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BALL BEARINGS

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## CHAPTER I

### PRINCIPLES OF BALL BEARINGS\*

The present chapter on ball bearings consists of an abstract of a paper by Mr. Henry Hess, presented before the American Society of Mechanical Engineers. It is of particular interest and value, because, although the field of usefulness of the ball bearing is as wide as the domain of mechanical engineering, but little information on ball bearings can be found in the engineering hand-books or text-books. Generally, the subject is dismissed with occasional reference. Sometimes a formula for carrying capacity is given, but this is usually wrong. There is, of course, much matter scattered through the technical press giving isolated experiences with a few bearings that happen to come within some one's observation. But, undoubtedly, the insufficient information on the elements of ball bearings is responsible for the directly contradictory statements to be found, and the generally accepted opinion that ball bearings are suitable only for comparatively light loads. This was the situation found by Prof. Stribeck, a German investigator, who has been experimenting with ball bearings for the German Small Arms and Ammunition Factories, of Berlin. The present chapter is a résumé of Prof. Stribeck's report, together with a number of notes based on Mr. Hess' own experience.

#### The Wear of Ball Bearings

Sliding bearings wear out by abrasion of the carrying surfaces, but ball bearings do not give out on account of wear. In fact, they do not wear. They may be ground out by admitting grit, but that is an abnormal condition for ball bearings, the same as it would be for sliding bearings. The only normal cause for the giving out of ball bearings is the stress on the material, when this stress exceeds certain limits. Lightly loaded bearings can be so designed as to eliminate this cause altogether, and to insure practical indestructibility. In heavily loaded bearings this condition may not be possible to realize within practical dimensions, but the proportions may be so chosen that the stress will not cause a break-down within the lifetime of any mechanism to which the ball bearing is applied. An important principle in the design of ball bearings is that the balls may be subjected to loads, increasing as the shape of the supporting surface more nearly becomes complementary to that of the ball. A ball running between races having a flat or straight line cross section will not support as great a load as if the section were curved, or in other words, if the balls were running in a groove. The groove, of course, must never have a curvature equaling that of the ball, as that would substitute sliding for rolling contact.

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\* MACHINERY, December, 1907, and January, 1908.

## Formulas for Loads on Ball Bearings

The frictional resistance of a ball bearing is lower the less the number of balls. Bearings should be designed to have between 10 and 20 balls. For this number of balls the equation

$$P_o = \frac{5}{z} P_b \quad (1)$$

should be used. In this equation

$P_b$  = total load on the bearing consisting of one row of balls, in kilograms.

$P_o$  = greatest load on one ball, in kilograms.

$z$  = number of balls.

The load carrying capacity of one ball is  $P_o = K d^2$ , in which

$d$  = the ball diameter, in eighths of an inch,

$K$  = a constant depending upon the material, and the shape of the ball supporting surface. This constant should vary between 3 and 5 for common materials for ball bearings, and between 5 and 7.5 for improved steel alloys.

As balls are usually made to English measurements, one-eighth inch has been selected as unit for the ball diameters.

From equation (1) we have

$$P_b = P_o \frac{z}{5}, \text{ and substituting } K d^2 \text{ for } P_o, \text{ we have}$$

$$P_b = K d^2 \frac{z}{5}.$$

Speed rotation, in as far as it is uniform, does not affect the carrying capacity of a ball bearing.\* This applies, however, only to radial bearings, but not to thrust bearings of the collar type. In these, the carrying capacity decreases with the increase of speed. Variations in load reduce the carrying capacity, the effect increasing with the amount of load change and the rapidity of the change. Uniformity of ball diameter is very essential as the calculated carrying capacity can be realized only if each of the balls sustains its share of the load. High finish on both ball and ball sustaining surfaces is also essential. The presence of grinding scratches will very materially cut down the load carrying capacity.

## Frictional Resistance of Ball Bearings

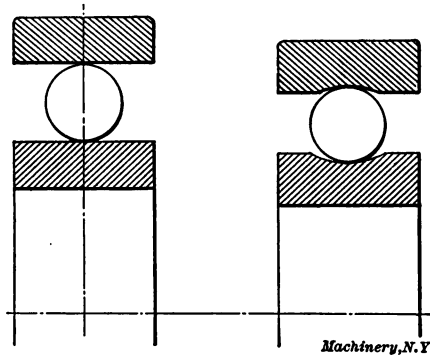
The frictional resistance of ball bearings has, by actual measurement, been found to vary from 0.0011 to 0.0095. These are the coefficients of friction referred to the shaft diameter, thus permitting direct comparison with coefficients of sliding friction. Ball bearings having a coefficient of friction materially above 0.0015 under the greatest allowable load should not be recommended, because they are too short-lived. The high resistance indicates the presence of too large an ele-

\* Some investigators do not agree to this conclusion. See Chapter II of this treatise.

ment of sliding friction. The coefficient of 0.0015 for a good ball bearing under its greatest load, independent of the speed within limits, will, however, rise to approximately 0.0030 under a reduction of the load to about 1/10 of the maximum.

#### The Requirements of a Good Ball for Ball Bearings

The requirements for a good ball are, in the first place, truth of shape and size. The permissible limit of error will vary with the character of the material. It is evident that where a ball is larger than the other balls in the bearing, it must be capable of a deformation sufficiently large so as to permit the others to carry part of the load, and for that reason, the smaller the deformation, the more accurate to size must the ball be. In the second place, a high degree of surface finish is essential. What is usually considered a very good finish is in a ball bearing totally inadequate. Grinding and polishing marks must not only not be discernable by the naked eye, but, if de-



Figs. 1 and 2. Examples of Simple Radial Bearings

tected with an ordinary pocket lens, the balls should be condemned. This, at least, is true of balls for bearings expected to have long life, and to carry heavy loads under high speed. The third condition for balls is that the material out of which they are made has an elastic limit as high as can be had. The uniformity of hardness throughout the mass of the ball is also very essential. General uniformity, in fact, is one of the most important factors, for it will not do to say that because some balls in a lot are better than the others, the design may be based on the poorer ones. Such reasoning would result in the better balls carrying more than their share of the load, producing too heavy a stress, and possibly breakage, of these balls. Lower quality, provided it is uniform, is better than such a condition. The limits of error in regard to the truth and size should not exceed 0.0001 of an inch.

#### Radial Bearings

Other things being equal, it is always best to arrange sustaining surfaces at right angles to the load direction. That gives a design of bearing such as shown in Fig. 1, but a better carrying capacity is to

be had from the modification shown in Fig. 2, in which the ball races have curved cross section instead of straight lines. The grooved races have the advantage of greater sustaining capacity, as has already been mentioned. Cutting a local groove from one side into the races, as in Fig. 3, for the purpose of assembling the balls between the two races, is common practice, but it is not good practice. So long as the loads are low enough, filling opening may be of no account, but at high speeds and loads this groove is objectionable, since then the

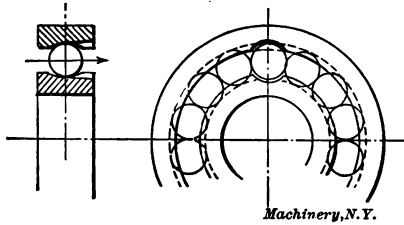


Fig. 3. Objectionable Filling Groove in Ball Bearing

catching of the balls at the junction of the filling groove results in damage to the balls, and through these to the race surface.

#### Thrust Bearings

In thrust bearings, of the type shown in Fig. 4, the requirement that the sustaining surface should be at right angles to the direction of the load is provided for. These bearings are frequently made with the surfaces *A* and *B* parallel. Providing that these surfaces are made truly parallel, that design is good, but in practice it is seldom possible

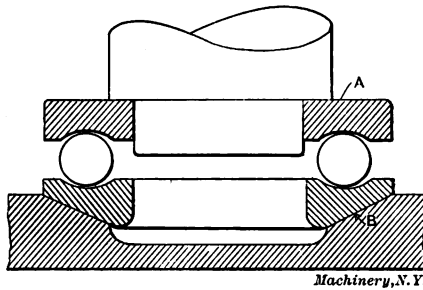


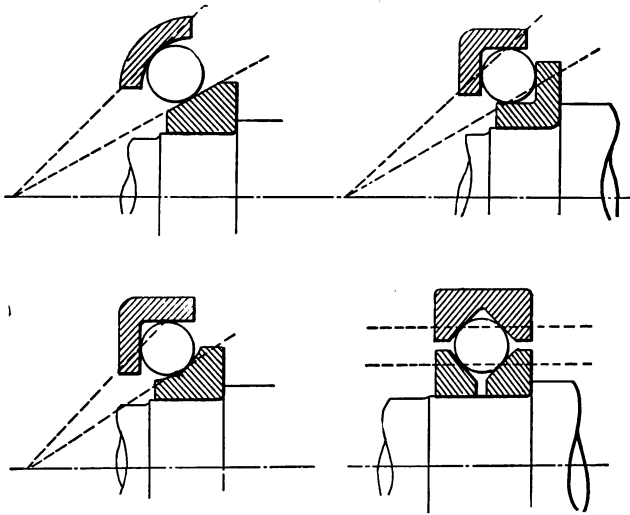
Fig. 4. Collar Thrust Bearing

to get these surfaces truly parallel, because even if such parallelism were possible of attainment, it could not be maintained under the deflections due to the load. It must be remembered that initial errors or deflections of a thousandth of an inch will cause the balls on one side to carry the entire load. For a given case this would demand bearings of larger size than otherwise necessary. By seating the lower collar on a spherical surface as shown at *B*, Fig. 4, this collar can adjust itself in such a way as to distribute the load over the entire number of balls. The speed at which these bearings are run enters

decidedly into the carrying capacity of this type of bearing. The utility of these bearings is greatly reduced when speeds exceed 1,500 revolutions per minute.

