POLE ON WHIST

FIFTH EDITION

THE

PHILOSOPHY OF WHIST

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THE

PHILOSOPHY OF WHIST

AN ESSAY ON THE SCIENTIFIC AND
INTELLECTUAL ASPECTS OF THE MODERN GAME

IN TWO PARTS

PART I
THE PHILOSOPHY OF WHIST PLAY

PART II
THE PHILOSOPHY OF WHIST PROBABILITIES

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"*Felix qui potuit rerum cognoscere causas.*"—VIRGIL.

"*Die ersten Gründe und Principien der Dinge aufzustellen ist doch wahrlich nichts leichtes.*"—HERDER.

FIFTH EDITION

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PREFACE.

In the face of the existence of such works as those of "Cavendish" and Clay, any new treatise professing to teach the complete Practice of Whist must certainly be superfluous.

This little work has no such pretension. It only aims at presenting what is already known under a new aspect, which may, as the author believes, bring out the character and merits of the Modern Game more prominently, and give it a higher reputation.

Taking the Art of Whist Play as set forth in the two works above named, it is proposed here to shew that this art is not, as most people imagine, a mere assemblage of detached empirical rules, but is, in reality, a compact and consistent logical system, of a highly intellectual and philosophical character; a carefully built-up structure, so to speak, remarkable not less for its unity of design than for its elaboration of detail.

There are, probably, many adepts in Whist-playing to whom this aspect of their favourite recreation may be novel, and who may be interested to examine the philosophical basis and the *raison d'être* of rules and maxims which they have been in the habit of practising as a matter of routine. It is not presumption to assume that by such an examination they may acquire, if not a surer command of their art, at least a higher estimation of its character

But it is in a humbler sphere that such a view of Whist Play is likely to be most useful—namely, as an aid to learners of the modern game, particularly in the earlier stages of their study. In the acquirement of all kinds of knowledge it is injudicious to offer to the beginner too much complexity of detail, which has the effect of discouraging him, and of making his progress difficult and irksome. His more advantageous course is to begin with simple and broad considerations, and then to pass on gradually to more intricate refinements.

Here, therefore, the philosophical view of Whist Play steps in most opportunely. The student will begin by making himself acquainted with the broad fundamental principles which underlie the whole system: following this up by the acquirement and practice of a few simple rules embodying their first development and most general applications; after which he will find himself in the most favourable position for perfecting himself in the art, by the study of the more comprehensive details to be found in "Cavendish" and Clay.

The philosophical analysis of Whist would be incomplete without a reference to the doctrine of Probabilities; and the fact that these probabilities enter so largely into the theory and the practice of Whist Play furnishes additional evidence of its scientific and intellectual character. The importance of this branch of the subject was well appreciated by Hoyle; but it seems afterwards to have dropped out of notice, or at least to have rested for a long period with only an occasional mention, more as a matter of curiosity than of utility. When, however, the recent consolidation of the structure of the Modern Game began to attract attention, it naturally led to a revival of the interest in the Probability questions: and for some years past such questions have been discussed, somewhat elaborately, by many Whist authorities, chiefly in the columns of *The Field*. The author has occasionally taken part in these discussions, and he has to thank the proprietor of that journal for the permission given him to reprint some of his papers. To these many additions have been made, and it is hoped that Part II. will be found, not only a novelty in Whist Literature, but a really useful example of the application of philosophical principles and processes, of a somewhat high order, to the theory and the practice of this noble game.

The Author has also to express his best acknowledgments to "Cavendish" for assistance kindly afforded to him in many ways.

ATHENÆUM CLUB, LONDON, S.W. *December*, 1883.

In the Fourth Edition, the whole has been revised, and some additions have been made.

The Author has again profited by the kind suggestions of "Cavendish," and he has also to thank Mr. W. H. Whitfield, of Trinity College, Cambridge, for a verification of the Table on p. 141, and for an important correction in the following chapter. Probability calculations are of a peculiarly subtle nature, and the notification of any inaccuracies that may be discovered will be esteemed a favour.

In compliance with a very general desire, there has been added a Rhymed Paraphrase of the "Elementary Rules."

September, 1886.

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PART I.

THE PHILOSOPHY
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PART I.

THE PHILOSOPHY OF WHIST PLAY.

CHAPTER I.

INTRODUCTORY.

It has often been remarked that the general popularity of Whist is strong evidence of its great merit as a game. And no doubt this popularity is very striking. In England, the place of its origin, it became popular in the best society immediately after it had taken a settled form, and during nearly a century and a-half that it has since been in course of development among British players, it has never lost its hold upon them; the love of it pervades all ranks and classes of people; numerous societies have been established expressly for its practice, and in nearly all the most important places of social rendezvous, both in the metropolis and in the provinces, it is played daily. But its popularity is not confined to this country. On the Continent it has become fully naturalized; the finest Whist-player that ever lived was a Frenchman, and some elaborate and thoughtful books on the subject are by foreign authors. It has, in fact, extended over the whole earth, and there is not a spot in which European civilization prevails, where Whist is not practised and prized. It is a singular but an unmistakeable testimony to the popularity of Whist, that in none of the books on the subject is it thought necessary to give any elementary description of the game; it is taken for granted that everybody, man, woman, or child, knows generally how to play Whist, the only object being to improve the mode of play; just as works on grammar assume a general knowledge of how to read and write, and only aim at improving the style of composition.

But many games somehow become popular which are but of indifferent merit; it is a much more certain proof of the high character of Whist that it forms a favourite recreation of persons of great intelligence, ability, and mental power. In domestic life, we generally find that the most thoughtful members of the circle practise the game with the most pleasure; but the evidence is much stronger if we notice the examples furnished every day, by the London Clubs, of the enthusiasm with which men of high culture devote themselves to the game. In a whist coterie at one of these institutions may be noticed cabinet ministers, ambassadors, peers, senators, statesmen, judges, magistrates, college professors, literary and scientific celebrities, and others of public reputation, who engage in the game with an earnestness that shows it is not an idle pastime, but a mental exercise in which they find real attraction. It is moreover a sign of the increased interest in the game, that the clubs are not now allowed to monopolise the good play, periodical afternoon meetings being organised for the practice of Whist in many private circles in the West End of London.

There must surely be something remarkable in the nature of a game which can offer such a wonderful interest for great minds—something far deeper than a mere application of a few superficial rules. The game must be essentially a *philosophical* one; it must possess features which afford a legitimate exercise for the reasoning powers, and bring into play the higher faculties of the mind.

It has seldom been viewed under this aspect, and it is the object of the present work to investigate it from this point of view. We shall endeavour to trace out what are the features which give the game of Whist such a high merit, and in what manner its practice furnishes such attractive elements of occupation for a high class of minds.

The enquiry will be essentially a philosophical one, and its results will embody what may fairly be called

THE PHILOSOPHY OF WHIST,

which accordingly has been chosen as our title.

It may perhaps be thought that the existence of the element of chance to such a large extent in

Whist tends to lower its intellectual character as compared with other games, such as chess, which engage only mental skill. Elaborate comparisons have been made of the interest offered in the two cases respectively, but the nature of the mental occupation is very different; and, so far as one can judge by experience and observation, no such inferiority exists. It should be understood that the element of chance, so far from standing in the way of intellectual exercise, is what chiefly gives the opportunity for it. It will be amply shown in the course of this work that the calculations, provisions, and speculations, arising out of the many uncertainties occurring in Whist-play, furnish the most important objects for scientific investigation, as well as the best inducements for the application of personal judgment and skill.

In reality, it is the happy combination of chance and skill that makes the game so generally attractive and popular. The influence of chance on the scoring is of two kinds—partly absolute, and partly dependent on the play. In the case of honours, the simple possession of certain cards counts directly towards game; but the result of a favourable chance distribution of cards for trick-making is largely dependent on skill in the management of them. It is very common for high cards to be lost, and it is one of the chief efforts of good players to make low cards win.

Complaint is sometimes made of the large influence of honours on success at Short Whist, and there have been proposals either to abolish the scoring of honours entirely, or to reduce their effect by counting two for the possession of four honours, and one for the possession of three. Such an alteration would give a great advantage to skill, but there is every probability it would damage the general popularity of the game, and would reduce the number of players. The love of speculation on chance events is undoubtedly a very strong natural impulse, and, accordingly, games of hazard are very common and often very attractive. It is the great merit of Whist that it embodies a large amount of this element of attraction, while at the same time it combines therewith ample scope for the exercise of the mind; and so far from the chance element being any drawback to the interest of the game, it is notorious that the best players are often those who most keenly enjoy the excitement it gives rise to. The vicissitudes caused by the variations of luck are in Whist almost as great as in games of pure chance; but still the effect of skill is perfectly substantial and real, and the cases are not unfrequent where a single trick won by good play may decide a rubber, and so influence materially the amount of loss or gain. The trial has often been made of reducing the value of the honours, but the game, deprived of the wonted sharp stimulus of the chance effects, has tended to become wearisome, and the change has never taken root, at least in this country.

It may be added that the restrictions intentionally introduced, such as the non-counting of honours at the score of four, and the preference given in all cases to the trick-score over the honour-score, give an increased importance to play, and often lead, in fact, to interesting and profitable applications of skill. Many a game, for example, has been cleverly won by tricks, when the opponents would have been out by honours if they could have scored them. All this suggests that the arrangements at present in vogue ought not to be lightly disturbed.

Turning now to the element of play, which is the only object of our study here, let us consider a little what the art of Whist-playing implies.

The game itself is of the very simplest construction; one of the simplest card games known. The pack, containing four suits of thirteen cards each, is distributed by chance among four players, two of whom are "partners" against a similar partnership of the other two. One player "leads" a card at pleasure, and the other players must, if they can, "follow" in the same suit, playing in regular order. The one who plays the highest card "wins the trick," and has to lead for the next one. One of the four suits, determined by hazard, is given a preference in value over the other three, and is called the "trump (triumph) suit"; and if a player is void of the suit led, he may, if he likes, play a "trump" which (if not over-trumped) will win the trick. The sole object of the play is to make tricks, all above six being scored to the side winning them; and an addition is made for certain high cards in trumps which may be held in the hands.

This is all; and it is no wonder that simple folk who have not studied the game should consider its play a very easy matter. But this is a delusion which it does not require much teaching to dispel. Let the four hands be exposed face upwards on the table, and let any one endeavour to find out, under that immensely simplifying condition, how to play the cards of one pair of hands to the best advantage in opposition to the other two hands. He will see that the cards admit of a great variety in the modes of treatment, and he will soon be brought up by perplexities and difficulties as to what is best to be done. This mode of play, by exposing all the cards, is not a mere fancy; it is a real game called "Double Dummy," and is often practised by good players, who know the difficulty of playing it well. Problems in Double Dummy are sometimes given as puzzles for solution, and one famed one especially, called the "Vienna Coup," is so difficult,* that any one who can solve it, even after long study, is considered deserving of great credit.

But the ordinary game is obviously far more difficult than this. The whole of the cards, except

* *See* CLAY'S "Treatise," p. 105. De La Rue & Co.

the player's own hand (*plus* the turn-up if he is not the dealer), are unknown to him, and for a considerable time he is absolutely in the dark as to what will be the result of his play, and whether it will turn out advantageous or disastrous to his side. If all the facts were before him, he would only have to exercise his judgment upon them; but now that so much is concealed, he has to take an entirely different line of action, and call to his aid a new set of considerations.

When all this is taken into account, the art of Whist-playing will immensely rise in estimation as an intellectual exercise.

It is worth while to take a brief review of what has been done to aid the acquisition of this art.

The Game of Whist was brought into its present form shortly before the middle of the last century, and a set of rules and directions for playing it were published by Hoyle in 1743. These were extremely good, but were intended chiefly to supplement personal instruction. They explain many interesting cases, and are remarkable for the attention paid in them to mathematical probabilities, in the treatment of which Hoyle appears to have been particularly skilled.

About 1800, Matthews's "Advice to the Young Whist-player" was published at Bath, where the game was much cultivated. The book was more comprehensive than Hoyle's, containing many improvements, resulting from the experience of half-a-century, and it is even now worthy of attentive study.

Whist continued to be practised, and the art of playing it became more and more developed. The introduction of Short Whist had made it more popular, and the long experience of adepts had led to the introduction of many improvements in detail; but nothing was done to bring the improvements into a systematic form, or to lay them clearly before the public, until about twenty years ago. Between 1850 and 1860 a knot of young men at Cambridge, of considerable ability, who had at first taken up Whist for amusement, found it offer such a field for intellectual study, that they continued its practice more systematically, with a view to its more complete investigation, and to the solution of difficult problems connected with it.* The little Whist School held together afterwards in London, and added to its numbers; and in 1862 one of its members brought out the original edition of the work now so well known under the name of "Cavendish."

This work gave, for the first time, a collection of the rules which constitute the art of Whist-playing according to the most modern and improved form of the game. It further stated briefly the grounds on which each rule was based, and it added a most useful feature entirely new to the art, namely, a number of illustrations of the application of the

* See several notices of their work in Part II.

rules by model hands, played throughout, with appropriate remarks and explanations. "Cavendish's" Treatise, in the enlarged and improved form of its later editions, may, it is believed, be accepted as the best authority on the practice of Modern Whist; particularly as its precepts have been corroborated, in a later work, by a player who has been admitted on all hands to hold the first rank in modern times—the late James Clay.

But the investigation of the art may be carried farther. In studying the structure of the modern game, as laid down by "Cavendish" and Clay, it has appeared to the author of the present work that a more comprehensive and extended basis would be found to underlie it than has generally been suspected; that, in fact, the rules of play, although often in appearance isolated and distinct, might be traced back to certain broad principles of origin, of a highly philosophical nature, and which, when clearly traced out, would give the game a new aspect and a more dignified character.

This view of the art of Whist play it is the object of the present work to explain.

Now, as an introduction to our further investigations, let us enquire, with some care, what is the nature of the mental processes which are comprehended in the play of a Whist hand?

The object of the play is to make tricks; sometimes the aim is to secure a certain definite number (required to win or save the game); at other times it is simply to gain as many as possible. What then has the player to do in order to accomplish these ends?

He finds himself holding a certain chance distribution of cards, in which, however, there will probably be very few, perhaps none, that can be relied on for trick-winning. It will therefore be evident that his chance of attaining his object will depend much more on the relative positions of the cards in the four hands respectively, than on the contents of his own hand taken alone. If the results of the play of a hand be examined, it will be found that out of the seven or more tricks made by one side, the majority have been won by means that could not, in the first instance, have been anticipated by either of the players.

Hence, at the commencement of a hand, the player is in the difficult position that he is in entire ignorance of what would be really the most advantageous thing for him to do. His first steps must be purely tentative, made blindfold; and (as a great French writer has justly remarked) whatever he does, most commonly proves afterwards to be *not* the best thing possible.

Untaught players are hopelessly helpless in this situation; they will either begin by playing out at once any master cards they hold (which is often ultimately very disadvantageous), or they will adopt some other plan, founded on very shallow motives, or perhaps dictated by pure caprice.

But the educated player pursues a line of policy based on higher grounds. He knows it is no rare thing, in the ordinary affairs of life, to be called on to take action in matters which are to a certain extent obscure; where, in fact, the ultimate results of his action may be as doubtful and unknown to him as that of the play of the first card of a Whist hand. But if he is a wise man he will not act at random. Although he cannot *foresee* what is the best thing to be done, he may at least, by careful consideration, form a judgment of what is *probably* the best thing to be done, when all the chances of what may occur are duly weighed. And if he follows this course, though his action may not succeed as he expects, he will have the satisfaction of knowing that he has made the attempt in the way most likely to conduce to success.

So with the early play of a hand at Whist. Although the player cannot *foresee* what will be the result of his play of particular cards, he may at least form a judgment of what may *probably* be the most likely course to be beneficial to himself and his partner, and the least likely to turn to the advantage of the adversaries. This invocation of *probability*, as a guide in obscure parts of the play, is what formed the great merit of Hoyle's teaching at the very birth of the game; and it is what now distinguishes the educated systematic player from the untaught beginner, or the more presumptuous (but not less ignorant) pretender, who glories in playing according to his own fancy.

It will hereafter be our business to show how the element of probability is applied to form rules of play in this first or tentative phase of the game.

But the state of things we have been considering does not last long. No sooner does the play begin than information is at once communicated as to the contents of the various hands, by means of what is termed the *fall of the cards.* In every trick the player sees three cards fall besides his own; and if he is duly observant, and is well acquainted with the ordinary modes of play, he will be able to draw, from each of these cards, inferences, more or less definite, as to where certain other cards do or do not lie. This knowledge increases rapidly, and by the time the hand is half played out, it generally happens that most of the important facts respecting the unplayed cards have become revealed.

No sooner does such knowledge dawn upon the player, than his position begins to change, and his course of action changes accordingly. For a long time, perhaps nearly to the end of the hand, he is not altogether relieved from the necessity of considering probabilities, but every item of information improves his means of judging what is best to be done, and in many cases his course, even when only a few facts have been made known, becomes clearly defined. At a later stage, when the hand is nearly exhausted, usually arrives the phase of the highest character—namely, that where personal skill and judgment are chiefly called into action. By that time opportunity will have been given for the knowledge of all the most important facts, and the capability of the player will be put to the test;—in the first place as to whether he has drawn the proper inferences from the data afforded him, and secondly, whether he is clever enough to take the best advantage of them.

It will thus be seen what a constantly varying demand is made on the attention and the skill of a player. Deschapelles, the great French writer, has a fanciful way of illustrating this: he likens the progress of a hand at Whist to the parabolic path of a shell thrown from a mortar, the seventh trick forming the apex of the curve. During the first half, corresponding to the rise of the projectile, the play is tentative, and the player is acquiring information, which in the latter or descending portion he has to apply. Or, more accurately, the information resembles the effect of gravity on the projectile, which, being zero at first, increases every moment till it brings the shell to the ground.

The exposition above given of the nature of Whist play will probably suffice to account for the great charm it presents to persons of high intellectual culture, and to justify its character as an eminently *philosophical* game. And there is no doubt that if the true nature of the attractiveness of Whist were more generally understood, the pleasure of playing it would be partaken of by great numbers of intelligent persons, who at present join in it without any clear understanding of what they are doing, and so lose its interest altogether.

It is our object in the following pages to spread this knowledge, and we propose to treat the subject in the following manner.

The great feature of the philosophical view of modern Whist is that the system of play is entirely based on *one broad fundamental principle*, which it will be therefore necessary, in the first instance, to enunciate and explain.

This being understood, we shall proceed to show how, by philosophical reasoning, this principle is applied to form practical rules for the conduct of the play. And it will have been made clear by the foregoing description that there are, in every hand, three distinct problems, or conditions of action, in regard to which guidance is required. These are—

The *tentative play*, which has to be followed at the beginning of the hand, while the player knows little or nothing as to any cards except his own.

The *means of acquiring information* as to the position of other cards, and the contents of the other hands.

The *modes of play determined by the information* thus obtained.

The first of these gradually merges into the third as the play proceeds; but as the two conditions involve different data and different motives of action, it is convenient to treat them separately.

CHAPTER II.

THE FUNDAMENTAL PRINCIPLE OF MODERN WHIST PLAY.

Our first duty is to explain and illustrate the great principle which underlies the entire structure of modern Whist play, and governs the whole of its rules.

The principle is one which, although most obvious and incontestable, has been unaccountably ignored, to a large extent, in Whist practice generally. It is founded on the relations existing between the players. According to the constitution of the game, the four players have not each an independent position—they are combined in pairs, two of them being what are termed *partners* against the other two, who are similarly combined in their opposition. It follows naturally from this arrangement that the interest of any one player is inseparably bound up with that of his partner, any advantages gained by either being common to both.

Now, there are two ways in which a partnership may be conducted, as we may see by a reference to the ordinary affairs of life. Take, for example, an ordinary business firm of two partners. Either the two persons may each conduct an entirely separate concern, merely putting their gains together: or they may both combine in the same establishment, each aiding the other in the joint work of carrying it on. There can be little doubt that, except perhaps in some special cases, the latter must for many reasons be the most advantageous, as it is by far the most general plan.

Or take another example of co-operation—namely, the alliance of two powers in warfare. They might act separately, each being guided by its own independent judgment; but this would be quite an exceptional proceeding. The ordinary way is to combine their forces and to act in concert. And no one can doubt that the junction and co-operation of two divisions of an army would give more powerful results than could be obtained by their divided action. It was the policy of Napoleon in 1815, according to the maxim *divide et impera*, to prevent such junction and co-operation, and had he succeeded there would have been no victory for us at Waterloo.

Applying this analogy to Whist, there may be two ways of treating the partnership relation. In the first place, each partner may play independently, in the manner which seems most suitable to his own hand—*i.e.*, he may endeavour to make as many tricks as he can by his own cards, leaving his partner to do the same, without reference to any idea of combination. This is the most primitive and simple view of the matter, and it is the one followed by players generally who have not studied in the modern school.

But it does not require much consideration to perceive that it is not the best view. In other alliances "Union is Strength;" it is advantageous for each partner to *aid* the other, and the principle is capable of application with most eminent advantage in the play of a Whist hand.

Examples of the benefit of this kind of co-operation will easily suggest themselves. One partner may have good cards in plain suits, while the other may have good trumps to enable him to make them; one partner may have several low cards of a long suit, and the other a few high ones to enable him to "establish" it; one partner may be short of a suit, and the other may be able to lead it to him to ruff; and so on in many ways.

In any of these cases the advantage of co-operation over isolated play is clear; but indeed such examples are hardly necessary, the proposition is so self-evident; and we may easily conceive that if the principle could be carried out to its full extent by each player seeing his partner's cards, so that *the two hands could be combined and played as one*, the advantage would be a maximum.

But it will be said that the condition of the player seeing his partner's cards cannot be carried out; and it may be reasonably asked, How, in the absence of this apparently essential condition, can the advantages of the co-operation be attained? The answer involves the key to the whole philosophy of modern Whist play. It is true, we may say to the inquirer;—at the commencement of the hand you know nothing, or next to nothing, of your partner's cards; but in the absence of information, you must *play in such a way as is likely to be best adapted to further the interests of the combination* when it may become known. And this can be easily done, seeing that philosophical reasoning enables us to devise rules expressly for this object. And these rules will have a treble purpose: they will be adapted not only to enable you to help your partner, but also to enable him to help you, and to enable you both to obstruct the opponents.

The nature and rationale of these rules will be hereafter explained; meanwhile it must be fully understood that the first and main principle which determines the precepts and practice of modern Whist is that of the *combination of the hands.* The modern game aims at carrying out this principle to the furthest extent possible. It forbids the player to consider his own hand apart from that of his partner, commanding him to treat both in strict union, and to make every step conducive to the joint interests of the pair.

Simple and obvious as this principle appears, it is only very lately, after a long-extended series of tentative approximations, that it has become fully recognised. The fact of the community of interests was of course always patent; but many of the earlier rules were either antagonistic to, or at least imperfectly fitted for, efficient combined play; and the tendency of the latest improvements has been either to abolish or to modify these, so as to make the combination of the hands the ruling principle—the great basis from which the whole play springs.

CHAPTER III.

THE TENTATIVE STAGE OF PLAY AT THE COMMENCEMENT OF THE HAND.

It will now be shown how the principle enunciated in the last chapter is carried out, so as to guide the player in the conduct of the hand. And it is necessary to begin with the condition which has been called the *tentative stage*—that is, the earliest phase of play, when the player knows little or nothing as to what the other hands contain. Here therefore the guidance given him must be in the shape of rules, so framed as to be applicable to the only data he has before him—namely, the cards of his own hand.

There are some very common mistakes as to the rules of Whist play, and it is as well to expose these at the outset. A certain class of persons, among whom we often find players of considerable skill, consider that the play of a hand is entirely a matter of individual judgment. It would be so if the player had full data to act upon, and towards the end of a hand, when the positions of the cards may be pretty well known, these persons often play very well. But they forget that during a large portion of the hand no sufficient data exist for judgment, properly so called, to be exercised upon, and what they under such circumstances call judgment must be really only caprice or fancy. Hence, at the beginning of a game such players often do things which, though they may sometimes accidentally turn out well, are more likely to do great mischief. In the absence of data the proper course is clearly to put all individual fancies aside, and to follow the rules which are dictated by philosophical reasoning as likely to do the most good, or the least harm.

Another class are content to accept rules of play, but they treat them as entirely matters of empirical authority. Their only idea of the value of a rule is, either that it has been published in some book, or has been given out by some person considered to be a good player. But there are many writings on Whist which are altogether untrustworthy and worse than useless, and there are many so-called good players who are but doubtful authorities on matters of theory. And even of the rules emanating from the highest quarters, such as the manuals of "Cavendish" or the dicta of Clay, it should be understood that they owe their chief value, not to the mere opinions of these eminent persons, but to the fact that they admit of being demonstrated by philosophical reasoning. It is quite true that many of the most valuable items of the Whist-code have been suggested by long experience, as collected and interpreted by men of acute penetration, and accurately-thinking minds;

but this is not of itself sufficient to warrant their reception without due enquiry. It is the spirit of the present age to "prove all things," and within the last few years the rules of Whist-playing have been submitted, like other branches of knowledge, to the test of strict philosophical investigation, and it is only by their passing this test that they can be implicitly relied on.

Then there is another class of players, who, in their confident reliance on the rules of play, misunderstand their nature by carrying them too far, and applying them to circumstances for which they are not suitable. Such rules, for example, as are based on the want of knowledge of the positions of the cards, obviously lose their applicability when these positions are known; and there are many other cases where the conditions that have given rise to the rule disappear, and where therefore the rule itself is no longer binding. Hence a good player ought to be acquainted not only with the rules themselves, but also with the reasoning on which they are founded, in order that he may be able to judge when they are not applicable as well as when they are.

It is for this reason that a philosophical investigation of the theory of the game becomes so advantageous, and indeed so essential to the development of really intelligent action. A mere superficial acquaintance with the verbal rules will, no doubt, be an advantage, as it will in many cases determine sound play; but it will never give that complete command over the infinitely varying situations of the game which will result from a knowledge of the *rationale* and the derivation of each formula.

Coming now to the deduction of rules for the play of the cards at the commencement of the hand, it will be evident, according to what has been said before, that the canon to direct their construction is—

That they must be of such a nature as will, MOST PROBABLY, *tend to carry out the grand principle of the combination of the hands of the player and his partner.*

Now, the way in which your play may further this object is two-fold; it may either help your partner, or it may give him information as to how he may help you. Either of these is a legitimate object to strive after; if one cannot be obtained, the other may be, and in many cases both may be combined.

Then again, some explanation may be given as to what is here meant by *probability*, and how it is to be determined. The most literal reference is to the mathematical determination of the probabilities of certain events happening, according to the "doctrine of chances," and this plays a large part in the formation of Whist rules and the solution of Whist problems, as we shall have occasion to see hereafter.

But this is by no means the whole of what is intended here. When we speak of an event being "probable," or of one event being "more probable than another," we also contemplate a mode of determination less technical and elaborate, namely, the exercise of logical reasoning. For there are many cases where mathematical calculation is not wanted, but where a simple logical discussion of the conditions will suffice to lead to the determination in view. It has been a bane of Whist writings that the authors have so frequently been unskilled in logical deduction, substituting for it mere dogmatical assertions of their own, or at most pretended conclusions which have no reasonable justification.

In the following investigation, both the mathematical and the logical modes of determining probabilities will be made use of.

THE OPENING LEAD.

The first rule a player has to learn is that which directs his opening lead. This is a most important step, for it not only directly influences the future fate of his own cards, but it is the step which is relied on as conveying the earliest and most comprehensive information to his partner. For this reason the determination of the opening lead must be well considered. A great many rules for first leads have been given by Whist writers at different times, varying materially from one another, and the result has been a good deal of confusion on the matter in the minds of imperfectly-taught players.

Let us consider a little what a player may do who finds himself obliged to lead, and has no guide but the cards he sees in his own hand.

The object of Whist play being to make tricks, an untaught player would be inclined to set about this as quickly as possible, in the most obvious ways; either by playing out any *master cards* he might hold, or in another mode very tempting to beginners, *i.e.*, by playing out a *single card* of a suit (if he had one) in order to trump the suit when anybody else led it again. But it will be shown hereafter that neither of these methods of proceeding is consistent with the policy of the combination principle, and the educated player will accordingly adopt another system, the advantages of which, though less immediate and obvious, are more certain and more comprehensive.

We must, for the present, exclude the contingency that the leader's hand may be strong in trumps, as that case will be the subject of future discussion. We will accordingly suppose that it contains only one, two, or three trumps. It will follow that among the other or "plain" suits there will be at least one of four or five or more cards. Such a suit is called a *long* suit, from its containing more than the average number of cards, and it has an inherent capability of trick-making which is very striking and important. To illustrate this, let us take an easy example:—Suppose I hold ace, king, and five small hearts, each other player having two. If I get the lead, and trumps are out, I can draw all the adversaries' hearts with my ace and king, and then all my five others, however small they are, will make tricks. Or, to vary the case, suppose I hold the knave and six small hearts, and suppose I have led small ones twice, which have brought out the ace, king, and queen, leaving, say, the ten in an adversary's hand. My long suit is then said to be "established," and if I can get the lead, I can "bring it in," as it is called, and may make tricks, not only with the knave, but with the three small ones remaining. It is easy to see from this what a great power a "long suit" may become, and although the cases cited are peculiarly favourable, the principle is the same in all. With even the least favourable case possible, namely four small cards, one will not unfrequently make a trick by virtue of the "long suit" capability.

Now the method of opening which has by far the greatest probability of furthering the combination of the hands, is by *leading the long suit*, with the ultimate view of establishing it and bringing it in. Hence the rule for plain suits—

Let your first lead be from the most numerous suit in your hand, or at least from a suit of not less than four cards. If you have two of the same number, you will naturally choose that consisting of the highest or "strongest" cards.

It must not be supposed that this is a mere dictum ; it is the object of the present work to show the philosophical foundation of every rule, and this rule is justifiable on the following grounds :—

1. The object aimed at, namely, that of causing small cards to make tricks, is in itself a very advantageous one; particularly as every trick so made may probably kill trick-making cards of the adversaries.

2. According to this rule, your first lead conveys of itself direct and positive information to your partner as to what is the most important component of your hand;—what is the chief object you are aiming at, and where you want help. And thus, by means of only one card shown, you at once set the combination principle in efficient action.

3. Neither the playing out of master cards from short suits, nor the lead of a single card, could be of any significance in calling for your partner's help; they could not by any reasonable or proper means be made to convey intelligible information to him which he could usefully act on.

4. Master cards in short suits may be safely allowed to wait. They will be almost sure to make to some one else's lead, and will *then* bring much more advantage than if led at first by the holder, as they may probably kill high cards of the adversaries, and they will give their holder additional leads at a later period, when such leads may be very valuable.

