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THE PRESCRIBING OF SPECTACLES

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BY

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ILLUSTRATED WITH DIAGRAMS

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PREFACE

This small work describes the practical methods of determining errors of refraction and errors of muscular balance, and gives, I hope, clear directions for prescribing the appropriate spectacles for their relief. The last fifty pages, comprising the Optical Section, give the mathematical solution of most of the optical problems that arise in dealing with the subject, as well as five tables that are frequently required.

I am deeply indebted to Dr. Duane for providing me with the results of his latest researches on accommodation, and to Dr. Tscherning for his last evaluation of the optical constants of the eye.

My little book, I may add, is not a slavish summary of the practice of others, and some of the methods detailed are original. This, I think, justifies its production.

Archibald Stanley Percival.

17, *Claremont Place, Newcastle-upon-Tyne,*
July, 1910.

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The Prescribing of Spectacles.

CHAPTER I.

INTRODUCTORY—ACCOMMODATION.

THE first spectacles of which we have any description were those made by Roger Bacon, towards the end of the thirteenth century. We learn that he ground and polished some glass that he obtained from Belgium, and thus with his own hands made the first convex reading glasses. Concave lenses were invented shortly afterwards.

There is reason, however, to believe that convex lenses were used as magnifying glasses in very early times, for Layard found in the ruins of Nimrud, near Nineveh, a convex lens of rock crystal; indeed, we might have inferred that magnifyers must have been used many years B.C. from the perfection of the carving of ancient gems. Aristophanes, in "The Clouds," 423 B.C., speaks of a crystal lens used as a burning-glass for lighting fires.*

Astigmatism was first discovered by Thomas Young,

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*ΣΤ. Ἤδη παρὰ τοῖσι φαρμακοπώλαις τὴν λίθον
ταύτην ἑόρακας τὴν καλὴν, τὴν διαφανῆ,
ἀφ᾽ ἧς τὸ πῦρ ἅπτουσι; ΣΩ. Τὴν ὕαλον λέγεις;
(ΝΕΦΕΛΑΙ, *ll.* 766-9).

Strepsiades : You know that sort of stone the oddment-dealers sell,
A pretty thing, and you can see through it quite well,
'Tis used for lighting fires ?
Socrates : The lens, you mean ? (*Rudd's Translation.*)

who, in the *Philosophical Transactions* for 1793, published an account of the asymmetry of his own eye, and attributed it to an oblique position of his lens.

His account is so accurate that we infer it would have been corrected by $-1{\cdot}7$ D cyl. axis 90°, and that it would be induced by a 16° rotation of his lens round its vertical axis. His astigmatism was peculiar in that it was certainly of lenticular origin, as it still persisted when his eye was plunged in water.

In 1827, the astronomer Airy corrected his defective sight by means of cylindrical lenses, and since that time spectacles for various purposes have been made, so that now they are expected to relieve not only errors of refraction, but also any slight want of balance that may be found in the ocular muscles.

The eye may be regarded as a sort of photographic camera, that produces upon the retina, by its refracting system, an inverted image of the object viewed. It is, moreover, able to focus objects at different distances. Normally, an eye at one moment can see distinctly objects as remote as the stars, and at another moment objects at a distance of ten inches or less. This focussing power of the eye is called its power of accommodation.

ACCOMMODATION.

Accommodation is brought into play by a contraction of the ciliary muscle, which renders the lens of the eye more convex, and therefore increases its power of refraction. A detective camera, of fixed length that is adjusted for distance, may be adapted for a distance of three feet by placing in front of its lens a convex glass of 3-ft. focus. Similarly, an eye which, in its state of rest, is adapted for parallel rays may, by exercising its accommodation, adapt itself to a distance of three feet. The accommodation exercised in this case is the power of a lens of focus 3 ft.

For example, a person of 25, who is able to see the stars distinctly when his accommodation is relaxed, will, on exerting his accommodation to the utmost, be able to see distinctly an object only four inches from his eye. In other words, the distance of his far point, or *punctum remotum* (R), is infinite (∞), while the distance of his near point, or *punctum proximum* (P), is 4 in. The amplitude of accommodation here is the power of a lens of 4-in. focus.

The metric system is invariably used in dealing with problems of refraction. A metre is 39·37 in., or roughly, 40 in., so that 4 in. is $\frac{1}{10}$ metre, 10 centimetres, or 100 millimetres. A lens of 4-in. focus, or $\frac{1}{10}$ metre, is called a lens of 10 dioptres. The stronger the lens, the shorter is its focal distance. A lens of focal distance 10 cm. is ten times stronger than a lens of focal distance 100 cm. Now a lens of focal distance 1 metre, or 100 cm., is called a dioptre, and when convex is written $+ 1$ D, i.e. $D = \frac{1}{F'}$, where F′ is the first focal distance expressed in metres. A $+ 4$ D lens means a convex lens, the first focal distance of which is $\frac{1}{4}$ metre or 25 cm., while a $- 5$ D lens is a concave glass, the first focal distance of which is $- \frac{1}{5}$ metre, or $- 20$ cm. It is clear that to express a lens by its power in dioptres is much more convenient than by its focal length.

The *amplitude* of accommodation is the greatest power of accommodation that can be exercised; it is found to decrease as age advances. We have found that the person of 25, referred to above, had an amplitude of accommodation of 10 dioptres, or 10 D. If the person had been 44 years old, it would have been probably found that he could only with the greatest effort have seen distinctly an object 25 cm. (or 10 in.) off. In that case his amplitude would have been indicated by the power of a lens of focal distance 25 cm., i.e., by a $+ 4$ D lens.

From elementary optics we know that if p denote the distance (considered positive) of the lens from the object, if q denote the distance of the lens from the image, and if F′ represent its first focal distance in metres, $\frac{1}{p} - \frac{1}{q} = \frac{1}{F'} =$ its power in dioptres.

Similarly, we say that the amplitude of accommodation or $A = \frac{1}{P} - \frac{1}{R} = \frac{1}{F'}$, i.e., the power of the lens in dioptres, where P is the distance (in metres) of the punctum proximum, and R that of the punctum remotum from the lens.

Suppose, for instance, a myopic patient presents himself who cannot see distinctly at a greater distance than 2 metres, but by exerting his utmost focussing power he can see an object at 12·5 cm., or $\frac{1}{8}$ metre from his eye. In such a case $R = 2$, $P = \frac{1}{8}$, therefore his amplitude is

$$A = \frac{1}{P} - \frac{1}{R} = 8 - {\cdot}5 = 7{\cdot}5 \text{ D.}$$

Let us take the case of a hypermetrope of $+ 3$ D. In this case, when accommodation is relaxed, in order that rays may come to a focus on his retina, they must originally be converging to a point $\frac{1}{3}$ metre behind his eye. Consequently his punctum remotum is negative or $R = -\frac{1}{3}$. Now if his punctum proximum is 12·5 cm., or $\frac{1}{8}$ metre distant,

$$A = \frac{1}{P} - \frac{1}{R} = 8 - (-3) = 8 + 3 = 11 \text{ D.}$$

It will be noticed that in these two illustrations we have measured P and R from the eye and not from the lens. Donders estimated the distances from the nodal point of the eye, but it is much more convenient to measure P and R from the anterior focal plane of the eye, the position in which spectacles for the relief of defective accommodation are placed. This is the method adopted by Dr. Duane, who says

" When we speak of a deficiency of accommodation of 6 D we mean one that will be compensated for by a lens of that strength placed at the anterior focus—not at the principal point—of the eye."

Influence of Age.—The position of the punctum remotum, or the static refraction of the eye, does not change until after the age of 50, when a slowly progressive " acquired hypermetropia " is found to occur, so that at 80 hypermetropia to the extent of + 2·25 D has developed, as is shown in the following table :—

Age	Acquired Hypermetropia	Age	Acquired Hypermetropia
55	·1 D	70	1·2 D
60	·3 D	75	1·75 D
65	·75 D	80	2·25 D

This " acquired hypermetropia " is no doubt due to the increased refractive index of the cortical layers of the lens. In youth the lens consists of a dense nucleus surrounded by concentric layers of a less dense material (*Fig.* 1). These peripheral layers may be considered as forming two diverging menisci, *a* and *b*, which enclose the nucleus and diminish its refractive power. When the cortical layers of the lens become more dense with age, the refractive power of these menisci becomes greater, and therefore the power of the whole lens becomes less.

A much more marked change is found to occur in the position of the punctum proximum : as age advances it gradually recedes, so that the amplitude of accommodation becomes less, as is indicated below. The diagram that is given in most of the books professes to be founded on Donders' observations, but recently, owing to the careful and laborious work of Dr. Alexander Duane and Dr. J. B. Thomas, his conclusions have been shown to be incorrect. Donders assumed emmetropia to be present without applying cycloplegic tests, and the number of cases he examined (less than 200) was too small for universally valid deductions to be made from them. Duane's results were published in a paper read in the Section of Ophthalmology of the American Medical Association, June, 1909. His Table 3 gives the maximum, minimum, and mean values of the accommodation found for every year

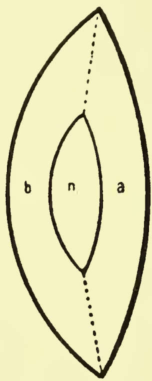

Fig **1.—Diagram of human lens, showing nucleus (n) enclosed by the two diverging menisci, (a) anterior, (b) posterior.**

from 10 to 61. It was based on an examination of 600 cases, and the results have been confirmed "by several hundred additional cases examined since." The diagram (*Fig.* 2) is a graphic representation of the values that he published in April, 1910, the thick line denoting the mean, the dotted line the maximum, and the spaced line the minimum values of A in dioptres. The near point is measured from the anterior focal plane of the eye (13·7 mm. in front of the cornea).

Dr. Duane has very kindly furnished me with the values that he has found since that date, which show a slightly

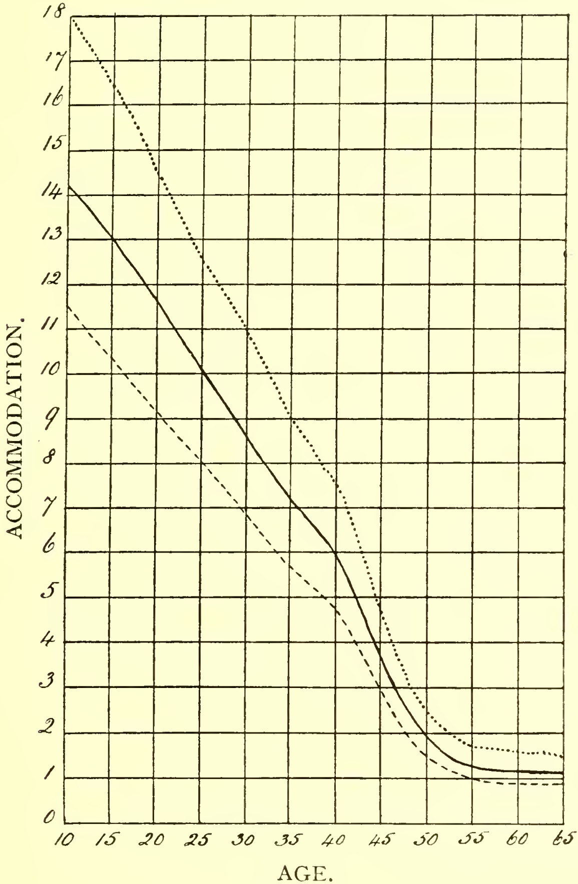

Fig. 2.

greater value for the maximum after the age of 30, and from this last table the following abridgement has been made, giving his results at intervals of five years.

AMPLITUDE OF ACCOMMODATION.

AGE	MEAN	MINIMUM	MAXIMUM	AGE	MEAN	MINIMUM	MAXIMUM
10	14·2	11·5	18·0	40	6·0	4·8	7·8
15	13·1	10·4	16·5	45	3·8	3·0	5·4
20	11·6	9·2	14·5	50	1·9	1·5	3·0
25	10·1	8·1	12·5	55	1·3	1·0	1·9
30	8·6	6·9	11·0	60	1·2	·9	1·8
35	7·2	5·7	9·3	65	1·1	·9	1·5

From childhood until the age of 40 the fall is nearly uniform, being ·29 D every year; from 40 to 48 the fall is more abrupt, the yearly decrement being ·44 D. After this age the descent becomes more gradual, so that after the age of 55 the amplitude of accommodation becomes practically stationary.

This loss of accommodative power is no doubt due to the increasing rigidity of the lens with age, so that the same contraction of the ciliary muscle no longer produces the same convexity of its surface.

Both these conditions, acquired hypermetropia and defective accommodation, should strictly be included under the term *presbyopia*. Donders, however, restricts this term to "the condition in which, as the result of years, the range of accommodation is diminished and the vision of near objects is interfered with," and he regards the commencement of presbyopia as the time at which the punctum proximum recedes to 8 Paris inches (i.e., 8¾ English inches) from the eye. Much difficulty and confusion has arisen from the use of this word presbyopia; for instance, in one excellent book the presbyopia at 70 is given as + 5·5 D, and at 80 as + 7 D. The reader might therefore give an old man of 80, who could see well at a distance, + 7 D spectacles for reading.

With these the unfortunate old gentleman would only be able to read a paper that was 6 ins. from his eyes, and besides this inconvenience, his internal recti would cause symptoms of great fatigue from the excessive convergence that would be necessary to direct both eyes to so close an object.

It will save endless confusion if we agree not to use the word presbyopia, and if we consider the distance at which the patient should hold his work. For playing the piano, for instance, the eyes should be held about 20 ins. ($\frac{1}{2}$ metre) away from the music; for reading, about 13 ins. ($\frac{1}{3}$ metre) is perhaps the ordinary distance, but short people with short arms will find it more convenient to read at a distance of 10 ins. ($\frac{1}{4}$ metre).

It is very important, then, to find the *working distance* at which the patient is going to hold his work. Further, we must find out his amplitude of accommodation, either by trial, or from his age and a reference to the table (*p.* 8). Now, although a patient may have 3 D of accommodation, it does not follow that he can use all his accommodation continuously; a man may be able to lift a 56 lb. weight, but it would be exceedingly irksome to carry such a weight for several hours at a time. In practice it will be found that $\frac{2}{3}$ of the accommodation can be used continuously by most people without fatigue (keeping $\frac{1}{3}$ in abeyance), and on this basis I have drawn up the following table for the working glass required at different ages for different distances :—

WORKING DISTANCE.

AGE	13 ins. $\left(\frac{1}{3}\text{ m.}\right)$	$11\frac{1}{4}$ ins. $\left(\frac{1}{3\cdot5}\text{ m.}\right)$	10 ins. $\left(\frac{1}{4}\text{ m.}\right)$	$8\frac{3}{4}$ ins. $\left(\frac{1}{4\cdot5}\text{ m.}\right)$
40	0	0	0	·5
45	$\cdot4\dot{6}$	$\cdot9\dot{6}$	$1\cdot4\dot{6}$	$1\cdot9\dot{6}$
50	$1\cdot7\dot{3}$	$2\cdot2\dot{3}$	$2\cdot7\dot{3}$	$3\cdot2\dot{3}$
55	$2\cdot1\dot{3}$	$2\cdot6\dot{3}$	$3\cdot1\dot{3}$	$3\cdot6\dot{3}$
60	2·2	2·7	3·2	3·7

It must be understood that the working glass must be added to that which fully corrects the patient for distance. As lenses are usually only made in strengths varying by a quarter of a dioptre, one would select the nearest equivalent glass. For instance, a myope of $-·5$ D, aged 60, who holds his work one foot from his eyes, will require $-·5\text{ D} + 2·75\text{ D} = +2·25\text{ D}$. Note that 2·75 is substituted for 2·7.

In every case one must find the highest convex (or the lowest concave) glass with which the patient can see distinctly the test types at 6 metres, and then the working glass is added to this to give the power of his reading glasses. In our clinical practice we pay no attention to that term of presbyopia which is called "acquired hypermetropia."

The table is very simply constructed. At the age of 50, $A = 1·9$ D, of which $\frac{2}{3}$, or $\frac{2}{3}$ of 1·9 D, i.e., 1·26 D are available for continuous use. For working at $\frac{1}{3}$ metre he will require $+3$ D of adjustment, of which he can use $+1·26$ D of his own accommodation; therefore he will require a glass of $+3\text{ D} - 1·26\text{ D} = +1·73\text{ D}$. Again, at the age of 60, $A = 1·2$ D, of which $\frac{2}{3}$, or ·8 D, are available; therefore for reading at $11\frac{1}{4}$ in. or $\frac{1}{3·5}$ metre's distance, he will require $+3·5\text{ D} - ·8\text{ D} = +2·7\text{ D}$.

Measurement of A.—When we have to measure the amplitude of accommodation (A) in a case, we must find the distance of the *punctum remotum* (R), and the distance of the *punctum proximum* (P). When the vision, even after correction by the requisite glasses, is very defective, it is clear that no accurate measurement of A can be made. The determination of R is called the determination of the static refraction, and will form the subject of the next chapter. This is the important problem that we always have to solve before prescribing any spectacles.

The determination of P may be effected in two ways :—

1. The most usual plan is to let the patient hold in his hand the small test type, and to note the shortest distance at which he can distinctly see the type with each eye separately. It will be found that he can only maintain his greatest accommodation for a few moments, so that the test need only occupy a minute or so.

If the patient be myopic, or if the amplitude be very great, this method is unreliable, as the difference of $\frac{5}{12}$ in. will introduce an error of 1 D in the determination of A, when the near point is only 4 in. off. For instance, in an emmetrope where P = 4 ins. or $\frac{1}{10}$ metre, A would be

10 D; for in emmetropia $R = \infty$, $\quad \therefore \frac{1}{R} = 0$

so A or $\frac{1}{P} - \frac{1}{R} = 10 - 0 = 10$ D;

but if P were $3\frac{7}{12}$ ins. or $\frac{1}{11}$ metre, A would be 11 D.

In all cases in which P is small, it is advisable to give concave glasses for the patient to wear, and to repeat the test. Suppose in the above case when − 3 D were worn P were found to be $\frac{1}{7}$ metre ($5\frac{5}{8}$ in.),

$$A = 7\text{ D} - (-3\text{ D}) = 10\text{ D}.$$

In fact in every case $A = \frac{1}{P} - G - \frac{1}{R}$ where P and R are measured in metres, and where G denotes the power of the lens worn by the patient during the test.

A difficulty will also arise if the patient be very hypermetropic, as in that case the distance of the *punctum proximum* may be inconveniently great; it will even be negative when only converging rays come to a focus on the retina.

Thus, suppose a hypermetrope of 8 D with A = 5 D. In such a case $R = -\frac{1}{8}$ metre, and $P = -\frac{1}{3}$ metre. The amplitude (A) could only be found by providing the patient with convex glasses. Suppose that + 5 D are given him; on now testing his near point, P will be found to be $\frac{1}{2}$ metre (or 20 ins.). Here $G = + 5$ D, and according to our formula $A = \frac{1}{P} - G - \frac{1}{R}$

$$A = 2 - 5 - (-8) = 5 \text{ D.}$$

I personally use for this test my modification of Landolt's Ophthalmo-dynamometer. It consists of a square black plate with a narrow vertical slit in the middle. On one side is Dr. Duane's standard test-object, a small white card 3 mm. $\times$ 1·25 mm. which is bisected by an engraved black line 3 mm. long by ·2 mm. thick; on the reverse side is a similar object, only double the size, for those whose visual acuteness is diminished. This is supported in a holder to the handle of which a steel tape-measure is attached. The tape is marked on one side in centimetres and on the other with the reciprocals of the metric lengths, i.e., opposite 10 cm. or $\frac{1}{10}$ metre is the number 10, opposite 20 cm. is the number 5, opposite 50 cm. is the number 2, and so on. On holding the tape against the temple of the patient and allowing him gradually to withdraw the frame from the eye that is being examined until the engraved line becomes distinct, P can be read off in centimetres, and on the reverse side we have $\frac{1}{P}$ or the dioptric value of the refraction exercised. It is essential that one eye should be examined at a time, the other being closed. If we wish to determine the distance of the punctum proximum from the eye, we must measure the distance of the frame from its first principal point, which is about 2 mm. behind the cornea; more frequently, however, we have to determine the power of the equivalent lens when placed in its usual position ($13\frac{3}{4}$ mm. in front of the cornea); in that case the distance must be measured from this position.

This little instrument will also be found to be very useful in determining the range of convergence (*p.* 91).

2. *Scheiner's Method.*—This depends on the phenomena

that are observed when a diaphragm with two pinholes is placed before the eye. The pinholes must be close together, not further apart than about $\frac{1}{8}$ in. (3 or 4 mm.), and one of the holes should be covered with a small piece of coloured gelatin. We will suppose the colour chosen to be red. The patient holds the diaphragm close to his eye, so that the pupil is immediately behind the two pinholes, and he is directed to look at a point of light, say the light reflected from a thermometer bulb. If his near point be 12 inches from his eye, and the point of light be at this distance, he will see a single point of light. If, however, his eye be nearer the point of light, he will see two points of light, one white and one red. The nearer the eye is to the object, the wider will be the separation of the images. When the red hole is placed on the right side of the pupil, the red image will appear to be on the left side of the white image, and *vice versa.* This is what is called *crossed* or *heteronymous diplopia.* The reason of this phenomenon will be evident from the adjoining diagram (*Fig.* 3). It will be seen that with a myopic patient if the point of light be beyond his far point, the red image will appear on the right side of the white one, in other words the diplopia will be *homonymous.*

In the diagram DD represents the diaphragm with its two pinholes A and B; behind it is the eye with its retina, if hypermetropic at H, if emmetropic at E, and if myopic at M. P is a point of light at the punctum proximum of an emmetropic eye. Diverging rays from P will be seen to reach the diaphragm D, and through the two holes at A and B two pencils will pass, being converged by the refracting system of the eye towards the focus p on the retina of the emmetropic eye. Therefore only one point will be seen by the emmetropic eye, when P is at its punctum proximum.

If P be within the distance of the punctum proximum of the eye, the rays from P will be so divergent that the eye will tend to bring them to a focus behind the retina. In other words, H will represent the retina in this case, and the two converging pencils will form images on the retina at *a* and *b*. They will really be confusion circles, but as the pencils transmitted through the holes are very thin, the images will be fairly distinct. These images will be projected outwards in the reverse direction, as are all retinal images.

The direction in which the object is seen is obtained by tracing a line from the retinal image through the nodal

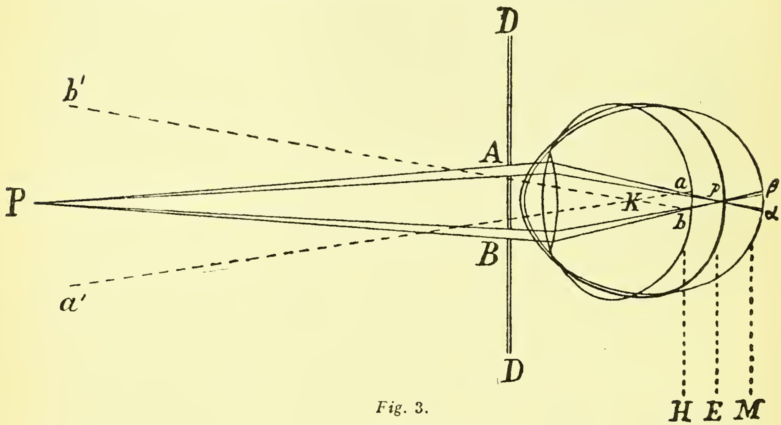

Fig. 3.

point K, so that the retinal images at *a* and *b* will be seen as two points of light at *a′* and *b′*. The diplopia will consequently be heteronymous.

If P be further from the eye than its punctum proximum, for which it is accommodated, the pencils will cross in the eye at *p* and will form small confusion circles at *α* and *β*. The retinal image *α* will be seen as an object in the direction of *b′*. In other words, two points of light will be observed, seen in the positions corresponding to the two apertures A and B. In this way *homonymous diplopia* will be produced.

I cannot recommend this method, as the patient cannot be trusted to always hold the two pinholes exactly in front of his pupil, and only one image will be seen, whatever the distance, when only one pinhole is opposite his pupil.

Paralysis of Accommodation.—When the amplitude of accommodation (A) is found defective for the age, a paresis or paralysis must be present. Paresis of the ciliary muscle will be found during convalescence from any debilitating illness, such as influenza, but in its most marked form it appears as a sequela of diphtheria. It frequently occurs after what has been diagnosed as follicular tonsillitis; it always affects both eyes, and it may be associated with paralysis of the soft palate, which is rendered evident by the nasal character of the speech. Dilatation of the pupil (iridoplegia) is usually absent; this distinguishes it from *ophthalmoplegia interna.* Objects often appear smaller than usual (micropsia), as the increased effort to exercise accommodation would lead the patient to think that the object is nearer than it really is, were it not that the retinal image is not increased. Hence objects appear smaller.

This, however, is not a complete explanation, as patients often say that objects seem to be small and very far away. Now objects of known size, if they appear small are presumably at a great distance; if they appear near, there will be a considerable amount of convergence exercised. In this case, as an unusual effort of accommodation is made while no excessive convergence is called into play, there is a confusion of mental judgment as to whether the objects are small or at a great distance. The final inference usually made is that the objects are both smaller and further away than they really are.

The prognosis in a case of diphtheritic paralysis is good, as it usually passes away of itself in one or two months; periodic instillations of a myotic such as pilocarpine will be found to hasten recovery.

CHAPTER II.

STATIC REFRACTION.

THE refraction of the eye when the accommodation is relaxed is called the static refraction.

If when the accommodation is completely relaxed the refraction of the eye is such that parallel rays come to a focus on the retina, the condition is called *emmetropia*. In other words, an emmetrope can see distinctly distant objects without exerting his accommodation, provided that no pathological lesion or functional incapacity exists. If parallel rays come to a focus behind the retina, the condition is *hypermetropia*, and if in front of the retina *myopia*.

The cause of *ametropia*, or the condition involving an error of refraction, is usually an excess or defect of the length of the eyeball in simple myopia or hypermetropia. Alterations of the curvature of the cornea are sometimes found, a flattening of the cornea causing "curvature hypermetropia," while "curvature myopia" is due to too pronounced a curvature. Astigmatism is caused almost entirely by a different amount of curvature of the cornea in different meridians ; it will be dealt with later on.

Sometimes, as in diabetes, an alteration of the refractive index of the media causes an error of refraction. Two other spurious forms of hypermetropia may be mentioned : aphakia, the condition of the eye when the lens has been removed ; and the suddenly acquired hypermetropia, due to detached retina, which however does not demand any treatment with glasses.

Tests of refraction may be divided into subjective and objective. Before applying any accurate subjective test it is most necessary to exclude all pathological errors except those due to errors of refraction. For this purpose nothing is better than the pinhole test, of which too little use is now made. We will, however, begin the description of the subjective tests with an account of an old-fashioned qualitative test that I personally have never found useful, although it is worth describing from its intrinsic interest.

SUBJECTIVE TESTS OF AMETROPIA.

Chromatic Test.—This depends upon the fact that the eye is not truly achromatic, blue light coming to a focus nearer the cornea than red light. Ordinarily this is of no inconvenience, but if a purple glass that only transmits red light and blue light is used, the chromatic aberration of the eye may be made evident. The patient is directed to look at a point of light through such a glass with one eye, the other eye being closed. In cases of hypermetropia the point of light will appear blue surrounded with a red ring, but when myopia is present the light will appear reddish and surrounded with a blue ring.

Pinhole Test.—A black diaphragm having a small perforation in its centre will be found in all boxes of trial lenses. The patient is first asked which line of the test types he can see with each eye separately. This being discovered, he is given the pinhole and told to hold it close to one eye and to look through the hole, while the other eye is closed. If his sight is improved, that eye quite certainly suffers from some error of refraction; if the sight is not improved, the failure of vision will be due to some other defect, and we may suspect that the transparency of the media or the retinal sensibility is defective.

The reason is obvious. The pinhole only gives passage to a very narrow pencil of rays, and therefore the size of the confusion circles on the retina must be smaller. It is true that less light will enter the eye, and therefore the brightness will be less; but the image, and consequently the vision in ametropia, will be much more distinct. If the diameter of the aperture be $\frac{1}{5}$ that of the pupil, the diameter of the circle of confusion on the retina will be $\frac{1}{5}$ that of the usual circle of confusion. Suppose, for instance, that owing to a refractive error, the rays diverging from a point of light come to a focus behind the retina, so that on the retina the converging pencil covers an area occupied by 100 cones at the macula; with a pinhole $\frac{1}{5}$ of the diameter of the pupil before it, the area covered will be $\frac{1}{25}$ smaller, and only 4 cones will be covered; consequently the acuteness of vision will be enormously increased.

Further, a test of the kind of refractive defect, and a rough estimate of its amount, may be made with this useful little device, although I do not recommend this plan for accurate results.

When the pinhole is held say 3 in. in front of an ametropic eye, and the patient moves it slightly up or slightly down, while his attention is directed to the distant test types (the other eye being of course closed), he will notice that the test types appear to move. If the motion is in the same direction as that of the pinhole, the vertical meridian of his eye will be myopic; if the motion is in the opposite direction, that meridian will be hypermetropic. Similarly, by moving the pinhole from side to side, the refraction of the horizontal meridian may be tested. If the motion in one direction is more extensive than in the other, astigmatism is present, the greatest error occurring in that meridian in which the motion is most rapid and extensive. Should the apparent motion of the test types be *with* the motion of the pinhole from above downwards, and *against* the motion from side to side, mixed astigmatism is present, a concave glass being required from above downwards and a convex glass from side to side. The presence of astigmatism of 1 D or so can be easily detected in this way.

On putting up before the eye (between it and the pinhole) the correcting glass, the test types will appear to be stationary, however the pinhole is moved. I do not think the test can be relied upon in any but the most intelligent patients to give results with less error than 1 D.

The explanation of the phenomena is obvious from a consideration of the diagram (*Fig.* 3).

The aperture is first held in the position A, and in a hypermetropic patient the image of the distant point tends to be formed at a; when the hole is depressed to B the image is at b. These retinal images are projected externally at a' and b'. Hence in hypermetropia, on moving the hole downwards, the object viewed appears to move upwards from a' to b'. If the eye be emmetropic, the retinal image is formed at p, wherever the aperture is; consequently the object appears stationary. In myopia the retinal image at α will be seen as an object near b', and that at β as an object near a', as explained above. Therefore, in myopia, the object will appear to move *with* the pinhole, and in hypermetropia *against* the pinhole.

Visual Acuteness.—The acuteness of vision is the function of the nervous mechanism of the eye and brain; the acuteness may be good although the refractive error be great; in such a case, on correcting the refractive error with glasses the vision will be of standard amount. The first point to decide is the answer to the question, What is *standard* vision?

In order that two points may be recognized as such by the eye, their retinal images must be separated by at least one unexcited retinal cone. Now the diameter of one macular cone subtends at K, the posterior nodal point of the eye, an angle of 26·55″. Therefore, the minimum visual angle is a little over 53″, in practice an angle of 1′ is assumed to be the minimum visible, i.e., we assume that two stars separated by an angular interval of less than 1′ would be seen as a single point of light. In some cases it will be found that the eye can separate two points that lie closer than this : the visual acuteness is then said to be greater than 1, but it is universally agreed to regard $1'$ as the standard value of the minimum visual angle.*

Snellen's test types are based on this principle. The line which should be read at 6 metres' distance consists of letters, each of which subtends an angle of $5'$, while the distinguishing characteristics of each letter subtend an angle of not less than $1'$. For instance, the central mark of the letter E subtends an angle of $1'$. The line above consists of larger letters which subtend the same angle $5'$ at a distance of 9 metres. Above this are lines of increasingly larger letters that a standard eye should read at distances of 12 m., 18 m., 24 m., 36 m., and 60 m. A patient who at 6 metres distance can only see the line which he should read at 12 metres is said to have a vision of $\frac{6}{12}$. If, on providing him with a correcting glass, he can read the 6-metre line, his visual acuteness is said to be $\frac{6}{6}$, or standard. It is assumed that rays proceeding from a point at a distance of 6 metres, or 20 feet, may be regarded as parallel ; they are so nearly parallel that the defect is quite negligible, so that a patient who has standard vision at 6 metres has also standard vision at an infinite distance.

As some letters are much easier to discern than others,

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* It might be thought that the resolving power of the eye should be dealt with in the same way as that of a telescope or a microscope, by taking account of the diffraction rings. It will be found that this method gives a lower limit than that indicated in the text, which depends upon the size of a macular cone. For instance, the diffraction formula for the minimum angle is $\theta = 1{\cdot}22\,\frac{\lambda}{a}$ or $\left(\frac{5}{a}\right)''$ where a is the aperture in inches, which is Dawes' rule for the resolving power of telescopes when light in the middle of the spectrum is considered. If the pupil be $\frac{1}{4}$ in. in diameter, of course the min. vis. would be an angle of $20''$. This refers to an emmetropic eye, and does not invalidate the statement made on p. 18 which deals with the improved definition that results from the use of a pinhole with an *ametropic* eye.

Landolt has provided us with some more accurate test types, formed of broken rings resembling the letter C; the gap subtending an angle of 1′ at the assigned distance of 5 metres. The sizes of the figures are so arranged that, at 5 metres distance, each of them corresponds to a different acuteness of vision (·1, ·15, ·2, ·3, ·4, ·5, ·6, ·7, ·8, ·9, 1, 1·25, 1·5, 1·75, 2 (*Fig.* 4).

