, 1896, where the losses were given as follows: c^2R loss, 3·75 k.w.; hysteresis and eddy loss, 7·83 k.w.)

When two coupled motors with constant induction factors have their armatures connected in series, the effect is as if there were one motor with an induction factor equal to the sum of the two. Thus two motors with induction factors of 6 and 4 placed in series will act as a single motor with an induction factor of 10, the resistance being the sum of the two resistances.

We have already seen that when two motors are connected in parallel, the combination acts as one motor with an induction factor equal to the mean of the two induction factors if the resistances are equal. Hence we have the following **possible combinations of two motors,** enabling us to get as many different speeds as there are combinations.

Let the induction factors be 6 and 4 respectively, the resistances equal, and the tension of the line 120 volts.

1. *A* only; maximum speed 1,200 r.p.m.
2. *B* only; maximum speed 1,800 r.p.m.
3. *A* and *B* in parallel; maximum speed 1,440 r.p.m.
4. *A* and *B* in series; maximum speed 720 r.p.m.

In this way we can get four different speeds by connecting up two motors in four different ways. If the induction factors were 3 and 5 we could get speeds of 600, 860, 1,200, and 2,000 r.p.m. on a tension of 100 volts.

If the torque load on the combined shaft is constant, the current from the line will be inversely proportional to the combined induction factor. Thus, in the case of the motors having induction factors of 6 and 4, if the current with $M=4$ is 100 amperes, the other currents would be 80, 67, and 40 amperes.

Hitherto we have supposed that the two motors were coupled mechanically so that they were obliged to rotate at the same speed. We shall now consider the case when the **motors** are **at liberty to rotate at different speeds.**

To fix ideas, suppose that the two motor shafts are in line, each fitted with a bevel wheel, facing the other, and gearing into an intermediate wheel, the axis of which is at right angles to the line of the motor shafts and free to rotate in a plane at right angles to that line. Such a mechanism, for instance, as is used in the Aron current meter.

The two motors are now free to rotate at different speeds, and the angular speed of the intermediate shaft depends on the mean of the two speeds if they rotate in the same direction, and on the difference if they rotate in opposite directions, and is measured by the number of revolutions per second that it makes in the plane at right angles to the main line of shafting, and not by the revolutions around its own axis. If the motors rotate at equal speeds in the same direction the intermediate shaft will rotate at the same speed as the two motor shafts. If the motors rotate at equal speeds in opposite directions, the intermediate shaft will not move.

The action of the motors may be determined graphically. In Fig. 19 take ag to represent the maximum current from the line through one motor, say 100 amperes. From any point *b* set off distances to represent values of the induction factor. From *a* set off distances to represent the speeds, on any given tension, for different values of M. Thus if the speed for an induction factor of 4 is represented by *af*, the speed for $M=8$ will be one half of *af*, and for $M=2$ twice *af*. For convenience, the scale of speeds thus constructed has been continued horizontally, the upper portion of the scale being cut off.

Suppose that motor A has $M=4$; join *ac*; then *ac* is a torque curve, since the torques are proportional to the currents. Let the torque load on A be represented by *dh*; the current in A is then *ah* amperes. Join *fg*, this is the speed curve for A, since *af* is the maximum speed of A. Continue *hd* to cut *fg* in *e*, the speed of A for this load is then given by *eh*.

Now in the arrangement that we have described the torque on the two motor shafts must be the same if the diameters of the three bevel wheels are equal. Hence if the resistance of the second motor, B, is the same as that of A, its speed for any load will be found by a similar construction as for A, taking into account the difference of the induction factors. Suppose that for B we have $M=6$. Join *vg*; this will be the speed curve of B. Take *pr* equal to *hd* cutting *ak* in *p*, produce *rp* to cut *vg* in *s*. The current in B is *ar*, and the speed of B is *rs*. If A and B are rotating in the same direction, the angular speed of the intermediate shaft, C, will be the mean between *rs* and *eh*. If A and B rotate in opposite directions the speed of C will be equal to the difference *rs* and *eh*.

By taking different values of M we can find a series of points giving the speeds of one motor at any fixed load;

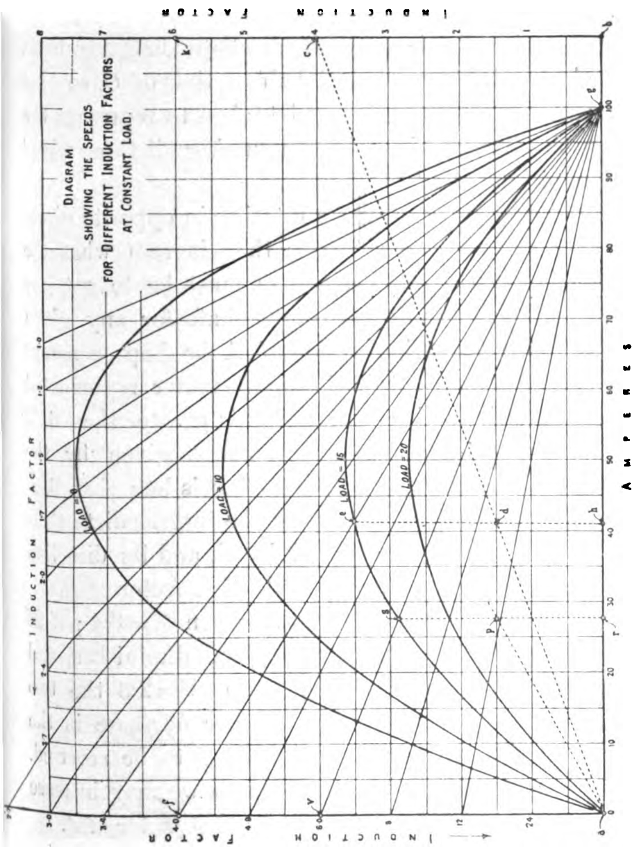


DIAGRAM
SHOWING THE SPEEDS
FOR DIFFERENT INDUCTION FACTORS
AT CONSTANT LOAD.
INDUCTION FACTOR
AMPERES
LOAD = 0
LOAD = 10
LOAD = 15
LOAD = 20

FIG. 19

these points lie on a curve passing through *a* and *g*. In the figure four such curves are taken for loads of 8, 10, 15, and 20, the load on *A* represented by *hd* being taken as 15.

For any given load on the intermediate shaft, the loads on *A* and *B* will be equal, and half of that of *C*, so that the numbers on the curves may be taken to represent the loads on the intermediate shaft, assuming all three wheel diameters to be equal.

Suppose that the motors are rotating in opposite directions. We can at once see from the diagram what the values of the two induction factors must be to get any required speed of the intermediate shaft for any given load. Let the induction factor of *A* be kept constant at $M=6$, with a constant load on each motor represented by 15. If the induction factor of *B* is greater than 6, *B* will run slower than *A* by an amount given by the diagram. If the induction factor of *B* is less than 6, *B* will run faster than *A*, the speed of the intermediate shaft *C* can thus be increased or diminished, and its direction reversed, simply by altering the induction factor of *B*.

The speed of *B* is a maximum when $M=3{\cdot}4$; if M is reduced below this value the speed diminishes, and therefore also the speed of *C*; when $M=2{\cdot}3$ the two speeds are again equal, and a further decrease in the value of M will cause the rotation of *C* to be reversed. When *B* has reached its maximum speed we may increase the speed of *C* by increasing the induction factor of *A*.

The student should compare the curve of speed with constant load and variable induction factor plotted on a base of current as in Fig. 19, with the curve of speed plotted on a base of induction factor as in Fig. 15.

An electric steering gear has been made on this principle by the Union Electricitäts Gesellschaft, a description of which may be found in the 'Elektrotechnische Zeitschrift,' 1897, No. 5. Two multipolar motors are connected by a differential driving gear to the main shaft operating the rudder, in such a way that when rotating at equal speeds in opposite directions they communicate no motion to the main shaft. The two armatures are connected in parallel, and the magnets in series with one another: a rheostat is placed in parallel with each magnet circuit. When the main shaft has to be moved, the induction factor of one motor is increased, and of the other is decreased, by altering the resistance in the rheostats in parallel with the magnets.

CHAPTER V

SERIES-WOUND MOTORS

WHEN the **magnets** of a dynamo are **connected in series with the armature,** the induction factor will vary with the current. In Fig. 20 distances measured along *ob* represent current in the armature, and also in the magnets. We will take 500 volts as the tension of the line, 5 ohms as the resistance of the motor, the maximum current being thus smaller than 100 amperes. Let the distance 01 represent 100 volts, 10 amperes, 100 r.p.m., 1,500 inch-pounds of torque, and an induction factor of 15. Let *oa* represent the tension of the line, *ob* the maximum possible current. Join *ba*, and produce it. We shall call *ba* the loss line, since the intercept *kc* of any ordinate between it and a horizontal line through *a* gives the volts lost in heat. On *ob* construct the induction curve. Thus, if for a current of 70 amperes, the induction factor is 67·5, we must set up on the point representing 70 amperes in the armature, a vertical distance equal to 4·5 inches, and this will be a point on the induction curve.

If there is no residual magnetism the induction factor is nothing when the current is nothing. When the current is reversed the induction factor will also be

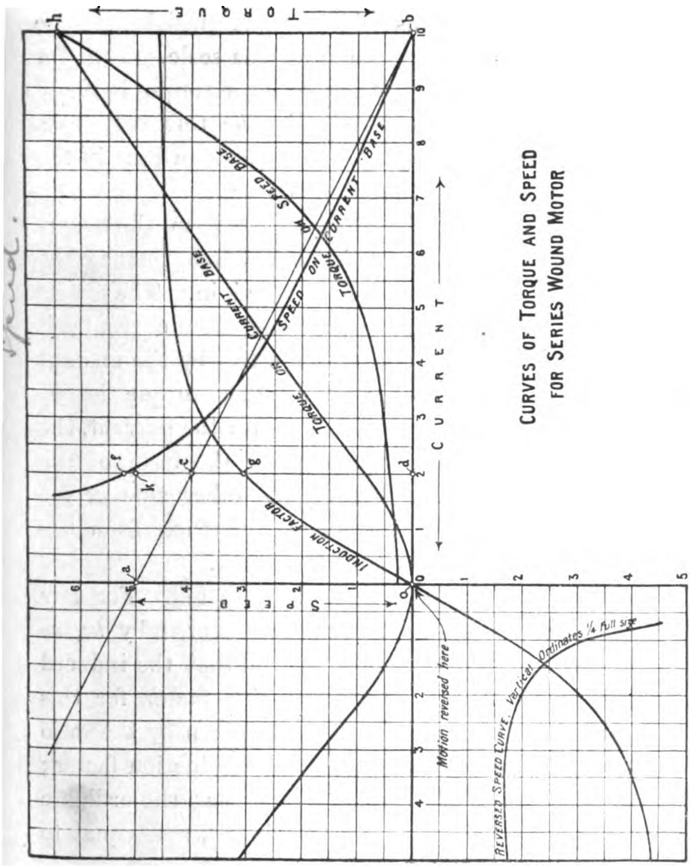

CURVES OF TORQUE AND SPEED
FOR SERIES WOUND MOTOR
TORQUE
SPEED
CURRENT
SPEED BASE
CURRENT BASE
SPEED ON CURRENT
SPEED ON TORQUE
TORQUE ON CURRENT BASE
INDUCTION FACTOR
Motion reversed here
REVERSED SPEED CURVE. Vertical Ordinates ¼ Full size

FIG. 20

reversed, the curve below the axis being similar to that above it. Construct the reversed induction curve.

The product of the current and the corresponding induction factor gives us the torque, which can be plotted on the current base on the given scale. Thus for the maximum current of 100 amperes the torque is 9,720 inch-pounds, set this off at the point *b*; this will be a point on the curve of torque on a base of current in the dynamo. Construct the rest of the curve.

We shall find that this curve differs from the torque curve of a motor of constant induction factor, since the axis of current is a tangent to it at the origin. The torque thus obtained is the total torque for any given current; it will be greater than that observed in a test by the amount required to overcome friction and other torque losses. Since the induction factor is reversed with the current, the sign of the torque remains unchanged, the form of the curve being the same as that on the other side of the origin. Here we see an important difference from the torque curve with constant induction factor.

The speed curve may now be constructed. For any current *od*, the heat-drop will be represented by *kc*, so that the motor must run at such a speed that the induced tension is equal to *cd*. The induction factor for this current is *dg*, the speed will therefore be given by the ratio of *cd*, the induced tension, to *dg*, the induction factor; using the given scale of speeds, we find that the ordinate of the speed curve at this point will be *df*, and equal to 520 revolutions per minute.

In this way the complete speed curve can be plotted; it will pass through the point *b*, and the axis of speed will be an asymptote. It differs in a marked way from the speed curve for a motor with constant induction factor, since the speed is infinite when the current is nothing.

When the current is reversed, the sign of the torque remains unchanged, hence the direction of rotation must change at the origin. The reversed speed curve may be plotted by taking the ordinates of the induction curve and the heat-drop as before. The dynamo is now acting as a generator, and sending a current into the line, but unlike the dynamo with constant induction factor, the change from motor to generator has been accompanied by a change in the direction of rotation. Motion is also reversed at the point *b* as with a dynamo of constant induction factor. The form of the speed curve is not symmetrical about the axis of speed. An inspection of the curve for reversed current shows that the speed first decreases and then increases.

In comparing the speed curve of a series-wound motor with that of a motor of constant induction factor, we see at once that the former gives much greater variations of speed for small loads. The line *ab* in the diagram would be the speed curve for a motor having a constant induction factor equal to that of the series-wound motor when running at a speed given by the intersection of its speed curve with the line *ab*. For loads greater than that corresponding to this speed, the behaviour of the two motors is not very different, but for smaller loads, the speed of the series-wound motor exceeds that of the other, the ratio of the speeds increasing to infinity when the load is nothing.

The energy curves for a series-wound dynamo will be similar to those for a dynamo with constant induction factor. The general form of the heat, line, and mechanical watt curves is not altered by the reversal of rotation or the change in the sign of the induction factor at the origin.

From the speed and torque curves we may construct a curve of torque on a speed base. In the diagram this curve is shown with the torques measured vertically, the maximum torque being *bh*, as in the original torque curve, the speeds being now measured horizontally from *b*, as origin, towards *o*. This curve gives us the speed at which the motor will run for any given load; we shall find it useful in determining the conditions under which a motor speeds up from rest; it also shows the changes in the speed consequent on a change in the load.

In a series-wound motor, the induction factor is a function of the current, and the torque is proportional to the product of the current and the induction factor, hence the torque varies as some power of the current always greater than one. For points on the induction curve near the origin, the induction factor increases in direct proportion to the current, so that the torque curve is here a parabola, the torque increasing as the square of the current. As the induction curve bends over, the torque curve changes in character and is slightly inflected, as a close inspection of the diagram will show, until when we have passed the bend and arrived at the straight upper portion of the induction curve, the torque increases at a rate greater than that of the current by an amount determined by the inclination of the induction curve to the horizontal.

The form of the torque curve of a series-wound motor has the very important practical result that large torques may be obtained without drawing proportionately large currents from the line. This is a great advantage when a motor is starting or when a sudden load is put on, but the motor must be properly designed in order to get the full benefit of this arrangement. For if the normal running point is high up on the induction curve, the induction factor will be only slightly increased when the heavy load is put on, and the current will be nearly in proportion to the torque. If, however, the points of normal and maximum load are so situated on the curve that the corresponding values of the induction factor are very different, then the current taken at the maximum load may be much smaller than it would be if the induction factor remained constant.

Take as an illustration the **G. E. 800 railway motor** made by the General Electric Company, and designed to exert a horizontal effort of 800 pounds on 33-inch wheels, when running at 9·4 miles an hour. The torque and speed curves of this motor, as obtained from actual test, are given in Fig. 21. The circles indicate observations. The resistance of the magnets and armature in series, when hot, was 1·245 ohms, being 0·805 for the magnets, and 0·44 for the armature. The induction curve is obtained from the speed curve by using the equation $M=\frac{E-cR}{n}$. This curve gives the induction factor for any current under normal running con ditions, and takes into account all effects due to armature, reaction and other losses tending to reduce the induction factor.

The curve of total torque is obtained from the equation $t=1{\cdot}41cM$. This curve gives the total torque for any current available for all purposes, including internal torque losses due to hysteresis, gear friction, &c.; it is greater than the torque actually observed by the amount of

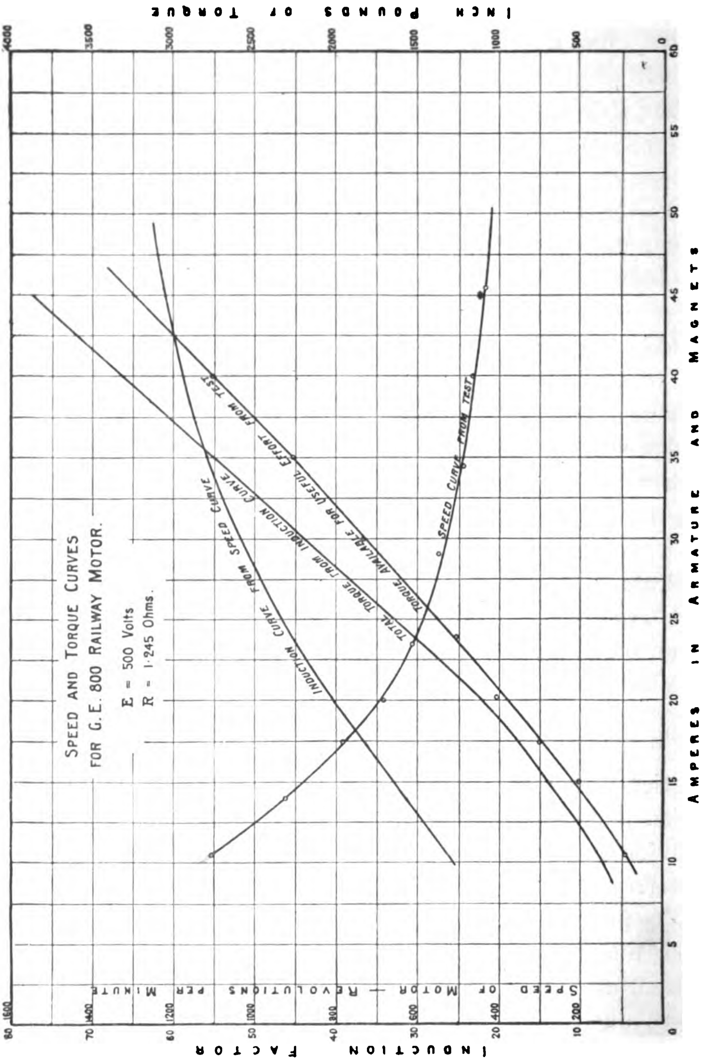

Speed and Torque Curves
for G.E. 800 Railway Motor.
E = 500 Volts
R = 1·245 Ohms.
Inch Pounds of Torque
Amperes in Armature and Magnets
Speed of Motor — Revolutions per Minute
Induction Factor
Torque available for useful effort from test
Total torque from induction curve
Induction curve from speed curve
Speed curve from test

Fig. 21

these losses, for when the motor is running under normal conditions, the frictional torque has to be deducted from the total torque, so that the torque measured at the rim of a brake wheel represents the total torque diminished by the amount of the torque losses. Hence when the torque curve obtained by a brake test is plotted, it will be found to lie below the curve of total torque as obtained from the equation $t = 1{\cdot}41\ cM$. In this chapter we shall take the speed curve as the basis for the construction of the curves of induction and total torque.

If the induction curve of **a series-wound motor** passes through the origin, we **cannot** make it **generate a current** by increasing the speed. This is seen from the form of the speed curve. For instance, it is not possible to utilise the energy of a car descending a grade by sending current into the line, as might be done if the motors were shunt wound. This is a consideration that may assume some importance in certain cases. Thus in the system of electric railroads radiating from the city of Baltimore, there are grades of one in sixty extending for a distance of as much as five miles. In order to run at fifteen miles an hour up such a grade with a seven-ton car, we should need about four horse-power for friction and track resistance, and eleven for the grade. On the descending journey, we should have to take current from the line only when starting up after a stop; for the rest of the time the motors would be cut out and the brakes on. If the motors are series wound, they are now useless either for restoring energy into the line, or for acting as brakes. If shunt-wound motors were used, a considerable proportion of the power required for an ascending car could be supplied by a descending car, the generating action of the motors meanwhile serving as a brake on the motion of the car.

The only condition under which a series-wound motor can be made to generate a current when running under ordinary circumstances is that the induction curve should not pass through the origin. This condition is frequently obtained in practice owing to the existence of **residual magnetism**. In Fig. 22 curves of torque and speed are given for a motor in which the induction curves for ascending and descending values of the currents are determined by the amount of hysteresis. In the figure the two induction curves are similar, but displaced horizontally.

The speed and torque curves for the ascending and descending portions of the induction curve are plotted in the usual way. There is a portion of the torque curve where the torque is negative, namely where the current and the induction factor are of opposite sign. Taking the descending portion *B*, we see that the vertical line drawn through the point *b*, where the current is equal to 10 amperes negative, is an asymptote to the speed curve, hence, when the current is nothing, the motor will be running at such a speed that the induced tension made up on the residual magnetism is equal to the tension of the line, and for speeds above this the motor will deliver current into the line, but this current cannot exceed 10 amperes.

If the magnets of a motor are made of steel, the residual magnetism may be considerable, and a current may be sent into the line when the motor is running at a high speed, but the current will be small. If, however, the motors are disconnected from the line, and have their armature terminals connected through a low resistance,

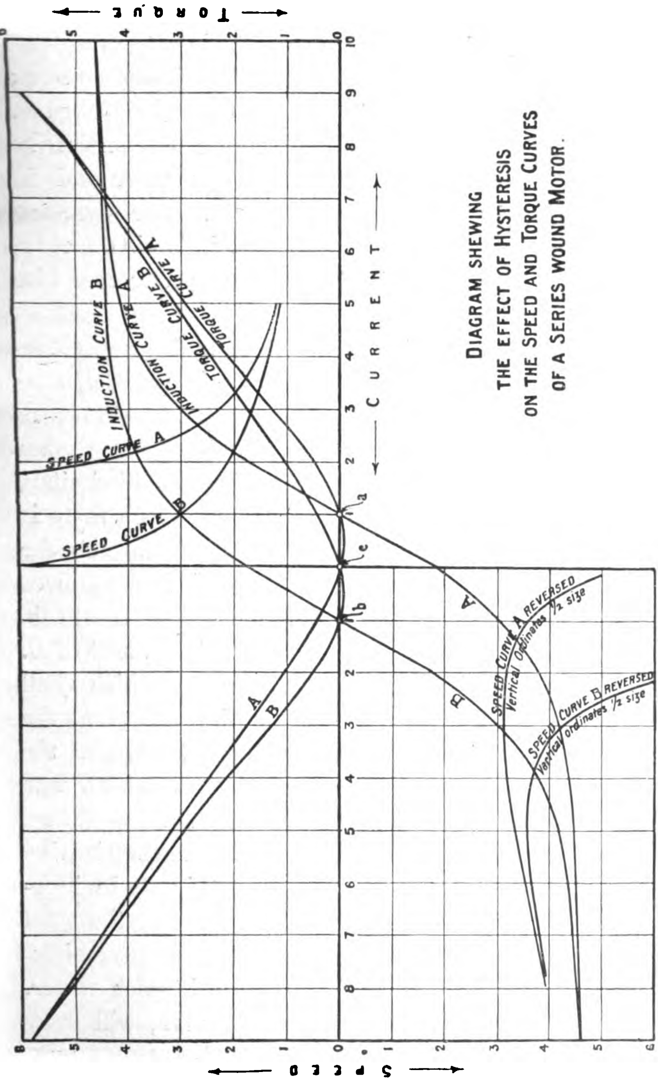


DIAGRAM SHEWING
THE EFFECT OF HYSTERESIS
ON THE SPEED AND TORQUE CURVES
OF A SERIES WOUND MOTOR.
TORQUE
SPEED
CURRENT
SPEED CURVE A
SPEED CURVE B
INDUCTION CURVE A
INDUCTION CURVE B
TORQUE CURVE A
TORQUE CURVE B
SPEED CURVE A REVERSED
Vertical Ordinates 1/2 size
SPEED CURVE B REVERSED
Vertical ordinates 1/2 size

FIG. 22

large currents can be obtained, the induced tension being made up on the residual magnetism. This is the method adopted in operating **electric brakes** for railway motors; in practice the induced tension seldom exceeds 10 volts. For the purpose of braking, the armature is completely disconnected from the line and from the magnets and is then short-circuited through a rheostat. The dynamo then acts as a generator with constant induction factor, and the retarding torque is given by the product of the induction factor and the current generated.

We shall now consider the action of **two series-wound motors** connected electrically in parallel, and **mechanically coupled** so that they both run at the same speed. The induction curves of the two motors are drawn in Fig. 23, and we shall suppose that they are dissimilar, but both pass through the origin, in other words, there is no residual magnetism in either motor. Construct the speed and torque curves as before. If the induction curve of A lies above that of B, the torque curve of A will lie above that of B, and the speed curve of A below that of B.

Construct the curve of combined speed, which shall represent the current from the line horizontally, and the speed of the combination vertically. Construct also the curve of combined torque, which shall give the total torque of the combination vertically, and the current drawn from the line horizontally. The speed of the combination is determined by the total load; setting this off in inch-pounds on the combined torque curve we find the total current, and thus obtain the speed from the combined speed curve; this gives us the current in each motor. Motor A, having the higher induction curve, will draw the smaller current for all loads. Unlike two shunt-

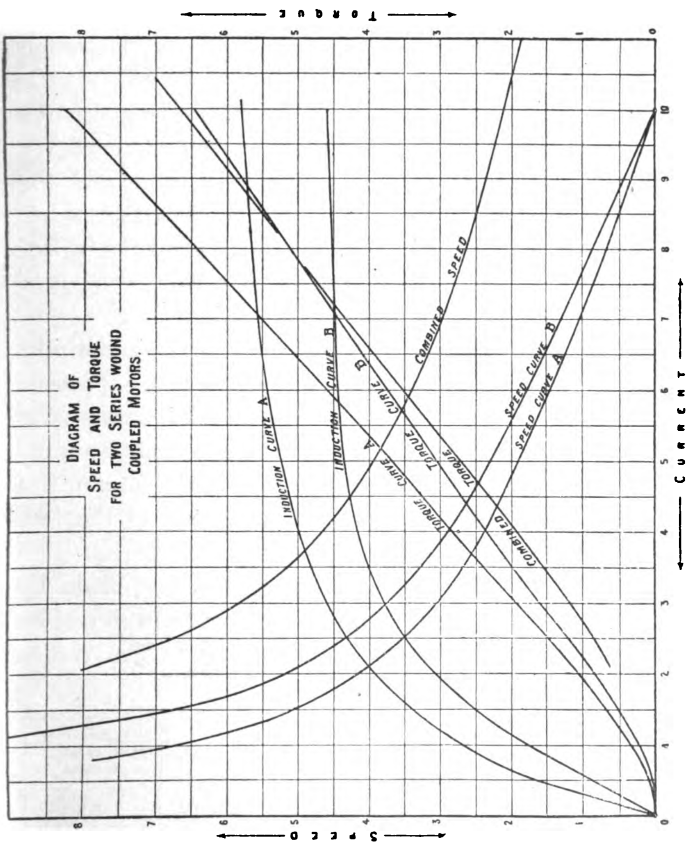


DIAGRAM OF
SPEED AND TORQUE
FOR TWO SERIES WOUND
COUPLED MOTORS.
TORQUE
SPEED
CURRENT
INDUCTION CURVE A
INDUCTION CURVE B
A
B
TORQUE CURVE
TORQUE CURVE
COMBINED TORQUE
COMBINED SPEED
SPEED CURVE A
SPEED CURVE B

FIG. 23

wound motors in parallel, **one of the motors will always be doing some of the work**, and cannot be made to generate current however light the load may be.

Since the motors are mechanically coupled, each will take its own share of the total load. For heavy loads the torque of *A* will be greater than that of *B*; the speeds being the same, the rates of working will be unequal. As the total load diminishes the torques become more nearly equal, until at a certain load they are the same, and for lighter loads the torque of *B* is greater than that of *A*. These results are shown in Fig. 24, which gives the difference of current, and hence of electrical horse-power, on a base of total current.

The difference between the two currents is nothing when the speed is nothing, if, as we have here supposed, the resistances are equal. As the speed increases the difference increases, and, in the case of the motors represented in the drawing, again decreases, but the form of the difference curve will depend upon the shapes of the two induction curves.

Fig. 25 gives the results of some tests made by Mr. H. S. Hering, on two 25 horse-power railway motors running a car under ordinary conditions. The values observed were as follows:

Line volts	Amperes, Motor No. 1	Amperes, Motor No. 2	Difference	Difference per cent. of total amperes
470	41·3	46·1	4·8	5·5
473	40·4	45·6	5·2	6·0
487	31·5	36·2	4·7	6·9
487	20·9	24·6	3·7	8·1
465	14·5	17·9	3·4	10·5

If this difference had been due to unequal resistances we should have found the difference increasing with the

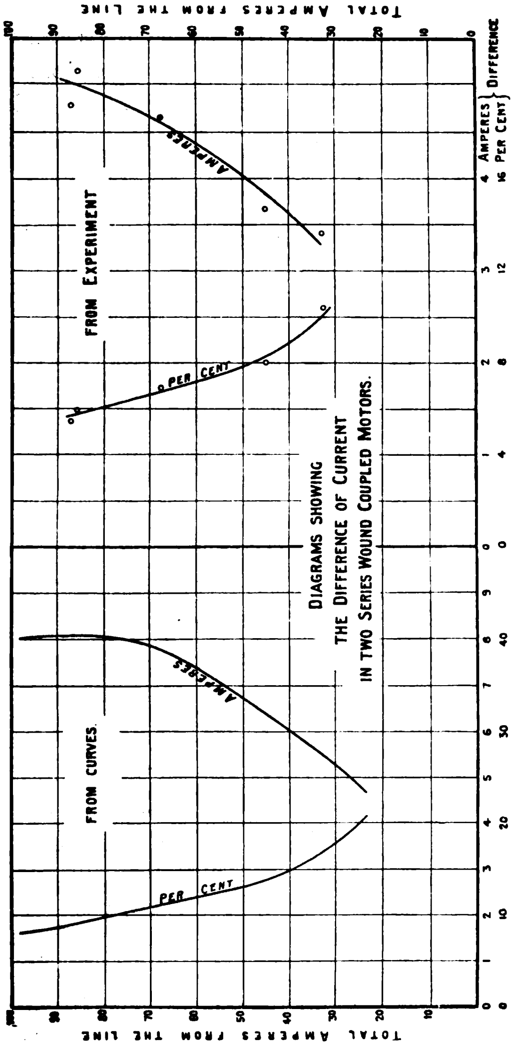

TOTAL AMPERES FROM THE LINE
FROM EXPERIMENT
AMPERES
PER CENT
FROM CURVES
AMPERES
PER CENT
DIAGRAMS SHOWING
THE DIFFERENCE OF CURRENT
IN TWO SERIES WOUND COUPLED MOTORS.
AMPERES } DIFFERENCE
PER CENT }
TOTAL AMPERES FROM THE LINE

FIG. 25

FIG. 24

total load, as may be seen by drawing the speed curves for two motors with equal induction curves but unequal resistances. If the wheels of the two motors were of unequal diameter due to wear, the motors would draw different currents, but not such as would account for the

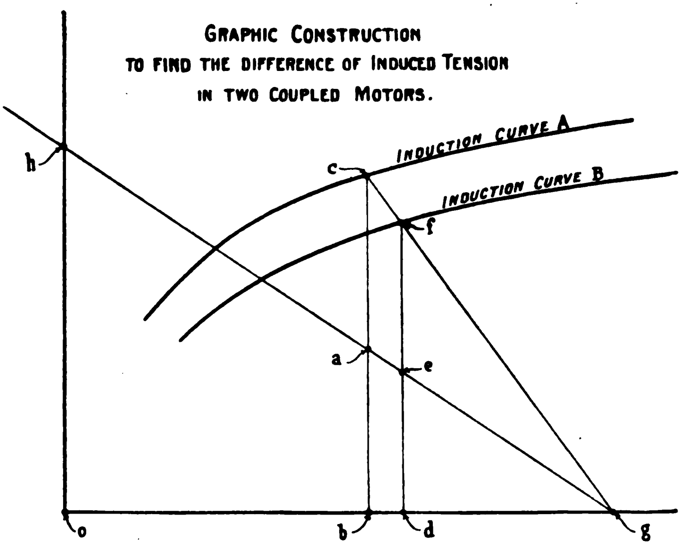


FIG. 26

results actually observed. The difference may with tolerable certainty be attributed to unequal induction curves.

When two series-wound motors are connected electrically in parallel, and coupled mechanically so as to run at the same speed, the induced tension in the two motors will be different if the induction curves are different.

Let A and B in Fig. 26 represent the two induction curves, and let *og* be the maximum possible current when the speed is nothing: from *g* draw any line *gfc* cutting the induction curves in *f* and *c*. Then *fd* and *cb* will represent the values of the induction factors in the two motors when *A* is drawing a current equal to *ob*, and *B* is drawing a current equal to *od*. Let *fd* and *cb* cut the loss line *hg* in *e* and *a*. Then, since *ab* and *ed* represent the induced tensions in the two motors, we see that the speed of *A* is *ab* divided by *bc*, and the speed of *B* is *de* divided by *df*, and these ratios are by construction equal to one another, hence the two motors are running at the same speed. We can then find the ratio of the induced tensions of two coupled series-wound motors in parallel, with unequal induction curves and equal resistances, by drawing lines from the point *g* cutting the curves at different angles, and the vertical ordinates at the points of intersection give the required ratio.

When two motors with unequal induction curves have no residual magnetisation, the difference between the induced tensions increases with the load and decreases with the speed, becoming nothing when the speed is infinite. If, however, there is residual magnetism in one or both of the motors, there is generally a certain speed for which the difference is greatest.

In Fig. 27, A B and A′B′ represent the magnets and armatures of two motors connected in parallel. If the points *a* and *c* are joined, we have conditions which may prove very troublesome. If the resistances of the magnet windings are equal in the two machines the currents in the magnets will be equal, so that whatever the motors are doing the current from the line will be equally divided between the two magnets. Suppose now that the induction curves given in Fig. 22 are those of the two motors, and that these represent an exaggerated

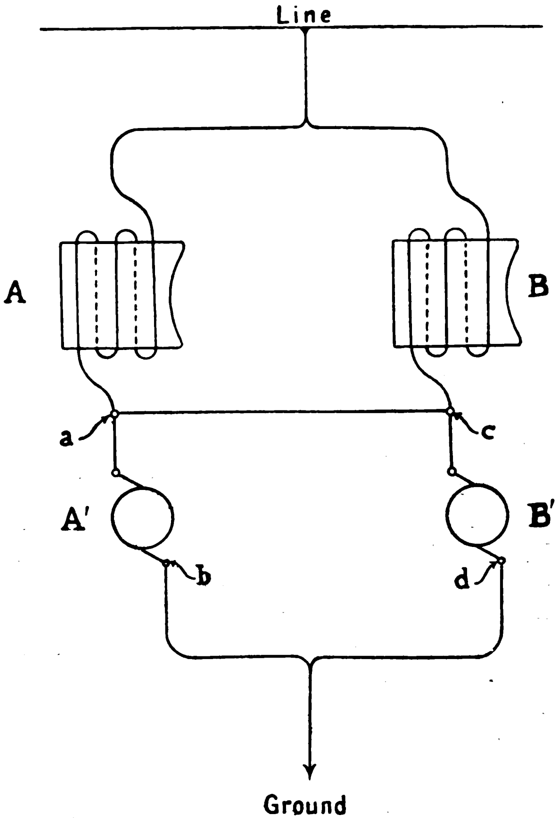


Fig. 27

difference due to hysteresis. We see that as the current in the magnets diminishes the difference between the induction factors increases. Now considering the two motors together, the current from the line will diminish as the speed increases, so that the difference between the induction factors will increase with the speed. If the induction factors were equal, the induced tensions would also be equal and the tension between *a* and *b* the same as that between *c* and *d*; if we increase the induction factor of one motor, say of *A*, and reduce that of the other, the tension between *a* and *b* is greater than that between *c* and *d*, since they are running at the same speed, and a current therefore flows from *a* to *c* and from *d* to *b*. For a certain difference in the induction factors *A* would be drawing no current and *B* would be doing all the work. Any further increase in the difference would cause *A* to generate current; and only so much more current would be drawn from the line as would be required to make up the increased heat losses.

If two motors driving a car are connected up in this way, the change in the relative values of the induction factors takes place automatically as the car speeds up. The result is that a large current circulates in the closed circuit *acdb*, quite independently of the work being done on the car. The difference in the induction curves may be very great, especially if the magnets are made of steel.

As soon as one motor begins to generate current the torque due to the current resists the rotation of the wheel to which that motor is geared, and when a certain speed is attained the wheel will be suddenly stopped.

When this difficulty is experienced it will be necessary either to alter the method of connecting the motors or to avoid the use of steel in the construction of the magnets.

We now come to the case where **two motors** are **connected electrically in series and** also **coupled mechanically** so that they both run at the same speed, each taking its own share of the total load.

The current is here the same in each motor, hence the heat-drop is the same if the resistances are equal, and since the speeds are the same the induced tensions will depend on the induction factors. If the induction curves do not coincide, the induced tensions will be proportional to the vertical ordinates of the two curves, and the line tension will be unequally divided between the two motors. In some experiments made by Mr. H. S. Hering the average difference in the terminal tensions of two street railway motors under test was six volts, the tensions being 242 and 248 volts for 41·8 amperes in each.

If the two induction curves coincide, the induced tensions will be equal, and the tension of the line will be equally divided between the two motors. Thus, when two similar motors are connected in series on a 500-volt line, the terminal tension will be 250 volts on each. If four motors are connected in series the terminal tension on each will be 125 volts. In railway practice two motors are frequently used together in this way and switched over from series to parallel, according to circumstances. It is often convenient to be able to represent graphically the result of this alteration in the connections. The change consists generally in doubling the terminal tension of the motors.

Fig 28. shows the curves of torque and current, both on a speed base, of the motor illustrated in Fig. 20. The curves represent the following conditions. (1) When the

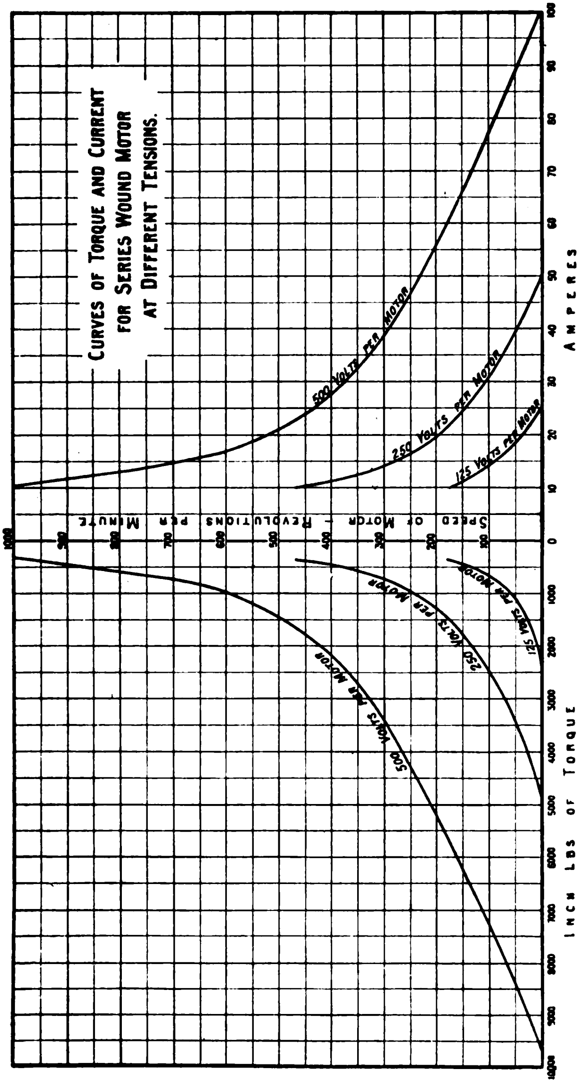

CURVES OF TORQUE AND CURRENT
FOR SERIES WOUND MOTOR
AT DIFFERENT TENSIONS.
500 VOLTS PER MOTOR
250 VOLTS PER MOTOR
125 VOLTS PER MOTOR
AMPERES
SPEED OF MOTOR - REVOLUTIONS PER MINUTE
INCH LBS OF TORQUE

FIG. 28

motor is placed by itself on the full tension of the line, 500 volts. (2) When the motor is in series with one other similar motor. (3) When the motor is in series with three other similar motors.