5. Trumping a short suit, if desired, generally comes about of itself more advantageously than by leading the single card, which of itself is, on independent grounds, a disadvantageous lead; it may kill a good card of your partner's without any compensating benefit to him or to you, and as the

probabilities are in favour of its establishing an adversary's suit, it would be playing their game.

6. It is urged against the long suit system, that the object aimed at more frequently fails than succeeds. This is true, as success usually requires not only the perfect co-operation of the partner, but also a fortunate arrangement of the cards. But the argument is worth nothing unless some disadvantage arises from the attempt if unsuccessful. This is quite the reverse of the fact; for (*a*) if the attempt fails, it does not stand in the way of the full realization of any other advantages the hand may possess; and (*b*) the system is so constituted as to do the least possible harm to either of the players using it, or good to their opponents; and indeed it offers generally the best obstructive tactics against the opposite party.

7. The long suit lead is almost always practicable. Leads on other principles are not. For example, you may have no master cards to lead out at once for trick-making, and no single card to lead out for trumping Some old authors recommend first leads from sequences, and other writers, more modern, from combinations which will leave tenaces to be led up to. But you may have no such cards in your hand. Hence all these fail in giving any definite information to your partner. Whereas it very rarely happens that you have not a long plain suit, and consequently your *invite*, as the French call it, to your partner is uniform and unmistakeable.

WHAT CARD OF THE LONG SUIT SHOULD BE LED.

Having shown that, as a matter of principle, the first lead in plain suits should be from the most numerous suit, there still remains a very important question—namely, *What card* of the suit should be led? This requires some consideration; but the philosophy of it is very clear, and can be easily explained.

The object being to *bring in* the suit, and so to make the small cards in it, the lead, on general principles, should be regulated with a view to the perfect *establishment* of the suit as early as possible. It would be easy to give rules with this view, and they would point generally to holding back the high cards. To take an instance: suppose you hold ace, king, five, four, three, two of hearts; if you were to play out the ace and king to begin with, there would be a high card still left in against you on the third round. The more proper way to establish it promptly would be to begin with a small card; it would then be an even chance that your partner might win the first trick, and, even if he did not, you could afford to lose it, seeing that afterwards, by leading your ace and king, you would probably clear everything away; or, in other words, by that play your suit might become established after the first round.

But a further consideration comes in which must not be lost sight of. The above reasoning assumes that you *will* bring in your long suit; but you must not forget that your design may fail from the strength in trumps being against you. And if this should happen in the above-mentioned case, the lead of a small card in the first instance would not be judicious. It would be better to lead out the ace and king at once. and make tricks with them while you can.

Now, considering that, as has been already said, the design of bringing in a long suit fails oftener than it succeeds, our philosophy will tell us that, although we are justified in making the attempt, we are *not* justified in doing it in such a way as to sacrifice tricks if we fail. Hence we get this rule—

In deciding what card to lead from the long suit, regard must be paid not only to the establishment of it, but also to the possibility of making tricks in it early, in case it should not be possible ultimately to bring it in.

On this maxim the ordinary rules for first leads in plain suits have been devised. They will be found fully explained in "Cavendish;" but it is our duty here to show briefly the logical basis of each.

As the consideration which determines the early lead of high cards is the chance of their being trumped if delayed, a preliminary question has to be philosophically solved—namely, what is the probability of a suit "going round" a certain number of times? This will vary according to the number the leader holds, and the mathematical process of solution will be found in Part II. of this work, Chapter IX. The result is given in the following table:—

PROBABLE NUMBER OF TIMES IN 100 LEADS THAT ANY SUIT WILL GO ROUND, THE LEADER HOLDING A GIVEN NUMBER OF CARDS OF THE SUIT.*

Number of cards of the suit held by the leader.........	4	5	6	7	8
	Times in 100 Leads.				
The suit will not go round once	4	8	12	20	31
It will go round once only...	28	40	54	66	69
,, ,, twice only..	57	52	34	14	0
,, ,, three times.	11	0	0	0	0
	100	100	100	100	100
Or, in other words, it will go round once or more...	96	92	88	80	69
Twice or more..................	68	52	34	14	0
Three times.....................	11	0	0	0	0

N.B.—It must be borne in mind that about once in three times when any player fails, it will be the leader's *partner*.

* These calculations suppose the cards to be *perfectly shuffled*, but this condition is often absent in practice; and the effect of insufficient shuffling is to make the suits go round *more frequently* than theory gives.

This may be easily applied to the most important cases of the first leads from hands containing high cards.

(*a*) *Ace and three small ones.*—Here there is no sufficient reason why the ace should be at once played out. The probability of its being trumped the second round is not great, and the general principle of holding it back may be safely preferred.

(*b*) *Ace and four small ones.*—Here the probability of the ace being lost second round is greater, and the general rule is, accordingly, to begin with it.

(*c*) *Ace, king, and two or more small ones.*—Here the advisability of leading out the ace and king is so evident as to need no demonstration.

(*d*) *King, queen, and two or more small ones.*—This case is not so clear as the last. Untaught players often lead a small one with the object, as they hope, of getting the ace out of the way. But this is a most illogical fancy. It is two to one that the ace lies with one of the adversaries. If it is with the second player, he ought not to play it if he has another card; and if it is with the fourth player, he will not play it unless forced to do so by a high card of the third hand. Besides, if it should come out the first round, it is improbable that king and queen will both pass in the second and third rounds without being trumped. Philosophy, therefore, dictates a bolder measure—*i.e.*, to play out the king; if your partner holds the ace, as will happen about once in three times, *cadit quæstio;* if an adversary holds it you will lose the king, but you will be much more certain of making at least one trick in the suit than by any other mode of play.

There is a pretty philosophical variation of this if you hold king, queen, and knave, with two others. Here, without losing any chance of trick-making, it is your interest to make an additional effort towards the *establishment* of the suit. For this purpose you want the ace early out of your way. If your partner holds it, he will not, if he can help it, put it on your king led; and you consequently lead the *knave*, which may induce him to do so, as, being so strong, you can afford to allow the two honours to fall together.

With the same object, if you hold king, queen, knave, and ten, you lead the *ten*.

(*e*) *Queen, knave, ten, and one or more small cards.*—Here there is no immediate probability of making a trick, and you may consequently adhere to the establishment policy. But there is an argument against beginning with a small one—namely, that by such a course, if your partner does not happen to have a good card, then a low card of the adversary's may win the trick, which would be a pity. You, having three high cards, can afford to guard against this by leading one of them (the queen), which is accordingly the usual rule. Your queen will force out one of the superior honours, and your knave another (if both are against you), and on the third round your suit will probably be established with the ten at the head.*

(*f*) *King, knave, ten, and one or more others.*—Same reasoning: begin with the ten.

(*g*) *Ace, queen, knave, and a small one.*—Here you combine the early trick-making with the establishment policy, by leading first the ace and then the queen, which will draw the king, if against you. If you hold *two* or more small ones, it is customary to lead the *knave* after the ace, instead of the queen, which should induce your partner to put on the king, if he has it, and so get it out of your way.

The above are the usual cases of special plain suit leads. In all others, you adhere to your establishment policy by opening with a small card. This is what the French call *l'invite; i.e.*, inviting your partner to show what he can do to aid you in the suit. The card led may either be the lowest (which may suffice for the learner), or it may be determined on the conventional *American* system, as described on page 56.

LEADING TRUMPS.

Hitherto we have ignored the trump suit, and treated of plain suits only; but the trump suit is so exceedingly important, that the philosophical

* For the case of *knave, ten, nine, and small cards*, see Part II., Chapter XXII.

principles of its management must now be taken carefully into consideration.

And we find at once a powerful light thrown on this subject by the results we have arrived at in regard to the opening lead. It is explained there that the object to be aimed at in the play of the hand is the bringing in of long suits; but the great obstacle to this is the chance of their being trumped by the enemy; and it naturally follows that it is the policy of the long-suit holder to get, if possible, his adversaries' trumps out of the way. Each party knows this very well, and consequently it often happens that the preliminary battle of the hand is fought on the trump suit, one of the parties endeavouring, by leading trumps, to disarm the other party and so to leave the field clear for the long suits to come in.

We have then to enquire, What do the laws of probabilities tell us as to leading trumps? One thing we may take for granted—namely, that in this battle, as in others, fortune favours *les gros bataillons:* the party who is strongest in trumps will be most likely to prevail, and what we have to enquire is, *How can we estimate the amount of strength* which will warrant an attack by a trump lead? This question we will proceed to discuss.

In the first place, it does not require much thinking to convince us that for this purpose *numerical* predominance in trumps is more efficient than high cards. If, for example, I hold five small trumps and my partner two, one of the adversaries possessing the four honours, four leads from our side will exhaust the enemy, and I shall have one trump left to enable me to bring in my own or my partner's long suit, which is the object to be attained.

Although, therefore, it would be absurd to say that high rank in trumps is no benefit, or is to be ignored, yet for our present purpose it is more material to consider the numerical strength only, and ascertain what amount in this will justify a trump lead.

And it must be observed that we must confine attention to the numerical strength in *one* hand, not in the two jointly; for the former determines the number of rounds, and consequently the exhausting power.

We must now call in mathematical probabilities to our aid, and the modes of calculating these are given in Part II., Chapter VII.

Having given the number of trumps held by one player, it can be found what is the probability of certain numbers being held by other players; and Tables, illustrating this, will be found on pages 134-5. It will be seen that the distribution depends largely on the position of the deal; but the general result may be expressed as follows:—

(1) If I happen to hold as many trumps as *seven* or more, the predominance is absolute.

(2) If I hold *six*, it is only about once in 100 times that any one else will hold an equal number.

(3) If I hold *five*, it is, in the worst case—*i.e.*, with an adversary's deal, 7 to 1 against his holding five or more; with the deal in my favour, it is 16 to 1 against it. Hence the predominance is so probable, that (barring any specially manifested reason to the contrary) we arrive at the rule that—

A trump lead is advisable when you hold five or more.

(4) If I hold *four* trumps, with an adversary's deal, it is an even chance that he may hold four or more; and if the deal is in my favour, he may do so about once in three times. Hence the rule that—

A lead of trumps from four must be made with great caution, being only justified by special inducements in regard to other cards.

(5) With *less than four* trumps, you evidently cannot exhaust them; but a lead of trumps from numerical weakness is sometimes made in the hope that the partner may have a good number. The probabilities affecting this will be seen from the tables above mentioned.

Say, for example, you hold only one trump: if your partner has had the deal, it is nearly an even chance that he will hold five trumps or more, and if you have good plain suits, you may be justified in taking the risk of the trump lead—particularly if your single trump is a tolerably high one, as it will strengthen your partner, and be in no danger of misleading him. This curiously justifies an instinct often shown by good players to lead a single trump, if it is a tolerably high one, and if they have otherwise good cards.

But if your partner has *not* dealt, or if you have more trumps than one, the tables show that your trump lead would be probably disadvantageous, or would at least only be warranted by very exceptional circumstances.

The element of the deal ought always to be considered in doubtful cases, as either favouring a lead of trumps if your partner is the dealer, or deterring from it if the deal has been with the adversaries.

It is sometimes urged that since the lead of trumps is intended for the purpose of bringing in long suits, or of protecting high cards, it ought not to be prescribed universally, but only on the condition precedent of the possession, by yourself or your partner, of good plain suits also. This precaution, however, in ordinary play is often impracticable; for the trump lead, to be efficacious, ought to be prompt, and to delay it till you have found out the contents of your partner's hand might probably defeat its object. But the precaution is, in reality, unnecessary. Suppose you have five trumps and *no* good cards; you may fairly presume on the probability that your partner will have something good in his hand; and if he has not, it is doubtful whether any other lead might not be more disadvantageous for you. It is impossible to reduce the probabilities of this case to mathematical calculation, but general experience, as represented by the opinion of the best authorities, appears to warrant the expediency of a trump lead from five or more, without waiting for the consideration of the other contents of the two hands. At any rate, the cases of disadvantage are so rare that they may be ignored in the expression of the general rule.

The question as to *what cards* should be led from the trump suit is a complicated one, involving too much detail to be explained here. The general maxim, however, regulating it is *to endeavour to establish the suit, and so to gain the perfect command of it, as early as you can.*

As a simple general rule, unless you have at least three very high cards, you begin with a small one. (See remark on *American Leads*, p. 56.)

FORCING.

It will be shown in Chapter V. that if your partner has manifested strength in trumps, your duty is carefully to avoid forcing him to trump a trick in a suit of which he is void, as this might do him great injury.

But a doubtful case often occurs. Suppose you find your partner void of a suit at an early period, before he has been able to give any clear intimation of what his hand contains; ought you then to force him if you get the lead? No doubt the temptation to do so is strong; too strong usually for inconsiderate players to resist. But much harm may be done thereby, and it is necessary to consider what course philosophical reasoning dictates on the point.

In the first place, if you are numerically very strong in trumps yourself, it is obviously good to force him before you lead trumps, and so to allow him to make tricks with his trumps instead of having them drawn. And if you are only moderately strong, say with four, no great harm can be done. Hence—

If numerically strong in trumps yourself, force your partner.

But suppose you are *not* numerically strong, say you have only three, or two, or one; then you should consider that the fact of his having renounced one plain suit increases the probability of his being strong in trumps, and as a matter of prudence, it is a general rule not to force your partner under these circumstances—*i.e.*,

If numerically weak in trumps yourself, refrain from forcing your partner until you are satisfied that he is not strong, and has no desire to lead them.

TRUMPING DOUBTFUL TRICKS.

Another question often arises whether, being second player, you ought to ruff what is called a "doubtful trick"—*i.e.*, a trick of which it is uncertain whether your partner holds the best card.

Ill-taught players are always much perplexed by this case, and are wont to enter into all sorts of (simply useless) speculations as to where the best card may lie. But our philosophy gives a far better guide. If you are weak in trumps they are only good for trumping, and you may use them unhesitatingly for that purpose. But, if you are numerically strong in trumps, they are so valuable that you ought not to waste one on the chance of its being an unnecessary sacrifice; in this case, too, your discard from a plain suit may be advantageous to you hereafter, and may give valuable information to your partner.

THE DISCARD.

When you renounce a suit and are not inclined to trump the trick, you have to "discard," and in the modern philosophical game it is important what suit you discard from.

The idea of untaught people is that you can best spare a card of your most numerous suit; but the philosophy we are expounding is quite antagonistic to this. The long suit is or may be (after trumps) the most valuable you have, and every card of it, even the smallest, may make a trick. Hence, you must discard on the contrary principle—*i.e.*, from a short or weak suit.

This is on the supposition that regulates the whole reasoning in this chapter—namely, that you have *no information* as to the other hands. When you *have* such information, the rule may be subject to change, as will be explained in Chapter V.

The discard is often of great use in giving your partner information as to your hand; suppose, hearts being trumps, your long suit is spades, and that you have not had the opportunity of leading it, and you throw away a club; your partner will be certain your suit is either spades or diamonds, and he will often be able to decide which, by the fall of other cards, or by his own hand. For this reason the rule ought to be rigidly adhered to. If, however, you have *already led* your suit, the object as regards the partner is gone, and you may use more latitude in the choice of your discard.

PLAY OF THE SECOND HAND.

The second hand will often have to play with very little knowledge of any hand except his own; and our philosophy must accordingly furnish him with some maxims for his guidance.

The first is one dictated by a simple consideration of his position in regard to the two players who have to follow him. Suppose a small card led, the third player must (as will be hereafter shewn) put on his best card of the suit, which the fourth hand will beat if he can. Generally speaking, therefore, the second hand is not called on to interfere; he may safely leave the matter to his partner, and merely let the trick pass, *putting on his lowest card*, and so saving any strength he may have. This is the general rule for the play of the second hand.

But there are some cases in which this rule may advantageously be departed from. Suppose you, being second hand, hold ace and king of the suit led, you should certainly not lose the opportunity of making them, but put one of them on.

Again, if you hold king and queen, by the same reasoning you should, if a small card is led, play one of them.

There are other cases of departure from the ordinary rule which are somewhat complicated, and which have lately been the subject of discussion. They will be found treated of in Part II., Chapters XX. and XXI.

THE LEAD FROM SHORT SUITS.

Cases will sometimes occur when, in consequence of some unfavourable previous fall of the cards, you will be unable to make your opening lead from a long suit of four or more, as the combination principle requires you to do.

In these cases you may be driven to make an unphilosophical, or, as it is technically called, a *forced lead*, from a short suit of three cards or less.

What card ought you then to lead from such a short suit?

Our philosophy will still supply you with an answer. The lead will be of no great good to yourself, but you must try not to deceive your partner into believing you are leading from a long suit; and an effective mode of doing this is to reverse the ordinary rule, and lead the *highest* instead of the lowest of the suit, which your partner, if he is observant, will soon discover. This rule is not arbitrary: it is founded on reason, for your high card will probably enable your partner to finesse, and will save him from losing a high card to no purpose, which he might do if you led the lowest.

If having three, the highest is an ace, king, or queen, you are justified in leading the lowest in the hope of afterwards making your high card, and to avoid the chance of strengthening the adversary.

THE PLAY OF SMALL AND INDIFFERENT CARDS.

All the previous cases have had to do with the play of cards which are of importance in regard to trick-making. But cases are continually arising where a card that has to be played is either so small as to be of no consequence in the trick, or is of equal value with others in the hand, so that it is indifferent as regards trick-making which of these small or indifferent cards is played.

Such cases must be philosophically provided for, and the rule which determines them springs from the same main principle as most others. It is this:—Although the mode of playing such cards may have no influence on the particular trick, yet it may be of the greatest importance, to the combination of the hands, by the *information it may give your partner;* and therefore it must be regulated with the view that he may rely on the inference to be drawn therefrom.

A few examples will explain the nature of such cases, and the proper mode of dealing with them.

Suppose a king is led, and I hold the ten and the four of the suit. It is reasonable and proper that I should throw away the card of least value—*i.e.*, the *four*, and not the ten, as the latter may be of use hereafter. My partner, taking it for granted that I play sensibly and carefully, will probably infer that the card I throw away is the lowest I have. But suppose I hold the *five and the four*, both of equal value to me, so that it is immaterial on grounds of general expediency which I play. What am I to do? May I throw away which I please, or should I adopt any rule in the matter? The answer is dictated by common sense. It would be most inexpedient that I should have the power of exercising caprice—sometimes doing one thing and sometimes another, which would puzzle my partner and prevent his drawing any trustworthy inference from my play. My natural course is therefore to follow the analogy of the former case, when the cards were wider apart—*i.e.*, to throw away the *lowest card* (the four, and not the five), from which my partner will infer as before, and rightly, that I have no card lower than the four in my hand. This practice, therefore, is established as obligatory, and must be followed with great care.

The same question arises, and the same principle applies also, in cases where the cards are not losing cards thrown away, but are valuable ones played for the purpose of winning a trick. Suppose I am fourth player, and hold the king, queen, and ten, the eight being in the trick against me. It is of course reasonable and proper I should *win with the lowest card possible*—that is, the ten. But suppose my cards are queen, knave, and ten, all three of equal value when in my hand; which should I play? Here analogy says I must play the *ten*, from which my partner will rightly draw the same inference—namely, that I have won with the lowest card possible. He will be certain, in both cases, that I cannot hold the nine. Hence it has been established as a general rule that, in playing *sequences* (except in leading them, when other considerations come in), the *lowest* card of the sequence must always be played; and the partner relying on this will draw his inferences accordingly.

In *leading* from a sequence, the rule is usually reversed, the *highest* card being led, for an obvious reason—*i.e.*, to prevent your partner from wasting the next highest card if he holds it. Thus, from queen, knave, and ten, you lead the queen to prevent your partner from putting the king on, which he might do if you led the ten. This rule, however, is subject to exceptions; for it may often happen that you *desire* your partner to put on the high card—as, for example, when you lead the ten from king, queen, knave, and ten, wishing to get the ace out of the way.

CHAPTER IV.

THE MEANS OF OBTAINING INFORMATION AS TO THE POSITION OF THE CARDS, AND THE CONTENTS OF THE DIFFERENT HANDS.

THE maxims laid down in the previous chapter are intended to guide the commencement of the play while the contents of the other hands are unknown. But we have now to consider the gradual dawning of information by the fall of the cards, and the changes which this information will lead to in the mode of play. And the first step is, to consider and explain by what means the information is gained.

This is a branch of Whist play the cultivation of which, to any large extent, is entirely modern. In the old treatises it was occasionally alluded to, the player being directed to do certain things "to inform his partner," and so on. But no great stress was laid on the general communication of information from one player to his partner, nor on drawing constant inferences from the play; nor does it seem to have occurred to the early writers what an amazing power lay latent in the full application of this principle. It is common at present to meet persons, thinking themselves tolerable Whist-players, who have no idea of learning anything from the partner's play as to what cards he holds. The utmost they will do is to remember the suit he leads first, in order to return it to him, and to notice when he renounces a suit, that they may lead it him to trump. If such persons are told that certain cards are known to lie in certain places, they think the knowledge can only have been acquired by looking over the hands!

It was only when the great principle of the *combination of the hands* became fully appreciated, that the importance of the communication between the partners began to be really understood. It was soon seen that, in order that the combination might be thoroughly carried out, each of the players must, in the first place, give his partner all possible information; and, in the second place, must carefully observe and interpret every intimation which his partner might be able to afford him. With this view, the intercommunication was made systematic and imperative; and in framing the modern rules this element of play has been specially borne in mind.

As a general principle, then, the player is bound to consider the communication of information to his partner as a matter of special importance, and must take every legitimate means of disclosing to him what cards he holds, and what are the chief aims in his play.

We say every *legitimate* means, because, of course, all means of conveying information must be such as are fully established and authorised.

Now, there are three well-established and authorised means by which a player can communicate information to his partner.

I. The simplest and by far the most comprehensive mode of conveying information, to an intelligent partner, is by following carefully the rules of play which are established as most expedient on general grounds; whether during the tentative stage (Chapter III.), or in response to some indication by the fall of the cards (Chapter V.) These rules are, or ought to be, familiar to all well-instructed Whist-players, and, if they are implicitly adhered to, an attentive partner cannot fail to draw abundant inferences from the play in which they are observed.

This will be evident when it is considered that, from the philosophical nature of the process of reasoning by which these rules are constructed, it follows that almost every mode of play must have a *condition precedent*, which specially dictates or calls for it; and hence, when an intelligent partner sees this mode of play used, he infers, or ought to infer, at once that the condition precedent is there.

For example: the first rule for the opening lead, implies the condition precedent that the suit led is the most numerous in the leader's hand, or that it consists of at least four cards, so that when your partner's first lead is from a plain suit, you should at once infer the existence of this condition.

Many examples of this kind of inference will be given hereafter.

II. But these rules are sometimes intentionally violated. A player may depart from them for several motives, and, among others, he may play irregularly with the special object of conveying information to his partner. For it is easy to conceive cases where cards played in an unexpected and abnormal way may imply something which it is desirable for the partner to know.

The commonest case of this is the modern practice of leading the king first, from ace and king, instead of the ace. It is an irregularity, the ace being the normal card. But if the king is not taken, and if the leader should have occasion to change his suit, his partner infers that he holds the ace also, and that knowledge may be of great value to the joint hands.

Another common irregularity is in discarding. If you hold, say, ace, king, queen, and knave of an unplayed suit, and have to discard from it, you throw away the ace, contrary to all rule. But your partner, if intelligent, will at once infer that you would not do so unless you had a sequence of cards of equal value still in your hand.

Another case of irregular play is holding up the turn-up card. Suppose, for instance, you have turned up the queen, and hold the king also; if you have to win a trick with one of them, the normal card to play is the queen, but in this case you play the king. This informs your partner you have both, which of course he could not know if you played the queen.

Several such devices are described by "Cavendish," and others may be imagined and practised, on the spur of the moment, for the purpose of enlightening an observant partner.

III. The third mode of communicating information is by *conventions.* It has been pre-arranged, with the knowledge and full concurrence of all the players, that certain modes of play are intended to have certain significations; and therefore when your partner sees you adopt any such mode of play, he draws from it the inference intended.*

The chief of them is the *signal for trumps.* In the combined game a player will often find it important to have trumps led promptly; and, as it may not be in his power to get the lead so soon as he wishes, he intimates his desire to his partner by a pre-arranged signal, which is given in a very simple

* There is some difference of opinion as to whether the conventions spoken of here constitute an innovation on the game, or are only logical extensions of analogical forms of play. But they are now so generally admitted in this country, that they must form part of any description of the game. There can be no question as to their *fairness*, if agreed to by all the players. (*See* "Fortnightly Review," April, 1879, p. 576.)

way—namely, by playing unnecessarily a higher card before a lower, when the usual custom would require the lower one to be played first. Thus, suppose a king and ace are led consecutively, and my two lowest cards of that suit are the seven and the three, the usual custom requires me to throw away first the three and then the seven; but if I reverse this, and play first the seven and then the three, I am understood to *call for trumps*, or *ask for trumps*, and it is my partner's duty to lead them for me as soon as he can get the opportunity.

This device was introduced some fifty years ago, and its origin and rationale may be thus explained. It is said to arise from an analogical extension of certain modes of play which may be illustrated by the three following cases:—

(*a*) Suppose I have a very strong hand of trumps, which are hearts; a strong suit of spades, a queen and one small club, and a knave and one small diamond. My adversary leads the king of clubs, to which I throw away the queen, in order to induce him to lead a trump. If he is an inexperienced player, he will very likely be tempted to do this; but if he is cautious, and does not fall into the trap, he will continue with the ace, to which I must play my small card. Then comes the *rôle* of my partner: he has, if observant, seen this manœuvre, and, if he is a good player, he will divine what was my object in playing the high card before the lower one, and will do for me, the first opportunity, that from which my enemy wisely abstained.

(*b*) Suppose it is my right hand adversary's lead, and that he begins by playing a small diamond. I, having knave and a small one, ought, according to the established rule, to play the latter; but, as I wish to get the lead in order to lead trumps, I depart from this rule and play the knave, hoping that the third player may have no bigger card in his hand. Suppose this does not succeed, the trick being won with the queen. On the next round, my partner seeing my small card fall, will ask himself what could be my motive for playing irregularly; and, if he is a good player, he can have little difficulty in guessing it, and will lead me a trump as soon as he can.

(*c*) Suppose this time it is my partner's lead, and that he has ace and king of clubs and ace and king of diamonds. He leads his two kings successively, to show me his strength; and to these I throw away the queen of clubs and knave of diamonds, instead of the small cards. He will then reason that my hand must be all—or nearly all—trumps and spades, or at any rate he will conclude (which is the fact) *that I wish him to think so*, and thereby to induce him to lead a trump, as obviously the best thing for both players.

These methods of play being perfectly justifiable and not uncommon, it appears to have occurred to some one that they might, by having a previous understanding to that effect, be made general, applying to the lowest cards as well as the highest, and so the "signal for trumps" came into use. It is no doubt a great power, and gives great advantages to those who use it; but it is by no means certain that, generally speaking, it has improved the game.*

It is the opinion of many good authorities that it requires somewhat greater strength to justify a call for trumps than an original lead of them. On this point refer to "Cavendish" and Clay.

Another conventional arrangement is what is called the *American* system of leading small cards. Formerly, when you had to make an *invite*, or an original lead of a small card from a long suit (see pp. 35, 40) it was customary to lead your lowest card. Some years ago "Cavendish" proposed that when the suit contained *five* cards, the card led should be the *penultimate*, or the lowest but one—an alteration which was largely adopted. Col. Drayson then proposed that from *six* cards the lead should be the lowest but two; but quite lately Mr. N. B. Trist, of New Orleans, has pointed out that all these varieties of *invite* may be brought into one category by a direction to lead the *fourth-best card*—*i.e.*, the fourth in rank from the top of the suit; and it has been shewn that, if this is done, the leader's partner may be able, not only to ascertain the number of cards

* *See* CLAY, Edition 1881, p. 111.

held, but to gather valuable information as to the rank of the higher ones among them. The system of *American Leads* has been fully explained in "Cavendish's" latest work, "Whist Developments."

A third conventional mode of play is called the *echo of the trump call.* If your partner leads trumps, or calls for them, and you hold *more than three* yourself, it is considered so important to convey the knowledge of this strength to your partner, that you are directed to "echo his call"—*i.e.*, to ask for trumps yourself.

This device was also introduced by "Cavendish," and is fully described in his earlier work.

From what has been above stated, it may easily be seen what a very large and copious opportunity of conveying information exists, if it is properly taken advantage of by both player and observer. In Clay's admirable work, the opening passages (Edition 1881, pp. 35–40) are entirely occupied with this subject, and may be read with great advantage. He begins with the question, "How am I to learn Whist?" and he gives a most interesting account of how he actually put the question in his own case, and of the process by which he was led to the answer. He devotes about three pages to analyzing the first two tricks of an imaginary hand, and pointing out what a large amount of information may be extracted from the fall of the cards in them, when all the available inferences are drawn.* This leads him to make use of the expression—

"*Whist is a language, and every card played an intelligible sentence.*"

The thought is not altogether new, for an old Spanish proverb says, speaking of cards and card-playing—

"*Que hablen cartas,*
Y callen barbas."

(Let the cards discourse, but the tongue be mute.)

But the idea is a very happy one, and has, since the introduction of the modern style of play, come into general use. It is customary to speak of the "conversation" between Whist partners with a meaning as definite as if the phrase referred to oral communication.

The philosophical bearing of all this on the conduct of the game must be well considered by both player and observer. Each of them has his duties in regard to it.

The player is bound to bear constantly in mind that every card he plays conveys a message to his partner as distinctly as if it were written in a letter; and that if he makes this message untrue by carelessly or unnecessarily violating the ordinary rules of play, even in so small a particular as the throwing

* We may, however, venture to take exception to all this being called "the way to *learn* Whist." Such an elaborate process of inference cannot be attempted by beginners. Whist must be *learnt* in much simpler ways; we should rather describe Clay's process as "The way to become a finished and fine player.

away of useless cards, he imperils the joint interests, and renders the great instrument of action (the combination of the hands) of uncertain effect. The necessity of regularity and care in the play is curiously illustrated by the character of *truthfulness* generally ascribed to it. The player who violates this regularity, by playing a card different from that which, as a matter of routine, his partner would expect him to play, is said to play "false," and the card so played is called a "false card." Clay says to such a person, You have told me, as plainly as Whist language can speak, such and such a fact deducible from your play. In no other position in life would you tell me that which is untrue; why do you do so here? He adds:

"The best Whist-player is he who plays the game in the simplest way, and who always bears in mind the great maxim, that *it is of more importance to give information to his partner than to deceive his adversary.*

"I value that player the most who never deceives me, and whose unvarying certainty enables me, as it were, to play his cards with almost the same knowledge of them as I have of my own.

"I hold in abhorrence the playing false cards."

That is, of course, playing them without a definite motive. False cards may often be played advisedly and properly, as is explained by "Cavendish" (Fourteenth Edition, pp. 102–104).