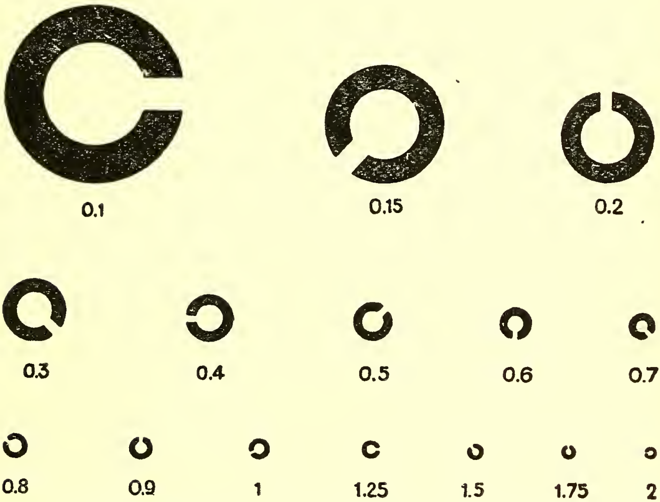

Fig. 4.—(In this figure the sizes of the test types have been reduced, so that the distance at which they should be viewed is 5 feet.)

When the patient cannot see the 60 m. line of Snellen's types at the prescribed distance of 6 m., he is allowed to approach the types until he can see the largest letter. If this occur at 3 m. we enter his uncorrected vision as $\frac{3}{60}$. When Landolt's types are used his vision would be noted as $\frac{3}{50}$ or ·06.

The system of radiating lines in the diagram (*Fig.* 5) is a most useful test for astigmatism, and is hence called the astigmatic fan. The method of using it is explained on *p.* 29; meanwhile it will be sufficient to say that if all these radiating lines appear *equally* distinct, astigmatism is probably not present.

We must carefully distinguish between the terms *vision* and *visual acuteness.* The first refers to uncorrected vision, while the second refers to the vision after correction. V = 1 should be termed *standard*, not *normal* vision, for it is by no means the average vision; misapprehensions often arise in the law courts from the use of the term *normal.*

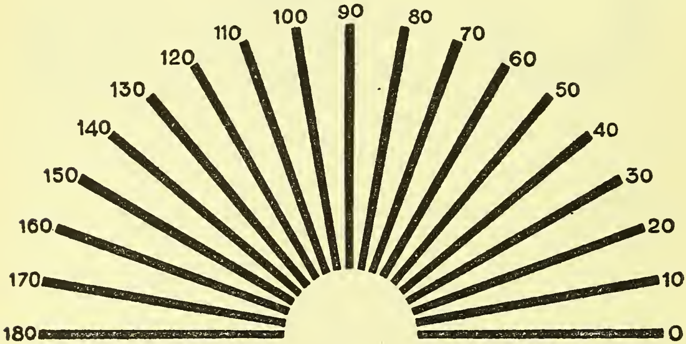

Fig. 5.

As we know that convex glasses magnify and concave glasses diminish the apparent size of objects, it would naturally be thought that the retinal images of a corrected hypermetrope would be larger than those of a corrected myope. It will be proved later (p. 125) that if the ametropia be axial this is not the case when the correcting glasses are placed in the anterior focal plane of the eye. Under these conditions, which commonly prevail, the retinal images are of precisely the same size as those of an emmetrope without glasses. Hence the results of all tests of visual acuteness in axial ametropia are strictly comparable.

Subjective Test of Visual Acuteness.—By the pinhole test we have found that there is some refractive error to correct. We therefore place the patient at 6 metres distance from the test types, which should be well illuminated, and placing the trial frame on his face we cover up his left eye with a diaphragm so as to exclude it. We then ask him to read the smallest type that he can see.

1. If he can read $\frac{6}{6}$ he must be either emmetropic or hypermetropic (or possibly very slightly astigmatic).

Holding up + ·5 D or + 1 D before his eye, if we find that he can still read $\frac{6}{6}$ he is certainly hypermetropic, and we go on trying stronger and stronger convex glasses until the 6 m. line is blurred. We then direct his attention to the astigmatic fan, and note if all the radiating lines appear equally distinct, or equally indistinct. If this is so, he is presumably not astigmatic in that eye. Suppose that with + 1·75 D the 6 m. line is just blurred, we note down that his manifest hypermetropia is + 1·5 D

or $$R.H^{m} = +1{\cdot}5\ D$$

when + 1·5 D is the strongest glass with which he can see $\frac{6}{6}$.

This is not, however, the total hypermetropia, for if the accommodation be paralysed by the instillation of gutt. atrop. sulph. 1 per cent three times a day for two or three days, a much stronger glass will be required to enable the patient to read $\frac{6}{6}$. We might find that + 4·5 D were required. It is clear that the total hypermetropia is + 4·5 D, of which + 3 D is *latent.* The hypermetropia of the patient has always been corrected by the accommodation, so that although the necessity for this is removed by offering him glasses, there remains a tonic contraction of the ciliary muscle which renders + 3 D latent. In such cases the best treatment is to prescribe lenses which correct all the manifest and one-third of the latent hypermetropia. It is true that such glasses will need changing for stronger ones in two or three years, as the tonic contraction of the ciliary muscle relaxes, but the glasses will be comfortable to wear from the first.

It will be found that the younger the patient the more of the total hypermetropia is masked or latent. In some cases the whole of the hypermetropia is latent, or even an apparent myopia may be induced by an excessive contraction of the ciliary muscle. Such a condition is often called *spasm* of accommodation, and will probably require a prolonged use of atropine instillations for treatment. A good indication of the presence of much latent hypermetropia is afforded by finding the position of the near point.

For instance, suppose a young girl of 15 presents herself with rather small pupils, and reddened eyelids. She complains that her eyelids feel heavy and that her eyes often ache and water. She has a nearly constant headache, which is worse in the evening after doing needlework. She is able to read $\frac{6}{6}$ with each eye separately, but she can also read the same line with $+ 4$ D. On testing her punctum proximum, we find that she cannot hold the type nearer than 8 ins. or $\frac{1}{5}$ metre. Now we know that at 15, A = 13 D (*see p.* 8). Therefore, if she have only $+ 4$ D of hypermetropia, she ought to have $+ 9$ D of accommodation available, and so we should expect her to read at $\frac{1}{9}$ metre or $4\frac{1}{2}$ ins. We suspect, then, that a great part (perhaps $+ 4$ D) of her hypermetropia is latent. We therefore instil atropine and defer her examination until another day. In this actual case I found on paralysis of her accommodation that her total hypermetropia was $+ 7{\cdot}5$ D. I ordered her to wear $+ 5$ D constantly, as that practically corrects all her manifest ($H^m = +4$ D) and $\frac{1}{3}$ of her latent H.

$$(H^l = H^t - H^m = +7{\cdot}5\text{ D} - 4\text{ D} = +3{\cdot}5\text{ D})$$

or $4\text{ D} + \frac{1}{3}$ of $3{\cdot}5\text{ D} = +5$ D approximately.

The note made in my notebook was

$$\text{R \& L. V}\tfrac{6}{6}\quad H^m + 4\,D\,\tfrac{6}{6}\quad H^t + 7{\cdot}5\,D.\quad A = 12{\cdot}5\,D.$$
$$\text{Ord.} + 5\,D.$$

The Fogging System.—It is a good plan, when testing a patient, to give him an over-correction at first. In hypermetropia, convex lenses of such strength are first put in the trial frames that the vision is reduced to, say, $\frac{6}{18}$. By slowly reducing the strength of these lenses, satisfactory vision of $\frac{6}{6}$ may be at last obtained with higher powers than would have been attained by gradually increasing the strength of the glasses. By this method we often succeed in getting a greater relaxation of the ciliary muscle, and so a higher amount of hypermetropia is made manifest.

After finding the highest convex glass with which the patient has the best vision with each eye separately, we should always try the vision with both eyes together, and see if a still higher glass will be tolerated.

For example, $\frac{6}{18}$ is read with each eye separately when $+ 4$ D lenses are put in the trial frames; on gradually reducing the strength to $+ 3$ D each eye can then read $\frac{6}{6}$. When both eyes are used together it is found that $\frac{6}{6}$ can still be read when $+ {\cdot}5$ D glasses are held before the trial frames.

The entry in the notebook should take the form

$$\left.\begin{array}{l} R\ H^m + 3\,D\ \tfrac{6}{6} \\ L\ H^m + 3\,D\ \tfrac{6}{6} \end{array}\right\} H^m + 3{\cdot}5\,D\ \tfrac{6}{6}$$

If no higher amount of hypermetropia is found by the other tests, these glasses may be prescribed.

2. Suppose that our patient's vision is below the "standard," and that he can only read the fourth line of the test types ($\frac{6}{18}$), and that this line is blurred with a weak convex glass. He must then be either myopic, astigmatic, or he may have spasm of accommodation.

The last condition, a high degree of latent H, is revealed by the position of the near point, by the instillation of atropine, and by the objective tests to be presently described.

If a concave glass improve his vision, and if the near point is closer to his eye than it should be at his age, we may assume him to be myopic, but he may also be astigmatic. Astigmatism, as before, must be first excluded by the use of the fan.

For example, a patient of 35 who is not astigmatic, sees with each eye $\frac{6}{18}$ and with $-$ 2 D he can see with each eye separately $\frac{6}{6}$. We find that his near point is about 5 in., or $\frac{1}{8}$ metre, distant. Now at 35 he should have 7 D of accommodation, therefore we might expect him to have $-$ 1 D of myopia, for $8 - 1 = 7$. In myopia the amplitude of accommodation is sometimes less than the normal amount, owing to the want of exercise of the ciliary muscle. On trying him with concave glasses we find that with $-$ 2 D he can see $\frac{6}{6}$ with each eye, and that with $-$ 1·5 D before both eyes together, he can still see $\frac{6}{6}$. We therefore enter in our note book

$$\left.\begin{array}{l} \text{R V } \frac{6}{18} \text{ M}^{\text{m}} - 2\text{ D } \frac{6}{6} \\ \text{L V } \frac{6}{18} \text{ M}^{\text{m}} - 2\text{ D } \frac{6}{6} \end{array}\right\} -1{\cdot}5\text{ D } \frac{6}{6}$$

and prescribe $-$ 1·5 D for his use.

In all cases of young myopes it is important to paralyse the accommodation before prescribing glasses, as otherwise one is almost certain to over-correct the myopia. To allow for the tone of the ciliary muscle one should add $-$ ·5 D to the glasses which give the best vision under atropine.

3. Should no spherical lenses materially improve the sight, the patient is almost certainly astigmatic.

ASTIGMATISM.

This may be either regular or irregular.

Irregular Astigmatism is usually due to the irregularities left on the cornea after ulcers and nebulæ, but it may be caused by conical cornea or some other defect. The diagnosis of any marked case is readily made in the following way. The patient, facing the window, is made to look in such a direction that the surgeon sees its reflected image on that part of his cornea that is behind the pupil. If the image is perfect, the surface of the cornea at this part is normal and regular. If the pupillary area of the cornea is irregular or abnormally curved, the reflected image of the window is distorted in shape. The vision will be exceedingly defective, and the examination of such cases is most tedious and unsatisfactory. Little or no help is given by objective tests, and great patience is required to discover the most suitable glasses. I think the best result can be obtained by the use of the stenopaic slit, which consists of a black diaphragm with a slit about 2 mm. wide in the middle line. Placing it first in the horizontal direction in the trial frames immediately before the pupil (the other eye being covered), we find out how much of the test types the patient can see, and the strongest convex (or the weakest concave) glass that gives him the best vision with the slit horizontal. We then repeat this procedure with the slit in the vertical direction, and again in oblique directions at angles of 45° and 135°. Great care must be taken that the slit in the vertical or oblique positions is exactly in front of the centre of the pupil. If the patient can see through the slit without tilting his head, it may be assumed that it is in the right position.

Suppose that, when the slit is horizontal, his best vision is attained with + 1 D. We then try, by altering the position of the slit a little one way or the other to see if his vision is improved. If we find that the exactly horizontal position is the best, we may regard + 1 D as giving the best correction in the horizontal meridian. Now, on rotating the slit to the vertical direction, we try again with various glasses. Suppose we find that − 4 D gives the best vision; we can then infer that the best glass we can order will be one that is convex to the extent of + 1 D in the horizontal direction, and concave to the extent of − 4 D in the vertical direction, i.e., a + 1 D sph. − 5 D cyl. plane axis horizontal.

However, on trying this glass, it is more than likely that the patient will not see so well as with a simple spherical lens and the slit. In such a case we can, if he prefer it, order a spherical lens with one of its surfaces blackened except for a central linear opening. Of course, with such a slit, the field of view is enormously diminished, but the patient may prefer better sight with diminished field. Patient trial is the only method by which even approximately satisfactory results can be obtained in irregular astigmatism.

Regular Astigmatism.—This occurs when the refraction of the eye is a maximum and a minimum in two meridians which are at right-angles to each other. It is usually due to the cornea having a different curvature in these two meridians. The cornea is never truly spherical even in emmetropia, but it may be regarded as resembling the top that one cuts off the breakfast egg. Now if one were to cut off, instead of the top, a piece from the side of the egg, it would resemble an astigmatic cornea. It would be more curved from side to side than from above downwards. Clearly, if the greatest curvature of the cornea be from above downwards, the greatest refraction would be in the vertical meridian, and the least in the horizontal meridian. This is called astigmatism "with the rule," and is the most common form. It is corrected by a cylindrical glass, the plane axis of which is in the emmetropic meridian.

The greatest curvature may be in an oblique or a horizontal meridian. This last condition is called astigmatism " against the rule," and commonly gives rise to more pronounced symptoms of asthenopia than the usual form.

A similar astigmatism of the lens sometimes occurs from an unequal contraction of the ciliary muscle ; it is always, I believe, in the reverse direction to that of the cornea, and appears to be Nature's attempt to correct the corneal astigmatism.

If a patient be astigmatic " with the rule," his horizontal meridian being emmetropic and his vertical meridian myopic, on looking at the letter T he will notice that the vertical limb of the letter is distinct, but the horizontal limb is blurred. The explanation of this phenomenon requires a little thought. A line is regarded as distinct when its margins are well defined, while the definition of its ends is of little importance. Therefore, on looking at a vertical line, all that is necessary to see it distinctly is to get good definition of its width, i.e., to have the horizontal meridian of one's eye properly focussed. If the vertical meridian of our patient's eye be myopic, it is clear that he will not be able to define clearly the height of a transverse line ; he will see its terminations clearly, but he will be unable to define its upper and lower margins.

On this principle is based the subjective test for astigmatism. The patient is directed to look at a system of lines radiating from a centre in directions varying from 0° to 180° (*Fig.* 5). The " Fogging " system is the best to adopt. The highest convex (or the weakest concave) glass is given that just enables him to distinguish one set of lines. The strength of this glass is reduced until he can see one of the lines distinctly, while the rest are indistinct, especially those at right angles to the distinct one. The distinct line gives the meridian of *greatest* curvature. Let us suppose it is the line at 80° from the horizontal. On now holding in front of the trial frames concave cylinders, with their plane axes at right angles to the 80° line (i.e., at an angle of 170°), of gradually increasing strength, we shall eventually find that cylinder with which the 170° line can be defined. This is the correction.

On putting the cylinder at this angle into the trial frames we try the effect of a weak convex or a weak concave glass held in front of the combination, and make any slight alteration with which he can read more of the test types. If he sees better with his head tilted to the right, he will require the cylinder slightly rotated to the right, and *vice versa*.

Astigmatism may be also tested with the stenopaic slit as described on *p.* 27, but it should be remembered that no test for astigmatism is reliable without the instillation of a mydriatic, and it is well not to waste too much time on subjective tests, as a more accurate determination can be made in a far shorter time by the objective tests.

Notation of Cylindrical Lenses. — This is still a vexed question; all, however, are unanimous on one point, "that the meridians of astigmatism should be measured and represented as the observer looks at the patient."

With regard to the angle notation, there is almost every possible diversity of opinion. Some, paying attention to the undoubted bilateral symmetry of the eyes towards the middle line, consider that the nasal extremities of the horizontal line of the trial frame should be the zero for each eye, and that the angles should be measured from this point by a radius vector rotating through an angle of 180° to the temple.* Here again there is room for

* This was the method submitted by the Commission at the International Congress held at Naples, 1909.

a difference of opinion which is a new source of trouble, some rotating the vector upwards and some downwards. Consequently it is necessary when prescribing for patients to indicate the angle by a line (oblique when needed) to show which method is adopted. It is exceedingly difficult for the printer to indicate these axes by sloping lines, so that it is very inconvenient for printed reports of cases.

In contradistinction to these ambiguities is the standard notation approved by the Optical Society in 1904, in which, for each eye, the radius vector is supposed to rotate

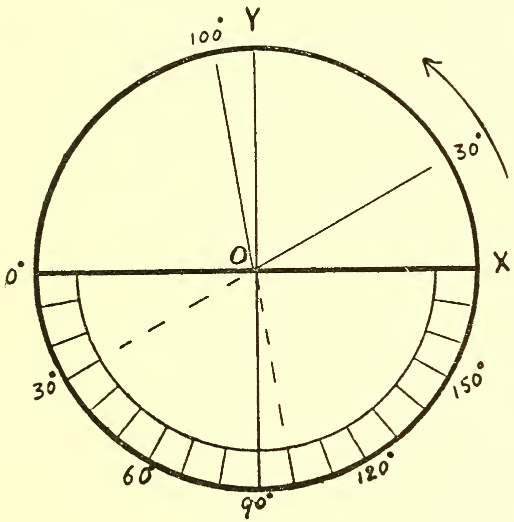

Fig. 6.

counter-clockwise from the horizontal position. This notation is easily understood from the adjoining diagram (*Fig.* 6, *see also Fig.* 5, *p.* 22). Angles of 30° and 100° mean angles measured counter-clockwise from the initial line OX. The trial frames which have the angles marked on the semicircular arc below the glasses must therefore be graduated counter-clockwise, starting from 0° on the observer's left.

This method is universally used by all mathematicians for positive angles, by all manufacturing opticians, and almost universally in the United States. Further, it avoids all source of error when using the astigmatic fan or the astigmometer, and takes away all the difficulty from printing reports of cases. When bilateral symmetry is present, the sum of the angles is always 180° (e.g., R 80° and L 100°), so there is no difficulty about this point.*

Transposition of Cylinders.—If an eye be hypermetropic in the vertical meridian to the extent of + 2 D, and to the extent of + 4 D from side to side, we can correct the error by either + 2 D sph. + 2 D cyl. ax. 90°, or by + 4 D sph. − 2 D cyl. ax. 0°. Clearly, the former glass would be the easier to make, and would be the lighter. The rule fcr transposing such a prescription is the following :—

The new spherical power will be the sum of the old spherical and cylindrical powers, while the new cylinder will have the same power as the old cylinder, but of opposite sign, and its axis will be at right angles to that of the old cylinder.

Thus + ·5 D sph. − 3 D cyl. ax. 15° is equivalent to − 2·5 D sph. + 3 D cyl. ax. 105°.

Now, although the last form would be heavier and would entail more grinding, its periscopic effect would be much greater if worn with the concave spherical surface next the eye. (*See p.* 51)

OBJECTIVE TESTS OF AMETROPIA.

There are several ready tests that we may apply with the ophthalmoscope to determine the nature of the ametropia. We will first describe these before dealing with the methods for precisely determining its amount.

* In this book the standard notation will be always adopted.

(*A*). *The Mirror at a Distance.*

(1) Let us suppose that the mirror is held about a metre from the patient in a dark room, and if, on throwing the reflected light into his eye, one can see the disc or some of the vessels without using the convex lens, one may be sure that there is a considerable error of refraction. If *myopia* be present, an inverted image of the patient's fundus will be formed at his far point, and when this happens to be on his side of the mirror, one will see a part of it at any rate.

In *emmetropia* no image will be formed, as the emergent rays will be parallel, and at the distance of one metre one will only be able to receive one beam of parallel rays, corresponding to a single point of his illuminated retina. In *hypermetropia* a virtual image will be formed behind the patient's head, and part of this erect magnified image will be seen.

If the observer now move his head (and the mirror) from side to side, the vessels that he sees will move in the same direction in cases of hypermetropia, as the image is erect, but they will move in the opposite direction in cases of myopia, as then the image is inverted.

(2) On slightly turning the mirror, the pupillary red reflex is seen to move *with* the mirror in myopia > -1 D, but against the mirror in hypermetropia. (*See* RETINOSCOPY, *p.* 36).

(*B*). *The Indirect Method.*—Let the ophthalmoscope be now brought about 20 inches from the patient, and on holding the convex lens close to the eye, an inverted image of his fundus will be seen in the usual way. Unless the observer be myopic, he will probably see the image by the indirect method more easily if he put up a $+3$ D lens behind his ophthalmoscope.

If he now slowly withdraw the lens from the patient's eye, he will notice that the image appears to increase in size in myopia, but to diminish in hypermetropia. The greater the change in size, the greater is the ametropia. If the disc is viewed, and, as the lens is slowly withdrawn from the eye, the image of the disc becomes more oval, astigmatism is present. The diameter that increases the most, indicates the meridian which is most myopic ; that which diminishes the most, is the most hypermetropic. If no change is noticed in the size of the disc on moving the lens, the eye must be nearly emmetropic.

The explanation of these points is given in the optical section of this book (*p.* 144).

When the convex lens is close to the eye, the image is greater in hypermetropia than in myopia.

Amount of Ametropia : Direct Method.—There are three conditions which must be fulfilled before this can be regarded as a satisfactory method. The accommodation both (1) of the patient, and (2) of the observer, must be relaxed, and (3) the ophthalmoscope must be held about $\frac{1}{2}$ in. from the patient's cornea, so that the correcting glasses that are required may be placed in the first focal plane of the patient's eye. The observer must first correct any existing ametropia in his own eye by an appropriate glass.

If the patient be *emmetropic* the light from his eye will emerge in parallel beams, and if the previous conditions are observed, a distinct image of his fundus will be seen, which will be blurred by the addition of the weakest convex glass.

If the patient be *myopic* the fundus will appear indistinct until the lowest necessary concave glass has been added.

Similarly, in *hypermetropia* the highest convex glass must be put up that will enable the observer to see the fundus distinctly.

In *astigmatism* a different glass will be required to get a distinct view of vessels running in different directions. Thus if + 3 D is the strongest glass with which the *horizontal* vessels are seen distinctly, + 3 D is the refraction in the *vertical* meridian. If the vertical vessels can be defined with + 5 D, this indicates the refraction in the horizontal meridian. This is much the readiest method of estimating a patient's refraction, but it requires a good deal of practice to be at all proficient at it. Even experienced ophthalmologists cannot attain the same accuracy in their results as can be reached by the next method described.

It is important to remember that, though the optic disc is the easiest object to examine, it is the very part to avoid in an estimate of a patient's refraction, as it corresponds to his blind spot. The macular region is the part of which the refractive error should be determined; but this is the very part in which there are no pronounced details to observe. It is true there are usually some horizontal vessels going towards the macula from the disc, but at the macula itself there are no vessels to be seen. Finally, in estimating astigmatism, it is most unlikely that the patient should happen to have two vessels near the macula at right angles to each other that lie exactly in the meridians of his astigmatism. I cannot therefore recommend this method, although it should be practised and checked by retinoscopy, as the ability once acquired is invaluable in estimating the amount of swelling present in a case of optic neuritis, or the forward displacement of a detached retina.

Should the observer be ametropic, he must deduct his own error of refraction from the correcting glass used behind the ophthalmoscope to determine the patient's ametropia. Thus if the observer be myopic to the extent of − 2 D, and find that he sees R fundus with − 1 D, L with + ·5 D,

$$\text{R ametropia is } -1\text{ D} - (-2\text{ D}) = +1\text{ D}.$$

$$\text{L ametropia is } +{\cdot}5\text{ D} - (-2\text{ D}) = +2{\cdot}5\text{ D}.$$

Retinoscopy.—This is far the most accurate and perfect way of objectively testing a patient's refraction. I shall therefore describe the method in full detail, as there are several minutiæ about its correct performance that do not seem to be generally known. The principle of the method is simple enough, as will be seen from the adjoining diagram (*Fig.* 7).

The light A is placed behind the patient's head 150 cm. distant from the concave mirror of focal length 25 cm.,

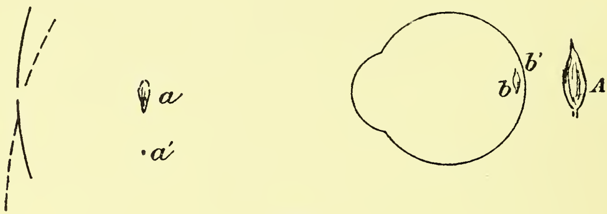

Fig. 7.

held by the observer. A real inverted image of the light will be formed at a, 30 cm. from the mirror, for

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}\therefore\frac{1}{q}=\frac{1}{25}-\frac{1}{150}=\frac{1}{30}.$$

The light from this image (a) will illumine a small area of the patient's retina at b; in fact, an inverted image of a will tend to be formed at b.

Now, on turning the mirror slightly downwards, the image at a will move downwards to a'. Consequently the illumined area on the patient's retina will move upwards to b' in every case, whether the patient be myopic or hypermetropic. If the patient's distance from the observer be less than that of his far point, the observer will see a magnified erect image of the patch of light through the patient's pupil. We will suppose that the observer is seated at the distance of 1 metre from the patient, so that, by leaning back a foot, the distance between his mirror and the patient's eye will be 133 cm., and by leaning forwards 8 in. the distance will be reduced to 80 cm. It is clear, then, that if the mirror is held at 1 metre's distance from the patient's eye, and, on turning the mirror, the light is seen to move in the reverse direction across the pupil, that eye must be hypermetropic, emmetropic, or myopic to a less extent than 1 D. If the myopia be greater than 1 D an inverted image of *b* will be formed at the patient's far point, which is situated somewhere between him and the observer. This inverted image of *b* is what the observer will see, and as it moves in the reverse direction to *b*, it will move in the same direction as the mirror.

Suppose a patient, whose eyes are fully dilated with atropine, keep his eyes fixed on the mirror, and, on turning the mirror from side to side, the red reflex is seen to move across the pupil in the same direction as the mirror; concave glasses are placed in the trial frames before the patient's eyes, until one is found that just reverses the movement of the light. It will be found that as the right correction is approached, the red reflex becomes brighter and the movements more rapid, so that after a little practice, one can judge from the first glance at an eye, by the appearance of the reflex and of its movements, whether the ametropia is high or not, i.e., whether to begin by putting in, say a 6 D or a 2 D lens. In this way much time may be saved. We will suppose in the given case that, from the appearance of the reflex, we decide on trying — 3 D first. The reflex looks much brighter and moves more quickly. On substituting a — 3·5 D lens, we find on leaning back that the light moves rapidly in the same direction as the mirror, but on leaning forwards the light moves against the mirror. We know, therefore, that the patient's far point is now more than 80 cm. and less than 133 cm. from his eye. On holding the mirror at one metre's distance, we find that the pupil is fully illuminated, but on slightly turning it the reflex suddenly disappears without our being able to say in which direction the light has moved. We may be sure that when this occurs the mirror is exactly at the far point of the eye, i.e., 1 metre, and therefore we say that, on adding — 1 D to the glass in the frames, that meridian of the eye will be corrected for distance, or infinity as it is often called. It will be found that, by adopting this procedure we can in most cases accurately determine the refraction of an eye with an error of less than ·25 D. For instance, in this case if we add + ·25 D to the — 3·5 D in the frames, we shall find that the light moves with the mirror at a distance of 133 cm. and also at that of 1 metre, but that at a distance of 80 cm. the movement of the light becomes indeterminate, showing that now — 1·25 D must be added to the glasses in the trial frames in order to correct the horizontal meridian of the eye for infinity ($80 = \frac{100}{1{\cdot}25}$). If, on turning the mirror from above downwards, exactly the same observations are made, we conclude that no astigmatism is present. We can then confirm our result by trying the patient with the test types when provided with — 4·5 D.

As $133{\cdot}\dot{3} = \frac{100}{{\cdot}75}$ it will be seen that, by varying the distance of the mirror from 133 cm. to 80 cm., one really has a range between ·75 D and 1·25 D; and, according to the position in which the indeterminate movement of the light occurs, one knows whether to add — ·75 D, — 1 D, or — 1·25 D.

In some cases the light seems to move "all ways." In conical cornea the illuminated part of the pupil appears to be surrounded with peculiar swirling shadows; when there is irregular astigmatism and spherical aberration, two shadows may appear on opposite sides of the pupil, and on tilting the mirror they move towards each other. This is called "the scissors movement"; it may be sometimes obviated by getting the patient to look in a slightly different direction, or by slightly inclining the mirror. In any case in which these anomalous shadows are troublesome, no such precisely accurate determination of the refraction can be made.

Aberration.—We know that if parallel rays fall on an ordinary spherical lens, the peripheral rays intersect at a point that is nearer the lens than the focus of the more central rays. The same thing usually occurs with the eye. Dr. Tscherning has devised a simple little instrument for subjectively testing the aberration of the eye. His "aberroscope" consists of a planoconvex lens on which is etched a micrometer scale in the form of little squares. Priestley Smith's "keratometer" serves the purpose admirably. A distant light is viewed through the instrument when it is 4 or 5 inches away from the eye; if the peripheral lines appear concave outwards ("pincushion distortion"), aberration is present; if the peripheral lines appear convex outwards ("barrel distortion"), there is an over-correction. Sometimes it will be found that the pupil shows an aberration in one part and an over-correction in another part.

Regular Astigmatism.—Suppose that when -3 D is put in the trial frames we see a bright band of light running across the pupil at an angle of about 10° or 15°, we may be sure that astigmatism is present and that this band denotes one of the axes of astigmatism. On replacing the -3 D by a $-3{\cdot}5$ D lens, and turning the mirror in the direction of this band (at an angle of 10° or 15°), suppose we find that the light moves with the mirror on leaning back, but against the mirror on leaning forwards, we discover as before that $-4{\cdot}5$ D must be the correcting glass for this meridian. On now turning the mirror at right angles to the band, we see a very distinct movement of the light with the mirror, i.e., in the meridian at an angle of 100° or 105°. The most usual and the easiest method to adopt is now to add concave spherical glasses until this meridian is corrected. The most accurate method, which has been ably advocated by Dr. Duane, is to add concave cylinders with their axes placed in the direction of the band of light, say 10°. Suppose one finds that with a − 2 D cylinder in this direction the movement of the light is reversed on leaning forwards, but that the movement is with the mirror on leaning back, one may conclude that − 2 D cyl. axis 10° fully corrects the astigmatism. If this is correct, one will find that, at the distance of 1 metre, the whole pupil will be illuminated, and on turning the mirror in any direction, the light will suddenly disappear ; on leaning backwards, the light will move with the mirror in all directions alike, and similarly, on leaning forwards, the light will move against the mirror in all directions alike.

If my correction of astigmatism is *wrong in amount* (e.g., if − 2·25 D cyl. instead of − 2 D cyl. is required), as I approach the patient from the distance of 133 cm. I shall, at a certain distance from him, find that in the meridian of 100° the light moves with the mirror, while in the meridian of 10° it moves against the mirror. When this happens, I simply change the strength of the cylinder until reversal takes place at just the same distance from the eye for all meridians alike.

If my cylinder is in the *wrong axis*, i.e., if it ought to be 15° instead of 10°, then, as I sweep the mirror from side to side or up and down, I shall notice that the light, instead of travelling along the same line as that in which I am moving my mirror, makes a skew or oblique movement, sliding off as it were to one side or the other. When this happens I shift the axis of my cylinder one way or the other until this obliquity of movement disappears.

Finally, if my *spherical lens* is at fault, then reversal takes place, evenly indeed in all meridians, but either too close to the eye or too far from it. I then alter the strength of the spherical accordingly, until reversal takes place at just one metre.

The last three paragraphs I have practically quoted from

Dr. Duane's paper on "The Systematic Use of Cylinders in making the Shadow Test" (*The Ophthalmic Record*, 1903).

There are a few further points that should be mentioned when dealing with retinoscopy.

1. For all accurate work, the patient should look directly at the mirror; for it is his macula that we wish to correct, not some other less sensitive part of his retina. This is especially important in cases of high myopia, when there is frequently a posterior staphyloma at the posterior pole, so that the glass which may correct his optic disc may differ from that required to correct his macula by two or three dioptres. Hence, atropine or homatropine should always be used unless contra-indicated, as otherwise the pupil will strongly contract when the light reflected from the mirror is viewed. However, for the convenience of the patient, he may be allowed to look above one's head until the final correction is almost attained.

2. It will be seen from the diagram (*Fig.* 7, *p.*36) that when the mirror is only a few inches away from the patient's eye, so that it is within its focal distance of 25 cm., the real image of the mirror will tend to be formed on the retina itself, and hence the light will move in the same direction as the mirror, even although the eye be highly hypermetropic.

3. When a plane instead of a concave mirror is used, all the movements are reversed, for only a virtual erect image is formed by it, and hence it will move up when the mirror is moved down; consequently, in hypermetropia the light moves with the mirror, and in myopia against the mirror.

My personal preference for the concave mirror in retinoscopy is based on my conviction that the troublesome swirling and cross shadows that are occasionally seen are less prominent with it than with the plane mirror. There are, however, many of great experience who habitually use the plane mirror, so I do not wish to speak too dogmatically on the subject.

The Ophthalmometer.—This is an instrument for measuring the corneal curvature in any meridian, and hence it is used to estimate the amount of corneal astigmatism present. Instruments of so many different patterns are now on the market that a very brief description of their principle must suffice. Two objects called "mires" are so placed that their images, reflected by the cornea, are viewed through a telescope, inside which is an arrangement for doubling the images seen. There are therefore four images visible, but the attention of the observer is to be fixed upon the two central images. First, the meridian of least refraction is found, by rotating the revolving disc or arc on which the mires are fixed until that position is found which shows the central images of the mires at their greatest distance. The position of the mires is now altered by turning the adjusting screw until their images just touch. The meridian of highest refraction (or greatest curvature) will be at right angles to this. In this position, if astigmatism be present, it will be found that the central images overlap, and by the extent of this overlapping, the corneal astigmatism can be directly read off. In most instruments, one of the mires is marked in steps, so that each step corresponds to one dioptre of astigmatism.

