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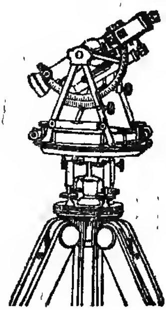
ENGINEERS'

# SURVEYING INSTRUMENTS;

## THEIR CONSTRUCTION AND USE.



BY I. O. BAKER, C. E.,
Professor of Civil Engineering, University of Illinois.

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# ENGINEERS

# SURVEYING INSTRUMENTS,

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### I.

### INTRODUCTORY.

It is proposed to give a series of articles on the best methods of constructing, adjusting, and using different engineering field instruments. The intention is to make the language so simple and clear that all can understand, and at the same time make it something more than an elementary discussion, and not a re-statement of matter found in ordinary text books and engineering manuals. It is not necessary to dwell upon the importance to the engineer of a knowledge of the best forms of construction and of a thorough understanding of the principles which govern the adjustments of his instruments, and of course the engineer should be an expert in handling his instruments.

### THE PLANE TABLE.

This instrument is not properly appreciated by the engineering profession. In a general way it is understood to be employed in the surveys, geodetical, topographical and geological, made by the general government, but it seems to be very well understood that the plane table is both useful and available in many cases in ordinary practice. It is frequently the best means of obtaining the plat and area of the irregular tracts that occur in ordinary land and city surveying; it is valuable in making plats of parks, cemeteries, mining property, system of ditches, etc. It has the great advantage of dispensing with all notes and records of the measurements, since they are platted as they are surveyed: it thus saves time and lessens the possibility of making mistakes.

It will be shown later how the simplest form of the plane table, combined with an ordinary transit or level, affords an easy, rapid, and accurate method of making preliminary surveys for railroads, for the tile or open ditch drainage, for water supply, etc.

1. Construction.—"In its simplest form, the plane table consists of a drawing board fixed on a rod, on which lines are drawn to represent the direction of any object as indicated by a ruler placed so as to point to the object. Any other parts are mere additions to render the operations more convenient and precise." The ruler is usually a scale of equal parts with a feather edge; at each end it carries a sight. The ruler and sight together are called the alidade. Fig. 2.

A very good plane table can be made very cheaply, if the engineer has a transit or leveling instrument, in which the upper part is detachable from the leveling screws. This requires a good drawing board, say 18 by 20 inches, to the under side of which is screwed a casting of iron or brass similar to Fig. 1. The conical portion, *a*, should be made to fit the socket from which the axis of the level or transit is taken; the cylindrical portion, *cc* should fit in the lower clamps.

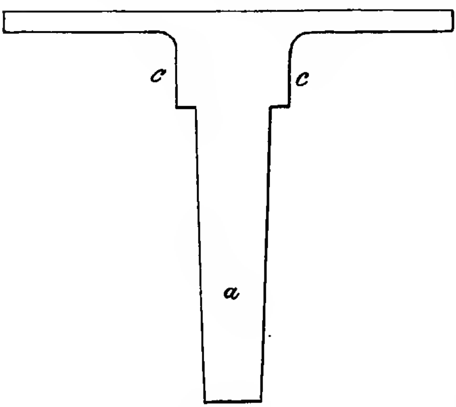


Fig. 1

2. In the best form of the plane table, such as are used on the elaborate topographical surveys conducted by the general govern-

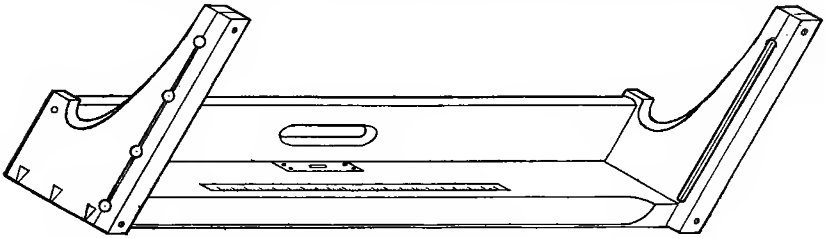


Fig. 2. The Alidade.

ment, a telescope with vertical arc and stadia hairs is substituted for the sights. These modifications add very considerably to the range and precision of the instrument, but it is too elaborate and expensive for many of the uses suggested above. A plane table of this form is virtually a transit in which the horizontal circle is replaced by the drawing board, the lines being drawn upon the paper instead of being read from the graduated limb. The principal advantages of this form of plane table may be obtained by using a transit with stadia hairs in connection with a drawing board and protractor. The form of the instrument used by the U. S. Coast and Geodetic Survey is illustrated and fully described, and the method of adjusting and using it explained in the reports of the above survey for 1865 and 1880. The article is also published separately, and is for sale by book dealers under the title, *The Plane Table and its uses in Topographical Surveying.*

3. Adjustments.—The adjustments are few and easily made, and having once been made are not likely to get out. They are; 1, the feather-edge of the ruler should be a straight line; 2, the sights should be perpendicular to the top of the board; 3, the table should be horizontal when the level bubble is in the middle of its race. The third is the rigorous requirement, but as the simpler form of plane table can be sufficiently accurate without any level, it is almost unnecessary to describe a method for testing this condition. However, to make the test, place the alidade on the table, and tip the table until the bubble is in the middle; reverse the alidade end for end; if the bubble is not in the middle the above condition is not satisfied. Even if the above condition is not satis-

fied, it may be adjusted so as to do perfectly accurate work. To adjust it, mark a point half way between its first and second position; when the bubble stands at this point the top of the board is horizontal.

4. Field Work.—Since there is such variety in the kind of work that may be done with the plane table, and in the method of doing it, it will not be possible to give any explanation of the method of doing field work in a short article. It is enough to say that the instrument is very flexible in its adaptation to different kinds of work, one valuable feature being that the work can generally be checked at every step. Gillespie's Land Surveying gives a few hints on the determination of areas with the plane table, and other methods will readily suggest themselves after a little practice.

5. Limits of Precision.—Since there is such variety in the kind of work, in the method of doing it, and in the conditions under which it is done, it is scarcely possible to state definitely the degree of precision to be obtained with the plane table.

The ordinary class work of the author's students in measuring the three angles of a triangle and four angles around a point, using a wooden alidade with slit and string for sights, gives an average error, for sighting, marking, and scaling off, of 1 minute and 44 seconds *per angle*; this is equivalent to a probable error of 88 seconds per angle. Under fair conditions the maximum error of an angle ought not to exceed one or two minutes. An angle can be measured by repetitions with a plane table with surprising accuracy.

A *plat* taken by progression or radix progression, to a scale of two inches to a chain, should close with a maximum error of about ¼ of a link per chain; with a little care this error may be much reduced and is often inappreciable. *Areas* should be found by the plane table with an average error of one in 5,000 or 6,000.

An engineer will find an instrument made as above very useful in many ways and worth many times all it costs.

## II.

### THE TELESCOPE.

1. A telescope consists, optically, of certain lenses which assist the eye in seeing distant objects; and, mechanically, of certain

parts which facilitate the use of the optical parts. The mechanical parts can best be discussed in connection with the instrument to which the telescope is applied, and in the present chapter only the optical parts will be considered.

No attempt at an elaborate discussion of the theory of the optical workings of the telescope will be made, but attention will be confined to such points as are needed by the engineer for the intelligent use of his instruments.

2. Construction.—There are two forms of the simple refracting telescope, usually known as the Galilean, and the astronomical. The last term is not a happy one, and *measuring* is suggested as being more appropriate, as will appear farther on.

The Galilean telescope consists of a double convex lens, called the object-glass, or simply objective, placed next to the object, and a double concave lens, called the eye-piece or ocular, placed near the eye. This form shows the object erect; an opera-glass is a good example.

The chief purpose of the objective is to increase the amount of light which reaches the eye from the object viewed. The sole object of the eye-piece is to magnify the thing looked at. The above form of telescope then assists the eye by its magnifying and light gathering power; but such a telescope would be useless for making precise measurements, since there is no means of indicating the exact point at which the telescope is sighted. It would not be inappropriate to call it a *seeing telescope.* The first telescope was of this form and took its name from the inventor, Galileo.

The *measuring* telescope consists of three essential parts; 1, a convex objective, which collects the rays of light and forms a bright inverted image of the object; 2, a convex eye-piece which is essentially a microscope, for viewing the image formed by the objective; and 3, two fine wires or spider threads, placed in the plane of the image, the intersection of which indicates the precise point sighted at. The objective collects the light, the eye-piece magnifies, and the cross-hairs indicate the point at which the telescope is directed. If such a telescope neither magnified nor increased the illumination, it would still be of great advantage in making measurements. This form shows the object inverted.

Both of the above skeleton forms of telescope are very imperfect. The methods of improving the optical qualities of these elementary forms will now be considered.

3. Objective.—A single lens used as an objective has the following defects. 1. The rays of light, which traverse a spherical lens near the edge, are refracted to a point nearer the lens than the rays which pass through the central portion; consequently the image is blurred. This deviation of the rays from the focus is called spherical aberration. 2. Rays of white light, after being separated by a single lens, are resolved into the colors of the prismatic spectrum; consequently the image will be disfigured by colored light. This deviation of the different colored rays is called chromatic aberration.

The objective of a telescope is rendered almost free from these defects by substituting for the single lens, a compound one, composed of a double-convex crown glass, and a concavo-convex flint glass lens; the two components have different refractive and dispersive powers, and by giving the four spherical surfaces proper curvatures, these defects are nearly eliminated.

Eye-Piece. A single lens used as an eye-piece possesses the same defects—spherical and chromatic aberration—as when used as an objective, but in a less degree. An eye-piece as a single lens has also the following defects; 1. The image of a flat object formed by a lens does not lie in a plane, but is concave towards the lens. This deviation of the image from a plane is termed aberration of sphericity, which is wholly separate and distinct from spherical aberration. The objective possesses this defect, but in so slight a degree as to be inappreciable; but in an eye-piece of a single lens, it is very serious. 2. A telescope with a single lens for an eye-piece has a limited field of view.

In both forms of telescope, the chromatic and spherical aberration, and the aberration of sphericity are nearly eliminated by substituting two plano-convex lenses for the single lens, which also increase the field of view.

Huyghen or Negative Eye-Piece.—This is a modification of the concave eye-piece of the Galilean telescope. It consists of two plano-convex lenses with their convex sides turned towards the objective. The relations of the eye lenses to each other, and to the objective, are shown in Fig. 3.

*P* is the object; *O*, the objective; *a* and *b* constitute the eye-piece which is placed between the objective and its principal faces; *F a* is known as the field lens, and *b* as the eye lens. The large

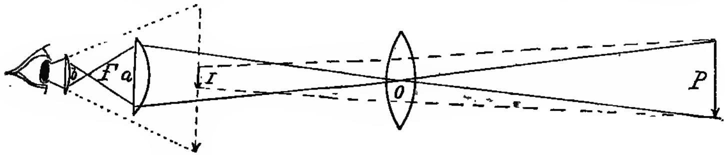



Fig. 3.

arrow at *I* represents the object as it appears through the telescope, the small one representing it as it would appear without the instrument. The angle which the large arrow at *I* subtends at the eye divided by the angle which the object subtends at the eye is equal to the magnifying power; that is, the magnifying power is equal to the ratio of the larger arrow at *I* to the small one.

Ramsden or Positive Eye-Piece.—This is a modification of the convex eye-piece of the measuring telescope. It consists of two plano-convex lenses with their flat sides towards each other. The relations of the eye lenses to the objective are shown in Fig. 4.

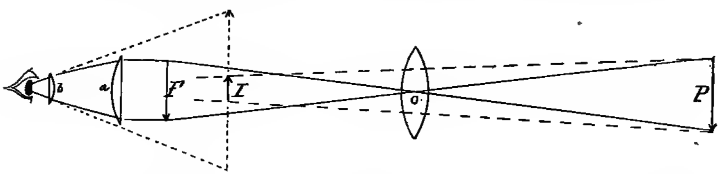



Fig. 4.

The nomenclature is as before. Notice that the eye lenses are farther from the objective than its principal focus. The objective forms an image at *F* which is viewed with the eye-piece. The magnifying power is the ratio of the two arrows at *I*, as before.

Notice that the essential difference between the positive and negative eye-piece is that only with the former is a real image of the object formed, and hence it is the only eye-piece with which cross-hairs can be used. Spider lines are sometimes placed in negative eye-pieces, for example in sextants, simply to indicate the middle of the field of view; they are but indirectly concerned in the accuracy of the observations. The negative eye-piece is better for simply seeing, while the positive is absolutely necessary for making precise measurements; the latter is always used in transits, levels, etc. Modifications of Ramsden's eye-piece, are frequently used which consist in the substitution of compound lenses instead of the single lenses shown in Fig.

4. Erecting Eye-Piece.—The erecting or terrestial eye-piece, in its most elementary form, consists of a convex lens placed between the eye and eye-piece of the measuring telescope, which inverts the image formed by the objective and shows the object erect. In its common form, it consists of a pair of plano-convex lenses, instead of the single lens as above; this pair is called the erecting piece.

The erecting eye-piece is inferior to the Ramsden or inverting eye-piece, owing to the loss of light occasioned by the two extra lenses. The inconvenience of the inversion of the object is easily overcome with a little practice. Furthermore, other things being equal, the telescope is shorter with inverting eye-piece; this is quite an important advantage in the transit. Most of the American engineering instruments, have an erecting eye-piece; but it would be a great improvement, if all were provided with the inverting eye-piece. It is said that the inverting eye-piece is more common in Europe than America.

Telescope Slide.—To assist in focusing the objective for different distances, the object glass is fastened in a tube which slides, with a rack and pinion, into the end of the main tube of the telescope. In instruments, provided with an inverting eye-piece, the objective is fixed and the cross hairs and eye-piece move together, to and from the objective. It is immaterial which form is used; the principle is the same in both, it being only important that the slide shall be straight and fit snugly.

To facilitate the focusing of the eye-piece upon the cross hairs, the ocular is provided with a similar slide. In some instruments the ocular is moved by a rack and pinion; but this is unnecessary and even worse than useless, for, having once adjusted the ocular for distinct vision of the cross hairs, it needs no change except for different observers, and it is better that it should not be easily moved. If no rack and pinion is provided, the ocular is moved in and out by hand with a screw-like motion.

CROSS HAIRS.—These are usually two spider threads, one vertical and the other horizontal, fastened to a ring which is adjusted in the tube by three or four screws. Lines ruled or etched upon a piece of thin glass, are sometimes used. The cross hairs are sometimes made of very fine platinum wire; can be drawn to the required fineness only by being previously surrounded by silver, which is removed by acid after the wire is drawn. Both spider threads and platinum wires have their advantages or disadvantages and as a consequence their advocates and opponents.

For the platinum wires, it is claimed that they are best because they are opaque; this is a desirable property when the cross hairs must be illuminated as in astronomical and mine surveying. Spider lines, particularly dark colored ones, can be illuminated pretty well. It is claimed also that wires are unaffected by the humidity of the atmosphere, and hence the line of collimation is not liable to change from this cause, an advantage which does not exist if the spider threads are properly stretched. Spider threads, on account of their fineness and cheapness, and the facility with which they are applied, will continue in the future, as in the past, to be used almost universally for cross hairs in engineering instruments.

STRETCHING THE CROSS HAIRS.—It does not require much time or skill to replace the cross hairs, the belief of many to the contrary notwithstanding. The ring which carries the cross hairs can be taken out, (having first taken out the eye-piece) by removing two opposite screws, and inserting a soft wooden stick of suitable size into one of the holes thus left open in the ring, which is turned sideways for that purpose, and then removing the other screws. Two scratches, at right angles to each other, will be found upon the face of the ring, into which the hairs are to be fastened.

The best spider lines are those of which the spider makes its nest. These nests are yellowish-brown balls which may be found hanging on the shrubs, etc., in the late fall or early winter. When found, the nest should be torn open and the eggs thrown out; if this is not done, the young spiders when hatched will eat the threads. The fibers next to the eggs are to be preferred on account of their darker color.

Draw out a single fiber and attach each end of it to as heavy a tack, brad, etc., as experiment shows it will support. Dampen the thread by breathing upon it, by holding it in a current of steam, or better, by dipping it in hot water. Support the ring in such a way that, when the thread is laid across it, the weights may hang freely down the sides, thus stretching the thread. The thread may be moved easily with a pin, and when in the proper position it can be fastened with wax, gum, etc., shellac varnish being the best for this purpose. The main point is to stretch it thoroughly and fasten it tightly. If the work is well done, the threads will remain straight, when the reticule is placed in a current of steam or even in hot water.

12. Testing the Telescope.—In buying an instrument it is very desirable to test the optical qualities of the telescope ; and the following directions are sufficient for such examination.

Chromatic Aberration.—To test for chromatic aberration, focus the telescope upon some bright object, either a celestial body or a white disk, and then move the object-glass slowly in and out. If, in the first instance, a light yellow ring is seen at the edge of the object, and in the second a ring of purple light, the object glass may be considered perfect; this proves that the most intense colors of the prismatic spectrum, orange and blue, are corrected.

13. Spherical Aberration.—To test this, cover the object-glass with a ring of black paper, so as to reduce the aperture about one-half; focus on some small and well defined object for distinct vision. Remove the ring of the paper and cover that part of the objective previously left open; then notice how much the object-glass must be moved in or out for distance vision. The amount of shift measures the spherical aberration of the objective; very little if any motion should be required to obtain a distinct view. Another test is to focus sharply upon some object, where the least motion of the objective should render the object indistinct. This last is not so good a test as the former, for the eye will change focus slightly to accommodate itself to the change of distance.

14. Flatness of Field.—The flatness of the field depends mainly upon the aberration of sphericity of the eye-piece. To test a telescope in this respect, draw a heavy lined square, six or eight inches on a side, on a sheet of white paper and fasten the paper to a flat board. Place this object at such a distance that the square shall nearly fill the field. Focus the telescope on it; then, if the sides appear perfectly straight, the telescope is perfect in respect to flatness of field.

A telescope which distorts the image perceptibly causes no error in common use, but is decidedly objectionable for stadia measurements, where two points of the field are used at the same time.

15. Size of the Field.—By the field of view is meant all those points which are visible at the same time through the telescope. The field of view depends upon distance between the objective and the image, and upon the diameter of the lenses of the eye-piece; but is independent of the size of the objective. Of course the wider the field of view the better, since width of field facilitates rapid working. The greater the magnifying power the less the field of view.

To determine the angular width of field, sight upon a rod, house, etc., and mark two opposite sides of the field; the distance between these points multiplied by 57.3 and divided by the distance from the rod to the objective is the angular width of field in degrees. Since the field varies with the distance, the latter should be stated.

16. Aperture of Objective.—By the aperture of the objective is meant the effective diameter of the object-glass, that is, that part of the objective which transmits light which finally reaches the eye. Usually, diaphragms are placed in the tube of the telescope, with a view to improve the optical workings of the lines; if the diaphragm is placed near the object-glass, it will cut off those rays which pass through the objective near the edge, thus improving the definition, without diminishing the field, but with a loss of illumination. If the diaphragm is placed in or near the eye-piece, it may diminish both illumination and field of view.

To find the real aperture, direct the telescope towards the sky, place a pointer in contact with the objective so that the pointer may be seen in the small illuminated circle which will be noticed at the small opening of the eye end when the head is drawn back a short distance from the telescope; move the pointer over the face of the object-glass until the point just appears, and measure the distance from the pointer to the edge of the object-glass. The real or clear aperture of the objective is equal to the diameter of the object-glass minus twice the above distance.

17. Illumination.—The brightness of an object as seen through a telescope depends upon: 1, the illumination of the object; 2, the relative sizes of pupils of the eye and the objective;

3, the polish and transparency of the lenses; and, 4, the magnifying power. In all telescopes light is lost by reflection from the surface of the lenses, and by imperfect transparency. The object-glass transmits a certain amount of light, which the eye-piece distributes over a larger or smaller area, according to the magnifying power of the telescope; therefore, the brightness of the image varies inversely as the square of the magnifying power. Since the brightness of view is an indispensable requisite of a good telescope, the magnification should never be excessively large, as it frequently happens that the telescope is used in viewing objects only faintly illuminated.

Since there can be no unit by which to measure the illuminating power of a telescope, we can only find the relative illumination. If two telescopes are to be compared as to light, they should stand side by side and be looked through at the same time, so as to be under the same atmospheric conditions. If, as night approaches, the two telescopes be placed side by side, and the same object be viewed through each, the one through which the object is longest visible has the best illumination.

Definition.—The definition of a telescope depends upon the accuracy of the curvature of the surfaces of the several lenses, and upon the coincidence of the axes of the component lenses of the objective and ocular. The reader should distinguish between illumination and definition. The lack of the former causes the image to be faint, the lack of the latter causes the image to be indefinite.

To test a telescope for definition, focus upon some small, well defined object; small, clear print is the best. If the print appears clear and well defined and fully as legible at 40 or 50 feet as if viewed with the naked eye at 6 or 8 inches (the best distance for distinct vision), the surfaces of the lenses are correct and well finished. Indistinctness may be caused by spherical aberration; therefore that should be tested for before definition.

