A COURSE

—IN—

ELEMENTARY

MECHANICAL DRAWING.

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BY

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MINNEAPOLIS, 1882.

MINNEAPOLIS:
L. KIMBALL & CO., PRINTERS, 25 & 27 SECOND STREET SOUTH.
1882.

COURSE IN

Elementary Mechanical Drawing.

DRAWING MATERIALS AND INSTRUMENTS.

Each Student will require the following Instruments on beginning the course, viz.:—

Half-a-dozen Sheets of Drawing Paper, a Drawing-board, a T-square, a Pair of Triangles, a Hard Pencil, a Right Line Pen, a Pair of Compasses with Pen, Pencil and Needle Points, a Pair of Plain Dividers, an accurate and finely divided Scale, a piece of India Ink, a Rubber, an Irregular Curve, and half-a-dozen Thumb-tacks.

These instruments and materials are all that are absolutely required up to the time of commencing tinting and shading, when a few other articles will be needed, which will be spoken of in their proper place.

Before purchasing, the following directions about the different instruments and materials should be noticed.

PAPER.—For all the drawings of this course, use Whatman's Imperial drawing paper. It comes in sheets of convenient size, and is well adapted to the work of the course. Six sheets will be enough up to the time of tinting.

DRAWING-BOARD.—Great care should be taken to secure a good drawing-board. The best boards are those made of thoroughly seasoned white pine, one inch thick, with cleats at the ends flush with the surface of the board. The most convenient size is twenty-three by thirty-one inches. This gives a small margin outside of a whole sheet of Imperial paper, which is twenty-two by thirty-inches.

One surface of the drawing-board must be plane, and the edge from which the T-square is used must be perfectly straight.

T-Square.—All horizontal lines in the drawings are made by the use of the T-square. The T-square should be used from the left-hand edge of the board, unless the person is left-handed, in which case it should be used from the right-hand edge. The upper edge of the blade only is to be used in drawing lines. The blade should be at least thirty inches long, and about two-and-one-half inches wide. The thickness should not be over an eighth of an inch. The head should be twelve or fourteen inches long, at least, in order that the blade may never be thrown out of line. By sliding the head up and down on the straight edge of the board, any number of parallel horizontal lines may be drawn. It is very important that the upper edge of the T-square be perfectly straight.

Triangles.—For making all vertical* lines, and all lines making the angles of thirty, forty-five and sixty degrees with the horizontal and vertical lines, triangles are used, sliding on the upper edge of the T-square. Two triangles are necessary, one forty-five degree and one thirty and sixty degree, as they are called from their angles. Each of these triangles has one right angle, and either can be used for drawing verticals. It is often convenient to have one triangle large enough for drawing quite long verticals, like border lines; but in lettering and in other small work a smaller one is much more convenient. It is therefore advisable to get a thirty and sixty degree triangle that has one of its rectangular edges about ten inches long, and to get a forty-five degree triangle much smaller.

Pencils.—All lines are to be made first with a hard pencil, and are afterwards to be inked. It is very important that the pencil lines be very fine and even, though they need not be very dark.

Ink will not run well over a soft pencil line, and it is impossible to do good work without making the lines fine. The best pencils for this work are Faber's H H H H and H H H H H H or some kind equally hard and even. The H H H H is recommended for beginners who are not accustomed to using a very hard pencil, but the H H H H H H is harder, and bet-

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* By vertical lines are meant lines parallel to the edge of the board from which the T-square is used, by horizontal those parallel to the upper edge of the T-square.

ter adapted for nice work. One of each kind will be amply sufficient for the work of the whole course. The pencil should be sharpened at both ends, at one end with a common sharp round point, and at the other with the lead of about the shape of the end of a table knife. The round point is to be used in lettering and in other small work, and the flat point in making long lines. The flat point will keep sharp much longer than a round point. Both points should be sharpened often by rubbing them on a piece of fine sand paper or on a very fine file. The flat point should always be used in the compasses with the edge perpendicular to the radius of the circle.

Right-line Pen.—In selecting a right-line pen care should be taken to get one with stiff nibs, curved but little above the points. If the nibs are too slender they may bend when used against the T-square or triangles, and the result will be an uneven line. If the nibs are too open there is danger of the ink dropping out and making a blot. If too little curved the pen will not hold ink enough; the nibs are more apt to be too open, however, than otherwise. The medium sized pens are best adapted for this work. The pen must have a good adjustment screw to regulate the width of the lines. The pens, as they are bought, are generally sharpened ready for use; but, after being used for a time, the ends of the nibs get worn down, so that it is impossible to make a smooth, fine line. When this occurs they should be sharpened very carefully on a fine stone. In order to have a pen run well two things are necessary, first the points must be of exactly the same shape and length, and both nibs must be sharp. In sharpening a pen, therefore, the first thing to be done is to even the points. This may be done by moving the pen with a rocking motion from right to left in a plane perpendicular to the surface of the stone while the nibs are screwed together. After the nibs are evened in this way, the points should be opened, and each nib sharpened by holding the pen at an angle of about thirty degrees with the surface of the stone, while it is moved in about the same manner as in sharpening a gouge. The point should be examined often with a lens.

Compasses.—The compasses must have needle points, with shoulders to prevent them from going into the paper below a certain depth. The needle point, when properly used, leaves a very slight hole in the centre of each circle; while the triangular point, with which the poorer instruments are provided, leaves a very large, unsightly hole, unless used with more than ordinary care. The pencil point should be one made to contain a small piece of lead only. All that has been said in regard to the right-line pen applies equally well to the pen point of the compasses. In using the pen point be sure that both nibs press equally on the paper, otherwise it will be impossible to make an even line. Both nibs may be made to bear equally by adjusting the joints in the legs of the compasses.

Dividers.—The dividers should be separate from the compasses, as it is very inconvenient to be obliged to change the points whenever the dividers are needed. The dividers have triangular points, which should be very fine, and of the same length. The legs of the dividers should move smoothly in the joint, and not hard enough to cause them to spring while being moved, The dividers are used principally for spacing off equal distances on lines, but are often used for taking measurements from the scale, especially when the same measurement is to be used on several different parts of a drawing.

Scale.—A very good scale, for this course, is one with inches divided into fourths, eights, sixteens, etc., on one edge; and into twelfths, twenty-fourths, etc , on the other. The first edge is very convenient for taking measurements, and for making drawings to a scale of one-half, one-fourth, etc.; but the second is better for drawing to a scale of a certain number of inches to the foot. Triangular scales are still better, but more expensive.

Ink.—India ink, which comes in sticks, is the best ink for general uses ; but the Higgins' ink, in bottles, is much more convenient for making line drawings. None of the ink that comes in bottles, however, is good for shading. If the India ink is used, an ink slab or saucer will be needed in addition to the instruments mentioned in the list. In grinding India ink, a small quantity of water is sufficient, and the ink should be ground until a very fine line can be made very black with one stroke of the pen. Ink will look black in the slab long before it is fit to use on a drawing. Ink should not be ground, however, so thick that it will not run well in the pen. The ink must be kept covered up or it will soon evaporate so much as to be too thick to run well.

Rubber.—Get a soft piece of rubber so as not to injure the surface of the paper in rubbing; what is known as velvet rubber is well adapted to the draughtsman's use.

Irregular Curve.—In selecting an irregular curve, one should be obtained which has very different curvature in different parts, in order to fit curves which cannot be drawn with compasses.

Thumb Tacks.—Thumb tacks should have good large heads, so firmly fastened on that they cannot get loose.

One can do much better in buying instruments, to buy them in separate pieces, each carefully selected, than to buy them in *sets*. It is very difficult to buy a *set* of instruments that will contain just what is required for this work, without buying many unnecessary pieces.

GENERAL DIRECTIONS FOR COMMENCING THE WORK.

Each plate of geometrical problems is to be made on a half sheet of the Imperial paper. The sheet should be folded over and cut with a sharp knife, but before cutting find out which is the right side of the paper. The right side of Whatman's paper may always be found by holding the sheet up to the light. When the name of the manufacturer can be read from left to right, the right side is the one toward the holder. The half that has not the name on it should be used first, while its right side is known; the right side of the other piece can be found in the way described, when it is to be used.

Place the paper on the drawing board so that two of its edges will be parallel to the upper edge of the T-square when in position on the edge of the board; and fasten it down with three thumb tacks in each of the long sides, placing each thumb tack within a quarter of an inch of the edge, in order that the holes may be cut off when the plate is trimmed. For convenience in working on the upper part of the plate, it is best to have the paper as near the bottom of the board as possible.

Begin each plate by drawing a horizontal line, with the use of the T-square, as near the thumb tacks at the top as possible. Fourteen inches below the first line, if the longest dimension is to be horizontal, draw another parallel to it, at the bottom of the paper, and by means of the larger triangle, draw vertical lines at the right and left of the paper, twenty-one inches apart. These lines are the limits of the plate, and are the ones that the plate is to be trimmed by when finished.

All of the plates that are to be drawn on a half-sheet must be of this size, twenty-one by fourteen inches, unless the paper is to be shrunk down, in which case the plates must be made somewhat smaller, as will be afterwards noticed.

All the plates are to have a border line one inch from the finished edge, except on the top, where the border is to be one and a quarter inches from the edge. This border should next be drawn by spacing off the proper distances from the lines just drawn, and drawing the border with T-square and triangles.

There are to be eight geometrical problems to each plate, and, for convenience in locating them, the space inside of the border lines, in the first five plates, should be divided into eight equal rectangles, four above and four below a horizontal line through the center. These last lines are not to be inked, but must be erased when the plate is completed.

The first five plates in the course are of geometrical problems. The problems that have been selected have many applications in subsequent work; and, moreover, the exact construction of them gives the best of practice for beginners in handling the different instruments. The construction of each problem is described in the text, with references to the plates; and each must be constructed according to the directions. The reasons for the different constructions, though necessarily omitted in the text, will be evident to every one who has a knowledge of plane geometry.

The geometrical problems are not to be drawn to scale, but they should be so proportioned that they will occupy about the same amount of space in the centre of each rectangle.

All of the lines must be made very fine and even; and great care must be taken to get good intersections and tangencies.

DIRECTIONS FOR LETTERING.

After the problems are pencilled they must be lettered to correspond to the plates in this pamphlet. Make all the letters on the plates of geometrical problems and elementary

PLATE A.

ABCDEFGHIJKLMN
OPQRSTUVWXYZ&
abcdefghijklmnopqrstuvwxyz.,:;
1234567890
ABCDEGJKMORSW
AE
Decker.

projections, like those given in Plate A. These skeleton letters are the simplest of all mechanical letters to construct, and, when well made, they are more appropriate for such work than if more elaborate. Make the small letters, in every case, two-thirds as high as capitals.

Before making a letter draw a small rectangle that will just contain the letter, and then construct the letter within the rectangle, as shown in plate A, using instruments wherever possible. The height of all the capital letters in the problems and in the general title at the top, is to be one quarter of an inch. The widths vary, and may best be found in each case, until practice renders it unnecessary, by consulting plate A. Great care must be taken in lettering to make all the lines of the letters of the same size, and in joining the curves and straight lines.

TITLES.

The title of each plate of geometrical problems must correspond to that given in plate I, except as to number. The titles of the projection plates will correspond to that of plate VI, except as to number, and the titles of all other drawings will be as indicated in the text. In constructing a title always work both ways from the central letter of the title, in order that the title may be symetrical, and over the center of the plate. In order to find the middle letter of the title, count the number of letters, considering the space between words as equal to that of a letter, and divide the number of spaces thus found by two; this will give the number of the middle letter from either end of the title. Construct this letter over the center of the plate, and then work both ways from this in the way just indicated. Make the letters in a word about an eighth of an inch apart, though the space will vary with the shape of the letter; and the space between words equal to that of an average letter with its spaces.

It is best, in all cases, to have the title before you in rough letters, to avoid making mistakes in working backwards from the middle letter. The titles at the top are to be made in capitals. The letters in the general title are to be a quarter of an inch high and a quarter of an inch above the border, and those in number of the plate of letters three-sixteenths of an inch high and the same distance above the general title.

The name of the draughtsman should be, in the first seven plates, at the lower left-hand corner, three-sixteenths of an inch below the border, and the date of completion in a corresponding position at the right. Make the date first, and commence the name as far from the edge, at the left, as the last figure of the date comes from the right-hand edge. Make the capitals in name and date three-sixteenths of an inch high.

Number the problems as they are in the plates, commencing the first letter of the abbreviations for problems in capitals, one-half an inch below, and half an inch to the right of the lines forming the upper right hand corner of the rectangle. The other letters of the abbreviations are to be small, and the numbers of the problems are to be marked with figures of the same height as the capitals.

Great pains must be taken in lettering the plates, as the general appearance of a drawing is very much affected by the arrangement and construction of the letters and titles. The directions here given apply to the plates of geometrical problems. Some modifications will be made in lettering the problems in projection ; but the remarks on the construction of the separate letters, and on the arrangment of the letters in a title, are general. After having had the practice in spacing and proportioning the skeleton letters, in the first seven plates, the student will be allowed to use other styles of letters on the remaining work. Care must be taken, however, to have the titles symetrical, and no letters on the plates of this course should be made over half an inch high.

INKING.

When the lettering is all done, a plate is ready to be inked. Before using the pen on the plate, be sure that it is in a condition to make a fine even line, by testing it on a piece of drawing paper, or on the part of your paper that is to be trimmed off. Be sure to have ink enough ground to ink the whole plate, as it is not best to change the ink while working on a plate, for the reason that it is nearly impossible to get the second lot of the same shade and thickness as the first. The arcs of circles should be inked first, for it is easier to get good intersections and tangencies by so doing, than it is if the straight lines are drawn first. *Make all the given lines and all the required lines in full ; but all the construction lines in fine*

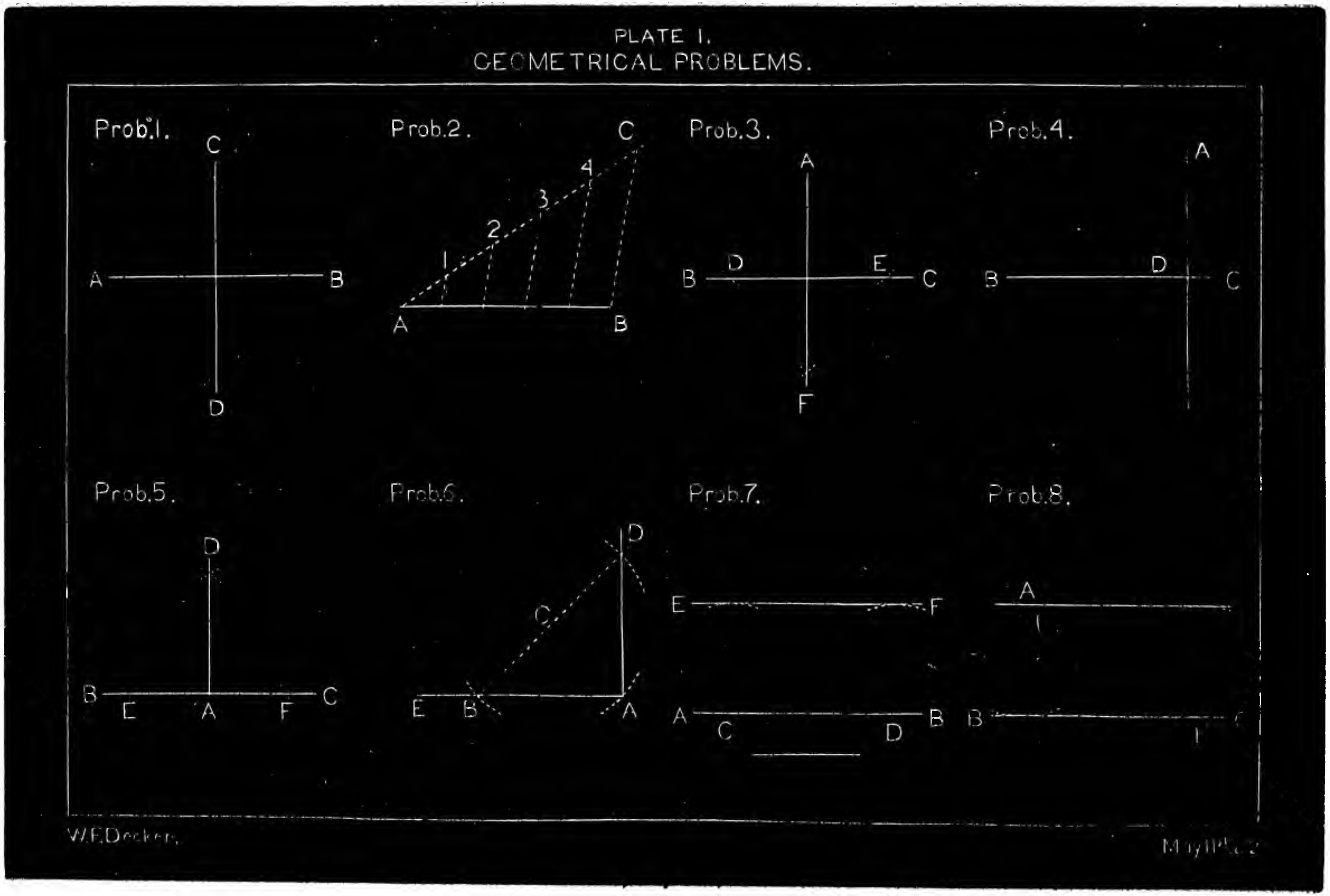


PLATE I.
GEOMETRICAL PROBLEMS.
Prob. 1.
Prob. 2.
Prob. 3.
Prob. 4.
Prob. 5.
Prob. 6.
Prob. 7.
Prob. 8.
W.F. Decker.

dots. Make all the lines in the geometrical problems as fine and even as possible. The border lines should be made a little heavier than the others. All the fine lines should be made, if possible, with one stroke of the pen. In order that an even line may be made, the pen must be held so that both nibs will bear on the paper equally ; and in order to do this, the T-square or triangle must be held a little way from the line, but parallel to it. The pen should be inclined slightly in the direction it is moved.

