3

1

NOTES ON

Elementary Mechanical Drawing

COURSE I.

UNIVERSITY OF MICHIGAN

1905

GEORGE WAHR
PUBLISHER AND BOOKSELLER
ANN ARBOR, MICH.

NOTES ON

Elementary Mechanical Drawing

COURSE I.

UNIVERSITY OF MICHIGAN

1905

GEORGE WAHR

PUBLISHER AND BOOKSELLER

ANN ARBOR, MICH.

PENCILING.

In the construction of the following problems familiarity with the instruments and accuracy in execution are the chief objects to be attained.

Before commencing any drawing the student should see that his instruments are in working order.

For mechanical drawing the pencil should be as hard as may be and still give a distinct line without creasing the paper. The lead must be perfectly smooth, having no grit or foreign substance, and tough enough to bear sharpening to a fine edge or point without breaking.

For freehand drawing pencils of various degrees of hardness are required for shading and outline work.

For right line work the pencils should be sharpened to an edge like a chisel, and the corners slighty rubbed off. This gives a point that wears much longer than the round one and is not so easily broken. The wood is cut away with a knife and the lead ground down upon a piece of fine emery paper.

In case the pencil drawing is to be inked and the work is to be made a finished production, the pencil should be a HHHHHH or one of equal hardness. When the pencil drawing is not to be inked but a tracing is to be made from it, a softer pencil of the grade HHHH should be used.

Care must be exercised in the use of the hard pencil, for if too much pressure is brought to bear upon it a crease is made in the paper which remains after the lead has been removed by the eraser.

In order to become an accomplished and reliable draftsman, it is absolutely necessary for the student to learn from the start to attain the accuracy in pencil construction which is desired in the finished drawing, having every line in its proper form, size, position and relation to every other line.

All corrections should be made in the pencil drawing and not left to be made during the process of inking in.

All lines are drawn from left to right and should be drawn lightly.

To draw a line between two points, place the pencil upon one of the points, bring the T square or triangle against it and at a distance from the other point equal to the width of the pencil point. Now move the pencil to the second point, using the straight-edge for a guide and keeping the pencil always parallel to its first position.

If it becomes necessary to erase a portion of the work, the particles of rubber left upon the paper should be removed with a clean piece of linen rag before attempting to ink the drawing.

INKING.

The pencil drawing completed, the next step is to examine the ruling or right line pen, and, if necessary, to place it in perfect working order preparatory to inking.

It is desirable for the student using a ruling pen for the first time to draw a number of lines at random on a loose sheet of paper using the triangles as guides, until he is able to draw them straight. At first thought this may seem easy, but to one unaccustomed to the use of the pen the drawing of a straight line will require much practice.

The lines are drawn by moving the pen from left to right, keeping it parallel to the first position, for if a lateral movement be given while tracing a line, the point of the pen will approach or recede from the guiding edge and the resulting line will be wavy.

In drawing lines with a right line pen, see that both blades bear with equal force upon the paper, being pressed lightly if the paper is smooth and with increasing pressure according to its roughness.

The pen should be inclined slightly in the direction of motion, and but sufficient pressure to guide it should be given against the straight edge, otherwise the blades will be forced together and a line of unequal width will be the result.

To lessen the danger of blotting, the guiding edge should be slightly removed from the line to be drawn so that the pen point in tracing the line will not come in contact with it.

Where several lines radiate from a point the lines should be drawn from the point, not toward it, allowing each line to dry before drawing the succeeding one in order to prevent a blot which is very likely to be made at the point.

As in the case of the right-line pen, the blades of the compass pen must bear evenly upon the paper, and to attain this the compass legs will have to be adjusted in the joints so that they will be perpendicular to the paper.

In describing arcs, allow only the weight of the compasses to bear upon the needle point, while a slight pressure may be given to the pen point, varying according to the surface of the paper.

The top of the compasses can be slightly inclined in the direction of the motion.

It is always better to describe the smallest of a number of concentric circles first.

Since it is easier to make a right line meet a curve than the reverse, do the compass work before the right-line work.

In erasing ink lines it is far preferable to use the steel eraser. After erasing rub the surface with a clean, smooth, hard substance to prevent the ink from spreading. The finger-nail does this very well.

The student should always strive to be neat, and should keep covered that portion of the drawing which he is not working upon at the time.

Fig. 1 (a). *To lay off on a given arc a length equal to a given right line.* Let BE be the arc and AB the given line. At any point of the arc, as B, draw a tangent BF. On this tangent lay off BA equal to the given line. Divide this length into four equal parts. From G, the point of division nearest to B, as center strike an arc through A cutting the arc BE in D. BD is the arc required.

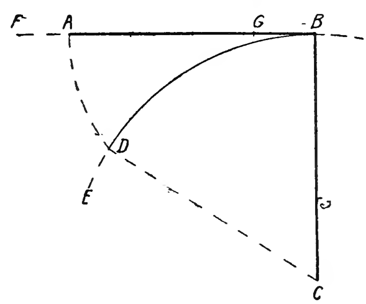


FIG. 1 (a).

If the line AB, when laid off on the arc, gives a distance which subtends an angle greater than 60° divide it into a number of equal parts such that each part when laid off on the arc would give a distance that would subtend an angle of 60° or less. Then step this distance off on the arc as many times as the line AB was divided into parts.

(b) *To rectify the arc of a circle.* Let AB be the given arc, and C the center. At either extremity of the arc, as B, draw the tangent BE. Bisect the chord AB in H and produce it to I, making BI equal to BH. From I as

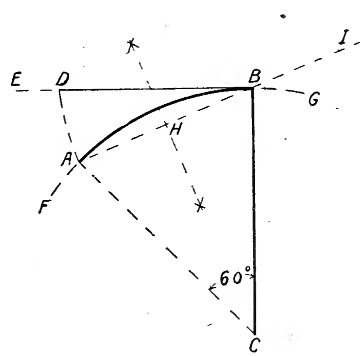


FIG. 1 (b).

center strike an arc through A cutting BE in D. BD is the length of the given arc.

If the arc AB subtends an angle greater than 60° divide it into a number of equal parts such that each part would subtend an angle of 60° or less. Rectify one of these parts and step the rectified length off on the tangent as many times as there are divisions in the arc.

Fig. 2. *To divide a given line into any number of equal parts.* Let AB be the given line and *6* the number of parts. Draw any line AC making a convenient angle with AB. On AC lay off six equal spaces of any length. Draw BC and through the points of division on AC pass parallels to BC. The intersections of these parallels with AB gives the required points of division.

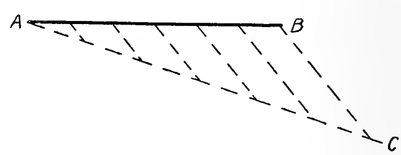


Fig. 2.

Fig. 3. *To construct a regular pentagon upon a given side.* Let AB be the given line. Bisect it at G by the perpendicular GD. At B erect the perpendicular BF equal to the given line. From G as center with radius GF strike an arc FH intersecting AB produced in H. From A and B as centers with radius equal to AH describe arcs intersecting at D. With A and D as centers and radius AB strike arcs intersecting at E.

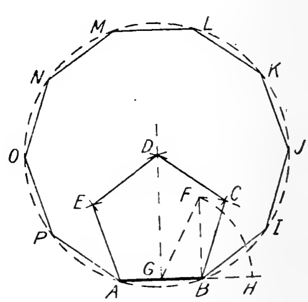


Fig. 3.

With B and D as centers and radius AB strike arcs intersecting at C. The points A, B, C, D, and E are the vertices of the required pentagon.

To construct a regular decagon upon a given side. Let AB be the given side. Find point D as above. From D as center strike a circumference through A and B. Step off the chord AB upon this circumference closing upon A. The points thus found are the vertices of the required decagon.

Fig. 4. *To inscribe a regular pentagon in a circle.* Let ADBC be the given circle. Draw any two perpendicular diameters, AB and CD. Bisect the radius JB at I. From I as center with radius IC strike an arc intersecting AB in K. From C as center with CK as a radius strike an arc cutting the circle in F. CF is one side of the required pentagon.

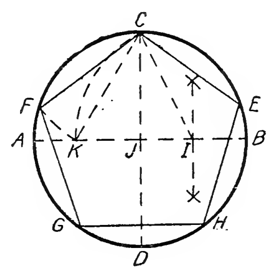


FIG. 4.

Fig. 5. *To construct an ellipse, having given the principal axes.* (Method here described is illustrated by left half of Fig. 5 (a) and is used to construct only the upper half of Fig. 5). Let AB and CD be the given axes. Through the points A, B, C, D, draw the rectangle EFGH. Divide AE into any number of equal parts, and AO into the same number. Number the points

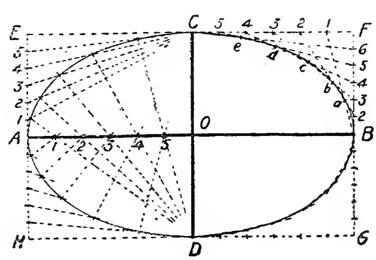


FIG. 5 (a).

of division from A toward E and from A toward O. From C draw a line to any point, as *4*, on AE and from D a line through *4* on AO. The intersection of these lines is a point on the curve. Other points are found in the same manner.

(Method here described is illustrated by left half of Fig. 5 (b) and is used to construct only the lower half of Fig. 5). Let AB and CD be the axes. Draw the rectangle EFGH as above. Draw AC and AD. From B draw any line B*3* cutting AC in *3* and from *3* a parallel to CD cutting AO at *3'*. Draw D*3'* intersecting B*3* at *3''*. This is a point of the curve. Other points are similarly found.

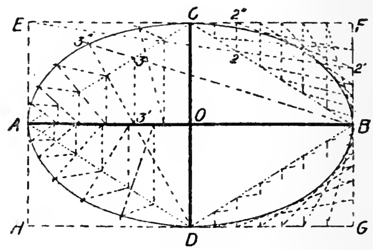


FIG. 5 (b).

Fig. 6. *To find a circle which shall circumscribe a given number of circles of a given size, the inscribed circles to be tangent to each other and to the circumscribing circle.*

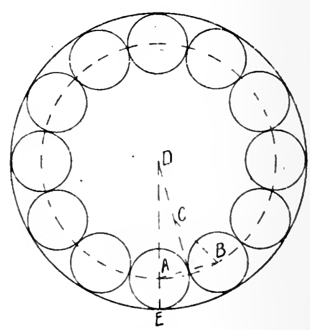


FIG. 6.

Let the number of circles be twelve and let the diameter be AB as shown in the figure. Upon this diameter as a side construct the equilateral triangle ABC. Draw CD bisecting AB. With C as center and CB as radius strike an arc cutting CD in D. With D as center and DE = DA + ½ AB as radius describe a circle, which is the required circumscribing circle. With D as center and DA as radius describe a circle, in which inscribe a regular dodecagon using AB as one side. The vertices of this polygon are the centers of the twelve inscribed circles.

Fig. 7. *To construct an ellipse, having given the principal axes.* Let AB and CD be the given axes. From C or D as center with radius equal to OA strike arcs cutting AB at F and F′. These are the foci of the curve. On FO take any points of division, as *1*, *2*, *3*, *4*, etc. Point *1* should be very near F and the spacing between points may increase toward O as shown. From F as center with radius equal to

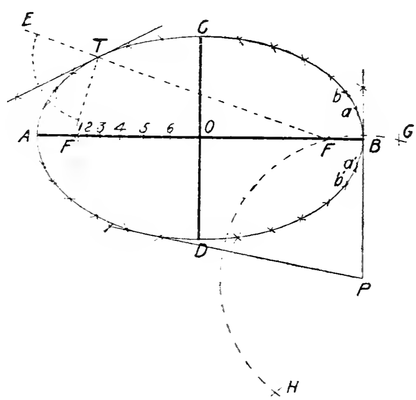


FIG. 7.