#### Angular Load Bearings

The shapes and modifications of angular load bearings are innumerable. Figs. 5, 6, 7, and 8 may be taken as typical instances of these bearings, representing 2-, 3-, and 4-point contacts. In order to secure rolling contact, the contact points of the balls on the races should be points on a cone of rotation whose apex lies in the center line of the shaft, or they may be points on the surface of an imaginary cylindrical roller that would be parallel to the shaft. The defect in all these



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Figs. 5, 6, 7 and 8. Examples of Angular Bearings

forms of bearings is their adjustable feature. This places them absolutely at the mercy of every one capable of handling a wrench. A bearing properly proportioned with reference to a certain load may be enormously overloaded by a little extra effort applied to the wrench, or, on the other hand, the bearing may be adjusted with too little pressure, so that the balls will rattle, and the results consequently be unsatisfactory. The prevalent idea that angular ball bearings can be adjusted to compensate for wear is erroneous. The wear will form a groove on the loaded side of the race, deepest at the point of maximum load, and adjusting the cone endwise will only cause the balls to be more tightly pinched between the sound portions of the races, which will most likely cause overload. The rough surface of the groove previously worn will attack the balls, and in due time the entire race and bearing will be destroyed.

Theoretically, it would seem that a radial bearing would be incapable of carrying thrust load, owing to the wedging of the balls between the races. In Fig. 9, however, is shown the condition of a ball bearing where the ball does not entirely fill the space between the races, if the bearing is not under load, and which, when under load, will assume the position shown. The ball does not come in contact with the race grooves where these are deepest, but so that the tangents to the race curvature at the contact points form angles with the line of thrust. A calculation of the amount of the wedging action in Fig. 9, with the radial freedom of ball permissible in these bearings, indicates an inadvisably large amount of wedging. Actual running tests, however, as well as a large fund of accumulated experience, have already proved that these bearings will carry much more thrust load than the calcu-

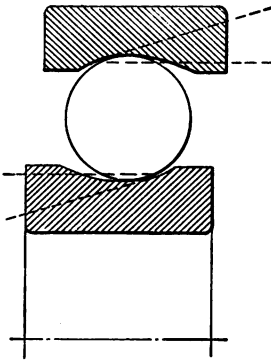


Fig. 9. Radial Bearing used as Thrust Bearing

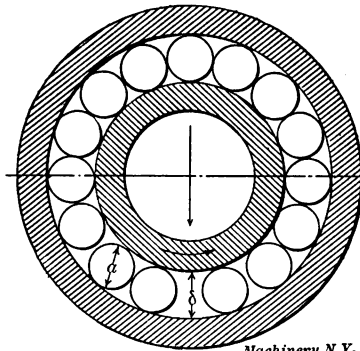


Fig. 10. Diagram used in Analyzing Condition of Sliding Friction

lation of the theoretical wedge angle indicates as possible. It is probable that the deformation which occurs at the point of ball contact, which results in small surface *areas* of contact, instead of mere *points*, has a mean tangent to the compression surface of greater inclination, and that the wedge is therefore more blunt. It has been experimentally determined that the thrust carrying capacity of the uninterrupted type of annular bearings is to the radial capacity as 0.1 is to 1, and may be as great as 0.25 to 1, the variation depending upon the ball diameter, race curvature, and number of balls. It has been experimentally found that speed has but a slight influence on the thrust carrying capacity of this class of ball bearings, and for speeds above 1,500 revolutions per minute, these radial bearings are more efficient thrust carriers than the collar type.

#### The Supposed Sliding Friction in Ball Bearings

Many designers have supposed that in ball bearings adjacent balls press against one another with considerable force. With the inner race in Fig. 11 running as indicated, the balls will also rotate as shown. The surfaces of the balls run in opposite directions, and therefore rub against one another. This is assumed to be a serious



defect by those who reason that these surfaces are in contact under pressure. The same general cure in innumerable forms, as shown in Fig. 12, has been proposed time and again. This cure consists in the provision of smaller balls interposed between the larger ones, so that all the contacting surfaces roll in the same direction relative to one another. This remedy is, however, fallacious, in that it brings about the very condition it seeks to avoid. If two large balls, Fig. 13, compress a smaller one between them, and the three have their centers connected by a straight line, they will retain their relative positions, but if the interposed ball has its center to one side, as in Fig. 14, then this ball will be forced outward. The resort to a cage for retaining the interposed roller or ball results in the latter being pressed against the sides of the cage, and keeps the ball in forcible sliding contact, the very thing that it was intended to avoid. In another design, Fig. 15, the interposed member is brought into contact with the race, and while the various balls have a rolling contact in relation to one an-

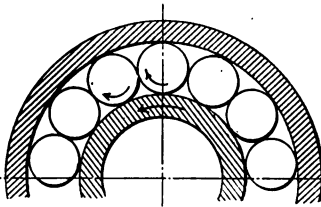
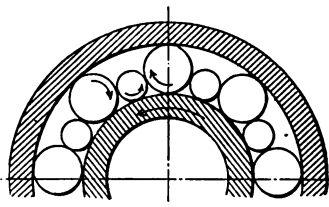


Fig. 11. Diagram showing Direction of Rotation of Balls, indicating Sliding Friction



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Fig. 12. Fallacious Means for avoiding Sliding Friction between Balls

other, the interposed member has a wrong direction with reference to the race against which it is forced, and thus a sliding contact is produced. All these designs are based on a failure to recognize that axiom in mechanics according to which a force whose direction is normal to the supporting surface has no component in any other direction.

Analyze the conditions in a ball bearing, and, referring to Fig. 10, suppose that the shaft is loading the inner race, and that the latter is fallaciously assumed to act as a wedge, forcing the balls at the bottom apart and consequently producing pressure between the balls at the top. In that case the space  $\delta$  must be smaller than the ball diameter  $a$ . The rotation of the inner race, however, will carry the balls around the bearing, and the diameter  $a$  is therefore forced through the smaller space  $\delta$ . To do this the ball must lift the inner race. The force to do this is impaired by the load and is equal to the rolling friction, and can, therefore, amount to but a fraction of that load. We would then have the absurd condition of this smaller force overcoming the larger original force. Were we to assume that the inner race is not raised by a ball in passing, but that the ball is compressed sufficiently to get through, it would mean the absurdity that the small force represented by the rolling friction would be sufficient to deform the ball.

## Correct Ball Bearing Mounting

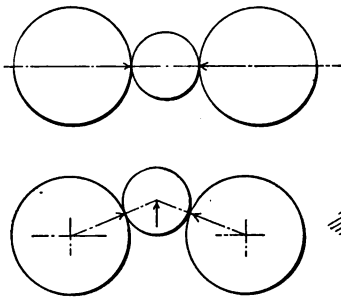
The following requirements are placed on correctly mounted ball bearings:

a. The proper size must be selected for the load and conditions in question. Rated capacities are usually for steady loads and speeds, but variations from these conditions demand a cutting down of the listed capacity.

b. Bearings must be lubricated. The often repeated statement that ball bearings can be run without a lubricant is not correct.

c. Bearings must be kept free from grit, moisture, and acid. No lubricants developing free acids should be permitted.

d. The inner race of the bearing should be firmly secured to the shaft. This can be done by a light driving fit, reinforced by binding the race between a substantial shoulder and a nut.



