FOURTH EDITION.

Revised December, 1901,

BY CAPT. E. dwin A. lvin ROOT, U. S. A.

ROOT'S

Military Topography and Sketching

PREPARED FOR USE IN THE
UNITED STATES INFANTRY AND CAVALRY SCHOOL.

REVISED AND ENLARGED BY THE DEPARTMENT OF ENGINEERING,
FORT LEAVENWORTH, KANSAS.
OCTOBER, 1896.

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December 27, 1906.

MEMORANDUM:

Lessons in "Military Topography and Sketching.—Root."

(Numbers of pages are inclusive.)

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Instructor in Military Topography.

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PREFACE.

The compilation of the following pages was undertaken, in accordance with the School Regulations, with a view to bringing together in one volume so much of the subject of Topography and Sketching as may be useful to Military Officers in their varied duties, and to cover the subject as taught from a number of books, lectures, notes, etc., in the Department of Engineering at the U. S. Infantry and Cavalry School.

The arrangement of the subjects (with slight modifications to adapt it to the time and instruments available at the school) is that recommended by the best authorities, as indicated in Chapter I., which is almost entirely made up of quotations from said authorities. It is also that which has been pursued in teaching the subject at the Infantry and Cavalry School for the past eight years.

The subject matter is in certain parts almost a verbatim compilation from different standard authorities, no claim being made for originality, though the subject is one capable of great development.

I wish particularly to acknowledge my indebtedness to Professor J. B. Johnson, C.E., of Washington University, St. Louis, author of "Theory and Practice of Surveying"; to Professor Cady Staley, C.E., of Case School of Applied Sciences, Cleveland, Ohio, author of "Gillespie's Land Surveying"; to Messrs. W. & L. E. Gurley, Troy, N. Y., authors of "Manual of Surveying Instruments"; and to Col. W. H. Richards, author of "Text-Book of Military Topography," for their cordial permission to make use of whatever I might find desirable in their respective works, of which permission I have made very liberal use.

I wish also to acknowledge the courtesy and assistance received, by letters and through works published from their

offices, from the Superintendent U. S Coast and Geodetic Survey, the Director U. S. Geological Survey, and the Commissioner of the General Land Office; also the Chief of Engineers, U. S. Army, for the excellent contoured maps of the battle-field of Gettysburg.

To Messrs. W. & L. E. Gurley, Troy, N. Y., manufacturers of drawing and surveying instruments, I desire to express my obligations and thanks for their kindness in furnishing me with their electroplates of instruments with which to so completely illustrate the text.

To Captain Wm. D. Beach, 3rd Cavalry, Instructor, and 1st Lieutenant Thomas H. Slavens, 6th Cavalry, Assistant Instructor, Department of Engineering, I wish to acknowledge my thanks for assistance and encouragement during the preparation of this work.

EDWIN A. ROOT,

1st Lieutenant 19th Infantry.

U. S. INFANTRY AND CAVALRY SCHOOL,
Fort Leavenworth, Kansas,
July 4, 1896.

PREFACE TO SECOND EDITION.

Another edition of "Military Topography and Sketching" having become necessary, it has been deemed wise by the author (who has been transferred to another station) to request the Department of Engineering of the Infantry and Cavalry School to undertake the task.

The first edition has undergone a year's criticism while it has been in use at this school, as severe and uncompromising a criterion as any to which a text-book can be subjected. The result is set forth in this edition.

The methods laid down have been given a very thorough and practical field test by successive classes in this institution, where a large amount of time is devoted to the more rapid methods of reconnaissance sketching, with considerable resulting skill, as witnessed by the ability of its graduates to map, in a single day, twenty miles of road with which they are totally unfamiliar. It is the desire of the author, Lieutenant E. A. Root, 19th Infantry, that the book be given the benefit of criticism and experience incident to its use here, and that, in succeeding editions, it be kept up to date by this department of instruction.

W. D. B.

Fort Leavenworth, Kansas,
September 30, 1896.

2nd U.S.G.

LIST OF BOOKS CONSULTED IN THE PREPARATION OF THIS WORK.

Appendices, U. S. Coast and Geodetic Survey Reports.

World's Fair Pamphlets, U. S. Coast and Geodetic Survey........................*T. M. Mendenhall, Supt.*

Manual of Topographic Methods, U. S. Geological Survey Monograph XXII........................*Gannett.*

U. S. General Land Office Surveying Instructions, 1894.

Smithsonian Geographical Tables.........*R. S. Woodward.*

U. S. Army Regulations 1889.

U. S. Engineers' Topographic Field Note Book.

Theory and Practice of Surveying............*J. B. Johnson.*

Gillespie's Land Surveying...................*Cady Staley.*

Davies' Surveying.........................*Van Amring.*

The Topographer, His Instruments and Methods.*L.M.Haupt.*

Manual of Surveying Instruments......*W. & L. E. Gurley.*

Stadia Surveying......................*Arthur Winslow.*

Manual for Railroad Engineers...............*Geo. L. Vose.*

Civil Engineering......................*W. J. M. Rankine.*

Engineers' Pocket Book......................*Trautwine.*

R. E. Aide-Memoire.

Topographic Drawing and Sketching, *Lt. H. A. Reed, U. S. Army.*

Map and Plan Drawing...........*C. Cooper King, R. M. A.*

Military Surveying ....*Capt. G. H. Mendall, U. S. Army.*

Notes on Military Surveying and Reconnaissance, *Lt.-Col. Wm. Paterson.*

Text-Book of Military Topography....*Col. W. H. Richards.*

Elementary Military Topography.........*Capt. J. Demagel.*

Rapid Sketching and Reconnaissance......*Capt. W. Verner.*

Military Topography ....*Capt. W. Verner.*

Military Sketching................*Hutcheson & McGregor.*

Range-Finders ....*Capt. J. G. D. Knight, U. S. Army.*

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Topographic Surveying.

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PART I.—TOPOGRAPHIC SURVEYING.

CHAPTER I.

GENERAL PRINCIPLES.

MILITARY TOPOGRAPHY is the *determination and representation*, by conventional signs and symbols, of such forms and features of a limited portion of the earth's surface as have special reference to its adaptability for military purposes. It includes undulations and accidents of ground, steepness of slopes, relative heights of commanding points, and all occurring incidents of water; the natural growths and character of ground, as forests, prairies, and desert lands; the features of culture and artificial structures of mankind, as crops, groves, canals, railways, roads, bridges, houses, etc., the whole forming a complete and useful picture.

The distinguishing characteristic of topography is the graphical representation, *on a plane surface*, of relative elevations and depressions, as they would appear to an observer looking vertically down from points above the plane.

Topography comprises so much of art that the question of personality enters largely into the facility of expressing it, and topographic surveys involve such varied processes, that it is difficult to make hard-and-fast rules to govern them. It has been said by one of the most distinguished scientists of our country, "that while but little more could be done to perfect the principles of other departments of mathematical science, topography was still susceptible of development and improvement." Each topographic position presents a problem as varied and often as difficult of solution as the phases of nature with which it has to deal.

*In topographic surveying,** as usually understood, all observations, measurements, drawings, etc., are more or less accurately made with suitable instruments by which perfect ideas are conveyed.

Future military operations will require reliable information for their execution, and topographic surveys and maps that do not meet this requirement will fail to supply a demand becoming more and more necessary. They should show everything that might have any influence on the result of a battle or the conduct of a march.

The ordinary civil maps of the country may be useful for such military purposes as planning campaigns and executing strategical movements, provided they show its general forms and features, and the locations of its principal cities, towns, highways, etc. But in planning a battle, in regulating a march, and in everything pertaining to minor operations of war, civil maps will seldom suffice. Whatever information they may contain, of a nature to be useful, will usually be deficient in details. They are generally made on too small a scale to be immediately available. Much that is of no special military value is shown, while many things that are of the greater military importance are omitted. Such maps may also be unreliable from changes having occurred since they were made. A slight fold in the ground, a trail or a small stream may be of the greatest tactical value, for a time, either for the offensive or defensive, and in proportion to the *tactical importance* of objects should the information concerning them be more detailed.

To obtain and report the information especially needed for

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*Surveying is a general term including all that is comprised in the art of making the field observations and measurements for determining lengths, directions, positions, sizes, shapes, figures, areas, volumes, or movements on the earth's surface; accompanying it are usually the notes of such observations and measurements, and all necessary calculations and drawings pertaining thereto. It is variously divided, according to its object, etc., into City, Land, Mining, Geodetic, Chorographic, Geographic, Hygrographic, Hypsometric, Orographic, and Topographic Surveying, and possibly others.

military purposes is one of the duties of military officers, and to be able to do this they must understand the principles of military topography, which is a most important branch of military art.

To be an expert topographer one must see everything as it actually exists and be able to represent it graphically, which requires a careful training of the senses and faculties in the performance of the work. He must first study and have experience with precise and accurate instruments, progressing step by step, inculcating accurate methods and results in his first efforts, thus gradually developing his faculties in correctly interpreting and representing what is seen.

Not everyone can become an expert topographer, no matter how diligently he tries, but much can be done toward understanding the principles of topography and being able to make use of the work of others.

Limit of error. It is a recognized and unavoidable fact in surveying that no measurement of any kind can ever be made absolutely exact. The most excellent instruments have some defects; the most careful observers are liable to some slight errors. The purpose of the survey determines the greatest amount of deviation from exactness allowable, which deviation is called the "limit of error."

As various methods of operation and various instruments upon which the exactness attainable greatly depends, may be used in making a survey, it is necessary for one to be familiar with all the causes of error, their relative amounts and importance, and how best to eliminate them. Every point that might arise during the progress of the work should be so thoroughly considered in the beginning as to enable a correct judgment in regard to it to be formed at once.

When neither time nor the necessary instruments are available for topographic surveying, then the information desired for military purposes must be obtained by topographic sketching. Circumstances having rendered this the only method possible to employ, it will from necessity have to suffice.

In topographic sketching, which is merely a less accurate method of *topographic surveying* than referred to above, the observations and measurements are generally made with instruments only approximately accurate, or they are simply estimated, and the representations are usually drawn free-hand.

To record his observations in time of war the military topographer will most frequently employ the method of sketching, especially in reporting upon the tactical capabilities of ground in anticipation of some military operation.

These may be sketches of roads previous to marching over them, sketches of outpost lines and vicinity, or sketches of positions contemplated to be occupied, etc.

The exigencies of service will require the work to be performed wi h rapidity, in order that the information may be submitted in time to be acted upon. Military characteristics must be rapidly analyzed and only objects of tactical importance reported. Especially must everything that would convey false impressions be avoided

To make an approximately accurate sketch under such conditions is more difficult than making a topographic survey with accurate instruments and unlimited time, and can only be acquired by careful practice in it.

No process has yet been devised by which one may learn to sketch accurately without first using instruments and taking measurements. The method adopted in the following pages is that pursued in teaching the subjects at the U. S. Infantry and Cavalry School, and endorsed by eminent topographers both in America and Europe. This consists in first examining the principles upon which large surveys are made and the instruments and methods used, in which every care is taken to obtain accurate work, before dealing with the more rapid and necessarily less accurate sketches executed for military or other purposes of a temporary character.

These principles are common to all methods of map or plan drawing, and it is only by first ascertaining and seeing the value of them that the rougher and more hasty methods can be understood and successfully applied. This is due not merely to the fact that these principles have been most exhaustively examined and worked out, but because they are very simple when understood and are applicable to every case

Triangulation. In topographic surveying the relative positions of widely separated points and the directions and lengths of the lines joining them are determined by the method known as *Triangulation.* This consists in obtaining a skeleton work of exact distances, directions, and elevations, on which to hang a more detailed survey of the country, by first dividing a portion of the earth's surface into a number of nearly equilateral triangles of as large size as possible, these in turn being subdivided into smaller ones, until enough points have been fixed by the vertices of the triangles, and their elevations determined, to permit an accurate representation of the forms and features being made.

First, a single line forming one side of one of the triangles is measured with extreme care. *Second*, the angles of the triangles are next measured and the distances between the connecting points computed, one from another, throughout the successive triangles, proceeding in regular order from the measured line, which is called the "base line." *Third*, the relative elevations of the points are determined by leveling.

It is usually desirable to know the *latitude* and *longitude* of one or more of the points of the system, though they are sometimes located in smaller surveys by a simple reference to some well-known point within the area surveyed. The azimuths of some of the lines are also determined.

Triangulation systems are of all degrees of magnitude and accuracy, depending upon the objects they are to serve, etc., the sole difference in the method of procedure being in the degree of accuracy with which the base-line and angles are measured and the location of the points determined.

In what are known as *primary systems*, where great areas are covered and the highest degree of accuracy is secured, base-lines from five to ten miles in length are accurately measured, and many refinements are introduced in the instruments used, in the methods employed, and in the reductions, which would be useless or needlessly expensive in smaller systems.

In *secondary* and *tertiary systems* the accuracy may vary from 1 in 50,000 to 1 in 5,000, with base-lines in the former from two to three miles in length, and in the latter from one-half to one and a half miles in length.

The large *primitive triangles* are most carefully constructed to fix the most prominent points, as any errors made at this time would be continually repeated. The angles of the triangles are measured many times and a series of most careful means secured. A net-work of smaller triangles is then measured, elevations determined, and the details of the ground filled in with the more portable instruments.

Until one understands the steps by which the most accurate work can be done, the rapid sketches cannot be properly made, and, if attempted, will be neither trustworthy nor effective from want of carrying out important principles. Hasty work implies only that the operator is sufficiently master of the subject to know when and where he can afford, without detriment to the work, to neglect certain points and pay more attention to others. A draughtsman may, if he has natural talent, produce an effective picture, but unless it is based on a knowledge of the principles of accurate surveying, it will be sure to fail in some important features.

The necessity in accurate work for measuring a base-line and for triangulation to determine principal points is equally necessary in sketching, whenever practicable. But one must know when he can dispense with these and trust to traversing the offsets only. Without a knowledge of the actual value and amount of work to be done by these methods, he cannot tell which is the readiest way of representing on paper in a limited time the information to be shown.

The use of accurate instruments trains the eye to see forms and features in their correct relative positions, sizes, and shapes, and to judge angles, slopes, and distances as they are.

Plotting the notes trains both the hand and the eye in the correct graphical representation of them. The actual instrumental measurement of distances in topographic surveying will assist one in estimating them in sketching. The practice in actually tracing contours with instruments, by which the shape of hill features, etc., is made evident, educates and assists in judging them by eye.

From the constructive point of view, a map is a sketch, corrected by locations. The work of making locations is *geometric*, that of sketching is *artistic*. This definition is applicable to all maps, whatever their quality or character. However numerous the locations may be, they form no part of the map itself, but serve only to correct the sketch, while the sketch supplies all the material of the map. The correctness of the map depends upon four elements: 1st, the accuracy of location; 2d, the number of locations per square inch of the map; 3d, their distribution; 4th, the quality of the sketching. The first element depends upon the economy of the work; the second and third upon the character of the country; the fourth upon the artistic ability of the topographer; this latter is the most important, and most difficult to meet.

The immediate object then in topographic surveying is to determine the locations of such a number of points on the earth's surface as will enable one to make a topographic map representing it.

The location of a point in a plane is known when its distance and direction from one or more given points of that plane are determined.

The location of a point anywhere is known when its horizontal distance and direction from, and its elevation above or depression below, one or more given points are determined.

The location of a straight line is known when its extreme points are located.

The distance of a point from a given point is determined by measuring the length of the horizontal projection of the straight line adjoining them.

The direction of a point from a given point is determined by measuring the horizontal angle between the straight line joining the points and a line of known direction through the given point.

The elevation of a point above or depression below a given point is determined by measuring the vertical distance between the level surfaces which contain them.

CHAPTER II.

DRAWING MATERIALS, INSTRUMENTS, AND THEIR USE.

DRAWING PAPERS.—The selection of the proper kinds of drawing papers very materially assists one in producing good work. Good drawing paper should be strong, be of uniform thickness and surface, stretch evenly, neither repel nor absorb liquids, admit of considerable erasing without detriment to its surface, should not become either brittle or discolored by reasonable exposure and age, and should not buckle when stretched or when inks or colors are applied to it.

Hot Pressed paper is used for fine line drawings. *Not Hot Pressed* for water colors. *Rough* for bold drawings and sketches.

Drawing paper may be obtained mounted on muslin.

Tracing Paper and *Vellum Cloth* are very thin and transparent, much used in copying maps and drawings, and in making blue prints. They are made in both sheets and rolls of various sizes and lengths.

Transfer Paper is a thin, tough paper, having one side covered with black, blue, vermilion, or other colored material. The colored face is laid next the paper on which the transfer is to be made, the copy placed on the back, and the lines followed with a stylus, which produces the transfer.

Profile Papers (Fig. 1), in sheets, rolls, or made into

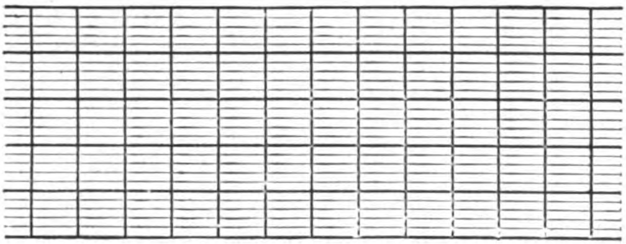


FIGURE 1.

books, are used in plotting profiles directly to scale. The distance apart of the horizontal line is usually from a fifth to a tenth of that of the verticals; this is because vertical distances are usually small as compared to horizontal, and in order to show small differences of elevation. It is called the exaggeration of the vertical scale.

Cross-Section Papers (Fig. 2), in sheets, rolls, or made

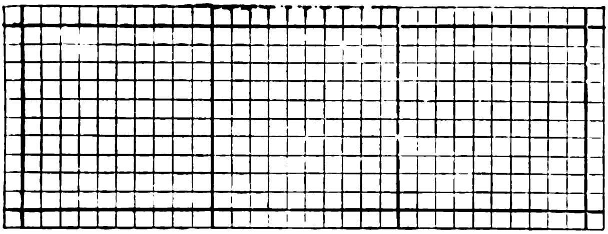


FIGURE 2.

into books, are also used in plotting directly to scale. The lines, both horizontal and vertical, are the same distance apart.

Both the above papers are ruled to parts of inches or metrically, and every fourth, fifth, or tenth line is made heavier for convenience in counting the spaces.

INKS.—Liquid indelible drawing inks are made in all colors and put up in bottles ready for use. Waterproof drawing inks are useful when tinting is to be done. India ink is, however, the best and most reliable. It comes in sticks (Fig. 4).

FIGURE 4.

and is prepared for use by pouring a little water in a saucer or ink-dish and then rubbing one end of the stick in it until a jet black is produced.

It is the most suitable ink for brush work. For drawings to be photographed the ink should be dead black. For those on transparent paper, to be reproduced by the blue process, it is recommended by some to add a few drops of crimson lake.

To erase ink lines, if still wet, first use blotting paper, then let the line dry; put the paper on a hard smooth surface and, with a needle point or fine-pointed sharp knife blade, carefully remove the line by lightly scratching it back and forth, disturbing the surface of the paper as little as possible; afterwards the rubber eraser is used over it and the surface of the paper rubbed smooth with bone, ivory, or the thumb nail.

DRAWING INSTRUMENTS.—In the selection and purchase of drawing instruments it is advisable to get only the best, even at a greater cost, although one who intends to use them only occasionally and to work with them without particular reference to the excellence of execution might get along with a cheaper grade than one anxious to accustom himself to doing the best and most accurate work.

The instruments most necessary in plotting notes of a survey are *thumb tacks* (Fig. 5), *pencils*, *pens*, *a straight-edged*

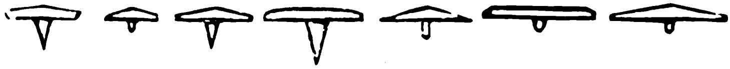


FIGURE 5.

ruler, *right-angled triangle*, *T-square*, *dividers*, *compasses*, *protractor*, and *a scale of equal parts*. Besides these, there are many others often conveniently used, but not absolutely essential.

PENCILS.—The best for plotting are the hard kinds corresponding to Faber's Siberian HHHH and HHHHHH, especially for drawing fine lines and marking points. For most kinds of work, a sharp-pointed pencil is used. For drawing long straight lines, a chisel-pointed pencil should be used to produce a line of uniform breadth. For sketching and filling in conventional signs, softer pencils are preferable, such as correspond to Faber's HB or H. To keep the points always in good condition, one should have a piece of fine emery- or fine sand-paper at hand for that purpose, being careful to remove any lead dust from the point before using. Much more depends upon the proper sharpening of a pencil, and afterwards keeping it so, than is commonly supposed. Most drawings to be inked are first constructed in pencil, the lines being made with as little pressure and as fine as is possible to show distinctly.

In erasing lead pencil lines, first use bread crumbs and then a chisel-edged piece of rubber.

PENS.—For ordinary free-hand work, common writing pens, as Gillott's 303 or Spencerian No. 1, may be used. For very fine work, the small crow-quill pen is better.

For drawing lines with the aid of a ruler or irregular curve, the Right Line or Drawing Pen (Fig. 7) is used. This

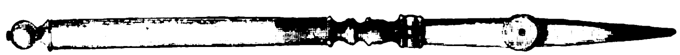


FIGURE 7.

consists of two steel points, one of which is hinged at its base to regulate the width of the line, and to facilitate the cleaning of the pen. The whole is fixed in a handle. In the larger sizes there is a needle point in the base of the handle which can be unscrewed from the points. The points should be well polished, of exactly the same length, rounded and moderately sharp, but not so much so as to cut the surface of or leave its impress on the paper.

To draw a straight line with a right line pen, the points are set so that a very faint streak of light can just be seen between them. Fill with ink by dipping a common pen or a narrow strip of paper in the ink and inserting it between the points, care being taken not to smear any on the outside. *The pen should never be filled by dipping it into the ink.* If the ink does not flow readily from the pen, pass the edge of a piece of paper between the points. Test the pen on a separate piece of paper to see if the line is of the required thickness and that the ink flows smoothly. India ink thickens very rapidly and clogs the pen, which must be frequently cleaned. and the edges of small pieces of paper be constantly passed between the points to insure smooth, clean lines. Having placed a ruler, beveled side next the paper, enough below the position of the desired line to enable the pen being held vertically on the position of the line, with the side of the pen resting lightly against the ruler, incline the pen about 30° in the direction of the movement, and with a steady, uniform motion draw the line, keeping both points touching the paper equally pressing against the ruler only hard enough to guide the pen, care being taken not to incline the pen in a direction perpendicular to the line. If a line is to be prolonged, a slight space should be left between the two parts, to be afterwards filled in with a common pen. Lines should be drawn from given points rather than towards them.

For drawing curved lines, free-hand, a curved pen (Fig. 8)

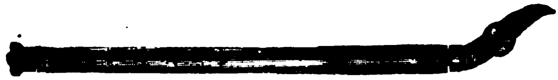


FIGURE 8.

is used. It differs from the right line pen only in the points being curved and fastened to a rod, which extends through and turns in the handle. By a nut on top, the pen may be fastened and used as a right line pen. In using it, the handle is held perpendicular to the paper and moved in the direction of the required curve which the pen follows. This is useful in putting in contours. When the points become dull from use, they may be resharpened on a fine-grained oil-stone.

The Border or Road Pen (Fig. 9) may be used for drawing

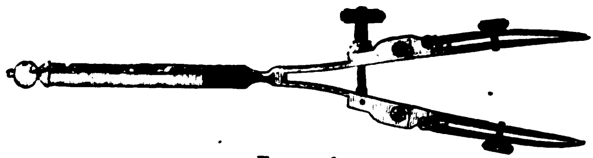


FIGURE 9.

a wide, heavy line by filling between the two pens with ink, or for drawing two lines at the same time by filling the two pens only.

Dotting Pens (Fig. 10) consist of a set of wheels with

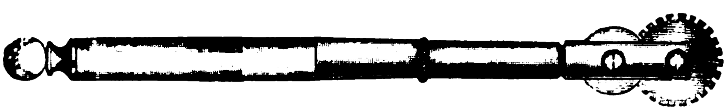


FIGURE 10.

teeth of different breadths for drawing evenly-spaced dotted or dashed lines.

Drawing pens should be carefully cleaned before being put away.

RULERS.—*The Straight-edged Ruler* (Fig. 11) may be

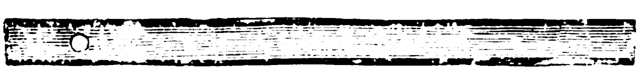


FIGURE 11.

made of hard wood, rubber, or steel; the edges should be perfectly straight and smooth. A convenient size is from 18 to 24 inches long by 2 inches wide, preferably with one edge beveled. Both wood and rubber warp, and their edges must frequently be tested. The simplest way of doing this is to draw a fine line connecting two points, distance apart the length of the ruler, then change the ruler end for end and, using the same edge, draw another line connecting the two points. If the two lines coincide throughout, the edge is straight; otherwise not.

For drawing a straight line between two points farther apart than the length of the ruler, two rulers may be laid side by side (Fig. 12), or a thread may be stretched between the

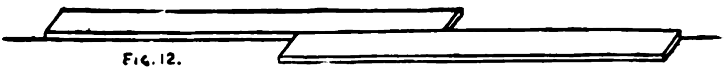


Fig. 12.

two points and other points marked under the thread at a less distance apart than the length of the ruler. A carpenter chalks his line and then snaps it, but to do this accurately requires the thread to be raised truly perpendicular to the surface, then released.

In determining the point of intersection of lines, the lines should extend a little distance each side of the point.

Irregular Curves (Fig. 13), of which there are many forms, are used in drawing occasional curves connecting straight lines, etc. To produce the effect of a smooth continuous line, the curve must be tangent to the line at the point of juncture.

FIGURE 13.

The Triangle (Fig. 14) may be made of the same material as the ruler and, for convenience, should have its longest side about 12 inches in length, with one right angle and two acute angles of 30° and 60° respectively. The other common form (Figure 15) has both acute angles 45°. The ruler and triangle together may be used for drawing parallels

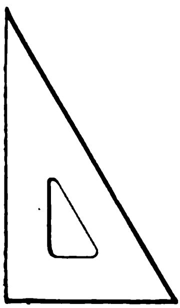


FIGURE 14.

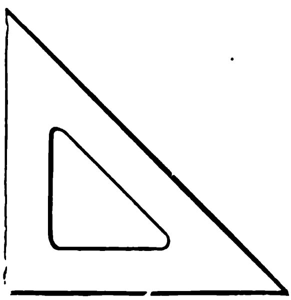


FIGURE 15.

and perpendiculars. Right angles should be constructed with compasses if the work is to be accurate.

To Draw Parallels.—Let it be required to draw through *a* (Fig. 16) a line parallel to BC. Place one edge of the triangle on the line BC, and an edge of the ruler against the edge DE of the triangle. Hold the ruler firmly in this position, then slide the triangle along the ruler in the direction of the point through which the parallel is to pass, keeping its edge against the ruler until the edge which passed through B and C is on the point, as at *a*. Hold it here and draw a line along this edge. This will be the required parallel. Likewise through any number of other points, as *b* and *c*, lines may be drawn parallel to a given line. This method is much used in the construction of scales.

To Draw Perpendiculars (Fig. 16).—Place the longest side of the triangle on the given line BC and an edge of the ruler against either of the other sides of it. Hold the ruler firmly in this position, then place the triangle with the third side against the ruler and the longest side will be perpendicular to the given line. Or (Fig. 17), place one of the shorter sides of the triangle against the given line AB, an edge of the ruler against the longest side, then the other shorter side will be perpendicular to the given line. By sliding the triangle along the ruler any number of parallel perpendiculars may be drawn, or, through any point on or off

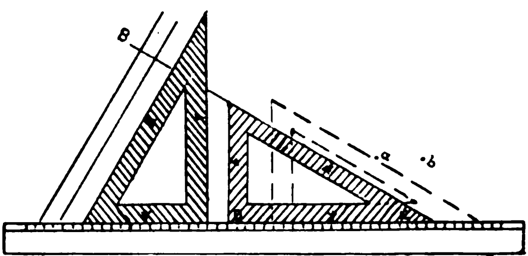


FIG. 16.

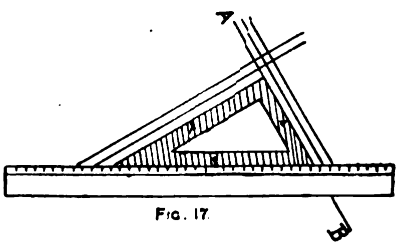


FIG. 17.

the given line, a perpendicular to the given line may be drawn.

***A T-Square* (Fig. 18) consists of a straight-edge, called a**

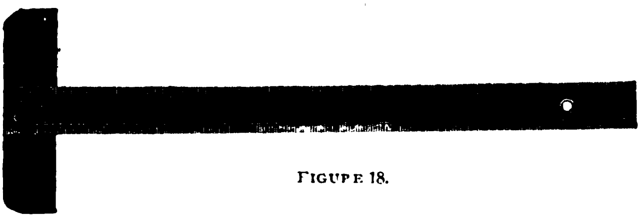


FIGURE 18.

blade, from 15 to 48 inches long, laid into a head at right angles to it; one face of the head being flush with the face of the blade, and the other face of the head extending beyond the blade. They are also made with what are called shifting heads (Fig. 19), so that the lower half of the head may be

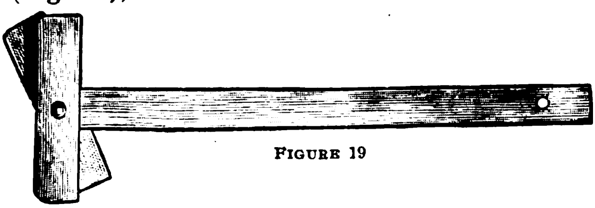


FIGURE 19

fixed at any desired angle with the blade; also with protractor head and vernier on blade (Fig. 20). T-squares are much

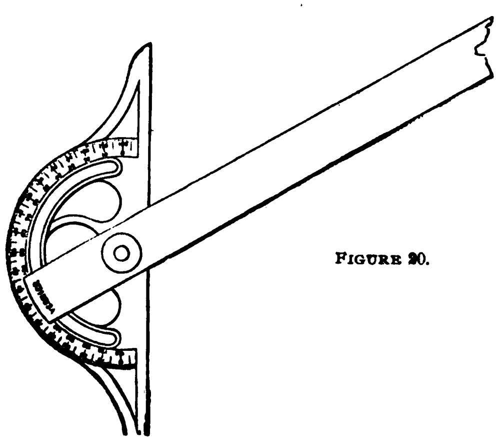


FIGURE 20.

used in drawing both parallels and perpendiculars, particularly at distances apart which could not be conveniently done with ruler and triangle.

To Draw Parallels (Figure 21).—Place one edge of the blade on the given line, then place an edge of a ruler against the outside of the head and, holding it firmly, slide the T-square along the guiding ruler until the edge of its blade passes through the required point *a*. Holding it here, draw a line along the edge and it will be the required parallel.

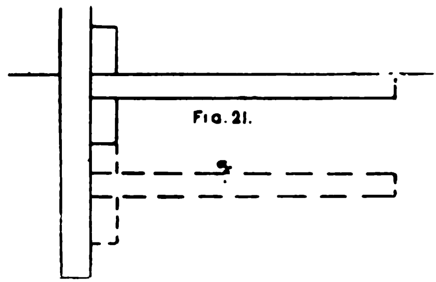


FIG. 21.

The T-square finds its greatest convenience when using a drawing-board on which the paper is fastened, and especially if the T-square is of the kind with the shifting head (Fig. 22). In this case an edge of the blade is placed on the given line, the head turned until the inner edge of the lower half of the head rests firmly along the edge of the drawing-board; then, by sliding the T-square along the edge of the drawing-board, lines parallel to the given line may be drawn along the edge of the blade. The object of one face of the head projecting beyond the blade is to form a shoulder which can be used against the edge of drawing-boards and tables while the blade lies flat on the paper. Parallels, to a line making any angle with the edge of the drawing-board, can thus be drawn.

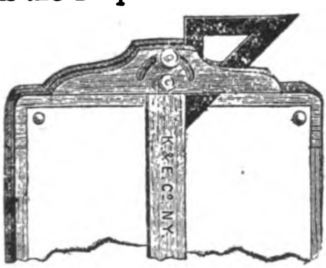


FIGURE 22.

Folding Parallel Rulers (Fig. 23) consist of two straight-

FIGURE 23.

edges connected near their ends by two metal strips of equal length, so that when the rulers are separated their edges remain parallel.

Rolling Parallel Rulers (Figure 24) consist of a heavy

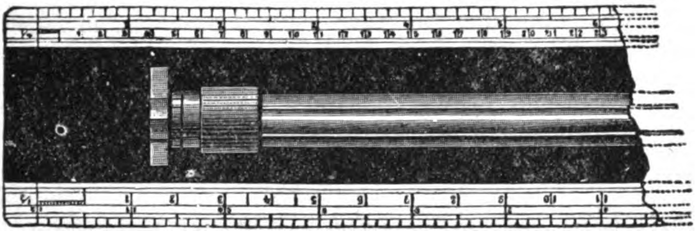


FIGURE 24.

straight-edge with rollers of equal diameter near the ends, by means of which it can be moved parallel to itself.

A Section Liner (Figs. 25 and 23) is an arrangement of springs and adjustable stops for attaching to a ruler and triangle, or a straight-edge by which the latter is carried over equally spaced intervals at each movement, for drawing parallels at equal distances apart. With practice the eye soon becomes accustomed to judging the equal spaces between the

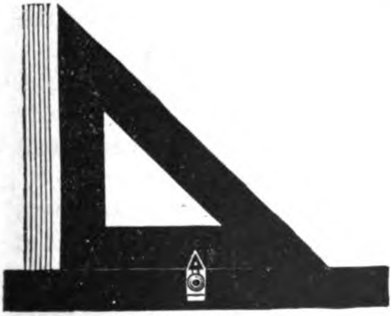


FIGURE 25.

parallels, but until it does so, in the absence of a section liner, one can make use of a graduated straight-edge and a triangle.

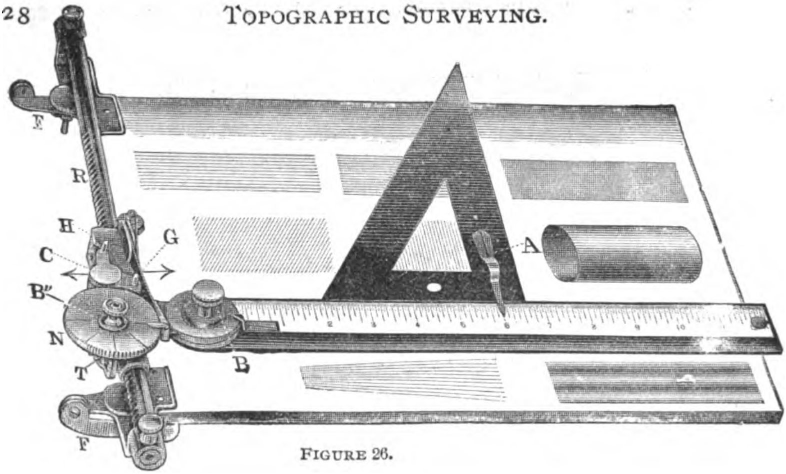


FIGURE 26.

as in Figs. 16 and 17, making an index on the triangle to coincide with the graduations on the rulers.

DIVIDERS (Fig. 27) consist of two legs movable about a

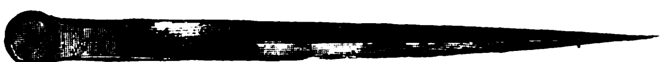


FIGURE 27.

point, so the legs can be opened and closed. The ends of the legs have fine points, which are used for spacing, laying off, measuring and comparing distances. In several varieties one point may be moved slightly toward the other for more accurate setting, by means of a screw on the side acting on a spring.

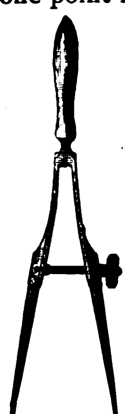


FIGURE 28.

Spring Bow Dividers (Fig. 28) consist of a single piece of steel shaped and bent into the form shown. They are used where very small distances are in question.

Proportional Dividers (Fig 29) have both ends of each leg pointed, and a movable point of rotation, such that the distance apart of one set of points may be in any ratio to that of the other set. They are used in copying, transferring, and comparing at different scales.

Consecutive distances on a line should be laid off from a single starting-point on it, and not one distance after another from the last point determined

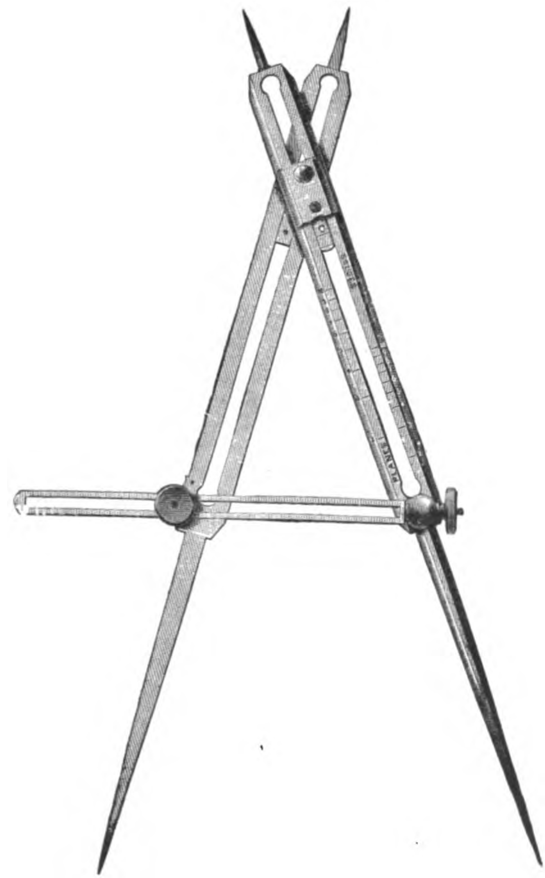


FIGURE 29.

Spring Bow Pens (Fig. 30) and *Spring Bow Pencils* (Fig. 31) are used for describing arcs of very small radius.

Other forms of the same are shown in Figs. 32 and 33; in the first two, the point and pen both turn in

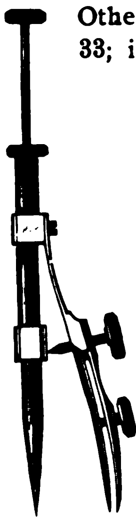


FIGURE 32.

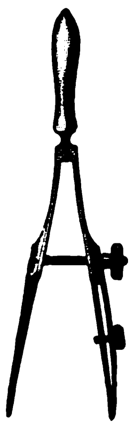


FIGURE 30.

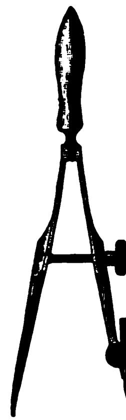
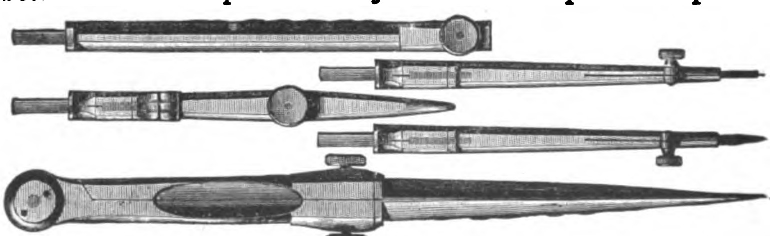


FIGURE 31.

FIGURE 33.

describing the arc; in Figure 32, the rod remains stationary while the pen or pencil turns around it.

COMPASSES (Fig. 34) are used to describe arcs of circles. They are made with joints in the legs so that the steel point may be removed and a line pen, pencil holder, or dotting pen may be inserted in its stead, and in connection with any of these a lengthening bar may be used. It describing arcs it is better to bend the parts at the joints until the points are per-



FIGURE

pendicular to the paper; then, holding the top of the compass between the thumb and forefinger, draw the curve with a continuous sweep. The lengthening bar is used between the pen or

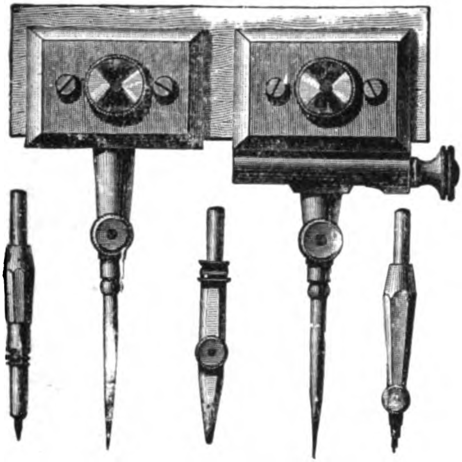
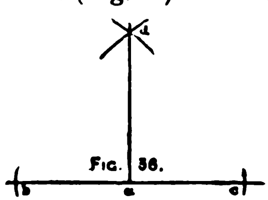


FIGURE 35.

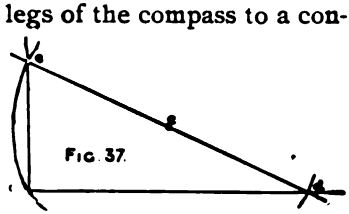
pencil holder and one leg of the compass in describing arcs larger than could be reached without it.

Beam Compass (Fig. 35) consists of two point holders for attaching to a bar of some kind to describe arcs with radii greater than the hand compass with lengthening bar will reach. The holders may be set any distance apart, and in them may be inserted points, line pen or pencil.

PROBLEMS WITH COMPASS.—*To erect a perpendicular to a given straight line at a given point on that line* (Fig. 36). From the given point *a* to a center, with any convenient radius, describe arcs intersecting the line at equal distances *b* and *c* on each side of the point. From the points of intersection of the arcs and line as centers, with a radius greater than the former radius, describe two arcs intersecting each other at *d;* join this point of intersection by a straight line with the given point, and it will be the perpendicular required.



To erect a perpendicular to a given straight line at the end of the line (Fig. 37). Spread the legs of the compass to a convenient radius. Place the pencil on the end of the line *a*, swing the needle point about half the radius length above the line to *c*. With this latter point as a center, describe an arc passing



through the end of the line and intersecting it in another point, *d*. Through this latter point and the center of the arc, draw the diameter of the circle and from the other end of the diameter *e* draw a straight line to the end *a* of the given line. This will be the perpendicular required.

To draw a perpendicular to a given straight line, from a given point without that line (Fig. 38). From the given point *a* as a center with a radius greater than the distance to the line describe an arc cutting the line in two points, *b* and *c*.

From these points as centers, with a radius greater than half the distance between them, describe arcs intersecting each other at *d* on the opposite side of the line from the given point; draw a straight line from the given point to the point of intersection *d* of the arcs and it will be the perpendicular required.

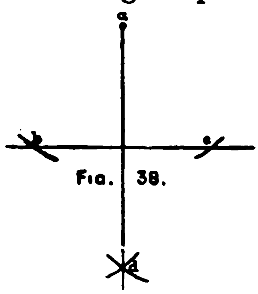


FIG. 38.

Through a given point, to draw a straight line parallel to a given straight line (Fig. 39). From the given point *a* as a center, with a radius greater than the distance to the line, describe an arc *cd* intersecting the line at *c*. From this intersection as a center, with the same radius describe another arc intersecting the line at *b* and passing through the given point *a*. Take the length of the chord of this arc from the point *a* to *b*, in the compass, and from the intersection *c* of the first arc with the line as a center, describe an arc on the same side of the line as the given point, intersecting at *d*. Through the point thus found and the given point draw a straight line *ad* and it will be the parallel required.

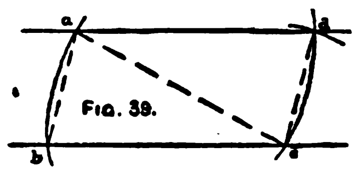


FIG. 39.

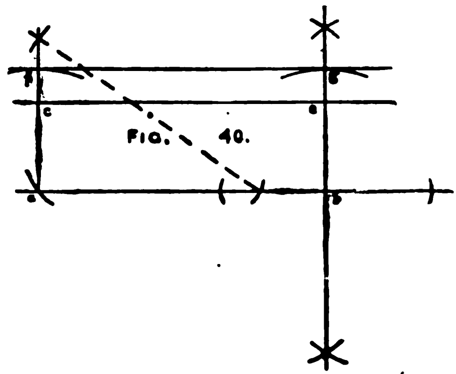


FIG. 40.

To draw a line parallel to a given line (Fig. 40). At any two points of the line, as *a* and *b*, erect perpendiculars, and on these lay off equal distances *ac* and *be* from the line. Through these points draw a straight line, and it will be a parallel to the given line; or, with any two points of the line as centers, describe arcs on the same side with the same radius *af*, *bg*, and draw a right line tangent to the arcs.

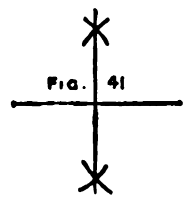


FIG. 41

To bisect a given straight line (Fig. 41). From the extremities of the line as centers, with a radius greater than half the length of the line, describe arcs intersecting on both sides of the line. Join points of intersection by a straight line and it will bisect the given line.

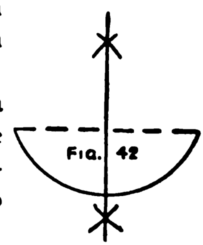


FIG. 42

To bisect a given arc (Fig. 42). Draw a straight line joining the extremities of the arc, this will be its chord; then draw a perpendicular bisecting this chord, which will also bisect the arc.

To bisect a given angle (Fig. 43). With the vertex *a* of the given angle as a center, describe an arc with any convenient radius intersecting the sides of the angle at *b* and *c*. Join the points of intersection by a straight line, construct the perpendicular *ad* bisecting it, which, if prolonged, will also bisect the angle.

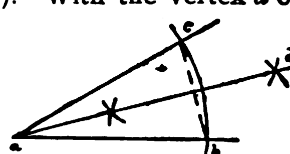


FIG. 43.

To construct an angle equal to a given angle (Fig. 44).

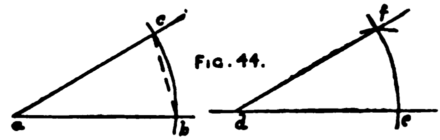


FIG. 44.

With the vertex *a* of the given angle as a center, with any convenient radius, describe an arc intersecting its sides at *b* and *c* and draw its chord. Draw a straight line, and, from a point *d* on it with the same radius, describe an arc as before, intersecting the line at *e*. From this point of intersection as a center, with a radius equal to the chord of the first arc, describe an arc intersecting the last arc at *f*. Join this point *f* and the point *d* on the line first used as a center, and the angle between the lines will be equal to the given angle.

To divide a given straight line into equal parts (Fig. 45). From one extremity *a* of the given line *ab*, draw an indefinite straight line *ac*, making any convenient acute angle with it. Set the legs of the compass at any convenient distance apart, and from the vertex *a* of the angle, on the indefinite line, lay off this distance as many times as there are to be equal parts in the given line. Join the last point *d* so found by a straight line with *b*, the other extremity of the given line, and through the points of division on the indefinite line draw parallels to *bd*. These parallels will divide the given line into the required number of equal parts.

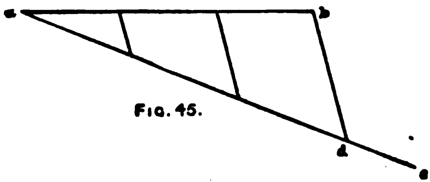


FIG. 45.

To divide a given straight line into parts proportional to given straight lines (Fig. 46). From one extremity *a* of the given straight line *ab*, draw an indefinite straight line *ac*, making any convenient acute angle with it. Beginning at the vertex of the angle, lay off in succession on the indefinite line the lengths *ad*, *de*, *ef*, *fg*, and *gi* of the given

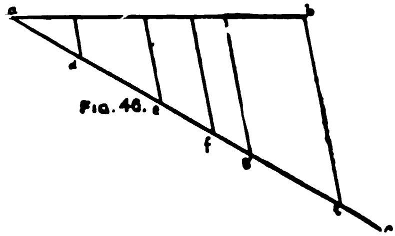


FIG. 46.

lines. Join the last marked point *i* by a straight line with *b*, the other extremity of the given line. Through the other marked points draw lines parallel to *ib*, and these parallels will divide the given straight line into parts proportional to the given parts.

PROTRACTORS are used in drawing and plotting to lay down and measure angles on paper, and are made rectangular, semi-circular, and whole circle.

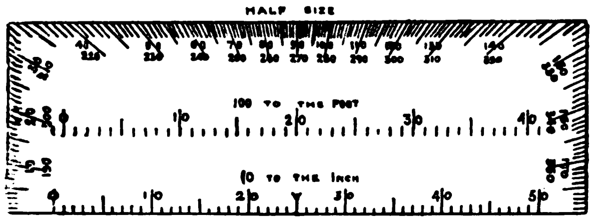


FIG. 47.

The rectangular protractors (Fig. 47) are made of wood, celluloid, and ivory, for ordinary use about 6 inches long and 2 inches wide. The middle of one side, corresponding to the diameter of a circle, is marked as the center, and on the other side and two ends are the degree and half degree divisions corresponding to the semi-circle. The divisions are numbered both ways from each end of the diameter from 0° to 180° or otherwise. It is nothing more than a semi-circular protractor cut down to a rectangular shape, and is used in the same manner as one. On some are scales of equal parts and a scale of chords.

The semi-circular protractor (Fig. 48), made usually of German silver, brass or horn, has its semi-circumference divided into degrees and half-degrees and numbered both ways from the extremities of the diameter from 0° to 180°. The middle of the diameter line is marked to indicate the center.

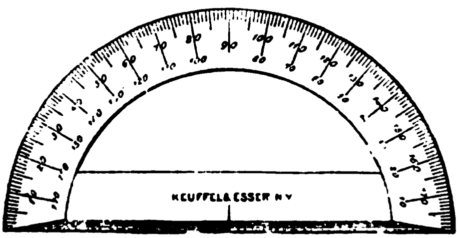


FIGURE 48.

Some protractors are graduated from 0 to 180 around the outside, and from 180 to 360 inside. These are particularly convenient in projecting angles of any size as in sketching with the box and prismatic compasses, as will be explained.

In whole circle protractors the graduations are from 0 to 360 around the circumference, the center of the circle being marked by a cross on a piece of horn.

Both semi-circular and whole circle protractors can be had with movable arms (Fig. 49) extending beyond the cir-

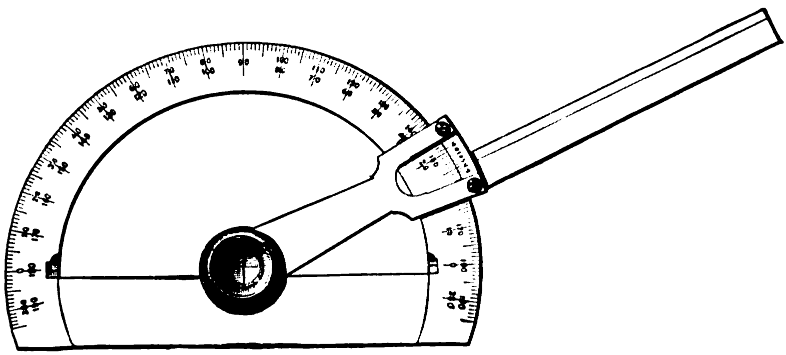


FIGURE 49.

cumference, and also with verniers reading as small as 1 minute, to increase the precision of plotting.

Protractors from 6 to 14 inches in diameter, printed upon drawing paper, are very convenient for plotting. Angles are plotted directly from the graduated circle on the paper.

In using the protractor to lay off an angle from a point on a straight line, lay the diameter on the line with the center at the point; then, with a sharp pencil, needle, or pin, make a dot on the paper opposite the required reading on the circumference. Remove the protractor and through the two points draw a straight line, and it will make with the given line the required angle.

TO PLOT ANGLES FROM THE SCALE OF CHORDS.—From the given point on the line as a center describe an arc with the chord of 60°* as a radius intersecting the line. From this point of intersection as a center, with a radius equal to the chord of the required angle, describe an arc intersecting the first arc, through which intersection and the given point draw a straight line, and it will make the required angle with the given line.

SCALES OF EQUAL PARTS.—There are paper, wood, ivory, and metal scales of equal parts manufactured for almost every conceivable purpose. Fig. 50 shows one style of flat ivory scale 6 inches long.

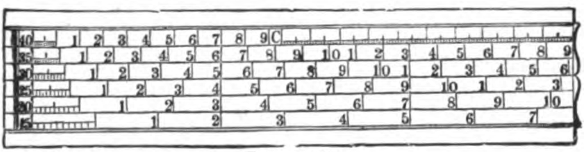


FIGURE 50.

The triangular scale (Fig. 51), made of boxwood or nickeled brass tubing, from 12 to 24 inches long, has six edges, all

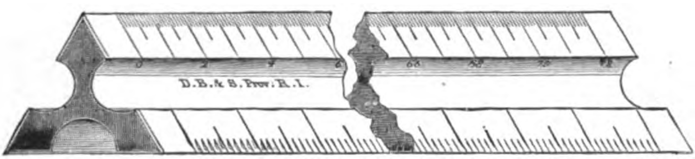


FIGURE 51.

graduated. Each edge is divided into inches, and the inches on the various edges into 10ths, 20ths, 30ths, 40ths, 50ths, and 60ths of an inch, and so numbered at the middle of the respective scales. Such scales, however, have but a limited use, on account of the great variety in the sizes of the scales and units of measure. Hence, in most cases it becomes necessary to construct the proper scale for the work in hand.

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* The chord of 60° is always equal to the radius and may be taken as 5, 10, or any number of inches. The chord of an angle is equal to twice the sine of half the angle multiplied by the assumed radius.

CHAPTER III.

SCALES OF MAPS.

A map or plot of a survey is usually a representation, on paper or other material, *on a reduced scale*, of the surface surveyed. It is necessary to thoroughly understand in the beginning the relation that is to exist between such map or plot and the surface, and the manner of intelligently representing such relation. This is done by what are called "*scales*."

The scale of a map is the ratio which any linear dimension of the map bears to the corresponding dimension of the surface represented.

In a topographic survey the first consideration is the purpose it is intended to subserve; knowing this, the next consideration is that of the scale and, relatively, the cost.

In such a survey the question of scale becomes a most important one in the economy of the work. In its practical operations, material objects and definite natural features can be more readily and accurately determined than what may be called the vertical measurements of the survey, for the obvious reason that the first mentioned are visible objects that can be observed upon, while the latter are imaginary and indefinite lines represented by contours, unless, as in special cases, the contour lines are "run out" by actual level stations. For such work the scale should not be so small as to embarrass the topographer by too much minutiæ in representation, nor so large as to affect the accuracy of his judgment of distance, proportion, and configuration of details that may not require special measurement.

Each subject of topography should be treated according to the character of the locality, and to each assigned its appropriate scale and method of execution. In a country of varied configuration and largely diversified detail, so full as to justify all the care and refinement possible with the larger scales, there may occur tracts of marsh, sandy waste, or woodland barren of topographic detail. Uniformity, however, requires a given scale, as a unit, to the whole survey. It is manifestly better, therefore, to apply the larger scale to the simpler topography than to jeopardize the elaboration of the more complex features by using an inferior scale. This scale or ratio may be expressed in one or more of three ways.

Thus, suppose a map is only one forty-eight-hundredth ($\frac{1}{4800}$) as long as the surface it represents; then it may be said that its scale is 1 *inch to* 400 *feet,* because in 400 feet there are 4,800 inches, and 1 inch on the map represents 4,800 inches on the ground.

Again, the ratio may be stated in the form of a fraction in which the numerator is a certain linear measurement on the map and the denominator the linear measurement of the surface represented by it, both being of the same denomination. *This fraction reduced to the form having unity for the numerator is called the "Representative Fraction"* and designated the R. F. For the scale assumed above it would be written, R. F. $\frac{1}{4800}$. This latter method of expressing the relation between the map and ground may be considered the general one, in which one unit of any kind on the map, as 1 inch, 1 foot, 1 yard, 1 meter, etc., represents 4,800 units of the same kind respectively on the ground. It is by means of the R. F. that maps are compared with each other as regards the sizes of the scales.

Thirdly, the scale of a map may be represented graphically by a straight line divided into equal parts, and at one extremity of each of the parts writing the number of units on the ground which are represented by a length on the map equal to the portion of the line considered. For the scale assumed it could be Fig. 52, in which the portion of the line

FIGURE 52.

to the right of the 0 one inch long is marked 400, as representing 400 feet; and $\frac{1}{4}$ inch to the left of the 0 marked 100, representing 100 feet, etc., each portion of the line having its origin at the point marked 0 and its other extremity at the number indicated.

As the inch is the common unit of measure in the United States by which the eye is accustomed to judge distances on paper, it is usual, when it can be so done, to state the relation as so many inches (on the map) to so many feet, yards, miles, etc. (on the ground). Thus, 3 inches to 1,000 yards, or 5 inches to 3 miles. Frequently, however, maps are drawn on scales, the denominators of the representative fractions of which are multiples of 10, as R. F. $\frac{1}{500}$, R. F. $\frac{1}{1000}$, R. F. $\frac{1}{25000}$, in which cases it is not always possible to express it as above.

The representative fraction for any scale expressed in the manner first indicated, as 3 inches to 1 mile, may be found by first writing it in the form of a fraction with the number of units indicated on the map for the numerator, and the number of units indicated on the ground for the denominator, thus $\frac{3 \text{ ins.}}{1 \text{ mile}}$, then reducing both terms to the same denomination, $\frac{3 \text{ ins.}}{1 \text{ mile}} = \frac{3 \text{ ins.}}{63360 \text{ in.}}$, because there are 63360 inches in 1 mile, and then reducing the fraction having unity for the numerator, $\frac{3 \text{ ins.}}{63360 \text{ ins.}} = \frac{1}{21120}$. R. F. $\frac{1}{21120}$. So for any other scale.

When comparing maps drawn on different scales, the larger the denominator of the R. F. the smaller the scale; that is, any specified dimension on the ground would be represented by a smaller dimension on the map constructed on the smaller scale than it would be on a map constructed on a larger scale.

On a given sheet of paper a greater area of ground can be represented with a small scale than could be represented with a larger scale.

If it is desired to construct a new scale a certain number of times larger than some other scale, one has only to divide the denominator of the given R. F. by the multiple of the new scale for the denominator of the new R. F., and conversely if the scale is to be reduced.

A graphical or linear scale (Fig 52), in accurate topographic work, should always be constructed on the sheet to contain the map, so that it may change in length with any change in the size of the map due to the effect of changes in temperature and moisture on the paper.*

The R. F. should also be given on all maps and plans to facilitate comparison with other maps having different scales and units of measure. When, however, maps are to be reproduced by photography on a larger or smaller scale, the R. F. should be covered so as not to show in the reproduced map, as it would not then be true. The new R. F. could be calculated and put on the reproduced map. The reproduced graphical scale would, however, still be the correct one for the new map.

The scale decided upon for any particular work will depend upon the amount of detail to be shown and the nature of the ground. In every case the rule governing is, *that the scale should be just large enough to express clearly all the details which it is desired to show.*

The degree of accuracy to which the work of sketching is carried on in the field depends largely upon the scale selected, since it would be useless to attempt to avoid errors that are

*It has been found from experiment that the variation between paper exposed alternately to damp open air and warm dry air of a room, and a boxwood scale is .012, and between paper and ivory, even greater. (Gillespie.) Hence, if a map is constructed on a certain scale with a boxwood or ivory scale and afterwards changes in size due to the weather, then all distances taken from it with the wood or ivory scale would be erroneous, which would not be the case if a graphical scale had been constructed on the paper at the time of making the map, and distances referred to it.

too small to appear upon the map, as well as useless to attempt to show details too small to be represented by the scale. If it be assumed that 0.01 of an inch is the smallest division appreciable to the eye, then the length applied to any scale will determine the limit of accuracy of the field plotting or the minimum size of objects that can be represented. Thus, if sketching at the scale of R. F. $\frac{1}{20000}$, 0.01 inch on paper will represent 200 inches or $16\frac{2}{3}$ feet on the ground; hence it will be useless to try to represent to scale objects or independent distances of less dimensions than $16\frac{2}{3}$ feet. At R. F. $\frac{1}{10000}$ it will be $8\frac{1}{3}$ feet. R. F. $\frac{1}{5000}$, 4.16 feet. R. F. $\frac{1}{1000}$, 10 inches. R. F. $\frac{1}{50}$, $\frac{1}{2}$ inch.

CONSTRUCTION OF SCALES.—The *Unit of Measure* of a scale is the linear quantity which the *primary* divisions of the scale read, as *feet*, *yards*, *meters*, *miles*, etc. The primary divisions may then be still further subdivided into other units or into fractional parts of the unit of measure.

A scale is ordinarily drawn about 6 inches long, but may be either longer or shorter, depending upon the size of the map. For reasons previously stated, *one inch* is assumed in all cases as being the numerator of the R. F. for purposes of constructing scales. *To construct a scale:* First, determine how many of the units of measure are represented by 1 inch. Thus, if the scale is to be R. F. $\frac{1}{21120}$ to read yards, then, because 21,120 inches are to be represented by 1 inch, as many yards will be represented by 1 inch as 21,120 inches $\div$ 36 inches $=586.6\frac{2}{3}$. Or, if the scale is to be 6 inches to one mile to read feet, then, because there are 5280 feet in a mile represented by 6 inches, there will be as many feet represented by 1 inch as $5280 \div 6 = 880$. Second, calculate how many of these units of measure will be represented by about the length in inches that you wish to make the scale; suppose it is desired to make the scale *about* 6 inches long. In the first case above, as 1 inch represents $586.6\frac{2}{3}$ yards, 6 inches will represent $6 \times 586.6\frac{2}{3}$ yards $=3520$ yards; and in the second case, 6 inches will represent 6×880 feet $=5280$ feet. Third, select the near-

At last!

est *exact* number of ten, hundred, or thousand units of measure to the number found in about the length you wish the scale, then substitute in the following proportion and solve.

$$\left\{\begin{array}{l}\text{The number of units}\\ \text{of measure represent-}\\ \text{ed by 1 inch.}\end{array}\right\} : \left\{\begin{array}{l}\text{The selected number of}\\ \text{ten, hundred, or thou-}\\ \text{sand units of measure.}\end{array}\right\} :: (1 \text{ inch}) : \left\{\begin{array}{l}x \text{ the required}\\ \text{length of scale}\\ \text{in inches.}\end{array}\right\}$$

This gives the length of the scale (to two places of decimals) in inches. In the first case, select say 3500 yards; then, substituting and solving, one gets $586.6\frac{2}{3}:3500::1:x=5.96$ inches.

In the second case, selecting 5000 feet, substituting, and solving, one gets $880:5000::1:x=5.68$ inches.

Having found the fourth term of the proportion, or x, lay off this length on a straight line, using a scale of equal parts of inches, and then divide this length of line into the number of equal parts most convenient for use.*

EXAMPLES.—I. Construct a scale of 1 inch to 400 feet to read yards. In this the *unit of measure* is yards. The scale is given in feet. Now 400 feet=133.33 yards, represented by 1 inch. Assuming 6 inches as about the desired length for the scale, it will represent 6 times 133.33 yards= 800.00 yards. 800 being an exact number of hundreds, it is unnecessary to use the proportion, but on a straight line (Fig. 54) lay off AB 6 inches long. From the end A of this line draw another straight line AC a little longer than AB, making any convenient acute angle with AB. On AC lay off 8 equal parts, 1, 2, 3, 4, 5, etc., and join the eighth one C with B. Now from the points on AC draw lines parallel to BC, cutting AB, when AB will be divided into 8 equal parts, each representing 100 yards and called the primary division. From

*The reason for selecting the nearest exact number of ten, hundred, or thousand units of measure to the number of units of measure found for about the desired length of scale, was because it is easier to divide the length of line obtained by the proportion into a certain number of equal parts than it would have been to attempt to divide the assumed length of the scale into the uneven number of units of measure that it would reprsent.

SCALE 1 inch to 400 feet. R.F $\frac{1}{4800}$

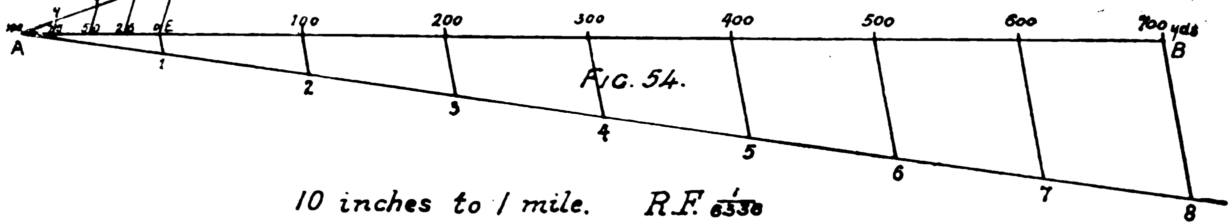


FIG. 54.

10 inches to 1 mile. R.F. $\frac{1}{6336}$

FIG. 55.

0.0396 inches to 1 chain. R.F $\frac{1}{20000}$.

FIG. 56.

12.672 inches to 1 mile. R.F. $\frac{1}{5000}$.

FIG. 57.

1 inch to 4.166 feet. R.F. $\frac{1}{50}$

FIG. 58.

A draw AD, making any convenient angle with AB. On this lay off 4 equal parts. Join D and E and from the points y draw lines parallel to DE, which will divide the left division into 4 equal parts, each representing 25 yards, and called secondary divisions. The zero of the scale is placed at E and the primary divisions are numbered to the right from it, while the secondary divisions are numbered to the left as shown. R. F. $\frac{1}{4800}$.

II. Construct a scale of 10 inches to 1 mile to read yards. Here the unit of measure is yards. The scale is given in miles. Now 1 mile $=1760$ yards, represented by 10 inches; hence 1 inch will represent as many yards as $1760 \div 10 = 176$. If 1 inch represents 176 yards, then 6 inches (about the right length for the scale) will represent $6 \times 176 = 1056$ yards. Select 1000 yards as being the nearest exact thousand to 1056, for use in the proportion, substituting in which gives $176 : 1000 :: 1 : x = \frac{1000}{176} = 5.68$. Draw a straight line 5.68 inches long (Fig. 55). Divide it into 10 equal parts (primary divisions), and each will represent 100 yards. Divide the left primary division into 5 equal parts (secondary divisions), each to represent 20 yards. R. F. $\frac{1}{6336}$.

III. Construct a scale of 0.0396 inches to 1 chain (Gunter's) to read meters. The unit of measure is meters of 39.37 inches each. The scale is in chains of 792 inches each, $792 \div 39.37 = 20.12$ meters. If 0.0396 inch represents 1 chain or 20.12 meters, then 1 inch will represent as many meters as $20.12 \div 0.0396 = 507.98$. If 1 inch represents 507.98 meters, then 6 inches will represent 6×507.98 meters $= 3047.88$ meters. Take 3000 meters as the nearest exact thousand to 3047.88 and substitute in the proportion, $507.98 : 3000 :: 1 : x$ $\frac{3000}{507.98} = 5.9$

Draw a straight line (Fig. 56) 5.9 inches long. Divide it into 6 equal parts, each representing 500 meters, for primary divisions, and the left into 5 equal parts for secondary divisions, each representing 100 meters. To find the R. F. $\frac{.0396 \text{ in.}}{1 \text{ chain}}$ $= \frac{.0396 \text{ in.}}{792 \text{ in.}} = \frac{1}{20000}$ or $\frac{1 \text{ in.}}{507.98 \text{ me.}} = \frac{1}{20000}$.

IV. Construct a scale of R. F. $\frac{1}{5000}$ to read feet. On this scale 1 inch répresents 5000 inches; $5000 \div 12 = 416.67$ feet. If 1 inch represents 416.67 feet, then 6 inches will represent 6×416.67 feet$=2500.00$ feet. Draw a straight line (Fig. 57) 6 inches long and divide into 5 equal parts for primary divisions of 500 feet, and the left division into 5 parts for secondary divisions of 100 feet.

To find an expression for any scale given by its R. F. in terms of any unit of measure, as miles, divide the number of inches in that unit of measure by the denominator of the representative fraction, thus 63,360 inches$=1$ mile, then $63{,}360 \div 5000 = 12.672$; hence, scale is 12.672 inches to 1 mile.

A diagonal scale is one constructed for reading either to a smaller number of the units of measure than could be done by the primary and secondary divisions, as feet, tenths, and hundredths of feet, or for reading on one scale different units, such as a distance given in yards, feet, and inches, or a distance in chains and links.

V. Construct a diagonal scale of R. F. $\frac{1}{50}$ to read yards, feet, and inches.

On this scale 1 inch represents 50 inches. 50 inches$=50 \div 36 = 1.39$ yards. If 1 inch represents 1.39 yards, 6 inches will represent $6 \times 1.39 = 8.34$ yards. Take 8 yards and substitute in proportion, $1.39 : 8 :: 1 : x = 5.76$ inches. Draw a line (Fig. 58) 5.76 inches long and divide into 8 equal parts, each representing 1 yard. If now the left division be divided into 3 equal parts, each will represent 1 foot. Now draw parallel to the line already drawn 12 equally distant parallel lines. From the points of the primary divisions already fixed, erect perpendiculars cutting the parallel lines. Divide the left division of the upper line into 3 equal parts and join each lower division with the next diagonally upper one to the left. On this scale can be taken any distance, even though it be in yards, feet, and inches, as 4 yards, 1 foot, 10 inches; 6 yards, 0 foot, 7 inches; 5 yards, 2 feet, 2 inches.

6.336 inches to 1 mile. R.F $\frac{7}{10000}$.

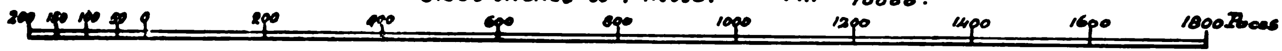


FIG. 59

0.0072 inches to 1 yard. R.F. $\frac{1}{5000}$.

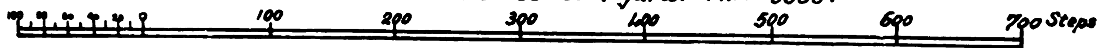


FIG. 60.

0.0079 inches to 1 metre. R.F. $\frac{1}{5000}$.

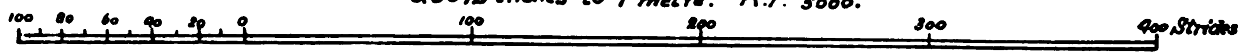


FIG. 61.

3 inches to 1 mile. R.F. $\frac{1}{21120}$.

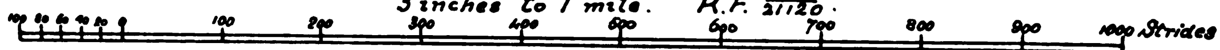


FIG. 62.

0.158 inches to 1 chain R.F. $\frac{1}{5000}$.

FIG. 63

3.6 inches to 1000 yards. R.F. 10000.

FIG. 64.

Had it been desired to make the scale read yards, feet, and tenths of feet, then only 10 parallel lines would have been drawn and the scale completed as shown.

VI. Construct a scale of 6.336 inches to 1 mile to read paces of 31 inches each. The unit of measure is paces of 31 inches. The scale is expressed in miles. 1 mile=63,360 inches=as many paces as $63{,}360 \div 31 = 2043.87$ paces. If 6.336 inches represent 2043.87 paces, 1 inch will represent as many paces as $\frac{2043.87}{6.336} = 322.57$ and 6 inches will represent 6×322.57 paces=1935.42 paces. Take 2000 paces and substitute in the proportion getting $322.57 : 2000 :: 1 \text{ inch} : x = 6.20$ inches. Draw a line (Fig. 59) 6.2 inches long and divide into 10 primary divisions of 200 paces each, and the left into 4 secondary divisions of 50 paces each. R. F. $\frac{1}{10000}$.

VII. A horse takes 545 steps in walking 500 yards. Construct a scale of 0.0072 inch to 1 yard to read the horse's steps. The unit of measure is the horse's steps. The scale is given in yards. If 0.0072 inch represents 1 yard, 1 inch will represent as many yards as $\frac{1}{0.0072} = 138.888$ yards; but as the horse takes 545 steps in 500 yards, he takes as many steps in 1 yard as $545 \div 500 = 1.09$, and in going 138.888 yards, or the distance represented by 1 inch, he takes 138.888×1.09 steps=151.388 steps. If 1 inch represents 151.388 steps, 6 inches represent 6×151.388 steps=908.33. Take 800 steps and substitute in proportion.

151.38 steps : 800 steps :: 1 inch : $x = 5.28$ inches. Draw a line (Fig. 60) 5.28 inches long and complete scale as shown. R. F. $= \frac{0.0072 \text{ inches}}{1 \text{ yard}} = \frac{0.0072 \text{ inches}}{36 \text{ inches}} = \frac{1}{5000}$.

Or, first reducing all to inches, if 0.0072 inch represents 1 yard or 36 inches, 1 inch will represent $36 \div 0.0072 = 5000$ inches, and, as the horse takes 545 steps in 500 yards or 18,000 inches, he takes one step in $18{,}000 \div 545 = 33.03$ inches, and in 5000 inches, which is represented by 1 inch, he will take as many steps as $5000 \div 33.03 = 151.38$.

VIII. A horse takes 120 strides in cantering 400 yards. Construct a scale of R. F. $\frac{1}{5000}$ to read strides cantering. The unit of measure is strides cantering. 1 inch represents as many yards as 5000÷36=138.89 yards, and if the horse takes 120 strides in 400 yards, in 1 yard he will take as many as 120÷400=0.3, and in going 138.89 yards he takes 138.89×0.3 =41.667 strides, the number represented by 1 inch. 6 inches would represent 6×41.667 strides=250.00 strides. Draw a line (Fig. 61) 6 inches long, divide into 5 equal primary divisions of 50 strides each and the left one into 10 secondary divisions of 5 strides each.

Or, reducing all to inches, if the horse takes 120 strides in 400 yards or 14,400 inches, he takes 1 stride in 14,400÷120= 120 inches, and in 5000 inches he takes 5000÷120=41.66⅔, the number represented by 1 inch.

IX. A horse plants his left fore foot 570 times in trotting 1 mile. Construct a scale of 3 inches to 1 mile to read strides. The unit of measure is strides trotting. 1 inch represents 570÷3=190 strides. 6 inches would represent 6×190 strides= 1140 strides. Take 1100 strides and substitute in proportion, 190 strides:1100 strides :: 1 inch : $x=\frac{1100}{190}$=5.79 inches. Draw a line (Fig. 62) 5.79 inches long and divide it into 11 primary divisions of 100 strides each, and the left one into 5 secondary divisions of 20 strides each.

X. A horse trots in 2 minutes a distance equal to 348 of his rider's paces of 31 inches each. Construct a scale of R. F. $\frac{1}{5000}$ to read minutes and 5 seconds.

The unit of measure is minutes. 348 paces of 31 inches each=10,788 inches, over which the horse trots in 2 minutes; hence in 1 minute he trots over 5394 inches. 1 inch represents 5000 inches; hence 1 inch will represent as many minutes as $\frac{5000}{5394}$ = 0.93 minutes, and 6 inches will represent 6×0.93 minutes=5.58 minutes. Take 5 minutes and substitute in proportion 0.93 minutes : 5 minutes :: 1 inch : $x=\frac{5}{0.93}$=5.38 inches. Draw a straight line (Fig. 63) 5.38 inches long, divide into 5 primary divisions of 1 minute each and the left hand division into 12 secondary divisions of 5 seconds each.

XI. A sketch has no scale on it. The distance between two points measured on the sketch is 3.6 inches. It is paced on the ground and found to be 1200 paces of 30 inches each. Construct the scale to read yards for the map and determine its R. F.

The unit of measure is yards. 1200 paces of 30 inches each=36,000 inches=1000 yards, represented by 3.6 inches. 1 inch represents as many yards as $\frac{1000}{3.6}$=277.77 yards, 6 inches would represent 6×277.77 yards=1666.62 yards. Take 1500 yards and substitute in proportion, 277.77 yards : 1500 yards :: 1 inch : x=5.40 inches. Draw a line (Fig. 64) 5.40 inches long and divide into 15 primary divisions of 100 yards each, and the left-hand division into 5 secondary divisions of 20 yards each. To determine R. F. $\frac{3.6}{36000}=\frac{1}{10000}$.

XII. A map is drawn on a scale of R. F. $\frac{1}{500}$ to a supposed length of pace of 32 inches. The length of pace is afterwards found to be only 29 inches. Construct a correct scale for the map to read yards and determine its R. F.

In a case of this kind, while the map itself may be correct, the scale of R. F. $\frac{1}{500}$ is not the correct scale for it, since for every pace of only 29 inches an actual representation of 32 inches on the scale of R. F. $\frac{1}{500}$ was plotted, whereas but 29 inches to a scale of some other R. F., as the map stands, should have been plotted. In other words, no distance measured on the map and applied to the scale R. F. $\frac{1}{500}$ will give the true measured distance on the ground between the same points. To determine the true scale it is only necessary to remember that as every pace of 29 inches was represented as 32 inches, the map as constructed will be larger than it would have been had the paces actually been 32 inches long, and consequently the true scale will be larger than R. F. $\frac{1}{500}$, or, as we have already seen, the denominator of its R. F. will be smaller than 500. A larger scale does not mean that more ground has been represented on the same size of paper than could be represented on a smaller scale, but that for the same piece of ground it takes a larger sheet of paper on which to represent it on a large scale than it would with a smaller scale. In the above case, for every pace of 29 inches we have plotted that pace 3 inches, to the scale of R. F. $\frac{1}{500}$, greater than we supposed, consequently the map is that proportion greater than represented by the scale of R. F. $\frac{1}{500}$. To determine the correct R. F. for the map, the proportion from the above reasoning is $29:32::\frac{1}{500}:x=\frac{1}{453.125}$, or $32:29::500:x=453.125$. Having found the true R. F. $\frac{1}{453.125}$, the scale is constructed as in Ex. IV.

If the conditions of the problem had been reversed and a 29-inch pace used while the actual length was 32 inches, then the map would have been smaller and the denominator of the true R. F. greater than 500, the proportion being $32:29::\frac{1}{500}:x=\frac{1}{551.724}$, or $29:32::500:x=551.724$.

XIII. A map 8 inches long by 7 inches broad represents 2.2317 acres. Construct its scale to read yards and feet. Determine its R. F. The map being 8 inches by 7 inches, contains 56 square inches. An acre contains 4840 square yards; hence 2.2317 acres will contain 2.2317 times 4840 square yards $=$ 10,801.428 square yards. If 56 square inches represents 10,801.428 square yards, 1 square inch will represent $\frac{1}{56}$ of 10,801.428 square yards $=$ 192.8826 square yards, 1 linear inch will represent as many linear yards as the square root of 192.8826 $=$ 13.89 yards. Now if 1 inch represents 13.89 yards, 6 inches will represent 6×13.89 yards $=$ 83.34 yards; take 80 yards and substitute in proportion, 13.89 yards : 80 yards :: 1 inch : x=5.76 inches. Construct scale as shown. To determine R. F. $\frac{1 \text{ inch}}{13.89 \text{ yards}}=\frac{1}{13.89\times 36}=\frac{1}{500.00}$.

XIV. A map on a scale of 3.168 inches to 1 mile is to be copied on a scale of .0079 inches to 1 meter. What ratio exists between the sizes of the maps? Which is the larger? The R. F. of the original map is $\frac{3.168 \text{ in.}}{1 \text{ mile}}=\frac{3.168 \text{ in.}}{63360 \text{ in.}}=\frac{1}{20000}$.

The R. F. of copied map is $\frac{.0079 \text{ in.}}{1 \text{ meter}}=\frac{.0079 \text{ in.}}{39.37 \text{ in.}}=\frac{1}{5000}$. By an examination of the R. F.'s, remembering what was said above, it is seen that the original map is to be copied on a scale four times as large.

Comparative or Equivalent Scales are those having the same R. F., but constructed to read different units of measure. Thus, on a foreign map the unit of measure might be meters or anything else, and if the R. F. were given, a scale of yards, feet, and inches, or chains and links, or any other familiar unit of measure could be constructed for the purpose of examining the map, and these scales, though of different units of measure, but of the same R. F., would be comparative scales. Figs. 57, 60, 61, and 63 are comparative scales of R. F. $\frac{1}{5000}$, and Figs. 59 and 64 of R. F. $\frac{1}{10000}$.

Having already constructed a scale of say R. F. $\frac{1}{10000}$, and it is desired to use it when plotting to a scale of R. F. $\frac{1}{5000}$, it is only necessary to multiply each measured distance by 2 and take it from the scale and *vice versa*. Thus any one scale may conveniently be used for another having the same unit of measure, provided the denominator of the R. F. of one is an exact multiple of the denominator of the R. F. of the other.

A vernier scale is an auxiliary scale of equal parts constructed on the secondary divisions of another scale of equal parts for reading and setting off fractional parts of the secondary divisions, and is a substitute for the diagonal scale. The principle of its construction is that each division of the vernier scale shall be the amount of the smallest reading desired larger than the divisions to which it is applied. Thus, if it is desired to construct a scale to read .01 of an inch, either a diagonal scale or a vernier scale may be used. The construction of the diagonal scale has been explained. To construct the vernier scale (Fig. 65), first draw

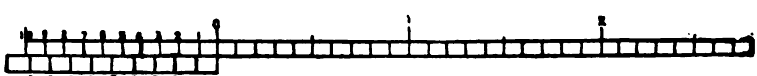


FIG. 65.

the scale of inches and divide each into tenths, then from the 0 of the scale to the left lay off a distance on the under side equal to 11 of the tenths. Divide this into 10 equal parts and each part will be .01 inch greater than the divisions of the scale, or .11 inches long. Starting from the 0 to the left, the first division will be distant .11 inches, the second .22 inches, etc. To take off from the scale any number of hundredths of an inch, as .07 inch, the distance between the two lines numbered 7 on the scale and vernier scale will be the required amount. If it is desired to take off .24 inches, place the point of the dividers on 4 of the vernier scale and move the other point towards the 0, 2 full divisions on the main scale for the other point of division. If desired to take off 2.85 inches, place one point of dividers on 5 of the vernier scale and move the other point in the direction of 0 over 8 full divisions of scale for the .8, then 2 inches further to the right for the entire distance, 2.85 inches.

TABLE OF EQUIVALENTS.

ON A SCALE OF $\frac{1}{21120}$, 1 SQUARE INCH REPRESENTS 70.859 ACRES.

1 inch represents 21120 inches÷12=1760 feet.	1 foot=12 inches÷21120=0.000568 inch.
1 inch represents 21120 inches÷36=586.66 yards.	1 yard=36 inches÷21120=0.0017 inch.
1 inch represents 21120 inches÷39.37=535.897 meters.	1 meter=39.37 inches÷21120=0.00186 inch.
1 inch represents 21120 inches÷792=26.667 chains.	1 chain=792 inches÷21120=0.0375 inch.
1 inch represents 21120 inches÷63360=0.333 mile.	1 mile=63360 inches÷21120=3 inches.

ON A SCALE OF $\frac{1}{20000}$, 1 SQUARE INCH REPRESENTS 63.72 ACRES.

1 inch represents 20000 inches÷12=1666.66 feet.	1 foot=12 inches÷20000=0.0006 inch.
1 inch represents 20000 inches÷36=555.55 yards.	1 yard=36 inches÷20000=0.0018 inch.
1 inch represents 20000 inches÷39.37=507.98 meters.	1 meter=39.37 inches÷20000=0.00197 inch.
1 inch represents 20000 inches÷792=25.25 chains.	1 chain=792 inches÷20000=0.0396 inch.
1 inch represents 20000 inches÷63360=0.316 mile.	1 mile=63360 inches÷20000=3.168 inches.

ON A SCALE OF $\frac{1}{10560}$, 1 SQUARE INCH REPRESENTS 17.715 ACRES.

1 inch represents 10560 inches÷12=880 feet.	1 foot=12 inches÷10560=0.00114 inch.
1 inch represents 10560 inches÷36=293.33 yards.	1 yard=36 inches÷10560=0.0034 inch.
1 inch represents 10560 inches÷39.37=267.648 meters.	1 meter=39.37 inches÷10560=0.00372 inch.
1 inch represents 10560 inches÷792=13.333 chains.	1 chain=792 inches÷10560=0.075 inch.
1 inch represents 10560 inches÷63360=0.1667 mile.	1 mile=63360 inches÷10560=6 inches.

ON A SCALE OF $\frac{1}{10000}$, 1 SQUARE INCH REPRESENTS 15.94 ACRES.

1 inch represents 10000 inches÷12=833.33 feet.	1 foot=12 inches÷10000=0.0012 inch.
1 inch represents 10000 inches÷36=277.77 yards.	1 yard=36 inches÷10000=0.0036 inch.
1 inch represents 10000 inches÷39.37=254.18 meters.	1 meter=39.37 inches÷10000=0.003937 inch.
1 inch represents 10000 inches÷792=12.63 chains.	1 chain=792 inches÷10000=0.0792 inch.
1 inch represents 10000 inches÷63360=0.158 mile.	1 mile=63360 inches÷10000=6.336 inches.

ON A SCALE OF $\frac{1}{5000}$. 1 SQUARE INCH REPRESENTS 3.985 ACRES.

1 inch represents 5000 inches+12=416.66 feet.	1 foot=12 inches+5000=0.0024 inch.
1 inch represents 5000 inches+36=138.89 yards.	1 yard=36 inches+5000=0.0072 inch.
1 inch represents 5000 inches+39.37=126.95 meters.	1 meter=39.37 inches+5000=0.0079 inch.
1 inch represents 5000 inches+792=6.313 chains.	1 chain=792 inches+5000=0.158 inch.
1 inch represents 5000 inches+63360=0.079 mile.	1 mile=63360 inches+5000=12.672 inches.

ON A SCALE OF $\frac{1}{500}$, 1 SQUARE INCH REPRESENTS 0.0398 ACRE.

1 inch represents 500 inches+12=41.66 feet.	1 foot=12 inches÷500=0.024 inch.
1 inch represents 500 inches+36=13.889 yards.	1 yard=36 inches+500=0.072 inch.
1 inch represents 500 inches+39.37=12.695 meters.	1 meter=39.37 inches+500=0.079 inch.
1 inch represents 500 inches+792=0.631 chain.	1 chain=792 inches+500=1.58 inches.
1 inch represents 500 inches+63360=0.0079 mile.	1 mile=63360 inches+500=126.72 inches.

ON A SCALE OF $\frac{1}{200}$, 1 SQUARE INCH REPRESENTS 0.00638 ACRE.

1 inch represents 200 inches+12=16.66 feet.	1 foot=12 inches+200=0.06 inch.
1 inch represents 200 inches+36=5.55 yards.	1 yard=36 inches+200=0.18 inch.
1 inch represents 200 inches+39.37=5.08 meters.	1 meter=39.37 inches+200=0.197 inch.
1 inch represents 200 inches+792=0.25 chain.	1 chain=792 inches÷200=3.96 inches.
1 inch represents 200 inches+63360=0.0032 mile.	1 mile=63360 inches+200=316.8 inches.

ON A SCALE OF $\frac{1}{50}$, 1 SQUARE INCH REPRESENTS 0.000398 ACRE.

1 inch represents 50 inches+12=4.166 feet.	1 foot=12 inches÷50=0.24 inch.
1 inch represents 50 inches+36=1.389 yards.	1 yard=36 inches÷50=0.72 inch.
1 inch represents 50 inches+39.37=1.269 meters.	1 meter=39.37 inches+50=0.79 inch.
1 inch represents 50 inches+792=0.0631 chain.	1 chain=792 inches+50=15.8 inches.
1 inch represents 50 inches+63360=0.00079 mile.	1 mile=63360 inches÷50=1267.2 inches.

The above scales are those used in Topographical Drawing and Sketching in the Department Work.

PROBLEMS.—1. Construct a scale of 0.072 inch to 1 yard, to read feet. R. F.?

2. Construct a scale of $\frac{1}{10000}$ to read yards. How many inches to mile?

3. Construct a diagonal scale of $\frac{1}{50}$ to read meters, decimeters, and 5 centimeters.

4. Construct a scale of $\frac{1}{10000}$ to read paces; 1 pace =31 inches.

5. Suppose a horse takes 160 strides trotting, in going 500 yards. Construct a scale of 3.168 inches to 1 mile to read trots. R. F.?

6. Suppose a horse takes 150 strides cantering in 500 yards. Construct a scale of 0.0012 inch to 1 foot to read canters. R F.?

7. Suppose a horse takes 1920 steps per mile. Construct a scale of 6.336 inches to 1 mile to read walks. R. F.?

8. Suppose a horse walks in 2 minutes 20 seconds a distance equal to 280 of his rider's paces (30 inches each). Construct a scale of R. F. $\frac{1}{5000}$ to read minutes and quarter-minutes.

9. Suppose a horse gallops in 1 minute 20 seconds a distance equal to 400 of his rider's paces (30 inches each). Construct a scale of R. F. $\frac{1}{500}$ to read seconds.

10. Suppose a horse trots a mile in 11 minutes 44 seconds. Construct a scale of R. F. $\frac{1}{10000}$ to read minutes and half-minutes.

11. A sketch has no scale. The distance between two points on sketch is 0.79 inch; on ground, 100 meters. Construct scale. R. F.?

12. A sketch has no scale. The distance between two points on sketch is 1.8 inch; on ground, 1200 paces (30 inches). Construct scale. R. F.?

13. A map is supposed to be drawn on a scale of R. F. $\frac{1}{10000}$ with 30-inch paces, which are afterwards found to be 33 inches. What is scale of map?

14. A map is supposed to be drawn on a scale of R. F.

$\frac{1}{500}$ with 34-inch pace, which is found to be 29 inches. What is scale of map?

15. A map is supposed to be drawn on a scale of R. F. $\frac{1}{10000}$ with horse trotting 150 yards per minute. He trots only 130 yards per minute. What is scale of map?

16. A map is supposed to be drawn on a scale of R. F $\frac{1}{10000}$ with horse galloping 250 yards per minute. He gallops 275 yards per minute. What is scale of map?

17. A map 6 inches long by 5 broad represents 478.2 acres. Construct its scale of chains (Gunter's). R. F.?

18. A map 20 inches long by 15 broad represents 577.896 square yards. Construct its scale of feet and inches. R. F.?

19. A plan of R. F. $\frac{1}{10000}$ is to be copied at R. F. $\frac{1}{5000}$. Original has $\frac{1}{4}$-inch squares. What will be ratio for copy?

20. A plan of 3.168 inches to 1 mile is to be copied at .0079 inch to 1 meter. What sized squares or ratio to use for original and copy?

CHAPTER IV.

VERNIERS.

A VERNIER is an auxiliary scale of equal parts, movable along the side of a main scale of equal parts (called the limb), for reading fractional parts of the divisions of the limb. A vernier is usually constructed by taking the length of a certain number of divisions of the limb and then dividing this length into one more or one less number of equal divisions, as a result of which each division *on the vernier* is slightly less or slightly greater in length than a division of the limb, equal to the value of a division on the limb divided by the number of divisions on the vernier. This small difference between the length of a division on the limb and the length of a division on the vernier is called the *smallest reading* or *least count.*

Take a scale of feet which is divided into hundredths of feet, and it is desired to construct a vernier so as to read to $\frac{1}{10}$ of the hundredths divisions, or $\frac{1}{1000}$ of a foot. The vernier must contain 10 divisions, and its length must be equal to either 9 or 11 of the hundredths divisions. (1) If 9 divisions are taken and divided into 10 equal parts, each division of the vernier will then be $\frac{1}{1000}$ of a foot shorter than a division of the limb. (2) If 11 divisions are taken and divided into 10 equal parts, then each division of the vernier will be $\frac{1}{1000}$ of a foot longer than a division of the limb.

A vernier of the first kind (Fig. 66) is found on leveling-

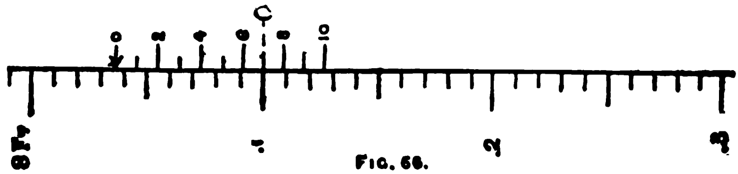


FIG. 66.

rods. The 0 line of the vernier marks the point on the limb which is to be read. If this coincides with a line on the limb, the reading would be entirely on the limb; if it were moved slightly, the 0 would leave the coincident line of the limb and the first, second, third, or some other line of the vernier would coincide with a line of the limb (in this case the 7th) showing that the 0 had passed on $\frac{7}{10}$ of the length of one of the smallest divisions of the limb, or .007 of a foot, which must be added to the reading of the last line (8.03 feet) passed on the limb by the 0 of the vernier, making a total reading of 8.037 feet. When, as in this case, the divisions on the vernier are smaller than the divisions of the limb, the graduations are numbered on both in the same direction and read in the same direction, and the vernier is called a *direct vernier*.

A vernier of the second kind is shown in Fig. 67, and is the one commonly found on mercurial barometers. The 0 of the vernier as before marks the point on the limb which is to be read. If the 0 of the vernier be moved forward from coincidence, in the direction of the numberings on the limb, the first, second, or some other line of the vernier, backwards from the direction in which the limb is numbered, will coincide with a line of the limb (in this case the 6th) showing that the 0 has passed on $\frac{6}{10}$ of the length of one of the smallest divisions of the limb, or $\frac{6}{100}$ of an inch, which must be added to the reading of the last line on the limb (30.2 inches) passed by the 0 of the vernier, making the total reading 30.26 inches. In this case the divisions on the vernier, being greater in length than the divisions on the limb, are numbered and read from the 0 in a reverse direction from the numberings on the limb, and the vernier is called a *retrograde vernier*.

The above verniers have been constructed on straight lines, but the same principles apply when constructed on curved lines. All the following verniers, on the instruments, are curves, but are here shown greatly enlarged and on straight lines for simplicity in explaining them.

EXAMPLES.—I. Figure 68 is a retrograde vernier on the Aneroid Barometer scale. This latter is for reading elevations and depressions in feet. The smallest divisions on the limb represent 10 feet. The length of 21 divisions of the limb is first divided into 20 equal parts, making the smallest reading 10 feet$\div$20=$\frac{1}{2}$ foot. The scale being small, the lines close together, and the smallest reading being smaller than justified, the length of each division on the vernier and the smallest reading are both doubled by erasing each odd numbered line, practically dividing 21 divisions of the limb into 10 equal parts. Each division of the vernier thus becomes 1 foot greater than *two* divisions of the limb, but neither the character of the vernier nor the method of reading it is at all changed by making the alteration. The reading being 2088 feet, the letter *C* indicating the coinciding line.