B*1* strike the arc of a circle near B. A similar arc may be struck near A with the same radius and F′ as center. From F′ with a radius equal to A*1* cut the first drawn arc in *a* and *a*′ which are two points of the required ellipse. From F with the same radius cut the arc previously described about F′ as center with radius B*1*, thus obtaining two more points of the required curve. In like manner with the radii B*2* and A*2* find the points *b* and *b*′ and also the corresponding points at the other extremity of the axis. The other points on the curve are found by the same method. Then with the irregular curve strike a fair curve through the points. This solution depends upon the principle that the sum of the distances of any point on the curve from the foci is a constant.

Where great accuracy is not desired, the same principle may be used by fastening a thread of length AB to pins at F and F′, and drawing a pencil along the loop thus formed, keeping the thread always taut.

To draw a tangent at any point on an ellipse. Let ACBD be the ellipse and T the given point. Draw TF and TF′, producing the latter to E. The bisector of angle ETF is the required tangent.

To draw a tangent to an ellipse from a point outside. Let ABCD be the ellipse and P the given point. From P as center strike an arc through F′, and from F as center with a radius equal to AB cut the first arc in G and H. Lines drawn from P bisecting arcs F′H and F′G are the required tangents.

Fig. 8. *To construct an hyperbola, having given the vertices and the foci.* Let A and B be the given vertices and F and F′ the given foci. Take any points on AB beyond F′, as *1*, *2*, *3*, etc.

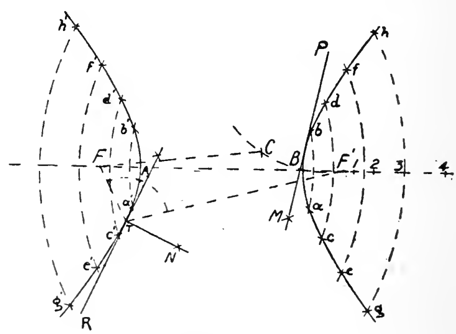


Fig. 8.

From F and F′ as centers with radius equal to A*1* strike the arcs *ab* and *a′b′*. From the same centers with a radius B*1* cut the first arcs in *a*, *b*, *a′*, and *b′*. In the same manner find the points *c*, *d*, *c′* and *d′*. The points thus located determine the curve.

To draw a tangent to an hyperbola at a given point of the curve. Let T be the given point. Join this point with the foci by the lines TF and TF′. Bisect the angles between these lines by the line TR which is the required tangent.

To draw a tangent to an hyperbola from a point outside the curve. Let P be the given point. From P as center strike an arc through F′. From F as center with radius equal to AB cut this arc in C. Bisect the arc F′C by the line PM. This is the required tangent.

Fig. 9. *To construct a parabola upon a given axis and base.* Let EG be the given base and MC the given axis. Take the vertex A upon MC, bisect EB in D, draw AD and perpendicular to it the line DC cutting MC in C. Lay off from A the distances AM and AF equal to BC. At M erect the perpendicular IJ to MC. F is the focus and IJ the directrix of the curve.

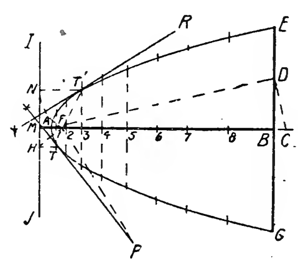


Fig. 9.

Divide the line AB in any points, as *1*, *2*, *3*, etc., and through these points erect perpendiculars to AB. From F as center with radius equal to M*1* strike arcs cutting the perpendicular through *1*; from F as center with radius equal to M*2* cut the perpendicular through *2*. Proceed in this manner, finding the points of intersection which are points of the required curve. This construction depends upon the principle that a parabola is the path of a point moving so that its distance from a point F is always equal to its distance from a given line IJ.

To draw a tangent to a parabola at a given point on the curve. Let T be the given point. Draw T′N perpendicular to IJ and T′F to the focus. Bisect the angle between these lines by the line T′R, which is the required tangent.

To draw a tangent to a parabola from a point outside the curve. Let P be the given point. From P as center with a radius PF strike an arc FH cutting IJ in H. Bisect this arc by the line TP, which is the required tangent.

Fig. 10. *To construct an ellipse, having given the principal axes.* Let AB and CD be the given axes. Upon each of these as diameter describe a circle. From the center draw any radius, as O*3*, cutting the circles at *3* and *3′*. Through *3* draw a line parallel to AB and through *3′* a line parallel to CD, giving at their intersection

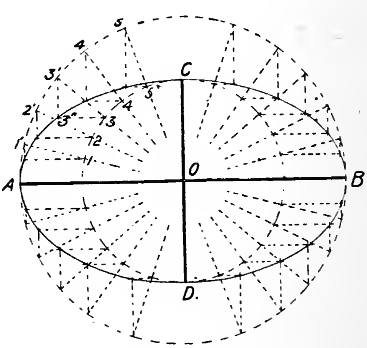


FIG. 10.

$3''$, a point on the curve. Other points are found in the same manner.

Fig. 11. *To construct the spiral of Archimedes.* Let O be the fixed point and O*a* the distance which the point recedes from O in making one complete revolution. From O as center describe the circumference of a circle of any convenient radius and divide this circumference and the line O*a* into the same number of equal parts.

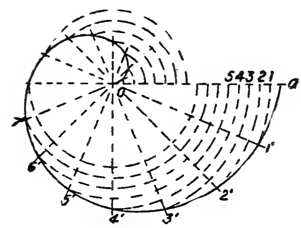


FIG. 11.

Through the points of division on the circle draw radii to O and through the points of division on O*a* strike circles from the center O. The circle from *1* cuts the radius O$1'$ at $1'$, the circle from *2* cuts the radius O$2'$ at $2'$, and so on, giving the points *a*, $1'$, $2'$, etc., which determine the curve.

Fig. 12. *To construct the Logarithmic spiral.* Let O be the fixed point and *a* a point on the curve. Draw O*a*, and by means of an auxiliary circle draw the radiating lines O$1'$, O$2'$, O$3'$, etc., through O. These lines must be separated by equal angles. From *a* let fall a perpendicular upon O$1'$, cutting it at *1*. From *1* let fall a perpendicular upon O*a* cutting the latter at *2*. In the same manner by perpendiculars alternately

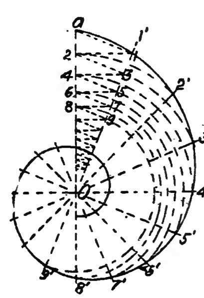


FIG. 12.

upon O$1'$ and O*a* find the points *3*, *4*, *5*, etc. From

O as center a circle through *1* cuts the line $O1'$, a circle from *2* cuts $O2'$, one from *3* cuts $O3'$, etc., giving points of the curve.

Fig. 13. *To construct an external epicycloid.* Let AB be the base and *acfi* the rolling circle. Let A be the point from which the curve springs. Divide

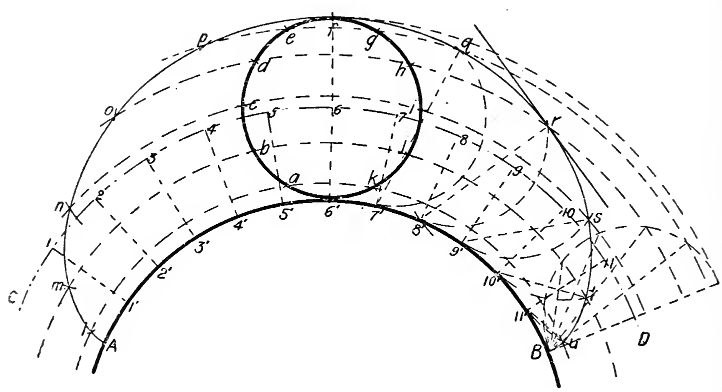


FIG. 13.

the circumference of the rolling circle into any number of equal parts. Draw a common tangent to the two circles at $6'$. By (*b*) Fig. 1 lay off on the tangent a length equal to $a6'$ and then by (*a*) Fig. 1 lay off on the base circle the length thus found giving a length of arc $5'6'$ which is equal to arc $a6'$. On the base circle from A to B lay off as many divisions as are on the rolling circle and make them equal in length to arc $5'6'$ thus giving points $1'$, $2'$, $3'$, etc. AB is therefore equal to the circumference *acfi*. Through the center of the rolling circle strike an arc CD parallel to the base. Draw lines from the center of the base circle through the points of division *1′*,*2′*,*3′*,etc., cutting CD in *1*,*2*,*3*,*4*, etc. Through the points of division on the rolling circle strike parallels to the base. From *1* as center with radius equal to that of the rolling circle strike an arc cutting the parallel through *a* at *l*; and from *2* as center with the same radius cut the parallel through *b* in *m*. In the same manner find the other points of the curve. The same points may be found by striking from *1′* as center, with radius equal to chord *a6′*, an arc cutting the parallel through *a* in *l*, and from *2′* as center with radius equal to the chord *b6′* and arc cutting the parallel through *b* in *m*. Other points are found in a similar manner.

To draw a tangent at a given point of an epicycloid. Let *r* be the given point. Through *r* draw a parallel to the base cutting the rolling circle in *h*. From *r* as center with a radius equal to the chord *h6′* strike an arc cutting the base in *8′*. Draw the line *8r′* and through *r* perpendicular to this line the tangent.

Fig. 14. *To construct a cycloid.* Let AB be the base and *acfi* the rolling circle. Let A be the point from which the cycloid springs. Lay off upon the

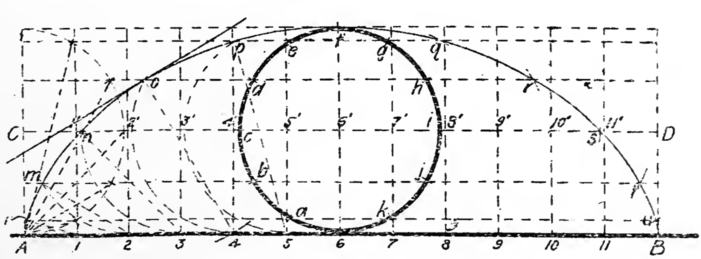


FIG. 14.

circle the equi-distant points *a*, *b*, *c*, *d*, *e*, etc., and through these points draw parallels to the base. From A lay off AB equal to the circumference of the rolling circle and divide it into as many equal parts as the rolling circle by the points *1*, *2*, *3*, etc. Use the method of Fig. 1, (*b*) to obtain the length *5-6* equal to *a6*. All the divisions on AB are equal to the division *5-6*. Through the center of the circle draw the line CD parallel to the base line and erect at A the line AC perpendicular to AB. From A, *1*, *2*, *3*, etc., erect perpendiculars to AB cutting CD in points C, *1'*, *2'*, *3'*, etc. From *1'* as center with radius equal to that of the rolling circle describe an arc *1l* cutting the parallel through *a* at *l*. From *2'* as center with the same radius cut the parallel through *b* at *m*, and with *3'* as center and the same radius cut the parallel through *c* at *n*. Continue in this manner, finding the points *o*, *p*, *f*, *q*, etc. The points *l*, *m*, *n*, etc., determine the cycloid. The same points may be found by letting fall from *1'*, *2'*, etc., perpendiculars upon the base to *1*, *2*, *3*, etc. From *1* as center with a radius equal to the chord of one division on the circle, as *a6*, cut the parallel through *a* in *l*. From *2* as center with radius equal to the chord of two divisions of the circle, as *b6* cut the parallel through *b* in *m*. Other points are found in a similar manner.

