PROBLEMS

IN

SURVEYING, RAILROAD SURVEYING AND GEODESY

WITH AN APPENDIX ON

THE ADJUSTMENTS OF THE ENGINEER'S TRANSIT AND LEVEL

BY

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FIRST EDITION
THIRD THOUSAND

NEW YORK
JOHN WILEY & SONS
LONDON: CHAPMAN & HALL, LIMITED
1912

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THE SCIENTIFIC PRESS
ROBERT DRUMMOND AND COMPANY
BROOKLYN, N. Y.

PREFACE.

Many of the problems in this book have been used for the past three years in the authors' classes. The exercises are, for the most part, progressive in order, although probably in many institutions they could not be given in their logical order on account of limited equipment.

Most of the problems in the first five chapters might well be given in a course in plane surveying; those in Chapter VI in connection with a course in railroad curves, and those in Chapter VII in geodesy. Problems in Chapter VIII may be given either in connection with the plane-surveying course or with geodesy, or, again, during the topographic survey.

Students at the University of Pennsylvania do most of the problems in this book, including a participation in the three surveys in Chapter IX. The problems are designed to require from two to four hours for their completion. The aim has been to make the work as practical as possible. Sufficient theory is given for a reasonable understanding of the problem. Specimen notes are in many cases given, but where they are not given the requirements are stated, so that no doubt is left as to the form of report desired.

With the large growth in the number of students in Civil Engineering at the University of Pennsylvania, need has been felt of a manual which would cover the field courses. It is to fill this need that this book is published, and with the hope that it may prove of service at other institutions.

"The Adjustments of the Engineer's Transit and Level," which is published separately, is for convenience given as an appendix.

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Assistance has been received from Prof. A. W. French of the Worcester Polytechnic Institute in connection with the problems on astronomy and on Problem D4.

H. C. I.
H. E. H.

PHILADELPHIA, PA., July 16, 1906.

NOTE TO FIRST EDITION, THIRD THOUSAND.

In this edition all known errors have been corrected and Table VIII, for the Reduction of Stadia Readings, has been added.

GENERAL DIRECTIONS FOR FIELD-BOOKS.

The points noted below are of so great importance that they are brought together under one heading, although attention will be called to many of them in the individual problems.

1. The date, personnel of party, and numbers of instruments should be given for each problem.

2. At the top of each double page the number of the problem and its title should be given.

3. Number each right-hand page. Place an index in the back of the book. The index should contain the number and title of the problem and the page on which it is given. Keep this index up to date.

4. All notes, descriptions, etc., must be in the Reinhardt style of lettering and well executed. No writing will be allowed in the book.

5. Notes must be entered in the field-book during the progress of the work, and in no case are they to be taken on small slips of paper and transferred to the book later.

6. Only hard pencils are to be used.

7. The student's name is to be placed in water-proof ink on the outside of the book.

8. *Unless notice is given to the contrary,* books are to be submitted at the end of the exercise with notes complete and corrections made. In no case is a student to leave the work without being excused by the instructor in charge of his section.

9. The necessary corrections and additions are to be made promptly, as required. The handling of this matter will, of course, depend on the view of the instructor in charge of the work, and detailed directions will accordingly be given by him.

10. Students are held responsible for all equipment issued

to them, and the attention of an instructor should at once be called to any damaged article.

The following list of "Don'ts" should be carefully read and the points noted:

a. **Don't** leave an instrument *unguarded in a street, road, or pasture, or in close vicinity to blasting.*

b. **Don't** lean an instrument up against a building or other object, but see that it is firmly set up on its legs.

c. **Don't** carry an instrument over your shoulder until you are out of doors, but carry it under your arm.

d. **Don't** strain any of the screws of an instrument, but remember that they are made of brass, and need to be clamped but lightly.

e. **Don't** attempt to force the tangent-screws to their extreme limit in either direction.

f. **Don't** pass in equipment which is liable to rust, until it has been thoroughly wiped off.

g. **Don't** fail to see, when you pass in an instrument, that it is centered and leveled; that the needle is raised; that sunshade and cover are with it; that object-glass is drawn back by focusing-screw; that plumb-bob cord is wound around the bob.

h. **Don't** forget to keep the needle raised when it is not in use.

i. **Don't** tie up tapes with plumb-bob cord, but see that they are securely tied up in the form of a figure 8 when they are without reels.

j. **Don't** try to throw out a tape as you would an engineer's chain. It won't work.

k. **Don't** stand directly back of your instrument when sighting through it, but place yourself in a *comfortable* position on the side, and bend your body sidewise.

l. **Don't** batter up the points of plumb-bobs.

m. **Don't** chop stones with the hatchet or brush-hook.

n. **Don't** throw equipment around in a careless manner, but remember that each article is an instrument of precision.

o. **Don't** fail to return stakes to the College when not used for permanent marks.

p. **Don't** forget to carry your magnifying-glass and adjusting-pin with you.

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PROBLEMS

IN

SURVEYING AND GEODESY

CHAPTER I.

CHAINING PROBLEMS.

PROBLEM A1. MEASUREMENT OF DISTANCES WITH THE STEEL TAPE OR ENGINEER'S CHAIN.

Party. Two men.

Equipment. 100-ft. steel tape or chain, two range-poles, set of marking-pins, hatchet, stakes, plumb-bobs and tacks.

Problem. *To measure the distance between two fixed points.*

Method. If no permanent monuments are available, set two stakes at least 1000 ft. apart. The difference in elevation of these stakes should be considerable, and the slope between them variable. Set a range-pole in back of one stake and measured toward it from the other stake.

In undoing the chain take the two handles in the left hand and the chain-bundle in the right hand. Throw the bundle in a direction opposite to that in which the measurement is to be made, the handles still remaining in the left hand. Do not use this method in undoing a tape. See *j* of "Don'ts." The head chainman then takes one handle and slowly walks in the direction of measurement, the rear chainman allowing the chain to slip through his hands, meanwhile straightening bent links and kinks. The rear chainman then lines the head chainman in by eye, and a pin is placed at the forward end of the chain, care being taken that the chain is horizontal. When the slope is steep, the chain must be broken at frequent intervals.

Measure the distance at least three times by plumbing and twice along the slope. After having measured along the slope, pace the distance four times, using a natural step, and then four times, using an assumed 3-ft. pace. In each case estimate the fractional pace at the end.

Computations. Compute on the right-hand page, for the distance obtained by plumbing, the ratio of the extreme range to the mean of the three measurements. The extreme range is the difference between the largest and the smallest measurement.

The measurement along the slope is to be used in computing the average length of pace. Find the length of pace and the number per hundred feet.

FORM OF NOTES.

(Left-hand page.)

MEASUREMENT OF A LINE.

Line.	Distance.	Direction.	Distance along slope.
A–B	1043′.4	Down	
B–A	1043′.5	Up	
A–B	1043′.3	Down	
B–A		Up	1044′.0
A–B		Down	1044′.1

LENGTH OF PACE.

Kind of pace.	Number.	Mean number.	Length of line.	Length of pace.	No. per 100 ft.
Natural..........	435.0				
	434.4				
	435.9				
	434.8	435.0	1044′.0	2′.4	41.7
Assumed 3-ft. pace..	355.0				
And so on.					

(Right-hand page.)

Oct. 7, 1905.
9.15 A.M.–12.45 P.M.
Party: J. B. Tustin, Head Chainman.
A. D. Hyman, Rear Chainman.

Equipment: Range-poles Nos. 8 and 10.
Chain No. 4.
Hatchet No. 6.
Instructor:

$$\text{Ratio extreme range to mean length} = \frac{0.2}{1043.4} = \frac{1}{5217}.$$

PROBLEM A2. MEASURING THE ANGLES OF A PENTAGON WITH TAPE OR CHAIN.

Party. Two or three men.

Equipment. Tape or chain, three range-poles, set of marking-pins, two plumb-bobs, hatchet, stakes, and a five-place table of logarithms.

Theory. If equal distances (d) are laid off on the two sides of an angle A, and the distance a between them measured then the sin $\frac{1}{2} A = \frac{a}{2d}$.

Method. Drive five stakes, forming a pentagon with angles from 90° to 120°, the sides being from 200 to 400 feet in length. Start at one station and place a range-pole at each adjacent station. Lay off equal distances on the two sides of the angle at this station, marking the points by pins, and measure the distance between these pins. The distance d should not be less than 25 ft. (The greater the distance the more accurate the results.) Sometimes it may be best to measure the supplement of the angle instead of the angle itself. Proceed in like manner at the other stations. Calculate the angles by the formula given above.

Measure the sides of the figure and assume one side as a meridian. From the computed angles find the bearings of the other sides. Compute the latitude and longitude differences and find the error of closure.

The *error of closure* $= \frac{\sqrt{(\text{Error Lat.})^2 + (\text{Error Long.})^2}}{\text{Perimeter}}$.

Express this as a fraction whose numerator is 1.

FORM OF NOTES.

(Left-hand page.)

Angle.	d.	a.	$\frac{a}{2d}$.	$\frac{1}{2}$ Angle.	Angle.
1	50′	76.90	.7690	50° 16′	100° 32′
2	50′	84.43	.8443	57° 36′	115° 12′
3	50′	81.02	.8102	54° 07′	108° 14′
4	50′	73.31	.7331	47° 09′	94° 18′
5	50′	87.49	.8749	61° 02′	122° 04′
			Sum of angles.......		=540° 20′
			Error.............		+ 0° 20′

(Right-hand page.)

Line.	Bearing.	Distance.	Lat. diff.	Long. diff.
1–2	Assumed N	280.60	+280.60	———
2–3	N 64° 48′			
	And so on			

Give a sketch of the field with the "error of closure"; also, party, date, equipment, etc., as usual.

PROBLEM A3. TESTING A TAPE OR CHAIN. (ROUGH METHOD.)

Party. Two men.

Equipment. Standard tape, chain or tape to be tested, spring balance, two plumb-bobs, thermometer, triangular scale, and a small piece of cardboard.

Problem. *To determine the absolute length of a steel tape or chain.*

Theory. For a change of one degree in temperature a steel tape will expand or contract 0.0000063 of its length.

The tension required to overcome the correction for sag is called the *normal tension.*

Method. This problem consists of two parts: First, to find the absolute length of the tape under given conditions; and, second, to find the normal tension.

First. To find the absolute length of the tape: Drive a brass tack in the floor of a passageway and make a fine scratch on it. Stretch out the standard tape and, when at a tension of 10 lbs., mark a point on a piece of cardboard at the 100-ft. graduation, the zero graduation coinciding with the scratch in the brass tack. Care should be taken that friction is eliminated. Note the temperature by placing the thermometer on the 50-ft. graduation of the tape.

If l = the absolute length of the tape at 62° Fahrenheit and 10 lbs. tension, and supported throughout, and
t = observed temperature,
then l' = length of base-line $= l + l(t - 62°)0.0000063$.

Apply the chain or tape to be tested to this base. Note the difference in length at 10 lbs. tension.

Call this difference d, and if t' = the temperature at time of second measurement and l'' = absolute length of tested tape, then $l' \pm d = l'' + l'\ (t' - 62°)\ 0.0000063$ or $l'' = l' \pm d - l'\ (t' - 62°)$ .0000063. (Strictly speaking, l' in the third term of the right-hand member should be l'', but the resulting error is inappreciable.) Make four trials of this test.

Second. To find the normal tension. With the chain or tape to be tested held clear of the floor, note the tension necessary to bring the end graduations the same distance apart as when the same tape was supported throughout with a tension of 10 lbs.

The plumb-bobs are used to bring the end-marks over the points on the floor.

Make four repetitions of this test.

FORM OF NOTES.

(Extending over both pages.)

Part.	Length of base. l'.	Tested tape. $l'-d$.	Temp.	Tested tape at stand. cond. l''.	Diff. d.	Mean error.	Pull.
1	100.005	99.903					
		99.903	74°	99.895	−0.102		
		99.902	74°.2	99.894	−0.103		
		99.903	74°	99.895	−0.102	−0.102	10 lbs.
2							25 "
							24.5 "
							25.5 "
					Normal tension		=25 "

On right-hand page give number of standard tape, together with its length; also, party, date, equipment, etc., as usual.

CHAPTER II.

LEVELING PROBLEMS.*

PROBLEM B1. DIFFERENTIAL LEVELING.

Party. Two men.

Equipment. Level, level-rod, and crayon or keil for marking turning-points.

Problem. *To find the difference in elevation of two points.*

Method. Set up and level the instrument so that a rod-reading may be taken on the first point. In this connection the following points are helpful: After bringing the plates into a position as nearly level as possible by changing the legs, place the bubble parallel to one pair of leveling-screws and bring the bubble to the center, remembering that both thumbs turn out, or both in, and that the bubble will go in the direction in which the left thumb moves. When the bubble is in the center, turn through 90° and level over the other set in a similar manner. Care should be taken that the screws do not bind, but they should be brought snug against the plate.

After taking a reading on the first point, have the rodman select a suitable turning-point in advance and read the rod on it. All turning-points should be taken on firm objects. The length of sights should be from 10 to 400 feet, depending on the natural conditions and the state of the atmosphere. The rod-reading which is used to find the H.I. is called a plus sight ($+S$); the reading made in order to determine the elevation of a point is a minus sight ($-S$); when the general direction is down-hill the plus sights should be small and minus sights large, when up-hill, *vice versa*.

To secure good results the following points should be observed in making the observations: First, see that the bubble is in the center, and then set the target, the rod being held plumb. After clamping the target and checking its position, again see that the bubble is in the center. Then record the reading after it has been checked by the levelman. In order to eliminate errors, the length of plus and minus sights at a set-up should be equal.

FORM OF NOTES.

(Left-hand page.)

+*S.*	H.I.	−*S.*	I. S.	Elev.	Sta.
6.629	106.629			100.00	B.M. 1
4.833	108.026	3.436		103.193	T.P.
0.669	102.183	6.512		101.514	T.P.
1.170	92.574	10.779		91.404	T.P.
1.636	84.346	9.864		82.710	T.P.
9.952	82.881	11.417		72.929	B.M. 2
11.616	93.176	1.321		81.560	T.P.
8.944	100.970	1.150		92.026	T.P.
6.360	105.411	1.919		99.051	T.P.
		5.421		99.990	B.M. 1
51.809		51.819		100.000	
		51.809		99.990	
		0.010	Check	0.010	

(Right-hand page.)

Describe each B.M. exactly opposite B.M. on left-hand page. Give party, date, equipment, etc., as usual.

PROBLEM B2. PROFILE LEVELING.

Party. Two men.

Equipment. Level, self-reading level-rod, steel tape and keil.

Problem. *To determine the profile of a line.*

Method. The line taken may be the center line of a street, a curb line, or a line of stakes representing the center line of a railroad. The points may be 50 or 100 feet apart.

The method of procedure is practically the same as that employed in differential leveling, except in the following particulars: Sights, which are called intermediate sights in this case, are taken on several stations from one set-up of the instrument.

After completing the profile of the line, check back by differential leveling to the original B.M. A bench-mark should always be left at the end of the line.

Plotting the profile. Plot the profile on profile-paper (Plate A), using a suitable scale, such as 200 feet to the inch horizontally and 20 feet to the inch vertically.

Form of Notes. The form of notes is the same as that given under differential leveling, except in this case the intermediate sight-column is used. In order to check the work, sum up the plus and minus sights on B.M.'s and T.P.'s, as in that problem.

PROBLEM B3. LAYING OUT A SIMPLE GRADE AND PLUNGING A GRADE.

Party. Three men.

Equipment. Level, self-reading level-rod, tape, hatchet, stakes, and keil.

Problem. *To set grade-stakes:* First, for a curb line; second, for reballasting a line of railroad; or, third, for the center line of a railroad. The problem will consist of the solution of the third case.

Theory. If the elevation of grade and ground are known at a given station, the difference of elevation can be obtained and marked on grade-stakes, or, if it is desired to put the top of the grade-stake at grade, as in the case of ballasting, this can be done if the H.I. is known.

Method. This problem will consist of two parts: First, to lay out the simple grade, and, second, to check the marking of the stakes by plunging the grade.

Simple grade. Run out a line from 200 to 400 feet in length, placing stakes 20 to 50 feet apart. Set the instrument up near the middle of the line and take a rod-reading on a B.M. (either actual or assumed). Then take rod-readings on the ground near each stake. Assume elevation of grade at the first station; then, knowing the rate of grade, the grade elevation at any other station may be found. The difference between grade elevation and surface will give the cut or fill at the point. Take a rod-reading on the top of each stake and find its elevation. Lower the stake until the cut or fill from its top is an even foot or an even foot and tenth of foot. Mark the cut or fill for top of stake.

Remark. The stakes may be lowered until the cut or fill will come out to even feet and inches instead of feet and tenths.

Plunging a grade. Set up over the first stake and bring a pair of leveling-screws in line with the stakes.

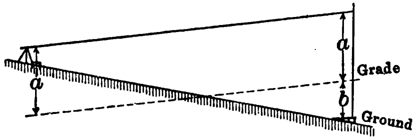
Let $a =$ the height of the cross-wires above grade at this station.

Place the level-rod on the top of the stake where the grade changes, and, with the target set at $a \pm b$ where

$b =$ the difference of elevation between the grade and top of stake at the second point,

bring the cross-wires to this reading by changing the leveling-screws.

Check the cut or fill as found in the simple grade from the top of each stake by comparing the rod-reading at the stake with the quantity $a \pm b$.



The figure given above is the case where the first stake is above grade and the second is below grade.

The student should be able to solve other cases as they arise.

FORM OF NOTES.

(Extending across both pages.)

SIMPLE GRADE.

Sta.	$+S.$	H.I.	$-S.$	Surface.	Grade.	Cut or fill on ground.	Elev. top of stake.	Cut or fill top of stake.

PLUNGING A GRADE.

Sta.	$+S.$	H.I.	$a \pm b.$	Rod-reading on top last stake $= a \pm b.$	Rod-reading intermediate stake.	Cut or fill top of stake.

Party, date, equipment, etc., are placed on right-hand page.

PROBLEM B4. ADJUSTMENTS OF THE Y LEVEL.

Party. Two or three men.

Equipment. Y level, level-rod, steel tape, hatchet, stakes, and adjusting-pins.

Theory. In this problem, as also in that of the adjustments of the transit, it is assumed that the student has had the theory of the adjustments in the classroom, and accordingly the description has been abridged. For convenience, however, a complete description of the adjustments is given in the Appendix.

Method of Making the Adjustments with Form of Notes. *To bring the axis of the bubble into the same plane as the axis of the telescope.* See Appendix, page 121. Note in the field-book the movement of the bubble when swung through a small angle, and state how is was corrected.

To make the line of collimation parallel to the axis of the bubble. See Appendix, page 121, for the indirect method, and page 127 for the direct method.

For the first part of the adjustment, note the amount that the line of collimation was out, illustrating by a sketch; and, for the second part of the adjustment, the movement of bubble in divisions of scale at each test, until entirely corrected. If the direct method or "peg adjustment" is used, illustrate by a suitable sketch, giving all rod-readings, including the corrected rod-reading for adjusting.

To make the axis of the bubble perpendicular to the vertical axis of the instrument. See Appendix, page 123. Note in the field-book the movement of the bubble when the telescope was revolved through 180°, and state how the error was corrected.

The title and number of problem, equipment, party, date, etc., should be given as usual. Both pages of the note-book may be used in this problem.

CHAPTER III.

COMPASS PROBLEMS.

PROBLEM C1. COMPASS PRACTICE.

Party. Two or three men.

Equipment. Surveyors' compass, two range-poles, hatchet, and stakes.

Problem. *To measure the angles of a hexagon by use of a surveyors' compass.*

Method. Drive six stakes, forming a hexagon with sides from 200 to 400 feet in length. Set the compass up over one station, and, with the declination arc set at zero, read the bearings of the lines 1–2 and 1–6 to the nearest five minutes; shift the declination arc by about 2° and read the new bearings of the lines. Set up at the next station and read the backward bearing of the line 2–1 and the forward bearing of the line 2–3. Shift the declination arc by the same amount and in the same direction as at the first station. Continue around the figure to the starting-point.

Always have the north end of the compass-box sighted along the line whose bearing is being measured, and read the north end of the needle.

Computations. Compute the interior angles of the hexagon, using the mean bearing of the backward and forward readings in each case. See notes.

FORM OF NOTES.

(Left-hand page.)

Line.	1st Reading.	2d Reading.	Mean 1st and 2d Readings.	Mean Bearing.	Mean Ang.
A–F	N 49° 45′ W	N 51° 50′ W	N 50° 47′ W	N 50° 51′ W	
A–B	N 23° 35′ E	N 21° 45′ E	N 22° 40′ E	N 22° 40′ E	73° 31′
B–A	S 23° 45′ W	S 21° 35′ W	S 22° 40′ W	S 22° 40′ W	
	And so on				
F–A	S 49° 45′ E	S 52° 05′ E	S 50° 55′ E	S 50° 51′ E	
			Summation of angles =		
			Error =		

(Right-hand page.)

Draw a sketch of the figure, giving the approximate relative positions of the stations, together with the direction of the meridian.

Party, date, equipment, etc., as usual.

PROBLEM C2. DECLINATION OF THE MAGNETIC NEEDLE.

Party. Two men.

Equipment. Surveyors' compass having a variation-plate, and range-pole.

Theory. The magnetic declination at any point is the horizontal angle between the magnetic meridian and the true meridian. The declination is subject to various changes; these are: the secular variation, the annual variation, the lunar inequalities, and the daily variation. Of these only the daily variation will be considered.

The needle is in its mean daily position at about 10.30 A.M. and 8 P.M. The range of movement extends from about 5 minutes in winter to nearly 12 minutes in summer. The daily variation changes slightly with the latitude. The corrections to be applied to the observed declination are given in Table VII.

Method. Set up and carefully level the compass over the south end of the true north and south line which has been previously determined. The north and south line should be located at a distance from any source of magnetic disturbance, such as trolley-wires, etc. With the declination arc set at zero and with the north end of the compass-box along the meridian, bring the needle to a zero reading by use of the tangent-screw of the declination arc. Note the reading on the declination arc and the time of the observation to the nearest minute. Note also the direction of the declination; for example, if the needle is west of north when the line of sight is on the true north and south line, then the declination is west, but the magnetic bearing of the true north and south line is northeast. Make at least ten independent observations and find the mean reading of the declination arc and the mean time corresponding to this mean reading. Apply the hourly correction to this mean declination reading, as given in Table VII.

FORM OF NOTES.

(Left-hand page.)

No. of reading.	Dec. arc.	Mean dec. arc.	Time.	Mean time.

(Right-hand page.)

Give the location of the true meridian and the usual information.

CHAPTER IV.

TRANSIT PROBLEMS.

PROBLEM D1. MEASUREMENT OF THE ANGLES OF A TRIANGLE.

Party. Two or three men.

Equipment. Transit, two range-poles, hatchet, stakes, and tacks.

Method. Drive three stakes, forming an approximate equilateral triangle with sides from 200 to 400 feet in length. Set up over each stake in turn, measuring the interior angles from four to ten times, as assigned by the instructor. The method of making an observation at a station is as follows: Set vernier *A* at any reading (near zero). Sight at one stake, clamp lower motion, and bring line of collimation to the tack by the lower tangent-screw. Read vernier *A* and vernier *B*. Unclamp upper motion and bring line of collimation to the tack in the other stake by upper clamp and tangent-screw. Read verniers *A* and *B*. Unclamp lower motion and again sight at the first stake, using the lower clamp and tangent-screw. Unclamp upper motion and proceed as before, finally reading verniers *A* and *B*.

Proceed in a similar way for as many measurements as may be desired. The observations at the other stations are conducted in a similar manner. Find the bearing of one of the lines to properly orient the figure.

Find the mean vernier readings. Then the differences between the successive mean readings will give several values of the angle. Take the mean of these values, which gives the probable value of the angle. These values should not differ by more than one minute of arc. Sum up the angles of the triangle and state the error.

Remark. In order to eliminate errors of adjustment and errors of observation, the "method of repetition," as given in Problem G5, is used in geodetic work.

This problem may be modified to suit the skill of the observer, by either increasing the number of repetitions of the angles or increasing the number of sides in the figure.

FORM OF NOTES.

(Left-hand page.)

Line.	Ver. *A*.	Ver. *B*.	Mean.	Diff.	Angle.

(Right-hand page.)

Give a sketch of the figure with the mean values of the angles. Party, date, equipment, etc., as usual.

PROBLEM D2. TRAVERSING WITH THE TRANSIT.

Party. Three men.

Equipment. Transit, two range-poles, 100-foot steel tape, set of marking-pins, hatchet, stakes, and tacks.

Problem. *To traverse a closed area or field.*

Method. Set six hubs, forming a hexagon with sides at least 250 feet in length. Set the transit up over one station. Set vernier *A* at zero and place line of sight on magnetic south. Clamp lower motion. Loosen the upper plate and sight on station 2, reading the azimuth on vernier *A*, and check by the needle; also sight at station 6 and obtain the azimuth of the line 1–6, checking by the needle. Now, set up at station 2, set backward azimuth of line 1–2 by use of vernier *A*, sight at station 1, and clamp lower motion; unclamp upper motion and read azimuth of line 2–3 and check it by the needle. The *backward azimuth* of a line is equal to the *forward azimuth* $\pm$ 180°. The backward azimuth should be set so that when the telescope is turned clockwise the plate-reading increases. Set up at stations 3, 4, 5, and 6 and proceed in a similar manner. Measure the sides of the figure.

Computations and Form of Notes. Calculate the latitude and longitude differences of the several courses and the error of closure. It is important to note that the bearings to be used in the computation are obtained from the azimuths, and not from the check needle readings.

Modification. The above problem may be modified by using the corners of a field whose sides can be directly measured.

Remark. The method of running a traverse by deflection angles might have been used in this problem.

FORM OF NOTES.

(Left-hand page.)

Inst. at.	Sight at.	Azimuth.	Distance.	Mag. Bearing.	Calculated Bearing.
1	South	0° 0′		S	
	2	4° 53′	238.6	S 4° 50′ W	S 4° 53′ W
2	1	184° 53′		N 4° 50′ E	
	3	106° 12′	245.2	N 73° 40′ W	N 73° 48′ W
And so on.					

(Right-hand page.)

Sides.	Lat. Diff.		Long. Diff.	
	+	−	+	−
1–2		237.73		20.31
2–3	68.41			235.46
And so on.				

$$\text{Error of closure} = \frac{\sqrt{\text{Error Lat.}^2 + \text{Error Long.}^2}}{\text{Perimeter}}.$$

Party, equipment, etc., as usual.

PROBLEM D3. SURVEY OF A CITY BLOCK.

Party. Four men.

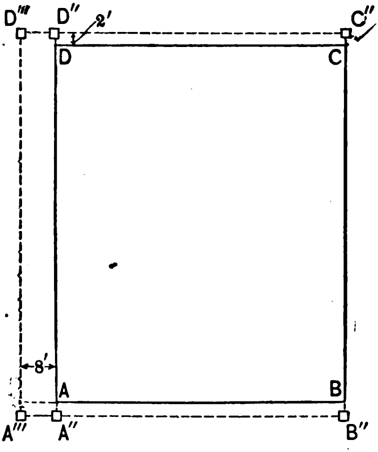
Equipment. Transit (reading to 20″, if possible), two range-poles, steel tape, plumb-bobs, hatchet, stakes, and the equipment for measuring a base-line, as given in Problem G1.

Method. Ascertain the relation of the various monuments in the block to the street lines.

Two cases may arise: First, when the bearings and lengths of the property lines may be obtained by direct measurement;

and, second, when the bearings and lengths must be obtained by use of offset lines.

In the first case, set the transit up over one corner monument. Sight at and measure the angle between the adjacent monuments. The number of repetitions of the angle will depend upon the kind of instrument used and the degree of accuracy desired. The lines between monuments should be measured by the first method of "Base-line Measurement," as given in Problem G1, and the true lengths of the lines found, as given in Problem G3. The remaining angles and sides of the block are found in a similar manner. The corrected bearings and lengths of the lines are found as described under "Computations" below.



In the second case, let $ABCD$ be a city block, with the monuments A'' B'' C'' D'' on line in one direction, and two feet in the street in the other direction. The property line AB may be found by measuring the distance between the monuments A'' and B'', which are situated 2 feet from the points A and B, and similarly for the line CD. The length of the line AD may be found by first setting a stake at A''', 8 feet from A'' and at about right angles to the line AD, and also a similar stake at D'''. Then set up at A''', sight at D''', turn 90°, and, if the line of collimation does not strike the point A'', shift the center of the transit along the line $A'''D'''$ until is does. Proceed similarly at D''', and then measure the length of the line between the two transit stations, which will be equal to the length $A''D''$. To obtain the interior angle at A, find the intersection of the line $A'''D'''$ with the line $A''B''$ by string intersection. Then set up at this point and measure the angle $D'''A'''B''$, which is equal to the angle A.

