MACHINERY'S REFERENCE SERIES

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No. 22—CALCULATIONS OF ELEMENTS OF MACHINE DESIGN

CONTENTS

CHAPTER I.

THE FACTOR OF SAFETY.

It is the custom among most firms engaged in the designing of machinery to settle upon certain stresses* as proper for given materials in given classes of work. These stresses are chosen as the result of many years of experience on their own part, or of observation of the successful experience of others, and so long as the quality of the material remains unchanged, and the service does not vary in character, the method is eminently satisfactory.

Progress, however, brings up new service, for which precedent is lacking, and materials of different qualities, either better or cheaper, for which the safe working stresses have not been determined, and the designer is compelled to determine the stress proper for the work in hand by using a so-called "factor of safety." The name "factor of safety" is misleading for several reasons. In the first place, it is not a factor at all, from a mathematical point of view, but is in its use a divisor, and in its derivation a product. In order to obtain the safe working stress, we divide the ultimate strength of the material by the proper "factor of safety," and in order to obtain this factor of safety we multiply together several factors which depend in turn upon the qualities of the material, and the conditions of service. So our factor of safety is both a product and a divisor, but it is not a factor. Then again, we infer, naturally, that with a factor of twelve, say, we could increase the load upon a machine member to twelve times its ordinary amount before rupture would occur, when, as a matter of fact, this is not so, at least not in a machine with moving parts, sometimes under load, and sometimes not subjected to working stresses. Still more dangerous conditions are met with when the parts are subjected to load first in one direction, and then in the other, or to shocks or sudden loading and unloading. The margin of safety is, therefore, apparent, not real, and we will therefore call the quantity we are dealing with the "apparent factor of safety," for the name factor is too firmly fixed in our minds to easily throw it off.

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* Throughout this chapter we will adhere to the following definitions:

A "stress" is a force acting within a material, resisting a deformation.

A "load" is a force applied to a body, from without. It tends to produce a deformation, and is resisted by the stress which it creates within the body.

A "working load" is the maximum load occurring under ordinary working conditions.

A "working stress" is the stress produced by the working load, statically applied.

The "safe working stress" is the maximum permissible working stress under the given conditions.

The "ultimate strength" of a material is its breaking strength in pounds per square inch, in tension. compression, or shearing, as the case may be.

The "total stress" is the sum of all the stresses existing at any section of a body.

Unless a stress is mentioned as a total stress, the number of pounds per square inch of section, sometimes called "the intensity of stress," will be meant.

Formula for Factor of Safety.

The apparent factor of safety, as has been intimated, is the product of four factors, which for the purpose of our discussion, we will designate as factors *a*, *b*, *c*, and *d*. Factors *b* and *c*, as will appear later, may be, and often are, 1, but none the less they must always be considered and given their proper values. Designating the apparent factor of safety by *F*, we have then

$$F = a \times b \times c \times d.$$

The first of these factors, *a*, is the ratio of the ultimate strength of the material to its elastic limit. By the elastic limit we do not mean the yield point, but the true elastic limit within which the material is, in so far as we can discover, perfectly elastic, and takes no permanent set. There are several reasons for keeping the working stress within this limit, the two most important being: First, that the material will rupture if strained repeatedly beyond this limit; and second, that the form and dimensions of the piece would be destroyed under the same circumstances. If a piece of wire be bent backward and forward in a vise, we all know that it will soon break. And no matter how little we bend it, provided only that we bend it sufficiently to prevent it from entirely recovering its straightness, it will still break if we continue the operation long enough. And similarly, if the axle of a car, the piston rod of an engine, or whatever piece we choose, be strained time after time beyond its limit of elasticity, no matter how little, it will inevitably break. Or suppose, as is the case with a boiler, that the load is only a steady and unremitting pressure. The yielding of the material will open up the seams, allowing leakage. It will throw the strains upon the shorter braces more than upon the others, thus rupturing them in detail. It is absolutely necessary, therefore, excepting in very exceptional cases, that we limit our working stress to less than the elastic limit of the material.

Among French designers it is customary to deal entirely with the elastic limit of the material, instead of the ultimate strength, and with such a procedure no such factor as we have been discussing would ever appear in the make-up of our apparent factor of safety. Although this method is rational enough, it is not customary outside of France, because many of the materials we use, notably cast iron, and sometimes wrought iron and hard steels, have no definite elastic limit. In any case where the elastic limit is unknown or ill-defined, we arbitrarily assume it to be one-half the ultimate strength, and factor *a* becomes 2. For nickel-steel and oil-tempered forgings the elastic limit becomes two-thirds of the ultimate strength, or even more, and the factor is accordingly reduced to 1½.

The second factor, *b*, appearing in our equation is one depending upon the character of the stress produced within the material. The experiments of Wohler, conducted by him between the years 1859 and 1870 at the instance of the Prussian government, on the effects of repeated stresses, confirmed a fact already well known, namely, that the repeated application of a load which would produce a stress less than the ultimate strength of a material would often rupture it. But they did more. They showed the exact relation between the variation of the load and the breaking strength of the material under that variation. The investigation was subsequently extended by Weyrauch to cover the entire possible range of variation. Cut of the mass of experimental data so obtained a rather complicated formula was deduced, giving the relation between the variation of the load (or rather the stress it produced), the strength of the material under the given conditions (which is generally known as the "carrying strength" of the material) and the ultimate strength. To Prof. J. B. Johnson, we believe, is due the credit of substituting for this formula a much simpler and more manageable one, which perhaps represents the actual facts with almost equal accuracy. Prof. Johnson's formula is as follows:

$$f = \frac{U}{2 - \frac{p'}{p}}$$

where f is the "carrying strength" when the load varies repeatedly between a maximum value, p, and a minimum value, p', and U is the ultimate strength of the material. The quantities p and p' have plus signs when they represent loads producing tension, and minus signs when they represent loads producing compression.

From what has just been said, it follows that if the load is variable in character, factor b must have a value,

$$b = \frac{U}{f} = 2 - \frac{p'}{p}.$$

Let us now see what this factor will be for the ordinary variations in loading.

Taking first a steady, or dead load, $p' = p$ and therefore $\frac{p'}{p} = \frac{1}{1} = 1$, and we have our factor,

$$b = 2 - \frac{p'}{p} = 2 - 1 = 1.$$

In other words, this factor may be omitted for a dead load.

Taking a load varying between zero and a maximum,

$$\frac{p'}{p} = \frac{0}{p} = 0,$$

and we have for our factor,

$$b = 2 - \frac{p'}{p} = 2 - 0 = 2.$$

Again, taking a load that produces alternately a tension and a compression equal in amount,

$$p' = -p \text{ and } \frac{p'}{p} = -1,$$

and we have, for our factor,

$$b = 2 - \frac{p'}{p} = 2 - (-1) = 2 + 1 = 3.$$

A fourth time, taking a load which produces alternately a tension and a compression, the former being three times the latter,

$$p = -3p' \text{ and } \frac{p'}{p} = -\tfrac{1}{3},$$

and we have for our factor,

$$b = 2 - \frac{p'}{p} = 2 - (-\tfrac{1}{3}) = 2 + \tfrac{1}{3} = 2\tfrac{1}{3}.$$

Recapitulating our results, we may say that when the load is uniform, factor $b = 1$; when it varies between zero and a maximum, factor $b = 2$; when it varies between equal and opposite values, factor $b = 3$; when the load varies between two values, p and p', of which p' is the lesser factor, $b = 2 - \frac{p'}{p}$.

The experiments which have been made upon the effects of variable loads have almost without exception been made upon mild steel and wrought iron. Designers are in need of data based upon the results obtained with bronze, nickel steel, cast iron, etc.

It has already been noted that a stress many times repeated will rupture a piece when that stress is greater than the elastic limit, but less than the ultimate strength. It is also known that the application of a stress will change the elastic limit of a material, often by a very considerable amount. A material has really two elastic limits, an upper and a lower one, the latter often being negative in value (*i. e.*, an elastic limit in compression). Between these two limits there is a range of stress, which we may call the elastic range of the material, and within which the material is, so far as we can discover, perfectly elastic. It has been assumed, therefore, that under the influence of the varying or repeated load, this elastic range takes on certain limiting values depending on the character of the variation. So long as the variation is confined within these limits, the piece is safe. If, however, the range of variation of the stress exceeds the elastic range of the material under the given conditions, the piece breaks down. In confirmation of this view of the case, it has been found that pieces long subjected to alternating stresses have an elastic limit of one-third their ultimate strength, while pieces subjected to either repeated tensions, or compressions, only, have an elastic limit of one-half their ultimate strength.

From lack of data we cannot speak with authority in this matter, but it is probable that for material whose elastic limit is other than one-half its ultimate strength, Prof. Johnson's formula, and considerations derived from it, no longer hold. It is more than likely that with fuller knowledge of the subject we will find that the facts of the case may be more truly expressed by the formula,

$$f = \frac{nU}{1 - \frac{p'}{p}(1 - n)}$$

where n is the ratio of the elastic limit to the ultimate strength.

The third factor, c, entering into our equation, depends upon the manner in which the load is applied to the piece. A load suddenly applied to a machine member produces twice the stress within that member that the same load would produce if gradually applied. When the load is gradually applied, the stress in the member gradually increases, until finally, when the full load is applied, the total stress in the member corresponds to this full load. When, however, the load is suddenly applied, the stress is at first zero, but very swiftly increases. Since both the load and the stress act through whatever slight distance the piece yields, the product of the average total stress into this distance must equal the product of the load into this same distance. In order that the average stress should equal the load, it is necessary that the maximum value of the stress should equal twice the load. In recognition of this fact, we introduce the factor $c = 2$ into our equation when the load is suddenly applied.

It sometimes occurs that not all of the load is applied suddenly, in which case the factor 2 is reduced accordingly. If one-half the load were suddenly applied, the factor would be properly 1½, and in general, if a certain fraction of the load, $\frac{n}{m}$, is suddenly applied, the factor is $1 + \frac{n}{m}$. Or, again, it may occur that friction, or some specially introduced provision, may prevent the sudden application of the load from having its full effect, in which case, if the amount of the reduction of this effect be known, or if it be possible to compute it, an appropriate reduction may be made in the value of this factor.

Sometimes, however, a load is applied not only suddenly, but with impact. In such a case it is highly desirable to compute the total stress produced by the load, and to substitute it for the load when obtaining the working section. Failing in this, it is necessary to make factor c more than 2, and sometimes as high as 10 or more. As an example of the possibilities arising in ordinary work, we may instance an elevator suspended by a wire rope of one square inch in section, and fifty feet long. If a truck weighing 500 pounds were wheeled over the threshold and allowed to drop two inches onto the elevator platform, a stress of over 10,000 pounds would be produced in the rope. Thus we see that in this very ordinary case arising in elevator service, this factor would need to be as much as 20.

The last factor, d, in our equation, we might call the "factor of ignorance." All the other factors have provided against known contingencies; this provides against the unknown. It commonly varies in value between 1½ and 3, although occasionally it becomes as great as 10. It provides against excessive or accidental overload, against unexpectedly severe service, against unreliable or imperfect materials, and against all unforeseen contingencies of manufacture or operation.

When we can compute the load exactly, when we know what kind of a load it will be, steady or variable, impulsive or gradual in its application, when we know that this load will not be likely to be increased, that our material is reliable, that failure will not result disastrously, or even that our piece for some reason must be small or light, this factor will be reduced to its lowest limit, 1½.

The conditions of service in some degree determine this factor. When a machine is to be placed in the hands of unskilled labor, when it is to receive hard knocks or rough treatment, the factor must be made larger. When it will be profitable to overload a machine by increasing its work or its speed in such a way as to throw unusual strains upon it, we are obliged to discount the probability of this being done by increasing this factor. Or again, when life or property would be endangered by the failure of the piece we are designing, this factor must be made larger in recognition of the fact. Thus, while it is 1½ to 2 in most ordinary steel constructions, it is rarely less than 2½ for a better grade of steel in a boiler. Even if property were not in danger of destruction, and the failure of the piece would simply result in considerable loss in output or wages, as in the case of the stoppage of a factory, it is best to increase this factor somewhat.

The reliability of the material in a great measure determines the value of this factor. For instance, in all cases where it would be 1½ for mild steel, it is made 2 for cast iron. It will be larger for those materials subject to internal strains, for instance for complicated castings, heavy forgings, hardened steel, and the like. It will be larger for those materials more easily injured by improper and unskillful handling, unless we know that the work will be done by skilled and careful workmen. It will be larger for those materials subject to hidden defects, such as internal flaws in forgings, spongy places in castings, etc. It will be smaller for ductile and larger for brittle materials. It will be smaller as we are sure that our piece has received uniform treatment, and as the tests we have give more uniform results and more accurate indications of the real strength and quality of the piece itself.

Of all these factors that we have been considering, the last one alone has an element of chance or judgment in it, except when we make an allowance for shock. In fixing it, the designer must depend on his judgment, guided by the general rules laid down.