To draw a tangent at any point on a cycloid. Let *o* be the point. Through *o* draw a parallel to the base cutting the generating circle in *d*. From *o* as center with radius equal to the distance *d6* cut the base in *4*. Draw *o4* and through *o* perpendicular to *o4* the tangent.

Fig. 15. *To construct a hypocycloid.* Let A*6*′B be the base and *acfi* the rolling circle. Let A be the point from which the curve springs. Lay off on the base an arc AB equal in length to the circumference of the rolling circle and divide this arc and the rolling circle into the same number of equal parts. Methods (*b*) and (*a*) Fig. 1 are necessary to accomplish this.

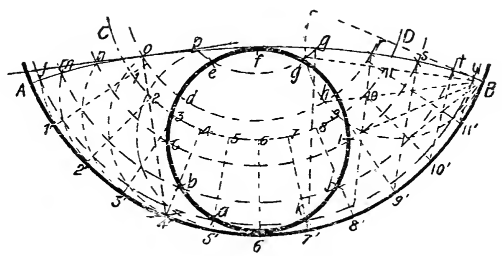


Fig. 15.

Through the center of the rolling circle draw CD parallel to the base, and through the points of division *1*′, *2*′, *3*′, etc., on AB, draw radii to AB cutting CD in *1*, *2*, *3*, etc. Through the points of division *a*, *b*, *c*, on the rolling circle strike parallels to the base. From *1* as center with the radius equal to that of the rolling circle strike an arc cutting the parallel through *a* in *l*; from *2* as center with the same radius cut the parallel through *b* in *m*. In like manner find other points. The same point may be found by striking an arc from *1*′ with radius equal to the chord *a6*′ cutting the parallel through *a* at *l*, and from *2*′ an arc with radius equal to *b6*, cutting the parallel through *b* in *m*.

Note:—Two rolling circles the sum of whose diameters is equal to the diameter of the base circle describe the same hypocycloid. When the diameter of the rolling circle is one-half that of the base circle the hypocycloid becomes a diameter of the base circle.

To draw a tangent at a given point of a hypocycloid. Let *o* be the given point. Through *o* draw a parallel to the base cutting the rolling circle in *d*. From *o* as center with a radius equal to the chord *d6'* strike an arc cutting the base in *4'*. Draw the line *o4'* and through *o* perpendicular to *o4'* the tangent.

Fig. 16. *To construct the involute of a circle.* Let *1*, *2*, *3*, *4*, *5*, *6*, be the given circle and O its center. Take any points *1*, *2*, *3*, etc., upon the circumference and let *1* be the point from which the curve springs, and from which the lengths of the arcs are measured. At *2*, *3*, etc., draw tangents to the circle toward *1*. On the tangent at *2* lay off a distance *2-2'* equal to the arc *1-2* by any convenient method, as that of (*a*) in Fig. 1. In like manner lay off on the tangent at *3* a length *3-3'* equal to the arc *1-3* and so continue, finding a sufficient number of points to locate the curve. The curve passes through *1*, *2'*, *3'*, *4'*, etc.

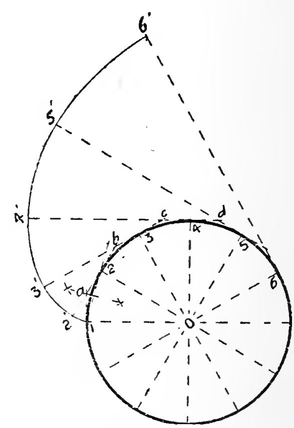


Fig. 16.

Fig. 17. *To construct an internal epicycloid.* Let the circle A$2'$ B$6'$ with center at O be the base and the circle *acfi* with center at C be the rolling circle. Divide the circumference of the rolling circle into any number of equal arcs—here twelve of length A*a*. Step off on the base circle as many arcs of the same length as arc A*a*. The equal arcs are obtained by methods (*b*) and (*a*) of Fig. 1. Since the rolling circle in this case is larger than the base circle the circumference of the former will go around the latter more than once and points $8'$, $9'$, $10'$, etc., will not fall on A, $1'$, $2'$, $3'$, etc., as they do in Fig. 17 which illustrates a special case. From the points of division $1'$, $2'$, $3'$, etc., upon the base circle draw lines through O and produce them beyond O to the circle through C, struck from the center O. Number the points thus found, *1*, *2*, *3*, etc. From O as center through the points of division, *a*, *b*, *c*, etc., on the rolling circle strike parallels to the base. From *1* as center with a radius equal to that of the rolling circle cut the parallel through *a* in *l*; from *2* as center with the same radius cut the parallel through *b* in *m*, and so on, finding the points on the curve. The same points may be found by striking from $1'$ as center with a radius equal to the chord *a*A an arc cutting the parallel through *a* in *l*, and

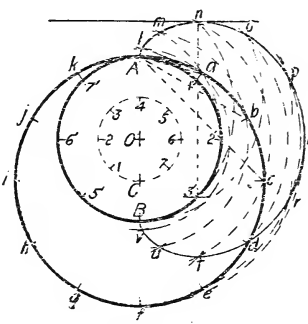


FIG. 17.

from $2'$ with radius equal to the chord bA an arc cutting the parallel through b in m.

After passing the point q it will be noticed that in striking the arc about 7 as a center with a radius equal to the rolling circle two points of intersection are found with the circle struck through e whose center is O. The point which lies on the desired curve will be that one nearest q, the point previously located. A like choice of points should be made where two intersections are obtained.

To draw a tangent at a given point on the internal epicycloid. Let n be the given point. Through n strike a parallel to the base cutting the rolling circle at c and from n as center with radius equal to the chord Ac cut the base in $3'$. Draw the right line $3'n$ and through n perpendicular to this line the tangent. The normal to the curve at n is $n3'$.

Fig. 18. *To construct the projections of the helix.* Let $a'l'$ be the axis and Oa the distance of the tracing point from the axis. Then will the projection of the axis upon its normal plane be found at O, and the helix will be projected in the circle $adgj$. Divide the circle into any number of equal parts, and let $a'b'$ be the distance which the tracing point moves in the direction of the axis while it describes an angle about

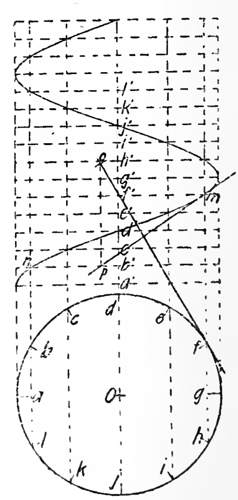


Fig. 18.

the axis subtended by one of the divisions of the circle. Lay off upon the axis a number of divisions equal to $a'b'$ and through these points of division draw perpendiculars to $a'l'$. Through the points of division on the circle draw parallels to $a'l'$ cutting these perpendiculars. If, now, the tracing point is seen at a and upon the line through a', when it has traveled to b it will be found upon the line through b', and will therefore be at the intersection of the lines through b and b'. In like manner it will pass through the intersections of the lines through c and c', etc. A fair curve drawn through these points is the projection upon the plane parallel to the axis.

To draw a tangent to a helix. Let m be the point, which is shown in the projection on the normal plane at f. Through any point of the helix below m draw a horizontal line, as at nb'. Through the intersection n draw a parallel to $a'l'$ cutting the circle in b. At f draw a tangent to the circle and lay off upon it a length fo equal to the arc fb. Through o draw a parallel to $a'l'$ cutting nb in p. Draw pm, which is the required tangent in the projection upon the plane parallel to the axis, as is of in the projection upon the normal plane.

LINE SHADING.

The work on line shading is illustrated by Figs. 19, 20, 21 and 22, and will be explained by the instructor.

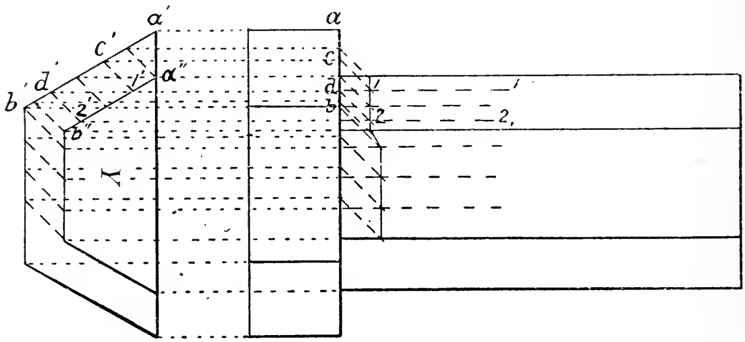


FIG. 19.

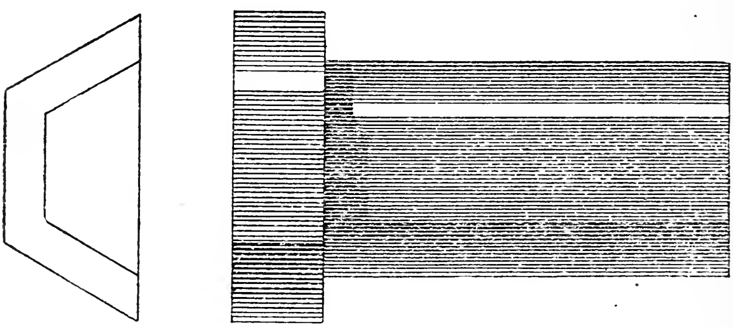


FIG. 20.

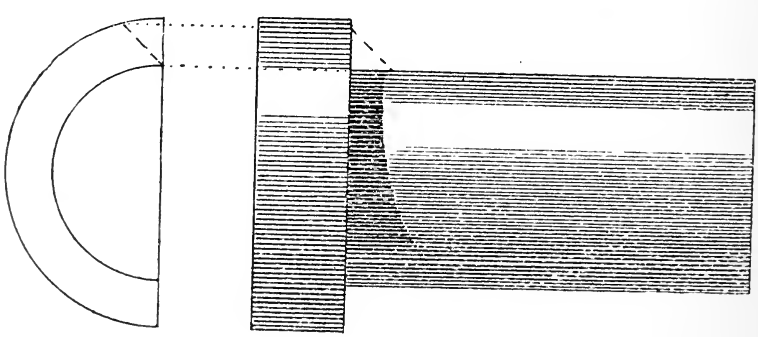


FIG. 21.

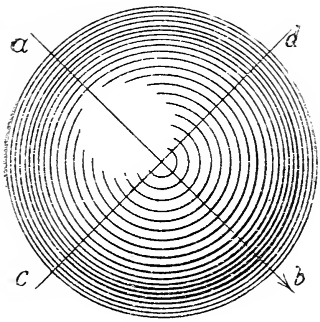


FIG. 22.

RIGHT LINE HATCHING.

Sections of objects are made to show hidden parts and forms which cannot be well represented in plan, front or side elevation. Where a section of an object is made, it is usually hatched, the style of hatching indicating the material of which the object is composed. The rectangles in Fig. 23 are hatched according to the conventional method.