II. Figure 69 shows the scale and verniers on the Abney Clinometer. The limb is a semi-circular arc divided into degrees and numbered both directions from 0 to 90°, beginning at the middle. The vernier is double—*i. e.*, there are two separate verniers, the right one for reading on the right half of the limb, and the left one for reading on the left half of the limb. The 0 being the same for both verniers. To construct these verniers, the length of 5 divisions of the limb is divided into 6 equal parts, making the *smallest reading* or *least count* equal to 1° or 60′$\div$6=10′. As the divisions of the verniers are smaller than the divisions of the limb, by the amount of the smallest reading, the verniers are direct, and the reading is 2° 20′, the right vernier being read, since the 0 of the vernier is to the right of the 0 of the scale and the second line to the right of the 0 of the vernier being the coinciding line of the vernier used; although the first line to the left of 30 on the left vernier is also in coincidence, it is not used. The coinciding line used and the 0 of the vernier must both be on the same side of the 0 of the limb.

III. Figure 70 shows the scale and verniers on the Architect's Level. The scale is a full circle graduated to

FIG. 67.

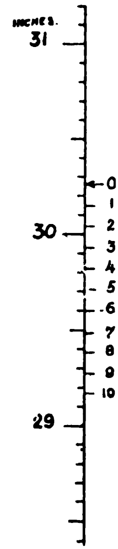


FIG. 68.

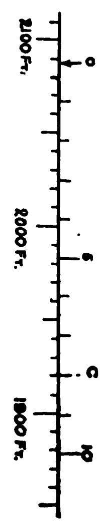


FIG. 69.

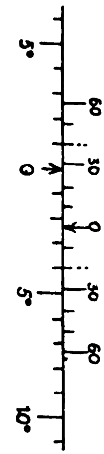


FIG. 70.

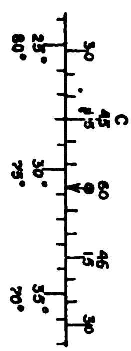


FIG. 71.

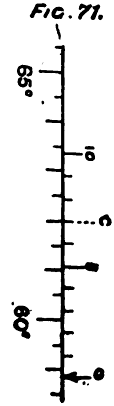


FIG. 72.

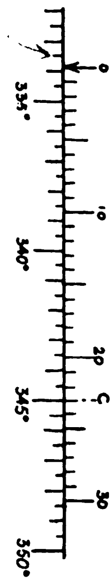


FIG. 73.

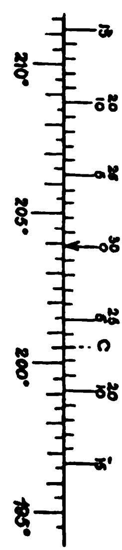


FIG. 74.

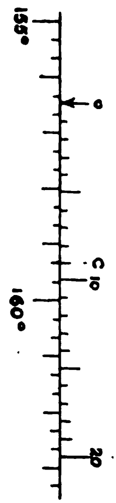


FIG. 75.

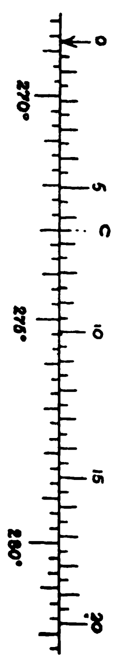


FIG. 76.

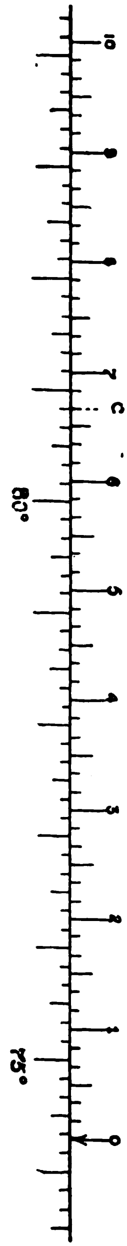
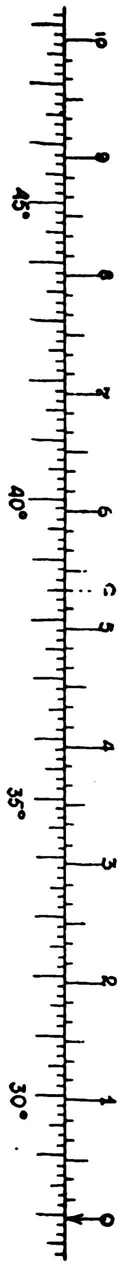


FIG. 77.



degrees, and numbered from 0 to 90° each way from the extremities of a diameter. (Two independent sets of limb numberings are shown in the figure for illustration only.) The vernier is a double one, but differs slightly from the preceding one. The length of 11 divisions of the limb is divided into 12 equal parts, making the smallest reading 1° or $60' \div 12 = 5'$. The verniers are direct. On account of the greater length which the verniers would have if placed side by side as in the preceding case, they are in this case superposed and require only one-half the space without impairing their efficiency. To read them, as in the figure, it is seen that the limb is numbered from left to right in the upper case, and that the 0 of the vernier is to the right of 30°; now to find how much, we start from the 0 of the vernier and go to the right, looking for a coinciding line, but do not find any between 0 and 30; jumping then from the 30 on the right end of the vernier to the 30 on the left end of the vernier, proceed from there to the right until coinciding lines are found, in this case the third from 30, which is 45; hence 45′ must be added to the 30°, making the reading 30° 45′. Had the vernier been on a quadrant of the circle numbered from right to left as in the lower case, then in finding the coinciding line of the vernier one would look along it to the left of the 0 to the 30 on the left, and, if not found there, jump to the 30 on the right and proceed to the left towards the 60 mark above the 0 until a coinciding line was found, the number of which must be added to the reading on the limb for the total distance of the 0 of the vernier from 0 of the limb, in this case 74° 15′. The lower row of numbers on the right half of the vernier and upper on left half constitute the right vernier, and the lower left and upper right the left vernier.

IV. Figure 71 is the scale and vernier of the arm protractor (Fig. 49). The limb is divided into half-degrees. The vernier is constructed by taking the length of 9 divisions of the limb and dividing it into 10 equal parts, the smallest reading being $\frac{1}{2}$ degree or $30' \div 10 = 3'$, and the vernier is direct.

When the protractor is numbered both ways a double vernier would be used, as in Figs. 69 or 70. The reading in the figure being 58° 51′.

V. Figure 72 shows a common form of direct verniers on some Compasses, Transits, and other instruments. The limb is divided to half-degrees. The length of 29 divisions is taken and divided into 30 parts, giving a smallest reading of 1′. As shown it reads 323° 53′.

VI. Figure 73 shows the same vernier, but doubled for reading both ways, as in Fig. 70, when the limb has two sets of numbers. Reading 203° 53′.

VII. Figure 74 shows the vernier on the Plane Table Alidade. The limb is divided to $\frac{1}{3}$ degrees or 20′. The length of 19 of these is divided into 20 equal parts, giving a smallest reading of 1′; reading 156° 29′.

VIII. Figure 75 is a scale and vernier on some transits, the divisions on the limb being 20′. The length of 39 divisions is divided into 40 parts, giving a smallest reading of 30″; reading 268° 46′ 30″. The numberings on this vernier, as on those shown in Figs. 76 and 77, are given for the full minute divisions, the number of $\frac{1}{2}$ minutes or 10 seconds to be added being found by counting.

IX. Figure 76 is the scale and vernier found on most sextants. The divisions of the limb are 10′ each. A length of 59 of these is divided into 60 parts, giving a smallest reading of 10″; reading 74° 16′ 40″.

X. Figure 77 is the vernier also found on some sextants. The divisions of the limb are 10′ each. A length of 119 of these is divided into 120 equal parts, giving the smallest reading 5″. This brings the lines too close together on the vernier, so the length of each division of the vernier and the smallest reading are doubled by erasing every odd numbered line, practically dividing 119 divisions of limb into 60 equal parts, making each division of the vernier less than two divisions of the main scale by 10″. Reading 27° 55′ 20″.

Figures 68 and 77 are apparent exceptions to the general rule about the divisions on verniers being less for direct, and greater for retrograde, than the divisions on the limb but from the manner of their construction it will be seen they are not. For *by a direct vernier* is to be understood one whose divisions are less than 1, 2, 3, or any other number of divisions of the limb by the amount of the smallest reading. *By a retrograde vernier* is to be understood one whose divisions are larger than 1, 2, 3, or any other number of divisions of the limb by the amount of the smallest reading.

ARC OF EXCESS.—On the sextant the divisions of the limb are carried beyond the 0 to the right. This is called the *Arc of Excess*. When the 0 of the vernier is on this part and it is desired to ascertain how far it is from the 0 of the limb, count the number of divisions of the limb passed over by the vernier 0, and to this add the fractional part of the next division passed over, to obtain which the vernier must be read backwards (as if it were numbered from left to right), or its actual reading must be subtracted from the value of the smallest divisions of the limb, for the reason that the reading of the vernier shows how far its 0 has gone past a division line of the scale if moving in the proper direction; and the remainder, after subtracting the reading from the value of a division of the limb, shows the distance forward to the next, or the amount passed over by the zero of the vernier on the arc of excess.

RULES.—*First. To find the "smallest reading" of the vernier, divide the value of the division on the limb by the number of divisions in the vernier.*

Second. Read forward along the limb to the last graduation mark passed by the zero of the vernier; then read forward along the vernier if direct, or backward if retrograde, until coincident lines are found. The number of this line on the vernier is the number of smallest reading units to be added to the reading made on the limb.

PROBLEMS.—1. A barometic scale is divided to $\frac{1}{20}$ inches. The vernier of 25 divisions covers 24 of the scale; a vernier of 20 divisions covers 41 of scale. What are the smallest readings? Are they direct or retrograde?

2. A scale is divided to $\frac{1}{10}$ inches. How many divisions must a vernier have to read to 0.005 of inches? How many divisions of scale will the vernier cover, and will it be direct or retrograde?

3. It is desired to graduate a scale and construct its vernier to obtain a smallest reading of $\frac{1}{2000}$ of a foot. What will be the scale and number of divisions of the vernier? Direct or retrograde?

4. A scale is graduated to $\frac{1}{100}$ of a foot. What will be the least count in the following cases. 10 of scale=11 of vernier? 10 of scale=9 of vernier? 11 of scale=10 of vernier? 9 of scale=10 of vernier? Which are direct and which retrograde?

5. A limb is divided to 15 minutes. Construct a vernier to read to 20 seconds.

6. The least count of a vernier is 20 seconds on a limb graduated to $\frac{1}{6}$ of degrees. How many divisions on vernier?

7. The least count of a vernier is 15 seconds. It covers 81 divisions of the limb. How many divisions on the vernier and what is the value of a division of the limb?

CHAPTER V.

INSTRUMENTS FOR MEASURING DISTANCES.

MEASURING THE BASE-LINE.—This is usually the first of the preparatory steps towards map-making. Upon the proper selection of the site for the base-line and its correct measurement depends all subsequent work of triangulation, since any error in the base-line will be multiplied many times.

(1) The site must be reasonably level; (2) the ends must be mutually visible; (3) a good view of the country must be obtainable from each end; (4) the base must be so situated as to form well-proportioned figures with the points observed.

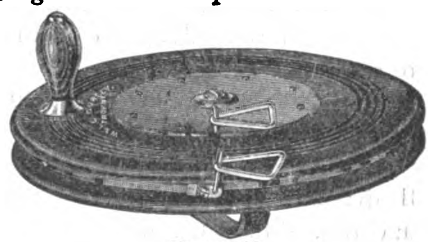


FIGURE 78.

WOODEN AND METAL RODS. — Various methods and instruments have been used for measuring base-lines. At first *wooden and metal rods* of various kinds were used, but they were found expensive and they afforded many possibilities of error, so are being superseded by other means.

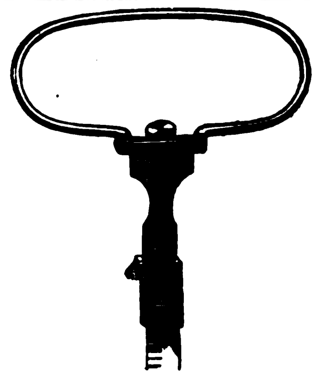


FIG. 78 a.

STEEL TAPES.—The most accurate, rapid, economical, and convenient method for this purpose, and the one that has come into general use, is with *steel tapes* (Fig. 78). They are made of thin steel ribbon in one piece from $\frac{1}{10}$ to $\frac{1}{2}$ inch wide and from 3 feet to 1000 feet long. The usual length for Engineering

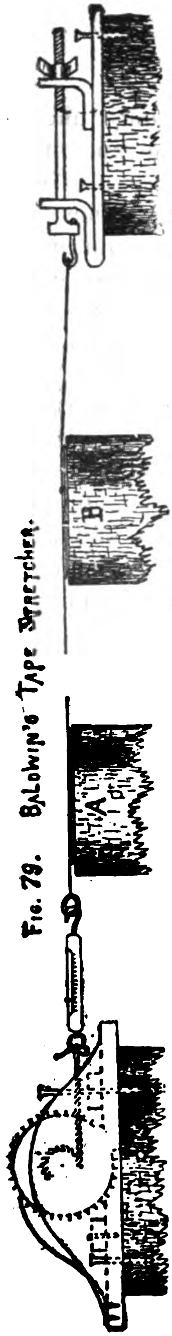


FIG. 79. BALDWIN'S TAPE STRETCHER.

work being 100 feet, and for Topographic work 300 feet. They may be obtained with almost any desired graduations, and with extras, as compensating handles with graduated scale for temperature (Fig. 78*a*), pocket thermometers, and spring balances containing a level-tube. Their lengths will vary with the temperature, which must be considered in very accurate work.

To obtain an accuracy of 1 in 5000, steel tapes may be used at all times and in all kinds of weather, being held and stretched by hand, the tapeman estimating the amount of pull, horizontal position and temperature.

To measure a line with an accuracy of 1 in 50,000, the tape should be tested under the same conditions of supports, tension, etc., that are to be used in the field. Marking-posts "A and B" (Fig. 79) are driven solidly into the ground, on the line, a distance apart equal to the length of the tape, with the tops at the height at which the tape is to be held. Zinc strips 1 to 2 inches wide are tacked on the tops of these posts, and on these the tape-lengths are marked with a steel point. Stakes are driven along the line to be measured at the distance apart at which the test was made (say 25 feet), with the front faces of the stakes in the line. On the front of the stakes nails are driven on the same level or at a determined grade for hanging hooks, the purpose being to support the tape. The tape is placed

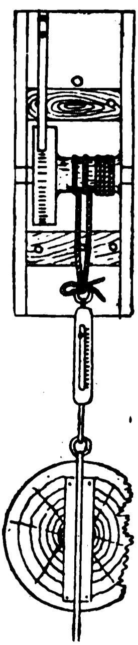


FIG. 79*a*.

on the hooks, which should be about 2 inches long, and the first graduation mark held at the scratch on the zinc on the first marking-post, the pull (about 15 pounds for a 100-foot tape) is applied at the other end of the tape, and, where the last graduation comes, is marked with the steel point on the zinc strip on the second post. Measure between the second and third marking-posts in the same way, and so on to the end of the line. The temperature should be taken at each measurement, and if different from that at which the test was made or the tape is standard, the proper correction should be applied. These measurements are best made in still, cloudy weather and when air and ground are at about the same temperature. If measured on a grade, it must be reduced to horizontal by multiplying by the cosine of angle of inclination.

The party should consist of *a chief*, who exercises a general supervision, marks the extremities of the tape and provides against errors; *a rear-tapeman*, who adjusts the rear end of tape to the contact mark, and carries and reads a thermometer; *a fore-tapeman*, who adjusts the forward end of the tape, exerts the required tension upon it, and carries and reads a thermometer; and *a recorder*.

The proper alignment is generally marked out before beginning the measurement. If a railway tangent is used as the site of the base-line, the tape is kept at a uniform distance from one of the rails, the ties forming the supports.

For obtaining the required tension, a spring balance is attached to the forward end of the tape, and a pull applied by various devices. One devised by Mr. H. L. Baldwin, Jr., of the Geological Survey, is very simple and easily made. Practically it is a small windlass or winch (Fig. 79*a*) fastened to a post beyond the marking-posts. For holding the rear end of the tape exactly at the contact mark, the tape is attached to a screw-bolt in a frame fastened to a post in rear of the marking-posts. The base should be measured at least twice, and the ends should, if possible, be permanently marked by stone monuments set in the ground, and the exact ends marked by a cross-cut in a copper bolt embedded in the head of the stones.

For greater accuracy than 1 in 50,000, other and greater precautions must be taken. Corrections must be made, not only for the effects of temperature, but also for elasticity, sag, pull, and alignment, both horizontal and vertical.*

The advantages of the Steel Tape, besides its accuracy, are its convenience and constant length (not being subject to changes of length due to wearing, stretching, and kinking like a chain).

The disadvantages are, however, that they are liable to break if stepped upon, or are carelessly used in high winds and allowed to coil and then jerked taut.

CHAINS.—*The ordinary Surveyor's or Gunter's Chain* (Fig.

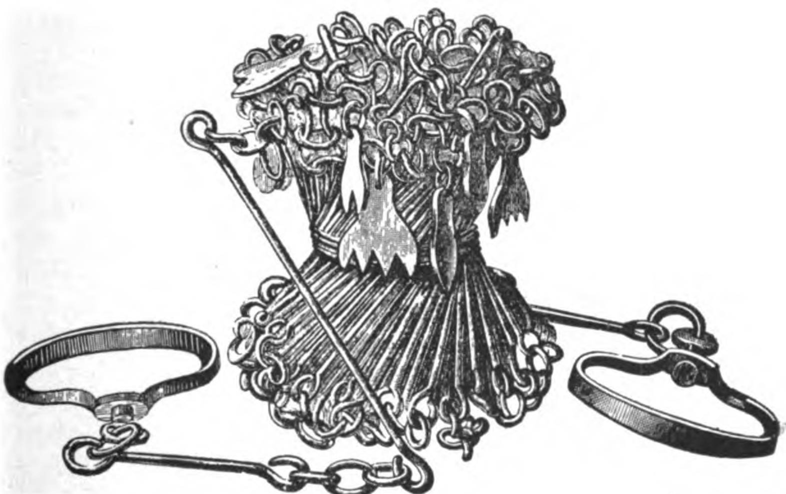


FIGURE 80

*In Geodetic Surveying the lengths of all triangulation lines are reduced to what they would be if projected upon a sea-level surface by radii passing through the extremities of the lines. But in Military Topography such reductions would seldom be made.

After the triangulation has been carried over a large district, the work is checked by so locating a side of a triangle or one side of the last triangle that it can be measured with the tape, and this length then compared with its computed length. If they agree, it proves the accuracy of the intermediate work. The side measured is called a *base of verification*.

80) is 66 feet long, composed of 100 links, each 7.92 inches long, connected with the other by 2 (or 3) rings and furnished with tally marks at the end of every 10 links. A link in measurement includes 1 ring at each end when joined by 2 rings (or $1\frac{1}{2}$ rings when joined by 3 rings). The handles are generally of brass and each forms a part of the end links, to which it is usually connected by a swivel and nut, by which the length of the chain may be adjusted.

The Tallies are of brass and have 1, 2, 3, or 4 notches as they are 10, 20, 30, or 40 links, respectively, from either end; the tally of the 50th link or middle of the chain is round, to distinguish it from the others. *The links* are made of iron or steel wire from $\frac{3}{32}$ to $\frac{5}{32}$ inches in diameter. The link connections are designed to avoid kinking and stretching as much as possible.

This chain, or a steel tape correspondingly graduated, is the one most generally used in land surveying, being convenient for determining areas in acres or distances in miles, the links being decimals of the whole chain and so entered in the notes and so regarded in calculations. Thus 10 square chains =1 acre, 80 chain lengths=1 mile. It is the one used in the Government Land Surveys and the one meant when used in deeds and other documents.

The Engineer's Chain differs from Gunter's in that the links are each 12 inches long instead of 7.92 inches. They are made either 50 feet or 100 feet long, with swivel handles, tallies, etc. The 50-foot chain having but 50 links, the counting, by its tallies, is done from one end in a single direction only. The degree of accuracy usually required for ordinary work with a chain of standard length is 1 in 1000.

Testing the Chain. The chain, like the tape, varies in length with changes in temperature, and in addition it changes in length from wear, flattening of rings, bending of links, etc. Hence, before using a chain it should be compared with a standard length of the United States Coast and Geodetic Survey laid off for that purpose and the chain adjusted to the true

8⁵ Ch... = 1,7...

length, if adjustable, or, if not adjustable and too long, marking the true length with a file on one of the handles, and if too short, noting the amount so as to apply it as a correction to all measured lines. The necessity of this care arises from errors being cumulative when made by using an incorrect measuring unit.

Laying off a Standard. If no standard is at hand, one should be laid off as follows: Two long stout stakes should be driven in the ground, reaching below the frost line, at a distance apart of 1 chain. In the top of these stakes should be driven nails marking the standard length. Stones may be sunken for the same purpose, with scratches to mark the standard length, or it may be marked on the water-table of a building, or coping of a wall.

For laying off the standard a steel tape is best, knowing the temperature and pull at which it is United States standard; a new chain from a reliable firm, with the same data, may also be used.

As both iron and steel expand with heat and contract with cold about 0.0000065 of their lengths for 1° F., if the tape or chain is used for *laying off a standard* at other temperatures than that at which it is standard, a correction must be applied to the measured length to obtain the true length. This correction is given by the formula, $C=0.0000065\ (Ts-Tu)\ L$; in which C=the correction to be applied to the measured length to give the true length, Ts=the temperature Fahrenheit at which the tape is standard, Tu=the temperature at which the tape is used, and L=the length of the tape. From the formula C is negative when the thermometer is higher than at standard or when $Ts<Tu$, because the measuring limit is too long (due to expansion), and positive when $Ts>Tu$, because the measuring unit is too short (due to contraction). This correction should always be applied when laying off a standard for future reference and comparison.

Measuring lines with an incorrect Chain. If, however, *lines are measured* with an incorrect chain, their true lengths may be obtained from the following proportion, provided it is possible to compare the chain with a standard and obtain the error in its length.

$$\text{The standard} : \left\{\begin{array}{l}\text{The true length of}\\ \text{the chain as given}\\ \text{by the standard.}\end{array}\right\} :: \left\{\begin{array}{l}\text{The measured length}\\ \text{in terms of incorrect}\\ \text{chains.}\end{array}\right\} : \text{True length.}$$

If the tape or chain was longer than the standard, the true length of the measured line will be longer than the recorded length; thus 100 feet : 101 feet : : 10 chains : 10.1 chains.

And if the tape or chain was shorter than the standard, *vice versa;* thus 100 feet : 99 feet : : 10 chains : 9.9 chains. In measuring a given line on the ground it should be borne in mind that if the chain is too long, the recorded distance will be too short, and *vice versa.*

To obtain correct areas from those calculated with measurements made with an incorrect chain, use the squares of the first members of the above proportions, and the areas for the second members.

All measurements with the tape or chain should be horizontal or be reduced to the horizontal if measured on a slope.

FIG. 81.

FIG. 81 a.

MARKING-PINS AND RINGS.—With each chain there are needed 11 marking-pins (Fig. 81), about 14 inches long, made of iron or steel wire, pointed at one end to enter the ground, and formed into a ring at the other, for convenience in handling. They can also be had loaded with a little mass of lead around the lower end (Fig. 81*a*), so as to answer as a plumb when dropped to the ground from the suspended end of the chain. Pieces of colored cloth should be tied in the rings, so they can be readily seen in grass, leaves, etc. The pins are carried on a large spring-catch ring.

TO FOLD AND OPEN CHAIN.—*The chain is folded* by taking the two middle links in one hand and folding towards the two ends simultaneously, laying each pair of links obliquely across those already folded, so as to give it the shape of an hour-glass, the handles coming outside as in Figure 80.

To open the chain, take it by the two handles in one hand and throw the chain out on the line with the other, at the same time retaining the hold on both handles.

CHAINING ON LEVEL GROUND. On level ground, the extremities of the line to be measured being marked by range poles or otherwise, two chainmen with the chain folded, 11 pins, note-book, etc., proceed to one extremity of the line. The fore-chainman with 11 pins, puts one pin down at the starting-point, takes one handle and draws the chain out on the line. The chain is then carefully examined its entire length to see that there are no kinks in it. The hind-chainman then holds the *outside* of his handle against the pin at the starting-point, and, sighting over the pin in the direction of the line, puts the fore-chainman in the line by motions of his hand or calling out "Right" or "Left." The fore-chainman, to facilitate this and to enable him to draw the chain taut at the moment of setting the pin, drops on his left knee and holds his handle in his right hand and braces his right forearm against the inside of his right knee. When accurately aligned, the hind-chainman calls out "Stick" or "Down," when the fore-chainman with his left hand forces the pin vertically in the ground on the *inside* of his handle and calls out "Stuck" or "Down." The hind-chainman then (and never until he hears this signal from the fore-chainman) pulls up his pin, puts it on his ring preparatory to proceeding, and picks up his handle. The fore-chainman then proceeds along the line, dragging the chain after him, until the hind-chainman, approaching the pin just set, calls out "Steady" or "Halt."

The chain is then again "lined in," a pin stuck, etc., as explained, and so on to the end of the line. Having passed over a distance of 10 chains, the fore-chainman, after signaling that he has stuck his pin in the ground, it being his last, calls out "Pins." The hind-chainman then pulls up his pin, goes forward with the 10 pins which he has picked up, and gives them to the fore-chainman, who counts them to see that none are lost and that he starts again with the right number, 10. The hind-chainman then records the 10 chains passed over.

The eleventh pin, the last one stuck, being left in the ground to serve as the starting-point, the fore-chainman picks up his handle and proceeds as before. On reaching the end of the line, the fore-chainman stops, holds his end of the chain against it, while the hind-chainman drops his end, comes up to the last pin stuck, counts the number of pins he has on his ring, records them as chains, then pulls the chain taut, notes the exact point opposite the pin, and, without removing the pin, counts the number of links to the end of the line by means of the tally tags, and records them as links; or all as chains and decimals; or hundreds, tens, and units of feet, depending on the chain used.

By the use of 11 pins, only every 10 chains need be recorded, always leaving 1 pin in the ground for a starting-point on the new tally. The position of hind-chainman being the more responsible, the more skilled of the two chainmen should take that position.

CHAINING OVER UNEVEN GROUND. In chaining over irregular ground two light rods or poles about 6 feet long are needed for aligning and to enable the elevated end of the chain to be held vertically over the point, and to assist in pulling the chain taut. Range poles 10 feet long are useful for ranging out a long line.

If the forward range pole cannot be seen at any time by the hind-chainman, then the fore-chainman must align himself on the pin of the hind-chainman and rear pole.

If no range poles are used and the line run by an instrument, then the instrument-man lines in the fore-chainman.

As the measured lines are to be horizontal measurements, when chaining over undulating ground the chain must be held horizontal by raising the down-hill end. For this purpose a staff is used (Fig. 82); the chain being raised along it until horizontal, a weighted pin or plumb is dropped, when an ordinary pin is stuck where it strikes. If the slope is very steep, a fraction only of the chain can be used at one time (the chain having been drawn out to its full length on the slope) until an entire chain's length has been measured. On account of it being almost impossible for the hind-chainman, even with a staff or plumb, to hold a point of the chain several feet vertically over a point on the ground, while the fore-chainman is pulling the chain horizontally and setting his pin, chaining up hill is much less accurate than chaining down.

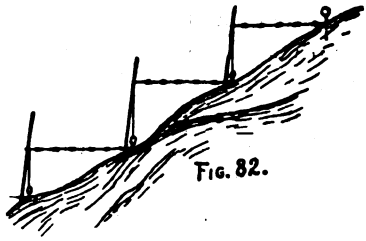


FIG. 82.

If an instrument for measuring the angles of the slopes is at hand, the chaining may be done on the surface and the proper corrections afterwards made in the chained distance to reduce it to the horizontal, either by multiplying the length by the cosine of the angle of slope or by reference to the following table:

TABLE OF REDUCTIONS FOR SLOPES.

ANGLE OF SLOPE IN DEGREES.	SUBTRACT FOR EACH 100 FEET.	RISE OR FALL IN 100 FEET HORIZONTAL.
1°	.015 foot.	1.75 feet.
2°	.061 foot.	3.49 feet.
3°	.137 foot.	5.24 feet.
4°	.244 foot.	6.99 feet.
5°	.381 foot.	8.75 feet.
6°	.548 foot.	10.51 feet.
7°	.745 foot.	12.28 feet.
8°	.973 foot.	14.05 feet.
9°	1.231 feet.	15.84 feet.
10°	1.519 feet.	17.63 feet.
11°	1.837 feet.	19.44 feet.
12°	2.185 feet.	21.26 feet.
13°	2.563 feet.	23.09 feet.
14°	2.970 feet.	24.93 feet.
15°	3.407 feet.	26.79 feet.
16°	3.874 feet.	28.67 feet.
17°	4.370 feet.	30.57 feet.
18°	4.894 feet.	32.49 feet.
19°	5.448 feet.	34.43 feet.
20°	6.031 feet.	36.40 feet.

When the line to be measured lies in a sloping plain this is more accurate than attempting to chain horizontally, on account of the several difficulties of holding the chain exactly horizontal, exerting the proper pull that the stretch may equalize the sag, and dropping the pin or plumb from the right point at the right time. In chaining on slopes, the elevated end of the chain is seldom held high enough to bring the chain horizontal.

RANGING OUT LINES.—In the description of the methods of measuring a line, it has been assumed as already marked out its entire length by range poles or otherwise. If, however, two points of a line are given and it is desired to prolong it, range poles are set up vertically at the two points, a man, if alone, goes in the direction in which the line is to be prolonged as far as he can still see both poles plainly, then, holding a plumb-line in front of him, moves sideways until he reaches a point where his line covers both poles, when he plants a range pole at the point indicated by dropping the plumb, and then proceeds to another point in prolongation of the line, plants a pole, and so on to the end of the line.

If the range poles are not to be left where placed, three men are required, two being on the line; the third ranges himself in, the first then goes beyond and ranges himself in, and so on.

If in a hilly country and it is desired to range out a line over a hill between points A and B (Fig. 85), in opposite valleys, neither of which can be seen from the other, two observers range themselves in near the top of the hill by *b* directing *a* until he is in line with the object *B*, then *a* from his position ranges in *b* with the object *A*, and so on until both are in the line.

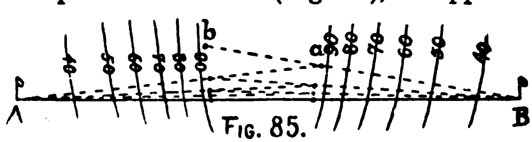


FIG. 85.

One observer with a long pole, rope, or chain could place himself on the line by observing each object in succession from the end of the pole farthest from the object observed, and moving the intermediate end into the line by a succession of approximations, until viewed from both ends the alignment is correct.

If across a valley between points A and B (Fig. 86), on opposite hills, an observer standing at A holds a plumb-line over it, and, bending down until the line is seen covering the distant point B, directs another person to place down the slope, poles p, p', p'', etc., in the same line with plumb-line and distant point.

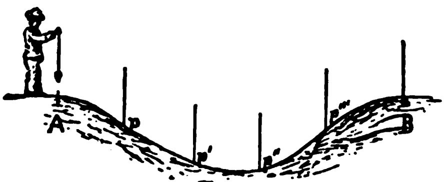


FIG. 86.

If through a wood (Fig. 78), range out a trial line AC as near the desired line AB as possible, until a point C is reached from which a perpendicular CB may be erected to the line. Measure this perpendicular, then move all intermediate stakes, a, b, c, etc., perpendicularly over their proportional amounts.

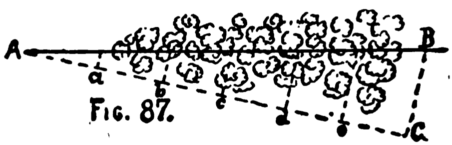


FIG. 87.

PASSING OBSTACLES.—In measuring lines, should objects, as houses, woods, ponds, etc., be in the line, means must be taken to pass them, at the same time measuring a line equal in length to the portion of the line omitted, and then getting on the line beyond the obstacle and proceeding as if it were not there.

By rectangular offsets. This is done by the application of some problem in geometry, as by rectangular offsets, or otherwise. Thus, range out the line up to the object (Fig. 88); then, by 2 equal rectangular offsets ab and cd long enough to clear the obstacle, range a line df past the object,

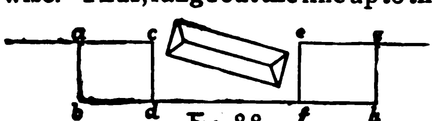


FIG. 88.

at the same time measuring it, and by two equal rectangular offsets *fe* and *hg* from this line, get back upon the original line and prolong it as before.

The simplest method of constructing with the tape or chain a perpendicular to a given straight line at its extremity is by the 3–4–5 rule. Form a triangle, with the tape or chain, with sides in the proportion 3, 4, and 5, as 15, 20, and 25 feet or links; place the angle opposite the longest or 25 side at the given point of the line, the mean or 20 side on the given line, and the shortest side in the direction of the desired perpendicular.

By equilateral triangle. Having prolonged the line up to the obstacle (Fig. 89), form the equilateral triangle *a* on the line, and prolong the side which will pass below the obstacle, measuring it; on this form another equilateral triangle *b*, and prolong the side to the original line prolonged, making it equal to the previous side; then on this form a third triangle *c*, and re-establish the line.*

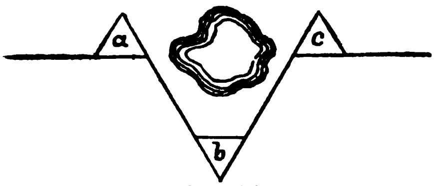


FIG. 89.

Substitutes for steel tapes and chains are linen tape-lines, metallic tape-lines (having metal tinsel wound around strands of the warp), rope after being well stretched, and rods of various materials.

ODOMETERS.—When traversing is done along roads, distances may be measured by counting the revolutions of a

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*For laying off an angle by means of a chain, the formula, Tan. $\frac{1}{2}$ A$=\frac{a}{\sqrt{4b^2-a^2}}$, (in which A is the angle sought, *b* the radius, and *a* the chord), may be used.

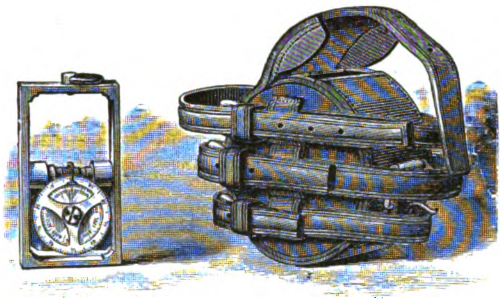

FIGURE 83.

wheel. For recording the revolutions, various automatic devices have been in use. The old form of odometer (Fig. 83), known as the pendulum, has been extensively used, but has not proven altogether satisfactory. The form now in general use, devised by Mr. E. M. Douglas of the Geological Survey, is the one known as the Douglas Positive Motion Odometer (Fig. 84). This form

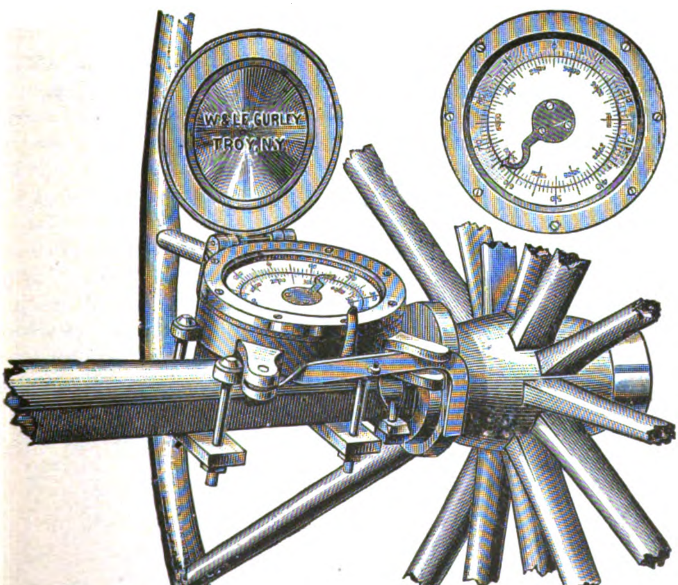

FIGURE 84.

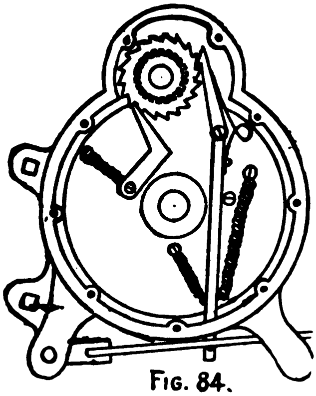

FIG. 84.

is the most trustworthy that has yet been devised,* but some topographers prefer to count the revolutions of the wheel directly, using an arrangemen which rings a bell at each revolution.

The circumference of the wagon wheel having been measured, the number of feet traveled can be at once ascertained by multiplying the circumference of the wheel by the number of revolutions made by it.

The most accurate method of getting this circumference is to divide some known distance on a road (a mile, for example), by the number of revolutions the wheel makes in passing over it. This practically compensates for the slip of the wheel on roads similar to that used.

Distances may also be measured with the Gradienter, and with the Transit and Stadia, to be explained hereafter.

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*The works are contained in a solid metal case, with glass covering the face of the dial. On the outer rim of the dial are 100 divisions read by an index securely attached to an under wheel with 1C0 cogs and carried forward one space by every outward movement of the steel lever shown. A wheel (Fig 84*a*), with 99 divisions and 99 cogs upon it, revolves under the index and on top of the wheel with 100 cogs, and makes a complete revolution and one division for every complete revolution of the index. The Odometer is fastened to the axle, as shown, a cam on the hub giving the outward motion to the steel lever.

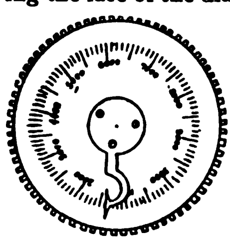

FIG. 84a.

CHAPTER VI.

EXPANSION OF A SURVEY.

The base-line having been measured, the next step is the expansion, which consists in the *selection of stations*, *erection of signals*, and the *measurement of angles*, keeping in mind the principle *to work from the whole to a part, from greater to less, and to enter on no small operations early in the work.*

THE SELECTION OF STATIONS.*—*The Stations*, which are

*Upon the proper selection of stations, together with the site for the base-line, depends, in a great measure, the value of the subsequent work. For this, experience and judgment are necessary. The reconnoitering officer must, in locating stations, consider the effects of the resulting triangles; compare the relative cost of cutting out lines and building high stations; foresee and avoid any disturbing influences or obstructions, etc. He usually provides himself with a pocket sextant for measuring angles between stations, a pocket compass for determin-

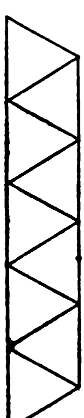

FIG. 91.

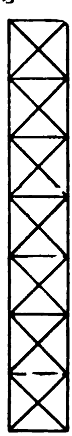

FIG. 92.

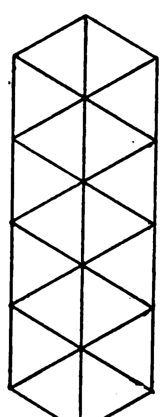

FIG. 93.

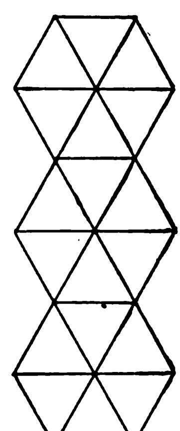

FIG. 94.

to be the vertices of the triangles, to be observed to and from, *must be so selected as to form strong figures*, in order to reduce to a minimum the errors which will creep in. No triangle should have an angle of less than 30°, nor more than 120°. The stations should be located on commanding ground, if possible, so as to be intervisible. They should be selected, as far as consistent with accuracy, with reference to the needs of the topographers—*i. e.*, so as to have at least two or more fall upon a single field sheet.

The triangulation may be extended by continually increasing the sides of the triangles as in Fig. 90, where AB is the measured base-line, C and D are observed upon giving a new base CD, from the ends of which E and F are observed upon giving a new base EF, and so on.

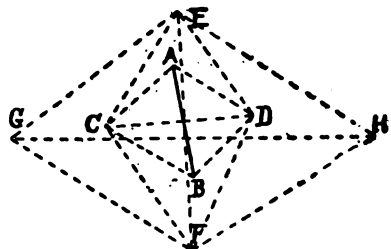

FIG. 90.

The triangulation stations are first plotted on the sheet that is to contain the finished map, and are usually surrounded by a triangle, thus △, which is read *triangulation station*. For

ing the direction of lines, an aneroid barometer for determining approximate elevations, a pair of field-glasses, and climbers or tackle for going up trees. Having located a station, it is temporarily marked by a flag on a pole. Having arrived near where a station would be desirable, the highest ground in the vicinity is ascended, and if there are trees, a house, tower, or other high object on it, one is climbed, and with the field-glasses a search is made for other stations. If no trees or other objects are available, ladders may be used, being held up by ropes.

In cases where the triangulation takes the form of narrow belts of figures, simple triangles or quadrilaterals are the most desirable. The least expensive and most rapidly executed would be as in Fig. 91. If the greatest attainable accuracy is desired, then Fig. 92 would be used. If the greatest area is to be covered for a given degree of accuracy and cost, then Figs. 93 or 94 would be used.

The reasons for using the above figures in different cases will be found in Appendix No. 15, Coast Survey Report of 1876.

"filling in" the details, the area is divided into small tracts embracing 2, 3, or more triangulation points, which are transferred to the *field sheets*, and all information in regard to them given to the one in charge of the details.

The plotting of the triangulation points consists in locating them to scale on the paper in their true relative positions.*

THE SIGNALS.—These should be high, conspicuous, and so made that the instrument can be placed precisely under them, at the same time being of the simplest and least expensive form that will serve the purpose. A pyramid of three or four timbers surmounted by a staff may be used in many cases, or a stout post set in the ground on the side of which a staff is held for observing upon and removed when using the instrument there. When necessary to raise the instrument above the ground to overlook surrounding obstacles, structures for supporting the instrument should be combined with the signals. These should consist of an interior structure for the support of the instrument and an outer structure with a platform for the observer. The two should be separate to avoid jarring the instrument.

To the signal staff is usually attached some kind of a target to make it more conspicuous, as red and white flags if seen against the ground, and red and green if seen against the sky.

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*The method employed depends on the extent of the earth's surface to be represented, the purpose of the survey, and the accuracy required. For very large areas on a single sheet, such as the U. S. Coast, Lake, and Geodetic Surveys include, the earth's sphericity must be considered and the polyconic projection of meridians and parallels be used. Points are plotted from computed latitudes and longitudes.

For areas of not more than 100 square miles, in which the effect of the earth's sphericity is so mall that it may be neglected, the rectangular projection of meridians and parallels may be used, and points plotted from polar coördinates (azimuth and distance) or from rectangular coördinates (latitude and departure). Of these the latter is better, as each point is independently located and cumulative errors are avoided.

When targets of the ordinary kind would be invisible by reason of distance, state of atmosphere, etc., a "heliotrope" is used. These are of various patterns, but all working upon the principle of projecting rays of light to a distant station by means of mirrors, as with "heliographs."

CHAPTER VII.

THE ENGINEER'S TRANSIT.

THE MEASUREMENT OF ANGLES.—Several instruments differing only in power and degree of accuracy have been used for measuring the angles in triangulation.

In topographic work undertaken for military purposes, the instrument most likely to be used in measuring angles is the Engineer's Transit (Fig. 95). This is the most accurate portable field instrument for the purpose, and with its various attachments it may be used for leveling, measuring distances, determining meridians, reading bearings, fixing grades, etc.

DESCRIPTION OF THE TRANSIT.—The essential parts of the Transit, as shown in the cut, are the *telescope* with its axis and two supports or standards, the *circular plates* with their attachments, the *sockets* upon which the plates revolve, the *leveling-head*, and the *tripod* on which the whole instrument stands

THE TELESCOPE is from 10 to 11 inches long, firmly secured to a horizontal axis having its bearings nicely fitted in the standards, and thus enabling the telescope to be moved either up or down, or revolved completely around if desired. The *object-glass* D (Fig. 96) is composed of two lenses, so as to show objects without color or distortion, and is placed at the end of a slide having two bearings, one at the end of the outer tube, the other in the ring C C, suspended within the tube by four screws, twc of which are shown in the cut. The object-glass is carried out or in by a pinion working in a rack attached to the slide, and thus adjusted to the distance of objects, either near or remote, as desired.

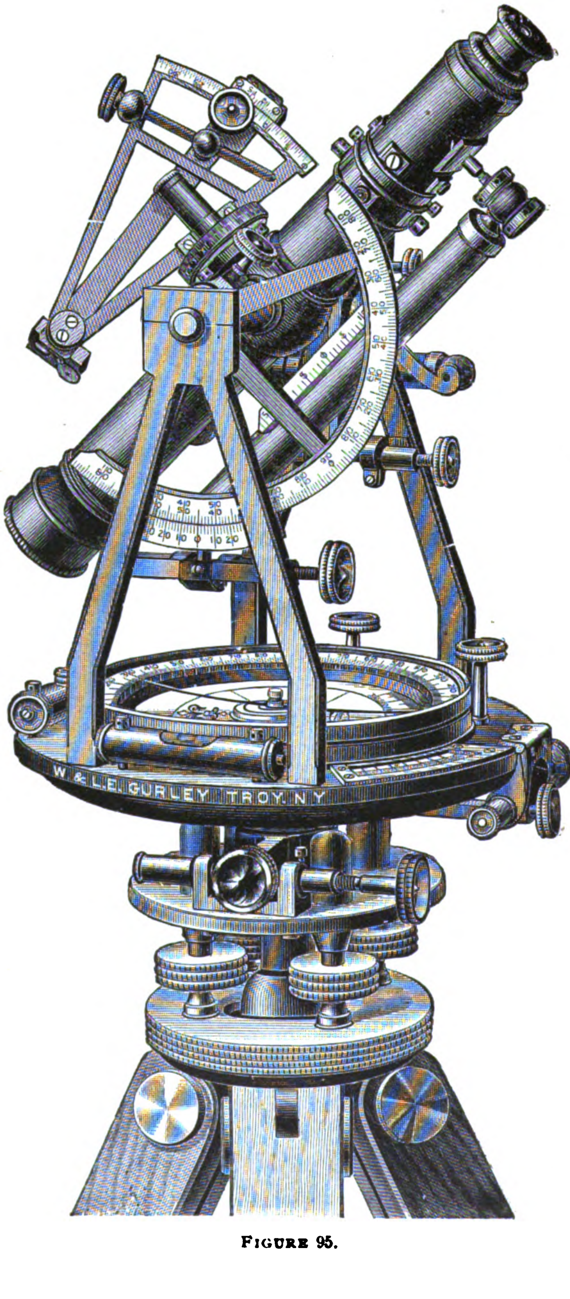

Figure 95.

The Eye-piece is made up of four lenses, which, beginning at the eye-end, are called respectively *the eye*, *the field*, *the amplifying*, and *the object* lenses, the whole forming a compound microscope having its focus in the plane of the cross-wire ring B B. The eye-piece is brought to its proper focus by a rack and pinion movement. This being an adjustment to suit the eye of the observer, when once made needs no further notice for the same person. Sometimes an eye-piece of two lenses only is employed; but this inverts the object seen and so has generally been discarded in American transits. Where it is desired to take vertical angles so great that the eye can not be placed under the telescope, a little cap on the end of the eye-piece is unscrewed and replaced by one containing a small prism, which reflects the image of the object at right angles and brings it to the eye of the observer (Fig. 96*a*). When used on the sun, a colored glass or darkener is interposed between the eye and prism.

FIGURE 96*a*.

The *Cross-wires* (Fig. 97) are two fibers of spider-web or very fine platinum wire, cemented into cuts on the surface of a metal ring, at right angles to each other, so as to divide the small open space in the center

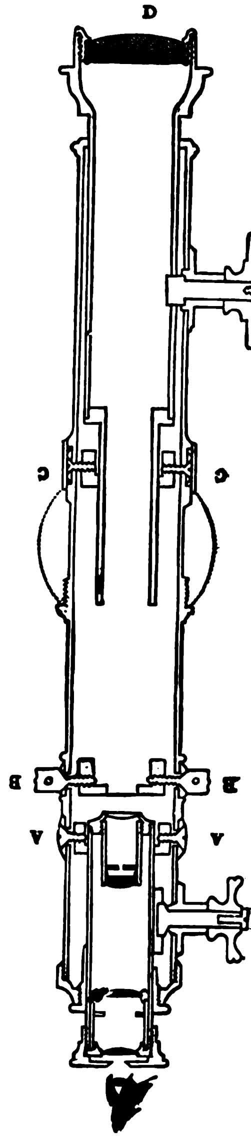

FIGURE 96.

into quadrants. The intersection of the wires forms a very minute point which, when it is adjusted in the axis of the telescope, enables the observer to fix it upon an object with the greatest precision.

This axis of the telescope (which is also the optical axis of the objective) is usually termed the *line of collimation*, and the operation of bringing the intersection of the wires into this axis is called the "*adjustment of the line of collimation.*"

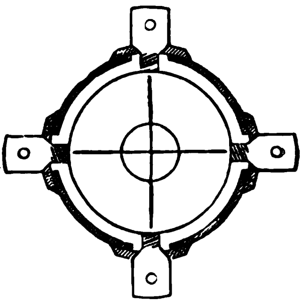

FIGURE 97.

The openings in the telescope tube are made considerably larger than the screws used in adjusting the cross-wires, so that, when the screws are loosened, the whole ring can be turned around for a short distance in either direction. The movable ring at A A (Fig. 96) is used to effect the centering of the eye-piece; and the one at C C is used to effect the adjustment of the object-glass slide.

The Stadia (Fig. 98) is a compound cross-wire ring, as shown, having three horizontal wires, of which the middle one is cemented to the ring as usual, while the others are fastened to small slides, held apart by a slender brass spring hoop, and actuated by independent screws, by which the distance between the two movable wires can be adjusted to include a given space, as 1 foot on a rod 100 feet distant. These wires will in the same manner include 2 feet on a rod 200 feet distant, or $\frac{1}{2}$ foot at a distance of 50 feet, and so on in the same proportion, thus furnishing a means of measuring distances.

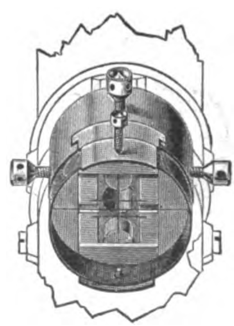

FIGURE 98.

The Supports or Standards of the Transit are firmly attached by their expanded bases to the upper plate, one of them having near the top a little movable box, actuated by a screw underneath, by which the telescope axis may be raised or lowered and thus made truly horizontal.

ATTACHMENTS OF THE TELESCOPE.—Although the instrument is sometimes used with a plain telescope, oftener it is provided with one or more attachments, as the vertical arc or circle, level, clamp and tangent, and solar attachment.

The Vertical Arc or Circle is firmly secured to the axis of the telescope, and, in the instrument shown, is 6 inches in diameter, divided on silver to degrees and half-degrees, and with its vernier reading to 1 minute. On some instruments the arc is replaced by a full circle.

The Level on the Telescope consists of a brass tube about 6½ inches long, each end of which is held between two capstan nuts connected with a screw or stem attached to the under side of the telescope tube. The vial enclosed in the tube is a little over 5 inches long and half an inch in diameter, ground on its upper interior surface so as to insure an even and sensitive bubble, the length of which is measured by the divided scale above, and thus determines when the bubble is brought into the center of its run.

The Clamp and Tangent consists of an arm, at one end encircling the telescope axis and at the other connected with the tangent or slow-motion screw; the clamp is fastened at will to the axis of the telescope by a clamp-screw inserted at one side of the ring,

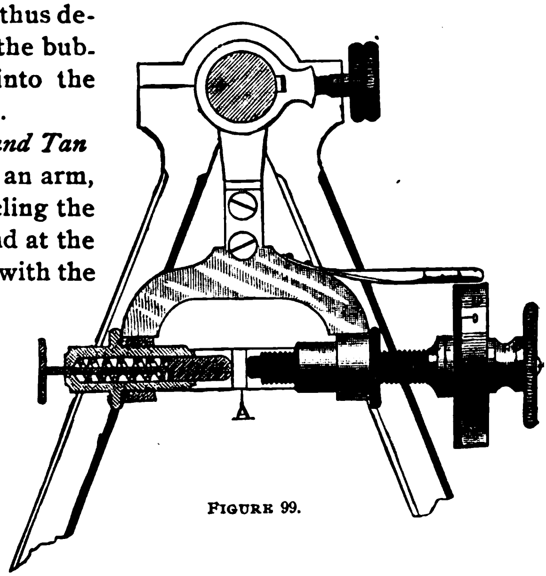

FIGURE 99.

and then by turning the tangent screw the telescope is slowly raised or lowered as desired. In the GRADIENTER, as shown in Fig. 99, a screw is attached to the semi-circular expanded arm of the ordinary clamp of the telescope axis. The screw is accurately cut to a given number of threads, and passing through a nut on one side of the arm, presses against a little stud A fixed to the inner surface of the right-hand standard. In the other side of the arm is inserted a hollow cylinder containing a pin actuated by a strong spiral spring, the end of the pin pressing against the side of the stud opposite that in contact with the screw. Care must be taken to see that the spring presses against the stud; otherwise there will be lost motion or a slight play. Near the other end of the screw, and turning with it, is a wheel or micrometer, the rim of which is plated with silver and divided into 50 or 100 equal parts. A small silver scale, attached to the arm and just above the wheel, is divided into spaces, each of which is just equal to one revolution of the screw, so that by comparing the edge of the wheel with the divisions of the scale, the number of complete revolutions of the screw can be easily counted on it.

The Solar Attachment, placed upon the cross-bar of the ordinary transit, consists essentially of a *polar axis* attached in the same vertical plane with the line of collimation of the telescope and perpendicular to it. Attached to the polar axis and revolving about it is a triangular frame carrying an arc, called the *declination arc*, graduated to quarter-degrees. At the angle of the frame opposite the declination arc a movable arm has its center of motion. On the other end of the arm is a vernier which reads to single minutes on the arc. The arm is moved by a tangent screw and has a clamp.

At each end of the arm is a rectangular block of brass in which is set a small convex lens having its focus on the surface of a little silver plate (Fig. 100) fastened by screws to the inside of the opposite block and forming between them two pencils of rays; on the surface of the plate are marked two sets

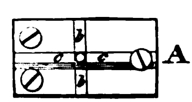

FIGURE 100.

of lines intersecting each other at right angles; of these *bb* are termed the hour lines, and *cc* the equatorial lines.

Surrounding the base of the polar axis is the *hour circle* graduated to 5 minutes of time and figured from I to XII. This is read by a small index fixed to the triangular frame and moving with it. When it is desired, the attachment can be removed from the telescope and packed in the case, and a protecting thin sheath put on the polar axis.

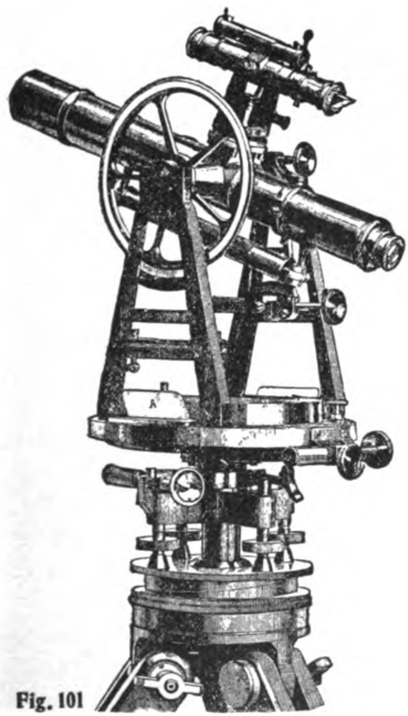

Fig. 101

Saegmuller's Solar Attachment (Fig. 101) consists essentially of a small telescope attached to the axis of the transit telescope and, used in connection therewith, performs the office of the solar compass.

THE CIRCULAR PLATES and their accompanying sockets are shown in Fig. 102. The upper plate A A, carrying the compass-limb, verniers V V, levels, telescope, etc., is screwed fast to the flange of the interior solid spindle, and is called the *alidade;* the lower plate or graduated limb B is fastened to the socket C, which encircles the spindle, and is in turn fitted to and turns in the hollow socket of the leveling head. All that portion of the instrument above the leveling head and which turns in the leveling head socket is sometimes called the *head of the instrument.*

THE COMPASS consists essentially of a circular box about 6 inches in diameter, covered by a glass to exclude dust and moisture, and containing a magnetic needle N (Fig. 102) of

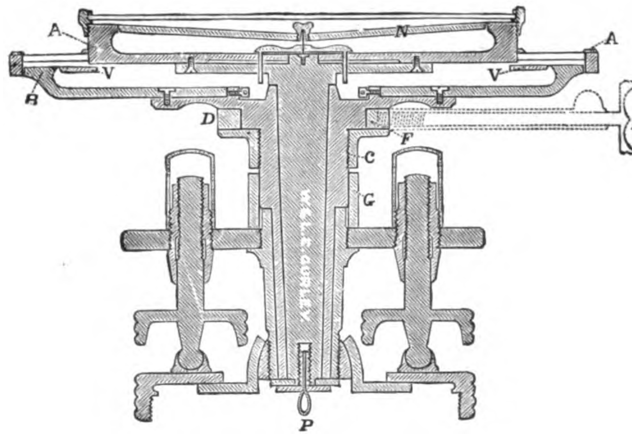

FIGURE 102.

best steel, pivoted upon a hardened pin in the center of the box, allowing the needle free play in a horizontal direction. The north end of the needle is usually marked in some way to distinguish it, and on the south end is a coil of small brass wire to balance it.

Under the needle is a little plate, which is raised by a screw outside the box, lifting the needle off the pivot when not in use and when being carried around, to avoid unnecessary wear on the pivot.

The circular rim of the box is silvered and graduated into degrees and half-degrees, the degree marks being extended down on the inner edge. The graduations are figured from 0° to 90° each way from each end of the diameter or zero line. On the interior of the circle described on the bottom of the box a short arc is graduated the same as the rim, and called the *declination arc*. On the outside of this same circle and opposite the declination arc is a double vernier reading to single minutes on the declination arc.

The graduated rim and the verniers have a slight independent circular motion, by means of a pinion, around the inner portion of the bottom of the box containing the declination arc, which latter remains fixed with reference to the line of sight. This motion is for setting off the declination of the needle, and can be clamped by a screw. One end of the zero line is called north, and generally indicated on the box with a *fleur-de-lis*, the other end south and marked with an S. The letter E (east) is placed 90° to the left of north and the letter W (west) 90° to the right of north, when looking from the south end of the zero line, because, at any place, the needle remains stationary while the line of sight and graduated circle revolve in taking bearings. Hence, if the line of sight points towards the northeast, the north end of the needle (which is the end always ready) will point to the quadrant between the letters N and E indicating at once the direction of the line of sight. Similarly for the other quadrants.

THE CLAMP AND TANGENT movement of the two plates or alidade has its tangent screw with opposing spring attached to the upper plate as shown in Fig. 95. The clamp, shown in Fig. 102, is a strong metal ring D F, moving easily around the socket C which carries the lower plate, and to which it is se curely fixed at will by the screw E, impinging upon a small segment F.

By this means the two circular plates are clampe d firmy together and moved slowly around each other, in either direction, by the tangent screw, or loosened at will and moved by the hand, the telescope being thus easily and accurately directed to the point of sight.

THE TWO LEVELS OR BUBBLE-TUBES are shown placed at right angles to each other so as to level the plate in all directions, and adjusted by turning the capstan-head screws at their ends by a small steel pin.

THE TWO DOUBLE VERNIERS V, V, attached to the upper plate are diametrically opposite to each other and serve to read the limb of the lower plate, around which they revolve.

THE LOWER PLATE OR LIMB B is graduated on its upper surface usually into degrees and half-degrees, and figured in two rows—viz., from 0° to 360°, and from 0° to 90° each way or in single series from 0° to 360°, or 0° to 180° on each side, If the vertical axis of the socket C does not pass through the center of the lower plate B which is attached to it, then the upper plate which revolves around it will have an eccentric motion, and if but one vernier were used the angles read would not be true; hence the two verniers are placed diametrically opposite and the average of the angles they record is used, which eliminates the effect of eccentricity. The verniers themselves may not be exactly 180° apart, but taking the mean of the two readings eliminates errors from this source also.

THE SOCKETS are compound. The inner solid spindle, to which the upper plate is attached, turns in the socket C when an angle is taken on the limb, but when the two plates are clamped together the socket C, and with it the whole head of the instrument, revolves in the socket of the leveling head.

THE LEVELING HEAD consists of two plates connected together by a socket having at its end a hemispherical nut fitting into a corresponding cavity in the lower plate. These two plates are inclined to each other or made parallel at will by four leveling screws, three of which are shown in Fig. 95. In leveling the instrument, diametrically opposite screws are grasped, one in each hand between the thumb and forefinger, and both turned at the same time either towards each other or from each other, turning the left thumb in the direction it is desired the bubble should move to come to the center.

THE CLAMP AND TANGENT movement of the leveling-head serves to turn the head of the instrument upon its sockets. Some instruments have a screw instead of the spiral spring in this movement, and when this is the case, care must always be taken to see that there is no lost motion between the two screws.

THE LOWER LEVELING PLATE is made in two pieces, of which the upper is screwed fast to the tripod and has a large opening in its center, in which the smaller lower one is shifted from side to side or turned completely around. This is called a "*shifting center*," and by means of it the plummet which hangs from P can be set precisely over a point without moving the tripod.

THE TRIPOD consists of three legs of wood attached to a head by bolts. The lower ends of the legs are fitted with brass shoes and iron points.

Extension tripods (Fig. 103) are very handy for use on uneven ground, where lengthening or shortening the legs at will would be necessary; also for shifting the head of the instrument over large distances to bring the plummet over a point without moving the tripod.

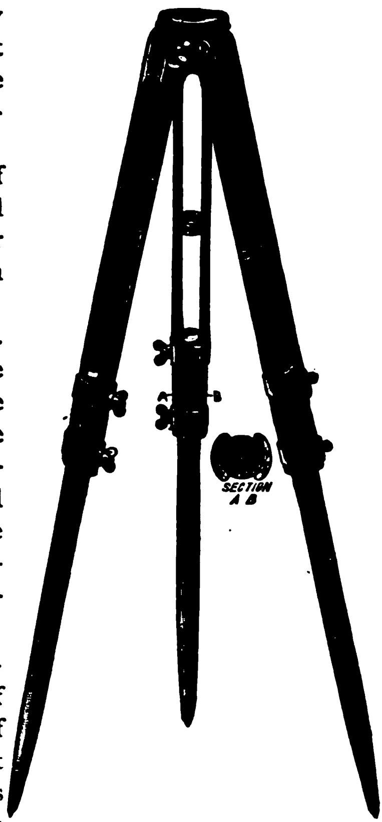

FIGURE 103.

PRECAUTIONS IN CLAMPING.—The different parts of the instrument being made of brass, bronze, and other soft metals, the greatest care is necessary not to strain any of the screws or force any of the movements. A slight tension only is required in clamping, as in all other working of screws. These precautions apply as well to all other instruments.

PLUMB-LINE AND BUBBLE-TUBE.—The plumb-line and bubble-tube are at once the most simple, universal, and essential of all appliances used in surveying and astronomical work. Without them neither the zenith nor the horizon could be effectually determined, and the determination of altitudes and of horizontal lines would be out of the question. Even azimuths, bearings, and horizontal angles require that the circle by which they are obtained shall be brought into a horizontal position.

The plumb-line is a vertical line pointing to the center of the earth, and a plane at right angles to this line is *for that point* a horizontal plane. As no two plumb-lines can be parallel, so no two planes can be parallel which are respectively horizontal at two different places on the earth's surface. Parallel horizontal planes can only be planes at different elevations (unless on opposite sides of the earth), all horizontal for a single point on the earth's surface.

A level surface is a surface (not a plane) which is at every point perpendicular to a plumb-line at that point. If the earth were covered with a fluid in a quiescent state, the surface of this fluid would be a level surface.

A bubble-tube is a round glass tube, bent or ground so that a longitudinal section of its inside upper surface is the arc of a circle. This tube is nearly filled with ether, the remaining space being occupied with ether-vapor, which forms the bubble, which latter always seeks the highest point of the tube.

Proposition I. If a bubble-tube be rigidly attached in a frame, as in a striding level, or under the telescope of a wye level, and if this frame be reversed end for end on the same two points of support, and the bubble remains in the same portion of the tube in both positions of the frame, whether it be the center of the tube or not, the points of support are in the same horizontal line.

The right line tangent to the inner upper surface of the bubble-tube, at the center of the bubble, is called *the axis of the bubble*, and is horizontal.

When the bubble is in the center of the tube, its axis is called the *axis of the bubble-tube.*

Proposition II. If a bubble-tube, like those on the plate of compass, and on transits and levels, be revolved about a supposedly vertical axis, and the bubble remains in the same portion of the tube during the entire revolution, the axis o revolution will be vertical, and *the axis of the bubble* will describe a horizontal plane.

Cor. 1. Similarly, if a bubble-tube be revolved 180° about a supposedly vertical axis, as in adjusting plate bubbles, and the bubble remains in the same portion of the tube in both positions, the axis of revolution lies in the vertical plane perpendicular to the axis of the bubble in the two positions. If the same test be made for any other two horizontal positions 180° apart (preferably 90 degrees from first position) and the bubble remains in the same portion of the tube, the axis of revolution lies in the vertical plane perpendicular to the two new positions of the axis of the bubble; hence it lies in the intersection of the two vertical planes and is itself vertical.

Cor. 2. If two bubble-tubes, preferably at right angles to each other, be rigidly attached to a plate or frame, as the plate bubbles of compass and transit, that revolves about a vertical axis, and if each bubble remains in the same portion of its tube in two positions 180° apart, the axis of revolution is vertical.

Proposition III. In all cases where a bubble-tube has been reversed 180° on the same points of support, or 180° about an axis, the angular difference between the two positions of the bubble is *twice* the angular deviation of the points of support from the horizontal, or of the axis from the vertical.

SENSITIVENESS OF THE BUBBLE.—A bubble is sensitive directly as the length of the radius of curvature. The longer the radius or flatter the curve, the more sensitive; the shorter the radius or the greater the curvature, the less sensitive. It is also sensitive in proportion to the length of the bubble itself, a long bubble settling more quickly and accurately than a short one.

ADJUSTMENTS OF THE TRANSIT.—Every instrument should leave the hands of the maker in complete adjustment, but all are so liable to derangement by accident or careless use, that everyone called upon to use any instrument should

(1) be familiar with the structure and use of all its parts, (2) be able to determine when it is in exact adjustment, (3) to locate the source of the error if not in adjustment, (4) to discuss the effect of any error of adjustment on the work in hand, and (5) to properly adjust all the adjustable parts, or make the proper corrections if not adjustable.

Principle of Reversion. The method of testing the accuracy of nearly every adjustment of instruments is by the application of the principle of "*reversion*," which apparently doubles a real error, and thus enables one to see it and more easily correct it.

The Adjustment of the Engineer's Transit are such as to cause: 1st, the head of the instrument to revolve in a horizontal plane about a vertical axis; 2d, the "line of collimation" to generate a vertical plane through the vertical axis when the telescope is plunged upon its horizontal axis; 3d, the axis of the telescope bubble-tube to be parallel to the line of collimation, thus enabling the instrument to do leveling; 4th, the vernier on the vertical circle to be so placed that its readings shall be the true altitude of the line of collimation. These four results are attained by making the following five adjustments.

1ST ADJUSTMENT.—*To adjust the levels—i. e.*, to make the plane of the plate-bubbles perpendicular to the vertical axis. Set up the instrument upon its tripod, as nearly level as possible, and, having unclamped the plates, turn the upper one until the levels are brought into positions parallel to the directions of diametrically opposite leveling screws; then with the thumb and first finger of each hand clasp the heads of two diametrically opposite screws, and, turning both thumbs in or both thumbs out, as may be required, turning the left thumb in the direction it is desired the bubble to move, bring the bubble to the center of the opening. Without moving the instrument, proceed in the same manner with the other pair of leveling screws to bring the other bubble to its center;

if in doing this the level first corrected be thrown out a little, bring it in again; when both are in place, turn the upper plate half-way around—*i. e.*, 180°; if the bubbles both come to the center, they would need no correction; but if not, with the adjusting pin turn the small screws at the end of the levels until the bubbles are brought back one-half of their displacement;* then bring the bubbles again into the centers with leveling screws, as at first, and repeat the operations until the bubbles will remain in the center during a complete revolution of the instrument, when the adjustment will be complete.

Sometimes the bubble-tube rests on a knife-edge under the middle, in which case the screw at one end must be loosened before the other is tightened.

This adjustment is important in measuring all horizontal and vertical angles. In the latter case the error may be equal to the amount of the inclination of the vertical axis, while in the former case it will be less if objects are nearly in same horizontal plane.

In all the following adjustments the first step is to set up

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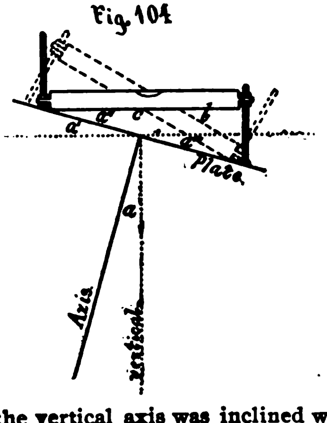
* To show that turning the upper plate half-way around *apparently doubles* the errors of adjustment of the bubble-tubes, or that the angle between the two positions of a bubble-tube (direct and reversed) is double the angle made by the axis of the bubble-tube with the upper plate, to which it should be parallel when in adjustment, assume the bubble-tube, drawn full in Fig. 104, as leveled in the original position, and that drawn broken as having been revolved 180° about the axis; then a=inclination of axis=a' =a''=a'''; b=angle between direct and reversed positions of bubble tube;

Fig. 104



$$b+c=a''+a'''+c;$$
$$\therefore\ b=a''+a'''=2a.$$

Correcting half the displacement of the bubble by means of the bubble-tube adjusting screws corrects the *whole* error of adjustment, but, by reversing the instrument, the amount the vertical axis was inclined was made to appear as an error of the bubble-tube; hence the necessity for releveling.

the instrument firmly and level carefully; hence it will not be repeated.

2D ADJUSTMENT.—*To Adjust the line of collimation—i. e.,* to make it perpendicular to the horizontal axis of the telescope. Bring the cross-wires into the focus of the eye-piece with the telescope turned towards the sky; determine if the vertical wire is plumb, by clamping the instrument firmly and applying the wire to the vertical edge of a building or plumb-line; should any deviation be manifested, loosen the cross-wire screws and by the pressure of the hands outside the tube move the ring around until the error is corrected.

Fix the point of intersection upon some well-defined point several hundred feet away, clamp the alidade and head, and assure the bisection by either tangent screw; plunge the telescope on its horizontal axis, and find or set some point in the opposite direction and at about the same distance from the instrument as the first object observed.

Great care should be taken in plunging the telescope that the position of the instrument upon the spindle is not in the slightest degree disturbed. Unclamp the alidade, turn the telescope around, and again direct it on the first object selected; clamp, and, having bisected this again, plunge the telescope on its horizontal axis and note if the vertical wire again bisects the second object observed. Should this happen, it will indicate that the wires are in adjustment and that the two points bisected and the center of the instrument are in the same straight line.

If not, the space which separates the wires from the second point observed will be double the deviation *of that point from a true straight line through the first point and center of the instrument,* or four times the real error of the wire from the optical axis of the telescope, since four observations have in reality been made, introducing the error at each observation. Figure 105 shows this; the line XY repre-

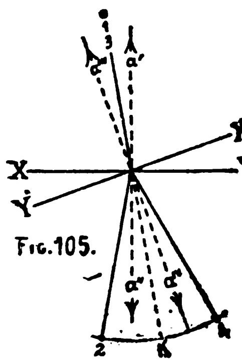


FIG. 105.

senting the horizontal axis of telescope during first two observations, a' and a'' the position of the optical axis, and the points marked 1 and 2 the points observed, the line X′ Y′ the position of the horizontal axis during the third, and fourth observations after turning nearly half way around the vertical axis, a''' and a'''' the positions of the optical axis and the points marked 3 and 4 the points observed. The real error of the wire, constant during the observations, is the deviation of 1 from a', or 2 from a'', and hence 4 from a'''', which is one-fourth of the distance from 2 to 4, while the straight line through 1 and the center of the instrument is midway between 2 and 4 at K; hence, in order to correct it, use the two capstan-head screws on the side of the telescope, these being the ones which affect the position of the vertical wire.

Remember that if the eye-piece gives an erect image, an inversion has taken place between the wires and eye, and therefore, in loosening one of the screws and tightening the other on the opposite side, the operator must proceed as if he were going to increase the error observed. Having in this manner moved back the vertical wire until by estimation one-quarter of the distance between 2 and 4 has been passed over, turn the instrument upon the first point observed, plunge the telescope, and, if the correction has been carefully made, the wires will now bisect the point K midway between 2 and 4; if not, repeat and continue correcting until the instrument will stand the test.

In figure 106, if c be on the axis of the telescope, and

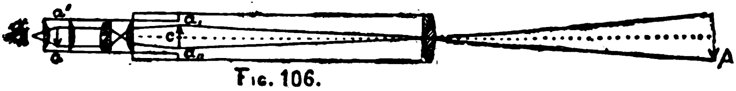


FIG. 106.

the intersection of the cross-wires be at $a_{,}$ above c, when $a_{,}$ is directed upon the point A of the arrow its image will be seen at *a* below the axis. The telescope in this case gives an erect image. On revolving the telescope on its own axis the intersection of the cross-wires will go to $a_{\prime\prime}$ below *c*, while its image will be at a' above the axis. In this latter position of the telescope the true displacement of the intersection of the cross-wires is below *c*, but apparently above it; hence, to correct it the cross-wire ring must be moved upward $\frac{1}{2}$ $a_{\prime}$ $a_{\prime\prime}$ in the direction of the apparent displacement.

In Fig. 107, if *c* be on the axis of the telescope, and the

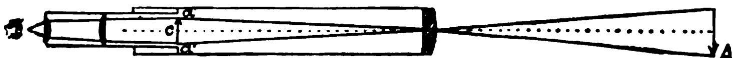


FIG. 107.

intersection of the cross-wires be at *a* above *c*, when *a* is directed upon the point A of the arrow its image will be seen at *a* above the axis. The telescope in this case gives an inverted image. On revolving the telescope on its own axis the intersection of the cross-wires will go to a' below *c*, as also its image. The apparent and true displacement in this case are in the same direction; hence, to correct it move the cross-wire ring up $\frac{1}{2}$ *a* a', in the opposite direction from the apparent displacement.

The importance of this adjustment appears in the measurement of horizontal angles between objects not in the same horizontal plane, but more particularly in prolonging a straight line by plunging the telescope; any error being doubled with each revolution.

3D ADJUSTMENT.—*To adjust the standards—i. e.*, to make the horizontal axis of the telescope perpendicular to the vertical axis of the instrument. In order that the intersection of the wires may trace a vertical line as the telescope is moved up or down, it is necessary that both the standards of the telescope should be of precisely the same height.

Having the line of collimation previously adjusted, set up the instrument in a position where points of observation, such as the top and base of a lofty spire or chimney, can be selected, giving a long range in a vertical direction.

Fix the intersection of the wires on the top of the object and clamp the plates and head, then bring the telescope down until the wires bisect some good point, either found or marked, at the base; unclamp the plates, turn the instrument half around, plunge the telescope, and fix the wires on the lower point; again clamp, and raise the telescope to the top object. If the wires intersect it, the vertical adjustment is correct; if they are thrown to either side, this would prove that the standard opposite that side was the higher, the apparent error being double that actually due to this cause. To correct it, where one of the bearings of the horizontal axis is movable, loosen the screws which hold on the cap of the standard, and raise or lower the block by the screw underneath the sliding-piece, bringing back the wire one-half its displacement from the first point observed, then tightening the cap screws. Repeat for a test.

If this method is not convenient, suspend a plumb-line 20 or 30 feet long, some 15 or 20 feet from the instrument; the weight suspended in a pail of water, and line hung from a rigid support; should be no wind; cord small and smooth. Set the line of sight on the cord at bottom. Clamp plates and head and turn up the telescope on the top of the cord. If the line of sight does not now strike the cord, raise or lower the adjustable end of the axis until it does. Repeat for a test. No reversion having taken place in this case, the apparent is the whole error. The importance of this adjustment is in the measurement of horizontal angles between points not in the same horizontal plane.

4TH ADJUSTMENT.—*To make the axis of the telescope bubble-tube parallel to the line of collimation.*

*1st method.** This consists in adjusting the bubble directly to the line of sight after the latter has been made horizontal. Drive two pegs on nearly level ground about 200 feet apart. Set the instrument so near one of them that when the rod is held upon it in a vertical position the eye end of

*Peg Adjustment.

the telescope will swing about half an inch from its face. Turn the eye end of the telescope upon the graduated face of the rod, the bubble being in the middle of its tube; look through the object end and set a pencil point on the rod at the center of the small field of view. Read the elevation of this point and call it a. Hold the rod on the distant peg and, with the bubble in the middle, set the target on the line of sight and call this reading b. Now carry the instrument to the distant peg, set it near it, read the elevation of the instrument as before, which reading call a'; carry the rod to the first peg and set the target on the line of sight, giving the reading b'. If the line of sight had been parallel to the axis of the bubble in each case, it would have been horizontal when the bubble was in the middle of the tube, and hence the difference between the a and b readings in each case would have been the difference in elevation of the pegs, and hence equal to each other—*i. e.*,

$$a-b=b'-a'=p \text{ (Fig. 108).}$$

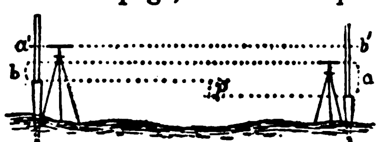


FIG. 108.

If the line of sight was not parallel to the axis of the bubble, however, then the differences of elevation of the two pegs as obtained by the two sets of observations, are not equal, and

$$(a-b)-(b'-a')=d \text{ (Fig. 109);}$$

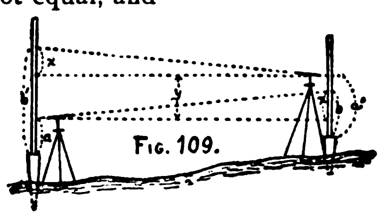


FIG. 109.

$$\begin{aligned}
\text{for } b&=x+(a'-y);\\
b'&=x+a+y;\\
a-b&=a-(x+a'-y);\\
&=a-x-a'+y;\\
b'-a'&=x+a+y-a';\\
\therefore (a-b)-(b'-a')&=(a-x-a'+y)-(x+a+y-a');\\
&=a-x-a'+y-x-a-y+a';\\
&=-2x;\\
&=d.
\end{aligned}$$

Now d is twice the deviation of the line of sight from the bubble axis for the given distance. If therefore the target be moved up or down, as the case may be, a distance equal to $\frac{1}{2}$

d, or x, then the line of sight may be brought to this position either by the leveling screws or the vertical tangent screw, and the bubble adjusted to bring it to the middle. The significant fact is that by moving the target $\frac{1}{2}d$ from its last position a truly horizontal line is established, and either the bubble or the line of sight can be adjusted to it after the other has been brought into a horizontal position.

The above equation may be written $(a+a')-(b+b')=d$, from which it may be seen at once that the line of sight inclines down when d is positive and up when negative.

Hençe, *add together the two heights of instrument and the two rod readings, subtract the latter from the former, and take $\frac{1}{2}$ the remainder. Move the target by this amount from the b reading, up when positive and down when negative.*

After adjusting, return to first peg, read height of instrument again, and the rod held on second peg, for a check, see if this new value of $(a-b)$ agrees with the adjusted value of $(b'-a')$. If not, adjust again.

2d method: First level the instrument carefully, and with the clamp and tangent movement to the axis, make the telescope horizontal, as nearly as possible, by eye; then, having the line of collimation previously adjusted, drive a stake at a convenient distance, say from 100 to 300 feet, and note the height cut by the horizontal wire, upon a rod set on top of the stake. Set another stake in the opposite direction and at the same distance from the instrument, and without disturbing the telescope, turn the instrument upon its spindle, set the rod upon the stake, and drive it in the ground, until the same height is indicated as in the first observation.

The tops of the two stakes will then be in the same horizontal line, however much the telescope may be out of level. Now remove the instrument from 50 to 100 feet beyond either of the stakes, but in line with both; again level the instrument, clamp the telescope as nearly horizontal as possible, and note the heights indicated upon the rod placed first upon the nearer and then upon the most distant stake. If both are the same, the telescope is level; if not, then with the tangent screw move the wire in the direction to decrease the error, as shown at the distant stake, until the two readings are the same, when the telescope will be horizontal. Being in this position, taking care not to disturb it, bring the bubble into the center by the little leveling nuts at the end of the tube, when the adjustment will be completed.

5TH ADJUSTMENT.—*To adjust the vertical circle—i. e.*, to make the vernier read zero when the line of sight is horizontal.

1st method: Having made the previous adjustment, bring the bubble to the center of the tube, and if the vernier does not read zero, loosen the capstan-head screws which fasten the vernier, and move the zero of the vernier to the zero of the circle. If the vernier is not adjustable, its reading will be the index error.

2d method: The zeros being placed together, the line of sight may be brought horizontal with the peg adjustment, by moving the horizontal wire by means of the screws on the telescope, and the bubble afterwards brought to the center of its tube. If the wires are moved after making the second adjustment, it must be tested to see if disturbed.

3d method: The zeros being placed together, with the telescope find or place some well-defined point from 100 to 500 feet distant, which is cut by the horizontal wire. Turn the instrument half-way round on its spindle, revolve the telescope on its axis, and, fixing the wire upon the same point as before, note if the zeros are again in line. If not, move the vernier over half the error, bring the zeros again into coincidence by the tangent screw, and proceed precisely as at first, until the error is entirely corrected, when the adjustment will be complete. This last method is only applicable with the full circle.

The last two adjustments are important in leveling operations and in measuring vertical angles.

To center the object-slide. The adjustments described are all that will ordinarily be required, though sometimes it may be necessary to *center the object-slide.*

The line of collimation being adjusted for objects from 300 to 500 feet distant, clamp the plates securely, and fix the vertical wire upon an object as distant as may be distinctly seen; then, without disturbing the instrument, move out the object-glass so as to bring the vertical wire upon an object as near as the range of the telescope will allow. Having this clearly in mind, unclamp the limb, turn the instrument half-way around, plunge the telescope, clamp the limb, and with the tangent-screw bring the vertical wire again upon the near object; then draw in the object-glass slide until the distant object first sighted upon is brought into distinct vision. If the vertical wire strikes the same line as at first, the slide is correct for both near and remote objects, and, being itself straight, for all intermediate distances.

If in error, proceed as follows: First, with the thumb and forefinger twist off the thin brass tube that covers the screws C, C. Next, with the screw-driver turn the two opposite screws C, C, loosening one and tightening the other, so as apparently to increase the error, making by estimation one-half the displacement. This done, readjust the line of collimation, then repeat the centering again, and so on until it will stand the test.

To center the eye-piece—i. e., to make the intersection of the cross-wires appear in the center of its field of view. This is accomplished by means of the four small screws at A, A, loosening one and tightening its opposite until the intersection of the cross-wires is seen in the center of the field of view.

Putting in new cross-wires. If a cross-wire becomes broken, the time required and the expense involved in sending the telescope to an instrument-maker to have it replaced may be saved to the operator if he can do it himself, and it is really a very simple thing to do. Having provided some kind of gum, wax, or varnish, and a piece of stiff wire 12 or 15 inches long, bend the latter in the shape of letter U with the prongs a couple of inches apart, or use a forked stick. Catch a spider, attach the end of his web to one prong, and while making him spin, wind up the web on the fork.

At each crossing fasten the wire to the fork with the wax or varnish, thus providing a large supply. After drying a few minutes they will be ready to use. The eye-piece is removed from the telescope, and two opposite screws from the cross-wire ring. The ring is turned 90°, a stick inserted in the screw-hole in its edge, the two remaining screws removed, and the ring taken out. The broken wire is removed, a new wire is laid across the ring in the scratches made for it, and stretched taut, then fastened with a drop of varnish or gum or piece of wax, care being taken not to get the wire out of the scratch by a side movement.

The eye-piece will answer for a magnifying glass for placing the wire, which latter should be small, round, and opaque. If old wires are used, they should be slightly steamed to restore their elasticity. Wires may be unwound from a cocoon, and shellac dissolved in alcohol makes a good varnish.

To adjust the compass. One or more of the following causes of error may exist in the compass. The point of the pivot not in the center of the graduated circle, the needle not straight, the zero line not in the vertical plane of sight.

To adjust the pivot. Find the position of the needle which gives the *maximum* difference of end readings, remove the needle, and bend the pivot at right angles to this position by one-half the difference in the extreme variation of end readings.

To straighten the needle. Set the north end exactly at some graduation mark and read the south end. If not 180° apart, bend the needle until they are. This assumes that the point of the pivot is in adjustment.

To determine whether the zero line lies in the plane of sight. Direct the telescope on an object and read the north end of the needle; then revolve 180° and sight to the same object and read. If the number of degrees read is the same as before, this will show that the line of sight passes through the center; otherwise, half the sum of the two readings will be the correct one.

The test of the delicacy of a needle is the number of horizontal vibrations which it will make in a certain arc before coming to rest.

TO USE THE TRANSIT.—It should be *set up firmly* by pressing the tripod legs into the ground.

The *center* must be brought exactly over the desired point by means of the plumb-line. The precision of centering an instrument over the point increases in importance inversely as the length of the triangulation lines; thus,

1 inch is equivalent to about 3 minutes of arc at 100 feet.

1 inch is equivalent to about 3 seconds of arc at one mile.

The *plates* should be brought as nearly horizontal as possible in setting up the transit, and afterwards made truly horizontal by means of the leveling screws and plate bubbles. The horizontal plate verniers should be lettered A and B.

The *objective end* of the telescope, in its normal position, is usually placed over the north end of the zero line of the compass; hence some engineers read azimuths, using vernier A starting with 0° at the north, and vernier B with 0° at the south; while others use vernier A starting with 0° at the south, and vernier B with 0° at the north. Whichever method is used should always be distinctly stated in the heading of the notes, so that no misunderstanding may occur.

Before beginning to take observations the *eye-piece of the telescope* should be moved in or out until the cross-wires appear distinct to the operator. The *object-glass* is focused by turning the pinion-head until the object sighted is seen clear and well defined, and the wires appear as if fastened to its surface. If, on moving the eye about, the wires appear to move on the object, the focusing of the objective is not exact and must be repeated until no such movement appears. This is particularly necessary in reading distances with the stadia rod and in leveling.

Needle check. The needle is used, as will be described under the compass, to give the bearings of lines, and as a rough check upon the angles obtained by the vernier and limb, but its employment is only subsidiary to the general purposes of the transit.

To measure a horizontal angle. Set up and level the transit exactly over the apex of the desired angle—*i. e.*, over the intersection of the lines whose difference of direction is desired. Turn the telescope upon one of the objects, preferably the left-hand, and clamp both horizontal movements By either tangent screw make the intersection of the cross-wires bisect the object, and read both verniers. Record the readings seperately as *Reading Vernier A*, *Reading Vernier B*. Unclamp the plates, *being careful not to disturb the lower clamp*, turn the telescope upon the other object, clamp the plates, and by the upper or alidade tangent screw make the intersection of the cross-wires bisect this object, and again read both verniers and record. The difference of the two vernier A readings should equal the difference of the two vernier B readings and be the value of the angle measured; but if these differences are not equal, then ½ their sum is taken as the value of the angle.

After setting up the transit and before pointing to the first object, many observers set vernier A to read zero and clamp it there, then make their pointing and bisection on the first object by the lower tangent movement, being careful not to disturb the alidade tangent screw until they are ready to unclamp the plates and turn upon the second object. This though in a way convenient, is not essential.

Should either vernier have passed the 360 degree mark in turning, then, before finding the difference of its readings, 360 must be added to its last reading.

Where great accuracy is necessary, the measurements are repeated on different portions of the graduated circle; thus, after measuring the angle as above, the head is unclamped below, the telescope turned back upon the first object, clamped and bisected by the lower tangent screw, without disturbing the first readings; then, unclamping the plates, the telescope again directed upon the second object and the angle determined as at first; having now measured the angle twice, ½ the sum thus found may be taken; or, finding the differences between the first and last readings of each vernier, take ½ of the mean for the angle, which should be the same as the other. This repetition may be continued any number of times, taking the mean of the differences of the first and last readings of each vernier and dividing by the number of times the angle has been observed for the measure of the angle.

When repeating angle measurements, better results are obtained by making about five pointings on each of the two objects, as follows: Point the telescope at the left-hand object, read both verniers; then by the upper motion bisect the right-hand object; unclamp lower motion and again bisect left-hand object, using lower motion only; again, by upper motion bisect right-hand object; keep on thus until five pointings on each object are had. At the *fifth* on the *right*-hand object, the verniers are read; one-fifth of the resulting angle will be the angle sought. After the fifth pointing on the right-hand object the work may be checked by reversing the operation, moving the telescope from right to left by the upper motion, and from left to right by lower motion; the fifth pointing at the left-hand object should give readings the same as those first recorded.

Objects not in same horizontal plane. If the objects between which the horizontal angle is desired *are not in the same horizontal plane*, they are reached by raising or lowering the telescope, which should move in a vertical plane. If the plate-bubbles are not in adjustment or do not show a vertical instrument axis, then the telescope will not revolve in vertical planes and the horizontal angle will be in error.

The angles of a triangle having been measured, their sum should equal 180°. If it does not, then they should be corrected. If all were measured with equal care, then ⅓ the difference between their sum and 180° is added to or subtracted from each, according as their sum was less or greater than 180°.

If each angle is the mean of a number of observations, the correction to be applied to each is inversely as the number of observations in obtaining the mean.

If the triangles have sides of 12 to 15 miles or more in length, there will be a slight spherical excess, which, however, may be neglected unless the greatest precision is required, as it amounts to only about 1″ for a triangle of 76 square miles and about 1′ for a triangle of 4500 square miles.

To measure vertical angles. If the vernier of the vertical arc or circle has been adjusted to read zero when the line of sight is horizontal, then the *vertical angle of any object* referred to the horizontal plane or its altitude will be determined by pointing the telescope to it and taking the reading. If the vernier has not been adjusted, then its index error must be determined and applied.

If the vertical *angle subtended by two objects* above the horizon be required the difference of the two readings will be the angle; but if one be above and the other below, then the sum will be the required angle.

The plate-bubbles must be watched to see that they indicate a vertical axis. If the vertical limb is a complete circle, then by taking the mean of readings with the telescope direct and revolved, errors of adjustment of the plate-bubble parallel to the telescope and of the vernier will be eliminated; otherwise, the exactness of the adjustments must be depended upon.

REFRACTION.—A ray of light in passing obliquely from a higher to a lower level through the air, the density of which increases towards the lower level, does not follow a right line, but a constantly increasing curve downward. The object reflecting the light is therefore not seen in its exact position, but on the *tangent* to the curved *ray* where the latter enters the eye or instrument, the effect of which is to make all distant objects appear higher than they really are. The *nearer* the object is to the horizon the *greater* the refraction, while an object in the zenith has no refraction.

Thus an object at S, as the sun (Fig. 173), viewed from A, would be seen in its true position on the right line AS if there were no refraction, but as soon as the ray enters the outer stratum of air at *b* it begins to curve downward and strikes the earth at B. Some other ray striking the outer stratum of air at *a* curves downward, striking the earth at A. It is along the tangent to this ray at A that the sun is seen, and appears to be situated at S′ instead of S.

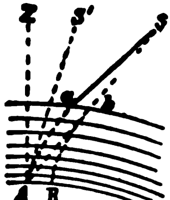


FIG. 173.

Temperature, moisture, wind, etc., which affect the density of the air, being variable, no exact rules for correcting refraction have been deduced, but tables have been prepared giving the mean refraction for the different degrees of elevation from the horizon in a vertical plane.

To clear an observation from refraction, it must be added to zenith distances and subtracted from altitudes.

To prepare data for setting off on an instrument to make an observation, refraction must be subtracted from true zenith distances to give apparent zenith distances and added to true altitudes to give apparent altitudes.

CURVATURE OF THE EARTH.—*The horizontal lines* observed with the spirit level are tangent lines to the earth's surface at the points of observation, being perpendicular to the plumb-line of the instrument, while a *true level surface* is a curve like the surface of a body of water. Points equally distant from the center of the earth are said to be on the same level. In Figure 174, BO, the apparent difference of level between L and O, is called "curvature" and is equal to the excess of the secant of the arc LO over the radius CO of the earth. From geometry $AB : LB :: LB : BO$, hence $LB^2 = BO \times AB$ or $BO\ (BO+2CO)$ and $BO = \frac{LB^2}{BO+2CO}$; BO, being so small compared to 2CO, may be neglected, and $BO = \frac{LB^2}{2CO}$. If LB=1 mile, $BO = \frac{1 \text{ mile}}{7916 \text{ miles}} = \frac{5280 \text{ feet}}{7916} =$ 0.667 feet $= \frac{63.360 \text{ inches}}{7916} =$ 8 004 inches. If LB=2 miles, BO=

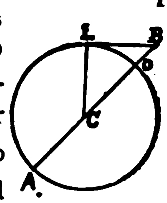


FIG. 174.

$\frac{4 \text{ miles}}{7916}=4\times0.667$ feet$=2.667$, feet$=4\times8.004$ inches$=32.016$ inches. If LB$=10$ miles, BO$=\frac{100 \text{ miles}}{7916}=66.7$ feet, from which it is seen that curvature is equal to the square of the distance between the places in miles divided by the diameter of the earth in miles. And that curvature in feet is equal to $\frac{2}{3}$ the square of the distance between the places in miles.

Curvature is always positive, and for angles of elevation apparent height is positive, tor angles of depression apparent depression is negative.

Sighting on C (Fig. 175) from A, we say it is BC higher than A, whereas it is$+$BC$+$BE (curvature) $=+$EC. Sighting on D, we say it is $-$BD lower than A, whereas it is $-$BD$+$BE$=$ $+$DE higher than A. On E we say it is $-$BE lower; it is $-$BE$+$BE$=0$ or on the same level. On F we say it is$-$BF lower it is $-$BF$+$BE$=-$EF lower.

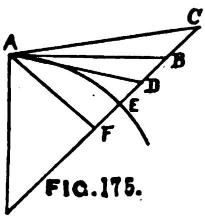


FIG. 175.

In leveling operations, an *approximation* is to deduct $\frac{1}{6}$ of curvature from itself before adding it, to clear it of refraction.