This **diagram** enables us **to find** at once **the speed for any given load with any given arrangement of motors**; the torque and current given is that per motor. Thus for a load of 3,000 inch-pounds per motor, each motor would run at 320 r.p.m. if in parallel; and at 70 r.p.m. if in series with a second motor. A car equipped with four motors with a load of 3,000 inch-pounds on each could not move if all four motors were thrown in series.

The curves in Fig. 28 for different tensions may be obtained graphically in the same way as the speed and torque curves were obtained in Fig. 20, or we may calculate the speeds for given current and given torque from formulæ. We know the induction factor for any given current, so that we can obtain the speeds for different values of E, the tension on the motor terminals.

The speed of a motor in series with one other similar motor is less than half the speed of the motor when placed direct on the full tension of the line by an amount depending on the load. The speed in parallel can be expressed thus:

$$n_p = \frac{E}{M} - \frac{cR}{M} \quad \text{....... ....(44).}$$

And the speed in series is

$$n_s = \frac{E}{2M} - \frac{cR}{M} \quad \text{... ....(45).}$$

So that we can put

$$n_s = \tfrac{1}{2} n_p - \tfrac{1}{2} \frac{cR}{M} \quad \text{.................(46).}$$

It is important to notice that these equations refer to conditions in which the load per motor remains the same, and when it is stated that the motor is connected in series with a second similar motor, it is assumed that the load on the first motor is unaltered, and that there is an equal load on the second motor.

This may be clearer if the equation is expressed in terms of the torque on the motor. Suppose that we have two motors. Let t be the torque in inch-pounds per motor, E the tension of the line, R the resistance of each motor. The induction factor corresponding to the torque t can be found from the induction curve, since for steady running, the torque determines the current and the induction factor; in other words, if the torque is given and we have the induction curve, we know what must be the induction factor of the motor when it is running at a uniform rate. Hence we can write

$$\text{Speed in parallel} = n_p = \frac{E}{M} - 0{\cdot}71\frac{Rt}{M^2} \quad \ldots\ldots(47).$$

also

$$\text{Speed in series} = n_s = \frac{E}{2M} - 0{\cdot}71\frac{Rt}{M^2} \quad \ldots\ldots(48).$$

We thus see that

$$n_s = \tfrac{1}{2}\left\{n_p - 0{\cdot}71\frac{Rt}{M^2}\right\} \quad \ldots\ldots\ldots\ldots\ldots(49).$$

so that the speed when the two motors are connected in series is less than half what it is when they are connected in parallel, by an amount depending on the load on each motor.

Generally, if there are x motors in series, the speed of each may be written

$$n = \frac{1}{x} n_p - \cdot 71 \frac{Rt}{M^2}\left(1 - \frac{1}{x}\right) \quad \text{...........(50)},$$

where n_p is the speed of each when placed alone on the full tension of the line, and t is the torque on each motor.

It may not perhaps at first sight be obvious why the induction factor remains the same when the motors are thrown in series. The reason is that **when the torque on the motor is constant**, the product of the current into the induction factor is constant, and independent of the tension on the motor terminals. Hence **the current per motor remains the same, whatever the arrangement of the motors may be.**

For instance, if the motors in the case quoted above were each taking 30 amperes when connected in parallel, they would each take 30 amperes when connected in series, the only difference would be a reduction in the speed and in the current drawn from the line. When the motors are in parallel the current from the line is 60 amperes, but when they are in series the current from the line is 30 amperes, since the current that passes through one motor also goes through the second motor. There is thus a saving of current by connecting the motors in series, but there is a corresponding reduction in the speed, and we see that the amount of this reduction is largely determined by the resistance of the motors. If the resistance were nothing the speed would be directly proportional to the tension on the motor terminals, and the speed in series would be exactly one half of that in parallel. With four motors in series the speed would then be one fourth of that of each motor in parallel, and so on.

It is possible that the term that has to be deducted on account of the load may be so large that when the motors are put in series they cannot run at all. Taking Equation 47 as giving the speed of a motor for any terminal tension E, we see that the speed will be nothing if $E = \cdot 71\frac{Rt}{M}$. Hence unless the tension on the motor when in series with a second motor is greater than $\cdot 71\frac{Rt}{M}$, the motors will not run when put in series. The limiting value of E may also be written

$$E = \cdot 71\frac{RTd}{2Mv} \quad \text{.....................(51).}$$

where T is the horizontal tractive effort, d is the diameter of the driving wheel in inches, and v the velocity ratio.

By the use of Equation 47 we can obtain curves giving the speed for different tensions with constant load. If we have the curves of total and useful torque and the induction curve of a motor, we can draw a set of curves that will give us a complete insight into the behaviour of the motor under different conditions.

Take the case of the G. E. 800 railway motor. The curves of torque and induction are given in Fig. 21; from these we can find the current required for any useful torque or tractive effort on wheels of given diameter, and also the corresponding value of M. Then by use of the equation, remembering that t is the total torque for each current, we can obtain two points on the speed line.

The results for this motor are plotted in Fig. 29. Vertical ordinates represent the tension at the terminals of the motor. Horizontal ordinates represent the speed

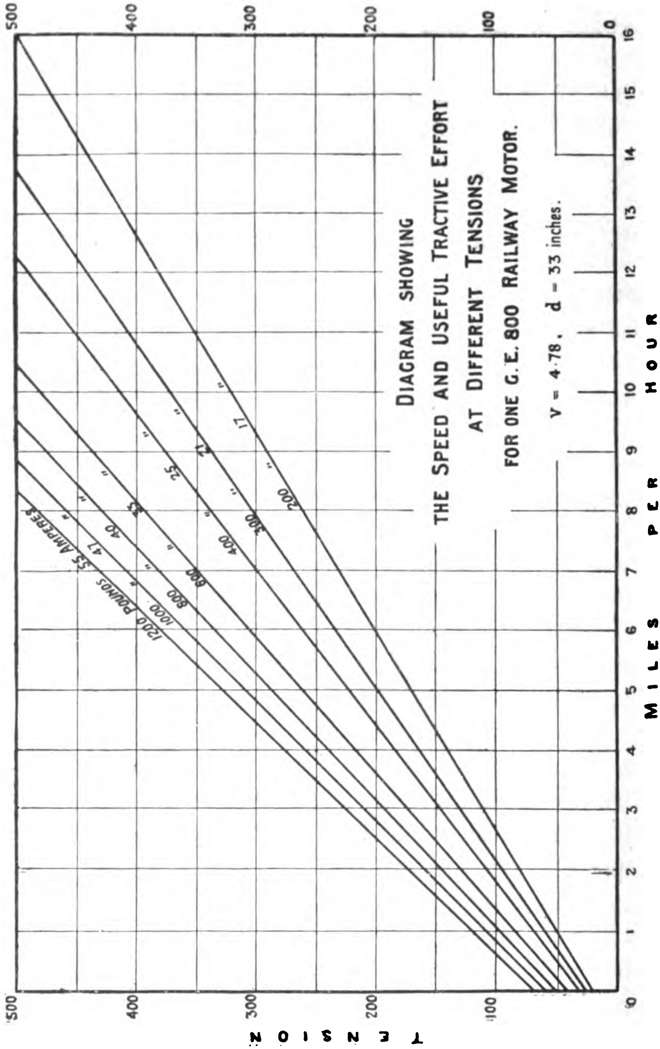


DIAGRAM SHOWING
THE SPEED AND USEFUL TRACTIVE EFFORT
AT DIFFERENT TENSIONS
FOR ONE G.E. 800 RAILWAY MOTOR.
V = 4·78, d = 33 inches.
MILES PER HOUR
TENSION
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
0 100 200 300 400 500
200 " 17 "
300 " 21 "
400 " 25 "
600 " 33 "
800 " 40 "
1000 " 47 "
1200 POUNDS 55 AMPERES

FIG. 29

of the car in miles per hour on 33-inch wheels. The intercept on the axis of tension is the term $\cdot 71\frac{RTd}{2Mr}$, giving the least tension that will move the car at the corresponding load. The tractive efforts specified in the figure are obtained from the curve of torque available for useful effort given in Fig. 21. The **diagram** enables us **to find** at once **the speed** of the motor **in miles an hour for any given load and terminal tension.**

Example 29.—A car is equipped with two of these motors, the tension of the line being 500 volts. If the load is 800 pounds of useful horizontal pull per motor, the speed, with the motors in parallel, will be 9·4 miles an hour. If the motors are now thrown in series the speed will be reduced to 4·2 miles per hour. If the car commences to ascend an incline of 3·5 per cent., the increased load per motor will be 1,200 pounds, and the speed up the incline will be 3·5 miles per hour. Suppose now that in consequence of the drop in the loads the tension of the line falls to 440 volts, the speed will fall to 2·9 miles per hour, the current from the line being 55 amperes, since the motors are in series. Should this speed be too low, the motors must be put in parallel. The terminal tension will then be 440 volts, the speed will rise to 7·2 miles per hour, while the current will increase to 110 amperes.

When **two motors**, A and B, are **connected in series** electrically, **but not coupled,** the load on the one will be mechanically independent of that on the other. The current is the same in each, hence for uniform motion in both motors the loads must be in the ratio of the induction factors. For if the load on one motor, say on A, is fixed, the current in A is fixed, for uniform motion; this current will be either larger or smaller than that required to balance the load of the other motor. In the first case, B would accelerate, the induced tension on B would increase, and the current would decrease. A would then be retarded, and finally stopped if the difference in the loads was sufficiently great. In the second case, B would be retarded and stopped.

If two similar motors driving a car are connected in series, the two induction factors are always the same. If one of them slips, the load on that motor is reduced, at the same time the mechanical connection previously secured by the adhesion of the rail is inoperative; the speed of the slipping motor will therefore increase, and with it the induced tension, hence the current will be decreased, and the torque assisting the motion of the non-slipping wheel, and therefore of the car, also decreased; if slipping continues, the car must stop. The **use of series-connected motors for railway work is therefore objectionable, unless the driving wheels can be coupled by connecting rods.**

If any number of motors are connected electrically in series, being independent mechanically, the condition of uniform running for all the motors is that the induction factor for each motor shall be proportional to its load. We have here the conditions under which it is possible to operate a number of **motors in series with a constant current,** each motor being mechanically independent of all the rest.

Let us suppose that we have several motors connected in series and a constant current passing in the circuit. Assume that one of the motors is at rest, the current being cut out by a short circuiting switch. When this motor is switched into the circuit it cannot move unless the torque produced by the current is greater than the load on the motor. If the induction factor can be increased by shunting some of the current through the magnets, any required acceleration can be obtained by adjusting the amount of current thus shunted. The motor will then speed up, and in doing so will diminish the current by its induced tension. If the generator is provided with a constant current regulator, this will not happen, and the current will be unaltered, the increased tension required to make up the terminal tension on the recently started motor being provided by the generator.

If the induction factor remained unaltered, and the load were constant, the motor would go on speeding up without limit. If, however, the motor is provided with a governor, the induction factor may be automatically reduced when any desired speed has been attained. If the governor is set for a certain speed, it will adjust the induction factor, say by shunting the magnets, so that if the speed is too high, the induction factor will be reduced, and the torque assisting the motion be made less than that due to the load; the motor will then be retarded. Similarly if the speed is too low, the induction factor may be increased.

If the load is reduced, the assisting torque due to the current will cause the motor to accelerate unless the governor reduces the induction factor. If the load is increased the governor will increase the induction factor, keeping the speed constant. If the load should increase beyond the ability of the governor to hold the speed, the motor will simply be retarded until it stops, without in any way affecting the other motors in the circuit. The motor may be set to run at any convenient speed: theoretically it will run equally well at any speed, since the terminal tension is not fixed.

The practicability of this system depends entirely upon the design of the automatic governor. Power transmissions on the constant-current principle are now being successfully worked with the aid of the Thury regulator, an account of which may be found in the 'Electrician' of March 19, 1897. This regulator will keep the speed variation within one-half per cent. from no load to full load. In Genoa, 1,400 horse-power is transmitted for a distance of twenty miles on this system.

Example 30.—At Chaux-de-Fonds, where the installation contemplates a tension of 14,400 volts being used over a distance of 30 miles, the current employed is 150 amperes. For a motor running at 600 r.p.m., with a load of 1,200 inch-pounds, M must be 56·8, and if $R=0{\cdot}003$, the terminal tension would be 57·2 volts, and the output would be 11·4 horse-power.

CHAPTER VI

EFFICIENCY

LET *af* in Fig. 30 represent the maximum current in a motor of R ohms resistance, and let *ab* represent the tension of the line to which it is connected. Complete the rectangle *afdb* and join *ad*. The vertical intercept between *af* and *ad* represents the watts taken from the line; this is nothing at *a*, and a maximum at *f*. Construct the curve of mechanical watts by taking the difference between the line watts and the heat watts, as explained in Chapter II.

The ratio of the mechanical watts to the line watts is the **efficiency of conversion**, and expresses the proportion of the electrical energy put into the motor that is converted into work. It is a maximum when the current is nothing, and is nothing when the current is a maximum. This is true for every form of continuous-current motor, whether shunt or series wound.

The mechanical watts is equal to cMn, where n is the revolutions per second, and M the induction factor. Since M is given by

$$M=\frac{E}{2n}\left\{1+\sqrt{\left(1-\frac{4Rw}{E^2}\right)}\right\} \quad \ldots\ \ldots\text{(52)}.$$

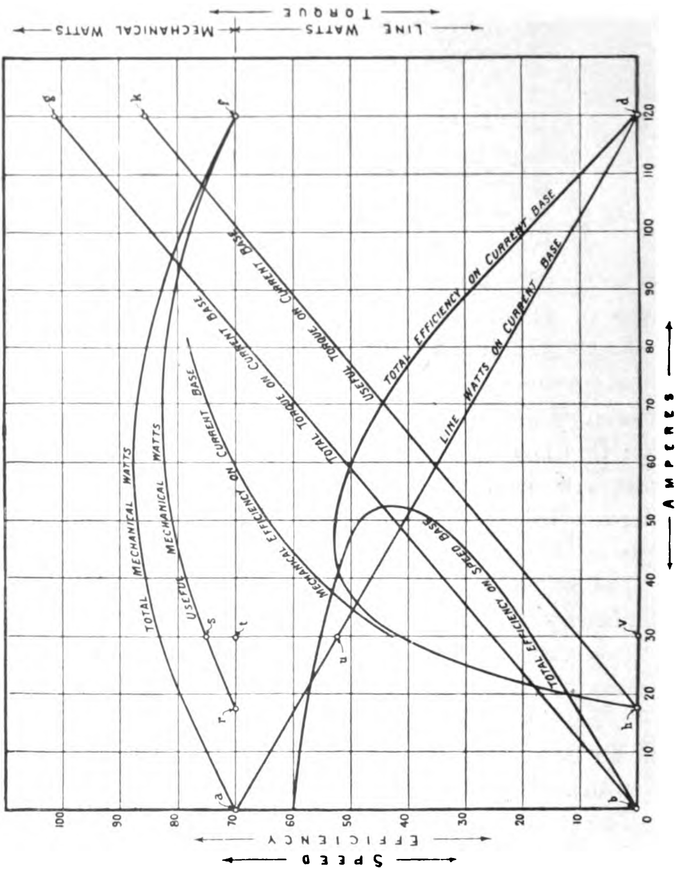

TORQUE
LINE WATTS
MECHANICAL WATTS
TOTAL MECHANICAL WATTS
USEFUL MECHANICAL WATTS
MECHANICAL EFFICIENCY ON CURRENT BASE
TOTAL TORQUE ON CURRENT BASE
USEFUL TORQUE ON CURRENT BASE
TOTAL EFFICIENCY ON CURRENT BASE
LINE WATTS ON CURRENT BASE
TOTAL EFFICIENCY ON SPEED BASE
EFFICIENCY
SPEED
AMPERES
a
b
d
f
g
k
h
r
s
t
u
v
0
10
20
30
40
50
60
70
80
90
100
110
120

Fig. 30

it follows that the efficiency of conversion can be expressed thus:

$$\eta_1 = \tfrac{1}{2}\left\{1 + \sqrt{\left(1 - \frac{4Rw}{E^2}\right)}\right\} \quad \text{.........(53).}$$

From this we see that for a given resistance and rate of working η_1 increases with the tension of the line. We may also write:

$$R = \frac{E^2}{w}(n_1 - n_1^2) \quad \text{...... ....(54).}$$

which shows that for a given efficiency and rate of working the resistance may increase as the square of the tension.

Example 31.—A railway motor has to be designed to drive a car at 20 miles an hour, with a total horizontal effort of 600 pounds. To find the resistance so that the efficiency of conversion shall be 80 per cent on a line of 500 volts tension. From Equation 54 we find that $R = 1{\cdot}67$ ohms. The result should be checked by showing that the current is 60 amperes, the line watts 30,000, and the heat watts 6,000, giving 24,000 for the mechanical watts, and an efficiency of 80 per cent.

Only a certain proportion of the total mechanical watts is available for useful effort. We shall denote by η_2 the ratio of the useful to the total mechanical watts, *i.e.* the **mechanical efficiency.** This gives us the ratio of the useful torque to the total torque for any current.

Let *bg* represent on any convenient scale the curve of total torque for different currents, *dg* being the maximum torque. Deduct from each ordinate of this curve the torque required to overcome all internal resistance to motion, such as friction, hysteresis, &c., and we shall get a second curve, *hk*, giving us the useful torque. In the figure we have assumed that the torque loss is constant, and we have taken it abnormally large in order to make the diagram clearer.

The mechanical efficiency is the ratio of the ordinates of the curves of useful and total torque. This efficiency is nothing at the point *h*, and increases with the total torque. If we divide the ordinates of the curve of mechanical watts in the ratio of the useful and total torques for each current, we shall get the curve of useful mechanical watts. We can also construct a curve of mechanical efficiency.

We have to make an arbitrary distinction in deciding what are to be counted as internal losses. In an ordinary belt connected motor it is obvious that these should include friction of bearings and brushes, hysteresis and eddy current losses, but when the motor is fitted with gearing as most railway motors are, it is not so easy to draw the line and say what losses are to be counted as internal losses. If the motor and its gearing form part of a self-contained equipment, the gear friction is usually counted in with the internal losses.

We have also assumed that the torque increases with the current, and have drawn the curve of total torque a straight line, as in a motor with constant induction factor. But all that follows is equally true for a series-wound motor, since the curves of heat and line watts are not affected by the variation of the induction factor.

The mechanical efficiency at different currents or loads may be found by the following method. Determine by a test the curve of useful torque, and also the speed curve. Deduce the induction curve from the latter, and from the induction curve get the curve of total torque, using the equation $t = 1{\cdot}41cM$. **The mechanical efficiency** is then **the ratio of the ordinates of the curves of useful and total torque.**

These curves have been given in Fig. 21 for the G. E. 800 railway motor. The curves of speed and useful torque are from actual tests, the curve of total torque is obtained from the induction curve, which is itself derived from the speed curve. We see that the mechanical efficiency at 40 amperes is 83·5 per cent., and at 20 amperes it is nearly 86 per cent.

The total torque at 40 amperes is 3,320 inch-pounds since M is 59, and the measured useful torque is 2,780, showing that the torque lost internally amounts to 540 inch-pounds. On 33-inch wheels with a velocity ratio of 4·78 the useful tractive for this current would be 800 pounds; this figure gives rise to the name of the motor. We cannot at present say how the torque loss is distributed; this must be determined by a separate test. We see from the curves that in this motor the torque loss increases with the total load.

In Fig. 31 similar curves are given for the motors used on the Baltimore and Ohio Railroad. In this case there is no gearing, so that the mechanical efficiency is much higher, being about 95 per cent. for 800 amperes. The torque loss increases with the load, but not to such an extent as with the geared motor.

The **torque loss in a gearless motor** may be divided into three parts as follows:—

(1) Friction.

(2) Eddy currents.

(3) Hysteresis.

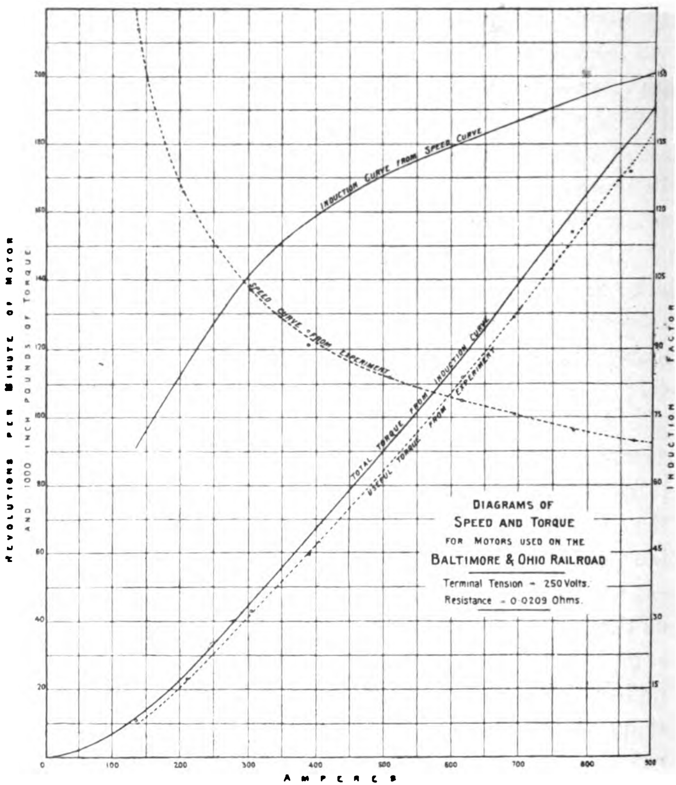

DIAGRAMS OF
SPEED AND TORQUE
FOR MOTORS USED ON THE
BALTIMORE & OHIO RAILROAD
Terminal Tension = 250 Volts.
Resistance = 0·0209 Ohms.
INDUCTION CURVE FROM SPEED CURVE
SPEED CURVE FROM EXPERIMENT
TOTAL TORQUE FROM INDUCTION CURVE
USEFUL TORQUE FROM EXPERIMENT
REVOLUTIONS PER MINUTE OF MOTOR
AND 1000 INCH POUNDS OF TORQUE
INDUCTION FACTOR
AMPERES
0
100
200
300
400
500
600
700
800
900
20
40
60
80
100
120
140
160
180
200
15
30
45
60
75
90
105
120
135
150

FIG. 31

The **friction** is made up of the friction of the brushes, of the bearings, and of the air. The second of these is the most important, and if there is no side thrust due to a belt or uneven pull of the magnets, will depend upon the weight of the armature.

It may not at first sight be obvious how the existence of **eddy currents** produces a torque tending to oppose the motion. But a little consideration will show that currents induced in any part of the armature will have the same effect as currents induced in the surface conductors, and will tend to stop the motion. We may imagine all the eddy currents to be concentrated in an imaginary conductor short circuited on itself and not connected to the surface conductors proper. The current in this conductor is wasted, but it will exert a torque tending to stop the motion, when there may be no current passing in the surface conductors. If m is the induction factor of the waste circuit, and r its resistance, the torque is given by $t=1{\cdot}41\frac{m^2n}{r}$, where n is the number of revolutions per second. Hence the eddy current torque varies as the speed, and therefore the watt loss varies as the square of the speed, m being constant.

A convenient way of separating the different torque losses is to run the machine by a second dynamo acting as a motor. Observe first the current required to run the motor alone, then couple on the dynamo to be tested and run it without excitation. The increased current in the motor will give the friction in the tested dynamo. Now excite the magnets; the increased motor current will then enable us to find the increased torque required to turn the armature of the tested dynamo due to hysteresis and eddy current torque.

A test of a 14 horse-power motor gave the following results :—Total torque loss at 1,500 r.p.m., 37 inch-pounds. Friction, 14 inch-pounds; hysteresis, 7 inch-pounds; whence we deduce 16 inch-pounds as the eddy current loss. This motor took 5·2 amperes when running at no load on a line of 125 volts tension. The value of M was 5.

Fig. 32 gives the losses in a 15 horse-power motor at different loads. The machine is a four pole belt type Crocker-Wheeler motor, shunt wound, and designed to run at 800 r.p.m. on a 230-volt circuit. The C^2R losses in the magnet winding given in this diagram are included in the total losses.

The **hysteresis** loss may be found by turning the armature slowly with a spring balance attached to a lever, and deducting for friction.

If the energy required to turn one cubic centimetre of iron through one complete cycle, in a magnetic field of given strength, is h ergs, the energy expended in rotating an armature containing v cubic centimetres, through one revolution, is $h\ s\ v$, if s is the number of complete cycles per revolution. If the armature be turned through n revolutions per second, the energy expended per second, that is, the rate of working, is $hsvn$, and the watts is equal to $hsvn\ 10^{-7}$. Tables of h for different values of B are given in most text-books on magnetism. If t is the torque in inch-pounds, $w = {\cdot}71\ t\ n$, hence we get :—

$$t = 1{\cdot}41 hsv 10^{-7} \quad \ldots\ldots\ldots\ldots\ldots (55).$$

Example 32.—A 10 pole dynamo has 621×10^3 cubic centimetres of iron in the armature. The maximum

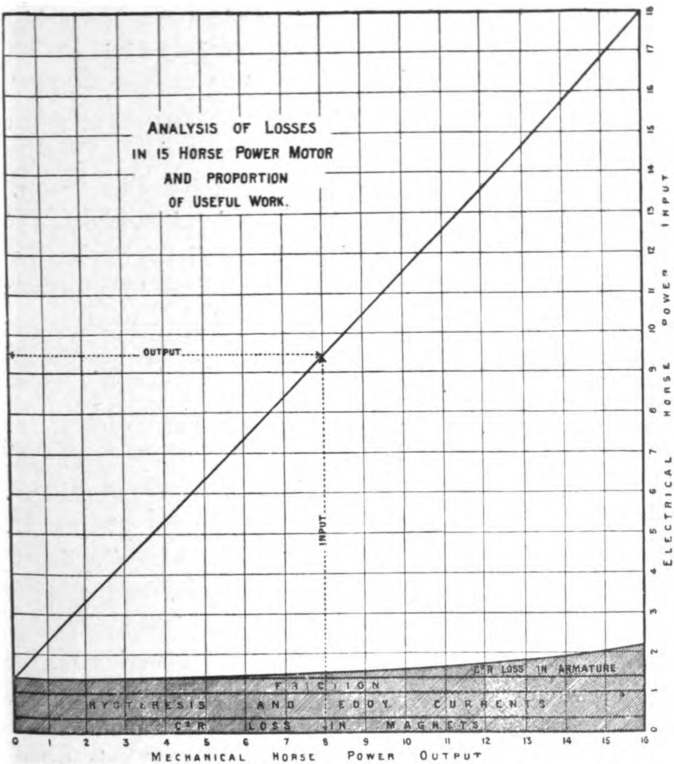

ANALYSIS OF LOSSES
IN 15 HORSE POWER MOTOR
AND PROPORTION
OF USEFUL WORK.
OUTPUT
INPUT
C²R LOSS IN ARMATURE
FRICTION
HYSTERESIS AND EDDY CURRENTS
C²R LOSS IN MAGNETS
MECHANICAL HORSE POWER OUTPUT
ELECTRICAL HORSE POWER INPUT

FIG. 32

intensity of magnetisation is 14,000 lines per square centimetre. Find the torque required to turn the armature when the magnets are excited. Suppose that from our knowledge of the quality of the iron used, we take h to be 9,000. The iron in the armature of a ten pole machine makes 5 complete cycles per revolution, giving $s=5$. Hence the required torque is 3,940 inch-pounds, and this must be continually exerted in order to turn the armature against the retarding torque due to hysteresis. The dynamo is an 800 k.w. Westinghouse Railway Generator.

When a motor is placed on a circuit without any external load, and allowed to run by itself, the load on the motor is the sum of all the retarding torques that we have been considering. Hence, if we observe the current and know the value of M, we can find the torque losses.

The point of intersection of the curve of useful torque and the axis of current gives us the value of the current the motor will take when running with no external load. Thus, in Fig. 30, this current is given by the intercept bh.

Example 33.—A motor when placed on a line of 100 volts tension runs at 1,240 r.p.m. and takes a current of 4 amperes. If the resistance is 0·1 ohm, find the torque losses. From the usual equation for the speed, we find the value of M to be 4·82, and since the current is 4 amperes, the torque is 27 inch-pounds. As the external load is nothing, the whole of this torque is expended in friction, hysteresis and eddy current loss.

The ratio of the useful mechanical watt output to the total electrical watt input is the **total efficiency** of the motor. If this ratio is plotted on the axis of the current, it will start from the point h, in Fig. 30; and rise to a maximum and fall again to nothing at the point d. The curve of total efficiency has also been plotted in the same figure on a speed base.

If η_3 is the total efficiency and t_f the torque loss we have :—

$$\eta_3 = \frac{\cdot 71\ \eta(t - t_f)}{cE} \quad \ldots\ldots\ldots\ldots\ldots (56).$$

Assuming that t_f is constant at all loads, and that M is constant, this becomes

$$\eta_3 = 1 - \frac{c_f}{c} - \frac{cR}{E} + \frac{Rc}{E} \quad \ldots\ldots\ldots\ldots (57).$$

where c_f is the current at no load. Differentiating with respect to c, and equating to nothing, we find that η_3 is a maximum when

$$c^2 = c_f c_x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (58).$$

where $c_x = \frac{E}{R}$. Inserting this value of c in Equation 57, we find that the maximum efficiency is

$$1 + \frac{c_f}{c_x} - \frac{2}{c_x}\sqrt{c_f c_x} \quad \ldots\ldots\ldots\ldots (59).$$

Example 34.—A motor with an internal resistance of 0·1 ohm takes 4 amperes when running on a line of 125 volts tension. The current for maximum efficiency is 70·7 amperes, the power is 8·35 k.w., and the total efficiency is 89 per cent.

The limiting value of c_f is $\frac{E}{R}$ since the current c_f cannot be greater than that given by $\frac{E}{R}$. When c_f has this value, the efficiency is nothing, since $\frac{Rc_f}{E}$ is unity. Hence we see that $2\sqrt{\frac{Rc_f}{E}}$ must be greater than $\frac{Rc_f}{E}$ and $\frac{Rc_f}{E}$ must be less than unity.

The expression 'total electrical watt input' does not here include the power used in exciting the magnets. In a shunt-wound machine this can be made anything we please by adjusting the number of turns. To find the maximum efficiency including the C^2R loss in the magnets, we must add the magnetising current c_m to the current from the line in Equation 56, and we shall find that the current for greatest efficiency is then given by

$$c_1^2 = (c_m + c_f)(c_m + c_x) \text{............} (60).$$

where c_1 is the whole current from the line including the magnetising current, and $c_x = \frac{E}{R}$. The maximum efficiency is then:

$$1 + \frac{c_f}{c_x} + 2\frac{c_m}{c} - \frac{2}{c_x}\sqrt{(c_f + c_m)(c_x + c_m)} \text{.........} (61).$$

Thus, if in Example 34 the magnetising current c_m is 2 amperes, the maximum efficiency is 86·8 and the current for this efficiency 86·6 amperes.

If M is kept constant and the tension of the line altered, the maximum efficiency and the most efficient current will increase with the tension.

Example 35.—If the motor in Example 34 be placed on a line of 250 volts tension, the torque losses and the resistance remaining the same, the maximum efficiency will be 91 per cent., and the corresponding current 100 amperes. Compare also Fig. 35.

If the torque loss is constant the current c_f required to make up the loss will vary inversely as the induction factor. Hence the higher the value of M the greater will be the efficiency and the smaller the current at which the efficiency is a maximum.

The larger we make M the slower the motor will run on a line of given tension, and the greater will be the torque for a given current. Since $w = ce$ the power at maximum efficiency will be nearly proportional to the current at maximum efficiency; hence, by increasing the induction factor we can increase the maximum efficiency, but in so doing we decrease the power at that efficiency.

Example 36.—If in Example 34 the induction factor is doubled, the torque loss remaining unaltered, the maximum efficiency will be increased to 92 per cent., the current for this efficiency will be 50 amperes, and the power 6·0 kilowatts.

The preceding equations assume that the induction factor remains constant. If M varies, as **in series-wound motors**, we may still assume that the torque losses remain the same at all loads, but we can no longer assume that c_f is always a measure of those losses. In a series-wound motor running without external load, the value of M is small, and hence the current c_f is large. As the load increases, the proportion of the total current required to make up the torque loss becomes smaller. Our equations assume that for all currents the torque loss is represented by c_f amperes. If then M is greater for any current than it was when the motor was running without external load, we must reduce c_f in our equations in the ratio of the value of M at this load to its value at no load.

Let us now compare the **efficiency curves of two motors** A and B **which have the same internal resistance and work on the same tension, but have different induction curves.** Suppose that for equal currents the values of M for A are greater than those for B.

If the torque losses are the same in both motors we notice first that the curve of total efficiency for A cuts the axis of current nearer to the origin than does that of motor B, since for the same torque the motor with the greater induction factor takes the smaller current. A will thus give a higher efficiency with light loads than B.

Looking now at the equation for the greatest efficiency, we see that for any current c_f is greater for B than for A, and $2\sqrt{\frac{Rc_f}{E}}$ must be greater than $\frac{Rc_f}{E}$, hence the maximum efficiency for B must be less than that for A. It follows that the motor with the higher induction curve has the greater maximum efficiency, and the greater mechanical efficiency at all loads.

Again, from Equation 58 we see that the current when the efficiency is highest increases with c_f. Hence the point of maximum efficiency for the motor with the lower induction curve will be shifted along the current axis, and the maximum efficiency for the motor with the higher induction curve will be reached at a smaller load than for the motor with the lower induction curve.

In Fig. 23 speed and torque curves were given for two motors of equal resistance, but with different induction curves. When the torque is large A runs quicker than

B, but when the torque is small A runs slower than B. Now the load on a railway motor when once started is comparatively light, so that we may take it that motors designed for this class of work will generally run slower in proportion as the induction curve is higher. In other words, for equal loads, wheel diameters, and velocity ratios, a motor must

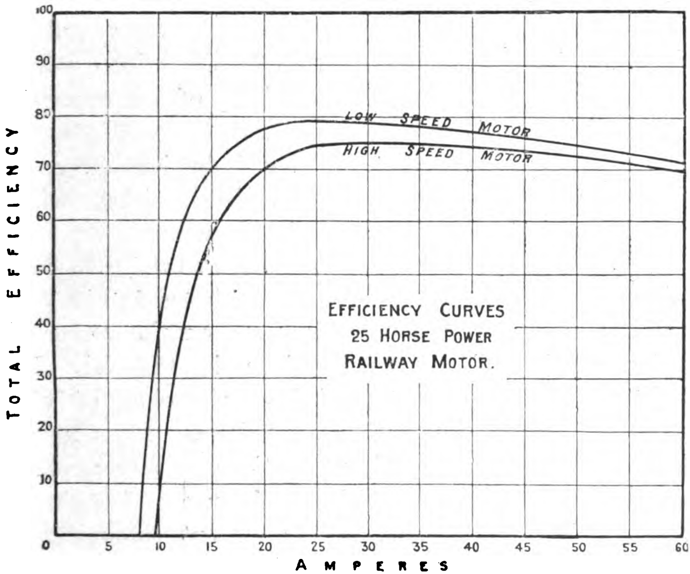


FIG. 33

have a low induction curve to run fast at light loads, and a high induction curve to run slowly. Hence the low-speed motor will have a higher efficiency than the high-speed motor. We must, however, remember that the terms 'high speed' and 'low speed' refer to the behaviour of the motors at comparatively light loads; the conditions may be reversed with heavy loads.

The efficiency curves for a motor with two magnet windings are given in Fig. 33; one is for a high induction curve and the other for a low induction curve. The difference due to the two windings is well shown.

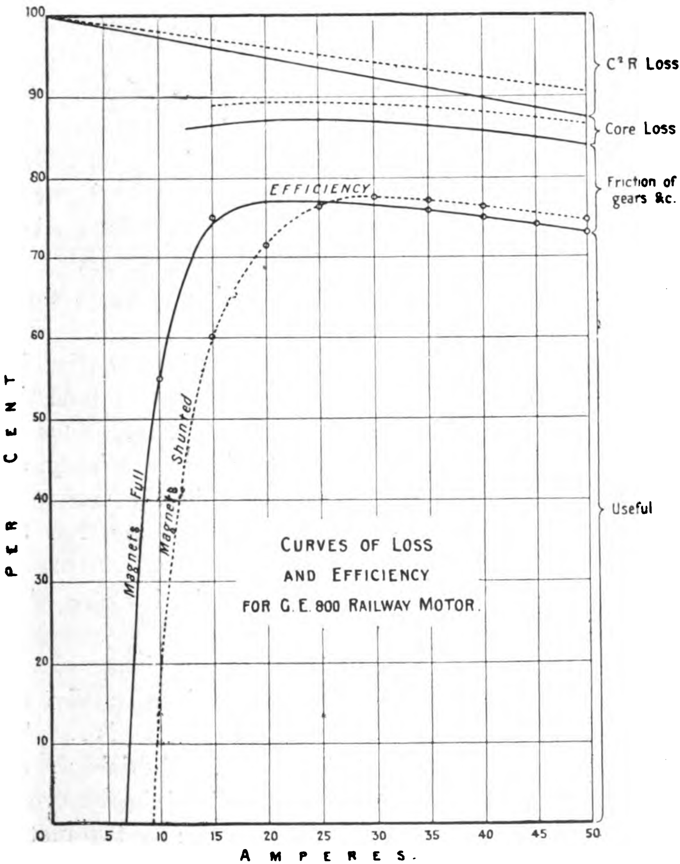
If the resistances of two motors are not equal, the advantage gained by the higher induction curve may be lost by the increase in the resistance.

Example 37.—A motor with a resistance of 1·245 ohms, running on a line of 500 volts tension, with $c_f = 2·5$ amperes, has a maximum total efficiency of 85 per cent. If the resistance is reduced to 1 ohm, the efficiency is increased to 86 per cent. If the induction curve is lowered, so that c_f is 3 amperes, the efficiency, with the reduced resistance, will be again 85 per cent.

This is illustrated by some tests made on the G.E. 800 railway motor, of which the results are plotted in Fig. 34. The curves marked 'magnets full' are for the motor of which the induction curve is given in Fig. 21, the resistance of the armature being 0·44 ohm, of the magnets 0·805 ohm—total, 1·245 ohms. The curves marked 'magnets shunted' are for the same motor, when the magnets are shunted with a resistance of 1·8 ohms, thus sending 30 per cent. of the current through the shunt. The tension of the line is 500 volts in each case.

