NEWNES

SLIDE-RULE MANUAL

F. J. CAMM

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NEWNES' SLIDE RULE MANUAL

BY

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PREFACE

THE slide rule is considered by many a mysterious instrument requiring years of study before it can be used with facility. I hope this book will show that it is a simple instrument which can be mastered in a few evenings of study, and which will save hours of laborious calculation. Almost every one engaged in engineering and the scientific professions needs to use a slide rule, and that is why its popularity has so greatly increased during the past twenty years. A knowledge of logarithms is, of course, necessary, since the graduations on the slide rule are themselves the logarithms of numbers. I show how extremely accurate results can be obtained by a little adjustment of the basic figures.

It is generally believed that the slide rule does not give results accurate beyond two places of decimals, but by a little care results accurate to three and four places can be obtained.

My thanks are due to Messrs. W. & F. Stanley & Co. Ltd., Fowler & Co., and J. Halden & Co. Ltd. for providing information relating to their special rules, and to W. A. Tuplin, D.Sc., for specially worked examples.

This volume has been prepared at the request of purchasers of my companion works, *Refresher Course in Mathematics*, *Mathematical Tables and Formulæ* and *Newnes' Metric and Decimal Tables.*

This third edition has been fully revised.

F. J. CAMM.

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CHAPTER I

Principle of the Slide Rule

We know that, by means of logarithms, the processes of multiplication and division are effected by addition or subtraction of the logarithms of the numbers to be multiplied or divided. By means of the sliding scale of the slide rule we are enabled to calculate in this way.

To illustrate the principle, take two ordinary rules calibrated in inches, as Fig. 1, equally graduated as shown. We can add

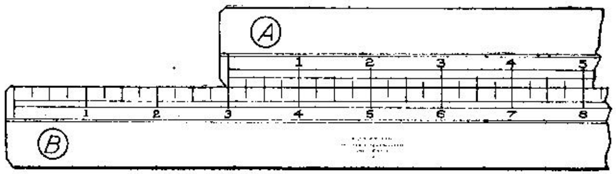


Fig. 1.—Two ordinary measuring rules used to illustrate the principle of the slide rule.

two dimensions together merely by sliding the rule along. Suppose we wish to add 1 inch to 3 inches. Slide scale A along until its left-hand end (o) coincides with 3 on Scale A. Opposite 1 on A read the answer, 4, on B. If we wish to add 1 and 5, slide zero on A to coincide with 5 on B, and opposite 1 on A read 6 on B, and so on. Reversing the process we can subtract the two numbers.

Now, a slide rule is engraved with the logarithms of numbers, and thus the spaces are not equally divided as they are on a rule; but by using the slides as in the example given we still perform the process of addition, and as the scales are logarithmic the answer will be the logarithm of the product, not the sum. Similarly, subtracting the two quantities would give the logarithm of the quotient, not the difference. Subtraction on the slide rule is equivalent to the process of division.

Thus, with the slide rule problems may be solved at sight.

The slide rule is particularly useful for extracting square roots, cube roots, etc. It may be used for multiplication, division, proportion, trigonometry, and so on.

Construction of the Slide Rule.—The average slide rule is 10 in. long, and it has four scales numbered respectively A, B, C, and D. It will be noted that A and D are on the top and bottom of the rule, while B and C are on the top and bottom of the slide. When the slide is full in, the divisions on C and D and those on A and B coincide. The graduation of the divisions of each of the scales is logarithmic; in other words, the distance from the starting-point of the scale to any other point on the scale represents the logarithm of that number. (See Fig. 2.)

It is important to remember that the A and B scales are divided into basic or primary divisions which represent the logarithm of numbers 1 to 100. Each of these divisions is sub-divided into ten secondary divisions, and from the left-hand side of the scales each of the secondary divisions is further sub-divided. It will be observed that as the divisions progress towards the right-hand side of the slide rule the primary divisions approach one another, so it is not possible to carry out extreme sub-divisions throughout the scale.

Similarly, the C and D scales are divided or graduated in conformity with the A and B scales with the exception that there are only ten primary divisions spaced over the same length of scale as the 100 on the A and B scale. The purpose of this is to permit a much extended sub-division providing greater possibility of accuracy.

It is important to note that the relationship between the two sets of scales is such that the figures on the A and B scales are the squares of those on the C and D scales. It therefore follows that the numerals on the C and D scales are the square roots of those on the A and B scales.

Nearly all slide rules are supplied with a sliding part known as the cursor. This usually consists of a piece of glass in a metal frame which slides along parallel to the scale, and having a hair-line engraved on its centre so that readings from one scale can be transferred to another scale. Some cursors, such as the Goulding, are lenses and magnify.

Multiplication.—Multiplication on the slide rule is carried out in the following way. It must be remembered that although the graduations extend from 1 to 100, larger numbers necessi-

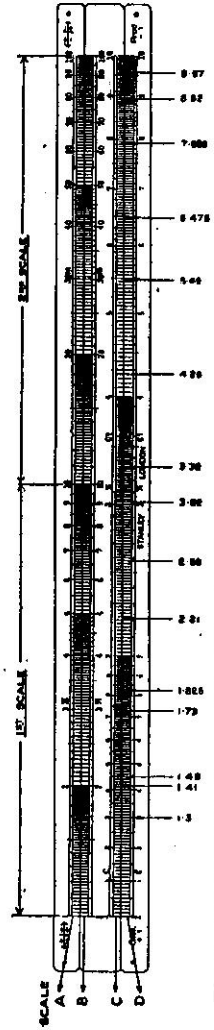


Fig. 2.—An exercise in the use of the slide rule. Set the cursor to the following points on the C and D scale: 1·3, 1·41, 1·45, 1·73, 1·825, 2·21, 2·58, 3·02, 3·32, 4·25, 5·455, 6·475, 7·925, 8·92, 9·57.

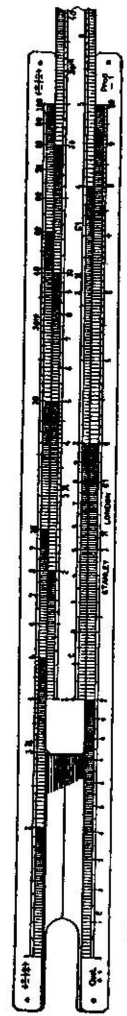


Fig. 3.—An example in division. Set 2 on C to 4 on D, and read 2 under left-hand index on D.

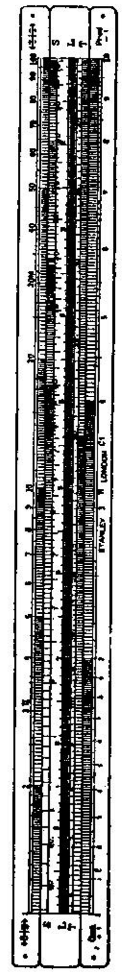


Fig. 4.—The slide reversed for sine and tangent calculations. The sine is read off on A over the given angle on S.

tate that each of the digits on the scale should be assigned a value proportionately greater, thus 1 can stand for 10, 100 for 1000, and so on, but once this value has been assigned, all of the other values on the scale must be proportionately increased for that particular calculation.

The index on the left hand of the slide is adjusted to coincide with the multiplicand on the rule, and opposite the multiplier on the slide will be read the product on the rule. This really amounts to adding the length equivalent to the log of the first number to the length corresponding to the log of the second number, and if there are more than two factors to be multiplied the result of the first is not read off, but the cursor is set to it and the slide index brought to the cursor setting, when the product can be read. This process continues irrespective of the number of factors. It is apparent that continuous progression of the slide from left to right would mean that the reading would eventually be beyond the limits of the scale, therefore the right-hand index may be used to set to the cursor, thus bringing the product within the compass of the scale. In fixing the position of the decimal point the usual rule regarding the number of digits applies; briefly, however, it should be remembered that the sum of digits in the factors, minus the number of times the slide has been projected to the right, indicates the number of digits in the product.

Division.—The process of division is carried out by subtracting the logs of the numbers, and it is therefore the opposite of multiplication. In division, therefore, the divisor set on the slide is brought to the dividend on the rule, and opposite the index on the slide will be the quotient on the rule. The total number of digits is found by adding the number of times the slide is projected to the right to the difference between the divisor and the dividend.

Proportion.—In proportion calculations bring the first term on the slide to the second on the rule and move the cursor to the third on the slide, reading the fourth on the rule.

Squares and Square Roots.—Squares and square roots are found by setting the cursor and reading from D to A or *vice versa*.

Cubes and Cube Roots.—Cubes and cube roots as well as higher powers and roots can be found by the use of tables of logarithms and antilogarithms.

Logarithms.—Logarithms can, however, be read on the back of most slide rules by setting the C index to the number whose log is required on scale D and multiplying the log by the power involved (or dividing, as the case may be). On some slide rules the D scale is superseded by a log/log scale, which means to say that the scale is a logarithm of a logarithm. This enables very high powers and roots easily to be obtained; thus to find the *n*th power set index on C to the number on D and against *n* on C read the answer on D. To find the *n*th root set the root index on C to the number on D and against the index on C read the answer on D.

Sines, Tangents, etc.—Sines and tangents are found by setting the slide in the same way as for finding the logarithm of a number. The values will be found marked on the back of the slide with the log values.

Examples in the Use of the Slide Rule.—The illustrations on p. 11 (Figs. 2 to 4) show one of the slide rules supplied by Messrs. W. F. Stanley & Co. Ltd., who supply slide rules of all types and for special purposes.

All numbers must, of course, be decimalised. The scales being arbitrary, the decimal point is ignored during calculation and can quite easily be located after the calculation is finished, either mentally or by means of a rough calculation.

10×20 could not be anything else but 200; in the same way, $1{\cdot}4 \times 1{\cdot}2$ is 1·68, and not ·168 or 168.

When setting the rule, the positions of 0·2, 2·0, 20, 200, are all the same. Thus it will be seen that figures of any magnitude can be handled by means of a slide rule.

It will be noticed that from 1 to 2 on the A and B scales, the rule is divided into fifty parts, *i.e.* it reads to 0·02, whereas from 9 to 10 it is divided into ten parts only. This is purely a matter of space. (Each portion of the rule is divided as finely as is consistent with legibility.) Finer readings must be estimated and results will be in accordance with the skill of the operator.

By this time the reader should have it firmly established in his mind that the letters A, B, C, and D indicate the various scales and should not have to stop to think which is which. The right and left indices should be equally familiar.

Having arranged our problem in slide-rule form, viz. as a question of multiplication and division or proportion, and decimalised all factors, first attempts at calculation should be confined to the two lower scales C and D.

Multiplication.—Example 1: Multiply 2×2, 2×3, 2×4, and 2×5.

Set the left index to 2 on D (see Fig. 2), and under 2, 3, 4, and 5 on C, read 3, 6, 8, and 10 on D. (See also Figs. 14 to 16.)

The reason for this is obvious, as owing to the displacement of the slide the logarithm of 2, represented by the distance the slide is moved to the right, is added to all these values which, therefore, are all multiplied by 2.

Example 2: Multiply $2\times2\times2=8$.

Set the left-hand index on C to 3 on D and the cursor to 2 on C, next set the left index to cursor and under 2 on C read 8 on D.

So far we have only used the left index for setting but this will not always be possible, as the slide will be moved so far to the right that the third factor will be off the rule altogether—in such cases the right index is employed.

Example 3: Multiply $3\times2\times3=18$.

Set the index to 3, cursor to 2, set right- or left-hand index to cursor and under 3 on C read 18 on D.

Multiplication may thus be summarised as follows:

$X\times Y\times Z$.

Right or left index (c) to X (d).
Cursor to Y (c).
Answer on (d) under Z (c).

$W\times X\times Y\times Z$.

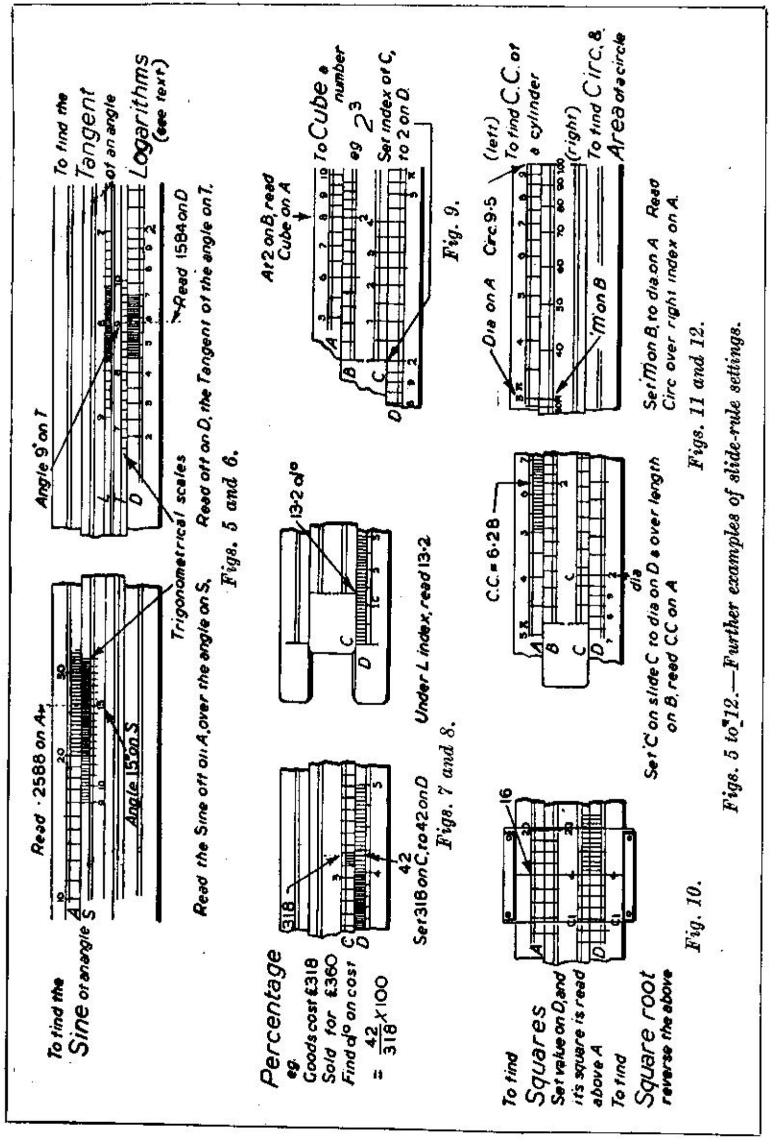
Right on left index (c) to W (d).
Cursor to X (c).
Right or left-hand index to cursor.
Cursor to Y (c).
Left or right-hand index to cursor.
Answer on (d) under Z (c).

It will be obvious that this sequence of operations can be continued for any number of factors, viz.: Index to first factor, Cursor to second, Index to cursor, Cursor to third factor, and so on.

Division.—A reversal of the foregoing method would obviously effect division.

Example 1: Divide 4 by 2.

Move the slide to the left until 2, the divisor, is opposite the left index of D, and under 4 on C you will find 2, under 3, 1·5, under 8, 4, and so on.



Figs. 5 to 12.—Further examples of slide-rule settings.

It is, however, more usual to set the divisor direct to the dividend as it saves a movement of the slide when we come to continuous multiplication and division.

Example 2 : Set 2 on C to 4 on D, and read 2 under left-hand index on D, similarly 3 into 6 goes twice, 4 into 8, etc. etc. (see Fig. 3).

Divide 3·75 by 2·5.

Set 2·5 on C to 3·75 on D, and under the left index of C find 1·5.

To facilitate the operation of setting the slide, set the cursor to one number first and bring the other number to the hair-line.

Combined Multiplication and Division.—$\left(\frac{a \times b \times c}{d \times e}\right)$.

Set *d* on C to *a* on D, set cursor to *b* on C, set *e* on C to cursor, and at *c* on C read answer on D.

Example 1 : $\frac{2 \times 4 \times 6}{3 \times 2}$.

Set 3 on C to 2 on D, set cursor to 4 on C. Set 2 on C to cursor, and at 6 on C read 8 on D.

Example 2 : $\frac{2{\cdot}75 \times 3{\cdot}44 \times 8{\cdot}4}{4{\cdot}3 \times 2{\cdot}8 \times 5{\cdot}5} = 1{\cdot}2$.

Set 4·3 on C to 2·75 on D, set cursor to 3·44 on C, set 2·8 on C to cursor, set cursor to 8·4 on C, set 5·5 to cursor, and under left index on C read 1·2 on D.

It will be obvious that the sequence of operations can be continued for any number of factors.

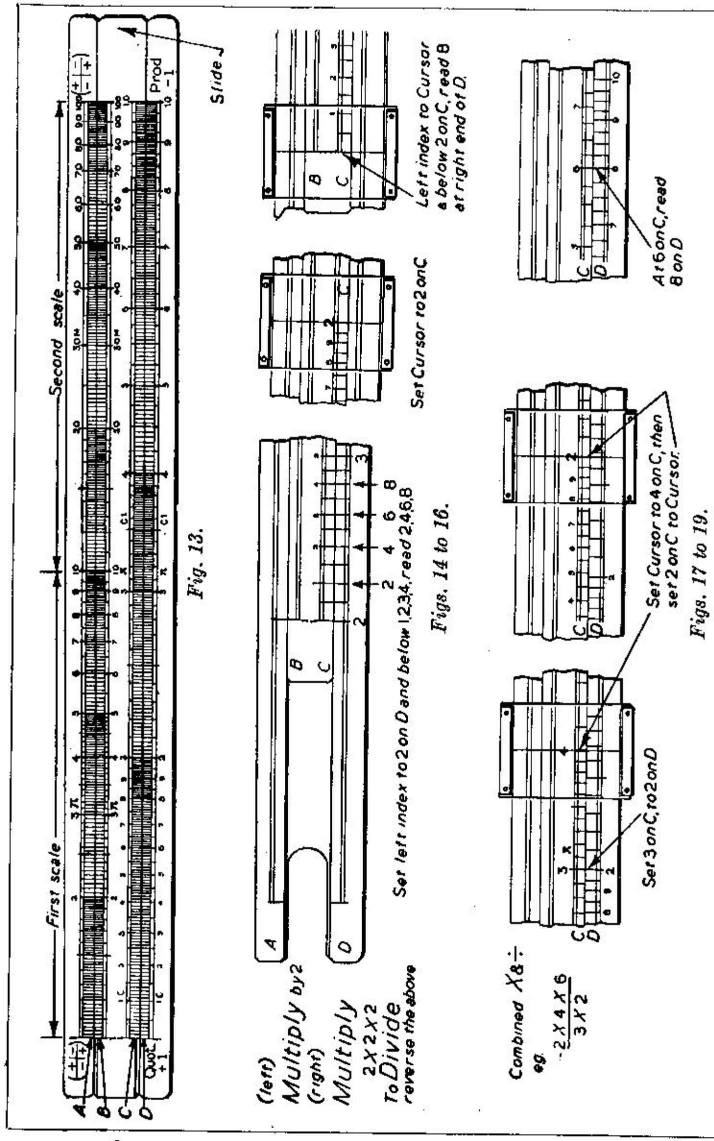
It will be further observed that the multiplication is done with the cursor and the division with the slide, the operations taking place alternately. (See Figs. 17 to 19.)

Ratio and Proportion.—The facility with which all calculations involving ratio and proportion can be done with the slide rule is one of its most valuable features. It is in this field that it makes its strongest appeal to the commercial man.

Under this heading we find :

Percentages.
Rates of exchange.
Insurance brokerage.
Cost and profit calculations, etc. etc.

Percentages.—Problem : What percentage of 840 are the following numbers : 336, 231, 73·5, and 47·25 ?



Figs. 13 to 19.—Examples in the use of the Slide Rule.

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Obviously the solution is to divide each figure in turn by 840 and multiply by 100 by moving the decimal point.

Procedure: Set 840 on C to each number in turn, reading the answer below the R index, viz. 40, 27·5, and 5·625 per cent. respectively.

Percentage of Profit.—If goods be purchased for £318 and sold for £360, what is the percentage of profit on cost and on selling price? Profit on sale, £42.

$$\text{Solution: } \frac{42}{318} \times 100.$$

Procedure: Set 318 on C to 42 on D and under L index read 13·2 per cent. (See Figs. 5 to 12.)

In the same way 360 on C is set to 42 on D and the percentage on selling price, viz. 11·7 per cent. will be found under the L index.

Problem: An article costs £5, what percentage must I add to gain a profit of $33\frac{1}{3}$ per cent. on the turnover?

Procedure: Where x=the percentage on turnover required, the percentage to be added=

$$\left(\frac{100}{100-x}\right).$$

Substituting for $x=\left(\frac{33{\cdot}3}{66{\cdot}6}\right)=50$ per cent.

Fifty per cent. of £5 is found by multiplying 5×5 and shifting the decimal point.

Squares and Square Roots.—To square a number we multiply its logarithm by 2 and the antilogarithm of this product is the required number.

This is done on the slide rule by using the two pairs of scales, AB and CD, in conjunction with one another.

As the two top scales A and B (1st and 2nd) are each half the length of the bottom pair of scales, C and D, any setting of the index on the C or D scale will be automatically squared on the A or B scale, as the index moves twice the distance proportionately (see Fig. 2).

Example: Immediately over 1·41 on D-C read 2 on A, over 1·73 on D-C read 3 on A, over 2 on D-C read 4 on A, and so on.

Thus we get the following rule:

If the cursor is set to any value on D, its square will be found directly above A. Conversely, if the cursor is set to any value on A, its square root will be found directly below on D.

To find the number of digits in the square of a number:

If the square is read off on the first half of scale A it will contain twice the number of digits in the original number, minus 1; if read off on the second half of scale A, there will be twice the number of digits in the original number.

Square Roots.—Where the number contains an odd number of digits the first half of scale A is used. For even numbers of digits use the second half of scale A. (See also Fig. 10.)

Cube and Cube Roots.—To cube a number, set either index of C as required to number on D, and at number on left half of scale B read cube on A.

Cube Roots.—Point off the number into groups of three, starting from the units place. If the first or left-hand group contains one figure, set the cursor to the number on the first half of scale A and adjust the slide so that reading at cursor on first half of scale B is equal to reading on D under left-hand index of C. This reading is the cube root.

If the first group contains two figures, set cursor to number on second half of scale A and proceed as before.

If the first group contains three figures, set cursor to number on first half of scale A and set slide so that reading at cursor on B is equal to reading on D under right-hand index of C.

Note.—The left-hand half of scale B must always be used for Cubes and Cube Roots.

To determine the number of digits in the cube of a number. If the first half of scale A is used with the slide projecting to the right, there will be three times the number of digits in the cube, less 2 ($3N-2$). If the second half of scale A is used with the slide projecting to the right there will be three times the number of digits in the cube, less 1 ($3N-1$).

If the first half of scale A is used with the slide projecting to the left, there will be three times the number of digits in the cube ($3N$).

To find the number of digits in the cube root of a number, point off the number into groups of three, starting from the decimal point if the number is a decimal. Then there will be one digit in the root for every section so pointed off. If the first group consists of one or two figures, it must still be counted as a complete section. If the number is a decimal there will be one 0 for every complete group of three 0's immediately following the decimal point. If required, 0's must be added to make up a complete section of the three figures. Thus 0·2=0·200, 0·0003=0·000300. This is to determine which scale is to be used, as explained above.

Logarithms.—On the reverse side of the slide will be found an evenly divided scale. The graduations are the mantissæ of the numbers on the C and D scales.

To find the log of a number set left-hand index of C to a number on D, and opposite the index line on the back of the body will be found the mantissa of number required.

The characteristic is found in the usual manner.

Sines and Tangents.—The upper graduations on the reverse side of the slide are the logs of the sines of angles, the lower graduations being the logs of the tangents of angles.