PROLONGING A STRAIGHT LINE.—The operations at each setting of the instrument are nearly the same as those in adjusting the line of collimation, and are made to eliminate any errors in the adjustments. Having two points of the line established:

1. Set up accurately over the forward point, putting one pair of leveling screws in the line.
2. Clamp the horizontal limb or head.
3. Level carefully and turn the telescope upon the rear point.
4. Re-level for the bubble that lies across the line.
5. Clamp the alidade and make the bisection on the rear point, plunge the telescope, and set a point in advance.
6. Unclamp the alidade and revolve it about the vertical axis till the telescope comes again on the rear point.
7. Re-level for the bubble across the line.

8. Clamp alidade, make bisection on rear point, plunge telescope again, and set a second point in advance beside the first one.

From what has already been stated, the point midway between these two should be in the same straight line as the rear point and instrument. Errors of adjustment are thus eliminated.

TO RUN A TRUE EAST-AND-WEST LINE—*i. e.*, PARALLEL OF LATITUDE.—A true east-and-west line is one which is at every point at right angles to a meridian passing through that point. It is, therefore, a constantly curving line, being always deflected toward the north in the northern hemisphere. A right line prolonged will be a great circle.

When using a transit without solar attachments, to run a true east-and-west line, starting at A, on a true east-and-west line (Fig. 176), run out a straight line by prolongation for some twelve miles' distance to B, making intermediate corrections northward for the points on the true parallel, and then offset the proper distance to C; set the transit again on the parallel at C, and either make a new observation for azimuth or carry the old azimuth forward, correcting it to agree with the new meridian, and run another line to D, making intermediate corrections northward, and then offset to E, etc.

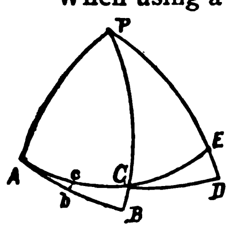


FIG. 176.

For this two tables are required: one to give the proper offset BC and DE from the prolonged right line AB to the parallel tangent to it at the initial meridians PA and PC, and the other to give the change in azimuth necessary to prolong the line from a new meridian when no new observation for azimuth can be obtained. The two tables—Appendix, Table VIII.—are combined in one. The angles there given are measured from the north point towards the point of tangency of the straight line with the parallel, which is the initial point from which the distances given in the table are measured (Fig. 177). The convergence of the meridians for the corresponding distances is 90° minus the angles given in this table. The offsets are always to be measured to the north from the tangent line in the northern hemisphere and south from it in the southern hemisphere.

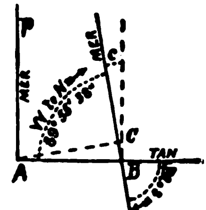


FIG. 177.

Having started from A (Fig. 177) due east in latitude 40° and run out a straight line for six miles to B, we find from the table that the true meridian is obtained by turning off from east to south or west to north, the angle 89° 55′ 38″, and the true position of the parrallel C at this point is 20.1 feet north. When 12 miles has been run out, the angle with the meridian is 89° 51′ 17″, and the position of the parallel C is 80.5 feet north.

TO MEASURE ANGLES OF DEFLECTION.—The supplement of the angle made by two lines, or the angle between a line prolonged and another, is called the angle of deflection, and is *measured* by setting up the instrument at the vertex of the angle, sighting the telescope upon one line, plunging the telescope on its horizontal axis, and then turning it upon the other line and reading the angle through which the telescope has been moved. Thus, in Fig. 110, set the transit at B, turn the telescope on A and read the verniers; plunge it, then turn on C and read; the difference of the readings will be the deflection angle. Move to C, D, etc., and go through similar operations.

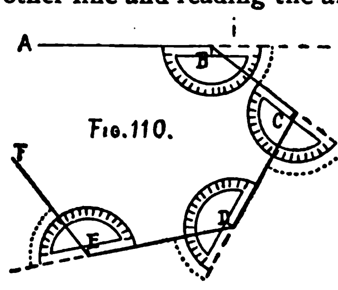


FIG. 110.

If the lengths of each course have been measured, they may be *plotted* by drawing a straight line and laying off to scale the distance from A to B. Place the center of a protractor at B, the diameter along AB and from the produced end mark the deflection angle for the direction to C, etc.

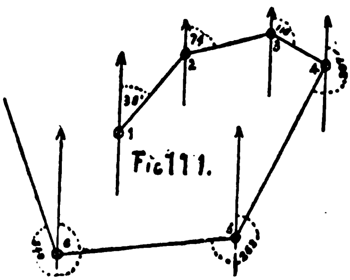
TRAVERSING.—In level country, if covered with forest, etc., triangulation is expensive and often impossible, and the control for maps must be obtained by traversing. A *traverse* consists of a series of consecutive straight lines—*i. e.*, a broken line, closing or not upon itself. They are largely used in topographic work proper for making miner locations, and in "filling in" details. *Traversing* consists in observing the directions and measuring the lengths of the consecutive straight lines forming the traverse. In observing the directions of the lines they may all be referred to the true meridian, in which case the angles are called the *azimuths* of the lines with respect to the meridian line, to distinguish them from the *bearings* as would be given by the compass. The direction line may also be the first line measured, or any other line desired, in which case the deviation of the different lines from it are simply referred to as *angles of deviation*. The stations at the ends of the lines are, when plotted, usually surrounded by a circle, thus ⊙.

If the *direction line is the true meridian*, one may begin to count all angles from 0° at the north around to the east 90°, south 180°, west 270°, to 360° at the north; or, starting with 0° at the south, around to the west 90°, north 180°, east 270°, south 360°. Different methods are employed for accomplishing practically the same results.

In all of them, however, the 0-180° line of the horizontal limb is first brought, at each station, into a line *parallel to the position it occupied at the first station*, with the zero end pointing in the same direction, and then the azimuth of the line observed.

The azimuth of a line taken from one end will differ from that taken from the other end by 180° plus or minus the convergence of the meridians at the extremities. In ordinary military requirements the convergence will be so small that no account will be taken of it. When taken looking in the direction in which the traverse proceeds or towards its forward end, it is called its *forward azimuth* or forward angle, and when taken looking back in the direction from which the traverse has proceeded, or towards the back end, it is called its *back azimuth* or back angle.

1st method: Starting at ⊙ 1 (Fig. 111) of the traverse, if the direction of the north-and-south line through it is known, the instrument is set up over the station, the zeros of vernier A and limb are made to coincide, and the plates clamped; the telescope is brought into the meridian pointing north, by the lower tangent screw. The plates are then unclamped, the telescope sighted down the traverse line to ⊙2, and vernier A read, 38°.



Move the instrument to ⊙2, being careful not to disturb alidade clamp, set up, and, plunging the telescope upon its horizontal axis, sight back upon ⊙1, using lower tangent screw; or, calculate the back azimuth of the line by adding 180° if the forward reading is less than 180°, or subtracting 180° if greater, then unclamp the plates, set the vernier to read this angle, 218°, clamp, and then direct the telescope in its normal position upon the back station by the lower tangent screw. The 0-180° line of horizontal limb will now be in a position parallel to what it was at ⊙1 and the 0 end of that line pointing in the same direction. That is, if the plates were now unclamped and the telescope pointed north, vernier A would read 0 on the limb. The instrument is now said to be *oriented*. If the telescope has been plunged upon its horizontal axis to orient, it is now plunged back to its normal position, the plates unclamped and the telescope directed upon ⊙3 by the plate tangent screw, and vernier A read, 74°. The instrument is moved to ⊙3 and the operations just described for ⊙2 repeated.

2d method: If the azimuths are counted from 0 looking south, around to the west 90°, etc., the steps are exactly similar; thus, set vernier A to read zero, clamp the plates, turn the head of the instrument until the telescope points south and clamp below, then unclamp above and direct telescope on $\odot$2, etc.

3d method: If referred to the first line of the traverse, it will be the direction of the line from $\odot$1 to $\odot$2. In this case the instrument is set up at $\odot$2, vernier A set at zero and the plates clamped. The telescope is then plunged upon its horizontal axis and sighted upon $\odot$1 by the lower tangent motion, then plunged back to its normal position, the plates unclamped and the telescope sighted upon the next station and the angle read, which will be the angle between the first line produced and the second line. The remainder of the operations at this and other stations are the same as already described.

The needle as a check. In traversing with the transit it is advisable to *use the needle as a rough check.* To do this, cut and graduate a ring half an inch wide, the size of the circular rim of the compass box. Number the graduations from 0° to 360° in the opposite direction to the numberings of the graduations on the horizontal limb. Paste this ring on the glass cover so the north end of the needle will be under its 0° when the telescope is pointed in the direction of 0° azimuth. With changes in the declination of the needle the ring must be reset. By freeing the needle at each setting of the instrument it should read the same as vernier A.

The *notes* should contain as a heading a general description of the line run, together with the date, name of party, etc.

Transit Traverse from——to——

Observer.

Date———— Recorder.

STATION.	AZIMUTH.	DISTANCE.	REMARKS.

Right-hand page for sketching.

TO PLOT THE TRAVERSE.—Assume a point on the drawing paper for $\odot$1. Through this draw a line to represent the meridian or direction line. Place the protractor with its 0–180° diameter on this line, its center at $\odot$1. Mark on the paper by a fine dot or pin-point the azimuth of $\odot$2, 38°. From $\odot$1 through this point draw a straight line and on it lay off from $\odot$1 the distance to $\odot$2. Through $\odot$2 draw a line parallel to the direction line through $\odot$1 and lay off the azimuth to $\odot$3, and so on to the end of the traverse (Fig. 111). If the traverse closes upon itself, the last line plotted should pass through $\odot$1 if the work has been accurate.

If the traverse has been run from one $\triangle$ to another $\triangle$, the end of the last course should be at the second $\triangle$. Since a higher degree of accuracy is possible in the field-work than in the plotting, extreme care should be taken in the latter, so that any failure to check within limits may not be in the plotting. If the discrepancy is too large, the traverse must be replotted. If from one $\triangle$ to another $\triangle$, *recompute and replot the line* joining them also. *Compare the plot and notes* and see if any probable error can be discovered, and if so, where and of what kind. If found, with the instrument in the field, obtain the correct information. If it does not appear that any error can be located further than the kind, as in azimuths, or in distances, then the traverse must be rerun for this alone. In traversing, *check azimuths* should be taken at each station to some one object which can be seen from all, and then, if the plotting does not close within limits, these check readings should be plotted. If some pass through one point and the remainder through another point, it is probable that the azimuth or distance of the course joining the two sets is in error.

TO DETERMINE THE TRUE MERIDIAN.—Polaris, popularly called the North Star, is not situated exactly in the North Pole of the heavens, but revolves around it at a distance of about 1° 16′ therefrom (Fig. 115). A complete revolution is made once in 23 hrs. 56 min. 4.09 sec. Hence it will appear on the true meridian once above the Pole (at 3) and once below (at 1), and also once at its extreme eastern position (at 2), and once at its extreme western position (at 4) every day. Any one of these four positions that is most convenient may therefore be used. Let it be when east or west of the Pole. The angular distance of Polaris from the North Pole when at its extreme eastern or western position is called the Azimuth of Polaris at elongation. When at elongation, the star apparently remains stationary for a short time, then begins to move backward, which is of advantage in using that position, since it is then not moving in azimuth.

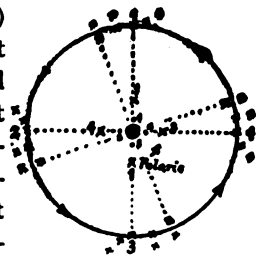
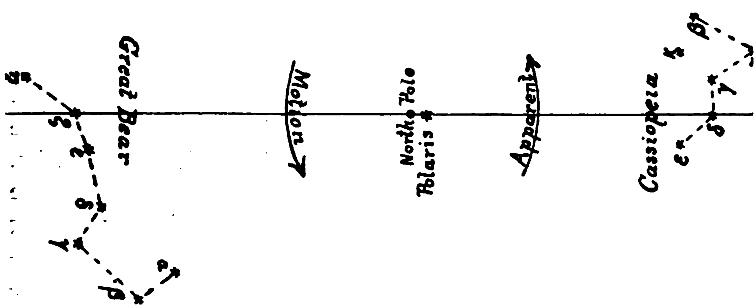


FIG. 115.



If, when facing north, the above diagram be held up with the *Great Bear* on the left, it will represent the configuration of the constellations, with *Polaris* near *eastern elongation* at midnight about July 10th; with the *Great Bear* below, it will show *Zeta* (Mizar) of the Great Bear, *Delta* of Cassiopeia and *Polaris* on the meridian, *Polaris* at *upper culmination,* at midnight about October 10th; with the *Great Bear* on the right, it will show *Polaris* near *western elongation* at midnight about January 10th; with the *Great Bear* above, it will show *Polaris* near *lower culmination* at midnight about April 10th.

Upper culmination occurs 5 h. 54.8 m. after eastern or before western elongation, and 11 h. 58 m. before or after lower culmination.

Lower culmination occurs 6 h. 3.25 m. after western or before eastern elongation.

Eastern elongation occurs 11 h. 49.6 m. before or 12 h. 6.5 m. after western elongation.

DETERMINING THE TRUE MERIDIAN WITH THE TRANSIT.—From Table III., Appendix, find the watch time of the most convenient elongation. From April to September, inclusive, eastern elongations occur at night and western elongations in the day time, and from October to March, inclusive, *visa versa.* Also from the Table of Azimuths of Polaris, when at elongation, Appendix, find his azimuth for the year and latitude of place of observation.

Having selected a piece of ground as level as possible, several hundred feet long, in the direction of the meridian, drive a stake at the south end, in the top of which drive a tack. About 15 or 20 minutes before the Pole-star is to reach its greatest eastern (or western) elongation, as found from the table, set the transit up exactly over the tack in the stake, and level carefully. Some arrangement must have been made for lighting the cross-wires during the observation by having an assistant hold a dark lantern so as to shine down the telescope withont obstructing the view, or some kind of a reflector of light must be used as in Fig. 112, or a bright piece of tin,

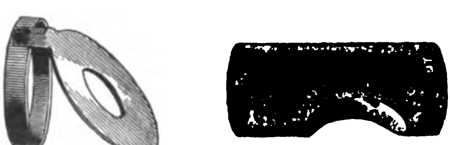


FIGURE 112.

or board covered with white paper, having in the center a hole smaller than the end of the telescope, but so large that the star can be seen through it when placed several inches in front of the telescope and illuminated.

Then bring the vertical wire upon the star and follow it by the tangent motion until it ceases to move laterally. Keeping the wire at this position, the star will soon appear to depart from it in the direction from which it came. Without disturbing any horizontal motion of the instrument, carefully lower the object end of the telescope and have an assistant drive a stake some 200 or 300 feet forward, "lining him in" with the telescope, sighting on a plumb-line. The assistant has a board with a hole in it, across which he holds the plumb-line and behind a lamp or lantern.

Being aligned, he drives a stake, in the top of which he drives a tack. Or a narrow slit, in a board covered with tissue paper, behind which a light is held, may be used by the assistant. The line passing through the tack *a* under the instrument and the tack *b* set in the forward stake makes with the true meridian an angle equal to the azimuth of the North Star at elongation.

Now multiply the natural tangent of the azimuth by the distance between the two stakes and lay off this result from *b* to *c*, on a perpendicnlar to *ab* on the side of the true meridian.

Or, turn off the azimuth on the horizontal limb to the west if an eastern elongation, or to the east if a western elongation, and the telescope will be in the true meridian. Have the assistant put a stake *c* in this line, and *ac* will mark the true meridian. The two may be used for a check.

CHAPTER VIII.

THE COMPASS.

DESCRIPTION.—Figure 113 shows the ordinary compass, like the one on the transit, mounted on a graduated horizontal limb, with the vernier reading to minutes, and accompanying clamp and tangent movement; it is also provided with a line of sight, which consists of two vertical slits in flat brass bars, with large circular apertures at intervals for more readily finding objects sighted.

One of these bars is attached to each end of the plate, the slit in it being in the vertical plane containing the needle pivot and the zero line. At the bottom and top of the "S" end sight are small eye-pieces for reading vertical angles on tangent scales on the edges of the "N" end sight. On the plate are two bubbles placed at right angles for leveling. A small round disk figured from 1 to 16, with an index turned by a milled head, is used for keeping tally in chaining. etc.

The whole is suitably mounted by being fitted to a slightly conical spindle and having on its outer end a ball confined in a socket so that it can be moved in any direction in leveling. The compass may be supported on a single pole, called a Jacob-staff, with proper mountings on top and a pointed iron shoe on the bottom for setting it firmly in the ground; or it may be supported on a tripod, with or without the usual leveling screws.

The compass in some form is one of the most valuable instruments used in military topography. It is simple in construction, convenient, and easy to use. It measures angles fairly reliably, although never exactly. Its needle always points in a known direction, to which the bearing of any line may be referred. This direction is, in a few places, true north and south. In most places, however, the needle makes an angle with the true north and south line, called *the declination of the*

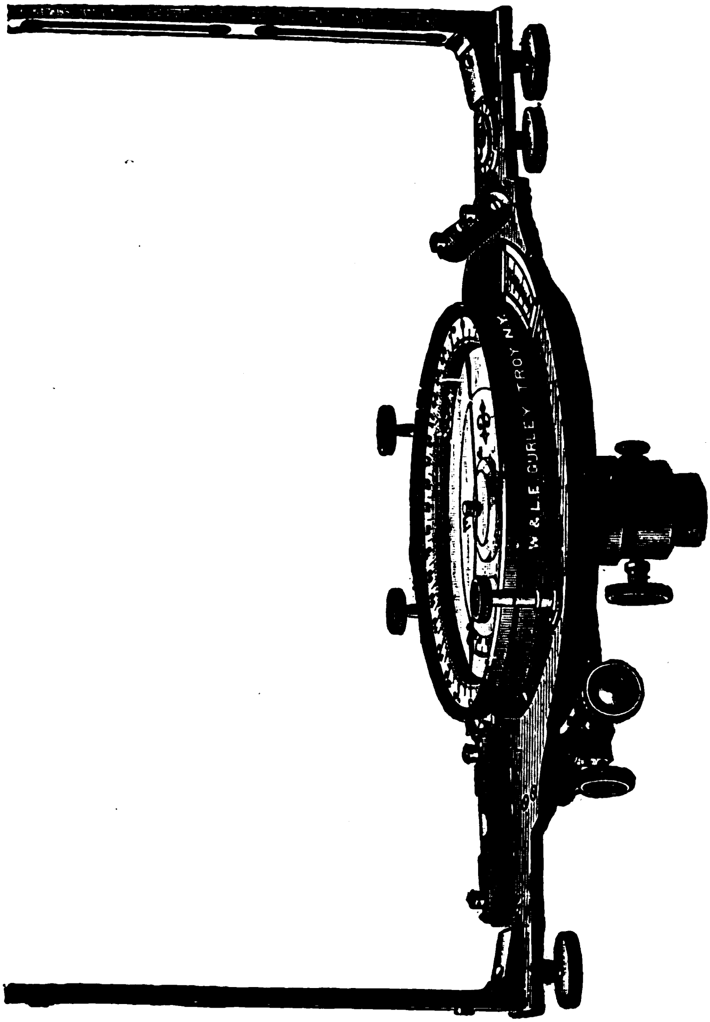


FIGURE 118.

needle, which can be easily determined for any particular place.

It is customary to refer the direction of the line to the north or south point, according to which is less than 90° from it.

If the line of sight and needle have the same direction, the reading will be zero, north and south. If now the north end of the zero line be turned towards the east, from either north or south, the north end of the needle, remaining stationary, will point to divisions to the left of N or S, as seen from the observer's position, in the direction of the letter E. Likewise if the line of sight be turned from the north or south towards the west, the north end of the needle will point to a division in the direction of the letter W. Hence the letter E or W which is nearest to the *north end of the needle* indicates on which side of the magnetic meridian lies the object that is sighted.

The direction of a line referred to the position of the needle or *magnetic meridian* is called its *magnetic bearing*. When referred to the *true meridian*, by correcting the magnetic bearing for the declination, it is called its *true bearing*.

In using the ordinary compass, always keep the north end of the zero line pointing in the direction of the line or object whose bearing is to be taken, and read the north end of the needle.

ADJUSTMENTS OF THE COMPASS.—*The levels.* This is the same as described for adjusting the plate bubbles on the transit.

The verticality of the sights. This may be tested by observing through the slits a fine plumb-line, after the compass is leveled. Should the plumb-line appear to cross the slit obliquely, the sight may be adjusted by filing off on its under surface on that side which seems the higher; or strips of paper may be inserted under the lower side until the slit follows the plumb-line.

To straighten the needle. Same as in Transit.

To straighten the pivot. Same as in Transit.

To determine whether the zero line lies in the plane of sight. Stretch a hair vertically in the middle of the slit of each sight and see if the hairs and the zero graduations of the circle lie in the same plane. The declination vernier must be set at zero in making this test.

To remagnetize the needle. It sometimes happens that the needle has lost its polarity or magnetism and will be sluggish in its movements and settle quickly. Then it must be remagnetized. This is done by holding an ordinary permanent magnet before one, and passing with a gentle pressure each end of the needle from the center to the extremity over that magnetic pole which attracts it, describing before each pass a circle of a foot or more, drawing the needle towards him. If the pivot becomes dulled, which will also cause the needle to be apparently sluggish, it must be unscrewed and sharpened on an oilstone by being placed in the end of a small stem of wood or a pin-vise and delicately twirling it with the fingers as it is moved back and forth at an angle of about 30° to the surface of the stone. When the point has been made so fine as to be invisible to the eye, it should be smoothed by rubbing it on the surface of a clean soft piece of leather.

CAUSES OF ERRORS IN THE COMPASS.—The attraction of the needle by foreign bodies, loss of magnetism, blunting of pivot, lack of means for reading the angles exactly, and variation of the position of the needle are causes of errors.

Local attraction. This may be discovered by taking both fore and back sights upon every line. If the bearings agree, they are assumed as correct; but if they do not agree, the fore sight must be taken over again. If the discrepancy still exists, it is probably due to the attraction of the needle at one of the stations by some foreign body, as iron, etc.

To determine which bearing is incorrect, or at which station the attraction exists, take the compass to some distant point and get the bearing of each of the stations, then go to each of these latter and get the bearing of the distant point, and see at which the fore and back sights disagree. Knowing this and the amount of the attraction, bearings taken from here can be corrected. Or, by taking the bearings of a line from intermediate points and others in prolongation of it some may be found to agree, which will be taken as the correct bearings. In traversing, if it is found that local attraction exists at a station, the angle made by the two courses with each other can be measured, and from the bearing of the back course that of the forward one can be determined. The local attraction will not affect the determination of this latter angle.

The presence of iron or steel on the observer, or near the instrument, or a chain near it, may cause local attractions. Rubbing the glass cover may magnetize it and attract the needle, which may be dissipated by touching it with the wet finger. Reading-glasses with covers of magnetic properties are to be avoided. Observations near railway tracks, water and gas pipes, electric wires, etc., are to be avoided if possible, or means taken to neutralize their effects, if unavoidable.

The end of the needle does not quite touch the divided circle and is of appreciable thickness; hence it is impossible to read its pointings precisely, and especially as the smallest divisions of the circle are half-degrees. Any intermediate position must be guessed and so be a source of error, nor does the vernier materially remove this source of error.

The declination of the compass is called *west* when the north end of the needle points to the west of the true meridian, and it is called *east* when the north end points to the east of the true meridian. The true meridian for any place is always the same fixed line, while the magnetic meridian is ever changing.

Variations of the declination. The direction in which the needle points with reference to the magnetic meridian is in a continuous, though very slight, state of change.

Daily variation. This is owing to the influence of the sun, which will, in summer, cause the needle to vary 10 or 15 minutes in a few hours when exposed to its fullest influence.

At about 10:30 A. M. and about 7 P. M. the needle has its true or mean declination, or is in the mean magnetic meridian. At about 8 A. M. it reaches its extreme eastern position from the mean, and at about 1:30 P. M. it reaches its extreme western position from the mean. The angle between its positions at 8 A. M. and 1:30 P. M. varies from 5′ to 15′, depending upon the seasons of the year. A similar but smaller movement takes place during the night (Fig. 114). Hence, not only the year and month in which important bearings are taken should be known, but also the hour of the day.

FIG. 114.

The mean of the extreme easterly and westerly positions in any one day approaches within half a minute of the mean position of the day. Since corrections to observed declinations to refer them to the mean of the day are generally very unsatisfactory, it is recommended to observe the declination for any one day at the epochs of the extreme easterly and westerly positions as given above, and to take the mean position as representing the declinations for that day.

Secular variations. It frequently becomes necessary to retrace old magnetic courses; to do this effectively one must know not only the magnetic declination at present and at the place in question, but he must also possess an accurate knowledge of the secular change of the declination in order to make proper allowance for its effect during the interval between the two epochs of the survey. The secular variation is the continuous increase or decrease of the mean declination at a place for a series of years. Thus at Paris in 1541 the declination was 7° E., and continued to increase until 1580, when it was 11⅛° E. It then began to decrease, and in 1666 was 0°, and in 1814 was 22° 34′ W., when it again began to return east, being between 16° and 17° W. in 1880.

In a greater part of the United States, since about 1800, all western declinations have been increasing and all eastern declinations decreasing, but at very different rates at different places. In a belt through the extreme northern part of Maine and New Brunswick the change, if any at all, is scarcely perceptible; as also in a belt through central Arizona, Nevada, western

Idaho, across Washington to Vancouver Island. To the west of this latter belt eastern declinations are increasing.

THE AGONIC LINE.—Between those places where the declination is west and those where it is east is a line of *no declination*—that is, the true and magnetic meridians coincide. Such a line in 1885 passed just east of Charleston, S. C., through central Ohio, west of Detroit, Mich., and near Mackinac Island. This is called the *Agonic Line.* Lines joining those places having the same declination are called *Isogonic Lines.* It is to be remembered that the north end of the needle always inclines towards this line of no declination, which is itself moving westward quite rapidly.

In the United States Coast and Geodetic Survey Report for 1882 are given formulæ expressing the Magnetic Declination at various places at any time, and a table of computed annual changes in declination for various places. These may be found in most standard works on surveying. All such are only approximate, however, and at the time and place of using a magnetic needle the true meridian should be first determined, and from this the declination.

The change in declination as one moves east or west from a place is about 1′ per mile.

It also happens sometimes that different compasses in perfect adjustment show different declinations at the same time and place, so that the declination as indicated by the compass to be used should be adopted.

TO DETERMINE THE TRUE MERIDIAN WITH THE COMPASS.—Select a site, on which to make the observation, that is level for several hundred feet to the north. Suspend a plumb-line about 15 or 20 feet long from a point of support, with a heavy weight attached swinging freely in a pail of water to prevent vibrating. At about 12 feet from the line and directly to the south of it, drive two posts about 6 feet apart into the ground in an east-and-west direction. On the sides of these posts about 4 feet above the ground nail two boards with their top edges smooth, at the same height and level. The compass may be fastened in a box support so as to be level when placed on the boards, or fastened by means of a Jacob-staff head to a weighted board so that it can be leveled in the usual manner; in both cases with its plumb-bob swinging below.

From the tables find the watch time of elongation, and the azimuth of Polaris for the year and latitude of place of observation, as previously described.

About 20 minutes before the time of elongation, place the compass on the boards and bring the *two sights*, *plumb-line*, and *star* into the same plane, by first bringing the rear-sight, plumb-line, and star into coincidence, then the rear-sight, front-sight and plumb-line. The star will be seen to be moving laterally and constantly getting out of the plane of the sights and plumb-line, and the compass must be moved accordingly, so as to keep all four in the same plane until the star ceases its lateral motion and appears to rise or descend on the plumb-line. At this instant the compass is made truly level and the positions of the sights, plumb-line, and star in the same plane is assured, after which the compass must not be disturbed. During the observation the plumb-line must be lighted by having an assistant hold a candle or lantern near it. The relative positions of the compass and plumb-line should be such as to have the star seen as near the point of support of the line as possible.

Drive a stake under the compass and put a tack in it under the plumb. Have a stake and tack set in line several hundred feet north, and another put in the true meridian as described for the transit. About 10:00 A. M. the next day set up the compass over the south stake and take the bearing of the true meridian, which will be the declination. If, as is the case in June and December, when the elongations occur between 1 and 3 oclock in the morning and afternoon, it is inconvenient to make the observations at those times, then add 5 hours 54.8 minutes to the time of eastern elongation for upper culmination, or 6 hours 03.25 minutes to western elongation for lower culmination and determine the meridian at those times.

At culmination the star is moving in azimuth at the rate of almost 1′ of arc in 1 minute of time, so the observation, to be even aproximately correct, must be made within a couple of minutes of its exact time of crossing the meridian, which is difficult to do.

DETERMINING MERIDIAN BY SUN AND PLUMB-LINE.—On a level surface raise a pole inclining northward. From its upper end suspend a plumb-line. From the point under the plumb-bob describe a number of arcs of circles of different radii on the north side. A couple of hours before noon watch the extremity of the shadow of the pole, and as it cuts the different arcs mark those points (Fig. 116).

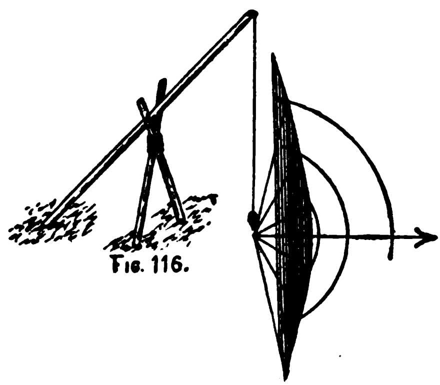


FIG. 116.

After noon the shadow will again cut the arcs on the opposite side of the meridian; mark these points also. Bisect the portion of each arc between the points marked, and take the mean of all. The line passing through the point under the plumb-bob and the middle of the arcs will be a true north-and-south line, and may be used for determining the declination.

TO SET OFF THE DECLINATION.—Place the compass with the south end of the zero line nearest the person as it would be used, and the zero of the declination arc and vernier coinciding. If the line of sight be brought into the true meridian pointing north, and if the declination of the place is east, the north end of the needle will lie to the east or right of the true meridian. Pay no attention to the position of the needle, but assume that the line of sight is in the true meridian, pointing north and looking at the 0° graduation of the circle at the north end of the zero line, turn the circle so that its zero graduation will move to the right or east of the line through the sights. If the declination is west, turn the circle so that its 0° graduation shall move to the left or west of the line through the sights. Or, if the declination is east, turn the 0° of venier towards the letter E, in the direction of the movement of the hands of a watch; if west, turn the 0° of the vernier towards the letter W, opposite to the movement of the hands of a watch. The declination having been set off, all bearings taken with compass are true bearings (neglecting the effects of diurnal variations of declination).

USE OF THE COMPASS.—The compass is used for determining the bearings of lines in traversing and the positions of objects in filling in details. If true bearings are to be used, either the declination should be set off or corrections made to all magnetic bearings to reduce them to true bearings. This can easily be done by first drawing a true and magnetic meridian in their relative positions and determinimg from them how the correction in any quadrant is to be applied.

To establish lines of given true bearings. To do this, either the declination must be set off or the magnetic bearing computed. Having turned the compass until the needle indicates the bearing, the line of sight will then be in the required direction.

To retrace old lines. In retracing old lines from bearings, allowance must be made for the secular variation of the declination during the interval of time between the former establishment and the retracing.

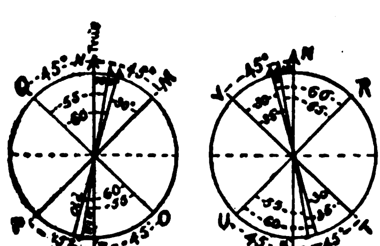
If the declination at the time of the former establishment and when retraced is *known*, the bearings for retracing can be computed and the line retraced.

If the former declination is *not known*, but the exact date of tracing is, then the declination can be ascertained by formula.

If the direction of the line is known, then the declination to be used in retracing can be determined by sighting on the line and turning the circle until the needle shows the given bearing.

If neither the direction of the line nor the former declination is known, then the declination to be used must be determined by running out a line of the given bearing and length, or until a point of the old line is seen on a perpendicular to the trial-line, and then determining the angle between the two lines for the declination. Measure on the trial-line the distance from the point of starting to the perpendicular, then the length of the perpendicular, and divide the latter by the former. With the quotient obtained, look in a table of natural tangents for the angle corresponding, which will be the amount of the declination to be used. If the trial-line lay to the right of the old line, then at the point of beginning, having the line of sight on the trial-line and the needle reading the given bearing, turn the line of sight to the left through the angle found between the two lines, then move the graduated circle until the needle reads the bearing of the line again, using the vernier of the declination arc to make the amount axact. If the trial-line lay to the left, then turn the line of sight to the right through the angle between the two lines, and move the graduated circle until the needle reads the bearing.

If the compass has no declination arc and it is desired to retrace old lines which were run when the declination was considerably different, the present bearings may be found by the following rules: *When the north end of the needle has been moving west, the present* N. E. & S. W. *bearings will be the change in declination plus the old* N. E. & S. W. *bearings; and the present* N. W. & S. E. *bearings will be the difference between the change and the old* N. W. & S. E. *bearings.* Thus (Fig. 117 *a*) true bearing of M=N 45° E, old bearing with declination 15° E=N 30° E, change in declination 5° westward, hence present bearing= 5°+30°=N 35° E; true bearing of P=S 45° W, old bear-

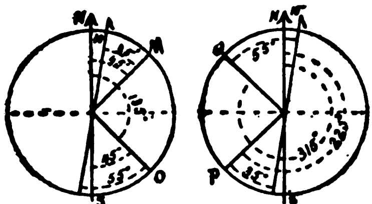


a FIGS. 117. b

ing S 30° W, present bearing=5°+30°=S 35° W; true bearing of O=S 45° E, old bearing S 60° E, present bearing=60° −5°=S 55° E; true bearing of Q=N 45° W, old bearing N 60° W, present bearing 60°−5°=N 55° W.

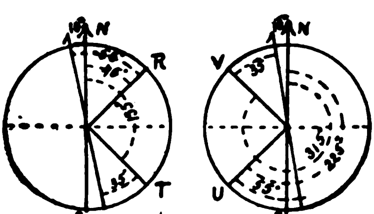
When the north end of the needle has been moving east, then the converse of the above obtains. In Fig. 117 *b* true bearing of R=N 45° E, old bearing with declination 15° W=N 60° E, change in declination 5° eastward, hence present bearing= 60°—5°=N 55° E; true bearing of U=S 45° W, old S 60° W, present=60°—5°=S 55° W; true bearing of T=S 45° E, old bearing S 30° E, present bearing=30°+5°=S 35° E; true bearing of V=N 45° W, old bearing N 30° W, present bearing=30°+5°=N 35° W.

If the compass has no declination arc and it is desired *to reduce the bearings of lines to azimuths from the north*, proceed as follows: If the declination is E., with N. E. bearings add the declination; with S. E. bearings subtract the bearings from 180° + the declination (Fig. 118 *a*). Using present bearings of same point as in Figs. 117, azimuth of M=35° +10°=45°; azimuth of O= (180°+10°)−55=135°; with S. W. bearings add the declination and 180°; with N. W.



a FIG'S. 118. b

bearings subtract the bearings from 360°+the declination. (Fig. 118 *b*) azimuth of P=35°+180°+10°=225°; azimuth of Q=(360°+10°)—55°=315°. If the declination is W., with N. E. bearings subtract the declination; with S. E. bearings subtract from 180° the sum of bearings and declination. (Fig. 118 *c*) azimuth of R =55°—10°=45°; azimuth of T=180° —(35°+10°)=135°; with S. W. bearings add 180° —the declination; with N. W. bearings subtract from 360°



c FIG'S. 118. d

the sum of bearings and declination. (Fig. 118 *d*) azimuth of $U = 55° + (180° - 10°) = 225°$; azimuth of $V = 360° - (35° + 10°) = 315°$.

The following is a convenient form for the notes of a compass survey:

COMPASS SURVEY.

.................................190...

OBSERVER..............................

CHAINMEN..............................

STATION.	POINT.	BACKSIGHT.	FORESIGHT.	DISTANCE.
....	....	....	....	....
....	....	....	....	....
....	....	....	....	....
....	....	....	....	....
....	....	....	....	....
....	....	....	....	....

CHAPTER IX.

THE PLANE TABLE.

DESCRIPTION.—*The plane table* (Fig. 119) consists of a *drawing-board* about 20x30 inches, mounted upon a tripod. It should admit *of being leveled, of being turned in azimuth,* and *of being clamped in any position.* On the drawing-board

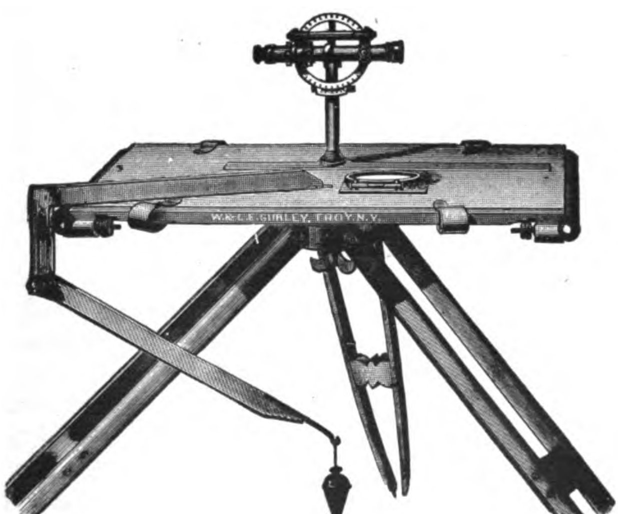


FIGURE 119.

is fastened a sheet of drawing paper. Upon this paper is plotted in miniature a representation of the country. *Directions are not read off* in degrees and minutes, but *plotted directly;* the instrument used being *the alidade,* which consists of a *bevel-edged ruler,* to which is attached for rough work *two raised sights,* as on the ordinary compass, and for the higher class of work a *telescope* which may be turned on a horizontal axis. The *line of sight* is usually, though not necessarily, parallel to the edge of the ruler.

The *telescope* carries a delicate *level* and has a *vertical arc* for measuring angles in the vertical plane, but has no horizontal motion independent of the ruler. It also has stadia wires for reading distances.

On the *ruler* are *two levels* at right angles, or *one round one*, for leveling the table.

With the instrumental outfit is usually a *declinator* consisting of a compass needle, swinging about 10° each side of the zero line, encased in a long narrow box, the longer sides of which are parallel to the zero line. By means of the declinator the magnetic meridian may be marked upon the paper and then the bearing of any line determined with a protractor. Instead of a declinator, a compass, set on a heavy base with two edges parallel to the zero line, is sometimes used, and on this base are usually two level tubes at right angles.

The paper used should be such as does not expand and contract differently in different directions under varying conditions of moisture. This is especially important in intersection work. If it expands and contracts uniformly in all directions, and the scale of the map is constructed upon the paper, it matters very little. To counteract unequal expansion, etc., two sheets may be mounted with cloth between them and with the grain at right angles. The paper may be attached in a variety of ways, as with thumb-tacks, with screws, or clamps for that purpose, with springs, or on rollers at each end of the table.

THE ADJUSTMENT OF THE TELESCOPIC ALIDADE are similar to those of the transit, and are:

(1)—For plate bubbles.

(2)—For line of collimation.

(3)—For telescope level.

(4)—For vertical circle.

USES OF THE PLANE TABLE.—The plane table, in some of its various forms, is the principal instrument used in "filling in" the details of a survey, when the principle points have been determined by the more precise method of triangulation.

All distances are plotted as soon as determined; hence no elaborate system of notes is kept.

From the primary triangulation sheet two or three points are located upon each plane-table sheet. Within these and depending upon them are then located a large number of points either by intersection, by traverse, or by both methods, forming a geometric framework upon which the sketching of the map depends. The work of making secondary and tertiary locations by intersection is done mainly by plane table. Locations by intersection should be carried as far as practicable; that is, all points which can be well located in this manner should be so located, in order to afford the most ample control for the traversing by which the intervening areas are to be filled in, since locations by intersection are more accurate, rapid, and economic.

Much misapprehension exists regarding the character and application of the plane table, from its being so little known For map-making it is the universal instrument. It is applicable to all kinds of country, to all methods of work, and to all scales. It is the most simple, direct, and economic of instruments.

SETTING UP THE PLANE TABLE.—The table is set over a point by grasping the nearest two legs of tripod, and with the knee extending the third until its foot reaches the ground at the proper distance from the point, then the other two feet are set in position. To bring a point on the paper exactly over the station on the ground, use is made of a U-shaped spring, or of a frame as shown in cut. The point of one leg is placed on the point on the paper and a plumb-line is suspended from the other end.

In taking up the table the nearest two legs are grasped, the table raised on the third leg, upon which the other two are closed, and the table placed on the shoulder.

THE SELECTION OF A BASE.—In using the plane table for determining points by intersection, a base-line must be accurately measured between two points over which the table may be set, and this base-line must be plotted to scale on the paper in practically the same relative position it occupies on the ground to be plotted. The selection of the site for the plane-table base-line should be governed by the following considerations, in addition to those for base-lines in general given in Chapter V: First, its position should be a central one. Second, it should be of a length proportioned to the area to be surveyed.

If the nature of the country is such that a base-line of a length proportioned to the extent of the survey cannot be found, the intersections may be extended as described under triangulation.

In triangulation work, several of the triangulation points are plotted to scale on the table sheet, the distances between them having been computed. These points and computed distances constitute stations and base-lines for beginning work with the plane table Any errors in measuring or plotting the base-line produce proportional errors in all other lines plotted.

ORIENTATION.—The chief and controlling condition in work with the plane table, and without which no accurate work can be done, is that it shall be *oriented;* that is, that at every station all lines joining points on the paper shall be parallel to the corresponding lines on the ground, or, that the table shall have at every station a position parallel to those it had at all others.

LOCATIONS BY INTERSECTION.—This consists in the location on the map or plot of points not occupied, by pointings from known and plotted points after the table has been oriented at each known point.

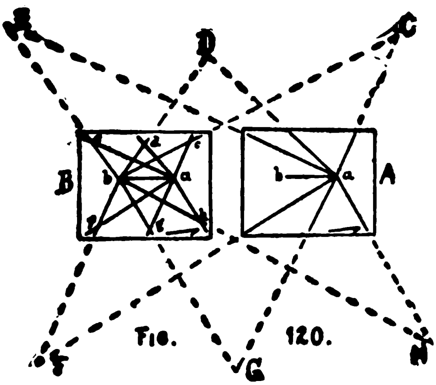


FIG. 120.

The base-line, or the stations A and B at its extremities (Fig. 120), having been plotted to scale on the paper in *a* and *b*, the table is set up over one of them, as A, approximately oriented with the plotted point *a* directly over the point A on the ground, and then leveled.

The beveled edge of the ruler is placed on the two plotted points *a* and *b* and the table revolved until B is bisected by the line of sight, with the point *b* towards it, then the table is clamped. The table is now said to be *oriented*, and it must be brought parallel to this position at every setting for this survey.

The ruler, with its beveled edge pivoted on *a*, is now revolved around towards the points C, D, E, F, G, H, and all other points the location of whose positions it is desired to fix by intersection, and a short line is drawn at the estimated distance of the point each time, and marked with a letter or number so as to distinguish it, keeping a note of the same for reference.

A pin or needle is sometimes stuck in the plotted station, against which the ruler is pivoted in taking sights, but this defaces the paper if many stations are occupied.

The table is then carried to B and set up approximately oriented with the point *b* over B, the end *a* of the plotted line *ab* pointing towards A, and then leveled. The beveled edge of the ruler is now placed on *ba*, and the table revolved until A is bisected by the line of sight, then clamped. In this position the table is *oriented*. The ruler, with its beveled edge pivoted on *b*, is now revolved around towards each of the points sighted from A, and the intersection of the two pointings on the same object marked, thus locating the true position, on the plot, of the point with reference to the base-line. Other points, seen from B, on which it is desired to intersect from subsequent stations, are now sighted and lines drawn, as at A. If the table be now taken to any of the points intersected from A and B, as C, and there set up and oriented by sighting back on A or B, and then sights be taken on the other points D, E, etc., and lines drawn, these lines should pass through the points already determined. By this means the previous work is *checked*, and this check should not be omitted for important points.

Precautions to be observed. When the ruler is directed upon an object, the extreme ends should be marked by *check dots*, so that it may be replaced with greater certainty by these than it could on the short line between the objects, if it should be desired to use such lines subsequently to orient the table. Angles of intersection less than 30° or greater than 150° should be avoided, if possible, or checked from other points. Preferably they should be between 60° and 120°, 90° being the best. To obtain such angles, the points intersected should not be at a much greater distance from either of the viewing stations than the distance between the latter; nor should the angle between the line joining the two viewing stations and the line joining either viewing station with the point sighted upon, be much greater than 90°.

Points unsuitably situated for intersecting from the ends of the base-line may be sighted from one end, with a view to intersection from more favorably situated points later. As soon as a point is determined, the details surrounding it should be drawn in.

USE OF THE TELESCOPIC ALIDADE WITH STADIA WIRES.—If the telescopic alidade with stadia wires is used and distances are read on the stadia rod, the distances so read are plotted to scale on the lines drawn towards the objects, thus fixing their positions at once. This is the usual method in filling in details.

While making the horizontal locations of points, their heights may also be measured, *relative to the point occupied*, by means of the vertical arc of the alidade and the telescope level. The telescope bubble is brought to the center and the index error determined. A sight is then taken on a point of the object as high above the ground as the telescope, the angle read, index error applied, and the elevation determined by solving a right-angled triangle by the formula $h=d$ tan. a, in which h=height, d=distance, and a=angle of elevation or depression.

The above determination of index error must be made for each point, as the table cannot be leveled, nor so maintained, with sufficient accuracy to dispense with it.

TRAVERSING WITH THE PLANE TABLE.—As previovsly stated, a traverse consists of a series of consecutive direction and distance measurements depending upon one another.

These lines should be connected wherever possible with triangulation points in order to check up accumulated errors. The relative extent to which triangulation and traversing can be applied depends principally upon the amount of relief of the surface and the prevalence of forests.

In Figure 121, suppose A and B to be two triangulation

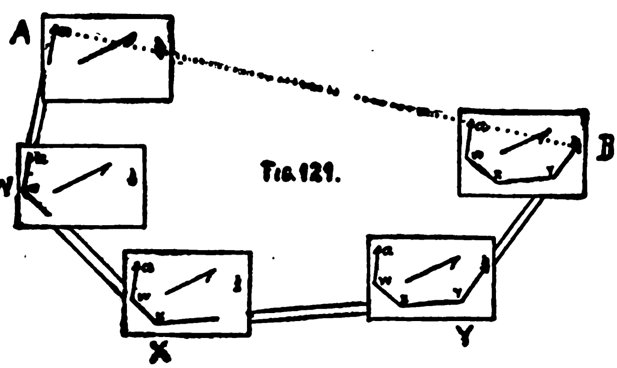


FIG. 121.

points plotted to scale on the paper, and it is desired to traverse from A to B by way of W, X, and Y, closing on B. The plane table is set up and leveled at A, the beveled edge of the ruler placed on *a* and *b*, the table turned until B is sighted, then clamped, thus orienting the table. The *declinator* or compass is now placed on the sheet and turned until the needle reads zero, then a line is drawn along an edge parallel to the zero line. The north end of this line, which is the magnetic meridian, is marked with a half arrow-head. The ruler, pivoted on *a*, is turned until W is bisected, then a line is drawn towards it. All details (omitted in figure) in the immediate vicinity, such as houses, fences, roads, etc., are sketched in to scale, their directions being determined with a ruler, and their distances measured with the tape, chain, or stadia, by pacing, or by estimation, according to circumstances. This completed, the measurement of the distance to W is commenced by whatever method has been previously decided upon, care being taken to follow the straight line AW. The details between A and W are sketehed in by the topographer, who stops, from time to time, as he arrives opposite these details. The distance he has traversed from A is laid off to scale on the line from *a*, the table oriented by sighting back on A, or by placing the side of the declinator on the meridian drawn and turning the table until the needle reads zero.

On reaching W, its distance from A is laid off to scale on the line drawn, thus locating W. The plane table is here set up, leveled, oriented by a back-sight on A, and clamped. The surrounding details are sketched in, and the direction from W to X determined. The topographer then proceeds to X, stopping from time to time to sketch in the details.

In traversing, *the distances are measured continuously from one station to another*, as from A to W, W to X, etc., and so plotted; thus, if an error is made in laying off any intermediate distance where one stops to sketch details, such error will not enter into the plotting of distances between stations. The measurements begin anew only at each change of direction.

Before leaving any station, *a visible mark* should be left to sight back upon from the forward station.

If, arriving at any station, any other station already located is visible, a *check* on the work may be had by sighting upon it after orienting the table. Thus, after arriving at B and orienting the table on Y, if the ruler is placed on *b* and *a*, and the line of sight passes through A, it proves the work is correct; but if not, the amount and direction of deviation show the amount of error introduced in the traverse, which may be here thrown out by orienting on A instead of Y, if the traverse is to continue from B.

Orienting by the needle. When traversing through woods, or along winding roads, or over undulating ground, it may often be more convenient to orient the table by means of the declinator or compass; hence the value of the magnetic meridian which was marked on the paper when beginning work.

The table might even be oriented at each station by the compass, instead of back-sighting, but this method would not be so accurate on account of magnetic disturbances of the needle, and the shortness of the needle line compared to the distance between *check dots.* Any single inaccuracy in orienting the table with the compass in a series of settings would produce less effect in the end, however, than a like error of the same amount at the same place made in orienting by back-sights. This will be understood by reference to the following figures:

FIG. 122.

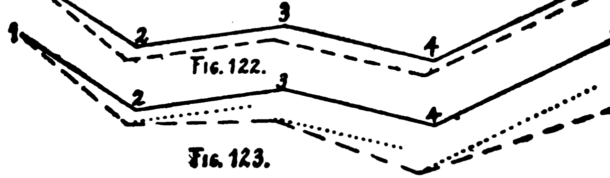


FIG. 123.

In Fig. 122, by using the compass to orient the table each time, no reference is had to the line 1–2; therefore the only error, if subsequent settings are correct, will be the first one made.

In Fig. 123, to orient the table at 2, reference must be had to the line 1–2, which being in error, the setting here will also be in error, as also at all the other points, not in themselves, perhaps, but because of an indirect reference back to 1–2.

LOCATION BY RESECTION.—This consists in finding the point on the map or plot which corresponds to the point on the ground over which the table is set, and is done by pointing to known and plotted points, after the table has been oriented. All problems under *Resection* consist of two parts—viz., the orientation and the resection.

In this determination the difficulty that arises is, *how to orient the table*

1st method: By use of the magnetic needle.

Orientation. If the magnetic meridian has been marked on the plot, then by using the compass the table may be oriented at any place where it may be set up, as described under *Traversing*.

Resection. The point occupied may be plotted by selecting two points (already plotted) about 90° apart, pivoting the ruler on the plotted position of one, sighting the object, and drawing a line back; then pivoting the ruler on the plotted position of the second object, sighting it, and drawing a line back, intersecting the previous line drawn; this intersection will be the plotted position of the point occupied.

Magnetic disturbances of the needle will, of course, affect the accuracy of this method, which may be tested by sighting, as before, on any other visible plotted point and seeing if the line passes through the point found. If not, it proves that the table was not exactly oriented.

2d method: By use of one known and plotted point and a line drawn from a plotted station through the unknown point.

If one sighting has been taken to a point from a station and a line drawn towards it, the point may be occupied with the table and its position plotted. Thus, from station S (Fig. 128) the object D was sighted and the line towards it drawn. The plotted position of D will be somewhere on this line, but its exact position is not known.

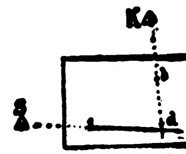


Fig. 128.

Orientation. The table is set up over D, leveled and approximately oriented. The edge of the ruler is placed on the line *ds* so that the check dots are just visible along the edge, and the table turned until S is bisected, when the table it oriented.

Resection. The ruler is then pivoted on the plotted point *k* of an object K that will give a good intersection and moved until K is bisected. Where the ruler intersects the line *sd* will be the plotted position of D. Resection upon any other plotted point will verify its position.

3d method: By use of two known and plotted points (the two-point problem).

1st. solution: Orientation. The occasion may arise where it is desirable to orient the table and begin work at a point from which only two known plotted points are visible. Let A and B (Figs. 129, 130, 131, and 132) be the two points, and *a* and *b* their plotted positions. To orient the table at C and determine its plotted position, select a fourth point D, that may be occupied, and so situated that the intersections from C and D on A and B make sufficiently large angles for good determinations. But the

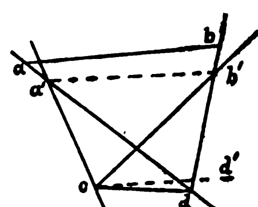


Fig. 129

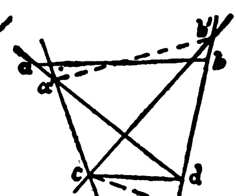


Fig. 130.

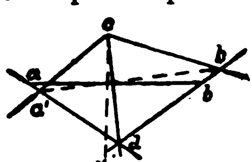


Fig. 131

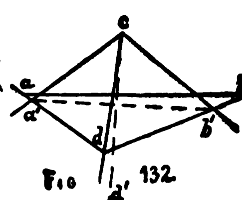


Fig. 132.

table approximately in position at D, by estimation or by compass, and draw the lines Aa and Bb intersecting d; through d draw a line towards C. Then set up at C and assuming the point c on the line dC, at an estimated distance from d, and putting the table in a position parallel to that which it occupied at D by the line cd, draw lines from c towards A and c towards B. These will intersect the lines dA and dB in the points a' and b', which form with c and d a quadrilateral similar to the true one, but erroneous in size and position.

The angle which the lines ab and $a'b'$ make with each other is the error in position. By now constructing through c a line cd' making the same angle with cd as that which ab makes with $a'b'$, and directing this line cd' to D, the table will be oriented. Or, to orient instead of transferring the angle of error by construction as above, proceed as follows: As the table stands, $a'b'$ is parallel to AB, but to orient the table, ab must be parallel to AB. Therefore, place the alidade on $a'b'$ and set a mark in that direction, then place the alidade on ab and turn the table until it again points to this mark; then ab will be parallel to AB and the table oriented.

Resection. The true plotted position of C may now be found by resection on A and B.

2d solution: Instead of performing the above operations on the actual plot-sheet, if a piece of tracing-linen or tracing-paper is at hand, it may be fastened on the table, and the auxiliary quadrilateral constructed upon it, by assuming a point d over D and drawing dA, dB, and dC, and completing as above. Then loosen the tracing and place the line $a'b'$ on ab, and with the alidade on ca', cb', or cd revolve the table until the line of sight comes on A, B, or D. The table will then be oriented.

3d solution: (Fig. 133). Set up the table at C as nearly oriented as can be done by eye, and resect upon A and B, intersecting at c'. The angle $ac'b$ being the true angle at C subtended by AB, the true point c must be on the circumference of the circle through $ac'b$. Construct this circle. Meas-

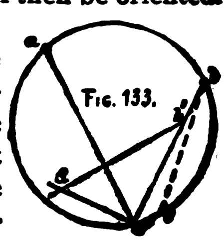


FIG. 133.

ure off a base CD at least half the length of CB at right angles, or nearly so, to *bc'* in the direction most convenient. Set up a signal at D and with the alidade draw the line *c'd*. Take the table to D, and by the line *dc'* sighting on C bring the table into a position parallel to that which it had at C. With the ruler pivoted on *d*, sight B and draw the line *db'* intersecting *c'b* at *b'*. *c'b'* is the plotted distance of C from B, which distance laid off on the circle as a chord from *b* will give *c*, the true plotted position of C. A fourth point may then be occupied, and by resection on A, B, and C the accuracy of the determination of C verified.

If the auxiliary point D can be taken *in range* with A and B, either between them or in prolongation of the line joining them, the table may be set up there and *oriented* on them, and then a line drawn towards C from any point on this range line on the plot. The table is then taken to C, *oriented* by the line just drawn and the position of *c* found by resection on A and B. One can place himself on the line joining two points, when he is between them, by observing them alternately from opposite ends of the ruler laid on the table and moving the whole table to one side or the other until the ruler viewed from either end bisects the objects.

4th method: By use of three known and plotted points (the three-point problem).

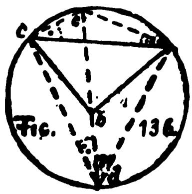


Fig. 136.

Orientation. The table may be at once oriented at any unknown point from which three known plotted points are visible, except when it lies on the circumference of a circle passing through the three known plotted points. In Figs. 134, 135, 136, and 137 let *a*, *b*, and *c* be the plotted positions of A, B, and C respectively, and D the unknown point. The table is set up over D and leveled. The ruler is set on the line *ca*, and *a* directed, by revolving the table, to its corresponding signal A and the table clamped. Then with the ruler pivoted on *c*, the middle signal B is sighted and the line *ce* drawn. The ruler is then set

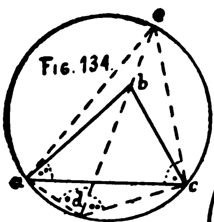


Fig. 134

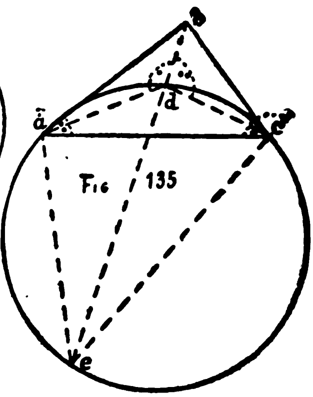


Fig. 135

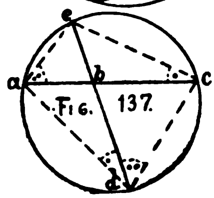


Fig. 137

on the line *ac*, and *c* directed, by revolving the table, to its corresponding signal C and the table clamped. Then, with the ruler pivoted on *a*, the middle-signal B is sighted and the line *ae* drawn. The point *e* (intersection of these two lines) will be in the line passing through the middle point and the point *d*. Set the ruler on the line *be*, direct *b* to the signal B by revolving the table and it will be *oriented.*

Resection. Clamp the table, pivot the ruler on *a*, sight A and draw the line *ad;* this will intersect the line *be* at the point *d* sought. Pivoting on *c*, sighting C will verify it. The angles *ace* and *ade* are subtended by the same chord *ae*, and *cae* and *cde* by the chord *ce*, hence *e* must fall on *db;* also, *f* being the point of intersection of *ac* and *de*, the triangles *adf* and *cef* are similar, and the triangles *cdf* and *aef* are similar, hence *d*, *f*, and *e* must be in a right line through *b*. (This is called Bessel's method by inscribed quadrilateral.)

A mechanical solution: Set up the table in any position over the point D, level, and clamp. Fasten a piece of tracing-linen on tracing-paper on the table, and assume a point for *d* in such a position that lines from it towards A, B, and C may be drawn long enough to pass through their plotted positions. Pivoting the ruler on *d*, take sightings on A, B, and C, and draw the lines of their directions. Unfasten the tracing-paper and turn it until the three lines pass exactly through the three points *a*, *b*, and *c*, when *d* will be in its true position and may be pricked through, the table be *oriented*, the work of plotting, etc., proceed.

CHAPTER X.

THE WYE LEVEL.

The determination of the relative elevations of triangulation and control points may be by *spirit-leveling*, or by *trigo-*

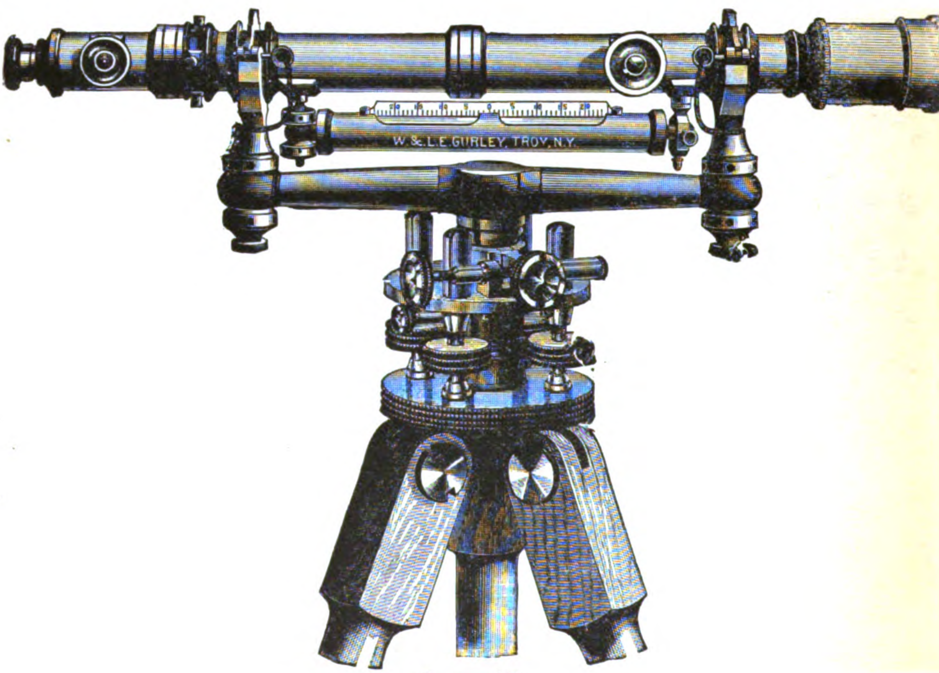


FIGURE 142.

nometrical leveling. For spirit-leveling, the ordinary engineer's spirit or wye level will answer most of the purposes of military topography, while for trigonometrical leveling, in which the vertical angles between the different points are determined, the transit with its vertical arc will be mostly used.

DESCRIPTION.—*The Wye Level* (Fig. 142 and Section), consists of a *telescope* having on each end a *ring* of bell-metal, turned very truly and both of exactly the same diameter. On these rings the telescope rests in *wyes*, in which it may be revolved, and clamped in any position by bringing down on the rings the *clips* of the wyes. The telescope is essentially the same as the one described on the transit, except that it is longer, and does not have the stadia wires.

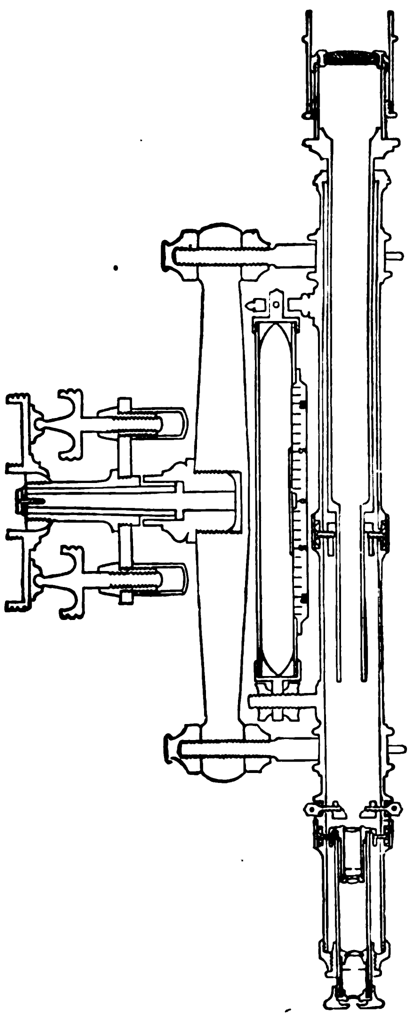


FIGURE 142.—Section of Wye Level.

The level or bubble-tube is encased in a brass tube-holder, which is attached to the under side of the telescope, and furnished at one end with a vertical movement, and at the other with a horizontal movement. Over the aperture of the brass tube is a scale graduated to tenths of an inch with every fifth division numbered from 0 at the center.

The wyes are the supports of the telescope, and each is held in the ends of the *level-bar* by two nuts, both adjustable with pins. The clip of one of the wyes usually has a pin or some arrangement for holding the telescope so the horizontal wire will remain horizontal. Connected with the level-bar is the head of the tripod-socket. The socket is compound, the interior of steel turning in one of bell-metal, which latter turns in the main socket of the leveling-head.

The upper part of the instrument, with the socket, may be detached from the leveling-head, and this latter may be unscrewed from the tripod-head.

The leveling-head and tangent screw are the same as those described on the transit.

ADJUSTMENTS OF THE LEVEL.—The objects to be accomplished in adjusting the level are: 1st, to bring the axis of the bubble-tube into the same vertical plane with the axis of the telescope; 2d, to make the line of collimation of the telescope parallel to the axis of the bubble-tube—*i. e.*, to make the line of collimation horizontal when the bubble is in the center of the tube when the telescope is revolved around the vertical axis.

1st adjustment: Turn the telescope until it is over a pair of diametrically opposite leveling-screws, and clamp it there. Revolve the telescope slightly in the wyes, the clips being open, until the level-tube is moved out of the vertical plane through the line of sight. Should the bubble run towards either end, it will show that the vertical plane through the axis of the bubble-tube is not parallel to the vertical plane through the axis of the pivot rings. To make them parallel, bring the bubble back nearly the whole of its displacement by the capstan screws of horizontal motion on the sides of one end of the level-holder. Revolve the telescope back with level underneath, relevel with leveling-screws, and repeat and correct until it will stand the test.

2d adjustment: *To make the line of collimation parallel to the axis of the bubble-tube.* This may be done directly by the Peg Adjustment, described under the Transit; or, *by*

(1) making the line of collimation parallel to the line joining the bottoms of the pivot rings, and then (2) the axis of the bubble-tube parallel to this latter line.

To do the *first*, set up the level and clamp it. Remove the wye pins and raise the clips. Bring the intersection of the cross-wires upon some well-defined point from 100 to 500 feet distant. Then revolve the telescope half over—*i. e.*, turn the bubble on top. If the intersection of the cross-wires remains on the point while the telescope is being turned, the line of collimation is in the axis of the pivot rings. If the intersection leaves the point, then by means of the capstan screws which hold the reticule or cross-wire ring move the intersection half-way back to the point, and by means of the leveling-screws bring the intersection the remaining half of its displacement back upon the point and repeat for a test, correcting one-half of the displacement by the capstan screws and the other half by the leveling-screws until the telescope can be revolved without the intersection leaving the point.

To do the *second*, bring the bubble to the center of its tube by the leveling-screws, then lift the telescope carefully out of the wyes, turn it end for end and replace it in the wyes. If the bubble returns to the center in this position, its axis is parallel to the bottoms of the rings. If it does not return to the center, correct one-half of its displacement by means of the nuts on the vertical screw on one end of the level-holder, and then bring the bubble the remainder of the way to the center of the leveling-screws, and turn end for end again, correcting as before until it will stand the test.

3d adjustment: To make the axis of the bubble-tube perpendicular to the vertical axis of the instrument, so that the bubble will remain in the center during an entire revolution of the instrument. With the telescope over a pair of opposite leveling screws, bring the bubble to the center, and revolve 180° horizontally. If the bubble does not remain in the center, bring it half-way back by the nuts on either end of the level-bar directly under one of the wyes. Relevel with the leveling-screws, and repeat for a test. Now try it over the other pair of leveling-screws, proceeding as before, changing to each pair of screws successively until it can be revolved entirely around without moving from the center.

RELATIVE IMPORTANCE OF THE ADJUSTMENTS.—The first can only affect observations when the telescope is slightly revolved in the wyes, which is usually provided against by some means for holding the telescope from revolving when the wye clips are fastened down.

The second adjustment is absolutely necessary and is the most important of all, as entering into every observation.

The third is made only as a matter of convenience in revolving the telescope around the vertical axis, so as not to have to stop and relevel for each observation, as when profiling, or contouring, etc.

LEVELING-RODS.—With the level there are used graduated rods of various patterns for measuring the vertical interval between the bench-marks, initial point, or turning-points, and the line of sight of the level.

They may be divided into two general classes of the *self-reading* or *speaking rods*, and the *target-rods*, depending upon the size of the divisions. The *first class* has such large graduations that the observer can read through the telescope the positions of the horizontal wire. The *second class* has such small graduations that at long distances the observer cannot read the positions of the of the horizontal wire, but has a rodman move a target up or down until the horizontal wire coincides with a certain line upon the target, when the rodman reads its position by means of a vernier on it. The front of the target should be so painted as to show a distinct line with which to bring the horizontal wire into coincidence Leveling rods are usually of well-seasoned wood, as cherry. maple, mahogany, or baywood.

The Philadelphia rod (Fig. 143) is composed of two

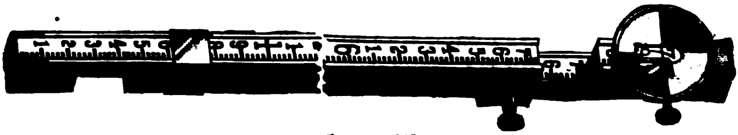


FIGURE 143.

parts sliding upon one another, and held together by clamps. It is graduated to hundredths of a foot and may be used as a speaking-rod, although it has a target with a vernier, which may be used if desired to read to thousandths of a foot.

The New York rod (Fig. 144), of two and three pieces, is graduated to hundredths of a foot and read by the vernier to thousandths of a foot. Up to 6.5 feet the target is moved up and down the rod until it is bisected by the horizontal wire, when it is clamped and its position read by the vernier on its face. For greater heights the target is clamped exactly at 6.5 feet, and the back of the rod moved up until the target is bisected, when the parts are clamped and the height of the target read by a vernier on the side of the rod, on numbers on the back part, which are figured from 6.5 feet at the target towards the bottom.

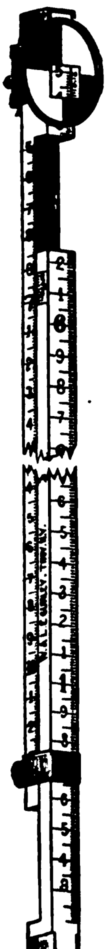


Fig. 144.

Speaking-rods may be made by taking a strip of board 4 to 6 inches wide, $1\frac{1}{4}$ to $1\frac{1}{2}$ inches thick, and 12 to 15 feet long, putting an iron shoe on the bottom, painting white, then graduating it to hundredths of feet beginning at the bottom, and painting devices, easy to read, with red or black paint. Figs. 145 and 146 show some devices used, on which the feet and tenths should be plainly numbered for reading at a long distance.

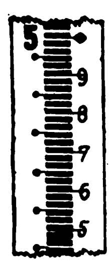


FIG. 145.

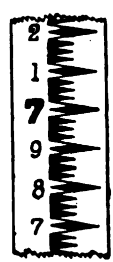


FIG. 146.

VERTICALITY OF ROD WHEN USING.—The rodman stands *behind* the rod to see the signals of the observer, and holds the rod as nearly vertical as possible. If he leans it sideways, the observer can detect it from its position with reference to the vertical wire in the telescope, and signals which way to lean it to be vertical. To tell whether the rod is inclined in the other directions, the rodman, after the target has been brought to the horizontal wire, inclines the rod slowly towards the instrument, then back towards himself, and if the target descends each time, it shows that the rod was vertical, and the observer signals him to hold it in the position where the target is the highest.* The rod not being held vertical would result in too great a reading. A plumb line or a pair of right-angle levels (Fig. 147) attached to the rod assists the rodman to hold it vertical.

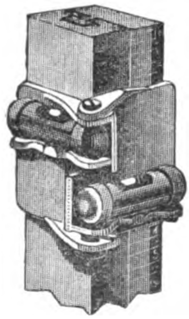


Fig. 147. In Position

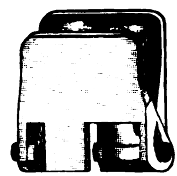


Fig. 147. Closed.

Various arrangements of targets have been devised for indicating to the observer when the rod is vertical. *Thompson's Target* (Fig. 148) does

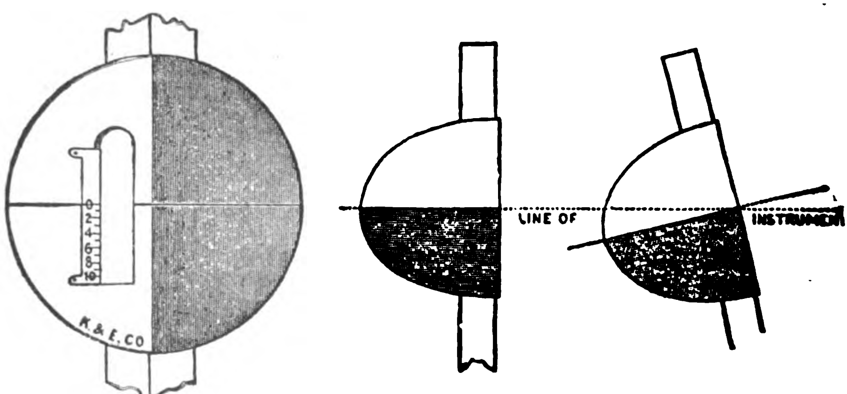


FIGURE 148.

this by having the horizontal dividing line carried over two surfaces placed at right angles to each other, thus showing a continuous unbroken line only when the rod is held vertical.

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*The rod should be held on the turning-pin so that the front edge of its base besets the pin, in order that when the rod is waved it will revolve about a line which is in the same plane with the graduated face of the rod.

TURNING-PIN.—The rodman carries a *turning-pin*, which is made of a piece of round bar-iron 6 to 8 inches long, one end turned down to a point, the other end rounded and having in it a ring or strap for withdrawing it from the ground and carrying.

THE DATUM-LEVEL is the level surface above or below which the heights of all points of a line or surface are measured. This is known as the *plane of reference* and *datum-plane*. When possible, the surface of the sea at mean tide is taken as the datum-level, or the results reduced to that level. Inland it is customary, in any piece of work, to assume an imaginary datum-plane so low that no point shall be lower (thus avoiding minus signs) by assuming a high elevation for the initial point.

BENCH-MARKS are more or less permanent objects whose elevations above datum are determined and recorded for future reference. The nature of the object selected for a bench-mark should be such that it will not change its elevation during the time that it is to be used. *Temporary* benches are sometimes made by driving two or three stakes into the ground and taking their mean elevation for the bench. *More permanent* benches may be stone steps, or a nail driven into a projecting spur of the spreading root of a tree, and the tree blazed and marked. *Permanent* benches may be made by inserting copper bolts in rock or selecting already constructed structures, as the water-tables of old masonry buildings, copings of foundation and retaining walls, or the piers and abutments of bridges, etc. Every bench-mark should have its exact location and description noted in such a manner that anyone could find it from the record. Bench-marks should be set near the beginnings and ends of lines run near △'s; and in long lines, benches should be set every half-mile or mile, also near the crossings of roads, rivers, and the tops and bottoms of ridges.

DIFFERENTIAL LEVELING.—When a line of levels is run for the sole purpose of establishing the heights of points, or for determining their relative elevations, the operation is called *differential leveling*.

The elevation of the point from which the leveling starts, called a *bench-mark*, being known or assumed, the leveling-rod is held vertically upon it, the level set up from 100 to 300 feet forward in the general direction, if possible, of the line to be run, the bubble brought to the middle of the tube, the line of sight turned upon the rod and the target moved up or down until the horizontal line across it coincides with the horizontal wire in the telescope, when it is clamped. The rodman then reads the height of the target and records it as a *back-sight*. The horizontal elements (line and wire) of the target and level being in the same horizontal plane, this rod reading also gives the height of the line of sight above the point upon which the rod was held, and, if added to the height of the point above datum, the *height of the instrument* above datum also becomes known. The rodman, after recording his reading, comes forward to the levelman, counting his steps as he does so. The levelman reads the rod and records it, then the two compare their readings for a check and any error is at once discovered and corrected. The rodman then passes on ahead the same number of paces it took him to reach the level, and there holds his rod vertical while the levelman turns the telescope upon it and directs him to raise or lower the target as before until the horizontal line and wire coincide. The rodman then clamps the target, reads its height, and records it as a *fore-sight*.

This reading of the rod being the distance of the point on which the rod is held below the line of sight, if it be subtracted from the *height of instrument*, the remainder will be the height of the point above the datum, a comparison of which with the height of the starting-point will give the difference of height or relative elevation of the two points. Since the *back-sight* was added to the height of the starting-point or bench-mark to determine the *height of the instrument*, it is called *plus-sight* (+) and the back-sight column in the record is so designated. And because the *fore-sight* was subtracted from the *height of instrument* to obtain the height of the fore-sight point, it is called *minus-sight* (—) and the fore-sight column is so designated. Had the back and fore-sights been added *algebraically*, or the less subtracted *arithmetically* from the greater and the sign of the greater been given to the remainder, it would be the difference of elevation, and indicate which was the higher. If the remainder were *plus*, it would indicate that the second or forward point was higher than the first or back point; if *minus*, that the forward point was lower than the back point. In general, it may be stated that a back-sight is always on a point of known elevation, while a fore-sight is always on one whose elevation is unknown.

If the points whose elevations are desired are so far apart or at such great differences of elevation as not to be determined by a single setting of the instrument midway between them, then for all intermediate rod positions, called *turning-points*, the rodman forces his turning-pin into the ground and holds his rod upon it (Fig. 149). Supposing the forward point above to have been on the turning pin, the levelman, after checking the rod reading, then goes forward as far as he can get an accurate sighting on the rod if on level ground, or until about 5 feet above if ascending, or 5 feet below if descending, sets up his level and repeats all the operations to include the checking of the fore-sight reading described above, and so on until the final point or points are reached, which then become *bench-marks*. The successive differences of elevation of the two points upon which the rod is held for each setting of the instrument can be determined as above, and then their alge-

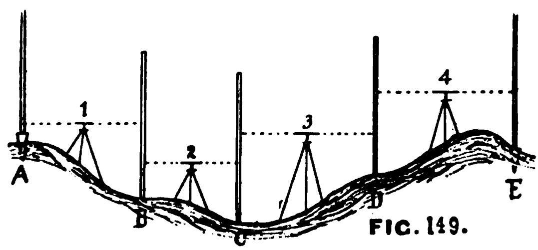


FIG. 149.

braic sum will be the difference of elevation of the terminal points. It is more simple, however, to find the sum of the *back-sights* and the sum of the *fore-sights* and to their difference give the sign of the greater for the difference of elevation of the terminal points. This difference of the sums added algebraically to the elevation of the bench-mark will give the elevation of the final point.

Use of sights of equal length. By the rodman proceeding as far in advance of the instrument each time as he was back of it, the lengths of the back and fore-sights at each setting of the instrument are made equal, which is necessary in this kind of leveling to avoid making corrections for curvature of the earth, and to eliminate instrumental errors, such as the line of sight not being horizontal when the bubble is in the middle of the tube. For if it inclines up or down on the back-sight, it will incline the same way and an equal amount on the fore-sight, if the lengths are equal, and so balance one another. If at any time it is impossible to make the two lengths of sights equal for a setting of the instrument, the inequality should be made up in subsequent observations by making the lengths of sights unequal in the oppcsite direction. This frequently occurs in leveling across a valley or ridge.

Use of long sights. In order that the work may progress as rapidly as possible, as long sights as may be consistent with accurate work should be used. These will depend upon the instrument used, and the state of the atmosphere, and will vary from 100 to 300 feet. If the air is tremulous and the position of the wire unsteady on the rod, shorter sights must be used. This is particularly the case in bright weather and over certain kinds of soil favorable to mirage. When the air is steady, as on cloudy days, etc., longer sights may be used.

The record. The record may be kept as shown in the accompanying forms. No distances are recorded unless desired for special reasons or when sights are unequal and it is intended to balance them.

DIFFERENTIAL LEVELING. (FIGURE 149.)

No. of Inst. Sta.	Distance.	Backsight. +	Foresight. —	Ele. of Bench.	Remarks.
				100.00	B. M. 8
1		1.037	8.436		B. S. on B. M. 8
2		3.475	5.834		
3		9.823	6.371		
4		7.826	4.879	96.641	F. S. on B. M. 9
		22.161	—25.520		
			+22.161		
			—3.359		

On Station.	Distance.	Back-sights. +	Fore-sights. —	Rise.	Fall.	Total Heights.
A		1.037	....			100 B. M.
B		3.475	8.436		7.399	92.601
C		9.823	5.834		2.359	90.242
D		7.826	6.371	3.452		93.694
E		.....	4.879	2.947		96.641 B. M.
		22.161	25.520	6.399	9.758	
					6.399	
					3.359	

Precautions. The level must be kept in adjustment by testing it every day. On whatever the rod is set, it must be so firm as not to change its elevation between the two sights upon it. The levelman must be certain that the bubble is in the middle of the tube when he makes his observation.

PROFILE LEVELING.—When it is desired to obtain the profile of the surface of the ground on certain lines, the level is used and the operation is called *profile leveling*. The line is chained and stakes driven every 100 feet, and at as many intermediate points as may be necessary to enable an accurate profile being drawn. The 100-foot stakes may be numbered and the intermediate stakes called pluses, as plus so many feet beyond the last numbered stake; or, all may be entered in the record by their distances from the starting-point. These intermediate or plus points are points of abrupt changes of slope, the tops of ridges or the bottoms of valleys, not touched by the 100-foot stakes driven, but are necessary points in the representation of the profile. The objects in profile leveling are to obtain the distances from, and the elevations above or depressions below, some fixed initial point, of the points where the stakes are driven. The level is set up on or near the line, as may be most convenient, as far forward from the initial point as the accuracy required will allow, and a reading taken upon the initial point, whose elevation is known, assumed, or determined from the bench-marks near by. This rod-reading, called a *back-sight*, added to the height of the initial point will give the *height of the instrument*. The rod is then held by the side of each of the stakes in succession and readings taken, called *intermediate-sights*, until the rodman has proceeded as far beyond the instrument as the initial point is back, when he puts his turning-pin into the ground and holds his rod upon it, and a reading is taken, called a *fore-sight*.

Each of the intermediate-sights, and the fore-sights on turning points subtracted from the height of instrument gives the elevation of the corresponding point of the surface of the ground. The turning-point may or may not be a point of the profile, being selected with reference to the best position for the next setting of the instrument. The instrument is then carried forward beyond the turning-point, set up, leveled, etc., a *back-sight* reading taken on the turning-point, added to the height of turning-point for a new height of instrument, and then the readings on and elevations of the intermediate points determined as before, and so continued until the extremity of the lines is reached, which is entered in the record as a *fore-sight*. For the *height of instrument* at any time the record should show one more *back-sight* than *fore-sights*, and this height, as shown by the notes, may be checked by adding the back-sights and fore-sights, taking the difference and adding it algebraically to the elevation of the initial point. The profile (Fig. 150) is then constructed to scale on profile paper, or otherwise, from the surface elevations of the points as found in the notes.

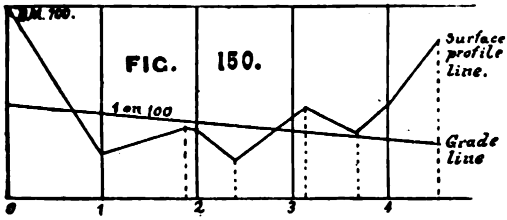


FIG. 150.

The back-sight and fore-sight readings for one setting of the level are not recorded on the same line, but both sights on one turning-point are on the same line. When turning-points are plotting-points of the profile, an entry is made in both F. S. and I. S. columns. When a B. M. is set during the work, the sight upon it is entered in I. S. column and not in F. S. column, which is reserved for turning-points.

The record. A form of record for profile and grade leveling follows; other forms are used, but this is a very convenient one inasmuch as numbers to be added or subtracted are found in adjacent columns.

RECORD FOR PROFILE LEVELING AND GRADE.

B. S. +	El. of T. P. & B. M.	F. S. —	H. I.	I. S. —	S. E.	Sta.	Remarks.	Ele. of Grade.	Cut or Fill.
0.636	100.00		100.636			B.M		90.00	-10.
				9.27	91.37	+50		89.50	-1.87
1.180	90.395	10.241	91.575			T.P.			
				6.95	84.63	1		89.00	+4.37
				4.18	87.40	+87		88.13	+0.73
				4.83	86.75	2		88.00	+1.25
				7.61	83.97	+40		87.60	+3.63
				.3.37	88.21	3		87.00	-1.21
69.79	90.685	0.890	97.664			T.P.			
				8.37	89.29	+13		86.87	-2.42
				11.02	86.64	+67		86.33	-0.31
				8.07	89.59	4		86.00	-3.59
	96.126	1.538		1.538	96.126	+45		85.55	-10.85

DETERMINATION OF GRADE.—Having constructed the profile, a grade line may be marked upon it and the amount of *cut* or *fill* at any and all points may be at once determined by measuring to scale the distance from the grade line to the surface profile line. The distances so found may then be marked upon the stakes, which is all that is necessary to enable the grade being established, if it is a simple ditch or trench, as for sewers, drains, etc. If the grade line is to be a road, however, and the sides of cuts and fills required to be given a certain slope, other stakes, called "slope stakes," must be set on the points marking the extreme lines of cuts or fills. The operation of determining their positions to fulfill the required conditions is called "cross section leveling." (See Chapter XXV.)

Should the required grade be determined upon before any leveling is done, as a grade of 1 on 100, and an elevation assumed for the initial point, then the amount of cut or fill may be determined during the leveling and marked upon the stakes at once, without constructing the profile. Referring to the Profile Record, the *elevation of grade* would be found for each profile point by adding to the height of the initial point, if an up-grade, or subtracting from the height of the initial point if a down-grade, the differences of elevation corresponding to the successive distances from the initial point of the profile points—*i. e.*, at 100 feet distance from B. M. the elevation of grade would be 1 foot below that at B. M. (or 89); at 200 feet, 2 feet below, etc.

These differences of elevation may be found by multiplying the tangent of the angle of grade by the horizontal distances from the initial point. The difference between the elevation of grade and surface elevation at any point would be the cut or fill.

CHAPTER XI.

METHODS USED IN "FILLING IN."

The methods and instruments that will most likely be employed in locating points for the control of a map in military topography having been explained, there remains to be considered the "filling in" of the details. For this purpose the same instruments may also be used.

The "filling in" consists principally in representing, by conventional signs and symbols, the undulations and accidents of ground, steepness of slopes, relative heights of commanding points, and all occurring incidents of water. A sufficient number of points for this purpose are located by their three coördinates—viz., direction, distance, and elevation. The representation of details between the determined points completes the survey, and draws upon the artistic skill, perception, and judgment of the topographer.

The method in general use for representing the configuration of the surface of ground is by "contours."

CONTOURS are the horizontal projections to scale, on the map, of the intersections (contour lines) of the surface of the ground by imaginary horizontal planes.*

These imaginary planes are taken at equal vertical intervals, as 1, 5, 10, 25, 50, or 100 feet.

Contours show not only relative heights of points, but also the shape of the ground by their curves, and the steepness of slopes by their distance apart.

*In the survey of large areas, where the curvature of the earth is considered, the imaginary lines projected are the intersections of the ground by imaginary level surfaces.

The usual method of explaining this is to assume a hill surrounded by a body of water, and to suppose the water to rise one foot (or the vertical interval chosen) at a time, until the hill is covered. The contour line defined by the edge of the water at each successive rise may be considered as the intersection of the surface of the ground by the imaginary plane, and the one to be represented by the contour on the map.

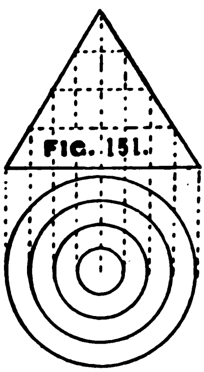


FIG. 151.

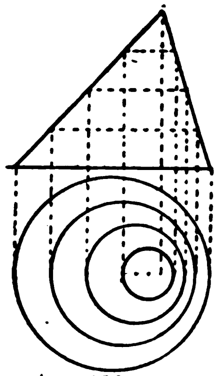
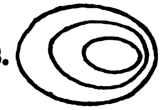


FIG. 152.

FIG. 153.



If it were desired to represent a *right cone* upon a horizontal surface (Fig. 151), the base would be shown by the largest or outer circle and the apex by the central dot. If the cone be cut by equidistant horizantal planes, the lines of intersection with the surface are shown upon the plane by the inner circles.

If the *cone is oblique* (Fig. 152), it will still be represented by circles, but they will not be concentric, but nearest to each other on the steepest side.

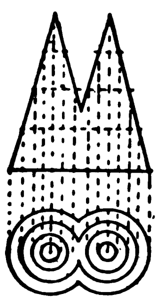


FIG. 154.

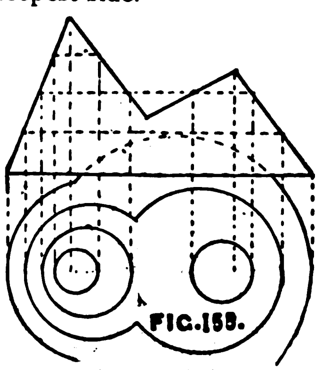


FIG. 155.

If the figure were half egg-shaped, then it would be represented something after the manner in Figure 153.

If two intersecting cones of the same size (Fig. 154), the bases will be shown by the two outer circles cutting each other, and the upper parts by the separate concentric circles. If the two cones are oblique and of different sizes, they will be represented as shown in Figure 155.

By a reference to the figures, which contain all the elements of contours, it will be seen that since the sections are regular the horizontal projections also are regular, but when a figure has an irregular surface like the ground, and is represented by this method, the lines of intersection will also be very irregular.

DEFINITIONS OF FORMS.—*Ridges or water-sheds*, exemplified by the ridges of roofs, are the highest lines of land separating lower land. In the case of ridges separating large drainage areas, there are usually smaller ridges projecting out, but connected with the main ridges.

Water-courses or thalwegs, exemplified by the valleys of roofs, are the lowest lines or parts of valleys or ravines, whether occupied by water or not. If water be present, the indication is unmistakable as regards the lowest line, and inferences may be drawn as regards the direction of surrounding slopes.

Varieties of ground. The infinite variety of forms and

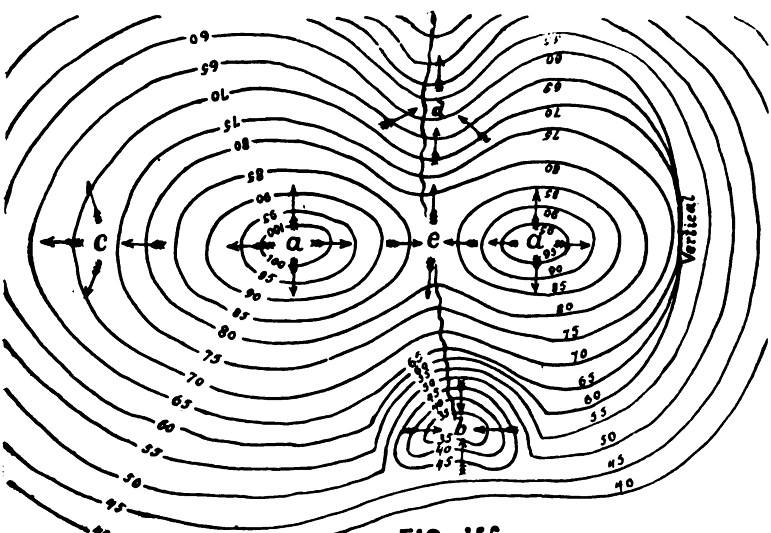


FIG. 156.

shapes of ground that may be found can be reduced to some half a dozen classes.

Thus, if one were standing at *a* (Fig. 156*), with the ground *falling from him in all directions*, it would be a peak, cone, hill, knoll, knob, or mound.

If at *b*, with the ground *rising from him in all directions*, it would be a crater, hollow, basin, pit, or hole.

If at *c*, with the ground *sloping from him on three sides and towards him on one*, it would be the end of a ridge, promontory shoulder, spur, or nose.

If at *d*, with the ground *rising from him on three sides and falling on the fourth*, it would be a valley, ravine, gorge, or cañon.

If at *e*, with the ground *falling from him on two opposite sides and towards him on the other two sides*, it would be a col, gap, or saddle.

If the ground is *level in all directions* from one, it would be a plateau, table-land, plain, or steppe.

Ground is said to be *undulating* or *rolling* when consisting of alternate gentle elevations and depressions.

Underfeatures are the minor features which spring from main features.

An examination of a correct contoured map will show:

1. That all points in a contour line have the same elevation above the datum-plane.

2. That where the contours are equally spaced the slope is uniform.

3. That when the contours are straight and equally spaced the ground is a sloping plane.

4. That the contours of a vertical surface lie on top of one another, as in palisades.

5. That if the lean, in rocky formations, is over the base, then only can contours cross.

6. That every contour closes upon itself or extends entirely across the map.

*The arrows show the direction of the flow of water.

7. That a contour coming to the bank of a stream turns up and crosses the bed under the water instead of crossing where it strikes the bank.

8. That contours cut at right angles all lines of greatest slope, and all ridge and valley lines.

9. That maximum ridge and minimum valley contours go in pairs—*i. e.*, that no single lower contour can lie between two higher ones, and *vice versa*.

10. That on water-sheds the contours are convex toward the foot or base of the slopes.

11. That in water-courses the contours are convex toward the sources of the streams.

Contours are designated by their heights above the datum-plane, which should always be written upon them to remove any doubt as to which are hills and which are hollows, which are ridges and which are valleys, etc.

There are two general methods of determining contour lines: *one* by determining them on the ground at once; the *other* by selecting points at the tops and bottoms of hills, and where the ground changes its slopes, on lines cutting the contours about at right angles, and then deducing the contours by proportion or interpolation.

By the *first method*, which is the more accurate, the height of one line is decided upon and that line run with a level of some kind, then the next higher or lower line is fixed and run in the same manner, and so on, thus locating points upon the different contour lines. The points, thus located, must be surveyed in some manner to enable one to plot the contours, This may be done with the compass and chain, plane table. or transit and stadia. In this mothod care should be taken to locate the contour points so as to facilitate the work and gain the greatest amount of information concerning the ground. They should be located on the ridge and valley lines, thus giving the points of maximum and opposite curvature of the contours.

If the ground to be contoured is a long narrow strip, section lines at intervals may be run across it on the greatest slope, and points located on these section lines at the heights of the contours. Points on the contour lines are then located between the section lines. If the ground is a broad area, the points on the boundary at the heights of the contour lines may be located and then section lines run across, and intermediate points located on them.

By the *second method*, profile lines may be run across in directions furnishing the greatest information, from which the points on the profile lines at the height of the contour lines may be determined and the contours plotted.

Or, the area may be divided into a number of equal squares, the heights of the corners of the squares determined, and the contours located from these elevations. In case of a single hill feature, lines may be run in various directions from a point on top, their azimuths or bearings observed, and on these lines the heights and distances of the critical points from the top determined, and the contours then deduced. The important points to be located in contouring are at the tops and bottoms of slopes, where a slope changes to a steeper slope, on water-shed and water-course lines, on cols and divides. The heights and locations of points having been determined, as above, the contour point or points of a desired intermediate height are found either by proportion or graphically.

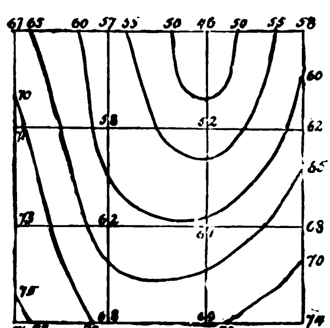


FIG. 157.

