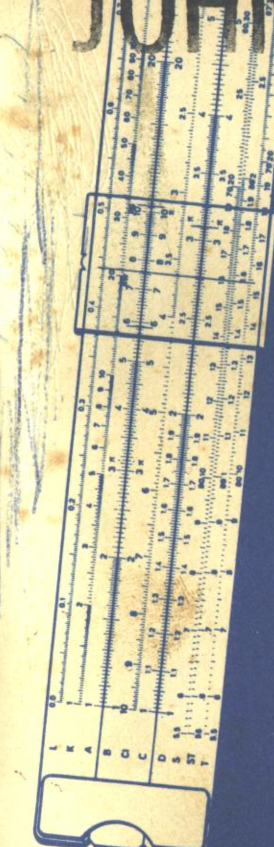
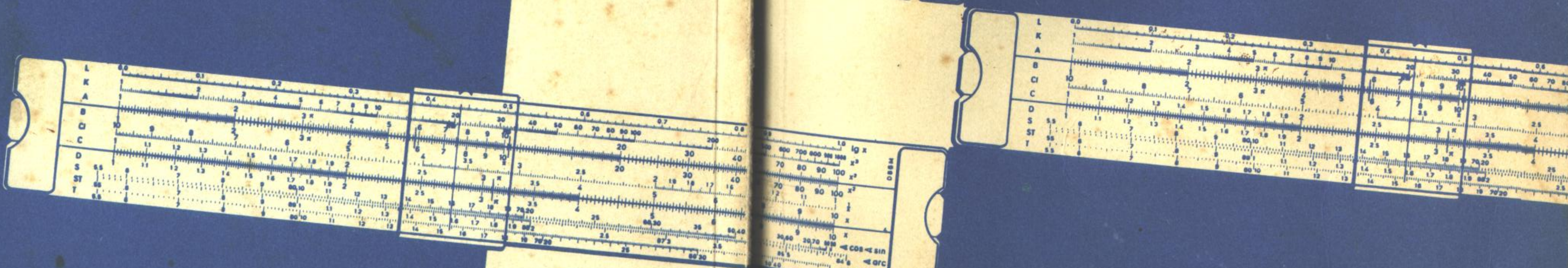
RICHARD STENDER
& K. K. McKELVEY

THE MODERN SLIDE RULE





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The Modern Slide Rule

DR RICHARD STENDER
& K. K. McKELVEY
B.SC. A.M.I.C.E.

The Modern Slide Rule

A MANUAL OF SELF-INSTRUCTION FOR
SCHOOLS, COLLEGES AND THE
PROFESSIONAL USER

THIRD EDITION

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Preface

This latest version of a manual which has been very favourably received all over the world is the result of a collaboration between the original author Dr Stender, who is primarily a mathematician and K. K. McKelvey who is a civil engineer. The result, it is hoped, will be found to possess both soundness in theory (though demanding no deep mathematical knowledge) and a highly practical approach to the varied uses of the Slide Rule.

Part I explains the working of the instrument even to those without previous knowledge of logarithms; Part II shows how to apply it to a great range of problems in engineering and other fields, and is devoted mainly to the log log and special scales; Part III consists of a large number of examples, many of them worked step by step.

Further examples have been added in this edition with a view to showing how the instrument will, if one troubles to study it, perform an astonishing variety of tasks—whether speeding or simplifying all kinds of calculations, checking figures obtained by other means, avoiding repetitive drudgery or rapidly tabulating the results of applying different values in a given formula. There is also more information on the work that can be done with the various advanced and special purpose instruments. This manual also describes in some detail the use, and also the usefulness of the so-called folded scales. These will no doubt soon become a normal feature of all slide rules.

Now that technical college and university lecturers tend more and more to assume the use of a slide rule where appropriate, some such manual as this is becoming a 'must' for science and engineering students, while the time-saving virtues of the instrument are being discovered by people in many branches of commerce and industry. There are a number of simple guides available; but the authors believe that the very little study needed to master more than the basic operations is eminently worth while. They therefore hope that this manual is simple enough for the beginner, but will carry him, if he is willing, quite a long way.

THE AUTHORS

Part I
The Modern Slide Rule

1 INTRODUCTION

Not so very long ago, anyone seen using a slide rule was in danger of being scoffed at and looked upon as a 'crank'. By others, the employment of a slide rule was regarded as a process akin to witchcraft. Slide rules, however, are not new. Samuel Pepys, the Diarist and 'Father of the Modern Navy', has an entry for 14th April, 1663, which reads: 'I walked to Greenwich, studying the slide rule for measuring of timber, which is very fine'.

Nowadays, to many people the slide rule has become as important as their pen or pencil. Its convenience and speed as an aid to calculations make it a 'must'. People in all walks of life use it: students, scientists, and technicians. Punters use it for working out permutations and combinations for pools entry coupons. Editors use it to estimate the size of a page or the number of words in a book; architects and engineers use it to make rapid estimates on a site; race-track fans use it to gauge the time and speeds of a heat; the gardener may use it to decide how many bulbs he needs, or the extent of his crop. There are countless possible uses for this handy little instrument.

Of course, the term 'slide rule' does not apply exclusively to one specific calculating tool; it is a general term given to any vernier system where calculations are performed by reading figures found on one scale suitably positioned against corresponding ones on an adjacent scale. Fundamentally, the scales of a slide rule are logarithmic, but different kinds of rules are made to enable specialized calculations to be performed, and some of these may have different bases.

If the student does not bring himself to understand the fundamental reasons why the answers he obtains are correct, there must always be uncertainty and lack of confidence in the results. To know, not merely what to do, but why one does it, should be the first aim.

A knowledge of advanced mathematics, however, is not essential to the use of a slide rule. To understand fully the theory of the instrument an acquaintance with logarithms is required, but this knowledge is not indispensable—many daily users of the slide rule could not even perform a calculation with the aid of tabulated logarithms. To get the most efficient results, however, an elementary knowledge of the use of logarithms is desirable. One further item of prior knowledge is essential—familiarity with the decimal system.

In any study it is necessary to decide on some starting point. We shall assume that the rules of simple arithmetical operations such as multiplication and division of numbers, expressed either as fractions or as decimals, are known, and that the student is also familiar with such things as the powers of numbers. Since, as has been said, logarithms form the basis of slide-rule mechanics, however, we shall first try to explain briefly, in simple non-mathematical language, what logarithms are.

2 LOGARITHMS SIMPLY EXPLAINED

Consider the series 2^2, 2^3, 2^4, 2^5, 2^6, etc. which, in mathematical shorthand, stands for the series

2×2	which multiplied out is				4
$2\times2\times2$	,,	,,	,,	,,	8
$2\times2\times2\times2$	,,	,,	,,	,,	16
$2\times2\times2\times2\times2$	,,	,,	,,	,,	32
$2\times2\times2\times2\times2\times2$	,,	,,	,,	,,	64

and 2^9 say, which when multiplied out:

$$2\times2\times2\times2\times2\times2\times2\times2\times2=512$$

From this it is clear that we can, if we wish, express certain numbers as powers of 2, i.e. as the result of multiplying 2 by itself a certain number of times.

This notion of using powers of simple numbers to express other numbers is the basis of logarithms. The index or the power to which the simple number must be raised or in other words, the number of times the 'base' must be multiplied by itself to equal a given number, is the logarithm of that number to the chosen base. Thus, in the easy example above, 512 has 9 as its logarithm to base 2.

As a base, however, 2 is not very convenient and in practice, two other systems are used, one in which the base is 10 and the other in which the base is ϵ, an indeterminate number which to five places of decimals has a value of 2·71828. The most commonly-used series is that based on

10, and the ϵ series, or 'Napierian' system, as it is called after the mathematician who developed it, will be left for the time being.

By analogy from the example to base 2, we have

$$10^1,\ \text{i.e. } 10 = 10$$
$$10^2,\ \text{i.e. } 10\times10 = 100$$
$$10^3,\ \text{i.e. } 10\times10\times10=1000$$
$$10^5,\ \text{i.e. } 10\times10\times10\times10\times10=100{,}000$$

and the powers or indices, 1, 2, 3 etc. are the logarithms, to base 10, of the numbers 10, 100, 1000 etc.

Naturally, all the quantities we are likely to have to work with are not simple integral powers of 10 such as 1000. The principle, however, applies to all numbers and we have therefore fractional indices.

Thus $10^{\frac{1}{2}}$ (or $\sqrt{10}$) has the approximate value 3·16
$10^{0\cdot3010}$,, ,, ,, ,, 2·0
and so on.

The student can see that whatever number is involved, it is possible to calculate the index or power to which the base 10 must be raised in order to equal that number. Such indices are tabulated to 4, 5, 6, 7 and 10 places of decimals, depending upon the accuracy required, in books of logarithms.

It is now possible to state two simple rules:

1. Any number can be written as the power of another number called the base, the index or power being called a logarithm of the original number, e.g. using base 10, $1000=10^3$ and $\log_{10} 1000=3$.

2. To perform multiplications and divisions it is necessary only to add or subtract the logarithms.

(*a*) To multiply together two numbers written as powers of the same base it is sufficient to add the indices of the powers, e.g. $100\times1000=10^2\times10^3$ which we can write as $10^{2+3}=10^5=100{,}000$
or, $\log_{10}100+\log_{10}1000=5=\log_{10}100{,}000$.

(*b*) To divide a number by another, each being expressed as a power of the same base, it is necessary only to subtract the index of the divisor from that of the dividend to obtain the quotient expressed as a power of the common base, e.g.

$$100\div10=10^2\div10^1=10^{2-1}=10^1=10$$

or, $\log_{10}100-\log_{10}10=1\ \log_{10}10$.

(*c*) To complete this we have to find a meaning for 10^0. Consider 10^3 divided by 10^3. We know that the quotient when any number divided by itself is 1.

Thus, $10^3/10^3 = 1$ or, using indices $10^{3-3} = 10^0 = 1$, and using logarithms: $\log_{10}10^3 - \log_{10}10^3 = 0 = \log_{10}1$.

Normally when dealing with logarithms to the base 10, it is unnecessary to write the base in; thus, $\log_{10}3$ is usually simply written as log 3.

It is now evident that for numbers between

1	and	10	the	logs	will	be	between	0	and	1
10	,,	100	,,	,,	,,	,,	,,	1	,,	2
100	,,	1000	,,	,,	,,	,,	,,	2	,,	3
1000	,,	10,000	,,	,,	,,	,,	,,	3	,,	4
10,000	,,	100,000	,,	,,	,,	,,	,,	4	,,	5

In other words, the logs of numbers between 1 and 10 will be wholly a decimal, between 10 and 100 they will be 1 followed by a decimal quantity, and so on.

Since the 'distance' between the logs of 0 and 1 is the same as that between the logs of, say 3 and 4, even although in one the 'distance' numerically is 9 and for the other 9000, it follows, by inference that for any number between 1 and 10, the decimal part of the log is the same as that for any number between 1000 and 10·000. This is in fact, so, e.g.

5 has a logarithm equal to 0·6990
and 5000 ,, ,, ,, ,, ,, 3·6990 (to four decimal places).
or in the terminology of indices,

$$10^{0\cdot6990} = 5$$
$$10^{3\cdot6990} = 5000$$

Thus, the logarithm of a number can be considered to be made up of two parts, the *characteristic*, or that part of the logarithm to the left of the decimal point, and the *mantissa*, or decimal portion.

The characteristic is invariably one less than the number of digits in the integral part of the number for which a logarithm is desired; thus, 10,000; 12,500; 68,999·27, all have a logarithmic characteristic of 4, the actual logarithms being 4·0000; 4·0969; and 4·8388 respectively (using four-figure logarithmic tables, i.e. to four places of decimals).

Not all the numbers for which logarithms are required are greater than unity. When the numbers are less than unity, the characteristic becomes negative. Here, the characteristic is greater by one than the number of 0's that follow the decimal point in the original number.

EXAMPLE

$$\text{Log}_{10}0\cdot106 = \bar{1}\cdot0253$$

Here, we look up 106 in the Tables, ignoring the decimal point and find 0·0253. The first figure of our number (0·106) is immediately after the decimal point and there are no 0's. But the characteristic, which will be negative in this case, is greater by one than the number of 0's and is therefore 1. It is negative and conventionally is written $\bar{1}$ (not −1), and is referred to as 'bar one'.

$\text{Log}_{10}0\cdot0106$ is found similarly. The mantissa is 0·0253, as before. There is one 0 after the decimal point and the characteristic will be $1+1=2$. Again, it is negative and will be written $\bar{2}$ and $\log_{10}0\cdot0106 = \bar{2}\cdot0253$. Similarly the logarithm of 0·00000106 would be $\bar{6}\cdot0253$.

If we now try to apply the second of the two rules indicated above we can perform the following operations:

arithmetically: $10{,}000 \times 12{,}500 = 125{,}000{,}000$
or in logs: $4\cdot0000 + 4\cdot0969 = 8\cdot0969$

If now we look up the mantissa or decimal part of the logarithm 0969 either backwards in logarithmic tables or, in the more common way, in antilogarithms, we get the value 125. Since the characteristic is 8, there will be nine integral digits to the answer, i.e. 125,000,000.

Similarly, to find $12{,}500 \times 0\cdot0106$ we have, using logarithms, $4\cdot0969 + \bar{2}\cdot0253 = 2\cdot1222$ and antilog $0\cdot1222 = 1325$ whence, since the characteristic is 2, there will be three integral digits and the answer is 132·5. In this example four-figure logarithms provided the exact answer, but usually the answer would be correct to only four figures.

If now we wish to divide two numbers, their logarithms are merely subtracted: Thus, $10{,}000 \div 12{,}500$ can be expressed logarithmically as $4\cdot0000 - 4\cdot0969 = \bar{1}\cdot9031$; and antilog $0\cdot9031 = 8000$, characteristic $\bar{1}$, answer 0·8000.

This then is the basis of a slide rule:

Imagine in the first example two scales engraved in such a manner that on one 10,000 corresponded to a length of scale say 4·0000 in. long (or 4 cm, or 4 ft), and on the other 12,500 corresponded to a length of 4·0969 in. (or 4·0969 cm, or 4·0969 ft). Then, if the two scales are placed end to end, their total length would be 8·0969 in. If the larger length were subtracted from the shorter, there would result a negative quantity of 0·0969 in. (or $-1\cdot000 + 0\cdot9031$, that is $\bar{1}\cdot9031$, expressed logarithmically). If then the scales are engraved so that a length of 8·0969 in. reads 125,000,000 and a length of −0·0969 in. (i.e. 0·0969 in. in the opposite direction to the origin) reads as 0·8000, then it is possible to determine the answers to the two sums—in one a multiplication, in the other a division, by simple reference to the graduations.

This illustration will serve to emphasize a very important point, namely that a slide rule is accurate only to certain mechanical limits. It consists basically of two scales placed one against the other so that their physical length is added or subtracted to give the answer. It is obviously unreasonable to be able to expect to measure 8·0969 in. without the aid of a very precise microscope and the nearest one is likely to get with the naked eye would be 8·1 in. or possibly 8·09 in. The answer would then be either 125,900,000 or 123,000,000. In other words, the slide rule can provide an answer which is correct to two, or at most, three places. At first sight this may seem far too little, but we shall see later that, if care is taken, this is often all that is necessary.

3 THE MATHEMATICAL BASIS OF SLIDE RULE CALCULATIONS

The slide rule may be regarded as the graphical counterpart of a Table of logarithms, the mantissas appearing on the graduations of the rule in the forms of segments of straight lines. This is best illustrated by converting the logarithms of the whole numbers from 1 to 10 into their equivalents in lengths, using as standard either 1 meter for the blackboard or 25 cm (or 10 in.) for a page in an exercise book. This gives the following Table:

log 1=0·0000	0·0 cm	0·0 cm	0·00 in.
log 2=0·3010	30·1 cm	7·5 cm	3·01 in.
log 3=0·4771	47·7 cm	11·9 cm	4·77 in.
log 4=0·6021	60·2 cm	15·1 cm	6·02 in.
log 5=0·6990	69·9 cm	17·5 cm	6·99 in.
log 6=0·7782	77·8 cm	19·5 cm	7·78 in.
log 7=0·8451	84·5 cm	21·1 cm	8·45 in.
log 8=0·9031	90·3 cm	22·6 cm	9·03 in.
log 9=0·9542	95·4 cm	23·9 cm	9·54 in.
log 10=1·0000	100·0 cm	25·0 cm	10·00 in.

Any other convenient length could be selected but 10 in. or 25 cm is frequently the actual physical graduated length of a slide rule.

By marking off these lengths on a straight line starting from a fixed point (labelled '1') we obtain a logarithmic scale.

Dimension-wise the scales represent logarithmic entities, while the accompanying figures, actual or assumed, give the corresponding numbers (anti-logarithms).

We see at a glance that a logarithmic scale has several characteristic features viz.: It starts with '1', the distances between each two points are unequal and they decrease towards the right (fig. 1).

If we make an exact duplicate of our first scale we possess the essentials of a primitive slide rule.

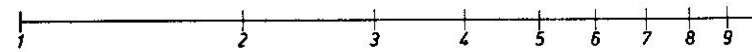


FIG. 1

The guiding principle is to carry out calculations mechanically by shifting one of the scales along the other in such a manner that we accomplish either the addition or the subtraction of the logarithms of two numbers without mental effort. *Generally speaking, the slide rule is an instrument for the geometric performance of arithmetical operations.* The following sketches (figs. 2 and 3) will make this clear.

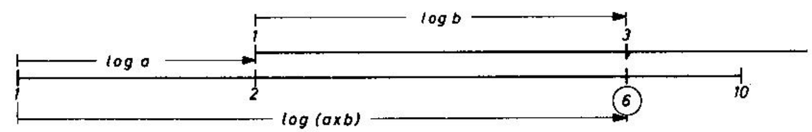


FIG. 2

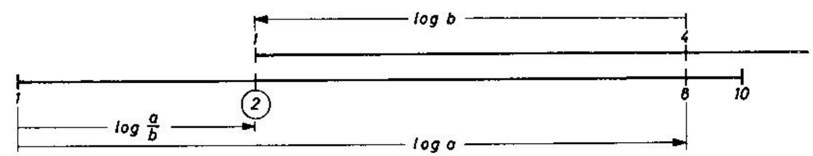


FIG. 3

These diagrams demonstrate how the examples

$$2 \times 3 = 6$$
$$\text{and } 8 \div 4 = 2$$

are solved by the addition or subtraction of their respective logarithmic lengths.

Here it will probably be argued that for such a simple problem there would be no sense in using any kind of instrument. The graduation certainly needs refinement. This can be achieved by proceeding as shown in fig. 4.

Let us reproduce along the y-axis the logarithmic scale of fig. 1 and graduate the x-axis into ten equal units. It is now possible to draw the curve for $y = f(x) = \log x$ which shows graphically the relationships between any number and its logarithm; and the logarithmic scale can be further subdivided. However, assuming that the learner has actually gone to all this trouble, he will realize that, at best, his effort has produced only an imperfect result. The commercially produced slide rule will give much more accurate results than his home-made scale.

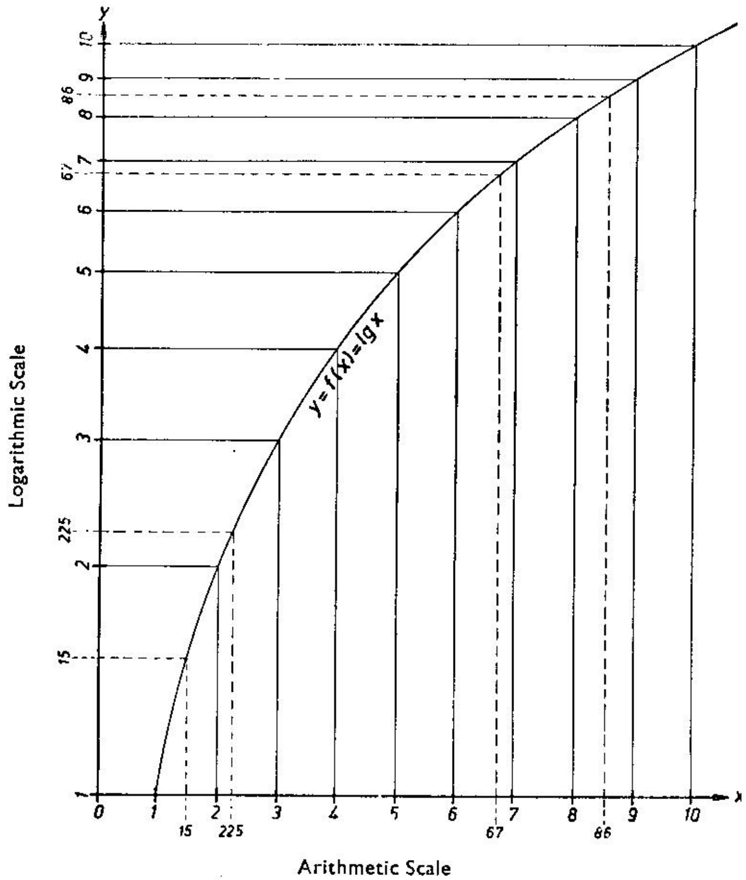


FIG. 4

4 DESCRIPTION OF A SIMPLE SLIDE RULE

The slide rule consists of three parts: a graduated frame called the *stock* or *body* of the instrument, a sliding part moving in it also graduated called the *slide*, and the *cursor.*

Older patterns of rule were made of wood, but wood changes its dimensions in sympathy with changes in atmospheric moisture and this lack of stability gives rise to inconvenience and tends to error in scale reading. The modern rule is generally made of a plastic resin, stable in all the conditions of temperature and humidity in which it is likely to be used. A central channel is formed longitudinally in the stock to accommodate the slide and has lateral grooves in which matching tongues on the edges of the slide can move.

The cursor is not absolutely essential to a slide rule, but is very convenient. Fine hair lines will be found engraved on that side of the glass nearest the rule, perpendicular to the scales (fig. 5).

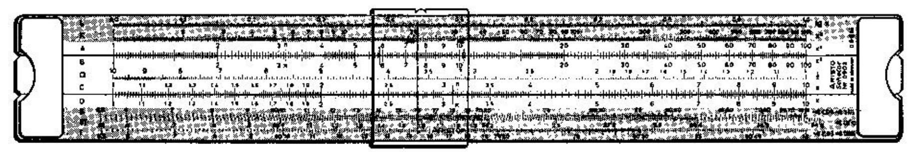


FIG. 5. Front face of Aristo Scholar 0903, 0903 VS and 0903 LL; a typical slide rule arrangement

There is a small air space between the hair-line on the cursor and the engraved scales and if the cursor is looked at sideways, it may appear to lie over a graduation which is, in fact, to the side of the mark it really covers. This effect is known as parallax.

In its simplest mathematical construction a slide rule contains scales generally termed A, B, C and D on its surface. To begin with we shall confine our attention to scales C and D, called the fundamental scales, where the unit of length is the distance between the first and the last line of the graduation (usually 25 cm or 10 in., depending on the manufacture). These letters are generally marked on all slide rules, A and B being those adjacent the top edge of the slide; C and D being along the bottom edge.

The distinguishing feature of a logarithmic scale is the fact that not only are the main lines of division spaced unequally but that also all intervals between them decrease from left to right. Between '1' and '2' we find ten secondary subdivisions bearing their respective numbers. These secondary intervals are again subdivided into ten tertiary intervals, so that in this part of the rule settings and readings can be made accurately to three figures. Between '2' and '4' each secondary interval is divided into only five parts, so that here numbers can be set accurately to two hundredths. Between '4' and '10' the tertiary intervals are only marked by one dividing line, so that each interval represents five hundredths of a number. When dealing with numbers of more than four digits, the numbers have to be reduced to three or four significant figures, e.g. 17·8742→17·87, or 8·6754 →8·68; although even then the numbers are 'accurate' only to one significant figure less, i.e. 17·87 could also stand for, say, 17·868, so that we only really know that 17·8 are true digits, anything else is estimated.

As we get more familiar with the scales we shall make a very important discovery. *Each line appearing on a scale represents not only its respective absolute value, but at the same time also any decimal variation thereof.* Since a setting really represents a mantissa we can make this deduction:

Any line, such as the one for 1–2–6 (this is the way we shall from now on express and think of a number, ONE–TWO–SIX, not one hundred and twenty-six) may just as well be read as 1·26, 12·6, 126, 1260, etc. or as 0·126, 0·0126, 0·00126, and so on. The decimal point is disregarded in slide rule work, and therefore all zeros preceding the first non-zero digit and those following the last non-zero digit in a number are discarded.

Initially we shall carry out calculations as if the numbers were whole ones without decimal fractions. Placing the decimal point in the answer presents no real difficulty. Sometimes the correct location is clear from the outset; often a simple mental assessment settles the question. It is good policy to begin every calculation with a rough estimate; this only need be close enough to remove any uncertainty about the decimal point.

Before embarking upon practical problems we must first acquire experience and skill in the correct setting and reading of numbers on the two fundamental scales C and D. This is best done by moving the cursor along the scales and paying close attention to the details of the graduations. This exercise is important. It is well-known that by far the greatest number of mistakes in slide rule reckoning occurs through lack of attention to the divisions and not through the user having failed to understand the working technique. So before going any further, let us practice setting the following numbers first by placing the hair-line of the cursor correctly against the D scale, then by placing the left-hand '1' on the C scale against the suitable graduation on D: 158, 10·1, 111·3, 172·5, 0·184, 2·09, 273, 0·289, 33·3, 367, 4110, 0·0482, 56·9, 7·22, 88·3, 9·81 and 9970. Further examples are readily imagined. It is essential to distinguish clearly between such notations as 100·1, 101 and 110!

5 MULTIPLICATION

The theory on which slide rule multiplication is based has already been discussed. It is the geometric addition $\log(a \times b) = \log a + \log b$ (fig. 6). We shall now work out the example $14{\cdot}92 \times 0{\cdot}283$.

First find the place for 1492 on scale D with the aid of the cursor and set the left end line of the C scale on the slide (the left index) against this place. The effect of this movement is that the result must appear under 283 of scale C, since the addition of the two logarithms has been accomplished by placing their respective two lengths end-to-end. We find that the result reads 4–2–2. We know from a quick mental check that it cannot mean anything but 4·22.

EXERCISES

	(*b*)		(*a*)		
	11·18	×	0·46	=	5·14
	175·1	×	0·39	=	68·3
	28·52	×	17·76	=	507
	376·1	×	2·27	=	854
	1·264	×	36·1	=	45·6

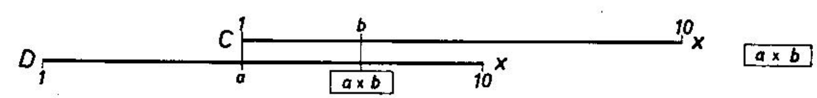


Fig. 6

It will soon be found that this method is apparently not quite perfect. In attempting to obtain $8{\cdot}23 \times 6{\cdot}41$, for instance, we discover that the product is not obtainable on D because at 6–4–1 the two scales are out of contact with each other. This obstacle is easily overcome. If the graduation of scale D were extended beyond its right end line (right index), or if a second rule were joined to the one we are using, we should be able to find the result by the method just described. The slide would penetrate exactly as far into the second rule as its left end is distant from the left index of the first rule. Thinking about it differently, we already possess such a 'second rule' if we imagine the protruding part of the slide inserted into the vacant part of the stock. All we therefore have to do, depending on the nature of the problem, is to use whichever of the two indexes is convenient; the left-hand '1' or the right-hand '10' on scale C (fig. 7). In our present example we push the slide over to the left, set its right index to 8–2–3 of scale D and find the answer available under 6–4–1. The product is 52·75. Figure 8 illustrates this procedure. The index changing movement is sometimes termed '*resetting the slide*'.

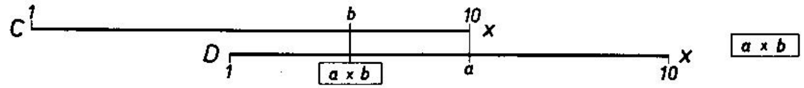


FIG. 7

EXERCISES

$69{\cdot}5 \times 5{\cdot}78 = 402$
$7{\cdot}82 \times 4{\cdot}66 = 36{\cdot}4$
$75{\cdot}2 \times 3{\cdot}96 = 298$
$18{\cdot}93 \times 0{\cdot}55 = 10{\cdot}41$
$732{\cdot}5 \times 0{\cdot}0086 = 6{\cdot}30$

In most cases it is possible to decide in advance whether the left or the right index has to be set to the multiplier. Part III contains numerous examples of multiplications occurring in everyday life and it is recommended that they should be worked out for practice before proceeding any further.

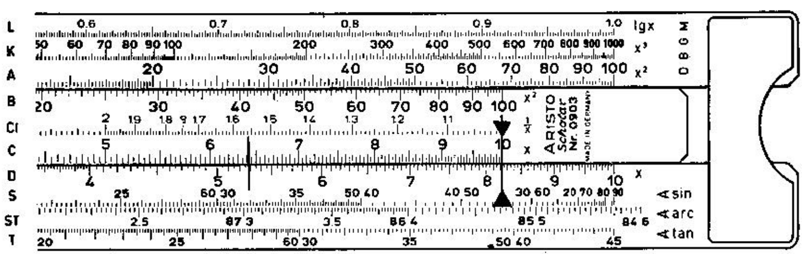


FIG. 8

It is worth digressing a moment to explain how the decimal point may be determined.