In using the compass pen, the joints of the compass legs should be so adjusted that the point will bear equally on both nibs.

The ink should be placed in the pens by means of a quill or a thin sliver of wood. The pen should never be dipped into the ink.

THE PLATES.

The plates in this pamphlet are given to show the arrangement and construction of the problems, but should not be followed as examples too closely, as mechanical difficulties make it necessary to use coarser lines in proportion to the size of the plates than should appear on the drawings.

GEOMETRICAL PROBLEMS.

Problem 1. To bisect a given line, A B, or to erect a perpendicular at the middle point of A B.

From A and B as centres, with a radius greater than one-half of A B, described two arcs intersecting at C, and two arcs intersecting at D. Join C and D by a straight line, it will bisect A B, and will be perpendicular to it.

Prob. 2. To divide a given line, A B into any number of equal parts, five for instance.

Draw a line, A C, making any angle with A B, and on A C set off *any* five equal distances, A 1, 1 2, 2 3, 3 4 and 4 C ; join C and B, and through 1, 2, 3 and 4, draw lines parallel to C B, these lines will divide A B into equal parts.

Prob. 3. To draw a perpendicular to a line B C, from a point A, without the line.

From A as a centre, and with any radius, describe an arc, cutting B C in D and E. From D and E as centres, describe two arcs intersecting in F. Join F with A.

Prob. 4. To draw a perpendicular to a line B C from a point A, nearly over one end, C, so that problem 4 cannot be used.

From any point B, on the given line as a centre, describe an arc passing through A. From some other point D, of B C, describe another arc passing through A. Join A with the other point of intersection of the arcs.

Prob. 5. To erect a perpendicular to a line B C, at a given point A, of the line.

Set off from A, the equal distances A E and A F, on either side. From E and F as centres, with any radius greater than A E and A F, describe two arcs, intersecting at D. Join D with A.

Prob. 6. To erect a perpendicular to a line A E, at a point A, at or near one end of A E, where problem 5 cannot be used.

From some point C, outside of A E, as a centre, and C A as a radius, describe an arc cutting A E in A and B. Draw a line through B and C, and produce it till it meets the arc again in D. Join D with A.

Prob. 7. To draw a line parallel to a given line A B, at a given distance from A B.

From two points C and D, of A B, which should not be too near together, describe two arcs, with the given distance as a radius. Draw a line E F tangent to these arcs.

Prob. 8. Through a given point A, draw a line parallel to a given line B C.

With A as a centre, describe an arc which shall be tangent to B C. With some point, F, of B C as a centre, and with the same radius as before, describe an arc. Draw through A a line tangent to the last arc.

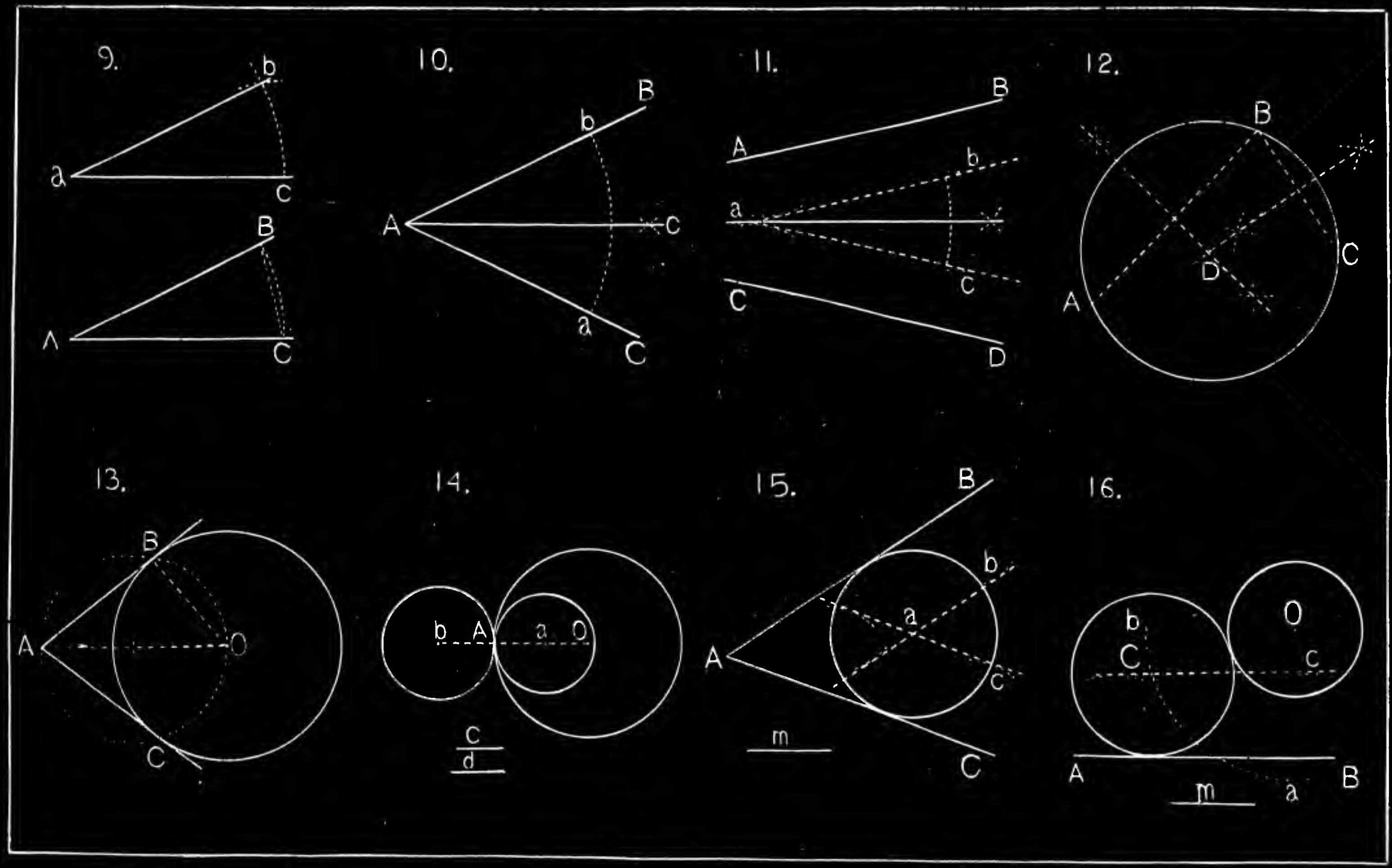
Prob. 9. To lay off an angle, at a given point a, on a given line a c, equal to a given angle B A C.

With A as a centre and any radius, describe an arc included between B A and A C. With a as a centre and the same radius, describe an indefinite arc. Lay off the chord b c equal B C from c on the arc b c. Join b with a.

Prob. 10. To bisect a given angle B A C, whose vertex A is within the limits of the drawing.

From A as a center describe an arc, cutting A B and A C in b and a respectively. From b and a as centres describe two arcs intersecting in c. Join c with A.

PLATE II.


9.
b
a
c
B
A
C
10.
B
b
A
c
a
C
11.
B
A
b
a
c
C
D
12.
B
C
D
A
13.
B
A
O
C
14.
b
A
a
O
c
d
15.
B
b
a
A
c
m
C
16.
b
O
C
c
A
B
m
a

Prob. 11. To bisect an angle B A—C D, whose vertex is not within the limits of the drawing.

Draw by Problem 7, two parallels, a b and a c to A B and C D respectively, and at the same distance from A B and C D; this distance must be such that a b and a c shall intersect. The problem is then reduced to one of bisecting b a c, which is done by problem 10.

Prob. 12. To pass the circumference of a circle through three points, A, B, C.

Draw the lines A B and B C. Bisect A B and B C by perpendiculars, by problem 1. With D, the intersection of these perpendiculars, as a centre, and D A as a radius, describe a circumference, it will pass through A, B and C.

Prob. 13. To draw two tangents to a circle, whose centre is O, from a point A, without the circle.

Join A with O; on O A as a diameter, describe a circle. Join the points B and C, in which the latter circle intersects the given one, with A. A B and A C will be the required tangents.

Prob. 14. To draw circles, with given radii, c and d, tangent internally and externally respectively to a circle whose centre is O, at a given point A.

First, internally. Join A with O. Lay off on A O from A, A a equal to c. From a as a centre, and A a as a radius, describe a circle.

Second, externally. Prolong O A, and lay off from A, A b equal to d. With b as a centre and A b as a radius, describe a circle.

Prob. 15. To draw a circle with a given radius m, tangent to two given lines A B and A C.

Draw a b and a c parallel to A B and A C respectively, and at a distance from them equal to m. With the point of intersection a, of a b and a c as a centre, and m as a radius, describe a circle.

Prob. 16. To draw a circle with a given radius m, tangent to a given circle O, and to a given line, A B.

With O as a centre and O a equal to m plus the radius of the given circle, as a radius, describe an arc, a b. Draw the line C c parallel to A B, and at a distance m from it, by problem 7. With C, the intersection of the arc and parallel, as a centre, and m as a radius, describe a circle.

Prob. 17. To draw a circle tangent to a given circle O, at a given point A, and to a given line B C.

Join O A ; at A draw the tangent A B, perpendicular to O A, and produce it till it meets B C at B. Bisect the angle A B C, by the line B a, by problem 10. Produce B a till it meets O A produced in D. With D as a centre, and D A, a radius, describe a circle.

Prob. 18. To draw a circle, tangent to a given line A B, at a given point C, and to a given circle O.

At C, draw D C, perpendicular to A B, by problem 5, and produce D C below A B till C a is equal to the radius of the given circle. Join a with O, and by problem 1, erect a perpendicular D b at the middle point of O a. With the intersection D, of D a and D b, as a centre, describe a circle.

Prob. 19. To draw a circle tangent to a given circle C, at a given point A, and to a given circle O.

Join A C, and produce it till A D is equal to the radius of the other circle. Join D with O, and bisect O D by a perpendicular E a, by problem 1. With E, the intersection of E a and A D produced as a centre, and E A as a radius, describe a circle.

Prob. 20. Given two parallels, B A and C D, to draw a reversed curve which shall be tangent to them at A and C.

Join A C. Bisect A C in 2, which will be the reversing point. Bisect A 2, and 2 C by perpendiculars, 1 F and 3 E. Draw A F and C E perpendicular to B A and C D, and with the intersection E of C E and 3 E, and the intersection F of 1 F, and F A, as centres, and radii, equal to E C or A F, describe two arcs.

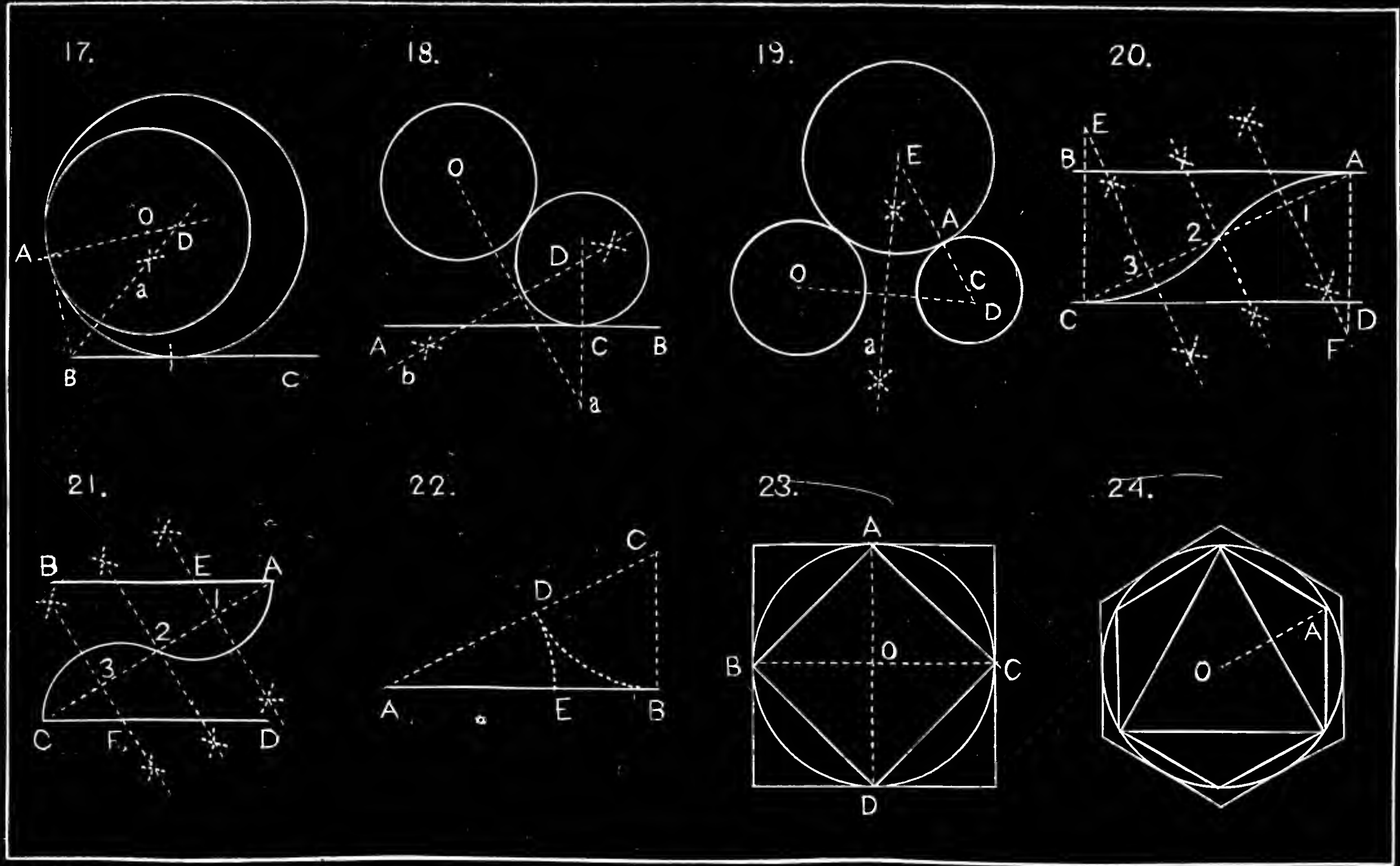
Prob. 21. Given two parallels, B A and C D, to draw a reversed curve, whose tangents at A and C shall be perpendicular to B A and C D.