By proportion. In Fig. 157 suppose the contours are to be at 5 feet vertical interval, and the squares 50 feet on a side. Beginning with the upper right-hand square, top line, to find the points where the 50- and 55-foot contours cross between the elevations of 46 and 58 feet; a difference of 12 feet elevation in 50 feet horizontal is found. The 50-foot contour point will be 4 feet above the 46-foot point, hence the proportion, 12 feet : 4 feet : : 50 feet : x=16⅔ feet; the 55-foot contour point will be found by 12 feet : 9 feet : : 50 feet : x=37.5 feet. From the 46-foot point towards the 58-foot point measure off 16⅔ feet for the position of the 50-foot contour, and 37.5 feet for the 55-foot contour. In the upper left-hand square, top line, there is a rise from 57 feet to 67 feet in 50 feet horizontal, hence the positions of points 60 feet and 65 feet above datum are found by the respective proportions, 10 feet : 3 feet : : 50 feet : x= 15 feet; 10 feet : 8 feet : : 50 feet : x=40 feet, from the 57-foot point. In the same manner the positions of the same points may be found from the higher points, as from the 58-foot and 67-foot points; thus, 12 feet : 8 feet : : 50 feet : x=33⅓ feet, and 12 feet : 3 feet : : 50 feet : x=12.5 feet, the distances of 50-foot and 55-foot points, respectively, from 58-foot point; and 10 feet : 7 feet : : 50 feet : x=35 feet, and 10 feet : 2 feet : : 50 feet : x=10 feet, the distances of 60-foot and 65-foot points, respectively, from the 67-foot point. In like manner the points where the contours cross all of the other sides of the squares are determined.

Graphically (Fig. 158). On a sheet of profile paper, plot the elevations of the determined points on each line and join them by straight lines, then measure the horizontal distances from these points to where the line connecting them crosses the lines representing the different contour planes; these will locate the positions of the contour points.

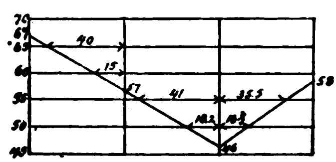


FIG. 158.

SKETCHING CONTOURS.—In order to plot contours truthfully, from the observations made, one must have a thorough knowledge of the ground as a help, which is best obtained by *sketches* of it made at the time the observations were taken. In making the sketch, first indicate the position of the ridge and valley lines, observe the curve of the contours at the tops of valleys and bottoms of slopes, sketching them in, in their true relative positions and distances, making the sketch as complete as possible in the field.

A scale of horizontal equivalents, or table corresponding to it, is necessary.

By *horizontal equivalent* is to be understood the horizontal distance on which a given rise or fall will occur at a given degree of slope. It is the horizontal base of a right-angled triangle of which the hypothenuse is the slope and the vertical side the rise or fall. Thus a rise of one foot on a 1° slope will occur in 57.3 feet horizontal*; a rise of two feet on a 1° slope will occur in 2 times 57.3 feet=114.6 feet; a rise of 10 feet on a 1° slope will occur in 10 times 57.3 feet=573 feet. As the slope becomes steeper it is evident that a given rise will occur in a correspondingly shorter horizontal distance; thus, a rise of 1 foot on a 2° slope will occur in $\frac{1}{2}$ of 57.3 feet =28.65 feet; a rise of 1 foot on a 3° slope will occur in $\frac{1}{3}$ of 57.3 feet=19.1 feet. If, instead of feet, any other unit of measure be taken, the same conditions will hold true; thus a rise of 1 yard, meter, pace, etc., on a 1° slope will occur in 57.3 yards, meters, paces, etc., horizontal, and on a 2° slope in $\frac{1}{2}$ of 57.3 yards, meters, paces, etc. The horizontal equivalent for any rise or fall on any degree of slope is given by the equation $H\,E=\frac{VI \times 57.3}{S}$, in which VI is the vertical interval (when speaking of contours, rise or fall) and S the slope in degrees.

The horizontal equivalents for a constant vertical interval for all different slopes, when tabulated, constitute a *table of horizontal equivalents;* thus the horizontal equivalent of

15 feet at 1°=859.5 feet, 15 feet at 2°=429.75 feet,
15 feet at 3°=286.5 feet, 15 feet at 4°=214.875 feet,
15 feet at 5°=171.9 feet, etc.

If these tabulated horizontal equivalents be laid off to the scale of the map, in succession, on a right line, there re-

*The natural cotangent of 1° being 57.29. While the above are only approximations, they are very close up to 20° or 25°. To obtain the H. E. more nearly exact, substract from the result, as obtained above, the degree multiplied by 0.006. This is exact to within less than 0.005 of a foot up to 50°.

sults a *scale of horizontal equivalents* from which may be taken at once the distance apart that contours should be drawn for any slope. Thus at 6 inches to 1 mile it would be.

1°	2°	3°	4°	5°

Such a scale may also be made by drawing, from a point on a horizontal line, radiating lines making different angles with it from 1° to 90°. Parallel to the horizontal line and distant from it the vertical interval between contours, to the scale of the map, draw another line intersecting the radiating ones. The distances on the parallel line, from the points of intersection with the radiating lines to the vertical, are the horizontal equivalents.

If a scale of horizontal equivalents be applied to a contoured map, the slope of the ground at any place can be determined in degrees.

As an *approximation* in hasty sketching, it is usual to use the number 60 instead of 57.3, calling the horizontal equivalent of 1 foot at 1°=60 feet, 1 foot on 2°=30 feet, etc.

Slopes are also expressed, in the form of fractions called *gradients*, by the ratio of the rise or fall to the horizontal; thus 1° slope $=\frac{1}{57.3}$, or roughly $\frac{1}{60}$; 2° slope $=\frac{1}{28.6}$, or $\frac{1}{30}$; 3° slope $=\frac{1}{19.1}$, or $\frac{1}{20}$, etc.

By means of the equation $HE=\frac{VI\times 57.3}{S}$, many problems relating to heights, distances, and slopes are solved by determining any two of the unknown quantities, substituting, and solving with reference to the third; thus, $S=\frac{VI\times 57.3}{HE}$ and $VI=\frac{HE\times S}{57.3}$.

RELATION BETWEEN VERTICAL INTERVAL AND SCALE OF MAP.—The same length of line on paper represents different distances on the ground on different scales; hence the same map distance between contours on different scales represents different slopes for the same vertical interval. Thus if a map distance of 1.3 inches between contours, with a vertical interval of 20 feet, represents a slope of 1° on a scale of 6 inches to a mile, it will represent a slope of $\frac{1}{2}$° on a scale of 3 inches to a mile, and a slope of 2° on a scale of 12 inches to a mile. *Hence, for the same vertical interval and the same map distance* between contours on different scales, the slopes vary directly as the scales. This fact may be made use of in sketching, as follows: if one already has a scale of horizontal equivalents constructed for 20 feet vertical interval on a scale of 6 inches to 1 mile and wishes to use it for the same vertical interval at 12 inches to 1 mile, he can divide by 2 the slopes as he reads them with his instrument, and take from the scale the horizontal equivalent corresponding to the quotient. If he wishes to use it at 3 inches to 1 mile, he multiplies by 2 the slope he reads and takes from his scale the horizontal equivalent corresponding to the product.

But the same map distance between contours on different scales may be made to represent the same degree of slope by varying the vertical intervals. Thus, the same map distance 1.3 inches on the three different scales may represent a 1° slope by making the vertical intervals 20 feet, 10 feet, and 40 feet, respectively. Hence, *for the same map distance between contours representing the same slope on different scales the vertical intervals vary inversely as the scales.* Use is made of this latter fact in some countries to have ordinarily only one scale of horizontal equivalents, called the *normal scale*, varying the vertical intervals between the contours to correspond to the scale.* If the scale of map is larger than the normal, the vertical interval is reduced; if smaller, it is increased, and the equivalent taken off the normal scale.

MEASURING DISTANCES WITH THE GRADIENTER.—If the value of the screw-thread is made such that a complete revolution of the screw will move the horizontal cross-wire of the telescope over a space of 1 foot on a rod at a distance of 100 feet from the center of the instrument, then when the screw is turned through 100 spaces on the graduated head, the wire

*The object of this is to educate the eye in reading and representing slopes quickly on different scales by fixing in the mind a particular horizontal equivalent for each degree of slope regardless of the scale of the map.

will pass over $\frac{50}{100}$ of a foot on the rod held at a distance of 50 feet and so on in the same proportion. This fact is made use of to measure distances.

On ground which is practically level, bring the line of collimation horizontal, then after one complete revolution of the screw, observe the distance, as 5.22 feet, passed over by the horizontal wire on the rod held at any point, which, multiplied by 100, gives the distance to the rod, as 522 feet. (Fig. 159.)

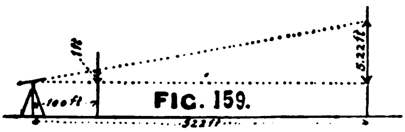


FIG. 159.

If the rod should happen to be an ordinary range pole with only whole foot divisions, then the method would be to turn the screw until the horizontal wire had passed over a number of exact feet on the rod, say 6, then read the number of complete revolutions on the scale, and hundredths on the ring, taken to do it, say 1.15—*i. e.*, 1 division on the scale and 15 divisions on the ring, then from the proportion, $1.15:100::6:x$, $x=522$—.

The rule: *Multiply the distance passed over by the horizontal wire on the rod by 100 and divide the product by the number of revolutions of the screw.* Could the rod, on slopes, be held exactly perpendicular to the first position of the line of sight, this same method could be used for determining distances on slopes, but this being impracticable, and as horizontal distances are generally desired, the rod is held vertical in all cases and a correction applied, since in such cases the distances obtained are too great, due to the two causes named. These corrections are the same as applied to stadia measurements under the same circumstances, and will be described under that head.

With the gradienter, grades may also be established with great facility, as in road work, by first leveling the instrument, bringing the telescope bubble to the center, and moving the graduated head (which is held by friction only) until its zero is brought to the edge of the scale, then turning off as many spaces on the head as there are hundredths of feet to the hundred in the grade to be established.

THEORY OF THE STADIA.—The theory of stadia measurements is based upon the corollary in geometry, that if any number of parallel straight lines be drawn cutting any number of other straight lines, they will divide the lines proportionally.

Thus in Figure 160, let GK, DF, AB, and A′B parallel lines be drawn cutting the lines MN, ST, and PR, then will $HO : GK :: EO : DF :: CO : AB$. If $C'O = CO$, then $A'B' = AB$; and in the similar triangles, $HO : GK :: EO : DF :: C'O : A'B'$.

If light be introduced into a dark chamber, box, or tube, through a very small hole, and this light falls upon a white screen therein, then an image of whatever reflects this light will be formed upon the screen in true relative perspective dimensions, no matter what may be the distances of the objects and screen from the small hole. To apply this fact, let a cap of tinfoil be placed over the objective end of a telescope, from which the lens has been removed. Make a small pin-hole through the center of the tinfoil cap. Then hold any object (as a stadia rod) at any distance in front of the telescope. The two stadia wires in the telescope will now include a certain portion of the image of the rod between them. The portion of the rod corresponding to this included portion of the image is called *the intercept on the rod*. And the distance of the rod from the pin hole will be found to be the same number of times the intercept on the rod as the distance of the wires from the pin-hole is times the distance between the wires.

In Figure 160, let O be the pin-hole, A′ and B′ the two

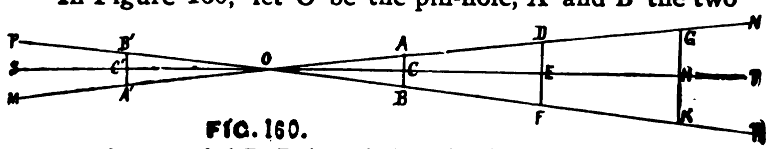


FIG. 160.

stadia wires, and AB, DF, and GK the intercepts on the rod when held at C, E, and H, respectively. It will be seen that A, D, and G are all on the line MN joining the wire A′ and the pin-hole O. And that B, F, and K are on the line PR joining the wire B′ and the pin-hole O, for the reason previously stated. Consequently CO, EO, and HO will be as many times AB, DF, and GK, respectively, as C′O is times A′B′. But C′O divided by A′B′ is constant for all the different positions of the rod, therefore the distance of the rod from the pin-hole is always that constant $\left(\frac{C'O}{A'B'}\right)$ number of times the intercept on the rod, no matter where the rod is held.

Now remove the tinfoil cap from the telescope and replace the objective lens. Focus the telescope upon an object (as a star) from which the rays are practically parallel (theoretically on an object an infinite distance away). In Fig. 161 let S

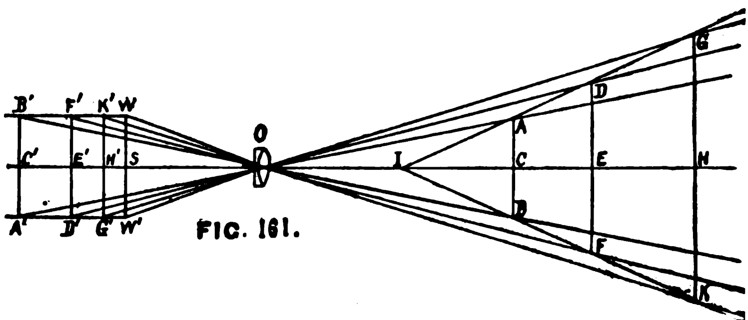


FIG. 161.

be the position of the image of the star. The distance SO of such image from the center of the lens is called the *principal focal length* of the lens and is a determinable constant for the lens. Let WW′ be the distance apart of the fixed stadia wires =K′G′=F′D′=B′A′.

Now focus the telescope on the rod held successively at H, E, and C. From Optics it is learned that the images will be situated at H′, E′, and C′, respectively. And the intercepts on the rod will be GK, DF, and AB, respectively.

For any position of the rod (as at H) there will be two similar triangles, GOK and G′OK′, one on each side of the lens, with their vertices at the center O of the lens. For all other positions of the rod there will be different sets of similar triangles. Between these sets of similar triangles there is no constant relation that can be used, because the quotients resulting from dividing the distances of the wires from the lens by the distance between the wires are different for every position of the rod. This will be understood by an examination of Fig. 161, where it will be seen that neither A, D, nor G is on the line passing through either of the others and the center of the lens. The same will be seen of B, F, and K. But by construction (and it can be proved mathematically) it can be shown, however, that a straight line can be drawn through A, D, and G, and one through B, F, and K, and these lines will intersect at I. Therefore CI : AB :: EI : DF :: HI : GK, from which we see that the distance of the rod from the point I is a constant number of times the intercept on the rod. Now it only remains to determine exactly where this point I is situated, and what this constant multiplier of the intercept on the rod is. The former has been found to be at a distance in front of the center of the lens equal to its principal focal length. By drawing lines through the center of the lens, parallel to the lines which intersect at I, it will be found that they pass through the wires when the lens is focused on an object an infinite distance away—that is, when the wires are at a distance from the lens equal to its principal focal length. The angles AIB and WOW′ are equal by construction and their opposite sides parallel, while the lines AB, DF, GK, and WW′ are parallels, cutting the sides of the angles; hence HI : GK :: EI : DF : CI : AB :: SO : WW′, the same relations that were obtained with the pin-hole. As SO and WW′ are both constant, then $\frac{SO}{WW'}$ is constant and is the multiplier sought of the intercepted space on the rod.*

* In Fig. 161 let SO $=f$, (principal focal length).

$\left.\begin{array}{l} H'O = f_1 \\ HO = f_2 \end{array}\right\}$, (conjugate foci).

GK $=s$, (intercept on stadia rod).

K′G′$=i$, (image of intercept).

From similar triangles we have

$$K'G' : GK :: H'O : HO, \text{ or } i : s :: f_1 : f_2 \quad \ldots \ldots (1).$$

The effects resulting from substituting the lens for the tinfoil cap are to change the positions of the images for different positions of the rod, consequently changing the angles formed between the lines through the wires and center of lens, also to separate the vertices of the similar triangles by a distance equal to the principal focal length of the lens.

TO MEASURE DISTANCES WITH THE STADIA.—In the construction of the Stadia by the makers, the wires are adjusted to include 1 foot on a rod at 100 feet from the center of the instrument and assumed to have a constant ratio for all other distances, but at no other distance will the part intercepted by the wires, and the distance be exactly in the proportion of 1 to 100, although the error of such supposition is so small for distances over 100 feet as to be neglected in some classes of work. In the *gradienter* the two sides of the triangle considered, are formed by the line of collimation on revolving the telescope and intersect in the center of the telescope on the horizontal axis; whereas in the *stadia* the sides of the angles pass through the stadia wires forming different angles for different distances; this results from the optical effects of the objective, hence it is inaccurate to consider the visual angle as constant and the apex as situated in the center of the telescope on the horizontal axis. But, as shown, it has been found that at a point situated at a distance in front of the objective of the telescope equal to its principal focal length, is the apex of the visual

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The law of lenses is

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f} \dots\dots (2).$$

Solving equations 1 and 2 with respect to $\frac{1}{f_1}$ and equating these values of $\frac{1}{f_1}$, we have $\frac{s}{if_2}=\frac{1}{f}-\frac{1}{f_2}$, from which is obtained

$$f_2=\frac{f}{i}s+f \dots\dots (3),$$

which shows that the distance of the rod from the *objective* is equal to the intercept on the rod multiplied by the constant ratio $\frac{f}{i}$ plus the constant f.

Calling the distance from the *center of the instrument* to the stadia rod d, and the distance from the objective to the center of the instrument c, we have $d=f_2+c$, from which we have $d=\frac{f}{i}s+f+c$.

angle which is constant for the spaces intercepted by the stadia wires on a rod held at different distances. Knowing this, it then becomes possible with the stadia to measure distances accurately from this apex, by the application of that principle of geometry used in the gradienter; then knowing the distance of this apex from the center of the instrument, located by the plumb-bob, the distance of any point from the center of the instrument can be accurately obtained.

The location of this apex is found by laying off from the center of the instrument or point of the plumb the sum of two distances called c and f; c being found by measuring the distance from the center of the horizontal axis of the telescope to the objective, when it is focused on a mean distance of expected readings, say 500 feet; and f, the principal focal length of the objective, being found by measuring from the plane of the cross-wires to the objective when focused upon an object at an infinite distance. If now a rod be held on level ground at 100 feet from this apex and the stadia wires be adjusted to intercept 1 foot on the rod, then at whatever other point the rod be held, its distance from this apex will be 100 times the intercepted space on the rod.

Thus with *adjustable stadia wires* it is possible to set them so that the ratio between the intercepted space on the rod and the distance of the rod from the apex may be made any desired number, and a rod of given length be used for any desired maximum distance.

If, however, the wires are cemented to the ring and *not adjustable*, and it is still desired to use a rod already graduated, as a leveling-rod, it will be necessary to determine the distances from the apex corresponding to the intercepted spaces on the rod.

If no rod is at hand, then it will be necessary to graduate one to suit the fixed wire interval.

Reduction to the horizontal. On slopes, with the Stadia as with the Gradienter, if the rod could always be held perpendicular to the line of collimation directed on a point of the rod as high as the instrument, the distance from the instrument to the rod could be obtained at once; but, this being impracticable and horizontal distances being desired, the rod is held vertically and read as on the level, then the necessary correction is applied, which correction depends upon the vertical angle of the line of collimation. These corrections have been calculated and put into a table for use. Table V., Appendix, gives the horizontal distances from the apex and the vertical elevations of points, computed from the proper formulæ for Stadia readings, which would correspond to 100 feet, yards, or meters, for all angles up to 30°. The use of the table is best explained by examples. Suppose the space intercepted on a rod, held on a hill, is just 1 foot. This on level ground would mean that the rod was 100 feet from the vertex, but the inclination of the line of collimation as shown by the vertical arc is 15° 12′. Looking in the table for the column of 15°, and following down until opposite 12′ of the left-hand column, we find the number 93.13, which is the horizontal distance from the apex to the rod. Likewise the difference of elevation is 25.30. If the distance of the apex from the center of the instrument is 1 foot, on level ground, on the slope of 15° 12′ it will be .96 foot, found at the bottom of page, opposite c=1.00 on the left. This must then be added to 93.13, giving 94.09 feet for the horizontal difference from the center of the instrument. Likewise for the distance of elevation there must be added to 25.30 the quantity .27, giving 25.57 feet for the difference of elevation. Had the reading, instead of being 1 foot, representing on the level 100 feet, been 5.79 feet, representing on the level 579 feet, it would have been necessary to multiply 93.13 by 5.79, giving 539.22+feet, to which would be added .96 foot, making 540.18 feet for the horizontal distance from center of instrument; and for the difference of elevation 25.30, multiplied by 5.79=146.487, to which would be added .27, making 146.757.

In the example, the ratio of the space intercepted to the distance of the rod from the apex was assumed as 100, and the reading multiplied by the numbers in the table. Should this ratio not be 100, then, after finding the distance represented on level ground by the reading, remove the decimal point two places to the left and proceed as above.* Except in very accurate work, the table is not generaly used to reduce the inclined to the horizontal distance when the angle of elevation of the line of collimation is less than 3°. If an error of 1 in 100 can be allowed, then the correction is not necessary for 5° 44′ or less. If the correction for the distance from the apex to the center of the instrument be also neglected, these two errors tend to compensate.

In obtaining the difference of elevation, the correction for elevation, due to the distance of the apex from the center of the instrument, may be omitted for angles under 6° if errors of $\frac{1}{10}$ of a foot are not important, as in side shots. But for elevations on the main line, as on at raverse line, this correction should always be included.

REDUCTION DIAGRAM.—Since the use of tables involves a multiplication each time, and since a table of varying distances and angles would be very voluminous, it is preferable to take out the elevations from a diagram. Such a diagram has been prepared, to be used in place of the table. It is arranged with both coördinates in feet, but can be used for both coördinates in meters, since the same unit is used for both. It will only be necessary to re-number the divisions to adapt it to the new scale.

This diagram has been prepared with great care, and is arranged to give distances to 500 yards or meters, or 1,500 feet, with elevations to 50 feet. For longer distances or higher elevations for a single pointing, the results may be obtained from the table. Elevations are taken from the diagram to

*Thus, the intercepted space on a leveling rod might be read 6.2 feet, which would be found from a prepared table to correspond to a distance of 579 feet from apex. Move the decimal point two places to the left in 579, giving 5.79, and multiply as explained.

the nearest tenth of a foot, with great readiness, as the smallest spaces are 2 millimeters square, and these correspond to two-tenths of a foot in elevation. It is of more convenient use than extended tables, and is just as accurate; the nearest tenth of a foot being quite as exact as one is warranted in recording elevations obtained in this manner.

Corrections to the distances read are also obtained from this diagram for large vertical angles.

ACCURACY OF STADIA MEASUREMENTS.—If the rod is properly graduated and always held vertically, the only error possible is that from reading the position of the wires, which is as apt to be too great as too small; hence such errors follow the law of compensating errors, which is that the square root of the total number of errors probably remains uncompensated.

This fact makes accurate linear measurements possible with the Stadia upon traverse lines, and the longer the line the more accurate the final result, since the *relative error* diminishes as the length of the line or number of sights increases; provided, the two lines are run under similar conditions as to length of sights, accuracy of work, etc.

In clear, steady atmosphere, for a given accuracy, much longer sights can be taken than when it is hazy and unsteady, or, for the same lengths of sights, a greater accuracy is obtainable in the former case than in the latter.

With care it is not difficult to reduce the error of closure on lines averaging from 1 to 2 miles in length to 1 in 1000 or 1 in 1200. Thus it is seen that distance measurements with the Stadia are as accurate as with a chain under ordinary circumstances, and much more accurate as well as more rapid than with a chain over rough, broken ground.

MAKING STADIA RODS.—Ordinary leveling rods with two targets may be used, but where much work is to be done it is preferable to make self-reading or speaking rods by which the observer reads the space intercepted by the stadia wires.

The varieties and shapes of the characters on the rods used are almost as numerous as those who use them; some few, however, having been found better than others, will be shown.

It has been found that it is better to keep the colors massed as much as possible, to prevent confusion and consequent difficulty in reading in hazy and unsteady atmosphere; large characters with points to indicate smaller divisions, being better than small detached ones. Wherever the wire falls there should be a white background on some part of the cross-section. The characters should not be carried to the edge of the board as there is danger of their becoming defaced.

The *graduation* should begin at the middle and extend the same distance towards each end, so that either end of the board can be placed on the ground, thus enabling the observer to find the height of his instrument on the rod, by a comparison with a similarly graduated staff he carries.

If it is intended to always have the same end of the rod on the ground, then the observed intercepted space may be at once divided, having previously put a fixed target at the top.

The wires should be exactly the same distance from the center wire. If it is desired to read greater distances than the length of the rod will permit, then use the middle and one extreme wire and double the distance read on the rod.

The stadia rod should be of well-seasoned, clear pine, 1 inch thick, 5 inches wide, and 14 feet long, The ends should be protected by iron shoes. Before marking, the board should receive several coats of white paint to make them thoroughly white. There should be no flange on the board to cast a shadow.

Having selected a place where a base-line can be measured with a tape or chain (which has been compared with a standard) over smooth, level ground, choose a time when the air is clear and steady. Set up the transit over one end of the base-line and level carefully. Lay off the distance $(c+f)$ in front of the plumb-bob to locate the apex of the visual angle and from this point lay off a base 600 feet long. Have one of two targets (which may be of cardboard with a black diamond painted on it and having a narrow horizontal slit along the center line through which to mark its position) fastened on the board about 6 feet from the top. Have the board held vertically on the stake, sight the *top* wire upon the *fixed* target and have an assistant set the *other target* where the *lower* wire cuts the board. Repeat the operations until satisfied of the correctness of the results. If they differ but by a small amount, take their mean. This gives the value of the intercepted space at 600 feet, and the spaces for all other distances will be proportional.

Measure the length of the board and find the middle. From this point lay off *toward each end* the intercepted space just found. If it is desired to read distances in feet, divide the intercepted space found into 6 equal parts corresponding

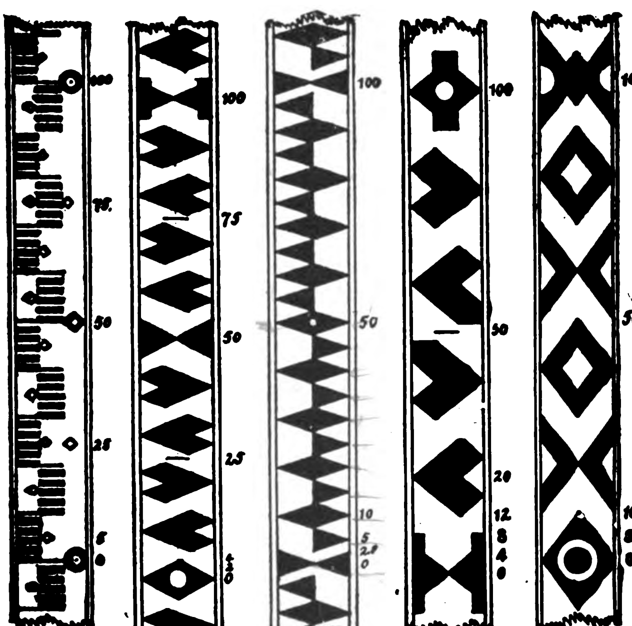

FIG. 163. FIG. 164. FIG. 165. FIG. 166. FIG. 167.

to 100 feet each, and each of these parts into other parts corresponding to the minimum number of feet to be read. If distances are to be read in yards, divide the intercepted space into two equal parts corresponding to 100 yards each, then each of these into parts corresponding to the minimum number of yards to be read. Continue these divisions to each end of the board. Having done this, construct the desired characters in pencil and afterwards paint them a coal black.

Figure 163 may be used where it is desired to read to single units of measure, either feet, yards, or meters. Figure 164 to two units. Figure 165 to two and one half units. Figure 166 to four units. Figure 167 to five units of measure. Some engineers insert a narrow strip of mirror on the 100 unit characters to facilitate reading.

On the government surveys the base is usually measured from the center of the instrument and its length is taken as about a mean of those which the stadia is intended to read; the result of which is to give all distances shorter than the base, too short, and all distances longer than the base, too long.*

Another method is to measure the base from the center of the instrument, as above, and then from a fixed point on the lower part of the rod, find the intervals that correspond to various distances, as 100 feet, 200 feet; 300 feet, etc., and mark these on the board, always keeping the lower wire on the fixed initial

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*Fig. 162, A being the center of the instrument, I the initial

FIG. 162.

point, AG the mean base, say 1,000 feet. If the rod so graduated be held at R, the space intercepted on rod is Ra, which corresponds to the position P=100 feet, a distance too short by K=PR=1.35 feet. If the rod be held at R', the space intercepted on the rod is R'c, which corresponds to the position P'=2,000 feet, too long by K=R'P=1.5 feet; the formula for the correction being $K=(c+f)\left(1-\frac{AP \text{ or } AP'}{AG}\right)$. The rod is graduated on the line AB, while the spaces intercepted are on the line IF.

point of the rod. Then each 100-foot space is subdivided according to the desired reading. If the lower wire is always set on the initial point, the reading gives the correct distance from the center of the instrument, except for distances less than 100 feet.

In topographic work there should usually be two rods exactly alike, to each instrument, and sometimes three and four are needed. After one rod is subdivided, the others of that set may be laid alongside and all fastened rigidly together; then by means of a T-square the remaining rods may be marked off.

The wire interval should be tested every few months by measuring a base as was done on graduation, and reading the rod on it, to see if it shows the true measured distance.

Points over which the instrument is set and from which distances are measured with the stadia are usually, in the notes and on plots, surrounded by a square, thus ⊡, and called stadia stations.

FIELD WORK.—*Small areas.* If the area to be surveyed is *small,* any point may be selected as the point of reference, and the survey referred to it as an origin. The transit is set up over the point, vernier A set at 0, and clamped. If the true meridian through the point is known, the telescope is placed in the meridian by the lower motion and clamped. The rodmen with the stadia rods then hold them on the critical points, the plates are unclamped, and the telescope turned in succession upon the rods; the distance, vertical angle, and azimuth are read and recorded each time.

In sighting, the horizontal wire of the telescope should be directed upon the rod at the height of the horizontal axis above the point over which the transit is set. This is determined by placing a visible mark, as a rubber band, or target, on the rods after the transit is leveled and before the rodmen start out, or the observer may have a staff 5 or 6 feet long, graduated the same as the rods, with which he determines the height of his instrument and on what point of the rod to sight. The latter method permits the rodmen to be examining the ground for points on which to hold the rod while the observer is setting up the transit. If the instrument occupies a central position, and distances can be read to all points on the boundary, a complete circuit is made around to the point of beginning, furnishing data for representing the surface.

Instead of sighting on critical points situated indiscriminately, section lines may be selected, especially if the ground is fairly uniform, and the critical points on these section lines observed.

The record may be kept on the left-hand pages as follows, the right-hand pages being reserved for sketching the locality, etc.

OBSERVER, W. V. SMITH.
RECORDER, C. P. JONES.

Date, Nov. 8, 1892.

At Central Point. Ht. of Inst. 480. Surf. Elev. 55.89.

OBJECT.	AZIMUTH. VER. A.	DISTANCE.	VERTICAL ANGLE.	DIFF. OF ELEVATION.	ELEVAT'N ABOVE DATUM.
B. M.	0°00′	225	—1°30′	—5.89	50.00
C. P.	30°00′	70	—0°50′	—1.0	54.9
C. P.	30°00′	223	+0°50′	+3.2	59.1
S. W. Cor.	43°35′	396	+2°35′	+17.82	73.71
C. P.	43°35′	250	+2°10′	+9.4	65.3
C. P.	75°00′	215	+3°30′	+13.1	69.0
N. W. Cor.	102°56′	370	+4°30′	+28.93	84.82
C. P.	135°00′	153	+2°12′	+5.8	61.7
C. P.	200°00′	220	+2°35′	+9.9	65.8
N. E. Cor.	221°52′	314	+3°35′	+19.69	75.58
C. P.	221°52′	155	+2°55′	+7.8	63.7
C. P.	258°00′	220	+3°00′	+11.5	67.4
S. E. Cor.	292°48′	390	+2°30′	+17.00	72.89
C. P.	292°48′	230	+1°15′	+5.0	60.9
C. P.	321°00′	150	—1°10′	—3.0	52.9

A description of the point over which the transit is set is first recorded as Central Point, △7, ⊡10, etc.; then the height of the instrument as given by the stadia rods, 480; then the elevation of the point above datum, if known, or its elevation as soon as determined from some near B. M. In the case above, the Central Point is on the true meridian through the B. M. on which the limb is oriented. Vernier A is at 0°, the distance read on the rod is 225 feet. The vertical angle—1° 30′ indicates that the B. M. is lower than the Central Point over which the instrument is set; this is found by the table to be 5.89 feet, which added to the height of the B. M., 50 feet, gives 55.89 feet for the elevation of Central Point, which is recorded in the heading. In the "Object" column is recorded a description of the points observed, as Bench Mark (B. M.), Contour Point (C. P.), intersection of roads, corners of field, triangulation station No. 7 (△7), stadia station No. 10 (⊡10), etc. Only the first four columns are used in the field, the last two being "filled in" in the reductions from the table or reduction-sheet.

Large areas. If the area is too large to be surveyed from a single point within, another instrument station is selected, a rodman holds his rod edgewise on it, and its azimuth is accurately determined by causing the vertical wire to bisect the edge of the rod. The rod is then turned, face to the instrument, and the distance and vertical angles read. The transit is then taken to the point selected for it, a rod is held on the station just left, on which the transit is oriented by the back azimuth or by plunging the telescope. The distance and vertical angle are again read from here for a check. The distance should be the same, and the vertical angle the same with the contrary sign. If the vertical angles are not the same, the mean will be the true angle. Other points are observed upon from this new station, as described, and so on until the entire area to be mapped has been surveyed. The positions of other important points or objects are determined in the same manner.

Tracing contours. To trace the actual contour lines on the ground, a point on one of the lines is fixed and the stadia rod held upon it.

The transit is set up at a point, about 3.5 feet lower, from which an extended view of the ground can be obtained. The height of the transit, after it is leveled and the telescope bubble is in the middle of the tube, must be such that the horizontal wire, when directed on the stadia rod held on the point, will cut the rod about a foot from the ground.

The limb of the transit must then be oriented, either in the true meridian, if known, through the point, or by the compass. The azimuth and distance of the contour point are then determined, and a mark of some kind put on the rod showing where the horizontal wire cuts it, the telescope bubble being in the middle of its tube. The rodman then proceeds around on the contour line selecting the critical points, and moving up or down hill, as directed by the observer, until the mark on the rod is cut by the horizontal wire of the telescope. The observer then reads the azimuth and distance of the point, and records it. As many other points on the same contour line, within the limits of good distance reading, are similarly located as may be necessary to plot it. The mark or target is then moved up on the rod the vertical distance apart of the contour planes, and points located in the same manner on the next lower contour. If the rod is 14 feet long and the contour planes 5 feet apart, three contour lines may be surveyed at the first setting of the instrument, one above and two below, and two at each subsequent setting, both below.

Having located the contour line as far below the transit as the length of the rod will permit, another transit position is selected, about 3.5 feet below the last contour line, and its azimuth and distance read. The transit is then moved to this point, set up and leveled, high enough to have the horizontal wire cut the rod, held on the last contour line, about a foot above ground. It is then oriented on its last position by its back azimuth or by plunging the telescope. The telescope bubble being in the middle of its tube, the point on the rod cut by the horizontal wire when the rod is held on the last contour line is noted, and the target moved up from this point the vertical distance apart of the contour planes and then the contour line next below the transit located as before, and so on.

By leaving marking stakes at the extreme points at each contour line, these lines may be made continuous, and extended for any distance and over any area, by beginning the continuous locations from these points; one instrument position for the beginning being located from some setting on the previous traverse, so as to make a connected whole.

Filling in triangulated survey. If triangulation points (primary and secondary) have been established and their elevations determined, as has been explained, the details are "filled in" on traverse lines, starting from and closing, if possible, on triangulation points.

The transit is set up over one of the △'s, the azimuth to one of the other △'s set off on the horizontal limb with vernier A, and the alidade clamped. The telescope is then directed upon that △ by the lower motion and clamped, thus *orienting* the limb in the true meridian. The azimuths of all pointings are now given by vernier A.

The height of the instrument above the △ is now measured on the stadia rod, and recorded in the notes. The rodmen go around holding their stadia rods on the critical points for contours, and on all other points desired for plotting. The form for the record is the same as already given. Before the transit is moved from the △ it is reoriented and releveled, as both may have been disturbed while taking side shots. A rodman selects a suitable place for the next station, and drives a stake into the ground, giving it a proper number, etc., for record, and drives a taller marking stake near it. The rod is held on the stake, edge towards the instrument for azimuth; the needle should also be read for a check as explained in traversing with transit. The rod is then turned face to the instrument while the distance and elevation are read. The transit is then moved to the new station, set up and leveled, and new height of instrument obtained and recorded. It is then oriented on the station just left by sighting on the edge of the rod held on it. The needle is again read for a check. The distance and elevation are again read and recorded. Observations for contour points and other details are then made and recorded, new stations selected, occupied, and the work continued until a △ is reached. A rod is held on this, exactly as has been explained for other points, and its azimuth, distance, and elevation read and recorded. The transit is then moved to it, set up, leveled, and height obtained. It is oriented on the back station, distance and vertical angle read for a check, then the telescope is turned on some other △ and the reading of vernier A, resulting from the traversing, compared with the known azimuth of that △. The elevations are checked by comparing the computed elevation of the △ with its elevation as determined with the level. The distances are checked by plotting them on the field sheet and seeing if they close all right, or by computing the latitudes and departures of the triangulation line and the traverse line separately and comparing the result.

If there is a difference in the reading of the fore and back distance and verticle angle between any two stations, in the heading of the forward station is recorded the mean difference of elevation as determined from these readings, and the elevation of the station is determined by using this mean difference.

If it is desired to start from any point near a traverse line and run a side line, a peg is driven there and connected with the traverse line as if it were to be immediately occupied, so as to make it unnecessary to occupy any station twice.

REDUCING NOTES.—The notes taken in the field during the day should be reduced and plotted in the evening. The difference of elevation column is filled in from the diagram or by using the table, and the elevation above datum column by adding algebraically the difference of elevation to the elevation of the instrument station. Elevations of contour points need be taken only to tenths of a foot, but elevations of station points should be obtained to hundredths of a foot. Where it is necessary to reduce the inclined distance read to the horizontal, as on ⊡5, it may be done by the table or diagram. Here the distance read was 548 feet, the horizontal being only 536.5 feet. As it was desired to subsequently start from ⊡5 to run a line, it was located while at ⊡3.

FORM OF RECORD.

OBSERVER, W. W. SMITH,
RECORDER, C. P. JONES.

Date, November 8, 1893.
At Central Point. Ht. of Inst. 480. Elev. 82.85.

Object.	Azimuth Ver. A.	Distance.	Vertical Angle.	Diff. of Elev.	Elev. above Datum.
At ⊡ 2 Hgt. of Inst. 460. Mean Diff. of Elev.—17.45. Elev. 65.4.					
⊡ 1	43°50′	428	+2°24′	+17.89	
C. P.	58°20′	327	—1°16′	—7.2	58.2
Cor. Fence	93°38′	564	+8°22′	+81.21	146.61
⊡ 3	165°29′	624	—4°18′	+34.76	
At ⊡ 3 Hgt. of Inst. 490. Mean Diff. of Elev. +33.66. Elev. 99.06.					
⊡ 2	345°29′	626	—3°10′	—32.55	
C. P.	217°42′	376	+2°46′	+18.1	117.16
⊡ 5	193°27′	548	+8°22′	+77.18	
		536.5			
⊡ 4	87°37′	644	—4°18′	—48.17	
At ⊡ 4 Hgt. of Inst. 480. Mean Diff. of Elev. —48.29. Elev. 50.71.					
⊡ 3	267°37′	643	+4°20′	48.42	

PLOTTING NOTES.—The stadia traverse is plotted exactly as has already been described.

Having plotted this and made it check, the next step is to plot the *side readings*. For this purpose semi-circular or whole-circle protractors may be used, the center being placed over the stations, oriented, and held in place by weights or otherwise. The azimuths of the side shots are then marked around the edge of the protractor. If a semi-circular protractor is used, the azimuths between 0° and 180° are first marked, then the protractor turned over and those between 180° and 360° marked. Through the points marked and the station, lines are drawn, and on them the distances laid off to scale. Where *many* side shots are to be plotted, more rapid methods are used, such as dividing the map sheet into small squares of 1000 or 5000 units on a side, *like cross-section paper*, then using an open whole-circle protractor, which can be centered over the station and properly oriented. The opening being greater than the longest side shot, a ruler, with a scale on its edge, has its 0 pivoted on the station and is revolved around to the different azimuths, and the distances laid off at once without drawing any lines.

Another method is by using a *semi-circular protractor* numbered in the reverse direction to the graduations of the horizontal limb. The diameter edge is graduated to the scale of the map and numbered each way from the center. The center is pivoted over the station. With this protractor, the direction of any point is given by the diameter when the azimuth number on the protractor coincides with the meridian through the station. The distance is laid off on the scale on the diameter. To orient the protractor at any forward station, the center is placed on the station, the diameter on the line joining the back station, and a dot marked at the azimuth reading will be a point on the meridian from which all azimuths at that station are measured.

Details. The side shots having been plotted, the details which they are to locate and the contours are then completed. If there is any doubt about these, the map should be taken to the ground and the contours sketched in, after points on them have been located by interpolation, as already explained.

FILLING IN WITH PLANE TABLE.—The telescopic alidade with stadia wires, and stadia rods, are used with the plane table for traversing and "filling in" details in a manner similar to what has been explained for the transit. The table with the attached sheet is analogous to the graduated horizontal limb of the transit. The traversing with and orienting the table have been explained. At any station the side shots are plotted in direction by the edge of the ruler, the distance is read by the stadia, and laid off to scale. Elevations are determined by observing the vertical angle and reducing. In fact, contours are more rapidly and accurately plotted with the plane table than by any other means. Single contour lines can be followed around and plotted by means of the plane table with as much facility and accuracy as the notes can be taken when using the transit. Every method that has been explained for contouring is applicable to the plane-table. It is almost the only method used in the Coast Surveys and Geological Surveys for contouring and "filling in" details, and is about the only practicable method of making hasty reconnaissance sketches.

NOTE.—The *only* way to impress the foregoing methods upon the mind of the student is for him *to actually do the work in the field*, with the various instruments, and afterward *to plot his notes to scale*.

CHAPTER XII.

SOLAR ATTACHMENTS AND THEIR USES.

To an observer on the earth, all the heavenly bodies appear to be situated on the inner surface of a great celestial sphere of infinite radius, of which the earth is the center. Fig. 168.

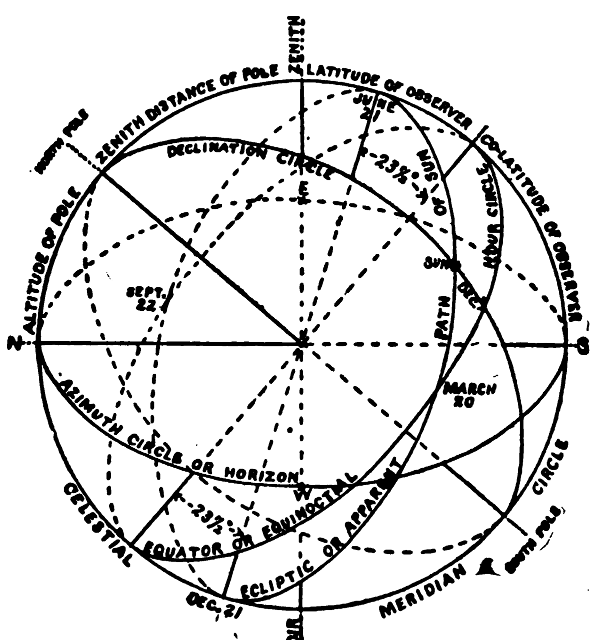

FIG. 168.

DEFINITIONS.—*The poles* are the points in which the earth's polar axis produced pierces this sphere.

The zenith of any place on the earth's surface is the point in which a plumb-line at that place, produced upward, pierces the sphere.

The nadir, the diametrically opposite point.

The equator or *hour circle* is the intersecting of this sphere by the plane of the earth's equator, this plane being perpendicular to the polar axis.

The horizon is the intersection of the sphere by a plane through the line of collimation of the telescope and perpendicular to the plumb-line.*

The ecliptic or *apparent path of sun* is the intersection of the sphere by the plane of the earth's orbit.

A meridian circle is the intersection of the sphere by a plane through the polar axis and the observer's place.

The sun's declination circle is the intersection of the sphere by a plane through the polar axis and the sun.

The meridian altitude of the sun is his angular distance (when on the meridian) from the horizon, measured on the meridian circle.

The declination of the sun is his angular distance from the equator, measured on the declination circle.

A meridian line is the intersection of the earth's surface by the plane of a meridian circle.

The latitude of a place on the earth's surface is its angular distance from the equator, measured on the meridian line, or angular distance of its zenith from the equator measured on the meridian circle, and is equal to the altitude of the celestial pole.

The co-latitude is the difference between 90 degrees and the latitude, equal to the zenith distance of the pole.

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*In reality it is the intersection by a plane through the center of the earth and perpendicular to the plum-line, but the distance between these two parallel planes is practically nothing compared to the great distance to their intersections with the celestial sphere, hence may be taken as the same plane. In measuring vertical angles of heavenly bodies from points on the earth's surface they are necessarily measured from the horizontal planes through the instrument considered as if they passed through the earth's center.

The longitude of a place is its angular distance, measured on the equator, east or west from the meridian of some place assumed as the starting meridian.*

The zenith distance of a heavenly body, when on a meridian, is its angular distance from the zenith and is measured on the meridian circle.

The hour angle of a heavenly body is the angular distance of its declination circle from the meridian circle, measured on the equator westward.

The apparent sun, from which is obtained *apparent time*, is the sun that appears in the heavens.

The mean sun is a fictitious sun in the equinoctial moving with the apparent sun's average yearly motion in right ascension.

The vernal equinox is the fixed point in the equinoctial where the sun crosses from south declination to north declination, about March 20th.

A day is the interval of time between the departure of a heavenly body, or point of the celestial sphere, from the meridian of a place, and its next return. It is an *apparent solar day* if the body is the apparent sun, a *mean solar day* if the body is the mean sun, and a *sidereal day* it the point is the vernal equinox. The interval between successive transits of fixed stars equals a sidereal day.

The right ascension of a heavenly body is the arc of the equinoctial from the vernal equinox eastward to the declination circle through the body. When the body is on the meridian of a place its right ascension is equal to the hour angle or sidereal time of the vernal equinox at that instant; hence, by observing the exact time of transit of a star whose right ascension can be obtained, the local sidereal time becomes known.

A mean solar day is longer than a sidereal day by about

*The meridians through Greenwich, Eng., and Washington, D. C., are usually taken as the starting meridians from which to measure longitude.

3 minutes 56 seconds, there being 366¼ sidereal days in a year.

1 mean solar day=1.00273791 sidereal days.

24 mean solar hours=24 hours 3 minutes 56.5554 seconds, sidereal time.

1 mean solar hour=1 hour 0 minute 9.8565 seconds, sidereal time.

1 sidereal day=0.99726957 mean solar day.

24 sidereal hours=23 hours 56 minutes 4.0906 seconds, mean solar time.

1 sidereal hour=59 minutes 50.1704 seconds, mean solar time.

In the back part of the Nautical Almanac are full instructions on how to convert sidereal intervals into mean solar intervals, and sidereal time into mean solar time.

The principal object of solar attachments is to furnish a ready means of determining a true meridian through the axis of the instrument, wherever set up, thus facilitating the running of meridian lines, parallels of latitude, and the measurement of angles referred to the true meridian. With no other instrument in common use can this object be attained.

THEORY OF SOLAR ATTACHMENTS.—*When the telescope is made horizontal* by its spirit level, the hour circle will be in the plane of the horizon, the polar axis will point to the zenith. and the zeros of the vertical arc and its vernier will coincide.

Now, *if the telescope be inclined*, the polar axis will descend from the direction of the zenith, the angle through which it moves being laid off on the vertical arc.

When the polar axis is made parallel to the earth's axis the vernier of the vertical arc will read the co-latitude of the place where the instrument is used.

If now the declination arm remains at zero and the lens be directed at the sun, his image will appear on the opposite plate (Fig. 100, p. 90), provided the instrument is used at the time of the equinox. When, however, the sun passes above or below the equator, his declination or angular distance from it, as given in the Ephemeris, can be allowed for and set off on the declination arc, and his image still be brought into position as before.

In order to do this, however, it is necessary, not only that the co-latitude and declination shall be correctly set off upon their respective arcs, but also that the instrument should be moved in azimuth until the polar axis points to the pole of the heavens, or, in other words, is placed in the plane of the meridian.

When the declination, corrected for refraction, is set off on the declination arc for any hour of the day, and the co-latitude on the vertical circle, and the image of the sun lies in the square, then the pencil of rays makes with the horizontal plates an angle equal to the apparent altitude of the sun at that time.

ADJUSTMENTS OF THE SOLAR ATTACHMENT.—*To adjust the equatorial lines and solar lenses.* First detach the declination arm from the arc by removing the clamp and tangent screws and the conical center with its two small screws, by which the arm is attached to the arc. The adjuster, which is a short bar furnished with every attachment, is then substituted for the declination arm, the conical center screwed into its place at one end and the clamp screw into the other, being inserted through the hole left by the removal of the tangent screw, thus securing the adjuster firmly to the arc. Now place the detached declination arm upon the adjuster, with the same side resting against the surface of the declination arc as before it was detached. Turn the instrument on its spindle so as to bring the solar lens to be adjusted in the direction of the sun, and raise or lower the adjuster on the declination arc until the image of the sun is brought between the equatorial lines on the opposite silver plate, the final position by the tangent screw. Then carefully turn the arm around its longitudinal axis until it rests upon the adjuster on the opposite edges of the rectangular blocks, and again observe the position of the sun's image.

If it remains between the lines as before, the lens and plate are in adjustment; if not, loosen the three screws which confine the plate to the block, and move the plate under their heads until one-half the error in the position of the sun's image is removed.

Repeat for a test, and correct until the image will remain in the same situation in both positions.

To adjust the other lens and plate, reverse the arm end for end on the adjuster, and proceed precisely as in the former case until the same result is attained.

In tightening the screws over the plate, care must be taken not to move the plate.

This adjustment now being complete, the adjuster is removed and the declination arm with its attachments replaced as before. Make this adjustment about noon, as the sun then, in a given length of time, has the least vertical motion.

To adjust the vernier of the declination arc. Set the vernier at zero and then raise or lower the telescope by the tangent screw until the sun's image appears exactly between the equatorial lines.

Having the telescope axis clamped firmly, carefully revolve the arm until the image appears on the other plate. If precisely between these lines, the adjustment is complete; if not, move the declination arm by its tangent screw, until the image will come precisely between the lines on the opposite plate: now remove one-half the apparent error by means of the tangent screw and the other half by loosening the two screws that fasten the vernier and placing the zeros of the vernier and limb in exact coincidence; now tighten the screws, and the adjustment is finished. This adjustment should be made about noon, for the same reason as the preceding one.

To adjust the polar axis. First level the instrument carefully by the long level of the telescope, using in the operation the tangent movement of the telescope axis in connection with the leveling-screws of the parallel plates until the bubble will remain in the centre during a complete revolution of the instrument upon its axis. Place the solar apparatus upon the axis and see that it moves easily around it; bring the declination arc into the plane of the telescope, and having the arc set at zero, place the adjusting or striding level (Fig. 169)

FIGURE 169.

on top of the rectangular blocks and bring the bubble of the level into the center by the two capstan-head screws under the hour arc, which are in line with the declination arc, loosening one end and tightening the other with the pin until the bubble is centered. Then turn the declination arc until it is at right angles to its former position, and, if out, bring the bubble to the center by the other pair of screws directly under the arc and in line with the horizontal axis of the telescope. Return the arc to the first position and, if necessary, repeat the operation of centering the bubble. To check the work, turn the arc 180°, bringing it again parallel to the telescope, and note the position of the bubble; if in the center, the polar axis is vertical in that direction; if not, make the correction and repeat the operation as before, taking care that the bubble under the telescope is always in the center, and the capstan screws brought to a firm bearing. Pursue the same course in adjusting the arc in the second position (over the horizontal axis of the telescope), and when completed the bubble will remain in the center during an entire revolution of the declination arc, showing that the polar axis is set at right angles to the level under the telescope, or truly vertical. The adjusting level in the above operation is assumed to be itself in adjustment; if not, it can be adjusted by reversing it upon a plane surface. It should be here noted that, as this is by far the most delicate and important adjustment, it should be made with the greatest care, the bubble kept perfectly in the center and frequently inspected in the course of the operation.

To adjust the hour circle. Whenever the instrument is set in the meridian, with the co-latitude and approximate declination set off and the image of the sun brought between the hour lines, the index of the hour circle should read local apparent time. If it does not, loosen the two flat-head screws on the top of the hour circle, and with the hand turn the circle around until it does so read, fasten the screws again, and the adjustment is complete.

To obtain mean or astronomical time, the correction of equation for the given day, as given in the Nautical Almanac, must be applied.

THE SAEGMULLER SOLAR ATTACHMENT (Fig. 101) consists essentially of a small telescope and level, the telescope being mounted in standards in which it can be elevated or depressed. The standard revolves around an axis, called the polar axis, which is fastened to the telescope axis of the transit instrument. The telescope, called the "solar telescope," can thus be moved in altitude and hour angle. It is provided with shade-glasses to subdue the glare of the sun, as well as a prism to observe with greater ease when the declination is far north. Two pointers attached to the telescope to approximately set the instrument are so adjusted that when the shadow of the one is thrown on the other the sun will appear in the field of view.

1st adjustment: Attach the "polar axis" to the main telescope axis in the center at right angles to the line of collimation. The base of this axis is provided with three adjusting screws for this purpose; by means of the level on the solar telescope this condition can be readily and accurately tested.

2d adjustment: Point the transit telescope—which instrument we assume to be in adjustment—exactly horizontally, and bisect any distant object. The bubble of the transit level will then be in the middle of the scale. Point the "solar telescope," also; *horizontally*, by observing the same object, and adjust its level to read zero, for which purpose the usual adjusting-screws are provided.

To use the attachment. 1. Take the declination ot the sun as given in the Nautical Almanac for the given day and hour, and correct it for refraction and hourly change. Incline the *transit telescope* until this amount is indicated by its vertical arc. If the declination of the sun is *north*, depress it; if *south*, elevate it. Without disturbing the position of the transit telescope, bring the solar telescope to a horizontal position by means of its level. The two telescopes will now form an angle which equals the amount of the declination.

2. Without disturbing the *relative* position of the two telescopes, incline them, and set the vernier to the co-latitude of the place.

3. Move the transit and the solar attachment around their respective vertical axes until the image of the sun is brought into the field of the solar telescope, and at the given hour accurately bisect the image; *the transit telescope is now in the meridian,* and the compass needle indicates its own declination at that time and place. The polar axis of the solar attachment will then point to the pole.

DIVISIONS AND KINDS OF TIME.—The sun apparently revolves from east to west around the earth in a circle of 360° once in 24 hours, or through 15 degrees in one hour of time.

The interval of time between two consecutive passages of the meridian of a place by the sun is called an *apparent solar day.*

The angular distance at any time of the sun from the meridian of a place measured on the equator to the west in hours, minutes, and seconds is called the *apparent solar time,* and the instant of the sun's being on the meridian is called the *apparent noon* of that place.

The lengths of the apparent solar days are not all equal, however, and it was found impossible to make watches and clocks correspond to them; therefore the mean of the lengths of all the apparent solar days in a year was adopted as the one to which watches could be regulated, and it is called the *mean solar day;* hence time carried by watches is called *mean solar time*, as the days by it are all of equal length, and 12 o'clock noon by the watch is called *mean noon.*

The difference at any instant between the apparent time, as given by the position of the sun with reference to the meridian, and the time given by a watch, is called *the equation of time.* Four times a year (in 1893 at Greenwich on April 15th at 4 A. M., on June 1st at 7 A. M., on August 31st at 6 P. M., and on December 24th at 1 P. M.), apparent and mean time were the same; at all other times they were different. In 1893 at Greenwich, apparent and mean noon differed on February 10th by 14 minutes 26 seconds; May 14th by 3 minutes 51 seconds; July 26th by 6 minutes 17 seconds; November 2d by 16 minutes 21 seconds.

It is to be understood that, although apparent and mean time may sometimes differ by as much as 16 minutes, the actual lengths of mean and apparent days do not differ by as much as 30 seconds, and very few approach that difference, some differing less than 2 seconds.

The difference between apparent and mean time for every hour in the year is given in the Nautical Almanac, and by it one may be changed into the other, and *vice versa*, by following the directions at the head of the column in which it is found.

Standard time. There is another division of time that has come into general use in the United States, known as Standard Railway Time. Very few places of importance now use local time, either apparent or mean, but instead the mean solar time of the 75th, 90th, 105th, or 120th degree meridian west of Greenwich, according to which one is nearest.

Hence, if Standard Railway Time is carried, it is easy to convert it into local mean time by adding or subtracting the difference in longitude, expressed in time, between the Standard Time meridian and that of the place of observation, depending upon whether the latter is east or west of the former.

Thus, if by the watch the Standard Time is 8 A. M., then the local mean time of a place 5° east of the Standard Time meridian will be 8:20 A. M., and of a place 5° west of it will be 7:40 A. M.

Local time. When local mean time is carried by a watch or chronometer it is the one most conveniently used in making observations on the sun out of the meridian, data for which is obtained from page II. of each month in the Nautical Almanac. But for making observations on the sun when on the meridian, as in making the adjustments of solar attachments, and determining time and latitude, local apparent time should be used, data for which is given on page I. of each month in the Nautical Almanac.

Arcs of great circles are reckoned in degrees, minutes, and seconds *of arc*, and also in hours, minutes, and seconds *of time;* thus 24 hours is equal to 360°, 1 hour is equal to 15°, 1 minute is equal to 15′, and 1 second is equal to 15″.

In civil reckoning of time the day begins at midnight, and hours are counted up to 12 at midday, and again up to 12 at the following midnight.

In astronomical reckoning the day begins at midday (12 hours after the civil day), and hours are counted up to 24, to the following midday. January 1st, astronomical time, is the afternoon of January 1st, civil time, and forenoon of January 2d, civil time.

Observations for time consist in observing the transit of the sun, or a star, across the meridian, noting the watch time and finding the difference between it and the calculated local mean or sidereal time of transit; the difference being the error of the watch.

The time of transit of the sun is obtained by taking the mean of the times of transit of the east and the west limbs.

When making use of stars it is usual to observe two (not circumpolar, as their apparent motion is too slow), so as to eliminate instrumental errors. From the Nautical Almanac, as explained therein, is obtained the mean time of transit of each of the two stars. The telescope is placed in the meridian (both motions in azimuth clamped) and then turned up to the *first* star and the exact time of its transit noted. The telescope is then plunged and the alidade revolved 180°, the telescope again placed in the meridian, raised to the altitude of the *second* star, and the exact time of its transit noted.

The mean of the errors of the watch is the error to obtain correct time.

For stars having high altitudes, the eye prism would have to be used.

The meridian may be established either by two points in it, one of which is occupied by the instrument and the other by the target, or by the azimuth of some other station.

LONGITUDE:—*By chronometers.* Finding the difference in longitude between two places is a more complicated problem than finding the latitude of a place. If it were possible to construct a timepiece to show *mean* or *sidereal* time without any variation, all that would be necessary would be to set it at the exact time at the initial station, carry it to the station whose longitude was desired, and there make an observation for time; then the error of the time-piece at the instant of the observation would be the difference in time, or the difference in longitude in time, which could be converted into difference in longitude in degrees, minutes, and seconds, but such perfection has not been obtained, and the rate of variation from mean or sidereal time has to be obtained and applied. Thus the error of timepiece (called chronometer) is noted on the *mean* or *sidereal* time on a fixed meridian on a given date. Assuming it to preserve a constant rate, it is carried to the place whose longitude is desired, an observation made for time, and the error at that instant is compared with the calculated error at the same instant at the meridian of departure. Navigators at sea depend chiefly upon this method, and it may be used for short distances on land, but cannot be depended upon for great differences.

By telegraph. The method now exclusively employed in geodetic operations, where possible, is to make a most accurate determination of time at both places to be compared, combined with a direct comparison of these times through the medium of the electric telegraph. The most simple one is for each observer to regulate his chronometer to local mean or sidereal time, and to note the time of any arbitrary break of the connecting circuit, by some prearranged signal.

Or, to telegraph from an eastern to a western station the instant of a fixed star's culmination at the eastern station, and conversely to telegraph to an eastern station the instant of culmination of the same star at a western station. The local time of both events being noted, the difference, as recorded at the same station, corrected for the rate of the timepiece, gives the difference of longitude.

Many improvements and refinements of this method are used for accurate determination.

Longitude by eclipses of Jupiter's satellites. At one time the eclipses of Jupiter's satellites were much used in determining longitude, but, since a satellite never enters the planet's shadow suddenly, because of its sensible diameter, the time from its first loss of light to its total extinction is quite perceptible. The same is true of emergence. Owing, also, to the difference in telescopes and eyes, this becomes a source of discrepancy in the times assigned by different observers for the beginning and ending of an eclipse. If, however, both be observed by the same person and with the same telescope, the half sum of the two times, as given by a properly regulated timepiece, will be that of apparent opposition, measurably free from error.

In the Nautical Almanac is given the mean time at Washington of the conjunctions of the satellites of Jupiter. If, then, the local mean time be noted of one of these on any particular day at a place whose longitude is desired, the Washington mean time at the same instant may be approximately obtained, and hence the difference of longitude.

THE SUN'S DECLINATION.—The sun apparently revolves about the earth once a year in the celestial ecliptic; and as the plane of the ecliptic makes an angle of 23° 28′ with the plane of the equator, the sun during the six months of the year from March 20th to September 22d is above the equator and its declination is north, reaching its maximum of 23° 28′ June 21st; and from September 22d to March 20th it is below and its declination south, reaching its maximum of 23° 28′ December 21st. On March 20th and September 22d, when crossing the equator, its declination is zero. The change in declination, however, is not constant, being as much as 59″.25 per hour when crossing the equator, and less than 1″ per hour when at its maximum points; therefore, when going to use a solar attachment, it is necessary to prepare beforehand a table of hourly declinations of the sun for each day.

Determining the declination. When the sun is on the meridian of Greenwich, England, at *apparent noon* of any day he will have a certain declination, which is given in the Nautical Almanac page I. of each month. At a place 15° west longitude from Greenwich it will be only 11 o'clock A. M. *apparent time*, but his declination for that hour will be the same as at Greenwich apparent noon. Now when the sun arrives at the 15° meridian, one hour of time will have passed and his declination will also have changed, which hourly change is also given in the Nautical Almanac. At a place 90° west longitude from Greenwich it will be only 6 o'clock A. M. *apparent time* when it is apparent noon at Greenwich, but the instant is the same as regards the declination of the sun, as given in the Nautical Almanac, and similarly for any other place whose longitude is known.

At Greenwich *mean noon* the sun has a certain declination each day, given on page II. of each month of the Nautical Almanac. At a place 15° west longitude it will have the same declination at 11 A. M. *mean time*, and at 90° west longitude the same at 6 A. M. *mean time*. To determine the declination for any other time of day the hourly change is applied as will be explained.

As the Nautical Almanac gives the declination of the sun in the heavens *unaffected by refraction*, it becomes necessary to *apply such an amount of refraction* to those declinations as will bring them up to the apparent heights before they are to be set off on the declination arc.

As before stated, before this instrument can be used at any given place it is necessary to set off, on the declination arc, the declination of the sun as affected by its refraction for the given day and hour, and, on the vertical circle, the co-latitude of the place where the observation is made.

The declination of the sun, given in the Nautical Almanac from year to year is calculated for both *apparent* and *mean noon* at Greenwich, England, together with its hourly change in declination. To determine it for any other hour at the same or any other place, a correction must be applied for the change in declination due to change in time, from hour to hour, and also for the difference in time due to the difference in longitude.

The longitude of any place in the United States can be found very nearly by reference to a good map, or it is the general practice now, where standard railway time is carried and is known not to differ more than half an hour from local time, to use that and make the corrections in declinations corresponding to it. Having thus the difference in time, the declination for a certain hour in the morning, which would be earlier or later as the longitude was greater or less, would be the same as that of Greenwich mean noon on the given day. Thus, supposing the observation be made at a place, say 5 hours earlier than at Greenwich, then the *declination* given in the almanac for the given day at mean noon, *corrected for refraction*, would be the apparent declination at the place of observation at 7 o'clock A. M.; this gives a starting-point.

To obtain the apparent declinations for the other hours of the day, take from the Almanac the declination for mean noon of the given day, and, as the declination is increasing or decreasing, add to or subtract from the declination of the first hour the difference for one hour, as given in the Almanac, which will give, when affected by the refraction, the apparent declination for the succeeding hour; proceed thus in making a table of the apparent declinations for every hour of the day. For example, suppose it were required to make out a table of declinations, corrected for refraction, for the different hours of May 1. 1893, at Fort Leavenworth, Kansas, in longitude 94° 51′ 7″ or 6 hours 19 minutes 24 seconds, and latitude 39° 21′ 24″.

Calling the longitude 6 hours 20 minutes, then the declination given in the Almanac for mean noon of May 1, 1893, at Greenwich, will be the declination at supposed place of observation at 5:40 A. M. of same day, and is N. 15° 13′ 54″.9; the change for one hour is +45″.04; the plus sign indicates that north declinations are increasing. Since 7 A. M. is about as early as the attachment can be reliably used, the sun by that time will have been changing his declination for 1 hour 20 minutes, or 1 ⅓ times 45″.04 = 60″.05, which added to his declination at 5:40 A. M. gives N. 15° 14′ 54″.95 for his declination at 7 A. M. To find the declinations for the remaining hours of the day and corresponding refractions, proceed as follows: Calling the latitude 40°, refer to the table of mean refractions and find the hourly amounts to be applied. 7 A. M. being the 5th hour.

Dec. at 7 A.M. N.	15° 14′54″.95	+ Ref. 5h 1′31″	= 15° 16′25″.95 = App. Dec. 7 A.M.
Add. Diff. for 1h	45″.04		
8 A. M.	15° 15′39″.99	+ Ref. 4h 55″	= 15° 16′34″.99 = App. Dec. 8 A.M.
	45″.04		
	15° 16′25″.03	+ Ref. 3h 40″	= 15° 17′ 5″.03 = App. Dec. 9 A.M.
	45″.04		
	15° 17′10″.07	+ Ref. 2h 32″	= 15° 17′42″.07 = App. Dec. 10 A.M.
	45″.04		
	15° 17′55″.11	+ Ref. 1h 27″	= 15° 18′22″.11 = App. Dec. 11 A.M.
	45″.04		
	15° 18′40″.15	+ Ref. 0h 27″	= 15° 19′ 7″.15 = App. Dec. 12 M.
	45″.04		
	15° 19′25″.19	+ Ref. 1h 27″	= 15° 19′52″.19 = App. Dec. 1 P.M.
	45″.04		
	15° 20′10″.23	+ Ref. 2h 32″	= 15° 20′42″.23 = App. Dec. 2 P.M.
	45″.04		
	15° 20′55″.27	+ Ref. 3h 40″	= 15° 21′35″.27 = App. Dec. 3 P.M.
	45″.04		
	15° 21′40″.31	+ Ref. 4h 55″	= 15° 22′35″.31 = App. Dec. 4 P.M.
	45″.04		
	15° 22′25″.35	+ Ref. 5h 1′31″	= 15° 23′56″.35 = App. Dec. 5 P.M.

At apparent noon, May 1, 1893, it will only be 11 hours 56 minutes 56.09 seconds mean time, hence the sun on the meridian will have the declination corresponding to that mean time instead of 12 M. mean time, as given in the table. What the exact declination for apparent noon is may be found from page I., or by subtracting from that given in the table the amount of change for 3 minutes 3.91 seconds (the difference between the times) which equals 2″.3 in declination giving a declination at apparent noon of 15° 19′ 4″.85.

From March 20th to June 21st add hourly differences in declination; from June 21st to September 22d subtract hourly differences; from September 22d to December 21st add hourly differences; from December 21st to March 20th subtract hourly differences.*

*If the observer's watch shows the standard time of Saint Louis (90th meridian) and the first method of determining the meridian is used, the table of declinations is computed for Saint Louis, not for the observer's longitude. When the observer's watch shows 9 A. M., it is 9 A. M. mean time at Saint Louis, not at the observer's position.

TO DETERMINE THE LATITUDE OF A PLACE.—In the explanation of the Table of Refractions it was stated that the refractions were calculated for latitudes at intervals of 2½ degrees, which is as near as required for finding the apparent declination to set off on the declination arc, but to use the instrument for determining the meridian it is necessary that the latitude of the place should be known accurately, in order to set off the co-latitude. If this is not known, it may generally be found from a map within the 2½ degrees for determining the apparent declination to set off in preparing the tables. Having done this, set off on the declination arc the apparent declination for 12 o'clock apparent noon of the given day. A few minutes before apparent noon set up the instrument and level carefully; set the declination arm at 12 o'clock on the hour ring and revolve the alidade in azimuth until the declination arm points towards the sun (Fig. 170). Set off the co-latitude approximately on the vertical circle, and clamp; then by means of the tangent screw bring the sun's image between the equatorial lines. As the sun continues to rise towards the meridian his image will descend; follow it with the tangent screw, keeping it between the equatorial lines, and by either lower tangent screw keep his image between the hour lines. On reaching the meridian his image will cease to descend and begin to rise. When this instant occurs, cease to follow it, and read the vertical circle, which will be the co-latitude of the place. The co-latitude thus found may be used with the instrument without regard to whether the vertical circle has an index error or not; but if the true co-latitude is used with it, any index error must be determined and properly applied in setting it off, as also in determining the true co-latitude by a meridian observation, as above.

FIG. 170.

By observations on circumpolar stars. The latitude of a place being equal to the altitude of the pole, measured at the place, the operation consists in simply observing the altitude of a circumpolar star at culmination, and correcting this altitude for refraction and for the pole distance of the star.

From a *table of culminations* find the time Polaris, or some other circumpolar star, crosses the meridian. About 15 or 20 minutes before this time set up the transit and level carefully. Set the horizontal wire of the telescope upon the star and follow it with the tangent screws until it reaches its highest or lowest point. Read the vertical angle and from it subtract the refraction corresponding to the reading, and then subtract if upper culmination, or add if lower culmination, the polar distance, and the result should be the latitude of the place.

Errors of adjustment of line of collimation, of vertical circle, and of plate-bubbles may be eliminated by determining the altitude of the star with the telescope direct about 5 minutes before culmination, then plunging the telescope and revolving alidade 180°, releveling, and again determining altitude, telescope reversed, by two readings. Then plunge telescope back to normal position, revolve alidade 180°, again relevel, and make another determination, telescope direct. Correct the mean of the four readings for refraction and pole distance as before.*

If the vertical arc of the transit is only 180°, an artificial horizon, as used with the sextant, may be employed, and an observation made, first to the star direct, and then to its image in the artificial horizon. The sum of the two observations will be double the apparent altitude of the star. Or, the first observation may be taken on the star direct, then two on its image in the artificial horizon, then another on the star direct, their sum giving four times the apparent altitude of the star, from which its altitude, and then the altitude of the pole or the latitude, may be obtained. The error due to an index error of the vertical circle will thus be eliminated.

*Errors of adjustment of the plate-bubble perpendicular to the line of sight and of the standards are not eliminated.

By meridian altitude of the sun, without solar attachment. The latitude of a place is equal to 90°—the meridian altitude of the sun+his declination (provided north declinations be considered positive and south negative). Or, when the observer and sun are upon the same side of the equator, the latitude =90°—the meridian altitude or the sun+the declination. When the observer or the sun are on different sides of the equator, the latitude=90°—the meridian altitude of the sun—the declination.

Hence, *to find the latitude by an observation on the sun*, make an observation for the altitude of his upper or lower limb when on the meridian at apparent noon, subtract meridian refraction, subtract his semi-diameter if on upper, or add if on lower limb, for the true altitude of the center, which latter subtract from 90°, and add the declination with its proper sign.

TO DETERMINE THE MERIDIAN WITH SOLAR ATTACHMENT.—Having now all the necessary data for using the instrument, to determine the meridian, or true north-and-south line at any time of the day, take from the table the apparent declination corresponding to the mean time when the observation will be made, and set it off on the declination arc; set off on the vertical circle the co-latitude of the place, clamp the horizontal plates at zero, revolve the whole head of the instrument until the telescope is *approximately* in the meridian, then with one hand turn the declination arm on the polar axis toward the sun, and with the other turn the whole head of the instrument until the image of the sun is brought between the equatorial and the hour lines; then clamp the head, and by means of lower tangent screw and movement about polar axis keep the image there until exact instant for which the declination is computed, when the telescope will be in the meridian.*

*Or another method is to set the hour circle at the hour for which the declination is computed, and clamp; set off the declination and co-latitude as before, and about ten minutes before the computed time bring the sun on the plate between the equatorial lines and keep it

When the instrument is accurately adjusted and leveled, and the corrected declination of the sun for the day and hour and the co-latitude of the place are set off on their respective arcs, the image of the sun cannot, at the given hour, be brought between the equatorial lines until the polar axis is placed in the plane of the meridian of the place, or in a position parallel to the axis of the earth, and the line of collimation of transit telescope is in the meridian. (See foot-note, p. 220.)

The slightest deviation from this position will cause the image to pass above or below the lines. Thus from the position of the sun in the heavens is obtained the true meridian with an accuracy corresponding to the accuracy of the adjustments and observation.

If the revolving arm be turned a little to one side of its proper position, a *false* image may appear in nearly the same position as that occupied by the true one. It is caused by the reflection of the true image from the surface of the arm. It can be distinguished by being much less bright and less clearly defined.

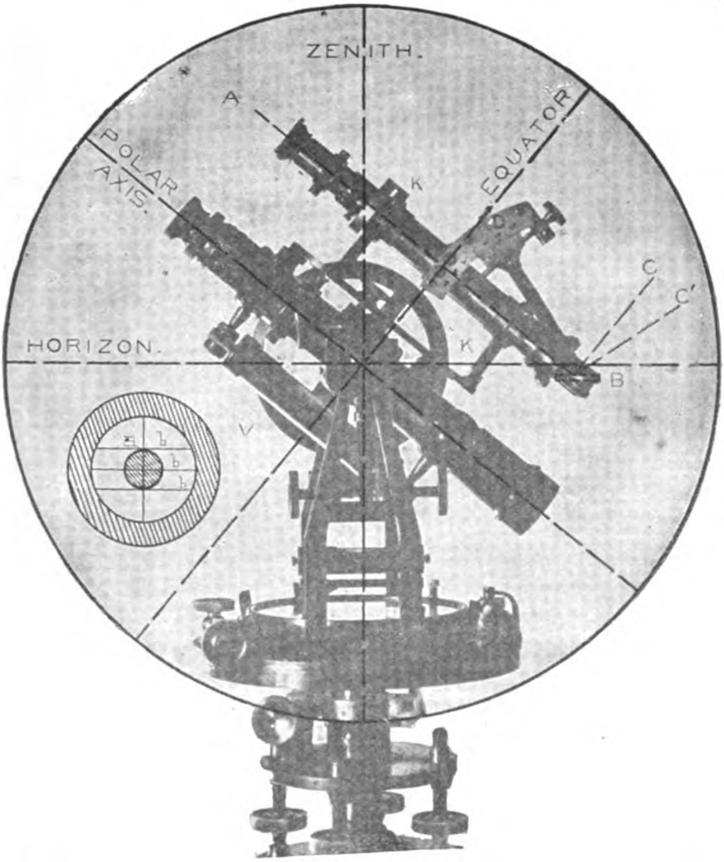
SMITH'S MERIDIAN ATTACHMENT.—*Description.* This attachment, shown in the figure and used in connection with the transit, consists of a small solar telescope free to revolve in the collars K, K. The collars are rigidly attached to the transit telescope, and therefore the only motion independent of the transit telescope possessed by the solar telescope is that of revolution about its longitudinal axis.

The amount of this revolution is recorded on the hour circle by an index on the upper collar K, the hour circle being a silvered ring graduated to ten minutes of time and rigidly attached to the solar telescope just above this upper collar.

The vertical limb of the transit is used as a latitude arc.

The declination arm is shown in the figure. Attached to the pivot end of this arm, and moving with it, is the re-

between them by moving the head of the instrument in azimuth by the lower tangent screw until the image appears between both the hour and equatorial lines—at this instant the telescope is in the meridian.



SMITH'S MERIDIAN ATTACHMENT.—B, reflector; K, K, collars; V, vertical limb, latitude arc; D, declination arc; A, B, C, C', solar line of collimation; a, hour wire; b, b, b equatorial wires. The line A B is parallel to the polar axis; and, as regards latitude, declination, and hour angle, is practically identical with it.

flector, so arranged that when the declination venier, properly adjusted, reads zero, the angle between the reflector and the optical axis of the telescope is 45°.

THEORY.—If the instrument, properly adjusted, be set in the meridian, and the latitude of the place set off on the latitude arc, the polar axis of the instrument will be parallel to the earth's polar axis.

If the declination vernier be set at zero, the portion of the line of collimation of the solar telescope from the reflector outward will make an angle of 90° with the optical axis of the solar telescope. And if the solar telescope be now revolved in its collars, this outward portion of the line of collimation will cut from the celestial sphere a circle parallel to and (on account of the infinite radius of the celestial sphere) coincident with the celestial equator.

If the apparent sun is on the equator at the time the solar telescope is being revolved, it will, when the telescope has been turned through the proper angle, be in the outward portion of the line of collimation, and its image will appear accurately on the cross-wires.

So, too, with the instrument as before, if the sun's declination for a particular day and hour, corrected for refraction, be set off on the declination arc and the solar telescope revolved, this outward portion of its line of collimation will cut from the celestial sphere a circle parallel to the equator and at a distance from it equal to the sun's declination for the selected time, corrected for refraction. When, therefore, the solar telescope has been revolved through the proper angle, its line of collimation will strike the sun and the image of the latter will appear accurately on the cross-wires.

If, however, the polar axis is not parallel to the earth's axis (when the instrument is properly adjusted and set up and the latitude set off on the latitude arc, this lack of parallelism occurs whenever the instrument is not in the meridian), the outward portion of the line of collimation no longer cuts from the celestial sphere the circles described, and the sun's image cannot be brought upon the cross-wires*

It therefore follows that, when the conditions mentioned regarding adjustment, setting up, latitude, and declination *are*

* There is one exception to this rule. The image *can* be brought upon the cross-wires when the telescope is no longer in the meridian; but in this position the transit telescope will point on the side of the sun opposite the meridian, and on the hour circle A. M. time will be used for P. M. observations, and the converse.

fulfilled, the instrument is in the meridian when the sun's image is brought accurately upon the cross-wires.

ADJUSTMENTS.—Before adjusting the attachment, see that the transit itself is in the most accurate adjustment possible.

To adjust the equatorial wires. Set up the instrument, and by the most convenient means bring the sun's image between the equatorial wires. Revolve the solar telescope slightly in its collars, so as to cause the image to traverse the field of view. If the image remains between the wires while traversing the field, the adjustment is correct.

If it does *not* so remain, loosen the screws holding the cross-wires diaphragm in place, and with the hand revolve the diaphragm until the image *will* remain accurately between the equatorial wires while traversing the field of view.

Errors from lack of this adjustment can be avoided by always centering the image on the hour wire as well as between the equatorial wires when making an observation.

To adjust the line of collimation of the solar telescope. Remove the two screws that fasten the base of the reflector frame to the solar telescope, and push the reflector to one side. Set the line of collimation of the transit telescope on some distant point, preferably a star (in which case the cross-wires must be illumined), and bring the line of collimation of the solar telescope upon the same point by means of the capstan screws carrying its diaphragm.

If no distant point is available, two points, the second above the first as far as the axis of the solar telescope is above that of the transit telescope, may be used.

To adjust the declination arc. Set up the instrument in the meridian, set off the latitude, and make a meridional observation of the sun, bringing its image accurately between the equatorial wires by means of the tangent screw on the declination arm. The difference between the reading of the declination vernier in this position and the corrected declination of the sun for the time of observation will be the index error of the arc.

Loosen the three small screws holding the arc in place, and remove this index error by shifting the arc until its vernier reads the corrected declination. Tighten the screws, being careful not to shift the arc from its corrected position.

If the arc is not adjustable, the index error must be applied each time the declination is set off.

To adjust the hour circle. Set up the instrument in the meridian, set off the latitude and the corrected declination. Then make an observation of the sun; being careful to center the image on the hour wire. If the index reads the local apparent time of the observation, the adjustment is correct. If not, mark a new index opposite the correct local apparent time of the observation.

TEST FOR ACCURACY.—Whenever the solar attachment is to be used, it should be tested by the method required of deputy surveyors by the manual of surveyor instructions issued by the Commissioner of the General Land Office. A complete description of the test is entered in the deputy's notes of survey.

*"August 28, 1890: In order to test the solar apparatus, by comparing the results of observations on the sun, made during A. M. and P. M. hours, with a *true meridian*, determined by observations on Polaris, I proceed as follows:

"At 4 hours 2 minutes P. M. local mean time, I set off 45°34′.5 on the latitude arc; 9°30′.5 N. on the declination arc; and mark the *true meridian* thus determined by the solar, by a cross on a stone firmly set in the ground, 5 chains north of the instrument.

"At 8 hours 56 minutes P. M. by my watch, which is 2 minutes fast of local mean time, I observe Polaris at *eastern elongation*, in accordance with instructions in the Manual, and mark the line thus determined, by a tack driven in a wooden plug set in the ground, 5 chains north of my station.

"August 29: At 6 A. M. I lay off the azimuth of Polaris, 1°49′.5 to the *west*, and mark the TRUE MERIDIAN thus determined, by cutting a small groove in the stone set last evening, on which the true meridian falls 0.2 inch *west* of the mark determined by the solar.

"At 8 hours A. M., I set off 45°34′.5 on the latitude arc; 9° 16′ N. on the declination arc; and mark the true meridian determined with the solar, by a cross on the stone already set 5 chains north of my station;

TO DETERMINE THE MERIDIAN.—*1st method:* Set up the instrument, set off the latitude and the corrected declination. A few minutes before the time of the observation bring the image into the field of view by moving the head of the instrument in azimuth and the solar telescope about its longitudinal axis. Clamp the head of the instrument. At the exact time of observation center the image by these two motions.

The telescope is then in the meridian.

THE TIME OF DAY, *when the meridian is known*, may be ascertained as described on page 226.

2d method: Set up the instrument, set off the latitude, the corrected declination, and, on the hour circle, the *local apparent time* of the observation. A few minutes before this selected time bring the image of the sun npon the equatorial wires by moving the head of the instrument in azimuth. Clamp the head of the instrument, and, avoiding all other motion, bring and keep the image accurately between the equatorial wires by turning the head of the instrument in azimuth by means of its tangent screw, until the image is centered upon the hour wire.

The telescope is then in the meridian.

REMARKS ON SOLAR INSTRUMENTS.—*First.* Solar instru ments should never be used between 11 A. M. and 1 P. M. for measuring azimuths, and preferably not between 10 A. M. and 2 P. M. if the best results are desired.

Second. The nearer noon the instrument is used the greater the errors in azimuth due to erroneous settings of declination or latitude, being as much as 10′, in latitude 40° at 11:30, or 12:30, for an erroneous setting of 1′.

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this mark falls 0.3 inch *west* of the *true meridian* established by the Polaris observation.

"The solar apparatus, by P. M. and A. M. observations, defines positions for true meridians, about 0′ 11″ *east* and 0′ 16″ *west* of the meridian established by the Polaris observation; therefore, I conclude that the adjustments of the instrument are satisfactory.

"The *magnetic* bearing of the *true meridian*, at 8 hours A. M., is N. 18° 10′ W.; the angle thus determined, reduced by the table, page 100, gives the *mean magnetic declination* 18° 04′ *east.*"

Third. At 6 o'clock A. M. and P. M., when the declination arm lies nearly at right angles to the meridian, a small change in the latitude will not appreciably affect the accuracy of the result.

Fourth. If the declination angle be erroneously set off and the co-latitude angle be also affected by *an equal error in the opposite direction*, then the two resulting errors in azimuth will tend to compensate. If, therefore, the declination angle be affected by an error, *and the latitude of the place then found by a meridian observation with the instrument*, *the error in declination will appear in the resulting co-latitude with the opposite sign.* In this way the effect of any constant error in the declination angle may be nearly eliminated.

The sun at a particular time having a fixed angular height in the heavens, we set this off on the instrument in two parts, one at the declination arc and the other on the latitude arc. Now, if we make an error and set off the declination *too great*, and an equal error *too small* in setting off the latitude, these errors will tend to balance each other and bring the line of collimation near its proper place, but will not do so *exactly* (because the planes of the two arcs are oblique to each other), except during the single instant when the sun is on the meridian, when the planes coincide and become one (Fig. 172). The declination and latitude angles bear a relation to each other similar to the two angles of a quadrant. If one is *too small*, the other will be *too large* by a certain amount, which errors become equal only when the sun is on the meridian.

FIG. 172.

Fifth. The best times of day for using the solar attachment are from 7 to 10 A. M. and from 2 to 5 P. M. So far as the instrumental errors are concerned, the greater the hour angle the better the observation, but when the sun is near the horizon the uncertainties in the refraction may cause unknown errors of considerable size.

Sixth. For a given error in the setting for declination or latitude, the resulting error in azimuth will have opposite signs in forenoon and afternoon.

As the sun at any particular time, say 9 o'clock A. M., has a fixed height in the heavens on any particular day, if the different angles are properly set off on the instrument, and the line of collimation turned upon the sun, its image will at that time be accurately centered between the proper lines on the silver plate, and the line of sight will lie in the true meridian; but suppose an error of 1′ too much be made in setting off the declination, then we will have set our line of collimation too high for the position of the sun at 9 o'clock, and it will not, when the instrument is in the meridian at 9 o'clock, center properly between the lines, but obliquely above; consequently, if we have clamped the hour circle at 9 o'clock, we must wait some time for the sun to reach the height that we have set off on the instrument for it before it will be properly centered. If we do center the image at 9 o'clock, it will be by turning our instrument slightly from the position it should occupy, and our line of sight will not lie in the meridian, but to the west of it, if looking south, or east of it, if looking north, by 1′.85 for latitude 40°. In the *afternoon* the converse will be true.

Seventh. If the adjustments are not carefully attended to, the error in the bearing of a line may be much greater when taken by the solar attachment than is likely to be made by the needle when there is no local attraction.

When the sun is on the equator, September 22d and March 20th, the declination arm will be perpendicular to the polar axis, and the zero of vernier will coincide with zero of declination arc. From March 20th to September 22d, the sun being above the equator and consequently above the equatorial plane, its rays will pierce the equatorial plane from above, and the relative positions of the declination arm, equatorial plane, polar axis, and horizontal plane will be as in Figure 170*a*. From September 22d to March 20th, the sun being below the equator, its rays will pierce the equatorial plane from below, and the relative positions of the declination arm, equatorial plane, polar axis, and horizontal plane will be as in Figure 170*b*. When the declination arc is graduated in but one direction from the zero of the scale, then it is necessary to have on each block a lens and silver plate, or, in other words, to have two lines of collimation; and it is necessary to revolve the declination arm from the position shown in Figure *a*, which is for north declination, to that shown in Figure *b*, for south declination. If, however, the declination arc is graduated in both directions from the zero of the scale then but one line of collimation is necessary.

THE TIME OF DAY, *when the meridian is known*, may be approximately ascertained with the solar attachment by setting the telescope in the meridian, then setting off the co-latitude and approximate declination and bringing the image of the sun between the hour lines by moving the declination arm in hour angle only; then the index on the hour circle will show apparent time, which can be reduced to mean time. It is best ascertained, however, when the sun is on the meridian, the time thus given being that of apparent noon, which can be reduced to mean noon by adding or subtracting the ***equation of time***, as the sun is slow or fast

CHAPTER XIII.

THE SEXTANT.

DESCRIPTION.—The Sextant (Fig. 178) is a hand instrument for measuring angles, up to 120°, subtended by any two objects, the angle being in the plane through the instrument and the two objects. It consists of the following parts—viz., a mirror I (Fig. 179) called the *index glass*, rigidly attached to a

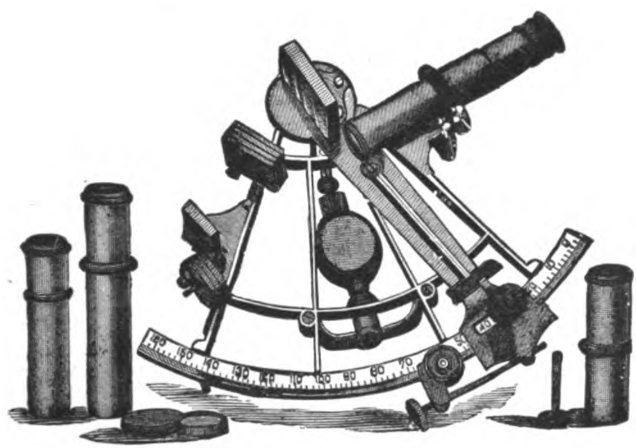

FIGURE 178.

movable arm A called the *index arm*, and a mirror H called the *horizon glass*, rigidly attached to the frame of the instrument. The lower half of the horizon glass is silvered while the upper half is clear. Both mirrors should be perpendicular to the plane of the graduated limb C.

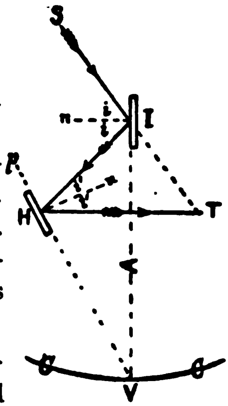

FIG. 179.

The arc of the limb is about 65° or 70° long, graduated to degrees, half degrees, etc., each half degree being numbered as a whole degree so the reading will be the angle subtended by the objects. On the extremity of the index arm is a vernier V. Opposite the horizon glass is a telescope T. Colored glasses are provided for neutralizing the sun's rays, and a magnifying glass for reading the vernier.

THEORY.—The principle of its construction is that a ray of light reflected at two plane surfaces in a plane normal to both is deviated from its orignal direction through an angle double that made by the two reflecting surfaces. Thus a ray of light from S to T being reflected at I to H, and again reflected at H to T, the angle STH between the original direction ST and the direction after the second reflection HT is double the angle IVH between the mirrors. For, drawing the normals n, n, to the mirrors and representing the angles of incidence and reflection at I by i and at H by i', as in the figure, in the triangle ITH the exterior angle

$$2i=2i'+T,$$
$$\therefore T=2i-2i',$$
$$=2(i-i'),$$

and in the triangle HVI the exterior angle

$$IHp=90°-i',$$
$$=HIV+V,$$
$$=(90°-i)+V,$$
$$\therefore V=(90°-i')-(90°-i)=i-i'$$
$$\therefore T=2V.$$

WHERE USED.—The sextant (Fig. 178) is the one principally used at sea in observing the altitude of the sun and lunar distances, to determine the latitude and longitude, where the unstable position of the mariner excludes the use of almost all other instruments. On land it is one of the most convenient, accurate, and generally useful instruments with which to obtain data for the solution of a variety of astronomical and other problems.

THE POCKET SEXTANT (Fig. 180) is the one most frequently used in reconnaissances, preliminary surveys and explorations.

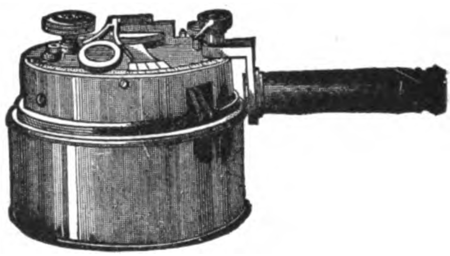

FIGURE 180.

Description. It is, in construction, exactly like the larger one, except that the mirrors and colored glasses are enclosed in a cylindrical box $2\frac{1}{2}$ to 3 inches in diameter and $1\frac{1}{4}$ to $1\frac{1}{2}$ inches high, with suitable openings in the side for exposing the mirrors. The index arm is worked by a milled-head screw and revolves on top of the box on which is the limb. The vernier reads to 1 minute. The telescope, when not needed, may be taken out, and a slide having a small peep-hole may be pushed over the opening. The adjustments are made by means of a key in holes provided on top and on the side of the horizon glass. The cover is screwed on the bottom of the box, serving as a kind of handle in making observations. The pocket sextant is used the same as the larger one.

The advantages of the pocket sextant are its accuracy and portability. When one's exact position is not material it can be readily used on horseback, in a tree, or wherever it may be necessary to take an observation. It is not affected by magnetic disturbances like the compass needle.

The disadvantages are, that it cannot be generally used in traversing and other surveying operations, but is limited to measuring minor angles in triangulation, finding one's place on a map, and determining heights and distances.

ADJUSTMENTS.—*1st adjustment: To make the index glass perpendicular to the plane of the graduated arc.* Test by setting the vernier at about the middle of the graduated arc, then holding the eye near the plane of instrument and looking into the index glass and at the arc, if the arc and its image appear to form one continuous arc, the adjustment is correct; if not, adjust by screws at the back until they appear continuous. In the pocket sextant the index glass is fixed to index arm and is not adjustable.

2d adjustment: To make the horizon glass perpendicular to the plane of the limb. To test, having made the previous adjustment, hold the instrument horizontally and sight on some well-defined distant vertical object, as the corner of a chimney, or, holding the instrument vertically, sight on the horizon and bring the direct and reflected images into coincidence; if they remain continuous when the instrument is tilted; or if by a sweep of the arm the reflected and the direct images pass accurately over each other, the horizon glass is perpendicular to the plane of the instrument; if not, it must be made so by the adjusting screws at the back. With the pocket sextant this adjustment is done with the key in the holes on top of the horizon glass.

3d adjustment: To make the line of collimation of the telescope parallel to the plane of the arc. Rest the sextant on a plane surface, pointing the telescope upon a well-defined point about 20 feet distant. Place two objects of equal height upon the extremities of the arc, that will serve to establish a plane of sight, parallel to the arc; two lead pencils of the same diameter will serve, but they should be of such height as to make this plane of sight the same height above the arc as the line of collimation of the telescope. If the line of collimation intersects the line defined by the two pencils, then the instrument is in adjustment, if it does not do so, then correct by the screws on the telescope holder.

4th adjustment: To correct the index error. Sight on some well-defined distant object (preferably a star), moving the index arm until the direct and the reflected images are coincident; if the vernier reads zero, the instrument is in adjustment; if not, the reading is the index error of the instrument and may be corrected, or simply noted and applied to all readings.