We see from the curves that the maximum efficiency of the shunted motor is higher than for the unshunted motor, in consequence of the reduced resistance of the former; if the resistances had been the same, the shunted motor would have had the smaller maximum efficiency.

Example 38.—Taking the resistance of the G.E. 800 motor as 1·245 ohms for the unshunted magnets, and



100
90
80
70
60
50
40
30
20
10
0
5
10
15
20
25
30
35
40
45
50
PER CENT
AMPERES.
C² R Loss
Core Loss
Friction of gears &c.
Useful
EFFICIENCY
Magnets Full
Magnets Shunted
CURVES OF LOSS
AND EFFICIENCY
FOR G. E. 800 RAILWAY MOTOR.

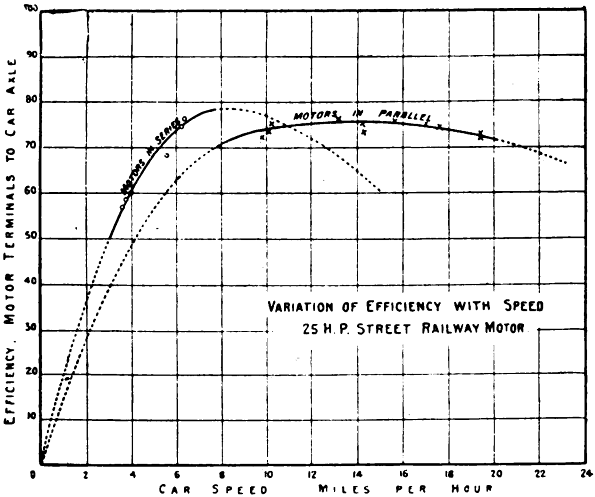
FIG. 34

c_f as 5·5 amperes. we find the maximum efficiency on 500 volts to be 76·7 per cent. If the magnets are now shunted, giving $R=1·0$ ohm, and $c_f=6·5$ amperes, we find the maximum efficiency to be 76·7 per cent. as before.

We must not forget that we are here assuming that the torque loss remains constant at all loads. From the torque curves in Fig. 21 we see that this is not strictly true. The torque loss actually increases with the total load. This will prevent our applying the equations strictly in such cases, but they may still serve as a general guide in determining the form of the efficiency curves under different conditions.

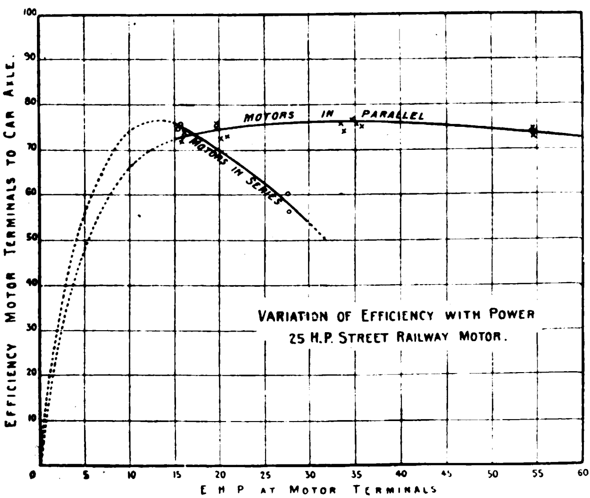
We have also assumed that the torque is proportional to the current; this is so in a motor with constant induction factor, but not in a series-wound motor. Thus in Example 37, the currents for maximum efficiency are by Equation 58, 47 and 57 amperes for the unshunted and the shunted motor respectively. We see from the curves in Fig. 33 that the currents are actually 25 and 33 amperes, the difference being due to the increase of M with the current.

Curves showing the variation in the efficiency of a 25 h.p. Westinghouse railway motor are given in Fig. 35. One curve shows the efficiency when the motor is running in parallel with a second similar motor, and driving a car weighing seven tons, the tension of the line being 500 volts. The other curve shows the efficiency when the two motors are connected in series on the same tension. Equation 58 shows that the series curve has been carried too high in the figure. The diagram is obtained from tests made by Mr. H. S. Hering.



EFFICIENCY. MOTOR TERMINALS TO CAR AXLE
100
90
80
70
60
50
40
30
20
10
0
2
4
6
8
10
12
14
16
18
20
22
24
CAR SPEED MILES PER HOUR
MOTORS IN SERIES
MOTORS IN PARALLEL
VARIATION OF EFFICIENCY WITH SPEED
25 H.P. STREET RAILWAY MOTOR

Fig. 35



EFFICIENCY MOTOR TERMINALS TO CAR AXLE.
100
90
80
70
60
50
40
30
20
10
0
5
10
15
20
25
30
35
40
45
50
55
60
E H P AT MOTOR TERMINALS
MOTORS IN PARALLEL
MOTORS IN SERIES
VARIATION OF EFFICIENCY WITH POWER
25 H.P. STREET RAILWAY MOTOR.

Fig. 36

The same results are plotted in Fig. 36, on a base of miles per hour. The maximum efficiency does not appear to be very different in the two cases, though the precise form of the whole curve is difficult to determine.

We have seen that when the speed of a car in feet per second, the tension of the line and the tractive effort are given, the ratio $\frac{Mv}{d}$ is fixed; we are, however, at liberty to take any values of M, v and d so long as this ratio is maintained. Since $c_f = \cdot 71 \frac{t_f}{Mv}$, we cannot reduce the value of c_f by simply increasing v, for in proportion as we increase v we must decrease M in order to get the proper speed. We can, however, decrease c_f by increasing Mv and d at the same time. For an increase of Mv will diminish c_f, and will also cause the car to run slower; we may then increase d and bring the speed up to that specified, without thereby increasing c_f, the useful current $c = \cdot 71 \frac{Td}{Mv}$ being unaltered. Hence in comparing the performance of two motors with different induction curves, if we assume that the wheel diameter remains the same in both, a car equipped with motors having high induction curves will run slower than one with lower induction curves, but with better efficiency. But if we are at liberty to alter the diameter of the wheel, we can increase the speed of the former, and make it run at the same speed as the latter, but at the same time with a higher efficiency. In practice, the wheel diameter is usually fixed for a given class of service; thus 33 inches is universally used for street railway work. When this is the case, and the velocity ratios are equal, the car equipped with motors having the higher induction curve will run the slower of the two.

Example 39.—The weight of the car of a passenger lift was balanced until the motor took the same current ascending and descending. The current at full speed was then 20 amperes, the tension at the terminals of the motor 120 volts, and the resistance 0·1 ohm. This current represents the torque loss at no load. A load of 1,600 pounds was now placed in the car, and lifted at the rate of 126 feet per minute, the motor then taking 73 amperes. The mechanical output is 4,560 watts, and the corresponding electrical input 8,760 watts, giving a total efficiency of 52 per cent. If we calculate the efficiency by Equation 57, taking $C=20$, we get an efficiency of 68 per cent., showing that the torque loss increases with the load. By inserting in Equation 57 the efficiency as found by experiment, we see that c_f is 32 amperes, showing that the torque loss increases by rather more than 50 per cent. from no load to full load. In this case the velocity ratio was 70, the induction factor 4·6, and the diameter of the rope drum 24 inches. If the induction factor and the diameter of the rope drum are both doubled, we get the same car speed on the same tension, but with increased efficiency. If at the same time we reduce the resistance, say to 0·05 ohm, we increase the efficiency still more. The value of c_f is reduced to 16 amperes, and the efficiency increased to 75 per cent.

The following **method for determining the efficiency of series-wound motors** at different loads will be found useful.

Two similar motors are geared together, the pinions of each meshing into one and the same intermediate gear wheel. One machine is run as a motor, and a load is put on the other acting as a generator, by means of a water or other rheostat. The two magnet circuits are in series, so that the same current passes in both, and the induction factors are equal. The full rated tension is maintained on the motor terminals.

The motors are first heated up to the normal working temperature, and the resistances taken; the test is then commenced at once. The readings observed are the terminal tension and current of motor and generator.

The difference between the line watts and the motor heat watts is equal to the mechanical watts in the motor. The sum of the electrical watt output of the generator and its heat watts is equal to the mechanical watts in the generator. Hence the total mechanical loss is the difference between the two mechanical watts, and assuming that the losses are equally distributed in the two machines, we get finally the torque loss in watts by halving this quantity.

If c_1 is the current in the motor, and c_2 that in the generator, E the terminal tension of the motor, and R the resistance in the motor circuit, w_t the losses of the two machines in watts, and t the torque losses in inch-pounds, then $1.41c_1M = 1{\cdot}41c_2M + t_l$ and $Mn = E - c_1R$, so that we have

$$w_t = (c_1 - c_2)(E - c_1R) \quad \ldots\ldots\ldots\ldots\ldots\ldots (62).$$

We can thus find the torque losses simply by observing c_1, c_2 and E, and it is not necessary to observe the terminal tension on the generator.

Example 40.—A railway motor when tested in this manner gave the following results. Motor terminal tension 500 volts, current 35 amperes, total resistance in motor circuit 1·245 ohms, resistance of motor armature and magnets 0·91 ohm, generator current 23·5 amperes.

Using Equation 62, we find at once that the torque losses in the two machines amount to 5,240 watts, or 2,620 watts each. The C^2R loss in the motor is 1,112 watts. The electrical input of the motor is 17,500, leaving 13,360 watts as useful mechanical output. We can now construct a table of energy expended in the motor.

	Watts	Per cent.
C^2R. loss	1,112	6·5
Torque loss	2,620	15·3
Mechanical output . . .	13,360	78·2
	17,092	100·0

The total efficiency of the motor is thus 78·2 per cent.

The results of a complete test of a G.E. 800 railway motor by this method are given in Fig. 34. The losses are shown in percentages of the whole input for different currents. The torque losses are divided into core loss, including hysteresis and eddy currents, and friction of gears, by a separate experiment, in which the motor armature was driven by a small motor on open circuit with separately excited magnets.

CHAPTER VII

ACCELERATION

LET the vertical and horizontal ordinates in Fig. 37 measure respectively torque in inch-pounds and revolutions per second. Let the curve *A* represent on the given scales the total torque of a motor at different speeds. At *o* where the torque is a maximum the speed is nothing. Let the curve *B* represent on the same scales the torque required to overcome all resistance to motion due to the load, friction of the gearing, hysteresis, &c. At the point *h* where the curves *A* and *B* intercept, draw a vertical line cutting the speed base in *g*. The speed at which the motor will run when uniform motion has been attained will then be given by *og*.

For if a curve *C* be constructed, whose vertical ordinates are the difference between those of curves *A* and *B*, the ordinate of the curve *C* gives the **torque available for acceleration** at any speed, being the difference between the total torque and that required to overcome resistance to motion. At *g* this difference is nothing, so that at this point the motor has ceased to accelerate, and is therefore running uniformly. The form of the curve *B* is determined mainly by the variation of the friction with the speed, since this constitutes the greater proportion of

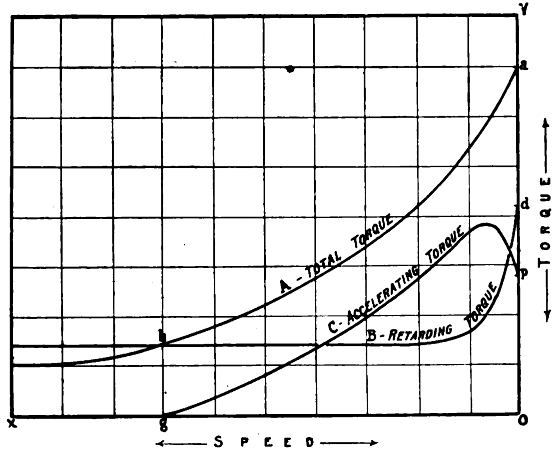

Y
a
d
p
A - TOTAL TORQUE
C - ACCELERATING TORQUE
B - RETARDING TORQUE
b
x
g
O
TORQUE
SPEED

FIG. 37

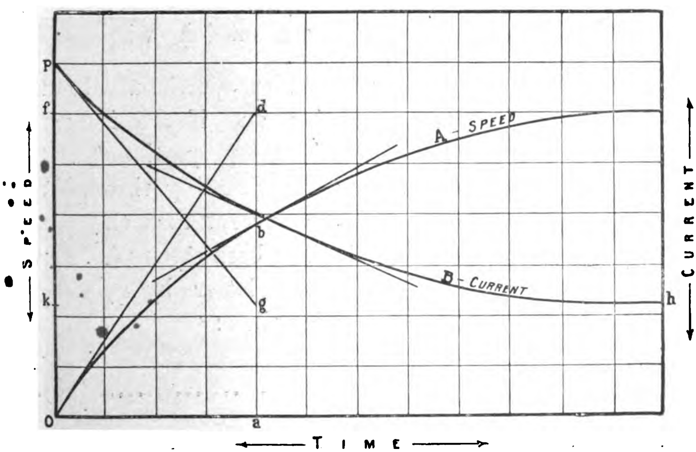

p
f
d
k
b
g
h
O
a
A - SPEED
B - CURRENT
SPEED
CURRENT
TIME

FIG. 38

the whole resistance to motion. Experiment shows that the friction is greatest at the moment of starting, and that after the first few revolutions it diminishes and continues very nearly constant for all speeds.

If the initial retarding torque *od* is greater than the maximum total torque *oa*, the motor cannot move. The current that is taken from the line when a motor is started depends simply upon the tension of the line and the resistance in circuit at the moment: if there is a heavy load on the motor, that is, if the resistance to motion is great, the motor will start up slowly, if at all; if the load on the motor is light, the motor will start up quickly. Thus **the load on the motor does not affect the current taken from the line at the moment of starting,** but only the rate at which the motor speeds up.

The acceleration at any instant is proportional to the accelerating force. If n is the speed of the motor in revolutions per second, the acceleration will be $\frac{dn}{dt}$. The accelerating force will be proportional to the torque available for acceleration, and if the induction factor remains constant, it will be proportional to the difference between the total current flowing through the motor at any moment, and the current that the motor will take when uniform motion has been attained. If we denote the former by c, and the latter by c_f, we can write the equation for acceleration thus:

$$\frac{dn}{dt}=k(c-c_f)\ldots\ldots\ldots\ldots(63).$$

where k is a constant depending, as we shall see later, upon the value of the induction factor, the moment of inertia of the mass to be moved, the velocity ratio, and the resistance of the motor.

If E is the tension of the line, R the total resistance in the circuit, for the present considered constant, and M the induction factor, we can write down the following equation, which we know must hold good at any instant:

$$E=Rc+Mn \qquad \text{(64)}.$$

Substitute in this equation the value of c obtained from Equation 63 and we have

$$\frac{dn}{dt}+\frac{kM}{R}n=\frac{k}{R}(E-Rc_f) \qquad \text{(65)}.$$

This equation is of the form $\frac{dy}{dx}+Py=Q$, the solution of which is

$$y=e^{-\int Pdx}\left\{\int e^{\int Pdx}Qdx+K\right\}$$

(See Perry's 'Calculus for Engineers,' p. 326.)

The solution of Equation 65 is then

$$n=e^{-\frac{kM}{R}t}\left\{\frac{E-Rc_f}{M}e^{\frac{kM}{R}t}+K\right\} \qquad \text{(66)}.$$

Now the expression $\frac{E-Rc_f}{M}$ is the final speed of the motor when it has ceased to accelerate. If this is n_f, we then get

$$n=n_f+Ke^{-\frac{kM}{R}t} \qquad \text{(67)}.$$

To find the value of the constant K, we know that when $t=o$, n is the initial speed, if we call this n_1, and put $n=n_1$

and $t=o$, we see that $K=n_1-n_f$. Hence Equation 67 can be written thus:

$$n=n_f+(n_1-n_f)e^{-\frac{kM}{R}t} \quad \ldots\ldots\ldots\ldots(68).$$

If we write a for $\frac{n_f-n_1}{n_f}$ and τ for $\frac{R}{kM}$ this becomes

$$n=n_f(1-ae^{-\frac{t}{\tau}}) \quad \ldots\ldots\ldots\ldots(69).$$

If n_1 is nothing so that the motor starts from rest, $a=1$, and we get

$$n=n_f(1-e^{-\frac{t}{\tau}}) \quad \ldots\ldots\ldots\ldots(70).$$

This equation tells us that if we switch on a motor of constant resistance R and induction factor M, the speed n any time t seconds after connection is less than the final speed n_f by a quantity which is itself a fraction of that final speed, the value of the fraction depending on the time and on a certain constant τ.

The act of switching on the motor really amounts to changing the tension on the motor terminals from nothing to E. If the tension on the motor is E_1 to begin with, it will already be running at a speed which we have called n_1. Hence if the tension of the line be changed so as to make the motor speed up towards a final speed n_f, the speed at any time t seconds after the moment of making the change will be given by the law expressed by Equation 69.

This equation will hold equally true if the tension of the line instead of being increased is diminished; in that case a is negative, and the speed at any moment is greater than the final speed by an amount given by the equation.

Equation 69 may be regarded still more generally. We have seen that the speed of a motor is given by $\frac{E-cR}{M}$, and that an alteration in any one of the quantities making up this expression will cause an alteration in the speed. The effect of an alteration in the tension of the line has already been considered. If the load is increased c will increase, and the speed of the motor will fall to that given by inserting the new value of c in the expression for n. Similarly, if the load is reduced, the motor will speed up according to the same law. During these changes the value of τ will remain unaltered.

If in Equation 70 we put $t=\tau$, we see that the speed is $\frac{e-1}{e}n_f$, or $0{\cdot}632n_f$, since $e=2{\cdot}718$. We find then that when the time from the start is given by $t=\tau$ the speed is a constant fraction of the final speed. The constant τ might then be defined as being the time from the start when the speed is 0·632 of the final speed. We shall call this constant **the Time Factor**. We only know at present that it varies directly as R, inversely as M, inversely also as the constant k, the value of which we have not yet determined.

By differentiating Equation 70 we obtain

$$\frac{dn}{dt}=\frac{n}{\tau}e^{-\frac{t}{\tau}} \quad \text{.....................(71).}$$

This gives us the value of the tangent to the curve at any moment, that is to say the acceleration. We see that the acceleration at the moment of starting up from rest, when $t=o$, is the final speed divided by the time factor. If the initial speed is not zero we have from Equation 69

$$\frac{dn}{dt}=\frac{an_f}{\tau}e^{-\frac{t}{\tau}} \text{ or } \frac{n_f-n_1}{\tau}e^{-\frac{t}{\tau}} \quad \ldots\ldots\ldots\ldots(72).$$

Thus when anything occurs to change the speed from n_1 to n_f, the initial rate of change of speed is $\frac{n_f-n_1}{\tau}$. This measures the sensitiveness of the motor to changes of load, and will guide us when the specification precludes more than a definite change of speed for certain brief and sudden changes in the running conditions.

In the cases considered hitherto in which the speed is caused to vary, the change has been made suddenly. If part of the resistance in the circuit be in the form of a rheostat, R may be reduced gradually as the motor speeds up, so that the current remains constant. This may be done by hand or automatically. The induction factor being constant, the acceleration is also constant. If the motor starts from rest, the current at the moment of starting is given by $c_1=\frac{E}{R}$, and if c_f is the frictional current we get at once the acceleration from Equation 63. The current will remain constant until the rheostat is all out; the speed at which this will occur is given by $n=\frac{E-c_1R}{M}$ where c_1 is the starting current and R the resistance of the motor only. The motor will now speed up in accordance with the law of Equation 70.

The following **graphical construction** will enable us **to obtain the form of the acceleration curve.**

In Fig. 38 take oa equal to the number of seconds in the time factor. Take *fo* equal to the final speed. Draw a vertical line through *a*, cutting a horizontal through *f* in *d*. Join *od*. Then *od* is the tangent to the acceleration curve at the origin, and *da/ao* will be the initial acceleration.

We know that when the time from the start is equal to the time factor the speed is 0·632 of the final speed. Take *ab* equal to this fraction of *ad*. The speed at *a* will then be represented by *ab*.

We know also that the acceleration when $t=\tau$ is equal to the initial acceleration divided by *e*. Draw a line through *b*, making an angle with the horizontal whose tangent is this fraction of *da/ao*, this will be the tangent to the curve at *b*.

In Fig. 38 curve *B* gives the current drawn from the line. Since the current at any instant is given by $c=\frac{E-Mn}{R}$, *n* being the speed at that instant, the equation to the current curve is

$$c=c_f+\frac{Mn_f}{R}e^{-\frac{t}{\tau}} \quad \text{.....................(79).}$$

The area of this curve gives the work done, since the tension of the line remains constant. If *ok* is the final current and *op* the initial current, then $c=ok+kp\times e^{-\frac{t}{\tau}}$. The tangent to the current curve at the origin is $\frac{kp}{\tau}$. If we take *ag* equal to *ok*, and join *pg*, this will be the tangent to the current curve at *p*. The current at $t=\tau$ is $ok+\frac{kp}{e}$, and the tangent to the curve here is the initial value of the tangent divided by *e*.

The speed at any instant is given by $n=n_f(1-ae^{-\frac{t}{\tau}})$. If the speed is constant for a short interval dt seconds, the number of revolutions made in this time is ndt, hence the total revolutions made between o and t seconds is given by

$$N=n_f\int_0^t(1-ae^{-\frac{t}{\tau}})dt \ldots\ldots\ldots\ldots\ldots(73).$$

Integrating this between the given limits, we have

$$N=n_f\left\{t-a\tau(1-e^{-\frac{t}{\tau}})\right\}\ldots\ldots\ldots\ldots\ldots(74).$$

If t is large compared with τ we may neglect the expression $a\tau e^{-\frac{t}{\tau}}$. Hence

$$N=n_f\left\{t-a\tau\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(75).$$

If D is the total distance travelled in feet, as in the case of a railway motor, and s_f the final speed in feet per second, we can write

$$D=S_f\left\{t-a\tau\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(76).$$

If θ is the circular measure of the angle turned through in t seconds, and if ω is the final angular velocity, we have

$$\theta=\omega\left\{t-a\tau\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(77).$$

This case is illustrated by the motion of a crane-stage or turret. Care must be taken to see that the term $a\tau e^{-\frac{t}{\tau}}$ can be safely neglected. In Equation 75 n_f is found from $n_f=\frac{E-c_fR}{M}$, where $1{\cdot}41c_fM$ is the torque in inch-pounds required to overcome the frictional and other resistances to motion which are supposed to remain constant.

In Equation 76 the speed S can be shown to be $\cdot 262 \frac{d}{v}$ of the revolutions per second of the motor, where d is the diameter of the car wheel.

Equation 76 may be conveniently written

$$t = \frac{D}{s_f} + a\tau \text{ .... } (78).$$

From these equations we can see the influence of the time factor, Equation 76 showing the reduction in the distance travelled when the time is given, and Equation 78 the increase in the time when the distance is given. A reference to Fig. 38 will make this clearer. The area between the acceleration curve and the time base measured from the origin to any given point oa, represents the distance travelled in that time. If the time factor were nothing the distance would be given by $n_f t$, that is, by the area $oadf$. The area $obdf$ represents the amount that has to be deducted to give the true distance travelled in a given time t, and is equal to $n\tau$, where n is the speed at t seconds. If the term $\tau e^{-\frac{t}{\tau}}$ is neglected, the amount deducted is $n_f\tau$.

We now have **to find the value of the constant** k in the expression for the time factor $\tau = \frac{R}{kM}$. From Equation 63 we see that k is a constant which multiplied into the accelerating current gives the acceleration in r.p.s. per second. If c_a is the accelerating current we can write:

$$\frac{dn}{dt} = kc_a \text{ .... } (80).$$

The acceleration a in f.p.s. per second produced by a force of T pounds acting on a mass of W tons is given by

$$a=\frac{32\cdot2}{2240}\frac{T}{W}=\frac{1}{69\cdot5}\frac{T}{W} \quad\ldots\ldots\ldots\ldots\ldots(81).$$

If the centre of gravity of this mass rotate in a circle of d inches diameter, and if the motion is derived from a motor with a velocity ratio of v, we have

$$T=2\cdot82\frac{Mv}{d}c_a \quad\ldots\ldots\ldots\ldots\ldots\ldots(82).$$

Inserting this value of T in Equation 81 we obtain the following:

$$a=405\cdot10^{-4}\frac{Mv}{d}\frac{c_a}{W} \text{ f.p.s. per second} \ldots\ldots(83.)$$

$$\text{or} \quad a=288\cdot10^{-4}\frac{v}{d}\frac{t_a}{W} \text{ f.p.s. per second} \ldots\ldots(84).$$

where t_a is the torque on the motor shaft available for acceleration.

Now one revolution of the motor corresponds to a motion of the rotating mass of $\frac{\pi}{12}\frac{d}{v}$ feet. Hence the acceleration of the motor is given by

$$a=0\cdot155M\frac{v^2}{d^2}\frac{c_a}{W} \text{ r.p.s. per second} \ldots\ldots(85).$$

Combining this with Equation 80 we get the value of k, namely

$$k=0\cdot155\frac{v^2}{d^2}\frac{M}{W} \quad\ldots\ldots\ldots\ldots\ldots\ldots(86).$$

The time factor can be expressed thus:

$$\tau = 6{\cdot}45\ RW \frac{d^2}{M^2 v^2} \text{ seconds} \quad \ldots\ldots\ldots\ldots(87).$$

When the motion of the motor shaft is communicated to the main shaft by a clutch, the motor may run at full speed before connection. Let the moment of inertia of the moving portion of the motor, the armature, pulley, &c., be I_1, and that of the main shaft with the rotating mass I_2. If the motor is running at n_1 r.p.s. before connection, the kinetic energy of the motor shaft will be $2\pi^2 n_1^2 I_1$. If the speed immediately after connection is n_2 r.p.s., the kinetic energy will be $2\pi^2 n_2^2 I_1$. The difference represents the kinetic energy of the main shaft immediately after connection, and this is $\frac{2\pi^2}{v^2} n_2^2 I_2$.

The reduced speed, n_2 of the motor shaft, is given by the equation

$$n_2^2 = \frac{I_1}{I_1 + \frac{1}{v^2} I_2} n_1^2 \ldots\ldots\ldots\ldots\ldots\ldots(88).$$

After connection the system will begin to accelerate, and the acceleration at any time can be found from Equation 69, with due regard to sign and to the value of τ. If the frictional torque is unaltered, the final speed of the motor will be the same as the initial speed. If the frictional torque has increased, we can find the new speed by inserting the new value of c_f in the equation for n_f.

We may have to design an equipment by which the main shaft can be started up without drawing more than a specified current of c_2 amperes from the line. The proper moment of inertia of the motor shaft is then given by the equation

$$I_1 = \frac{n_2^2}{v^2(n_1^2 - n_2^2)} I_2 \quad \ldots\ldots\ldots\ldots\ldots(89).$$

where n_2 is given by

$$n_2 = \frac{E - c_2 R}{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots(90).$$

Example 41.—The moving mass of a lift is perfectly balanced and weighs 1,500 pounds. This weight is moved by ropes wound on a rope drum 48 inches in diameter. The drum is driven by a motor connected to it with velocity ratio of 70, connection being made by a clutch which we may suppose to act without slipping, but with a certain amount of elastic giving. The induction factor of the motor is 5, and the resistance 0·1 ohm. The frictional current is 10 amperes, and the motor runs at 24·8 r.p.s. on a line having a tension of 125 volts. We have to design an equipment by which we can throw on the clutch and start the lift without drawing more than 40 amperes from the line. When the motor is drawing 40 amperes, the speed will be 24·2 r.p.s., hence by Equation 89 the moment of inertia of the motor shaft must be 0·004 of the moment of inertia of the main shaft. If we neglect the moment of inertia of the rope drum and wheel, in comparison with that of the moving weight, we have $I_2 = \frac{Wr^2}{g}$; W is here the weight of 1,500 pounds, g is 32·2, and r the radius of gyration of the moving mass is two feet. Hence $I_2 = 186{\cdot}5$. The moment of inertia of the motor shaft must therefore be 0·742, or the equivalent of 95·5 pounds acting at a distance of 6 inches from the centre of the shaft.

The conditions of the preceding example are to some extent realised in the case of thoce **lifts** which are **driven by belting**. The motor is always running, and when the lift has to be started, the belt is thrown over from a loose onto a fast pulley. In practice, the belt always slips and stretches; if it did not do so it would be impossible to work a lift on this principle, on account of the shock occasioned to the gearing.

The rotating parts of a motor shaft generally have a moment of inertia sufficiently great to assist materially in starting, without any special alteration in the design. In order to make use of the principle to the extent that the theory indicates as possible, an elastic coupling would have to be contrived that would take up the shock and limit the acceleration of the car.

The **slipping** of the belt might be utilised to effect a smooth start without using an elastic coupling. When a belt or coupling slips, the torque transmitted is limited to that at which slipping takes place. If the slipping torque is constant, the acceleration of the main shaft is constant, and so is the retardation of the motor shaft. The conditions then are as if a definite load—the slipping torque—were thrown on the motor. The motor shaft will be retarded, and the main shaft accelerated until the two speeds are equal, and slipping will then cease. The amount of retardation and acceleration will depend on the moment of inertia of the two shafts. The slipping of the belt in the belt-driven lift thus assists the start by limiting the transmitted torque to that at which slipping takes place.

In establishing Equations 82 to 87 we assumed that the whole mass to be moved was situated so that its centre of gravity rotated in a circle of *d* inches diameter. In the case of a lift it is easy to see that the weight of the car is acting at a point on the rim of the rope drum, so that the mass that has to be moved may be considered as being virtually situated at a point *d* inches from the centre of the drum. Also, the mass of the drum or other rotating part may without difficulty be conceived as acting at the rim of the drum, the mass thus acting being, of course, such as will give the same moment of inertia as the drum itself.

When, however, we come to the case of a motor driving a railway car it is not so obvious that the mass that has to be accelerated is now that of the car, and that the weight W in Equation 84, for instance, is the weight of the car. A reference to Figs. 13 and 14 may make this clearer. We have already shown that the dynamical conditions of the two cases here illustrated are identical, and that while in Fig. 13 the force T acts vertically through the rope, in Fig. 14 the same force acts horizontally on the car axle. Hence in the first case the mass that has to be accelerated is that hanging at the end of the rope, while in the second it is that of the car itself.

We shall now give a **graphic solution of the acceleration problem,** by which we shall be able to take account of any variations that may exist in the value of the induction factor.

We shall take as an example the motors used on the City and South London Railway. There are two motors on each locomotive, the armatures being placed directly on

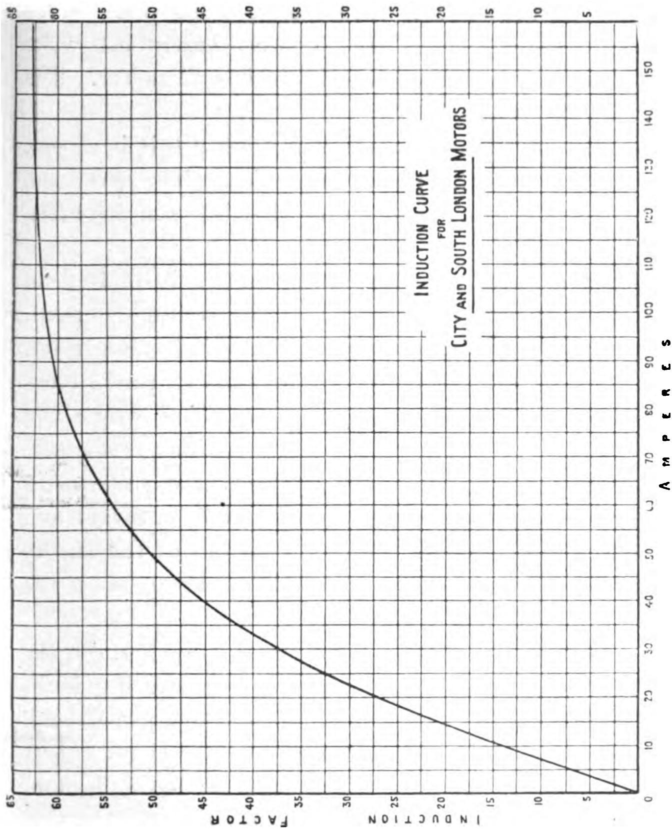

INDUCTION CURVE
FOR
CITY AND SOUTH LONDON MOTORS
AMPERES
INDUCTION FACTOR

FIG. 39

the axles without gearing. The motors are connected permanently in series, and are also series wound. Their resistances are: magnets, 0·087 ohm; armature, 0·3 ohm; total resistance of each motor, 0·387 ohm. We shall take the tension of the line as being 400 volts throughout. The wheels are 27 inches diameter. The weight of the train is 35 tons.

A starting rheostat is used, and when this is all out, the motors are left to speed up in series. The maximum current per motor, and hence from the line, will be taken as 150 amperes.

The induction curve for these motors is plotted in Fig. 39, the horizontal ordinates representing current, and the vertical ordinates values of the induction factor. We must first find the values of the total torque in inch-pounds, and plot them on a speed base. The torque and speed can be obtained from the usual equations, remembering that since the motors are in series, the value of E is 200 volts. The speed can be reduced from revolutions per second of the motor to feet per second of the car by multiplying by the factor $262 \cdot 10^{-3} d$, where d is 27 inches.

The results are plotted as a curve, A, in Fig. 40, the origin being at o, horizontal ordinates representing inch-pounds of torque, and vertical ordinates feet per second. When the speed is nothing, the current is 150 amperes per motor, the total torque is 13,280 inch-pounds, and remains constant until the rheostat is all out. At this moment the speed of the car is found to be 16 feet per second, using the ordinary equation for the speed in r.p.s., putting $R = 0 \cdot 387$, and $C = 150$. The curve is thus straight as far as the point where the speed is 16 feet per

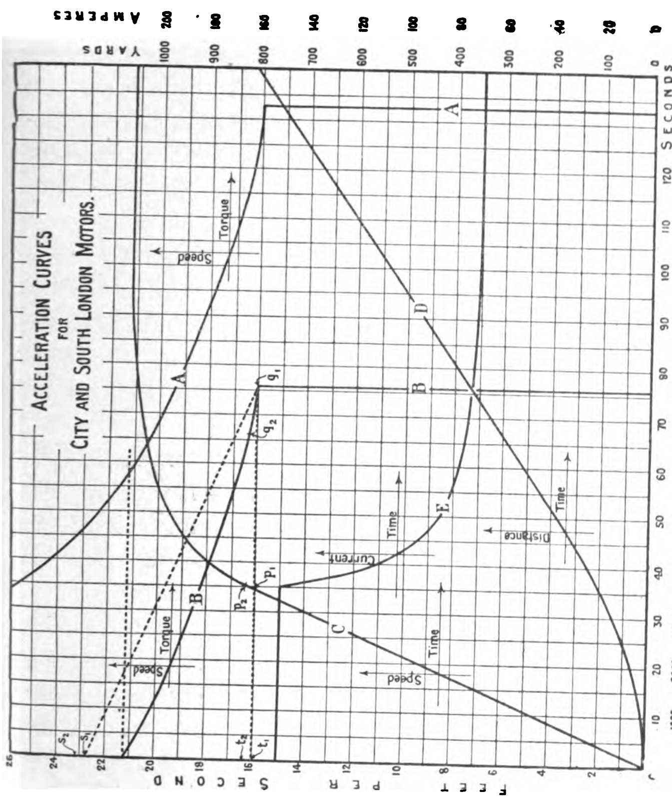

ACCELERATION CURVES
FOR
CITY AND SOUTH LONDON MOTORS.
AMPERES
YARDS
SECONDS
INCH-LBS. OF TORQUE
FEET PER SECOND
Speed
Torque
Current
Time
Distance
A
B
C
D
E

FIG. 40

second; the torque will then rapidly diminish as the car speeds up; we may plot the curve as far as 26 feet per second.

From the records of actual performance we find that the average current at full speed is 70 amperes, giving a retarding torque of 5,760 inch-pounds, equal to a tractive effort of 42,000 pounds on 27-inch wheels. This includes all forces opposing the motion. Since the weight of the train is 35 tons, this corresponds to a tractive effort of 19·7 pounds per ton on wheels of 33 inches diameter.

Construct a curve *B* giving torque available for acceleration, by deducting from the horizontal ordinates of curve *A* an amount equal to 5,760 inch-pounds; this curve will cut the axis of torque at $t = 7{,}610$ inch-pounds, and the axis of speed at 21·2 f.p.s. This gives us the final speed of the car, which is equivalent to 14·5 miles per hour.

Curve *B* shows that the accelerating torque is constant from the start up to 16 f.p.s., that it then rapidly diminishes, until at 21·2 f.p.s. it becomes nothing; the car then ceases to accelerate, and uniform speed is attained.

We here assume that the retarding torque remains constant at all speeds. This is not strictly correct, but it is probably not far from the truth. If the values of the retarding torques for different speeds can be obtained from experiments, the true form of the curve of accelerating torque can be deduced. No experiments of this kind have been made with these motors; we shall therefore take the average value, and assume it to remain constant at all speeds.

Since the weight of the train is 35 tons, each motor will have to accelerate 17·5 tons. Using the Equation 84 we find the initial acceleration to be 0·463 f.p.s. per second. As the speed when the rheostat is all out is 16 f.p.s., the acceleration will remain constant for 34·6 seconds. Plot the first portion of curve C showing the speed in f.p.s. on a base of seconds. Let the point p_1 represent the time when the rheostat is all out. Draw a line at right angles to op_1, passing through the point q_1, and cutting the speed base in s_1. Draw a horizontal line passing through q_1 and p_1 cutting the speed base in t_1. Then q_1t_1 is the accelerating torque at the point p_1 on the acceleration curve, and the tangent at the point p_1 is at right angles to the line drawn through the points q_1 on the torque curve and s_1 on the speed base.

We have then a general construction for drawing the acceleration curve. Take the point p_1, where the acceleration ceases to be constant, draw a horizontal line to cut the torque curve in q_1 and the speed base in t_1. Measure t_1s_1 along the speed base, join q_1s_1, and from p_1 draw a line at right angles to q_1s_1, and continue this line for a period during which the acceleration may be assumed to be constant, say to the point p_2. Now draw another horizontal line through p_2, cutting the torque curve in q_2, and the speed base in t_2; set off from t_2 along the speed base a distance t_2s_2 equal to t_1s_1; join s_2q_2 and draw a line from p_2 at right angles to q_2s_2, extend this line to a point p_3, and so on. In the triangle $q_2s_2t_2$, $s_2t_2 = s_1t_1$, and is a constant quantity, and q_2t_2 is the accelerating torque, it follows that the line drawn through p_2 at right angles to q_2s_2 will be the tangent to the curve at p_2; for the tangent at any point of the acceleration curve represents the acceleration at that point, and the tangent at p_2 is equal to the tangent of the angle $q_2 s_2 t_2$, that is to say, is proportional to the accelerating torque, and therefore to the acceleration at p_2.

In this way we can construct the whole of the acceleration curve, taking care to keep the points $p_1 p_2$ closer together as the curve becomes flatter. The curve will finally approximate to the speed of 21·2 feet per second.

From the curve C we can obtain the distance travelled at any time. Draw a vertical line through any given time, cutting the acceleration curve; the area enclosed between this line, the curve, and the time base, will be the distance travelled from the start. Find these distances for successive seconds, and then plot them as a curve D on the time base, taking vertical ordinates as distances travelled.

From the acceleration curve we can deduce the current curve E, showing on a base of seconds the current taken from the line. This will begin at 150 amperes, and will be straight for 34·6 seconds, when it will rapidly slope down and approximate to the uniform value 70 amperes. The area of this curve up to any point on the time base will give the energy drawn from the line.