In the best instruments, a scale is provided which enables one to read off in millimetres the radius of curvature of the cornea in the meridian observed. As a scientific instrument it is admirable for determining the curvature of the cornea, and hence for determining the corneal astigmatism. Its advantages may be summarized in this way :—

1. Economy of time. The examination should not take longer than one minute for each eye.
2. A mydriatic is unnecessary.
3. In aphakia it is extremely useful.
4. It is also useful in determining the meridian of

greatest corneal curvature before performing a keratotomy upon the eye. If the meridian of greatest curvature be vertical, a superior or inferior keratotomy will tend to reduce the astigmatism; if the greatest curvature be horizontal, a lateral keratotomy would be preferable.

Its disadvantages are :—

1. It determines only the corneal astigmatism, so that the lenticular element is entirely neglected. It would therefore have entirely failed to detect the astigmatism in Dr. Young's eye (*p.* 2).

2. The astigmatic reading given by the instrument does not give the exact cylinder required, as the reading gives the cylinder that would, if placed in contact with the cornea, correct the corneal astigmatism.

As the glasses will be worn 13·75 mm. in front of the cornea, an allowance will have to be made for the position of the correcting lens (*p.* 126). It will be found that, whenever myopia is present, the correcting glass must be higher than that given by the reading of the instrument, whereas in hypermetropia it must usually be lower. Tables of these necessary corrections are generally provided with the instrument.

So many different tests of ametropia have now been given, that the reader may feel rather perplexed as to which to adopt, and in what order to conduct the examination. I usually first test the patient's visual acuteness with the test types as on *p.* 23, and find the position of his near point. However, if I cannot readily get his vision up to the standard amount, I at once instil homatropine, unless contra-indicated, with a view to testing him by retinoscopy. After two or three instillations of homatropine every quarter of an hour, the pupils will be widely dilated, and the accommodation will be practically paralyzed in an hour's time. By the next day the effects of the homatropine will have almost passed away. The solution that I use is made according to this prescription :—

℞	Homatr. Hydrobrom.	.. ..	gr. viij
	Cocain. Hydrochlor.	.. ..	gr. iv
	Chloretone	.. .. ..	gr. ij
	Aq. Destill.	.. .. ..	ad ℥j

Chloretone acts as an antiseptic, and prevents fungus from growing in the solution, and cocaine, in addition to its other effects, widens the palpebral opening owing to its stimulation of the sympathetic which innervates Müller's palpebral muscles. This is a great advantage in some elderly people who have a difficulty in raising their upper lids. To avoid any desiccation of the cornea, it is well to enjoin the patient to keep his lids closed for a few minutes after the instillation.

With young patients, however, in whom I suspect spasm of accommodation, I always use atropine. There are two objections to atropine : (1) It will take about a fortnight for the power of accommodation to be recovered after its use, and (2) There is always a danger of inducing an attack of glaucoma in patients above the age of 40. In patients who have a shallow anterior chamber or increased tension it is advisable for this reason to avoid even the use of homatropine, and to trust to the ophthalmometer, and to retinoscopy with undilated pupils, in association with the usual subjective tests. However, in almost all such cases a moderate dilatation of the pupil can be obtained without risk by the instillation of 1 per cent cocaine or 3 per cent euphthalmine.

Practically, however, I never rely on any determination of a patient's refraction without the instillation of a cycloplegic, and therefore I always use one unless there are definite contra-indications.

Treatment.—*Hypermetropia.*—When a cycloplegic has been used so that the total H has been determined, the best method is to prescribe lenses which correct all the manifest and one-third of the latent H.

In cases in which the manifest error of refraction has not been determined before the instillation of the mydriatic, it will usually be sufficient to deduct 1 D from the total H found in the case of atropine, and about ·75 D when homatropine is used.

Should the spectacles be worn *constantly, or only for near work*? So long as $\frac{6}{6}$ can be seen quite comfortably, without any symptoms of asthenopia or headache, I do not insist on the glasses being worn constantly, but I consider them necessary for all near work. When the glasses improve the distant vision, they should be worn constantly.

In all cases of convergent concomitant strabismus (C C S), the full correction should be worn constantly, and never more than 1 D taken off for the atropine used.

Myopia.—In all cases of high myopia, there is a great tendency for it to increase, so special instructions should be given to the patients. As the working distance of myopes is usually too close, their eyes are strongly converged and rotated downwards, to see the book or the work that is below them. Now, if we think of the muscle that is chiefly employed to turn downwards an eye that is already directed inwards, we remember that it is the superior oblique. When this muscle contracts, especially if there is any synergic action of the inferior oblique, we see that the eyeball must be compressed laterally by these encircling muscles, which have no intra-capsular ligaments to keep off their injurious pressure from the eyeball. It is clear that any lateral pressure will tend to increase the length of the eyeball, and hence to increase the myopia, to cause posterior staphyloma, and even retinal detachment. The work should therefore be raised to the level of the eyes and held as far off as is convenient, say 33 cm. Books should be placed on a suitable book-rest, or held in the hands with the elbows resting on the arms of a chair, and all writing should be done on a sloping desk, with the illumination coming over the left shoulder, so that the letters as they are written may be distinctly seen.

Many authorities habitually under-correct myopia. I give the full correction under homatropine, unless the myopia is extreme, above — 15 D, as I find that the effort to see distinctly with an under-correction, strains the eyes far more than when the full correction is given, and I have never found any reason to alter my practice when sufficient care is taken to avoid the action of the superior oblique muscle.

Astigmatism.—The whole of the astigmatism found by retinoscopy under homatropine should be corrected by appropriate cylinders, and the glasses should be worn constantly. If a patient is entirely free from headaches and symptoms of asthenopia, and if his distant vision is only triflingly defective without his spectacles, this rule need not be rigorously insisted on, and he may be allowed to wear them only for close work. When the astigmatism is oblique the correction will be almost always uncomfortable at first ; if the discomfort continues for more than three weeks, it is well to examine the case again. Patients will sometimes return complaining of discomfort with their astigmatic correction, when it will be found that their spectacle frames have become bent, so that the setting of the axis is imperfect. All that is necessary in such a case is to send them to the optician to have the frames readjusted.

Anisometropia.—In this condition the refraction of the two eyes is different, and three principal varieties are found : (1) Those who have binocular vision. (2) Those who use their eyes alternately : this is usually the case where one eye is slightly myopic while the other is hypermetropic. Such patients use one eye for close work and the other for distance. (3) Those who only use one eye, the other being amblyopic.

1. In this class, the difference of refraction will most probably be small, and when distant vision is improved by the glasses, the full correction should be given to each eye. It is commonly stated that if the difference be greater than 1·5 D, the full difference cannot be comfortably worn. This I have not found to be true; after perhaps a fortnight of discomfort, patients are usually delighted with a full correction, even when the difference is 3·5 D or 4 D. The discomfort does not arise from any difference in size of the retinal images, for as we have seen in corrected axial ametropia, the size of the retinal image is that of a normal emmetrope. (For proof, *see pp.* 123-5.) The trouble is brought about in this way. Suppose a patient requires R + 3 D, L + 6 D, and that, on looking straight forwards with these, he sees well. On ranging his eyes from side to side, and especially on looking downwards through the glasses, he will complain of discomfort. This is easily understood when it is remembered that, on looking through an eccentric part of a lens, it will have a prismatic effect that will increase with the power of the lens. On looking at a distant object 15° below the horizontal plane, his eyes will have to be depressed more than this amount—his right eye 16° 15′ and his left eye 17° 44′—to see the object singly. This is more than he can do, and therefore he complains of a vertical diplopia owing to this difference of level (1° 29′) between the two eyes (*p.* 144).

It is clear, then, that a fully corrected anisometrope must learn to turn his head and not his eyes; when he has acquired this habit the discomfort will disappear. For reading, he must have another pair of spectacles that are properly centred for that purpose, the optical centre of each lens being 6·5 mm. downwards and 2·5 mm. inwards. As the lateral effect is much less annoying than the vertical displacement, I have sometimes found this difficulty overcome by ordering pince-nez instead of spectacles, as by altering the position of the pince-nez on his nose, he can adjust them for reading so that his fixation lines pass through the optical centres of his glasses.

For a discussion of the prismatic effects of lenses I must refer the reader to the Optical Section of this book.

If the patient cannot learn to move his head instead of his eyes, he is sometimes satisfied with a bifocal lens for his most defective eye. A prescription for the above case would be + 3 D for each eye, but over the centre of the left lens a thin convex wafer of + 3 D about 8 mm. wide is to be cemented. In this case, for nearly central vision each eye will be fully corrected, while for eccentric vision there will be no unequal prismatic effect.

It is usual in practice to order as near the full correction as the patient can tolerate.

2. In these cases we must try to find out what is best for the patient. If provided with a correction, he will in the first place have to learn to use it, and to put up with the initial discomfort, and then he must do stereoscopic exercises for a long time, so as to obtain valuable binocular vision. If the patient is satisfied with his present condition, and if he refuses to train his binocular sense, I merely order the full correction for reading and close work. I have not found one such person with patience enough to persist in his binocular training, to get any result that was worth the time and trouble it cost.

3. Similarly in these cases, unless the patient is young, the training of an amblyopic eye is very tedious, and the rate of improvement steadily diminishes, so that few will be found who will practise their amblyopic eye for the years that are required to get a really satisfactory result. However, this should always be urged. It is a good plan

to order the correction and insist on their eating their meals with the better eye covered up. It is important that the exercise should not be too exhausting to the eye, and that it should be regular, frequent, and not too long continued.

Aphakia.—After the crystalline lens has been removed, the eye, if previously emmetropic, requires a strong convex glass in order to see distinctly. As the refracting system of the eye is now entirely different from that of an ametropic eye, the tests of visual acuteness are no longer comparable with that of normal eyes. Indeed, it is shown on *p.* 125 that the retinal image in an aphakic eye is 1·33 times larger than in a normal eye. Hence vision of $\frac{6}{9}$ in a corrected aphakic would really only correspond to a visual acuteness of $\frac{6}{12}$ in a normal eye.

The lens is sometimes removed in cases of high myopia, and in such cases it is often useful to know what strength of glass will be required after the operation. Assuming that the curvature of the cornea has undergone no alteration, I have found the following simple formula very useful. The proof will be found on *p.* 122.

$$D' = \frac{25 + D}{2 - (\cdot 01)\,D}$$

where D′ will be the new glass if D has been the power of the old lens. For instance, a myope of − 18 D will require after operation about + 3 D for distance.

$$\text{For} \quad D' = \frac{25 + (-18)}{2 - \cdot 01\,(-18)} = \frac{25 - 18}{2 \cdot 18} = \frac{7}{2 \cdot 18} \backsimeq +3 \cdot 2^{*}$$

Tilting the Lens.—It will be noticed that myopic patients often wear their pince-nez inclined, and say that they see better with them than through spectacles of the same strength. It will be found in such cases that the patients have some astigmatism axis 0°, and by tilting

* The sign $\backsimeq$ denotes approximate equality.

their glasses, they utilize the cylindrical effect of the inclined lens. This is occasionally of service to the practitioner in cases in which the cylinder is weak, with its axis horizontal, and the spherical lens is of high power. I have drawn up the following table, showing the effect of tilting a 10 D lens through different angles. The method of obtaining this table is explained on p. 129, The index of refraction is taken to be 1·52.

It is almost always found that cataract patients, after the operation, require a convex cylinder axis 0°, which in a few months must be decreased in strength. For hospital patients, it is a great advantage to tilt their spherical glasses downwards. When their astigmatism diminishes, the correction may be simply given by bending the legs of the spectacles upwards. The glasses cost less originally, are lighter, and last longer. When the glass to be tilted is not 10 D, but some other number such as 9 or 11, the figures must be multiplied by ·9 or 1·1, in fact always by the tenth part of the power of the glass.

For instance, suppose an aphakic requires + 11·5 D sph. + 1·5 D cyl. ax. 0°. We see that this result can be nearly obtained by tilting a + 11 D lens 20°,

$$\text{for } 1{\cdot}1 \times 10{\cdot}41 = 11{\cdot}45 \text{ D}$$

$$\text{and } 1{\cdot}1 \times 1{\cdot}379 = 1{\cdot}5 \text{ D cyl. approximately.}$$

We can order him then + 11 D sph. to be inclined downwards 20° as though for reading.

Suppose a myopic patient of − 20 D sph. − 2·5 D cyl. presents himself, complaining of the weight of his glasses. We shall probably serve him well by ordering − 19 D inclined 20°; for the figures in the table multiplied by 1·9 show that by this means we obtain the effect of about − 19·78 D and − 2·62 D cyl. ax. 0°.

TILTING SPHERICAL 10 D LENSES. $\mu = 1{\cdot}52$.

OBLIQUITY	SPHERICAL	CYLINDRICAL
10° ..	10·1007	·3140
15° ..	10·2284	·7344
20° ..	10·4102	1·3791
25° ..	10·6492	2·3156
30° ..	10·9497	3·6499
35° ..	11·3168	5·5485

Periscopic Lenses.—With the ordinary biconvex lenses, in order that the wearer may see lateral objects clearly, it is necessary for him to move his head, keeping his eyes fixed opposite the centre of the glass. This is due to the fact that when an eccentric portion of a lens is used so that the incident light is oblique, the lens acts as a sphero-cylinder, and hence the image of the object is blurred. Periscopic lenses are used to enable the wearer to see more clearly when he moves his eyes from side to side; this is a great advantage to him when playing such games as tennis, or indeed at all times. The form of the glass must be that of a meniscus with its concave surface facing the cornea; there is a prevailing idea that the concavity should theoretically be of the same curvature as that of the cornea, but this is absolutely untrue. In another section (*p.* 132) I give the method by which I have calculated the curvature that theoretically should be given to each surface of the meniscus required to form a distinct image of an object situated 25° on either side of the middle line. As far as I know, Dr. Ostwalt was the first to publish work on this subject in *v. Graefe's Archiv.* xlvi. 3. The tables that I give below I have worked out according to my method, and for simplicity I have given the nearest dioptric power of each surface of the meniscus. It will be seen that the glasses in the opticians' shops, commonly called periscopic, are not nearly deep enough. Some patients may prefer a cheaper and perhaps a more elegant-looking lens to that of the true periscopic form; but those who wish for the best results should be given glasses of the shape indicated in the tables.

There is one point that requires notice. Since the principal points of a diverging meniscus are one or two millimetres outside the glass on its concave or ocular side, the meniscus acts as if it were a lens placed slightly closer to the eye: in other words, the meniscus need not be quite so strong as the correcting biconcave lens. Similarly, the principal points of a converging meniscus are a little in front of its convex surface, and so the meniscus acts as a biconvex placed a little further from the eye, and consequently, when used for distance, the meniscus need not be quite so strong as the correcting biconvex (*see pp.* 126-7). This effect, however, is so slight that it makes practically no difference in the prescription.

In the tables, the figures given form a definite image on the retina for a range of view bounded by a solid angle of 50°, except in those cases where the figures are enclosed in brackets, in which case the best attainable result is obtained, but the distinct field of view is not quite so great.

Consideration of the periscopic effect will often enable us to determine the best form for a case of mixed astigmatism. For instance, suppose a patient requires + 2·5 D sph. — 5 D cyl. ax. 0°. It is clear that a better prescription would be — 2·5 D sph. + 5 D cyl. ax. 90°, and instructions should be given to the optician that the concave spherical surface is to be worn next the eye.

In this table a due allowance has been made for the different thicknesses of the lenses; the centre of motility of the eye is assumed to be 27 mm. behind the meniscus, the radius of the pupil is taken to be its average value, 1·6 mm., and the index of refraction of the glass used is taken as 1·52. When the figures are enclosed in brackets, the field of view is somewhat less than that bounded by a solid angle of 50°.

TABLE OF PERISCOPIC LENSES.

POWER	SURFACE FACING THE LIGHT	SURFACE FACING THE EYE
+ 20 D	(+ 30 D)	(− 10 D)
+ 15 D	(+ 27 D)	(− 12 D)
+ 12 D	(+ 24 D)	(− 12 D)
+ 10 D	(+ 22 D)	(− 12 D)
+ 8 D	+ 18 D	− 10 D
+ 6 D	+ 15 D	− 9 D
+ 4 D	+ 12 D	− 8 D
+ 2 D	+ 8 D	− 6 D
− 2 D	+ 5 D	− 7 D
− 4 D	+ 4 D	− 8 D
− 6 D	+ 3 D	− 9 D
− 8 D	+ 2 D	− 10 D
− 10 D	Plane	− 10 D
− 15 D	(Plane)	(− 15 D)

Bifocal Lenses.—The upper parts of these lenses are adapted for distant vision, while the lower parts are adapted for reading and close work. In the original form, "the straight split bifocal," or "Franklin," the line dividing the two lenses is horizontal and in the middle. In the "Uni-bifocal," the reading segment is actually ground on the surface of the distance-glass. A segment of the distance-glass is cut out in the "Improved Franklin," and a similarly shaped reading segment is fitted in. In the "Kryptok," a concavity is ground in the lower part of the distance-glass, in which is fused a glass of higher refractive index; the whole surface is then ground to the requisite curvature, and the lower portion being of higher refractive index, has the required higher power for reading. The most usual form, the "Perfection," is that in which a thin convex wafer is cemented on to the lower part.

Great credit is due to the optician for making such artistic spectacles, but many will be found to fail in practice, as some of the essential requisities have not been fulfilled. It will be well, therefore, to consider in detail what these requirements are.

1. *Size.*—The smaller the reading segments are, the greater will be the field of view for distance, so we must determine how large it is necessary for them to be. I have long insisted that they are usually made much too large, and of the wrong shape.

No one will care to see a line of print more than 6 in. long without moving his head; indeed, unless the glasses were periscopic, he would only see indistinctly on turning his eyes behind his glasses more than this amount. Now the distance (k) of the centre of motility of the eye from the lens is 27 mm., and if we assume the work to be held at a distance of 13 in. it is clear that the reading segment need never be more than $\frac{6}{13}$ of 27 mm. Similarly, one need never see clearly more than 4 in. in height without moving one's head, so that the height of the segment need not exceed $\frac{4}{13}$ of 27 mm. Theoretically, the upper margin of the reading portion should be a horizontal line, and not the convex curve that is usually seen. We may say, then, that the reading portion should be rectangular in shape and need never be more than 9 mm. $\times$ 12·5 mm. in size. These dimensions will enable the wearer to have a much larger field of view for distance.

2. *Position.*—As the eyes when reading always converge, and are usually depressed 13° or 14°, the optical and geometrical centre of the reading combination should be 2·5 mm. to the nasal side and 6·5 mm. below the centre of the distance glass, while the upper margin should not be more than 2 mm. below the mid-horizontal line.

Many people complain of a difficulty in going downstairs, as the steps below appear blurred, for they can only see them through the reading segments. Such a difficulty is easily obviated by not allowing the segment to extend to the bottom of the spectacle frame. It will be found that if a narrow strip 2 mm. or 3 mm. wide of the distance-glass at the very bottom be uncovered by the rectangular reading wafer, a view will be obtained of a width of about 7 in. or 8 in. at a distance of 6 feet, so that the step below can be quite easily seen. As the ordinary spectacle glass is 37 mm. $\times$ 28 mm., a rectangular wafer 9 mm. in height can be cemented to it 3 mm. from the bottom, so that its upper margin is 2 mm. below the centre of the distance glass.

3. *Optical Centre.*—It is important that the optical centre of the reading combination should coincide with the geometrical centre of the segment. The fact that bifocals are so commonly unsatisfactory to anisometropes is due to carelessness in this particular. Suppose for instance that the distance correction is

$$R + 3\,D,\ \ L + 6\,D,$$

and that normally centred wafers of $+ 3$ D are cemented on to the glasses in their appropriate positions: it will be found on viewing an object (at $\frac{1}{3}$ m.) 15° below the horizontal plane that the right eye must be depressed 16° 23′ and the left eye 17° 52′ to see the object. This would entail a difference of level of 1° 29′ between the two eyes, which is of course impossible to exist normally.

A correction can be practically made with the help of the following simple formula. Let V denote the vertical decentration of the wafer, i.e., the distance between its optical and geometrical centre, and let D and *d* denote the dioptric strengths of the distance glass and the wafer respectively; then, if the centre of the wafer is to be 6·5 mm. below the mid-horizontal line $V = -\dfrac{D}{d}\ 6{\cdot}5$ mm.

Similarly, if H denote the horizontal decentration of the wafer, $H = -\dfrac{D}{d}\ 2{\cdot}5$ mm. In the above case for the right eye $V = -\frac{3}{3} \times 6{\cdot}5$, $H = -\frac{3}{3} \times 2{\cdot}5$, while for the left eye $V = -\frac{6}{3} \times 6{\cdot}5$, $H = -\frac{6}{3} \times 2{\cdot}5$.

Hence, the right wafer must be decentred 6·5 mm. downwards and 2·5 mm. inwards, while the left wafer must be decentred 13 mm. downwards and 5 mm. inwards, for both V and H carry negative signs. Both wafers will therefore have thick edges below and on their inner sides.

Artistic opticians prefer to send out spectacles furnished with wafers as thin as possible. I, personally, have always insisted on the horizontal decentration being attended to, but I have never had any complaints when the vertical decentration was modified, as long as the angle of depression of each eye is kept the same. For instance, I think no harm would result if the optical centre of the right wafer were normally centred as far as the vertical displacement is concerned (i.e., if it were kept in its mid-horizontal line), and if the left wafer only were decentred downwards 6·5 mm. The formula for this modification when D and D′ represent the stronger and weaker glass respectively is $V = -\frac{D - D'}{d}\ 6{\cdot}5$.

In the above case $D = 6$, $D' = 3$, and this decentration (− 6·5 mm.) of the left wafer will maintain the visual lines in the same plane when they are depressed 13° 32′.

In the case of bifocals for myopia the wafers must be decentred upwards and outwards, so that they have the effect of prisms with their bases upwards and outwards.

The numbers 6·5 and 2·5 are of course average numbers, and are obtained in this way. If a denote the usual angle of depression of the eyes, then v, the downward displacement of the optical centre for reading, is given by $v = k \tan a$, and $27 \times \tan 13\frac{1}{2}^\circ = 6{\cdot}5$.

Similarly, for reading at $\frac{1}{3}$ metre's distance $h = k \tan 3$ m.a. Now as the value of 3 m.a. depends on the interocular distance (*see p.* 61), which varies from 52 mm. in children to 64 mm. or more in adults, h also varies. In the tables (*pp.* 153-4) $h = 2{\cdot}44$, as the interocular distance is taken to be 60 mm. If $h = 2{\cdot}5$ the i.o. distance is 61·4 mm.

CHAPTER III.

FAULTY TENDENCIES AND DEVIATIONS OF THE OCULAR MUSCLES.

PRISMS.

AS prisms are so largely used in the diagnosis and in the treatment of these faulty tendencies, it is advisable first to describe their clinical properties. They have several other interesting physical properties which it is not necessary to specify, as the ophthalmic surgeon makes no use of them. Indeed, the prisms he employs have such an acute angle that the only property which concerns him is that of altering the direction of the incident light.

A square or rectangular prism is a wedge of glass presenting two rectangular "refracting surfaces" which meet at an acute angle called the "apical angle." The line of junction is called the "edge," and the opposite part or "base" presents a very narrow rectangular surface. If such a square prism be trimmed into a circular or elliptical shape, the "edge" is the tangent to the thinnest part, and the "base" is the tangent to the thickest part. The line that passes through the centre at right angles to these two tangents is the "base-apex" line. In prisms of elliptical shape for spectacle frames, if the base-apex line is oblique, the thinnest part of the glass will not represent the apex, nor will the thickest part represent the base of the prism, but the base-apex line will be that drawn at right angles to the two parallel tangents at the thickest and thinnest parts of the glass. In fact, if the base-apex line be oblique, it will be less inclined to the vertical than the line joining the thickest and thinnest parts of the glass.

The path of light transmitted through a prism is always deviated towards the base of the prism. The amount of this deviation depends primarily upon the apical angle of the prism and upon the refractive index of the glass of which it is composed, although it depends to some extent, upon the tilting of the prism with reference to the incident light.

The glass most commonly used in England for spectacles has a refractive index (μ) equal to about 1·52; for that used in America $\mu = 1·54$. It is found that the deviation induced by a prism is a minimum when the light passes symmetrically through it, or when the angle of incidence is equal to the angle of emergence,* in other words, when the path of light in the prism is parallel to the base of the prism. When all the angles concerned are small, the approximate formula, $D = (\mu - 1) A$, may be regarded as true,* where A denotes the apical angle of the prism and D its deviation. When A is not greater than 10°, even when the angle of incidence is as much as 20°, the formula gives a result well within 5 per cent of the truth. We shall assume, therefore, for practical purposes that this formula gives the true deviation of any prism that is used by an ophthalmic surgeon.

Nomenclature.—There are now six different methods of denoting prisms, and hence some confusion has arisen. It will be necessary to describe all the methods, and I shall definitely give the reasons for my own preference.

1.—The prisms in most trial cases are marked with the degrees of their *apical angles.*

This method has nothing to support it except the convenience of the optician. Prisms are used for the *effect* that they produce, and they should therefore be

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* For proof, see my *Optics*, pp. 150–154.

named according to this effect, just as lenses are named according to their power. It is useless to know either the curvature of the surfaces of a lens or the apical angle of a prism, unless we also know the refractive index of the glass used. Even when we know μ it requires a small calculation to find its deviation, and then in the prescription to the optician, it would be necessary to give the apical angle as well as the μ of the glass required for the prism.

2.—The prisms are indicated by their angles of *minimum deviation*. This is an excellent method, and in prescriptions is always denoted by the small letter *d*. Thus a prism of 3° *d* means a prism that exerts a minimum deviation of 3°, and it is left to the optician to make it up of any crown glass he pleases, according to the formula $A = \frac{D}{\mu - 1}$; thus if his glass has $\mu = 1{\cdot}54$ the apical angle must be about 5° 33′, if $\mu = 1{\cdot}52$ the apical angle must be 5° 46′. It does not matter to the surgeon or to the patient which is supplied him, as long as the desired effect is produced.

3.—The *Metre Angle*.—This is the most convenient unit for measuring the function of convergence, but it is ill adapted for the measurement of prisms. It is the angle through which each fixation line sweeps in moving from parallelism to view a point in the middle line one metre distant from the centre of motility of the eye. The angle that the fixation line of each eye passes through in viewing a point half a metre distant is 2 m.a. (two metre angles), and so on. Hence, the number of metre angles of convergence exerted is numerically equal to the number of dioptres of accommodation exercised by an emmetrope in viewing the point at the given distance. Thus, when reading at a third of a metre's distance, 3 D of accommodation and 3 m.a. of convergence are exercised.

This is the only unit by which charts of the relationship of convergence to accommodation can be plotted out. There are, however, several weighty objections to it.

(*a*). Its size is different in different individuals, for clearly it depends on the interocular distance (*Fig.* 8). The angle NMO = BOM = $\sin^{-1}\frac{BM}{OM}$. Thus, when

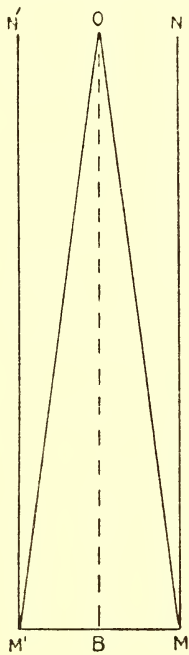

Fig. 8.

OM = 1 m., if M′M, the interocular distance, be 58 mm., BM = 29 mm., one metre angle (1 m.a.) has the value $\sin^{-1}$ ·029 or 1° 40′; but if M′M = 64 mm., BM = 32, so 1 m.a. = $\sin^{-1}$ ·032 or 1° 50′.

(*b*). There is a difficulty with multiplication, for 5 m.a. is not five times the size of 1 m.a., if 1 m.a. = $\sin^{-1}$ ·032 or 1° 50′, 5 m.a. = $\sin^{-1}$ ·16 = 9° 12·4′, not 9° 10′.

(*c*). A minute inaccuracy is also involved by the fact that the distance MO for convergence is measured from the centre of motility, while the distance of O for accommodation is measured from the first principal point of the eye.

4.—The *Centune* or *Percentage System*.—We owe to Dr. Maddox this very convenient term for naming a class which contains three subdivisions, but the units of all of them are one percentage angles, that is to say, the unit prisms all produce a deviation of 1 centimetre at a distance of 1 metre.

The unit *arc centune* is the angle subtended at 1 metre by 1 centimetre of arc ; in the *tangent centune* the length of one centimetre is measured on the straight line perpendicular to the base of 1 metre's length, while in the *sine centune* the metre's length is measured along the slanting radius vector, and a perpendicular of one centimetre is dropped from the extremity of this radius vector to the base.

Two of these centune angles have been used under other names for several years.

(*a*). *Prism Dioptre* or *Tangent Centune*.—This unit was proposed by Mr. C. F. Prentice, and is designated by an upright delta ($\triangle$). One prism dioptre is therefore the angle whose tangent is ·01 or $34'\cdot37643$, $10\ \triangle = \tan^{-1}\cdot1$ or $5^\circ\ 42'\cdot6355$, it is not equal to ten times the value of $1\ \triangle$. This is a great disadvantage to the unit. Its charm consists in the delightful simplicity that is brought into problems of decentration by its use. The prismatic effect of decentring a lens is represented by Dc, where c is the decentration in centimetres and D is the dioptric strength of the lens. Thus a -3 D lens decentred $\frac{1}{2}$ cm. outwards is equivalent to a -3 D lens associated with a negative or abducting prism of $-1\frac{1}{2}\ \triangle$.

(*b*). *Centrad* or *Arc Centune*.—This unit was proposed by Dr. Dennett, and is the hundreth part of a radian, i.e., ·5729578° or 34′·37747, and is denoted by a reversed delta (▽). With this unit there is no difficulty about multiplication, e.g., 10 ▽ is ten times one centrad. The difference between the two units is exceedingly minute, and, on examination, the apparent superiority of the prism dioptre for decentration problems is found not to hold. For, as Dr. Maddox has pointed out, owing to spherical aberration, the effect of decentration is rather more than is indicated by the tangent formula (*see Optical Section, p.* 139). Hence, as with the prism dioptre, when N denotes the number of centrads, $N = Dc$ or $\frac{Dl}{10}$, where l is the number of millimetres that a lens of power D is decentred; and of course $l = \frac{10\,N}{D}$. A convex lens of + 5 D combined with an *adducting* prism of 2 ▽ is precisely equivalent to the convex lens decentred *outwards* 4 mm. Note that *adducting* prisms and decentration *outwards* are both considered *positive.*

For very rough approximations, the number indicating the apical angle of the prisms in the ordinary trial cases may be regarded as the number of centrads of deviation that they induce.

The centrad, again, has a very convenient approximate relation to the metre angle. If M denote the number of metre angles and c denote the interocular distance in centimetres, while N denotes the number of centrads,

$$M \risingdotseq \frac{2N}{c} \quad \text{or} \quad N \risingdotseq M\frac{c}{2}$$

Again, as 4° = 6·981 centrads (*Table, V p.* 157) 1° is nearly $\frac{7}{4}$ centrads.

The American Ophthalmological Society has adopted the centrad as its official unit, and although in this country it makes its way but slowly, I hope it will be universally adopted before very long. For its many conveniences I think it is far the best unit that has been devised.

(*c*). *Sine Centune.*—A railway gradient of one in a hundred would be an angle of one sine centune or $\sin^{-1} \cdot 01$ or $34' \cdot 37815$. It will be seen that it is a trifle greater than one centrad and the prism dioptre, but there is the same difficulty in multiplying it that we found with the prism dioptre. The only advantage it has is the absolute accuracy of the above formula for comparing metre angles and centrads when sine centunes are substituted for centrads.

It will be noticed that one centune of each of these three varieties may be regarded as equal to $34 \cdot 377'$, and that, therefore, the distinction between them may be thought unnecessary. The difference between them, however, is very apparent when a hundred centunes of each variety is considered. Thus $100 \bigtriangleup = 45°$, $100 \bigtriangledown = 57° \cdot 296$, but 100 sine centunes $= 90°$.

Resultant Prisms.—We sometimes wish to give a correction for a faulty tendency upwards as well as inwards or outwards; in such cases it is necessary to know how to correct both faulty tendencies of the eye with a single prism. Let θ be the prism required to correct the horizontal deviation, and let ϕ be that for the vertical deviation, then both errors will be corrected by a prism of deviation D set at an angle ρ with the horizontal line, if

$$\tan^2 D = \tan^2\theta + \tan^2 \phi, \text{ and if } \tan\rho = \frac{\tan\phi}{\tan\theta} *$$

As the prisms that can be worn continuously are of such feeble strength, no great error is introduced by replacing the tangent of the angle by the angle, and hence Dr. Maddox's simple device is practically quite accurate enough. "Draw a horizontal line as many inches long as there are centunes (or degrees) in the deviating angle of the horizontal prism ; note which end of this line represents

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* For proof, *see* my *Optics*, pp. 167–8.

the edge of the prism, and erect thereon a vertical line as many inches long as there are centunes (or degrees) in the deviating angle of the vertical prism to be compounded, should its edge point upwards. If its edge be directed downwards, the vertical line should be dropped downward instead. In either case the number of inches between the free ends of these two lines equals to a close approximation the number of centunes (or degrees) in the required resultant prism." That is, the length of the hypothenuse represents the strength of the resultant prism, while the angle ρ is given by the inclination of the hypothenuse to the horizontal line. Should the unit be the prism dioptre, it is clear that $D^2 = \theta^2 + \phi^2$ and $\rho = \frac{\phi}{\theta}$ exactly, when all the angles are measured in prism dioptres or tangent centunes, but only a negligible error is introduced by using arc centunes or centrads.