But, on the other hand, all this precaution is useless unless the partner, for whose benefit it is taken, is also thoroughly alive to the advantages of the intercommunication. He too has his duties.

The language is addressed, the message of the card is sent, to him, and he must educate himself to accurate observation and prompt drawing of inferences, in order to be able to observe the message and to appreciate its signification, otherwise he may lose favourable opportunities offered him by his partner of improving their joint position.

It may be worth while, in order to show what the opportunities and the duties of the observer are, to point out some of the more obvious of the communications that may be made to him, and which he is bound to notice and act upon.

(A) Take in the first place your partner's first lead, and see what are the probable messages conveyed to you by this single card.

1. If he leads a trump, he tells you he is strong in trumps, and wants them drawn.

2. If he leads a plain suit, he tells you he is not very strong in trumps, but that the suit led is the best in his hand, that he holds four or more cards in it, and wants to establish it, and bring it in.

3. If his first lead in it is the king, he tells you he has either ace or queen also, perhaps both.

4. If it is the ace, followed by a small one, he tells you he has not the king, or queen with knave, but that he had originally five or more.

5. If it is the ace followed by the queen, he tells you he has the knave also and one

small card. But if the ace is followed by the knave, he tells you he holds also the queen and two or more small cards.

6. If it is the queen, you infer he holds also the knave and ten.

7. If it is the knave, you infer he may hold king and queen with two others.

8. If he leads the ten, you infer he has most probably also the king and knave.

9. If he leads a small card, he tells you he has none of the above combinations which require him to lead a high one.

10. If he adopts the American system, and leads a small card, he tells you he has three higher ones, and enables you very soon to find out what lower ones he holds also.

(B) Secondly, suppose he is returning your suit. In this he is acting on information given by you (see Chapter V.) You will see what his mode of play probably tells you.

1. If he does not lead out the master card, he tells you he does not hold it, and, if not in your own hand, you therefore know it is against you.

2. If he leads any card, afterwards dropping a lower one, he tells you he has then no more of the suit.

3. If he leads any card and afterwards drops a higher one, he tells you he has still another left.

4. If he returns your plain suit at once without shewing you his own previously, you may infer he has no good long suit in his hand, but prefers playing your game.

(C) In what may be called accidental leads of your partner.

1. If he leads a losing card of a suit of which he knows you have none (you not having yet given him any intimation of your strength), you may infer generally he is strong in trumps himself, or that he sees an opportunity for a cross ruff.

2. If, when he has the power, he refrains from attempting to force you, he tells you he is weak in trumps.

3. If he leads a card of doubtful significance, such as a knave, ten, or nine, you should consider the possibility of its being a forced or unnatural lead.

(D) When your partner is second player.

1. If he trumps a doubtful trick, he tells you he is weak in trumps.

2. If he refrains from doing so, he tells you either that he has no trump, or that he has four trumps at least, or else some good ones guarded.

3. If he plays a high card, the rules for the play of the second hand will tell you the object, and will enable you to draw the proper inference.

4. If he plays a low card, he tells you that is the smallest in his hand; unless he is calling for trumps.

(E) If he is third player.

You generally infer he has played the highest he has, or at least the lowest of a head sequence; but this is sometimes modified by finessing. If he plays the ace he tells you he has not either king or queen.

(F) If he is fourth player.

1. You know he will win the trick as cheaply as he can, and that he has therefore nothing between the card he plays and the one against him—*e.g.*, if third hand has played the nine and your partner takes it with the king, he tells you he has not either ten, knave, or queen.

2. If he cannot take a moderately high card against him, you see his weakness, and must provide against it as well as you can.

(G) Where your partner is in any position except leader.

1. Any card that he plays tells you generally he has not the next below.

2. When he discards, in ordinary circumstances he tells you, generally, that the suit he plays is his shortest or weakest.

3. But if strength of trumps has been declared against him, he tells you, generally, that

the suit he discards is his longest and best protected, and that he wishes you to lead it for him.

4. If he plays unnecessarily a higher card before a lower one, he is calling for trumps.

5. If he "echoes" when you are leading trumps, or calling for them, he originally held more than three.

6. If he refuses to trump a trick that is certainly against him, he either has no trumps, or is strong in them, and wants them led.

8. If he discards the second best card of a suit, he has probably no more of it.

7. If he discards the best of any suit, he tells you either that he has only trumps and winning cards left, or that he has the next best of the suit discarded, and the full command.

A good player will take such advantage of these and other inferences that he will often, when only a few tricks remain to be played, "count the hands" of some or all of the other players, which may give him a great advantage.

The foregoing considerations have referred solely to the communication between the two partners; but it is obvious that all the steps that are taken by any player to give information to his partner are equally available for the information of the two adversaries, if they watch closely for them. The "conversation of the cards," or the "language of

Whist" must be spoken aloud, and may be heard all round the table.

A question obviously arises on this—namely, whether the fact, that the information is open to the adversaries, is of sufficient weight to be an argument against the free communication of it to the partner? Opinions have varied on this point; but the more authoritative view, as derived from the experience of the best players, is, very decidedly, that the advantage of informing your partner is considerably greater than the attendant disadvantage of informing the adversaries, and ought consequently to regulate the play. In fact, if the policy were changed to one of universal concealment, everybody trying to deceive and mislead everybody else, it is difficult to conceive how the game could be brought into any rational form at all.

At the same time, there are occasions when the difficulty must be borne in mind, and may have a legitimate influence on the play. If, for example, you have a bad, unobservant partner, it is obvious that many of your efforts to inform him may be purely wasted, and may only do you harm by being taken advantage of by clever observant adversaries. In such a case, or even if a good partner is found so weak as to be powerless for good or harm, it would be more expedient to withhold than to give information, and you would be perfectly justified in modifying your tactics. These, however, are purely practical matters of detail, which hardly enter into the scope of this book.

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CHAPTER V.

ACTION ON THE INFORMATION OBTAINED.

We now come to the last phase of the hand—the descending curve of the parabola—when the player has to act, not as at first, blindly or tentatively, on rules dictated by mathematical probability or logical speculation, but to a considerable extent on facts that have been disclosed to him as to the cards remaining in the various hands.

Here comes into play the exercise of personal skill; for the combinations of Whist are so varied, that the particular mode of action which will be best adapted to the latter portion of a hand must be left largely to the judgment of the player.

There are, however, some situations where the information received dictates certain modes of play. These can be easily defined, and it will be found that in all such cases the line of conduct is consistently founded on the same great principle that has ruled the previous phase of play—namely, the effective *combination of the hands* of the two partners.

PLAY OF THE THIRD HAND.

When you have to play third hand your partner has led, and you have therefore obtained important information as to the state of his hand. Let us see therefore how the application of philosophical principles will guide you.

We may ignore exceptional leads, and suppose your partner to have led from his most numerous plain suit. If he leads a high card, you will, knowing the rules in Chapter III., have no difficulty in understanding what to do.

If he begins with the ace, you of course throw to it a low card; and the same if he begins with king. In the latter case, if you are void of the suit, you must not trump it, because he either holds the ace himself, or wants to get it out of his way to establish his suit, a desire you must not oppose.

If he leads the queen, you of course will not put on the king, nor must you put on the ace, as this is a "finesse" always prescribed; understanding, however, that you play the ace afterwards as soon as you can, to get it out of your partner's way.

The leads of knave, ten, or nine, you holding higher cards, lead to some rather complicated considerations, which will be found explained in more detailed works; they are ruled on the same principles.

But it will more frequently happen that your partner's first lead will be a small card, and here you have a greater range of choice as to your play.

The principles will clearly point out your duty. You know that your partner has, in all probability, led from a long suit, and you must take it for granted he wants you to help him to establish it and bring it in. When he led a high card, *he* was taking the initiative in the establishment of his suit; when he leads a small one, he asks *you* to take the initiative, and your course is therefore more important to him. The French call this lead *l'invite;* your partner inviting you to show what you can do to aid him.

How, then, can you best do this? The answer is easy. You must recollect that his lead of a small card shows he has not already the full command of the suit, to obtain which he wants to get certain high cards out of the way. The mode, therefore, in which you can best help him is, *by playing the highest card you have.* If it wins the trick, all well and good; if it is taken by the fourth hand, there is still a valuable result obtained, for two high cards (if you have been able to play a high one) are thus got out of your partner's way at once. Suppose, for instance, he leads from king, knave, and small ones, and that you put on the queen, which is taken by the ace; your partner's suit is thus established in one round only, a result that may be very profitable to him.

If you happen to hold queen and ace, you are at liberty to finesse the queen, playing out the ace afterwards as above stated;—but generally you must comply with the principle by putting on the highest card you have.

RETURN OF THE LEAD.

But you have a further duty to perform—in regard to what you now know to be your partner's long suit—*i.e.*, when a suitable opportunity arises, you have to help him further in regard to it, by leading it again for him, or, as it is called, *returning* it. This is not only a benefit to *him*, but it is also a considerable advantage to *you*, as supplying you with a useful object for a lead.

Now you must apply to our philosophical theory to guide you how to return your partner's lead with the best advantage to the combined hands.

The key to the problem is the consideration of what your partner desires—namely, to *establish* his suit. Suppose you had originally three cards of it, say the ace, knave, and four; your partner having led a small one, you win with the ace—which of the two others, the knave or the four, ought you to lead in returning? You must recollect that, to carry out your partner's object of establishing the suit, the high cards have to be got out of his way as early as possible; and you will facilitate this object much better by returning the knave than the four. The knave will probably be sacrificed, but this will be for your partner's benefit. His best card may be the queen, when your knave will force out the king, and leave the suit established with the queen as master card.

Whereas, if you led the four, your partner's queen might be taken by the king, and then your partner's suit would *not* be established, as your knave (then the best card) would stand in his way, or "block his suit," as it is termed.

It is a general rule, of which the above is an example, that if you are short in your partner's long suit, the best use you can make of your high cards is to sacrifice them, if necessary, for him. This is called "strengthening" play, and the import of the term will be evident when it is considered that the removal of every high card out of the way increases the value of those below it, by promoting their rank—a process tending directly towards the prompt establishment of the long suit, which is the object desired.

Now, suppose you have ace, knave, and *two* small ones—say four and three. After winning your partner's lead of the small card with the ace, which ought you to return? Here you have a right to consider your own hand, as you are fairly strong in the suit yourself, and you should return the three; for the sacrifice of the knave is not required, and it cannot block your partner's suit, as you have still a small one left to give him after you have played the knave to the third round.

Hence, the rules have been laid down—

In returning your partner's suit, if you have only two left, return the highest; if more than two, the lowest.

This rule is also very valuable as giving information to your partner, who is of course much interested to know where the cards of his suit lie. If after you return the knave he sees you drop the four, he will know you have no other. But if after you return the three he sees you play the four, he will know you have still at least one of the suit left. For this reason philosophy requires you, as a matter of principle and uniformity, to adhere to the rule even though the original object, dependent on the value of the cards, may no longer exist. Suppose, for example, you held originally ace, five, and four, after winning with the ace you must return the five; if you held ace, five, four, and three, you must return the three.

It is generally to be recommended that before returning your partner's lead you should open your own suit, if you have a moderately good one, in order to inform your partner; for otherwise, when he has the lead again, if he cannot go on with his own suit, he may be obliged to lead at random, which is often the bane of Whist play.

The rules for the return of the lead imply, of course, that the lead has been a normal one, from a long suit; if you have reason to think it has been a "forced" lead, it must not be returned, or at least not in the same way.

COMMAND OF THE LONG SUIT.

There is a rule intimately connected with the preceding, but in a more general form, namely—

Do not retain the best card of your partner's long suit. The reason is obvious; the establishment of the suit, which he aims at, implies obtaining the full command, which, if you keep the best card, he cannot do.

The converse rule,—

Keep the command of an adversary's suit as long as you conveniently can, only requires enunciation as a matter of form. You have, however, to consider that if you hold a master card too long, it may be lost by trumping; and for this reason it is usual, under ordinary conditions, not to hold it up beyond the second round.

It may be added that "getting out of your partner's way" in complicated cases becomes almost a special art, and full directions on it will be found in the works of "Cavendish," "Laws and Principles" pp. 90-93, and "Whist Developments" p. 51.

MANAGEMENT OF TRUMPS.

So soon as you have acquired information as to the position and intentions of any of the players in regard to trumps, your duties become very important, and must be carefully regulated by philosophical considerations, or you may not only lose the chance of doing much good, but may do a great deal of positive mischief.

In the first place, suppose your partner *leads trumps*. You then infer, by the considerations in Chapter III., that he wants to get them out; and it is your duty to help him in this object. Hence it is clear that, if you win the first trick in them, or if, losing it, you get the lead again anyhow before your partner, *you are bound to return trumps immediately*. This is one of the most imperative of all modern Whist rules. The card to return is determined by the considerations already given.

Again, your partner, not having the opportunity of leading, may *call for trumps;* and, as will be clear from what has been said, you are bound to lead trumps in answer to the call the first opportunity. The answer to the call is considered as equivalent to a return lead, and the cards to play are determined on the same principle. If you have less than four, you begin with the highest; if four or more, with the lowest; unless you hold the ace, when, for the sake of getting two rounds certain, you play it out, even at the risk of misleading your partner as to the number you hold.

But the knowledge that your partner is strong in trumps brings other duties upon you, as the philosophical theory will easily explain.

The importance of numerical strength in trumps has been made so evident in Chapter III., that it will readily be understood what a disadvantage a loss of one of them must be. Hence, under the above condition, it would be most improper to damage your partner's prospects and intentions by "forcing" him to trump a trick when he is short of a suit, and you must carefully avoid the risk of doing so.

On the other hand, conversely, when one of your *adversaries* has declared strength in trumps, your policy is, of course, to force him whenever you can.

THE DISCARD.

Chapter III. contains the philosophical principle regulating the discard so long as you are in ignorance of the contents of the hands, in which case you are directed to discard from your shortest or weakest suit.

But when you obtain information, this rule may possibly be influenced thereby, the case turning on where the *strength of trumps* may be declared to lie. If this strength is *on your side*, then the above rule still remains good. But if the strength of trumps is declared to be with the *adversaries*, then the following reasoning comes in :—The object of the original rule was to preserve your long suit, in case it might be brought in ; but if the strength of trumps is against you, this object may be given up as impracticable, and you must change your tactics to a *defensive* policy. Now, it often happens that in your short suits you may have, say a king guarded, or a queen or knave doubly guarded ; and it would be a pity to destroy the guard for the sake of an unattainable object, as the guarded card might make a trick and perhaps save the game, or it might be still more useful to obstruct the long suit of one of the adversaries. It would be less disadvantageous to throw away *a small card of your long suit*, as you would even then have probably more left than would be of use to you.

But the discard is of so much importance as a means of information to your partner, that if you adopt this policy you must be consistent, and adopt it in *all cases* of adverse strength in trumps, whether or not you have the guarded cards that rendered it necessary; and hence it has, in modern play, become a rule, and a very proper one;

Whenever the strength in trumps is clearly shown to lie with your adversaries, you must reverse the ordinary rule in regard to your first discard, which must in this case be from your longest and best protected suit, or at least from the suit which you wish your partner to lead you.

In ordinary cases a lead of trumps, or a call for trumps on the part of the adversaries, may be sufficient to determine the question of strength; but it is not always so, for it may happen that your own side may turn out the stronger, in which case the ordinary rule should be adhered to. See explanations on this point in "Cavendish," pp. 98, 99.

LEADING TO YOUR PARTNER'S DISCARD.

It has been shown, in this Chapter and in Chapter III., how your partner may, before leading, communicate to you the knowledge of what is his suit, by the medium of the *discard*, and it may often happen that you get the opportunity in this way of leading his suit for him before he can open it himself.

In this case you must treat your play as a return of the lead, although it is really anticipatory of it—*i.e.*, you must adopt *strengthening* play, leading him out your highest if you have not more than three.

It is, in fact, the same case as a response to a call for trumps; your partner "calls" for a certain plain suit, and you must lead it to him somewhat as you would answer a trump signal.

QUESTIONS OF POSITION.

Another obvious use of information obtained is, to take advantage of any disclosures that have been made, as to certain positions of cards that appear to offer opportunities of making tricks with comparatively low cards.

Suppose, for example, your right-hand adversary has shown, in a previous trick, that he has no high hearts; if you lead a heart it affords your partner the opportunity of winning the trick with a comparatively small card. Hence the rule, that *it is good to lead up to a weak hand.*

If, on the contrary, the indications have shown that any player is strong in a suit, it is obviously not good to lead up to him, for you may thereby sacrifice good cards of your partner's; therefore—

Avoid leading up to a strong hand.

The converse rule—

It is good to lead through a strong hand, and bad to lead through a weak one, is generally well-founded, but is not capable of such definite proof as the others. In fact, we begin here to approach the boundary of the region of personal skill and judgment, in regard to which it is impossible to give definite rules.

CHAPTER VI.

CONCLUSION.—ADVICE TO STUDENTS.

FROM what has been said in the foregoing chapters, an idea may now be formed of what qualifications are necessary to constitute a really good Whist-Player.

In the first place, he must have a competent understanding and appreciation of the general philosophical system which forms the basis of Modern Whist Play. He must have a fair knowledge of the application thereto of the calculations of probabilities, and must intelligently enter into the nature of the logical reasoning by which the various problems of the case are determined and solved.

Secondly, he must have a thorough acquaintance with the codes of rules which have been drawn up for practical use, and such a familiarity with them as shall ensure their prompt application. It is not sufficient that he should know these as mere forms; he must understand their origin, derivation, and objects, in order to be able to distinguish the cases where they are strictly applicable from those where they may fail, or may be of doubtful propriety. And his knowledge must extend, not merely to the important rules themselves, but to all the shades of detailed modification in their practice which have been dictated by acute reasoning, or by extended and careful observation and experience.

Thus much may be called "Book-learning," which, though not sufficient of itself to make a good player, is absolutely necessary to enable any one to merit that title. Beyond this, personal qualifications come into play.

Thirdly, the player, if he is to take high rank, must be able to give the most careful attention to the cards as they fall, and must possess the faculty of drawing rapid and accurate inferences as to the facts deducible therefrom. And he must devote particular attention, in this way, to the intercommunications between himself and his partner, so as to carry out, to the fullest extent, the combination of the hands.

And lastly, he must bring to bear on the game, at every stage, a sound and clear judgment, so as to take the best advantage of its ever-changing positions and accidents. He must extend this judgment even to the personalities of his partner and his opponents, taking the measure of their knowledge and skill, and adapting his play accordingly. He must be able to perceive when critical positions arise, and to meet them by exceptional or irregular play; and he must have the ability by an occasional dexterous "coup" to effect some desired result which, to an ordinary player, would have been quite unattainable.

To do all this perfectly is no light matter, and it is no wonder that the effort to do it, in a more or less perfect degree, should present a powerful attraction for intelligent minds. There is, probably, no intellectual recreation, the exercise of which so well repays in interest the trouble spent upon it; and there is certainly none in which the degree of perfection attained furnishes a better test of the mental powers. An eminent statesman once said that he would be content to choose a prime minister by his Whist-playing.

But, large as is the demand that Whist, when finely played, may make on the ability and talent of the player, it must not be supposed that its acquisition, to a more moderate extent, involves any great difficulty; for it is one of the peculiar attributes of the philosophical game that, from its systematic structure, a moderate proficiency in it is very easy to acquire. It is infinitely easier than the groping in the dark which untaught persons are accustomed to use, and which is neither intelligible nor teachable. So easy are the first principles of the art, that we have often met with intelligent learners, who, though quite young, could go through the formalities of the game with tolerable correctness, and who made very good and satisfactory partners.

It is one of the objects of this book to aid those who earnestly aspire to a knowledge of the game in its best form, and we cannot do better than conclude by offering a few words of guidance to them.

First, then, we say to the student, you must be convinced that you have something to learn. It is the want of appreciation of this truth that accounts for such a general prevalence of bad play. People fancy they can become good players by mere practice, which is a great mistake; they only move on in one eternal blundering round. The philosophical game has been the result of years upon years of elaborate thought and incessant experiment, and you can no more arrive at it by your own limited experience than you could become acquainted with modern scientific astronomy by watching for a few weeks the apparent motion of the stars. And, further, if you have already learnt and practised Whist on the erroneous principle of considering merely your own hand, you must wipe out all that, and make a *tabula rasa*, on which true knowledge can be inscribed.

Your mind, being thus clear from any of the false notions that prevail among untaught players, will easily be impressed with the importance of the great fundamental principle of the combination of the hands, which, through the entire study of the game, is always to be kept steadfastly in view. And you will, of course, begin by making yourself familiar with the rules founded on it. But in doing this, it is highly desirable to confine your attention, in the first instance, to the simplest and broadest precepts, and not to distract the mind at the outset by too much detail. To aid in this, it may be useful here to put down a few simple Elementary Rules which are of the most importance to beginners. They do not pretend to be anything like a complete code, but if fully mastered for ready application in practice, they will at once enable their possessor to make a fair beginning as a Whist player on the proper system.

SIMPLE ELEMENTARY RULES CHARACTERIZING THE PHILOSOPHICAL GAME OF WHIST.

GENERAL PRINCIPLES.

Remember that the great fundamental principle of Philosophical Whist is the *Combination of the Hands* of the two partners.

For which reason it is of the highest importance that you should watch, and draw inferences from, the fall of your partner's cards, with the view of gaining information as to his hand; and that you should play your own cards very carefully, in order to give him information as to yours.

THE OPENING.

Let your first plain-suit lead be from your *longest suit*, which gives your partner the most ample and positive information.

In this suit—

IF YOU HOLD	LEAD
Ace and king	King, then ace.
King and queen	King.
Ace, queen, knave	Ace, then queen.
Queen, knave, ten	Queen.
King, knave, ten	Ten.
Ace, and four or more small ones	Ace.

In other cases, lead the *lowest*.

RETURN OF YOUR PARTNER'S SUIT.

If you have not more than two left, return the highest; if more, the lowest. But in any case get rid of the commanding card.

MANAGEMENT OF TRUMPS.

If you hold five trumps, lead them; and if they contain an honour, call for them.

If your partner leads trumps, it is imperative that you return them the first opportunity.

If he calls for them, you must lead them for him as early as you can; if you hold three or less, play out your best; if more than three, your lowest.

Do not force your partner if he has shown strength in trumps, or if (being in ignorance of this) you are weak in them yourself.

But force a strong adverse trump hand whenever you can.

Do not trump a doubtful trick second hand if you have four or more trumps; if you have less do so.

SECOND HAND.

Generally play your lowest card.

But if you hold ace and king, or king and queen, play the lowest of them.

THIRD HAND.

Generally play your highest. But with ace and queen you should finesse the queen, playing out the ace afterwards.

SECOND, THIRD, OR FOURTH HAND.

Always play the lowest card of a sequence.

DISCARDING.

As a general rule discard from your shortest suit.

But if strength of trumps is declared against you, reverse the rule and discard from your most numerous one.

IN ALL POSITIONS.

Avoid playing "false" cards, and be very careful in playing even the smallest cards, lest you may deceive your partner.

These rules, simple and short as they are, will, when firmly impressed on the mind, so as to be ready for prompt use, put you in possession of the main points of the modern system, and when their spirit is once acquired and appreciated, the great portion of the learning is done. You will then be surprised to find how different the game has become from that of the uninitiated; how much more intellectual, how much more interesting, and how much more simple and easy. All the blindfold hap-hazard stumbling has disappeared; everything you do has now a clear meaning and object, and you are enabled to appreciate the play of a hand as no longer a mere jumble of chance events, but as a connected series of combinations guided by intelligence and design.

Practise as much as possible with good players; but do not be turned aside from correct play by unsound criticism, or by unfavourable results, both which you will have to encounter. Neither be discouraged by finding you have made mistakes and played badly, for even good players are liable to this at times. Clay, who associated with far better players than you will be at this stage, used to say that he won more by his adversaries' mistakes than by his own skill.

It is very essential to draw a proper distinction between errors of different kinds. This is very seldom done in the wranglings and disputes one hears after the play of a hand, when an unfortunate partner is reproached for bad play with no kind of qualification. There are three clearly distinct kinds of errors which may be (and constantly are) made, namely—errors of *form;* errors from *want of attention;* and errors from *mistaken judgment.* By errors of form are meant contraventions of the established book rules, such as playing false cards, leading wrongly, returning the wrong cards, not returning trumps, forcing the partner improperly, and so on; these errors are the least excusable, whether they arise from ignorance, which ought to be corrected, or perversity, which ought to be reproved. Errors from want of attention, such as trumping your partner's best card, or omitting to take advantage of any sort of intimation that has been clearly given by the fall of the cards, should be viewed more leniently, as the power to keep up the attention varies very much in different persons, and even in the same person at different times. Errors in judgment, which consist in not doing the best thing possible in difficult or doubtful situations, are the most excusable of all, seeing that there is generally room in these cases for difference of opinion, and that it often requires a fine player to see at the right moment what is the best thing to be done.

Do not take undue advantage of the statement made in all good Whist-books, that rules ought sometimes to be departed from. This is true; but to judge correctly when and how such departure should be made is one of the attributes of the very best players. There is no greater nuisance at a Whist-table than a young player who, though he knows what the rules of good play are, is constantly breaking them because he thinks he can do something better. And, conversely, there is no more satisfactory partner than one who, even with a little self-denial to his own inclinations, keeps his play within the accustomed bounds, and so renders it fully trustworthy.

At an early stage of practice your attention will naturally be directed to the question of *memory*. In regard to this subject there is a vast amount of misunderstanding among the public in general. Most people who do not play Whist, and do not care to take the trouble of learning, excuse themselves by saying "they have no memory for it." They imagine that the great art in playing consists in remembering every card that falls; and it is not uncommon to hear somebody spoken of as a very fine player because he can tell the four cards left in the hands before the last trick. All this is pure delusion. The necessity of remembering all the cards that fall is a fiction: no one attempts to do it, or needs to do it. The effort of memory required for fairly good playing is very moderate indeed, and such as no one need despair of being able to apply, when the game is learnt systematically. For it is one of the best features of philosophical play, that it immensely simplifies the exercise and application of the mnemonic faculty, by showing *to what points it is most important to direct attention.*

This may be easily gathered from the explanations given in previous chapters, which suggest the following hints:—

1. In the modern game the trumps occupy the most prominent place, and your first effort of memory must therefore be directed to these; begin by *counting* them, quite positively and distinctly, as they fall, and you will soon take a special pleasure in finding your mental "thirteen" correspond with the fall of the last trump upon the table.

2. The next thing to attend to is to notice and recollect the fall of the *high* trumps. Try always to remember the play of the four honours; and if you then extend your memory to the ten and the nine, you will go as far as the general run of moderate players pretend to.

3. After the trump suit, the most important thing to you is *your own long suit.* Let this therefore have your next attention. It is better, at first, not to attempt actually to *count* the cards falling of any suit except trumps; but you will find that the counting may be done almost instinctively, by certain indications that you will soon be familiar with. For example: suppose you have four cards of your suit; if it goes round three times, you will at once know you hold the thirteenth; if one player fails the third time, then some one besides yourself has one left for the fourth round; and so on. Again, suppose you hold five cards, and the suit

having gone round twice, three cards fall the third time; then you should know that you hold the only two left; if two only fall the third round, there is still one besides your own to fall on the fourth round; and so on.

4. Then you must try to remember the fall of the *highest cards* in your suit, in order to know whether you possess the full command, or whether there may be a master card still in your way.

5. After this, the next thing in importance is *your partner's long suit*, which you note by his first lead, and have to return to him.

These things do not require any extraordinary memory, and if you accomplish them you will do very well. Any surplus power you have may be then employed on the general observation of the fall of the cards, and the meaning of the inferences to be drawn from them.

You should bear in mind that a *habit of observation* is much more important than memory; for when people complain that they do not recollect the fall of certain cards, they imply that they did not attentively observe them when they fell. If a player really *observes* that the queen of hearts is played to a trick, he is hardly likely to forget it a minute later. The books tell you to "keep your eyes on the table," and not on your own cards; but that is not enough. You must *pay attention* to the cards as they fall; and if you do this, your memory will repay you for your trouble.

It is best to trust to your natural memory only, and not to adopt any artificial contrivances for aiding it, such as putting certain cards in particular places, and so on. All such mnemonic arrangements do mischief; even the plan—so common with inferior players—of placing the trumps always in the same part of the hand, is simply childish; for a player who cannot recollect what is the trump suit, merely shows he is taking no interest in the game. The only kind of arrangement really to be recommended is the careful sorting and counting of your cards at the beginning of the hand: this not only facilitates correct play, but may also aid your recollection of how the suits have gone.

In practising the simple rules above laid down, you will gradually discover that they admit of a good deal of amplification and extension in points of detail, which, after a time, you must make yourself acquainted with. For this purpose you must consult the works of "Cavendish" and Clay, where all details are fully and carefully explained, and your mastering these will advance you another and a very great step in sound play.

You may then be supposed to have become thoroughly familiar with the philosophical system; you will speak the language of the game with fluency and correctness; and, if you have cultivated your habits of observation and inference, you may be credited with a facile interpretation of it when it is spoken to you. You may then turn your attention to the *accidents* of play, which have the object of taking advantage of particular situations. You will find plenty of examples of these in "Cavendish's" published "Hands," and many others in Hoyle and Matthews, which you may at this period study with advantage. And here you will find the field gradually opening for your personal skill; your knowledge of system has already made you a sound trustworthy player, and you have then to advance into the first rank, and mount as high in it as your ability and your genius will allow you. You must not expect to get on too fast, as you may be years before you really excel; but do not be discouraged, as you are in the right way, and with reasonable effort you cannot help improving.

This fact is one of the most remarkable characteristics of the philosophical game. The old unscientific mode of play admits of no progress. Even a man of superior ability, after playing by it for years, finds his skill limited to the mere practice now and then of a few clever strokes in an accidental way, and has no more general power over the management of a hand than he had when he first began. But when the play is founded on system, every year's practice must make its working more perfect and more comprehensive, and must open a surer way for the attainment of the highest aims.

PART II.

THE PHILOSOPHY OF WHIST PROBABILITIES

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CHAPTER I.

INTRODUCTORY.

In the foregoing Part of this work, mention has frequently been made of the mathematical probabilities affecting certain combinations or certain events, and these probabilities have in many cases been made use of in order to determine the modes of play. It need hardly be said, to those who play Whist, that the combinations occurring therein admit of a most extraordinary extent and variety; but the possibility of applying scientific investigation to these combinations, and of deducing therefrom results of practical application, is much less generally known.