Join A C. Bisect A C in 2, which will be the reversing point. Bisect A 2, and 2 C by perpendiculars 1 E and 3 F. With the intersection E, of 1 E and B A, and the intersection F, of 3 F and D C, as centres, and radii equal to E A or F C, describe two arcs.

Prob. 22. To divide a given line A B in extreme and mean ratio.

Bisect A B by problem 1. At one extremity B, erect a perpendicular B C, and lay off on it B C, equal to one-half of A

PLATE III.


17.
A
O
D
a
B
C
18.
O
D
A
b
C
B
a
19.
E
A
O
C
D
a
20.
E
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A
1
2
3
C
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F
21.
B
E
A
1
2
3
C
F
D
22.
C
D
A
E
B
23.
A
O
B
C
D
24.
O
A

B. With C as a centre and B C as a radius, describe an arc, cutting A C at D. With A as a centre, and A D as a radius, describe an arc, cutting A B in E. Then E will divide A B in extreme and mean ratio.

Prob. 23. Given a circle O, to inscribe and circumscribe squares.

Through O draw two diameters, B C and A D, perpendicular to each other. Join their extremities, A, B, C and D, for the inscribed square. To circumscribe a square, draw lines through A and D parallel to B C, and through B and C, parallel to A D.

Prob. 24. Given a circle O, to inscribe and circumscribe regular hexagons, and to inscribe a regular triangle.

Lay off the radius O A, six times as a chord on the circumference, for the inscribed hexagon. For the circumscribed hexagon, draw parallels to the sides of the inscribed figure, which shall be tangent to the circle.

Join the alternate points of division of the circle for the inscribed regular triangle.

Prob. 25. Given a circle O, to inscribe and circumscribe regular octagons.

By problem 23, obtain the sides A B, and so on of an inscribed square. Bisect these chords by perpendiculars, and thus the arcs subtended by them. Joins the points C, &c., with the vertices of the square for the inscribed octagon. For the circumscribed octagon, proceed as in circumscribing the regular hexagon in problem 24.

Prob. 26. Given a circle O, to inscribe and circumscribe regular decagons.

By problem 22, divide the radius O B, in extreme and mean ratio at b. Apply the larger portion O b, ten times to the circumference as a chord. Circumscribe a regular decagon from the inscribed, as in the last two problems

Prob. 27. Given a circle O, to inscribe and circumscribe regular pentagons.

Obtain the vertices of an inscribed regular decagon by problem 26, and join alternate vertices. Proceed for the circumscribed pentagon, as usual.

Prob. 28. To construct a regular polygon with a given number of sides, five for instance, the sides to be of a given length A B.

On A B as a radius describe a semi-circle. Divide the semi-circumference into five equal parts, A 4, 4 3, and so on. Omitting one point of division 1, draw radii through the remaining points and produce them. With 2 as a centre, and A B as a radius, describe an arc cutting B 3 produced in C; B 2 and 2 C will be two sides of the polygon. With C as a centre, and A B as a radius, describe an arc cutting B 4 produced in D; C D will be another side. Continue this construction; the last point should come at A.

Prob. 29. On A B and C D as major and minor axes, to construct an ellipse.

We proceed on the principle that the *sum* of the distances of any point of an ellipse from the foci is equal to the major axis. We must first fix the position of the foci. From C as a centre, and O B as a radius, describe two arcs, cutting A B in a and b, these are the foci. To apply the principle just mentioned, take the distances from any point, as c of A B to A and B as radii, and a and b as centres. By describing arcs above and below A B, and using both radii from each centre, four points of the ellipse will be obtained. Other points are obtained by taking other points on A B, and proceeding in the same way. Connect the points found in this way by using the irregular curve. In using the irregular curve always be sure to have it pass through at least three points.

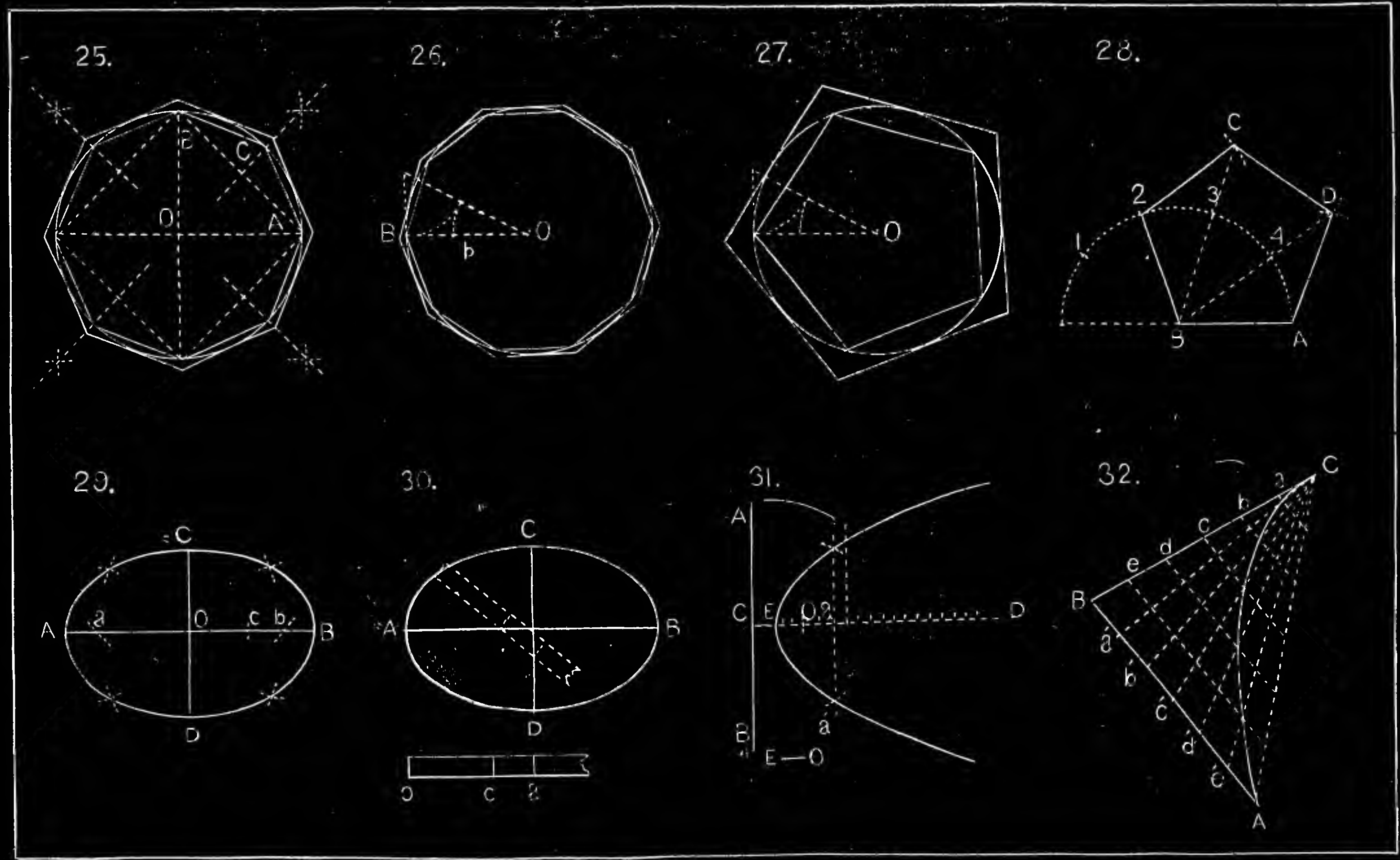
Prob. 30. On A B and C D as major and minor axes, to construct an ellipse; another method.

On the straight edge of a slip of card board or paper, set off three points o, c, a, the distance o a being equal to the given semi-major axis, and o c to the semi-minor. Place the slip in various positions such that a shall always rest on the minor, and c on the major axis. The various positions marked by the point o will be points of the ellipse.

Prob. 31. To construct a parabola, given the focal distance O E.

We proceed on the principle that the distance of any point from a line A B, called the directrix, is equal to its distance from a certain point called the focus. Draw the indefinite line A B, for the directrix, and C D perpendicular to it. From C, lay off C E and E O each equal to the focal distance. The point O is the focus. Draw a number of perpendiculars to C D at various points. To find the points in which the parabola

PLATE IV.



intersects any one of them as a a', describe an arc with O as a centre, and a C, the distance from that perpendicular to C as a radius. E is the point of C D through which the curve will pass.

Prob. 32. To construct a parabola between the points C and A on two given lines C B and A B, and tangent to the former.

Divide C B into any number, as five, equal parts, C a, a b, b c, &c., and B A into the same *number* of equal parts, B a', a' b', &c. Connect C with a', b', c', d' and A. Draw from a, b, c and d, lines parallel to A B. The points of the curve will be those in which the parallel from d meets C d', the parallel from c meets C c', that from b, C b', and so on.

Prob. 33. To construct a hyperbola, having given the distances A O and a O, on the horizontal axis, from the centre to either vertex, and from the centre to either focus.

In the hyperbola, the *difference* of the distances of any point from the foci is equal to the distance between the vertices, as in the ellipse the sum. Lay off from O the equal distances A O and O B to the vertices, and the equal distances O a and O b to the foci. To obtain any point of the curve, take any point on the axis as c ; with c A and c B as radii, and a and b as centres, describe four pairs of intersecting arcs, as in the ellipse ; the points of intersection will be points of the hyperbola. By taking other points on the axis, other points of the curve will be obtained in the same manner.

Prob. 34. To construct a curve similar to a given curve B A C, and reduced in a given ratio, one-half for instance.

Draw some indefinite line, a centre-line, if possible, in the given curve, as A D. On A D, lay off a number of distances ; at the points of division, erect perpendiculars to meet the curve above and below A D. Draw an indefinite line a d, and on it lay off distances bearing respectively to those laid off on A D, the given ratio. Through these points of division draw perpendiculars, and lay off on them above and below a d, distances bearing the given ratio to those on the perpendiculars to A D.

Prob. 35. To describe a given number of circles, six, for instance, within a given circle O, tangent to each other and to the given circle.

Divide the circumference of the given circle into twice as many parts as the number of the circles to be described, 1 2, 2 3, &c. To obtain the first circle draw a tangent to the given circle at 1, 1 a; produce O 2 to 1 a, at a. Lay off on O a, from a inwards the distance 1 a to b, since tangents to a circle are of equal length. At b draw a perpendicular to O a, meeting O 1 in c. With c as a centre and c 1 as a radius describe a circle. From O as a centre and O c as a radius, describe a circle intersecting the alternate radii O 3, O 5, &c., in points which will be centres of the required circles.

Prob. 36. To construct a mean proportional to two given lines A D and D B.

With the sum of these lines as a diameter, describe a semicircle A C B. At the point D, between the two lines, erect a perpendicular, meeting the circumference at C. D C will be the mean proportional required.

Prob. 37. To divide a line a b into the same proportional parts as a given line A B is divided by the point C. Draw a b parallel to A B, and draw lines through A and a, B and b, till they meet in d. Draw C d, the point c will divide a b into the same proportional parts as C divides A B.

Prob. 38. To draw a tangent to the ellipse O, at a given point A. Find the foci F and F' as in problem 29. Draw A F and A F' and bisect the angles between these lines by problem 10. Draw A C at right angles to A B. It will be the required tangent.

Prob. 39. Given a square A B C D, to cut off the corners so as to form a regular octagon.

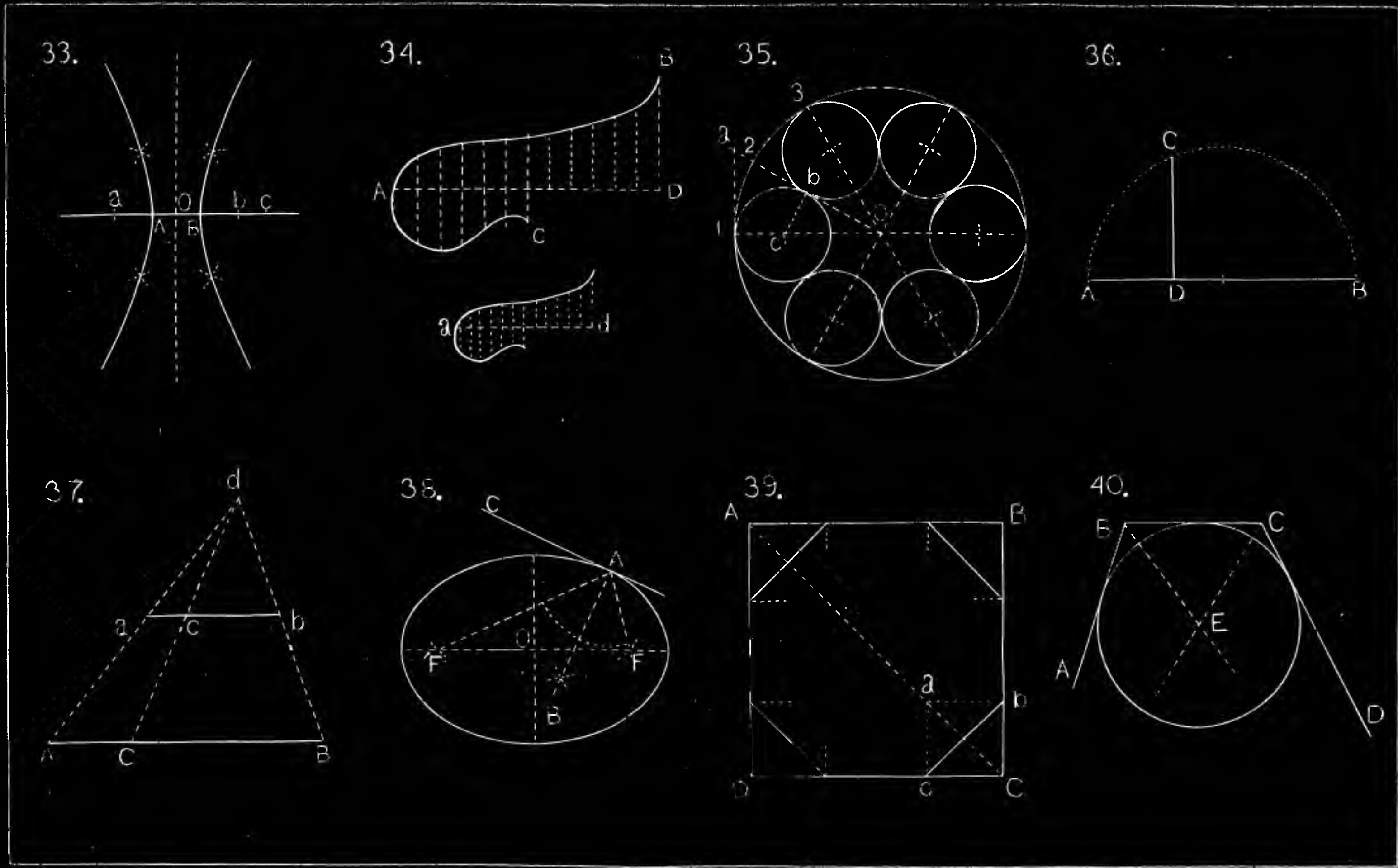
Draw the diagonal A C. From A, lay off A a, equal to A D. From a draw a c and a b, parallel to the sides of the square. Join b c, which will be one of the sides of the octagon. To obtain other vertices, lay off from the other vertices of the square distances equal to C c.