Should we wish to determine the effect of an oblique prism in the vertical and horizontal directions, we have $\tan \phi = \tan D \sin \rho$, and $\tan \theta = \tan D \cos \rho$ or, practically, $\phi = D \sin \rho$, and $\theta = D \cos \rho$. Thus a prism of 4 centrads set at an angle of 30° with the horizontal is equivalent to two prisms, one of ϕ centrads, edge upwards, and another of θ centrads, edge outwards, where $\phi = 4 \sin 30° = 2\ \nabla$, and $\theta = 4 \cos 30° = 3{\cdot}464\ \nabla$. (*Table V*, *p.* 157.)

Clinical Properties of Prisms.—The position of objects as seen by the eye, depends upon the direction in which the light from the object enters the eye. Hence, if a prism be held before one eye, the other being closed, the light that enters the eye will have been deflected towards the base of the prism, and consequently the eye looking in the direction of the entering rays will be deflected towards the edge of the prism. If the object viewed be distant, it will appear to be displaced towards the edge of the prism in such a way that its angular displacement will be equal to its angle of deviation.

When both eyes are used and a prism is held before one of them, the result is different.

(*A*). *When the object is distant.*

1. If the prism be too strong for its effect to be overcome by the ocular muscles, diplopia will result. If, for instance, when both eyes are viewing some distant object, a prism causing 5° of deviation be held before the right eye, edge out, two objects will be seen; the left eye will see the object in its true position, but the right eye, being unable to turn itself outwards to this amount, will judge from the direction of the entering rays of light that the object has been displaced to the right by an angular displacement varying from about 5° to 3°. The images will seem to approach each other with every attempt of the eye to overcome the effect of the prism.

2. If a weaker prism, say one of 3°, be used, as the ocular muscles can diverge to this extent, only one object will be seen, but it will appear to be displaced $1\frac{1}{2}°$ towards the edge of the prism. Normally the eyes can diverge 4°, i.e., each eye can rotate outwards 2° from parallelism, so that when a 3° prism is held before the right eye, edge out, each eye diverges $1\frac{1}{2}°$, and then a ranging movement of both eyes $1\frac{1}{2}°$ to the right, enables the image of each eye to be formed on the macula. As the direction of an object when viewed binocularly is judged from the cortical innervation of the ranging centre, the object is assumed to be $1\frac{1}{2}°$ to the right.

(*B*). *When the object is close to the observer.* The displacement is not so great as the deviation of the prism, even when only one eye is used, as can be easily seen by drawing a diagram. The deviation of the prism is indicated by the ratio of the arc of displacement of the object to its distance from the prism; whereas the deviation of the eye when only one is used is indicated by the ratio of the arc of displacement of the object to its distance from the eye.*

1. If a prism, too strong to be overcome by the ocular muscles be used, as before, two objects will be seen, whose distance apart varies with each attempt of the eyes to overcome the prism.

2. If a weaker prism be used, the object appears to be displaced through half the displacement that occurs when the other eye is closed. The object will also appear to be closer to the observer when *adducting* prisms (edges in) are used, but further off when the experiment is made with *abducting* prisms (edges out). This is due to the fact that the distance of an object is estimated almost entirely by the cortical innervation of the converging centre. When the object is close, some estimate of its nearness is made from the amount of accommodation exercised to see it distinctly; in this case the two functions are opposed, and so the position assigned to the object is not quite that due to the estimate given by the convergence exercised.

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* Let L denote the arc of displacement, p the distance of the object from the prism, $p + k$ the distance of the object from the centre of motility of the eye. Then if D denote the deviation of the prism, and θ the deviation of the eye,

$$\mathrm{D} = \frac{\mathrm{L}}{p} \text{ and } \theta = \frac{\mathrm{L}}{p + k}$$

$$\text{But } \frac{\mathrm{L}}{p + k} = \frac{\mathrm{L}}{p}\left(1 - \frac{k}{p + k}\right)$$

$$\therefore \theta = \mathrm{D}\left(1 - \frac{k}{p + k}\right)$$

When the prism is in a spectacle frame worn in the usual position, $k = 27$ mm., a little more than an inch.

Hence if $p + k = 12$ inches, $\theta \backsimeq \mathrm{D}\frac{11}{12}$ and $\mathrm{D} - \theta \backsimeq \frac{1}{12}\mathrm{D}$,

i.e., the effect of the prism is less than its deviation by the fraction whose denominator is the distance of the object from the eye in inches approximately. This is a simple practical rule we owe to Dr. Maddox. This is of course true whether the base-apex line of the prism be horizontal or vertical.

Prisms are often used in the consulting-room to determine the amount of convergence and of divergence that can be exercised by the eyes when they are focussed for a given point. As we shall see, it is important in many cases to find this range of convergence when the accommodation is exercised for a distance of $\frac{1}{3}$ metre, and again when the accommodation is relaxed. In this case it is obvious that the prisms do not test the strength of the muscles, but the relative range of convergence.

The practical points which result from this discussion may be briefly summarized thus :—

1. The effect of a single prism before one eye is equivalent to the effect of two prisms of half the strength, one before each eye.

Of course, if horizontal deviations are under consideration, the two prisms must be placed both edges inwards; or both edges outwards, as the case may be ; when vertical displacements are being dealt with, one must be placed edge upwards, the other edge downwards.

As chromatic and prismatic aberrations are minimized when weak prisms are used, it is always better to prescribe a weak prism for each eye rather than one strong prism for one eye.

2. Adducting or positive prisms are placed edges inwards; abducting or negative prisms edges outwards.

3. *Relieving Prisms*.—For the relief of diplopia, prisms, if ordered, must be placed so that their edges are in the direction of the deviation. Thus, if the eyes deviate outwards, the edges of the prisms must be placed outwards. In other words, the bases of the prisms must be on the side of the inefficient muscles ; but it is simpler always to think of the edges of the prisms towards the direction of the eyes. So we have for the relief of *heteronymous diplopia*, *edges outwards*, of *homonymous diplopia edges inwards* ; in other words, for divergence, abducting or

negative prisms ; for convergence, adducting or positive prisms.

4. The adequacy of muscles is tested by noting the extent of their ranging movements. The relative range of convergence and supravergence is tested by the strength of the prisms that can be overcome.

The following simple experiment proves that the estimation of the direction of an object is due to the innervation of the "ranging centre," while the estimation of its distance is due to the innervation of the "converging centre." It is described in Dr. Maddox's book, *The Ocular Muscles*, p. 89. Place a tiny piece of white paper on the black cover of a book, and holding it about 6 inches from the eyes look at the piece of paper intently. Now suddenly cover the right eye. The paper will appear to move to the right, and also to recede to a distance.

At first, both eyes were converging to a point at a distance of 6 inches ; on covering the right eye it is found to be less fatiguing to the nervous system to relax the convergence exercised and to innervate the ranging centre to the right. As a proof of this, the right eye will be seen to move to the right behind the cover. The object, therefore, appears to move to the right owing to the ranging innervation to the right, and it appears to recede owing to the diminution of the convergence innervation. It is not due to any afferent impulse of "muscular sense" from the state of tension of the muscles in the left eye ; for, as the left eye remains absolutely fixed in position, the tension of its muscles is unaltered.

DEVIATIONS.

These may be either manifest—squints—or latent. We owe to Dr. Stevens the term *heterotropia* to include all manifest squints, and *heterophoria* to include all latent deviations, i.e., those deviations which only manifest themselves when single binocular vision is rendered impossible. *Orthophoria* is the term used for perfect muscular balance. The subjective tests of these conditions are the more important, as they admit of much more accuracy and refinement than the objective tests ; although we may be sometimes misled by faulty statements of the patient. We shall consider vertical deviations first, as they are the easiest to understand and to correct.

The terms *hypertropia* and *hyperphoria* denote an upward deviation or an upward tendency respectively. Now, except in paralytic cases, it is generally impossible to determine which eye is at fault, so that the term right hyperphoria means that the right eye tends to assume a direction above that of the left, and it is fortunately unnecessary, for purposes of treatment, to determine whether the right eye tends upwards or the left eye tends downwards. It is always important to test for concomitancy i.e., whether the deviation remains the same in all positions of the head. Vertical deviations give rise to much greater discomfort than convergence defects of the same amount, so that it is most important to examine the vertical equilibrium as accurately as possible before proceeding to the examination of the convergence function. The first case of hyperphoria I ever had, now more than twenty years ago, puzzled me extremely, because I found the convergence range so defective. Eventually I discovered the hyperphoria, and found, on correcting this with a suitable prism, that the convergence range was then perfect.

Symptoms.—The most characteristic symptom of heterophoria, though sometimes absent, is periodic diplopia. The unconscious effort to overcome a heterophoria in the interests of single vision may occasion asthenopia, headache, giddiness, and all the symptoms of depression one finds in neurasthenia. When the tired nervous centres give up the effort to maintain single vision, diplopia ensues, and possibly a visible squint ; this, however, usually lasts for only a short time, and then single vision is re-established.* Patients will often acquire the habit of temporarily closing one eye; indeed, the facial muscles of one side of the face are sometimes affected with nervous twitching movements that are entirely cured with an appropriate prismatic correction.

In hyperphoria I have sometimes found a peculiar but characteristic symptom, the inability to read near type (J_1) with both eyes, although each eye separately can read J_1 quite easily. No such difficulty is found with the distant type; $\frac{6}{6}$ is read fluently with both eyes together, and yet there is no defect of accommodation. The explanation of this curious symptom is probably the vertical diplopia which frequently recurs. As the angle of separation is so small, the images of the printed letters overlap when close to the eyes, and hence cause great confusion; whereas, at a distance the vertical separation will be greater, so that less inconvenience is occasioned, or one image may be suppressed. I have already alluded to the anomalies of convergence induced by hyperphoria. There is often present a tendency to convergence (esophoria) at a distance, but a tendency to divergence (exophoria) at reading distance.

It is, however, most important to remember that heterophoria may exist to a marked degree without giving rise to any symptoms whatever; in such cases it is unnecessary to order prisms, or to subject the patient to any of the various operations that have been devised for its relief. The symptoms of the affection are, however, so protean, that the cause is much more likely to be overlooked than to be overtreated.

In describing the tests for this condition I shall give them in the order in which I usually make them. As will be seen, they are chiefly subjective. I consider it

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* The giddiness due to heterophoria may be easily recognized by this peculiarity, that it ceases at once when one eye is closed.

absolutely essential that all errors of refraction should be corrected before subjecting the patient to these tests. Care must be taken that there is no difference of level between the optical centres of the glasses.

(*A*).—STATIC TESTS.

These tests determine the position of the eyes when at rest.

1. **Screen Tests.** (*a*) *Objective.*—The test object should be a small electric light on a dark black velvet background; but practically a small circular white dot in the centre of a blackened target of cardboard will answer quite satisfactorily in most cases. The patient is placed 6 metres distant from this target, with his head erect and his eyes looking straight forwards at the test object. The surgeon now suddenly covers the right eye with a card, and notices whether this right eye moves in any direction when covered.

If the right eye *alone* moves when covered, while the left eye remains fixed on the test object, *heterophoria* is present. If the right eye moves outwards, exophoria; if inwards, esophoria; if upwards, right hyperphoria; if downwards, left hyperphoria.

If *both* eyes move when the right eye is covered, the *left* eye is squinting, or *heterotropia* is present.

If *neither* eye moves, either orthophoria is present or the right eye is squinting; in the latter case, the deviation is easily seen on covering the left eye. If orthophoria be present, on repeating the test with a prism of 4 $\bigtriangledown$ held before the left eye, first edge in, and then edge out, it will be found that the covered eye deviates an *equal* amount, first outwards and then inwards, owing to the 4 $\bigtriangledown$ exophoria or esophoria induced by the prism.

Now, after holding the card over the right eye for at least a minute, suddenly remove it, when a corrective movement in the opposite direction will be seen. This movement of "redress" is usually much more easily seen than the movement of deviation just described.

Dr. Duane has added a refinement to this test by holding prisms of different strengths before the right eye in the appropriate positions until the movement is unobserved. This will probably be found to be the case with two or three prisms, e.g., 4 ▽, 5 ▽, and 6 ▽, then the heterophoria is probably of 5 ▽.

Test for *concomitancy* by repeating the test with the patient's head in different positions. Thus, if right hyperphoria be present, repeat the test with the patient's head inclined forwards, and again when tilted backwards. If the deviation is the same in each of these positions, simple (or concomitant) hyperphoria is present; if it is much more marked in one position than another, paralytic hyperphoria is diagnosed. For instance, if the deviation is most marked when the patient's face is directed upwards and to the right, his eyes must be directed downwards and to the left; as this is the direction in which the greatest deviation occurs, there must be some weakness of the right superior oblique (or of the left inferior rectus if left hyperphoria be present. *See p.* 80).

When oblique deviations occur, decompose them into their vertical and horizontal components. Correct the vertical component first, and then study the horizontal component.

(*b*) *Subjective.—Apparent movement* of the object, sometimes conveniently, although somewhat loosely, called *parallax.*

When the right eye behind the screen is deviating inwards, on suddenly transferring the screen to the left eye, the object will appear to make a jump to the right. This is called by Dr. Duane *homonymous parallax*, as it is analogous to *homonymous diplopia*, for instead of the double images being seen simultaneously they are seen in succession. Its amount may be tested in the same way with prisms, by finding the strength of prism that will abolish this apparent movement. This test I personally consider unreliable, for in the first place it requires an intelligent patient for its perception, and as the movement of redress is in the opposite direction to that of the deviation, it is sometimes this movement of redress which is noticed, and not the primary jump, so that confusion may easily result. I may say that Dr. Duane has himself pointed out these sources of error.

The screen test is sometimes used with the object at a distance of $\frac{1}{3}$ metre or so, but I prefer to use Harman's diaphragm test for near vision.

2. **Corneal Images.** *Objective.*—Mr. Priestley Smith and Dr. Maddox are the ablest advocates of this exceedingly useful method of detecting slight deviations; the reader is referred to Dr. Maddox's book on *The Ocular Muscles* for a fuller account of the procedure. The patient is directed to look at the central hole in the mirror of the ophthalmoscope, with which the surgeon, being about a foot distant, throws the light first on one cornea and then on the other. The corneal reflection of the illuminated mirror appears as a small bright spot of light which "maps out with sufficient precision for clinical purposes, the point in each cornea which is traversed by the visual line." It is important (*a*) To keep the patient's attention fixed on the mirror, as then the visual line of the patient coincides with that of the surgeon; (*b*) To judge of the position of the bright spot with reference to the cornea and not to the pupil, which is usually eccentric.

In normal eyes these corneal reflections occupy *symmetrical* positions in the two corneæ, nearly always slightly to the inner side, for a reason that will be given later.

If there is *asymmetry*, heterotropia is present. In this way the distinction between real and apparent squint can be at once made.

The test for *concomitancy* is made by turning the patient's head in different directions and noting, while he is still fixing the mirror, whether the position of the corneal image in the squinting eye is unchanged ; if there is an alteration in its position, the squint is nonconcomitant or paralytic.

3. **Maddox Rod.**—The mounted row of red glass rods and the tangent scales that were introduced by Dr. Maddox are too well known to need any detailed description. It is the simplest, most rapid, and most reliable of any subjective test for detecting any want of balance in the ocular muscles. It is assumed that the tangent scales are affixed to the wall at the end of the consulting-room in the form of a cross, one horizontal, and the other vertical, with a small light at the zero point of intersection of these scales.

(*a*). *Vertical.*—The rods are placed vertically in the right cell of the trial frames, and a green glass is placed in the hinder cell on the left. The patient, armed with these trial frames at 6 metres' distance, is directed to look attentively at the light. His right eye will see a red horizontal line, while his left eye will see the light coloured green. The eyes, being thus dissociated, will take up a position of rest; and if the right eye has a tendency to deviate upwards, it will be revealed by the displacement downwards of the red line below the level of the light. The patient can at once say through which figure on the vertical scale the red line passes. This tells us at once the amount in degrees of deviation of the hyperphoria.

If the red line passes exactly through the light, there is no hyperphoria. Should the patient have such deficient visual acuity that he cannot distinguish the figures on the scale, we place a 5° d. prism exactly horizontally in the anterior cell of the trial frames before the left eye. The patient will tell us that the red line occupies the same place, but on slowly rotating the prism edge downwards we shall find one position in which the red line passes through the light. On now reading off the angle at which the edge of the prism is pointing, we can at once determine the extent of the hyperphoria. Suppose, for instance, that the edge of the prism points to 30°, we know from *p.* 65 that $\phi = D \sin 30°$, and on referring to *Table V*, *p.* 157, we see that $\sin 30° = \cdot 5$,

$$\therefore \text{ the right eye tends to deviate upwards } 2\tfrac{1}{2}°.$$

If Dr. Maddox's prism verger is at hand, even this little trouble is avoided; we have only to turn the milled head until the red line passes through the light, and then the deviation can be directly read off from the index on the right celluloid scale.

I personally always test for hyperphoria first, and find that much time is saved by rotating a strong prism in this way instead of holding up different prisms until one is found that corrects the defect. The statements of the patient can always be checked by noting whether they are consistent with the prism used to correct or to overcorrect the displacement.

The test for concomitancy is very easily made by the patient tilting his head forwards and backwards, and noting if the deviation of the red line remains the same.

(*b*). *Horizontal.*—The glass rods are now placed horizontally in the trial frames, when the patient will see a vertical red line with his right eye, and he is asked through which figure on the horizontal scale this red line passes. This gives the amount of deviation: if it is to the right (the black figures) the case is one of esophoria, if to the left (the red figures) of exophoria. If the red line is continually shifting its position, uncorrected anisometropia or paresis is possibly present. This dancing of the red line is much more frequently complained of when testing horizontal deviations than vertical deviations. When the patient cannot read the figures, a similar procedure with a rotating prism must be undertaken. The prism must be now set vertically, and rotated until the red line passes through the light; then, on reading off the angle at which the edge points, and remembering that $\theta = D \cos \rho$ and referring to the *Table, V, p.* 157, we find the amount of the horizontal defect.

Dr. Maddox has also provided us with convenient scales for measuring the horizontal deviations at a distance of 10 inches ($\frac{1}{4}$ metre); but it must be remembered that a fair amount of exophoria is physiological at this distance, at least 3° or 4°. I myself have 8° of exophoria at $\frac{1}{4}$ m. without any symptoms. Esophoria for near vision usually needs correcting.

4. Harman's Diaphragm Test. *Subjective.* — For testing hyperphoria at close distances I prefer this test. The instrument consists of "a length of wood like a flat ruler 44 cm. long, which is fitted with a rack at one end to receive the test cards, and a screen measuring 9 × 6 cm. fixed at 11 cm. from the rack. In this screen a hole is cut 1·7 cm. square." If a test card with the letters A B C D E F G is inserted in the rack, the patient will see through the hole in the screen A B C with his right eye, E F G with his left eye, and the letter D with both eyes.

When right hyperphoria is present, the letters D E F G will be seen above the level of the letters A B C D seen with the right eye.

The correction is made by placing a prism of the requisite strength, edge upwards, before the right eye that will bring the letters in one line.

The normal physiological exophoria is well seen by a doubling of the letter D and a separation of the left half from the right half of the series of letters.

If esophoria is present, the middle letters overlap so that perhaps all that is seen distinctly is A B F G.

It is a most simple and efficient test for binocular vision, and is useful for the detection of malingerers, but this use is outside the scope of this book.

Although the description of these tests has seemed so long, it will be found that they only take a short time to carry out in practice.

(*B*).—DYNAMIC TESTS.

These tests determine the ability of the eyes to move. We must distinguish between a paralysis of one or more muscles and a paralysis of an "associated movement." Thus, a paralysis of the movement of convergence is easily distinguished by the complete freedom of the ranging movements from a paralysis of both internal recti. We will first briefly consider the paralysis of one or more ocular muscles, although it has less to do with the purpose of this book than the anomalies of the associated movements.

Ocular Paralyses.—The symptoms are:

1. *Limitation of movement* of the affected eye in some cardinal direction. The cardinal directions are up, down, in, and out. As the eyes move together in one of these directions, one eye lags more and more behind the other, producing a continually increasing deviation. This gives rise to

2. *Diplopia*, which is most marked in the cardinal direction of the action of the affected muscle; the false image is displaced as the healthy muscle would displace the eye.

Dr. Maddox has pointed out the inaccuracy of the common aphorism, that the affected muscle is the one which physiologically turns the eye in the direction of greatest diplopia. The superior rectus, for instance, turns the eye up and in; but when paralyzed, the greatest diplopia is up and out. The introduction of the word *cardinal* corrects the aphorism so that it is always true.

3. *Altered Position of the Head.* The face looks in the direction of the greatest diplopia, so that the eyes may be

directed towards the single vision area. In paralysis of the right superior rectus, the diplopia is greatest in the right superior area ; hence, if the patient directs his face upwards and to the right, he will be least inconvenienced by his diplopia, as for an object immediately in front of him he will have to direct his eyes downwards and to the left.

This gives us the key by which we may solve all these problems. We must first consider the line of action of the muscles. The external and internal recti of course act in the horizontal direction ; the superior or inferior rectus elevates or depresses the eye simply (i.e., without any torsion) when it is turned 27° outwards ; the inferior or superior oblique would simply elevate or depress the eye if it could be turned 51° inwards.

Hence, both eyes will be turned to the right by the action of the right external rectus and the left internal rectus ; when they are in this "eyes right" position, both eyes will be elevated by the action of the right superior rectus and the left inferior oblique almost entirely.

Consequently, the twelve muscles of the eyes may be divided into three groups of four each, four moving the eyes laterally, four upwards, and four downwards. Each group is divided into two pairs, one muscle of each pair being in the right eye and one in the left eye.

1. LATERAL	*a.* Right turners	R. ext. rect. and L. int. rect.
	b. Left turners	L. ext. rect. and R. int. rect.
2. ELEVATORS	*a.* Eyes right	R. sup. rect. and L. inf. oblique
	b. Eyes left	L. sup. rect. and R. inf. oblique
3. DEPRESSORS	*a.* Eyes right	R. inf. rect. and L. sup. oblique
	b. Eyes left	L. inf. rect. and R. sup. oblique

Each of these six pairs of muscles consists of Graefe's "True Associates," and are easily remembered by Dr. Maddox's mnemonic that their names are the most contrary possible, e.g., right external rectus and left internal rectus; or left superior rectus and right inferior oblique.

It is clear then that if we consider the patient's field of view to be divided into *right* superior, external and inferior areas, and *left* superior, external and inferior areas, and if we find the area in which the greatest diplopia occurs, the condition must be due to paralysis of the "same named muscle or the most cross-named muscle." As the diplopia is only considered in the cardinal directions, any source of error from a pre-existing heterophoria is eliminated.

Thus, if the maximum vertical diplopia occurs in the right superior area of the patient's field, he must have a paralysis of the same-named muscle, i.e., the right superior rectus, or of the most cross-named muscle, i.e., the left inferior oblique.

It only remains to distinguish which eye is affected. By screening one eye, while the object is held in the area of maximum diplopia, the corresponding image disappears. "The image that lies farthest in the direction of increasing diplopia belongs to the paralyzed eye."

If the diplopia is increased in more than one of these directions, an affection of more than one muscle is indicated.

Spasm.—The overaction of one muscle, say the right external rectus, would, on a cursory examination, resemble a paralysis of its opponent, the right internal rectus, or possibly of the left external rectus; but the distinction would be easily made. Its characters would be a *sudden* and *temporary* deviation of one eye, and absence of any marked heterophoria (revealed by Maddox's rods) in the intervals between the attacks. Normal movements in the other eye of course exclude paralysis of the left external rectus. When a sudden temporary deviation of one eye occurs, it is nearly always due to a heterophoria, or latent squint, which becomes manifest when the nervous centres are exhausted. I believe I have only seen one case of this very rare condition, a spasm of the right external rectus occurring during a meningitis, afterwards followed by a paralysis of the right sixth.

For a fuller and more detailed account of ocular paralyses I would refer the reader to Dr. Maddox's *Ocular Muscles*.

Associated Movements.—Those in the horizontal direction are somewhat difficult, and we shall have to devote some time to their study. The associated movements in the vertical direction, supra- and infra-vergence, are simpler, and therefore we shall describe the tests for these movements first. These tests determine what has been called "the breadth of fusion," i.e., the strength of the prism, edge up and edge down (or edge in and edge out), that can be overcome by the eyes. For this purpose Dr. Maddox has provided us with an invaluable instrument, called the prism verger, that enables the tests to be carried out much more quickly and more accurately than was possible before.

The prism verger consists of a frame in which two prisms, each of 6° d. are so mounted as to be simultaneously rotated in opposite senses by turning a milled head.

Vertical Deviations. *Supravergence.*—Adjust the prisms in the verger so that the edges of both point to the patient's left, and, placing the instrument on his face, direct his attention to the test object previously described, i.e., a round white spot in the centre of a blackened target. On now rotating the milled head clockwise until two white spots are seen by the patient, and noting the reading of the instrument, we find the limit of the supravergence of the left eye; on then rotating the milled head counter-clockwise we similarly find the limit of supravergence of the right eye. Normally, about 1° d. is the limit of supravergence for each eye. The actual breadth of fusion is in this case unimportant; the important point is the difference between the supraverging power of the eyes. In true orthophoria, the right eye will be able to move up above the level of its fellow to exactly the same extent as the left eye will be able to move up. Should there be a difference, half the difference determines the amount of hyperphoria. This test is very useful in those cases in which the rod test is unsatisfactory owing to the dancing of the line of light.

Thus, suppose the right eye could move up 2° d., while the supravergence of the left eye was $\frac{1}{2}$° d.,

Since $\frac{1}{2}(2 - \frac{1}{2}) = \frac{3}{4}$, there must be $\frac{3}{4}$° d. of hyperphoria in the right eye.

Treatment of Hyperphoria.—If hyperphoria is found, and there are no symptoms whatever attributable to it, do not correct it. If there are symptoms, it is important to distinguish between concomitant hyperphoria and paretic hyperphoria.

In *paretic* hyperphoria the most satisfactory course is to exercise the weak muscle by ordering appropriate prisms for occasional use. In the above case of $\frac{3}{4}$° d. right hyperphoria, when the eyes were directed straight forwards, suppose that, on depressing the eyes, the hyperphoria was increased especially in the " eyes left " position, we should diagnose paresis of right superior oblique. We should then order $\frac{1}{2}$° d. prism, edge down, before the right eye. This will exercise the depressing muscle of that eye, and if we direct the patient when wearing the spectacles to turn his face rather to the right, so that he adopts the " eyes left " position, we can be sure that it will be chiefly the right superior oblique that is exercised. It will be sufficient at first if he wear these glasses for half an hour after each meal ; as soon as they cause diplopia they must be taken off. Probably in two or three weeks it will be found that his supra- and infra-verging powers are equal, and that he is cured of his paralytic hyperphoria. Of course, if any internal treatment such as potassium iodide is indicated, it should not be neglected.

Dr. Maddox has suggested the use of circular prisms mounted in special frames, so that the position of the prisms in the frames can be altered, if necessary, at each visit to the surgeon. When the edges of the prisms are both directed towards the left, they will have no vertical effect on the eyes; but by rotating one prism a trifle upwards and the other a trifle downwards, the required amount of exercise can be given to the weak muscle.

In *concomitant* hyperphoria the same method of treatment may be adopted; I have never myself found it of any permanent use, and therefore I regard the condition as incurable (probably congenital), and only order relieving prisms. These have been universally satisfactory, and although one would expect the hyperphoria to increase, so that stronger prisms would be required from time to time, I have not found this to be the case.

At least three-fourths of the whole amount of hyperphoria should be corrected; if this is not enough to relieve the symptoms, I do not hesitate to correct the whole defect found by the distance test, whether static or dynamic. From the note on *p.* 67 it will be seen that if reliance were placed on the near test, an over-correction would be given for distance. In relieving prisms, the edge of the prism must point in the direction of the deviation; thus, for right hyperphoria the right prism must point edge upwards and the left edge downwards. When a refractive error needs correction, it is usually more convenient to order the glasses to be decentred to the amount required to give the prismatic relief by the formula $l = \frac{10\ \mathrm{N}}{\mathrm{D}}$ where N denotes the number of centrads required for the relief of the hyperphoria, and l the amount of decentration in millimetres. It is customary to regard a prism with its edge down (or its base up) as a positive prism. Consequently, the equivalent of a $+ 4$ D lens combined with a $1\ \nabla$ prism, edge up (i.e., $-1\ \nabla$) is $\frac{-10}{4} = -2{\cdot}5$ mm.; the lens must therefore be decentred downwards 2·5 mm.; the optical centre being 2·5 mm. below the geometrical centre of the lens. Before definitely ordering this or any other prismatic correction, it is advisable to let the patient wear the prisms in the waiting-room for at least twenty minutes in order to be sure that they relieve the discomfort from which he suffers.

Points to remember about Hyperphoria.—

1. Always correct all refractive errors before testing for hyperphoria.

2. Never correct hyperphoria unless symptoms of the defect are discoverable.

3. Small errors such as $\frac{1}{2}°$ d. may cause great discomfort; although high errors such as 3° d. or 4° d. may cause no inconvenience.

4. Treat concomitant hyperphoria by relieving prisms, the edges of the prisms in the direction of the deviation; treat paretic hyperphoria at first by exercising prisms, their *bases* in the direction of the deviation. These should be worn three or four times a day, and only when the paresis is found to be incurable should relieving prisms be ordered, and then they should be only just strong enough to relieve the symptoms of discomfort.

5. Let the patient wear the correction for twenty minutes before finally ordering it.

6. Note that a prism edge down is regarded as a positive prism, also that decentration upwards is regarded as the positive direction. If these two points are borne in mind no difficulty will be experienced in using the formula $l = \frac{10\,N}{D}$.

Horizontal Deviations.—Before dealing with this intricate subject, we must define certain lines and angles that are of importance in understanding certain conditions, and the literature dealing with them.

The Visual Line and the Fixation Line.—In *Fig.* 9 the line AK′ K″ φ'' represents the optic axis, F the fovea, and O the object viewed. What is called the *visual line* consists, strictly speaking, of two parallel lines, one OK′ drawn from O to the first nodal point, the other K″ F

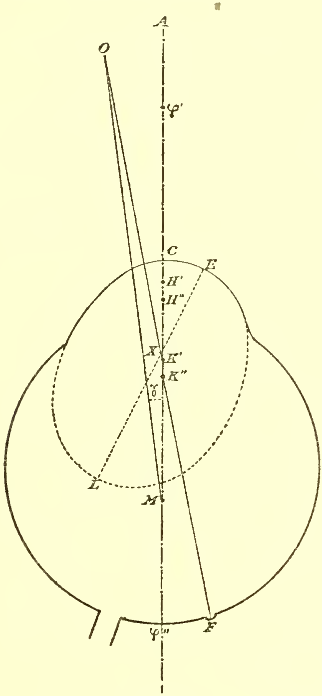

Fig. 9.

drawn from the second nodal point to the fovea. It will be noticed that the visual line cuts the cornea on the inner side of the optic axis in the diagram. This is always the case in emmetropia and hypermetropia. Thus, if a hypermetrope view some distant object so that his visual lines are parallel, his optic axes will diverge. This divergence is less in myopia ; in extreme degrees of myopia the optic axes may even converge.

It is found that slight movements of the eyeball may be considered as movements of rotation about a fixed centre M, called the centre of motility, which is usually about 13·4 mm. behind the cornea.

The *fixation line* is the line MO joining the centre of motility and the object viewed.

The angles α, β, γ, *and* κ.—The angle α is now used in two different senses. The angle α of Landolt is the term used to denote the angle OXE between the visual line and EL, the major axis of the corneal ellipsoid. We now know from the investigations of Dr. Tscherning and Dr. Sulzer that the shape of the cornea is not that of an ellipsoid of revolution and that it has no axis of symmetry.

The angle α of Donders was the angle between the axis of the cornea and the visual line. Now, as Donders assumed wrongly that the axis of the cornea coincided with the optic axis, it is clear that his angle α is AK′O.

In order to avoid this confusion, the angle AK′O is called by Dr. Brubaker the angle β. I do not know whether he was the originator of this term, but it seems to me the best to use, as it has no ambiguous meaning.

The angle γ is the angle AMO between the fixation line and the optic axis.

The angle κ is the angle that the ~~fixation~~ visual line makes with that normal to the cornea that passes through the pupillary centre. This angle is a concession to hurried workers, as the centre of the pupil is much easier to see than the point where the optic axis cuts the cornea. It is not, however, to be recommended, as the pupil is always eccentric, being usually placed to the inner side, and it is not uncommon to find that the pupillary opening is asymmetrical in the two eyes.

All these angles are considered positive when the visual and fixation lines lie to the inner side of the optic axis, which is nearly always the case except in high myopia. The variation of these angles in different cases depends of course on the situation of the fovea (F).

Apparent Squint.—It is obvious that, with a high angle β, there will be an appearance of external squint. The diagnosis is at once made by examining the corneal images with the ophthalmoscope; when these images are symmetrically placed, while the patient is gazing at the mirror, there is no real squint.

As we have already described the static and dynamic tests of the external and internal recti, it only remains to consider the dynamic tests of the associated movements of convergence and divergence. As I have already said, it is most important to first correct all refractive errors and any hyperphoria that may be present, before investigating the function of convergence.

As especial precautions must be taken that no experimental error is introduced by any displacement of the centres of the trial glasses, it is well to consider what means we have to avoid this error. We will first find how we can so adjust the trial lenses that their optical centres are exactly traversed by the visual lines of the patient.

Centring the Glasses.—It is usually only considered necessary to see that the centres of the correcting glasses are opposite the centres of the pupils; but this is not accurate work. The centres of the glasses must be traversed by the visual lines of the patient, and, owing to the variability of the angle β, the visual lines may pass by the extreme inner margin of the pupil, or they may not pass through the pupil at all. It is here that the method of corneal images renders us such invaluable service when a Maddox localizer is used. The localizer with its two sights is placed before one eye in the trial frames, and the surgeon alters the width of the frame until the corneal reflection is seen to be intersected by the sights of the localizer. The distance is read off in millimetres, and then the other eye is tested in the same way. It will be found that this measurement is an exceedingly troublesome one to make, and fortunately it is quite unnecessary before testing the convergence of the eyes. It is true that, if we needed to know the actual deviation of the fixation lines, the utmost accuracy in centring the glasses would be required, as an error of half a millimetre in each eye when -10 D are worn would entail a total error of convergence of 1 ∇. For the purpose, however, of ordering relieving glasses, we need only know what will give the requisite relief, without investigating the actual effect which the prescribed glasses have on the ocular fixation lines.