The science of probabilities has never been a popular one, for the reason that the popular mind does not easily grasp the peculiar and exceptional reasoning on which it is founded. Other branches of philosophy, which really are deeper, offer to the unlearned a more intelligible appearance. Physics, mechanics, chemistry, astronomy, are all based on certain laws, the existence and nature of which involve nothing at variance with our usual notions of possibility; but the idea of defining laws to determine the action of pure chance is one against which the mind naturally rebels. If we tell a man that we know the distance from the earth to the moon, he has no difficulty in conceiving that we may have some intelligible means of measuring it, though unknown to him; but when we say that, if he draws a card at random from the pack ten thousand times, we will undertake that about 192 of those times it shall be the ace of spades, he can form no idea of any reasonable cause to produce such an effect, except perhaps some suspected legerdemain. Yet that the action of chance in such cases may be reduced to certain laws, and may be predicted by mathematical calculation with a great approach to accuracy, is established beyond doubt.

The general idea of using such calculations in card games appears to have arisen towards the end of the seventeenth century. A certain Dr. Arbuthnot published, in 1692, a book "On the Laws of Chance; or, a Method of Calculation of the Hazards of Game, plainly demonstrated and applied to games at present most in use." Among these applications were two problems on the situation of honours at "Whisk," and on the odds at certain scores.

James Bernouilli, who died in 1705, wrote a celebrated Latin work called the "*Ars Conjectandi*," which, however, was not published till 1713; this also contained the solution of several problems relating to games of chance, and it was noticed in an English work published in 1729 by Richard Seymour, author of the well-known "Court Gamester," who says,—

"All games of chance are governed so much by an *Art of Conjecturing*, that it may be certainly determined by calculation how much the player shall win or lose after being a long time at play."

De Moivre (of whom Newton used to say, "Go to him; he knows mathematics better than I do") published in 1718 his great work, "The Doctrine of Chances; or, a Method of Calculating the Probabilities of Events in Play." It contains calculations on many games, and among them several on Whist, treating of the probability of making tricks, and of holding trumps and honours, the odds at certain scores, etc.

When, twenty-five years later, the immortal Edmond Hoyle consolidated the game, he paid particular attention to this matter. He was not satisfied with laying great stress on the use of probabilities in the celebrated book that he used for teaching, but he took the trouble to write a special Essay on the subject. The work is rare, and the copy before us is entitled—

"An Essay towards making the Doctrine of Chances easy to those who understand vulgar Arithmetic only. To which are added some useful tables on annuities for lives, etc., etc.

By Mr. Hoyle. A new edition, corrected. London: Printed for T. Osborne, in Gray's Inn; and R. Baldwin, at the Rose, in Paternoster Row. 1764. Price Two shillings and six pence."

It contains, in the first place, some general explanations regarding combinations and permutations, and it then gives, explained in full arithmetic, the modes of calculating various problems referring to Picquet, All-Fours, Whist, Dice, Lotteries, and Annuities. The problems for Whist, explained in this way, are but few and simple, and many of them are only matters of curiosity. The odds at certain scores, and the chances of holding honours, are stated without any explanation of the calculations; so that, as regards this game, Hoyle's book hardly bears out the expectation that might be raised by its title.

In 1843 an excellent little book on Whist was published in Vienna, which contained a very fair Essay on Whist Probabilities; and about twenty years later some few calculations were given on the same subject in a small English *brochure* under the name of "Reuben Roy." But although the calculus of probabilities is so intimately connected with Whist, it has never hitherto been made a feature in any modern English treatise on the game.

The object of this Part of the present work is to explain generally the modes of calculation used in questions of this nature, and to apply them to a considerable number of Whist problems of interest and importance.

And it is proposed to add a somewhat novel feature to these calculations—namely, to give, in regard to some of them, an account of various experimental trials, made for the purpose of ascertaining (so far as a limited number of observations can do) to what extent the predictions due to calculation are borne out in the actual practice of Whist play.

Many eminent philosophers have thought it worth their while to ascertain, by actual experiment, how far the probabilities of certain events, as predicted by theory, have been verified by the results actually obtained in practice. Buffon, for example, tossed a penny some thousands of times, carefully recording the results, and Professor De Morgan repeated his experiments (see *Athenæum*, June 23, 1866). The throwing of dice, the drawing of coloured balls from a bag, and many other expedients, have been resorted to, which were capable of both theoretical computation and practical trial; a favourite one among mathematicians being to let a short stick fall at hazard on a boarded floor, and to notice how often in a given large number of trials it would fall across the joints of the boarding. It is curious that the investigation of this problem involves, as an element of the probability, the ratio between the diameter and the circumference of the circle, and its determination by experiment has been facetiously called by Mr. De Morgan "squaring the circle by pitch and toss."

But Whist combinations offer a greater variety than the more commonly adopted chance problems, and form therefore a better means of checking and illustrating the application of the theoretical laws.

CHAPTER II.

ELEMENTARY PRINCIPLES.

THE usual mode of expressing the probability that an event will happen in a particular way, is in the form of a fraction, the denominator of which represents the total number of ways (all supposed equally probable) in which the event may occur, and the numerator of which represents the number, out of these, which will give the result required. For example, the probability of drawing a king from a pack of cards is expressed by the fraction $\frac{4}{52}$ $(=\frac{1}{13})$; for either of fifty-two cards may be drawn, and there are four of these which will give the result required.

Now, this probability fraction, which it will be convenient to denote generally by $\frac{N}{D}$, has the sense of a *prediction*. It implies that out of D occurrences of the event, it ought theoretically to occur N times in the way referred to. Thus, in fifty-two drawings, we ought to draw a king four times, or in thirteen drawings we ought to draw a king once. The question then naturally arises—Will this prediction be verified in practice? The answer most immediately presenting itself is in the negative; for if any one tries the experiment by drawing a card thirteen times from the pack, it will be quite uncertain whether he may draw a king once, or twice, or three or four times, or not at all. But let us make a simple alteration in the form of the fraction, by increasing both numerator and denominator in a certain proportion, which, as every schoolboy knows, does not alter its real value. Say that we make it $\frac{100}{1300}$. This will mean that if we draw a card from the pack 1300 times, 100 times of these it ought to come a king. Now, if this be tried, although it may not come exactly true, it will probably be much nearer the truth, proportionally, than the former trial; and if we increase the numbers still more, making the fraction, say $\frac{1,000,000}{13,000,000}$, the accordance between theory and experiment, though it may not even yet be absolutely exact, may be expected to approach much nearer than before—so near, in fact, as to amount to a correspondence between the two in a practical sense of the word.*

Thus, though the mathematical prediction may, and probably will, fail in a small number of trials, yet as we continue them longer the divergence will constantly tend to become less and less, till it

* In 20,000 cuts for deal, as mentioned in Chapter XVIII., one player's side won 9,852 times, and lost 10,148 times, the theoretical probability giving in each case 10,000.

reaches such insignificant proportions that it may be neglected altogether, and the prediction will be practically confirmed.

It is a curious question how this comes to pass, considering how capricious and irregular the phenomena of chance are; it is indeed very remarkable that, by simply taking a large number of cases together, we should find them marshal themselves into an order and regularity which one would suppose quite foreign to their nature and essence. We do not offer any explanation of the fact, for such an explanation would involve metaphysical questions on which logicians are far from agreed; and it is by no means certain that the thing even admits of any explanation which is reasonably intelligible. All that can be said is, we know from experience that it is so. There are great varieties of chance events constantly going on in common life which are sufficient to prove the proposition; but, as these often require careful investigation to trace the results from them, there is always a great interest attaching to more direct experiments, as tending to bring the proof more immediately home to the popular mind. If anybody would be at the trouble of drawing a card thirteen millions of times, and keeping an exact account of how many times he drew a king, the fact, simple as it is, would be considered a scientific note worthy of enduring record, and forming a valuable contribution to the practical demonstration of the correctness of the theoretical laws.

Many of the results hereafter given are of this character, and are made out with this object; only they will derive more interest from the great variety of the conditions, and from the more elaborate nature of the calculations they are intended to illustrate and confirm.

Recurring to the probability fraction $\frac{N}{D}$, it may be further remarked that we may give to D any value we please (altering N accordingly), and we may thus adapt the prediction to any number of cases selected for trial.

Thus, suppose it is desired to know how often in 10,000 deals, we ought to have any named card, say the knave of diamonds, turned up? The probability of the event being expressed by $\frac{1}{52}$, we have to put 10,000 in the denominator, which, keeping the same value of the fraction, will give $\frac{192}{10,000}$ nearly; that is, it ought to occur about 192 times.

This probability fraction may also be easily converted into the ordinary form of what are called "odds," by recollecting that the odds will be always $(D-N)$ to N against the result occuring, or N to $(D-N)$ in its favour. Thus, the probability of drawing a king from the pack is $=\frac{1}{13}$; here $N=1$ and $D=13$, so that the odds against it will be $(D-N)$ to N—*i.e.*, 12 to 1.

The converse proposition is often useful, namely, to convert "odds" into the form of probabilities. The rule for this is, that if it be m to n against anything happening, or n to m in its favour, the probability of its occurrence is $\frac{n}{m+n}$. Thus, if we know that the odds are 6 to 1 against any given horse winning the Derby, the probability, scientifically expressed, that that horse will win is $\frac{1}{6+1}=\frac{1}{7}$.

We shall further have occasion to make use of certain rules which affect *combinations* of probabilities. These are as follows:—

The probability of the *concurrence* of two independent events is equal to the *product* of their separate probabilities. Thus, if I draw a card from each of two separate packs, the probability that I shall draw a king from each is $\frac{1}{13}$; hence the probability that I shall draw a king from *both* is $=\frac{1}{13}\times\frac{1}{13}=\frac{1}{169}$, or 168 to 1 against it.

Then, again, the probability that one out of several events will happen (it being a condition that not more than one can happen, which makes them what are called "conflicting" events), is equal to the *sum* of their probabilities. Thus, the probability that a card I draw from a pack will be either a king or a queen will be $=\frac{1}{13}+\frac{1}{13}=\frac{2}{13}$.

The united chances of all the different ways a thing may happen (provided one of them *must* happen) are equivalent to *certainty*, which is expressed in the language of probabilities by unity or 1. Thus, if I draw a card from a pack, it is a certainty that I shall draw either a spade, a heart, a club, or a diamond. The probability of drawing a spade is $\frac{13}{52}=\frac{1}{4}$; a club, $\frac{1}{4}$; a heart, $\frac{1}{4}$; and a diamond, $\frac{1}{4}$. These four added together are = 1. This result is always obtained by adding together the chances of all the possible ways of a thing happening, and hence certainty is expressed by 1.

If it is an *even chance* that an event will or will not happen (as in tossing a penny, where it is an even chance whether a head or a tail may come), each chance is the half of certainty—consequently an even chance is expressed by the fraction $\frac{1}{2}$, or the decimal 0·5.

CHAPTER III.

PROBABILITY OF WINNING THE RUBBER AFTER HAVING WON THE FIRST GAME.

We may now apply these rules to a simple case of very common occurrence in Whist, respecting which there is much misunderstanding; this is *the probability of winning the rubber after having won the first game.* The general opinion seems to be that the odds in its favour are 5 to 2 (equal to a probability of $\frac{5}{7}$). It has often been stated that this opinion is incorrect; but these odds are still often taken.

Let us therefore enquire, according to the principles laid down, what the probability really is. It will of course be affected in some degree by the comparative skill of the players, as well as by the fact of who has the first deal in the second game; but we will for the present ignore these, and assume that each party has an equal chance of winning any game not already commenced. We will call the players who have won the first game AB, and their opponents YZ.

The question then is, What is the probability of one of two things occurring:—either (1) of AB winning the game about to be played; or (2) of their losing this, and winning the next one? Either of these will obviously cause them to win the rubber; and, by our rule, to find the probability of one of them occurring, we must first find the probability of each separately, and then add the results together.

1. The probability of AB winning the game about to be played must be taken $=\frac{1}{2}$, according to the assumption we have made of either party having an even chance of winning either game.

2. To find the probability of the other element is a little more complicated, as it involves the *concurrence* of two conditions, *i.e.*, losing the first game and winning the second.

Now, by our hypothesis, the probability of each of these is $=\frac{1}{2}$; and by our rule the probability that they, being independent, will both concur, will be $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$.

Finally, the sum of these two probabilities is $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$, which expresses the true probability of AB winning the rubber.

Converting this into the shape of odds, we find it gives 3 to 1 in favour of AB, or 6 to 2 instead of 5 to 2, the odds most commonly laid.

The problem may be solved more simply by putting the question in another way, thus: What is the probability of YZ winning the rubber, they having lost the first game?

In order to win the rubber they must win two games successively. Now, the probability of their winning each of these separately is $= \frac{1}{2}$, and the probability of the concurrence of both independent events is $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, giving the odds 3 to 1 against them, as found by the former mode of calculation. And we may check these results by observing that, since one of the two parties *must* win the rubber, the sum of the probabilities of each must be equal to certainty, which is evidently true, as $\frac{3}{4} + \frac{1}{4} = 1$.

In the above calculation we have ignored the two disturbing elements of deal and superior skill, but we will now endeavour to include them; remarking, however, at the same time, that the data are imperfect, and that the results obtained must only be considered as a rough approximation, introduced rather for illustration than as absolutely to be relied on.

First as to the *deal.* The odds in favour of the dealer's winning the game about to commence have been variously estimated by different authorities; but they will be found fully treated of in Chapter XVII. According to the elaborate and valuable series of experiments and calculations there given, the odds on the game, at love-all, come out at about 11 to 10 in favour of the deal. This gives the probability of the dealer's side winning the second game of the rubber $= \frac{11}{21}$, and of their losing it $= \frac{10}{21}$. The probability of the third game may be taken $= \frac{1}{2}$ as before, as it is unknown how the first deal of it may fall.* Working this out on the same plan as before, but substituting the amended values, we obtain for the probability of the winners of the first game winning the rubber:

	Prob.	Odds.
With the deal at the commencement of the second game †	0·762	$3\frac{1}{4}$ to 1
Against the deal	0·738	$2\frac{3}{4}$ to 1

Secondly, as to the disturbing effect of superior *skill*. This is a much more difficult thing to reduce to figures than the effect of the deal, but it may suffice here, for the sake of illustration, to take it at a maximum, for two very good against two very bad players, which may be assumed about the same as the odds on the deal, viz., 11 to 10. This would give, approximately, for the probability of winning a game about to commence,

With skill and deal	$= \frac{23}{42}$
With either, ignoring the other	$= \frac{22}{42}$
With either, against the other	$= \frac{21}{42}$
Against either, ignoring the other	$= \frac{20}{42}$
Against both skill and deal	$= \frac{19}{42}$

Working the problem out again with these data, we obtain the results in the following table.

* This is not strictly correct, for "Cavendish" (*Field*, Oct. 2, 1869) has shown that the chances are in favour of a game occupying an odd number of deals.

† "Cavendish" gives

£3 : 3 : 0 to £1 : 0 : 0 on the deal.
£2 : 17 : 0 to £1 : 0 : 0 against the deal.

PROBABILITY OF THE RUBBER BEING WON BY THE WINNERS OF THE FIRST GAME.

	Probability.		Odds in favour.
(1) Ignoring both deal and skill	0·750	...	3 to 1
(2) With equal skill on both sides, but making allowance for the position of the deal at commencement of second game—			
With the deal	0·762	...	$3\frac{1}{4}$ to 1
Against the deal	0·738	...	$2\frac{3}{4}$ to 1
(3) Ignoring the position of the deal, but allowing for a difference of skill equivalent to the odds of 11 to 10—			
With the skill	0·773	...	$3\frac{1}{2}$ to 1
Against the skill	0·726	...	$2\frac{2}{3}$ to 1
(4) Allowing for both deal and skill—			
With both skill and deal	0·784	...	$3\frac{3}{4}$ to 1
With skill against deal	0·762	...	$3\frac{1}{4}$ to 1
Against skill with deal	0·738	...	$2\frac{3}{4}$ to 1
Against both skill and deal	0·713	...	$2\frac{1}{3}$ to 1

The general correctness of these calculations has been verified by experience. A very eminent Whist-player has kindly enabled the author to state the result of thirty years' experience, comprising between 30,000 and 40,000 bets. These were all made with either deal or skill, or both, in favour, but never against both skill and deal. The result gives the odds on the average over 3 to 1, the theoretical average of the three cases being 0·766, or about $3\frac{1}{4}$ to 1.

We have further been favoured, through "Cavendish," with the actual records, kept for this purpose, of a considerable number of rubbers, which have given the following results. The winners of the first game are denoted always by AB, although they were not always the same individuals.

First Game. Won by	Second Game. Won by	Third Game. Won by	Number of Rubbers.
AB	AB	—	528
AB	YZ	AB	260
Rubbers won by AB			788
AB	YZ	YZ	236
Total rubbers played			1024

No record was kept of either deal or skill; probably they were equally distributed, and may be ignored. On this assumption, out of these 1024 rubbers the winners of the first game ought, according to the calculated probability of $\frac{3}{4}$, to have won 768; the actual fact was, that they won 788.

A further example is given in Mr. Perceval's data, quoted in Chapter XVIII. His results, with skill, but ignoring the deal, corresponded to the probability 0·764; against skill, to 0·742. The value of skill, here, comes out less than assumed in the above calculations.

It is clear, therefore, both from theory and experiment, that the taker of a bet of £5 to £2 on the rubber, after the first game, is at a considerable disadvantage, unless in the case where both deal and skill are against the winners of the first game. The fair odds are £3 to £1, which should be increased if both deal and skill are in favour, and diminished in the contrary case.

CHAPTER IV.

PROBABILITY OF A PLAYER HOLDING CERTAIN CARDS.

We now pass on to cases of probabilities involving various arrangements and combinations of the cards; and as the first example, we may take a problem that is of continual application in actual play, and which is generally stated as follows:—

At the Commencement of the Hand, what is the probability of your Partner holding certain Cards which you do not hold?

It must be remarked that although, in this statement of the question, "your partner" is selected for the sake of convenience, the calculation holds equally good for either of the adversaries.

This problem was investigated by Hoyle in his work on Chances, and the results he obtained have been since stated in several Whist treatises; but the example is an instructive one, and worth repeating.

The probability is affected by the position of the deal, on account of the influence of the trump card.

We will take first the simplest case, namely, where the deal is with the known hand, and will afterwards show the effect of variation in this particular.

The calculations may be worked out for any number of cards, but it will be sufficient for our purpose to give them first for one, and then for two.

For One Card.

Here the question is, What is the probability of your partner holding a given card, say the ace of hearts, which you do not hold?

Suppose the cards of the three unknown hands laid out singly on the table face downwards, and each card numbered, as in the following diagram:—

14, 15, 16, 17, 18, 19. 20, 21, 22, 23, 24, 25, 26
Partner's Hand.

Left Adversary's Hand: 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1

Right Adversary's Hand: 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39

Known Hand.

Now, first, it is clear that the ace of hearts may be any one of these thirty-nine unknown cards;

and there is no reason why it should be any one in preference to another. We have, in fact, thirty-nine possible events, all equally probable, and hence in our fraction $\frac{N}{D}$ the denominator, D, = 39.

Then, secondly, if it happens to be either of those numbered 14 to 26, it will be in the partner's hand; and hence, out of the thirty-nine possible events, there are thirteen that will give the result required, and we have the numerator, N, = 13.

Therefore the probability that the ace of hearts lies in the partner's hand will be $= \frac{13}{39} = \frac{1}{3}$; or 2 to 1 against it.

For Two Cards.

This form of the problem is more complicated, as it involves four cases. Let us call the two given cards A and B respectively. Then the enquiry is, What are the probabilities of the partner holding—

1. Both A and B;
2. A alone;
3. B alone;
4. Neither of them.

Now there are two easy, and at first sight plausible, ways of finding these probabilities, which it will be well to look at, as they afford a useful lesson as to the care and consideration that chance problems require, and show how easy it is to fall into error with them.

First, it may be said that, as the probability of holding *one* card has been already found, this will be the same thing as the chance of holding either A or B singly, namely $= \frac{1}{3}$ for each; and further, that the chance of holding both must be the *concurrence* of these chances $= \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$. Hence, the probabilities of the first three cases being known, the chance of the fourth would be found by deducting their sum from unity, $= \frac{2}{9}$. But this calculation is erroneous, because the chance of holding one only out of two given cards is not the same as that of holding a single given card; for the former case really involves, in addition to the condition of holding the card A, the extra condition of *not holding* the other card B. Hence the same rule will not apply.

Another mode of calculating the problem has more pretension. It may be said there are nine possible ways in which the two cards may lie, thus:—

Left opponent's hand.	Partner's hand.	Right opponent's hand.
A B	—	—
A	B	—
A	—	B
B	A	—
B	—	A
—	A B	—
—	A	B
—	B	A
—	—	A B

Here there is *one* case which will give *both* cards in partner's hand; two which will give A alone, two B alone, and four neither; whence it might be argued that, by this rule:—

Probability of holding both	$=\frac{1}{9}$
Probability of holding A only	$=\frac{2}{9}$
Probability of holding B only	$=\frac{2}{9}$
Probability of holding neither	$=\frac{4}{9}$

This is nearer the truth than the former computation, but it is still wrong, for the reason that *all the nine ways are not equally probable.* The chance of both cards being in any one hand is less than that of any of the other modes of distribution. Hence the rule fails.

The correct way of working the problem is to adopt the method followed for the single card. Reverting to the diagram given for that case, we have to find—first, in how many different ways the two cards may lie; and, secondly, how many of these ways will favour the particular cases whose probabilities we wish to know.

The card A may obviously be either of the 39. Let it first be No 1. Then B may be either No. 2 or No. 3 or No. 4, and so on to 39, giving 38 different ways.

Next let the card A be No. 2. Here B may be either No. 1, or No. 3, or No. 4, &c., to 39, also giving 38 ways. Making A No. 3, we also get 38 ways, and so on, for the whole 39 different places where we may suppose A to lie.

Hence the total number of different ways in which the two cards may be disposed will be $= 39 \times 38 = 1482$.

We have now to enquire how many of these ways will favour the four several cases named, in regard to the partner's hand.

First, as to both cards. Let A be No. 14; then B may be either of the 12 others in that hand; and by repeating this process for every other of the 13 positions of A in that hand, we get $13 \times 12 = 156$ varieties of position, both cards being in that hand. The probability of this is therefore $= \frac{156}{1482} = \frac{6}{57}$.

Next, as to the probability of holding A only. Let A, as before, be No. 14, then B may be either No. 1, No. 2, &c., to No. 13, or No. 26, 27, &c., to 39—in all 26 positions of B for one of A. Repeating this for all the 13 positions of A, we have $13 \times 26 = 338$, and the probability of the partner's holding A alone becomes $= \frac{338}{1482} = \frac{13}{57}$. The same result also obtains for B alone.

Finally, to find the chance of neither of the cards being in the partner's hand, we must make A either of the other 26 cards (1 to 13, and 27 to 39), to each of which there will be 25 corresponding positions of B; so that the number of different ways this can happen will be $26 \times 25 = 650$, and the probability $= \frac{650}{1482} = \frac{25}{57}$.

To check these results we may consider that the partner *must* either hold both, or A alone, or B alone, or none; wherefore the sum of the whole of the probabilities ought to be certainty. We accordingly find $\frac{6}{57}+\frac{13}{57}+\frac{13}{57}+\frac{25}{57}=\frac{57}{57}=1$. The chances are, therefore, as follows, ignoring the turn-up card:—

	Probability.	Odds.
Probability that your partner will hold both of two named cards, A and B, which are not in your own hand	$\frac{6}{57}$	$8\frac{1}{2}$ to 1 against.
That he will hold A only...	$\frac{13}{57}$	44 to 13 against ($3\frac{1}{2}$ to 1 nearly).
That he will hold B only...	The same.	
That he will hold neither of them	$\frac{25}{57}$	32 to 25 against (5 to 4 nearly).
That he will hold either A or B ($\frac{13}{57}+\frac{13}{57}$)	$\frac{26}{57}$	31 to 26 against (6 to 5 nearly).
That he will hold either A or B, or both ($\frac{26}{57}+\frac{6}{57}$)...	$\frac{32}{57}$	32 to 25 in favour (5 to 4 nearly).
(The same probabilities will apply to the hand of either adversary).		

The author undertook a series of experiments to see how nearly these probabilities came out in practice, by dealing out and noting 741 hands. The results came very close, as will be seen in the following table; they are given for every batch of 57 hands, to show the variations of luck. The cards were not played, and may be taken as thoroughly shuffled. After each deal the observer took up one hand, fixed on two cards not in it, and then examined the opposite hand for them.

NUMBER OF TIMES, IN EACH BATCH OF 57 DEALS, THAT THE PARTNER HELD,

	Both A and B	Either A or B alone.	Neither.
By calculation	6	26	25
In experiments	6	18	33
	6	30	21
	9	23	25
	4	25	28
	9	21	27
	5	27	25
	2	30	25
	5	27	25
	8	23	26
	5	29	23
	5	25	27
	4	27	26
	6	32	19
Total by experiment	74	337	330
Total by calculation	78	338	325

The influence of the turn-up (or any other card being *known*) on the partner's or opponents' hands, is easily found by noting it as one of the cards in the diagram, corresponding to the hand it is in, and then simply excluding it from the calculation. The effect will be shown by the following comparative table, where the probabilities are given in decimals :—

Probability of your partner holding.	Your own deal.	Your partner's deal.	Opponents' deal.
Both	0·105	0·094	0·112
A alone	0·228	0·222	0·231
B alone	0·228	0·222	0·231
Neither	0·439	0·462	0·426
	1·000	1·000	1·000
A or B	0·456	0·444	0·462
A or B or both	0·561	0·538	0·574

It must be observed that all these calculations only hold good at the commencement of the hand. As the play advances the data are altered, not only by the alterations of the numbers of cards in each hand, but also by the fact that, by the indications of play, some of the cards in the other three hands must become *known*. It is impossible to lay down any rules for these contingencies; the calculated probabilities will always be some guide, but they must not be implicitly trusted to when they no longer apply.

CHAPTER V.

RULES FOR COMBINATIONS AND PERMUTATIONS, WITH EXAMPLES.

In the following problems we shall often have occasion to use some of the formulæ for combinations and permutations. It would lead us too far from our object to attempt to explain these; it must therefore suffice simply to state them.

The most useful one is the following, which is taken from "Todhunter's Algebra:"

To find the number of combinations of n things taken r at a time. This is given in algebraical terms–

$$=\frac{n\,(n-1)\,(n-2)\quad.\quad.\quad.\quad.\quad.\quad.\quad(n-r+1)}{1.2.3\;.\;.\;.\;.\;.\;.\;r}$$

The way this becomes applicable to questions of the kind we are now considering is as follows:

Suppose we have a given number of cards before us, which number we will call n; and suppose we wish to select out of these, for any particular purpose, a certain smaller number, which we will represent by r. Then the number of different ways in which we may make the selection, without ever getting, twice over, the same combination of cards selected, will be found by the formula above given.

This formula, for facility of future reference, may be called the "formula of selection." A few examples will make its application easily intelligible.

Suppose we place before us the four kings; let us enquire in how many ways we could take two kings from them into our hand without repeating the same combination. Here $n = 4$ and $r = 2$, and the rule will give $\frac{4 \times 3}{1 \times 2} = \frac{12}{2} = 6$ ways, as anybody may convince himself by trying. This is well known to be the number of pairs that can be made with four cards at Cribbage.

At Cribbage, again, if we happen to have four fives shown, how many fifteens can we count out of them? Here $n = 4$, and, as we must have three of the cards to make each fifteen, $r = 3$. Hence the number of ways we can get three cards without using the same combination twice will be by the rule $\frac{4 \times 3 \times 2}{1 \times 2 \times 3} = 4$, scoring eight for the fifteens.

Again, let it be asked in how many different ways we may take 4 spades out of the suit of 13. Here $n = 13$, and $r = 4$, therefore the required number $= \frac{13 \cdot 12 \cdot 11 \cdot 10}{1 \cdot 2 \cdot 3 \cdot 4} = 715$.

Another example may be taken from the game of Écarté. Suppose it be asked in how many different ways your opponent's hand may be constituted? The total number of cards used is 32; of these six are known—namely, your own five and the turn-up, leaving 26 for the opponent's hand to be chosen from. Here, therefore, $n=26$ and $r=5$; and the formula becomes

$$\frac{26 \,.\, 25 \,.\, 24 \,.\, 23 \,.\, 22}{1 \,.\; 2 \,.\; 3 \,.\; 4 \,.\; 5} = 65{,}780.$$

Finally, having given the whole pack of 52 cards, let us ask in how many ways we may take a hand of 13 out of them, without ever repeating the same combination of cards. Here $n=52$, and $r=13$; and our formula becomes

$$\frac{52 \,.\, 51 \,.\, 50 \ldots\ldots\ldots 40}{1 \,.\; 2 \,.\; 3 \ldots\ldots\ldots 13}$$

which being worked out gives the large number

635,013,559,600.

This, therefore, is the *number of different hands which any single player at Whist may obtain;*—a number we shall have occasion to refer to hereafter.

And while we have it before us, we may as well allude to a kindred problem, which, though it is of no practical use, has often excited great curiosity, and upon which there has been some discussion—

namely, the question, *How many different combinations can be held by four persons playing at Whist?*

The true mode of solution was pointed out in *The Field* of June 10, 1865, and is as follows:—

Let the four players be called A, B, C, and D, and let us for the present ignore the turn-up card.

In the first place, we have already found how many different hands may be held by one player, say the one called A; this is the large number of 12 figures above given.

Now, it is clear that for every one of these hands held by A, the player B may hold any hand which contains no card of A's—that is to say, B can hold any hand which can be taken out of the remaining 39 cards. Referring therefore again to our formula of selection, we have in this case $n = 39$ and $r = 13$. So that the number of different hands B may hold will be $= \frac{39 \cdot 38 \cdot 37 \ldots\ldots 27}{1 \cdot 2 \cdot 3 \ldots\ldots 13}$, which when worked out gives the number

8,122,425,444.

Thirdly, while A and B hold any of these hands, C may hold any hand which can be made out of the remaining 26 cards; this we find $= \frac{26 \cdot 25 \cdot 24 \ldots\ldots 14}{1 \cdot 2 \cdot 3 \ldots\ldots 13}$, amounting to

10,400,600.

We may ignore the fourth player (D) altogether, as it is clear that after the other three hands are provided for, there can be only one combination left for him.

The total number of different ways, therefore, in which the 52 cards may be distributed among the four players, will be the product of the three large numbers above arrived at, and which amounts to the number

53,644,737,765,488,792,839,237,440,000.

This number, however, does not represent all the varieties that might occur in Whist. They would be increased by considering the trump card, as, in the same distribution of the hands, a different card being turned up would give rise to a different variety of game. A further variety would also be caused by the interchangeability of the respective positions of the four hands. But it is needless to go into these *minutiæ;* the above number is so vast that it conveys no intelligible idea to our minds, beyond that of the practically infinite variety of which the game is capable.

CHAPTER VI.

PROBABILITY OF A PLAYER HOLDING A GIVEN NUMBER OF CARDS OF A GIVEN SUIT.