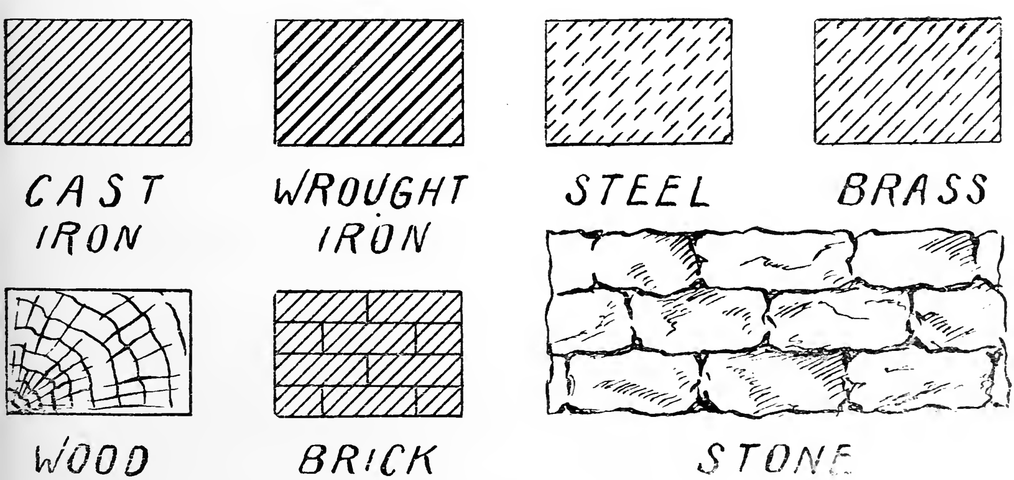


FIG. 23.

TEETH OF GEARS.

In connecting the moving parts of machines, whether the motion be rotary or rectilinear, where an absolutely invariable velocity ratio is desired the connection is most frequently made by means of gears. In the past these gears have usually been built by using hard wood teeth in frames of wood or iron, or have been cast in iron of the desired form. At the present time wooden gears have passed out of date and cast gears, that is, those in which the finished form is taken from the mold, are only used in cheap machinery, where the speed is slow and economy of power transmission is a secondary consideration, and in an occasional example of a gear of very large size, which cannot readily be handled for milling or planing. The process of cutting gears has been so cheapened that in all machine work the cut gears are used, there being great economy in power transmission and also in weight, since the more perfect action of the cut gears gives a large increase in their working strength and consequently permits the use of far lighter gears than could be used in the cast form.

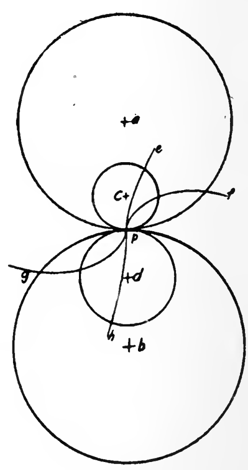


FIG. 24.

If two cylinders, *a* and *b*, Fig. 24, having fixed parallel axes are placed so as to be in contact, and if one, as *a* be set in rotation, the other will also rotate, and they will have a rolling contact. Their angular velocity ratio will be inversely as their radii, and will remain constant so long as the surface friction at their line of contact is sufficient to overcome the resistance offered by *b* due to friction and the work it is performing. If the load on *b* is increased to a sufficient amount *a* will slip over *b* to a greater or less extent and the velocity ratio will not be constant.

It is to avoid this slipping that teeth are placed upon the rims of the cylinders to engage or mesh with each other. It is evident that in order to secure a uniform velocity ratio the teeth must have a certain definite form, and it is with the determination of this form that we are concerned. In order to take up the discussion of gear teeth the following definitions of terms are necessary. Fig. 25 shows an end view of a portion of two gears in

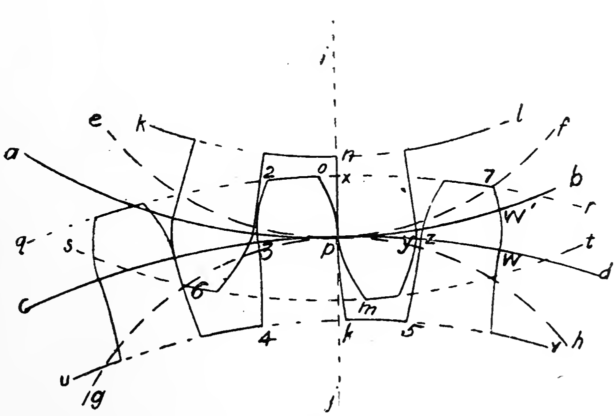


FIG. 25.

action. The two cylinders show in the two circles *ab* and *cd*. These cylinders are ***pitch cylinders*** and the circles are ***pitch circles.*** They are imaginary surfaces and lines, but they are eminently necessary, since the construction of the entire gear depends upon them. The point *p* is the ***pitch point,*** lying always upon *ij*, the ***line of centers.*** The distance *pw* or *pw'* is the ***circular pitch.*** It is the distance from a point on the pitch circle on one tooth to a point similarly situated on the adjacent tooth. Formerly this was universally used in giving the size of the teeth of gears. At present it has almost gone out of use, having been superseded by diametral pitch. ***Diametral pitch*** is a ratio. It is the quotient obtained by dividing the number of teeth in the gear by the diameter of the pitch circle of the gear in inches. The following holds true for the numerical values of the two. On any gear, the product of the diametral pitch by the circular pitch equals 3.1416. Hence—

Diametral pitch 1 is equivalent to circular pitch 3.1416″.

Diametral pitch 2 is equivalent to circular pitch 1.5708″.

Diametral pitch 4 is equivalent to circular pitch 0.7854″, etc.

Also circular pitch 1″ is equivalent to diametral pitch 3.1416.

Also circular pitch 2″ is equivalent to diametral pitch 1.5708.

Also circular pitch ½″ is equivalent to diametral pitch 6.2832, etc.

The length of the tooth is limited by a circle *qr* called the ***addendum or point line***, and by the circle *vu* called the ***dedendum or root line.*** ***Addendum*** and ***dedendum*** are the distances from the pitch circle to the addendum and dedendum line respectively. The dedendum is made somewhat larger than the addendum, and the difference is called the ***clearance.*** When circular pitch is used the addendum is 0.3 of the pitch and the dedendum 0.4, leaving the clearance 0.1, or one-third of the addendum. When diametral pitch is used the addendum is the reciprocal of the pitch in inches, and the dedendum $\frac{5}{4}$ the addendum, leaving the clearance ¼ the addendum. The line *op* is the ***face*** of the tooth and *pk* is the ***flank,*** O2 is the ***point*** and *4k* the ***root.*** The width *p3* in cut gears is made one-half of the circular pitch, and it is equal to the space *pz*. In cast gears the tooth is slightly less than the space and the difference, shown at *yz*, is the ***side clearance or back-lash.*** This is usually made equal to about $\frac{1}{25}$ of the circular pitch. The following table gives in a convenient form the usual dimensions:

DIAMETRAL PITCH = P. CIRCULAR PITCH = p.

Addendum $\frac{1}{P}$ or $\frac{3}{10}$ p.

Dedendum $\frac{5}{4} \frac{1}{P}$ or $\frac{4}{10}$ p.

Clearance $\frac{1}{4} \frac{1}{P}$ or $\frac{1}{10}$ p.

Height of Tooth $\frac{9}{4} \frac{1}{P}$ or $\frac{7}{10}$ p.

Thickness of Tooth $\frac{8\pi - 1}{16 P}$ or $\frac{48}{100}$ p.

Space $\frac{8\pi + 1}{16 P}$ or $\frac{52}{100}$ p.

Backlash $\frac{1}{8}$ $\frac{1}{P}$ or $\frac{4}{100}$ p.

Face of Gear $8 \frac{1}{P}$ or $2 \frac{1}{2}$ p to 3 p.

For the Involute System the height of tooth is frequently taken as $2 \frac{1}{P}$ or $\frac{2}{3}$ p.

If in Fig. 24 we have our two cylinders *a* and *b* and we consider a section of each normal to the axis, we may consider these sections to be by planes which are co-incident but which move independently. The point of contact of the circles is at *p*. If, now, we place two other circles, *c* and *d*, tangent to the first two at *p*, and consider that all centers are fixed, we have the necessary arrangement to obtain the required form for our tooth. For if we take the points of *c* and *d* which are now at *p* for tracing points, and set the circles in motion so that they roll together without slipping and *a* is turning clock-wise, then the tracing point on *c* will mark on the moving plane of *a* the curve *pe* and simultaneously on *b* the curve *pf*. If we turn *a* counter-clockwise *d* will trace on *a* the curve *pg*, and on *b* the curve *ph*. From the method of generation it is evident that as *a* and *b* roll together *pg* and *ph* will always have a point in common, as will *pe* and *pf*. If, then, we take for our tooth curve on *a* the line *gpe* and on *b* the line *hpf* it is manifest that they will give a uniform velocity ratio between *a* and *b*. This solution is known as the cycloidal solution, since the tooth curves are all forms of the cycloid, epi-cycloid, or hypo-cycloid. *pg* and *pf* are epi-cycloids, *ph* and *pe* are hypo-cycloids, while if either *a* or *b* became infinite in radius, so that the circle became a right line, the curves generated upon it would both be cycloids. The circles *a* and *b* are called the pitch circles, and their radii determine the velocity ratio of their axes *a* and *b*. They are always tangent at *p*, the pitch point. The circles *c* and *d* are called ***rolling or generating circles;*** they are also called the ***lines of action,*** since the point of contact between *gpe* and *hpf* is always on *c* or *d*. They may be of any size and are not necessarily of the same size, but practically they must lie within certain limits. The controlling condition is this, that *any two tooth curves which are to work together must be generated by the same line of action.* For any pair of gears which are always to work together the rolling circles should be comparatively large in order to decrease the loss by friction in the action of the teeth and to bring a larger number of teeth in action at once. The largest rolling circle that is commonly used is one whose diameter is equal to the radius of the pitch circle within which it rolls. The hypo-cycloid generated will be a diameter of the pitch circle, and the teeth will have what is known as radial flanks. As the size of the rolling circle is decreased, the tooth becomes broader at the base and narrower at the point, consequently stronger, but the frictional loss in action is increased and a smaller number of teeth is in action at a given time. In Fig. 25 the point of contact moves along the curve *g63p7f* when *cd* drives *ab* to the right. It cannot come upon a tooth of *ab* until it crosses the addendum or point circle *st* at *6* and it runs off the end of a tooth of *cd* where it crosses the point circle *qr* at 7. The distance *6-7* is the ***arc of action,*** and evidently increases as the radii of the rolling circles *ef* and *gh* are enlarged. The portion *6p* is the ***arc of approach*** and *p7* is the ***arc of recession.***

When a number of gears are to be constructed so that any two of them may work together they are said to be interchangeable. In this case the same rolling circle must be used throughout; for if the gears A, B, and C in Fig. 26 are to work interchangeably, then must A work with B and C, and B must work with C. Then, of the curve *mpn* on A, the portion *pn* is the epicycloid generated by E when A works with B, and by F when A works with C. Hence E and F must be of the same size. Similarly of the curve *op'q* the portion *p'q* must be generated by D when B and A are working together, and by F when

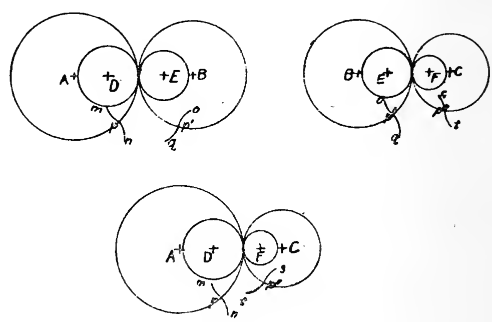


FIG. 26.

B and C are in mesh. Hence D and F must be of the same size and it is evident that the same rolling circle must be used throughout for a set of interchangeable gears.