The curves in Fig. 41 record the results of tests made on the South London Railway with the motors previously described. They are constructed from data given in the 'Proceedings of the Institution of Civil Engineers,' vol. cxii. p. 246. The track is not level, as shown by the outline of the grades, which is drawn on a time base, so that the position of the train on any grade can be seen at once. The distances travelled in 80 and 120 seconds are

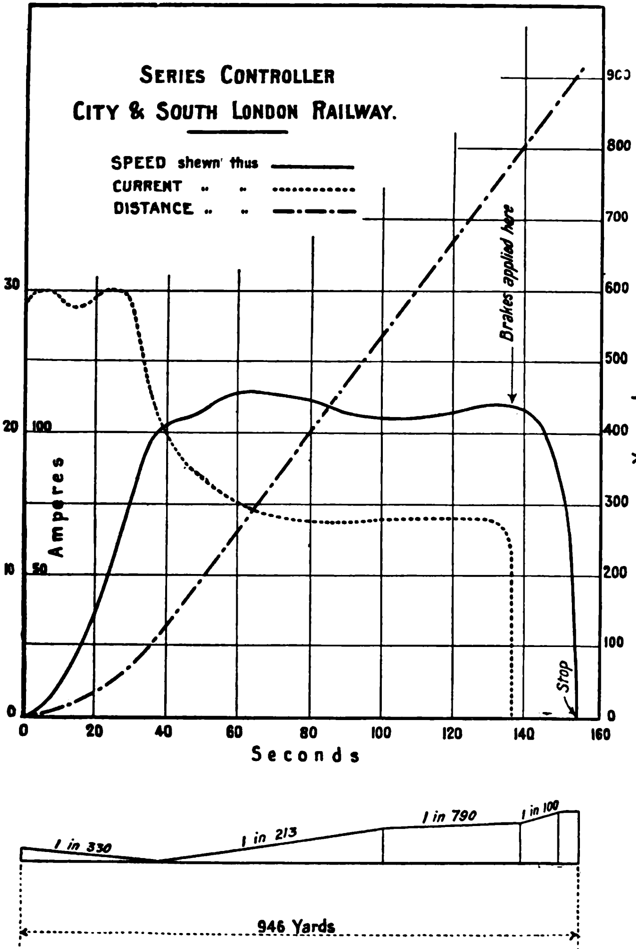

SERIES CONTROLLER
CITY & SOUTH LONDON RAILWAY.
SPEED shewn thus
CURRENT " "
DISTANCE " "
Brakes applied here
Stop
Amperes
Yards
Seconds
0
20
40
60
80
100
120
140
160
10
30
50
200
300
400
500
600
700
800
900
1 in 330
1 in 213
1 in 790
1 in 100
946 Yards

FIG. 41

400 and 670 yards respectively, while the first 100 yards is covered in 36 seconds. The weight of the train is 35 tons.

The down grade at the station exit has been constructed to assist the locomotive in starting the train. In this case the acceleration is increased by about 20 per cent.

CHAPTER VIII

THE FORCE FACTOR

THE method of rating a motor in horse-power gives us no indication of its ability to accelerate, although the function of accelerating may be the most important that it is called upon to perform. We shall find it convenient to be able to describe a motor in terms of a unit that shall be independent of speed and that will tell us to what extent a motor is able to accelerate under given conditions. This must be a force unit and not a power unit.

If a pulley of d centimetres diameter is placed on the shaft of a dynamo of induction factor M, carrying a current of c amperes, the tangential force at the rim of the pulley is given by

$$T = \frac{1}{\pi d} Mc 10^7 \text{ dynes} \quad \ldots\ldots\ldots\ldots\ldots (91).$$

If $d = \frac{1}{\pi} 10^7$ centimetres, this may be written

$$T = Mc \text{ dynes} \quad \ldots\ldots\ldots\ldots\ldots\ldots (92).$$

The force of a dynamo may thus be defined as a force of Mc dynes at the rim of a pulley 10^7 centimetres in circumference. We shall call Mc **the force factor** of the dynamo.

Since M is equal to c, the induced volts, divided by n, the number of revolutions per second, it follows that $Mc = \frac{ce}{n}$, but ce is the work done per second, hence Mc is the work done per revolution. If then we are given Mc and M, we can find the work done per second, that is the rate of working, or the power, by simply multiplying the product Mc by the number of revolutions per second.

The ratio of the watts to the revolutions per second, sometimes called the 'mass factor,' has been used as a basis for the comparison of dynamos, its true significance not, however, being perceived. The fact seems to have been overlooked that the ratio of the induced volts to the revolutions per second is a constant, so long as the number of useful lines per pole remains unaltered, being, in fact, what we have termed the induction factor.

While the force factor and the so-called 'mass factor' are one and the same thing, the latter is expressed in a way involving the idea of speed, and consequently of power, while the former indicates the real nature of this ratio, showing that it is independent of speed, and therefore not a power unit, much less a mass unit, but a force unit.

Example 42.—A four-pole dynamo with armature parallel connected, giving $p=1$, has 440 surface conductors, with $16{\cdot}1 \times 10^6$ lines per pole. The induction factor is 77 and the force factor for 600 amperes is $77 \times 600 = 46{\cdot}2$ kilodynes; the power at 450 r.p.m. is $46{\cdot}2 \times 7{\cdot}5 = 346$ kilowatts. The dynamo is a General Electric four-pole railway generator.

Example 43.—A ten-pole dynamo with armature parallel connected has 1,440 surface conductors and

$28 \cdot 6 \times 10^6$ lines per pole. The induction factor is 412. The force factor for 1,500 amperes is $412 \times 1{,}500 = 618$ kilodynes, and the power at 80 r.p.m. is 825 kilowatts. The dynamo is a Westinghouse ten-pole railway generator.

Example 44.—The motors in use on the Liverpool Overhead Railway and on the City and South London Railway are rated at about the same horse-power, but on actual test one of the Liverpool motors can exert a pull equal to that of two of the South London motors, when equal current is passing in all three.

The motors on the Liverpool railway are designed to run in parallel, two motors to each car, while those on the South London Railway are designed to run permanently in series, two motors also to each car. If the tension of the line, the speed, the diameter of the driving wheel, and the horse-power were the same in the two cases, the induction factor of the Liverpool motors would have to be twice that of the South London motors. Suppose the values to be 120 and 60, then for 100 amperes the force factors would be twelve and six kilodynes respectively. Hence each of the Liverpool motors would give twice the draw-bar pull of one of the South London motors for the same current. The wheels on the Liverpool cars have a diameter of 33 inches, while those on the South London cars are 27 inches diameter, so that the respective draw-bar pulls for 100 amperes would be (taking the values of M as given above) for one South London motor, 630 pounds, and for one Liverpool motor, 1,030 pounds.

We must not infer from what has been said that the Liverpool equipment would be necessarily more efficient than the South London equipment when running at full speed, because, although for equal speed and draw-bar pull the current per motor in the former would be half that in the latter, yet since the South London motors are in series, the current from the line would be the same.

Example 45.—A motor that will draw a car at thirteen miles an hour, with a total tractive effort of 800 pounds and a mechanical efficiency of 85 per cent., would be rated as a 24 horse-power motor. Two such motors would drive a 10-ton car up a 5 per cent. grade with a train resistance of 20 pounds per ton. These motors, however, would not be able to start the car on this grade. They could drive the car when once started, but if it should stop on the grade, they could not start it again. In order to start up on the grade we do not need greater power, but greater force. Let us find the force factor required to start up in 10 seconds. The final speed of 13 miles an hour is equivalent to 19·1 feet per second; to make up this speed in 10 seconds, the acceleration must be 19·1 f.p.s. per second, assuming uniform acceleration throughout. From Equation 21 we find that if $E=500$, $v=4{\cdot}78$, and $d=33$, M must be 41·7 and the final current 47 amperes. Inserting these values in Equation 83, remembering that each motor has to accelerate half the car, we find the accelerating current to be twenty amperes, so that the total current from the line at the start is 67 amperes. The motors then must be capable of carrying this current, so that the force factor must be 2,790 dynes. Thus, to drive the car up the grade we need a force factor of 1,960 dynes, and in order to be able to start on the grade we have to increase the force factor by 42 per cent.

Example 46.—A motor has to be designed for a lift which will raise an unbalanced weight of 2,000 pounds at 200 feet per minute, the tension of the line being 125 volts, the velocity ratio 75, the friction 1,400 pounds at the rim of the rope drum, the diameter of which is 36 inches. The resistance of the motor is 0·05 ohm. Inserting these values in Equation 20, we find the induction factor to be 4·45 and the final current 130 amperes. The force factor required simply to drive the lift is thus 578 dynes. Since the total pull at the rim of the rope drum is 3,400 pounds, this motor would develop 20 horse-power when running at full speed.

If the lift had to act always at full speed, it would be sufficient to describe the motor as a 20 horse-power motor; but this would give no information as to the ability of the motor to accelerate, since at the moment of starting the horse-power is nothing. We have then to find the force factor required to start up the car, say, in two seconds. To get up a speed of 200 f.p.m. in two seconds we require an acceleration of 1·67 f.p.s. per second. If we suppose that the total mass to be moved is five tons, the accelerating current as given by Equation 83 is 22 amperes, making a total current at the start of 152 amperes. The force factor is therefore 676 dynes, or 17 per cent. greater than that required to raise the car when once started.

Suppose now that we had to reduce the time of starting by one half, and to start up in one second. One way of stating the case would be to say that we needed a 'more powerful' motor, but so long as the maximum load, the friction, and the final speed remain the same, the maximum horse-power required remains unaltered, so that we do not need more power, but more force; we require a motor with a higher force factor.

We cannot alter M so long as the final speed is specified, so we must double the accelerating current, making the total current at the start 174 amperes. The motor would then be described as having an induction factor of 4·45, with an armature capable of carrying 174 amperes safely—in other words, the force factor must be 775 dynes, or 15 per cent. greater than that required to start up in two seconds.

Example 47.—A train and locomotive weighing 780 tons have to mount a grade of 0·8 per cent. at 10·7 miles per hour. Friction and other retarding forces amount to nine pounds per ton. The tension of the line is 625 volts. Four gearless motors are used, permanently connected in series, each having an internal resistance of 0·0209 ohm. Each motor has to move 195 tons at a tension of 156·2 volts. The draw-bar pull per motor for the grade is 3,490 pounds; for friction, 1,755 pounds; allowing 95 per cent. mechanical efficiency, we get 5,500 pounds as the required tractive effort per motor; inserting this value together with those for E, R and S in Equation 21, we find the value of $\frac{M}{d}$ to be 2·32, v being unity. Take $M=144$, $d=62$ inches. When running at full speed, each of these motors would be doing work at the rate of 158 horse-power, so that the total horse-power of the locomotive would be 632. This gives us, however, no indication of the ability of the locomotive to accelerate. If a start has to be effected on the grade in 40 seconds, the final speed being 10·7 miles an hour, or 15·7 feet per second, the acceleration must be 0·393 f.p.s. per second. Inserting this value in Equation 83, we find the current required for acceleration to be 810 amperes, while that required for friction can be shown to be 850 amperes, making a total current at the start of 1,660 amperes and a force factor of 239 kilodynes.

The weight of a dynamo, if the induction factor is constant, is mainly determined by the current that it has to carry, and may be taken as increasing very nearly in direct proportion to the current.

On the other hand, the speed of a dynamo is limited by its weight, since for any given type of machine, the weight determines the diameter of the armature, and therefore the speed for the maximum permissible peripheral velocity, so that if a number of machines of the same type have to be made giving the same induced tension, the heavier machines must have the higher induction factors in order to give the same induced volts at lower speeds.

If the power of a dynamo is given, and the induced tension, either as motor or generator, the current is then fixed, being the quotient of the power by the tension. The current being fixed, the minimum weight is also fixed, for any given type of machine. Having obtained the minimum weight, we can then find the induction factor, as we know the highest permissible speed for a machine of this weight. The induction factor is obtained by dividing the given tension by the maximum speed. We thus get the lightest dynamo that will give the required power at the given tension.

For equal currents, the weight of a dynamo increases with the induction factor, but not in direct proportion; since for any given type the ratio of the induction factor to the weight increases slightly with the weight.

Hence for equal force factors the weight will increase slightly with induction factor.

In Fig. 42 the force factors of a complete set of four-pole dynamos of the same type are plotted on a base of weight. The numbers opposite each machine indicate the rated power in kilowatts. The force factors are plotted vertically in kilodynes. All these machines are for 125 volts.

Example 48.—To find the lightest dynamo of this type that would give a power of 40 k.w. with 125 volts induced tension. The current is 320 amperes. The weight of a machine to carry this current must be 4,250 pounds, and a dynamo of this weight has to run at 960 r.p.m. and must have an induction factor of 7·81 in order to give the required induced tension at the maximum permissible speed. This is the lightest dynamo of this type that will give the specified power at the given induced tension. The force factor of this machine will be 2·5 kilodynes. We might have taken an induction factor greater than 7·81. Suppose we took $M=10$. The current would be unaltered, but the speed would be reduced to 750 r.p.m. and the weight would be increased to 6,250 pounds.

We have already seen that a motor must have a larger force factor at the moment of starting than when giving the maximum power for which it is designed, and that when the speed at the maximum power is fixed, the increased force factor at the start must be obtained by increasing the current if M is constant. Hence a motor will be heavier in proportion as the force factor at the start is greater.

Example 49.—The motor in Example 48 has a force factor of 2·5 kilodynes when developing its maximum

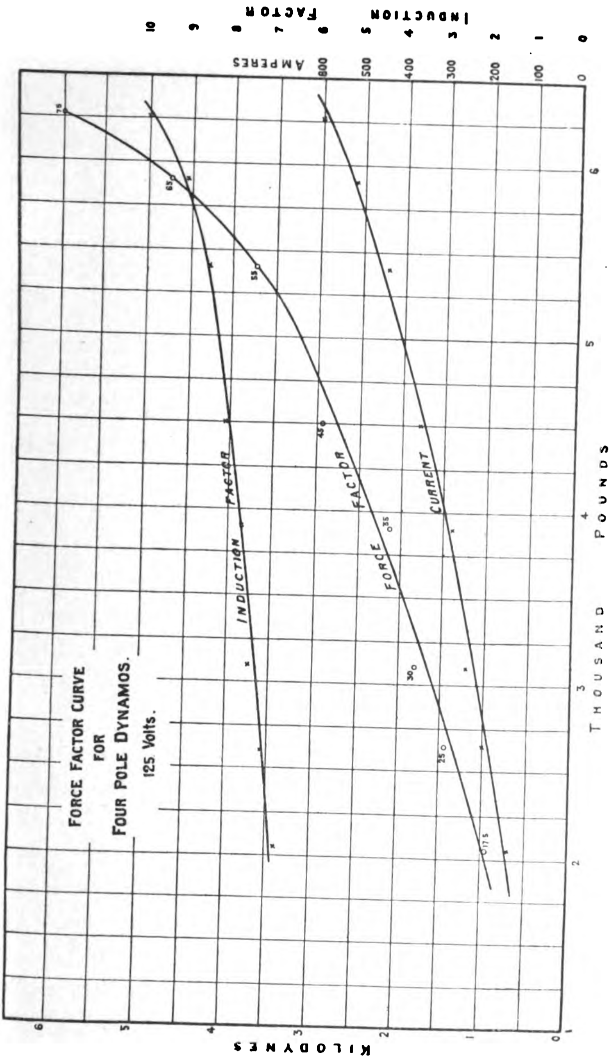

FORCE FACTOR CURVE
FOR
FOUR POLE DYNAMOS.
125 Volts.
INDUCTION FACTOR
FORCE FACTOR
CURRENT
INDUCTION FACTOR
AMPERES
KILODYNES
THOUSAND POUNDS

FIG. 42

power. If a force factor of 3·5 kilodynes is required at the start and M is constant, the motor must be capable of carrying 450 amperes, and will weigh about 5,300 pounds.

The force factor curves afford a convenient comparison between dynamos of different types. Thus in Fig. 43, force factor curves are given for the four-pole type already referred to, and for a set of two-pole dynamos of 125 volts induced tension. Since equal force factor and equal speed give equal power, we see that a four-pole dynamo weighs less than a two-pole dynamo of the same power, and that for equal weight the four-pole machine carries more current than the two-pole machine, but runs at a slower speed, because the ratio of the diameter to the width of the armature is greater in the four-pole than in the two-pole type.

The saving in weight is due to the fact that the four-pole type carries more current per pound than the two-pole type. It might appear that since the four-pole type has to run at a slower speed for the same weight than the two-pole type, it is at a disadvantage, and that if it could run quicker there might be a still further saving in weight. But this is not really so.

A given power means a given current for given induced tension. It is the current that determines the weight, and it is here that the four-pole type has the advantage, carrying more amperes per pound. Now the weight determines the speed for each type, and the speed determines the induction factor. If a machine of any type and weight could run quicker, it would simply mean that the induction factor might be less, but this would not lighten the machine, since the weight is determined by the current. The only difference would be that the force factor would be less.

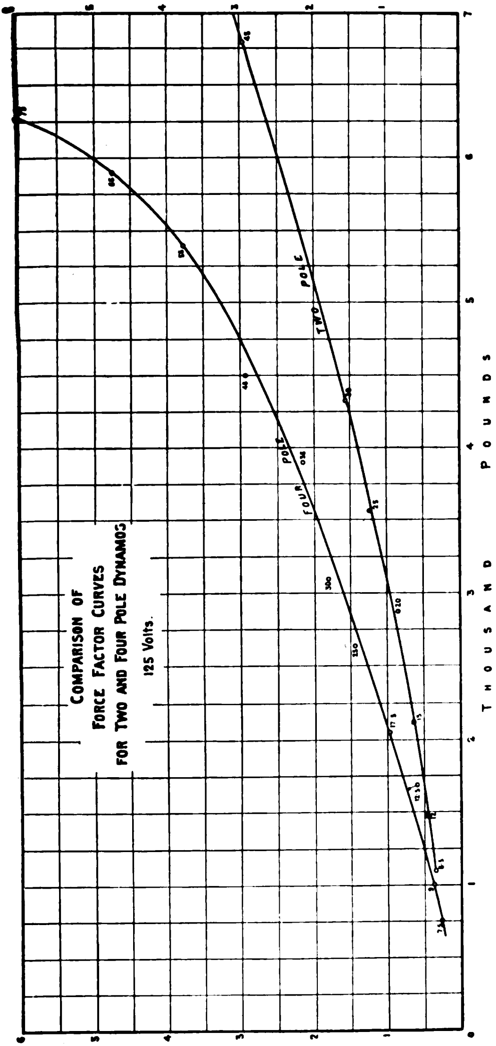

COMPARISON OF
FORCE FACTOR CURVES
FOR TWO AND FOUR POLE DYNAMOS
125 Volts.
TWO POLE
FOUR POLE
THOUSAND POUNDS
KILODYNES

FIG. 43

In Fig. 44 force factor curves are given for different types of dynamos. Curve *A* is for a set of four-pole belt-driven railway generators, and curve *C* for a set of six-pole direct connected railway generators, made by the General Electric Company. Curve *B* is for a set of four-pole belt-driven railway generators made by the Walker Manufacturing Company. The power in kilowatts is stated opposite each machine. The dynamos are all for 550 volts. The current can be obtained by dividing the power in each case by the tension, and the induction factor can then be deduced by dividing the force factor by the current.

In Fig. 45 the weights and force factors of a complete set of direct connected railway generators made by the General Electric Company are given. Particulars of these machines will be found in the table. The letters *A*, *B*, *C* refer to the method of winding the armature.

It is instructive to take different machines designed to carry equal currents and see how the weight varies with the induction factor. Also to take equal induction factors and see how the weight varies with the current, and to take equal force factors and see how the weight varies with the induction factor.

The force factor curve is really the combination of a number of different force factor curves for the different types, six-, eight- and ten-pole, only a certain portion of each being used in practice. In the diagram the current curve is assumed to be straight, and the curve of induction factor is obtained by dividing the force factor by the current for each weight.

Example 50.—To find the lightest dynamo for 500 kilowatts. The current at 550 volts tension is 910 amperes, and the weight from the current curve is there-

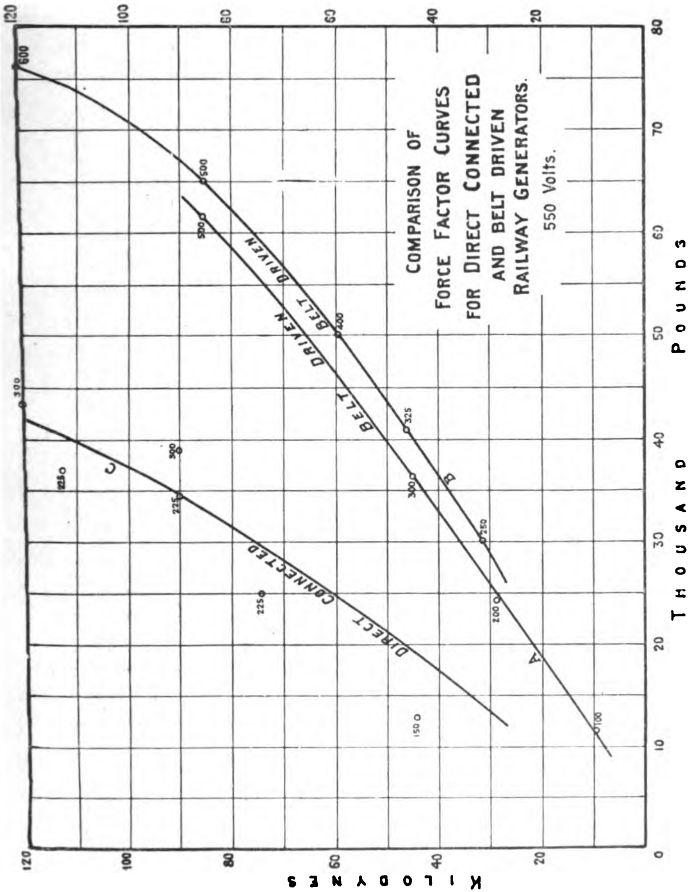
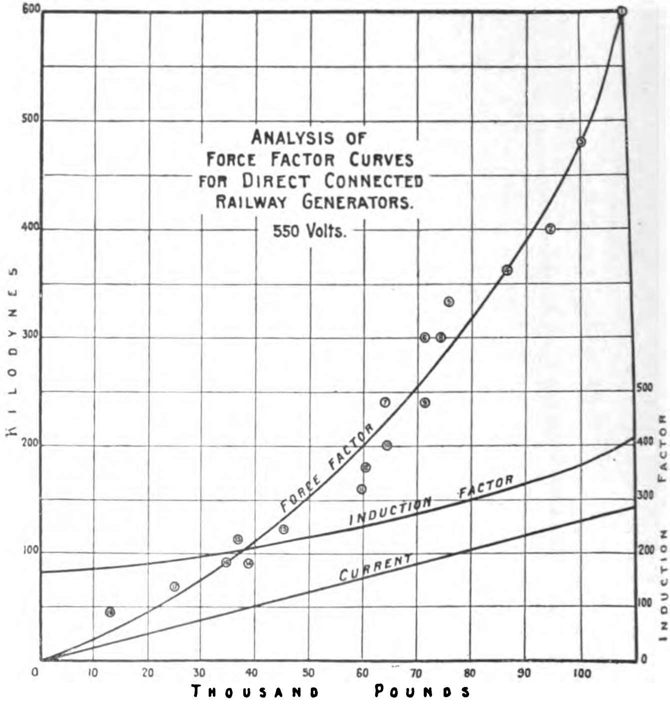

COMPARISON OF
FORCE FACTOR CURVES
FOR DIRECT CONNECTED
AND BELT DRIVEN
RAILWAY GENERATORS.
550 Volts.
BELT DRIVEN
BELT DRIVEN
DIRECT CONNECTED
A
B
C
THOUSAND POUNDS
KILODYNES

FIG. 44



ANALYSIS OF
FORCE FACTOR CURVES
FOR DIRECT CONNECTED
RAILWAY GENERATORS.
550 Volts.
FORCE FACTOR
INDUCTION FACTOR
CURRENT
KILODYNES
INDUCTION FACTOR
THOUSAND POUNDS
0
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600

FIG. 45

fore 69,000 pounds. The speed for this weight is 122 r.p.m. Hence the induction factor must be 270. A reference to the table shows that the lightest dynamo for this power is actually 64,000 pounds.

DIRECT CONNECTED RAILWAY GENERATORS

No. of poles	No. in diagram	Rated Power K.W.	Current amperes	Speed r.p.m.	Induction factor	Force factor K.D.	Weight 1000 lbs.
10 B	1	800	1450	80	412	600	110·0
10 B	2	—	—	120	275	400	94·4
10 C	3	—	—	100	330	480	100·9
10 B	4	500	910	75	440	364	87·1
10 B	5	—	—	90	367	334	76·0
10 B	6	—	—	100	330	300	71·3
10 A	7	—	—	125	264	240	64·0
8 A	8	400	725	80	412	300	74·2
8 A	9	—	—	100	330	240	71·4
8 A	10	—	—	120	275	200	64·3
6 B	11	—	—	150	220	160	59·7
6 A	12	300	545	100	330	180	60·4
6 A	13	—	—	150	220	120	43·9
6 A	14	—	—	200	165	90	39·1
6 A	15	225	410	120	275	113	37·0
6 A	16	—	—	150	220	90	34·3
6 A	17	—	—	200	165	67·5	25·1
6 A	18	150	270	200	165	44·5	13·1

CHAPTER IX

CONTROL

THE action of a motor during the time in which it is being brought up to speed from a state of rest is determined by the **method of control.** When there is a single motor a rheostat is used, the resistance of which can be varied by a switch worked automatically or by hand. The maximum resistance of the rheostat must be such that when connected in series with the motor at rest, the current from the line does not exceed the maximum safe current that the motor can carry. If the torque due to this current is greater than the frictional and other torque resisting the motion, the motor will begin to speed up, and the current will decrease. We may, however, keep the current constant by reducing the resistance as the motor speeds up. This process can be carried on until the resistance is cut out completely, after which we cannot prevent the current from getting less; the motor will then speed up according to the law expressed by Equation 69.

When two motors are used we find more than one possible method of control. If we take as an example a motor-car driven by two motors, there are the following alternatives:—

I. The motors may be started and run in parallel.

II. The motors may be started and run in series.

III. The motors may be started in series and run in parallel.

To compare the relative advantages of these methods we shall take a practical case and work out the results for each method.

Suppose, then, that we have given the following data: A motor-car weighing 40 tons is driven by two gearless motors; the wheel diameter is 33 inches; the maximum speed is to be 30 miles per hour; the retarding torque is 200 inch-pounds per ton of load; the track is straight and level; the tension of the line is 500 volts; the minimum resistance per motor is 0·4 ohm; the maximum current per motor is 200 amperes; the maximum current from the line is 200 amperes; the induction factor is to be constant. We shall divide the process into steps, each being distinguished by a different law of acceleration.

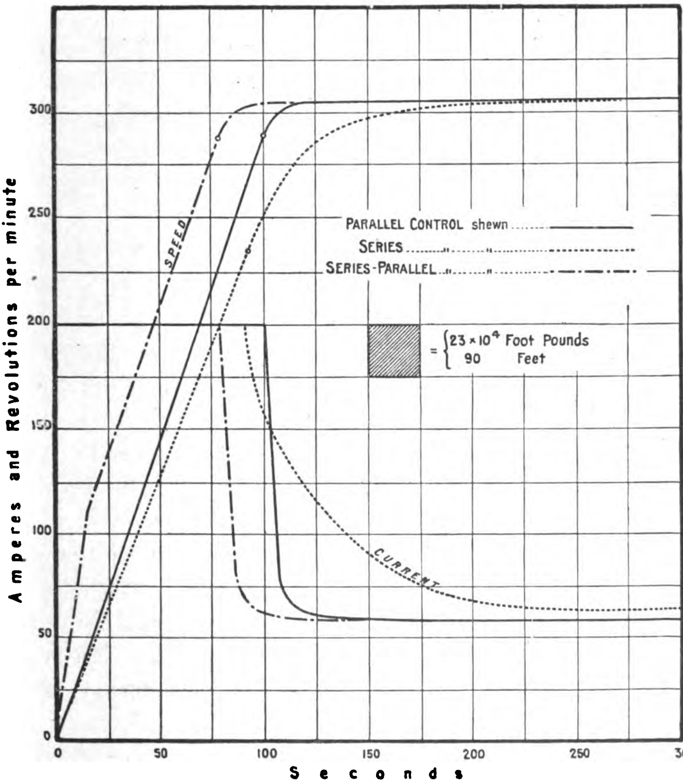
CASE I. **Motors started and run in parallel.**—Since the specification requires that the motors shall drive the car at a maximum speed of 30 miles per hour, with a frictional torque of 200 inch-pounds per ton of load, the induction factor must be determined by these data. The car weighs 40 tons, therefore the load per motor is 4,000 inch-pounds of torque. As the wheels are 33 inches diameter, and the velocity ratio unity, a tractive effort of 242 pounds per motor is required. First suppose that the resistance of the motor is as small as possible—namely, 0·4 ohm. Using Equation 21, we find that $M=95{\cdot}8$.

STEP I.—Starting rheostat in circuit; constant acceleration. The motors are in parallel, the maximum permissible current is 200 amperes, and the current per motor is 100 amperes. As the resistance of the motor is

0·4 ohm, the resistance of the rheostat on the first step of the controller must be 4·6 ohms. The current used in overcoming the friction which we suppose to be constant at all speeds, is 29·6 amperes. The current available for acceleration is therefore 70·4 amperes. Inserting this in Equation 83 we obtain the initial acceleration—namely, 0·415 f.p.s. per second. If the resistance is taken out as the car speeds up, the current being kept constant, the acceleration will be constant until the resistance is all out. The speed is then given by the consideration that at that moment the motor is taking 100 amperes, and the resistance in circuit is that of the motor only, that is to say, 0·4 ohm. Inserting these values in the speed equation, we get 41·5 f.p.s., or 288 revolutions per minute. The time from the start is 100 seconds, and the distance travelled is 2,075 feet. These results are shown in Fig. 46. Throughout this step the current per motor remains equal to 100 amperes.

STEP II.—Starting rheostat all out; motors speeding up according to law of Equation 69; acceleration gradually diminishing.

From Equation 87 we find the time factor to be 6·1 seconds. Since the final speed is 306 r.p.m., and the speed when the resistance is all out is 288 r.p.m., *a* is 0·058, so that the speed at 6·1 seconds on Step II. is 300 r.p.m., and the current per motor has fallen to 52·5 amperes. Now to find the distance travelled during this step, say in 80 seconds from the point when the rheostat was all out. We see that if the motor had attained its full speed immediately on Step II., the distance travelled would have been 3,520 feet; but we must deduct from this an amount $s_f a T$, or 15 feet, so that the whole distance



300
250
200
150
100
50
0
0
50
100
150
200
250
300
Amperes and Revolutions per minute
Seconds
SPEED
CURRENT
PARALLEL CONTROL shewn
SERIES
SERIES-PARALLEL
= {23 × 10⁴ Foot Pounds
90 Feet

FIG. 46

travelled in 3 minutes from the start is 5,580 feet. The distances travelled are plotted in Fig. 47. It is noticeable how short a time elapses on Step II. before the final speed is practically reached, and how rapidly the current then diminishes.

We might suppose that a greater initial acceleration could be attained if the resistance at the start were wholly in the motor itself. If we look at the equation for M, we see that the greatest possible resistance that the motors can have is given by the equation $R=\frac{E^2}{8TS}$, when E, T, and S are given. If R is greater than this, the equation for M is insoluble. In the case before us the limiting value for R is 4·3 ohms. If we adopt this value and the current is not to exceed 100 amperes per motor, we cannot dispense with a rheostat. Suppose that we make our motors of 4·3 ohms each, and have besides a starting rheostat of 0·7 ohm. The expression under the root vanishes, and we find the value of the induction factor to be 49·1, that is to say, M must have this value if the car is to run at the specified speed with the given load. The current required to overcome friction is now 57·2 amperes, leaving 42·8 amperes available for acceleration. The torque available for acceleration is then 2,960 inch-pounds, and the initial acceleration is 0·13 f.p.s. per second, or about one-third of its former value; so that, putting aside the question of the decreased efficiency when running at full speed, we see that the acceleration is nearly one-third of what it is when the resistance is 0·4 ohm. It appears then that an increase in the resistance of the motor involves a decrease in the value of M, and consequently in the acceleration.

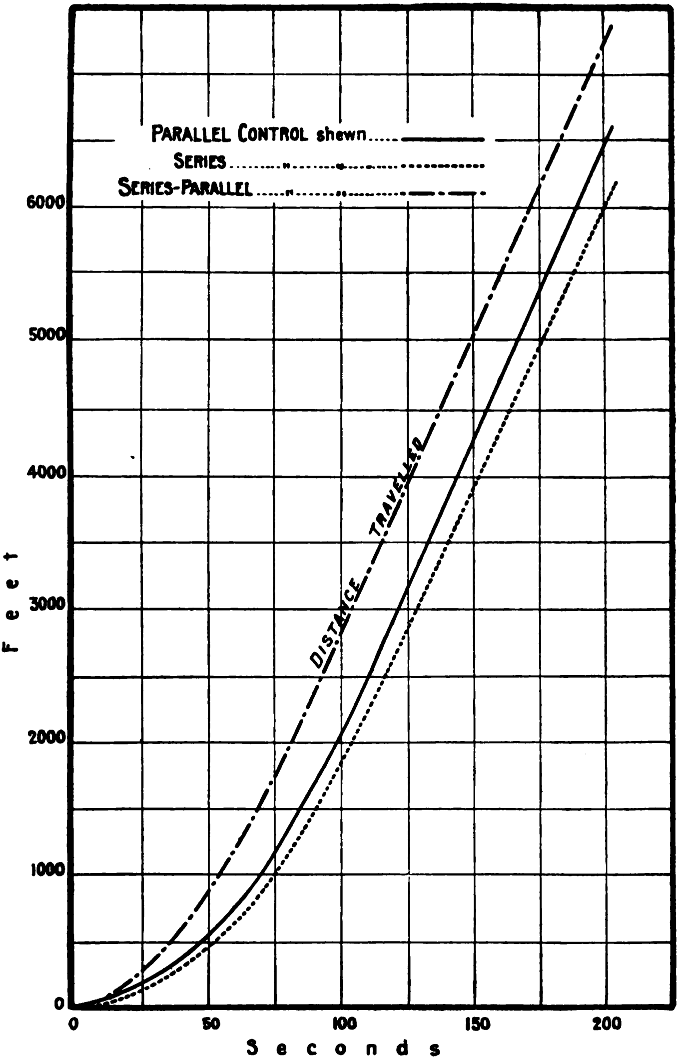

PARALLEL CONTROL shewn
SERIES
SERIES-PARALLEL
DISTANCE TRAVELLED
6000
5000
4000
3000
2000
1000
0
Feet
0
50
100
150
200
Seconds

Fig. 47

Returning to the case when the motors have the lowest possible resistance, we can calculate how much of the energy taken from the line is lost in the rheostat. The total energy expended on the first step is $500 \times 100 \times 100$ watt-seconds, or 369×10^4 foot-pounds, per motor. This is expended in three ways.

1. In accelerating the car. At the end of 100 seconds the car is found to be moving at 41·5 feet per second; taking half the weight, we find the kinetic energy, $\frac{Wv^2}{2g}$, to be 120×10^4 foot-pounds. 2. In heating the rheostat and motor. The mean resistance during 100 seconds is 2·7 ohms. The energy used in heating is thus 199×10^4 foot-pounds. 3. In overcoming friction. This involves exerting a force of 242 pounds through a distance of 2,075 feet, giving 50×10^4 foot-pounds. We can then construct a table of energy expended for one motor, thus:—

Kinetic energy . .	120×10^4	foot-pounds	32·5 per cent.
Heating . . .	199×10^4	"	54·0 "
Friction . . .	50×10^4	"	13·5 "
Total . .	369×10^4		100·0

An inspection of Fig. 47 shows the influence of the acceleration period upon the distance travelled in a given time. If the distance travelled is considerable, the rate at which the car speeds up will have but a small influence on the distance covered in a given time; but if the distance is small, the time occupied in getting up speed becomes of great importance.

CASE II. **Motors started and run in series.**— The terminal tension on each motor is now 250 volts, and the resistance 0·4 ohm. We find from Equation 21 that in order to comply with the specification the motors must have an induction factor of 44. The current required to overcome friction will be 64·4 amperes. Since the motors are in series each will now carry the full current from the line, namely, 200 amperes; hence the current available for acceleration is 135·6 amperes.

STEP I.—Motors in series. Starting rheostat of 1·7 ohms in circuit. Considering one motor and half the weight of the car, we get from Equation 83 an initial acceleration of 0·367 f.p.s. per second. The speed when the rheostat is all out is 232 r.p.m., the time occupied is 91 seconds, and the distance travelled 1,518 feet.

STEP II.—Motors still in series. Rheostat all out. Acceleration diminishing according to Equation 69. We have $\tau = 29{\cdot}1$ seconds, $a = 0{\cdot}24$. Hence in fifteen seconds the speed is 262 r.p.m., and the current is 145 amperes. At 40 seconds the speed is 280 r.p.m. and the current 110 amperes; at 90 seconds the speed is 302·6 r.p.m. and the current is 70 amperes. These results are also shown in Figs. 46 and 47.

We see that this method compares badly with that of Case I., both in respect of the distance travelled in a given time and of the energy expended. The rheostat is all out at a lower speed, and the high time factor causes the curve to bend over as the car speeds up. The final current for two motors is 64·4 as compared with 59·2 amperes in Case I.

If we consider the car as 'started' when the motors have attained a speed 300 r.p.m., or 29·5 miles per hour, we can find with a planimeter the area of the curve giving the work done when this speed is attained. The car is started in 160 seconds; the total energy taken from the line is then 972×10^4 foot-pounds; in the first case the car is started in 107 seconds, and the energy is 753×10^4 foot-pounds.

CASE III. **Motors started in series and run in parallel.**—This is known as the 'Series-Parallel' method. Since the motors are to run at full speed in parallel they will have the same induction factor as in Case I.; hence $M=95{\cdot}8$.

STEP I.—Motors in series. Starting rheostat of 1·7 ohms in the circuit. Frictional current 29·6 amperes. Current available for acceleration 170·4 amperes. The initial acceleration is then 1·005 f.p.s. per second. The speed of the motors when the rheostat is all out is 106·4 r.p.m. The time on Step I. is 15·3 seconds, and the distance travelled is 118 feet.