Prob. 40. To draw a circle, tangent to three straight lines, A B, B C, and C D.

Bisect the angles B and C, by problem 10. Produce the bisecting lines till they meet at E. With E as a centre and the distance to either line as a radius, describe an arc.

A special case of this problem, when the three lines form a closed figure, gives a circle inscribed in a triangle

PLATE V.



PROJECTIONS.

If we wish to represent a solid body by drawing, and, at the same time, to show the true dimensions of that body, we must have two or more views, or *projections*, of it on as many different planes. Take for example a cube. In order to show it in a drawing, we must have views of more than one face, in order to show that the body has three dimensions. We will consider this cube to be behind one plate of glass and below another, and in such a position that two of its faces are parallel to these plates, which are respectively vertical and horizontal. Now suppose that perpendiculars are dropped from every corner of the cube to each of these plates. The points where these perpendiculars pierce the surfaces of the plates, are called, respectively, the vertical and horizontal projections of the corners of the cube. If these points be joined by lines, corresponding to the edges of the cube, we shall have in this case, exact figures of the two faces of the cube that are parallel to these plates. These two figures are called, respectively, the vertical and horizontal projections of the cube, according as they are on the vertical or horizontal plates. In this way we may get two views of any solid object, supposing it to be in such a position as that of the cube, in the case just noticed, with reference to two plates of glass, which we will now call the *vertical and horizontal planes of projection.*

If the object has a third side very different from the two shown in this way, we may consider it to be projected on a third plane perpendicular to the two others, and on the side of the object to be represented.

A fourth side may in the same way be represented on a fourth plane; but three projections are generally all that are needed to show even very complicated objects; and in most cases two projections, a vertical and a horizontal, are all that are necessary, lines on the opposite faces being shown by dotted lines on the projections of the faces toward the planes.*

* We have considered the planes of projection to be in front and above the object to be represented, but drawings are often made with the planes behind and below. It is, however, believed that the method given is better for practical use. Details are often shown as projected on oblique planes, as planes parallel or at right-angles to the axis of an object.

As it is not convenient to have two or more separate drawings of an object on different planes, as would be necessary if we were to represent the projections of the body in their true positions; we may consider that the body has been projected in the manner indicated, and that the planes of projection have been revolved about their intersections so as to bring them all into the horizontal plane, with the end views, if any, on the right and left of the vertical projection, and the horizontal projection above the vertical. In this way we may bring all the different views or projections into the plane of the top surface of the drawing paper; and by representing the intersections of the planes of projection by lines, we may show all the projections in their true relative position in one drawing.

The line that represents the intersection of the vertical and horizontal planes of projection, is called the *ground line.* The ground line, as well as the other lines of intersection of the planes of projection, is often omitted in actual drawings.

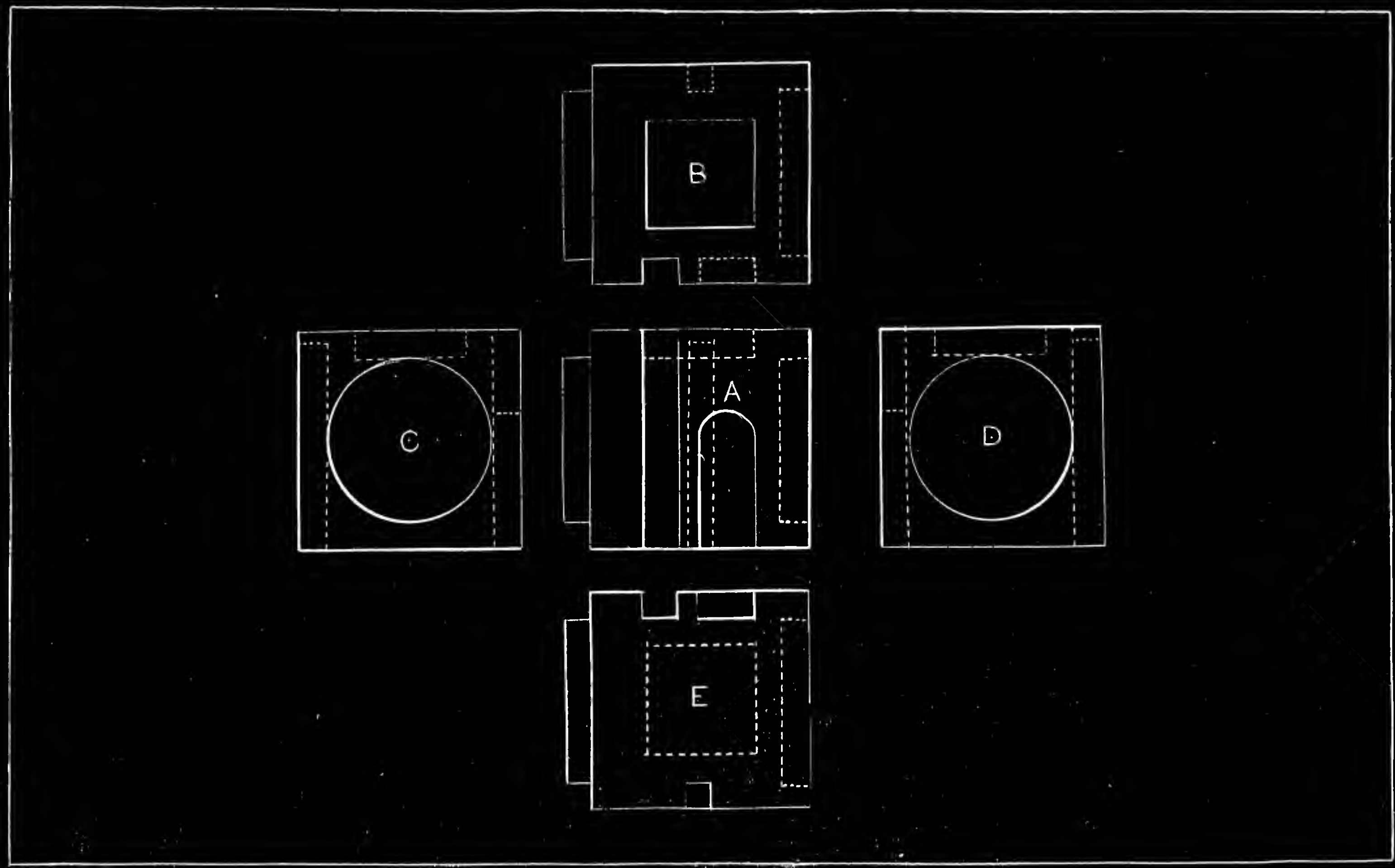
It will appear on consideration of the method of projection that the distances of the projections of any point from the ground line show the true position of the point in space, with reference to the planes of projection. Suppose, for example, the horizontal projection of a point to be one inch above the ground line, and the vertical projection to be two inches below the same line, this shows that the true position of the point in space is one inch back of the vertical plane and two inches below the horizontal plane. Moreover, it may easily be demonstrated that the two projections of a point always lie in a common perpendicular to the ground line.

As lines are determined by locating points in them, the principles just given apply in getting the projections of any figure that can be represented by lines. In the problems in projection, following, the ground lines must be drawn and the points located in the manner just indicated.

An object may be in any position whatever with reference to the planes of projection; but for convenience the body is usually considered to be in such a position that the vertical projection will show the most important view of the object, such, for example, as the front of a building.

In representing an object of this kind in projection, the front of the object is usually considered parallel to the vertical plane of projection.

PLATE B.



The vertical projection of an object is called its *elevation*, and the horizontal projection, its *plan.* The other projections are called *end views*, or *sections*, according as they represent an end or some part cut by a plane passing through the object.

By the method of projection just explained each projection represents the view of the object a person would have were the eye placed on the side of the object represented by the projection and at an infinite distance from it. When an object is viewed from a finite distance it is seen in perspective and not as it really is. Projections show an object as it really is, and not as it appears in perspective. Projections are therefore used to represent bodies in their true form and are employed as working drawings, in which a body to be constructed is represented as it would appear in projection when finished.

Plate B shows the projections of a cubical object having two slots in its front face, a rectangular depression in the top, a cylindrical depression in the right hand face, a cylindrical projection on the left and a slot running nearly through from the bottom on the back face. A is the *elevation*, B the *plan*, C the *left hand end view*, D the *right hand end view*, and E the *bottom view.*

This plate is not to be drawn, but is given to illustrate this method of projection. By a a careful study of this plate it may readily be seen how the lines of an object will appear in the several projections.

LINE SHADING.

In order to give the projections of a body the appearance of relief, the light is supposed to come from some particular direction, and all lines that separate light faces from dark ones are made heavy.

The direction of the light is generally taken for convenience at an angle of forty-five degrees from over the left shoulder as the person would stand in viewing the projections, or in making the drawing; and in all cases the projections of this pamphlet are to be shaded with the light so taken.

It will be readily seen on considering the direction of the light, that the elevation of a solid rectangular object with plane faces in the common position, will have heavy lines at the lower and right hand sides, and that the plan will have heavy lines on the upper and right hand sides.

When a body is in an oblique position with reference to the planes of projection, the heavy lines of the projections may be determined by using the forty-five degree triangle on the T-square. If we apply this triangle to the T-square, so that one of its edges inclines to the T- square at an angle of forty-five degrees upward, and to the right this edge will represent the horizontal projection of a ray of light; and by noticing what lines in the plan of the object this line crosses, it may readily be seen what faces *in the elevation* will receive the light and what faces will be in the shade. By applying the triangle so that an edge will make an angle of forty-five degrees downward and to the right, this edge will represent the vertical projection of a ray of light, and by applying it to the elevation, the faces *in the plan* that will be in light and shade may be determined.

Where the limiting line of a projection is an element of a curved surface as in the elevation of a vertical cylinder, that line should not be shaded. The plan of the vertical cylinder, which is a circle, should be shaded, for the circumference is an edge separating light from dark portions of the object. In this case the darkest shade of the line should be where the diameter, that makes an angle of forty-five degrees to the right with the T-square cuts the circumference above, and the lightest part should be where this diameter cuts the circumference below. The dark part should taper gradually into the light part.

Lines that separate parts of a body that are flush with each other, as in joints, should never be shaded, and when a line that would otherwise be shaded, rests on a horizontal plane as in the first positions of the following problem, it should not be shaded.

The shaded lines of a projection need not be very heavy if the light lines are made, as they should be, as fine as possible.

Plate B is shaded correctly for the light at forty-five degrees from over the left shoulder, and a study of it will illustrate all that has been stated as to the proper lines to be shaded and the relative widths of the light and heavy lines.

THE PLATES OF PROJECTIONS.

The plates of projections are to be of the same size as those of geometrical problems, but for convenience in showing three positions of each object adjacent to each other, the top of these plates will be taken at one of the short edges, and each plate will contain nine problems in the position shown in the cuts. The border should be drawn first, leaving a margin of one and a quarter inches at the top as in the previous plates. Divide next the space within the border line into nine equal rectangles, by drawing two vertical and two horizontal lines. Draw a ground line in each rectangle two inches and a half below the top, making the ends of the ground lines within a quarter of an inch of the vertical lines dividing the space. At a distance of two inches and three-quarters below the ground line draw a broken horizontal line, as shown in the plates.

The objects projected in the following problems are all supposed to rest on a horizontal plane below the horizontal plane of projection and behind the vertical plane of projection. The ground line, as has already been noticed, represents the intersection of the two planes of projection before the vertical plane is revolved into the plane of the horizontal. The broken line below represents the intersection of the vertical plane with the plane on which the body rests.

In order to get a good conception of the position of the object, suppose that the body rests on the drawing table, and that a plate of glass be held above and parallel to the table, and another plate be held in front and vertical. The position of the object in relation to the planes represented by the plates of glass will be the same as that of the cube, which we considered in explaining projections in general.

The ground line and the broken line below will represent in this case, respectively, the intersections of the plates of glass with each other and with the top surface of the drawing table. A sheet of paper may now be put in place of the horizontal plate of glass, and it will represent the revolved position of the planes precisely as they are in the drawing.

In the descriptions of the problems, G L refers to the ground line, or intersection of the planes of projection, and G' L' refers to the line of intersection of the vertical plane of projection with the plane on which the body rests.

The plates must be lettered to show the general title, and the numbers of the problems, as shown in Plate VI. The letters used in describing the problems, however, need not be drawn.

All lines that would not be seen from the position indicated by the projection in question, must be indicated by fine dots. The problems in the finished plates must be shaded according to the directions above.

PROBLEMS IN PROJECTION.

Problem 1. To construct the projections of a prism one and a quarter inches square at the base and two and a quarter inches in height, of whose faces one rests on a horizontal plane, and one is parallel to the vertical plane of projection. Draw the square, A B C D, equal to the top face of the prism, above G L, with C D one-quarter of an inch from G L and parallel to it. Draw from C and D lines perpendicular to G L, and prolong them below until they intersect G′ L′. Measure off the height of the prism from G′ L′, and draw a horizontal line for the top line of the elevation. The rectangle, E F G H, formed below the line last drawn will be the elevation of the prism, and the square above G L will be the plan. No dotted lines will appear in this problem, as all the lines on the opposite sides of the object will be covered by the full lines in front. Shade according to the directions above.

Prob. 2. To revolve the prism of Problem 1 through a given angle about an edge through H, so that the planes parallel to the vertical plane shall remain so.

Locate H on G L as far to the right in its rectangle, as H, in Problem 1, is from the border line. As the revolution has been parallel to the vertical plane, the *elevation* will be unchanged in form and dimensions, but will be inclined to G L. Lay off G H, making the given angle of revolution with G′ L′. Complete E F G H, on G H as base. Since the body has revolved parallel to the vertical plane, the horizontal projections of lines perpendicular to the vertical plane as A C and B D, have not changed in length, but those of lines parallel to the vertical plane, as A B and C D, will be shortened.

Considering these facts, and that the two projections of a point are in the same line perpendicular to G L, the following is seen to be the construction of the plan:

Prolong A B and C D of Problem 1, indefinitely to the right. As E F G H represent the same points in both elevations, erect perpendiculars from each of these points in Problem 2, intersecting the indefinite lines just drawn, for the plans of the same points. A B C D and I J K L will be the bases of the prism in its revolved position. K L is to be dotted because not seen. Shade as directed above.*

Prob. 3. To revolve the prism, as seen in its last position, through a horizontal angle, that is about a line through H, perpendicular to the horizontal plane.

As the revolution is parallel to the horizontal plane, the *plan* is changed only in position, not in form or dimensions.

Therefore, draw the plan of Problem 2 inclined to G L at the angle of revolution, taking L in L D produced from Problem 2.

Now as each point of the body revolves in a horizontal plane, its vertical projection will move in a straight line parallel to G L. Hence we make the following construction for the elevation: In the case of any point, as B, in the plan, draw a perpendicular from this point to G L, and from F, which is the elevation of B in Problem 2, draw an indefinite line parallel to G L. The intersection of these lines gives the elevation of the point in its revolved position. Proceed in the same way with all the other points.

It will be noticed that in the three positions of the body just taken the plan is drawn first in Problem 1, the elevation first in Problem 2, and the plan first in Problem 3. The reasons for so proceeding are evident from the constructions.

This order will hold true in all the problems following.

Prob. 4. To construct the projections of a square prism of the same size as in Problem 1, whose lower base is on a horizontal plane, and whose lateral faces make angles of forty-five degrees with the vertical plane.

Lay out the plan, A B C D, a square, with its sides at forty-five degrees to G L, and with C one-quarter of an inch above it. Draw perpendiculars from its vertices to G L, and prolong them below G L, as in Prob. 1, for the vertical edges in the elevation. Join the extremities of these equal vertical lines.

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*Nothing more will be said about shading, but it is to be understood that each projection is to be shaded, with the light taken from over the left shoulder at an angle of forty-five degrees with the horizontal plane. The shading is a very important point in these problems.

Prob. 5. To revolve the prism of Prob. 4, through a given angle, about the lower right-hand corner, so that the vertical edges shall revolve in planes parallel to the vertical plane of projection.