Magnification. The power of a telescope, or degree of magnification, depends upon the relative focal length of the objective and eye-piece. Mathematically any power can be given to any telescope; but, in practice, it is limited by the effects of loss of light, size of field and imperfection of the lenses. For rapid work, the exact focusing necessary with high powers is a drawback, since a small change in distance requires a corresponding change in focus. The magnifying power varies slightly with the distance to the object; but, fortunately, the exact magnifying power of the telescopes on engineering instruments is not required. Several methods of measuring the magnifying power are given in Chauvenet's Practical Astronomy, Vol. I. Articles 7-13, of which the two following are the simplest.

1*st Method*. "Direct the telescope in daytime towards the open sky; near the eye-piece and a little beyond it, a small illuminated circle will be seen, which is nothing more than the image of the objective opening of the telescope. Let the diameter of this circle be measured by a very minutely divided scale of equal parts; then the magnifying power is equal to the quotient arising from dividing the diameter of aperture of the object glass by the diameter of this illuminated circle." The chief difficulty in this method lies in the exact measurement of the diameter of the small illuminated circle.

2*nd Method*. "Let a staff which is very boldly divided into equal parts of heavy lines, be placed vertically at any convenient distance from the telescope, for example, 150 feet. While one eye is directed towards the staff through the telescope, the other may observe the staff by looking along the outside of the tube. One division of the staff will be seen by the eye at the eye-piece to be magnified, so as to cover a number of divisions of the staff, and this number, which is the magnifying power required may be observed by the other eye looking along the tube." A little difficulty may be experienced on the first trial of this method, but with a few trials it becomes very easy.

PARALLAX.—This is an apparent movement of the cross-hairs in reference to the object sighted at, caused by a real movement of the eye of the observer. It shows that the image and cross-hairs are not in the same plane.

To make this adjustment, direct the telescope toward the sky, or throw it out of focus so that no object can be distinguished in the field, then move the ocular in or out until the cross-hairs can be seen very distinctly. When the cross-hairs are properly focused little specks of dust will be seen on them. Next direct the telescope to the object, keeping the attention fixed upon the cross-hairs, so that the eye shall not change focus to accommodate itself to the position of the object; move the objective in or out until the object appears very sharply defined; then move the eye back and forth sidewise and note whether the cross-hairs and object alter their relative position. If the cross-hairs appear to move with the eye, they are farther from the eye than the image, and therefore the objective should be moved nearer the object; for remember, first, that the farther object appears to move with the eye, and second that the farther the object from the objective, the nearer is the image. In practice this is more easily done by trial than described. When properly focused there should be absolutely no movement of the cross-hairs with reference to the object. Of course a telescope should be accurately adjusted for parallax before it is used in making precise measurements.

The writer has incontestable evidence for believing that many of the errors of instrument work are due to parallax; this source of error is more serious as the work becomes more accurate. The difficulty is usually in not adjusting the hairs for distinct visions for the normal or natural focus of the eye. The eye perceives things best at a distance of 6 or 8 inches, and this is known as the normal focus. When using a telescope the eye should be at the normal focus, the vision then being better and the fatigue less. The eye is continually changing focus to accommodate itself to the different distances of the objects viewed, hence care should be taken that the cross-hair focused for distant visions be at normal focus, which is secured by following the directions as above, particularly if the eye be closed for a moment before pronouncing the adjustment correct. Having secured the proper focus, it need not be changed except for the change of focus of the eye with advancing age and for different observers; the rack and pinion for focusing the cross-hairs is worse than useless.

Care. If the objective becomes dusty, brush it off with a fine camel hair brush, or rub it off with a piece of soft, clean chamois skin or a piece of old linen or silk, taking care to use a clean spot for each rub. Unnecessary rubbing of the lenses should be avoided, since it will destroy the fine polish upon which depends the sharpness and brilliancy of the image. Dust upon the glasses is not as objectionable as a thin, almost imperceptible film of grease; therefore, the lenses should never be touched with the fingers. When the lenses become very dirty, wash them with alcohol.

In removing the cell containing the objective, care should be taken to screw it back exactly to its former position, else the adjustment of the line of sight is thereby destroyed. The component lenses of the objective should never be taken apart, nor removed from the cell containing them, since they may not be returned to their former position, thus disturbing the adjustments. Dust should carefully be kept from the inside of the telescope tube, as it will get on the lenses and cross-hairs. When not in use the eye-piece and object-glass should be covered by their caps.

The following definition and explanations will be found useful farther along.

THE OPTICAL CENTER of a lens is, " A point so situated that any ray of light passing through it will undergo equal and opposite reflection on entering and leaving the lens. It will therefore, be found where a line joining the extremities of two parallel radii of the opposite surfaces, intersects the optical axis of the lens." For a double-convex lens it is always within the surface of the lens. For a planor-convex, or a plano-concave, the optical center will be at the inte section of the axis with the curved surface.

In lenses of long focal length, such as are used for the objectives of telescopes, the deviation caused by the thickness of the lens may be neglected, as the thickness is very small compared with the distance to the object; therefore, it is assumed that the rays which pass through the optical center are not deviated at all. This leads to the simple definition that a lens is a mathematical point which allows a great deal of light to go through it; the better the lens the more nearly is this condition realized. This "*point*" is the optical center, as defined ab ove.

## VERNIERS.

### III

I. PRINCIPLES.—A vernier is a short scale moveable by the side of a longer scale, by which subdivisions of the longer scale may be measured. The scale to be subdivided is called a limb. A division of the vernier is a little shorter or longer than a division of the main scale or limb. The small difference is the space which is measured by the vernier.

A vernier may be constructed by taking a length equal to any number of parts of the limb, and dividing it into a number of equal parts, one more or one less than the number into which the same length on the limb is divided.

For example, the limb shown in Fig. 5, is a scale of inches divided into tenths, the vernier beside it is capable of measuring to hundredths. Notice that the spaces of the vernier are equal to nine of the limb. Therefore each space of the vernier is equal to 0.1 of $0.9 = 0.09$ of an inch; it is 0.01 of an inch shorter than a space of the main scale.

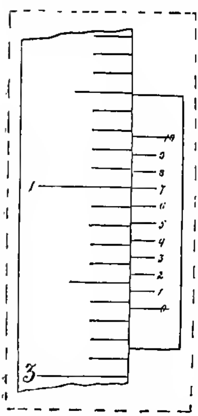


Fig. 5.

The first mark of the vernier falls short of a mark on the limb by 0.01 of an inch; the second falls short by 0.02 of an inch, and so on. Therefore if the vernier be moved slowly forward the successive coincidence of a line on the vernier with one on the limb will indicate successive advances of the vernier, each equal to 0.01 of an inch. If the lines of the vernier are numbered as in Fig. 5, the number on the vernier of the line which coincides will indicate the amount that the zero of the vernier has passed a division of the limb.

2. Direct and Retrograde Verniers.—In the above illustration the spaces on the vernier are shorter than those on the limb; the supposed motion was in the direction of the graduation of the limb, and the successive lines of the vernier came into coincidence also in the direction of the graduation; the numbering on the limb and also on the vernier increased in the same direction. Therefore Fig. 5, is a *direct* vernier.

If the spaces on the vernier are larger than those of the limb and the vernier is moved in the direction of the graduation of the limb, the successive coincidence will occurin the direction opposite to the motion andalso opposite to the direction of the graduation on the limb; therefore,the lines on the vernier should be numbered in an opposite direction to those on the limb. Such an arrangement is a *retrograde* vernier; see Fig. 7.

The direct vernier is much more common; it is shorter and more convenient to read. A retrograde vernier is used when the lines of a direct vernier would be inconveniently close together.

3. Least Count.—The least count of a vernier is the difference

in length between a space on the limb and one on the vernier. To find the least count of a vernier, *i. e.* to determine how small a distance it can measure, let $l =$ the length of a division on the limb; $v =$ a division on the vernier; and $n =$ the total number of spaces on the vernier; then by the principle of the vernier, $n\,l = n\,v \pm l$, solving which gives, $l - v = \frac{l}{n}$, the least count.

For example in Fig, 5, $l = 0.1$, and $n = 10$; hence the least count equals $\frac{l}{n} = \frac{0.1}{10} = 0.01$

The above formula expresses a very important relation; it is the key to reading all verniers. Notice that the least count of the vernier is equal to the smallest division on the limb divided by the number of spaces on the vernier. In practice, it is not necessary to count $n$; it is indicated by the numbering on the vernier itself. For example, if the limb is divided to half degrees and there are thirty spaces on the vernier, which would be indicated by the end line being numbered thirty, the vernier reads to one-thirtieth of a half degree or to minutes.

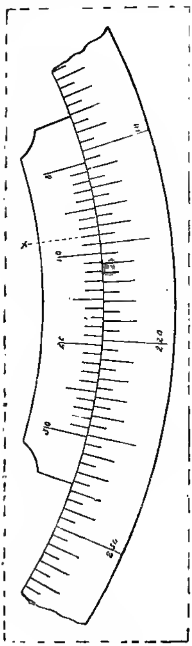
Notice that the above relation is true for both direct and retrograde verniers.

4. To Read a Vernier.—Look at the zero line of the vernier; if it coincides with a division of the limb, the number of that line on the limb is the correct reading, the vernier divisions not being required. But if, as usually happens, the zero of the vernier comes between two divisions of the scale, note the nearest division on the limb next less, and then look along the vernier till a line is found which exactly coincides or forms a straight line with some line on the limb. The number of this line on the vernier is the distance between the zero of the vernier and the next lower division of the limb, and must be added to the reading taken from the limb.

A number of examples will now be given to further illustrate these general principles. The student should draw the limb and scale on separate slips of paper of card board, and move one beside the other until he can read them in any position.

5. The vernier shown in Fig. 6, is the one used on the New York levelling rod.

The main scale is divided to feet, tenths and one hundredths; ten divisions of the vernier are equal to nine of the limb; therefore, the vernier reads to tenths of one hundreth, or to one thousandths. The vernier as drawn reads 3 feet, 0 tenths, three hundreths, and seven thousands, or 3,037 feet.



'Fig. 7.

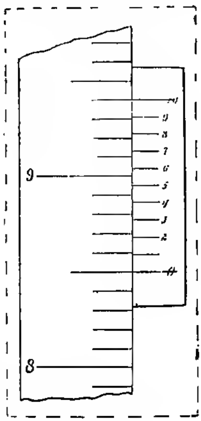



Fig. 6.

6. Fig. 7, shows part of a circle graduated to degrees and half degrees; the vernier has thirty parts, therefore it reads to single minutes. The reading is 210° 30′ + 8′ = 210° 38′. This is a very common vernier for engineering instruments.

7. The graduation of transits usually has two rows of numbers, one increasing to the right and the other to the left; in this case there are two verniers, one for each series of numbers. Such an arrangement is shown in Fig. 8. Each vernier is like the one described in the preceding paragraph, and is read in the same way. The reading of the left hand vernier is 181° 30′ + 12′ = 181° 42′ with the outer numbers, and 1° 42′ with the inner. The right hand vernier reads 167° 12′.

There is sometimes a doubt as to which vernier should be read. First notice whether the vernier divisions are larger or smaller than those on the limb. If the spaces on the vernier are the smaller, it is a *direct* vernier, and that vernier should be read the numbers of which increase in the *same* direction as those on the limb. If the spaces on the vernier are the larger, it is a *retrograde* vernier, and that vernier should be read the numbers of which increase in the *opposite* direction to those on the limb.

Or the vernier to be read can always be determined as follows; move the vernier, until say, the ten mark of one vernier and the twenty of the other coincides each with a line on the limb; look at the zero, estimate the reading, and then read the vernier that agrees most nearly with the estimated reading.

In Fig. 8, the numbers on the vernier are inclined in the same direction as the numbers on the limb to which the vernier belongs. Instruments are not made in this way, but such an innovation would be a great improvement.

8. Fig. 9, is another form of double vernier; it is often applied to the compass, to be used in setting the declination. Notice that this form is only half as long as the double vernier of Fig. 8.

It is used where there is not space for the longer one and is called a double folded vernier. The lower numbers on one side of the zero and the upper ones on the other side constitute a vernier. The proper vernier to be read, in any given case, can be determined by either of the rules of the preceding article. The vernier as drawn, reads $2° 30' + 12 = 2° 42'$ to the right. Notice that the reading of the vernier is $12'$ not $17'$.

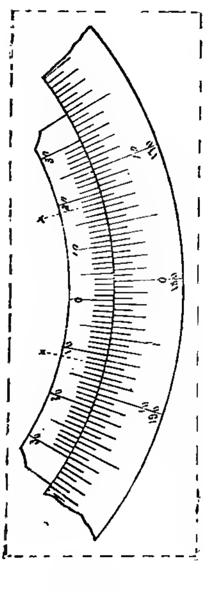


Fig. 8.

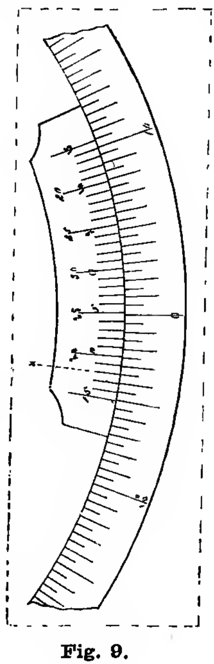


Fig. 9.

9. Practical Hints.—To determine exactly which lines coincides best, notice the next line on either side and see whether they fall short equal amounts. When several lines on the vernier appear to coincide equally with

several lines of the limb, take the middle line. When no line coincides, but one line on the vernier is on one side of a line on the limb, and the next line on the vernier is as far on the other side of it, the true reading is midway between the readings indicated by these two lines. If the graduation is very accurate and the lines fine, it is possible, by this method, to estimate the reading to half, or even to thirds, of the least count.

It frequently happens that the instrument is to be used to lay off a number of equal angles, as for example, 1°; the vernier is then to be set each time at some particular mark. If it is a double vernier, set the zero line to coincide each time, and notice whether the lines next on either side fall short an equal amount; if it is a single vernier, set, say, the fifteen line to coincide, and note the agreement on both sides. This process is considerably more accurate than setting the end line of the vernier to coincide each time.

The most frequent error in reading a vernier is to omit part of the reading of the limb; for example, in Fig. 7, forgetting to record the half degree from the limb.

## IV.

### THE TRANSIT.

1. This is the name usually given to instruments in common use among American engineers for measuring horizontal and vertical angles. It is sometimes, but incorrectly, called a theodolite. The theodolite is the name given by British engineers to their favorite portable instrument, which is capable of performing the same work as the transit. The essential difference between the transit and the theodolite is that in the former the telescope can transit or turn completely over, while in the latter it cannot. The telescope of the theodolite can be reversed only by lifting it out of its supports and replacing it end for end, which is a very imperfect substitute for the revolution of the telescope of the transit. The transit is sometimes called an engineer's transit, or railroad transit, to distinguish it from an astronomical transit. The transit was invented and first made by a Philadelphia firm in 1831, previous to which time the English theodolite and the magnetic compass, sometimes provided with a full circle graduation by which angles could be read independently of the needle, were the common angle instruments.

The great value of the transit as an instrument of precision is due to the telescope, by which great precision in sighting is obtained, and to the graduated circle with its vernier, by which angles can be read with ease and accuracy. All other parts are to facilitate the use of these two.

2. THE TRIPOD.—The manner of connecting the leg with the head needs attention. The leg should not be placed between two lugs or ears which are fastened to the plate, for in case the leg wears or shrinks there is no adequate means of making it fit. Drawing the lugs together by a screw, bends the plate, and even then only partially remedies the evil. An excellent form is that in which the leg is made of two pieces that bear upon opposite sides of a lug cast upon the plate. Sometimes the leg is in one piece and slotted at the top, which is also very good. Another good form is that in which the leg is not open at the top but bears upon only one side of the lug. In the three forms last mentioned, any looseness is taken up by a thumb-nut. Sometimes the leg is inserted in a metal cap which is then fastened to the tripod head; this is better than putting the leg directly between two metal ears, but it is not as good as any of the others.

To set the tripod, notice 1st, that to allow the position of the plumb-line, the legs must be swung on their pivots but not sidewise; and 2nd, that to put the plate level, the legs must be swung sidewise but not on their pivots. These two points, though small in themselves, are worth remembering as a means of saving time and labor and also of preventing unnecessary wear and strain on the leveling screws owing to faulty construction of the leveling appliances. With many instruments the last point is a very important one.

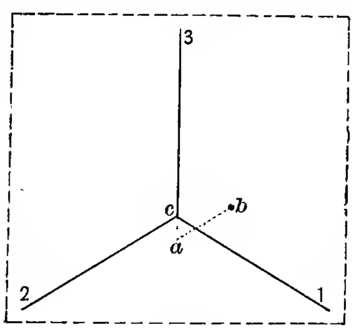


Fig. 10.

Attention to the following principles also will save much time and hard labor in setting the tripod. Let the lines radiating from $c$, Fig. 10, represent lines joining the feet of the tripod legs with the point over which the plumb bob is to be placed; *b* is the position of the plumb bob. It is desired to move the plumb bob from *b* to *c*. Press the leg 1 into the ground until the bob swings into the line 3 *c* prolonged; then force 3 in until it swings to *c*. In general, press one of the legs into the ground until the plumb bob swings to the opposite side of the point from one of the other legs; then press that leg in until the plumb bob arrives at the center.

The tripod should be set firmly, but a great deal of time and effort is frequently wasted in forcing the legs into the ground needlessly. Setting the tripod is an operation that must be repeated many times, and the beginner should learn to do it quickly and easily.

In case the plumb-bob is lost and only a rough piece of some heavy substance can be had, the instrument may still be plumbed down accurately by holding a second plumb line before the eye in such a position that the eye shall be in the same plane with the two lines; then, without moving the eye, mark a line under the instrument in the plane. Repeat the operation at 90 degrees from the first position.

The intersection of these two is the desired point. The writer heard of a railroad surveying party waiting ten hours while the transitman sent for a plumb bob, which was almost as bad as being "delayed two days by a hornet's nest."

3. Leveling Screws.—Some instruments are provided with three leveling or foot screws, and others with four. The instrument can be leveled more quickly with three than with four; besides three are more delicate, and are less liable to strain and damage the instruments. The best instruments never have but three screws.

A very serious defect in many instruments with four leveling screws, is that the lower ends of the screws are above the center of the ball and socket joint which fastens the upper part of the instrument to the tripod. When the one pair of screws is being used to level the plate, the upper part of the instrument must revolve about a line parallel to, but below a line joining the feet of the pair not in use; therefore the instrument cannot revolve without causing the screws not in use to bind, and it can turn only by causing the feet of the screws to slip on the lower plate. Besides the annoyance in leveling the instrument, this binding tends to bend the screws and warp the plates and the slipping defaces the instrument by cutting spherical holes in the lower plate. Placing the feet of the screws in small cups obviates the holes in the upper face of the lower plate but increases the objections in the other and more important respects.

The defect may be remedied by bringing the center of the ball and socket joint into the plane of the feet of the foot screws. As there is no ball and socket joint with three leveling screws, this defect can not exist in that form of construction. A spiral spring fastens the instrument to the tripod. Since nearly all instruments possess the above defect, it is very necessary that the instrument should be leveled approximately by manipulating the tripod legs as already described.

Whatever the number, there should be no looseness between the screw and the nut. This looseness is particularly objectionable in a leveling instrument or in a transit used to measure vertical angles. The best instruments have a split nut with a clamp-screw by which to adjust for wear.

Some instruments are provided with an arrangement for approximately leveling the instrument very quickly without manipulating the foot-screws. This device is especially suitable for leveling instruments; but by using a little care in setting the tripod, it can be dispensed with easily. All such additions are an advantage in the greater convenience, and a disadvantage in the increased weight, complexity and cost.

4. Graduation.—The lines of the graduation should be uniform, and as small as possible and still be legible. In the best instruments the graduation is upon solid silver.

The most common graduation for the horizontal circle of engineers' transits, is a circle about 6 inches in diameter divided to one-half degree with a vernier reading to minutes. The degrees are usually numbered in two rows, one like the compass and another from 0° to 360°. The field work is most simple and least liable to error if only the latter numbering is used. See remarks on the transit verniers.

The verniers on the horizontal circle are sometimes placed 90 from the line of sight, in which case the observer must change his place between sighting the telescope and reading the vernier; sometimes the verniers are placed immediately under the telescope, in which case the observer can read the vernier and sight the telescope without changing his position, although the telescope must be revolved before the vernier can be read. The latter position is preferable, especially in confined positions as in underground surveying, etc. An intermediate position would be still better. For convenience of reference, the verniers should have some distinguishing mark, say *A. B. C.* etc., upon their faces.

5. Centers or Vertical Axes.—Usually there are two concentric vertical axes, the verniers, telescope, etc., turning about the inner, and the graduation revolving about the outer one. As ordinarily used, the outer axis is useful only in enabling the observer to shift the graduation so as to begin each time at zero. The inner one is made more carefully than the outer. These two axes are often called "centers" and sometimes "compound centers." For work not requiring great accuracy, but demanding a light portable instrument, these axes are made quite short and are called "flat centers." The more accurate instruments have "long centers."