Someone may ask at this point, why, if we introduce a factor for the elastic limit, do we also introduce a factor for repeated loads? It may be argued that if we keep the stress within the elastic limit, no harm will be done, no matter how often the load be repeated, and they are right. However, with a dead load acting upon a piece and straining it to its elastic limit, we have as a margin of safety the difference between its elastic limit and its ultimate strength. But when the load is a repeated load, of the same amount as before, the piece has no margin of safety, unless its section be increased, and it does not have the same margin of safety as it had in the first place, until its section is doubled.

Examples of Application of Formula.

It remains to illustrate the method outlined for developing an "apparent factor of safety" by some practical examples. Let us take first the piston rod of a steam engine. It will be of forged steel, of simple form and reasonable size. The elastic limit will presumably be slightly more than one-half the ultimate strength, so factor $a = 2$. The rod will be in alternate tension and compression many times a minute and factor $b = 3$. The steam pressure will be applied suddenly (in a great many engines, on account of compression, only a part of this load is applied suddenly) and factor $c = 2$. And since the material is reliable, and the service definite and not excessively severe, factor $d = 1\frac{1}{2}$. Then,

$$F = 2 \times 3 \times 2 \times 1\tfrac{1}{2} = 18.$$

Taking next a steam boiler, our factor $a = 2$ as before. While the load in reality varies between zero and a maximum, since the load is steady in operation, and gradually applied, it is correct to make factor $b = 1$ and factor $c = 1$. Although we have an exceptionally reliable material, corrosion is likely to occur, and failure would be disastrous to life and property, so factor $d = 2\frac{1}{2}$ or 3, depending upon the workmanship.

Then, $F = 2 \times 1 \times 1 \times 2\frac{1}{2}$ (or 3) $= 5$ (or 6).

For our last illustration we will take the rim of a cast iron flywheel for a steam engine. Factor $a = 2$, factor $b = 1$, and factor $c = 1$, for the load which is due to centrifugal force is constant. However, the material is the most unreliable with which the designer has to deal. It is probably spongy, and has great internal stress resulting from the cooling. It would be easy and profitable to increase both the power of the engine and the strain in the rim, by speeding it up. In ordinary cases we would make factor d equal to 3 or 4, but in this case the stress in the rim increases, not with the speed, but with the square of the speed, and it is entirely proper to make factor $d = 10$. So we have

$$F = 2 \times 1 \times 1 \times 10 = 20.$$

Table of Factors of Safety.

The following table may be helpful in assisting the designer in a proper choice of the factor of safety. It shows the value of the four factors for various materials and conditions of service, and will give helpful hints to the young designers as to what factors to use under similar circumstances.

Class of Service or Materials.	Factor a	Factor b	Factor c	Factor d	F
Boilers	2	1	1	2¼-3	4½- 6
Piston and connecting rods for double-acting engines	1½-2	3	2	1½	13½-18
Piston and connecting rod for single-acting engines	1½-2	2	2	1½	9 -12
Shaft carrying bandwheel, fly-wheel, or armature	1½-2	3	1	1½	6¾- 9
Lathe spindles	2	2	2	1½	12
Mill shafting	2	3	2	2	24
Steel work in buildings	2	1	1	2	4
Steel work in bridges	2	1	1	2½	5
Steel work for small work	2	1	2	1½	6
Cast iron wheel rims	2	1	1	10	20
Steel wheel rims	2	1	1	4	8
Materials.	**Minimum Values.**				
Cast iron and other castings	2	1	1	2	4
Wrought iron or mild steel	2	1	1	1½	3
Oil tempered or nickel steel	1½	1	1	1½	2¼
Hardened steel	1½	1	1	2	3
Bronze and brass, rolled or forged	2	1	1	1½	3

CHAPTER II.

WORKING STRENGTH OF BOLTS.

Doubtless most mechanics have heard of the rule in use in many drafting offices, "Use no bolts smaller than ⅝-inch diameter, unless space or weight is limited." Or perhaps they may have heard pretty much the same thing stated in another way, namely, that a man will twist off a ½-inch bolt, trying to make a steam-tight joint. It is a matter of common experience among mechanics that a bolt has to be strained a good deal in order to make a tight packed joint, and that bolts must not only be made large enough to properly sustain the load due to the steam or water pressure, but to sustain this initial stress as well.

Bolts subject to tension are called upon for two different classes of service. Either they serve to hold two heavy and rigid flanges together, metal to metal, or they serve to compress a comparatively elastic packing, in order to make a joint steam-tight. In either case the bolt is under a considerable initial tension, due to the strain of screwing up, and hence the advisability of not making it smaller than ⅝ inch diameter. When the flanges are pressed together iron to iron, they are much more unyielding than the bolts. Hence when the bolts are screwed up, they are stretched a good deal more than the flanges are compressed. If we assume that the flanges are so heavy and unyielding that they cannot be compressed at all, the bolt is virtually a spring, and in order to produce in it a stress greater than the initial stress, we must pull so hard on the flanges as to separate them.

The truth of this statement may be seen by referring to Fig. 1. The bolt shown clamps together the two flanges, and the nut is screwed down so tight that the bolt is stretched 1/1000 inch. We will assume that the bolt is of such a size that the stress produced in it by this elongation is 1,000 pounds. If so, the flanges are pressed together with a force of 1,000 pounds. Supposing now that we pull the flanges apart in the manner shown by the arrows, with a force of 500 pounds. We

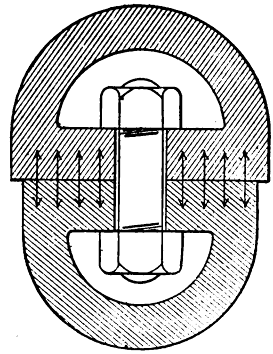


Fig. 1

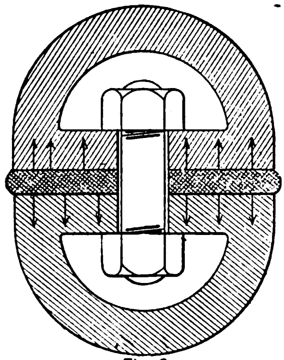


Fig. 2

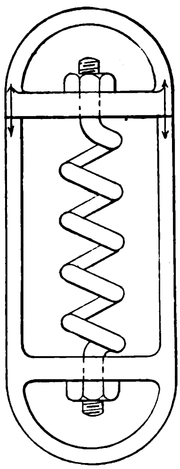


Fig. 3

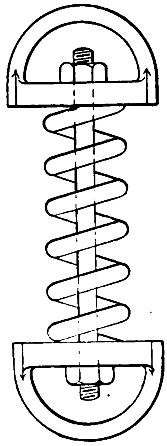


Fig. 4

Machinery, N Y

Figs. 1 to 4. Illustrations of Stresses in Bolts.

cannot produce a greater stress in the bolt than 1,000 pounds until we stretch it a little more than it is stretched already. We cannot do this unless we separate the flanges, and it will take a pull of over 1,000 pounds to do that. Although the pull of 500 pounds adds nothing to the stress in the bolt, it does diminish the pressure between the flanges, which will be now the pressure holding them together, less the force pulling them apart, or 500 pounds. Exactly the same effects would have been noted had we chosen any other force than 500 pounds, provided it was less than 1,000 pounds. The stress in the bolt would not have been increased, but the pressure between the flanges would have been diminished by exactly the amount of the force applied.

On the other hand, supposing that we apply a force of 2,000 pounds to separate the flanges, we will find that the bolt will stretch under this load 2/1000 inch, allowing the flanges to separate by only half that amount, and the pressure between them is nothing. It follows then that the stress in the bolt is now 2,000 pounds. If we had chosen any other force greater than 1,000 pounds, it would have been sufficient to separate the flanges, and the stress in the bolt would have been equal to the force applied. In other words, we find that the stress in the bolt is always either the initial stress, or else the force tending to separate the flanges, and it is always the greater of the two.

If, however, we place a piece of packing between the faces of the flanges, we find it is the packing rather than the bolt that is elastic. On tightening up the nut the packing will be compressed say 1/100 inch. The stress in the bolt we will again assume to be 1,000 pounds. Applying a force of 500 pounds in the same manner as before, as shown in Fig. 2, we will not stretch the bolt very much in comparison to the amount by which we have already compressed the packing. Hence the packing will maintain its pressure against the flanges with almost undiminished force. We have simply added the 500 pounds to the 1,000 pounds stress already in the bolt. Exactly the same thing occurs when the force is increased to 2,000 pounds. The bolt will not give sufficiently to materially reduce the pressure due to the elasticity of the packing, and the stress in the bolt is the initial stress, plus the stress due to the force tending to separate the flanges.

The principles involved in the above discussion may be more easily understood by a reference to the illustrations, Figs. 3 and 4. The yielding members in Figs. 1 and 2 are represented in Figs. 3 and 4 as springs. A few moments consideration of the forces acting in each case will convince one of the truth of these two rules:

1. When the bolt is more elastic than the material it compresses, the stress in the bolt is either the initial stress or the force applied, whichever is greater.

2. When the material compressed is more elastic than the bolt, the stress in the bolt is the sum of the initial stress, and the force applied.

Some experiments were made at the mechanical laboratories of Sibley College, Cornell University, some years ago, to determine the initial stress due to screwing up the bolts in a packed joint in an effort to get it steam-tight. The tests were made with ½, ¾, 1, and 1¼-inch bolts. Twelve experienced mechanics were allowed to select their own wrenches, and tighten up three bolts of each size in the same way as they would in making a steam-tight joint. The bolts were so connected in a testing machine that the stress produced was accurately weighed. The wrenches chosen were from 10 to 12 inches long in the case of the ½-inch bolts, and ranged up to 18 and 22 inches long in the case of the 1¼-inch bolts. Thirty-six tests were made with each size of bolt, and while the results were not very close together in all cases, it was shown that the stress in the bolt due to screwing up varies about as its diameter, and that the stress produced in this way is often sufficient to break off a ½-inch bolt, but never anything larger.

Now since the stress varies about as the diameter of the bolt, and the area varies as the square of the diameter, it is evident that the larger the bolt is, the greater the margin of safety it will have. If the stress in a ½-inch bolt is equal to its tensile strength, the stress in a 1-inch bolt will be about one-half its tensile strength, and in a 2-inch bolt, one-quarter of its tensile strength. These are very low factors of safety, especially in the case of the sizes commonly used. When we come to add the stress due to the force tending to separate the flanges, there is an exceedingly small margin left, which is in many cases absolutely wiped out by any sudden increase of pressure due to water hammer, or some similar cause. If, however, we are to use the same factors of safety in designing the bolting for packed joints as we do in designing the other parts of machinery, we would use nothing smaller than 1¼-inch bolts under any circumstances, and generally bolts ½ inch or so larger. Such a proposition as this seems ridiculous in the light of successful practice, and so the writer was moved some time ago to investigate a great many flanged joints, some successful and some otherwise, with a view to obtain if possible some rule for proportioning the bolts so that they can always be relied upon.

From this investigation it was found that we may take for the "working section" of a bolt in a joint *its area at the root of the thread, less the area of a ½-inch bolt at the root of the thread times twice the diameter of the given bolt, in inches.* This working section must be sufficient to sustain, with a liberal factor of safety, the stress due to the steam load, or other force tending to separate the flanges. The largest unit stress, found by dividing the stress due to the load on the bolt produced by the steam pressure, or other such cause, by the working section of the bolt, is about 10,000 pounds per square inch. Let us take as an example of the application of this rule the case of an inch bolt. Its area at the root of the thread is 0.550 square inch. Twice its diameter in inches is 2. The area of a ½-inch bolt at the root of the thread is 0.126 square inch. If from 0.550 square inch we subtract 2×0.126 square inch, the result, 0.298 square inch, is the working section of the 1-inch bolt. At 10,000 pounds to the square inch this bolt will sustain a stress of not quite 3,000 pounds, in addition to the stress due to screwing up.

There is reason, although not very sound, for this allowance. It has already been noted that a ½-inch bolt will sometimes be twisted off in screwing it up to make a steam-tight joint. It has also been noted that an inch bolt will have twice the initial stress due to this cause that a ½-inch bolt will. Therefore if we could divide the area of the inch bolt into two parts, 0.252 square inches of it would be strained to the breaking limit, resisting the initial stress, and the rest of the area, 0.298 square inches, would be free to tend to the other stresses that might come upon it. As a matter of fact, we cannot so divide the area, so the reasoning is not very sound, but inasmuch as the rule corresponds to the best practice in this regard, while theoretically more perfect rules would give us excessive and undesirable diameters, it seems better to use it than to adopt the familiar method of using a high factor of safety, and paying no attention to the initial stress. The latter method invariably leads one to grief, unless one is familiar

TABLE I. WORKING STRENGTH OF BOLTS.