Remark. The method given above is general, and the student should be able to solve any problem which may arise.

Computations and Plotting. The lengths of the measured lines may be obtained by making reductions similar to those in Problem G3.

Compute the latitude and longitude differences in the usual manner, using a six-place table of logarithms, and distribute the error of closure by the proper rule. Finally, compute the corrected bearings and lengths of the courses. Having made a plot of the block by the method of co-ordinates, see that all the items enumerated in Appendix III of Raymond's Surveying are contained on the drawing.

Form of Notes. On the right-hand page give a sketch of the block, showing the location of all monuments and stakes with the values of the measured angles. On the left-hand page the notes are similar to that given in Base-line Measurement, Problem G1, and the notes for the Measurement of the Angles are as given in Problem D1.

PROBLEM D4. STAKING OUT A BUILDING.

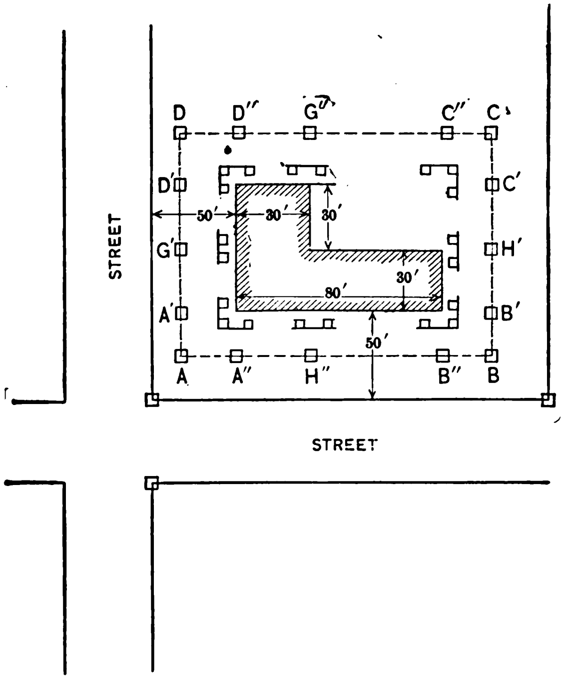
Party. Three men.

Equipment. Transit, steel tape (reading to hundredths), two range-poles, hatchet, stakes, tacks, and plot of the lot, showing location and dimensions of the building.

Problem. *To establish the corners of a building, having given the dimensions of the building and its location with respect to the property lines.*

Method. Set a hub at the point A, which is determined by being at the intersection of lines parallel to and at a distance of from 12 to 20 feet from the two sides of the building. Set a similar stake at the point B, and then turn 90° and set the point D, located at the same distance from the corner as A and B. Set up at D and locate the point C at right angles to the line AD, and at a distance from it equal to *AB*. Compare the distance *BC* with *AD* as a check. For convenience in obtaining the corners of the building and for lining-in the sides, set the stakes shown in the sketch which are marked with primes and seconds.

The main rectangle *ABCD*, the sides of which have been carefully measured and checked, lies entirely outside of both the batten boards and the excavation. These stakes serve to preserve the lines and may be used at any time to check



the position of the nails in the batten boards. The batten boards are located as shown in the sketch.

Instead of locating the stakes as described above, they might be placed at the corners of the building. The lines might then be thrown up onto the batten boards which are placed at some definite elevation and enough outside of the building to prevent their being disturbed during the excavation.

Note. In setting stakes for the location of a culvert, the center line of the culvert is first established and used as the reference line. The number of auxiliary stakes depends upon the form of the culvert. These stakes may be set at a specified distance from the neat line.

There is no fixed rule for locating the stakes of a building or other structure.

Modification. This problem may be modified, if desired by the instructor, by changing the dimensions of the building and its location with respect to the property lines. The angle between the property lines may also differ from 90°. In this case one side of the building may be located parallel to one property line and one corner at a certain distance from the other property line.

Notes. On the right-hand page give a plan of the structure and the location of all stakes. Give the party, date, equipment, etc., as usual.

A brief description of the method used in locating the stakes may be given on the left-hand page.

PROBLEM D5. TOPOGRAPHIC SURVEY OF A SMALL AREA BY TRANSIT, LEVEL AND TAPE.

Party. Three men.

Equipment. Transit, level, self-reading level-rod, steel tape, set of marking-pins, hatchet, and stakes.

Survey. Divide the area into squares or rectangles, starting with any convenient base-line. (One boundary-line of the area may be taken.) The transit and tape are used for laying out the lines, and the level (or transit) for obtaining elevations, using a self-reading rod to hundredths of a foot.

Find the elevations of the corners of these figures above a given or an assumed datum. If the area is not too large the elevations may all be obtained from one set up.

The degree of accuracy of the work will depend upon the following conditions: First, the size of the squares and the accuracy with which they are laid out. Second, the accuracy of the level work.

Plotting. The squares may be laid out to any convenient scale and the elevations placed in their proper positions. The elevations of the points after the grading has been completed may be written in red. If desired, the contours may be found by interpolation.

Remark. This is a very convenient method for making the survey of the site for a reservoir or dam, a viaduct or a borrow-pit.

Form of Notes. On the right-hand page give a sketch showing the arrangement of the squares or rectangles. For convenience, the stations along the base-line may be called *A*, *B*, *C*, *D*, etc., and, along the perpendiculars, 0, 1, 2, 3, etc. Any point is given by its co-ordinates as *B*–2, *C*–3, etc.

On the left-hand page the common form of level notes for obtaining the elevations of stations can be used, see Problem B2.

PROBLEM D6. ADJUSTMENTS OF THE ENGINEER'S TRANSIT.

Party. Two or three men.

Equipment. Engineer's transit, steel tape, level-rod, hatchet, stakes, and adjusting-pins.

Theory. See the Appendix.

Method of Making Adjustments with Form of Notes. Use both pages of the field-book, giving data with regard to party, equipment, etc., in the upper part of the right-hand page.

I. *To make the axes of the plate levels perpendicular to the vertical axis of the instrument.* Bring both bubbles to the center, revolve 180°, and correct one-half the error by the leveling-screws and the other half by raising or lowering one end of the bubble-tube. Each bubble should be adjusted separately by placing it parallel to a pair of leveling-screws.

1st trial.	2d trial.	3d trial.	Remarks.
2 div.	0.4 div.	0.0 div.	Bubble parallel to telescope.
1.5 div.	0.3 div.	0.0 div.	Bubble perpendicular to telescope.

II. *To make the line of collimation revolve in a vertical plane when the telescope is turned on its horizontal axis.*

(*a*) *To make the line of collimation perpendicular to the horizontal axis of the telescope.* See Appendix, Fig. 8. Set the instrument up on nearly level ground at a point *O*, where a clear sight may be had of a point A a few hundred feet away. Bring the line of collimation to it, plunge the telescope, and if the line of collimation is not in adjustment it will mark a point B. Unclamp the upper motion and turn the telescope in azimuth until it again strikes the point A. Plunge the telescope, and the line of collimation will now strike the point C. Bring the line of collimation to the point E by changing the reticule. The point E is one-quarter of the distance C–B from C.

Give a sketch similar to Fig. 8 in the Appendix and state the approximate distances O–A, O–D, and B–C for the different trials. State how the adjustment was made.

(*b*) Fig. 9 of Appendix. *To make the horizontal axis of the telescope perpendicular to the vertical axis of the instrument.* Set the instrument up about 20 feet from a building and level carefully. Select as high a point on the building as possible and bring the intersection of the cross-wires to it. Swing the telescope down and set a point near the base of the building. Plunge the telescope, turn in azimuth, and again sight at the upper point. Depress the telescope and mark another point B. Adjust for one-half the error by raising or lowering one end of the horizontal axis.

In the field-book give a sketch similar to Fig. 9, with the approximate height of P above the lower point and the approximate distance to the building. Also give the error for the different trials and state how the adjustment was made.

III. *To make the axis of the telescope bubble parallel to the line of collimation.* Use either the first or second method given in the Appendix, as directed by the instructor.

In the field-book give a sketch similar to Fig. 10 or 11, with all rod-readings and distances. Give the corrected rod-reading used in making the adjustment. State how the adjustment was made with the check on the work.

IV. *To make the vernier of the vertical circle read zero when the line of collimation is horizontal.* Bring the telescope bubble to the center, and if the vernier of the vertical circle is adjustable bring it so that its zero coincides with that on the limb. If the vernier is not adjustable, note the amount and sign of the angular error, which is called the "index error."

PROBLEM D7 DETERMINATION OF STADIA CONSTANTS.

Party. Three men.

Equipment. Engineer's transit, two stadia-rods, steel tape, set of marking-pins, and engineer's scale.

Problem. *To determine the constants f, c, and $\frac{f}{i}$ for any particular instrument.*

Theory. The space intercepted on a rod by the stadia wires of an instrument varies directly as the distance from a point $(f+c)$ in front of the instrument.

Method. Select a practically level stretch of ground and set up the transit over a pin. With the engineer's scale measure f (which is the distance from the center of the objective to the cross-wires when focused on an object at an infinite distance away); also measure c (which is the distance from the center of the objective to the center of the instrument when focused on a point varying from 200 to 1000 feet away). Lay off the distance $(f+c)$ from the center of the instrument, setting a pin at the point. With this as a starting-point, lay off a base-line 400 to 1000 feet long, marking the stations by pins. Then, with the telescope level, or not depressed more than to make the upper wire come on a tenth graduation, place the rod at different points (none less than 50 feet from the instrument), and have the instrument man read the wire interval and record it. The chainmen measure the distance to the nearest tenth of a foot from the preceding pin. When all readings are taken, the chainmen and instrument-man will exchange notes.

Instead of placing the rod so that the distances will measure tenths of feet, it might be held at even foot points.

Calculate the value of $\frac{f}{i}$ for each distance, which equals the distance divided by the wire interval. Give the mean value of $\frac{f}{i}$.

FORM OF NOTES.

(Left-hand page.)

Number.	Distance.	Wire interval.	$\frac{f}{i}$

(Right-hand page.)

Give value of f, c, and $f+c$; party, date, equipment, etc., as usual.

PROBLEM D8. TRAVERSING WITH THE STADIA.

Party. Three men.

Equipment. Transit, two stadia-rods, 5-foot rod, hatchet, stakes, and tacks.

Problem. *To traverse a closed area or a field with the transit and stadia.*

Method. The method of procedure is the same as that given in Problem D2, except that the distances are measured by the stadia instead of by tape or chain.

The observations at a station are conducted as follows: The backward readings at a station, consisting of the distance, vertical angle, and bearing, are checked before the forward readings are taken. If these readings do not check, consult the instructor.

To take the forward readings, set off the backward azimuth on the plate and sight at the preceding station, using the lower motion; unclamp the upper motion and sight at the next station, setting first for azimuth, having the narrow side of the rod turned toward the instrument; then turn the rod around and bring the upper or lower wire to an even foot mark, and read the wire interval and record it; find the height of the cross-wires above the ground, and set the middle wire at this reading on the rod when held at the next station; finally, read the azimuth, bearing, and vertical angle.

Modification. As in Problem D2.

Computations. Reduce the measurements to the horizontal, and find the difference of elevation of the various stations by the regular formulas for inclined sights as given in the surveying text-books. Use the mean readings of vertical angles and wire readings in the formulas. Carry the horizontal distances to the nearest foot, and differences in elevation to the nearest tenth of a foot. Sum up the elevations and state the error. The error of closure may be found as in Problem D2.

FORM OF NOTES.

(Left-hand page.)

Inst. at.	Sight at.	Azimuth.	Wire Int.	Vert. Ang.	Magnetic Bearing.
1	South	0° 00′			S
	2	32° 20′	134′.0	+0° 26′	S 32° 15′ W
	6	106° 31′	200′.0	−0° 28′	N 73° 30′ W
2	1	212° 20′	134′.0	−0° 25′	N 32° 30′ E
	3	111° 05′	142′.0	+0° 15′	N 68° 50′ W
And so on.					

(Right-hand page.)

Sides.	Horizontal dist.	Diff. of elevation.
1–2	135′.0	+1.00
2–3	143′.0	+0.60
And so on.		

Error in diff. of elevation = +0.1 foot.

Error in azimuth = 0° 1′.

Party, date, equipment, etc., as usual.

CHAPTER V.

MISCELLANEOUS PROBLEMS.

PROBLEM E1. ELEVATIONS BY USE OF THE ANEROID BAROMETER.

Party. Two men.

Equipment. Two aneroid barometers, two thermometers and watches or mercurial barometer, aneroid barometer and thermometers.

Theory. The use of the barometer to determine differences of elevation depends upon the supposition that the atmosphere is composed of a series of layers of air which decrease uniformly in density as the altitude increases. Practically this is not true, and a correction must be applied for temperature and humidity.

A common formula used is:

$$\text{Difference in elevation (in feet)} = 62737 \left(\log \frac{30}{B_1} - \log \frac{30}{B}\right)$$

where B and B_1 are the barometer readings at the lower and higher points in inches. Table I gives the quantity 62737 $\log \frac{30}{B}$. This value is for a mean temperature of 50°; for any other mean temperature the difference in elevation is corrected by the quantity in Table III, this correction being multiplied by the approximate difference in elevation of the two points as obtained from Table I.

Problem. *To determine the difference in elevation of several points by use of* (*a*) *two aneroid barometers and* (*b*) *an aneroid and a mercurial barometer.*

(*a*) Owing to the greater cost and non-portability of the mercurial barometer, two aneroid barometers are sometimes used. One aneroid is left in the office and is read by the office man, who notes the heights of the readings, the temperature, and the time. The field aneroid is taken to the several points; the reading, the temperature, and the time are taken. Before going into the field, note the readings of both the field and office aneroids when at the same elevation, and also note the time. Then go in succession to the several points whose elevations are desired, read the barometer and thermometer, and note the time, finally returning to the office. Again read the office and field barometers, and note the temperature and time.

(*b*) When the mercurial barometer is used as reference, its attached thermometer is read and the mercurial reading reduced to 32° by use of Table II. The elevation obtained by reducing this reading is practically the absolute elevation of the point above sea-level, and after the difference in elevation of the office and field points has been found, the absolute elevations of the field points may be determined. When two aneroids are used, the *relative* differences in elevation are found.

The matter following refers more particularly to (*a*) above, but the method of procedure in (*b*) is similar after having reduced the mercurial column to 32°.

Computations. The difference in the readings of the field and office barometers does not remain a constant quantity, i.e., it is not the same at the beginning of the work as it is at the end. In order to eliminate error, as far as possible, from this source, the following methods are pursued.

First. When the observations extend over a time interval of from seven to eight hours. Find the difference of the field and office barometer readings, first, at the beginning, and, second, at the end of the work. Find the difference of these two quantities. Divide this difference by the number of hours between beginning and end of work. Then, having the difference at the beginning and end, and knowing the rate of change of the difference between the two times, a table can be constructed giving the difference of readings of the two barometers for the same elevation at any given time. For a given observation in the field add this correction, and use this corrected reading in connection with the office reading for the given time to get the approximate difference in elevation of the points. Multiply this approximate difference in elevation by the temperature coefficient, as obtained from Table III, and apply the result to the approximate difference in elevation. This will give the true difference in elevation.

Second. When the time interval between first and last readings is only one or two hours. Find the difference of the field and office barometer readings at the beginning and end of the work. Compute the mean difference. Apply this mean difference to the field readings to reduce both barometer readings to the same plane of reference, and proceed with reductions as before.

The example shown is one which was solved in the field and should be clearly understood after reading the text.

FORM OF NOTES.

RECONNAISSANCE SURVEY, LOWER LINE R.R., EAGLES MERE, PA., JUNE 8, 1906.

OFFICE ANEROID

Time.	Temp.	Reading.
8.11	67°	28.210
8.26	67°	28.209
8.41	67°	28.205
....	..	......
.....	..	......
11.26	69	28.220
.....	..	......
.....	..	......
12.26	72	28.200
12.30	72	28.200

FIELD ANEROID.

Station.	Reading.	Corr. Field Aneroid.	Elev. Corr. to Field An.	Elev. Corr. to Off. An.	Corr.	True Diff. of Elev.
Office 8.11 67°	27.970	28.210	1676	....	...	....
△2 8.35 66°	28.004	28.244	1643	1679	+1	+37
....	......	......	....	....	...	....
△15 11.30 69°	28.036	28.242	1645	1667	+1	+23
....	......	......	....	....	...	....
△27 12.18 70°	28.010	28.204	1682	1683	0	+1
Office 12.30 72°	28.006	28.200	1686	....	...	....

TABLE FOR REDUCTION OF FIELD READINGS TO DATUM OF OFFICE READINGS.

Time.		Correction.
8.11	(28.210 – 27.970)	= 0.240
9.00		0.236
9 30		0.230
10.00		0.224
10.30		0.218
11.00		0.212
11.30		0.206
12.00		0.200
12 30	(28.200 – 28.006)	= 0.194

PROBLEM E2. PLANE-TABLE PRACTICE.

Party. Three men.

Equipment. Plane table (including alidade, compass and level-box, plumbing-arm and bob), steel tape, stadia-rods, hatchet, and stakes. Also sheet of drawing-paper 20″ by 20″, engineer's scale, triangles, and irregular curve.

Definitions. The method of *radiation* consists in locating a point by azimuth and distance from a given station.

The method of *intersection* consists in the location of a point by knowing the direction of the point from the two ends of a given base-line.

The method of *traversing* is similar to that pursued in traversing with the transit, except that the notes are plotted directly in the field.

The three-point problem. If three points are plotted on paper, the position of any station (within the limits of the drawing) may be found by placing a piece of tracing-paper on the table; then assume any convenient point on the tracing-paper, and draw lines from this point toward the three field points. Shift the tracing-paper until the three lines traverse the three plotted points. The position of the intersection of the three lines then determines the position of the instrument with respect to the three plotted points.

Problem. *The survey of an area by use of the three given methods.*

Method. Lay off a base-line from 200 to 500 feet in length. Assume a convenient scale and plot this base-line. Set the plane table up over one station and orient it with respect to the other station. Locate any desired objects by an angle and distance (radiation) or by drawing lines of indefinite length towards the objects. Then set up over the other end of the base-line, and having oriented the table, check the position of the points obtained by the method of radiation by the method of intersection. Make a complete plot of the structures or natural objects within the enclosure. The property lines may be obtained by the method of radiation or intersection or by the method of traversing, starting with one boundary-line as the base-line.

If it is desired to set up over any point in the area whose position on the drawing is not known, the instrument may be set up over the point, and the corresponding point on the paper found by use of the three-point problem.

If only two points have been located on the drawing, the location of a third point may be found by use of the *two-point problem*. For a solution of this problem see any work on surveying.

Notes. Enter in the field-book the problem, party, equipment, and date. Give a brief statement of the work performed in the field. The drawing should be carefully and accurately made and submitted with the field-book.

PROBLEM E3. USE OF THE SEXTANT.

Party. One or two men.

Equipment. Sextant and several range-poles.

Problem. *To measure the angles between permanent objects or to close the horizon, using range-poles with varicolored flags as sights.*

Theory. The principle upon which the sextant is constructed is that if a ray of light is reflected successively between two plane mirrors the angle between the incident and reflected rays is double the angle of the mirrors.

Method. Set six range-poles about a central point so that they will subtend angles approximately 60° in magnitude and be at least 200 feet from the central point. Measure each of the angles at least six times by use of the sextant.

The method of procedure is as follows: Hold the plane of the instrument in the plane of the observer's eye and the two points sighted at. Point the telescope at the fainter object, and bring the image of the other object directly over it by moving the vernier arm. Clamp the movable arm, and bring the two objects to exact coincidence by use of the tangent-screw. If the brighter object is to the right of the fainter one the instrument must be direct, and if to the left, the instrument must be held upside down.

To secure good results the following points should be noted: (1) The index error should be applied (see next problem). (2) The instrument should be held as near the central point as possible at each observation. (3) Unless the objects are in the same horizontal plane the angles will not sum up to 360°.

Modification. The sun's altitude may be measured by use of the sextant. When the sun is on the meridian, the measured altitude, corrected for refraction, index error, etc., may be determined, and knowing the declination of the sun, the colatitude of the place may be found.

FORM OF NOTES.

(Left-hand page.)

Stat.	Vernier.	Index error.	Corrected Angle.	Mean Angle.

(Right-hand page.)

Give a sketch showing the location of the stations. State the error and give party, date, equipment, etc., as usual.

PROBLEM E4. THE ADJUSTMENTS OF THE SEXTANT.

Party. Two men.

Equipment. Sextant.

Method. The adjustments of the sextant are four in number: 1st. To make the plane of the index-mirror perpendicular to the plane of the arc. 2d. To make the plane of the horizon-glass perpendicular to the plane of the arc. 3d. To make the axis of the telescope parallel to the plane of the arc. 4th. To determine the index error.

1. *To make the plane of the index-mirror perpendicular to the plane of the arc.* Place the arm of the instrument about in the middle of the arc, and set the instrument on a stable support. Place the eye so that the direct and the reflected image of the arc scale may be seen joining each other in the index glass. If they do not form one straight arc, tip the mirror either backward or forward by use of the adjusting-screw under the mirror. It may be necessary to slip some paper or tracing-cloth under the arm attached to the mirror in order to make the sextant hold this adjustment.

2. *To make the plane of the horizon-glass perpendicular to the plane of the arc.* This observation is best carried out by observation on a star. Place the arm so that it reads zero on the arc; hold the instrument so that the plane of the arc is vertical, and slide the arc backward and forward by slow motion. If the instrument is out of adjustment in this respect, the direct and reflected images will not coincide, but the reflected image, as the arm moves, will be slightly above or below the direct image. Adjust the horizon-glass so that the two images coincide when the reflected passes the direct in moving the arm.

3. *To make the axis of the telescope parallel to the plane of the arc.* Prepare two blocks which are equal in height and the same as the height of the center of the telescope above the plane of the arc. Place the blocks on the arc and set a point on a wall about 25 to 30 feet from the instrument by sighting over the blocks. Move the sextant slightly, so that the point may be seen through the telescope. If the center of the telescope traverses the point, the center of the telescope being as high above the plane of the arc as the two blocks are, the axis of the telescope is parallel with the line of the blocks, and therefore parallel with the plane of the arc. If the axis of the telescope does not traverse the point, adjust the collar attached to the telescope until the axis does traverse the point. Some sextant telescopes are equipped with cross-hairs, which are at the same distance from the axis of the telescope and which may be made parallel to the plane of the arc. When making the adjustment by means of this telescope, place it in the collar of the sextant and make the cross-hairs parallel to the plane of the arc. Sight on an object, such as

a slender rod, some distance away, and make the images stand one over the other on one wire. Then move the sextant so that the images are on the other wire. If they stand one directly over the other, as before, the instrument is in adjustment If they do not, adjust the collar by trial until they hold the same relative position when seen on both wires.

4. *To determine the index error.* When the direct and the reflected image of a distant object, as a star, are seen one superimposed on the other, the zero of the vernier should coincide with the zero of the scale. If they do not coincide, the distance from one zero to the other is called the "index error." The sextant is equipped with adjusting-screws for making the two coincide, but it is better to adjust the instrument so that this error is small, and then note whether the error is "on arc" or "off arc," and apply the index error to the readings of all angles taken with that particular instrument. The sextant is graduated for several degrees on the lower side of zero, so that this index error can be read on both sides of the zero-mark. The index correction is "on arc" when it is on the main scale, and is then *minus;* it is "off arc" when it is on the extension of the scale, and is then *positive.* The best object to use in making this adjustment is a star, but a range-pole may be set up at a distance of several hundred feet and used for making the adjustment.

Notes. In making the adjustment of a particular instrument, give a record of the readings on the several trials; in the third adjustment give the distance between the two lines of sight on the wall (in inches), until they are made to coincide. Make six determinations of the index error, and take the mean of the six readings as the index error. State whether it is on arc or off arc.

PROBLEM E5. SENSITIVENESS OF LEVEL-VIALS.

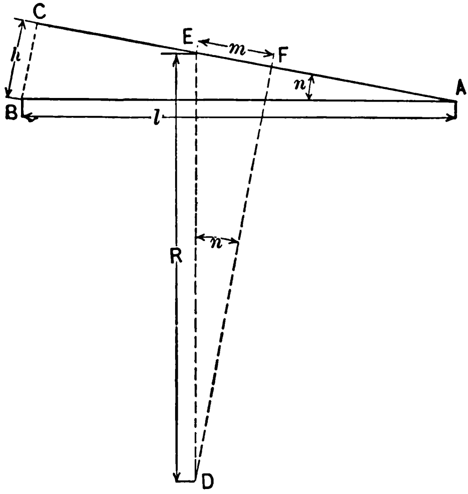
Party. Two men.

Equipment. Level-trier and level-vial to be tested, or transit or level, with steel tape, leveling-rod, and plumb-bob.

Problem. *To determine the radius of curvature of a level-vial: first, by the level-trier, and, second, by the optical method.*

Practical application. To secure good results in leveling, solar and stadia work, it is essential that the different bubbles be fairly sensitive. Many instrument-makers use level-vials of a very low degree of sensitiveness. An instrument must many times stand in one position for an extended interval of time and will get out of level. The plate-levels should be sensitive enough to make this readily apparent. The required degree of sensitiveness for the level-tube depends upon the character of the work. Prof. L. S. Smith of the University of Wisconsin recommends a sensitiveness for the long bubble of the telescope of 20 seconds for one-tenth inch of arc, and for the plate-levels a sensitiveness of 30 seconds for one-tenth inch of arc.

Theory. *First, by level-trier.* The constants of the level-trier must be given or found. These are the values for the length of the arm and the pitch of the screw.



In the figure given above, the line AB represents the first position of the arm of the trier, and CA the second position. The movement of the center of the bubble is represented by the distance EF, and FD is the radius of the bubble-tube. The angle $CAB = n$ is found from the proportion.

$$\frac{n}{360°} = \frac{h}{2\pi l} \quad \text{or} \quad n = \frac{180°\, h}{\pi l} \text{ (in degrees),}$$

or

$$n = \frac{206{,}265\, h}{l} \text{ (in seconds).}$$

Then the angular value of one division of the tube

$$=\frac{n}{\text{number of divisions the bubble moves.}}$$

The radius of curvature is found from the proportion

$$\frac{2\pi R}{m}=\frac{360^\circ}{n},$$

or

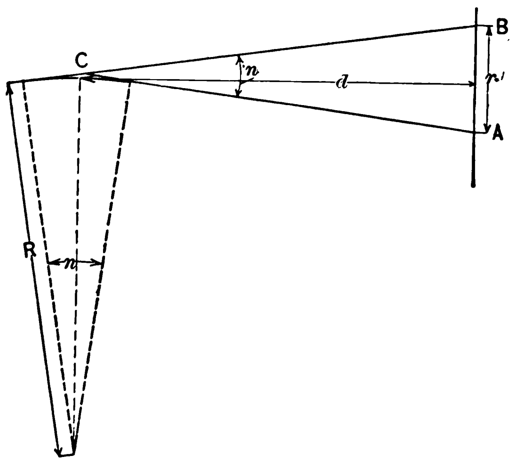
$$R=\frac{m(206,265)}{n};$$

or, substituting the value of n given above, we have

$$R=\frac{ml}{h}.$$

The same equation could be obtained from the two similar triangles ABC and EDF.

Second or Optical Method. This method can be used in determining the radius of curvature of the bubbles on the transit or level.



In the figure let d represent the distance from the center of the instrument to the rod, AB the difference in rod-readings $=r$, n the total angular movement of the bubble, and $R=$ the radius of the tube.

Then the triangle CAB is practically a right triangle, and the horizontal distance may be considered the hypothenuse of the triangle. Then

$$\sin n = \frac{d}{r},$$

or, as n is a small angle,

$$n \sin 1'' = \frac{r}{d},$$

or

$$n = \frac{r}{d \sin 1''}.$$

Then R is found from the proportion

$$\frac{R}{d} = \frac{m}{r},$$

or

$$R = \frac{md}{r}.$$

The four quantities should be in the same unit m (denoting the movement of bubble), being measured with an engineer's scale.

Method. *First, by level-trier.* Place the level-trier on a stable pier and put the level-tube or instrument to be tested on the trier so that the axis of the bubble is parallel to the arm. Bring the bubble so that its two ends can be read either by use of the scale on the level-trier or by the graduation on the tube itself. With the transit or level the bubble should be brought into view by use of the leveling-screws, but if a detached bubble is used it must be accomplished by changing the position of the trier. With the bubble near one end of the tube, note the readings of its two ends and the reading of the micrometer-screw. Turn the micrometer-screw through a certain number of divisions, and again note the position of the two ends and the reading of the screw. Continue in this manner until the bubble is near the other end of the tube. Then turn the screw a few divisions further in the same direction, and finally backward to the same micrometer readings as used before. This operation is done to determine the lagging of the bubble.