In many problems, final calculations reduce to a form such as

$$\frac{4{\cdot}37 \times 586{\cdot}2 \times 28{\cdot}6}{32 \times 575}$$

As all students will realize, such expressions may contain common factors which should be taken out as a matter of course. Often, however, the expression will still be awkward to handle even after all such possible simplification. To solve, we may proceed by one of several ways. A useful method of approximation and one that lends itself well to slide rule work is to factorize in terms of 10.

Consider the second term in the numerator of our example, 586·2. This we can regard as $5{\cdot}862 \times 10 \times 10$. Similarly, the third term in the numerator, 28·6 can be written as $2{\cdot}86 \times 10$. Applying similar reasoning to the terms of the denominator, we have: $32 = 3{\cdot}2 \times 10$, and 575 as $5{\cdot}75 \times 10 \times 10$. Let us write the whole expression in this amended form:

$$\frac{4{\cdot}37 \times 5{\cdot}862 \times 10 \times 10 \times 2{\cdot}86 \times 10}{3{\cdot}2 \times 10 \times 5{\cdot}75 \times 10 \times 10}$$

This clearly lends itself to simplification by taking out the common factor 10. However, it is a good habit to write such fractions in a special way, removing the 10's into a separate part of the fraction, thus:

$$\underset{(a)}{\frac{4{\cdot}37 \times 5{\cdot}862 \times 2{\cdot}86}{3{\cdot}2 \times 5{\cdot}75}} \times \underset{(b)}{\frac{10 \times 10 \times 10}{10 \times 10 \times 10}}$$

Looking at part (*a*), we see that we have *reduced each term to the condition in which there is only a single digit before the decimal point*. By this artifice we can easily get an approximation to the value of the whole expression, i.e. by rounding off to one significant figure. Thus, $(a) \times (b)$ becomes very approximately

$$\frac{4 \times 6 \times 3}{3 \times 6} \times 1 = 4$$

If we now use the slide rule (for method *see* §8) on the original fraction, the answer appears at first sight to be 398, but we know from approximate calculation that the required value is about 4, hence the actual answer is 3·98.

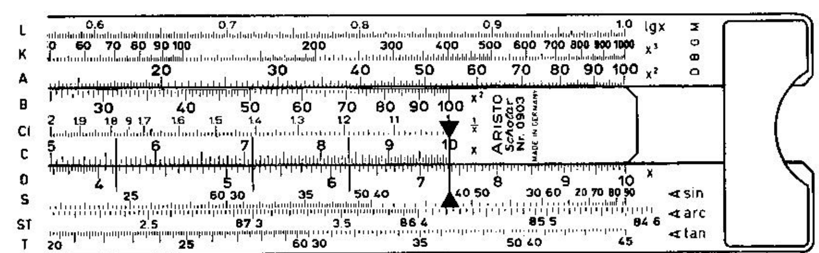


FIG. 9

For tabulations the slide rule is the ideal instrument as in most cases the required values can be read off directly with only one setting of the slide. Let us illustrate this by means of a problem occurring frequently in practice: knowing that 1 h.p. = 746 watts = 0·746 kilowatts, we require to find other equivalent values.

This is a typical instance of repeated multiplication with one constant factor. All we have to do is to set the appropriate left- or right-hand index of the slide over 7–4–6 of D and use the cursor to obtain the following table of equivalents. Figure 9 illustrates the last three settings.

H.P.	18	25	32	44	56	71	84
Kilowatts	13·44	18·66	23·9	32·9	41·8	53·0	62·7

6 CONTINUED PRODUCTS

The usefulness of the slide rule will become more and more apparent as the tasks become more and more complicated, for instance, when dealing with multiplications in which more than two factors are involved. By using the method we have just learned we can get the final result without having to pay attention to any intermediate results, especially if the cursor is used.

EXAMPLE $2{\cdot}2 \times 3{\cdot}7 \times 5{\cdot}1 \times 4{\cdot}1 = 170{\cdot}2.$

$2{\cdot}2 \times 3{\cdot}7$ is done first, the product being marked for reference by moving the cursor; i.e. '1' on C scale is placed against 2·2 on D and the hairline on the cursor is placed against 3·7 on C. The product could now be read off from scale D but this need not be noted, the cursor has done it for us. We now place '10' on C against the hairline (which actually marks the first product) and move the cursor against 5·1 on C. This last operation is repeated with the cursor moved this time to 4·1 on C and the answer can be read off: 1702; a rapid mental check tells us that the actual result is nearer 200 than 2000, i.e. 170·2 to be precise.

Later in this text we shall learn how resetting the slide can be partly avoided by using other scales to increase the speed and accuracy of our computations (*see* pp. 24 and 73).

7 DIVISION

As explained earlier, division by use of the slide rule is carried out by subtracting one logarithmic length from another.

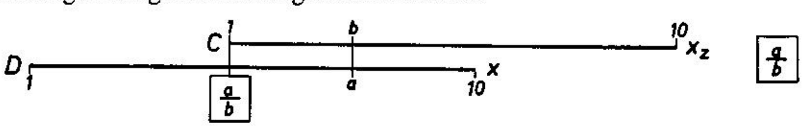


FIG. 10

The formula $\log a/b = \log a - \log b$ is applied geometrically (fig. 10).

Let us calculate the example

$$61{\cdot}4 \div 12{\cdot}85$$

After aligning 6–1–4 on D with 1–2–8–5 on C with the assistance of the cursor, if desired, we find the quotient 4–7–8 on D under the left index of the slide, as shown in fig. 11(*a*). By a rough estimate we know that the answer can only be 4·78. In this case we have read the result under the left slide index.

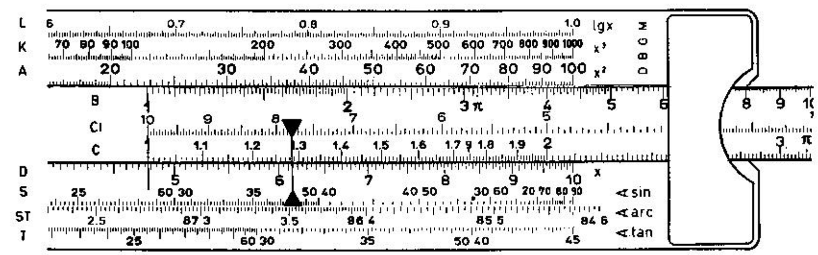


FIG. 11(*a*)

EXERCISES

$693 \div 15{\cdot}8 = 43{\cdot}9$

$55{\cdot}25 \div 3{\cdot}25 = 17{\cdot}0$

$48{\cdot}4 \div 116 = 0{\cdot}417$

$46{\cdot}40 \div 1{\cdot}685 = 27{\cdot}5$

$211{\cdot}9 \div 128{\cdot}4 = 1{\cdot}650$

We shall soon observe that the quotient is not always available under the left index of the slide. This arises, for instance, in solving 139·2 ÷ 0·642. Here again we can imagine that the result would be found on a second rule, so placed against the first that the two graduations run consecutively to each other. The slide would penetrate the body of the second rule by the exact portion of length by which its other end is drawn out of the first rule.

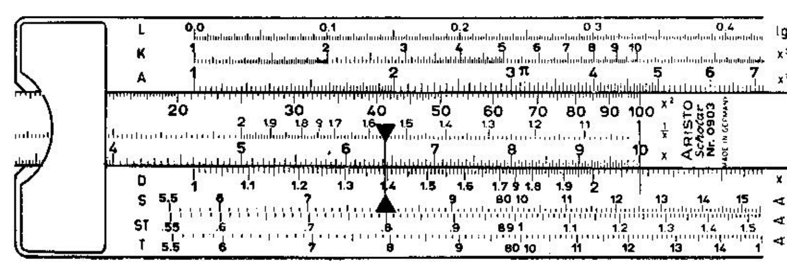


FIG. 11(*b*)

It follows that the right index would occupy the identical place where the left index is situated. For this reason we need only to look under the right index of the slide for the answer. It is not even necessary, in this example, to move the slide as when multiplication is involved. In our present problem we find 2–1–7 under the right index, as shown in fig. 11(*b*). By a rough estimate we know that the result must be 217.

EXERCISES

13·34 ÷ 0·613 = 21·8
176·2 ÷ 51·6 = 3·41
1287 ÷ 4·93 = 261
3·15 ÷ 7·5 = 0·420
6120 ÷ 8·52 = 718

Note on Use of 'Folded Scales' CF and DF

Considerably greater speed and accuracy is possible if use is made of the so-called folded scales. These, labelled CF and DF are similar in design and mode of operation to the C and D scales, except that their graduations are displaced sideways relative to those of C and D. This transfers the 1 to a position in the middle of the graduation, where it constitutes at the same time the beginning and the end of the scale. The section to the right of this centre index is a reproduction of the first part of a normal fundamental scale C or D, and the section to the left of it, the end part. For computations involving the use of the folded scales, it is necessary to bring the cursor to bear simultaneously on the CF and DF and C and D scales. This may be done if these scales are on the rear face of the rule, either by transferring the cursor to the rear face or by using a double faced cursor.

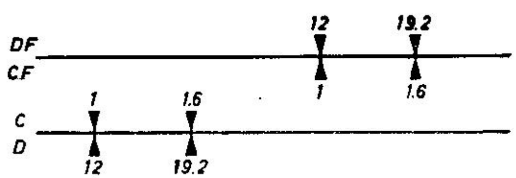


FIG. 12. Readings identical on C and D and CF and DF

The problem 12 × 1·6 can be solved with the CF and DF scales by setting the index 1 of CF to 12 of DF. This brings out the important fact that, simultaneously, the left index of the C scale has performed a corresponding motion equal to the value 12 on D (fig. 12).

Now take a glance along the two pairs of scales and observe that the ratio of any number on one scale to the number opposite on its companion

scale is constant throughout the entire system C and D and CF and DF. From this it follows that (1) the initial setting for the multiplication can be made with either pair of scales, and (2) where the readability of one pair of scales ends, the other pair always supplies the answer. This, of course, applies to all problems.

Take a second example 8 × 7 = 56 and let us assume that a large variety of other values has to be multiplied by 8 also. Note that, when the first setting is made with CF and DF, the effect is that there is no need to choose between the two slide indexes. All products of 8 multiplied by any other factor are now readable on C and D or CF and DF, respectively.

EXERCISES

18 × 0·285 = 5·13 (on D)
18 × 7·8 = 140·4 (on DF)

Multiplications involving the constant π are particularly easy and convenient to perform, since π on CF and DF is permanently located opposite the indexes of C and D, respectively, as a constant factor. If, for instance, the diameter 65 in. of a circle is set by cursor on the D scale, the circumference, 204 in., appears directly under the hairline on DF, and vice versa, of course. This is because the 'break' about which a typical scale C or D has been split so as to reassemble it to form CF or DF, occurs at the value of π.

In division, the use of the folded scales is equally advantageous, the numerator is read on DF opposite the denominator on CF (i.e. in their logical order), and the quotient appears opposite the respective index of CF or C.

8 COMBINED MULTIPLICATION AND DIVISION

In the preceding examples we have had to deal with problems concerning either multiplication or division. More frequently, however, we are confronted with problems in which both operations occur together. In this type of sum the usefulness of the slide rule becomes still more convincing. One clear advantage lies in the fact that we do not have to pay attention to intermediate results in a calculation and, further, that two operations can be performed with one setting of the slide. To make this clear let us work out the following illustrative example:

$$\frac{427 \times 0{\cdot}524}{0{\cdot}091}$$

It is always preferable to begin with the division because this very often saves one slide movement. Hence we align 4–2–7 of scale D with the line corresponding to 9–1 on C. We know that the quotient is under the right index of the slide. While its value does not interest us in the least, we know the slide to be in the very position in which we require it to be for the next step, i.e. the multiplication by 5–2–4. This means that we simply read off 2–4–6 under 5–2–4 of C without touching the slide (fig. 13).

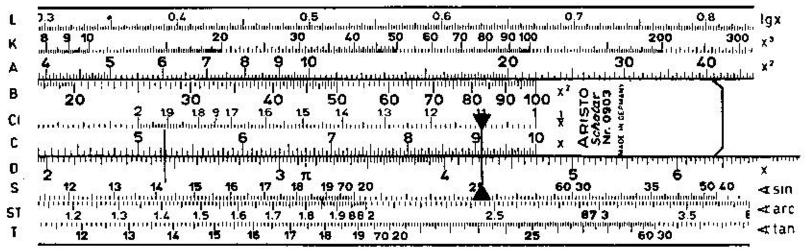


FIG. 13

By a rough approximation we conclude that 2460 is the correct result. Briefly expressed: Set C 9–1 over D 4–2–7 and read the answer under C 5–2–4 on D. Figure 13 illustrates this graphically.

Rule: In problems of the type $a \times c/b$ division comes first! (*fig.* 14). *This often saves one setting of the slide. Multiplication follows immediately. The intermediate result may be ignored.*

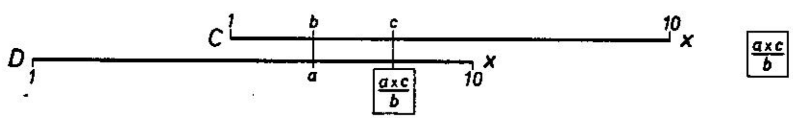


FIG. 14

EXERCISES

$$\frac{539 \times 5{\cdot}8}{0{\cdot}72} = 4340 \qquad \frac{4{\cdot}58 \times 182}{666} = 1{\cdot}252$$

$$\frac{8{\cdot}7 \times 340}{9{\cdot}5} = 311 \qquad \frac{293 \times 0{\cdot}95}{5060} = 0{\cdot}0550$$

$$\frac{30{\cdot}8 \times 2{\cdot}5}{7{\cdot}04} = 10{\cdot}94 \qquad \frac{0{\cdot}237 \times 1745}{33{\cdot}3} = 12{\cdot}42$$

In dealing with computations containing several factors both in the numerator and in the denominator such as $(a \times c \times e)/(b \times d)$ the method previously explained is generally applicable, as for instance in the example

$$\frac{58{\cdot}3 \times 215 \times 3{\cdot}46}{41{\cdot}6 \times 7{\cdot}41}$$

After dividing 583/416 we carry out the multiplication by 215 without moving the slide. After setting the cursor hairline over the result, we simply draw 741 of the slide under the hairline and immediately read off 1–4–0–7 under C 3–4–6. An approximate estimate establishes that 140·7 is correct.

EXERCISES

$$\frac{216 \times 0{\cdot}454 \times 62{\cdot}2}{20{\cdot}35 \times 3{\cdot}09} \rightarrow 9\text{–}7 = 97{\cdot}0$$

$$\frac{115{\cdot}4 \times 4{\cdot}19 \times 63{\cdot}6}{0{\cdot}458 \times 198} \rightarrow 3\text{–}3\text{–}9 = 339$$

$$\frac{8580 \times 15{\cdot}45 \times 2{\cdot}86}{22{\cdot}1 \times 17{\cdot}6 \times 7{\cdot}32} \rightarrow 1\text{–}3\text{–}3\text{–}2 = 133{\cdot}2$$

Even though we follow the '*division comes first*' rule, we shall occasionally be confronted with the fact that a multiplication is impossible without slide movement. The problem $(65{\cdot}6 \times 7{\cdot}25)/(3{\cdot}88)$ is a case in point.

Having done the required division we shall find that the slide projects so far to the right of the rule that 7–2–5 is 'out of contact' with scale D. We already know the remedy consists in resetting the slide so that the right index takes the place of the left index. This is easy to do with the cursor and without stopping to read the intermediate result. The final result 122·6 is now available on D. Alternatively, the folded scale may be used (*see* p. 24).

We may thus conclude that, while our 'division comes first' rule may not simplify the task in every instance, and we may sometimes have to make a resetting of the slide, it can still be considered preferable to the other method in which the slide must always be reset twice. (*See also* p. 73 on the use of Folded Scales.)

EXERCISES

$$\frac{218 \times 0{\cdot}29}{0{\cdot}084} = 753$$

$$\frac{45{\cdot}6 \times 8{\cdot}25}{2{\cdot}83} = 132{\cdot}9$$

$$\frac{45{\cdot}8 \times 114{\cdot}6}{8{\cdot}43} = 623$$

In more complicated problems of this type it is sometimes possible to rearrange the order of the factors for greater convenience; thus, consider the computation of:

$$\frac{4{\cdot}3 \times 69 \times 7{\cdot}1 \times 0{\cdot}436}{7{\cdot}29 \times 3{\cdot}3 \times 0{\cdot}834}$$

If this is tackled as written by the normal division-multiplication-division sequence, considerable slide movement will be needed. By rearranging the order of the factors, slide movement may be reduced, thus:

Redisposing, we have:

$$\frac{7{\cdot}1}{7{\cdot}29}\times\frac{69}{0{\cdot}834}\times\frac{0{\cdot}436}{3{\cdot}3}\times 4{\cdot}3$$

This need, of course, only be done in the head; it is not necessary to re-write the fraction. The guide to rearrangement is to pair off numerators and denominators so that (always remembering that in slide-rule work we ignore the importance of the decimal point when performing the mechanical operations of moving slide and cursor) the first digits of each number are as nearly the same as possible.

In the case of combined multiplication and division it often happens when using the C and D scales only that the slide will have to be reset to obtain the product and the advantage of starting with the division is lost. The computation can, however, always be completed on CF and DF without slide manipulation. Better still, start with the division on CF and DF. The multiplication can then often be done with both pairs of scales, in other cases with the one or the other pair.

9 PROPORTIONS

Mathematically speaking, proportions are merely another form of the unitary method dealt with in the preceding chapter.

From the proportion

$$\frac{a}{b}=\frac{c}{x} \text{ or } a:b=c:x$$

we derive the equation of products

$$ax=bc$$

and further

$$x=\frac{b\times c}{a}$$

Solving for x we set a of C over b of D. The answer, that is x, is available under c of C on D as diagrammatically shown in fig. 15.

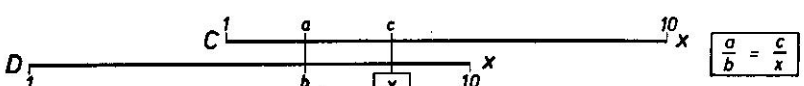


FIG. 15

Mathematically considered, we find that the formula:

$$\log x-\log b=\log c-\log a$$

or $$\log\frac{x}{b}=\log\frac{c}{a}$$

is equivalent to $$\frac{x}{b}=\frac{c}{a}$$

so that $$\frac{a}{b}=\frac{c}{x}$$

i.e. the physical arrangement of the slide rule is such as to present the appearance of the fractional equation.

Rule: The narrow gap between slide and body may be assumed to represent the line dividing the terms of a fraction. Thus, in setting the fraction a/b by use of the scales C and D, we instantly obtain the relationship c/x farther along. With the slide in this position we can also read all other fractions corresponding to the same ratio.

In fact, we actually have the proportions before our eyes. No matter how we may transform our proportion, the clarity remains the same.

The principle of proportion may be called 'the golden rule of slide rule technique' because unitary, or 'rule of three' problems are an important part of every-day arithmetic and it is here that the slide rule proves a most useful helpmate.

In the example: $21{\cdot}4:37{\cdot}5=430:x$

$$x=754$$

EXERCISES

$11{\cdot}1:24{\cdot}9=560\ :x$	$x=1256$
$48{\cdot}4:27{\cdot}6=\ x:670$	$x=1175$
$210\ :x\ =13{\cdot}7:29{\cdot}2$	$x=448$

Conversion of angles in degrees into radians:

Formula: $360°=2\pi\approx 6{\cdot}28$ thence $180°=\pi\approx 3{\cdot}14$

Degrees	360	25	40·5	53·2	121	105	57·3	68·4	73·9	88·1
Radians	6·28	0·436	0·707	0·929	2·11	1·833	1·000	1·194	1·290	1·538

i.e. proceed in this last case as follows: Set 360 on C against 628 on D. Now move the cursor along the rule and read off the answers on D for each number of degrees on C. No further movements of the slide are required.

Using the CF and DF scales in proportional problems will be found useful because it will no longer be necessary to reset the slide when a ratio falls beyond the reading range of C and D, the CF and DF scale then take over automatically.

10 GAUGE POINTS

Certain factors of constant value, such as the relationship between h.p. and electrical energy, occur frequently in calculations. Generally, the accurate setting of such factors requires more than ordinary care because of the nature of the quantities involved. When a factor is in regular use in calculations, it may be marked on the appropriate scale or scales and is then known as a gauge point. The most simple example is the value of π, numerically equal to 3·14159 . . . the relation between the diameter of a circle and its circumference. The gauge point π is generally marked on all four main scales of any slide rule.

Another gauge point which is useful is a constant $c=\sqrt{(4/\pi)}$ which is useful in determining areas of circles. This is sometimes printed on the C and D scales at the appropriate place, viz. 1·128, but more often it takes the form of a suitably-marked secondary hairline on the cursor (*see* p. 35 for further description).

Different slide rules may have different gauge points marked upon them: thus, a slide rule destined primarily for use by electricians may be marked with a gauge point for h.p. or 746 watts. In some slide rules a second hairline is shown on the upper right-hand side of the cursor. This is distant by the factor 0·746 from it so that h.p. conversions may be read by reference to either one line or the other over scale A. Other gauge points are frequently included for easy conversion between British/U.S/metric weights and measurements.

In §19 the value of $\rho=\text{arc } 1°=\pi/180°=0{\cdot}01745$ is found to be convenient and this value may be marked on all CI, C and D scales (*see* pp. 55 and 62).

11 SQUARES

Our next step will be to explore the upper two scales, labelled A and B, which will be called the scales of squares. This designation discloses their purpose. Upon examination we notice that the A scale (B is identically graduated) may be considered as made up of two equal portions with a centre-line at '10'. One part, to the left-hand end of the rule, is divided from 1 to 10, the other is figured from 10 to 100. The main intervals, 1–2, 2–3 . . . 8–9–10 in the left-hand portion are equal in length, each to each, to the intervals 10–20, 20–30 . . . 80–90–100 of the part on the right-hand.

This we should expect, since we already know that the scales represent lengths proportional to the logarithms to base 10 of the natural numbers 1–10 and the mantissa or decimal portion of the logarithm of 2 is the same as the mantissa of the logarithm of 20. The main intervals of both portions of the A and B scales are, however, shorter than those of the C and D scales and the subdivisions are less detailed.

For A or B, each secondary interval between 1 and 2 or 10 and 20, respectively, is subdivided into fifths, so that each line of the tertiary graduation is equal to two hundredths between the printed '1' and '2', and to two tenths between '10' and '20'. Moreover we find that between '2' and '5' or '20' and '50' there are only two intervals in each tenth, so that each interval represents five hundredths or five tenths, respectively. Between '5' and '10' or '50' and '100' the lines of the secondary graduations are so close together that the tertiary graduation has to be omitted altogether. Thus, the third figure will here have to be set by estimation.

Let us analyse the mathematical background for A and B. In the upper scales one half of the length of the graduations has been adopted as unit length. It follows that, when numbers a and d on the body scales A and D are brought into correspondence (this is also true for the scales B and C of the slide), which is easily done by means of the cursor, we have

$$\log a = 2 \log d$$

which means

$$a = d^2$$

Let us check this: With the index marks vertically in line run the cursor so that the centre hairline is over 4 on A. The figure on D beneath the hairline is 2. Run the cursor along to 9 on A and find 3 on D under the hairline. Put the cursor on 49 on A and read 7 on D. Thus, the intervals on scales A and B are proportional to the squares of numbers on scales C and D. Conversely, if we find a number on A we can read, with the cursor, its square root on D. Scales B and C, on the slide, are in an identical relationship.

EXAMPLE. Set for $27{\cdot}4^2$ and read 7–5–1 as shown in fig. 16. By a rough estimate we know that the result must be 751.

EXERCISES

$17{\cdot}2^2=296$	$346^2=119\,700$
$19{\cdot}8^2=392$	$\pi^2=9{\cdot}87$
$0{\cdot}285^2=0{\cdot}0812$	$2{\cdot}96^2=8{\cdot}76$
$0{\cdot}393^2=0{\cdot}154$	$9{\cdot}81^2=96{\cdot}2$
$7{\cdot}85^2=61{\cdot}6$	$10{\cdot}5^2=110$
$5{\cdot}41^2=29{\cdot}3$	$76{\cdot}2^2=5810$

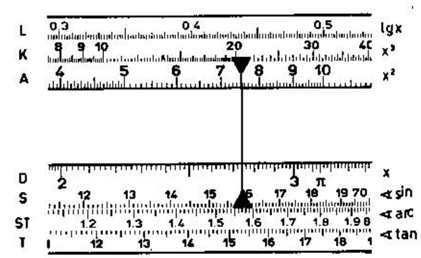


FIG. 16

12 SQUARE ROOTS

Extracting the square root of a number is the reverse process of squaring. In finding a root by use of the slide rule we simply proceed therefore, in the reverse order. Since below any number on scale A its square root appears on D, we have only to bring the cursor hairline upon the given number on A to read the corresponding root under the hairline on D. Figure 17 shows this for $\sqrt{\pi}=1{\cdot}772$.

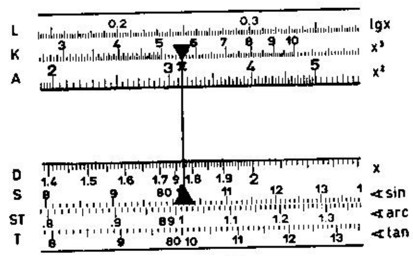


FIG. 17

As we have seen, squaring by slide rule is a purely mechanical operation, The reverse process, extracting a root, requires us to determine beforehand in which of the two halves of the scale A we must set the radicand. Indeed, a quick inspection will show us that we can read 81 either on the left-hand portion of scale A or the right-hand portion. Similarly, we could locate 810 and 8100 on either portion. If we wish to take out the square root on D, however, we clearly have no such freedom of choice since, under 81 on the left-hand part of scale A we read 2·84 on D while under 81 on the right-hand scale we read 9 on D.

One-digit numbers must obviously be set in the left half and two-digit numbers in the right half. Three-digit numbers, on the other hand, are again set on the left and four-digits on the right, and so on. The general rule is: *To extract the square root of a number with an odd number of digits before the decimal point set the number on the left half of A and read the root below on D. When dealing with numbers having an even number of places, use the right half.*

For square roots of numbers smaller than 1 (decimal fractions) we have this supplementary rule: *Divide the number into groups of two digits, starting from the decimal point, ascertain by inspection whether the first group that does not consist entirely of zeros has an odd or an even number of significant figures and apply the general rule.* So, for instance, 0·01'7 has an odd number, but 0·00'18' has an even number of places.

EXERCISES

$\sqrt{7}$	$=2{\cdot}65$	$\sqrt{0{\cdot}86}$	$=0{\cdot}927$
$\sqrt{70}$	$=8{\cdot}37$	$\sqrt{27{\cdot}2}$	$=5{\cdot}22$
$\sqrt{529}$	$=23{\cdot}0$	$\sqrt{123}$	$=11{\cdot}09$
$\sqrt{1\ 273}$	$=35{\cdot}7$	$\sqrt{4491}$	$=67{\cdot}0$
$\sqrt{23\ 470}$	$=153{\cdot}2$	$\sqrt{0{\cdot}017}$	$=0{\cdot}1304$
$\sqrt{0{\cdot}56}$	$=0{\cdot}748$	$\sqrt{0{\cdot}0018}$	$=0{\cdot}0424$
$\sqrt{0{\cdot}043}$	$=0{\cdot}208$	$\sqrt{1024}$	$=32{\cdot}0$
$\sqrt{3{\cdot}72}$	$=1{\cdot}929$	$\sqrt{9{\cdot}81}$	$=3{\cdot}13$

Problems of the form $\sqrt{(a^2+b^2)}$ occur frequently in practice. The task consists in computing the root of the sum of two squares. In slide rule practice, so as to avoid having to write down intermediary calculations, we use the following method of transformation

$$x=\sqrt{(a^2+b^2)}=a\sqrt{\left\{1+\left(\frac{b}{a}\right)^2\right\}}$$

in which the greater number is called b.

Accordingly in

$$x=\sqrt{(32^2+48^2)}=32\sqrt{\left\{1+\left(\frac{48}{32}\right)^2\right\}}$$

we set the left index of C over D 3–2 and place the cursor over 4–8 on D. Immediately above we find the quotient of 48/32. In this example it is 1·5, and on the next higher scale, that is scale B of the slide, we find the square of this quotient viz. $(48/32)^2=2{\cdot}25$.

We now increase this number, mentally, by 1 thus getting 3·25 and then set the cursor to the new value on scale B. The required value of $x=57{\cdot}7$ can be read on the fundamental scale D.

EXERCISES

$a=3{\cdot}6$	$b=6{\cdot}2$	$x=7{\cdot}17$
$a=5{\cdot}1$	$b=8{\cdot}6$	$x=10{\cdot}0$
$a=2{\cdot}9$	$b=9{\cdot}1$	$x=9{\cdot}55$
$a=12{\cdot}4$	$b=14{\cdot}8$	$x=19{\cdot}31$
$a=24{\cdot}2$	$b=31{\cdot}7$	$x=39{\cdot}9$

13 COMPUTATIONS INVOLVING SQUARES AND SQUARE ROOTS

The slide rule is an excellent aid in computing expressions containing squares or square roots. In dealing with problems of this kind it is necessary to understand fully the relationship existing between the four scales, also the fundamental operations involved.