To find the sine of an angle, reverse slide so that the trigonometrical scales are uppermost and the end divisions on the slide coincide with the end divisions on the body (see Fig. 4). Then the sine is read off on A over the given angle on S.

Results on the second half of scale A are prefixed by a decimal point only, results on first half of scale A being preceded by an 0.

To find the tangent of an angle, reverse slide as explained above. Then read off on D the tangent of the given angle on T; the result found being wholly decimal. For tangents of angles of less than 5 deg. 43 min. use the scale of sines prefixing a cypher to the result.

For tangents of angles above 45 deg. use the formula:

$$\text{Tan. } A = \frac{1}{\text{Tan. } (90^\circ - A)}$$

The sign $\left\langle \begin{array}{c|c} + & - \\ \hline - & + \end{array} \right\rangle$ found at the ends of the A scale is a reminder of the method used in ascertaining the number of figures in a result.

π is given on both A-B and C-D scales.

The gauge marks c and c1 are used in calculating the contents of cylinders, viz. set c on C to diameter on D, and over length on B read cubic contents on A. c1 is used when the slide has to be pulled out more than half way.

The gauge point M is used in finding the circumference and area of the curved surface of a cylinder, viz. set M on B to diameter on A, the circumference being read over 1 on B, and the area of the curved surfaces over the length on B.

CHAPTER II

The Principle of Logarithms

As, in order to understand the use of slide rules correctly, a knowledge of logarithms is necessary, in this chapter I deal with the general principles.

By means of logarithms (logs. for short) we are able to carry out the operations of multiplication, division, involution, and evolution (not, be it noted, of addition or subtraction). A knowledge of logarithms is quite essential before the slide rule can be mastered. The slide rule itself considerably augments the rapid calculation which logarithms provide. I shall deal here first with common logarithms leaving Naperian logarithms or Hyperbolic logarithms until later. Common logarithms are those calculated to base 10.

In logarithms the processes of multiplication and division are converted into those of addition and subtraction. Thus, to multiply numbers together, their logarithms are added, and to divide them their logarithms are subtracted. Let us first define what a logarithm is, and then obtain an idea of how logarithms are applied.

The logarithm of a number, to a given base, is the index of the power to which the base must be raised to produce the aforesaid number. Here is a table of the number 3 raised to various powers:

$3^1=3$	$3^7=2187$
$3^2=9$	$3^8=6561$
$3^3=27$	$3^9=19683$
$3^4=81$	$3^{10}=59049$
$3^5=243$	$3^{11}=177147$
$3^6=729$	$3^{12}=531441$

Suppose we wish to multiply 27 by 19683. We can proceed by the lengthy way, as follows:

$$\begin{array}{r} 19683 \\ 27 \\ \hline 137781 \\ 393660 \\ \hline 531441 \end{array}$$

By means of the table of logarithms, however, we look up the logarithm of the two numbers, add the two logarithms together, and then consult the table to ascertain which number corresponds to the logarithm so obtained. Thus, from the table we see that—

$$27=3^3, \text{ and } 19683=3^9$$

The two logarithms are therefore 3 and 9 which, added together, equal 12.

Consulting the table we see that $3^{12}=531{,}441$.

Observe that we have not multiplied the quantities to obtain the answer.

The table above is calculated to the base 3. In other words, it contains the index of the power (1, 2, 3, . . . 12) to which the base (3) must be raised to produce the numbers (3 to 531,441).

If we wished to divide two of the numbers in the table we should subtract their logarithms. For example, $19683 \div 27$. By ordinary methods this is found to be 729.

But the logarithm of 27 (from the table) is seen to be 3, and the logarithm of 19683 is 9. Subtracting the logarithms, $9-3=6$; the answer therefore to $19683 \div 27$ is $3^9-3^3=3^{9-3}=3^6$. From the table we see that $3^6=729$, the answer.

These are simple examples of the principle of logarithms.

Common logarithms are, however, calculated to the base 10.

It is obvious that in multiplication and division by logarithms the numbers to be multiplied or divided will never be an exact power of 10. If they were, logarithms would not be needed, for we should merely add noughts to the multiplicand according to the number of digits in the multiplier. Thus, $10{,}000 \times 100 = 1{,}000{,}000$.

So let us make another table of powers of 10.

$1{,}000{,}000=10^6$	$\cdot 1=\frac{1}{10}=\frac{1}{10^1}=10^{-1}$
$100{,}000=10^5$	$\cdot 01=\frac{1}{100}=\frac{1}{10^2}=10^{-2}$
$10{,}000=10^4$	$\cdot 001=\frac{1}{1000}=\frac{1}{10^3}=10^{-3}$
$1{,}000=10^3$	$\cdot 0001=\frac{1}{10{,}000}=\frac{1}{10^4}=10^{-4}$
$100=10^2$	$\cdot 00001=\frac{1}{100{,}000}=\frac{1}{10^5}=10^{-5}$
$10=10^1$	$\cdot 000001=\frac{1}{1{,}000{,}000}=\frac{1}{10^6}=10^{-6}$
$1=10^0$	

The Characteristic and the Mantissa.—It is obvious that any number between, say, 1,000,000 and 100,000 would have an index or characteristic between 5 and 6, and it is the decimal part of the index for which we require tables of logarithms. This decimal part of the index or characteristic is called the mantissa. Thus the logarithm of a number consists of two parts—the index or characteristic, and the mantissa.

Now the characteristic (to base 10) of any number can quite easily be found by observation.

The characteristic of any number greater than unity is always 1 less than the number of digits in the number.

Thus in the table above :

1,000,000 has 7	digits,	and the	characteristic is	6
100,000 has 6	,,	,,	,,	5
10,000 has 5	,,	,,	,,	4
1,000 has 4	,,	,,	,,	3
100 has 3	,,	,,	,,	2
10 has 2	,,	,,	,,	1
1 has 1	,,	,,	,,	0

The characteristic of any number less than unity is greater by 1 than the number of noughts which follow the decimal point.

The index or characteristic of a number less than unity is also negative ; this is indicated by placing the negative sign or bar over the figure, as in the table.

Here are a few examples :

Index	or characteristic	of	547·062	is	2
,,	,,	,,	47·062	,,	1
,,	,,	,,	7·062	,,	0
,,	,,	,,	·7062	,,	$\bar{1}$
,,	,,	,,	·07062	,,	$\bar{2}$
,,	,,	,,	·007062	,,	$\bar{3}$
,,	,,	,,	·0007062	,,	$\bar{4}$

The characteristic of 100 is 2, and of ·01, $\bar{2}$. The characteristic of any number over 100 but less than 1000 will also be 2, plus the mantissa found from a table of logarithms. The characteristic of any decimal from ·1 to ·9 will always be $\bar{1}$, the characteristic of any decimal from ·01 to ·09 will be $\bar{2}$, the characteristic of any decimal from ·001 to ·009 will be $\bar{3}$, and so on.

The numbers, whether whole or decimal, or whether consisting of a whole number and a decimal, are treated as whole numbers for the purpose of extracting the mantissa from the tables.

The characteristic of a number which is a decimal only, and has no whole numbers before the decimal point, is always negative. The characteristic of a number consisting of whole numbers and decimal will always be positive, and decimal part in this latter case is ignored when determining the characteristic by inspection. Thus, in the number 4890·375, we ignore the decimal ·375, and as the whole number contains 4 digits the characteristic will be 3. We must, however, take into account both the whole number and the decimal when finding the mantissa from the tables. We, in fact, proceed as if the decimal point did not exist.

It is important to remember that the mantissa is always positive; and the mantissa of a number, irrespective of whether it is a decimal or contains a whole number and a decimal, contains the same digits. It is only the characteristic which varies, according to the number. For example, the logarithm of 101 is 2·0043; the logarithm of 1·01 is 0·0043, of ·101 it is $\bar{1}$·0043, and so on. The logarithm of all numbers from 1 to 9·9999 . . . will consist of decimals only.

Use of Log. Tables.—Some tables of logarithms are calculated to four places of decimals, and these are sufficiently accurate for most purposes. By using such tables we are only able to extract the logarithms of the first four figures of numbers; hence, if we wish to extract the logarithm of 605·125 we must abbreviate the number by the method now described.

In this case we should extract the logarithm of 605·1.

For example, 3341·2817025 for all practical purposes can be shortened to 3341·282 rejecting the remainder of the decimal. Such a shortened result is said to be approximately correct to three decimal places or to three significant figures. There is a rule concerning this shortening process. *If a rejected or discarded decimal is 5 or over, 1 is added to the next figure to the left.* Thus, in the decimal given above, 7 being greater than 5 is rejected, and the figure 1 to the left of it is increased to 2. Another method of decimal approximation which has been

approved internationally is to make the decimals even. That is to say, in the case of (for example) the number 39·455, we should shorten this to 39·46 (adding 1 for the 5 which is dropped or for any number over 5). In the case of a number such as 39·445 we should merely shorten this to 39·44. Making the decimals even, it is claimed, gives a closer average result.

If the result were required approximately correct to one decimal place, we should shorten 3341·2817025 to 3341·3. If required correct to one, two, three, etc., decimal places, the decimals beyond are merely discarded, thus:

3341·282
3341·28
3341·3

are correct to three, two, and one decimal places respectively.

If the decimal is purely fractional and contains a number of noughts after the decimal point, at least one significant figure must, of course, be left. For example, in the decimal fraction ·0000063192 we may shorten it only to ·000006.

If the number, however, was, say, 605·175 we should (as 7 is more than 5) call it 605·2. Again, if the expression were 605·045 we should write it as 605·0, because ·045 is less than ·05 and is therefore nearer to ·0 than ·1.

Find the logarithm of 126·5. The characteristic is 2, and we look up the first two figures of the number (12) in the first column of the table of logarithms. We next run our eye along that line and read beneath the third figure of the number (6), 1004; continuing, under 5 in what is known as the difference column, we read 17. We add 17 to 1004, making 1021. Therefore, 126 equals $10^{2·1021}$.

Find the logarithm of 146·0.	Answer, 2·1644.
Find the logarithm of 17·53.	Answer, 1·2437.
Find the logarithm of 1·399	Answer, 0·1459.
Find the logarithm of ·1621	Answer, 1·2098.
Find the logarithm of ·01788.	Answer, 2·2524.

The logarithms represent the power to which the base 10 must be raised to produce the number.

This description and these examples are given to indicate how to extract the logarithms of numbers from tables, for the object of carrying out the processes of multiplication, division, involution, and evolution, when we have large and unwieldy quantities with which to deal, or when we have a large number of calculations to make. We have seen that in order to multiply in logarithms, we merely add the logarithms.

I will now give some examples. Multiply 165·3 by 144·6.

The logarithm of 165·3 is found to be 2·2183, and the logarithm of 144·6 is 2·1602. Add these together :

$$\begin{array}{r} 2{\cdot}2183 \\ 2{\cdot}1602 \\ \hline 4{\cdot}3785 \end{array}$$

We now look up the table of antilogarithms (ignoring the characteristic) and we find that ·3785 corresponds to 2391. We now have to fix the position of the decimal point. We have seen that the characteristic is always 1 less than the number of digits. As the characteristic is 4, there will be 5 figures before the decimal point in the answer. Therefore, the answer is 23910·0, which is approximately correct, because if we multiply 165·3 by 144·6 in the ordinary way we shall obtain 23902·38.

Multiply 789·36 by 284·87. As we are using four-figure tables we must contract each of the two quantities to four figures. Thus 789·36 becomes 789·4, and 284·87 becomes 284·9.

The log. of 789·4 is 2·8973
The log. of 284·9 is 2·4547

Adding 5·3520

Antilog. of ·352=2249.

As the characteristic is 5 there will be six places before the decimal point, and the answer is therefore 224,900·0, which is approximately correct.

In the two preceding examples the characteristic is positive. In cases where the characteristic of one quantity is positive and in the other negative, slightly different treatment is necessary. For example : Multiply 37·65 by ·0135.

Log. 37·65=1·5758
and log. ·0135=$\bar{2}$·1303

Adding $\bar{1}$·7061

Here it will be seen that the positive characteristic added to the negative produces $\bar{1}$.

Antilog. ·7061=5083.

As the characteristic is $\bar{1}$ there will be no ciphers before the decimal point.

Answer : ·5083.

Similarly, when adding two negative characteristics. Suppose the two logs. are $\bar{2}$·3064 and $\bar{5}$·0913.

Adding $\bar{2}$·3064
$\bar{5}$·0913

$\bar{7}$·3977

Antilog.=2499.

Answer : ·000000249.

The treatment is different in cases where the addition of the two mantissæ provides a "carry over."

For example, in adding the following two logarithms :

$$\begin{array}{r} 2{\cdot}3164 \\ \bar{3}{\cdot}9112 \\ \hline \text{Adding}\quad {\cdot}2276 \end{array}$$

Here it will be seen that, in adding the 9 and 3 of the mantissa, there is a positive 1 to carry to the left of the decimal point. (We have already seen that the mantissa is always positive.)

Adding this 1 to the positive 2, we obtain 3, which cancels out the negative $\bar{3}$. Another example : Add together log. $\bar{3}$·1654 and $\bar{1}$·9733.

$$\begin{array}{r} \bar{3}{\cdot}1654 \\ \bar{1}{\cdot}9733 \\ \hline \bar{3}{\cdot}1387 \end{array}$$

The two negative characteristics added together equal $\bar{4}$, and as there is a positive 1 carried from the addition of the mantissæ, the characteristic of the addition becomes $\bar{3}$, because $\bar{4}+1=\bar{3}$.

Division by Logarithms.—Division by logarithms is effected by subtracting the logarithm of the divisor from that of the dividend, the result of the subtraction being the logarithm of the quotient.

Example : Divide 37·65 by 19·01.

Log. 37·65=1·5758
Log. 19·01=1·279

Subtracting : 0·2968

Antilog.=1981.

As the characteristic is 0, there will be one digit (0+1) before the decimal point, and the answer is 1·981.

Divide 5·065 by ·0015.

Log. 5·065=0·7046
Log. ·0015=$\bar{3}$·1761

Subtracting : 3·5285

Antilog. of ·5285=3377.

As the characteristic is 3, there will be four figures before the decimal point. Note that the characteristic of the log. being subtracted is changed from negative to positive.

Answer : 3377·0.

In subtraction, change the sign of the characteristic being subtracted and add.

Involution.—Logarithms may be used to perform the functions of involution and evolution. The rule is : *To evaluate the power of a number (as $7{\cdot}5^3$), multiply the logarithm of the number by its index, and the result is the logarithm of the number required.* Consult the table of antilogarithms for the number corresponding.

Example : Find the number which equals $7{\cdot}5^3$.

Log. 7·5=0·8751
Multiply by index 3   3

2·6253

Antilog. ·6253=4220.

Characteristic is 2, therefore there will be three figures before the decimal point.

Answer : 422·0 (actually 421·875).

Evaluate $\cdot0735^5$.

Log. ·0735=$\bar{2}$·8663
Multiply by index 5

$\bar{6}$·3315

Antilog. ·3315=2145.

Characteristic is $\bar{6}$, so there will be five ciphers after the decimal point. Answer, ·000002145.

Remember that the mantissa is always positive, so in multiplying, the carry over from the decimal part will be positive, and must be subtracted if the characteristic is negative. In the above example there was 4 to carry ; $5\times\bar{2}=\overline{10}$, and subtracting the 4 leaves $\bar{6}$.

When the index of a number consists of several figures, and the number is less than unity (the characteristic thus being negative) the whole logarithm must be converted into a negative number before multiplying by the index.

Example : Evaluate $\cdot735^{-3{\cdot}75}$.

Log. ·735=$\bar{1}$·8663
=−1+·8663
=−·1337 (·8663 has been subtracted from 1)

− ·1337
Multiply by index −3·75

6685
9359
4011

·501375

Antilog. ·5014=3173

Answer : 3·173.

Remember : *When negative signs are multiplied together the result is positive ; but when the logarithm is positive and the index negative, the product is negative. In the latter case the mantissa must be made positive before referring to the table of antilogarithms.*

Example : Evaluate $(7{\cdot}5)^{-1{\cdot}5}$.

$$\text{Log. } 7{\cdot}5 = 0{\cdot}8751$$
$$-1{\cdot}5 \times 0{\cdot}8751 = -1{\cdot}31265$$

Here the mantissa is negative, and it is made positive in the following way :

$$-{\cdot}31265 = \bar{1}{\cdot}68735$$
$$\text{Therefore } -1{\cdot}31265 = \bar{2}{\cdot}68735$$
$$\text{Antilog. } {\cdot}6874 = 4868$$

Answer : ·0468.

The expression $(7{\cdot}5)^{-1{\cdot}5}$ can, of course, also be written $\frac{1}{7{\cdot}5^{1{\cdot}5}}$ and by working it out in this form verification of the above result can be obtained.

Evolution.—The root of a number can be obtained by dividing the logarithm of the number by the required root.

Example : Evaluate $\sqrt[3]{28{\cdot}06}$.

$$\text{Log. } 28{\cdot}06 = 1{\cdot}4481$$
$$\text{Divide by the root } 3 = {\cdot}4827$$
$$\text{Antilog. } {\cdot}4827 = 3{\cdot}039$$

Answer : 3·039.

The beginner finds no difficulty in evolution by logs. when characteristic and mantissa are both positive. When the characteristic is negative it must be adjusted so that the logarithm is exactly divisible by the root.

Example : Find the cube root of ·625.

$$\text{Log. } {\cdot}625 = \bar{1}{\cdot}7959.$$

The characteristic is negative and the mantissa positive in this case. Therefore we must add the smallest number to the characteristic to make it divisible by 3. We, hence, add $\bar{2}$ to $\bar{1}$, making $\bar{3}$. To preserve balance we must add 2 to the mantissa ; thus $\bar{1}{\cdot}7959$ now becomes $\bar{3}+2{\cdot}7959$.

Dividing by 3 the expression becomes—

$$\bar{1}{\cdot}93196.$$
$$\text{Antilog. } {\cdot}0932 = 8551.$$

Answer : ·8551.

The adjustment of the characteristic is, of course, performed mentally once the principle is mastered.

It is important to remember that, when dividing a logarithm by a number which is greater than the first figure in the

mantissa, a cipher must be added. For example : Find the seventh root of 4.

$$\text{Log. } 4 = {\cdot}6021$$
$$\tfrac{1}{7}({\cdot}6021) = {\cdot}08601$$
$$\text{Antilog. } {\cdot}08601 = 1219$$

Answer : 1·219.

Here it will be seen that 7 will not divide into 6, so a cipher is added : 7 divided into 60 gives 8, and so on.

Find the 5th root of ·009, or $\cdot 009^{\frac{1}{5}}$.

$$\text{Log. } {\cdot}009 = \bar{3}{\cdot}9542$$
$$\tfrac{1}{5}(\bar{3}{\cdot}9542) = \tfrac{1}{5}(\bar{5}+2{\cdot}9542)$$
$$= \bar{1}{\cdot}5908$$
$$\text{Antilog. } {\cdot}5908 = 3897$$

Answer : ·3897.

The principle of logarithms was discovered by Napier, but Naperian logarithms (sometimes termed hyperbolic or natural logarithms, as distinct from common logarithms calculated to base 10), are calculated to a base which is the sum of the series :

$$1+\frac{1}{2}+\frac{1}{2\times3}+\frac{1}{2\times3\times4}+\frac{1}{2\times3\times4\times5}\ \cdots\cdots$$

The sum of this series to seven places of decimals is 2·7183818, denoted by the symbol e. To convert common into Naperian or hyperbolic logarithms, multiply by 2·3026 or, more accurately, 2·30258509.

To convert Naperian into common logarithms, multiply by ·4343. . . .

To convert logarithms calculated to base a to logarithms of base b proceed algebraically, letting x=log. of number calculated to base a, and y=log. of number calculated to base b.

$$\text{Let N} = \text{the number}$$
$$\text{Then N} = a^x$$
$$\text{and N} = b^y$$
$$\text{Therefore } a^x = b^y$$
$$\text{and } b = a^{\frac{x}{y}}$$
$$\frac{x}{y} = \log.\ a^b$$
$$\frac{y}{x} = \frac{1}{\log.\ a^b}$$

Common logarithms were calculated in this way from Naperian logarithms.

CHAPTER III

USING THE SLIDE RULE

A COMMON failing among slide-rule users, even of some experience, is uncertainty as to the position of the decimal point in the "answer." Quite often the slide rule is used to determine only the figures in the answer, and the decimal point is "placed" in a second calculation. This practice is both slow and risky.

First, it is always desirable to use the lower scales (the C and D scales) in the interest of accuracy. The beginner is sometimes tempted to use the upper scales (which extend from 1 to 100) because a simple multiplication can always be carried out by moving the slide to the right of its central position, whereas when the lower scales are used it is sometimes necessary to move the slide to the right of the central position and sometimes to the left. In practice this point is far outweighed by the higher accuracy afforded by the lower scales.

The rules for determining the position of the decimal point resolve themselves into the following simple forms:

Multiplication.—The number of figures before the decimal point in the product is equal to the *sum* of the numbers of figures before the decimal point in the quantities being multiplied *unless* the product appears to the *right* of the original position of the cursor in which case it is *reduced* by 1.

Division.—The number of figures before the decimal point in the quotient is equal to the number of figures before the decimal point in the original quantity *minus* the number of figures before the decimal point in the divisor, unless the quotient appears to the *left* of the original position of the cursor, in which case the number so determined is *increased* by 1.

Multiplication or division on the slide rule should always be accompanied by mental addition and subtraction, so that the position of the decimal point in the result is known even before the figures are read from the scale. This applies particularly when a calculation involves a number of such steps, as, for example, in evaluating

$$\frac{16{\cdot}1\times25{\cdot}3\times0{\cdot}063}{247\times3{\cdot}3\times0{\cdot}179}$$

It may be noted that in the case of a decimal the equivalent to the "number of figures before the decimal point" is *minus* the number of 0's between the decimal point and the first figure. For example, 0·0031 is classed as −2.

In dealing with logarithms, the "characteristic" is one less than the number of figures before the decimal point. When using the slide rule, this fact is best forgotten as it may lead to some mental confusion.

Accuracy of Slide Rule.—The slide rule is obviously an instrument of limited accuracy, whereas the corresponding arithmetical processes may, if desired, be carried out with perfect accuracy. What must be remembered, however, is that for many engineering calculations the slide rule is accurate enough. In other words, the figures upon which the calculation is based often include unavoidable errors, probably much greater than those introduced by the slide rule, and so its defects are unimportant.

Modification of Formulæ.—It sometimes happens that modification of a formula will make it possible for the slide rule to evaluate it in some particular case with much higher accuracy than could be done at first.

Thus, if it be required to evaluate $\frac{17{\cdot}313}{13{\cdot}792}$, direct division by slide rule gives 1·256, but the last figure 6 is uncertain, and might be read as 5 or 7. Higher accuracy, with little extra time, can be achieved by writing

$$\begin{aligned}\frac{17{\cdot}313}{13{\cdot}792}&=\frac{13{\cdot}792+17{\cdot}313-13{\cdot}792}{13{\cdot}792}\\&=1+\frac{3{\cdot}521}{13{\cdot}792}\\&=1+0{\cdot}2555\\&=1{\cdot}2555\end{aligned}$$

and this is correct to $\pm0{\cdot}0002$.

The principle involved here is to subtract unity from the given quantity, and then to use the slide rule to evaluate the remaining fraction.