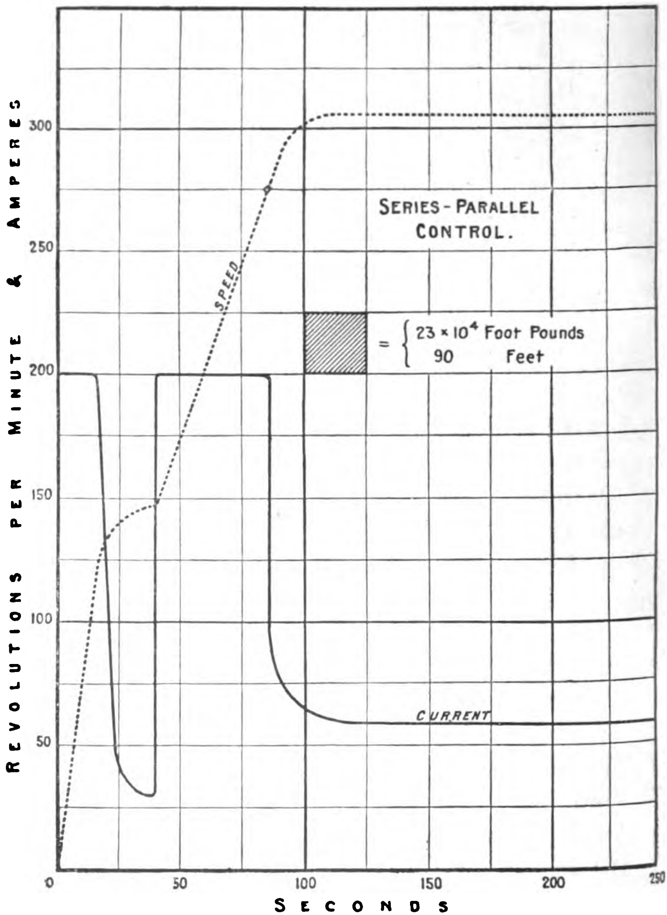
STEP II.—Motors in parallel. Rheostat again in circuit. On switching over to parallel the current from the line must not exceed 200 amperes. If X is the resistance in series with each motor at the beginning of Step II. we have, since the speed is the same as it was immediately before switching over, $\frac{250-200\times{\cdot}4}{M}=\frac{500-100\times X}{M}$ or $X=3{\cdot}3$ ohms. As the internal resistance is 0·4 ohm, we shall need a rheostat of 2·9 ohms in series with each motor. The current per motor will then be 100 amperes. The speed of the motors when this resistance is all out is 288 r.p.m., as in Case I. The acceleration here is 0·415 f.p.s. per second, the time occupied is 63 seconds, and the distance travelled 1,789 feet.

STEP III.—Rheostat all out. Acceleration diminish ing according to Equation 69. This step will commence at 78 seconds from the start. The form of the curves will be the same as those in Case I. The car will be started in 87 seconds, and the energy taken from the line in that time is 605×10^4 foot-pounds. These results are shown in Figs. 46 and 47.

This method is better than either of the former methods. The gain is made on the first step, where the acceleration is high. This step, however, lasts only for 15 seconds, as the motors speed up so quickly that the rheostat is soon all out.

If at the end of the first step we allow the motors to speed up in series they will attain a final speed of 149 r.p.m., and this speed will be reached in about 25 seconds, or 40 seconds from the moment of starting. We may connect the motors in parallel at any moment while they are speeding up in series. Thus in Fig. 48 they are shown to have been connected in parallel just at the moment when full speed in series has been reached. They now speed up as in Fig. 46, full speed is reached in 95 seconds, with an expenditure of energy of 497×10^4 foot-pounds. It thus appears that there is a saving of energy in allowing the motors to attain full speed in series, but there is a loss of time amounting to 15 seconds. The energy saved is not increased if the throwing over into parallel is delayed for any number of seconds, but the loss of time is proportionately increased.

The motors may be connected in parallel at the moment when the acceleration on Step I. is the same as that on Step II.; this point can be found in Fig. 48 by drawing a tangent to the rounded portion of Step I., parallel to the acceleration curve on Step II. There would



REVOLUTIONS PER MINUTE & AMPERES
300
250
200
150
100
50
0
50
100
150
200
250
SECONDS
SPEED
SERIES-PARALLEL CONTROL.
= { 23 × 10⁴ Foot Pounds
90 Feet
CURRENT

FIG. 48

then be no abrupt change in acceleration, and therefore no shock, as there is in the case illustrated in the figure. The car would then be started in 80 seconds, with an expenditure of 562×10^4 foot-pounds. This is, therefore, the best method from the point of view of the time occupied. If the motors are allowed to speed up in series after this, there is a small gain in energy but a loss in time.

The results of the previous calculations may for convenience be arranged in tabular form as follows :—

—	Parallel	Series	Series—Parallel
Induction factor . .	95·8	44·0	95·8
Time factor . . .	6·1	29·1	6·1
Final current, total from the line	59·2	64·4	69·2
Initial acceleration, f.p.s. per second	0·415	0·367	1·005
Initial accelerating torque, inch-pounds	9520	8400	23000
Time occupied in starting, seconds	107	160	80
Distance travelled in starting, yards	770	1450	700
Energy expended in starting whole car, foot-pounds	753×10^4	972×10^4	562×10^4

We shall now illustrate the three methods of control here described by diagrams of current and acceleration obtained in practice.

In Fig. 49 curves are given showing the variations in the current during the process of starting up a car weighing 6·25 tons by two different methods of control. The dotted line is the current curve obtained with the parallel method. The first step is irregular, owing to the way in which controlling apparatus is handled. The second step is, on the whole, clearly shown. The full line gives the current curve obtained by starting up the same car by the series-parallel method. The gradual decrease of the current during the second stage of the first step is clearly marked, the irregularities being due to the uneven

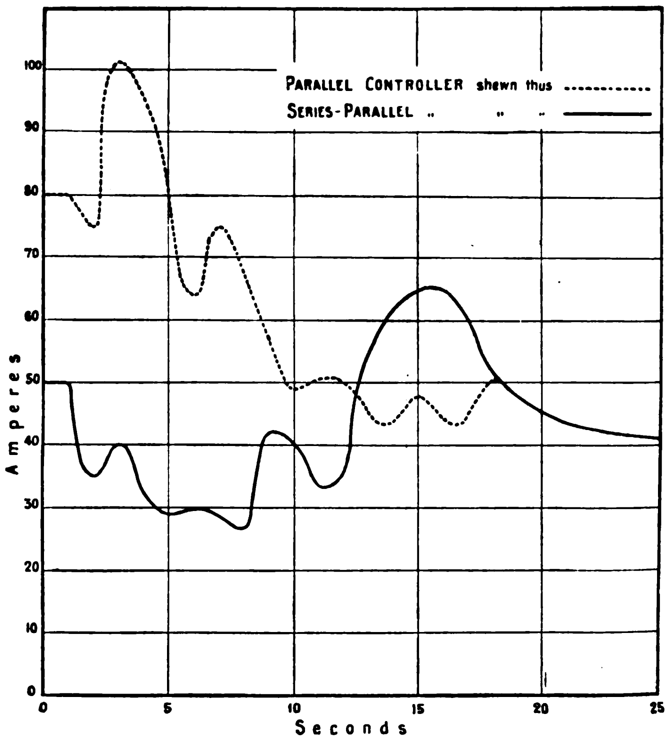


FIG. 49

handling of the controller. The hump in the curve at nine seconds from the start is due to a step in the controller which weakens the magnets, thereby increasing the current momentarily, and slightly increasing the speed. There is no advantage in this, as it merely extends the time of speeding up under a low acceleration, and involves complications in the controller connections. The motors are thrown into parallel at eleven seconds from the moment of starting. At this point the car has almost ceased to

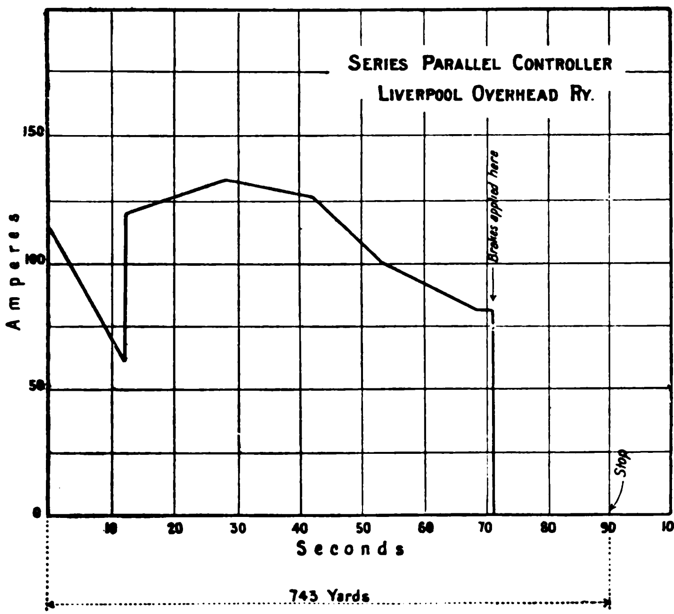


FIG. 50

accelerate, so that it is subjected to a sudden shock which would be avoided if the motors were not permitted to speed up in series. Step II. is slightly rounded off, but Step III. is well shown.

In these experiments the parallel method gives a quicker start with a greater expenditure of energy. We must not assume that the parallel method necessarily takes a greater current than the series-parallel method. We may adjust the starting rheostat so as to take the same current in both cases, but the parallel method would then occupy a longer time in starting. Neither of these curves represents the best start that might be made with the two different controllers, but they afford a fair comparison and show the superiority of the series-parallel method.

In Fig. 50 a curve is given showing the current taken by the motors on the Liverpool Overhead Railway. In the test of which this curve is a record, the weight of the train was 39 tons, and the average tension of the line 484 volts. For further particulars the reader is referred to Mr. Thomas Parker's paper in the 'Proceedings of the Institution of Civil Engineers,' Vol. CXVII.

This railway is equipped with motor cars each carrying two series-wound motors controlled on the series-parallel method. No gearing is used, and the wheels have a diameter of 33 inches. The curve gives the total current from the line for one train. The notch in the curve represents the period during which the motors are speeding up in series. When the current was cut off and the brakes put on, the train came to a standstill in 19 seconds.

We may notice that these motors have a very high internal resistance, amounting to as much as 1·04 ohms each, made up as follows:—armature resistance 0·67 ohm; magnet resistance 0·37 ohm. The object of making the resistance so high appears to have been to get as much as possible of the resistance at starting in the form of convolutions on the armature, the supposition being that if the starting rheostat could be entirely dispensed with, a much greater starting torque could be obtained.

The specification, however, called for a definite maximum speed—namely, 30 miles per hour, and we have already seen that when the final speed is given, the greater the internal resistance, the smaller will be the induction factor, and consequently the smaller will be the torque for any given current. The induction factor of these motors with 100 amperes in the series-wound magnets is 172, giving a tractive effort of 1,460 pounds per motor on 33-inch wheels. If the resistance had been, say, 0·387 ohm per motor, the induction factor—to run at the given speed—might have been 184 at 100 amperes, giving a tractive effort of 1,560 pounds. The **high resistance** thus **involves a loss** of over 6 per cent. **in the starting torque**. Some motors recently designed for this line have a resistance of 0·83 ohm (*vide* Paper by Mr. S. B. Cottrell published in the 'Electrician' of October 9, 1896).

The motor cars of the City and South London Railway are each equipped with two series-wound motors controlled on the series principle. Diagrams of current and acceleration, the results of experiment, have been given in Fig. 41. The weight of the train is 35 tons; the tension of the line is 400 volts. The motors are gearless, with an internal resistance of 0·387 ohm each. As these motors have to make up their speed in series, the induction factor will be less than half that of the Liverpool motors. The value of this factor for 100 amperes is 61 (see Fig. 39) and the tractive effort is 640 pounds, the wheels being 27 inches in diameter.

If we take the friction to be the same as for the Liverpool motors—namely, 20 pounds per ton on 33-inch wheels, the tractive effort required to overcome this friction will be 430 pounds. This leaves us with only 210 pounds per motor for accelerating, as compared with 1,030 in the Liverpool motors. This difference is due to the fact that the friction absorbs so large a proportion of the total tractive effort. Speaking roughly, we may say that two of the Liverpool motors in series will give for the same current twice the pull of two of the South London motors in series, and that the accelerations will show a still greater difference in proportion as the friction is increased. If the friction, instead of being 20 pounds per ton on 33-inch wheels, is 10 pounds per ton, the acceleration of the South London motors is increased from one-fifth to one-third that of the Liverpool motors.

Figs. 51 and 52 contain the records of tests made on the Buffalo and Niagara Falls Electric Railway by Messrs. H. P. Curtiss and H. O. Pond. A full account of these experiments may be found in the 'Street Railway Review' for July 1896. We may here note the following data:—

The cars on which the tests were made were 36 feet in length and weighed 13·6 tons. Each car is equipped with four G. E. 800 motors, one on each axle. The velocity ratio is 4·78 and the wheel diameter 33 inches. The motors are controlled on the series-parallel method in pairs, thus: Step I., two in series, two in parallel. Step II. four in parallel. The average tension of the line at the car was 550 volts. The track where the tests were made was straight and level.

The vertical ordinates of the current curves give the total current for the car, so that when in series the current per motor may be taken as half, and when in parallel, as one-fourth of the ordinate of the current curves.

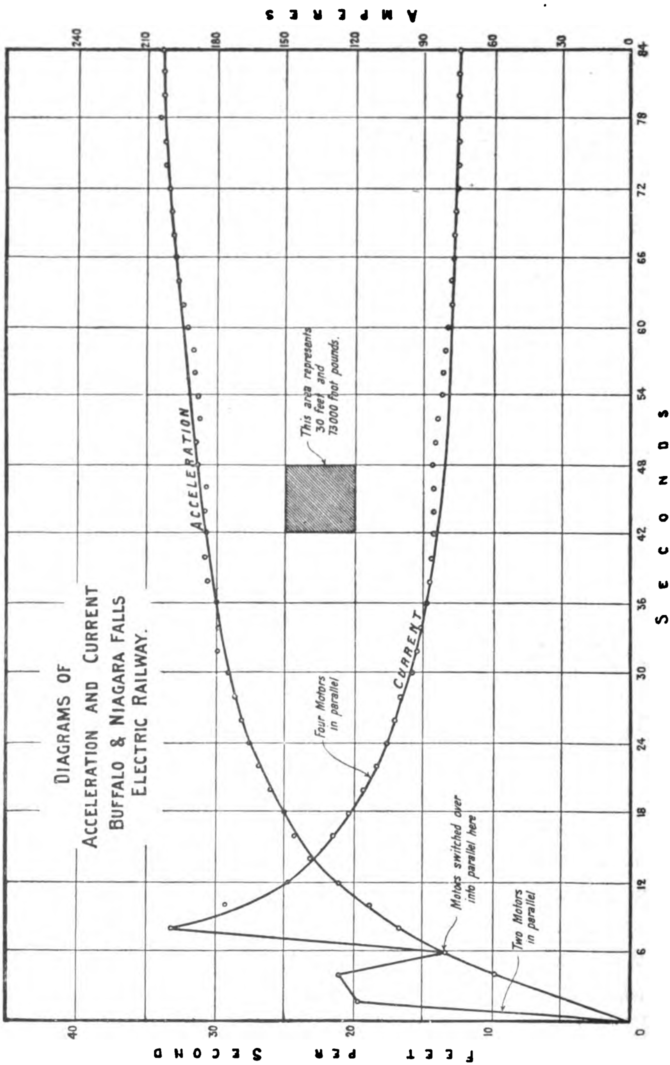
The vertical ordinates of the acceleration curves give the speed of the car in feet per second. The base of both acceleration and current curves gives the time in seconds, measured from the moment of starting.

The curves of induction factor and torque for the G. E. 800 motor have been given in Fig. 21, and may be taken as representing very nearly the action of the motors used on the Buffalo and Niagara Falls line.

Fig. 51 gives the record of a test arranged to effect a continuous start. The point at which the motors are thrown over into parallel can only be detected from the current curve, the acceleration curve being unbroken by any sudden change of curvature. This means that the start was effected without shock. The dip in the current curve at six seconds from the start shows that the motors were allowed to speed up in series for a short time, but not long enough to cause a shock in the transition to parallel. In other words, the change over was made when the acceleration in series had diminished to the value of the acceleration on the parallel connection.

The current per motor at the start was 60 amperes, but the full current was not thrown on at once, a period of about two seconds elapsing before the maximum current per motor is reached. If we take the initial friction to be twice as great as the running friction, the current must rise to 72 amperes before the car will begin to move. The friction decreases to the running value directly motion commences.

When the motors are thrown into parallel connection the current from the line rises from 120 to 200 amperes, while the current per motor decreases from 60 to 50 amperes. The form of the current curve as the motors



DIAGRAMS OF
ACCELERATION AND CURRENT
BUFFALO & NIAGARA FALLS
ELECTRIC RAILWAY.
ACCELERATION
CURRENT
This area represents
30 feet and
13000 foot pounds.
Four Motors
in parallel
Motors switched over
into parallel here
Two Motors
in parallel
AMPERES
0
30
60
90
120
150
180
210
240
SECONDS
0
6
12
18
24
30
36
42
48
54
60
66
72
78
84
FEET PER SECOND
0
10
20
30
40

FIG. 51

speed up is well shown. The final current is 18 amperes per motor.

The mean current for the first four seconds from the moment when the circuit is made appears to be 75 amperes. This would give us a total torque per motor (see Fig. 21) of 2,430 inch-pounds. Deducting the frictional torque corresponding to a current of 18 amperes per motor —namely, 755 inch-pounds—we have 1,675 inch-pounds available for acceleration. Since the weight to be accelerated is 3·4 tons per motor, the acceleration is 2·05 f.p.s. per second. The acceleration curve has been drawn as if the acceleration were constant from the moment of making the current; this is not strictly correct. The curve should cut the time base about one second from the origin, but it gives a fairly accurate measure of the mean initial acceleration, which by measurement appears to be about 2·1 f.p.s. per second.

Fig. 52 is the record of a test in which the motors were allowed to speed up in series. The current taken does not appear to have been quite so much as in the case represented in Fig. 51. The form of the current curve is well shown, both with the series and with the parallel connection, and indicates the jerk experienced when the motors are thrown into parallel, the acceleration at this point being greater than at the moment of starting. The energy required to attain a speed of 35 f.p.s. is less than that required by the method illustrated in Fig. 51.

The diagrams show a considerable increase in the current taken from the line at the moment when the motors are thrown into parallel connection. It is instructive to inquire if this increase is necessary, and to

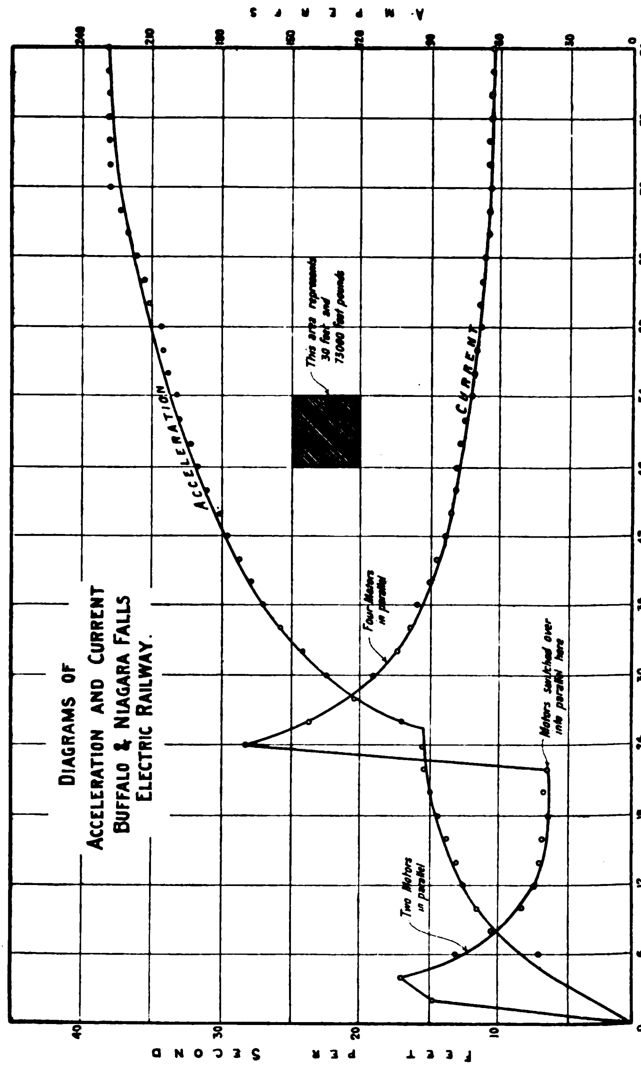

Diagrams of
Acceleration and Current
Buffalo & Niagara Falls
Electric Railway.
Acceleration
Current
This area represents
30 feet and
73000 foot pounds
Four Motors
in parallel
Two Motors
in parallel
Motors switched over
into parallel here
Amperes
0
30
60
90
120
150
180
210
240
Seconds
0
6
12
18
24
30
36
42
48
54
60
66
72
78
84
Feet per Second
10
20
30
40

Fig. 52

find to what extent the motion of the car is affected by the amount of current taken at this point.

Let us take the case of a car equipped with two G. E. 800 motors, whose induction and torque curves are given by the diagrams in Fig. 21. Suppose that the car weighs 10 tons, that it runs on 33-inch wheels, with a velocity ratio of 4·78. Let the tension of the line be 500 volts, and the resistance per motor 1·245 ohms. Suppose, also, that the frictional resistances are 325 inch-pounds of torque per motor, so that the car would take 24 amperes when running at full speed with the motors in parallel. If the maximum current per motor is 50 amperes, we see from the torque curve in Fig. 21 that the corresponding torque would be 3,700 inch-pounds. Deducting the frictional torque, we have 3,375 inch-pounds available for acceleration, from which we find the acceleration to be 2·8 f.p.s. per second. The results are plotted in Fig. 53.

The motors will be in series up to the point a; after this, three cases have been taken; the first assumes 50 amperes per motor when in parallel, the second 35, and the third 25 amperes. The corresponding current curves give the total current from the line in the three cases. The acceleration when in parallel ceases to be constant in each case at the point f. The areas of the acceleration curves have been integrated with a planimeter, and the distances travelled plotted in three curves, of which the vertical ordinates represent feet. From these curves we can obtain the time required to cover any distance, and also the corresponding energy for each of the three maximum currents, 100, 70, and 50 amperes, lettered in the table A, B, and C respectively. If we take distances

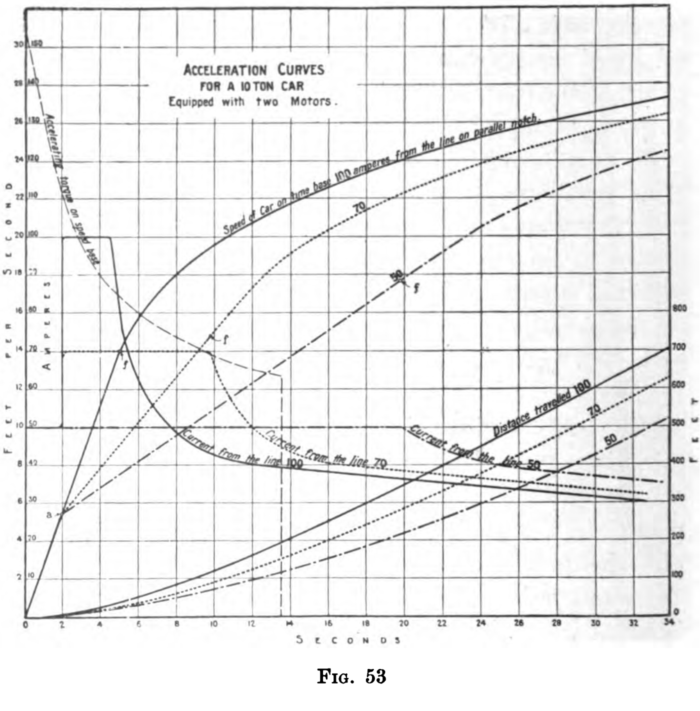

ACCELERATION CURVES
FOR A 10 TON CAR
Equipped with two Motors.
Speed of Car on time base 100 amperes from the line on parallel notch.
Accelerating torque on speed base.
Current from the line 100
Current from the line 70
Current from the line 50
Distance travelled 100
FEET PER SECOND
AMPERES
SECONDS
FEET

FIG. 53

of 300, 400, 500, and 600 feet, we get the results shown in the following table :—

Distance travelled. Feet	Time occupied. Seconds			Energy used. 1,000 foot-pounds		
	A	B	C	A	B	C
300	18	21	24	361	423	438
400	22	25	29	417	477	505
500	26	29	33	465	528	563
600	30	33	37	506	575	610

We see that when the distance is greater than 300 feet, the difference in time between the 100 and the 50 ampere arrangement is constant, and equal to seven seconds. The actual difference in the energy increases slightly with the distance, the smaller current involving a greater expenditure of energy; but the percentage difference remains very nearly the same at all distances.

Since the difference in time for a given distance becomes constant, it follows that the difference in energy must also become constant. This would be clearer if the current curves were continued to the point at which they coincide; we should then see that the difference in the shape of these curves was only at the beginning, and that an increase in the distance travelled simply means adding an equal area to each.

We may think of two cars on parallel lines started up at the same moment, one taking 100 and the other 50 amperes. The former will start off quicker, but in time they will both attain the same speed, and the former will then keep seven seconds ahead of the latter. The difference in the energy for any given distance is evidently the difference occasioned during the period of starting.

When the stops are frequent it becomes worth while to consider the saving of time effected by taking a large current at the start; but in suburban traffic this consideration would be of small importance. On the other hand, we must remember that it is mainly the peaks in the current curves for individual cars that are responsible for the irregular load at the power house, and that the advantages of a steady demand will probably outweigh any gain that might result from a small saving in the time of starting.

CHAPTER X

TIME CURVES

WE shall now consider to what extent we can improve the conditions of starting by a suitable choice of induction factor, velocity ratio, and diameter of driving-wheel.

The problem presents itself in two ways. 1. The final speed is specified, and we have to design an equipment that shall start up from rest and attain a certain fraction of the final speed in the shortest time for a given current from the line. 2. The final speed is not specified, and we have to design an arrangement that shall start up from rest and cover a given distance in a given time with the least possible expenditure of energy.

We may here restate the equation for the initial acceleration:—

$$a = 405 \cdot 10^{-4} \frac{Mv}{d} \frac{c_a}{W} \text{ f.p.s. per second} \quad \ldots(93).$$

In this equation c_a is the current available for acceleration, and W the weight in tons per motor. If the maximum current per motor is given, c_a can be found by deducting from it c_f the current required to overcome the frictional and other retarding forces.

For the present we shall assume that the induction factor remains constant. First let us take the **case where the final speed** S in miles an hour **is specified.** We have

$$\frac{Mv}{d}=\frac{E}{11 \cdot 2S}\left\{1+\sqrt{\left(1-\frac{7 \cdot 96RTS}{E^2}\right)}\right\} \quad \ldots\ldots(94).$$

From this we see that when R, T, S, and E are given, the ratio $\frac{Mv}{d}$ is fixed. Referring now to Equation 93, we find that under these conditions the only way of increasing the acceleration is to increase c_a. When the maximum current is fixed we can do this only by reducing the value of c_f, and this is given by the equation

$$c_f=\frac{E}{2R}\left\{1-\sqrt{\left(1-7 \cdot 96\frac{RTS}{E^2}\right)}\right\} \quad \ldots\ldots(95).$$

Hence since E, R, T, and S are all fixed we cannot alter c_f. We conclude then, that when the final speed, the value of T, and the maximum current are fixed, the initial acceleration is also fixed.

As an illustration we may take the case of the motors used on the Baltimore and Ohio Railroad. The conditions are as follows:—A freight train weighing 780 tons has to start from rest on a grade of 0·8 per cent. and move up the incline at 10·7 miles per hour. The train is to be drawn by a locomotive equipped with four motors, which are permanently connected in series across a 625-volt line. The motors are gearless, the internal resistances when hot, are for the armature 0·0154, for the magnets 0·0055, total 0·0209 ohm. The maximum permissible current is

1,800 amperes. The tractive effort required for the train is 9 pounds per ton of load.

The given weight includes that of the locomotive, and the tractive effort required for the latter is to be reckoned in the same way as for the train; each motor has then to draw 195 tons with a tension of 156·2 volts.

The draw-bar pull per motor for the grade is 3,490 pounds, and for friction 1,755 pounds, making altogether 5,245 pounds. If we allow 95 per cent. as the mechanical efficiency of the motors, the value of T to be inserted in Equation 94 is 5,500 pounds; using the given values of R, S, E, and v, we find that $\frac{M}{d} = 2{\cdot}33$. We have thus a wide range of possible values of M and d, any of which would satisfy the required conditions, but whatever values are taken we shall find the time of starting up to be the same.

We might, for instance, take $M = 150$ and $d = 64$ inches, the friction current would then be 795 amperes, and the accelerating current 1,005 amperes. From Equation 93 we find the acceleration to be 0·49 f.p.s. per second. The speed when the rheostat is all out is 13·3 f.p.s., and the time occupied on this step is, therefore, 27 seconds. The time factor is 4·8 seconds from Equation 87. If we consider the start to have been effected when the speed is 0·98 of the final speed, the time must be increased by 2τ, say by 10 seconds, so that the train will have been started in 37 seconds from rest. Nothing is gained by increasing the induction factor or the diameter of the driving-wheels. The values actually used are: $M = 144$ at 800 amperes, and $d = 62$ inches.

The induction curve for these motors has been given in Fig. 31. The value of M at 1,800 amperes is probably about 165, but the test was not carried up to this point.

Fig. 54 gives the current curve on a time base, as actually observed, and is taken from a paper by Mr. L. H. Parker in the 'Street Railway Journal' for March 1896. At 0·98 of the final speed the current is 930 amperes; in the figure this is reached at 45 seconds from first making the current. But we see that eight seconds was occupied in increasing the current from nothing to 800 amperes, which is that required to overcome friction, so that the train could not begin to move until 8 seconds after the current was first made, giving us 37 seconds for the actual start.

We shall now consider the **case where** the final speed is specified, and **the value of** T, instead of being fixed as before, **depends upon the value of** d. This is the case in motor cars, where the resistance to motion consists in a torque t_f at the car axles to which the motors are geared. Since $T=\frac{2}{d}t_f$ we can reduce the value ot T by increasing the size of the driving-wheel, the value of t_f the frictional torque, remaining constant. By putting $2\frac{t_f}{d}$ for T in Equation 95 we see that the ratio $\frac{Mv}{d}$ increases slightly with an increase of d for a given speed. The frictional current is given by the equation

$$c_f=\frac{E}{2R}\left\{1-\sqrt{\left(1-\frac{15\cdot92RSt_f}{E^2d}\right)}\right\} \;\ldots\ldots(96).$$

From this we see that c_f decreases with an increase in d,

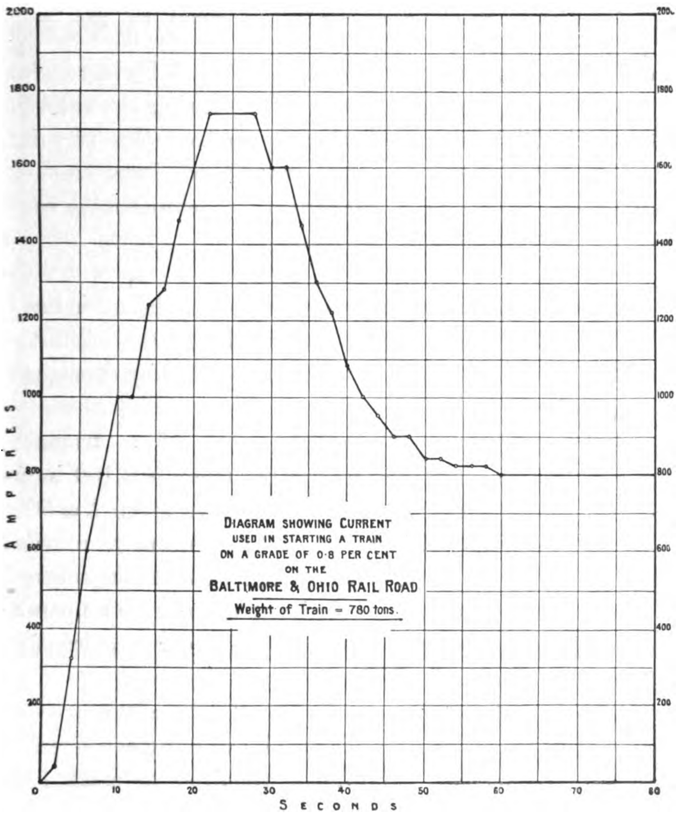


DIAGRAM SHOWING CURRENT
USED IN STARTING A TRAIN
ON A GRADE OF 0·8 PER CENT
ON THE
BALTIMORE & OHIO RAIL ROAD
Weight of Train = 780 tons.
AMPERES
SECONDS
0
10
20
30
40
50
60
70
80
200
400
600
800
1000
1200
1400
1600
1800
2000

FIG. 54

but is independent of v, in other words, we can reduce the current required to overcome the frictional resistance by increasing the wheel diameter but not by altering the velocity ratio. Since, however, $\frac{Mv}{d}$ is given by the final speed, when d has been fixed we can reduce M by increasing v, *i.e.* we can lighten the motor by increasing the velocity ratio, although we cannot thereby reduce the running current. We must also note that if we increase d we must at the same time increase M or v to maintain the ratio $\frac{Mv}{d}$ constant.

In street railway equipments v is usually taken as large as possible, to reduce the weight of the motor, while the size of the driving wheel is generally limited by structural considerations.

Example 49*a*.—Take the case of a tramcar. Suppose that the frictional torque per motor is 8,100 inch-pounds at the car axles; and that we have given $E=500$, $R=1{\cdot}25$, $M=60$, $v=4{\cdot}78$, $d=33''$; maximum current from the line 60 amperes, $W=8$ tons, series-parallel control.

Then we get Step I. Frictional current 20 amperes; accelerating current $60-20=40$ amperes; acceleration 3·52 f.p.s. per second; time 1·5 seconds.

STEP II.—Accelerating current $30-20=10$ amperes; acceleration 0·88; time to full speed, assuming acceleration constant, 10 seconds; total time required to start 11·5 seconds.

Now suppose that we put on 54 instead of 33-inch wheels. To get the same final speed as before, 14·3 f.p.s., or 9·75 m.p.h., we must make $v=7{\cdot}81$. The frictional current is now 12 amperes, so that the accelerating current on the first step is 48 amperes, and the acceleration 4·3 f.p.s. per second, and the time 1·2 seconds. On the second step the acceleration is 1·61, the time 5·65 seconds, and the total time of starting 7 seconds. So that by putting on larger wheels and increasing v we have reduced the time of starting from 11·5 seconds to 7 seconds.

It appears, then, that when the resistance to motion consists of frictional torque at the axles, as in motor cars, we can reduce the time of starting by putting on larger wheels and increasing v or M; but that if the resistance to motion is in the form of draw-bar pull, as with locomotives, we cannot quicken the start by increasing the weight of the motor and the diameter of the driving-wheels.

The reduction of the frictional and other resistances to motion is thus **of great importance**, for we can thereby reduce the frictional current and have a greater proportion of the total current available for acceleration.

Let T be the total tractive effort produced by a given current, T_f the frictional and other resistances of a train weighing W_1 tons. The acceleration will be $\frac{T-T_f}{69{\cdot}5W}$ f.p.s. per second.

If the frictional resistances are reduced to $\frac{1}{\mu}T_f$, we can get the same acceleration as before with a train weighing W_2 tons, where W_2 is given by the equation

$$W_2=\frac{\frac{T}{T_f}-\frac{1}{\mu}}{\frac{T}{T_f}-1}W_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots(97)$$

This equation tells us by how much we can increase

the weight of a train without reducing the acce
when the resistance to motion has been diminishe
ratio of μ to 1. It appears that for any given re
of the frictional resistances, the possible increase
weight diminishes with the acceleration.

Example 50.—The trains on the Liverpool head Railway were made up of two cars, each wei 19 tons, giving 38 tons as the weight of the train. train was driven by two motors taking 160 amper the start when in parallel, or 80 amperes per motor. resistance to motion appeared to be 410 pounds per m from the record of the current taken by the motors w running at a uniform speed. The question arose as whether an additional car could be put on each train if resistance to motion were reduced by the use of roller be ings, the acceleration and the maximum current from t line remaining unaltered. Tests were made showing that train fitted with roller bearings offered only one-fifth of th resistance to motion at the moment of starting, of that of train with ordinary bearings. This gave $\mu=5$. Experi ment showed that the tractive effort per motor for 80 amperes was 1,100 pounds. Using these values in Equation 97, we find that the weight of the train may be increased to 56 tons with unaltered acceleration. At the moment of starting the motors are connected in series, and then switched over into parallel; this affects the result slightly in favour of the roller bearings, the time of covering the first 400 yards being, by calculation, 57 seconds with ordinary, and 52 seconds with roller bearings. If no other alteration is made, the final speed will be increased by the introduction of the new bearings from 26 to 48 miles an hour.

We have now to consider the **case where the final**

speed is not given, but may be made anything we please.

Suppose that the distance D in feet that has to be covered and the accelerating current per motor are given. The problem then before us is to determine the values of M, v, and d, so that the distance may be covered in the shortest time.

We may divide the whole period of motion into two parts, that of acceleration and that of uniform speed. We shall assume for the present that the parallel method of control is used, and that the motor or motors speed up with uniform acceleration until full speed is reached.

In Fig. 55 let hb represent the time occupied in accelerating, ba the final speed, and hg the time occupied in completing the given distance; the area $hafg$ will then represent the whole distance travelled.

We know from Equation 93 that the acceleration varies inversely as $\frac{d}{Mv}$. We may express this by the equation

$$\frac{ab}{bh}=k_1\frac{1}{\beta} \quad \text{.....................(98)},$$

where k_1 is a constant and $\beta=\frac{d}{Mv}$.

The final speed is given by the equation

$$s_f=262\times 10^{-3}\times\frac{d}{v}\times\frac{E-c_fR}{M}\text{ f.p.s.} \quad \text{......(99)},$$

where E is the tension of the line and c_fR the internal drop at full speed. If c_fR is specified, the final speed will be independent of variations of c_f, which will, as we have seen, decrease slightly when d is decreased. In other words, if we are at liberty to fix the drop by adjusting the

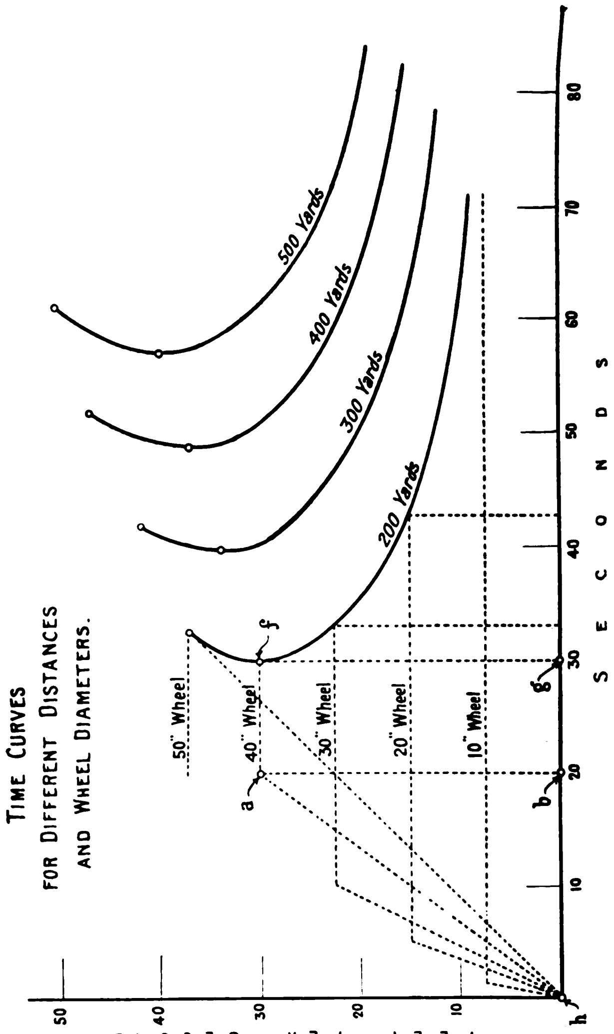

TIME CURVES
FOR DIFFERENT DISTANCES
AND WHEEL DIAMETERS.
500 Yards
400 Yards
300 Yards
200 Yards
50" Wheel
40" Wheel
30" Wheel
20" Wheel
10" Wheel
a
b
f
g
h
10
20
30
40
50
60
70
80
SECONDS
FEET PER SECOND

FIG. 55

value of R, the final speed will vary directly as the first power of β. We may express this as follows:

$$ab = k_2\beta \quad \text{.....................(99)},$$

where k_2 is a constant.