Figs. 13 and 14. Analysis of Sliding Friction

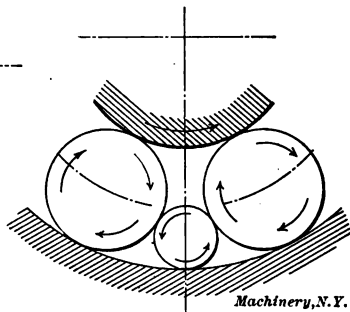


Fig. 15. Fallacious Means for Avoiding Alleged Sliding Friction

e. The outer race must have a sliding fit in its seat.

These conditions should under all circumstances be adhered to, and a failure to do so will result in very unsatisfactory bearings. The two following conditions are frequently disregarded, and while the disregard of these conditions is not so serious as of those mentioned before, it is safe engineering to follow them, and a disregard of them is a standing invitation to trouble.

f. Thrusts should always be taken up whether in the same or opposite directions by the same bearing.

g. Bearings should never be dismembered, or at least never more than one at a time. That will avoid all danger of mixing the balls from different bearings; the balls from different bearings are very apt to vary more than is permissible for the individual bearing.

## Illustrations of Correct Mounting

The development of ball bearings being so recent is probably the cause that so little information as to correct mountings for various conditions is available. The experience of those best informed has been that faulty mountings are very general, and for this reason illustrations have been given of correct mountings, going more into details

than would be considered necessary for a more familiar mechanical subject.

Fig. 16 shows a bearing in which the inner race has a light driving fit on the shaft, and is securely clamped between a shoulder on the shaft and a nut. The shoulder on the shaft should be high enough to get a firm grip on the surface of the side of the race. It is good practice to make this shoulder about one-half as high as the race thickness, perhaps a little less for large bearings and a little more for small bearings. The outer race has a tight sliding fit in the housing, so that the bearing as a whole may be able to respond to relative shifting of the shaft and housing without being subjected to end thrust through the balls.

Fig. 17 shows a radially loaded shaft held against endwise motion in either direction. This bearing is capable of carrying thrust load in

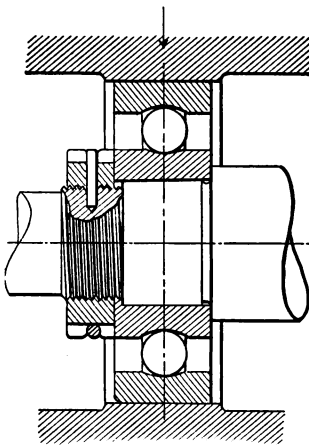


Fig. 16. Free Mounting for Radially Loaded Bearing

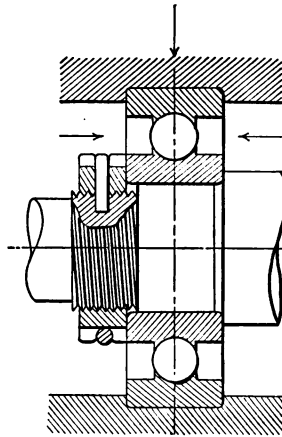


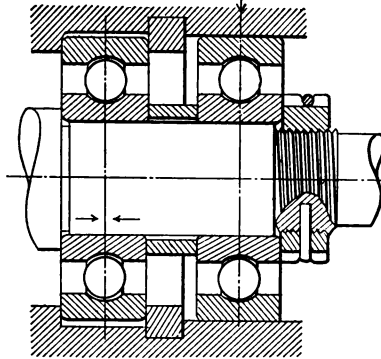
Fig. 17. Radially Loaded Bearing held against Longitudinal Motion

either direction, but never more than one bearing on the same shaft should be held in this manner. This bearing differs from the preceding mounting only therein that it has the outer race secured between shoulders in the housing. This arrangement and the preceding one are usually found combined on the same shaft, which is then held endwise at one point only, so that temperature changes, or deflections of the shaft, can cause no cramping.

Fig. 18 shows separate radial bearings for radial and thrust loads. This type of bearing is used when it is desirable to take thrust load on bearings of the radial type, although the space available does not permit of a single radial bearing of sufficient diameter to take both loads. One bearing is then mounted entirely free circumferentially so as to take the radial load, while the other bearing is mounted between shoulders, and takes the thrust load.

Figs. 19 and 20 show thrust loads on a collar bearing in one direction only. Here the stationary race is provided with a spherical seat

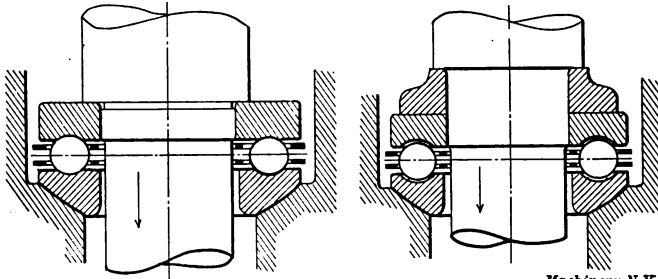
so that it will distribute the load over the complete circle of balls. In order to permit compensating shifting, the fixed seat must be radially free of the shaft and of the housing. The shoulder on the shaft should be large enough so as not to permit any bending strains on the rotating race. When it is inconvenient to provide a large enough shoulder on the shaft, a washer can be inserted between the shoulder and the race, as shown in Fig. 20. In Fig. 21 is simply shown a modification of



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Fig. 18. Separate Radial Bearings for Radial and Thrust Loads

the bearings in Figs. 19 and 20. This bearing takes thrust loads in two directions on two collar bearings. Fig. 22 shows an arrangement by which thrust load in two directions may be taken up by a single collar bearing. This arrangement is one which economizes space, cost of bearings, and number of parts. Fig. 23 shows an arrangement where a radial bearing is used for taking the radial thrust, and a collar bearing is used for taking the end thrust. Fig. 24 shows an



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Figs. 19 and 20. Thrust Load in One Direction on Collar Bearings

arrangement for taking up radial load as well as thrust load in two directions, these loads being carried on one radial bearing and two collar bearings. In this design attention may be called to the distance piece inserted for binding the inner race of the radial bearing against the shoulder of the shaft.

It is occasionally inconvenient to arrange the bearing so that the parts of the inner races can be clamped to the shaft, or it may be

desirable to have a shaft sliding through the bearing. In such cases a sleeve may be introduced on which the inner race of the bearing is firmly clamped endwise, the shaft simply resting in this sleeve. This gives a long bearing to the shaft, which would not be possible if the shaft was directly mounted in the ball race, because the peening effect of the vibrating loads would, even if the race itself was prolonged, be

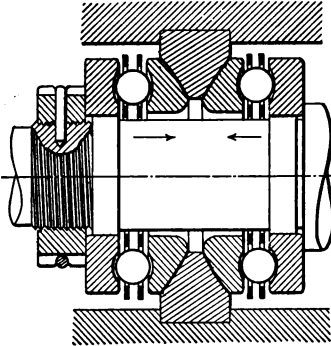


Fig. 21. Thrust Load in Two Directions on Two Collar Bearings

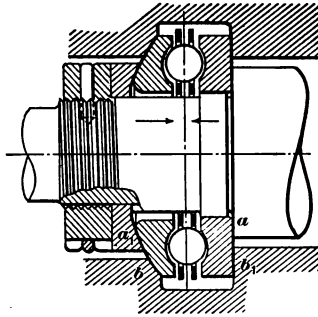


Fig. 22. Thrust Load in Two Directions on One Collar Bearing

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concentrated on a narrow zone of the shaft. A bearing of this kind is shown in Fig. 25.

In Fig. 26 is shown a bearing which is intended for shafting which may not be fully to standard size. The inside of the ball race is tapered, and a split bushing, tapered on the outside as shown, will,

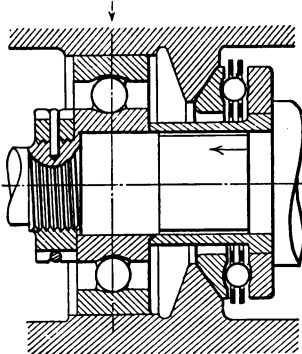


Fig. 23. Radial Load, and Thrust Load in One Direction

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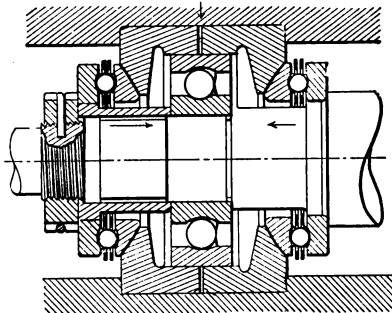


Fig. 24. Radial Load, and Thrust Load in Two Directions

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when tightened, bind the shaft as well as the race to it, and will compensate for all variations in size. The nut should not be used to draw the bushing in, but should merely act as a lock to hold it in place after it has been driven home with a soft hammer.