In this problem, as in Prob. 2, the revolution will be such that the vertical projection will be changed only in position. Lay off, then, E F, making with G L the given angle of revolution, and draw the elevation of Problem 4 on E F as a base. Draw the plan, as in Problem 2, by erecting perpendiculars from each point in the elevation, and by drawing indefinite lines from corresponding points in the plan of Problem 4. The intersection found in this way will be the points in the plan. The revolution being the same as in Problem 2, the reasons given there apply to this case as well.

Prob. 6. To revolve the prism of Problem 5 through a given horizontal angle about E.

Draw the plan of Problem 5, inclined to G L, at the given angle of revolution locating the plan of the point E, on the same horizontal line as it is in Problem 5. Get each point on the elevation, in the same way that the points in Problem 3 are found, by finding the intersections of perpendiculars and horizontals, drawn respectively from the plan, and from the elevation of Problem 5, remembering that the points revolve horizontally, and do not change vertically.

The projections of the point A in Problems 4, 5 and 6, are indicated by the dotted lines on the cuts. The other points in Problem 6 are found in precisely the same way.

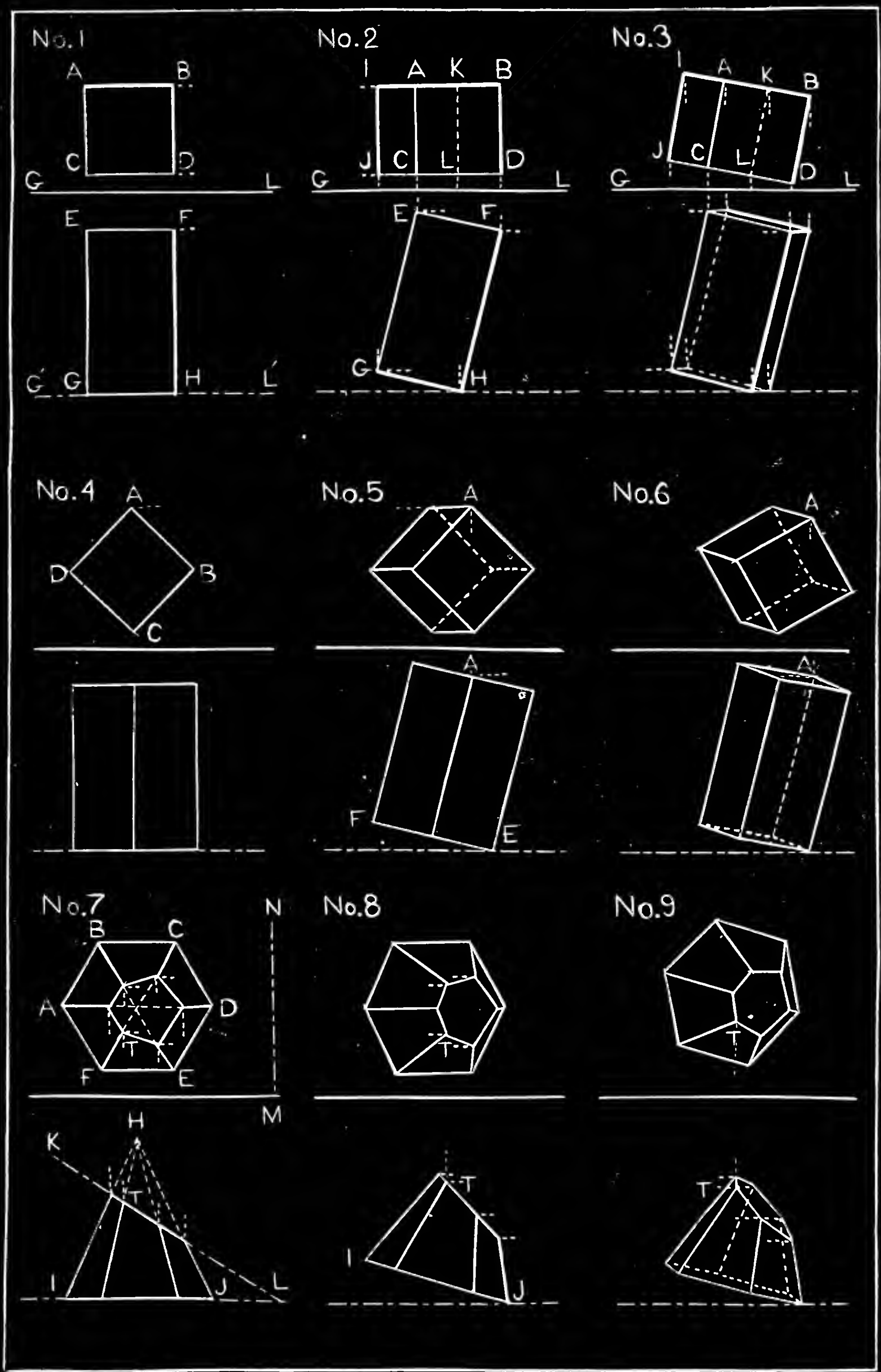
Prob. 7. To construct the projections of a regular hexagonal pyramid, and the projections of a section of that pyramid made by a plane which is perpendicular to the vertical plane.

The height of the pyramid is to be the same as that of the prism in Problem 1, and the diameter of the circumscribing circle of the base is to be two inches.

The part of the pyramid above the section is to be represented by dotted lines and the lower part, or frustum, in full lines.

To find the projections of the pyramid, draw a regular hexagon, A B C D E F, above G L, with lines joining the opposite vertices for the plan of the pyramid; draw perpendiculars from the vertices to G L. The intersections of these perpendiculars with G L will be the elevations of the corners of the base. Erect a perpendicular from the center of the elevation

PLATE VI,
PROJECTIONS.



of the base, and on it measure off the vertical height to H, join H with the points on the base for the elevations of the edges.

To draw the projections of the section, assume L K making an angle with G L, and cutting the pyramid as shown in the plate. Draw M N perpendicular to G L, and vertically above L. K L represents the intersection of the cutting plane with the vertical plane; and M N, its intersection, with the horizontal plane.

These lines are called the *traces* of the cutting plane, and must be represented by broken lines like G L. The elevation of the section will be the part of K L included between the limiting edges of the pyramid. and is to be shown a full line.

The plan of the section is found by erecting perpendiculars to G L from the points where K L cuts the elevations of the edges of the pyramid, and by finding where these perpendiculars intersect the plans of the same edges. T represents the plan and elevation of one point in the section.

Prob. 8. To revolve the frustum of the pyramid in Problem 7 through a vertical angle about J.

Draw J I equal to the same line in Problem 7, and making a given angle with G′ L′. Construct, on J I, an elevation like the one in Problem 7.

Each point in the plan may be found, as in the second positions of the prisms, by erecting perpendiculars from the points on the elevation, and finding their intersections with horizontals drawn from the plans of the same points in Problem 7. T shows the plan and elevation of the point T in Problem 7.

Prob. 9. To revolve the frustum of Problem 8 through a given horizontal angle.

Draw the plan like that of Problem 8, but making the given angle of revolution with G L.

Each point in the elevation may be found by drawing perpendiculars and horizontals respectively, from points in the plan, and from corresponding points in the elevation of Problem 8. The revolution of the point T is indicated by the dotted lines.

Prob. 10. To construct the projections of a regular octagonal prism, one of whose bases is in a horizontal plane.

Lay out a regular octagon, one inch and three-quarters between parallel sides, for the plan; and from its vertices draw verticals to G L, and produce them below G L, until they intersect G′ L′, for the vertical edges of the prism. Make the top line of the elevation three-eighths of an inch below G L.

Prob. 11. To revolve the prism of Problem 10, parallel to the vertical plane, through a given angle.

Construct, as usual, the vertical projection, differing only in position from that in Problem 10. In the case of any point as T, to find its plan, erect a perpendicular to G L from the elevation of the point and draw a horizontal from the corresponding point in the plan of Problem 10. The bases of the prism in this position will be equal octagons though not regular.

Prob. 12. To revolve the prism of Problem 11 through a given horizontal angle.

Draw the plan like that in Problem 11, making the given angle of revolution with G L. In the case of each point, to obtain the elevation, drop a vertical from the plan of the point, and draw a horizontal from the corresponding point in the elevation of Problem 11. The bases in the elevation will be equal octagons, in which the parallel sides are equal.

The point T is the same point that is marked in Problems 10 and 11.

Prob 13. To construct the projections of a given right circular cylinder, whose lower base is in a horizontal plane.

Draw a circle of the given radius, fifteen-sixteenths of an inch, for the plan. The elevation of a right cylinder in this position is evidently a rectangle.

Drop perpendiculars from the right and left hand limits of the plan for the limiting elements in the elevation. Make the elevation of the same height, and the same distance below G L, as in Problem 10.

Prob. 14. To revolve the cylinder of Problem 13 through a given angle parallel to the vertical plane.

The elevation will be a rectangle inclined at the angle of revolution. In the same way that the regular octagon in Problem 11 was shortened in one direction, remaining no longer regular, the circle in Problem 13 will be shortened horizontally and we shall have an ellipse.

This ellipse is constructed by points. Returning to Problem 13, divide the semi-circumference above the horizontal diameter into any number of equal parts. It is not absolutely essential that these parts be equal; but, for convenience, they are usually so taken. Drop verticals from each of these points of division intersecting the semi-circle below and also the bases of the cylinder in the elevation. Returning to Problem 14, space off on the two bases in the elevation divisions equal to those on the corresponding bases in Problem 13.

To get, now, the plan of either base, erect verticals from the points in the elevation of that base, and draw horizontals from the two points corresponding in Problem 13.

The point marked T in problem 14 is the second point from the right in Problem 13.

It will be observed that in this case each point in the elevation corresponds to two points in the plan. There will evidently be the same number of lines across the plans of the bases in Problem 14, as there are in Problem 13, and they will be of the same length in both cases, but slightly nearer together in Problem 14 than in Problem 13. To complete the plan, after getting the bases, draw horizontals tangent to the two bases above and below. These will be the limiting elements of the cylinder in the plan, but will correspond to no line given in the elevation.

Prob. 15. To revolve the cylinder of Problem 14 through a given horizontal angle.

Draw the plan of Problem 14, making the given angle of revolution with G L. This can best be done by drawing a center line in Problem 14, and then drawing a corresponding line making the given angle with G L.

Erect, then, perpendiculars to this last line equal in length and corresponding in position to the lines crossing the bases of Problem 14. The ellipses will be exactly like those in Problem 14. For the elevation, proceed as in Problem 12, by dropping verticals from points in the plan, and finding their intersections with horizontals from corresponding points in the elevation of Problem 14. The construction lines for the point T are given in the plate. The two bases in the elevation will be ellipses, projections of the ellipses in the plan. Draw the limiting elements tangent to these ellipses at the right and left.

Prob. 16. To construct the projections of the frustum of a right cone, whose base is in a horizontal plane.

Draw a circle of the same radius as in the cylinder above for the plan of the base. Drop verticals from the right and left-hand limits of this circle intersecting G′ L′, which should be the same distance below G L as in problem 13, for the elevation of the base. Drop another vertical from the center of the circle, and on it measure the same height from G′ L′ as that of the cylinder above. This will give the elevation of the apex of the complete cone, the plan of which is the center of the circle above. Join this point with the two ends of the elevation of the base for the limiting elements in the elevation.

The upper base of the frustum in this case, is formed by a a plane, cutting the cylinder, perpendicular to the vertical plane of projection, and making an angle with the horizontal plane. The cutting plane is given by its traces on the plate. This upper base will be an ellipse, as is every section of a cone made by a plane that does not cut the base of the cone.

The elevation of this upper base will evidently be that part of the vertical trace of the cutting plane included between the limiting elements.

To get the plan of this base, proceed as follows: Divide the straight line, representing the elevation of this base, into any number of equal parts, and through these points of division draw horizontals as shown on the plate.

The distances of these points from the axis of the cone are evidently equal to the lengths of the horizontals drawn through these points in the elevation, and limited by the axis and the limiting elements. Hence, to get any point, like A, in the plan, erect a vertical from the elevation of that point; and with O the plan of any point in the axis, as a center, and the horizontal M N through the point in the elevation as a radius, describe arcs intersecting the vertical, A and the point above it, are both found by using the same radius, getting the intersections above and below with the vertical from the point in the elevation.

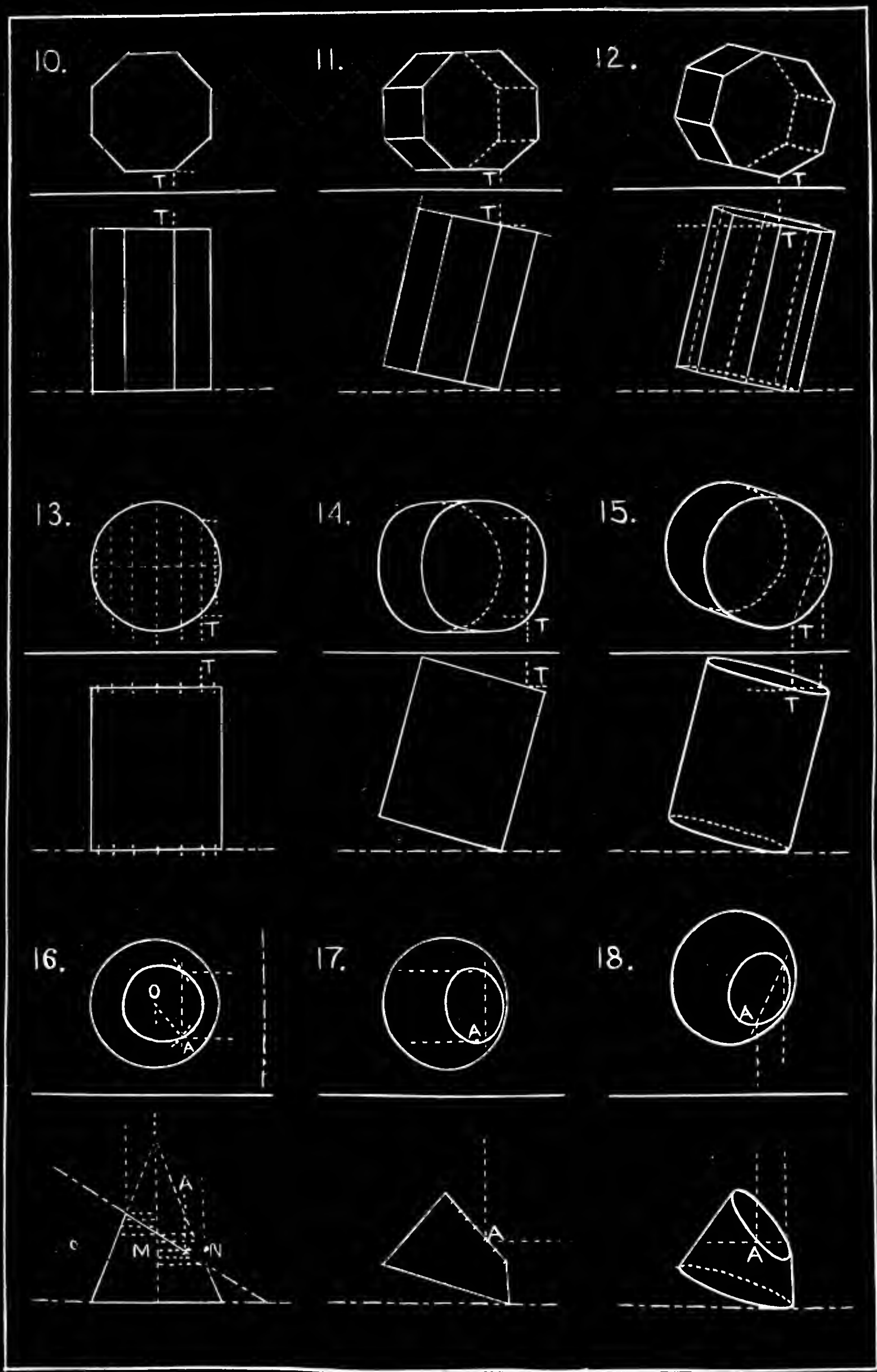
Prob. 17. To revolve the frustum of Problem 16 through a given angle parallel to the vertical plane.