6. Tangent Screws.—With the unaided hand the telescope cannot easily be made to cover or bisect the exact point sighted at; to assist in thus directing the telescope, the instrument is provided with a clamp and tangent or slow motion screw.

Clamps are made in a variety of ways, but all consist essentially of a contrivance by which a piece may be connected with the axis or rim of the graduation by simply tightening a screw. The clamp is connected with the vernier plate or the tripod, as the case may be, by a screw which is always nearly tangent to the direction of motion. When the screw is turned, the two parts of the instrument rotate slowly with reference to each other. No description can give an adequate understanding of these parts; they must be seen and examined to be comprehended.

A perfect tangent screw should have a very smooth motion, free from lost motion or "back lash," a great durability and an even wear so that the screw will never have lost motion in one part and move hard in another. Lost motion is the common defect of tangent screws and also the source of great annoyance to the engineer, besides often being the cause of serious errors in the field. Several forms of tangent screws will now be described.

The oldest and most imperfect of all is that known as the English or stiff tangent screw, in which the screw works through a post in the clamp and against a collar in a post attached to the plate, both parts being free to turn about a vertical axis. The defects of this form of construction, are, 1, if the screw is not perfectly straight and true, it will bind during one part of the revolution; 2, the nut and collars should be the same height above the plate, and to allow for errors of workmanship, the hole in the nut is often made too large and then tightened until it fits, thus touching in only two points, and therefore soon wears loose; and 3, the parts are so arranged that, if the movements are adjusted so as to prevent looseness, the friction will be so great as to interfere with the freedom of the motion.

Another form has a long spiral spring between a collar on the screw and the nut, which takes up the back lash. On trial this has proved defective, since the spring must be made long enough and strong enough to act in every portion of the length of the screw, and the alternate opening and closing weakens it so that in a short time it fails to remove the back play.

Another form, which has the same objections, allows the screw to abut against the clamp, a spring keeping the clamp in contact with the screw. Even under the most favorable conditions, tangent movements of this class are objectionable, because the spring causes a strain between the parts which may change with changes of temperature and which is liable to cause motion or derange the levels.

One of the best forms of tangent screws is that commonly known as the Gambey tangent movement, first made by the celebrated instrument maker, Gambey, of Paris. As commonly made, the nut is a split sphere which is confined like any ball joint. Instead of the collar, as in the English form, another split spherical nut, with a thread of a different pitch, is used. This construction provides for all necessary motions and all probable imperfections of the screw. The only objection to this form is that, since the nut is short the screw will wear most near the middle, and when the nut is tightened to remove the lost motion from the small part of the screw, it will work too hard on portions less worn, therefore the screw can be turned only a few revolutions without having lost motion in one part or a great friction in another.

In the genuine Gambey tangent movement, the halves of the spherical nuts are confined between springs; consequently, it does not possess the above defect, and is all that could be desired.

The latest tangent movement is an improvement of the counterfeit Gambey. It consists of "a long spiral cylinder nut, from the interior of which two-thirds of the threads have been removed; into half the recess thus left in the nut is nicely fitted a cylinder follower, with the same length of screw thread as the nut; this follower is fitted with a key, which prevents it from turning in the recess, but allows motion in the direction of its length. A strong spiral spring is placed in the remaining half of the recess, between the fixed nut and the movable follower; this spring always has tension enough to force the follower and fixed thread in contrary directions, and thus remove any lost motion that may occur in the screw. It will be observed that with this construction the spring remains in a state of rest, instead of closing and opening, as has been the case in all other application of springs." The spiral spring takes up the wear of the screw, and the great length of the nut secures nearly uniform wear over the whole screw. These long nuts may be confined like any ball joint, or may be attached by gimbals, thereby allowing free motion even though the screw should not be straight.

Sometimes two opposing or butting screws are used to get rid of lost motion; but these are objectionable since two hands must be employed in using them. For several reasons they are not at all suitable for the upper motion, but on account of their steadiness are often used for the lower motion. If a stiff, flat spring were attached to the side of the lug against which the tangent screw bears, the setting could be finished with one screw. Such a spring is a very useful addition.

7. The magnifying power of the telescope and the least count of the vernier should be proportioned to each other, so that the least perceptible movement of the vernier will cause sufficient motion of the cross-hairs on the object to be easily noticed through

the telescope; and *vice versa*, the least noticeable motion of the cross-hairs on the object should cause a perceptible change of the vernier. Since the horizontal circle is usually (and properly) the more accurate, this condition applies more especially to the horizontal circle than to the vertical. A higher power, or a smaller count of the vernier, than required by the above conditions is detrimental, the former causes an unnecessary loss of light; and the latter a waste of time in reading the vernier. A similar relation should exist between the magnification and the level under the telescope, and also between the magnification and the plate levels. Many instruments are very faulty in some or all of these particulars.

8. An engineer should have a good instrument and then take delight in keeping it in good condition. It is not desirable to take the instrument apart unnecessarily, for even if the fittings are perfect, it requires considerable care to put them together properly. A little dust or dirt in a joint or bearing, or a screw left loose or tightened too hard, may damage the instrument and cause errors in its use.

There are a great many small screws about a transit and the general tendency is to overstrain them. This is especially true of the cross-hair screws; all straining of these screws, beyond that necessary to insure a firm seat is more apt to cause the instrument to lose than retain the adjustment. Over-straining the leveling screws bends the plates and wears the screws unnecessarily. If the leveling or tangent screws get to working hard, take them out and brush with soap and water; the nuts can be cleaned by screwing a thin piece of soft wood through them. The tangent screws are only intended to complete the settings; they should never be used except to give a slight movement.

The telescope slide and the center should be examined occasionally; if there is any fretting or cutting, take the piece out and burnish down the rough place with any smooth hard tool, as the back of the blade of a pocket-knife. Very little, if any, oil or grease should be used upon bearings exposed to the dust; probably powdered plumbago is best for exposed bearings, and rendered ox-marrow, or watch oil, for bearings not exposed.

8. Practical Hints.—The beginner should not neglect the preliminary matters of planting the tripod, bringing the plumb-bob

over the point, and leveling the instrument. There is great difference in the skill and rapidity with which different persons will set up the transit; generally there is an unnecessary waste of time and hard labor. A very neat way of doing it is as follows: tighten the screws in the upper end of the legs, until friction will just hold them whenever placed, open them until they make an angle of about 30° with the vertical; let down the plumb-bob, and set the instrument over the point, a gentle pressure on the legs of the tripod brings the plummet over the point, and a slight movement of the leveling screws brings the plate level.

Some engineers seem to think that the harder the tripod legs are focused into the ground, the tighter the leveling screws are, and the tighter the instrument is clamped, the more accurate the work, but the contrary is more nearly true. An instrument keeps it adjustments better and works more kindly, when handled delicately.

If the instrument is not firm, examine the tripod-head, and the iron shoes on the legs, to see that they are not loose; no instrument can stand firm with any looseness in these parts. The clamps and tangent screws should be examined to see that they are not loose. The instrument may slip on the lower plate, owing to the leveling screws not being tight enough.

If the observer has clearly in mind what he is trying to do, and thoroughly comprehends the effect of possible errors, in his instrument, he can save much time and trouble, thus leaving himself free to give his attention where it will do the most good. For exsample, a great deal of time is often wasted in getting the instrument precisely over the point; the case should vary inversely as the distance of the object sighted at; an inch may cause an error of three seconds if the object is a mile away, but an error of three minutes at 100 feet. The position of the instrument from the point, with reference to the object sighted at, affects the value of the resulting erro.r

It is a waste of time to level up accurately, each time, regardless of the kind of work to be done; if only the horizontal angles between points on the same level, are to be found the instrument can be leveled with sufficient accuracy by the eye alone; if only vertical angles between points in the same vertical are desired, only the level parallel to the telescope need be read. On the other hand, in measuring a horizontal angle between a high and a low point, the level perpendicular to the telescope should be very carefully attended to. So in all instrumental work, there are certain operations which should be carefully attended to while to attend equally carefully to others would be only a waste of effort.

10. Nothing need be said here concerning the adjustment of the instrument or its ordinary manipulation.

MEASURING ANGLES ACCURATELY.—It sometimes happens that an engineer desires an angle with the utmost accuracy. There are two methods of making the observations, when extreme accuracy is desired; they are *by series* and *by repetition*. One or the other of these methods is always used in measuring the principal angles of the geodetic triangulation. The principles involved are useful in less accurate work.

BY SERIES.—Sight upon the first station, and read both verniers; this eliminates eccentricity. Sight upon the next station to the right, and read as before; continue on around the horizon, reading upon each station, and close by reading upon the first station again; if the last reading is the same as the first, it proves that the instrument did not slip or get moved. Reverse in altitude and azimuith, move the lower motion a little to eliminate personal bias, an read upon the first station; proceed around the horizon towards the left, reading upon each station and closing upon the first. The reversal in azimuith and altitude eliminated eccentricity of line of sight, error of telescope slide, and inclination of horizontal axis. Reversing the direction around the horizon eliminated any twist of the tripod. Shifting the horizontal circle diminished the posibilities of accidental error of graduation. The above observations constitute one "set."

To secure greater accuracy by increasing the number of observations and also to eliminate periodic errors of graduation, shift the horizontal circle an aliquot part of the distance between verniers, and take another set. The amount that the circle should be shifted between sets is equal to the distance between verniers divided by the number of sets to be taken. The mean of the observed values is true angle.

By Repetition.—Sight upon the first station, read both verniers; with the upper motion, turn to the next station, and read as before, (this last reading is only for a check); with the lower motion turn back to the first station, the reading remaining unchanged; then unclamp above, and turn forward again to the second station. The angle will now have been measured a second time, but on a part of the circle adjoining that on which it was first measured, the second beginning where the first ended. This operation may be repeated any number of times; read the circle after the last sighted upon the second point; the difference of the first and last readings, divided by the number of repetitions gives the angles more precisely than a single observation would. Notice that the vernier need be read only at the begining and end, although the second reading, as above, is a valuable check in determining how many time 360° should be added to the last reading, in case the vernier has passed 0°. Next reverse in altitude and azimuth, and measure the angle as before, beginning however at the second station. This eliminates all error of adjustments and reduces the error of observation by increasing their number. Of course, the mean of the observed values is assumed to be the true angle.

Comparison of Methods.—Both methods seem be to about perfect, as far as the elimination of errors of adjustments, of graduation, and of observation is concerned. The method by series is preferred by most observers, for triangulation work; its peculiar advantages can be fully realized only with the precise instruments used in that kind of work. With ordinary engineering instruments, there is an obvious limit beyond which it is useless to multiply observations by this method.

The method by repetition was once a great favorite with the best engineers, especially the French, for triangulation work; but the improvements in the manufacture of angle instruments, has given preference for the most accurate work, to the method by series. However, the method by repetition is peculiarly suited to the precise measurement of angles with a coarsely-divided circle, as, for example, a common engineer's transit. The principle of this method is certainly very beautiful, but its accuracy is largely dependent upon the freedom with which the instrument turns on its centers, and upon the stability of the clamp and tangent screws. Under ordinary conditions, the limit of this method is reached after a few repetitions.

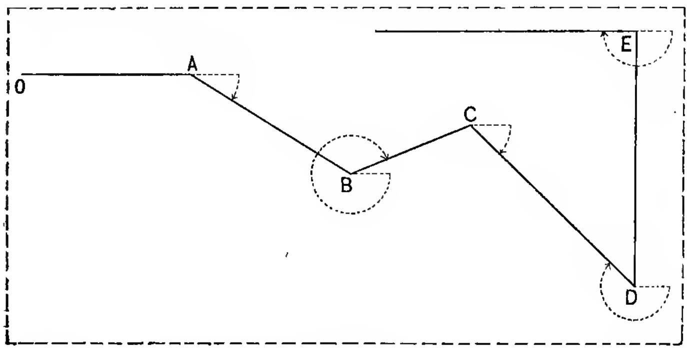
Transit Surveying.—Unfortunately, there is no generally accepted method of doing transit work, either for the field work or for recording the results. Three methods of more or less general use among engineers will be considered. The first, for want of a better name, we will call the *angle method*; the second can appropriately be called the *quadrant method*; and the third could appropriately be called the *full circle method*; but as this method of surveying has been called *traversing*, we will use that term and not attempt to introduce a new one.

1. Angle Method.—This method consists in measuring and recording the angle which each line makes with the preceding one. The angle measured may be the one included between the two lines, or the angle between the second line and the first produced. In the latter case, the telescope is sighted along the first course, the vernier read, and the telescope transited; the telescope now points in the direction of the first line produced, it is next turned to the second line, and the vernier read; the difference of the readings, *i.e.* the angle swept over, is equal to the angle between the first course produced and the second. This angle is sometimes called the angle of deflection.

2. Quadrant Method.—The distinguishing characteristic of this method is the manner of designating the observed horizontal angles with reference to the quadrant in which the enclosing lines belong. The angles are read and recorded as bearings, just as in compass surveying. The meridian through the first station is obtained by reading the needle, or by sighting upon some line whose bearing is known, or it may be assumed; the object of such surveys is generally not so much to get the *true* bearings, as to get the relative bearings accurately. The bearing of any succeeding line is found by measuring the angle which it makes with the preceding line produced, and adding it to or subtracting it from the bearing of the preceding line. This method is doubtless a relic of the time when the compass was the common angle instrument. The 0° to 90° numbering on the horizontal circle is especially useful in this system.

3. Traversing.—Traverse surveying, or running a traverse, or simply traversing, is conducting a survey in such a way that the readings of the plate will show the angles which each line of the survey makes with any chosen reference line. Although the essential principles of this method are used to a limited extent, there is

no agreement as to the details of the process. Even the term *traversing* is not always used as defined above. This method could appropriately be called surveying by the back azimuth, or surveying by the back angle, or surveying by carrying the azimuth.




Traversing.

To run a traverse, set the instrument up over the first station, *i.e.*, the end of the first course. For the present we will assume that the first side is to be the reference line, *i.e.*, the meridian of the survey. Set vernier *A* at 0°, clamp the upper motion, and turn the telescope upside down, with the lower motion, direct the line of sight to the other end of the first course; transit the telescope, loosen the upper motion; sight upon the next station, and read vernier *A*. Move to the next station, loosen the lower motion,

| Station. | Back-Sights | Fore-Sights | Dist. | Remarks. |
|---|---|---|---|---|
| *A* | 0° 00′ | 35° 52′ | | Angles read from ver. *A*. |
| *B* | 35° 52′ | 340° 31′ | | |
| *C* | 340° 31′ | 41° 08′ | | |
| *D* | 41° 08′ | 270° 00′ | | |
| *E* | 270° 00′ | 180° 00′ | | |

level up, invert the telescope, and glance at the reading to see that it has not changed; sight upon the last station; using the lower motion. Invert the telescope, loosen the upper movement, sight upon the next station, and read. Proceed in like manner for any number of lines. Each reading is the angle between its corresponding course and the first one. Below are the notes and plan of a traverse survey, on preceding page.

The instrument was first at *A*, the line 0*A* being regarded as the first course. The vernier was set at zero, and a back sight taken to 0′; it is recorded opposite *A*, in the back sight column. The telescope was then transited, the upper motion loosened, and the telescope directed to *B*; the reading of this line, 35° 52′, being recorded as in the table. After the instrument is removed to *B*, and the back sight taken up on *A*, the vernier is to be read to make sure that it has not been changed; the record of this reading is made in the back sight column. The writing of this down will be evidence that the reading of the vernier was checked; in actual work this will be found to be an important check against errors, as, for example, turning the wrong tangent screw or reading the wrong vernier. The angles marked in the diagram are the corresponding angles of the fore sight column in the table.

Instead of making the first course the reference line, as above, we may take any other line, as the meridian or reference line. In this case, the instrument is first set up at the point of beginning, the vernier set as zero, and the first back sight made in the direction of the meridian; the remainder of the field work and the record is as before.

It makes little or no difference whether the first back sight is made towards the north or south end of the meridian; but a note in the remarks column should show which way it was made.

This method of using the transit is specially convenient in railroad surveying.

Comparison of Methods.—In the time required for the field work, there is very little difference between the three methods; the second and third require less than the first. The quadrant method is simple and easy enough to understand; but, in field work, in the computations, and in the plotting, there will be found many drawbacks and many opportunities for errors, which do not exist in the full circle system. One of the chief objections to the quadrant method is that four different directions are designated by the same angle numerically. As the advantages of the full circle system are better understood, it will be more fully adopted; as yet it is only used to a limited extent.

SOURCES OF ERRORS.—The sources of the errors of transit work may be classified as follows: 1. Errors in setting up; 2. of sighting; 3. of manipulation; 4. of adjustment; 5. of reading.

1. The plates of the instrument may not be placed horizontal and the instrument may not be set up over the point about which the angle is desired or over the point previously sighted at; in anything like good work, these errors will be inappreciable.

2. ERRORS OF SIGHTING.—If a flag-pole is sighted at, it may not be vertical; therefore, sight as low upon it as possible. The intersection of the cross-hairs may not exactly cover the point; this is due to lack of care or to parallax in the instrument, and in either case, the remedy is obvious. It is not enough to bring some point of the vertical hair over the point, for the hair may not be truly vertical.

ERRORS OF MANIPULATION.—The wrong tangent screw may be turned; this is a fruitful source of error, and one difficult to discover and impossible to correct. If the tangent screw has back lash, the instrument must be handled so as to prevent it; error from this cause sometimes is produced by the instrument turning on the ball and socket joint owing to the foot screws not being screwed up tight enough. The upper part of the instrument should move so freely as not to twist the tripod.

ERRORS OF ADJUSTMENT.—The points to be considered are eccentricity, equality of standards, straightness of telescope slide, and adjustment of cross-hairs. The eccentricity is always small and easily eliminated by reading two verniers. The equality of the standards can effect only the horizontal angles between high and low points, and in ordinary practice it will be inappreciable.

The straightness of the slide and the adjustment of the cross-hairs are closely related; neither will produce error either in measuring horizontal angles between points equal distances from and equal distances above vertical angles between points or below the instrument, or in measuring equally distant from the instrument. An error of adjustment of the line of collimation produces an error of double amount in traversing, or in prolonging a line by back and foresights. If great accuracy is desired in either of these operations, the telescope should be reversed on each alternate back-sight and fore-sight. All errors of adjustment can be eliminated by reversing, making a second observation and taking the mean.

Errors of Reading.—Errors may be produced by reading the wrong vernier, the wrong end of a double vernier, the wrong row of numbers, or by reading 28° instead of 32°, etc., or by forgetting to add the ½ degree of the line to the reading of vernier and recording 20′, instead of 50′ etc.

Limits of Precision.—It is a little difficult to get sufficient data for a satisfactory discussion of this subject, without making observations specially for this purpose, which is not desirable. The sophomores, '86, in the prosecution of the ordinary class work in topographical surveying, measured the angles of ten triangles; the sums of the three angles should in each case equal 180°. The length of the sides varied between 400 and 1,200 feet; the conditions as to time, targets, etc., were about those of actual practice. The instrument read to minutes, and certainly was not the best.
errors enumerated in the preceding article were involved. This work gives the probable error of a single direction = 32 seconds; the probable error of an angle = 50″; the probable error in the sum of three angles of a triangle = 86 seconds. The maximum error in the closing of a triangle was 3½ minutes.

The same class in measuring the four angles around a point and the three angles of a triangle, using chaining pins for targets, with sights about 100 feet long, obtained results about half as large as those above. The results of traversing, with flag-poles for targets and sights varying between 200 feet and 800 feet, gave results a little greater than half those of the preceding paragraph. This is all the data at hand to show the degree of accuracy attainable, but in a general way it is believed that the above results are not exceptional.

A transit is sometimes used to determine areas. The legitimate errors in the balancing of the latitudes and departures in transit surveying can be discussed as in compass surveying; the formulas deduced for that case are applicable to transit surveying. Such a discussion shows that ordinary work with the tansit is proportionally as accurate as the best chaining. Therefore, in finding areas by the transit, the greatest care must be given to the chaining.

# V.

## THE LEVEL.

1. Qualities Desired in a Leveling Instrument.—The main qualities to be secured in a leveling instrument are delicacy of level, stability and defining and magnifying power in the telescope. The optical qualities of the telescope have already been discussed, but it may not be amiss to repeat that the optical qualities should be correctly proportioned to the sensitiveness of the level bubble. Of two levels in the writer's possession, one had a magnification of 17 and a radius of the bubble of 84 feet, and the other a magnification of 27 and a radius of 22 feet; the simple expedient of changing the bubbles improved both instruments very much. A third instrument is inferior to the other two in definition, although it has a bubble of 165 feet radius.

The stability of the instrument depends very much upon the leveling screws and the manner of connecting the instrument with the tripod-head. This point was discussed under the transit; but we repeat that three screws give greater stability than four; and, in either case, the distance between opposite screws should be as great as possible. This item is of greater importance in the level than in the transit. It seems to be self-evident that the center of gravity of the instrument should be as near to the tripod-head as possible, but this is a common defect.