It is important to realize that the prisms that produce a given deviation on the uncorrected eye, will produce quite a different effect when correcting glasses are worn, (*p.* 140). I have calculated some tables, *pp.* 150-4, that will show the difference that convex and concave glasses make on this deviation, but in practice they will rarely be found useful if the simple method that I recommend be adopted.

Place the correcting glasses in a trial frame, of which the distance between the centres is known ; suppose that this distance is 60 mm., quite irrespective of the actual distance between the visual lines of the patient, and test the patient's convergence. Allow me to anticipate the results of this test, so that finally he is found to require abducting prisms of 1 ∇ before each eye ; the distance between the centres of the glasses can be immediately written down according to the formula $l = \frac{10\ \text{N}}{\text{D}}$. In this case, as the prism is abducting, N = -1. Suppose that the patient required R $+5$ D, L $+4$ D, and was wearing these in the trial frames. Clearly, the right glass must be decentred $\frac{10\ (-1)}{5}$ or -2 mm., i.e., 2 mm. inwards, while the left glass must be decentred

$\frac{10\,(-1)}{4}$ or $-2{\cdot}5$ mm., i.e., 2·5 mm. inwards. As the centre of each lens in the trial frames was 30 mm. from the mid-nasal line, the optical centre of the right glass in the spectacles must be $30-2$ or 28 mm., while that of the left glass must be $30-2{\cdot}5$ or 27·5 mm. from the centre of the nose. It does not matter in the least what the distance between his visual lines is, this is his correction. Opticians always measure the interpupillary distance; and it is well, for cosmetic reasons, that the geometrical centres of the glasses should be opposite the pupils, and any competent optician can be trusted to carry out such a prescription perfectly, whatever may be the distance between the patient's eyes.

For instance, if in the above case the interpupillary distance were 64 mm., the half of this is 32 mm. The right glass is to be decentred $28-32$ or -4 mm. (inwards), and the left glass $-4{\cdot}5$ mm. (inwards). The optician may consider his own convenience whether it is easier for him to furnish decentred glasses as specified, or to grind $+5$ D spherical surfaces on a prism of $-2\,\triangledown$ for the right eye, and spherical surfaces of $+4$ D on a prism of $-1{\cdot}8\,\triangledown$ for the left eye, for

$$N = \frac{lD}{10}, \text{ and } -2\,\triangledown = \frac{-4 \times 5}{10}, \text{ and } -1{\cdot}8\,\triangledown = \frac{-4{\cdot}5 \times 4}{10}.$$

When the difference of refraction of the two eyes is greater than $+1$ D, it may happen that the equivalent prism, as found by this method, that is used before one eye is considerably greater than that used before the other. In such cases it is advisable in accordance with the rule given above (p. 68) to divide the total prismatic effect equally between the two eyes. In the above case, the total prism required being $-3{\cdot}8\,\triangledown$, the same result would be attained by supplying a prism of $-1{\cdot}9\,\triangledown$ to each eye; or by decentring the right glass $\frac{-19}{5}$ or $-3{\cdot}8$ mm. and the left glass $\frac{-19}{4}$ or $-4{\cdot}75$ mm. (inwards).

CONVERGENCE.

We will suppose then that our patient has had his hyperphoria and his refractive errors corrected, and we now proceed to test his convergence.

Minimum Convergence or Divergence.—The patient is directed to fix a small light, or one of the smaller letters of the test types at 6 metres' distance, while the surgeon with different prisms (held edge out) discovers the strongest abducting prisms that the patient can overcome. As soon as this limit is passed, he will see two objects. For this purpose the verger is most convenient, as with it the test can be very expeditiously made, which is of importance, as all these tests are very tiring to the patient. Normally about 4° d. edge out can be overcome with single vision, i.e., 2° d. each eye. The result may be noted as $-$ 2° d. (The angle is given in degrees of deviation, as at present the verger is not marked in centrads).

Relative Range of Convergence or Breadth of Fusion at 6 metres.—By rotating the milled head of the verger in the clockwise direction, a series of adducting prisms are practically placed before the eye, and we in this way find what is the maximum convergence that the eyes can exercise when the accommodation is relaxed. It is necessary, in executing this test, that the patient should fix his attention on the smallest letter that he can see at 6 metres' distance, and warn us as soon as the letter becomes indistinct, or becomes double. Suppose it is found, when great care is taken that the small letters can still be distinctly seen, that the eyes can overcome 7° d.; if the patient is allowed to exercise his accommodation, a very much greater amount of convergence is possible, but this will be discovered by his failing to read the small type. The result is noted—breadth of fusion at distance $-2°$ d. to $+ 3\frac{1}{2}°$ d. The negative sign shows the amount of divergence, while the positive sign indicates convergence.

The note does not mean that his fixation lines diverge and converge to exactly this extent; the actual amount they can move depends on the glasses that he is also wearing; the note merely implies that he can overcome these prisms when wearing his correcting glasses 60 mm. apart, and that practically is all one need know.

Relative Range of Convergence at $\frac{1}{3}$ Metre.—A small test object is now placed at $\frac{1}{3}$ metre from the patient's eyes, i.e., about 31 cm. from the bridge of his nose, and the same method of examination is used. I find my simple little dynamometer (*p.* 12) very convenient for the purpose. This will test the range of convergence when 3 m.a. of convergence and 3 D of accommodation is exercised, as the patient is still assumed to be wearing his distant correction. With the verger, his minimum convergence is found under these conditions: we will suppose that he can just maintain single vision of the slit, and define the engraved line when the verger indicates $-10°$ d., i.e., single vision can be maintained for a moment when a $-5°$ d. prism is held before each eye. By insisting on the patient's saying the moment that the fine line appears blurred, we have some guarantee that exactly 3 D of accommodation is being maintained. On now rotating the milled head of the verger in the clockwise direction, his maximum convergence under these conditions is noted. If the indication on the verger is 7° d., each eye can overcome $3\frac{1}{2}°$ d., and we make the note that his *breadth of fusion under these conditions* is $-5°$ d. to $+3\frac{1}{2}°$ d. (As 3 m.a., say 5° d. of convergence, are already exercised, the observation shows that when exerting 3 D of accommodation he can maintain parallelism, or each eye can converge $8\frac{1}{2}°$ d. (5 m.a.) if the error introduced by his correcting glasses be neglected).

The amount of divergence under these conditions will usually be found to be greater than the amount of convergence in normal individuals. This explains why exophoria at reading distance is commonly observed, but it is a somewhat remarkable fact that the position of rest as indicated by the Maddox scale does not usually, in my hands, occupy the middle of the range, but tends to be rather on the divergent side. For instance, in the above case, where the range was -5° to $+3\frac{1}{2}^\circ$, the position of rest, instead of being at the mid-point $\frac{1}{2}(-5+3\frac{1}{2})$ or $-\frac{3}{4}^\circ$, was at -2° d. It is important to execute each part of this test as rapidly as possible, with a rest between the tests for divergence and convergence, as it is exceedingly fatiguing to the patient. For this reason it is not very reliable, as one can never be sure that the patient is exerting himself to the utmost; however, when the patient's statements are reliable, I consider it the most valuable test of all. I personally rarely make the next test, except in cases of squint, before operation, when it is most important to indicate whether advancement or tenotomy should be performed. However, many experts place great reliance on the test for the maximum convergence as an indication of the prismatic correction that should be given them to wear. I will discuss the matter presently. I may say that I always use a trial frame in which the trial lenses are 60 mm. apart, and I use the abbreviations C to denote correcting glasses and R_c to denote the range of convergence for each eye, so that, after noting the refraction, I denote the result of the two previous tests by CR_c at 6 m. -2° to $+3\frac{1}{2}^\circ$ and CR_c at $\frac{1}{3}$ m. -5° to $+3\frac{1}{2}^\circ$.

Absolute Maximum of Convergence.—As the near point of convergence is usually about 4 cm. from the bridge of the nose, and as an error of only 2·5 mm. in this situation entails an error of 1° d. in the amount of convergence for each eye, it will be seen that some special precautions must be taken to obtain anything like an accurate result. The first essential is that the object fixed should be seen distinctly; this is of course impossible for a hypermetrope, or even for an emmetrope whose near point of accommodation is about 25 cm. off. We must therefore give him such glasses as will place his near point (punctum proximum) at $\frac{1}{3}$ metre, and then, with the help of the tables (p. 153), we shall be able to get an approximate result. Suppose a hypermetrope of $+ 7$ D, with say 5 D of accommodation, is furnished with $+ 5$ D lenses, his near point or punctum proximum will then be at $\frac{1}{3}$ metre, for $5 + 5 - 7 = 3$. On putting these lenses in the trial frames, and directing him to fix the slit in the dynamometer, we add $+ 8°$ d. prisms to each eye, and also use the verger. If we find that he can still see the slit single when the verger indicates 10°, we know that each eye is able to overcome at least an adducting prism of $8 + 5$ as well as the initial convergence of 3 m.a. (or 5°), that is, about $13° + 5°$ or 18°. On referring to *Table II p.* 153, we can get a rather more accurate result, for we see that an adducting prism of 59′ with a $+ 5$ D lens entails a convergence of 4 m.a. or 6° 53′, and that each 57′ added causes an additional convergence of 1° d. We have then a convergence of 4 m.a. $+ \frac{13° - 59'}{57'} = 6°\ 53' + 12°\ 39' =$ 19° 32′. This of course would only be correct if the interocular distance were 60 mm., when the glasses would be properly centred. However, in practice we should obtain a different result each time that we examined the patient, according to the effort that he made to overcome the diplopia, so that little reliance can be placed on this test even with the utmost precautions to ensure accuracy. As I said before, in my opinion it is of no use in prescribing glasses.

Treatment of Abnormalities of Convergence.—I feel that I cannot repeat too often the caution about prescribing prisms unnecessarily when there are no symptoms that suggest their employment. It is quite common to find a slight esophoria for distance, and sometimes a marked exophoria for reading, without any symptoms being occasioned.

If symptoms arise in *paretic* cases, exercising prisms should be tried, just as in cases of paretic hyperphoria.

In *concomitant* cases I rely on the determination of the relative range of convergence at 6 metres, and on that found at $\frac{1}{3}$ metre for the amount of correction, when the experiment with the Maddox scales shows a marked deviation from the normal. Of course, if the Maddox scales show only a slight deviation within physiological limits, these tedious later tests need not be executed.

If there are symptoms and a marked *esophoria* for distance, I add $\frac{1}{3}$ of the total range for distance to the minimum convergence found, and give prisms of that strength. If possible, I get the patient to try the prism with the correcting glasses for twenty minutes or so before finally ordering his spectacles, so that any slight alterations may be made if necessary.

For instance, suppose that a patient have at 6 metres $R_c = -1^\circ$ to $+7^\circ$, $\frac{1}{3}$ of the range is $\frac{1}{3}$ of 8° or $2{\cdot}\dot{6}^\circ$, and this added to -1° is $1{\cdot}\dot{6}^\circ$. I should therefore try him with adducting prisms of $1^\circ\ 40'$ for each eye, added to his correcting glasses in the trial frame for 15 or 20 minutes. If these seemed to be successful, we must find at what distance the centres of his glasses must be to be equivalent to these prisms. A $1^\circ\ 40'$ prism $= 2{\cdot}9\ \nabla$. If he was wearing R $+ 3$ D, L $+ 4$ D, since $l = \dfrac{10\ \mathrm{N}}{\mathrm{D}}$, for the right glass, $l = \dfrac{29}{3} = 9{\cdot}\dot{6}$ mm. and for the left glass $l = \dfrac{29}{4} = 7{\cdot}25$ mm. outwards; that is to say, the optical centre of the right glass in the spectacles must be $30 + 9{\cdot}\dot{6}$ or $39{\cdot}\dot{6}$ mm., while that of the left glass must be $30 + 7{\cdot}25$ or $37{\cdot}25$ mm. from the centre of the nose. The same result will be found whatever the distance between his visual lines, whether 56 mm. or 66 mm. The optician can be usually trusted to execute the order neatly. If preferred, explicit instructions can be given, e.g., geometrical centres 64 mm.; right glass to be decentred 7·6 mm. outwards; left glass 5·25 mm. outwards.

Should symptoms of *exophoria* be present, we subtract from the maximum relative convergence $\frac{1}{3}$ of the relative range, and try an abducting prism of that strength.

For instance, if a patient had at 6 metres R_c -4 to 0, the strength of the abducting prism would be

$$0 - \tfrac{1}{3} \text{ of } 4 = -1{\cdot}3^\circ \text{ (or } -2{\cdot}3\ \nabla) \text{ before each eye.}$$

The equivalent decentration can, as before, be found by the formula $l = \frac{10\,N}{D}$; in this case the value of N is $-2{\cdot}3$.

Similarly, for near vision at $\frac{1}{3}$ m. I give a prism that is expressed by the sum of $\frac{1}{3}$ of the range at $\frac{1}{3}$ metre, and his minimum convergence *at that distance* in cases of *esophoria*. If troublesome *exophoria* be present, I subtract $\frac{1}{3}$ of his relative range from the maximum relative convergence *at $\frac{1}{3}$ metre*, and having tried the prisms, order his glasses to be decentred to the requisite extent as before.

For instance, if a patient while wearing his correction were still found to have troublesome esophoria at $\frac{1}{3}$ m., and if it were then found that his relative range at $\frac{1}{3}$ m. as tested by the verger showed a divergence of 2° and a convergence of 7°, his $R_c = 9^\circ$. Now as $\frac{1}{3}\, R_c = 3^\circ$ and his minimum relative convergence is -2°,

$$\text{Min.} + \tfrac{1}{3}\, R_c = -2^\circ + 3^\circ = +1^\circ.$$

We therefore must give him a $+1^\circ$ prism (i.e., an adducting prism) for each eye. If after the usual preliminary trial these glasses seem satisfactory, we proceed to make out the corresponding prescription. Suppose that his correction is

R + 2 D sph. + 3 D cyl. ax. 90°,
L + 1 D sph. + 4 D cyl. ax. 90°;

clearly in the horizontal direction the refractive value of each glass is $+ 5$ D, and as $1^\circ = 1{\cdot}745\ \nabla$,

$$l \text{ or } \frac{10\text{ N}}{\text{D}} = \frac{17{\cdot}45}{5} = 3{\cdot}49 \text{ mm.}$$

Each glass must therefore be decentred outwards (as l is positive) 3·49 mm., so the distance between the optical centres of the glasses must be $60 + 6{\cdot}98 = 66{\cdot}98$ mm. This is all that we need note on this prescription form that bears the power of the glasses required.

Points to remember about Convergence Defects :—

1. Always correct hyperphoria and all refractive errors before testing for this variety of heterophoria.

2. Never correct unless symptoms call for it.

3. For *esophoria* give Min. $+ \frac{1}{3}\ R_c$.
For *exophoria* give Max. $- \frac{1}{3}\ R_c$.

The symbols Min. and Max. are to be understood as meaning the *relative* minimum and the *relative* maximum convergence found at the distance for which the glasses are to be used.

4. If the relative convergence range is defective, squinting exercises should be enjoined in exophoria ; exercising prisms (abducting) in esophoria.

5. Note that an *adducting* prism (edge in) is a positive prism, and that decentration *outwards* is regarded as the positive direction.

These rules are such, that the prisms ordered are the weakest that are likely to relieve the symptoms, so, if found necessary, one need not hesitate to order rather stronger ones. If none but prisms of 4° d. afford any assistance, an advancement operation must be considered.

The method that I have just given is not that usually recommended ; it is customary to measure the absolute range of convergence, and then consider that not more than $\frac{1}{3}$ of this total range can be exercised continuously without fatigue. This sounds a simple rule, but as I have pointed out in describing the test for the maximum convergence, there are very great difficulties in finding the total range. Further, I consider the rule illogical, as the amount of convergence that can be exercised depends entirely upon the amount of accommodation that is exerted at that time. In fact, it must be the relative, not the absolute range that must be considered. I admit that my rule is only empirical, and that some patients may require rather more than $\frac{1}{3}$ of their relative range to enter into the calculation, but I have found the method exceedingly useful in practice, and I think that the following charts will justify my contention.

It is beyond the province of this book to deal with the subject of squints, but I may remind the reader that if esophoria at a distance is present, it is well to fully correct any hypermetropia and to rather undercorrect myopia, and when exophoria is present to give rather weaker convex and stronger concave glasses.

RELATION OF CONVERGENCE TO ACCOMMODATION.

Donders was the first to study this relation, and to give charts for cases of hypermetropia, emmetropia, and myopia, showing how much they differed from each other. Nagel published several charts of great interest, and by introducing the metre angle, simplified their construction and made them much more readily intelligible. Few people have paid much attention to them since, so I here reproduce some of the charts that I have made out, as I feel that more light is thrown on the proper correction of esophoria and exophoria by the study of this relationship than in any other way.

We will begin by considering the first chart (*Fig.* 10), which may be taken as a normal chart of a person with ·5 D of hypermetropia.

The figures along the horizontal line to the right indicate the metre angles of convergence exercised; thus 0 denotes no convergence, or parallelism of the fixation lines, 3 denotes 3 m.a., or convergence of the fixation lines to a point $\frac{1}{3}$ metre distant. The space to the left of the point 0 denotes negative convergence or divergence. We see at once from the figure, that Dr. Campbell could diverge 1 m.a. and could converge 15 m.a.

The vertical figures under the heading Accommodation may require some explanation; the figures represent the reciprocals of the distances for which the eyes are accommodated; thus 0 represents the state of the eyes when accommodated for infinite distance, 3 when accommodated for $\frac{1}{3}$ metre, and so on. While her fixation lines were diverging 1 m.a., Dr. Campbell could see clearly with either + ·5 D or − 2 D, i.e., she could exert accommodation sufficient to see an object $\frac{1}{2}$ metre distant, so the upper dotted line begins at the point marked 2. When her fixation lines were parallel, she could see clearly with either a + ·5 D or a − 3 D lens, hence the upper dotted line rises to the point 3, while the lower dotted line continues at the level of ·5 below the horizontal line.

When the test object was held at $\frac{1}{3}$ metre, the diverging prisms that could be overcome showed that parallelism could be momentarily attained, and also converging prisms of such strength that they corresponded to a total convergence of 8 m.a., while distinct vision of the test object at $\frac{1}{3}$ metre was still maintained.

In this way the relation of the two functions has been traced, and is represented by the chart, showing that the maximum of convergence is 15 m.a., and that the maximum of accommodation is denoted by 8·5, which, when added to the ·5 D of hypermetropia, represents 9 D of accommodation.

The horizontal lines evidently give the relative convergence for each dioptre of accommodation exercised. Thus at the point 3 (representing + 3·5 D of

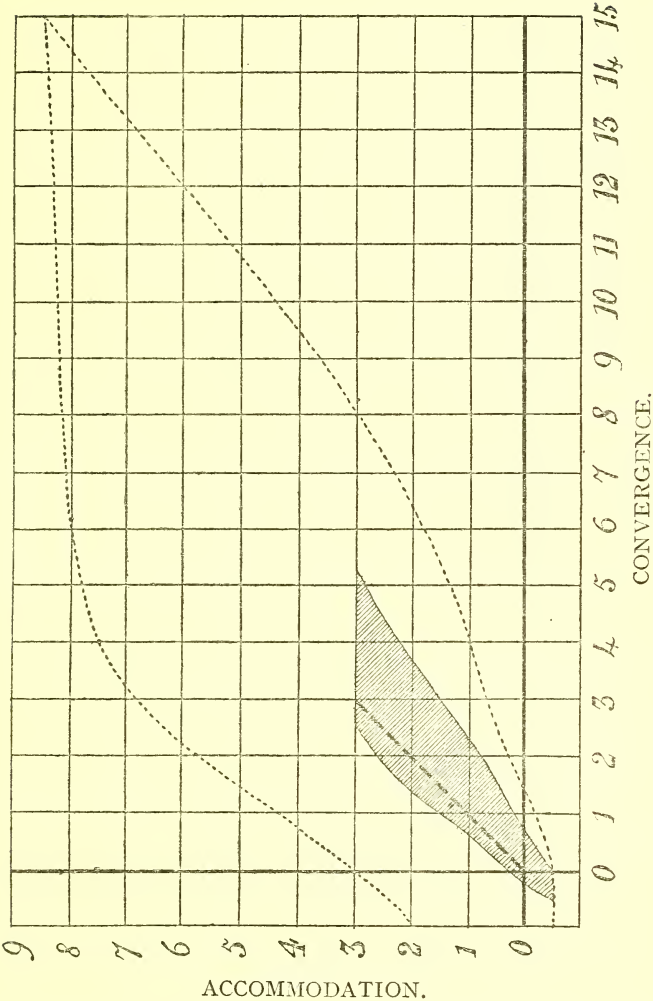

Fig. 10.—Chart of Dr. Mabel Campbell (House Surgeon at the Newcastle-on-Tyne Eye Infirmary) showing an excellent range of convergence. It is a little peculiar in showing no exophoria at $\frac{1}{3}$ m., the range at A 3 C 3 being from $-$ 3 to + 5 (0 to + 8 on the chart). It is more usual to find the negative part of the range at this distance greater than the positive part.

her accommodation) her range of convergence was from 0 to 8 m.a., at 7·5 (+ 8 Ac.) her range of convergence was from 4 m.a. to 13·5 m.a.

Similarly, the range of accommodation to a given convergence is given by the vertical lines. Thus, when exerting 4 m.a. of convergence, her range of accommodation is denoted by the line from 7·5 to 1.

It is obvious, then, that the amount of convergence that can be exercised depends upon the amount of accommodation that is exerted at the same time. Consequently, I contend that it is only the relative range of convergence that can be considered in treating defects of convergence.

Some, laying stress on the fact that in many cases faulty tendencies, as revealed by the rod test, do not give rise to any symptoms, are inclined to disregard them in all cases, and confine their attention to the exact correction of refractive errors. Others urge the correction of every anomaly as soon as it is discovered, attributing to it almost any ache and pain, or indeed anything else that the patient may suffer from. Clearly, if the relation between the two functions is unfitted for present requirements, and if there is no sufficient faculty of adaptation that can be brought into play by training, we should make the glasses suit the patient, instead of vainly attempting to make the patient suit the glasses.

To illustrate what I mean by the faculty of adaptation, I append the chart of an active man of 24 who came to me for glasses (*Fig.* 11). I found simple hypermetropia of + 6 D; but his manifest binocular hypermetropia was only + 3 D. When lenses stronger than + 3 D were given him his eyes diverged. In fact, this hypermetrope had learnt to accommodate without converging, just as most myopes learn to converge without accommodating. The chart shows what discomfort the full correction would cause him until he could adapt himself to the new conditions. Fortunately, most patients are able to adapt themselves to the glasses ordered them, even though these embody a full correction or something

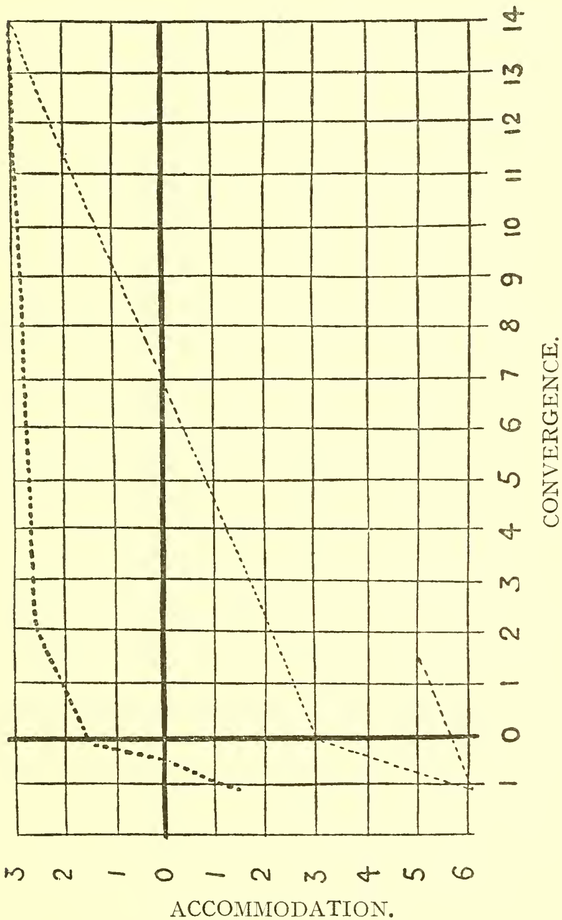

Fig. 11.

very near it. Thus, although I ordered for my patient + 5 D, he only complained of discomfort for the first few days, and then professed himself to be much pleased with the glasses. On his second visit, a fortnight afterwards, I found that instead of diverging when wearing the + 5 D correction, he could not only maintain parallelism of his fixation lines, but he could overcome adducting prisms of 2° 30′ before each eye (i.e., convergence of 1·5 m.a.). This is indicated by the lower short dotted line. It would have been worse than foolish to have ordered prismospheres for this patient when he had such good adaptive power, but some patients will be found who have no such capacity for adapting themselves to new conditions. This is the reason why new spectacles are so often troublesome at first.

When hypermetropes cannot acquire the art of accommodating without converging, the familiar concomitant squint occurs, and in some cases it will be found that the correction of the hypermetropia at once cures the squint. As this faculty of adaptation varies so much in different people, it appears that the peculiarities of each case must be considered before ordering prismospheres.

Now the requirements of most people will be satisfied if they can see clearly and without discomfort both distant objects and those at $\frac{1}{3}$ metre. Clear binocular vision will be possible if the diagonal from 0 to 3 lie within the figure bounded by the dotted lines, and possible only on this condition; but this does not necessarily imply comfort. What then defines this *area of comfort?*

The Area of Comfort.—From a careful examination of the notes and charts that I have taken of different patients, I have come to the conclusion that this area occupies about the middle third of the relative ranges between the limits of 0 and 3 m.a. of convergence. Above 3 m.a. this definition does not hold good, but we need not trouble ourselves about the discomfort of patients who persist in holding books too close to them when provided with proper glasses. In the next charts, this area of comfort is shaded; whenever I have found the diagonal to lie outside this area of comfort, I have heard complaints of asthenopia.

Numerous other suggestions have been made as to the principle on which corrections should be given.

I have mentioned Landolt's suggestion that $\frac{1}{3}$ of the absolute range of convergence can be exercised continuously without fatigue, and I have given my reasons against its adoption. I have tried this method, and have found that it has frequently failed to give relief.

Some have aimed at putting the absolute far point of convergence in its normal position. Others have tried altering the position of the absolute near point. Neither of these methods commend themselves, as the total range of convergence may be anything from 9 m.a. to 16 m.a. without giving rise to any symptoms.

My definition of the area of comfort was published in the *Ophthalmic Review* in 1891, and since that time I have seen nothing to alter in my original communication. Some of my original diagrams are given below.

It is clear that, by prisms or by operative interference, we can alter the position of the zero line of convergence as we will. By ordering abducting prisms of 1 m.a. we can displace this line one division to the left, or with adducting prisms we can displace it towards the right.

Similarly, with the help of spherical lenses we have complete control over the position of the horizontal zero line.

The principle that I adopt is sufficiently clear from a study of these diagrams. Whenever the patient has not sufficient adaptive power to suit his new correcting glasses, after a sufficient trial, I so alter them that the diagonal from 0 to 3 falls within the area of comfort.

EXAMPLES.—(1) Mrs. F., a neurotic emmetrope, aged 25, suffered from asthenopia that was found to be due to an abnormal relative range of convergence. Her chart (*Fig.* 12) shows that with her accommodation relaxed she could diverge 2 m.a. and could also just maintain parallelism. When 3 D of accommodation was exerted her range of convergence was from $-\cdot 5$ m.a. to 4 m.a. The diagonal was about $\cdot 5$ m.a. on the positive side of her area of comfort. She had been given $+1$ D glasses for reading, which only aggravated her difficulty : this can be easily seen from the chart, as the $+1$ D lenses would practically lower the zero horizontal line one step, so that when exercising 2 D of accommodation she would have to exert her utmost limit of convergence to see

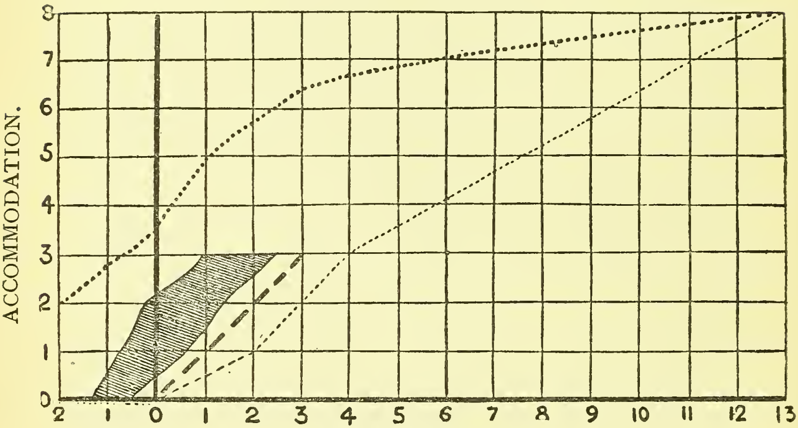

Fig. 12.

at $\frac{1}{3}$ metre. Clearly -1 D would have been much more serviceable. I ordered her simple abducting prisms ($-1°$ d.) for constant use. These have been quite satisfactory in relieving her of pain, and, what is perhaps better evidence of their benefit, the puffiness of the lids and the conjunctivitis have disappeared and shown no tendency to recur. The abducting prisms ordered, just brought the diagonal within the shaded area of comfort.

(2). The next chart (*Fig.* 13) is that of a man, aged 29, with about $\cdot 5$ D of hypermetropia, who had a very contracted range of convergence. At first, finding that he had 7 D of accommodation, I thought that reading glasses ought to be quite unnecessary, although he had obtained some relief from wearing + 1 D reading glasses. The chart explains this observation, for on lowering the zero horizontal line one step, it is seen that the summit of the diagonal would be in the centre of the area of comfort, being at the point of intersection of 2 of accommodation and 3 of convergence. This case undoubtedly ought to have been given exercising prisms to increase his convergence range, but as he absolutely refused

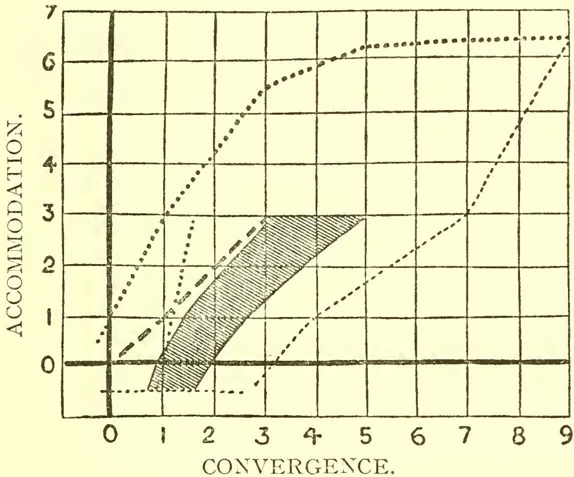

Fig. 13.

to undertake orthoptic exercises I gave him adducting prisms of 1° d. associated with the correction of his refractive errors. The result has been completely satisfactory ; although I feel sure that, with exercise, it would have been unnecessary for him to wear glasses at all. It is clear from the chart that if immediate relief were required, adducting prisms of 1 m.a. (+1° 43′) should have been ordered, but part of the esophoria was intentionally left to be corrected by the patient's own efforts.

When tested with a Maddox rod, the line of repose almost exactly bisected the area of comfort with the test light held at different distances, from 6 metres to $\frac{1}{3}$ metre ; when, however, the test light was held at 6 metres and his accommodation excited by concave glasses, the line of repose occupied the position shown by the nearly vertical dotted line from 1 m.a. to 1·5 m.a., showing the very great influence that the estimate of distance had upon this subject, though in my own case its effect is hardly noticeable.

(3). The next chart (*Fig.* 14) is very peculiar ; it is that of a lady of pronounced neurotic type, who had been subject to headaches all her life, true migraine and occasional attacks of giddiness. These attacks came on when she looked at

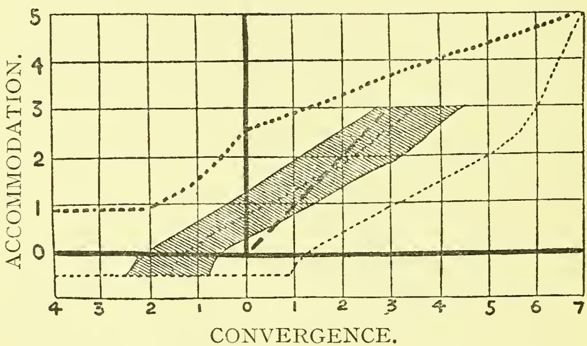

Fig. 14.

distant objects quite as much as when she attempted to do needlework. Her error of refraction was R + ·25 D sph. + ·5 D cyl. ax. 130°. L + ·5 D sph. + ·5 D cyl. ax. 10°.

She had visited many distinguished specialists, who had all found practically the same error of refraction. As will be seen from her chart, she had 5 D of accommodation, which corresponded with her age, 39. Her range of convergence was from − 4 m.a. to 7 m.a., and it is seen that, with glasses, the diagonal is outside the area of comfort for distance ; while it is only just within this area at reading distance. Clearly an advancement of her internal recti, while correcting her exophoria for distance, would give her most troublesome esophoria for reading. The line of repose almost bisected the area of comfort, showing exophoria for distance and esophoria for reading.