It appears then that by varying the diameter of the driving-wheel, the induction factor and the velocity ratio, we can vary the way in which the given distance is covered. If, for instance, we put on a small wheel, we shall get a high acceleration and a low final speed, which will soon be reached, and most of the distance will be covered at full speed. If, on the other hand, we use a large wheel, we shall get a low acceleration and a high final speed, which will be reached only after a considerable lapse of time, perhaps not at all, and most or all of the distance will be covered during the accelerating period. Between these extremes there is a certain value of β that will enable us to cover the given distance in the shortest time; this we now proceed to determine.

The area *hafg* may be expressed thus:

$$D = \tfrac{1}{2}\frac{k_2^{\,2}}{k_1}\beta^3 + k_2\beta \times bg \quad \text{............(100)}.$$

We have also $bh = \frac{k_2}{k_1}\beta^2$, and $bg = \frac{D}{k_2\beta} - \frac{1}{2}\frac{k_2}{k_1}\beta^2$; hence the time occupied is given by

$$t = hb + bg = \frac{D}{k_2\beta} + \tfrac{1}{2}\frac{k_2}{k_1}\beta^2 \quad \text{........(101)}.$$

To find the value of β that makes this a minimum, differentiate and equate to nothing; this gives us $\beta^3 = \frac{k_1}{k_2^{\,2}}D$.

But $\frac{bg}{bh} = \frac{k_1}{k_2^2}\frac{D}{\beta^3} - \frac{1}{2}$; hence we find that $bg = \frac{1}{2}bh$. The given distance will then be covered in the shortest time when the distance travelled during the process of acceleration is equal to that travelled at full speed, the time of acceleration being two-thirds of the whole time.

Substituting for k_1 and k_2 their values as given by Equations 93 and 99, we get

$$\left(\frac{d}{Mv}\right)^3 = 0{\cdot}59\,\frac{D}{W}\,\frac{c_a}{(E - c_f R)^2} \quad \ldots\ldots(102).$$

We thus see that when a train of weight W tons has to be started from rest and moved through a distance of D feet, the tension of the line being E volts, the accelerating current c_a amperes, and the internal drop when running at full speed $c_f R$ volts, **the time occupied** in covering the given distance **is least when** the equipment is so designed that **half the distance is covered during the period of acceleration**. In order to secure that this may be so, the ratio $\frac{d}{Mv}$ must be that given by Equation 102. If the ratio $\frac{d}{Mv}$ is greater or less than that given by this equation, the time occupied will be greater than it need be, and to reduce the time we shall have therefore to use a greater accelerating current.

In Fig. 55 time curves for different distances are drawn on a base of seconds. The dotted lines give the acceleration curves for wheels of different diameters and equal values of M and v. Thus we see that when D is 200 yards, a 40-inch wheel will cover the distance in the shortest time, while a 20-inch wheel will take about 10 seconds longer. The condition assumed is, that with a 40-inch wheel the acceleration is 1·5 f.p.s. per second; the final speed is 30 feet per second, M, v, c_a, and W, being the same in all cases.

We here assume that we are dealing with one motor, or that if two motors are used, they are connected in parallel throughout. If there are two motors, connected first in series, and then in parallel, the accelerating current from the line remaining the same throughout, that is, the accelerating current per motor when in series being twice what it is in parallel, the best result is not obtained when half the distance is travelled during acceleration, but when about two-thirds the distance is thus covered. It follows that the best diameter for the series-parallel method of control is rather larger than that for the parallel method, the ratio of the cubes of the diameters being very nearly as the square root of two to one. If then we wish to find the best diameter for the series-parallel method of control, we may substitute 0·834 for 0·59 in Equation 102. The case when the accelerating current per motor is the same throughout the whole process of acceleration is discussed later on.

Equation 102 shows that for any value of Mv there is a certain value of d that will cover the given distance in the shortest time. If d be made larger or smaller than this, the time occupied will be increased. We shall find, however, that there is a considerable range of values above and below the best value, for which the time occupied differs but little from the shortest time.

While the best value of $\frac{d}{Mv}$ enables us to cover the distance in the shortest time, it does not do so with the least expenditure of energy. The larger the value of $\frac{d}{Mv}$ the smaller is the acceleration, and the greater the final speed; hence by increasing $\frac{d}{Mv}$ we lengthen the period during which the motor is accelerating, and during which the maximum current is being drawn from the line. As this period is smallest with the smallest value of $\frac{d}{Mv}$ the energy expended in covering the given distance will decrease with $\frac{d}{Mv}$, so that if $\frac{d}{Mv}$ is greater than the best value, the time is longer and the expenditure of energy in accelerating greater, while if $\frac{d}{Mv}$ is less than the best diameter, the time is also longer, but the expenditure of energy in accelerating is less. Hence in order to save energy during the process of acceleration, we should increase the ratio $\frac{Mv}{d}$ as much as possible, so long as we do not thereby lose too much time in starting.

We must now consider the **effect of changes in the values of Mv and d upon the total current taken from the line**. The current required to overcome the resistance to motion may be expressed thus: For motor cars, where the resistance may be considered as a constant torque of t_f inch-pounds on the car axles, we have $c_f = \cdot71 \frac{t_f}{Mv}$, showing that c_f is independent of d. For locomotives, however, where the resistance to motion may be considered as a constant pull on the draw bar of T pounds, $c_f = \frac{Td}{2\cdot82Mv}$, showing that c_f increases with d.

Now when c_a is given, we can reduce the total current from the line by reducing c_f. Hence, in both motor cars and locomotives, we should make Mv as large as possible, to reduce c_f, and then insert in Equation 102 the values of M and v that have been chosen, and find the best diameter. This will enable us to cover the given distance in the least possible time. In choosing values of M and v we must remember that the weight of the motor increases with M.

In the case of a motor car, we cannot further reduce the frictional current c_f by decreasing d. We can, however, thereby diminish the time during which the motor is drawing the maximum current from the line. This will, of course, increase the whole time occupied, since the diameter found from the equation gives the least time. It may, however, be worth while to sacrifice a few seconds if we can reduce the time during which the maximum current is being drawn from the line.

In the case of a locomotive, we can save current in two ways by decreasing d; first because c_f decreases with d, and secondly because the period of acceleration is thereby shortened, as with motor cars. The reduction of the diameter of the driving-wheel is therefore of greater importance in the case of locomotives than of motor cars.

Example 51.—A motor car weighing 40 tons is equipped with two motors controlled on the series-parallel method, each having a resistance of 0·4 ohms. The tension of the line is 500 volts, the frictional torque 6,600 inch-pounds on each axle, the maximum current per motor, and from the line 200 amperes, and the distance to be covered 700 yards.

The following table gives the time occupied and the energy expended in covering the given distance under different conditions :—

D yards	M	d inches	c_f amperes	t seconds	foot-pounds whole car	r	
700	150	20·0	31	129·0	36×10^5	1	
700	150	33·0	31	89·2	38×10^5	1	
700	150	40·0	31	83·5	45×10^5	1	
700	150	44·2	31	82·0	51×10^5	1	Best diameter for $M=150$
700	150	48·0	31	82·4	57×10^5	1	
700	150	56·0	31	85·6	63×10^5	1	Full speed not reached
700	100	27·3	46	88·1	56×10^5	1	Best diameter for this M
700	150	44·2	31	82·0	51×10^5	1	Best diameter for this M
700	200	60·7	23	79·5	48×10^5	1	Best diameter for this M
700	150	44·2	31	82·0	51×10^5	1	
700	150	93·5	15·5	77·5	46×10^5	2	

We assume that the accelerating current from the line remains the same, hence the best diameter is obtained from Equation 102 by using the numerical constant 0·834. The time occupied has been worked out by taking each step separately.

In obtaining these results we have assumed the accelerating current from the line to be constant throughout the period of acceleration. Thus in the case when the diameter is 44·2 inches, $c_f=31$ amperes; the accelerating current per motor when the motors are in series is 169 amperes, the acceleration is 1·161 f.p.s. per second, the distance travelled is 74 feet, and the time is 11 seconds. When the motors are in parallel the accelerating current per motor is 69 amperes, the acceleration is 0·473, the distance travelled is 1,309 feet, and the time is 51·7 seconds. Full speed is 37·6 feet per second, and the rest of the distance, 717 feet, is covered at this speed in 19 seconds, making in all 82 seconds.

Example 52.—A lift has to be designed to start from rest and raise an unbalanced weight of 1,500 pounds through 15 feet; the frictional torque on the drum axle is known to be 8,000 inch-pounds; the total mass to be moved is 2·5 tons; the tension of the line is 125 volts; the accelerating current is 25 amperes; the drop at full speed is not to exceed 10 volts; the diameter of the rope drum is 36 inches, and the velocity ratio is fixed at 70. To find the induction factor that will cover the distance in the shortest time. Using Equation 102, we find that the induction factor has to be 2·72. The acceleration is 2·14 f.p.s. per second, full speed is 5·7 feet per second, and the total time is 4 seconds. The torque on the drum axle due to the unbalanced load of 1,500 pounds, together with the frictional torque, make up 35,000 inch-pounds, giving a final current of 131 amperes. As the drop is limited to 10 volts, the resistance of the motor must be 0·0765 ohm. The total current at the start will be 156 amperes.

If an induction factor of twice the above value were taken, we could reduce the final current to 65·5 amperes. The time would be increased to 5·6 seconds. For $M=8{\cdot}16$ the running current would be 43·7 amperes, and the time 10 seconds.

These results are tabulated below:

M	Acceleration	Final speed	Total current to start	Time of covering 15 feet
2·72	2·14	5·70	156	4·0
5·44	4·28	2·85	90	5·6
8·16	6·42	1·90	69	10·0

Example 53.—A crane has to lift a weight of half a ton from rest to a height of 20 feet. The induction factor of the motor is 60; the velocity ratio is 5; the frictional torque on the drum axle is 5,000 inch-pounds; the tension of the line is 500 volts; the accelerating current must not exceed 40 amperes; the drop at full speed is to be 50 volts. To find the diameter of the drum, and the value of R, so that the weight may be lifted in the shortest time.

Equation 102 gives at once 50 inches as the best diameter. We then find the acceleration to be 19·4 f.p.s. per second. Full speed is given by equation $n_f = \frac{E - c_f R}{M}$; since $c_f R = 50$ volts, we can deduce the value of the final speed; it is 19·7 feet per second; the time of acceleration is 1·01 seconds; the distance covered in that time is 10 feet; the remainder of the distance is covered in 0·51 second, making the whole time 1·52 seconds.

The torque on the drum axle, due to the weight, is 28,000 inch-pounds, which added to the frictional torque makes the total torque 33,000 inch-pounds; the final current is 78 amperes, so that the resistance of the motor has to be 0·64 ohm.

CHAPTER XI

DESIGN OF RAILWAY MOTORS

WE shall now proceed **to determine the best arrangement for starting up from rest and covering a given distance in a given time.**

If we are at liberty to adjust R so that the drop at full speed is independent of M, v, and d, the best arrangement, if M is constant, will be that in which the motor accelerates for two-thirds of the given time, and covers one-half of the given distance during the period of acceleration.

We have then two conditions given by the two processes of accelerating and running at full speed; the first is deduced from the formula $s=\frac{1}{2}ft_1^2$, from which by substituting the value of the acceleration in terms of M, v, and d, remembering that $s=\frac{1}{2}D$ and $t_1=\frac{2}{3}t$, we obtain the equation

$$\frac{Mv}{d}=55\cdot5\frac{DW}{c_{a}t^2} \quad \text{..............(103).}$$

This gives us the best value of $\frac{Mv}{d}$ in terms of the accelerating current c_a. We have, however, to satisfy the condition that one-half the distance is covered, at full speed, in one-third the given time; substituting the value of the final speed in terms of M, v, and d, we get

$$\frac{Mv}{d}=0\cdot1747\frac{t}{D}(E-c_fR) \quad \text{........ (104).}$$

The greatest possible value of $\frac{Mv}{d}$ is when R is nothing; hence the least possible accelerating current that would satisfy the conditions is given by the equation

$$c_{a} = 318 \frac{D^2 W}{Et^3} \quad \text{..................(105).}$$

If we determine the value of c_{a} by this equation, and insert it in Equation 103, we shall find that the running conditions require that R should be nothing. We must then make c_{a} a little larger than this.

Example 54.—A car weighing 40 tons has to be equipped with two motors controlled on the parallel method, operating on a line having a tension of 500 volts. The frictional and other resistances retarding the motion amount to 4,950 inch-pounds of torque on the car axles. The motors have to be designed to start up from rest and cover 400 yards in 80 seconds.

The least possible accelerating current is given by Equation 105, where D is expressed in feet, and W in tons. Inserting the given values we get $c_{a} = 35{\cdot}8$ amperes. As the motors must be of some finite resistance, put c_{a} equal to 40 amperes, and use this value in Equation 103; we find that the best value of $\frac{Mv}{d}$ is 5·2.

If the motors are to be gearless the induction factor is thus found to be as 5·2 times the wheel diameter. To make the motors as light as possible, take d as small as possible, say 30 inches, then M becomes 156. If gearing is introduced the motors can be made lighter; thus if $v = 4$, and the wheel diameter is 60 inches, the induction factor is 78. We shall assume v to be unity.

We now find that the acceleration is 0·421 f.p.s. per second, and that the motor will accelerate for 53·4 seconds; the distance travelled in this time is 600 feet; full speed is given either by 53·4 times 0·421, or by three halves of D/t, both giving 22·5 feet per second, or 15·3 miles per hour, and the distance covered at this speed in the remainder of the 80 seconds is 600 feet.

The frictional current is 22·5 amperes: knowing the final speed we can determine the drop, which is 54 volts; hence the resistance of the motor must be 2·4 ohms. This is more than it need be. By taking $c_{a} = 38$ amperes, we find the resistance to be 1·4 ohms. We may of course make it anything we please.

Taking the accelerating current then as 38 amperes, the value of M will be 164·4, and c_f will be 21·4 amperes; the starting rheostat must be arranged to carry 59·4 amperes, and the motor must be designed to carry this current for the whole period of acceleration.

Suppose, now, that instead of using the best diameter we had taken $d = 40$ inches, M being the same as before. Full speed would not be reached, and the car would take 87 seconds to cover the given distance. If, on the other hand, we had used a motor with a higher induction factor than the best, say $M = 200$, the acceleration would be 0·540 f.p.s. per second, but the final speed would fall to 18·7 feet per second, or 12·8 miles per hour, and the car would take 81·6 seconds to cover the 400 yards, so that there would be no gain by increasing the weight of the motor.

We have seen that the best arrangement is that in which one-half the given distance is covered at full speed in one-third of the given time. The final speed is therefore given by the equation

$$s_f = 1\cdot5\,\frac{D}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 106.$$

Now $\frac{D}{t}$ is the mean speed; hence for the best arrangement the final speed is 50 per cent. greater than the mean speed. We have here the practical limit of the application of this principle—namely, the limit of speed, since the use of the best diameter involves high speeds for long distances. If, in Example 54 for instance, the time was kept fixed at eighty seconds, and the distance to be travelled increased to 800 yards, the final speed would increase to 30·6 miles an hour. This is shown in the following table:—

Time. Seconds	Distance. Yards	Mean speed. Miles an hour	Final speed. Miles an hour
80	400	10·2	15·3
80	800	20·4	30·6
80	1,200	30·2	45·9
80	1,600	40·8	61·2
80	2,000	51·0	76·5

In working out this problem we assume that when more than one motor is used the motors are connected in parallel. We have now to see how these equations apply when two motors are connected in series at the start.

When connected in parallel throughout, the acceleration is uniform up to full speed. **If the motors are connected in series at starting,** we get the same acceleration as before with less current from the line.

Thus in Example 54, an accelerating current of 40 amperes per motor gives an acceleration of 0·421 f.p.s. per second; if the motors are in parallel, we require 80 amperes from the line to accelerate the whole car. If, however, the motors are in series, we only require an accelerating current of 40 amperes from the line. Hence the series-parallel arrangement enables us to get the same acceleration as before with half the accelerating current from the line, as long as the motors can be held in series. Our calculations will then be unaltered except in this, that during the time that the motors are in series the accelerating current is half that given by Equation 105. In the case quoted, the motors will remain in series for 25 seconds, during which time the current from the line will be 59·4 instead of 118·8 amperes as with the parallel connection.

It is of course always possible to reduce the time occupied in covering any distance by increasing the accelerating current, without altering the general design. Thus in Example 54, if the accelerating current were doubled, we should cover the 400 yards in five-sixths of the previous time, or 67 seconds, with $M=156$ and $d=30$ inches. The accelerating current would then be 72 amperes. We could, however, obtain an equally good result with less expenditure of current, either by putting on wheels 52 inches in diameter, or by inserting gearing, $v=1·73$, keeping M unaltered; in either case we should cover the 400 yards in 67 seconds with only 62 amperes of accelerating current.

Thus far we have assumed that the induction factor remains constant throughout the whole period; we must now consider the **effect of the variation in the value of M due to series winding.**

Let us take the case of a tramcar weighing 10 tons, driven by two motors working on a line having a tension of 500 volts. Suppose that we have to design an arrangement by which the car will start up from rest and travel 500 feet in 30 seconds. The motors are to be series wound.

First find the least possible accelerating current per motor by Equation 107, remembering that $W=5$ tons. We get $c_a=29{\cdot}5$ amperes. Take 30 amperes to allow for the resistance of the motor. From Equation 103 we find that the best value of $\frac{Mr}{d}$ is $5{\cdot}15$. For the present we may take $r=4{\cdot}78$ and $d=33$ inches, giving $M=35{\cdot}5$.

The maximum speed is 25 feet per second or 17 miles an hour. If the frictional and other resistances retarding the motion amount to 3,580 inch-pounds of torque on the car axle, the corresponding current will be 15 amperes, and the resistance of each motor must therefore be 0·6 ohm.

The initial acceleration will be 1·25 f.p.s. per second, and the current of 45 amperes will be constant until the starting rheostat is all out, at which point the speed of the motor will be given by $n=\frac{500-45\times 0{\cdot}6}{35{\cdot}5}=800$ r.p.m. The speed of the car will therefore be 24·2 feet per second. Thus we see that if the induction factor is constant, the acceleration can be maintained constant up to a speed of 97 per cent. of final speed; after this point the motor will speed up according to the law already given in Chapter VII.; the error involved in assuming that the acceleration is constant up to full speed will be small, and

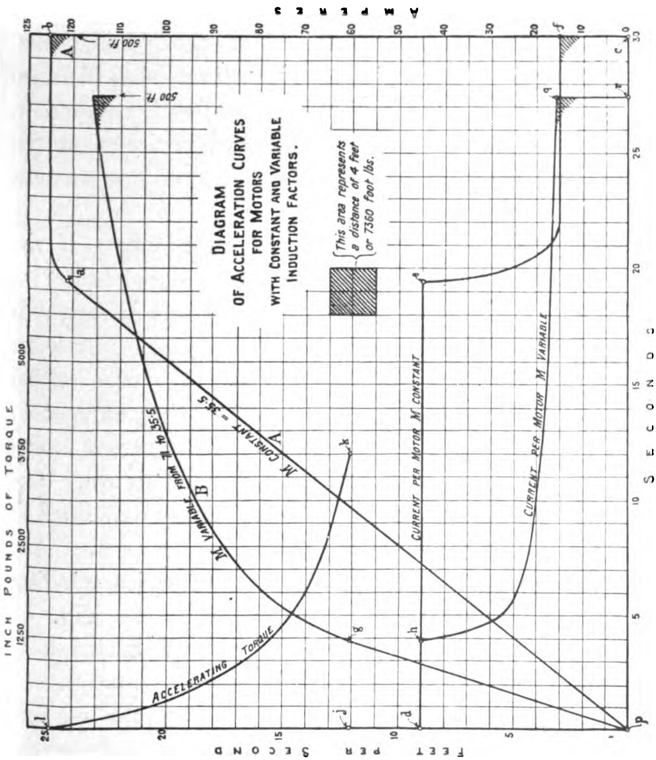


DIAGRAM
OF ACCELERATION CURVES
FOR MOTORS
WITH CONSTANT AND VARIABLE
INDUCTION FACTORS.
This area represents a distance of 4 feet or 7360 foot lbs.
INCH POUNDS OF TORQUE
AMPERES
FEET PER SECOND
SECONDS
500 ft.
ACCELERATING TORQUE
M CONSTANT = 35.5
M VARIABLE FROM 71 to 35.5
CURRENT PER MOTOR M CONSTANT
CURRENT PER MOTOR M VARIABLE

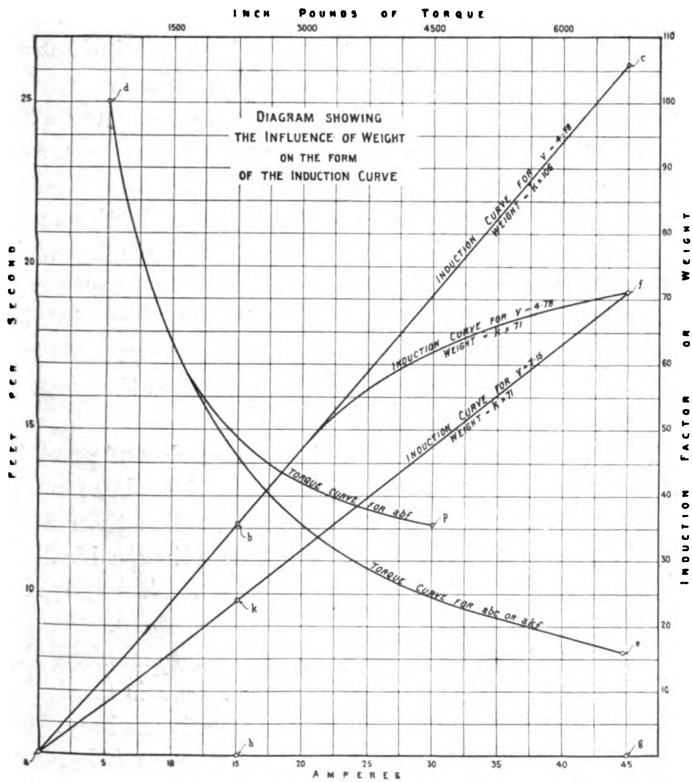
FIG. 56

the actual distance covered will be very nearly equal to that calculated.

These results have been plotted in Fig. 56. The acceleration is constant from p to a, and the speed is practically constant from a to b. The area $oabc$ represents 500 feet. The car accelerates for two-thirds of the whole time, namely 20 seconds, during which time the distance covered is 250 feet; the remaining 250 feet is covered in 10 seconds. The error due to the assumption that the acceleration is constant up to full speed does not amount to one foot of distance.

This is the form of the acceleration curve if the induction factor remains constant and equal to 35·5 throughout the whole period; the given distance is then covered in the given time with the least possible accelerating current. Any other values of v or d would require a longer time or a greater accelerating current to cover the distance in the given time. The current curve for one motor has been plotted in the same figure. The area of the curve $pdefc$ represents the energy expended in covering the given distance.

We shall now consider the influence of series winding on the acceleration and current curves. In Fig. 57 let values of the current be measured horizontally and values of M vertically. Take ah equal to 15 amperes, and set up hb equal to 35·5 on the vertical scale. Then b is a point on the induction curve of the motor. Produce ab to cut a vertical line drawn through g in the point c, where ag is 45 amperes. Then gc, equal to 106 of M, is the greatest possible induction factor the motor can have for 45 amperes. For the induction curve of a series-wound motor cannot be convex to the current



INCH POUNDS OF TORQUE
1500
3000
4500
6000
DIAGRAM SHOWING
THE INFLUENCE OF WEIGHT
ON THE FORM
OF THE INDUCTION CURVE
INDUCTION CURVE FOR V = 4.78
WEIGHT = K × 106
INDUCTION CURVE FOR V = 4.78
WEIGHT = K × 71
INDUCTION CURVE FOR V = 7.15
WEIGHT = K × 71
TORQUE CURVE FOR abf
TORQUE CURVE FOR abc OR akf
FEET PER SECOND
INDUCTION FACTOR OR WEIGHT
AMPERES

Fig. 57

axis, but may be a straight line, if no part of the iron in the magnetic circuit is magnetised over the bend of the magnetisation curve. Now in this case the induction curve must pass through the point b, for the motor must have $M=35\cdot5$ for 15 amperes; hence the greatest possible value of M for 45 amperes is found by making the induction curve a straight line passing through the point b.

Our calculations hitherto have shown us that the motor must have an induction factor of 35·5 at 15 amperes, and that the maximum current at starting must be 45 amperes. We have not, however, determined what must be the value of the induction factor for 45 amperes. All we know is that if the induction factor is constant for all currents and equal to 35·5, we shall cover the given distance in the given time. We shall see later on, that the greater we can make the induction factor for 45 amperes the greater will be the economy of starting.

It is clear that there are an infinite number of possible induction curves all passing through the point b, but having different values of M for 45 amperes, all less than 106. Any one of these curves would comply with the specification as to time and distance, but none of them would be so good as the line *abc* from the point of view of economy.

We have seen in a previous chapter, that when the maximum current to be carried by a motor is fixed, the weight W increases nearly with M, the maximum induction factor. We shall assume that $W=kM$, where k is some constant. Hence of all possible induction curves that might be chosen, *abc* will give the best results, but will involve the greatest weight.

In practice the weight of the motor will be limited; let us suppose that the limit is such that the greatest permissible induction factor for 45 amperes is 71, or twice that for 15 amperes. Our induction curve then has to pass through the point *f*.

It might now appear that we are obliged to adopt a design that would not give us the best results, on account of the limit of weight imposed, but this is not the case if we are at liberty to adjust the values of the velocity ratio and of the wheel diameter.

Neglecting the heat drop at full speed, we see from Equation 104 that the induction factor hb can be written

$$hb = 0{\cdot}1747 \frac{Etd}{Dv} \quad \ldots\ldots\ldots\ldots\ldots(107).$$

While if T is the retarding force in pounds at the car axle we have

$$ah = \frac{Td}{2{\cdot}82v}\,\frac{1}{hb} = 2{\cdot}03\,\frac{TD}{Et} \quad \ldots\ldots\ldots(108).$$

Hence the inclination θ of the induction curve to the horizontal, as given by the ratio of bh to ha, can be expressed thus:—

$$\theta = 0{\cdot}086\,\frac{E^2t^2}{TD^2}\,\frac{d}{v} \quad \ldots\ldots\ldots\ldots\ldots(109).$$

When E, t, D, and T are fixed, we can reduce this inclination by increasing v or by diminishing d. We can thus make the best induction curve go through any required point on the vertical through g by properly adjusting the values of v and d.

If in this example we increase the velocity ratio in the proportion of 106 to 71, we get $v = 7{\cdot}15$; with this value of v, and with $d = 33$ inches, the best induction curve gives a maximum value of M equal to that required by the weight limit. We shall thus not only fulfil the conditions as regards time and distance, but we shall do so in the most economical way.

We may here notice that the inclination of the induction curve to the horizontal may be written

$$\theta = \frac{4\pi}{10} pAgS10^{-8} \quad \ldots\ldots\ldots\ldots\ldots(110).$$

Where p is the number of polar divisions of the armature connected in series, A the number of surface conductors, S the number of turns per pole, each carrying the whole current, and g the permeance of each polar gap in centimetres.

It will however generally happen that the weight limit requires a velocity ratio that is practically unattainable even with the largest values of d. We have here the same problem that occupied our attention in a former chapter, namely, to get the greatest ratio of v to d. For single reduction spur gearing this ratio is limited by the clearance between the car axle and the ground. If we take a 33-inch wheel we could not make v as much as 7·15, the value required to get the best results. We shall here suppose that the largest possible value of v is 4·78. We shall then have a bent induction curve, such for instance as that given by the line *abf*. In what follows we shall take this as the induction curve of the motor.

Construct the curve *dp* of total torque on a speed base, measuring torque horizontally and speed vertically. By deducting the retarding torque, 750 inch-pounds on the motor shaft, from each horizontal ordinate of this curve we get the curve of torque available for acceleration. This has been drawn at *kl* in Fig. 56.

Draw pg to represent an acceleration of 3·12 f.p.s. per second. This is the acceleration for 45 amperes per motor with $M=71$. The total torque is now 4,500 inch-pounds, and the whole retarding torque is 750 inch-pounds, so that the accelerating torque is more than twice what it was when M was 35·5. The speed of the car when the starting rheostat is all out is 12·1 f.p.s. The acceleration curve can now be constructed from the torque curve by the method described in Chapter VII., and continued until the area included is equal to 500 feet. Draw dh at 45 amperes and construct the current curve from the speed curve. The energy expended is represented by the area $pdhqr$.

A comparison of the curves for constant and variable induction factor shows us in the first place how great a saving of energy is effected by the use of the series winding, the area of the current curve for the series-wound motor being 0·6 of that for the motor with constant induction factor.

In the second place we see that there is a small saving of time effected by using series winding. The amount of this saving will depend on the shape of the induction curve We may generally assume that in a well-designed motor the use of series winding will save about 5 per cent. in time. Hence we may use in our equations a value of t about 5 per cent. greater than the time actually specified, and thus obtain a smaller value for the starting current.

The current curves for the two types of motor are plotted in Figs. 58 and 59. At the point f the whole of the energy is being expended in heat. The heat loss at any point may be calculated by taking the corresponding

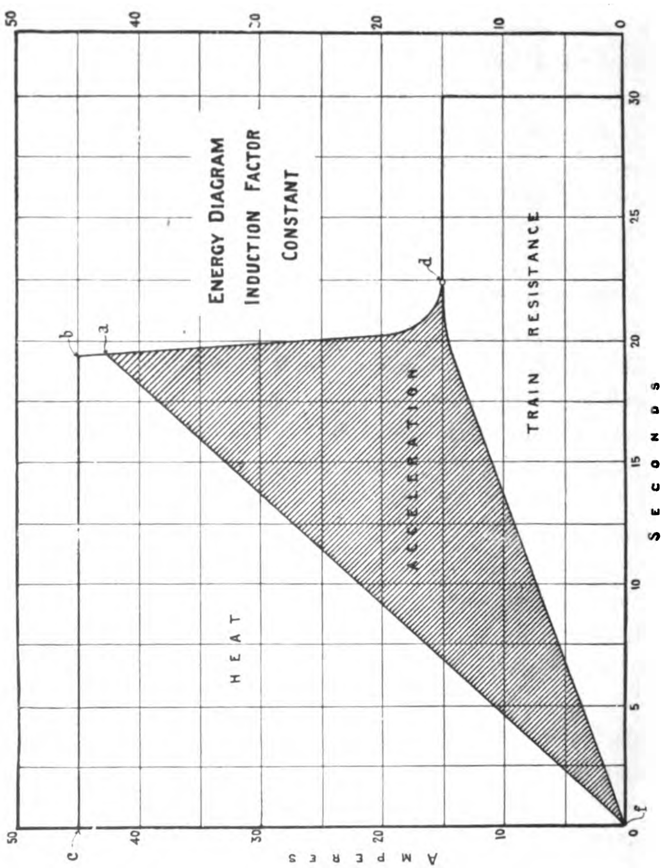

ENERGY DIAGRAM
INDUCTION FACTOR
CONSTANT
HEAT
ACCELERATION
TRAIN RESISTANCE
SECONDS
AMPERES
0
5
10
15
20
25
30
40
50
a
b
c
d
f

FIG. 58

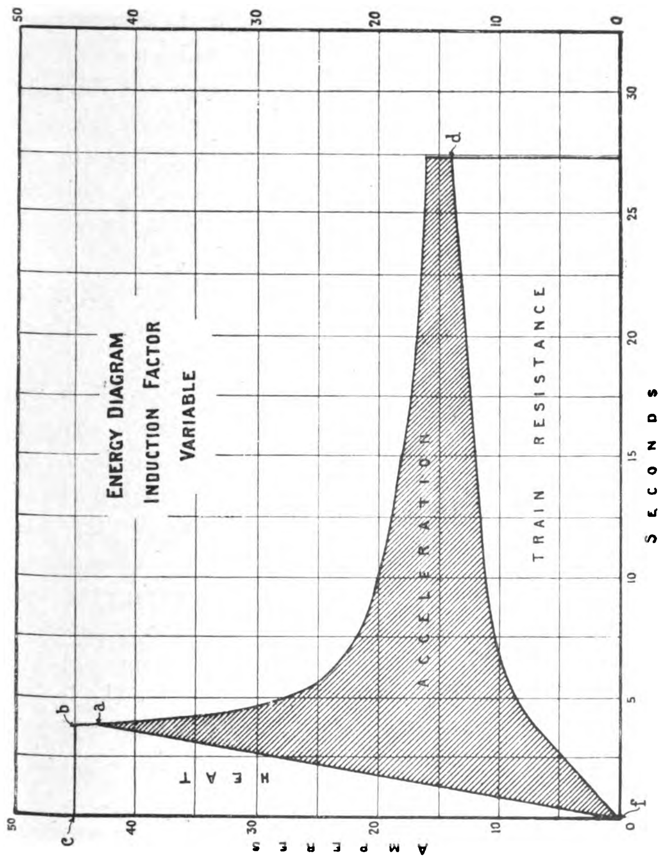

ENERGY DIAGRAM
INDUCTION FACTOR
VARIABLE
HEAT
ACCELERATION
TRAIN RESISTANCE
SECONDS
AMPERES
a
b
c
d
f
0
5
10
15
20
25
30
0
10
20
30
40
50

Fig. 59

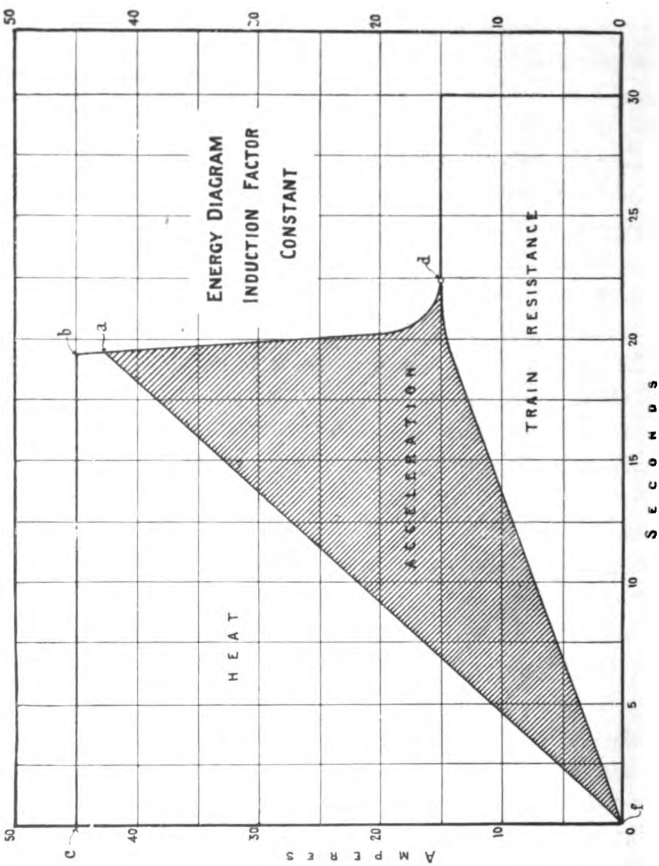
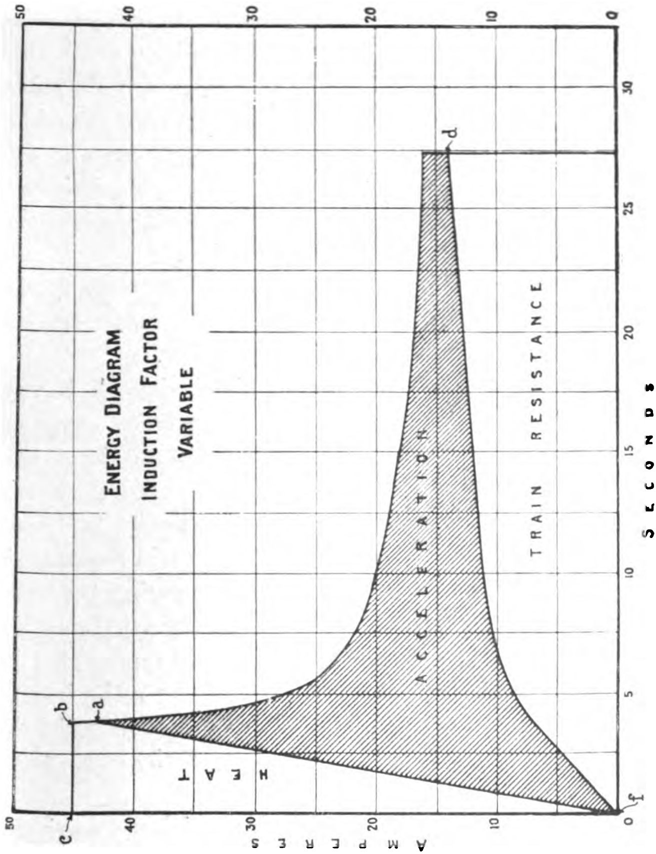

ENERGY DIAGRAM
INDUCTION FACTOR
CONSTANT
HEAT
ACCELERATION
TRAIN RESISTANCE
SECONDS
AMPERES
a
b
c
d
f
0
5
10
15
20
25
30
40
50

FIG. 58



ENERGY DIAGRAM
INDUCTION FACTOR
VARIABLE
HEAT
ACCELERATION
TRAIN RESISTANCE
SECONDS
AMPERES
0
5
10
15
20
25
30
10
20
30
40
50
a
b
c
d
f

FIG. 59

speed and finding the resistance in the circuit, and then multiplying this by the square of the current. If the heat watts is divided by the tension of the line we obtain the part of the total current that represents the loss due to heat. In Figs. 58 and 59 this current has been set off from a horizontal line passing through 45 amperes; the points obtained lie on a straight line passing through the origin and a point *a*, where *ab* represents the heat loss when the current of 45 amperes is passing through the resistance of the motor only. The area *fabc* represents the heat loss during the period of constant acceleration.

Beyond the point *a* the current used to make up the heat loss is very small compared with the whole current, and is neglected in the following discussion.

The heat loss can be predetermined, since it is very nearly one half of the area of the current curve up to the point at which the starting rheostat is all out. There is not much difference in the final speed of the two motors, the kinetic energy is, therefore, nearly the same. The energy expended in overcoming the train resistance is the same; this should be checked by comparing the areas marked train resistance in the two diagrams. Hence the difference in energy expenditure is nearly represented by the difference of the heat areas. We thus see the importance of setting back as far as possible the point at which the starting rheostat is all out. This is what the series winding effects for us.

If t_1 is the time in seconds during which the maximum current c_1 is flowing, **the energy, H, expended in heat** is nearly $\frac{1}{2} Ec_1t_1$. If s_1 is the speed at which the starting rheostat is all out, $s_1 = \frac{k_1}{M}$, and $\frac{s_1}{t_1} = k_2M$, being the acceleration; hence $t_1 = \frac{k_1}{k_2} \cdot \frac{1}{M^2}$, and $H = \frac{1}{2} \frac{k_1}{k_2} \cdot \frac{Ec_1}{M^2}$. Inserting the values of k_1 and k_2 we have

$$H = 3{\cdot}23\,(E - c_1 R)\frac{Ec_1}{c_a}\,\frac{d^2}{v^2}\,\frac{W}{M^2} \;\ldots\ldots\; (111),$$

where c_1 is the total starting current, and c_a the current available for acceleration. Hence H varies inversely as $c_a M^2$. Now the larger we make M at the start, the larger will be the current c_a, since the retarding torque is constant. It is thus of great importance to increase M at the start.