However good the other parts of the instrument may be, the accuracy of the work depends upon the sensitiveness of the bubble. It must be remembered that, although a sensitive bubble may not remain exactly stationary, it will still give better results than a sluggish one, which shows no movement when the instrument is slightly displaced.

2. Sensitiveness of Bubble.—The sensitiveness depends upon the radius of curvature, or what amounts to the same thing, upon the distance the bubble moves for any change of inclination.

To measure the sensitiveness, proceed as follows:—Bring the bubble nearly to the center and sight upon a rod held vertically. Raise or lower one end of the level, by operating the foot screws, until the bubble moves about as far to the other side of the center, and sight at the rod again. Let $h$ = the difference of rod readings;

$D$ = the distance from the instrument to the rod; $m$ = the distance the bubble moved; $d$ = the length of one division of the scale: $n$ = the number of divisions bubble moved, $m = nd$; $I$ = the change of inclination of the line of sight; $R$ = radius of curvature of the level tube. Then, in Fig. 1, we have, from the approximately similar triangles $A\ B\ C$ and $O\ P\ Q$. $R = \frac{D}{h} m = \frac{D}{h} nd$,

and $\tan I = \frac{h}{D}$; approximately $I'' \tan 1'' = \frac{h}{D}$

hence $I'' = \frac{h}{D \tan 1''}$ $I''$ is the value in seconds of arc of $n$ division of the scale, consequently the angular value of one division in seconds

$$= \frac{h}{n\ D \tan 1''} = \frac{h}{0.000005\ n\ D.}$$

The sensitiveness of a bubble is generally stated by giving the angle corresponding to a movement of an inch. Good engineer's levels have a motion of 200″ to 120″ per inch, or a radius of curvature of 80 to 140 feet. Levels of precision have a motion of about 75″ to 100″ per inch, or a radius of curvature of 400 to 160 feet.

3. Different Forms of Levels.—Leveling instruments may be grouped in three classes. The first includes all instruments that can be adjusted by reversals; the common wye or Y-level is a representative of this class. The second includes all that can not be adjusted by reversals; the "dumpy level," is a type form. The third includes all instruments whose errors of adjustment may be wholly eliminated by a system of double observations; the instruments of this class are usually called "levels of precision," and are used only where extreme accuracy is aimed at.

The distinguishing characteristic of the Y-level is that the telescope may be revolved about its own axis, and turned end for end in its bearings; the only advantage of this construction being to facilitate the adjustment of the instrument. "The Y-level is easily adjusted and nearly always needs it." It is the favorite with American engineers.

The "dumpy level" is the name given to that form of leveling instrument in which the telescope is attached to the bar in such a way as not to admit a rotation around the axis of the telescope, nor allow of reversion end for end. The telescope is usually inverting, and therefore shorter than the one commonly used on leveling

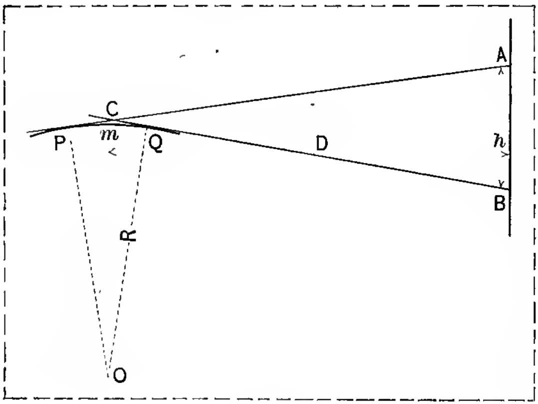



Fig. 1.

instruments, hence the name dumpy; however, this construction can be employed with an erecting as well as with an inverting telescope. English engineers use this form exclusively.

Without doubt the American engineers could profitably follow the English in using the dumpy to the exclusion of the Y-level. As ordinarily made, the former has the advantage of an inverting telescope as has been discussed; it is also more simple and compact. If equally well made, it is capable of doing as accurate work as the more elaborate and more expensive Y-level. The only disadvantage charged against the dumpy is that it is not so easily adjusted, although it is admitted that when once adjusted it retains the adjustment very much better than the Y-level. The last is a very important advantage, for unless an instrument is in adjustment the work is nearly certain to be inaccurate.

The writer firmly believes that the dumpy is as quickly and as easily adjusted as the Y-level. Finally there are a few particulars connected with the manufacture of the instrument in which the general design of the dumpy is superior to the Y-level; as these would probably be inappreciable, except in very accurate work, they will not be discussed here.

Target Rods.—There are two classes of leveling rods: 1, target rods, these in which the line of sights is always directed to a movable target, the position of which is read by the rod-man; and 2,

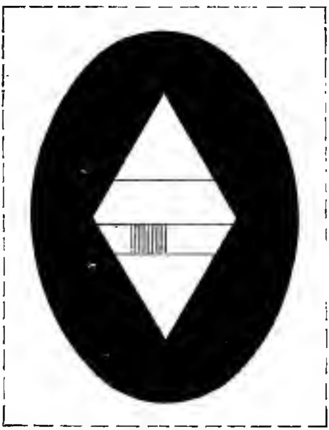


Fig. 2.

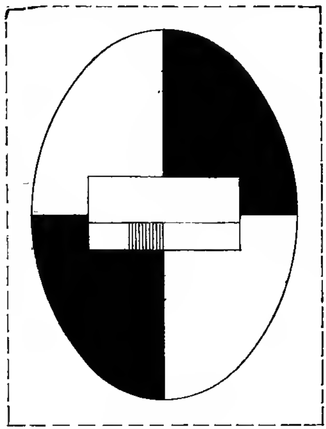


Fig. 3.

self-reading or speaking rods, those in which the graduation is such that the position of the line of sight can be read with the telescope. The well known New York rod is an example of the first-class. The Philadelphia rod may be used either as a target or a self reading rod.

The target is a piece of brass or iron which can be moved up or down the rod, or clamped in any position. It carries a scale or vernier to sub-divide the least space on the rod. The face of the target should be painted of such a pattern that it may be precisely bisected by the horizontal cross-hairs. Some of the many varieties are given in Figs. 2, 3 and 4.

Fig. 2. is the target of the New York rod; the design is not good, because the cross-hairs may be above or below the middle of the target by its full thickness, as magnified by the eye-piece, without the error being perceptible.

Fig. 3, depends upon the nicety with which the eye can determine whether a line bisects an angle; this can be done very accurately by noticing the length and position of the two points formed on either side of the hair.

Fig. 4, depends upon the accuracy with which the eye can bisect a space.

Black and white are best for visibility; but red and white are most easily distinguished among trees, shadows, etc., and red gives the stronger contrast with the cross-hairs. Probably red and white are the best for Fig. 2, and black and white for Fig. 4.

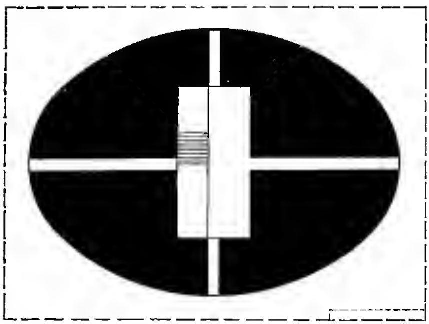


Fig. 4.

5. Self-Reading Rods. The most common rod of this class is the Philadelphia rod. The graduation is in feet and tenths, the figures indicating feet are red, the tenths black. The figures are six-hundredths in height, placed with centers over the marks; sometimes the bottom of the figures is placed on the line, in which case the figures may be eight-hundredths high. To make a reading greater than 7 feet, without a target, the rod must be extended to its greatest length.

A few of the many patterns which have been proposed for self-reading rods are shown in Figs. 5 to 8.

The advantage of the form of graduation of which Fig. 5 is a type, is distinctness and visibility. The angles of the figures indicate fractions of tenths; the figures may be placed with centers or with bottoms, on the lines. The graduation given is suitable for short sights; for longer ones the figures are made larger, and only those for the even tenths are marked.

Fig. 6 shows the Francis rod, which is claimed to be a complete graduation to hundredths without visual division, and can be read without counting. The foot marks are red and larger than the tenths. It is best adapted to short sights.

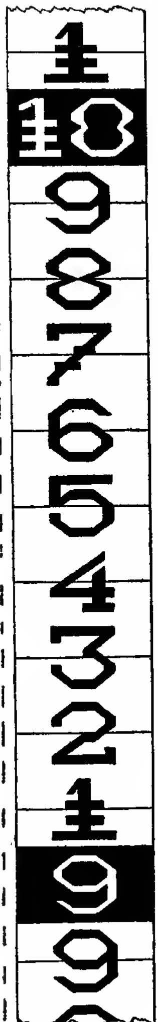


Fig. 5.

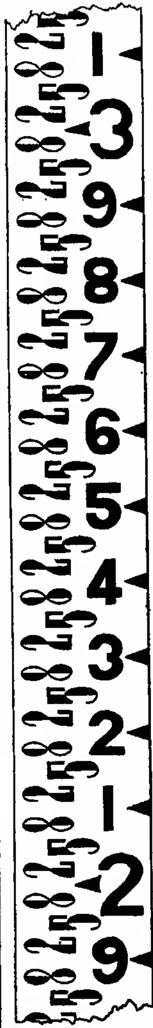


Fig. 6.

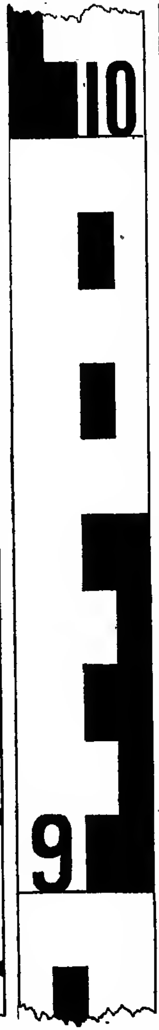


Fig. 7.

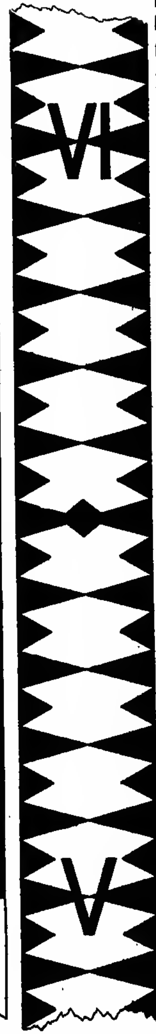


Fig. 8.

In its essential features, Fig. 7 is the favorite British self-reading level rod. The graduation is to hundredths. Although the one shown is better than the typical English rod, it cannot be considered a good one for the same reason for which the usual target to the New York rod is condemned in a previous paragraph.

The principle of the Texas rod, Fig. 8, is of frequent application in making self-reading rods. It is superior to that of Fig. 7 in that it obviates the error due to the thickness of the cross-hairs. The points indicate tenths, but it can be read by estimation to hundredths; it is possible to make the reading much more accurately by dividing the oblique side of the triangle, than could be done when the rod is graduated according to the principle of Fig. 7. This form of graduation reduces the difficulty of counting a number of small divisions, and also the possibility of gross mistakes.

Any pattern suitable for a stadia rod, samples of which will be given in the next article, can be used in making a self-reading leveling rod. The pattern may be painted or stenciled directly upon the wood, or it may first be drawn or painted upon paper, and then fastened on the rod with varnish or any glue not soluble in water. Strips of cloth or paper containing scales for this purpose are sold by dealers in engineering stationery. The graduation will keep cleaner and last much longer, if the face of the rod is recessed slightly to receive it. The rod can be made of a light board hinged in the middle, so that it may be folded to protect the face and for convenience of transportation. It is held open by a button or bolt on the back. Stencils can be cut out of a sheet of writing paper which has been varnished or oiled.

6. Target *vs.* Self-Reading Rods.—Probably the former are the more common now, but as the advantages of the latter are becoming better understood, they are being more generally used. The chief advantage of self-reading rods is the saving of time. Setting the target to some exact point in accordance with directions given from a distance, is a tedious process at best. After a little familiarity with the pattern of the self-reading rod, the height of the line of sight upon the rod can be read very quickly.

On the other hand, target rods must of necessity be capable of greater accuracy than self-reading ones, but the difference in accuracy is not so great as might at first seem; the accuracy of a level determination depends upon a number of things of which the reading of the position of the line of sight upon the rod is one of the least important, all of the others being independent of the kind of rod.

Reading the position of the target to 1,000th is unnecessary and useless, unless all the other parts of the work are equally precise. The thing to be sought is corresponding accuracy in all parts of the work. If several independent readings of a rod be made upon the same point, the difference between the various readings will probably be considerably larger than the probable error of reading a self-reading rod.

In precise leveling self-reading rods are used, and the position of two or three hairs, generally the latter, are read to reduce the error of reading; the mean of these is used as the rod reading. For a single observation, a target rod is more accurate than a self-reading one; but three observations, as above, are probably more accurate than a single observation upon a target, and can be made in about the same time.

7. Adjustments.—Nothing need be said of the ordinary adjustments; they are fully described in all text-books and manuals, but the following section, which is specially applicable to the dumpy level, is not so common but that it may be given here.

8. Two Peg Method.—To adjust the dumpy level, proceed as follows: Drive two pegs into the ground, say 400 feet apart, set the instrument exactly half way between them, and carefully determine the difference of level in the ordinary way. A line joining the two positions of the target is a level line, and the difference between the readings is the true difference of level however much the instrument may be out of adjustment. Next set the instrument very near one of the pegs and re-determine the difference of level. If the second difference is the same as the first, the instrument is in adjustment. If the differences are not the same, raise or lower the line of sight, by manipulating the foot-screws, until the difference between the two readings is the same as the difference first obtained; the line of sight is then horizontal. Without altering the inclination of the line of sight, raise or lower one end of the level tube until the bubble is in the middle; the instrument is then in adjustment.

For example, let $A$ and $B$, Fig. 9, be the two points. When the instrument is at $c$, midway between $A$ and $B$, the targets are at $a$ and $b$: $ab$ is a level line. The true difference of level between $A$ and $B = Aa - Ab$. When the instrument is at $D$, the targets are at $c$ and $d$, and the apparent difference of level is $de = (Cc - Bb) - (Ad - Aa)$. If the line of sight were horizontal the target would be at $f$; therefore $df$ is the connection.

$$df = de \frac{A\,D}{A\,B} = de \left( \frac{2ac + B\,D}{2\,A\,c} \right)$$

If $D$ is between $a$ and $B$, the quantity $B\,D$ must be subtracted.

The above is the only adjustment absolutely necessary with the dumpy level; but to save the trouble of leveling up every time the instrument is turned on its vertical axis, it is desirable to adjust the supports of the telescope so that the bubble will stand in the middle when the instrument is revolved on its vertical axis. This can be done by the method of adjusting plate levels.

9. Sources of Error.—For convenience of discussion, we will classify errors of leveling as follows:—1, Instrumental errors; 2, Rod errors; and 3, Errors of observations. In general a clear comprehension of all the sources of error, their amounts and the means of avoiding them, will be of great service in indicating the

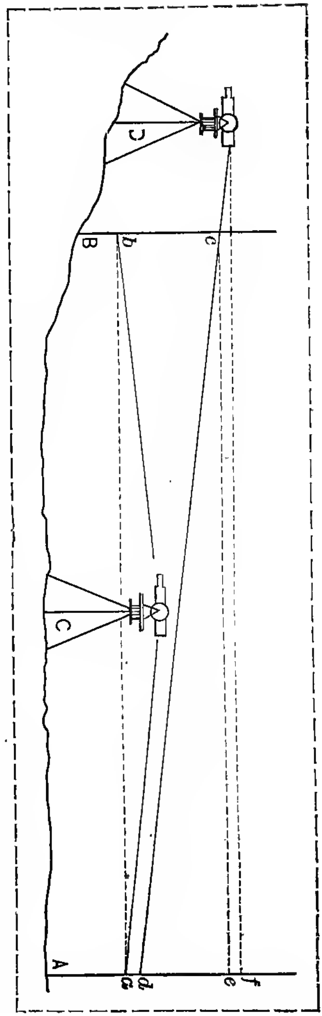


Fig. 9.

care necessary to secure a given degree of accuracy, and particularly in leveling is this true. In no other form of surveying, with the possible exception of chaining, is it as necessary to distinguish between cumulative and compensating errors, as in leveling. An apparently inappreciable cumulative error may in the course of a series of observations amount to more than a much larger, but compensating error. The effect of compensating errors is reduced nearly to zero simply by multiplying the number of observations, but cumulative errors should be avoided entirely, or observations made by which they may be corrected. The observer should avoid errors which usually occur in a single direction; but he need not always take the greater care to avoid errors which are as liable to be negative as positive. A clear comprehension of all the sources of error, their amounts, and the means of avoiding them, will be of great service in indicating the care necessary to secure a given degree of accuracy.

INSTRUMENTAL ERRORS.—The chief error of this class is lack of adjustment in the instrument; the one essential adjustment is that the line of sight should be parallel to the level. The ordinary method of adjusting the Y-level is too well known to be repeated here, but every Y-level should be tested by the two peg-method as already described before being regarded as a Y-level; that is, a Y-level should be carefully adjusted in every particular and then tested by the two peg-method, and if this test shows it to be in adjustment then it may be regarded as a Y-level and ever afterwards adjusted as such, but if this test shows it not to be in adjustment the instrument is nothing more than a dumpy level and must be adjusted accordingly, and the telescope must not be reversed in the Y's. If a Y-level in adjustment by the ordinary method, is found to be out when tested by the two peg-method, the error may be due to any one of several things; the rings may not be of the same size, the Y's may not have the same angle or may not squarely face each other, the optical center of the objective may not lie in the axis of the rings, the direction of motion of the objective may not be in the line of the axis of the rings. The relation of these elements is too complicated to be discussed here, but it is only proper to say that none of these defects are likely to occur to an extent appreciable in ordinary work for which the engineer should thank the accuracy of American machinery and the good workmanship of American mechanics, rather than the design of the instrument. All errors from any of the above sources are compensating and will be entirely eliminated by setting the instrument midway between the turning points.

There are a few minor instrumental errors, which though small in themselves and not occuring frequently may have an appreciable effect in very accurate work as has been demonstrated practically; they are (1) the setting of the instrument on its vertical axis, (2) the setting of the tripod legs into the ground in sandy ground or in ground which is thawing, and the heaving of the legs in spongy or clayey soil, (3) the sun heating and expanding one Y more than other. They are all cumulative errors.

Rod Errors.—The principal rod error is in not holding the rod vertical, and is greater for a large rod reading than for a small one; it is compensating, and may be eliminated by attaching a level to the rod or by waving the rod. With a good rodman this error should be inappreciable in ordinary work. With telescoping target-rods when extended, the slipping of the upper piece after the target has been pronounced correct and before the vernier has been read, is a source of error, The target itself may slip, but this is not so probable, because of its less weight.

Another source of error is the setting of the turning point, due in loose or sandy soil, to its own weight or to the impact of setting the rod upon it. The resulting error is cumulative. The remedy, in the first case, is to use a long peg, or rest the rod upon a triangular plate with the corners turned down slightly or with spikes on the underside. This foot plate with a convex button attached, on which to place the rod, is better for all cases than a peg. Whatever the turning point, the rod should never be dropped upon it.

Finally, another small rod error is the error in the graduated length. This affects only the total difference of elevation between the two points. This is a much more important source of error with the numerous home-made self-reading rods now in use, than with the rods made by regular instrument-makers.

Errors of Observation.—The principal error is in reading the position of the bubble; an error may be made in reading it, or it may be read before it has stopped moving. If the sensitiveness of the bubble has been determined as described the levelman can easily find the error on the rod corresponding to a slight movement of the bubble; he knows then how carefully he must read the result to obtain the particular degree of accuracy aimed at.

Another important source of error is the moving of the bubble after it has been centered and before the sighting has been made. This movement of the bubble may be caused by its being read before it has come to rest, by disturbing the instrument by stepping near the tripod legs, by turning the instrument slightly in azimuth, or by raising or lowering one end of the telescope in focussing, or by the action of the sun or wind. The bubble should be re-read after the target is nearly adjusted, or, with a self-reading rod, after the reading has been made and before the rodman is signaled to move on. These two probably constitute the chief sources or error in leveling operation.

In many Y-levels there is no adequate means of keeping the horizontal hair horizontal and if it is not horizontal any observation not taken at the intersection of the cross-hairs will be erroneous. Since it is very tedious to bring the vertical hair to coincide with the rod at each sighting, and since the telescope is liable to get revolved in the Y's, it is almost certain that this error is frequently a great source of inaccuracy in leveling. It is compensating. To eliminate this error some instrument-makers fasten the telescope in the Y's, in such a manner as to prevent any rotation; others place a mark upon the collar of the telescope and another upon the Y; by noticing to see that these lines coincide, the leveler may be certain that the hairs are correct. This improvement can be added to any instrument by riveting a lug on the side of one of the clips and cutting a notch to match in the lip of the ring or collar on the barrel of the telescope. The work can be done in any machine shop. It is impossible to tell when the target is exactly behind the target or the point on the rod covered by the hair. Because a target rod is read to thousandths is no evidence that it is correct to that limit. This error is compensating, and varies with the form of target or kind of rod, distance and size of cross-hairs.