The ***smallest gear*** that can be used in this system is given by some authors as one of *twelve teeth*, and by others as one of *thirteen*, but the former is more commonly used. Hence for any set of interchangeable gears the rolling circle is taken equal to one-half the size of the smallest pitch circle of the set, and if a set is required which may be extended to include gears of any number of teeth the smallest gear in the set is considered to have twelve teeth, the rolling circle is taken ½ the size of this pitch circle, and this method gives what is known as the *general solution*.

To construct the drawings for a pair of spur gears, then, the following is the usual process. The velocity ratio and the distance between the axes being known, find the diameters of the pitch circles from the following equation:—

$$r_1 = \frac{a}{a+b}\,d, \text{ and } r_2 = \frac{b}{a+b}\,d;$$

in which r_1 and r_2 are radii of the pitch circles, d is the distance between the axes, and $a:b$ is the velocity ratio.

Draw the line of centers and lay off upon it at the proper distances the centers and the pitch point. Draw the pitch circles. Determine the pitch to be used, and from the pitch and pitch circles find the number of teeth upon each wheel; divide each pitch circle into as many equal parts as there are teeth in the wheel, beginning for convenience at the pitch point. From these points of division lay off each tooth, taking notice that the tooth of one wheel must come opposite the space of the other. From the table of proportions determine the addendum and dedendum, and strike in the point and root circles. Assume the rolling circles that are to be used. Construct the curves for faces and flanks of the teeth. Place a sheet of transparent celluloid over the drawing, and stick a fine needle through it into the center of the pitch circle. With a needle point trace upon this sheet the tooth curve on the wheel to whose center it is fastened, extending this curve some distance beyond the point and root circles. Remove the celluloid and with a keen knife cut out the curve, thus making a templet. Pin it again to the center of the pitch circle, and with the templet strike in the face curve on one side of each tooth of the wheel. Turn the templet over, and in the same manner strike in the other face of each tooth. Each wheel is constructed in the same manner. In cutting the templet care should be taken that the edges are left smooth and that it exactly fits the curve from which it is taken. An error commonly made by beginners is at the point where the curve crosses the pitch circle. At this point both portions of the curve come tangent to the radius; this is an important point that should not be overlooked. When the teeth are constructed the rim, arms, and nave are drawn. The face of the gear is usually about $\frac{8}{P}$ for diametral pitch, or $2\frac{1}{2}\,p$ to $3\,p$ for circular pitch. Where there is room on the shaft the nave is made from 25% to 50% greater than this in depth, that is, in the direction of the axis. The size of the key is given by Unwin empirically as follows:—

$$b = \tfrac{1}{4}\,d + \tfrac{1}{8}''; \; t = \tfrac{1}{2}\,b;$$

where d = diameter of shaft, b = breadth of key, t = the mean thickness of the key. In fitting this, ⅔ of the thickness is cut in the nave and ⅓ in the shaft. Unwin says that the thickness of the metal of the nave should be

$$\tfrac{1}{2}'' + 0.4\,\sqrt[3]{p^2\,r},$$

in which p is the circular pitch, and r the radius of the wheel; and the length should be at least three times the thickness. It is more common in practice to roughly proportion the nave to the diameter of the shaft upon which the wheel is fixed. If d is the diameter of the shaft the boss, or enlarged portion of the shaft for receiving the wheel would be 1.17 d, the nave thickness ½ d and the nave length 1.63 d. The older forms of arms were commonly a cross or a T, but at present the best gears have arms of elliptical or segmental cross-section, the longer axis being in the plane of the wheel. In the segmental section, Fig. 27, the thickness is one-half the breadth, in the elliptical section it is four-tenths. The number of arms is largely arbitrary and the calculation of their cross-section according to formulæ given by Reuleaux and Unwin do not give satisfactory results. At the nave the breadth should be about thirty per cent. greater than at the rim.

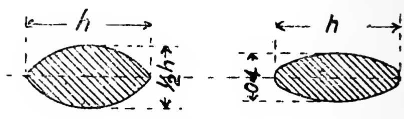


Fig. 27.

In the best practice the rim is made of the form shown in Fig. 28. The dimensions shown are in terms of the circular pitch.

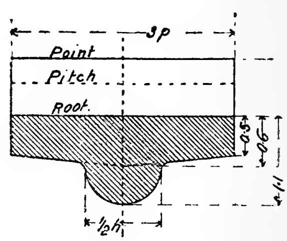


Fig. 28.

THE INVOLUTE SOLUTION.

In Fig. 29 let *ab* and *cd* be two pitch circles, having their point of tangency at *p* and their line of centers *ij*. Draw a line *rs* making a convenient angle with *ij*, and from the centers of the pitch circles let fall perpendiculars upon *rs* at *k*

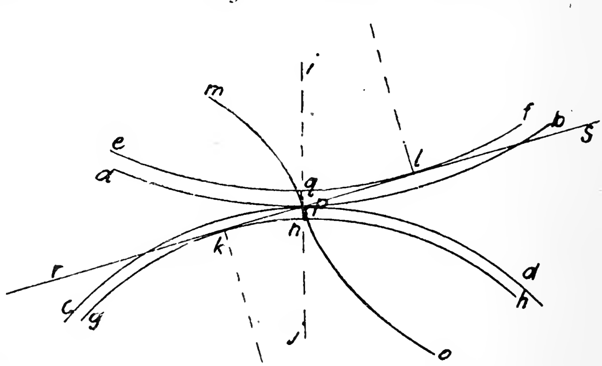


Fig. 29.

and *l*. Draw the circles *ef* and *gh* through *k* and *l*. Now, if *ef* rotates with *ab* and *gh* with *cd* it is evident that the linear velocity of a point on *ef* is equal to that of one on *gh*, and the line *rs* will roll upon *ef* and *gh* without sliding. If a point of *rs* be taken as a tracing point it will describe upon the plane of *ab* the involute of the circle *ef* and upon the plane of *cd* the involute of *gh*. Since these curves are generated simultaneously by the motion of a single point they have always a point in common, and since *ab* and *cd* are rolled together to give the required motion for generating the curve, these involutes may be used as tooth curves to give a uniform angular velocity ratio. The line of action is *rs*, and for cut gears is taken so that it makes with the tangent to *ab* and *cd* at *p* an angle whose sine is 0.25, or 14° 27′. For cast gears it is taken so that this angle is 15°. *ef* and *gh* are the ***base circles or generating circles.***

To design an involute gear, Fig. 30, draw the pitch circle and divide it as in the cycloidal method. Draw the line of action through the pitch point making an angle of 75° with the radius and let fall upon it from the center of the gear a perpendicular. Describe a circle upon this perpendicular as radius. This is the generating circle, and from it is generated the involute which is the tooth curve. Since this curve comes down to a cusp at the point where it touches the generating circle, no contact between the teeth is possible below this line. However, for the sake of clearance it becomes necessary to cut the space lower than this line, and the side of the

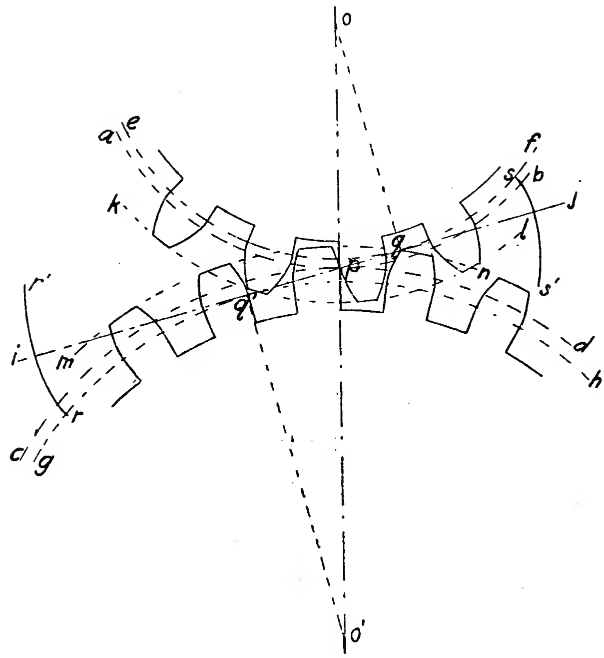


FIG. 30.

tooth is made radial below the generating circle. It must not be forgotten that there is no contact upon a tooth below the base circle but that this extra depth is merely for clearance.

In his Odontics Grant makes the following statement in regard to the length of tooth permissible upon the involute gear. *When two gears are in action, the teeth of one cannot sweep over the point at which the line of action is tangent to the base circle of the other.* If the tooth is extended beyond this point interference will occur, and the wheels will not work at all, or else the points on one will undercut the flanks on the other. Fig. 31 shows this effect, where the rack tooth has undercut the tooth of the pinion. Where the wheels are large this limitation does not affect the length of the tooth, but on small gears it is often the controlling condition in determining it. Where it is possible,

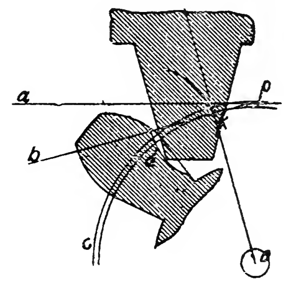


FIG. 31.

the tooth extends above and below the pitch circle exactly as in the cycloidal system; that is, the addendum is $\frac{1}{P}$ and the dedendum 25% greater. The involute rack tooth takes a special form. The tooth curve becomes a right line perpendicular to the line of action. This is the simplest possible form of the tooth curve, and is shown in Fig. 32.

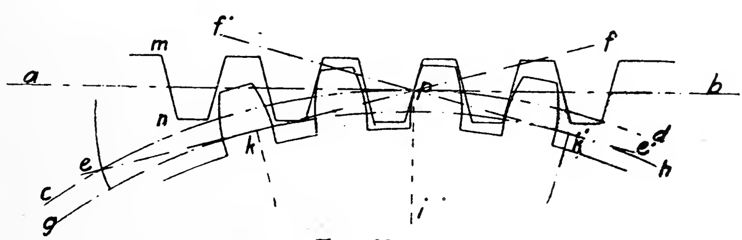


FIG. 32.

THE BEVEL GEAR.

The foregoing forms of teeth are used only when the axes of the gears are parallel. When they are not parallel, they may lie in the same plane or not. In case they are in the same plane they will intersect, and may be connected by the ordinary bevel gearing. If they are not in the same plane they cannot intersect, and may either be connected by an intermediate shaft and two pairs of bevel gears, or they may be directly connected by spiral gears, worm gears, or spiral bevel gears. The simplest case is that in which the axes intersect. The normal surface, which before was a plane, is here a cone having the same axis as the pitch cone, and generated by a line which is perpendicular to the generating line of the pitch cone. Thus, in Fig. 33 if *ab* and *ac* are the axes intersecting at *a*, and *af* is the element along which the pitch cones are tangent, then *af* revolved about *ac* generates the pitch cone, and *gf* perpendicular to *af*, revolved about *ac* generates the normal cone. So also *af* and *fh* generate the pitch and normal cones on the axis *ab*. The teeth are cut off at the smaller end by similar cones generated by *mk* and *lk* revolving about *ab* and *ac* respectively.