Diameter of Bolt, inches.	Area at Root of Thread, square inches.	Working Section, square inches.	Strength of Bolt, 5000 pounds Stress.	Strength of Bolt, 6,000 pounds Stress.	Strength of Bolt, 7,000 pounds Stress.	Strength of Bolt, 8,000 pounds Stress.	Strength of Bolt, 10,000 pounds Stress.	Strength of Bolt, 12,000 pounds Stress.
½	.126	0	0	0	0	0	0	0
⅝	.202	.044	220	264	308	352	440	528
¾	.302	.113	565	678	791	904	1,130	1,356
⅞	.420	.200	1,000	1,200	1,400	1,600	2,000	2,400
1	.550	.298	1,490	1,788	2,086	2,384	2,980	3,476
1⅛	.694	.411	2,055	2,466	2,877	3,288	4,110	4,932
1¼	.893	.578	2,890	3,468	4,046	4,624	5,780	6,936
1⅜	1.057	.710	3,550	4,260	4,970	5,680	7,100	8,520
1½	1.295	.917	4,585	5,502	6,419	7,336	9,170	10,504
1⅝	1.515	1.105	5,525	6,630	7,735	8,840	11,050	13,260
1¾	1.746	1.305	6,525	7,830	9,135	10,440	13,050	15,660
1⅞	2.051	1.578	7,890	9,468	11,046	12,624	15,780	18,936
2	2.302	1.798	8,990	10,788	12,586	14,384	17,980	21,576
2¼	3.023	2.456	12,280	14,736	17,192	19,648	24,560	29,472
2½	3.719	3.089	15,445	18,534	21,623	24,712	30,890	37,068
2¾	4.620	3.927	19,635	23,562	27,489	31,416	39,270	47,124
3	5.428	4.672	23,360	28,032	32,704	37,376	46,720	56,064
3¼	6.510	5.690	28,450	34,140	39,830	45,520	56,900	68,280
3½	7.548	6.666	33,330	39,996	46,664	53,328	66,660	79,992

by long experience with the proper working stress to use with each size of bolt.

It will be found that for ordinary sizes of bolts the above rule works out in about the following form:

$$S = f\ (0.55\ D^2 - 0.25D)$$

where S = the strength of the bolt when used in a packed joint,
D = the diameter of the bolt in inches,
f = the safe working stress in pounds per square inch.

This formula is simple to use, and not difficult to remember. It must be borne in mind that it is only approximate, and not exact. As an example of its use, we will take the case of the inch bolt again. Using a working strength of 10,000 pounds per square inch it will be found that

$$S = 10{,}000\ (0.55 - 0.25) = 3{,}000.$$

As the sizes of the bolts become greater, the formula gives results lower than they should be. It is very nearly correct for the common sizes of bolts, and on the safe side for the uncommon sizes.

Table I on the opposite page has been prepared, giving the diam-

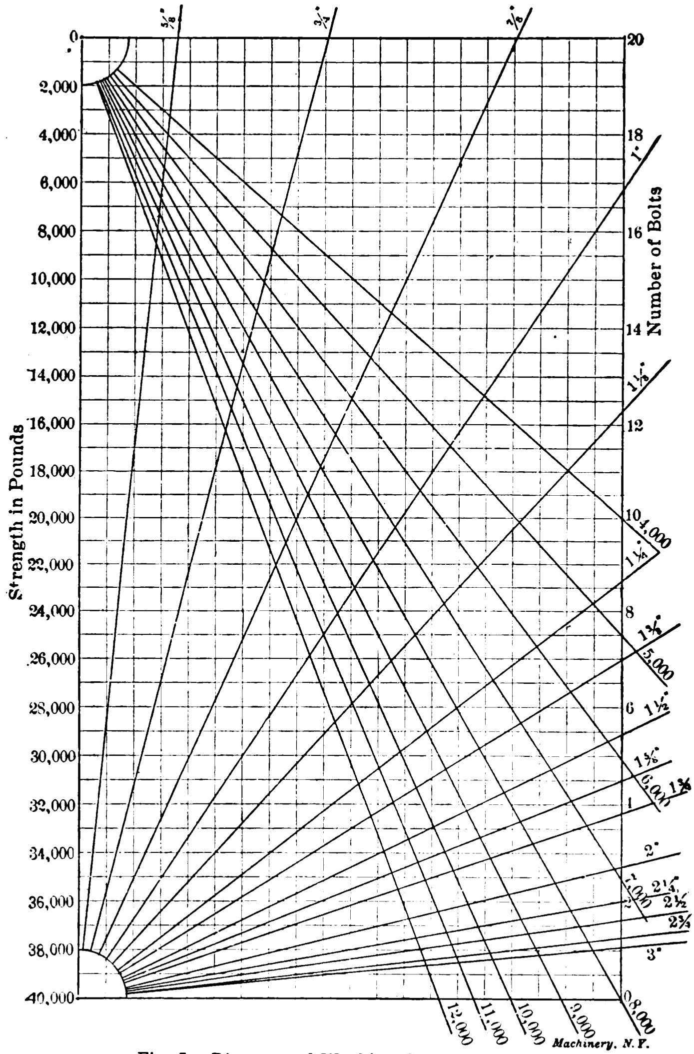


Fig. 5. Diagram of Working Strength of Bolts.

eters, least areas, working sections, and strengths of different sizes of bolts with U. S. standard threads. Thus from the table we find that the area of a 1¼-inch bolt, at the root of the thread, is 0.893 square inch. Its working section is 0.578 square inch, and its strength

at 8,000 pounds per square inch working stress is 4,624 pounds. As an example of the use of the table, let us design the bolting of a valve chest 8 inches wide and 12 inches long. Let us assume that the steam pressure is 100 pounds per square inch, and that ten bolts will be needed. The total load on the ten bolts will then be 8 × 12 × 100, or 9,600 pounds. The load per bolt is 960 pounds. Assuming a working stress of 6,000 pounds, we find that a ⅞-inch bolt is necessary.

The diagram, Fig. 5, gives the strength of any number of bolts, of any given size, with any required working stress when used in a packed joint. Supposing that it is required to find the strengh of 20 ¾-inch bolts when used with a working stress of 6,000 pounds to the square inch. Finding the figure "20" at the right-hand side of the chart, we follow horizontally to the left on the heavy line, until we reach the diagonal line marked ¾ inch. We then descend the vertical line which intersects the line ¾ inch at the same point as does line 20, until this vertical line intersects the diagonal line marked 6,000. We then follow the horizontal line which intersects line 6,000 at this point, to the left-hand edge of the chart, where the figures adjacent indicate that the answer is 13,500 pounds. If we check the answer from the table we will find that the strength of a ¾-inch bolt at 6,000 pounds working stress is 678 pounds, and therefore the strength of 20 of them is 13,560 pounds.

In designing flanged joints it must be remembered that an unlimited number of bolts cannot be crowded into a flange. The largest number of bolts that it is possible to use in a flanged joint and still have room to turn the nuts with an ordinary wrench is equal to the diameter of the bolt circle, divided by the diameter of the bolts, both in inches. A greater number of bolts than this can be used if necessary but a special form of wrench must be provided. The number of bolts generally used is about $D - 2\sqrt{D} + 8$, where D is the diameter of the interior of the pipe or cylinder in inches. For ordinary pressures this does not crowd the bolts too closely, although it puts them close enough together so that the flange will not leak under steam. The number of bolts actually taken for any flange is usually the nearest number divisible by four. For instance, for a water chamber of 60 inches diameter, the number of bolts obtained from the formula is $60 - 2\sqrt{60} + 8$, or 52½. The number of bolts actually taken might be 52 or 56, probably 52.

For our last problem let us take a rather extreme case. We will suppose the case of the water chambers of a high-pressure mining pump, 30 inches internal diameter, and subject to a pressure of 500 pounds per square inch. The number of bolts taken will be $30 - 2 \times \sqrt{30} + 8$, or taking the nearest number exactly divisible by four, 28 bolts. The area of the 30-inch circle is 0.7854×30^2, or 706.86 square inches. The total load on all the bolts due to the water pressure is 706.86 × 500, or 353,430 pounds. It will be noted that the diagram which we have already used does not extend above 40,000 pounds strength, but by multiplying both the number of pounds strength and the number of bolts by 10, the effective range can be increased to 400,000 pounds strength and 200 bolts. Taking, then, 35,300 instead of 353,000 at the left-hand edge of the chart, we follow to the right to the intersection with the diagonal line marked 8,000, then ascend the vertical line passing through this intersection till it meets horizontal line 2.8, we find that this point falls between the radial lines marked 1¾-inch and 2 inches, thus indicating that 28 bolts 1¾-inch diameter are not strong enough, and 28 bolts 2 inches diameter are stronger than is necessary. In fact the vertical line we have been following intersects the line marked 2 inches at the horizontal line 2.4, indicating that 24 2-inch bolts would be required.

Stresses on Bolts Caused by Tightening of the Nuts by a Wrench.

An interesting discussion on the stresses thrown upon bolts by the tightening of the nut by a wrench appeared in the *Locomotive*, July, 1905, and it may be considered proper to include the substance of this discussion in this chapter. While it is impossible to make any accurate computation of the tensile stress that is thrown upon a bolt by tightening a nut on its end, says the author of the article referred to, it is possible to obtain a roughly approximate estimate of that stress, when the nut is tightened under given conditions.

Let us suppose that a given screw is provided with a nut, which is to be turned up solidly against some resisting surface, so as to throw a tensile stress on the screw. Let the nut be turned by means of a wrench whose effective length is L inches. When the nut has been brought up pretty well into place, let us suppose that a force of P pounds, when applied to the end of the wrench in the most effective manner, will just move it. The work done by the man at the wrench, per revolution of the nut under these circumstances, is found by multiplying the force P by the circumference of the circle described by the end of the wrench. The wrench being L inches long, the circumference of this circle is $2\pi L$ inches, where $\pi = 3.1416$. Hence the work performed by the workman, per revolution, is $2\pi LP$ inch-pounds. Let us assume, for the moment, that the screw runs absolutely without friction, either in the nut, or against the surface where the nut bears against its seat. Then the work performed by the workman is all expended in stretching the screw, or deforming the structure to which it is attached. Hence, if the screw has n threads per inch of its length, and T is the total tension upon it in pounds, the work performed may also be expressed in the form $T \div n$; for in one turn the screw should be drawn forward $1 \div n$ inch, against the resistance T. Under the assumed conditions of perfection, the two foregoing expressions for the work done must be equal to each other. That is, we should have $2\pi LP = T \div n$, or

$$T = 2\pi nLP.$$

from which we could calculate the tension, T, on the bolt, if the screw were absolutely frictionless in all respects.

We come, now, to the matter of making allowances for the fact that in the real screw the friction is very far from being negligible. The actual tension that the given pull would produce in the bolt will be smaller than the value here calculated, and the fraction (which we will denote by the letter E) by which the foregoing result must be multiplied in order to get the true result is called the *efficiency* of the screw. The efficiency of screws has been studied both experimentally and theoretically; but the experimental data that are at present available are far less numerous than might be supposed, considering the elementary character and the fundamental importance of the screw in nearly every branch of applied mechanics. In the *Transactions* of the American Society of Mechanical Engineers, Volume 12, 1891, pages 781 to 789, there is a paper on screws by Mr. James McBride, followed by a discussion by Messrs. Wilfred Lewis and Arthur A. Falkenau, to which we desire to direct the reader's attention. In this place Mr. Lewis gives a formula for the efficiency of a screw of the ordinary kind, which appears to be quite good enough for all ordinary purposes, and which may be written in the form

$$E = 1 \div (1 + nd),$$

where d is the external diameter of the screw. If we multiply the value T, as found above, by this "factor of efficiency," the value of T, as corrected for friction, becomes

$$T = \frac{2\pi nLP}{1 + nd}$$

As an example of the application of this formula, let us consider the case in which a workman tightens up a nut on a two-inch bolt, by means of a wrench whose effective length is 50 inches, the maximum effort exerted at the end of the wrench being, say, 100 pounds. A standard two-inch bolt has 4.5 threads per inch; so that in this example the letters in the foregoing formula have the following values: $n = 4.5$; $L = 50$ inches; $P = 100$ pounds; $d = 2$ inches; and π stands for 3.1416. Making these substitutions, the formula gives

$$T = \frac{2 \times 3.1416 \times 4.5 \times 50 \times 100}{1 + 4.5 \times 2} = \frac{141{,}372}{10} = 14{,}137 \text{ pounds.}$$

That is, the actual total tension on the bolt, under these conditions, is somewhat over 14,000 pounds, according to the formula. As another example, let us consider a screw 1.5 inch in external diameter, with the nut set up with the same force and the same wrench as before. A standard screw of this size has six threads to the inch, so that the formula gives in this case

$$T = \frac{2 \times 3.1416 \times 6 \times 50 \times 100}{1 + 6 \times 2} = \frac{188{,}496}{13} = 14{,}500 \text{ pounds, approx.}$$

Comparative Strength of Screw Threads.

A subject nearly related to the working strength of bolts is the comparative strength of screw threads. There has been considerable discussion from time to time among mechanics as to which of the three forms of thread, V, square, and Acme, is the strongest against shear.

The following report of tests undertaken by C. Bert Padon at the James Millikin University, Decatur, Ill., to settle this question, with the idea of determining as nearly as possible with the means at hand just what relation these styles of thread bear to each other, will, therefore, prove of interest.