ARC OF EXCESS.—The graduations on the arc are continued some 10° to the right of the 0 of the scale; this is called the "*arc of excess.*" If, when the mirrors are parallel, the 0 of the vernier is on the arc of excess, then all angles will be measured from this point and will be read too small and the index error must be added, since the zero is "off" the scale. If, however, the zero is "on" the scale when the mirrors are parallel, angles will be read too great and the index error must be subtracted. In reading the index error "off" the arc it must be remembered that the amount is the distance on the scale from its zero passed over by the zero of the vernier, but as the vernier is only constructed to read in one direction, when reading the vernier off the arc the number of least reading units must be subtracted from the smallest reading of the scale and this remainder added to the reading on the scale for the index error.

TO USE THE SEXTANT.—*To measure any angle*, the instrument is held in one hand in the plane of the two objects. The telescope is directed towards the *fainter object* through the unsilvered portion of the horizon glass. With the other hand the index arm is moved until the other object, seen by double reflection in the lower part of the horizon glass, is brought into exact coincidence with the object seen direct. The reading of the vernier is the required angle. If the fainter object is to the right, the instrument will have to be held upside down.

If the horizontal angle between two objects of different elevations is desired, some object is found by a plumb-line, or otherwise, directly above or below each and in a horizontal plane through the instrument, and then the angle measured.

If the angular distance between the two objects *is very small*, then the angle between each and some third object in line with them may be measured and the difference taken. If greater than the range of the instrument, the sum of the angles between each and an intermediate object will be the angle required.

Where one of the objects is near the observer, it is better to sight at this one directly and bring the distant one into coincidence by reflection, to reduce the effect of *parallax* as much as possible, parallax being the angle subtended at the reflected object by the distance between the observer's eye and the center of the index glass.

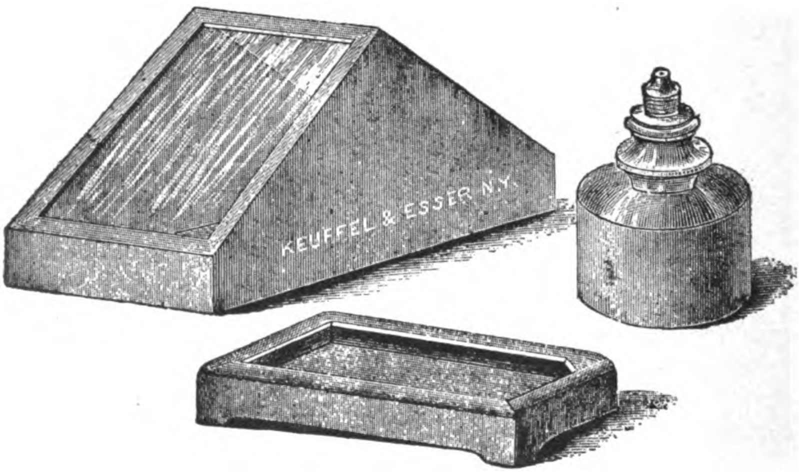

FIGURE 181.

Artificial horizon. With the sextant on land there is used, in observing altitudes, what is called an artificial horizon (Fig. 181). This may be mercury, oil, or water placed in a small shallow dish so as to furnish a horizontal reflecting surface. A glass cover is ordinarily used to protect the surface from wind and dust in making the observation. The angle obtained is double the altitude of the object, since it is the angle at the eye subtended between the object and its image in the artificial horizon.

Suppose (Fig. 182) O a distant object, as the sun, moon, or star, E the eye, and H the artificial horizon. A ray from O is received at E, a parallel ray strikes the surface of H and is reflected to E, giving the appearance of the object O at O′. Now with the sextant looking directly at the reflection of the object in the artificial horizon and bringing the double reflection of the object into coincidence with it, the angle OEO′ is measured, which is double the altitude.

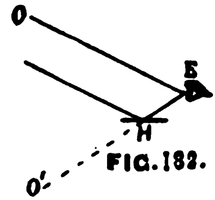

FIG. 182.

In making observations upon the sun and moon it is to be remembered that the artificial horizon inverts as well as the telescope, and since it is preferable to measure its altitude by getting the altitude of the upper or lower extremity of the vertical diameter and adding or substracting the semi-diameter for the altitude of the center, attention must be paid to the appearances of the images in the instrument. After getting the images in contact, if, by moving the arm outward, the lower image passes over the other, then the lower limbs were in contact, since the double reflected image was below. If, however, they separate, then the upper limbs were in contact, for the image reflected from the mercury finally appears erect to the eye, while the image from the mirrors is inverted and was above, and by moving the arm it is the one affected.

Measuring depressions and low altitudes. In measuring low altitudes and depressions some means must be arranged for establishing a vertical plane. This is frequently done by stretching a string about 3 feet above the artificial horizon and so placing the eye that when looking through the telescope vertically down on the string it will hide its reflection, then bringing the double reflection of the object into coincidence with the string. The difference between the measured angle and 90° will be the angle of elevation or depression.

Latitude with the Sextant. The double altitude of a star is measured directly by bringing the direct and the reflected images into coincidence. Then take ½ the double altitude, correcting for refraction and pole distance, for the latitude.

The altitude of the sun when on the meridian may be observed with either the mariner's or the pocket sextant, and from it the latitude of the place be determined as previously explained.

PROBLEMS.—1. *Resection with the Sextant.* Having three visible points, A, B, and C, plotted on a map in *a*, *b*, and *c* to find one's place *x*. (Fig. 183). Observe with the sextant, at X, the angles AXB and BXC. Since through any three points, as A, X, and B, or B, X, and C, not in a straight line, a circle may be drawn, and since an angle at the center (O or O′) of a circle is double an angle on the circumference at *x*, including the same arc of the circumference *ab* or *bc* as the angle at the center, the unknown point *x* may be found and plotted

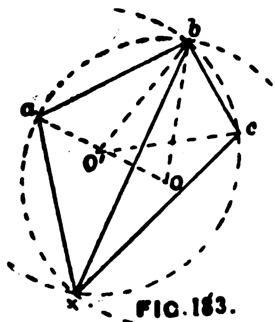

FIG. 183.

The operations consist in finding the centers O and O′, drawing the circles, whence one of their intersections will be the plotted point of X. To find the center O, double the angle AXB, subtract result from 180°, divide remainder by 2; at *a* and *b* lay off from the line *ab* this quotient *bao* and *abo* and the intersection of the sides will be O. To find O′, double BXC, subtract result from 180°, divide remainder by 2, and lay off at *b* and *c* from the line *bc* the quotient, *cbo′* and *bco′* the intersection of the sides being O′. The sum of the three angles of a triangle being 180°, double the angle observed at X being equal to the angle at the center O, then the other two angles of the triangle, as at *a* and *b*, must each be equal to the quotient laid off.

Should either of both of the angles observed at X be greater than 90° (Fig. 184), the center of the circle to be constructed will be on the opposite side of the chord, joining the plotted points, from the required point X. To find the centers in such cases,

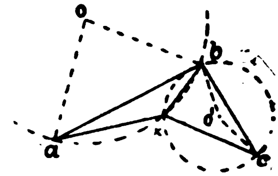

FIG. 184.

use the supplement of the observed angles, double, subtract from 180°, etc., and construct as explained.

2. *To set off a perpendicular to a line.* Set the index at 90°. Hold the sextant over the point of the line. When looking along the line, find or have set a stake coinciding by reflection with the line and it will be in the required perpendicular. Similarly, *to find where a perpendicular, let fall from a point without, intersects the line.* With the index at 90°, walk along the line until some point of it is found where the direction point of the line and the given point coincide.

3. *To measure the distance to an inaccessible object.* Let B (Fig. 185) be the inaccessible object whose distance from A is desired.

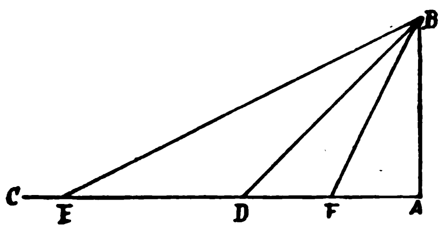

FIG. 185.

At A find with the sextant a distant object C in a line perpendicular to AB. Then set the sextant at 45° and move along AC until the point D is found where A and B coincide. Measure AD; it is equal to AB. With the sextant set at 26° 34′ AB will be ½ AE, and with it set at 63° 26′ AB will be twice AF. The following table gives a few angles at which to set the sextant, and corresponding multipliers of base to obtain the required distance:

14° 2′ = ¼.	18° 26′ = ⅓	26° 34′ = ½	45° = 1.
63° 26′ = 2.	71° 34′ = 3.	75° 58′ = 4.	

4. *To find the height of an object on level ground.* Suppose AB to be a vertical object the height of which is desired. and AC level ground. Make a mark on the object at the height of the eye, set the index at 63° 26′, 45°, or 26° 34′, and move back until the mark and the top of the object coincide in the sextant, when the height above the mark will be twice, equal to, or one-half the distance moved back, and, adding the height of the eye, the height of the object is obtained.

5. *To find the height of an object on level ground, but inaccessible at the base.* (Fig. 186). Find a point D where the top of the object A and a point B, the height of the eye, coincide in the sextant set at the angle of 26° 34′. Mark the point D. Set the sextant at 45° and move towards the object on the line BD until A and B again coincide. The point reached will be C; mark it and measure CD; it will be the height AB, to which add the height of the eye for the height of the object.

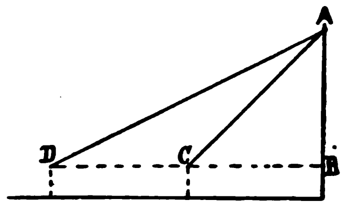

FIG. 186.

If for any reason it is impossible to use the point C in the line BD, but it is possible to find a corresponding point in a line at right angles to BD, then at D observe the angle subtended by AB (Fig. 187). Find some distant object E in a line perpendicular to BD. Set the sextant at the complement of the angle observed at D and move along the line DE until the point C is reached, where B and D coincide. Measure DC and it will be equal to AB; to this add the height of the eye for the height of the object. This method is the more general one and is independent of fixed angles.

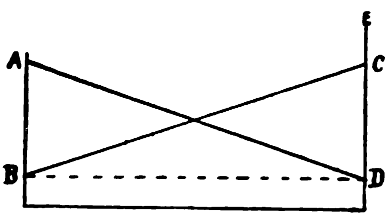

FIG. 187.

6. *To measure the distance between two points, both inaccessible.* (Fig. 188). At any point A measure the angle BAC. Set the sextant at ½ this angle and move back from A keeping aligned on C till B and C again coincide, then DA will equal BA. Similarly find E such that AE will equal AC, then DE will equal BC.

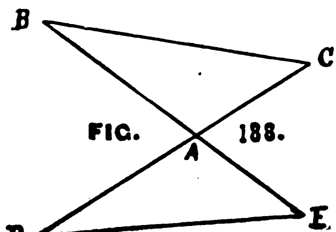

FIG. 188.

CHAPTER XIV.

THE ANEROID BAROMETER.

DESCRIPTION.—For determining approximate differences of elevations of points on reconnaissances and explorations, and sometimes in traverse work, the *aneriod barometer* (Fig. 189) is used. It consists of a flat cylindrical box of thin elastic metal with a corrugated top which communicates with an index through a train of mechanism. The box is nearly exhausted of air before being sealed, only enough being left in to resist or compensate, by its expansion, the increased pressure of the air on the greater surface of the box at higher temperatures. In some aneroids one of the levers is made of two metals (brass and steel) which expand and contract differently. This compensation for temperature simply refers to the instrument itself, freeing it from errors arising from changes of temperature, and in no way refers to the difference of temperature at the different points of observation, which must always be taken into account.

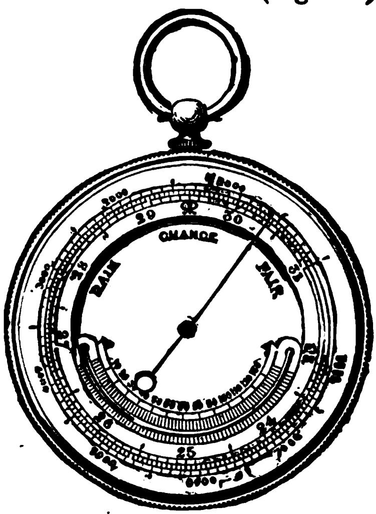

FIGURE 189.

The index moves over a dial having graduated on it either a fixed scale of inches and a revolving altitude scale; or, both the inch and altitude scales are fixed, and there is a revolving vernier scale. The usual altitude scale is a gradually diminishing one, but to enable a vernier to be used the interior action of the instrument has to be adjusted so as to give accurate readings upon a uniform altitude scale, and the scale of inches, which is usually uniform, has to be made progressive.

Several sizes are made, the 2½-inch being the most satisfactory. Owing to its extreme delicacy, it is a very uncertain instrument and should be used for only small differences of elevation and small intervals of time. Its indications should be checked by reference to known elevations whenever opportunity is afforded during the day, and at the beginning and ending of each day's work.

USE OF THE ANEROID.—What the aneroid actually does is to weigh the pressure of the atmosphere at the time of reading it in terms of a mercurial column expressed in inches and decimals; hence before the dial is graduated it is compared with a standard mercurial barometer. It does this by the rising and falling of the corrugated top, under different pressures, which rise or fall is multiplied several hundred times before being communicated to the point of the index. Since by means of *barometric formulæ* relative elevations may be obtained with the mercurial barometer after making certain corrections and reductions in the readings for *temperature, humidity, latitude, and gravity,* so the readings of the aneroid can be used in the same way for the determination of relative elevations, which is its principal use. Since the pressure of the air at any place varies considerably at different times from various causes, though no difference of elevation has taken place, all changes in readings cannot therefore be due to changes in elevation. But if two barometers, which have been adjusted and compared, be read at the same time at two points not too distant, of different elevation, under the same conditions, etc., then from these readings the difference of elevation of the points may be determined very closely. If the points be very distant, a long series of observations must be made to clear the results of local changes before the difference of elevation can be obtained.

Airy's Tables are prepared to show differences of elevation corresponding to different readings for a mean temperature of 50° F.; hence, if the mean temperature differs much from 50°, to determine the difference of elevation of points, the temperatures at the two points must be added together, and if the sum is greater than 100° F., the difference of elevation as obtained must be increased by its $\frac{1}{1000}$ part for every degree in excess; if less than 100° F., it must be diminished by its $\frac{1}{1000}$ part for every degree less.

A convenient formula under altitudes of 3,000 feet, giving approximate differences of elevations without the use of tables, is $D = 55{,}032\frac{H-h}{H+h}$ for a mean temperature of 55° F., in which H and h are the barometric readings in inches. For other temperatures apply $\pm \frac{1}{435}$ of itself for each degree above or below.*

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* When using but one instrument, Mr. Chas. A. Ashburner, Geologist of the Second Geological Survey of Pennsylvania, sometimes used the method originated by him of passing in the forenoon between stations as rapidly as possible, stopping at a number of them for half an hour or so, reading the barometer on arriving and leaving, and in the afternoon returning over the same route, repeating the operations. The difference of the two readings at any station indicates the rate of change for that time. From these isolated rates of change, on the assumption that changes between stops were regular, he constructed a continnous correction curve for the day on profile paper (Fig. 190),

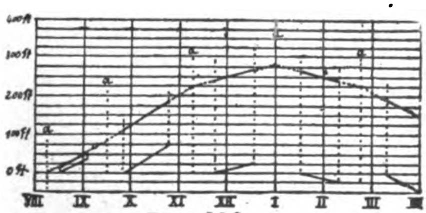

FIG. 190.

E.A.R.

from which he obtained, by scaling, the probable corrections to be made to the reading, due to changes in atmospheric pressure, to obtain the correct altitudes or differences of elevations. For constructing the curve the *scale of time* is taken horizontal, the interval of time between the verticals being 30 or 60 minutes. The *scale of feet* (difference between readings during stops) is taken vertical, the inter-

Precautions. In using the aneroid it should be kept in the case so as to be protected from sudden changes of temperature, the influence of the heat of the hand, or body, or sun; before reading, it should be swung backwards and forwards, or the glass cover tapped, to overcome any friction of its parts; it should always be held, preferably horizontal, at a constant height from the ground when being read.

Advantages. The aneroid is more portable and indicates changes of atmospheric pressure more quickly than a mercurial barometer, for which reasons it is particularly adapted to reconnaissance and exploration surveys.

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val between horizontal lines being 5, 10, or 25 feet. The changes at each stop are first plotted from the zero horizontal line between the times they were made; thus, if at a stop between 8:30 and 9:06 A. M. a difference of reading of 25 feet occurred, on the zero line at 8:30 o'clock a straight line would start and rise to 25 feet on the 9:06 o'clock vertical, and so for all other stops. The hour distances between stops are then bisected (*a*) and verticals erected at these points. Beginning where the first bisecting vertical intersects the zero horizontal, a line is drawn parallel to the profile of the change at first station to the second bisecting vertical, then from here another line is drawn, parallel to the profile of the change at second station, to the third bisecting vertical, and so on. The profiles of the changes first plotted are then projected vertically upwards on the broken line just drawn between the bisecting verticals, and the extremities of the profiles thus projecting upwards are then connected by a curved line to represent the changes in the barometer readings between stops due to changes in the atmosphere. The corrections to be made to the barometer's reading at any station may now be obtained directly from the correction curve.—*Haupt.*

PART II.

Topographic Sketching.

CONTENTS.

PART II.—TOPOGRAPHIC SKETCHING.

CHAPTER XV.

MEASUREMENT OF DISTANCES.

It is only after considerable study and practical experience with instruments of precision that the topographer can hope to make a comparatively accurate graphical projection (or sketch) without their aid. Before proceeding to the explanation of topographic sketching, it will be necessary to consider the instruments used in and the methods of measuring distances, angles, and elevations, in the same order as in more accurate work.

TELEMETERS OR RANGE-FINDERS.—These instruments are called telemeters (distance-measurers) and range-finders indiscriminately. If used to determine *distances*, when making a *survey or reconnaissance*, they are called telemeters; but if used to determine *ranges* on the field of battle *for firing*, they are called range-finders.

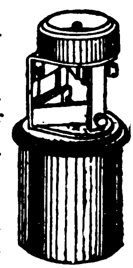
The importance of determining accurately the distances to objects in sketching is becoming more and more necessary. The estimation of distances by eye, with even approximate accuracy, over familiar ground and at short ranges is a difficult matter. To do so over varied and unknown ground and at longer ranges is so difficult and gives results so unreliable as to be almost useless for practical purposes.

The following points are desirable in Telemeters to make them applicable for all military purposes: cost moderate, construction simple, absence of delicate manipulation, not easy to get out of adjustment but capable of being re-adjusted, easy to learn to use; not easily damaged but easily repaired, weight light, size small to be held and used in the hands, giving ranges quickly either directly or by a simple multiplication or division, not requiring over two men, requiring short base, capable of being used kneeling or lying down.

Telescope Telemeters, consisting of *micrometer eye-pieces* fitted to ordinary telescopes, are useless for practical purposes and are classed as optical curiosities.

The *principle* upon which the following range-finders are constructed is that of measuring the base and the two adjacent angles of a triangle, and then partially solving the triangle, the distance or range being the required side.

The Pratt consists of a triangular frame holding two pairs of mirrors capable of being adjusted to certain angles by means of adjusting screws. On top of the frame is attached a small compass. In the back of the frame, between the two pairs of mirrors, is a small rectangular opening for sighting through. The frame is screwed for use on top of a cylindrical case, in which it is carried when not in use. On the bottom of the case is a small ring for attaching a plumb-line. The upper pair of mirrors is set at an angle of 45°. The lower pair is generally set at 44° 17′ 02″, the tangent of twice which is 40, though it may be set at one-half of other angles, as 87° 08′ 15″, 88° 05′ 27″, or 88° 51′ 15″, whose tangents are 20, 30, or 50, respectively. Weight is 2.1 ounces.



PRATT.

To adjust the instrument: Set up a transit over a marked stake. Find two small well-defined distant objects, or set range poles at 90° from each other; also two objects, or set range poles at 88° 34′ 04″ from each other. Stand over the transit stake with the instrument and by means of the adjusting-screw of the upper pair of mirrors bring the two objects 90° apart into coincidence, one seen through the opening and one by reflection; then by means of the lower adjusting-screw bring the two objects 88° 34′ 04″ apart into coincidence. Repeat for a test.

The principle of its construction is similar to that of the sextant—*i. e.*, that by reflection at two plane surfaces, a ray of light is deviated through an angle double that made by these surfaces.

The instrument requires occasional testing, as a change of only 3′ in the angle of a mirror will introduce an error of $7\frac{1}{2}$ per cent of the length of the range. The necessity for frequently testing the adjustments is a fault. The mirrors are too small. The effect of temperature, although undetermined, is to change the angles between the mirrors.

In determining a distance with this instrument there are used two fixed angles, and a base whose ratio to the distance required is also fixed.

FIG. 1.

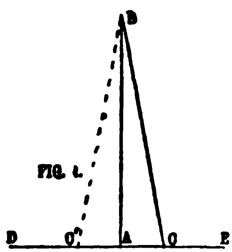
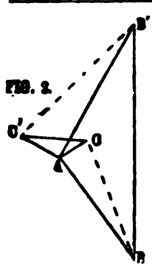


FIG. 2.



To determine a distance A B (Fig. 1), it is held vertically over A; the operator, looking toward B directly through the opening between the pairs of mirrors, turns it until a well-defined line on some distant object D (called the *direction point*) is seen by reflection in the upper pair of mirrors in coincidence with some well-defined line of B. The angle subtended by these two lines will be twice that between the mirrors, or 90°. Then placing a stake vertically in the ground at A, he moves off to the right towards E, keeping aligned on A and D, until a point C is reached at which, looking at D through the opening, the lines on B and D before selected again coincide (B by reflection) in the lower pair of mirrors. Measure AC and multiply by the tangent of twice the angle at which the lower pair of mirrors is set, 40 in case above, and the product will be the distance sought. Care must be taken that vertical objects appear vertical when reflected.

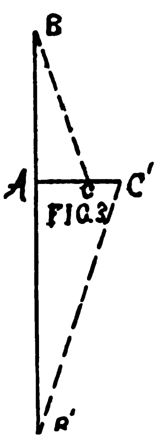
A base to the left of A may, if more convenient, be used by taking a direction point to the right, as E, and looking into that mirror of each pair not looked into with the base to the right; or, D may be viewed directly and B seen by reflection.

By observing the caution to select the ***distant*** object **D** as the direction point, very slight departures from the line AD in moving to C will not materially affect the angle BCD; were D near A, the result would be different.

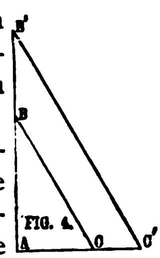
The movement from A away from D enables the observer to align himself on A and D.

It may also be used for determining the *distance between two inaccessible objects*, as B and B′ when standing at A.

1st. When the observer is *not* on BB′ (Fig. 2, p. 245) proceed as if to determine the distance AB, as previously explained, simply marking the extremity C of the base AC, without measuring it. Then proceed as if to determine the distance AB′, again simply marking the extremity C′ of the base AC′; now measure CC′, and multiply by the multiplier for which the instrument is set, and the product will be the distance BB′. In cases like this, where the observer is not on the line BB′, it is necessary that the direction of both the bases AC and AC′ be taken to the same side of A—that is, both to the right or both to the left of A when facing the distant station. In other cases this is not essential.



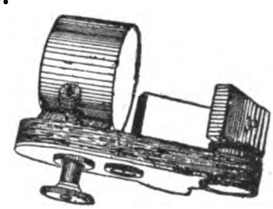
2d. When the observer is *on the line* between B and B′ (Fig. 3), the sum of the bases AC and AC′ is used with the multiplier for the distance BB′. If on the line, but below both B and B′ (Fig. 4), then the difference of the two bases is used with the multiplier.



With two observers, two instruments, and a tape-line, the distance of a *moving object* may be determined. The two observers face each other. One holds the end of the tape and brings the other and object into coincidence in the upper pair of mirrors by moving to right or left. The other observer, by moving backwards or forwards, brings the first observer and the object into coincidence in the lower mirror; when by concert of action both have the coincidence at the same time, the distance between them is the base to use with the multiplier.

GREEN

The Green consists of two pairs of mirrors mounted in a frame as shown, one pair, marked 1 on the back below the two screws seen in cut, being set at 45°, and the other pair, marked 2, being set at 44° 25′ 37.5″. It is also provided with a ball-and-socket joint and a strap for fastening on the wrist. This instrument is used in the same manner as the Pratt. The multiplier, however, is 50 instead of 40.



GORDON.

The Gordon consists of a horizontal bed on which are mounted two small mirrors 2″×1″.2, both vertical and capable of adjustment like the glasses of a sextant; it has a ring for attaching it to an object-glass of a binocular, the field of which is thus half covered by one of the mirrors.

This mirror can be moved slightly around a vertical axis by a nut below the horizontal plate which turns a cam, operating on a lever; a graduated disk underneath gives the reading. The disk is graduated from 0 to 250 and revolves 360°, while the mirror revolves but 3°, from 45° to 48°. Weight, 2.5 pounds.

To determine the distance AB (Fig. 1, page 245), set the disk at 0 and so as to give increased readings as it revolves. This sets the mirrors at 45°. With them above the horizontal plate at A, sight over the mirror at a distant direction point D (Fig. 9), and find a well-defined line or point of it which coincides with a line of B, the latter seen by reflection. Then move from A towards D to a point C′ on the line AD; the length AC′ should increase as AB increases. At C′, again sighting towards D, turn the disk

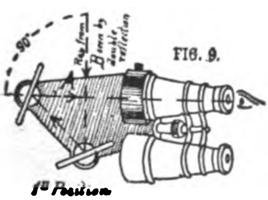


FIG. 9.

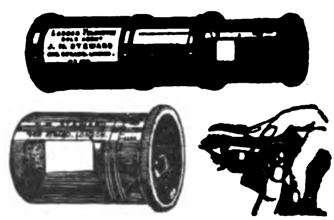
until the lines B and D are again brought into coincidence. Take the reading of the disk and measure AC′. Divide the latter by the disk reading and multiply the quotient by a number corresponding with the disk reading, which is found in a table furnished with each instrument.

A base to the right may be used by sighting towards B and finding E by reflection.

The principle of its construction is similar to that of the sextant. With one fixed angle, 90°, a varying obtuse angle DC′B and a varying base are used. By taking certain of the varying angles the ratio of base to distance may be fixed as in the Pratt.

A single observer must virtually select two objects on which to align himself as he moves from A to C′.

The Labbez consists of two plane mirrors set in a cylindrical tube, the front end of which is partially, and the rear end entirely open.



LABBEZ.

The principle of its construction is similar to that of the sextant. The front or horizon glass has a movement, around a vertical axis, of 4° by means of a small toothed wheel on the front end. The rear or index glass has a movement of 5° by turning the front end of the tube. With both glasses at 0°, the angle between them is 45° and may be varied between 43° and 47° by moving the toothed wheel. Keeping the toothed wheel at 0° and turning the front end of the tube, the angle will vary between 45° and 50°.*

* Were the mirror set at 47½° at A, with D′ for the direction point, the instrument should read 1,809.5 yards for a distance AB of 1,800 yards, and 504.5 yards for AB of 500 yards using 80 yards base. The reason for this, having moved the toothed wheel 2½° until B when reflected coincides with D′, and having moved 80 yards to C′, if now D′ be sighted again, an object at B′ should coincide with it, B′C′ being parallel to AB. To bring B into coincidence again with D′ the index mirror must be moved over ½ the angle BC′B′, or practically the angle

Accompanying it is a line *30 yards* in length, used for measuring the base. The ranges corresponding to the various positions of the index-glass are marked on the cylinder of the revolving end. This instrument is made in two sizes, to both of which telescopes may be attached. Distances may be determined with them from 250 to 3,000 and 5,000 yards, respectively. Weight of smaller size with telescope, $6\frac{1}{2}$ ounces; with both line and case, 11 ounces.

To determine a distance AB (Fig. 5), open the slide on the front end. Set both mirrors at 0. Look through the instrument over the mirrors for a direction point D coinciding with the reflection of B. If none such be available, either set one at any distance over 60 yards, or move the toothed wheel on front end to right or to left until a direction point D′ or D″ is found. Fasten one end of the line at A, move 30 yards toward the direction point to C, C′, or C″, where, facing the direction point as at A, turn the head of the cylinder until coincidence with B is again obtained. The required distance in *yards* is then read off the cylinder.

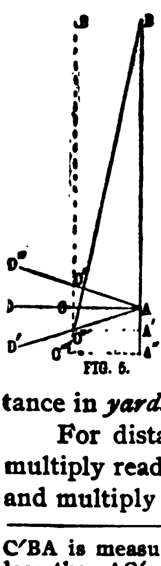


FIG. 5.

For distances over 1,000 yards, use 60-yard base and multiply readings by 2; over 2,000 yards, use 90-yard base and multiply readings by 3.

C′BA is measured. Now C′A′, a perpendicular to BA prolonged, is less than AC′, or less than 30 yards; produce BA and BC′ to A″ and C‴, respectively, the distance A″C‴ being 30 yards long and perpendicular to BA. Then will AA″ be the error in distance corresponding to the angle C′BA. With a base of 1 to 100, the maximum error is said to be about 1 in 30; with a base of 1 to 50, about 1 in 70; so the errors due to the toothed wheel are within the maximum errors of the instrument.

BA″ : C‴A″ :: BA′ : C′A′ or $d+E : b :: d+b \sin. a : b \cos. a$; hence

$$E = \frac{d \pm b \sin. a}{\cos. a} - d,$$

in which E = error, d = true distance, b = base, a = angle by which b departs from being perpendicular to d; upper sign for D′, lower for D″.

The Watkin Mekometer consists of two rectangular aluminum boxes, one for each end of the base. Each box has two mirrors inside, and is fitted with a removable handle, a small telescope, and on top a folding flap on which is an ivory strip. Accompanying each box is a reel on which are wound two cords, each $12\frac{1}{2}$ *yards* long, and having a spring hook on each end. One instrument,

known as "the right" or "No. 1," is the larger and views the object directly through an opening in the end, and the ivory of "the left" or "No. 2" by double reflection from the two mirrors through an opening in the left side. A drum on the right side, engraved with distances in *yards*, when revolved, slightly changes the angle of the mirrors inside and allows an exact coincidence of the object and the ivory of "the left." The reading on the drum opposite the fixed index gives the distance in *yards*. Weight $1\frac{1}{2}$ pounds. "The left" or "No. 2" instrument has no drum and the angle of the mirrors is fixed at 45°. With it the object is viewed directly through the telescope, and the ivory of "the right" by double reflection from the mirrors through an opening in the right side. Coincidence is obtained by moving backwards or forwards. Weight 1 pound.

To determine a distance AB (Fig. 1, p. 245), both observers screw in the handles, hook the ends of their cords together, and separate until 25 yards apart (2 cords), then attach the other ends of their cords to the hooks on the handles. The flaps are raised. The left observer, "No. 2," places himself in the right angle at A by moving backwards or forwards until the object seen by direct vision is coincident with the ivory of "No. 1" seen by double reflection. The right observer, "No. 1," at C, then brings the object and the ivory of "No. 2" into coincidence by turning the drum, and reads off the distance. If the distance of the object is greater than 1,500 yards, a base of 50 yards (4 cords) is used, and the readings on the drum doubled. If the distances are short and space restricted, a base of 12½ yards (1 cord) is used, and the readings on the drum halved.

If the object observed is moving, "No. 2" swings his body, backwards if the object is moving from right to left, or forwards if from left to right; and keeps exact coincidence while "No. 1" is obtaining coincidence with the drum of his instrument.

Of the mirror instruments described, the Pratt and Green can be considered as having fixed angles.*

WELDON.

A couple of prism instruments, having fixed angles, will now be described.

THE WELDON† (original pattern) has three glass prisms. Two of these prisms, one having an angle of 90°, the other 88° 51′ 15″, are cemented together back to back in a ring ½ inch thick and

*A pocket sextant in adjustment or of known index error may be used for determining distances by setting it successively at the same angles as are used in the Pratt and Green.

†A ray of light passing from air into glass is refracted (bent from its original direction) and makes in the glass a less angle with the normal (perpendicular) to the surface of entrance than it did in the air. Again, no ray of light lying in the plane perpendicular to the face of a prism will pass, by two refractions only, through any prism whose prism angle is greater than twice the greatest possible angle which the ray can make, *in the glass after refraction*, with the normal, but will be reflected at the inside surface of the second face. This angle, called the *critical angle*, is about 42° for glass. Since this angle of 42° is the greatest angle which a refracted ray in a prism can make with the normal after it has entered, it follows, as emerging rays follow the same law regarding critical angle as entering rays, that it is also the greatest angle of incidence which a refracted ray in a prism can make with the normal and still emerge from the prism. Hence, if a ray of light enters and is refracted at one face of a prism and then falls on another face of it at an angle with the normal to that face greater than 42°, it can not pass out, but must be reflected by that face. Further-

$1\frac{1}{8}$ inches in interior diameter and revolve on a diameter of the ring. The third has an angle of 74° 53′ 15″, and is cemented in the handle. The shape of this instrument admits of its being carried in the pocket. It weighs but 4 ounces, is not subject to derangement, and gives greater clearness of image than the mirror instruments.

To determine a distance AB (Fig. 6, p. 253). Standing at

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more, if a third face of the prism be silvered, incident rays at any angle will be reflected by it.

As two of the prisms have angles greater than twice 42°, the refracted rays used suffer other changes of direction in the prisms, due to reflection. Though the angle of the third prism is less than twice 42°, the frame intercepts the passage of those rays which would suffer double refraction only.

Now follow a ray of light from any object B (Fig. 10). Striking the outside of the first face of the prism, it enters, and is refracted in the prism, strikes the inside of the second face, but at an angle with the normal greater than the critical angle; hence, instead of passing out again, it is reflected by the second face to the silvered third face, where it is again reflected back to the inside of the second face, but at a less angle with the normal than the critical angle; hence it can pass out of the prism into the air, where it is again refracted, making the same angle with the normal to the second face that it did with the normal to the first face before entrance. Should a ray strike the first face normal to it and emerge normal to the second face, it would have been deviated through an angle equal to the angle of the prism. Should it not strike the first face normal to it, and still be required to emerge from the second deviated through an angle equal to an angle of the prism, it must, when emerging from the second face, fall on that face at an angle of incidence equal to its angle of refraction at the first face, and in its intermediate passage have been reflected at the second and third faces. Hence, these latter two faces must make an angle of half that of the prism. Of the 90° prism, either angle at the base is 45°. Of the second and third prisms, only the angle at the base between the second and third faces is half the prism angles. The angle between the first and third faces, being non-available, is blackened. These partially covered first faces are the ones which must be turned towards the object from which rays are to be deviated from the prisms.

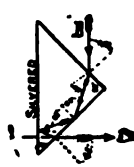


FIG. 10.

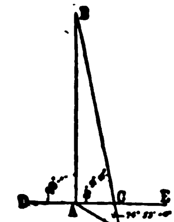


FIG. 6.

A, face towards D at right angles to and to the left of AB. Revolve the ring containing the two prisms until the silvered face of the 90° prism is parallel to the case ring. Turn up the hinges so that the images may be continuous. Hold the instrument with the 90° angle towards the eye (Fig. 7). Notice what well-defined line of B (Fig. 6), seen in the left face

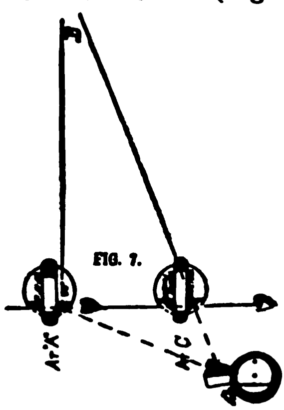
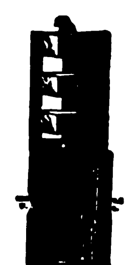


FIG. 7.

of the prism, coincides with a similar line of D, seen directly above or below the prism. Place a stake at A, move backwards aligned on AD, at the same time revolving the prism ring over in the case ring until the edge of the 88° prism is towards the eye and the partially blackened first face is towards B. Stop at the point C where the same coincidence is obtained with this prism as with the first. Measure the distance AC and multiply by 50, the tangent of 88° 51′ 15″, and the product will be the required distance. Should the base be very long, or the ground such as to prevent measuring AC directly, the points A and C having been marked by stakes, the observer moves backward on the line BC until he reaches a point, as F, where, holding the back of the handle towards B and the partially blackened face of the prism towards A, the coincidence is obtained of A in the prism and C seen either above or below. Measure CF and it will be one-fourth of AC or $\frac{1}{200}$ of AB.

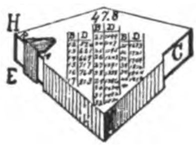


Weldon Range Finder.

When using the Weldon, on revolving it slightly above a vertical axis, some images will be seen to move past very quickly. These must neither be used or confounded with those that remain steady. They result from surface reflection.

The distance between two inaccessible points may be obtained as with the Pratt.

In the improved Weldon, the three prisms are held in a small rectangular lid, having openings between them for sighting through. It has also a small magnetic needle and a spirit-level. The average error of this is said to be only 2½ to 3½ per cent.



SOUCHIER

The Souchier consists of a 5-sided prism of glass, about ⅜-inch thick, enclosed in a metal case, excepting at the two points C and E, called windows, the latter being provided with a sliding shutter, which half closes it. On one face of the cover is engraved a table, giving opposite certain lengths of bases the corresponding distances, and a multiplier to be used for lengths of bases not found in the table. The instrument weighs 1.5 ounces. The principle involved is that of refraction of rays of light passing from air into glass and out again. The angles of the prism are supposed to be, at C, 67° 30′; F, 90°; E, 177° 50′; H, 69° 40′; top, 135°; but they are not always exact; hence the multiplier to be used is given with each instrument. On account of the large angle at E, the angles BC′D and BAD differ by about 1° 10′; hence the multipliers vary between 47 and 52.

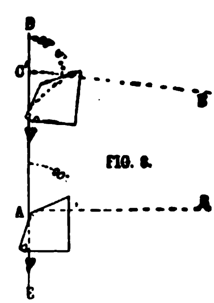


FIG. 8.

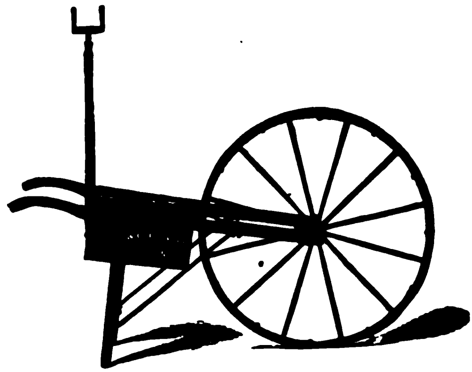
To determine a distance AB (Fig. 8). Standing at C′, face the direction point D. Hold the instrument in the left hand between the thumb and first two fingers, all well bent, the prism horizontal, with uncovered window C toward the object B. Place the shutter over the right half (A) of the window E. Looking in the left half (R) of this window, find a line of the image of B by reflection, coinciding with a similar line of some object D seen above or below the prism. This measures the angle DC′B. Place a stake at C′, move backwards, aligned on C′D, until some point as A is reached, where by looking in the right half (A) of the window E, the shutter having been moved over the left half (R), the coincidence of the same lines of B and D again occurs. This measures the angle DAB. Measure AC' and look in table for the distance corresponding to it; or, if not found there, multiply it by the multiplier of the instrument, the product being the distance. It may be used for find ing the distance between two inaccessible objects.

The most serious objection to range-finders like the Pratt, Green, Weldon and Souchier, requiring bases which bear a fixed ratio to the range, is that for long ranges a long base is required; and that it may happen on arriving at the end of the base fixed in length and direction, that some intervening object prevents any view of the object whose range is desired. The advantage is in being able to quickly determine the distance between two inaccessible objects, which is peculiarly the property of fixed-angle telemeters.

The great advantage in range-finders like the Gordon, Labbez, and Mekometer, is in allowing some latitude in choice of the length or the direction of a base, or both, while in all fixed-angle instruments the base is proportional to the range.

A use to which telemeters may be put is to measure short distances, such as the width of a river, across which to throw a bridge. In military sketching, the taking of offsets from traverse lines can be performed with considerable accuracy with the right-angled prisms or mirrors and with them one can place himself exactly on the line connecting two distant objects when he is between them. For the latter purpose, on looking into the two faces of the right-angled prism successively the objects and third point should seem to coincide. If they cross, he must advance; if they diverge, he must retire.

THE WHEELBARROW ODOMETER OR PREAMBULATOR is, as its name indicates, an apparatus similar to a wheelbarrow, used for measuring distances when wheeled along a road or path by hand. The wheel is large and light. The number of revolutions is counted by an odometer or otherwise, and, when multiplied by the circumference of the wheel, gives the distance passed over. One form has a box between the handles for carrying necessary instruments, an upright staff with a vernier pocket compass with a $3\frac{1}{2}$-inch needle for taking bearings, and a positive motion odometer for counting the revolutions of the wheel. All metal work is of brass, so as not to affect the compass.



It is especially adapted to places where wagons can not be taken.

CYCLOMETERS are instruments which record the number of revolutions of a wheel of a bycicle, or give the distance directly.

PEDOMETER.—This is a small instrument, about the size and form of a watch, used for recording the number of steps taken in walking, from which the distance traveled can be computed. By means of a small weighted lever, which descends with every step, motion is communicated to a mechanism composed of a train of wheels, and the number of steps is recorded on a dial by pointers.

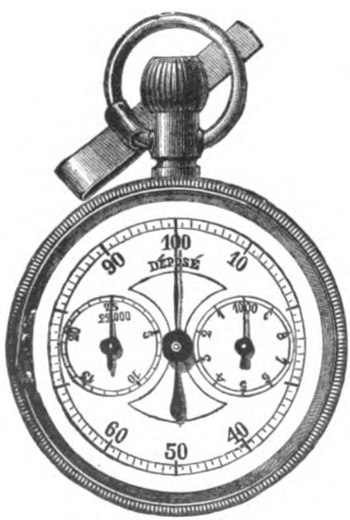
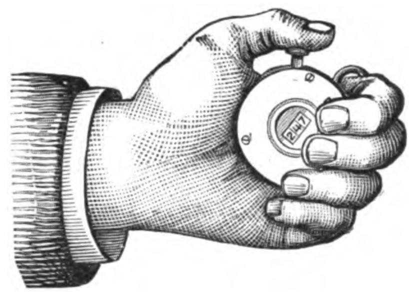


FIGURE 195.

There is another pattern arranged to indicate distances up to 12 miles in $\frac{1}{4}$-mile units, and another up to 50 miles in 80-yard units. The hand advances in proportion to the length of the step, after being adjusted to this length of step of the bearer by an adjusting-screw.

By the use of an instrument called a Tallying Register (see figure), the record of strides may be kept by the topographer without danger of making mistakes, such as dropping or adding one or more hundreds. Each stride should be recorded as it is taken; after practice, the impulse necessary to make the instrument record becomes almost involuntary.



PACING.—The average length of a person's pace having been determined by walking at an easy, natural, uniform pace over a measured distance a number of times, it can be used for measuring distances to within 2 or 3 per cent of the truth. The average step is about 31¾ inches long. The length of the military pace in quick time is 30 inches, in double time 36 inches. On slopes the horizontal projection of the step is usually shorter than the step on level ground, whether one goes up or down hill. One should always take his natural step.

If distances are measured on sloping ground with what is known to be a full-length pace, then deductions must be made to reduce them to corresponding desired horizontal distances. This deduction is about 1 pace from every 250 paces on a 5° slope, 1 pace from every 63 paces an a 10° slope, 1 pace from every 29 paces on a 15° slope, and 1 pace from every 17 paces on a 20° slope.*

A better method, however, would be for one to ascertain the exact horizontal length of his pace by actual trial over measured distances on slopes of different degrees; then by an observation with an instrument or by estimation ascertain the

*As extreme error is only about 3 per cent (the limit to be counted on in ordinary pacing), it is not customary to apply this correction in ordinary small surveys.

degree of the slope to be paced and multiply the number of paces by the length of pace on that slope. Distances of varying lengths over level and sloping ground of varying conditions are measured with a chain or tape and stakes driven at the extremities. An observer passes along one of these lines, having selected before starting an object in prolongation of the line joining the stakes so as to enable him to walk in a straight line, since two direction points are necessary for one to do this. He counts his paces, being careful that they are of the length he would naturally take in walking. Arriving at the other stake, he will record his results with any other observations that may have influenced the length of his pace. He then returns over the same line in the same manner and continues the practice until satisfied with his results. Then he does the same over each of the other lines, until finally he has a record of the length of his paces under all conditions, and can construct scales for any work, or make any necessary corrections.

Care must be taken not to permit the approach to a stake to influence the length of pace, the tendency being to alter it to conform either to the known or esitmated distance.

Reliable pacing is limited to slopes not greater than 12° to 15°.

Horse paces. The lengths of a horse's paces at the different gaits can be determined by riding him over measured distances a number of times and taking a mean of the results of the trials. This furnishes a very satisfactory method of measuring distances when sketching mounted, by counting the number of paces* and referring them to a scale of paces constructed from the data so obtained. The length of pace will vary for different horses; hence the necessity for determining it for the horse used, and when done it will be found to be

*In practice it has been found more convenient to count strides, instead of steps or paces—*i e.*, to count the number of times the same foot strikes the ground; hence 1 stride equals 2 paces (or steps), either mounted or on foot.

quite constant. The average of 30 troop horses twice over a measured distance of half a mile, up and down hill, was 965 steps walking, or about 33 inches each, and 680 steps trotting, or about 47 inches each. The U. S. Drill Regulations gives the average step of a horse at a walk as 0.916 yards; at a trot as 1.22 yards.

The U. S. Drill Regulations (Cavalry) gives the maneuver walk at the rate of 4 miles an hour, or 1 mile in 15 minutes, or $117\frac{1}{3}$ yards in one minute. The trot at the rate of 8 miles an hour, or 1 mile in $7\frac{1}{2}$ minutes, or $234\frac{2}{3}$ yards in a minute; the canter at 8 miles per hour; the gallop at 12 miles per hour, or one mile in 5 minutes, or 352 yards a minute; the full gallop at 16 miles an hour.

TIME.—Distances are often measured by the time taken to pass over them at a known uniform speed. If sketching on horseback, this method is very useful. Having ascertained the time it takes a horse to pass over a measured distance by a series of trials, scales reading minutes and fractions can be constructed for plotting.

If the sketcher has provided himself with a *stop-watch* which he can fasten on his wrist (see Fig. 208), he can by this means do very satisfactory work, without any of the mental effort required in trying to keep count of paces and without the danger of dropping whole hundreds of paces, which is very commonly done.

SOUND.—Sound travels through the air at a rate of about 1,130 feet per second at a temperature of 70° Fahr. For each degree higher add, lower subtract, $1\frac{1}{7}$ feet. If the wind is blowing in or against the direction the sound travels, its velocity also must be added or subtracted. If the wind blows obliquely, the amount to be added or subtracted will be its velocity into the cosine of the angle it makes with the direction from which the sound comes. Consequently if an observer at one point fires a gun, or makes any other sound, and at the same time a visible signal, and an observer at another point notes the time in seconds (by a stop-watch or otherwise) taken for the sound to travel, he can approximate very closely to the distance between the observers.

ESTIMATION.—The most inaccurate, yet the most generally used, method of measuring distances for filling in details in hasty sketching is that of estimation by the eye. The art of quite accurately estimating distances can be acquired by careful practice, and it is of the greatest importance to the sketcher, as most distances up to 100 yards are estimated. This is done by a mental comparison with certain known distances, or it may be done on the principle of similar triangles by holding a rule at a certain known distance in front of the eye and seeing how great a space on the rule the object covers; from this its distance may be deduced, provided the size of the object be known.

The degree of approximation attainable after proper practice in the estimation of distances precludes the probability of an error of more than 10 per cent up to 300 yards, of more than $12\frac{1}{2}$ per cent up to 600 yards, of more than $16\frac{2}{3}$ per cent up to 1200 yards.

In determining distances by sight the estimate is based upon the distinctness with which the object can be seen, upon its apparent height when its dimensions are known, and upon a comparison of the extent of the ground between it and the observer with some other known distance which is either within view or so distinctly impressed upon one's memory as to serve accurately as a unit of measure. The distinctness with which any object at any particular distance is visible varies considerably with different men; hence no inflexible rule can be expressed. Actual practice must determine for every individual. As the atmospheric conditions and nature of the background greatly affect the degree of visibility of objects, the practice should be conducted in different varieties of weather and along lines variously situated with reference to the sun and any surrounding hills or woods, particular attention being paid to all these circumstances.

When the light shines directly on objects, or when they are light-colored, or when they are seen against a light background, their details are more clearly visible, and they appear nearer than they really are. So also if the observer's back be towards the sun, or the observation be made in winter when the air is dry and clear, or else just before or after a rain, or if the ground be level and of a uniform tint, or if it rise towards the object, the distance will appear less than it really is.

Under the reverse conditions the distance will appear greater. The tendency in looking for an elevation down to a lower level is to over-estimate the true distance and over-estimate the degree of slope, while in looking upward to a height it is just the reverse. On a wide plain of uniform color, if the eye be arrested by no intermediate points, the estimate will be generally too short.

CHAPTER XVI.

MEASUREMENT OF DIRECTIONS.

In Topographic Sketching the instrument ordinarily used to measure directions is some form of hand compass.

THE BOX COMPASS.—*Description. The rectangular box compass* consists of a circular brass box, from 2 to 3 inches in diameter, containing a needle and graduated circle or card, sunken its depth in a rectangular block of wood from ½ to ¾ inches thick, with a hinged cover. To prevent unnecessary wear, a lever and pin are so arranged that the needle is lifted from its pivot whenever the lid is closed.

FIGURE 196.

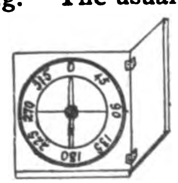


FIG. 197.

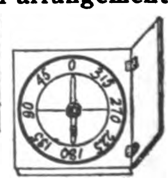


FIG. 198.

Or, as more frequently made, a cylindrical hole about ½ inch deep is bored in the block of wood. In the center of this hole is a pivot on which the needle is poised free to move. Around the edge on a plane with the needle is fastened a thin, flat, graduated ring. The usual arrangement for lifting the needle is also provided.

The card or ring is graduated into 360°, from 0° to the right 90°, etc., clockwise (Fig. 197), or from 0° to the left 90°, etc., contra-clockwise (Fig. 198). In one pattern the 0°—180° line is parallel to the hinged edge of the box. An edge of the lid, when raised at right angles to the block, is used for the line of sight. In another (Fig. 196) the line of sight is at right-angles to the hinged edge. A mark on the lid in prolongation of the 0°—180° line being used as the line of sight.

Use. To take a bearing with the first-mentioned pattern, raise the lid at right angles to the box and hold the latter horizontal. The lid being generally hinged on the right-hand side of the 0°—180°, the compass is held in the right hand at several inches in front of the eye. With the zero end of the 0°—180° line directed towards the object, sight along an edge of the lid at the object, watching the oscillations of the needle, which may be checked by pressing on the pin for raising it. The needle must be free, however, at the time of settling. When it has settled, read the north end of the needle. The mean of two or three readings may be taken as the correct one. With the second pattern, hold the compass with the lid towards the object and bring the line on the lid in line with the object and its reflection on the glass over the pivot, then read the north end of the needle. In both these patterns the line of sight and graduated ring both move around the needle, which remains stationary in taking bearings; hence, when figured from 0 at the north to the left (Fig. 198), the readings as given by the north end of the needle are the angles from the magnetic meridian at the north around to east, south, and west. When figured from 0 at the north to the right (Fig. 197), the readings of the north end of the needle are the angles from the magnetic meridian at the north around to the west, south, and east.

THE PRISMATIC COMPASS (Fig. 199).—*Description.* This compass differs from the box compass described, in that the graduated disk or card is fastened on top of the needle with the 0°—180° line coincident with it. The 0 of the disk is usually placed over the south end of the needle and the 180° division over the north end. On the south end of the box is a sight-vane with a prism which reflects the graduations on the disk up to the eye, while the eye at the same time sees the object observed through a slit in this sight, and the vertical hair in the leaf-sight on the north end. When the leaf-sight is folded down it raises the needle off the pivot. The swing of the needle is checked by pressing on a little button under the leaf-sight which presses a spring against the edge of the disk. On some prismatic compasses there is a mirror on the leaf-sight for reflecting elevated or depressed objects to the eye, and on the prism sight there are colored glasses to be interposed when observing the sun.

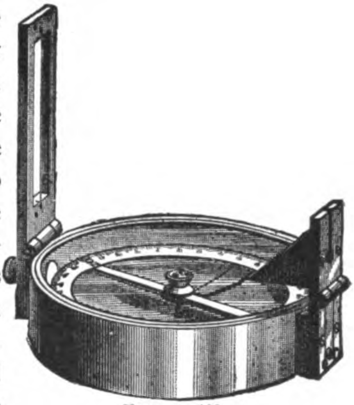


FIGURE 199.

Use. To take a direction with a prismatic compass, turn the prism sight up, slide the prism up or down until the graduations are distinctly seen through it; raise the front-sight; then, holding the prism sight near the eye, sight through the slit above the prism and bring the vertical hair in the leaf sight on the object, and when the needle comes to rest, read the division on the disk which the hair appears to cut. With the prismatic compass, on account of the disk being fastened on top of the needle and remaining stationary while the line of sight revolves around, the graduation of the disk under the eye is read; hence, in order to read zero when sighting north, the 0 of the disk is usually placed over the south end of the needle. If the disk is graduated from 0 clockwise (Fig. 197, page 262), the different readings will be angles from the magnetic meridian to the east, south, west, etc., but if graduated contra-clockwise (Fig. 198, page 262), the readings will be angles from the magnetic meridian to the west, south, east, etc.

Since in both the box and prismatic compasses the disk may, by accident or design, be so shifted that the 0—180° line may have any direction with the line of sight, there is an absolute necessity for always determining the direction of the true north by that compass before using it, in order to place a true meridian upon the plot.

THE PLOTTING DIAGRAM.—*To plot any angle taken with a box compass* graduated from 0° to 360°, the simplest method, requiring no calculation or study, is to first make on a piece of paper a diagram consisting of two straight lines intersecting at right angles. Mark one end of one line 0° and the other end of it 180°; place the box compass on the paper with the line of sight parallel to this line, the eye-end of the line of sight at the 180° end of diagram line. Revolve the paper and compass together until the north end of the needle comes to zero; holding the paper in this position, turn the line of sight to the right through 90° and read the north end of the needle; if this be 270°, which it would be if graduated as in Fig. 197 (p. 262); mark 270° on the end of the line to the right of the 0—180° line, and 90° on the opposite end (Fig. 200); if it reads 90° as when using Fig. 198 (p. 262), *vice versa* (Fig. 201).

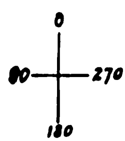


FIG. 200

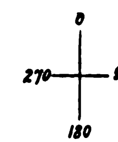


FIG. 201.

With a box compass in which the 0—180° line is parallel to the line of sight, and whose graduations are from 0 at the north around *to the right* (clockwise), then when looking *north*, by a reference to the diagram (Fig. 200) it will be seen that the north end of the needle will read zero; when looking *east*, the needle will read 270°; when looking *south*, 180°; when looking *west*, 90°. If graduated *to the left* (contra-clockwise), then when looking *east* (Fig. 201), the needle will read 90°; when looking *south*, 180°; when looking *west*, 270°. Frequently persons who are left-handed hold the compass in the left hand with the line of sight on the left of the 0—180° line and read the north end of the needle. By so doing, if graduated to the right, they read: north, 180°; east, 90°; south, 0°; west 270°. If graduated to the left, they read: north, 180°; east, 270°; south, 0°; west, 90°. But if they construct a diagram as described above, and revolve it 180°, as they have done with the compass, there need be no confusion from this change.

To plot any angle taken with a prismatic compass, the prism or eye-end of the line of sight is placed at the 180° end of the diagram line. The paper and compass are revolved until the 0 of the card comes under the prism. The paper is held in this position and the compass turned to the right through 90° and the number on the card under the prism placed on the end of the line to the right of the 0 of diagram.

With the prismatic compass in which the 0 is over the south end of the needle and the graduations are to the right (clockwise), it will read: *east*, 90°; *south*, 180°; *west*, 270°, as in Fig. 201. If graduated to the left (contra-clockwise), it will read: *east*, 270°; *south*, 180°; *west*, 90°, as in Fig. 200.

If, with either a box or a prismatic compass, a sight be now taken on a true meridian line looking north and the reading of the north end of the needle or number under the prism be plotted from the point of intersection of the diagram lines in the proper quadrant, a glance at the diagram will give the true relations and directions of all lines sighted with that compass. By a reference to this diagram the proper quadrant in which a line with any reading should be plotted is at once seen.

If plotting compass readings is carried on in the field simultaneously with the observations, this diagram and the direction of the true meridian should be put in one corner of the plot, always in sight. The observance of this rule will be the means of preventing many errors.

Plotting compass readings. A protractor may be used in two ways for plotting: *first*, (Figs. 202 and 203), by placing the center on the station point with the diameter along the 0—180° line through it and making a dot opposite the direction of the line on the edge of the protractor, then removing the protractor and drawing a line from the station through the dot; or, *second* (Figs. 204 and 205), by placing the center on the station and the direction of the line on the edge on the 0—180° line through the station, and drawing a line along the diameter of the protractor from the station for the direction. Which method will be preferred depends upon the compass used, the direction of its graduations, and those of the protractor.

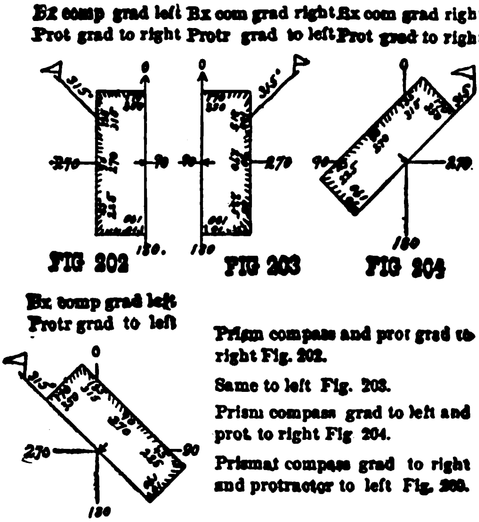


FIG 202 FIG 203 FIG 204

FIG 205

In using a *box* compass, and the protractor is graduated in the *opposite* direction to the compass disk, the *first* method will be the easier; but if the graduations on compass disk and protractor are in the *same* direction, then the *second* method will be the easier in both cases.

In using a *prismatic* compass with disk graduated in the same direction as the protractor, the first method is easier; if the disk and protractor are graduated in opposite directions, the second method is easier.

The *paper* should be prepared for plotting by drawing fine parallel lines about $\frac{1}{4}$ or $\frac{1}{8}$ of an inch apart, to represent the magnetic meridians to which all the readings refer. One end of these should be marked with a half arrow-head or N before beginning to plot, to avoid making errors.

If the *first method* described above is used, the protractor is laid on the paper with its center at the point from which the line is to be drawn and its diameter edge parallel to the meridian lines, one of which will not be over $\frac{1}{8}$ or $\frac{1}{6}$ of an inch from it. The diagram of the compass readings having been constructed, as described, reference to it at once shows whether the graduated edge of the protractor is to be laid to the right or to the left of the diameter or meridian line, and in which quadrant the line should be drawn.

If the *second method* of protracting is used, then through each station a meridian line is drawn before beginning to protract angles from it, the diagram showing the direction the diameter side of the protractor should take from the station point.

Resection with compass. With a compass one may find the place on a map corresponding to his position on the ground by "resection," when two distant plotted objects are visible. Thus, from the position, take the directions of the two objects, and with the protractor at the plotted position of each object plot their directions and produce them backwards until they intersect; this point will be the place. The accuracy of the resection depends upon securing a good angle at the point of intersection. This operation is useful in finding a convenient starting-point in sketching or for checking the accuracy of a traverse.

CHAPTER XVII.

MEASUREMENT OF SLOPES.

ABNEY LEVEL AND CLINOMETER (Fig. 206).—*Description.* This consists of a line of sight, a level-tube and arm attached, and a graduated arc. The body in which the line of sight is defined is square, so that it may be placed upon a surface and its slope determined by bringing the bubble to the

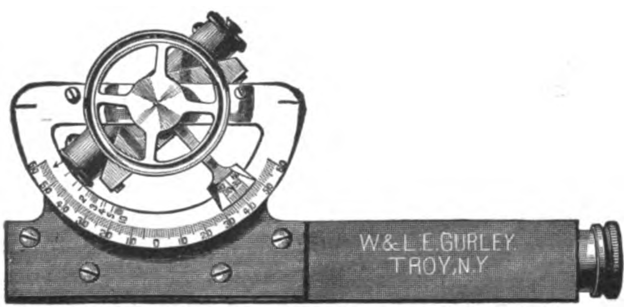


FIGURE 206.

center of its tube. Directly under the center of the bubble-tube is an opening in the body, and inside, occupying one-half the body, is a mirror facing the eye-end at an angle of 45° with the line of sight. A horizontal wire extends across the middle of the body in front of the mirror. When the instrument is held horizontal and the bubble brought to the center, on looking through the body the wire appears to bisect the bubble, seen by reflection in the mirror.

Use. When the line of sight is directed upon any object and the bubble brought to the center of its tube, or until the wire bisects its reflected image, the slope of the line of sight may be read on the graduated arc, which is divided to degrees and numbered each way from the zero to 60° or 90°. On the double vernier (Fig. 69, p. 61), on the end of the arm the smallest reading is 10′.

When the vernier is set at zero, the instrument may be used as a hand level to find points at the same elevation as the eye.

The graduations on the inner edge of the limb are the denominators, unity being the numerators of fractions expressing slopes, as $\frac{1}{5}$, $\frac{1}{10}$, etc. The graduation in coincidence with the front edge of the arm is read.

Adjustment: To test for index error. Place the instrument on top of a fence-post, sight to the top of another one several hundred feet distant, bring the bubble to the center, and read the vernier. Go to the other post, sight back, bring bubble to center, and read. One-half the difference of the readings will be the index error to be applied to all readings, being careful to observe whether it is to be added to or subtracted from angles of elevation, and *vice versa.* If the first observation was 2° elevation and the second 3° depression, the index error is $\frac{1}{2}$° to be added to all elevations and subtracted from all depressions.

To make the adjustment. If where it can be done, place it upon a smooth inclined surface, bring the bubble to the center, and read the vernier; reverse it end for end in the same place, bring bubble to center again, and read vernier. If in adjustment, the two readings will be the same; if not, $\frac{1}{2}$ the difference will be the index error. Apply the index error to one of the readings, set the vernier at that reading, place on the inclined surface, and bring the bubble to the center by the screws at the end of the level-tube. Repeat for a test.

SLOPE BOARD (Fig. 207).—*Description.* A substitute for the clinometer may be arranged on the field drawing board. If one edge is straight, this may be used as the line of sight. On the back of the board construct a graduated semi-circle with its diameter parallel to the line of sight, or paste it on a graduated semi-circular protractor printed or constructed on paper. The graduations should be numbered in degrees each way from the center. Then if a plummet or pendulum be suspended from the center of the diameter, and the line of sight be inclined, the reading of the arc opposite the thread or wire, will be the degree of inclination, for the thread retains a vertical position, while the board and attached arc are inclined.

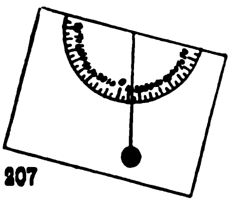


FIG 207

Use. To determine *a slope*, bring the line of sight parallel to it, with the plummet free of the board, then carefully tilt the board until the thread lies on the arc, secure it with the finger and read off the degrees. To determine *a level line*, with the thread passing through the 0 mark, stick a pin against one side of the thread, then hold the board up, until the thread just touches the pin, and sight along the edge. In order to bring the upper edge of the board in line with the slope, it may be necessary to stand back above it until the prolongation of the surface strikes the eye, or to kneel or lie down until the eye is on a level with the grass or some object judged to be at the height of the eye when the slope is taken to it.

CHAPTER XVIII.

CONVENTIONAL SIGNS AND SYMBOLS.

The conventional signs and symbols adopted for representing forms and features on a map are such as to suggest, if possible, the objects for which they stand. Ordinarily no effort will be made to show lights and shadows on military maps, but when they are shown the light is supposed to come from the upper left-hand corner at an angle of 45° with the horizontal.

WITHOUT COLORS.—*Forests* of deciduous trees, except oaks, are represented by signs suggesting irregular projections of trees with bushy tops. In representing oaks, the loops have their points out instead of in as shown.

Perennial trees, as pines, firs, etc., are represented by stars.

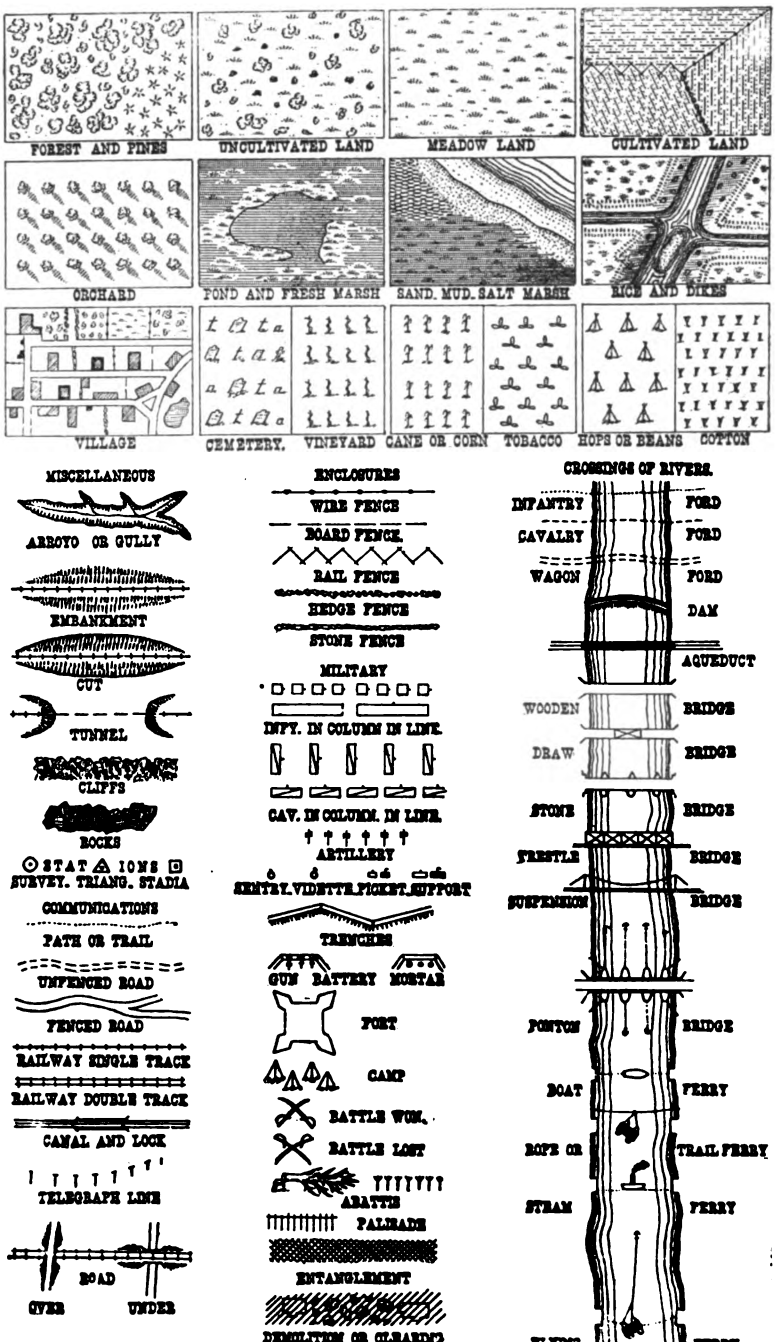
Uncultivated land, which is neither cleared nor forest, is represented by the signs for small trees, grass, and rocks if present.

Meadows or cleared land by signs suggesting tufts of grass, composed of 5 or 7 short lines, the middle one being longest; the bases straight and parallel to the bottom of the map; the tufts evenly but not too thickly distributed and not in lines; dots may be added to produce pleasing appearance.

Cultivated land by signs suggesting furrows, consisting of alternate broken and dotted lines, the breaks short but not opposite each other, the dots fine and close together.

Orchards by regularly distributed trees, sometimes shaded.

Marsh by parallel lines and tufts of grass; if of fresh water, the lines are broken and filled in with the tufts; if of salt water, the lines are continuous and the tufts made over them.


FOREST AND PINES
UNCULTIVATED LAND
MEADOW LAND
CULTIVATED LAND
ORCHARD
POND AND FRESH MARSH
SAND. MUD. SALT MARSH
RICE AND DIKES
VILLAGE
CEMETERY.
VINEYARD
CANE OR CORN
TOBACCO
HOPS OR BEANS
COTTON
MISCELLANEOUS
ARROYO OR GULLY
EMBANKMENT
CUT
TUNNEL
CLIFFS
ROCKS
STAT IONS
SURVEY. TRIANG. STADIA
COMMUNICATIONS
PATH OR TRAIL
UNFENCED ROAD
FENCED ROAD
RAILWAY SINGLE TRACK
RAILWAY DOUBLE TRACK
CANAL AND LOCK
TELEGRAPH LINE
ROAD
OVER
UNDER
ENCLOSURES
WIRE FENCE
BOARD FENCE
RAIL FENCE
HEDGE FENCE
STONE FENCE
MILITARY
INFY. IN COLUMN IN LINE
CAV. IN COLUMN. IN LINE
ARTILLERY
SENTRY.VIDETTE.PICKET.SUPPORT
TRENCHES
GUN BATTERY MORTAR
FORT
CAMP
BATTLE WON
BATTLE LOST
ABATTIS
PALISADE
ENTANGLEMENT
CROSSINGS OF RIVERS.
INFANTRY FORD
CAVALRY FORD
WAGON FORD
DAM
AQUEDUCT
WOODEN BRIDGE
DRAW BRIDGE
STONE BRIDGE
TRESTLE BRIDGE
SUSPENSION BRIDGE
PONTON BRIDGE
BOAT FERRY
ROPE OR TRAIL FERRY
STEAM FERRY
FERRY

Ponds by equidistant lines parallel to the bottom of the map, suggesting still water.

Streams and large bodies of water by lines suggesting waves along the shore. The high- and low-water levels by distinct shore-lines. If streams are not large enough to be water-lined, they are represented by a full sinuous line.

Dry runs by dashes and three dots between.

Sand and *gravel* by dots.

Mud by short dashes.

Buildings on small scales are usually shown as rectangular blocks; on large scales, by the shape of the building in outline, and filled with fine diagonal lines, the outline for wooden buildings being very fine, for brick heavy, and for stone very heavy.

Streets of towns and villages should be shown as they are.

Crops. Although signs are often employed to represent different crops, features, etc., it is not safe to trust their doing so without the name attached. Their employment is more for pictorial effect than otherwise.

Arroyos or gullies have their outlines sharply marked, and hatching lines are added to represent the slopes or wash of the earh.

Embankments or fills have the highest outlines sharply marked and hatching lines for the slopes or fall of earth.

Cuts have the cutting or highest line sharply marked and hatching lines for the slopes. The cuts up to the mouth of tunnels are shown. The tunnel itself is shown by broken lines.

WITH COLORS.*—The use of colors on hastily executed military maps will ordinarily be limited to a very few in number (four or five), and they will be applied with colored pencils which can be carried in the field. The forms of the various conventional signs are similar to those already described.

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*For a complete description of the use of water-colors on maps, and how to apply them, the student is referred to "Topographical Drawing and Sketching," by 1st Lieut. Henry A. Reed, U. S. A.

Wooden buildings, fences, telegraph poles, etc., are drawn in *brown*.

Brick buildings with a *light red border*, stone buildings with a *heavy red border*, all filled with a *light flat tint of red*.

Masonry bridges, stone fences, railroads, etc., in *red*.

Wire fences, metal bridges, and water in *blue*.

Dry runs by broken and triple-dotted *blue* lines.

Earth and sand in *yellow ochre*.

Cultivated land ruled with *brown*, over a flat tint of *yellow ochre*.

Trees, grass, and vegetation in *green*.

Rocks in *brown*.

Contours in *red* continuous lines of constant thickness, except every fifth, which is sometimes made heavier.

Lettering is usually done in *red*.

HILL SHADING.—A method much used in the past for shading hills was by short strokes, called *hachures*, drawn either perpendicular or parallel to the steepest slope, the thickness and number of which were regulated by a *scale of shade* according to the degree of slope, the steepest being darkest. As it has the effect of covering up details and was difficult to properly produce, it is being superseded by *lead shading*, which is more easily and rapidly executed and does not destroy details.

In *lead shading* the object is to produce a *transparent shade*, darkest where the ground is steepest. Its purpose is to give body and expression to the map. In this, rays of light are supposed to be vertical; hence horizontal surfaces are lightest or white. On slopes, the rays striking obliquely, the illumination is less bright, decreasing with the increase of the slope, being least for angles of 35° or upwards; hence slopes are shaded, the shade being denser as the slope is steeper.

How produced. Lead dust from a soft pencil is scraped on a piece of paper, lumps being carefully avoided. The map is placed on a smooth hard surface. With a piece of chamois skin folded into a firm point some of the lead is taken up and applied to the map by sweeping movements in the direction of contours, endeavoring to produce broadly the desired effect as regards depth of shade, the first application of lead being on the portions to be the darkest, representing the steepest slopes, fading away to the lighter portions. Care must be taken to avoid hill-tops and parts which are to remain light.

When the broad or general effect of shade has been produced, a fresh point is folded and the map rubbed with it, forcing the lead into the grain of the paper, blending the different shades and removing any mottled appearance; a fresh point is again folded, the blending continued, and so on until the desired effect is produced. Some portions, where very steep, may now require to be again darkened with more lead and other portions brightened with a pencil eraser and again blended. Very dark shades cannot be rubbed too hard without removing too much of the lead. The shading of abrupt rocky cliffs, etc., may be finished with a pencil. To bring out a small portion of high light, a hole of the required shape is cut in a piece of paper, held over the place, and the lead removed with an eraser.

Exaggerating the contrast of light and shade to produce *relief* at the expense of truth is to be avoided. Water-sheds may, however, be a little lighter than contiguous water-courses. Colors should be applied after the shading. The shading which has been unavoidably carried beyond the border can be removed by placing a piece of paper with the edge on the dividing line and using an eraser.

Lead shading may be fixed with a spray of gum arabic water.

CHAPTER XIX.

FINISHING MAPS.

LETTERING.—The final appearance of a well-drawn map may be anything but pleasing, as a result of poor or unsuitable lettering. The letters should be simple, neat, and of a size proportioned to the scale of the map, their position, and the objects to which they refer. Fancy letters are out of plaee and script should never be employed. When lettering on a map or sketch will not obscure valuable details, it is to be preferred to reference numbers, which are sometimes necessary, however, in conjunction with marginal notes or a separate report.

Styles used. The letters which are most useful are the **GOTHIC**, ROMAN CAPITALS, Roman lower-case, *ITALIC CAPITALS*, *Italic lower-case*. A very common style of lettering maps and other drawings is styled "Round Writing," in which all lines are made with a stub pen thus:

A B C D E F G H I J K L M N O P Q

R S T U V W X Y Z.

a b c d e f g h i j k l m n o p q r s t u v w

x y z. 1 2 3 4 5 6 7 8 9 0.

Another style, known as the "Marking Alphabet," is very easily and quickly acquired.

Marking Alphabet.

ABCDEFGHIJKLMN

OPQRSTUVWXYZ&

abcdefghijklmnopqrst

uvwxyz

When done. As a rule, lettering is done after the features have been drawn, but in pen drawing, large bodies of water, forests, etc., are lettered before the signs are drawn, to avoid blurring. In general, the lettering is parallel to the bottom of the map, but rivers, roads, etc., are lettered in the direction of their courses and so as to be most easily read.

The title. This is usually placed within the border in one of the corners of the map. The middle points of all the lines should be in a vertical line. The name of the locality is generally the most important and most prominent word, but should not exceed in height $\frac{1}{80}$ the length of the short side of the border. The letters of the other words may vary in size according to the importance of the words.

The name of the draftsman, date of the survey, and like information, appear in small letters.

The scales. Below the title or in some other convenient place should be drawn a graphical scale of distances, with a legend of "So many inches to the mile," etc, and, if desired, the representative fraction. In a contoured map a statement of the vertical interval between contours and a graphical scale of horizontal equivalents is added.

The meridian line or needle several inches long must always be drawn, as it determines the relative directions of all objects represented. When the survey has been made with a compass, the *magnetic meridian* is drawn, and also the *true meridian*, if the declination of the compass is known. These should be two lines intersecting near their middle, making an angle with each other equal to the declination. The true meridian is drawn with a full arrow-head, and the magnetic meridian with a half arrow-head on the side of the declination. When the map is colored, it is well to make the arrow-head red to draw attention to it. The needle should be assigned a prominent place on the map.

The border. The map is generally arranged so as to be enclosed by a rectangle, the sides being in north-and-south and east-and-west directions, with the top of the map towards the north, whenever practicable. The border generally consists of two lines, the interior one light, the exterior one $\frac{1}{200}$ of the small side of the rectangle in width, with an equal space between the lines. The heavy outer line may be filled in with a brush or shading pen.

CHAPTER XX.

MAP-READING.

Map-reading is the art of grasping quickly the correct meaning of a military or other map, and is only acquired by a careful study of the principles of topography and the ground, thus developing a just appreciation of the relative heights of hills, the steepness of slopes, and the shapes of features as shown by the contours and other symbols.

Every officer should be thoroughly competent to make use of maps both for strategical and tactical purposes, and should possess a good eye for ground, one of the greatest gifts, from a military point of view, a soldier can possess. He should be able, with the aid of a map, to move to any place pointed out, and to occupy it in any manner ordered. It is impossible for any soldier to discuss the simplest problem in connection with tactics or field fortifications without taking into full consideration the topographic aspect of the case, and having recourse to maps; hence the ability to read a map readily and to make use of it intelligently is one of prime importance to an officer.

AIDS IN MAP-READING.—*A thorough acquaintance with the conventional signs and symbols* is necessary, as well as the *ability to appreciate distances to scale.*

As an aid to learning map-reading, one may take a map of a piece of ground to some commanding spot on it, and there study how to identify his position on it, how to recognize the different forms and features, as water-sheds, water-courses, cols, etc., and how the contours curve for the different shapes and slopes they represent. To do this, it is first necessary to note the scale of the map, and to impress on the mind the spaces on the map corresponding to certain distances on the ground.

Orienting a map on the ground. The map is spread out, and, if possible fastened to a drawing-board or box lid, and then oriented.

This may be easily done as in orienting the plane table if one has a *magnetic compass* and the map has a magnetic meridian on it. If the map has a true meridian and the declination of the compass is known, a magnetic meridian can be drawn.

If no compass is at hand, and the exact spot occupied is not located on the map, it may be found by the *mechanical method* previously described; and then, if any distant conspicuous point or object can be identified on the map, it can be oriented by placing the edge of a ruler on the plotted points of the position occupied and the distant object, and the whole map turned until the distant object is sighted. By now pivoting the ruler on the plotted position of the point occupied, objects may be sighted, their distances estimated, and their plotted positions recognized.

Finding one's place on the map. If the map is oriented by the compass, but the position occupied is not plotted, it can be found by resection on two known plotted points that can be recognized.

One of the difficulties in making use of maps is, that they may not be corrected up to date, and time may be lost in trying to find roads that have ceased to exist, or in searching in vain for important buildings, railways, etc., that have been built since the map was drawn.

READING MAPS.—To read a map, first look to the *lie* and *direction* of the streams; note how the *water-courses are generally found to trend*, as this will indicate the lowest levels; next observe where the *features of ground project towards the streams*, as this shows the positions of water-sheds and spurs; *where they recede* shows the valleys.

If the heights of the contours are indicated, showing elevations above some datum plane, there can be no mistaking hills for hollows; the relative heights of different points are found by the differences of the numbers of the contours on which the points lie.

If the contours are not numbered, then the existence of water must be one's guide in distinguishing between water-sheds and water-courses. The lowest part of a country is usually occupied by the largest body of running water. From this the tributary streams may be traced upwards to their sources, thus locating the valleys, in which the contours curve outward from the main stream. The water-sheds lying between the valleys may next be distinguished by the contours curving towards the main stream. Having found the water-sheds, next locate cols or saddles connecting the different features. If the contours are not numbered and the points whose relative heights are desired lie on adjacent features, count the number of *contour intervals* to each of them, either from the col which connects the features, or the water-course which separates them, and multiply their difference by the interval between the contours. As *contour intervals are counted* and not contours, the first, or lowest, contour at the col or water-course, where the counting begins, must be considered as 0 and not 1.

SECTIONS.—One test of the understanding of map-reading is the ability to make *sections* of the ground on any required line. In fact, the studying of a map largely consists in mentally making sections along different lines.

A section is the representation of the intersection of the surface of the ground by a vertical plane. If one imagines the surface to be cut by a vertical plane, and the rest of the ground nearest him to be removed, the result would be to expose a certain irregular profile, which is represented in the construction of sections (Figures 191, 192, 193).

The horizontal distances are generally made on the same scale as the map, while the heights are usually exaggerated a number of times in order to show more strikingly the changes of slope which, in a true section, would be barely perceptible

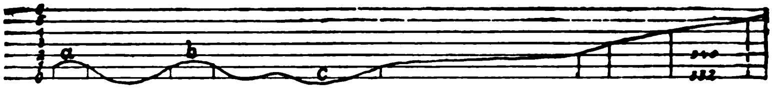


FIG. 191. SECTION thro' hills 336 and 452 north. Heights to Distances as 15 to 1.

FIG. 192. SECTION from J. CRAWFORD to hill 388 east. Hgts to Dsts as 15 to 1.

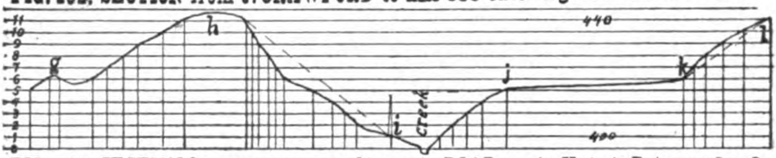


FIG. 193. SECTION from s. w. corner of town to ROAD west. Hgts to Dsts as 15 to 1.

on the scale of the map. The exaggeration of the vertical to the horizontal scale should always be stated under the section as "Heights to Distances as 5 to 1," or whatever it may be.

To draw a section on a given line of a contoured map.

1st method: Lay a straight-edged piece of paper along the section line and dot along the edge the exact points where the line cuts the contours, and also any intermediate point where the slope changes, as on water-sheds and in water-courses. Number these points 0, 1, 2, 3, etc., corresponding to the relative height of the contour of each, calling the lowest contour 0, or number them with the heights above datum. Care must be taken that every time the same contour is cut by the section line the dots are numbered the same.

Now draw lines parallel to each other, at a distance apart equal to the vertical interval between the contours to the exaggerated scale, to represent the contour planes, and number these from the bottom up 0, 1, 2, 3, etc., or beginning with the lowest contour height, corresponding with numbers of the dots. Place the edge of the piece of paper which was laid on the section line along the bottom line and dot off the points marked thereon, and from each of these points raise perpendiculars to the parallel line having the same number as the point from which each is drawn; then join the tops of the perpendiculars.

2d method: For purposes of map-reading, a simpler method may be employed. On a strip of paper a little longer than the section to be drawn and 3 or 4 inches wide, draw lines parallel to the long edge at equal distances of about $\frac{1}{8}$ or $\frac{1}{10}$ of an inch, and other lines at the same distance apart perpendicular to these. Place the edge of the paper on the section line, and from each of the points of intersection with the contours draw a vertical line till the horizontal line which represents the level of the contour is reached; then join these points.

ELEVATIONS.—*An elevation* of the ground shown by contours is the representation of an orthographic projection of it on a vertical section plane. The observer's eye is always supposed to be exactly at the same level as the spot observed and looking at it perpendicularly to the section plane. The section occupies the foreground; points behind the plane higher than the section will appear above it, and those on the same level and lower will be hidden. The outlines of the hills nearest the front will first be drawn after the section, then the other outlines in succession backwards. Any details, as roads, houses, etc., which would be visible, may also be shown in their proper positions in the elevations (Fig. 194). The locations of the various points are found by drawing from them lines perpendicular to the section plane until they inter-

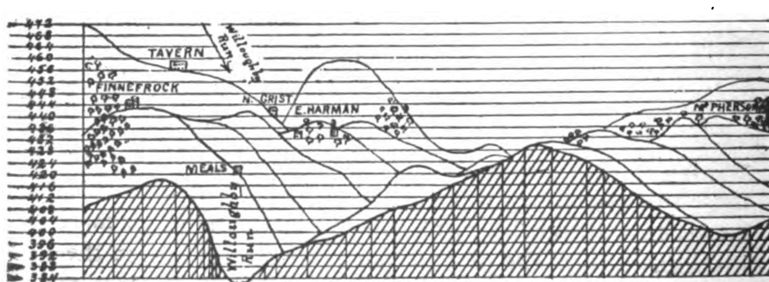


FIG. 194. SECTION on s. w. edge of map and ELEVATION north. Hg to Ds as 16 to 1.

sect the horizontals whose numbers correspond to the heights of the points.

PROBLEMS.—Map-reading includes the following problems:

Determining the direction of the flow of streams.

Determining the location of water-courses, water-sheds, and cols.

Numbering the contours.

Determining the visibility of one point from another.

Determining the horizon visible from a given point.

Determining where a given line pierces the ground.

Calculating the height of a point just visible from another point, etc.

In this determination the height of the observer is not taken into account, as undulations may occur between contours and not be shown, while walls, fences, brushwood, etc., may also intervene.

To determine the various features.

Problem 1: Place arrow-head on meridian line of Gettysburg map, near marsh. Place arrow on all streams, showing direction of flow. Nnmber all contours and hill-tops. Trace all contours whose references are divisible by 20, in red. Construct scale of distance to read feet, also scale of horizontal equivalents.

To determine the visibility of one point from another.

Problem 2: Let it be required to determine whether *i* (Fig. 193) is visible from *h*. From the section drawn on this line it will be seen that the points are mutually visible, because all of the ground between them lies below the broken line *hi*, and the general section is concave. This same fact can be recognized from the contours, without drawing a section, because they are closer together at the top of the slope than at the bottom. If the ground had the shape shown between *k* and *l*, neither point would be visible from the other, because the ground passes above the broken line *lk*, and the general section is convex, which fact can be told from the contours lying closer together at the bottom of the slope than at the top.

Hence the visibility of one point from another depends upon the general concavity or convexity of the section of ground between the points, which can always be determined by drawing a section, but there are simpler processes of arriving at the same results. The only difficulty in this is to determine *what point*, if any, may intercept the view. Thus, between *k* and *i*, the first part of the section may be gentle, as between *k* and *j*, the intercepting point being *j*, where the slope becomes steeper; or some knoll, spur, or water-shed may intervene, as *h* between *g* and *i*. The slope of the ground between the observer and intercepting point, as the slope *lkjih*, does not effect the visibility of points beyond, as *g*, but the degree of slope of *hg* beyond the intercepting point *h* as compared with the slope *hl* of the right line joining the first two points does affect the visibility. If the intercepting point *b* (Fig. 191) is at the level of the observer at *a*, no lower point *c* beyond is visible.

Since a section across a ridge is convex, points on opposite sides, as *g* and *i*, are invisible. A section across a valley is concave; hence points on opposite sides are generally visible, as *h* and *j*, the exception being when the first part is very gentle, as *kj*, and grows steeper to the water-course, as from *j* to *i*.

Problem 3: Referring to the map: Is the stream, where the fence crosses it near H. McDonald's, visible from the road from Gettysburg where same fence meets it? By an inspection of the map it is seen that the section is convex and the stream invisible, the intercepting point being where the fence crosses contour 428 just north of hill 432. For, from the road to this point is a slope of nearly 3 contour intervals in 0.6 of an inch, while from this point to the stream is a slope of nearly 7 intervals in 0.8 of an inch; hence the slope from the intercepting point to the stream is steeper than the line from the road to the intercepting point, and the stream is invisible.

Problem 4: Is the letter S on the meridian line visible from hill 460 west of Seminary Ridge? The contours being close together at top and separating towards S, the ground is concave and S is visible, there being no intercepting point.

Problem 5: Was Hagy's visible to Wilkinson's Battery on hill 404 northwest of Thomas Scott's? By drawing a line from hill 404 to Hagy (house nearest H) it is seen that hill 400 may intercept the view. It is one contour lower in about 2 inches, while the house is one contour lower than hill 400 in about 1½ inches; hence invisible.

Problem 6: Were Davis' Mississippi troops crossing the bridge over Willoughby Run north of Spangler's visible to the 20th New York troops on hill 432 west of E. Harman's? The hill north of 432 may intercept the view. This is 3 contours lower in about 1½ inches, while the bridge is 4 contours lower than the intercepting hill in about 3 inches; hence the bridge is not hidden by this hill-top.

Another method is as follows: Since the intercepting hill is 3 contours lower than the point ofobservation, ⅓ of the horizontal distance between them will be the horizontal equivalent for 1 contour interval of the right line joining them. If this horizontal equivalent be now applied on the line to the bridge from the intercepting point, as many times as there are contour intervals between it and the bridge, and the last application does not reach the bridge, it shows the latter is not hidden by the hill-top. If the last application extends beyond the bridge, it would be hidden by the hill-top.

To find the point where a line of given slope from a given point pierces the ground.

Problem 7: Find the point of piercing the ground of the right line just touching the hill 460 in woods on Seminary Ridge and the hill 452 due west. The line has a slope to the west of 2 contours between the hills, a distance of 1.1 inches. The horizontal equivalent of 1 contour interval will be .55 inch. Beginning at the western hill, lay off this distance and compare each time with height of ground. The end of first length is 448 in height and is over contour 440, so is above ground; in the second length it pierces hill 448, the end 444 being over 440 contour; the third length 410 is over 432; the fourth length 436 is over 404; the fifth length 432 is over 408 west of Willoughby Run; the sixth length 428 is over 422; the seventh length 424 is under 432, hence it must have pierced the ground somewhere; ½ its length, being 426, reaches midway between the contours 424 and 428, hence here is where it pierces.

So the point where any line pierces the ground, starting from a given point with a given slope, can be determined.

To find the visible horizon from a point is to determine the line separating all the visible from the invisible portions of the ground with regard to that point. To do this a number of radiating lines are drawn from the point of observation and along these the portions of ground which are visible are determined by the methods explained above. The points determined as separating the visible and invisible are then joined, forming the horizon, which if the map were large enough would be a closed line perhaps with loops.

In a limited portion of a country it is evident that there may be several visible horizons.

Problem 8: What was the visible horizon of Lane's 7th N. C. at 4 P. M., when in depression on south edge of map 2½ inches east of Willoughby Run, height 400? By drawing radiating lines and determining what portions of them are over seen and what over unseen ground, the horizon will be found to run irregularly along the ridge through the word Hagerstown almost up to the hill 480 east of J. Forney's, then back again along Seminary Ridge.

Problem 9: If the horizon is to be determined from a high hill, as Benner's Hill, height 452, the problem becomes more complicated. A point of Seminary Ridge south of Q. McMillan's is just visible, McMillan's being hidden by the ridge west of East Cemetery Hill, the south part of which is hidden by ridge of East Cemetery Hill; the line down the latter extends to contour 428, when it divides: one part going west to the creek, crosses, then tends southwest up the slope to ridge at about 448, down ridge, across col, over hill in town, down ridge towards Stevens' Run; another part winds off from 428 around to the southeast, off the map, on again, down the ridge to 408, across Rock Creek, etc. There are other portions.

To calculate the height of a point just visible.

Problem 10: An object at *i* (Fig. 193) is just visible from *k*, over the point *j*; what is the height of the object? The distance of *j* from *k* is 928 feet and its fall 1 contour (4 feet). The distance of *i* from *k* is 1,533 feet; hence by similar triangles its fall is $928:1533::4:x=66$ feet; but *i* is 5 contours or 20 feet below *k*, while the top of the object is only 6.6 feet below *k*; hence the object is $20-6.6=13.4$ feet high. Or, since the slope of the line from *k* to *j* is 4 on 928, or 1 on 232, then the fall from *j* to the object on this line will be as many feet as 232 is contained times in $605=2.6$ feet. Now *i* being 4 contours or 16 feet below *j*, and the top of the object 2.6 feet below *j*, the object must be $16-2.6=13.4$ feet high.

Problem 11: The ridge of a house in Gettysburg on the 3d contour below the number 412 is just visible from contour 480 of East Cemetery Ridge; how high is the house? The intervening hill is distant 2,300 feet from the observer and 36 feet lower; the house is 2,860 feet distant; hence by similar triangles $2300:2860::36:x=44.8$ feet lower than the observer. But the contour on which the house stands (420) is 60 feet lower; therefore the house is $60-44.8=15.2$ feet high.

Problem 12: Where and what is the steepest gradient on the Hagerstown road? Applying the scale of horizontal equivalents it is found on the east slope of Seminary Ridge and just west of Willoughby Run, about 7.5 degrees or 1 on about 7.6.

CHAPTER XXI.

COPYING MAPS.

COPYING SAME SIZE.—Maps may be copied the same size—

1st. By fastening a piece of tracing-paper or tracing-linen on the map and tracing over the lines with a pencil, or with pen and ink. If tracing-linen is used, the lines are drawn on the glazed side. and if to be tinted, the colors are applied on the back.

2d. By fastening the drawing-paper on the map and holding both against a window, or a pane of glass in a frame, so situated as to receive a strong light on the back· then trace the lines.

3d. By *transfer paper* as previously described.

4th. By dividing the map to be copied into a certain number of equal squares or rectangles, with sides from ½ inch to 2 inches, depending upon the amount of detail to be copied and the accuracy required. The paper on which the copy is to be made is then divided into squares or rectangles of the same size, which are numbered the same on both.

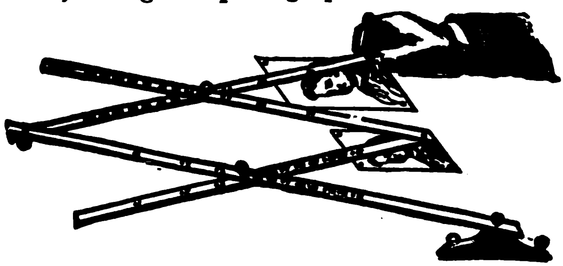
The points where the different prominent lines on the map, as roads, rivers, etc., intersects the sides of the different squares are marked on the corresponding squares on the paper to contain the copy and then properly joined. These being drawn in, the other details are "sketched in" in their proper positions. For greater accuracy within the squares, points may be located by perpendiculars from the sides, or by the intersection of arcs from two corners of the square.

In copying contours, draw on the map lines of greatest slope on water-sheds and in water-courses; draw these on the copy, marking on them where the contours cut, and then join them with the proper curves.