We may now consider a problem which admits of a wide range of chance results—namely, *What is the probability of holding a given number of cards of a given suit?*

To avoid the disturbing effect of the turn-up card, the suit must be named before the deal is concluded. For example, what is the probability, before the trump is turned up, of my holding m hearts, and no more—*i.e.*, m hearts exactly?

Referring to the general fraction of probability, $\frac{N}{D}$, and recollecting that any combination of thirteen cards is equally likely to occur, the probability of any given player holding exactly m hearts is

$$= \frac{\text{No. of hands possible which will contain exactly } m \text{ hearts}}{\text{Total number of hands possible.}}$$

We have already found the denominator of this fraction in Chapter V. ; it amounts to

635,013,559,600.

This is a general denominator, which we shall have to use often.

Next, as to the numerator, which is more troublesome to get at. We will take a special case as an example, as this will illustrate the process more clearly.

Let it be asked what is the probability of holding three hearts. We have then to find, for the numerator, how many different hands are possible, each containing three hearts. The hand consists of two portions—viz., the three hearts and the ten other cards—and each of these divisions must be considered separately.

First, in how many ways may we get three hearts? For this we must use the "formula of selection." As there are thirteen hearts in all, and we have to select three from them, $n = 13$ and $r = 3$, and the formula becomes $= \frac{13 . 12 . 11}{1 . 2 . 3}$.

Secondly, in how many ways may we get the other ten cards? Here there are thirty-nine cards, not hearts, left to select from, whence $n = 39$ and $r = 10$, which gives $= \frac{39 \cdot 38 \ldots\ldots 30}{1 . 2 \ldots\ldots 10}$.

Now as each separate combination of the hearts may exist along with any one of the combinations of the other part of the hand, these two results must be multiplied together, so that the number of different hands a player may get—each containing three hearts, and no more—will be $= \frac{13 . 12 . 11}{1 . 2 . 3} \times \frac{39 \cdot 38 \ldots\ldots 30}{1 . 2 \ldots\ldots 10}$, which being worked out gives a number very near 181,900,000,000, and this is the numerator sought.

Thus the probability of any particular player holding exactly three cards of any suit named before the deal, is, in round numbers $= \frac{181,900,000,000}{635,000,000,000}$ or $= \frac{287}{1000}$ nearly—*i.e.*, he ought to hold them 287 times in 1000 deals.

The following table shows the results for every number of cards calculated in this way:—

Probability of a given Player holding any given Number of Cards of a particular Suit, named before the Deal is concluded.

Cards.					
0	will be held by him about once in			80 deals	
1	,,	,,	80 times in	1000	,,
2	,,	,,	206 ,,	,,	,,
3	,,	,,	287 ,,	,,	,,
4	,,	,,	239 ,,	,,	,,
5	,,	,,	125 ,,	,,	,,
6	,,	,,	once in	24	,,
7	,,	,,	,,	114	,,
8	,,	,,	,,	860	,,
9	,,	,,	,,	12,100	,,
10	,,	,,	,,	243,000	,,
11	,,	,,	,,	11,000,000	,,
12	,,	,,	,,	1,250,000,000	,,
13	,,	,,	,,	635,000,000,000	,,

These have not been tested by experiment, for it affords more variety to direct attention specially to the trump suit, after the trump card is turned up. The calculations and experiments on this will follow in the next chapter.

CHAPTER VII.

PROBABILITY OF A CERTAIN NUMBER OF TRUMPS BEING HELD BY THE DEALER AND BY THE NON-DEALERS RESPECTIVELY.

THE peculiarity of this case, as distinguished from the last, is that one particular player (the dealer) has a *certainty* of getting one card of that suit (the turn-up), which puts him in a different category from the three non-dealers as regards the probabilities of holding cards in it. Hence the cases of dealer and non-dealer must be treated separately, and we will take, as before, the number 3 as an example in each case.

Dealer.—Having one card certain, he has to receive twelve other cards out of fifty-one, and the total number of ways he may get these will be $\frac{51 \cdot 50 \ldots\ldots 40}{1 \cdot 2 \ldots\ldots 12} = 158,750,000,000$, which is the denominator of the probability fraction.

To find the numerator, we must consider that he may get two other trumps (three in all), out of the twelve remaining to be disposed of, in $\frac{12 \cdot 11}{1 \cdot 2}$ different ways, and that the ten other cards in his hand may be obtained in $\frac{39 \cdot 38 \ldots\ldots 30}{1 \cdot 2 \ldots\ldots 10}$ ways as before, the product of which two sums will amount, when worked out, to about 41,958,000,000; so that the dealer's probability of holding three trumps, including the turn-up card, will be $= \frac{41,958,000,000}{158,750,000,000}$, or about $\frac{264}{1000}$.

Non-dealer.—As regards any non-dealer, the number of cards available is 51, and out of these the total number of hands he may get is $= \frac{51 \cdot 50 \ldots\ldots 39}{1 \cdot 2 \ldots\ldots 13}$, or about 476,266,700,000, the denominator of the fraction.

To find the numerator proceed as before, recollecting that there are 12 trumps, and 39 other cards to select from. The number of different hands he may get containing three trumps will thus be $= \frac{12 \cdot 11 \cdot 10}{1 \cdot 2 \cdot 3} \times \frac{39 \cdot 38 \ldots\ldots 30}{1 \cdot 2 \ldots\ldots 10}$, which comes out about 139,160,000,000. Hence the probability of a non-dealer receiving three trumps will be $= \frac{139,860,000,000}{476,266,700,000}$, or about $\frac{294}{1000}$.

In this way the probabilities have been calculated for all the various numbers of trumps, both for dealer and non-dealer, and the results of the calculation will be found in the following tables.

To test these calculated probabilities, the author undertook a series of observations on the actual contents of the various hands in 1000 deals, made specially for the purpose. Great care was taken to have the cards well shuffled each deal; but, as they were not played, there was no tendency for them to arrange themselves into suits. After shuffling and cutting, the four hands were dealt and a trump turned up, just as in actual play, after which the number of trumps in each hand was noted down, the cards were shuffled together, and another deal was made in like manner.

The following tables show the comparative results.

NUMBER OF TRUMPS HELD BY THE DEALER, AS DETERMINED BY THEORY AND BY EXPERIMENT.

	Number of times held in 1000 deals. As predicted by calculation.	As found by experiment.
One trump and no more	25	27
Two trumps ,,	126	115
Three ,, ,,	264	284
Four ,, ,,	293	295
Five ,, ,,	192	175
Six ,, ,,	77	80
Seven ,, ,,	19	22
Eight ,, ,,	3	2
Nine and more	1	0
	1000	1000

NUMBER OF TRUMPS HELD BY THE NON-DEALERS, AS DETERMINED BY THEORY AND BY EXPERIMENT.

	Number of times held in 3000 deals. As predicted by calculation.	As found by experiment.
No trumps	51	53
One trump and no more	294	288
Two trumps ,,	700	695
Three ,, ,,	882	886
Four ,, ,,	660	661
Five ,, ,,	306	316
Six ,, ,,	90	84
Seven ,, ,,	16	16
Eight ,, ,,	1	1
Nine and more	0	0
	3000	3000

It will be noticed that in the second table the correspondence is closer than in the first, as might be naturally expected from the larger number of cases observed. In either table, however, the general agreement is remarkable. It must be explained that in the second table the 3000 data are not all *independent* observations, from the fact that three hands were always combined together in one deal, which would make them to some extent dependent on each other. They are, however, strictly what they profess to be—3000 *bonâ fide* observations on as many non-dealing hands.

The author has further tested the *total number* of trumps held in the 1000 deals by each player. The determination of this by theory is very easy; we have merely to multiply the number in the first column of either of the foregoing tables by the number of trumps it applies to, and to add the results together, which will give the total number that ought to be held in so many hands.

The following table shows the comparison be tween theory and experiment:—

TOTAL NUMBER OF TRUMPS HELD IN 1000 DEALS BY EACH PLAYER, AS DETERMINED BY THEORY AND BY EXPERIMENT.

	As predicted by calculation.		As found by experiment.
First hand	3060	....	3110
Second hand	3060	....	3119
Third hand	3060	....	2957
Fourth hand, or dealer ...	3820	....	3814
	13,000	....	13,000

Or which is the same thing—

AVERAGE NUMBER OF TRUMPS HELD IN EACH HAND.

By non-dealers...............	3·060	....	3·062
By dealers.....................	3·820	....	3·814

The correspondence here also is tolerably near, the discrepancy being in the worst case only about 3 per cent. But, after all, 1000 deals is not a large number to test probabilities by.

It is seen from this that, on the average of 1000 deals, one player might get $3\frac{1}{3}$ less per cent. than his proper share, which, calculating $5\frac{1}{2}$ points to a rubber, and assuming that the number of trumps formed an index to the success of the play, would be equal to a disadvantage of 1-5th or 1-6th of a point per rubber. On 100 deals it is found that the number will sometimes vary above 12 per cent. from the average, which would imply, on the same hypothesis, a disadvantage of 2-3rds of a point per rubber. In smaller numbers of deals it would, of course, tell more still.

The following is an altered, and perhaps more useful, form of the above Tables :—

PROBABLE NUMBER OF TRUMPS HELD BY A DEALER OR A NON-DEALER.

	Number of times in 1000 deals. Dealer.	Non-Dealer.
No trump	0	17
One trump or more	1000	983
Two trumps or more	975	885
Three ,,	846	951
Four ,,	585	375
Five ,,	292	137
Six ,,	100	35
Seven ,,	23	5
Eight ,,	4	—
Nine ,,	1	—

The question as to the probable number of trumps held often takes, in practice, the following form, which it will therefore be desirable to consider.

Holding a given number of trumps myself, I wish to form an idea (with a view to judging of the expediency of leading them) how many some other player may probably hold?

The answer to this will depend largely on the position of the deal; we will take a few examples to show the method of calculation.

(*a*) First, suppose it has been my deal, and that I hold five trumps. What is the probability of one of the other players (to be specified) also holding five?

Here there are eight trumps and 31 cards of other suits, 39 in all, available for his hand; therefore

$$D = \frac{39 \cdot 38 \ldots\ldots 27}{1 \cdot 2 \ldots\ldots 13},$$

$$\text{and } N = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \times \frac{31 \cdot 30 \ldots\ldots 24}{1 \cdot 2 \ldots\ldots 8},$$

$$\text{which gives } \frac{N}{D} \text{ about} = \frac{54}{1000}.$$

(*b*) Secondly, suppose I, not having dealt, hold five trumps; what will be the probability of the dealer holding five?

He has 12 unknown cards, in addition to the turn-up, and we have to calculate the probability of his holding four more trumps among these.

There are seven trumps, and 31 cards of other suits available = 38 in all; and we have, consequently,

$$D = \frac{38 \cdot 37 \ldots\ldots 27}{1 \cdot 2 \ldots\ldots 12},$$

$$\text{and } N = \frac{7 \cdot 6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3 \cdot 4} \times \frac{31 \cdot 30 \ldots\ldots 24}{1 \cdot 2 \ldots\ldots 8},$$

$$\text{which gives } \frac{N}{D} = \frac{102}{1000} \text{ nearly.}$$

(*c*) Thirdly, suppose I, as before, not having dealt, hold five trumps, what is the probability that another (specified) non-dealer will also hold five?

Here, omitting the turn-up, there are seven trumps and 31 other cards available; whence we have

$$D = \frac{38 \,.\, 37 \,.\, \ldots\ldots \,.\, 26}{1 \,.\, 2 \,.\, \ldots\ldots \,.\, 13},$$

$$\text{and } N = \frac{7 \,.\, 6 \,.\, 5 \,.\, 4 \,.\, 3}{1 \,.\, 2 \,.\, 3 \,.\, 4 \,.\, 5} \times \frac{31 \,.\, 30 \,.\, \ldots\ldots \,.\, 24}{1 \,.\, 2 \,.\, \ldots\ldots \,.\, 8},$$

$$\text{which gives } \frac{N}{D} = \frac{30}{1000}.$$

A number of cases most likely to be useful in practice have been calculated out in this way, and the results are given in the two following tables:—

If I, being the dealer, hold	*Then the number of times, in 100, the undermentioned number of trumps, exactly, should be held by any other (specified) player is*			
	FOUR TRUMPS.	FIVE TRUMPS.	SIX TRUMPS.	SEVEN TRUMPS.
1 trump	29	22	10	3
2 trumps	32	18	7	2
3 ,, ...	26	13	4	1
4 ,, ...	22	9	2	—
5 ,, ...	17	5	1	—
6 ,, ...	12	3	—	—
7 ,, ...	7	1	—	—

If I, not being dealer, hold	*Then the number of times, in 100, the undermentioned number of trumps, exactly, should be held by other players is*							
	By the Dealer.				By any (specified) Non-Dealer.			
	Four Trumps.	Five Trumps.	Six Trumps.	Seven Trumps.	Four Trumps.	Five Trumps.	Six Trumps.	Seven Trumps.
0 trump	27	30	20	8	29	23	11	3
1 ,, ...	29	28	16	5	29	19	8	2
2 trumps............	31	24	11	3	27	14	5	1
3 ,, ...	31	20	7	2	23	10	2	—
4 ,, ...	30	15	4	1	19	6	1	—
5 ,, ...	26	10	2	—	13	3	—	—
6 ,, ...	21	6	1	—	8	1	—	—
7 ,, ...	14	3	—	—	4	—	—	—

These tables shew what a very important effect the position of the deal has in the distribution of the trumps; a consideration often overlooked or much underrated.

Applications of these tables are shewn in pp. 37 to 39.

CHAPTER VIII.

PROBABILITY OF THE DISTRIBUTION OF THE TRUMPS IN THE FOUR HANDS; AND OF THE DISTRIBUTION OF THE FOUR SUITS IN ANY ONE HAND.

THIS problem is one much more complicated. The thirteen cards may be distributed among the four players in 39 different combinations of numbers. Eleven of these, involving more than 8 trumps in the hand of one player, have a very small probability. The other 28 will be seen from the table presently to be given.

To illustrate the mode of calculation, we will take an example, "What is the probability of the trumps being divided among the four hands, in the numbers 5, 4, 3, and 1 respectively?"

In this case we may ignore any effect from the turn-up card, the mode of calculation being adapted to any given suit indifferently. Call the suit hearts, and let us identify the four players as A, B, C, D respectively. The calculation has now five steps:—

1. The probability that A will hold exactly 5 hearts has been already determined — namely, $\frac{125}{1000}$.

2. What is now the probability that B will hold exactly 4 hearts. Supposing A's hand already to be taken in one of its combinations with 5 hearts and 8 other cards, B has only 8 hearts and 31 other cards available, 39 in all. Hence the total number of his possible combinations is $\frac{39 \cdot 38 \cdots\cdots 27}{1 \cdot 2 \cdots\cdots 13}$; and the number of combinations which will give him 4 hearts is

$$\frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4} \times \frac{31 \cdot 30 \cdots\cdots 23}{1 \cdot 2 \cdots\cdots 9}$$

3. C has now 4 hearts and 22 other cards to take from; and, proceeding on the same plan, we find the probability of his holding exactly 3 hearts is

$$\text{Denominator} = \frac{26 \cdot 25 \cdots\cdots 14}{1 \cdot 2 \cdots\cdots 13}$$

$$\text{Numerator}\ldots = \frac{4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3} \times \frac{22 \cdot 21 \cdots 13}{1 \cdot 2 \cdots 10}$$

D may be ignored, as after the three other players have taken their cards, he has only one combination left for him.

4. Now, to give the result we want, all the above three events must *concur;* the probability of which, by a well-known rule, is the product of the three multiplied together. Hence the probability that A will hold five hearts, B four, and C three (leaving D with one) is found by multiplying together the three results above obtained.

5. But we have still another thing to consider. We have found a certain probability that

A given player,	A,	will hold	5	hearts.
,,	B,	,,	4	,,
,,	C,	,,	3	,,
,,	D.	,,	1	,,

But it is just as probable that

B	will have	5
A	,,	4
C	,,	3
D	,,	1

and, in fact, there are twenty-four ways in which, by this sort of variation, the same result may occur, all of which are equally likely.

We must therefore here apply another rule, which teaches us that *the chance of one out of several events occurring, when all are equally probable, is equal to the sum of the probabilities of all;* so that we have to multiply the previous result by 24, which gives about $\frac{129}{1000}$ for the general probability of the hearts being distributed in these numbers.

The table about to be given comprises, however, another element—*i.e.*, *the distribution of the four suits in any one hand.* The calculation of this is easy. Take the same numbers as in the last example, 5, 4, 3, 1, and in the first instance let us identify the suits, as 5 hearts, 4 spades, 3 diamonds, 1 club. The denominator of the probability fraction will be as before, 635,000,000,000.

The number of ways we can obtain

$$\text{5 hearts is } = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 1287$$

$$\text{4 spades is } = \frac{13 \cdot 12 \cdot 11 \cdot 10}{1 \cdot 2 \cdot 3 \cdot 4} = 715$$

$$\text{3 diamonds is} = \frac{13 \cdot 12 \cdot 11}{1 \cdot 2 \cdot 3} = 286$$

$$\text{1 club is } = \frac{13}{1} = 13$$

and hence, as any one of these may exist with all the others. the number of hands in which they all exist will be the product of all multiplied together, which, when worked out, comes = 3,421,000,000 nearly, and the probability of holding the several numbers of the suits named respectively will be about $\frac{1}{186}$.

But our problem does not bind us to the identification of the suits for each number, and by leaving this free we can obtain, by changing the suits about to the different numbers, 24 different combinations, all equally probable, of which any one will answer our conditions; hence, multiplying by 24, we have the probability of the general numeral distribution $=\frac{129}{1000}$.

It will be noticed that this is exactly the result arrived at in the case of the distribution of trumps, and it is capable of demonstration that the *probability of any given distribution of a named suit in the four hands is the same as the probability of the same distribution of the four suits in one hand.*

On these principles the whole of the varieties of combinations have been calculated; and to check the calculation, 1000 hands have been experimentally observed. The results are shown in the following Table.

The first column shows the experimental results of the arrangements of the trumps in the four hands for 1000 deals; the second column those of the experimental arrangement in 1000 hands (250 deals) of the four suits in one hand; the third contains the sum of the two, being together equivalent to 2000 observations: and the fourth shows the calculated number.

Probabilities of Different Combinations of Cards in Hands at Whist.

Combinations.	EXPERIMENTAL RESULTS. Arrangement of Trumps in the four Hands. Number of times occurring in 1000 deals.	Arrangement of Suits in one Hand. Number of times occurring in 1000 hands.	Sum, giving the Result of 2000 observations.	CALCULATED RESULTS for 2000 times.
4 4 4 1	32	24	56	60
4 4 3 2	220	187	407	431
4 3 3 3	114	108	222	211
5 5 3 0	8	6	14	18
5 5 2 1	37	40	17	63
5 4 4 0	14	10	24	25
5 4 3 1	117	127	244	259
5 4 2 2	96	110	206	212
5 3 3 2	157	159	316	310
6 6 1 0	0	2	2	1½
6 5 2 0	7	5	12	13
6 5 1 1	8	8	16	14
6 4 3 0	17	14	31	26
6 4 2 1	47	58	105	94
6 3 3 1	32	24	56	69
6 3 2 2	53	63	116	113
7 6 0 0	0	0	0	—
7 5 1 0	1	4	5	2
7 4 2 0	2	6	8	7
7 4 1 1	2	6	8	8
7 3 3 0	2	7	9	5
7 3 2 1	26	15	41	38
7 2 2 2	5	9	14	10
8 5 0 0	—	1	1	—
8 4 1 0	—	2	2	1
8 3 2 0	2	2	4	2
8 3 1 1	1	3	4	2½
8 2 2 1	—	0	0	4
Various small probabilities of combinations not enumerated.				1
	1000	1000	2000	2000

It will be seen that generally, notwithstanding the extremely wide and diversified range of the probabilities, the correspondence between theory and experiment is very close. It must be remembered that for such comparatively small probabilities as many of them are, the number of observations is too limited for closer correspondence between theory and experiment to be expected.

CHAPTER IX.

PROBABILITY OF THE NUMBER OF TIMES THAT A SUIT WILL GO ROUND.

THE Table in the last chapter furnishes a convenient means of solving a problem which has a constant application in Whist-play—namely, as to the number of times that a suit will probably "go round." Suppose, for example, a given player holds four hearts. What is the probability that the suit will go round once, twice, or three times, without any player failing?

We must first find how often, in a certain number of deals, a given player ought to hold four hearts. This may be easily deduced from the combinations in the table which contain the figure 4. Under the combination 4 3 3 3, occurring 211 times in 2000, a given player will hold four $211 \times \frac{1}{4}$ times; under the combination 4 4 3 2 he will hold them $431 \times \frac{2}{4}$ times; under the combination 4 4 4 1 he will hold them $60 \times \frac{3}{4}$ times, and so on. To avoid fractions we may take 8000 deals, when we shall get the following:—

Combination.	Number of Times in 8000 Deals when a Given Player will hold Four Hearts under each Combination.
4 4 4 1	186
4 4 3 2	862
4 4 5 0	50
4 3 3 3	211
4 5 3 1	259
4 5 2 2	212
4 6 3 0	26
4 6 2 1	94
4 7 2 0	7
4 7 1 1	8
4 8 1 0	1
	1910

Now, in each combination that contains the figure 0, some player will fail the first round. These combinations are—

4 4 5 0	occurring	50	times.
4 6 3 0	,,	26	,,
4 7 2 0	,,	7	,,
4 8 1 0	,,	1	,
		84	,,

For each combination that contains the figure 1 (and not 0) the suit will go round once, but somebody will fail the second round. These combinations are–

4	4	4	1 occurring	....	180	times.
4	5	3	1 ,,	....	259	,,
4	6	2	1 ,,	....	94	,,
4	7	1	1 ,,	....	8	,,
					541	,,

For each combination that contains the figure 2 (and not 1 or 0) the suit will go round twice only. These combinations are—

4	4	3	2 occurring	....	862	times.
4	5	2	2 ,,	....	212	,,
					1074	,,

For the remaining single combination, the suit will go round three times. This combination is—

4 3 3 3 occurring .... 211 times.

Hence we get the following results:—

The Four Suit will Go Round	Out of 1910 Cases.	Out of 1000 Cases.
Three times	211	110
Twice only	1074	563
Once only	541	283
Not once	84	44
	1910	1000

The same result may be obtained, though with more labour, by the direct method of calculation, as follows:—

Suppose the given player, A, holds four hearts and 9 cards of other suits.

The probability of any number of hearts being held by other players than A may be calculated in a mode that has been explained on p. 137, and, allowing for the possible permutations, we find—

Probability that the other Three Players will hold the Combination				
5	4	0		=0·026
6	3	0		=0·014
7	2	0		=0·004
8	1	0	(say)	=0·000
			Probability that the suit will not go round once	=0·044
4	4	1		=0·094
5	3	1		=0·136
6	2	1		=0·049
7	1	1		=0·004
			Probability that it will go round once only ...	=0·283
4	3	2		=0·452
5	2	2		=0·111
			Probability that it will go round twice only ...	=0·563
3	3	3		=0·110
			Probability that it will go round three times ..	=0·110

This example will suffice to show the modes of calculation. The problem has been worked out also on the data of three, five, six, seven, and eight cards held by a given player, and the result is shewn in the following Table.

NUMBER OF TIMES THAT A SUIT WILL PROBABLY GO ROUND, WHEN A GIVEN PLAYER HOLDS A GIVEN NUMBER OF CARDS OF THAT SUIT.

Number of cards held by the given player	3	4	5	6	7	8
	Times in 1000 Deals.					
The suit will not go round once	25	44	76	122	200	315
It will go round once only	191	283	400	537	657	685
It will go round twice only	508	563	524	341	143	0
It will go round three times	276	110	0	0	0	0
	1000	1000	1000	1000	1000	1000
Or the result may be differently stated as follows:—						
The suit will go round once or more	975	956	924	878	800	685
Twice or more	784	673	524	341	143	0
Three times...........	276	110	0	0	0	0

N.B.—These results suppose *perfect shuffling* after each deal. Imperfect shuffling will cause the suits to go round oftener.

They also apply, strictly, to plain suits only, as they ignore the disturbing effect of the turn-up card, which gives to the dealer the probability of holding a larger number of trumps than he would otherwise have. To make the Table applicable to trumps, the leader must also be considered the dealer.

CHAPTER X.

PROBABILITY OF WINNING ANY GIVEN NUMBER OF POINTS IN A RUBBER.

THIS problem is of a different character from the preceding ones, inasmuch as it involves to some extent the element of skill in play. If two good players are matched against two bad ones, the former will not only, in the long run, win more rubbers, but these rubbers will consist of a somewhat higher number of points than those of the latter. Notwithstanding this, however, they will still show a fair correspondence with the probabilities.

A rubber consists of two games won by the same party, but these games have different values, according to the manner in which they are won. A "treble" game counts three, a "double" game two, and a "single" game one. The values of the two games won are added together, adding also two points as a bonus for winning the rubber; from this sum is then deducted the value of the game, if any, won by the adversaries; the remainder is the rubber score. It is evident this may vary from 1 to 8 points; thus

$$\text{Minimum} = 1 + 1 + 2 - 3 = 1$$
$$\text{Maximum} = 3 + 3 + 2 - 0 = 8$$

with all intermediate values. The games may be permuted in sixty-three different ways, giving several modes of producing each value of rubber. For example, a rubber of two points may be produced by six different permutations of games, and one of four points by fifteen, while eight points can only be produced by one combination.

The first element in the calculation is the probability of the different values of each game; this is only to be determined by experience. We will call the probability of the occurrence of a single game $= \alpha$; a double $= \beta$: a treble $= \gamma$; where $\alpha + \beta + \gamma = 1$.

We have next to trace these through all their possible combinations; a very long business, of which we will only give two examples for illustration.

First, what is the probability of the same party winning first a treble and then a double game, giving a rubber of 7 points?

Here the probability of either party winning the first is $= \gamma$; but the probability of the *same party* winning the second will be $\frac{\beta}{2}$: and the probability of their concurrence, which will also be the probability of this way of obtaining a rubber of 7 points, will be $= \frac{\beta\gamma}{2}$.

To take another case: what is the probability of the occurrence of three games in the following order?

First game. One party, A, wins a single.

Second. The opponents, B, win a double.

Third. The party A win a treble. The whole giving a rubber of 4 points for A.

Here, since A may be either party indifferently, the probability of winning the first single is $=\alpha$; but, as the winners of the two others must be identified, the probability for them will be $=\frac{\beta}{2}$ and $\frac{\gamma}{2}$ respectively; and the probability of the whole combination, or of one of the ways of obtaining a rubber of 4 points, will be $=\frac{\alpha\beta\gamma}{4}$.

But, keeping all other elements as they are, we may suppose the third game to be won by B instead of A, which will give B a rubber of 6 points. The probability will be the same, but it expresses the chance of one of the modes of obtaining a rubber of another value.

On this principle the whole of the sixty-three permutations have been calculated; the probabilities for every value of rubber have been added together; and the result is given in the following table.

The values of α, β, and γ are subject in some measure to the influence of skill and deal, but may be determined approximately from a large number of observations. Those made by the author and others give, approximately, $\alpha = 0{\cdot}24$, $\beta = 0{\cdot}23$, $\gamma = 0{\cdot}53$.* The third column of the table shows the

* Mr. Perceval's observations on a very large number, subsequently obtained, give—

$$\alpha = 0{\cdot}23560$$
$$\beta = 0{\cdot}23844$$
$$\gamma = 0{\cdot}52618.$$

predicted number of rubbers of each value in a total of 1000; the last two columns show the number resulting from actual records of play. The column A is deduced from 4124 rubbers played by the author and others; the column B, from 10,000 rubbers played by Mr. Perceval. The correspondence is not so close as in former cases, owing, no doubt, to the disturbing effect of skill; but still, considering the complicated nature of the calculation, its general corroboration is worthy of notice.

PROBABILITY OF THE DIFFERENT VALUES OF RUBBERS AT SHORT WHIST.

No. of Points in each Rubber.	Probability.	Number of each in 1000 Rubbers. As predicted by calculation.	As found in practice. A	B
1	$\frac{\alpha^2\gamma}{2}$	15	14	14·8
2	$\frac{\alpha^2\beta}{2}+\alpha\beta\gamma$	36	31	40·9
3	$\frac{\alpha^3}{2}+\frac{\beta^2\gamma}{2}+\alpha\beta^2+\alpha\gamma^2$	99	102	95·7
4	$\frac{\beta^3}{2}+\frac{\alpha^2}{2}+\alpha^2\beta+\alpha\beta\gamma+\beta\gamma^2$	142	136	143.6
5	$\frac{\gamma^3}{2}+\frac{\alpha\beta^2}{2}+\alpha^2\gamma+\beta^2\gamma+\alpha\beta$	195	183	197·3
6	$\frac{\beta^2}{2}+\frac{\beta\gamma^2}{2}+\alpha\beta\gamma+\alpha\gamma$	216	223	214·4
7	$\frac{\alpha\gamma^2}{2}+\beta\gamma$	156	163	155·2
8	$\frac{\gamma^2}{2}$	141	148	138·1
		1000	1000	1000

If we multiply the number of points in each case by its probability, and add the whole together, we shall obtain the *average* value of the rubber. With the values named above, this comes out = 5·44.

The actual average value deduced from statistics of upwards of 33,000 rubbers, recorded by various authorities, is about 5·43.

When good and inferior players play together, the former will realise an average value from 5·5 to 5·7, the latter from 5·3 to 5·45 points per rubber.

CHAPTER XI.

AVERAGE VALUES OF GAMES AND RUBBERS.

In Chapter X. the average value of a rubber, taking the rubbers all round, is given as = 5·44 points.

But it is worth while to extend the calculation, and enquire—

(1) What is the average value of a single game?

(2) What is the average value of a rubber of two games?

(3) What is the average value of a rubber of three games?

When an event is capable of occurring with several different values, then its probable *average* value will be found by multiplying each value into the probability of that value occurring, and adding the whole of the products together. For example: suppose the event may have either of the values A, B, or C, and must have one of them; then if the probabilities of these are $= a$, b, and c respectively, the probable *average* value of the event will be $= Aa + Bb + Cc$.

(1) Now to apply this to the average value of a single game.

This may have three different values, viz., either 1, 2, or 3 points; and it has been stated in Chapter X. that the probabilities of these are 0·24, 0·23, and 0·53 respectively.

Hence the average value of a single game is

$$= 1 \times 0·24 + 2 \times 0·23 + 3 \times 0·53 = 2·29 \text{ points.}$$

(2) To find the average value of a rubber of two games.

Two average games will give 4·58 points, to which we have to add 2 points for winning the rubber, giving a value = 6·58 points.

(3) To find the average value of a rubber of three games.

This consists of two games won = 4·58, less one game lost, which gives 2·29, and adding as before two points for the rubber, gives a value = 4·29.