Each of the three forms was tested under two different conditions. First, a screw and nut of each form was made with threads all the same outside diameter, 15/16 inch, and with both screw and nut of the same axial length, 17/32 inch, and of the same material, the grade of steel commonly known in the shop as "machine steel." These three samples are shown at *a*, *b*, and *c* in Fig. 6, in which *a* is the V-thread, *b*, the Acme thread, and *c*, the square thread. In the second test all three screws were of the same root diameter, about ⅝ inch, and were

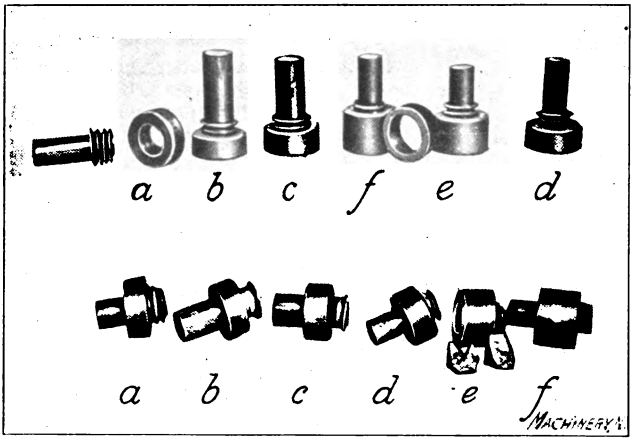


Fig. 6. Test Pieces used for Finding the Comparative Strength of Screw Threads.

all made of gray cast iron, while the nuts were of machine steel. The length of the thread helix in each screw was such that each of the samples would present the same shearing area, the assumption being that they would shear at the root diameter of the screw since the screw was made of the weaker material. The different thicknesses of the nuts to suit the length of the helix required for this will be noticed in the halftone at *d*, *e*, and *f*, which show respectively the V-thread, Acme, and square samples. All the threads were made a snug fit, with the threaded length of the screw exactly the same as the thickness of the nut. The diameter of the shank was less than the root diameter of the thread in each case. The screws had all 6 threads per inch.

In the cut the upper row shows the samples before testing, while the lower row shows the nature of the failure of each sample under test. A 50,000-pound Olsen machine was used. The nuts were supported on the ring shown with sample *f*, to allow room for the screw to drop through the nut when it failed, while pressure was applied at the top of the shank, which was carefully squared. The shank of the Acme thread screw *e* in the second set of three samples was not strong enough to withstand compression, but crushed before the thread gave way, at a pressure of 29,300 pounds. The fragments of the broken shank are shown. The screw was afterwards pushed through with a short piece of steel rod, failing at 29,600 pounds pressure. Table II gives the results of the test. As will be seen from the table, the Acme, or 29-de-

TABLE II. RESULTS OF TESTS OF SHEARING STRENGTH OF SCREW THREADS.

Sample.	Style of Thread.	Material.		Thickness of Nut.	Diameter of Screw.	Breaking Load in pounds.
		Screw.	Nut.			
Threads same outside diameter and all 6 threads per inch.						
a	Sharp V	M. S.*	M. S.	$\frac{17}{32}$	$\frac{15}{16}$	29,980†
b	Acme	"	"	$\frac{17}{32}$	$\frac{15}{16}$	34,090‡
c	Square	"	"	$\frac{17}{32}$	$\frac{15}{16}$	23,880‡
Threads same root diameter, ⅝ inch, and same area of section to resist shear. All are 6 threads per inch.						
d	Sharp V	C. I.*	M. S.	$\frac{1}{2}$	0.914	20,450‡
e	Acme	"	"	$\frac{13}{16}$	0.792	29,600‡
f	Square	"	"	1	0.792	25,550‡

* M. S. stands for Machinery Steel; C. I. for Cast Iron.
† Threads bent over in both screw and nut.
‡ Sheared at root of thread.

gree thread, makes the best showing in each case. The V-thread sample, *a*, evidently could not have failed in the way described without expanding the nut enough to allow the distorted threads to slip by each other. In this case, then, the thickness and strength of the nut play an important part. If the hole had been tapped in a larger piece of metal, it is difficult to believe that the thread would have failed by shearing, or in any other way, at a pressure less than that sustained by the Acme thread.

CHAPTER III.

FLANGE BOLTS.

The calculations required for determining the number and size of bolts necessary to hold down a pillar crane are very instructive. The illustrations herewith, Figs. 7 to 9, show three examples of bolts used in this manner—that is, a series of bolts equally spaced around a circular flange intended to resist overturning. The first shows a pillar crane where the load has a tendency to overturn the pillar; the second, a radial drill where the pressure on the drill has a tendency to overturn the column, and the third a self-supporting chimney, where the wind pressure has an overturning effect.

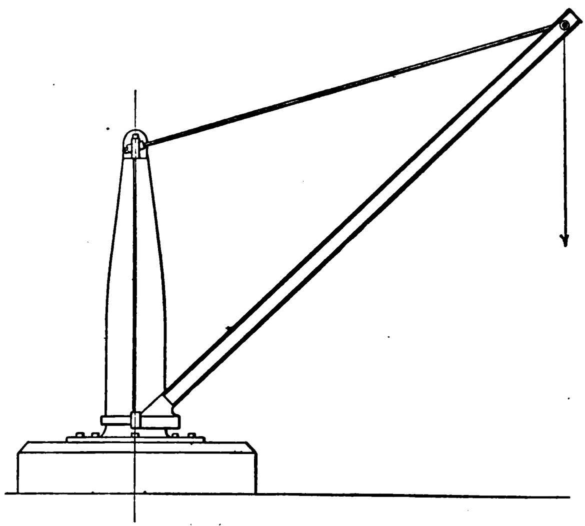


Fig. 7. Jib Crane; Load has a Tendency to Overturn.

It will be noted that there are two elements—one of tension due to the strain in the bolts, and one of compression due to the compression set up in the foundation. To exaggerate matters, suppose we were to place a layer of soft wood between the flange of the crane and the foundation. It is evident that the load would have a tendency to stretch the bolts on the side opposite the load and also to sink that part of the flange nearest the load, into the wood as in Fig. 10. The neutral axis would be a line drawn through the point where the flange and the foundation separate and at right angles to the direction of the load. On one side of this line we have the compression element due to the foundation, the bolts on this side having no value whatever. Starting at this neutral line and running the other way, we note that each bolt has a different value. To find the total value of the bolts, which constitutes our problem, we must add up these different values, and in consequence must know the position of the neutral axis.

If instead of coming in contact with the foundation or bed-plate, the flange was supported by studs as shown in Fig. 11, we would have half of the studs in compression and the other half in tension, and the neutral axis would pass through the center of the bolt circle. If the flange had an annular surface inside of the bolts upon which to rest, as in Fig. 12, the neutral axis would lie somewhere inside of the larger circumference of this annular bearing surface as indicated. If conditions were as in Fig. 13, the neutral axis would be somewhere

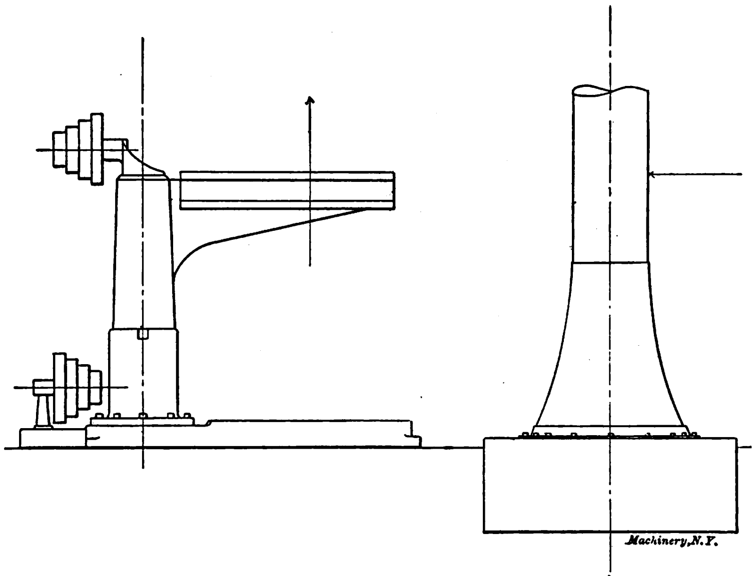


Fig. 8. Radial Drill; Pressure of Feed Tends to Overturn.

Fig. 9. Wind Pressure Tends to Overturn Chimney.

between the bolt circle and the outside circumference of the flange, or possibly tangent to the bolt circle. Let us first determine the total bolt values for certain given positions of the neutral axis, and later look into the factors that control the position of this axis.

Referring to Fig. 10 it will be evident that the amount each bolt is stretched, and therefore the stress it resists, varies directly as its distance from the neutral axis. It will be further noted that the moment of any one bolt as regards the neutral axis is directly proportional to the square of its distance from this axis, because the moment of any bolt is the product of the force it exerts, and the distance through which it acts. Consequently, if we could easily determine the value of the mean square, as we surely can, we will then only have to multiply it by the number of bolts to obtain the sum of the squares.

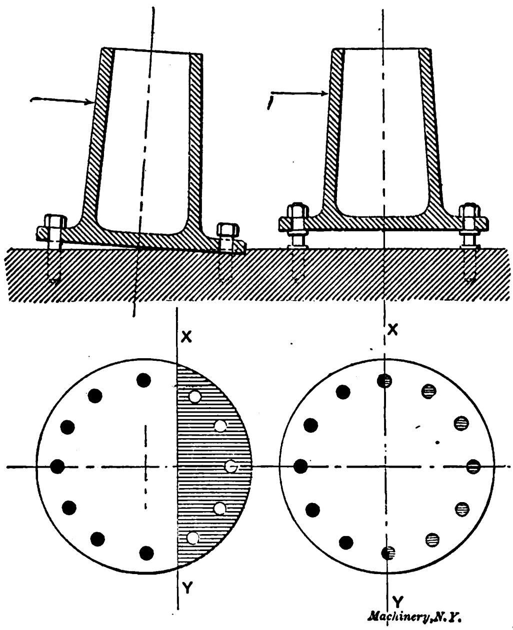
Consider six bolts, as in Fig. 14, spaced equidistant on a circle of radius = 1. Let the maximum stress in any bolt be 8,000 pounds, and take the neutral axis as being tangent to the bolt circle. Hence we have the following:

TABLE III. SIX BOLTS.

Bolt No.	Distance.	Square of Distance	Stress.	Moment.
1...........	2.00	4.00	8,000	16,000
2...........	1.50	2.25	6,000	9,000
3...........	.50	.25	2,000	1,000
4	....	....	.....	......
5...........	.50	.25	2,000	1,000
6...........	1.50	2.25	6,000	9,000
Totals......	....	9.00	.....	36,000

This gives a value for the mean square 9.00 ÷ 6 = 1.50. If the radius were twice as great, the mean square would, of course, be four times as great. This table, therefore, indicates that the

Mean square $= 1.50\,R^2 = \frac{3}{8}\,D^2$ (1)



Figs. 10 and 11. Location of Neutral Axis under Varying Conditions.

The total of these square values represents the moment of inertia of the set of bolts, and if we multiply the sum by the maximum stress and divide it by the distance of the point at which that stress acts, *viz.*,

D, we obtain the moment of resistance just as we do in figuring the strength of a beam in flexure. Hence we have the following:

Moment of inertia = No. of bolts × mean square = 1.50 R^2N = ⅜ D^2N, and

$$\text{Moment of resistance} = \frac{1.50\, R^2\, N\, S}{D} = \tfrac{3}{8}\, N\, D\, S \qquad (2)$$

where S is the maximum total stress in any bolt.

TABLE IV. TWELVE BOLTS.

Bolt No.	Distance.	Square of Distance.	Stress.	Moment.
1..........	2.000	4.000	8,000	16,000.0
2..........	1.866	3.482	7,464	13,928.3
3..........	1.500	2.250	6.000	9 000.0
4..........	1.000	1.000	4,000	4,000.0
5..........	.500	.250	2,000	1,000.0
6..........	.134	.018	536	71.8
7..........	.....	.....	.....	....
8..........	.134	.018	536	71 8
9..........	.500	.250	2,000	1,000.0
10..........	1.000	1.000	4,000	4,000.0
11..........	1.500	2.250	6,000	9,000.0
12..........	1.866	3.482	7.464	13,928.3
Totals......	.....	18.000	.....	72,000.2

TABLE V. TWENTY-FOUR BOLTS.

Bolt No.	Distance.	Square of Distance.	Stress.	Moment
1	2.000	4.000	8 000	16,000
2–24.......	1.966	3.865	7 864	15 461
3–23.......	1.866	3.482	7,464	13,928
4–22.......	1.707	2.914	6,828	11,655
5–21.......	1.500	2.250	6,000	9,000
6–20.......	1.259	1.585	5,036	6,340
7–19.......	1.000	1.000	4,000	4 000
8–18.......	.741	.549	2,964	2,196
9–17.......	.500	.250	2 000	1,000
10–16.......	.293	.086	1,172	343
11–15.......	.134	.018	536	72
12–14.......	.034	.001	136	5
Totals......		36.000	....	144,000

Applying this to Fig. 14 we have ⅜ × 6 × 2 × 8,000 = 36,000, which is verified by Table III where the moment of each bolt is computed separately.

