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No. 18

SHOP ARITHMETIC FOR THE MACHINIST

BY ERIK OBERG.

SECOND EDITION—TEN THOUSAND COPIES

CONTENTS

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INTRODUCTION.

In the following, some of the most common shop subjects requiring simple calculations have been treated, and special efforts have been made to treat each subject as simply as possible, so that the present treatise may be of service to those, particularly, who have not previously acquired a great amount of knowledge about handling figures, and who are not familiar with mathematical expressions and usages. In order to fix the processes and rules more firmly in the reader's mind, examples have been given in almost all instances, and in many cases a number of similar examples have been given, so as to permit the repetition of the same calculation a number of times. All formulas in the first part of the pamphlet have been written out in words, as this gives a better idea about what the formula actually means, at least to those not familiar with handbooks. Mathematical signs have also been avoided in the text to a certain extent, and the corresponding words have been written out in full. In short, all precautions have been taken to present the methods in as plain and simple language as possible. Many text-books deal with principles rather than with specific examples, and to a person who is not used to solving problems of the kind that are met with in the machine shop, it is often difficult to apply the principles involved to each particular case. The purpose of this book has therefore been to select the most common specific cases, and show directly how the principles are applied.

While the subject in hand has been treated to accommodate the requirements of those who demand a book that is plain and simple, it has been necessary to presuppose fundamental knowledge in regard to the use of numbers in calculations, that is, the reader must be fairly competent to add, subtract, multiply, and divide whole numbers and decimals, and also have some fundamental ideas of the use of common fractions. If such knowledge has been acquired, no difficulty will be experienced in making use of the rules and formulas given.

It is assumed that the reader is familiar with the common mathematical signs, + (plus) which signifies addition, — (minus) which signifies subtraction, × (times) which signifies multiplication, and ÷ (divided by) which signifies division, as well as with the sign = (equals) which is put between quantities which are equal to one another to signify this condition. But it may be appropriate to call attention to the different methods commonly used for indicating division, as these may not be clear to all. Usually, as we already have said, in arithmetic, division is indicated by the sign ÷, so that we have, for instance,

$$12 \div 3 = 4.$$

A more common method in technical works, however, is to simply write the dividend as the numerator of a fraction and the divisor as the denominator, thus:

$$\frac{12}{3} = 4.$$

In that case the fraction indicates a division. This system will be followed in many of the following formulas, and it should therefore be remembered that *the line between the numerator and denominator in a fraction always indicates a division, the numerator to be divided by the denominator.*

The actual division, however, is not necessarily worked out in every case where division is thus implied. When two divisions are multiplied together, cancellation, and the following operations of addition or subtraction, may make the actual numerical work very simple.

Although knowledge of common fractions is presupposed, as mentioned, it may be well at this point to mention the rules for multiplication and division of common fractions, as in the following many operations of this kind must be made. Two fractions are multiplied by multiplying numerator by numerator and denominator by denominator, (*numerator* being the *upper*, and *denominator* the *lower* quantity in a fraction). For instance, let it be required to multiply ¼ by ⅜. We have then,

$$\frac{1}{4} \times \frac{3}{8} = \frac{1 \times 3}{4 \times 8} = \frac{3}{32}$$

If the numbers to be multiplied contain whole numbers, these are first converted into fractions. Let it be required to multiply 1¼ by 3¼. We have then

$$1\frac{1}{4} \times 3\frac{1}{4} = \frac{5}{4} \times \frac{13}{4} = \frac{65}{16} = 4\frac{1}{16}$$

Division is simply the reverse of multiplication. The number which is to be divided is called the *dividend*, and the number by which we divide is called the *divisor*. If one number is to be divided by another, we simply invert the divisor, and *proceed as in multiplication.* To invert the divisor means that we place the denominator as numerator, and the numerator as denominator, as, for instance, 3/8 inverted is 8/3. Suppose that we wish to divide ¾ by 7/16. We have then,

$$\frac{3}{4} \div \frac{7}{16} = \frac{3}{4} \times \frac{16}{7} = \frac{48}{28} = 1\frac{20}{28} = 1\frac{5}{7}$$

If the number to be divided contains a whole number besides a fraction, we first convert this to a fraction, and then proceed as before. Suppose that we wish to divide 2¼ by 3¾. We have then,

$$2\frac{1}{4} \div 3\frac{3}{4} = \frac{9}{4} \div \frac{15}{4} = \frac{9}{4} \times \frac{4}{15} = \frac{36}{60} = \frac{3}{5}$$

After this introduction, we are ready to take up some of the most commonly occurring shop problems, and apply to them the principles of general arithmetic.

CHAPTER I.

FIGURING TAPERS.

In all circular or round pieces of work, the expressions "taper per inch" and "taper per foot" mean the taper on the *diameter*, or the difference between the smaller and the larger diameter of a piece, measured one inch or one foot apart, as the case may be. Suppose in Fig. 1 that the diameter at *A* is one inch, and the diameter at *B*, one and one-half inch, and that the distance or dimension between *A* and *B* is 12 inches or one foot. This piece, then, tapers one-half inch per foot, because the difference between the diameter at *A*, one inch, and at *B*, one and one-half inch, is one-half inch. In Fig. 2, the diameter at *C* is 7/16 inch, and at *D*, ½ inch, and the distance between *C* and *D* is one inch. This piece of work, therefore, tapers 1/16 inch per inch. Tapers may also be expressed for other lengths than one inch and one foot. In Fig. 3, the diameter at *E* is 1⅛ inch, and at *F*, 1 9/32 inch, and the dimension from *E* to *F* is 5 inches. This piece of work, therefore, tapers 5/32 inch in 5 inches, the difference between 1 9/32 and 1 1/8 being 5/32.

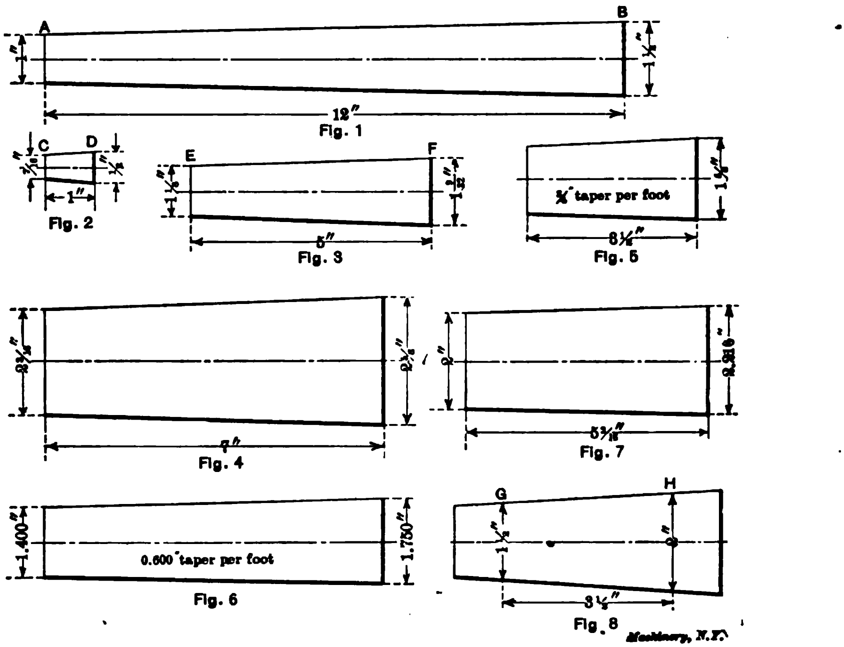
If we know the taper in a certain number of inches, as for instance, that the taper in 5 inches is 5/32, it is easy to find the taper per inch. It is clear that the taper per each inch is one-fifth of what the taper is in 5 inches. We only divide the taper in 5 inches by 5, and we get the taper per inch. In this case, dividing 5/32 by 5 would give us 1/32. If we now want to find the taper per foot we only have to multiply the taper per inch by 12. It is clear that the taper per foot, or the taper in 12 inches, is 12 times the taper in one inch. In this case, therefore, the taper per foot would be equal to 12 times 1/32, or ⅜ inch.

The problems met with in regard to figuring tapers may be of three classes. In the first place we may have given us the figures for the large and small end of a piece of work, and the length of the work, as in Fig. 4, and we want to find the taper per foot. In the second place we may know the diameter at one end, the length of the work, and the taper per foot, as in Fig. 5, and we want to find the diameter at the other end of the work. In the third place we may know the required diameters at both ends of the work, and the taper per foot, as in Fig. 6, and we want to find the dimension between the given diameters, or the length of the piece. We will now treat each of these problems in detail.

1. *To find the taper per foot when the diameters at the large and small ends of the work, and the length, are given.*

Referring to Fig. 4, the diameter at the large end of the work is 2⅝ inches, the diameter at the small end, 2 3/16 inches, and the length of the work 7 inches. The taper in 7 inches is then equal to the difference between 2⅝ inches and 2 3/16 inches, or 7/16 inch. The taper in one inch equals 7/16 divided by 7, or 1/16 inch; and the taper per foot is 12 times the taper per inch, or 12 times 1/16, which equals ¾ inch. The taper per foot in our case in Fig. 4, then, equals ¾ inch.

If the dimension between the small and the large diameter is not expressed in even inches, but is 5 3/16 inches, for instance, as in Fig. 7, the procedure is exactly the same. Here the diameter at the large end is 2.216 inches and at the small end 2 inches. The taper



Figs. 1 to 8.

in 5 3/16 inches is, therefore, 0.216 inch. This is divided by 5 3/16 to find the taper per inch.

$$0.216 \div 5\frac{3}{16} = 0.216 \div \frac{83}{16} = 0.216 \times \frac{16}{83} = 0.0416.$$

The taper per inch, consequently equals 0.0416 inch, and the taper per foot is 12 times this amount, or almost exactly ½ inch.

Expressed in a formula, if all dimensions given are in *inches*, the previous calculation would take this form:

$$\text{taper per foot} = \frac{\text{large dia.} - \text{small dia.}}{\text{length of work}} \times 12.$$

It makes, of course, no difference if the large and small diameters are measured at the extreme ends of the work or at some other place on the work, provided the length or distance between the points where the diameters are given, is stated. In Fig. 8, the smaller and larger diameters are given at certain distances from the ends of the work, but the dimension from *G* to *H* is given, and the figuring is carried out exactly as if the work were no longer than between *G* and *H*. The following examples will tend to show how the figuring of the taper per foot enters in actual shop work.

Example 1.—Fig. 9 shows the blank for a taper reamer. The diameters at the large and small ends of the flutes, and the length of the fluted part, are stated on the drawing. It is required to find the taper per foot in order to be able to set the taper turning attachment of the lathe.

Referring to the figures given in Fig. 9, the difference in diameters at the large and small ends of the taper is 15/64 inch. This divided

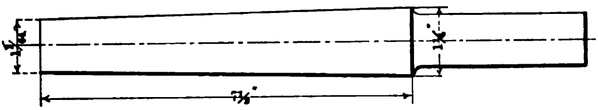
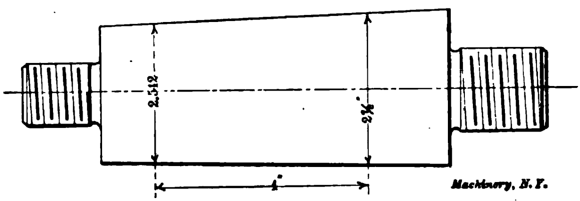


FIG. 9



Machinery, N.Y.

FIG. 10

Figs. 9 and 10.

by the length of the flute, 7½ inches, gives us the taper per inch. This we find to be 1/32. The taper per foot is 12 times the taper per inch, or, in this case, then, ⅜ inch. The taper attachment of the lathe is, therefore, set to the ⅜-inch graduation, and the taper turned will be according to the diameters given on the drawing.

Example 2.—Fig. 10 shows a taper clamping bolt, entering into the design of a special machine tool. As seen from the cut, the drawing calls for a diameter of 2⅞ inches a certain distance from the large end of the taper, and for a diameter of 2.542 inches a distance 4 inches further down on the taper. The taper in 4 inches is then 2⅞ inches minus 2.542 inches, or 0.333 inch. The taper in one inch equals this divided by 4, or 0.0833. The taper per foot is 12 times the taper per inch, or 12 times 0.0833, which equals one inch, almost exactly. The taper to which to turn the bolt in Fig. 10 is thus one inch per foot.

2. *If the diameter at one end of the taper is given, and also the length of the work and the taper per foot, to find the diameter at the other end of the work.*

Referring to Fig. 5, the diameter at the large end of the work is 1⅝ inch, the length of the work is 3½ inches, and the taper per foot is ¾ inch. We now want to find the diameter at the small end. In this case we simply reverse the method employed in our previous problems, where we wanted to find the taper per foot. In this case we know that the taper per foot is equal to ¾ inch. The taper in one inch must be one-twelfth of this, or ¾ inch divided by 12, which equals 1/16 inch. Now, the taper in 3½ inches, which we want to find in order to know what the diameter is at the small end of the work, must be 3½ times the taper in *one* inch, or 3½ times 1/16, which equals 7/32. The taper in 3½ inches, then, is 7/32 inch, which means that the diameter at the small end of a piece of work, 3½ inches long, is 7/32 inch smaller than the diameter at the large end. The diameter at the large end, according to our drawing, is 1⅝ inch. The diameter at the small end, being 7/32 inch smaller, is therefore 1 13/32 inch.

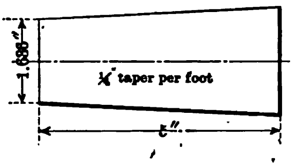


Fig. 11.

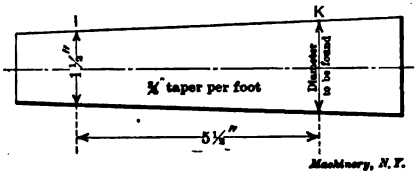


Fig. 12.

Expressed in a formula, the previous calculation would take this form:

$$\text{dia. at small end} = \text{dia. at large end} - \left(\frac{\text{taper per foot}}{12} \times \text{length of work}\right).$$

If we now take a case where the diameter at the small end is given, as in Fig. 11, and the diameter at the large end is wanted, the figuring is exactly the same, except of course, we *add* the amount of taper in the length of the work to the small diameter to find the large diameter. When the large diameter is given, we *subtract* the amount of taper in the length of the work to find the small diameter. This is so self-evident that no difficulties ought to be experienced on this account.

Referring again to Fig. 11, where the small diameter is given as 1.636 inch, the length of the work as 5 inches, and the taper per foot as ¼ inch, how large is the large diameter of the work? If the taper per foot is ¼ inch, the taper per inch is ¼ divided by 12 which equals 0.0208, and the taper in 5 inches consequently 5 times 0.0208, or 0.104 inch. The diameter at the large end of the work, which we are figuring, is, then, 0.104 inch larger than the diameter at the small end. The diameter at the small end is given on the drawings as 1.636 inch; adding 0.104 inch to this, we get 1.740 inch as the diameter at the large end.

Expressed in a formula, the previous calculation would take this form:

$$\text{dia. at large end} = \text{dia. at small end} + \left(\frac{\text{taper per foot}}{12} \times \text{length of work}\right)$$

It may again be well to call attention to the fact that it makes no difference whether the large and small diameters are figured at the extreme ends of the work or at some other points, as long as the diameter to be found is located at one end of the length dimension, and the diameter stated on the drawing on the other. Thus, in Fig. 12 the diameter stated at *I* is given a certain distance up on the taper, and the diameter at *K*, which is wanted, is not at the end of the taper. But the dimension 5½ is given between the points *I* and *K*

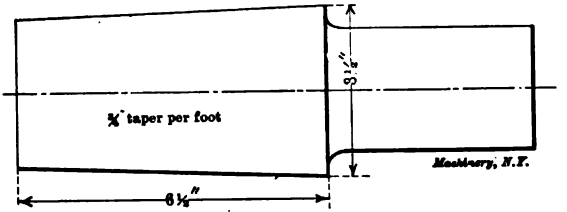


Fig. 13.

where these diameters are to be measured, and in figuring, one may reason as if the work ended at *I* and *K*, the diameter at *I* being the small diameter, the diameter at *K*, the large diameter, and 5½ inches the total length of the work. The following examples of direct practical application to shop work will prove helpful in remembering the principles outlined.

Example 1.—Fig. 13 shows a taper tap, the blank for which is to be turned. The diameter at the large end of the threaded part is 3½ inches, as given on the drawing, the length of the thread is 6½ inches, and the taper per foot is ¾ inch. We want to find the diameter at

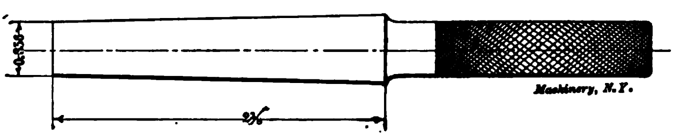


Fig. 14.

the small end, in order to measure this end and ascertain that the tap blank has been correctly turned.

The taper per foot being ¾ inch, the taper per inch is ¾ divided by 12, or 1/16 inch. The taper in 6½ inches is 6½ times the taper in one inch, or 6½ times 1/16 inch, which equals 13/32 inch. The taper in 6½ inches being 13/32 inch means that the diameter at the small end of the tap blank is 13/32 inch smaller than the diameter at the large end. The diameter at the small end is, therefore, 3 3/32 inches.