We may check these results as follows:—Experience shows that rubbers of 2 and of 3 games occur with about equal frequency. Hence the average value of a rubber should be $= \frac{6·58 + 4·29}{2} = 5·44$, which agrees with the determination in Chapter X.

CHAPTER XII.

EXPECTATION OF THE PARTIES AFTER ONE GAME OF A RUBBER HAS BEEN PLAYED.

THE following problem has often been mooted, and differing opinions have been held on it.

Suppose the first game of the rubber to be already won by a party A, what is the value of the expectation of that party as regards winning the rubber?

The value of the game already won may be omitted from consideration; the expectation required is in addition to this.

There are several ways of solving the problem; the shortest is to assume the games at their probable *average* value, as given in Chapter XI.

Suppose, therefore, that every game to be played will have the value of 2·29 points, there will be three ways in which the rubber may be decided, viz.:—

1. A may win the second game, in which case he wins, in addition to his former one, 2·29 points, *plus* the rubber-point = 4·29; the probability of this occurring is = $\frac{1}{2}$.

2. A may lose the second game and win the third. In this case, in addition to his first game,

he wins the rubber point = 2. The probability of this combination = $\frac{1}{4}$.

3. A may lose both the second and third games, by which (taking no account of the first game) he will lose 4·58 + 2 = 6·58. The probability of this is also $\frac{1}{4}$.

Now, reverting to the rule for averages, we find that the probable *average* value of the event (which must occur in one of these three ways) will be

$$=(4{\cdot}29 \times \tfrac{1}{2}) + (2 \times \tfrac{1}{4}) - (6{\cdot}58 \times \tfrac{1}{4}) = 1.$$

That is to say, the value of the expectation of the winners of the first game, as regards winning the rubber, is = one point in addition to the value of the game already won. Thus, if I and my partner have already won, say a treble, a couple of strangers who propose to take our places and play the rubber out, ought to pay us the value of 4 points for the privilege. Or, if we break up, the opponents ought to pay us the same sum.

There is a popular notion that the expectation is equal to $\frac{3}{4}$ of the rubber points (= $1\frac{1}{2}$ point); this appears to be founded on the well known fact that it is 3 to 1 in favour of A's winning the rubber. But the inference is erroneous, because, although A has an expectation of winning $1\frac{1}{2}$ point, he has also an expectation of *losing* $\frac{1}{2}$ a point, and the latter must be deducted from the former. To put it more clearly, suppose A to win (as he ought in the long run to do) three rubbers out of four at £1 points he will, as regards the rubber-points, win

£6 and lose £2, there being a gain of £4, or one point on each rubber.

The expectation as regards the values of the games to be played is nil; the probability of winning and losing by these being equal; it is only the addition for the rubber that comes into calculation.

CHAPTER XIII.

THE PROBABILITY OF HOLDING HONOURS

This is a problem of constant interest, and it is one to which De Moivre and Hoyle, at an early period, devoted attention. In Hoyle's "Essay towards making the Doctrine of Chances Easy," pp. 37–8, is this passage :—

First Quære. What is the chance that the dealers at Whist have Four Honours?

Answer. 27 to 2 against them.

Second. What is the chance that the oldest hands have Four Honours?

Answer. 23 to 1 against them.

Third. What is the chance that either side have Four Honours?

Answer. Nearly 8 to 1 against them.

Fourth. What is the chance that the dealers have Three Honours?

Answer. 13 to 7 against them.

Fifth. What is the chance that the oldest hands have Three Honours?

Answer. 20 to 7 nearly against them.

Hoyle does not explain his calculations, and probably took his results from De Moivre; they are correct, and have been repeated in all editions of his book down to the present day.

We have mentioned, at p. 9, a small body of students, who, some years ago, undertook to investigate various scientific Whist problems. Among other things, this "Little Whist School" re-calculated the probability of holding honours, and the result, which was published by "Cavendish" in *The Field* of July 3, 1869, was as follows:—

In 1666 hands the dealer's side should hold

		Times
4	honours	115
3	,,	468
2	,,	650
1	,,	364
0	,,	69
		1666

These numbers agree essentially with those of De Moivre; and they both apply to the *probability before dealing;* for of course when once the turn-up card is disclosed the case is entirely changed. But it is a matter of considerable interest to know what is the probability of holding honours *after* dealing; indeed this is the most practical form of the problem, and the one that most frequently presents itself to players; and as this case does not appear to have been investigated, we propose to consider it here.

The problem will be twofold, dependent on whether the turn-up is an honour or not an honour.

1. *What is the probability of either side holding honours after an honour is turned up?*

The dealer has already one honour, and we have to calculate the probability that he and his partner, in their combined hands, will hold either 3 others, 2 others, 1 other, or no other.

The dealer has 12 unknown cards, and his partner has 13. There are, therefore, 25 cards to be considered; and they must be selected out of 51. Hence the total number of combinations of cards they can contain will be

$$=\frac{51 \cdot 50 \cdot 49 \ldots\ldots 27}{1 \cdot 2 \cdot 3 \ldots\ldots 25}.$$

This therefore is the denominator D, for all the following expressions.

(*a*) Probability of dealer and his partner holding 3 honours beside the turn-up.

These 3 cards can only be got in one way, but they may be combined with 22 other cards, selected out of 48. Here therefore,

$$N=\frac{48 \cdot 47 \ldots\ldots 27}{1 \cdot 2 \ldots\ldots 22},$$

so that the probability required

$$=\frac{N}{D}=\frac{23 \cdot 24 \cdot 25}{51 \cdot 50 \cdot 49}=\text{about } \frac{1}{9}.$$

(*b*) Probability of their holding 2 honours beside the turn-up.

Two honours can be got out of the three remaining in the pack in 3 ways, and each of these may be combined with 23 cards out of 48.

$$\text{Hence } N = 3\,\frac{48\,.\,47\,\ldots\ldots\,26}{1\,.\,2\,\ldots\ldots\,23},$$

$$\text{and } \frac{N}{D} = 3\,\frac{24\,.\,25\,.\,26}{51\,.\,50\,.\,49} = \text{about } \frac{3}{8}.$$

(*c*) Similarly for one honour beside the turn-up,

$$N = 3\,\frac{48\,.\,47\,\ldots\ldots\,25}{1\,.\,2\,\ldots\ldots\,24},$$

$$\text{and } \frac{N}{D} = 3\,\frac{25\,.\,25\,.\,26}{51\,.\,50\,.\,49} = \text{about } \frac{39}{100}.$$

(*d*) For no honour beside the turn-up. Here there are 25 cards to combine out of 48, so that

$$N = \frac{48\,.\,47\,\ldots\ldots\,24}{1\,.\,2\,\ldots\ldots\,25},$$

$$\text{and } \frac{N}{D} = \frac{24\,.\,25\,.\,26}{51\,.\,50\,.\,49} = \text{about } \frac{1}{8}.$$

The result, brought into comparable figures, is as follows :—

Dealer's side will hold	Times in 1000.
4 honours	110
3 ,,	375
2 ,,	390
1 ,,	125
	1000

The probability of the dealer's side holding honours of course determines that of their being held by the opposite party.

(2) *What is the probability of either side holding honours after a card below an honour has been turned up?*

It will suffice to explain briefly the difference between this case and the last. Here the total number of possible combinations, D, is the same as before; the numerators differ.

(*a*) Probability of there being 4 honours among the 12 unknown cards of the dealer, and the 13 of his partner.

The 4 honours can be got in only one way, but they will be combined with 21 cards selected out of 47. Hence

$$N = \frac{47 \cdot 46 \ldots\ldots 27}{1 \cdot 2 \ldots\ldots 21} \text{ and}$$

$$\frac{N}{D} = \frac{22 \cdot 23 \cdot 24 \cdot 25}{51 \cdot 50 \cdot 49 \cdot 48} = \frac{1}{20} \text{ nearly.}$$

(*b*) Probability of 3 honours. These can be got in 4 ways, and

$$N = 4\frac{47 \cdot 46 \ldots\ldots 26}{1 \cdot 2 \ldots\ldots 22}.$$

$$\frac{N}{D} = 4\frac{23 \cdot 24 \cdot 25 \cdot 26}{51 \cdot 50 \cdot 49 \cdot 48} = \frac{6}{25} \text{ nearly.}$$

(*c*) Probability of 2 honours. These can be got in 6 ways, and

$$N = 6\frac{47 \cdot 46 \ldots\ldots 25}{1 \cdot 2 \ldots\ldots 23}.$$

$$\frac{N}{D} = 6\frac{24 \cdot 25 \cdot 25 \cdot 26}{51 \cdot 50 \cdot 49 \cdot 48} = \frac{39}{100}$$

(*d*) Probability of 1 honour. This can be got in 4 ways. Hence

$$N = 4\frac{47 \cdot 46 \ldots\ldots 24}{1 \ldots\ldots\ldots 24}$$

$$\frac{N}{D} = 4\frac{24 \cdot 25 \cdot 25 \cdot 26}{51 \cdot 50 \cdot 49 \cdot 48} = \text{about } \frac{26}{100}.$$

(*e*) Probability of 0 honours. Here

$$N = \frac{47 \cdot 46 \ldots\ldots 23}{1 \cdot 2 \ldots\ldots 25}$$

$$\frac{N}{D} = \frac{23 \cdot 24 \cdot 25 \cdot 26}{51 \cdot 50 \cdot 49 \cdot 48} = \text{about } \frac{3}{50}.$$

So that in this case

Dealer's side will hold	Times in 1000.
4 honours	50
3 ,,	240
2 ,,	390
1 ,,	260
0 ,,	60
	1000

The mode of calculating De Moivre and Hoyle's original problem is easily deduced from the foregoing, and may be added here.

(3) *What is the probability, before the deal, of either side holding honours?*

The dealer's side will certainly have one trump; the probability that it will be an honour is $= \frac{4}{13}$, and that it will not be an honour $= \frac{9}{13}$.

First, assume it to be an honour, then the probability of dealer's side holding 4 honours has been found $= \frac{23 \cdot 24 \cdot 25}{51 \cdot 50 \cdot 49}$. And if we multiply this by $\frac{4}{13}$ we get the probability of dealer's side holding 4 honours in this way.

But, secondly, it may not be an honour; then the probability of holding 4 honours has been found $= \frac{22 \cdot 23 \cdot 24 \cdot 25}{51 \cdot 50 \cdot 49 \cdot 48}$, and if we multiply this by $\frac{9}{13}$ we get the probability of their getting 4 honours in this way.

As there are therefore two ways of getting 4 honours, the probability of holding them by either one way or the other, will be the sum of the two probabilities—*i.e.*,

$$= \left(\frac{4}{13} \times \frac{23 \cdot 24 \cdot 25}{51 \cdot 50 \cdot 49}\right) + \left(\frac{9}{13} \times \frac{22 \cdot 23 \cdot 24 \cdot 25}{51 \cdot 50 \cdot 49 \cdot 48}\right) = \frac{115}{1666}.$$

Calculating in a similar way, the probability of the dealers holding 3 honours will be

$$= \left(\frac{4}{13} \times \frac{3 \cdot 24 \cdot 25 \cdot 26}{51 \cdot 50 \cdot 49}\right) + \left(\frac{9}{13} \times \frac{4 \cdot 23 \cdot 24 \cdot 25 \cdot 26}{51 \cdot 50 \cdot 49 \cdot 48}\right) = \frac{468}{1666}.$$

Of their holding 2 honours,

$$= \left(\frac{4}{13} \times \frac{3 \cdot 25 \cdot 25 \cdot 26}{51 \cdot 50 \cdot 49}\right) + \left(\frac{9}{13} \times \frac{6 \cdot 24 \cdot 25 \cdot 25 \cdot 26}{51 \cdot 50 \cdot 49 \cdot 48}\right) = \frac{650}{1666}.$$

Of their holding 1 honour,

$$=\left(\frac{4}{13}\times\frac{24 \cdot 25 \cdot 26}{51 \cdot 50 \cdot 49}\right)+\left(\frac{9}{13}\times\frac{4 \cdot 24 \cdot 25 \cdot 25 \cdot 26}{51 \cdot 50 \cdot 49 \cdot 48}\right)=\frac{394}{1666}.$$

Of their holding 0 honours,

$$=0+\left(\frac{9}{13}\times\frac{23 \cdot 24 \cdot 25 \cdot 26}{51 \cdot 50 \cdot 49 \cdot 48}\right)=\frac{69}{1666}.$$

The following Table brings the whole of the results together in a form convenient for comparison :—

PROBABILITY OF HOLDING HONOURS.

	Before the Deal.	After Dealing. Honour turned up.	After Dealing. Smaller Card turned up.
	Times in 1000 Deals.		
Dealer's side—			
Will hold four by honours	69	110	50
Will hold two by honours	281	375	240
Non-dealer's side—			
Will hold four by honours	41	0	60
Will hold two by honours	219	125	260
*Honours will be divided**	390	390	390
	1000	1000	1000

* This result (that the turn-up card makes no difference in the probability of honours being divided) has been thought strange and improbable; and it certainly seems, at first sight, hardly credible that the non-dealers' side are just as likely to hold two honours after an honour has been turned up against them as they were before. But the calculations are positive and trustworthy on the point. The explanation

is that the probability in question depends not only on the number of honours available for the non-dealers, but also on the number of other cards that must be associated with them, and on the total number of cards from which their hands have to be selected; and in the three cases in question it happens that variations occur in these elements which counteract each other. As regards the dealers' side, the turning up of an honour does not either increase or diminish their chance of holding two honours exactly; but it very much increases their chance of holding three or four honours.

This apparent anomaly, and some others connected with it, have attracted the curiosity of some authorities skilled in the theory and application of probability calculations; and their investigations have fully confirmed the correctness of the results stated in the text.

CHAPTER XIV.

PROBABILITY OF ANY NUMBER, OUT OF A SERIES OF EVENTS, HAPPENING IN ONE WAY, WHEN EACH EVENT MAY HAPPEN IN EITHER OF TWO WAYS.

THE object of this chapter is to explain a case which applies to many Whist probability problems — namely: Suppose there are a series of events, each of which may occur in either of two ways, with an even chance for each way; what is the probability that a certain number out of the series will occur in one way, and the remainder the other way?

Perhaps the simplest example is tossing a coin. Suppose we toss a penny ten times: What is the probability that we shall get 6 heads and 4 tails? Or, in $m+n$ tosses, what is the probability of getting m heads and n tails?

To illustrate the nature of the solution let us take a very simple case, and see what may happen. Suppose I toss a penny three times. There may be

0	heads and	3	tails
1	,,	2	,,
2	,,	1	,,
3	,,	0	,,

and we have to find the probability of each of these.

Here, the total number of ways in which the three tosses may occur are as follows:—

T	T	T	....	*a*
T	T	H		
T	H	T	....	*b*
H	T	T		
H	H	T		
H	T	H	....	*c*
T	H	H		
H	H	H	....	*d*.

That is, eight in all, and this is therefore the denominator of the probability fraction. There is one way (*a*) which will give 3 tails, shewing N = 1 for that probability; there are 3 ways (*b*) of getting 1 head and 2 tails, and 3 ways (*c*) of getting 1 tail and 2 heads; for each of these therefore N = 3; and for 3 heads there is only one way (*d*)—*i.e.*, N = 1. So that

Probability of	0	heads and	3	tails	=	$\frac{1}{8}$
,,	1	,,	2	,,	=	$\frac{3}{8}$
,,	2	,,	1	,,	=	$\frac{3}{8}$
,,	3	,,	0	,,	=	$\frac{1}{8}$
				Total	=	1

Now if this process be extended, and its rationale carefully examined, it will be found to indicate the mode of dealing with the problem generally. It must suffice here to state the results. In dealing with $m+n$ tosses it will be found that the denominator of the probability fraction—*i.e.*, the total number of combinations possible,

$$= 2^{m+n}.$$

The numerator is a more complicated fraction; it represents the number of ways in which the results of the $m+n$ tosses may be permutated, all giving the condition of m heads and n tails. This number (see "Todhunter's Algebra," p. 287) will be expressed by

$$\frac{\lfloor m+n}{\lfloor m \; \lfloor n}$$

The sign $\lfloor m$ (called *factorial* m) is an abbreviation for

$$1 \cdot 2 \cdot 3 \cdot 4 \cdot \cdot \cdot \cdot \cdot \cdot \cdot \, m,$$

and so on for the other quantities.

Hence the Rule,—

In a series of $m+n$ events, each of which may occur in two ways, with an even chance for each way; the probability of m events happening in one way and n events happening the other way, is

$$=\frac{\lfloor m+n}{\lfloor m \; \lfloor n \times 2^{m+n}}.$$

Thus, in 10 tosses of a penny, the probability that there will be 6 heads and 4 tails will be

$$\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10}{(1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6) \times (1 \cdot 2 \cdot 3 \cdot 4) \times 2^{10}}$$

$$=\frac{210}{1024}.$$

The calculation of the factorials for large numbers is very laborious; but it may be facilitated by a logarithmic table published in the "Encyclopædia Metropolitana," Vol. II., p. 486.

CHAPTER XV.

PROBABILITY OF WINNING OR LOSING ANY NUMBER OF RUBBERS AT WHIST, OUT OF A CERTAIN SERIES.

THIS is one of the most interesting applications of the formula in the last chapter.

Suppose, after playing 1000 rubbers, I find I am, on the whole, a winner of 50 rubbers—*i.e.*, I have won 525 and lost 475. I am in doubt whether to ascribe this to luck or to skill in play. It is of course quite *possible* that, the players being perfectly equal, luck may have favoured me as much as this, or even more; and I wish to form some opinion how far this is *probable*. If the probability is somewhat large, I may conclude that the result shows nothing decisive as to my skill; but if, on the other hand, there is only a small probability that luck will have favoured me to this extent, I shall have a right to assume that the result implies superior play.

Now the formula in the last chapter supplies the means of determining this, for it will give the probability of the event, on the assumption of its being due to pure chance.

The following is an example worked out for 20 rubbers. It will give a general idea how the probabilities vary for the different numbers gained and lost. The probabilities at the two ends of the scale are so minute they may be considered as practically *nil.*

PROBABILITY OF WINNING OR LOSING ANY NUMBER OF RUBBERS IN TWENTY RUBBERS PLAYED; ASSUMING IT AN EVEN CHANCE WHETHER ANY SINGLE RUBBER IS WON OR LOST.

Win.	Lose.	Probability — *i.e.*, number of times this result should occur in 1000 series.
20	0	0·0
19	1	0·0
18	2	0·2
17	3	1·1
16	4	4·7
15	5	15
14	6	37
13	7	74
12	8	120
11	9	160
10	10	176
9	11	160
8	12	120
7	13	74
6	14	37
5	15	15
4	16	4·7
3	17	1·1
2	18	0·2
1	19	0·0
0	20	0.0
		1000

The above figures apply to the exact numbers given; but there is another way of stating the question which is perhaps more to the purpose—*i.e.*, what is the chance of my winning not a certain definite number, but *at least* that number? To answer this, all that is necessary is to add together the probability of the exact numbers and of all above it. Thus the probability of my winning exactly fifteen is expressed by 15 in 1000 (or 17 to 3 against it), but the probability of my winning *at least* fifteen is $(15 + 4 \cdot 7 + 1 \cdot 1 + 0 \cdot 2) = 21$ in 1000 (or about $11\frac{1}{4}$ to 3 against it).

Twenty is, however, a very small series, and in proportion as we increase the numbers played, we tend to *eliminate* the effect of luck—*i.e.*, to lessen materially the chance of any definite proportion of gain. For example, the probability of winning by luck a balance of at least one rubber in 10 played will be—

If we play	20 rubbers	=	3 to 2 against it.
,,	100 ,,	=	$4\frac{1}{2}$ to 1 ,,
,,	1000 ,,		above 500 to 1 against it

The following are some general results for a large number.

IN 1000 RUBBERS PLAYED.

Probability of winning 500 and losing 500......					$\frac{1}{40}$
Probability of winning a balance of			10	or more	$\frac{1}{2\cdot6}$
,,	,,	,,	20	,,	$\frac{1}{3\cdot65}$
,,	,,	,,	30	,,	$\frac{1}{5\cdot5}$
,,	,,	,,	40	,,	$\frac{1}{9\cdot2}$
,,	,,	,,	50	,,	$\frac{1}{16\cdot5}$
,,	,,	,,	60	,,	$\frac{1}{31}$
,,	,,	,,	70	,,	$\frac{1}{65}$
,,	,,	,,	80	,,	$\frac{1}{140}$
,,	,,	,,	90	,,	$\frac{1}{285}$
,,	,,	,,	100	,,	$\frac{1}{536}$

The probability of *losing* is the same as that of winning.

CHAPTER XVI.

PROBABILITY OF WINNING TRICKS.

It has often been a matter of interest to know what is the probability that either side would make or lose any given score by cards; or, in other words, would win any given number of tricks out of the thirteen; but this problem has always eluded accurate calculation, and must do so, for two reasons. In the first place it is influenced materially by the *deal*, which must give an advantage to the party who has it, an advantage moreover so varying in its value as to render its influence incapable of any positive estimation. And then, secondly, the element of volition and skill in play must have an effect on the probability which is very large, but which is also not amenable to calculation.

The only way in which calculation has been brought to bear on this problem, has been by ignoring these two elements—*i.e.*, in the first place allowing nothing for any advantage by the deal; and in the second place assuming that the winning of tricks is simply the work of chance. It is supposed that each trick is an independent event, and that it is an equal chance which of the two parties may win it.

On this simplified hypothesis the calculation was undertaken by De Moivre. It is effected by aid of the rule laid down in the previous chapters, in the following way:—

Let AB and YZ be the two pairs of players. Then the problem is, What is the probability

		Tricks.		Tricks.
that	AB will win	13,	which is the same thing as YZ winning ...	0
or that	,, ...	12	,, ...	1
	,, ...	11	,, ...	2
	,, ...	10	,, ...	3
	,, ...	9	,, ...	4
	,, ...	8	,, ...	5
	,, ...	7	,, ...	6

The first of these is obviously the same probability as that of tossing head 13 times running, which is known to be

$$=\frac{1}{2^{13}}=\frac{1}{8192}$$

For the others we must adopt the formula

$$\frac{\lfloor m+n}{\lfloor m \lfloor n \times 2^{m+n}}$$

where $m+n=13$, and m and n are the two numbers in any line of the above statement of the question.

For example: the probability of AB winning 12, and YZ winning 1, will be

$$=\frac{1\,.\,2\,.\,3\,\ldots\ldots\,13}{(1\,.\,2\,.\,3\,\ldots\ldots\,12)\times 1\times 2^{13}}=\frac{13}{8192}$$

The probability that A B will win 7, and Y Z will win 6,

$$=\frac{1 \cdot 2 \cdot 3 \ldots\ldots 13}{(1 \cdot 2 \cdot 3 \ldots\ldots 7) \times (1 \cdot 2 \cdot 3 \ldots 6) \times 2^{13}} = \frac{1716}{8192}$$

and so on for the other numbers.

The result of these calculations comes out as follows:—

PROBABILITY OF WINNING ANY GIVEN NUMBER OF TRICKS, EACH TRICK BEING ASSUMED AN INDEPENDENT EVENT, AND EQUALLY LIKELY TO BE WON BY EITHER PARTY.

	In 8192 Deals.	Reduced to 1000 Deals.
A B will Win or *Y Z will Lose*	Times	
7 by cards	1	0·1
6 ,,	13	1·6
5 ,,	78	8·3
4 ,,	286	35·0
3 ,,	715	87·5
2 ,,	1287	156·3
The odd trick	1716	210·
A B will Lose or *Y Z will Win*		
The odd trick	1716	210·
2 by cards	1287	156·3
3 ,,	715	87·5
4 ,,	286	35·
5 ,,	78	9·5
6 ,,	13	1·6
7 ,,	1	0·1
	8192	1000

This appears to be the only result which can be arrived at by mathematical computation, and it no doubt bears a resemblance to the truth. But, as has been said already, it omits the very important conditions of deal and volition; and there is no means, probably, of arriving at a notion of the effect of these except by data procured from actual trial.

The "Little Whist School," already mentioned, undertook this laborious work, and an account of their investigations was published by "Cavendish" in the *Field* of July 3, 1869, from which the following details are abstracted.

The problem was attacked by a series of experiments on a large scale. Four players, all with the requisite amount of scientific knowledge of the game to play according to general principles and to avoid egregious errors, and all also about equally matched, undertook the task of playing out 1666 hands to the end, whether the game was won or not at an earlier part of the hand. The results were carefully noted and tabulated, and are shown in the following table. The number 1666 was chosen for a particular reason having reference to the probability of holding honours; but, for facilitating comparisons, a column has been added reducing it to the proportions in 1000.

EXPERIMENTAL RESULTS OF TRICKS ACTUALLY MADE.

	Honours held by Dealer's side.					Total in 1666 Deals.	Reduction to 1000 Deals.
	IV	III	II	I	0		
Dealer's side won—							
7 by cards ...	3	2	—	—	—	5	3·0
6 ,, ...	10	9	4	—	—	23	13·8
5 ,, ...	16	38	7	—	—	61	36·6
4 ,, ...	21	59	28	1	—	109	65·4
3 ,, ...	28	89	74	14	1	206	123·3
2 ,, ...	21	88	108	40	2	259	155·1
Odd trick ...	13	97	139	52	2	303	182·1
						966	579·3
Non-dealer's side won—							
Odd trick ...	2	51	133	61	2	249	150·1
2 by cards ...	1	22	80	58	12	173	104·1
3 ,, ...	—	8	53	70	17	148	88·5
4 ,, ...	—	5	18	37	16	76	45·6
5 ,, ...	—	—	6	22	10	38	22·8
6 ,, ..	—	—	—	9	5	14	8·4
7 ,, ...	—	—	—	—	2	2	1·2
						1666	1000

This throws a great deal of light on the effect of the two disturbing elements before named.

In the first place, it shows that the *deal* is a great advantage, not only as regards honours, but as regards trick-making. The dealers won the odd trick 182 times in 1000 deals, the non-dealers only 150 times. The dealers made a score by cards 580 times in 1000 deals, the non-dealers made a score by cards only 420 times. This result is startling, as the casual impression is that the *lead* is of more advantage than the deal for the odd trick. But the explanation lies in the great superiority as regards the holding of trumps, which (see Chapter VII.) always attends the dealer's side.

Secondly, the effect of volition in the play may be best shown by taking the average of the tricks made by the two sets of players, so as to eliminate the effect of the deal, as follows :—

COMPARISON OF CALCULATION AND EXPERIMENT, SHOWING THE EFFECT OF VOLITION IN THE PLAY.

	Calculated Results, showing the Effect of Chance only.	Actual mean Results, showing the Effect of Volition in the Play. (Deal eliminated.)
A B will win–		
7 by cards	0·1	2
6 ,, ...	1·6	11
5 ,, ...	9·5	30
4 ,, ...	35·	55
3 ,, ...	87·5	106
2 ,, ...	156·3	130
The odd trick..............	210·	166
Y Z will win—		
The odd trick	210·	166
2 by cards	156·3	130
3 ,, ...	87·5	106
4 ,, ...	35·	55
5 ,, ...	9·5	30
6 ,, ...	1·6	11
7 ,, ...	0·1	2
	1000	1000

This comparison is very curious. It shows that the interference of volition in the play has the effect of considerably increasing the larger trick-scores at the expense of the smaller ones. And this is what might reasonably be expected, seeing that the effect of the skilful management of a good hand is to make it produce more tricks than would be due to the simple natural predominance of the cards it contains. By chance, a slam would be obtained only once in 4096 times; by the effect of skilful play it is obtained once in 250 times.

It is curious that, with two by honours, 3 by cards and 1 by cards were more frequent than 2 by cards, both on the dealer's and on the non-dealer's side. This anomaly ("Cavendish" says) puzzled the investigators for some time, as the other numbers ran in a regular order of increase or decrease; but it was at last accounted for as the natural effect of skilful volition in play. With two by honours, the players would frequently strive for 3 by cards, and if they saw 3 by cards was not to be had, they would make certain of the odd trick, not risking anything for 2 by cards. It is true that the opponents would also strive to prevent the score of the trick or of 3 by cards, but they, having the inferiority in trumps, would succeed less often than their more favoured adversaries. Hence the possession of three honours was accompanied by a score of 2 by cards less frequently than it would have been had each hand been played without reference to these considerations.

CHAPTER XVII.

ON THE PROBABILITY OF WINNING A GAME AT VARIOUS POINTS OF THE SCORE.

THIS probability, which is technically called the "Odds at Whist," has always excited interest. Its calculation was partially attempted at a very early period, and data on the point are given in many Whist Books, but often much by guess-work, and sometimes very erroneously.

The problem is, supposing the scores of the two parties to stand at certain named points, what is the probability of either of them winning the game?

It will be obvious, from what has been said in previous Chapters, that the *deal* is a very important element in the problem, and therefore it is necessary to state in the question which side has the deal. This may be conveniently done by affixing a D to the statement of the score. Thus, supposing one side to score four, and the other side three, if the former have the deal in the ensuing hand, the score is called 4 D to 3. If the latter, the score is designated as 4 to 3 D.

Now the calculation of these probabilities was undertaken by the "Little Whist School" as a sequence to the experiments described in the last chapter, which formed a basis for them; and they were published by "Cavendish" in the *Field* of August 28th, and October 2nd, 1869. The following is the substance of what is there given:—

Score 4 *D to* 4.

To win this it is necessary to secure the odd trick. Now it is seen by the table on p. 178 that the probability of the dealer's side winning the odd trick exactly is $= \frac{303}{966}$. But to this we must add also the probability of winning two tricks, and three tricks, and so on up to seven tricks; in fact we must take the probability of making *any score by cards*, which is $= \frac{966}{1666}$. This is equivalent to odds of about 4 to 3 in the dealer's favour.

Score 4 *D to* 3.

This is more troublesome to calculate. We must first state all the possible ways in which the dealer's side can win the game, then find the probability of each, and finally add all these probabilities together.

The dealers may win in three ways.

1. They may win the odd trick or more, as in the last case: the probability being $=\frac{966}{1666}$;

2. Or they may (*a*) lose the odd trick exactly, and hold 4, 3, or 2 honours, making the score four-all with the deal against them, and then (*b*) win the trick in the next hand. Here are two steps—

(*a*) The probability of the dealers losing the odd trick exactly when they hold 4, 3, or 2 honours, is found from the table $=\frac{2+51+133}{1666}=\frac{186}{1666}$.

(*b*) The probability that they will then (not having the deal) win the odd trick or more $=\frac{700}{1666}$.

These two events must combine, and the combined probability is $=\frac{186 \times 700}{(1666)^2}=\frac{78}{1666}$ nearly.

Hence the probability of winning the game by one or other of these events is equal to the sum of their probabilities $=\frac{966+78}{1666}=\frac{1044}{1666}$. This is equivalent to odds of about 5 to 3 in favour of the dealer.