Ball bearings should always be enclosed so that lubricant will not be lost by leakage, and so that foreign matter will be excluded. Fig.

27 shows an efficient way of enclosing a ball bearing without using any packing. At the end where the shaft passes out of the enclosure, a flange should be bored out about 0.020 inch larger in diameter than the shaft. This flange should be separated into two lips by an angular groove, either cored or bored, as shown at A. These lips should not

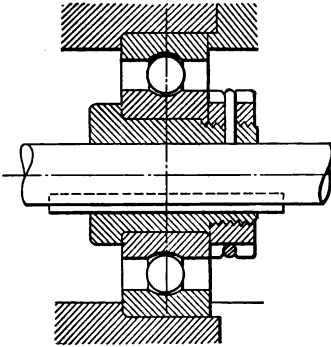


Fig. 25. Shaft Free in Longitudinal Direction in Inner Race

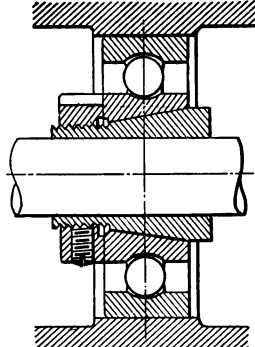


Fig. 26. Adapter Bearing for Shafting varying from Standard Size  
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be less than  $\frac{1}{8}$  inch wide and should have sharp edges. The groove should be provided with a hole or narrow slot B at its lowest point to communicate with the bearing oil space. The groove itself should have a width of not less than  $\frac{3}{16}$  inch, and a depth of about  $\frac{5}{16}$  to  $\frac{3}{8}$  inch, and should not be filled with packing material. Fig. 28 shows

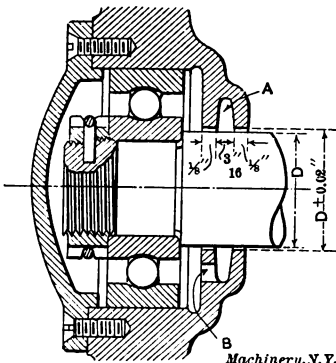


Fig. 27. Example of Enclosed Bearing  
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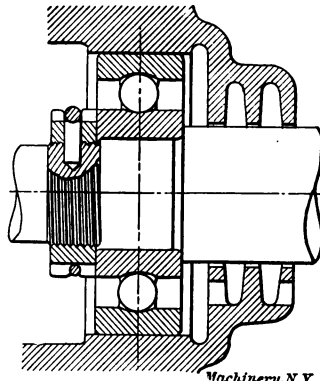


Fig. 28. Bearing Enclosure with Double Groove  
Machinery, N.Y.

an arrangement of a similar kind, excepting that here is introduced a second groove and a third lip. This arrangement is employed where water may be occasionally encountered, and will prevent its entrance. What little may find its way past the outer lip into the outer groove is soon drained out again through the holes provided.

Where much grit is encountered, as in grinding machinery, a packing may be necessary, and filling the outer grooves with a fairly consistent grease will provide such a packing without introducing friction. A bearing of this kind is shown in Fig. 29. A grease cup of the spring loaded piston type will automatically maintain the integrity of the packing. In some cases felt ring packings may be used, but these

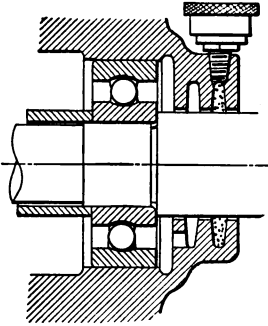


Fig. 29. Enclosed Bearing with Grease Packing

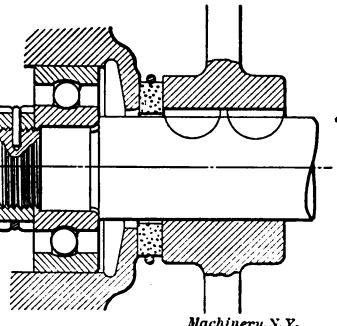


Fig. 30. Felt Ring Packing

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ought to be soaked in good soft paraffine, and a spring wire ring should be placed around the felt washer so as to force the outer edges of the washer outward, which will cause the felt to come into more intimate contact on the sides. Felt washers may also be applied as shown in Fig. 31. Here the washer is tapered on one or on both sides, and the sealing of the enclosure is made entirely against the sides of the sur-

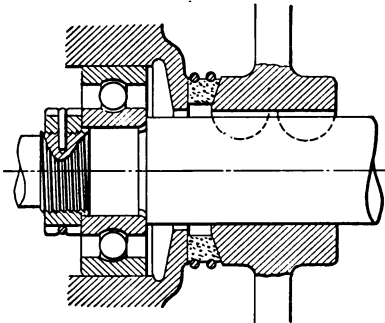


Fig. 31. Angular Felt Ring Packing

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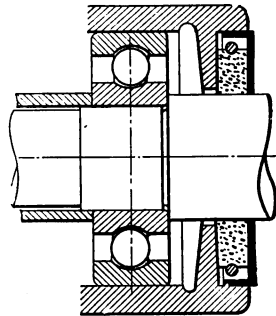


Fig. 32. Modification of Felt Ring Packing in Fig. 30

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faces against which the felt washer bears. The felt washer is pressed inwards by means of springs on the outside. The two modifications shown in Figs. 30 and 31 are intended to be inserted between the faces of a stationary boss and a rotating hub. The modification in Fig. 32, however, is enclosed entirely within one or the other. A felt ring is set into a counterbore and held in place by a light metal cap sprung into position.

## CHAPTER II

### DESIGN AND CONSTRUCTION OF BALL BEARINGS\*

Mechanical friction is always a resisting force, whether utilized to effect the stability of structures, or to transmit motion from one part of a machine to another, or even to enable us to walk. In producing these effects, friction is essential and desirable, and means are often taken to increase it; such, for example, as the covering of leather transmission belts with adhesive substances and the covering of icy pavements to prevent slipping. On the other hand, friction is injurious in preventing relative motion of different parts of a machine and causing waste of energy. Under these conditions, friction is a serious disadvantage and its elimination a problem of grave importance. Any discussion relating to ball bearings, therefore, bears directly on the question of eliminating friction, and indirectly on the question of saving fuel and oil, and preventing the wear and tear of machinery.

One way of minimizing the effects of friction suggested itself to the man who first applied wheels to a road vehicle. It must have been apparent to him that a vehicle would wear better and require less effort to move it if some means were provided to roll it along instead of having to drag it over the ground. This condition of affairs may be exaggerated by assuming that in one case we have two toothed racks in mesh and we try to move one relatively to the other; in the second case, that we replace one of the racks by a gear wheel and move it over the rack by means of a trunnion. The limit of the load required to effect motion in the first case, is that which will break the teeth, and in the second case, the effort required to overcome the comparatively small weight of the wheel and the friction between the meshing teeth. A comparative idea of the advantage gained by substituting rolling for sliding friction may be obtained approximately by taking two smooth blocks of steel, lubricating the surfaces in contact, and then tilting them; one block will begin to slide over the other when the angle through which they are tilted is between 8 and 9 degrees. If now we replace the oil between the two blocks by a number of steel balls, we find on tilting the blocks, that they will begin to slide over each other when the angle is about 0.08 degree. Again, if we take two smooth horizontal blocks of steel, weighing, say 1 pound, it will require a force of approximately 2.7 ounces to move one over the other if the surfaces in contact are lubricated. If, instead of oil, we use steel balls between the blocks, only 0.027 ounce will be required to effect the motion. The only question raised against the use of the concentric ball bearing for stationary machinery and the cheaper

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\* MACHINERY, May, 1909.



grades of automobiles, is in regard to first cost. Aside from this, no one questions the relative advantage of the two-point ball bearing over the plain bearing, or even the roller bearing or cup and cone. In regard to ultimate cost, it may be safely said that the two-point bearing is the most efficient and cheapest. It only remains, regarding the ball bearing as a friction eliminator, to consider some of the inherent mechanical defects found in ball bearings at present on the market. These defects may be put under the following heads: Side slots in the rings for assembling the balls; small number of balls in races

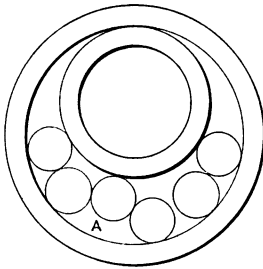


Fig. 33

Method of Assembling a Ball Bearing

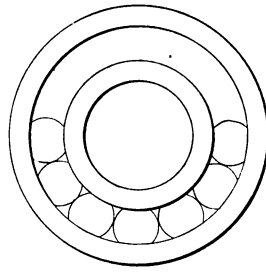


Fig. 34

owing to absence of side filling slots; separators or spacers made of several disconnected pieces.