With long sights there is another source of error, the curvature of the earth and the effect of the refraction of the atmosphere. The combined effect of these two items is to make the target when 225 feet distant appear one-thousandth of a foot high; this correction varies as the square of the distance. If the line of sight passes near the earth or over a body of water, the effect of refraction may vary considerably and consequently the above correction is only approximate. Ordinarily this error is compensating; and will usually be eliminated by setting the instrument midway between turning points.

Such blunders as an error of one foot or one-tenth in reading the rod, or recording the foresight in the back-sight column, or vice versa do sometimes occur; unfortunately they are not small, and in ordinary leveling there is no check against them which is practicable.

Limits of Precision.—A great diversity of opinion exists as to what should be called accurate leveling, so diverse that nothing would be gained by citing them. Let us first consider the degree of precision attained in what is know as "precise leveling," by skillful observers, with the best instruments andplenty of time.

According to the theory of probabilities, the final error of a series of observations each of which is subject to error, and as liable to be greater as to be smaller than the true result, will vary as the square root of the number of observations. For example, if in measuring a mile the error is 1 foot, it is probable that the error for four miles will be 2 feet. This is true only when the error of each observation is as liable to be plus as minus, when the only errors are compensating ones. In the above illustration, if the chain were too short, the error in the second distance *due to this cause* would be four times as much as in the first distance. In leveling, a method should be adopted which will eliminate all cumulative errors; and therefore, since only compensating errors remain, the final error should vary as the square root of the distance. The final error due to cumulative errors will vary as the distance, and as it is highly improbable that all cumulative errors will be entirely eliminated, the final error will be a little larger than proportional to the square roof of the distance. The precision of leveling is usually assumed to vary as the square root of the distance.

It is pretty well established that the error of the most accurate leveling will be 0.005 feet $\sqrt{\text{distance in miles}}$, and even with the greatest care it may be as much as 0.020 feet $\sqrt{\text{distance in miles}}$. The first result may be taken as the extreme degree of accuracy attainable. To accomplish this, requires skillful observers, the best instruments, and plenty of time; ordinarily there are not more than three or four hours of the day on which this class of work can be done, and as a general average not more than one or two miles can be made per day.

It is impossible to establish a limit for work less accurate than the best, the conditions under which it may be done are too diverse. Results of leveling are often given of apparently greater accuracy than the above, but an occasional accurate result, probably more largely due to good fortune than good management, gives no indication as to what results may be regularly expected; naturally it is the most accurate result that is reported. However, the difference in precision between ordinary leveling and precise leveling, is not as great proportionally as the difference in care, time, etc.; a little increase in accuracy costs a very great increase of effort.

A line of ordinary levels was run on the bank of the Mississippi river, * and checked upon the benches of the precise levels, with an average error of .011 feet $\sqrt{\text{distance in miles}}$. This was about the conditions under which a preliminary railroad survey is made, but is probably more accurate than such surveys usually are.

In some of the branches of the A. T. & S. F. R. R., the instructions were to re-run the line whenever the difference between the levels on construction and location was .03 feet between benches about 2,000 feet apart; this is equivalent to limiting the maximum admissible error to .048 feet $\sqrt{\text{distance in miles}}$.

The amount of work that can be done by an observer in a day, is a point about which there has been much debate, and which will never be settled. Leveling is of different kinds, for different purposes, with instruments of different powers and delicacy, on different ground, and with varying atmospheric conditions. Professor J. B. Johnson, who has had large experience in levels of precision on the U. S. Lake Survey and on the Mississippi river, states, that "with a wye level and a target rod, a single instrument should duplicate 30 miles per month, with no greater error than 0.05 feet $\sqrt{\text{distance in miles}}$, or with a level of precision and speaking rod, do the same work with a limit of 0.02 feet $\sqrt{\text{distance in miles}}$."

* Communicated privately to the writer.

PRACTICAL HINTS.—BEST LENGTH OF SIGHT.—The length of sight is limited by the power of the telescope and the atmospheric condition. It has just been seen that some errors increase directly and others indirectly as the number of sights taken in a given distance. In view of these facts, it is generally assumed that, for the most accurate work, the rod should be at least 100 feet and never more than 400 feet, from the instrument, for ordinary work; the best length of sight is thought to be 300 to 400 feet, while the greatest should never exceed 500 to 600 feet.

It is very desirable that at each setting of the instrument the length of the back-sight and the fore-sight should be equal; for, as has been seen, there are a number of errors which are proportional to the length of sight, but which cancel each other when the distances are equal. This is a very important point and should always be kept in mind. When stakes are set at regular intervals, there is no difficulty in determining the length of sights, and making them equal; in other cases, the distance can be easily determined by the principle of the stadia, as will be explained in the next chapter. All levels should be provided with two extra horizontal cross-hairs for this purpose.

In ascending or descending a hill, it is nearly impossible, and always very tedious, to make the back-sight and fore-sight equal. As the rod is about twice as high as the instrument, the down-hill sights will be about twice the length of the up-hill ones. When the ground renders sights of unequal length unavoidable, keep notes of the distance, and, as soon as possible, take sights with corresponding inequalities in the contrary direction. When approaching a long incline make part of this compensation in advance.

RECIPROCAL LEVELING.—In crossing a river, it is absolutely necessary that the back-sight and fore-sight should differ considerably; also other somewhat similar cases occur, to which the principles of reciprocal leveling are applicable. The method of procedure is very simple.

Establish a bench upon both sides of the river and determine the difference of level; move the instrument to the other side, and re-determine the difference of level. If the sights were taken in quick succession, the mean of the two results is the true difference of level. Simultaneous observations with two instruments would be still better.

The surface of the water can not be assumed to be level, except there is no current, or the line joining the two benches is perpendicular to the current and there is no wind.

In ascending or decending a steep hill, it is desirable, for speed, that the line of sight should strike as near as possible to the bottom of the rod on the up-hill side, and to the top of the rod on the down-hill side. In selecting the position of the instrument corresponding to this condition, set the instrument up lightly, turn the telescope in the right direction, bring the bubble approximately to the middle by manipulating the tripod legs, and sight along the outside of the telescope. Even this rude observation will be valuable as showing whether the instrument should be moved up or down the hill. It will save considerable time.

With a little practice, the same observation may be made by drawing the tripod legs together and using them as a Jacob's staff; then the bubble can speedily be brought to its proper position by simply inclining the whole instrument.

If the up-hill rod is too near to be focused on, set a little to one side. In short intermediate sights for which the telescope cannot be focused, it is sufficient to sight by the bottom of the Y's or by the side of the telescope.

If the line passes over a stream with steep high banks, or over a narrow, deep gorge or valley, establish a turning point on the farther side by reciprocal leveling; then, to find the depth of the opening, level down the bank without much regard to equality in length of sights or other refinements. This will usually be all that is necessary and is much quicker than leveling down one bank and up the other.

Instruments are often provided with sun shades to prevent the sun from troubling by shining into the telescope; if the metallic shade is not at hand, make one by rolling up a piece of paper and gumming, pinning or tieing it together, or springing a rubber band around it; it is easier and better than holding the hat or note-book over the objective.

If the instrument has once been leveled, and the bubble is found to have moved a little, bring it back with a slight pressure of the fingers, Finally, in closing at noon or night, be careful to set halfway between the last two turning points; on resuming work, set near one of these points, and re-determine their difference of level; the same difference of level each time, affords an excellent check upon the adjustments of the instrument.

## VI.

### THE STADIA.

1. An Italian engineer (Porro) in about 1820, was the first to suggest the determination of distance in surveying by a visual angle and a rod. He proposed the name stadia; this term is applied to the instrument as a whole, although its derivation would indicate that it referred more properly to the rod. The instrument is more properly, but less frequently, called a telemeter.

The principles involved have long been well known, and have been applied in gunnery and military reconnoisance; but it is only lately that they have been used in engineering. Even now the stadia has not come into as general use as its merits demand. It was used first in Switzerland in 1836 in making a topographical survey of that country. It seems not to have been introduced into America until nearly thirty years afterward; and has not been employed to any considerable extent except in topographical surveys carried on by the U. S. Government. It is more generally used in Europe than in the United States, although it is rapidly coming into use here.

It is peculiarly adapted to topographical surveying, for it possesses the double advantage of giving both the horizontal and vertical co-ordinates, and this, too, by the most rapid method.

2. Principle.—In all its forms the stadia depends upon the principle of the similarity of triangles. A simple form of the stadia is used in the familiar method of determining the distance of a man by measuring on a rule held at arm's length the space covered by his height. There are several forms of this simple device, but none of them are of any practical value in surveying, owing to the impossibility of focusing the eye for two distances at the same time, and second, the indistinctness of the farther object.

To employ the principle of the stadia with a telescope, it is necessary to introduce two parallel cross-hairs and observe the amount of the rod intercepted between the two hairs. The distance from the instrument to the rod varies as the space on the rod intercepted between the two hairs. The hairs may be horizontal and the rod vertical, or vice versa; the former is generally preferred. Also the hairs may be fixed, the variable distance between them being measured on the rod; or the hairs may be movable and be set to cover always the same two points on the rod, the varying distance between the hairs being measured by a graduated screw. The fixed hairs are cheaper, more simple, more accurate, and in every way better; it is the one generally used.

3. Adjusting the Hairs.—Any measuring telescope can be used as a stadia by adding a horizontal hair, but it is much better to add two extra ones, one on each side of the ordinary one. With the stadia hairs thus placed, the field of view is symmetrical about the center, and the added hairs interfere less with the ordinary uses of the telescope.

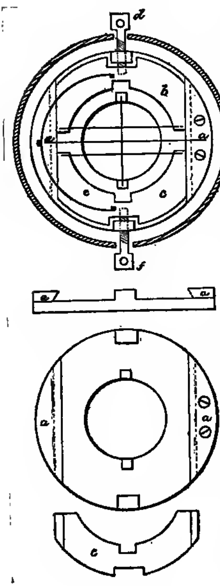


Fig. 1.

In fixing the hairs, three conditions must receive attention; they should be parallel, equally distant from the central one, and at a suitable distance from each other. The various instrument makers have different ways of making the hairs adjustable for convenience in satisfying the above conditions.

One of the most common of these is shown in the accompanying sketches. The first shows the reticule in place in the telescope tube; the others show details. The screws *d* and *f* can be operated from the outside of the telescope tube, and move (with reference to the ring *a*), the slides *b* and *c*, which carry the stadia hairs, *e* is a bent spring to take up lost motion in the screws *d* and *f*. The ring *a*, and all parts connected therewith, is adjusted in the telescope tube by the ordinary cross-hair screws, which are not shown. By means of the screws *d* and *f*, the distance of the stadia wires from the central wire, and from each other, can be varied at will. Lines are made upon the slides $b$ and $c$ to assist in placing the hairs parallel to each other.

The objections to this construction are, first, that it is expensive; second, that the projecting heads of the adjusting screws are liable to be struck and turned in handling the instrument or in carrying it through brush, and so produce a serious error with no adequate means of detecting it; and third, since a spring must be inserted to take up lost motion, the screws have a tendency to work loose.

4. Fig. 2, shows a much better way of making the hairs adjustable; $a$, $b$, $c$, and $d$ are small wire plugs which are free to turn, being held only by friction; the shaded portion is in the plane of the face of the ring, the unshaded portion projects, say $\frac{1}{8}$ of an inch above. The cross-hairs are to be stretched in the line of the centers of $a$ and $d$, and $c$ and $b$, and fastened at the outer edge of the ring; then, by turning the plugs, the hairs will be moved toward or from the central one, according to which side of the plug is toward the center hair. The wires may easily be made to fit tight enough not to work loose, and still turn freely enough for the above adjustment.

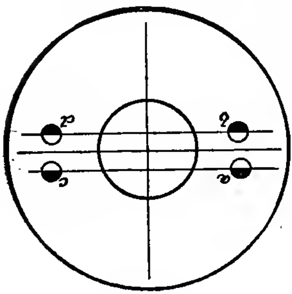



Fig. 2.

With telescopes having inverting eye-pieces, which are much the best for stadia work, the wires can be turned by removing the eye-piece and without taking the reticule out of the telescope tube. With an erecting eye-piece, the hairs can be adjusted without removing the ring from the telescope tube, by making a little wrench (by cutting a kerf with a hacksaw in the end of a small strip of brass) especially for the purpose, and operating it through a hole in the telescope tube, also made for the purpose and which can be closed by a band; or the ring may be removed from the tube to adjust the stadia hairs. The telescope does not need to be collimated to test the relative position of the stadia hairs.

This method provides a way of making the hairs parallel to each other, and allows a variation in the distance between the central and each outside hair equal to the diameter of the plug. It is comparatively inexpensive.

Although the conditions specified above, for the position of the hairs, are the rigorous ones, farther on it will be shown that no appreciable error will be produced if they are only approximately satisfied. For example, the condition that the hairs should be parallel, is only necessary in case the observation is not made exactly on the vertical hair. Also if the stadia hairs are not symmetrically about the central one, it only produces an error in the vertical angle. Finally, it is immaterial what distance the hairs are apart, provided the rod is graduated to correspond. Hence it is possible to attach simply two extra hairs in the ordinary way to any measuring telescope, and thus fit it for stadia work. We will produce a formula to meet the case in which the rod is already graduated, and does not agree with the distance between the hairs.

5. The maximum distance between the hairs is limited by the size of the field of view. A magnifying power of ten will generally allow a field of view of 10° and for greater powers proportionally less. Hence for a telescope which magnifies ten times, the aperture may be 10° as viewed through the eye-piece, the real visual angle being 1° and the distance between the hairs $\frac{1}{57}$ of the focal length of the objective. This would be about equivalent to a ratio of space on the rod to the corresponding distance from the instrument of 2 to 100. This ratio might be used for short sights, but for long sights it would require a rod too long to be easily handled.

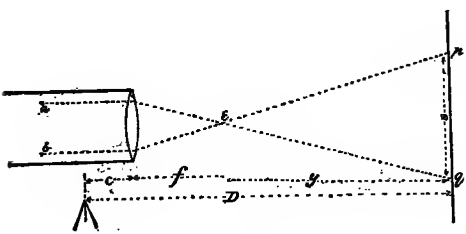



Fig. 3

On the other hand, if the hairs are very close together, the interception on the rod will be too small to be read accurately. All things considered, the best width is probably that which makes 1 ft. on the rod correspond to 100 ft. on the ground.

6. Formula for a Horizontal Line of Sight and a Vertical Rod.—In Fig. 3, let $a$ and $b$ represent the stadia hairs, and $i =$ the distance between them; let $s =$ the distance on the rod $p\ g$, intercepted between the hairs; let $f =$ the focal distance of the objective; let $e$ represent a point at a distance $f$ in front of the optical center of the objective, or $e$ is the outer focus of the objective; $c =$ the distance from the plumb line of the instrument to the optical center of the objective; $y =$ the distance from the outer focus $e$, to the rod: $D =$ the distance from the instrument to the rod.

From the principles of optics, all rays of light which pass through $e$, become parallel to each other after passing through the objective; therefore, there is some point $p$, which will emit a single ray of light that will pass through $e$ and after traversing the objective, will strike the cross hair $a$. If the telescope is focused for th point $p$, the objective will bring all rays emitted by $p$ to a focus at $a$; hence it is immaterial whether we consider the real course of the rays, or assume that all the light from $p$ passes along the lines $g\ e\ a$. Similarly for the point $g$. Therefore, the diagram shows correctly the geometrical relations between the variables involved in the determination of distance by a visual angle with a telescope.

From Fig. 3 we easily get $s : y :: i : f$, from which

$$y = \frac{f}{t} s \tag{1}$$

and

$$D = \frac{f}{i} s + c + f \tag{2}$$

Notice that $\frac{f}{i}$ is a constant co-efficient peculiar to each instrument, also that $s$ varies with the distance to be determined, and that it is proportional to the distance from the rod to the outer focus of the objective.

It is now necessary to show how the numerical value of the terms $\frac{f}{i}$ $c$ and $f$ can be found. To find $f$ focus the instrument on an object, say half a mile away, and measure with an ordinary rule the distance from the cross-hairs to the middle of the thickness of the objective. To find $c$ measure the horizontal distance from the vertical of the instrument to the middle of the thickness of the objective; $c$ is not strictly constant in instruments in which the cross-hairs are fixed and the objective movable, but if the values of $c$ and $f$ are found within an inch it is more than sufficient.

To find the co-efficient $\frac{f}{i}$, mark a point on the ground under $e$ (Fig. 3), by measuring a distance $= (c + f)$ from the plumb line of the instrument, and set a rod, say 100 ft., in front of this point; this distance $= y$ in equation (1). Sight through the instrument and have an assistant mark the point on the rod covered by each hair, the distance between these points correspond to $s$ (1), we have

$$\frac{f}{i} = 100\ s.$$

If the space on the rod be then divided into this number of spaces, and the same graduation be carried throughout the length of the rod, then the number of divisions on the rod included in the visual angle will always be equal to the distance from the rod to the front focus of the objective.

If $R =$ the number of divisions included in the visual angle, equation (2) then becomes

$$D = R + f + c \tag{3}$$

The use of the preceding equation demands that each value of $\frac{f}{i}$ shall have a rod graduated to correspond. It frequently happens, from one cause or another, that it is desired to use a rod whose divisions are too great or too small; it then becomes necessary to multiply the reading by a co-efficient to correct it. Let $K =$ this co-efficient. To find $K$, measure $D - (f + c)$ for some particular case, and observe the corresponding $R$: then $K$ should have such a value as will make $K\,R$ numerically equal to

$$D - (f + c), \text{ or } K = \frac{D - (f + c}{R} \tag{3}$$

then becomes

$$D = K\,R\,(f + c).$$

8. THE ROD.—The rod must be light and handy for transportation. A board about 1-in. thick and 4 or 5 wide does very well;

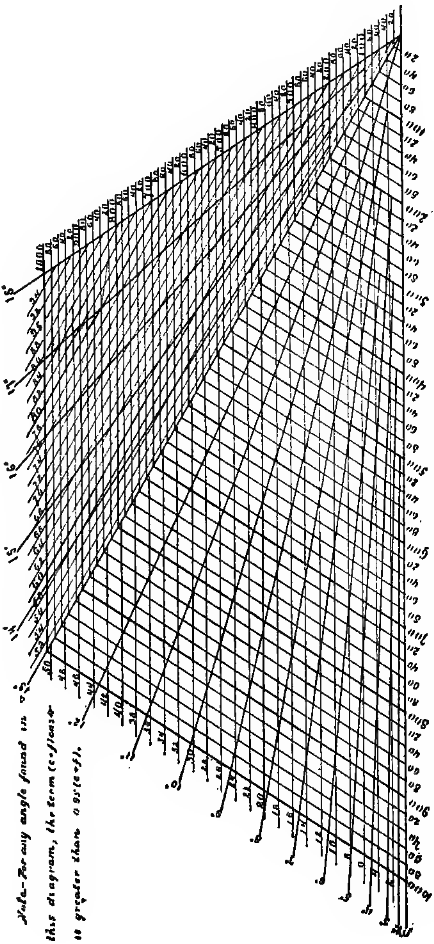



Plate 1.—Stadia Reduction Diagram for Horizontal Listances.

This diagram gives the term $R \sin.^2 \Theta$. For any angle found, the term $(c + f) \cos. \Theta$ is greater than $0.95 (c + f)$.

it may be stiffened by screwing a strip edgewise on the back. The rod may be hinged in the middle, folded for ease of transportation and for the protection of the graduation; when in use, it may be kept upright by a button or a bolt on the back. It should be provided with a disk level, or a short plumb, for keeping it vertical. A handle is very useful in holding the rod steady.

As in leveling, there are two kinds of stadia rods, self-reading and target rods. The latter require two targets; one of them is sometimes permanently fixed to the rod to save the trouble of setting two targets, but, when the vertical co-ordinate is desired, this adds more complication than it saves. Target rods may be a little more accurate, but they are certainly very much less convenient. Self-reading stadia rods are generally preferred.

9. The principles involved in the graduation of stadia rods are much the same as those discussed for self-reading level rods; the only difference is that the latter are generally read at short distances, while it is frequently necessary to read the former at long distances. Hence, in graduating a stadia rod, visibility is of first importance.

It is generally stated that a red disk on a white ground can be seen by the unaided eye at about 6,000 times its own diameter. A black disk can be seen farther, and a black line still farther. A good telescope would increase this distance nearly in proportion to its magnifying power. To be on the safe side and allow for the loss of light in the telescope and for unfavorable atmospheric conditions, we will assume that a black line on a white ground will always be visible, to the unaided eye, at 4,000 times its least dimensions; experiments seem to show that this is safe. Therefore, the greatest distance, $L$, at which a mark of width, $w$, will be visible through a telescope magnifying $M$ times is

$$L = 4{,}000\, w\, M; \tag{5}$$

and the length of rod, $l$, required for the sight, $L$, is determined by

$$l = L\frac{S}{D} = 4{,}000\, w\, M\frac{S}{D} = L\frac{i}{f} \text{ very nearly,} \tag{6}$$

in which $D$ is the distance on the ground corresponding to the space, $S$, on the rod; $\frac{S}{D}$ is usually about $\frac{1}{100}$ (7)

Also the smallest difference in distance perceptible $= \frac{D}{S} d$ in which $d$ is the value of the smallest division of the graduation. These formulas, together with a knowledge of the field of view, the magnifying power of the telescope, and the longest probable sight, are useful in determining the distance apart of the hairs and in graduating the rod.