Second or Optical Method. Set up the instrument and lay off any convenient base-line. With the bubble near one end of the tube, take a rod-reading, noting at the same time the readings of the ends of the bubble. Bring the bubble to the other end of the tube by use either of the tangent-screw, leveling-screws, or micrometer-screw (in the precise level), and take another rod-reading, again noting the readings of the two ends of the bubble. Make any desired number of observations, and find the angular value of one division of the bubble and the radius of curvature.

The mean radius of curvature is obtained in this way. If a test for *uniformity of curvature* is desired, observations should be made over the entire length of the tube.

FORM OF NOTES FOR THE LEVEL-TRIER.

(Left-hand page.)

Number.	Microm. Readings.	Bubble Readings.		Differences.		Length of Bubble.
		Left end.	Right end.	Left end.	Right end.	
1	60	20.0	2.1	...	...	....
2	80	18.2	4.2	1.8	2.1	22.4
3	100	16.0	6.2	2.2	2.0	22.2
4	120	13.9	8.6	2.1	2.4	22.5
5	140	11.7	10.8	2.2	2.2	22.5
6	160	9.6	13.0	2.1	2.2	22.6
7	180	7.2	15.4	2.4	2.4	22.6
8	200	5.0	17.8	2.2	2.4	22.8
9	220	2.8	20.0	2.2	2.7	22.8
9	220	2.5	20.3	...	...	22.8
8	200	4.6	18.1	2.1	2.2	22.7
7	180	7.2	15.6	2.6	2.5	22.8
6	160	9.5	13.3	2.3	2.3	22.8
5	140	11.7	11.2	2.2	2.1	22.9
4	120	13.8	9.1	2.1	2.1	22.9
3	100	15.9	7.0	2.1	2.1	22.9
2	80	18.1	4.8	2.2	2.2	22.9
1	60	20.5	2.5	2.4	2.3	23.0
				35.0	35.7	

Mean = 2.21 divisions.

(Right-hand page.)

Length of arm of level-trier No. 1 = 17″.92.

Pitch of screw, 60 per inch. Therefore one division of disk raises arm $\frac{1}{6000}$ of an inch.

$$n = \frac{206{,}265 \times \frac{20}{6000}}{17.92} = 38''.37,$$

and one division $= \frac{38''.37}{2.21} = 17''.3.$

$$\text{Radius in feet} = \frac{ml}{h} = \frac{0.221 \times \frac{17.92}{12}}{\frac{20}{6000}} = 99.0.$$

Jan. 20, 1906.

Equipment: Level-trier No. 1.

Transit: T. 4.

Party: { ....
....

FORM OF NOTES FOR OPTICAL METHOD.

Number.	Rod-reading.	Bubble.		Differences.		Diff. of rod-readings.
		Left end.	Right end.	Left end.	Right end.	

On the right-hand page find the angular value of one division of the bubble and the radius of curvature. Give party, date, equipment, etc., as usual.

PROBLEM E6. THE POLAR PLANIMETER.

Party. One or two men.

Equipment. Polar planimeter, set of drawing instruments triangles, scale, and sheet of paper about 15 by 15 inches.

Problem. *To check the constants of the planimeter and afterwards to find the area of an irregular figure.*

Theory. In tracing a closed figure with the fixed point of the planimeter outside the figure, the area of the figure is equal to the roll of the wheel times the length of the arm. When the fixed point is inside the figure, the area of the zero circumference must be added.

Method. Draw a four-inch square, a six-inch square, a circle of four-inch diameter, and a circle of six-inch diameter, designating them as figures 1, 2, 3, and 4, and so placed that when the fixed point of the planimeter is inside the six-inch figures the rolling wheel will stay on the paper. Set the slide index opposite the 10□″ graduation on the bar. This means that the area in square inches traversed equals ten times the number of revolutions of the wheel, or, in the formula $A = hnc$, hc becomes 10, whence $A = 10n$.

Traverse each figure with the fixed point outside. In each case one student begins with a certain reading a. After traversing the figure once he notes the reading b. The other student starts with this reading and ends with the reading c, when the figure has been traversed again. To find "area by planimeter" take the mean in columns 5 and 6 for each figure and multiply this mean by 10.

Take similar readings on the six-inch circle with the fixed point inside. Before entering the values $(b-a)$ and $(c-b)$, add to each the number on top of the bar just over the 10 □″ graduation. This number is one-tenth the area of the zero circle. Then if Z denotes the area of the zero circle,

$$A=10n+Z=10\left(n+\frac{Z}{10}\right).$$

Note that $(b-a)$ is *negative* if motion is *clockwise*; also, that the wheel will turn more than ten times in going around the six-inch circumference. With the index still set at the 10 □″ graduation, find the distance from the fixed point to the tracing-point when the wheel only slides for circumferential motion. This is the radius of the zero circle. Designate this reading as R, and record it on the right-hand page of the field-book. Then $\frac{\pi R^2}{10}$ should be a rough check of the number on top of the bar.

Assume that the four-inch square represents a plot of ground, the scale being $\frac{1}{8}$ inch $=1$ foot. Set the index at 400 □′ $\frac{1}{8}''=1'$. This means that if the scale be $\frac{1}{8}''=1'$, A in square feet $=400n$.

Enter this computation as 1′.

From the formula $A=hnc$, determine the circumference of the wheel by traversing a known area with any convenient value of h ($h=$ length of arm), h being measured with a decimal scale. Record on the right-hand page.

Area of an Irregular Figure. Having obtained the area of an irregular figure by calculation, check it by the planimeter.

The plot of any survey made during the year may be traversed with the planimeter, the calculated area being checked by the area obtained with the planimeter; for instance, the survey of a farm where one or more sides are bounded by streams.

FORM OF NOTES.

(Left-hand page.)

Figure.	a.	b.	c.	$(b-a)$.	$(c-b)$.	Area by plan.	True area.

(Right-hand page.)

Party, date, equipment, etc., also other matter previously noted

CHAPTER VI.

RAILROAD SURVEYING PROBLEMS.

PROBLEM F1. SIMPLE CURVE.

Party. Three men.

Equipment. Transit, 100-foot steel tape, two range-poles set of marking-pins, hatchet, two plumb-bobs, stakes, and tacks.

Problem. *To lay out a simple curve with a transit and tape, having given the degree of curve, the central or intersection angle, and the station of the P.I.*

Method. Compute the tangent distance and a complete table of deflections for staking out the curve from the *P.C.*, applying the check that the last deflection should equal ½ *I*. Set a hub at the *P.I.* and lay off the tangent distances in suitable directions, making the angle *I* with each other. Set up the transit at the *P.C.*, and after having checked the angle *P.I.–P.C.–P.T.*, stake out the curve to an even station to be designated, placing a hub at this station. Set the transit up at this station and stake out the remainder of the curve. To find the tangent to the curve at this station, sight at any transit station on the curve, with the deflection of that station set off on the proper side of zero; plunge the telescope and continue as if at the *P. C.* of the curve. Reference-stakes should be placed at uniform distances to the left of all transit stations and should give on the front of stake the number of the station, whether hub is *P.C.* or *P.T.*, and on the back the degree of curve and its direction. All other stations should be marked from top to bottom.

Note the error in line and distance between the *P.T.* which was first set and that found by running around the curve. This should be a reasonably small error, depending on the length of the curve and other conditions. Keep the *P.T.* as first located.

Remark. The above problem may be modified to suit conditions which arise in actual practice.

FORM OF NOTES.

(Left-hand page.)

Station.	Descr. of Curve.	Defl. Ang.	Calc. Bear.	Mag. Bear.

(Right-hand page.)

Give brief computations for the curve; also the usual information.

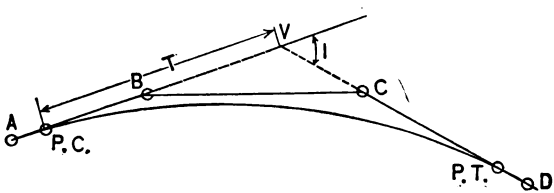
PROBLEM F2. SIMPLE CURVE, *P.I.* INACCESSIBLE.

Party. Three men.

Equipment. Transit, 100-foot steel tape, two range-poles set of marking-pins, hatchet, two plumb-bobs, stakes, keil, and tacks.

Problem. *Given two tangents and any number of intervening traverse lines (one traverse line is used in this problem), to connect the tangents by a simple curve of given degree, the P.I. being inaccessible.*

Method. Solve the triangle *VBC* for *VB* and *VC*, knowing the deflection angles at *B* and *C* and the chord *BC*. Assume *D*, calculate *T*, and lay off the difference between *T* and



BV to locate the *P.C.* and *T* and *VC* to locate the *P.T.* Line for the hubs which determine the *P.C.* and the *P.T.* is given by the transit set up at stations *B* and *C.*

Example. Given $BC=230.0$ feet, angle $B=10°\ 05'$, angle $C=7°\ 35'$, $D=4°\ 00'$, and station of $B=22+50$.

Answer. $T=222.64$, $BV=100.02$, $VC=132.69$, sta. $P.C.=21+27.38$.

Remark. The above example may be modified either by changing the given data or by increasing the number of traverse lines. In the latter case there will be one triangle to solve for each traverse line.

Notes. Similar to those in the preceding problem for the left-hand page, and for the right-hand page a sketch similar to that given above, together with the principal computations; also error, party, date, equipment, etc., as usual.

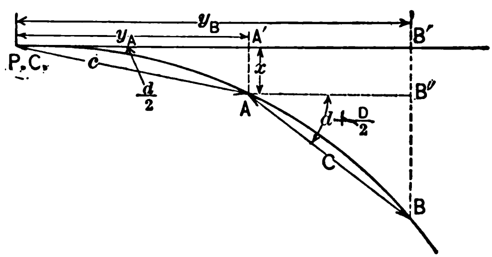
PROBLEM F3. LOCATION OF A SIMPLE CURVE BY USE OF TAPE ALONE.

Party. Three men.

Equipment. Two range-poles, steel and cloth tapes, set of marking-pins, plumb-bobs, hatchet, and stakes, also a transit to set the *P.C.* and the *P.T.* if so directed by the instructor.

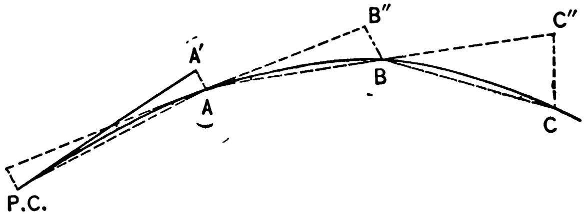
Problem. *First, to locate the curve by offsets from the tangents, and, second, to locate the curve by offsets from the chords produced.*

Theory. *Offsets from the tangents.* In the figure the distance AA' $(=x)$ is equal to $c \sin \frac{d}{2}$, where $d=$ central angle



for the chord c. $y_A=c \cos \frac{d}{2}$. For the point B the angle $B''AB=d+\frac{D}{2}$, where D equals the degree of curve, and the distance $BB''=100 \sin\left(d+\frac{D}{2}\right)$. The distance $B'B=B'B''+B''B$. The distance $AB''=100 \cos\left(d+\frac{D}{2}\right)$, and the distance $y_B=y_A+AB''$. The distances x and y for other points may be found similarly.

Offsets from the chords produced. In the figure the distance $AA'=\frac{c^2}{2R}$. A distance equal to AA' is laid off at the $P.C.$,



and the line joining this point and A is a common tangent to the curve at the point A. The distance BB'' is equal to $\frac{100^2}{2R}$ if $AB=100$ and the angle $AB''B=90°$. The distance $CC''=2BB''$ and $BC''=BC=100$ feet. Similarly for other points.

In order to determine the direction of the following tangent, this method is reversed.

Method. Having set the $P.C.$ and $P.T.$, locate the points on the curve, first, by use of tangential offsets. The angles at A' and B' are right angles, and the offsets in every case are obtained by use of the cloth tape. The points A', B', C', etc., are lined in by eye. If the curve is long, there will be a saving of time by locating the last part of the curve from the $P.T.$ end instead of prolonging the first tangent beyond the $P.I.$

Second, locate the curve by offsets from the chords produced. When the curve begins with a subchord, the first two points are located by offsets from the tangents. The method of procedure is evident from an inspection of the figure. The chord offset for a full station is twice the tangential offset for a full station. A given point can best be located by an offset from the chord produced when the preceding chord is a full station in length. When this is not the case, locate by offset from the tangent at the preceding station.

FORM OF NOTES.

(Left-hand page.)

Sta.	Point.	Descr. of Curve.	Tang. Offset.	x.	y.

(Right-hand page.)

Party, date, equipment, etc., as usual. Also give main part of the computations if not directed to the contrary.

PROBLEM F4. COMPOUND CURVE.

Party. Three men.

Equipment. Same as in simple curve.

Problem. *To locate a compound curve connecting two tangents, having given the degrees and central angles of the two arcs and the P.I.*

Example. Given:

$$D_l = 3° 00'$$
$$D_s = 5° 00'$$
$$I_l = 7° 30'$$
$$I_s = 15° 00'$$

Sta. $P.I. = 73 + 50$.

The sharper arc is near the $P.C.$

Answer. $T_l = 311.9$, $T_s = 245.1$, $L_l = 250.0$, $L_s = 300.0$, $P.C. = 71 + 04.9$.

Method. From the given data compute the tangent distances. Select a suitable point for V and locate the $P.C.$ and $P.T.$ Then, moving the instrument to the $P.C.$, run in the curve to the $P.C.C.$ (point of compound curve). Set up at the $P.C.C.$, and sight at the $P.C.$ with the deflection of the $P.C.C.$ laid off on the plate in the proper direction; plunge the telescope and turn to zero. Proceed then as in any simple curve.

Modification. Instead of the above problem, one of the following cases might be taken: First, given I, R_l, R_s, and one T (within limits). Second, I, T_l, T_s, and one R (within limits). Third, I, I_l, T_l, R_l, or I, I_s, T_s, R_s.

Form of Notes. On the left-hand page give a table of deflections, with full description of curve, etc., as in problem F1. On the right-hand page give the principal parts of the computations, errors in line and distance, party, date, equipment, etc.

PROBLEM F5. THE SEARLES SPIRAL.

Party. Three men.

Equipment. Transit, 100-foot steel tape, two range-poles, set of marking-pins, hatchet, two plumb-bobs, stakes, keil, and tacks.

Theory. The Searles spiral is a multiform compound curve, the central angle of the first chord being made the common increment of increase for the successive chords.

Two conditions govern the selection of the particular spiral to be used: first, the amount of superelevation required and the rate of running it out; and, second, the degree of curve on the $(n+1)$ chord should nearly equal the degree of the simple curve.

Method. Having given I and D, select the spiral (by use of the table in Searles) and calculate T_s. Set the transit up at the *P.I.* and set *P.S.*$_1$ and *P.S.*$_2$ (points on the main tangent corresponding to the *P.C.* and *P.T.* of a simple curve). Set up at the *P.S.*$_2$, and, by use of the table in Searles for deflections, lay out the spiral. Set up at the *P.S.*$_1$ and lay out the other spiral. Set up at the *P.C.S.* and check the angle between the common tangent to the curve at this point and the *P.T.S.*; this angle should equal $\frac{1}{2}(I-2s)$, where $I=$ the total central angle of the curve, and $s=$ central angle of the spiral. The direction of the common tangent is found by laying off the angle "deflection from auxiliary tangent," found in Table II of Searles, under "instrument at n." In order that the zero reading shall correspond to the auxiliary tangent, care must be taken that this deflection is set off on the proper side of zero.

Modification. This problem may be modified to suit any one of the various problems given in Searles' "Railroad Spiral."

The method pursued in the field will be practically the same as given above for all cases.

Form of Notes. The form of notes is essentially the same as that given in Problem F1. The deflection angles for the spiral are given opposite their proper stations, and the elements of the spiral are given in the description of curve column.

On the right-hand page record the errors in line and distance. Also give party, date, equipment, etc., as usual.

PROBLEM F6. TURNOUT FROM AN EXISTING TRACK.

Party. Three men.

Equipment. Transit, 100-foot steel tape, two range-poles, set of marking-pins, hatchet, two plumb-bobs, stakes, and tacks.

Problem. *To locate a turnout from an existing main track.*

Theory. The lead is the distance measured along the main track from the switch-point to the frog-point. If a stub switch is used, this is equal to $2gn$, where g = gauge and n = number of frog; this considers the turnout curve as extending from the switch-point to the frog-point.

It may be proved that in a split switch, where the turnout curve extends from the heel of the switch to the point of frog, the lead is equal to $l+\frac{g-t}{\tan \frac{1}{2}(F+S)}$, where l = length of switch-rail, g = gauge, t = distance gauge to gauge at heel of switch, F = frog-angle, and S = switch-angle.

The radius of the curve connecting the center line of the turnout curve opposite the frog-point with the center line of parallel track is found from the equation

$$R_2=\frac{g}{2}+\frac{p-g}{\text{vers } F},$$

where p is the perpendicular distance between center lines of the two parallel tracks.

Method. The problem will be divided into two parts: first, where the existing track is straight, and, second, where it is curved.

First. Having selected the location of the frog-point, set a stake at this point at a distance of $\frac{1}{2}$ gauge from the gauge-line and in the direction in which the turnout is to go. Then at a convenient distance (from 200 to 400 feet) set a second stake at a distance of ½ gauge from the gauge-line. The line between these stakes will be parallel to the main track. Set up over the stake near the frog-point, and sight at the second stake with the frog-angle laid off, so that when the vernier reads zero the line of collimation will be on the common tangent to the curve. Then proceed as in the location of a simple curve, except that 25-foot chords are used. The switch-point is located by laying off the lead. The length of the lead depends upon whether a stub or a split switch is used. In the case of a split switch, the lead depends upon the length of the switch-rail, the distance gauge to gauge at the heel of the switch, the number of the frog, and whether the turnout curve is considered as extending from the heel of the switch to the point of the frog or from the heel of the switch to the toe of the frog. The values for these quantities differ greatly in practice, and a table of split-switch leads would have to be of considerable length to be of any practical service. The standard leads for any particular road may be used.

The lead curve is usually located by the eye of the section foreman, although some roads give diagrams or tables for locating the curved rail by offsets from the main track.

Second. It has been proved that the lead from a curved main track for a certain frog number is the same as that from a straight track with the same frog number, and that the degree of the turnout curve equals $D_m + D$, depending on whether the center of the turnout curve is on the same side as the center of the main track or on the opposite side. When the centers are on opposite sides and the turnout is outside the main track, the degree of turnout curve $= D - D_m$, where $D =$ degree of turnout curve from straight track, corresponding to a given frog number, and $D_m =$ degree of main track.

Form of Notes. Same as in Simple Curve

In the description of curve column the quantities F and E (frog-angle and lead) should be given.

Brief computations should be given on the right-hand page.

PROBLEM F7. VERTICAL CURVE.

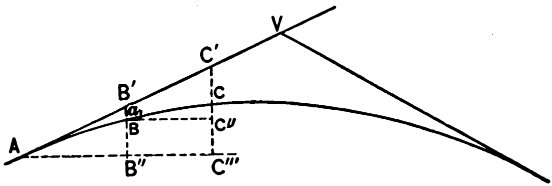
Party. Three men.

Equipment. Level, self-reading level-rod, tape, hatchet, stakes, and keil.

Problem. *To connect two grades by a vertical curve.*

Theory. The curve which is most commonly used for connecting two grades is the parabola, and the notation used in the case of the simple curve as respects *P.C.*, *P.T.*, and *P.I.* may be applied to the vertical curve.

Assuming the origin of co-ordinates at the *P.C.*, the offsets from the tangent to the curve vary as the squares of the distances from the *P.C.*



In the figure the offset a_1 at the first station equals $\frac{g-g'}{4n}$, where g equals the first grade and g' equals the second grade and n equals one-half the length of the vertical curve in stations. The offset at the second station equals $4a_1$, at the third station $9a_1$, etc. Theoretically, the measurements should be made along the tangent and at right angles to the tangent, but there is no practical error for ordinary grades by making the measurements horizontally and vertically.

The elevation of points on the vertical curve may be computed in one of the following ways: First, compute the elevations of points on the tangent and apply the offsets a_1, $4a_1$, etc., giving them the correct sign. Second, find the elevations of the points B, C, etc., above the horizontal line AC'''. Third, find the elevation of B above A, C above B, etc. The length of the vertical curve depends upon the rate of change of grade. In general, vertical curves in sags should be longer than those at summits.

It is preferable to have the intersection of the grades come at an even station, but if this is impossible the offsets at equal intervals from the *P.I.* may be computed. Where the even stationing is used, the offsets at the even stations are found by multiplying a_1 by the squares of the distances in stations from the *P.C.* or *P.T.* It has been found in practice that the best results are secured when a_1 is equal to or less than 0.25 feet, n being taken in all cases as a whole number of stations.

Method. The method of procedure is similar to that given in Problem B3. The stakes are placed 20 to 100 feet apart. The elevations of the grade and the ground are found with respect to a given datum and the cut or fill marked on the center stakes. The elevations of the successive points, instead of changing by a constant amount, as in a simple grade, change by a variable amount, as determined above.

Form of Notes. The form of notes is the same as that given in Problem B3, Simple Grade. Brief computations for determining the points upon the vertical curve may be preserved on the right-hand page.

PROBLEM F8. CROSS-SECTIONING.

Party. Three men.

Equipment. Hand-level, 50-foot tape, 5-foot rod, and a Philadelphia or a plain leveling-rod. Instead of the equipment noted, a set of cross-section rods may be used. This set of rods consists of one 10-foot rod, similar to a plain leveling-rod, but graduated only to tenths, and a 10- or 12-foot rod with a level-bubble set in the center and graduated to tenths.

Problem. *To cross-section for a certain distance on each side of a line of stakes.* This problem is of common occurrence in a preliminary survey of a line of railroad or in double-tracking a line of road. In the former the hand-level is more commonly used. In the latter both hand-level and cross-section rods are used, but cross-section rods are the more rapid.

Method. First, by use of the Locke hand-level. Having given the surface elevation of the station, the height of the eye is found (corresponding to the usual H.I.), and any desired contour may be found by reading on the rod a quantity equal to the difference in elevation between the eye and the contour. The distance of the contour from the center line is obtained by use of the tape. To obtain the second contour each side of the center line, the observer stands at the first contour point, and, by resting the hand-level on the 5-foot rod, obtains a zero reading on the rod if the ground rises, or a 10-foot reading if the ground falls, the contour interval being five feet. This method is pursued until any desired number of contours are obtained. Should any other contour interval be used, a corresponding change must be made in the rod-reading.

The method given above is that used in making the contour map for preliminary surveys. Instead of finding any even foot contour, the rod may be held at any break of grade and the elevation of this point and its distance out from the center determined.

Second, by use of the cross-section rods. If the ground falls, place the zero end of the rod containing the bubble at the center stake, and note the vertical rod-reading at the other end. Proceed similarly to any other desired distance. If the ground rises, the height of the 10-foot end of the rod is obtained by taking the reading at the center stake. If the ground is irregular, the elevation at any point may be obtained by making a vertical reading at that point and noting the horizontal rod-reading. This method is extensively used in taking the cross-sections for double-tracking a line of road. The sections are taken with respect to the center line and the top of rail. The point 2.35 feet (for gauge 4′ 8½″) is marked on the rod, and when this point is held at the gauge-line the zero end of the rod will be on the center of the existing track. In this class of work the slope-stake may be set more rapidly by the use of these rods than by the usual method. See Problem F9.

Form of Notes. The form of notes is similar to that given in the next problem. The data is given in the form of a fraction, the numerator of the fraction being the distance from the center line to the point where the reading is taken, and the denominator the distance above or below either the elevation of grade at the center point or the ground at that point.

In preliminary survey work, instead of taking the notes in field-books, the contours may be plotted in the field on cross-section paper, and these sheets left in the office, and the contours either transferred to the preliminary map or the sheets fastened together in their correct relative positions and a paper location made.

PROBLEM F9. SETTING SLOPE-STAKES.

Party. Three men.

Equipment. Level, self-reading level-rod, 50-foot cloth tape, hatchet, and stakes.

Problem. *To set the slope-stakes for a line of railroad, having given the grade elevation at one station and the rate of grade.*

Theory. For a level section, the distance from the center to the slope-stake on either side is equal to $\frac{b}{2}+sc$; where b

equals the width of road-bed, c equals the center cut or fill, and s equals the slope ratio. In a *cut*, when the ground slopes transversely from the center, the distance to the slope-stake is either more or less than that given above, depending on whether the ground rises or falls. In a *fill* the *reverse* is true.

The grade-rod, which is constant for a given station (where only one set-up is necessary), is equal to the difference between the height of instrument and the grade elevation.

The figures opposite illustrate two cases: first, when the height of instrument is above grade, and, second, when the height of instrument is below grade. In the first case the cut at any station is equal to grade-rod minus the rod-reading (in fill it equals the rod-reading minus the grade-rod). In the second case the fill at any station is equal to the rod-reading plus the grade-rod.

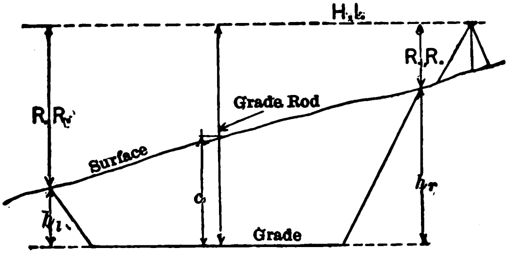
Method. The following quantities are assumed:

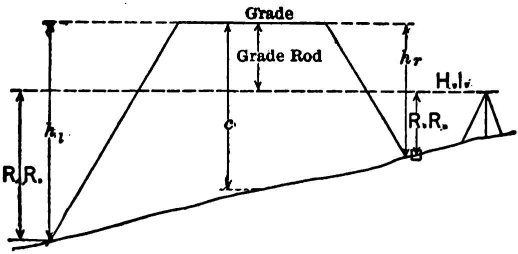
$$b=\frac{+20\ \text{(cut)}}{-18\ \text{(fill)}};\ s=1\tfrac{1}{2}\ \text{to}\ 1.$$

Drive a center line of stakes 50 feet apart and from 200 to 500 feet in length. Set the instrument up so that a maximum

FORM OF NOTES.

Station.	H.I.	−S.	Surface.	Grade Elev.	Grade-rod.
2+50		6.6	94.0	101.3	−0.6
2+00		4.9	95.8	100.3	0.4
1+50		4.0	96.7	99.3	1.4
1+00		2.6	98.1	98.3	2.4
0+95		2.5	98.2	98.2	2.5
0+50		2.0	98.7	97.3	3.4
0+00	100.68	1.4	99.3	96.3	4.4





L.	*C.*	*R.*
$\frac{21.0}{-8.0}$	$\frac{0}{-7.3}$	$\frac{19.2}{-6.8}$
$\frac{17.1}{-5.4}$	$\frac{0}{-4.5}$	$\frac{15.0}{-4.0}$
$\frac{13.5}{-3.0}$	$\frac{0}{-2.6}$	$\frac{12.3}{-2.2}$
$\frac{10.0}{0}$	$\frac{0}{-0.2}$	$\frac{9.3}{-0.2}$
$\frac{10.6}{+0.4}$	$\frac{0}{0.0}$	$\frac{9.3}{-0.2}$
$\frac{13.3}{+2.2}$	$\frac{0}{+1.4}$	$\frac{10.0}{0}$
$\frac{16.0}{+4.0}$	$\frac{0}{+3.0}$	$\frac{13.6}{+2.4}$

Give also the usual information in a suitable place.

number of stations may be observed. Find the grade-rod at the first station and the center cut or fill at that station. With this as a basis, calculate the distance to the slope-stakes, assuming the section as level. Then estimate the difference of elevation between the center stake and a point about this distance from the center. With this quantity, calculate the distance from the center. Send the rodman out this distance and take a rod-reading at the point. Compute the distance out corresponding to this rod-reading. If the rod is out this distance from the center stake, the position of the slope-stake is determined. If not, assume a point between the computed distance out and the distance the rod is actually out, and, placing the rod at this point, proceed as above until the computed distance out checks with the actual distance out.

In passing from cut to fill or *vice versa*, three sections must be taken unless the grade-line runs at right angles to the center line. These sections are taken at stations on the center line opposite the points where the toe of the excavation slope meets the surface of the ground and at the grade-point station. The points on grade at a distance of $\frac{b}{2}$ for cut from the center are found, because the material of excavation is usually paid for instead of that in embankment.