#### Methods of Assembling Ball Bearings

There are employed at present but two general methods of assembling a full or partly full ball bearing. One of these is to arrange the balls and races as shown in Fig. 33, and after placing in the crescent-shaped space A, as many balls as can be freely dropped in, the inner ring is shifted, leaving an annular space with the races about half filled with balls, as shown in Fig. 34. The remaining balls are then

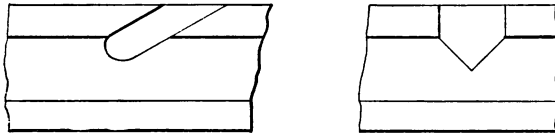


Fig. 35. Two Forms of Filling Slots

sprung in through slots in the sides under comparatively high pressure. This process is used extensively by Fichtel & Sachs (F. & S.); Standard Roller Bearing Co. (S.R.B.); and every other manufacturer of ball bearings, with the exception of three or four. The master patents for this process are owned by Fichtel & Sachs. Two forms of these filling slots are shown in Fig. 35. It is claimed by all those who employ the filling slot, that after the balls are assembled, they have practically a continuous race and that the bearing is as perfect as if the slot never existed. It may be said, however, that the existence of

a filling slot as a defect is indirectly recognized by every bearing manufacturer. If it were possible to use radial bearings to carry only a purely radial load, almost any ball bearing, no matter how assembled, would give satisfactory service, provided the materials were fairly good and the bearing not overloaded; but, in many instances, radial bearings are used to carry, simultaneously, ordinary radial loads and comparatively excessive thrust loads. This applies particularly to radial bearings used in automobile front wheels, and to some extent in the rear wheels as well. Under these conditions, the inner and outer rings are relatively displaced, as shown in Fig. 36, and it is evident that if a bearing having a side filling slot be subjected to excessive end thrust, the balls are forced into the slot, a treatment which is manifestly not very good for either balls or races. Of course, the extent to which this pinching of the ball in the slot will take place will depend on the depth of the filling slot. The Fichtel & Sachs Co. overcome this difficulty to some extent by employing a slot which is

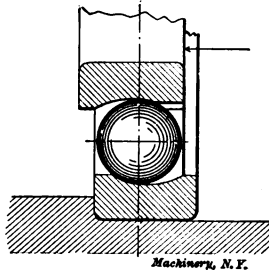


Fig. 36. Displacement of the Rings of a Radial Bearing when it is subjected to an End Thrust

not quite as deep as the ball races. The balls are then forced through these slots under a pressure which is much higher than the thrust loads that can possibly be put on the bearings in practice. This is at least defensible practice, provided that the materials employed are of excellent quality. A bearing may be made without a filling slot, but in that case the limit to the number of balls which may be inserted is set by what will go into the crescent-shaped space *A*, Fig. 33. The balls are disposed around the circumference and some form of separator or spacer used to keep them apart. It has often been stated by those who employ this process, that there is no need of completely filling the races with balls; in other words, that a ball bearing which is only partly filled with balls, is equal to or better than one which is completely filled. One might almost reason instinctively that such statements are erroneous. That they are actually so will be shown.

#### Capacity of Ball Bearings

The permissible load in kilograms which a two-point ball bearing having one row of balls, will carry, is given by the following formula given by Professor Stribeck, and already referred to in the previous chapter:

$$P = \frac{K d^2 z}{5} \tag{2}$$

where  $K$  is a constant depending on the properties of the material, the form of the ball race and the angular speed of the bearing; it may also be regarded as a factor of safety;  $z$  is the number of balls; the factor 5 takes account of the fact that only part of the balls carry the radial load;  $d$  is the diameter of the ball, taking  $\frac{1}{8}$  inch as the unit. For example:

- If diameter of ball is  $\frac{1}{8}$  inch,  $d = 1$
- If diameter of ball is  $\frac{1}{4}$  inch,  $d = 2$
- If diameter of ball is  $\frac{7}{16}$  inch,  $d = 3.5$
- If diameter of ball is  $\frac{5}{8}$  inch,  $d = 5$ , etc.

From the above it is seen that if we take two bearings having the same form of groove and the same dimensions for both the balls and races, then if  $P_1$  and  $P_2$  are the carrying capacities of the two bearings, and  $z_1$  and  $z_2$  the corresponding number of balls, we have for the same angular speed

$$P_1 = \frac{K d^2 z_1}{5}$$

$$P_2 = \frac{K d^2 z_2}{5}$$

from which we then obtain  $\frac{P_1}{P_2} = \frac{z_1}{z_2}$ ; or, in words, the carrying ca-

pacities of the two bearings are directly proportional to the number of balls. It is seen, therefore, that for a given bearing, other things being equal, the full type has a greater carrying capacity than that only partly filled with balls.

While it is seldom stated that the carrying capacity of a radial bearing is some function of the angular speed, it will be seen that such is the case, since, if the speed is zero, the carrying capacity of the bearing is the rupture load of the balls; this load is far greater than the rated capacity of the bearing. For ball bearings made of high-grade material and accurately machined,  $K$  has the following approximate values for steady loads and uniform speeds.

Revolutions per Minute	Values of $K$
10 .....	20
150 .....	18
300 .....	15
500 .....	10
1000 .....	7.5
1500 .....	5

From these figures, it will be seen that a given bearing will carry only one-fourth the load at 1500 R. P. M. that it will at 10 R. P. M. Since manufacturers employ the same material for all their bearings, and make the ball races of approximately the same form, it would be

a simple matter to calculate the carrying capacities after adopting a value of  $K$  for a definite speed of rotation. There are some manufacturers, however, who assume high values of  $K$  in order to rate their carrying capacities high. There is no particular harm in this, provided the loads are not excessive. These values of  $K$  even vary between the full and silent type of the same size bearings. It may be remarked, however, that the values of  $K$  may be the same for all the bearings, and that the capacities are calculated for different angular speeds, but no reason can be seen for such practice. For example, there is no reason why a manufacturer should rate the carrying capacity of his full type bearing, at say, 10 R. P. M., and the silent type of the same size at 300 R. P. M.

To determine the value of  $K$  used in rating the capacities, equation (2) is written in the form

$$K = \frac{P}{0.44 d^2 z} \quad (3)$$

where  $P$  is now given in pounds.

From equation (2), it is readily seen that the load per ball is lower in the full type than in the silent type. It may be argued from this that a full ball bearing will wear better than the other, but it is here necessary to remark that with the better grades of bearings, provided that the load is almost purely radial and within the limits set down in the catalogues, and the mountings made in accordance with the manufacturers' instructions, such a thing as wear is practically unknown, whether the bearing be of the full or silent type. But, as stated above, the load on the radial bearing is never free from end thrust; where this is excessive, the full bearing will give better service than the silent type, since end thrust is taken up by all the balls in the bearing, while a radial load is carried by between one and four balls, depending upon the type of bearing. Hence, everything else being equal, the full bearing is more satisfactory.

In regard to the third defect mentioned, it may be said that the employment of the non-continuous spacer, or one made up of several disconnected pieces, is very poor practice. Such a spacer has been used extensively by the D. W. F. Co., but is now being abandoned for a one-piece bronze separator. The non-continuous separator is also used by a well-known American manufacturer, but its use is incidental to the accomplishment of another end, which will be mentioned immediately.

#### "Frictionless" Ball Bearings

Notwithstanding that the frictional losses in a full ball bearing are very small, so small in fact as to be negligible, many attempts have been made to eliminate what little friction there is. One noteworthy example is that of the bearing just referred to. Here a series of alternately large and small balls is used, as shown in Fig. 37. This example is noteworthy because it actually increases the friction which it is designed to eliminate, and introduces other serious disadvantages not found in the full bearing, or the silent type bearing, having a one-piece

separator. Referring to Fig. 38, it will be seen that if the inner race rotates in the direction indicated, the balls will rotate as shown by the small arrows, and in the same relative direction as the outer race. There is in this case practically no sliding, except at the point of contact of every two adjacent balls where, it is seen, the ball surfaces are moving in opposite directions. At first glance, we may be led to infer that, since the balls are in contact and rubbing against each other, there must be an appreciable frictional loss and consequent wear and tear. This inference is without foundation, since the balls are not in contact under pressure, except in the case of those balls which are not carrying the load; then the pressure is the weight of the ball; this is negligible in comparison with the normal load on the bearing. Referring to Fig. 39, we see that if no load is placed on the inner ring, the balls will come together if the weight of the inner ring is less than that of the balls; but the moment we apply a load to the inner ring,

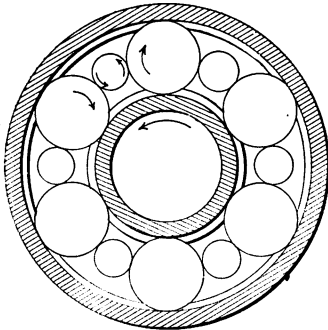
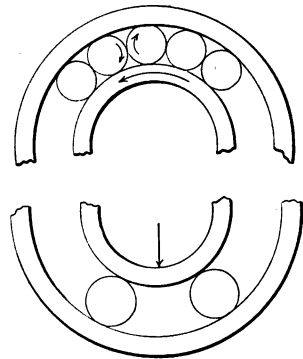


Fig. 37. Bearing Designed to Eliminate Friction



Machinery, N. Y.

