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GIRDERS FOR ELECTRIC OVERHEAD CRANES

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CHAPTER I

PRELIMINARY CONSIDERATIONS

The primary consideration when designing an overhead crane lies in selecting the correct type of main girders, since not only the general efficiency of the crane is affected by this question, but, as the girders usually represent the bulk of the machines, the subsequent success in competition largely depends upon the selection of an economical type. Coincident with the question of type is that of the factor of safety, or the working stress. Practically all crane girders are now built of steel sections made by the open hearth acid process, and usually specified to possess a tensile strength of from 28 to 32 tons per square inch.

A great variation of opinion exists on the question of working stress;

TABLE I. TYPE OF GIRDERS USED FOR DIFFERENT LOADS AND SPANS

R S B = beam section.
S W P = single web girder with lateral bracing girders.
B G = ordinary box girders.
L G = lattice girders, preferably Warren type.

Load in Tons.	Span in Feet.					
	30	40	50	60	70	80
5	RSB	RSB	SWP	SWP	LG	LG
10	RSB	SWP	SWP	SWP	LG	LG
15	RSB	SWP	SWP	SWP	LG	LG
20	RSB	SWP	SWP	SWP	LG	LG
25	BG	BG	BG	BG	LG	LG
30	BG	BG	BG	BG	LG	LG
40	BG	BG	BG	BG	LG	LG
50	BG	BG	BG	BG	LG	LG
60	BG	BG	BG	BG	LG	LG
75	BG	BG	BG	BG	LG	LG
100	BG	BG	BG	BG	LG	LG

while many cranes are made having girders stressed to only four tons per square inch and even less, others will be found having above seven tons stress. Taking into consideration the fact that a general factor of safety of five seems to be usual and most desirable in crane work, it is necessary to limit the stress to one-fifth of the maximum breaking strength of the steel, which gives approximately 5½ tons per square inch.

There are four types of girders commonly used, *viz.*, box-plate girders, single web-plate girders, braced girders, and rolled beam girders. Each will be considered independently.

The simplest form of girder for spans up to about 40 feet is the

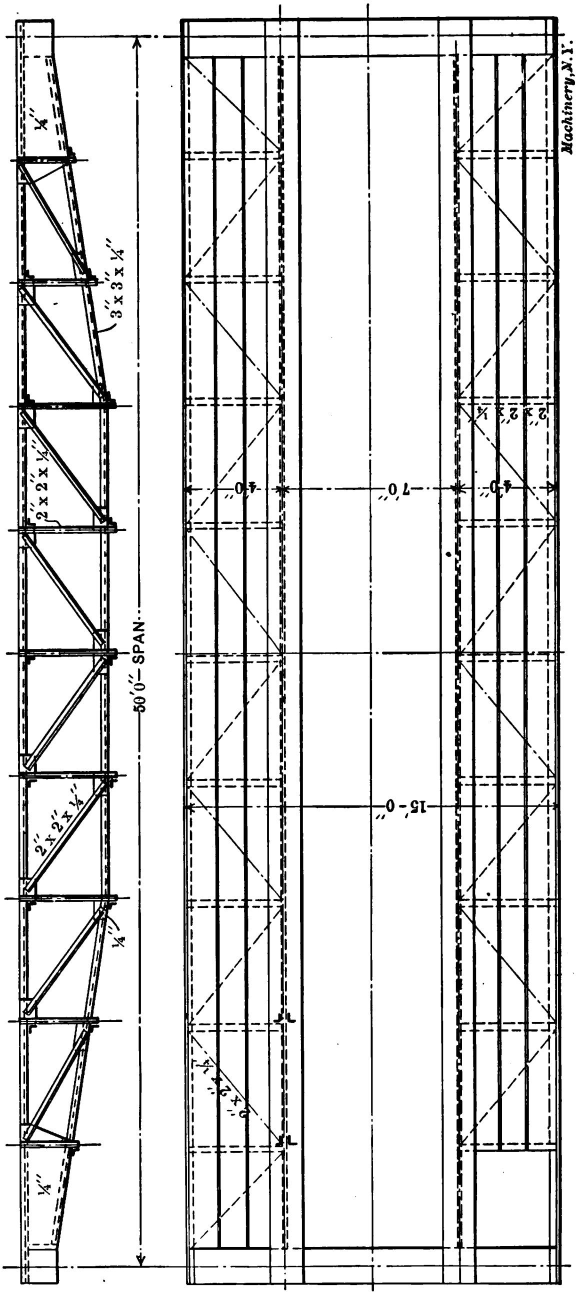


Fig. 1. Web-plate Girder with Braced Platform Girder, added to insure Sufficient Strength for Wide Spans

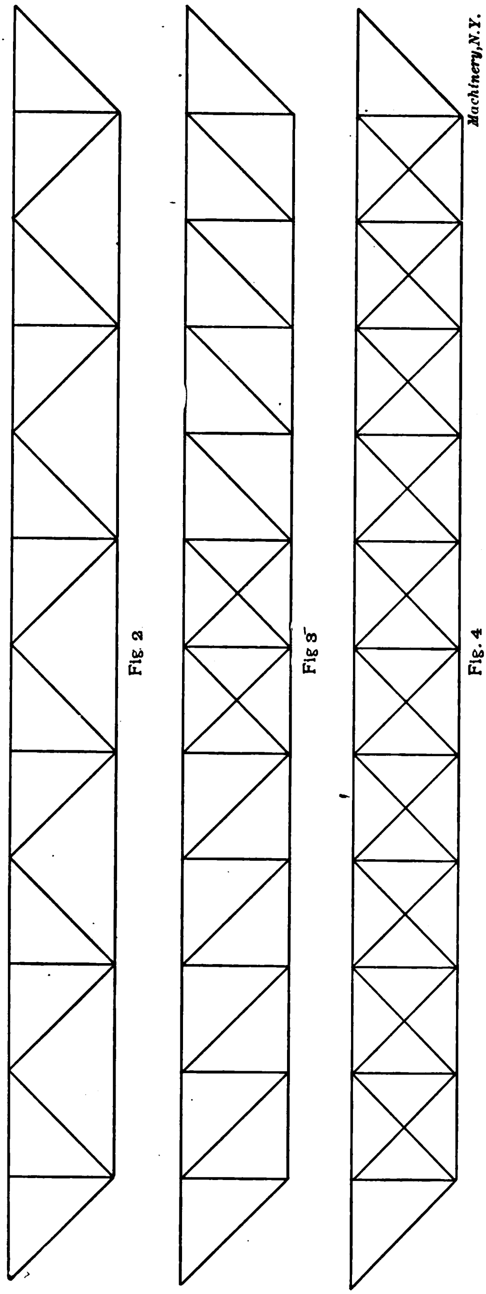
rolled steel joist, and for light loads it is undoubtedly the cheapest type. The effective range of span and load for beam girders will be seen from Table I, which has been compiled to show the type of girder generally considered suitable for a given span and load.

For cranes up to 15 tons with spans too great for joists but not more than 65 feet, the best form of girder is the single web type. Up to 40 feet span, providing the traveling speed is not very high, these girders can be made to carry themselves and a light platform, if the flanges are moderately wide, but above 40 feet span these girders will be found weak laterally, and a subsidiary braced platform girder of light construction should be added, the same being braced horizontally to the main girder, as shown in Fig. 1. Although this type has been found somewhat costly, there is no doubt it is stiffer laterally than a box girder, and the single web girder always possesses the advantage of permitting inspection and painting, thereby avoiding deterioration from corrosion, such as occasionally takes place in box girders. For cranes above 15 tons and for spans up to 65 feet, however, the box girder is considered the best and cheapest type that can be used. The sections and proportions required by such loads generally ensure the girder being stiff enough to carry the platforms and cross-shaft without causing any lateral distortion.

For cranes up to and including 4 tons, above 40 feet span and for all cranes from above 65 or 70 feet span, braced girders are the most economical. They are cheaper to make, and the reduced weight of the crane effects a saving in the power required for traveling, and may possibly reduce the scantlings of the runway girders. The most important question concerning braced girders lies in the adoption of the correct system of bracing, of which there are three designs in use, *viz.*, the Warren, the Linville and the latticed, as shown in Figs. 2, 3 and 4, respectively.

In point of cost, weight, and general convenience, the Warren type is the most suitable for the ordinary form of traveler. It has fewer joints and members, and gives satisfactory results in all respects. When the rolling load is large in proportion to the structural load, as is invariably the case with cranes, the Linville type requires so much counterbracing in the center, that it practically results in a latticed girder pure and simple, which, although frequently adopted, is heavier and more costly than the Warren type.

In order to treat the subject completely, it is proposed to consider the details of each type of girder, *i. e.*, solid, web, and latticed, independently, and although some parts may be a repetition, the arrangement will be more convenient to the designer.



Figs. 2 to 4. Warren, Linville and Latticed Girders

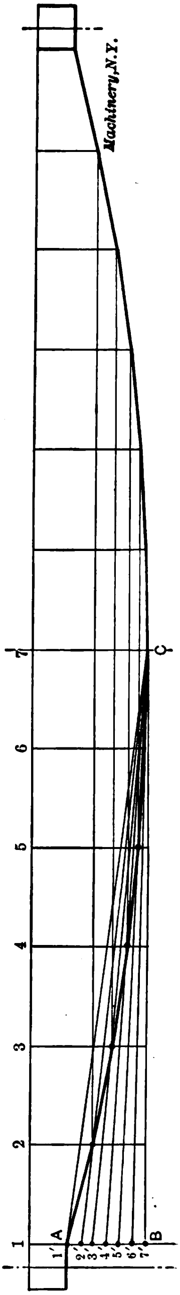


Fig. 5. Laying out a Girder of the A Type, Fig. 9

CHAPTER II

SINGLE WEB, BOX AND BEAM GIRDERS

The preliminary calculation concerning the strength of girders of the above types is principally that of finding the bending moment in the ordinary way. This quantity should include the forces due to the rolling load of the weight and crab combined, the structural load due to the weight of the girder itself and the platform and cross-shaft; and, if the driving motor is in the center, allowance must be made for this also.