Similarly, the slide rule is used in the ordinary way to show that $\frac{55{\cdot}314}{18{\cdot}052}=3{\cdot}065$, but the last figure is uncertain. This result is equal to about 3, and in order to attain a more accurate result the given quantity is therefore reduced by 3, thus

$$\frac{55{\cdot}314}{18{\cdot}052}=\frac{3\times18{\cdot}052+55{\cdot}314-3\times18{\cdot}052}{18{\cdot}052}$$
$$=3+\frac{55{\cdot}314-54{\cdot}156}{18{\cdot}052}$$
$$=3+\frac{1{\cdot}158}{18{\cdot}052}$$
$$=3{\cdot}0641$$

and this is correct to $\pm0{\cdot}0002$.

Again $\frac{11{\cdot}413}{2{\cdot}912}=3{\cdot}92$ and, as this is a little *less* than 4, the difference between the original quantity and 4 is determined thus:

$$\frac{11{\cdot}413}{2{\cdot}912}=\frac{4\times2{\cdot}912+11{\cdot}413-4\times2{\cdot}912}{2{\cdot}912}$$
$$=4+\frac{11{\cdot}413-11{\cdot}648}{2{\cdot}912}$$
$$=4-\frac{0{\cdot}235}{2{\cdot}912}$$
$$=4-0{\cdot}0807$$
$$=3{\cdot}9193$$

and this is correct to $\pm0{\cdot}0002$.

These methods, it will be noted, include first a quick determination of the result by straightforward slide-rule working and then a more accurate determination by a different method. The total time occupied is less than would be required for long division, and, in addition to this, the two-way method affords the valuable safeguard of an independent check, so that an accidental error is fairly sure to be noticed.

The same principle of using the slide rule to calculate the difference between the desired quantity and some slightly different quantity that can be determined quickly and accurately may be used in other cases. For example, it may be desired to evaluate the expression $\frac{a}{\sqrt{(a^2+b^2)}}$ in a case where b is small compared with a. To adapt this expression to accurate slide-rule calculation in such circumstances it is transformed thus:

$$\frac{a}{\sqrt{(a^2+b^2)}}=\sqrt{\frac{1}{\left(1+\frac{b^2}{a^2}\right)}}=\left[1+\left(\frac{b}{a}\right)^2\right]^{-\frac{1}{2}}$$

By use of the binomial theorem, this last quantity is expressed in a series of terms thus:

$$\frac{a}{\sqrt{(a^2+b^2)}}=1+(-\tfrac{1}{2})\left(\frac{b}{a}\right)^2+\frac{(-\frac{1}{2})(-\frac{3}{2})}{1\times2}\left(\frac{b}{a}\right)^4\ \ldots,\text{ etc.}$$

If $\frac{b}{a}$ is much less than unity, successive terms in this series become rapidly smaller, and only the first one or two need be considered. If, for example, $a=14{\cdot}72$ and $b=2{\cdot}31$, the original expression is

$$\frac{14{\cdot}72}{\sqrt{(14{\cdot}72^2+2{\cdot}31^2)}}=\frac{14{\cdot}72}{\sqrt{222{\cdot}35}}=\frac{14{\cdot}72}{14{\cdot}91}=0{\cdot}987$$

but there is some doubt about this, because the 14·91 might be 14·90 or 14·92.

Using the modified form of the expression, its value is

$$1+(-\tfrac{1}{2})\frac{(2{\cdot}31)^2}{14{\cdot}72}+\frac{(-\frac{1}{2})(-\frac{3}{2})}{1\times2}\ \frac{(2{\cdot}31)^4}{14{\cdot}72}\ \ldots\ldots,\text{ etc.}$$
$$=1-0{\cdot}0123+0{\cdot}000227\ \ldots\ldots,\text{ etc.}$$
$$=0{\cdot}9879.$$

This is correct to $\pm0{\cdot}0001$ for the terms considered, and it is clear from the rapid reduction in passing from the first to the third that all others will be negligible.

Application of the binomial theorem succeeds here, because it is used for the expansion of an expression of the form $(1+x)^n$ where x is much smaller than unity. This means that succeeding terms diminish rapidly, so that all beyond the first few are negligible. If this were not the case the method would be useless, as the whole expansion involves an infinite number of terms.

For convenience of reference the expansion of $(1+x)^n$ is given here in general terms. It may be noted that the more general form $(a+x)^n$ is equal to $a^n[1+(x/a)]^n$, so that if x/a is small the theorem can still be usefully employed in this way.

$$(1+x)^n=1+nx+\frac{n(n-1)x^2}{1\times2}+\frac{n(n-1)(n-2)}{1\times2\times3}x^3+\ \ldots,\text{ etc.}$$

Succeeding terms are easily added. The power of x in any term is 1 more than that of the preceding term. The numerator of the coefficient includes all the terms of the preceding coefficient and an extra one that is 1 less than its predecessor. The denominator of the coefficient includes all the terms of the preceding coefficient and an extra one that is 1 more than its predecessor.

Some common occurring examples of this application of the binomial theorem are given below for convenient reference :

$$\frac{1}{1+x}=1-x+x^2-x^3+x^4- \ldots\ldots, \text{ etc.}$$

$$\sqrt{(1+x)}=1+\frac{x}{2}-\frac{x^2}{8}=\frac{x^3}{16}-\frac{5}{128}x^4+ \ldots\ldots, \text{ etc.}$$

$$\frac{1}{\sqrt{(1+x)}}=1-\frac{x}{2}+\frac{3x^2}{8}-\frac{5x^3}{16}+\frac{35}{128}x^4- \ldots\ldots, \text{ etc.}$$

$$(1+x)^2=1+2x+x^2 \text{ (complete).}$$

$$\frac{1}{(1+x)^2}=1-2x+3x^2-4x^3+5x^4- \ldots\ldots, \text{ etc.}$$

$$(1+x)^{\frac{3}{2}}=1+\frac{3x}{2}+\frac{3x^2}{8}-\frac{x^3}{16}-\frac{3x^4}{128}- \ldots\ldots, \text{ etc.}$$

$$1/(1+x)^{\frac{3}{2}}=1-\frac{3x}{2}+\frac{15x^2}{8}-\frac{35x^3}{16}+\frac{315x^4}{128}- \ldots\ldots, \text{ etc.}$$

By this method, slide-rule working is reduced to a simple multiplication and the unavoidable error in reading is confined to the final result. It is only one-twentieth of the possible error by the obvious method.

Similarly such expressions as (d^3-e^3), (d^4-e^4), and so on, may be transformed to give higher accuracy in slide-rule computation, and, for convenience, a number of the more commonly occurring cases are tabulated below :

$$a^2-b^2=(a-b)\ (a+b)$$
$$a^3-b^3=(a-b)\ (a^2+ab+b^2)$$
$$a^4-b^4=(a-b)\ (a+b)\ (a^2+b^2)$$
$$a^5-b^5=(a-b)\ [a^4+b(a+b)\ (a^2+b^2)]$$

Trigonometrical Quantities.—The slide rule is not always used as freely as it might be for evaluating trigonometrical functions of angles. This is largely the result of lack of practice, but once that has been overcome, and confidence attained,

the slide rule within its limits of accuracy is much quicker and easier to use than are trigonometrical tables.

It has some limitations, but they can be overcome without much trouble. For example, it does not show the tangent of any angle less than 5° 43′, but it does show the sines of angles down to 0° 34′, and for angles between that and 5° 43′ the tangent may, for most purposes, be regarded as equal to the sine.

More accurately, for angles less than 5° 43′

$$tan\ A=sin\ A+\frac{sin^3 A}{6}$$

For example, the slide rule shows that sin 4°=0·0695 and therefore, using the formula given above,

$$\begin{aligned} tan\ A &= 0{\cdot}0695+\frac{(0{\cdot}0695)^3}{6} \\ &= 0{\cdot}0695+0{\cdot}00006 \\ &= 0{\cdot}06956. \end{aligned}$$

It will be realised, however, that the last figure in 0·0695 (read as the sine of 4°) cannot be guaranteed, so that this correction is a fine one.

For all angles less than 0° 34′ the sine and tangent may be taken as equal to the number of minutes divided by 3440.

Another range in which the slide rule fails to give accurate direct readings is for cosines of small angles. This defect is overcome by making use of the trigonometrical transformation.

$$cos\ A=1-2\ sin^2\frac{A}{2}$$

This is true for all values of A. For the case $A=10°$, the relation becomes

$$\begin{aligned} cos\ 10° &= 1-2\ sin^2 5° \\ &= 1-2x\ (0{\cdot}087)^2 \\ &= 1-0{\cdot}01514 \\ &= 0{\cdot}98486. \end{aligned}$$

Again, it may be desired to determine the secant of a small angle. At first sight this appears to be determinable by taking the reciprocal of the cosine, calculated as above. It will be realised, however, that the degree of accuracy in the value of the secant obtained in this way would be much lower than that of the cosine. In the particular case of 10°, the cosine is

0·98486 (correct to fourth figure), but it is impossible to set the slide rule to anything finer than 0·985 for this purpose and the secant is then $1{\cdot}015 \pm 0{\cdot}001$.

An alternative is to make use of the transformation

$$\text{Sec. A} = \sqrt{(1+tan^2 A)}$$
$$= 1 + \frac{tan^2 A}{2} - \frac{tan^4 A}{8} + \ldots\ldots, \text{etc.}$$
$$\text{For A} = 10^\circ$$
$$\text{Sec. } 10^\circ = 1 + \frac{tan^2 10^\circ}{2} - \frac{tan^4 10^\circ}{8} + \ldots\ldots, \text{etc.}$$
$$= 1 + \frac{(0{\cdot}1763)^2}{2} - \frac{(0{\cdot}1763)^4}{8} + \ldots\ldots, \text{etc.}$$
$$= 1 + 0{\cdot}01552 - 0{\cdot}00012 + \ldots\ldots, \text{etc.}$$
$$= 1{\cdot}01540$$

and this may be accepted as correct to the fourth figure after the decimal point.

These random examples of application of elementary mathematics to increase the utility of the slide rule are not by any means exhaustive, but familiarity with them will often be found useful and will also suggest how similar methods may be developed to deal with other forms of slide rule calculation.

It is wise to purchase a good quality slide rule with clear calibrations. Some of the cheaper models warp badly, making movement of the slide difficult, and accurate reading almost impossible.

For pocket use the circular watch-type of slide rule is handy and quite accurate. The Halden Calculex and the Fowler circular slide rules are good examples of this type.

For those whose eyesight does not permit them easily to distinguish the scale markings, magnifying cursors are available, such as the Goulding.

CHAPTER IV

Examples in the Use of Slide Rules

So far as the vast majority of engineering calculations are concerned, the slide rule is used for rapid multiplication and division; it can be used for more uncommon arithmetical processes, but the two simple operations mentioned comprise the bulk of its work.

It is wrong to suppose that the slide rule is an advanced mathematical device, for on the contrary its whole object is to simplify arithmetical calculation. Unfortunately, one's first sight of the slide rule hardly conveys that impression, for its array of four closely graduated scales (and even more than that in some specially well-equipped forms of the instrument) certainly tends to overawe anyone who is viewing it for the first time. In practice only two of the scales are at all commonly used, and 95 per cent. of ordinary slide-rule work could be carried out equally well on a much simpler instrument than the standard four-scale type.

Basic Principle.—The slide rule replaces multiplication of numbers by addition of lengths. If we wish to multiply two numbers, say 2·2 and 3·1, the slide rule does it by adding together two lengths corresponding respectively to 2·2 and 3·1 and indicating from the sum of the lengths that the product of 2·2 and 3·1 is 6·82. To make this possible, the scales are graduated in a special manner. The distance from 1 to 2 is not the same as from 2 to 3, although the difference between 1 and 2 is the same as the difference between 2 and 3. On the slide rule the distance from 1 to 2 is the same as the distance from 2 to 4, because the ratio of 1 to 2 is the same as the ratio of 2 to 4. This relationship applies over any part of the scales; they are graduated so that equal distances between marks correspond to equal ratios between the numbers associated with the marks.

Decimal Point.—Confining attention to the C and D scales (see Fig. 20) of the slide rule, it will be noticed that the graduations are numbered only from 1 to 10. The slide rule is actually capable of dealing with any number whatever, and

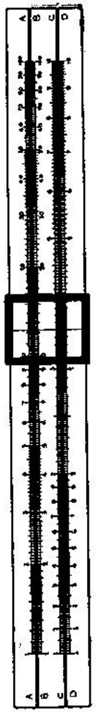


Fig. 20.—Standard four-scale slide rule. The scales are lettered A, B, C, and D, reading from the top downwards.

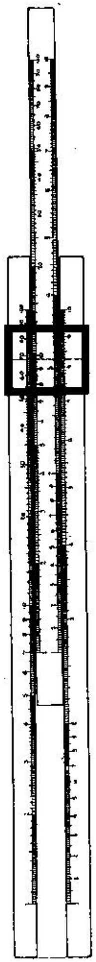


Fig. 21.—Setting for multiplying 2·64 by 3·12. 1 mark on C against 2·64 on D. Hair line of cursor on 3·12 on C.

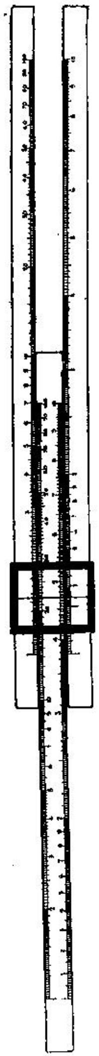


Fig. 22.—Setting for multiplying 2·64 by 4·7. 10 mark on C against 2·64 on D. Hair line of cursor on 4·7 on C.

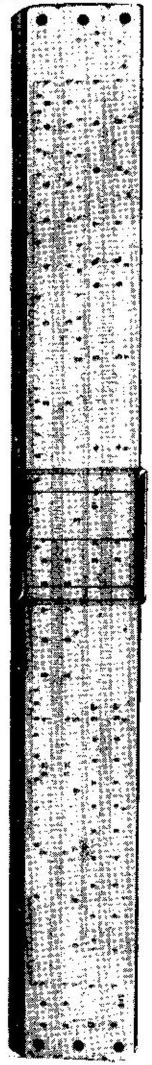


Fig. 23.—A typical Slide Rule.

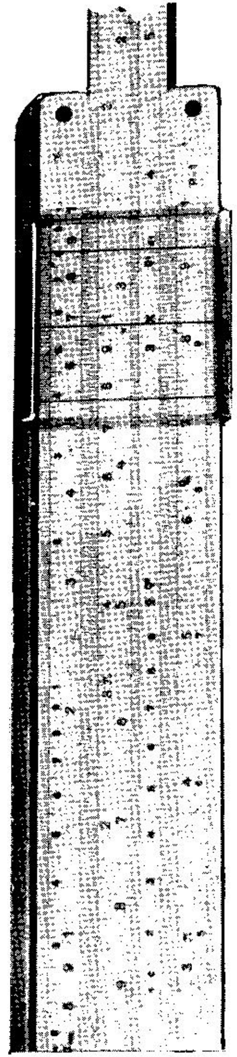


Fig. 24.—The Cursor setting illustrated by Fig. 21.

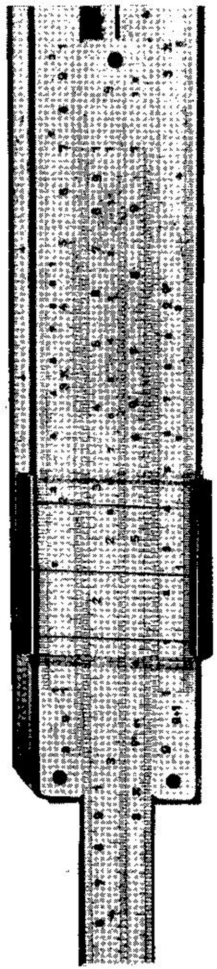


Fig. 25.—The cursor setting illustrated by Fig. 22.

this is done by using the simple 1 to 10 scale in conjunction with a mental note of the position of the decimal point in the number under consideration.

Thus the number 35·3 is associated with the graduation 3·53 on the slide rule and with the mental note "2," which is the number of figures in front of the decimal point in 35·3. One first of all imagines that the decimal point follows the first figure in the given number, locates the slide-rule graduation on that assumption, and then bears in mind the actual number of figures before the decimal point. As another example, 1012·0 is regarded as 1·012 with a decimal point indicator "4" because there are four figures before the decimal point. Also :

0·81 is regarded as 8·1 with "point-number" 0.
1·56 is regarded as 1·56 with "point-number" 1.
78·5 is regarded as 7·85 with "point-number" 2.

If the number is less than 1 and contains a group of 0's immediately after the decimal point, the point-number is *minus* the number of 0's. Thus, 0·00685 is regarded as 6·85 in conjunction with −2.

Those familiar with logarithms will recognise a similarity between this point-number and the "characteristic" of a logarithm. The two things are not quite the same, because the characteristic of a logarithm is *one less* than the number of figures before the decimal point in the given number. To avoid any chance of confusion it is well to forget all about logarithms when actually using the slide rule and to remember instead certain operating rules, given below, for deciding the position of the decimal point in the result of any slide-rule calculation.

Mechanical Construction.—The construction of the slide rule as two main parts, one carrying the A and D scales, and the other the B and C scales, is devised to provide a convenient means of adding lengths together. Thus, if we wish to multiply 2·64 by 3·12, we do it by adding together the lengths corresponding to 2·64 and 3·12. The extreme left-hand graduation (marked "1") on the C scale is brought into line with the 2·64 mark on the D scale (Fig. 21).

The cursor (or "runner") is a metal frame carrying a transparent sheet marked with a hair-line parallel to the graduations. For this calculation the cursor is set so that its hair-line coincides with the 3·12 mark on the C scale. In that position

the distance of the hair-line from the "1" mark at the left-hand end of the D scale is the sum of the distances corresponding to 2·64 and 3·12 respectively, and, because of the mode of graduation, it corresponds to the product of those numbers. The hair-line intersects the D scale between the 8·20 and 8·25 marks and the reading is estimated at 8·23.

The central sliding member makes it possible to set the two lengths to be added "end to end," and the cursor transfers the reading from the C scale to the D scale and indicates the product of the two numbers concerned.

Position of Decimal Point.—If it is required to multiply 2·64 by 4·7, the foregoing procedure fails because 4·7 on the C scale lies to the right of the "10" mark on the D scale, and can therefore yield no reading on that scale. The procedure to be adopted in a case of this sort is to re-set the C scale so that the "10" (instead of the "1") coincides with the "2·64" on the D scale. The "4·7" mark on the C scale then lies in line with the "1·24" mark on the D scale, but this is obviously too small to be the correct answer (Fig. 22). This brings us to the question of settling the position of the decimal point in the answer, for "1·24" might mean 1·24 or 12·4 or 12·400 or 0·0124, or 1·24 multiplied by any power of 10. Actually, it will be obvious that the true answer is 12·4, *i.e.* 1·24 with two figures before the decimal point.

Writing after each number the associated "point-number," the foregoing operations may be summarised, thus :

$$2{\cdot}64\ (1)\times 3{\cdot}12\ (1) = 8{\cdot}23\ (1)$$
$$2{\cdot}64\ (1)\times 4{\cdot}7\ (1) = 12{\cdot}40\ (2)$$

Every multiplication operation on the slide rule belongs to one or other of these classes, and the rule for determining the number of figures before the decimal point in a product embodies alternative clauses. It may be written :

(*a*) If the "10" mark on the C scale is used in setting it, the point-number for the product is the sum of the point-numbers for the quantities multiplied.

(*b*) If the "1" mark on the C scale is used in setting it, the point-number for the product is *one less than* the sum of the point-numbers for the quantities multiplied.

The first of the two examples given above is covered by **Case** (*b*) and the point-number for the product is thus

$(1+1)-1=1$. The other example is covered by Case (*a*), and the point-number for the product is $1+1=2$.

Repeated Multiplication.—The rules and procedure given above apply to each stage in the operation of multiplying a series of numbers together. Thus (extending the first example), if it were desired to evaluate $2{\cdot}64\times3{\cdot}12\times7{\cdot}79\times0{\cdot}0061$, the basis of the procedure is indicated by the following stages :

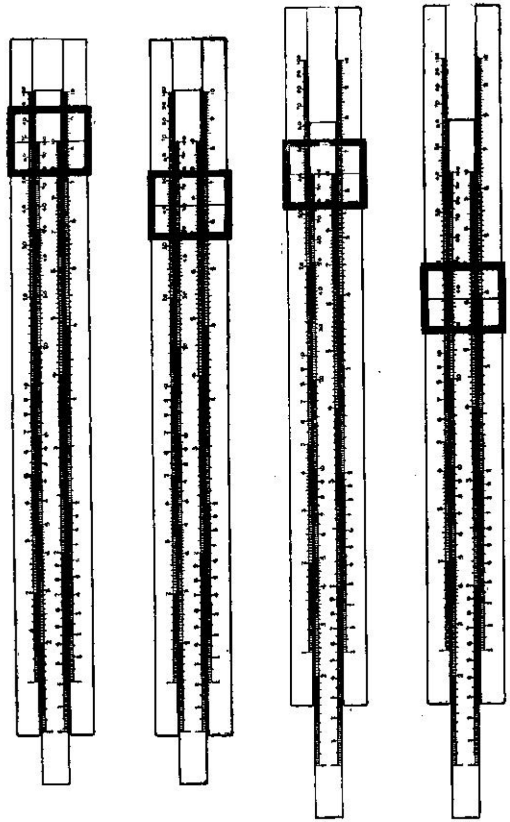
$$2{\cdot}64\ (1)\times3{\cdot}12\ \ (1)\ =\ 8{\cdot}23\ \ (1)$$
$$8{\cdot}23\ (1)\times7{\cdot}79\ \ (1)\ =64{\cdot}1\ \ (2)$$
$$64{\cdot}1\ \ (2)\times0{\cdot}0061\ (-2)=\ 0{\cdot}391\ (0)$$

It is not necessary to write down anything when performing this calculation. The cursor, situated at 8·23 on scale D after the first operation, is in the right position for locating the setting mark on scale C when the second step begins (Fig. 23).

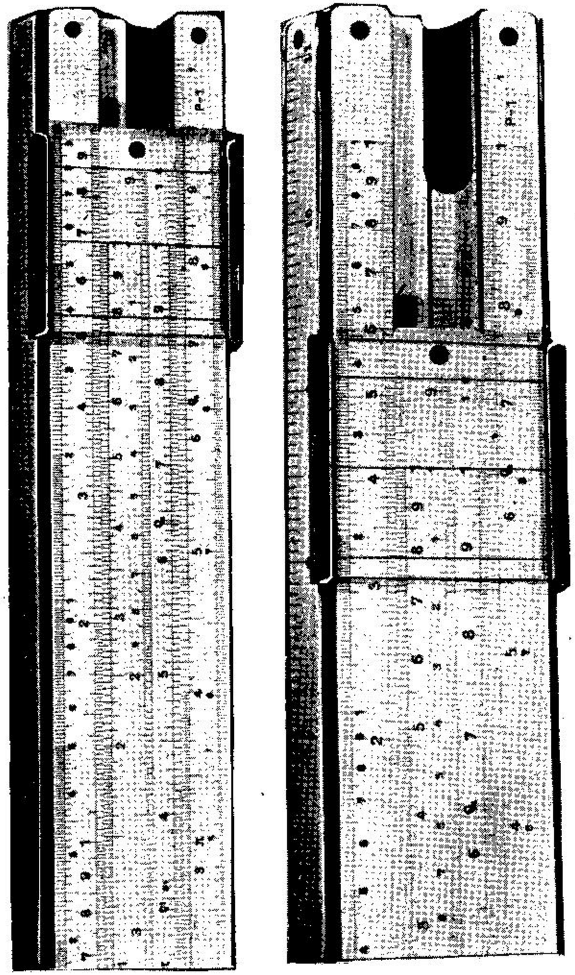
It is found that to bring the answer to step 2 on to the D scale, the " 10 " on the C scale must be lined up with the hair-line on the cursor, and so the point-number for the next multiplication is decided by Case (*a*). Leaving the C scale in that position, the cursor is moved so that its hair-line coincides with " 7·79 " on the C scale, and the point-numbers (1) and (1) for 8·23 and 7·79 are added together to give (2) as the corresponding point-number for the second product. This is indicated on scale D as 64·1, but there is no need to note the fact during this calculation.