The numerator of the fraction is the distance from the center line to the slope-stake, and the denominator is the cut or fill at that point.

The signs conform to the usage in analytic geometry, the grade-line being the x axis. When the ground does not slope uniformly from the center to the slope-stake, the points of change of grade are noted.

CHAPTER VII.

GEODETIC PROBLEMS.

PROBLEM G1. MEASUREMENT OF A BASE-LINE ALONG THE SLOPE.

Party. Six to eight men.

Equipment. Transit, level, level-rod, standard tape, two range-poles, two thermometers, spring-balance, hatchet, package of zinc strips, stakes, and tacks.

Problem. *To find the distance between two fixed monuments by measuring along the slope.*

Method. Carefully align a series of hubs (by use of the transit), placing them at even stations or at changes in the slope. The hubs are driven practically flush with the ground, and a zinc strip is tacked on each. With the tape at any desired tension and the zero graduation coinciding with the first monument, scratch a mark on the zinc strip of the first hub. Note the temperature at the 25- and 75-foot points. Proceed in a similar manner to the second monument, noting the distance from the last even station to that point. Find the difference in elevation of the stakes by use of the level. As many measurements should be made as the time available will permit.

The absolute length of the line is found in the way described in Problem G3. The correction due to sag is omitted.

FORM OF NOTES.

(Left-hand page.)

Sta.	Temp.	Elev.	Diff. Elev. = h.	h^2.	Tension.

On the right-hand page give the location of the permanent hubs, the absolute length of the standard tape, the party, date, equipment, etc. The level notes are preferably kept in a separate book, and the elevations of the points transcribed in the base-line book.

PROBLEM G2. MEASUREMENT OF A BASE-LINE WITH TAPE SUSPENDED.

Party. Eight men—transitman, levelman, rodman, head and rear chainmen, two temperature men, and stakeman.

Equipment. Transit, level, level-rod, standard tape, two range-poles, two special base-tripods, two thermometers, two spring-balances, stakes, hatchets, and tacks.

If a 300-foot tape is used, two pieces of lath or two old tripods may be used to support the tape at the 100- and 200-foot points.

The base-tripods above mentioned are similar to those shown in "Johnson's Surveying," in the description of base-line measurement by Jäderin's method.

If there is a millimeter scale on one end of the tape, a knife-edge may be inserted in the top of the base-tripod; if there is no millimeter scale, a zinc strip is tacked on the top of each base-tripod, and the ends of the tape marked by scratches made with a sharp knife.

Problem. *To measure the distance between two fixed monuments with the tape suspended.*

Method. Before beginning the measurement of the base-line, the knife-edge of the base-tripod must be placed over the first monument. This condition is secured as follows: By use of the transit, set a stake at right angles to the base-line and about ten feet from the first monument. With the transit set up over this stake and with the plates level, bring the line of collimation to the point on the monument. Raise the telescope slightly and bring the knife-edge to coincide with the line of collimation. Set the second base-tripod approximately at the right point with respect to line and distance. Then, with the zero graduation coinciding with the knife-edge of the first tripod when at the desired tension, note the scale-reading, and its sign at the second tripod; also note the temperatures, if a 100-foot tape is used, at the 25- and 75-foot graduations.

If a 300-foot tape is used, the temperatures are taken at the 100- and 200-foot graduations. By use of the level take rod-readings on each tripod, including the intermediate ones, if they are used.

Carry the tripod which was over the first monument ahead on the line, and proceed in the manner described above for the rest of the line.

Care should be taken that a tripod is not moved until its elevation has been determined and a forward one set.

When the end of the line is reached, a base-tripod is placed over the monument in the manner previously described for the first monument. The plus station should be recorded. As many measurements may be made as are desired.

The absolute length of the line may be found in the way described in Problem G3. All the corrections there noted are applied.

Form of Notes. Same as in the previous problem.

PROBLEM G3. BASE-LINE REDUCTIONS.

Equipment. Table of squares and a table of logarithms.

Theory. To find the true length of a line, certain corrections must be applied to the measured length. The corrections which must be applied depend on the degree of accuracy required. When the tape is suspended, corrections are applied for sag, temperature, grade, pull, and absolute length. When the tape is supported throughout, the correction for sag drops out.

Field Notes. The forms of field notes are those given in Problems G1 and G2. The reductions may be given in concise form on the last page of these notes.

The formulas for the different corrections will be given in order. It is assumed that the student is familiar with the derivation of these formulas.

Reductions. *Correction for sag.* The formula used is $C_s = -\frac{L}{24}\left(\frac{wd}{P}\right)^2$, where L is the length of the line, w is the weight of a unit length of tape, d is the distance between supports, and P is the field-tension used. Where the line is not an even number of hundred feet, the correction must be computed for the even number of stations, and then a second correction found for the substation.

To determine the area of cross-section and the weight of a unit length of the tape, first weigh the tape with the reel, and then, by weighing the reel, obtain the weight of the tape. Having the weight of the tape, and knowing the weight of a cubic foot of steel (=490 lbs.), we can obtain the area of cross-section and the weight of unit length.

Temperature correction. This correction is found from the formula $C_t = \alpha L(t-62°)$, where $\alpha = 0.0000063$ and t is the mean of all thermometer readings taken.

Grade correction. This correction is found from the formula $C_g = -\frac{\Sigma h^2}{2l}$, where h is the difference in elevation of the two ends of the tape, and l equals the length of the tape (practically 100 feet). A table of squares is used in finding the numerator of this fraction. The formula given can only be applied where the distance along the slope, between successive hubs, is equal to the full tape-length. For any other length a separate correction must be found by substituting this length for l in the formula $\frac{h^2}{2l}$.

Correction for pull. The formula used is $C_p = \frac{(P-P_0)L}{SE}$, where P = the tension used in the field, P_0 = the tension at which the tape is standard, S = the area of cross-section, and E = the modulus of elasticity = 28,000,000 lbs.

Correction for absolute length. This correction must be obtained by having the tape standardized, either by the National Bureau of Standards, at Washington, D. C., or some competent firm. The correction to be applied for each application of the tape is the difference between the absolute length of the tape and its length as given by the graduations. Due regard must be paid to the sign of this correction.

Tabulation. The above corrections should be tabulated, and the resultant correction applied to the measured length.

PROBLEM G4. TESTING A TAPE. PRECISE METHOD

Party. Two men.

Equipment. Standard tape or bar, tape to be tested, spring-balances with some means of adjusting the position of the ends of the tape, and one or two micrometer microscopes.

Problem. *To find the absolute length of a tape.*

Method. *First,* suppose that the standard of reference is a 100-foot bar, the absolute length of which is known at standard conditions. Apply the tape to be tested to this bar, and note the distance between the end graduations on the tape and those on the bar when the tape is stretched at a certain tension and as much of the friction eliminated as possible. The distance between the graduation on the tape and that on the bar is found by observing the number of micrometer divisions on the micrometer-head, and knowing the exact value of one division. Before this distance is measured, the tape must be so arranged by the stretching apparatus at each end that the distance between the two end graduations is within the range of the micrometer microscope. Owing to the many different forms of stretching apparatus in common use, the arrangement cannot be briefly described.

Note that temperature readings are not necessary if the metals are of like composition.

Second, if the tape is to be tested when suspended, some form of support and stretching apparatus must be designed for each end, and at an elevation sufficient to clear the floor when under the least tension used.

The standard tape, and the tape to be tested, are placed side by side, and the distances between the end graduations are measured as in the first case. Knowing the absolute length of the standard tape, the length of the other may then be found. If the same tension is used on both tapes, the correction for sag will be eliminated, provided the two tapes are of the same cross-section. If they are not of the same cross-section, the sag correction must be applied. As in the first case, no temperature readings are required.

Notes. Give data with regard to the standard bar or tape, and also the tape which is being tested. Record the micrometer readings at the two ends. Also give the general information common to all problems.

PROBLEM G5. TRIANGULATION BY THE METHOD OF REPETITION.

Party. Two or three men.

Equipment. Transit, striding-level, and magnifying-glass. (The transit reading to 20 seconds, if possible.)

Where the angles to be measured are between permanent objects, no other equipment is necessary. If the angles to be measured are those of a triangulation system, signals, etc., must be provided.

Problem. *To measure the angles between several permanent objects by the method of repetition, the angles being taken so as to close the horizon.*

Theory. When an angle is measured first with the telescope direct, and then with the telescope inverted, the errors arising from the line of collimation not being perpendicular to the horizontal axis of the telescope, and the horizontal axis not being perpendicular to the vertical axis, are eliminated. The errors due to eccentricity and graduation are eliminated by reading both verniers and by measuring the angles on different parts of the limb. It has been found that the best results are secured when the number of repetitions is such that the angle multiplied by the number of repetitions is nearly equal to 360°.

Method. The method of repetition is explained in Problem D1. In that problem, however, the telescope was kept direct throughout the work, and the angles were measured only in one direction.

The problem given here differs from that just mentioned in the following particulars: First, the angles are measured with telescope direct, starting from the left and turning to the right, and then, with the telescope inverted, measuring from the right to the left. Second, in measuring an angle, the verniers A and B are read on the first point, then on the second point, and, finally, again when the number of repetitions has been completed. The difference between the first and last readings, divided by the number of repetitions, is equal to the mean angle. This mean angle should check with the angle obtained by single measurement, to within the smallest reading of the vernier. The calculations are carried out for telescope both direct and inverted, and the mean of the two values taken for the final value of the angle.

Each angle around the horizon is measured in this way, and the sum of the several angles should equal 360°.

Remark. The vernier reading, when pointing on the first station, should read an odd number of degrees and minutes.

Form of Notes. Same as in Problem D1, except that another column should be added for stating whether telescope was direct or inverted.

A complete description of the objects sighted at should be given on the right-hand page, illustrating by sketch if to advantage.

PROBLEM G6. THEODOLITE WITH MICROMETER MICROSCOPES. "RUN OF THE MICROMETER."

Party. Two men.

Equipment. Direction theodolite which may be read to single seconds.

Theory. One of the most important parts of observing with a theodolite is the correct reading of the scale. In the engineers' transit this is accomplished with sufficient accuracy (10″ to 1′) by using a vernier. Greater precision is required in primary triangulation or in astronomical observations, therefore the micrometer microscope is used.

The micrometer microscope is an instrument for measuring accurately the motion of a cross-hair over the field of a microscope of low power. The cross-hairs are stretched on a frame, to which there is rigidly attached a fine accurately cut screw. They are placed at such a distance apart as will allow a bright line to show between the two hairs and the graduation, if the graduation is placed so as to bisect the distance between them. The screw is cut so that an even number of revolutions of the head will move it over certain divisions on a comb-scale, which is placed rigidly in the box, but which may be adjusted to conform with the readings on the main scale, if necessity requires. The faces of the fixed and movable frames are put as closely together as possible, so that the comb-scale and the cross-hairs may be simultaneously placed in the common focal plane of the objective and eyepiece, therefore avoiding parallax.

The main scale is generally divided into 5′ spaces, and is read to 5′ directly, then to even minutes by passing the hairs over the space between the last 5′ division and the zero of the comb-scale. The fractional part of the minute is obtained from the micrometer-head. The micrometer-head is divided into 60 parts, therefore single seconds can be read and tenths of seconds approximated. Owing to imperfections in the graduations of the 5′ spaces on the main scale, to lost motion in the micrometer-box, to inaccuracies in the pitch of the screw, and also reading and temperature, the head, if passed from the zero of the comb to the 5′ graduation preceding the zero, and then to the succeeding 5′ graduation, will not give exactly 300″. The difference between this reading of the 5′ space and 5′ is the run of the micrometer, and it is the purpose of the problem, after learning how to read the micrometer exactly, to correct the reading for this error, or "run of the micrometer."

Before taking a series of readings, the following adjustments must be carefully tested (in the presence of the instructor):

(*a*) If the eyepiece is not properly focused, carefully twist it to the right and left in its socket until the divisions on the main scale and the cross-hairs are sharply defined. This necessitates good light. Do not try to read one micrometer in good light and the other in poor light.

(*b*) It is necessary that an even number of turns of the screw be equivalent to a given space. To test this, measure the image of the space with the screw. If the image is too small, the objective must be brought nearer the object, and the cross-hairs moved farther away. Opposite motion of the parts must be made if the image is too large. The tubes carrying the objective and the micrometer-box permit this, but should not be taken apart without the aid of the instructor.

(*c*) The zero of the comb-scale should be brought into coincidence with the cross-threads when the micrometer head reads zero.

(*d*) The micrometer microscopes should be placed 180° apart. To test this, set one at 0° and bring a graduation-line to bisect the thread interval; then the other micrometer may be brought to bisection on the other line opposite. This is corrected by moving the adjusting-drum, which protrudes from the left side of the micrometer-box.

The microscope is an inverter; therefore the graduations, which really increase to the left, appear to increase to the right, i.e., the graduation last passed appears to be on the left of the observer, when it is really on the right.

To read the position of the zero, follow the instructions given below. Note the last-numbered degree-mark passed by the zero of the comb-scale, and count the number of 5′ spaces until the one is reached just preceding the zero of the comb-scale. Carefully note the proper degree and last 5′ mark passed by the zero of the comb as it was moved in sighting on the mark whose position is being read. Put these quantities in column 2 of the form of notes given. To read the number of minutes and seconds, move the wires until the zero of the comb-scale is directly between the wires (the micrometer-head should read zero), then move the cross-hairs apparently to the left and so that the numbers increase on the micrometer-head. Carefully move the cross-hairs until the graduation last passed bisects the distance between the cross-hairs. Then read the number of minutes passed over by the cross-hairs by noting the number of points on the comb-scale which have been passed by the right-hand wire in its moving from the zero position, and read the number of seconds on the micrometer-head. Reduce this odd number of minutes and seconds to seconds and call it b (backward reading).

If n = number of full turns in the backward reading, and
o = number of seconds read on the micrometer-head,
then

$$b \text{ (in seconds)} = n \times 60 + o.$$

Now turn until the next succeeding graduation is reached. Read only the number of seconds registered on the micrometer-head and call this number p. Then add this number of seconds to the ($n \times 60$ seconds) of the backward reading and call it the forward reading, or

$$f \text{ (in seconds)} = \text{forward reading} = n \times 60 + p.$$

The total number of seconds read between the two main-scale divisions (5′) should be 300″, but it will almost always be slightly smaller or greater than this number, i.e., there will not be exactly 5 turns of the micrometer-head between the graduations.

The difference from $300'' = b - f$, and will vary for different parts of the scale, and for the same part at different times owing to changing conditions of the instrument, etc. Let

$$\Delta = 300 + (b - f)$$

and

$$D = 300$$

and

$$d = \Delta - D,$$

also m = mean reading $= \frac{b+f}{2}$ = mean observed position of zero line of micrometer as referred to actual graduations.

The problem now is to apportion d (which equals $b-f$) so that the two discordant readings b and f may be made equal; therefore making $\Delta = D$.

There are two conditions which may arise.

First. When the space Δ is measured greater than the mean number of micrometer divisions between successive graduations, i.e., 300. Then the backward reading b is given too great by b alone, and the forward reading is too small by f alone.

Therefore b must be numerically decreased by a quantity depending directly upon the distance of the zero line from the graduation last passed, diminished by $\frac{d}{2}$ and inversely as D.

Second. If the space Δ is measured less than 300″, b is too small by b alone and f is too large by f alone, and the corrections will have opposite signs to the first condition. Let

c = correction to backward reading,
c' = correction to forward reading.

Then

$$\frac{c}{d} = \frac{b - \frac{d}{2}}{D},$$

or

$$c = \frac{\left(b - \frac{d}{2}\right)d}{D};$$

also

$$\frac{c'}{d} = \frac{\left(D - \frac{d}{2}\right) - f}{D},$$

or

$$c' = \left[\frac{\left(D - \frac{d}{2}\right) - f}{D}\right]d.$$

The backward reading must be numerically decreased by c, and the forward reading, f, must be numerically increased by the quantity c', if the measurement comes under the first case; but if the measurement comes under the second case, the opposite is true.

Or when Δ is too small, if M = the true value of backward and forward reading, then

$$M=\frac{b+c+f-c'}{2},$$

and by substituting values for c and c' we have

$$M=m+\frac{md}{D}-\frac{d}{2},$$

and when Δ is too large

$$M=\frac{b-c+f+c'}{2},$$

or by substitution of the values for c and c'

$$M=m+\frac{d}{2}-\frac{md}{D}.$$

For use in reducing observations we can apply the equation

$$M=m+\frac{d}{2}-\frac{md}{D}=m+r,$$

and by giving d its proper sign we will have the correct value of r, which is the "run of the micrometer."

d is minus when f is greater than b, and is plus when b is greater than f.

It should be here noted that d is the total error which is accumulated by moving the cross-wires over a 5′ space, and is called the "error of run," and that the correction which is applied to the mean reading is the "run of the micrometer" for that reading.

FORM OF NOTES.

Microm	Scale Read.	b.	f.	$\frac{b+f}{2}$.	$\frac{d}{2}-\frac{md}{D}$.	Corrected Reading.	Corrected Total Reading.
A	24° 35′	75″.2	76″.2				
B		75″.6	76″.8				
		75″.4	76″.5	75″.95	−00″.27	75″.68	24° 36′ 15″.68

PROBLEM G7. MEASUREMENT OF ANGLES OF TRIANGLE BY DIRECTION INSTRUMENT. METHOD OF INDEPENDENT MEASUREMENTS.

Party. Four men.

Equipment. 10-inch theodolite reading to single seconds, with necessary tripods and equipment consisting of: (1) For triangles whose sides are less than 1500 feet in length, two brass plumb-bobs, two tripods to which plumb-bobs may be attached, and three permanent monuments, preferably of metal, on which there are the customary reference-points. (2) For sides considerably longer than 1500 to 2000 feet, permanent monuments as above, and two range-poles or sight-rods with stands, which will hold them plumb when placed on the reference-points.

Problem. *The three angles of the triangle are to be measured.* The longer the sides of the triangle (up to a reasonable limit), the more satisfactory will be the results. It is better to use the range-poles at a greater distance than the plumb-bobs at a comparatively short distance, owing to the amplitude of vibration of the bobs.

Method. Set up the instrument over one of the monuments. Level it very carefully, preferably by use of the striding-level. (1) Sight on one of the range-poles and read both micrometers. (2) Sight on the other range-pole and read both micrometers. (3) Transit the telescope and turn the alidade 180°. (4) Sight on the last point and read both micrometers, and finally (5) Sight on the first point and read both micrometers.

Then shift the circle $\frac{180°}{4}$ and repeat the observations.

Four combined readings will be obtained after correcting for run of the micrometer, according to the form of notes given on the next page.

Certain errors occur in all observations, namely, instrumental errors and errors arising from other sources. By proper arrangement of the observations the errors due to instrumental imperfections may be eliminated. In order to eliminate as far as possible extra-instrumental errors, the following instructions should be adhered to.

(1) The theodolite must be sheltered from the sun and wind, and while setting the tripod see that the tripod-head thumb-screws are loose until the legs are firmly planted.

(2) The tripod should be firmly set, and the instrument should rest freely without being clamped to the head. After leveling the instrument, tighten the foot-screws.

(3) The instrument should be accurately centered.

(4) Care should be taken to record correctly the degrees and minutes of an observed angle.

(5) Combined measures should be taken in order to eliminate the error due to twist.

(6) Parallax of cross-wires should be eliminated by changing the focus of the eyepiece.

PROGRAM.

Instrument at A.
Sight on B and read both micrometers.
Sight on C and read both micrometers.
Transit telescope and turn alidade 180°.
Sight on C and read both micrometers.
Sight on B and read both micrometers.

Shift circle by $\frac{180°}{4}$ and carry same program throughout.

FORM OF NOTES.

(Extending across both pages.)

Instr. at Sta. A. Fauth 10″ Theodolite. Party:—
Date. 1 div. micro. head = 1″.

Sight on.	In-stru.	Micro.	b.	f.	$\frac{b+f}{2}=m$	$\frac{d}{2}-\frac{md}{D}$	M.	True Read	Angle.
B	Dir.	A ° ′	—	—					
		B	—	—					
			Mean b	Mean f	—	—	—	—	
C		A ° ′	—	—					° ′ ″
		B	—	—					
			Mean b	Mean f	—	—	—	—	
C	Rev.	A ° ′	—	—					
		B	—	—					
			Mean b	Mean f	—	—	—	—	° ′ ″
B		A ° ′	—	—					
		B	—	—					Take mean
			Mean b	Mean f	—	—	—	—	of four combined Readings.

On page preceding the record of angles give an accurate sketch of the station occupied with offsets to objects in the vicinity; also plot a rough diagram of the pointings.

PROBLEM G8. PRECISE-LEVEL CONSTANTS.

Party. Three men.

Equipment. Precise level, precise leveling-rod (metric), steel tape, and plumb-bob.

Theory. The constants of a precise level consist of the following: (1) the angular value of one division of the bubble, (2) the inequality of the pivot-rings, and (3) the angular value of the wire interval. To these might be added the absolute lengths of the leveling-rods. The adjustments, which should be examined frequently, are (4) the error in the line of collimation, and (5) the error of the striding-level. The derivation of the respective formulas will be given in connection with the different tests.

Method. (1) *The angular value of one division of the bubble.* This is found by the optical method as given in the second part of Problem E5.

(2) *The inequality of the pivot-rings.* This test will be made so that information will also be secured for the fifth part of the problem, i.e., the error of the striding-level. Place the instrument on a concrete foundation and, with the telescope and striding-level in their normal positions, bring the bubble to the center by use of the leveling-screws and micrometer-screw. Note the position of the two ends of the bubble, remove the striding-level and replace it on the wyes. Again note the position of the two ends of the bubble. Remove the striding-level, turn the telescope end for end and replace it in the wyes. Replace the striding-level in its direct position and note the reading of the two ends of the bubble. Reverse the striding-level and again note the position of the ends of the bubble.

One-half the motion of the bubble when the striding-level is direct and when reversed gives the error of the striding-level and covers test number five. The movement of the center of the bubble when the telescope is direct and when reversed (with the striding-level in the same position) is twice the error of the pivot-rings. What is desired is the *effect* of the *error* in the *pivot-rings on* the *line* of *collimation.* If the pivot-rings are of unequal diameter, the line of collimation (if in adjustment), will be the axis of a cone when the telescope is revolved in the wyes, and the real error in the line of collimation will be one-quarter of the apparent error caused by the rings. This can easily be seen by use of a sketch. If the striding-level is not in adjustment, its error will be eliminated by observing with the striding-level both direct and reversed. The error caused by the inequality of the pivot-rings and the inclination of the bubble is found in terms of divisions of the bubble-tube. Its value in seconds of arc is found by using the angular value of one division of the bubble as found in test number one. This error should be given as a certain amount in a certain horizontal distance. The sign of the correction should be given, i.e., whether the line of collimation points upwards or downwards when the telescope is normal and the striding-level normal.

(3) *Angular value of the wire interval.* This test is made in a manner similar to that used in the determination of the stadia constant as given in Problem D7.

If a metric rod is used, the base-line should be measured in the same unit.

(4) *The error of the line of collimation.* The same base-line may be used as in tests one or three. Without leveling the instrument, sight at the rod and read the three wires. Invert the telescope, i.e., rotate it 180° in the wyes, and again read the three wires. Take one-half the difference of the mean of the first three wire readings and the last three wire readings. This gives the error of the line of collimation for the length used as the base. This may be expressed as a certain part of a foot in 100 feet, or as a certain number of dcms. in 100 dcms.

(5) *Error of the striding-level.* The method of procedure is the same as outlined in test two.

FORM OF NOTES.

Test 1.

The form of notes is the same as that given in Problem E5.

Test 2.

(Notes extending across both pages.)

Teles.	Strid. level.	Bubble.		Center of bubble.
		Left end.	Right end.	

Motion of bubble for alt. positions.	Motion of the bubble for corresponding positions of the striding-level.	½ motion of bubble for reversal.

On the right-hand page give the information outlined in the method for Test 2.

Test 3. *Angular value of the wire interval.*

The form of notes is that given in Problem D7.

Test 4. *Error of the line of collimation.*

FORM OF NOTES.

Wire read. Tel. direct.	Wire read. Tel. inv.	Mean read. Tel. direct.	Mean read. Tel. inv.	½ diff. for inversion.

On the right-hand page give the information outlined in the method for Test 4.

Test 5. *Error of the striding-level.*

The form of notes is the same as in Test 2.

PROBLEM G9. RUNNING A LINE OF PRECISE LEVELS.

Party. Five men.

Equipment. Precise level, two precise leveling-rods, two steel turning-points, umbrella, and two hatchets or mallets.

Problem. *To run a line of precise levels between two bench-marks.*

Theory. With the Y or dumpy levels the adjustments are made as nearly correct as possible, and care is then taken that the bubble is in the center at each observation. The backsight and foresight for each set-up are made of equal length, so that the errors of adjustment, refraction, etc., are eliminated. With a precise level the field observations are conducted with a view of eliminating as far as possible the errors of adjustment and observation, but as the errors which arise are relatively greater than with the ordinary types of level, the different constants are determined so that corrections may be applied for excess of length of backsights over foresights.

The field observations as given below are made with a view of eliminating the error of the line of collimation and the error of the striding-level, even though the lengths of the backsights and foresights are unequal. The correction on account of the inequality of the pivot-rings must be applied if the backsights and foresights are unequal.

Place the instrument so that a sight may be taken on the first bench-mark. Level up carefully by use of the foot-screws. Sight on the rod, and bring the bubble to the exact center by use of the micrometer-screw below the eyepiece. Take the reading on the bench-mark. Then turn the instrument and take a reading on a turning-point, the bubble being brought to the center by the micrometer-screw. Remove the striding-level, turn it end for end, and, having inverted the telescope in the Y's, replace the striding-level. Take a second reading on the turning-point with the bubble in the center. Turn the telescope and take another reading on the bench-mark, the bubble being again in the center.

In carrying out these observations the following points should be carefully noted. First, that the mean of the three wire readings is obtained, and if it does not check with the middle wire reading within one millimeter, the three readings are discarded. Second, the backsights and foresights should be made nearly equal by pacing, the wire interval being carried along in pencil, and the last set is chosen so that the total backsight and foresights will be equal. Third, the rods should be held perfectly plumb. Fourth, the turning-points should be exceptionally stable. Fifth, the instrument should be at all times protected from the wind and sun by the umbrella.

Carry out these observations from bench-mark to bench-mark in a direct line, and then start from the last bench-mark and return to the first by a reverse line, the turning-points on the reverse line not being the same as on the direct line.

FORM OF NOTES.

(Extending across both pages.)

Direct Line.							B.M.....to B.M.....
Date.		Backsights.					
Point.	Rod No.	Thread Readings. 1	2	3	Mean.	Thread Dist.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	
B.M..	1	24.174	26.006	27.838			Description of
		24.176	26.008	27.840			B.M. with ele-
		24.175	26.007	27.839	26.007	3.664	vation.
T.P. 1	2						
T.P. 7	2	17.520	19.104	20.690			
		17.516	19.098	20.684			B.S. +156.357
		17.518	19.101	20.687	19.102	3.169	F.S. −112.384
					156.357	26.547	D. = + 43.973

The above shows the form for first double page of precise level-notes. The second will contain the corresponding foresights. In the lower right of this should be placed computations indicating difference in length between sum of B.S. and sum of F.S. Thus:

Thread Dist.
F.S. = 26.550
B.S. = 26.547

Ex. F.S. = 0.03 m.

In the above form of notes one double page is used for backsights and a second for foresights. The thread distances should be computed as the work progresses, so that the last set-up in each line may be chosen so as to equalize backsights and foresights. The description of the B.M. is given on the right-hand page directly opposite B.M. on the left-hand page.

On the right-hand page of the "direct-line backsights" give the computations for difference in elevation as found by the direct line.

On the right-hand page of "direct-line foresights" give the computations for finding the difference in length due to the excess of the B.S's. over the F.S's.

The third and fourth double pages give notes for the reverse line, in all respects similar to the above except that on the last double page the closing error is noted. Party, date, equipment, etc., as usual are given on the first right-hand page.

PROBLEM G10. TESTING A TELESCOPE.

Party. Two men.

Equipment. Engineer's transit and level, self-reading leveling-rod, two stakes, and special cards for making the tests.

Problem. Certain imperfections may exist in the telescopes of instruments used in conducting surveys or geodetic operations. It is the purpose of this problem to determine if they exist in a specified instrument.

The imperfections may be divided into two classes:

First. Defects which exist originally in the material from which the instrument, and particularly the lenses, is made.

These may be called inherent imperfections.

Second. Defects which depend upon the manner in which the instrument is constructed.