I ordered abducting prisms of $1^\circ\ 30'$ to be added to her distance glasses, and + 1 D added to the distance correction (but without prisms) for near work. It will be seen that these glasses brought both extremities of the diagonal to nearly the middle of the area of comfort, the upper extremity being lowered one step, while the lower one was displaced backwards 1 m.a. The result of this prescription was quite satisfactory, as she was able to continue reading and working without fatigue, and was almost entirely free from her old headaches.

All the above cases were entirely free from any trace of hyperphoria, and I submit that they afford good evidence in support of my contention as to the definition of the *area of comfort*. It will be seen that the directions given on *p.* 96 are really only another way of expressing what is more clearly and graphically shown in these charts. Many will find it more convenient in practice to use a graphic method. For instance, in the case given on *p.* 94 where $R_c = -1$ to $+7$ for distance, they might draw a line 8 cm. long and divide it into three equal parts, then the middle third will represent the area of comfort. On now marking the point which indicates parallelism with 0, the prism required will be that denoted by the distance between the figure 0 and the beginning of the area of comfort. In this case, as 0 is 1 centimetre and the area of comfort begins at $2\frac{2}{3}$ cm., from the commencement of the line, the distance between these two points is $1\frac{2}{3}$ cm.: consequently, prisms of $1^\circ\ 40'$ (edges in) are given to the patient to wear for twenty minutes as a preliminary trial.

In conclusion, I would say that although few patients will require such a complete examination as is here suggested, yet it is well to investigate the relationship of these functions of convergence and accommodation whenever symptoms still persist after the correction of any refractive errors and hyperphoria that may exist.

Orthoscopic Lenses.—Dr. Scheffler has suggested the use of prismosphores that should relieve exactly the same amount of convergence and accommodation; such glasses he called orthoscopic.

Thus a patient of 50 requiring + 2 D glasses for reading would be given prismosphores of + 2 D + 2 m.a., i.e., prisms of 2 m a. edges outwards combined with the + 2 D lenses.

Now we know that as age advances, it is only the amplitude of accommodation that diminishes, while the amplitude of convergence remains unchanged, so it is evident that orthoscopic lenses are based on a wrong principle. A patient having only 3 D of accommodation will have to exert as much effort to maintain 1 D, as a patient with 9 D of accommodation would exert to maintain 3 D. In each case the relative range of convergence would normally be the same, about 7·5 m.a.

The following are the various purposes for which prisms may be ordered for patients.

1. *Exercising* prisms in cases of paresis, and when the near point of convergence is too distant.

2. *Relieving* prisms in cases of concomitant heterophoria, and in paretic cases that do not improve with exercise.

3. *Cosmetic* prisms when a blind eye deviates. It is sometimes useful to order for, say, a slight outward deviation of a blind eye, a prism, edge in, as by that means the deviation of the eye is concealed.

CYCLOPHORIA.

Very occasionally it will be found that symptoms arise from the torsion of one or both eyes about an antero-posterior axis. The condition is very readily recognized with a double prism. Two prisms of 2° d. united at their bases, are held before one eye, so that the horizontal line of junction bisects the pupil. When a horizontal line is viewed through this double prism, two parallel lines are of course seen by this eye, and the horizontal line in its normal position,—that is between the previous two parallel lines,—is seen by the other eye. In cases of cyclophoria, the middle line seen by the naked eye will appear tilted and no longer parallel to the other two lines. Of course, the eye will be twisted in the reverse direction to the line.

A slight amount of cyclophoria is often found in near vision, but it rarely occasions any symptoms. Dr. Savage is perhaps the greatest authority on this difficult subject ; I would refer the reader to his papers and to Dr. Maddox's *Ocular Muscles.*

As to treatment, little can be done, but Dr. Duane's suggestion sounds eminently reasonable : To use two of Maddox's rods, one before each eye, and to gradually rotate them in opposite directions while endeavouring to keep the line of light from doubling. This will exercise the muscles that tend to twist the eyes about their antero-posterior axis.

In some rare cases of oblique astigmatism, it will be found that the axis of the cylinder must be set at one angle for near vision and at a different angle for distant vision, or troublesome symptoms of cyclophoria will occur. In such cases two pairs of spectacles must be ordered, one for reading and the other for distance.

CHAPTER IV.

OPTICAL SECTION.

A SOUND and thorough knowledge of elementary optics is absolutely necessary for the explanation and comprehension of the rules and formulæ given in the preceding chapters, and for the solution of the numerous other problems that may present themselves to the practitioner. As the standard works on optics will hardly meet his requirements, I have thought it well to give a brief résumé of the principal problems that arise, and the method of their solution.

I shall assume in this section that the reader is familiar with the elements of the subject.

1. The meaning of the algebraic notation of signs. In this section the direction of the incident light is considered positive, and in these diagrams this direction will be from cornea to retina. Similarly, lines measured from below upwards will be considered positive. Lines in the opposite direction will be regarded as negative.

The distance PH measured from the object P to a point H (in a lens or mirror) is positive, and is denoted by p. Similarly QH is measured from the image Q to the point H and is denoted by q.

2. The first principal focus F_1 is the point from which incident rays must originate in order that, after refraction through the medium, they may emerge parallel. The point F_2 is the second principal focus towards which incident parallel rays converge after refraction through the medium. The first focal distance F' or $F_1 H$ and the second focal distance F'' or $F_2 H$ are both measured *towards* the point H.

3. The index of refraction $\mu = \frac{V_1}{V_2}$ is the ratio of the velocities of light in the first and the second medium. From this it follows that, as light after reflection is travelling in the same medium but in the reverse direction, its velocity must be the same in absolute amount, but it must have a negative value. Hence the expression

$$\mu = \frac{V_1}{V_2} \text{ becomes in reflection } \frac{V_1}{-V_1} = -1.$$

Consequently, all the optical formulæ relating to refraction at a single spherical surface can be converted into those of reflection at a mirror by giving to μ the value -1.

For instance, $\mu = \frac{\sin\phi}{\sin\phi'}$ becomes in the case of reflection $-1 = \frac{\sin\phi}{\sin\phi'}$, or $\phi = -\phi'$, which shows that the angle of reflection ϕ' is measured in the reverse direction from the normal to that of the angle of incidence ϕ.

Again, for $\frac{\mu}{q} - \frac{1}{p} = \frac{\mu-1}{r}$ and $F'' = -\mu F'$ we have in reflection $\frac{-1}{q} - \frac{1}{p} = \frac{-1-1}{r}$ or $\frac{1}{p} + \frac{1}{q} = \frac{2}{r}$, and $F'' = F'$.

The formulæ for eccentric pencils incident on a single refracting surface are

$$\frac{\mu\cos^2\phi'}{V_1} - \frac{\cos^2\phi}{U} = \frac{\mu}{V_2} - \frac{1}{U} = \frac{\sin(\phi-\phi')}{r\sin\phi'}$$

where U, V_1, and V_2 represent the distances of the eccentric portions of the spherical surface from the source of light U, from the first focal line V_1, and from the second focal line V_2. The corresponding formulæ for reflection are obtained from this expression by substituting -1 for μ and $-\phi$ for ϕ'.

$$\text{e.g.,}\quad \frac{-\cos^2\phi}{V_1} - \frac{\cos^2\phi}{U} = \frac{-1}{V_2} - \frac{1}{U} = \frac{\sin 2\phi}{-r\sin\phi} = \frac{2\sin\phi\cos\phi}{-r\sin\phi}$$

$$\text{or}\quad \frac{1}{V_1} + \frac{1}{U} = \frac{2}{r\cos\phi} \text{ and } \frac{1}{V_2} + \frac{1}{U} = \frac{2\cos\phi}{r}$$

These illustrations are merely given to show that, if the refraction formulæ are known, the reflection formulæ can be immediately deduced from them.

4. I also assume that the reader knows that in the case of a lens $F' = - F''$, and that in all cases, whether of refraction or reflection, $\frac{F'}{p} + \frac{F''}{q} = 1$. It is at once apparent that this becomes $\frac{F}{p} + \frac{F}{q} = 1$ or $\frac{1}{p} + \frac{1}{q} = \frac{1}{F}$ when reflection is considered, and that when we are considering refraction in lenses the expression is identical with $\frac{1}{p} - \frac{1}{q} = \frac{1}{F'}$ or $= -\frac{1}{F''}$. Further, the relation of the height of an image to the height of an object is given in all cases by the formula $\frac{i}{o} = \frac{F'}{F'-p} = \frac{F''-q}{F''}$.

Cardinal Points.—Gauss has shown us how to extend the use of these formulæ to any refracting system, formed of any number of media bounded by centred spherical surfaces, by finding the position of two points called principal points, towards which the distances of all the other points are measured. In a thick double convex lens, the two principal points H_1 and H_2 are situated on the axis of the lens within its substance. For instance, by applying Gauss's method we find the first principal point H_1 is 2·308 mm. behind the anterior surface of the lens of the eye, and the second principal point H_2 is 1·385 mm. in front of the posterior surface of the lens. (*See Fig.* 15). The focal length ϕ of the lens *in situ* is 51·34 mm.; we will denote by ϕ' the distance of H_1 from the first principal focus, and by ϕ'' the distance of H_2 from the second principal focus, and as the refractive index of the aqueous and vitreous humour is practically the same, we know that $\phi' = - \phi''$. As an example of the way in which the cardinal points of a complex system may be determined, we will take that furnished by the eye.

Let f' denote the distance from the first (or anterior) focus of the cornea to its apex A_0, ; similarly, let f'' denote the distance of A_0 from the second or posterior focus. The distance from the object P to A_0, or PA_0, is represented by p, and the distance from the image Q to A_0, or QA_0, is q_1. For the first or corneal refraction

$$\frac{f'}{p} + \frac{f''}{q_1} = 1 \quad \therefore \; q_1 = \frac{pf''}{p-f'}.$$

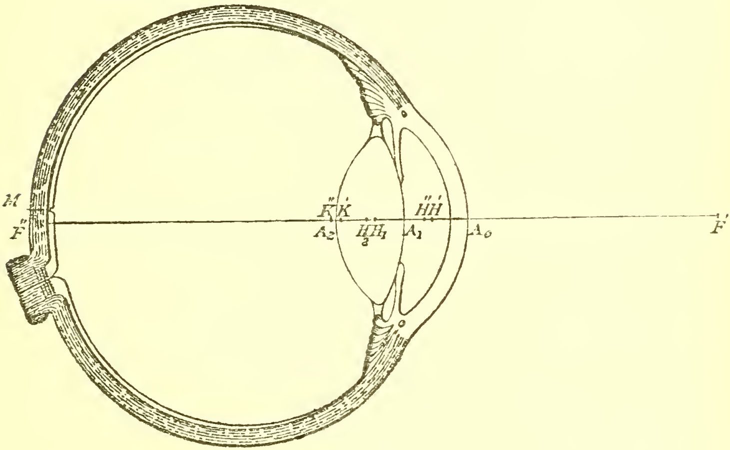

Fig. 15.

Now this image at Q serves as the object for the lens of the eye ; let the distance QH_1 be denoted by p'. Then if $A_0H_1 = t$, $p' = q_1 + t$, for $QH_1 = QA_0 + A_0H_1$.

Now, for the second or lenticular refraction we have

$$\frac{\phi'}{p'} + \frac{\phi''}{q} = 1 \quad \therefore \quad p' = \frac{q\phi'}{q-\phi''}.$$

But $p' = q_1 + t = \frac{pf''}{p-f'} + t \quad \therefore \quad \frac{q\phi'}{q-\phi''} = \frac{pf''}{p-f'} + t,$

or $pq(t+f''-\phi')-p(f''\phi''+t\phi'')+q(f'\phi'-tf')+tf'\phi''=0$ (1).

It is required to find the positions of the points H′ and H″, the first and second principal points of the system, so that $\frac{F'H'}{PH'} + \frac{F''H''}{QH''} = 1$ when F′ and F″ are the first and second foci of the system.

Let $H'A_0$, the distance between the first principal point of the system and that of the first spherical surface, be denoted by h', and let h'' denote the distance $H''H_2$ between the second principal point of the system and the second principal point of the lens.

Then $PH' = PA_0 + A_0H' = PA_0 - H'A_0 = p - h'$.
and $QH'' = QH_2 + H_2H'' = QH_2 - H''H_2 = q - h''$.
and let F′H′ and F″H″ be denoted by F' and F''. It is required then to find the values of h', h'', F' and F'' in the expression $\frac{F'}{p-h'} + \frac{F''}{q-h''} = 1$, so that it shall be identical with equation (1), i.e.,

$$pq - p\,(h'' + F'') - q\,(h' + F') + h'h'' + F'h'' + F''h' = 0$$

is identical with

$$pq\,(t + f'' - \phi') - p\,(f''\phi'' + t\phi'') + q\,(f'\phi' - tf') + tf'\phi'' = 0.$$

By comparing coefficients we find that this is the case

$$\text{if } t + f'' - \phi' = \frac{f''\phi'' + t\phi''}{h'' + F''} = \frac{tf' - f'\phi'}{h' + F'} = \frac{t\,f'\phi''}{h'h'' + F'h'' + F''h'},$$

or calling this expression D

$$\text{if } h'' + F'' = \frac{f''\phi'' + t\phi''}{D}, \text{ if } h' + F' = \frac{tf' - f'\phi'}{D},$$

$$\text{and if } h'\,h'' + F'\,h'' + F''\,h' = \frac{t\,f'\phi''}{D},$$

$$\text{i.e., if } h' = \frac{t\,f'}{D}, \text{ if } h'' = \frac{t\phi''}{D}, \text{ if } F' = \frac{-f'\phi'}{D} \text{ and if } F'' = \frac{f''\phi''}{D}.$$

These values are clearly consistent, and they are therefore the solutions required. In the case of the eye t or $A_0H_1 = 5{\cdot}908$ mm., $f' = 23{\cdot}11$ mm., $f'' = -\ 30{\cdot}91$ mm., and we have given above 51·34 mm. as the value of ϕ'.

$\therefore$ D $= -$ 76·342 mm., and consequently
h' or $H'A_0 = -1{\cdot}789$ mm., h'' or $H''H_2 = 3{\cdot}973$ mm.
$F' = 15{\cdot}54$ mm. and $F'' = -20{\cdot}79$ mm.

In the diagram (*Fig.* 15) the positions of the cardinal points of the eye are clearly indicated.

As we shall in future always measure the distances of the foci, of the object and of the image to their respective principal points in every complex system, we get the formula $\frac{F'}{p} + \frac{F''}{q} = 1$ universally true. The following table gives the values of the ocular constants according to Dr. Tscherning's latest researches. The figures are taken from the *Encyclopédie francaise d'Ophthalmologie*; it will be noticed that they differ considerably from the table of constants that he first published, which referred to one eye that he most carefully measured, but which unfortunately was an abnormal eye. The figures here given represent as closely as possible the values of the constants in an average emmetropic eye.

TSCHERNING'S CONSTANTS.

CORNEA	Radius of curvature	−7·8 mm.
	Index of refraction of aqueous and vitreous humours	1·3375
	Anterior focal distance $f' =$	23·11 mm.
	Posterior focal distance $f'' =$	−30·91 mm.
	Depth of the anterior chamber ..	3·6 mm.
LENS	Thickness	3·9 mm.
	Radius of anterior surface ..	−10 mm.
	Radius of posterior surface	6 mm.
	Index of refraction with reference to surrounding media	1·0743
	Distance of 1st principal point from anterior surface or A_1H_1	2·308 mm.
	Distance of 2nd principal point from anterior surface or A_1H_2	2·515 mm.
	Focal distance	51·34 mm.
	Power	19·5 D

Reference to the diagram (*Fig.* 15) will make the letters used in this and the next table quite clear. A_1 and A_2 denote the anterior and posterior surfaces of the lens, H_1 and H_2 are the principal points of the lens. H' and H'' are the principal points, and K' and K'' are the nodal points of the combined system.

When the lens has been removed, that is in an aphakic eye, f' and f'' represent the first and second focal distances of the eye respectively.

Cardinal Points of the Average Emmetropic Eye.

$H'A_0$ ..	− 1·789 mm.	$H'H''$ ..	·353 mm.
$H''A_0$ ..	− 2·142 mm.	$K'K''$ ..	
$K'A_0$ ..	− 7·04 mm.	$H'K'$ ..	5·25 mm.
$K''A_0$ ..	− 7·39 mm.	$H''K''$ ..	
$F'A_0$ ..	13·75 mm.	$F'H'$ ..	15·54 mm.
..		$K''F''$ ..	
$F''A_0$ ..	− 22·93 mm.	$F''H''$ ..	− 20·79 mm.
		$K''F'$ ...	

Donders' Eye.—It will be noticed that the two principal points and the two nodal points lie very close together, so that without introducing much error we may regard them as coinciding in one point H and in one point K. On this principle Donders has given us a simplified schematic eye consisting of a single refracting medium ($\mu = \frac{4}{3}$) presenting a single spherical surface, the radius of which = − 5 mm. The principal point H is at the vertex of the spherical surface, and the nodal point K is at its centre.

The first principal focus $F'H$ of this reduced eye is 15 mm., and the second principal focus $F''H$ is − 20 mm.

The simplicity of Donders' eye makes it exceedingly convenient for all rough calculations, and the error introduced by using it is never large.

When we want to draw the image formed by a complex system, we must first indicate the position of the four cardinal points just found, and through them draw lines at right angles to the axis to represent the principal planes and the focal planes. The diagram (*Fig.* 16) represents these planes in the eye. Now the directions of an incident ray and its corresponding emergent ray cut the two principal planes in two points situated on the same side and at the same distance from the axis.

Therefore if S be a luminous point in the first focal plane, and SJ_1 be one of its rays, viz., that parallel to the axis, it must proceed to J_2 on the second principal plane, and then be deflected towards F''. Similarly any other ray

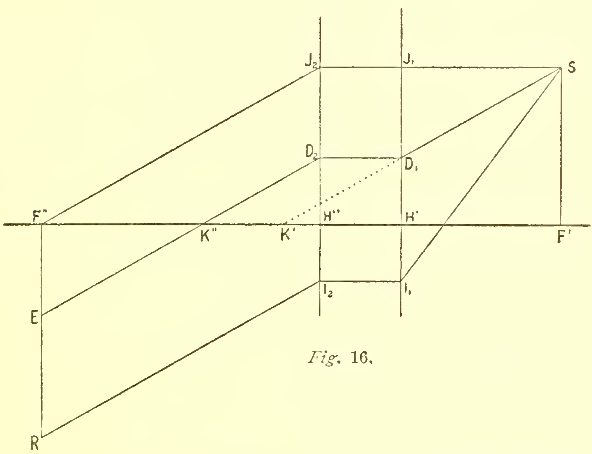

Fig. 16.

such as SI_1 must pass to I_2 and then emerge parallel to J_2F''.

The Nodal Points.—These are two most important points, added by Listing to the cardinal points of a refracting system. In order to find them we have merely to draw from S a line SD_1K' parallel to J_2F'', and then from D_2 the point corresponding to D_1 draw $D_2K''E$ parallel to J_2F''. Then K' and K'', the two points on the axis, are the nodal points. For an incident ray SK' emerges after traversing the system in the direction K″E which is parallel to S K′.

As the sides of the $\triangle$ F′SK′ are parallel to the sides of the $\triangle$ H″J$_2$F″, and as F′S = H″J$_2$, $\therefore$ K′F′ = F″H″ or F''. Also the $\triangle$ EF″K″ = $\triangle$ D$_1$J$_1$S, as the sides are parallel and as EF″ = D$_2$J$_2$ = D$_1$J$_1$, $\therefore$ K″F″ = SJ$_1$ = F′H′ or F'. It is also clear that K″K′ = D$_2$D$_1$ = H″H′.

In every system in which the initial and the final media have the same refractive index (i.e., when F″H″ = H′F′), the nodal points K′ K″ coincide with the principal points H′H″.

Fig. 17 shows how useful these points are. To find the image of AB, we join AK′ and draw K″*a* parallel to it, and then either draw AJ″ parallel to the axis and J″F″*a* through F″, or we draw AF′I$_1$ through F′ and draw I$_1$I$_2$*a* parallel to the axis. In either case *a* is the point of intersection of the line with that through the nodal point.

It is easily seen that $\frac{i}{o}$ or $\frac{ab}{\mathrm{AB}} = \frac{b\mathrm{K}''}{\mathrm{BK}'}$

The distances bK″ and BK′ are usually denoted by g'' and g', so that $\frac{i}{o} = \frac{g''}{g'}$.

Again $\frac{i}{o}$ or $\frac{ab}{\mathrm{J''H''}} = \frac{\mathrm{F''}b}{\mathrm{F''H''}} = \frac{\mathrm{F''H''} - b\mathrm{H''}}{\mathrm{F''H''}} = \frac{F'' - q}{F''}$;

and also $\frac{i}{o} = \frac{\mathrm{I'H'}}{\mathrm{AB}} = \frac{\mathrm{F'H'}}{\mathrm{F'B}} = \frac{\mathrm{F'H'}}{\mathrm{F'H'} - \mathrm{BH'}} = \frac{F'}{F' - p}$.

Note that the expressions on the right-hand side being negative in this case, show that the image *i* is inverted.

Since $\frac{i}{o} = \frac{F'}{F' - p} = \frac{F'' - q}{F''}$, $(F' - p)\,(F'' - q) = F'F''$ (1)

i.e., $F''F'' - F'q - F''p + pq = F'F''$.

or $F'q + F''p = pq$ i.e., $\frac{F'}{p} + \frac{F''}{q} = 1$.

Correction of Ametropia.—We must consider in

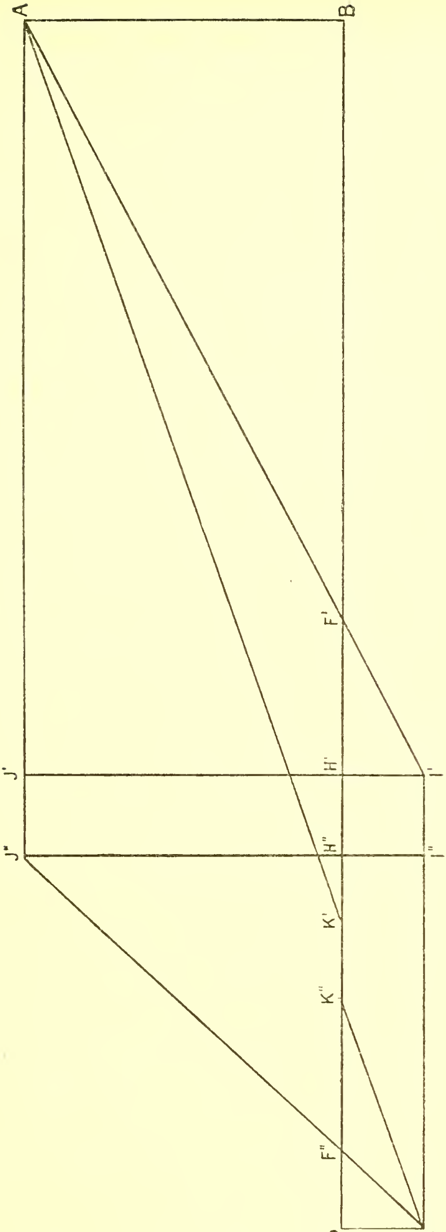

Fig. 17.

detail the common or axial form of ametropia and the means that we have for its correction.

Let us take the case of a myopic eye l mm. too long, i.e., its retina is placed l mm. behind the principal focus of the eye, say at Q. (*See Fig.* 18.)

Let q or QH″ denote the distance of the second principal point (H″) of the eye from the retina, and let the focal distances F′H′, F″H″ be denoted as usual by F' and F''. Then $q - F''$ is a negative quantity denoting the distance (QF″) of F'' from the retina

$$\therefore\ q - F'' = -l \ \text{(or} - \text{F}''\text{Q)}.$$

Let P denote the *punctum remotum*, light from P will, on entering the eye, converge to a focus on the retina at Q. In order that distinct images of distant objects may be formed on the retina, a lens must be placed somewhere (between P and the cornea), such that incident parallel rays, after traversing the lens, diverge as if they had come from P. It will be shown presently that there is a peculiar advantage in placing the lens in the first focal plane of the eye, i.e., about half an inch in front of the cornea.

Let p = PH′. Now if the lens be placed in the plane at F′, the second focal distance (f'') of the lens must be PF′ or PH′ − F′H′, i.e., $p - F'$ or $f'' = p - F'$. But from (1) (*p.* 118) we know that $(p - F')\,(q - F'') = F'F''$.

$$\therefore\ p - F' = \frac{F'F''}{q - F''} \text{ or } f'' = \frac{F'F''}{-l}.$$

On substituting the values given in the table (*p.* 116) for $F'F''$, we get $f'' = \dfrac{(15 \cdot 54)\,(-20 \cdot 79)}{-l} = \dfrac{323}{l}$ mm.

Now the power of the lens in dioptres is $\dfrac{1}{f''}$ in metres or $-\dfrac{1000}{f''}$ in millimetres. $\therefore$ D $= \dfrac{-l}{\cdot 323}$ or $l = -\cdot 323$ D.

Examples.—(1) Required, the lens to correct an eye which is 3·23 mm. too long.

Here $l = 3{\cdot}23 \quad \therefore \ D = \frac{-3{\cdot}23}{{\cdot}323} = -10.$

(2) Required, the length of an eye which needs $+ 4$ D for its correction.

$$l = -{\cdot}323\,\mathrm{D} = -{\cdot}323 \times 4 = -1{\cdot}292 \text{ mm.}$$

The eye is therefore too short by 1·292 mm., i.e., it is 22·93 — 1·3 or 21·63 mm. long, presuming, of course, that the ametropia is due to an axial defect solely.

This is the principle on which all differences of level are measured with the ophthalmoscope. Note that it is absolutely necessary the instrument should be held in the first focal plane of the eye, and then each dioptre used corresponds to a difference of ·323 mm.

Aphakia.—When the lens of the eye is removed, it is reduced to a simple refracting system with one principal point on the cornea, and $F' = 23{\cdot}11$ mm. while $F'' = -30{\cdot}91$ mm.

Consequently in aphakia $f'' = \frac{F'F''}{-l} = \frac{(23{\cdot}11)\,(-30{\cdot}91)}{-l} = \frac{714}{l}$,

$$\therefore \ \mathrm{D} \text{ or } -\frac{1000}{f''} = \frac{-l}{{\cdot}714}, \text{ and } l = -{\cdot}714 \text{ D.}$$

When using this formula, the lens must be placed in the first focal plane of the aphakic eye, i.e., 23·11 mm. (nearly an inch) from the cornea. When the ophthalmoscope is held in this position, we can apply this formula for estimating differences of level in the fundus. The previous formula leads to totally incorrect results.

For instance, a detached retina that required $+ 4$ D more than the disc would be raised nearly 3 mm.

$$\text{for } l = -{\cdot}714 \times 4 = -2{\cdot}856 \text{ mm.}$$

This formula is useless in determining the glass required to correct aphakia, for spectacles are never worn an inch from the eyes. We must therefore find a more correct expression.

In aphakia we will assume that the correcting glasses are worn in the usual place, i.e., 13·75 mm. in front of the cornea; if we denote this distance by d, the focus of the correcting lens as on *p*. 120 must be $f'' = p - d$, and when D′ represents its power in dioptres

$$D' = -\frac{1000}{p-d} \text{ or } \frac{1000}{d-p} \text{ and } \frac{F'}{p} = 1 - \frac{F''}{q} = \frac{q-F''}{q}.$$

$$\therefore \; p = \frac{F'q}{q-F''} \qquad \therefore \; D' = \frac{1000\,(q-F'')}{d\,(q-F'')-F'q}.$$

Now in *emmetropia* the distance of the cornea from the retina or $q = -22·93$ mm. and F'' in aphakia is $-30·91$, ∴ in emmetropia $q - F'' = 7·98$ mm. Let us call it the constant C.

Then in emmetropia

$$D' = \frac{1000\,C}{d\,C - F'q} = \frac{7980}{(13·75)\,(7·98) + (23·11)\,(22·93)},$$

$$\therefore \; D' = \frac{7980}{109·7 + 529·9} \backsimeq 12·5.$$

This is a higher value than that usually given for the glass required after removing the lens, but we must remember that according to Donders there is about 1 D of acquired hypermetropia at the age of 67.

In axial ametropia due to the eye being l mm. longer than the normal length (i.e., 22·93 mm.), and remembering that $l = -·323$ D where D was the old correcting glass,

$$D' = \frac{1000\,(C-l)}{d\,(C-l) - F'\,(q-l)} = \frac{7980 + 323\,D}{109·7 + 529·9 - ·323\,D\,(23·11 - 13·75)},$$

$$D' = \frac{7980 + 323\,D}{639·6 - 3·023\,D} = \frac{24·958 + 1·01\,D}{2 - ·00945\,D} \backsimeq \frac{25 + D}{2 - ·01\,D}.$$

Examples.—(1) A myope corrected by — 15 D has his lens removed. Find his approximate correcting glass after the operation, assuming that the curvature of his cornea is unchanged.

$$D' \backsimeq \frac{25 - 15}{2 + ·15} = \frac{10}{2·15} \backsimeq + 4·65\text{ D}.$$

(2) A hypermetrope of $+ 6$ D after operating for cataract will require $D' \backsim \dfrac{25 + 6}{2 - {\cdot}06} = \dfrac{31}{1{\cdot}94} \backsim + 16$ D.

Correcting Lens in First Focal Plane.—The advantage that is gained by placing the correcting lens in the first focal plane of the eye is that, when so placed, the retinal image of a distant object formed by the combined system is of precisely the same size as that which would be formed by an emmetropic eye. This renders all tests of visual acuteness strictly comparable to each other, whatever the correction required, provided

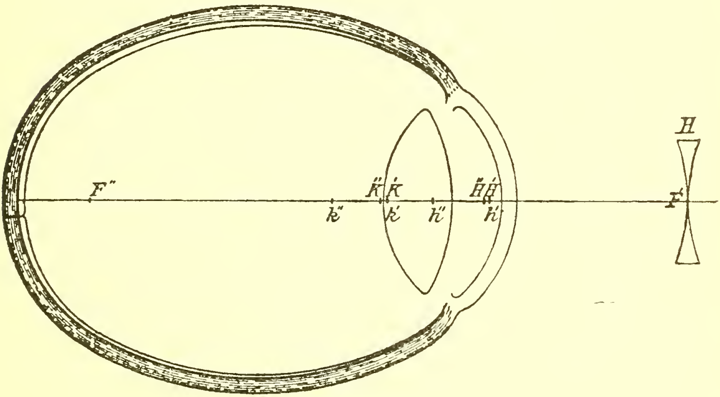

Fig. 18.

always that the ametropia is axial. Consider an eye l mm. too long (*Fig.* 18), with the thin correcting lens H of focus f in the first focal plane F′. Then if h', h'' be the two principal points of the complete system, we know from what has been said about cardinal points that to find h'

$$h'\mathrm{H} = \frac{f'\,(\mathrm{HH}')}{\mathrm{HH}' + f'' - F'} = \frac{f'F'}{F' + f'' - F'} = -\,\mathrm{F}',$$

$\therefore$ the first principal point h' of the system coincides with the first principal point H′ of the eye.

Also $h''\text{H}'' = \dfrac{F''\,(\text{HH}')}{\text{HH}' + f'' - F'} = \dfrac{F''F'}{F' + f'' - F'} = \dfrac{F''F'}{f''} = -l$ (*p.* 120).

The negative sign shows that the second principal point h'' is situated l mm. on the retinal side of the point H″.

It will be found that the first principal focal distance of the system is $\dfrac{-f'F'}{f''} = F'$ and the second principal focal distance is $\dfrac{f''F''}{f''} = F''$.

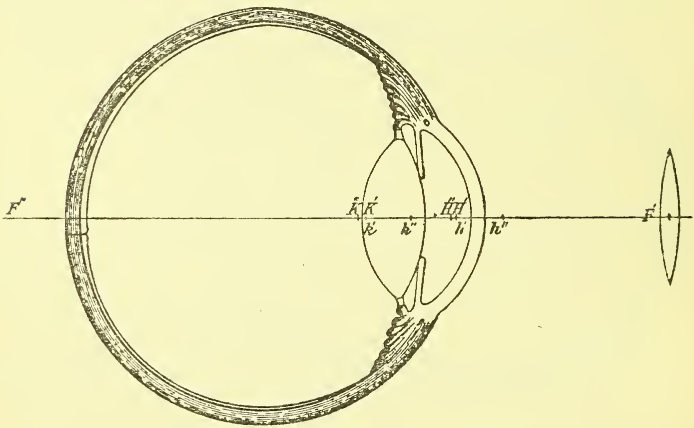

Fig. 19.

The principal focal distances of the system have therefore precisely the same values as they have in an emmetropic eye, whereas the position of the second principal point and of the second nodal point is changed, as is indicated by the letters below the optic axis of the figure.

Similarly in axial hypermetropia (*Fig.* 19), the addition of the convex correction in F′ causes no alteration in the position of the first principal and focal points, but h'' is displaced l mm. forwards, when the eye is l mm. too short, which places the second focus on the retina.

Aphakia.—We have seen that after an operation for cataract the correcting lens is no longer placed in the first focal plane of the aphakic eye, and consequently when F'' is placed on the retina, all the other cardinal points have their position changed. In a previously emmetropic eye, for instance, the second principal point h'' will be nearly 4·75 mm. in front of the cornea, while h' will be 12·31 mm. behind the lens, or 1·44 mm. in front of the cornea, and $F_a' = 20{\cdot}69$ mm., while $F_a'' = -27{\cdot}68$ mm.

Size of the Image.

1. *In Corrected Ametropia.*—Clearly, as $\frac{i}{o} = \frac{F'}{F' - p}$, and as no change has been made in the values of p or F', the size of the image in axial ametropia, which has been corrected by a glass in the first focal plane, is the same as that in an emmetropic eye, for $\frac{i}{i'} = \frac{F'}{F'} = 1$; hence the tests for visual acuteness are comparable.