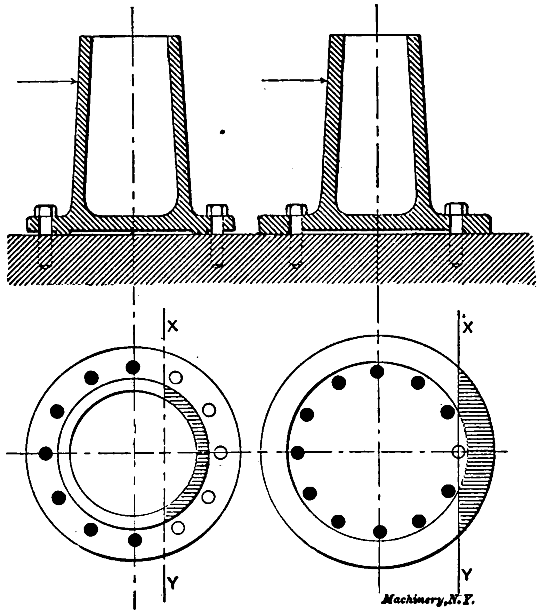
Similarly we may take twelve bolts, and considering that the maximum stress on any bolt is 8,000, the distance to, and stress in, each bolt are as given in Table IV.

By equation (2) we have

Moment of resistance = ⅜ NDS = ⅜ × 12 × 2 × 8,000 = 72,000, which

agrees with the result found by computing the moment of each bolt separately, as Table IV shows. The value of the mean square is by equation (1) equal to 1.5 R^2, and the table gives in this case 18 ÷ 12 = 1½. This table, then, verifies our formulas for both mean square and total moment exerted by the twelve bolts.

For twenty-four bolts the results are the same, and Table V on the previous page is given to show that the formulas are applicable to any number of bolts.



Figs. 12 and 13. Location of Neutral Axis under Varying Conditions.

$$\text{Moment} = \tfrac{3}{8}\, NDS = \tfrac{3}{8} \times 24 \times 2 \times 8{,}000 = 144{,}000.$$

$$\text{Mean square } 1.5\, R^2 = \frac{36}{24} = 1.5.$$

The foregoing applies only where the neutral axis is tangent to the bolt circle, but knowing what the moment of a series of bolts is when the neutral axis is in this position, it is a simple matter to determine the moment for any other known position.

Referring to Fig. 16, let the neutral axis have the position *XY*. It will be evident that the moment depends upon the mean square of a series of distances, which are composed of two parts, *viz.*, a constant ϕ and a variable such as *a*, *b*, *c*, *d*. Hence for the total of the squares we have

$$(\phi + 0)^2 + (\phi + a)^2 + (\phi + b)^2 + (\phi + c)^2 + \ldots$$

which may be written $N\phi^2 + 2\phi\,(a + b + c + \ldots) + a^2 + b^2 + c^2 + \ldots$

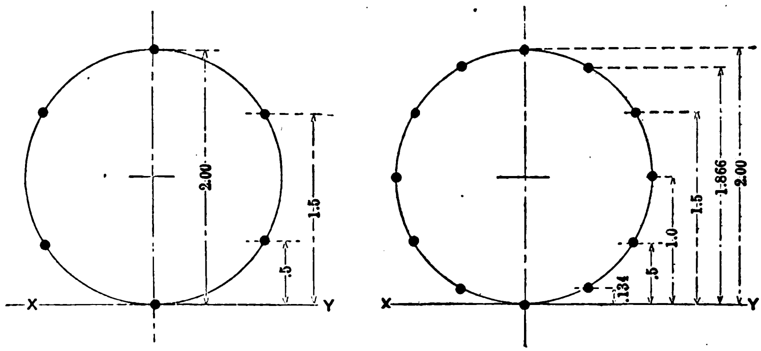
Referring to Fig. 16 it will be seen that the average of 0 and $f =$ radius; a and $e =$ radius; b and $d =$ radius, etc., which means that the sum of $a + b + c \ . \ . \ . = NR$, which may be written for the second term of the previous expression. For the third term we may write $a^2 + b^2 + c^2 \ . \ . \ . = \frac{3}{8} ND^2$ by equation (1) which we have already outlined.

Hence we may write for the sum of the squares

$$N\phi^2 + N\phi D + \tfrac{3}{8} ND^2.$$

To obtain the moment of resistance we must divide this by the distance of the point of maximum stress from the neutral axis and multipy it by the maximum stress. Therefore

$$\text{Moment of resistance} = \frac{N(\phi^2 + \phi D + \frac{3}{8} D^2)\, S}{\phi + D} \qquad (3)$$



Figs. 14 and 15. Finding the Stress on the Bolts for Six and Twelve Bolts.

When the neutral axis lies inside of the bolt circle we have $(0 - \phi) + (a - \phi) + (b - \phi) + (c - \phi) + \ . \ . \ .$ which may be written $N\phi^2 - 2\phi\,(a + b + c + \ . \ . \ .) + a^2 + b^2 + c^2 + \ . \ . \ .$ and for the moment we have

$$\text{Moment of resistance} = \frac{N(\phi^2 - \phi D + \frac{3}{8} D^2)\, S}{D - \phi} \qquad (4)$$

The only remaining factor to determine is the position of the neutral axis so that we can apply the above formula. In the first place it would be well to point out certain conditions that render this somewhat uncertain. In these, as in most all bolt calculations, the initial strain set up in a bolt by tightening the nut cannot be definitely determined. Then again, the assumption that each bolt is strained directly in proportion to its distance from the neutral axis necessitates that the flange be absolutely rigid. While a heavy cast iron flange with a large fillet, and possibly a few stiffening ribs, is about as rigid as anything we might find in construction work, yet it is not absolutely rigid. Finally we might mention the weight of the structure or pillar that is borne by the flange. This factor has a tendency to increase the element of compression and decrease the element of tension to a slight extent.

It is, however, much more practical and advisable to determine the position of the neutral axis as closely as possible than to attempt to determine these several uncertain quantities. The formula will at best give uniformity of results, and if experience points out that our results are correct in one case, they will also be correct for other cases when they apply to similar conditions.

It is an accepted fact that in all cases of flexure the neutral axis

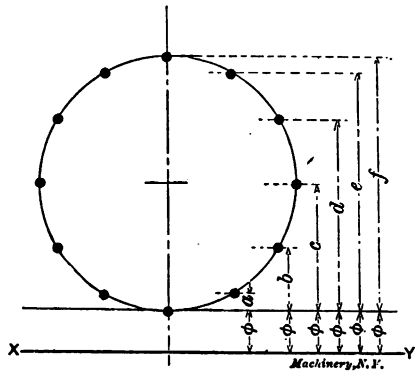


Fig. 16. Finding the Stress on the Bolts when the Neutral Axis is Outside the Bolt Circle.

passes through the center of gravity of the section. This means that in Figs. 10, 11, 12 and 13, the moment of the shaded area in compression on one side of the line would exactly balance the moment of the bolt areas on the other side, provided, of course, that the same material were used throughout. It would therefore seem that the practical method to locate this neutral axis would be to lay out the bolts and that part of the flange in contact with the foundation and find the center of gravity, making allowance for the fact that the weight per unit of area of tension and compression areas should be taken as proportional to their respective stresses per square inch.

CHAPTER IV.

FORMULAS FOR DESIGNING RIVETED JOINTS.

In designing a riveted joint it is first necessary to know the pressure per square inch and the diameter of the cylinder, or the thickness of the metal.

In the following formulas the notation below is used:

t = thickness of the plate,
P = pressure to be resisted by 12 inches of the joint,
D = diameter of the cylinder, in inches,
a = pressure per square inch,
S = ultimate shearing strength of rivet or plate,
p = pitch of rivets,
f = factor of safety = ratio of bursting pressure to working pressure,
T = tensile strength of the plate,
d = diameter of the rivet hole,
B = bearing value of the plate,
l = distance from center of rivet to the edge of the plate,
b = diagonal pitch,
e = efficiency of the joint,
n = number of rows of rivets,

The value of some of the above letters are as follows:

S = 0.75 to 0.80 of the tensile strength of the plate, for a rivet in single shear; a rivet in double shear is taken as 1¾ times one in single shear. As the rivets of a joint are protected from deterioration while the plates are thinned by wear, the shearing strength of a rivet is frequently taken as equal to the tensile strength. Also, in determining the shearing value of a rivet from the tensile strength of the plates, if iron rivets are being used with steel plates, the shearing value of the rivet must be determined from the tensile strength of iron, and not from the tensile strength of steel.

f = 6, for cylinders of moderately good materials and workmanship. The following additions should be made for structural defects when they exist, *viz.*, an addition of 25 per cent when the rivets are not good and fair in the girth seams; 50 per cent if the rivets are not good and fair in the longitudinal seams; 100 per cent if the seams are single riveted; and 200 per cent when the quality of materials or workmanship is doubtful or unsatisfactory.

T = for steel plates about 55,000 to 60,000 pounds per square inch; for wrought iron about 45,000 pounds per square inch. The tensile strength of wrought iron plates across the grain is on an average 10 per cent less than along the grain.

$B = \frac{3T}{2}$ for ordinary bearing, and, $2T$ for web bearing.

The formulas apply to joints having only one pitch.

If the thickness, *t*, of the plate is known

$$d=\frac{\sqrt{t\times 92}}{8}+\frac{1}{16} \qquad (5)$$

$$p=\frac{d^2\times 0.7854\times S\times n}{t\times T}+d \qquad (6)$$

A riveted joint is twice as strong against circumferential rupture as against longitudinal rupture. Therefore, a cylinder which requires a double riveted lap joint for the longitudinal seams will only require a single riveted lap joint of the same diameter and pitch for the circular seams.

$$P=6aD \qquad (7)$$

Now choose a trial value, *d*, for the diameter of the rivet hole; commercial rivets vary by 1/16 inch up to ⅞ inch, more commonly by ⅛ inch; ⅝ inch, ¾ inch, ⅞ inch, and 1 inch being the most frequently used. Remember that the cold rivet is 1/16 inch diameter less than the hole, and that the diameter of the hole must be greater than the thickness of the plate, otherwise the punch will not be likely to endure the work of punching.

Substitute the chosen value of *d* in the following equations until the proper pitch is found. Six diameters of the rivet is the maximum pitch for proper calking, owing to the liability of the plates to pucker up when being calked.

$$p=\frac{9.4248d^2S}{Pf} \qquad (8)$$

for single riveted lap joints.

$$p=\frac{18.8496d^2S}{Pf} \qquad (9)$$

for double riveted lap joints and single riveted butt joints with two cover plates.

$$p=\frac{37.6992d^2S}{Pf} \qquad (10)$$

for double riveted butt joints with two cover plates.

Notice that twice the result found by (8) is equal to the result found by (9), and that four times the result found by (8) is equal to the result found by (10).

Having now the pitch and diameter of the rivet, try the percentage of strength, or efficiency, of the plate, by,

$$e=\frac{p-d}{p}, \qquad (11)$$

and if the result is not satisfactory, try a new diameter of rivet and find its corresponding pitch as before.

The strength or efficiency of a well designed single riveted joint may be 56 per cent; of a double riveted joint 70 per cent; and of a triple riveted joint 80 per cent of that of the solid plate.

In determining the pitch of rivets and the efficiency of joints with punched holes, the larger diameter of the punched hole should be used in determining the efficiency, and the smaller diameter, or the diameter of the rivet, should be used in determining the bearing value, etc., of the rivet.

$$t = \frac{f \times P \times p}{12 \times T(p - d)} \qquad (12)$$

Now check the pitch, diameter and thickness by substituting these values in (6).

If the rivet fills the hole, and is well driven, there is no bending moment exerted on it unless it passes through several plates. Practical tests have shown that rivets cannot be made to surely fill the holes if the combined thickness of plates exceeds 5 diameters of the rivets.

Butt joints are generally used for plates over ½ inch in thickness. Where one cover plate is used on a butt joint, its thickness is 1⅛ times the thickness of the plate. Where two cover plates are used each should be about ⅝ of the plate thickness.

Now check the diameter, thickness and pitch for crushing by

$$\frac{12dtB}{P} = \text{or} > Pf \qquad (13)$$

for single riveted joint.

$$\frac{24dtB}{P} = \text{or} > Pf \qquad (14)$$

for double riveted joint.

The distance from the center of the rivet to the edge of the plate after being beveled for calking should be $1\frac{1}{2}d + \frac{1}{8}$ inch. Check by

$$l = \frac{fPp}{24tS} \qquad (15)$$

and if the result is greater than $1\frac{1}{2}d$, use it, adding ⅛ inch.

The diagonal pitch of rivet of a seam having several rows of rivets, all of the same pitch, is generally equal to 0.75 to 0.80 of the straight pitch, and should not be less than

$$b = \frac{(p \times 6) + (\textit{dia. of rivet} \times 4)}{10} \qquad (16)$$

Diagonal Seams.

The ratio of strength, R, of an inclined or diagonal seam to that of a straight seam, or ordinary longitudinal seam, may be found by

$$R = \frac{2}{\sqrt{\text{cos of angle of inclination} \times 3 + 1}} \qquad (17)$$

Rivet Material.

It is necessary to make the rivets of the same material as the plates to prevent corrosive wasting from galvanic action. That is, iron rivets should be used with iron plates, steel rivets with steel plates, and copper rivets for copper plates.

Elastic Limit of Riveted Joints of Steam Boilers.

The riveted seams of a steam boiler should cease to be steam tight for some time before the internal pressure is equal to the elastic limits of the plate. If a boiler were stretched beyond the elastic limit of the material, the rivet holes would become stretched and the joints of the plates would be disturbed, resulting in large leakage from the rivet holes and seams.