Instead of defacing the map by drawing the squares on it, a pane of glass with the proper size squares ruled on it, or a frame with a fine silk thread stretched from side to side forming the proper size squares, can be laid on the map.

5th. By photography in the regular way, or by making the map translucent with oil or otherwise, then placing the printed side next the sensitive side of a plate and exposing to light, etc., thus obtaining a negative from which any number of copies may be made in a variety of ways.

6th. By using the pantograph. This is an instrument



consisting of four pieces of wood about $\frac{1}{8}$-inch thick, $\frac{3}{4}$-inch wide, and from 18 to 36 inches long, fastened together with movable joints so as to always form a parallelogram. To use it, a tracing-point is made to travel over the outlines of the map; a pencil is so connected with the tracing-point that it is always in a straight line with the tracing-point and with a fixed center, and always at a distance from that center bearing a given constant ratio to the distance of the tracing-point from the center; the pencil draws the outlines of a copy of the map in the given ratio.

REDUCING AND ENLARGING MAPS.—Maps may be enlarged or reduced by photography, or with the pantograph, but the only method usually available for officers is by means of squares or rectangles. The original will be divided as explained, and the paper to contain the copy will be prepared with the same number of squares or rectangles having sides bearing the required ratio to those of the original. The details are then copied to the enlarged or reduced scale.

The distinction must be thoroughly understood between enlarging to two, three, or any other times the size (area), and to two, three, or any other times the scale. Likewise for reductions. Thus a map six inches square containing 36 square inches enlarged to *two times the size or area* will contain 72 square inches and will be 8.485 inches square; while the same map enlarged to *two times the scale* will contain 144 square inches and be 12 inches square.*

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* In enlargements in terms of size or area, the relation of the sides is found as follows: n inches on the map will be represented on the copy double the size by $n\sqrt{2}$, and on the copy three times the size by $n\sqrt{3}$, etc. In reductions, n inches on the map will be represented on the copy one-half the size by $\frac{n}{\sqrt{2}}$, and on the copy one-third the size by $\frac{n}{\sqrt{3}}$, etc.

CHAPTER XXII.

METHODS OF FIELD WORK.

GENERAL IDEA.—Before beginning the actual work of sketching a piece of ground it will be an advantage to proceed to some elevated portion of it, from which a good view can be had, and there obtain a general idea of the shape of the forms and features, and a good mental picture of the appearance they will present in plan.

BASE-LINE.—Some convenient piece of level ground without obstructions is next selected, over which is measured, or paced, several hundred yards for a base-line, from the extremities of which the principal objects on the ground can be seen.

INTERSECTIONS.—Then from both ends of this base-line the directions of all the prominent objects on the ground to be sketched are taken with the compass; if using the plane table, their positions are fixed by intersection. These objects may be spires of churches, chimneys of houses, a flag-staff, gate-post, isolated trees, etc., all of which, when correctly determined, serve as stations from which to start to fill in the details.

TRAVERSING.—Having completed this, the next operation is *traversing* the roads and other lines and sketching in the details, such as houses, fences, streams, bridges, woods, railroads, etc.

Traversing should be performed with as few changes of direction as possible. Thus, instead of following the center of a winding road by a number of short courses, it should be traversed from side to side with as few and as long lines as possible, the directions being taken on the most distant points visible from each station.

In traversing across undulating ground, one is liable, on descending into hollows, to lose sight of the point towards which he is pacing. Foreseeing this, he should look for some object, as a bunch of grass, bush, or other mark, on the alignment, so situated that it will remain in view at such times, thus preserving the direction of the course. When a traverse line crosses a stream, cutting, or anything narrow that interferes with measurement, the distance to the further side should be estimated and the pacing resumed there.

Obstacles, such as ponds, houses, etc., situated on the traverse line may be passed by triangular or rectangular offsets as already described.

When "filling in" the details of ground on which stations have been fixed by triangulation, intersection, or otherwise, the traverse generally commences at one of these stations and is carried up to and "closed" on another as soon as possible. If there is a large error in closing, it is best to traverse back from the closing station, by which the error may be discovered and corrected. If the error of closure is not large, it may be distributed among the last few courses. The traverse may then be started anew and carried forward to another station. The scale at which the notes are to be plotted must be constantly borne in mind, and the notes not burdened with offsets and objects too small and unimportant to be represented.

The forward measurements are commenced anew at each station, but all intermediate measurements are inclusive from the back station.

The *principal rule* to bear in mind in arranging the work is to make the operations as large as possible, using only long traverse lines and long offsets, in preference to changing the directions frequently to sketch in details.

Secondary offsets, perpendicular to the principal offsets, may have to be made in order to sketch in some details.

It is by means of offsets from traverse lines that most of the details of the ground are filled in. Thus, to locate a stream or irregular fence, the direction of a course close to and parallel to its general direction is observed, and perpendicular offsets are measured or estimated to the points which best define its forms. To sketch in a winding road through a field, traverse a line parallel to its general direction and determine its sinuosities by offsets.

To sketch in a winding road or stream which *cannot be traversed*, draw the direction to some distant point of it, then estimate the forward distances along this line to the successive points where offsets would be taken, if traversing along this direction line, and then estimate the lengths of the imaginary offsets at the successive points and plot them in.

Offsets to two points of a straight line, as a railroad, fix its position with reference to the traverse line.

Offset measurements are all inclusive from the traverse line, and not from one object to another, if they happen to be on the same offset line.

Before leaving the traverse line to pace or measure an offset, a mark should be left at the spot, so that the forward measurements may be resumed there.

The notes should include as much of the country as can be well examined on each side of the traverse line, depending somewhat on the scale and the time available. But nothing will be recorded nor sketched that is not seen or at least known.

HILL FEATURES.—Lastly, the *hill features* are sketched in. A beginner can perform only one of these operations of sketching at a time, and is obliged to divide them as above. After some experience, however, they may all be combined, so that no part of the ground need be visited twice.

The first operation of selecting and measuring the base-line and of locating points from its ends does not differ, except in the accuracy of the instruments used, from what has been already described.

TRAVERSING WITH COMPASS AND TOPOGRAPHIC FIELD NOTE BOOK.—This consists in measuring the lengths and observing the directions of the straight lines forming the *traverse*, and determining the positions of adjacent objects with reference to these lines; the measurements and observations being entered in the Field Note Book, from which the plan is subsequently plotted.

Description of book. The Engineer Department Topographic Field Note Book, specimen pages of which are shown in plates "a" and "b," is a book $8\frac{1}{2}$x12 inches when open; on each left-hand page are ruled five columns, from top to bottom; the center one, $\frac{8}{10}$ inch wide, marked "Courses and Distances," is used for recording only the forward directions of the traverse lines and the forward distances along these lines. On each side of the center column is one, $\frac{1}{2}$ inch wide, in which are recorded the offsets or perpendicular distances of objects from the traverse line. The marginal columns, nearly 2 inches wide, are for remarks. The right-hand pages, which are used for sketching as the traverse proceeds, are ruled into square inches, have a blank protractor printed in the center, and scales of 8ths and 10ths of inches at the bottom.

Recording. The note-book, in travering and sketching, should be so held that the *top will always be in the forward direction* of the traverse line, thus having the book approximately oriented, and care should be taken that objects to the right and left are recorded in their respective "Offsets" columns and sketched on the proper side of the drawn traverse line. All records and sketching will begin at the bottom of the page and be carried up to the top.

The notes should be of such character as to be readily understood and plotted by another. The figures should be legibly printed and the entries should not be crowded.

To avoid confusion and mistakes, all figures should be marked, as yards, paces, etc., if linear; or with the signs for degiees and minutes, if of direction.

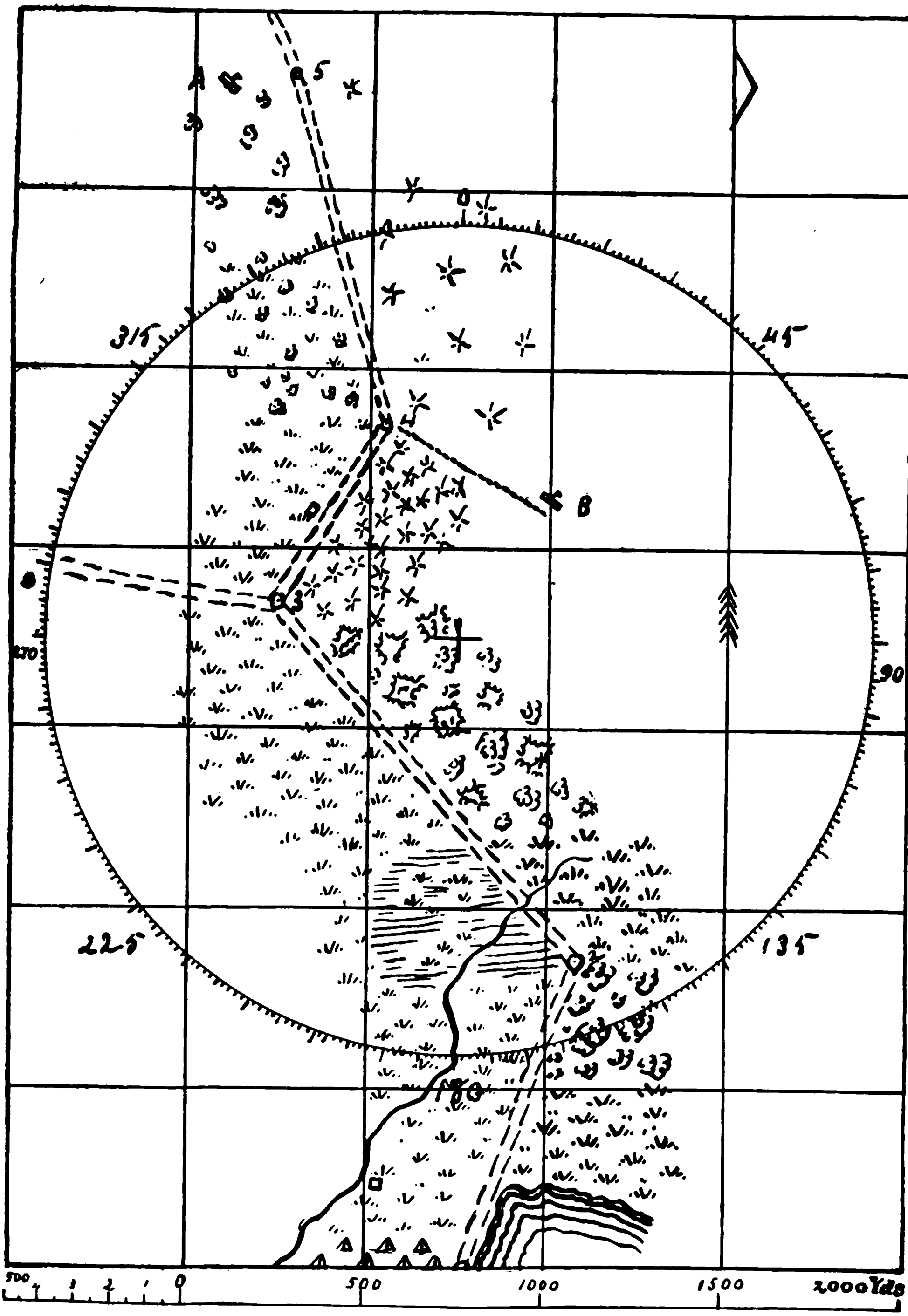
The point of beginning, as well as each station where the course changes direction and is observed by the compass, is marked by a dot with a circle around it, in the center column, and designated with its proper number by its side.

PLATE A.

Remarks Left.	Offsets Left.	Courses and Distances	Offsets Right.	Remarks Right.
Picket A.	200	⊙5. / 1000.		Steep slope towards enemy
Scattered trees				Road steep. Scattered pines
Cleared land.		345° / ⊙4 / 600	500	Trail to Picket B. bearing 120°
Wooden house	35	300		
Dirt road, unfenced, to Newton, dist 8m, Bears 280°		30° / ⊙3. / 1300		Woods (pine)
Meadow / Marsh ends.		400		Woods (mostly oak) / Meadow ends
Forded Creek.		200		Stream flows 220°
Marshy.		320° / ⊙2. / 900		Meadow / Woods and road good
Creek	115	600		Woods
Creek stone house on left bank	165	250		Meadow / Left lake shore: Bearing of edge of lake 100°
Meadow.		20° / ⊙1.		Left camp followed unfenced road parallel to lake shore.

Bearings taken with Box compass graduated contra-clockwise.

Distances in Yards.
0° of Compass at North.



A
5
B
315
45
270
90
225
135
180
500
3
2
1
0
500
1000
1500
2000 Yds

After the last forward distance of a course is recorded, a line is drawn just above it across the distance column, then the new station sign and number are entered, and above these the new direction.

The entries in the "Offsets" and "Remarks" columns are made on the same horizontal line as the record of the forward distances in the center column of the points of observation.

In the columns of "Remarks" are recorded the directions of objects too distant to be located by offsets, and such other remarks as may be pertinent.

Sketching. The sketching, which is to be merely an assistance in subsequently plotting accurately, should be performed as the traverse proceeds and should be as full as possible. The blank protractor, printed on the right-hand page, should be marked to correspond to the *protracted readings* of the compass used, as described for marking the diagrams, and the record on the left-hand page should be made exactly as observed; then a line through this reading and the center of the protractor will be parallel to and indicate the direction of the course. The meridian line should always be put on the sketch in such direction and the sketch begun at such point as to permit of putting as much of the traverse as possible on each page.

Having taken the first forward direction, and recorded it in the center column just above the station sign 1, a line is drawn on the right-hand page, from the point selected for the first station, parallel to a line from the center of the printed protractor to the number corresponding to the direction of the course. This will be the plotted direction of the first course. The surrounding details, by offsets, are then recorded and sketched in to scale, either by measurement or estimation. Pacing or measuring forward, being careful to preserve the alignment of the object sighted with the compass, is then begun and continued until the sketcher arrives opposite some offset which he wishes to note, when the distance to this point from station 1 is recorded in the center column at the estimated distance to scale from the station. On the same horizontal line in the "Offsets" column, on the right or left of the center column, depending on which side the object is situated, is recorded the perpendicular distance of the object from the traverse line, and in the column of "Remarks" such information is added as may seem necessary. On the right-hand page, at the distance of the offset from the station point to scale, on the line drawn a dot is made, and on a line through this dot perpendicular to the course the object noted is sketched in at its offset distance. This completed, the pacing forward is resumed, beginning with the number recorded from the station up to this point and not with 0. This is continued until the point sighted or the end of the first course is reached, when the total distance from the station is recorded at its distance to scale in the center column. A line is then drawn across the column just above this last recorded distance, and the station sign made and numbered just above the line.

The total distance to this point is then laid off to scale on the line representing the course on the sketch, a dot made, surrounded by a circle and numbered.

The next forward direction is observed and recorded, the surrounding details recorded and sketched in, the pacing to the next station begun, the counting beginning again at zero, and so continued to the end of the traverse.

To plot the notes from the Field Book on the final sheet that is to contain the map, plot from the point for the first station the first forward direction with a protractor, and on this lay off to scale the total measured distance to the second station. From here plot the second direction and lay off the total distance to the third station, and so on in succession to the end of the traverse. If this latter has been run from one already determined point to close on another, it can be determined by the above method whether it closes satisfactorily, before proceeding to plot in the details. Next return to the first station, lay off on the courses the intermediate forward distances recorded in the center column, from each back station, and from these points the corresponding offset distances perpendicular to the traverse line, thus locating the positions of details, such as the sides of a road, if the traverse has been along one, houses, ponds, etc. For laying off the offsets, a scale should be constructed on the edge of a card and numbered each way from the center. The position of the paper should be shifted so as to always keep the direction of each course on it corresponding to the direction of the center column in the Field Book, to avoid laying off offsets on the wrong side of the traverse. Constant reference should be made to the rough sketch on the right-hand page.

A modification of the foregoing method and one requiring greater skill, that is applicable when no scales have been constructed, when neither note-book nor protractor is available and the field work must be done rapidly, consists in using a sheet of cross-section or ordinary ruled paper folded bellows-like, the sketch beginning in the middle of the last fold and running forward through the book, which is held side-wise, the ruled lines being perpendicular to the traverse. Work done in this manner must be plotted afterwards to scale, data for the construction of which may be obtained at any time previous to plotting, so that but a brief description of the field work will be given, as follows:

Standing at the initial point, take the bearings of all roads and plot them by eye on the last page of the folded sheet while holding the paper before you so that its length is in the general direction of the route to be traversed, then write the bearings on the lines so drawn and sketch or write in such detail as is required.

Traverse to the second station and plot your position approximately to scale—*i. e.*, call four of the intervals between lines 100 strides, whether at a walk or trot, or a minute if time scale is used. Sketch in the usual details, such as houses, fences, contours, streams, bridges, etc., and be sure to number the successive stations consecutively, *writing on edge of page opposite each station its exact distance counting from previous station.*

Do not use compass for slight changes of direction, but depend on the eye after orienting by the back station. Estimate steepness of all slopes and sketch in contours as accurately as practicable. After completing the field work, a fair copy of the map is made, plotting the recorded steps or time accurately to scale, and protracting such angles as were measured.

TRAVERSING WITH COMPASS AND DRAWING-BOARD.—This consists in observing the directions and measuring the lengths of the courses of a traverse, and determining the positions of adjacent objects and plotting them accurately at once to scale on a sheet of paper in the field, instead of entering them in a note book to be subsequently plotted.

Preparing paper. A piece of drawing-paper, of a size depending upon the length and shape of the traverse and the scale of the drawing, is fastened upon a small drawing-board, which is carried in the hand, and should be turned so that the forward direction of a course corresponds in direction with the line on the ground which it represents. The same principles which govern one in taking notes when traversing with a note-book are applicable when traversing with the drawing-board. The paper should be ruled, before starting out, with parallel lines from 1/3 to 1/4 of an inch apart and one end marked with a half arrow-head to indicate north. A diagram of the compass readings should be made in one corner.

Fixing initial station on paper. If the traverse is not already fixed by known and plotted points, then to determine at what point on the paper to begin plotting to include the greatest amount, some knowledge of its directions and length should be had; otherwise a decision must be made after taking the first direction and referring it to the diagram. Thus, if south be taken at the upper part of the paper, and the first direction is only a few degrees from the south and it is known that the general direction of the traverse is to the southwest, then the starting-point would be fixed near the lower left-hand corner. The sketcher must exercise his judgment in properly placing his work.

Field work. Having decided upon the point for the first station, the first direction is protracted by a line lightly drawn, the surrounding details accurately plotted and sketched in, and the pacing or measuring to the next station begun, the sketcher stopping from time to time to take offsets and sketch in details as he arrives opposite them.

The position of an object too distant to fix by an offset may be determined by two sightings taken to it at different points of the traverse. Unnecessary prolongation of lines should be erased to avoid confusion. All points marking the end of the different courses or stations should be surrounded with a circle to avoid the common error of laying off distances from the wrong point. As previously stated, the counting of paces, after halting to sketch or measure an offset, is resumed at the number reached at the time of stopping, and not at zero. On starting out from a new station in a new direction, the counting begins again at zero.

Sketching with the compass and drawing-board should be conducted as with the plane table, without tripod, in which the compass takes the place of the alidade for determining the directions of objects. Although protracting them is a more troublesome and less accurate process than drawing them with the alidade, both are intended to accomplish the same result in the end.

Or, instead of using the compass for determining the directions of objects to be subsequently plotted, it may simply be used to orient the board and keep it in this position, while the directions of objects are obtained by sighting along the ruler, as will be explained in sketching without instruments.

These lines should not extend clear across the board from the point of observation, but a short line is drawn at about the estimated distance of the object, and then marked to identify it. It will be well to have a meridian line on each side of the sketch, so the compass can be under the eye while holding the board to sketch, no matter which side is next the body. Should there be local attraction at any point, it can be detected by sighting to any already plotted object, after orienting the board with the compass. If such be found, the board may be oriented by a back-sight, as already explained, and the work proceed. Such attraction would probably not be noticed until too late to make correction if directions are observed with the compass and plotted with the protractor. While sighting, the board should neither be strapped to the person nor hung against the body by a string around the neck, but should be held so as to be readily turned in any required direction, by resting it upon the forearm, grasping the edge firmly with the fingers and pressing it against the arm and body, or by holding it upon the tips of the fingers of one hand.

TRAVERSING WITH THE FIELD SKETCHING-CASE.—*Description. The field sketching-case** (Fig. 208) is a small pat-

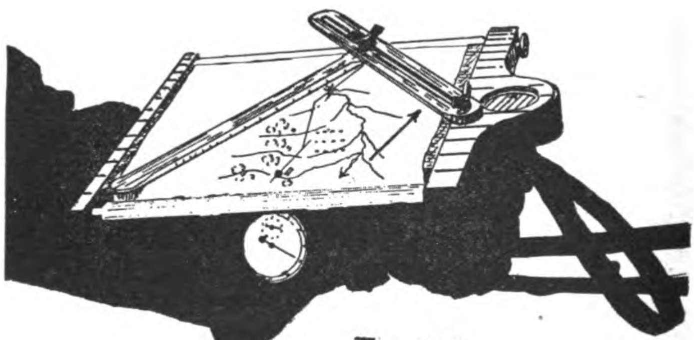


Fig. 208.

tern of the old-style plane table drawing-board in which the compass for orienting the table was "set in" on one side, and the paper fastened on rollers at the ends. It is about $8\frac{1}{8} \times 10\frac{1}{4}$ inches in size, has a roller and clamp on each side for holding the paper, and a compass sunken flush in the head. On the

* In nearly its present form and size it is the invention of Col. W. H. Richards, for many years Professor of Military Topography at the Staff College, Sandhurst, England, and author of "Text-Book of Military Topography."

glass of the compass box is marked a *meridian line* with an arrow-head for the north end; the needle must always be brought, by turning the case, to coincide with this meridian line, with its north end under the arrow-head, whenever plotting a direction.

Instead of the alidade of the plane table, there is used an ordinary straight-edged ruler of some kind, or, as shown in the cut, an arm attached to the head, and a sliding universal ruler, on the edge of which is marked the working scale.

On the under side of the case is a semi-circular protractor and plumb for use in connection with the foot of the case in determining the degree of slopes, as described for the slope-board. To use it, sight along the foot-piece, holding the case so the pendulum will swing freely till oscillations cease; then tilt case to the right until pendulum rests on arc, and read. There is also a strap on the back for holding the case in the hand if used on foot, or for fastening it to the bridle wrist, if used on horseback.

As thus constructed, it has, after a fashion, all the apparatus of a plane table, except the levels, and is used in every respect like the plane table. With it the making of rapid sketches is greatly simplified and facilitated. The degree of accuracy which it is possible to attain with it by a little care and attention is quite remarkable.

To put on paper. The case will hold a strip of drawing-paper about 7 inches wide by 3 or 4 feet long, which is put on by first making a fold across each end about ⅓ of an inch wide, truly perpendicular to the length. Holding the case in front of the body, with the compass to the right (normal position), pass one end of the paper down between the board and the roller farthest from the person, bring it up outside the roller, and put the fold on the end in the slit in the roller; holding the end there, turn the roller until all but about 8 or 10 inches of the paper is rolled up, then clamp the roller and stretch a rubber band from the opposite side of the compass head across the roll of paper to keep it from uncoiling. Pass the other end of the paper across the top of the board, down between the board and the roller next the person, up outside, and place the fold in the slit in the roller, and roll it up until the paper lies flat and smooth across the board; then clamp and remove the band.

To draw the meridian. The compass box can be turned around in the case in order that the meridian line on the glass may be given such a direction with reference to the sketch that the *general* direction of the traverse will lie near the central line of the paper. This is determined by holding the case in normal position, laying the ruler across the middle of the two rollers, and turning the case until the end over the far roll of paper points in the general direction of the traverse. The compass box is revolved, without changing the position of the case, until the meridian line is directly over the needle, with the arrow-head over the north end.

A meridian line is next drawn on the paper parallel to that the glass, and the north end marked corresponding to the arrow-head.

A number of lines are drawn on the glass cover parallel to the meridian line to assist in drawing the meridian line on the paper, by placing the edge of the ruler parallel to one of these lines. This operation, it will be observed, is the same as marking a meridian line on the plane table sheet with a compass or declinator to orient the table by, as described. After this, the compass box must not be moved so long as the traverse will remain on the paper.

If the traverse or route be indicated on a *furnished map*, the meridian line can be set in the required relation to the sketch. To do this, the magnetic meridian, if not already on the map, is drawn on it, the sketching-case with the drawing-paper on it is laid on the map so that its length shall correspond with the general direction of the traverse line, while the compass box is turned until the meridian on the glass is parallel to the magnetic meridian on the map.

Field work. To begin the traverse, stand at the first station, face exactly towards the second station, bring the case level in front of the center of the body and revolve it until the meridian line on the glass is exactly over the needle, arrowhead over the north end. The oscillations of the needle may be checked by tilting the case slightly, but the case must be level and the needle free when the direction is finally taken.

Next bring the point from which the line is to be drawn in front of the center of the body by moving the arm, but without turning the case. Lay the ruler on the paper with one edge on the point and direct the other end of the ruler towards the object ahead, glancing at the meridian line to see that it still corresponds with the needle, and that the latter is free; place the little finger of the pencil hand on the ruler, if it is not weighted or otherwise arranged not to slip; then draw the line or make a dot at the further end, and afterwards join the points by a straight line. If, on glancing at the meridian line, it does not correspond exactly with the needle, it may be made to do so by a slight movement of the wrist. The alignment of the ruler on the object may be done by looking at the object intently; then, by dropping the eyes without moving the head, trace an imaginary vertical line through the object and point on the paper, with which make the edge of the ruler through the point coincide, or by fastening one end of a thread, with a bullet on the other end, to the cap visor in front of the eye.

With the case still oriented, sightings are next taken to all prominent objects too distant to be located by offsets.

Traversing towards the second station is then begun, stopping from time to time to locate and sketch in details to the right and left as they become perpendicular to the traverse line.

On reaching the second station the length of the first course is laid off to scale, the case is oriented by revolving it until the meridian line on the glass comes over the needle, the direction of the second course, from the second to the third station, is drawn as before, and the work carried on in the above manner to the end.

If the sketching-case is used *on horseback*, the operations are exactly the same as described above. To draw a direction, the horse is turned facing exactly in the direction of the object to be sighted, the case oriented, and the line drawn as before. A horse will generally stand still long enough for this.

As the sketching approaches the far roller, the sketch is rolled up on the near roller and fresh paper brought upon the board.

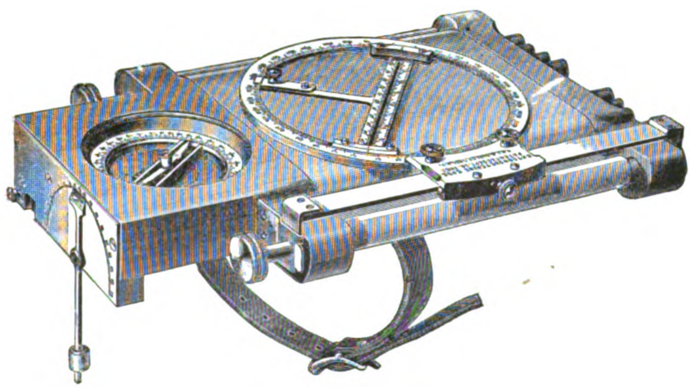
Should the traverse, either because of a change in its general direction or because the meridian line was not given a proper direction, run off the side of the paper, sketching is stopped and a line drawn there across the paper. The case is turned in the new general direction of the traverse, as was done on commencing, the meridian line on the glass turned until over the needle, thus altering it to suit the new direction, and a new meridian line drawn on the fresh portion of paper parallel to the meridian line on the glass. The sketch is commenced in the center of the paper and 2 or 3 inches above the line drawn. If the running off is due to some *local* change of direction, and the general direction of the traverse still coincides with the central line of the paper, the meridian will usually remain unchanged.

Instead of making the alterations of meridian at the end of a course where there is a change of direction, it is better to proceed along the next course for a distance a little greater than the distance to which offsets are sketched in, and there make the change, thus avoiding the duplication of details in the smaller angles between the courses at the station, and the omission of others in the larger angle.

These alterations of the meridian may have to be made several times, but the necessity therefor should be avoided as much as possible by proper arrangement.

To finish the sketch. If it has been necessary to change the meridian, the paper is cut across the lines where the changes were made, the corresponding points of stopping and recommencing the sketch are made to coincide by sticking a pin through them into a board, the pieces are turned until their meridian lines are parallel, then firmly pinned in this position and both cut through with a sharp knife, passing through the coinciding points. The two pieces are then united by a strip of paper pasted on the back along the cut edges. When all are thus united, the traverse will follow along the middle of the irregularly shaped strip. This may then be mounted, if desired, on another piece of paper by its edges, and placed under a heavy weight while drying,

The Batson Sketching-Case differs from the one described in having on its upper surface a revolving and sliding graduated circular protractor carrying a small revolving alidade with



scales, all of which may be raised to an upright position; and on one end a pendulum clinometer, and in the other end holes for carrying pencils. The general principles of the use of this sketching-case are the same as those already described.

Sketching-pad. For the field sketching-case, satisfactory results have been obtained by using a pad of suitable size, of good paper, backed with heavy cardboard, the paper being attached on all four edges. On the back of the pad is fastened a loop of leather for carrying and holding it while sketching. On one edge of the pad is rigidly attached by clamps a compass in a wooden head and the rulers, giving it the appearance of the sketching-case (Fig. 208), without the rollers. For a clinometer, the ruler is used as a plumb in connection with a properly graduated line or arc on the opposite side of pad from the compass.

As thus arranged, the sketching-pad is used in every respect as has been described for the sketching-case, except that as each sheet of the pad is filled, it is torn off and put in the pocket, and another one filled, etc.

TRAVERSING WITHOUT INSTRUMENTS.—*Making the scale.* A sheet of paper is fastened on a piece of smooth board or lid of a box, and a straight-edged piece of wood is provided for a ruler.

The sketcher decides on about the *scale* he wishes to use, then takes a strip of paper of about the length he calculates will represent say 800 to 1,600 paces and folds it three or four times, thus dividing it into 8 or 16 equal parts, each representing 100 paces; these he uses or draws them on the edge of a card for laying off his distances. The smaller distances than 100 paces he can divide by estimation. He can also estimate his offsets in his paces and use the same scale or construct a scale of yards from his scale of paces already made. The representative fraction corresponding to his scale can afterwards be determined when he has access to a scale of equal parts.

Field work. The system to be followed is that which has been already described for traversing with the plane table when orienting by back-sights instead of by the compass. The legs of the plane table have to be dispensed with and the board is laid as nearly level as possible upon the ground. Stations and prominent objects are located by intersection, the ruler being pivoted on the point on the sketch representing the position of the board on the ground. *The ruler is aligned* on the different objects whose directions are desired by standing a little distance behind the board and holding the lead pencil vertically several inches in front of the eye and moving sideways until it is in the vertical plane through the object and station point, then noting which way and how much to revolve the ruler, making the alteration, sighting again and altering until aligned; or a stone tied to a string, forming a plumb-line, may be nsed for determining the vertical plane through the object and station point, and aligning the ruler. The surrounding details are sketched in, then the traversing forward to the next station is begun.

When stopping at any time to sketch in details as he arrives opposite them, or on arriving at a station, the sketcher *orients his sketch* on the back station by placing his ruler along the plotted direction of the course on the sketch, and then, standing behind the board, he determines the vertical plane through the back station and the plotted point of his position, and moves the whole board until the edge of the ruler coincides with the pencil or string defining this plane. Having the board oriented, sightings may be taken to important objects as before. If at a station where a change of direction is to be made, the forward direction is drawn and the traversing continued. Whenever possible, the board should be oriented on the most distant visible objects whose positions have already been fixed on the sketch, rather than by the short back courses. The edge of the ruler is laid on the two plotted points of the distant object and sketcher's position and the board turned until the ruler points to the distant object. The whole operation is nothing more than crude plane table work, and one who can use the plane table will find little difficulty in applying the same principles here.

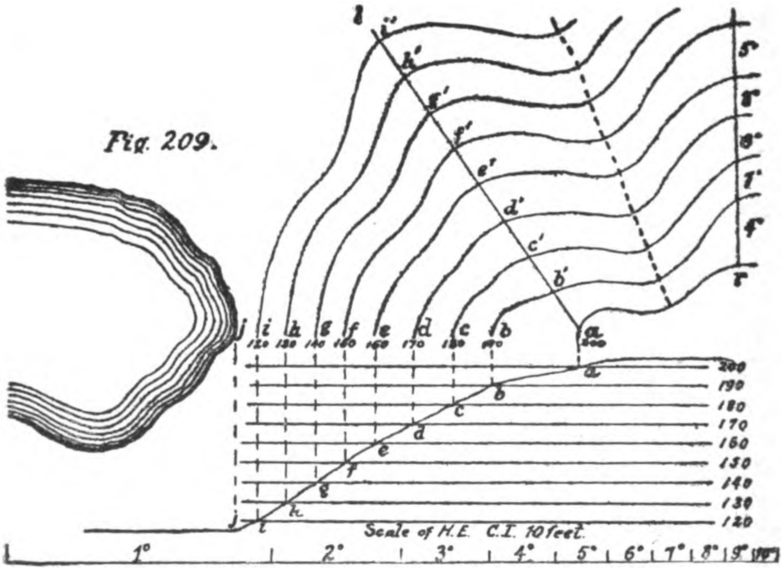
The true meridian may be approximately laid down on the sketch by determining it at noon from the sun as previously explained.

The sketcher can at any time find his position by resection as with the plane table under the same conditions.

SKETCHING HILL FEATURES.—Contours carry the greatest amount of information with the least amount of drawing in the field, but to sketch them intelligently it is necessary to have a clear idea of the shape of the features and a good mental picture of the appearance they will present in plan.

Supposing stations to have been intersected, and houses, woods, streams, roads, fences, and other details to have been sketched in by traversing, the operations of sketching the hill

Fig. 209.



features will be much simplified since the directions of water-sheds and water-courses, etc., can often be recognized by reference to the details already drawn, without other measurements.

Suppose the sketch to be made on a scale of R. F. $\frac{1}{5000}$ and hill features to be shown by contours at 10 feet contour intervals. Prepare a table of H.E.'s as follows: 10 feet at 1°=573 feet, 2°=286 feet, 3°=191 feet, 4°=143 feet, 5°=115 feet, 6°=95 feet, 7°=82 feet, 8°=72 feet, 9°=64 feet, 10°=57 feet, or draw a scale of the same.

To sketch a spur one might, when standing at A on the ground (Fig. 209), whose height above datum has been determined, or assumed, to be 200 feet, observe the slope with his clinometer to be 3° in the direction of a pond. Referring to his table or his scale of horizontal equivalents for 10 feet C. I., he would find the H. E. for 3° to be 191 feet. He paces 191 feet down to B and plots 191 feet to scale on his sketch or lays off the H. E. for 3° from his scale of H. E., fixing the contour point *b*.

Change of slope at contour. Supposing the slope changes at B, a contour point, he observes it to be 7°, the H. E. of which is 82 feet. He paces 82 feet to C and lays off from *b* the H. E. for 7° from his scale, fixing the contour point *c*. The slope remaining the same for some distance, he continues pacing down distances of 82 feet, stopping to plot each on his sketch; or, what is simpler, he paces from C down to E, where the slope again changes, and finds he has paced 246 feet, or 3 times 82 feet, and he lays off three distances of 82 feet, fixing the contour points *c*, *d*, and *e*. At E he observes the slope to be 9° to the pond, already sketched in, so he merely measures off the H. E. of 9°=64 feet as many times as it will go into the distance, it being unnecessary to pace the distances, thus fixing the points *f*, *g*, *h*, and *i*.

The line along which he has descended is termed a *section line.*

When standing at A, he also observes with his clinometer that on the section line AL the slope of 6° is uniform for some 250 yards, so he marks off in that distance the H. E. of 6°=95 feet, fixing the points b', c', d', etc., at the same levels as *b*, *c*, *d*, etc., and draws the contours joining these points.

With his clinometer he also finds points on the same level as A, from which he can observe the slopes and set off other points on the same levels as *b*, *c*, etc., and join them to these, paying attention to the form of the intermediate ground.

Change of slope between contours. In the preceding discussion, the slopes were supposed to change exactly on the contour points B and E, but this would seldom be the case in actual practice; nor can any exact spot be said to be the point of change, as the slopes change gradually from one to another. By practice, however, a mean can be struck and the difficulty overcome.

1st method: Suppose (Fig. 210) the first slope *ab* to be 5°, the H. E. of which is 115 feet. Pacing 115 feet from *a*, he arrives at *r*, which he lays off from *a* on his sketch with his scale, fixing *r*. The slope being the same beyond *r*, he continues pacing to *b*, about where he judges it to change. The distance *rb* being 58 feet, or ½ of 115 feet, he has descended ½ a contour interval, or 5 feet. The new slope he observes to be 3°, on which he must descend ⅛ a contour interval to reach the next contour point. The H. E. of 3° being 191 feet, he paces ½ that distance, or 95 feet, and locates *s*. From here he paces 128 feet to *c*, where the slope again changes to 6°.

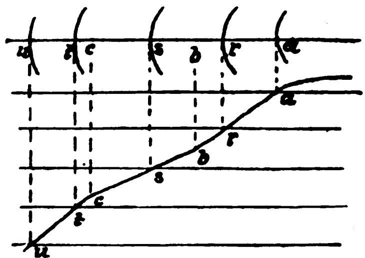


Fig. 210.

Now 128 feet is the H. E. of ⅝ the contour interval of 3°; hence he has descended 6⅝ feet from *s* and he must descend ⅓ of a contour interval on the 6° slope to arrive at the next contour point. The H. E. of 6° being 95 feet, ⅓ will be 32 feet, which he proceeds to pace, and lays it off on his map, locating *t* and so on.

2d method: This process is, however, long and troublesome, but after some experience one becomes accustomed to judging about where the positions of the different points *s* and *t* come, 10 feet below the preceeding one, then measures the slope to them with his clinometer and paces down a distance equal to the H. E. corresponding to it, thus making allowance for any slight change of slope in observing it. This is of course easy when the slope is concave in section, as from *a* to *c*, but should it be convex, as from *s* to *u*, it would be more difficult, though after some practice one may form an idea of the approximate position of the contour point, if not too convex, and observe the slope accordingly. If the section be very convex, this might be impossible and the first method explained have to be used.

Contour Working Table. The operation of sketching hill features is much facilitated by the use of the *Contour Working Table*, which is a personal table filled out by each sketcher after he has determined the length of his stride. Assuming the length of the sketcher's stride to be 60 inches, the following table gives the number of strides (to the nearest tenth of a stride) for slopes from 1° to 12°, inclusive, and for vertical intervals from 1 foot to 15 feet, inclusive. Similarly for any other length of stride the table may be constructed by finding, from the formula HE $=\frac{VI \times 57.3}{s}$, the number of strides for a slope of 1° and a vertical

CONTOUR WORKING TABLE FOR VARYING SLOPES.

Length of stride 60 inches.

NUMBER OF STRIDES.						CORRESPONDING VERTICAL INTERVALS.
SLOPE 1°	SLOPE 2°	SLOPE 3°	SLOPE 4°	SLOPE 5°	SLOPE 6°	
11.5	5.7	3.8	2.9	2.3	1 9	1 Foot.
22.9	11.5	7.6	5.7	4.6	3 8	2 Feet.
34.4	17.2	11.5	8.6	6.9	5.7	3 Feet.
45.8	22.9	15.3	11.5	9.2	7 6	4 Feet.
57.3	28.7	19.1	14.3	11.5	9.6	5 Feet.
68.8	34.4	22.9	17.2	13.8	11.5	6 Feet.
80.2	40.1	26.7	20.1	16.0	13.4	7 Feet.
91.7	45.8	30.6	22.9	18.3	15.3	8 Feet.
104 1	51.6	34.4	25.8	20.6	17.2	9 Feet.
114.6	57 3	38.2	28.7	22.9	19.1	10 Feet.
126.1	63.0	42.0	31.6	25.2	20.1	11 Feet.
137.5	68.8	45.8	34.4	27.5	22.9	12 Feet.
149 0	74.5	49.7	37.2	29.8	24.8	13 Feet.
160 4	80 2	53.5	40.1	32.1	26.7	14 Feet.
171.9	86.0	57.3	43.0	34.4	28.7	15 Feet.
SLOPE 7°	SLOPE 8°	SLOPE 9°	SLOPE 10°	SLOPE 11°	SLOPE 12°	
1 6	1.4	1 3	1.1	1.0	1.0	1 Foot.
2.3	2.9	2.6	2.3	2.1	1.9	2 Feet.
4.9	4.3	3.8	3.4	3.1	2.9	3 Feet.
6.6	5.7	5.1	4.6	4.2	3.8	4 Feet.
8.2	7.2	6 4	5.7	5.2	4.8	5 Feet.
9.8	8.6	7.6	6.9	6.3	5.7	6 Feet.
11.5	10.0	8.9	8.0	7.3	6.7	7 Feet.
13.1	11.5	10.2	9.2	8.3	7.6	8 Feet.
14.7	12 9	11.5	10.3	9.4	8.6	9 Feet.
16.4	14.3	12.7	11.5	10.4	9 6	10 Feet.
18.0	15 8	14.0	12.6	11.5	10.5	11 Feet.
19 6	17.2	15.3	13.8	12.5	11 5	12 Feet.
21.3	18.6	16.6	14.9	13.5	12 4	13 Feet.
22.9	20.1	17 8	16.0	14.6	13.4	14 Feet.
24.6	21 5	19.1	17.2	15 6	14.3	15 Feet.

interval of 1 foot; dividing this result by 2, 3, 4, etc., the number of strides for the same vertical interval and a slope of 2°, 3°, 4°, etc.; will be found. Multiplying these results by 2, 3, 4, etc., the vertical columns are filled in.

To use the table. Suppose the sketcher, while sketching with a contour interval of 15 feet, after having gone down a 2° slope a distance of 63 strides from the last contour point, comes to a change of slope. He looks down the column headed "Slope 2°" until he comes to the number of strides taken, and then horizontally across to the corresponding number of feet in the column headed "Corresponding Vertical Interval" and finds that he has descended 11 feet. He has therefore a further descent of 4 feet to make before he reaches the next contour point. He observes the new slope and finds it to be 6°. Looking in the column headed "Slope 6°," and opposite 4 feet in the column headed "Corresponding Vertical Interval" he finds that he must take 7.6 strides to reach the next contour point. On a uniform slope this table may be used in connection with the scale of strides in precisely the same manner as a *table* of horizontal equivalents.

In case of a *knoll* rising in front of a sketcher as he descends from the high ground (Fig. 211), he continues to the lowest contour g in the saddle, then with his clinometer finds where this contour would strike the knoll at g', paces across to it, then over the knoll until he finds g'' on the same level as g and g', from which point he resumes his locations of points for contours h, i, etc.

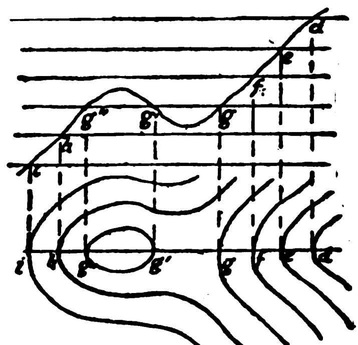


Fig. 211.

It is better to *begin contouring from high ground* and follow the section lines down hill, because a more extended view of the ground can be obtained, the formation of the features can be more plainly observed, and the directions of water-sheds and water-courses can be more easily determined. Cases may arise where one will have to *start* from low ground, as, for example, when a bench-mark or large body of water furnishes a convenient level from which to start. Even in this case, however, the high ground is reached as soon as possible and the work continued from there.

When one halts on a contour to sketch it in, he should generally face about and orient his sketch, and then put in the contour to his right and left as far as it is in view. He may at first have to search the ground with his clinometer set at 0° in order to recognize his level, or even back down hill until his eye is on a level with the contour point, but with practice he will soon learn to dispense with this assistance.

Before drawing contours, it is a good plan to *indicate by dotted lines* the estimated positions of water-courses (Fig. 209) which bound the feature. These can usually be seen and their distances judged on either side from the water-sheds.

In sketching, the distances of features up to 100 yards are usually estimated, and one should have impressed upon his mind the space, according to his scale, which such small distances will occupy, and not have to refer to his scale each time.

When the *features are very large* and the position of a contour can be observed to the right and left for a distance of a quarter of a mile or more, it is well to take its direction with the compass, sketching its distance by estimation, subject to correction later.

The section lines should follow the water-sheds, if possible, as more or less of a view is obtained from them to the water-courses on each side. With this end in view, it may be necessary to alter the direction of the section lines from time to time, so as to follow them, since water-sheds seldom run in straight lines. In many cases the direction can be referred to distant objects which have already been sketched in, thus avoiding the necessity for observing it with a compass. Thus, it is noticed that the section line *ab* (Fig. 212) passes a little to the left of the house T, already sketched in, and so it is drawn. At *b* it changes and passes through the corner of the fence at K, and is so drawn. At *c* it again changes, passing to the left of the pond R. From *h* it passes to the right of the orchard, at *i*

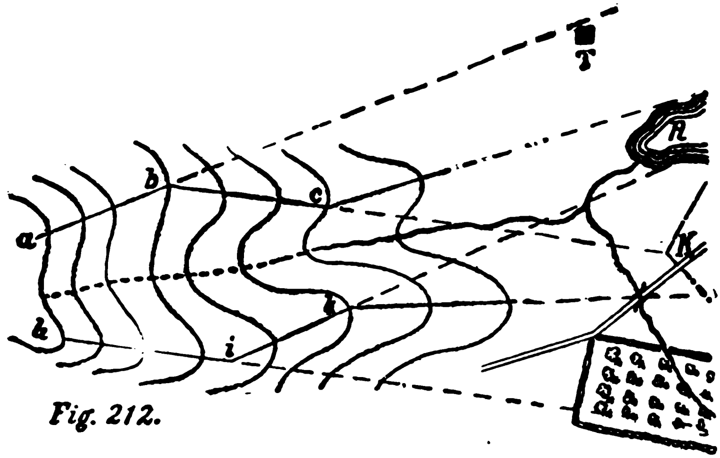
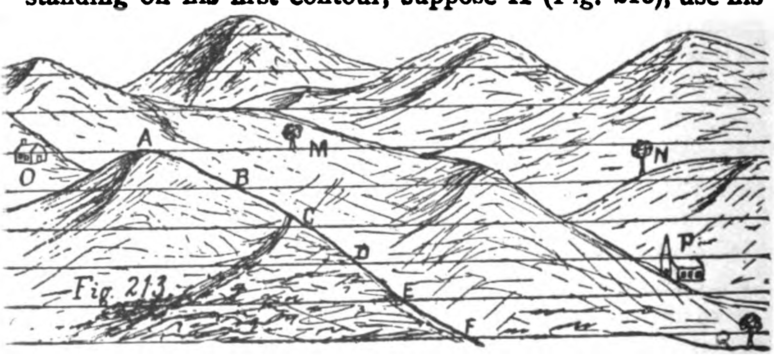


Fig. 212.

changes to the right of the pond, and at *k* it passes through the bridge. This is sufficiently accurate, in sketching, for the purpose of tracing the direction of the imaginary line of a water-shed.

Reference points. The preceding descriptions have dealt with some of the processes of sketching *single* features only. If there are a number of such features to be sketched, and it is desired to make the contours of all unite and be continuous, *one must arrange to begin sketching the contours of each feature at some level common to the others.* To do this, he may, when standing on his first contour, suppose A (Fig. 213), use his



clinometer as a level and find several conspicuous objects on the different features within a half mile or so, on the same level as the point from which he is going to begin. For instance, he may find that the foot of the tree M on one feature, the top of N on another, and the eaves of the house O on another are all on the same level with A.

These points M, N, and O are called *reference points*, and a note made of them on the margin of the sketch for future use. At any time while sketching, if A, M, N, or O happens to be in sight, he can place himself on their level by setting his clinometer at 0° and moving up or down hill while sighting the visible reference point until the clinometer indicates that he has reached such a level. Having done this, he can then find his place on the sketch by *resection* and commence sketching in the contours of the feature he is on, both above him and below him, with the certainty that they will join on the same numbered ones of adjacent features and be continuous for all.

When the hills are of considerable height, *reference points* should be found and noted at every 50 or 100 feet elevation, in order to furnish the sketcher with a ready means of placing himself on a known level at any point where he may be, instead of his having to go back to the level at which he began, which might be either impracticable or inconvenient. Thus, suppose the positions of the contours to have been found at B, C, D, E, F, etc., on the first section line AF. On arriving at D, and looking around with the clinometer set at 0°, the ridge of the church P is seen to be on the same level, and a note is made of it; when arriving at F, the underside of the leafy part of the tree Q is found to be on a level with it, which is recorded, and so on.

By thus selecting *reference points* at different levels, some one of them will probably be in view from almost any part of the field, thus furnishing a means at all times and at any place for beginning to sketch the contours so they will be continuous.

In some places there may be natural levels common to the whole ground, presenting well-defined crests and so nearly level as to furnish a contour common to the whole section, and very convenient for starting from to sketch in the lower contours.

When sketching hill features *in bad weather or when time is not available for doing so with accuracy*, they may still be shown with some degree of truth, provided the traversing has been performed, or one is able to find his place at any time. He visits some of the most prominent features (Fig. 214), finds

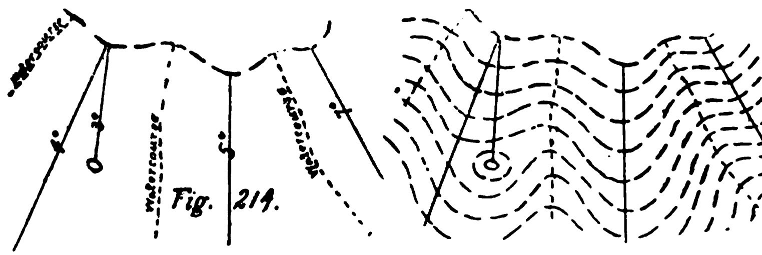


Fig. 214.

his place by *resection*, marks the forms as well as he can on the sketch, and writes on them the vertical angles which he observes with his clinometer to some one central feature or object which is or can be located on the sketch. If the positions of the water-courses are already drawn, or if he can draw them, then he can sketch in his contours later from the data thus obtained.

Sometimes small but important features lie between contours, untouched by them; they may be shown by dotted lines to distinguish them from the contours.

Form lines. Should time not be available for sketching contours common to the whole area, or the country be much wooded, or the sketch be of such a nature (as, for example, a road sketch) as to make it impossible to follow the methods described, the hill features may still be shown by what are known as *form lines*, which consist of contours at the given contour interval for each feature, but which are not drawn to join those of adjacent features, nor to show the same levels above a given datum. They simply refer to the features which they represent and have no connection with those of adjacent features.

SKETCHING MOUNTAINS.—On account of the difficulties in measuring distances by pacing, the preceding methods must be modified somewhat in sketching mountains.

The base-line. In arranging for the intersections for control points, the base is usually measured at the foot of the mountains. Where the slopes terminate in deep ravines, however, from which no view can be obtained, it becomes necessary to go up higher, perhaps even on top, or on a water-shed where a more suitable base can be found. In this work a good telemeter will be of great assistance, for with it a base may be measured between convenient points, though the ground between be rough and filled with obstacles. And if several trials are made and the *mean* taken for the measurement, a closer approximation to its true length would probably be obtained than could be done by pacing on fairly level ground. Or, from a single point the distance could be measured between two distant points which were considered desirable for stations.

Location of points. From the ends of the base, the highest and most sharply defined peaks would be located by intersection. From these, less important knolls, etc., can be located by resection during the progress of the work.

Vertical interval. On account of great differences of level occurring in small horizontal distances, *the vertical interval* between contour planes must be considerably increased. In the survey of the Rocky Mountains, contour intervals have been as much as 250 feet. In other cases contour intervals have been 50, 100, and 200 feet, depending upon the character of the mountains.

The *levels of important and accessible points* may be obtained by using the aneroid barometer, and the *less important and inaccessible ones* by measuring the vertical angles from some one or more points whose elevations have been already determined.

The clinometer may be used *to determine the difference of level between two points, when tho horizontal distance between* *them is known.* Thus, suppose two points have been plotted and the vertical angle of one is observed 5° elevation from the other. Suppose the contour interval 15 feet and the distance between the two points is found to be 1,350 feet. From the table of horizontal equivalents, it will be found that a difference of level of 15 feet occurs at 5° every 171.9 feet; hence one point is above the other $1350 \div 171.9 = 7.85$ contour intervals $= 117.75$ feet.

Instead of making the above calculation, if the horizontal equivalent for 5° as found on the scale to be applied to the distance between the two points, it will be found to go almost 8 times, indicating *the number* of contours, but their *grouping* will be arranged according to the steepness of the slope in different parts. If the slope is steepest near the top, the contours will be grouped closer together there and farther apart near the bottom.

The accuracy of this method depends upon the accuracy of the plotted distance between the two points and the angle as taken with the clinometer. Should an opportunity occur for observing the angle of depression from the other point, the number of contours between them could be checked.

This application of the clinometer is particularly useful in very hasty work (from any cause, as bad weather, want of time, etc.) as described.

COMBINATION OF THE DIFFERENT STEPS.—Having practiced the different operations of sketching *separately*, and having become proficient in them, one can *combine these operations*, and not have to visit any portion of the ground twice. Thus, beginning at a point from which an extended view can be obtained, the directions of a few prominent distant objects, such as houses, trees, etc., will be determined, either plane table fashion or with the compass, and drawn on the sketch with the names of the objects written on the margin. Selecting another good point at some distance for the other end of a base, he can, if he has a telemeter, measure the distance to it; otherwise he proceeds at once with the traversing towards it, selecting as long courses as possible. Arriving at any point from which a good intersection of one of the distant points can be had, he determines its position by observation and intersection. Thus the operations of sketching the details close at hand, and the hill features, and the fixing of distant points are carried on simultaneously.

The great advantage of the use of *reference points* is now made evident. Having made note of several of these on beginning the sketch, one can, with his clinometer, place himself on a contour at any time and sketch the feature on which he may be with the knowledge that its contours can be joined with truth to those of adjoining features.

This system must, however, be followed with great care and judgment, as there is no means of checking the accuracy of the work in the first operations, and any error will be carried on and will enter all subsequent work. But, if the first part be carefully executed, a number of useful points can be obtained by reference to which, later on, the accuracy may be checked.

As a matter of economy of time, *one must learn to distinguish between the relative importance of different objects, and to devote proportionate time and attention to them.* In fact, this is essential in all kinds of topographic work.

Time may be so limited that all possible in the way of hill-sketching would be the indication by a few touches of the general position of the principal features. By means of these indications and a good memory, one should be able to produce a faithful representation of his recollection of the ground, its steepness being graphically shown by the contours being placed according to the scale of horizontal equivalents, which should be drawn on the sketch.

The sketch should be drawn with a hard lead pencil, to avoid obliterating any of the details when applying the shading. An eraser must not be used on the sketch if the paper is damp or wet.

LANDSCAPE SKETCHING.—A landscape sketch may sometimes be the only means of showing a portion of ground of which the sketcher succeeds in getting a view, though he cannot, as when it is occupied by an enemy, attempt measurements or observations of any kind. In such sketches prominence should be given to such objects as are likely to play an important part in an attack or defense, as villages, farms roads, streams, artillery positions, etc. The sketch should be as much in outline as possible, with nothing exaggerated or misplaced.

The point of the compass from which the landscape sketch of an object was made should be stated, as also the estimated distance.

The ability to make a good landscape sketch is a natural talent, but something may be learned of it with a few simple aids:

1. To obtain an idea of the relative sizes and perspective of objects, as seen from a certain point, get a photographic negative from there. From the negative make *a blue print*, take the blue print to the point, and with a lead pencil trace the outlines on the blue print just as if you were making an original sketch on a clean sheet of paper, constantly referring to the landscape to get the connection between it as seen and as it appears on the blue print and as traced. Having traced all the outlines, etc., on the blue print, put it in a weak solution of ammonia and fade out all the color, then wash in clear water, and there will remain only the pencil outline on the white paper.

2. Another aid to learning is to flow a *pane of window glass* with photographer's transparent negative varnish, or other preparation, to produce a surface upon which pencil lines may be drawn. Fix the pane of glass in a vertical position, and, placing the head behind it at such a point as to get a view, through the glass, of the landscape to be sketched, proceed to draw in the outlines in pencil on the varnished surface, being careful not to move the head from its position until the sketch is completed.

3. Another aid is to take an *ordinary picture frame*, stretch fine threads across, forming suitable size squares, and fix it in a vertical position in front of one. On a sheet of paper similarly divided into squares, locate and sketch objects as seen in the different squares in the frame, being careful, when locating the objects, to always have the eye at the same point in front of the frame.

4. As one becomes more proficient he may dispense with these and sketch *directly on his paper*, at first, perhaps, drawing a horizontal line across his paper, to represent his visible horizon, at the relative height on the sketch that it occupies in the landscape, and through the middle of this horizon line drawing a vertical line. Then looking intently for a short time at an object centrally located, and without moving the head, suddenly raise the paper in a vertical position in front of the eye, the horizon line coinciding with the visible horizon, the vertical line through the object, and mark the position of the object on the vertical line. Other points in the field are similarly marked, always holding the sheet in the same position with the first point marked covering the central object each time, and at the same distance from the eye. After a sufficient number of points have been thus located, the intermediate points may be filled in by the eye. For this, distances may be measured from located objects, on the pencil held at the distance from the eye that the paper was held.

Marginal sketches. It has always been an admitted advantage if a good pictorial representation of ground could be made in addition to mapping it. And small free-hand sketches may be made on the margin of maps opposite the objects they represent, to illustrate the description of such objects. These might be of *bridges*, or a *gate* where a trail leads to a ford, etc. Such sketches enable one to recognize the point at once, when its identification by means of the map might cause delay. Or the distant view of a remarkable clump of trees, church spire, or other object might assist the superior officer in recognizing the locality on his arrival. There should always be a good reason, however, for adding such sketches to a map, other than their pictorial effect or to show the skill of the sketcher.

The use of hand cameras is advocated for these purposes, and while it does not yet possess the rapidity so essential to reproducing views at once, it is rapidly advancing towards such a state, and much valuable information will probably be obtained in the future from the use of photography in the field.

COMBINED SURVEYS.—*I. In case of a very large tract, which has not been previously mapped, when time and instruments are available for accurate work*, the operations may be carried on by a number of parties simultaneously.

Control Points. A number of control points, consisting of conspicuous and easily recognized objects or targets throughout the tract, are located by primary triangulation. These are then carefully plotted on one large sheet for the basis of the survey, and by this means the general accuracy of the map is secured, independent of the minor errors that may be made in the component surveys, executed with less accurate instruments.

The tract is then divided into parts, bounded by prominent features, natural limits, or by established lines. The triangulation points contained in each part are "pricked off" on a field sheet and the calculated distances and directions of the lines between the points recorded on it. These field sheets are then given to different surveyors.

These control points should be so situated as to have at least two or three in each part, thus obtaining one well-established reference line. From these control points, the positions of interior points will be obtained by secondary triangulation. Triangulation points beyond the actual limits of his portion may also be given to each surveyor, by means of which he may determine interior points either by resection, triangulation, or by intersection.

Filling in. The details of the ground are then *filled in* by some of the methods already explained.

Each surveyor sketches the *hill features* in contours at the same contour interval, but not necessarily from the same level. In a combined survey for military purposes, all that is necessary is, that the shapes and slopes of the features should be correctly represented in each portion, which can be done without the contours meeting. To make them do so would be difficult to arrange, and the temptation to make them meet when correcting the margins should be avoided. The details of roads, rivers, etc., should meet accurately.

Should a magnetic compass be used in "filling in," each surveyor will determine its declination from an observation on the line joining the triangulation points, so as to lay down a magnetic meridian on his portion, in its proper relation to these points.

Finishing the sheets. When finishing up the sheets in ink, a margin of half an inch around each is left in pencil. The work of the best surveyors will not always agree where roads, fences, streams, etc., cross their mutual boundaries; hence a certain amount of alteration will be required in these parts. These alterations will be made in pencil while both sheets are adjusted side by side, on the general triangulation sheet, *by means of the given control points.*

The application of lead shading is peculiarly suited for producing uniformity in a combined survey. After all are joined, a little blending of the margins only is required.

Each surveyor will be instructed as to the size and style of lettering and coloring to be used.

The officer, charged with the whole, distributes the work and combines the surveys. If he has a number of officers assisting him, he may divide the tract into groups, of three or four parts each, giving to each officer tracings of the original triangulation of his group.

Grouping the sheets. When the field sheets of a group are finished, except the margins in pencil, the officer in charge of the group adjusts them *by means of the given triangulation points*, fastens them down firmly on a table with pins along their mutual borders. With a sharp knife held vertically, he cuts through both sheets at one cut, which will be made close along the boundaries, so as to leave the road, river, or whatever it may be, in one survey only. The necessary alterations are then made in pencil, the sheets separated and afterwards inked, except around the outer edges of the group. The sheets of the group are then again adjusted and joined by narrow strips of paper pasted on the back. The different groups as thus completed are, by the officer in charge of the whole, then adjusted beside each other on the general triangulation sheet. The group sheets are pinned down, cut through, the alterations made in pencil, separated and inked, and afterwards adjusted and fastened together.

Should there be any large discrepancies in the margains of adjoining surveys, the error should be discovered and corrected on the ground.

II. If a map of the tract is at hand, and it is desired to resurvey it for the purpose of showing its tactical capabilities and recent alterations, but neither time or means to triangulate it are available, different methods would be adopted.

The officer in charge would cut out from the map the groups into which the tract is to be divided, and hand them over to the officers in charge of the groups, who further subdivide them into parts bounded as much as possible by roads, streams, etc.

Each surveyor fastens his portion of the map on his sketching-board, and studies the general map so as to recognize the limits of his part, and to impress on his memory the relation of his part to the whole, and the direction and position of rivers, water-sheds, etc.

Each should also draw on his part a magnetic meridian, so he may recognize its relation to his work done with the compass. He should also determine the declination of his compass so as to put a true meridian on his part.

The parts being finished as before, they are adjusted by the officers in charge of groups, by means of the true or magnetic meridians being made parallel, with the north points in the same direction. The chief difficulty in combining them is that errors cannot be at once noticed and traced to the part in which they exist, as in the case where the tract is triangulated.

Even with the best surveyors, the errors and failures in meeting will generally be considerable, and much judgment is required in the arrangement and execution, and in combining them. After the parts are adjusted, they are cut, corrected, etc., as explained above.

III. When triangulation is impracticable and no map is available, a general route or road through the entire tract is selected to form the boundary between the sketches made on both sides of it (Fig. 215), the outer boundary being the limits of the tract. The compass bearing of a line perpendicular, or nearly so, to the general direction of the road, is selected as the lateral boundary of each part, and each surveyor takes the reading of his own compass on this line.

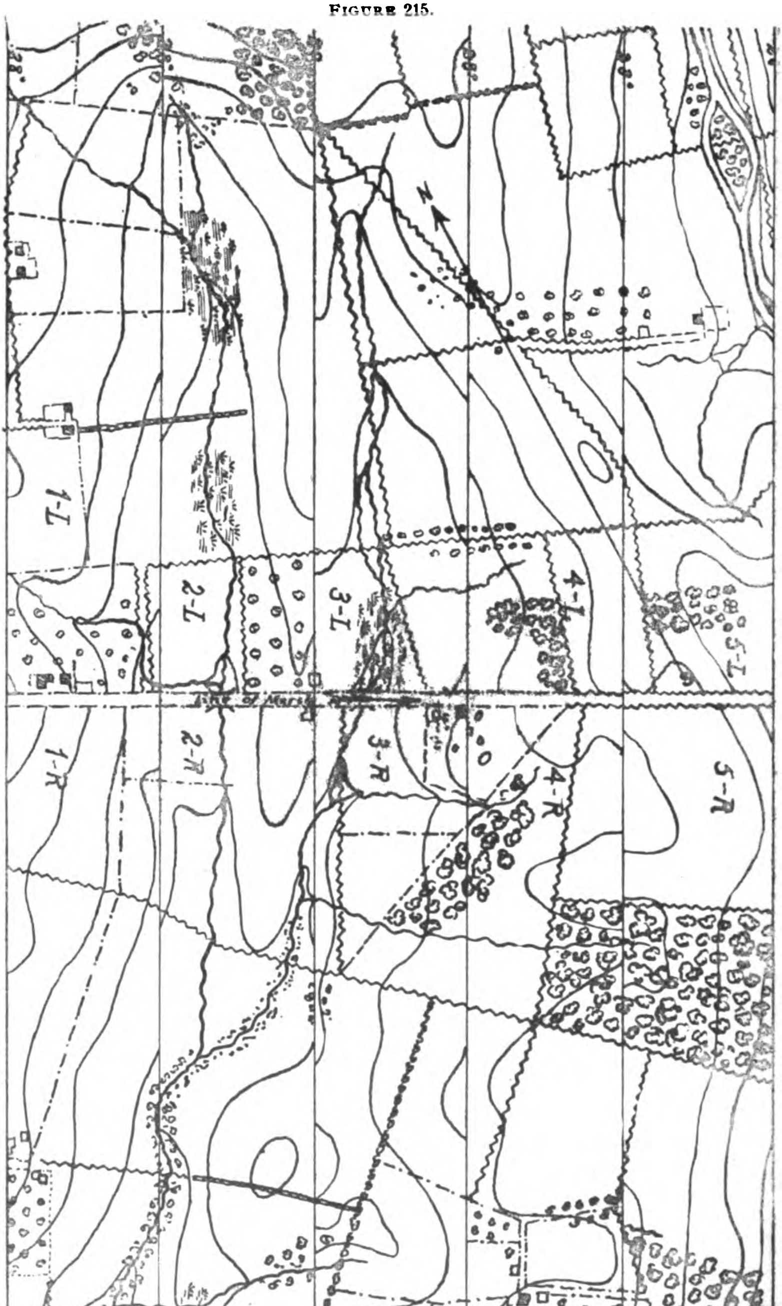
The tract is then divided along the road into parts of as nearly equal width as possible, the points of division being marked by conspicuous natural or artificial objects, and each part on the right and left assigned to a surveyor.

The officer in charge will walk along the road with the surveyors and leave two at each point of division, one to sketch on the right and the other on the left of the road.

On being shown the limits of his part of the tract, as marked on the road, each surveyor will find by trial an object in the distance having the direction of the dividing line as shown by his compass, and will carefully traverse the line in this direction the entire distance to which his sketch is to be carried from the road.

Each one of a pair will be made especially responsible for the accuracy of *one* of his lateral boundaries, and for the accuracy of the road between them. Thus, if the road traversed the tract from west to east, the lateral boundaries being north and south, each might be made responsible for the western

FIGURE 215.



boundary of his part, and he will devote particular attention to this line and trust to the man east of him to traverse his eastern boundary accurately. In this way a much greater general accuracy may be obtained than if each were to employ his time in sketching every part of his ground with equal care. In combining the sketches, the necessary alterations are made only on the side of each on which the least care was bestowed.

CIVIL MAPS AS A BASIS FOR MILITARY TOPOGRAPHICAL MAPS.—Occasions will often arise, especially in a country like our own, where small-scale civil maps are quite common and military maps scarce, when the ability to use the civil map as a basis or control for a military map may be of the utmost importance. This result may be attained in various ways; one of the simplest, whose value has been tested, is as follows:

Preliminary work. Having secured the best civil map (county, state, coast, or geological survey) that is attainable of the tract of country desired, enlarge it on tracing-cloth, by one of the methods described in Chapter XXI., to ½ inch or 1 inch to the mile, showing, as in the original, roads, streams, towns, railroads, school-houses, churches, etc. Make two nigrosine (or any other black-and-white process) copies from this tracing and mount them on cardboard, Draw meridians about 3 inches apart across one of these mounted maps and then cut it into rectangles of a size convenient for carrying. (In doing this, care should be exercised in order that as long stretches of road as possible may be left on the small sheets.)

Field work. Having selected from among the small sheets those showing the routes for the day, the sketcher, either on horseback, on a bicycle, or in a vehicle, travels over the roads, making constant comparison of the terrain with his map. He records no distances and takes no compass directions, except such as may be necessary to orient his map or locate his position on it; he *jots down* in black pencil on his skeleton map, after estimation of distances and directions, all details of military importance, such as the location and extent of forests, new roads and railroads, telegraph lines, new bridges, school-houses, churches; *obliterates* from his sketch such features as have ceased to exist, and *changes* such as he finds to be wrong.

While accurate contouring will usually be impracticable, the general configuration of the terrain should be shown by form lines, as in road-sketching.

A brief *road report* would also be of value.

Finishing the map. On returning at night, a fair copy of the field notes taken during the day is made in colored pencil or colored inks upon the uncut skeleton map, while important data from the road reports is embodied in a legend.

The process outlined is perfectly applicable to a *combined survey*, although in this case the fair copy from all the field sheets should be made by one person. The fair copy, in both cases, may be made by any draughtsman, while skill and judgment are prime requisites to the sketcher in the field work. A good sketcher can, if necessary, fill in from 30 to 40 miles of road in a day.

CHAPTER XXIII.

MAP-PLOTTING FROM DATA.

It will often happen, owing to lack of time or to inclement weather, that a sketcher can make but few notes. This data, if judiciously taken, may be the basis of a good map.

Cases may arise where one topographer will have to translate and plot the notes of another. In such a case, if the notes have been clearly and legibly written, a useful map may be produced. To do this, the topographer must understand the meaning and significance of the various terms employed, and be able to express them by conventional signs.

The construction of maps from data is also an important aid in learning map-reading.

As an example of the foregoing, let it be required to make a contoured plot from the following data:

Problem 1: The pickets of an outpost occupy a ridge whose general direction has a bearing of 120° with a *prismatic compass* graduated thus:

180
N
90 270
S
0

The enemy is in a direction bearing 20°.

The ridge has two prominent hill-tops, A and B, with a col between them. Hill-top A is the more northerly.

Parallel to the ridge and 2,000 feet in rear of it flows a stream for 1,500 yards, where it empties into a lake.

The head of the stream is on a level with the top of a 60-foot pine standing on the bank of the lake into which the stream flows.

From the point A the following observations are made:

Bearing of head of stream, 256°;
Vertical angle to head of stream, −2°;
Slope uniform;

and from B the following are made:

Bearing of mouth of stream, 200°;
Vertical angle to mouth of stream, −4°;
Slope uniform.

Ridge terminates in an abrupt cliff 500 feet beyond B.

The col is midway between A and B and 76 feet below A. From the col the slope toward the stream is slightly concave and heavily wooded.

The lake is assumed as datum.

Scale to be 1 inch=1,000 feet. Contour interval 20 feet.

Problem 2: A stream winds through a valley in a direction bearing 215 degrees for a distance of 1,000 yards. Its average fall is $\frac{1}{30}$.

From the head of the stream, with a *box compass* graduated thus:

	0 N	
90		270
	S 180	

The bearing of A is 245 degrees, the angle of elevation 4 degrees, slope concave; and from the mouth of the stream it bears 357 degrees, the bearing of C being 60 degrees.

From A the bearing of C is 120 degrees.

From C the bearing of B is 33 degrees.

The angle of elevation of B from the head of the stream is 4 degrees.

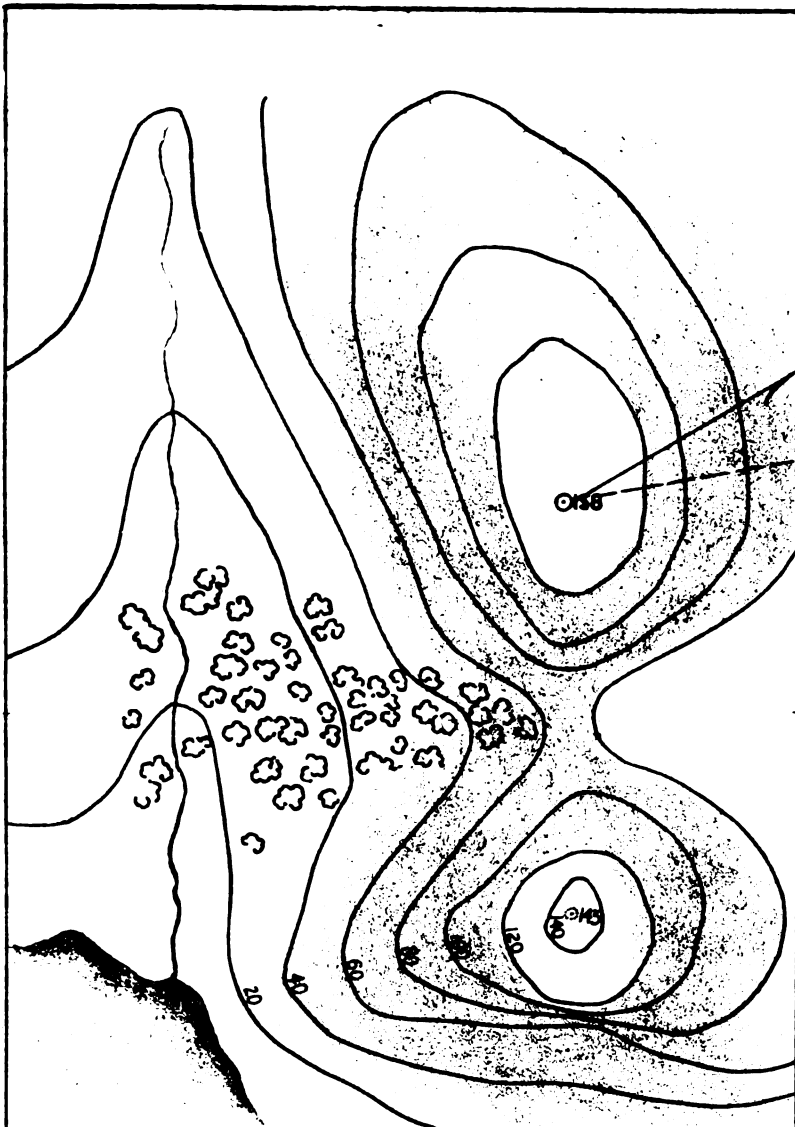
C and B are each 40 feet higher than the head of the stream.

The col between B and C is 55 feet above sea-level.

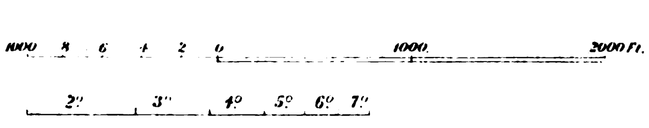
Required a contoured plot from the above data. Scale, 6 inches to 1 mile; contour interval, 20 feet.

Problem 3: A, B, and C are hill-tops overlooking a lake between them.

The bearing of this lake from A is 15 degrees, of B is



Problem 1.



135 degrees, and of C is 255 degrees, with a *prismatic compass* graduated thus:

180
N
90 270
S
0

The hills A, B, and C are respeetively 70 feet, 130 feet, and 90 feet above datum level, the lake being 30 feet above it.

The clinometer shows the slope to A, B, and C to be $1\frac{1}{2}$ degrees, 4 degrees, and 2 degrees, respectively.

Required a contoured plot from the above data. Scale 6 inches to 1 mile; contour interval, 20 feet.

CHAPTER XXIV.

MILITARY RECONNAISSANCE.

Military reconnaissance is the process of obtaining information of a military character; 1st, *about the enemy*, his movements, numbers, dispositions, etc.; 2d, *about the country for tactical purposes*, as its nature and resources, its communications and facilities for the movements of troops, etc.

Reconnaissance of the *enemy* is a subject of Security and Information. Reconnaissance of the *country* belongs to both Topography and Tactics.

As indicated in Chapter I., though existing maps may be available, it may be necessary, in order to make them useful for military purposes, to supplement or correct the information contained in them by a *topographic reconnaissance*.

This will usually consist of two parts, *the sketch*, and a *written report or description* accompanying it, giving details which could not be shown on the sketch without crowding or confusing it. Complete information can be given only by the two together.

Time is, as a general rule, limited, even if the difficulty is not still further increased by the actual presence of the enemy. Hence, rapidity and accuracy in sketching and in taking notes are indispensable, as well as coolness and good judgment. The order for the reconnaissance will indicate its object, the extent of country to be embraced, the time allowed, together with any special instructions, which will be the guide as to the nature and amount of details to be sketched and described, and the objects to which particular attention is to be paid.

THE SKETCH.—The sketch should contain all that can be shown without crowding or confusion; remarks being made in the margin, clear of the sketch, and connected with the object to which they refer by a light line.

Drawing. The minimum of drawing is done in the field, the sketch being afterwards finished up as highly as time permits.

Details. The smaller the scale, the fewer details can be shown, time being wasted in trying to show too much.

Towns. On very small scales, in representing towns, it is only necessary to distinguish the largest and most important buildings, to show the side streets correctly, and afterwards fill up the remainder with the conventional signs for buildings, grouping them to conform to the general shape of the towns.

Roads. While roads, etc., cannot be represented at their proper width on small scales, being shown in the field by a single line, they should be drawn about $\frac{1}{20}$ of an inch wide.

Fences. In a fenced country, it is allowable to generalize details of fences in the less important parts, and simply represent enclosed fields of the average size, according to scale.

Distant places. Lines may be drawn from different points in the direction of important places some miles distant, their names being written on these lines, the intersections of which fix the places in direction and distance.

Lie of the country. Attention must be paid not only to what is near, but also to the general lie of the country, to prominent distant features, etc. The positions of streams may be recognized by trees growing along the banks.

Heading sketch. The sketch should be headed, signed, and dated on the face, by the officer making it.

THE REPORT.—*Reference numbers*. Before commencing to write the report, every place on the sketch to which reference will be made in the report, except names given on sketch, should be marked with a colored numeral and surrounded by a circle, beginning at the bottom and numbered consecutively to the top, for the purpose of identification.

Writing report. The rule in writing reports is to make it legible and not to repeat what is already given in the sketch.

Names of places. Names of places should be written phonetically, as pronounced by the inhabitants, in addition to their spelling as given on the maps of the country.

Information. Only relevant and thoroughly tested information should be given. The sources of all other must be stated, as hearsay, or estimated, etc., if reported. Reports should be governed solely by the object of the reconnaissance, as information relevant in one case might be useless in another. In framing reports, the ability and judgment of an officer are shown as much in what he omits as in what he writes. It requires more thought and care to compose terse and well-condensed than lengthy, diffuse reports.

Terms used. General terms, such as large, small, wide, narrow, etc., should not be used, but instead the exact numbers and dimensions should be given. The terms *north, south*, etc., should be used instead of *right* and *left*, except in case of streams, where the *right bank* and *left bank* are always considered to be those on the right and left hand of the observer when facing down stream.

Orders. A copy of the order and instructions in accordance with which the reconnaissance was made should always be attached on the last page of the report. These may exonerate the officer from blame if circumstances should materially change during the execution of the reconnaissance, or from what was supposed to exist when the order was issued.

Briefing and attaching report. The report should be briefed, signed, and dated, the same as the sketch, by the officer making it, so that if it becomes detached they may be rejoined. The report is attached to the right edge of the sketch by a strip of mucilaged paper on the back, with the bottom edges of the two even. If the sketch is longer than the height of the report sheet, the sketch is first folded over even with the top of the report and then the whole folded to official size, the last page of the report being the first page of the whole.

Blank forms. A form for the report, for each kind of information, will usually be furnished, giving the headings of each branch, and in some cases a summary of the details embraced by the headings. Only those headings would be reported which come under the object of the reconnaissance.

RECONNAISSANCE OF A ROAD.—Reconnaissances of roads are usually made with a view to the movement of troops over them, hence precede such movements by one or more days. The work is usually done on horseback. If in an enemy's country, the reconnoiterer is accompanied by an escort of cavalry for protection and to seek additional information.

The object of the reconnoiterer is to produce a sketch of the route by which the troops may move, reporting on the general nature of the country, with the difficulties to be met with in marching, etc.

The sketch. This is made on a *scale* of about 1, 2, or 3 inches to a mile, using either horses' paces or a time scale for distances. A graphical scale of miles, yards, or feet and a magnetic meridian must always be constructed on the sketch before handing in.

The *sketch is begun* at the bottom of the paper and carried upwards in the order of march, and is in the nature of a traverse either with compass, field sketching-case, or without instruments. When the sketch will take the form of a long, narrow strip, *the meridian lines* must be arranged diagonally, if necessary, so the road may occupy the central line of the sheet.

The sketcher seldom leaves the *main road*, except to go down a cross-road a short distance to examine a bridge or ford by which it crosses a stream, or to a near hill-top to get a distant view, etc.

The *adjacent country*, as far as visible from the road, about $\frac{1}{4}$ or $\frac{3}{4}$ of a mile to the right and left, should be sketched in.

The *general direction* of cross-roads is merely referred to traversed road, except where seen for some distance, when their direction may be observed.

In sketching a *winding road* (unless using the field sketching-case), it will generally be sufficient to take the bearing of the long reaches, sketching in the intermediate portions by eye.

Details are sketched in by offsets, as already explained.

Hill features, the grades on the road, etc., are shown by *form lines*, no attempt being made to make the contours of adjacent features join and be continuous, or to refer them to a common datum. The slopes are read with the clinometer, or, after practice, simply estimated.

All *commanding ground* within artillery range should be located by intersection.

Free-hand sketches may be used as explained.

The Report. The information required about the road may be either *tactical* (relating to maneuvering capabilities) or *statistical* (relating to the population, supplies, accommodations, etc.).

Both kinds of information might be required at the same time of a single officer and embodied in one report, but it is usual to separate them and assign only one to a single officer.

The tactical information most usually required is here classified.

HEADINGS.	SUMMARY.	EXPLANATIONS AND REMARKS.
I. THE ROADWAY.	*Construction.*	Whether macadam, corduroy, plank, gravel, clay, or earth. Whether worked or formed by traffic.
	Width.	Of the roadway proper. 9 feet is the minimum for infantry in fours, cavalry in twos, wagons or guns in column; 16 feet for a double column, or to permit passing.
	Condition.	First its *present* condition, then whether rains would affect it and how; whether any other circumstances might alter its condition, such as soil, nature of drainage, adjacent ground, etc. The length of any bad parts should be stated, and the way of avoiding them, if possible.
	Materials for repair.	Such as piles of stones on the roadside, loose stones from near quarries, gravel, timber, or brushwood for fascines, etc.
	Grades.	On hilly roads with a grade steeper than 3°, or $\frac{1}{20}$, the slope should be stated. For short distances artillery can go up 15°, or about $\frac{1}{4}$; heavy wagons 8°, or about $\frac{1}{7}$; for steeper grades extra horses must be attached.
	Defiles.	Any narrow part, as the street of a village, a cutting, etc., together with its length, width, and height of sides, and whether it can be widened, or the sides accessible, etc.
	Fences.	If on the sides of a road, the kind, as stone, hedge, board, picket, wire, rail, etc., and the distance between them. Whether they are an obstacle to movement or afford cover from fire.
II. BRIDGES	*Variety.*	Whether a draw, swing, suspension, cantilever, trestle, truss, arch, ordinary or foot-bridge, etc., in detail. If an arch, number of, whether elliptical, semi-circular, or segmental. The *span* is the horizontal distance from pier to pier. Whether strong enough to bear guns. Bridges of small span, easily demolished or repaired, need not be so minutely described.

	Material.	Whether of steel, iron, wood, stone, or brick, or a combination of several.
	Length, breadth, and height above water.	Length of bridge, length of approaches, breadth between wing walls. If bridge passes over the road, the clear height above the road should be given.
	Piers.	Describe in detail as to material, thickness, number, distance apart.
	Fords near.	State exact position; the length and breadth; ordinary depth; nature of bottom; whether liable to shift or deepen; velocity of current; means of destroying or repairing; condition of approaches, roads, or paths. See *Fords* in Reconnaissance of Rivers.
III. RATE OF MARCH.	*Anything that might retard the usual marching rate.*	Such as steep grades, rocky places, deep mire, heavy sands, etc. In every case the probable rate of marching to be given.
IV. TOWNS AND VILLAGES.	*Description.*	Whether *salient* (extending lengthwise of road), *broadside* (extending across the road to right and left), or *circular.* Farms of importance under this head.
	Construction.	Material of the houses, inflammable or not.
	Streets.	Whether paved (giving kinds, condition, etc.) or not. Breadth, straight or tortuous, etc.
	Enclosures.	Kinds of, whether gardens, orchards, etc., and walls or fences by which surrounded.
	Principal buildings.	Such as churches, manufactories, public buildings, large dwellings, telegraph and telephone offices, etc. Size of each; whether suitable for barracks, hospitals, etc. Position of each, if not shown on sketch, with reference to approaches.
	Defensibility.	Whether commanded from neighboring heights; materials for barricading and making abattis; whether tools for entrenching are available in any quantity.
V. COUNTRY.	*General description.*	Of what is not shown on sketch. Anything that limits the view, etc., as woods, nature of them, impassable or not.

	Nature of soil and cultivation.	Whether rocky, sandy, clayey, marshy etc. Character of cultivation, direction of rows of high crops, as corn, vineyards, etc., kinds of fences by which covered.
	Practicability for different arms.	Whether movement across country or parallel to the road would be possible for any or all arms.
VI. RIVERS, STREAMS OR CANALS.	*Width. Depth. Velocity. Bottom. Banks. Fords. Locks. Inclined planes. Aqueducts.*	The summary here given applies only to the points where the road crosses a river, stream, or canal. Of the banks, the command of one over the other would be given. Important rivers would usually be separately reconnoitered. See Reconnaissance of Rivers.
VII. HALTING-PLACES.	*Extent. Proximity of water.*	The amount of space, the amount and kind of drinking water, for men and animals, are the chief points. Whether part of command could pass from rear to front, etc.
VIII. CAMPING-GROUNDS.	*Force for which available. Water, wood, grass, etc.*	This only when required by orders.
IX. POSITIONS.	*On or near road for advance or rear guard.*	According to special instructions.
	Any favorable for enemy from which he could observe road.	Enemy's position reported on as seen from road.
X. LATERAL COMMUNICATIONS.	*Construction, width, condition, etc.* See I.	These relate to roads or trails crossing or joining the road being reconnoitered.
XI. RAILROADS.	*Gauge. Single or double track.*	Also, name if attainable, and to and from where it runs. Stores and rolling stock available.
XII. TELEGRAPH LINES.	*Number of wires. Kind of poles.*	The connecting points if obtainable.

The statistical information desired would relate almost exclusively to supplies, accommodation, and population, and would probably be entirely a written report on a form prepared for that purpose.

I. In the first column would be the name of the place reported on; or, in case the sketch was made, its reference number. The order of reference numbers and names should be the same as on the sketch—that is, the highest number or most distant place would first be described at top of report, the starting-point at the bottom.

II. Distance from starting-point and from place to place are both reported, unless the report accompanies a sketch, when they may be omitted.

III. The number of houses and the population may generally be obtained from the corporate authorities. If not, the number of houses may be approximated by estimating the number in each street from the estimated length of street and the average size and compactness of the houses on each.

The population may be estimated at an average of about 5 to every house.

IV. In estimating the accommodation, one man per yard of front of the house may be allowed if only one room deep, two men per yard of front if two rooms deep, etc. If house has more than one story, multiply by number of stories high, less one for the family. If estimated by rooms separately, one man per yard of length may be allowed if not over 15 feet wide, two men per yard of length if between 15 and 25 feet wide, 3 men per yard if width exceeds 25 feet. These would be only temporary accommodations.

If all houses could not be calculated as above, then they might be divided into classes according to size, and a house of each class examined and the accommodation estimated, from which that of the entire place could be obtained.

In estimating the accommodation of barns and sheds for horses, if desired, one horse per 5 feet of length may be allowed if not over 24 feet wide, and two horses per 5 feet of length if over 24 feet.

V. Under supplies, the names of persons from whom they may be obtained are reported; also the number of grocery stores, butcher shops, blacksmith shops, hotels, and any other places where edibles and drinkables are to be obtained.

VI. Under transportation, the names of persons from whom wagons, horses, etc., can be obtained are reported; also the number and kind obtainable from each.

VII. Under water would be stated whether drinking water is obtainable from reservoirs, cisterns, wells, springs, etc.; the amount and quality; also water supply for animals.

To calculate the supply obtainable from a small stream, multiply together the mean depth, width, and velocity per minute, all in feet, for the cubic feet per minute. To reduce cubic feet into gallons of 231 cubic inches, multiply by 7.5.

In permanent camps men require at least 5 gallons each per day and horses 10 gallons.

RECONNAISSANCE OF RAILROADS.—Railroads play an important part in modern warfare. By their means troops and supplies are transported with rapidity from one point to another. Railway centers and junctions become important strategic points. In the reconnaissance of a railroad, if a sketch were required, it would be made upon the same principles as the sketch of an ordinary road, but would also include plans on a larger scale of the important centers and yards. On these plans each pair of tracks would be shown by a single thick line.

The report would be divided into two parts: *1st, The Line*; *2d, The Stations*, and the information under each about as follows:

1st. The Line.

1. The names of places through which it passes, and other lines with which it connects.
2. The general character of the country through which it passes.
3. Whether single or double track; gauge in feet and inches (4 feet 8½ inches standard gauge), and general condition.
4. Description of rails, whether iron or steel; weight per yard; how laid, on chairs, tie-plates or not; how fastened, by fish-plates, angle-irons, etc.
5. Description of ties, material, distance apart, whether laid on ballast (kind) or not.
6. Description of bridges, kind, material, length of spans; best method of destroying, etc.
7. Locations and lengths of sidings, and if there are conveniences for loading and unloading troops, animals, and supplies.
8. Gradients, cuttings, embankments, viaducts, tunnels.
9. Practicability of marching troops along the line.
10. Places, other than at sidings or stations, where troops could be loaded or unloaded.

2d. Stations.

1. General description as to size, material of which built, whether situated on the level, on an embankment, or in a cutting, and whether adapted to defense.
2. Description of freight-houses, round-houses, and other buildings.
3. The number of cross or junction lines.
4. The lengths and breadths of platforms, facilities for erecting temporary ones.
5. Direction of approaches and widths of entrances to the station.

6. Space outside where troops could be formed and for camping in vicinity.
7. Supply of water for engines, and for troops and animals.
8. Amount of rolling stock, locomotives, and cars of different kinds and capacity.
9. Stores of different kinds, as rails, ties, fuel.
10. Number of telegraph lines, location of batteries and materials for repair.

RECONNAISSANCE OF RIVERS.—The object of the reconnaissance of a river is generally for the purpose of ascertaining how it may be crossed by an army, or to oppose the crossing of it by an enemy.

The sketch. This must usually be made on foot, and generally on a scale of about 2 or 3 inches to a mile.

If the river can be followed on either bank, it would be sketched by traversing, points on the opposite bank being fixed by intersections or offsets.

The sketch should include as much of the country as can be seen from the banks; and towns, woods close to it, roads and trails leading to it, and all commanding heights within artillery range, should be carefully shown.

It will seldom happen that the same bank can be followed for the whole length, so the officer must be accompanied by a boat, to enable him to cross, and traverse on the other bank when necessary.

The report. If the river is *parallel to the general line* of operations, it would be reconnoitered with a view to showing how connection could be maintained between columns on both banks, either during the march or for action. Such a river is the best protection from an enemy's enterprise on a flank, as it is only necessary to sieze the boats and guard the bridges and fords to secure it.

The information in such a case would be as follows:

HEADINGS.	SUMMARY.	EXPLANATIONS AND REMARKS.
I. THE VALLEY.	*General description.*	Whether broad or narrow; marshy, rocky, wooded; proximity of heights on one or both banks. Much the same information as under V. THE COUNTRY in road reports. As much of this to be given on sketch as possible.
II. THE STREAM.	*Navigable or not.*	Length of navigable portions; obstacles to navigation, to be given.
	Tidal or not.	
	Width.	
	Average depth.	The deepest parts in winding rivers will be found near the concave bank.
	Velocity.	Obtained by noting how many seconds it takes a floating object to go a certain number of feet. 0.7 the number of feet per second is approximately the number of miles per hour. The strongest current will usually be in the center of the river.
	Liability to overflows.	Learned from inquiries. Extent. Time of year of usual large rises.
	Bottom.	Whether rocky, gravelly, sandy, or muddy; whether even or irregular.
III. THE BANKS.	*Nature and heights.*	Whether precipitous, and if so, how high; marshy or not; lined with trees or not; what cover they afford; what facilities for crossing, or forming ramps to fords.
	Command.	Of one bank over the other, in feet.
	Towpaths.	
IV. TRIBUTARIES.		General description of each to be given.
V. ISLANDS.		Position; size; whether woody, marshy, cultivated, etc. Command over either bank; distance from, to banks.
VI. BRIDGES	*In detail.*	As given in road report.

VII. FORDS.	*In detail.*	As given in road report. Location may be ascertained by questioning; by roads or trails leading to it from either bank; often by houses near; by dropping down in boat with sounding line of required length attached when weight touches bottom, a ford may be looked for. May be expected in parts where river widens, or obliquely across from one convex bank to another. Fords practicable for cavalry about 4 feet; infantry 3 feet; guns and wagons 2.5 feet.
	Other points.	Where crossing might be made practicable by military bridges or other means. Also if river freezes over to bear crossing, and for how long.
VIII. BOATS.	*In detail.*	Whether regular boats, ferries, or flying bridges, with full description as to kind, size, capacity, etc. Number of men that can be carried each trip and time required. Whether materials for rafts are near.
IX. LOCKS.	*In detail.*	Length; width; fall of water; time required to fill and empty; whether exposed to distant artillery fire. Position of, should be shown on sketch.