I. $(a \times b)^2$

To compute the square of a product we simply multiply as usual with scales C and D. Over the product on D we find its square on A.

EXERCISES
$$(19{\cdot}6 \times 4{\cdot}15)^2 = 6620$$
$$(66 \times 8{\cdot}9)^2 = 345000$$
$$(1{\cdot}57 \times 5{\cdot}98)^2 = 88{\cdot}1$$
$$(7{\cdot}58 \times 3{\cdot}14)^2 = 566$$
$$(4{\cdot}67 \times 0{\cdot}85)^2 = 15{\cdot}76$$
$$(2{\cdot}1 \times 0{\cdot}76 \times 94)^2 = 22500$$

II. $\sqrt{(a \times b)}$

To find the square root of a product we can carry out the multiplication using scales C and D, set the product in the appropriate half of A and so obtain the root from D, as we have learned. Another method of solution is to do the multiplication using scales A and B. This we have neglected to practise for the reason that we cannot expect from them the same degree of accuracy as that obtained by using scales C and D. The advantage of multiplying with A and B consists in eliminating the need for resetting the slide. In choosing this method for roots we must be careful to set the given number in the correct half of A. It may happen that we have to change from one half to the other.

EXERCISES
$$\sqrt{(3{\cdot}72 \times 4{\cdot}46)} = 4{\cdot}07$$
$$\sqrt{(4{\cdot}12 \times 53{\cdot}6)} = 14{\cdot}86$$
$$\sqrt{(0{\cdot}73 \times 6{\cdot}54)} = 2{\cdot}19$$
$$\sqrt{(4{\cdot}62 \times 3{\cdot}81)} = 4{\cdot}20$$
$$\sqrt{(94{\cdot}2 \times 0{\cdot}037)} = 1{\cdot}869$$
$$\sqrt{(62{\cdot}7 \times 0{\cdot}83 \times 1{\cdot}22)} = 7{\cdot}97$$

III. $(a/b)^2$

When a problem involves squaring a quotient, the solution is again exceedingly simple. After using scales C and D for the division we can at once read the answer on A over either the left or the right index of the slide.

EXERCISES
$$\left(\frac{6{\cdot}41}{4{\cdot}62}\right)^2 = 1{\cdot}925 \qquad \left(\frac{0{\cdot}445}{24{\cdot}3}\right)^2 = 0{\cdot}000335$$
$$\left(\frac{33{\cdot}6}{12{\cdot}01}\right)^2 = 7{\cdot}83 \qquad \left(\frac{8{\cdot}92}{0{\cdot}0346}\right)^2 = 66{\cdot}500$$
$$\left(\frac{67{\cdot}30}{0{\cdot}875}\right)^2 = 5920 \qquad \left(\frac{124{\cdot}5}{288{\cdot}4}\right)^2 = 0{\cdot}1864$$

This method can be applied very profitably to all calculations involving circular areas.

For this purpose we make use of the constant c which is derived as follows:

$$c = \sqrt{\frac{4}{\pi}} = \frac{2}{\sqrt{\pi}} = 1{\cdot}128$$

This gauge point is sometimes inscribed on the C and D scale with a special symbol.

By using this value, slide rule computations of circle areas take the following course:

We know that area $$A = r^2\pi = \frac{d^2}{4}\pi$$

or $$A = \frac{d^2}{4/\pi}$$

Since $\frac{4}{\pi} = c^2$, we can also write

$$A = \frac{d^2}{c^2} = \left(\frac{d}{c}\right)^2$$

The procedure is made easier still with rules equipped with a so-called three-line-cursor. The value of the constant c is introduced in the following way: Over scales A and B and over scales C and D respectively to the left or right of the centre hairline a short line has been etched into the cursor. With a 25-cm slide rule the actual distances will be $12{\cdot}5 \log c^2 = 25 \log c$.

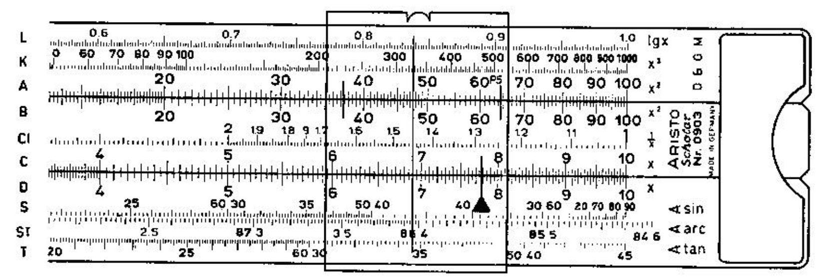


FIG. 18

Assuming that we require the area of a circle with a diameter $d = 77{\cdot}8$ in., we set the short line to this value on the fundamental scale D and read the area 4750 in.² directly under the centre hairline on A without moving the slide. In fig. 18 this is demonstrated graphically.

For the inverse operation we proceed as follows with reference to scales C and D. To determine the diameter d of a circle of area $A=750$ cm², we set the centre hairline on 750 on scale A and read off d on scale D under the short right-hand hairline, as shown in fig. 19.

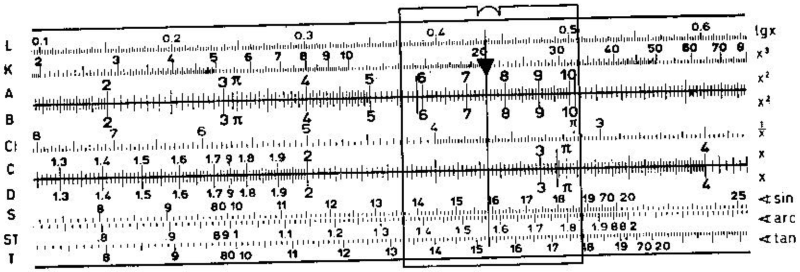


FIG. 19

We could also have set the short left-hand hairline to 7–5–0 on A, in which case the reading would be under the centre line at 3–0–9 of scale D.

To form a table of circle areas and diameters the left short line is set to the left index of B and, simultaneously, the centre line to the left index of A. In this position of the slide all values of scale C are divided by c, so that for any diameter d to which the cursor centre line is set on C, the square of the quotient $(d/c)^2$ viz. the area A appears on scale A.

EXERCISES

d	18	27·3	5·38	437	12·4	9·6	26·8	38·2	6·19
A	255	585	22·7	150 000	120·8	72·4	564	1146	30·1

In certain isolated instances we may discover that the area is not available on A under the left short line. If this happens we have the following alternative: After setting the centre hairline over the diameter d, we move the slide just far enough to bring the left index of B under the left short line of the cursor and read the answer on A over the right index of the slide. So, for example, we obtain an area of $A=88{\cdot}9$ in.² for diameter $d=10{\cdot}64$ in.

The above exceptions solely arise when the diameter falls within the short range of 1 to 1–1–2–8 on D.

IV. $\sqrt{a/b}$

In finding roots of quotients we again have the choice of two procedures. The first consists in performing the division with scales C and D and

setting the number obtained in the appropriate half of scale A. The second method makes use of the upper scales, where we often save one cursor movement at the expense, of course, of a slight loss in accuracy of the quotient. In every case we must carefully establish in which half of the scale the number should be set. (*See* rule §12, p. 32.)

EXERCISES

$$\sqrt{\frac{84{\cdot}7}{19{\cdot}6}}=2{\cdot}08 \qquad \sqrt{\frac{131{\cdot}3}{0{\cdot}888}}=12{\cdot}16$$

$$\sqrt{\frac{3{\cdot}14}{0{\cdot}078}}=6{\cdot}34 \qquad \sqrt{\frac{40}{3{\cdot}14}}=3{\cdot}57$$

$$\sqrt{\frac{9{\cdot}81}{11{\cdot}75}}=0{\cdot}914 \qquad \sqrt{\frac{1239{\cdot}4}{18{\cdot}73}}=8{\cdot}13$$

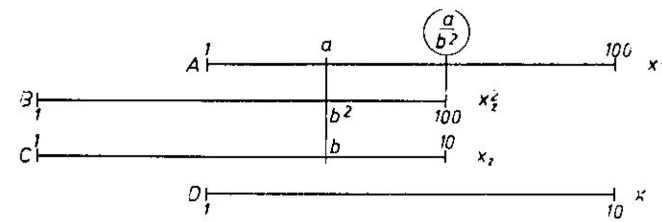


FIG. 20

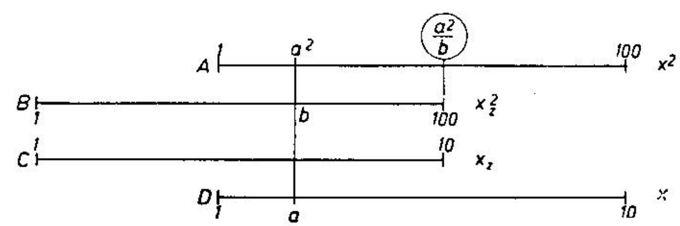


FIG. 21

Solutions for the group of problems a/b^2; a^2/b; $\sqrt{a/b}$; $a/\sqrt{b}$ are graphically presented in the diagrams of figs. 20, 21, 22 and 23 which are self-explanatory. As has been pointed out earlier in this text, however, it is important to give careful attention to the setting on scale A when dealing with problems involving square roots.

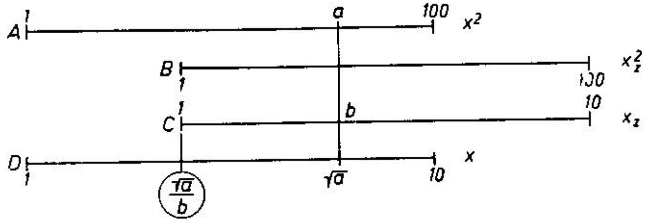


FIG. 22

V. a/b^2

EXERCISES

$$\frac{5{\cdot}35}{1{\cdot}79^2}=1{\cdot}67 \qquad \frac{71{\cdot}3}{4{\cdot}17^2}=4{\cdot}10 \qquad \frac{5{\cdot}17}{2{\cdot}28^2}=0{\cdot}995 \qquad \frac{337}{11{\cdot}17^2}=2{\cdot}70 \qquad \frac{22{\cdot}9}{0{\cdot}073^2}=4300$$

VI. a^2/b

EXERCISES

$$\frac{3{\cdot}46^2}{1{\cdot}782}=6{\cdot}72 \qquad \frac{57{\cdot}2^2}{13{\cdot}8}=237 \qquad \frac{3{\cdot}14^2}{8{\cdot}37}=1{\cdot}178 \qquad \frac{78{\cdot}3^2}{1{\cdot}7}=3610 \qquad \frac{9{\cdot}16^2}{288}=0{\cdot}291$$

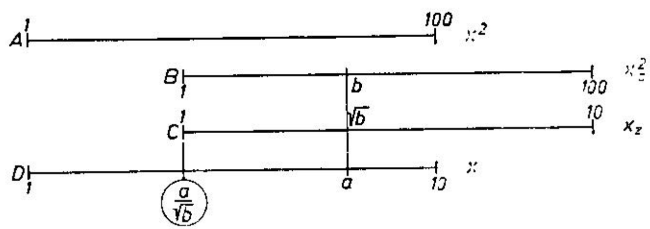


FIG. 23

VII. $\sqrt{a}/b$

EXERCISES

$$\frac{\sqrt{435}}{6{\cdot}24}=3{\cdot}34 \qquad \frac{\sqrt{13{\cdot}97}}{0{\cdot}053}=70{\cdot}5 \qquad \frac{\sqrt{5{\cdot}83}}{17{\cdot}34}=0{\cdot}1392 \qquad \frac{\sqrt{88{\cdot}3}}{5{\cdot}26}=1{\cdot}786 \qquad \frac{\sqrt{251}}{28{\cdot}7}=0{\cdot}552$$

VIII. $a/\sqrt{b}$

EXERCISES

$$\frac{27{\cdot}8}{\sqrt{12{\cdot}68}}=7{\cdot}81 \qquad \frac{1{\cdot}042}{\sqrt{66{\cdot}01}}=0{\cdot}1283 \qquad \frac{3}{\sqrt{2}}=2{\cdot}12 \qquad \frac{9{\cdot}81}{\sqrt{32{\cdot}57}}=1{\cdot}719 \qquad \frac{578}{\sqrt{173{\cdot}5}}=43{\cdot}9$$

IX. $a\times b^2$

This product can best be handled by starting with the squaring operation. Squaring is effected by setting either the left or the right index of the slide over b on D and then multiplying the square b^2 in the usual manner with the scales A and B, i.e. read answer under hairline at a on B. If a greater degree of accuracy is required, we can change to the multiplication $a\times b\times b$ using fundamental scales C and D.

EXERCISES

$$31{\cdot}5^2\times 1{\cdot}92=1\,905$$
$$3{\cdot}06^2\times 7{\cdot}18=67{\cdot}2$$
$$0{\cdot}583^2\times 21{\cdot}7=7{\cdot}38$$
$$785^2\times 0{\cdot}136=83\,800$$
$$8{\cdot}59^2\times 4{\cdot}48=331$$
$$6{\cdot}85^2\times 3{\cdot}14=147$$

X. $a\sqrt{b}$

Here the course to be followed is quite clear: We first determine the root with due regard to the now familiar rule (*see* p. 32), then multiply with a, using scales C and D.

EXERCISES

$$13{\cdot}65\times\sqrt{22{\cdot}6}=64{\cdot}9$$
$$3{\cdot}5\times\sqrt{2}=4{\cdot}95$$
$$2{\cdot}7\times\sqrt{1{\cdot}73}=3{\cdot}55$$
$$54{\cdot}5\times\sqrt{15{\cdot}9}=217$$
$$22{\cdot}1\times\sqrt{319}=395$$

As a further test, which recalls some of the exercises just performed, let us find the volume of the expression

$$\sqrt{\left(\frac{\pi\times 22{\cdot}9\times 12{\cdot}67^2}{63{\cdot}2\times 0{\cdot}056}\right)}$$

Evaluate first the expression within the brackets.

Step 1. Cursor to gauge point π on D.
2. Bring 632 on C under hairline.
3. Cursor to 229 on C.
4. Bring 56 on C under hairline.
5. Cursor to right-hand index of C.
6. Bring left-hand index of C under hairline.
7. Cursor to 1267 on C.
8. Read value on B = 1605 (thus 160·5 is the square of 12·67).
9. Under 1605 on C read 3255 on D.

Now, before we can use the relationship between the A and D scales to find the square root, we must first fix the decimal point by approximation.

The expression within the brackets can be written

$$\frac{3{\cdot}14\times 2{\cdot}29\times 1{\cdot}604}{6{\cdot}32\times 5{\cdot}6}\times\frac{10\times 10\times 10\times 10\times 10}{10}$$

or, again approximating $(3\times 2\times 2)/(6\times 6)\times 10^4$ (after cancelling 10's) which gives 3333 and indicates that there will be four figures before the decimal point.

Step 10. We therefore take our value as 3255 and as this has an even number of digits, we set it on the right-hand part of scale A.

Step 11. Under the centre hairline of the cursor read 57 on scale D. This is the required answer. Note that with Step 10, the accuracy of reading 3255 is lost, so that a quicker and better method would be as follows:

1. Cursor to gauge point π on A (left).
2. Bring 632 on B under hairline (left).
3. Cursor to 229 on B (right).
4. Bring 56 on B under hairline (left).
5. Cursor to 1267 on C.
6. Bring left index AB under hairline.
7. Under 10 of B read 57 on D.

14 CUBES

If, in the group of problems $a\times b^2$, we substitute a for b we get the process of cubing. In the example $5{\cdot}29^3$ we do the squaring first then, set the right index of the slide on the result. The answer 1–4–8 is then read off on A over 5–2–9. Hence the cube is 148 (fig. 24), since only this number would

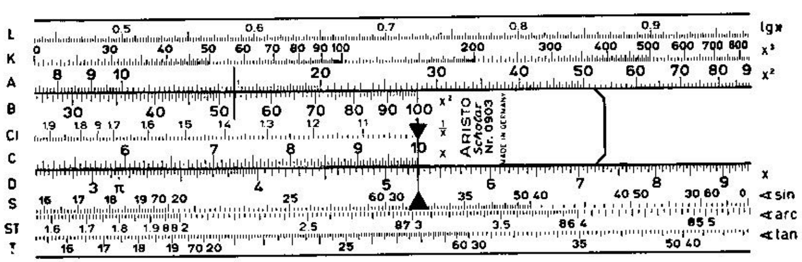


FIG. 24

be reasonable. We will soon get used to deciding by a rough approximation whether the right or the left index has to be employed in the second multiplication.

We will now show a more convenient method of cubing by using the special scale, termed the K scale or cube scale. In many slide rules, this is located directly above the scale of squares A. Sometimes this K scale is printed on a coloured background for the double purpose of clear differentiation from the other scales and to indicate that the novice in slide rule work does not have to depend on this scale if he prefers other methods of solution. The cube scale K consists of three equal segments, placed end-to-end, each one-third the length of a normal 25 cm scale. The graduations of each segment are identical. Numerically the entire triple graduation covers the range 1 to 1000, the logarithmic unit of length of each segment being contracted to one-third of a normal 25 cm scale (or $8\frac{1}{3}$ cm). Since there are few subdivisions on the K scale, the accuracy is diminished.

The general structure of the graduation, however, is the familiar one, and its correct interpretation is, by now, simple routine to the practised eye of the pupil.

Let d be some random value set on D. Then, by following the cursor hairline, we find another value k on K and can establish, for a slide rule 25 cm long, the relation

$$8\tfrac{1}{3}\log k = 25\log d$$
$$\log k = 3\log d$$
$$k = d^3$$

where d is the number and k is its cube.

In other words: for any number brought under the cursor line on D we can read its cube on K.

EXERCISES

$1{\cdot}82^3 = 6{\cdot}03$	$0{\cdot}123^3 = 0{\cdot}00186$
$4{\cdot}64^3 = 99{\cdot}9$	$0{\cdot}39^3 = 0{\cdot}0593$
$8{\cdot}43^3 = 599$	$0{\cdot}77^3 = 0{\cdot}457$
$12{\cdot}4^3 = 1907$	$0{\cdot}0215^3 = 0{\cdot}00000994$
$21{\cdot}7^3 = 10{,}220$	$0{\cdot}0275^3 = 0{\cdot}0000208$
$68{\cdot}1^3 = 316{,}000$	$0{\cdot}0628^3 = 0{\cdot}000248$

15 CUBE ROOTS

When the problem consists in finding the cube root of a given number the foregoing procedure is, of course, applied in reverse. After placing the cursor hairline over the radicand in the 'correct' section of scale K, we obtain the cube root by following the hairline to D. It has to be pointed out, however, that strict attention should be given to the correct digit count for the radicand as this decides in which third of the K scale the setting has to be made. There is no difficulty in this respect when the radicand lies between 1 and 1000, since it will then correspond directly with the numbering on K.

To find the cube roots of numbers outside the range 1 to 1000, we divide the radicand into groups of three digits to left and right of the decimal point. The number of figures in the foremost group (or respectively in the first group after the decimal point not entirely consisting of zeros) defines whether the setting on scale K is in the 1st, 2nd or 3rd sections numbered from the left.

EXERCISES

$\sqrt[3]{8{\cdot}49} = 2{\cdot}04$	$\sqrt[3]{0{\cdot}036} = 0{\cdot}330$
$\sqrt[3]{555} = 8{\cdot}22$	$\sqrt[3]{0{\cdot}043} = 0{\cdot}350$
$\sqrt[3]{163{\cdot}7} = 5{\cdot}47$	$\sqrt[3]{0{\cdot}00108} = 0{\cdot}1026$
$\sqrt[3]{927} = 9{\cdot}75$	$\sqrt[3]{7300} = 19{\cdot}4$
$\sqrt[3]{0{\cdot}167} = 0{\cdot}551$	$\sqrt[3]{10} = 2{\cdot}154$
$\sqrt[3]{356} = 7{\cdot}09$	$\sqrt[3]{32{\cdot}5} = 3{\cdot}19$
$\sqrt[3]{1127} = 10{\cdot}41$	$\sqrt[3]{0{\cdot}078} = 0{\cdot}427$
$\sqrt[3]{0{\cdot}836} = 0{\cdot}942$	$\sqrt[3]{97{\cdot}4} = 4{\cdot}60$
$\sqrt[3]{260{\cdot}9} = 6{\cdot}39$	$\sqrt[3]{5{\cdot}93} = 1{\cdot}81$
$\sqrt[3]{163{\cdot}7} = 5{\cdot}47$	$\sqrt[3]{12{,}490} = 23{\cdot}2$
$\sqrt[3]{636} = 8{\cdot}60$	

16 SCALE OF RECIPROCALS

The CI (C inverted) scale is an exact counterpart of scale C with the exception that its graduation runs in the opposite direction, namely from right to left (*see* fig. 25).

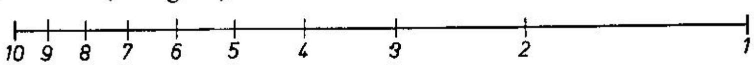


FIG. 25

We shall first inquire into the mathematical relationship of the numbers printed on the two scales when the slide is 'in the neutral position', that is when their indices coincide. By inspection we can see that any number d multiplied by the corresponding number c_1 equals 10. In other words $\log d + \log c_1 = \log 10$. In plain numerals without regard to the decimal point, as agreed upon, this would give

$$\log d + \log c_1 = \log 1$$
$$\text{so that} \quad d \times c_1 = 1$$
$$\therefore \quad c_1 = 1/d$$

It will now be clear that for any value on scale D we find its reciprocal on scale CI.

The example in fig. 26 demonstrates that $(1/0{\cdot}206) = (4{\cdot}85)$. A rough estimate decides the correct place for the decimal point when its location is not clear from the outset.

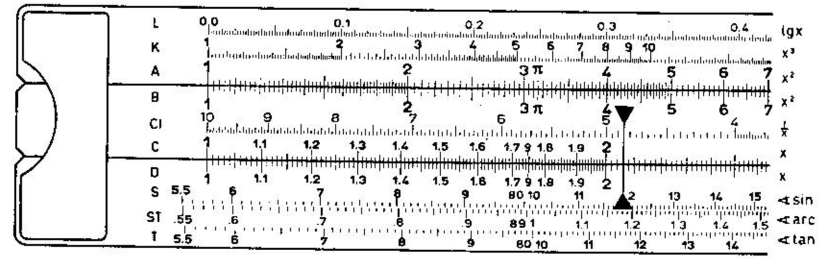


FIG. 26

EXERCISES

$\frac{1}{7{\cdot}29} = 0{\cdot}1372$	$\frac{1}{0{\cdot}082} = 12{\cdot}20$
$\frac{1}{0{\cdot}348} = 2{\cdot}87$	$\frac{1}{0{\cdot}222} = 4{\cdot}44$
$\frac{1}{38{\cdot}3} = 0{\cdot}0261$	$\frac{1}{55{\cdot}6} = 0{\cdot}01799$
$\frac{1}{273} = 0{\cdot}00366$	$\frac{1}{3{\cdot}7} = 0{\cdot}270$

Similarly the reciprocals of squares can be read directly when the slide is 'in the neutral position'. In looking for $(1/5{\cdot}2^2)$ we set the cursor to 5–2 on CI and find the answer 0·037 on A. It is also easy to obtain the reciprocals of square roots. In dealing with $(1/\sqrt{9{\cdot}81})$ we simply reverse the foregoing procedure by setting the radicand 9–8–1 on scale A and reading the answer 0·319 on CI. (As previously, when dealing with square roots, there is an odd number of digits to the left of the decimal place, hence the radicand will be sought for in the first, or left-hand section of A.)

EXERCISES

$\frac{1}{1{\cdot}26^2} = 0{\cdot}630$	$\frac{1}{\sqrt{2}} = 0{\cdot}707$
$\frac{1}{2{\cdot}24^2} = 0{\cdot}1993$	$\frac{1}{\sqrt{5}} = 0{\cdot}447$
$\frac{1}{3{\cdot}84^2} = 0{\cdot}0678$	$\frac{1}{\sqrt{13}} = 0{\cdot}277$
$\frac{1}{0{\cdot}56^2} = 3{\cdot}19$	$\frac{1}{\sqrt{46{\cdot}5}} = 0{\cdot}1466$
$\frac{1}{0{\cdot}725^2} = 1{\cdot}902$	$\frac{1}{\sqrt{0{\cdot}615}} = 1{\cdot}275$

Scale CI does more than merely supply reciprocals. We shall easily realize that a multiplication done after the pattern used with the C and D scales, amounts to a division when CI takes the place of C. Conversely, a division by one of the CI values is equivalent to a multiplication. Since we know from earlier experience in the course of our studies that a division can always be carried out without the inconvenience of 'resetting the slide', it is often practicable to change a multiplication into a division. From a mathematical point of view the problem $a \times b$ is changed to the division

$$\frac{a}{1/b} \text{ or } a \div \frac{1}{b}$$

Consider the equation $7{\cdot}84x = 0{\cdot}652/0{\cdot}384$; to solve, convert this mentally to

$$\frac{x}{1/7{\cdot}84} = \frac{0{\cdot}652}{0{\cdot}384} \qquad \left(\text{i.e.} \frac{x}{c_1} = \frac{d}{c}\right)$$

then: Step 1. Cursor to 652 on D.

2. Draw 384 on C under cursor.
3. Under 784 on CI read 0·216 on D.

This method involves but one movement of the slide (fig. 27).

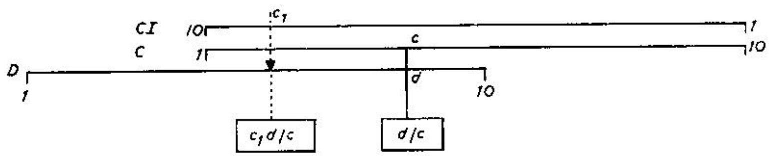


FIG. 27

Fig. 28 gives the solution for $38{\cdot}7 \times 12{\cdot}71 = 492$.

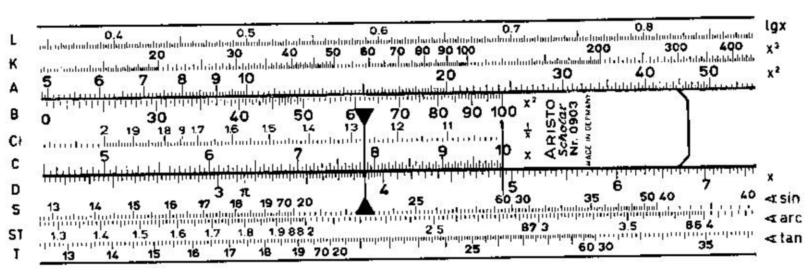


FIG. 28

EXERCISES

426·1	×	0·824	=	351
3·69	×	7·23	=	26·7
567	×	4·18	=	2370
0·047	×	107·6	=	5·06
0·01947	×	48·25	=	0·939
0·358	×	7·49	=	2·68
5·92	×	1852	=	10,960
2·718	×	13·9	=	37·8

Scale CI is also useful when some constant value d is to be divided by several divisors to produce a table of quotients. The procedure consists in setting either index of the slide to d on scale D and reading the required quotients on D by successive moves of the cursor along scale CI without moving the slide. Only in some rare cases it may become necessary to reset the slide. By use of this method we can, for instance, find the ordinates of the rectangular hyperbola

$$y = \frac{d}{x} = \frac{3.68}{x}$$

x	1·5	2·1	2·5	3·2	3·9	4·6	7·2
y	2·45	1·752	1·472	1·150	0·944	0·800	0·511

Moreover the frequently recurring problem $a/(b \times c)$, or division by two divisors, can be dealt with in a simplified form. In the example $82{\cdot}5/(87 \times 66)$ the first step is the division of 82·5 by 87 using scales D and C, followed by multiplication of the quotient (whose value we may ignore) by the reciprocal 1/66. This means that we must look for the answer under the place for 6–6 of scale CI, where we find the result 0·01437 on the D scale.

The most remarkable time-saving feature of the CI scale is demonstrated in continued multiplication. In dealing with $a \times b \times c$ we first transform this expression to suit slide rule requirements, i.e. we write

$$\frac{a}{1/b} \times c.$$

Computation can usually be carried out with only one setting (compare also fig. 27) for examples such as

$$2{\cdot}61 \times 5{\cdot}71 \times 3{\cdot}24 = 48{\cdot}3.$$

Figure 29 shows in detail how the answer is obtained.

EXAMPLE. To solve $1{\cdot}827 \times 2{\cdot}68 \times 88 \times 0{\cdot}436 \times 360$

work as $$1{\cdot}827 \times \frac{1}{1/88} \times 2{\cdot}68 \times \frac{1}{1/0{\cdot}436} \times 360$$

Step 1. Cursor to 1827 on D.
2. Draw 88 on CI under hairline.
3. Cursor to 268 on C.
4. Draw 436 on CI under hairline.
5. Cursor to 360 on C.
6. Under hairline read significant figures of answer on D 676.
7. Locate decimal point. Answer 67,600.

Only two movements of the slide were required.