The practice of making some allowance for impact forces is neglected by the majority of crane builders, while, on the other hand, when work of this nature is undertaken by bridge builders, one finds as much as 50 per cent being added to the actual rolling loads to

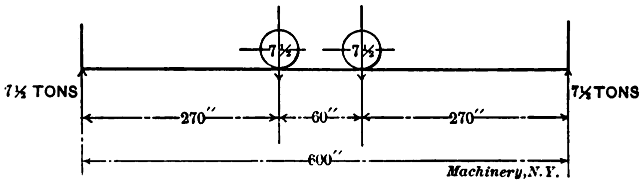


Fig. 6. Example for Calculating Bending Moment in Crane Girder

cover supposed impact forces. That some allowance should be made appears quite consistent, particularly in high speed cranes, but there seems to be no definite rule for this. Generally speaking, the working stresses of crane girders, say 5 to 6 tons per square inch, provide a margin for small additional impact forces. No allowance will be made in the following calculations for impact stress, but such allowance could easily be added if considered necessary in any particular case.

If the crab is symmetrically built, the rolling load may be considered as being divided equally over the four wheels. By making this allowance, it will be seen, as far as the rolling load is concerned, that the effective span of the girder is shortened by a distance equal to the center distance of the crab wheels. This can better be seen by reference to the bending moment diagram in Fig. 7, where it will be seen that the maximum bending moment from the rolling load occurs at *A A*, and is equal to the reaction at either support multiplied by the distance from that support to the center of the crab wheel. This quantity is also the bending moment at the center. To find the bending moment due to the structural load, it is usual to treat the latter as an evenly distributed load, where $M_b = \frac{WL}{8}$; similarly a traveling motor in the center of the span must be considered as a concentrated load, where $M_b = \frac{WL}{4}$.

A bending moment diagram might be drawn combining the whole of the above forces, but the same result can be found more quickly directly by figures, as shown in the following example.

Find the total bending moment of a 25-ton crane, 50-foot span, weight of crab 5 tons, centers of wheels 5 feet. Approximate weight of one girder and platform, etc., 5 tons. Traveling motor in the center: weight ¾ ton. (See Fig. 6.)

$$\text{Bending moment, rolling load} = 270 \times 7\tfrac{1}{2} = 2{,}025 \text{ inch-tons}$$

$$\text{Bending moment, structural load} = \frac{5 \times 600}{8} = 375 \text{ inch-tons}$$

$$\text{Bending moment, traveling motor} = \frac{3 \times 600}{4 \times 4} = 112.5 \text{ inch-tons}$$

$$\text{Total} = 2{,}512.5 \text{ inch-tons}$$

When the maximum bending moment has been found, the depth of the girder must be considered. Modern practice generally makes this

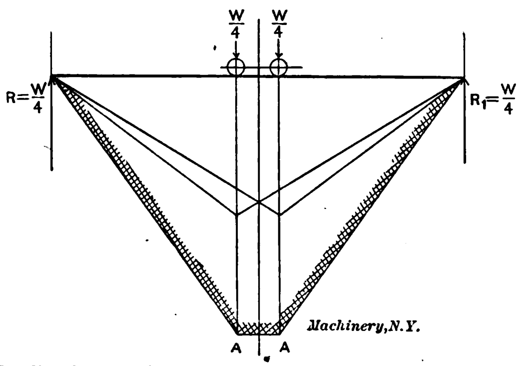


Fig. 7. Bending Moment Diagram, showing how the Effective Span of Girder is shortened by the Center Distance of the Crab Wheels

quantity the nearest even dimension equal to 1/15 or 1/16 of the span. In the case of heavy cranes, however, it is more economical to increase the depth of the girders than to make the flanges abnormally heavy, so that in the case of 100-ton overhead cranes of moderate span, the best proportion is about 1/12 of the span. An exception to the above rules occurs in the case of all short span cranes, where the depth becomes a matter of convenience.

In determining the section of the girder it has, until quite recently, been common practice to totally ignore the value of the webs to resist bending, the flange area alone being taken into consideration. This practice is open to some question; it seems that if the webs are stiffened in the usual way, they are of such value as to allow the whole section of the girder being taken into account, but in any case, the webs do not add much to the modulus. In order to calculate the strength, the moment of resistance or modulus of the section must be found; and this must be equal to the bending moment divided by the working stress F in the girder:

$$\frac{M_b}{F} = Z.$$

When finding the required modulus, the section has to be assumed, preferably by comparison, after the depth has been fixed. Allowance

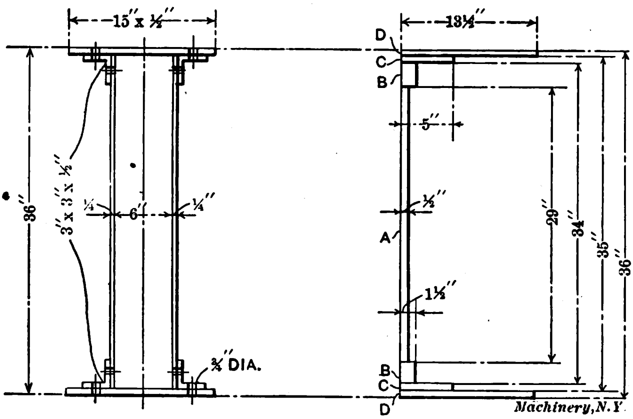


Fig. 8. Proposed Section of Girder, drawn to convenience its Calculation

should be made for the rivet holes in the flanges to be ⅛ inch larger than the size of rivet. It is generally found convenient to draw the proposed section as shown in Fig. 8.

Before the modulus itself can be found, the moment of inertia of the section must be calculated. This quantity, for rectangular symmetrical sections taken about the neutral axis, is equal to $1/12\ b\ h^3$, when b = breadth and h = height of rectangle.

Referring to Fig. 8, it will be understood that, owing to its irregular shape, each rectangle A, B, C, and D must be treated independently.

The total moment of inertia of the section can, therefore, be stated as follows:

$$I = \frac{(36^3 - 35^3) \times 13\frac{1}{2} + (35^3 - 34^3) \times 5 + (34^3 - 29^3) \times 1\frac{1}{2} + 29^3 \times \frac{1}{2}}{12}$$

= 8,600, approximately.

It is well known that the modulus or moment of resistance of a symmetrical section is equal to the moment of inertia divided by the distance from the neutral axis to the extreme outer edge of the section; consequently the modulus Z of the section $= \frac{8600}{18} = 478.$

Tables II and III give approximate values of various box and single web sections suitable for crane work.

Although box girders have been made with 3/16-inch web plates, it cannot be considered good practice to use plates less than ¼ inch thick, owing to the small margin allowed for deterioration from rust. For box girders on cranes up to and including 20 tons, ¼-inch webs have been repeatedly used and show no signs of buckling with the ordinary arrangement of stiffeners. The same applies to 5/16-inch

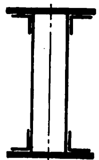

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TABLE II. GENERAL DIMENSIONS OF BOX GIRDERS FOR ELECTRIC OVERHEAD CRANES, AND MODULUS OF GIRDER SECTION

Depth.	Flange, Inches	Angles, Inches.	Webs, Inches.	Modulus.
2'-3'	12 x ⅜	3 x 3 x ⅜	¼	230
2-3	12 x ½	3 x 3 x ½	¼	292
2-6	12 x ⅜	3 x 3 x ⅜	¼	273
2-6	12 x ½	3 x 3 x ½	¼	308
2-9	12 x ⅜	3 x 3 x ⅜	¼	314
2-9	12 x ½	3 x 3 x ½	¼	380
3-0	15 x ⅜	3 x 3 x ⅜	¼	378
3-0	15 x ½	3½ x 3½ x ½	5/16	530
3-6	15 x ⅜	3½ x 3½ x ⅜	¼	500
3-6	15 x ½	3½ x 3½ x ½	5/16	653
4-0	18 x ½	3½ x 3½ x ⅜	5/16	797
4-0	18 x ⅝	3½ x 3½ x ½	5/16	940
4-6	18 x ½	3½ x 3½ x ½	5/16	998
4-6	18 x ⅝	4 x 4 x ½	5/16	1118
5-0	21 x ¾	4½ x 4½ x ¾	⅜	1809
5-0	21 x 1	4½ x 4½ x ¾	⅜	2057

webs for 30- to 50-ton cranes; above this size ⅜-inch plates are recommended.

In the case of single web girders, ¼-inch plates have been used for cranes up to 7 tons, increasing to 5/16 inch up to 20 tons, and ⅜ inch or more, above, as required. When the stiffeners are placed outside, T-sections are generally used, but for cranes up to 10 tons, 2½- to 3-inch angles are amply strong enough, and also convenient for fastening the brackets. The various formulas for pitch of stiffeners used in bridge practice do not seem to agree with the results found in crane girders, since they usually make the centers of stiffeners unnecessarily close. The average practice is to place the stiffeners about 5 feet apart, but it is better to reduce this dimension to 4 feet 6 inches for girders up to 2 feet 6 inches deep. The practice of placing channels or Z-bars inside, instead of using outside T-stiffeners, generally necessitates some hand riveting; otherwise this is a neat and strong type.