Defects of the first class include:

(*a*) Spherical aberration.
(*b*) Flatness of field.
(*c*) Chromatic aberration.

Defects of the second class include:

(*d*) Definition.
(*e*) Illumination.
(*f*) Magnification.
(*g*) Centering.
(*h*) Size of field of view.

Theory. (*a*) *Spherical Aberration.* The rays of light which impinge upon a lens near its edge are refracted more than those which pass through the center. Therefore the rays from a point do not come to a single focus, but for every point on the object there will be shown many points on the image; consequently the image will be blurred. This is corrected by making the object-glass of two different materials, flint and crown glass, which have different refractive and dispersive powers, and then carefully grinding them.

(*b*) *Aberration of Sphericity.* The flatness of field depends mainly upon the aberration of sphericity of the eyepiece. The image of a flat object will be concave towards the lens. This defect is more liable to cause error in the case of a stadia measurement than in the measurement of elevation when only one wire is read.

(*c*) *Chromatic Aberration.* When white light passes through a simple convex lens it is resolved into the colors of the spectrum. As the different colors have different indices of refraction they will come to a focus at different points; violet nearest the lens and red farthest away. By the use of two lenses we can make the resulting lens achromatic as regards any two colors, i.e., by using an object-glass composed of a double-convex lens of crown glass and a flint diverging lens we may correct for the two more intense colors, red and violet, which have the greatest effect on the eye.

(*d*) *Definition.* Definition in a telescope depends upon the accuracy of the curvature of the surfaces of the several lenses and upon their centering.

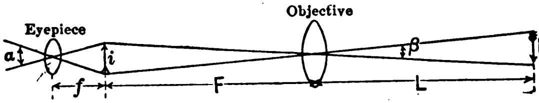
(*e*) *Illumination.* Illumination is a comparison between the amount of light conveyed by a lens from a point to the eye and the amount conveyed from the point to the eye without the lens. In good telescopes the ratio is about 85 per cent., and often falls to 70 per cent. It depends upon the aperture of the object-glass, upon the magnification, and upon the capacity of the lenses for transmitting light.

(*f*) *Magnification.* The magnifying power of a telescope is the ratio of the angular size of an object as seen through the telescope to its angular size as seen by the naked eye.

In our discussion let

I = size of object;
L = distance of object from lens;
i = size of image;
F = principal focal distance of objective;
f = principal focal distance of the eyepiece.

Then in the figure below:



From similar triangles

$$\frac{I}{i}=\frac{L}{F},$$

or

$$i=\frac{F}{L}I;$$

also $\frac{i}{f}$ = circular measure of angle of image as seen at eye, or

$$=\frac{F}{L}\times\frac{I}{f};$$

but the angle subtended by the object seen without the telescope is

$$\frac{I}{L} \text{ (when } F+f \text{ is small in comparison to } L).$$

Hence the magnification M

$$=\frac{FI}{Lf}\div\frac{I}{L}=\frac{F}{f}.$$

This may be put in the form

$$\frac{F}{f}=\frac{F}{i}\times\frac{i}{f}=\frac{\tan\alpha}{\tan\beta}=\frac{\alpha}{\beta} \text{ (as angles are small).}$$

This is seen to be the ratio of angular magnitude as seen by the telescope to the angular magnitude as seen by the eye.

(*g*) *Centering.* In a perfectly centered telescope the principal axes of the several lenses should fall in the same straight line. If they do not, the image will be blurred and indistinct.

(*i*) *Size of field of view.* The field of view is the extent of space which one can see upon looking through the telescope of the instrument. For convenience it is expressed in angular measure, i.e., the angle subtended by a diameter of the circle of the field of view. It is independent of the size of the object-glass and varies directly with the size of the eyepiece.

Method. The detects for which the telescope will be tested will be given in order, one or two being omitted.

(*a*) *Spherical Aberration.* Place a circular piece of paper over one-half the effective area of the object-glass (by wetting the paper and sticking it on the object-glass). Focus the telescope on some well-defined point some distance away (small print at 50 to 100 feet). Now remove the circular paper and place an annular piece over the portion which was uncovered at first. Focus on the same print. The amount that the telescope was moved is a measure of the spherical aberration.

Describe the method of making the test, and state the results obtained in the field-book.

(*b*) *Flatness of Field.* To test a telescope for this, draw with India-ink a square 6 to 8 inches on a side and with very heavy lines. Place the square at such a distance as will make it include nearly the whole field of view after focusing; then gradually move away from the instrument until the square includes about one-half of the field of view. If the lines appear straight the telescope has a flat field, and if curved the field is not flat.

(*c*) *Chromatic Aberration.* To test for chromatic aberration, sight the telescope at a white disk (about one and a half to two inches in diameter) in a black field; then move the object-glass slowly in and out. If in the first instance a light yellow ring is seen at the edge of the disk and in the second place a ring of purple light, the object-glass may be considered perfect in this respect, as this proves that the most intense colors are corrected.

(*d*) *Definition* in a telescope depends upon the accuracy of curvature of the surfaces of the several lenses and upon their centering, and may be tested by focusing the telescope on small, clear print at a distance of from 20 to 100 feet. Note if the print is sharply defined.

(*f*) *Magnification. First method.* Find two well-defined objects which can be seen symmetrically with reference to the vertical cross-hair of the transit when looking through the object end of the instrument to be tested. Focus for parallel rays. Set up a transit exactly back of the telescope to be tested, so that the two instruments are directly in line, and measure the angle subtended by the objects as seen through the two telescopes (by moving cross-hair of transit by tangent-screw of upper motion, and reading the angle on the horizontal circle). Call this reading α. Take three observations and find the mean of the three.

Then remove the telescope which is being tested and set the transit so that the center of transit comes where eye-end of telescope tested was. Measure the angle between the same objects, calling the reading β. Then

$$M = \text{the magnification} = \frac{\frac{1}{\alpha}}{\beta} = \frac{\beta}{\alpha}.$$

When the test is made out of doors, it may be best done by using, as the objects, two stakes placed about 400 feet from the instrument and at a suitable distance apart. It is to be noted in this test that the transit may be considered as a part of the eye.

Second method. Sight on a self-reading leveling-rod at about 30 to 40 feet from the instrument to be tested. With one eye look through the telescope to be tested, and with the other look along the barrel of the telescope. Select a unit division on the rod as seen through the telescope and count the number of divisions of the same as seen by the naked eye which come within the division as seen through the telescope. Take three readings and find the mean value. This will give a check on the magnifying power as derived by the first method.

(*g*) *Centering.* This is the coincidence of the optical axes of the different lenses. To make the test, fix a white paper disk about one-eighth inch in diameter in the center of a black surface, and focus on it at a distance of from 30 to 40 feet. If the image of the disk when a little out of focus is surrounded by a uniform haze, the centering is good.

Notes. Describe fully in the field-book the method of procedure in making the several tests, together with the conclusions arrived at concerning the particular instrument. Give party, date, equipment, etc., as usual. Both pages of the field-book may be used in this problem.

PROBLEM G11. REDUCTION TO CENTER.

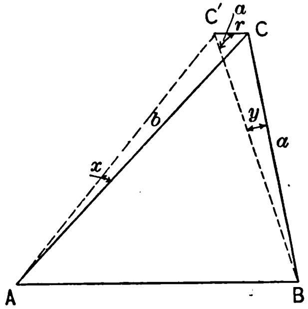
Equipment. Six- or seven-place logarithmic tables.

Problem. *To find the true angles at a station when the instrument must be set up over an eccentric station.*

Theory. In the small figure below the angle at C is found from the formula

$$C = C' + \frac{r}{\sin 1''}\left(\frac{\sin (C' + \alpha)}{b} - \frac{\sin \alpha}{a}\right).$$

The different quantities in this formula are shown on the sketch



Example. The figure given on the next page represents a part of a triangulation system which was expanded from the measured base-line South Sta.—North Sta.

The station at City Hall was eccentric. The data needed for three of the angles about this point is as follows:

1. Angle Clark—City—Boynton.

   The measured angle $C' = 107° 29' 44''.16$.

   $CC' = r = 14.44$ feet.

   Angle $CC'B = \alpha = 76° 16' 10''.00$.

   Log. distance Boynton to City Hall = 3.620889.

   Log. distance City Hall to Clark = 3.850521.

2. Angle Clark—City—Newton.

   The measured angle $C' = 54° 55' 02''.09$.

   α = same as in first angle.

   The log. of the distance City Hall to Clark is given under the first angle. The distance City Hall to Newton is computed by use of the measured angles City—Clark—Newton and Clark—Newton—City, which are respectively equal to $48° 55' 15''.79''$ and $76° 09' 32''.83$.

3. Angle Boynton—City—Newton.

   The measured angle $C' = 52° 34' 40''.42$.

   angle $\alpha = 131° 11' 12''.09$.

The other angles and distances required in the solution have been previously given.

Computations. From the given data construct suitable figures and compute the three angles.

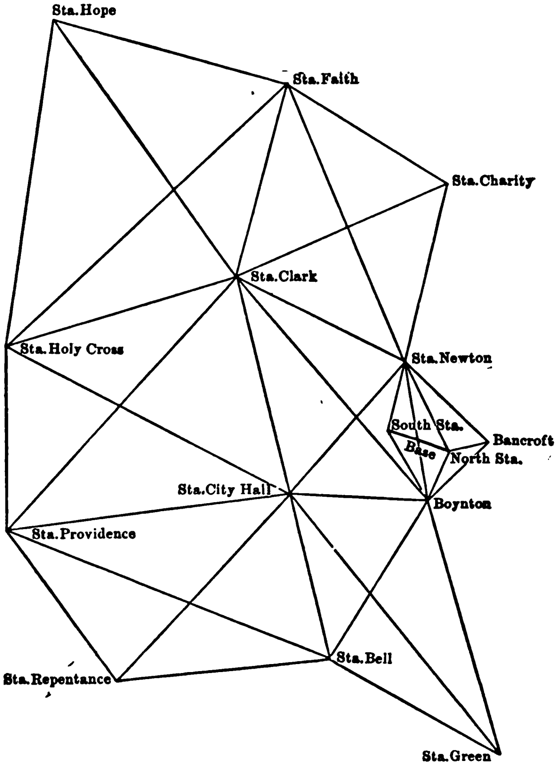
The work may be best carried out by finding the log. of the quantities outside the parenthesis and adding it to the log. of each fraction inside the parenthesis in turn, and looking up the numbers corresponding. The corrections should be carried to hundredths of a second.

$$\log. \sin 1'' = 4.685575.$$

Answers.

Angle Clark—City—Boynton $= 107° \ 22' \ 09''.13$.
Angle Clark—City—Newton $= 54° \ 55' \ 01''.22$.
Angle Boyton—City—Newton $= 52° \ 27' \ 06''.27$.

Report. The computations in this problem may be placed in the field-book in condensed form.



CHAPTER VIII.

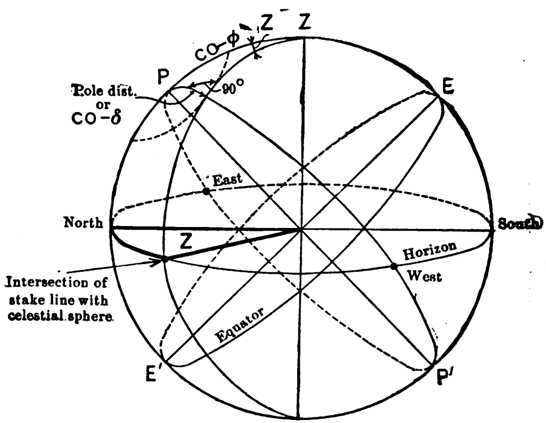
ASTRONOMICAL PROBLEMS.

PROBLEM H1. DETERMINATION OF AZIMUTH BY OBSERVATION ON POLARIS AT ELONGATION.

Party. Two men.

Equipment. Engineer's transit with reflector, small lantern for illuminating the cross-wires, plumb-bob (with some method of making it visible), or a wooden box with a vertical slit cut in it and an ordinary lantern.

Theory. Polaris appears to revolve around the north pole once in 23 hours and 56 minutes. The determination of azimuth by observation on Polaris is best made when the star is at elonga-



tion, because its motion at this time is entirely vertical and the determination of latitude is made at culmination because its motion is then entirely horizontal. If the observation is made when the star is at elongation, the angle at Z in the figure given above must be laid off in the proper direction. This angle is obtained from the right triangle and equals

$$\frac{\sin \text{pole distance}}{\cos \text{latitude}}.$$

The pole distance is obtained from the solar ephemeris, and the latitude either by making an observation on Polaris or the sun when at culmination, or by taking it off a reliable map.

Method. Set up the transit over one point in the line whose azimuth is to be determined, and level up carefully about 20 minutes before elongation. Set vernier *A* at zero and follow the program given in the notes. The movement in azimuth for 10 minutes either side of the time of elongation is inappreciable when using an engineer's transit reading only to minutes. The observations are made with telescope direct and then inverted in order to eliminate instrumental errors. Throughout the observations the lower motion is clamped.

PROGRAM AND FORM OF NOTES.

Instrument.	Time.	Sight on.	Vernier A.	Vernier B.	Mean.
Direct		Target			
Reverse		"			
"	4m. before	Star			
"	2m. "	"			
Direct	2m. after	"			
"	4m. "	"			
"		Target			
Reverse		"			

Compute the azimuth of Polaris at elongation by use of the formula previously given. Apply this in the proper direction to the mean angle as above obtained. The result will be the azimuth of the line referred to the north.

Time of Elongation. Most of the text-books in surveying give tables for time of elongation for a certain year and latitude, with rules for determining the time of elongation for other years and latitudes. The method of calculating the time of elongation is as follows:

If $\alpha_{\odot}$ = right ascension of the mean sun at Greenwich mean noon, and L = longitude of the place at which the observation is taken, then the right ascension of the mean sun at noon for the place of observation $= \alpha_{\odot L} = \alpha_{\odot} + L(9.865 \text{ seconds})$. If α_s = right ascension of Polaris on the given day, then $\alpha_{\odot L} - \alpha_s =$

sidereal interval of star's culmination before the sun's transit and $24-00-00-(\alpha_{\odot L}-\alpha_s)$ = the sidereal interval from sun's transit to the next culmination of star, and this result $\mp t$(= sidereal interval from noon to $\frac{\text{eastern}}{\text{western}}$ elongation), and mean time = sidereal time − (9.856 sec.) × (sidereal time). Finally reduce mean time to standard time by applying the difference between standard and mean time at the place in question.

Example. *To find the time of elongation of Polaris for July 14, 1906, at Philadelphia.*

$$\begin{aligned} \alpha_{\odot L} &= 7 \text{ h. } 29 \text{ m. } 43 \text{ s. } + 5.01\ (9.865 \text{ s.}) \\ &= 7 \text{ h. } 30 \text{ m. } 32 \text{ s.} \\ \alpha_s &= 1 \text{ h. } 25 \text{ m. } 35 \text{ s.} \\ \hline \alpha_{\odot L}-\alpha_s &= 6 \text{ h. } 04 \text{ m. } 57 \text{ s.} \end{aligned}$$

Star again passes meridian in 24 h. 0 m. 0 s.

Sidereal interval from sun's transit to next culmination of star = 17 h. 55 m. 03 s.

To determine the hour-angle t of star's elongation, use the formula $\cos P = \frac{\tan\phi}{\tan\delta}$.

$$\begin{array}{lr} \phi = 39^\circ\ 58'\ 02'' & \log\tan = 9.923309 \\ \delta = 88^\circ\ 48'\ 02'' & co\log\tan = 8.320919 \\ \hline & \log\cos P = 8.244228 \end{array}$$

$$P = 88^\circ\ 59'\ 40'' = 5 \text{ h. } 55 \text{ m. } 59 \text{ s.} = t,$$

then

$$\begin{array}{l} 17 \text{ h. } 55 \text{ m. } 03 \text{ s.} \\ -5 \text{ h. } 55 \text{ m. } 59 \text{ s.} \\ \hline 11 \text{ h. } 59 \text{ m. } 04 \text{ s.} = \text{sidereal time of eastern elongation.} \end{array}$$

Mean time interval = 11 h. 59 m. 04 s. − 1 m. 58 s. (= 11.99 × 9.856 s.) = 11 h. 57 m. 06 s.

Standard time fast 39 s.

Standard time of eastern elongation = 11 h. 57 m. 45 s. or 2 m. 15 s. before 12 (midnight) on July 14.

To compute the azimuth of the star at elongation, use the formula $\sin Z = \frac{\sin \text{co-}\delta}{\sin \text{co-}\phi} = \frac{\cos \delta}{\cos \phi}$.

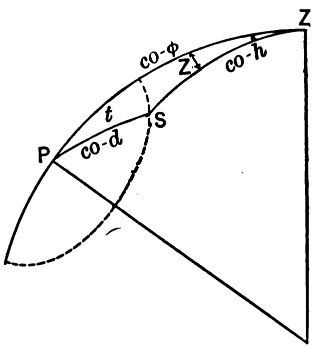
$$\begin{array}{lll} \phi = 39^\circ\ 58'\ 02'' & \log \cos \phi = & 8.320825 \\ \delta = 88^\circ\ 48'\ 02'' & \log \cos \delta = & .115537 \\ \hline & \log \sin Z = & 8.436362 \\ & Z = & 1^\circ\ 33'\ 54'' \end{array}$$

PROBLEM H2. DETERMINATION OF AZIMUTH BY OBSERVATION ON POLARIS AT ANY HOUR-ANGLE.

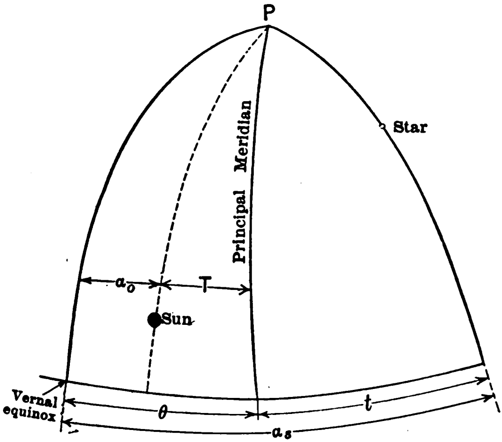
Party. Two men.

Equipment. Engineer's transit with reflector, small lantern for illuminating the cross-wires, plumb-bob with some method of making it visible, or a wooden box with a vertical slit cut in it and an ordinary lantern.

Theory. The determination of azimuth by an observation on Polaris at any hour involves, 1st, reading the angle several times between the star and a mark used as a point of reference and noting the time of each observation; 2d, the solution of an oblique spherical triangle knowing two sides and an included angle.



In the triangle SPZ the side PZ = the co-latitude, and the side PS = the co-declination. The angle at $P = t$ = the hour-angle of the star. ϕ, which = the latitude of the place, is known (it may have been determined by observation on Polaris at either upper or lower culmination), and δ, = the declination of Polaris, is taken from the solar ephemeris. The angle t may be readily found, if a sidereal chronometer is used, by multiplying the time by 15 to reduce to degrees, etc., but if a watch is used and only the right ascension of the sun at mean noon in Washington is known, then the reduction for t is more extended.



$\theta \pm t = \alpha_s$

$\theta = \alpha_s \mp t$ according whether the star is $\begin{smallmatrix}\text{east}\\\text{west}\end{smallmatrix}$ of the meridian. In the sketch the star is shown east of the meridian.

or $\mp t = \theta - \alpha_s$ (1)

α_s, which is the right ascension of the star, can be taken directly from the Nautical Almanac for the given day.

θ, which is the sidereal time of observation, can be found directly from the sidereal chronometer; but if a watch is used, the mean local time will have to be reduced to mean time and then to sidereal time.

In the sketch $\alpha_{\odot}$ = the right ascension of the mean sun at the time of observation, and T = hour-angle at the time of observation.

Then $$\theta = \alpha_{\odot} + T.$$

Suppose that the sidereal time of mean noon at Greenwich is given for each day in the year and $= V_{\odot}$, i.e., the angle between the sun and the vernal equinox is given for a certain time. Then as the sidereal time increases by 9.856 seconds every hour over the mean time (as there is one more sidereal day in the year than solar day), then $\alpha_{\odot} = V_{\odot} + T(9.856 \text{ sec.})$ for the hour-angle T.

For any other longitude $= L$.

$$\theta = V_{\odot} + (T + L)9.86 \text{ sec.} + T,$$

or finally, from (1),

$$\text{Hour-angle} = \mp t = V_{\odot} + (T + L)9.86 \text{ sec.} + T - \alpha_s.$$

To reduce to degrees multiply by 15.

By spherical trigonometry, the triangle PZS, shown in the first sketch, may be solved, after finding the angle t, by the following equations:

$$\tan \tfrac{1}{2}(S + Z) = \cot \frac{t}{2} \frac{\cos\left(\dfrac{\text{co-}\delta - \text{co-}\phi}{2}\right)}{\cos\left(\dfrac{\text{co-}\delta + \text{co-}\phi}{2}\right)}$$

and $$\tan \tfrac{1}{2}(S - Z) = \cot \frac{t}{2} \frac{\sin\left(\dfrac{\text{co-}\delta - \text{co-}\phi}{2}\right)}{\sin\left(\dfrac{\text{co-}\delta + \text{co-}\phi}{2}\right)}.$$

Then $$Z = \tfrac{1}{2}(S + Z) - \tfrac{1}{2}(S - Z).$$

Method. At any convenient time set up the transit as described in the preceding problem, and follow the program given below. Throughout the observation the lower motion is clamped. The instrument is not set on zero either when sighting on the mark or on the star. Time will be saved by taking the observations in the order given.

FORM OF NOTES.

Inst.	Time.	Sight.	Horizontal angle readings.			
			Ver. A.	Ver. B.	Mean.	Angle.
R		Mark	125° 30′ 30″	−30′ 30″	−30′ 30″	
D		Mark	125° 30′ 00″	−30′ 00″	−30′ 00″	
D	9h. 34m. 05s.	Star	225° 40′ 00″	−40′ 00″	−40′ 00″	
D	9h. 35m. 00s.	Star	225° 39′ 00″	−39′ 00″	−39′ 00″	100° 09′ 15″
R	9h. 52m. 44s.	Star	225° 38′ 00″	−38′ 00″	−38′ 00″	
R	9h. 54m. 00s.	Star	225° 37′ 30″	−37′ 30″	−37′ 30″	
R		Mark	125° 29′ 30″	−29′ 30″	−29′ 30″	
D		Mark	125° 29′ 30″	−29′ 30″	−29′ 30″	100° 08′ 15″

Find from these notes the average angle between the star and the mark; also the average of the four times of observation. Use the average time for finding the hour-angle, and use the angle between the star and the mark in the final calculations for determining the angle between the line and the true north and south line.

Computations. July 17, 1906, Philadelphia.

$$\text{R. A. Polaris} = \alpha_s = 1\text{ h. } 25\text{ m. } 37\text{ s.}$$
$$V_\odot = 7\text{ h. } 37\text{ m. } 36\text{ s.}$$
$$(T+L)9.856\text{ s} = 2\text{ m. } 25\text{ s.}$$
$$T = 9\text{ h. } 43\text{ m. } 57\text{ s.}$$
$$\theta = 16\text{ h. } 82\text{ m. } 118\text{ s.}$$
$$\alpha_s = 1\text{ h. } 25\text{ m. } 37\text{ s.}$$

$$t = 15\text{ h. } 58\text{ m. } 21\text{ s.}$$
or
$$= 239° 35' 15''.$$
$$\phi = 39° 58' 02'',$$
$$\delta = 88° 48' 02''.$$

Substitute the above values of t, ϕ, and δ in the equations given, and find $\frac{1}{2}(S+Z)$ and $\frac{1}{2}(S-Z)$, and finally Z. Use this in connection with the angle between the star and the mark to find the azimuth of the line.

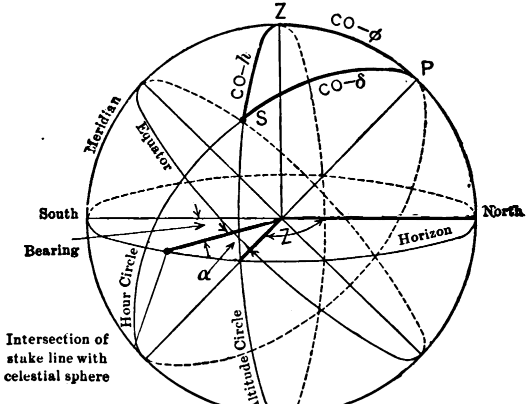
The time in this problem was obtained from a sidereal chronometer. If a watch is used, reduce the standard time to mean time, and mean time to sidereal time. This sidereal time is then used in the solution.

PROBLEM H3. DETERMINATION OF AZIMUTH BY DIRECT SOLAR OBSERVATION.

Party. Two men.

Equipment. Engineer's transit with full vertical circle, prismatic eyepiece with colored glass, and solar ephemeris.

Theory. For a solution of this problem it is necessary to know either the three sides of the spherical triangle *SPZ* in the figure below, or two sides, *SP* and *ZP*, and the hour-angle or apparent time, which is the angle at *P*. In the solution of the first case, the three sides are obtained in the following manner. The side *ZP* is equal to the co-latitude, the latitude



being taken from a map or being obtained from observation on Polaris at upper or lower culmination. The side *SP* is equal to the co-declination, which is obtained by use of the solar ephemeris, knowing the time. The side *ZS* is equal to the co-altitude, and is obtained by measuring the altitude and correcting for refraction, index error, etc. In the second case the sides *ZP* and *SP* are known and the angle *ZPS*. The side *ZS* can be found and also the angle *Z*. Knowing the angle *Z*, the azimuth of the sun can be found, being $=180° \pm Z$. The first method, being the more general one, is given.

Method. Set the transit up over one end of the line whose azimuth is desired. Level up with extreme care, using the telescope bubble, and if the vernier of the vertical circle does not have an attached bubble, note the index error. Set vernier A at zero and sight on the hub at the other end of the line. Unclamp the upper motion, turn the telescope to the sun and bring the sun's image into the quadrant so that it appears to run onto both wires. Note the time to the nearest minute, and read the vertical and horizontal circles. Invert the telescope and bring the sun's image into the quadrant, where it seems to recede from both wires. Note the time, and read the vertical and horizontal circles.

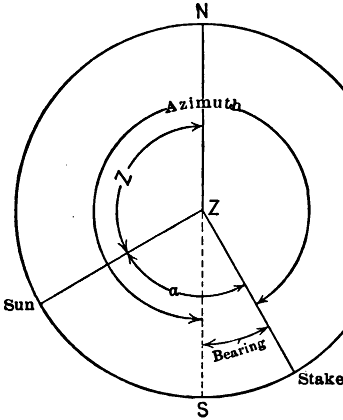
Time of making observation. To secure reliable results, the observation must be made when the sun is between two and four hours from the meridian.

Modification. If unable to get a pair of observations on account of the sun being obscured, the one observation is used. In this case the altitude is corrected for the sun's semi-diameter before the reductions are made, and the computed azimuth is then corrected by the quantity $\frac{\text{sun's semi-diameter}}{\text{cos. altitude}}$. If the instrument does not have a complete vertical circle, this method *must* be used.

Calculations. Find the mean altitude and the mean plate-reading. Correct the apparent altitude for refraction (see Table IV). Knowing the time, the declination is found as follows: The sun's apparent declination and its hourly change for Greenwich mean noon are given in the Nautical Almanac. For any other place it may be obtained if the longitude of the place and the mean time are known.

For example, the difference of time between Greenwich and Philadelphia is five hours. Therefore the declination for two o'clock at Philadelphia is equal to the declination for Greenwich mean noon on the day in question, plus 7 times the hourly change (if the declination is increasing). If the observations are made when the sun is from two to four hours from the meridian, the error caused by the difference between mean local time and standard time will be inappreciable when an engineer's transit reading only to minutes is used.

Then, having the altitude, the declination, and the latitude, the co-functions may be found, and finally the angle at Z, by use of the formula $\sin^2 \frac{1}{2}Z = \dfrac{\sin(s-\text{co-}h)\sin(s-\text{co-}\phi)}{\sin \text{co-}h \sin \text{co-}\phi}$, where $h=$ the true altitude, $\phi=$ the latitude, and $s=\frac{1}{2}(\text{co-}h+\text{co-}\phi+\text{co-}\delta)$.



Finally, the azimuth of the line may be found by use of the angle α. This angle is the angle between the stake and the sun, and is measured to the right, starting from the stake. It is the angle recorded in the "horizontal circle" column in the notes below, provided that zero was first set on the stake.

The figure given above shows one position of the reference-line and sun. A general equation might be deduced connecting the azimuth with the quantities α and Z, but it is best to construct a figure for each problem as it arises. The data in the notes below do not conform to the above figure.