To construct the drawings for a pair of bevel gears it is necessary to show the projections of the gears upon a plane parallel to their axis, a projection of each gear upon a plane perpendicular to its axis, and the development of the intersection of the teeth with the normal cones. The quantities usually given are the angle between the axes, the pitch, and the number of teeth upon each gear, from which the pitch diameters can be found. The pitch is measured at the intersection of the pitch cone with the normal cone, and on the larger ends of the teeth. If we have given *ab* and *ac* as axes of a pair of bevel gears intersecting at *a*, with radii *df* and *ef*, having 24 and 18 teeth of diametral pitch 4, *ae* is laid off equal to *df* and *ad* equal to *ef*, and *df* and *ef* drawn to form the parallelogram *adfe*. The diagonal *af* of this parallelogram is the element of tangency of the pitch cones, and *gh*, drawn through *f* perpen-

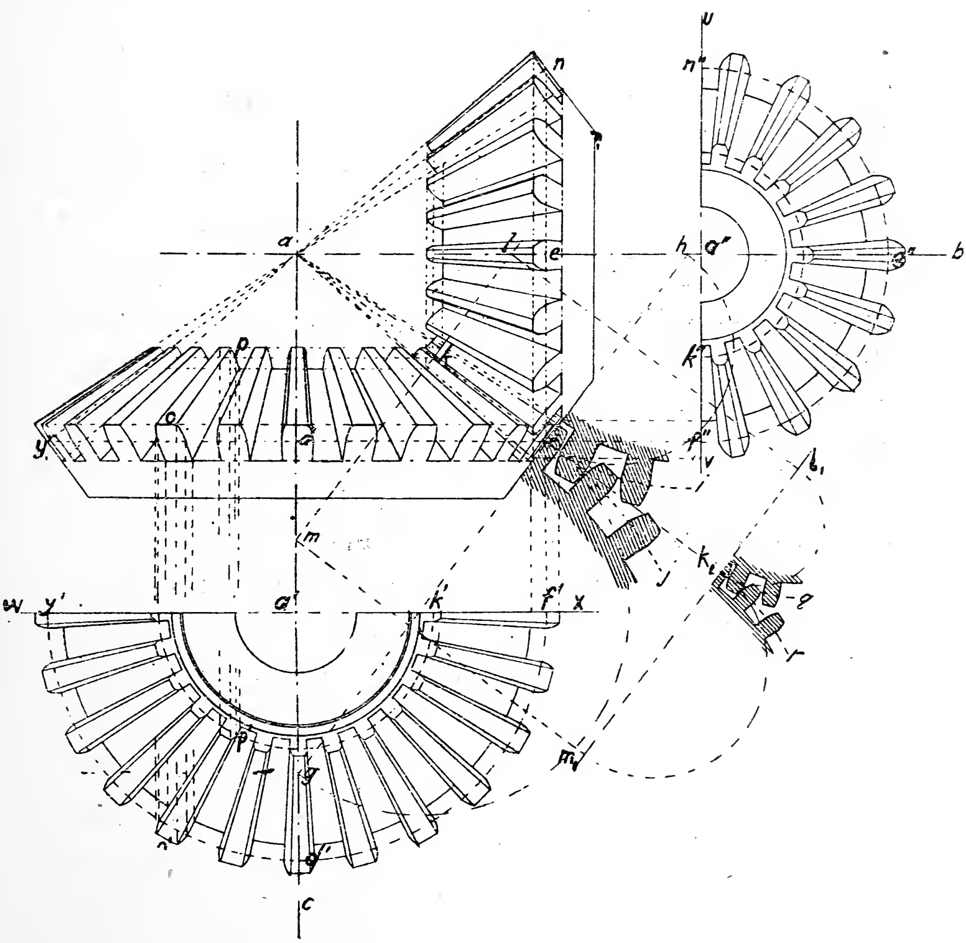


FIG. 33

dicular to af, gives the elements gf and fh which generate the normal cones. The length of the teeth which is equal to $\frac{8}{P}$ is laid off from f toward a, giving k, and through this point ml is drawn parallel to gh, giving the elements mk and lk of the inside normal cones. Assume uv perpendicular to ab and project f and k upon it giving f'' and k'', and from a'' as center strike circles through these points. These are the pitch circles for the outside and inside ends of the teeth on the smaller gear. In a similar manner assume wx perpendicular to ac, project f and k upon it at f' and k', and from a' as center strike in the pitch circles for the larger gear.

To develop the normal cone.—From g and h as centers strike the pitch circles fj and fi, and upon these pitch circles construct a pair of ordinary spur gears as shown in the shaded figure. These are the developments of the outside ends of the teeth. In a similar manner construct a pair of spur gears upon the radii mk and lk, transferring this line to the position m_1 l_1 to avoid confusing the drawing. The same method of construction must be used as in the outer cones. The form kqr thus found is the development of the inside ends of the teeth.

To construct the projections of the gear upon the planes normal to the axes.—The teeth on fi are the development of the outer ends on the smaller gear. Carry the point and root circles in this development to the intersection with gf, and project the points thus found upon *uv* at *1* and *2*. Through *1* and *2* strike circles about the center a''. These are the point and root circles in the normal projection, and since the projection is parallel to the plane of the circles they are in their true size. Divide the pitch circle $f''e''n''$ into a number of parts equal to the number of teeth and through these points of division draw the center lines of the teeth to a''. On these center lines lay out the form of teeth, taking the width on the point, pitch, and root circles from the development *fi*. In a similar manner construct the projection of the inside ends of the teeth, working from the development *kq*. Connect the points of the outer ends with the corresponding points on the inner ends of the teeth by right lines which must, if extended, pass through a''. The construction for the projection $y'd'f'$ is similar to that of $f''e''n''$, the width of the teeth and the size of the point and root circles being found from the development *fj* and *kr*.

To construct the projection upon a plane parallel to the axes.—The projections of the axes are *ab* and *ac*. Through the intersection of the point and root circles on *fi* with *gh* draw right lines perpendicular to *ab*, and through the corresponding point on *fj* draw right lines perpendicular to *ac*. The projection of any tooth, as *op*, $o'p'$, is found by projecting the points from the point, pitch, and root circles at $o'p'$ to the corresponding lines at *op*. In this projection all elements of the conical surfaces must pass through *a*.

THE SCREW.

If a plane figure, such as the quadrilateral *rmos* in Fig. 34 be given a uniform motion of rotation about an axis in its plane and at the same time a uniform motion of translation in the direction of that axis, the figure will describe in space that solid which is known as a screw thread. From the nature of the motion it is evident that each point in the plane figure generates a helix, and that the pitch of all these helices is the same, it being the distance which each point moves along the axis while the plane rotates through one complete revolution, or 360°.

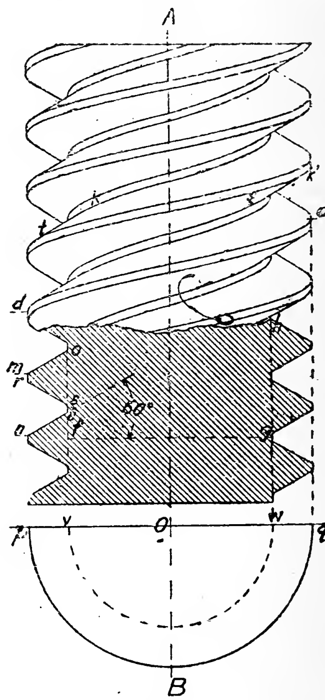


Fig. 34.

The pitch of these helices is also the pitch of the screw. The point *d* in making one complete revolution about the axis AB passes through *e* to the point *c*, therefore *cd* is the pitch of the screw. If this is the same as the distance *mn* or *md* between two adjacent threads the screw has a single thread, if it is twice as great as *mn* it is a double thread screw, and if three times as great, a triple thread screw.

The most common screw is the single thread, but double and triple threads are not uncommon.

The form of the generating figure *rmos* gives the name to the thread. If it be a square it is a square thread, and the sides of the square will be to the pitch in the ratio of 1: an even number, as 1:2, 1:4, 1:6. In the single square thread screw the side of the square is one-half the pitch. The square thread is used to a considerable extend in screws that receive a great deal of wear, since it wears better and works with less friction than does the triangular thread, although, on the other hand it is not so strong.

The most common form of thread is the triangular. In England the Whitworth triangular thread is used and in America the Seller's or U. S. Standard. In the Seller's thread the lines *rs* and *mo* each make an angle of 60° with the axis, and the lines *rm* and *su* are right lines parallel to the axis and equal in length to one-eighth *mn*. In the Whitworth thread the lines *rs* and *mn* inclose an angle of 55° and the right lines *mr* and *us* are replaced by circular arcs.

To make the drawings for a screw assume the axis AB and upon it a center O, from which strike the circle *p*B*q* which is the plan or end view. Through *p* draw *pc* parallel to the axis and lay off upon it spaces equal to the height of the thread, *mn*. Lay off below each of these points a distance equal to one-eighth of *mn*, and draw the lines corresponding to *mo* and *rs*. From the intersection of *rs* and *nu* lay off toward *n* a distance equal to one-eighth *mn* giving *u*, and through this point draw *ov* parallel to AB. Draw the circle *vw* and the lines *wh* and *qe* parallel to AB. If the screw is to have a single or triple thread, project the points from *pc* to *wh* and from *vo* to *qe*; if a double or quadruple thread, project the points from *pc* to *qe* and from *ov* to *wh*.

Construct a helix of amplitude *pq* and of pitch equal to that of the screw, and with it connect the points on *pc* with those on *qe*. Construct a second helix of amplitude *vw* and of the same pitch as the first and with it connect the points on *ov* with those on *wh*. On the top side of the thread draw the line *tk* tangent to the two helices at *t* and *k*, and on the lower side draw *t′k′* in a similar manner. The curve from *e* to *i* is similar to that at *t′k′*. The only line of the original section *rmos* which remains visible is the line *rm* and a small portion of *su*.

The standard sizes and numbers of threads as adopted by the U. S. Navy are given in the following table. This is the standard now universally used in the U. S. The letters refer to the dimensions shown in Fig. 35.

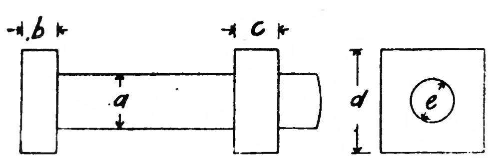


FIG. 35.