If the river is *perpendicular to the general line* of operations, it is necessary to concentrate at one or more points, in order to cross, and to deceive the enemy as to one's real intentions. In which case it might be used as a defensive obstacle, and the report would contain, in addition to the above, the following information

X. APPROACHES.	*Nature of.*	And points at which access by them may be barred by troops or obstacles.
XI. HEIGHTS.	*Command and distance.*	On either bank within artillery range.
XII. INUNDATIONS.	*Points and means of effecting.*	
XIII. POINTS SUITABLE FOR COVERING PASSAGE OF TROOPS TO OPPOSITE BANK.	*Straight, reentrant or salient.*	

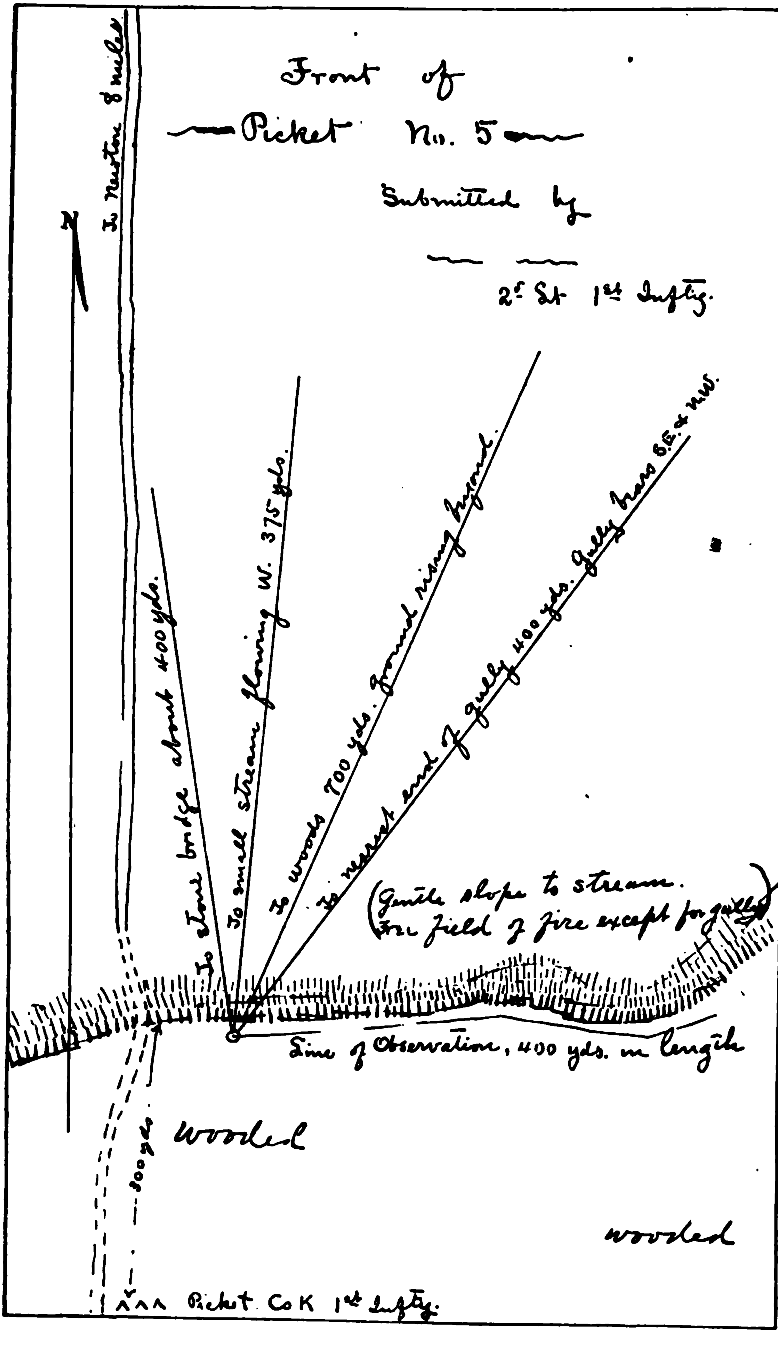
RECONNAISSANCE OF OUTPOSTS.—An officer commanding an outpost will naturally avail himself of the readiest means at hand of becoming familiar with the ground in his own immediate front. He will, of course, ride along the line of sentries or videttes to make personal observations, and take mental note of the steepness of slopes, the depth and direction of gullies, the location of impassable obstacles, and, above all, the position of roads over which the enemy may advance. To impress this on his mind and to have a means at hand for refreshing his memory, he may direct the commander of each picket to submit a sketch of the ground in his own front. As a rule, it may be assumed that the sketcher will find it impracticable to traverse the ground he is to sketch; in other words, that the line of sentinels or videttes will be the base from which the ground in front must be plotted.

The sketch. There are two methods of doing this work.

1st method: This, while it hardly merits the title of a sketch, is shown below and consists merely in drawing radiating lines from some prominent point on the line of observation toward the various points to which reference is to be made, and then writing such data as may be necessary along these lines. (See figure). Telemeters are valuable aids in this work, although a ruler and pencil are the only instruments absolutely necessary.

2d method: The only other way of making the sketch where circumstances forbid traversing, is to measure as accurately as possible (usually by pacing) a base-line along the front, and then, using an ordinary drawing-board or box-lid as a plane table, locate the points in front by intersection. The instruments required in this class of work are a box compass, a ruler, and a clinometer or slope board. It has been found in practice that, while not absolutely essential, an improvised tripod, made by lashing three sticks together as shown, is of material assistance in securing rapidity and accuracy. The successive steps in this work are as follows:

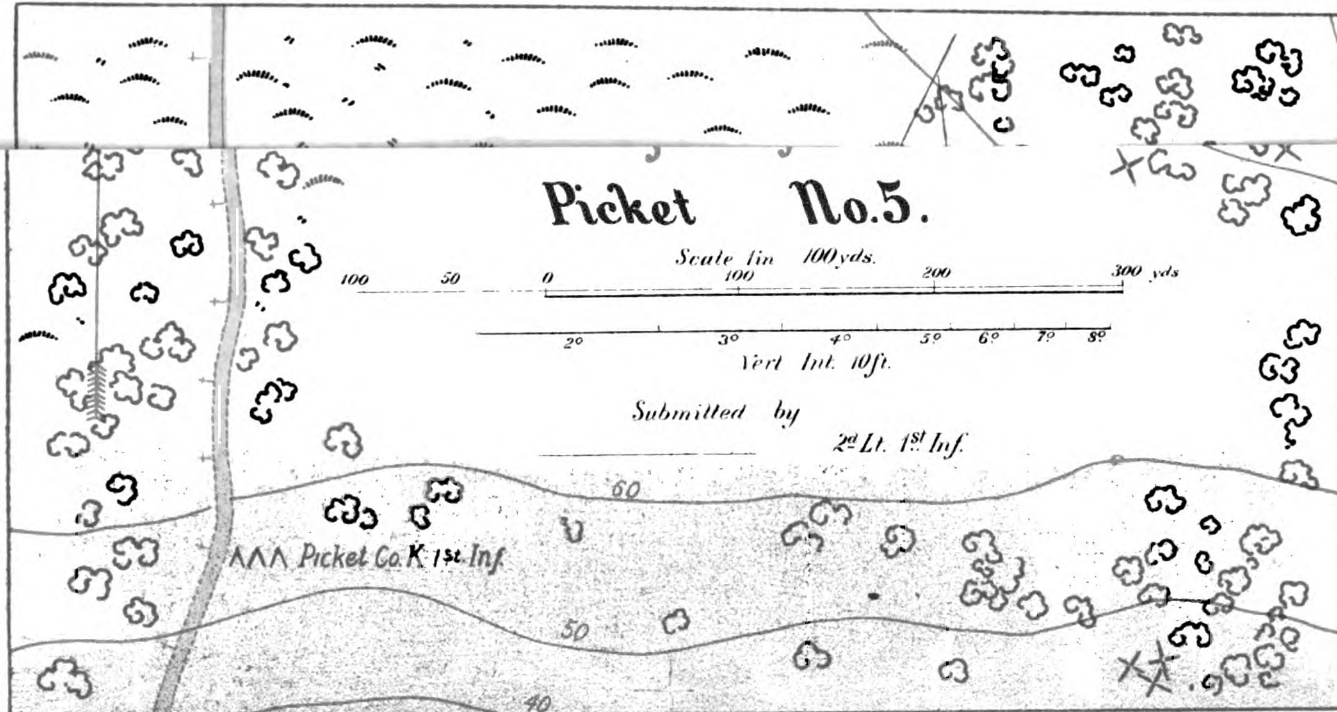
Construct a scale of yards, of horizontal equivalents and


Front of
Picket No. 5
Submitted by
2d Lt 1st Infty.
To Newton 8 miles
N
To stone bridge about 400 yds.
To small stream flowing W. 375 yds.
To woods 700 yds. Ground rising beyond
To nearest end of gully 400 yds. Gully bears S.E. & N.W.
(Gentle slope to stream.
Free field of fire except for gully)
Line of Observation, 400 yds. in length
wooded
800 yds.
wooded
Picket Co K 1st Infty.

of strides (the first two only should appear on the completed map) at such an R. F. that the map will be as large as the paper will allow (usually 6 inches to 1 mile, with a contour interval of 15 feet).

Select a base on or near the line of observation such that the important points to be plotted can be seen from the ends of the base, if possible. Go to one end of this base, and, having driven a pin into the board at the assumed origin, lay the ruler against it and draw an indefinite right line on the paper in the direction it is desired the base should lie. Revolve the board until this line points to the other end of the base; the board is now oriented and must not be moved until all sightings to be made at this point are completed.

Place the compass on the board and turn the former until the north end of the needle is at the N of the graduated circle; then draw a right line along the edge of the box and mark the north end of it. Pivot the ruler around the pin, and, sighting to prominent or important points, draw indefinite lines toward them, being careful to select definite points on which to sight; write along each line the point to which drawn. Refer to the compass occassionally to see that the sketch is still oriented. Pace to the other end of the base, and, after laying off its length to scale, and driving a pin into the plotted position of the other end, place the ruler against the two pins and orient by a back-sight, checking by the compass. Should the base be short, the orientation by the compass will be the more accurate. Pivoting the ruler around the second pin, take sightings on the same points that were observed from the first station; the intersections of the corresponding lines will


Picket No.5.
Scale 1 in 100 yds.
100
50
0
100
200
300 yds
2°
3°
4°
5°
6°
7°
8°
Vert Int. 10 ft.
Submitted by
2d Lt. 1st Inf.
60
50
40
ΛΛΛ Picket Co. K 1st Inf.

be the plotted positions of these points. Having thus located as many points as may be desired to fix the positions of roads, buildings, marshes, streams, gullies, etc., sketch them in by eye. Locate the important and the more distant points with the greatest care, slighting the "filling in" if any part must be slighted.

In regard to sketching in contours, it should be borne in mind that ridges and valleys *must* be located and their directions plotted before contouring is commenced. The readiest method of acquiring skill in this variety of contouring is to locate the prominent points of the sketch *first*, and then to go back to the initial point and begin contouring.

Assume, for example, in the map shown, that the important points, including a point *d* in the creek bottom and several reference points, have been located in the manner described (construction lines are left to show the method), and that the sketches has returned to station *a*. He notices from this station that rather a prominent valley extends to the creek in the direction *ad*. This direction he plots, and, finding the general angle of depression to *d* to be 2°, he applies the horizontal equivalent for 2° as many times as it will go (in this instance four and something over, showing that the difference of level is between 40 and 50 feet). He further notes that the actual slope is 3° for about 150 yards from the top, and then more gentle the remainder of the distance. With this data, and having the ground itself to refer to, the contours crossing *ad* can be quite accurately sketched in for some distance to the right and left. Pacing along the base toward *c*, a ridge is encountered extending nearly to the creek. The sketcher stops, plots his position *b*, indicates on the map the general direction of the ridge, and then prolongs the contours already started, making them convex toward the creek, where they cross the ridge-line. Arriving at *c*, it is noticed that the line of sight of the clinometer used as a level cuts the lower branches of a tree near *a* at an estimated height of 12 feet; this shows *c* to be one contour higher than *a*, and it is drawn accordingly. After a little practice, it is perfectly feasible to carry the contouring along at the same time with the original plotting.

The map should be finished as shown, and, if necessary, a report submitted with it calling attention to features that were omitted, such, for example, as artillery positions too distant to be plotted to scale. The position of the picket and of all roads in rear of the line of observation should be accurately shown.

RECONNAISSANCE OF POSITIONS.—*A defensive position* is one affording protection from the shot and observation of an enemy, and, at the same time, commanding the ground in front, within range.

In order to reconnoiter and report intelligently on a position, it will be necessary to bear in mind the following principles and conditions which a good position should fulfill:

(*1.*) *A position should conform to the special tactical requirements of the occasion, and should be such as to favor the use of the relatively strongest arm.*

(*2.*) *It should be made impossible for the enemy to obtain natural cover during his advance. In other words, the position should have a "free field of fire."*

(*3.*) *The defenders should be protected from the fire and view of the enemy naturally, or by cover so arranged as not to interfere with counter-attacks.*

(*4.*) *The advance of the enemy should be hindered by obstacles so arranged that he may be checked while under the fire of the defenders.*

(*5.*) *The depth and communications should be such that the defenders may freely move from one point of the position to another, while the contrary should obtain with respect to the enemy's ground in front.*

A position which has all the above advantages will seldom be found, nor will a commander, as a rule, have the opportunity of choosing his position and there awaiting attack, but will have to make the best of the ground on which the turn of events has found him, fortunate if within a few miles he can find such as will give his troops any advantage.*

The best way of proceeding in the reconnaissance of a position is to begin at about 4,000 yards "in front" of the position, so as first to reconnoiter it from the point of view of the attack; moving thence towards the position, thus considering its capabilities from both the assailants' and defenders' points of view.

The following points would be reconnoitered:

(1.) *The ground in front.* (From the limit of effective artillery range up to the effective range of rifle.)

**Slopes admitting of maneuvers:*

From 0° to 5°	1.	Infantry can move in formations.
	2.	Cavalry can move with order; its charge more effective up hill than down.
	3.	Artillery fire more effective down than up hill.
5° to 10°	1.	Close movements difficult.
	2.	Charge possible up hill only, and for a short distance.
	3.	Moves with difficulty; effectual and constant fire ceases.
10° to 15°	1.	Can move but a very short distance in order.
	2.	Can trot but a short distance up hill, and walk down.
	3.	Moves with great difficulty; its fire ceases.

Slopes which may be ascended and descended singly:

15° to 20°	1.	Cannot move in order; can only fire singly with effect.
	2.	Can ascend at a walk, and descend obliquely.
20° to 25°	1.	Can move in extended order only.
	2.	Can ascend and descend obliquely, one by one.
25° to 30°	1.	Can move in extended order only and very slowly.
	2.	As before, when the slope is of soft earth, but with great difficulty.

Slopes which may be climbed:

30° to 35°—1. Lightly equipped, slowly in extended order.

35° to 40°—1. As before, with help of their hands.

40° to 45°—1. Men accustomed to hilly country may climb, holding on by their hands, but with danger of falling.

The nature of the surface, as well as the steepness, affects the capabilities of ground for the movements of troops and affects the three arms differently. Infantry can ascend a steep slope easier if the surface is rocky and rugged than if it is smooth turf. For cavalry the latter is easier.

Positions favorable for artillery of assailant; undulations; obstacles; cover; points on or in front of position where the fire of the defense might tell effectively on assailants; the slopes; communications; defiles; bridges; fords; etc.

(2.) *The approaches.* (Ground within effective rifle range of position.)

Same as (1) with regard to rifle fire; parts accessible or inaccessible; also points that may be used as advance posts.

(3.) *Main line of position.* (To be marked on sketch.)

Command; extent of front; artillery positions; hedges, fences, walls, to be prepared for defense; positions of shelter trenches; cover for firing line, supports and reserves; points of support (woods, farms, etc.) in detail; nature of soil; lateral communications, work necessary to improve them.

(4.) *The flanks.*

Natural obstacles, if any, as small woods, steep slopes, rivers, etc. If none, points that may be occupied by artillery, or defensive posts in prolongation of main line, in front or rear of flanks.

(5.) *The interior.*

Depth; cover; slopes; position for second line and cavalry; communications; points of support; nature of soil.

(6.) *The ground in rear.*

Lines of retreat; obstacles; positions for rallying line, if required.

The sketch. A *good sketch* is far preferable to a written description of a position, for one who can read a map. And however pressed for time one may be, *he should try* to convey as much information by *a sketch* as circumstances will permit. The scale should be not less than about 6 inches to a mile, to show certain details more accurately than could be done on a smaller scale.

The report. This will, of course, explain all points in which the sketch is deficient. Frequently the officer will be given a government map of the country, and be required to simply *draw up a report* furnishing all the tactical information in which the map may be deficient, or to make a copy on the required scale and then on arriving on the ground fill in the details not marked and correct those that have changed. It is not the business of the reconnoitering officer to suggest the proper method of occupying a position, unless specially ordered to do so. He will merely give the result of his observations, explain the points of weakness and strength, general capabilities, and suitability for the action of the three arms, the means which exist for obstructing an assailant's advance and retarding his pursuit, nature of communications, etc., and leave the superior officer to draw his own conclusions.

The report would be made under the following heads:

I. THE POSITION.—1, description of all that is not already explained by the sketch concerning its form; 2, the direction in which it faces; 3, its flanks; 4, any remarkable or dominating point in it; 5, the condition of the ground, whether soft and heavy for movement or the reverse; 6, probable direction of the enemy's attack, and reasons for this conclusion; 7, wood and water in case it is intended to camp or bivouac on the ground.

II. ADVANTAGES.—1, good positions for artillery, with wide and distant range commanding approaches and flanks; 2, good points of support on the front and flanks; 3, good cover for infantry, with ground in front suitable for the development of rifle fire; 4, cover for the supports and reserves; 5, communications in a lateral direction; 6, communications to the front, and the directions in which a counter-attack could be delivered; 7, communications to the rear, 8, flanks well posted; 9, extent suitable for the force; 10, natural obstacles to the enemy's advance.

III. DISADVANTAGES.—The reverse of the above. 1, limited range for artillery, or the ground affording better positions for enemy's artillery; 2, no favorable grounds on which to rest flanks; 3, no cover for firing line, or in such position that the fire would not be effective; 4, no cover in convenient places for the supports and reverses; 5, inconvenient or poor communications; 6, extent unsuitable to strength, or its nature to the composition of defending force; 7, cover for the enemy during his advance.

IV. THE COUNTRY IN THE VICINITY.—1, general description of the surrounding country beyond the limits of the sketch; 2, positions or strong points on the line of retreat, at which pursuit might be checked as farms, towns, etc.; 3, distant view to be obtained.

V. COMMUNICATIONS.—Description of roads, trails. etc., both parallel and perpendicular to the position; also those from the front towards the flanks.

VI. BRIDGES.—In detail as in road reports.

VII. RIVERS.—As in road reports, particularly as to points of passage and nature of banks.

VIII. WOODS.—1, its extent and form; 2, communications and paths through it; 3, average circumference of trees; 4, nature of undergrowth and its penetrability; 5, means to be adopted for the defense of the edges nearest the enemy; 6, whether timber is suitable for abattis to be placed at the salients and most exposed portions; 7, whether parts of the edge flank other parts; 8, whether the woods is quite isolated, or is connected with neighboring woods by scattering timber; 9, whether there are any folds in the ground, or any bank that would give shelter to an enemy close in front of it; 10, same in rear of the wood whence fire might be brought to bear on its near edge to prevent the enemy from issuing from it, in case he should succeed in capturing it.

IX. FARMS AND VILLAGES.—1, whether compact or straggling; 2, exterior surrounded by walls or fences suitable for defense; 3, whether fences beyond would cover enemy in his advance; 4, means of retarding advance; 5, whether commanded by artillery positions on enemy's or defender's side; 6, full details concerning buildings suitable for a réduit, as churches, etc.

X. FENCES,—Full details of those shown on sketch which it is proposed to utilize for defense.

XI. OBSTRUCTIONS, DEMOLITIONS, TRENCHES, AND OTHER WORKS.—Details of these to be submitted on a separate report if required, showing kinds, time required, number of reliefs, number of men in each relief, tools required, etc.

(Note example of position sketch and report.)

CHAPTER XXV.

LAYING OUT ROADS.

SELECTION OF SITE.—*Use of maps.* If a contoured map of the country is available, that which is probably the best line may be selected from it. An idea of a possible site may be obtained from any existing map. On account of drainage, water-sheds are more advantageous than either water-courses or hill-sides.

Ruling points. The road should be so located as to require the least amount of labor and expense, due regard being paid to direction and gradients. There will usually be some feature whose passage will be difficult. The part of this feature that can be passed most easily will constitute a *ruling point,* provided it is not too far out of the direct line of the road. A col in a chain of mountains would be such a point.

Secondary points. Between ruling points other points of less importance are chosen. These are called *secondary points* and in their selection there is much latitude. Ruling and secondary points are usually determined by an inspection of the ground. After this inspection, the possible lines would be compared and the best one selected.

LAYING OUT THE GRADE.—This may be done with a pair of dividers on a contoured map, or with the clinometer, slope board, or transit on the ground.

On a contoured map the maximum gradient is specified in the instructions. Take off on the dividers, from the scale of horizontal equivalents, the distance corresponding to the maximum slope. Should the distance between contours be greater than this, the road can have any desired direction. Should the distance be less, the road must be run obliquely between contours so as to make the distance between them equal to or greater than that corresponding to the maximum slope.

On the ground. Standing at one point of the road, direct an assistant to move up or down the hill until he comes to a point on the *desired slope*, as indicated by the instrument used. A picket is driven into the ground at this point and, if required, its bearing from the preceding point taken. The maximum gradient should be used only where absolutely necessary, a smaller one being chosen when practicable, so as to make the gradient as uniform as possible throughout. The work of marking the center grade line is continued until a ruling point is reached. Should the line surveyed run above or below the ruling point, the pickets will have to be shifted in order to equalize the gradient throughout, or, if this be not possible, a new line between ruling points will have to be run.

Leveling to grade between pickets. This is done with the clinometer, slope board, or transit. The instrument is taken to one picket and set to read the slope to the next picket. An assistant then moves along the line between pickets and at the points of change of slope drives in stakes and marks on them the amount of cut or fill as indicated by a rod or staff carried for this purpose. *a* and *b* (Fig. 215) are the positions of the pickets at points of grade. The position of the target on the

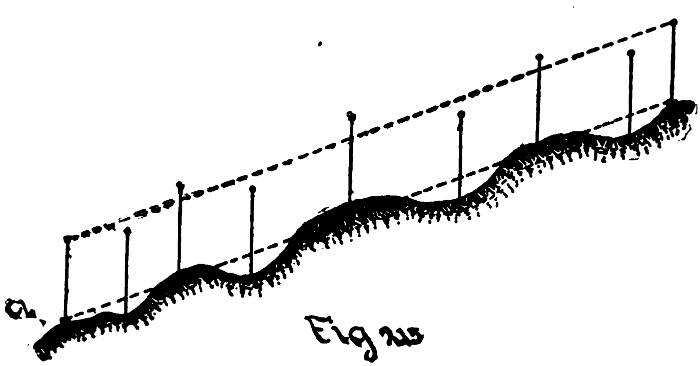


Fig 215

rod held at the intermediate points shows the amount of cut or fill.

LAYING OUT THE SECTION.—In rough work this may be done with any instrument that can be used as a level; for accuracy the wye level is necessary. This operation is sometimes called *cross-section leveling*.

The required width of the road is measured off horizontally from the center line and the exact edges marked by means of stakes or by scoring the ground.

The required slope of the road bed from center to sides is usually specified in the instructions. If not specified, it must be assumed and the difference of level between the center and side stakes computed. The amount of cut or fill at any point

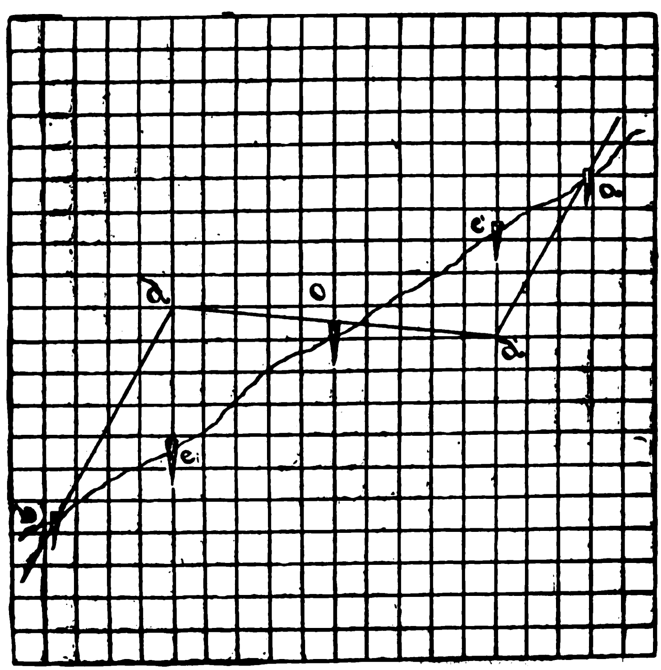


Fig. 216.

along the central grade line being known, the amount at the sides corresponding to it can be computed and marked on the side stakes.

Side slopes. To locate the slope stakes. Having the readings on the center and side stakes, a rough profile of the ground should be constructed on cross-section paper carried for the purpose. The location of the stakes for the side slopes is then determined as follows: Suppose *ab* (Fig. 216) to be the profile showing the natural surface of the ground as determined by the level, *o* being the position of a central picket. Plot *d* and *d'* on the diagram, their position having been computed as explained. Draw *dd'*; then from *d* and *d'* draw *db* and *d'a*, having an inclination corresponding to the required side slopes. The points where these last lines intersect the line *ab* will be the plotted positions of the bottom and top of the required slopes respectively.

The horizontal distances corresponding to *eb* and *e'a* on the diagram will be the required distances to lay off from the stakes at *e* and *e'* to determine the bottom and top of the side slopes. Stakes are driven at the points *a* and *b*.

Sections should be made about every thirty yards, or as required by the conformation of the ground.

CHAPTER XXVI.

COMPUTING AREAS.

LONG OR LINEAR MEASURE.

12 inches....1 foot.
3 feet.......1 yard=36 inches.
5.5 yards ...1 rod=16.5 ft.=198 inches.
40 rods ......1 furlong=220 yds. =660 feet.
8 furlongs ..1 mile=320 rods=1760 yds.=5280 ft.=63,360 ins.
3 miles.....1 league.
6 feet.......1 fathom.
120 fathoms...1 cable's length.
6080 feet. .....1 nautical mile or knot.
Russian verst=3500 feet.
Spanish vara=32.8748 inches.
California vara=33.372 inches.
1 Gunter's chain=4 rods=66 ft.=792 inches=100 links.
80 Gunter's chains=320 rods=1 mile.

SQUARE OR LAND MEASURE.

144 sq. inches..1 square foot.
9 square ft...1 sq. yd.=1296 sq. inches.
30.25 sq. yds..1 sq. rod, perch or pole=272.25 square feet.
40 square rods, perchs or poles... ....1 rood=1210 sq. yards=¼ acre.
4 roods......1 acre=160 sq. rods =4840 sq. yds.=43,560 sq. ft.
625 sq. links=1 sq. rod, perch or pole.
16 sq. rods=1 sq. ch.=10,000 sq. links.
2.5 sq. chs.=1 rood=40 sq. rds. =25,000 square links.
4 roods=1 acre=10 sq. chs.=160 sq. rds.=100,000 square links.
640 acres=1 sq. mile=6400 sq. chs.
A section of land is 1 mile square =640 acres=3,097,600 sq. yds. =27,878,400 sq. ft.
A square acre is 208.71 ft. on each side.
A square half-acre is 147.581 ft. on each side.
A square quarter-acre is 104.355 ft. on each side.

HOW LAND IS REGARDED.—In computing areas, land is regarded as a horizontal plane bounded by horizontal lines, however rolling, uneven or broken the surface may be; consequently, if divided into any number of parts, the sum of the areas of all the parts will equal the area of the whole, which would not be the case if the areas of the actual surfaces of the parts were measured, and that of the entire piece computed from the bounding lines.

HOW AREAS ARE EXPRESSED.—If the measurements are taken with a Gunter's chain, the areas are usually expressed in acres, roods and perches; but if the measurements are taken with an Engineer's 100-foot chain, the areas are usually expressed in acres and decimals of an acre. By a reference to the above table of *Square Measure* it will be seen how an area, if expressed in any terms of Gunter's chain, as square links, may be reduced to any other terms, as acres, by dividing by 100,000; or roods, by dividing by 25,000; or square chains, by dividing by 10,000; etc.

I. AREAS COMPUTED BY THE USE OF THE CHAIN ONLY.

THE AREA OF A PIECE OF GROUND in the form of a *square*, *rectangle* or *parallelogram* is found by multiplying the base by the altitude, and reducing it to any terms desired, by the above table.

If the piece is in the form of a *triangle*, then the area will be one-half the product of the base by the altitude; or by the formulæ, $Area = \sqrt{s(s-a)(s-b)(s-c)}$, in which $s = \frac{1}{2}(a+b+c)$.

If in the form of a *trapezoid*, the area will be one-half the sum of the two parallel sides multiplied by the perpendicular distance between them.

If in any other form that can be divided into triangles, do so, find the area of each triangle separately and take their sum for the area of the whole.

If in the form of a circle, multiply the square of the radius by 3.1416.

If in the form of an ellipse, multiply the product of the semi-axes by 3.1416.

II. AREAS BY THE USE OF THE COMPASS AND CHAIN.

Having divided the piece up into triangles, measure two sides and the included angle of each triangle and compute the area of each triangle by the formula, $Area = \frac{1}{2}\,a\,b \sin C$, in which a and b are the two sides and C the included angle.

III. AREAS FROM BEARINGS AND LENGTHS OF THE BOUNDARY LINES.

In ordinary land surveying, simply for the purpose of locating the boundary lines and computing areas, the compass and Gunter's chain have been generally used since the first settlement of the country, and are so recognized in describing land in purchases and sales, and in the courts and official records. Owing to the errors liable to be made with the compass and the difficulties of reading bearings to a degree of accuracy desired, the *Transit* is sometimes used instead, where the land is valuable, and the angles measured with it; and afterwards, if so desired, the bearings of the courses can be computed after having determined the bearing of one course sufficiently accurate. This is not necessary, however, to determine the area.

In the field work, the surveyor usually begins at some prominent corner of the field and follows the boundary lines entirely around the field to the point of beginning, taking the bearings with the compass (fore and back) and measuring the length with the chain of each of the bounding lines. It is immaterial whether the field be kept on the right hand or left hand in going around it. If the compass cannot be set upon the boundary lines and the chaining done on them, then equal rectangular offsets are made, and the bearings and lengths of corresponding parallel lines measured.

If there are fences, streams, houses, etc., in the field that it is desired to locate and plot, their bearings can be taken, of the fences and streams where they intersect the boundary lines, and of the houses and other objects from the corners, when visible.

Taking the bearing of a single object in the field, as a large tree, visible from every corner, is to be strongly recommended as furnishing a means of detecting errors that may be made in reading or recording the bearings of the courses or in the chaining.

For if these bearings are all plotted on the map, they should all intersect in one point, if no errors have been made; but if an error has been made in the bearing or length of a course, the bearing of the tree or other object from the end of this course will not pass through the common point when plotted and it will show at once where there has been made an error of some kind, either in the field measurements or notes, or in the plotting.

The form for the field notes, given under the description of the compass, can be used, or better still, if a sketch is to be made, a form similar to that described in the Engineer's Field Note Book, beginning at the bottom of the left-hand page and using the right-hand page for a sketch made at the time of taking the notes. The advantage of this being, that when looking in the forward direction of a course with the note book open before him, the surveyor sees the columns in his book lying just as the course does, and offsets made to the right and left are entered on the right and left of the center column.

Field Notes.—Compass Survey. Dec. 24, 1901.

Observer.—Henry Arnold.

Chainmen.—William Jones and Frank Smith.

STATION.	POINT.	BACKSIGHT.	FORESIGHT.	DISTANCE.	REMARKS.
$\odot$1 (B)	$\odot$2		N76°45'E	3.75 chs.	
$\odot$2 (C)	$\odot$1	S76°45'W			
$\odot$2	$\odot$3		N34°45'E	4.32	
$\odot$3 (D)	$\odot$2	S34°45'W			
$\odot$3	$\odot$4		N50°15'W	5.86	
$\odot$4 (E)	$\odot$3	S50°15'E			
$\odot$4	$\odot$5		S48°30'W	6.55	
$\odot$5 (A)	$\odot$4	N48°30'E			
$\odot$5	$\odot$1		S41°45'E	5.06	
$\odot$1	$\odot$5	N41°45'W			

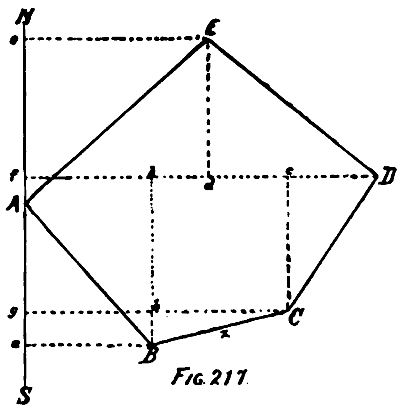


FIG. 217.

The figure *BCDEAB* in Fig. 217 is a plot of the Field Notes, beginning at *B* as station 1 and following around through *C*, *D*, etc., to *B* again.

Having plotted the Field Notes, draw a meridian through the most easterly or westerly corner, in this case NS through the most westerly station *A*. From each of the other stations draw lines perpendicular to this meridian, as *Ee*, *Df*, etc., and on them let fall perpendiculars as *Cc*, *Ed*, etc.

Now, by reference to the figure, it will be seen that the Area $ABCDEA = aBCg + gCDf + fDEe - aBA - AEe$.

From Geometry the area of $aBCg = \left(\frac{aB+gC}{2}\right) \times bB$; hence, $2aBCg = (aB+gC) \times bB$; and likewise $2gCDf = (gC+fD) \times cC$; and $2fDEe = (fD+eE) \times Ed$; and $2aBA = (aB) \times Aa$; and $2AEe = (eE) \times eA$; therefore $2ABCDEA = (aB+gC) \times bB + (gC+fD) \times cC + (fD+eE) \times Ed - (aB) \times Aa - (eE) \times eA$.

In this equation the quantities in parentheses are known as the *double meridian distances* of the courses *BC*, *CD*, *DE*, *AB*, and *EA* respectively, and are all positive, and the multipliers are known as the *difference of latitudes* of the same courses respectively, *bB*, *cC*, and *Ed* being positive, while *Aa* and *eA* are negative, all of which may be easily obtained from the data in the Field Notes, as will be shown.

Referring to Fig. 217 again, it will be observed that the surveyor in passing from *B* to *C* goes north a distance equal to *Bb;* this distance *Bb* is the *difference of latitude* of the two stations at the extremities of the course, and is called the difference of *latitude of the course*, or briefly the latitude, and from Geometry is equal to the *cosine* of the angle *bBC* multiplied by the length *BC*, or *the cosine of the bearing of the course* *multiplied by the length of the course.* And *the latitude of any course is equal to the cosine of the bearing of the course into its length.* If the *forward bearing* of a course is northward, its *latitude* is called a *northing*, and is given a + sign in the computations (see *bB*, *cC*, and *Ed* above); if the *forward bearing* is southward, the *latitude* is called a *southing*, and is given a — sign (see *Aa* and *eA* above). Having a table of *natural cosines*, find in it the cosine of the bearing of the course and multiply it by the length of the course and the product will be the *latitude* of the course, to which give the + or — sign according as it is a northing or southing; or having tables of logarithmic cosines and logarithms of numbers, find the logarithm of the cosine of the bearing and the logarithm of the length of the course, add them together and look in the table of logarithms for the number corresponding to the sum, which will be the latitude of the course.

A due east-and-west course has no difference of latitude.

Likewise the surveyor in passing from *B* to *C* departs a distance east from *B* equal to *bC*. This distance *bC* is called *the departure of the course BC*, and is equal to the *sine of the bearing of the course multiplied by the length of the course*, as the departure of any course is equal to the sine of its bearing into its length.

If the forward bearing of a course is *east*, its *departure* is called an *easting* and is given a + sign; if *west*, its *departure* is called a *westing* and is given a — sign. Having a table of *natural sines*, the value of the *sine* for the bearing of any course can be taken out, and multiplied by the length of the course will give the *departure* of that course, to which the proper sign must be given; or with tables of *logarithmic sines* and *logarithms of numbers*, the departures may be found similarly to the method of determining latitudes. A due north-and-south course has no departure. The *meridian distance* of *B* is the distance *aB*; the meridian distance of *C* is the distance *gC*; and so the meridian distance of any point is its perpendicular distance from the reference meridian NS. The meridian distance of a point, as A, on the reference meridian is 0. The ***meridian distance of the course*** BC is the perpendicular distance of its middle point x from the reference meridian NS, midway between a and g, and *the meridian distance* of any course is the *meridian distance of its middle point.*

The *double meridian distance* (D. M. D.) of any course is, therefore, equal to twice *its meridian distance*, or equal to the sum of the meridian distances of its two extreme points.

The D. M. D. of the course AB is, therefore, equal to aB, which is also its *departure;* and of EA is equal to eE, which is its departure; and so the D. M. D. of any course, one extremity of which is on the reference meridian, is equal to its departure.

Calling AB the first course in the computation of the area, and BC the second, and so on around, it is seen that the D. M. D. of the first course, AB, is equal to its *departure* aB. That the D. M. D. of the second course, BC, [equal to $aB+(gb+bC)$] is equal to the D. M. D. of the first course (aB), plus the departure of the first course ($aB=gb$), plus the departure of the course itself (bC). That the D. M. D. of the second or any other course (except the two as AB and EA meeting on the reference meridian) is equal to the D. M. D. of the preceding course, plus the departure of that course, plus the departure of the course itself. Of CD it is equal $gC+fD=(gC+aB)+bC+cD$, since $(aB=fk)+(bC=kc)+cD=fD$.

Therefore, comparing the quantities, in the equation beginning $2ABCDEA=$, with those in the preceding explanations, it will be seen that *twice the area of the field is equal to the algebraic sum of the products of the double meridian distance if each course into its own latitude*, being careful to observe the signs. In stating that *north latitudes* and *east departures* are *positive* and *south latitudes* and *west departures* are *negative*, these terms are used in their algebraic sense, so that if a positive D. M. D. and a negative departure are added together their algebraic sum will be the numerical difference, with the sign of the greater. And so the algebraic sum of the products above will be the numerical difference between the sum of the positive products and the sum of the negative products.

A check on the correctness of the work is, that the D. M. D. of the last course should be equal to its departure.

Had the surveyor gone around the field with it on his right instead of on his left, the signs of the latitudes and departures would be changed, but the numerical values would be the same.

HOW TO COMPUTE THE AREA.—Understanding the above, the surveyor can now proceed to compute his area from his field notes as follows:

Rule a FORM B and enter therein the Stations, as given by the letters on the plot, Fig. 217, and the Bearings and Distances of the courses; then with a table of natural cosines and sines proceed to find the latitudes and departures of each course as explained, and enter them in the FORM A, north latitudes in the + column, and south latitudes in the — column; and east departures in the + column, and west departures in the — column. Thus, for the first course AB, bearing S 41° 45′ E, length 5.06 chains.

Natural cosines 41°45′	.7461	Natural sines 41°45′	.6659
Length of course....	5.06	Length of course...	5.06
	44766		39954
	37305		33295
Product=Lat........	3.775266	Product=Dep.......	3.369454

The course being southward, the lat. is negative; hence enter 3.78 in the – lat. column of FORM A.

The course being eastward, the dep. is positive; hence enter 3.37 in the + dep. column of FORM B. Proceed in the same manner for all the other courses.

If the bearings of the courses are read only to the nearest 15′ of arc, then the work of computing the latitudes and departures may be much simplified by using the *Traverse Table*, in which is given the latitude and departure corresponding to bearings that are expressed in degrees and quarters of a degree, from 0° to 90°, and for every length of course from 1 to 100 computed to two places of decimals. The latitudes and departures so tabulated were obtained by using the formulas and method just explained; viz., lat.=course×cos. of bearing; dep.=course×sine of bearing. When the courses are longer than 100, as 175, take out the lat. and dep. for 100 and then for 75 and add them together. If the length of a course is expressed decimally, as 5.06, take out the lat. and dep. for 5, then for 6 and move the decimal point, in the latter, two places to the left and add to those for 5.

As the compass can be read to closer than 15′, and if the mean is taken when fore and back sights differ only slightly, the results will seldom be expressed in quarter degrees, and when the transit is used, the azimuths will be read to the nearest minute, and the Traverse Table could not then be used in all cases.

If using logarithmic tables, rule a FORM A, enter the courses, at tops of columns noting whether *northing* or *southing* for latitudes, and whether *easting* or *westing* for departures, with their proper signs; then from the log. tables take the *log. cos.* and *log. sin.* of the first course and the *log. of its length* and enter them in FORM A as shown*. Do this for each course. Then add each pair of logarithms for the *log. lat.* and *log. dep.*, and enter the table of logarithms of natural numbers and find the number corresponding to each *log. sum* and write it under it. Having found them all, enter them in their proper columns opposite their respective courses in FORM B.

FORM A for Computing Latitudes and Departures.

	COURSE AB.	COURSE BC.	COURSE CD.	COURSE DE.	COURSE EA.
	S —	N +	N +	N +	S —
Log. cos. (lat.)	9.8727	9.3602	9.9147	9.8058	9.8213
Log. dist.	.7042	.5740	.6355	.7679	.8162
Log. lat.	.5769	.9342	.5502	.5737	.6375
Latitude.	—3.78	+0.86	+3.55	+3.75	—4.34
	E +	E +	E +	W —	W —
Log. sin. (dep.)	9.8234	9.9882	9.7559	9.8858	9.8745
Dog. dist.	.7042	.5740	.6355	.7679	.8162
Log. dep.	.5276	.5622	.3914	.6537	.6907
Departure.	+3.37	+3.65	+2.46	—4.51	—4.91

FORM B for Computing Areas from Bearings and Distances of Bounding Lines.

STA.	COURSES. BEARINGS	COURSES. DIST.	LATITUDE. N+	LATITUDE. S—	DEPARTURE. E+	DEPARTURE. W—	BALANCED. LAT.	BALANCED. DEP.	D.M.D. +	DOUBLE AREA +	DOUBLE AREA —
		chs.									
A	S41°45′E	5.06		3.78	3.37		—3.79	+3.36	3.36		12.73
B	N76°45′E	3.75	0.86		3.65		+0.86	+3.64	10.36	8.91	
C	N34°45′E	4.32	3.55		2.46		+3.54	+2.46	16.46	58.27	
D	N50°15′W	5.86	3.75			4.51	+3.74	—4.53	14.39	53.82	
E	S48°30′W	6.55		4.34		4.91	—4.35	—4.93	4.93		21.45
		25.54	8.16	8.12	9.48	9.42				121.00	34.18
			8.12		9.42					54.18	

Error in Lat. = .04 Dep. = .06

Error of closure in links = $\sqrt{4^2+6^2} = 7.21$ links.

Error of the survey = $\frac{7.21}{2554}$ = 1 in 354.

2 | 86.82
43.41 sq. chs.
4.341 acres,
or 4 A. 1 R. 14 P. 375 sq. lks.

*This FORM answers equally well when the survey has been made with a transit and the bearings read to minutes, by noting in which quadrant the different azimuths lie, as shown in Figs. 200 and 201; or if measured from 0° at the south to 90° west, etc., or 0° at the south to 90° east, etc., make similar diagrams, and mark in the different quadrants the proper signs for the latitudes and departures lying in them. Thus in Fig. 200 azimuths between 270° and 360° will have lat. and dep. both +; azimuths between 180° and 270° will have lat. —, and dep. +, between 180° and 90° lat.—, dep.+; between 90° and 0° lat+, dep.—; and similarly for other diagrams

Having found the latitudes and departures of all the courses and entered them in FORM B, find the sum of each column. As the surveyor, in traversing around the field back to the point of beginning, traveled as far south as he had gone north, then the sum of the south latitudes ought to be equal to the sum of the north latitudes, but in reality they seldom or never will be so, on account of the impossibility of ever making any measurements exact, and the difference between the sums will show the errors made in Latitude, or the distance the plot lacks of closing in a north-and-south direction. So also should the sum of the west departures be equal to the sum of the east departures, but neither will they be so, and their difference will be the error in Departures, or the distance the plot lacks of closing in an east or west direction, and the square root of the sum of the squares of these errors will equal the distance, in a straight line, that the whole plot lacks of closing, which is called the *error of closure;* and the ratio of the error of closure to the total length of the whole perimeter, as found from the Field Note and expressed as an error of 1 link, chain, etc., in so many links, chains, etc., is called the *error of the survey.*

THE LIMIT OF ERROR.—The limit of this error *allowable* depends upon the importance of the survey, or value of the land surveyed. In ordinary land surveying it should never be greater than 1 in 300, and generally not that great; while in city surveying it should not be greater than 1 in 1000, and average less than 1 in 5000.

BALANCING THE SURVEY.—Having computed the latitudes and departures of all the courses and determined the errors, the next step is to *Balance the Survey*, which consists in distributing the error in latitude among the latitudes of all the courses, and the error in departure among the departures of all the courses, in proportion to the length of each course, being careful to *add* the correction in each case when applied to the deficient column and to *subtract* it when applied to the excess column.

A reference to the example in FORM B will show how the error of 4 links in latitudes was distributed, by comparing the numbers in the column of Balanced Latitudes with those in the columns under Latitudes; and the same for the error of 6 links in departures.

This distribution of the errors does not necessarily correct them, but as has been aptly said, "only humors them in"; and cases might arise where such a distribution would make the error in area larger than the error in the perimeter.

Two methods are used for this distribution of errors, both based on the same principle, that they are due as much to erroneous bearings as chaining; one by means of applying the corrections graphically to the plotted courses by the formulæ

$$\text{Sum of all the courses} : \text{Error of closure} :: \text{Course 1} : \text{Error for course 1}$$

and

$$\text{Sum of all the courses} : \text{Error of closure} :: \text{Sum of a course and preceding courses} : \text{Error for course considered;}$$

thus moving all the corners of the plot except the first a slight amount to bring the end of the last course so as to fall on the point of beginning of the first course.

This has changed the lengths and bearings of all the courses, and the new lengths and bearings are taken from the plot with the scale and protractor, and used in computing the area. The other method, which is more exact, is by calculation, using the formula

$$\text{Sum of all the courses} : \text{Any given course} :: \text{Total error of lat. or dep.} : \text{Error of lat. or dep. of given course;}$$

each correction being so applied as to diminish the total error. A modification of the above formula is to weight each of the courses according to the relative difficulties in measuring the bearings and distances, selecting some one course as standard and weighting it 1. Then multiply each course by the weight given it, and use these multiplied lengths in the above formula, thus:

Sum of all the . Any given .. Total error of . Error of lat. or
mult'd courses . mult'd course .. lat. or dep. . dep. of given course

Should the angles or bearings have been measured with a transit or solar compass, then the assumption would be that the errors were all in the chaining, and the above formulæ would not be used, but the following:

The arithmetical sum . Any given .. Total error . Error of
of all the latitudes . latitude .. in latitudes . given lat. ;

and similarly for departures, substituting departures in the formula for latitudes. If the courses had been weighted according to the difficulties in chaining, then the latitudes and departures would be multiplied by the weights of their courses, and these weighted results used in the formula.

When conditions of the survey render it probable that errors were more likely to have occurred on certain courses than others, then the corrections can properly be applied to those courses.

When the *error of the survey* is so large as to indicate a re-survey, the notes, computations, and plotting should all be carefully re-examined for errors in them, or for indications that may point to where errors were likely to be found, and thus save some of the work of a complete re-survey.

THE DOUBLE MERIDIAN DISTANCES.—The balanced latitudes and departures having been determined and placed in FORM B, the next step is to determine the D. M. D. of each course, as explained when considering the plot of the notes, Fig. 217. If the reference meridian is taken through the most easterly or westerly corner of the field, whether the corner at which the surveyor began his work or not, then all the D. M. D.'s will be positive. Referring to FORM B, it is seen that the D. M. D. of A equals its dep., 3.36; of B equals D. M. D. of A, 3.36+dep. of A, 3.36+dep. of B, 3.64=10.36; of C equals D. M. D. of B, 10.36+dep. of B, 3.64+dep. of C, 2.46=16.46; of D equals D. M. D. of C, 16.46+dep. of C, 2.46+dep. of D, (—4.53)=14.39; of E equals D. M. D. of D, 14.39+dep. of D, (—4.53)+dep. of E, (—4.93)=4.93.

Having determined the Double Meridian Distances of all the courses, the last step consists in finding double the areas of the trapezoids and triangles into which the plot has been divided, the algebraic sum of which double areas will be equal to double the area of the field.

The double areas of these trapezoids and triangles are found by multiplying the Double Meridian Distances of each course into its corresponding Balanced Latitude, observing that when the Latitude is + the double area will be algebraically +, and when the Latitude is − the double area will be algebraically —. Make these multiplications and place the products in one of the last two columns. Find their sums, and then their numerical difference, which will be double the area of the field, when divided by 2 will give the area.

If the measurements were made with a Gunter's chain, as in the problem, then the area will be given in square chains, and dividing by 10 gives it in acres and decimals; then multiplying the decimal part by 4 and pointing off will reduce this decimal part of an acre to roods and decimals; multiplying this decimal part by 40 and pointing off will reduce it to perches and decimals; and multiplying this decimal part by 625 will reduce it to square links.

Had a 100-foot chain or tape been used to measure distances and the measurements been expressed in feet, the area would have been obtained in square feet and could have been reduced to acres and decimals by dividing by 43560, the number of square feet in one acre. Then if the decimal part of acres be multiplied by 4840 and pointed off, it will be in square yards and decimals; and this decimal multiplied by 9 will reduce it to square feet.

If after computing the area it is discovered that the chain or tape used was not of standard length, the area may be corrected by the proportion given in the front part of the book under the description of the chain.

COMPUTING AREAS FROM THE RECTANGULAR COÖRDINATES OF THE CORNERS.

As the area of the plot, Fig. 217, was divided into a number of trapezoids and triangles, and their areas separately computed and added together to obtain the area of the whole, so any right-lined figure may be divided into a series of right trapezoids and triangles, by letting fall from the corners perpendiculars to any fixed line called the *base*, and the triangles may be considered as trapezoids having one parallel side reduced to zero. Then the area of the whole will be the algebraic sum of the areas of the trapezoids.

The application of the above is, that frequently a crooked stream or irregular shore line of a lake or pond occurs on a boundary line, and it is then customary to run as long lines as possible near said irregular boundaries, and from these *base-lines* measure the short perpendicular offsets to the stream or lake, at intervals frequent enough to define the shore line quite accurately, and then compute the areas of the separate trapezoids thus formed, and adding them together for the whole area between the base-line and shore line. It having been determined that the area of a figure bounded by a continuous or a broken curve is approximately equal to that of a right-line figure whose vertices are contained in that curve. By properly locating the vertices the approximation may be made as closely as desired.

As the base-line and perpendicular offsets constitute a series of rectangular coördinates for the trapezoids, and if there are any considerable number of such trapezoids lying consecutively with their bases on one straight line, as in Fig. 218, the area of the series may be found by the following rule:

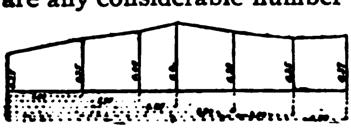


FIG. 218.

RULE A.—*Multiply the distance along the base-line of each intermediate ordinate (or offset) from the first, by the difference between the two adjacent ordinates (or offsets), always subtracting the following offset from the preceding offset, in the order* *along the broken line. Also multiply the distance of the last ordinate from the first by the sum of the last two ordinates. Divide the sum of these products by two.* Thus Fig. 218, the area equals $\frac{1}{2}$ $[1.0(.25-.40)+1.75(.35-.45)+2.25(.40-.39)+3.00(.45-.35)+3.80(.39-.37)+4.50(35+37)]=\frac{1}{2}$ $[1.0(-.15)+1.75(-.10)+2.25(.01)+3.00(.10)+3.80(.02)+4.50(.72)]$ $=1.6567$ sq. ch. If that portion of the broken line forming the upper side of the last trapezoid is perpendicular to the base-line, the rule still holds good, the area of that trapezoid being zero. If the upper side of last trapezoid returns toward the origin, the last trapezoid will be algebraically negative and must be so added, or numerically subtracted. Hence, the above rule is true for all complications of the broken line, even if composed of progressive, retrogressive and perpendicular elements.

Even suppose the broken line by a series of directions returns upon itself at the point of beginning, thus forming an enclosed area or polygon, as for example the one considered in Fig. 217, when Rule A can be made to apply. Thus, Fig. 219 let *ABCDEA* be that irregular line forming any polygon. Assume some point, as *O*, as the point of origin of the *rectangular coördinates*, to which refer all the corners of the field, and draw the lines *OY* parallel to NS, Fig. 217, and *OX* perpendicular to it. Join *O* and *A* and consider it as the first and last line of the broken series. Then proceed to find the area of the polygon *OABCDEAO*, by the rule, which will be found to equal the area of the polygon *ABCDEA*. As it is imma-

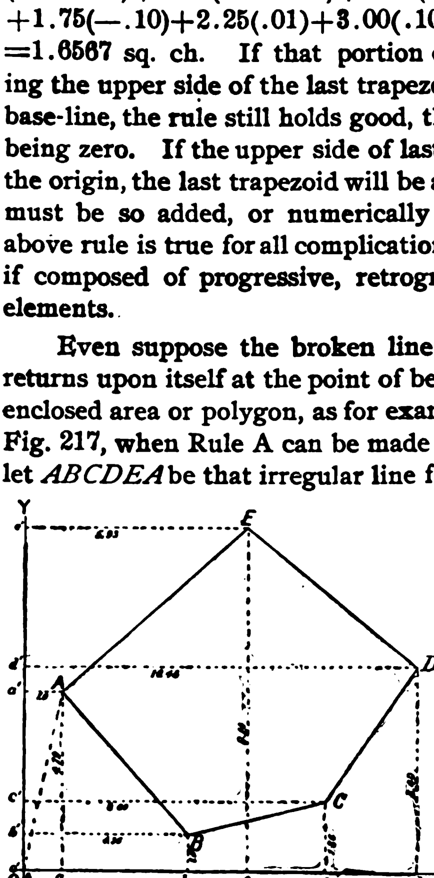


FIG. 219.

terial where the point O is located, let it be assumed for convenience of using the problem Fig. 217 at the intersection OY, one chain distant from A, and OX, one chain distant from B. Adding unity to the balanced latitude and departure, in FORM B, of the course AB, the rectangular coördinates of each of the corners of the field can be obtained and one can proceed to find the area as follows: Representing the distances of the ordinates of the corners of the broken line $OABCDEAO$, measured on OX from O, as o, a, b, c, d, e, a, o, which are called the abscissas of the corners; and the lengths of the ordinates of the corners by o', a', b', c', d', e', a', o' respectively, and applying the rule would give $\frac{1}{2}[a(o'-b')+b(a'-c')+c(b'-d')+d(c'-e')+e(d'-a')+a(e'-o')+o(a'-o')]$; but o and o' both being zero, the last term reduces to zero. Substituting the values for the letters gives $\frac{1}{2}[1(0-1)+4.36(4.79-1.86)+8.00(1-5.40)+10.46(1.86-9.14)+5.93(5.40-4.79)+1(9.14-0)+0(4.79+0)]$ $=\frac{1}{2}[(-1.0000)+12.7748+(-35.20)+(-76.1488)+3.6173+9.14]=\frac{1}{2}[(-112.3488)+25.5321]=\frac{1}{2}(-86.8167)=43.40835$ sq. chs.

If the distances of the ordinates are taken on the line OY from O, and the lengths of the ordinates on OX from O, and apply the rule, the result will come out just the same; thus, $\frac{1}{2}[a'(o-b)+b'(a-c)+c'(b-d)+d'(c-e)+e'(d-a)+a'(e-o)+o'(a+o)]=\frac{1}{2}[4.79(0-4.36)+1(1-8.0)+1.86(4.36-10.46)+5.40(8.00-5.93)+9.14(10.46-1)+4.79(5.93-0)+0(1+0)]=\frac{1}{2}[(-20.8844)+(-7.0000)+(-11.3460)+11.1780+86.4644+28.4047]=\frac{1}{2}(126.0471-39.2304)=\frac{1}{2}(86.8167)=43.40835$ sq. chs., the same result obtained by the method of Double Meridian Distances. Hence, the following modification of the wording of Rule A has been made for finding the area of any polygon whose corners are fixed by rectangular coördinates.

RULE B.—*Multiply the abscissa (or ordinate) of each corner by the difference between the ordinates (or abscissas) of the two adjacent corners; always making the subtraction in the same direction around the polygon. Half the sum of the products is the area of the polygon.*

The application of the above method and rule is of frequent occurrence, where, as in mining and coal regions, many claims, grants, etc., have been surveyed and the corners fixed in position by being referred to some established point by rectangular coördinates run in a due north-south and east-west direction.

And in making a topographical survey locating points by their coördinates, being careful to so locate all corners of fields, etc., at any time afterwards one can plot in any field if it has not already been plotted; and when its area is wanted, it is only necessary to take the rectangular coördinates of its corners and proceed as above, without the necessity of following around its boundary measuring the courses.

SMITHSONIAN

GEOGRAPHICAL TABLES.

TABLE 1.

FOR CONVERTING U. S. WEIGHTS AND MEASURES.*

CUSTOMARY TO METRIC.

LINEAR.

	Inches to millimetres.	Feet to metres.	Yards to metres.	Miles to kilometres.
1 =	25·4001	0·304801	0·914402	1·60935
2 =	50·8001	0·609601	1·828804	3·21869
3 =	76·2002	0·914402	2·743205	4·82804
4 =	101·6002	1·219202	3·657607	6·43739
5 =	127·0003	1·524003	4·572009	8·04674
6 =	152·4003	1·828804	5·486411	9·65608
7 =	177·8004	2·133604	6·400813	11·26543
8 =	203·2004	2·438405	7·315215	12·87478
9 =	228·6005	2·743205	8·229616	14·48412

CAPACITY.

	Fluid drams to millilitres or cubic centimetres.	Fluid ounces to millilitres.	Quarts to litres.	Gallons to litres.
1 =	3·70	29·57	0·94636	3·78543
2 =	7·39	59·15	1·89272	7·57087
3 =	11·09	88·72	2·83908	11·35630
4 =	14·79	118·29	3·78543	15·14174
5 =	18·48	147·87	4·73179	18·92717
6 =	22·18	177·44	5·67815	22·71261
7 =	25·88	207·02	6·62451	26·49804
8 =	29·57	236·59	7·57087	30·28348
9 =	33·27	266·16	8·51723	34·06891

SQUARE.

	Square inches to square centimetres.	Square feet to square decimetres.	Square yards to square metres.	Acres to hectares.
1 =	6·452	9·290	0·836	0·4047
2 =	12·903	18·581	1·672	0·8094
3 =	19·355	27·871	2·508	1·2141
4 =	25·807	37·161	3·344	1·6187
5 =	32·258	46·452	4·181	2·0234
6 =	38·710	55·742	5·017	2·4281
7 =	45·161	65·032	5·853	2·8328
8 =	51·613	74·323	6·689	3·2375
9 =	58·065	83·613	7·525	3·6422

WEIGHT.

	Grains to milligrammes.	Avoirdupois ounces to grammes.	Avoirdupois pounds to kilogrammes.	Troy ounces to grammes.
1 =	64·7989	28·3495	0·45359	31·10348
2 =	129·5978	56·6991	0·90719	62·20696
3 =	194·3968	85·0486	1·36078	93·31044
4 =	259·1957	113·3981	1·81437	124·41392
5 =	323·9946	141·7476	2·26796	155·51740
6 =	388·7935	170·0972	2·72156	186·62088
7 =	453·5924	198·4467	3·17515	217·72437
8 =	518·3914	226·7962	3·62874	248·82785
9 =	583·1903	255·1457	4·08233	279·93133

CUBIC.

	Cubic inches to cubic centimetres.	Cubic feet to cubic metres.	Cubic yards to cubic metres.	Bushels to hectolitres.
1 =	16·387	0·02832	0·765	0·35239
2 =	32·774	0·05663	1·529	0·70479
3 =	49·161	0·08495	2·294	1·05718
4 =	65·549	0·11327	3·058	1·40957
5 =	81·936	0·14158	3·823	1·76196
6 =	98·323	0·16990	4·587	2·11436
7 =	114·710	0·19822	5·352	2·46675
8 =	131·097	0·22654	6·116	2·81914
9 =	147·484	0·25485	6·881	3·17154

1 Gunter's chain = 20·1168 metres.
1 sq. statute mile = 259·000 hectares.
1 fathom = 1·829 metres.
1 nautical mile = 1853·25 metres.
1 foot = 0.304801 metre, 9·4840158 log.
1 avoir. pound = 453·5924277 gram.
15432·35639 grains = 1 kilogramme.

The only authorized material standard of customary length is the Troughton scale belonging to this office, whose length at 59°·62 Fahr. conforms to the British standard. The yard in use in the United States is therefore equal to the British yard.

The only authorized material standard of customary weight is the Troy pound of the Mint. It is of brass of unknown density, and therefore not suitable for a standard of mass. It was derived from the British standard Troy pound of 1758 by direct comparison. The British Avoirdupois pound was also derived from the latter, and contains 7,000 grains Troy.

The grain Troy is therefore the same as the grain Avoirdupois, and the pound Avoirdupois in use in the United States is equal to the British pound Avoirdupois.

The British gallon = 4.54346 litres. The British bushel = 36.3477 litres.

The length of the nautical mile given above and adopted by the U. S. Coast and Geodetic Survey many years ago is defined as that of a minute of arc of a great circle of a sphere whose surface equals that of the earth (Clarke's Spheroid of 1866).

* Issued by U. S. Office of Standard Weights and Measures, and republished here by permission of Superintendent of Coast and Geodetic Survey.

FOR CONVERTING U. S. WEIGHTS AND MEASURES.

METRIC TO CUSTOMARY.

LINEAR.

	Metres to inches.	Metres to feet.	Metres to yards.	Kilometres to miles.
1 =	39·3700	3.28083	1·093611	0·62137
2 =	78·7400	6·56167	2·187222	1·24274
3 =	118·1100	9·84250	3·280833	1·86411
4 =	157·4800	13·12333	4·374444	2·48548
5 =	196·8500	16·40417	5·468056	3·10685
6 =	236·2200	19·68500	6·561667	3·72822
7 =	275·5900	22·96583	7·655278	4·34959
8 =	314·9600	26·24667	8·748889	4·97096
9 =	354·3300	29·52750	9.842500	5·59233

CAPACITY.

	Millilitres or cubic centimetres to fluid drams.	Centilitres to fluid ounces.	Litres to quarts.	Decalitres to gallons.	Hectolitres to bushels.
1 =	0·27	0·338	1·0567	2·6417	2·8377
2 =	0·54	0·676	2·1134	5·2834	5·6755
3 =	0·81	1·014	3·1700	7·9251	8·5132
4 =	1·08	1·353	4·2267	10·5668	11·3510
5 =	1·35	1·691	5·2834	13·2085	14·1887
6 =	1·62	2·029	6·3401	15·8502	17·0265
7 =	1·89	2·367	7·3968	18·4919	19·8642
8 =	2·16	2·705	8·4535	21·1336	22·7019
9 =	2·43	3·043	9·5101	23·7753	25·5397

SQUARE.

	Square centimetres to square inches.	Square metres to square feet.	Square metres to square yards.	Hectares to acres.
1 =	0·1550	10·764	1·196	2·471
2 =	0·3100	21·528	2·392	4·942
3 =	0·4650	32·292	3·588	7·413
4 =	0.6200	43·055	4·784	9·884
5 =	0·7750	53·819	5·980	12·355
6 =	0.9300	64·583	7·176	14·826
7 =	1·0850	75·347	8·372	17·297
8 =	1·2400	86·111	9·568	19·768
9 =	1·3950	96·875	10·764	22·239

WEIGHT.

	Milligrammes to grains.	Kilogrammes to grains.	Hectogrammes to ounces avoirdupois.	Kilogrammes to pounds avoirdupois.
1 =	0·01543	15432·36	3·5274	2·20462
2 =	0·03086	30864·71	7·0548	4·40924
3 =	0·04630	46297·07	10·5822	6·61387
4 =	0·06173	61729·43	14·1096	8·81849
5 =	0·07716	77161·78	17·6370	11·02311
6 =	0·09259	92594·14	21·1644	13·22773
7 =	0·10803	108026·49	24·6918	15·43236
8 =	0·12346	123458·85	28·2192	17·63698
9 =	0·13889	138891·21	31·7466	19·84160

CUBIC.

	Cubic centimetres to cubic inches.	Cubic decimetres to cubic inches.	Cubic metres to cubic feet.	Cubic metres to cubic yards.
1 =	0·0610	61·023	35·314	1·308
2 =	0·1220	122·047	70·629	2·616
3 =	0·1831	183·070	105·943	3·924
4 =	0·2441	244·094	141·258	5·232
5 =	0·3051	305·117	176·572	6·540
6 =	0·3661	366·140	211·887	7·848
7 =	0·4272	427·164	247·201	9·156
8 =	0·4882	488·187	282·516	10·464
9 =	0·5492	549·210	317·830	11·771

WEIGHT — (*continued*).

	Quintals to pounds av.	Milliers or tonnes to pounds av.	Kilogrammes to ounces Troy.
1 =	220·46	2204·6	32·1507
2 =	440·92	4409·2	64·3015
3 =	661·39	6613·9	96·4522
4 =	881·85	8818·5	128·6030
5 =	1102·31	11023·1	160·7537
6 =	1322·77	13227·7	192·9044
7 =	1543·24	15432·4	225·0552
8 =	1763·70	17637·0	257 2059
9 =	1984·16	19841·6	289·3567

By the concurrent action of the principal governments of the world an International Bureau of Weights and Measures has been established near Paris. Under the direction of the International Committee, two ingots were cast of pure platinum-iridium in the proportion of 9 parts of the former to 1 of the latter metal. From one of these a certain number of kilogrammes were prepared, from the other a definite number of metre bars. These standards of weight and length were intercompared, without preference, and certain ones were selected as International prototype standards. The others were distributed by lot, in September, 1889, to the different governments and are called National prototype standards. Those apportioned to the United States were received in 1890 and are in the keeping of this office.

The metric system was legalized in the United States in 1866.

The International Standard Metre is derived from the Mètre des Archives, and its length is defined by the distance between two lines at 0° Centigrade, on a platinum-iridium bar deposited at the International Bureau of Weights and Measures.

The International Standard Kilogramme is a mass of platinum-iridium deposited at the same place, and its weight in vacuo is the same as that of the Kilogramme des Archives.

The litre is equal to a cubic decimetre, and it is measured by the quantity of distilled water which, at its maximum density, will counterpoise the standard kilogramme in a vacuum, the volume of such a quantity of water being, as nearly as has been ascertained, equal to a cubic decimetre

Table 3.

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000.\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
1	1000.000	1	1	1.0000	1.0000	0.00000
2	500.000	4	8	1.4142	1.2599	0.30103
3	333.333	9	27	1.7321	1.4422	0.47712
4	250.000	16	64	2.0000	1.5874	0.60206
5	200.000	25	125	2.2361	1.7100	0.69897
6	166.667	36	216	2.4495	1.8171	0.77815
7	142.857	49	343	2.6458	1.9129	0.84510
8	125.000	64	512	2.8284	2.0000	0.90309
9	111.111	81	729	3.0000	2.0801	0.95424
10	100.000	100	1000	3.1623	2.1544	1.00000
11	90.9091	121	1331	3.3166	2.2240	1.04139
12	83.3333	144	1728	3.4641	2.2894	1.07918
13	76.9231	169	2197	3.6056	2.3513	1.11394
14	71.4286	196	2744	3.7417	2.4101	1.14613
15	66.6667	225	3375	3.8730	2.4662	1.17609
16	62.5000	256	4096	4.0000	2.5198	1.20412
17	58.8235	289	4913	4.1231	2.5713	1.23045
18	55.5556	324	5832	4.2426	2.6207	1.25527
19	52.6316	361	6859	4.3589	2.6684	1.27875
20	50.0000	400	8000	4.4721	2.7144	1.30103
21	47.6190	441	9261	4.5826	2.7589	1.32222
22	45.4545	484	10648	4.6904	2.8020	1.34242
23	43.4783	529	12167	4.7958	2.8439	1.36173
24	41.6667	576	13824	4.8990	2.8845	1.38021
25	40.0000	625	15625	5.0000	2.9240	1.39794
26	38.4615	676	17576	5.0990	2.9625	1.41497
27	37.0370	729	19683	5.1962	3.0000	1.43136
28	35.7143	784	21952	5.2915	3.0366	1.44716
29	34.4828	841	24389	5.3852	3.0723	1.46240
30	33.3333	900	27000	5.4772	3.1072	1.47712
31	32.2581	961	29791	5.5678	3.1414	1.49136
32	31.2500	1024	32768	5.6569	3.1748	1.50515
33	30.3030	1089	35937	5.7446	3.2075	1.51851
34	29.4118	1156	39304	5.8310	3.2396	1.53148
35	28.5714	1225	42875	5.9161	3.2711	1.54407
36	27.7778	1296	46656	6.0000	3.3019	1.55630
37	27.0270	1369	50653	6.0828	3.3322	1.56820
38	26.3158	1444	54872	6.1644	3.3620	1.57978
39	25.6410	1521	59319	6.2450	3.3912	1.59106
40	25.0000	1600	64000	6.3246	3.4200	1.60206
41	24.3902	1681	68921	6.4031	3.4482	1.61278
42	23.8095	1764	74088	6.4807	3.4760	1.62325
43	23.2558	1849	79507	6.5574	3.5034	1.63347
44	22.7273	1936	85184	6.6332	3.5303	1.64345
45	22.2222	2025	91125	6.7082	3.5569	1.65321
46	21.7391	2116	97336	6.7823	3.5830	1.66276
47	21.2766	2209	103823	6.8557	3.6088	1.67210
48	20.8333	2304	110592	6.9282	3.6342	1.68124
49	20.4082	2401	117649	7.0000	3.6593	1.69020
50	20.0000	2500	125000	7.0711	3.6840	1.69897
51	19.6078	2601	132651	7.1414	3.7084	1.70757
52	19.2308	2704	140608	7.2111	3.7325	1.71600
53	18.8679	2809	148877	7.2801	3.7563	1.72428
54	18.5185	2916	157464	7.3485	3.7798	1.73239

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000\cdot\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
55	18.1818	3025	166375	7.4162	3.8030	1.74036
56	17.8571	3136	175616	7.4833	3.8259	1.74819
57	17.5439	3249	185193	7.5498	3.8485	1.75587
58	17.2414	3364	195112	7.6158	3.8709	1.76343
59	16.9492	3481	205379	7.6811	3.8930	1.77085
60	16.6667	3600	216000	7.7460	3.9149	1.77815
61	16.3934	3721	226981	7.8102	3.9365	1.78533
62	16.1290	3844	238328	7.8740	3.9579	1.79239
63	15.8730	3969	250047	7.9373	3.9791	1.79934
64	15.6250	4096	262144	8.0000	4.0000	1.80618
65	15.3846	4225	274625	8.0623	4.0207	1.81291
66	15.1515	4356	287496	8.1240	4.0412	1.81954
67	14.9254	4489	300763	8.1854	4.0615	1.82607
68	14.7059	4624	314432	8.2462	4.0817	1.83251
69	14.4928	4761	328509	8.3066	4.1016	1.83885
70	14.2857	4900	343000	8.3666	4.1213	1.84510
71	14.0845	5041	357911	8.4261	4.1408	1.85126
72	13.8889	5184	373248	8.4853	4.1602	1.85733
73	13.6986	5329	389017	8.5440	4.1793	1.86332
74	13.5135	5476	405224	8.6023	4.1983	1.86923
75	13.3333	5625	421875	8.6603	4.2172	1.87506
76	13.1579	5776	438976	8.7178	4.2358	1.88081
77	12.9870	5929	456533	8.7750	4.2543	1.88649
78	12.8205	6084	474552	8.8318	4.2727	1.89209
79	12.6582	6241	493039	8.8882	4.2908	1.89763
80	12.5000	6400	512000	8.9443	4.3089	1.90309
81	12.3457	6561	531441	9.0000	4.3267	1.90849
82	12.1951	6724	551368	9.0554	4.3445	1.91381
83	12.0482	6889	571787	9.1104	4.3621	1.91908
84	11.9048	7056	592704	9.1652	4.3795	1.92428
85	11.7647	7225	614125	9.2195	4.3968	1.92942
86	11.6279	7396	636056	9.2736	4.4140	1.93450
87	11.4943	7569	658503	9.3274	4.4310	1.93952
88	11.3636	7744	681472	9.3808	4.4480	1.94448
89	11.2360	7921	704969	9.4340	4.4647	1.94939
90	11.1111	8100	729000	9.4868	4.4814	1.95424
91	10.9890	8281	753571	9.5394	4.4979	1.95904
92	10.8696	8464	778688	9.5917	4.5144	1.96379
93	10.7527	8649	804357	9.6437	4.5307	1.96848
94	10.6383	8836	830584	9.6954	4.5468	1.97313
95	10.5263	9025	857375	9.7468	4.5629	1.97772
96	10.4167	9216	884736	9.7980	4.5789	1.98227
97	10.3093	9409	912673	9.8489	4.5947	1.98677
98	10.2041	9604	941192	9.8995	4.6104	1.99123
99	10.1010	9801	970299	9.9499	4.6261	1.99564
100	10.0000	10000	1000000	10.0000	4.6416	2.00000
101	9.90099	10201	1030301	10.0499	4.6570	2.00432
102	9.80392	10404	1061208	10.0995	4.6723	2.00860
103	9.70874	10609	1092727	10.1489	4.6875	2.01284
104	9.61538	10816	1124864	10.1980	4.7027	2.01703
105	9.52381	11025	1157625	10.2470	4.7177	2.02119
106	9.43396	11236	1191016	10.2956	4.7326	2.02531
107	9.34579	11449	1225043	10.3441	4.7475	2.02938
108	9.25926	11664	1259712	10.3923	4.7622	2.03342
109	9.17431	11881	1295029	10.4403	4.7769	2.03743

Table 3.

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000 \cdot \frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
110	9.09091	12100	1331000	10.4881	4.7914	2.04139
111	9.00901	12321	1367631	10.5357	4.8059	2.04532
112	8.92857	12544	1404928	10.5830	4.8203	2.04922
113	8.84956	12769	1442897	10.6301	4.8346	2.05308
114	8.77193	12996	1481544	10.6771	4.8488	2.05690
115	8.69565	13225	1520875	10.7238	4.8629	2.06070
116	8.62069	13456	1560896	10.7703	4.8770	2.06446
117	8.54701	13689	1601613	10.8167	4.8910	2.06819
118	8.47458	13924	1643032	10.8628	4.9049	2.07188
119	8.40336	14161	1685159	10.9087	4.9187	2.07555
120	8.33333	14400	1728000	10.9545	4.9324	2.07918
121	8.26446	14641	1771561	11.0000	4.9461	2.08279
122	8.19672	14884	1815848	11.0454	4.9597	2.08636
123	8.13008	15129	1860867	11.0905	4.9732	2.08991
124	8.06452	15376	1906624	11.1355	4.9866	2.09342
125	8.00000	15625	1953125	11.1803	5.0000	2.09691
126	7.93651	15876	2000376	11.2250	5.0133	2.10037
127	7.87402	16129	2048383	11.2694	5.0265	2.10380
128	7.81250	16384	2097152	11.3137	5.0397	2.10721
129	7.75194	16641	2146689	11.3578	5.0528	2.11059
130	7.69231	16900	2197000	11.4018	5.0658	2.11394
131	7.63359	17161	2248091	11.4455	5.0788	2.11727
132	7.57576	17424	2299968	11.4891	5.0916	2.12057
133	7.51880	17689	2352637	11.5326	5.1045	2.12385
134	7.46269	17956	2406104	11.5758	5.1172	2.12710
135	7.40741	18225	2460375	11.6190	5.1299	2.13033
136	7.35294	18496	2515456	11.6619	5.1426	2.13354
137	7.29927	18769	2571353	11.7047	5.1551	2.13672
138	7.24638	19044	2628072	11.7473	5.1676	2.13988
139	7.19424	19321	2685619	11.7898	5.1801	2.14301
140	7.14286	19600	2744000	11.8322	5.1925	2.14613
141	7.09220	19881	2803221	11.8743	5.2048	2.14922
142	7.04225	20164	2863288	11.9164	5.2171	2.15229
143	6.99301	20449	2924207	11.9583	5.2293	2.15534
144	6.94444	20736	2985984	12.0000	5.2415	2.15836
145	6.89655	21025	3048625	12.0416	5.2536	2.16137
146	6.84932	21316	3112136	12.0830	5.2656	2.16435
147	6.80272	21609	3176523	12.1244	5.2776	2.16732
148	6.75676	21904	3241792	12.1655	5.2896	2.17026
149	6.71141	22201	3307949	12.2066	5.3015	2.17319
150	6.66667	22500	3375000	12.2474	5.3133	2.17609
151	6.62252	22801	3442951	12.2882	5.3251	2.17898
152	6.57893	23104	3511808	12.3288	5.3368	2.18184
153	6.53595	23409	3581577	12.3693	5.3485	2.18469
154	6.49351	23716	3652264	12.4097	5.3601	2.18752
155	6.45161	24025	3723875	12.4499	5.3717	2.19033
156	6.41026	24336	3796416	12.4900	5.3832	2.19312
157	6.36943	24649	3869893	12.5300	5.3947	2.19590
158	6.32911	24964	3944312	12.5698	5.4061	2.19866
159	6.28931	25281	4019679	12.6095	5.4175	2.20140
160	6.25000	25600	4096000	12.6491	5.4288	2.20412
161	6.21118	25921	4173281	12.6886	5.4401	2.20683
162	6.17284	26244	4251528	12.7279	5.4514	2.20952
163	6.13497	26569	4330747	12.7671	5.4626	2.21219
164	6.09756	26896	4410944	12.8062	5.4737	2.21484

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000.\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
165	6.06061	27225	4492125	12.8452	5.4848	2.21748
166	6.02410	27556	4574296	12.8841	5.4959	2.22011
167	5.98802	27889	4657463	12.9228	5.5069	2.22272
168	5.95238	28224	4741632	12.9615	5.5178	2.22531
169	5.91716	28561	4826809	13.0000	5.5288	2.22789
170	5.88235	28900	4913000	13.0384	5.5397	2.23045
171	5.84795	29241	5000211	13.0767	5.5505	2.23300
172	5.81395	29584	5088448	13.1149	5.5613	2.23553
173	5.78035	29929	5177717	13.1529	5.5721	2.23805
174	5.74713	30276	5268024	13.1909	5.5828	2.24055
175	5.71429	30625	5359375	13.2288	5.5934	2.24304
176	5.68182	30976	5451776	13.2665	5.6041	2.24551
177	5.64972	31329	5545233	13.3041	5.6147	2.24797
178	5.61798	31684	5639752	13.3417	5.6252	2.25042
179	5.58659	32041	5735339	13.3791	5.6357	2.25285
180	5.55556	32400	5832000	13.4164	5.6462	2.25527
181	5.52486	32761	5929741	13.4536	5.6567	2.25768
182	5.49451	33124	6028568	13.4907	5.6671	2.26007
183	5.46448	33489	6128487	13.5277	5.6774	2.26245
184	5.43478	33856	6229504	13.5647	5.6877	2.26482
185	5.40541	34225	6331625	13.6015	5.6980	2.26717
186	5.37634	34596	6434856	13.6382	5.7083	2.26951
187	5.34759	34969	6539203	13.6748	5.7185	2.27184
188	5.31915	35344	6644672	13.7113	5.7287	2.27416
189	5.29101	35721	6751269	13.7477	5.7388	2.27646
190	5.26316	36100	6859000	13.7840	5.7489	2.27875
191	5.23560	36481	6967871	13.8203	5.7590	2.28103
192	5.20833	36864	7077888	13.8564	5.7690	2.28330
193	5.18135	37249	7189057	13.8924	5.7790	2.28556
194	5.15464	37636	7301384	13.9284	5.7890	2.28780
195	5.12821	38025	7414875	13.9642	5.7989	2.29003
196	5.10204	38416	7529536	14.0000	5.8088	2.29226
197	5.07614	38809	7645373	14.0357	5.8186	2.29447
198	5.05051	39204	7762392	14.0712	5.8285	2.29667
199	5.02513	39601	7880599	14.1067	5.8383	2.29885
200	5.00000	40000	8000000	14.1421	5.8480	2.30103
201	4.97512	40401	8120601	14.1774	5.8578	2.30320
202	4.95050	40804	8242408	14.2127	5.8675	2.30535
203	4.92611	41209	8365427	14.2478	5.8771	2.30750
204	4.90196	41616	8489664	14.2829	5.8868	2.30963
205	4.87805	42025	8615125	14.3178	5.8964	2.31175
206	4.85437	42436	8741816	14.3527	5.9059	2.31387
207	4.83092	42849	8869743	14.3875	5.9155	2.31597
208	4.80769	43264	8998912	14.4222	5.9250	2.31806
209	4.78469	43681	9129329	14.4568	5.9345	2.32015
210	4.76190	44100	9261000	14.4914	5.9439	2.32222
211	4.73934	44521	9393931	14.5258	5.9533	2.32428
212	4.71698	44944	9528128	14.5602	5.9627	2.32634
213	4.69484	45369	9663597	14.5945	5.9721	2.32838
214	4.67290	45796	9800344	14.6287	5.9814	2.33041
215	4.65116	46225	9938375	14.6629	5.9907	2.33244
216	4.62963	46656	10077696	14.6969	6.0000	2.33445
217	4.60829	47089	10218313	14.7309	6.0092	2.33646
218	4.58716	47524	10360232	14.7648	6.0185	2.33846
219	4.56621	47961	10503459	14.7986	6.0277	2.34044

TABLE 3.

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
220	4.54545	48400	10648000	14.8324	6.0368	2.34242
221	4.52489	48841	10793861	14.8661	6.0459	2.34439
222	4.50450	49284	10941048	14.8997	6.0550	2.34635
223	4.48431	49729	11089567	14.9332	6.0641	2.34830
224	4.46429	50176	11239424	14.9666	6.0732	2.35025
225	4.44444	50625	11390625	15.0000	6.0822	2.35218
226	4.42478	51076	11543176	15.0333	6.0912	2.35411
227	4.40529	51529	11697083	15.0665	6.1002	2.35603
228	4.38596	51984	11852352	15.0997	6.1091	2.35793
229	4.36681	52441	12008989	15.1327	6.1180	2.35984
230	4.34783	52900	12167000	15.1658	6.1269	2.36173
231	4.32900	53361	12326391	15.1987	6.1358	2.36361
232	4.31034	53824	12487168	15.2315	6.1446	2.36549
233	4.29185	54289	12649337	15.2643	6.1534	2.36736
234	4.27350	54756	12812904	15.2971	6.1622	2.36922
235	4.25532	55225	12977875	15.3297	6.1710	2.37107
236	4.23729	55696	13144256	15.3623	6.1797	2.37291
237	4.21941	56169	13312053	15.3948	6.1885	2.37475
238	4.20168	56644	13481272	15.4272	6.1972	2.37658
239	4.18410	57121	13651919	15.4596	6.2058	2.37840
240	4.16667	57600	13824000	15.4919	6.2145	2.38021
241	4.14938	58081	13997521	15.5242	6.2231	2.38202
242	4.13223	58564	14172488	15.5563	6.2317	2.38382
243	4.11523	59049	14348907	15.5885	6.2403	2.38561
244	4.09836	59536	14526784	15.6205	6.2488	2.38739
245	4.08163	60025	14706125	15.6525	6.2573	2.38917
246	4.06504	60516	14886936	15.6844	6.2658	2.39094
247	4.04858	61009	15069223	15.7162	6.2743	2.39270
248	4.03226	61504	15252992	15.7480	6.2828	2.39445
249	4.01606	62001	15438249	15.7797	6.2912	2.39620
250	4.00000	62500	15625000	15.8114	6.2996	2.39794
251	3.98406	63001	15813251	15.8430	6.3080	2.39967
252	3.96825	63504	16003008	15.8745	6.3164	2.40140
253	3.95257	64009	16194277	15.9060	6.3247	2.40312
254	3.93701	64516	16387064	15.9374	6.3330	2.40483
255	3.92157	65025	16581375	15.9687	6.3413	2.40654
256	3.90625	65536	16777216	16.0000	6.3496	2.40824
257	3.89105	66049	16974593	16.0312	6.3579	2.40993
258	3.87597	66564	17173512	16.0624	6.3661	2.41162
259	3.86100	67081	17373979	16.0935	6.3743	2.41330
260	3.84615	67600	17576000	16.1245	6.3825	2.41497
261	3.83142	68121	17779581	16.1555	6.3907	2.41664
262	3.81679	68644	17984728	16.1864	6.3988	2.41830
263	3.80228	69169	18191447	16.2173	6.4070	2.41996
264	3.78788	69696	18399744	16.2481	6.4151	2.42160
265	3.77358	70225	18609625	16.2788	6.4232	2.42325
266	3.75940	70756	18821096	16.3095	6.4312	2.42488
267	3.74532	71289	19034163	16.3401	6.4393	2.42651
268	3.73134	71824	19248832	16.3707	6.4473	2.42813
269	3.71747	72361	19465109	16.4012	6.4553	2.42975
270	3.70370	72900	19683000	16.4317	6.4633	2.43136
271	3.69004	73441	19902511	16.4621	6.4713	2.43297
272	3.67647	73984	20123648	16.4924	6.4792	2.43457
273	3.66300	74529	20346417	16.5227	6.4872	2.43616
274	3.64964	75076	20570824	16.5529	6.4951	2.43775

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000 \cdot \frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
275	3.63636	75625	20796875	16.5831	6.5030	2.43933
276	3.62319	76176	21024576	16.6132	6.5108	2.44091
277	3.61011	76729	21253933	16.6433	6.5187	2.44248
278	3.59712	77284	21484952	16.6733	6.5265	2.44404
279	3.58423	77841	21717639	16.7033	6.5343	2.44560
280	3.57143	78400	21952000	16.7332	6.5421	2.44716
281	3.55872	78961	22188041	16.7631	6.5499	2.44871
282	3.54610	79524	22425768	16.7929	6.5577	2.45025
283	3.53357	80089	22665187	16.8226	6.5654	2.45179
284	3.52113	80656	22906304	16.8523	6.5731	2.45332
285	3.50877	81225	23149125	16.8819	6.5808	2.45484
286	3.49650	81796	23393656	16.9115	6.5885	2.45637
287	3.48432	82369	23639903	16.9411	6.5962	2.45788
288	3.47222	82944	23887872	16.9706	6.6039	2.45939
289	3.46021	83521	24137569	17.0000	6.6115	2.46090
290	3.44828	84100	24389000	17.0294	6.6191	2.46240
291	3.43643	84681	24642171	17.0587	6.6267	2.46389
292	3.42466	85264	24897088	17.0880	6.6343	2.46538
293	3.41297	85849	25153757	17.1172	6.6419	2.46687
294	3.40136	86436	25412184	17.1464	6.6494	2.46835
295	3.38983	87025	25672375	17.1756	6.6569	2.46982
296	3.37838	87616	25934336	17.2047	6.6644	2.47129
297	3.36700	88209	26198073	17.2337	6.6719	2.47276
298	3.35570	88804	26463592	17.2627	6.6794	2.47422
299	3.34448	89401	26730899	17.2916	6.6869	2.47567
300	3.33333	90000	27000000	17.3205	6.6943	2.47712
301	3.32226	90601	27270901	17.3494	6.7018	2.47857
302	3.31126	91204	27543608	17.3781	6.7092	2.48001
303	3.30033	91809	27818127	17.4069	6.7166	2.48144
304	3.28947	92416	28094464	17.4356	6.7240	2.48287
305	3.27869	93025	28372625	17.4642	6.7313	2.48430
306	3.26797	93636	28652616	17.4929	6.7387	2.48572
307	3.25733	94249	28934443	17.5214	6.7460	2.48714
308	3.24675	94864	29218112	17.5499	6.7533	2.48855
309	3.23625	95481	29503629	17.5784	6.7606	2.48996
310	3.22581	96100	29791000	17.6068	6.7679	2.49136
311	3.21543	96721	30080231	17.6352	6.7752	2.49276
312	3.20513	97344	30371328	17.6635	6.7824	2.49415
313	3.19489	97969	30664297	17.6918	6.7897	2.49554
314	3.18471	98596	30959144	17.7200	6.7969	2.49693
315	3.17460	99225	31255875	17.7482	6.8041	2.49831
316	3.16456	99856	31554496	17.7764	6.8113	2.49969
317	3.15457	100489	31855013	17.8045	6.8185	2.50106
318	3.14465	101124	32157432	17.8326	6.8256	2.50243
319	3.13480	101761	32461759	17.8606	6.8328	2.50379
320	3.12500	102400	32768000	17.8885	6.8399	2.50515
321	3.11527	103041	33076161	17.9165	6.8470	2.50651
322	3.10559	103684	33386248	17.9444	6.8541	2.50786
323	3.09598	104329	33698267	17.9722	6.8612	2.50920
324	3.08642	104976	34012224	18.0000	6.8683	2.51055
325	3.07692	105625	34328125	18.0278	6.8753	2.51188
326	3.06748	106276	34645976	18.0555	6.8824	2.51322
327	3.05810	106929	34965783	18.0831	6.8894	2.51455
328	3.04878	107584	35287552	18.1108	6.8964	2.51587
329	3.03951	108241	35611289	18.1384	6.9034	2.51720

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000 \cdot \frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
330	3.03030	108900	35937000	18.1659	6.9104	2.51851
331	3.02115	109561	36264691	18.1934	6.9174	2.51983
332	3.01205	110224	36594368	18.2209	6.9244	2.52114
333	3.00300	110889	36926037	18.2483	6.9313	2.52244
334	2.99401	111556	37259704	18.2757	6.9382	2.52375
335	2.98507	112225	37595375	18.3030	6.9451	2.52504
336	2.97619	112896	37933056	18.3303	6.9521	2.52634
337	2.96736	113569	38272753	18.3576	6.9589	2.52763
338	2.95858	114244	38614472	18.3848	6.9658	2.52892
339	2.94985	114921	38958219	18.4120	6.9727	2.53020
340	2.94118	115600	39304000	18.4391	6.9795	2.53148
341	2.93255	116281	39651821	18.4662	6.9864	2.53275
342	2.92398	116964	40001688	18.4932	6.9932	2.53403
343	2.91545	117649	40353607	18.5203	7.0000	2.53529
344	2.90698	118336	40707584	18.5472	7.0068	2.53656
345	2.89855	119025	41063625	18.5742	7.0136	2.53782
346	2.89017	119716	41421736	18.6011	7.0203	2.53908
347	2.88184	120409	41781923	18.6279	7.0271	2.54033
348	2.87356	121104	42144192	18.6548	7.0338	2.54158
349	2.86533	121801	42508549	18.6815	7.0406	2.54283
350	2.85714	122500	42875000	18.7083	7.0473	2.54407
351	2.84900	123201	43243551	18.7350	7.0540	2.54531
352	2.84091	123904	43614208	18.7617	7.0607	2.54654
353	2.83286	124609	43986977	18.7883	7.0674	2.54777
354	2.82486	125316	44361864	18.8149	7.0740	2.54900
355	2.81690	126025	44738875	18.8414	7.0807	2.55023
356	2.80899	126736	45118016	18.8680	7.0873	2.55145
357	2.80112	127449	45499293	18.8944	7.0940	2.55267
358	2.79330	128164	45882712	18.9209	7.1006	2.55388
359	2.78552	128881	46268279	18.9473	7.1072	2.55509
360	2.77778	129600	46656000	18.9737	7.1138	2.55630
361	2.77008	130321	47045881	19.0000	7.1204	2.55751
362	2.76243	131044	47437928	19.0263	7.1269	2.55871
363	2.75482	131769	47832147	19.0526	7.1335	2.55991
364	2.74725	132496	48228544	19.0788	7.1400	2.56110
365	2.73973	133225	48627125	19.1050	7.1466	2.56229
366	2.73224	133956	49027896	19.1311	7.1531	2.56348
367	2.72480	134689	49430863	19.1572	7.1596	2.56467
368	2.71739	135424	49836032	19.1833	7.1661	2.56585
369	2.71003	136161	50243409	19.2094	7.1726	2.56703
370	2.70270	136900	50653000	19.2354	7.1791	2.56820
371	2.69542	137641	51064811	19.2614	7.1855	2.56937
372	2.68817	138384	51478848	19.2873	7.1920	2.57054
373	2.68097	139129	51895117	19.3132	7.1984	2.57171
374	2.67380	139876	52313624	19.3391	7.2048	2.57287
375	2.66667	140625	52734375	19.3649	7.2112	2.57403
376	2.65957	141376	53157376	19.3907	7.2177	2.57519
377	2.65252	142129	53582633	19.4165	7.2240	2.57634
378	2.64550	142884	54010152	19.4422	7.2304	2.57749
379	2.63852	143641	54439939	19.4679	7.2368	2.57864
380	2.63158	144400	54872000	19.4936	7.2432	2.57978
381	2.62467	145161	55306341	19.5192	7.2495	2.58092
382	2.61780	145924	55742968	19.5448	7.2558	2.58206
383	2.61097	146689	56181887	19.5704	7.2622	2.58320
384	2.60417	147456	56623104	19.5959	7.2685	2.58433

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000 \cdot \frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
385	2.59740	148225	57066625	19.6214	7.2748	2.58546
386	2.59067	148996	57512456	19.6469	7.2811	2.58659
387	2.58398	149769	57960603	19.6723	7.2874	2.58771
388	2.57732	150544	58411072	19.6977	7.2936	2.58883
389	2.57069	151321	58863869	19.7231	7.2999	2.58995
390	2.56410	152100	59319000	19.7484	7.3061	2.59106
391	2.55754	152881	59776471	19.7737	7.3124	2.59218
392	2.55102	153664	60236288	19.7990	7.3186	2.59329
393	2.54453	154449	60698457	19.8242	7.3248	2.59439
394	2.53807	155236	61162984	19.8494	7.3310	2.59550
395	2.53165	156025	61629875	19.8746	7.3372	2.59660
396	2.52525	156816	62099136	19.8997	7.3434	2.59770
397	2.51889	157609	62570773	19.9249	7.3496	2.59879
398	2.51256	158404	63044792	19.9499	7.3558	2.59988
399	2.50627	159201	63521199	19.9750	7.3619	2.60097
400	2.50000	160000	64000000	20.0000	7.3681	2.60206
401	2.49377	160801	64481201	20.0250	7.3742	2.60314
402	2.48756	161604	64964808	20.0499	7.3803	2.60423
403	2.48139	162409	65450827	20.0749	7.3864	2.60531
404	2.47525	163216	65939264	20.0998	7.3925	2.60638
405	2.46914	164025	66430125	20.1246	7.3986	2.60746
406	2.46305	164836	66923416	20.1494	7.4047	2.60853
407	2.45700	165649	67419143	20.1742	7.4108	2.60959
408	2.45098	166464	67917312	20.1990	7.4169	2.61066
409	2.44499	167281	68417929	20.2237	7.4229	2.61172
410	2.43902	168100	68921000	20.2485	7.4290	2.61278
411	2.43309	168921	69426531	20.2731	7.4350	2.61384
412	2.42718	169744	69934528	20.2978	7.4410	2.61490
413	2.42131	170569	70444997	20.3224	7.4470	2.61595
414	2.41546	171396	70957944	20.3470	7.4530	2.61700
415	2.40964	172225	71473375	20.3715	7.4590	2.61805
416	2.40385	173056	71991296	20.3961	7.4650	2.61909
417	2.39808	173889	72511713	20.4206	7.4710	2.62014
418	2.39234	174724	73034632	20.4450	7.4770	2.62118
419	2.38663	175561	73560059	20.4695	7.4829	2.62221
420	2.38095	176400	74088000	20.4939	7.4889	2.62325
421	2.37530	177241	74618461	20.5183	7.4948	2.62428
422	2.36967	178084	75151448	20.5426	7.5007	2.62531
423	2.36407	178929	75686967	20.5670	7.5067	2.62634
424	2.35849	179776	76225024	20.5913	7.5126	2.62737
425	2.35294	180625	76765625	20.6155	7.5185	2.62839
426	2.34742	181476	77308776	20.6398	7.5244	2.62941
427	2.34192	182329	77854483	20.6640	7.5302	2.63043
428	2.33645	183184	78402752	20.6882	7.5361	2.63144
429	2.33100	184041	78953589	20.7123	7.5420	2.63246
430	2.32558	184900	79507000	20.7364	7.5478	2.63347
431	2.32019	185761	80062991	20.7605	7.5537	2.63448
432	2.31481	186624	80621568	20.7846	7.5595	2.63548
433	2.30947	187489	81182737	20.8087	7.5654	2.63649
434	2.30415	188356	81746504	20.8327	7.5712	2.63749
435	2.29885	189225	82312875	20.8567	7.5770	2.63849
436	2.29358	190096	82881856	20.8806	7.5828	2.63949
437	2.28833	190969	83453453	20.9045	7.5886	2.64048
438	2.28311	191844	84027672	20.9284	7.5944	2.64147
439	2.27790	192721	84604519	20.9523	7.6001	2.64246

TABLE 3.