The cursor in its last position forms the starting-point for the next stage, which is carried out by setting the C scale to multiply by 0·0061. It is again found that if the mark " 1 " on the C scale is set to coincide with the hair-line, the mark " 6·1 " on that scale is beyond the right-hand end of the D scale. Consequently, it is necessary to set mark " 10 " on the C scale to coincide with the hair-line, and the cursor is then moved so that the hair-line coincides with mark " 6·1 " on the C scale. The point-number for 0·0061 is (−2), and this, added to the number (2) for the previous product 64·1, gives (0) as the point-number for the answer, this again being decided by Case (*a*).

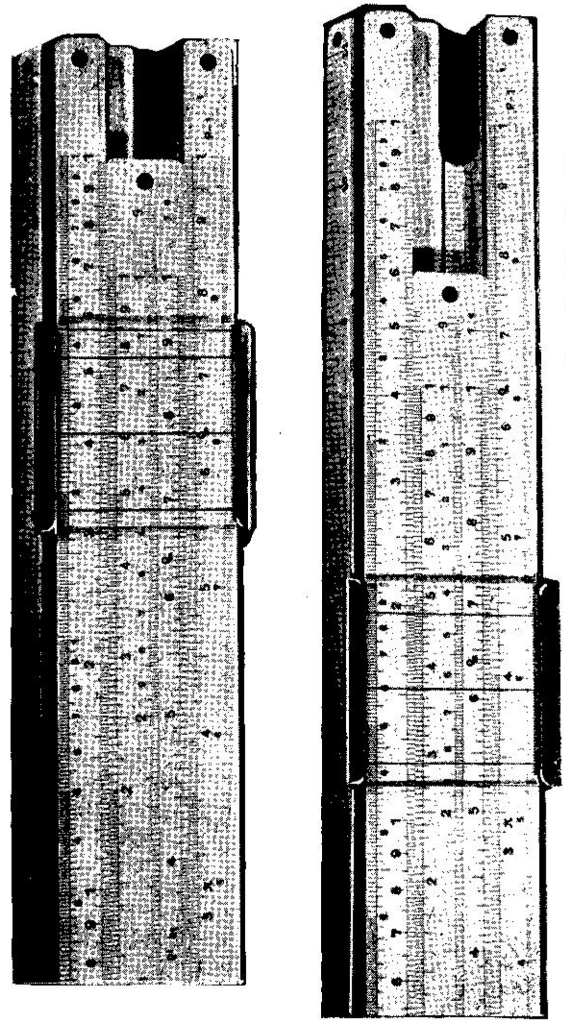
The hair-line in its final position lies about half-way between 3·90 and 3·92 on Scale D. Accepting this as 3·91, and using the point-number (0), the final product is seen to be 0·391. (The point-number (0) means that there are no figures before the decimal point and no 0's immediately behind it.)



Figs. 26 to 29.—Settings in calculating 2·64×3·12×7·79×0·0061. Reading from top to bottom: 1st, C-scale set with 10-mark against D's 8·23 (product of 2·64 and 3·12). 2nd, hair-line of cursor set on 7·79 on C. 3rd, C-scale set with 10-mark under hair-line of cursor. 4th, hair-line of cursor set on 6·1 on C.



Figs. 30 and 31.—Photographic illustrations of the first two stages of the settings shown by Figs. 26 to 29.



Figs. 32 and 33.—The second two stages of the setting shown by Figs. 26 to 29.

The practice of mentally noting successive point-numbers whilst carrying out the mechanical operations of moving slide and cursor has everything to commend it. It is quicker than the alternative method (often advised) of leaving the determination of the position of the decimal point until all slide-rule movements have been made, and it is much less liable to error.

Division.—The arithmetical operation of division is the converse of multiplication, and in using the slide rule for division the procedure is the reverse of that used for multiplication. Arithmetical division is a slightly more troublesome operation than arithmetical multiplication, but in using the slide rule the reverse is the case. It is slightly easier to divide than to multiply.

The reason is that when multiplying on the slide rule, it may be necessary to use either the " 1 " mark or the " 10 " mark on the C scale, and some small amount of thought is required to decide which. Usually one guesses whether the position of the answer will require the " 1 " setting or the " 10 " setting, and if the guess proves to be incorrect there is a loss of time whilst the C scale is being re-set. In dividing, however, there are no such alternatives, and the position of the C scale is decided with certainty.

For example, suppose it were required to divide 197 by 6·35. The procedure is to set the hair-line of the cursor on the 1·97 line on scale D (noting " point-number " 3), and then to move scale C so that the 6·35 mark (point-number 1) coincides with the hair-line. The quotient is then indicated on scale D by either the " 1 " mark or the " 10 " mark of scale C; one or other of them must be on scale D, and it is this fact that eliminates any alternative position of scale C. In this case it is found that the " 10 " mark on scale C lines up with the " 3·1 " mark on scale D, and this (modified according to the point-number) is the required quotient. (See Fig. 34.)

The rule for deciding the point-number in a quotient is :

(a) *If the " 10 " mark on the C scale indicates the quotient on the D scale, the point-number for the quotient is the point-number for the dividend minus the point-number for the divisor.*

(b) *If the " 1 " mark on the C scale indicates the quotient on*

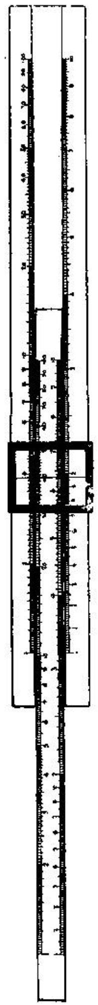


Fig. 34.—Setting for dividing 197 by 6·35. Hair-line of cursor on 1·97 on D. 6·35 on C set under hair-line.

Fig. 35.—Setting for dividing 36·3 by 0·0235. Hair-line of cursor on 3·63 on D. 2·35 on C set under hair-line.

Fig. 36.—Setting for extracting cube root of 565. Hair-line of cursor on 5·65 on A. 10 mark on C on 8·26 on D.

the D scale, the point-number for the quotient is one more than *the point-number for the dividend minus the point-number for the divisor.*

The example given above is covered by Case (*a*) and the point-number for the quotient is thus $(3-1)=2$. The quotient is therefore 31 (Fig. 35).

As a further example, let it be required to divide 36·3 by 0·0235. The hair-line of the cursor is set on the 3·63 mark on scale D and the point-number (2) for 36·3 is mentally noted. Scale C is then moved so that the 2·35 mark coincides with the hair-line, and the point-number (−1) for 0·0235 mentally noted. It is then found that the " 1 " mark on scale C lies between the 1·54 and 1·55 marks on scale D and the reading may be estimated as 1·544. Because the " 1 " mark indicates the quotient, its point-number is decided by Case (*b*) and is therefore

$$[2-(-1)]+1=4,$$

and the quotient is therefore 1544, *i.e.* four figures before the decimal point.

Example of Multiplication and Division.—As a further example to illustrate the procedures described for multiplication and division, let it be required to evaluate :

$$\frac{253\times3{\cdot}71\times58{\cdot}3}{26{\cdot}5\times7{\cdot}19\times89{\cdot}8}.$$

Operation 1. $253\times3{\cdot}71$.

C scale set with 1 coinciding with 2·53 on D scale. Cursor set with hair-line coinciding with 3·71 on C scale. Point-number for product$=3+1-1=3$.

Operation 2. *Previous product* $\times\,58{\cdot}3$.

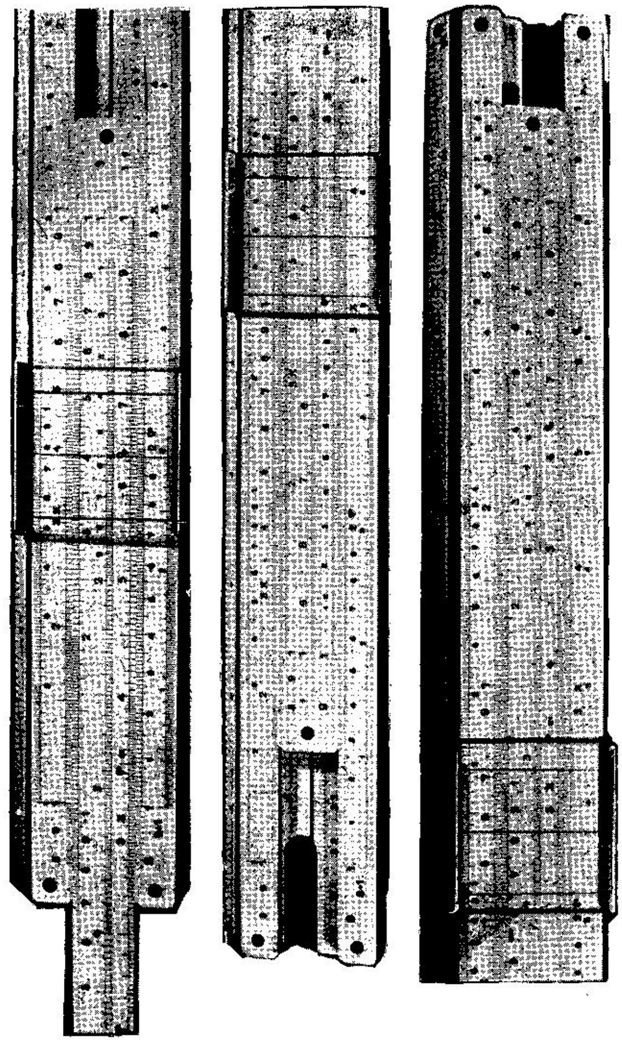
C scale set with 10 coinciding with hair-line on cursor. Cursor set with hair-line coinciding with 5·83 on C scale. Point-number for product$=3+2=5$.

Operation 3. *Previous product* $\div\,26{\cdot}5$.

C scale set with 2·65 mark coinciding with hair-line on cursor. Quotient indicated by 1 line on C scale. Point-number for quotient$=5-2+1=4$.

Operation 4. *Previous quotient* $\div\,7{\cdot}19$.

Cursor set with hair-line coinciding with 1 mark on C scale.



Figs. 37 to 39.—The settings given by Figs. 34 to 36 shown photographically.

C scale set with 7·19 mark coinciding with hair-line on cursor.

Quotient indicated by " 10 " line on C scale.

Point-number for quotient=4−1=3.

Operation 5. *Previous quotient*÷89·8.

Cursor set with hair-line coinciding with 10 mark on C scale.

C scale set with 8·98 mark coinciding with hair-line on cursor. Quotient indicated by 10 mark on C scale.

Point-number for quotient=3−2=1.

Final result=3·2.

Accuracy.—For several of the settings in the operations just described (although the word " mark " was used for convenience), it is necessary to estimate sub-divisions of the graduations on the slide rule. Because of the unequal spacing of the numerals, the sub-divisions vary along the length of the rule so as to avoid excessively fine graduation at one end and excessively coarse graduation at the other. From 1 to 2 the distance between adjacent marks represents 0·01 and steps of about 0·002 can be estimated. From 2 to 4 the steps are 0·02 and from 4 to 10 they are 0·05 ; in each case, steps of about one-fifth the marked steps can be estimated.

The error in any one setting or reading may be as much as two-tenths of 1 per cent. After a calculation requiring 10 settings (such as that just described) the error may therefore be as much as 2 per cent. This, however, is unlikely, as the probability is that the various errors will tend to balance one another.

Between 1 and 2, the divisions at 1·1, 1·2, 1·3, etc., are numbered 1, 2, 3, etc. This is apt to be a little misleading when dealing with numbers such as 1·13, as one may confuse it with 1·3. Furthermore, in the early stages of one's acquaintance with the slide rule, it may not be realised that the small 5 (say) refers to 1·5 and not to 5 itself.

Extraction of Square Root.—The square root of a number is extracted by setting the hair-line of the cursor at the appropriate mark on the A scale, when the reading of the hair-line on the D scale gives the answer. The central slide carrying the B and C scales is not used. The A scale extends from 1 to 10 and from 10 to 100, so that there are two choices for the initial setting ; the point-number decides which is to be used.

If the point-number of the given quantity is even, the hair-line is set between 10 and 100 on the A scale and the point-

number of the square root is half the point-number of the given quantity.

Thus to extract the square root of 4,750 (point-number 4), the hair-line is set at 47·5 on the A scale, in which position it reads 6·89 on the D scale, and since the point-number of the root is half the original point-number (*i.e.* $\frac{1}{2}\times4=2$), the required square root is 68·9.

Similarly the square root of 0·26 (point-number 0) is found by setting the hair-line to 26 on the A scale and is indicated at 5·1 on the D scale. Since the point-number of the root is $\frac{1}{2}\times0=0$, the root is 0·51.

Again, the square root of 0·0072 (point-number −2) is indicated at 8·48 and the actual root is 0·0848 (point-number −1).

If the point-number of the given quantity is odd, the hair-line is set between 1 and 10 on the A scale and the point-number of the square root is half of 1 plus the point-number of the given quantity.

Thus, to extract the square root of 475 (point-number 3), the hair-line is set at 4·75 on the A scale, in which position it reads 2·18 on the D scale, and the point-number of the square root is $\frac{1}{2}(1+3)=2$, so that its value is 21·8.

Similarly the square root of 0·026 (point-number −1) is found by setting the hair-line to 2·6 on the A scale and is indicated at 1·613 on the D scale. Since the point-number of the root is $\frac{1}{2}(1-1)=0$, the root is 0·1613.

Extraction of Cube Root.—The operation of extracting the cube root of a number by means of the slide rule is more difficult than any yet described. As in the operation of extracting the square root, the point-number of the given quantity decides the exact procedure.

Case (*a*). *Point-number zero or divisible by* 3 (*e.g.* 3, 0, −3).

Set the hair-line of the cursor on the appropriate mark *between* 1 *and* 10 on the A scale.

Starting from the position in which the A and B scales coincide, move the slide *to the left* until the " 10 " on the C scale first indicates on the D scale the same reading as that of the hair-line on the B scale.

This reading is the required cube root and its point-number is one-third of that of the given quantity.

The difficulty in this operation arises from the fact that as the slide moves, the reading of the C10 mark on the D scale and the reading of the hair-line on the B scale are both changing. Furthermore, it should be noted that there is more than one position of the slide that will give the specified relation between the readings. The correct one is the *first* that occurs as the slide is moved from its central position towards the left.

As an example, let it be required to determine the cube root of 565. (The point-number is 3 and so the method described is applicable. See Fig. 36.)

The hair-line is set to the 5·65 mark on the A scale. The slide is moved to the left, so that the 10 mark on the C scale indicates quantities decreasing from 10 on the D scale, and the hair-line indicates quantities increasing from 5·65 on the B scale. Eventually the former reading would drop to about 7·5 whilst the other would rise to 10, so that the D scale reading, at first more than the B scale reading, is finally less than it. At some intermediate point the readings must be equal, and this is found to occur at 8·26.

The original point-number is 3 and so that of the cube root is 1 and the cube root is thus 8·26.

Similarly $\sqrt[3]{0{\cdot}565}=0{\cdot}826$ and $\sqrt[3]{0{\cdot}000565}=0{\cdot}0826$.

Case (*b*). *Point-number equal to* 1 *plus a multiple of* 3 (*e.g.* 4, −2, 1).

Set the hair-line of the cursor on the appropriate mark *between* 1 *and* 10 on the A scale.

Starting from the position in which the A and B scales coincide, move the slide *to the right* until the " 1 " on the C scale first indicates on the D scale the same reading as that of the hair-line on the B scale.

This reading is the required cube root and its point-number is one-third of 2 plus the original point-number.

As an example, let it be required to determine the cube root of 6,850. (The point-number 4 is equal to 1+3×1, and so the method described is applicable. See Fig. 40.)

The hair-line is set to the 6·85 mark on scale A. As the slide is moved from its central position to the right, the D-scale reading of the " 1 " mark on C increases from 1, while the hair-line reading on the B scale falls from 6·85. It is found that the readings become equal at 1·9.

The point-number of the cube root is one-third of 2 plus 4, or $\frac{2+4}{3}=2$, and so the required cube root is 19.

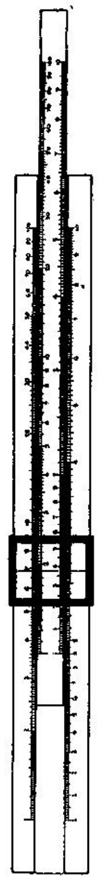


Fig. 40.—Setting for extracting cube root of 6850. Hair-line of cursor on 6·85 on A. 1 mark on C on 1·9 on D.

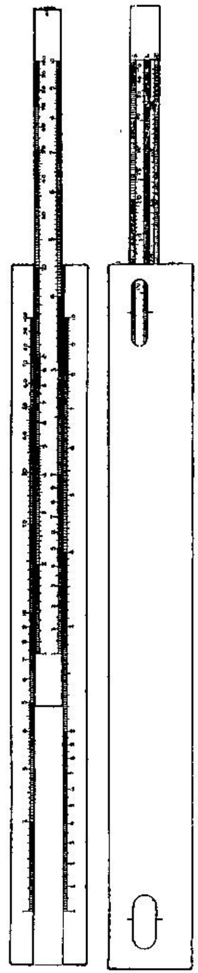


Fig. 41.—Two views of slide rule. Setting for determining logarithm of 2·7. (Above) 1 mark on C on 2·7 on D. Cursor not required. (Below) Right-hand end of back of rule set as above. Index on lower side of slot reading 4·32 on L scale.

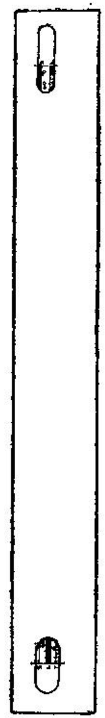


Fig. 42.—View of complete back of slide rule showing both ends of S scale, both ends of L scale, and left-hand of T scale. Reading points of logarithmic and trigonometrical scales.

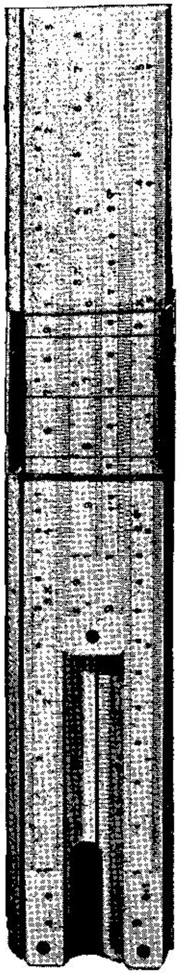


Fig. 43.—The cursor setting illustrated in Fig. 40.

Fig. 44.—Back view of Slide Rule.

If it were required to determine the cube root of 0·00685, the same procedure would be followed, because the point-number is -2, which is equal to $1+3\times(-1)$. The point-number of the cube root is $\frac{2+(-2)}{3}=0$, and the cube root itself is therefore 0·19.

Similarly $\sqrt[3]{6{\cdot}85}=1{\cdot}9$.

Case (c). *Point-number equal to 2 plus a multiple of 3* (*e.g.* 5, -1, -7).

The procedure is as for Case (b) except that the hair-line is set between 10 and 100 on the A scale and the point-number of the cube root is one-third of 1 plus the original point-number.

As an example, let it be required to determine the cube root of 37,300 (point-number 5). The hair-line is set at 37·3 on the A scale, and the slide is moved to the right until it is found that with the " 1 " on the C scale coinciding with 3·34 on the D scale the hair-line is coinciding with 3·34 on the B scale. The point-number of the cube root is $\frac{1+5}{3}=2$, and the cube root is thus 33·4.

Similarly $\sqrt[3]{0{\cdot}0373}=0{\cdot}334$.

Check Calculation.—In view of the alternatives that exist when extracting the cube root, it is desirable always to check the result by cubing it and comparing with the original quantity. The cubing is effected by two successive multiplications of the cube root by itself, and not merely by an inversion of the process just described.

Extraction of Other Roots.—Certain higher roots may be extracted by repetition of the processes just described. Thus the fourth root is the square root of the square root. The sixth root is the cube root of the square root, and so on.

In general, however, extraction of other roots must be effected by use of logarithms, and the logarithmic scale on the back of the slide is useful for this purpose. It is not so accurate as even four-figure tables of logarithms, but is somewhat quicker to use.

Logarithms.—The logarithm of a number (to base 10) is determined by setting the " 1 " mark on the C scale in line with the D scale mark corresponding to the number. Care being taken not to move the slide from this position, the rule is inverted, when the index mark on the lower side of the slot at the right-hand end of the back of the rule will be found indicating 10 times the mantissa, or decimal part of the logarithm, on the L scale. The characteristic of the logarithm is 1 less than the point-number.

It should be particularly noted that, unlike other scales on the rule, the L scale reads from *right to left*, between 0 and 10 in equal steps (Fig. 41).

For example, with the 1 mark on the C scale in line with 2·7 on the D scale, the index mark reads 4·32 on the L scale. The mantissa is thus 0·432. The characteristic is 1 less than the point-number of the given quantity, so that, for example,

$$\begin{aligned} \log.\ 2{\cdot}7 &= 0{\cdot}432 \\ \log.\ 270 &= 2{\cdot}432 \\ \log.\ 0{\cdot}027 &= 2{\cdot}432. \end{aligned}$$

Evaluation of Fractional Powers.—The commonest need for the use of logarithms in connection with the slide rule arises when dealing with fractional powers of numbers. Thus if $x=16^{2{\cdot}3}$ the procedure is based on

$$\log.\ x = 2{\cdot}3 \log.\ 16.$$

Evaluating the logarithm by the method first described, the index mark reads 2·04 on the L scale, so that the mantissa is 0·204, and since the point-number is 2, the characteristic is 1. The logarithm of 16 is thus 1·204, and this is noted.

Then $\log.\ x = 2{\cdot}3 \times 1{\cdot}204$.

The product of 2·3 and 1·204 is evaluated by the normal procedure for multiplication on the slide rule and is found to be 2·77. This is the logarithm of x, and the value of x must be determined from it by an inversion of the process of determining logarithms.

The slide is set so that the logarithm index mark registers on the L scale 10 times the mantissa (0·77), *i.e.* 7·7. The " 1 " mark on the C scale then indicates 5·88 on the D scale, and as the characteristic of the logarithm 2·77 is 2, the point-number for x is 3 and x is therefore 588. (In other words 588 is the " antilogarithm " of 2·77.)

Evaluation of Fractional Powers of Fractions.—The procedure just described is exactly the normal one employed when using logarithms, except that the L scale on the slide rule is used instead of the table of logarithms, and the multiplication of the

logarithm by the index is effected on the slide rule instead of by using logarithms. The slide rule method is less confusing because it makes smaller use of logarithms as such.

Those readers familiar with the logarithmic procedure will remember that it is particularly awkward when it is used to evaluate a fractional power of a quantity less than 1. This is because the characteristic of a fraction is a negative quantity, and after multiplication of the logarithm by the index, a little reshuffling is necessary to make the characteristic into a whole number. When the slide rule is used, this difficulty can be avoided because it is easy to determine the reciprocal of the given number, to raise it to the specified power, and then to take the reciprocal of the result.

For example $(0{\cdot}5)^3=(\frac{1}{2})^3=\dfrac{1}{2^3}=\frac{1}{8}=0{\cdot}125.$

In this simple example we avoid the necessity for cubing 0·5 by finding its reciprocal 2, cubing that (8), and finally finding the reciprocal (0·125) of the cube.