As shown in Fig. 9, there are two forms of fish bellied girders in use. The lower boom in both cases is polygonal, that in *A* having a side or flat for each division made by the stiffeners, while that in *B* has three straight cuts only, and is, therefore, cheaper; the general opinion is that both look equally well when erected. To find the varying depths in girder *A*, it is best to draw a parabola in the usual way, as shown in Fig. 5. The divisions 1 to 7 represent the centers of the stiffeners, and the distance *A B* must be divided into the same

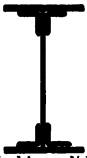

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TABLE III. GENERAL DIMENSIONS OF SINGLE WEB GIRDERS FOR ELECTRIC OVERHEAD CRANES, AND MODULUS OF GIRDER SECTION

Depth	Flange, Inches.	Angles, Inches.	Webs, Inches.	Modulus
2′-3″	12 x $\frac{3}{8}$	3 x 3 x $\frac{3}{8}$	$\frac{1}{4}$	202
2-3	12 x $\frac{1}{2}$	3 x 3 x $\frac{1}{2}$	$\frac{5}{16}$	271
2-6	12 x $\frac{3}{8}$	3 x 3 x $\frac{3}{8}$	$\frac{1}{4}$	238
2-6	12 x $\frac{1}{2}$	3 x 3 x $\frac{1}{2}$	$\frac{5}{16}$	282
2-9	12 x $\frac{3}{8}$	3 x 3 x $\frac{3}{8}$	$\frac{1}{4}$	272
2-9	12 x $\frac{1}{2}$	3 x 3 x $\frac{1}{2}$	$\frac{5}{16}$	315
3-0	15 x $\frac{3}{8}$	3 x 3 x $\frac{3}{8}$	$\frac{5}{16}$	340
3-0	15 x $\frac{1}{2}$	$3\frac{1}{2}$ x $3\frac{1}{2}$ x $\frac{1}{2}$	$\frac{5}{16}$	467
3-6	15 x $\frac{3}{8}$	$3\frac{1}{2}$ x $3\frac{1}{2}$ x $\frac{3}{8}$	$\frac{5}{16}$	448
3-6	15 x $\frac{1}{2}$	$3\frac{1}{2}$ x $3\frac{1}{2}$ x $\frac{1}{2}$	$\frac{3}{8}$	583
4-0	18 x $\frac{1}{2}$	$3\frac{1}{2}$ x $3\frac{1}{2}$ x $\frac{3}{8}$	$\frac{3}{8}$	705
4-0	18 x $\frac{5}{8}$	$3\frac{1}{2}$ x $3\frac{1}{2}$ x $\frac{1}{2}$	$\frac{3}{8}$	849
4-6	18 x $\frac{1}{2}$	$3\frac{1}{2}$ x $3\frac{1}{2}$ x $\frac{1}{2}$	$\frac{3}{8}$	881
4-6	18 x $\frac{5}{8}$	4 x 4 x $\frac{1}{2}$	$\frac{3}{8}$	1002
5-0	21 x $\frac{3}{4}$	$4\frac{1}{2}$ x $4\frac{1}{2}$ x $\frac{3}{4}$	$\frac{7}{16}$	1631
5-0	21 x 1	$4\frac{1}{2}$ x $4\frac{1}{2}$ x $\frac{3}{4}$	$\frac{1}{2}$	1917

number of equal divisions as that on each side of the center line. Drop verticals from 2, 3, and 4, etc., and join *C2′*, *C3′*, *C4′*, etc. The point of intersection between these two lines gives the depth of the girder at that point. Although the above method gives a well-shaped girder, it is not theoretically correct, since the parabola should be set off from the top flange. When this is done, however, the ends of the girder have to be parallel for a certain distance, and this causes extra trouble in planing the webs, without any economy in material. In the case of type *B*, the correct parabola should be laid out, and the flats laid off to suit.

For girders up to 40 feet span which are going to be shipped in one piece, it is possible and preferable to have the flange plates and angles in one length, and thereby avoid the joints. When, however, joints are necessary in the flanges, it is the practice of some makers to allow 25 per cent extra section for the rivets, and the flange joint plates and angles should be of the same section as the flanges themselves, at least.

Fcr cranes traveling at a moderately high speed, that is, anything over 200 feet per minute, the lateral stresses due to suddenly stopping the load require consideration. If, for example, the 25-ton crane

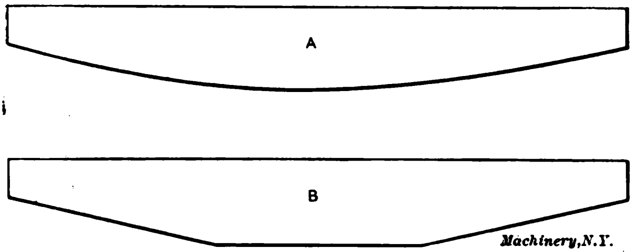


Fig. 9. Types of Fish-bellied Girders in Use

previously referred to travels at 300 feet per minute (5 feet per second) under full load, the momentum of the load and crab at full speed will be $\frac{W v^2}{2 g}$, or

$$\frac{30 \times 5^2}{64.4} = 11.6 \text{ foot-tons.}$$

It is difficult to assume what would be the least distance that the crane would travel before coming to rest after the current had been shut off and the brake had been applied, but a minimum of five feet has been found satisfactory, and under this condition the average horizontal force on the two girders would be $\frac{11.6}{5} = 2.3$ tons, or 1.15

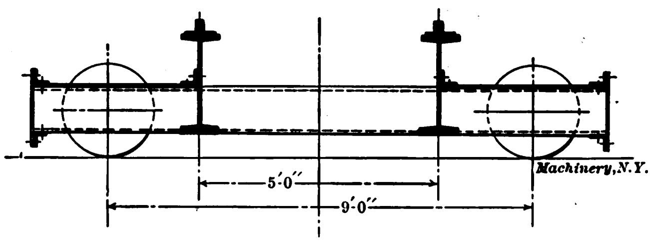


Fig. 10. Method of Stiffening Joist Girders

ton per girder. This would be the concentrated effort, but there is also the distributed effort due to the girder itself, which will be found by the above formulas to be about 0.95 ton per girder.

In order to avoid possible distortion from the concentrated load, one must assume that it is carried by the upper part of the girder only, that is, the flange plate, angles and about 18 inches of the webs. The bending moment from the concentrated load, in the above example, allowing for the spacing of the crab wheels, is $0.57 \times 270 = 154$ inch-tons.

The modulus of the upper flange taken horizontally is 45.7; therefore, the stress is $\frac{154}{45.7} = 3.4$ tons per square inch.

The distributed load is carried by the full depth of the girder, and, allowing for the horizontal modulus of the whole of the section, gives a stress equal to about 0.7 ton per square inch, bringing the total lateral stress up to $3.4 + 0.7 = 4.1$ tons per square inch.

The total lateral stress should not exceed four tons per square inch under the above conditions, since the ratio between flange width and

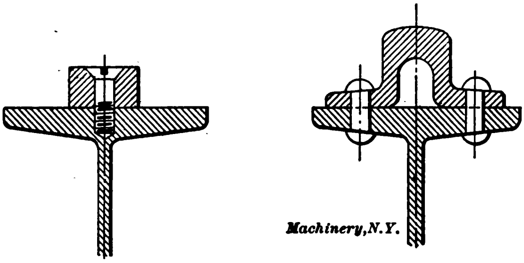


Fig. 11. Methods of Attaching the Rail to the Beam Girder

span, which is usually about 1 to 40, is large, and the girder is, therefore, more easily deflected.

By referring to Table I it will be seen that rolled steel beams can be used for cranes of varying capacity up to 40 feet span. The most economical and only really practicable method of using steel beams for moderate-speed and high-speed cranes is to attach a steel chequer-plate platform to both girders in such a manner as to provide the necessary lateral stiffness. Fig. 10 shows a typical form of this arrangement.

There are two common methods of fixing the rails on the beams, either by riveting on a bridge section rail, or screwing on a flat bar, as shown in Fig. 11. Both methods are equally satisfactory.

CHAPTER III

BRACED GIRDERS

Reference was made on page 5, Chapter I, to the three types of braced girders commonly in use, of which the Warren type is the most suitable for ordinary traveler work. The details of construction of this type differ somewhat according to the size and span of the crane, but the nature and magnitude of the stresses, which have to receive primary consideration, are found by the same methods in all cases.

In order to become thoroughly familiar with this type of girder, it is best to study its construction "anatomically." Fig. 12 shows the outline of the construction of a Warren girder consisting of a compression flange *AA'*, and a tension flange *ADD'*. These flanges are kept in position by the diagonal struts *BD*, *BE*, *CE*, *CF*, etc., which

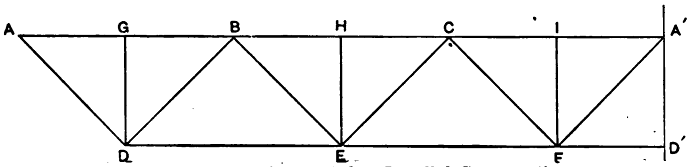


Fig. 12. Warren Type Girder, Parallel Construction

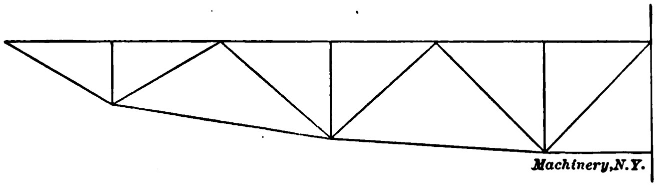


Fig. 13. Warren Type Girder, Fish-bellied Construction

are subjected alternately to tensile and compressive stresses as the position of the load varies. Apart from the compression in the top flanges, due to the maximum bending moment, there is an additional force, due to the bending moment in the top flange caused by the load of the crab wheels. In order to minimize this quantity the vertical members *GD*, *HE*, *IF*, etc., are added, thereby reducing the effective spans in the top flange by one-half. Girders of this type are made either parallel (Fig. 12), or fish-bellied, as shown in Fig. 13. The former have a satisfactory appearance and do not need to be made so heavy at the ends as the latter; they are cheaper to make, owing to the fact that the lower flange does not need to be bent, and one set of templets will in some cases suit all the diagonals. The fish-bellied form is, however, often preferred and is sometimes most convenient and will, therefore, be considered also.

The stresses in the various members may be found either by moments or by diagrams. The former method is somewhat tedious and not often adopted, except perhaps for finding the maximum flange stresses for comparison purposes, or to check the diagrams. If the stress diagrams are carefully drawn, the forces given will be sufficiently accurate for practical purposes. It is not within the scope of this treatise to prove the methods employed, since such can be done by referring to the larger works on girder construction, the principal object being to take an example of each particular type and show the quickest methods of obtaining those results which directly concern the designer.