10. Any self-reading leveling rod can be used as a stadia rod; but it is best to omit the numbers, for it is easier to count the divisions included in the visual angle than to read both hairs and subtract. If the graduation of the rod consists of a great number of very small divisions, they become wholly indistinct at a distance greater than $L$ as above, and at even shorter distances, it is very confusing to count them. Also, it is best to keep the colors as much together as possible, else when the air is unsteady the figures will run together to such an extent that it will be impossible to distinguish them. For these reasons, it is much better to make the graduation marks larger, and secure the smaller divisions by subdividing the larger ones by estimation; the rod can then be used at very great distances, although at a sacrifice of precision. This method by estimation is almost, or quite, as accurate as when the smaller divisions are marked; it is often claimed that very small subdivisions can be estimated more exactly than read by a direct graduation.

The adjoining figures show a few of the many forms proposed for stadia rods. Fig. 4 is a rod used on the U. S. Coast Survey; the vertical distance from $a$ to $b$ corresponds to 10 ft. on the ground from $b$ to $d = 4$ ft. and by dividing $b\ d$ into quarters by estimation, the rod may be read to single feet. The other designs are read in a similar manner. Fig. 5 was used on the late U. S. Lake Survey. The other figures are added to show the facility with which designs may be made. Nearly every one who uses a stadia, has a design of his own which he thinks is the best.

11. Position of Rod for Inclined Line of Sight.—Formulas (1) and (2) were deduced on the assumption that the central visual ray was horizontal and the rod vertical, but this is not sufficiently general. These formulas would be sufficient, if the observations were made with a leveling instrument; but the telescope of a level is too limited in its range to secure the full advantage of the prin-

ciple of the stadia. In all that follows, it will be assumed that a transit is used.

Formula (3), $D = R + (f + c)$, may be used with an inclined line of sight, provided the rod is held perpendicular to the central visual ray; in this case $D$ is no longer horizontal distance from the instrument to the rod; but is the oblique distance from the horizontal axis of the telescope to the point on the rod covered by the central visual ray. It then becomes a question whether, with an inclined line of sight, the rod should be held perpendicular to the central visual ray, or vertical.

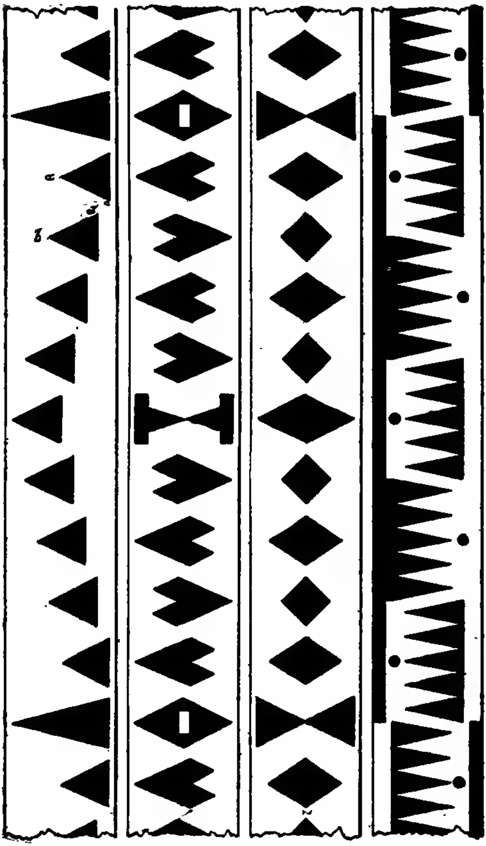


Fig. 4. Fig. 5. Fig. 6. Fig. 7.

The position of the rod perpendicular to the line of sight may be determined by a telescope, or a pair of sights, attached at right angles to the rod, which is directed toward the observing telescope by the rodman. The verticality of the rod may be determined by attaching a plumb-line or a level.

Some prefer the rod perpendicular to the line of sight, but this position involves serious difficulties; first, it is not easy to hold the rod steady in this position; second it is not always possible for the rodman to see the telescope, especially at long distances or great vertical angles, or when undergrowth, etc., intervenes. Without going into the details it is sufficient to state that the formulas for computing the horizontal and vertical co-ordinates of the point are more simple when the rod is held vertical than when it is perpendicular to the line of sight.

FORMULAS FOR INCLINED LINE OF SIGHT AND VERTICAL ROD.—Let $\theta$ = the angle of the central visual ray with the horizontal. To measure $\theta$, a third horizontal hair should be placed half-way between the other two, or rather stadia hairs are to be added on opposite sides and equally distant from the ordinary horizontal one; $\theta$ is then determined as any other vertical angle by reading the vertical circle. $\theta$ will generally be small. Let $2\infty$ = the visual angle, = $B\,o\,D$, Fig. 8; $2\infty$ is always small, 35 may be taken as its maximum value. $A\,E$ is the actual intercept, and $B\,D$ the value it would have if the rod were held perpendicular to the central visual ray. It is desired to find a relation between $A\,E$ and $B\,D$.

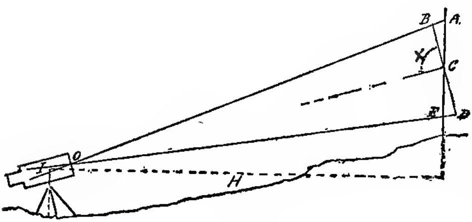



Fig. 8.

The angle $C\,B\,A = 90^\circ + x$; and, since $x$ is very small, $B\,C = A\,C \cos\theta$ nearly. Similarly $C\,D\,E = 90^\circ - x'$, and $C\,D = C\,E . \cos\theta$ Hence $(A\,C + C\,E) \cos\theta = A\,E \cos\theta = B\,D$. Notice that the two approximations tend to neutralize each other; the final error involved is much less than the error of observing $A\,E$ [2].

Since $B\,D$ is the intercept perpendicular to the line of sight, it is the rod reading corresponding to the distance $I\,C$. If we repre-

[2] If the side hairs are equally distant from the central one, the true relation is $B\,D = A\,E \cos\theta\,(1 - \tan^2\theta \tan^2 d)$; and if the side hairs are not equally distant, the upper angle being $x'$ and the lower, $x$ the true relation is

$$A\,E = C\,B \frac{\cos x'}{\cos(\theta + \infty')} + C\,D \frac{\cos x}{\cos(\theta - \infty)}$$

sent the reading on the vertical rod by $R$, we have for the oblique distance $I\,C$.

$$D = R \cos \theta + (f + c) \qquad (7)$$

13. Horizontal Distance.—Let $H =$ the horizontal distance from the center of the instrument to the vertical through the foot of the rod: $H = I\,P$, Fig. 8. $I\,P = I\,C \cos \theta = D \cos \theta$, and

$$H = R \cos^2 \theta + (c + f) \cos \theta \qquad (8)$$

$$\text{or } H = R - R \sin^2 \theta + (c + f) \cos \theta \qquad (8')$$

The second form is preferable, for it is always better to compute a correction to a quantity than the quantity itself. Notice that since $(c + f)$ is always small, and $\cos \theta$ nearly 1', $(c + f) \cos \theta$ may always be taken equal to $(c + f)$, and often be omitted entirely. Finally, $\sin^2 \theta$ is so small that the whole operation of reducing an observation by (8') may be performed mentally.

14. Vertical Distance.—To determine the vertical co-ordinate of the point upon which the rod is set, the middle hair must be sighted upon a point of the rod at a known distance from its foot. The simplest way of accomplishing this is to provide the rod with a movable target, which is to be set at a distance from the foot of the rod equal to the height of the horizontal axis of the telescope above the reference point. If the instrument is set over the reference point, the target can easily be set by placing the rod at the side of the instrument; if the instrument is not over the point, set the rod on it, bring the line of sight horizontal, and sight the target in. To determine the height of any subsequent point, set the middle hair upon the target, and read the angle from the vertical circle. This method of pointing, besides being very simple in itself, very much simplifies the formula for the vertical co-ordinate.

Let $V =$ the height of the point on which the rod is placed above the reference point; notice that for the above method of adjusting the target, $V$ is equal to the distance of the target above the axis of the telescope. Then $C\,P$, the distance sought, $= V = I\,C \sin \theta$; substituting the value of $I\,C$ from (7)

$$V = R \cos \theta \sin \theta + (c + f) \sin \theta \qquad (9)$$

$$V = R \tfrac{1}{2} \sin 2\,\theta + (c + f) \sin \theta \qquad (9)$$

$(c + f) \sin \theta$ may generally be omitted. Notice that $V$ will be + or —, according as $\theta$ is an angle of elevation or depression.

15. Reducing the Field Notes.—This consists in finding the horizontal and vertical distance from the observed reading and

angle of observation. This can be done by using formulas (8) and (9). Although the formulas are in a very convenient form for computation, it would be very tedious and slow to solve both equations for each observation. It is a great saving of labor and time, if the results are computed once for all and tabulated.

Notice that $R \cos^2 \theta$ and $R \frac{1}{2} \sin 2\theta$ are independent of the instrument and of the unit of linear measurement used, and can be tabulated for all cases; $(c+f) \cos \theta$ and $(c+f) \sin \theta$ depend upon both the instrument and the unit, and must be computed for each special case, but as they are so small they can be given in a brief supplemental table.

The result may be expressed in an arithmetical table or a geometrical diagram.

Arithmetical tables are capable of greater accuracy; but they must be either very extended and therefore inconveniently large, or brief and give results by interpolation which is slow and tedious. On the other hand, it is urged against geometrical diagrams, that, to be accurate, they must be drawn to a large scale, and are therefore large and unwieldy. It is believed that, with properly constructed diagrams, the reductions can be made without sacrificing much, if any, accuracy and with greater facility than by the use of arithmetical tables.

In constructing a table, we may tabulate $R \cos^2 \theta$ from equation (8′) and $R \frac{1}{2} \sin 2\theta$ from equation (9′) for different values of $R$ and $\theta$, in which case the horizontal and vertical distances can be taken directly from the table; or, we may tabulate only $\cos^2 \theta$ and $\frac{1}{2} \sin 2\theta$ for different values of $\theta$, in which case the horizontal and vertical distances are found by multiplying the reading $R$, by the tabulated factor. The first method would be the better, if it did not require such voluminous tables.

16. Geometrical Tables.—There is a variety of methods of constructing diagrams for reduction of stadia observations, but as modifications of those proposed by Prof. S. W. Robinson[3] are believed to be the best they only will be explained. The tables on pages 65, 66, 67, 68, give the value of $\sin 2\theta$ in the column marked H, and the value $\frac{1}{2} \sin 2\theta$ in the column marked V.

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[3] Journal Franklin Institute, Vol. 49, pp. 80-1; and do. Vol. 51, pp. 15-6.

| $\theta$ | 0° | | 1° | | 2° | | 3° | | 4° | | 6° | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | H. | V. | H. | V. | H. | V. | H. | V. | H. | V. | H. | V. |
| 0' | .0000 | .0000 | .0003 | .0174 | .0012 | .0349 | .0027 | .0623 | .0049 | .0696 | .0076 | .0868 |
| 2 | .0000 | .0006 | .0003 | 0180 | .0013 | .0356 | .0028 | .0528 | .0049 | .0702 | .0077 | .0874 |
| 4 | .0000 | .0012 | .0003 | .0186 | .0013 | .0360 | .0029 | 0634 | .0060 | .0707 | .0078 | .0880 |
| 6 | .0000 | .0017 | .0004 | .0192 | .0013 | .0366 | .0029 | .0640 | .0051 | .0713 | .0079 | .0886 |
| 8 | .0000 | .0023 | .0004 | .0197 | .0014 | .0372 | .0030 | .0546 | .0052 | .0719 | .0080 | .0891 |
| 10 | .0000 | .0029 | .0004 | .0203 | .0014 | .0378 | .0030 | .0552 | .0063 | .0725 | .0081 | .0897 |
| 12 | .0000 | .0035 | .0004 | .0209 | .0015 | .0384 | .0031 | .0557 | .0054 | .0730 | .0082 | .0903 |
| 14 | .0000 | .0041 | .0004 | .0215 | .0015 | .0390 | .0032 | .0563 | .0064 | .0736 | .0083 | .0908 |
| 16 | .0000 | .0046 | .0005 | .0221 | .0016 | .0395 | .0032 | .0569 | .0056 | .0742 | .0084 | .0914 |
| 18 | .0000 | .0052 | .0005 | .0227 | .0016 | .0401 | .0033 | .0575 | .0056 | .0748 | .0086 | .0920 |
| 20 | .0000 | .0058 | .0005 | .0232 | .0017 | .0407 | .0034 | .0580 | .0057 | .0763 | .0086 | .0926 |
| 22 | .0000 | .0064 | .0005 | .0238 | .0017 | .0413 | .0035 | .0586 | .0058 | .0759 | .0087 | .0931 |
| 24 | .0000 | .0070 | .0006 | .0244 | .0017 | .0418 | .0035 | .0692 | .0069 | .0765 | .0088 | .0937 |
| 26 | .0000 | .0076 | .0006 | .0260 | .0018 | .0424 | .0036 | .0598 | .0060 | .0771 | .0089 | .0943 |
| 28 | .0001 | .0081 | .0007 | .0256 | .0018 | .0430 | .0037 | .0604 | .0061 | .0776 | .0090 | .0948 |
| 30 | .0001 | .0087 | .0007 | .0261 | .0019 | .0436 | .0037 | .0609 | .0062 | .0782 | .0091 | .0954 |
| 32 | .0001 | .0093 | .0007 | .0267 | .0019 | .0442 | .0038 | .0615 | .0062 | .0788 | .0092 | .0960 |
| 34 | .0001 | .0099 | .0008 | .0273 | .0020 | .0148 | .0039 | .0621 | .0063 | .0794 | .0093 | .0966 |
| 36 | .0001 | .0105 | .0008 | .0279 | .0021 | .0453 | .0039 | .0627 | .0064 | .0799 | .0094 | .0971 |
| 38 | .0001 | .0110 | .0008 | .0285 | .0021 | .0459 | .0040 | .0633 | .0065 | .0805 | .0095 | .0977 |
| 40 | .0001 | .0116 | .0009 | .0291 | .0022 | .0465 | .0041 | .0638 | .0066 | .0811 | .0097 | .0983 |
| 42 | .0002 | .0122 | .0009 | .0297 | .0022 | .0471 | .0042 | .0644 | .0067 | .0817 | .0098 | .0988 |
| 44 | .0002 | .0128 | .0009 | 0302 | .0023 | .0476 | .0042 | .0650 | .0068 | .0822 | .0100 | .0994 |
| 46 | .0002 | .0134 | .0010 | .0308 | .0023 | .0482 | .0043 | .0656 | .0069 | .0828 | .0101 | .1000 |
| 48 | .0002 | .0139 | .0010 | .0314 | .0024 | .0488 | .0044 | .0661 | .0070 | .0834 | .0102 | .1005 |
| 60 | .0002 | .0145 | .0010 | .0320 | .0024 | .0494 | .0045 | .0667 | .0071 | .0840 | .0103 | .1011 |
| 52 | .0003 | .0151 | .0011 | .0326 | .0025 | .0499 | .0045 | .0673 | .0072 | .0845 | .0104 | .1017 |
| 54 | .0003 | .0157 | .0011 | .0331 | .0026 | .0505 | .0046 | .0678 | .0073 | .0861 | .0105 | .1022 |
| 56 | .0003 | .0163 | .0011 | .0337 | .0026 | .0511 | .0047 | .0684 | .0074 | .0867 | .0107 | .1028 |
| 68 | .0003 | .0168 | .0011 | .0343 | .0027 | .0517 | .0048 | .0690 | .0076 | .0863 | .0108 | .1034 |
| 60 | .0003 | .0174 | .0012 | .0349 | .0027 | .0522 | .0049 | .0696 | .0076 | .0868 | .0109 | .1040 |

For all values of $\theta$ found on this page we may take $\begin{cases} (c+f)\cos\theta = (c+f) \\ (c+f)\sin\theta = 0 \end{cases}$

| θ | 6° |  | 7° |  | 8° |  | 9° |  | 10° |  | 11° |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  | H. | V. | H. | V. | H. | V. | H. | V. | H. | V. | H. | V. |
| 0 | .0109 | .1040 | .0149 | .1210 | .0194 | .1378 | .0245 | .1545 | .0302 | .1710 | .0364 | .1873 |
| 2 | .0110 | .1045 | .0150 | .1215 | .0195 | .1384 | .0247 | .1551 | .0304 | .1716 | .0366 | .1878 |
| 4 | .0112 | .1051 | .0151 | .1221 | .0197 | .1389 | .0248 | .1556 | .0306 | .1721 | .0368 | .1884 |
| 6 | .0113 | .1057 | .0153 | .1226 | .0199 | .1395 | .0250 | .1562 | .0308 | .1726 | .0371 | .1889 |
| 8 | .0114 | .1062 | .0154 | .1232 | .0200 | .1401 | .0252 | .1567 | .0310 | .1732 | .0373 | .1895 |
| 10 | .0115 | .1068 | .0156 | .1238 | .0202 | .1406 | .0254 | .1573 | .0312 | .1737 | .0375 | .1900 |
| 12 | .0117 | .1074 | .0157 | .1243 | .0203 | .1412 | .0256 | .1578 | .0314 | .1743 | .0377 | .1905 |
| 14 | .0118 | .1079 | .0158 | .1249 | .0205 | .1417 | .0257 | .1584 | .0316 | .1748 | .0379 | .1911 |
| 16 | .0119 | .1085 | .0160 | .1255 | .0207 | .1423 | .0259 | .1589 | .0318 | .1754 | .0382 | .1916 |
| 18 | .0120 | .1091 | .0161 | .1260 | .0208 | .1428 | .0261 | .1595 | .0320 | .1759 | .0384 | .1921 |
| 20 | .0122 | .1096 | .0163 | .1266 | .0210 | .1434 | .0263 | .1600 | .0322 | .1765 | .0386 | .1927 |
| 22 | .0123 | .1102 | .0164 | .1272 | .0212 | .1440 | .0265 | .1606 | .0324 | .1770 | .0388 | .1932 |
| 24 | .0124 | .1108 | .0166 | .1 77 | .0213 | .1445 | .0267 | .1611 | .0326 | .1776 | .0391 | .1938 |
| 26 | .0126 | .1113 | .0167 | .1283 | .0215 | .1451 | .0269 | .1617 | .0328 | .1781 | .0393 | .1943 |
| 28 | .0127 | .1119 | .0169 | .1288 | .0217 | .1456 | .0271 | .1622 | .0330 | .1786 | .0395 | .1948 |
| 30 | .0128 | .1125 | .0170 | .1294 | .0218 | .1462 | .0272 | .1628 | .0332 | .1792 | .0397 | .1954 |
| 32 | .0129 | .1130 | .0172 | .1300 | .0220 | .1467 | .0274 | .1633 | .0334 | .1797 | .0400 | .1959 |
| 34 | .0131 | .1136 | .0173 | .1305 | .0222 | .1473 | .0276 | .1639 | .0336 | .1803 | .0402 | .1964 |
| 36 | .0132 | .1142 | .0175 | .1311 | .0224 | .1479 | .0278 | .1644 | .0338 | .1808 | .0404 | .1970 |
| 38 | .0133 | .1147 | .0176 | .1317 | .0225 | .1484 | .0280 | .1650 | .0340 | .1814 | .0407 | .1975 |
| 40 | .0135 | .1153 | .0178 | .1322 | .0227 | .1490 | .0282 | .1656 | .0342 | .1819 | .0409 | .1980 |
| 42 | .0136 | .1159 | .0180 | .1328 | .0229 | .1495 | .0284 | . 661 | .0345 | .1824 | .0411 | .1986 |
| 44 | .0137 | .1164 | .0181 | .1334 | .0231 | .1601 | .0286 | .1666 | .0347 | .1830 | .0414 | .1991 |
| 46 | .0139 | .1170 | .0183 | .1339 | .0232 | .1506 | .0288 | .1672 | .0349 | .1835 | .0416 | .1996 |
| 48 | .0140 | .1176 | .0184 | .1345 | .0234 | .1512 | .0290 | .1677 | .0351 | .1841 | .0418 | .2002 |
| 50 | .0142 | .1181 | .0186 | .1350 | .0236 | .1517 | .0292 | .1683 | .0353 | .1846 | .0421 | .2007 |
| 52 | .0143 | .1187 | .0187 | .1356 | .0238 | .1523 | .0294 | .1688 | .0355 | .1851 | .0423 | .2012 |
| 54 | .0144 | .1193 | .0189 | .1361 | .0239 | .1528 | .0296 | .1694 | .0358 | .1857 | .0425 | .2018 |
| 56 | .0146 | .1198 | .0190 | .1367 | .0241 | .1634 | .0298 | .1699 | .0360 | .1862 | .0428 | .2023 |
| 58 | .0147 | .1204 | .0192 | .1373 | .0243 | .1640 | .0300 | .1705 | .0362 | .1868 | .0430 | .2028 |
| 60 | .0149 | .1210 | .0194 | .1378 | .0245 | .1545 | .0302 | .1710 | .0364 | .1873 | .0432 | .2034 |