Example 2.—Fig. 14 shows a taper gage for a standard Morse taper No. 1. The diameter at the small end is 0.356 inch, the length of the gage part is 2⅜ inches, and the taper per foot 0.600 inch. We want the diameter at the large end, in the first place in order to know what size stock to use for the gage, and later for measuring this diameter when turned, to see that the taper turned is correct.

A taper of 0.600 per foot, gives us a taper of 0.050 per inch. In 2⅜ inches the taper equals 2⅜ times 0.050, or 0.119 inch. This added to the diameter at the small end gives us the diameter at the large end: $0.356 + 0.119 = 0.475$ inch.

Example 3.—Fig. 15 shows a taper bolt used as a clamp bolt. The diameter 3¼ inches is given 3 inches from the large end of the taper. The total length of the taper is 10 inches. The taper is ⅜ inch per foot. We want to find the diameters at the extreme large and small ends of this piece.

We will first find the diameter at the large end. The taper per foot being ⅜ inch, the taper per inch equals 1/32 inch. The taper in 3 inches is consequently 3/32. This added to 3¼ inches will give us the diameter at the large end, which is 3 11/32 inches.

To find the diameter at the small end, subtract the taper in 10 inches, which is 10 times the taper in one inch, or 10 times 1/32, which

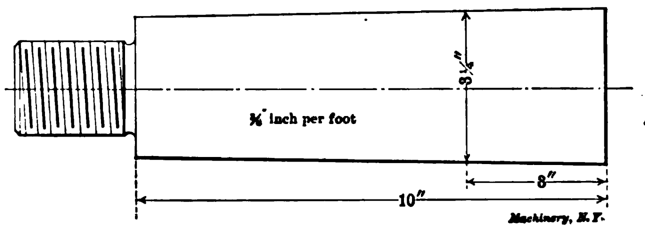


Fig. 15.

equals 5/16, from the diameter 3 11/32 inches at the large end. This gives us the diameter at the small end 3 1/32 inches.

We can also find the diameter at the small end without previously finding the diameter at the extreme large end. The total length of the taper is 10 inches, and the dimension from where the diameter 3¼ inches is given to the large end is 3 inches. Consequently, the dimension from where the diameter 3¼ inches is given to the small end is 7 inches. The taper in one inch was 1/32 inch; in 7 inches, therefore, 7/32 inch. The diameter at the small end of the work is 7/32 inch smaller than 3¼ inches, or 3 1/32 inches, the same as found previously when we figured from the extreme large diameter of the taper.

3. *To find the distance between two given diameters on a tapered piece of work, if the taper per foot is known.*

Referring to Fig. 6, if the diameters at both ends of a tapered piece are known, together with the taper per foot, it is required to find the length of the work. Assume that the diameter at the large end of the piece is 1.750 inch, and at the small end, 1.400 inch. The taper per foot is 0.600 inch. How long is this piece of work required to be, in order to have the given diameters at the ends, with the taper stated? We know that the taper per foot is 0.600 inch. The taper per inch is then 0.600 divided by 12, or 0.050 inch. The difference in diameters between the large and the small ends of the work is $1.750 - 1.400$, or 0.350 inch, which represents the taper in the length of the work.

Now, we know that the taper is 0.050 inch in *one* inch. How many inches does it then require to get a taper of 0.350 inch? This we find by seeing how many times 0.050 is contained in 0.350, or, in other words, by dividing 0.350 by 0.050, which gives us 7 as answer. This means that it takes 7 inches for a piece of work to taper 0.350 inch, if the taper is 0.600 per foot. The length of the work consequently is 7 inches in the case referred to.

Expressed in a formula the previous calculation would take the form:

$$\text{length of work} = \frac{\text{dia. at large end} - \text{dia. at small end}}{\text{taper per foot} \div 12}$$

The taper per foot divided by 12, as given in the formula above, of

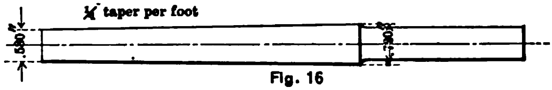


Fig. 16

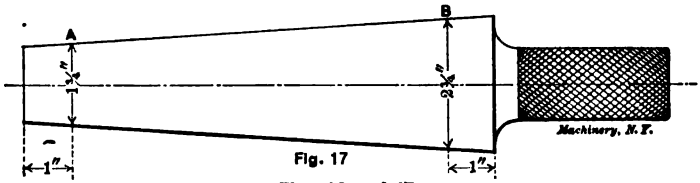


Fig. 17

Figs. 16 and 17.

course simply represents the taper per inch. The formula may therefore be written:

$$\text{length of work} = \frac{\text{dia. at large end} - \text{dia. at small end}}{\text{taper per inch}}$$

A few examples of the application of these rules will make their use in actual shop work clearer.

Example 1.—A taper reamer, Fig. 16, for standard taper pins, having ¼ inch taper per foot, is to be made. The diameter at the large end of the flutes is wanted to be 0.720 inch. The diameter at the point of the reamer must be 0.580 inch, in order to accommodate the longest taper pins of this size made. How long should the fluted part of the reamer be made?

The taper per foot is 0.250 inch, and the taper per inch, consequently, 0.250 divided by 12, or 0.0208 inch. The taper in the length of reamer required is equal to the difference between the large and the small diameter, or 0.720 — 0.580 equals 0.140 inch. This amount of taper divided by the taper in one inch gives the required length of the flutes. Thus, 0.140 divided by 0.0208 equals 6.731, which represents the length of flutes required. This dimension is nearly 6¾ inches, and, being a length dimension of no particular importance, it would be made to an even fractional part of one inch.

Example 2.—In Fig. 17 is shown a taper master gage intended for inspecting taper ring gages of various dimensions. The smallest diameter of the smallest ring gage is 1¾ inch, and the largest diameter of the largest ring gage is 2¾ inches. The taper per foot is 1½ inch. It is required that the master gage extends one inch outside of the gages at both the small and the large ends, when these are tested. How long should the gage portion of this piece of work be?

The taper per foot is 1½ inch, which is equivalent to ⅛ inch taper per inch. The total taper from *A* to *B* in Fig. 17 is 2¾ minus 1¾, or one inch. Therefore, as the taper per inch, ⅛, is contained in the taper of one inch in the distance from *A* to *B* exactly 8 times, the dimension from *A* to *B* is 8 inches. The gage extends one inch beyond *A* and *B*, respectively, at either end, and the total length of the gage is, therefore, 10 inches.

Rules for Figuring Tapers.

If we formulate the previous discussion for figuring tapers into rules, these may be stated as follows:

1. If the taper per foot is known, the taper per inch is found by *dividing the taper per foot by* 12.

2. If the taper per inch is known, the taper per foot is found by *multiplying the taper per inch by* 12.

3. To find the taper per foot, when the diameters at the large and small ends and the length of the taper are given, *subtract the small diameter from the large, divide the remainder by the length of the taper, and multiply the result by* 12.

4. To find the diameter at the small end when the diameter at the large end, the length of the taper, and the taper per foot are given, *divide the taper per foot by* 12, *multiply the result by the length of the taper, and subtract the resulting dimension from the diameter at the large end.*

5. To find the diameter at the large end when the diameter at the small end, the length of the taper, and the taper per foot are given, *divide the taper per foot by* 12, *multiply the result by the length of the taper, and add the resulting dimension to the diameter at the small end.*

6. To find the dimension between two given diameters of a piece of work, when the taper per foot is given, *subtract the diameter at the small end from the diameter at the large end, and divide the remainder by the taper per foot divided by* 12.

7. To find how much a piece of work tapers in a certain length, when the taper per foot is given, *divide the taper per foot by* 12, *and multiply the result by the dimension of the certain length in which the taper is required.*

CHAPTER II.

SETTING OVER TAIL-STOCK FOR TAPER TURNING.

When the tail-stock of the lathe is set over for turning taper, in cases where no taper turning attachment is available, the amount to set over the tail-stock can be determined, if the taper per foot of the work, and the length, are known. Suppose a piece of work is 7½ inches long, as shown in Fig. 18, and that the taper per foot is ¾ inch. We first require to know how much the work tapers in 7½ inches. According to our previous discussion, Rule 7, page 12, we find that the work tapers ¾ divided by 12, times 7½, or 15/32 inch in 7½ inches. The purpose of setting over the tail-stock is to make the front of the work come parallel with the travel of the lathe carriage,

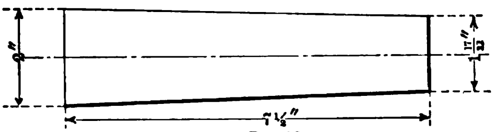


Fig. 18

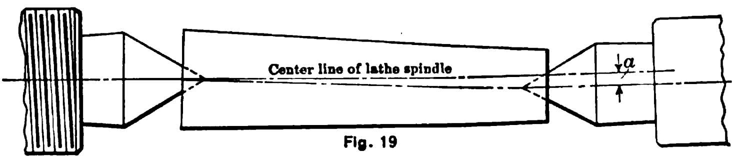


Fig. 19

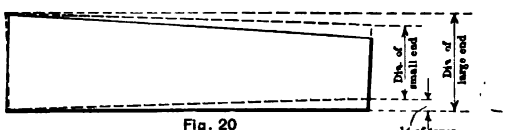


Fig. 20

Machinery,

Figs. 18 to 20.

or, which is the same, parallel with the center line of the lathe spindle, as shown in Fig. 19. It is clear, upon examination of Fig. 20, where the dotted lines indicate the original position of the work, that in order to get the front side of the work parallel with the center line of the spindle, its small end must be moved forward a distance equal to one-half the taper in the length of the work. This forward movement is accomplished by moving the tail-stock over an amount equal one-half of this taper, as shown at *a*, Fig. 19. In the case in Fig. 18, where, as we have found, the taper in the length of the work is 15/32 inch, the tail-stock should be moved 15/64 inch sideways in relation to the spindle of the lathe.

If the diameters at both the large and the small ends of the work are given, we can determine the amount to set over the tail-stock without knowing the taper per foot, because all we need to know is the amount of taper in the length between the centers of the lathe. In Fig. 21, for instance, the diameter at the large end of the work is 1½ inch, and the diameter at the small end is 1¼ inch. The amount to set over the tail-stock will be one-half of the difference between 1½ and 1¼, or ⅛ inch.

If part of the work is turned straight, and part of it turned taper, as shown in Fig. 22, we must determine the amount of taper in the *whole* length of the work, and then set over the tail-stock one-half of this amount. In Fig. 22 the piece of work shown is 1⅜ inch at the small end of the taper. It is then turned taper for 4 inches, and is 1⅝ inch in diameter at the large end of the taper. It is then turned

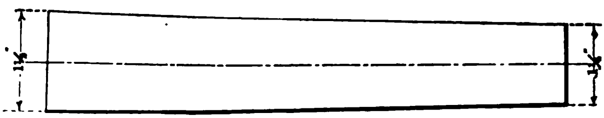


FIG. 21

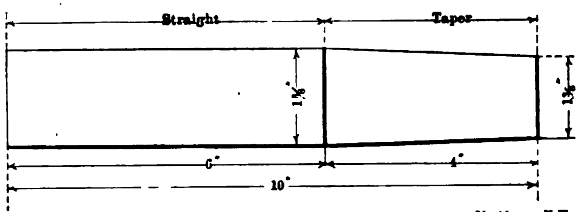


FIG. 22 Machinery, N.Y.

Figs. 21 and 22.

straight for the remaining 6 inches, the total length of the work being 10 inches. In this case we must first find out what the amount of taper would be in 10 inches, if the whole piece had been turned taper, with the same taper as now required for 4 inches. The taper in 4 inches is 1⅝—1⅜, or ¼ inch. The taper in one inch is consequently 1/16 inch, and in 10 inches ⅝ inch. The amount to set over the tail-stock is one-half of this, or 5/16 inch.

If in the case in Fig. 22, the diameter at the small end had not been given, but the taper per foot of the tapered part been stated instead, the diameter at the small end would first have been found according to Rule 4, page 12, and then the taper in the total length of the work, and the amount to set over the tail-stock, would have been found exactly as indicated above.

If we state what has previously been said in formulas, we would get, for the case when the taper per foot and the length of the work are known:

$$\text{amount to set over tail-stock} = \frac{1}{2} \times \left(\frac{\text{taper per foot}}{12} \times \text{length of work} \right)$$

For the case in Fig. 21, we would have:

$$\text{amount to set over tail-stock} = \frac{1}{2} \times \left(\text{dia. at large end} - \text{dia. at small end.}\right)$$

Finally, for the case in Fig. 22, we would have:

$$\text{amount to set over tail-stock} = \frac{1}{2} \times \left(\frac{\text{dia. at large end of taper} - \text{dia. at small end}}{\text{length of taper}} \times \text{total length of work.}\right)$$

For those who prefer rules in words to formulas, the following rules have been formulated:

1. To find the amount to set over the tail-stock for work tapering for its full length, when the taper per foot and length of the work are known, *divide the taper per foot by* 12, *multiply the result by the length of the work, and divide this result, in turn, by* 2.

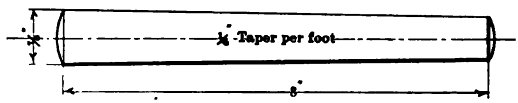


FIG. 23

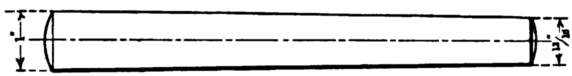


FIG. 24

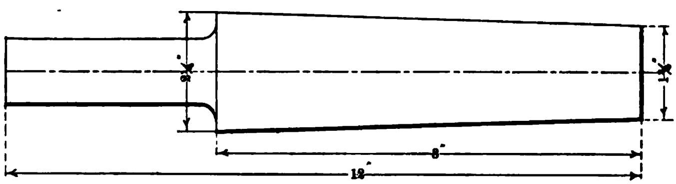


FIG. 25

Machinery, N.Y.

Figs. 23 to 25.

2. To find the amount to set over the tail-stock for work tapering for its full length, when the diameters at the large and small ends are known, *subtract the small diameter from the large, and divide the remainder by* 2.

3. To find the amount to set over the tail-stock for work partly tapered and partly straight, when the diameters at the large and small ends of the taper, the length of the taper, and the total length of the work are known, *subtract the small diameter from the large, divide the remainder by the length of the taper, multiply the result thus obtained by the total length of the work, and finally divide by* 2.

The following examples will help to give a clear idea of the application of these rules.

Example 1.—The taper pin shown in Fig. 23 is 8 inches long, and tapers ¼ inch per foot. How much should the tail-stock be set over when turning this pin?

Dividing the taper per foot by 12 gives us 0.0208. Multiplying this figure (which represents the taper per inch) by 8 gives us 0.166 as the taper in 8 inches. Dividing this by 2 gives us the amount required to set over the tail-stock. This amount, then, is 0.083 inch.

Example 2.—Another taper pin, Fig. 24, is 1 inch in diameter at the large end, and 13/16 inch at the small end. How much should the tail-stock be set over for turning this pin?

The total taper of this pin is found by subtracting the diameter at the small end, 13/16 inch, from the diameter at the large end, 1 inch. This gives us a remainder of 3/16. One-half of this amount, or 3/32 inch, represents the amount which the tail-stock should be set over.

Example 3.—A taper gage, as shown in Fig. 25, is to be turned by setting over the tail-stock. The diameter at the large end of the taper is 2¼ inches, the diameter at the small end is 1¾ inch, the length of the taper, 8 inches, and the total length, 12 inches. How much should the tail-stock be set over?

Subtracting the diameter at the small end, 1¾ inch, from the diameter at the large end, 2¼ inches, gives us a taper of ½ inch in 8 inches. Dividing ½ by 8, gives us the taper in one inch, which is 1/16 inch. Multiplying this with the total length of the work, 12 inches, gives us ¾ inch, which, divided by 2, gives us, finally, the required amount to which to set over the tail-stock. This latter is, therefore, set over ⅜ inch.

CHAPTER III.

CUTTING SPEEDS AND FEEDS.

There is a certain mathematical relation between the diameter of the work turned in a lathe or on a boring mill (or the diameter of the drill, or hole drilled, in a drill press), the number of revolutions, and the cutting speed of the work or tool. This relationship is simple, and can be easily explained.

The cutting speed of a tool is the speed with which the tool passes over the surface of the work operated upon, counted in so many feet per minute. Thus, if the point of a lathe tool passes over the surface of a casting turned in the lathe at a rate of 40 feet per minute, this figure expresses the cutting speed. Of course, the tool point is really stationary, and it is the casting surface that passes by the tool point, but it is customary to say that the "tool passes over the work," as it actually does in a shaper, for instance.

The feed of a tool is its sideways motion for each revolution or stroke of the work. Thus in a lathe, if the feed is 1/16 inch, it means that for each revolution of the work the tool moves along the lathe bed 1/16 inch, so as to cut a chip 1/16 inch wide. In a planer, the feed would mean the amount the tool-carrying head is moved sideways for each complete stroke of the table or platen.

Cutting Speeds.

The problems which meet the machinist in regard to cutting speeds may be divided up in two groups: If the diameter of the work is known (which, of course, can always be, at least approximately, measured), and a certain cutting speed is required, how many revolutions per minute ought the work to make; and, if the diameter of the work and the number of revolutions are known, what is the cutting speed? We will now deal with each of these problems in the order they come.