In designing girders of this type there are three distinct processes, as it were, to be gone through: 1. Draw an outline of the proposed girder, fixing the depth and number of bays; 2. Find the stresses which occur with the load in various positions in each member; 3. Select suitable sections to withstand the various stresses found.

Let Fig. 15 represent the outline of a Warren girder for a 15-ton

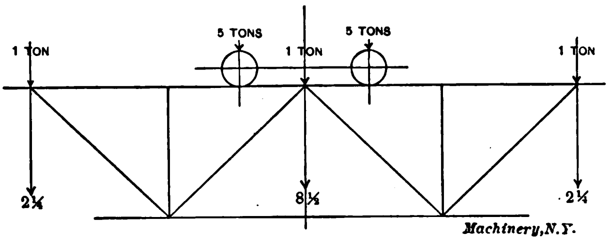


Fig. 14. Enlarged Portion of Girder in Fig. 15

crane of 72-foot span, weight of crab 5 tons, and centers of runners 6 feet. The most economical depth of these girders in relation to the span has been found to be about 1/12, so that in the present example the depth may be taken as 6 feet. It is preferable (but not essential) to divide the girder into an even number of bays on the top flange. No definite rule can, however, be given for the angle of the diagonals, which may be found to vary from 45 degrees in the case of light cranes to 60 degrees in those of heavier construction, but it is not economical to make the angle much *less* than 45 degrees. Other things being equal, the principal object is to have as few members as possible, and this result is generally gained by making the included angle of the diagonals as large as reasonable. There is, however, a limit to econcmy in this direction, which is reached when the span of the unsupported lengths of the top flange become so long as to require abnormally heavy sections to resist the combined bending moment from the crab wheels and compression in the girder itself. This quantity can only be settled by trial or comparison. In the example it will be seen that the top flange has been divided into six bays of 12-foot centers, the unsupported length being reduced to 6 feet by the insertion of the vertical struts. From the above figures it will be seen that the angle of the diagonals is 45 degrees, which represents a fair average for girders of this size.

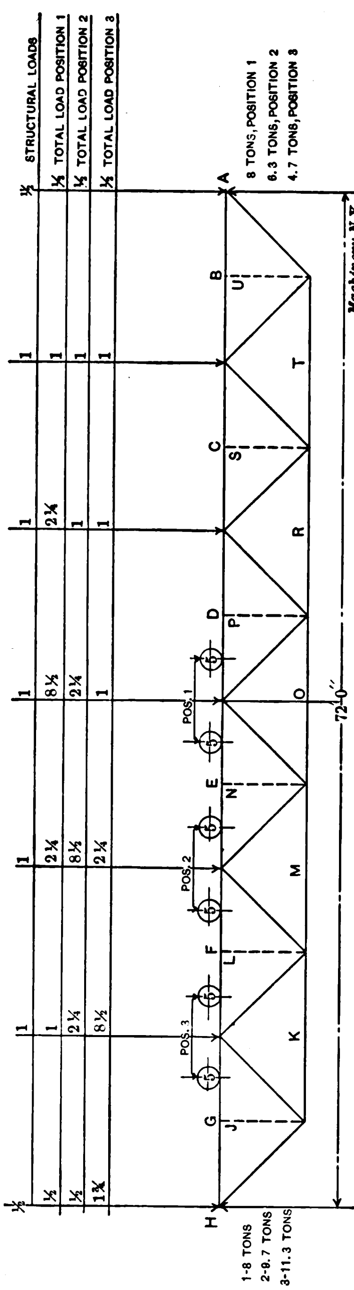


Fig. 15. Warren Crane Girder, 72-foot Span

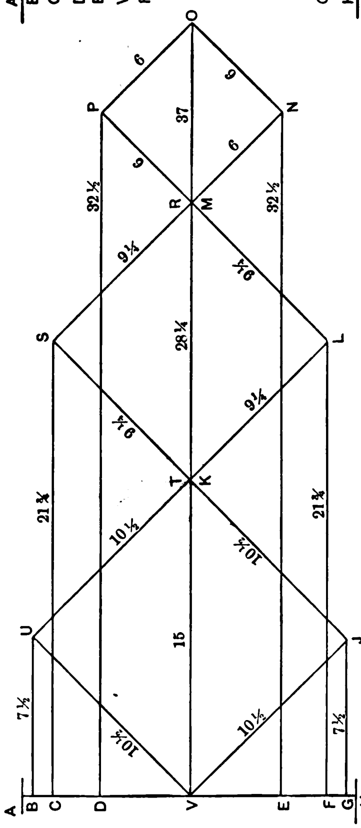


Fig. 16. Stress Diagram for Crane Girder in Fig. 15, Load in Center

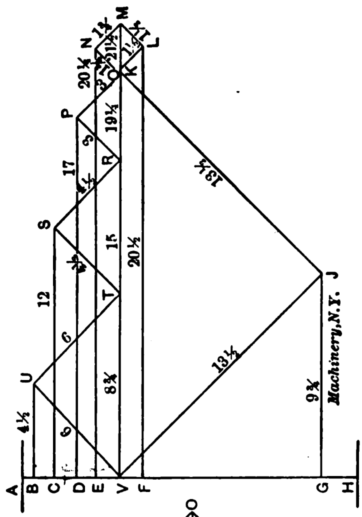


Fig. 17. Stress Diagram for Crane Girder in Fig. 15, Load in Position 3

The outline of the proposed girder is now complete, but before proceeding it is necessary to call attention to the fact that when possible the line drawn through the center of gravity of the various members should intersect at the same point, as in the case of the outline diagrams from which the stress diagrams are drawn. This precaution is necessary in order to minimize the secondary forces, which, although of no importance in small cranes, are sometimes considerable in heavy work; the girders and templets are also much more easily "set off" under these conditions. Before the stress diagram can be drawn, the loading of the girder must be considered, and it will be necessary to assume the weight of one girder together with its platform and cross-shaft.

In the present case this quantity may be taken at 6 tons, and since the girder will be of practically uniform construction, is equal to one ton per bay. The loading on the various bays from the crab wheels must be found by assuming the crab to be in the position shown in Fig. 14, where it would give the greatest reaction on any diagonals.

The effect of the above loads can be found from the skeleton diagram shown in Fig. 15. The first diagram is drawn for the loads as they occur when the crab is at the center of the girder. In reaching the loads from the crab wheels onto the apices of the various diagonals, no notice is taken of the intermediate vertical struts. This is not quite correct, since the vertical members convey part of the load direct to the bottom flange, but, as the inaccuracy is of little importance for small girders, it is simpler to eliminate the vertical struts from the diagrams altogether.

The first diagram for the load in the center is commenced by drawing the line AH, Fig. 16, which is the load or base line of the diagram. From A set off the distance AB equal to 0.5 ton at any convenient load scale, say, ¼ inch = 1 ton. This quantity represents the structural load acting directly over either abutment. Similarly, make BC = 1.0 ton, CD = 2.25 tons, DE = 8.5 tons, EF = 2.25 tons, FG = 1.0 ton and GH = 0.5 ton. Bisect AH at V, and draw a horizontal line through this point, as shown. The distance VA, or VH (8 tons) will be equal to the reactions at either point of support. The loads AB and GH are only set off on the load line to make the reactions complete, because, since they are directly over the abutments, they do not have any direct influence on the stresses of the structure itself.

From V draw a line VU parallel to the diagonal members in the skeleton diagram, producing same until it intersects a horizontal line drawn from B at U. Then the force in VU can be read off this line to the same scale as that adopted for the load line. Similarly draw UT, TS, SR, etc., parallel to their respective members in the skeleton diagram.

When the diagram has been drawn on one side, the opposite side may be duplicated if required, since the loading is symmetrical. When the diagram is completed, the letters on the force diagram correspond to those given on the skeleton. Thus, the maximum tension in the lower flange occurs at the center and is equal to VO, while the maxi-

TABLE IV. SAFE LOADS IN TONS FOR ANGLE IRONS WHEN USED AS STRUTS

F = fixed ends R = round ends Factor of safety = 5

Size of Angle, Inches	Safe Loads in Tons for Different Length of Strut													
	4 Feet		5 Feet		6 Feet		7 Feet		8 Feet		9 Feet		10 Feet	
	F	R	F	R	F	R	F	R	F	R	F	R	F	R
$6 \times 6 \times \frac{5}{8}$	32.3	19.4	31.4	18.8	30.2	18.1	28.9	17.3	27.4	16.4	25.9	15.5	24.1	14.5
$6 \times 6 \times \frac{3}{4}$	38.3	23.0	37.2	22.3	35.8	21.5	34.2	20.5	32.5	19.5	30.6	18.4	28.5	17.1
$5 \times 5 \times \frac{1}{2}$	21.1	12.7	20.2	12.1	19.1	11.5	17.9	10.7	16.5	9.9	15.1	9.1	13.6	8.2
$5 \times 5 \times \frac{5}{8}$	26.0	15.6	24.9	14.9	23.5	14.1	22.1	13.3	20.4	12.2	18.6	11.2	16.8	10.1
$4\frac{1}{2} \times 4\frac{1}{2} \times \frac{1}{2}$	18.5	11.1	17.4	10.4	16.3	9.8	15.0	9.0	13.4	8.0	12.0	7.2	10.6	6.4
$4\frac{1}{2} \times 4\frac{1}{2} \times \frac{5}{8}$	22.7	13.6	21.5	12.9	20.1	12.1	18.4	11.1	16.6	10.0	14.7	8.8	13.1	7.9
$4 \times 4 \times \frac{1}{2}$	15.8	9.5	14.7	8.8	13.4	8.0	12.0	7.2	10.6	6.4	9.2	5.5	8.1	4.8
$4 \times 4 \times \frac{5}{8}$	19.4	11.6	18.1	10.9	16.6	10.0	14.7	8.2	13.0	7.8	11.3	6.8	9.9	5.9
$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$	10.2	6.1	9.2	5.5	8.2	4.9	7.1	4.3	6.1	3.7	5.2	3.1	......	......
$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$	13.3	8.0	12.1	7.3	10.6	6.4	9.2	5.5	7.9	4.7	6.8	4.1	......	......
$3 \times 3 \times \frac{5}{16}$	6.9	4.1	6.0	3.6	5.1	3.1	4.3	2.6	......	......	......	......	......	......
$3 \times 3 \times \frac{3}{8}$	8.1	4.9	7.0	4.2	6.0	3.6	5.0	3.0	......	......	......	......	......	......
$3 \times 3 \times \frac{1}{2}$	10.6	6.4	9.2	5.5	7.8	4.7	6.5	3.9	......	......	......	......	......	......
$2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$	4.1	2.5	3.4	2.1	2.6	1.6	......	......	......	......	......	......	......	......
$2\frac{1}{2} \times 2\frac{1}{2} \times \frac{5}{16}$	5.0	3.0	4.1	2.5	3.3	2.0	......	......	......	......	......	......	......	......
$2\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$	5.9	3.5	4.9	2.9	3.9	2.3	......	......	......	......	......	......	......	......