Let it be required to evaluate $(0{\cdot}034^{0{\cdot}85})$.

Operation 1. *Determine the reciprocal of* 0·034 *by dividing* 1 *by* 0·034.

Divide 1 by 0·034 by setting the 3·4 mark on the C scale to coincide with the " 1 " mark on the D scale, when the " 10 " mark on the C scale indicates 2·94 on the D scale. Since the point-number of 1 is 1 and of 0·034 is −1, and since the 10 mark on the C scale indicates the quotient, the point-number of the quotient is $1-(-1)=2$, and the quotient (the reciprocal of 0·034) is 29·4.

Operation 2. *Determine the logarithm of* 29·4.

Set the " 1 " mark on the C scale to coincide with the 2·94 mark on the D scale when the index mark on the back of the slide rule will be found to read 4·69 on the L scale. Hence the mantissa is 0·469, and since the point-number of 29·4 is 2, the logarithm is 1·469.

Operation 3. *Multiply the logarithm of* 29·4 *by the index* 0·85.

This is a plain multiplication of 1·469 by 0·85 and the usual slide-rule process shows the product to be 1·249.

Operation 4. *Determine the antilogarithm of* 1·249.

Set the slide so that the index mark on the back of the slide rule indicates on the L scale ten times the mantissa, *i.e.* $10\times0{\cdot}249=2{\cdot}49$. In that position, the " 1 " mark on the C scale indicates 1·774. Since the characteristic of 1·249 is 1, the point-number for the antilogarithm is one more than this, *i.e.* 2, and the antilogarithm is therefore 17·74.

Operation 5. Determine the reciprocal of 17.74.

This operation is similar to Operation 1, and it is found that $\frac{1}{17\cdot74}=0\cdot0563$.

Hence $(0\cdot034)^{0\cdot85}=0\cdot0563$.

Trigonometrical Functions.—No special difficulty arises in determining the values of trigonometrical functions of angles from the slide rule. It is simply a handy alternative to the usual tables of values of sine, cosine and tangent, and its accuracy, in general, is lower than that of four-figure tables (Fig. 42).

When the slide rule is inverted, it is seen that the slot at the left-hand end reveals three scales, marked S, L and T on the slide. The L scale has already been dealt with when dealing with logarithms. The upper side of the slot has an index-mark to register on the S scale, and there is also an index-mark for the S scale on the upper side of the slot at the right-hand end.

The S scale is marked in degrees and fractions of a degree over a range extending from 34 mins. to 90 deg. If the slide is set so that the left-hand index-mark coincides with (say) the 5 mark on the S scale, and the slide rule is inverted, the " 1 " mark on the A scale indicates on the B scale 100 times the sine of 5 deg. This is seen to be 0·087. If the right-hand index-mark is used, the 100 mark on the A scale indicates on the B scale 100 times the sine of the angle concerned. Note that the top scale (the S scale) on the back of the rule is associated with the top scales (A and B) on the front of the rule.

For angles between about 70 deg. and 90 deg. the markings on the S scale are very close together, and it is impossible to obtain an accurate value of the sine by direct reading. To improve matters in this range the following formula may be used:

$$\sin P=1-2\sin^2\left(45^\circ-\frac{P}{2}\right).$$

For example, let it be required to evaluate sin 75 deg. Here $P=75$ deg. and so 45 deg. $-\frac{P}{2}=7\cdot5$ deg. Setting the slide so that the left-hand index-mark on the back of the rule indicates

7 deg. 30 min. on the S scale, and inverting the rule, it is seen that

$$\sin 7\cdot5 \text{ deg.}=\frac{13\cdot05}{100}=0\cdot1305.$$

Hence $\sin 75 \text{ deg.}=1-2\times(0\cdot1305)^2$.

By applying normal slide-rule working to evaluate the square, it is seen that sin 75 deg. = 0·9659.

For angles less than 34 min., the sine of the angle is equal to the number of minutes divided by 3440.

Determination of Cosine.—The slide rule possesses a separate scale to indicate the cosine, but this is no great hardship, because $\cos A=\sin(90 \text{ deg.}-A)$, and so the S scale can be used to determine cosines.

For example, cos 41 deg. = sin 49 deg., and from the S scale used as described above, this is 0·755.

For angles between about 0 deg. and 20 deg. the cosine is more accurately determined by using the relation—

$$\cos P=1-2\sin^2\frac{P}{2}.$$

Thus

$$\begin{aligned}\cos 12^\circ&=1-2\sin^2 6^\circ\\&=1-2\times(0\cdot1042)^2\\&=0\cdot9782.\end{aligned}$$

Determination of Tangent and Cotangent.—To determine the tangent of an angle, set the slide so that the index-mark on the lower side of the left-hand slot at the back of the rule coincides with the T scale mark corresponding to the angle. Then the " 1 " mark on the D scale indicates on the C scale 10 times the tangent of the angle. Also, the " 10 " mark on the C scale indicates the cotangent of the angle on the D scale.

Thus, setting the 20 mark on the T scale in line with the index-mark on the back of the rule, the 1 mark on the D scale indicates 3·64 on the C scale so that

$$\tan 20^\circ=\frac{3\cdot64}{10}=0\cdot364.$$

Also the 10 mark on the C scale indicates 2·75 on the D scale, so that cot 20 deg. = 2·75.

The T scale has no readings less than 5 deg. 43 min. For angles less than this the tangent is very nearly equal to the sine, and so the method described for determining that function can be used in this case also.

For angles between 45 deg. and 90 deg., which do not appear on the T scale, use the formula

$$\tan P = \cot (90° - P)$$
$$\text{and } \cot P = \tan (90° - P)$$
$$\text{Thus } \tan 68° = \cot 22° = 2{\cdot}475$$
$$\cot 81° = \tan 9° = 0{\cdot}1583.$$

General.—This chapter outlines simple and reliable methods of using the slide rule in the most straightforward way for ordinary arithmetical operations. Although not so accurate, in general, as four-figure logarithms, the slide rule is very much quicker and easier to use, and its general standard of accuracy is quite high enough for most engineering calculations. It is possible to perform certain calculations on the slide rule in special ways that lead to a much more accurate result than is attainable by the straightforward procedure. An account of some of these artifices has already been given.

Note.—In some slide rules the S and T scales are not situated in the positions described in this chapter. The correct method of using any particular rule can soon be determined by setting it at a few angles such as 30 deg. and 60 deg., for which the trigonometrical functions are well known.

The "Q" Gauge Points.—Some slide rules (see photographic illustrations in this Chapter) have gauge points marked Q″, Q′, and Q″ on the C scale and Q″ and Q″ on the D scale. These relate to circular measure. Q′ relates to angles and represents: 3437·74, the number of minutes in a radian, value $\frac{180 \times 60}{\pi}$.

Q″ is for determining the functions of small angles. The number of seconds in radians is set at 206265 which= $\frac{180 \times 60 \times 60}{\pi}$.

The Q with the suffix is set to position, 636620 which= $\frac{200 \times 100 \times 100}{\pi}$ which equals the number of seconds in radians in the decimal or centesimal (French) system.

CHAPTER V

Circular Slide Rules

There are many special forms of slide rule, and circular or pocket calculators have several advantages over the straight type. The underlying principle is, like slide rules generally, based upon the properties of logarithms as previously explained. These operations are made mechanical by moving scales, either straight or circular, logarithmic distances being marked on them so that distances can be added or subtracted from one another. The usefulness of such rules is increased by the addition of special scales, which enable squares, cubes, roots, etc., to be read off directly. The longer the scale the more accurately can calculations be made and read. Although the straight slide rule is in general use, it has certain disadvantages which the circular calculator overcomes. For example, with a straight rule it often happens that the result of a particular multiplication of division cannot be read on the scale, which extends the full length of the rule, owing to the slide being drawn out too far. For this reason one side usually consists of two duplicate scales, and the majority of calculations are made on a scale only half the length of the rule. This, of course, applies to rules less than 10 in. long. Sometimes trouble is experienced with tight slides, or slack slides, according to the weather conditions.

I shall deal in this chapter with two of the best-known circular pocket slide rules—the Fowler and the Halden Calculex. These very handy and accurate rules somewhat resemble a watch in design, and are conveniently carried in the waistcoat pocket.

The advantage of circular calculators such as the Fowler is that in any calculation the scales are always the full-length of the circumference, and they do not stick. Being operated by gearing they can be easily and accurately set, and further, since the scales are protected with glass covers in a sealed metal case they are always clean and available for use indoors, out-of-doors, and under all climatic conditions. Such calculators give the accuracy of a 10-in. slide rule. Various types of Fowler's pocket calculators for science students, mechanics, draughtsmen, and engineers are available. In type H here illustrated, the front dial comprises five scales—multiplication and division, reciprocals, logarithms, square roots (extending over two circles), and sines of angles. The back dial is a scale of cubes and cube roots. In type R, the outer scale is complete in a single circle as in Type H, and can be used for multiplication and division in the same way. The six remaining circles constitute one scale 30 in. long, and for many calculations is equal to a straight slide rule 60 in. long. This six-scale is used in the same way as the single outer scale, though this latter is convenient for finding the precise circle on which to read the result. The back dial is a scale of cubes and cube roots. Type R is useful for draughtsmen, or when calculations are required to be made with greater accuracy than is possible in Type H, where the length of the scale is a single circumference. (See Fig. 45.)

Other slide rules supplied by Fowler & Company include their long-scale calculator which is particularly useful for engineers, draughtsmen, and science students; their "Magnum" long-scale calculator which is a large model intended for office use or for those with poor eyesight, and the "12–10" calculator which is primarily designed for architects, contractors, surveyors, builders, timber merchants, and craftsmen.

In Type RX the front dial is the same as in Type R. The back dial is the same as used on the front of Type H. This is a very useful combination for engineers and draughtsmen, and by many preferred to Type R, as it gives squares, roots, sines, tangents, logs, and reciprocals from the back dial by reading, while multiplication and division can be done on the front dial with the long or the short scale.

The following instructions relate to the Fowler Type H calculator:

Multiplication.—$a \times b \times c \times \ldots\ldots$, etc. Use Scale 1.

Set a under datum F by nut X.
Set cursor E at 1·0 by nut Y.
Set dial by nut X till b is under cursor E.
Read $a \times b$ under datum F.

If there is a third factor, c:

Set cursor again at 1·0 by nut Y.
Set dial by nut X till c is under cursor E.
Read $a \times b \times c$ under datum F, and so on for any number of factors.

Division.—$\frac{a}{m}$. Use scale 1.

Set a under datum F by nut X.
Set cursor E to m by nut Y.
Set dial by nut X till 1·0 comes under cursor E.
Read $\frac{a}{m}$ under datum F.

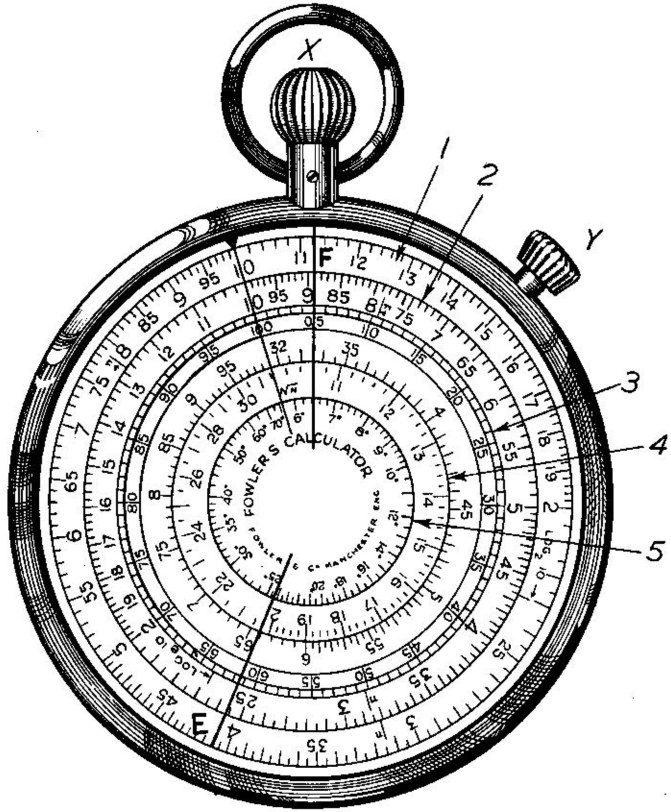


Fig. 45.—The Fowler Circular Pocket Slide Rule.

Fractions.—If fraction of form $\frac{a\times b}{m}$ continue thus :

Set dial by nut X till b is under cursor E.

Read $\frac{a\times b}{m}$ under datum F.

If fraction of form $\frac{a\times b}{m\times n}$ continue thus :

Set cursor by nut Y to n.

Set dial by X till 1·0 is under cursor E.

Read $\frac{a\times b}{m\times n}$ under datum F, and so on for any number of multipliers or divisors.

The numerator of the fraction must contain one more factor than the denominator. If there are not enough multipliers or divisors in the fraction used, the figure 1 is to be used as often as required. In recording the final result of a calculation the position of the decimal point, if one is required, is determined by inspection, as with all logarithmic rules.

The datum F is only used for first multiplier and final answer.

The cursor E is turned only for divisors.

The dial is turned only for multipliers.

Squares.—a^2. Squares may be obtained by multiplication. $a^2=a\times a$.

Alternative method :

Use Scales 1 and 4 and read direct.

Set a on Scale 4 under datum F.

Read a^2 on Scale 1.

Square Roots.—$\sqrt{a}$. Use Scales 1 and 4.

Set a on Scale 1 under datum F.

Read $\sqrt{a}$ on *outer* edge of Scale 4 if a contains an *even* number of figures.

Read $\sqrt{a}$ on *inner* edge of Scale 4 if a contains an *odd* number of figures.

Cubes.—a^3. This may be got by multiplication. $a^3=a^2\times a$. Use Scales 1 and 4.

Set a on Scale 4 under datum F by nut X.

Read a^2 on Scale 1 under datum F.

Set cursor E at 1·0 by nut Y.

Set a on Scale 1 under cursor by nut X.

Read a^3 on Scale 1 under datum F.

If the instrument is fitted with a cube root back dial, a^3 may be got by direct reading, thus :

Set a number a on one of the three inner scales under datum.

Read a^3 on outer scale under datum F.

Cube Roots.—$\sqrt[3]{a}$. If the instrument is fitted with a cube-root back dial the simplest method is to use the dial.

Set a on outer circle under datum F.

Read $\sqrt[3]{a}$ on one of the three inner scales under datum F.

If the instrument is not fitted with cube-root dial the cube root may be got by using Scales 1 and 3 of the front dial. Since, if $x=\sqrt[3]{a}$, log. $x=\frac{1}{3}$ log. a. This method enables the nth power or nth root to be obtained.

Natural Sine or Log. Sine.—Use Scales 1, 3, and 5.

Set number of degrees in angle on Scale 5 under datum F.

Read natural sine on Scale 1.

Read log. sine on Scale 3.

Natural Cosine or Log. Cosine.—Cosine is deduced from sine.

Cosine of an angle equals sine of its complement.

Example : Cosine 66°=sine (90−60)°=sine 30°.

Tangents and Cotangents.—

$$\text{Tangent}=\frac{\text{sine}}{\text{cosine}}. \qquad \text{Cotangent}=\frac{\text{cosine}}{\text{sine}}.$$

Circumference of a Circle.—Use Scale 1.

Set diameter under datum F by nut X.

Set cursor E at 1·0 by nut Y.

Set dial by nut X till 3·1416 (π) on Scale 1 is under cursor.

Read circumference under datum F.

Area of a Circle.—Use scales 1 and 4.

Set diameter on Scale 4 under datum F by nut X.

Set cursor E at 1·0 by nut Y.

Set dial by nut X till ·7854 $\left(=\frac{\pi}{4}\right)$ is under cursor.

Read area on Scale 1 under datum F.

Common Logarithms.—Use Scales 1 and 3.

Set number on Scale 1 under datum F.

Read mantissa of log. of number under cursor on Scale 3.

Characteristic of log. is determined in usual way.

Hyperbolic Logarithms.—Hyperbolic log.: common log. × 2·30. 2·30 is marked (log. 10) on Scale 1.

Reciprocals.—Scale 2.

This scale is very convenient for obtaining values of expressions such as $\frac{1}{a}$, $\frac{1}{a^2}$, $\frac{1}{\sqrt{a}}$, $\frac{1}{\sin a}$, $\frac{1}{\cos a}$, etc., by simply reading the values on Scale 2, instead of Scale 1.

The Halden Calculex.—This is another type of slide rule made up in circular form. The circular metal dial on which the scales are engraved consists of two pieces, an outer ring and an inner disc, the logarithmic scales being engraved on the inner edge of the outer ring and on the outer edge of the disc. (This disc corresponds to the " slide " of an ordinary slide rule, and can be revolved by thumb-nuts attached to the centre.)

The Calculex consists of two of these dials, placed back to back, each of which is protected by a glass (or celluloid) disc, the whole being enclosed in a German-silver ring. On the inner surface of each of the glasses is engraved a fine " cursor " line —the glasses are moved by thumb pressure on their faces and arranged so that they can be revolved independently of the centre dial. The edges are bevelled so as to carry more comfortably in the pocket.

Scales.—On the Front Face (on which the name is printed) are the two logarithmic scales for multiplication, division, proportion, etc. These scales are the ordinary slide rule scales in circular form: the range 1–10 occupying one complete circle, there is thus no need for a second duplicate scale as the end of the scale being coincident with the beginning makes an endless series of logarithmic scales, and numbers can thus be read off from one to the infinite without interruption.

Concentric with the outer logarithmic scale A is a scale of logarithms read with the cursor in the usual manner. On the inner disc are the scales of the square roots of the B scale (logarithmic scale on the outer edge of the inner disc); this scale is continuous, but the range 1–10 occupies two complete circles; the figures commence on the inner circle, go once round to the starting-point, and thence continue on the outer circle once round, again returning to the inner circle, and so on, *ad infinitum*.

Back Face.—The scales A and B for inverse proportion take the place of the front multiplying scales A and B, and are manipulated in the same manner. The Scale of Angles takes

the place of the scale of logarithms, and a scale of cube roots takes the place of the square root scale on the front face, and is continuous, similar to the latter scale, but the range 1–10 occupies three circles.

Users of the Slide Rule will appreciate the ease with which

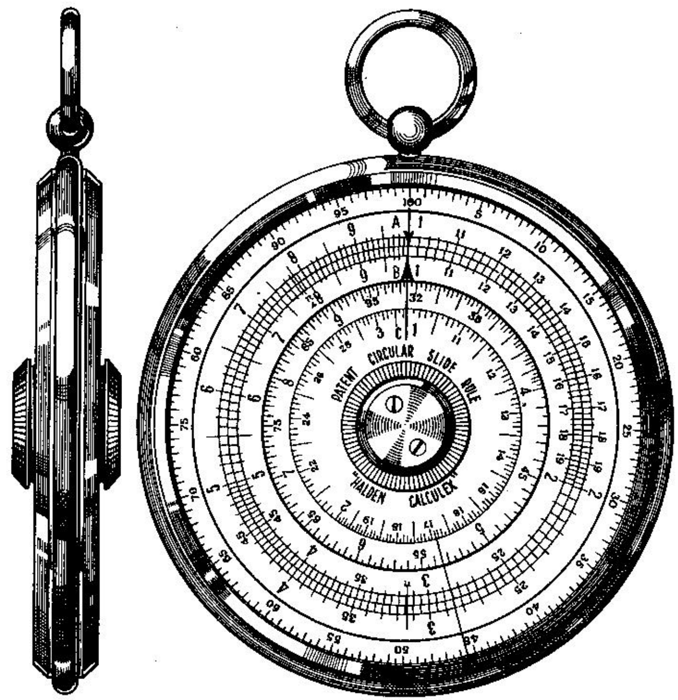


Fig. 46.—The Halden Calculex.

both square and cube roots (especially the latter) can be read off on the Calculex.

Explanation.—Before attempting any calculation upon the slide rule it is necessary to thoroughly understand the reading and value of the figures, both in decimals and whole numbers.

In all calculations, we ignore the decimal point until the finish and then place it (thus fixing the number of digits in the answer) by the methods described later.

The figures shown on the dial may have any value assigned to them, thus 7 may be ·007, ·07, ·7, 7, 70, 700, etc., and this will be more clearly seen by counting round the dial several times in sequence. For clearness, set the large arrows (front face) together, then, starting on the calculating scales A and B, at the top we read 1, following round in a clockwise direction we find 2, roughly at 4 o'clock; the figures 11, 12, 13–19 may be read as 1·1, 1·2–1·9. Reading on we find 3, near 6 o'clock, then 4, 5, 6, 7, 8, and 9 in sequence, and it should be noted that the successive intervals get smaller and smaller.

Following on round we reach 1 again, which now reads as 10; then 11, 12, 13, etc., up to 2, which now becomes 20; the large division (each divided again into halves) may be read as 21, 22, etc., up to 25 (marked, and thence to 3, 4, and 5, which are now 30, 40, and 50. It will now be found that between 50 and 60 there are only 10 divisions, which may be read as 51, 52, etc., to 60, and thence on past 70, 80, and 90 to 1, which now reads as 100.

Going round again, the small divisions are now 102, 104, 106, etc., to 11, which now reads 110, 12 becomes 120, and so on to 2, which is now 200.

The small divisions are now 205, 210, 215, etc., to 25, which is now 250.

3 becomes 300, 4 becomes 400, and so on until we reach 1, which is now 1000.

Again, starting at 1 as before, and reading backwards, we should read ·9, ·8, ·7, etc., until we get back to 1 again, which would then be ·1, and if we go round again we should have ·09, ·08, ·07, etc., and the small divisions between would have corresponding values, thus, ·099, ·098, ·097, etc.

Multiplication (Front Face).—Set the arrow on B to one factor on A, and over the other factor on B read the answer on A.

Example: 3×4.

Set the arrow on B to 3 on A, and over 4 on B you read 12 on A. It will also be seen that the two scales A and B form a multiplication table of the number to which the arrow B is pointed.

Note also that the same operations would hold good for 3×40, 30×40, $\cdot3\times4$, etc., the only difference being the position of the decimal point.

If there are three factors to multiply, proceed as above with the first two factors. Set arrow on B to the product and read result on A opposite third factor on B.

Another way—on inverse scale, set the first factor on A to second on B. Turn over to front face and read result on B opposite third factor on A.

Example: $6\times2\times5$.

On inverse scale, set 6 on A to 2 on B. Turn over and read 60 on B opposite 5 on A.

To fix the position of the decimal point: the best and easiest way to do this is by inspection or mental calculation, thus—

3×4	is obviously	12	and not	1·2 nor 120.
3×40	"	120	"	12 nor 1200.
30×40	"	1200	"	120 nor 12,000.
$\cdot3\times\cdot4$	"	·12	"	1·2 nor ·012.

For those who prefer a rule the following is given:

The number of figures in the product equals the number of figures in the two factors added together.

The number of figures in the quotient equals the number of figures in the dividend less those in the divisor, when the divisor comes anywhere between the two arrows from the right of A1; but if it comes between the arrows from the left, then add 1, being the reverse of that in multiplication. 0's between decimal point and significant figures count as negative digits as before.

Example: $5950\div70=85$.

The divisor comes between the arrows from the right of A1, and as there are four figures in the dividend and two in the divisor, $4-2=2$ figures in the quotient.

Example: $-852\div12=71$.

Here the divisor is found from between the two arrows from the left of A1, therefore $3-2=1+1=2$ figures in the quotient.