1. *The diameter of the work turned in a lathe or boring mill (or the diameter of the drill, or drilled hole, in a drill press) and the required cutting speed are known. How many revolutions per minute should the work make?*

Assume that the diameter of the work is 5 inches, and the required cutting speed 40 feet per minute. When the diameter of a piece of work is known, its circumference equals the diameter times 3.14. Therefore, the circumference of the work in hand is 15.7 inches. It is evident, that for each revolution of the work the length of its circumference passes by the tool once. Thus, for each revolution 15.7 inches passes by the tool. As the cutting speed is expressed in feet, this length should also be given in feet, and not in inches, when we figure. To transform 15.7 inches into feet we divide by 12, thus obtaining 1.308 feet as the circumference of the work. How many revolutions, each represented by 1.308 feet, does it require to get a cutting speed of 40 feet? This we evidently get by finding how many times 1.308 feet is contained in 40 feet, or, in other words, by dividing 40 by 1.308. Carrying out the division gives us 30.6 revolutions per minute, as required to obtain a cutting speed of 40 feet per minute with a piece of work 5 inches in diameter.

The calculation carried out above is expressed, shortly, by the formula

$$\text{number of revolutions per minute} = \frac{\text{cutting speed in feet per minute}}{\left(\begin{matrix}\text{diameter of} \\ \text{work in inches}\end{matrix} \times 3.14\right) \div 12}$$

A few examples may tend to make this formula clearer.

Example 1.—A tool steel arbor is turned, using an ordinary carbon steel for turning. The diameter of the arbor is 2 inches. The cutting speed, in feet per minute, ought to be about 18 feet. How many revolutions ought the work to make per minute?

The diameter of the work, 2 inches, multiplied by 3.14, gives us a circumference of 6.28 inches. This circumference, expressed in feet, is obtained by dividing 6.28 by 12, getting 0.523 as a result. The cutting speed per minute, 18 feet, divided by the circumference (or the distance traveled for *one* revolution) gives us the number of revolutions. Thus 18 divided by 0.523, which equals 34.4, represents the proper number of revolutions for a piece of work 2 inches in diameter, being cut at a rate of 18 feet surface speed per minute.

Example 2.—A brass rod, 1 inch in diameter, is being turned. The proper cutting speed for this material is 100 feet per minute. How many revolutions should the work make?

The circumference of the work in inches is 3.14, and expressed in feet, 3.14 divided by 12, or 0.262. The cutting speed, 100 feet, divided by 0.262 gives us 382 revolutions, approximately, as the proper number of revolutions.

Example 3.—A ½-inch drill, cutting cast iron, may cut at a speed of 40 feet per minute. How many revolutions ought it to make?

The circumference in feet equals ½ times 3.14 divided by 12, or 0.131. This divided in 40 gives us approximately 305 as the proper number of revolutions.

2. *The number of revolutions which the work makes in a lathe or boring mill, or the number of revolutions of the tool in a drill press, and the diameter are known. What is the cutting speed?*

Assume that the work is a tool steel stud, 4 inches in diameter, and revolving at a speed of 16 revolutions a minute. The circumference of the work is figured as before, and transformed into feet. The circumference, in feet, equals, in this case, 4 times 3.14 divided by 12, or 1.05 foot. This is the distance traversed by the tool for each revolution. For 16 revolutions the distance traversed is evidently 16 times greater, or 16 times 1.05, which equals 16.8. As the work makes 16 revolutions a minute, and during that time the tool traverses 16.8 feet on its surface, that means that the cutting speed is 16.8 feet per minute.

The calculation carried out above is expressed by the formula:

$$\text{cutting speed in feet per minute} = \frac{\text{dia. of work in inches} \times 3.14}{12} \times \text{number of revolutions per minute}$$

The following examples will tend to make clear the use of this formula:

Example 1.—A cast iron pulley, the rim of which is being turned, revolves in the lathe at a rate of 6½ revolutions per minute. The diameter of the pulley is 23 inches. What is the cutting speed?

The circumference of this pulley in feet equals 23 times 3.14 divided by 12, which is 6.02. Multiplying the circumference by the number of revolutions, 6½, gives the cutting speed. We have, then, 6.02 times 6.5, equals 39.13, which is the cutting speed in feet.

Example 2.—A one-inch drill, cutting tool steel, revolves at a rate of 60 revolutions per minute. What is the cutting speed?

The circumference of the drill, in feet, is 3.14 divided by 12, or 0.262. This multiplied by the number of revolutions, 60, gives us 15.72 feet as the cutting speed of the drill.

Rules for Calculating Cutting Speeds.

What has been previously stated in formulas, may be expressed in rules as follows:

1. To find the number of revolutions per minute, when the diameter of work (or drill) in inches and the cutting speed in feet per minute are known, *multiply the diameter by* 3.14, *and divide the result by* 12; *then divide the cutting speed by the figure thus obtained.*

2. To find the cutting speed in feet per minute, when the diameter of the work (or drill) in inches, and the number of revolutions per

minute are given, *multiply the diameter by 3.14 and divide the result by 12; then multiply the result thus obtained by the number of revolutions per minute.*

Feed of Cutting Tool.

The feed of the cutting tool enters but little in the calculations that a machinist would be required to make. The most common question into which the feed enters is the time required for turning or planing a certain piece of work. In this case both the cutting speed, or the number of revolutions per minute, and the feed would have to be considered.

Suppose for instance that a tool steel arbor is 2 inches in diameter; that its length is 10 inches; that the cutting speed of the work is 18 feet per minute; and that the feed of the cutting tool along the work is 1/16 inch per revolution. How long time would it require to take one cut over the surface of the work?

We first find from our discussion on the cutting speed, and from Rule 1 on previous page, that the revolutions per minute of the work equal 34, approximately. As the tool feeds forward 1/16 inch for each revolution of the work, it is fed forward 34/16 or 2⅛ inches in 34 revolutions, or in one minute. The time required to traverse the whole length of the work, 10 inches, will evidently be found by finding how many times 2⅛ is contained in 10 inches, or, in other words, by dividing 10 by 2⅛. The result of this division is 4.7 minutes. It would therefore take 5 minutes, approximately, to traverse the work once with the cutting speed and feed mentioned.

Expressed in a formula, our calculation would take this form:

$$\text{time required for one cut over the work} = \frac{\text{total length of work (or length of cut)}}{\text{revolutions per minute} \times \text{feed per revolution}}$$

Expressed as a rule, the formula takes this form:

To find the time required for one complete cut over the work, when the feed per revolution, the total length of the cut, and the number of revolutions per minute are given, *divide the total length of the cut by the number of revolutions per minute multiplied by the feed per revolution.* If the cutting speed is given, originally, instead of the number of revolutions, find the latter number first from Rule 1, page 18.

CHAPTER IV.

TRAINS OF GEARS.

Suppose that we have two shafts *A* and *B*, as shown in Fig. 26, and that we want to connect these shafts by gears so that the shaft *A* is making one revolution while shaft *B* makes three. In order to do this we must place a gear on *A* having three times the number of teeth as compared with the number of teeth in the gear on *B*. The stud with the larger gear will always run slower than the stud with a smaller gear. Suppose that we have 90 teeth in the gear on *A*. The gear on *B* must then only have 30 teeth, because the gear on *A* was to have three times the number of teeth in the gear on *B*. Each time the gear on *B* turns around one complete revolution, it engages 30

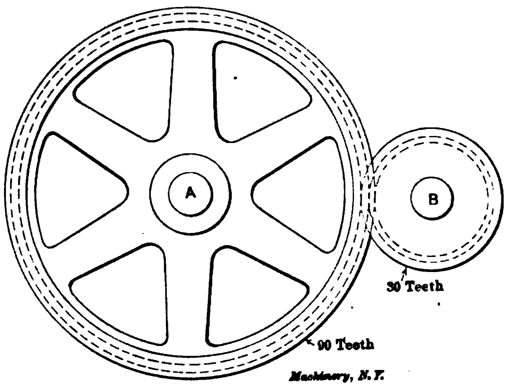


Fig. 26.

teeth in the gear on stud *A*. It is then plain that it must turn around three complete revolutions in order to engage all the 90 teeth of the gear on *A*, or, in other words, turn around three times in order to turn the gear on *A* once. The numbers expressing the relationship between the number of times one gear revolves, to the corresponding number of times the other revolves, is called the *ratio* of the gearing. Thus, in the present case, the ratio is 3 to 1, one gear revolving 3 times, while the other revolves once.

When the ratio of the speed of studs or shafts is given, it is possible to find the gears which will cause the gears to run at the required speed. In Fig. 27, suppose that shafts *C* and *D* are required to run in a ratio of 5 to 1, that is, shaft *C* is to revolve 5 times, while shaft *D* revolves once. What gears should we select to make the shafts run as required? If we place a 20-tooth gear on shaft *C*, the gear on shaft *D* should have 5 times as many teeth, or 100. Then it will revolve but once, when the gear on *C* revolves 5 times. Suppose

that we put gears with the number of teeth mentioned on the shafts. Now the distance between the shafts may be such that the gears would not reach together. We can then place an intermediate gear *E*, mounted on stud *F*, in the train, which meshes with the gears on both *C* and *D*. *The intermediate gear simply transmits motion from the gear on C to the gear on D, and does not have any influence on the relative speed ratio.* When the intermediate gear is inserted, the gear on *C* still revolves 5 times while the gear on *D* revolves once.

If we placed a number of intermediate gears, *E*, *F*, and *G*, in the train, as in Fig. 28, the result would still be the same, the gear on *C* would turn 5 times while the gear on *D* turned once, as long as the number of teeth in the gear on *D* is 5 times the number of teeth in the gear on *C*.

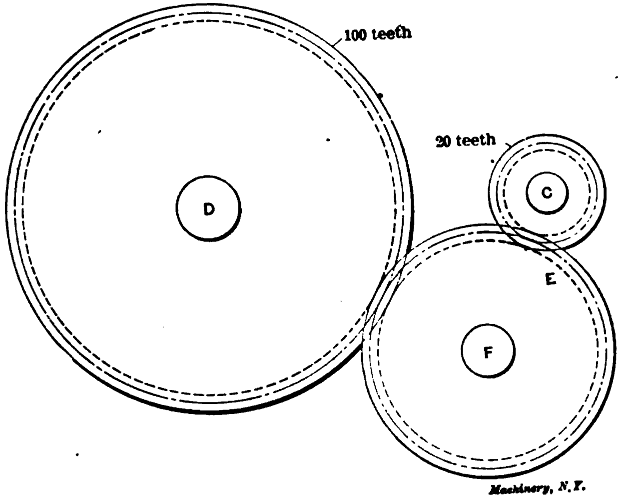


Fig. 27.

In order to prove this, let us assume that in Fig. 28, as before, the gear on stud *C* has 20 teeth, and the gear on stud *D*, 100 teeth, so that consequently the stud *C* makes 5 revolutions, while stud *D* makes one. The intermediate gears, *E*, *F*, and *G*, have 50, 40, and 40 teeth, respectively, as shown in the cut. Now, when the gear on *D* turns around once, the gear *G* must turn 2½ times $(100/40 = 2\frac{1}{2})$. The gear *F*, having the same number of teeth as gear *G*, makes one revolution while *G* makes one, and consequently also turns 2½ times while the gear on *D* turns once. The gear on *E*, having 50 teeth, turns 4/5 of a revolution while gear *F* revolves once $(40/50 = 4/5)$, and consequently, while *F* makes 2½ revolutions, gear *E* makes $2\,1/2 \times 4/5 = 5/2 \times 4/5 = 2$ revolutions. Thus *E* turns twice while the gear on stud *D* revolves once. Finally, the gear on *C* turns 2½ times to each revolution of gear *E* $(50/20 = 2\frac{1}{2})$, or 5 times to 2 revolutions of *E*. But 2 revolutions of *E* correspond, as we have seen, to one revolution of the gear on stud *D*; consequently, the gear on stud *C* makes 5 revolutions to one of the gear on stud *D*, which, as we previously found, is also the case if these two gears had been connected directly without any intermediate gearing.

Principles of Compound Gearing.

Compound gearing consists of a train of gears in which certain gears are placed in pairs on the same stud as shown at *A*, Fig. 29, and turn together, one of the gears, *D*, being driven by another gear *B*, and the other gear, *E*, in turn driving a gear *C*. The simplest, and most common case of compound gearing consists of four gears, as shown in the cut referred to. Suppose that the stud, *F*, carrying gear *C*, Fig. 29, is required to revolve 6 times while stud *G*, carrying gear *B*, revolves

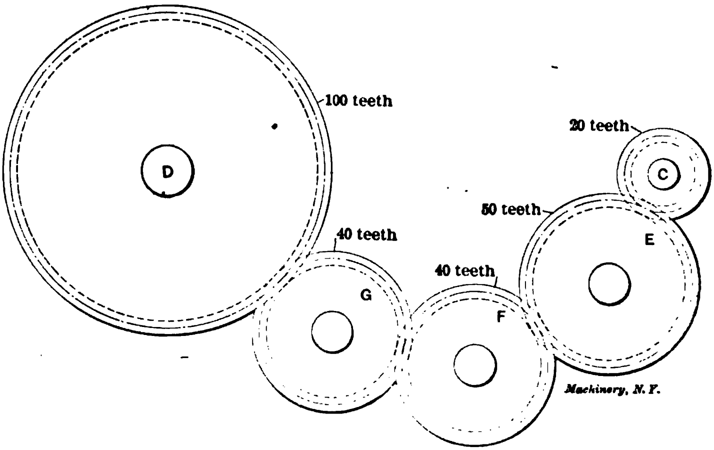


Fig. 28.

5 times. The ratio in speed of these two shafts would then be 6 to 5. Gears are not available so that studs *F* and *G* can be directly connected, or to make it possible to put in an intermediate gear to transmit the motion directly. In such cases the gears are compounded, that is, the train of gears is made up of two sets of gears, in each of which sets one gear is the driving and one the driven gear. In Fig. 29, *B* and *E* are driving gears, and *C* and *D* are driven gears.

Each of these two sets of gears has a speed ratio of its own, which combined with the ratio of the other set gives the total ratio of the whole system. Referring to our specific case in Fig. 29, if gear *B* has 90 teeth, and gear *D*, 45 teeth, then gear *D* revolves two times while gear *B* revolves once, or, as our ratio between *C* and *B* is 6 to 5, let us say that gear *D* turns around 10 times while *B* turns around 5 times. Gear *E* is placed on the same stud as gear *D* and, therefore, also turns 10 times while *B* turns 5 times. Gear *E* drives gear *C*, which is required to turn 6 times to the 5 times of gear *B*. It, therefore, must also turn 6 times to the 10 times of gear *E*. If, then, gear *E* has 60 teeth and gear *C*, 100 teeth, this requirement will be filled. We now have gear *C* turning 6 times while gear *B* turns 5 times.

How could we have found the gears for this compound gearing directly by calculation?

The ratio required was 6 to 5, or if written as a fraction, 6/5. We can multiply numerator and denominator in a fraction by the same number without changing its value. We can also divide up the numerator and denominator in factors, and multiply each pair of factors with the same number. Thus we have:

$$\frac{6}{5} = \frac{2 \times 3}{1 \times 5} = \frac{(2 \times 45) \times (3 \times 20)}{(1 \times 45) \times (5 \times 20)} = \frac{90 \times 60}{45 \times 100}.$$

Here we have, then, the gears used in our train. The gears in the

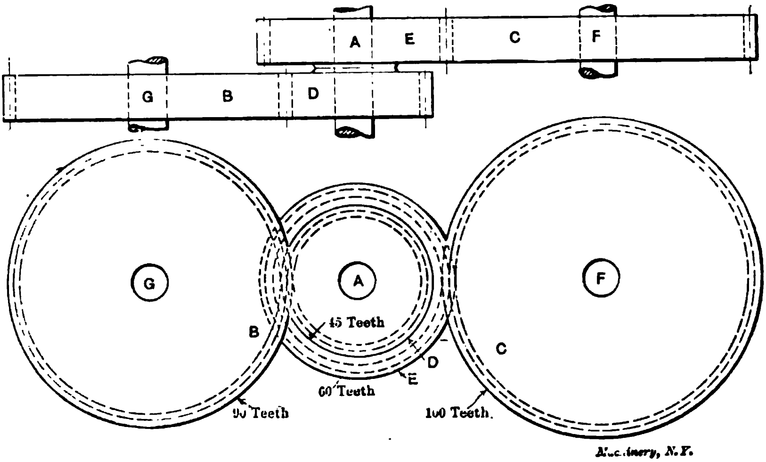


Fig. 29.

numerator are the driving gears *B* and *E*. The gears in the denominator are the driven gears *D* and *C*.

We may formulate a rule from the foregoing for finding gears for transmitting motion, in general, when the speed ratios between the driving and driven shafts are known.

1. *Place the number of revolutions of the driven shaft in the numerator, and the corresponding number of revolutions of the driving shaft in the denominator of a fraction* (or, in general, write the ratio in the form of a fraction), *and multiply the numerator and denominator with the same number, until a new fraction is obtained having numerator and denominator expressing suitable numbers of teeth for the gears.* The gear represented by the new numerator is the driving gear, and that represented by the new denominator is the driven gear.