Construct, as in all the preceding similar cases, the elevation of the preceding problem, making the given angle with G L.

Get the plan of the lower base precisely, as in getting the bases of the cylinder in Problem 14.

To get the plan of the upper base, erect verticals from points

PLATE VII,



in the elevation corresponding to those marked in Problem 16, and find the intersections of horizontals from the plans of the same points in Problem 16.

In case the frustum were turned through a larger angle than that shown in the plate, limiting elements would show tangent to the two ellipses in the plan.

Prob. 18. To revolve the frustum of Problem 17 through a given horizontal angle.

Construct the plan of Problem 17; making the given angle with G L, in a manner similar to that employed in constructing the plan of the cylinder in Problem 15.

Get the points in the two bases by dropping verticals from the points in the plan, and finding the intersections of horizontals from corresponding points in the elevation of Problem 17. Draw the limiting elements tangent to the ellipses found in this way.

The orthographic projections of an ellipse are always ellipses, the circle and the straight line being special cases of ellipses.

PRACTICAL APPLICATIONS OF PROJECTION

*Roof Truss.—*Scale one half an inch to the foot.*

In this case, the two sides being symmetrical, an elevation of one half and a section through A B, will show every part, and are therefore chosen as the best views for the working drawing.

First, lay off the dotted horizontal line B C, which is twenty-five feet in length. Then at E erect the vertical D E making E B eight feet and E D nine feet, reducing each to the proper scale.

Join C and D. These lines are the center lines of the principal timbers in the truss and of the iron rod D E.

Draw next the timber of which C B is the center, fifteen inches deep, leaving the end near C unfinished until the rafter is drawn. The other two timbers of which the center lines have been drawn are twelve inches deep. Draw the lines parallel to the center lines. The line forming the joint at D is

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* The Scale of a drawing is the ratio that the lines on the drawing bear to the actual lengths of the lines on the object. The Scale should always be stated on a drawing; and may be given as a fraction like ½, ¼ etc., or it may be stated as a certain number of inches to the foot.

found by joining the intersections of the lines of the timbers. The joint above C is formed by cutting in three inches in a direction perpendicular to the upper edge of C D and joining the end of this perpendicular to the intersection of the lower line of C D with the upper line of C B.

The hatched pieces near C, H and D, are sections of the purlins, long pieces resting on the truss and supporting the rafters. These purlins are cut into the rafters and into the truss one inch, with the exception that the one near C is not cut into the truss. These should next be drawn, rectangles ten inches by six inches. The center line of the one near C is a prolongation of the short line of the joint at C, the one near D is three inches below the joint at the top, and the third is half way between the other two.

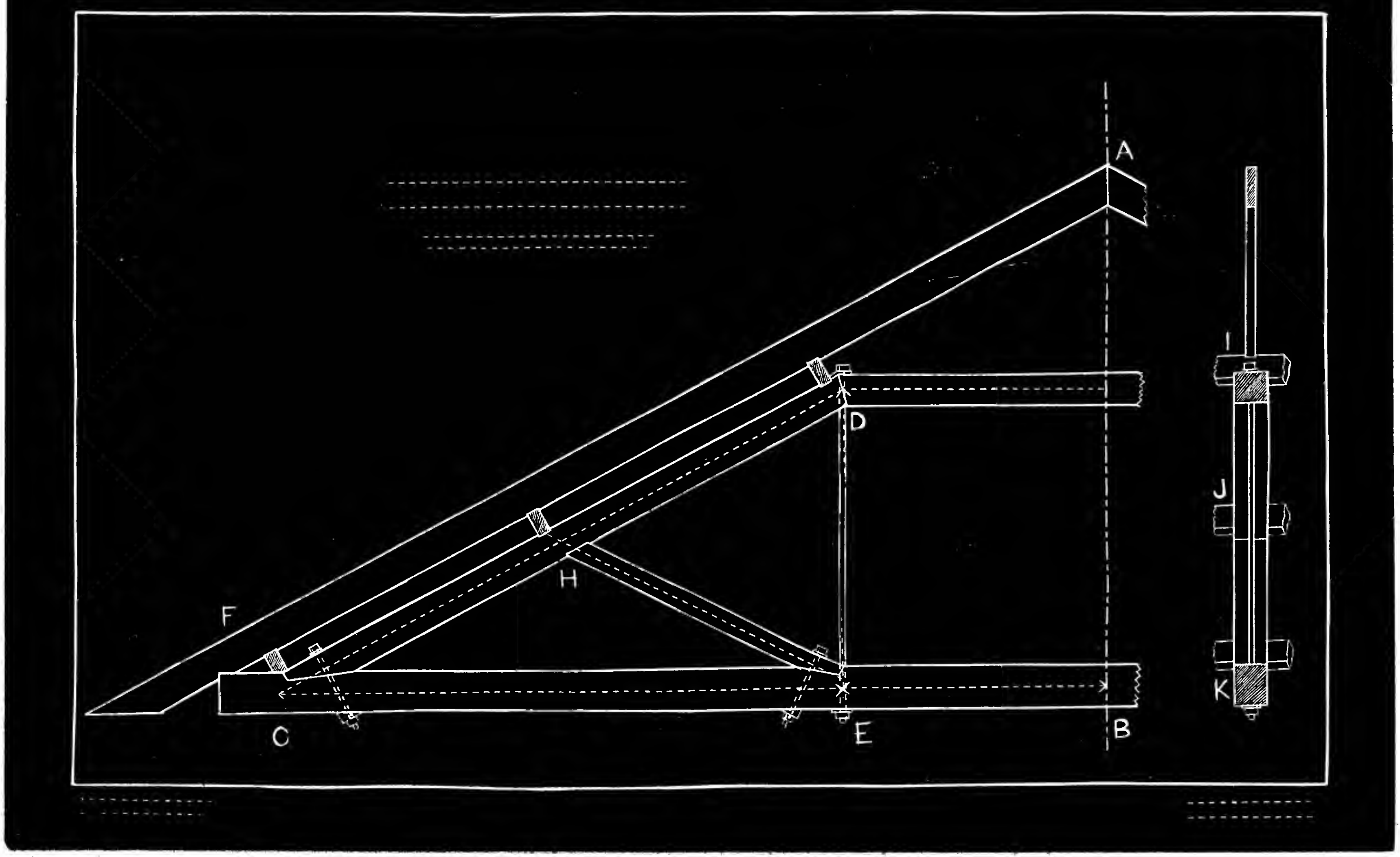
The center line of H E should start from the lower part of the center line of the middle purlin, and the top edge should meet the top edge of C B in D E. With one half the depth of H E, four inches, as a radius describes an arc with the point where D E meets the upper line of C B as a center. Draw the center line through the point indicated above, and tangent to this arc. Cut in at E one third the depth of H E, or two and two-thirds inches and, at H, one inch, in the way shown in the plate.

Draw next the rafter, F A. twelve inches deep and eight inches from the top of C D. Cut into the rafter a horizontal distance of six inches for the end of the beam C B, and make the end of the rafter in line with the bottom of C B.

Make a short end of the rafter on the other side as shown at A, and show the horizontal pieces broken off as shown in the plate. Make the rod, D E, an inch and a half in diameter with washer and nut at the lower end, and with a head and an angle plate running to the purlin at the upper. Make the washer and angle plate one inch thick, the washer four inches in diameter and the nut and head according to the standard. The short bolt near E is one inch in diameter and the head, nut and washer of two-thirds the size those on in D E. The short bolt near C is precisely like the one just described, make the angular washers at the bottom of the same diameter and at right angles to the bolts as in the other washers.

The sectional view at the right is formed by projecting lines

PLATE VIII



across from the elevation just drawn and measuring off the proper widths. The timbers of the truss proper are all twelve inches wide, the rafter is three inches wide and the purlins are broken off so as to show about two feet in length of each.

Great care must be taken in inking not to cross lines, those nearest the observer in any view should be full lines, and those hidden by them should be broken off or dotted.

Hatching, as it is called, is a method of representing a surface cut, as in a section, and it is done by drawing fine parallel lines at an angle of forty-five degrees with the vertical and very near together.

In case two different pieces joining are cut, the lines should be at right angles to distinguish the two sections. There are no such cases in this example however. It is very important that a hatched surface shall look even, and this can only be effected by making all lines of the same width, and the same distance apart.

The title, which is given in italics at the beginning of this description, may be placed within the border as indicated by the dotted lines.

The student will be allowed to choose any mechanical letters for the title, but the heights must be three-eighths of an inch in the words Roof Truss, which must be in capitals, and the letters in the words indicating the scale are to be one-half as high. The scale should be put on thus: *Scale* $\frac{1}{2}'' = 1'$, one dash indicating feet, and two dashes inches.

Stub End of a Connecting Rod.

The two projections chosen to represent this object are a front elevation, and a longitudinal section through A B. In this case these projections show the different parts much more clearly than they could be shown in plan and elevation.

The section at the right of the front elevation shows what would be seen from that position were the part on the right of A B removed. The set screw that holds the key is shown as though not cut through A B. This example illustrates the necessity of hatching to distinguish the cut portions from those beyond. It also shows the proper method of representing the different pieces, shown in section, by lines running in different directions on adjacent pieces.

The dimensions to be used on the full sized drawing are marked in inches on the cut. The arrow heads on either side of the dimension marked represent the limits of the dimension. It will be noticed that some of the dimensions at the top are diameters, whilst others are radii.

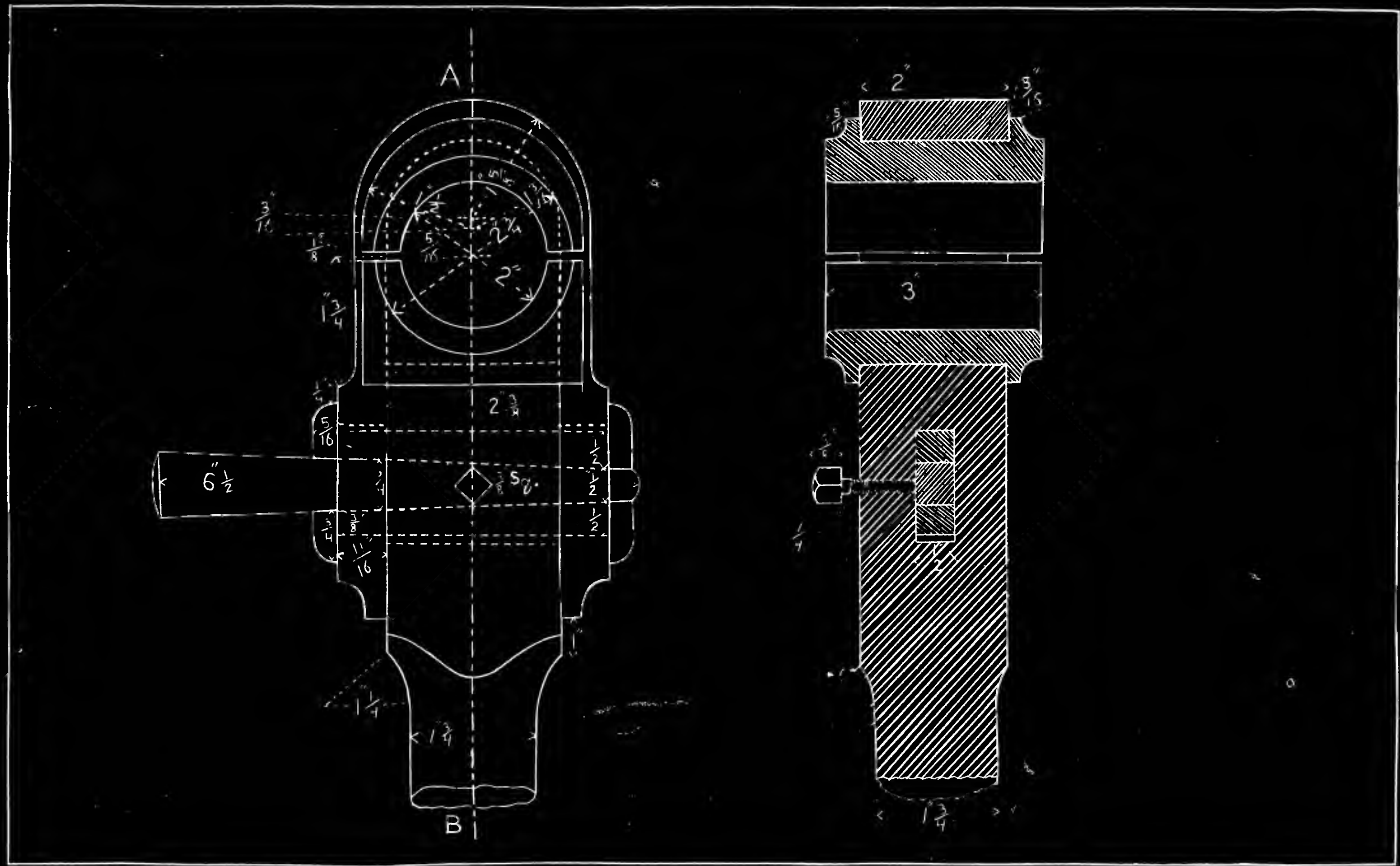
Divide the space within the border on a half-sheet of Imperial by two vertical lines making three equal spaces. Use these lines as center lines of the two projections. Draw the front elevation first. Commence by assuming on the line A B, the center of the inner circle, near the top, and describe the two concentric circles, about this center with the given radii. The common center of these circles should be taken far enough below the border at tho top to leave about the same amount of margin above and below when the elevation is completed. Take a point on A B, five-sixteenths of an inch above the first, and about it as a center, describe the semicircle of the upper limit of the brass bushing, using a radius of one inch and a half. From this same center describe the dotted semicircle with the radius indicated. One-half an inch above the first point on A B, take a third point as the center of the semicircle of the top of the strap.

Notice, in the cut, what lines are tangent to the semicircles just drawn, and proceed with the drawing, making each dimension as indicated. No dimensions are marked on the irregular curve near the bottom, which shows the intersection of the flat face of the rectangular end of the rod with the cylindrical part below; but such an unimportant line does not require to be so carefully drawn to dimensions as the others, and it is sufficient to draw it as nearly as possible like that in the cut, taking great care to have the two sides exactly alike. The horizontal lines in the section may all be projected from corresponding points in the front elevation. All the horizontal distances are indicated on the section, but the vertical distances, being the same as in the front elevation, are purposely omitted. The most difficult part of the work in this drawing is to make the hatching even. Use a sharp pen and make all the lines of the same width and the same distance apart.

Projections of Screws.

The thread of a screw may be considered to be generated by a section moving uniformly around a cylinder, and at the same

PLATE IX.



time uniformly in a direction parallel to the axis of the cylinder. Plate X shows the true projections of a V-threaded screw at the left, and of a square threaded screw at the right.

V-Threaded Screw.—Commence by describing a semicircle with a radius of one inch and a half, as shown in the outer dotted circle in the plan. This will be the half plan of the outer part of the thread. Drop verticals from the outer limits of the semi-circle for the limiting lines of the V threads in the elevation. The projections of the head and an outline of a section of the nut should next be drawn. The standard dimensions of heads and nuts are expressed by the following formulae, in which d is the outside diameter of the screw, h the thickness of the head or nut and D the distance between the parallel sides of the head or nut: $D = 1\frac{1}{2} d + \frac{1}{8}''$, $h = D \div 2$.

Construct the projections of the heads and nuts according to this standard, and show the hexagonal head finished as in the plate. The short arcs that cut off the corners are described with the middle of the lower line of the nut as a center, and the longer arcs bounding the top faces of the head are described with the middle point af the lower line of each face as a center. The top of the head is a circle, as shown in the plan.