2. *In Corrected Aphakia* the size of the image has practically been increased in the ratio

$$\frac{F_a'}{F'} = \frac{20{\cdot}69}{15{\cdot}54} \backsimeq 1{\cdot}33.$$

Hence the image in corrected aphakia is about one-third larger than in an emmetropic eye. This explains why it is inadvisable to remove a cataract from one eye, if the sight of the other eye be good, for the unfortunate patient will be unable to fuse two images of such different sizes.

Further, the tests for visual acuteness are no longer comparable to the results in emmetropia. Indeed, the test type result of $\frac{6}{9}$ in aphakia would only correspond to a visual acuteness of $\frac{6}{12}$ in emmetropia.

Position of the Correcting Lens.—The effect of a lens on the eye will vary according to its distance. We have discussed the effect of a lens placed in the usual position (at F′); we must now consider its effect when its position is changed.

Myopia.—Let P denote the punctum remotum, and let PF′ $= p$, that is the distance from P to the first focal plane of the eye, or the usual position of the lens. Then, if d or LF′ denote the distance that the lens L is removed from its usual position, PL or $p - d = f''$ or $-f'$, the focal length of the lens required for *distance*.

For instance, using Donders' eye for simplicity, consider a myope of -10 D for whom $p = 100$ mm. If the lens were removed 9·1 mm. away from his eye, it would be found that a stronger concave glass was required for his correction; for $-f' = p - d = 100 - 9{\cdot}1 = 90{\cdot}9$. But $D = \frac{1000}{f'}$ or $\frac{1000}{-90{\cdot}9}$ or -11 D. Therefore, removing a concave glass from the eye has the effect of diminishing its strength, and as it makes the value of F′ due to the combination smaller, it reduces the size of the image. Another way of expressing this fact, is to say that the posterior nodal point is displaced towards the retina. In the above case it will be easily found that the retinal image is about $\frac{10}{11}$ of its previous size.

For *near* vision the same reasoning applies, unless $d > p$. In such a case it will be found that the lens must be convex, and the image therefore inverted.

Suppose a myope of -20 D for whom $p = 50$ mm., and whose working distance (w) is 250 mm. from the first focal plane at F′. Let us find what lens would be required if held 150 mm. from F′.

Here $\frac{1}{w-d} - \frac{1}{p-d}$ or $\frac{1}{250-150} - \frac{1}{50-150} = \frac{1}{f'}$,

$$\frac{1}{f'} = \frac{1}{100} + \frac{1}{100} = \frac{1}{50}, \text{ or a lens of } +20 \text{ D.}$$

This is of more than theoretical interest, for some high myopes prefer to hold their books upside down, and to use a convex lens in this way to form an inverted image of the print.

Hypermetropia.—In hypermetropia p is always negative, and therefore $p - d$ is a negative quantity which increases with d.

For *distant* vision as before, $p - d = -f'$. Consequently f' must be a positive quantity, and D or $\frac{1000}{f'}$ must diminish as d is increased. In other words, removing a convex glass from the eye has the effect of increasing its strength when used for distance, and the effect of magnifying the image. For *near* vision it will be found that removing the glass has the effect of strengthening or weakening its power according as $p + w - d$ is positive or negative. For $\frac{1}{f'} = \frac{1}{w - d} - \frac{1}{p - d} = \frac{p - w}{pw - d(p + w - d)}$.

Here $p - w$ and pw must always be negative quantities; if $p + w - d$ is positive, the value of f' must be increased (i.e., the power D must be diminished) and *vice versa*.

Whenever $p + w - d$ is positive, removing a convex glass virtually increases its strength. (For *distant* vision when $w = \infty$, the expression is of course positive.)

The following examples will make this clear:—

1. Suppose a hypermetrope of $+ 4$ D ($p = - 250$ mm.), who has no power of accommodation, and whose working distance (w) is 250 mm.

With the correcting lens in the normal position,

$$\frac{1}{f'} = \frac{1}{w} - \frac{1}{p} = \frac{1}{250} - \frac{1}{-250} = \frac{1}{125},$$

$\therefore$ a $+ 8$ D lens will be needed.

If now the lens is removed 50 mm. ($d = 50$), obviously $p + w - d$ has a negative value, viz., $- 50$, and so the power of the lens must be increased to correct him.

$$\frac{1}{f'} = \frac{1}{w-d} - \frac{1}{p-d} = \frac{1}{200} - \frac{1}{-300} = \frac{5}{600},$$

$$\therefore \quad D = \frac{50}{6} = 8\cdot\dot{3}.$$

Consequently, removing the lens has the effect of diminishing its strength.

2. Let us now take the case of an aphakic needing + 11 D for distance ($p = -90\cdot9$) when the lens is in the usual place. On now moving the glass 21·3 mm. (about $\frac{5}{6}$ in.) away, $d = 21\cdot3$.

Then
$$\frac{1}{w-d} = \frac{1}{f'} + \frac{1}{p-d},$$

or
$$\frac{1}{w-21\cdot3} = \frac{1}{90\cdot9} + \frac{1}{-90\cdot9-21\cdot3} = \frac{21\cdot3}{10199},$$

$$\therefore \quad \frac{1}{w-21\cdot3} = \frac{1}{478\cdot6} \quad \therefore \quad w = 478\cdot8+21\cdot3 = 500\cdot1 \text{ mm.}$$

Consequently, by removing the + 11 D lens 21·3 mm., the eye has been adapted for $\frac{1}{2}$ metre, or the lens has now the effect of a + 13 D.

In this way aphakics requiring strong convex glasses can alter the "focus of their glasses" by a slight movement of the frames. Note that in this case $p + w - d$ is positive, being $-90\cdot9 + 500\cdot1 - 21\cdot3$, or $+387\cdot9$.

Patients who are wearing convex glasses that are too weak for them, are often noticed reading with their spectacles at the end of their nose. A popular but, as we have seen, an erroneous explanation of this habit is that by so doing they virtually increase the strength of the glass. This cannot occur unless $p + w - d$ is positive; practically this effect is not obtained with glasses under the strength of + 6 D or so. The true explanation is, that by wearing their glasses in this way they get larger though more indistinct retinal images.

Tilting of the Lens.—If a spherical lens be so inclined that the plane of the glass makes an angle with the incident wave front of light, the refracted pencil is astigmatic, so that when the incident pencil is centric it is refracted as though through a spherocylinder. As the pupil only allows a very narrow pencil to enter the eye, this property of tilted lenses is often of service to those patients who cannot afford spherocylinders. The formula for oblique centric pencils through a lens is the following:

$$\frac{\cos^2\phi}{V_1} - \frac{\cos^2\phi}{U} = \frac{1}{V_2} - \frac{1}{U} = \frac{\sin(\phi-\phi')}{\sin\phi'}\left(\frac{1}{R_1} - \frac{1}{R_2}\right),$$

where U, V_1, and V_2 represent the distances of the principal point of the lens from the source of light, the first focal line, and the second focal line. When the correction is to be used for distance, $U = \infty$, and this formula suffices:

$$\frac{\cos^2\phi}{V_1} = \frac{1}{V_2} = \frac{\sin(\phi-\phi')}{\sin\phi'}\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

Or, since $\overline{\mu - 1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f''} = \frac{-D}{1000}$ the expression in dioptres will be $\cos^2\phi\, D_1 = D_2 = \frac{\sin(\phi-\phi')}{\sin\phi'} \cdot \frac{D}{\mu-1}$.

where D_1 and D_2 represent the power in the two meridans of the lens D tilted the angle ϕ round the axis of its least refraction. Since this is equivalent to a spherical glass of dioptric strength D_2 combined with a cylindrical lens of dioptric strength D_c,

$$D_c = D_1 - D_2 = D_2\left(\frac{1-\cos^2\phi}{\cos^2\phi}\right) = D_2 \tan^2\phi.$$

We have then $D_2 = \frac{\sin(\phi-\phi')}{\sin\phi'}, \frac{D}{\mu-1}$, and $D_c = D_2 \tan^2\phi$, which are the simplest forms for calculation.

For instance, the value of D_2 in the case of a $+ 10$ D lens inclined at an angle of 30° can be calculated or found from the table on *p.* 51 to be 10·9497. Now $\tan 30° = \frac{1}{\sqrt{3}}$

$\therefore\ \tan^2 30° = \frac{1}{3} \qquad \therefore\ D_c = \frac{1}{3}(10{\cdot}9497) = 3{\cdot}6499.$

It will be noticed that the value of D_2 varies slightly with the value of μ, which on *p.* 51 is taken to be 1·52.

Two Cylinders with the Axes Asymmetrical.—It is sometimes required to find the effect of two cylinders in juxtaposition, the axes of which do not coincide. If two cylinders of + 4 D are placed together with their plane axes coinciding, the effect is of course that of a + 8 D cylinder; if their axes are at right angles to each other, the effect is that of a + 4 D spherical lens. When, however, the axes are at an angle of 30°, it will be found that the resulting effect is nearly equivalent to + ·54 D sph. + 6·93 D cyl. In this place I shall simply give the formulæ, as the method of obtaining them is rather long and troublesome.

Let α be the angle which one cylinder C_1 makes with the horizontal line, and let β be the angle which the other cylinder C_2 makes with the horizontal line, and let γ be the angle at which the resulting cylinder is to act, and let D be the power of the resulting spherical lens.

Then $C_1 \sin 2\alpha + C_2 \sin 2\beta = C_3 \sin 2\gamma,$ (1)

and $C_1 \cos 2\alpha + C_2 \cos 2\beta = C_3 \cos 2\gamma,$ (2)

$$\therefore \tan 2\gamma = \frac{C_1 \sin 2\alpha + C_2 \sin 2\beta}{C_1 \cos 2\alpha + C_2 \cos 2\beta}. \qquad (3)$$

Further, $C_3 = \dfrac{C_1 \sin 2\alpha + C_2 \sin 2\beta}{\sin 2\gamma},$ (4)

and $2D = C_1 + C_2 - C_3,$ (5)

where D is the power of the spherical equivalent of the combination.

Examples.—(1) What is the spherocylindrical equivalent of two + 4 D cylinders, one of which is horizontal and the other is at an angle of 30°?

Here $\alpha = 0$, and $2\beta = 60°$.

From (3) $\tan 2\gamma = \dfrac{C_2 \sin 60°}{C^1 + C_2 \cos 60°} = \dfrac{2\sqrt{3}}{4+2} = \dfrac{1}{3}\sqrt{3} = \tan 30°$

$$\therefore \gamma = 15°.$$

From (4) $C_3 = \dfrac{C_2 \sin 60^\circ}{\sin 30^\circ} = 4\sqrt{3} = 6{\cdot}9282$

and

$2D = C_1 + C_2 - C_3 = 8 - 6{\cdot}9282 = 1{\cdot}0718 \therefore D = {\cdot}5359.$

$\therefore$ the combination is equivalent to about + ·54 D sph. + 6·9 D cyl. axis 15°.

(2). How can a − 4·5 D cyl. be combined with a − 5·5 D cyl. so as to have the effect of a − 4 D sph. − 2 D cyl. axis horizontal?

Here $D = -4$, $C_1 = -4{\cdot}5$, $C_2 = -5{\cdot}5$, $C_3 = -2$, and $\gamma = 0$; it is required to find α and β.

Then from (1) $-4{\cdot}5 \sin 2\alpha = 5{\cdot}5 \sin 2\beta$

and from (2) $-4{\cdot}5 \cos 2\alpha = 5{\cdot}5 \cos 2\beta - 2$

Squaring and adding, $20{\cdot}25 = 30{\cdot}25 - 22 \cos 2\beta + 4,$

$\therefore \cos 2\beta = \frac{7}{11} \qquad \therefore 2\beta = 2n\pi \pm 50^\circ\ 28'\ 44'',$

and $-4{\cdot}5 \cos 2\alpha = 3\frac{1}{2} - 2 = 1\frac{1}{2},$

$\cos 2\alpha = -\frac{1}{3} \qquad \therefore 2\alpha = 2n\pi \pm 109^\circ\ 28'\ 16''.$

But from (1) if $\sin 2\alpha$ is positive, $\sin 2\beta$ is negative,

$\therefore$ if $2\alpha = 109^\circ\ 28'\ 16''$

$\alpha = 54^\circ\ 44'\ 8''$, and $\beta = -25^\circ\ 14'\ 22''$, or $154^\circ\ 45'\ 38''$.

Consequently the − 4·5 D cyl. must be set at 54° 44′ 8″ and the − 5·5 Dcyl. at 154° 45′ 38″.

The Refractive Value of an Oblique Cylindrical Lens in any meridian. Let θ be the angle made by the meridian with the plane axis of the cylindrical lens, and let C denote its dioptric strength in its working axis, then if C_θ is its dioptric strength in the required meridian, $C_\theta = C \sin^2 \theta$. This formula follows at once from the expression for the radius of curvature of the ellipse. It is often required in problems of decentration, e.g., + 6 D cyl. axis 60° has the effect in the horizontal meridian of $6 \sin^2 60^\circ = 6 \times \frac{3}{4} = +4{\cdot}5$ D and in the vertical meridian of $6 \sin^2 30 = 6 \times \frac{1}{4} = +1{\cdot}5$ D.

Periscopic Lenses.—The object of these lenses is to enable the wearer to see distinctly when he is looking through an eccentric part of his spectacles. We must therefore give the lens such a shape that the small pencil of rays entering his pupil does not form a confusion circle on his retina larger than the sectional area of a macular cone, even when he is looking through an eccentric portion of the glass. The table on *p.* 53 gives the curvature, or rather the dioptric value, of the two surfaces of the lens, which will allow a range of view of a solid angle of 50° fulfilling this condition.

The process by which these values were obtained is the following (see my paper on Periscopic Lenses, *Knapp's Archives of Ophthalmology*, Vol. xxx, No. 5, 1901) :—

Let a and b represent half the height and width of an eccentric pencil emerging from the lens at P and reaching two focal lines at F_1 and F_2. Let the position of the circle of least confusion be at D, and let its radius be k, the square of which is represented in the figure. Further, let H′ represent the first principal plane of the eye, and let R denote the radius of the equivalent pupil in this plane. Then it is required to construct the lens in such a fashion that the retinal image of this circle of confusion at D shall not be larger than the sectional area of a macular cone, the radius of which is ·001 mm.

We will first find an expression for R, the radius of the equivalent pupil in the first principal plane of the eye at H′ (*Fig.* 15). Imagine a line drawn from F″ touching the pupillary margin of the iris and cutting the principal plane at J, then H′J is R ; if y be the radius of the pupil at A_1.

$$\frac{R}{y} = \frac{H'F''}{H'F'' - H'A_1} = \frac{21{\cdot}14}{21{\cdot}14 - 1{\cdot}81}.$$

This gives the value of R for an emmetropic eye ; when the eye has D of ametropia its length is increased or decreased by a quantity l which we know is $-$ ·323 D.

$$\therefore \quad R = \frac{21{\cdot}14 \pm l}{19{\cdot}33 \pm l}\, y = \frac{21{\cdot}14 \mp {\cdot}323\,D}{19{\cdot}33 \mp {\cdot}323\,D}\, y.$$

The size of the pupil varies, but perhaps the average value of y is 1·6 mm.

Now from a consideration of *Fig.* 20, where a, b, represents the plane of the glass at P, the principal plane of the eye being at H′, it is evident that

$$\frac{a}{R} = \frac{F_1P}{F_1P - H'P} \text{ and } \frac{b}{R} = \frac{F_2P}{F_2P - H'P}.$$

Let F_1P, F_2P and $H'P$ be denoted by v_1, v_2 and s, and let DP, the distance of the lens from its circle of confusion, be x.

$$\text{Then} \quad \frac{v_1}{a} = \frac{v_1 - s}{R} \text{ and } \frac{v_2}{b} = \frac{v_2 - s}{R}. \qquad (1)$$

$$\left.\begin{aligned} &\text{Again} \quad \frac{F_1P}{a} \text{ or } \frac{v_1}{a} = \frac{DF_1}{k} = \frac{DP - F_1P}{k} = \frac{x}{k} - \frac{v_1}{k} \\ &\text{and} \quad \frac{F_2P}{b} \text{ or } \frac{v_2}{b} = \frac{F_2D}{k} = \frac{F_2P - DP}{k} = \frac{v_2}{k} - \frac{x}{k} \end{aligned}\right\} \qquad (2)$$

$$\therefore \text{ on addition } \frac{v_1}{a} + \frac{v_2}{b} = \frac{1}{k}(v_2 - v_1),$$

$$\text{and on substituting from (1) } \frac{v_1 - s}{R} + \frac{v_2 - s}{R} = \frac{1}{k}(v_2 - v_1),$$

$$\therefore \quad k = \frac{R(v_2 - v_1)}{v_1 + v_2 - 2s}. \qquad (3)$$

$$\text{Again, from (2) } \frac{1}{k}(x - v_1) = \frac{v_1}{a} \text{ or } \frac{v_1 - s}{R}$$

$$\text{and } \frac{1}{k}(v_2 - x) = \frac{v_2}{b} \text{ or } \frac{v_2 - s}{R}$$

$$\therefore \quad \frac{R}{k} = \frac{v_1 - s}{x - v_1} = \frac{v_2 - s}{v_2 - x},$$

$$\therefore \quad xv_2 - xs - v_1v_2 + sv_1 = v_1v_2 - sv_2 - xv_1 + xs$$

$$\text{or } x\,(v_1 + v_2 - 2s) = 2v_1v_2 - s\,(v_1 + v_2)$$

$$\text{or } x = \frac{2v_1v_2 - s(v_1 + v_2)}{v_1 + v_2 - 2s}. \qquad (4)$$

Now this circle of confusion at D of radius k may be

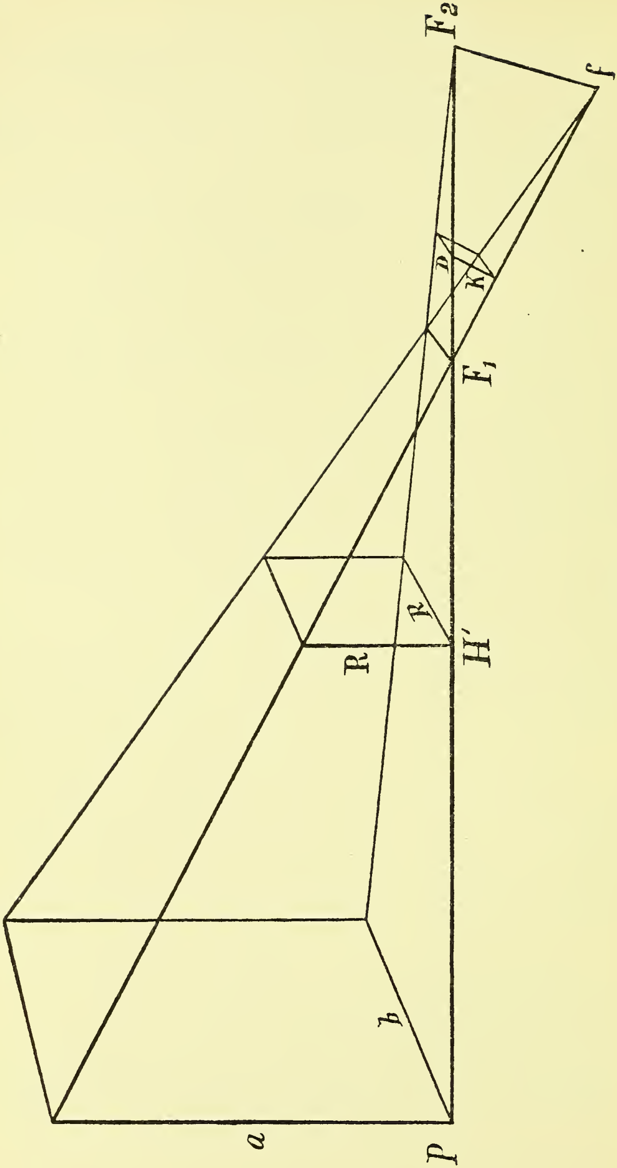

Fig. 20.

regarded as the object of which a retinal image of radius r will be formed.

$$\text{Consequently } \frac{r}{k} = \frac{F'}{F'-p}.$$

But as the lens is placed in the first focal plane, PH′ = F', so $s = -F'$ and $F' - p$ = PH′ − DH′ = PD = $-x$.

$$\therefore \quad r = \frac{sk}{x} = \frac{sR(v_1 - v_2)}{2v_1v_2 - s(v_1 + v_2)} \quad \text{or} \quad \frac{R(v_1 - v_2)F'}{2v_1v_2 + F'(v_1 + v_2)}.$$

We must now find an expression for v_1 and v_2.

We will consider the case for parallel rays inclined at an angle α with the axis. Let M be the centre of motility of the eye, which is directed towards a distant point S at the angle α. C_1 and C_2 are the centres of curvature of the first and second surfaces of the lens, and EFQK is the course of the light refracted at the first surface, $\angle C_1EM$ being ϕ and $\angle C_1EQ$ being ϕ'. It will be seen that in this figure (*Fig.* 21) all the angles are in the clockwise direction.

To find ϕ and ϕ'.

$$\frac{\sin\phi}{\sin\alpha} = \frac{\sin C_1EM}{\sin AME} = \frac{\sin C_1EM}{\sin EMC_1} = \frac{C_1M}{C_1E}$$

Let $A_1A_2 = t$, $A_2M = k$, C_1E or $C_1A_1 = r_1$, AME $= \alpha$. Then $C_1M = C_1A_1 + A_1A_2 + A_2M = r_1 + t + k$,

$$\therefore \sin\phi = \frac{r_1 + t + k}{r_1}\sin\alpha = \mu\sin\phi'.$$

To find ψ' and ψ.

Let $C_2FQ = \psi'$; produce FQ to K and from C_2 and C_1 drop the perpendiculars C_2N_2 and C_1K, and draw C_2N parallel to FK.

Now

$KN = N_2C_2 = C_2F \sin C_2FN_2 = r_2 \sin \psi'$,

$KC_1 = C_1E \sin C_1EK = r_1 \sin \phi'$,

$NC_1 = C_1C_2 \sin C_1C_2N = C_1C_2 \sin AQE$.

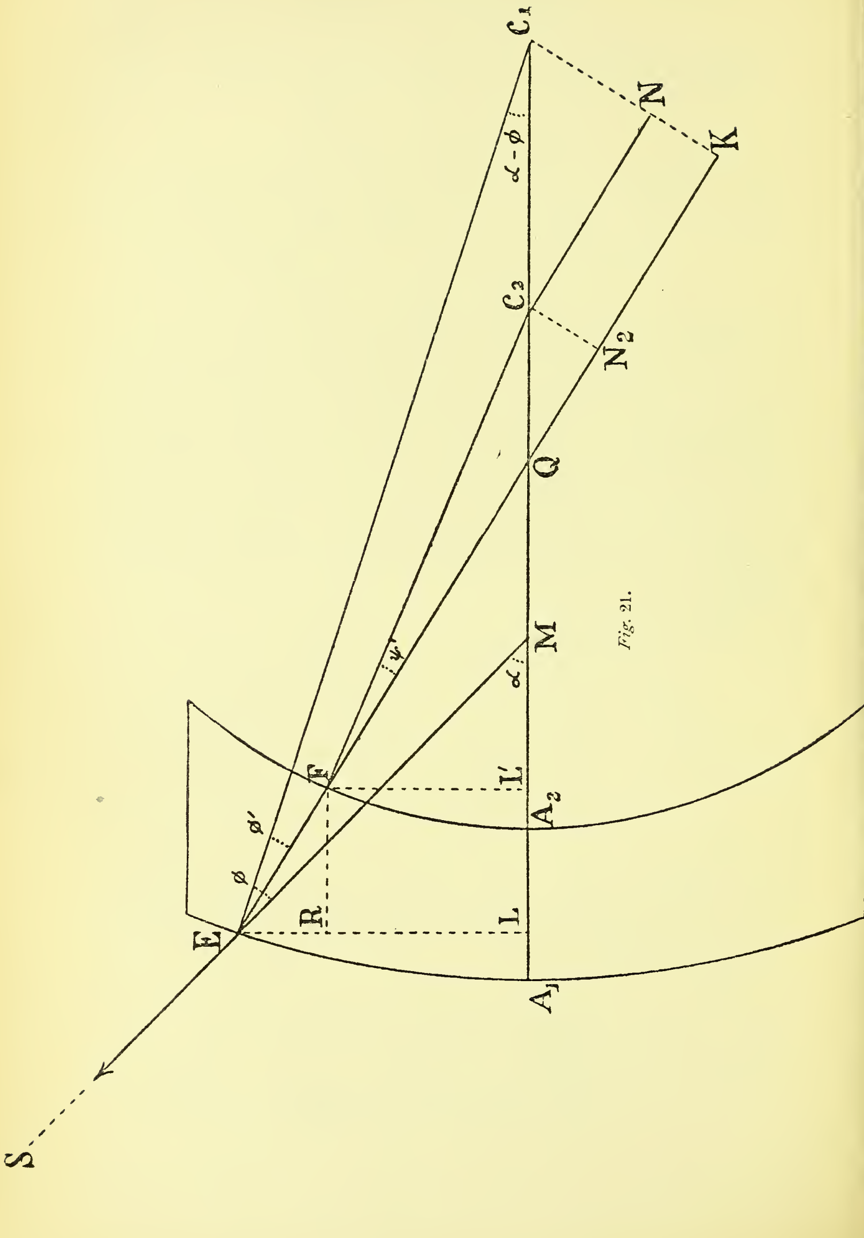

Fig. 21.

But $\quad AQE = QC_1E + C_1EQ = \alpha - \phi + \phi'$

and $\quad C_1C_2 = r_1 + t - r_2.$

$$\therefore \ NC_1 = (r_1 + t - r_2) \sin (\alpha + \phi' - \phi).$$

and $\quad KN = KC_1 - NC_1,$

$$\therefore \ r_2 \sin \psi' = r_1 \sin \phi' - (r_1 + t - r_2) \sin (\alpha + \phi' - \phi)$$

$$\therefore \sin \psi' = \frac{r_1 \sin \phi' - (r_1 + t - r_2) \sin (\alpha + \phi' - \phi)}{r_2} = \frac{1}{\mu} \sin \psi.$$

To find E F.

From E and F drop the perpendiculars EL, FL′ to the axis, and through F draw FR parallel to the axis.

Since

$RE = EF \sin RFE = EF \sin AQE = EF \sin (\alpha + \phi' - \phi).$

and $LE = EC_1 \sin LC_1E = - r_1 \sin (\alpha - \phi).$

and $LR = L'F = FC_2 \sin L'C_2F = FC_2 \sin (AQF - C_2FQ)$

$$= - r_2 \sin (\alpha + \phi' - \phi - \psi')$$

and $RE = LE - LR.$

$$EF \sin (\alpha + \phi' - \phi) = - r_1 \sin (\alpha - \phi) + r_2 \sin (\alpha + \phi' - \phi - \psi'),$$

$$\text{or} \quad EF = \frac{r_2 \sin (\alpha + \phi' - \phi - \psi') - r_1 \sin (\alpha - \phi)}{\sin (\alpha + \phi' - \phi)}.$$

Now parallel rays on entering the lens will be refracted at the first surface towards two focal lines at f_1 and f_2.

Let f_1 E and f_2 E be denoted by v' and v''. We know

$$\text{that} \quad \frac{\mu \cos^2 \phi'}{v'} - \frac{\cos^2 \phi}{U} = \frac{\mu}{v''} - \frac{1}{U} = \frac{\sin (\phi - \phi')}{r_1 \sin \phi'}$$

Here $U = \infty$, so we have $\dfrac{\cos_2 \phi'}{v'} = \dfrac{1}{v''} = \dfrac{\sin (\phi - \phi')}{r_1 \sin \phi}.$

At the second surface the distance $U_1 = f_1F$ or $f_1E + EF = v' + EF$

and $U_2 = f_2F \quad$ or $f_2E + EF = v'' + EF$

$$\text{and} \ \frac{\cos^2 \psi}{\mu v_1} - \frac{\cos^2 \psi'}{U_1} = \frac{1}{\mu v_2} - \frac{1}{U_2} = \frac{\sin (\psi' - \psi)}{r_2 \sin \psi}.$$

Hence v_1 and v_2 can be determined, and we have only to substitute their values in the previous formula,

$$\frac{R\,(v_1 - v_2)\,F'}{2v_1v_2 + F'\,(v_1 + v_2)},$$

to obtain the radius of the retinal confusion circle.

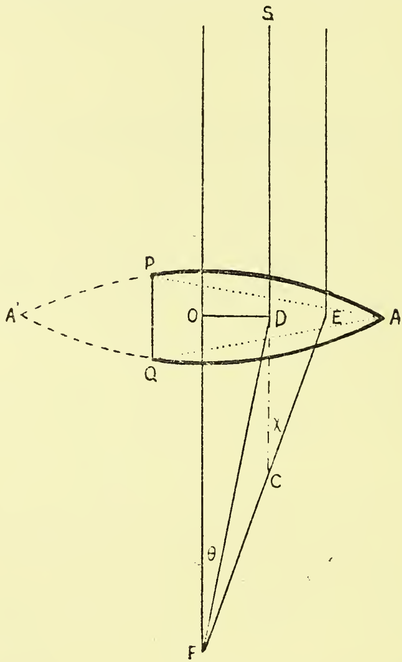

Fig. 22.

Deviation of Prismospheres. — In *Fig.* 22, AA′ represents a lens with its optical centre at O. The part APQ may be regarded as a decentred lens with its geometrical centre at D, or as a combination of a prism (marked in dotted lines) with a convex lens. Such a combination is called a prismosphere. A narrow pencil of parallel rays incident in the direction SD will be deflected towards F″, and if θ be the angle of deflection, θ = OFD. The distance DO (or l) of the optical centre from the geometrical centre of the lens, is called its decentration ; in this case it is inwards or negative.

Clearly $\tan\,\theta = \frac{\text{OD}}{\text{FO}} = \frac{-\,l}{f''}$, or if the dioptric strength of the lens be D, $\tan\,\theta = \frac{\text{D}l}{1000}$. When θ is small, no great error is introduced by replacing $\tan\,\theta$ by θ

Then $\theta = \text{D}\,l\,\tan^{-1}\,{\cdot}001 = \text{D}\,l \times 3'437$.

Now if the unit of angular measurement be Mr. Prentice's Prism Dioptre (symbol $\triangle$), that Dr. Maddox has conveniently designated the tangent centune, as it is the angle the tangent of which is ·01, it is clear that the number of prism dioptres or $\text{N}\triangle = \frac{1}{10}\,\text{D}l$, or is D$c$ where c represents the decentration in centimetres. The centrad (·01 radian, symbol $\bigtriangledown$) has practically the same value as the prism dioptre (for $1\bigtriangledown = 34'37747$ and $1\triangle = 34'37642$), and as it is more suited for multiplication, I think this unit is far the better. Dr. Maddox has pointed out that the slightly greater value of the centrad is in its favour when determining the decentration by this formula as, owing to spherical aberration, the effect of decentration is slightly greater than that indicated by the diagram and expressed by the formula. It is, then, rather more accurate to use the formula $\text{N}\bigtriangledown = \frac{1}{10}\,\text{D}l$. The decentration of a + 5 D lens required to be equivalent in effect to that of an adducting prism of, say, $2\bigtriangledown$ is given by the formula $l = \frac{10\ \text{N}}{\text{D}}$ in this case $l = \frac{20}{5} = 4$ mm. outwards.

Note that as we have agreed to regard *adducting* prisms as positive and *abducting* prisms as negative, it is consistent with this convention to regard decentration *outwards* positive, and decentration *inwards* negative.

Effects of Prismospheres on the Eye (*A*) Horizontal.—If we now imagine an eye placed behind the prismosphere, we shall find that the deviation of the eye it causes is quite different from the deviation of the course of light it produces. Suppose that C is the centre of motility of an eye behind the prismosphere in *Fig.* 22. If the eye were previously viewing some distant object in the direction of S, after the interposition of the prismosphere, its fixation line would have to rotate through the angle D C E or χ, which is obviously greater than the angle C D F, that is equal to O F D or θ. When a concave lens is used, it will be found that $\chi < \theta$; it is consequently most important to determine the clinical effect that a prismosphere has upon the eye behind it.

Let us consider the case of a concave lens with its second principal focus at F (*Fig.* 23), the position of the lens being represented by a principal plane MD that passes through its optical centre O. Let C denote the centre of motility of the eye situated DC (or k mm.) behind the lens. Then the decentration is DO or l. (In *Fig.* 23 DO is negative or measured inwards.) Let P be an object situated in the middle line between the eyes; a virtual image of P will be formed at Q. The eye therefore must be directed towards Q when viewing the object P through the decentred lens. If CD represent the direction of the fixation line of the eye when viewing an object immediately in front of it, it must rotate through the positive angle DCE or χ.

As the lens is supposed to be in its normal place, in the first focal plane of the eye, and as the centre of motility of the eye is usually about 13·4 mm. behind the cornea, DC or $k = 13{\cdot}4 + 13{\cdot}75 \backsimeq 27$ mm. Let MD or m denote half the interocular distance, say 30 mm., and let PM, QS, and FO be denoted by p q and f'' as usual.

Then $SO = \frac{QS}{PM} MO = \frac{q}{p} (MD - OD) = \frac{q}{p} (MD + DO)$

or $SO = \frac{q}{p} (m + l)$.