2. If compounding of the gears is necessary or advisable, *divide up both numerator and denominator in the fraction. giving the ratio, in two factors, and multiply each pair of factors* (one factor in the numerator and one in the denominator making "one pair") *by the same numbers, until gears with suitable numbers of teeth are found.*

The most common application of trains of gearing is that occurring in lathes, and used for thread cutting. We shall, therefore, apply the principles of trains of gears to this case. But it should be clearly understood that the same principles hold good, no matter what machine the gears are applied to, and that the fundamental condition which determines the number of teeth in the gears connecting two shafts is the ratio of speed, that is, the number of turns made by one gear to a certain number of turns made by another.

Pitch and Lead of Screw Threads.

The terms pitch and lead of screw threads are often confused, and particularly in the case of multiple threaded screws does this confusion cause difficulties. Before we therefore enter upon the subject of figuring change gears for the lathe for cutting screw threads, it may be well to make clear the real meaning of the words "pitch" and "lead," and their relation to the number of threads per inch.

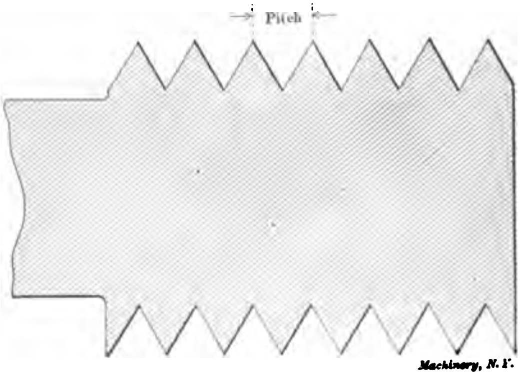


Fig. 30.

The *pitch* of a screw thread is the distance from the top of one thread to the top of the next, as shown in Fig. 30. No matter whether the screw has single, double, triple, or quadruple thread, the pitch is always the distance from the top of one thread to the top of the next. Often, though improperly, the word "pitch" is used in the shop to denote "number of threads per inch." We hear of screws having 12 pitch thread, 16 pitch thread, etc. This is not correct usage of the word pitch, and only tends to cause unnecessary confusion.

The *lead* of a screw thread is the distance the screw will move forward in a nut if turned around one full revolution. It is clear that for a single-threaded screw the pitch and the lead are equal, as the screw would then move forward the distance from one thread to the next if turned around once. In a double-threaded screw, however, the screw will move forward two threads, or twice the pitch, so that in a double-threaded screw the lead equals twice the pitch. In a triple-threaded screw the lead equals three times the pitch, and so forth.

The lead may also be expressed as being the distance from center to center of the *same* thread, after this thread has made one turn around the screw. In the single-threaded screw the *same* thread is the next thread to the one first considered. In a double-threaded screw there are two threads running side by side around the screw, so that the *same* thread here is the second one from the one first considered. In a triple-threaded screw it is the third one, in a quadruple-threaded, the fourth, and so forth. However we consider this, we still see that the lead and pitch are alike for a single-threaded screw, that the lead is twice the pitch for a double-threaded, and three times for a triple-threaded, as already stated. The actual relationship is very plainly shown in Fig. 31, where are shown parts of three screws with Acme threads, the first single-threaded, the second double-threaded, and the last triple-threaded.

The main point to remember, however, is that in *any kind of a*

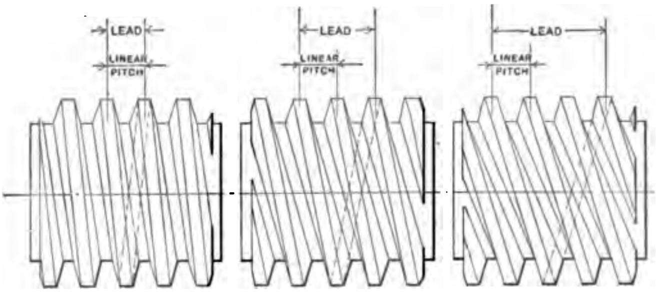


Fig. 31.

screw, the lead is the distance which the screw will move forward in a nut if turned around one revolution.

In this connection it may be appropriate to give the rules and formulas for the relation between the lead and the number of threads per inch. If there are 8 threads, single, in one inch, the lead is evidently ⅛ inch. This we found, mathematically, by dividing *one* by 8, which is the number of threads per inch. The formula, therefore, is

$$\text{lead} = \frac{1}{\text{number of threads per inch}}$$

This formula, expressed in words, says: *The lead of a screw equals one divided by the number of threads per inch.*

Confusion is often caused by indefinite designation of multiple thread screws. The most common way to state the lead and the class of thread is perhaps to say ¼ *inch lead, double*, which means a screw with a double thread, which, when cut, has the lathe geared for four threads per inch, but each thread is cut only to a depth corresponding to eight threads per inch. The same condition is also expressed by: 4 *threads per inch, double.* These two ways of expressing the number of multiple threads are both correct, but the expression which ought to be used in order to avoid misunderstanding under any circumstances would be: ¼ *lead*, ⅛ *pitch, double thread.*

CHAPTER V.

LATHE CHANGE GEARS.

While the principles and rules governing the calculation of change gears are very simple, they, of course, presuppose some fundamental knowledg of the use of common fractions. If such knowledge is at hand, the subject of figuring change gears, if once thoroughly understood, can hardly ever be forgotten. It should be impressed upon the minds of all who have found difficulties with this subject that the matter is not approached in a logical manner, and is usually grasped by the memory rather than by the intellect. Before answering the question in regard to any rules for figuring change gears, let us therefore analyze the subject. The lead-screw *B* of the lathe (see Fig. 32) must be recognised as our first factor, and the spindle as the second. If the lead-screw has six threads per inch, then, if the lead-screw makes six revolutions, the carriage travels one inch, and the thread-cutting tool travels one inch along the piece to be threaded. If the spindle makes the same number of revolutions in a given time as the lead-screw, it is clear the tool will cut six threads per inch. In such a case the gear *D* on the spindle stud *J*, and gear *E* on the lead-screw, are alike. If the spindle makes twice the number of revolutions of the lead-screw, the spindle revolves twelve times while the tool moves one inch, and consequently twelve threads per inch will be cut. But in order to make the spindle revolve twice as fast as the lead-screw, it is necessary that a gear be put on the spindle stud of only half the number of teeth of the gear on the lead-screw, so that when the lead-screw revolves once the spindle stud gear makes two revolutions. The conditions governing the relationship between the number of teeth in gears and the number of revolutions of the studs on which they are mounted were, as we remember, explained in the last chapter.

Simple Gearing.

Suppose we wish to cut nine threads per inch with a lead-screw of six threads per inch, as referred to above. Then the six threads of the lead-screw correspond to nine threads on the piece to be threaded, which is the same as to say that six revolutions of the lead-screw correspond to nine revolutions of the spindle; or in other words, one revolution of the lead-screw corresponds to 1½ of the spindle. From this it is evident that the gear on the lead-screw must make only one revolution while the spindle stud gear makes 1½. Thus, if the lead-screw gear has, for instance, 36 teeth the gear on the spindle stud should have only 24; the smaller gear, of course, revolving faster than the larger. If we express what has been previously said in a formula we have:

$$\frac{\text{threads per inch of lead-screw}}{\text{threads per inch to be cut}} = \frac{\text{teeth in gear on spindle stud}}{\text{teeth in gear of lead-screw}}$$

Applying this to the case above, we have:

$$\frac{6}{9} = \frac{24}{36}$$

The values 24 and 36 are obtained by multiplying 6 and 9, respectively, by 4. By multiplying both the numerator and the denominator by the same number, we do not change the proportion. As a general rule we may then say that the change gears necessary to cut a certain number of threads per inch are found by placing the number of threads in the lead-screw in the numerator, the number of threads to be cut in the denominator, and then multiply numerator as well as denominator by the *same* number, by trial, until two gears are obtained, the number of teeth of which are both to be found in the set of gears accompanying the lathe. The gear with the number of teeth designated by the

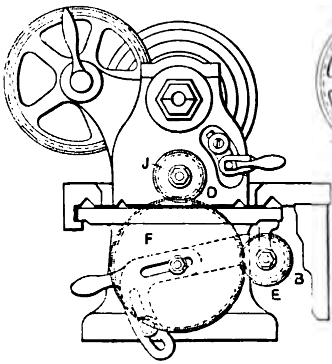


Fig. 32.

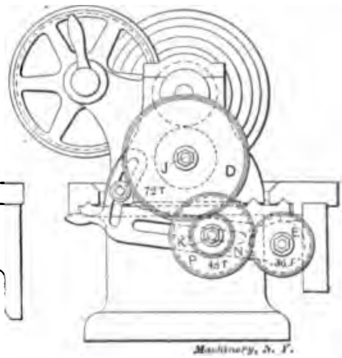


Fig. 33.

new numerator is to be placed on the spindle stud (at *J*, Fig. 32), and the gear with the number of teeth corresponding to the denominator on the lead-screw *B*.

A few examples of this will more clearly explain the rule. Suppose the number of teeth of the change gears of a lathe are 24, 28, 32, 36, and so forth, increasing by 4 teeth up to 100. Assume that the lead-screw is provided with 6 threads per inch, and that 10 threads per inch are to be cut. Then,

$$\frac{6}{10} = \frac{6 \times 4}{10 \times 4} = \frac{24}{40}$$

By multiplying both numerator and denominator by 4, we obtain two available gears with 24 and 40 teeth, respectively. The 24-tooth gear goes on the spindle stud, and the 40-tooth gear on the lead-screw. Assuming the same lathe and gears, let us find the gears for cutting $11\frac{1}{2}$ threads per inch, this being the standard number of threads for certain sizes of pipe thread. Then,

$$\frac{6}{11\frac{1}{2}} = \frac{6 \times 8}{11\frac{1}{2} \times 8} = \frac{48}{92}$$

It will be found that multiplying with any other number than eight would, in this case, not have given us gears with such number of teeth as we have in our set with this lathe. Until getting accustomed to figuring of this kind, we can, of course, only by trial find out the correct number by which to multiply numerator and denominator. The number of teeth in the *intermediate* gear *F*, Fig. 32, which meshes with both the spindle stud gear and the lead-screw gear, is of no consequence.

Lathes with Reduction Gearing in Head-stock.

In some lathes, however, there is a reduction gearing in the headstock of the lathe, so that if equal gears are placed on the lead-screw and the spindle stud, the spindle does not make the same number of revolutions as the lead-screw, but a greater number. Usually in such lathes the ratio of the gearing in the head-stock is 2 to 1, so that with equal gears the spindle makes two revolutions to one of the lead-screw. This is particularly common in lathes intended for cutting fine pitches or, in general, in small lathes. In figuring the gears this must, of course, be taken into consideration. As the spindle makes twice as many revolutions as the lead-screw with equal gears, if the ratio of the gears be 2 to 1, that means that if the head-stock gearing were eliminated, and the lead-screw instead had twice the number of threads per inch as it has, with equal gears the spindle would still revolve the same as before for each inch of travel along the piece to be threaded. In other words, the gearing in the headstock may be *disregarded, if the number of threads of the lead-screw is multiplied by the ratio of this gearing.* Suppose, for instance, that in a lathe the lead-screw has eight threads per inch, that the lathe is geared in the head-stock with a ratio of 2 to 1, and that 20 threads are to be cut. Then

$$\frac{2 \times 8}{20} = \frac{16}{20} = \frac{16 \times 4}{20 \times 4} = \frac{64}{80}$$

which two last values signify the number of teeth in the gears to use.

Sometimes the ratio of the gearing in the head-stock cannot be determined by counting the teeth in the gears, because the gears are so placed that they cannot be plainly seen. In such a case, equal gears are placed on the lead-screw and the spindle stud, and a thread cut on a piece in the lathe. The number of threads per inch of this piece should be used for the numerator in our calculation instead of the actual number of threads of the lead-screw. The ratio of the gearing in the head-stock is equal to the ratio between the number of threads cut on the piece in the lathe and the actual number of threads per inch of the lead-screw.

Compound Gearing.

The cases with only two gears in a train referred to are termed simple gearing. Sometimes it is not possible to obtain the correct ratio excepting by introducing two more gears in the train, which, as has previously been said, is termed compound gearing. This class of gearing is shown in Fig. 33. The rules for figuring compound gearing are exactly the same as for simple gearing, excepting that we must divide both our numerator and denominator into two factors, each of which are multiplied with the same number in order to obtain the change gears.

Suppose a lathe has a lead-screw with six threads per inch, that the number of the teeth in the gears available are 30, 35, 40, and so forth, increasing by 5 up to 100. Assume that it is desired to cut 24 threads per inch. We have then,

$$\frac{6}{24} = \text{ratio.}$$

By dividing up the numerator and denominator in factors, and multiplying *each pair of factors by the same number*, we find the gears:

$$\frac{6}{24} = \frac{2 \times 3}{4 \times 6} = \frac{(2 \times 20) \times (3 \times 10)}{(4 \times 20) \times (6 \times 10)} = \frac{40 \times 30}{80 \times 60}$$

The last four numbers indicate the gears which should be used. The upper two, 40 and 30, are driving gears, the lower two, with 80 and 60 teeth, are driven gears. Driving gears are, of course, the gear *D*, Fig 33, on the spindle stud, and the gear *P* on the intermediate stud *K*, meshing with the lead-screw gear. Driven gears are the lead-screw gear, *E*, and the gear *N* on the intermediate stud meshing with the spindle stud gear. It makes no difference which of the driving gears is placed on the spindle stud, or which of the driven is placed on the lead-screw.

Suppose, for a final example that we wish to cut 1¾ threads per inch on a lathe with a lead-screw having six threads per inch, and that the gears run from 24 and up to 100 teeth, increasing by 4. Proceeding as before, we have

$$\frac{6}{1\frac{3}{4}} = \frac{2 \times 3}{1 \times 1\frac{3}{4}} = \frac{(2 \times 36) \times (3 \times 16)}{(1 \times 36) \times (1\frac{3}{4} \times 16)} = \frac{72 \times 48}{36 \times 28}$$

This is the case directly illustrated in Fig. 32. The gear with 72 teeth is placed on the spindle stud *J*, the one with 48 on the intermediate stud *K*, meshing with the lead-screw gear. These two gears (72- and 48-teeth) are the *driving* gears. The gears with 36 and 28 teeth are placed on the lead-screw, and on the intermediate stud, as shown, and are the *driven* gears.

Fractional Threads.

Sometimes the lead of the thread is expressed by a fraction of an inch, instead of stating the number of threads per inch. For instance, a thread may be required to be cut having a ⅜-inch lead. In such a case the expression "⅜-inch lead" should first be transformed to "number of threads per inch," after which we can proceed in the same way as has already been explained. To find how many threads per inch there is when the lead is stated, we simply find how many times the lead is contained in one inch, or, in other words, we divide *one* by the given lead. Thus *one* divided by 3/8 gives us 2 2/3, which is the number of threads per inch of a thread having ⅜-inch lead. To find change gears to cut such a thread we would proceed as follows:

Assume that the lead-screw has 6 threads per inch, and that the change gears run from 24 up to 100 teeth, increasing by 4. Proceeding to find the gears as before, we have:

$$\frac{6}{2\frac{2}{3}} = \frac{2 \times 3}{1 \times 2\frac{2}{3}} = \frac{(2 \times 36) \times (3 \times 24)}{(1 \times 36) \times (2\frac{2}{3} \times 24)} = \frac{72 \times 72}{36 \times 64}.$$

The rule for finding the number of threads per inch, when the lead is given, may be expressed by the formula:

$$\text{number of threads per inch} = \frac{1}{\text{lead of thread}}$$

which is simply a reversal of the formula given on page 25.

What has been said in the foregoing in regard to the figuring of change gears for the lathe may be summed up in the following rules:

1. To find the number of threads per inch, if the lead of a thread is given, *divide one by the lead.*

2. To find the change gears used in simple gearing, when the number of threads per inch on the lead-screw, and the number of threads per inch to be cut are given, *place the number of threads on the lead-screw as numerator and the number of threads to be cut as denominator in a fraction, and multiply numerator and denominator with the same number until a new fraction results representing suitable number of teeth for the change gears.* In the new fraction, the numerator represents the number of teeth on the spindle stud, and the denominator, the number of teeth in the gear on the lead-screw.

3. To find the change gears used in compound gearing, *place the number of threads per inch on the lead-screw as numerator, and the number of threads per inch to be cut as denominator in a fraction, divide up both numerator and denominator in two factors each, and multiply each pair of factors* (one factor in the numerator and one in the denominator making "a pair") *by the same number, until new fractions result representing suitable number of teeth for the change gears.* The gears represented by the numbers in the new numerators are driving gears, and those in the denominators are driven gears.

CHAPTER VI.

SPEED OF PULLEYS.

The principle applied to gearing in regard to the ratio between the speeds of two shafts, may be directly applied to the question of sizes of pulleys, with the only difference that we here deal with the number of inches to the diameter of the pulley instead of the number of teeth in the gear. Suppose that a shaft is required to make 300 revolutions per minute, and that this shaft is driven from a line-shaft making 180 revolutions per minute, as shown in Fig. 34. The pulley on the line-shaft is already in place, and is 15 inches in diameter. What diameter should the pulley on the shaft making 300 revolutions per minute be made?