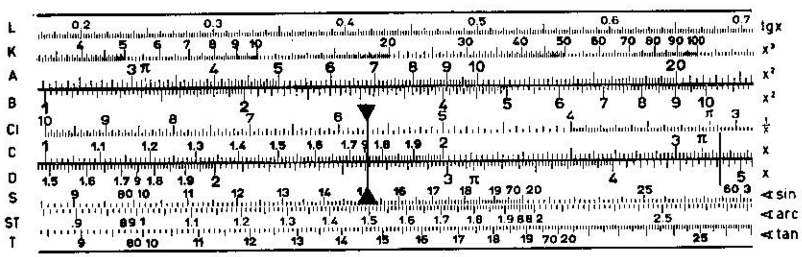


FIG. 29

EXERCISES

0·467	× 2·63	× 66·6	= 8–1–8	=	81·8
36·9	× 4·51	× 54·3	= 9–0–3	=	9030
2·15	× 8·26	× 3·14	= 5–5–8	=	55·8
0·526	× 0·144	× 1·945	= 1–4–7–3	=	0·1473

17 SOLVING EQUATIONS BY SLIDE RULE

By way of introduction to this chapter it may be stated that every kind of equation, including those of higher degrees can be rationally solved with the aid of the slide rule.

(*a*) In dealing with *equations of the first degree* with two unknowns, we proceed as follows

(1) $3{\cdot}54x + 1{\cdot}97\ y = 36{\cdot}46$ | $\frac{2{\cdot}11}{1{\cdot}97}$ Step 1. suitably multiply (1) by $\frac{2{\cdot}11}{1{\cdot}97}$ so as to obtain (3)

(2) $4{\cdot}87\ x - 2{\cdot}11\ y = 9{\cdot}62$

(3) $3{\cdot}79\ x + 2{\cdot}11\ y = 39{\cdot}05$

(4) $8{\cdot}66\ x = 48{\cdot}67$ Step 2. add (2) and (3) to get (4).

(5) $x = 5{\cdot}62$ Step 3. whence (5).

Leaving the slide in this position, we continue by computing $3{\cdot}79\ x = 21{\cdot}3$ hence

$$2{\cdot}11\ y = 39{\cdot}05 - 21{\cdot}3$$
$$2{\cdot}11\ y = 17{\cdot}75$$
$$y = 8{\cdot}41$$

This procedure is particularly useful and is a considerable short-cut where cumbersome coefficients have to be dealt with. In practice these undoubtedly occur more frequently than whole numbers.

EXAMPLE

$$4{\cdot}25\ x - 7{\cdot}15\ y = 19{\cdot}82 \quad \left|\ \frac{1{\cdot}56}{7{\cdot}15}\right.$$
$$9{\cdot}22\ x + 1{\cdot}56\ y = 79{\cdot}2$$
$$0{\cdot}93\ x - 1{\cdot}56\ y = 4{\cdot}32$$
$$10{\cdot}15\ x = 83{\cdot}52$$
$$x = 8{\cdot}23$$

therefore

$$7{\cdot}63 - 1{\cdot}56\ y = 4{\cdot}32$$
$$y = 2{\cdot}12$$

(*b*) In dealing with *quadratic equations* of the usual form

$$x^2 + px + q = 0$$

it is convenient to recall Viëta's formula, which states that the roots x_1 and x_2 are related as follows

$$x_1 + x_2 = -p$$
$$x_1 \times x_2 = q$$

The product of the roots is thus known since it is the constant term in the equation, so that we can also determine the signs of the roots. Our next step is finding the two factors x_1 and x_2 whose sum equals the prescribed magnitude $-p$. We accomplish this by systematic inquiry. Let us take the example

$$x^2 + 4{\cdot}58\ x - 11{\cdot}98 = 0$$

At a glance we see that the two roots must have different signs and that the absolute value of the negative root must exceed the value of the positive root. We can easily form a complete table of pairs of numbers whose respective products are 11·98 by use of scale CI, as previously practised. For this purpose we set the left index of the slide over the line corresponding to 1–1–9–8 on scale D. Then each pair of values on D and CI under the cursor hairline $\{x_1 x_2 = x_1 \div (1 - x_2)\}$ are factors of the required product.

It would be senseless, however, to go about this trusting to chance. Our search must follow a systematic course and the first step in the computation should be an estimate of the approximate value. Since the algebraic sum must be $-4{\cdot}58$, we can quickly see by inspection of the relative positions of CI and D that the roots are definitely near -6 and 2, respectively. It is advisable to draw up a simple testing table in the following manner:

x_1	$-6{\cdot}00$	$-6{\cdot}50$	$-6{\cdot}40$	$-6{\cdot}44$
x_2	$+1{\cdot}99$	$+1{\cdot}84$	$+1{\cdot}87$	$+1{\cdot}86$
x_1+x_2	$-4{\cdot}01$	$-4{\cdot}66$	$-4{\cdot}53$	$-4{\cdot}58$

It is unlikely that the first trial will yield the right figures. In the second attempt we have obviously gone too far for x_1, whereas the third endeavour is just short of our aim. It is the fourth effort that gives the roots we have been looking for.

The next example may serve for practice:

$$x^2-5{\cdot}05\,x+5{\cdot}85=0$$

x_1	$+2{\cdot}00$	$+1{\cdot}75$	$+1{\cdot}80$
x_2	$+2{\cdot}92$	$+3{\cdot}34$	$+3{\cdot}25$
x_1+x_2	$+4{\cdot}92$	$+5{\cdot}09$	$+5{\cdot}05$

Check: $1{\cdot}80\times3{\cdot}25=5{\cdot}85$, which is correct!

EXERCISES

$x^2-\ 6{\cdot}61\,x+10{\cdot}16=0$ $(x_1=+2{\cdot}43;\ x_2=+4{\cdot}18)$
$x^2+\ 6{\cdot}96\,x+\ 7{\cdot}23=0$ $(x_1=-1{\cdot}27;\ x_2=-5{\cdot}69)$
$x^2+11{\cdot}03\,x+27{\cdot}1\ =0$ $(x_1=-73{\cdot}4;\ x\ =-3{\cdot}69)$
$x^2-\ 3{\cdot}06\,x-42{\cdot}4\ =0$ $(x_1=+8{\cdot}22;\ x_2=-5{\cdot}16)$

(*c*) *Cubic equations* should be reduced to the form

$$x^3-ax+b=0$$

and then transformed for slide rule workability into

$$x^2+\frac{b}{x}=a$$

Using the example $\quad x^3-25{\cdot}6\,x+46{\cdot}7=0,$

we have $\quad x^2+\dfrac{46{\cdot}7}{x}=25{\cdot}6$

so that their treatment is analogous to that for quadratics.

Here too we use the inverted scale, setting its right index over 4–6–7 on scale D. Now each pair of values on D and CI multiplied together gives the product b. But we still require to know the value of x^2. This, in turn, is found under the same cursor line on A.

Once again we move our cursor systematically and draw up a working table, viz.

x	3	2·5	2·4	2·3	4	3·8	3·6	3·5
x^2	9	6·25	5·76	5·29	16	14·44	12·96	12·25
$\frac{b}{x}$	15·57	18·68	19·46	20·30	11·68	12·29	12·97	13·34
$x^2+\frac{b}{x}$	24·57	24·93	25·22	25·59	27·68	26·73	25·93	25·59

It will not take us long in this case to ascertain that $x_1=2{\cdot}3$ and $x_2=3{\cdot}5$. Consequently, and in accordance with Viëta's formula for roots, we obtain

$$x_3=-(x_1+x_2)=-5{\cdot}8$$

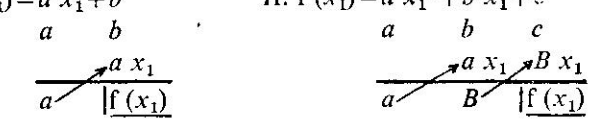
EXERCISES

$x^3-67{\cdot}8x\ +103{\cdot}5\ =0$ $[x_1=+1{\cdot}59;\ x_2=+7{\cdot}32;\ x^3=-\ 8{\cdot}91]$
$x^3-98{\cdot}3x\ +361{\cdot}2\ =0$ $[x_1=+4{\cdot}8;\ x_2=+6{\cdot}6;\ x^3=-11{\cdot}4]$
$x^3-\ 5{\cdot}93x+\ \ 3{\cdot}05=0$ $[x_1=+0{\cdot}54;\ x_2=+2{\cdot}12;\ x^3=-\ 2{\cdot}66]$
$x^3-36{\cdot}43x+\ 77{\cdot}1\ =0$ $[x_1=+4{\cdot}3;\ x_2=+2{\cdot}6;\ x^3=-\ 6{\cdot}9]$

(*d*) For solving *equations of the third and higher degrees* Newton's formula for approximation can often be employed. We shall recall that in this method a so-called correction term $\delta=-[f(x_1)]/[f'(x_1)]$, is inserted, in which x_1 represents the first approximation. In the computation of the values of a function, on the other hand, we shall resort to Horner's method which is explained below.

After writing down the coefficients we multiply by the value of x in the direction of the arrows and add up the numbers in the vertical columns.

I. $f(x_1)=a\,x_1+b$ II. $f(x_1)=a\,x_1^2+b\,x_1+c$



because: $B\,x_1=(a\,x_1+b)\,x_1=a\,x_1^2+b\,x_1$

III. $f(x_1) = a\,x_1^3 + b\,x_1^2 + c\,x_1 + d$

a	b	c	d
	$a\,x_1$	$B\,x_1$	$C\,x_1$
a	B	C	$\mid f(x_1)$

a	b	c
	$a\,x_1$	$B'\,x_1$
a	B'	$\mid f'(x_1)$

because:

$$B = a\,x_1 + b$$
$$C = (a\,x_1 + b)\,x_1 + c = a\,x_1^2 + b\,x_1 + c$$
$$Cx_1 = a\,x_1^3 + b\,x_1^2 + c\,x_1$$
$$B' = B + a\,x_1 = 2a\,x_1 + b$$
$$B'x_1 = 2a\,x_1^2 + b\,x_1$$
$$f'(x_1) = 3a\,x_1^2 + 2b\,x_1 = c$$

IV. $f(x_1) = a\,x_1^4 + b\,x_1^3 + c\,x_1^2 + d\,x_1 + e$

a	b	c	d	e
	$a\,x_1$	$B\,x_1$	$C\,x_1$	$D\,x_1$
a	B	C	D	$\mid f(x_1)$
	$a\,x_1$	$B'\,x_1$	$C'\,x_1$	
a	B'	C'	$\mid f'(x_1)$	

EXAMPLE $f(x) = x^3 - 4{\cdot}21\,x^2 + 9{\cdot}2\,x - 38{\cdot}73 = 0.$

Assuming that the first rough approximation has given $x_1 = +4$.

1	$-4{\cdot}21$	$+\ 9{\cdot}20$	$-38{\cdot}73$
	$+4$	$-\ 0{\cdot}84$	$+33{\cdot}44$
1	$-0{\cdot}21$	$+\ 8{\cdot}36$	$\mid -5{\cdot}29 = f(x_1)$
	$+4$	$+15{\cdot}16$	
1	$+3{\cdot}79$	$\mid +23{\cdot}52 = f'(x_1)$	

Then by Newton's formula

$$x_2 = x_1 - \frac{f'(x_1)}{f'(x_1)}$$

$$x_2 = 4 - \frac{-\ 5{\cdot}29}{+\ 23{\cdot}52}$$

$$x_2 = 4 + 0{\cdot}225$$

$$x_2 = 4{\cdot}2$$

Let it be assumed that we have increased the accuracy by one decimal place and proceed by use of this closer approximation to find x_2.

1	$-4{\cdot}21$	$+\ 9{\cdot}20$	$-38{\cdot}73$
	$+4{\cdot}2$	$-\ 0{\cdot}042$	$+38{\cdot}50$
1	$-0{\cdot}01$	$+\ 9{\cdot}16$	$\mid -\ 0{\cdot}23 = f(x_2)$
	$+4{\cdot}2$	$+17{\cdot}60$	
1	$+4{\cdot}19$	$\mid +26{\cdot}76 = f'(x_2)$	

$$x_3 = 4{\cdot}2 - \frac{-\ 0{\cdot}23}{+26{\cdot}76}$$

$$x_3 = 4{\cdot}2 + 0{\cdot}0086 \qquad x_3 = \underline{+4{\cdot}21}$$

In this instance the advantage of slide rule computation consists in our being able to perform the various multiplications with the same factor and, therefore, generally with only one setting of the slide.

EXERCISES

$x^3 - 3\,x - 4 = 0$	$[x_2 = +2{\cdot}20]$	$[x^3 = +2{\cdot}196]$
$x^4 + 8\,x^3 - 800 = 0$	$[x_2 = +4{\cdot}1]$	$[x^3 = +4{\cdot}05]$
$x^5 - 18{\cdot}3\,x^3 + 25{\cdot}6\,x + 36{\cdot}4 = 0$	$[x_2 = +1{\cdot}7]$	$[x^3 = +1{\cdot}74]$

18 THE MANTISSA SCALE L

In the present chapter we shall get our second introduction to those parts of our rule which are frequently tinted, in contrast to the white portion which has so far been the main object of our studies in the more elementary stages of this course. Surveying the scale marked L it will strike us as peculiar that the dividing lines in the graduation are all equidistant. The same standard of length (25 cm) which was chosen for the logarithmic scales C and D is here divided into 10 primary decimal parts bearing their respective numerals. Each primary subdivision is, in turn, split up into 50 further subdivisions, the lines for every fifth of which are slightly longer than the others. This means that each interval is equivalent to 0·002 (two thousandths) of the total length. Settings and readings can be carried out accurately to one thousandth without eye-strain.

The relationship between this scale L and the fundamental scale D is based upon the following mathematical concepts:

Since the scales C and D were originally drawn by plotting consecutive numbers logarithmically, it is obvious that the reverse is true, i.e. a direct measurement of metric distance along the scales will provide a logarithmic length. Direct readings are to two parts in 1000, that is, to three decimal places. Hence for any number set on scale D we find its logarithm on scale L. In reversed order we can set the logarithm on L and read the antilogarithm on D. The scale L is sometimes referred to as a scale of logarithms. This name is not quite correct and 'mantissa scale' is more accurate, since we can only read the mantissas without the characteristic. We can, in short, regard our slide rule as a substitute for a three-figure Table of logarithms.

Figure 30 shows the setting for

log 7·2 = 0·857

EXERCISES

log 128·5	= 2·109
log 0·296	= 0·471 − 1
log 44·9	= 1·652
log 9·51	= 0·978
log 1878	= 3·274
log 0·063	= 0·799 — 2
log 65·6	= 1·817

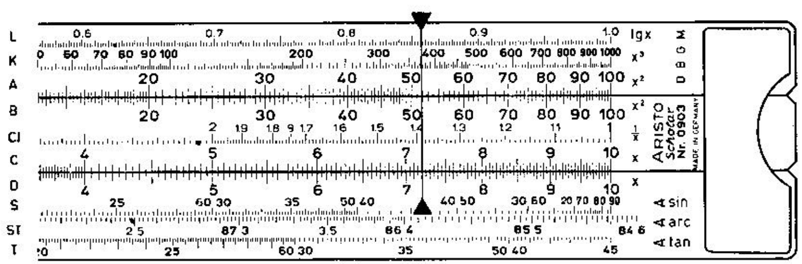


FIG. 30

For the reverse procedure we shall consider the following example. Let the given logarithm be log x = 2·620. Its mantissa 0·620 gives for the antilogarithm the digits 4–1–7. The characteristic of the given logarithm is 2, hence we know that the antilogarithm must have 2 + 1 = 3 digits. The required logarithm therefore is 417.

EXERCISES

log x = 1·896	x = 78·7
log x = 3·255	x = 1790
log x = 0·300–1	x = 0·1995
log x = 2·802	x = 634
log x = 0·917–2	x = 0·0826
log x = 4·628	x = 42,500
log x = 1·997	x = 99·3
log x = 2·028	x = 106·7

In practice this process of finding logarithms is applicable to problems involving powers and roots to any index. To clarify this we shall solve some simple examples which can be checked by another method. Computing, for instance, $2{\cdot}34^4$ we look up and find log 2·34 = 0·369. A quick mental multiplication gives 4 × log 2·34 = 4 × 0·369 = 1·476.

Antilog 1·476 = 30, hence too $2{\cdot}34^4 = 30$.

Checking this is a simple matter of twice squaring 2·34. The result of this operation is $2{\cdot}34^4 = 5{\cdot}48^2 = 30$.

Computing $1{\cdot}86^3$ we obtain x = antilog (3 log 1·86)

log 1·86 = 0·269
3 log 1·86 = 3 × 0·269 = 0·807
x = 6·42

By squaring and multiplying we get $1{\cdot}86^3 = 6{\cdot}43$ (Check!).

In dealing with $\sqrt[3]{46{\cdot}8} = x$ we solve for x = antilog ($\frac{1}{3}$ log 46·8), thus

log 46·8 = 1·670
$\frac{1}{3}$ log 46·8 = 0·557
x = 3·61

Checking by the usual method for extracting the cube root: $\sqrt[3]{46{\cdot}8} = 3{\cdot}60$. There is, as can be seen, a negligible difference in the third digits of the two answers, which is to be expected, due to 'slide rule accuracy'.

In another type of problem such as $\sqrt[3.6]{268}$, we first establish that the mantissa is 0·428. After prefixing the characteristic we have log 268 = 2·428. By use of scales D and C we now perform the division by 3·6, reading the quotient (1/3·6) log 268 = (2·428/3·6) = 0·674 under the right index of the slide. With this mantissa set on L we obtain the answer 4·72 on scale D.

Problems involving higher powers are likely to arise in computations involving the compound interest formula

$$S = P\,(1+i)^n$$

EXAMPLE. To find the amount to which a principal of £1480 will accumulate after 9 years at compound interest of 4 per cent per annum:

Set log 1·04 = 0·017, multiply mentally
9 log 1·04 = 9 × 0·017 = 0·153.
∴ antilog 0·153 = 1·423, hence by slide rule
1480 × 1·423 = £2105

Solving by use of a Table of logarithms we would obtain S = £2106·50.

EXERCISE

Find the amount to which a principal of $775 accumulates after 7 years at interest of 5 per cent compounded annually.

$$\log 1{\cdot}05 = 0{\cdot}021$$
$$7 \log 1{\cdot}05 = 0{\cdot}147$$
$$775 \times 1{\cdot}403 = 1087$$

Answer $1087.

This method can, of course, be used only in approximate computations. This is due to the fact that every error in reading the logarithm, however slight, accumulates in the subsequent operations, so that the final result must necessarily be affected. This problem also arises in lengthy computations using four-figure Tables, when it is customary to use seven- or ten-place logarithmic Tables for greater accuracy.

19 THE SINE SCALE S

The scales marked S, ST and T give sine and tangent values. They are related to the fundamental scales C and D. On the S scale the tenfold values of angles are reproduced in the form of logarithmic lengths, comprising the angles from 5·5° to 90°. The subdivisions of the degrees are in decimals, this arrangement being preferred in modern engineering and science. Between 5·5° and 20° the graduation progresses by tenths of one degree, between 20° and 80° the lines are separated by intervals of 0·2°, and between 80° and 90° only the whole degrees are marked to avoid crowding the scale. The left-hand index of the D scale coincides with 5·74° on S. This is so because the sine of an angle of 5·74° is exactly 0·1. When angles expressed in degrees and minutes have to be dealt with, we first have to convert the minutes into two-place decimal fractions by mental arithmetic, thus: $58^\circ\ 26' = 58\tfrac{26}{60}{}^\circ = 58{\cdot}43^\circ$. This is a necessary procedure for all slide-rule work, and in this text we are concerned only with the decimal method of notation for degrees and angles. Where many such computations are required it is best to prepare a conversion Table. We may say that when the sine scale S is used in conjunction with the fundamental scale D the slide rule supplies us with a sine Table and, coincidentally, with a cosine Table. For the use of this convenient Table we generally need to move only the cursor. Thus:

sin 30° = 0·500 (i.e. cursor to 30 on S, read 5 on D)

EXERCISES

sin 6·1° = 0·1063	sin 52·7° = 0·795
sin 12·6° = 0·218	sin 66·8° = 0·919
sin 22·4° = 0·381	sin 71·3° = 0·947
sin 36·9° = 0·600	sin 80° = 0·985
sin 47·2° = 0·734	sin 84·1° = 0·995
	sin 88° = 0·999

For small angles, i.e. those smaller than 5°, we have the relation $\sin \alpha \approx \text{arc}\ \alpha$. The arc itself is usually determined by multiplying the angle by $\rho = \text{arc}\ 1^\circ = \pi/180^\circ = 0{\cdot}01745$. The symbol ρ for this gauge-point factor is often printed on all three scales C, D and CI. The amount of possible error is $< 1{\cdot}55$ per thousand.

The special scale ST is used for the sines as well as the tangents of small angles, meaning angles between 0·574° and 5·74° in the slide rule's system. As a matter of fact, this special ST scale is a trigonometric scale divided in radian measure, but numerated in degrees. Here is the reason: the sines of angles below 5·74° are the same for most practical purposes, as the tangents of like angles. Since the numerical value of an angle expressed in radians lies somewhere between that of its sine and that of its tangent, we arrive, when considered jointly, at an even closer approximation for the two functional values obtained with the ST scale.

After setting the given small angle on the ST scale, its sine or tangent is found under the hairline on D. Bear in mind that in this case we must prefix 0·0 . . . to the readings, because the sines and tangents of all angles in this range lie between 0·01 and 0·10.

EXERCISES

sin 0·7° = 0·01222	sin 3·6° = 0·0628
sin 1·5° = 0·0262	sin 4·8° = 0·0838
sin 2·3° = 0·0401	sin 5° = 0·0873

In a similar manner we obtain the cosine value by using the relation

$$\cos \alpha = \sin (90^\circ - \alpha).$$

It will be observed that the process for setting sines becomes less accurate towards the right end. Where the sine values are greater than 0·985 (i.e. for angles between 80° to 90°), readings are not very accurate. This is so because the slide rule, seen as a graphical Table of logarithms, is a three-figure Table, whereas in the example referred to the differences in the 4th and the 5th decimal places affect the magnitude of the angle. In such a situation it is practicable to use the formula derived from Pythagoras' theorem

$$\sin \alpha = \sqrt{(1 - \cos^2 \alpha)}$$
$$\sin \alpha = \sqrt{\{1 - \sin^2 (90^\circ - \alpha)\}}$$

This computation can be simplified further by means of a formula of approximation. For small values of x the notation $\sqrt{(1-x)} \approx 1-(x/2)$ is generally applicable. In our example we could write

$$\sin \alpha \approx 1 - \tfrac{1}{2} \sin^2 (90° - \alpha).$$

Assuming that the sine of 84·1° is required to a fine degree of accuracy, we look up sin 5·9 and find above this value on A the square of $\sin^2 5{\cdot}9° = 0{\cdot}0106$. This gives for $\sin 84{\cdot}1° = 1 - 0{\cdot}00530 = 0{\cdot}99470$. Checking by means of a 5-figure Table of logarithms, we obtain exactly the same value.

For the computation of angles whose sines and cosines are known, the process is, of course, reversed.

EXERCISES

$\sin \alpha = 0{\cdot}1444$	$\alpha = 8{\cdot}3°$	$\sin \alpha = 0{\cdot}0836$	$\alpha = 4{\cdot}8°$
$\sin \alpha = 0{\cdot}269$	$\alpha = 15{\cdot}6°$	$\sin \alpha = 0{\cdot}0668$	$\alpha = 3{\cdot}83°$
$\sin \alpha = 0{\cdot}370$	$\alpha = 21{\cdot}7°$	$\sin \alpha = 0{\cdot}0595$	$\alpha = 3{\cdot}4°$
$\sin \alpha = 0{\cdot}586$	$\alpha = 35{\cdot}9°$	$\sin \alpha = 0{\cdot}0412$	$\alpha = 2{\cdot}36°$
$\sin \alpha = 0{\cdot}816$	$\alpha = 54{\cdot}7°$	$\sin \alpha = 0{\cdot}0289$	$\alpha = 1{\cdot}65°$
$\sin \alpha = 0{\cdot}927$	$\alpha = 68°$	$\sin \alpha = 0{\cdot}0158$	$\alpha = 0{\cdot}9°$

When a given trigonometric function is given to 4 or 5 places, we can still solve by slide rule, if we reverse the method described above. Since $\sin \alpha \approx 1 - \frac{1}{2} \sin^2 (90° - \alpha)$ it follows that

$$\tfrac{1}{2} \sin^2 (90° - \alpha) = 1 - \sin \alpha$$

$$\sin (90° - \alpha) = \sqrt{\{2\,(1 - \sin \alpha)\}}$$

$$90° - \alpha = \frac{\sqrt{\{2\,(1 - \sin \alpha)\}}}{\rho}$$

$$\alpha = 90° - \frac{\sqrt{\{2\,(1 - \sin \alpha)\}}}{\rho}$$

For instance: $\sin \alpha = 0{\cdot}99905$.

Then $1 - 0{\cdot}99905 = 0{\cdot}00095$

and $\sqrt{(2 \times 0{\cdot}00095)} = 0{\cdot}0436$

$= \text{arc } 2{\cdot}5° = \text{arc } (90° - \alpha)$.

So that $\alpha = 87{\cdot}5°$.

The slide rule procedure for this should be as follows:

Step 1. Right-hand index of B on to 95 on A. This has to be multiplied by 2: either 2 or 20 on B may be used for doing this; assess the result mentally and check the pairs of digits after the decimal point: the first group without zeros has an even number of significant figures, therefore the right-hand portion of B scale is the relevant one.

Step 2. Cursor to 20 on B.

Step 3. Read 2·5 on ST; alternatively read 0·0436 on D and divide by $\rho = 0{\cdot}1745$ on C to get arc 2·5° on D.

The usefulness of the slide rule again becomes particularly convincing when the sine rule enters into a problem. The shape of the notation $a/\sin \alpha = b/\sin \beta = c/\sin \gamma$ corresponds exactly to the order in which settings appear on the slide-rule scales. Here again the 'golden rule' of slide-rule computation comes into practical operation, fig. 31(*a*).

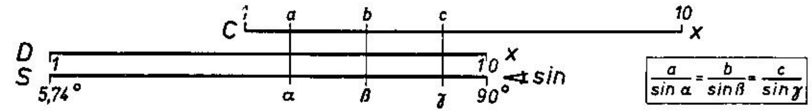


FIG. 31(*a*)

Given one side $a = 12{\cdot}8$ in. and two angles $\alpha = 54{\cdot}5°$ and $\beta = 76{\cdot}1°$ in a triangle, so that $\gamma = 49{\cdot}4°$. To find the other sides, set 1–2–8 on C over the respective angle 54·5° on S. Then we immediately obtain on C the side $b = 15{\cdot}3$ in. over β and the side $c = 11{\cdot}9$ in over γ, fig. 31(*b*).

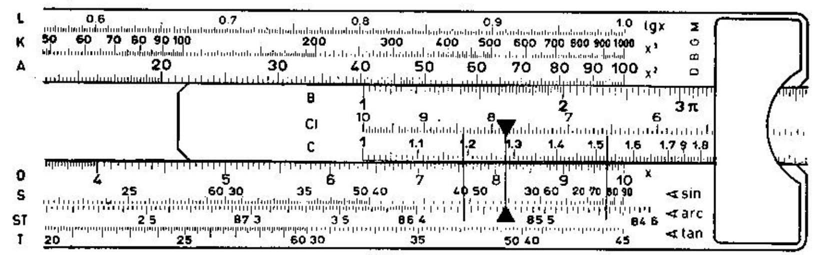
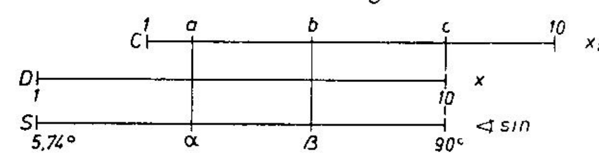


FIG. 31(*b*)

In the same manner right-angled triangles can be evaluated by use of the sine rule when the slide rule is set as fig. 32 shows.

FIG. 32



By forming a simple table and listing therein the given values, then that pair of values which fills one of the vertical columns is our initial ratio and the unknown elements are now easily found by use of the cursor.

$a=$	$b=$	$c=$
$\alpha=$	$\beta=$	$\gamma=$

EXERCISES FOR A RIGHT-ANGLED TRIANGLE

Given	$a=12$ ft	$c=13$ ft	*To find*	$\alpha=67\cdot4°$

(i.e. ratio $12\div13$ on C÷D scales, answer α is read against right-hand index of C scale on S scale).