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000.\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
440	2.27273	193600	85184000	20.9762	7.6059	2.64345
441	2.26757	194481	85766121	21.0000	7.6117	2.64444
442	2.26244	195364	86350888	21.0238	7.6174	2.64542
443	2.25734	196249	86938307	21.0476	7.6232	2.64640
444	2.25225	197136	87528384	21.0713	7.6289	2.64738
445	2.24719	198025	88121125	21.0950	7.6346	2.64836
446	2.24215	198916	88716536	21.1187	7.6403	2.64933
447	2.23714	199809	89314623	21.1424	7.6460	2.65031
448	2.23214	200704	89915392	21.1660	7.6517	2.65128
449	2.22717	201601	90518849	21.1896	7.6574	2.65225
450	2.22222	202500	91125000	21.2132	7.6631	2.65321
451	2.21730	203401	91733851	21.2368	7.6688	2.65418
452	2.21239	204304	92345408	21.2603	7.6744	2.65514
453	2.20751	205209	92959677	21.2838	7.6801	2.65610
454	2.20264	206116	93576664	21.3073	7.6857	2.65706
455	2.19780	207025	94196375	21.3307	7.6914	2.65801
456	2.19298	207936	94818816	21.3542	7.6970	2.65896
457	2.18818	208849	95443993	21.3776	7.7026	2.65992
458	2.18341	209764	96071912	21.4009	7.7082	2.66087
459	2.17865	210681	96702579	21.4243	7.7138	2.66181
460	2.17391	211600	97336000	21.4476	7.7194	2.66276
461	2.16920	212521	97972181	21.4709	7.7250	2.66370
462	2.16450	213444	98611128	21.4942	7.7306	2.66464
463	2.15983	214369	99252847	21.5174	7.7362	2.66558
464	2.15517	215296	99897344	21.5407	7.7418	2.66652
465	2.15054	216225	100544625	21.5639	7.7473	2.66745
466	2.14592	217156	101194696	21.5870	7.7529	2.66839
467	2.14133	218089	101847563	21.6102	7.7584	2.66932
468	2.13675	219024	102503232	21.6333	7.7639	2.67025
469	2.13220	219961	103161709	21.6564	7.7695	2.67117
470	2.12766	220900	103823000	21.6795	7.7750	2.67210
471	2.12314	221841	104487111	21.7025	7.7805	2.67302
472	2.11864	222784	105154048	21.7256	7.7860	2.67394
473	2.11416	223729	105823817	21.7486	7.7915	2.67486
474	2.10970	224677	106496424	21.7715	7.7970	2.67578
475	2.10526	225625	107171875	21.7945	7.8025	2.67669
476	2.10084	226576	107850176	21.8174	7.8079	2.67761
477	2.09644	227529	108531333	21.8403	7.8134	2.67852
478	2.09205	228484	109215352	21.8632	7.8188	2.67943
479	2.08768	229441	109902239	21.8861	7.8243	2.68034
480	2.08333	230400	110592000	21.9089	7.8297	2.68124
481	2.07900	231361	111284641	21.9317	7.8352	2.68215
482	2.07469	232324	111980168	21.9545	7.8406	2.68305
483	2.07039	233289	112678587	21.9773	7.8460	2.68395
484	2.06612	234256	113379904	22.0000	7.8514	2.68485
485	2.06186	235225	114084125	22.0227	7.8568	2.68574
486	2.05761	236196	114791256	22.0454	7.8622	2.68664
487	2.05339	237169	115501303	22.0681	7.8676	2.68753
488	2.04918	238144	116214272	22.0907	7.8730	2.68842
489	2.04499	239121	116930169	22.1133	7.8784	2.68931
490	2.04082	240100	117649000	22.1359	7.8837	2.69020
491	2.03666	241081	118370771	22.1585	7.8891	2.69108
492	2.03252	242064	119095488	22.1811	7.8944	2.69197
493	2.02840	243049	119823157	22.2036	7.8998	2.69285
494	2.02429	244036	120553784	22.2261	7.9051	2.69373

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000 \cdot \frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
495	2.02020	245025	121287375	22.2486	7.9105	2.69461
496	2.01613	246016	122023936	22.2711	7.9158	2.69548
497	2.01207	247009	122763473	22.2935	7.9211	2.69636
498	2.00803	248004	123505992	22.3159	7.9264	2.69723
499	2.00401	249001	124251499	22.3383	7.9317	2.69810
500	2.00000	250000	125000000	22.3607	7.9370	2.69897
501	1.99601	251001	125751501	22.3830	7.9420	2.69984
502	1.99203	252004	126506008	22.4054	7.9476	2.70070
503	1.98807	253009	127263527	22.4277	7.9528	2.70157
504	1.98413	254016	128024064	22.4499	7.9581	2.70243
505	1.98020	255025	128787625	22.4722	7.9634	2.70329
506	1.97628	256036	129554216	22.4944	7.9686	2.70415
507	1.97239	257049	130323843	22.5167	7.9739	2.70501
508	1.96850	258064	131096512	22.5389	7.9791	2.70586
509	1.96464	259081	131872229	22.5610	7.9843	2.70672
510	1.96078	260100	132651000	22.5832	7.9896	2.70757
511	1.95695	261121	133432831	22.6053	7.9948	2.70842
512	1.95312	262144	134217728	22.6274	8.0000	2.70927
513	1.94932	263169	135005697	22.6495	8.0052	2.71012
514	1.94553	264196	135796744	22.6716	8.0104	2.71096
515	1.94175	265225	136590875	22.6936	8.0156	2.71181
516	1.93798	266256	137388096	22.7156	8.0208	2.71265
517	1.93424	267289	138188413	22.7376	8.0260	2.71349
518	1.93050	268324	138991832	22.7596	8.0311	2.71433
519	1.92678	269361	139798359	22.7816	8.0363	2.71517
520	1.92308	270400	140608000	22.8035	8.0415	2.71600
521	1.91939	271441	141420761	22.8254	8.0466	2.71684
522	1.91571	272484	142236648	22.8473	8.0517	2.71767
523	1.91205	273529	143055667	22.8692	8.0569	2.71850
524	1.90840	274576	143877824	22.8910	8.0620	2.71933
525	1.90476	275625	144703125	22.9129	8.0671	2.72016
526	1.90114	276676	145531576	22.9347	8.0723	2.72099
527	1.89753	277729	146363183	22.9565	8.0774	2.72181
528	1.89394	278784	147197952	22.9783	8.0825	2.72263
529	1.89036	279841	148035889	23.0000	8.0876	2.72346
530	1.88679	280900	148877000	23.0217	8.0927	2.72428
531	1.88324	281961	149721291	23.0434	8.0978	2.72509
532	1.87970	283024	150568768	23.0651	8.1028	2.72591
533	1.87617	284089	151419437	23.0868	8.1079	2.72673
534	1.87266	285156	152273304	23.1084	8.1130	2.72754
535	1.86916	286225	153130375	23.1301	8.1180	2.72835
536	1.86567	287296	153990656	23.1517	8.1231	2.72916
537	1.86220	288369	154854153	23.1733	8.1281	2.72997
538	1.85874	289444	155720872	23.1948	8.1332	2.73078
539	1.85529	290521	156590819	23.2164	8.1382	2.73159
540	1.85185	291600	157464000	23.2379	8.1433	2.73239
541	1.84843	292681	158340421	23.2594	8.1483	2.73320
542	1.84502	293764	159220088	23.2809	8.1533	2.73400
543	1.84162	294849	160103007	23.3024	8.1583	2.73480
544	1.83824	295936	160989184	23.3238	8.1633	2.73560
545	1.83486	297025	161878625	23.3452	8.1683	2.73640
546	1.83150	298116	162771336	23.3666	8.1733	2.73719
547	1.82815	299209	163667323	23.3880	8.1783	2.73799
548	1.82482	300304	164566592	23.4094	8.1833	2.73878
549	1.82149	301401	165469149	23.4307	8.1882	2.73957

TABLE 3.

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000.\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
550	1.81818	302500	166375000	23.4521	8.1932	2.74036
551	1.81488	303601	167284151	23.4734	8.1982	2.74115
552	1.81159	304704	168196608	23.4947	8.2031	2.74194
553	1.80832	305809	169112377	23.5160	8.2081	2.74273
554	1.80505	306916	170031464	23.5372	8.2130	2.74351
555	1.80180	308025	170953875	23.5584	8.2180	2.74429
556	1.79856	309136	171879616	23.5797	8.2229	2.74507
557	1.79533	310249	172808693	23.6008	8.2278	2.74586
558	1.79211	311364	173741112	23.6220	8.2327	2.74663
559	1.78891	312481	174676879	23.6432	8.2377	2.74741
560	1.78571	313600	175616000	23.6643	8.2426	2.74819
561	1.78253	314721	176558481	23.6854	8.2475	2.74896
562	1.77936	315844	177504328	23.7065	8.2524	2.74974
563	1.77620	316969	178453547	23.7276	8.2573	2.75051
564	1.77305	318096	179406144	23.7487	8.2621	2.75128
565	1.76991	319225	180362125	23.7697	8.2670	2.75205
566	1.76678	320356	181321496	23.7908	8.2719	2.75282
567	1.76367	321489	182284263	23.8118	8.2768	2.75358
568	1.76056	322624	183250432	23.8328	8.2816	2.75435
569	1.75747	323761	184220009	23.8537	8.2865	2.75511
570	1.75439	324900	185193000	23.8747	8.2913	2.75587
571	1.75131	326041	186169411	23.8956	8.2962	2.75664
572	1.74825	327184	187149248	23.9165	8.3010	2.75740
573	1.74520	328329	188132517	23.9374	8.3059	2.75815
574	1.74216	329476	189119224	23.9583	8.3107	2.75891
575	1.73913	330625	190109375	23.9792	8.3155	2.75967
576	1.73611	331776	191102976	24.0000	8.3203	2.76042
577	1 73310	332929	192100033	24.0208	8.3251	2.76118
578	1.73010	334084	193100552	24.0416	8.3300	2.76193
579	1 72712	335241	194104539	24.0624	8.3348	2.76268
580	1.72414	336400	195112000	24.0832	8.3396	2.76343
581	1.72117	337561	196122941	24.1039	8.3443	2.76418
582	1.71821	338724	197137368	24.1247	8.3491	2.76492
583	1.71527	339889	198155287	24.1454	8.3539	2.76567
584	1.71233	341056	199176704	24.1661	8.3587	2.76641
585	1.70940	342225	200201625	24.1868	8.3634	2.76716
586	1.70648	343396	201230056	24.2074	8.3682	2.76790
587	1.70358	344569	202262003	24.2281	8.3730	2.76864
588	1.70068	345744	203297472	24.2487	8.3777	2.76938
589	1.69779	346921	204336469	24.2693	8.3825	2.77012
590	1.69492	348100	205379000	24.2899	8.3872	2.77085
591	1.69205	349281	206425071	24.3105	8.3919	2.77159
592	1.68919	350464	207474688	24.3311	8.3967	2.77232
593	1.68634	351649	208527857	24.3516	8.4014	2.77305
594	1.68350	352836	209584584	24.3721	8.4061	2.77379
595	1.68067	354025	210644875	24.3926	8.4108	2.77452
596	1.67785	355216	211708736	24.4131	8.4155	2.77525
597	1.67504	356409	212776173	24.4336	8.4202	2.77597
598	1.67224	357604	213847192	24.4540	8.4249	2.77670
599	1.66945	358801	214921799	24.4745	8.4296	2.77743
600	1.66667	360000	216000000	24.4949	8.4343	2.77815
601	1.66389	361201	217081801	24.5153	8.4390	2.77887
602	1.66113	362404	218167208	24.5357	8.4437	2.77960
603	1.65837	363609	219256227	24.5561	8.4484	2.78032
604	1.65563	364816	220348864	24.5764	8.4530	2.78104

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000.\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
605	1.65289	366025	221445125	24.5967	8.4577	2.78176
606	1.65017	367236	222545016	24.6171	8.4623	2.78247
607	1.64745	368449	223648543	24.6374	8.4670	2.78319
608	1.64474	369664	224755712	24.6577	8.4716	2.78390
609	1.64204	370881	225866529	24.6779	8.4763	2.78462
610	1.63934	372100	226981000	24.6982	8.4809	2.78533
611	1.63666	373321	228099131	24.7184	8.4856	2.78604
612	1.63399	374544	229220928	24.7386	8.4902	2.78675
613	1.63132	375769	230346397	24.7588	8.4948	2.78746
614	1.62866	376996	231475544	24.7790	8.4994	2.78817
615	1.62602	378225	232608375	24.7992	8.5040	2.78888
616	1.62338	379456	233744896	24.8193	8.5086	2.78958
617	1.62075	380689	234885113	24.8395	8.5132	2.79029
618	1.61812	381924	236029032	24.8596	8.5178	2.79099
619	1.61551	383161	237176659	24.8797	8.5224	2.79169
620	1.61290	384400	238328000	24.8998	8.5270	2.79239
621	1.61031	385641	239483061	24.9199	8.5316	2.79309
622	1.60772	386884	240641848	24.9399	8.5362	2.79379
623	1.60514	388129	241804367	24.9600	8.5408	2.79449
624	1.60256	389376	242970624	24.9800	8.5453	2.79518
625	1.60000	390625	244140625	25.0000	8.5499	2.79934
626	1.59744	391876	245314376	25.0200	8.5544	2.79657
627	1.59490	393129	246491883	25.0400	8.5590	2.79727
628	1.59236	394384	247673152	25.0599	8.5635	2.79796
629	1.58983	395641	248858189	25.0799	8.5681	2.79865
630	1.58730	396900	250047000	25.0998	8.5726	2.79934
631	1.58479	398161	251239591	25.1197	8.5772	2.80003
632	1.58228	399424	252435968	25.1396	8.5817	2.80072
633	1.57978	400689	253636137	25.1595	8.5862	2.80140
634	1.57729	401956	254840104	25.1794	8.5907	2.80209
635	1.57480	403225	256047875	25.1992	8.5952	2.80277
636	1.57233	404496	257259456	25.2190	8.5997	2.80346
637	1.56986	405769	258474853	25.2389	8.6043	2.80414
638	1.56740	407044	259694072	25.2587	8.6088	2.80482
639	1.56495	408321	260917119	25.2784	8.6132	2.80550
640	1.56250	409600	262144000	25.2982	8.6177	2.80618
641	1.56006	410881	263374721	25.3180	8.6222	2.80686
642	1.55763	412164	264609288	25.3377	8.6267	2.80754
643	1.55521	413449	265847707	25.3574	8.6312	2.80821
644	1.55280	414736	267089984	25.3772	8.6357	2.80889
645	1.55039	416025	268336125	25.3969	8.6401	2.80956
646	1.54799	417316	269586136	25.4165	8.6446	2.81023
647	1.54560	418609	270840023	25.4362	8.6490	2.81090
648	1.54321	419904	272097792	25.4558	8.6535	2.81158
649	1.54083	421201	273359449	25.4755	8.6579	2.81224
650	1.53846	422500	274625000	25.4951	8.6624	2.81291
651	1.53610	423801	275894451	25.5147	8.6668	2.81358
652	1.53374	425104	277167808	25.5343	8.6713	2.81425
653	1.53139	426409	278445077	25.5539	8.6757	2.81491
654	1.52905	427716	279726264	25.5734	8.6801	2.81558
655	1.52672	429025	281011375	25.5930	8.6845	2.81624
656	1.52439	430336	282300416	25.6125	8.6890	2.81690
657	1.52207	431649	283593393	25.6320	8.6934	2.81757
658	1.51976	432964	284890312	25.6515	8.6978	2.81823
659	1.51745	434281	286191179	25.6710	8.7022	2.81889

TABLE 3.

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000 \cdot \frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
660	1.51515	435600	287496000	25.6905	8.7066	2.81954
661	1.51286	436921	288804781	25.7099	8.7110	2.82020
662	1.51057	438244	290117528	25.7294	8.7154	2.82086
663	1.50830	439569	291434247	25.7488	8.7198	2.82151
664	1.50602	440896	292754944	25.7682	8.7241	2.82217
665	1.50376	442225	294079625	25.7876	8.7285	2.82282
666	1.50150	443556	295408296	25.8070	8.7329	2.82347
667	1.49925	444889	296740963	25.8263	8.7373	2.82413
668	1.49701	446224	298077632	25.8457	8.7416	2.82478
669	1.49477	447561	299418309	25.8650	8.7460	2.82543
670	1.49254	448900	300763000	25.8844	8.7503	2.82607
671	1.49031	450241	302111711	25.9037	8.7547	2.82672
672	1.48810	451584	303464448	25.9230	8.7590	2.82737
673	1.48588	452929	304821217	25.9422	8.7634	2.82802
674	1.48368	454276	306182024	25.9615	8.7677	2.82866
675	1.48148	455625	307546875	25.9808	8.7721	2.82930
676	1.47929	456976	308915776	26.0000	8.7764	2.82995
677	1.47710	458329	310288733	26.0192	8.7807	2.83059
678	1.47493	459684	311665752	26.0384	8.7850	2.83123
679	1.47275	461041	313046839	26.0576	8.7893	2.83187
680	1.47059	462400	314432000	26.0768	8.7937	2.83251
681	1.46843	463761	315821241	26.0960	8.7980	2.83315
682	1.46628	465124	317214568	26.1151	8.8023	2.83378
683	1.46413	466489	318611987	26.1343	8.8066	2.83442
684	1.46199	467856	320013504	26.1534	8.8108	2.83506
685	1.45985	469225	321419125	26.1725	8.8152	2.83569
686	1.45773	470596	322828856	26.1916	8.8194	2.83632
687	1.45560	471969	324242703	26.2107	8.8237	2.83696
688	1.45349	473344	325660672	26.2298	8.8280	2.83759
689	1.45138	474721	327082769	26.2488	8.8323	2.83822
690	1.44928	476100	328509000	26.2679	8.8366	2.83885
691	1.44718	477481	329939371	26.2869	8.8408	2.83948
692	1.44509	478864	331373888	26.3059	8.8451	2.84011
693	1.44300	480249	332812557	26.3249	8.8493	2.84073
694	1.44092	481636	334255384	26.3439	8.8536	2.84136
695	1.43885	483025	335702375	26.3629	8.8578	2.84198
696	1.43678	484416	337153536	26.3818	8.8621	2.84261
697	1.43472	485809	338608873	26.4008	8.8663	2.84323
698	1.43266	487204	340068392	26.4197	8.8706	2.84386
699	1.43062	488601	341532099	26.4386	8.8748	2.84448
700	1.42857	490000	343000000	26.4575	8.8790	2.84510
701	1.42653	491401	344472101	26.4764	8.8833	2.84572
702	1.42450	492804	345948408	26.4953	8.8875	2.84634
703	1.42248	494209	347428927	26.5141	8.8917	2.84696
704	1.42045	495616	348913664	26.5330	8.8959	2.84757
705	1.41844	497025	350402625	26.5518	8.9001	2.84819
706	1.41643	498436	351895816	26.5707	8.9043	2.84880
707	1.41443	499849	353393243	26.5895	8.9085	2.84942
708	1.41243	501264	354894912	26.6083	8.9127	2.85003
709	1.41044	502681	356400829	26.6271	8.9169	2.85065
710	1.40845	504100	357911000	26.6458	8.9211	2.85126
711	1.40647	505521	359425431	26.6646	8.9253	2.85187
712	1.40449	506944	360944128	26.6833	8.9295	2.85248
713	1.40252	508369	362467097	26.7021	8.9337	2.85309
714	1.40056	509796	363994344	26.7208	8.9378	2.85370

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000.\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
715	1.39860	511225	365525875	26.7395	8.9420	2.85431
716	1.39665	512656	367061696	26.7582	8.9462	2.85491
717	1.39470	514089	368601813	26.7769	8.9503	2.85552
718	1.39276	515524	370146232	26.7955	8.9545	2.85612
719	1.39082	516961	371694959	26.8142	8.9587	2.85673
720	1.38889	518400	373248000	26.8328	8.9628	2.85733
721	1.38696	519841	374805361	26.8514	8.9670	2.85794
722	1.38504	521284	376367048	26.8701	8.9711	2.85854
723	1.38313	522729	377933067	26.8887	8.9752	2.85914
724	1.38122	524176	379503424	26.9072	8.9794	2.85974
725	1.37931	525625	381078125	26.9258	8.9835	2.86034
726	1.37741	527076	382657176	26.9444	8.9876	2.86094
727	1.37552	528529	384240583	26.9629	8.9918	2.86153
728	1.37363	529984	385828352	26.9815	8.9959	2.86213
729	1.37174	531441	387420489	27.0000	9.0000	2.86273
730	1.36986	532900	389017000	27.0185	9.0041	2.86332
731	1.36799	534361	390617891	27.0370	9.0082	2.86392
732	1.36612	535824	392223168	27.0555	9.0123	2.86451
733	1.36426	537289	393832837	27.0740	9.0164	2.86510
734	1.36240	538756	395446904	27.0924	9.0205	2.86570
735	1.36054	540225	397065375	27.1109	9.0246	2.86629
736	1.35870	541696	398688256	27.1293	9.0287	2.86688
737	1.35685	543169	400315553	27.1477	9.0328	2.86747
738	1.35501	544644	401947272	27.1662	9.0369	2.86806
739	1.35318	546121	403583419	27.1846	9.0410	2.86864
740	1.35135	547600	405224000	27.2029	9.0450	2.86923
741	1.34953	549081	406869021	27.2213	9.0491	2.86982
742	1.34771	550564	408518488	27.2397	9.0532	2.87040
743	1.34590	552049	410172407	27.2580	9.0572	2.87099
744	1.34409	553536	411830784	27.2764	9.0613	2.87157
745	1.34228	555025	413493625	27.2947	9.0654	2.87216
746	1.34048	556516	415160936	27.3130	9.0694	2.87274
747	1.33869	558009	416832723	27.3313	9.0735	2.87332
748	1.33690	559504	418508992	27.3496	9.0775	2.87390
749	1.33511	561001	420189749	27.3679	9.0816	2.87448
750	1.33333	562500	421875000	27.3861	9.0856	2.87506
751	1.33156	564001	423564751	27.4044	9.0896	2.87564
752	1.32979	565504	425259008	27.4226	9.0937	2.87622
753	1.32802	567009	426957777	27.4408	9.0977	2.87679
754	1.32626	568516	428661064	27.4591	9.1017	2.87737
755	1.32450	570025	430368875	27.4773	9.1057	2.87795
756	1.32275	571536	432081216	27.4955	9.1098	2.87852
757	1.32100	573049	433798093	27.5136	9.1138	2.87910
758	1.31926	574564	435519512	27.5318	9.1178	2.87967
759	1.31752	576081	437245479	27.5500	9.1218	2.88024
760	1.31579	577600	438976000	27.5681	9.1258	2.88081
761	1.31406	579121	440711081	27.5862	9.1298	2.88138
762	1.31234	580644	442450728	27.6043	9.1338	2.88195
763	1.31062	582169	444194947	27.6225	9.1378	2.88252
764	1.30890	583696	445943744	27.6405	9.1418	2.88309
765	1.30719	585225	447697125	27.6586	9.1458	2.88366
766	1.30548	586756	449455096	27.6767	9.1498	2.88423
767	1.30378	588289	451217663	27.6948	9.1537	2.88480
768	1.30208	589824	452984832	27.7128	9.1577	2.88536
769	1.30039	591361	454756609	27.7308	9.1617	2.88593

TABLE 3.

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000.\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
770	1.29870	592900	456533000	27.7489	9.1657	2.88649
771	1.29702	594441	458314011	27.7669	9.1696	2.88705
772	1.29534	595984	460099648	27.7849	9.1736	2.88762
773	1.29366	597529	461889917	27.8029	9.1775	2.88818
774	1.29199	599076	463684824	27.8209	9.1815	2.88874
775	1.29032	600625	465484375	27.8388	9.1855	2.88930
776	1.28866	602176	467288576	27.8568	9.1894	2.88986
777	1.28700	603729	469097433	27.8747	9.1933	2.89042
778	1.28535	605284	470910952	27.8927	9.1973	2.89098
779	1.28370	606841	472729139	27.9106	9.2012	2.89154
780	1.28205	608400	474552000	27.9285	9.2052	2.89209
781	1.28041	609961	476379541	27.9464	9.2091	2.89265
782	1.27877	611524	478211768	27.9643	9.2130	2.89321
783	1.27714	613089	480048687	27.9821	9.2170	2.89376
784	1.27551	614656	481890304	28.0000	9.2209	2.89432
785	1.27389	616225	483736625	28.0179	9.2248	2.89487
786	1.27226	617796	485587656	28.0357	9.2287	2.89542
787	1.27065	619369	487443403	28.0535	9.2326	2.89597
788	1.26904	620944	489303872	28.0713	9.2365	2.89653
789	1.26743	622521	491169069	28.0891	9.2404	2.89708
790	1.26582	624100	493039000	28.1069	9.2443	2.89763
791	1.26422	625681	494913671	28.1247	9.2482	2.89818
792	1.26263	627264	496793088	28.1425	9.2521	2.89873
793	1.26103	628849	498677257	28.1603	9.2560	2.89927
794	1.25945	630436	500566184	28.1780	9.2599	2.89982
795	1.25786	632025	502459875	28.1957	9.2638	2.90037
796	1.25628	633616	504358336	28.2135	9.2677	2.90091
797	1.25471	635209	506261573	28.2312	9.2716	2.90146
798	1.25313	636804	508169592	28.2489	9.2754	2.90200
799	1.25156	638401	510082399	28.2666	9.2793	2.90255
800	1.25000	640000	512000000	28.2843	9.2832	2.90309
801	1.24844	641601	513922401	28.3019	9.2870	2.90363
802	1.24688	643204	515849608	28.3196	9.2909	2.90417
803	1.24533	644809	517781627	28.3373	9.2948	2.90472
804	1.24378	646416	519718464	28.3549	9.2986	2.90526
805	1.24224	648025	521660125	28.3725	9.3025	2.90580
806	1.24069	649636	523606616	28.3901	9.3063	2.90634
807	1.23916	651249	525557943	28.4077	9.3102	2.90687
808	1.23762	652864	527514112	28.4253	9.3140	2.90741
809	1.23609	654481	529475129	28.4429	9.3179	2.90795
810	1.23457	656100	531441000	28.4605	9.3217	2.90849
811	1.23305	657721	533411731	28.4781	9.3255	2.90902
812	1.23153	659344	535387328	28.4956	9.3294	2.90956
813	1.23001	660969	537367797	28.5132	9.3332	2.91009
814	1.22850	662596	539353144	28.5307	9.3370	2.91062
815	1.22699	664225	541343375	28.5482	9.3408	2.91116
816	1.22549	665856	543338496	28.5657	9.3447	2.91169
817	1.22399	667489	545338513	28.5832	9.3485	2.91222
818	1.22249	669124	547343432	28.6007	9.3523	2.91275
819	1.22100	670761	549353259	28.6182	9.3561	2.91328
820	1.21951	672400	551368000	28.6356	9.3599	2.91381
821	1.21803	674041	553387661	28.6531	9.3637	2.91434
822	1.21655	675684	555412248	28.6705	9.3675	2.91487
823	1.21507	677329	557441767	28.6880	9.3713	2.91540
824	1.21359	678976	559476224	28.7054	9.3751	2.91593

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000.\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
825	1.21212	680625	561515625	28.7228	9.3789	2.91645
826	1.21065	682276	563559976	28.7402	9.3827	2.91698
827	1.20919	683929	565609283	28.7576	9.3865	2.91751
828	1.20773	685584	567663552	28.7750	9.3902	2.91803
829	1.20627	687241	569722789	28.7924	9.3940	2.91855
830	1.20482	688900	571787000	28.8097	9.3978	2.91908
831	1.20337	690561	573856191	28.8271	9.4016	2.91960
832	1.20192	692224	575930368	28.8444	9.4053	2.92012
833	1.20048	693889	578009537	28.8617	9.4091	2.92065
834	1.19904	695556	580093704	28.8791	9.4129	2.92117
835	1.19760	697225	582182875	28.8964	9.4166	2.92169
836	1.19617	698896	584277056	28.9137	9.4204	2.92221
837	1.19474	700569	586376253	28.9310	9.4241	2.92273
838	1.19332	702244	588480472	28.9482	9.4279	2.92324
839	1.19190	703921	590589719	28.9655	9.4316	2.92376
840	1.19048	705600	592704000	28.9828	9.4354	2.92428
841	1.18906	707281	594823321	29.0000	9.4391	2.92480
842	1.18765	708964	596947688	29.0172	9.4429	2.92531
843	1.18624	710649	599077107	29.0345	9.4466	2.92583
844	1.18483	712336	601211584	29.0517	9.4503	2.92634
845	1.18343	714025	603351125	29.0689	9.4541	2.92686
846	1.18203	715716	605495736	29.0861	9.4578	2.92737
847	1.18064	717409	607645423	29.1033	9.4615	2.92788
848	1.17925	719104	609800192	29.1204	9.4652	2.92840
849	1.17786	720801	611960049	29.1376	9.4690	2.92891
850	1.17647	722500	614125000	29.1548	9.4727	2.92942
851	1.17509	724201	616295051	29.1719	9.4764	2.92993
852	1.17371	725904	618470208	29.1890	9.4801	2.93044
853	1.17233	727609	620650477	29.2062	9.4838	2.93095
854	1.17096	729316	622835864	29.2233	9.4875	2.93146
855	1.16959	731025	625026375	29.2404	9.4912	2.93197
856	1.16822	732736	627222016	29.2575	9.4949	2.93247
857	1.16686	734449	629422793	29.2746	9.4986	2.93298
858	1.16550	736164	631628712	29.2916	9.5023	2.93349
859	1.16414	737881	633839779	29.3087	9.5060	2.93399
860	1.16279	739600	636056000	29.3258	9.5097	2.93450
861	1.16144	741321	638277381	29.3428	9.5134	2.93500
862	1.16009	743044	640503928	29.3598	9.5171	2.93551
863	1.15875	744769	642735647	29.3769	9.5207	2.93601
864	1.15741	746496	644972544	29.3939	9.5244	2.93651
865	1.15607	748225	647214625	29.4109	9.5281	2.93702
866	1.15473	749956	649461896	29.4279	9.5317	2.93752
867	1.15340	751689	651714363	29.4449	9.5354	2.93802
868	1.15207	753424	653972032	29.4618	9.5391	2.93852
869	1.15075	755161	656234909	29.4788	9.5427	2.93902
870	1.14943	756900	658503000	29.4958	9.5464	2.93952
871	1.14811	758641	660776311	29.5127	9.5501	2.94002
872	1.14679	760384	663054848	29.5296	9.5537	2.94052
873	1.14548	762129	665338617	29.5466	9.5574	2.94101
874	1.14416	763876	667627624	29.5635	9.5610	2.94151
875	1.14286	765625	669921875	29.5804	9.5647	2.94201
876	1.14155	767376	672221376	29.5973	9.5683	2.94250
877	1.14025	769129	674526133	29.6142	9.5719	2.94300
878	1.13895	770884	676836152	29.6311	9.5756	2.94349
879	1.13766	772641	679151439	29.6479	9.5792	2.94399

Table 3.

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000 \cdot \frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
880	1.13636	774400	681472000	29.6648	9.5828	2.94448
881	1.13507	776161	683797841	29.6816	9.5865	2.94498
882	1.13379	777924	686128968	29.6985	9.5901	2.94547
883	1.13250	779689	688465387	29.7153	9.5937	2.94596
884	1.13122	781456	690807104	29.7321	9.5973	2.94645
885	1.12994	783225	693154125	29.7489	9.6010	2.94694
886	1.12867	784996	695506456	29.7658	9.6046	2.94743
887	1.12740	786769	697864103	29.7825	9.6082	2.94792
888	1.12613	788544	700227072	29.7993	9.6118	2.94841
889	1.12486	790321	702595369	29.8161	9.6154	2.94890
890	1.12360	792100	704969000	29.8329	9.6190	2.94939
891	1.12233	793881	707347971	29.8496	9.6226	2.94988
892	1.12108	795664	709732288	29.8664	9.6262	2.95036
893	1.11982	797449	712121957	29.8831	9.6298	2.95085
894	1.11857	799236	714516984	29.8998	9.6334	2.95134
895	1.11732	801025	716917375	29.9166	9.6370	2.95182
896	1.11607	802816	719323136	29.9333	9.6406	2.95231
897	1.11483	804609	721734273	29.9500	9.6442	2.95279
898	1.11359	806404	724150792	29.9666	9.6477	2.95328
899	1.11235	808201	726572699	29.9833	9.6513	2.95376
900	1.11111	810000	729000000	30.0000	9.6549	2.95424
901	1.10988	811801	731432701	30.0167	9.6585	2.95472
902	1.10865	813604	733870808	30.0333	9.6620	2.95521
903	1.10742	815409	736314327	30.0500	9.6656	2.95569
904	1.10619	817216	738763264	30.0666	9.6692	2.95617
905	1.10497	819025	741217625	30.0832	9.6727	2.95665
906	1.10375	820836	743677416	30.0998	9.6763	2.95713
907	1.10254	822649	746142643	30.1164	9.6799	2.95761
908	1.10132	824464	748613312	30.1330	9.6834	2.95809
909	1.10011	826281	751089429	30.1496	9.6870	2.95856
910	1.09890	828100	753571000	30.1662	9.6905	2.95904
911	1.09769	829921	756058031	30.1828	9.6941	2.95952
912	1.09649	831744	758550528	30.1993	9.6976	2.95999
913	1.09529	833569	761048497	30.2159	9.7012	2.96047
914	1.09409	835396	763551944	30.2324	9.7047	2.96095
915	1.09290	837225	766060875	30.2490	9.7082	2.96142
916	1.09170	839056	768575296	30.2655	9.7118	2.96190
917	1.09051	840889	771095213	30.2820	9.7153	2.96237
918	1.08932	842724	773620632	30.2985	9.7188	2.96284
919	1.08814	844561	776151559	30.3150	9.7224	2.96332
920	1.08696	846400	778688000	30.3315	9.7259	2.96379
921	1.08578	848241	781229961	30.3480	9.7294	2.96426
922	1.08460	850084	783777448	30.3645	9.7329	2.96473
923	1.08342	851929	786330467	30.3809	9.7364	2.96520
924	1.08225	853776	788889024	30.3974	9.7400	2.96567
925	1.08108	855625	791453125	30.4138	9.7435	2.96614
926	1.07991	857476	794022776	30.4302	9.7470	2.96661
927	1.07875	859329	796597983	30.4467	9.7505	2.96708
928	1.07759	861184	799178752	30.4631	9.7540	2.96755
929	1.07643	863041	801765089	30.4795	9.7575	2.96802
930	1.07527	864900	804357000	30.4959	9.7610	2.96848
931	1.07411	866761	806954491	30.5123	9.7645	2.96895
932	1.07296	868624	809557568	30.5287	9.7680	2.96942
933	1.07181	870489	812166237	30.5450	9.7715	2.96988
934	1.07066	872356	814780504	30.5614	9.7750	2.97035

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000.\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
935	1.06952	874225	817400375	30.5778	9.7785	2.97081
936	1.06838	876096	820025856	30.5941	9.7819	2.97128
937	1.06724	877969	822656953	30.6105	9.7854	2.97174
938	1.06610	879844	825293672	30.6268	9.7889	2.97220
939	1.06496	881721	827936019	30.6431	9.7924	2.97267
940	1.06383	883600	830584000	30.6594	9.7959	2.97313
941	1.06270	885481	833237621	30.6757	9.7993	2.97359
942	1.06157	887364	835896888	30.6920	9.8028	2.97405
943	1.06045	889249	838561807	30.7083	9.8063	2.97451
944	1.05932	891136	841232384	30.7246	9.8097	2.97497
945	1.05820	893025	843908625	30.7409	9.8132	2.97543
946	1.05708	894916	846590536	30.7571	9.8167	2.97589
947	1.05597	896809	849278123	30.7734	9.8201	2.97635
948	1.05485	898704	851971392	30.7896	9.8236	2.97681
949	1.05374	900601	854670349	30.8058	9.8270	2.97727
950	1.05263	902500	857375000	30.8221	9.8305	2.97772
951	1.05152	904401	860085351	30.8383	9.8339	2.97818
952	1.05042	906304	862801408	30.8545	9.8374	2.97864
953	1.04932	908209	865523177	30.8707	9.8408	2.97909
954	1.04822	910116	868250664	30.8869	9.8443	2.97955
955	1.04712	912025	870983875	30.9031	9.8477	2.98000
956	1.04603	913936	873722816	30.9192	9.8511	2.98046
957	1.04493	915849	876467493	30.9354	9.8546	2.98091
958	1.04384	917764	879217912	30.9516	9.8580	2.98137
959	1.04275	919681	881974079	30.9677	9.8614	2.98182
960	1.04167	921600	884736000	30.9839	9.8648	2.98227
961	1.04058	923521	887503681	31.0000	9.8683	2.98272
962	1.03950	925444	890277128	31.0161	9.8717	2.98318
963	1.03832	927369	893056347	31.0322	9.8751	2.98363
964	1.03734	929296	895841344	31.0483	9.8785	2.98408
965	1.03627	931225	898632125	31.0644	9.8819	2.98453
966	1.03520	933156	901428696	31.0805	9.8854	2.98498
967	1.03413	935089	904231063	31.0966	9.8888	2.98543
968	1.03306	937024	907039232	31.1127	9.8922	2.98588
969	1.03199	938961	909853209	31.1288	9.8956	2.98632
970	1.03093	940900	912673000	31.1448	9.8990	2.98677
971	1.02987	942841	915498611	31.1609	9.9024	2.98722
972	1.02881	944784	918330048	31.1769	9.9058	2.98767
973	1.02775	946729	921167317	31.1929	9.9092	2.98811
974	1.02669	948676	924010424	31.2090	9.9126	2.98856
975	1.02564	950625	926859375	31.2250	9.9160	2.98900
976	1.02459	952576	929714176	31.2410	9.9194	2.98945
977	1.02354	954529	932574833	31.2570	9.9227	2.98989
978	1.02249	956484	935441352	31.2730	9.9261	2.99034
979	1.02145	958441	938313739	31.2890	9.9295	2.99078
980	1.02041	960400	941192000	31.3050	9.9329	2.99123
981	1.01937	962361	944076141	31.3209	9.9363	2.99167
982	1.01833	964324	946966168	31.3369	9.9396	2.99211
983	1.01729	966289	949862087	31.3528	9.9430	2.99255
984	1.01626	968256	952763904	31.3688	9.9464	2.99300
985	1.01523	970225	955671625	31.3847	9.9497	2.99344
986	1.01420	972196	958585256	31.4006	9.9531	2.99388
987	1.01317	974169	961504803	31.4166	9.9565	2.99432
988	1.01215	976144	964430272	31.4325	9.9598	2.99476
989	1.01112	978121	967361669	31.4484	9.9632	2.99520

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

n	$1000.\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	log. n
990	1.01010	980100	970299000	31.4643	9.9666	2.99564
991	1.00908	982081	973242271	31.4802	9.9699	2.99607
992	1.00806	984064	976191488	31.4960	9.9733	2.99651
993	1.00705	986049	979146657	31.5119	9.9766	2.99695
994	1.00604	988036	982107784	31.5278	9.9800	2.99739
995	1.00503	990025	985074875	31.5436	9.9833	2.99782
996	1.00402	992016	988047936	31.5595	9.9866	2.99826
997	1.00301	994009	991026973	31.5753	9.9900	2.99870
998	1.00200	996004	994011992	31.5911	9.9933	2.99913
999	1.00100	998001	997002999	31.6070	9.9967	2.99957
1000	1.00000	1000000	1000000000	31.6228	10.0000	3.00000

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9	Prop Parts 1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
N.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Parts.								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
N.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

NATURAL SINES AND COSINES.

Natural Sines.

Angle.	0′	10′	20′	30′	40′	50′	60′	Angle.	Prop. Parts for 1′.
0°	.0000 00	.0029 09	.0058 18	.0087 27	.0116 35	.0145 44	.0174 52	89°	2.9
1	.0174 52	.0203 6	.0232 7	.0261 8	.0290 8	.0319 9	.0349 0	88	2.9
2	.0349 0	.0378 1	.0407 1	.0436 2	.0465 3	.0494 3	.0523 4	87	2.9
3	.0523 4	.0552 4	.0581 4	.0610 5	.0639 5	.0668 5	.0697 6	86	2.9
4	.0697 6	.0726 6	.0755 6	.0784 6	.0813 6	.0842 6	.0871 6	85	2.9
5	.0871 6	.0900 5	.0929 5	.0958 5	.0987 4	.1016 4	.1045 3	84	2.9
6	.1045 3	.1074 2	.1103 1	.1132 0	.1160 9	.1189 8	.1218 7	83	2.9
7	1218 7	.1247 6	.1276 4	.1305 3	.1334	.1363	.1392	82	2.9
8	1392	.1421	.1449	.1478	.1507	.1536	.1564	81	2.9
9	1564	.1593	.1622	.1650	.1679	.1708	.1736	80	2.9
10	.1736	1765	.1794	.1822	.1851	.1880	.1908	79	2.9
11	1908	.1937	.1965	.1994	.2022	.2051	.2079	78	2.9
12	2079	.2108	.2136	.2164	.2193	.2221	.2250	77	2.8
13	.2250	.2278	.2306	.2334	.2363	.2391	.2419	76	2.8
14	.2419	.2447	.2476	.2504	.2532	.2560	.2588	75	2.8
15	.2588	.2616	.2644	.2672	.2700	.2728	.2756	74	2.8
16	2756	.2784	2812	.2840	.2868	.2896	.2924	73	2.8
17	.2924	.2952	.2979	.3007	.3035	.3062	.3090	72	2.8
18	.3090	3118	.3145	.3173	.3201	.3228	.3256	71	2.8
19	.3256	3283	.3311	.3338	.3365	.3393	.3420	70	2.7
20	.3420	3448	.3475	.3502	.3529	.3557	.3584	69	2.7
21	3584	.3611	.3638	.3665	.3692	.3719	.3746	68	2.7
22	3746	.3773	.3800	.3827	.3854	.3881	.3907	67	2.7
23	.3907	.3934	.3961	.3987	.4014	.4041	.4067	66	2.7
24	4067	.4094	.4120	.4147	.4173	.4200	.4226	65	2.7
25	.4226	.4253	.4279	.4305	.4331	.4358	.4384	64	2.6
26	.4384	.4410	.4436	.4462	.4488	.4514	.4540	63	2.6
27	.4540	.4566	.4592	.4617	.4643	.4669	.4695	62	2.6
28	.4695	.4720	.4746	.4772	.4797	.4823	.4848	61	2.6
29	.4848	.4874	.4899	.4924	.4950	.4975	.5000	60	2.5
30	.5000	.5025	.5050	.5075	.5100	.5125	.5150	59	2.5
31	.5150	.5175	.5200	.5225	.5250	.5275	.5299	58	2.5
32	.5299	.5324	.5348	.5373	.5398	.5422	.5446	57	2.5
33	.5446	.5471	.5495	.5519	.5544	.5568	.5592	56	2.4
34	.5592	.5616	.5640	.5664	.5688	.5712	.5736	55	2.4
35	.5736	.5760	.5783	.5807	.5831	.5854	.5878	54	2.4
36	.5878	.5901	.5925	.5948	.5972	.5995	.6018	53	2.3
37	.6018	.6041	.6065	.6088	.6111	.6134	.6157	52	2.3
38	.6157	.6180	.6202	.6225	.6248	.6271	.6293	51	2.3
39	.6293	.6316	.6338	.6361	.6383	.6406	.6428	50	2.3
40	.6428	.6450	.6472	.6494	.6517	.6539	.6561	49	2.2
41	.6561	.6583	.6604	.6626	.6648	.6670	.6691	48	2.2
42	.6691	.6713	.6734	.6756	.6777	.6799	.6820	47	2.2
43	.6820	.6841	.6862	.6884	.6905	.6926	.6947	46	2.1
44	.6947	.6967	.6988	.7009	.7030	.7050	.7071	45	2.1
	60′	50′	40′	30′	20′	10′	0′	Angle.	

Natural Cosines.

NATURAL SINES AND COSINES.

Natural Sines.

Angle.	0′	10′	20′	30′	40′	50′	60′	Angle.	Prop. Parts for 1′.
45°	.7071	.7092	.7112	.7133	.7153	.7173	.7193	44°	2.0
46	.7193	.7214	.7234	.7254	.7274	.7294	.7314	43	2.0
47	.7314	.7333	.7353	.7373	.7392	.7412	.7431	42	2.0
48	.7431	.7451	.7470	.7490	.7509	.7528	.7547	41	1.9
49	.7547	.7566	.7585	.7604	.7623	.7642	.7660	40	1.9
50	.7660	.7679	.7698	.7716	.7735	.7753	.7771	39	1.9
51	.7771	.7790	.7808	.7826	.7844	.7862	.7880	38	1.8
52	.7880	.7898	.7916	.7934	.7951	.7969	.7986	37	1.8
53	.7986	.8004	.8021	.8039	.8056	.8073	.8090	36	1.7
54	.8090	.8107	.8124	.8141	.8158	.8175	.8192	35	1.7
55	.8192	.8208	.8225	.8241	.8258	.8274	.8290	34	1.6
56	.8290	.8307	.8323	.8339	.8355	.8371	.8387	33	1.6
57	.8387	.8403	.8418	.8434	.8450	.8465	.8480	32	1.6
58	.8480	.8496	.8511	.8526	.8542	.8557	.8572	31	1.5
59	.8572	.8587	.8601	.8616	.8631	.8646	.8660	30	1.5
60	.8660	.8675	.8689	.8704	.8718	.8732	.8746	29	1.4
61	.8746	.8760	.8774	.8788	.8802	.8816	.8829	28	1.4
62	.8829	.8843	.8857	.8870	.8884	.8897	.8910	27	1.4
63	.8910	.8923	.8936	.8949	.8962	.8975	.8988	26	1.3
64	.8988	.9001	.9013	.9026	.9038	.9051	.9063	25	1.3
65	.9063	.9075	.9088	.9100	.9112	.9124	.9135	24	1.2
66	.9135	.9147	.9159	.9171	.9182	.9194	.9205	23	1.2
67	.9205	.9216	.9228	.9239	.9250	.9261	.9272	22	1.1
68	.9272	.9283	.9293	.9304	.9315	.9325	.9336	21	1.1
69	.9336	.9346	.9356	.9367	.9377	.9387	.9397	20.	1.0
70	.9397	.9407	.9417	.9426	.9436	.9446	.9455	19	1.0
71	.9455	.9465	.9474	.9483	.9492	.9502	.9511	18	0.9
72	.9511	.9520	.9528	.9537	.9546	.9555	.9563	17	0.9
73	.9563	.9572	.9580	.9588	.9596	.9605	.9613	16	0.8
74	.9613	.9621	.9628	.9636	.9644	.9652	.9659	15	0.8
75	.9659	.9667	.9674	.9681	.9689	.9696	.9703	14	0.7
76	.9703	.9710	9717	.9724	.9730	.9737	.9744	13	0.7
77	.9744	.9750	.9757	.9763	.9769	.9775	.9781	12	0.6
78	.9781	.9787	.9793	.9799	.9805	.9811	.9816	11	0.6
79	.9816	.9822	.9827	.9833	.9838	.9843	.9848	10	0.5
80	.9848	.9853	.9858	.9863	.9868	.9872	.9877	9	0.5
81	.9877	.9881	.9886	.9890	.9894	.9899	.9903	8	0.4
82	.9903	.9907	.9911	.9914	.9918	.9922	.9925	7	0.4
83	.9925	.9929	.9932	.9936	.9939	.9942	.9945	6	0.3
84	.9945	.9948	.9951	9954	.9957	.9959	.9962	5	0.3
85	.9962	.9964	.9967	.9969	.9971	.9974	.9976	4	0.2
86	.9976	.9978	.9980	.9981	.9983	.9985	.9986	3	0.2
87	.9986	.9988	.9989	.9990	.9992	.9993	.9994	2	0.1
88	.9994	.9995	9996	.9997	.9997	.9998	.9998	1	0.1
89	.9998	.9999	.9999	1.0000	1.0000	1.0000	1.0000	0	0.0
	60′	50′	40′	30′	20′	10′	0′	Angle.	

Natural Cosines.

Table 8.

NATURAL TANGENTS AND COTANGENTS.

Natural Tangents.

Angle.	0′	10′	20′	30′	40′	50′	60′	Angle.	Prop. Parts for 1′.
0°	.0000 0	.0029 1	.0058 2	.0087 3	.0116 4	.0145 5	.0174 6	89°	2.9
1	.0174 6	.0203 6	.0232 8	.0261 9	.0291 0	.0320 1	.0349 2	88	2.9
2	.0349 2	.0378 3	.0407 5	.0436 6	.0465 8	.0494 9	.0524 1	87	2.9
3	.0524 1	.0553 3	.0582 4	.0611 6	.0640 8	.0670 0	.0699 3	86	2.9
4	.0699 3	.0728 5	.0757 8	.0787 0	.0816 3	.0845 6	.0874 9	85	2.9
5	.0874 9	.0904 2	.0933 5	.0962 9	.0992 3	.1021 6	.1051 0	84	2.9
6	.1051 0	.1080 5	.1109 9	.1139 4	.1168 8	.1198 3	.1227 8	83	2.9
7	.1227 8	.1257 4	.1286 9	.1316 5	1346	.1376	.1405	82	3.0
8	.1405	.1435	.1465	.1495	.1524	.1554	.1584	81	3.0
9	.1584	.1614	.1644	.1673	.1703	1733	.1763	80	3.0
10	.1763	.1793	.1823	.1853	.1883	.1914	.1944	79	3.0
11	.1944	.1974	.2004	.2035	.2065	.2095	.2126	78	3.0
12	.2126	.2156	.2186	.2217	.2247	.2278	.3309	77	3.1
13	.2309	.2339	.2370	.2401	.2432	.2462	.2493	76	3.1
14	.2493	.2524	.2555	.2586	.2617	.2648	.2679	75	3.1
15	.2679	.2711	.2742	.2773	.2805	.2836	.2867	74	3.1
16	.2867	.2899	.2931	.2962	.2994	.3026	.3057	73	3.2
17	.3057	.3089	.3121	.3153	.3185	.3217	.3249	72	3.2
18	.3249	.3281	.3314	.3346	.3378	.3411	.3443	71	3.2
19	.3443	.3476	.3508	.3541	.3574	.3607	.3640	70	3.3
20	.3640	.3673	.3706	.3739	.3772	.3805	.3839	69	3.3
21	.3839	.3872	.3906	.3939	.3973	.4006	.4040	68	3.4
22	.4040	.4074	.4108	.4142	.4176	.4210	.4245	67	3.4
23	.4245	.4279	.4314	.4348	.4383	.4417	.4452	66	3.5
24	.4452	.4487	.4522	.4557	.4592	.4628	.4663	65	3.5
25	.4663	.4699	.4734	.4770	.4806	.4841	.4877	64	3.6
26	.4877	.4913	.4950	.4986	.5022	.5059	.5095	63	3.6
27	.5095	.5132	.5169	.5206	.5243	.5280	.5317	62	3.7
28	.5317	.5354	.5392	.5430	.5467	.5505	.5543	61	3.8
29	.5543	.5581	.5619	.5658	.5696	.5735	5774	60	3.8
30	.5774	.5812	.5851	.5890	.5930	.5969	.6009	59	3.9
31	.6009	.6048	.6088	.6128	.6168	.6208	.6249	58	4.0
32	.6249	.6289	.6330	.6371	.6412	.6453	.6494	57	4.1
33	.6494	.6536	.6577	.6619	.6661	.6703	.6745	56	4.2
34	.6745	.6787	.6830	.6873	.6916	.6959	.7002	55	4.3
35	.7002	.7046	.7089	.7133	.7177	.7221	.7265	54	4.4
36	.7265	.7310	.7355	.7400	.7445	.7490	.7536	53	4.5
37	.7536	.7581	.7627	.7673	.7720	.7766	.7813	52	4.6
38	.7813	.7860	.7907	.7954	.8002	.8050	.8098	51	4.7
39	.8098	.8146	.8195	.8243	.8292	.8342	.8391	50	4.9
40	.8391	.8441	.8491	.8541	.8591	.8642	.8693	49	5.0
41	.8693	.8744	.8796	.8847	.8899	.8952	.9004	48	5.2
42	.9004	.9057	.9110	.9163	.9217	.9271	.9325	47	5.4
43	.9325	.9380	.9435	.9490	.9545	.9601	.9657	46	5.5
44	.9657	.9713	.9770	.9827	.9884	.9942	1.0000	45	5.7
	60′	50′	40′	30′	20′	10′	0′	Angle.	

Natural Cotangents.

NATURAL TANGENTS AND COTANGENTS.

Natural Tangents.

Angle.	0′	10′	20′	30′	40′	50′	60′	Angle.	Prop. Parts for 1′.
45°	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44°	5.9
46	1.0355	1.0416	1.0477	1.0538	1.0599	1.0661	1.0724	43	6.1
47	1.0724	1.0786	1.0850	1.0913	1.0977	1.1041	1.1106	42	6.4
48	1.1106	1.1171	1.1237	1.1303	1.1369	1.1436	1.1504	41	6.6
49	1.1504	1.1571	1.1640	1.1708	1.1778	1.1847	1.1918	40	6.9
50	1.1918	1.1988	1.2059	1.2131	1.2203	1.2276	1.2349	39	7.2
51	1.2349	1.2423	1.2497	1.2572	1.2647	1.2723	1.2799	38	7.5
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37	7.9
53	1.3270	1.3351	1.3432	1.3514	1.3597	1.3680	1.3764	36	8.2
54	1.3764	1.3848	1.3934	1.4019	1.4106	1.4193	1.4281	35	8.6
55	1.4281	1.4370	1.4460	1.4550	1.4641	1.4733	1.4826	34	9.1
56	1.4826	1.4919	1.5013	1.5108	1.5204	1.5301	1.5399	33	9.6
57	1.5399	1.5497	1.5597	1.5697	1.5798	1.5900	1.6003	32	10.1
58	1.6003	1.6107	1.6212	1.6319	1.6426	1.6534	1.6643	31	10.7
59	1.6643	1.6753	1.6864	1.6977	1.7090	1.7205	1.7321	30	11.3
60	1.7321	1.7437	1.7556	1.7675	1.7796	1.7917	1.8040	29	12.0
61	1.8040	1.8165	1.8291	1.8418	1.8546	1.8676	1.8807	28	12.8
62	1.8807	1.8940	1.9074	1.9210	1.9347	1.9486	1.9626	27	13.6
63	1.9626	1.9768	1.9912	2.0057	2.0204	2.0353	2.0503	26	14.6
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25	15.7
65	2.1445	2.1609	2.1775	2.1943	2.2113	2.2286	2.2460	24	16.9
66	2.2460	2.2637	2.2817	2.2998	2.3183	2.3369	2.3559	23	18.3
67	2.3559	2.3750	2.3945	2.4142	2.4342	2.4545	2.4751	22	19.9
68	2.4751	2.4960	2.5172	2.5386	2.5605	2.5826	2.6051	21	21.7
69	2.6051	2.6279	2.6511	2.6746	2.6985	2.7228	2.7475	20	23.7
70	2.7475	2.7725	2.7980	2.8239	2.8502	2.8770	2.9042	19	
71	2.9042	2.9319	2.9600	2.9887	3.0178	3.0475	3.0777	18	
72	3.0777	3.1084	3.1397	3.1716	3.2041	3.2371	3.2709	17	
73	3.2709	3.3052	3.3402	3.3759	3.4124	3.4495	3.4874	16	
74	3.4874	3.5261	3.5656	3.6059	3.6470	3.6891	3.7321	15	
75	3.7321	3.7760	3.8208	3.8667	3.9136	3.9617	4.0108	14	
76	4.0108	4.0611	4.1126	4.1653	4.2193	4.2747	4.3315	13	
77	4.3315	4.3897	4.4494	4.5107	4.5736	4.6382	4.7046	12	
78	4.7046	4.7729	4.8430	4.9152	4.9894	5.0658	5.1446	11	
79	5.1446	5.2257	5.3093	5.3955	5.4845	5.5764	5.6713	10	
80	5.6713	5.7694	5.8708	5.9758	6.0844	6.1970	6.3138	9	
81	6.3138	6.4348	6.5606	6.6912	6.8269	6.9682	7.1154	8	
82	7.1154	7.2687	7.4287	7.5958	7.7704	7.9530	8.1443	7	
83	8.1443	8.3450	8.5555	8.7769	9.0098	9.2553	9.5144	6	
84	9.5144	9.7882	10.0780	10.3854	10.7119	11.0594	11.4301	5	
85	11.4301	11.8262	12.2505	12.7062	13.1969	13.7267	14.3007	4	
86	14.3007	14.9244	15.6048	16.3499	17.1693	18.0750	19.0811	3	
87	19.0811	20.2056	21.4704	22.9038	24.5418	26.4316	28.6363	2	
88	28.6363	31.2416	34.3678	38.1885	42.9641	49.1039	57.2900	1	
89	57.2900	68.7501	85.9398	114.5887	171.8854	343.7737	∞	0	
	60′	50′	40′	30′	20′	10′	0′	Angle.	

Natural Cotangents.

MEAN REFRACTION.

Apparent altitude.	Refraction		Apparent altitude.	Refraction.		Apparent altitude.	Refraction.		Apparent altitude.	Refraction.		Apparent altitude.	Refraction.	
° ′	′ ″	″	° ′	° ″	″	° ′	′ ″	″	° ′	′ ″	″	°	″	″
0 0	34 54.1		7 0	7 19.7		14 0	3 47.4		28 0	1 48.2		42	64.0	
10	32 49.2	124.9	10	7 10.5	9.2	20	3 42.1	5.3	20	1 46.7	1.5	43	61.8	2.2
20	30 52.3	116.9	20	7 1.7	8.8	40	3 37.0	5.1	40	1 45.3	1.4	44	59.7	2.1
30	29 3.5	108.8	30	6 53.3	8.4	15 0	3 32.1	4.9	29 0	1 43.8	1.5	45	57.7	2.0
40	27 22.7	100.8	40	6 45.1	8.2	20	3 27.4	4.7	20	1 42.4	1.4	46	55.7	2.0
50	25 49.8	92.9	50	6 37.2	7.9	40	3 22.9	4.5	40	1 41.0	1.4	47	53.8	1.9
1 0	24 24.6	85.2	8 0	6 29.6	7.6	16 0	3 18.6	4.3	30 0	1 39.7	1.3	48	51.9	1.9
10	23 6.7	77.9	10	6 22.3	7.3	20	3 14.5	4.1	20	1 38.4	1.3	49	50.2	1.7
20	21 55.6	71.1	20	6 15.2	7.1	40	3 10.5	4.0	40	1 37.1	1.3	50	48.4	1.8
30	20 50.9	64.7	30	6 8.4	6.8	17 0	3 6.6	3.9	31 0	1 35.8	1.3	51	46.7	1.7
40	19 51.9	59.0	40	6 1.8	6.6	20	3 2.9	3.7	20	1 34.5	1.3	52	45.1	1.6
50	18 58.0	53.9	50	5 55.4	6.4	40	2 59.3	3.6	40	1 33.3	1.2	53	43.5	1.6
2 0	18 8.6	49.4	9 0	5 49.3	6.1	18 0	2 55.8	3.5	32 0	1 32.1	1.2	54	41.9	1.6
10	17 23.0	45.6	10	5 43.3	6.0	20	2 52.5	3.3	20	1 30.9	1.2	55	40.4	1.5
20	16 40.7	42.3	20	5 37.6	5.7	40	2 49.3	3.2	40	1 29.8	1.1	56	38.9	1.5
30	16 0.9	39.8	30	5 32.0	5.6	19 0	2 46.1	3.2	33 0	1 28.7	1.1	57	37.5	1.4
40	15 23.4	37.5	40	5 26.5	5.5	20	2 43.1	3.0	20	1 27.6	1.1	58	36.1	1.4
50	14 47.8	35.6	50	5 21.3	5.2	40	2 40.2	2.9	40	1 26.5	1.1	59	34.7	1.4
3 0	14 14.6	33.2	10 0	5 16.2	5.1	20 0	2 37.3	2.9	34 0	1 25.4	1.1	60	33.3	1.4
10	13 43.7	30.9	10	5 11.2	5.0	20	2 34.5	2.8	20	1 24.3	1.1	61	32.0	1.3
20	13 15.0	28.7	20	5 6.4	4.8	40	2 31.9	2.6	40	1 23.3	1.0	62	30.7	1.3
30	12 48.3	26.7	30	5 1.7	4.7	21 0	2 29.3	2.6	35 0	1 22.3	1.0	63	29.4	1.3
40	12 23.7	24.6	40	4 57.2	4.5	20	2 26.8	2.5	20	1 21.3	1.0	64	28.2	1.2
50	12 0.7	23.0	50	4 52.8	4.4	40	2 24.3	2.5	40	1 20.3	1.0	65	26.9	1.3
4 0	11 38.9	21.8	11 0	4 48.5	4.3	22 0	2 21.9	2.4	36 0	1 19.3	1.0	66	25.7	1.2
10	11 18.3	20.6	10	4 44.3	4.2	20	2 19.6	2.3	20	1 18.3	1.0	67	24.5	1.2
20	10 58.6	19.7	20	4 40.2	4.1	40	2 17.4	2.2	40	1 17.4	0.9	68	23.3	1.2
30	10 39.6	19.0	30	4 36.3	3.9	23 0	2 15.2	2.2	37 0	1 16.5	0.9	69	22.2	1.1
40	10 21.2	18.4	40	4 32.4	3.9	20	2 13.0	2.2	20	1 15.6	0.9	70	21.0	1.2
50	10 3.3	17.9	50	4 28.7	3.7	40	2 10.9	2.1	40	1 14.7	0.9	71	19.9	1.1
5 0	9 46.5	16.8	12 0	4 25.0	3.7	24 0	2 8.9	2.0	38 0	1 13.8	0.9	72	18.8	1.1
10	9 30.9	15.6	10	4 21.4	3.6	20	2 7.0	1.9	20	1 12.9	0.9	73	17.7	1.1
20	9 16.0	14.9	20	4 18.0	3.4	40	2 5.1	1.9	40	1 12.0	0.9	74	16.6	1.1
30	9 1.9	14.1	30	4 14.6	3.4	25 0	2 3.2	1.9	39 0	1 11.2	0.8	75	15.5	1.1
40	8 48.4	13.5	40	4 11.3	3.3	20	2 1.4	1.8	20	1 10.3	0.9	76	14.5	1.0
50	8 35.6	12.8	50	4 8.1	3.2	40	1 59.6	1.8	40	1 9.5	0.8	77	13.4	1.1
6 0	8 23.3	12.3	13 0	4 4.9	3.2	26 0	1 57.8	1.8	40 0	1 8.7	0.8	78	12.3	1.1
10	8 11.6	11.7	10	4 1.8	3.1	20	1 56.1	1.7	20	1 7.9	0.8	79	11.2	1.1
20	8 0.3	11.3	20	3 58.8	3.0	40	1 54.4	1.7	40	1 7.1	0.8	80	10.2	1.0
30	7 49.5	10.8	30	3 55.9	2.9	27 0	1 52.8	1.6	41 0	1 6.3	0.8	81	9.1	1.1
40	7 39.2	10.3	40	3 53.0	2.9	20	1 51.2	1.6	20	1 5.5	0.8	82	8.1	1.0
50	7 29.2	10.0	50	3 50.2	2.8	40	1 49.7	1.5	40	1 4.7	0.8	86	4.1	4.0
7 0	7 19.7	9.5	14 0	3 47.4	2.8	28 0	1 48.2	1.5	42 0	1 4.0	0.7	90	0.0	4.1

APPENDIX.

TABLE I.—Mean Refractions.

APPARENT ALTITUDE.	REFRACTION. *minus.*	APPARENT ALTITUDE.	REFRACTION. *minus.*
25°	0° 2′ 4.2″	38°	0° 1′14.4″
26	1 58.8	39	1 11.8
27	1 53.8	40	1 9.3
28	1 49.1	41	1 6.9
29	1 44.7	42	1 4.6
30	1 40.5	43	1 2.4
31	1 36.6	44	1 0.3
32	1 33.0	45	0 58.1
33	1 29.5	46	0 56.1
34	1 26.1	47	0 54.2
35	1 23.0	48	0 52.3
36	1 20.0	49	0 50.5
37	1 17.1	50	0 48.8

TABLE II.—Corrections for Curvature and Refraction.

D	h_2	D	h_2	D	h_2	D	h_2
Miles.	*Feet.*	*Miles.*	*Feet.*	*Miles.*	*Feet.*	*Miles.*	*Feet.*
1.0	0.6	5.5	17.3				
1.1	0.7	5.6	18.0	3.6	7.4	8.1	37.6
1.2	0.8	5.7	18.6	3.7	7.8	8.2	38.6
1.3	1.0	5.8	19.3	3.8	8.3	8.3	39.5
1.4	1.1	5.9	20.0	3.9	8.7	8.4	40.5
1.5	1.3	6.0	20.6	4.0	9.2	8.5	41.4
1.6	1.5	6.1	21.3	4.1	9.6	8.6	42.4
1.7	1.7	6.2	22.0	4.2	10.1	8.7	43.4
1.8	1.9	6.3	22.8	4.3	10.6	8.8	44.4
1.9	2.1	6.4	23.5	4.4	11.1	8.9	45.4
2.0	2.3	6.5	24.2	4.5	11.6	9.0	46.4
2.1	2.5	6.6	25.0	4.6	12.1	9.1	47.5
2.2	2.8	6.7	25.7	4.7	12.7	9.2	48.5
2.3	3.0	6.8	26.5	4.8	13.2	9.3	49.6
2.4	3.3	6.9	27.3	4.9	13.8	9.4	50.7
2.5	3.6	7.0	28.1	5.0	14.3	9.5	51.7
2.6	3.9	7.1	28.9	5.1	14.9	9.6	52.8
2.7	4.2	7.2	29.7	5.2	15.5	9.7	53.9
2.8	4.5	7.3	30.5	5.3	16.1	9.8	55.1
2.9	4.8	7.4	31.4	5.4	16.7	9.9	56.2
3.0	5.2	7.5	32.2	5.5	17.3	10.0	57.3
3.1	5.5	7.6	33.1				
3.2	5.9	7.7	34.0				
3.3	6.2	7.8	34.9				
3.4	6.6	7.9	35.8				
3.5	7.0	8.0	36.7				

TABLE III.—Local Mean Times (astronomical, counting from noon, and from 0 to 24 hours) of the Elongations and Culminations of Polaris in the year 1893, latitude + 40° north, longitude six hours west from Greenwich.

DATE.	EAST ELONG.		UPPER CULM.		WEST ELONG.		LOWER CULM.	
1893.	Hour	Min.	Hour	Min.	Hour	Min.	Hour	Min
Jan. 1	0	37.2	6	32.0	12	26.8	18	30.0
Feb. 1	22	30.8	4	29.6	10	24.4	16	27.6
Mar. 1	20	40.4	2	39.1	8	33.9	14	37.1
Apr. 1	18	38.3	0	37.0	6	31.8	12	35.1
May 1	16	40.5	22	35.2	4	34.0	10	37.2
June 1	14	38.9	20	33.7	2	32.4	8	35.7
July 1	12	41.4	18	36.2	0	34.9	6	38.2
Aug. 1	10	40.0	16	34.8	22	29.6	4	36.8
Sept. 1	8	38.5	14	33.3	20	28.1	2	35.3
Oct. 1	6	40.8	12	35.6	18	30.4	0	37.6
Nov. 1	4	39.0	10	33.8	16	28.6	22	31.8
Dec. 1	2	40.8	8	35.6	14	30.4	20	33.5

For other days of each month subtract 3.94 minutes for each day from that of 1st. Thus for 2d subtract 3.94 min.; for 3d subtract 7.88 min.; for 4th subtract 11.82 min., etc.

For other years than 1893 add to the above 0.25 min. for every additional year; also add

0.0 min. if the year is the first after leap-year.
0.9 min. if the year is the second after leap-year.
1.7 min. if the year is the third after leap-year.
2.6 min. if the year is leap-year before March 1st.
Subtract 1.2 min. if the year is leap-year after March 1st.

For longitude correction add 0.16 min. for each hour east of the 6 hour, and subtract 0.16 min. for each hour west of the 6 hour meridian.

For other latitudes between 25° and 50° north, *add* to time of west elongation 0.13 min. for every degree south of latitude 40°; and *subtract* 0.18 min. for every degree north of latitude 40°; reverse these signs for corrections to the times of east elongations.

The year 1900 will not be a leap-year for dealing with the dates before and after March 1st of that year. The 20th century begins after December 31, 1900.

TABLE IV.—Azimuths of Polaris (from the north pole) at elongation, between 1895 and 1910, for different latitudes between 25° and 49° north.

Latitude.	1895		1896		1897		1898		1899		1900		1901		1902	
°	°	′	°	′	°	′	°	′	°	′	°	′	°	′	°	′
25	1	22.9	1	22.5	1	22.2	1	21.8	1	21.5	1	21.2	1	20.8	1	20.5
26		23.6		23.2		22.9		22.5		22.2		21.8		21.5		21.1
27		24.3		24.0		23.6		23.3		22.9		22.5		22.2		21.9
28		25.1		24.7		24.4		24.0		23.7		23.3		23.0		22.6
29		25.9		25.5		25.2		24.8		24.5		24.1		23.8		23.4
30	1	26.8	1	26.4	1	26.0	1	25.7	1	25.3	1	24.9	1	24.6	1	24.2
31		27.6		27.3		26.9		26.5		26.2		25.8		25.5		25.1
32		28.6		28.2		27.9		27.5		27.1		26.7		26.4		26.0
33		29.6		29.2		28.8		28.5		28.1		27.7		27.3		27.0
34		30.6		30.2		29.9		29.5		29.1		28.7		28.4		28.0
35	1	31.7	1	31.3	1	30.9	1	30.6	1	30.2	1	29.8	1	29.4	1	29.0
36		32.9		32.5		32.1		31.7		31.3		30.9		30.5		30.1
37		34.1		33.7		33.3		32.9		32.5		32.1		31.7		31.3
38		35.3		35.0		34.6		34.2		33.8		33.4		33.0		32.6
39		36.7		36.3		35.9		35.5		35.1		34.7		34.3		33.9
40	1	38.1	1	37.7	1	37.2	1	36.8	1	36.4	1	36.0	1	35.6	1	35.2
41		39.6		39.1		38.7		38.3		37.9		37.5		37.1		36.7
42		41.1		40.7		40.3		39.8		39.4		39.0		38.6		38.2
43		42.7		42.3		41.9		41.4		41.0		40.6		40.2		39.8
44		44.4		44.0		43.6		43.1		42.7		42.3		41.8		41.4
45	1	46.2	1	45.8	1	45.4	1	44.9	1	44.5	1	44.0	1	43.6	1	43.2
46		48.2		47.7		47.2		46.8		46.3		45.9		45.5		45.0
47		50.2		49.7		49.2		48.8		48.3		47.9		47.4		46.9
48		52.3		51.8		51.3		50.9		50.4		49.9		49.5		49.0
49		54.5		54.1		53.6		53.1		52.6		52.1		51.7		51.2

Latitude.	1903		1904		1905		1906		1907		1908		1909		1910	
°	°	′	°	′	°	′	°	′	°	′	°	′	°	′	°	′
25	1	20.1	1	19.8	1	19.4	1	19.1	1	18.7	1	18.4	1	18.1	1	17.7
26		20.8		20.5		20.1		19.8		19.4		19.1		18.7		18.4
27		21.5		21.2		20.8		20.5		20.1		19.8		19.4		19.1
28		22.2		21.9		21.6		21.3		20.9		20.5		20.1		19.8
29		23.0		22.7		22.4		22.1		21.7		21.3		20.9		20.5
30	1	23.9	1	23.5	1	23.1	1	22.8	1	22.4	1	22.1	1	21.7	1	21.3
31		24.7		24.4		24.0		23.6		23.2		22.9		22.5		22.2
32		25.6		25.3		24.9		24.5		24.1		23.8		23.4		23.1
33		26.6		26.2		25.9		25.5		25.1		24.7		24.3		24.0
34		27.6		27.2		26.9		26.5		26.1		25.7		25.3		25.0
35	1	28.7	1	28.3	1	27.9	1	27.5	1	27.1	1	26.8	1	26.4	1	26.0
36		29.8		29.4		29.0		28.6		28.2		27.9		27.5		27.1
37		30.9		30.5		30.1		29.7		29.3		29.0		28.6		28.2
38		32.2		31.8		31.4		31.0		30.6		30.2		29.8		29.4
39		33.5		33.1		32.7		32.3		31.8		31.4		31.0		30.6
40	1	34.8	1	34.4	1	34.0	1	33.6	1	33.2	1	32.8	1	32.4	1	32.0
41		36.2		35.8		35.4		35.0		34.6		34.2		33.8		33.4
42		37.7		37.3		36.9		36.5		36.0		35.6		35.2		34.8
43		39.3		38.9		38.5		38.1		37.6		37.2		36.8		36.3
44		41.0		40.5		40.1		39.7		39.2		38.8		38.4		37.9
45	1	42.7	1	42.3	1	41.8	1	41.4	1	40.9	1	40.5	1	40.1	1	39.6
46		44.6		44.2		43.7		43.2		42.7		42.3		41.9		41.4
47		46.5		46.0		45.6		45.1		44.6		44.2		43.7		43.3
48		48.6		48.1		47.7		47.2		46.7		46.3		45.8		45.3
49		50.7		50.2		49.8		49.3		48.8		48.4		47.9		47.4

TABLE V. Horizontal Distances and Elevations for inclined stadia readings.

Minutes.	0° Hor. Dist.	0° Diff. Elev.	1° Hor. Dist.	1° Diff. Elev.	2° Hor. Dist.	2° Diff. Elev.	3° Hor. Dist.	3° Diff. Elev.
0	100.00	0.00	99.97	1.74	99.88	3.49	99.73	5.23
2	"	0.06	"	.80	.87	.55	.72	.28
4	"	0.12	"	.86	"	.60	.71	.34
6	"	0.17	.96	.92	"	.66	"	.40
8	"	0.23	"	.98	.86	.72	.70	.46
10	"	0.29	"	2.04	"	.78	.69	.52
12	"	0.35	"	.09	.85	.84	"	.57
14	"	0.41	.95	.15	"	.90	.68	.63
16	"	0.47	"	.21	.84	.95	"	.69
18	"	0.52	"	.27	"	4.01	.67	.75
20	"	0.58	"	.33	.83	.07	.66	.80
22	"	0.64	.94	.38	"	.13	"	.86
24	"	0.70	"	.44	.82	.18	.65	.92
26	99.99	0.76	"	.50	"	.24	.64	.98
28	"	0.81	.93	.56	.81	.30	.63	6.04
30	"	0.87	"	.62	"	.36	"	.09
32	"	0.93	"	.67	.80	.42	.62	.15
34	"	0.99	"	.73	"	.48	"	.21
36	"	1.05	.92	.79	.79	.53	.61	.27
38	"	1.11	"	.85	"	.59	.60	.33
40	"	1.16	"	.91	.78	.65	.59	.38
42	"	1.22	.91	.97	"	.71	"	.44
44	99.98	1.28	"	3.02	.77	.76	.58	.50
46	"	1.34	.90	.08	"	.82	.57	.56
48	"	1.40	"	.14	.76	.88	.56	.61
50	"	1.45	"	.20	"	.94	"	.67
52	"	1.51	.89	.26	.75	.99	.55	.73
54	"	1.57	"	.31	.74	5.05	.54	.78
56	99.97	1.63	"	.37	"	.11	.53	.84
58	"	1.69	.88	.43	.73	.17	.52	.90
60	"	1.74	"	.49	"	.23	.51	.96
c=0.75	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05
c=1.00	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06
c=1.25	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08

TABLE V.—*Continued.* Horizontal Distances and Elevations for inclined stadia readings.

Minutes.	4° Hor. Dist.	4° Diff. Elev.	5° Hor. Dist.	5° Diff. Elev.	6° Hor. Dist.	6° Diff. Elev.	7° Hor. Dist.	7° Diff. Elev.
0	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2	"	7.02	.23	.74	.90	.45	.50	.15
4	.50	.07	.22	.80	.88	.51	.48	.21
6	.49	.13	.21	.85	.87	.57	.47	.26
8	.48	.19	.20	.91	.86	.62	.46	.32
10	.47	.25	.19	.97	.85	.68	.44	.38
12	.46	.30	.18	9.03	.83	.74	.43	.43
14	"	.36	.17	.08	.82	.79	.41	.49
16	.45	.42	.16	.14	.81	.85	.40	.55
18	.44	.48	.15	.20	.80	.91	.39	.60
20	.43	.53	.14	.25	.78	.96	.37	.66
22	.42	.59	.13	.31	.77	11.02	.36	.72
24	.41	.65	.11	.37	.76	.08	.34	.77
26	.40	.71	.10	.43	.74	.13	.33	.83
28	.39	.76	.09	.48	.73	.19	.31	.88
30	.38	.82	.08	.54	.72	.25	.29	.94
32	"	.88	.07	.60	.71	.30	.28	13.00
34	.37	.94	.06	.65	.69	.36	.27	.05
36	.39	.99	.05	.71	.68	.42	.25	.11
38	85	8.05	.04	.77	.67	.47	.24	.17
40	.34	.11	.03	.83	.65	.53	.22	.22
42	.33	.17	.01	.88	.64	.59	.20	.28
44	.32	.22	.00	.94	.63	.64	.19	.33
46	.31	.28	98.99	10 00	.61	.70	.17	.39
48	.30	.34	.98	.05	.60	.76	.16	.45
50	.29	.40	.97	.11	.58	.81	.14	.50
52	.28	.45	.96	.17	.57	.87	.13	.56
54	.27	.51	.94	.22	.56	.93	.11	.61
56	.26	.57	.93	.28	.54	.98	.10	.67
58	.25	.63	.92	.34	53	12.04	.08	.73
60	.24	.68	.91	.40	51	.10	.06	.78
c=0.75	0.75	0.06	0.75	0 07	0.75	0.08	0.74	0.10
c=1.00	1.00	0.08	0.99	0 09	0.99	0.11	0.99	0.13
c=1.25	1.25	0 10	1.24	0.11	1.24	0.14	1.24	0.16

TABLE V.—*Continued.* Horizontal Distances and Elevations for inclined stadia readings.

Minutes.	8° Hor. Dist.	8° Diff. Elev.	9° Hor. Dist.	9° Diff. Elev.	10° Hor. Dist.	10° Diff. Elev.	11° Hor. Dist.	11° Diff. Elev.
0	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2	.05	.84	.53	.51	.96	.16	.34	.78
4	.03	.89	.52	.56	.94	21	.32	.84
6	.01	.95	.50	.62	.92		.29	.89
8	.00	14.01	.48	.67	.90	.32	.27	.95
10	97.98	.06	.46	.73	.88	.37	.25	19.00
12	.97	.12	.44	.78	.86	.43	.23	.05
14	.95	.17	.43	.84	.84	.48	.21	.11
16	.93	.23	.41	.89	.82	.54	.18	.16
18	.92	.28	.39	.95	.80	.59	.16	.21
20	.90	.34	.37	16.00	.78	.65	.14	.27
22	.88	.40	.35	.06	.76	.70	.12	.32
24	.87	.45	.33	.11	.74	76	.09	.38
26	.85	.51	.31	.17	.72	.81	.07	.43
28	.83	.56	.29	.22	.70	.86	.05	.48
30	.82	.62	.28	.28	.68	.92	.03	.54
32	.80	.67	.26	.33	.66	.97	96.00	.59
34	.78	.73	.24	.39	.64	18.03	95.98	.64
36	.76	.79	.22	.44	.62	.08	.96	.70
38	.75	.84	.20	.50	.60	.14	.93	.75
40	.73	.90	.18	.55	.57	.19	.91	.80
42	.71	.95	.16	.61	.55	.24	.89	.86
44	.69	15.01	.14	.66	.53	.30	.86	.91
46	.68	.06	.12	.72	.51	.35	.84	.96
48	.66	.12	.10	.77	.49	.41	.82	20.02
50	.64	.17	.08	.83	.47	.46	.79	.07
52	.62	.23	.06	.88	.45	.51	.77	.12
54	.61	.28	.04	.94	.42	.57	.75	.18
56	.59	.34	.02	.99	.40	.62	.72	.23
58	.57	.40	.00	17.05	.38	.68	.70	.28
60	.55	.45	96.98	17.10	.36	.73	.68	.34
c=0.75	0.74	0.11	0.74	0.12	0.74	0.14	0.73	0.15
c=1.00	0.99	0.15	0.99	0.16	0.98	0.18	0.98	0.20
c=1.25	1.23	0.18	1.23	0.21	1.23	0.23	1.22	0.25

TABLE V.—*Continued.* Horizontal Distances and Elevations for inclined stadia readings.

Minutes.	12°		13°		14°		15°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25 00
2	.65	.39	.91	.97	.12	52	.27	.05
4	.63	.44	.89	22.02	.09	.58	.24	.10
6	.61	.50	.86	.08	.07	.63	.21	.15
8	.58	.55	.84	.13	.04	.68	.18	.20
10	.56	60	.81	.18	.01	.73	.16	.25
12	.53	.66	.79	.23	93.98	.78	.13	.30
14	.51	.71	.76	.28	.95	.83	.10	.35
16	.49	.76	.73	.34	.93	.88	.07	.40
18	.46	.81	.71	.39	.90	.93	.04	.45
20	.44	.87	.68	.44	.87	.90	.01	.50
22	.41	.92	.66	.49	.84	24.04	92 98	.55
24	.39	.97	.63	.54	.81	.09	95	.60
26	.36	21.03	.60	.60	.79	.14	.92	.65
28	.34	.08	.58	.65	.76	.19	.89	.70
30	.32	13	.55	.70	.73	.24	.86	.75
32	.29	.18	.52	.75	.70	.29	.83	.80
34	.27	.24	.50	.80	.67	.34	.80	.85
36	.24	.29	.47	.85	.65	.39	.77	.90
38	.22	.34	.44	.91	.62	.44	.74	.95
40	.19	.39	.42	.96	.59	.49	.71	26.00
42	.17	.45	.39	23.01	.56	.55	.68	.05
44	.14	.50	.36	.06	.53	.60	.65	.10
46	.12	.55	.34	.11	.50	.65	.62	.15
48	.09	.60	31	.16	.47	.70	.59	.20
50	.07	.66	.28	.22	.45	.75	.56	.25
52	.04	.71	.26	.27	.42	.80	.53	.30
54	.02	.76	.23	.32	.39	.85	.49	.35
56	94.99	.81	.20	.37	.36	.90	.46	.40
58	.97	.87	.17	.42	.33	95	.43	.45
60	.94	.92	.15	.47	.30	25.00	.40	.50
c=0.75	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0 20
c=1.00	0 98	0 22	0.97	0 23	0 97	0.25	0.96	0.27
c=1.25	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34

TABLE VI.—A Table of Mean Refractions in Declination.

To apply on the declination arc of Solar Attachment of either Compasses or Transits.

HOUR ANGLE.	DECLINATIONS.								
	FOR LATITUDE 25°.								
	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
0 Hour.	05"	10"	15"	21"	27"	33"	40"	48"	57"
2	08	14	19	25	31	38	46	54	1'05
3	12	18	24	30	37	44	58	1'04	1 18
4	23	29	35	45	58	1'03	1'16	1 31	1 52
5	49	59	1'10	1'24	1'52	2 07	2 44	3 46	5 43
	FOR LATITUDE 27° 30'.								
0 Hour.	08"	13"	18"	24"	30"	36"	44"	52"	1'02"
2	11	16	22	28	34	41	49	1'00	1 10
3	17	22	28	35	42	50	1'00	1 11	1 26
4	28	35	42	50	1'00	1'11	1 26	1 43	2 09
5	54	1'05	1'18	1'34	1 54	2 24	3 11	4 38	8 15
	FOR LATITUDE 30°.								
0 Hour.	10"	15"	21"	27"	33"	40"	48"	57"	1'08"
2	14	19	25	31	38	46	54	1'05	1 18
3	20	26	32	39	47	55	1'06	1 19	1 36
4	32	39	46	52	1'06	1'19	1 35	1 57	2 29
5	1'00	1'10	1'24	1'52	2 07	2 44	3 46	5 43	18 06
	FOR LATITUDE 32° 30'.								
0 Hour.	13"	18"	24"	30"	36"	44"	52"	1'02"	1'14"
2	17	22	28	35	42	50	1'00	1 11	1 26
3	23	29	35	43	51	1'01	1 13	1 28	1 47
4	35	43	51	1'01	1'13	1 27	1 46	2 13	2 54
5	1'03	1'15	1'31	1 53	2 20	3 05	4 25	7 36	
	FOR LATITUDE 35°.								
0 Hour.	15"	21"	27"	33"	40"	48"	57"	1'08"	1'21"
2	20	25	32	38	46	55	1'05	1 18	1 35
3	26	33	39	47	56	1'07	1 21	1 38	2 00
4	39	47	56	1'07	1'20	1 36	1 59	2 32	3 25
5	1'07	1'20	1'38	2 00	2 34	3 29	5 14	10 16	
	FOR LATITUDE 37° 30'.								
0 Hour.	18"	24"	30"	36"	44"	52"	1'02"	1'14"	1'29"
2	22	28	35	42	50	1'00	1 12	1 26	1 45
3	29	36	43	52	1'02	1 14	1 29	1 49	2 16
4	43	51	1'01	1'13	1 27	1 49	2 14	2 54	4 05
5	1'11	1'26	1 44	2 10	2 49	3 55	6 15	14 58	

TABLE VI.—*Continued.* **A Table of Mean Refractions in Declination.**

HOUR ANGLE.	DECLINATIONS.								
	FOR LATITUDE 40°.								
	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
0 Hour.	21"	27"	33"	40"	48"	57"	1'08"	1'21"	1'35"
2	25	32	39	46	52	1'06	1 19	1 35	1 57
3	33	40	48	57	1'08	1 21	1 38	2 02	2 36
4	47	55	1'06	1'19	1 36	1 58	2 30	3 21	4 59
5	1'15	1'31	1 51	2 20	3 05	4 25	7 34	25 18	
	FOR LATITUDE 42° 30'.								
0 Hour.	24"	30"	36"	44"	52"	1'02"	1'14"	1'29"	1'49"
2	28	35	39	50	1'00	1 12	1 26	1 45	2 11
3	36	43	52	1'02	1 13	1 29	1 49	2 17	2 59
4	50	1'00	1'11	1 26	1 44	2 10	2 49	3 55	6 16
5	1'16	1 36	1 58	2 30	3 22	5 00	9 24		
	FOR LATITUDE 45°.								
0 Hour.	27"	33"	40"	48"	57"	1'08"	1'21"	1'39"	2'02"
2	32	39	46	52	1'06	1 19	1 35	1 57	2 29
3	40	47	56	1'07	1 21	1 38	2 00	2 34	3 29
4	54	1'04	1'16	1 33	1 54	2 24	3 11	4 38	8 15
5	1'23	1 41	2 05	2 41	3 40	5 40	12 02		
	FOR LATITUDE 47° 30'.								
0 Hour.	30"	36"	44"	52"	1'02"	1'14"	1'29"	1'49"	2'18"
2	35	42	50	1'00	1 12	1 26	1 45	2 01	2 51
3	43	51	1'01	1 13	1 28	1 47	2 15	2 56	4 08
4	56	1'09	1 23	1 40	2 05	2 40	3 39	5 37	11 18
5	1'27	1 46	2 12	2 52	4 01	6 30	16 19		
	FOR LATITUDE 50°.								
0 Hour.	33"	40"	48"	57"	1'08"	1'21"	1'39"	2'02"	2'36"
2	38	46	55	1'06	1 18	1 35	1 57	2 28	3 19
3	47	56	1'06	1 19	1 36	2 29	2 31	3 23	5 02
4	1'02	1'14	1 29	1 48	2 16	2 58	4 18	6 59	19 47
5	1 30	1 51	2 19	3 04	4 22	7 28	24 10		

Explanation of the Table of Refractions.—The table is calculated for latitudes between 25° and 50° at intervals of 2½°, that being as near as is required.

The declination ranges from 0 to 20° both north and south, the + declinations being north, and — south, and is given for every five degrees, that being sufficiently near for all practical purposes.

The hour angle in the first column indicates the distance of the sun from the meridian in hours, the refraction given for 0 hours being that which affects the observed declination of the sun when on the meridian, commonly known as meridional refraction; the refraction for the hours just before and after noon is so nearly that of the meridian that it may be called and allowed as the same.

When the table is used, it must be borne in mind that when the declination is north or + in the table, the refraction is to be added; when the declination is south or —, the refraction must be subtracted.

It will be noticed that the refraction in south or — declination increases very rapidly as the sun nears the horizon, showing that observations should not be taken with the sun, when south of the equator, less than one hour from the horizon.

TABLE VII.—Pole Distances of Polaris.

1895.	1896.	1897.	1898.	1899.	1900.
1° 15′ 08″	1° 14′ 49″	1° 14′ 30″	1° 14′ 11″	1° 13′ 71″	1° 13′ 33″

TABLE VIII.—Azimuths of the Tangents, and Offsets in feet, to the Parallel.

Latitude.	1 MILE.		2 MILES.		3 MILES.		4 MILES.	
	Azimuth.	Offset.	Azimuth.	Offset.	Azimuth.	Offset.	Azimuth.	Offset.
30°	89°59′30″	0.39	89°59′00″	1.54	89°58′30″	3.47	89°58′00″	6.17
35	24	0.47	58 47	1.87	11	4.20	57 34	7.47
40	16	0.56	33	2.24	57 49	5.03	06	8.95
45	08	0.67	16	2.66	24	5.99	56 32	10.65
50	58 58	0.79	57 56	3.17	56 54	7.13	55 53	12.68
	5 MILES.		6 MILES.		7 MILES.		8 MILES.	
	Azimuth.	Offset.	Azimuth.	Offset.	Azimuth.	Offset.	Azimuth.	Offset.
30°	89°57′30″	9.64	89°57′00″	13.88	89°56′30″	18.89	89°56′00″	24.67
35	56 57	11.68	56 22	16.81	55 45	22.89	55 09	29 89
40	22	13.98	55 38	20.11	54 55	27.40	54 11	35 78
45	55 40	16 64	54 48	23 96	53 56	32 61	43 04	42 59
50	54 51	19.80	53 49	28.52	52 47	38 82	51 45	50 70
	9 MILES.		10 MILES.		11 MILES.		12 MILES.	
	Azimuth.	Offset.	Azimuth.	Offset.	Azimuth.	Offset.	Azimuth.	Offset.
30°	89°55′30″	31 23	89°55′00″	38 55	89°54′30″	46 65	89°54′00″	55.52
35	54 32	37 83	53 56	46.71	53 20	56 62	52 43	67.26
40	53 28	45.29	52 44	55.91	52 00	67.65	51 17	80.51
45	52 12	53 91	51 20	66 55	50 28	80 53	49 36	95.84
50	50 43	64 17	49 41	79 22	48 39	95.86	47 37	114.08

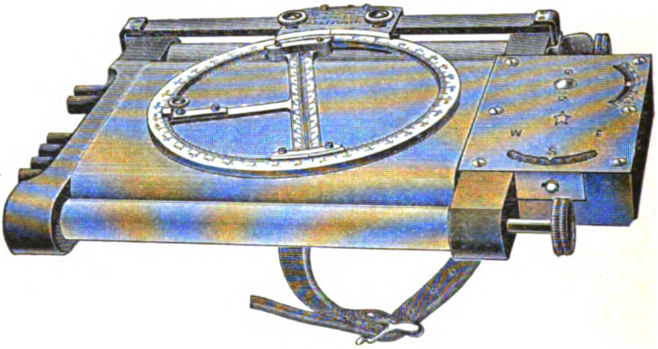
Interpolate for offsets for other latitudes. For offsets for half-mile points, take one-fourth the offset for a point twice the distance of the half-mile point from the tangential point.

Thus, the offset for 3½ miles=¼ the offset for 7 miles.

INDEX.

The Batson Sketching-Case.

(IMPROVED PATTERN, MARCH, 1902.)



The engraving shows the Batson Sketching-Case designed for the use of Civil and Military Engineers and Surveyors in reconnaissance and topographical surveys. It was given an extensive and successful trial in 1898 and 1899 in Cuba and the Philippines as well as in the United States.

This instrument is a small drawing-board having upon its upper surface a movable graduated circle, carrying a small alidade with scales, and at one end of the board a compass, and a clinometer.

The Drawing-board is made of wood, and is provided with rollers which carry the paper for recording observations. Friction-brakes hold the rollers so that the paper is held down snugly to the board and prevented from uncoiling.

Six holes at the end of the board opposite the compass afford receptacles for the colored pencils used in topographical sketching.

The protractor is held in position by a carrier which slides upon a bar attached to the wooden end-pieces shown in the cut. The construction of the carrier allows the protractor to be turned or, if desired, to be clamped by means of two set-screws, as shown.

The protractor can also be lifted to an upright position by pulling back the spring-catch at the end of the carrier-bar.

The alidade turns within the graduated circle, and, with it, forms the protractor.

The paper for use with this instrument is six inches wide, and thirty to forty inches is found to be a convenient length.

The sketching case is fitted with a strap for carrying on the forearm, and, if desired, is provided with a short, light staff, or a plain tripod, for use in taking bearings on reference points and on objects which it is desirable to locate with more accuracy than can be done by holding the case in the hand.

A sole leather case, having a pocket for the instrument and another for sketches and extra paper, and fitted with lock and shoulder-strap, is provided with each instrument.