As the belt over the two pulleys runs at the same speed as the

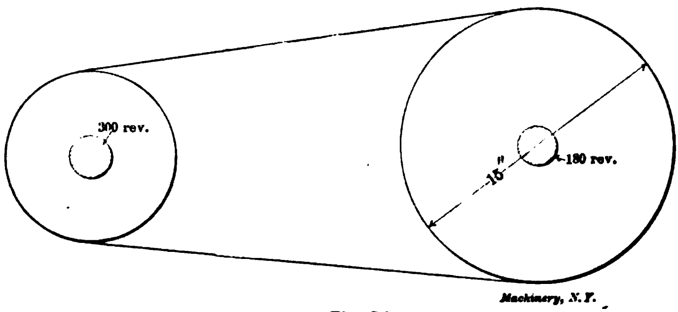


Fig. 34.

circumference of the pulleys, it is clear that the circumferences of both pulleys run at the same speed. The pulley running a fewer number of revolutions consequently must be larger, in order that its circumference may run at the same speed as the circumference of the pulley running faster in regard to number of revolutions. The circumference depends directly upon the length of the diameter, because we know that the circumference equals the diameter times 3.1416. But as the factor 3.1416 would enter in the case of both pulleys, it is not necessary to carry it along in the calculation. We can figure with the diameter directly, substituting the diameter for the circumference, so to say. *For the obtaining of our ratio of speed between the pulleys, the diameters serve the same purpose in figuring as the circumferences*, because the ratio between the diameters is the same as the ratio between the circumferences. Now, the circumference of the pulley making 180 revolutions, and having a diameter of 15 inches, passes through a distance of 180 times its circumference in one minute, or $180 \times 15 \times 3.1416$. The circumference of the pulley making 300 revolutions must pass through the same distance, for as we have said, the belt causes the circumferences to run equally fast. Therefore, for *each* revolution the latter pulley must pass through the distance 180 × 15 × 3.1416 divided by 300. This then would be the circumference of the smaller pulley, or its diameter times 3.1416. We can therefore write

$$\frac{180 \times 15 \times 3.1416}{300} = \text{diameter of small pulley} \times 3.1416.$$

As we said before, 3.1416 enters as a factor in both cases, and we can therefore cancel it. Then we have

$$\frac{180 \times 15}{300} = \text{diameter of small pulley} = 9 \text{ inches}.$$

From this we can formulate a rule for all figuring of pulleys:

The number of revolutions of one shaft multiplied with the diameter of the pulley on the same shaft, divided by the number of revolutions of the second shaft, gives the diameter of the pulley of the second shaft.

We may also write this as a formula, thus:

$$\frac{\text{diameter of pulley on}}{\text{second shaft in inches}} = \frac{\text{number of revolutions of first shaft} \times \text{diameter of pulley on first shaft in inches}}{\text{number of revolutions of second shaft}}$$

If we know the sizes of the pulleys, and the number of revolutions of one shaft, and want to find out the number of revolutions of the other shaft, the method is very similar to that used for figuring change gears. Evidently this should be so, because the diameters of the pulleys simply express the ratio of speeds of the shaft. Suppose one pulley is 12 inches in diameter, and another 20, and the shaft with the 12-inch pulley makes 180 revolutions. How many revolutions does the other pulley make?

Our ratio is $\frac{20}{12}$. Now multiply the numerator, 20, with a number giving 180 as result. This number is 9. Multiply the denominator by the *same* number, and we get 9 times 12 equals 108 as the number of revolutions of the second pulley. In other words, we have

$$\frac{20}{12} = \frac{20 \times 9}{12 \times 9} = \frac{180}{108}$$

It should be understood that the number of revolutions are in a *reverse* ratio to the ratio of the diameters, that is,

$$\frac{\text{diameter of first pulley}}{\text{diameter of second pulley}} = \frac{\text{revolutions of second pulley}}{\text{revolutions of first pulley}}$$

In the first ratio (of diameters), the diameter of the first pulley is the numerator and that of the second pulley is the denominator, but in the second ratio (of revolutions), the number of revolutions of the second pulley is the numerator, and that of the first pulley is the denominator.

A few examples may tend to make this clearer and fix the procedure more firmly in the mind of the reader.

Example 1.—A line-shaft for a few grinding machines is required to run at 320 revolutions per minute, and to be driven from a main line-shaft, running at 200 revolutions per minute. The pulley on the main line-shaft is already in place, and is 24 inches in diameter. What diameter ought the pulley on the grinding machine line-shaft to be?

We find this diameter directly from our rule on page 32 saying that the number of revolutions of one shaft times the diameter of the pulley on this shaft, divided by the number of revolutions of the second shaft, gives us the diameter of the pulley of the second shaft. Thus

$$\frac{200 \times 24}{320} = \frac{4800}{320} = 15.$$

The diameter of the pulley on the grinding machine line-shaft, therefore, should be 15 inches.

Example 2.—The pulley of a machine tool is 8 inches in diameter. The driving pulley on the line-shaft is 34 inches in diameter. It is known that the line-shaft makes 120 revolutions per minute. How many revolutions per minute does the 8-inch pulley on the machine make?

The diameters of the pulleys are in *inverse ratio* to the number of revolutions, that is, as already said,

$$\frac{\text{diameter of first pulley}}{\text{diameter of second pulley}} = \frac{\text{revolutions of second pulley}}{\text{revolutions of first pulley}}$$

From this we have, then, in our present example,

$$\frac{8}{34} = \frac{8 \times 15}{34 \times 15} = \frac{120}{510}.$$

The 8-inch pulley, consequently, makes 510 revolutions per minute.

Example 3.—The largest step of a cone-pulley in the countershaft of a machine is 12 inches in diameter. The smallest step is 6 inches. The largest step of the cone pulley on the machine is 10 inches, the smallest, 4. If the countershaft runs 300 revolutions per minute, which are the highest and lowest speeds of the spindle of the machine on which the cone pulley is mounted?

The largest step on each respective pulley runs with the smallest step of the other. Therefore, proceeding exactly as if we had two sets of pulleys, one set 12 and 4, and one 6 and 10 inches in diameter, we find by figuring exactly as in our previous example:

$$\frac{4}{12} = \frac{4 \times 75}{12 \times 75} = \frac{300}{900},$$

and

$$\frac{10}{6} = \frac{10 \times 30}{6 \times 30} = \frac{300}{180}.$$

The smallest speed is therefore 180 revolutions per minute, and the largest 900 revolutions.

CHAPTER VII.

INDEXING MOVEMENTS FOR MILLING MACHINE.

The figuring of indexing movements for the dividing head of the milling machine is a subject which many mechanics think complicated, although it really is very simple. The index head is constructed with a worm and worm-wheel mechanism, the worm being on the crank turned when indexing, and the worm-wheel being mounted on the index spindle to which the work is attached. The worm-wheel has 40 teeth, so that turning the crank around one full revolution, which also turns the worm one revolution, moves the worm-wheel one tooth, or one-fortieth of its circumference. In the same way, to turn the worm-wheel and the spindle on which it is mounted around one full revolution, we must turn the index-crank 40 revolutions. If we thoroughly understand this, the figuring of indexing movements is very easy. Suppose that we want to mill a hexagon. We then want to turn the work one-sixth of a revolution for each side milled. As it requires 40 revolutions of the index-crank to turn the index-spindle once, it evidently requires only one-sixth of that number to turn the index-spindle one-sixth revolution. Consequently, the index-crank should be turned around $\frac{40}{6} = 6\ 2/3$ revolutions for milling a hexagon. That is, we first turn the crank around 6 times, and then, by means of the index-plate, we turn it 2/3 of a revolution. This would mean 12 holes in an 18-hole circle, for instance, as 12 is two-thirds of 18.

Suppose we should want to mill a piece of work having 8 sides, regularly spaced. The indexing for each space is found by dividing 40 by 8. Then, $\frac{40}{8} = 5$, represents the number of turns of the index-crank for each side indexed. If we want to cut 9 flutes, regularly spaced, in a reamer, we must turn the crank $40/9 = 4\ 4/9$ revolutions to index for each flute. The 4/9 of a revolution would correspond to 8 holes in the 18-hole circle, because $8/18 = 4/9$.

In order to find which index circle to use, and how many holes in the index circle to move for a certain fractional turn of the index-crank, numerator and denominator of the fraction expressing the fractional turn are multiplied by the same number, this number being so selected that the denominator in the new fraction equals the number of holes in some index circle. The new numerator then expresses *how many* holes in this circle the crank is to be moved. Suppose that we want to index for 12 flutes in a large tap. We first divide 40 by 12 to find the number of turns of the index-crank required. Writing out this division as a fraction, and carrying out the calculation, gives us:

$$\frac{40}{12} = 3\frac{4}{12} = 3\frac{1}{3}.$$

The fractional turn required is 1/3 of a revolution. Now multiply, according to the rule given, the numerator and denominator of this fraction by a number so that the denominator equals the number of holes in some index circle. Multiplying with 6 would give us

$$\frac{1 \times 6}{3 \times 6} = \frac{6}{18}$$

in which fraction 18 is a number expressing the number of holes in the index circle to use, and 6 the number of holes the crank has to be moved in this circle to turn one-third of a revolution.

Suppose, that instead of having given a certain number of times which the work is to be indexed to complete one revolution (as 80 for an 80-tooth gear; 6 for a hexagon nut, etc.), we instead had given a certain number of degrees which it is required that the work be turned before taking another cut by the milling cutter.

As there are 360 degrees in a complete circle or turn (one right angle is 90 degrees, and the whole circle, of course, consists of four right angles), and as 40 turns of the index crank are required for one revolution of the work, *one* turn of the index crank must correspond to $360/40 = 9$ degrees. Then, if one complete turn of the index crank equals 9 degrees, it is clear that if we index in the 18-hole circle, 2 holes must correspond to one degree. This is, therefore, the fundamental principle or rule for indexing angular movement. *Two holes in the 18-hole circle equals a movement of one degree of the work.*

Suppose that we wish to index 11 degrees. We first, then, divide the number of degrees by 9 to find how many complete turns the index crank should make; and the number of degrees left to turn when we have completed our full turns are indexed by taking two holes in the 18-hole circle for each degree. In this case $11/9 = 1\ 2/9$, which gives us the answer that we must turn the crank one full revolution, and then index 2 degrees more, or 4 holes in the 18-hole circle.

It is evident that one hole in the 18-hole circle represents 1/2 degree, when two holes represent one degree. Should it be required to index only 1/3 degree, this may be done by using the 27-hole circle. If two holes in the 18-hole circle, which is 1/9 of a turn, equals one degree, three holes in the 27-hole circle, which also is 1/9 of a turn, must also equal one degree; and if a 3-hole movement equals one degree, a 1-hole movement in that circle must equal 1/3 degree. Therefore, if we wish to index the work through an angle of 48 degrees 40 minutes (there are 60 minutes to one degree, so that 40 minutes equals 2/3 degree), we simply turn the crank five complete turns for 45 degrees ($5 \times 9 = 45$), and we have then 3 degrees 40 minutes, or 3 2/3 degrees left. In the 27-hole circle 3 degrees correspond to 9 holes, and 2/3 degree to 2 holes, according to what we have just said. Consequently, we turn the crank 11 holes, in all, further in the 27-hole circle to complete the angular movement of 48 degrees 40 minutes.

CHAPTER VIII.

CHANGE GEARS FOR MILLING SPIRALS.

The method for the figuring of change gears for cutting spirals on the milling machine, is, in principle, exactly the same as that used for figuring change gears for the lathe, but it will be necessary to shortly refer to the construction of the mechanism for connecting the index head spindle and the feed-screw to make perfectly clear the fundamental ideas governing the selection of change gears. In Fig. 35, *A* is the feed-screw of the milling machine; and *B* is the gear placed on this feed-screw, commonly called the feed-screw gear. This gear meshes with the gear *C*, placed on the stud *D*, from which, in turn, motion is imparted to the worm in the index head and from the worm to the worm-wheel and the index spindle. The gear *C* on the stud *D* is called the worm gear, because it directly operates the movement of the worm. This expression "worm gear" should not be confused with the worm-wheel, which is placed on the index spindle. The case shown in Fig. 35 is one of simple gearing. In Fig. 36 is shown a case of compound gearing. Here *B* still represents the feed-screw gear, *E* is a gear on the intermediate stud, meshing with gear *B*, and gear *F* is another gear on the same intermediate stud, meshing with gear *C*, thus transmitting motion from the feed-screw to the stud *D* by compounding the gears.

The figuring of change gears for the milling machine consists simply in the selection of the proper gears, *B* and *C*, used in a simple train, as in Fig. 35, or gears *B*, *E*, *F*, and *C*, as used in a compound train of gears, as shown in Fig. 36.

In order to figure change gears for the lathe we remember that it was necessary to first know the number of threads per inch in the lead-screw. Knowing that, we knew how many revolutions the lead-screw had to make to move the carriage and the thread tool one inch along the work. In the case of the milling machine we must know *how far the table travels while the index spindle makes one complete revolution, when gears B and C, Fig. 35, have an equal number of teeth.* This distance is the constant which we use in figuring the change gears, the same as we used the number of threads per inch of the lead-screw in figuring change gears for the lathe. This constant, which may be different for different milling machines, is called the *lead of the milling machine.* We will now see how this constant is found.

Suppose, for instance, that in a milling machine one revolution of the worm gear, *C*, Figs. 35 and 36, will produce exactly one revolution of the shaft on which the worm is placed, that is, one revolution of the index crank, and suppose that equal gears are placed on the feed-screw and on the stud *D*, so that one revolution of the feed-screw produces exactly one revolution of the gear *C* and stud *D*. Then, if the feed-screw revolves one revolution, the milling machine table will advance a distance equal to the lead of the feed-screw, because, as we have said before, the lead of the screw is the distance which it will advance in one turn.

Now, when the table of the milling machine moves forward a distance equal to the lead of the feed-screw, or a distance equal to one thread in the feed-screw, the feed-screw turns one revolution, and gear *C* also turns one revolution, the worm-shaft and the index crank turn one revolution, and, there being 40 teeth in the worm-wheel which is mounted on the index head spindle, this worm-wheel, with its spindle, will turn 1/40 of one revolution. To make one complete revolution of the index head spindle, the feed-screw would have to be revolved as many times as there are teeth in the worm-wheel, each revolution,

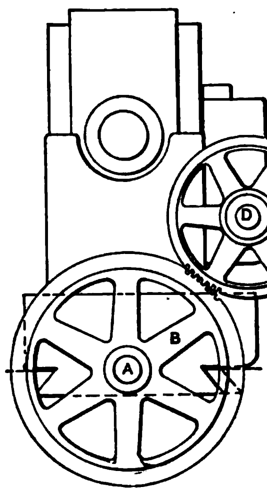


Fig. 35.

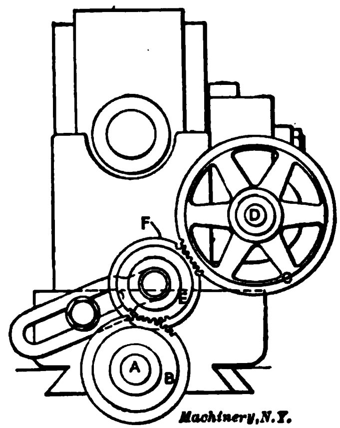


Fig. 36.

as we have seen, moving it one tooth. The distance which the milling machine table will advance, when the index head spindle revolves one complete revolution, is, as we have said, the lead of the machine. This distance evidently equals the distance that the feed-screw advances the table in one revolution (which is the lead of the screw) times the number of revolutions made. If we assume that the feed-screw has a lead of 1/8 inch, and that there are 40 teeth in the worm-wheel on the index head spindle, the feed-screw, when the gears *B* and *C* are equal, will have to turn 40 times, in order to move the index head spindle around once, and the distance the table will then advance will be 40 times 1/8 inch, or 5 inches. In this particular case, 5 inches is then the *lead of the machine*. If now a piece of work had been affixed to the index spindle, it is clear that this piece of work would have made one complete revolution, while the milling machine table advanced 5 inches, and that the lead of the spiral cut on the work would have been 5 inches.

A general rule for finding the lead of a milling machine may be stated as follows: *To find the lead of a milling machine, place equal gears on stud D, Fig. 35, and on the feed-screw, and multiply the number of revolutions made by the feed-screw in order to produce one revolution of the index head spindle, by the lead of the feed-screw.*

Suppose, for an example, that it is necessary to make 40 revolutions of the feed-screw in order to turn the index head spindle one complete revolution, when gears *B* and *C*, Fig. 35, are equal, and that the lead of the feed-screw of the milling machine is ¼ inch, then the lead of the machine equals 40 × ¼ inch, or 10 inches.

The rule just given is general, and will apply even if the number of teeth in the indexing worm-wheel were different from that in standard indexing heads, because, in the rule no consideration is taken of the number of teeth of the worm-wheel directly, but simply the number of turns made by the feed-screw to correspond to one turn of the index spindle itself.