$a=8\cdot0$ cm	$c=8\cdot9$ cm	$\alpha=64°$
$b=5\cdot5$ in.	$c=7\cdot3$ in.	$\beta=48\cdot9°$
$b=23\cdot8$ in.	$c=82\cdot6$ in.	$\beta=16\cdot75°$
$b=42\cdot5$ m	$c=87$ m	$\beta=29\cdot2°$
$a=7\cdot3$ ft	$c=9$ ft	$\alpha=54\cdot2°$
$a=2\cdot7$ yd	$c=11\cdot2$ yd	$\alpha=14\cdot0°$
$b=4\cdot7$ cm	$c=22\cdot8$ cm	$\beta=11\cdot9°$

Given	$c=35\cdot3$ ft	$\alpha=50°$	*To find*	$a=27\cdot0$ ft
	$c=5\cdot3$ in.	$\beta=58°$		$b=4\cdot5$ in.
	$c=6\cdot1$ cm	$\beta=25\cdot3°$		$b=2\cdot6$ cm
	$c=45$ mm	$\beta=10\cdot5°$		$b=8\cdot2$ mm
	$c=67$ in.	$\alpha=75°$		$a=65$ in.
	$c=78$ ft	$\alpha=35\cdot6°$		$a=45\cdot4$ ft
	$c=15\cdot6$ in.	$\alpha=21°$		$a=5\cdot6$ in.

Given	$a=12\cdot5$ in.	$\alpha=27\cdot5°$	*To find*	$c=27\cdot1$ in.
	$a=24\cdot7$ cm	$\alpha=41\cdot9°$		$c=37\cdot0$ cm
	$b=25\cdot9$ in.	$\beta=38°$		$c=42\cdot1$ in.
	$b=7\cdot8$ ft	$\beta=21\cdot3°$		$c=21\cdot5$ ft
	$b=15\cdot3$ yd	$\beta=12\cdot4°$		$c=71\cdot2$ yd
	$a=53$ mm	$\alpha=73°$		$c=55\cdot4$ mm
	$b=40$ ft	$\beta=60\cdot5°$		$c=46$ ft
	$a=6\cdot5$ cm	$\alpha=48\cdot5°$		$c=8\cdot7$ cm

EXERCISES FOR NON-RIGHT-ANGLED TRIANGLES

Given { $b=7\cdot2$ ft, $\beta=15\cdot5°$, $\gamma=65\cdot3°$ } *To find* { $\alpha=99\cdot2°$, $c=24\cdot5$ ft, $a=26\cdot6$ ft }

$\beta+\gamma=80\cdot8°$

Given { $c=31\cdot8$ in., $\gamma=28\cdot7°$, $\beta=54\cdot8°$ } *To find* { $b=54$ in., $\alpha=96\cdot5°$, $a=65\cdot8$ in. }

$\gamma+\beta=83\cdot5°$

The group of problems in which two sides and the angle opposite the longer side are given is equally easy to solve by the slide rule with the aid of the sine rule.

EXERCISES

Given { $a=14\cdot4$ in., $c=11\cdot2$ in., $\alpha=58\cdot6°$ } *To find* { $\gamma=41\cdot6°$, $\alpha+\gamma=100\cdot2°$, $\beta=79\cdot8°$, $b=16\cdot6$ in. }

Given { $b=2\cdot3$ ft, $c=7\cdot5$ ft, $\gamma=68\cdot3°$ } *To find* { $\beta=16\cdot6°$, $\beta+\gamma=84\cdot9°$, $\alpha=95\cdot1°$, $a=8\cdot0$ ft }

Given { $a=26\cdot5$ cm, $b=43\cdot2$ cm, $\beta=81°$ } *To find* { $\alpha=37\cdot3°$, $\alpha+\beta=118\cdot3°$, $\gamma=61\cdot7°$, $c=38\cdot5$ cm }

When two sides of a triangle and the angle between them are given, the cosine rule is normally applicable. This rule, as we know, means interrupting the calculation when logarithms are used. We shall show a way of overcoming this difficulty in slide-rule work and how to solve problems of this kind by means of the more elegant sine rule in accordance with the principle of proportion. The following illustration will explain the procedure.

EXAMPLE 1. When the three given elements in a triangle are $a=7\cdot2$ in.; $b=8\cdot9$ in. $\gamma=61\cdot5°$, then we can state the following relationships

$$\text{I. } \frac{\sin\alpha}{\sin\beta}=\frac{a}{b}=\frac{7\cdot2}{8\cdot9} \qquad \text{II. } \alpha+\beta=180°\gamma-=118\cdot5°.$$

We know that opposite the values 7·2 and 8·9 on C different pairs of angles can be found on S depending on the position of the slide C. Only one such combination of angles will satisfy equation I. So, by manipulation of the slide, we look for a pair of angles that will satisfy equation II. This should not be done in an aimless manner but by following a definite plan. It is obvious that $\alpha<\beta$, so we try $\alpha=50°$ first. After three or four further attempts we have the right angle, as the following Table shows.

α	50°	49°	49·2°
β	71°	69°	69·3°
$\alpha+\beta$	121°	118°	118·5°

if $y = a \times \sin x$,

$y = 0{\cdot}52 \times \sin 0{\cdot}96$

then $y = 0{\cdot}52 \times \sin 55°$

whence $\underline{y = 0{\cdot}426.}$

A shorter method uses scale ST. To change degrees to radian measure, and vice versa, set the cursor to the angle on the ST scale and read the equivalent in radians on D, and vice versa.

Considering that both scales have decimal subdivisions, the foregoing applies not only to the small angles actually readable on the ST scale, but also to their decimal variants; that means to all possible angles. Earlier slide rules had a setting mark ρ, inserted in the D scale.

$$\rho = \frac{\pi}{180°} = 0{\cdot}01745.$$

This mark has now become superfluous.

20 THE TANGENT SCALE T

The Tangent Scale T on the slide rule body supplies a three-figure Table of tangents and cotangents. It will be noticed that the logarithms of the tangents of the respective angles are projected in the form of line segments, and, further, that all angles from 5·5° to 45° are included. Looking up a Table of trigonometric functions we find that tan 5·71° is exactly 0·1, and of the angle 45° we know that its tan equals 1 so that these values correspond respectively to the left and right extremities of the C and D scales. In the range 5° to 30° the subdivisions represent tenths of one degree. Between 30° and 45° each interval of the graduation represents 0·2° (or one fifth of one degree), so that the odd tenths will have to be located by sight.

For reading we use the cursor hairline to bring values on scale T into coincidence with D.

EXERCISES

tan 36·2° = 0·732	tan 10·3° = 0·1817
tan 6·2° = 0·1086	tan 28·8° = 0·550
tan 43·3° = 0·942	tan 31·7° = 0·618
tan 25·3° = 0·473	tan 38·2° = 0·787
tan 19·8° = 0·360	tan 42·5° = 0·916

When the given angle $\alpha > 45°$, the relation $\tan \alpha \times \cot \alpha = 1$ provides a convenient alternative.

By this formula we have

$$\tan \alpha = \frac{1}{\cot \alpha} \qquad \text{e.g. } \tan 70° = \frac{1}{\tan 20°} \quad (\textit{see fig. 33})$$

$$\tan \alpha = \frac{1}{\tan (90° - \alpha)} \qquad \tan 70° = 2{\cdot}75$$

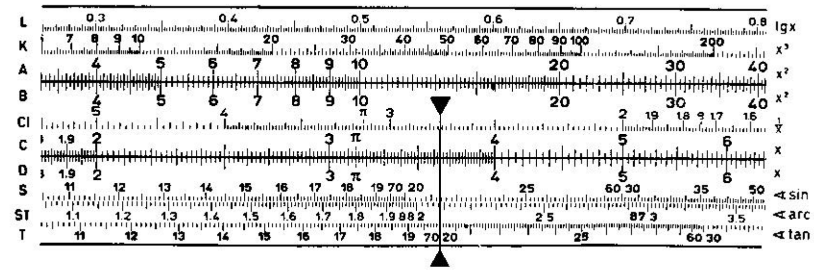


FIG. 33

In slide rule technique this means that after reaching tan 45° = 1, at the right-hand extreme, we now use the T scale, but counting from right to left for angles tan 45° = 1 up to tan 84·29° = 10, and *read the functions on the* CI *scale.*

For convenience, there are frequently overprinted a set of red numerals running right to left from 50° to 80°.

If the function is a tangent, the colour rule is Black for Black and Red for Red, i.e. T scale black onto D scale black; or T scale red onto CI scale red.

If the function is a cotangent, the colour rule is Black for Red or Red for Black, i.e. T scale black onto CI scale red, or T scale red onto D scale black.

EXERCISES

tan 50·2° = 1,200	tan 48·5° = 1·130
tan 63·0° = 1·963	tan 83·7° = 9·060
tan 81·4° = 6·610	tan 67·3° = 2·390
tan 58·9° = 1·658	tan 75·5° = 3·870

When $\alpha < 5{\cdot}71°$ we make use of the formula $\tan \alpha° \approx \text{arc } \alpha° = \rho \times \alpha°$. Alternatively, we can once again use the ST scale.

The determination of small angles from their given tangents is in every respect the same as that for the sine function.

EXERCISES

tan 3·7° = 0·0647	tan 2·5° = 0·0437
tan 4·2° = 0·0734	tan 1·7° = 0·0297
tan 3·0° = 0·0524	tan 0·8° = 0·0140

When α is greater than 84·29°, the correct setting is

$$\tan \alpha = \frac{1}{\tan (90° - \alpha)} \text{ and we switch from scale ST to scale CI.}$$

EXERCISES

tan 87° = 19·08	tan 89·7° = 191
tan 88·20 = 31·8	tan 89·9° = 573

When the problems to be solved are formulated in reverse order to the foregoing, that is, when the tangents are given and the corresponding angles are to be computed, we have likewise to distinguish, by mental assessment, between the four possibilities just discussed, and the computation must be performed accordingly

EXERCISES

tan α = 0·800	α = 38·7°	tan α = 2·07	α = 64·2°
tan α = 0·477	α = 25·5°	tan α = 4·10	α = 76·3°
tan α = 0·171	α = 9·7°	tan α = 5·85	α = 80·3°
tan α = 0·708	α = 35·3°	tan α = 1·15	α = 49·0°
tan α = 0·220	α = 12·4°	tan α = 1·402	α = 54·5°
tan α = 0·1069	α = 6·1°	tan α = 9·5	α = 84·0°

In the rare cases where the tangent of the angle is of exceptional magnitude, for instance, when tan α = 26, so that tan (90° − α) = 1/26 we obtain 90° − α = 2·2°, hence α = 87·8°.

Let the two sides in a right-angled triangle be a and b, respectively; we may then write

$$\tan \alpha = \frac{a}{b} \text{ and } \tan \beta = \frac{b}{a}$$

We can also decide to make use of the principle of proportion in problems of this type, and we either form

$$\frac{a}{\tan \alpha} = \frac{b}{1\ (= \tan 45°)} \quad \text{or} \quad \frac{b}{\tan \beta} = \frac{a}{1\ (= \tan 45°)}$$

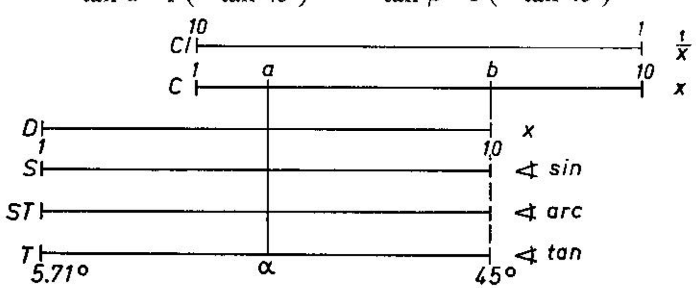


FIG. 34

Here we make it a rule to compute with that angle, either α or β, which is smaller than 45°.

In fig. 34 the initial ratio is set as shown and the other values read directly although it may sometimes be necessary to reset the slide.

EXERCISES

Given:	a = 4·8 in.	b = 5·7 in.	*To find:* α = 40·1°
	b = 7·3 cm	a = 8·9 cm	β = 39·4°
	a = 75·9 ft	b = 97·6 ft	α = 37·9°
	a = 27·85 in.	b = 61·0 in.	α = 24·6°
	b = 25·7 mm	a = 83 mm	β = 17·25°
	a = 12 ft	b = 58 ft	α = 11·7°
Given:	a = 13·8 ft	b = 7·3 ft	*To find:* α = 62·1°
	a = 25·6 in.	b = 18·2 in.	α = 54·6°
	a = 52 mm	b = 24·4 mm	α = 64·8°
	b = 9·3 cm	a = 6·5 cm	β = 55·1°
	a = 76·2 yd	b = 19·1 yd	α = 75·9°
Given:	b = 28·7 in.	α = 37·1°	*To find:* a = 21·7 in.
	b = 18·4 ft	α = 42·5°	a = 16·9 ft
	b = 4·7 in.	α = 26·9°	a = 2·38 in.
	b = 26·3 yd	α = 8·7°	a = 4·03 yd
	b = 12·3 m	α = 18·3°	a = 4·07 m
Given:	b = 15·2 in.	α = 67·5°	*To find:* a = 36·8 in.
	a = 21·8 in.	β = 77·7°	b = 100 in.
	b = 72·7 yd	α = 48·3°	a = 81·7 yd
	a = 7·3 in.	β = 45·7°	b = 7·5 in.
	b = 8·7 cm	α = 51·3°	a = 10·86 cm
	a = 44·5 ft	β = 82·3°	b = 329 ft
Given:	a = 37·5 in	α = 42·7°	*To find:* b = 40·7 in.
	b = 19·4 ft	β = 30·2°	a = 33·4 ft
	a = 22·2 mm	α = 19·7°	b = 62·0 mm
	a = 7·8 yd	α = 28·7°	b = 14·25 yd
	a = 4·6 ft	α = 15·7°	b = 16·4 ft
	a = 1·72 in.	α = 11·8°	b = 8·25 in.
Given:	a = 17·5 ft	α = 59·2°	*To find:* b = 10·42 ft
	a = 19·3 cm	α = 53·8°	b = 14·15 cm
	a = 23·4 in.	α = 61·3°	b = 12·81 in.
	b = 27·6 yd	β = 63·3°	a = 13·88 yd
	b = 9·6 in.	β = 71·5°	a = 3·21 in.
	b = 41·40 ft	β = 81·2°	a = 6·41 ft

Part II

More Advanced Slide Rules

21 RULES WITH LOG LOG SCALES

The student who has diligently worked through the earlier lessons and the examples given in Part III will by this time have acquired some reasonable dexterity and confidence in the use of the standard model slide rule.

More elaborate designs are available for the assistance of those proficient in general work and who have to solve special types of problems frequently.

We have seen how to use the log scale for the solution of problems such as $3{\cdot}42^{1\cdot 63}$, or $\sqrt[1\cdot 2]{3{\cdot}42}$, etc., by using the equivalent forms $1{\cdot}63 \times \log 3{\cdot}42$ and $1/1{\cdot}2 \log 3{\cdot}42$ respectively. Such computations would be much easier if we could eliminate the multiplication and division by means of the ordinary face-scales, and solve entirely by adding or subtracting lengths on the face scales. For this, we should have to work the problems as $\log 1{\cdot}63 + \log (\log 3{\cdot}42)$, for example. That is, we should need a scale (or scales) giving the logarithms of logarithms. We could call such a scale a 'log log scale' and this term is in fact used to describe such a scale incorporated on certain makes of rule. The student will on reflection realize that, whereas in the ordinary log scale we were concerned with the mantissa only, a log log scale *must* provide for the log of a complete log, i.e. characteristic *and* mantissa. This results in the log log scale being arranged in two or three sections; for example, we may find above scale A a log log scale graduated between 1·1 and 2·9, while beneath the D scale is a continuation, giving values between 2·9 and 100,000. In this design, both log log scales are used with reference to the D scale.

In still other designs, the log log scales can be used in conjunction with scale B which, since it is in two parts, extends the range by permitting the operator to choose the part of scale B appropriate to his data.

For students of engineering technology it will be interesting to note that, whereas the normal log scale gives mantissae referred to base 10, the log log scales are, practically without exception, calibrated to the base e, or Napierian logs, e being the indeterminate constant 2·7183. . . .

As a result, most arrangements of the log log scales enable us to determine easily the Napierian (or hyperbolic) logarithm of any number.

In several patterns of rules, the log log scales are arranged in sections such that the tenth root of any number *n* found on the lower scale is immediately found over *n* on the upper scale. The converse obviously applies —numbers on the lower scale are the tenth powers of the corresponding numbers on the upper scales.

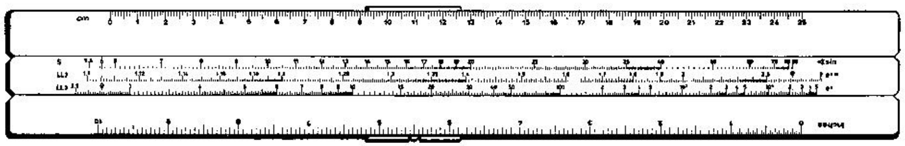


FIG. 35

In other patterns the back of the slide (fig. 35) carries an additional sine scale as well as a log log scale in two sections, LL2 and LL3 (or LL_2 and LL_3), covering the range 1·1 to 50,000. The second sine scale facilitates the solution of problems in spherical trigonometry. When using these new scales insert the slide back to front.

The term 'log ln' scales is sometimes applied to the LL2 and LL3 scales because logarithms to the base 10 of logarithms to the base e are plotted using 10 inches as unit length, the conventional scale length of all standard-size slide rules; the metric equivalent is generally 25 cm.

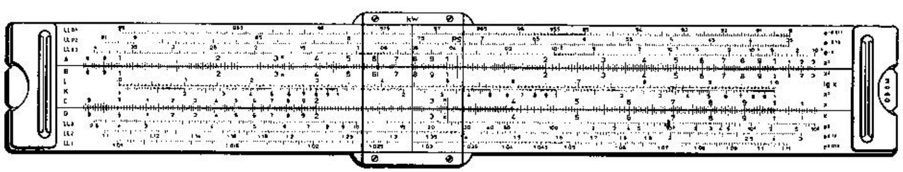


FIG. 36

When greater accuracy is required, the log log scales are extended still further to three or six lengths of the rule. The first portion, generally labelled LL_1 reads from $1{\cdot}01 = e^{0{\cdot}01}$ to $1{\cdot}105 = e^{0{\cdot}1}$. The second section labelled LL_2 goes from 1·105 to 2·718 = e; and the third called LL_3 from 2·718 to $22{,}000 = e^{10}$. (*See* fig. 36.) On another part of the rule there are a second set of log log scales generally labelled LL_{01} reading from $e^{-0{\cdot}01} = 0{\cdot}99$ to $e^{-0{\cdot}1} = 0{\cdot}9048$, LL_{02} reading from $e^{-0{\cdot}1}$ to $e^{-1{\cdot}0} = 0{\cdot}3679$, and finally LL_{03} which comes $e^{-1{\cdot}0}$ down to $e^{-10{\cdot}0} = 0{\cdot}0000454$ (with subdivisions to 0·00003!).

22 RAISING NUMBERS TO ANY POWER: A^x

Raising a number to any power depends on the logarithmic theorem

$$\log a^x = x \log a$$

or, converting both sides of this expression into their respective logarithms we have:

$$\log \ln a^x = \log x + \log \ln a$$

This means that the original logarithm of the exponent *x* set on the D scale has to be increased by the log ln value of the base *a* taken from one of the log ln scales, in order to obtain the log ln value of the power. Consequently, then, finding powers with the LL scales is similar to the simple operation of multiplication with the fundamental scales C and D.

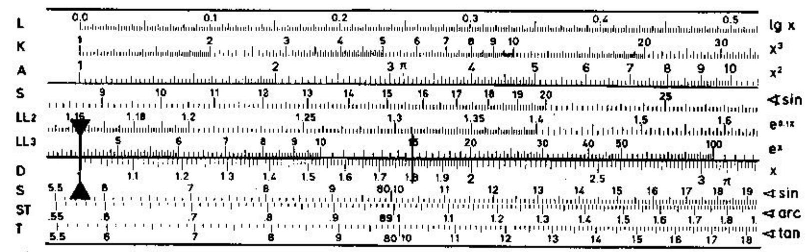


FIG. 37

Figure 37, part of a rule where a slide of the type shown in fig. 35 has been inserted, explains how the example $4{\cdot}5^{1{\cdot}8} = 15$ is worked out by slide rule. It will be easy to see that now the slide rule may be regarded as a Table for reading whatever further powers of the number 4·5 may be required. For exponents smaller than 1 we must, in the present example, simply pass to the adjacent LL_2 scale for the answer, e.g. $4{\cdot}5^{0{\cdot}18} = 1{\cdot}311$.

It is important to observe that the log ln scales supply all values with the decimal point given in its correct place. This signifies that, contrary in character to the fundamental scales, the readings here are immutable as to their real value. Hence the answer to the above problem can only be interpreted as 15 and not, alternatively, 1·5 or 150, etc.

In the example $1{\cdot}8^{5{\cdot}5} = 25{\cdot}4$ (fig. 38) the computation begins with the right index of the D scale aligned to the LL_2 scale, and the result will be taken from LL_3. This switch-over from one scale to the neighbouring one will always be necessary when the respective setting takes place over the right index of D in consideration of the fact that a power whose exponent is greater than 1 cannot be smaller than its base.

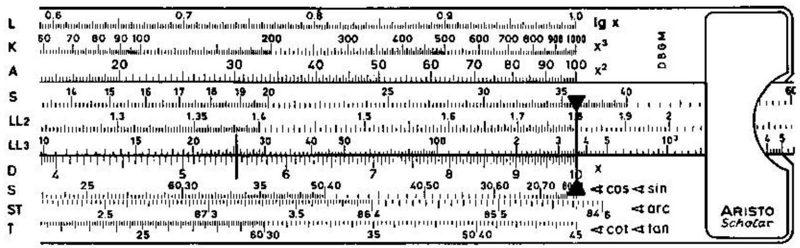


FIG. 38

EXERCISE

$4{\cdot}22^{1{\cdot}92} = 15{\cdot}8$ (Set on LL_3, read over D on LL_3)
$1{\cdot}82^{2{\cdot}5} = 4{\cdot}48$ (Set on LL_2, read over D on LL_3)
$2{\cdot}57^{1{\cdot}72} = 5{\cdot}07$ (Set on LL_2, read over D on LL_3)
$2{\cdot}57^{0{\cdot}172} = 1{\cdot}176$ (Set on LL_2, read over D on LL_2)

It is obvious that when the slide and body indices coincide (rule 'closed') we can regard the slide rule as a table for supplying the functional values of $y=e^x$.

$y=e^x$

x	y
0·35	1·419
2	7·39
π	23·1
7	1100

23 FINDING ROOTS TO ANY INDEX: $\sqrt[x]{a}$

Roots are extracted with the log log scales using the inverse process for finding a power.

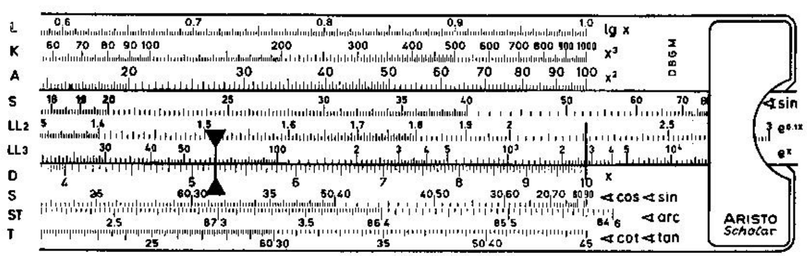


FIG. 39

The course to be taken for the solution of $\sqrt[5{\cdot}2]{62} = 2{\cdot}21$ is demonstrated in fig. 39. After aligning the exponent 5·2 on the D scale to the radicand 62 on the LL_3 scale we can read the answer 2·21 on LL_2 over the right index of the D scale.

EXERCISES

$\sqrt[1{\cdot}48]{247} = 41{\cdot}5$ (Set on LL_3, read over D on LL_3)
$\sqrt[2{\cdot}25]{6{\cdot}48} = 2{\cdot}29$ (Set on LL_3, read over D on LL_2)
$\sqrt[0{\cdot}87]{24{\cdot}1} = 38{\cdot}8$ (Set on LL_3, read over D on LL_3)

If a rule having an extended log log scale is used, the foregoing operations have to be slightly modified, but are basically identical. The advantage is that problems can be handled which involve any power of numbers <1, roots to any index of radicands <1, powers with negative exponents and logarithms to a base <1; in fact any computations involving powers, roots and logarithms to any exponent, index or base.

24 LOGARITHMS TO ANY BASE: $\text{LOG}_x a$

Finding logarithms may be defined as the second reversal of the process of raising a number to a power. From this we can conclude that logarithms to any base can be computed by reversing the order in which the readings take place, i.e. by setting the base on the LL scale over the right or left index of the D scale, as the problem may require.

When the rule is 'closed', i.e. when the mark 'e' of LL_3 is opposite the left index of D, we can read off the natural logarithms to the base e which have such an important part in many branches of science and engineering.

EXAMPLES (*a*) $\log_e 330 = 5{\cdot}80$ (*b*) $\log_e 2{\cdot}2 = 0{\cdot}788$

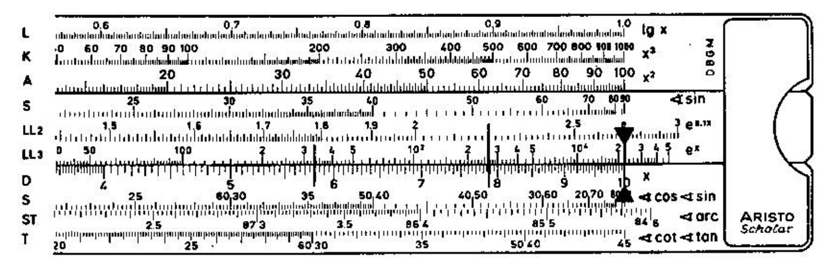


FIG. 40

It is interesting to note that when the value 10 of the LL_3 scale is used as base and set opposite the left index of the D scale, the slide rule may be regarded as a Table of common logarithms. This adjustment is worth remembering as it serves as a visual demonstration by which the learner may at any time reconstruct the slide rule procedure applicable to logarithmic problems, since $\log_{10} 100 = 2$ is an expression with which every student of mathematics is well acquainted. Furthermore this setting gives us a clear picture of the processes of raising $10^2 = 100$ and finding $^2\sqrt{100} = 10$.

EXERCISES $\log_{10} 1{\cdot}77 = 0{\cdot}2480$ $\quad \log_{10} 1{\cdot}236 = 0{\cdot}0920$

$\log_{10} 20 = 1{\cdot}301$ $\quad \log_{10} 9{\cdot}81 = 0{\cdot}992$

25 RULES WITH A PYTHAGORAS SCALE

The inclusion of a Pythagoras scale $\sqrt{(1-x^2)}$ on some rules, *see* fig. 41, will be found very convenient, since by its help the sines of angles $>50°$ and the cosines of angles $<40°$ can be read with greater precision.

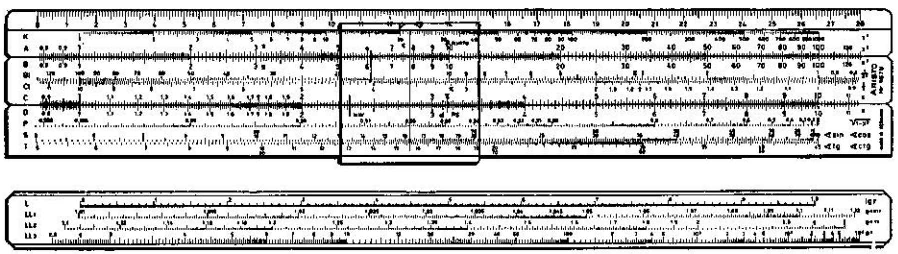


FIG. 41

This section reads from 0·995 to 0 for values of $x=0{\cdot}1$ to 1 respectively and is sometimes labelled the P scale. The scale is based upon the Pythagoras theorem which states that for a right-angled triangle with hypotenuse 1 the relationship $x^2+y^2=1$ holds good. This can be re-written as

$$y = \sqrt{(1-x^2)}$$

Thus, for instance, if we examine the relationship $\sqrt{(1-0{\cdot}6^2)} = 0{\cdot}8$, we can look up 0·6 on P and find 0·8 on D. Since the P scale is in fact a reciprocal scale to D the converse operation is also possible, i.e. looking up 0·6 on D we find 0·8 on P.

Consider the relationship $\sin^2\alpha + \cos^2\alpha = 1$. This can be re-written as $\sin\alpha = \sqrt{(1-\cos^2\alpha)}$ or $\cos\alpha\sqrt{(1-\sin^2\alpha)}$ which by reason of the reciprocity of the co-function enables sines and cosines of the same angle to be read simultaneously on either the P or D scales.

EXAMPLES (*See* fig. 42 where the Aristo 'MultiRietz' is used.) In this rule cosines are written in red on the sine scale by virtue of the relationship $\cos\alpha = \sin(90° - \alpha)$.

(*a*) $\begin{cases} \sin 81{\cdot}5° = 0{\cdot}98902 \\ \cos 81{\cdot}5° = 0{\cdot}1478 \end{cases}$ On P where the cursor matches the red 81·5° on S. Appears simultaneously on D.

(*b*) $\begin{cases} \cos 11{\cdot}81° = 0{\cdot}9788 \\ \sin 11{\cdot}81° = 0{\cdot}2047 \end{cases}$ On P when the cursor matches the black 11·81° on S. Appears simultaneously on D.

(*c*) $\begin{cases} \cos 11{\cdot}81° = 0{\cdot}979 \\ \sin 11{\cdot}81° = 0{\cdot}205 \end{cases}$ When making the setting with the red 11·81° on S the result is less precise.