For values of $\theta$ on this page $\begin{cases} (c+f)\cos\theta = (c+f) \\ (c+f)\sin\theta \text{ varies between } 0.1\,(c+f)\ \&\ 0.2\,(c+f \end{cases}$

| $\theta$ | 12° | | 13° | | 14° | | 15° | | 16° | | 17° | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | H. | V. | H. | V. | H. | V. | H. | V. | H. | V. | H. | V. |
| 0 | .0432 | .2034 | .0506 | .2192 | .0585 | .2347 | .0670 | .2600 | .0780 | .2650 | .0855 | .2796 |
| 2 | .0435 | .2039 | .0509 | .2197 | .0588 | .2352 | .0673 | .2506 | .0763 | .2655 | .0858 | .2801 |
| 4 | .0437 | .2044 | .0511 | .2202 | .0591 | .2368 | .0676 | .2510 | .0766 | .2659 | .0861 | .2806 |
| 6 | .0439 | .2050 | .0513 | .2208 | .0594 | .2363 | .0679 | .2515 | .0769 | .2664 | .0865 | .2810 |
| 8 | .0442 | .2055 | .0516 | .2213 | .0596 | .2368 | .0682 | .2620 | .0772 | .2669 | .0868 | .2815 |
| 10 | .0444 | .2060 | .0519 | .2218 | .0699 | .2373 | .0685 | .2526 | .0775 | .2674 | .0871 | .2820 |
| 12 | .0447 | .2066 | .0521 | .2223 | .0602 | .2378 | .0687 | .2530 | .0778 | .2679 | .0874 | .2825 |
| 14 | .0449 | .2071 | .0524 | .2228 | .0605 | .2383 | .0690 | .2535 | .0782 | .2684 | .0878 | .2830 |
| 16 | .0451 | .2076 | .0527 | .2234 | .0607 | .2388 | .0693 | .2540 | .0785 | .2689 | .0881 | .2834 |
| 18 | .0454 | .2081 | .0529 | .2239 | .0610 | .2393 | .0696 | .254 | .0788 | .2894 | .0884 | .2839 |
| 20 | .0456 | .2087 | .0532 | .2244 | .0613 | .2399 | .0699 | .2560 | .0791 | .2699 | .0688 | .2844 |
| 22 | .0459 | .2092 | .0534 | .2249 | .0616 | .2404 | .0702 | .2555 | .0794 | .2704 | .0691 | .2849 |
| 24 | .0461 | .2097 | .0537 | .2254 | .0619 | .2409 | .0706 | .2560 | .0797 | .2709 | .0894 | .2854 |
| 26 | .0464 | . 103 | .0540 | .2260 | .0621 | .2414 | .0708 | .2565 | .0800 | .2713 | .0898 | .2858 |
| 28 | .0466 | .2108 | .0542 | .2265 | .0624 | .2419 | .0711 | .2670 | .0804 | .2718 | .0901 | .2883 |
| 30 | .0469 | .2113 | .0545 | .2270 | .0627 | .2424 | .0714 | .2575 | .0807 | .2723 | .0904 | .2668 |
| 32 | .0471 | .2118 | .0548 | .2275 | .0630 | .2429 | .0717 | .2580 | .0810 | .2728 | .0908 | .2873 |
| 34 | .0473 | .2124 | .0550 | .2280 | .0633 | .2434 | .0720 | .2585 | .0813 | .2733 | .0911 | .2677 |
| 36 | .0476 | .2129 | .0553 | .2285 | .0636 | .2439 | .0723 | .2590 | .0818 | .2738 | .0914 | .2882 |
| 38 | .0478 | .2134 | .0556 | .2291 | .0638 | .2444 | .0726 | .2595 | .0819 | .2743 | .0918 | .2887 |
| 40 | .0481 | .2139 | .0558 | .2296 | .0641 | .2449 | .0729 | .2600 | .0822 | .2748 | .0921 | .2892 |
| 42 | .0483 | .2145 | .0561 | .2301 | .0644 | .2455 | .0732 | .2605 | .0826 | .2762 | .0924 | .2896 |
| 44 | .0486 | .2150 | .0564 | .2306 | .0647 | .2460 | .0735 | .2610 | .0829 | .2767 | .0928 | .2901 |
| 46 | .0488 | . 165 | .0568 | .2311 | .0650 | .2465 | .0738 | .2615 | .0832 | .2762 | .0931 | .2906 |
| 48 | .0491 | .2160 | .0569 | .2316 | .0653 | .2470 | .0741 | .2620 | .0835 | .2767 | .0935 | .2911 |
| 50 | .0493 | .2166 | .0572 | .2322 | .0656 | .2475 | .0744 | .2625 | .0839 | .2772 | .0938 | .2915 |
| 52 | .0496 | .2171 | .0575 | .2327 | .0658 | .2480 | .0747 | .2630 | .0842 | .2777 | .0941 | .2920 |
| 54 | .0498 | .2176 | .0577 | .2332 | .0661 | .2486 | .0750 | .2635 | .0845 | .2781 | .0945 | .2925 |
| 56 | .0501 | .2181 | .0580 | .2337 | .0664 | .2490 | .0754 | .2640 | .0848 | .2766 | .0948 | .2930 |
| 58 | .0504 | .2187 | .0583 | .2342 | .0667 | .2495 | .0757 | .2645 | .0852 | .2791 | .0951 | .2934 |
| 60 | .0506 | .2192 | .0585 | .2347 | .0670 | .2500 | .0760 | .2650 | .0855 | .2796 | .0955 | .2939 |

For values of $\theta$ on this page $\begin{cases} (c+f)\cos\theta > .95\,(c+f) = (c+f) \\ (c+f)\sin\theta \text{ varies between } 0.2\,(c+f)\ \&\ 0.3\,(c+f). \end{cases}$

| θ | 18° | | 19° | | 20° | | 21° | | 22° | | 23° | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | H. | V. | H. | V. | H. | V. | H. | V. | H. | V. | H. | V. |
| 0 | .0955 | .2939 | .1050 | .3078 | .1170 | .3214 | .1284 | .3346 | .1403 | .3473 | .1527 | .3597 |
| 2 | .0953 | .2944 | .1054 | .3083 | .1174 | .3218 | .1288 | .3350 | .1407 | .3477 | .1531 | .3601 |
| 4 | .0952 | .2948 | .1057 | .3087 | .1177 | .3223 | .1292 | .3354 | .1411 | .3482 | .1535 | .3605 |
| 6 | .0955 | .2953 | .1071 | .3092 | .1181 | .3227 | .1296 | .3359 | .1416 | .3486 | .1539 | .3609 |
| 8 | .0969 | .2958 | .1074 | .3097 | .1185 | .3232 | .1300 | .3363 | .1420 | .3490 | .1544 | .3613 |
| 10 | .0972 | .2962 | .1078 | .3101 | .1189 | .3236 | .1304 | .3357 | .1424 | .3494 | .1548 | .3617 |
| 12 | .0976 | .2957 | .1082 | .3105 | .1192 | .3241 | .1308 | .3372 | .1428 | .3498 | .1552 | .3621 |
| 14 | .0979 | .2972 | .1085 | .3110 | .1196 | .3245 | .1312 | .3376 | .1432 | .3502 | .1556 | .3625 |
| 16 | .0982 | .2975 | .1089 | .3115 | .1200 | .3249 | .1316 | .3380 | .1436 | .3507 | .1560 | .3629 |
| 18 | .0986 | .2981 | .1092 | .3119 | .1204 | .3254 | .1320 | .3384 | .1440 | .3511 | .1565 | .3633 |
| 20 | .0989 | .2985 | .1096 | .3124 | .1207 | .3258 | .1323 | .3389 | .1444 | .3511 | .1659 | .3537 |
| 22 | .0993 | .2990 | .1099 | .3128 | .1211 | .3253 | .1327 | .3393 | .1448 | .3519 | .1573 | .3641 |
| 24 | .0995 | .2995 | .1103 | .3133 | .1216 | .3267 | .1331 | .3397 | .1451 | .3523 | .1577 | .3645 |
| 26 | .1000 | .3000 | .1107 | .3138 | .1219 | .3272 | .1335 | .3401 | .1456 | .3627 | .1582 | .3649 |
| 28 | .1003 | .3004 | .1111 | .3142 | .1223 | .3276 | .1339 | .3406 | .1460 | .3531 | .1585 | .3653 |
| 30 | .1007 | .3009 | .1114 | .3147 | .1227 | .3280 | .1343 | .3410 | .1465 | .3536 | .1590 | .3657 |
| 32 | .1010 | .3014 | .1118 | .3151 | .1230 | .3285 | .1347 | .3414 | .1459 | .3540 | .1594 | .3661 |
| 34 | .1014 | .3019 | .1122 | .3156 | .1234 | .3289 | .1351 | .3418 | .1473 | .3644 | .1599 | .3555 |
| 36 | .1017 | .3023 | .1125 | .3160 | .1238 | .3293 | .1355 | .3423 | .1477 | .3548 | .1603 | .3669 |
| 38 | .1021 | .3028 | .1129 | .3165 | .1242 | .3298 | .1359 | .3427 | .1481 | .3552 | .1607 | .3573 |
| 40 | .1024 | .3032 | .1133 | .3169 | .1246 | .3302 | .1363 | .3431 | .1485 | .3556 | .1611 | .3677 |
| 42 | .1028 | .3037 | .1136 | .3174 | .1249 | .3307 | .1367 | .3435 | .1489 | .3560 | .1616 | .3680 |
| 44 | .1032 | .3041 | .1140 | .3178 | .1253 | .3311 | .1371 | .3440 | .1493 | .3554 | .1520 | .3584 |
| 46 | .1035 | .3046 | .1144 | .3183 | .1257 | .3315 | .1375 | .3444 | .1498 | .3568 | .1624 | .3688 |
| 48 | .1039 | .3051 | .1147 | .3187 | .1261 | .3320 | .1379 | .3448 | .1502 | .3572 | .1629 | .3692 |
| 50 | .1042 | .3055 | .1150 | .3192 | .1255 | .3324 | .1383 | .3452 | .1505 | .3576 | .1633 | .3696 |
| 52 | .1045 | .3050 | .1155 | .3196 | .1269 | .3328 | .1387 | .3457 | .1510 | .3580 | .1537 | .3700 |
| 54 | .1049 | .3065 | .1159 | .3201 | .1273 | .3333 | .1391 | .3451 | .1514 | .3585 | .1641 | .3704 |
| 56 | .1053 | .3069 | .1163 | .3205 | .1277 | .3337 | .1395 | .3465 | .1518 | .3589 | .1646 | .3708 |
| 58 | .1056 | .3074 | .1166 | .3209 | .1280 | .3341 | .1400 | .3469 | .1523 | .3593 | .1650 | .3712 |
| 60 | .1060 | .3078 | .1170 | .3214 | .1284 | .3346 | .1403 | .3473 | .1527 | .3597 | .1654 | .3716 |

For values of $\theta$ on this page $\begin{cases} (c+f)\cos\theta > 0.9\ (c+f)\text{; therefore we may take} \\ (c+\ \ )\cos\theta = (c+f) \\ (c+f)\ \sin\theta \text{ varies between } 0.3\ (c+f)\ \&\ 0.4\ (c+f). \end{cases}$

Table for $(c+f)\sin\theta$

| $c+f$ \ $\theta$ | 5° | 10° | 15° | 20° |
|---|---|---|---|---|
| 1.00 | .09 | .17 | .26 | .34 |
| 1.50 | .13 | .26 | .39 | .51 |
| 0.40 | .04 | .07 | .10 | .15 |

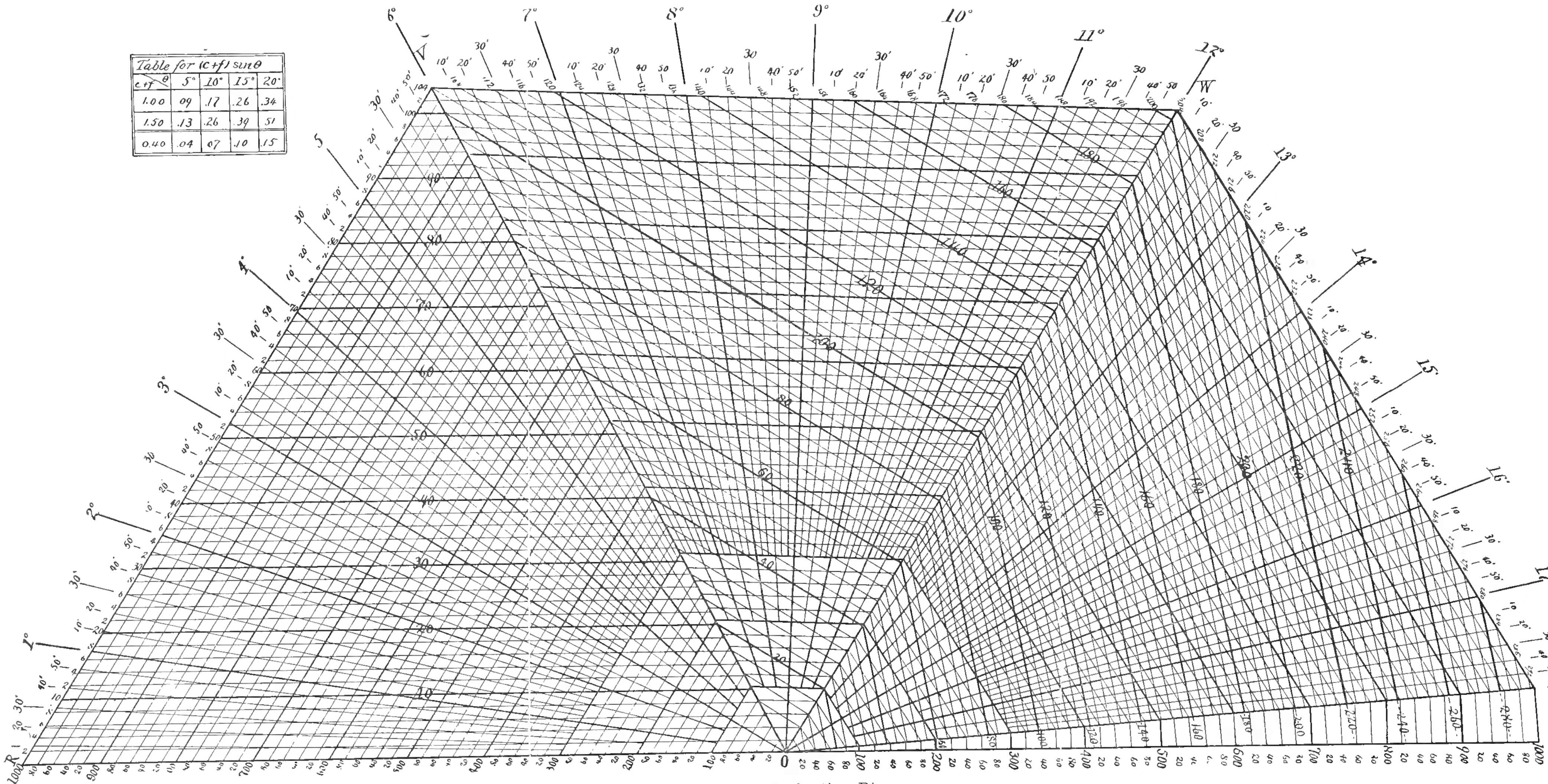


Plate 2.-Stadia Reduction Diagram.

17. Diagram for Vertical Distance.—We will employ equation (9,) and for the present consider only the term $R\frac{1}{2}\ sin\ 2\ \theta$. Draw any line, as $O\ R$, Fig. 9, (Plate 2), and on it construct any scale of equal parts to represent the observed value of the reading $R$; for convenience, we will assume 1,000 as the maximum value of $R$, which will be sufficient for most cases. Through the point $R$, $R$, draw another line $R\ V$, making the angle $V\ R\ O$ about 60°; on this line lay off another scale of equal parts, making the unit of this scale, say ten times, greater than that of $R\ O$. Put $R = 1{,}000$ and $\theta = 1°$ and compute the value of $R\frac{1}{2}\ sin.\ 2\ \theta$; from the corresponding point of the scale of $R\ V$, draw a line to $O$, and mark it 1°, as in the diagram. In a similar manner, draw lines corresponding to $\theta = 2°$, 3°, 4°, etc., and such fractions of degrees as may be desired. Complete the diagram by drawing lines, parallel to $R\ O$, through the principal points of division of $R\ V$.

If the construction of the diagram, with the proportions as above, be continued much beyond 5° or 6°, the intersections will become too oblique for accuracy. This difficulty may be met by continuing the scale of $R\ V$ along $V\ W$, the angle $O\ V\ W$ being about 60°. The lines of the scale by which $V$ is determined will no longer be parallel to the line along which $R$ is measured. The position where the line for $V = 100$ intersects the line $V\ O$ is found by the diagram as already explained; to find where the line for $V = 100$ intersects $W\ O$, use the equation $V = R\frac{1}{2}\ sin.\ 2\ \theta$, putting $V = 100$, $\theta = 12°$ (the degree represented by $O\ W$), and solve for $R$. For the example cited $R = 490.7$; hence the line for $V = 100$ in the triangle $V\ O\ W$ must be drawn from its intersection with $V\ O$ to a point on $O\ W$ corresponding to $R = 490.7$. Parallel to the line drawn as above, draw other lines through the intersections of the elevation lines of the triangle $R\ O\ V$ with $O\ V$[4].

This process may be continued until the probable maximum value of $\theta$ has been reached, or until the diagram completes the circle.

If the scale of $R\ V$ is made smaller, a greater number of degrees will be included, but the diagrams will be less accurate. For convenience in using the sections $V\ W$, etc., repeat the numbering of $R\ O$ along $V\ O$, $W\ O$, etc.

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[4]This ingenious device was proposed by one of the author's students, Alonzo Courtney; Prof. Robinson meets the difficulty by constructing separate diagrams.

Concerning the second term of (5′), $(c+f) \sin.\theta$, we notice that it is always small and varies but slightly for a change of a whole degree in $\theta$; therefore calculate the values of $(c+f) \sin.\theta$, and place them in the supplemental tables; as on the diagram. For values of $(c+f)$ which differ from those given, the correction may easily be interpolated with ample accuracy. If the particular value of $(c+f)$ is known, the correction may be written outside of the diagram between the radii for the different degree.

To use this diagram, find on $O\ R$ the number corresponding to the reading of the stadia; then follow a line from this point parallel to $R\ V$ or $V\ W$, etc., until it intersects the line drawn from $O$, which represents the number of degrees in the observed vertical angle; then follow a line parallel to $O\ R$ or $O\ V$, etc., to the scale $R\ V\ W$; the intersection on this scale, plus the small correction $(c+f) \sin.\theta$, is the vertical co-ordinate sought.

18. Diagram for Horizontal Distance.—It is best to use equation (4′), and find the correction, $R \sin.^2\theta$, which is to be subtracted from $R$. The construction of the diagram for this case is similar to the previous one. The completed diagram is shown in Fig. 10, (Plate 1). The correction, $R \sin.^2\theta$, as found from this table is to be subtracted from $R$, and the term $(c+f) \cos.\theta$, as found from the little table on the diagram, added, to obtain the required horizontal distance.

These diagrams are very carefully drawn, and can be used in reducing actual stadia readings.

19. Sources of Error in Stadia Work.—Stadia work is subject to many of the errors discussed in the articles on the transit[5] and the level[6]; the additional errors peculiar to the stadia are, 1, inclination of the rod; 2, error in the value of $\frac{f}{i}$; and 3, error of observation.

Inclination of Rod.—To investigate the effect of a slight inclination of the rod, let $R =$ the reading when the rod is vertical; $R' =$ the reading when the rod makes an angle $v$ with the vertical.

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[5] Engineering News. Vol. 17. p. 305-6.

[6] Engineering News. Vol. 17. p. 197-9.

Then $R = R' \cos. v$ very nearly, and the error $= R - R' = R (1 - \cos. v) = R \sin.^2 \frac{v}{2} = \frac{R}{7000} v^2$, in which $v$ is in degrees. Notice that the error increases as the square of the inclination, which shows the necessity of some means of plumbing the rod. Without some means of keeping the rod vertical, this would undoubtedly be the principal source of error in stadia work. On the U. S. Lake Survey, an inclined support was used to steady the rod and prevent its inclining to or from the instrument; a light tripod was also sometimes used for the same purpose.

VALUE OF $\frac{f}{i}$. If the value of $\frac{f}{i}$, or the corresponding unit on the rod is not correctly determined, the results will be incorrect, although the relative position of points will be all right. The correct value of $\frac{f}{i}$ is easily found in the beginning, and with the methods of attaching the hairs described in Sec. 4 there is but little danger of its changing.