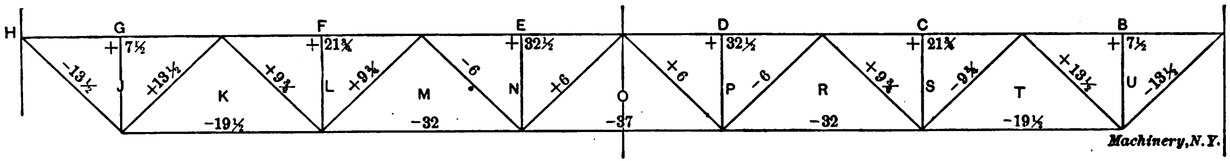


Fig. 18. Stresses in Crane Girder in Fig. 15, as determined by Stress Diagrams

mum force in the diagonals occurs at *U* or *J* and equals *VU*, *VJ*, *UT*, and *JK*. The various stresses occurring in each member have been scaled off and written in the skeleton diagram for reference.

In the case of small cranes, it will generally be found most economical in practice to make all the diagonals of one, or, at the most, two sections, since the difference between the maximum and minimum loads is not great, and, when this is done, it is only necessary to draw one diagram in the center as shown in order to get the maximum flange stresses, and another with the load in a position near the end as shown in Fig. 17 to obtain the maximum diagonal stresses. When, however, the girders for cranes above 20 tons are being designed, it is better practice to find the maximum stresses occurring in each member and proportion the sections to suit; and this will be done in the present case, in order to show the method used.

In drawing the diagram, Fig. 19, for position 2, Fig. 15, the load line will be drawn as in position 1, with the order of loading, in accordance with the re-distribution of loads as in the skeleton dia-

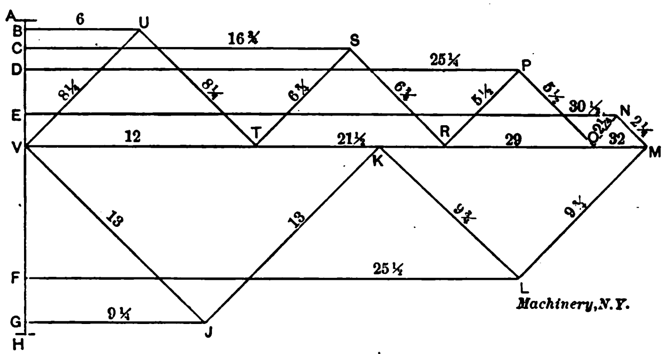


Fig. 19. Stress Diagram for Crane Girder in Fig. 15. Load in Position 2

gram, Fig. 15. The reactions at the abutments from both the rolling and structural loads must be found in the usual way, and set off on the load line as shown. From *V* draw a horizontal line and then draw the diagonal stress lines as in the previous example. If the diagram is correctly drawn, the last force line will join at the starting point *V*, thereby checking itself, but it should be borne in mind that unless the position of *V* is exactly to scale, the diagram will not close and cannot be considered correct.

The stresses are found when the load is in position 3, as shown by the diagram, Fig. 17; this process could be repeated for any number of bays.

Since there is no definite relation between the centers of the crab wheels and the pitch of the diagonals, it is difficult to say beforehand whether the loads in the positions already considered give the maximum stresses on the diagonals; generally speaking, they do, but in heavy cranes it is better to make certain by drawing another set of diagrams for other positions. Having found the stresses arising from the loads in the various positions, it is convenient to record the highest stress found on any diagram for any particular member, on another skeleton diagram, as shown in Fig. 18.

Under certain conditions, it might have been preferable to make the above girder fish-bellied in form, as previously referred to, and, in order to make this treatise complete, the diagrams shown in Figs. 20 and 21 have been drawn to suit the altered design. The diagrams

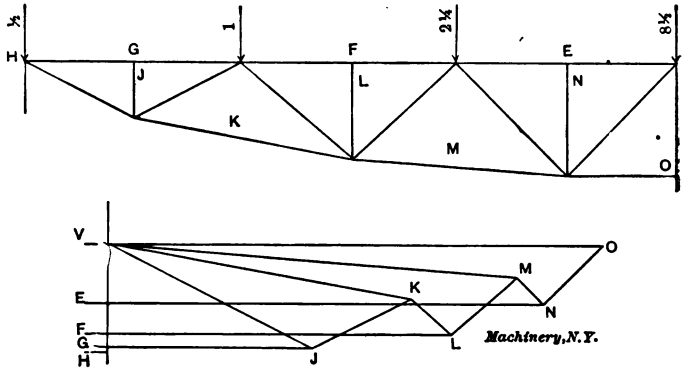


Fig. 20. Stress Diagram of Fish-bellied Girders

are constructed in precisely the same manner as in the previous example, the only exception being that the force lines for the lower flange must be drawn parallel to their corresponding members. By comparing these diagrams with those for the parallel girders, it will be seen how unsuitable, comparatively speaking, the fish-bellied girder is, for at the point where the diagonals have the maximum stress,

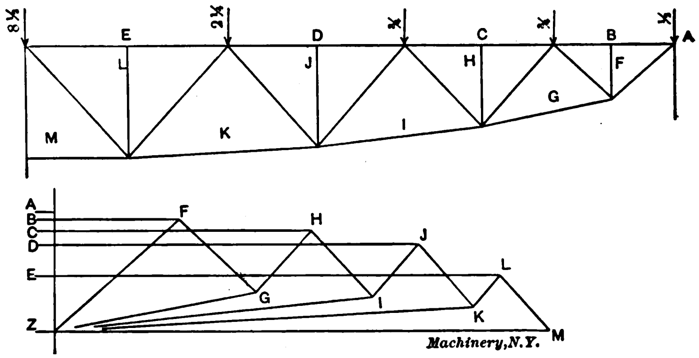


Fig. 21. Length of Bays at End of Fish-bellied Girders Shortened to equalize Stresses in Struts

they are inclined at the smallest angle, and consequently receive the greatest possible stress which, generally speaking, is so heavy that sufficient rivets cannot be put into a suitable strut, and web plates, or very large gusset plates, must be used. The stresses in the end struts may be minimized by shortening the length of the bays at the ends, as shown in Fig. 21.

The corners or sets on the lower flange do not form a parabolic line as may be the case in a plate girder, but are more a question of practical judgment, the main object being to make the end bays as deep as possible. In details of construction this girder is practically the same as the parallel type.

When the stress sheet is finished, the designer may pass on to the first operation of selecting suitable sections to withstand the strains. There are four different types of girder sections commonly in use, as shown in Figs. 22 to 25. The type shown in Fig. 22 is suitable for cranes up to 3 tons capacity, and over 40 feet span, when no platforms are specified, as in the case of small cranes worked from the floor level. By placing the channel forming the top flange horizontal, as shown, the girder receives the necessary lateral stiffness in

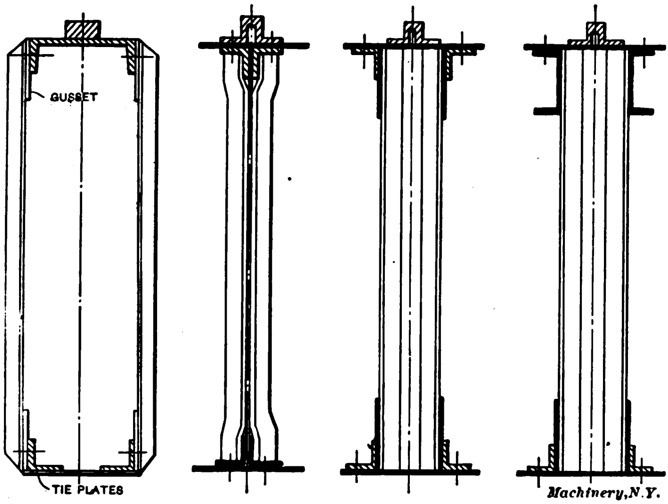


Fig. 22 Fig. 23 Fig. 24 Fig. 25

Types of Girder Sections in Common Use

the right place, and, since the wheel pressures are light, there is always sufficient strength to resist bending in the other direction.

When platforms are required, and for all cranes above 3 tons and up to 20 tons, of more than 65 feet span, the construction shown in Fig. 23 has been found suitable. Lateral strength is given to this type by the addition of braced platform girders. For cranes of 25 tons capacity and upwards, and over 65 feet span, the box-latticed types shown in Figs. 24 and 25 are most suitable. The channel construction shown in the top flange of Fig. 25 generally becomes necessary for cranes of 40 tons capacity and upwards. When these types are used, it is cheaper to make the girders strong enough laterally, and attach ordinary platform brackets, as shown, althought this arrangement is not suitable for high speeds. The type of girder used in the example given will be the same as shown in Fig. 23.

The scantlings of the bottom flange can be determined without difficulty, since no lateral stiffness has to be provided for in the girder itself. The width of flanges becomes principally a question of convenience, adding a rail and web to suit the top flange, as shown in Fig. 26. It will be seen that in this case the rail has been riveted on continuously in such a manner that it can be regarded as a useful part of the section.