Proportion (Front Face).—Set the first term on B to the second term on A, and over the third term on B read the answer on A.

Example: As 6 : 42 : : 9 : X

Set 6 on B to 42 and over 9 on B read 63 on A.

When the scale is set to any proportion it gives a complete range of proportion all in the same ratio.

Example: If a man can finish an article in 3 minutes (easily), how many should he do in a day of 10 hours?

Set 3 on B to 600 minutes on A, and over the arrow or 1 on B read the answer 200 on A.

Inverse Proportion (Back Face).—When more requires less, and less requires more.

Example: If 10 men will do certain work in 18 days, how many should do the same work in 6 days?

Set 10 on B to 18 on A, and under 6 on A read the answer 30 on B.

Also see the examples given under "Levers, Speeds of Pulleys, etc."

When the scale is set it gives a complete range of factors having a common product.

Square Roots (Front Face).—Set the cursor to any number on B, and read the root of same under the cursor on Scale C.

If the number has an odd number of digits, read the roots on the centre circle, but if it has an even number of digits then read the root on the larger circle.

Example: Find $\sqrt{60}$.

Set the cursor to 60 on B, and as the figures are even, read 7·75 under the cursor on the larger circle of C.

Example: Find $\sqrt{530}$.

Set the cursor to 530 on B, and as the figures are odd, read 23 under the cursor on the smallest scale of C.

The following table shows the relative values of Scales B and C:

From	1 to	10	on B read units on smallest circle C.
From	10 to	100	on B read units on larger circle C.
From	100 to	1,000	on B read tens on smallest circle C.
From	1000 to	10,000	on B read tens on larger circle C.

Cube Roots (Back Face).—The figures on this scale read in the reverse direction to the square roots on the Front Dial.

Set the cursor to any number on Scale B, and its cube root will be found under the cursor on Scale C.

Example: Find $\sqrt[3]{162}$.

Set the cursor to 162 on B, and read the answer 5·45 on the largest circle of C.

The relative values of Scales B and C.

From	1 to	10	on B read units on smallest circle C.
From	10 to	100	on B read units on middle circle C.
From	100 to	1,000	on B read units on largest circle C.
From	1000 to	10,000	on B read tens on smallest circle C.

Squares and Cubes of Numbers.—Set the cursor to the number on C, and read the answer on B.

Example: Find the cube of 30.

Set the cursor to 30 on C (Back Face), and read 27,000 on B.

To Multiply or Divide Squared Numbers.—Example: $4{\cdot}25^2 \times 3$.

Set 1 on C to 3 on A, and over 4·25 on C read 54·2 on A.

Example: $8^2 + 16$.

Set 8 on C to 16 on A, and under 1 on A read 4 on B.

Find $\sqrt[4]{95}$ = the sq. root of the sq. root.

Set the cursor over 95 on B, and read the sq. root 9·74 on C. Set cursor to 9·74 on B, and read 3·12 the 4th root on C.

Find the 4th power of 2·45.

Set the cursor to 2·45 on C, and read 6 on B. Set cursor to 6 on C, and read 36 on B.

Find $\sqrt[9]{95}$ = the cube root of the cube root.

In extracting cube roots it must be remembered that the figures run from left to right on this scale.

Set cursor over 95 on B, and read the cube root 4·56 on middle scale. Set cursor to 4·56 on B, and read cube root 1·66 on smallest circle.

Logarithms (Front Face).—Set the cursor to any number on Scale A, and read the logarithm radially opposite to it on the outer scale.

Example: Find log. 15.

Set the cursor over 15 on A, and read ·176 on outer scale, this is the mantissa of the log. The characteristic must be determined by the ordinary rule for logarithms.

To convert common logs. to hyp. logs. $\times 2{\cdot}3$.

To convert hyp. logs. to common logs. $\times {\cdot}43$.

Trigonometrical Ratios (Back Face): *Sine.*—Set the cursor to the angles in the outside circle, and read its value under the cursor on A.

Cosine.—Take the sine of the complement of angle.

Tangent.—Tan. $A = \dfrac{\text{Sin. } A}{\text{Cos. } A}$.

Another method: Set 1 on B to the angle on the outside circle (Sine Scale) and under the complement (90 − angle) on the Sine Scale read Tan. on B.

Example: Find Tan. 30°.

Set 1 on B to 30° on Sine Scale and under 60° (90−30) read ·577 on B.

Cotangent.—Cot. $A = \frac{\text{Cos. } A}{\text{Sin. } A}$.

Another way: Set 1 on B to the complement of the angle on the outside circle, and under the given angle on this circle read Cot. on B.

Secant.—Take cosec. of the complement of angle.

Cosecant.—Set 1 on B to 1 on A. Opposite any angle on outside circle read cosec. on B.

To reduce Vulgar Fractions to Decimals.—Set the denominator on B to the numerator on A, and the arrow will point to the required decimal on A.

Example: Reduce $\frac{5}{8}$ to decimals.

Set 8 on B to 5 on A, and the arrow points to ·625, the answer on A.

Example: Reduce $\frac{5}{32}$ to decimals.

Set 32 on B to 5 on A, and read ·156 on A opposite the arrow 1 on B.

Decimals to Common Fractions.—Find common fraction of ·875.

Set 1 on B to ·875 on A and read $\frac{7}{8}$ on A and B.

To Convert Weights and Measures into Decimals.—Set 8, 16, 32, or 64 inches—12 for feet or 36 for yards—upon B to 1 on A, and over any number of fractions on B read decimal equivalents on A.

Example: Find decimal equivalents of an inch in 8th, 16th, or 32nd.

Set the denominator 8 on B to 1 on A, and over the numerator, say 4, on B read ·5 on A; also over 5 read ·625, and over 6 read ·75, etc. The same rule applies to 16ths, 32nds, or any other fraction.

In the above manner any weights, measures, or money may easily be converted into decimals by using 12 shillings, 20 pounds, 16 for lbs., 112 for cwts., etc.

Example: What is the decimal equivalent of 8d.?

Set 12 on B to 1 on A, and over 8 on B read ·667, the answer on A.

Example: What is the decimal equivalent of 12s.?

Set 20 on B to 1 on A, and over 12 on B read ·6 on A.

Rectangles into Equivalent Squares.—Set one side on B to arrow A, under other side on A read area on B and side of square on C.

Example: Find equivalent square of 8 in. × $\frac{1}{2}$ in.

Set 8 on B to arrow on A, and opposite ·5 on A read 4 (area) on B and 2 (side of sq.) on C.

Another method (Back Face): Set the length on A to the width on B, move the cursor line round until the reading on A is same as on B. This occurs at 2 (which is side of square required).

The area, 4, will be found under the 1 on either A or B.

Equal readings on A and B also occur at 6·3, this is side of square equivalent to 8×5 or 80×·5, and the area 4 under a row is, of course, 40. Which of the two is the correct one must be decided by mental calculation.

Diagonal of any Square.—Set 70 on B to 99 on A, and over the side of any size square on B read the diagonal of same on A.

Rectangles of any Proportions, all the same area.—Set 1 on B (on inverse scale) to the required area on A, and any measurement on B multiplied by that opposite to it on A=the same area.

Example: A tank to contain a certain quantity must have 66 ft. bottom area, and can only be 5 ft. 6 in. wide; what will its length be?

Set (on inverse scale) 1 on B to 66 on A, and opposite 5·5 read 12 ft., the answer.

To Find Area of Surface.—Set g.p. (from table below) on B to length on A, and opposite breadth on B read area on A.

Dimensions.	Result in				
	Sq. in.	Sq. ft.	Sq. yds.	Acres.	Sq. miles.
In. × in.	1	144	1296	..	..
Ft. × in.	·83	12	108	..	..
Ft. × ft.	·0694	1	9	..	..
Yds. × ft.	..	·33	3	..	..
Yds. × yds.	..	·11	1	4840	..
Rods × rods	..	..	..	160	..
Chs. × chs.	..	..	..	10	6400

To Find Area of Polygon.—Set 1 on C to the constant in the following table on A, and over the length of one of the sides in ins. or ft. on C read area in same measured on A.

No. of sides, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Constant, ·433, 1·0, 1·72, 2·6, 3·63, 4·83, 6·18, 7·7, 9·36, 11·2.

Example : Find the area of an octagon, the length of one side being $2\frac{1}{2}$ in.

Set 1 on C to 4·83 on A, and over 2·5 on C read 30·25 on A.

Circles, Areas, etc.—Set 1 on C to ·7854 on A, and over any diameter on C read area on A, or *vice versa*.

Areas and Circumferences of Circles.—Set π (3·14) on B to 2·5 on A, and under any area on A find its circumference on C or *vice versa*.

Circles and Squares of Equal Area.—Set 7 on B to 6·2 on A, and over any diameter, of circle on B read the side of square equal to the same area on A, or *vice versa*.

Diameter in inches, area in square feet.—Set 144 on B to ·7854 on A, and over any diameter in inches on C read area in sq. ft. on A. 13·55 on C=1 sq. ft. on A.

Areas of Circles and Inscribed Squares.—Set 7 on B to 11 on A, and over area 0 inscribed square on B read area of circle on A.

Circumference .	3·14	A	Circumference .	6·28	A
Diameter .	1	B	Radius .	1	B
Side of inscribed square .	7	A	Side of square .	7	A
Diameter .	9·9	B	Diameter=to sq.	7·9	B

Over any diameter on B read circumference, or side of sq. on A, or *vice versa*.

Cubic Contents—*Prisms.*—Set 1 on B to length on A, set cursor to width on B. Set g.p. (below) on B under cursor, and find result on A opposite depth on B.

Wedges.—Find volume of prism of same length, breadth, and thickness, and divide by 2.

Pyramids.—Same procedure, but divide by 3.

Dimensions.	Result in				
	Cub. in.	Cub. ft.	Cub. yds.	Gal.	Bushels.
In.×in.×in. . .	1	1728	46656	276	2208
Ft.×in.×in. . .	·833	144	3883	23	184
Ft.×ft.×in. . .	·0695	12	324	1·92	15·3
Ft.×ft.×ft. . .	·0058	·1	27	·16	1·27

Cylinders and Pipes.—Set g.p. (below) on B to length on A. Read result on A opposite diameter on C.

Cones.—Find volume of cylinder of same diameter and length and divide by 3.

Dimensions.	Results in				
	Cub. in.	Cub. ft.	Cub. yds.	Gal.	Bushels.
In. length×in. diameter	1·27	220	198	351	280
Ft. length×in. diameter	1·06	183	1642	29·2	234
Ft. length×ft. diameter	739	1·27	11·5	·204	1·62

Spheres.—Set g.p. (below on B) to diameter on A, and over diameter on C read volume on A.

Dimensions.	Result in				
	Cub. in.	Cub. ft.	Cub. yds.	Gal.	Bushels.
In. diameter . .	1·91	3300	29970	528	4220
Ft. diameter . .	·00111	1·91	17·19	·306	2·45

MECHANICAL POWERS

Inclined Planes, force to support weight upon.

Set height of plane on B to length of same on A, and under weight on A read force on B, also over arrow read ratio.

Example : What force will be required to support 500 lb. on an incline 30 in. long by 5 in. rise ?

Set 5 in. on B to 30 in. on A, and under 500 lb. on A read 83·5 lb. on B. Also read as 1 on B is to 6 on A=the ratio of either the force to the weight, or the length of incline to the height.

When the ratio of either force to weight or length of incline to height is decided—

Then set, say, 1 on B to 6 on A, and under any length on A read height of incline on B. Also under any weight on A read the force to sustain same on incline. This does not include friction.

Power of Screws.—Set the pitch of screw in decimal parts of an inch on B to the circumference described by the force in inches on A, and under any weight on A read the force to turn the screw on B. Also the ratio of weight to force over 1 on B.

Example : What force is required at the end of a lever to equal a pressure of 400 lb. by a screw with a $\frac{3}{4}$-in. p. and a lever 15 in. long ?

Set ·75 on B to (15 in.=) 94 in. cir. on A, and under 400 lb. on A read 3·2 lb. force required on B. Also the ratio of force and speed 1 on B to 125 on A.

This does not include friction (allow, say, 30 per cent.).

Levers and Wheels.—Assuming the weight or resistance to be overcome by a lever or wheel to be 300 lb. and force to balance same 50 lb.—

Set 50 lb. on B to 300 on A (the arrow points to the ratio of either force or speed as 1 is to 6), and under any length of lever or radius of wheel on A read length of short arm or radius of pinion on B.

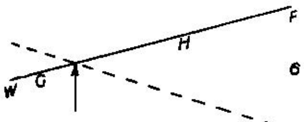


Fig. 47.

For all levers $\frac{GW}{H}=F$.

G=distance fulcrum to weight (W).
H=distance fulcrum to force (F).

Example : If the distance from the fulcrum of a lever to the weight is 3 in. and the weight to be raised is 200 lb., what length should the long arm of lever be with a force of 30 lb. ? Also what force would be required if the long arm were 24 in. long ?

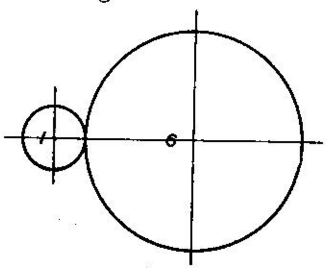


Fig. 48.

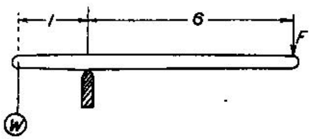


Fig. 49.

Figs. 47 to 49.—Diagrams illustrating lever and wheel calculations.

(Back Face) Set 3 in. on B to 200 lb. on A, and under 30 lb. on A read 20 in. length of lever on B, and over 24 in. length of lever on B read 25 lb. on A.

HYDRAULICS

Weight and Volume of Water.—When any of the following combinations are set upon the calculator, scales A and B form a complete table of either weights and volumes or volumes and weights.

Ft. c.	1	on A	Imperial pints	1·76	on A
Lb.	62·5	on B	Litres	1	on B
Ft. c.	61	on A	Cubic in.	104	on A
Cwt.	34	on B	Pints	3	on B
Tons	1	on A	U.S. gal.	1·2	on A
Ft. c.	36	on B	Imperial gal.	1	on B
Gal.	100	on A	Cubic in.	61	on A
Ft. c.	16	on B	Litres	1	on B
Gal.	224	on A	Litres	50	on A
Tons	1	on B	Imperial gal.	11	on B
Pounds	10	on A	Lb. fresh water	39	on A
Imperial gal.	1	on B	Lb. sea water	40	on B

Volumes of Square or Cylindrical Tanks, Pipes, etc., in cubic in., cubic ft., or gal. (See Cubic Contents, pp. 76 and 77.)

Discharge of Pipes.—Set g.p. on B to velocity on A, and read discharge on A opposite diameter on C.

Velocity and Diameter.	Discharge in				
	Cub. in. per min.	Cub. ft. per min.	gal. per min.	gal. per hr.	tons per hr.
Ft. per sec. × in. diameter	177	305	420	700	312
Ft. per min. × in. diameter	106	183	292	420	188
Ft. per min. × ft. diameter	729	127	204	340	152

For discharge of oils in tons per hour—multiply above results by specific gravity.

Pressure per sq. in. due to Head.—Set 60 ft. on B to 26 lb. on A, and over any head in ft. on B read pressure in lb. per sq. in. on A.

Force per sq. ft. against a plain surface, at right angles to the motion—

Set arrow to ·976 on A, and over any velocity in ft. per second on C read pressure in lb. per sq. ft. on A (8 ft.=62·5 lb.).

Discharge of Pump.—Set 35·3 on B to length of stroke in in. on A and over any diameter of barrel on C read gal. per stroke on A × number of strokes=gal. per minute. An allowance of 10 to 40 per cent. must be made for loss by leakage.

Ice, weight of, at 32°, 9 ft. cube=520 lb.

Set 9 ft. c. on B to 520 lb. on A, and over any number of ft. c. on B read weight in lb. on A.

MACHINERY

Gear Wheels.—To find the diameter.—Set circumferential pitch in inches on B to 3·141 on A, and under number of teeth on A read diameter on B.

To find the circumferential pitch.—Set diameter in inches on B to number of teeth on A, and under 3·14 on A read pitch on B.

To find number of teeth.—Set circumferential pitch in inches on B to 3·14 on A, and over diameter in inches on B read number of teeth on A.

To find the diameter of a pair of wheels, centres and speed given.—Set the revolutions of both wheels added together on B to twice the distance of centres apart on A, and over revolutions of wheels on B read diameters on A.

Example: Two shafts, Y and Z, 36 in. apart, Y runs 80 revolutions, and is required to drive Z 120 revolutions. What will be the diameters of the wheels?

Set 120+80=200 on B to 36×2=72 on A, and over the revolutions of each shaft on B read $28\frac{3}{4}$ and $42\frac{1}{4}$, the required diameters on A.

Pulleys and Wheels.—On inverse scale set diameter of pulley or number of teeth in wheel on B to its speed or revolutions on A, and under any speed of driving shaft on A read diameter of driving pulley or number of teeth in wheel on B.

Example: A machine has a 16-in. pulley, which must be driven 160 revolutions per minute from a shaft running at 95 revolutions. What size pulley will be required on same?

On inverse scale set 16 in. on B to 160 on A, and under 95 on A read 27 in. on B.

Belts.—Set 382 on B to the revolutions of pulley per minute on A, and over diameter in inches on B read speed of belt in ft. on A.

Horse-power.—Set 660 on B to speed of belt in ft. per minute on A (see above rule), and over width of belt in inches on B read h.p. on A.

Width of, for a given h.p.—Set as above, and under h.p. on A read width of belt in inches on B.

Double belts: instead of 660, use 38 for g.p.

Shafting, h.p. of—For ordinary mill work:

Set g.p. on B to cube of shaft diameter on A, and over any number of revolutions per minute on B read h.p. of A.

Example: What is the h.p. of a 3-in. shaft, running revolutions per minute bending moment=$\frac{1}{2}$ twist?

Set g.p. 57 on B to 27 (3^3) on A, and over 120 revolutions on B read 56·9 h.p. on A.

Diameter of Shaft.—Set revolutions of shaft per minute on B to h.p. on A, and over g.p. on B read cube of shaft's diameter in inches on A, the root of which equals shaft's diameter.

Gauge Points (corresponding to 9000 lb. sq. in. stress):

For twisting only	35·6
For twisting plus bending (=$\frac{1}{2}$ twist) .	57
For twisting plus bending (=1 twist) .	86
For twisting plus bending (=$1\frac{1}{4}$ twist) .	118

Span Between Bearings.—Span in ft. $K\sqrt[3]{D^2}$.

On inverse scale, set D (in.) on A to D (in.) on B. Read $\sqrt[3]{D^2}$ on C opposite 1 on A. Set this figure on to K on A and read Span (ft.) on B opposite 1 on A.

K (for shafts without pulleys) . . .	=6
(for shafts with ordinary number of pulleys).	=5
(for shafts heavily laden) . . .	=4

BEAMS AND CANTILEVERS

To Find Bending Moment.—Set g.p. on B to length of span in ft. or in. on A, opposite load in lb. or tons on B, read bending

6

moment on A in lb.-in., lb.-ft., tons-in., or tons-ft., according to units used.

G.P.'s.—	Cantilever fixed one end,	load at other	1
	Cantilever fixed one end,	load distributed	2
	Beam, supported both ends,	load at centre	4
	Beam, supported both ends,	load distributed	8
	Beam, fixed both ends,	load at centre	8
	Beam, fixed both ends,	load distributed	12

Example: What is bending moment on a cantilever 10 ft. long loaded with 500 lb. at end ?

Set 1 on B to 10 ft. on A, and opposite 500 on B read 5000 lb. ft. on A.

What is bending moment on a beam 12 ft. long supported at each end, and carrying uniformly distributed load of 900 lb. ?

Set 8 on B to 12 on A, and opposite 900 on B read 1350 lb. ft. on A.

Cutting Speed.—Set 3·82 on B to diameter (in.) on A, and opposite r.p.m. on B read cutting speed (ft. per minute) on A.

Time on Job.—Set r.p.m. on B to length of the job on A, and opposite cuts per in. on B read time (minutes) on A.

STEAM AND PETROL ENGINES

Indicated Horse-power.—Set the diameter of cylinder on C to g.p. on A. Now set the number found on B under the piston speed in ft. per minute on A to 1 on A, and under any mean steam pressure in lb. per sq. in. on A read i.h.p. on B.

G.P. for double-acting steam engine	42
G.P. for single-acting steam engine	84
G.P. for 2-stroke petrol engine	84
G.P. for 4-stroke petrol engine	168

Example: A steam engine has an 8-in. cylinder with a piston speed of 300 ft. per minute and a mean steam pressure of 40 lb. per sq. in.

Set 8 on C to 42 on A, and under 300 ft. on A find 45·6, which set to 1 on A, and under 40 lb. on A read 18·2 h.p. on B, or under 45 lb. read 20·5 h.p., etc.

Taxable H.P. (or Treasury Rating) of Car.—Set g.p. 2·5 on B to number of cylinders on A. Read h.p. on A opposite cylinder bore on C.

Speed of Motor Car, Train, or Ship.—Set seconds per mile on scale B to g.p. 36 on scale A. Read miles per hour on A by arrow on B. (To read knots, set seconds per mile on B to g.p. 313 on A.)

Brake Horse-power.—Set 1 on B to the i.h.p. found above on A. Read B.H.P. on A opposite mechanical efficiency on B.

Expansion of Steam in Cylinder : *On Inverse Scale.*—Set the initial pressure of steam in lb. per sq. in. on B to the distance travelled by the piston at the point of cut-off on A, and under any other distance travelled by piston on A read the pressure of steam in cylinder in lb. per sq. in.

Example: The initial pressure in a cylinder at the time of cut-off is 75 lb. per sq. in., and the point of cut-off is 6 in.

On Inverse Scale.—Set 75 lb. on B to 6 in. on A, and under 10 in. read 45 lb.; 25 in., 18 lb., etc.

Heating Surface of Boiler Tubes.—Set 3·82 on B to diameter (in.) on A, opposite total length (ft.) on B. Read area in sq. ft. on A.

PUMPING ENGINES

Set diameter of pump in inches on C to the effective steam pressure in lb. on A, and under the pressure on pump in lb. per sq. in. on A read diameter of cylinder on C.

Example: An 8-in. pump is required to work against a pressure of 78 lb. per sq. in., the effective steam pressure is 15 lb., what should be the diameter of the cylinder ?

Set 8 on C to 15 lb. on A, and under 78 lb. on A read 18·25 in. diameter on C.

For efficient working this must be increased from 15 to 30 per cent. according to circumstances. Therefore, set arrow (100) on B to area 262 in. on A, and over, say, 125=25 per cent. on B read area 327=20·4 in. diameter on A.

Head in ft. × ·434 or yds. × 1·3=lb. per sq. in.

ELECTRICITY

Ohm's Law :

$$C=\frac{E}{R},\ R=\frac{E}{C},\ E=RC.$$

C=Current, E=E.M.F., R=Resistance.

These equations are of the form $\frac{a}{b}=x,\ ab=x.$

Operations for solving them are given on p. 105, equations 1 and 2.

$$C=\frac{NE}{R},\ R=\frac{NE}{C},\ N=\frac{CR}{E}.$$

E=E.M.F. per cell. N=No. of cells.

These are of the form $\frac{ab}{c}=x$. (See equation 7, p. 106.)

Current through parallel Resistances: *On Inverse Scale.*—Set joint resistance on B to total current on A, and opposite resistance of separate branches on B read current on A.

Example: 20, 50, and 100 ohms in parallel have 6 amperes through them. What current will pass each branch?

Joint resistance of 20, 50, and 100=12·5.