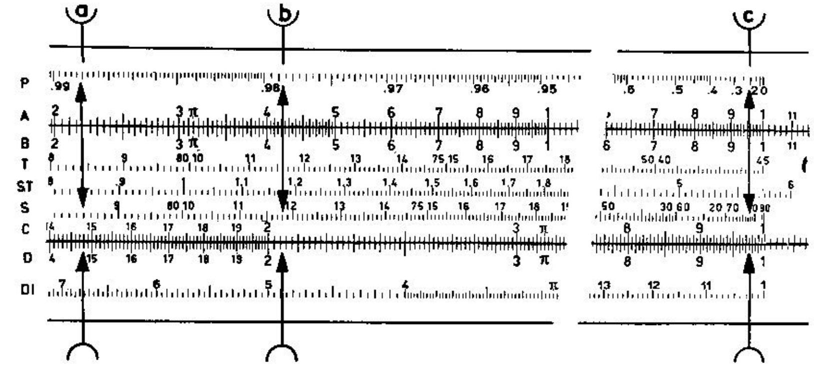


FIG. 42

An interesting use for the P scale is found in the more accurate extraction of roots; an example will illustrate this.

$$\sqrt{91{\cdot}28} = \sqrt{(100-8{\cdot}72)} = \sqrt{\{100\,(1-0{\cdot}0872)\}} = 10\,\sqrt{(1-0{\cdot}0872)}$$

Whence against 872 on A we find 0·9554 on P so that

$$91{\cdot}28 = 10 \times 0{\cdot}9554 = \underline{9{\cdot}554.}$$

The scale is also convenient for solving right-angled triangles where we have the following relationship:

$$c^2 = a^2 + b^2$$
$$b^2 = c^2 - a^2$$

therefore

$$b = \sqrt{(c^2 - a^2)}$$
$$= c\sqrt{\left(1 - \frac{a^2}{c^2}\right)}$$

If we let $a = 3{\cdot}4$ in. and $c = 5{\cdot}6$ in., then

$$b = 5{\cdot}6\sqrt{\left\{1 - \left(\frac{3{\cdot}4}{5{\cdot}6}\right)^2\right\}} = \underline{4{\cdot}45 \text{ in.}}$$

26 THE FOLDED SCALES CF, DF AND CIF

What, exactly, is a 'folded' scale? If one of the scales C or D were cut at the line for $\pi = 3{\cdot}14$ and the end of the tail section then joined to the tip of the curtailed original scale the result would be a so-called folded scale.

From its only index in the near centre position the graduation is progressive to π on the right and retrogressive, to π again, on the left.

By means of this arrangement of the scales every setting of the C and D scales is also automatically made with the CF and DF scales. A particularly useful improvement in slide-rule work is possible since the combination has two definite conjugate functions. (*See* fig. 43, also Note p. 24.)

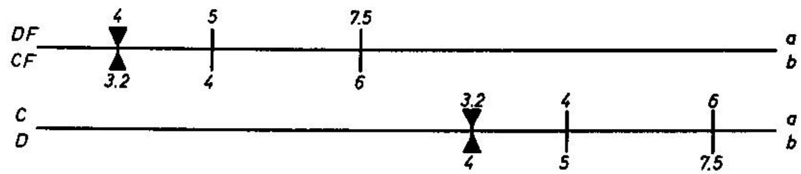


FIG. 43

One of these is to avoid the trouble of having to 'reset' the slide in multiplication when the point is reached where C and D lose contact with each other. In such a situation the reading range is augmented indefinitely by the simple process of switching by cursor over to CF and DF. CF and DF take over where C and D break off. Observe: Scale C on the slide is located over scale D on the body whereas scale CF on the slide is situated below scale DF on the body. In the Aristo 'MultiRietz' slide rule, to guide the eye in switching from one pair of scales to the other, the scales on the slide are printed on a yellow background. Thus, yellow continues on yellow, white on white.

Only when more than half of the slide length projects beyond the rule will the readability be incomplete. But practised slide-rule users will always avoid this situation instinctively by making the initial setting with the proper index. Even such considerations can be disregarded if the user makes it a strict rule to *start every computation with the* CF *and* DF *scales*.

The second advantage of the folded scales is that, since the DF scale overlaps D and CF overlaps C by the value π, multiplication or division by π is performed automatically by setting the cursor on the D scale and reading under the cursor line on the DF scale or proceeding in the reverse order, respectively.

The CIF scale is, of course a folded scale of the reciprocal CI scale so that complete mobility is assured.

Let us suppose that we wish to find the value of some function of the type ax, where a is a constant. The use of the folded scales CF and DF together with the normal C and D scales will reduce the work to a succession of cursor movements.

EXAMPLE. Find $y=ax$ when $a=4{\cdot}57$ and x has the values 0·23, 1·36, 1·78, 3·2, 4·92, 5·85, 6·7, 11·38 and 14·05.

Place the index of C over 4·57 on D, choosing the right-hand index so that the maximum length of slide remains in the body of the rule. We can now read off with the cursor the required products.

1. Cursor to 0·23 on C in the normal way which gives 1·052 on D.
2. The next value 1·36 is not available on D without moving the slide, therefore run the cursor to 1·36 on CF and read 6·32 on DF which is the required product.
3. Cursor to 1·78 on CF gives 8·15 on DF.
4. Factors $x=3{\cdot}2$, 4·92, 5·85 and 6·7 are available on C, giving 14·62, 22·5, 26·7 and 30·6 respectively.
5. Factors $x=11{\cdot}38$ and 14·05 are used in conjunction with the CF and DF scales, to give 52·0 and 64·3 as the products.

The reverse operation, that of division, is performed likewise. Suppose we have the function x/a with values as previously; then the constant divisor, a is found on C against the index of D which leaves the maximum amount of slide within the body of the rule. The series of divisions can then be performed as before using either the C or CF scales and reading onto D or DF.

If the problem is one of the form $y=a/x$ with a as a constant dividend then we proceed as before but read from CI and CIF onto D or DF respectively.

From the foregoing it will be appreciated that there is perfect freedom for the operator to use the folded scales in conjunction with the normal C, D and CI scales. Many slide movements can frequently be avoided.

EXAMPLE (*a*) Required the product $8{\cdot}3\times638\times41{\cdot}2\times0{\cdot}73$.

Step 1. Cursor to 83 on DF.
Step 2. Bring 638 on CIF under hairline.
Step 3. Cursor to 412 on C.
Step 4. Bring 73 on CI under hairline.

Read answer 159,300 on D under L.H. index of C.

EXAMPLE (*b*) Evaluate $\dfrac{8{\cdot}3\times638\times41{\cdot}2\times0{\cdot}73}{165\times0{\cdot}32\times1{\cdot}08}$

Step 1. Find 83 on DF.
Bring 165 on CF under hairline.
(This gives a quotient of 504 on D under R.H. index of C.)
Step 2. Cursor to 638 on C.
(Product 322 on D.)

Step 3. Bring 32 on C under hairline.
(Quotient 1005 under left index of C.)
Step 4. Cursor 412 on C under hairline.
(Product 415 on D.)
Step 5. Bring 108 on C under hairline.
(Quotient 384 on D.)
Step 6. Cursor to 73 on CF.
Read answer 2810 on DF under centre index.

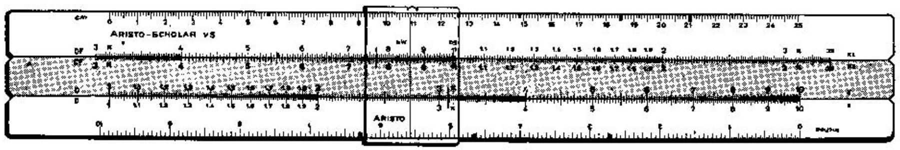


FIG. 44

In some rules the CF and DF scales are printed on the underneath of the slide together with corresponding C and D scales on the underside of the stock. In order to operate, it is necessary to re-position the cursor on the under face (fig. 44). Some scales have a double cursor.

27 SPECIAL-PURPOSE INSTRUMENTS

Special rules are manufactured by all makers of slide rules to meet the requirements of particular professions.

DYNAMO AND MOTOR EFFICIENCY SCALES

Electrical engineers are so frequently called upon to compute the relations between the input and output of power to dynamos and motors, that scales incorporating the constant quantities used in such calculations have been developed. Several makers include such scales and enable efficiencies to be read directly. These special scales used to be laid out on the floor of the groove in the stock and were read by means of knife-edges at the ends of the slide. Nowadays, they are generally on the front face of the rule.

Associated with the dynamo-efficiency scale is another of considerable value to many engineers—a scale facilitating the calculation of the fall in potential in electrical circuits.

RULES FOR COMMERCIAL CALCULATIONS

Although it is at once apparent that the normal arithmetic of interest, discount, foreign exchange conversion, etc. can be carried out with the standard type of rule, or even better with log log scales, the necessary level of accuracy cannot always be attained with a standard rule. To avoid making rules inconveniently long, special 'commercial' rules are available. In these, the normal scales, nominally 10 in. in length, are divided, two lengths being laid out to give the equivalent of a 20 in. rule, similar to the device adopted for the LL_1, LL_2 scales. Additional scales give percentages and 'Exchange' equivalents and, often, an auxiliary scale on the edge of the rule gives shillings and pence as decimal fractions of £1. Such rules are of the greatest value to students faced with percentages, currency, pricing and interest problems, etc.

LONG-SCALE INSTRUMENTS—CYLINDRICAL RULES

The thoughtful reader will have realized that the accuracy to which a slide rule can be read is governed, ultimately, by the length of the scales. Standard pattern rules up to 20 in. are made, but this length (or over) is inconvenient on a rigid rule of the flat stock and slide type. To make possible the use of much larger scales, two main types of instrument have been developed. The first of these is the Fuller Calculator, fig. 45.

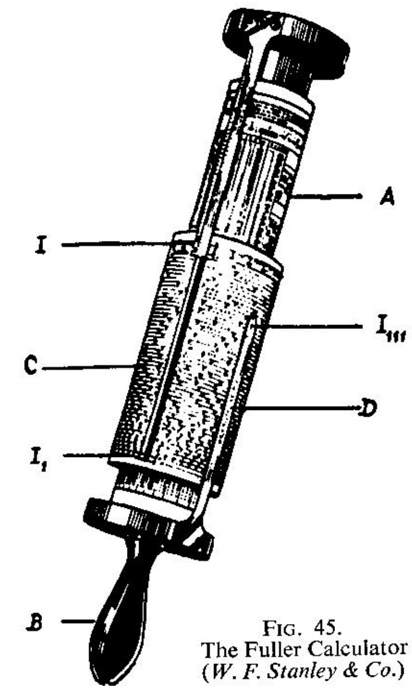


FIG. 45.
The Fuller Calculator
(*W. F. Stanley & Co.*)

In this we have a main cylinder *A* carried by the handle *B*. The hollow cylinder *C* is in sliding contact with *A*, and around *C* is laid out an engraved scale, as a helix, approximately 500 in. in length. A brass pointer, fixed (*D*) bearing two index marks I and I_1 and a further index point I_{111} enable references to be made to the graduations on *C*. *D* is carried on a core which can be revolved with *A* and pulled in and out of it.

Various models are made, to include log log scales, trigonometrical scales, etc. and the accuracy is such that 4 figures can usually be read and a fifth estimated.

A cheaper and very popular instrument, also employing spiral scales, is the Otis King Calculator, fig. 46. Here we have three main parts—the cylinder, the holder, and the cursor. The holder has engraved upon it a scale of length 66 in., while the cylinder bears two scales, giving numbers from 1–100, of total length twice that of the holder.

FIG. 46. Otis King's Calculator

The cylinder can be pushed in or out of the holder and revolved within it. The cursor, bearing the two arrow points (index marks), which is a close sliding fit on the holder, can be slid up and down, and revolved, to bring the arrow points into coincidence with the terms being handled.

The greater length of scale promotes accuracy, and the mutual arrangement of the scales has certain advantages over the normal flat calculating rule.

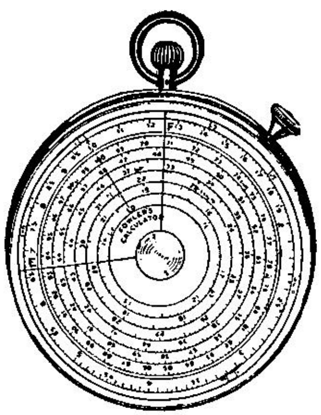


FIG. 47. A circular slide rule—Fowler's Calculator

WATCH-TYPE INSTRUMENTS

Although the 10-in. flat slide rule is a convenient and accurate instrument, it cannot comfortably be carried in the pocket. Reduction in length to a 5-in. scale produces an instrument which, although still extremely useful, has its accuracy reduced by comparison with 10-in. scales. Watch-type instruments have therefore been developed to provide computors capable of being carried in the pocket. A typical example is shown in fig. 47. In this the scales are laid out flat, in a circle.

The outer scale is fixed, and an inner, movable line is provided to work with the fixed scale. Glass discs, which can be revolved in the main outer frame carrying the fixed scale, bear cursor lines.

Although the operations necessary to complete calculations with these instruments are usually greater in number than with the 10-in. rule, many find the convenience of easier transportation a great point in favour of circular-scale instruments.

28 ON THE ACCURACY OF THE SLIDE RULE

In general, there are two sources of error to be considered in relation to the accuracy of the results obtained by the use of the slide rule: (1) The accuracy of the graduations of the scales; (2) the accuracy with which settings are made and results read off by the operator.

The first class of error is purely a mechanical one. It can be shown, by methods beyond the scope of this work, that (*a*) the order of accuracy is calculable, and (*b*) that the error is constant throughout the scales. On the C and D scales, the error is almost exactly 1/1000, so that, theoretically, we should be able to set with an accuracy or error of one tenth of one per cent. On the A and B scales, the mechanical error is 1/500.

The second class of error is not predictable. It depends largely upon the ability of the operator to subdivide a space visually and as the spaces vary in width throughout the scales, a varying and incalculable factor of error is introduced. It is obvious that one person is better at 'drawing' or (transcribing proportional sizes) than another.

Taking first the D scale, the student will have little difficulty in reading three figures direct and in making a very close estimate of the fourth, in the interval 1–2. Between 2 and 4, two figures are read immediately, the third is marked on the rule if even, and can be accurately estimated if odd. From 4 to 10 the first two figures are marked and the third, unless it is 5, is to be estimated with progressive uncertainty. Skill and accuracy in visual estimation come with practice. Following diligent and intelligent practice, the general level of efficiency to be attained will be between 0·15 and 0·2 per cent. When practising, it is best to compare the results with known values, e.g. many handbooks give areas tabulated for circles of given diameter or radius.

To give the student confidence, let us compare the results obtained for a given case (*a*) by extended arithmetic (*b*) with the slide rule, and (*c*) by four figure logs.

Evaluate $$\frac{1{\cdot}0015 \times 2{\cdot}063 \times 0{\cdot}998}{(0{\cdot}997)^2}$$

If we can assume that these values are exact, we can with much labour calculate the arithmetical result as 2·07439 to six significant figures. Careful setting of the slide rule will give 2·07 and by four figure logs we have 2·075. With the slide rule therefore we have an error of 0·00439 and with logs an error of 0·00061, neither of which is usually of serious dimensions. Particularly is this so when we realize that in practical circumstances it is unlikely that we shall be able to secure 'readings' or dimensions to more than 4 figure accuracy. It follows that to show an answer to six significant figures when the original data is necessarily limited to four significant figures is misleading besides being wrong in principle. It follows also that slide-rule results, obtained with careful setting and reading of the instrument, will in general afford a satisfactory degree of accuracy.

When, by refinement of instruments used and special technique, we do have measurements correct to more than 3 digits and a result, involving such figures, is wanted to a high degree of accuracy, then neither the slide rule nor four figure logs are suitable.

For certain types of calculation, however, it is possible to raise the order of precision of the slide-rule results. Two methods will be mentioned here. The first is rather obvious. Consider the product of $7{\cdot}0\times 8{\cdot}3$. Setting these factors on the rule, using for preference scales C and D, we read as the product 58·? on the D scale. The cursor is at some indefinite point beyond the 8 but we cannot closely estimate the value of the decimal fraction. We can, at sight, mentally multiply 7 by 0·3 to get 2·1 and will thus establish the last figure as 0·1 to enable us to write $7{\cdot}0\times 8{\cdot}3$ as 58·1. Note, however, that this ruse can indicate the value of only the *last* digit in the product. Care must be taken to observe this point, otherwise error will be introduced. To make this clear, try $7{\cdot}6\times 8{\cdot}3$. On the rule, we read 63·?. If we multiply $0{\cdot}3\times 0{\cdot}6$ to obtain 0·18, our answer might seem to be 63·18. Actually it is 63·08; i.e. let us make due allowance for the decimal points.

The second method of increasing the accuracy of slide-rule working is rather more complicated and makes use of the Binomial Theorem. This enables us to write an expression such as $(a+b)^x$ as an expansion.

Thus,

$$(a+b)^2=a^2+2ab+b^2$$
$$(a+b)^3=a^3+3a^2b+3ab^2+b^3$$
$$(a+b)^4=a^4+4a^3b+6a^2b^2+4ab^3+b^4$$

We can use such an expansion to some advantage where the order of precision of the data we are given, justifies this.

Consider finding the value of $\sqrt{16{,}235}$. From scale A we at once find an approximate root, 127·4. Let us assume that the root is of the form $(a+b)^2$, in which $a=127{\cdot}4$.

Then $(127{\cdot}4+b)^2=16{,}235$. Expanding the left-hand side, we have:

$$127{\cdot}4^2+(2\times 127{\cdot}4)\times b+b^2=16{,}235$$

The value of b will clearly be very small and b^2 still smaller. We will ignore b^2, therefore, and put

$$127{\cdot}4^2+(2\times 127{\cdot}4)\times b=16{,}235$$

whence

$$16{,}230{\cdot}76+254{\cdot}8\times b=16{,}235$$
$$254{\cdot}8b=16{,}235-16{,}230{\cdot}76$$

and

$$b=4{\cdot}24/254{\cdot}3$$
$$=0{\cdot}0166$$

We can now write $\sqrt{16{,}235}$ as 127·4+0·0166, i.e. 127·4166 to four places of decimals of which three are probably correct. There is, in general, little advantage in determining b^2 as a further correction although this could be done if desired.

The Binomial expansion can also be used to raise the precision in calculating squares or cubes.

Consider the problem of finding accurately $(16{\cdot}3)^2$. Write this is in the form $(16+0{\cdot}3)^2$ which is the general form $(a+b)^2$. This last, in the expanded form is $(a^2+2ab+b^2)$ and we may thus determine $(16{\cdot}3)^2$ as $16^2+2\times 16\times 0{\cdot}3+0{\cdot}3^2$, i.e. 266+9·6+0·09, which adds up to 265·69.

By direct setting of 16·3 on scale D, our nearest reading on scale A, accurate enough for most practical purposes, is 265·5.

Trigonometrical scales can be read with varying degrees of accuracy. If much work has to be done, the greatest use should be made of trigonometrical transformations. Handling complicated expressions can often be done by rearranging the data into a form involving other ratios. Frequently for instance, a difficult calculation with a cosine may be much simpler when expressed as a secant function; or a cotangent may replace a tangent, and so on.

Certain types of problems deserve particular care in manipulation. Careful thought and reasoned procedure can do much to raise the precision of slide-rule work.

Consider the problem, typical of some encountered in electrical work:

$$y=\sqrt{\left\{\left(\frac{82{\cdot}8}{0{\cdot}85}\right)^2-82{\cdot}8^2\right\}}$$

This obviously could be solved by one of several methods, e.g.

METHOD A

(i) Divide 82·8 by 0·85, square the quotient and note it down.
(ii) Square 82·8 and subtract from square of quotient obtained in (i).

(iii) Set difference of squares on scale A and read result D (i.e., the square root of the expression under the $\sqrt{}$ is obtained).

If we are very careful in our setting, we shall obtain a result 51+? (apparently, but not exactly, 51·1). As several terms cannot be read on the scales and have to be squared, then read, and reset after a subtraction, a considerable chance of error has been introduced.

METHOD B

For increased accuracy, we recall that $a^2-b^2=(a+b)\ (a-b)$. The use of this artifice will help us here, avoiding the squared terms.

(i) Find the value of $82{\cdot}8 \div 0{\cdot}85$ which is 97·4.

(ii) Solve the expression as $\sqrt{\{(97{\cdot}4)^2-(82{\cdot}8)^2\}}$

i.e., $\sqrt{\{(97{\cdot}4+82{\cdot}8)\ (97{\cdot}4-82{\cdot}8)\}}$

$$=\sqrt{(180{\cdot}2\times 14{\cdot}6)}$$

$$=\underline{51{\cdot}3}$$

The answer by extended arithmetic is 51·32. The dodge described in *Method B* has therefore raised the precision and simplified the working.

The artifice given in §25 for solving roots using the Pythagoras scale can also be used even if such a scale is not available. That is, the number for which the root is required is suitably factorized and expanded so as to obtain a problem lending itself to a summation of individually more accurate parts.

In other instances the number can be split into two others, one of which at least can easily be factorized, e.g.

$\sqrt{752}=\sqrt{(64\times 1{\cdot}175)}$	(set 64 on C over 752 on D).
$=\sqrt{(8^2\times 1{\cdot}083^2)}$	(1 on B against 1175 on A, read 1083 on D).
$=8{\cdot}66$	(cursor to 8 on C).
or better $=8{\cdot}664$	(multiplying out mentally).

If we had attempted to read from 752 on A scale onto D scale we should have been much less certain of the answer.

A final word of caution should be made. It is sometimes thought that the degree of accuracy to which a slide rule can be read depends largely upon the keenness of eyesight of the user. Frequently magnifying glasses are used with this in mind, and the manipulator expects to be able to estimate values to 5 or even 6 places of decimals. This is quite false. Parallax errors are increased, the thickness of the graduation lines become important and the mechanical imperfections of the rule become more apparent. Such aids are useful only as an assistance in preventing eye fatigue and do not normally assist in achieving greater accuracy.

At best, four figures are certain on the lower portion of the C and D scales and three in the upper zones, and the last digit in both instances is estimated. The only improvement which can be made on this is to resort to a mathematical ruse such as those we have just shown, or to use rules with extended scales.

Part III

Examples in Practical, Everyday Mathematics

The respective Sections of Part I are given in parentheses

29 EXAMPLES ON MULTIPLICATION, DIVISION AND COMBINED OPERATIONS (§§5, 6, 7, 8, 9)

1. Given 1 ft = 30·5 cm
   9·84 ft = <u>3·00 m.</u>

   i.e. set 10 on C against 305 on D, hence under 984 of C find 300 on D (similarly for any other required amount).

2. 1/64 in.^{2} = 10·08 mm^2
   37/64 in.^{2} = <u>3·73 cm^2.</u>

3. The exchange rate of £ into Swiss francs is £1 per 12·23 Swiss francs. Find the value of £28·6 in Swiss francs. Answer: <u>350 Swiss francs.</u>

4. The area of a rectangle is equal to the base multiplied by the height: $A = l \times h$.
   A floor is to be covered with linoleum. The dimensions of the room are 15·5 ft × 14·2 ft. How many square feet of linoleum are required?
   Answer: <u>220 ft^2.</u>

5. Find the area of a triangle:
   Given the base $b = 8{\cdot}6$ in. $A = (b \times h/2)$
   and the height $h = 3{\cdot}5$ in.

   Answer: <u>$A = 15{\cdot}05^2$.</u>

6. One nautical mile equals 1·852 km. Convert 3·2, 10·8 and 16·3 nautical miles into km. Answer: <u>5·93; 20·0; 30·2 km.</u>

7. Calculate the mass M of a body of known volume V and density d:

V	d		Answer
425 cm³	11·3	g/cm³ (lead) $\therefore V \times d = M =$	4·8 kg
6·65 dm³	2·8	g/cm³ (marble)	= 18·62 kg
0·650 litres	1·83	g/cm³ (sulphuric acid)	= 1·190 kg
64 m³	1·293	g/dm³ (air)	= 82·8 kg

8. Conversion of velocities given in ft/sec into m.p.h. Factor for the conversion 0·682 i.e. multiply ft/sec by this factor to get m.p.h. or vice versa.

e.g.	Aeroplane	$v = 475$ ft/sec	= 324 m.p.h.
	Train:	$v = 95$,,	= 64·8 ,,
	Car:	$v = 82$,,	= 55·9 ,,

9. To calculate the circumference of circles from their given diameters:

$$C = \pi \times D$$

place 1 (or 10) on C scale against π on D scale to get the following Table. *Note D* is read on C scale and *C* on D scale, i.e. the symbols bear no relation to the lettering on the scale.

D	14·2	17·2	19·1	21·8	29·6	35·0	46·2	6·6	8·4	10·5 in.
C	44·6	54·0	60·0	68·5	93·0	110·0	145·1	20·7	26·4	33·0 in.

This exercise provides another good example for using the slide rule when compiling a Table. Set the left index to the mark π on scale D to read the first five answers. Then reset the slide as usual for the remaining problems. The cursor is useful in determining the results quickly and accurately.

10. Calculation of distances traversed by a body in uniform motion. Given are the velocity v and the time needed t.

$$s = v \times t$$

Let $v = 1·40$ ft/sec and $t = 480$ sec,	then $s = 672$ ft.
,, $v = 4·80$ ft/sec and $t = 210$,, ,	,, $s = 1008$ ft.
,, $v = 51$ ft/sec and $t = 34·8$,, ,	,, $s = 1775$ ft.
,, $v = 331$ in./sec and $t = 17$,, ,	,, $s = 5630$ in.

11. Express an acceleration of 4 m.p.h./sec in ft/sec.² Factor for the conversion 1·467

Answer: $a = 5·87$ ft/sec.²

12. Percentages:

13·8% of \$290 = \$40·00
2·08% of \$812 = \$16·89
0·75% of \$244 = \$ 1·83

13. The wattage of an electric light bulb is 40. Calculate the current in amperes (A), if the voltage is 220.

$$\text{Current} = \frac{W}{V} = \frac{40}{220}\ A = 0·182\ A$$

14. Determine the current driving an electric motor of 1 h.p. output, if $\frac{1}{5}$ of the input is lost during the conversion into mechanical energy.

1 h.p. = 746 W = $\frac{4}{5}$ of the electrical energy used;

$$\text{the loss is therefore: } \frac{746}{4}\ W = 186·5\ W$$

The energy required to drive the motor is the sum 746 + 186·5 = 932·5 W. Assume that the voltage is 220, then:

$$\text{the current } I = \frac{932·5}{220}\ A = 4·24\ A$$

15. An electric iron uses 500 W. Calculate the current flowing through the iron if the voltage is 220.

$$I = \frac{500}{220}\ A = 2·27\ A$$

16. A given quantity of water is found to have a mass of 7·50 lb, and its volume is 0·1202 ft³. The density is then

$$d = \frac{7·50}{0·1202}\ \text{lb/ft}^3 = 62·4\ \text{lb/ft}^3$$

17. The current I which can be obtained from a battery is given by:

$$I = \frac{E}{R_a + R_i}$$

where E is the volume, R_a the external, and R_i the internal resistance of the battery.
$E = 4·0$ V and $R_1 = 0·1\ \Omega$. Calculate I if R_a has the following values: 5, 8·2, 15·8, 120 and 960 Ω.

Answers: $I = 0·784$, 0·482, 0·251, 0·0333 and 0·00417 A.

These can be obtained by one of two ways:

(*a*) 1. Set cursor at 4 on D.
2. Mentally add 0·1 to each value of R_a and set this value on C under hairline.
3. Read answer under approximate index.

(*b*) 1. Set index of C against 4 on D.
2. Mentally add 0·1 to each value of R_a and move cursor to this value on CI scale.
3. Read answers under hairline on C.

Method (*b*) requires only two moves of the slide as against five using method (*a*).

18. In working out sizes of structural steel members it is necessary to check the stress *f*, using *M*, the moment to *Z*, the modulus of section.

$$f=\frac{M}{Z}=\frac{130 \text{ tons in.}}{13{\cdot}22 \text{ in.}^3}=\underline{9{\cdot}8 \text{ tons/in.}^2}$$

19. In thermodynamics the quotient $\gamma=(c_p/c_v)$ represents the ratio of the specific heats of a gas under special conditions.

For air we find that this quotient $=\dfrac{0{\cdot}238}{0{\cdot}170}=\underline{1{\cdot}40}$

20. To express minutes of arc in terms of fractions of degrees.

18°22′	31°7′	20°41′	57°19′	12°34′	72°56′	95°12′	162°46′
18·367°	31·117°	20·683°	57·317°	12·567°	72·933°	95·200°	162·767°

i.e. divide the minutes by 60 or multiply by the reciprocal 1/60.