FORM OF

Tel.	Time.	Hor. Circle.	Vert. Circle.	Refraction.	h.
Direct	9.29	228° 37′ 20″	52° 14′ 00″		
Rev.	9.31	230° 8′ 40″	53° 17′ 00″		
Mean	9.30	229° 23′ 00″	52° 45′ 30″	−0′ 44″	52° 44′ 46″
Direct	9.38	230° 38′ 00″	53° 44′ 40″		
Rev.	9.40	232 10′ 00″	54° 42′ 50″		
Mean	9.39	231° 24′ 00″	54° 13′ 45″	−0′ 42″	54° 13′ 03″

Also give on the right-hand page brief computations for the solution of the spherical triangle.

PROBLEM H4. DETERMINATION OF AZIMUTH BY USE OF THE SOLAR ATTACHMENT.

Party. Two men.

Equipment. Transit with solar attachment and the solar ephemeris, and a table of declinations and refraction corrections computed for a series of observations varying in time from 3 to 10 minutes, depending upon the skill of the observer.

Method. Set the transit up over one point of the line whose azimuth is to be determined. Level up carefully, using the long telescope bubble. Set vernier A of the horizontal plate at zero, and sight at the second point in the line with the upper motion clamped. Unclamp the upper motion, and having laid off the sun's apparent declination on the declination arc and the co-latitude on the vertical circle, turn the alidade in azimuth and the attachment about its polar axis until the sun's image appears between the lines of the silver disk. The telescope will then point south, and the azimuth of the mark may be read on the plates. This applies to the Burt attachment.

If the Saegmuller attachment is used, the vertical circle of the main telescope must be used in laying off the declination. The co-latitude is laid off as with the Burt attachment (not from the declination reading). In the final setting of the instrument, the angle between the lines of sight of the small and the large telescope is equal to the apparent declination, and the main telescope is in the plane of the equator, the object end pointing south. After taking a series of six readings

NOTES.

δ	Z.	True Azimuth.	
+23° 15′ 18″	106° 55′ 22″	57° 32′ 22″	June 14, 1905. Transit T. 15 used, which reads to 20″ both horizontally and vertically.
+23° 15′ 19″	108° 55′ 52″	57° 31′ 52″	.... Transitman. .... Recorder. $\phi=41°\ 27'$.

with the instrument in this position, turn the alidade 180° and set off the colatitude as before. The eyepiece end is now pointing south. Take a series of six readings as before. The mean of the twelve readings will be the true azimuth of the line, the instrumental errors having been eliminated by the reversal.

Computations. The declination is obtained in a manner similar to that given in the preceding problem. The apparent declination is found by applying the refraction correction obtained from Table VI. When using the attachment, the refraction correction is a function of the latitude of the place, the horizontal angle, and the declination.

FORM OF NOTES.

(Left-hand page.)

Time.	Inst.	Decl.	Refr. Corr.	App. Decl.	Azim.	Mean Az.
2.55	Direct	21° 43′	+0′ 33″	21° 44′	18° 51′	
....	"	......	......	......	......	
....	"	......	......	......	......	
3.27	"	21° 43′	+0′ 47″	21° 44′	18° 50′	18° 50′ 30″
3.41	Reversed	21° 43′	+0′ 47″	21° 44′	161° 11′	.
....	"	......	......	......	......	
....	"	......	......	......	......	
3.51	"	21° 43′	+0′ 47″	21° 44′	161° 12′ 20″	161° 11′ 30″

(Right-hand page.)

Date.
Party.
Equipment.
$\phi=41°\ 27'$.
co-$\phi=48°\ 33'$.
Index error $=0°\ 20'$.
Calculation for declination.
Mean azimuth $=18°\ 49'\ 30''$.

PROBLEM H5. DETERMINATION OF LATITUDE WITH THE STAR AT CULMINATION, STAR BEING NORTH OF THE ZENITH.

Party. Two men.

Equipment. Transit with reflector and watch which has the correct standard time.

Theory. The watch time of culmination is first found, and then the transit is set up and the observations made during five minutes before and after the time of culmination.

To find the time of culmination. If the right ascension of the mean sun at Greenwich mean noon, and the longitude of the point of observation are known, and if $\alpha_{\odot}$ = right ascension of mean sun at Greenwich mean noon, and L = longitude of the place of observation, then $\alpha_{\odot L}$ = right ascension of mean sun at mean noon at the place of observation, whose longitude is $L = \alpha_{\odot} + L(9.865 \text{ sec.})$, if west longitude. If α_s is the right ascension of the star, then $\alpha_{\odot L} - \alpha_s$ = sidereal interval of star's culmination before sun's transit, and the sidereal interval to next culmination (lower) = 12 hrs. $\pm (\alpha_{\odot L} - \alpha_s)$.

$$+\text{when } \alpha_{\odot L} < \alpha_s,$$
$$-\text{when } \alpha_{\odot L} > \alpha_s,$$

or, reducing to mean time interval,

$$= [12 \text{ hrs.} \pm (\alpha_{\odot L} - \alpha_s)] - \{[12 \text{ hrs.} \pm (\alpha_{\odot L} - \alpha_s)][9.865 \text{ sec.}]\}.$$

Then reduce the mean time to standard time by applying the difference between mean time and standard time at the place in question.

Method. Set up the transit at least 15 minutes before time of culmination, and carefully level the instrument by use of the long telescope bubble. Follow the star with the horizontal wire till about five minutes before culmination. Read the vertical circle. Reverse the instrument, level it and set on the star. Read the vertical circle. Continue this program for five minutes either side of culmination. Find the mean of the vertical-circle readings, and subtract the refraction correction. This gives the altitude of the star above the horizon. From the Nautical Almanac find the co-declination of the star

($\mp$ if at $\begin{smallmatrix}\text{upper}\\ \text{lower}\end{smallmatrix}$ culmination) and apply to the altitude. The result will be the latitude of the place.

Note. This method is to be used when the instrument reads only to single minutes. When a transit reading to 20″ or closer is used, the exact time of culmination is computed and the observation made at this time.

Remark. The latitude may be found by making an observation on Polaris when it crosses a true north and south line which has been previously determined.

Form of Notes. Both right- and left-hand pages of the field-book may be used in this problem. Give calculations for finding the time of culmination, and also the reduction for the latitude of the place.

Example. *To find the time of culmination of Polaris on July 16, 1906, at Philadelphia.*

Longitude = 5 h. 0 m. 39 s. W. of Greenwich.

$$\alpha_{\odot} = 7 \text{ h. } 29 \text{ m. } 43 \text{ s.}$$

and

$$\alpha_{\odot L} = 7 \text{ h. } 29 \text{ m. } 43 \text{ s.} + 5.01\ (9.865 \text{ s.})$$
$$= 7 \text{ h. } 30 \text{ m. } 32 \text{ s.}$$
$$\alpha_{s} = 1 \text{ h. } 25 \text{ m. } 35 \text{ s.}$$
$$\alpha_{\odot L} - \alpha_{s} = 6 \text{ h. } 04 \text{ m. } 57 \text{ s.}$$

12 hrs. $-(\alpha_{\odot L} - \alpha_s)$ = time of lower culmination = 5h. 55 m. 03 s.

Lower culmination, reduced to mean time, = 5 h. 55 m. 03 s. -5.92×9.865 s. = 5 h. 54 m. 05 s.

Standard time fast 39.0 s.

Standard time of lower culmination = 5 h. 53 m. 26 s. or 5.53.26 P.M., July 16, 1906.

Polaris will, of course, be invisible at this time, and if an observation is desired when it is at culmination, it must be taken 11 h. 59 m. 04 s. later when at its upper culmination.

PROBLEM H6. DETERMINATION OF LATITUDE WHEN THE SUN IS ON THE MERIDIAN.

Party. Two men.

Equipment. Transit with prismatic eyepiece and colored glass.

Theory. The local apparent time of the sun's transit is 12 M. The mean solar time, however, which is the time according to the motion of a fictitious sun, is different from the apparent time. Therefore a correction, called the equation of time, has to be applied to the apparent time to give the mean solar time of the sun's transit. But clock time is standard time, and standard time varies from 0 to 30 minutes from the mean solar time. Therefore find the difference of longitude in degrees between the place of observation and the standard meridian (for which the time of the place is correct). Divide by 15, and apply this correction to the mean local time to get the standard or clock time of transit. Then, knowing the standard time of the sun's transit, the instrument may be set up shortly before this time and the altitude of the sun determined. This altitude corrected for refraction, index error, and semi-diameter gives the true altitude of the sun's center. Then $90° -$ altitude $=$ the zenith distance, and the zenith distance $\pm$ sun's declination $=$ latitude of the place.

Method. Level the transit carefully by use of the long telescope bubble, and point at the sun so that the sun's image approaches the horizontal wire. Follow the sun until the watch (which has been set with a standard timepiece) registers within 2 or 3 minutes of the time of culmination. Make the cross-wire tangent and read the circle. Reverse the telescope and relevel the instrument. Again point on the sun. Make the horizontal wire tangent and read the circle. Both readings should be taken as near the clock time of culmination as possible, and in no case more than five minutes from this time.

Note. If a sextant and artificial horizon are used, the double altitude of the sun is obtained. The observation is made as near the exact time of culmination as possible. The index error should be applied to the measured angle, and the corrections for refraction and semi-diameter applied.

The above method gives a fair value of the latitude and may be used by the engineer in the field without tedious reductions for circummeridian altitudes. If a more exact value is needed, a treatise on astronomy should be consulted.

The notes appended were taken without regard to the mean local time of the sun's transit. The instrument was set up over one end of a line whose azimuth had been determined. The telescope was then pointed to the meridian, and the altitude of the lower limb of the sun determined to the nearest 20 seconds as it passed the vertical wire. The corrections were then applied and the latitude found.

OBSERVATION FOR LATITUDE.

Apparent alt.		69° 42′ 20′	Eagles Mere, Pa., May 30, 1906.
Index corr.	+	0° 20′	Index error = 0° 20′.
Sun's semi-dia.	+	15′ 51″	
Ref. corr.	−	0′ 24″	
Declination.		21° 42′ 25″	$\phi = 90° - (h + I + \text{semi-dia.} - r - \delta)$.
co-ϕ.		48° 35′ 22″	
ϕ		41° 24′ 38″	

CHAPTER IX.

SURVEYS.

PROBLEM 11. CONDUCTING A FARM SURVEY.

Party. Six to eight men.

Equipment. Transit, two range-poles, steel and cloth tapes, set of marking-pins, hatchet, stakes, and tacks.

Problem. *It is assumed that the corners of the farm are known and that the area and a plot are desired.*

Method. Place two stakes on or along a boundary-line so that they are intervisible. Set up over the second stake and find the magnetic bearing of the line and its length. Assume that this bearing is correct. Set a forward hub at or near another corner of the field. Measure the deflection angle to this point and compute the bearing of the line. Observe the magnetic bearing of the line as a check on the computed bearing. Measure the distance between the successive points. Set up at the succeeding station and set a forward station in a similar manner. At each station take deflection angles and distances to each property corner, or to any desired point, such as a fence-line, etc. The accuracy of the work depends upon the care taken in chaining and measuring the angles, provided that the instrument is carefully set up, centered, and leveled. In this connection the following points should be noted. A check on the angular work may be had by doubling all important angles, always reading the first angle before doubling. The measurement obtained by chaining may be checked by using a range-pole as a stadia-rod. If it is necessary to have a very large party on the work, two sets of chainmen may measure the main lines of the traverse. In case the boundary-line is irregular, offsets from the transit line are taken at frequent intervals. These offsets are taken at right angles to the transit line, the interval between successive offsets depending upon the regularity of the boundary-line in question.

Property Survey. If it is possible to run the traverse line on the boundary-lines themselves, the *corrected* bearings and lengths of the courses, found in the manner described below, will be the property lines of the farm. If, however, the traverse line does not form the boundary-line of the farm, then a description can only be written after the traverse is balanced, and the lines computed by trigonometrical methods. It is presumed that enough information has been taken to give a solution.

Computations. Compute the bearings or azimuths of the courses and obtain the latitude and longitude differences. Sum up the latitude and longitude differences and find the error of closure. Distribute the error by the proper rule for balancing. Having distributed the error, find the corrected bearings and lengths of the courses.

Compute the double longitudes of the courses and the double area.

FORM OF NOTES FOR FIELD WORK.

(Left-hand page.)

Sta.	Defl. Ang.		Dist.	Mag. Bearing.	Calc. Bearing.
	L.	R.			

(Right-hand page.)

Give sketches, date, party, equipment, etc., as usual.

COMPUTATIONS.

Course.	Bearing.	Length.	Long. Diff.		Lat. Diff.		Double Long.	Double Area.
			+	−	+	−		

The balanced latitude and longitude differences are placed in the proper columns in red ink. The computations may either be placed in the field-book or on the standard size paper used by the department.

Organization of the Farm Survey. Transitman, head chainman, rear chainman, stakeman, back flag, recorder, and two tapemen.

The members of the party perform duties very similar to those in the railroad survey (Problem 13). Stakes are set, however, only at transit stations. The two tapemen either take offsets to the irregular boundary or check the measurement of the traverse lines made by the regular chainmen.

PROBLEM 12. CONDUCTING A STUDENTS' HYDROGRAPHIC AND TOPOGRAPHIC SURVEY.

The methods described below have, for the most part, been used for many years on the Summer Survey of the University of Pennsylvania. The directions given may be used with slight modification on any similar survey.

The work of the survey may be divided into three parts:

A. The Triangulation System.

B. The Hydrographic Survey.

C. The Topographic Survey.

There is no sharp line of distinction between the different parts of the work.

A. The Triangulation System.

A triangulation system is used as the basis of the work. If it is deemed advisable the triangulation system may be omitted, and reliance placed on a single triangle, or on closed traverses with the stadia. In such a case, care should be taken that sufficient checks on the work are secured.

The triangulation may be divided into six parts, as follows:

1. The Reconnaissance and Location of the Triangulation Stations.
2. The Measurement of the Base-line.
3. The Determination of the Azimuth of the Base-line.
4. Measurement of the Angles.

5. Adjustment of the Angles; Computations of the Lengths of the Sides; Plotting.

6. Elevations of the Triangulation Stations.

1. The Reconnaissance and Location of the Triangulation Stations. Choose a suitable spot for the location of the base-line, and so select the triangulation stations that one line near the end of the system may be used as a check-base. Pick out the stations so that as many angles in the different quadrilaterals may be measured as possible. The best angles are those between 60° and 120°, and small angles should be avoided if possible. The reason for this is that the sides of the triangles are computed by use of the sine formula, and the sines of small angles change very rapidly.

The instruments which will be of service in the reconnaissance are a prismatic compass or pocket sextant, to give a rough value of the angles before actually setting the stakes, and a steel tape for referencing the points. Range-poles with flags should be taken on the reconnaissance, so that before the stakes are actually driven there may be no doubt as to the intervisibility of the points.

Heavy oak stakes, $2'' \times 2'' \times 8''$, may be used for the stations. After the stakes have been driven the triangulation tripods are placed over the points, and the range-poles or sights made plumb.

2. The Measurement of the Base-line. The principal base-line should be 2000 feet long if possible, and the site should be nearly level. The method of procedure is the same as that given in Problem G1. and the length of base is found as described in Problem G3. A check-base should be carefully measured and compared with the computed length.

3. The Determination of the Azimuth of the Base-line. The azimuth may be determined either by direct solar observations or by use of the solar attachment. Detailed directions for making the observations are given in Problems H3 and H4. The latitude may be obtained from any reliable map of the region, or by making an observation in the way described in Problem H5 or H6. The method of determining azimuth by use of the solar attachment will, as a rule, give the azimuth more quickly. The direct solar method, however, requires a less expensive equipment and would be more frequently used by the student after leaving college.

4. Measurement of the Angles. The form of sight used will depend on the size of the system. A cheap form of tripod can be made by using a triangular piece of wood about 7″ on a side, and with three legs fastened to it by hinges. A hole a little larger than the diameter of a range-pole allows the pole to be placed over the tack in the stake. The legs of the tripod are about 30″ long, and 4″ wide at top and 1½″ at the bottom. A tripod holding a range-pole is placed at each of the triangulation stations, and the rod is made plumb and secure. The angles of the system are measured in the way described in Problem D1, or, if plenty of time is available, according to the method in Problem G5. The form of notes is the same as in those problems.

Better results will be secured if one party measures all the angles of the system; but if the class is large this will, of course, be impossible. Time will be saved in the end if there are not more than three parties on the measurement of the angles.

Strict orders should be given that when a range-pole is off a stake it is to be laid on the ground, and when the party has completed the work at the station, the range-pole is to be carefully replaced.

5. Adjustment of the Angles; Computation of the Lengths of the Sides; Plotting. After the angles and the base-line have been measured and the azimuth of the latter determined, the angles in the different triangles are adjusted and the azimuths of the various lines of the system computed. The error in any triangle is distributed equally to each angle in the triangle, preference being given to the more important triangles in the system. Should the error in the measured angles of any triangle be greater than one minute, they should be remeasured.

Compute the lengths of the different sides of the triangulation system, starting with the measured base and using the adjusted angles. The sine formula is used. Whenever possible, compute a side through two triangles. Finally, compare the length of the check-base as found by computation with that found by actual measurement.

Having computed the lengths of the different sides of the triangulation system, find their latitude-and-longitude differences, and plot the system to a convenient scale by use of a plotting-table which has been previously prepared. The reference meridian is taken through the most westerly point in the system.

6. Elevations of the Triangulation Stations. Lines of differential levels should be run from the lake to the different triangulation stations. Finally, the absolute elevations of the stations are found. The lake surface may be assumed at a certain elevation, or if the elevation of a bench-mark with respect to mean sea-level is known, it will be preferable to use this elevation.

The method of procedure and form of notes are given in Problem B1.

B. The Hydrographic Survey.

The hydrographic survey consists of two parts:

1. Locating the Sounding-stations and the Shore-line.
2. Making the Soundings.

1. Locating the Sounding-stations and the Shore-line. When the triangulation stations are situated at some distance from the shore, additional stations should be located for use in connection with the sounding work. These stations may either be located by a stadia traverse connecting with the triangulation system, or by sights from two or more triangulation stations. They should be located so that good range-lines may be obtained. The shore-line may be located in several different ways. Two of the quicker ways are either by stadia from sounding or triangulation stations or by an angle read from two or more stations.

2. Making the Soundings. The soundings may be located in any one of the ways given in the text-books in surveying. A method used for some years at the University of Pennsylvania, is given below.

Range-lines are laid out nearly perpendicular to the course of the river, their distance apart depending upon the size and depth of the body of water. In general a distance of 200 to 400 feet will be best. The soundings are taken at distances not greater than 50 feet. The points are located by two transits on shore, set up at adjacent sounding-stations. As a check on the shore readings, the third angle in the triangle is read by a sextant in the boat. In addition, the boat is kept as nearly on the range-line as possible.

A code of signals should be adopted so that there will be no loss of time. A signal should be given at the moment a sounding is taken and the three angles observed. The time and depth is also noted. The watches of members of the party should be compared at the beginning and end of the work.

Soundings up to 15 feet may be made with a pole. For depths from 15 to 50 feet an ordinary clothes-line with an attached sash-weight may be used. The foot and five-foot points should be suitably designated.

Three note-books are used for the work, two on shore and one in the boat, and are designated as "Shore Angles A," "Shore Angles B," and "Soundings." The following form of notes for the first two books is used.

Station.	Range.	Time.	Angle.
Sounding-sta. *d*	IX		
	1 2	9.17	50° 40′

The form of notes for the "Soundings" book is as follows:

Range.	Time.	Depth.	Sextant.
IX 1 2	9.17	15.0	49° 58′

A complete description of all stations occupied, together with the method of numbering the ranges and soundings, should be given in *each* book. The points should be numbered consecutively. It is particularly important that all data with regard to this part of the work should be clear.

The notes should be plotted the following evening so that any work which fails to check may be repeated the following day.

C. The Stadia Survey.

A traverse is run, starting from one triangulation station and either closing on another or on some other point whose location has been previously determined. The method of field work and the form of notes are given in Problem D8. In addition to the notes of the traverse line, "side shots" are taken to any desired points, such as buildings, street-corners, and contour points.

Care should be taken in selecting points for stadia stations and contour points. A few contour points well chosen will be of much more service than many more taken thoughtlessly.

As explained in the Organization of the Survey of the University of Pennsylvania, given below, a map is drawn in the field in connection with the work, and errors are therefore quickly found. The area is divided into a certain number of parts and a stadia party placed on each part. Each party has a protractor field-sheet, on which there is located at least one triangulation station and the directions of several others.

The reduction to the horizontal and the difference of elevation are found by the use of either Colby's or Webb's slide-rules. A special sketch-book, such as Heany's or Turner's, may be used, the right-hand page having a protractor sheet upon which the contours and objects are roughly sketched. When a map is not made in the field, these sketches will be of great value.

In addition to the directions which were given in Problem D8, the following points should be noted: If true azimuth is used for the work, when the line of collimation passes through 360° or 0°, the bearing should nearly check with the known magnetic declination of the place. If the transit has adjustable stadia wires, they should be examined; and if the wires are fixed, a correction should be applied to all main readings, if the constant differs materially from 100. Particular care should be taken that the transit is level, and all main readings made before taking side shots.

The Finished Map. After the different protractor sheets are completed in pencil, they are assembled and pasted together in their proper order. The map is then inked in and the shoreline and triangulation system plotted. Finally, the map is cleaned, tinted, and a title made.

Organization of the Survey.

The organization given below has been used with success for the past two years at the University of Pennsylvania.

Stadia Party.

1. Transitman.
2. Recorder.
3. Rodman.
4. Topographer.
5. Rodman.
6. Computer.

Triangulation Party.	7. Transitman.
	8. Recorder.
	9. Tripod man.
Triangulation-station Party.	7*a*. Instrument man.
	8*a*. Tripod man.
	9*a*. Stakeman.
Base-line Party.	10. Transitman.
	11. Recorder.
	12. Rodman.
	13. Head Chainman.
	14. Rear Chainman.
	15. Temperature man.
	16. Temperature man.
	17. Stakeman.
	18. Levelman.
Sounding Party.	19*a*. Transitman.
	19*b*. Transitman.
	20*a*. Recorder.
	20*b*. Recorder.
	21. Sextant man.
	22. Recorder.
	23. Rodman.
	24. Sounding-man.
	25. Rower.
	26. Instructor's Oarsman.
Solar Observations.	27. Transitman.
	28. Recorder.
	27*a*. Transitman.
	28*a*. Recorder.
Level Party.	29. Levelman.
	30. Rodman.
	31. Note-keeper.

A brief outline of the duties of the different positions is given below. It is assumed that the reader has read the matter in the first part of this problem.

Stadia Party. Transitman. The transitman is in charge of the party, and he should endeavor to keep all the men busy to the best advantage. He does all instrument work, the more important parts being checked by the recorder. All information with regard to the traverse line should be taken before any side shots are made. Care should be taken in checking azimuths, distances, and vertical angles on the main line. The rodmen are instructed as to the points upon which sights are desired.

The Recorder assists the transitman and topographer in every way possible. He should be alert and not require the transitman to repeat the information taken. A sharp lookout is kept for errors. The note-book should be complete in every detail. The form of notes is similar to that given in Problem D8, and a special sketch-book similar to Turner's or Heany's may be used.

Rodmen. The extent of area covered depends to a large degree on the foresight of the rodmen. The transitman should not be kept waiting for sights. The forward rodman picks out the new stadia station. All side shots should be selected with a view to giving the maximum amount of information. In wooded country it will be necessary for the rodmen to also act as axemen.

Topographer. The method of plotting notes in the field presents many advantages over the old method. Errors are quickly discovered, and it is believed that there is a direct saving in time. A protractor sheet is provided, giving the location of at least one triangulation station and the directions of several others. The azimuths are laid off by use of the graduations on the protractor sheet and transferred by triangles to any desired part of the drawing. Great care must be taken in this work. If any appreciable error is made, it may result in a reconstruction of a large part of the sheet. The buildings and natural features are sketched in, and as soon as a sufficient number of contour points have been located, the contours are sketched in. By making the map in the field a better representation of the contours is secured.

The topographer provides his own drawing instruments.

Computer. The computer makes the reduction to the horizontal and finds the difference in elevation of the points and their absolute elevations. He should give attention to the signs of the vertical angles read by the transitman, and promptly call attention to any apparent error.

Some form of slide-rule may be used, such as Colby's or Webb's; but if a slide-rule is not available, a table from the back of one of the text-books in surveying may be used.

The computer should aid the progress of the survey in every way possible, and when going through a wooded country, where but few side shots are taken, he should assist in the clearing of underbrush.

The Triangulation Party. The Transitman does all instrumental work, the vernier readings, however, being checked by the recorder. The method of measuring the angles has been given in the first part of this problem.

Men may interchange positions sometime during the half-day, if it is thought advisable by the instructor.

The Recorder. The form of notes is given in Problem D1. The recorder assists the transitman and checks the vernier readings.

The tripod man is responsible for the placing of all tripods and sights. They should be carefully guarded and made plumb by use of a bob.

The Triangulation-station Party. The three men in this party have no fixed duties, but work together with a view to getting the best location for the triangulation stations. The use of the different instruments is given under The Reconnaissance and Location of the Triangulation Stations.

Base-line Party. The transitman lines the chainmen in from one end of the base-line. It is not necessary that the intermediate stakes be exactly in line, an error of from one to three inches being allowable.

The recorder notes the thermometer readings and the other data as outlined in Problem G1. He also, with other members of the party to be designated, makes the reductions in the way described in Problem G3. This is done either at the end of the field work for the half-day, or in the evening, as thought best by the instructor in charge of the party.

The rodman works with the levelman and gives readings on the stakes after they have been set and the marks scratched on the zinc strips. Care should be taken that the stakes are not disturbed in any way.

Chainmen and Stakeman. The chainmen, together with the stakeman, measure the length of the line. With the zero end of the tape on the first point, a stake is driven on line and practically flush with the ground at the 100-foot end. The tension should be approximately correct, so that when the exact point is marked it will come on the zinc strip which is tacked to the top of the stake. Finally, a scratch is made on the zinc strip with a sharp knife-edge.

At this instant the temperature men take readings at the 25- and 75-foot points. Care should be taken that as much friction is eliminated as possible, that the tape is straight, and that the tension is correct. Stakes are placed at every change of grade and their elevations obtained. It is not necessary, however, to make scratches on these stakes, and their pluses need be taken only to the nearest tenth of a foot.

Temperature Men. As previously noted, thermometer readings are taken at the 25- and 75-foot points. The thermometers should be placed near the tape and at all times kept away from the body. The duties of these positions are necessarily not arduous, and men should be alert in seeing that the work is not hindered.

The most uncertain correction in base-line work is the temperature correction, and if the greatest accuracy is desired, the work should be done on a cloudy day or in a mild rain, as the temperature at such times is nearly constant.

Levelman. The elevations of the different stakes are taken so that a correction may be applied for slope. Readings are made to hundredths, and the elevations and differences of elevation are obtained. These are transcribed to the base-line book at a convenient time. The form of notes conforms to that in profile leveling. The corrections which are applied to the measured length of the line are outlined in Problem G3.

Sounding Party. The soundings are taken in the way described in the first part of this problem.

Transitmen and Recorders. These four men are stationed on shore. The two transits are sighted at each other, with the zeros on the plates set and sights taken on the range-pole in the boat at the time of making the sounding. Readings need be taken only to the nearest five minutes, and accordingly, after the zeros have been set, the vernier need not be used. The form of notes has already been given. The time is noted first and then the angle is read.

Sextant Man and Recorder. The sextant man measures the angle between the two transit stations on shore at the moment the signal is given. The adjustments of the sextant should have been previously tested. The form of notes used by the recorder has already been given. The records are made in the following order: time, depth, sextant reading, and number of pointing in the range. The description of the shore stations and the range, with all other data, should be entered as soon as opportunity offers.

Rodman and Sounding-man. The rodman gives the signal when the sounding is to be taken and holds the range-pole for the line. The sounding-man notes the depth, care being taken that a vertical measurement is secured.

The Rower should maintain an even stroke and keep on the range line as nearly as possible.

Solar Observation Parties. Two parties are given in the organization, the first using the direct method and the second using the solar attachment. Detailed directions of the methods used in making the observations are given in Problems H3 and H4. These problems should be carefully studied in advance. The observations may be conducted from one end of the base-line, but if there are several parties auxiliary lines may be used connection being finally made to the base-line, so that the number of observations for the azimuth of this line may be increased.

Level Party. Levelman and Rodman. The elevations of the triangulation stations and stadia stations are desired. The work is that of differential leveling described in Problem B1, and the precautions noted there should be strictly observed. Unless directed to the contrary, readings will be taken by use of the target. In railroad profile leveling, however, an open rod should be used and readings taken to hundredths.