Figs. 38 and 39. Diagrams showing Action of Balls in a Full Bearing

which is in excess of the weight of the balls, there is no pressure between the balls; if there were, they would be together. While it may seem to some that this matter is dilated upon to an undue extent, the discussion is nevertheless warranted; this is done in order to correct an impression which seems to be general with those unfamiliar with ball bearing phenomena.

To overcome this imaginary ball friction, this bearing is provided alternately with large and small balls. Referring to Fig. 37, it will be seen that since the small balls are not in contact with the inner race, they are not constrained to rotate in the same direction as the large balls; the small balls, in fact, rotate in the opposite direction, and hence, between every two adjacent balls, there is practically pure rolling; that is, the negligible friction between the balls is eliminated—but at a serious sacrifice. First, since only the large balls are in contact with the inner and outer races, only these balls are useful in carrying the load. As stated in the foregoing, the carrying capacity (in pounds) of a radial bearing is  $0.44 K d^2 z$ , so that a bearing of this

type will carry a load of  $0.44 K d^2 \times 6$ , since 6 is the maximum number of balls that can be inserted in the bearing. A similar bearing when completely filled will hold about 9 of the large balls; hence, we see that the bearing completely filled with large balls will carry three pounds to every two of the bearing having alternately large and small balls. Second, to sustain the small balls, it is essential that some form

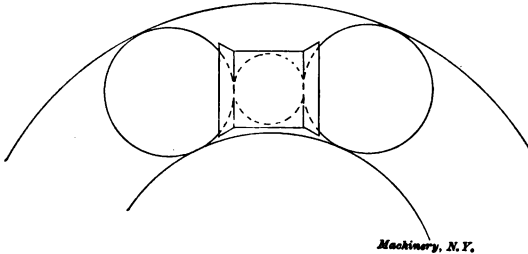


Fig. 40. Type of Separator used in the Bearing shown in Fig. 37

of separator be used. The separator used by the manufacturer of this bearing has the form shown in Fig. 40. It will be seen that the ends of the separator are in contact with the balls, and that although the friction in this case is admittedly very small, it is considerably greater than in a similar full type ball bearing; the friction is further increased, owing to the fact that the makers of this bearing state that

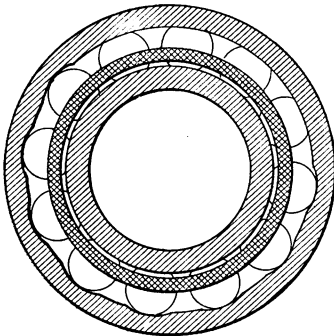


Fig. 41. Bearing with Outer Ring Grooved to permit the Use of a One-piece separator.

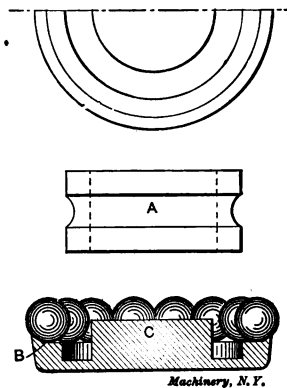


Fig. 42. Inner Race and Section through One-half the Cup used in Assembling the R. B. F. Radial Bearing

their bearings do not require oil. In addition, if one of the large balls should break, the bearing falls apart. This defect is a very serious one, and is common to all the bearings which have non-continuous separators.

As pointed out above, all other things being equal, the carrying capacities of two bearings are directly proportional to the number of

balls; hence, it is evident that under all conditions of service a bearing that is completely filled with balls is to be preferred to one that is only partly filled, provided that the side filling slot can be done away with. Owing to the apparent impossibility of introducing a sufficient number of balls to completely fill the race, most manufacturers have made use of the filling slot, while a few have taken the alternative step of avoiding the filling slot by employing fewer balls. The extent to which some manufacturers may go in order to gain the advantage of a large number of balls, as well as to avoid the non-continuous separator is shown in Fig. 41. This bearing, made by the Auto-Machinery Co. ("A. M.") of Coventry, England, has eight filling slots. The main object of this, however, will be best seen from the following



Fig. 43. View Showing Method of Assembling the R. B. F. Bearing. After the Outer Race is heated with Oil, the Balls and the Inner Race are Forced into Place, after which the Assembling Cup is removed

abstract of an article in *The Automotor Journal*, January 25, 1908, describing this bearing:

"One of the principal difficulties in connection with the use of a cage, is its interference with the process of assembling the component parts. If the balls are inserted separately, they present no great difficulty, but when surrounded by a cage, special provisions are necessary. Some firms have for this reason divided the cages of their bearings, so that they can be fitted in place afterwards, but the Auto-Machinery Co. dislike the use of a divided cage, and have thus evolved a method of inserting the balls and their cage *en bloc*." After the outer ball race is circumferentially grooved, "it has a series of slots cut across its inner surface. These slots, however, are cut on one side only, and do not reach quite to the center of the groove; moreover, they are not provided around the entire circumference. This object is to facilitate the insertion of the balls *en bloc* with their cage \* \* \*"

The advantage of inserting the balls and their one-piece separator as a complete unit, cannot, however, be readily seen. If there is an advantage, it has been gained, in this case, by a great sacrifice.

#### The R. B. F. Radial Bearing

We shall now describe the R. B. F. radial bearing made by the Société Française des Roulements à Billes. This bearing is unique in that it has none of the defects discussed above. There are no filling slots either in the full or silent type. The full type contains as many balls as it is possible to get into the races. The silent type has a continuous separator. In the assembling of the R. B. F. radial bear-

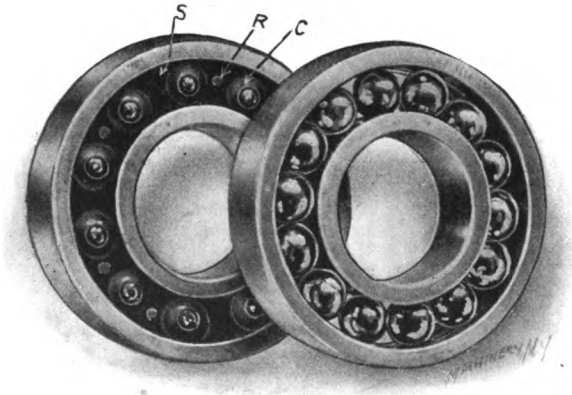


Fig. 44. Radial Bearings with and without Retainer

ing, two similar steel cups are used of the form shown in Fig. 42. After placing the inner race *A* of the bearing over the central core *C* of one of the cups, the balls are laid around the groove *B*, and the inner race and balls are then covered by the second cup. The outer ring is now heated in a bath of oil, and placed over the assembled ball and cup unit as shown in Fig. 43. This complete unit is now placed in a press and the balls forced into position. After the outer ring has cooled down to normal temperature, it is impossible to detect the slightest axial play. This method of assembling a full ball bearing is an excellent check on the quality of the bearing, since, if the balls are improperly heat treated, they are either deformed or broken, as the case may be, when pressure is applied to force them into the outer race, and are, therefore, rejected. The spacer or separator used by the R. B. F. Co. consists, as will be seen from Fig. 44, of two steel stampings *S*, each having a number of saucer-shaped cavities *C* which serve to engage the balls and keep them apart. The two halves of the spacer are kept apart by a number of distance pieces, and are fastened together by means of rivets *R*. Sufficient room is allowed be-



tween the cavities and the balls to provide for a liberal film of grease or oil, so that there is never metal to metal contact under normal running conditions.