21. Conversion of lengths:

Knowing that 39·4 in. = 1 metre (or 1/1000 of a km).
since there are 1760 yards to a statute mile we have

$$1 \text{ statute mile}=\frac{1760\times 3\times 12}{39{\cdot}4\times 1000}=\underline{1{\cdot}609 \text{ km.}}$$

22. A force F_2 of 1·26 lb is acting on one arm of a lever at a distance $l_2=45$ in. from the fulcrum. Calculate the weight F_1 required to maintain equilibrium which must be applied to the other arm at a distance $l_1=55$ in. from the fulcrum.

$$F_1\times l_1=F_2\times l_2$$

$$F_1=\frac{1{\cdot}26\times 45}{55}\text{ lb}=\underline{1{\cdot}03 \text{ lb.}}$$

Alternatively calculate as a proportion:

$$\frac{F_1}{l_2}=\frac{F_2}{l_1}$$

23. Determine the resistance *R* of a copper wire of length $l=4{\cdot}850$ km, of sectional area $A=0{\cdot}75$ mm² and of specific resistance $\rho=0{\cdot}017$ ohms (Ω) per metre of wire of 1 mm² cross-section.

$$R=\rho\times\frac{l}{A}=0{\cdot}017\times\frac{4{\cdot}850}{0{\cdot}75}\ \Omega=\underline{110\ \Omega}$$

24. A resistance coil of 25 Ω has to be wound, using constantan wire of 0·57 mm² sectional area. Find the length of wire required if the specific resistance of constantan wire is 0·49 Ω/metre of wire of cross-sectional area of 1 mm².

$$l=\frac{R\times A}{\rho}=\frac{25\times 0{\cdot}57}{0{\cdot}49}\text{ metres}=\underline{29{\cdot}1 \text{ metres}}$$

25. The resistances in a network are $R_1=25{\cdot}2\ \Omega$ and $R_2=72{\cdot}5\ \Omega$ respectively. A current I_1 of 4·9 A flows through R_1.
Determine the current in R_2.

$$I_2=\frac{R_1\times I_1}{R_2}=\frac{25{\cdot}2\times 4{\cdot}9}{72{\cdot}5}\ A=\underline{1{\cdot}7\ A}$$

26. The simple interest accruing on a capital sum in time *t* days can be calculated from the formula:

$$S=\frac{P\times p\times t}{100\times 365}=\frac{P/100\times t}{365/p}$$

where *S* is the interest
P is the principal
p is the percentage per annum
t is the time in days.

The slide rule can be used very advantageously for interest computations. (*a*) To find the interest accruing in 49 days on £1306 invested at 2½ per cent:

$$\frac{13{\cdot}06\times 49}{146}=£4{\cdot}38=\underline{£4\ 7s.\ 7d.}$$

(*b*) To find the interest accruing in 28 days on £425 invested at 2¾ per cent <u>17*s*. 11*d*.</u>

(*c*) To find the interest accruing in 7 days on £1280 invested at 2¼ per cent <u>11*s*. 1*d*.</u>

(*d*) To find the interest accruing in 298 days on £19,250 invested at 4 per cent. £628

(*e*) To find the interest accruing in 98 days on £140 15*s*. 9*d*. invested at 5 per cent £1 17*s*. 0*d*.

27. The power from a stream of water is given by

$$W = \frac{62 \cdot 4\ Qh}{550}$$

where Q is the volume discharged per sec and h is the difference in elevation between the input and output water levels.

For example: $W = \frac{62 \cdot 4 \times 6 \cdot 53 \times 22}{550}$ h.p. $= \underline{16 \cdot 3 \text{ h.p.}}$

28. A force of 75 lb acts on a body weighing 45·5 lb. The body being suspended by a fixed pulley. Determine the acceleration a with which the body will move.
The effectual force is (75 − 45·5) lb = 29·5 lb

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$29 \cdot 5 \times 32 = 45 \cdot 5 \times a$$

$$\therefore\ a = \frac{29 \cdot 5 \times 32}{45 \cdot 5} \text{ ft/sec}^2 = \underline{2 \cdot 075 \text{ ft/sec}^2.}$$

29. The capacitance of a parallel plate condenser is given by:

$$C = \frac{\epsilon\,(n-1)\,A}{4\pi d}$$

where $\epsilon = 2 \cdot 6$, $A = 8 \cdot 8$ cm², $n = 7$, $d = 0 \cdot 03$ cm

$$C = \frac{2 \cdot 6 \times 6 \times 8 \cdot 8}{4 \times 3 \cdot 14 \times 0 \cdot 03} \text{ cm} = \frac{2 \cdot 6 \times 8 \cdot 8 \times 6}{0 \cdot 12 \times 3 \cdot 14} \text{ cm} = \underline{364 \text{ cm.}}$$

30. The power output of a motor is given by:

$$N = \frac{2\pi\ r\ n\ (F_2 - F_1)}{33{,}000}$$

where r is the radius of the wheel around which the brake is applied
n the number of revolutions of the motor
F_2 the applied force
F_1 the weight applying the force.

Determine N, if $r = 2 \cdot 2$ in.
$n = 3500$
$F_2 = 18 \cdot 4$ lb
$F_1 = 9 \cdot 8$ lb

$$N = \frac{6 \cdot 28 \times 2 \cdot 2 \times 3500 \times 8 \cdot 6}{33{,}000} \text{ h.p.}$$

$$= \underline{12 \cdot 6 \text{ h.p.}}$$

31. An aluminium wire of cross-sectional area $A = 0 \cdot 048$ cm² is stretched by a force of 14·3 kg. The original length of the wire is 9·68 m. Determine the elongation of the wire, if the modulus of elasticity E for aluminium is 680,000 kg/cm².
Applying Hooke's law:

$$l = \frac{14 \cdot 3 \times 968}{0 \cdot 048 \times 680{,}000} \text{ cm} = \underline{0 \cdot 425 \text{ cm.}}$$

30 EXAMPLES ON PROPORTIONS (§§1, 7)

32. To convert inches into centimetres. 1 inch = 2·54 cm.

cm	66	30·5	48·3	55·9	78·7	96·5	114·2	132	155	208·3
in.	26	12	19	22	31	38	45	52	61	82

i.e. set 2·54 on C against index of D and move slide along C; read answers on D.

33. To convert measures of atmospheric pressure.
29·8 in. of mercury = 14·7 lb/in.²

in. of mercury	30·0	30·3	29·5	28·9	27·8	26·2	24·6
lb/in.²	14·8	14·95	14·55	14·26	13·72	12·93	11·89

i.e. set up the ratio 29·8/14·7 on the C and D scales (or, come to that, the ratio 14·7/29·8) and read off the answers.

34. Conversion of degrees Centigrade (Celsius) into degrees Fahrenheit

$$\frac{C}{100} = \frac{F - 32}{180}$$

C°	100	22·3	19·4	31·0	15·9	14·0	11·3	10·5
F°	212	74	67	88	61·5	58	54	51·5

35. It was found that a sample of an aluminium alloy weighing 87·760 oz contained 82·400 oz Al, 0·40 oz Mg, 0·45 oz Mn, 0·280 oz Si and 4·230 oz Cu.

Determine the percentage composition of the alloy.
Answer: 93·90% Al, 0·46% Mg, 0·51% Mn, 0·32% Si, 4·82% Cu.

36. In many technical and commercial procedures, it is necessary to determine both percentages and cumulative percentages. For example, supposing we obtain by a succession of sieving operations the proportional weights of soil particles in a sample of earth weighing 204 g. Then the following Table could be drawn up:

		Percentage by weight	*Cumul. weight*	*Cumul. Percentage*
Particles >1·6	cm weight 28 g	13·70	28 g	13·7
>0·8	15	7·36	43	21·4
>0·4	9·6	4·70	52·6	25·8
>0·2	7·2	3·53	59·8	29·3
>0·1	6·0	2·94	65·8	32·2
>0·05	13·8	6·76	79·6	39·1
>0·025	17·2	8·42	96·8	47·5
>0·012	23·0	11·28	119·8	58·6
>0·006	19·7	9·66	139·5	68·5
>0·003	21·3	10·45	160·8	78·6
<0·003	43·2	21·20	204·0	100·0
	204·0	100·00		

37. In a Wheatstone bridge three resistances R_2 and R_3 and R_4 have the values 61·9, 26·8 and 38·1 Ω respectively. Find the value of the unknown resistance R_x.

$$R_x : R_2 = R_3 : R_4$$
$$R_x : 61{\cdot}9 = 26{\cdot}8 : 38{\cdot}1$$
$$\therefore\ R_x = \underline{43{\cdot}5\ \Omega}$$

38. Two gear wheels have 24 and 81 teeth respectively. The larger wheel rotates with a speed of 38 rev/min. Determine the number of revolutions of the smaller wheel.

$$24 : 81 = 38 : x$$
$$\therefore\quad x = \underline{128{\cdot}3\ \text{rev/min.}}$$

39. If a man earns $58·40 per week, how long must he work to earn $369·40?

$$58{\cdot}4 : 369{\cdot}4 = 1 : x$$
$$\therefore\qquad x = \underline{6\tfrac{1}{3}\ \text{weeks.}}$$

40. Students of chemistry will find the slide rule particularly useful when working out stoichiometrical calculations:

(*a*) Determine the volume of acetylene that can be obtained at N.T.P. from 42 g of pure calcium-carbide.

$$CaC_2 + 2\ H_2O = C_2H_2 + Ca\ (OH)_2$$

64 g CaC_2 will produce 22·4 litres of C_2H_2 at N.T.P.
42 g CaC_2 will produce x litres of C_2H_2 at N.T.P.

$$\therefore\ x = \frac{42 \times 22{\cdot}4}{64}\ \text{litres} = \underline{14{\cdot}7\ \text{litres}}$$

(*b*) Determine the amount of hydrogen sulphide produced at N.T.P. from 6·8 g iron sulphide

$$FeS + 2\ H\ Cl = Fe\ Cl_2 + H_2S$$

87·9 g FeS will produce 22·4 litres H_2S at N.T.P.
6·8 g FeS will produce x litres H_2S at N.T.P.

$$\therefore\ x = \frac{6{\cdot}8 \times 22{\cdot}4}{87{\cdot}9}\ \text{litres} = \underline{1{\cdot}73\ \text{litres}}$$

(*c*) Determine the amount of sodium chloride that can be produced from 33·6 g NaOH and hydrochloric acid

$$NaOH + H\ Cl = Na\ Cl + H_2O$$

40 g NaOH will produce 58·46 g Na Cl
33·6 g NaOH will produce x g Na Cl

$$\therefore\ x = \frac{33{\cdot}6 \times 58{\cdot}46}{40}\ \text{g} = \underline{49{\cdot}1\ \text{g.}}$$

(*d*) Determine the amount of sulphuric acid required to produce 2150 g of nitric acid from sodium nitrate

$$Na\ NO_3 + H_2SO_4 = NaHSO_4 + HNO_3$$

98 g H_2SO_4 are required to produce 63 g HNO_3
x g H_2SO_4 are required to produce 2150 g HNO_3

$$\therefore\ x = \frac{2150 \times 98}{63}\ \text{g} = \underline{3340\ \text{g.}}$$

In the next two examples it is necessary to reset the slide of the rule.

(*e*) Determine the amount of metallic copper which is produced on pouring hydrogen over 11·2 g hot cupric oxide, assuming that all the cupric oxide is converted into copper.

$$Cu\ O + H_2 = Cu + H_2O$$

79·6 g Cu O will produce 63·6 g Copper
11·2 g Cu O will produce x g Copper

$$\therefore\ x=\frac{63{\cdot}6\times 11{\cdot}2}{79{\cdot}6}\ \text{g}=\underline{8{\cdot}95\ \text{g.}}$$

(*f*) Determine the amount of hydrogen produced at N.T.P. from 25 g of zinc and sulphuric acid.

$$Zn+H_2SO_4=ZnSO_4+H_2$$

65·38 g Zn will produce 22·4 litres H_2 at N.T.P.
25 g Zn will produce x litres H_2 at N.T.P.

$$\therefore\ x=\frac{25\times 22{\cdot}4}{65{\cdot}38}\ \text{litres}=\underline{8{\cdot}57\ \text{litres}}$$

In many of the above problems the reciprocal scale could be used with advantage. Also do not forget the 'golden rule' of the slide rule, i.e. the rule is set up in the same pattern as the question is posed.

31 EXAMPLES ON SQUARES AND SQUARE ROOTS
(§§11, 12, 13)

41. Determine the areas of the following squares.

Side length:	23	28	32	38	46	68	85 in.
Area:	529	784	1024	1444	2120	4620	7230 in.^{2}

i.e. hairline on D, answer on A.

42. Calculation of the standard deviation by the formula of statistics:

$$\sigma=\sqrt{\left(\frac{d_1^2+d_2^2+d_3^2+\ \ldots\ldots\ d_n^2}{n-1}\right)}$$

In this expression, d_1, d_2, d_3, etc. are the deviations of individual values from the arithmetic mean and n is the number of measurements.

The standard deviation σ in an example =

$$\sqrt{\left(\frac{0{\cdot}0134^2+0{\cdot}0291^2+0{\cdot}0427^2+0{\cdot}1084^2+0{\cdot}0689^2+0{\cdot}0328^2+0{\cdot}0213^2+0{\cdot}0187^2}{n-1}\right)}$$

$0{\cdot}0134^2=0{\cdot}0001796$
$0{\cdot}0291^2=0{\cdot}0008470$
$0{\cdot}0427^2=0{\cdot}0018230$

$0{\cdot}1084^2=0{\cdot}011750$
$0{\cdot}0689^2=0{\cdot}004750$
$0{\cdot}0238^2=0{\cdot}001076$
$0{\cdot}0213^2=0{\cdot}000454$
$0{\cdot}0187^2=0{\cdot}000350$
$\Sigma=0{\cdot}0212296$
then $\Sigma/7=0{\cdot}003033$
whence $\sigma=\underline{0{\cdot}0551.}$

This is a laborious calculation, however, since everything has to be written down, but it is still a little quicker than using logs.

43. Draw up a Table showing values of the function $y=(2\ x\pi)^2$, when x has the values 1, 2, 3, 4, 5, 6.

x	1	2	3	4	5	6
y	39·5	158	355	630	986	1421

44. Determine the moment of inertia I of a solid cylinder of mass M, radius r and of any axial length, revolving about its own axis by the formula

$$I=\tfrac{1}{2}\ M\ r^2$$

For example: $I=8{\cdot}4\times 2{\cdot}7^2$ lb ft^2
$I=\underline{62{\cdot}2\ \text{lb ft}^2}$

i.e. index of C against 2·7 on D, answer on A, opposite 8·4 on B.

45. The losses due to eddy currents in a sample of iron of volume V dm^3 and of thickness d mm. is calculated from the formula:

$$N_w=\frac{V\times B^2\times f^2\times d^2}{5\times 10^9}$$

If V=7·75 dm^3; f=50; B=15,400 and d=0·5 mm.

$$N_w=\frac{7{\cdot}75\times 15{,}400^2\times 50^2\times 0{\cdot}5^2}{5\times 10^9}\text{W}$$
$$=\underline{230\ \text{W}}$$

1. By inspection it is obviously simplest if the equation is divided through by 5, then neglecting temporarily all decimals we have $N_w=775\times 154^2\times 5^3\times$(10 to some power).
2. Cursor over 5 on D, read 125 on K (=5^3).
3. Index of B against 125 on A (=1×5^3).
4. Hairline on 154 on C (=154^2 on B).
5. Move index of slide under hairline (=$5^3\times 154^2$ on A).

6. Move cursor to 775 on B $(=\times 775)$.
7. Read 230 on A; by quick mental check, it is apparent that this is the required answer (or, if preferred, 10 required to be raised to the power -7 in the equation of step 1).

46. The numerical value of the geometric mean of two quantities is the square root of the product of these quantities.

$$\text{If } h=\sqrt{(p\times q)}$$

calculate h for the following values of p and q

p	3·53	4·05	1·62	11·8	0·78	0·94
q	5·82	7·15	4·94	9·3	2·92	0·69
h	4·54	5·37	2·83	10·5	1·51	0·805

Place the index of B against the value of p on A and read answer under hairline on D with cursor over q on B. By preliminary inspection it is obvious that the result will be somewhere mid-way between the values of p and q so that it is easy to decide which section of B to work in.

47. Torricelli determined the velocity v of a liquid discharged from a pipe and found that

$$v=\sqrt{(2\,g\,h)}$$
$$\text{If } g=32 \text{ ft/sec}^2$$
$$\text{and } h=7{\cdot}3 \text{ ft}$$
$$v=\sqrt{(64\times 7{\cdot}3)} \text{ ft/sec}^2=8\sqrt{7{\cdot}3} \text{ ft/sec}^2$$
$$\underline{21{\cdot}6 \text{ ft/sec}^2}$$

Note that the factorization of $\sqrt{64}$ into the value of 8 is done quite automatically on the slide rule. To solve $8\sqrt{7{\cdot}3}$, place 10 on C against 8 on D, read off answer on D under hairline placed over 7·3 on B; to solve $\sqrt{(64\times 7{\cdot}3)}$, place 100 on B against 64 on A (which brings the slide into identically the same position) thence as before.

48. The wave length of a vibration in an open circuit according to Thomson:

$$\lambda=2\pi\sqrt{(L\times C)}$$

(λ, L and C are measured in cm)

Calculate λ, if $L=770$ cm and $C=285$ cm

$$\lambda=2\,\pi\sqrt{(7700\times 285)}$$
$$\lambda=2\,\pi\sqrt{(2{,}190{,}000}=9300 \text{ cm}=\underline{93 \text{ metres.}}$$

If an approximate value for the root can first be assessed mentally (obviously a number somewhere between 285 and 7700, probably about 1500), then

1. Right-hand index of B against 77 on A
2. Cursor scans first near 285 on right-hand side of B, value on D is too great. ∴ proceed to 285 on left hand side, this gives 148 on D which is obviously the desired root (when corrected for decimal place).
3. Then multiply out by $2\,\pi$, using C – D scales.

49. In a right-angled triangle, with sides a, b and c, $a=\sqrt{(c^2-b^2)}$
To calculate a if c and b are known, we write: $a=\sqrt{\{(c+b)\times(c-b)\}}$
Let $c=15{\cdot}8$ in., and $b=9{\cdot}3$ in.
Then $c+b=25{\cdot}1$ in. and $c-b=6{\cdot}5$ in.

$$\therefore a=\sqrt{(25{\cdot}1\times 6{\cdot}5 \text{ in.})}=\underline{12{\cdot}8 \text{ in.}}$$

50. The area of a triangle with the sides a, b and c and the semi-perimeter $s=\frac{1}{2}(a+b+c)$ as found by Hero of Alexandria:

$$A=\sqrt{\{s\,(s-a)\,(s-b)\,(s-c)\}}$$
$$\text{If } a=13{\cdot}5 \text{ in.,}$$
$$b=16{\cdot}4 \text{ in.}$$
$$\text{and } c=21{\cdot}7 \text{ in.}$$
$$A=\underline{110{\cdot}6 \text{ in.}^2}$$

51. To calculate the volume V of a cylinder from its area of cross-section

$$F=\left(\frac{d}{c}\right)^2 \qquad \left(c^2=\frac{4}{\pi}, \textit{ see } \text{page } 30\right)$$

Then $V=\left(\frac{d}{c}\right)^2\times h$, where h is the height of the cylinder.

d (in.)	2·8	4·6	6·8	18·5	4·44	6·35	7·78
h (in.)	7·8	52	13·7	29·5	18·8	7·4	30·4
V (in.^{3})	48	864	497	7930	291	234	1445

e.g. 1. Left-hand index of C against 28 on D.
2. Cursor to 78 on B.
3. Read answer under c line over A.

52. To calculate the weights in kg of 1000 metres length of wire from the formula:

$$W=\left(\frac{d}{c}\right)^2\times l\times d$$

Thickness in mm	0·5	1·0	1·5	2·5	3·5
Steel wire (d=7·85) kg	1·54	6·17	13·9	38·5	75·5
Copper wire (d=8·9) kg	1·75	6·99	15·7	43·7	85·6

53. The volume of a circular cone is given by $V=\left(\frac{d}{c}\right)^2\times\frac{h}{3}$

Let $d=5{\cdot}2$ in., and $h=4{\cdot}4$ in.
then $V=\underline{31{\cdot}2\text{ in.}^3}$

54. The surface area of a sphere is given by $A=\left(\frac{d}{c}\right)^2\times 4$.

Calculate A if d is known.

d (in.)	5·6	34·6	4·72	40·7	78·3	9·15	58·1
A (in.^{2})	96·5	3760	70	5200	19,300	263	10,600

55. The volume of a sphere $V=\frac{4\pi r^3}{3}$, or $V=\frac{\pi d^3}{6}$; when using the slide rule to calculate the volume of a sphere, this formula is best transformed into $V=\frac{\pi d^2}{4}\times\frac{2}{3}d$ or

$$V=\left(\frac{d}{c}\right)^2\times\frac{2}{3}d$$

$$\therefore V=\left(\frac{d}{c}\right)^2\times\frac{d}{1{\cdot}5}$$

d (in.)	5·6	14·8	7·9	3·78	9·8	0·27	44·8
V (in.^{3})	92	1700	258	28·3	493	0·0103	47·000

56. To find the square roots of the following fractions:

$$\sqrt{\frac{3}{4}}=\underline{0{\cdot}866} \qquad \sqrt{\frac{1}{8}}=\underline{0{\cdot}354}$$

$$\sqrt{\frac{2}{3}}=\underline{0{\cdot}817} \qquad \sqrt{\frac{2}{9}}=\underline{0{\cdot}471}$$

$$\sqrt{\frac{5}{6}}=\underline{0{\cdot}913} \qquad \sqrt{\frac{7}{12}}=\underline{0{\cdot}764}$$

$$\sqrt{\frac{4}{7}}=\underline{0{\cdot}756} \qquad \sqrt{\frac{17}{31}}=\underline{0{\cdot}740}$$

e.g. 3 on B against 4 on A read answer on C opposite right index.

57. The period T of a simple pendulum is given by the expression

$$T=2\pi\sqrt{\frac{l}{g}}$$

Let $l=56$ cm, and $g=981$ cm/sec^2

then $T=6{\cdot}28\sqrt{\frac{56}{981}}\text{ sec}=\underline{1{\cdot}5\text{ sec.}}$

58. The time of vibration T of a hoop about an axis at its circumference is given by

$$T=2\pi\sqrt{\frac{2r}{g}}$$

For example $T=6{\cdot}28\times\sqrt{\left(\frac{2\times 2}{32}\right)}$ sec, if $g=32$ ft/sec.^{2}

$$T=\underline{2{\cdot}22\text{ sec.}}$$

59. To calculate the length of a simple seconds pendulum:

$$1=\frac{g}{\pi^2}=\frac{32}{3{\cdot}14^2}\text{ ft}=3{\cdot}24\text{ ft}$$

$$\text{or}\quad 1=\frac{g}{\pi^2}=\frac{9{\cdot}81}{3{\cdot}14^2}\text{ cm}=\underline{99{\cdot}4\text{ cm.}}$$

60. The velocity of an electromagnetic wave is given by

$$v=\frac{c}{\sqrt{K\mu}}$$

where K is the dielectric constant of the medium (paraffin $K=2{\cdot}10$), μ is the permeability and c is the velocity of light.

$$c=186{,}000\text{ miles/sec}$$

$$v=\frac{186{,}000}{\sqrt{2{\cdot}10\times 120}}\text{ miles/sec}$$

$$v=\underline{11{,}720\text{ miles/sec}}$$

61. The superelevation in mm of the outer rail of a circular railroad curve is found by applying the formula

$$h=11{\cdot}8\times\frac{v^2}{R}$$

where v is the speed of train in km/hr and R the radius of curvature in m. To calculate h, when $v=89$ km/hr and $R=1300$ m

$$h=11{\cdot}8\times\frac{89^2}{1300}\text{ mm}=\underline{72\text{ mm.}}$$

62. To prepare a graph for $y=\frac{3\cdot5}{\sqrt{x}}$

Find y, when x has the values 1, 2, 3, 4, 5 and 6

x	1	2	3	4	5	6
y	$\pm3\cdot5$	$\pm2\cdot47$	$\pm2\cdot02$	$\pm1\cdot75$	$\pm1\cdot57$	$\pm1\cdot43$

Place cursor over 3·5 on D, move slide so that 1, 2, 3, etc. on B come under hairline, answers opposite C index on D.

63. The output of an electric furnace is given by

$$N=R\times I^2$$

Let $R=21\cdot7\ \Omega$ and $I=4\cdot8$ A
then $N=21\cdot7\times4\cdot8^2$ W

$$\therefore N_e=\underline{500\text{ W.}}$$

64. The energy equivalent of an electron having a mass of $9\cdot11\times10^{-28}$ g

is $E=M\,c^2$
where $E=9\cdot11\times10^{-28}\times(3\times10^{10})^2$ ergs
i.e. $E=\underline{8\cdot20\times10^{-7}\text{ ergs.}}$

65. The kinetic energy W of a body moving through space is given by the formula:

$KE=\frac{W\,v^2}{2g}$, where W is the weight of the body in lb, g the acceleration of gravity and v the velocity in ft/sec,

for instance, $KE=\frac{3\cdot6\times55^2}{64}$ ft lb

$$=\underline{170\text{ ft/lb.}}$$

66. (*a*) The horizon visible from the deck of a ship h m above the water-line is given by

$$d=2\cdot08\sqrt{h}\text{ nautical miles}$$

Calculate d, when h has the values 10, 20, 30, 33, 37·5, 40 and 45 metres.

h	10	20	30	33	37·5	40	45	metres
d	6·58	9·30	11·40	11·95	12·74	13·16	13·95	nautical miles

Notice how the slide rule operation compares here with Example 62.

(*b*) The dip of the horizon which can be calculated with the aid of spherical geometry, is given by:

$x=1\cdot8\sqrt{h}$ minutes of arc.
If $h=2000$ metres
$x=1\cdot8\sqrt{2000}=80\cdot5'$
$\therefore x=\underline{1°20'30''=1\cdot342°}$

(*c*) The distance in miles d which can be seen from the altitude h feet above sea level is given by:

$$d=\sqrt{\frac{3h}{2}}$$

For example: $d=\sqrt{(1\cdot5\times49\cdot50)}$ miles
$=\underline{8\cdot62\text{ miles.}}$

67. The diameter d of a circle of known area F can easily be calculated from the formula $F=\left(\frac{d}{c}\right)^2$, since: $d=c\sqrt{F}$

F (in.^{2})	428	228	518	411	2·09	6·29	2140
d (in.)	23·3	17·03	25·7	22·9	1·63	2·83	52·2

32 EXAMPLES ON CUBES AND CUBE-ROOTS (§§14, 15)

68. The volume V of a sphere of radius r may be found by the formula:

$$V=\frac{4\pi\,r^3}{3}$$

since $\frac{4\pi}{3}=4\cdot19$

$$V=4\cdot19\times r^3$$

If r is given, r^3 is found first by use of scale K.

r (in.)	7·2	3·3	8·5	1·74	6·77	20·4	2·4
r^3 (in.^{3})	373	35·9	614	5·27	310	8490	13·8
V (in.^{3})	1563	150·3	2570	22·1	1298	35,600	57·8

69. The effort N required of an aircraft engine to keep the aircraft flying at a velocity v is given by:

$$N = c_w \frac{\sigma}{2} F v^3,$$

where c_w is a resistance value of the motor,

$\rho = \frac{\gamma}{g}$ the density of air in kg/m³,

and F the area of the face

If $c_w = 0{\cdot}033$, $\rho = 1/8$ kg/m³, $F = 25{\cdot}8$ m³, $v = 72$ m/sec

then $N = \frac{0{\cdot}033 \times 25{\cdot}8 \times 72^3}{2 \times 8 \times 75}$ h.p. (metric)

$$\underline{N = 265 \text{ h.p. (metric)}}$$

70. To calculate the radius r from the volume V of a sphere.

$$V = 4{\cdot}19\, r^3$$

$$\therefore r = \sqrt[3]{\frac{V}{4{\cdot}19}}$$

V (in.³)	57·9	327	1·556	3·270	34,600	01·88
V/4·19 (in.³)	13·82	78·0	372	780	8270	0·448
r (in.)	2·40	4·27	7·19	9·21	20·2	0·765

71. The diameter d of a cast-iron transmission shaft required to transmit N h.p. during n revolutions can be calculated from the empirical formula:

$$d = 14{\cdot}5 \sqrt[3]{\frac{N}{n}}$$

If $n = 144$ and $N = 15$ h.p. (metric)

then $d = 14{\cdot}5 \sqrt[3]{\frac{15}{144}}$ cm $= 14{\cdot}5\ \sqrt[3]{0{\cdot}1042}$ cm $= 14{\cdot}5 \times 0{\cdot}47$ cm

$$\underline{d = 6{\cdot}82 \text{ cm.}}$$

33 EXAMPLES SHOWING THE ADVANTAGES OF THE INVERTED SCALE CI (§16)

72. Conversion of metric weights and measures into British units.

1 metre	= 3·28 ft	↔	1 ft = 0·305 metre
1 metre	= 1·094 yd	↔	1 yd = 0·914 metre
1 kilogramme	= 0·01968 cwt	↔	1 cwt = 50·8 kilogrammes

i.e. if, for instance, we place the hairline on 328 of the reciprocal scale we find 305 on C.