Note-keeper. The authors have found that less errors arise where there are three men in the level party. There is also a direct saving of time where there are three men in the party instead of two.

The note-keeper checks all rod-readings, particular care being taken that + and − sights are not interchanged, that bench-marks are fully referenced and described in the note-book, and that all the data mentioned in Problem B1 is given. The success of the work depends to a large degree on the way the notes are kept.

Miscellaneous Work. At different stages of the survey slight modifications in the organization of the parties may be desirable; but the organization as given is flexible, and positions may be omitted or combined as deemed advisable, a proper change in the equipment being made.

PROBLEM 13. METHOD OF CONDUCTING A RAILROAD SURVEY.

No fixed practice is common on American railways, but it is believed that the methods given below are those in most common use.

The different parts of a railroad survey are:

A. The Reconnaissance.

B. The Preliminary Survey.

C. The Location Survey.

A. The Reconnaissance Survey.

The reconnaissance survey is of an area rather than of a line. It consists of an examination of the country in order to determine which of several probable routes will justify the making of a preliminary survey.

An approximate value of the maximum grade is desired, and for this purpose the approximate length of line and the approximate elevations of points on the line must be determined. The general direction of the line is also desired. As a rule, only pocket instruments are used for this work. The prismatic compass gives the bearings of the courses; the pedometer, odometer, or some similar instrument gives the distances, and the aneroid barometer the elevations.

A large section of the New England and Middle States, as well as certain sections of the rest of the country, have been covered by the U. S. Geological Survey, and topographical maps have been issued. These may be obtained at a nominal cost from the Director of the Geological Survey, Washington, D. C. Information may be secured from him as to the section of the country already covered. For sections of the country covered by these maps a large part of the work of reconnaissance is precluded. Any other maps of the country which are available should also be brought into service.

The form of notes for the reconnaissance is as follows:

Sta.	Azimuth.	Distance.	Aneroid Reading.	Elevation.	Remarks.

The method of obtaining the elevations of points by use of the aneroid barometer is explained in Problem E1.

The right-hand page of the note-book contains sketches of the topographical features and data with regard to the contours. Side shots should be taken to points which would influence the location of the line.

The elevations are reduced the following night and a map drawn. This map is used in deciding where the preliminary surveys will be run.

B. The Preliminary Survey.

The preliminary survey may be made either by the two-instrument method or by the stadia method. The stadia method of making the survey is similar to that in which any stadia survey is made. The stadia method has not as yet been used to a very great extent. The two-instrument method will be used in this survey.

The two-instrument method gets its name from the principal instruments used, namely, the transit and the level. A transit line is located approximately where the final line is to come. Stakes are placed at intervals of one hundred feet, and hubs are placed at transit stations. The bearings and lengths of the courses are determined. The level party follows the transit party and secures a profile of the line. The elevations of the stations are used by the topography party in locating the contours.

The transit party by the aid of the level party keeps on ground which will give approximately the desired grade. Should it be found at any time that the line is either too high or too low, the party goes back and starts again at some convenient point.

A map and profile is made and the final line located, either by "paper location" or in the field by use of this map.

It should be here noted that the preliminary line consists of a series of broken lines and angles and is supposed to be near where the located line will come, but it *may be* several feet from the final line. This line is merely used as a reference-line for locating the contours, and by use of which the located line is found, giving supposedly the best combination of tangents, curves, and grades.

The form of notes for the transit work is as follows:

	Defl. Angle.		Mag. Bearing.	Calc. Bearing.	Remarks.
	L.	R.			

The right-hand page is used for sketching.

The level notes are the same as in profile leveling.

The contours may either be sketched in a field-book to a convenient scale, or on separate sheets of cross-section paper fastened to a small drawing-board.

The duties of the different positions on the survey are given under Organization of the Railroad Survey.

C. The Location Survey.

After the preliminary map is completed a "paper location" is made. The directions of the tangents are fixed and the probable degrees of curve decided upon. Offsets are taken from stations on the preliminary line to the new tangents, and these tangents are prolonged to intersection, the intersection angles measured and the curves located.

The *P.C.*'s and the *P.T.*'s of the curves are located from the *P.I.*, and stakes are placed at each even station, both on curves of less than 5° and on tangents. The common method of curve location (Problem F1) is usually used, but to lessen the amount of clearing, the methods outlined in Problem F3 may be used after the *P.C.* and *P.T.* of the curve have been set.

The level party follows the transit party and secures a profile of the line. The final grade-line is established and the elevations of grade at the different stations computed. The line is then "slope-staked," according to the method given in Problem F9.

Special surveys for the location of structures are then made. A compass or transit party, followed by a level and cross-section party, locates the watershed line so that the size of the waterways may be computed.

The forms of notes for the transit, level, and slope-stake work have already been given in Problems F1, B2, and F9. The duties of the members of the various parties are outlined below.

Organization of the Railroad Survey.

Transit Party.	R. 1. Transitman.
	R. 2. Head Chainman.
	R. 3. Rear Chainman.
	R. 4. Stakeman.
	R. 5. Back Flag.
	R. 6. Axeman.
	R. 7. Note-keeper.
	R. 8. Head Flag.
Level Party.	R. 9. Levelman.
	R. 10. Rodman.
	Note-keeper. (Similar to position 31 in other survey.)
Cross-section Party.	R. 11. Levelman.
	R. 12. Rodman.
	R. 13. Topographer.
	R. 14. Assistant.
Slope-stake Party.	R. 15. Levelman.
	R. 16. Rodman.
	R. 17. Tapeman.
Reconnaissance Party.	R. 18. Station man.
	R. 19. Compass man.
	R. 20. Pacer.
	R. 21. Office man.
	R. 22. Note-keeper.
	R. 23. Aneroid man.

A brief outline of the duties of the different positions will be given.

The Reconnaissance Party is given last because it is not advisable for any one man to spend more than a half-day or a day on this work, and, accordingly, the positions most frequently used are given first.

No fixed method of procedure can be given for the work of reconnaissance, but the arrangement of the party above will enable men to get more out of the work than if they simply go along without being required to do anything in particular. Two or more men may be assigned to all positions in this party, except the fourth and sixth. The reconnaissance of the area is supposed to have been done, and the work then consists in running a line with the hand instruments previously noted. It is desired to find the approximate grade of the line and to secure general information as to the country traversed.

The Station Men go ahead of the party and pick out the stations. A range-pole with a flag is used so that a sight can be taken by the compass man. The station men remain at the station until the rest of the party arrives.

The Compass Man reads the bearings or azimuths of the lines by use of the prismatic compass.

The Pacer finds the lengths of the lines between stations by pacing, the length of pace having been previously determined.

Office Man and Aneroid Man. The aneroid man takes readings at the different stations chosen by the station men. The aneroid is read in the office, before starting, in connection with the office aneroid, and again upon the return to headquarters. The office man notes the reading of the office aneroid at intervals of fifteen minutes until the party returns. The form of notes is given in Problem E1. The readings of the aneroid in the field are taken by the recorder in the regular notes. When the reductions for elevations are made, the method explained in Problem E1 is used, the elevations being placed in the field-book used by the office man.

Transit Party. The duties of the transit party in the preliminary and location surveys are substantially the same. The work does not progress as rapidly in the location survey as in the preliminary on account of the curve work.

The Transitman does all transit work. In the preliminary survey he lines in the chainmen and axemen, measures the angles, and notes the bearings of the lines as a check on the angular work. It is advisable to double all angles on the main line.

In wooded country the transitman also keeps the notes; but in open country, where more notes are taken, it may be advisable to have a recorder. The forms of notes for both the preliminary and location transit work are given in the first part of this problem. On the location the transitman and the other men in the party compute the curves. Every man in the party should carry a curve book.

The Head Chainman carries the forward end of the tape, and after line has been given by the transitman, a stake is driven by the stakeman. The tape should be held plumb and should be free from kinks. It is not customary on the preliminary survey to mark a point on the stake, but in location larger stakes are, as a rule, used, and the exact point is marked.

The Rear Chainman holds the zero end of the tape at the even station; or if at a plus station, the plus is held at the station, the 100-foot end being ahead. A plumb-bob is used when necessary. The tape should be held over the point and no pressure exerted on the stake. The rear chainman checks the numbering of the stations. On curve work he should stand on the outside of the curve.

The Stakeman drives and numbers the stakes. Reference-stakes are placed to the left of all transit stations on curves, and should give on the front of the stake the number of the station, whether hub is *P.C.* or *P.T.*, and on the back the degree of curve and its direction. In the preliminary survey the stakes are placed on the side to which the line deflects. All other stakes should be marked from top to bottom, and the numbering should be visible, coming from the zero end of the line. The letter *P* is placed in parenthesis on all preliminary stakes, and *L* in parenthesis on all location stakes. The *P.C.* and *P.T.* of each curve should be referenced from two or more stakes outside the excavation or embankment.

The Back Flag gives sights on all rear stations. The rod should be carefully plumbed for each sight by balancing it between the thumb and first two fingers of each hand. The transitman should not be kept waiting for sights.

The Axeman clears the line of brush as directed by the transitman. In wooded country two or more men act as axemen. In open country the position may be changed to that of either head flag or note-keeper.

The Note-keeper relieves the transitman of the duty of keeping notes. This position may be filled in open country, where progress is rapid, and omitted in a wooded district. See the outline of the duties of the transitman for the requirements of the position.

The Head Flag in the preliminary survey chooses the forward stations. The position may, as a rule, remain unfilled, except in open country, where it is essential that progress be very rapid.

The Level Party. The Levelman's and Rodman's duties are similar to those outlined in Problem B2. In all cases a return to the original B.M. should be made. An open rod is used, and readings are taken to hundredths on benches and turning-points and to tenths on intermediate stations. Suitable bench-marks should be established at intervals of about 1500 feet. Wherever possible on the location levels, readings should be taken on preliminary bench-marks. It will prove an advantage to have a note-keeper in the party as outlined in the preceding survey.

Cross-section Party. Contours are located by use of the Locke hand-level, rods, and tape, as outlined in Problem F8.

The Levelman, after finding the height of eye, stands at the station and sends the rodman out on one side until a five-foot contour is located. He then goes to this contour point and finds the next contour, and so on for any desired number. For instance, if the elevation of the station occupied is 512.8, the height of eye 5.4, and the 515 contour is desired, the rodman goes out so that the reading 3.2 is secured. The levelman then goes to this point and finds the next five-foot contour by use of the five-foot rod, the reading being zero if the ground rises as in this case. Points where the contours cross the center line are also found.

The Rodman holds the rod so that the desired reading may be obtained. He holds the zero end of the tape, the box and fifty-foot end being held by the levelman.

The Topographer sketches the contours to a convenient scale, either in a field-book or on cross-section paper. In addition to sketching the contours, it is also advisable to give the *total* distance from the center line to each contour point.

The Assistant's position will, as a rule, remain unfilled, except in rough country. He will assist in the measurement of distances and in clearing.

Slope-stake Party. The methods used by this party have been given in Problem F9. The surface elevations and grade elevations of the stations should be given and also the width of road-bed and side slopes.

The Levelman reads the rod (to tenths only) and keeps the notes. The form of notes is the same as that given in Problem F9. If care was taken in checking the location levels, the elevations of the stations may be assumed as correct, and a separate set-up may be made at each station, and the time necessary for running another line of levels thus saved.

The Rodman and Tapeman find the location of the slope-stakes in the way described in Problem F9. In student work it is better that the computations be made by these men rather than by the levelman. The levelman, however, records the data and gives the center cut or fill at the station and the grade rod.

APPENDIX.

ADJUSTMENTS OF THE ENGINEER'S TRANSIT AND LEVEL.

INTRODUCTION.

There is probably no other part of the subject of surveying which is so hard for a student to understand thoroughly as that of the adjustments of the principal instruments with which he has to deal. This difficulty can mostly be accounted for by the fact that he relies on memory for the correct methods and order of adjustment, and does not realize that they are founded on common-sense principles which may be illustrated by simple sketches. It has been the practice of the writer to have his students actually *make* nearly all the adjustments, rather than simply to *test* them without touching the screws; and to require a written report on the work together with the proper figures showing just what was done. That this is the better way to handle the subject there can be no doubt, as was evidenced by the experience of a former student during a summer vacation. This student was rodman for a much older man, and during the progress of the work there was occasion to adjust the instrument. The levelman was meeting with great difficulty in making the adjustments, and upon looking into the matter it was found that, not having a field book at hand, he was relying on his memory, and did not make the adjustments in the proper order. The student saw at once the cause of the trouble, and by the use of a sketch soon convinced his superior that he was wrong.

It is greatly to be deplored that there are still some engineers who do not have the "nerve" to adjust an instrument; who, if it gets out of adjustment, send it to an instrument-maker as soon as possible. One engineer whom the writer encountered said that he preferred to have an instrument put in adjustment and then let the screws rust in. Teachers should not be content to have their students simply *test* the adjustments, but should require each adjustment to be actually *made*. The following description is an endeavor to make the subject a simple one.

PRELIMINARIES.

It is assumed that the reader understands how to focus the eye-piece in order to prevent parallax, and how to focus the objective when sighting at an object; also that he knows how to level his instrument. In this connection the following points are helpful: Secure a firm set-up, and if the spot selected is sloping, place two legs of the instrument down hill. After bringing the plate into a position as nearly level as possible by changing the legs, place the bubble parallel to one pair of leveling screws and bring the bubble to the center, remembering that both thumbs turn out or both in, and that the bubble will go in the direction in which the left thumb moves. When the bubble is in the center, turn through 90° and level over the other set in a similar manner (this is not necessary in leveling a transit, as that instrument has two plate levels). Care should be taken that the screws do not bind, but they should be brought snug against the plate. Before proceeding to the adjustments a few definitions and fundamental principles will be given.

FUNDAMENTAL PRINCIPLES.

The Bubble Tube. Fig. 1. The spirit level consists of a glass tube nearly filled with ether or some other liquid, the remaining part of the tube being filled with vapor. The tube is bent or ground, the longitudinal section being the arc of a circle. The bubble being lighter than the liquid will always seek the highest part of the tube. When the bubble is in the middle of the tube, the line tangent to it is called the *axis of* *the bubble.* By the use of this simple device we are enabled to obtain a horizontal line. The top surface of the tube is

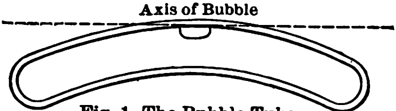


Fig. 1. The Bubble Tube

graduated, or else there is a scale above it so that we can determine when the bubble is in the center.

The Reticule. Fig. 2. The reticule is a brass ring upon which the cross-wires are fastened. These wires are either very fine platinum wires or spider lines, one being vertical and the other horizontal. (Two additional horizontal wires

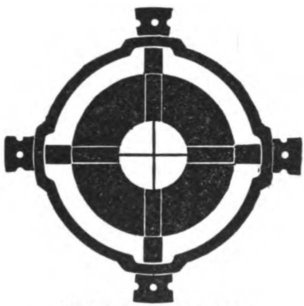


Fig. 2. The Reticule

are employed for stadia work.) This ring is centered in the barrel of the telescope by four capstan-headed screws, the screws being *threaded* into the *reticule ring.*

Line of Collimation. This is defined as the line joining the optical center of the object glass and the intersection of the cross-wires. The line of collimation is changed by shifting the reticule.

Vertical Axis. The vertical axis of the instrument is the line through the center of the spindle, and its position is changed by the leveling screws.

THE ADJUSTMENTS OF THE Y LEVEL.

A cross-section of the Y level showing the principal parts is given in Fig. 21 in Raymond's Surveying, and in Fig. 13 in Johnson's Surveying. It may also be found in almost any instrument maker's catalogue, and therefore will not be given here.

In the Y level, the telescope rests in the Y's, and can be taken from them by loosening and raising the clips. For all practical purposes we may consider that the telescope rests on two points, one at each end of the telescope, and this line is called the bottom line of the Y's. Then by reference to Fig. 3, the principal elementary lines of the Y level are:

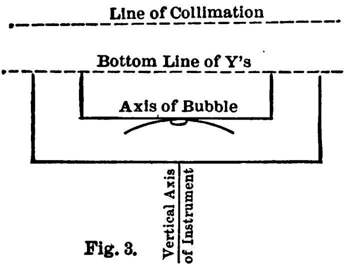


Fig. 3.

(1) the line of collimation; (2) the axis of the bubble; (3) the vertical axis of the instrument; and, for the purposes of adjustment, (4) the bottom line of the Y's; (5) a line joining the centers of the Y's, called the axis of the Y's. The adjustment of the Y level consists of the following:

I. To bring the axis of the bubble into the same plane as the axis of the telescope so that the bubble will remain in the center even though the telescope is turned slightly in the Y's

II. To make the line of collimation parallel to the axis of the bubble, in order that the line of collimation may be horizontal when the bubble is in the center.

III. To make the axis of the bubble perpendicular to the vertical axis of the instrument, in order that the bubble may remain in the center when the telescope is revolved around the vertical axis.

I. To bring the Axis of the Bubble into the same Plane as the Axis of the Telescope. Level up, loosen the clips, and turn the telescope through a small angle of 10° or 15°. If the bubble goes to one side, bring it *entirely* back by use of the capstan-headed screws on the side of the bubble tube. These screws are called the *azimuth* screws. Fig. 4 illustrates the

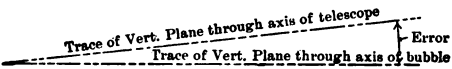


Fig. 4.

adjustment. This adjustment is not of very great importance with the later makes of instruments, as the telescope is not free to turn in the Y's, being held in place by a pin in one of the clips, which fits into a notch cut into one of the rings on the telescope.

II. To make the Line of Collimation Parallel to the Axis of the Bubble. This adjustment is made in two different ways: (1) by the *indirect* method, and (2) by the *direct* method.

(1) The *indirect* method consists first, of making the line of collimation (line 1) parallel to the bottom line of the Y's (line 4), in which case, assuming that the pivot rings which rest in the Y's are of equal diameter, in revolving the telescope in the Y's it will be the axis of a cylinder; but in case the pivot rings are not of the same diameter, due to wear, etc., it will be the axis of a cone, and will not be parallel to the bottom line of the Y's, and under these circumstances the method fails; and *second*, of making the axis of the bubble (line 2) parallel to the bottom line of the Y's. When these two things have been accomplished the line of collimation and the axis of the bubble will be parallel, since they are both parallel to the bottom line of the Y's.

To make the line of collimation parallel to the bottom line of the Y's. Set up the instrument and place a nail or mark a point on a fence or building at a distance of about 250 or 300 feet, at the point where the intersection of the cross-wires strikes. Unfasten the clips, and having clamped the horizontal motion, turn the telescope over in the Y's (not end for end), and find whether the intersection of the cross-wires still strikes the point or not. Suppose that it does not, and that the intersection with respect to the horizontal wire strikes on the point A, Fig. 5, and the point B after turning the telescope over in the

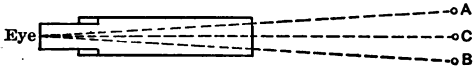


Fig. 5.

Y's. The correct position of the point of intersection is half-way between the two points, and the line of collimation must be brought to this point by changing the reticule. It is to be remembered that the reticule must be moved in a direction *opposite* to that which appears to be the right one, unless the telescope is an inverting one, as in the dumpy level and in some transits. In our case the reticule must be moved down (appears as if it must be moved up), and this is accomplished by loosening the upper capstan screw and tightening the lower one until the intersection of the wires strikes the point C half-way between A and B. To do this each screw must be turned so that the *near* side moves to the right, the upper nut being of necessity moved first in this case as we wish to lower the reticule. The adjustment with respect to the vertical wire is made in a similar way, the side screws being the ones considered. Each part must be tested until it is known to be correct. The adjustment with respect to the horizontal wire is the more important.

It should be appreciated by the student that unless care is exercised in handling the instrument in this and in the following adjustments, the errors which appear are due to his own unskilful use of the instrument, and are not due entirely to the instrument's being out of adjustment.

To make the axis of the bubble parallel to the bottom line of the Y's. Having brought the telescope exactly over a pair of leveling screws, loosen the clips, and clamp the horizontal motion. Bring the bubble to the exact center by the use of the leveling screws. Take the telescope out of the Y's, turn it end for end and replace it carefully. If the bubble runs to one end, that end of the tube is too high, and it must be lowered and the bubble brought half-way back. To do this the lower screw on the level tube must be loosened (by turning the near side to the left), and the upper one tightened. Bring the bubble the rest of the way back by the use of the leveling screws. Repeat the operation until it is correct. The screws at the top and bottom of the bubble tube are called the *altitude*

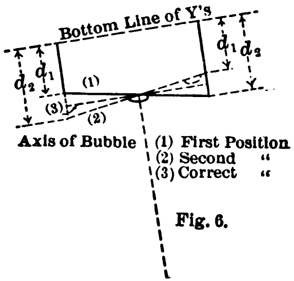


Fig. 6.

screws. Fig. 6 shows why the *apparent* error is twice the *real* error, and no further explanation is needed.

As previously stated, the above method of adjusting the line of collimation and the axis of the bubble depends upon the pivot rings being of the same diameter, in which case the line of collimation is the axis of a cylinder, and will be parallel to the bottom line of the Y's. In case they are not of the same diameter, this method fails, and the direct method must be employed.

(2) The *direct* method or "*peg adjustment.*" This method consists in adjusting the bubble directly to the line of sight. The indirect method is the one usually used with the Y level, but the "peg adjustment" must be used for the dumpy level and for the transit. This adjustment will accordingly be given with the transit adjustments entitled "*To make the axis of the telescope-bubble parallel to the line of collimation.*"

III. To make the Axis of the Bubble perpendicular to the Vertical Axis of the Instrument. The axis of the bubble has already been made parallel to the bottom line of the Y's, and the bottom line of the Y's must be made perpendicular to the vertical axis of the instrument. If this is done we can revolve the telescope around horizontally, and the bubble will remain in the center. The adjustment is simply one for convenience, as it does away with the necessity of releveling at every sight taken. The method of adjustment is as follows: Level the instrument over both sets of screws, and more carefully over the set where the telescope is left, and turn through 180° until over the same set of screws. If the bubble runs to one end bring it half-way back by the large capstan nuts at the bottom of the Y's, and the rest by the leveling screws. Remember that to lower one end of the line of Y's, and the end of the bubble at that Y, the upper screw must be *loosened* and

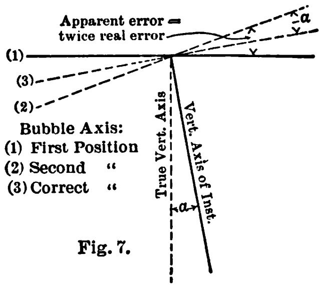


Fig. 7.

the lower one *tightened*. Fig. 7 shows the position of the different lines, and requires no particular explanation.

FIELD NOTES ON ADJUSTING THE LEVEL.

It is suggested that the student state in his own words just how each adjustment was made, using the number of the adjustments as given above. For I note the movement of the bubble when swung through a small angle, and how corrected. For II (a) record the amount that the line of collimation was out, illustrating by a sketch; and for the second part of this adjustment, the movement of bubble in divisions of scale, at each test, until entirely corrected. If the direct method, or "peg-adjustment," is used, illustrate by a suitable sketch, giving all rod readings, and corrections for adjusting. For III, note the movement of the bubble when telescope was revolved through 180°, and state how it was corrected.

The title of problem, equipment, party, etc., should be given as usual on the right-hand page of the note book. Both pages may be used for the above.

THE TRANSIT.

The elementary lines in the transit are: (1) the axes of the plate levels; (2) the vertical axis of the instrument; (3) the horizontal axis of the telescope; (4) the line of collimation; and (5) the axis of the telescope bubble. The adjustments consist of the following:

I. To make the axes of the plate levels perpendicular to the vertical axis of the instrument. II. To make the line of collimation revolve in a vertical plane when the telescope is turned on its horizontal axis. III. To make the axis of the telescope bubble parallel to the line of collimation. IV. To make the vernier of the vertical circle read zero when the line of collimation is horizontal.

I. To Make the Axes of the Plate Levels Perpendicular to the Vertical Axis of the Instrument. This adjustment is the same in principle as that in the level, of making the axis of the bubble perpendicular to the vertical axis, and is illustrated in Fig. 7. Bring both bubbles to the center, revolve 180°, and correct one-half the error by the leveling screws and the other half by raising or lowering one end of the bubble tube. The screws are turned in a direction previously described. Each bubble should be adjusted separately by placing it parallel to a pair of leveling screws. When this adjustment is made the bubble will remain in the center during a complete revolution, and the axis of revolution will be vertical.

II. To Make the Line of Collimation Revolve in a Vertical Plane when the Telescope is Turned on its Horizontal Axis. This adjustment consists of two parts: (*a*) To make the line of collimation perpendicular to the horizontal axis of the instrument; and (*b*) to make the horizontal axis of the telescope perpendicular to the vertical axis of the instrument.

(*a*) *To make the line of collimation perpendicular to the horizontal axis of the telescope.* When this condition exists, a straight line may be obtained by setting up over one point, sighting at another, plunging the telescope and lining in a point. The method of adjustment is as follows, referring to Fig. 8: Set the instrument up on nearly level ground, at a point O, where a clear sight may be had of a point A, a few hundred feet away, and bring the line of collimation to it. Plunge the telescope, and if the line of collimation is not perpendicular to the horizontal axis of the telescope, it will mark a point B. Revolve the telescope by unclamping the upper motion and sight again

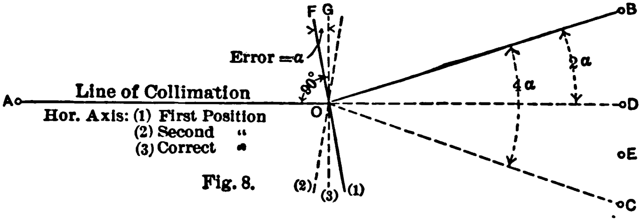


Fig. 8.

on the point A. Plunge the telescope and the line of collimation will now strike the point C. A point D, half-way between B and C, will be in the straight line AO. To *adjust* the line of collimation, the reticule is moved so that the cross-wires intersect on a point E, the distance CE being one quarter of the distance CB. It is easily seen by reference to Fig. 8 why the angle COB is four times the true error, the angle FOG.

(*b*) *To make the horizontal axis of the telescope perpendicular to the vertical axis of the instrument.* The adjustment is as follows, referring to Fig. 9: Set the instrument up firmly about twenty feet from a building, and level carefully. Select as high a point (represented by P) on the building as possible, and bring the intersection of the cross-wires to it. Swing the telescope down and set a point A near the base of the building. Plunge the telescope, revolve it through 180°, and again sight at the upper point. Depress the telescope and mark another point B. If the horizontal axis of the telescope is truly horizontal, the points A and B will coincide. If not, adjust for one-half the error by raising or lowering one end of the horizontal axis. Fig. 9 shows why the apparent error AB, is twice the real error AC, and needs no particular explanation. It is important that the intersection of the wires be set on the true position, which is half-way between A and B, and that the raising or lowering of the horizontal axis be done while pointing at the *upper* point. In the latter position a movement of the axis is easily seen by the changing position of the wires. If done with the telescope nearly level, no noticeable change is

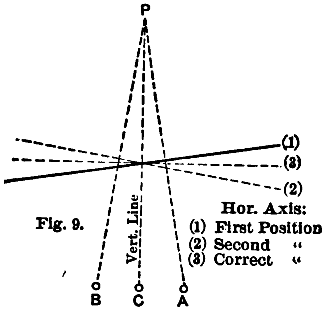


Fig. 9.

seen, and the correct amount of adjustment is not easily gauged. This adjustment is quite commonly called the adjustment for *height of standards*, and consists in making the horizontal axis of the telescope parallel to the horizontal plane through the axis of one of the plate levels, the axis of the plate level having previously been made perpendicular to the vertical axis of the instrument. This adjustment is sometimes made by comparing with a long plumbline, the weight of which rests in a pail of water.

III. To Make the Axis of the Telescope-Bubble Parallel to the Line of Collimation. When the adjustment is made, the instrument may be used as a level. There are two common methods of making this adjustment.