As a preliminary guide in assuming a suitable section, the designer may select such sizes as will give an area which will correspond to not more than from 2 to 3 tons per square inch. The depth of the web varies according to the load, but is generally proportioned to suit the riveting of the diagonals. When the section has been assumed in this manner, the next step is to find the modulus in the usual way.

Draw the section full size, and from the vertical line *AB*, drawn parallel to the center of the section, set off the net section, as shown in Fig. 26. Determine the center of gravity in each piece, together with its area. When these particulars have been fixed, the center of gravity of the whole mass may be found in the ordinary way, by multiplying the area of each piece by the distance from *AC* to its center of gravity, and dividing the sum of these products by the total area of the section, thus:

Section 1.	$2.2500 \times 0.5625 =$	1.2656
2.	$1.1250 \times 1.6875 =$	1.8984
3.	$1.2187 \times 2.4375 =$	2.9707
4.	$3.8437 \times 2.8125 =$	10.8105
5.	$1.5937 \times 3.1875 =$	5.0800
6.	$1.9687 \times 4.6875 =$	9.2285
7.	$3.0000 \times 7.0000 =$	21.0000

Complete area = 15 square inches; sum of moments = 52.2527. Then

$$\frac{52.2527}{15} = 3.48,$$

or about $3\frac{1}{2}$ inches from *A* to the center of gravity and neutral axis of the section.

The next step is to find the moment of inertia of each half as divided by the neutral axis.

The moment of inertia of a section taken about its base is equivalent to $1/3\ b\,h^3$; therefore the moment of inertia of the section shown will be found as follows:

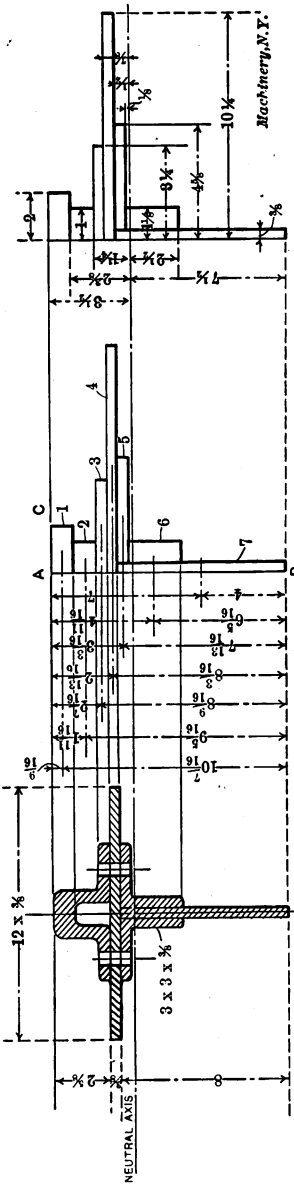
$$1/3\ [\ (3.5^3 - 2.375^3) \times 2 + (2.375^3 - 1.25^3) \times 1 + (1.25^3 - 0.875^3) \times 3.25 + (0.875^3 - 0.5^3) \times 10.25 + (0.5^3 - 0.125^3) \times 4.625 + (\tfrac{1}{8}^3 \times 1.125)\ ] = \frac{80.7}{3} = 26.9 = \text{moment of inertia of the upper half.}$$

The moment of inertia of the lower part can be found in a similar manner, as follows:

$1/3\ [\ (7.5^3 \times 0.375) + (2.5^3 \times 0.75)\ ] = 56.6 =$ moment of inertia of lower half.

Total moment of inertia of section $= 26.9 + 56.6 = 83.5$.

This quantity, divided by the distance from the neutral axis to the upper or lower outer edge of the section,,will give the compression and tension moduli, respectively, thus:



Figs. 26 and 27. Lay-out for Calculating Section Modulus of Girder Section

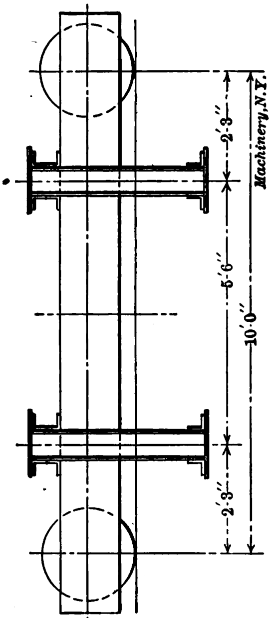


Fig. 29. Arrangement of End Carriages of Crane shown in Outline in Fig. 28

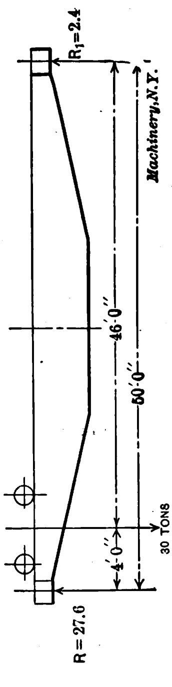


Fig. 28. Diagram for Calculating Wheel Pressure on End Carriages

$$\frac{83.5}{3.5} = 23.8 = \text{compression modulus.}$$

$$\frac{83.5}{7.5} = 11.13 = \text{tension modulus.}$$

It is also necessary to know the maximum bending moment in the unsupported part of the top member, which, in the present case, occurs when one wheel of the crab is in the center of the bay. It is difficult to say how much benefit is due to the fact that the unsupported parts of the top flange form a more or less continuous girder. The results due to taking the bending moment as being equal to $\frac{WL}{6}$ give satisfaction; the bending moment, in this case, is therefore equal to

$$\frac{5 \times 72}{6} = 60 \text{ inch-tons.}$$

The stress in the section from this load alone will be:

$$\frac{60}{23.8} = 2.5 \text{ tons per square inch, compression.}$$

$$\frac{60}{11.13} = 5.4 \text{ tons per square inch, tension.}$$

The maximum compression in the members from the load is equal to about 2.2 tons per square inch, and to this quantity the compression found above must be added, making the total maximum compression $= 2.2 + 2.5 = 4.7$ tons per square inch.

The above method is approximate and only suitable for the comparatively small girders used in crane work, and is not directly applicable to large bridge girders.

The stresses from bending alone should be kept as low as possible, in order to minimize the possibilities of deflection, which detracts from the value of the member as a strut. The safe working stress of this member, taken as a strut, pure and simple, and fixed in the plane of the joints, is generally taken as equal to about 4½ tons per square inch for a section having a radius of gyration of from 1/30 to 1/40 of the unsupported length. In fixing the size of the struts, some consideration should be given to the practical economy effected by using as few sections as possible, so that, although in the case of heavy cranes it is advisable to select sections to suit the varying stresses, it is better to select say, two sections equal to the maximum stresses and use these throughout for girders of moderate size.

Angle sections are generally used both for diagonal and vertical struts, and in order to avoid continual calculations, the values given in Table IV will be found useful. The safe loads given in this table are calculated by Gordon's formula. The factor of safety is five.

Generally speaking it is safe to consider that the struts have the ends fixed, in the plane of the rivets, but free in the opposite direction. Matters may be more nearly equalized between these two conditions if a small tie plate is used to tie the two sections forming one strut together. When, however, the work throughout is light, it is safer to take all struts as having free ends, and thereby avoid the possibility of flexure in different directions. A sufficient number of rivets should be allowed for at the joints to limit the stress to 5 tons per square inch in shear, and 8¾ tons per square inch of bearing.

Generally speaking, the last bay should be plated in, in order to stiffen the end joint and provide sufficient section to meet the shearing stresses.

It is seldom necessary to make any extra provision for shearing stresses at the various flange joints for small and medium sized girders, but for large girders these stresses should always be calculated and checked.

CHAPTER IV

END CARRIAGES, WHEELS, AXLES AND BEARINGS

In order to calculate the strength of the end carriages, it is only necessary to find the maximum wheel pressure occurring when the crab and load are at the extreme end of the span. Take, for example, a 25-ton crane with a 5-ton crab, where the minimum distance from center of load to center of track is 4 feet, as shown in Fig. 28.

The maximum reaction R from the traveling load will be $\frac{46 \times 30}{50}$ $= 27.6$ tons, or 13.8 tons per wheel. Added to this is the wheel pres-

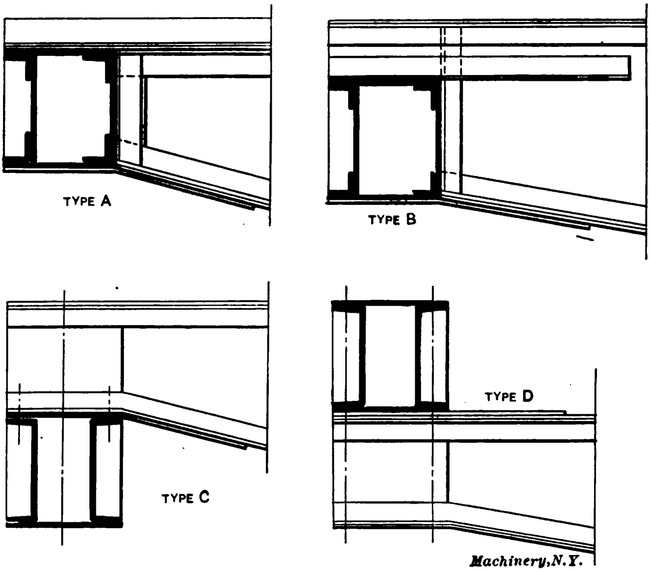


Fig. 30. Various Methods in Use for Attaching the Main Girders to the End Carriages

sure due to the weight of girders, end carriages, etc., which is practically divided over the four wheels, and in the present case would be about 14 tons, or $3\frac{1}{2}$ tons per wheel, making the total maximum wheel pressure $13.8 + 3.5 = 17.3$ tons.

It is also necessary to know the center distances of the main girders and of the traveling wheels. The former distance is decided by the requirements of the crab, which depend to a large extent on the height of lift, and consequent amount of rope which has to be coiled on the barrel. There is no economy in cramping this dimension, and it is always a convenience to be able to take a moderately long lift without altering the standard patterns. The centers of the traveling wheels should not be less than 1/5 of the span, for electric travelers, particularly those traveling at high speeds, this proportion having been found the most suitable to resist cross twisting.