If it is now perfectly clear that the lead of the machine means the distance which the table of the milling machine must move forward in order to turn the work placed on the index head spindle around one complete revolution, with equal gears, then we see that if we want to get a spiral that is twice as long as the lead of the machine, we must place gears on the feed-screw and on the stud *D* of such size that the indexing spindle will only turn half a revolution while the table moves forward a distance equal to the lead of the machine. Suppose, for instance, that we want to cut a spiral, having a lead of 20 inches, that is, making one complete turn in a distance of 20 inches, and that the lead of the milling machine is 10 inches. Then, while the table moves forward 20 inches, we want the indexing spindle to turn once. In order to make the table move forward 20 inches, when the indexing spindle turns around once, the feed-screw evidently must turn twice as many times as it did in the case when the table only moved 10 inches for one turn in the index spindle, in which case we had equal gears on the feed-screw and on stud *D*. Here we then have two studs or shafts, *A* and *D*, where we wish that the one should turn twice as fast as the other. The ratio between the speeds, then, is 2 to 1, which means that the feed-screw, which is required to turn twice while the stud *D* turns once, must have a gear that has only half the number of teeth of the gear placed on stud *D*.

If the lead of the machine be 10 inches, and the lead required to be cut on a piece of work is 30 inches, then it would be necessary to have the ratio between the gears 3 to 1, which, of course, is the same as the ratio between the lead of the machine and the lead of the spiral to be cut (30 to 10 equals 3 to 1). We can therefore express the rule for finding the change gears by a simple formula:

$$\frac{\text{lead of spiral to be cut}}{\text{lead of machine}} = \frac{\text{number of teeth in gear on worm stud } (D, \text{ Fig. 35})}{\text{number of teeth in gear on feed-screw}}$$

Expressed as a rule, this formula would read: *To find the change gears to be used in a simple train of gearing, when cutting spirals on a milling machine, place the lead of the spiral in the numerator and*

the lead of the milling machine in the denominator of a fraction, and multiply the numerator and denominator with the same number, until a new fraction is obtained in which the numerator and denominator give suitable numbers of teeth.

As an example of the above rule, we will take the case of a milling machine in which we have found that there are 4 threads per inch on the feed-screw, and that 20 revolutions of the feed-screw are necessary to make the index spindle turn one complete revolution when having equal gears on *A* and *D*, Fig. 35. On a machine of this kind, assume that it is required to cut a spiral the lead of which is 12 inches. The first thing for us to do is to find the lead of the machine. This, as we have already said, is equal to the revolutions of the feed-screw necessary to turn the index spindle one revolution, multiplied by the lead of the feed-screw. As the feed-screw has 4 threads per inch, the lead of the feed-screw is ¼ inch, and this, multiplied by 20, gives us $\frac{1}{4} \times 20 = \frac{20}{4} = 5$. To find our gears we now place the lead of the spiral in the numerator of a fraction and the lead of the machine in the denominator, and multiply both numerator and denominator with the same number until we get a new fraction in which the numerator and denominator express a suitable number of teeth. Following this rule, we have then:

$$\frac{12}{5} = \frac{12 \times 6}{5 \times 6} = \frac{72}{30}.$$

The gear with 72 teeth is placed on stud *D*, which, of course, is required to revolve slower than the feed-screw, in order to cut a spiral which is 12 inches, when the spiral cut with equal gears is only 5 inches, or equal to the lead of the machine. The gear having 30 teeth is placed on the feed-screw. If it should be necessary to put an intermediate gear between the gear on the feed-screw and the gear on stud *D*, the number of teeth in this intermediate gear would have no influence on the ratio of the speeds of feed-screw *A* and stud *D*, but would simply serve the purpose of transmitting motion from the one gear to the other. In Chapter IV this matter of the intermediate gear was treated at length, and an example given showing that even a series of intermediate gears did not influence the ratio of speeds of two shafts.

Compound Gearing.

If it is not possible to find a set of two gears that will transmit the motion required, it will be necessary to use compound gearing. In this case the manner of getting the compound gears is exactly the same as that of getting compound gearing for the lathe. We have already explained that the lead of the spiral to be cut is placed in the numerator of the fraction, and the lead of the milling machine in the denominator. We then divide up this numerator and denominator in two factors, the same as we did in the case of figuring lathe change gears, and having divided them up in two factors, we multiply each two of these factors by the same number, exactly as before, thus getting the gears we require. As an example, let us suppose that the lead of a certain machine is 10 inches, and that we wish to cut a spiral the lead of which is 60 inches. We then have:

$$\frac{60}{10}=\frac{6\times 10}{2\times 5}=\frac{(6\times 15)\times(10\times 8)}{(2\times 15)\times(5\times 8)}=\frac{90\times 80}{30\times 40}.$$

The gear having 90 teeth is placed on the stud *D*, and meshes with the 30-tooth gear *F* (see Fig. 36) on the intermediate stud; on the same intermediate stud is then also placed the gear having 80 teeth, which is driven by the gear having 40 teeth placed on the feed-screw. This makes the gears having 90 and 80 teeth the driven gears, and the gears having 30 and 40 teeth the driving gears, the whole train of gears being driven from the feed-screw of the table.

In general, for compound gearing it may be well to remember the rule given by the formula:

$$\frac{\text{lead of spiral to be cut}}{\text{lead of machine}}=\frac{\text{product of driven gears}}{\text{product of driving gears}}$$

CHAPTER IX.

SQUARE AND SQUARE ROOT.

The expressions "square" and "square root" often occur in technical formulas, and to one unfamiliar with these names, and the mathematical operations which they signify, as well as the signs by which they are indicated, it may appear that difficult mathematical operations are involved. But this is not the case.

The square of a number is simply the product of that number multiplied by itself. Thus, the square of 2 is $2\times 2=4$, and the square of 3 is $3\times 3=9$. Similarly, the square of 177 is $177\times 177=31{,}329$. Instead of writing 177×177, it is common practice to signify this operation 177^2, which is read "177 square," and simply means that 177 is multiplied by itself. Thus we have $5^2=5\times 5=25$, and $27^2=27\times 27=729$. The "2" at the upper right-hand corner is called *exponent*. Most mechanical and engineering handbooks have tables which give the squares of all numbers up to 1,000, so that by means of such a handbook it is unnecessary to figure out the squares of the numbers there given when required, by actual multiplication. The square of numbers is very much used in solving many of the problems occurring in the machine shop.

The square root of a number is that number which, if multiplied by itself, would give the given quantity. Thus, the square root of 4 is 2, because 2 multiplied by itself equals 4. The square root of 25 is 5, and so forth. It will be noticed at once that the square root is simply the reverse of the square, so that if the square of 25 is 625, then the square root of 625 is 25. The mathematical sign for the square root is $\sqrt{\ }$. Thus, $\sqrt{4}=2$, $\sqrt{16}=4$, and so forth. The engineering handbooks give tables of the square roots of all numbers up to

1,000 or more, so that the process of actually figuring the square root, which is rather complicated, and which space does not permit to deal with here, has been omitted.

In the same way as we write $2^2 = 4$ we can write $2^3 = 8$, in which case $2^3 = 2 \times 2 \times 2 = 8$, the exponent indicating how many times the number given is to be repeated as factor. Similarly, $4^3 = 4 \times 4 \times 4 = 64$. The expression 4^3 is read "4 cube" or "the third power of 4." The expression 2^5 would mean that two is to be repeated as factor five times; thus, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. The expression 2^5 is read "the fifth power of 2."

In the same way as the square root means the reverse of square, so cube root means the reverse of cube (or "third power"), that is, the cube root of a number is the number which, if repeated as factor three times, would give the number given. Thus, the cube root (or "third root") of 27 is 3, because $3 \times 3 \times 3 = 27$. We see, of course, that if the cube of a number, as 5, is 125 ($5 \times 5 \times 5 = 125$), then the cube root of 125 is 5. The sign for the cube root is $\sqrt[3]{\ }$. Thus, $\sqrt[3]{8} = 2$ (because $2 \times 2 \times 2 = 8$), and $\sqrt[3]{27} = 3$ (because $3 \times 3 \times 3 = 27$). Similarly $\sqrt[3]{125} = 5$, and $\sqrt[3]{1728000} = 120$.

If we want the number which repeated as factor four times gives a given number, we must get the "fourth root" or $\sqrt[4]{\ }$. Thus, $\sqrt[4]{81} = 3$, because $3 \times 3 \times 3 \times 3 = 81$. Similarly we write the fifth root, $\sqrt[5]{\ }$; and we write $\sqrt[5]{32} = 2$, because $2 \times 2 \times 2 \times 2 \times 2 = 32$.

After this explanation, formulas reading $\frac{3^2 \times \sqrt{4}}{\sqrt{36}}$ do not any longer look so mysterious.

If we simplify, according to the meanings given to squares and square roots, we have:

$$\frac{3^2 \times \sqrt{4}}{\sqrt{36}} = \frac{9 \times 2}{6} = \frac{18}{6} = 3.$$

Similarly,

$$\frac{\sqrt[3]{27} + \sqrt[3]{216}}{3^2} = \frac{3 + 6}{9} = \frac{9}{9} = 1,$$

and

$$\frac{5^2 + 4^2}{5^4} = \frac{25 + 16}{625} = \frac{41}{625}.$$

Right Angle Triangles.

One of the most common applications of the squares of numbers is that of the right angle triangle, as shown in Fig. 37, in which angle *A* is a right, or 90 degree angle. If we assume the lengths of the sides to be 5 inches, 4 inches, and 3 inches, respectively, as indicated on the cut, we find that

$$5^2 = 4^2 + 3^2, \text{ or } 25 = 16 + 9.$$

This holds good for all right angle triangles. The square of the side *opposite* the right angle equals the sum of the squares of the sides *including* the right angle.

Suppose, for instance, that in Fig. 38 we know the length of the two sides including the right angle to be 6 inches and 8 inches, and we want to know the length of the side *BC* opposite the right angle. Then the sum of the squares of the two given sides which include the right angle equals the square of the side opposite the right angle, and the square root of this number must be the length of *BC*. Thus

$$6^2 + 8^2 = 36 + 64 = 100.$$

The square of the length *BC* is then 100, and the length itself is the square root of 100, or $\sqrt{100} = 10$. The side *BC* is therefore 10 inches long.

If we knew that *BC* was 10 inches, and *AB* 8 inches, and wanted to find *AC*, we would subtract the square of *AB* from the square of *BC* to find the square of *AC*, or

$$10^2 - 8^2 = 100 - 64 = 36 = \text{square of } AC.$$
$$\sqrt{36} = 6 = \text{length of } AC.$$

A little thought, of course, at once convinces us that if the sum of two numbers equals a third, then if one of the first numbers is sub-

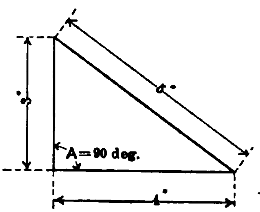


Fig. 37.

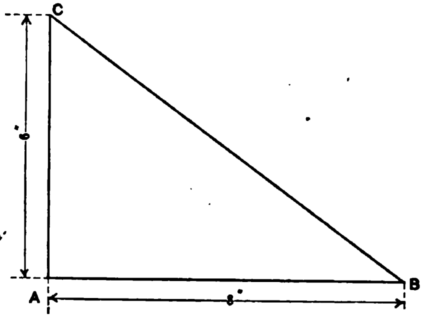


Fig. 38.

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tracted from the third, the remainder equals the other of the given numbers. Thus, in Fig. 38, we have the square of *AB* plus the square of *AC* equals the square of *BC*; and the square of *BC* minus the square of *AB* equals the square of *AC;* and, finally, the square of *BC* minus the square of *AC* equals the square of *AB*. In this case, where the sides are 10, 8, and 6 inches, respectively, we then have

$$8^2 + 6^2 = 10^2, \text{ or } 64 + 36 = 100.$$
$$10^2 - 8^2 = 6^2, \text{ or } 100 - 64 = 36.$$
$$10^2 - 6^2 = 8^2, \text{ or } 100 - 36 = 64.$$

As general formulas we may write this,

$$AB^2 + AC^2 = BC^2, \text{ or } BC = \sqrt{AB^2 + AC^2}$$
$$BC^2 - AB^2 = AC^2, \text{ or } AC = \sqrt{BC^2 - AB^2}$$
$$BC^2 - AC^2 = AB^2, \text{ or } AB = \sqrt{BC^2 - AC^2}$$

These last formulas say, that no matter what the actual numerical values of the sides *AB*, *AC*, and *BC*, the relationship between their squares, as expressed by the formulas, holds true for any right angle triangle. This carries us directly in on the subject of the use of formulas for expressing mathematical and engineering facts and data.

CHAPTER X.

USE OF FORMULAS.

As we have said before, if the three sides in a right angle triangle are 10, 8, and 6 inches long, respectively, we know that

$$10^2 = 8^2 + 6^2.$$

But if we write the formula in this manner, although it may be true for this particular triangle, there is no indication of that the same relationship holds good for any right angle triangle. If, however, we substitute the numbers giving the lengths of the sides by the letters a, b, and c, we have

$$a^2 = b^2 + c^2,$$

and this formula expresses the rule given for any right angle triangle, where the length of the side opposite the right angle is a, and the length of the sides including the right angle b and c. *The letters simply stand in place of the figures that would be applied in each particular case.*

Formulas serve the purpose of expressing shortly and precisely some general law or relationship in calculations. Each letter stands for a certain quantity, and when we figure any special case we put the figures for this case into our formula, and figure as usual. Some examples will make this clear.

If we return to the question of figuring tapers, we have on page 6 the formula

$$\text{taper per foot} = \frac{\text{large dia.} - \text{small dia.}}{\text{length of work}} \times 12.$$

Now this formula can be expressed far more simply by putting letters in place of the various quantities given. Let, for instance,

t = taper per foot,
a = large diameter of work,
b = small diameter of work, and
l = length of work.

Then our formula would take the form:

$$t = \frac{a - b}{l} \times 12.$$

If we apply this formula to the example shown in Fig. 9, we have directly,

$$t = \frac{1\frac{1}{4} - 1\frac{1}{64}}{7\frac{1}{2}} \times 12 = \frac{\frac{15}{64}}{7\frac{1}{2}} \times 12 = \left(\frac{15}{64} + 7\frac{1}{2}\right) \times 12 = \frac{15}{64} \times \frac{2}{15} \times 12 = \frac{1}{32} \times 12 = \frac{3}{8}.$$

As t stands for the taper per foot, this, consequently equals ⅜ inch. It is seen that the dimensions for the large and small ends of the work, the length, etc., are simply put *in place of the letters* in the formula, and then simple arithmetic is applied. Formulas are very easy to use if this is understood. It is only necessary to know exactly what quantity each letter stands for, and then put the numerical value of that quantity into the formula.

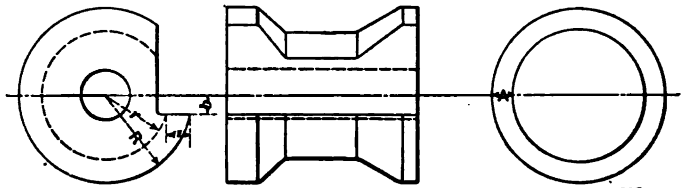
The formula for finding the number of teeth in the change gears for a lathe may, for instance, be written,

$$\frac{l \times a}{c \times a} = \frac{T}{t},$$

in which formula

l = number of threads per inch on lead-screw,
c = number of threads to be cut,
a = the number by which numerator and denominator is multiplied to find the number of teeth in change gears,
T = number of teeth in spindle stud gear, and
t = number of teeth in lead-screw gear.

Suppose now that we want to cut 16 threads per inch in a lathe having a lead-screw with 6 threads per inch, and let the number with



FORMING TOOL

END VIEW OF PIECE TO BE FORMED

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Fig. 39.

which we multiply numerator and denominator be 6. Then, by putting these figures in place of the letters in the formula we have:

$$\frac{6 \times 6}{16 \times 6} = \frac{36}{96} = \frac{T}{t}.$$

Consequently T, or the number of teeth in the spindle stud gear, is 36, and t, or the number of teeth in the lead-screw gear, is 96.

A useful application both of the use of formulas and of the square and square root of numbers, is found in the problems occurring when figuring forming tools.

Formulas for Circular Forming Tools.

When laying out circular forming tools, such as shown in Fig. 39, the cutting edge, as is well known, must be located a certain amount below the horizontal center line of the tool, in order to provide for sufficient clearance for the cut. On account of this, the actual differences of diameters in the piece of work to be formed cannot be directly copied in the forming tool. The distance A in the piece to be formed must equal the distance a on the forming tool, but as this latter distance is measured in a plane a certain distance b below the horizontal plane through the center of the forming tool, it is evident that the differences of diameters in the tool and the piece to be formed are not the same. A general formula may, however, be deduced, by use of elementary geometry, by means of which the various diameters of the forming tool may be determined if the largest (or smallest) diameter of the tool, the amount that the cutting edge is below the center, and, of course, the diameters of the piece to be formed, are known.

If R = the largest radius of the tool,

a = difference in radii of steps in the work, and

b = amount cutting edge is below center,

then, if r be the radius looked for,

$$r = \sqrt{(\sqrt{R^2 - b^2} - a)^2 + b^2}$$

If the smaller radius r is given and the larger radius R sought, the formula takes the form:

$$R = \sqrt{(\sqrt{r^2 - b^2} + a)^2 + b^2}$$

Suppose, for an example, that a tool is to be made to form the piece in Fig. 40. Assume that the largest diameter of the tool is to be 3 inches, and that the cutting edge is to be ¼ inch below the center of the tool. Then the diameter next smaller to 3 inches is found from the formulas given by inserting the given values: R = 1½ inch,

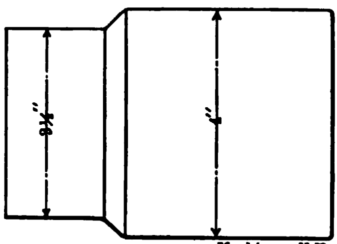

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Fig. 40.

b = ¼ inch, and a = ¼ inch (half the difference between 4 and 3½ inches; see Fig. 40).