The elastic limit of riveted joints of wrought iron and mild steel is as follows:

Best quality of mild steel, 32,000 to 34,000 pounds per square inch.

Ordinary quality of mild steel, 28,000 to 30,000 pounds per square inch.

Best quality of wrought iron, 24,000 to 26,000 pounds per square inch.

Ordinary quality of wrought iron, 20,000 to 22,000 pounds per square inch.

Weight of Seams or Riveted Joints of Cylinders.

The weight of seams of cylinders varies according to their proportions and must be calculated in each particular case. A rough approximation of the weight of riveted seams may, however, be obtained by increasing the weight of the cylinder by 1/6, if formed with single riveted circumferential seams and double riveted longitudinal seams; and by 1/5, if formed with double riveted circumferential seams and triple riveted longitudinal seams.

Gripping Power of Rivets.

When two plates are fastened together by properly proportioned and well closed rivets, the frictional adhesion of the plates depends upon the longitudinal tension of the rivets. The adhesion of the plates or their resistance to sliding, per square inch of sectional area of the rivets, is in a general way equal to 2/9 of the ultimate tensile strength of the rivet.

Punched Holes.

The distressing effect on the plate due to punching may generally be neutralized by countersinking ⅛ inch in width around the rivet hole with a reamer. All rivet holes shall be so accurately spaced and punched that when several parts are assembled together, a rivet 1/16 inch less in diameter than the hole can generally be entered hot into any hole. In the better class of plate work it is now the practice to drill rivet holes in plates after the plates are in place, so that the holes are sure to be fair. In some cases the holes are punched to a smaller diameter, and then drilled out to final size after the plates are in place. In either case the plates are afterwards separated, and the burr left by the drill removed.

The effect of clearance between the punch and die is to produce a conical hole in the plate. The punched plates are generally arranged with the large ends of the holes outside or the small ends together.

Comparative Strength of Boiler Joints.

An interesting fact about riveted joints, which will prove instructive to discuss more fully, is that the stress in the second row of rivets always amounts to more than that in the first row. This is the case when a triple joint is used, having a narrow outer butt strap and a wide one inside, and when the pitch in the second row is half the pitch of the first, and all rivets have the same diameter. We will here show how to calculate the stress of the shell plate at both rows of rivets. Take the joint shown in Fig. 17, *i. e.*: shell, ⅝ inch, rivets, 1 1/16

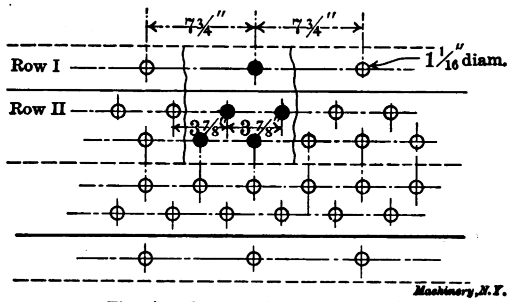


Fig. 17. Joint to be Investigated.

inch = 1.06 inch, about; radius of shell, 29 inches; pitch, 7¾ inches; pressure, 200 pounds per square inch.

Row I. Pull along one pitch = 7.75 × 29 × 200 = 45,000 pounds.

Length of plate = 7¾ — 1 1/16 = 6.68 inches.

$$\text{Tearing of plate} = \frac{45{,}000}{6.68 \times 0.625} = 10{,}780 \text{ pounds per square inch.}$$

$$\text{Shearing of rivets} = \frac{45{,}000}{9 \times \frac{\pi}{4} \times 1.06^2} = 5{,}650 \text{ pounds per square inch.}$$

Row II. Pull in second row of rivets is 45,000 pounds less the amount taken away by rivet in (I); that is, the amount transmitted in row (I) through one rivet to the butt straps.

$$45{,}000 - 5{,}650 \times \frac{\pi}{4} \times 1.06^2 = 45{,}000 - 5{,}000 = 40{,}000 \text{ pounds.}$$

Length of plate = 7¾ — 2 × 1 1/16 inch = 5⅝ = 5.625 inches.

$$\text{Tearing of plate} = \frac{40{,}000}{5.625 \times 0.625} = 11{,}380 \text{ pounds per square}$$

inch or about 5½ per cent more than in row (I).

To avoid this there are two methods possible; one of them is shown in Fig. 18. Use the same pitch at row (I), but increase the pitch at rows (II) and (III), all rivets remaining the same diameter.

Row I. Pull along one pitch $= 7.75 \times 29 \times 200 = 45{,}000$ pounds.

$$\text{Tearing of plate} = \frac{45{,}000}{6.68 \times 0.625} = 10{,}780 \text{ pounds per square inch.}$$

$$\text{Shearing of rivets} = \frac{45{,}000}{7 \times \frac{\pi}{4} \times 1.06^2} = 7{,}275 \text{ pounds per square inch.}$$

$$\text{Factor of safety} = \frac{38{,}000}{7{,}275} = 5.22.$$

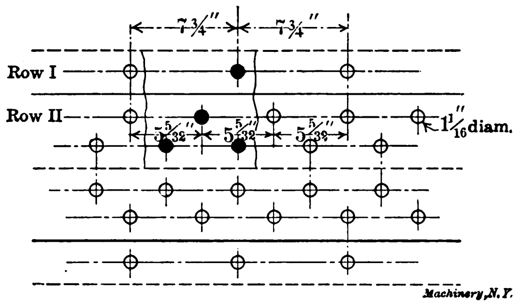


Fig. 18. Joint Re-designed to give Less Stress in Row II than in Row I.

Row II. Pull along one pitch $= 45{,}000 - 7{,}275 \times \frac{\pi}{4} \times 1.06^2 = 38{,}575$ pounds.

Length of plate $= 7.75 - 1.5 \times 1.06 = 7.75 - 1.6 = 6.15$ inches.

$$\text{Tearing of plate} = \frac{38{,}575}{6.15 \times 0.625} = 10{,}050 \text{ pounds per square}$$

inch or 7 per cent less than in row (I).

A second method, shown in Fig. 19, consists in increasing the pitch and diameter of rivets in the first row, or using smaller rivets in the second and third rows. Of course, this is somewhat awkward, on account of it being necessary to change the riveting tools (but on the European continent this is the usual practice) for the two sizes of rivets. If, however, we keep the 1 1/16-inch rivets in the first row, and use 15/16-inch rivets in the second and third rows, we get:

Row I. Pull along one pitch $= 7.75 \times 29 \times 200 = 45{,}000$ pounds.

$$\text{Area of rivets} = \left(1 \times \frac{\pi}{4} \times 1.06^2\right) + \left(8 \times \frac{\pi}{4} \times 0.94^2\right) =$$

$$0.883 + 5.550 = 6.433 \text{ square inches.}$$

Length of plate $= 7\tfrac{3}{4} - 1\ 1/16 = 6.68$ inches.

$$\text{Tearing of plate} = \frac{45{,}000}{6.68 \times 0.625} = 10{,}780 \text{ pounds per square inch.}$$

$$\text{Shearing of rivets} = \frac{45,000}{6,433} = 7,000 \text{ pounds per square inch.}$$

Row II. Pull = 45,000 — 0.883 × 7,000 = 38,820 pounds.

Length of plate = 7.75 — 2 × 15/16 = 5.875 inches.

$$\text{Tearing of plate} = \frac{38,820}{5.875 \times 0.625} = 10,580 \text{ pounds per square}$$

inch or 1¾ per cent less than in row (I).

If, instead of using smaller diameter rivets in the second and third rows, we keep 1 1/16-inch rivets, but increase the diameter of rivets in the first row to 1 3--1/16 inch, and also the pitch to give the same percentage, similar results would be obtained. In a triple butt joint with straps of equal width, the stress in the second row would always

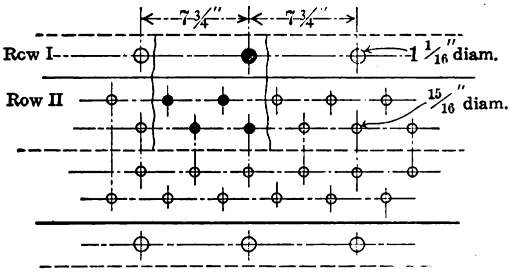


Fig. 19. Joint in which the Stresses are Nearly Equalized.

be less than in the first row; on this account, therefore, it is unnecessary to make any calculations of row (II).

English Practice.

In England it is customary to use higher working stresses than in the United States; while here plates are used with a tensile strength of 55,000 pounds per square inch, with a factor of safety of 5, they use there plates of not less than 60,000 pounds, allowing a factor of safety of 5 for double butt joints, and a factor of safety of 4½ for triple butt joints. In England they never use iron rivets, but always steel rivets, with a shearing strength of 50,000 pounds per square inch, and a factor of safety of 5, which equals 10,000 pounds per square inch, under pressure. It is also their rule to take the diameter of the steel rivets from $1.1\sqrt{T}$ to $1.2\sqrt{T}$, where T equals thickness of plate in inches; so that in the previous case they would have used $1.2\sqrt{0.625} =$ 15/16 inch for the diameter of the rivets, and the riveting as shown in Fig. 18.

CHAPTER V.

CALCULATING THE STRENGTH OF A MOUTHPIECE RING AND COVER.

There are thousands of digesters, vulcanizers and other similar vessels in use working under considerable pressure. Accidents to these, particularly the bursting of the head or of the ring to which it is clamped, are almost as common as boiler explosions, and oftentimes do considerable damage and sometimes result in the loss of life. There are one or two points relating to the problem of designing vessels of this kind which do not always receive proper attention from the men

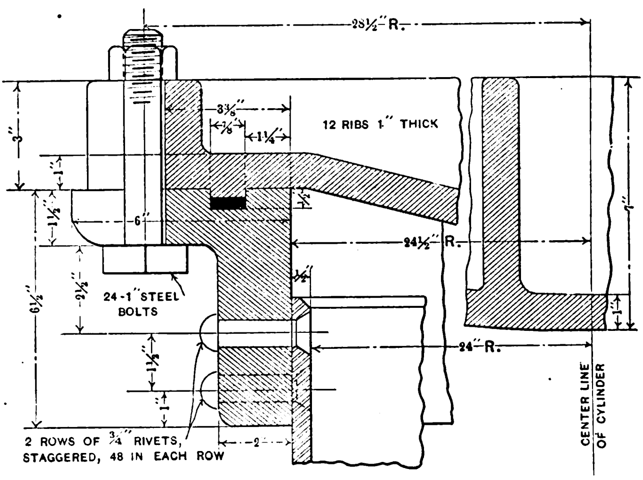


Fig. 20. Design of Mouthpiece Ring and Cover.

responsible for the calculations involved, and it is with the object of calling attention to some of these points that we give herewith the calculations made for figuring the strength of a cover and mouthpiece ring.

Fig. 20 shows the essential features of the design. The body of the cylinder itself is a welded steel tube 4 feet in diameter, ½ inch thick, and about 7 feet long. To this is riveted a mouthpiece ring, presumably of cast iron, having slots for 24 one-inch steel bolts by which the cover is made fast. The important dimensions are shown. No other information being at hand, the material of the cover is taken as cast iron, while the shell is supposed to be made of steel having a tensile strength about equal to that of boiler plate. The following data as to the strength of the materials are assumed:

Factor of safety	5
Cast iron, ultimate tensile strength....................	20,000 pounds
Steel shell, ultimate tensile strength....................	55,000 pounds
Rivets, ultimate shearing strength.....................	40,000 pounds
Rivets, ultimate bearing strength......................	90,000 pounds
Steel bolt, working tensile stress.....................	4,000 pounds

Working pressure to which vessel is subjected, 60 pounds per square inch.

The blueprint from which these details were taken calls for a testing pressure of 125 pounds per square inch. On this question something will be said later.

The ways in which it is possible for this structure to fail are almost too numerous to catalogue. A rapid inspection, however, shows the following as being the only ones which we need to consider:

First, bursting of the cylinder head.

Second, rupture of cover bolts.

Third, failure of rivets from shearing.

Fourth, failure of mouthpiece ring from tensile stresses in lower edge of the hub.

In considering failure from the first cause, the cover may be treated the same as the cylinder head of an engine would be. The formulas given in Kent's Handbook for determining the thickness of cylinder heads may be used; a number of different ones will be found there. Taking, for instance, Thurston's rule, the first one given:

$$t = \frac{Dp}{3,000} + \frac{1}{4}$$

in which D is the diameter of the circle in which the thickness is taken, p is the maximum working pressure per square inch, and t is the thickness of the head. Substituting the known values in this equation we have

$$t = \frac{52 \times 60}{3,000} + \frac{1}{4} \text{ inch} = 1.040 + 0.250 = 1.290 \text{ inch.}$$

The diameter taken is, roughly, the diameter of the gasket. The result, 1.290 inch, is found to be somewhat greater than the figure given on the sketch, but to the cover there shown is added the strengthening effect of the heavy ribs provided; the cover with these should be entirely satisfactory for a working pressure of 60 pounds. The crowning shape of this part also adds to its strength.