A section of a standard V-thread is an equilateral triangle all the angles of which are sixty degrees, hence the outlines of the sides of the elevation may be drawn by means of the thirty degree triangle used on the T-square.

Before drawing these triangles, however, the *pitch* must be determined. The pitch of a screw is the distance from any point on a thread to another point on the same thread on a line parallel to the axis. The pitch is usually expressed by stating the number of threads to the inch. This screw has two threads to the inch, therefore the pitch is one-half an inch. This represents the advance in the direction of the axis during one revolution.

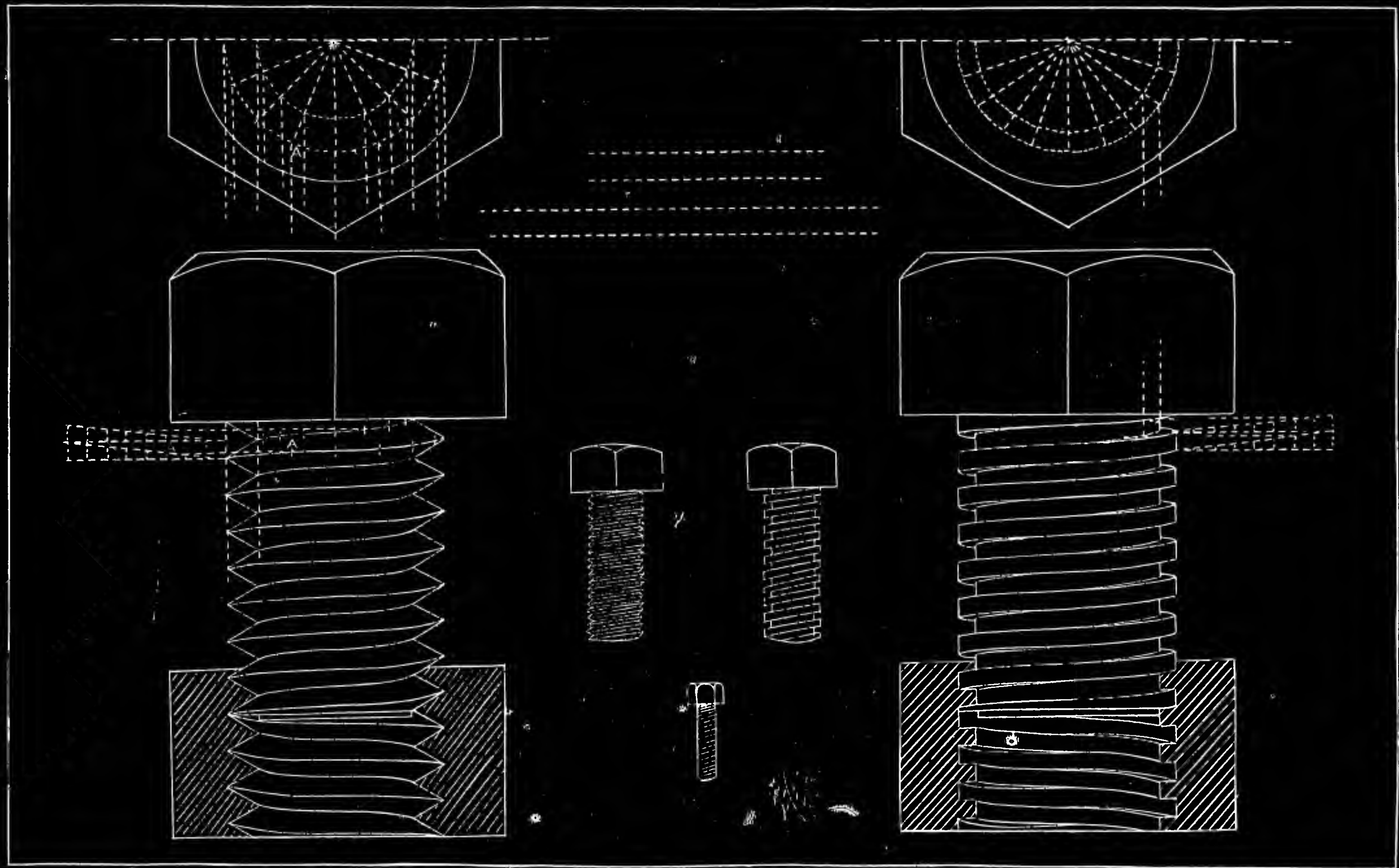
Lay off, then, on the limiting line at the left, distances of one-half an inch, commencing at the bottom of the head.

Through these points draw lines as indicated above, making a series of triangles. The inner intersections of these lines will be in a vertical line, which, projected up, gives the radius of the inner dotted semicircle in the plan. This semicircle is a half plan of the bases of the threads. As the thread advances a distance equal to the pitch in a whole revolution, it is evident that in a half revolution the advance will be equal to half the pitch; therefore commence on the right hand limiting line with the first space a quarter of an inch, and from this point on, make the spaces equal to the pitch. Describe a series of triangles on this side in the same way as before.

Every point in the generating triangle describes a helix as it revolves about and at the same time moves in the direction of the axis of the screw. It is evident that the helices described by the vertices of the triangle will be the edges and intersections of the threads. The manner of getting the projections of these lines will now be described. The plans of these helices will be the circles which have just been obtained and which are shown in the plate in dotted lines. Draw from the outer vertex of one of the triangles representing the edges of the threads, an indefinite line toward the left as shown in the plate. Divide the semicircle above into a number of equal spaces, eight at least, and draw radii to these points of division. Lay off the same number of equal divisions on the indefinite line, and at the last point erect a perpendicular equal in length to one-half the pitch. Join the end of this line with the right hand end of the horizontal line, forming a triangle. Erect verticals from each point of the division of the horizontal line. To find any point, like A, in the helix forming the edge of the threads, drop a vertical from one of the divisions of the semicircle, and find where it intersects a horizontal drawn from a corresponding point on the diagonal line of the triangle at the left, counting the same number of spaces from the right on the diagonal line as the point taken on the semicircle is from the left. As many points may be found this way as there are on the semicircle. Join these points by using the irregular curve.

The points in the helices at the bases of the threads may be found in the same way as shown by the dotted lines, the equal divisions of the semicircle in this case being where the radii of the center semicircle cut this one. The reason for this construction will be plain on considering that the equal spaces on the arc, represent equal angles of revolution of the generating triangle; and the distances between the horizontals drawn from the points of division of the diagonal line, represent the equal rates of advance in the direction of the axis.

PLATE X



As the curves at the edges of the different threads are all alike, a pattern should be made, from thin wood, of the one constructed, and this should be used to mark all the long curves of the screw and nut. Another curve should be made for the inner helices.

The helices will evidently be continuous from one end of the screw to the other, but the dotted lines which would show the parts on the back side are left out in order that the drawing may not be confused by too many lines.

In the plate, the screw is shown as entering only a short distance into the nut which is shown in section below.

The threads of the nut are the exact counter parts of the threads of the screw; but as the threads on the back side of the nut, only are shown in the section, the curves run in the opposite direction. A small cylindrical end is shown on the bottom of the screw. This represents the end of the cylinder on which the thread is wound.

Square Threaded Screws. The square threaded screw is generated by a square revolving about the cylinder and at the same time moving in a direction parallel to the axis. In the square single threaded screw the pitch is equal to the width of a space and the thickness of a thread, measured in a direction parallel to the axis.

Draw the projections of the head and nut of the same dimensions as in the V-threaded screw. Lay off a series of squares, the sides of which are equal to one-half the pitch, on the two edges of the screw, and find the points in the helices as in the example preceding. It should be observed that the long curves show in their full lengths, and the short ones only show to the center in the screws, whilst in the nut the opposite is true.

The smaller screws near the center of the plate, show how V and square threaded screws are after represented when so small that the construction of the helices is impracticable. The construction only varies from the larger ones inasmuch as the curves are replaced by straight lines.

Below there is shown still another method of representing very small screws, either V or square threaded, and the projection of a hexagonal head with face parallel to the plane of projection.

Below is given a table of the Franklin Institute, or United States standard proportions for screw threads. This table is given here that it may be conveniently referred to whenever screws and nuts are to be drawn. A real V thread is often used, but a thread very similar, having a small flat part, in section, at the outside of each thread and a similar flat part between the threads, is becoming more common. The dimensions of such a thread are given in the following table, where diameter of screw means the outer diameter, diameter of core, the diameter of the cylinder on which the thread is wound, and width of flat, the width of the flat part just described. The four columns at the right relate to the nuts and bolt heads.

PROPORTION OF SCREW THREADS, NUTS AND BOLT HEADS.

SCREWS.				NUTS AND HEADS.			
				HEXAGONAL.		SQUARE.	
DIAMETER OF SCREW.	THREADS PER INCH	DIAMETER OF CORE.	WIDTH OF FLAT.	OUTSIDE DIAMETER.	INSIDE DIAMETER.	DIAGONAL.	HEIGHT OF HEAD.
1/4	20	.185	.0062	9/16	1/2	11/16	1/4
5/16	18	.240	.0070	11/16	19/32	13/16	19/64
3/8	16	.294	.0078	25/32	11/16	31/32	11/32
7/16	14	.344	.0089	43/48	25/32	1 1/16	25/64
1/2	13	.400	.0096	1	7/8	1 1/4	7/16
9/16	12	.454	.0104	1 7/64	31/32	1 5/16	31/64
5/8	11	.507	.0113	1 7/32	1 1/16	1 1/2	17/32
3/4	10	.620	.0125	1 7/16	1 1/4	1 3/4	5/8
7/8	9	.731	.0140	1 21/32	1 7/16	2 1/32	23/32
1	8	.837	.0156	1 7/8	1 5/8	2 5/16	13/16
1 1/8	7	.940	.0180	2 3/32	1 13/16	2 1/2	29/64
1 1/4	7	1.065	.0180	2 5/16	2	2 27/32	1
1 3/8	6	1.160	.0210	2 1/2	2 3/16	3 1/10	1 3/32
1 1/2	6	1.284	.0210	2 3/4	2 3/8	3 3/8	1 3/16
1 5/8	5 1/2	1.389	.0227	2 15/16	2 9/16	3 5/8	1 9/32
1 3/4	5	1.490	.0250	3 3/16	2 3/4	3 29/32	1 3/8
1 7/8	5	1.615	.0250	3 13/32	2 15/16	4 3/16	1 15/32
2	4 1/2	1.712	.0280	3 5/8	3 1/8	4 7/16	1 9/16
2 1/4	4 1/2	1.962	.0280	4 1/16	3 1/2	4 31/32	1 3/4
2 1/2	4	2.175	.0310	4 1/2	3 7/8	5 1/2	1 15/16
2 3/4	4	2.425	.0310	4 29/32	4 1/4	6	2 1/8
3	3 1/2	2.628	.0357	5 3/8	4 5/8	6 9/16	2 5/16

DRAWING FROM ROUGH SKETCHES.

Plate XI is given to illustrate the method of making rough sketches of an object from which a finished drawing is to be made. The rough sketches here shown are of a large valve such as is used on large water pipes.* This example has been chosen because it is symmetrical with reference to the center line. In such a case as this, it is obviously unnecessary to make complete sketches of the whole object. Enough of the plan of the object is given above to make a complete plan from, in the drawing. The sketch of the elevation below shows all that is necessary for that.

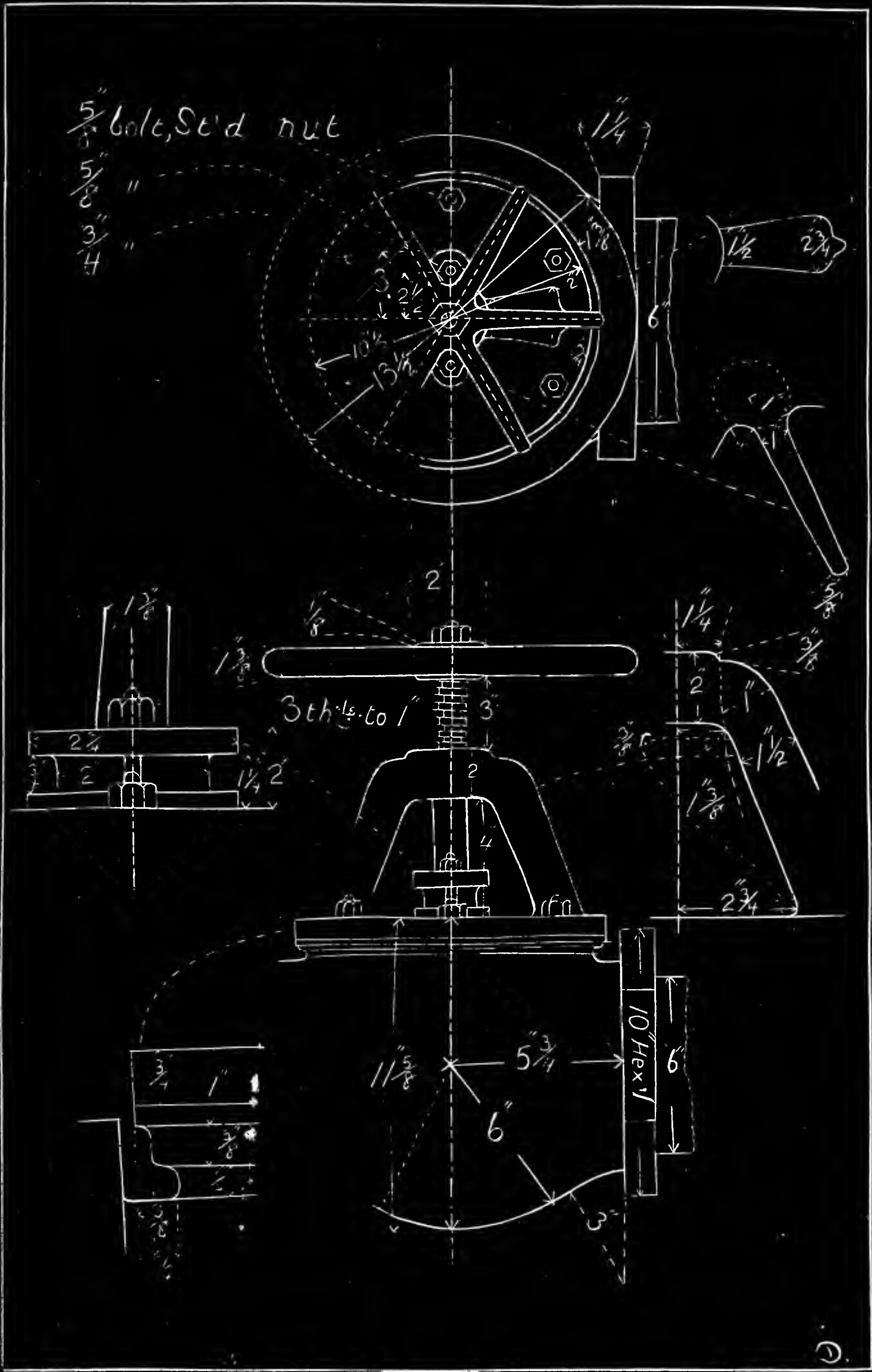
In making a rough sketch decide what projections will best represent the object, and get in such a position as to see the object as nearly as possible as it will appear in the projection, changing the position of observation for the sketches of the different projections. It must be borne in mind that the view a person has of an object while sketching is a *perspective* view and allowance must be made for the way it will appear in projection. Sketches similar to the projections of the objects are better than perspective sketches to work from. The sketches should be made in the same relative position that they will appear in the projection drawing. Be sure to represent every line of the object in the sketches, excepting the cases where symmetrical parts may be drawn from a sketch of one part, and indicate all the dimensions by plain figures and arrow heads, *taking all the dimensions possible from some well defined line like a center line or a bottom line.*

If any of the details on the principal sketch are too small to contain the figures of the dimensions make enlarged sketches aside from the other as indicated in the plate. All that is necessary to be known about a nut is the diameter of the bolt; the nut may then be constructed according to the standard.