In the example just given above, the end carriage would be arranged as shown in Fig. 29. It will readily be seen that the maximum bending moment occurs at the center of the main girders, where it is equal to the wheel pressure multiplied by the distance from the center of the traveling wheel to the center of the girders $= 17.3 \times 27 = 467.1$ inch-tons; stressing the material up to 5½ tons per square inch, giving a maximum factor of safety of 5, the modulus required would be

$$\frac{467.1}{5.5} = 85.$$

Rolled steel channels can be used for cranes up to about 30 tons when reinforced with flange plates. When channels are not convenient,

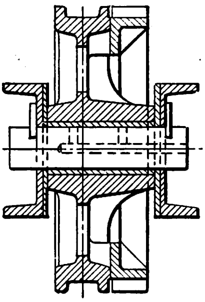


Fig. 31. Traveling Wheel with Bushing running loose on Pin fixed to the Framing

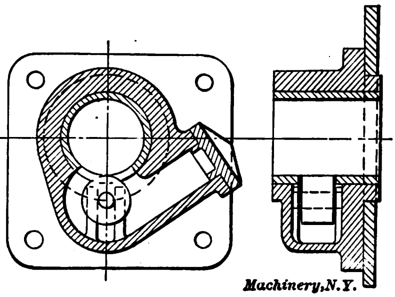


Fig. 32. Self-lubricating Axle Boxes for Crane Traveling Wheels

plate and angle sections are adopted, consisting of web plates and angles. Some makers substitute bracing for the webs between the girders, but it is doubtful whether any economy is effected by so doing.

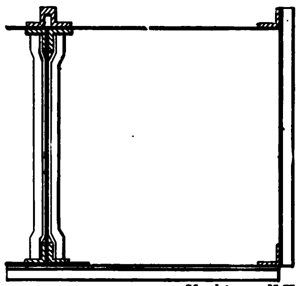
The method of attaching the main girders is a detail of some importance, the various systems commonly in use being shown in Fig. 30. Type A forms a very neat connection and is particularly suited for cranes up to about 15 tons. The rail and top flange plate should be run across the top of the carriage as shown, and a substantial gusset ought to be fixed to the lower flange to stiffen the joint. The two sides of the end carriage are tied together by heavy diaphragms, as shown, and this arrangement to some extent ensures the outer member of the carriage taking some of the weight. The tendency for the inner members to take the full load is, however, one reason why this construction is often avoided for heavy cranes, in favor of the method shown as type B, which is the strongest and most satisfactory form for fixing the girders. It will generally be found in this design that the shallowest construction that can be employed at the ends of the girder is sufficiently strong to carry the whole load, even if it were concentrated at the extreme end of the girder, and this fact alone ensures both members of the end carriages getting an equal load, and at the same time does not encroach too much on the head room, for which reason the construction shown in type C is seldom adopted. The attachment illustrated in type D is strong and inexpensive, but occasionally has the fault of limiting the end travel of the crab.

Wheels, Axles and Bearings

The most convenient sizes of traveling wheels taken from practice seem to be

18 inches diameter for cranes up to 4 tons.
21 inches diameter for cranes up to 7 tons.
24 inches diameter for cranes up to 25 tons.
30 inches diameter for cranes up to 25 tons and above.

The 18- and 21-inch wheels should be made of cast steel, in order to



Machinery, N.Y.

Fig. 33. Section of Girder suitable for Light Cranes of Long Spans

withstand the wear on the tread. The 24-inch wheels are sometimes made of cast steel also, but general practice inclines to steel-tired wheels for this and larger sizes.

The strains due to shrinking both in the tire and center appear to be very great, and it is for this reason that the centers are made heavy to avoid cracking. A very convenient position for the driving spur is to bolt it directly to the traveling wheel, and is a better arrangement than casting it on the center. Some makers key the wheel onto the axle, outside the carriage, so that it can be easily removed; this arrangement, however, rather interferes with a neat connection for the platform at the ends. When the spur is attached directly to the wheel, allowance should be made for the consequent unequal loading on the sides of the end carriages.

Some makers bush the traveling wheels and run them loose on pins fixed to the framing, as shown in Fig. 31. This method allows the wheels to be easily withdrawn for repairs, and simplifies the lubrication, which, however, should consist of grease instead of oil, owing to the fact that the latter is apt to run out when the bushes have worn slightly oval.

The diameter of the axles must be sufficient to resist bending from a distributed load, and at the same time have enough bearing area to keep the pressure below 1,000 pounds per square inch. Large cranes, and, in the case of some makers, all cranes, are fitted with self-lubricating axle boxes, of the type shown in Fig. 32. The roller shown in this illustration is of cork, with a hardwood spindle. The slot in the casting requires to be well made, or occasionally the roller may become jammed and ineffective.

Platforms, Brackets, and Traveling Gear

The design and arrangement of the platforms and brackets either improves or detracts from the appearance of a crane, to a considerable extent. A variety of opinions seems to exist on the question of platforms, and while one engineer requires platforms on both sides of a one-ton traveler, another may be satisfied with one platform for a 25-ton crane. In the case of light high-speed travelers, it has been found both convenient and economical to have only a small platform over the cage, to stand on when examining and oiling the crab, the longitudinal

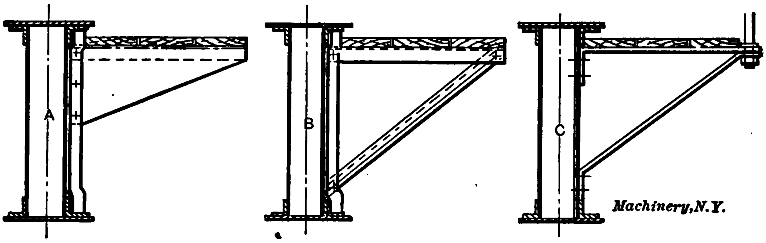


Fig. 34. Types of Platform Brackets

shaft and axles being oiled from below, or from a platform fixed to the shop end wall.

If, however, a platform must be fixed to cranes having light lattice or single web plate girders with over forty feet span, it will be found that the section necessary to carry the load itself is seldom sufficient to withstand the torsional stress due to the platform without deflecting laterally. The best means of overcoming this difficulty, which practically applies to all sizes of cranes with single web girders, is to attach a light subsidiary lattice girder to the main girder by horizontal bracing, as shown in Fig. 1, Chapter I. The section employed can be very light. The diagonals, or tension bars, are formed of light angles in preference to flat bars, to ensure rigidity and freedom from vibration; since they are strong enough to take compression, they obviate the necessity of counterbracing the central bays.

A section of a special girder is shown in Fig. 33. This has been found eminently suitable for light cranes of long span, traveling at a high speed. In this design it will be seen that the top flange is really a chequer-plate platform, one side being riveted to the web and the other to the lattice bracing, forming the front, the lower side also being braced. The increased width of the top flange gives a lateral stiffness sufficient to withstand the most severe strains.

The various forms of platform brackets in use are shown in Fig. 34. The advantage of each type of bracket is a matter of opinion, but it is generally considered, however, that type *A* is the simplest and best for plate girders, and more in accordance with the type of bracket which is employed to carry the traveling motor. Type *B* is formed of flat bars about 3 × ¾ inch and makes a very suitable bracket for box lattice girders, where it can be bolted onto the vertical members of the structure.

Timber is generally employed as a platform, on account of its lightness and cheapness; it also gives the best foothold. When cranes are working in the presence of fire, ¼-inch steel chequer plates are generally used in place of timber, and carried on steel brackets in the same manner.

A type of platform much employed on the European continent is made up of perforated steel screen plate, usually about 3/16 inch thick, which possesses the advantage of lightness and good appearance, while the cost is only very slightly in advance of rolled chequered plates. This type is, however, most suitable for those cranes having lattice braced platform girders, which give the close support a plate of this description requires.

The traveling motor ought, theoretically, to be placed in the center of the drive. Cranes up to 50 feet span are, however, sometimes driven with the motor fixed at one end, but since it adds very little to the cost, it is best practice to place the motor in the center for all cranes over 30 feet span.

Whenever possible, the gearing ought to be confined to two reductions, and in order to do this, the motor should not be speeded above 600 revolutions per minute. Ordinary plummer blocks form a suitable type of bearing for the cross shaft. The bearings are usually placed on about 10-foot centers.

When using flanged couplings for this shaft in conjunction with timber platforms, there is always the disadvantage of either using somewhat high plummer blocks, or cutting the platform away to clear the flange, and this difficulty can best be avoided by using ordinary split muff couplings.

APPENDIX

APPROXIMATE FORMULAS FOR BEAMS AND CRANE GIRDERS

In determining the size of a beam to carry a uniformly distributed load, it is customary to use the tables in the various steel companies' hand-books as follows: The total load and span are given. The size is tentatively found by selection from the load tables; the weight of the beam thus provisionally determined is calculated and added to the load; and then a second reference is made to the tables to see that the total capacity is not exceeded.

A more rapid, and closely approximate method for determination without tables is here developed. It is founded on the well-known fact that I-beams and plate girders resemble open beams or trusses. In the ordinary truss with uniformly distributed load, the horizontal stress at the center of either chord is approximately the total weight times the span, divided by 8 times the height. Applying this to an I-beam flange, we have for 16,000 pounds fiber stress:

$$\frac{Wl}{8\,h} = 16{,}000 \times \text{ area of flange.}$$

As the area of the flange is practically one-third the area of the cross-section of the beam:

$$\frac{Wl}{8} = \frac{A}{3} \times 16{,}000\,h.$$

Transforming the left-hand member of this equation to inch-pounds, we have:

$$\frac{2{,}000\,W \times 12\,l}{8} = \frac{A \times 16{,}000\,h}{3}$$

The assumption that the area of the flange is one-third the area of the cross-section of the beam is, of course, only approximately correct. The area of the flange varies in proportion to the total area of the beam from 0.26 for the larger sizes of standard I-beams to 0.37 for the smaller sizes. It should also be remarked at the outset that the approximate formulas in the following apply only to the minimum or standard sizes of I-beams.