Now let us reverse the deal, making the

Score 3 *D to* 4.

The dealers here can win the game in three ways.

1. By winning the odd trick or more, holding 4 honours or 3 honours. The probability of this may be got out from the Table, and is found $=\frac{494}{1666}$.

2. By winning 2 by cards or more, holding 2, 1, or 0 honours. This also may be got out from the Table $=\frac{279}{1666}$.

3. By (*a*) winning the odd trick exactly, holding 2, 1, or 0 honours (which will make the score 4 to 4 D), and then (*b*) winning the trick, not having the deal. Here are two steps—

(*a*) The probability of this is found from the Table $=\frac{193}{1666}$.

(*b*) The probability of this $=\frac{700}{1666}$. The probability of the combination is $=\frac{193 \times 700}{(1666)^2}=\frac{81}{1666}$.

Hence the entire probability of the dealers winning the game in this case is $=\frac{494+279+81}{1666}$ $=\frac{854}{1666}$, which is equivalent to odds of about 21 to 20 in their favour.

In this manner the probabilities were worked out by the "Little Whist School," with great skill and labour, for every possible state of the score throughout the game, and the result is given in the following Table.

The "shortened odds" are not scientifically exact; they are only approximations in low figures convenient for use in practice.

TABLE OF PROBABILITIES OF WINNING THE GAME AT ANY SCORE.

Score.	Number of times in 1666 games that Dealer's side will win.	Shortened odds in favour of Dealer's side
		About
4 D to 4	966	4 to 3
,, 3	1044	5 ,, 3
,, 2	1100	6 ,, 3
,, 1	1169	7 ,, 3
,, 0	1231	8 ,, 3
3 D to 4	854	21 to 20
,, 3	943	5 ,, 4
,, 2	1034	3 ,, 2
,, 1	1114	2 ,, 1
,, 0	1182	7 ,, 3
2 D to 4	758	6 to 7
,, 3	805	15 ,, 16
,, 2	905	7 ,, 6
,, 1	989	4 ,, 3
,, 0	1064	7 ,, 4
1 D to 4	637	2 to 3
,, 3	681	2 ,, 3
,, 2	789	9 ,, 10
,, 1	880	10 ,, 9
,, 0	962	4 ,, 3
0 D to 4	547	1 to 2
,, 3	591	1 ,, 2
,, 2	700	3 ,, 3
,, 1	791	9 ,, 10
,, 0	874	11 ,, 10

It is interesting to compare the effect of the deal at different scores. Thus, at

0	D to	0	it is	11	to	10	on the deal.
1	,,	1	,,	10	,,	9	,,
2	,,	2	,,	7	,,	6	,,
3	,,	3	,,	5	,,	4	,,
4	,,	4	,,	4	,,	3	,,

In all cases when one side is at 4 the deal makes a considerable difference. The most remarkable instance is 4 to 2 D, at which score the odds are only 7 to 6 against the deal; while at 4 D to 2 they are 2 to 1 in favour of the deal. The great difference is due to the fact that it is 4 to 3 on the deal for one trick or more; this is the fundamental base on which all the calculations rest; it enters over and over again into all of them.

The odds in favour of the first dealer winning the rubber (see "Cavendish" in the *Field*, Oct. 2, 1869) are about 25 to 24.

The odds stated in books differ materially from those above given. Hoyle, adapted to Short Whist, gives—

1 D to 0	....	5	to	4
2 ,, 0	....	5	,,	3
3 ,, 0	....	5	,,	2
4 ,, 0	....	2	,,	1
2 D to 1	....	4	to	3
3 ,, 1	....	2	,,	1
4 ,, 1	....	7	,,	4
3 D to 2	....	3	to	2
4 ,, 2	....	3	,,	2
0 ,, 0	....	21	,,	20

Clay gives—

0 D to 0		....	5 to 4
1 ,,	0	....	11 ,, 8
0 ,,	1	....	even
2 ,,	0	....	13 to 8
0 ,,	2	....	8 ,, 11
3 } 4 } ,,	0	....	2 ,, 1
0 ,,	3 or 4	....	8 ,, 15
Odds for the deal on the rubber, at love-all		......	6 to 5

He, however, only professes to give these as "current odds"—*i.e.*, odds usually adopted for betting in Whist circles,—stating that they are not exact. Indeed, before the great experimental determination of the trick probabilities above alluded to, no sufficient data existed to found any correct estimate of these odds upon.

De Moivre made the odds for the scores of 8 against 9 at Long Whist 25 to 18; but it must be recollected that at the score of 8 the parties could, if they held 2 by honours, "call" the game without playing.

Mr. Clay (p. 36) gives a curious estimate of the advantage of the deal. He says :—

"It is an even bet that the dealer has 2 points or more. For the purpose of this bet it is held that the dealer has two points although he may not be able to score them, *i.e.*, the bet is won if the dealer and his partner hold 2 by honours, although the adversaries being game by tricks, such honours are not scored. This bet is very slightly in favour of the layer."

It is worth while to test this statement, which can easily be done by the table on p. 178. It will there be seen that in 1666 hands the dealers made 2 or more tricks 663 times; and that when they did not make 2 by cards they held 2 or 4 by honours 199 times. Hence, in 1666 hands the bet was won (663 + 199 =) 862 times, giving odds in its favour of 862 to 804 or about 1·07 to 1.

This is, just as Mr. Clay says, "slightly in favour of the layer."

Now as no such materials for the calculation of the chances had been published when Mr. Clay's statement was made, the estimate must have been found solely by long experience;—and this remarkable corroboration shews how acute the writer's observation must have been; and also proves by a very unexpected test, the trustworthiness of the data given in the table.

CHAPTER XVIII.

STATISTICS AND CALCULATIONS ON THE VALUE OF SKILL AT WHIST.

It is very desirable to ascertain what value may be attached to skill in play at Whist; but considering the large entrance of chance into the game, the problem is very difficult of solution.

The voluntary power we have over the results is in reality compounded of two elements; first, the *system* of play; secondly, the *personal skill* applied. These two elements are quite distinct, for it often happens that we meet with skilful, clever players who do not adopt the modern philosophical system of play; and it is an interesting question what the respective values of the two elements of advantage may be.

Some years ago, the author of this work, being desirous to test this, instituted a series of experiments by playing a large number of rubbers with a partner on the philosophical system, in opposition to two adversaries who were in the habit of playing independently. They knew nothing of the philosophy of Whist, but played the old-fashioned independent game in a very tolerable way. They would always play out early their aces and kings;

would be delighted to force the partner under all circumstances; would lead a single card with a strong trump hand, and so on; but still they would count the trumps, would generally recollect the best cards in, and would neglect no advantage that their notion of the game allowed. In some cases one of them was acquainted with the combined system, but never both. The author's partner was in all cases a young learner, who was sufficiently acquainted with the combined system to play it soundly and correctly in its general rules, but had had very little practice or experience with good players; and the play was purposely made as simple and easy as possible. It may therefore be fairly assumed that the table fairly represented combined as against independent play.

The result was as follows:—Out of 891 rubbers played in this way, the combined play scored 2607 points; the independent play 2110 points, giving a balance of 497 points, or a little more than *half a point per rubber in favour of the combined system* over the independent treatment of the hands.

Turning now to the element of personal skill, it is much more difficult to get at statistics that are to be relied on. A statement has been made, on the authority of Deschapelles, that the *maximum* value of personal skill to its possessor, assuming that all the players at the table adopt the same system, may be estimated at about *a quarter of a point* per rubber. But this was at

Long Whist, in which play told more than it does now.

The "Little Whist School," already often mentioned, undertook, as a part of their scientific investigations, an experiment of a very novel and interesting kind, to test the value of skill.

Two rubbers were formed in two adjoining rooms; two very good players (good in both system and personal skill) being partners against two indifferent players in each room. The cards were dealt in one room, and note being taken of each hand, the same cards were reversed in the other room (those held by the good players being now given to the bad ones, and *vice versâ*, the order of the hands being preserved), and the hands were played again. At the end of each hand the result was noted for comparison.

In room A six rubbers were played, comprising 33 hands. The good players in this room held very good cards, and they won four rubbers out of the six; in points a balance of 18. In room B the good players had of course the bad cards; the 33 hands in this room made up *seven* rubbers, and the good players won three out of the seven, losing 7 points on the balance. The difference, therefore, was 11 points in favour of skill, or nearly *one point per rubber.*

A comparison of tricks only, excluding honours, shewed some curious results. In 7 out of the 33 hands the score by cards in each room was the same—*i.e.*, the same hand being played over twice, the result was the same at both tables. In 18 hands the score by cards was in favour of the superior players; in 8 hands in favour of the inferior. In one of these hands the "bad" players won 2 by cards at one table, and 3 by cards at the other, having changed cards, be it remembered, shewing that bad play sometimes succeeds.

The grand result was this: That at both tables the superior players gained a majority of tricks. In room A the good players with the good cards won 19 by tricks; in room B the good players with the bad cards won 2 by tricks.

The above result corresponds fairly with the maximum amount previously estimated; but the conditions assumed in both cases are so exceptional that such a high advantage cannot be expected to obtain in practice; and it is more useful to enquire what results are given by actual experience in ordinary play. We are able to quote a few data of the kind.

"Cavendish" has given some particulars of his own experience in the *Field* of Feb. 5, 1870. He there states that his records of 15,000 rubbers shew the balance of rubbers won by him to be $2\frac{3}{4}$ per cent. on the whole number played, and the average gain one-fifth of a point per rubber. He has since informed the author that in a second 15,000 the gain was rather less—namely, 2 per cent. of rubbers won, and about one-sixth of a point per rubber.

The author has been favoured with some data compiled from an elaborate series of Whist statistics kept with great care and detail, during many years, by Mr. Spencer Perceval. They embody the results of 20,000 rubbers, played by himself in several clubs, usually with either very good or moderately good players; and they are of much interest on many grounds.

The 20,000 rubbers consisted of 50,011 games, and counted 108,377 points, being an average of 5·4189 points per rubber.

9989 rubbers were won in two games, 10,011 rubbers required three.

In 15,062 cases the winners of the first game won the rubber; in 4938 they lost it.

Out of the 50,011 games 26,309 were trebles; 11,922 were doubles; and 11,780 were single games.

In cutting for the first deal of the rubber it was won by S. P. or his partner 9852 times, and lost by them 10,148 times; the difference against them being thus 296, or about 1.48 per cent. on the whole number of cuts.

In the separate games, the first deal fell to S. P. or his partner 24,445 times, and to the adversaries 25,556 times, giving an adverse difference of 1111; of which 296 arose from the cutting, and 815 from the course of play.

The results of the play were as follows:—

	Won by S. P.	Lost by S. P.	Winning Balance.
Rubbers of 2 games	5199	4790	409
Rubbers of 3 games—			
1st game won by P.	2620	2414	206
,, lost ,, ...	2524	2453	71
Total Rubbers	10,343	9657	686
Points, total	56,314	52,063	4251
Points per Rubber	5·445	5·391	
Treble Games	13,584	12,725	859
Double ,, ...	5994	5928	66
Single ,, ...	5975	5805	170
Total Games	25,553	24,458	1095
Rubbers in which S. P. or his partner had the first deal...	5186	4666	520
Rubbers in which the adversaries had the first deal......	5157	4991	166
	10,343	9657	686
Games in which S. P. or his partner had the first deal...	13,084	11,361	1723
Games in which the adversaries had the first deal.....	12,469	13,097	– 628
	25,553	24,458	1095

In 10,233 rubbers he won the first game; and in 7819 of these he afterwards won the rubber, losing it 2414 times.

In 9767 rubbers the adversaries won the first game; and in 7243 of these they afterwards won the rubber, losing it 2524 times.

It is instructive now to examine these figures and see what indications they give of the character of the play.

The most prominent result is the winning balance of 686 rubbers, or 3·43 per cent. on the whole number played. It is of course *possible* this might be due to chance, but there is a very heavy probability against it. It would be 4½ to 1 against even players winning 3 or more per cent. in a single 1000 rubbers, and the probability in 20,000 would be enormously diminished.

The balance of points gained amounts to 0·2125, or above one-fifth of a point per rubber, which appears to be about the maximum attained by a single player in actual practice.

This evidence is confirmed by the fact that the average value, in points, of the rubbers won is 0·054 points higher than that of the rubbers lost. This also is known to be an unfailing test of skill.

Then, taking the games singly, we find a winning balance of 1095 in 50,011, or above 2 per cent.

It has been already mentioned that the deal was adverse, both in respect to the first deal of the rubbers and to that of the single games. The figures shew nevertheless that the disadvantage was in a considerable degree overcome by the effect of skill.

Another evidence is derived from the statistics of what are called the "long" and the "short" odds. It is shewn in Chapter III. that after winning the first game the probability of winning the rubber, omitting the effect of deal and skill, is 0·75. The deal in this case is not recorded, and may be taken as equal, but the actual result is—

For S. P. $\frac{7819}{10233} = 0{\cdot}764$; and for the adversaries $= 0{\cdot}742$. This advantage may reasonably be attributed to superiority of play.

On the whole, therefore, it is not only possible to trace clearly, in these statistics, that the player who kept them must have had in the long run a considerable superiority over the average of the players opposed to him, but an intelligible estimate can be formed of the value that personal skill in the game may practically attain.

The reduction of the results on the games (according to a mode suggested by Mr. Perceval) gives the following probabilities :—

PROBABILITY OF WINNING A SINGLE GAME.

With deal and skill	0·535
With deal, even skill.................	·523
With deal, against skill	·512
Ignoring both deal and skill	·500
With skill, ignoring deal	·511
With skill against deal	·488
Against both deal and skill	·465

According to this, the odds on the game would be—

For the deal, about	11 to 10
For skill, about	21 to 20

This advantage of the deal agrees exactly with that given in former chapters, and affords almost a demonstration of the correctness of the odds as calculated by the "Little School."

In a similar manner may be calculated the—

Probability of Winning the Rubber.

With first deal and skill	0·526
,, even skill	·509
,, against skill	·492
Ignoring both deal and skill	·500
With skill ignoring deal	·517
,, against deal	·508
Against both deal and skill	·474

Odds on the Rubber.

For the first deal, about	31 to 30
For skill, about	16 to 15

The value of the deal here comes out a little less than formerly estimated.

In another series of 18,000 rubbers, to which the author has access, the balance of rubbers won amounted to 598, or 3·27 per cent., and the balance of points to 3521, or an average of 0·196, or very nearly one-fifth of a point per rubber. The rubbers won averaged 0·032 point more in value than those lost. Here again is evidence of skill, though less prominent than in the last case.

From these investigations it will be seen that the result of skill, in ordinary Whist play, is but small. Measured by the points won, it is about $3\frac{3}{4}$ per cent.; by the rubbers won, a little over 3 per cent.; and by the games won, about $2\frac{1}{4}$ per cent. on the whole number played. Whereas the effects of luck will often cause such quick and such enormous vicissitudes of gain and loss, as apparently to swamp altogether the character of the play. At the same time, the advantages of system and skill are undoubtedly real and substantial, and, in the long run, will be sure to tell.

It is often asked, What is the probability of winning the rubber when you have one game and four points of another game already scored against love? This is deduced as follows:—

With the Deal.

	In 1666 times.
You should win the second game	1231
You should lose it 435 times, but you should win the third game	217
You should win the rubber	1448

Odds in favour, nearly $6\frac{3}{4}$ to 1.

Against the Deal.

You should win the second game	1119
You should lose it 547 times, but you should win the third game	273
You should win the rubber	1392

Odds in favour, nearly 5 to 1.

CHAPTER XIX.

PROBABILITY OF HOLDING CERTAIN GOOD OR BAD HANDS.

THERE is a legend that many years ago Lord Yarborough offered a bet of 1000 to 1 against any given player's holding a hand at Whist in which there should be no card higher than a nine; and such a hand is in consequence called a "Yarborough." Questions have been raised as to the correctness of the odds offered; and it may be interesting to show the mode by which the calculation is made, and at the same time to extend it to other cases of "good" or "bad" hands.

It is, of course, impossible to give any accurate definition of what constitutes a good or a bad hand without taking the trump suit into account, but the fifteen cases in the following table will furnish a graduated scale sufficiently definite to give an idea of the probabilities.

The hands there mentioned comprise two different classes, and we will take an example of each.

First take the "Yarborough" hand. What is the probability of a given player being dealt a hand which shall contain no card higher than a nine?

According to the usual rule, if D represents the total number of different hands that the player may receive, and N = the number of possible varieties of combinations which will give the hand in question, then:

$$\text{Probability of holding the hand} = \frac{N}{D}.$$

The value of D is given in Chapter V.:

$$= 635{,}013{,}559{,}600.$$

To find N we proceed as follows: The number of cards in the pack from which this hand may be taken is 32, and, according to the known formula of selection, the number of possible varieties of hands of 13 that may be taken out of it will be

$$= \frac{32 \cdot 31 \cdot 30 \ldots\ldots 20}{1 \cdot 2 \cdot 3 \ldots\ldots 13},$$

This when worked out comes, in round numbers, to about

$$347{,}760{,}000.$$

$$\text{Hence the value of } \frac{N}{D} \text{ will be} = \frac{347{,}760{,}000}{635{,}013{,}559{,}600},$$

and the probability of the player's getting the "Yarborough" hand is therefore about

$$\frac{1}{1826},$$

giving an odds of 1825 to 1 against it.

We see, therefore, that Lord Yarborough's bet was a very safe one for him, and a disadvantageous one for the person taking it.

The calculations in the table for the hands containing no card higher than an 8, or a 10, are made in the same manner.

The other class of hands, containing cards higher than a 10, require somewhat different treatment, and we will take for an example a hand containing four cards above a 10.

We must first consider these special four cards. In the whole pack there are 16 cards higher than a 10, and the number of ways we may get four out of these will be

$$= \frac{16 \cdot 15 \cdot 14 \cdot 13}{1 \cdot 2 \cdot 3 \cdot 4}.$$

Then each of these varieties has to be combined with 9 other cards, which must be selected out of 36 in the pack; and the number of ways this may be done is

$$= \frac{36 \cdot 35 \cdot 34 \cdot \ldots \cdot 28}{1 \cdot 2 \cdot 3 \cdot \ldots \cdot 9}.$$

These two results must be multiplied together, giving the value of N, and taking the same value for D as before, the probability, $\frac{N}{D}$, of getting this hand becomes

$$= \frac{1}{3 \cdot 706},$$

or about 8 to 3 against it.

The other corresponding cases are calculated in a similar way; and the table will be intelligible without further explanation. The results are given approximately, to avoid too much complication.

PROBABILITY OF A GIVEN PLAYER HOLDING HANDS AS FOLLOWS:

Contents of hand.	Probability.	Probable number of times he will hold the hand in 10,000 deals.	Approximate odds against.
No card so high as an 8...	Very small	0	Very large
No card higher than an 8	$\frac{1}{17000}$	0	17.000 to 1
,, ,, ,, 9 (Yarborough)	$\frac{1}{1826}$	6	1825 to 1
,, ,, ,, 10	$\frac{1}{274}$	36	273 to 1
One card higher than a 10	$\frac{1}{31.8}$	314	31 to 1
Two cards ,, ,,	$\frac{1}{8.83}$	1133	8 to 1
Three ,, ,,	$\frac{1}{4.46}$	2240	7 to 2
Four ,, ,,	$\frac{1}{3.706}$	2698	8 to 3
Five ,, ,,	$\frac{1}{4.8}$	2083	15 to 4
Six ,, ,,	$\frac{1}{9.52}$	1050	17 to 2
Seven ,, ,,	$\frac{1}{28.5}$	351	28 to 1
Eight ,, ,,	$\frac{1}{129}$	77	128 to 1
Nine ,, ,,	$\frac{1}{940}$	11	940 to 1
Ten ,, ,,	$\frac{1}{11100}$	1	11,100 to 1
Eleven, twelve, or thirteen ,,	Very small	0	Very large
		10,000	

CHAPTER XX.

CALCULATIONS AFFECTING THE PLAY OF THE SECOND HAND.

In Chapter III., treating of the play of the second hand, it was stated that there were some complicated exceptions to the ordinary simple and logical rule of playing the lowest card. These have been among the most difficult and obscure points of Whist play, and it is proposed here to see what light can be thrown on them by philosophical reasoning according to the principles of the modern game.

There are two classes of cases in which it has been the practice for the second hand to play high cards: these are—1. In "covering" an honour led; 2. When a small card is led, and the second hand holds certain combinations of high cards. We will consider these two classes in order.

I.

COVERING AN HONOUR WITH AN HONOUR.

A general impression has prevailed among Whist players, almost from time immemorial, that *if an honour is led the second hand should cover it with a bigger honour if he has one.*

The rule is not absolute, but it is practised very commonly, and it is necessary to distinguish between the different varieties of cases in which it may apply.

We will premise that in all cases the lead is supposed to be a normal original lead from a long suit, according to the established rules.

King led.

This may be briefly dismissed, as no second player holding the ace need hesitate to put it on.

Queen led.

If second player holds both king and ace, there is no reason against covering.

If he holds the ace without the king, the reasonable rule is also to put it on; for the only argument against it would be the probability of his partner's holding king single, which would occur very seldom.

If he holds the king without the ace, the usual rule tells him that if he has three or more small cards he should play his lowest; but that if he has only two or one, he should adopt the "honour on honour" rule, and cover the queen with the king. This is one of the rules which we propose to investigate, in order to see whether the covering does really give any advantage over the normal plan of playing "the lowest second hand."

For this purpose we must examine the various combinations of the cards, and observe the effect of "covering," and "not covering," in each case; noting how many tricks will be made by each party, by the two modes of play respectively. In doing this, as it is impossible to imitate the continually varying accidents of actual play, it will be necessary to adopt simplified conditions. We will, in the first place, suppose the play confined to the suit in question, having no regard to trumping; secondly, we will limit the attention to the three first tricks of the suit; and, thirdly, we will assume the lead for each of these tricks to lie with the same player who led originally This simplified plan will serve to test the general principle more clearly than if accidental irregularities were allowed to interfere.

We will denote the players in the way adopted by "Cavendish"—viz., A, leader; Y, second player; B, third player; Z, last player.

Case No. 1.—A holds queen, knave, ten, and two small cards, and leads the queen. (With one small card the general result will be the same.)

Y is assumed to hold king and *two* small cards.

This leaves the ace and four small cards to be distributed between B and Z.

The following table shews the various modes of distribution and the results of the two different modes of play.

B holds	Z holds	If Y covers, tricks won by		If Y does not cover, tricks won by	
		AB	YZ	AB	YZ
Ace alone, or with 1 small ...	3 or 4 small.........	3	—	2	1
Ace, with 2, 3, or 4 small	2, 1, or 0 small ...	3	—	3	—
4 small	Ace alone	2	1	1	2
3, 2, 1, or 0 small ..	Ace, with 1, 2, 3, or 4 small.	1	2	1	2

Here, therefore, there are ten different cases of distribution; in seven of them it is immaterial whether Y covers or not, but in three he gains one trick by not covering.

If the advantages were sometimes on one side and sometimes on the other, it would be necessary to consider the probabilities of each; but, as the advantage is all on one side, this addition is not required. It is, however, right to state that the advantageous cases are not of very frequent occurrence, and that therefore the advantage is not material. It will suffice to say that "covering" is no benefit over the normal rule.

Case No. 2.—In this case we will suppose the second player to hold the king and *one* small card only, leaving ace and four small cards for B and Z.

B holds	Z holds	If Y covers, tricks won by		If Y does not cover, tricks won by	
		AB	YZ	AB	YZ
Ace alone	5 small...............	3	—	2	1
Ace, with any number of small cards ...	....	3	—	3	—
5 small	Ace alone	2	1	1	2
——	Ace, with any number of small cards ...	2	1	2	1

Here, again, there is an advantage in not covering; but the probabilities of its occurrence are very small.

Knave led.

This gives greater variety. The knave, as an original long-suit lead, may be either from king, queen, knave, and two others, or from knave, ten, nine, and one or two others.* The second hand may accordingly hold one or more honours under several conditions.

(*a*) He may hold ace, king, and queen, or ace and king. In either of these cases the "honour on honour" play admits of no question.

(*b*) He may hold ace and queen; here it is of no use putting on the queen, as the king must be

* See remarks on this lead in Chapter XXII.

behind him, and the usual and reasonable practice is to put on the ace.

(*c*) He may hold king and queen. Here the usual practice is to cover the knave with the queen; but the propriety of this has been questioned, as the ace must lie either with the third or fourth player.

(*d*) He may hold the ace without any other honour; in which case he has a right to assume that the lead is from king, queen, knave, and two others, and ought to play his ace, or it may be trumped the second round.

(*e*) He may hold the king or the queen, without any other honour. In either of these cases, it has usually been the practice to cover the knave holding one or two small cards, but not to cover holding more than two. But when the propriety of covering is tested, in the same way as above described,* it is found that in no case does covering with the higher honour present any trick-making advantage, while there are many combinations in which the non-covering appears preferable.

These results as to covering only apply, however, to an original lead from a strong suit: when the lead can be identified as a forced or a strengthening one, the best authorities consider that the covering is advantageous and should be adhered to.

* Details will be found in the *Field* of May 3, 1884.

II.

PLAY OF THE SECOND HAND WHEN A SMALL CARD IS LED.

When a small card is led, it has been customary for the second player to depart from the normal rule of playing the lowest in certain cases where he holds certain high cards. It is desirable to investigate these, and see what justification there is for the exceptions; and it will be convenient to consider the cases of holding one, two, and three high cards respectively.

(A) Holding One Honour.

If the honour is accompanied by more than one small card, it may make, if the ordinary rule of playing the lowest be adhered to. The cases for possible exception are, when the honour is accompanied by only one small card.

Knave, with one small card.—Here the knave, if put on, might sometimes make, but it would expose the weakness of your hand and enable the original leader to finesse in the second round. This disadvantage is considered to forbid the irregularity.

Queen, with one small card.—In this case, the queen, if put on, would more frequently make, and save a higher card of the partner's, whereas, by playing the small one, the queen would be lost; but the same objection is considered to apply here. If, however, the ten or nine is led (probably from king, knave, &c.) the custom is to put on the queen.

King, with one small card.—This case needs a long discussion, which is given in Chapter XXI.

(B) Holding Two High Cards.

Ace and King.—Playing one of these second hand is clearly reasonable, in order to make your good cards early.

King and Queen.—A similar reason applies here. You may indeed lose one of the honours, but you make sure of the second round. By putting on a small one you may still lose the honour, and then get your best card trumped, so making no trick in the suit.

Queen and Knave.—This is a case involving much more complicated considerations. The usual rule is that, if you have only *one* small card, you play the knave; but if more than one, you follow the normal play of your lowest card.

It is difficult to find any good justification for playing the knave in this case. The reasons given are usually vague and unsatisfactory. There is, however, a very plausible and simple reason for *not* doing so—namely, that one better card at least must be behind you. If the third player has it, your knave is lost; if the fourth player has it, your knave is unnecessary.

It is needless to go through all possible combinations; it will suffice to say that in the great majority of them it is of no consequence, as regards trick-making, whether the knave or the small card be played. The following are the exceptions:—

A	Y	B	Z	Probability.	
Kg with 3 small	Qn, Knv, & 1 small	3 small	Ace with 2 small ...	0·022	Here a trick is gained by putting on the knave.
Kg with 4 small	Qn, Knv, & 1 small	2 ,, ...	Ace with 2 small ...	0·018	Here a trick is gained by putting on the knave.
Kg with 4 small	Qn, Knv, & 1 small	3 ,, ...	Ace with 1 small ...	0·011	Here, by putting on knave, Y Z make the first and second tricks; by putting on small one, they make the first and third.
Kg with 5 small	Qn, Knv, & 1 small	2 ,, ...	Ace with 1 small ...	0·007	Here, by putting on knave, Y Z make the first and second tricks; by putting on the small one, they would make the first and third, if the latter is not lost by trumping.
Favourable to playing the knave				0·058	
Small ...	Qn, Knv, & 1 small	Ace with 2 or more small	Kg single	0·010	Here a trick is lost by putting on the knave, it being provided in the calculation that A B shall make the 10.
,, ...	Qn, Knv, & 1 small	King with 2 or more small	Ace	0·044	
Unfavourable to playing the knave				0·054	

Now, here it will be seen that there are some combinations in which it is advantageous to put on the knave and some where it is disadvantageous. It is necessary therefore to estimate the probabilities of each, and we may take the first of them as an example to shew how this may be done.

The contents of the second hand are known, and the problem is to find the probability of the given distribution in the hands of A, Y, and Z. And first as to the *numbers* in each hand. Call the suit hearts. A has led from four or more, and the probability, Y having three, that A has led from four only, may be found by the table in the "Philosophy of Whist," Chapter VIII. = 0·53. Then we have to find the probability that the other six hearts are equally divided between B and Z, which may be found by the rule in Chapter XIV. = 0·313. We have then to consider the *rank* of the cards. The probability that A has the king is $= \frac{4}{10}$; that he has not the ace $= \frac{6}{9}$. And since, then, the latter must lie with Y or Z, and each of them having the same number of cards, the probability that Z has it $= \frac{1}{2}$. Hence, as all these must combine, the probability of the combination $= 0·53 \times 0·313 \times \frac{4}{10} \times \frac{6}{9} \times \frac{1}{2} = 0·022$.

In this way have been calculated, approximately, the probabilities entered in the fifth column of the above table; and it will follow from them that out of 1000 hands where the second player has the cards named, there will be approximately—

58	where it will be	advantageous to put on the knave.
54	,,	disadvantageous.
888	,,	indifferent.
1000		

But there are some further considerations. The original leader will probably not lead from a numerous suit of small cards if he hold an equally numerous suit containing an honour; and since the unfavourable chances are all leads from small cards, this consideration should reduce them. But when the leader holds king with three small cards, he may also hold another plain four-suit, in which there are two or more honours; and in this case we may presume he would choose the latter, which will reduce the favourable probabilities. These corrections have been calculated, and the result gives—

52	advantageous.
41	disadvantageous.
907	indifferent.
1000	

shewing that the usual play, of putting on the knave to a small card led, presents a very slight advantage, namely, about one trick in 100 hands, assuming all the conditions of the calculations to apply.

It must however be carefully borne in mind that this reasoning applies to plain suits only; with trumps it fails, because the leader will often play a small card when he holds both ace and king. Hence in trumps, the second hand, having queen, knave, and one small one, should put on the knave as the best way to stop the lead, and make at least one trick in the suit.

Knave and ten.—Here the ordinary rule is the same as in the last case, *i.e.*, holding one small one, to put on the ten.

There does not appear to be any combination where there is a *disadvantage*, in trick-making, from this course; usually it is a matter of indifference. There is, however, one combination when it is an advantage, namely :—

A	Y	B	Z
Ace, queen, with 2 small.	Knave, ten, and 1 small.	3 small.	King with 2 small.