The strength of the bolts to resist rupture will next be considered. The inside diameter of the gasket is 4 feet 3½ inches, or 51½ inches, and the area of a 51½-inch circle is about 2,100 square inches. With a pressure of 60 pounds per square inch this gives a total load on the head equal to 2,100 × 60 = 126,000 pounds. Since there are 24 cover bolts the pressure sustained by each cover bolt will be 126,000 pounds divided by 24, or 5,250 pounds, the amount due to the steam pressure. The area of a 1-inch United States standard bolt at the bottom of the thread is about 0.55 square inch. The fiber stress in the bolt due to the steam pressure will then be 5,250 ÷ 0.55 = 9,550 pounds, about. This figure in itself is well within the safe limit for steel of the quality from which such bolts are usually made. We have, however, to reckon with a number of other factors. We have, for instance, to consider the old question as to whether there is any greater tension on the cover bolts after the steam has been turned on above the initial tension due to the tightening of the cover. With the elastic gasket used it can be shown that the steam pressure will be added to the tension produced by setting up the bolts, which will thus have to be stronger than they would if a metal to metal joint were provided. For a full discussion of the question of the stresses in cover bolts the reader is referred to Chapter II, and also to a paper read by Carl Hering before the Engineers' Club of Philadelphia, January, 1906. Considering that these bolts will be tightened by comparatively inexperienced men, opened and closed a number of times a day, and are certain at some time to be overstrained, and that the constant use to which they are subjected will tend in time to weaken the material through fatigue, it is not at all advisable to put a stress of more than 4,000 pounds per square inch on these bolts. It is suggested that the diameter of these bolts be increased to 1¼ inch and that their number be increased to 36. We would then have for the tension of each bolt 126,000 ÷ 36 = 3,500 pounds, and since the area of a 1¼-inch bolt at the root of the thread is about 0.89 square inch, the stress on the bolt will be 3,500 ÷ 0.89 = 3,930 pounds per square inch. This is none too low, taking into account the elastic gasket and the possibility of abnormal tightening through the occasional use of a pipe extension to the wrench.

Calculation for the strength of the rivets in shear is very simple. There are 96 of these rivets, so that each of them bears as its part of the load on the cover an amount equal to

$$\frac{24.5^2 \times \pi \times 60}{96} = 1{,}180 \text{ pounds, about.}$$

This amount divided by 0.44, the area of a ¾-inch ring, gives a shearing stress of 2,680 pounds, a figure which need never cause the slightest anxiety. The bearing value of the rivet will be proportionately low.

The last question to be considered, that of the tensile stress in the lower edge of the hub of the ring, is discussed at length in *The Locomotive*, issue of July, 1905, published by the Hartford Steam Boiler Inspection and Insurance Co. This cause of failure was, until recently, a rather obscure one. The engraving, Fig. 21, shows the action which causes the deformation. There is an upward pull of the cover bolts at P with a downward pressure of the gasket at Q, and a further downward pull at S due to the pressure of the steam on the bottom of the vessel. These three forces, working together, tend to turn the ring inside out, as we might say, elevating the outer edge and depressing the inner edge, and thus expanding the lower portion of the hub. From this distortion the principal stress is that of tension in the hub. The way in which the part fails under these circumstances is shown in Fig. 22. "Hub cracks" are introduced running from the lower edge up into the body of the ring, sometimes passing through the rivet holes and sometimes avoiding them. The formula given in *The Locomotive* for determining the maximum tensile stress at this point is as follows:

$$F = \frac{(mNE + LD)\ (h - a)}{6.2832\ (I - a^2A)}$$

in which F = the tensile stress per square inch,
m = the distance from the gasket to the bolt circle,
N = the total number of the cover bolts,
E = the excess of the actual tension on each cover bolt above

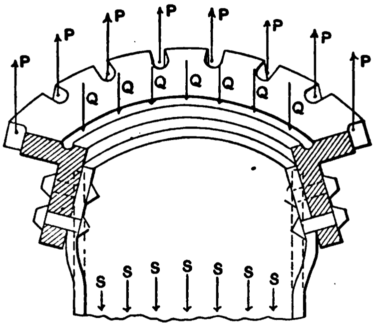


Fig. 21. Stresses on the Ring.

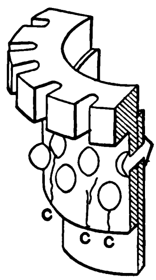


Fig. 22. Usual Manner of Failure.

that due to the steam load (1,200 pounds is suggested in the article referred to),
L = total steam load,
D = the distance from the inner edge of the ring to the bolt circle,
h = height of the ring,
a = the distance from the center of gravity of the ring section to the face of the ring,
I = the moment of inertia of the ring section about axis OX (see Figs. 23 and 24),
A = area of the ring section.

Those letters which refer to dimensions will be found in Fig. 23, where a diagrammatical sketch of the ring section is given. The quantity of the denominator $(I - a^2A)$ amounts to the same as the moment of inertia of the section about the neutral axis. It is put in the form given for convenience in calculating, the issue of *The Locomotive* referred to having a table of moments of inertia of rectangles provided for the purpose. No explanation need be given here of the methods of finding the center of gravity and moment of inertia of a section. This will be found discussed in any text book dealing with the strength of materials.

Drawing the diagram shown in Fig. 23 and for the sake of simplicity risking the leaving out of the gasket groove, we find the following values:

$A = 15$ 1/16 square inches,

$a = 2.91$ inches,

I (about axis OX) $= 184.6$.

Substituting the known values in the given formula we have

$$F = \frac{(2\tfrac{5}{16} \times 24 \times 1200 + 126{,}000 \times 4)\ (6.5 - 2.91)}{6.2832\ (184.6 - 2.91^2 \times 15\tfrac{1}{16})} = 5.600 \text{ pounds.}$$

Twenty thousand pounds was taken as a safe figure for the tensile strength of cast iron. This is none too high, especially if great care

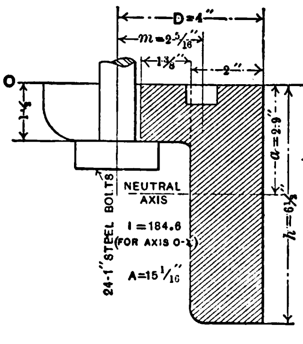


Fig. 23. Data for Original Ring.

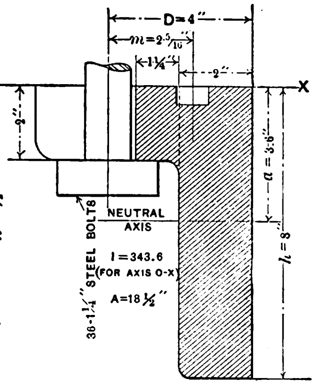


Fig. 24. Suggested Section.

is not taken in the selection of the iron and the inspection of the casting after it is completed. With a factor of safety of 5 we have 4,000 pounds as the safe figure for a working tensile strength. The results of our calculation would thus show that the stresses in the ring are high enough to be dangerous. To give the additional strength necessary the section shown in Fig. 24 is suggested. The hub has been made 1½ inch longer, and the thickness of the flange has been increased about ½ inch. This latter change was made both to keep the parts in good proportion so far as looks are concerned, and from the fear, as well, that the ring might fail by breaking at the corner of the gasket groove. The possibility of this would be a rather difficult thing to calculate with assurance, but good judgment would seem to indicate that the casting is none too strong at this point. Repeating the same operation on this enlarged section that we went through in calculating the strength of the smaller section, also now considering 36 bolts instead of 24, as already suggested, we have

$A = 18\tfrac{1}{2}$ square inches,

$a = 3.6$ inches,

I (about axis OX) $= 343.6$.

Substituting the known values in the equation as before, we have

$$F = \frac{(2\tfrac{5}{16} \times 36 \times 1200 + 126{,}000 \times 4)\ (8 - 3.6)}{6.2832 \times (343.6 - 3.6^2 \times 18.5)} = 4{,}070 \text{ pounds.}$$

This figure, while a little large, may be considered safe, perhaps, if a good casting from a good quality of iron is used.

The value of E used above is that recommended in the discussion from which the formula was taken, namely 1,200 pounds. This is arbitrarily selected, and although it would seem somewhat low in view of the possibilities for excessive strain afforded by the wrench and pipe combination, the boiler insurance company referred to has found that the formula, as given, is rather on the side of safety. The large bolts suggested for the improved section are favorable for reducing the excess pressure, since the workman is not liable to overstrain a large bolt in the same proportion that he would a smaller one.

It would be unwise to conclude this chapter without some reference to the testing pressure called for on the blueprint previously referred to. All the parts have thus far been figured out for a working pressure of 60 pounds. If this really is to be the *maximum* working pressure, and the parts have been proportioned with this figure in view, it is an exceedingly unwise thing to do to test the vessel at a pressure greatly in excess of this; 75 or 80 pounds at least should never be exceeded in testing the structure. Damage is often done by careless use of excessive pressures in testing, these injuries sometimes not showing at the time, but being disastrous later on. If the pressure in use will occasionally run up to a figure approaching 125 pounds per square inch that is another matter, and the whole design should be altered to make this possible without straining the parts beyond what they are able to bear.

CHAPTER VI.

KEYS AND KEYWAYS.

It is not very common in practice to determine the dimensions of keys by calculation, but rather according to the results of experience, so that great differences between the sizes used by different machine builders are not uncommon. Twenty years ago, however, a collection was made of the various key standards, and a system of average dimensions was founded on this basis. These dimensions, having stood the test of time, can be utilized as a basis for the examination of the strain to which keys are exposed. If we assume that the narrow side of the key alone has to take up the moment of rotation, then the strain of these narrow sides must be about the same as the strain of the material in the shaft itself. The narrow sides are subjected to the specific superficial pressure p, while the tension k in a shaft of the

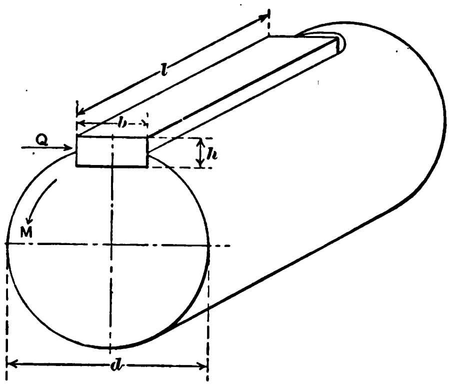


Fig. 25. Shaft with Ordinary Rectangular Key.

diameter d is produced by the moment of rotation M. (See Fig. 25.) The lateral surface pressure Q on the key is therefore

$$Q = \frac{M}{\frac{d}{2}} = \frac{\pi}{8} d^2 k = 0.4\, d^2 k \text{ (approximately).} \qquad (18)$$

This pressure has to be taken up by half the narrow side of the key, and therefore

$$0.4 d^2 k = \frac{h}{2} lp \qquad (19)$$

The length l of the key is usually about 1 or $1\frac{1}{2}d$, the value $l = d$ being the average minimum. The superficial pressure p should not be allowed to exceed 17,000 pounds per square inch. The strain of rotation k should be taken at a lower value than in the case of shafts exposed to a pure twisting strain, since keyed shafts are almost invariably subjected to a high bending strain at the same time by the pull of belting, the pressure of gear teeth, etc. Consequently k may be taken from 2,800 to 5,600 pounds per square inch, or an average of 4,200 pounds to the square inch.

By substituting the values $k = 4{,}200$, $p = 17{,}000$, and $l = d$ in equation (19) we have approximately $h = 0.2d$. The key should therefore be sunk into the shaft and hub to a depth equal to 1/10 of the shaft diameter in each case, the depth being measured at the side of the key and not at the center.

The ordinary key offers a resistance to twist on the broad and narrow sides, the manner in which the strain is distributed between them being illustrated in Fig. 26. When the hub and shaft undergo a rela-

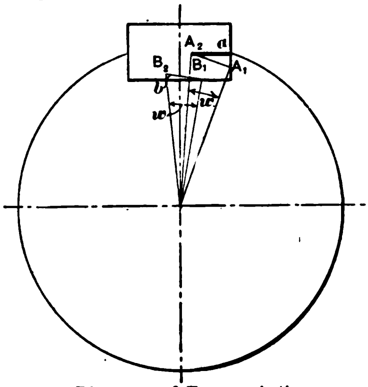


Fig. 26. Diagram of Forces Acting on Key.

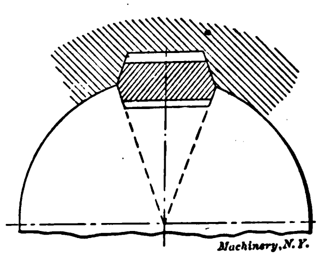


Fig. 27. Proposed Form of Key, Equalizing the Radial and Tangential Pressure.

tive displacement through the angle w, the point A_1 on the narrow side moves toward A_2 and the point B_1 on the broad side toward point B_2. This results in a compression of the material to an extent indicated by a on the narrow side and by b on the broad side, the latter distance being about 1/6 of the former. The resistance to twist about the actual grooved surface for an equal strain on the material is proportionate to these two distances calculated on the relative dimensions of the two effective surfaces of the groove. For medium key dimensions this proportion is about 1 to 3½, or in other words, the narrow sides are exposed to more than three times the twist of the broad sides. A key of the usual form, that is, slightly tapered and driven in place, takes up little or no strain on its narrow sides until the twisting force comes into play, but a very slight twist between the hub and shaft, resulting from slight changes in form in the broad sides, will bring the narrow sides into action. Whether the changes formed on the broad side exceed the elastic limit depends entirely on the care with which the groove has been cut and the key fitted. For these reasons the desire to secure both radial and tangential tension in one and the same key has led to the form shown in Fig. 27. Such a key would not be very difficult to make, the slots being given a considerable radial taper.