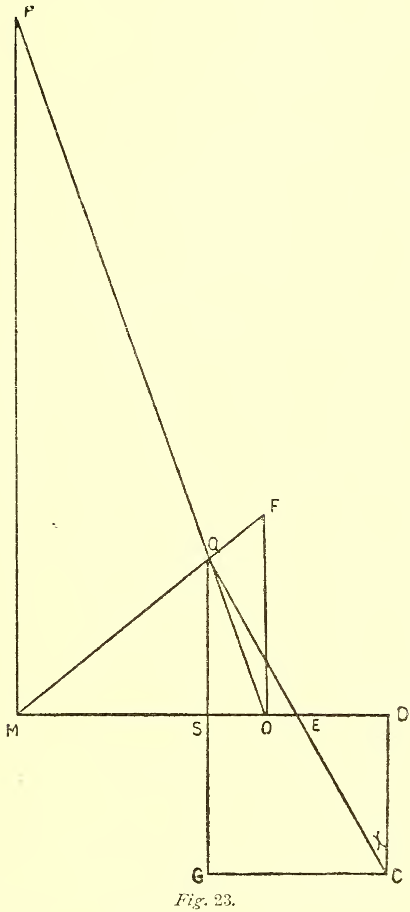

Fig. 23.

And GC or SD = SO + OD = SO − DO.

$\therefore \quad GC = \frac{q}{p}(m + l) - l = \frac{q}{p} m + l\left(\frac{q}{p} - 1\right)$.

But since $\frac{1}{p} - \frac{1}{q} + \frac{1}{f''} = 0, \qquad \frac{q}{p} - 1 = \frac{-q}{f''},$

$$\therefore \quad \text{GC} = \frac{q}{p}\, m - l\frac{q}{f''}.$$

Now $\chi = \text{DCE} = \text{GQC}.$

$$\therefore \quad \tan\chi = \frac{\text{GC}}{\text{QG}} = \frac{\text{GC}}{q+k} = \frac{q}{q+k}\left(\frac{m}{p} - \frac{l}{f''}\right)$$

When $p = \infty$, $q = f''$, and so in that case $\tan\chi = -\frac{l}{f''+k}$.

This is the formula for the effect of a prismosphere on an eye when viewing distant objects. As f'' is always greater than k, it is clear that when the decentration is towards the left or negative, and a concave lens is considered, the eye must rotate to the left, or χ is positive; when a convex lens is used, f'' is negative, and therefore the eye rotates to the right. For reverse rotations of the eye, l must be positive, i.e., the decentration must be to the right.*

Since $\frac{1}{f''} = \frac{-\text{D}}{1000}, \qquad f'' = \frac{-1000}{\text{D}},$

$$\therefore \quad \tan\chi \text{ or } \frac{-l}{f''+k} = \frac{\text{D}l}{1000 - k\text{D}},$$

when distant vision is considered.

For near vision, since $q = \frac{pf''}{p+f''}$, when the incident pencil is not too oblique, the expression for

---

* As it is convenient to regard convergence as positive, we must make a special convention to be consistent with this usage. In convergence the right eye rotates counter-clockwise, while the left eye rotates clockwise; so, if we regard the diagram as referring to the right eye, all the ordinary conventions hold good. Therefore we must consider that what has gone before refers to the right eye, and notice that decentration inwards increases, and decentration outwards diminishes, convergence when a concave glass is used; the reverse is the case when a convex glass is used. The formulæ will give the correct sign to the angle χ in every case if decentration outwards is considered positive.

$$\tan \chi \text{ or } \frac{q}{q+k}\left(\frac{m}{p} - \frac{l}{f''}\right) \text{ becomes } \frac{pf''}{pf''+k(p+f'')}\left(\frac{m}{p} - \frac{l}{f''}\right),$$

$$\text{or} \quad \frac{mf'' - lp}{f''(p+k)+kp}, \quad \text{i.e.,} \quad \frac{-1000\,m - lp\mathrm{D}}{-1000\,(p+k)+kp\mathrm{D}},$$

$$\text{so that} \quad \tan \chi = \frac{m + l\,p\,\mathrm{D}\,(\cdot 001)}{p + k - k\,p\,\mathrm{D}\,(\cdot 001)}.$$

The tables given on *pp*. 151-4 have been calculated from these formulæ by assigning the values 27 mm. to k and 30 mm. to m. The interocular distance is usually 60 mm. or nearly $2\frac{3}{8}$ in. As the table is calculated for metre angles and the object in *Tables II* and *III* is at such a distance that 3 m.a. of convergence are required, its distance from the centre of motility of the eye is $\frac{1}{3}$ metre, consequently $p + k$ is about 331·98 mm., and p about 304·98 mm. (a foot from the spectacles).*

(*B*). **Vertical.**—In the same way it can be shown, that the angle of depression θ when viewing an object P situated an angle α below the horizontal plane of the eye through an eccentric portion of a lens, is different from α.

The same significance is attached to the letters, but of course in this case the letter m is no longer a constant but a variable, MD or $(p+k)$ tan α and χ is replaced by θ.

$$\tan \theta = \frac{q}{q+k}\left(\frac{\mathrm{MD}}{p} - \frac{l}{f''}\right) = \frac{q}{p}\,\frac{p+k}{q+k}\tan \alpha - \frac{ql}{f''(q+k)}.$$

If there is no decentration, $l = 0$, and

$$\tan \theta = \frac{q}{p} \cdot \frac{p+k}{q+k} \tan \alpha, \quad \text{or} \quad \frac{f''\,(p+k)}{pf'' + k\,(p+f'')} \tan \alpha.$$

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* In my *Optics* the sign of the expression is different, as in that place I regarded lines measured towards the left as positive. The figures given in the tables must be regarded as only approximately accurate, as no allowance has been made for the obliquity of the incident pencil.

From this expression $\frac{\tan \theta}{\tan \alpha} = \frac{q}{q + k}\left(1 + \frac{k}{p}\right)$.

Now if P be very distant, $p = \infty$, $q = f''$,

so that $\tan \theta = \frac{f''}{f'' + k} \tan \alpha$.

Practically these expressions mean that a deviation of θ in the eye will be induced by a prism of deviation α when combined with a lens of focal length f''.

Suppose, for instance, that the right eye of a patient tended downwards 1° ; when wearing his distance glasses, say $-$ 10D, he would not require a 1° but a 1·27° prism to be combined with his spectacles.

For taking k, as always, to be 27 mm., since f'' here is 100 mm.,

$$\tan 1^\circ = \frac{100}{127} \tan \alpha \qquad \therefore \quad \alpha \fallingdotseq 1 \cdot 27^\circ.$$

The calculation is much more troublesome in the first case when the object P is close at hand, but according to the method advocated (*p.* 84) in testing for hyperphoria, the correcting prism is found by trial whilst the patient is wearing the glasses needed for his refractive errors. Hence all this trouble is avoided, and all one has to do is to order the equivalent decentration.

Objective Tests of Ametropia.—When the eye is examined with the ophthalmoscope by the indirect method, it was stated on *p.* 33 that on removing the lens from the eye in myopia the size of the image of the fundus was increased, but that in hypermetropia it was diminished. The adjoining diagram will explain this peculiarity.

As the fundus in *myopia* lies behind the focus of the eye, it will, when illuminated, form a real inverted image (I) in front of the eye, without the lens (O), say at P′ its punctum remotum (*Fig.* 24).

On interposing a convex lens of 2½ in. focus (63 mm.) near the eye at O, the emergent light will converge still more, so as to form an image at Q. This inverted image (i) is what is seen with the ophthalmoscope. In the diagram, the incident light reflected from the ophthalmoscope is passing from right to left, whereas the light from the fundus is travelling from left to right; we will regard this last as the positive direction.

Let us imagine that $-9 \cdot 5$ D of myopia is present, then from *p.* 120 we know that the eye is $9 \cdot 5 \times \cdot 323$ mm. (say 3 mm.) longer than normal. Taking Donders' eye for simplicity, we know that CA = 5 mm., and in this myopic eye CP = -18 mm., whereas $AP' = F' + \frac{1000}{9 \cdot 5} = 15 + 105$ i.e., $AP' = 120$ mm., so $CP' = 125$ mm.

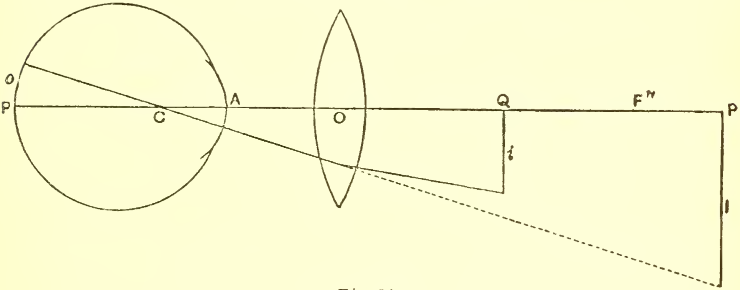

Fig. 24.

Hence $$\frac{I}{o} = \frac{CP'}{CP} = \frac{125}{-18}.$$

Now if f' be the first focal distance of the convex lens, $f' = 63$ mm.

and $$\frac{i}{I} = \frac{f'}{f' - p} = \frac{f'}{f' - P'O} = \frac{f'}{f' + AP' - AO}$$

$$\frac{i}{I} = \frac{63}{63 + 120 - AO} = \frac{63}{183 - AO},$$

$$\therefore \quad \frac{i}{o} \text{ or } \frac{I}{o} \cdot \frac{i}{I} = -\frac{125}{18} \cdot \frac{63}{183 - AO}.$$

It is clear that if AO be increased the value of i must be increased, and *vice versa*.

If $AO = 8$, $\quad \frac{i}{o} = -\frac{125}{2} \cdot \frac{7}{175} = -2\frac{1}{2}$,

if $AO = 58$, $\quad \frac{i}{o} = -\frac{125}{2} \cdot \frac{7}{125} = -3\frac{1}{2}$.

The negative sign shows that the image is inverted.

In *emmetropia*, the cone of light originating from P will emerge parallel to the axis at A, and hence will come to a focus at F″, the second principal focus of the lens. Similarly, the cone of light from the upper part of the fundus will, on emerging from the eye, proceed as a parallel beam inclined downwards until it meets the lens, when it will converge to a focus in the lower part of the second focal plane, i.e., to a point below F″. It is unnecessary to give the figure, but the reader is urged to draw a diagram for himself; he will then see that the line joining the lower part of the image and O, will be parallel to the rising line from C to the upper part of the fundus, and that therefore $\frac{i}{o} = \frac{F''O}{PC} = \frac{-63}{15} = -4\frac{1}{5}$. This is the magnification of the fundus of an emmetropic eye as seen by the indirect method with an ophthalmoscope. It is obvious that whatever be the distance AO, the image is always of the same size, for it is always formed at the second focal plane of the lens.

In *hypermetropia*, as the fundus lies nearer the cornea than the second focus of the eye, the refracting media of the eye must act as an ordinary magnifying glass, so that a virtual erect image of the fundus will be formed, say, at P″ behind the eye. The reader is expected to draw a diagram for himself. Suppose that the eye is hypermetropic to the extent of $+9{\cdot}5$ D, then the eye must be 3 mm. too short, so that $CP = -12$ mm., and $AP'' = F' + \frac{1000}{-9{\cdot}5} = 15 - 105 = -90$ mm. As before, CA $= 5$ mm. so that CP$'' = -85$ mm.

$$\therefore \quad \frac{\mathrm{I}'}{o} = \frac{\mathrm{CP}''}{\mathrm{CP}} = \frac{85}{12},$$

and with the same convex lens of focal length 63 mm.

$$\frac{i}{\mathrm{I}'} = \frac{f'}{f'-p} = \frac{f'}{f'-\mathrm{P}''\mathrm{O}} = \frac{f'}{f'-(\mathrm{P}''\,\mathrm{A}+\mathrm{AO})} = \frac{63}{63-90-\mathrm{AO}}$$

$$= -\frac{63}{27+\mathrm{AO}}$$

$$\therefore \quad \frac{i}{o} \text{ or } \frac{\mathrm{I}'}{o} \cdot \frac{i}{\mathrm{I}'} = -\frac{85}{12} \cdot \frac{63}{27+\mathrm{AO}}.$$

It is obvious that if AO be increased the denominator is increased, and therefore the value of i must be diminished.

Thus if AO $= 36$ mm $\quad \frac{i}{o} = -\frac{85}{12} \cdot \frac{63}{63} = -7\frac{1}{12}$

but if AO $= 58$ mm. $\quad \frac{i}{o} = -\frac{85}{12} \cdot \frac{63}{85} = -5\frac{1}{4}.$

A glance at the diagram (*Fig.* 24) will explain the point mentioned on *p.* 33. It was said that when the observer reflecting the light from the mirror at a distance sees an image of the fundus without a lens, either hypermetropia or myopia is present. If myopia, the vessels of the fundus appear to move in the opposite direction to the observer's head, for in this case clearly the image is an inverted one, being formed at P$'$. If the vessels move in the same direction, the image must be an erect one situated behind the patient's eye; in other words the eye must be hypermetropic.

Minimum Visual Angle.—This, as explained on p. 19, is twice the angle subtended at the posterior nodal point by one macular cone (diameter ·002 mm.). If this be represented by o in *Fig.* 24, tan $= \frac{o}{\mathrm{CP}}$, but in emmetropia, from *p.* 16 we know that CP or K$''$F$'' = 15{\cdot}54$ mm.

$$\therefore \quad \tan \alpha = \frac{\cdot 002}{15{\cdot}54} = \tan 26{\cdot}55''.$$

The minimum visible is therefore $53{\cdot}1''$.

APPENDIX.

SERVICE REGULATIONS.

THE following Service Regulations are given in Mr. Hartridge's *Refraction of the Eye.*

Regulations for Candidates for Commissions in the Army.

A candidate must be able to read at least $\frac{6}{36}$ with each eye separately without glasses, and this must be capable of correction with glasses up to $\frac{6}{6}$ in one eye and $\frac{6}{12}$ in the other ; he must also be able to read No. 1 of the near type with each eye without the aid of glasses.

Squint, colour-blindness, or any serious disease of the eye renders the candidate ineligible.

Navy.

A candidate must be able to read $\frac{6}{6}$ with each eye, and the near type at the distance for which it is marked, without glasses.

Colour-blindness, squint, or any disease of the eye disqualifies.

Indian Civil Service.

A candidate must be able to read $\frac{6}{9}$ with one eye and $\frac{6}{6}$ with the other, with or without correcting lenses.

Any disease of the fundus renders the candidate ineligible. Myopia, however, with a posterior staphyloma, may be passed if the ametropia do not exceed 2·5 D, and the candidate has a visual acuteness equal to that stated above.

Indian Medical Service.

The candidate must have a visual acuteness of $\frac{6}{6}$ in one eye and $\frac{6}{12}$ in the other. Hypermetropia and myopia must not exceed 5 D, and then with the proper correction the vision must come up to the above standard.

Astigmatism does not disqualify a candidate, provided the combined spherical and cylindrical glass does not exceed 5 D, and the visual acuteness equals $\frac{6}{6}$ in one eye and $\frac{6}{12}$ in the other. Colour-blindness, ocular paralysis, or any active disease of the fundus renders the candidate ineligible.

A nebula of the cornea will not disqualify the candidate if he is able to read $\frac{6}{12}$ with this eye and $\frac{6}{6}$ with the other.

Public Works.

Candidates for the departments of Public Works, Survey, Forest, Telegraph, Railways, Factories, and Police of India must pass the following eyesight tests.

If myopic, the defect must not exceed 2·5 D, and with this glass the candidate must read $\frac{6}{9}$ in one eye and $\frac{6}{6}$ in the other.

If myopic astigmatism is present, the vision must reach the above standard with correcting glasses, and the combined spherical and cylindrical glass must not exceed 2·5 D.

In hypermetropia and hypermetropic astigmatism an error of 4 D is permissible, provided that with this glass $\frac{6}{9}$ is read with one eye, and $\frac{6}{6}$ with the other.

A corneal nebula with vision of $\frac{6}{12}$, and $\frac{6}{6}$ in the other eye will not disqualify the candidate.

Colour-blindness, any disease of the eye, or paralysis of one of the muscles of the globe, will disqualify.

English Railways.

There is, unfortunately, no uniform standard for our railways ; each company has its own standard, in many cases a very low one ; every engine-driver should have at least $\frac{6}{12}$ in each eye without glasses, and normal colour vision.

Tables I, II, and III.

In these tables the glasses are supposed to be accurately centred for distance, so that if my method of testing the patient is adopted they will be rarely used, except in those cases in which one wishes to find the near point of convergence. An example of this mode of use has been given on p. 93. If, however, the usual procedure of testing the patient be adopted, they will be found exceedingly useful, as the tables give the effect of the prisms or decentration on a patient's fixation lines that are 60 mm. apart.

Table I is the table for distance. A hypermetrope of + 9 D is found to have say $1\frac{1}{2}$ m.a. of exophoria for distance, of which 1 m.a. needs correction. On referring to *Table I* we see that under 9 and opposite to 1 m.a. are the figures 1° 18′ and 2·5. This shows us that abducting prisms of 1° 18′ d. will relieve a divergence of 1° 43′·1 in his fixation lines, or that the same result may be attained by decentring his lenses 2·5 mm. inwards.

TABLE I.

	1	2	3	4	5	6	7	8	9	10	12
4 m.a. 6° 53·5′	6° 44′ 117·6	6° 33′ 57·2	6° 22′ 37·0	6° 11′ 27·0	6° 20·9	5° 49′ 16·9	5° 37′ 14·0	5° 25′ 11·8	5° 14′ 10·2	5° 3′ 8·8	4° 41′ 6·8
3 m.a. 5° 9·8′	5° 2′ 87·9	4° 50′ 42·7	4° 44′ 27·3	4° 38′ 20·2	4° 28′ 15·6	4° 20′ 12·6	4° 13′ 10·5	4° 5′ 8·9	3° 56′ 7·6	3° 47′ 6·6	3° 30′ 5·1
2 m.a. 3° 26·4′	3° 21′ 58·5	3° 15′ 28·4	3° 11′ 18·4	3° 4′ 13·4	2° 59′ 10·4	2° 53′ 8·4	2° 48′ 7·0	2° 42′ 5·9	2° 38′ 5·1	2° 31′ 4·4	2° 20′ 3·4
1 m.a. 1° 43·1′	1° 40′ 29·2	1° 38′ 14·2	1° 35′ 9·2	1° 32′ 6·7	1° 29′ 5·2	1° 27′ 4·2	1° 24′ 3·5	1° 20′ 2·9	1° 18′ 2·5	1° 16′ 2·2	1° 10′ 1·7
1°	58′ 16·98	57′ 8·26	55′ 5·35	53′ 3·89	52′ 3·02	50′ 2·44	49′ 2·02	47′ 1·71	45′ 1·47	44′ 1·27	41′ 9·83
1 ∇	33′ 9·73	33′ 4·73	32′ 3·06	31′ 2·23	30′ 1·73	29′ 1·40	28′ 1·16	27′ ·98	26′ ·84	25′ ·73	23′ ·56
0											
1 ∇	35′ 10·27	36′ 5·27	37′ 3·60	38′ 2·77	39′ 2·27	40′ 1.94	41′ 1·70	42′ 1·52	43′ 1·38	44′ 1·27	45′ 1·10
1°	1° 2′ 17·93	1° 3′ 9·20	1° 5′ 6·29	1° 7′ 4·84	1° 8′ 3·96	1° 10′ 3·38	1° 11′ 2·96	1° 13′ 2·65	1° 14′ 2·41	1° 16′ 2·22	1° 20′ 1·93
1 m.a. 1° 43·1′	1° 46′ 30·8	1° 49′ 15·8	1° 51′ 10·8	1° 54′ 8·3	1° 57′ 6·8	2° 5·8	2° 3′ 5·1	2° 6′ 4·6	2° 8′ 4·1	2° 11′ 3·8	2° 16′ 3·3
2 m.a. 3° 26·4′	3° 32′ 61·7	3° 38′ 31·7	3° 44′ 21·7	3° 48′ 16·6	3° 53′ 13·6	3° 59′ 11·6	4° 6′ 10·2	4° 11′ 9·1	4° 16′ 8·3	4° 21′ 7·6	4° 32′ 6·6
3 m.a. 5° 9·8′	5° 19′ 92·8	5° 27′ 47·6	5° 36′ 32·6	5° 44′ 25·0	5° 52′ 20·5	6° 17·5	6° 8′ 15·3	6° 16′ 13·7	6° 26′ 12·5	6° 35′ 11·5	6° 52′ 10·0
4 m.a. 6° 53·5′	7° 6′ 124·1	7° 18′ 63·7	7° 30′ 43·6	7° 41′ 33·5	7° 54′ 27·4	8° 2′ 23·4	8° 13′ 20·5	8° 25′ 18·4	8° 37′ 16·7	8° 49′ 15·4	9° 8′ 13·3

CONVEX.
Divergence, decentration inwards.
Convergence, decentration outwards.

CONCAVE.
Divergence, decentration outwards.
Convergence, decentration inwards.

Table II is used in cases of hypermetropia when the object is $\frac{1}{3}$ metre distant. It will be noticed that normally, when 3 m.a. of convergence are exercised, the lenses must be decentred inwards $-$ 2·44 mm. if the interocular distance is 60 mm. (When the interocular distance is 64 mm., the decentration must be 2·6 mm. inwards.)

An aphakic requiring $+$ 12 D for reading at $\frac{1}{3}$ metre can only maintain convergence of 2 m.a. We see that opposite 2 m.a. in the column 12 are the figures $-$ 3° and $-$ 4·4, so that abducting prisms of 3° d., or a decentration inwards of 4·4 mm., will relieve this defect.

Table III is similarly used in cases of myopia when the object is $\frac{1}{3}$ metre distant; as before, when 3 m.a. of convergence are exercised, the glasses must be decentred 2·44 mm. inwards.

A myope requiring $-$ 8 D who can only maintain 2 m.a. of convergence when reading at $\frac{1}{3}$ metre, must have his glasses decentred outwards 2·5 mm., or he must have abducting prisms of 1° 9′ combined with his glasses.

The tables should be used whenever it is required to know the actual relative range of convergence of the fixation lines; but, as we have pointed out, this is unnecessary for the prescription of proper spectacles, as all we need know is the strength of the prisms that measure the range, without troubling ourselves about the actual effect that these prisms have on the fixation lines.

TABLE II. (CONVEX).

	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+12
0 m.a.	− 5° 37′ − 98·4	− 5° 37′ − 49·2	− 5° 37′ − 32·8	− 5° 37′ − 24·6	− 5° 37′ − 19·7	− 5° 37′ − 16·4	− 5° 37′ − 14·1	− 5° 37′ − 12·3	− 5° 37′ − 10·9	− 5° 37′ − 9·8	− 5° 37′ − 8·2
1 m.a. **1° 43·1′**	− 3° 48′ − 66·5	− 3° 51′ − 33·7	− 3° 54′ − 22·7	− 3° 56′ − 17·2	− 3° 59′ − 13·9	− 4° 3′ − 11·8	− 4° 5′ − 10·2	− 4° 8′ − 9·0	− 4° 10′ − 8·1	− 4° 13′ − 7·4	− 4° 19′ − 6·3
2 m.a. **3° 26·4′**	− 1° 59′ − 34·5	− 2° 4′ − 18·1	− 2° 10′ − 12·6	− 2° 16′ − 9·8	− 2° 21′ − 8·2	− 2° 27′ − 7·1	− 2° 32′ − 6·3	− 2° 38′ − 5·7	− 2° 43′ − 5·3	− 2° 49′ − 4·9	− 3° − 4·4
3 m.a. **5° 9·8′**	− 8′ − 2·44	− 17′ − 2·44	− 25′ − 2·44	− 34′ − 2·44	− 42′ − 2·44	− 50′ − 2·44	− 59′ − 2·44	− 1° 7′ − 2·44	− 1° 15′ − 2·44	− 1° 24′ − 2·44	− 1° 41′ − 2·44
4 m.a. **6° 53·5′**	1° 43′ 29·9	1° 32′ 13·3	1° 21′ 7·8	1° 9′ 5·0	59′ 3·4	47′ 2·3	36′ 1·5	24′ ·89	13′ ·43	2′ ·05	− 20′ − ·5
Diff. **1** m.a.	1° 50′ 31·9	1° 46′ 15·5	1° 44′ 10·1	1° 41′ 7·4	1° 38′ 5·7	1° 35′ 4·6	1° 33′ 3·9	1° 30′ 3·3	1° 27′ 2·8	1° 24′ 2·45	1° 19′ 1·9
1°	1° 4′ 18·5	1° 2′ 9	1° 5·9	59′ 4·3	57′ 3·3	56′ 2·7	54′ 2·2	52′ 1·9	51′ 1·64	49′ 1·43	45′ 1·1
1 ∇	36′ 10·6	36′ 5·2	35′ 3·4	34′ 2·5	33′ 1·9	32′ 1·5	31′ 1·3	30′ 1·1	29′ ·94	28′ ·82	26′ ·64

TABLE III (CONCAVE).

	− 1	− 2	− 3	− 4	− 5	− 6	− 7	− 8	− 9	− 10	− 12
0 m.a.	− 5° 37′ 98·4	− 5° 37′ 49·2	− 5° 37′ 32·8	− 5° 37′ 24·6	− 5° 37′ 19·7	− 5° 37′ 16·4	− 5° 37′ 14·0	− 5° 37′ 12·3	− 5° 37′ 10·8	− 5° 37′ 9·8	− 5° 37′ 8·2
1 m.a. **1° 43·1′**	− 3° 43′ 64·9	− 3° 40′ 32·0	− 3° 37′ 21·1	− 3° 34′ 15·6	− 3° 31′ 12·3	− 3° 29′ 10·1	− 3° 26′ 8·6	− 3° 23′ 7·4	− 3° 21′ 6·5	− 3° 17′ 5·7	− 3° 12′ 4·7
2 m.a. **3° 26·4′**	− 1° 48′ 31·3	− 1° 42′ 14·8	− 1° 36′ 9·4	− 1° 31′ 6·6	− 1° 25′ 5·0	− 1° 20′ 3·9	− 1° 14′ 3·1	− 1° 9′ 2·5	− 1° 3′ 2·0	− 59′ 1·7	− 47′ 1·1
3 m.a. **5° 9·8′**	8′ − 2·44	17′ − 2·44	25′ − 2·44	34′ − 2·44	42′ − 2·44	50′ − 2·44	59′ − 2·44	1° 7′ − 2·44	1° 15′ − 2·44	1° 24′ − 2·44	1° 41′ − 2·44
4 m.a. **6° 53·5′**	2° 5′ − 36·5	2° 17′ − 19·9	2° 28′ − 14·3	2° 39′ − 11·5	2° 50′ − 9·9	3° 1′ − 8·8	3° 12′ − 8	3° 23′ − 7·4	3° 35′ − 7	3° 47′ − 6·6	4° 8′ − 6·0
Diff. **1** m.a.	1° 54′ − 33·5	1° 58′ − 17·1	2° 1′ − 11·7	2° 3′ − 9·0	2° 6′ − 7·3	2° 9′ − 6·3	2° 12′ − 5·5	2° 15′ − 4·9	2° 17′ − 4·4	2° 20′ − 4·1	2° 26′ − 3·5
1°	1° 6′ − 19·5	1° 8′ − 10	1° 10′ − 6·8	1° 12′ − 5·2	1° 13′ − 4·3	1° 15′ − 3·6	1° 16′ − 3·2	1° 18′ − 2·8	1° 20′ − 2·6	1° 21′ − 2·4	1° 25′ − 2·1
1 ∇	38′ − 11·2	39′ − 5·7	40′ − 3·9	41′ − 3·0	42′ − 2·4	43′ − 2·1	44′ − 1·8	45′ − 1·6	46′ − 1·5	47′ − 1·4	49′ − 1·2

Table IV.—Squares of numbers from 10 to 100.

The vertical column on the left gives the digits in the tens' place, while the horizontal column at the top gives the digits in the units' place. Thus the square of 21 is 441, and the square of 76 is 5776. The table is also of use for indicating the square roots of numbers approximately, if we remember that one decimal place in the square root corresponds to two decimal places in the square.

Thus, as the square root of 2209 is 47, the square root of 22 is nearly 4·7.

TABLE IV.

	0	1	2	3	4	5	6	7	8	9
1	100	121	144	169	196	225	256	289	324	361
2	400	441	484	529	576	625	676	729	784	841
3	900	961	1024	1089	1156	1225	1296	1369	1444	1521
4	1600	1681	1784	1849	1936	2025	2116	2209	2304	2401
5	2500	2601	2704	2809	2916	3025	3136	3249	3364	3481
6	3600	3721	3844	3969	4096	4225	4356	4489	4624	4761
7	4900	5041	5184	5329	5476	5625	5776	5929	6084	6241
8	6400	6561	6724	6889	7056	7225	7396	7569	7744	7921
9	8100	8281	8464	8649	8836	9025	9216	9409	9604	9801

TABLE V.—TRIGONOMETRICAL RATIOS, ETC.

If the number of degrees is under 45, the names of the ratios are given at the top of the table. Thus sin 10° is ·17365, and tan 20° is ·36397.

If the number of degrees is over 45, the names of the ratios are given at the foot of the table. Thus sin 65° is ·90631 and cos 76° is ·24192. The number of centrads corresponding to each degree up to 90° is given in a separate column.

For intermediate values the rule of proportional parts may be used., e.g., find sin 20° 40′.

The difference for 60′ between 20° and 21° is ·01635

The difference for 40′ or $\frac{2}{3}$ of 60′ is ·01090

sin 20° = ·34202

sin 20° 40′ = ·35292

TABLE V.

Angle		Sine	Cosine	Tangent	Cotangent	Secant	Cosecant		
Degrees	Centrads								
0°	0	0	1	0	∞	1	∞	157·080	90°
1°	1·745	·01745	·99985	·01746	57·2900	1·00015	57·2987	155·334	89°
2°	3·491	·03490	·99939	·03492	28·6363	1·00061	28·6537	153·589	88°
3°	5·236	·05234	·99863	05241	19·0811	1·00137	19·1073	151·844	87°
4°	6·981	·06976	·99756	·06993	14·3007	1·00244	14·3356	150·098	86°
5°	8·727	·08716	·99619	·08749	11·4301	1·00382	11·4737	148·353	85°
6°	10·472	·10453	·99452	·10510	9·51436	1·00551	9·56677	146·608	84°
7°	12·217	·12187	·99255	·12278	8·14435	1·00751	8·20551	144·862	83°
8°	13·963	·13917	·99027	·14054	7·11537	1·00983	7·18530	143·117	82°
9°	15·708	·15643	·98769	·15838	6·31375	1·01247	6·39245	141·372	81°
10°	17·453	·17365	·98481	·17633	5·67128	1·01543	5·75877	139·626	80°
11°	19·199	·19081	·98163	·19438	5·14455	1·01872	5·24084	137·881	79°
12°	20·944	·20791	·97815	·21256	4·70463	1·02234	4·80973	136·136	78°
13°	22·689	·22495	·94737	·23087	4·33148	1·02630	4·44541	134·390	77°
14°	24·435	·24192	·97030	·24933	4·01078	1·03061	4·13357	132·645	76°
15°	26·180	·25882	·96593	·26795	3·73205	1·03528	3·86370	130·900	75°
16°	27·925	·27564	·96126	·28675	3·48741	1·04030	3·62796	129·154	74°
17°	29·671	·29237	·95630	·30573	3·27085	1·04569	3·42030	127·409	73°
18°	31·416	·30902	·95106	·32492	3·07768	1·05146	3·23607	125·664	72°
19°	33·161	·32557	·94552	·34433	2·90421	1·05762	3·07155	123·918	71°
20°	34·907	·34202	·93969	·36397	2·74748	1·06418	2·92380	122·173	70°
21°	36·652	·35837	·93358	·38386	2·60509	1·07115	2·79043	120·428	69°
22°	38·397	·37461	·92718	·40403	2·47509	1·07853	2·66947	118·682	68°
23°	40·143	·39073	·92050	·42447	2·35585	1·08636	2·55930	116·937	67°
24°	41·888	·40674	·91355	·44523	2·24604	1·09464	2·45859	115·192	66°
25°	43·633	·42262	90631	·46631	2·14451	1·10338	2·36620	113·446	65°
26°	45·379	·43837	·89879	·48773	2·05030	1·11260	2·28117	111·701	64°
27°	47·124	·45399	·89101	·50953	1·96261	1·12233	2·20269	109·956	63°
28°	48·869	·46947	·88295	·53171	1·88073	1·13257	2·13005	108·210	62°
29°	50·615	·48481	·87462	·55431	1·80405	1·14335	2·06267	106·465	61°
30°	52·360	·50000	·86603	·57735	1·73205	1·15470	2·00000	104·720	60°
31°	54·105	·51504	·85717	·60086	1·66428	1·16663	1·94160	102·974	59°
32°	55·851	·52992	·84805	·62487	1·60033	1·17918	1·88708	101·229	58°
33°	57·596	·54464	·83867	·64941	1·53987	1·19236	1·83608	99·484	57°
34°	59·341	·55919	·82904	·67451	1·48256	1·20622	1·78829	97·738	56°
35°	61·087	·57358	·81915	·70021	1·42815	1·22077	1·74345	95·993	55°
36°	62·832	·58779	·80902	·72654	1·37638	1·23607	1·70130	94·248	54°
37°	64·577	·60182	·79864	·75355	1·32704	1·25214	1·66164	92·502	53°
38°	66·323	·61566	·78801	·78129	1·27994	1·26902	1·62427	90·757	52°
39°	68·068	·62932	·77715	·80978	1·23490	1·28676	1·58902	89·012	51°
40°	69·813	·64279	·76604	·83910	1·19175	1·30541	1·55572	87·266	50°
41°	71·559	·65606	·75471	·86929	1·15037	1·32501	1·52425	85·521	49°
42°	73·301	·66913	·74314	·90040	1·11061	1·34563	1·49448	83·776	48°
43°	75·049	·68200	·73135	·93252	1·07237	1·36733	1·46628	82·030	47°
44°	76·794	·69466	·71934	·96569	1·03553	1·39016	1·43956	80·285	46°
45°	78·540	·70711	·70711	1·00000	1·00000	1·41421	1·41421	78·540	45°
		Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	Centrads	Degrees
								Angles	

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