73. Kirchhoff's law states that in a network of resistance

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

Calculate R, if $R_1 = 3{\cdot}5$, $R_2 = 2{\cdot}5$, $R_3 = 4{\cdot}8$ and $R_4 = 8{\cdot}0\ \Omega$

Then $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$

$= (0{\cdot}286 + 0{\cdot}400 + 0{\cdot}208 + 0{\cdot}125)\ \Omega^{-1}$

$= 1{\cdot}019\ \Omega^{-1}$

$\frac{1}{R} = 1{\cdot}019\ \Omega^{-1}$

$\therefore R = 0{\cdot}981\ \Omega$ or

$\underline{R \approx 1\ \Omega.}$

74. For a lens of focal length f, the distance u of the object from the lens and the distance v of the image from the lens is given by the formula:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

f can therefore be calculated if u and v are known:

u	23·1	39·4	51·4	11·4	34·6	1·28
v	9·5	2·2	16·7	7·8	42·5	0·31
(1/u)	0·0433	0·0254	0·0194	0·0875	0·0289	0·781
(1/v)	0·1052	0·455	0·0598	0·128	0·0325	3·230
(1/f)	0·1485	0·4804	0·0792	0·2155	0·0524	4·011
f	6·73	2·08	12·63	4·64	19·1	0·25

75. If the radii of curvature of a lens are r_1 and r_2, and μ is the refractive index of the glass, the focal length of the lens can be calculated from the formula:

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

If $\mu = 1{\cdot}56$, $r_1 = 5{\cdot}2$ in. and $r_2 = 2{\cdot}8$ in.

$(1/f) = 0{\cdot}56\ (1/5{\cdot}2 + 1/2{\cdot}8)$ in.

$(1/f) = 0{\cdot}56\ (0{\cdot}192 + 0{\cdot}357)$ in. $= 0{\cdot}56 \times 0{\cdot}559$ in. $= 0{\cdot}307$ in.

$\therefore \underline{f = 3{\cdot}26 \text{ in.}}$

76. The amount of heat absorbed or diffused by a body is obtained from the product of the mass, the specific heat and the temperature difference before and after the heat exchange has taken place.

$$Q = M\ c\ t$$

Calculate the quantity of heat required to increase the temperature of air in a room of 61 dm³ from 12·5 to 20·0° C.

$$Q = 61 \times 1{\cdot}29 \times 0{\cdot}239 \times 7{\cdot}5 \text{ cal} = \underline{141 \text{ cal.}}$$

Normal procedure to multiply this out means:

1. Place 10 on C against 61 on D (slide movement $2\frac{1}{8}$ in.)
2. Cursor to 75 on D
3. 10 on C under hairline (,, ,, $1\frac{1}{4}$ in.)
4. Cursor to 239 on D
5. 1 on C under hairline (,, ,, $3\frac{3}{4}$ in.)
6. Cursor to 129 on D answer on C.

A total of 3 slide movements covering $7\frac{1}{8}$ in. were required.
Using the reciprocal scale, the procedure would be:

1. Hairline to 61 on D
2. Slide under hairline to 239 on CI (slide movement $1\frac{5}{8}$ in.)
3. Cursor to 129 on C
4. Slide to 75 on CI (,, ,, $\frac{1}{8}$ in.)

A total of 2 slide movements covering $1\frac{3}{4}$ in. were required.
To reduce the amount of movement which has to be made by the slide when using the CI scale, the secret is to pair off numbers and reciprocals of numbers which occur in the same zone of the slide. In this instance 129 on C is physically very close to 75 on CI.

77. The surface area of a cone-shaped roof of a tower is given by:

$$A = r\pi s$$

Calculate A, if $r = 4{\cdot}60$ yd and $s = 12{\cdot}5$ yd.

$$A = \underline{181 \text{ yd.}^2}$$

78. In a calculation of thermal efficiency it is necessary to evaluate the expression

$$\frac{1037{\cdot}6 \times 8750}{869 \times 14405} \times 100$$

Solve as follows:

1. Left-hand index of C to 10376 on D (i.e. to 1038) $(=1 \times 1038)$
2. Cursor to 869 on CI $\left(=1038 \times \frac{1}{869}\right)$

3. 875 on CI to hairline $\left(=1038 \times \frac{1}{869} \times \frac{1}{1/875}\right)$
4. Cursor to left-hand index of C.
5. Draw 14405 on C to hairline.

Alternatively, using DF and CF:

1. Cursor to 1038 on DF
2. 869 on CF to hairline
3. Cursor to 875 on CF.
4. 114 on CF to hairline.

Answer is <u>72·5 per cent</u> on D under right-hand index of C.

Note that it proved to be better to manipulate as in Steps 4 and 5, rather than to continue using the inverted scale, since this would have meant moving the slide a considerable distance.

79. The area of an ellipse whose major axis is a and whose minor axis is b, is found by $A = a\,b\,\pi$.
Calculate A, if $a = 4{\cdot}2$ in. and $b = 2{\cdot}7$ in.

$$A = 4{\cdot}2 \times 2{\cdot}7 \times 3{\cdot}14 \text{ in.}^2 = \underline{35{\cdot}6 \text{ in.}^2}$$

80. The area of parabolic segment $A = (4/3)\ a\ y_a$.
Calculate A, if $a = 4{\cdot}2$ in. and $y_a = 2{\cdot}9$ in.

$$A = 1{\cdot}333 \times 4{\cdot}2 \times 2{\cdot}9 \text{ in.}^2 = \underline{16{\cdot}24 \text{ in.}^2}$$

81. What is the distance in km covered by a ship in one day if her speed is 28 knots?

$$s = 28 \times 24 \times 1{\cdot}852 \text{ km} = \underline{1245 \text{ km.}}$$

82. Find the weight of a sheet of steel $a = 7$ ft 2 in. long $b = 3$ ft 2 in. wide and $c = \frac{1}{4}$ in. thick, if specific gravity d is 0·283

$$W = 86 \times 38 \times \frac{1}{4} \times 0{\cdot}283 \text{ lb} = \underline{229 \text{ lb.}}$$

83. The increase in the depth of submergence of a ship which is being loaded is given by:

$$h = \frac{Q}{qd}$$

Where Q is the load in tons (metric), q the cross-section of the ship at the water level and d is the specific gravity of water.

Calculate h if $Q=1440$ tons

$q=620$ m²

$d=1{\cdot}03$ tons/m³ (the specific gravity of sea water)

then $$h=\frac{1440}{620\times1{\cdot}03}\text{ metres}=\underline{2{\cdot}26\text{ metres.}}$$

84. The external voltage resistance of an electric circuit may be variable. Determine the current flowing through the network. Applying Ohm's law:

$$I=\frac{E}{R}$$

Let $E=220$ V and $R=250$, 312, 355, 402 and 496 Ω.

R	250	312	355	402	496	Ω
I	0·880	0·705	0·620	0·547	0·443	A

34 EXAMPLES USING THE MANTISSA SCALE L (§18)

85. The intensity of radiation from a 'black body' is calculated from the Stefan-Boltzmann law: $I=\sigma\,T^4$ where σ is a constant depending on the units used for expressing I and T. Taking T as the absolute temperature of the sun, the intensity of radiation of its surface will be

$$I=1{\cdot}36\times10^{-12}\times6150^4\text{ cal/sec/cm.}^2$$
$$I=\underline{1946\text{ cal/sec/cm.}^2}$$

Against 615 on C find 0·789 on L. Then 3·789 × 4 = 15·1556. Reversing, we have 0·1556 on L giving 143 on C which multiplied by 136 gives the answer to the appropriate decimal place.

86. The air in the cylinder of a diesel motor is compressed adiabatically according to Poisson's law:

$$\frac{T_2}{T_1}=\left(\frac{V_1}{V_2}\right)^{0{\cdot}4}$$

where T_1 is the initial temperature in degrees K

V_1 is the initial volume in cm³

V_2 is the final volume in cm³.

Determine T_2 if $T_1=293$°K (20°C), $V_1=600$ cm³ and $V_2=44$ cm³ Dividing $V_1/V_2=13{\cdot}62$ (compression ratio), read off on L the

mantissa 0·134. Then $\log\left(\frac{V_1}{V_2}\right)=1{\cdot}134$. This number is multiplied by 0·4 and 0·4536 is obtained.

The mantissa is again set by the left index and in the same position of the slide below 293 we find the result for $T_2=833$°K

∴ the final temperature = $\underline{560°\text{C}}$.

87. The interval is given by $x=\sqrt[12]{2}$
Find x.

Find log 2 = 0·301 and calculate $\frac{1}{12}\times\log 2=0{\cdot}0251$.

This value is again set on the mantissa scale and on D we read off the value for $x=\underline{1{\cdot}06}$.

88. Determine the rate of interest on a capital of £280 which has been accumulating compound interest for 7 years and has thereby increased to £344.

$$S=P\,(1+i)^n$$
$$1+i=\sqrt[n]{\frac{S}{P}}$$
$$i=\sqrt[n]{\frac{S}{P}}-1$$
$$i=\sqrt[7]{\frac{344}{280}}-1=\sqrt[7]{1{\cdot}23}-1$$
$$\log 1{\cdot}23=0{\cdot}09$$
$$\tfrac{1}{7}\log 1{\cdot}23=0{\cdot}013$$
$$1+i=1{\cdot}03$$
$$i=\underline{3\text{ per cent}}$$

89. Determine volume of 1 dm³ of gas which has been compressed adiabatically from atmospheric pressure to 12 atmospheres

$p_2\times V_2^{1{\cdot}4}=p_1\times V_1^{1{\cdot}4}$ $p_1=1$ atmosphere and $V_1=1$ litre

$p_2\times V_2^{1{\cdot}4}=1$ $V_2^{1{\cdot}4}=(1/p_2)$ $V_2=\sqrt[1{\cdot}4]{(1/p_2)}=\sqrt[1{\cdot}4]{0{\cdot}0833}$

$$\log V_2=\frac{\log 0{\cdot}0833}{1{\cdot}4}=\frac{0{\cdot}92-2}{1{\cdot}4};\ \log V_2=\frac{1{\cdot}721-2{\cdot}8}{1{\cdot}4};\ \log V_2=(1{\cdot}23-2)$$

$$\therefore V_2=\underline{0{\cdot}170\text{ dm.}^3}$$

Note that if C and D are lined up, the CI scale may be used in conjunction with the L scale, thus facilitating operations.

35 EXAMPLES USING THE SINE SCALE S (§19)

90. A ladder of length l ft is leaning against a wall. The top of the ladder is h ft from the ground. Determine the angle the ladder makes with the ground, if $l=3{\cdot}75$ and $h=2{\cdot}12$ ft.

$$\sin\theta=\frac{h}{l}=\frac{2{\cdot}12}{3{\cdot}75}$$
$$\therefore\ \theta=\underline{34{\cdot}4^\circ}.$$

91. The gradient of a road is often defined as the ratio of the difference between two levels and the length of the road between these two levels. Determine the gradient if the difference in levels h is 56 ft and the length s is 500 ft.

$$\sin\theta=\frac{h}{s}=\frac{56}{500}$$
$$\therefore\ \theta=\underline{6{\cdot}4^\circ}.$$

92. The gradient of a mountain railway is 2·6 per cent. Determine the inclination of the railway with the horizontal.

$$\sin\theta=0{\cdot}0260$$
$$\therefore\ \theta=\underline{1{\cdot}5^\circ}.$$

93. Determine the vertical component of a force F making an angle θ with the horizontal.

$$F_y=F\sin\theta$$

Let $F=21{\cdot}8$ lb and $\theta=49{\cdot}5^\circ$, then

$$F_y=21{\cdot}8\ \text{lb}\times\sin 49{\cdot}5^\circ$$
$$\therefore\ F_y=\underline{16{\cdot}6\ \text{lb}}.$$

94. The radius r of a section parallel to the earth's equator is given by:

$$r=R\cos\varphi$$

where R is the average radius of the earth
and φ the latitude of the section.

Determine r, if R is 3960 miles, and $\varphi=53{\cdot}5^\circ$

$$\cos 53{\cdot}5^\circ=\sin(90^\circ-53{\cdot}5^\circ)=\sin 36{\cdot}5^\circ$$
$$r=3960\ \text{miles}\times\sin 36{\cdot}5^\circ$$
$$\therefore r=\underline{2360\ \text{miles}}.$$

95. The area of a triangle is half the product of two adjacent sides and the sine of the angle included by them.

$$A=\frac{1}{2}\,b\,c\sin\alpha$$

Let $b=5{\cdot}4$ in., $c=8{\cdot}3$ in. and $\alpha=36{\cdot}7^\circ$
then $A=(1/2)\times5{\cdot}4\times8{\cdot}3\ \text{in.}^2\times\sin 36{\cdot}7^\circ$
$=2{\cdot}7\times8{\cdot}3\ \text{in.}^2\times\sin 36{\cdot}7^\circ$
$=22{\cdot}4\ \text{in.}^2\times\sin 36{\cdot}7^\circ$
$\therefore A=\underline{13{\cdot}38\ \text{in.}^2}$

96. The slope angle of a certain kind of dry sand is 32°. If the height h of the cone is 18 ft, determine the side length c of it.

$$\sin\theta=\frac{h}{c}$$
$$c=h/\sin\theta$$
$$=(18\ \text{ft}/\sin 32^\circ)$$
$$=\underline{34\ \text{ft}}.$$

97. The effective power P of an alternating current is equal to the product of the effective voltage E, the effective current I and the power factor of the circuit $\cos\theta$.

$$P=E\,I\cos\theta$$

Determine P, if $I=2{\cdot}5$ A, $E=220$ V and $\theta=37{\cdot}5^\circ$

$$P=\underline{436\ \text{W}}$$

98. A wattmeter registered 7200 when an alternating current of 11·5 A and 380 V is flowing through it. Determine the phase shift.

$$\cos\theta=\frac{P}{\sqrt{(3)}\,IE}=\frac{7200}{1{\cdot}732\times11{\cdot}5\times380}$$
$$\therefore\ \theta=\underline{18^\circ}.$$

99. The law of refraction, discovered by Willebrod Snell is formulated by

$$\mu=\frac{\sin i}{\sin r}$$
$$\therefore\ \frac{\sin i}{\sin r}=\frac{\mu}{1}\ \text{or if } \mu=1{\cdot}61 \text{ (for flint glass)}$$
$$\frac{1}{\sin r}=\frac{1{\cdot}61}{\sin i}$$

After setting the ratio $\dfrac{1{\cdot}61}{\sin i}$ on scales S and C, find r on S.

i	42°	58°	27·5°	10·25°	90°
r	24·6°	31·8°	16·7°	6·35°	38·4°

100. From the last computation we can easily deduce the angle of total reflection which is to be calculated for a few other values of μ.

	water	crown glass	carbon disulphide	diamond	alcohol
μ	1·33	1·52	1·63	2·42	1·36
r	48·7°	41·2°	37·8°	24·4°	47·4°

101. In a triangle ABC, BC is 8·6 in., $\alpha=65°$, $\beta=36°$; find the length of the side AC.

$$\frac{b}{\sin\beta}=\frac{a}{\sin\alpha}$$

$$b=\frac{8{\cdot}6\text{ in.}\times\sin 36°}{\sin 65°}$$

$$\therefore\ b=\underline{5{\cdot}58\text{ in.}}$$

102. Two forces $F_1=28$ lb and $F_2=36$ lb are acting on a body forming an angle $\theta=124°$ between them. Find the magnitude and direction of the resultant of F_1 and F_2.
Apply the sine rule $\measuredangle F_1R=75{\cdot}5°$; $\measuredangle F_2R=48{\cdot}9°$

$$\therefore R=\underline{30{\cdot}7\text{ lb}}$$

103. Determine the height of a mountain $h=\dfrac{a\sin\gamma\sin\delta}{\sin(\delta-\gamma)}$,

where $a=162$ ft, $\gamma=21{\cdot}2°$, $\delta=24{\cdot}4°$

$$h=\frac{162\text{ ft}\times\sin 21{\cdot}2°\times\sin 24{\cdot}4°}{\sin 3{\cdot}2°}$$

$$=\frac{162\text{ ft}\times 0{\cdot}361\times 0{\cdot}413}{0{\cdot}0558}$$

$$=\underline{434\text{ ft.}}$$

104. Determine the displacement y in harmonic motion $y=a\sin(2\pi t/T)$
where $a=1{\cdot}56$
$T=2{\cdot}8\times10^{-2}$ sec
$t=1\times10^{-2}$ sec

$$y=1{\cdot}56\sin\frac{6{\cdot}28\times1}{2{\cdot}8}$$

$$=1{\cdot}56\times\sin 2{\cdot}245$$
$$=1{\cdot}56\times\sin 128{\cdot}5°$$
$$=1{\cdot}56\times\sin 51{\cdot}5°$$
$$=\underline{1{\cdot}221}$$

105. Determine the direction cosines in a position vector:

$$\cos\alpha=\frac{x}{\sqrt{(x^2+y^2+z^2)}};\ \cos\beta=\frac{y}{\sqrt{(x^2+y^2+z^2)}};\ \cos\gamma=\frac{z}{\sqrt{(x^2+y^2+z^2)}}$$

where $x=7$, $y=2$ and $z=5$

Then $\cos\alpha=\dfrac{7}{\sqrt{78}}$; $\cos\beta=\dfrac{2}{\sqrt{78}}$ and $\cos\gamma=\dfrac{5}{\sqrt{78}}$

$\alpha=\underline{37{\cdot}6°}$ $\beta=\underline{76{\cdot}9°}$ $\gamma=\underline{55{\cdot}5°}$

36 EXAMPLES USING THE TANGENT SCALE T (§20)

106. A chimney of height $h=38$ ft casts a shadow of length $s=45$ ft. Determine the sun's angle of altitude at this time.

$$\tan\theta=\frac{h}{s}=\frac{38}{45}$$

$$\therefore\theta=\underline{40{\cdot}2°.}$$

107. The equation of a straight line is:

$$y=\frac{7}{11}x+5$$

Determine the angle θ of slope

$$\tan\ \theta=\frac{7}{11}$$

$$\therefore\ \theta=\underline{32{\cdot}5°}$$

108. The Cartesian coordinates of a point P are $x_1=4{\cdot}3$ and $y_1=5{\cdot}8$; a line is drawn through the origin to the point. Determine the angle θ of slope

$$\tan\theta=\frac{y_1}{x_1}=\frac{5{\cdot}8}{4{\cdot}3}$$

$$\therefore\ \theta=\underline{53{\cdot}5°}$$

109. The base of a roof is 8·8 yd wide and the ridge is 5·7 yd high. Determine the angle of slope of the sides.

$$\tan\theta=\frac{\text{height}}{\text{base}/2}=\frac{5\cdot70}{4\cdot40}$$
$$\therefore\ \theta=\underline{52\cdot3^\circ}$$

110. Determine the angle θ through which the outside of a circular curve of roadway has to be raised in order to compensate the centrifugal force acting on vehicles driven over the same at 68·4 m.p.h. The radius of curvature of the road is 984 yd = 2752 ft.

$$\tan\theta=\frac{v^2}{gr}$$
$$68\cdot4\ \text{m.p.h.}=33\cdot4\ \text{yd/sec}=100\cdot2\ \text{ft/sec}$$
$$\tan\theta=\frac{100\cdot2^2}{32\cdot2752}$$
$$\therefore\theta=\underline{6\cdot7^\circ}$$

111. Determine the height h of a cloud, if the vertical reflection of a search light is observed from a distance $d=1500$ yd, the angle of elevation being 38·5°.

$$\tan\theta=\frac{h}{d}$$
$$h=1500\ \text{yd}\times\tan 38\cdot5^\circ$$
$$\therefore\ h=\underline{1193\ \text{yd.}}$$

112. The top of a church tower is observed from a point 124 yd from the base of the tower. The angle of elevation $\alpha=52\cdot4^\circ$. Determine the height of the tower.

$$h=124\ \text{yd}\times\tan 52\cdot4^\circ$$
$$\therefore\ h=\underline{161\ \text{yd.}}$$

113. The diameter of a screw is 1½ in. and the pitch has 6 threads/in. Determine the angle of lead.

$$\tan\alpha=\frac{\frac{1}{6}}{\frac{3}{2}\pi}=\frac{1}{9\pi}=0\cdot0354$$
$$\therefore\ \alpha=\underline{2\cdot03^\circ}$$

114. A balloon flying at a height of $h=1800$ ft is observed at an angle of elevation of 55°. What is the horizontal distance of the balloon from the observer?

$$x=\frac{1800\ \text{ft}}{\tan 55^\circ}$$
$$=\underline{1260\ \text{ft.}}$$

115. Determine L from $\tan\delta_L=\frac{R}{\omega L}$ (inductive coil)

if $\delta_L=3^\circ$, $R=2\cdot5\ \Omega$ and $\omega=314$.

$$L=\frac{R}{\omega\tan\delta_L}$$
$$=\frac{2\cdot5}{314\times0\cdot0524}\ \text{henrys}$$
$$=\underline{0\cdot152\ \text{henry.}}$$

116. Determine δ_C from $\delta_C=\omega\ RC$ (capacitor)

if $R=47\cdot5\ \Omega$, $c=10\times10^{-6}$ farads and $\omega=314$.

$$\tan\delta_C=314\times47\cdot5\times10\times10^{-6}$$
$$\tan\delta_C=0\cdot1491$$
$$\therefore\ \delta_C=\underline{8\cdot5^\circ}$$

117. Determine A in the formula (variable capacitor)

$$A=(R^2-r^2)\tan(\varphi/2),$$
if $R=7\cdot8$ in., $r=5\cdot2$ in. and $\varphi=75^\circ$
$$A=(R+r)(R-r)\tan(\varphi/2)$$
$$=13\times2\cdot6\times\tan 37\cdot5^\circ$$
$$=\underline{25\cdot9\ \text{in.}^2}$$

37 EXAMPLES USING THE FOLDED SCALES (§26)

118. Find the value of 167 × 5·9 × 0·83.
Step 1. Cursor to 167 on D.
Step 2. Bring 5·9 on CI under hairline.
Step 3. Cursor to 0·83 on CF.
Step 4. Under hairline read 818 on DF = Answer.

The total movement of the slide was about ⅛ in. By the usual method, using scales C and D the total movement would have been 2¼ in.

119. The surface of a cylinder is given by the product of the radius times the height times π. Assume the radius to be 4·6 in. and the height 12·5 in.

Step 1. Set 1 on C against 4·6 on D.
Step 2. Read the answer 180·6 in. on the DF scale with the cursor over 125 on C.

120. The volume of an ellipsoid with half-axes a, b and c is given by:

$$v=(4/3)a\ b\ c\ \pi$$

If $a=12{\cdot}5$ in.; $b=7{\cdot}8$ in.; $c=5{\cdot}1$ in.
Then $V=\underline{2080 \text{ in.}^3}$

38 EXAMPLES USING LOG LOG SCALES (§21)

The calculation of pressures and volumes, involving changes of state according to the laws $PV^n=$Constant, is a field of activity in which the log log scales save much time and tedium.

121. Taking first a simple problem of terminal temperature in an adiabatic expansion, when T_2 is to be found from $520\ (14)^{0{\cdot}4}$
Step 1. With the cursor set right-hand index of C over 14 on LL_3.
Step 2. Cursor to 0·4 on scale C.
Step 3. Under hairline read 2·875 on LL_3.
Step 4. 520 can then be multiplied by 2·875, in the usual way on scales C and D to obtain

$$T_2=\underline{1495°\text{R}}$$

122. If $T_2=783\times\left(\frac{3{\cdot}40}{0{\cdot}36}\right)^{0{\cdot}372}$

we have
Step 1. Set 0·36 on C over 3·4 on D to obtain 9·45.
Step 2. Evaluate $(9{\cdot}45)^{0{\cdot}372}$ by:
(*a*) setting left-hand index of C over 9·45 on LL_3,
(*b*) cursor to 0·372 on C;
(*c*) under hairline read $(9{\cdot}45)^{0{\cdot}372}=2{\cdot}305$ on LL_2.
Step 3. Then $T_2=783\times2{\cdot}305$ (scales C and D), i.e.

$$\underline{1810°\text{K}}$$

123. We sometimes require to find γ from the expression

$$\frac{T_a}{T_b}=\left(\frac{V_b}{V_a}\right)^{\gamma-1}$$

Suppose this is, in a given instance

$$\frac{540}{293}=\left(\frac{5}{1}\right)^{\gamma-1}$$

The left-hand side is reduced by division on scales C and D to 1·843 then $1{\cdot}843=5^{\gamma-1}$

Step 1. Set left-hand index of C to 5 on LL_3.
Step 2. Cursor to 1·843 on LL_2.
Step 3. Under hairline read 0·380 on scale C.

$$\text{so } \gamma-1=0{\cdot}38 \text{ whence } \gamma=\underline{1{\cdot}38}$$

124. An apparently more complicated example, also issuing from an adiabatic expansion, is:

$$\frac{860}{560}=\left(\frac{100}{20}\right)^{\gamma-1/\gamma}$$

$$\text{i.e. } 1{\cdot}535=5^{\gamma-1/\gamma}$$

Step 1. Set left-hand index of C over 5 on LL_3.
Step 2. Cursor to 1·535 on LL_2.
Step 3. Under hairline read 0·266 on C.

Step 4. So $\frac{\gamma-1}{\gamma}=0{\cdot}266$ whence $\gamma=\frac{1}{1-0266}$

$$\text{and } \gamma=\underline{1{\cdot}36}$$

125. Consider $V_b=4\times\left(\frac{14}{50}\right)^{1/1{\cdot}3}$ ft.^{3}

This can be written $\frac{4}{3{\cdot}57^{\ 1/1{\cdot}3}}$

To evaluate the denominator of the fraction

Step 1. Set cursor to 3·75 on LL_3.
Step 2. C under hairline (or 1/1·3 on CI).
Step 3. Read 2·63 on LL_2 under index of C.

Step 4. V_b is then $4\times\frac{1}{2{\cdot}63}$ i.e. $\underline{1{\cdot}52 \text{ ft.}^3}$

AIDE MEMOIRE

Problem	*Method*	*Answer*
1. Multiplication $a \times b$	§5, p. 19: a on D + b on C	opp Cursor at b
2. Continued Products $a \times b \times c$	§6, p. 22: a on D + b on C, cursor, c on C, cursor	opp Cursor at c
3. Division $a \div b$	§7, p. 22: a on D − b on C	opp index
4. Combined multiplication and division $\frac{a \times b}{c}$	§8, p. 25: a on D − c on C + b on D *Rule: division comes first*	opp cursor at b
5. Proportions $\frac{a}{b} = \frac{c}{d}$	§9, p. 28: a on D over b on C *Rule: gap between slide and stock corresponds to fraction dividing line*	c on D occurs over d on C
6. Squares	§11, p. 30: a on D	a^2 on A
7. Square Roots	§12, p. 32: a^2 on A *Rule: when there are an odd number of digits in front of decimal point, or an odd number of noughts after it (in the case of a decimal) use L.H. of* A. *Conversely, use R.H.*	a on D
8. Cubes	§ 14, p. 40: d on D	d^3 on K

Problem	Method	Answer
9. Cube Roots	§ 15, p. 41: d^3 on K *Rule: bracket into groups of three figures. The number of digits in the L.H. group (for a whole number) or (in the case of a decimal) in the first L.H. group not entirely zeros, defines the setting in 1st, 2nd or 3rd section of* K, *from left.*	d on D
10. Reciprocals: (i) $1/a$ (ii) $1/a^2$	§16, p. 42: Cursor to a on C=D Cursor to a on CI	(or $1/a$ on CI vice versa) $1/a^2$ on A=B
11. Logarithms	§18, p. 51: Cursor to a on D	mantissa of a on L
12. Sines: Sin $\theta = a$ (i) $\theta > 5{\cdot}74°$ (ii) $\theta < 5{\cdot}74°$	§19, p. 54: Cursor to θ on S Cursor to θ on ST *Note: prefix* 0·0 *to value read off.*	 a on D a on D
13. Tangents tan $\theta = a$ (i) $\theta = 5{\cdot}71°$ to 45° (ii) $\theta < 5{\cdot}71°$	§20, p. 62: Cursor to θ on T Cursor to θ on ST *Note: prefix* 0·0 *to values read off*	 a on D a on D
(iii) $\theta = 45°$ to 84·29°	Cursor to θ on T, slide neutral	a on CI
(iv) $\theta > 84{\cdot}29°$	Cursor to θ on ST, slide neutral	a on CI
14. Log logs	§21, p. 62: n on D	log n on LL
15. Numbers to any power a^x	§22, p. 69: a on LL against index on D	a^x against x on D

Problem	Method	Answer
16. Any Roots $\sqrt[x]{a}$	§23, p. 70: x on D against a on LL	$\sqrt[x]{a}$ against index on D
17. Logs to any base $\text{Log}_x{}^a$	§24, p. 71: set x on LL over index of D	$\text{Log}_x{}^a$ against a on LL
18. Pythagoras' scales $y = \sqrt{(1-x^2)}$	§25, p. 72: x on P	Y on D
19. Folded Scales	§26, p. 73 also p. 24: settings of C, D and CI *Rule: start computations with* CF *and* DF *scales*	Identical with settings of CF, DF and DIF
20. Determination of decimal point	p. 20: (i) Factorize in terms of 10 (ii) Round off each term to one significant figure (iii) Calculate approximate value mentally, or on slide rule	 True value is close to approximate value
21. Slide needs resetting	See items 10 and 19. Consider using either CI scales of CF, DF scales	Resetting may be avoided

TO THE READER

Author and publisher would welcome suggestions towards future editions of this book, or the pointing out of any misprint or obscurity. Please write to the Technical Editor, Macmillan and Co. Ltd., Saint Martin's Street, London WC 2.