CHAPTER VII.

TOGGLE-JOINTS.

The toggle-joint, while one of the simplest mechanisms to construct, is quite as difficult to understand as many of the more complicated movements. In Figs. 28 and 29 are shown the two simplest forms in which the toggle-joint appears. In the first instance the force is supposed to be applied at F to overcome a pressure at P. In the second figure the right-hand arm is extended so as to form a handle to which the force is applied in a direction at right angles to the arm. It should be noted that while this mechanism is called a "toggle-joint," it is really nothing more nor less than a crank and connecting-rod,

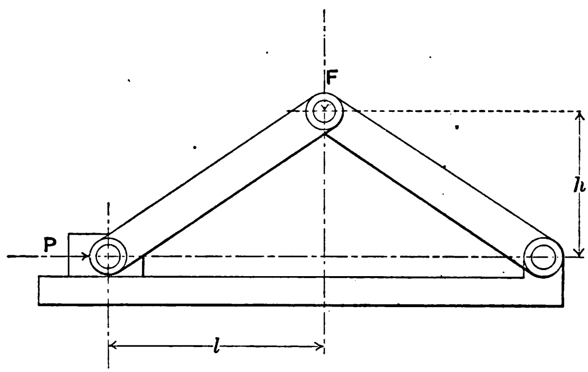


Fig. 28. Example of Simple Form of Design in which the Toggle-Joint Appears.

of which the cross-head is at P, and the connecting-rod from P to E, the right-hand arm corresponding to the crank.

The problem is generally to find how great a resistance at P will be overcome by a force applied at F; and as the resistance that can be overcome at P for a given applied force increases as the two arms approach a straight line, no calculation can be made until the positions of the arms are known.

Instantaneous Center.

All cases of the toggle-joint can be easily solved by what is known as the principle of *instantaneous centers.* This principle is simple, and is clearly illustrated in Figs. 30 to 33, which apply to the two forms of toggle-joint shown in Figs. 28 and 29.

In any machine, simple or complex, no matter what its construction, the force applied, multiplied by the distance through which it acts, must equal the resistance overcome by the machine, multiplied by the distance through which it is moved. The principle of the instantaneous center affords us the means of finding the relative distances moved by the points where the force is applied and the resistance overcome.

In Fig. 30 *ad* and *cd* are the arms of the toggle-joint. What we call the instantaneous center is at *o*. It is located at the intersection of the perpendiculars to the lines along which the two ends of the arm *ad* move, this being the arm upon which forces *F* and *P* act. Thus, the end *a* moves in a horizontal line at right angles to line *ao*, and the end *d*, which is guided by the arm *cd*, and travels about the center *c*, moves for the instant at right angles to line *do*. The point of intersection *o* of lines *ao* and *do* is the instantaneous center.

The reason why this point is given the name of "instantaneous center" is because, if we consider the movements of the ends of the arm and the forces *F* and *P* for an instant, that is, for an infinitesimal time, they will be exactly the same as though the forces were rotating

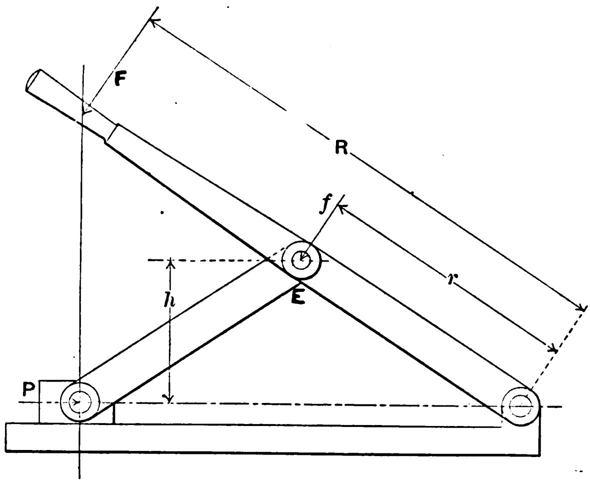
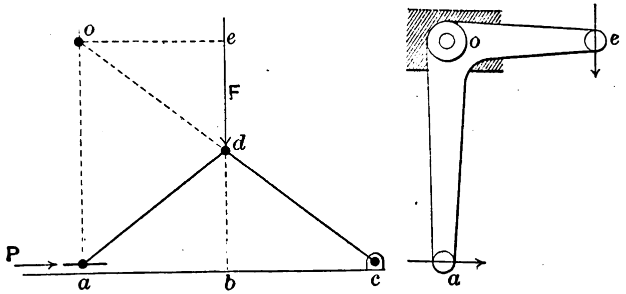


Fig. 29. Another Example of Simple Design in which the Toggle-Joint Principle is Employed.

about the center *o* for that instant. To make this clearer, Fig. 31 has been drawn. This represents a bell-crank lever with arms *eo* and *ao* corresponding to the lines designated by these letters in Fig. 30. The axis *o* corresponds to the position of the instantaneous center of Fig. 30. Now it is plain, that if the lever be moved an exceedingly small distance about center *o*, the movements of points *e* and *a* will be precisely the same as the movements of forces *F* and *P* in the actual toggle-joint.

For example: Suppose it were found that for the position of the toggle-joint shown in Fig. 30, a downward push of 0.001 inch at *d* produced a movement at *a* of 0.002 inch. Also, suppose the lever in Fig. 31 to be constructed as directed, with the center-lines of its arms corresponding to *eo* and *ao* in Fig. 30. It will then be the case that a downward movement of 0.001 inch at *e* will move point *a* 0.002 inch, just as in the toggle-joint.

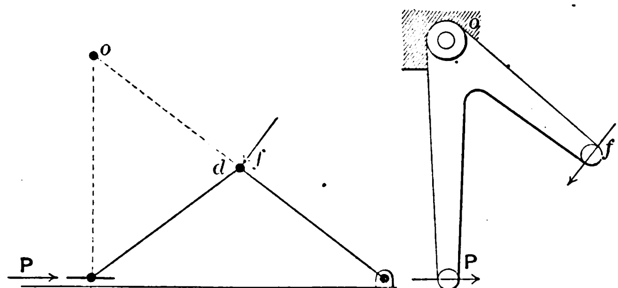
Since the movements of the extremities of the two arms of a lever are proportional to the lengths of the arms, it makes the calculation of any toggle-joint very simple to first find the instantaneous center about which an equivalent lever may be assumed to turn, and then



Figs. 30 and 31. Analysis of Principles Involved in Design Fig. 28.

make the calculations as though based upon the lengths of these lever arms.

Basing our calculations, now, upon the respective lengths of the lever arms, it ought to be clear from the reasoning given above, or even without that reasoning, that if the lever in Fig. 31 is in balance, the force at *e* multiplied by the length of the arm *eo* will equal the force at *a* by the length of the arm *ao*. Returning to Fig. 30, this is



Figs. 32 and 33. Analysis of Principles Involved in Design Fig. 29.

equivalent to saying that $F \times eo = P \times ao$. To locate point *o* conveniently, erect at point *a* the perpendicular to the direction of force *P*, and continue *cd* until it intersects the perpendicular at *o*.

Transposing our formula, we now have

$$P = \frac{F \times eo}{ao}$$

When, as is often the case, the two arms of the toggle are of equal length, then *eo* will be equal to one-half *ac*, or *ac* and *ao* will equal twice *bd*. Substituting *h* for *bd* in Fig. 28 and *l* for *ab*, we shall then have, for a toggle-joint with equal arms, like that in Fig. 28,

$$P = \frac{F \times l}{2h}$$

Referring to Fig. 29, this case is best solved by first neglecting the handle *F*, and assuming the toggle-joint to be composed of the linkage *afc* as in Fig. 32. Here the force *f* acts at right angles to the arm *cd*. It rotates about the center *o* with a radius *fo*, and *P* rotates about *o* with a radius *ao*, as indicated in Fig. 33. Therefore, $f \times do = P \times ao$, or,

$$P = \frac{F \times do}{ao}$$

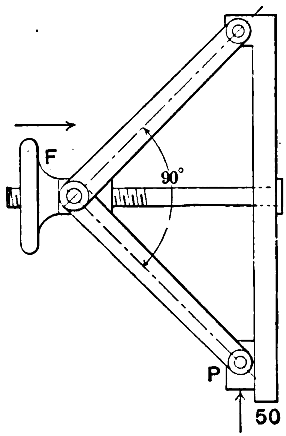


Fig. 34. Toggle-Joint Design where Pressure is Exerted by Handwheel and Screw.

With equal arms, $do = dc = r$ in Fig. 29, and $ao = 2 \times h$. Hence, for equal arms, as in Fig. 29,

$$P = \frac{f \times r}{2h}$$

Now, taking into account the increased leverage afforded by the handle, with the force acting at *F*, we have $f \times r = F \times R$. Or,

$$f = \frac{F \times R}{r}$$

Combining this with the equation above, the effect of force *F* upon *P* is found to be,

$$P = \frac{F \times R}{2h}$$

Double Toggle-Joint.

In most presses in which a screw and toggle-joint are used, the latter is usually made in the form of a double toggle-joint, as shown in Figs. 35 and 36. The question is often asked whether such an arrangement is twice as powerful as a single joint, and to make this point clear let us first take up the joint and screw of Fig. 34.

Assume for illustration that the two arms are of equal length and at an angle of 90 degrees with each other, and that a force *F* of 100 pounds is applied by means of the hand-wheel. With the proportions and position assumed, it is evident that a small movement of the joint at *F* will produce twice as much movement at *P*, and consequently only half as much resistance, or 50 pounds, can be overcome at *P*.

In Fig. 35 the same proportions and positions of the parts are used as in Fig. 34. While the action of these different joints can easily be demonstrated, whatever the proportions, it is simpler to take the positions shown, because the relative movements of the parts can be seen at a glance. In Fig. 35 a right- and left-hand screw is used of the

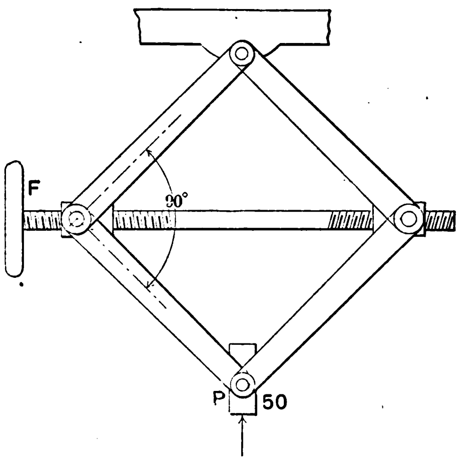


Fig. 35. Double Toggle-Joint.

same pitch as in Fig. 34, and one turn of the hand-wheel will therefore advance each one of the toggle joints and also the point *P* just as far as the corresponding parts were advanced in Fig. 34, and no farther. It will, therefore, take just the same pull on the hand-wheel to overcome 50 pounds at *P* as in Fig. 34, but as each joint takes half the strain, there will be only 50 pounds tension in the rod between the joints instead of 100 pounds as before.

In Fig. 36 the case is somewhat different. Here the rod is threaded at one end only, of the same pitch as before, and the hand-wheel screws on the threaded part, drawing the two parts of the joint together. One turn of the hand-wheel will advance the hand-wheel itself a distance, relative to the screw, equal to the pitch. Each side of the toggle-joint will be advanced a distance equal to half the pitch, and point *P* will be moved twice this amount, or a distance equal to the pitch, or the same distance that the hand-wheel moves along the screw. Hence, if the hand-wheel produces a force of 100 pounds, a resistance of 100 pounds can be overcome at *P*, or twice as much as in Fig. 35. The stress in the rod will, of course, be 100 pounds.

To summarize, one inch horizontal movement of the hand-wheel in

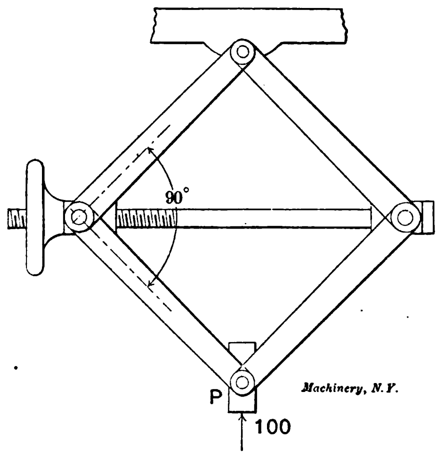


Fig. 36. Alternative Design of Double Toggle-Joint.

Fig. 34 will produce two inches movement at *P*; one inch movement in Fig. 35 will accomplish the same result, and hence the resistance overcome will be the same; but one inch movement of the wheel in Fig. 36 will produce the same movement, or one inch at *P*, and this form of toggle-joint has twice the power, but half the motion, of the other two.