#### Radial Bearings as Thrust Carriers

While ball bearing manufacturers, as a rule, are desirous of limiting the use of radial bearings to their normal function, it has nevertheless become the practice among bearing users to employ the radial bearing to take axial thrust in addition to the normal radial load. To meet this demand, the bearing manufacturer has been obliged to stretch a point and suggest means whereby the radial bearing can also be used as a thrust bearing. To this end, it is considered good practice where two bearings are mounted on the same shaft, to make the inner ring of both bearings a light driving fit on the shaft. The outer ring of one bearing is then made a sliding fit in its seat and

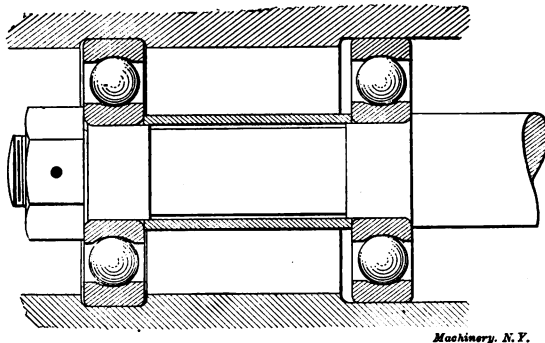


Fig. 45. Radial Bearings Arranged to take Axial Thrust

clamped up far enough to allow it from 0.5 millimeter (0.0196 inch) to 1 millimeter (0.0394 inch) axial play; the outer ring of the other bearing is also made a sliding fit, but is allowed considerable axial play. The first bearing takes the radial load and end thrust, and the second bearing, a purely radial load only. This arrangement, shown diagrammatically in Fig. 45, is a preventive against the wedging of the bearings, and is an assurance of long life. It is not uncommon, however, to find ball bearing users who are bent on carrying out their own ideas in regard to mountings. It is from these that most complaints come.

It may be said here that although the arrangement just described is considered good practice, it is decidedly poor practice to use a radial bearing to take the end thrust of a shaft having a low speed of rotation. This applies, particularly, where the end thrusts are excessive, as in automobile front wheels. This is the real cause of the complaints heard on every hand as to the unsuitability of two-point bearings in automobile front wheels, and has led many to adopt some form of inclined roller or cup-and-cone bearing, both of which are far inferior to the two-point bearing. The remedy in these cases

is very simple. The trouble can be effectively eliminated by making the outer rings of the two bearings a sliding fit and allowing them considerable axial play, so that they cannot possibly take end thrust. This thrust is then taken on a thrust bearing or collar placed somewhere on the shaft, usually between the other two. If the speeds of rotation are very high, say above 1,000 R. P. M., there is no harm in taking thrust on a radial bearing, since at high speeds, radial bearings are better thrust carriers than thrust bearings.

**Thrust Bearings**

For the determination of the permissible load of a thrust bearing, a relation analogous to equation (2) is used. The load  $P$  in kilograms is

$$P = K d^2 z \tag{4}$$

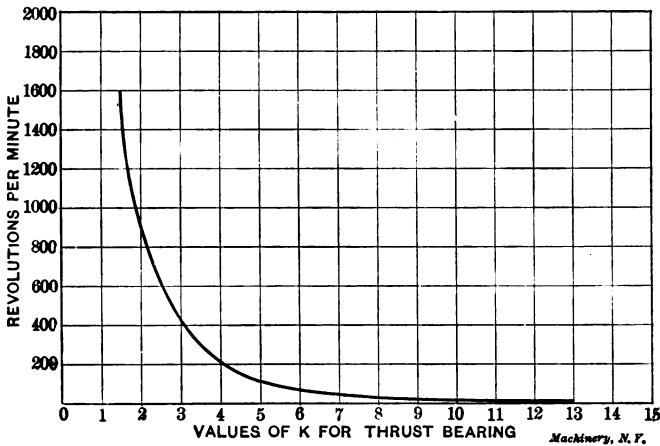


Fig. 46. Diagram showing how the Constant  $K$ , which is used in the Formula for Determining the Capacity of Thrust Bearings, varies with the Angular Speed

This is for steady loads and uniform speeds. This equation is the same as (2), except that the factor 5 drops out, since it is here assumed that all the balls are effective in carrying the load. The curve Fig. 46 shows how  $K$  varies with the angular speed. The values of  $K$  for speeds usually given in catalogues are shown in the following table:

Revolutions per Minute	Values of $K$
10	12.5
150	4.5
300	3.5
500	3
1000	2
1500	1.5

For parts that have very little motion, such as crane hooks,  $K$  may be taken as high as 18 to 20. For very high speeds above 1500,

the ordinary thrust bearing is practically useless for taking end thrusts. Centrifugal force, at these speeds, plays a very important part. The balls are driven toward the outer edge of the race, and against the walls of the ball retainer. To avoid this condition, use is

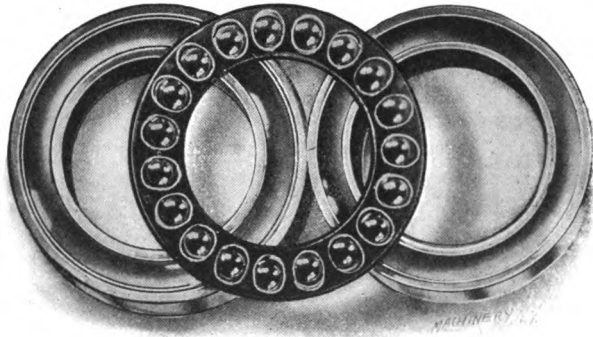


Fig. 47. Upper and Lower Races for Thrust Bearing, and Balls with Retainer having Elliptical Holes

made of a retainer with elliptical holes as shown in Fig. 47. This retainer also permits the shaft to deflect without pinching. It is used in America by the Société Française des Roulements à Billes, and the Hess-Bright Manufacturing Co., who are the only American licensees under the patent. The manner in which the permissible load varies

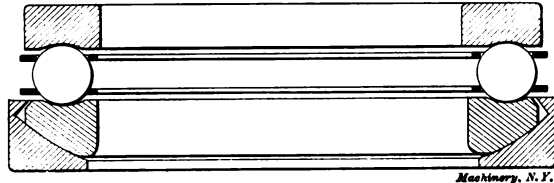


Fig. 48. Section through Thrust Bearing, the Lower Race of which is made in Two Parts

with the speed is seen from the following table constructed for a particular bearing.

Revolutions per Minute	Load in Pounds
10 .....	11,000
150 .....	3,740
300 .....	2,640
500 .....	2,420
1000 .....	1,760
1500 .....	1,540

Every shaft, no matter how short, is deflected under load; this deflection, although too small to be seen in many cases, can at least be calculated. In order to take the end thrust of such a shaft, it is customary to provide a thrust bearing having the lower surface of the

bottom race of spherical form. The seat for this bearing is machined to the same form. This permits the shaft to deflect and the bearing to move as a complete unit with the shaft. It assures that the plane of the bearing will always be perpendicular to the axis of the shaft, and that the load will be uniformly distributed among all the balls. The use of such a bearing has two disadvantages: First, in the machining of the seat, a special fixture is required to allow the cutting

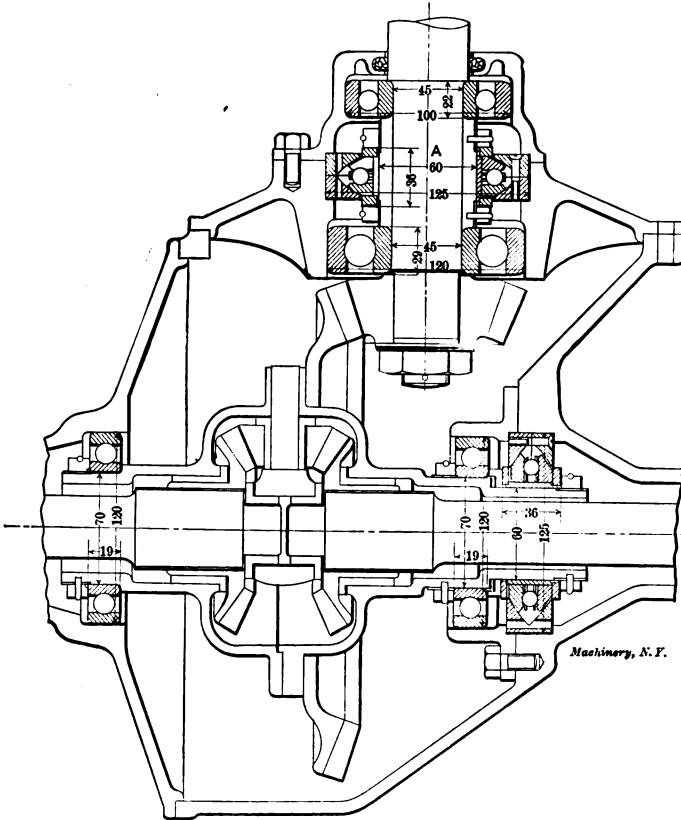


Fig. 49. Automobile Rear Axle and Differential Gear equipped with Double Thrust Bearing

tool to swing over the required radius of curvature of the seat; second, the material of which the races are made is hardened, while the seat for the bearing is machined out of ordinary unhardened material, so that appreciable wear will take place in a comparatively short time. There need be no objection to this bearing, however, if the speeds of rotation are very low, or the loads are very light. To overcome these disadvantages, the Société Française des Roulements à Billes supplies a thrust bearing having the lower race made of two parts, one

floating within the other as shown in Fig. 48. These two parts are made of the same material and heat treated in the same way, so that there is practically no wear even when the speeds of rotation are high and the loads great. All that is necessary in mounting the bearing, is to machine an ordinary flat seat; no special tools or fixtures are required.