In I-beams it is found, closely enough for our present calculations, that $A = h - 2$.

[More accurately A varies between $h - 0.68$ and $h - 2.74$ for different sizes of I-beams.]

Introducing this value of A, reducing, and solving for h, we have:

$$h = \sqrt{0.56\ \ Wl + 1} + 1.$$

The quantity 1 under the vinculum is so small as to be negligible.

The coefficient 0.56 needs further modification to allow for the average weight of the beam itself, which value is taken as equivalent to $3.5\,h - 10$ per foot; also a correction for using h as the distance between extreme fibers instead of the distance between the center of gravity of the flanges, and an allowance for the strength of the web. This changes the coefficient to 0.6, giving as a final formula:

$$h = \sqrt{0.6\,Wl} + 1.$$

This is so simple that it can be solved mentally. Thus h may be determined without reference to the tables for all preliminary work. A trial in comparison with the tables will be convincing proof of its accuracy. It applies *only to the minimum sections*, as these are always used if possible.

BEAMS OF MINIMUM SECTION

h = height in inches, l = span in feet, W = load in tons.

Shape	Maximum Fiber Stress	For Uniformly Distributed Load	For Center Load
I-beam.............	16,000	$h = \sqrt{0.6\,lW} + 1$	$h = \sqrt{1.2\,lW} + 1$
Channel...........	16,000	$h = \sqrt{lW} + 1$	$h = \sqrt{2lW} + 1$
Z bar..............	16,000	$h = \sqrt{lW}$	$h = \sqrt{2lW}$
L (equal legs).....	16,000	$h = \sqrt{lW} + 1.3$	$h = \sqrt{2lW} + 1.3$
T (equal legs).....	16,000	$h = \sqrt{lW} + 1$	$h = \sqrt{2lW} + 1$
Plate girders.......	15,000	$h = \sqrt{0.5\,lW}$	$h = \sqrt{lW}$

Similar formulas have been deduced for minimum sections of channels, Z-bars, angles with equal legs, and T's with stem and flange equal; also for plate girders of the ordinary dimensions tabulated in Carnegie Steel Co.'s hand-book, as shown in the table above.

The formulas reduce to this simple form because the relation between weights and areas of beams, if plotted, closely approximates a straight line.

Incidentally, it may be stated that a closer approximation is given by the following formulas:

Weight per foot of I-beams, minimum section $= 0.07\,h^2 + 1.7\,h$ pounds.

Area of I-beams, minimum section $= 1.1\,h - 3$ square inches.

The average weight per foot of plate girders of standard dimensions (Carnegie Steel Co.) in pounds, is

$$\text{weight} = 0.1\,h^2 + 2\,h.$$

Memorizing the formulas which are most useful to the work in hand enables one to make mental calculations without reference to tables.

Another handy approximate rule, *good for all standard weights* of I-beams, and depending on l in feet, W in tons, and w (weight of beam per foot) in pounds being known, comes from the following well-known relation: When depth of beam in inches equals span in feet, then, weight of beam per foot in pounds equals load in thousands of pounds; other loads vary inversely as the length of span.

Then, changing the load from pounds to tons, it follows that

$$\frac{w\,h}{2\,l} = W, \text{ and}$$

$$h\,w = 2\,l\,W.$$

As w depends on h, a value of h must be selected by trial to balance the equation.

It has now been shown that for I-beams of the minimum sections, which are commonly used, with a fiber stress not exceeding 16,000 pounds per square inch, and properly braced sideways at distances not exceeding twenty times the width of the flange, the depth of a beam, h, is closely determined by the formula $h = \sqrt{1.2\,l\,W} + 1$, where l is the span in feet and W the center load in tons; and approximately for center loads for the standard plate girders tabulated in the Carnegie hand-book, with a fiber stress not exceeding 15,000 pounds per square inch, $h = \sqrt{l\,W}$; but for the ordinary crane beam, and similar uses which do not admit of side supports, the above formulas require modification.

They become with center loads: For beams, $h = \sqrt{1.2\,l\,W} + 1 + 0.004\,l^2$; for girders, $h = \sqrt{l\,W} + 0.004\,l^2$.

Of course these are based on the supposition that the beams are sufficiently long not to fail by the crippling of the web—a case which almost never occurs in practice.

These formulas give a ready means for determining the size required for a given load and span without the use of tables and the usual trial-and-error methods; they are especially useful in the field, and for preliminary work. Up to the point where the length of the beam does not exceed seventy times the flange width, they will be found sufficiently accurate. Above this point the error slowly increases, but always on the safe side.

As with the original formulas, the correction given is not purely empirical, but is deduced from accepted laws. The result is surprisingly simple when the long road required to reach it is considered. The method may be of interest.

The manner of determining the proper coefficient for reducing the usual load to prevent undue strains in the compression flange when the beam is considered as a column, is well known; and different authorities give slightly varying figures. The Carnegie table is as follows:

When the length of a beam does not exceed 20 times the flange width, the coefficient for safe load = 1.

When the ratio does not exceed 30, coefficient = 0.9.
When the ratio does not exceed 40, coefficient = 0.8.
When the ratio does not exceed 50, coefficient = 0.7.
When the ratio does not exceed 60, coefficient = 0.6.
When the ratio does not exceed 70, coefficient = 0.5.

Changing the ratio to feet, and keeping the width in inches, this is equivalent to the following:

When the ratio, l in feet, w in inches $= 1.7$, coefficient $= 1$.
When the ratio, l in feet, w in inches $= 2.5$, coefficient $= 0.9$.
When the ratio, l in feet, w in inches $= 3.3$, coefficient $= 0.8$.
When the ratio, l in feet, w in inches $= 4.2$, coefficient $= 0.7$.
When the ratio, l in feet, w in inches $= 5.0$, coefficient $= 0.6$.
When the ratio, l in feet, w in inches $= 5.8$, coefficient $= 0.5$.

We now wish to substitute for w its equivalent value in terms of h. By plotting, and deducing an approximate formula for the curve, we find that $w = 1.4\sqrt{h}$ very nearly, this being a simple parabolic curve which closely averages the plotted points.

Substituting in the above, we have, for the first case,

$$\frac{l}{1.4\sqrt{h}} = 1.7. \quad \text{Reducing} \frac{l}{\sqrt{h}} = 2.4, \text{ and}$$

$$\frac{l^2}{h} = 5.7.$$

Applying this method to the other ratios, as well, we have:
When ratio $l^2/h = 5.7$, coefficient of load $= 1$.
When ratio $l^2/h = 12.2$, coefficient of load $= 0.9$.
When ratio $l^2/h = 21.2$, coefficient of load $= 0.8$.
When ratio $l^2/h = 34.7$, coefficient of load $= 0.7$.
When ratio $l^2/h = 49.0$, coefficient of load $= 0.6$.
When ratio $l^2/h = 67.0$, coefficient of load $= 0.5$.

Plotting the above, it will be found that the straight line formula, coefficient $= 1 - 0.008$ ratio, gives practically the same coefficients.

Therefore, coefficient for safe load, $C = 1 - \frac{0.008\,l^2}{h}$.

$$C = \frac{h - 0.008\,l^2}{h}$$

Now let W_1 represent the load in our original formula for plate-girders, *i. e.*, $h = \sqrt{l\,W_1}$.

Applying our coefficient for correction we have approximately,

$W_1 = \frac{W}{C}$, and $h = \sqrt{\frac{l\,W}{C}}$. Squaring, and clearing of fractions,

$h^2\,C = l\,W$. Substituting the value of C determined above:

$$\frac{h^2(h - 0.008\,l^2)}{h} = l\,W, \text{ and } h^2 - 0.008\,l^2h = l\,W.$$

Completing the square, and solving for h:

$$h - 0.004\,l^2 = \sqrt{l\,W + (0.004\,l^2)^2}.$$

The quantity $(0.004l^2)^2$ is so small in comparison with $l\,W$ as to be negligible, and we have finally, $h = \sqrt{l\,W} + 0.004\,l^2$, as first given.

By a similar process for I-beams, we have approximately:

$$h = \sqrt{1.2\,l\,W} + 1 + 0.004\,l^2.$$

It is easy to point out discrepancies in the extreme cases of this method of averaging by simple curves, but the final tests show that the errors of the approximations are relatively small. The resulting formula is so simple that it can be solved mentally, and is close enough for all but construction work. A few comparative examples with extreme lengths are appended, to show their accuracy; shorter lengths give even more accurate determinations.

For this purpose, a size of beam and span are assumed, the safe loads taken from the tables and corrected for center load, and the usual correction applied for "no lateral support," giving the net load for this condition. Then for comparison with this usual method, assuming this same net load and span to be given, the process is reversed, and the size of beam determined by the formula.

From the tables, reduced to center load:

24-inch I-beam, 36 feet span, 12.9 tons load.

$w = 7$ inches,

$36 \times 12 \div 7 \doteq 62$,

$C = 0.6$,

$12.9 \times 0.6 = 7.7$ tons net safe load.

By formula:

$h = \sqrt{1.2 \times 36 \times 7.7} + 1 + (0.004 \times 36^2) = 18.3 + 1 + 5.2 =$ 24.5 inches, required height of beam.

Similar extreme examples for smaller beams give the following results:

Beam =	18 inches	12 inches	6 inches	4 inches
$W =$	6.5 tons	3.3 tons	1.0 ton	0.4 ton
$l =$	36 feet	20 feet	30 feet	20 feet
$w =$	6 inches	5 inches	3 inches	2.66 inches
$\frac{12\,l}{w} =$	72	72	80	90
$C =$	0.5	0.5	0.4	0.3
$WC =$	3.3 tons	1.6 ton	0.4 ton	0.12 ton

Calculated by the formula, the required heights of the beams, in inches, are the sums of the following:

12	7.6	3.1	1.5
1	1	1	1
5.2	3.6	1.6	1.6
18.2	12.2	5.7	4.1

In practical solutions, all decimals of feet and tons may be disregarded, using the nearest whole number, and the results obtained mentally will give the same beam. A few decimals were retained in the examples for the purpose of showing how closely the formula applies.