First Method. Fig. 10. Drive two stakes on nearly level ground, from 200 to 400 feet apart. Set the instrument up near one stake, so that when the rod is placed on it the eye-end of the telescope will swing within an inch of its face. Level the instrument and sight through the object-glass end of the telescope, and note the rod reading, calling it a. Then take a reading on the other stake, being careful that the bubble is in the center, and call this reading b. Then supposing that the line of collimation is not horizontal when the bubble is in the center, but is out by an angle α which makes an error e in the distance between the stakes. Then the difference in elevation of the two stakes is equal to $a-(b-e)$. Remove the instrument to a similar position near the other stake, repeat the observations, calling the readings a' and b', and the difference in elevation is now $(b'-e)-a'$. If the instrument is in adjustment, e becomes zero, and $(a-b)$ should equal

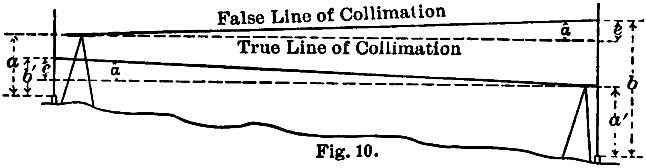


Fig. 10.

$(b'-a')$; but if it does not, equate the two values for the difference in elevation of the two stakes, and solving for e, we get $e=\frac{1}{2}\{(b'-a')-(a-b)\}$. Apply this with its correct sign to the last rod reading, bring the line of collimation to the center of the target by the use of the tangent screw on telescope, and then bring the bubble to the center of the tube by the use of the altitude screws as previously described. Repeat the observation until the work checks sufficiently well. In the figure given the line of collimation was inclined upwards.

Note. The level vial might have been left in the same position, and the correction made by changing the line of collimation by shifting the reticule.

Second Method. Fig. 11. Drive two stakes, A and B, at a distance of 300 to 400 feet apart, and on slightly rising

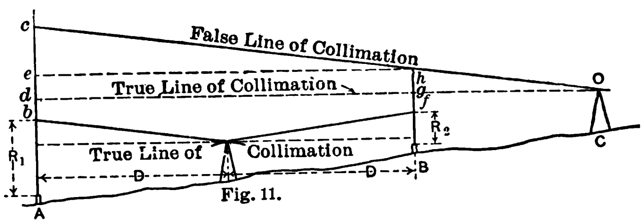


Fig. 11.

ground. Set the instrument up *exactly* half-way between them. Take rod readings on the two stakes, making sure that the bubble is exactly in the center at each sight. If these readings are designated by R_1 and R_2, then $(R_1 - R_2)$ will be the *true* difference in elevation of the stakes (unless the instrument is otherwise very badly out of adjustment), since the two stakes are equidistant from the instrument, and the bubble is in the center for each sight. Then take the instrument to a point C back of B and in line with A and B, making the distance BC some exact fraction of AB, say 50 ft. if the distance AB is 250 ft. With two leveling screws in the direction of the line AC, and the bubble in the center, take rod readings on B and A, which may be represented by B*h* and A*c*. The line O*d* will represent a horizontal line through the point O, and the correct position of the line of collimation, and *cd* will be the error in the distance CA. Draw the horizontal line *he* through *h*, and then *ce* is the error in the distance AB, which we know, since $(R_1 - R_2)$ is the true difference in elevation of the stakes, which, if the instrument is in adjustment, should be the same, no matter where the instrument is set up. $(Ac - Bh) - (R_1 - R_2)$ = the error in the distance AB, or *ce* in this case. As the instrument is now set up at O, what is wanted is the error *cd* in the distance AC. This is obtained from the similar triangles *hec* and O*dc*, by the proportion.

$$cd : ce :: Od : he\,(=BA), \quad \text{or} \quad cd = \frac{ce \times Od}{he}.$$

Apply this correction to the rod reading at the stake A, set the target and bring the line of collimation to it and the bubble to the center, as described in the first "peg-adjustment." The case represented is one where the line of collimation is inclined upwards; but when this line is inclined down $(R_1 - R_2)$ will be greater than $(Ac - hB)$, and the correction must be added to the rod reading at the point A.

Note. This second method is preferred by many engineers, as it is considered difficult to get the height of the cross-wires in the first method by sighting through the object-glass end of the telescope. This defect may be overcome by placing the instrument *back of each stake* at a distance of about eight feet (shortest distance for which the telescope can be focussed) and then taking the rod readings in the usual manner.

The first method has the advantage of requiring no measurement of distances. In the second, several readings should be taken on each stake, and the mean taken to counteract errors of setting the target. If permanent and unyielding monuments are set at the points A and B, and their difference of elevation accurately obtained, then to make the adjustment only the one set up at the point C is necessary.

IV. To Make the Vernier of the Vertical Circle read Zero when the Line of Collimation is Horizontal. Having made the axis of the telescope-bubble parallel to the line of collimation, bring the bubble to the center, and if the vernier of the vertical circle is adjustable bring it so that its zero coincides with that on the limb. If the vernier is not adjustable, note the *amount* and *sign* of the angular error, which is called the "index-error," and apply it to all vertical angles.

FIELD NOTES ON ADJUSTING THE TRANSIT.

See note on adjusting the level previously given. In adjustment I, note the error of adjustment in divisions of scale, and state how it was corrected. In II (*a*) give a sketch showing what the errors were and the distances between the stakes, and state how corrected. In II (*b*) state the error, the distance to the building (approx.) and the approximate height of the upper point above the lower point. For III, see the similar adjustment in the level. For IV note the amount and sign of the correction and state whether it was corrected or not. General data as in the level adjustment.

THE DUMPY LEVEL.

The main difference between the dumpy level and the Y level is that in the dumpy level the telescope cannot be taken from the Y's. There are then only the first three lines in the Y level. The two adjustments are as follows:

A. *Making the axis of the bubble perpendicular to the vertical axis of the instrument.* The principle of this adjustment is the same as II in the Y level, and is illustrated by Fig. 7. One-half the error is corrected in this case by raising or lowering one end of the level vial. (In the Y level, made by the large Y nuts).

B. *To make the line of collimation parallel to the axis of the bubble.* These lines must be compared directly with each other by use of either one of the peg tests (Figs. 10 and 11), as previously given, and the *line of collimation* must be moved to be parallel with the axis of the bubble.

TABLE I.

ELEVATIONS CORRESPONDING TO BAROMETRIC READINGS.*

B.	A.	Diff. for .01
Inches	Feet	Feet
18.0	13,918	−15.1
18.1	13,767	15.0
18.2	13,617	14.9
18.3	13,468	14.9
18.4	13,319	14.7
18.5	13,172	14.7
18.6	13,025	14.6
18.7	12,879	14.6
18.8	12,733	14.4
18.9	12,589	14.4
19.0	12,445	14.3
19.1	12,302	14.2
19.2	12,160	14.2
19.3	12,018	14.1
19.4	11,877	14.0
19.5	11,737	13.9
19.6	11,598	13.9
19.7	11,459	13.8
19.8	11,321	13.7
19.9	11,184	13.7
20.0	11,047	13.6
20.1	10,911	13.5
20.2	10,776	13.4
20.3	10,642	13.4
20.4	10,508	13.3
20.5	10,375	13.3
20.6	10,242	13.2
20.7	10,110	13.1
20.8	9,979	13.1
20.9	9,848	13.0
21.0	9,718	12.9
21.1	9,589	12.9
21.2	9,460	12.8
21.3	9,332	12.8
21.4	9,204	12.7
21.5	9,077	12.6
21.6	8,951	12.6
21.7	8,825	12.5
21.8	8,700	12.5
21.9	8,575	12.4
22.0	8,451	12.4
22.1	8,327	12.3
22.2	8,204	12.2
22.3	8,082	−12.2
22.4	7,960	

B.	A.	Diff. for .01.
Inches	Feet	Feet
22.4	7,960	−12.2
22.5	7,838	12.1
22.6	7,717	12.0
22.7	7,597	12.0
22.8	7,477	11.9
22.9	7,358	11.9
23.0	7,239	11.8
23.1	7,121	11.7
23.2	7,004	11.7
23.3	6,887	11.7
23.4	6,770	11.6
23.5	6,654	11.6
23.6	6,538	11.5
23.7	6,423	11.5
23.8	6,308	11.4
23.9	6,194	11.4
24.0	6,080	11.3
24.1	5,967	11.3
24.2	5,854	11.3
24.3	5,741	11.2
24.4	5,629	11.1
24.5	5,518	11.1
24.6	5,407	11.1
24.7	5,296	11.0
24.8	5,186	10.9
24.9	5,077	10.9
25.0	4,968	10.9
25.1	4,859	10.8
25.2	4,751	10.8
25.3	4,643	10.8
25.4	4,535	10.7
25.5	4,428	10.7
25.6	4,321	10.6
25.7	4,215	10.6
25.8	4,109	10.5
25.9	4,004	10.5
26.0	3,899	10.5
26.1	3,794	10.4
26.2	3,690	10.4
26.3	3,586	10.3
26.4	3,483	10.3
26.5	3,380	10.3
26.6	3,277	−10.2
26.7	3,175	

B.	A.	Diff. for .01.
Inches	Feet	Feet
26.7	3,175	−10.2
26.8	3,073	10.1
26.9	2,972	10.1
27.0	2,871	10.1
27.1	2,770	10.0
27.2	2,670	10.0
27.3	2,570	10.0
27.4	2,470	9.9
27.5	2,371	9.9
27.6	2,272	9.9
27.7	2,173	9.8
27.8	2,075	9.8
27.9	1,977	9.7
28.0	1,880	9.7
28.1	1,783	9.7
28.2	1,686	9.7
28.3	1,589	9.6
28.4	1,493	9.6
28.5	1,397	9.5
28.6	1,302	9.5
28.7	1,207	9.5
28.8	1,112	9.4
28.9	1,018	9.4
29.0	924	9.4
29.1	830	9.4
29.2	736	9.3
29.3	643	9.3
29.4	550	9.2
29.5	458	9.2
29.6	366	9.2
29.7	274	9.2
29.8	182	9.1
29.9	91	9.1
30.0	00	9.1
30.1	−91	9.0
30.2	181	9.0
30.3	271	9.0
30.4	361	9.0
30.5	451	8.9
30.6	540	8.9
30.7	629	8.8
30.8	717	8.8
30.9	805	−8.8
31.0	−893	

* Taken from Appendix 10, Report of U. S. Coast and Geodetic Survey for 1881.

TABLE II.

REDUCTION OF MERCURIAL COLUMN TO 32° F.*

Temp.	Inches.										
	26	26.5	27	27.5	28	28.5	29	29.5	30	30.5	31
45	−.038	−.039	−.040	−.041	−.041	−.042	−.043	−.044	−.044	−.045	−.046
46	.041	.042	.042	.043	.044	.045	.045	.046	.047	.048	.049
47	.043	.044	.045	.046	.046	.047	.048	.049	.050	.050	.052
48	.045	.046	.047	.048	.049	.050	.051	.051	.053	.053	.054
49	.048	.049	.050	.050	.051	.052	.053	.054	.055	.056	.057
50	.050	.051	.052	.053	.054	.055	.056	.057	.058	.059	.060
51	.052	.053	.054	.055	.056	.057	.058	.059	.060	.061	.063
52	.055	.056	.057	.058	.059	.060	.061	.062	.063	.064	.066
53	.057	.058	.059	.060	.061	.062	.064	.065	.066	.067	.068
54	.059	.060	.062	.063	.064	.065	.066	.067	.068	.070	.071
55	.062	.063	.064	.065	.066	.068	.069	.070	.071	.072	.074
56	.064	.065	.066	.068	.069	.070	.071	.073	.074	.075	.077
57	.067	.068	.069	070	.071	.073	.074	.075	.076	.078	.080
58	.069	.070	.071	.073	.074	.075	.076	.078	.079	080	.082
59	.071	.072	.074	.075	.076	.078	.079	.080	.082	.083	085
60	.074	.075	.076	077	.079	.080	.082	.083	.084	.086	.088
61	.076	.077	.078	.080	.081	.083	.084	.086	.087	.089	.091
62	.078	.079	.081	.082	.084	.085	.087	.088	.090	.091	.094
63	.080	.082	.083	.085	.086	.088	.089	.091	.092	.094	.096
64	.082	.084	.086	.087	.089	.090	.092	.094	.095	.097	.099
65	.085	.086	.088	.090	.091	.093	.095	.096	.098	.099	.102
66	.087	.089	.090	.092	.094	.095	.097	.099	.101	.102	.105
67	.089	.091	.093	.095	.096	.098	.100	.101	.103	.105	108
68	.092	.093	.095	.097	.099	.101	.102	.104	.106	.108	.110
69	.094	.096	.098	.099	.101	.103	.105	.107	109	.110	.113
70	.096	.098	.100	.102	.104	.106	.107	.109	.111	.113	.116
71	.099	.101	.102	.104	.106	.108	.110	.112	.114	.115	.119
72	.101	.103	.105	.107	.109	.111	.113	.115	.117	.118	.122
73	.103	.105	.107	.109	.111	.113	.115	.117	.119	.121	.124
74	.106	.108	.110	.112	.114	.116	.118	.121	.122	.124	.127
75	.108	.110	.112	.114	.116	.118	.120	.122	.125	.127	.130
76	.110	.112	.114	.117	.119	.121	.123	.125	.127	.129	.133
77	.113	.115	.117	.119	.121	.123	.126	.128	.130	.132	.136
78	.115	.117	.119	.121	.124	.126	.128	.130	.133	.135	.138
79	.117	.119	.122	.124	.126	.128	.131	.133	.135	.137	.141
80	.119	.122	124	.126	.129	.131	.133	.136	.138	.140	.144
81	.122	.124	.126	.129	.131	.133	.136	.138	.141	.143	.147
82	.124	.126	.129	.131	.134	.136	.138	.141	.143	.146	.149
83	.126	.129	.131	.134	.136	.139	.141	.143	.146	.148	.152
84	.129	.131	.134	.136	.139	.141	.144	.146	.148	.151	.155
85	.131	.134	.136	.139	.141	.144	.146	.149	151	.154	.158
86	.133	.136	.138	.141	.144	.146	.149	.151	.154	.156	161
87	.136	.138	.141	.143	.146	.149	.151	.154	156	.159	.163
88	.138	.141	.143	.146	.149	.151	.154	.156	.159	.162	.166
89	.140	.143	.146	.148	.151	.154	.156	.159	.162	.164	.169
90	.143	.145	.148	.151	.153	.156	.159	.162	.164	.167	.172

* Compiled from R. S. Williamson's "Use of the Barometer."

TABLE III.

CORRECTION COEFFICIENTS TO BAROMETRIC ELEVATIONS FOR TEMPERATURE AND HUMIDITY.*

$t+t'$.	C.	Diff. for 1°	$t+t'$.	C.	Diff. for 1°.	$t+t'$.	C.	Diff. for 1°.
0°	−0.1025	10.9	60°	−0.0380	10.7	120°	+0.0262	10.6
10°	− .0915	10.9	70°	− .0273	10.7	130°	+ .0368	10.4
20°	− .0806	10.8	80°	− .0166	10.8	140°	+ .0472	10.3
30°	− .0698	10.6	90°	− .0058	10.7	150	+ .0575	10.2
40	− .0592	10.6	100°	+ .0049	10.7	160°	+ .0677	10.2
50°	− .0486	10.6	110°	+ .0156	10.6	170°	+ .0779	10.0
60°	− .0380		120°	+ .0262		180°	+ .0879	

* Compiled from Tables I and IV, Report of U. S. Coast and Geodetic Survey for 1881.

TABLE IV.

MEAN REFRACTIONS (BESSEL).

TRUE FOR BAROMETER AT 30″; TEMPERATURE 50° F.

Alt.	Refr.	Alt.	Refr.	Alt.	Refr.	Alt.	Refr.
0° 00′	34′ 54″	6° 30′	7′ 54″	15° 00′	3′ 34″	27° 00′	1′ 54″
0 30	29 19	7 00	7 24	16 00	3 20	28 00	1 49
1 00	24 38	7 30	6 57	17 00	3 08	29 00	1 45
1 30	20 51	8 00	6 33	18 00	2 57	30 00	1 41
2 00	18 19	8 30	6 12	19 00	2 48	35 00	1 23
2 30	16 00	9 00	5 53	20 00	2 39	40 00	1 09
3 00	14 22	9 30	5 35	21 00	2 31	45 00	0 58
3 30	12 48	10 00	5 19	22 00	2 23	50 00	0 49
4 00	11 45	11 00	4 51	23 00	2 16	60 00	0 34
4 30	10 40	12 00	4 27	24 00	2 10	70 00	0 21
5 00	9 52	13 00	4 07	25 00	2 04	80 00	0 10
5 30	9 07	14 00	3 49	26 00	1 59	90 00	0 0
6 00	8 28						

TABLE V.

ERRORS IN AZIMUTH FOR ONE MINUTE ERROR IN DECLINATION OR LATITUDE.

No. of Hours from Noon.	For One Minute Error in Declination.			For One Minute Error in Latitude.		
	Lat. 30°.	Lat. 40°.	Lat. 50°.	Lat. 30°.	Lat. 40°.	Lat. 50°.
h. m.	Min.	Min.	Min.	Min.	Min.	Min.
0 30	8.85	10.00	12.90	8.77	9.92	11.80
1 00	4.46	5.05	6.01	4.33	4.87	5.80
2 00	2.31	2.61	3.11	2.00	2.26	2.70
3 00	1.63	1.85	2.20	1.15	1.30	1.56
4 00	1.34	1.51	1.80	0.67	0.75	0.90
5 00	1.20	1.35	1.61	0.31	0.35	0.37
6 00	1.15	1.30	1.56	0.00	0.00	0.00

TABLE VI.

REFRACTIONS IN TERMS OF LATITUDE, HOUR-ANGLE, AND SUN'S DECLINATION.

A TABLE OF MEAN REFRACTIONS IN DECLINATION TO BE USED WITH THE SOLAR ATTACHMENT.

Apply to the Declination as Found in the Ephemeris.

Declinations.

For Latitude 20°.

Hour-angle.	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
0 h	0″	05″	10″	15″	21″	27″	33″	40″	48″
2	03	07	13	18	24	30	36	44	52
3	06	13	18	24	30	36	44	52	1′ 02
4	17	22	28	35	42	50	1′ 00	1′ 11	1 26
5	39	47	57	1′ 07	1′ 20	1′ 37	2 00	2 32	3 25

For Latitude 30°.

Hour-angle.	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
0 h	10″	15″	21″	27″	33″	40″	48″	57″	1′ 08″
2	14	19	25	31	38	46	54	1′ 05	1 18
3	20	26	32	39	47	55	1′ 06	1 19	1 36
4	32	39	46	52	1′ 06	1′ 19	1 35	1 57	2 29
5	1′ 00	1′ 10	1′ 24	1′ 52	2 07	2 44	3 46	5 43	13 06

For Latitude 40°

Hour-angle.	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
0 h	21″	27″	33″	40″	48″	57″	1′ 08″	1′ 21″	1′ 39″
2	25	32	39	46	52	1′ 06	1 19	1 35	1 57
3	33	40	48	57	1′ 08	1 21	1 38	2 02	2 36
4	47	55	1′ 06	1′ 19	1 36	1 58	2 30	3 21	4 59
5	1′ 15	1′ 31	1 51	2 20	3 05	4 25	7 34	25 18	

For Latitude 50°

Hour-angle.	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
0 h	33″	40″	48″	57″	1′ 08″	1′ 21″	1′ 39″	2′ 02″	2′ 36″
2	38	46	55	1′ 06	1 18	1 35	1 57	2 28	3 19
3	47	56	1′ 06	1 19	1 36	2 29	2 31	3 23	5 02
4	1′ 02	1′ 14	1 29	1 48	2 16	2 58	4 18	6 59	19 47
5	1 30	1 51	2 19	3 04	4 22	7 28	24 10		

For Latitude 60°.

Hour-angle.	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
0 h	48″	57″	1′ 08″	1′ 21″	1′ 39″	2′ 02″	2′ 36″	3′ 33″	5′ 23″
2	54	1′ 04	1 17	1 33	1 54	2 24	3 12	4 38	8 15
3	1′ 03	1 15	1 30	1 51	2 20	3 04	4 24	7 31	24 44
4	1 18	1 34	1 56	2 28	3 18	4 50	8 53		
5	1 45	2 11	2 50	3 57	6 21	15 32			

TABLE VII.

AMOUNT AND VARIATION OF THE MAGNETIC NEEDLE FROM ITS MEAN DAILY POSITION.

The letters E and W indicate which side of the mean position the needle points.

Season and Position in Latitude.	Local Mean Time: Morning Hours.						
	6 h.	7 h.	8 h.	9 h.	10 h.	11 h.	12 h.
December, January, February:	Min.	Min.	Min.	Min.	Min.	Min.	Min.
Latitude 37° to 49°........	0.7 E	1.1 E	1.9 E	2.2 E	1.5 E	0.1 W	1.8 W
Latitude 25° to 37°........	0.1 W	0.1 E	1.0 E	2.0 E	2.2 E	1.1 E	0.5 W
March, April, May:							
Latitude 37° to 49°........	2.6 E	3.8 E	4.4 E	3.5 E	1.2 E	1.6 E	3.8 W
Latitude 25° to 37°........	1.6 E	2.8 E	3.3 E	2.6 E	1.1 E	0.6 W	1.9 W
June, July, August:							
Latitude 37° to 49°........	4.0 E	5.6 E	5.7 E	4.5 E	1.7 E	1.6 E	4.1 W
Latitude 25° to 37°........	2.4 E	4.0 E	4.2 E	2.9 E	0.5 E	1.6 W	2.8 W
September, October, November:							
Latitude 37° to 49°........	1.8 E	2.6 E	3.1 E	2.5 E	1.0 E	1.5 E	3.3 W
Latitude 25° to 37°........	0.9 E	2.1 E	2.6 E	2.1 E	0.6 E	0.9 E	2.1 W

Season and Position in Latitude.	Local Mean Time: Afternoon Hours.						
	0 h.	1 h.	2 h.	3 h.	4 h.	5 h.	6 h.
December, January February:	Min.	Min.	Min.	Min.	Min.	Min.	Min.
Latitude 37° to 49°........	1.8 W	2.9 W	2.8 W	2.1 W	1.3 W	0.7 W	0.2 W
Latitude 25° to 37°........	0.5 W	1.5 W	1.8 W	1.6 W	1.0 W	0.4 W	0.1 W
March, April, May:							
Latitude 37° to 49°........	3.8 W	4.8 W	4.6 W	3.8 W	2.5 W	1.4 W	0.7 W
Latitude 25° to 37°........	1.9 W	2.6 W	2.8 W	2.4 W	1.6 W	0.9 W	0.5 W
June, July, August:							
Latitude 37° to 49°........	4.1 W	5.6 W	5.6 W	4.6 W	3.0 W	1.4 W	0.6 W
Latitude 25° to 37°........	2.8 W	3.2 W	3.1 W	2.4 W	1.5 W	0.8 W	0.4 W
September, October, November:							
Latitude 37° to 49°........	3.3 W	4.0 W	3.4 W	2.3 W	1.2 W	0.6 W	0.1 W
Latitude 25° to 37°........	2.1 W	2.3 W	1.9 W	1.2 W	0.7 W	0.4 W	0.2 W

Table VIII.

REDUCTION OF STADIA READINGS

TO

HORIZONTAL DISTANCES

AND TO

DIFFERENCES OF ELEVATION.

This table was computed by Professor Arthur Winslow,
State Geologist of Missouri.

TABLE VIII.

STADIA REDUCTIONS FOR READING 100.

Minutes.	0°		1°		2°		3°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0′	100.00	.00	99.97	1.74	99.88	3.49	99.73	5.23
2	"	.06	"	1.80	99.87	3.55	99.72	5.28
4	"	.12	"	1.86	"	3.60	99.71	5.34
6	"	.17	99.96	1.92	"	3.66	"	5.40
8	"	.23	"	1.98	99.86	3.72	99.70	5.46
10	"	.29	"	2.04	"	3.78	99.69	5.52
12	"	.35	"	2.09	99.85	3.84	"	5.57
14	"	.41	99.95	2.15	"	3.90	99.68	5.63
16	"	.47	"	2.21	99.84	3.95	"	5.69
18	"	.52	"	2.27	"	4.01	99.67	5.75
20	"	.58	"	2.33	99.83	4.07	99.66	5.80
22	"	.64	99.94	2.38	"	4.13	"	5.86
24	"	.70	"	2.44	99.82	4.18	99.65	5.92
26	99.99	.76	"	2.50	"	4.24	99.64	5.98
28	"	.81	99.93	2.56	99.81	4.30	99.63	6.04
30	"	.87	"	2.62	"	4.36	"	6.09
32	"	.93	"	2.67	99.80	4.42	99.62	6.15
34	"	.99	"	2.73	"	4.48	"	6.21
36	"	1.05	99.92	2.79	99.79	4.53	99.61	6.27
38	"	1.11	"	2.85	"	4.59	99.60	6.33
40	"	1.16	"	2.91	99.78	4.65	99.59	6.38
42	"	1.22	99.91	2.97	"	4.71	"	6.44
44	99.98	1.28	"	3.02	99.77	4.76	99.58	6.50
46	"	1.34	99.90	3.08	"	4.82	99.57	6.56
48	"	1.40	"	3.14	99.76	4.88	99.56	6.61
50	"	1.45	"	3.20	"	4.94	"	6.67
52	"	1.51	99.89	3.26	99.75	4.99	99.55	6.73
54	"	1.57	"	3.31	99.74	5.05	99.54	6.78
56	99.97	1.63	"	3.37	"	5.11	99.53	6.84
58	"	1.69	99.88	3.43	99.73	5.17	99.52	6.90
60	"	1.74	"	3.49	"	5.23	99 51	6.96
$c+f=$ .75	.75	.01	.75	.02	.75	.03	.75	.05
$c+f=$ 1.00	1.00	.01	1.00	.03	1.00	.04	1.00	.06
$c+f=$ 1.25	1.25	.02	1.25	.03	1.25	.05	1.25	.08

TABLE VIII.

STADIA REDUCTIONS FOR READING 100.

Minutes.	4°		5°		6°		7°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2	"	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4	99.50	7.07	99.22	8.80	68.88	10.51	98.48	12.21
6	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.26
8	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43
14	"	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94
32	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22
42	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
$c+f=$ .75	.75	.06	.75	.07	.75	.08	.74	.10
$c+f=$ 1.00	1.00	.08	.99	.09	.99	.11	.99	.13
$c+f=$ 1.25	1.25	.10	1.24	.11	1.24	.14	1.24	.16

TABLE VIII.

STADIA REDUCTIONS FOR READING 100.

Minutes.	8°		9°		10°		11°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0′	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34
$c+f=$ .75	.74	.11	.74	.12	.74	.14	.73	.15
$c+f=1.00$	.99	.15	.99	.16	.98	.18	.98	.20
$c+f=1.25$	1.23	.18	1.23	.21	1.23	.23	1.22	.25

TABLE VIII.

STADIA REDUCTIONS FOR READING 100.

Minutes.	12°		13°		14°		15°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0′	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60
26	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15
48	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
$c+f=.75$	.73	.16	.73	.17	.73	.19	72	.20
$c+f=1.00$	.98	.22	.97	.23	.97	.25	.96	.27
$c+f=1.25$	1.22	.27	1.21	.29	1.21	.31	1.20	.34

TABLE VIII.

STADIA REDUCTIONS FOR READING 100.

Minutes.	16°		17°		18°		19°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0′	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2	92.37	26.55	91.42	28.01	90.42	29.44	89 36	30.83
4	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87
6	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14	92.19	26 84	91.22	28.30	90.21	29.72	89.15	31.10
16	92.15	26.89	91.19	28.34	90.18	29.76	89 11	31.15
18	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33
26	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56
36	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09
60	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
$c+f=$ .75	.72	.21	.72	.23	.71	.24	.71	.25
$c+f=1.00$	.96	.28	.95	.30	.95	.32	.94	.33
$c+f=1.25$	1.20	.36	1.19	.38	1.19	.40	1.18	.42

TABLE VIII.

STADIA REDUCTIONS FOR READING 100.

Minutes.	20°		21°		22°		23°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0′	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14	83.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34	87.66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38	87.58	32.98	86.41	34.27	85.19	35.52	83.93	36.73
40	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
$c+f=$ .75	.70	.26	.70	.27	.69	.29	.69	.30
$c+f=$ 1.00	.94	.35	.93	.37	.92	.38	.92	.40
$c+f=$ 1.25	1.17	.44	1.16	.46	1.15	.48	1.15	.50

TABLE VIII.

STADIA REDUCTIONS FOR READING 100.

Minutes.	24° Hor. Dist.	24° Diff. Elev.	25° Hor. Dist.	25° Diff. Elev.	26° Hor. Dist.	26° Diff. Elev.	27° Hor. Dist.	27° Diff. Elev.
0'	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92
30	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16
44	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19
46	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45
$c+f=.75$	.68	.31	.68	.32	.67	.33	.66	.35
$c+f=1.00$	.91	.41	.90	.43	.89	.45	.89	.46
$c+f=1.25$	1.14	.52	1.13	.54	1.12	.56	1.11	.58