In order to assemble the two parts of the lower race, the bottom portion is heated in oil, and the upper portion forced into position under pressure. The advantage of this arrangement is, that the lower race, although made up of two parts, can be handled as a complete unit. An exterior view of this race is shown in Fig. 47.

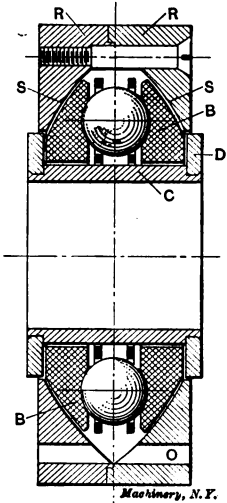


Fig. 50. Bearing Designed for Axial Thrust in Either Direction

Bearing for End Thrust in Two Directions

It is often desired to take the end thrust on a shaft in both directions, as for example, where bevel or worm gearing is used, or on a marine propeller shaft. To do this, it is usual to employ two distinct thrust bearings. Such an arrangement has the disadvantages spoken of above in connection with the single thrust bearing. To permit of end thrust being taken in both directions, the Société Francaise des Roulements a Billes, manufactures a bearing of which a section is shown in Fig. 50. The bearing consists of two outer rings *R*, held together by screws, and provided with a number of oil holes *O*. The inner surfaces of these rings are machined to a spherical form to accommodate the outer surfaces *S* of the rings *B*. The rings *D* are forced on the collar *C* after expansion by heat in the manner described in connection with the

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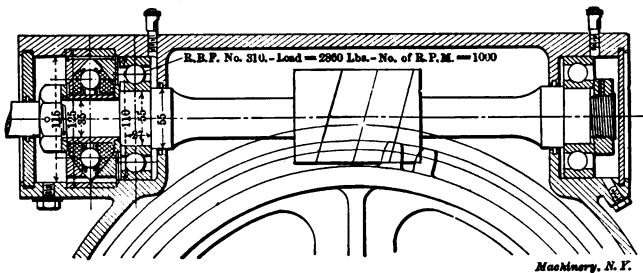
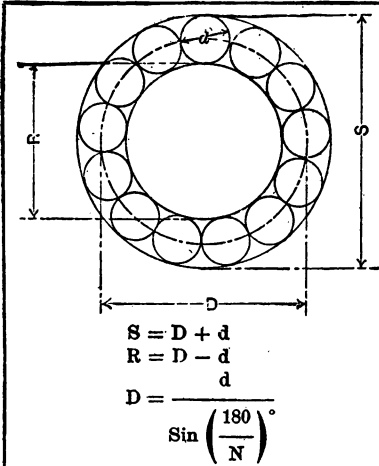


Fig. 51. Double Thrust Bearing applied to a Worm Shaft

other products of the R. B. F. Co. Every working part of the bearing is enclosed and it is entirely self-contained. One application of this bearing is shown in Fig. 49 in connection with an automobile rear axle and differential, and Fig. 51 shows an application to a worm and wheel.



**FORMULAS FOR BALL AND ROLLER BEARINGS.**

**N** = number of balls.  
**S** = diameter of enveloping cylinder.  
**d** = diameter of balls.  
**R** = diameter of enveloped cylinder.  
**D** = diameter of circle through center of balls.

**FORMULAS.**

$$S = \frac{d}{\sin\left(\frac{180}{N}\right)^\circ} + d = d \left( \frac{1}{\sin\left(\frac{180}{N}\right)^\circ} + 1 \right)$$

$$R = \frac{d}{\sin\left(\frac{180}{N}\right)^\circ} - d = d \left( \frac{1}{\sin\left(\frac{180}{N}\right)^\circ} - 1 \right)$$

**CONSTANTS FOR USE IN ABOVE FORMULAS.**

N	$\left(\frac{180}{N}\right)^\circ$	$\sin\left(\frac{180}{N}\right)$	$\frac{1}{\sin\left(\frac{180}{N}\right)^\circ}$	$\frac{1}{\sin\left(\frac{180}{N}\right)^\circ} + 1$	$\frac{1}{\sin\left(\frac{180}{N}\right)^\circ} - 1$
5	36°	.58779	1.7012	2.7012	0.7012
6	30°	.50000	2.0000	3.0000	1.0000
7	25° 42' 51.4"	.43387	2.3048	3.3048	1.3048
8	22° 30'	.38268	2.6131	3.6131	1.6131
9	20°	.34202	2.9238	3.9238	1.9238
10	18°	.30902	3.2360	4.2360	2.2360
11	16° 21' 48.3"	.28173	3.5495	4.5495	2.5495
12	15°	.25882	3.8637	4.8637	2.8637
13	13° 50' 46.1"	.23932	4.1786	5.1786	3.1786
14	12° 51' 25.7"	.22252	4.4940	5.4940	3.4940
15	12°	.20791	4.8097	5.8097	3.8097
16	11° 15'	.19509	5.1258	6.1258	4.1258
17	10° 35' 17.6"	.18375	5.4422	6.4422	4.4422

**FORMULAS FOR BALL AND ROLLER BEARINGS. (Continued.)**

N	$\left(\frac{180}{N}\right)^\circ$	$\text{Sin}\left(\frac{180}{N}\right)^\circ$	$\frac{1}{\text{Sin}\left(\frac{180}{N}\right)^\circ}$	$\frac{1}{\text{Sin}\left(\frac{180}{N}\right)^\circ} + 1$	$\frac{1}{\text{Sin}\left(\frac{180}{N}\right)^\circ} - 1$
18	10°	.17365	5.7588	6.7588	4.7588
19	9° 28' 25.2"	.16459	6.0755	7.0755	5.0755
20	9°	.15653	6.3925	7.3925	5.3925
21	8° 34' 17.1"	.14904	6.7095	7.7095	5.7095
22	8° 10' 54.5"	.14231	7.0267	8.0267	6.0267
23	7° 49' 33.9"	.13617	7.3439	8.3439	6.3439
24	7° 30'	.13053	7.6613	8.6613	6.6613
25	7° 12'	.12533	7.9787	8.9787	6.9787
26	6° 55' 23"	.12054	8.2963	9.2963	7.2963
27	6° 40'	.11609	8.6138	9.6138	7.6138
28	6° 25' 42.8"	.11196	8.9314	9.9314	7.9314
29	6° 12' 24.8"	.10812	9.2491	10.2491	8.2491
30	6°	.10453	9.5668	10.5668	8.5668
31	5° 48' 23.2"	.10107	9.8931	10.8931	8.8931
32	5° 37' 30"	.09801	10.2030	11.2030	9.2030
33	5° 27' 16.3"	.09505	10.5208	11.5208	9.5208
34	5° 17' 35.9"	.09225	10.8402	11.8402	9.8402
35	5° 8' 33"	.08963	11.1570	12.1570	10.1570
36	5°	.08716	11.4731	12.4731	10.4731
37	4° 51' 53.5"	.08481	11.7911	12.7911	10.7911
38	4° 44' 12.6"	.08258	12.1095	13.1095	11.1095
39	4° 36' 55.3"	.08047	12.4270	13.4270	11.4270
40	4° 30'	.07846	12.7456	13.7456	11.7456

**Formulas and Tables for Ball or Roller Bearings**

On pages 30 and 31 are given formulas and tables for determining the dimensions of the enveloping and enveloped cylinder in ball or roller bearings, when the diameter of ball or roller and the number of balls or rollers are known. These formulas and tables are compiled by W. B. Chapin, New Orange, N. J., and were published in *MACHINERY'S Data Sheet Supplement*, No. 25, October, 1903. For additional formulas and tables relating to ball bearings, see *MACHINERY'S Data Sheet No. 56*, April, 1906.