In the previous example the ratio of the induction factors is 2 to 1, and the ratio of the accelerating currents is 1·3 to 1, giving 5·2 as the ratio of the heat losses. By actual measurement of the diagrams this ratio is 5·25.

When the current representing the heat loss has been deducted from the total current at any instant, the remainder represents the expenditure of energy in producing acceleration and overcoming train resistance. The proportion of these two can be obtained from the curve of total torque, since that tells us how much is being used for accelerating and how much for overcoming train resistance at any speed. The curves *od* in Figs. 58 and 59 have been constructed in this way, thus dividing the remaining area of the current curve into one portion, shaded in the figures representing the energy expended in acceleration, and a second portion representing the expenditure of energy in overcoming the train resistance. The latter of these two areas is equal in the two figures, while the acceleration energy is a little greater for the constant induction motor, the final speeds being 25 and 23·2 feet per second respectively.

The following table shows the expenditure in foot-pounds in the two cases:—

	Constant Induction Factor	Variable Induction Factor
For acceleration . .	109×10^3	$88{\cdot}7 \times 10^3$
For train resistance .	109×10^3	$109{\cdot}0 \times 10^3$
For C^2R loss . . .	169×10^3	$32{\cdot}2 \times 10^3$
Total . . .	387×10^3	$229{\cdot}9 \times 10^3$

We must remember that in this example the expression 'train resistance' means all resistances opposing the motion, including those due to the friction of the gearing and the torque lost in the motor itself.

The advantage of series winding thus consists mainly in a saving of energy. There is, however, a limit to the amount of energy saved. Thus with the highest possible induction factor, the C^2R loss is 13×10^3 foot-pounds. Hence there is a limit of weight beyond which it will not be worth while to go with a view to reducing the heat loss. This is shown by the following table, where the weights are taken as proportional to the induction factors, the heat losses inversely as M^2, and the efficiency as the ratio of the sum of the friction and acceleration to the total energy:—

Maximum current. Amperes	Maximum induction factor, or weight	C^2R loss 10^3 foot-pounds	Efficiency
45	35·5	169	54
,,	40	102	66
,,	50	65	75
,,	60	45	82
,,	70	33	86
,,	80	25	89
,,	90	20	91
,,	106	13	94

may here inquire as to the **effect of using o values of v and d than those given by Ec .tion 103.**

n increase in the value of d will have the same effect as a decrease in the value of v; if then we take three cases, each having $v=4{\cdot}78$, d being respectively 24·3, 33, and 40 inches, we shall get the same results as if we took a constant diameter of 33 inches, and velocity ratios of 6·50, 4·78, and 3·94 respectively.

In Fig. 60 curves of accelerating torque are drawn for velocity ratios 6·50, 4·78, and 3·94, d being 33 inches in each case. The curve for $v=4{\cdot}78$ is the same as curve kl in Fig. 56. In constructing these curves with different values of v we must remember that an increase in v increases the initial acceleration, but diminishes the final speed and also diminishes the speed when the starting rheostat is all out.

In calculating the initial acceleration we must also bear in mind that the torque on the motor shaft required to overcome the frictional torque on the car axle increases inversely as v. Thus when v is 4·78 the maximum total torque on the motor shaft is, as we have seen, 4,500 inch-pounds, from which we have to deduct 750 to balance the frictional resistance, leaving 3,750 available for acceleration at the start, the acceleration in f.p.s. per second is then found by Equation 85 to be 3·12 f.p.s. per second. Now when the velocity ratio is 3·94, the total torque on the motor shaft is 4,500 as before, but from this we must now deduct 910 to overcome the friction on the axle, leaving us with 3,590 inch-pounds available for acceleration. Using this value of t_a in Equation 85 and putting $v=3{\cdot}94$, we find the acceleration to be 2·46.

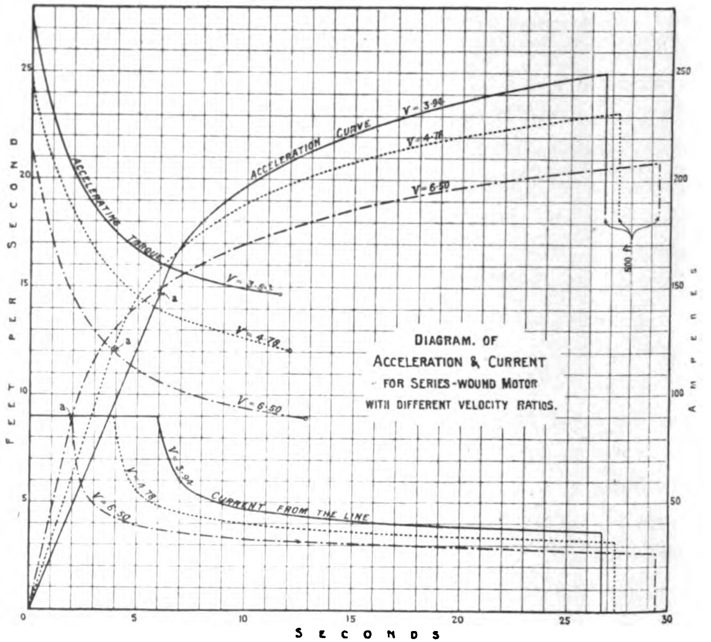


DIAGRAM. OF
ACCELERATION & CURRENT
FOR SERIES-WOUND MOTOR
WITH DIFFERENT VELOCITY RATIOS.
ACCELERATION CURVE
ACCELERATING TORQUE
CURRENT FROM THE LINE
V=3.94
V=4.78
V=6.50
500 ft
FEET PER SECOND
AMPERES
SECONDS
0
5
10
15
20
25
30
50
100
150
200
250

FIG. 60

The acceleration curves are obtained from the torque curve by the graphic method, and are continued until the enclosed area represents 500 feet. The corresponding current curves are also shown.

We see that as the velocity ratio increases the energy expended, given by the area of the current curve, decreases, but the time occupied increases. Thus we gain half a second by putting on a 40-inch wheel at the expense of a considerable increase in the energy used. On the other hand, we save energy by putting on a 24-inch wheel, but at the expense of two seconds of time. There is a minimum limit to the time in which the distance can be covered, which in this case is about 26 seconds, but the increase in the size of the wheel for each fraction of a second gained becomes greater as this limit is approached.

Hence if we have allowed for the saving of time effected by the use of series winding, the given distance will be covered in the given time, and any other values of v or of d than those given by Equation 103, will involve either a longer time or a greater expenditure of energy.

We shall complete our discussion of this example by considering to what extent our **results** are **affected by the use of the series-parallel controller.**

We can adopt either one of two methods. We may take the same current from the line as before—namely, 90 amperes, the whole of this current going through each motor in series, or we may take the same current per motor as before—namely, 45 amperes, the current from the line being thus halved as long as the motors remain in series. Let us compare these two methods.

Suppose that the induction factor of each motor is increased to 74 when 90 amperes is passing, the total torque will be 9,400, less 750 for friction, giving an acceleration of 7·2 f.p.s. per second; this can be maintained until a speed of 4·8 f.p.s. is reached, when the motors must be thrown into parallel. The rest of the curve will be the same as before; if plotted, it will show a gain of one second in covering the given distance.

If on the other hand we allow only 45 amperes per motor, we obtain the same acceleration curve as in Fig. 56, but as we are able to keep the motors in series until a speed of 5·7 f.p.s. has been reached, the expenditure of energy up to that point is halved, the total expenditure of energy being slightly less than by the first method.

We see then that the only advantage in taking 90 amperes per motor is a gain of one second in time. Since the motors used in the first method have to be designed to carry twice as much current as those used in the second, the latter method is to be preferred.

As an illustration of the application of the principles of this chapter to the heavier class of railway work, we will take the Metropolitan Elevated Railroad of Chicago.

Particulars of this railway will be found in a paper by Mr. M. H. Gerry, published in the 'Proceedings of the American Institute of Electrical Engineers' for 1897.

The rolling stock consists of motor cars and passenger cars. The former measure 47 feet in length and when fully loaded weigh 62,000 pounds. They are mounted on locomotive trucks, having 33-inch wheels, with a velocity ratio of 3·18. One truck of each motor car is equipped with two motors.

The passenger cars are 47 feet in length, having trucks fitted with 30-inch wheels, and when fully loaded weigh

46,000 pounds. Trains of two, three, and four cars are made up according to the demands of the traffic at different hours. We shall consider a train of one motor and three passenger cars, weighing in all 90 tons.

The maximum grade on one out of the four lines radiating from the power house is 0·75 per cent., ascending for 2,350 feet and descending for 1,950 feet; other grades do not exceed 0·03 per cent. The average distance between stations is 2,000 feet. E=500 volts.

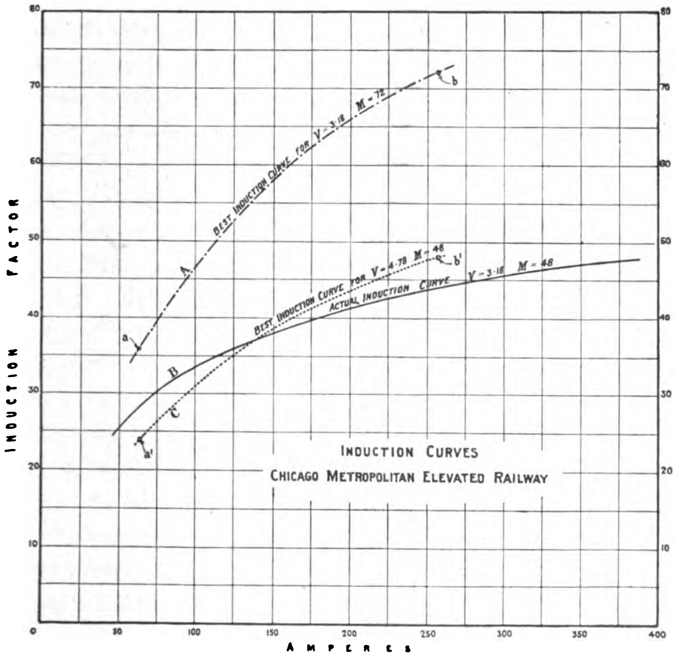
We will take the case of two stations separated by 2,500 feet of level track. The time table requires that this distance shall be covered in 100 seconds from start to stop. Experience shows that the brakes may be counted upon to stop a train of four cars weighing 90 tons, running at 25 miles an hour, in 500 feet in 20 seconds, leaving 2,000 feet to be covered in 80 seconds. Using Equation 104, remembering that the weight per motor is 45 tons and taking the drop at full speed to be 5 volts, we obtain the relation $\frac{Mv}{d}=3{\cdot}46$. Inserting this value in Equation 103 we find that $c_{a}=226$ amperes.

The wheels now in use have a diameter of 33 inches; we will adopt this size. The velocity ratio is 3·18; for the present we will take this value. Using the relation already found, we get $M=35{\cdot}9$.

Since 1,000 feet must be covered in two-thirds of 80 seconds, the maximum speed is 37·5 f.p.s., or 25·6 miles per hour, or 13·8 r.p.s. of the motor. To find the resistance of the motor we must know the amount of frictional and other torque opposing the motion. For this purpose we can make use of the record of the current taken when running at a uniform speed on a level with the existing equipment. Taking the average of a number of tests, it appears that when running on a level at a uniform speed, the motors take 75 amperes. The induction curve of these motors is given in Fig. 61, and for this current $M=30$, giving us a total torque on the motor shaft of 3,180 inch-pounds, the speed being 15·1 miles per hour. Experiments previously made with these motors showed that with 75 amperes the torque available for useful effort was 2,340 inch-pounds, giving 73·4 per cent. mechanical efficiency. Now 2,340 inch-pounds is equivalent to 450 pounds of horizontal pull on 33-inch wheels, with $v=3{\cdot}18$. Hence the train resistance is 450 pounds per motor, or 10 pounds per ton.

Assuming that the retarding resistances are the same for all speeds, we see that the torque on the motor shaft at full speed is 3,180, and since $M=35{\cdot}9$ the current when running at full speed is 63 amperes per motor. Since the drop at full speed has been fixed at 5 volts, the resistance of the motor must be 0·0795 ohm.

We have thus found one point on the induction curve, namely, $M=35{\cdot}9$ for 63 amperes. Suppose now that the limit of weight imposed gives the maximum value of M at 72 and that we are not at liberty to increase v or diminish d. With this value of M the current required to overcome the retarding torque is 31·5 amperes, so that the total current at the start must be $226+31{\cdot}5$, or say 257 amperes. This gives us a second point on the induction curve, namely, $M=72$ for 257 amperes. These points are plotted at a and b in Fig. 61. We will suppose that A is



INDUCTION CURVES
CHICAGO METROPOLITAN ELEVATED RAILWAY
BEST INDUCTION CURVE FOR V=3·18 M=72
BEST INDUCTION CURVE FOR V=4·78 M=48
ACTUAL INDUCTION CURVE V=3·18 M=48
A
B
C
a
b
a'
b'
INDUCTION FACTOR
AMPERES
0
50
100
150
200
250
300
350
400
10
20
30
40
50
60
70
80

FIG. 61

the best curve that can be obtained for $M=72$, and $v=3\cdot18$, while B is the induction curve for the motors that are actually employed on this line.

The acceleration and current curves obtained with motors having induction curves represented by A and B respectively, are given in figs. 62 and 63. The dotted lines give the curves for the motors in use on the Chicago elevated railroad, as obtained by actual experiment. For details of the way in which the experiment was carried out, the reader may consult the paper by Mr. Gerry already referred to.

The weight of the train was estimated at 90 tons, the track was level, and the mean tension at the train was 504 volts. The brakes were applied at the end of 77 seconds, when 1,930 feet had been covered, and the remaining distance of 570 feet, making up the total of 2,500 feet, was covered in 27 seconds, making the total time 104 seconds.

The acceleration curve for the motors with induction curve A in Fig. 61, is plotted on the same scale as the experimental curve. The initial acceleration is 1·27 f.p.s. per second; the motors speed up in series to 8·7 feet per second, and then in parallel with the same acceleration to 18 feet per second; the remainder of the curve is obtained graphically from the torque curve.

The total torque at the start is 26,100 inch-pounds; after deducting 3,180 for retardation we have left 22,920; allowing 90 per cent. mechanical efficiency we get 20,600 inch-pounds available for acceleration. The curve of accelerating torque is plotted, points on it being obtained from the induction curve A in Fig. 61. Point b is given, for we know that at full speed the train is moving at 37·5 feet per second or 25·6 miles an hour. The highest speed actually attained is 23·6 miles an hour.

The acceleration curve has been continued up to the point when 1,930 feet has been covered, corresponding to the point at which the brakes were put on in the experiment; the curve gives 76 seconds as the time occupied, compared with 77 seconds in the test. The time taken to cover 2,000 feet is 78 seconds by the curve, being 2·5 per cent. less than the time calculated, the difference being due to the use of series winding.

In Fig. 63, vertical ordinates represent current from the line, the dotted curve giving the results of the test of which the acceleration curve is given in Fig. 62. The calculated current curve is shown by a full line; the motors are in series for 7 seconds, during which time the current is 257 amperes; they are in parallel, taking 257 amperes each, or 514 from the line, for about 7 seconds longer; the current then rapidly decreases: points on the curve are found from curve *A* in Fig. 61.

The irregularities in the experimental curve are the results of the uneven handling of the controller. We see that the motors were taking about 380 amperes each at the start, and were allowed to speed up in series for about 10 seconds after the starting resistance was taken out. When thrown into parallel the current per motor is about 330 amperes per motor, or 660 from the line. The form of the current curve as the motors speed up in parallel is well shown. More careful manipulation of the controller would have effected a better start. The dip in the acceleration curve during the period from 10 to 40 seconds might have been avoided by not allowing the motors to speed up in series.

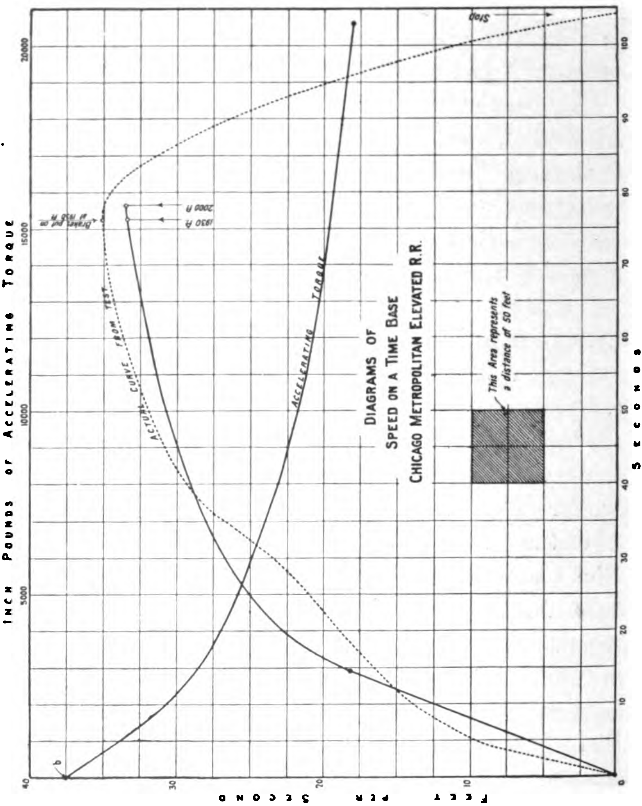

INCH POUNDS OF ACCELERATING TORQUE
5000
10000
15000
20000
FEET PER SECOND
40
30
20
10
0
SECONDS
10
20
30
40
50
60
70
80
90
100
DIAGRAMS OF
SPEED ON A TIME BASE
CHICAGO METROPOLITAN ELEVATED R.R.
ACTUAL CURVE FROM TEST
ACCELERATING TORQUE
1930 ft
2000 ft
Stop
This Area represents
a distance of 50 feet
b

FIG. 62

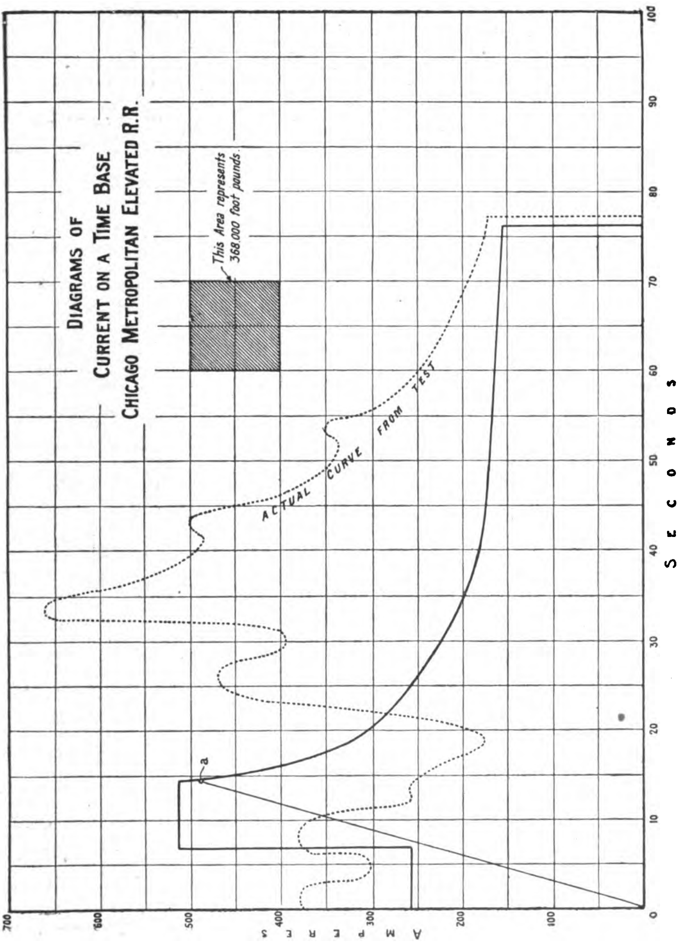

DIAGRAMS OF
CURRENT ON A TIME BASE
CHICAGO METROPOLITAN ELEVATED R.R.
This Area represents 368,000 foot pounds.
ACTUAL CURVE FROM TEST
a
SECONDS
0
10
20
30
40
50
60
70
80
90
100
AMPERES
0
100
200
300
400
500
600
700

FIG. 63

When we compare the energy expended by the two methods, we see that the results of the test give a much greater expenditure than that indicated as necessary by the calculations. The maximum speed attained in the test was 35 feet per second, the kinetic energy is thus 380×10^4 foot-pounds. The train resistance is 900 pounds, giving an expenditure of energy of 174×10^4 foot-pounds throughout the distance of 1,930 feet. Hence the total energy expended as work is 554×10^4 foot-pounds.

The area of the dotted current curve in the figure represents the total expenditure of energy in the experiment; this area measured with a planimeter is found to represent 963×10^4 foot-pounds. The difference between the work done and the observed expenditure of energy, amounting to 409×10^4 foot-pounds, is mainly represented by the energy lost in heating the resistances. If we allow a mechanical efficiency averaging 85 per cent., the heat loss amounts to 311×10^4 foot-pounds.

The maximum speed for the calculated curve is 33·5 feet per second, giving 350×10^4 foot-pounds of kinetic energy; the energy required to overcome the train resistance is 174×10^4 foot-pounds, giving a total of 524×10^4 foot-pounds of work done. Assuming an average efficiency of 85 per cent. we get a total torque loss of 92×10^4 foot-pounds. We have already seen how to estimate the heat loss, and know that it is represented, within a small error, by the area of the current curve above the line *oa* in the figure; this area is 67×10^4 foot-pounds, hence the total energy required, as obtained by calculation, is 683×10^4 foot-pounds, as compared with 963×10^4 obtained with the existing motors. The area of the calculated current curve in

Fig. 63 gives 675×10^4 foot-pounds, or about 3 per cent. less than that estimated. These results are represented below in tabular form.

	Total energy required From test	Calculated
For acceleration . .	380×10^4	350×10^4
For train resistance . .	174×10^4	174×10^4
For torque losses . .	98×10^4	92×10^4
For C^2R loss . . .	311×10^4	67×10^4
Total . . .	963×10^4	683×10^4

The difference in the energy expended is almost wholly accounted for by the difference in the heat loss. The existing motors require 40 per cent. more energy to operate the train under the given conditions than those whose induction curves have been calculated, and the maximum current from the line is 28 per cent., and the maximum current per motor 48 per cent. higher than appears necessary.

The force factor required to start is, by calculation, 257×72 or 18·5 kilodynes. If a hyperbola be drawn in Fig. 61 having $MC = 18{\cdot}5$ kd., it will cut curve A at $c = 257$, and curve B at $c = 385$ amperes. We thus see why the existing motors have to take nearly 50 per cent. more current to start than is necessary.

We have supposed that the limit of weight fixes the maximum induction factor at 72, for 257 amperes. The motors in use have a maximum induction factor of 48. For this value of M the best economy is obtained when $v = 9{\cdot}75$, the induction curve is then straight. Suppose that the consideration of clearance limits the value of v to 4·78, the induction curve must then be of the form C in Fig. 61, whose vertical ordinates bear to those of curve

A the ratio of 4·78 to 3·18. The expenditure of energy will then be the same as for the motors with induction curve A.

The **economy of working** may be expressed in terms of the energy required to move the given weight through the given distance. This may be stated in terms of **watt-hours per ton mile.** Thus in the previous example, the trains can be worked at the required speed with an expenditure of 60·5 watt-hours per ton mile; the distance, of course, includes that in which the brakes are on. The actual energy expenditure is 85·5 watt-hours per ton mile.

The results of our investigation may be summed up as follows:—There are three forms of expenditure of energy involved in carrying a train of given weight through a given distance in a given time. (1) The work done in overcoming **train resistance.** This depends on the distance, and can only be reduced by increasing the mechanical efficiency of the motors. (2) The work done in producing **kinetic energy.** This increases as the square of the final speed. Equation 103 gives us the least possible final speed for the given conditions, and thus the least possible expenditure of energy in accelerating. (3) The energy expended in **heat.** Equation 111 shows that this increases as the square of $\frac{d}{Mv}$. Equation 103 gives us the least possible value of $\frac{d}{Mv}$, if M is to be constant. If, however, series winding is used, the heat loss may be reduced to any desired extent by increasing M, but at the expense of an increase in the weight of the motor.

CHAPTER XII

ARMATURE REACTION

THE magnetisation curves of a dynamo can be found by substituting for the ordinary brushes a pair of thin steel brushes insulated from one another and touching the commutator at a small angular distance apart. If these brushes are connected to the terminals of a voltmeter, and the armature rotated at a uniform speed when the magnets are excited, the reading on the voltmeter will measure the rate at which the conductors on the armature, included between the two exploring brushes, are cutting lines of force. If the brush holder is moved round the commutator, the voltmeter reading will vary with the intensity of the magnetisation measured across the surface in which the conductors on the armature are moving.

If the readings observed are plotted vertically on a line along which distances represent successive positions of the exploring brushes, we shall obtain a curve whose ordinates represent the intensity of magnetisation round the armature. The curve in Fig. 64 marked 'magnets only' is such a curve. The magnetisation is nothing at points *a* and *b*, and is uniform under the poles, the sign of the curve being different under adjacent poles. The actual values of the voltmeter readings will depend upon the angular width of the exploring brushes; hence while this method gives us the form of the magnetisation curve, we must make a separate experiment to find its area in terms of lines of force.

The area of each polar portion of this curve represents N, the total useful lines of force per pole, passing through the armature. If we observe the speed and the total induced tension between a and b as measured in the usual way, we can find N from Equation 8.

The only variable in the expression for the induction factor is the term N, and this is represented by the area of the magnetisation curve; it follows that any change in the area of this curve involves a change in the value of the induction factor. We shall now discuss the **influences tending to alter the area** of this curve.

The intensity of magnetisation at any point h produced by a wire say at k, in Fig. 64, can be found by considering the lines of force in the magnetic circuit passing through h, due to i amperes at k. The lines of force passing through one square centimetre at h will circulate in some path round k, the form of which need not concern us if we assume that the greater part of its reluctance consists of the air gap which is crossed twice; if for the present we neglect all other reluctance but this, we can write down the lines per square centimetre at h thus:

$$H = \frac{4\pi}{10} \frac{i}{2\delta} \quad \text{..................(112).}$$

where δ is the width of the gap in centimetres, and i is the number of amperes in the wire at k.

This equation shows that the intensity due to i amperes at the point k is uniform between k and the tip of the pole, that the intensity on one side of the wire is equal but

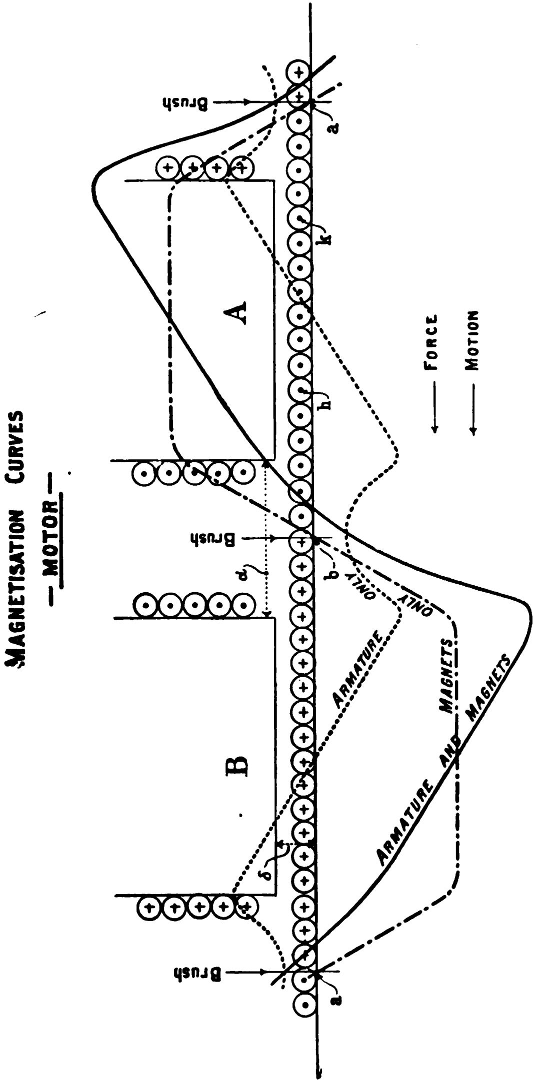

Magnetisation Curves
— Motor —
A
B
Brush
Brush
Brush
a
a
b
k
h
d
δ
Force
Motion
Armature only
Magnets only
Armature and Magnets

Fig. 64

of opposite sign to that on the other side, and that it is independent of the position of the wire under the pole.

We can now pass from the case of a single wire to that of a number of wires placed side by side on the surface of the armature. Consider first the wire at the point k lying under the pole A. Suppose that the dynamo is acting as a motor and that the current in this wire is coming towards us, the intensity of magnetisation at each point along the gap due to the current i flowing in the wire at k is then given by Equation 112. Now all the conductors lying to the right of h will produce an effect at h of the same sign and amount as the wire at k. Hence if there are s_1 conductors between h and the right-hand pole-tip, the intensity of magnetisation produced at h by all these conductors will be given by

$$H=\frac{4\pi}{10}\;\frac{s_1 i}{2\delta} \quad \ldots\ldots\ldots\ldots\ldots\ldots(113).$$

where i is the current in each conductor.

The conductors lying to the left of h, between h and the left-hand pole-tip, will produce an effect at h of opposite sign to those on the right of h, the intensity being given by simply changing s_1 to s_2 in Equation 112, where s_2 is the number of conductors lying between h and the left-hand pole-tip. The resultant intensity at h is the difference between these two intensities, and may be written

$$H=\frac{4\pi}{10}\;\frac{2s_3 i}{2\delta} \quad \ldots\ldots\ldots\ldots\ldots(114).$$

where s_3 is the number of conductors lying between h and the centre of the pole.

This equation shows that the intensity due to the current in the armature is nothing when $s_3=o$, *i.e.* at the centre of the pole, and increases to a maximum at the tip of the pole. If we denote as before by A the number of conductors counted round the surface of the armature, and by ϕ the angular breadth of the pole-piece subtended at the centre of the armature, we can put $\frac{A\phi}{360}$ for $2s_3$, so that **the intensity at either pole-tip** will then be given by

$$H_p = \frac{1}{573} \frac{iA\phi}{\delta} \qquad \ldots\ldots\ldots\ldots(115).$$

This is the expression for the intensity of magnetisation in lines per square centimetre under the pole-tips produced by a current of i amperes flowing *in each* of the A conductors round the armature; if the machine has two poles the current from the line will be $2i$; δ is the width of the air gap from iron to iron measured in centimetres.

The curve of magnetisation due to the armature can be drawn by calculating the value of H under the tip by Equation 115, and then drawing a straight line as that in Fig. 64, marked 'armature only,' through the centre of the pole. We shall call the two curves thus found the curves of magnet and armature magnetisation.

For points along the surface of the armature outside the pole the magnetisation due to the armature will diminish rapidly on account of the increased reluctance of the magnetic circuit. The intensity at each point may be found by changing 2δ in Equation 115 into $\delta+x$, where x is the distance between any point and the tip of the pole; if a is the distance between the tips of two adjacent poles, the intensity at a point midway between them is twice that due to the action of the wires under one pole only, so that the intensity at this point may be expressed by

$$H_b = \frac{1}{143 \cdot 2} \cdot \frac{iA\phi}{\delta + \frac{a}{2}} \quad \text{............(116).}$$

We are thus able to complete the curve of armature magnetisation, and find that it cuts the axis at points on the armature at the centre of each pole, and that it rises to a maximum at the tips of the poles, and bends down again to a minimum, *but does not cut the axis*, at points midway between the tips of the poles.

Figs. 64 and 65 show the curves of armature magnetisation for a motor and for a generator. In each case the motion is in the same direction, also the induced tension and the magnetisation due to the magnets. The only difference is in the direction of the current in the armature; this depends upon whether the tension impressed on the terminals of the armature is greater or less than the induced tension.

The student should reason out for himself the signs of each curve of magnetisation in these figures. The direction of the current in the wires is shown by dots and crosses, the dots representing currents coming towards us, and the crosses representing currents going from us. Looking at the pole *A* across the air gap, the lines of force will go from us; this is shown as positive by the uniform curve above the axis. In the next pole this direction is reversed, the magnetisation at *b* being nothing; *b* and *a* will be midway between the pole-tips if, as we here suppose it to be, the field is symmetrical.

If we assume the direction of motion, we are able to determine the direction of the induced tension, which will

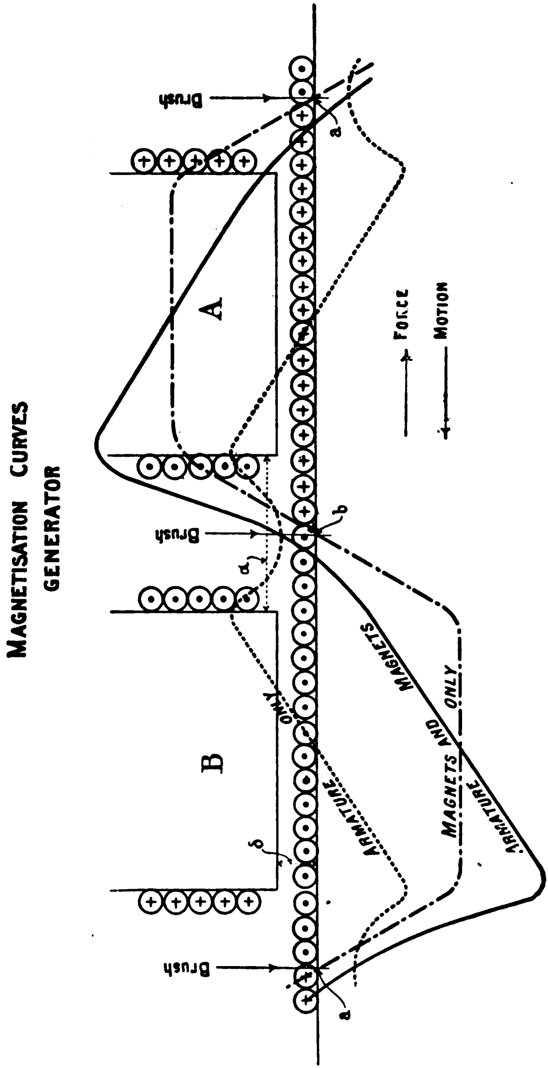

MAGNETISATION CURVES
GENERATOR
Brush
Brush
Brush
A
B
a
b
a
α
δ
Force
Motion
MAGNETS ONLY
ARMATURE ONLY
MAGNETS AND ARMATURE

Fig. 65

be the same in both motor and generator. This direction we can find by an application of Fleming's rule; pointing the first finger of the right hand in the direction of the lines of force; the thumb in the direction of motion; the second finger gives the direction of *the induced tension,* though not necessarily of the current; now if the tension impressed on the dynamo terminals is greater than the induced tension, the current will flow against this tension, that is towards us, as shown by the dots in the conductors, under pole *A* in Fig. 64 and in the reverse direction under pole *B*.

Knowing the direction of the current in the armature, we see that the lines of force caused by this current must flow in a counter-clock-wise direction under pole *A* for the motor, and must, therefore, be humped up at the back of the pole; this gives us the proper sign for the curve of armature reaction, positive behind and negative in front of each pole.

If the tension impressed on the terminals of the dynamo is less than the induced tension, as in the case of a generator, then the current will flow in the direction of the induced tension, or from us, as shown by the crosses in the wire in Fig. 65 under pole *A*, and in the reverse direction in the wires under pole *B*.

When the dynamo is acting either as motor or generator the magnetising effect of the armature is superposed upon that due to the magnets., If the resultant effect can be obtained by adding the ordinates of the armature curve to that of the magnet curve when of the same sign, and subtracting them when of different sign, we shall obtain curves like those in Figs. 64 and 65 marked 'armature and magnets,' whose areas would be equal to those due to the magnets only. We should in fact be simply taking away a certain number of lines of force from one part of the pole and adding them to another.

We have already seen that the value of the induction factor is unaltered so long as the area of the magnetisation curve remains unchanged; hence if the assumption made above is correct, we ought to find that the torque for a given current in the magnets increases in direct proportion to the current in the armature. We know, however, that this is not true, but that the torque observed generally decreases as the current in the armature increases. The explanation of this is that the magnetisation curve is not distorted symmetrically, but is reduced at one pole-tip by a greater amount than it is increased at the other.

We have here in fact two magnetising forces, the one that of the ampere-turns on the magnets; the other that of the ampere-turns on the armature. These both act upon a magnetic circuit common to the magnets and the armature. We assumed that we might neglect all the reluctance of this common portion except that of the air gap, whereas strictly the magnetisation produced by adding the two magnetising forces in the common circuit ought to be found in the usual way for a magnetic circuit consisting partly of air and partly of iron. The result will depend upon the degree of saturation of the iron in the neighbourhood of the pole-tips.

The pole-tip under which the effects are added will be the one where the loss of area takes place, especially if the pole is tapered and extended, while the result under the other tip may generally be obtained by simply subtracting the magnetising forces.

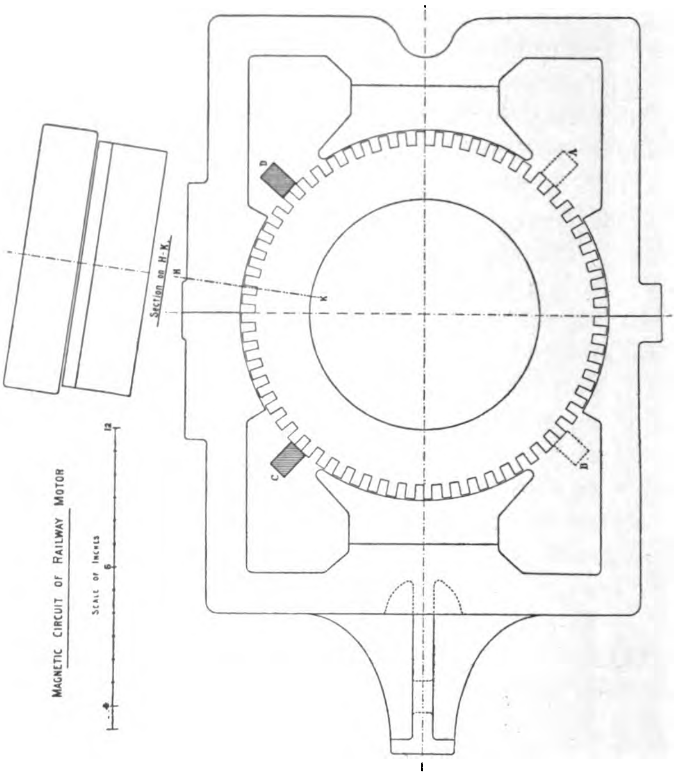

MAGNETIC CIRCUIT OF RAILWAY MOTOR
SCALE OF INCHES
0
6
12
Section on H-K.
H
K
A
B
C
D

FIG. 66

Fig. 66 gives the general dimensions of a railway motor with a slotted armature, designed to carry 24 amperes at full load.

The outside diameter of the armature is 16″, the inside diameter is 10″, the length of the armature 10¾″. The armature has 60 slots, each 5/8″ deep and 13/32″ across; the teeth are 14/32″ across. The single gap from iron to iron is 9/64″, the bore of the poles is 16″ and 9/32″; the angular breadths of the poles are 9″ and 5/8″ and 8″ and /4″ respectively, and the length parallel to the shaft 11½″.