Here, by putting on the ten, Y Z make the first and third trick, whereas by playing the small one they make the first only. The probability of this case is about $\frac{1}{80}$, and we should have, in 1000 hands, about

12 where it would be advantageous to put on the ten.
988 ,, indifferent.
———
1000

Ten and nine.—Here, again, with only one small one, the play is a matter of indifference, with the exception of one combination, as follows :—

A	Y	B	Z
Queen, knave, and 2 small.	Ten, nine, and 1 small.	3 small.	Ace, king, and 1 small.

In this case a trick is gained by putting on the nine, and the probability of their combination is about $\frac{1}{420}$. In 1000 hands there will be, say, about

$2\frac{1}{2}$ where it would be advantageous to put on the nine.
$997\frac{1}{2}$,, indifferent.
————
1000

It will now be evident that in no case is there any great advantage in adopting the exceptional mode of play; and it is a question whether the small advantage that exists is not more than counterbalanced by the exposure of the second hand in the adversary's suit. For example, the lead is from king, nine, and small ones. Second hand has knave and ten and one small card, and plays the ten. On the third round the leader knows he must not *finesse* the nine, even if strong in trumps, and that by putting on the king he will secure the fall of the knave; or, if he has the lead on the third round, he knows the knave will fall.

Had the small card been first played the original leader would be in the dark as to the position of the knave, and might *finesse* the nine, or might hesitate to continue the suit. Similar cases might be put with regard to the play of the nine by the second hand, holding ten, nine, and one small one.

Certainly the normal rule of the lowest second hand is simpler, and it has the advantage of avoiding the great trouble that so often happens by the high card, followed by a lower one, being mistaken for a trump call, a single case of which might do more mischief than could be compensated for in hundreds of hands by the minute advantage of the abnormal play.

(C) Holding Three High Cards.

The following are the chief exceptional cases in this class:—

Ace, King, Queen, etc.—The play here of the queen requires no justification.

King, Queen, Knave, etc.—In this case the play of the knave is justified on the same ground as that of the queen, holding king and queen only.

Queen, Knave, Ten, etc.—Here we come to a doubtful case. The ordinary rule says play the ten. But with queen and knave we have seen that the play of the knave is of doubtful propriety, and it is by no means clear how the addition of the ten should alter the argument. It is certain that at least one card higher than the queen is behind you; and by playing the small one you must (bar trumping) make at least two tricks in the suit, however the cards may lie.

Knave, Ten, Nine, etc.—The argument here is the same, justifying the play of the small one.

Ace, King, Knave, etc.—We have seen that, with ace and king, it is generally advisable to play the king; and the addition of the knave does not alter the case, unless with strength in trumps you might finesse the knave on the assumption that the lead was from the queen, which is about an even chance. The small card here would not be justifiable, as it might enable the third player to win the trick very cheaply.

Ace, Queen, Knave, etc.—Here again the small card would not be advisable for the reason just given. The knave is clearly the right card, the chances being much more than two to one that the king does not lie with the third player. It may be asked, Why play the knave here, and not in the last case, where it would have an equal chance of winning? The answer is that in the former case you can make certain of the first trick by playing the king, and still hold the ace over the queen, a condition quite different from the present combination.

Ace, Queen, Ten, etc.—This is a somewhat difficult case. The usual rule is to play the queen; but with ace, queen, and cards under the ten, the acknowledged rule is to play the smallest, by which (bar trumping) you will certainly make *both* ace and queen. The question is, therefore, Why should the possession of the ten alter the rule? We can only conceive the motive to be to prevent the third player winning with the nine or a smaller one. But to enable him to do this the leader must hold both king and knave, when the proper card for the second player to put on would be the ten. This would occur only about once in four times; therefore in three times out of four the ten would be wasted, which would be a pity, as it might be of considerable use if the king or the knave, or both, had come out from the *third or fourth hand* in the first trick. And even suppose the first trick to be won cheaply by the third hand (occurring once in eight times), the second player is afterwards put in a most advantageous position over the original leader, entirely blocking his suit every time it is led. It appears, therefore, the small card is the proper play for this combination. The difference between the play in this and the preceding case is, that by playing knave from ace, queen, knave, you have a much better chance of winning the first trick without damaging the subsequent advantageous positions of the ace and queen.

King, Knave, Ten, etc.—Here again the usual rule is to play the ten, but the case is very analogous to the last. In about three cases out of four the ace or queen, or both, will be behind you, and in these the ten would be thrown away. If the leader holds ace and queen, you will make three tricks (bar trumping) by putting on the ten, but you *may* make four by putting on a small card. Hence the small one seems the best play.

We may now collect into one table the results of the foregoing investigation :—

Suggested Play of the Second Hand in Plain Suits, to Normal Leads.

Card led.	Second Hand Holds.	Second Hand Plays.
King	Ace	Ace
Queen	Ace, king	King
,,	Ace	Ace
Knave	Ace, king, queen	Queen
,,	Ace, king	King
,,	Ace, queen	Ace
,,	Ace	Ace
9	King with only one small card	King
10 or 9	Queen with only one small card	Queen
Small card	Ace, king	King
,,	King, queen	Queen
,,	Ace, king, queen	Queen
,,	King, queen, knave	Knave
,,	Ace, king, knave	King
,,	Ace, queen, knave	Knave
In all other cases		Small card.

There is no novelty in any part of this table except as regards the last line; the new point being that we have relegated to that simple old rule thirteen cases which hitherto have been treated as exceptional, and have had special rules for their play.

It is right to state that although the arguments for these changes have received favourable consideration, they cannot all be said to be adopted by Whist-players generally.

It must, however, be borne in mind—

1. That it is of high importance to students of Whist that the rules for their guidance should be as simple and clear as possible.

2. That the general rule for the second hand—namely, to play the lowest—is exceedingly simple and clear; but it has been hitherto complicated by a great number of exceptions, which are very puzzling to students to master and to remember.

3. That if any of these exceptions, when philosophically investigated, are found to have but slight or uncertain justification, it is better that students should ignore them, whatever advanced players may do.

CHAPTER XXI.

PROBABILITIES AFFECTING THE PLAY OF THE KING AND A SMALL CARD, SECOND HAND.

THIS curious problem has furnished a never-ending topic for discussion ever since Whist was invented, and it appears likely to continue to do so as long as Whist survives in its present form. There is now a tolerably concurrent opinion as to what it is best to do; but in none of the discussions that have taken place has there been any serious attempt to apply to the case the *laws of probabilities*, which are in modern days so well understood as bearing on Whist problems.

The following is a contribution to the discussion of the question, based on the doctrine of chances, which we will begin by applying to the very first and simplest element of the case, namely, the probable position of the ace.

When a small card is led, where is it most probable that the ace lies? What does the doctrine of chances tell us as to the likelihood of its being in the hand of the leader, or of the third player, or of the fourth player? Almost all writers on the subject dismiss this question very summarily, by assuming it is equally likely to be in either of the three hands; or, in technical language, that the probability of the ace lying in either of the three hands is $= \frac{1}{3}$. But a more careful consideration will show that this cannot be so, inasmuch as there are disturbing circumstances which destroy the simplicity of the assumption. Let us investigate how the matter really stands.

The only datum is the fact that a low card is led; and from this very small datum the second hand has to deduce the probability of the ace being in the hand of either the first, third, or fourth player. The problem is an intricate and difficult one, but it admits of an approximate solution.

We must of course assume that the leader follows the ordinary rules of Whist, as generally accepted; and, if we consider what these are, we shall find they will guide us to a mode of solving the problem.

The rules for leading differ considerably for plain suits and for trumps, and we must therefore treat the two cases separately.

A. *Plain Suits.*

In the first place, we assume that the player leads from the most numerous plain suit in his hand, with four as the minimum. And, secondly, we know that the card he leads from that suit will be determined by certain combinations he may hold.

For example,

(*a*) If he holds ace with four or more others, he will begin with the ace.

(*b*) If he holds A., Q., Kn., he will begin with the ace.

(*c*) If he holds Q., Kn., 10, he will begin with the Q.

(*d*) If he holds Kn., 10, 9, he may begin with the Kn. (See Chapter XXII.)

(*e*) In all other combinations we may assume he will begin with a small card.

The problem is therefore first to find how many times, in say 1000 hands, he will get the condition *e* (which is the actual fact in the hypothesis); and secondly, in how many of these he will hold the ace. This result, divided by 1000, will represent the entire probability that when a small card is led, the ace lies in the leader's hand.

But the probability of condition *e* can only be found by first ascertaining separately the probabilities of *a*, *b*, *c*, and *d*, and adding them together, which is therefore a necessary preliminary step.

We need not go through the whole of the calculations, which are very elaborate; we will only indicate generally the means by which they are made.

One element is, What is the probability of a player holding a certain given number of cards in a suit? This may be determined by modes previously given.

Another element is, assuming the leader to hold a certain number of cards, what is the probability of his holding one of the combinations before named, *a*, *b*, *c*, or *d*. The manner of calculating this may be best illustrated by an example. Suppose he holds, before leading, five cards of the suit. As we know that king and a small one lie in the second hand, the leader has 11 cards available for his combinations, and the total number of combinations possible is $= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 462$.

The number of possible combinations containing the ace is $= \frac{10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} = 210$; therefore the probability of his holding the ace is $= \frac{210}{462}$ or $\frac{5}{11}$.

The number of possible combinations containing Q., Kn., 10, and not ace, is $= \frac{7 \cdot 6}{1 \cdot 2} = 21$, therefore the probability of holding these is $= \frac{21}{462}$.

The number containing Kn., 10, 9, and not A. or Q., is $= \frac{6 \cdot 5}{1 \cdot 2} = 15$, prob. $\frac{15}{462}$.

Hence the probability of holding one of these $= \frac{210 + 21 + 15}{462} = \frac{246}{462} = \frac{41}{77}$. In other words, out of 77 times when he leads from 5 cards of a plain suit, he will lead an honour 41 times and a small card 36 times.

A third element is, supposing him to have led a small card from a number which admits of his holding the ace, what is the probability that he does hold it? The mode of calculating this has been explained above; only it must be remembered that he has one card less than before, and that there is one card less available for the combinations. Thus if he leads a small card from 5 trumps the chance of his holding the ace is $= \frac{4}{10}$ and not $\frac{5}{11}$. (It should more correctly be $= \frac{83}{210}$, as we have to exclude one combination A., Q., Kn., but $\frac{4}{10}$ is near enough.)

Proceeding on these principles, and working out all the necessary calculations, we arrive at the following results, which are given in round numbers, not attempting a useless affectation of accuracy.

In 825 normal first leads of plain suits, about 500 will be from 5 or more. But calculating the chances of the leader's beginning with an honour, as already explained, we find that there are only about 200 cases out of the 500 when he will begin with a small card. In the remaining 325 leads he will lead from 4 cards; in 25 of these he will lead an honour, and in 300 he will begin with a small card.

Hence we arrive at a very important fact—namely, that in 200 out of 500 leads of a small card, the leader *cannot* hold the ace; but he *may* hold it in the remaining 300 leads.

We have, therefore, secondly to estimate in how many of these cases the ace will lie in the leader's hand, and this is easily done. He has led from four, and has 3 cards remaining, there being 10 cards available for his combinations.

Excluding A., Q., Kn., the probability of his holding the ace is $=\frac{35}{120}$, which may be taken in round numbers $=\frac{3}{10}$. In other words, he will hold the ace about 90 times in those 300 leads. Hence in 1000 total leads of a small card in plain suits, the leader will hold the ace about 180 times, or, the probability of his holding it is = 0·18.

In the remaining 820 cases the ace will lie between the third and fourth players, and, as there is no reason why either of these should have any preference over the other, the probabilities stand thus :—

If I hold king and a small card in Plain Suits, and a small one is led to my right, it is probable that in 1000 *times the ace will lie*

	Times.
With the leader	180
,, 3rd hand	410
,, 4th hand	410
	1000

This it will be seen is a very different result from the ordinary assumption, that the ace is equally likely to lie in either of the three hands, the reason being a very obvious one—namely, that the lead of a small card of itself implies the absence of the ace in a very large number of the possible leads.*

According to this, if the king were played second hand, it would fall to the adverse ace 410 times out of 1000; while it would escape the adverse ace only 180 times. In 410 times it would be indifferent

* If the *nine* is led, as an original long suit lead, the leader must hold the ace, and consequently the king ought to be put on. The same may be said if the eight is led, the second player holding the nine or ten.

as regards that trick whether it was put on or not. This, so far as it goes, would be an argument in favour of the ordinary rule of *not* putting on the king. The king put on would win the trick (supposing fourth player's ace to be guarded) nearly 6 times out of 10.

We now go on to consider further the case of

B. *Trumps.*

The question of the probable position of the ace assumes a totally different character when trumps are led, owing to the different nature of the rules on which the leader decides what particular card he will lead. And then, in trumps, the probability depends, to some extent, on whether the lead is from the dealer or from a non-dealer, and is moreover modified by the position and rank of the turn-up card. It would be out of the question to calculate the effect of the variations of these or other disturbing elements; it must suffice to obtain an estimate which shall be a general approximation to the truth.

Adopting the same process of investigation as before, we have first to consider the number of cards that may be led from. The general rule (in the absence of information from the fall of the cards) is not to lead trumps from less than five. An early trump lead from four is only considered advisable when there is either great strength in them (when a small card would not be led as a rule), or when there is fairly good strength in all other suits; or under some special circumstances of an exceptional character. We might, therefore, in treating the problem generally, ignore the lead from four trumps altogether; at any rate it will suffice to make a small allowance for it.

Then, secondly, we have to consider the rules determining the particular card led. In trumps, with ace and four or five small ones it is not usual (as in plain suits,) to begin with the ace; with six others, probably the rule is to do so.

With A., Q., Kn.; or Q., Kn., 10; or Kn., 10, 9; the leader would begin with an honour.

The third step, finding the probability of holding the ace from a small card led, requires no further remark.

Calculating on the same principles as before, but under the varied conditions, it will be found, as a general approximation, that for every ten leads of a small trump, the leader will hold the ace about four times; so that,

If I hold king and a small card in Trumps, and a small one is led to my right, it is probable that, in 1000 *times, the ace will lie*

		Times
With the leader		400
,,	3rd hand	300
,,	4th hand	300

Here again we have a very different result from the ordinary assumption that the ace is equally likely to lie in either of the three hands, the reason being also very obvious, namely, that the leader has probably more trumps than any one else (the ace being therefore more likely to be among them), and also that the rules of the game are more favourable in this case to his keeping up the ace in his hand.

According to this, if the king were played second hand, it would fall to the adverse ace 300 times out of 1000, and would escape the adverse ace 400 times. In 300 times it would be indifferent, as regards that trick, whether it were put on or not. The king would win the first trick 7 times out of 10.

This, so far as it goes, might furnish an argument for putting on the king in trumps;—and it is singular that this difference between the practice in trumps and plain suits should have been recommended by Clay, although the reason he gives for it is questionable. He was a very clever and experienced player, but not an equally good logician, and it is quite probable that he may have astutely seen through all this complicated argument, without having a clear idea how it might be explained. At the same time there is a good deal to be said on the question independently of the position of the ace.

We now go a step further, by considering the *Combination of the First and Second Rounds.*

The foregoing investigation bears on the first round only; but we may, carrying out the same principles, consider what will happen the second round; this gives us the opportunity of seeing the effect of the alternative play of the small card.

To do this it becomes necessary, however, to introduce the queen, which may lie in either of the three hands. We get, in this way, a number of different combinations, which will be found in the tables below, and we have to calculate the probability of each combination, both in plain suits and trumps. The probability will depend on the number of cards held by each player respectively; but we may take a few common cases as examples.

We will begin with the *Trump Suit*, and will take the most usual case of a trump lead, where the leader has five, and the third and fourth players three each. The calculations are made in the following manner.

The leader, playing a small card, will hold the ace about 400 times in 1000 leads. He has then 3 unknown cards, and as the third and fourth players have each the same number, it is about an equal chance that the queen lies in either of the hands.

The third player will hold the ace 300 times; but as he has then only two other cards, while the leader has four, and the fourth player three, the queen will lie with the third player $300 \times \frac{2}{9}$ times, with the leader $300 \times \frac{4}{9}$ times, and with the fourth player $300 \times \frac{3}{9}$ times. And similarly for the fourth player. The construction of the table will be obvious from this explanation.

LEAD OF A SMALL CARD FROM FIVE TRUMPS; THE THIRD AND FOURTH PLAYERS HOLDING THREE EACH.

Leader.	Second hand.	Third hand.	Fourth hand.	Number of times this combination will occur in 1000 cases.	King played first round.			Small card played first round.		
					irst trick.	Second trick.	Tricks made for each combination.	First trick.	Second trick.	Tricks made for each combination.
AQ	Kg	—	—	133	1	0	133	½	0	66
A	Kg	Q	—	134	1	0	134	0	0	—
A	Kg	—	Q	133	1	0	133	1	0	133
—	Kg	AQ	—	67	0	0	—	0	0	—
Q	Kg	A	—	133	0	0	—	0	1	—
—	Kg	A	Q	100	0	1	100	0	1	133
—	Kg	—	AQ	67	1	1	134	1	1	100
Q	Kg	—	A	133	1	1	266	1	1	134
—	Kg	Q	A	100	1	1	200	1		266
										200
				1000						
Total tricks made in two rounds of 1000 cases							1100			1032

The meaning of the fraction $\frac{1}{2}$ in the first combination is that it is an equal chance whether third or fourth hand holds the best card, and consequently, in half the cases one party will win the trick, in the other half the other party will win it.

The general result is, that in the two first rounds of 1000 small card leads of trumps, the second player will win 1100 tricks if he puts on the king, and 1032 if he begins by putting on the small card. According to this, therefore, there would seem to be a very slight advantage in playing the king; but it must be remarked that no notice is taken here of the disadvantage of this play in exposing the weakness of the king-holder's hand, and in enabling the original leader to finesse against him if the suit is returned by the third player. This is probably the great argument, after all, which should turn the scale.

This Table also enables us to solve another problem, namely, the probability of making *at least one trick* in the suit, supposing this is, for any reason, a special object to be obtained. It will be found that

By playing the king in 1000 leads the player will make at least one trick...	800 times.
By playing the small card he will make at least one trick	733 times.

showing a slight advantage in playing the king. Of course, it is assumed that the previous fall of the cards has given no very positive indication.

With *Plain Suits* the case assumes a different aspect, according to the number the leader held, and we give two tables to show this. In the first he is supposed to lead a small card from five (when he cannot hold the ace), the third and fourth players having three each; in the second he leads from four, when he will hold the ace 300 times out of the 1000. In the latter table we have supposed the third player to hold three and the fourth player four cards, which gives a slight preference to the play of the king; reversing these the advantage would turn to the play of the small card.

PLAIN SUIT LEAD OF A SMALL CARD FROM FIVE; THE THIRD AND FOURTH PLAYERS HOLDING THREE EACH.

Leader.	Second hand.	Third hand.	Fourth hand.	Number of times this combination will occur in 1000 cases.	King played first round.			Small card played first round.		
					First trick.	Second trick.	Tricks made for each combination.	First trick.	Second trick.	Tricks made for each combination.
—	Kg	AQ	—	111	0	0	—	0	0	—
Q	Kg	A	—	222	0	0	—	0	1	222
—	Kg	A	Q	167	0	1	167	0	1	167
—	Kg	—	AQ	111	1	1	222	1	1	222
Q	Kg	—	A	222	1	1	444	1	1	444
—	Kg	Q	A	167	1	1	333	1	1	333
				1000						
Total tricks made in two rounds of 1000 cases							1166			1388

Plain Suit Lead of a Small Card from Four; the Third Player holding Three and the Fourth Player Four Cards.

Leader	Second hand	Third hand.	Fourth hand.	Number of times this combination will occur in 1000 cases.	King played first round. First trick.	King played first round. Second trick.	King played first round. Tricks made for each combination.	Small card played first round. First trick.	Small card played first round. Second trick.	Small card played first round. Tricks ma for each combination.
AQ	Kg	—	—	69	1	0	67	½	0	33
A	Kg	Q	—	100	1	0	100	0	0	—
A	Kg	—	Q	133	1	0	133	1	0	133
—	Kg	AQ	—	67	0	0	—	0	0	—
Q	Kg	A	—	100	0	0	—	0	1	100
—	Kg	A	Q	133	0	1	133	0	1	133
—	Kg	—	AQ	133	1	1	267	1	1	267
Q	Kg	—	A	133	1	1	267	1	1	267
—	Kg	Q	A	133	1	1	267	1	1	267
				1000						
Total tricks made in two rounds of 1000 cases							1234			1200

Taking the probabilities all round, it would seem that the advantage, in plain suits, shows rather in favour of playing the small card, which is, of course, enhanced to some extent by the avoidance of the exposure of the hand.

If the king-holder wishes particularly to make a single trick, the result of the two modes of play is that, with the lead from five, there is an advantage in playing the small card; with four it does not much matter.

The result of the foregoing investigations may be summed up as follows:—

In plain suits the advantage, on the ground of the strict probabilities, is with the play of the small card in the first round; and this is in some degree enhanced by the exposure of the hand consequent on the play of the king.

In trumps, there appears, on the ground of the strict probabilities, a slight advantage for the play of the king; but this is largely outweighed by the increased disadvantage of the exposure, and the greater probability of adverse finessing.

If it be desired to win the first trick (to stop the lead or otherwise), the king put on will win it, on the average, in trumps, 7 times out of 10; in plain suits, rather less than 6 times out of 10.

If it be desired to win at least one trick in the suit, there appears a little advantage in playing the king in trumps, and the small card in plain suits; but the difference is not great either way.

It is very satisfactory to find that these results fully corroborate, in their more general and more important particulars, the opinions arrived at by eminent authorities on entirely different and independent grounds.

CHAPTER XXII.

ON THE LEAD FROM KNAVE, TEN, NINE, AND SMALL CARDS.

HOLDING knave, ten, nine, and small cards, it has been customary generally to lead the knave. It is proposed here to inquire into the expediency of this practice.

In the first place, it must be observed that there is a considerable difference between this and the apparently analogous case of queen, knave, ten. In the latter there are only two higher honours to be extracted before the suit is cleared; and, if these are both against you, the queen and knave will draw them, leaving then the suit established with the ten. But with knave, ten, nine, there are three higher honours to be got rid of; and, if they are all in the opponents' hands, the establishment of the suit by leading the high cards cannot be relied on.

In the face of this difficulty, the lead of the knave must probably have been favoured by the old-fashioned practice of the second hand covering it with another honour—a practice which often caused the knave lead to bring out two, or sometimes three, other honours in the first trick, thereby much promoting the result desired. It has also been feared that to begin with a small card might have the effect of enabling the first trick to be won too cheaply.

But now, assuming that the second player no longer considers himself bound to put honour on honour, the case is changed, and it may be well to inquire what is likely to be the result of the alternative lead—namely, that of a small card. If such a lead will bring out one of the superior honours, it will, while it prevents the trick from being won cheaply, contribute essentially to the clearing of the suit, without the expenditure of one of the leader's high cards.

The probability of doing this will depend on the various ways in which the three superior honours may be divided among the second, third, and fourth players (called Y, B, and Z respectively).

There are 27 ways in which this division may take place, and these may be classed in three different categories, according to the results that they will give rise to in the first trick.

In the first place, under the following modes of division, it is certain that two honours will fall :—

CASES IN WHICH TWO HONOURS WILL FALL IN THE FIRST TRICK TO A SMALL CARD LED.

Second Player, Y	Third Player, B	Fourth Player, Z
Kg, Qn	Ace	—
Ace	Qn	Kg
Kg	Qn	Ace
Qn	Kg	Ace
—	Kg, Qn	Ace
—	Ace, Qn	Kg
—	Kg	Ace, Qn
—	Qn	Kg, Ace

In the first of these cases your partner wins the trick, catching an honour of the adversary Y. In the other cases the adversaries win the trick, and an honour of your partner's is lost. It must, however, be borne in mind that this lost honour only takes the place of your own knave, which would be equally lost if led on the usual plan.

Secondly, in the following modes of division it is certain that at least one honour will fall :—

CASES IN WHICH ONE HONOUR WILL FALL TO A SMALL CARD LED.

Y	B	Z
Ace, Kg, Qn	—	—
Ace, Kg	Qn	—
Ace, Kg	—	Qn
Kg, Qn	—	Ace
Ace	Kg	—
Ace	Kg, Qn	—
Ace	Kg	Qn
Kg	Ace, Qn	—
Kg	Ace	Qn
Qn	Ace, Kg	—
Qn	Ace	Kg
—	Ace, Kg, Qn	—
—	Ace, Kg	Qn
—	Ace	Kg, Qn

In four of these cases the adversaries win the trick; in ten it is won by your partner.

The third category is the doubtful one :—

CASES IN WHICH IT IS NOT CERTAIN WHETHER AN HONOUR WILL FALL TO A SMALL CARD LED.

Y	B	Z
Ace, Qn	—	Kg
Ace	—	Kg, Qn
Kg	—	Ace, Qn
Qn	—	Ace, Kg
—	—	Ace, Kg, Qn

In these cases it is possible that no honour may fall, and that the trick may be won by Y or Z with a card lower than the nine. But to produce this result in any case, three things must concur:—

1. Y's honours must be guarded.

2. Z's must also be guarded.

3. Either Y's lowest card or Z's highest small card must be bigger than the best card held by B.

If either of these conditions fail, an honour must fall.

It is hardly possible to reduce these chances to figures; but one may form an empirical judgment that the fall of an honour will be more probable than the contrary.

The matter therefore stands thus:—

There are 27 modes in which the three superior honours may be divided. In 8 of these two honours will fall by leading a small card; in 14, one honour at least will fall; and in the remaining 5 it is probable, but not certain, that an honour will fall.

The author has attempted to check this proposition by actual trial, in the way already explained, and has obtained the following results:—

In 100 leads of a small card from knave, ten, nine, and one small card, one honour at least fell 93 times. In 100 leads when the leader held two small cards, one honour at least fell 91 times; and when he held three small cards, one honour at least fell 95 times. This is a fair practical corroboration of the expectation on theoretical grounds.

Now, in the small percentage of cases where an honour will not fall, the three superior honours are all against the leader; and it is doubtful whether in these cases the lead of the knave would be any more advantageous than that of the small card.

If the above reasoning is correct, therefore, the lead of a small card presents the following advantages:—

1. It forms the best and most certain way of furthering and hastening the establishment of the suit.

2. It relegates the case of knave, ten, nine, etc., to the large simple category of small-card leads.

3. It much simplifies the play of the partner.

4. And finally, it enables the leader to form more easy, more positive, and more useful inferences from the first trick than the lead of the knave can do.

It was formerly the custom to lead the ten from ten, nine, eight, and small cards. This has now been abandoned except in trumps, and the alteration above proposed is only an extension of the change.

In this, as in the doubtful cases in Chapter XX., it must be explained that the proposed alteration cannot be said to be yet generally adopted; but it may be recommended to students, as simplifying the play without any appreciable disadvantage to them.

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On pages 82 and 83.

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Where the Rules are derived (as expressed by the name)
From strict calculation and reasoning pure;
A logical structure that 's perfect and sure.

And here a broad Principle comes into view,
Which underlies all that the Rules have to do;
The two Partners' hands must be treated as one,
And played in conjunction; not each as alone.

To further this object, in each situation,
The Partners must both *gain and give information;*
And to help them to work with the best-contrived tools,
Is the aim of these short ELEMENTARY RULES.

THE OPENING

In opening your game, you must clearly be told,
To shew to your Partner what strength you may hold;
And hence, if your Trumps are of not high repute,
You must make your *First Lead from your longest Plain Suit.*
And lest your endeavours to win should be marred,
You must carefully learn how to lead the *right card.*

With *Ace, King,* and others, the *King* is first led;
Then follow with Ace, it need hardly be said.

If you hold *King and Queen*, then the *King* is the play,
To get the opposing Ace out of the way.

With *King, Queen, and Knave*, lead the *King* if you 've four,
But the *Knave* is the lead if you chance to have more.

With *Ace, Queen, and Knave, Ace*, then *Queen* we should have,
But with more than four cards, after Ace play the *Knave*.

With *ten, Knave, and Queen*, lead the *Queen* out; but when
You have *ten, Knave, and King*, you begin with the *ten*.

With *Ace and four small ones*, the *Ace* should begin,
To give the best chance that a trick it may win.

If none of these specialties come in your way,
You are told by old writers a *small card* to play;
But, following the plan of the smart Yankee Nation,
You may lead your *fourth best*, 't will give much information.

RETURN OF YOUR PARTNER'S SUIT.

In returning your Partner's Suit, mind what you do;
If you 've *only two* left, lead the *best* of the two;
But the *lowest's* the play if you 've *more* in your hand,
Taking care that you do not retain the command.

MANAGEMENT OF TRUMPS.

In the Modern Whist Game, the perfection of art,
The Trumps always play the most prominent part;
And be it observed that their *numbers* give weight,
And make them prevail, though their rank be not great.

Hence with *five or more Trumps* your supremacy 's clear,
And your course is to *lead them* at once, without fear.
Or, if there 's an honour, the *Call* you display,
A signal your Partner is bound to obey.

When your *Partner leads Trumps*, 'tis a terrible crime
If you do not *return them* without loss of time;
And if his *Trump Signal* developed you see,
You must *lead him your best* if you 've not more than three.

You should *not force your Partner*, your Trumps being *few;*
For if he is strong you much mischief may do.
But if 'tis an *enemy's* Trumps that are strong,
Then force him by all means; he can't stand it long.

If with *more than three* Trumps you should be Second Player,
Don't trump doubtful tricks; for your Trumps you can't spare;
But with three Trumps or less, your decision is quick
To put one of them on, and straightway win the trick.

SECOND HAND.

A good Second Player will mostly decide
To put on his *lowest*, whate'er may betide;
But having *King, Ace*, or *King, Queen* in his hand,
He will play one of *these* cards, the trick to command.

THIRD HAND.

The play of the Third Hand is easily told,
As the rule is to put on the *best card* you hold;
But having *Ace, Queen*, you *finesse* with good grace,
Provided you afterwards play out the Ace.

SECOND, THIRD, OR FOURTH HAND.

The Rules as to *Sequences* must be well taught,
If not there are few things with more evil fraught;
So if you 're not leading, 'tis thus the Rules say,
Of the Sequence of cards you the *lowest* must play.

DISCARDING.

So important it is to have Rules well provided,
E'en throwing away is by principle guided.
As a general maxim, tho' none of the newest,
You discard from the Plain Suit of which you hold *fewest;*
But if *Trumps are against you*, you alter your play,
And then from your *best* suit throw freely away.

IN ALL POSITIONS.

On the whole let your Whist play be careful and straight,
Be as strict with your small cards as if they were great;
And do not play false ones without proper cause,
So you 'll merit your Partner's discerning applause.

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