Often a few words of description written on the sketch as, in the case of a bolt, four threads to the inch, will describe a part sufficiently to one acquainted with the standard proportions of such common pieces as screw bolts, etc.

*A complete drawing of such an object should show the internal parts, but as the object of this plate is simply to illustrate the method of sketching, the internal arrangement is not shown.

PLATE XI.



5/8" bolt, St'd nut
5/8" "
3/4" "
1 1/4"
1 1/2
2 3/4
6"
10 1/2
13 1/2
1 3/8
3th'ds to 1"
3"
2
2 1/4
1 1/4
2 3/4
1 1/2
1 3/8
11 5/8
5 3/4
6"
10" Hex 1
3"
D.

One unaccustomed to making sketches, is apt to omit some dimensions, and too great care cannot be taken to have every part of the object clearly indicated in some way on the sketch.

TINTING AND SHADING.

At this point of the work the following additional materials will be needed:

A set of water colors, a nest of cabinet saucers, a camels hair brush, a bottle of mucilage and brush, and a small glass for water.

Water Colors.—Winsor & Newton's water colors in "half-pans" are recommended . The set should contain the following colors:—Burnt Sienna, Raw Sienna, Crimson Lake, Gamboge, Burnt Umber, Indian Red and Prussian Blue. If the bottled ink has been used for the previous work, a stick of India ink will also need to be purchased. All the conventional colors used to represent the different materials may be mixed from the simple ones given in this list.

Saucers.—The "nest" should contain six medium sized saucers.

Brush.—The camel hair brush should have two points, and should be of medium size.

Mucilage.—The mucilage needs to be very thick, as it is used in shrinking down the heavy drawing paper. The ordinary mucilage in bottles is not fit for this use, and it is recommended that each person buy the Gum Arabic, and dissolve it in a bottle of water, using it as thick as it will run.

Water Glass.—This glass is for holding clean water with which the colors are mixed. Any small vessel will answer this purpose, but a small sized tumbler is the most convenient.

DIRECTIONS FOR SHRINKING DOWN PAPER.

Whenever a drawing is to be tinted it must be shrunk down in order that it may not wrinkle after tinting. To shrink down the paper proceed as follows: Lay the edge of the T square parallel to an edge of the paper, and about five-eighths of an inch from it; and turn up the paper at right angles, making a sharp edge where the paper is bent up by pressing it hard against the edge of the T square with the thumb nail or a knife blade. Turn up all the edges in this way so that the paper will resemble a shallow paper box. The corners need not be cut, but must be doubled over so that all the edges of the paper will stand nearly perpendicular. After this is done the paper should be turned over so as to rest on the upturned edges, and dampened very slightly with a sponge on the back. Every part of the paper must be dampened except the upturned edges which must be kept dry in order that the mucilage may stick. No water should be left standing on the sheet when it is turned over.

The paper should next be turned over and placed so that two edges at right angles may correspond when turned down to two edges of the drawing board. The other side should then be thoroughly wet. The mucilage should next be applied to the dry edges as rapidly as possible. The two edges that correspond to the edges of the drawing board should first be turned down, great care being taken to leave no wrinkles in these edges nor in the corner between them. The other edges should then be turned down, the same care being taken to leave no wrinkles either in the edges or corners. The edges must be kept straight, and, if there are no wrinkles left in the edges, the paper will come down smooth when dry, no matter how much wrinkled while wet. The natural shrinkage of the paper is sufficient without stretching. The edges should be pressed down smooth with the back of a knife or the thumb nail, and the paper should be allowed to dry slowly.

Considerable practice may be necessary before the paper can be shrunk down successfully, but if the directions above are followed closely there need be no difficulty. The paper must be dampened evenly, and the mucilage must be put on evenly and abundantly. Great care must be taken not to drop any mucilage on the middle of the drawing board, and not to get any beyond the dry edge of the paper. Otherwise the paper may be stuck down so as to make trouble in cutting the plate off when finished.

THE PLATES IN TINTING AND SHADING.

Plates A and B, on the walls of the Mechanical Drawing Room at the University, contain the most common forms that are brought out by shading in ordinary mechanical drawings and the most common conventional colors used in working drawings.*

Plate A, which is shaded altogether with India ink is to be done first.

Shrink down a half sheet of Imperial paper, and mark it inside of the edges so that it may be, when cut off, twenty by thirteen and a half inches. Lay out a border one inch inside of the lines just drawn and draw the outlines of the figures with a very sharp pen, using the best of ink, and making the lines as fine as possible. The figures must be drawn of the same size, and arranged in the same way as in the wall plates. The border and the letters, which are are to correspond with those in the wall plates, should not be drawn until the plate is shaded. The dimensions of the figures need not be put on to the finished plates at all.

After inking in the figures, the plate should be washed over with clean water to take out any surplus ink and to leave the paper in better condition for the water shades. The paper should be sopped very lightly with a sponge and a large quantityof water should be used. After washing the paper allow it to dry slowly. If the paper is dried in the sun it will get so warm that the shades will dry too rapidly. When the paper is down smooth and dry, it should be placed on the drawing table slightly inclined in one direction in order that the ink or water color may always flow in one direction.

Take a saucer half full of clean water and by rubbing the wet brush on the end of a stick of ink mix enough India ink to make a shade no darker than that in (*c*), on wall plate. A small piece of paper should be kept to try the shades on before putting them on the plate. Mix the ink thoroughly with one end of the brush before applying to the paper. One end of the brush should always be used in the ink or tint while the other end is kept clean for blending.

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*These colored plates could not conveniently be placed in this pamphlet, and in cases where access cannot be had to the wall plates here mentioned, it is recommended that the instructor make similar ones for the use of the students. Plate A contains five rectangles in the upper row the first three of which are to be plain shades, and the other two are to be blended. The lower row of figures in this plate contains plans and elevations of the following figures in the order named, a prism, pyramid, cylinder, cone and sphere.

Plate B contains, in the upper row, circular figures tinted to represent the conventions for cast-iron, wrought-iron, steel and brass; and in the lower row four square figures with the conventional colors for copper, brick, stone and wood.

With considerable ink in the brush but not nearly all it will hold, commence at the top line of (*a*), and follow it carefully with the first stroke. Before the ink dries at the top, lay on the ink below by moving the brush back and forth, using enough ink in the brush so that it will flow gradually, with the help of the brush, toward the bottom. The lines must be followed carefully at first, and the brush should not be used twice over the same place. In following a line with the brush get in such a position that the forearm will be perpendicular to the direction of the line, Do not paint the shades on but allow them to flow quite freely after the brush. In shading or tinting there is great danger of making clouded places and "water lines" unless the greatest of care is taken in using the brush. If the brush is used over a shade that is partly dry it will make it clouded. And if the edge of the shade is allowed to dry before finishing, a "water line" is produced where the new shade is joined to the old.

In finishing up a figure the ink should be taken up with the brush so that it will not spread beyond the lines. The sun shonld never be allowed to shine on the paper, as it will dry it too fast. A damp day is better for tinting or shading than a dry one for the reason that the drying is then very slow. The shades of (*a*), (*b*) and (*c*) are all plain. Commence on (*a*), and while it is drying put a coat on (*b*). To determine when a shade is dry look at it very obliquely, and if it does not glisten it is ready for another coat. Put four coats on (*a*), two on (*b*) and one on (*c*).

Blending.—A varying shade, such as is noticed in viewing a cylindrical object, may be obtained by blending with India ink. This operation of blending is employed in bringing out the forms of objects, as seen in the lower figures of plate A.

The figures (*d*) and (*e*) are for practice in blending before applying to the solid objects below. Begin (*d*) by laying on a flat shade about an eighth of an inch wide, using but little ink; and when nearly dry take the other end of the brush, slightly moistened in clean water, and run it along the lower edge of the shade blending downward. When this is entirely dry lay on another plain shade a little wider than the first, and blend it downward in the same way. Use but little water and lay on the shade in strips, always commencing at the the top line. When finished the lower part will have had but one coat whilst the upper part will have had several. Blend (*c*) in the same way as (*d*), but use narrower strips of tint in order to make more contrast between the top and bottom.

Shading Solids.—When a solid object is placed in a strong light coming principally from one direction, a strong contrast will be noticed between the shades of the different portions, and these shades serve to reveal the shape of the object much more clearly than when it is placed in diffused light only. For this reason, as well as from the fact that the laws of the shades of an object in light from one direction are very simple, the shades in a drawing are usually made to correspond to those of a body where the light comes from a single window. In all cases, however, it is assumed that there is a certain amount of diffused light, such as is always present in a room lighted by a single window, aside from the strong beam of light that comes directly through the window.

1. *The shades of an object are always in greater contrast when the object is near the eye than when far away.*

2. *The lightest portion of a cylinder, cone or sphere is where the direct light strikes the object perpendicularly, and the darkest portion of the same is where the light strikes tangent to the object, the shade varying gradually between these parts.*

The facts just given may easily be proved by holding a body in the light and noticing the shades.

These facts we will assume as the principles that govern the shading of the following objects.

In view of the above principles the first thing to be determined, after assuming the direction of the light, is where the lightest and darkest parts will be, and what parts are near to the observer and what parts are farthest away. In all the following cases we shall assume the light to come from over the left shoulder, making the angle of forty-five degrees with both the vertical and horizontal planes of projection.

The Prism.—By the use of the forty-five degree triangle on the T square, draw the arrows as shown on the plan. The points where these touch the plan show where the direct light will strike by the prism. By dropping verticals from these points we see that one will fall behind the elevation and one in front, showing that the vertical edge near the right separates the light from the dark portions of the prism The light will nowhere strike the prism perpendicularly, but it will strike that face nearest the left the most directly of any, and it will, of course, be the lightest face of the prism. The front face will be a little darker and the right-hand face, being lighted only by diffused light, will be much darker than either of the other two.

The plan showing only the upper base, receives light at the same angle as the front face, and will have the same shade, which should be about the same as on the plate, and not blended. Considering the principle that the contrast is less between light and shade at a distance, we know that the outside parts of the faces on the right and left will tend to assume nearly the same shade as they recede from the observer, consequently the light face should be blended slightly toward the right, and the dark face on the right should also be blended toward the right, making the former darker toward the outside and the latter lighter toward the outside.

THE PYRAMID.—The light and dark portions are found in the same way, and the elevation is shaded almost precisely like the prism. The top recedes slightly and the contrast there should be slightly less than at the bottom where it is nearer the eye. The faces in the plan recede very rapidly, and the greatest contrast must be at the top. The upper right hand part receives only diffused light.

THE CYLINDER.—The dotted lines show on the plate the method of finding the lightest and darkest portions. Use the forty-five degree triangle on the T-square so as to draw the two diagonal radii as shown Where the one on the right cuts the lower semi-circumferencs is the darkest point and where the other cuts the same on the left is the lightest point. These points projected down will give the lightest and darkest lines on the elevation., Blend quite rapidly both ways from the dark line and toward the right from the left hand limiting element. The shades near the limiting elements should be about alike on the two sides.

The shade of the plan should be the same as that of the plan of the prism.

THE CONE —The instructions given for shading the cylinder with those given for shading the pyramid apply to this figure. Great care must be taken to bring out the vertex in the plan.

THE SPHERE.—It will be evident on consideration that the darkest portion of the sphere is a great circle, the plane of which is perpendicular to the direction of the light; but, as this great circle is not parallel to the planes of projection, its projections are both ellipses. There can evidently be but one point where the light can strike the sphere perpendicularly, and that is where the radius parallel to the direction of the light meets the surface.

To find the lightest points in plan and elevation, join the centers of the plan and elevation by a vertical, draw a diameter of the plan upward and to the right at an angle of forty-five degrees and of the elevation downward and to the right. Draw a line using the forty-five degree triangle from the point where the line joining the centers cuts the circumference of the plan perpendicular to the diameter drawn in the same. Where it intersects the same will be the lightest point in the plan; a line similarly drawn gives the lightest point in the elevation as shown in the plate. The ellipse, which is the dark line of the object, crosses the two diameters drawn on the plan and elevation, just as far from the centers of each as the light points are from the same. These points may be laid off from the centers by means of the dividers. The shades of the plan and elevation of the sphere are exactly alike, but the position of the light and dark portions are different, as seen in the plate.

Commence by laying on a narrow strip of shade about the form of a crescent over the dark point on the diameter and conforming as closely as may be to an arc of the ellipse. Add other and wider strips of the same general form, and blend each toward the light point and toward the outside. Great pains must be taken with this to get the correct shades and the two exactly alike.

Put on all the lines and letters shown on the plate, making the dotted lines and arrows very fine.

Tinting.—Plate B contains the conventional tints for the following materials: Cast Iron, Wrought Iron, Steel, Brass, Copper, Brick, Stone and Wood. The square figures are three inches on a side, and the circular figures have diameters of the same length. These colors are more difficult to lay on evenly than the India ink shades, but what has been said about the application of ink shades applies to them. Great pains must be taken to have the paper in good condition, and to keep the colors well mixed. Enough color should be mixed to finish the figure as it is impossible to match the colors exactly. Wash the brush thoroughly before commencing a new figure. The plates are to be lettered like the wall plate, the initials at the bottom standing for the colors used. Below are given the materials to be used in each convention. The exact proportions of these can best be found by experiment, comparing the colors with those on the wall plate. A number of thin coats, well laid on, generally look more even than when the tints are laid on in single coats.

For *Cast Iron*, use India ink, Prussian Blue and Crimson Lake; for *Wrought Iron*, Prussian Blue and India ink; for *Steel*, Prussian Blue; for *Brass*, Gamboge, Burnt Umber and Crimson Lake; for *Copper*, Crimson Lake and Burnt Umber; for *Brick*, Indian Red; for *Stone*, India ink and Prussian Blue; for *Wood*, Raw and Burnt Sienna. The convention for the body of the wood is made by laying on a light coat of Raw Sienna, and the Grain is made by applying the Burnt Sienna, after the first is dry, with the point of the brush, blending slightly in one direction.

SHADE LINING.

Shade lining is a method of representing the shades of an object by a series of lines drawn on the projections so as to produce the same general effect as when blended with India ink. This effect is produced by making the lines very fine and at a considerable distance apart on the light portions, and quite heavy and near together on the dark portions. This method of shading is often employed in uncolored drawings to bring out the forms of parts that might not otherwise be clearly understood, and often to give a drawing a fine finished appearance.

*Plates C and D on the walls of the Drawing Room contain the figures that generally require to be shaded in ordinary drawings. Each of these plates is made on a quarter sheet of Imperial paper, and of the size shown.

*Plate C contains the following figures in plan and elevation, three cylinders, ranging from one fourth of an inch to an inch in diameter; a hexagonal prism, and a concave cylindrical surface. Plate D contains, in plan and elevation, a frustum of a pyramid, a frustum of a cone and a sphere. Each of these plates is made on a quarter sheet of Imperial, with considerable margin outside of the figures.

These plates are made smaller than the others for the reason that each figure contains a great many lines.

It is recommended that a half sheet of Imperial paper be shrunk down, and that this be divided when the plates are finished.

The dimensions of each figure are marked on the wall plates but need not be marked on the finished plates.

The figures should all be penciled in of the dimensions indicated, and in the positions shown in the plates. The limiting lines of the figures to be shade lined should be drawn as fine as possible, and, on these practice figures, had better not be inked until the shade lines are drawn.

The light and dark portions are found as in Plate A, and as the shades to be represented are the same as in that plate, reference is made to the remarks on shades in the description of the same.

No description can be of so much value as a thorough study of the wall plates. Notice carefully the gradation of the shade lines on each projection. In Plate C, the shade lines are all parallel; but in Plate D, they are neither parallel nor of the same width throughout.

In Figure 8 the light lines are made full circles, with the lightest point in each projection as a center. The middle portions only of dark lines are made with the compasses, the ends being finished free hand. In shade lining the pen must be kept very sharp and the ink must run well. Put on all the arrows and dotted lines shown on the wall plates.