Set 12·5 on B to 6 on A, and opposite 20, 50, and 100 on B read 3·75, 1·5, and ·75 amperes.

Kilowatts and H.P.—Set 1·34 on B to 1 on A, opposite H.P., on B read kilowatts on A.

BUILDING

Brickwork.—1 rod=272 ft. supr. 1½ brick thick=306 cub. ft. =11⅓ cub. yds.=4350 stock bricks in mortar=15 tons or 110 lb. per cub. ft.

To reduce brickwork to standard thickness, 1½ brick thick, or to ft. cube:

Bricks Thick	½	1	..	..	..	..	1½	..	..	..	..	2	2½	3
Inches Thick	4½	9	10	11	12	13	13½	14	15	16	17	18	22½	27
Ft. Supr. 1 Rod	816	408	366	334	306	282	272	262	245	230	216	204	163	136
Ft. Supr. on A	27	27	27	27	27	27	27	27	27	27	27	27	27	27
Ft. Standard on B	9	18	20	22	24	26	27	28	30	32	34	36	45	54
Ft. Supr. on A	24	24	24	24	24	24	24	24	24	24	24	24	24	24
Ft. Cube on B	9	18	20	22	24	26	27	28	30	32	34	36	45	54
No. Bricks on A	214	428	272	521	568	618	640	662	720	759	806	854	1065	1280
Ft. Supr. on B	40	40	40	40	40	40	40	40	40	40	40	40	40	40

When any of the above combinations are set together on scales A and B, the latter form a complete table of ft. supr. and ft. standard or ft. supr. and ft. cube, etc.

Example: Reduce 42, 57, 130 ft. supr. 18-in. work to standard work 1½ brick thick.

Set 36 on B to 27 on A (see Table), and below any ft. supr. on A read ft. standard on B thus—

Ft. supr. 18-in. thick on A	27	42	57	130
Ft. standard 13½ in. on B	36	56	76	173

Ft. supr. to ft. cube may also be found in same manner, also ft. supr. and number of bricks.

WEIGHT OF MATERIALS PER FT. CUBE

Concrete	120–140	Marble	170
Earth	80–120	Portland Stone	511
Granite (Aberdeen)	163	Sand	118
Limestone (Purbeck)	162	Slate, Welsh	180

USEFUL TABLES

Measurement

Ft. cube	45 on A
Ft. standard	40 on B
Yds. cube	34 on A
Rods standard	3 on B

No. of Bricks

1000 bricks	27 on A
Yds. cube	70 on B
No. bricks	355 on A
Ft. cube	25 on B

Nogging

Bricks flat	900 on A
Yds. supr. brick nogging	20 on B
Bricks on edge	600 on A
Yds. supr. brick nogging	20 on B

Arches

Bricks	200 on A
Ft. supr. gauged arches	20 on B

Paving

Bricks on edge	1040 on A
Yds. supr. of paving	20 on B
Paving bricks laid flat	720 on A
Yds. supr. of paving	20 on B

USEFUL TABLES—*continued*

Paving—*continued*

Paving bricks on edge .	1640 on A
Yds. supr. of paving	20 on B
10-in. paving tiles	260 on A
Yds. supr. of paving	20 on B
12-in. paving tiles .	180 on A
Yds. supr. paving .	20 on B
Dutch clinker on edge .	2800 on A
Yds. supr. of paving	20 on B

Mortar

Brickwork c. ft. .	87 on A
Mortar in same .	20 on B
1000 bricks .	3300 on A
Loads mortar .	2 on B

Weight

Cube ft. . .	56 on A
Cwts. brickwork .	55 on B
Rods standard .	1 on A
Tons brickwork .	15 on B

Prices.—When any of the following combinations are set on scales A and B they form a complete table of the factors in question:

£ per		1·
Pence per ft. supr.	3 bricks thick	1·77
,, ,, ,,	$2\frac{1}{2}$,, ,,	1·47
,, ,, ,,	2 ,, ,,	1·18
,, ,, ,,	$1\frac{1}{2}$,, ,,	·883
,, ,, ,,	12 in. (or cub. ft.) thick	·785
,, ,, ,,	1 brick thick	·59
,, ,, ,,	$\frac{1}{2}$,, ,,	·294
Shillings per cub. yd.		1·77

Example: Find weight of 85 cub. ft. of brickwork in cwts. Set 55 on B to 56 on A (see Useful Tables), and under 85 on A read 83·5 cwt. on B.

Example: Find number of bricks required in 84 cub. ft. Set 25 on B to 355 on A, and over 84 on B read 1195 on A.

Land Measure.—Length and breadth (or depth and frontage) to acres. Set depth on B to g.p. on A, then read:

(i) Frontage per acre on A opposite 1 on B.
(ii) Acreage of plot on B opposite frontage on A.
(iii) £ per acre on A opposite £ per unit frontage on B, or *vice versa.*

Chains × chains,	g.p.= 10.
Rods × rods,	g.p.= 160.
Yards × yards,	g.p.= 4840.
Ft. × ft.,	g.p.=43560.

Other Useful Figures.—When any of the following combinations are set on scales A and B they form a complete table of the factors in question:

£ per acre	1·0
Shilling per sq. yd. . . .	·00412
Pence per sq. yd. . . .	·0494
Shillings per sq. rod . . .	·125
Pence per sq. rod . . .	1·5

1. Calculation of Co-ordinate of a survey.

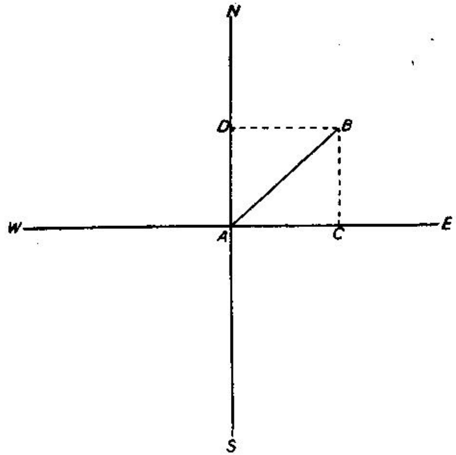


Fig. 50.—Calculation of Co-ordinate of a Survey.

Co-ordinate of point *B*.
Line *A–B*. N. 46° E. 234 links.
Latitude *A–D* $=234 \times \cos DAB$.
$=234 \times \cdot 695 = 162 \cdot 6$ links.
Departure *B–D* $=234 \times \text{sine } DAB$.
$=234 \times \cdot 719 = 168 \cdot 3$ links.
Lat. 162·6 N. Dep. 168·3 E.

This can be calculated to one decimal place with the circular rule.

2. Given a roadway dipping 1 in 6 (say), to find dip in degrees and inches per yard and deduction of links per chain to convert dip distance to horizontal distances. The horizontal distance *A–B* is of course equivalent to the departure (in previous example), *i.e.* *C–B* × Angle of dip.

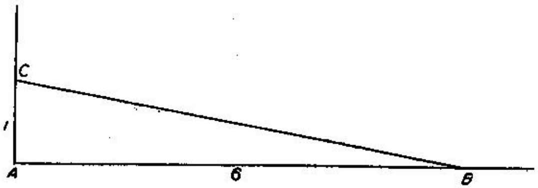


Fig. 51.—Finding angle of dip.

In connection with angles it is important to remember the following trigonometrical functions :

$$\begin{cases}\text{Sine} = \dfrac{\text{Perp.}}{\text{Hyp.}} \\ \text{Cosecant} = \dfrac{\text{Hyp.}}{\text{Perp.}}\end{cases} \quad \begin{cases}\text{Cosine} = \dfrac{\text{Base}}{\text{Hyp.}} \\ \text{Secant} = \dfrac{\text{Hyp.}}{\text{Base}}\end{cases} \quad \begin{cases}\text{Tangent} = \dfrac{\text{Perp.}}{\text{Base}} \\ \text{Cotangent} = \dfrac{\text{Base}}{\text{Perp.}}\end{cases}$$

It will be seen that some of the ratios are reciprocals.

Timber, Deals, etc.—To reduce feet run of any section to Petrograd Standard = 1440 ft. $1\frac{1}{2}$ in. × 11 in.

Width in Inches.	Thickness in Inches.		
	$2\frac{1}{2}$ in.	3 in.	4 in.
6	1584	1320	990
$6\frac{1}{2}$	1462	..	..
7	1357·7	1131·4	858
8	1188	990	742
9	1056	880	660
10	950·4	792	594
11	864	720	540
12	792	660	495

Set 1440 ft. (=Petrograd Standard) on B to number of feet run according to section on A (see Table), and below any number of feet of that section on A, read equivalent number of feet in Petrograd Standard on B.

Example : Reduce 2600 ft. 3 in. by 7 in. to Petrograd Standard.

Set 1440 on B to 1131 on A, and under 2600 ft. on A find 3310 ft. standard on B.

Read feet run on A and standard on B.

Feet Run per Square (on inverse scale).—Set 6 in. on B to 200 ft. run on A and over any other width in inches on B read ft. run per square on A, over 8 in. read 150 ft., over 12 in. 100 ft., etc.

Number of Squares in a Standard, any thickness (on inverse scale).—Set 3 in. on B to 6·6 on A, and over any other thickness read square per standard on A.

Feet Run per Square Yard (on inverse scale).—Set 6 in. on B to 18 ft. run on A, and over any number of inches in width on B read ft. run per sq. yd. on A.

Square Yards per Standard (on inverse scale).—Set 1 in. on B to 220 sq. yds. on A, and over any other thickness on B read sq. yds. per standard.

Feet Run of any Section in One Cubic Foot (on inverse scale).—Set 9 in. on B to 16 ft. on A, and over any number of sq. in. in any section on B read ft. run per cubical ft. on A ; over 6 in. on B read 24 on A, and over 8 in. read 18 ft. on A.

Cubic Feet in a Given Number of Feet Run of any Section.—On front face set 144 on B to the length in ft. run on A, and over the area of section in sq. in. on B read cub. ft. on A.

Example : How many cub. ft. are there in 136 ft. $4\frac{1}{2}$ in. by 2 in. ?

Set 144 on B to 136 on A, and over 9 in. area on B read $8\frac{1}{2}$ in. cubic ft. on A.

Also the cub. ft. for any other area of the same length . Over $4\frac{1}{2}$ in. read $4\frac{1}{2}$ in., ft., c., etc.

Round Timber.—To find the contents of round timber in cub. ft. the length is usually multiplied by the square of the 1–4th of the mean girth with an allowance of from 1–10th to 1–20th made for bark. This gives a result 21·5 per cent. low.

Rule.—Set the g.p. on C to the length on A, and over the girth in inches on C read cub. ft. on A.

	Allowance.	Quarter Girth.	Whole Girth.
Oak, old thick bark .	$\frac{1}{10}$=	12·66	50·64
Oak, young thin bark .	$\frac{1}{12}$=	12·55	50·2
Elm, pine, fir . .	$\frac{1}{15}$=	12·43	49·72
Ash, beech . . .	$\frac{1}{20}$=	12·3	49·2
No bark . . .	..	12	48

Example : Find contents of an elm 30 ft. long, 60 in. girth.

For whole girth : set 51·48 on C to 30 ft. on A, and over 60 in. on C read 40·8 ft. on A.

For $\frac{1}{4}$ girth.—Set 12·87 on C to 30 ft. on A, and over 15 in. (=$\frac{1}{4}$ girth) on C read 40·8 ft. on A.

Hewn Timber and Scantlings.—Given length, breadth, and thickness. Cubic contents may be calculated by cubic contents formulæ on p. 76.

Example : Find cubic contents in 20 ft. of timber 16 in.× 18 in.

Set 1 on B to 20 on A. Set cursor to 18 in. on B.

Set g.p. 144 on B under cursor and find result on A opposite 16 in. on B=40 cub. ft.

If section in sq. in. be given and length in ft., set 144 on B to length on A, and opposite section on B read contents in cub. ft. on A.

Example: Take same as above, 16 in.×18 in.=288 sq. in.

Set 144 on B to 20 ft. on A, and over 288 on B read 40 cub. ft. on A.

Wallpaper.—Number of pieces of ordinary English paper required to cover a room.

Set the height of room in ft. on B to 60 on A, and under the measurement round walls in ft. on A read number of pieces of paper required on B.

For French paper add 50 per cent.

Allowance must be made for doors and windows.

Example : The height of a room is 11 ft., and measurement round same is 68 ft. How many pieces will be required ?

Set 11 ft. on B to 60 ft. on A, and under 68 ft. on A read 12·5 pieces on B.

Slates.—Weight and No. per sq. ft., allowing 3 in. lap :

Name and Size.	No. per sq.	Sq. yds. per 1200.	Cwt. per 1200 Average.	A=No. of Slates. B=Sq. ft.
Duchess, 24×12 .	115	116	60	A 80 B 70
Marchioness, 22×11 .	138	97	60	A 80 B 58
Countess, 20×10 .	170	78	40	A 85 B 50
Viscountess, 16×10 .	220	60	35	A 90 B 41
Ladies, 16×8 . .	275	48	25	A 55 B 20
Small Ladies, 14×8 .	328	40	18	A 95 B 29
Doubles, 13×7 . .	412	32	15	A 70 B 17

Example : Find the number of Duchess slates to cover 860 sq. ft.

Set 70 on B to 80 on A (see Table), and under 860 sq. ft. on B, read 983 on A.

Example : Find the weight of the above slates.

Set 60 on B to 1200 on A, and opposite 983 on A read 49·2 cwts. on B.

STRENGTH OF MATERIAL

Crane Chains—Wrought Iron.—Example : Find the breaking-strain of a $\frac{1}{2}$ in. chain. The diameters of chains advance by 16ths of an inch in decimals.

Set 8-16ths diameter on C to 7·5 tons breaking-strain on A, and over any other diameter on C read its breaking-strain in tons on A. Over 9 read 9·49. Over 12($\frac{3}{4}$) read 16·89, etc.

Admiralty Proof Strain.—Set 8-16ths on C to 3 tons on A.

Safe Working Strain.—Set 8-16ths on C to 1·5 tons on A.

Stud-linked Cable Chains.—Set 8-16ths in. on C to 6·75 tons b.s. on A, and over any other diameter on C read its breaking-strain in tons on A.

Admiralty Proof Strain.—Set 8 on C to 4·5 tons on A.

Safe Working Strain.—Set 8 on C to 2·25 tons on A.

Ropes, White Manilla : Breaking-strain.—Set 3 in. cir. on C to 2·8 tons on A, and over any other cir. on C read its breaking-strain on A.

Working Load.—Set 3 in. cir. on C to 11·2 cwt. on A, and over any other cir. on C read safe load in cwts. on A.

Weight per Fathom.—Set 3 in. cir. on C to 1·6 lb. on A, and over any other cir. on C read weight in lb. per fathom on A.

Tarred Russian : Breaking-strain.—Set 3-in. cir. on C to 2·2 tons breaking-strain on A, and over any other cir. on C read breaking-strain on A.

Working load in cwt.=the square of the circumference, and over 3-in. cir. on C read 9 cwt. on B, etc.

Weight per Fathom.—Set 3-in. cir. on C to 2·2 lb. on A, and over any other cir. on C read lb. per fathom on A.

Wire Ropes (Bullivants) :

	Breaking Strain. Tons.	Working Load. Tons.
Crucible steel . .	24·75	4·12
Crucible steel—best sel.	26·5	4·4
Spec. mild Plough steel	29	4·8
Spec. extra Plough steel	31·75	5·3

To find either the breaking-strain, working load, or weight per fathom :

Set 3-in. cir. on C to any of the above g.p.'s on A, and over any other cir. on C read strength of rope on A, either for breaking-strain, working load, or weight per fathom.

Weight per Fathom.—6 strands 7 wires each=8·5 lb., and 6 strands 6 wires each=7·8 lb. for ropes of 3-in. circumference.

Stresses in Round Rod, Bolts, Wire, etc.—Set diameter of rod or wire (for bolts take diameter at bottom of thread) on C to g.p. on A and read stress on A for any load on B.

Load lb. ; stress tons per sq. in. g.p.=568
Load tons ; stress tons per sq. in. g.p.=127
Load lb. ; stress lb. per sq. in. g.p. =127

Suspension and Tie Rods, Wrought Iron.—Safe load at 7 tons per sq. in.

Round Bars.—Set 8 (eighths of an inch) on C to 5·5 tons safe load on A, and over any other diameter in eighths on C read safe load on A.

Square Bars.—Set 8 on C to 7 tons on A, and over side of square in eighths on C read tons safe load on A.

Bars, Round or Square, Compared.—Strength, weight, or area.

Set 1 on B to ·7854 on A. Then B gives area of square bar, A area of round bar, and C size of bar (diameter or side of square). Thus, opposite 2·5 on C will be found 6·25 on B (area of $2\frac{1}{2}$ square bar) and 4·9 on A (area of $2\frac{1}{2}$ round bar).

Also over the weight of a 2-in. round bar wrought-iron= 10·49 lb. per foot on B, read weight of 2-in. square bar=13·36 lb. per ft. on A. Also read relative tensile strengths in same manner.

In other words, a square bar is 25 per cent. stronger (tensile) and heavier than a round bar.

Ultimate Tensile Strength of Materials.—Per sq. in. :

	Lbs.	Tons.		Lb.	Tons.
Cast-iron . . .	17,000	7·5	Manganese Bronze .	60,000	27
Wrought-iron bars .	54,000	24	Muntz Metal .	49,000	22
Wrought-iron plate .	48,000	21·5	Ash . .	15,000	6·7
Mild steel . . .	70,000	31	Beech .	12,000	5·4
Copper bolts . .	36,000	16	Elm . .	13,000	5·8
Copper sheet . .	30,000	13·4	Pine . .	12,000	5·4
Copper wire . .	58,000	26	Leather .	4,200	1·88
Gun metal . .	32,000	14·1			
Phos. bronze . .	35,000	15·7			

Rule.—Set 1 *on B to strength of material in above Table on A, and over any area in inches on B read breaking in lb. or tons per sq. in. on A.*

WEIGHTS OF MATERIALS

Round and Square Bars : *Rule.—For a given length of any diameter at one setting. Set the g.p. on B* (see Table, p. 96) *to the length on A, and over diameter or side of square bar on C read weight on A. For weight per ft. take length=1 ft.*

Example : Find the weight per ft. of any diameter round iron.

Set the g.p. 381 on B to 1 (or any length) on A, and over any diameter on C read weight on A. See following example :

Scale A—To	1	2·62	10·5	23·6	42	=lb.
Scale B—Set 381						
Scale C—		1 in.	2 in.	3 in.	4 in.	=in. diameter.

Rule.—For any length of a given size of bar at one setting. Set the side or diameter on C to the g.p. on A (see Table), *and under any length on A read weight on B.*

Example : Find weight of 3-in. round bar, wrought-iron, any length.

Set 3 in. on C radially opposite the g.p. 281 on A, and under any length on A read weight on B as follows :

Scale A—To 381		1	2	3	4	=ft.
Scale B—	↕	23·6	47·2	70·8	94·4	=lb.
Scale C—Set 3 in.						

If any dimensions contain fractions convert them into decimals (see Rule, p. 74).

Weight of Globes, Spheres, etc.—Set the g.p. on B to the diameter on A, and over the diameter on C read the weight on A.

Example : Find the weight of a 6-in. cast-iron ball.

Set the g.p. 733 on B to 6 in. on A, and over 6 in. on C read 29·5 lb. on A.

Flat Bars, Rectangular Castings or Forgings.—Weight by Table on page 96.

Set 1 on B to length on A, set cursor to width on B. Set g.p. on B under cursor and find result on A opposite depth or thickness on B.

Example : Find weight of 100 ft. 3 in. × $\frac{3}{4}$ in. steel bar.

Set 1 on B to 100 on A. Set cursor to 3 in. on B. Set 294 on B under cursor and read 767 lb. on A opposite ·75 on B.

Hexagon Bars, Weight, etc.—Find weight of square or round bar of thickness=to hexagon across flats by rule, p. 94. Then haxagon bar=sq. bar × ·8661 or round × 1·103.

The above gives weights for any thickness at same setting. For weights per foot take length=1 ft.

Gauge Points for any other Material.—Set any of the following constants on B to the weight or capacity of 1 cubic ft. of the material on A, and under 1 on A read the required g.p. on B. FFF=1 on B. FII=144 ; III=1728. FI=183·3 ; II=22. F=19 ; I=33.

Pipes or Cylinders, Weight of.—Weight of pipe=weight of solid cylinder of outside diameter minus weight of solid cylinder of inside diameter.

G.P. :

Brass	363	Cast-iron	408
Copper	331	Wrought-iron	381
Gun metal	346	Lead	257

Set the g.p. on B to 1 ft. or any length on A, and over outside and inside diameter on C read weights of solid cylinders on A. The difference between them equals the weight of pipe or cylinder, which can be read off on the scale by the cursor.

For two flanges of cast-iron pipe reckon as 1 ft. of pipe.

Example : Find weight of 2 ft. of $4\frac{1}{2}$-in. cast-iron pipe $\frac{1}{2}$-in. thick.

Set 408 on B to 2 ft. on A, and over 4·5 on C read 99 lb. on A, and over 5·5 on C read 148 lb. on A, 148−99=49 lb.

Any other diameter and thickness may be calculated with the same setting.

Weight of Sheet Metal, etc.—Set 833 on B to length (ft.) on A, set cursor to width (ft.) on B. Set FII g.p. (Table, p. 96) on B under cursor and find result on A opposite thickness (inches on B).

Example : Find weight of a sheet of steel 16 S.W.G. 6 ft. g.p. 294 on B under cursor, and read 31·5 lb. on A opposite ·064 in. (=16 S.W.G.) on B.

Set 833 on B to 6 on A. Set cursor to 2 on B. Set g.p. 294 on B under cursor and read 31·5 lb. on A opposite ·064 in. (=S.W.G.) on B.

For weight per sq. ft. : take length and width=1 ft.

Gauge Points for Weight, etc., of Various Materials.—

Material.	Per Cub. Ft.	Square.			Cylinder.		Globe.	
		FFF.	FII.	III.	FI.	II.	F.	I.
Aluminium, cast, lb.	160	625	9	108	114	137	119	209
Brass, lb.	505	198	285	343	362	435	377	655
Copper, lb.	552	181	26	313	331	398	346	6
Gun metal, lb.	528	189	273	327	346	416	362	630
Iron, cast, average, lb.	450	222	32	385	406	488	425	733
Iron, wrought, aver. lb.	480	208	3	36	381	458	397	689
Lead, lb.	712	141	202	243	257	310	268	464
Steel, lb.	490	204	294	353	374	449	390	673
Tin	[illegible]	217	311	374	396	477	411	717
Zinc	437	229	330	395	419	502	435	756
Water, at lb.	62·5	16	231	277	294	352	306	528
Water, at cwts.	·557	18	258	310	329	395	343	593
Water, at tons	·0279	358	516	619	656	789	684	118

FFF = ft. × ft. × ft.
FII = ft. long × in. sq.
III = in. × in. × in.
FI = ft. long × in. diameter.
II = in. long × in. diameter.
F = ft. diameter.
I = in. diameter.