Then

$$r = \sqrt{(\sqrt{(1\tfrac{1}{2})^2 - (\tfrac{1}{4})^2} - \tfrac{1}{4})^2 + (\tfrac{1}{4})^2} = \sqrt{(\sqrt{\tfrac{35}{16}} - \tfrac{1}{4})^2 + \tfrac{1}{16}} = \frac{5.017}{4}$$

= 1.254 inch.

While the formula looks complicated, by means of a table of squares the calculations are easily simplified and can be carried out in three or four minutes. The value of r being 1.254 inch, the diameter to make the smaller step of the forming tool will be 2.508 inches, instead of 2½ inches exact, as would have been the case if the cutting edge had been on the center line.

A parenthesis ( ) used in a formula indicates that all the operations of figuring inside of the parenthesis are to be performed first, before we go further. Thus, for instance,

$$\left(\sqrt{\frac{25}{4}} + \frac{1}{2}\right)^2 + 3 = \left(\frac{5}{2} + \frac{1}{2}\right)^2 + 3 = \left(\ 3\ \right)^2 + 3 = 9 + 3 = 12.$$

It is common in formulas to leave out the multiplication sign (×)

between two letters and write them together without any sign at all. Thus ab means $a \times b$, and the expression

$$\frac{2APLN}{33,000} = \frac{2 \times A \times P \times L \times N}{33,000}$$

Using Decimal Equivalents Instead of Common Fractions.

It seems to be the general impression that it is easier and quicker to use the decimal equivalent of a fraction, instead of the fraction itself, when multiplying or dividing. There are very few cases, however, where the calculation can be made simpler by this substitution, and the results obtained are invariably less correct, because all the decimals which are necessary to correctly express the value of the fraction are, as a rule, not used, and when multiplying, an original error of only one-half thousandth inch may finally cause serious errors in close work.

An example will plainly illustrate this assertion. In a certain problem the formula reads:

$$R = \frac{a^2 + (b - c)\, b}{2\,(a - \frac{1}{2}c)}.$$

Now suppose that $a = 11/16$, $b = 1\frac{1}{2}$, and $c = \frac{3}{4}$. Then, if the formula is written with decimal equivalents substituted for the letters, and these equivalents are given to the thousandth of an inch, as is most common, the formula would read,

$$R = \frac{0.687^2 + (1.5 - 0.75)\,1.5}{2\,(0.687 - 0.375)}$$

Proceeding we find

$$R = \frac{0.472 + 1.125}{0.624} = \frac{1.597}{0.624} = 2.559.$$

If instead of using decimal equivalents for the fraction originally given in the problem we use the fractions themselves, we would write

$$R = \frac{\left(\frac{11}{16}\right)^2 + \left(1\frac{1}{2} - \frac{3}{4}\right)1\frac{1}{2}}{2\left(\frac{11}{16} - \frac{3}{8}\right)}$$

Simplifying this expression we find

$$R = \frac{\frac{121}{256} + 1\frac{1}{8}}{\frac{5}{8}} = \frac{\frac{121}{32} + 9}{5} = \frac{409}{160} = 2.556.$$

We notice in the first place that the denominator 0.624 ought to have been 0.625 or $\frac{5}{8}$, and further, the final result shows a difference of 0.003 inch, which is enough to spoil many a job which may not even be required to be of extreme accuracy. This error is all due to the seemingly small original error of writing 0.687 instead of 0.6875.

CHAPTER XI.

USE OF TABLES OF SINES, COSINES, TANGENTS AND COTANGENTS.

The figuring of angles the average machinist usually looks upon as something above his capacity, and regards the knowledge of this mathematical process with more or less awe. But simple cases of the figuring of angles from given formulas are very easy, indeed; in fact, these cases are often much simpler than many ordinary arithmetical problems in the shop which the machinist tackles with success. All that is necessary is a table of sines, cosines, tangents, and cotangents; after having found the figures corresponding to a given angle from the table, the whole thing resolves itself to a case of simple multiplication or division.

Often, in technical papers, the reader will find himself confronted by such formulas as, for instance,

$$A = \frac{27}{\cos 36 \text{ deg.}}.$$

Of course, it is impossible to figure out how much A is from this formula, unless the expression "cos 36 deg." (read: cosine of 36 degrees) can be transformed and expressed in plain figures. But if we know how much "cos 36 deg." is expressed in plain figures, then we can immediately divide 27 by this value, and thus find the value of A. Suppose that A stands for the length of one side in a triangle, and that the expression "cos 36 deg." equals 0.80901. Then,

$$A = \frac{27}{0.80901} = 33.37.$$

The tables of sines, cosines, tangents, and cotangents simply serve the purpose of giving in figures the values of these expressions for different angles. The angles are given in degrees and minutes. A right angle is 90 degrees (90°), and each degree is further subdivided into 60 minutes (60′). The four expressions: sine, cosine, tangent, and cotangent, which are used to designate certain numerical values, to be found from the tables, are called the *functions of the angle*. These functions or numerical values equal a definite amount for each different angle. On pages 48, 49, 50, and 51 will be found tables giving the values referred to for all degrees and for every ten minutes (1/6 of a degree). The four expressions sine, cosine, tangent, and cotangent are abbreviated "sin," "cos," "tan," and "cot," respectively.

The tables of sines, cosines, etc., are read the same as a railroad time-table. It will be noticed that at the top of the tables on pages 48 and 49 the heading reads "Table of Sines," and at the bottom of the same tables it says "Table of Cosines." At the top of the two

TABLE OF SINES.

Read degrees in left-hand column and minutes at top.
Example: sin 7° 10′ = .12475.

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00291	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02326	.02617	.02908	.03199	.03489	88
2	.03489	.03780	.04071	.04361	.04652	.04943	.05233	87
3	.05233	.05524	.05814	.06104	.06395	.06685	.06975	86
4	.06975	.07265	.07555	.07845	.08135	.08425	.08715	85
5	.08715	.09005	.09295	.09584	.09874	.10163	.10453	84
6	.10452	.10742	.11031	.11320	.11609	.11898	.12186	83
7	.12186	.12475	.12764	.13052	.13341	.13629	.13917	82
8	.13917	.14205	.14493	.14780	.15068	.15356	.15643	81
9	.15643	.15930	.16217	.16504	.16791	.17078	.17364	80
10	.17364	.17651	.17937	.18223	.18509	.18795	.19080	79
11	.19080	.19366	.19651	.19936	.20221	.20506	.20791	78
12	.20791	.21075	.21359	.21644	.21927	.22211	.22495	77
13	.22495	.22778	.23061	.23344	.23627	.23909	.24192	76
14	.24192	.24474	.24756	.25038	.25319	.25600	.25881	75
15	.25881	.26162	.26443	.26723	.27004	.27284	.27563	74
16	.27563	.27843	.28122	.28401	.28680	.28958	.29237	73
17	.29237	.29515	.29793	.30070	.30347	.30624	.30901	72
18	.30901	.31178	.31454	.31730	.32006	.32281	.32556	71
19	.32556	.32831	.33106	.33380	.33654	.33928	.34202	70
20	.34202	.34475	.34748	.35020	.35293	.35565	.35836	69
21	.35836	.36108	.36379	.36650	.36920	.37190	.37460	68
22	.37460	.37730	.37999	.38268	.38536	.38805	.39073	67
23	.39073	.39340	.39607	.39874	.40141	.40407	.40673	66
24	.40673	.40939	.41204	.41469	.41733	.41998	.42261	65
25	.42261	.42525	.42788	.43051	.43313	.43575	.43837	64
26	.43837	.44098	.44359	.44619	.44879	.45139	.45399	63
27	.45399	.45658	.45916	.46174	.46432	.46690	.46947	62
28	.46947	.47203	.47460	.47715	.47971	.48226	.48481	61
29	.48481	.48735	.48989	.49242	.49495	.49747	.50000	60
30	.50000	.50251	.50503	.50753	.51004	.51254	.51503	59
31	.51503	.51752	.52001	.52249	.52497	.52745	.52991	58
32	.52991	.53238	.53484	.53730	.53975	.54219	.54463	57
33	.54463	.54707	.54950	.55193	.55436	.55677	.55919	56
34	.55919	.56160	.56400	.56640	.56880	.57119	.57357	55
35	.57357	.57595	.57833	.58070	.58306	.58542	.58778	54
36	.58778	.59013	.59248	.59482	.59715	.59948	.60181	53
37	.60181	.60413	.60645	.60876	.61106	.61336	.61566	52
38	.61566	.61795	.62023	.62251	.62478	.62705	.62932	51
39	.62932	.63157	.63383	.63607	.63832	.64055	.64278	50
40	.64278	.64501	.64723	.64944	.65165	.65386	.65605	49
41	.65605	.65825	.66043	.66262	.66479	.66696	.66913	48
42	.66913	.67128	.67344	.67559	.67773	.67986	.68199	47
43	.68199	.68412	.68624	.68835	.69046	.69256	.69465	46
44	.69465	.69674	.69883	.70090	.70298	.70504	.70710	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COSINES.

Read degrees in right-hand column and minutes at bottom.
Example: cos 56° 20′ = .55436.

TABLE OF SINES.

Read degrees in left-hand column and minutes at top.
Example: sin 56° 20 = .83227.

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	.70710	.70916	.71120	.71325	.71528	.71731	.71934	44
46	.71934	.72135	.72336	.72537	.72737	.72936	.73135	43
47	.73135	.73333	.73530	.73727	.73923	.74119	.74314	42
48	.74314	.74508	.74702	.74895	.75088	.75279	.75471	41
49	.75471	.75661	.75851	.76040	.76229	.76417	.76604	40
50	.76604	.76791	.76977	.77162	.77347	.77531	.77714	39
51	.77714	.77897	.78079	.78260	.78441	.78621	.78801	38
52	.78801	.78979	.79157	.79335	.79512	.79688	.79863	37
53	.79863	.80038	.80212	.80385	.80558	.80730	.80901	36
54	.80901	.81072	.81242	.81411	.81580	.81748	.81915	35
55	.81915	.82081	.82247	.82412	.82577	.82740	.82903	34
56	.82903	.83066	.83227	.83388	.83548	.83708	.83867	33
57	.83867	.84025	.84182	.84339	.84495	.84650	.84804	32
58	.84804	.84958	.85111	.85264	.85415	.85566	.85716	31
59	.85716	.85866	.86014	.86162	.86310	.86456	.86602	30
60	.86602	.86747	.86892	.87035	.87178	.87320	.87462	29
61	.87462	.87602	.87742	.87881	.88020	.88157	.88294	28
62	.88294	.88430	.88566	.88701	.88835	.88968	.89100	27
63	.89100	.89232	.89363	.89493	.89622	.89751	.89879	26
64	.89879	.90006	.90132	.90258	.90383	.90507	.90630	25
65	.90630	.90753	.90875	.90996	.91116	.91235	.91354	24
66	.91354	.91472	.91589	.91706	.91821	.91936	.92050	23
67	.92050	.92163	.92276	.92388	.92498	.92609	.92718	22
68	.92718	.92827	.92934	.93041	.93148	.93253	.93358	21
69	.93358	.93461	.93565	.93667	.93768	.93869	.93969	20
70	.93969	.94068	.94166	.94264	.94360	.94456	.94551	19
71	.94551	.94646	.94739	.94832	.94924	.95015	.95105	18
72	.95105	.95195	.95283	.95371	.95458	.95545	.95630	17
73	.95630	.95715	.95799	.95882	.95964	.96045	.96126	16
74	.96126	.96205	.96284	.96363	.96440	.96516	.96592	15
75	.96592	.96667	.96741	.96814	.96887	.96958	.97029	14
76	.97029	.97099	.97168	.97237	.97304	.97371	.97437	13
77	.97437	.97502	.97566	.97629	.97692	.97753	.97814	12
78	.97814	.97874	.97934	.97992	.98050	.98106	.98162	11
79	.98162	.98217	.98272	.98325	.98378	.98429	.98480	10
80	.98480	.98530	.98580	.98628	.98676	.98722	.98768	9
81	.98768	.98813	.98858	.98901	.98944	.98985	.99026	8
82	.99026	.99066	.99106	.99144	.99182	.99218	.99254	7
83	.99254	.99289	.99323	.99357	.99389	.99421	.99452	6
84	.99452	.99482	.99511	.99539	.99567	.99593	.99619	5
85	.99619	.99644	.99668	.99691	.99714	.99735	.99756	4
86	.99756	.99776	.99795	99813	.99830	.99847	.99863	3
87	.99863	.99877	.99891	.99904	.99917	.99928	.99939	2
88	.99939	.99948	.99957	.99965	.99972	.99979	.99984	1
89	.99984	.99989	.99993	.99993	.99998	.99999	1.0000	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COSINES.

Read degrees in right-hand column and minutes at bottom.
Example: cos 7° 10′ = .99218.

TABLE OF TANGENTS.

Read degrees in left-hand column and minutes at top.
Example: tan 7° 10′ = .12573.

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00290	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02327	.02618	.02909	.03200	.03492	88
2	.03492	.03783	.04074	.04366	.04657	.04949	.05240	87
3	.05240	.05532	.05824	.06116	.06408	.06700	.06992	86
4	.06992	.07285	.07577	.07870	.08162	.08455	.08748	85
5	.08748	.09042	.09335	.09628	.09922	.10216	.10510	84
6	.10510	.10804	.11099	.11393	.11688	.11983	.12278	83
7	.12278	.12573	.12869	.13165	.13461	.13757	.14054	82
8	.14054	.14350	.14647	.14945	.15242	.15540	.15838	81
9	.15838	.16136	.16435	.16734	.17033	.17332	.17632	80
10	.17632	.17932	.18233	.18533	.18834	.19136	.19438	79
11	.19438	.19740	.20042	.20345	.20648	.20951	.21255	78
12	.21255	.21559	.21864	.22169	.22474	.22780	.23086	77
13	.23086	.23393	.23700	.24007	.24315	.24624	.24932	76
14	.24932	.25242	.25551	.25861	.26172	.26483	.26794	75
15	.26794	.27106	.27419	.27732	.28046	.28360	.28674	74
16	.28674	.28989	.29305	.29621	.29938	.30255	.30573	73
17	.30573	.30891	.31210	.31529	.31850	.32170	.32492	72
18	.32492	.32813	.33136	.33459	.33783	.34107	.34433	71
19	.34433	.34758	.35084	.35411	.35739	.36067	.36397	70
20	.36397	.36726	.37057	.37388	.37720	.38053	.38386	69
21	.38386	.38720	.39055	.39391	.39727	.40064	.40403	68
22	.40403	.40741	.41080	.41421	.41763	.42104	.42447	67
23	.42447	.42791	.43135	.43481	.43827	.44174	.44522	66
24	.44522	.44871	.45221	.45572	.45924	.46277	.46630	65
25	.46630	.46985	.47341	.47697	.48055	.48413	.48773	64
26	.48773	.49133	.49495	.49858	.50221	.50586	.50953	63
27	.50953	.51319	.51687	.52056	.52427	.52798	.53170	62
28	.53170	.53544	.53919	.54295	.54673	.55051	.55430	61
29	.55430	.55811	.56193	.56577	.56961	.57347	.57735	60
30	.57735	.58123	.58513	.58904	.59297	.59690	.60086	59
31	.60086	.60483	.60881	.61280	.61681	.62083	.62487	58
32	.62487	.62892	.63299	.63707	.64116	.64528	.64941	57
33	.64941	.65355	.65771	.66189	.66608	.67028	.67451	56
34	.67451	.67875	.68301	.68728	.69157	.69588	.70021	55
35	.70021	.70455	.70891	.71329	.71769	.72211	.72654	54
36	.72654	.73100	.73547	.73996	.74447	.74900	.75355	53
37	.75355	.75812	.76272	.76733	.77196	.77661	.78129	52
38	.78129	.78598	.79070	.79544	.80020	.80498	80978	51
39	.80978	.81461	.81946	.82434	.82923	.83415	.83910	50
40	.83910	.84407	.84906	.85408	.85912	.86419	.86929	49
41	.86929	.87441	.87955	.88473	.88992	.89515	.90040	48
42	.90040	.90569	.91099	.91633	.92170	.92709	.93252	47
43	.93252	.93797	.94345	.94896	.95451	.96008	.96569	46
44	.96569	.97133	.97700	.98270	.98843	.99420	1.0000	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COTANGENTS.

Read degrees in right-hand column and minutes at bottom.
Example: cot 56° 20′ = .66607.

TABLE OF TANGENTS.

Read degrees in left-hand column and minutes at top.
Example: tan 56° 20′ = 1.5013.