a	*No. of threads*	*b*	*c*	*e*	*d*
$\frac{1}{4}$	20	$\frac{1}{4}$	$\frac{1}{4}$	0.185	$\frac{1}{2}$
$\frac{5}{16}$	18	$\frac{19}{64}$	$\frac{5}{16}$	0.24	$\frac{19}{32}$
$\frac{3}{8}$	16	$\frac{11}{32}$	$\frac{3}{8}$	0.294	$\frac{11}{16}$
$\frac{7}{16}$	14	$\frac{25}{64}$	$\frac{7}{16}$	0.344	$\frac{25}{32}$
$\frac{1}{2}$	13	$\frac{7}{16}$	$\frac{1}{2}$	0.4	$\frac{7}{8}$
$\frac{9}{16}$	12	$\frac{31}{64}$	$\frac{9}{16}$	0.454	$\frac{31}{32}$
$\frac{5}{8}$	11	$\frac{17}{32}$	$\frac{5}{8}$	0.507	$1\frac{1}{16}$
$\frac{3}{4}$	10	$\frac{5}{8}$	$\frac{3}{4}$	0.62	$1\frac{1}{4}$
$\frac{7}{8}$	9	$\frac{23}{32}$	$\frac{7}{8}$	0.731	$1\frac{7}{16}$
1	8	$\frac{13}{16}$	1	0.837	$1\frac{5}{8}$
$1\frac{1}{8}$	7	$\frac{29}{32}$	$1\frac{1}{8}$	0.94	$1\frac{13}{16}$
$1\frac{1}{4}$	7	1	$1\frac{1}{4}$	1.065	2
$1\frac{3}{8}$	6	$1\frac{3}{32}$	$1\frac{3}{8}$	1.16	$2\frac{3}{16}$
$1\frac{1}{2}$	6	$1\frac{3}{16}$	$1\frac{1}{2}$	1.284	$2\frac{3}{8}$
$1\frac{5}{8}$	$5\frac{1}{2}$	$1\frac{9}{32}$	$1\frac{5}{8}$	1.389	$2\frac{9}{16}$
$1\frac{3}{4}$	5	$1\frac{3}{8}$	$1\frac{3}{4}$	1.491	$2\frac{3}{4}$
$1\frac{7}{8}$	5	$1\frac{15}{32}$	$1\frac{7}{8}$	1.616	$2\frac{15}{16}$
2	$4\frac{1}{2}$	$1\frac{9}{16}$	2	1.712	$3\frac{1}{8}$
$2\frac{1}{4}$	$4\frac{1}{2}$	$1\frac{3}{4}$	$2\frac{1}{4}$	1.962	$3\frac{1}{2}$

a	*No. of threads*	*b*	*c*	*e*	*d*
$2\frac{1}{2}$	4	$1\frac{15}{16}$	$2\frac{1}{2}$	2.176	$3\frac{7}{8}$
$2\frac{3}{4}$	4	$2\frac{1}{8}$	$2\frac{3}{4}$	2.426	$4\frac{1}{4}$
3	$3\frac{1}{2}$	$2\frac{5}{16}$	3	2.629	$4\frac{5}{8}$
$3\frac{1}{4}$	$3\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{4}$	2.879	5
$3\frac{1}{2}$	$3\frac{1}{4}$	$2\frac{11}{16}$	$3\frac{1}{2}$	3.100	$5\frac{3}{8}$
$3\frac{3}{4}$	3	$2\frac{7}{8}$	$3\frac{3}{4}$	3.317	$5\frac{3}{4}$
4	3	$3\frac{1}{16}$	4	3.567	$6\frac{1}{8}$
$4\frac{1}{4}$	$2\frac{7}{8}$	$3\frac{1}{4}$	$4\frac{1}{4}$	3.798	$6\frac{1}{2}$
$4\frac{1}{2}$	$2\frac{3}{4}$	$3\frac{7}{16}$	$4\frac{1}{2}$	4.128	$6\frac{7}{8}$
$4\frac{3}{4}$	$2\frac{5}{8}$	$3\frac{5}{8}$	$4\frac{3}{4}$	4.256	$7\frac{1}{4}$
5	$2\frac{1}{2}$	$3\frac{13}{16}$	5	4.480	$7\frac{5}{8}$
$5\frac{1}{4}$	$2\frac{1}{2}$	4	$5\frac{1}{4}$	4.730	8
$5\frac{1}{2}$	$2\frac{3}{8}$	$4\frac{3}{16}$	$5\frac{1}{2}$	4.953	$8\frac{3}{8}$
$5\frac{3}{4}$	$2\frac{3}{8}$	$4\frac{3}{8}$	$5\frac{3}{4}$	5.203	$8\frac{3}{4}$

DIRECTIONS FOR PLATES IN DRAWING I.

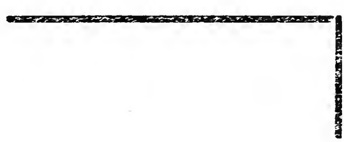
GENERAL.

All plates in this course shall be of the *size 12'' x 15''.*

Each plate shall have a border line removed ½'' from the edge all around; thus making the *border rectangle 11'' x 14''.* Width of border line, $\frac{1}{30}$''.

In the upper right hand corner of each plate, and $\frac{1}{16}$'' from the upper border line shall be placed the *plate number* in upright capital letters ⅛'' high, thus:

PLATE 23.



the letter P being 1¼'' from the corner of the border rectangle.

All plates, excepting those traced from given copies, shall bear the *name* of the draftsman at the lower right hand corner in capital letters ⅛'' high and $\frac{1}{16}$'' below the lower border line, the name to begin 2'' from the right end of border line.

Similarly the *date* of the completion of the plate shall be placed at the lower left hand corner, beginning ¼'' from the left end of border. The date shall be of the form 1905–4–26, in the order, *year*, *month*, *day*.

Unless otherwise instructed, each plate shall first be done *in pencil* and submitted to the instructor for approval or corrections. If approved, he will place the word "*Ink*" on the margin. When the plate is finished, it shall again be submitted to the instructor *before* the word "*Ink*" is cut off; and if approved he will place his signature upon the plate, which shall then be trimmed to the proper size and handed in.

Do not submit any pencilled drawing for approval until the preceeding plate has been handed in.

One plate from each set will be retained as the property of the University.

Plate 1. (upright letters).

Secure a sheet of the *ruled paper* to the board, so that the upper edge is parallel to the upper edge of the T square blade. Draw the border rectangle so that the right and left ends are equally distant from the ends of the blue lines (they are nearly, but not exactly, 13″ long). Draw a light pencil line through the middle of the plate, perpendicular to the blue lines. Fill the space occupied by the blue lines, with a system of 22½° lines about 2″ apart, to be used as guide lines for the inclination of the letters.

Read pages 13–20 in Reinhardt's Lettering Book. Note carefully the formation of each letter and numeral.

The plate consists of ½ row each of the letters and numerals in the order given in the text-book, omitting lower case *l*, and *i*, capital I, and numerals 1 and 0, and adding & (see pp. 21 and 35).

Put the letters about ⅓″ apart and begin each line at the extreme left end of the blue lines.

Make about 5 letters of each kind with a 4H pencil, submit for approval, and do the others directly in ink, using Gillott's No. 303 pen and Waterproof India Ink.

In order to avoid soiling the lettering plates, the space not in use should be kept covered by a cloth or sheet of clean paper; or the hand may rest upon a blotter, while lettering.

PLATE 2. (UPRIGHT TEXT).

Read pp. 20–22, Reinhardt. Beginning with the chapter on *Capital Letters*, p. 16. copy the text in upright letters until the plate is filled. Place the caption in the middle of the first line. Indent all paragraphs ¾″. Observe instructions as to spacing. The letters and numerals are correctly shown in Reinhardt's Plate 1, lines 6, 7 and 8.

PLATE 3. (INCLINED LETTERS).

Read pp. 5–13, Reinhardt. Make a half row each, of the upright letters and numerals, in the order given in the text-book, omiting *l*, *i*, I. 1 and 0 as in Plate 1.

Plate 4. (inclined text).

Use $22\frac{1}{2}^{\circ}$ guide lines as in Plate 1.

Copy caption and text beginning with "*Inclined Lettering*," p. 5. The letters and numerals are correctly shown in Reinhardt's Plate 1, lines 1, 2 and 3.

Plate 5. (inclined text).

Copy text beginning with "*Lower Case Letters*," p. 7, Reinhardt.

Plate 6. (inclined text).

Copy text beginning with "*Numerals*," p. 11, Reinhardt.

Plate 7. (numerals).

Make ten lines of *inclined* numerals, 1 2 3 4 5 6 7 8 9 &, placing seven groups on a line and separating the groups by an extra space. Fill the next ten lines in a similar way with *upright* numerals. Fill the next five lines with *inclined Roman* numerals I II III IV V VI VII VIII IX X, placing four groups on a line.

Fill the last four lines in a similar way with *upright Roman* numerals.

Plate 8. (table).

Use *unruled* paper. Copy one page of a five-place logarithmic table, placing it in a heavy border rectangle 11″ x 8″. Above the border of the table put: COMMON LOGARITHMS OF NUMBERS in inclined capital letters ¼″ in height.

Make the longest dimension of the table correspond to the longest dimension of the plate and arrange the whole exactly in the middle of the plate.

PLATE 9. (FRACTIONS).

Use ruled paper. Make fractions $\frac{1}{16}$, $\frac{1}{8}$, $\frac{3}{16}$, $\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{7}{16}$ . . . . 1, $1\frac{1}{16}$, $1\frac{1}{8}$, etc., increasing the value of each fraction by $\frac{1}{16}$. See Reinhardt's Plate 1.

PLATE 10. (TITLES).

Read pages 26–31, Reinhardt. Reproduce the six titles of Plate IX, Reinhardt, *double size*, on unruled paper.

Draw two light pencil lines parallel to, and 3¾″ from the left and right border lines. Make the titles symmetrical with respect to these lines, and space the titles uniformly with respect to upper and lower border lines and with respect to each other.

The titles in the lettering book are not exactly symmetrical. The student will correct this in his plate.

PLATE 11. (PRACTICE WITH COMPASS AND RULING PEN).

Read pages 3–6 of Notes.

Use a 6H pencil for all mechanical drawings that are to be inked. The pencil must be kept sharp, and the lines made fine and light so that the exact point where two or more lines intersect can easily be distinguished.

Construct within the border rectangle the outlines shown in the figure (see blackboard sketch or blue-print) drawing all straight lines with the T square head on the *left* edge of the drawing board; vertical and oblique lines being drawn with the triangles fixed against the *upper* edge of the T square blade.

Take great care that (*b*) (*c*) and (*d*) are perfect three inch squares; otherwise the final results will be far from accurate.

Part (*a*) consists of six groups of lines most frequently used in mechanical drawings. Each group consists of three lines ¼″ apart. Distance between groups, ½″. The pencil lines may be drawn light and continuous. When inked in, the first line of each group shall be *light*, the second *medium*, and the third *heavy*, as shown on the blue-prints.

Group.

1. ——————————————full line
2. ——— ——— ——— ——— broken line
3. — — — — — — — — — — — dash-dotted line
4. ....point-dotted line
5. ——— . ——— . ——— . ——— broken and dotted
6. ——— — — ——— — — ——— long and short dashes.

Length of long dashes, aprox. ½″

" " short dashes, " $\frac{1}{10}$″

Spaces, " $\frac{1}{10}$″

Part (*b*) is an exercise for the T square and triangles. Draw the diagonals and diameters. From the left end of the horizontal diameter, *ah*, lay off distances $ab = \frac{1}{2}''$, and $bc = \frac{3}{8}''$ (See blackboard sketch). Lay off similar distances from the right end of the horizontal diameter. Through *a* draw lines above and below making 75°, 60°, 45° and 15° respectively with the horizontal diameter *ah*. Through *b* and *c* draw 60° and 15° lines. From the points *g*, *d*, *e* and *f*, where these several lines cross the diagonal, draw lines making 15°, 30°, 45°, and 75° with *ah*. Repeat this for the other three quadrants.

Note.—Whenever several lines converge to a point, as in this figure, it is best, *when pencilling*, to leave a clear space around the point in order that its exact position may not be lost. The pencil lines need not be brought closer than $\frac{1}{16}''$ from the point. *When inking*, these lines will be brought to the point if the drawing requires it; but if they are merely auxiliary lines or construction lines, they should be terminated about $\frac{1}{8}''$ from the point.

Part (*c*). Exercises in drawing arcs tangent to right lines. Use T square, triangles and bow-compasses. Draw diagonals and diameters. Divide a horizontal and vertical side into half inch spaces. Through the points of division, with 45° triangle, draw lines parallel to the diagonals. Through the points *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, etc., as centers, strike tangent arcs, as shown in the sketch. Repeat for the remaining quadrants. The small arcs should all be drawn without changing the radius. Similarly for the large arcs.

Part (*d*). Arcs tangent to arcs. Use T square, triangles and bow-compasses. Lay off *ab*, *ae*, etc., equal to ½″. Through *e*, *b*, etc., draw with T square and triangles, lines *en*, *bk*, etc., parallel to the sides of the square. These lines should intersect in the diameter at *h*. With *k*, *h*, *n*, etc., as centers and ½″ radius, describe circles. With *h*, *o*, etc., as centers, and a radius equal to ½ of *oh* describe circles which are tangent to one another at *p*. With *k*, *h*, *n*, etc., as centers, and radius *rn*, describe the small circles.