Table of Gauges.					
S.W.G.	in.	S.W.G.	in.	S.W.G.	in.
7/0	·500	3	·252	14	·080
6/0	·464	4	·232	16	·064
5/0	·432	5	·212	18	·048
4/0	·400	6	·192	20	·036
3/0	·372	7	·176	22	·028
2/0	·348	8	·160	24	·022
0	·324	9	·144	26	·018
1	·300	10	·128	28	·0148
2	·276	12	·014	30	·0124

WEIGHTS AND MEASURES

Avoirdupois Weight

Dr.	16 A
Oz.	1 B
Oz.	16 A
Lb.	1 B
Lb.	14 A
St.	1 B
Lb.	28 A
Qrs.	1 B
Lb.	112 A
Cwts.	1 B
Cwts.	20 A
Tons	1 B

Long Measure

In.	12 A
Ft.	1 B
Ft.	3 A
Yds.	1 B
Ft.	6 A
Fathom	1 B
Yds.	5·5 A
Rods	1 B
Yds.	1760 A
Miles	1 B
Rods	320 A
Miles	1 B

Surveying Measure

Links	100 A
Ft.	66 B
Ft.	66 A
Chains	1 B
Chains	10 A
Furl.	1 B
Chains	80 A
Miles	1 B
Furl.	8 A
Miles	1 B
Miles	19 A
Admty. kts.	16·5 B

Square Measure

Sq. in.	144 A
Sq. ft.	1 B
Sq. ft.	9 A
Sq. yds.	1 B
Sq. yds.	60·5 A
Sq. rods	2 B
Sq. yds.	4840 A
Acres	1 B
Sq. rods	160 A
Acres	1 B
Acres	640 A
Sq. miles	1 B

Ale and Beer Measures

Dry Measures

Gills	4 A
Pints	1 B
Gals.	9 A
Firkins	1 B
Pints	8 A
Gals.	1 B
Gals.	6·25 A
Cub. ft.	1 B
Pints	2 A
Quarts	1 B
Gals.	54 A
Hogsheads	1 B
Cub. in.	34·6 A
Pints	1 B
Bushels	7 A
Cub. ft.	9 B
Quarts	4 A
Gals.	1 B
Gals.	8 A
Bushels	1 B
Cub. in.	276 A
Gals.	1 B
Bushels	8 A
Quarters	1 B

Example : To 8 pints on A set 1 gal. on B (see Table) : then opposite any number of gallons on B, read pints on A, or *vice versa*.

COMMERCIAL

Interest or Discount : *Rule.—For given principal and rate of interest but varying periods at one setting. Set arrow on B to principal on A, and opposite rate of interest (or discount) on B* *read interest (or discount) per annum on A. Set cursor to this and bring 365 days on B to cursor, then opposite any number of days on B read interest (or discount) on A.*

Example : To find the interest on £3500 at 4 per cent. for 90 days.

Set the arrow on B to 35 (hundred pounds) on A, and over 4 per cent. on B read £140 for one year on A. Then set 365 days on B to £140, the interest on A, and over 90 days on B read the answer £34, 10s. on A.

The shillings are only approximate, being in the decimal part of a pound it is difficult to read the exact amount.

Example : To find the discount on £560 at $2\frac{1}{2}$ per cent.

Set the arrow to £560 on A, and over the rate of discount, 50s. on B, read the answer 280s. on A.

Set 365 on B to 280 on A, and opposite any number of days on B read discount on A.

Rule.—For given period and rate of interest, but varying principals. Set 365 on B to rate of interest on A, and opposite number of days on B read interest on £100 on A. Mark with cursor and set 1 on B to it. Read interest on A opposite any principal on B.

Find the interest on £32, £48, £60, and £80 at $7\frac{1}{2}$ per cent. for 160 days.

Set 365 on B to 7·5 on A, and opposite 160 on B read £3·28 on A. Mark with cursor and set 1 on B to it. Read :

A	1·05	1·58	1·97	2·63
	32	48	60	80

To find the selling price of goods to realise a certain profit (as percentage on cost).

Set 100+the percentage for profit on B to 100 on A, and under the cost price for any amount on A read the selling price with profit on B, or *vice versa*.

Example : Find the selling prices of goods to realise 15 per cent. profit.

Set 100 percentage of profit=115 on B to 100 on A ; and under any cost price on A read selling price on B at 15 per cent. profit, under £50 cost price read £57, 10s. selling price, etc.

For profits as percentage of return. Set 100 on B to 100 − percentage of profit, and proceed as before.

Percentage of Loss.—Example : If 70 tons of coal are consigned to me and I only receive 63 tons, what percentage do I receive, and what is the loss ?

Set the arrow or 100 on B to 70 tons on A, and under 63 on A read 90 per cent. received on B, and from 90 to the arrow on B equals the loss, 10 per cent.

Percentage of Gain.—Set the arrow or 100 on B to the nett amount on A, and under the gross amount on A read gain per cent. on B, counting from the arrow.

Example : If 80 lb. of goods gain 8 lb.=88 lb. during the process of manufacture, what is the percentage of gain ?

Set the arrow on B to 80 lb. on A, and under 88 on A read 10 per cent. on B, counting from the arrow on B to 88 on A.

To find the Percentage deducted from any Amount.—Set the gross amount on B to the nett on A, and read the discount on A from 100 on A to the arrow on B.

Example : A cheque for £76 is received in settlement of an account for £80. What was the discount ?

Set 80 on B to 76 on A and from 100 on A to the arrow on B=5 per cent., the answer on A.

To find Dividends.—Example : Set the total debts on B to the assets on A, and over each debt on B read the dividend for same on A.

Set debts=	£240	80	65	70	20	5=debts.
	95	31·6	25·73	27·7	7·9	1·98=dividend.

To divide any Amount into Proportional Parts.—Set the total number of parts on B to the amount on A, and over the number of parts on B read the value of each on A.

Example : Divide £174 into 3, 1, 2, and 5=11 parts. Set 11 on B to 174 on A, and over the parts on B read the values on A.

A	15·8	31·6	47·4	79
B	1	2	3	5

Compound Proportions.—Example : In the manufacture of an article certain ingredients are used in the following proportions : 2, 7, 13, 40, 62 lb.=124 lb. altogether. It is required to make 5 cwt. at one time in exactly the same proportions.

Set 124 lb. on B to 560 lb. on A, and over the proportions on

B read the quantities required on A to make up 5 cwt. as follows :

A	560	9	31·5	58·5	181	280	=560 lb.
B	124	2	7	13	40	62	=124 lb.

Carpets, Linoleum, etc.—To find the number of yards required to cover a floor.

Set 3 on B to the length in feet on A, and over any number of widths required on B read yards on A.

Another way : Set g.p. on B to the length in feet on A, and over any width in feet on B read number of yards of carpet on A.

Allowance must be made for matching patterns, from half to one pattern for each length of carpet.

G.P.'s for carpet $21\frac{1}{2}$ in. wide=5·375
G.P.'s for carpet 27 in. wide=6·75
G.P.'s for carpet 36 in. wide=9
G.P.'s for lino. 6 ft. wide =18

Salary, Wages, etc.

Shillings per week	10 A	Shillings per day	4 A
£ per annum	26 B	£ per annum	73 B
Pence per day	23 A	Pence per day	6 A
£ per annum	35 B	Shillings per month (30 days)	15 B

Pence per Hour to Shillings per Week.—Set 12 on B to number of hours worked on A and read shillings per week on A opposite pence per hour on B.

HIDES, VALUE OF

To find the value of hides according to their average yield in pelt found in practice.

On inverse scale set the weight of hide on B to the price in pence per lb. on A, and over the weight of pelt on B find the cost of same per lb. on A.

If certain hides are 4d. per lb., the cost of pelt will be as follows :

Hides.	Pelts.	Cost of Pelt.	
80 lb. yield	68 lb.	4·7	Average 4·7d.
74 lb. yield	$68\frac{1}{2}$ lb.	4·32	
90 lb. yield	71 lb.	5·08	

To find the relative value of hides, set the weight of pelt on B to the average cost of same (say 4·7) on A, and over the weight of hide on B find its relative value per lb. on A, as follows :

Pelt.	Hide.	
68 lb.	from 80.	Relative value, 4d.
$68\frac{1}{2}$ lb.	from 74.	Relative value, 4·32d.
71 lb.	from 90.	Relative value, 3·71d.

Averaging.—If 70 butts (leather) weigh 1960 lb., what is the average ?

Set 70 on B to 1960 lb. on A, and the arrow on B points to the answer, 28 lb. each on A.

Tanning Material, Value of.—If a sample of tanning material at 7s. per cwt. produces 30 per cent. of strength, what strength should another sample at 6s. per cwt. show under the same conditions to be of equal value ?

Set 7 on B to 30 degrees on A, and over 6s. on B read 25·7 degrees on A.

Or under any degrees of strength on A read its comparative value in shillings on B.

CLOTH TRADE

Weight of Warp of Weft.—

$$\frac{\text{Width (in.)} \times \text{thds. per in.} \times \text{length (36-in. yds.)}}{840 \times \text{counts.}}$$

Set 840 on B to width (in.) on A. Set cursor to thds. per in. on B. Set counts on B under cursor and read weight in lb. on A opposite length (yds.) on B.

Weight per Sq. Yd. or Sq. Metre.—Set length (36-in. yds.) on B to weight of roll (lb.) on A. Set cursor to g.p. on B. Set width (in.) on B under cursor. Read weight on A opposite 1 on B.

Oz. per sq. yd., g.p. =195
Grams per sq. metre, g.p. =195

PRICE OF GOODS

By Weight

	Per	
Pence	oz.	6 A
Shillings	lb.	8 B
Shillings	oz.	5 A
£	lb.	4 B
Pence	lb.	6 A
Shillings	st.	7 B
Pence	lb.	6 A
Shillings	gr.	14 B
Pence	lb.	7·5 A
Shillings	cwt.	70 B
Pence	lb.	7·5 A
£	cwt.	3·5 B
Shillings	lb.	2·5 A
£	cwt.	14 B
Pence	st.	9 A
Shillings	cwt.	6 B
Shillings	st.	5 A
£	cwt.	2 B
Shillings	st.	5 A
£	ton	40 B
Shillings	cwt.	6 A
£	ton	6 B

By Measure

	Per	
Pence	pt.	3 A
Shillings	gal.	2 B
Pence	pt.	2 A
Shills.	firkins	12 B
Pence	qt.	3 A
Shills.	bushel	16 B
Pence	qt.	3 A
Shillings	gal.	1 B
Pence	qt.	3 A
Shills.	bushel	8 B
Shillings	gal.	5 A
£	bushel	2 B
Shillings	gal.	20 A
£	barrel	36 B
Shills.	bushel	5 A
£	qr.	2 B

By Length

	Per	
Pence	in.	6 A
Shillings	ft.	6 B
Pence	in.	1 A
Pence	ft.	4 A
Shillings	yd.	1 B

By Number

Pence, each	1 A
Shills., per doz.	1 B
Pence, each	3 A
Shills., per score	5 B
Pence, each	3 A
Shills., per 100	25 B
Pence, each	5 A
Shills., per gr.	60 B

Example: To 3s. per pint on A set 24s. per gal. on B (see Table), then over any other price per gal. on B read price per pint on A.

Pence as Decimal of Shillings, also Inches as Decimal of Foot.

1d.	2d.	3d.	4d.	5d.	6d.
·083	·166	·25	·333	·416	·5
7d.	8d.	9d.	10d.	11d.	
·583	·666	·75	·833	·916	

EXCHANGE RATES, ETC.

To convert price of goods in foreign currency and measure to price sterling and British measure, given the rate of exchange.

Set exchange rate on scale B to conversion factor on scale A. Opposite foreign price on B read price sterling on A.

Example: Goods at 250 francs per kilo. Exchange rate, 50. Find price, £ per ton, on A.

Answer: £50·15=£50, 3s.

Example: Goods at 50 cents per sq. ft. Exchange rate 4·40 (=440 cents) to £1. Find price, £ per sq. yd.

Set 440 on B to 9 (sq. ft. to sq. yd.) on A. Opposite 50 on B read £ per sq. yd. on A.

Answer: £1·22=£1, 4s. 5d.

SUNDRY CALCULATIONS

Falling Bodies.—To find the distance fallen in a given time.

Set 1 on C to 16·1 on A, and over any time falling in seconds on C read space fallen through in ft. on A. Over 5 seconds on C read 402 ft. on A, and over 7 read 790 ft.

To find the velocity acquired by a falling body in ft. per second at any time during its fall.

Set 1 on B to 32·2 on A, and over any number of seconds on B read the velocity in ft. per second on A. Over the 4th second on B read 128 ft. on A, and over the 5th second 160 ft. velocity per second.

Sound.—Sound travels about 370 yds. per second, or 1 mile in 4·75 seconds.

Set 1 mile on B to 4·75 seconds on A, and under 9½ seconds read 2 miles on B, also under 19 seconds read 4 miles, etc.

Or set 370 yds. on B to 1 second on A, and under 4 seconds on A read 1480 yds., and under 7 read 2600 yds. on B.

Wind.—Pressure of, in lb. per sq. ft.

Set 40 miles per hour on C to 6·4 lb. per sq. ft. on A, and over miles per hour on C read lb. per sq. ft. on A, over 50 miles per hour read 10 lb. per sq. ft., etc.

Velocity in ft. per second=miles per hour.

Set 44 ft. per second on B to 30 miles per hour on A, and over any number of ft. per second on B read miles per hour.

Pendulum, Vibration of.—A pendulum 39·138 in. long will make 1 vibration per second in London.

To find the time taken by pendulums of any other length to make 1 vibration:

Set the arrow or 1 on B to 39·14 on A, and under any other length on A read time on C. Under 20 in. on A read ·715 sec. on C; 22 in. = ·75, 30 in. = ·875, 36 in. = ·96, 48 in. = 1·108, etc.

Centrifugal Force.—Set g.p. 2933 on B to the weight in lb. on A. Set cursor to the revolutions on C, bring 1 on B to cursor, and over radius in ft. on B read centrifugal force in lb. on A.

Example: Find the centrifugal force of 40 lb. making 130 revolutions per minute in a circle of 2 ft. radius.

$$F=\frac{W\ Rad.\ R^2}{2933}.$$

Set 2933 on B to 40 lb. on A. Now bring cursor to 130 revolutions on C. Set 1 on B to cursor, and over 2 ft. on B read 461 lb. centrifugal force on A.

Temperature.—To convert Centigrade to Fahrenheit. Set 10 on A to 18 on B and add 32 to reading on B opposite Centigrade reading on A.

To convert Fahrenheit to Centigrade. Subtract 32 from Fahrenheit and read Centigrade on A opposite this figure on B.

FOREIGN MEASURE

The Metric System.—This system is now adopted in Austria, Germany, Greece, Holland, Italy, Norway, Portugal, Spain, Sweden, and Switzerland.

Lineal

Millimetres	63·5	A
16th inches	40	B
Millimetres	63·5	A
8th inches	20	B
Metres	12	A
Feet	39·37	B
Metres	36	A
Yards	39·37	B
Metres	11	A
Fathoms	6	B
Metres	40·2	A
Rods	8	B
Kilometres	140	A
Miles	87	B

Lineal—*continued*

Millimetres	76	A
Inches	3	B
Centimetres	76	A
Inches	30	B

Square

Sq. centimetres	200	A
Sq. inches	31	B
Sq. metres	6·5	A
Sq. feet	70	B
Sq. metres	71	A
Sq. yards	85	B

Cubic

Cub. centimetres	360	A
Cub. inches	22	B
Cub. metres	1·7	A
Cub. feet	60	B
Cub. metres	23	A
Cub. yards	30	B

Capacity

Litres	17	A
Pints	30	B
Litres	50	A
Imperial gal.	11	B
Litres	290	A
Bushels	8	B

Weights

Grammes	6·48	A
Grains	100	B
Grammes	5·3	A
Drachms	3	B
Grammes	85	A
Ounces	3	B
Kilogrammes	2·95	A
Pounds	6·5	B
Kilogrammes	2·95	A
Cwts.	6	B
Kilos	60	A
Tons	59	B

Example: To 50 litres on A set 11 imperial gal. on B (see above), then over any number of gal. on B read litres on A, or *vice vresa*.

A few equations which can be solved by the Calculator in one setting. Use front face unless otherwise specified.

$$a\times b=x \quad . \quad . \quad . \quad (1)$$

Set 1 on B to a on A, read x on A opposite b on B.

Another way (back face): Set a on A to b on B, read x on A opposite 1 on B.

$$\frac{a}{b}=x \quad . \quad . \quad . \quad (2)$$

Set b on B to a on A, read x on A opposite 1 on B.

Another way (back face): Set 1 on B to a on A, read x on A opposite b on B.

$$a^2 \times b = x \quad . \quad . \quad . \quad (3)$$

Set a on C to 1 on A, and under b on A read x on B.

$$a \times b \times = x \quad . \quad . \quad . \quad (4)$$

On back face: Set a on B to b on A, and on front face under c on A read x on B.

$$\frac{b}{a^2} = x \quad . \quad . \quad . \quad (5)$$

Set a on C to b on 4, and over 1 on B read x on A.

$$\sqrt{a^3 \times b} = x \quad . \quad . \quad . \quad (6)$$

On back face: Set a on C to b on A, and on front face under 1 on A read x on C.

$$\frac{a \times b}{c} = x \quad . \quad . \quad . \quad (7)$$

Set c on B to a on A, read x on A opposite b on B.

Another way (back face): Set a on B to b on A and under c on A read x on B.

$$\frac{a \times b^2}{c} = x \quad . \quad . \quad . \quad (8)$$

Set b on C to c on A, and under a on A read x on B.

$$\frac{(a)^2}{(b)} = x \quad . \quad . \quad . \quad (9)$$

Set b on B to a on A, and over 1 on B find quotient on A (or over same number on C, read x on B).

$$(a \times b)^2 = x \quad . \quad . \quad . \quad (10)$$

Set 1 on B to a on A and over b on B find product, and over same number on C read x on B.

$$\frac{a^3}{b} = x \quad . \quad . \quad . \quad (11)$$

On back face: Set a on C to 1 on A, and under b on A read x on B.

$$\sqrt{a} = x \quad . \quad . \quad . \quad (12)$$

Under a on B read x on C.

$$a = x \quad . \quad . \quad . \quad (13)$$

On back face: under a on B read x on C.

$$\sqrt{\frac{a}{b}} = x \quad . \quad . \quad . \quad (14)$$

Set a on B to b on A, and under 1 on A read x on C.

$$\sqrt[3]{\frac{a}{b}} = x \quad . \quad . \quad . \quad (15)$$

On back face: Set a on B to 1 on A, and under b on A read dividend on B, and the root or x on C.

$$\sqrt{a \times b \times c} = x \quad . \quad . \quad . \quad (16)$$

On back face: Set a on B to b on A, and on front face under c on A read x on C.

$$\sqrt{a^2} \times b = x \quad . \quad . \quad . \quad (17)$$

Set a on C to 1 on A, and under b on A read x on C.

$$\sqrt[3]{a \times b} = x \quad . \quad . \quad . \quad (18)$$

On back face: set a on B to b on A, and under 1 on A read x on C.

$$\sqrt{a \times b} = x \quad . \quad . \quad . \quad (19)$$

Set a on B to 1 on A, and under b on A read x on C.

$$\sqrt{\frac{a \times b}{c}} = x \quad . \quad . \quad . \quad (20)$$

On back face: Set a on B to b on A, and under c on A find dividend on B, and under same number on b, front face, read x on C.

$$\sqrt[3]{\frac{a \times b}{c}} = x \quad . \quad . \quad . \quad (21)$$

On back face: Set a on B to b on A, and under c on A read x on C.

$$\sqrt[3]{\frac{a^2}{b}}=x \quad . \quad . \quad . \quad (22)$$

Set a on C to b on A, and on back face under 1 on A read x on C.

$$\frac{a\sqrt{b}}{c}=x \quad . \quad . \quad . \quad (23)$$

Under b on B find the root, then on back face set this number on B to a on A, and under c on A read x on B.

Quadratic (back face).—Example:

$$x^2 \pm 4x = +21 \quad . \quad . \quad . \quad (24)$$

Set 1 on B to 21 on A. Read the roots where the difference of the figures on A and B=4.

The roots are +7 and −3, or −7+3.

Example: $x \pm 5x = -6$.

Set 1 on B to 6 on A. Read the roots where the sum of the figures on A and B=4.

The roots are −3 and −2 or −3 and −2.

Note.—If the right-hand side is + the roots will have opposite signs and the difference must be taken in reading the "Calcux." The sign of the lesser is the same as that of the co-efficient of "x." If the right-hand side is −, the roots will have like signs, either both + or both −, hence the sum is taken in reading the "Calculex." The sign is opposite to the co-efficient of "x."

To find the price per ft. of timber from the price per standard.

Set 99 on B to price in pounds per standard on A and over area in sq. in. on B read price per ft. in pence on A.

Example: Find the price per ft. of 9×3 timber at £22 per standard.

Set 99 on B to £22 on A and over 27 sq. in. on B read 6d. per ft. on A.

USEFUL GAUGE POINTS

Known Value on Slide.	Required Value on Rule.	Set to On Slide.	Set to On Rule.
Square inches	Square centimetres	31	200
Square feet	Square metres	140	13
Square miles	Square kilometres	112	290
Acres	Hectares	42	17
Cubic inches	Cubic centimetres	36	590
Cubic feet	Cubic metres	106	3
Square, side	Square, diagonal	70	99
Inches	Centimetres	50	127
Inches, eights	Millimetres	40	127
Feet	Metres	292	89
Yards	Metres	35	32
Miles	Kilometres	87	140
Circle, diameter	Circle, circumference	223	710
Circle, diameter	Circle, side of inscribed square	99	70
Circle, diameter	Circle, side of equal square	79	70
Circle, diameter	Circle, side of equal equilateral triangle	72	97
Circle, circumference	Circle, side of inscribed square	40	9
Circle, area	Inscribed, square area	300	191
Pounds per square inch	Atmospheres	485	33
Pounds per square inch	Water, head, feet	13	30
Pounds per square inch	Water, head, metres	33	25
Pounds per square inch	Inches, mercury gauge	25	51
Inches, water gauge	Pounds per square inch	360	13
Inches, water gauge	Inches, mercury gauge	14	1
Inches, mercury	Atmospheres	30	1
Atmospheres	Kilos per square centimetre	89	92
British thermal unit	Calories	250	63
British thermal unit per pound	Calories per kilogramme	9	5

USEFUL GAUGE POINTS—*continued*

Known Value on Slide.	Required Value on Rule.	Set to On Slide.	Set to On Rule.
Foot Pounds	Kilogrammetres	340	47
Horse-power	Force de cheval	72	73
Pounds per H.P. ..	Kilos per cheval	300	134
Horse-power per hour ..	Kilowatts (B.T.U.) ..	134	100
Watts	Horse-power	5	·0067
Pounds per square foot ..	Kilos per square metre ..	87	425
Pounds per lineal foot ..	Kilos per lineal metre ..	41	61
Pounds per lineal mile ..	Kilos per kilometre ..	71	20
Pounds per cubic foot ..	Kilos per cubic metre ..	39	625
Cubic feet of water ..	Weight in pounds ..	17	1060
Cubic feet of water ..	Gallons (imperial) ..	17	106
Gallons of water	Weight, kilos	108	490
Pounds of water (fresh) ..	Pounds of water (sea) ..	38	39
Feet per second	Metres per minute ..	7	128
Feet per second	Miles per hour	22	15
Feet per minute ..	Miles per hour	264	3
Yards per minute ..	Miles per hour	88	3
Miles per hour	Metres per minute ..	12	322
Knots	Metres per hour	33	38
Pounds per square inch ..	Kilogrammes per square centimetre	128	9
Cubic feet	Litres	3	85
Cubic yards	Cubic metres	51	39
Gallons	U.S. Gallons	5	6
Bushels	Cubic metres	110	4
Ounces (Avoirdupois) ..	Grammes	67	1900
Ounces (Avoirdupois) ..	Kilogrammes	670	19
Pounds (Avoirdupois) ..	Kilogrammes	280	127
Hundredweights	Kilogrammes	5	254
Tons	Tonnes	62	63

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