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44
46	1.0355	1.0415	1.0476	1.0537	1.0599	1.0661	1.0723	43
47	1.0723	1.0786	1.0849	1.0913	1.0977	1.1041	1.1106	42
48	1.1106	1.1171	1.1236	1.1302	1.1369	1.1436	1.1503	41
49	1.1503	1.1571	1.1639	1.1708	1.1777	1.1847	1.1917	40
50	1.1917	1.1988	1.2059	1.2131	1.2203	1.2275	1.2349	39
51	1.2349	1.2423	1.2496	1.2571	1.2647	1.2723	1.2799	38
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37
53	1.3270	1.3351	1.3432	1.3514	1.3596	1.3680	1.3763	36
54	1.3763	1.3848	1.3933	1.4019	1.4106	1.4193	1.4281	35
55	1.4281	1.4370	1.4459	1.4550	1.4641	1.4733	1.4825	34
56	1.4825	1.4919	1.5013	1.5108	1.5204	1.5301	1.5398	33
57	1.5398	1.5497	1.5596	1.5696	1.5798	1.5900	1.6003	32
58	1.6003	1.6107	1.6212	1.6318	1.6425	1.6533	1.6642	31
59	1.6642	1.6753	1.6864	1.6976	1.7090	1.7204	1.7320	30
60	1.7320	1.7437	1.7555	1.7674	1.7795	1.7917	1.8040	29
61	1.8040	1.8164	1.8290	1.8417	1.8546	1.8676	1.8807	28
62	1.8807	1.8940	1.9074	1.9209	1.9347	1.9485	1.9626	27
63	1.9626	1.9768	1.9911	2.0056	2.0203	2.0352	2.0503	26
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25
65	2.1445	2.1609	2.1774	2.1943	2.2113	2.2285	2.2460	24
66	2.2460	2.2637	2.2816	2.2998	2.3182	2.3369	2.3558	23
67	2.3558	2.3750	2.3944	2.4142	2.4342	2.4545	2.4750	22
68	2.4750	2.4959	2.5171	2.5386	2.5604	2.5826	2.6050	21
69	2.6050	2.6279	2.6510	2.6746	2.6985	2.7228	2.7474	20
70	2.7474	2.7725	2.7980	2.8239	2.8502	2.8770	2.9042	19
71	2.9042	2.9318	2.9600	2.9886	3.0178	3.0474	3.0776	18
72	3.0776	3.1084	3.1397	3.1715	3.2040	3.2371	3.2708	17
73	3.2708	3.3052	3.3402	3.3759	3.4128	3.4495	3.4874	16
74	3.4874	3.5260	3.5655	3.6058	3.6470	3.6890	3.7320	15
75	3.7320	3.7759	3.8208	3.8667	3.9136	3.9616	4.0107	14
76	4.0107	4.0610	4.1125	4.1653	4.2193	4.2747	4.3314	13
77	4.3314	4.3896	4.4494	4.5107	4.5736	4.6382	4.7046	12
78	4.7046	4.7728	4.8430	4.9151	4.9894	5.0658	5.1445	11
79	5.1445	5.2256	5.3092	5.3955	5.4845	5.5763	5.6712	10
80	5.6712	5.7693	5.8708	5.9757	6.0844	6.1970	6.3137	9
81	6.3137	6.4348	6.5605	6.6911	6.8269	6.9682	7.1153	8
82	7.1153	7.2687	7.4287	7.5957	7.7703	7.9530	8.1443	7
83	8.1443	8.3449	8.5555	8.7768	9.0098	9.2553	9.5143	6
84	9.5143	9.7881	10.078	10.385	10.711	11.059	11.430	5
85	11.430	11.826	12.250	12.706	13.196	13.726	14.300	4
86	14.300	14.924	15.604	16.349	17.169	18.075	19.081	3
87	19.081	20.205	21.470	22.904	24.541	26.431	28.636	2
88	28.636	31.241	34.367	38.188	42.964	49.103	57.290	1
89	57.290	68.750	85.939	114.58	171.88	343.77	∞	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COTANGENTS.

Read degrees in right-hand column and minutes at bottom.
Example: cot 7° 10′ = 7.9530.

following pages the heading reads "Table of Tangents," and at the bottom of these pages is the legend "Table of Cotangents." At the top of the tables themselves we find that the heading of the extreme left-hand column reads "Deg." and then the following columns are headed 0′, 10′, 20′, and so forth. If we look at the bottom of the pages, we find the same legends *under* the columns, but reading from the extreme right-hand column to the left. The purpose of this will be immediately explained.

When we wish to find the sine or tangent for a given angle, we first find the number of degrees in the extreme left-hand column of the respective tables, and then the number of minutes at the top of the columns. We then follow the column over which the number of minutes is given downward until we come to the figure in line with the given number of degrees. This figure is then the numerical value which we call the sine or the tangent, as the case may be, for the given angle. If the angle is given in even degrees, with no minutes given, the corresponding value will be found opposite the number of the degrees in the column marked 0′ (the sign ′ indicates minutes, just as the sign ° indicates degrees).

The cosines and tangents of angles are found in the same tables as the sines and tangents, but when we want these quantities we read the tables *from the bottom up*. The number of degrees we find in the extreme right-hand column, and the number of minutes at the bottom of the columns. This is the reason that we read the legends "Cosines" and "Cotangents" at the bottom of the pages, because these functions are read from below and up. If the number of minutes given should not be an even number, as 10′, 20′, 30′, etc., but 26′, for instance, it is, for nearly all calculations in the shop, near enough to take the figures given for the nearest given number of minutes, being in this case, then, for 30 minutes.

If we now return to our formula

$$A = \frac{27}{\cos 36 \text{ deg.}},$$

we find in the table on page 49, opposite 36 degrees in the right-hand column and in the column marked 0′ at the bottom, the figures 0.80901. This is the cosine for 36 degrees, and should be placed in the formula instead of the expression "cos 36 deg." We have, then, as we have already shown,

$$A = \frac{27}{0.80901},$$

which we can calculate by simple division.

Suppose the angle had been 36 degrees 20 minutes, instead of 36 degrees even. We then would have read the figures in the column marked 20′ at the bottom, and opposite 36 degrees we would have found 0.80558. A few more examples will tend to make the use of these tables clearer.

In a right angle triangle, as shown in Fig. 41, the side *BC*, which is opposite the right angle, equals the side *AB* divided by the cosine of the angle β, included between these two sides. Expressed as a formula this rule would be:

$$BC = \frac{AB}{\cos\beta}.$$

In a particular case let us assume that AB equals 6 inches and the angle β is 37 degrees. How long is the side BC?

According to our formula, $BC = \frac{6}{\cos 37 \text{ deg.}}$. The cosine for 37 degrees we find from the table to be 0.79863. Then $BC = \frac{6}{0.79863} = 7.51$ inches.

In order to become familiar with the use of the tables, let us find the sine for 56 degrees 20 minutes, or, as it is written in formulas, sin 56° 20′. The "sines" are found by reading the tables from the top. Find 56 in the tables of sines in the left-hand column and read off the value 0.83227 in the column marked 20′. This is the sine 56° 20′.

Find the tangent for 56 degrees 20 minutes, or, as it is written, tan 56° 20′. Exactly in the same way as we found the sine, we find the tangent in the tables headed "Tangents" to be 1.5013.

The cotangent for the same angle (cot 56° 20′) is found by reading the table from the bottom, first finding 56 in the right-hand column, and the required value in line with this figure 56 in the column marked 20′ at the bottom. The cotangent of 56 degrees 20 minutes we thus find to be 0.66607.

These tables are constantly used when figuring triangles. In every triangle, if we know the length of *all the three sides, or of two sides and the measure in degrees of one angle*, or of *one side and two angles*, we can figure the other sides or angles by means of certain formulas into which these angular functions enter.

Suppose, for instance, that we call the three sides in a right angle triangle a, b, and c, as shown in Fig. 42, and the angles opposite those sides A, B, and C. The angle A, of course, is a right or 90-degree angle. Then, for all right angle triangles these formulas hold true:

$$a = \frac{b}{\cos C}; \qquad a = \frac{b}{\sin B};$$

$$a = \frac{c}{\cos B}; \qquad a = \frac{c}{\sin C};$$

$$b = a \cos C; \qquad b = a \sin B;$$

$$b = c \tan B; \qquad b = c \cot C;$$

$$c = a \cos B; \qquad c = a \sin C;$$

$$c = b \tan C; \qquad c = b \cot B.$$

It will be remembered that expressions such as $c \cot C$ mean simply $c \times \cot C$.

By means of the formulas given above, and a table of sines, cosines, etc., either of the sides in a right angle triangle may be found when one side and one angle, besides the 90-degree angle, are known. If two sides are known, but no angle outside of the 90-degree angle, the third side may be found from the formulas on page 42, and the angles found from the formulas:

$$\sin B = \frac{b}{a}; \qquad \cos B = \frac{c}{a};$$

$$\tan B = \frac{b}{c}; \qquad \cot B = \frac{c}{b};$$

$$\sin C = \frac{c}{a}; \qquad \cos C = \frac{b}{a};$$

$$\tan C = \frac{c}{b}; \qquad \cot C = \frac{b}{c}.$$

Thus, suppose that in Fig. 42 $b = 4$ inches and $c = 3$ inches.

Then $a = \sqrt{(4)^2 + (8)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$ inches.

To find angle b, write

$$\sin B = \frac{b}{a} = \frac{4}{5} = 0.800,$$

and from our table of sines, finding 0.800 in the column marked 10′ at

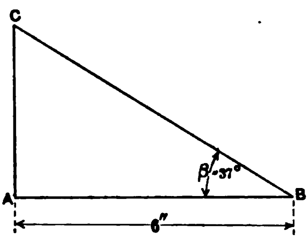
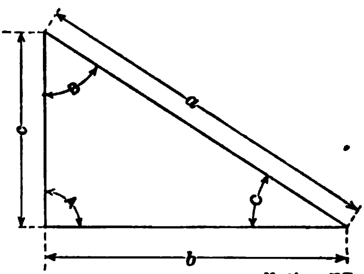


Fig. 41.



Machinery, N.Y.
Fig. 42.

the top of the page, we find that the angle B is 53 degrees 10 minutes, very closely. Angle C is found from formula

$$\cos C = \frac{b}{a} = \frac{4}{5} = 0.800.$$

This angle is then 36 degrees 50 minutes as read from below in the tables of cosines at the bottom.

For the finding of any angle or side in *any* triangle when the requisite number of sides or angles are given, the chart on page 55, and the tables of angular functions are used. The whole process consists merely in finding the corresponding value of the function of the angle in the table, insert it into the formula, and figure as if the whole were simple arithmetic. This simple process of figuring angles embodies the principles of what is called, in mathematics, *Trigonometry*. But complicated names do not, as we have seen, necessarily imply complicated processes of figuring.

We will figure an example to show the use of the chart on page 55.

CHART FOR SOLUTION OF TRIANGLES.*

Parts Given	Parts to be found.					
	$a=$	$b=$	$c=$	$\angle A$	$\angle B$	$\angle C$
$a-b-c$				$\frac{b^2+c^2-a^2}{2bc}=\cos A$	$\frac{a^2+c^2-b^2}{2ac}=\cos B$	$\frac{a^2+b^2-c^2}{2ab}=\cos C$
$b-c-\angle A$	$\sqrt{b^2+c^2-2bc\cos A}$				$\frac{b\sin A}{c-b\cos A}=\tan B$	$\frac{c\sin A}{b-c\cos A}=\tan C$
$a-c-\angle B$		$\sqrt{a^2+c^2-2ac\cos B}$		$\frac{a\sin B}{c-a\cos B}=\tan A$		$\frac{c\sin B}{a-c\cos B}=\tan C$
$a-b-\angle C$			$\sqrt{a^2+b^2-2ab\cos C}$	$\frac{a\sin C}{b-a\cos C}=\tan A$	$\frac{b\sin C}{a-b\cos C}=\tan B$	
$a-b-\angle A$			$\frac{a\sin C}{\sin A}$		$\frac{b\sin A}{a}=\sin B$	$180°-(A+B)$
$a-b-\angle B$			$\frac{b\sin C}{\sin B}$	$\frac{a\sin B}{b}=\sin A$		$180°-(A+B)$
$a-c-\angle A$		$\frac{a\sin B}{\sin A}$			$180°-(A+C)$	$\frac{c\sin A}{a}=\sin C$
$a-c-\angle C$		$\frac{c\sin B}{\sin C}$		$\frac{a\sin C}{c}=\sin A$	$180°-(A+C)$	
$b-c-\angle B$	$\frac{b\sin A}{\sin B}$			$180°-(B+C)$		$\frac{c\sin B}{b}=\sin C$
$b-c-\angle C$	$\frac{c\sin A}{\sin C}$			$180°-(B+C)$	$\frac{b\sin C}{c}=\sin B$	
$a-\angle A-\angle B$		$\frac{a\sin B}{\sin A}$	$\frac{a\sin C}{\sin A}$			$180°-(A+B)$
$a-\angle A-\angle C$		$\frac{a\sin B}{\sin A}$	$\frac{a\sin C}{\sin A}$		$180°-(A+C)$	
$a-\angle B-\angle C$		$\frac{a\sin B}{\sin A}$	$\frac{a\sin C}{\sin A}$	$180°-(B+C)$		
$b-\angle A-\angle B$	$\frac{b\sin A}{\sin B}$		$\frac{b\sin C}{\sin B}$			$180°-(A+B)$
$b-\angle A-\angle C$	$\frac{b\sin A}{\sin B}$		$\frac{b\sin C}{\sin B}$		$180°-(A+C)$	
$b-\angle B-\angle C$	$\frac{b\sin A}{\sin B}$		$\frac{b\sin C}{\sin B}$	$180°-(B+C)$		
$c-\angle A-\angle B$	$\frac{c\sin A}{\sin C}$	$\frac{c\sin B}{\sin C}$				$180°-(A+B)$
$c-\angle A-\angle C$	$\frac{c\sin A}{\sin C}$	$\frac{c\sin B}{\sin C}$			$180°-(A+C)$	
$c-\angle B-\angle C$	$\frac{c\sin A}{\sin C}$	$\frac{c\sin B}{\sin C}$		$180°-(B+C)$		

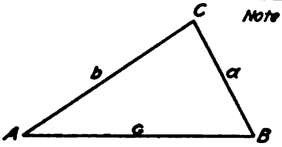


Fig. 1.

Note 1:– By means of the table any part of an oblique triangle may be found when any three other parts are given, with the following exception:

Given two sides and the angle opposite one of them; then, if the side opposite is less than the adjacent × the sine of the angle, the triangle is impossible; or if the side opposite = the adjacent × the sine of the angle, the triangle is a right triangle; or if the side opposite is less than the adjacent but does not come under the above, the triangle is capable of two solutions and can be drawn as in Fig. 2 as well as in Fig. 1.

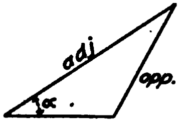


Fig. 2.

Note 2:– In some cases two steps are necessary to solve, as for example, having given sides a and b and angle A, to find c: The formula reads $c=\frac{a\sin C}{\sin A}$ but angle C must first be derived from $C=180°-(A+B)$, and the same applies to other angles in certain cases as is apparent above.

* Reproduced from MACHINERY'S Data Sheets, February, 1908. Contributed by E. A. Johnson.

Assume that in a triangle as shown in Fig. 43, one side c is known to be 6 inches long. The angle A equals 50 degrees and B is 68 degrees. How long are sides a and b, and how many degrees is angle C?

The parts given are side c and angles A and B. We, therefore, first find in our chart the line containing the formulas corresponding to our problem. We find that the extreme left-hand column is headed "Parts given," and following it down we finally find "$c — \angle A — \angle B$,'

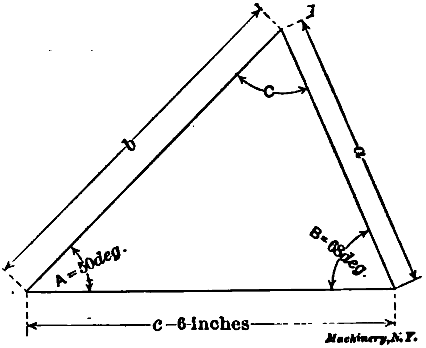


Fig. 43.

in which the sign $\angle$ simply stands for "angle." In line with this in the table we find the formulas for finding the sides a and b, and the angle C ($\angle C$), as follows:

$$a = \frac{c \sin A}{\sin C}; \quad b = \frac{c \sin B}{\sin C}; \quad C = 180° - (A + B).$$

Substituting in these formulas the values for c, A, and B, we have:

$$a = \frac{6 \times \sin 50 \text{ deg.}}{\sin C}; \quad b = \frac{6 \times \sin 68 \text{ deg.}}{\sin C}.$$

In these formulas, however, we find that sin C is required. The angle C is found from the third formula given:

$$C = 180° - (50° + 68°) = 180° - 118° = 62 \text{ degrees.}$$

We have then,

$$a = \frac{6 \times \sin 50 \text{ deg.}}{\sin 62 \text{ deg.}}; \quad b = \frac{6 \times \sin 68 \text{ deg.}}{\sin 62 \text{ deg.}}.$$

We now find, from our tables of sines, the value of the expressions "sin 50 deg.," "sin 62 deg.," and so forth. These values we put in place of these expressions in our formulas, and obtain:

$$a = \frac{6 \times 0.76604}{0.88294} = 5.206,$$

$$b = \frac{6 \times 0.92718}{0.88294} = 6.301.$$

We have thus found that the sides a and b equal 5.206 and 6.301 inches, respectively, and the angle C, 62 degrees.

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