If accuracy is to be expected great care must be taken in laying off the required distances, and in taking centers at the *exact* points of intersection of the lines. The graphite in the compasses must be hard and sharp.

Full and dash-dotted lines are to be used as shown. Parts (*b*) (*c*) and (*d*) are to be inked in *medium weight* lines.

Part (*e*) consists of six concentrix circles ¼″ apart, the largest having a radius of 2″. The lines are all medium weight, and corresponding to the six forms of part (*a*) in order; the outer circle being a full line.

Part (*f*) consists of twelve concentrix circles in pairs. The space between any two lines is equal to the width of the heavier line. The larger spaces between the pairs are ¼″. The widths of the lines beginning with the outer circle are heavy, light; medium, light; two light lines; two light lines again; light, medium; light, heavy.

PLATE 12. (GEOMETRICAL PROBLEMS).

Fig. 1. With center at (½″, 7″),—i. e., ½″ from *left* border line, and 7″ from *lower* border line, —and radius 2½″, describe the upper right hand quadrant of a circle. With center at (5¾″, 10½″), describe an arc tangent to the first arc. Connect the two centers by a right line, and at the point of tangency construct the common tangent, perpendicular to the line of centers. From the point of tangency lay off on the smaller circle an arc whose chord is 2¼″. Lay this arc off first on a right line, and then on the larger arc, by the methods of Figs. 1 (*a*) and 1 (*b*).

Fig. 2 (*a*). Divide a line 3″ long into 7 equal parts by the method of Fig. 2. Place the line parallel to the upper border, its left end being at (6¼″, 9½″). (*b*). Divide a line 3″ long into two parts proportional to the side and diagonal of a square, by method of Fig. 2. Place left end of line at (6¼″, 6½″).

Fig. 3. Construct a regular decagon upon a side 1″ in length. Show the circumscribed circle in a light broken line. Put left end of given side at (11″, 6¼″).

Fig. 4. In a circle of 1½″ radius, inscribe a

regular pentagon. (Fig. 4). Center of circle at (2″, 3¼″).

Fig. 5. Construct an ellipse with axes 5¼″ and 3″, by the method of Fig. 7. Center of ellipse at (6¾, 3¼″).

Fig. 6. Inscribe ten circles of ¾″ diameter in another circle. Center of lowest circle at (12″ 2″).

Note.—In all geometrical problems of this and succeeding plates, make *given lines heavy*, ($\frac{1}{30}$″); *required lines medium*, and *construction lines light, dash-dotted* as in the preceeding plate. Place FIG. 1, FIG. 2, etc., in a convenient place in capitals ⅛″ high.

Construction lines, when drawn in pencil, should be *full*, not broken.

Letter each figure as in the text-book.

Plate 13. (geometrical problems).

Fig. 7. Construct an ellipse by the trammel method. Axes 4″ and 2½″. Center of ellipse at (2½″, 8¼″). Construct two tangents from a point without at (4⅝″, 9¼″), and a tangent from a point on the ellipse 1⅛″ from left border line.

Fig. 8. Construct the two nappes of an hyperbola. Distances between vertices 1¼″. Each focus is ½″ from the corresponding vertex. Left hand focus at (6″, 8¼″). Construct two tangents to the left hand nappe from a point without, at (6¾″, 9″) and a tangent from a point on the right hand nappe, 7½″ from lower border.

Fig. 9. Construct a parabola on a 3″ base, the vertex being 3¼″ to the left of the base.

Put vertex at (10″, 8¼″). Construct two tangents from a point at (11″, 6¾″), and a tangent at a point on the curve, 9″ from lower border line.

Fig. 10. Construct an ellipse with horizontal axis 3½″, and vertical axis 5½″ long. Center of ellipse at (2½″, 3½″). Construct the upper half by the method shown in the left half of Fig. 5 (*a*). Construct the lower half by the method shown in the left half of Fig. 5 (*b*).

Fig. 11. Construct one complete convolution of a spiral of Archimedes, with origin at (6½″, 2¾″). The curve passes through a point 3″ to the right of the origin. Find sixteen points on the curve.

Fig. 12. Construct one convolution of a logarithmic spiral with origin at (12½″, 2¾″). The curve passes through a point 2½″ to the left of the origin.

PLATE 14. (ROULETTES).

Fig. 13. Construct a cycloid, with a tangent at some point. Radius of rolling circle = ⅞″. Center of rolling circle at (3½″, 9″).

Fig. 14. Construct an epicycloid with tangent. Radius of base circle = 3″. Radius of rolling circle = ⅞″. Center of base circle at (3½″, 2″).

Fig. 15. Construct a hypocycloid with tangent. Radius of base circle = 3″; Radius of rolling circle = ⅞″. Center of base circle at (3½″, 4″).

Fig. 16. Starting with the highest point on a circle of 1⅛″ radius, construct an involute, carrying it through 180°. Center of circle at (9″, 8½″).

Fig. 17. Construct an internal epicycloid. Radius of base circle, 1⅛″. Radius of rolling circle, $1\frac{17}{30}$″. Center of base circle at (8½″, 3½″).

Fig. 18. Construct the vertical and horizontal projections of one complete convolution of a helix. Diameter, 2¾″. Curve rises 3″ for each convolution. Center of horizontal projection at (12⅛″, 2″). Determine 16 points on the curve.

PLATE 15. (LINE SHADING).

Fig. 19 is a shaded hexagonal prism with hexagonal abacus (Figs. 19 and 20). Put the center of the auxiliary semi-circumference which circumscribes the half hexagon at (2″, 8″). The radius is 1¾″. The distance a' a'' is $\frac{7}{16}$″. Parallel to a' a'' and ½″ from it, draw the left vertical line of the abacus. Complete the drawing of the abacus, making it ⅞″ thick. Length of prism is 5½″. Determine the outline of the shadow. It will be convenient to draw the shade lines the following distances apart.

For the light surface, $\frac{1}{12}$″.
" " medium " $\frac{1}{16}$″.
" " dark " $\frac{1}{20}$″.
" " shadow " $\frac{1}{20}$″.

Do not put shade lines in pencil.

Fig. 20 is a shaded cylinder with cylindrical abacus. (Fig. 21) Center of semi-circumference is at (2″, 3″). Diameter of abacus is 3½″. Diameter of cylinder is 2¾″. Thickness of abacus is ⅞″. Length of cylinder is 5½″.

Do not put shade lines in pencil.

Fig. 21. Read the text on Right Line Hatching, p. 25. The methods of "hatching" are shown in Fig. 23. The upper left hand corner of the cast-iron rectangle is at (9½″, 9¾″). The six small rectangles are each 1″ x 1¼″ and are separated from each other by ¼″. The stone rectangle is 1½″ x 4¼″. It is placed directly below the other six, and separated by ½″.

Fig. 22 is a sphere shaded by circular arcs. The radius of the sphere is 1¾″, the center being at (11½″, 2½″). See Fig. 22.

Do not draw the circles for shading in pencil, but indicate the points along one of the 45° lines, through which the circles are to be drawn. Great care must be taken not to wear a large hole in the paper at the center of the circle.

Plate 16. (tracing).

Read pp. 22–26, Reinhardt. Trace the plate found in the envelope and entitled "Slating Ts and Ls."

Stretch a sheet of tracing cloth over the plate with *dull* side up. To make the cloth take the ink evenly, sprinkle powdered chalk, talcum or magnesium upon the surface, and rub with a clean dry cloth.

In tracing, each figure should be completed before going to the next; for if the draftsman is unable to complete the plate at one sitting, he will find that the cloth has shrunk during the night, and that the lines drawn upon the cloth do not exactly cover the lines of the original.

Whenever open holes are indicated by black circles, the outlines of these circles should first be drawn with the bow compasses and then filled in with the free-hand pen.

Put the *plate number* in the usual place; but the *name and date* in the rectangle at the lower right hand corner of the plate.

The tracings required to be done by the student in this course are intended to give him an idea as to how mechanical drawings are made and dimensioned. To this end the student should carefully study each drawing until he is able to explain it when called upon to do so.

Plate 17. (tracing).

Tracing of "Lathe Apron Casting," on *smooth* side of cloth.

Plate 18. (tracing).

Tracing of "Split Sleeve and Collar," on dull side.

PLATE 19. (TRACING).

Tracing of "Pedestal Bearing," on glossy side. A full sized model of this may be found in Room 418.

PLATE 20. (TRACING).

Tracing of "Beams and Connections." Use dull side of cloth.

PLATE 21. (TRACING).

Tracing of "Bottom Chord Joint" (glossy side). A full sized model of this may be found in Room 415.

PLATE 22. (TRACING).

Tracing of "Column" (dull side).

PLATE 23. (CYCLOIDAL CUT GEARS).

Read pages *26 to 36* in notes. A cycloidal cut spur gear of 12 teeth and 8″ diameter meshes on the *left* with an annular gear, diameter 22″; and on the *right* with a rack.

Put radial flanks on pinion, and use the same rolling circle throughout.

The thickness of the rim should be approximately one-half the circular pitch.

Show 6 teeth on annular gear, and 5 on rack. Put center of pinion at (7″, 6″). Show all constructions and all important dimensions. Construct full size. In a rectangle at lower right hand corner, place title, as in tracings.

In some other convenient place, put.

No. of teeth on Annular Gear,——
" " " " Pinion,——
Dia. Pitch,——
Vel. Ratio,——

Plate 24. (involute cut gears).

Read pages *36 to 39* of the notes. An involute cut spur gear of 20 teeth and 16'' diameter meshes with a pinion, and the pinion with a rack on the right. Angular velocity ratio, 3:4. Draw to scale ½'' = 1''.

Show only half of the larger wheel, its center being at (2¼'', 6''). Larger wheel has four arms; smaller one has none. Breadth of arm, 1¾''.

At the left, show a section through the axis of the larger wheel. Face, 2''; Nave, 2½''. Thickness of rim, 1¼''. Dia. of shaft, 1¾''. Boss, 2''. Thickness of nave, ⅞''.

Give all important dimensions, and arrange title and notes as in Plate 23.

Note.—Cut off teeth at interference point as explained on page 38, and make height of teeth $\frac{2}{P}$ as nearly as possible.

Plate 25. (involute bevel gears).

Read pages 39–43 of the notes. Construct a pair of involute bevel gears. Angle between axes 90°. Dia. pitch $\frac{5}{4}$. Teeth 15 and 21.

Show the projections of a *half* of the smaller gear below, and a little more than a *quadrant* of the larger gear to the right. Scale, ½″ = 1″. Place intersection of axes at (4″, 9½″).

Make title and notes according to taste.

PLATE 26. (SCREWS AND BOLTS).

Read pages 44–46 of the text. Construct:

(*a*). A U. S. Standard screw thread cut on a bar 6″ long and 2¾″ diameter. Thread *double* and right-handed. Pitch 2″. Show section of stud as in Fig. 34.

(*b*). A similar stud with *square* thread.

(*c*). A U.S.Standard bolt with hexagonal head, and nut on bolt. Dia. 1″. Length under head, 4″. Single, *right handed* thread. See *table* for other dimensions. Show threads by straight lines instead of helical curves. Show top view of nut removed from bolt.

(*d*). A *left handed* bolt and nut (U. S. Standard), showing threads by heavy and light lines. Dia. ¾″. Length under head 4″. Omit top view of nut in this figure.