The armature is ring-wound; there are 120 commutator bars, 6 turns per bar; hence $A=720$. Each turn consists of two wires, No. 16 *B* and *S*., wires in parallel, thus making 12 wires per bar; the wires of two bars are placed in one slot, giving 24 wires per slot. The armature is series-connected with two brushes, giving $p=2$. The resistance of the armature with carbon brushes is 1·08 ohms when hot.

The two magnets have 810 turns each, and these are connected three in parallel, 270 in series, so that the current per wire in the magnets is one-third of the main current. The resistance of the magnet winding thus connected is 0·69 ohm, making the whole resistance of the motor with magnets and armature in series 1·77 ohms.

Figs. 67, 68, and 69 give the results of **experiments** made in the Electrical Engineering Laboratory of McGill University **to ascertain the amount of the armature reaction** in this motor.

The magnets were first excited with 4 amperes per turn, and the armature rotated by means of a belt on a pulley keyed to the shaft, but no current allowed

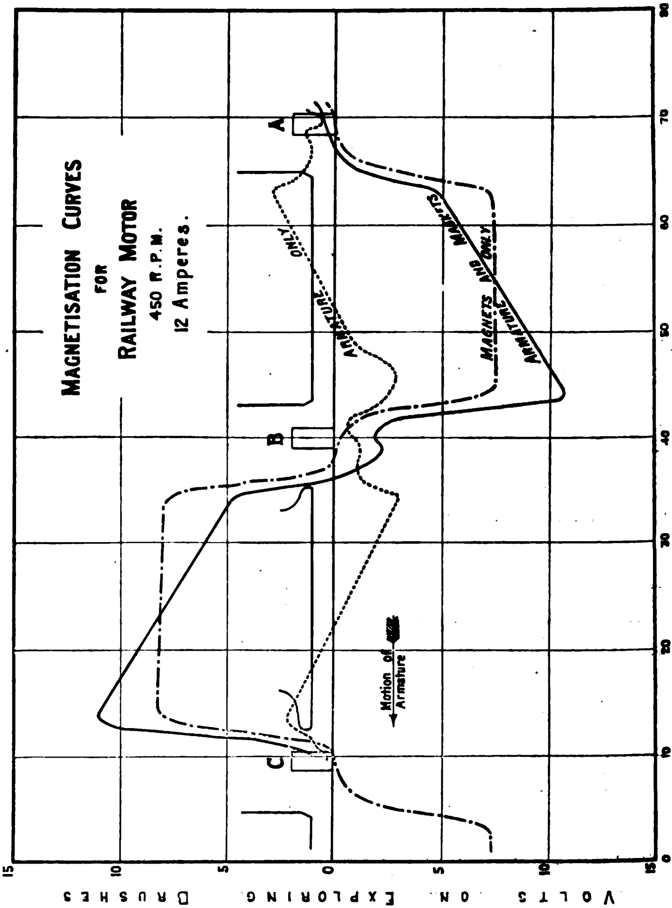

MAGNETISATION CURVES
FOR
RAILWAY MOTOR
450 R.P.M.
12 Amperes.
A
B
C
ARMATURE ONLY
MAGNETS ONLY
MAGNETS AND ARMATURE
Motion of Armature
VOLTS ON EXPLORING BRUSHES
15
10
5
0
5
10
15
0
10
20
30
40
50
60
70
80

FIG. 67

to pass in the armature. The magnetisation curve was then obtained by using exploring brushes in the way already described. As the armature is series-connected with two brushes the exploring brushes could be passed freely round the surface of the commutator crossing the positions virtually, though not actually, occupied by a brush. In the figures, A and B represent the positions thus virtually occupied by brushes; when brush A or brush B is mentioned we shall refer to the brush diametrically opposite to the positions A and B.

The magnet current was then broken and a current of 12 amperes passed through the armature; the armature was rotated as before, and the curve of armature reaction obtained. The same currents were then passed in magnets and armature at the same time, the armature was rotated as a generator in the direction shown by the arrows, and the resultant curve obtained. Each of these experiments was repeated with the full load of 24 amperes, and with 36 amperes, half overload.

The width of the brushes is given by the fact that they cover two commutator bars and two insulations, or an angular breadth of six degrees. Fig. 66 shows that they were not well set, and this is borne out by the form of the magnetisation curves. In all cases when the magnetisation due to the magnets only was being observed, the brushes were removed from their holders.

There was no sparking at all in any of the 12 ampere experiments, although there was a field under both the brushes of the wrong sign for commutating, particularly under brush B, where the field due to the magnets is also of the wrong sign.

With the full load of 24 amperes, there was no spark-

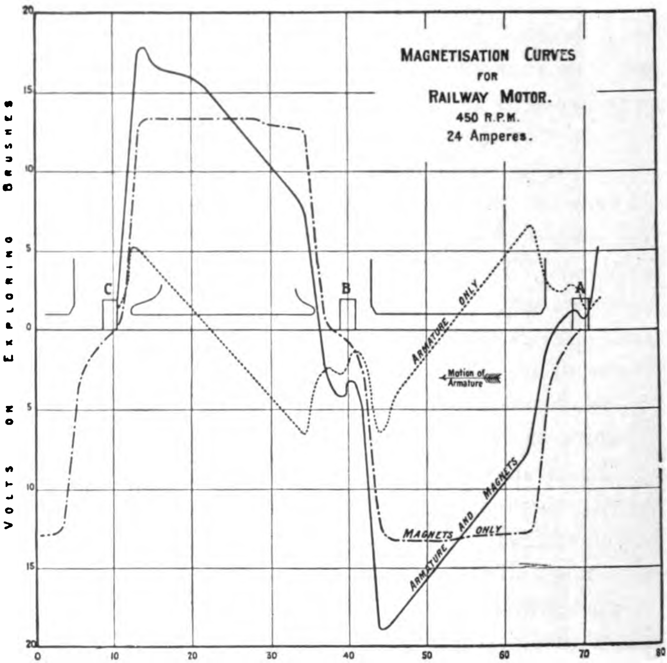

MAGNETISATION CURVES
FOR
RAILWAY MOTOR.
450 R.P.M.
24 Amperes.
VOLTS ON EXPLORING BRUSHES
C
B
A
ARMATURE ONLY
Motion of Armature
MAGNETS ONLY
ARMATURE AND MAGNETS
20
15
10
5
0
5
10
15
20
0
10
20
30
40
50
60
70
80

FIG. 68

ing with magnets only, and none with armature only, although under brush B the field is considerable, and of the wrong sign for commutation. Using Equation 116 we can calculate the intensity of this field. We know that A is 720, i is 12, ϕ is 68°, taking the larger of the two poles, δ is 0·358, and a is 9·0 cm. Hence, H is 850 lines per square cm.; this is the calculated strength of the field under the brush due to 24 amperes in the armature only. The maximum ordinate of the magnet curve in Fig. 68 represents 5,000 lines per square cm.; using this scale, we see from the diagram that the mean ordinate actually found by experiment under the brush, with 24 amperes in the armature only, was 770, calculation giving 850.

The calculated value of the intensity of the field due to the armature under the tips of the tapered pole, is 2,850 from Equation 115, while the intensity observed, taking the mean of the two, was 2,250. The difference here is probably due to the tapering of the pole-tips. Taking the square pole, with an angular width of 62° we find that the calculated value for the armature reaction under the tips is 2,600, while the observed value is 2,500.

Fig. 68 gives the results of experiments with 50 per cent. overload, namely with 36 amperes in the magnets and in the armature. There was no sparking when the magnets only were excited. With the current in the armature only there was a slight sparking under brush B, none under brush A. With current in both magnets and armature, brush B sparked badly, but there was no sparking under A.

We see here clearly the effect of the saturation of the

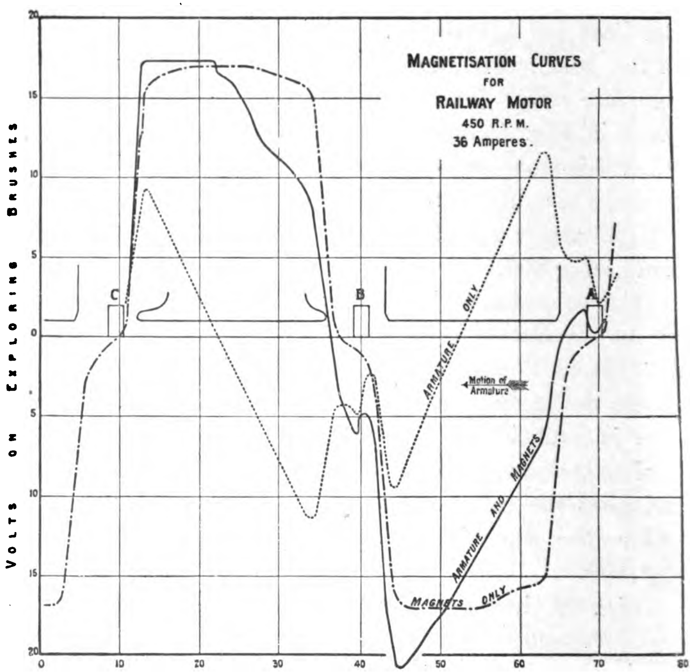

MAGNETISATION CURVES
FOR
RAILWAY MOTOR
450 R.P.M.
36 Amperes.
VOLTS ON EXPLORING BRUSHES
C
B
A
ARMATURE ONLY
Motion of Armature
ARMATURE AND MAGNETS
MAGNETS ONLY
0
10
20
30
40
50
60
70
80
20
15
10
5
0
5
10
15
20

FIG. 69

tapered pole-tips. Under the tapered pole, where the two fields are added, the intensity of magnetisation due to the magnets is hardly increased at all by the superposition of the armature field, at the same time the field under the pole where the effects are subtracted is reduced by the full amount of the armature magnetisation. Hence the area of the resultant curve is considerably less than that due to the magnets only, and the induction factor with 36 amperes in the magnets is therefore reduced by making this current also pass through the armature.

The mean value of the field under the square pole with 36 amperes in the armature is 5,900 lines per square cm. The calculated value of the armature reaction under the tip is 3,900, the mean of the values actually observed at the two tips is 3,750.

The intensity of the field under the brush due to the armature is 1,100 by Equation 116; experiment shows it to be actually rather over 1,300. The unsymmetrical position of the brushes, and the fact of one pole-tip being square and the other tapered would account for some discrepancy between the calculated and observed values, bearing in mind the assumptions that were made in arriving at the equations used.

Experiments were made with this dynamo to find by actual measurement the values of the induction factor for different currents in the magnets and armature.

The machine was driven as a generator by a belt as before; the first experiment consisted in passing a current through the magnets only, and observing the volts at the brushes at a fixed speed. The results obtained are plotted as a curve in Fig. 70. Horizontal ordinates give current in the magnets, the sum of three turns in parallel; vertical ordinates giving values of the induction factor.

The second experiment consisted in passing the same current in the armature as in the magnets, running the machine at a fixed speed as a generator, and observing the volts at the brushes. The resistance was taken when the machine was hot, the heat-drop allowed for, and the values of the induction factor deduced and plotted. The following table shows the alteration in the value of the induction factor as actually observed in the speed test, and as calculated from the area of the magnetisation curves.

Current in amperes	M Armature unloaded from speed test	M Armature loaded from speed test	Armature unloaded		Armature loaded	
			From speed test	From area of curves	From speed test	From area of curves
			Per cent.	Per cent.	Per cent.	Per cent.
12	31·2	31·2	100	100	100	100
24	51·4	49·3	165	175	158	171
36	60·6	56·0	195	220	180	182

In comparing the results of the speed test with those obtained by mechanically integrating the areas, we must remember that the voltmeter placed at the main brushes to read the terminal tension practically integrates the areas of the magnetisation curves lying under all the four poles, taking account of the way in which these curves cut the axis; whereas in Figs. 67, 68 and 69 we have only two out of four areas, and cannot assume that the other two are the same as those we have determined experimentally.

Let us suppose that the railway motor illustrated in Fig. 66 has to give a total horizontal pull of 500 pounds

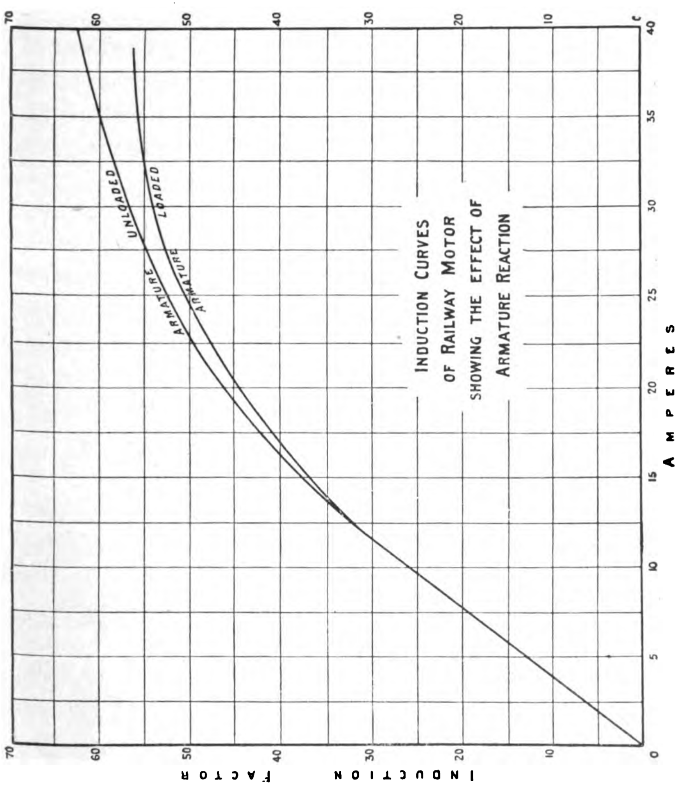

INDUCTION CURVES
OF RAILWAY MOTOR
SHOWING THE EFFECT OF
ARMATURE REACTION
UNLOADED ARMATURE
LOADED ARMATURE
AMPERES
INDUCTION FACTOR
0
5
10
15
20
25
30
35
40
10
20
30
40
50
60
70

FIG. 70

when running a tramcar at 11 miles an hour, on a line having a tension of 500 volts, the gear ratio being 4·78, and the wheel diameter 33″. From Equation 21 we find that the induction factor has to be 51·3, and that the current when running at full speed will be 24 amperes. We have **to determine the number of surface conductors** so that at full load the magnetisation under the tip of the pole due to the armature reaction is not greater than 0·52 of that due to the magnets, or

$$H_g = \frac{1}{\cdot 52} H_p = \frac{1}{298} \frac{iA\phi}{\delta}.$$

Since the armature is series-connected, the current per conductor is 12 amperes; taking the larger pole of the two, we find ϕ to be 68°, while δ is 0·358 cm. Inserting these values in our equation we find that $\frac{A}{H_g} = 0{\cdot}132$. If $\frac{A}{H_g}$ is greater than 0·132, the reaction will be too great; if it is smaller, the reaction will be less than the amount allowed.

Now since $M = pAN10^{-8}$ and $p = 2$, we see that $A = \frac{M}{2N} \times 10^8$. The area of the large pole is 714 square cm.; hence $N = 714H_g$, and we may write $AH_g = 36 \times 10^5$.

Now A must be a multiple of the number of bars on the commutator, so that

If $A = 4 \times 120 = 480$, $H_g = 7{,}500$, and $\frac{A}{H_g} = 0{\cdot}064$.

If $A = 6 \times 120 = 720$, $H_g = 5{,}000$, and $\frac{A}{H_g} = 0{\cdot}144$.

If $A = 8 \times 120 = 960$, $H_g = 3{,}750$, and $\frac{A}{H_g} = 0{\cdot}256$.

We see that the smaller the ratio $\frac{A}{H_g}$ the greater is H_g, and consequently the greater is the winding required on the magnets. We must then make H_g as small as possible, consistent with having the ratio $\frac{A}{H_g}$ as nearly as possible equal to 0·132. We may then take $A=720$, so that $H_g=5{,}000$, $H_p=2{,}600$, and the useful lines per pole, N, is $3{\cdot}56 \times 10^6$.

The conditions under which motors have to work generally demand that the brushes shall be set once for all, and operate sparklessly when the armature is running in either direction. The brushes must therefore be placed at the points where the magnetisation due to the magnets is nothing. It follows from this that the magnetisation at the centre of the brush is at all times simply that due to the armature reaction.

Now in order **to commutate sparklessly**, each coil as it comes under the brush, and is there short-circuited, ought to find itself in a magnetic field of such a sign as will tend to check the current flowing in it. In the case of a motor, the current flowing in any coil is flowing there in opposition to the tension induced in the coil, *i.e.* the induced tension tends to check the current. Hence the proper sign in which to commutate a coil in a motor armature is that of the magnetic field in the gap through which the coil has just been passing.

A reference to Fig. 64 will show that the sign of the armature magnetisation under the brush in a motor is the same as that due to the magnets in the gap ahead of the brush, and is consequently always of the wrong sign for commutation. It is clear, then, that sparkless commutation has to be accomplished in spite of the fact that the brush is always in a magnetic field of the wrong sign.

The fact that sparkless commutation can be obtained under these conditions must be attributed to the little understood action of the carbon brush, which is now almost universally used in motors. There are, however, limits to the practical use of this form of brush, but those limits must be determined by what is going on under the brush itself, and not by the magnetic condition under the pole-tip, where in practice the brushes are never placed.

Equation 115 gives us the value of the intensity of magnetisation under the pole-tip, and it is a common rule of design to allow such an armature load as will make this intensify half of that due to the magnets; the effect of this would be to reduce the intensity under the tip behind the brush in a motor to one half of what it would be if there were no current in the armature. (See a paper by Mr. W. B. Esson in the 'Journal of the Institution of Electrical Engineers,' Vol. XX.)

Equation 116, however, tells us that the intensity of magnetisation under the brush does not depend simply upon the width of the gap, but upon the distance between adjacent pole-tips, and that we can diminish it by increasing a without altering δ.

By combining Equations 115 and 116, we find that H_b, the magnetisation under the brush, can be expressed in terms of H_p, that under the pole-tip due to the reaction of the armature, in the form :—

$$H_b = \frac{8}{2+\rho} H_p \quad \ldots\ldots\ldots\ldots\ldots(117).$$

where ρ is the ratio of a, the space between the tips, to δ the width of gap.

Suppose that the magnetisation in the gap due to the magnets is 6,000 lines per square centimetre, $a=9{\cdot}0$ cm., $\delta=0{\cdot}3$ cm. Let the armature be designed to carry such a load that the magnetisation at the pole-tip due to reaction, given by Equation 115, is half that due to the magnets, *i.e.* 3,000; the magnetisation under the brush as given by Equation 116 will be 750. This will be of the wrong sign for commutating, and sparkless commutation will not be assisted, but, on the contrary, will be hindered by this magnetisation.

Suppose that the dynamo works satisfactorily under these conditions owing to the use of carbon brushes. If now we had **to make the armature carry twice the former load,** we should have to double the width of the gap to obtain a reaction at the pole-tip, as before, of an intensity half that due to the magnets, supposing the latter to remain unaltered. The magnetisation under the brush would, however, be much increased, in fact, very nearly doubled, the actual value being 1,410.

Instead of doubling the width of the gap we might increase the value of a, so as to have the same magnetisation under the brush with twice the armature load, δ being unaltered; to do this we should have to rather more than double the space between the pole-tips, the new value a being 18·6 cm. It is true that the effect of the magnets at the tip behind the brush would be reduced to nothing, but this would not affect the sparking conditions, since the magnetisation under the brush remains the same as before. The value of a may be increased by cutting the pole pieces at the tips without altering their relative positions, and in fact anything tending to increase the magnetic reluctance between the point where the brush is placed and the tip of the pole, will diminish the intensity at the brush and thus assist commutation.

It is important that the point where the magnetisation due to the magnets is nothing should not shift when the load increases. Any dissimilarity in the form of adjacent pole-tips, such as a straight radial face on one and a tapered horn on the other—as in Fig. 66—will cause this point to shift with the load, owing to the unequal saturation of the pole-tips, and this will cause the brushes to spark at a load that might be carried easily if the magnetic field were symmetrical.

A motor with fixed brushes is always running with too much forward lead, hence a current will flow in the coil that is being short-circuited under the brush. This current will be of the same sign as that which has to be commutated, and the amount of the current may be very large. There will then be a certain number of surface conductors, corresponding to the coils that are being short-circuited, carrying currents considerably greater than the rest of the conductors; these currents will give rise to circular magnetic fields of their own, especially if the conductors carrying them are imbedded in slots, and will distort the curve of magnetisation under the brushes. The curves given in Figs. 67, 68 and 69 show the hump in the magnetisation curves under the brushes due to this action.

From what has been said it would appear possible to reduce the amount of the magnetisation due to the magnets when running with full armature current, without increasing the tendency to spark. This is, in fact, the case, provided the magnetisation due to the magnets is perfectly symmetrical. The reason why a weakening of the magnets is supposed to increase the liability to spark is because such weakening usually takes place in some way tending to disturb the symmetry of the field, as, for instance, when a portion of the winding on one magnet is short-circuited. If a well-designed motor is driven at full speed by some external means, there should be no sparking when the magnets are cut out and the armature is carrying its maximum current. On the other hand, the magnetisation due to the magnets is often so unevenly distributed that sparking is actually produced when the magnets are excited, although the brushes may run sparklessly with no current in the magnets and full current in the armature.

When a coil as *ab* in Fig. 71 has its segments equally covered by the brush, the resistance in the two circuits *acf* and *bdf* will be equal, and if the currents flow according to Ohm's law, equal portions of the main current pass through each segment, and there is no current in the coil itself, so that when the coil is half-way across the brush the current in the coil is reduced to nothing. As the coil moves, the current in *bd* increases, and that in *ac* decreases, until when segment 1 is about to leave the brush, the whole of the main current is flowing down *bd*, and commutation will be effected without sparking.

In practice, however, there is a **tendency of the current** in the coil **to persist in flowing**, and this tendency is directly proportional to the rate at which the current is changing in the coil. Now if the current is changing at the rate we want it to change, that is, if it is completely reversed during the time that the coil is under the brush, which is the condition of sparkless commutation, then we can say at once what is the amount of this tendency to persist in flowing.

The current will not vary in the way we require unless we can neutralise the tendency to persist; we have then a measure of the necessary neutralising action, since it

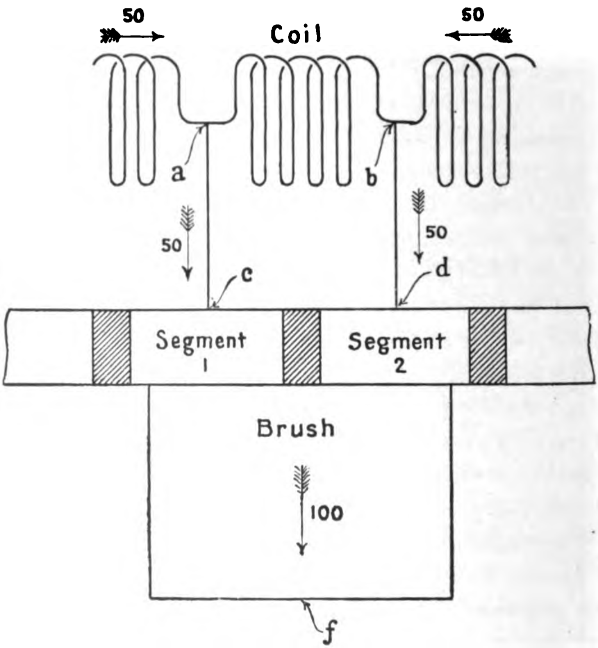


Fig. 71

must exactly balance the tendency to persist. If we apply this neutralising action we can come back to the original case, when we supposed there was no tendency to persist, and we shall then obtain the desired sparkless commutation.

The tendency of a current to persist in flowing in a circuit is called the tension due to self-induction. If $\frac{dc}{dt}$ is the rate at which the current is changing, the tension opposing the change, which is the measure of the tendency to persist, can be written:

$$e = L\frac{dc}{dt}10^{-9} \text{ volts}..............(118).$$

where dc is measured in amperes, and L is a constant called the co-efficient of self-induction, here measured in henries, the practical unit of self-induction.

If S is the number of turns in the coil, and p the permeance of the magnetic circuit surrounding the coil, measured in centimetres, $L = 4\pi S^2 p$. Further, if θ is the angular width of the brush minus that of one insulation, and if the armature is rotating at n revolutions per second, we have $\frac{dc}{dt} = \frac{720ni}{\theta}$, since the current has to be changed from $+i$ to $-i$ during the time in which the forward tip of any one segment moves through an angular distance of θ degrees.

We can balance the tension due to self-induction by causing the coil to pass at the same time in a magnetic field giving rise to an induced tension in the coil of equal amount but of opposite sign. If H is the intensity of this field per square centimetre, the lines cut by the coil during the time of commutation will be $\frac{Hl\theta\pi d}{360}$, where l is the length of the armature parallel to the shaft and d its diameter in centimetres. These lines are cut in time $\frac{360}{\theta}n$ seconds, hence the induced tension is $\pi dHln10^{-8}$ volts.

Equating this to the tension due to self-induction, we have

$$H = 288\frac{S^2ip}{\theta dl} \quad \text{.................(119).}$$

This equation gives us a measure of the tendency to spark, and will serve as a guide when, as with carbon brushes, there are other influences affecting sparkless commutation besides that of the magnetic field in which the brush is placed.

We see that the tendency to spark increases directly as the square of the number of turns per coil, as the current per wire, and as the permeance of the magnetic circuit surrounding the coil; it also varies inversely as the angular width of the brush, as the diameter, and as the length of the armature. It is evident that in a slotted armature the tendency to spark will be much greater than in a surface-wound armature.

PROBLEMS

1. A four-pole dynamo has to be designed with an induction factor equal to 8. The armature is connected in series, with 480 surface conductors. Find the useful lines per pole. $0{\cdot}83 \times 10^6$.

2. A bipolar dynamo has $4{\cdot}2 \times 10^6$ useful lines per pole, and 220 surface conductors. Find the torque for a current of 75 amperes. 980.

3. A bipolar dynamo has 300 surface conductors. The torque for 54 amperes, as measured by the method described on p. 8, is 846 inch-pounds, including that due to friction and hysteresis, which amounts to 78 inch-pounds. Find the useful lines per pole. $3{\cdot}36 \times 10^6$.

4. A direct connected generator is running at 400 r p.m. when the terminal tension on open circuit is 250 volts. Find the torque on the shaft for 500 amperes in the armature. 26,400.

5. A four-pole railway generator has its armature connected in parallel with 440 surface conductors. The internal resistance is 0·04 ohm. When running at 450 r.p.m. the output is 600 amperes at 553 terminal volts. Find the number of useful lines per pole. $17{\cdot}5 \times 10^6$.

6. A ten-pole railway generator has its armature connected in parallel with 1,440 surface conductors, and is to deliver 1,500 amperes at 550 terminal volts when running at 80 r.p.m. Find the number of useful lines per pole, (1) considering the internal resistance, which is 0·012 ohm, (2) neglecting the resistance. $29{\cdot}6 \times 10^6$ and $28{\cdot}7 \times 10^6$.

7. A dynamo gives 900 terminal volts on open circuit at 850 r.p.m. Find the maximum possible speed when placed as a motor on a 300-volt circuit with the same magnet strength as before. 284 r.p.m.

8. A motor with an internal resistance of 0·03 ohm is connected to a 100-volt circuit. Find the speed when there is a load of 5,400 inch-pounds on the shaft. 924 r.p.m.

9. What is the maximum possible horse-power of a motor with an internal resistance of 0·8 ohm when running on a 100-volt circuit? 4·2.

10. Find the induction factor of a motor that will run at 480 r.p.m. on a tension of 500 volts, and give a torque of 3,280 inch-pounds. The internal resistance is 0·8 ohm. 58·5.

11. Find the induction factor of a motor that will give 12 horse-power at 1,500 r.p.m. on a tension of 100 volts. The resistance is 0·1 ohm. 3·6.

12. A four-pole street railway motor has a series-connected armature with 624 surface conductors and a resistance of 1·2 ohm. Find the number of useful lines per pole required to run at 800 r.p.m. with 1,200 inch-pounds of torque on a 500-volt line. $2{\cdot}83 \times 10^6$.

13. Check the results of Problems 11 and 12 by showing that the line-watts is equal to the sum of the heat-watts and the mechanical watts.

14. A main shaft has to run at 240 r.p.m. with a load of 4,600 inch-pounds, and is driven by a motor with a 10-inch pulley belted to a 48-inch pulley. The friction of the motor, &c., amounts to 500 inch-pounds on the motor shaft. The tension of the line is 150 volts, and the resistance of the motor 0·07 ohm. Find the induction factor. 7·28.

15. Check the result of Problem 14 for input and output of energy.

16. A four-pole motor which has its armature connected in series and 480 surface conductors, raises an unbalanced lift weighing 1,500 lbs. at 240 f.p.m. The rope

drum has a diameter of 48 inches. The velocity ratio is 70. The tension of the line is 125 volts. The internal resistance of the motor is 0·1 ohm. Find the number of useful lines per pole. $0·55 \times 10^6$.

17. A motor has to raise a weight of 1,500 lbs. at 200 f.p.m. The frictional torque is 430 inch-pounds on the motor shaft. The rope drum is 36 inches in diameter. The velocity ratio is 75. The tension is 125 volts, and the resistance of the motor 0·05 ohm. Find M. 4·47.

18. A tramcar has to be driven at 13 miles an hour by two four-pole motors connected in parallel on a 500-volt line. The armatures are series-connected with 960 surface conductors. The total tractive effort per motor at full speed is to be 800 lbs. The resistance is 1·3 ohm per motor. The wheels have a diameter of 33 inches. The velocity ratio is 4·78. Find the area of each polar surface in square centimetres if the magnetisation in the gap is fixed at 5,000 lines per square centimetre. 435 sq. cm.

19. A gearless motor has to exert a total tractive effort of 800 lbs. when running at 12 miles an hour on 33-inch wheels. The tension of the line is 500 volts, and the resistance of the motor 1·5 ohm. Find the induction factor of the motor. 214.

20. Find the current taken by the motor in Problem 19. If gearing with $v = 4$ is introduced, without altering any other conditions, what is the current?

43·7 amperes in both cases.

21. A tramcar is equipped with two motors, each having $M = 60$ and $R = 1·5$ ohm. If $E = 500$ volts, $v = 4·8$, and $d = 33$ inches, find the speed in miles an hour (1) in parallel, (2) in series, when the total load per motor is 700 lbs. horizontally. 9·4 and 4·2.

22. A motor car weighing 60,000 lbs. is driven by two gearless motors, connected in parallel, at 16 miles an hour on a level. The tension of the line is 500 volts, and the motors take 20 amperes each. The resistance of each motor

is 1·0 ohm. The driving wheels are 33 inches in diameter. If the frictional resistances remain the same, find the speed up a grade of 1 in 20. 12·7 miles an hour.

23. A car weighing 30,000 lbs. is driven by two geared motors designed to run in parallel at 15 miles an hour on a level when the frictional resistance is 600 lbs. per motor. If $E = 500$ volts, $v = 4{\cdot}78$, $d = 33$ inches, and $R = 1{\cdot}5$ ohm per motor, find the steepest grade the car can ascend with the motors in parallel. 12·4 per cent.

24. A motor car weighing 20,000 lbs. is driven by two geared motors, connected in parallel, at 13 miles an hour on a level when the total horizontal pull is 800 lbs. per motor, the tension of the line being 500 volts. If the load is increased by that due to a grade of 1 in 10, find the speed when the motors are connected (1) in parallel, (2) in series. $R = 1{\cdot}3$ ohm per motor, $d = 33$ inches, $v = 4{\cdot}78$.

10·7 and 3·3.

25. A motor car weighing 40,000 lbs. has to be driven by two bipolar motors at 12 miles an hour up a grade of 1 in 15. The frictional resistances amount to 10 lbs. per 1,000, including the motor losses. $E = 500$ volts, $R = 1{\cdot}0$ ohm per motor, $v = 5$. The poles each have a gap area of 967 square centimetres. Find the magnetisation in the gap if the number of surface conductors is 520. 8,000 per sq. cm.

26. Two shunt-wound motors are mechanically coupled and connected in parallel on a 500-volt line. If their induction factors are 71 and 70, and the resistance of each is 0·4 ohm, find the total mechanical horse-power when one motor is doing all the work. 11·7.

27. Two shunt-wound motors are mechanically coupled and connected in parallel on a tension of 140 volts. One motor has an induction factor of 7·5, and a resistance of 0·3 ohm; the other has an induction factor of 8·7, and a resistance of 0·5 ohm. Find the total torque on the shaft when the motors are working at the same rate.

5,720 inch-pounds.

28. A railway generator is being tested by the electrical method described on p. 87. The line tension is 550 volts, the speed is 400 r.p.m., the generator output is 500 amperes, the resistance of the generator is 0·04 ohm, and that of the motor 0·03 ohm. If the current from the line is 48 amperes, and the torque losses are equal in the two machines, find the torque loss in the generator. 782 inch-pounds.

29. A motor is tested by Hopkinson's method. The total mechanical output is 45 horse-power when running at 900 r.p.m. on a tension of 125 volts. The resistance of the motor and of the generator used in the test are both equal to 0·015 ohm. The torque input measured on the belt is 600 inch-pounds. What is the torque loss in the motor? 191 inch-pounds.

30. A generator is being tested by the method described on p. 87. The terminal tension is 250 volts, the current output 800 amperes, and the current from the line 92 amperes. The resistance of each armature is 0·0058 ohm. If the power used in magnetising the magnets of the generator is 5·2 k.w., find its total efficiency. 92·5 per cent.

31. A railway motor is tested by the method described on p. 144. The terminal tension on the motor is 500 volts, the current 44 amperes, the resistance of the motor magnets and armature 0·72 ohm. The generator current is 34 amperes, and the resistance of the generator magnets in series with the motor circuit 0·43 ohm. Find the efficiency of the motor. 83 per cent.

32. A motor generator runs at 550 r.p.m. on a tension of 588 volts. The current in the motor is 64 amperes, and that in the generator 280 amperes. The motor resistance is 0·175 ohm. The induction factor of the motor is five times that of the generator. Find the total frictional losses in the two machines. 705 inch-pounds.

33. A motor has an induction factor equal to 6, an internal resistance of 0·08 ohm, and torque losses amounting to 76

inch-pounds. Find the most efficient current and the efficiency for that current. 116 amperes and 85 per cent.

34. The armature of an eight-pole motor contains 646,000 cubic centimetres of iron. If the hysteresis loss is 8,600 ergs per cubic centimetre per cycle, find the current required to turn the armature against the hysteresis torque, the induction factor being 68. 33·2 amperes.

35. A train weighing 40 tons is driven by two gearless motors designed to run at 20 miles an hour in parallel on a tension of 500 volts when the resistance to motion is 495 inch-pounds of torque per ton. The driving wheels are 33 inches in diameter. The resistance of the motors is 0·3 ohm each. The maximum current per motor is to be 200 amperes. If the acceleration is uniform up to full speed, how many seconds will be spent in covering the first 200 yards from rest? Assume M to be constant and the parallel method of control. 31·5 seconds.

36. A train weighing 600 tons is drawn by four gearless motors at 10 miles an hour up a grade of 0·7 per cent., the frictional resistance being 12 lbs. per ton and the tension of the line 500 volts. Find the total current required to start up on the grade with an acceleration of 0·5 f.p.s. per second, if the motors are connected permanently in series and M is constant. 1,845 amperes.

37. A motor has $M = 8{\cdot}1$, $R = 2{\cdot}4$ ohms. The armature, with an attached fly-wheel, weighs 1,500 lbs., the radius of gyration being 9·2 inches. The frictional and other resistance to motion amounts to 280 inch-pounds, and is constant. The motor is switched on to a line of 100 volts tension and left to run. How many revolutions will it make in two minutes? 365.

38. A motor is running on a line of constant tension with a constant load. The speed has to be varied by varying M with a rheostat in the magnet circuit, which is a shunt on the main circuit. Show that the change of speed for any

given change of M decreases as the resistance in the armature circuit increases.

39. A crane has to lift a weight of 5 tons from rest through a distance of 22 feet. M is constant, and equal to 16·4 ; $R = 1·3$ ohm. The diameter of the rope drum is 36 inches, and $v = 65$. The friction amounts to 380 inch-pounds of torque on the motor shaft. The tension of the line is 225 volts. Find the time required to cover the given distance if the maximum current is limited to 160 amperes.

86 seconds.

40. A weight of 2 tons has to be lifted by a motor working on the principle described in p. 157. $E = 120$ volts, $R = 0·15$ ohm, $M = 18$, $v = 95$, and $d = 24$ inches. The torque due to friction is 254 inch-pounds on the motor shaft. Find the weight of a fly-wheel, of radius of gyration 6 inches, that must be placed on the motor shaft, so that on connecting the clutch the current drawn shall not exceed 50 amperes.

18·1 pounds.

41. Same data as in Problem 40. Find the weight of the fly-wheel, of radius of gyration equal to 6 inches, so that the speed shall not fall below that at which the motor runs when raising the weight at a uniform speed. 36 lbs.

42. A crane has to lift a weight of 20 tons through 15 feet in 35 seconds from rest. $E = 500$, $v = 80$, $M = 62$ and is constant. The drop at full speed is to be 20 volts. The frictional resistances amount to 10 per cent. of the load. Find the diameter of the chain drum and the resistance of the motor. 25·4 inches and 0·228 ohm.

43. A motor with an induction factor of 75 and an internal resistance of 0·02 ohm is running at full speed on a 500-volt line, with a torque on the shaft equal to 63,500 inch-pounds. A fly-wheel weighing 1·6 ton, with radius of gyration 8 feet, is mounted on the motor shaft. If the tension of the line drops 10 per cent., find in how many seconds the speed will have dropped the same per cent. Neglect the moment of inertia of the armature. 4·1 seconds.

44. A turret weighing 20 tons, with a radius of gyration of 7 feet 6 inches, is rotated by a motor with a gear ratio of 300, the induction factor being constant and equal to 8, and the resistance being 0·2 ohm. The frictional torque is 338 inch-pounds on the motor shaft. The tension of the line is 80 volts. What is the shortest time in which the turret can be turned through 240 degrees if the maximum current does not exceed 40 amperes? 24·4 seconds.

45. The bascules of the Tower Bridge each weigh 1,070 tons, and have a radius of gyration of 40 feet. The arc of rotation is 82 degrees, and the time 90 seconds. Motion is derived from a motor shaft with a velocity ratio of 360. The friction may be estimated at 200 inch-pounds of torque per ton on the main bearings. The motion can be stopped in 12 degrees in twenty seconds. If the tension of the line is 100 volts, and the drop at full speed 4 volts, find the value of M and the maximum current, the motor being shunt wound.

$M = 64$ and $C = 31·4$ amperes.

46. The trains on the Central London Railway weigh 145 tons, and are each hauled by a locomotive equipped with four gearless motors, with driving wheels 42 inches in diameter. The tension of the line is 500 volts. The frictional retardation may be taken at 7·5 lbs. per 1,000. The drop at full speed is not to exceed 5 volts, and the mechanical efficiency is to average 95 per cent. The motors are started two in parallel, and changed over to four in parallel for full speed. The time schedule requires that 2,000 feet shall be covered in 75 seconds from rest. Find the current and induction factor at full speed, and the force factor required to start.

Each motor must have $M=136$ for 70 amperes, and a maximum force factor of 40·8 kilodynes.

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