ERRORS OF OBSERVATIONS.—The principal item under this head is the inaccuracy in estimating the position of the hair on the rod; the amount of this error depends upon the size of the space on the rod, corresponding to a unit on the ground, and upon the form of graduation. For those rods with which the sides of the hairs are observed, this error is still further increased by the uncertainty due to the thickness of the hairs, and to any inequality in their thickness, as magnified by the eye-piece; this element of uncertainty does not exist with the graduations shown in Figs. 4—8.

Imperfect focusing, either of rod or hairs, is a source of error, because it is only when both are in focus at the same time that the assumed relations exist. If the rod is not in focus, the image covers too much space, which makes the distance too small. If the cross hairs are not in focus, the distance will be read too small or too great according as the hairs are on one side or the other of the focus. If there is no parallax in the telescope, there will be no error from this cause.

The indistinctness of the image due to an unsteadiness of the atmosphere produces an error by making the image too large, and therefore, the distance too small. The only remedy is to wait for better atmospheric conditions.

Another somewhat common error is miscounting the reading so as to make errors of 10, 1000, etc., feet. These errors may be prevented by care, or checked by double readings; or, instead of making an entirely new reading, the two halves of the visual angle may be read and their sum used to check the reading of the two outside hairs.

20. LIMITS OF PRECISION.—With reasonable care a high degree of accuracy can be attained in stadia measurement. The degree of precision is dependent upon the magnifying power of the telescope, the length of sight, and the ratio of the space on the rod to the corresponding space on the ground. It is sometimes claimed that stadia measurements are more accurate than chaining, but from the nature of the principles involved, it cannot be so for equally favorable conditions; nevertheless, under many circumstances, the stadia is more accurate than chaining.

The following result seem to have been obtained with an ordinary engineer's transit; they are not selected, but are all the results of actual practice that could be found. In comparing results we must distinguish between the error of simply finding the distance by the stadia, and the final error of a series of courses, the lengths of which were determined by the stadia; the latter involves the error of measuring the angles as well as the distances.

To show the degree of precision obtained in measuring horizontal distance, we have the following: Three measurements, each of six distances, from 50 to 500 ft. made to test the accuracy of the stadia measurements,[7] the readings being made with targets, show an average probable error for a single sight of 1 in 4,000.[8] On the U. S. Lake Survey, three measurements of a base line, gave errors of 1 in 1000; 1 in 1635 and 1 in 1888.[9]

Only the following can be found concerning the accuracy with which vertical distances can be determined. "Courses have been run one to six miles, over heights of 150 to 200 feet in which the

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[7]Van Nostrand's Engineering Magazine. Vol. 30, p. 319.
[8]Van Nostrand's Engineering Magazine. Vol. 30, p. 476.
[9]Jour. Franklin Inst. Vol. 49, p. 74.

final error in height ranged from 0 to 1.5 feet, with no more than ordinary care."[9]

Concerning the final error of a series of courses, the lengths of which were determined by the stadia, we have the following: "The stadia was used for getting the topography of some densely wooded timber land in the summer of 1863. The courses were so run as to connect points of triangulation from one to four miles apart. The distances from point to point along the courses, ranged from 100 to 500 feet. The latitudes and departures were subsequently computed with the object of finding the error of stadia measurement. The results obtained were about 1 in 800, 1 in 1,000. and 1 in 1,100." [10]. On the U. S. Lake survey, in computing the coordinates of stadia work for 1875, the average amount discovered in 141 lines, varying between 3,200 feet and 22,000 feet (mean 8,080 feet), when compared with lines determined by triangulation or chaining, was found to be 1 in 649; the maximum limit of error was put at 1 in 300.[11] In a survey of Red river, conducted by the U. S. Army Engineers and extending over about 70 miles, the stadia work and chaining were checked at 8 points along the line, and showed an average difference of 1 in 430.[12]

21. Practical Hints.—The many advantages of stadia measurements in surveying need not be dwelt upon; they are self-evident to those acquainted with the principles. The general ease and quickness of telescopic measurement has always been recognized. In broken country, the stadia can be readily used where the use of the train is not practicable at all. The stadia is specially applicable to topographical surveying, to the topographical work of a railroad survey, and to determining lengths of sights in leveling.

It could be profitably employed in ordinary land surveying; the trouble of training chainmen, or of trusting work to inexperienced chainmen, would be avoided, and the delays of chaining would be saved. Instead of two chainmen only, one rodman is needed, and his duties are very simple; thus all the important work could be done by the surveyor himself. This would secure greater accuracy where most needed, in the measurement of the distances. It would require the substitution of a telescope for the ordinary compass-

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[10] Jour. Franklin Inst. Vol. 49, p. 74.
[11] Final Report U. S. Lake Survey, p. 34.
[12] Report of Chief Engineer, U. S. A., 1873, p. 638.

sights, or, better the employment of a transit instead of a compass, and the reading of the angles independent of the needle.

The stadia may also be used for under-ground surveying, where chaining is peculiarly disagreeable and difficult; in under-ground work, a box lighted on the inside, with a glass front on which the graduation is painted, is substituted for the rod.[13]

22. To obviate the difficulty of estimating a fraction for both hairs, set one of them at the beginning of a division; this will produce a little error in the vertical co-ordinate, but generally so little as to be inappreciable. With long sights or small angles, it is still more convenient and also sufficiently accurate, to set one of the hairs upon one of the more prominent divisions; as the even foot-mark, etc.; by remembering that each hair may be moved either of two ways and moving the telescope so as to produce the least displacement, this method of placing one of the hairs at an even division can frequently be used without any error, and thus greatly acilitate the work.

## APPENDIX.

### LOCAL ATTRACTION IN LAND SURVEYING.

It is very common to hear the remark that the common compass could not be used because of local attraction. It is well known that there are many localities in which the needle is deflected by the attraction of iron, etc.; but the object of this article is to show that no matter how much the local deflection, the common compass can be made to give as accurate results as though there was no such disturbing influence.

This method of using the compass seems not to be either understood or practiced by surveyors generally; and this, even though the essential features of the method are given, rather obscurely, perhaps, in a book which is very generally used, "Gillespie's Land Surveying," page 143. This is the more surprising when it is known that instead of using the simple compass in this very easy way, the complicated, expensive, and it is believed, less accurate, solar compass has been employed. The much maligned magnetic compass is an instrument capable of much greater precision than it is generally credited with; with the solar compass the case is nearly reversed.

[13] Jour. Franklin Inst. Vol. 55, pp. 384-7.

CASE I.—*When only the area is wanted.*—Place the compass at each corner of the field in succession and determine the bearing of the two lines meeting in each station. Call the sight made in the direction in which the surveyor goes around the field a *fore-sight*, and that made in an opposite direction a *back-sight*, keep one end of the box front on fore-sights and the other end front on back-sights; but if one sight of the compass consists of a slit and the other of a hair, the same end must necessarily be kept next to the eye, therefore read the letters from one end of the needle for fore-sights, and from the opposite end for back-sights, and the degrees from the same end. When all the bearings have been taken, the record will be similar to the first three columns of the following table, with the exception of the small figures written above each bearing which will be explained presently.

| Station. | Back Sight. | Fore Sight. | Correction. |
|---|---|---|---|
| $N$ | $S\ 78\quad 00\ E$ | $N\ 16\quad 05'\ W$ | 0 |
| $Q$ | $N\ \overset{16}{16}\quad \overset{05}{45}\ W$ | $N\ \overset{80}{81}\quad \overset{30}{10}\ W$ | $40'\ F$ |
| $S$ | $N\ \overset{80}{80}\quad \overset{30}{05}\ W$ | $S\ \overset{86}{46}\quad \overset{10}{35}\ W$ | $25'\ B$ |
| $U$ | $S\ \overset{46}{46}\quad \overset{10}{35}$ | $S\ \overset{83}{82}\quad \overset{00}{35}\ E$ | $25'\ B$ |
| $V$ | $S\ \overset{83}{82}\quad \overset{00}{25}\ E$ | $S\ \overset{78}{77}\quad \overset{00}{25}\ E$ | $35'\ B$ |

Owing to changes in the declination, it is not probable that every fore-sight will agree with its corresponding back-sight. However, if the two bearings at any station have been taken in a short time of each other, the notes may be corrected so as to wholly eliminate the effect of any change of the needle.

Since the assumption is that only the area is required, it is immaterial where we commence to correct the angles. Therefore we may assume the bearings taken at any station, say $N$, are correct. The back-sight from the next station, say $Q$, (it is immaterial whether the angles are connected in the order in which they were surveyed or not), differs from the fore-sight from $N$ by 40′, therefore the bearings at $Q$ are in error 40′, to correct which we may conceive that the needle should be moved 40′ in a direction from the north toward the east, *i. e.*, in the direction in which the hands of a watch move. It makes no difference, in this matter, whether we consider *N*, *E*, *S* and *W* in their true relations or in their reversed position, as given by the face of the compass. We will assume them to be in their true relations; and briefly say that the correction at *Q* is 40′ forward. This is the correction which is to be applied to the fore-sight at *Q*, and is written in the column headed "correction." The corrected fore-sight from *Q* is 80° 30′; the back-sight from *S* should agree with this, but there is a difference of 25′. Therefore the correction at *S* is 25′ backward; and the corrected fore-sight is 46° 10′. The corrections and corrected angles for the other stations are formed in a similar manner, and the corrected bearings written above the observed value, as in the table.

The agreement of the last corrected fore-sight with the first corrected back-sight is a valuable check on the accuracy of the work, and as a rule, this difference should not exceed five or ten minutes.

Case II.—*When the area and also the true bearings are desired.*—The case requires one of two things, either 1: That several successive back-sights shall agree with their corresponding fore-sight before either have been connected; or 2, that there shall be a true meridian which can be connected with a corner of the field by lines whose bearings are to be found as those of the boundary lines. In the first instance, it may safely be assumed that, since a number of the fore-sights and back-sights agree, there has been no change in the middle between those stations; therefore the observed bearings may be assumed to be the true bearings. Having some of the correct bearings, if there are any stations at which the back-sight and fore-sights do not agree, they may be corrected as in Case I.

If the declination is not set off on the compass, it may be applied as a correction to those stations at which the back and fore-sights corresponded as above,and the correction carried on to those stations at which a difference was found.

In the second instance as above, the true bearings of the several lines can be found in succession beginning at the meridian. The following example will illustrate this method: *Z* is not a corner of the field but a point on a known meridian; the line *Z Q* is run to determine the declination at *Q*, a corner of the field. All of the remaining stations are corners of the field.

| Stations. | Back Sights. | Fore Sights. | Corrections. |
|---|---|---|---|
| *Z* | $N\,\overset{0}{3}°\ \overset{00}{50}\ W$ | $N\,\overset{13}{69}\ \overset{10}{20}\ E$ | 3° 50′ F |
| *Q* | $N\,\overset{73}{69}\ \overset{10}{15}\ E$ | | 3° 55 F |
| *Q* | $N\,\overset{14}{10}\ \overset{40}{45}\ E$ | $S\,\overset{93}{89}\ \overset{40}{45}\ W$ | |
| *T* | $S\,\overset{93}{89}\ \overset{40}{35}\ W$ | $S\,\overset{43}{39}\ \overset{5}{00}\ W$ | 4° 05 F |
| *U* | $S\,\overset{43}{39}\ \overset{5}{10}\ W$ | $N\,\overset{14}{10}\ \overset{45}{40}\ E$ | 3° 55 F |

Notice that the first two lines of the above record are preliminary to the survey of the field, and are required only to find the declination at *Q*. Having found the declination or correction at *Q*, the bearings are corrected as in the previous case. The back-sight from *Q* to *U*5′ from the fore-sight at *U* to *Q*, which shows that there was an error or inaccuracy of 5′ in reading the angles.

Undoubtedly the method of correcting with a meridian as just explained, is more exact than the previous method, although the latter is more convenient and shorter and is the one which will generally be used in practice. However if the bearings show local attraction at a number of stations the first method of Case II cannot be applied while the second will give strictly correct results whatever the number of stations at which local attraction exists or whatever its amount.

The back-sights should always be taken, although it may be nearly certain that no local attraction exists, for this is a check against possible local deflection of the needle, and also against errors in reading the needle.

Sources of Error.—The error of compass work may be grouped in one of the following classes: 1st. Instrumental errors. 2nd. Instrument or flag-pole not being set at the right place. 3rd. Errors of chaining. The first have been discussed already.† The second

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are small and easily remedied. The flag-pole should be set vertical but, as a precaution, sight as low on it as possible.

The instrumental errors are due either to lack of adjustment, or to errors in the practice. In using any instrument, it is a good rule to adjust it carefully in all particulars and then use it in such a way as to eliminate any residual errors of adjustment, or, in other words, adjust it carefully and then use it as though it were not adjusted. In using a compass, guard against errors of coincidence of magnetic and geometrical axes of the needle, and also of straightness of needle, by reading always the same end of the needle. Sighting nearly horizontal will reduce the possibility of error due to lack of perfectness in the adjustment of the levels.

To guard against the possibility (1) that the magnetic axis of the needle may not coincide with its geometric axis, (2) that the zero of the vernier may not coincide with the line of sight, and (3) errors in straightness of the needle, determine the declination by setting the compass upon a true meridian, sighting along it, and then moving the vernier until the needle reads zero. This also provides against errors in charts or tables giving the declination, and also against changes in the declination since the chart was made. This observation should be made about 10 A. M., or 6 P. M., as then the effect of the daily variation is nearly zero.

It is very important that the three sources of error just mentioned should be eliminated. For one or the other or all of these reasons the bearing of a line as read by several instruments at the same time and place, and by the same person will often differ 10 to 15 minutes. For an example of five instruments which differed 20 minutes, see the Fifth Annual Report of the Ohio Society of Surveyors and Civil Engineers, page 71; the same page gives an account of four instruments which differed 31 minutes. Notice that these instruments were read at the same time and place, but by different men. On page 74 of the above report, is an account of six needles made specially for this purpose, as nearly alike as possible, read under identically the same conditions in every respect, which differed 10 minutes. In all work in which the location of a line is to be determined by its recorded bearing, the above precaution is exceedingly important.

The writer believes that we shall never be able to follow all the excentricities of the variation of the declination, but he also believes that the method of using the compass here advocated, together with the precautions mentioned above, is practically independent of all such variations.

The most common error in the practice is reading N. for. S., etc., and 28° for 32°, etc. The only check or remedy is carefulness. After the needle has come to rest, it should be tapped gently to destroy the effect of any adhesion to the pivot.

LIMITS OF PRECISION.—The error of taking a bearing is made up of the error of sighting, of sluggishness of the needle, and error of reading the needle. The last is the greatest. With a compass graduated to half a degree, the angles can be estimated to the nearest five minutes, and the maximum error of a bearing should not exceed ten minutes. The average probable error of a bearing for a class of ten, as deduced from the error in closing the angles around a field, was 1.64 minutes. That the average probable error is 1.6 minutes show that bearings can be read certainly to the nearest five minutes. For the purpose of this record, the same men read the bearing of a flag-pole at one, two and three hundred paces, with probable errors of $2\frac{2}{3}$, $3\frac{1}{3}$ and $3\frac{1}{2}$ minutes respectively, the sun and wind being in the observer's face while sighting. This would seem to indicate that, under favorable circumstances, the probable error of a bearing should be less than two minutes, and under unfavorable conditions it may be more than three minutes. The angular error of closing a field is equal to the square root of twice the number of the sides, multiplied by the error of a single bearing.

The error of an area found by compass surveying is made up of the error of chaining, of reading the bearings, and of computations. Land surveyors in actual practice, find that the chaining as ordinarily done is much the greatest source of error. With proper care the error of chaining should be much less than the error of the bearings. Since one in fifty-seven corresponds to 1°, the above maximum error of 10 minutes corresponds to an error of one in three hundred and forty-four, and the probable error of 1.64 corresponds to one in two thousand, which is greater than that of chaining under the same conditions. Since the computations are self-checking at every step, there is little probability of any blunder in this part of the work; and as the work may be carried to any desired number of decimals, the inaccuracies of the computations can be made much less than those of the field work; therefore we conclude that the accuracy of the area depends mainly upon the accuracy of reading the needle. An area found by compass surveying should be true to within three or four thousand.

BALANCING THE WORK.—When the angles are measured as above, an answer can be given to the question, which is frequently asked, viz:—How great a difference between the sums of the + and — latitude and departure is admissible?

Let $C$ = the linear error of closing due to the chaining; $C$ is equal to the perimeter of the field ($= P$) divided by the distance $= d$) in which the error of chaining s a unit, that is

$$C = \frac{P}{d}$$

Let $A$ = the linear error due to the measurement of the angles. The difference between the last fore-sight and the first back-sight is the angular error of measuring the angles; we will assume this difference to be in minutes and represent it by $a$. It cannot be known how this error occurred; whether it occurred all in one sight, whether it occurred equally among the sides, or among the sides in proportion to their length. But as the last is most probable, and also as it gives the largest linear error in closing, we will assume the error $a$ to have occurred among the sides in proportion to their lengths. Hence to reduce $a$ to its linear equivalent, we must multiply it by the length of the perimeter ($= P$) and divide it it by the distance at which a unit subtends an angle of one minute, or

$$A = a \frac{P}{3438} = \frac{3\,a\,P}{10{,}000} \text{ nearly.}$$

Let $E$ = the total error, *i. e.* the error due to chaining and measuring the angles. Notice that the actual error, $E$, is the hypothenuse of a right angled triangle of which the differences of the latitudes and departures are the other sides. Hence, if $L$ = the difference of the + and — latitudes, and $D$ = equals the same for the departures, $E = \sqrt{D_2 + L_2}$.

By the theory of probabilities. we know that

$$E = \sqrt{A^2 + C^2} = \sqrt{\left(\frac{P}{d}\right)^2 + \left(\frac{P\ a}{3438}\right)^2} = P\sqrt{\left(\frac{1}{d^2} + \frac{a^2}{12000000}\right)} \text{ nearly.}$$

Equating these two values of $E$, we get

$$D^2 + L^2 = P^2 \left(\frac{1}{a^2} + \frac{a^2}{12000000}\right)$$

We may simplify the matter a little further by finding a relation between $D$ and $L$. Assume that the sum of the latitude $= n$ times the sum of the departures, $n$ is easily determined from the computations, or it can be estimated in the field or from the plot with sufficient accuracy. From the theory of probabilities, we know that $L = \sqrt{n}\,D$; then the above formula becomes

$$D^2 + L^2 = (1 + n)\,D^2 = P^2 \left(\frac{1}{a^2} + \frac{a^2}{12000000}\right)$$

To illustrate the method of applying this let us assume that in surveying a field whose perimeter is ten chains the difference between the first back-sight and the last fore-sight was 10′; we will also assume that the conditions were such that we might expect an error of 1 in 2000 in chaining. The above formula then becomes.

$$D^2 + L^2 = 10^2\left[\left(\frac{1}{2000}\right)^2 + \frac{10^2}{12000000}\right] = \frac{1}{40000} + \frac{1}{1200} = .00086, \text{ or}$$

$$(1 + n)\,D^2 = P^2 \left(\frac{1}{a^2} + \frac{a^2}{12000000}\right) = .00086 \text{ chains.}$$

If the field is twice as long north and south as east and west, $n = 2$, and hence $D = .017$ chains or 1.7 links. This shows that the error in the departures should be about 1.7 links. The error in the latitude should be $1.7\sqrt{n} = 1.7\sqrt{2} = 2.4$ links. Results much greater than these show an error in the work other than the usual inaccuracy.

Finally, by reversing the problem, knowing the error of closing the angle, and the errors in balancing the latitudes and departures, we may reverse the above formulæ and compute the error of chaining.

The writer does not wish to be understood as advocating that the compass is the best instrument for land surveying, or that it is sufficiently exact; he only wishes to call attention to a better method of using an instrument that is not without merits, but which can never become an instrument of the highest precision.

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## ACCURATE CHAINING.

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Chaining is a very simple operation, and doubtless, everyone thinks he knows all about it. Many there are who will claim that they can do absolutely accurate chaining. But the results of inaccurate chaining in the past is a continual annoyance to the present generation of surveyors and engineers, and the day of unnecessarily inaccurate work is probably not yet passed. The following remarks are intended to apply specially to land surveying.

It goes without saying that the means used in the past have been rude, and the results correspondingly inaccurate; but, owing to the increase in value of land, and the reflex action of the general advancement in other directions, there seems to be a growing desire for increased accuracy in the methods of land surveying. To assist in this laudable reform is the purpose of this article, although increased accuracy in chaining is but one of the steps leading to the desired end.

Before considering the several errors to which chaining is liable, it will be well to notice that, in all measuring operations, the observer should carefully distinguish between two classes of errors, viz:—*compensating* errors, or those which are as likely to be plus as minus, and tend to balance each other; and *cumulative* errors, or those which always have the same sign, and affect the final result in the same way. This distinction is very important. The observer should avoid errors which usually occur in a single direction, but he need not always take the greatest care to avoid errors which are as liable to be negative as positive. An apparently inappreciable but cumulative error may, in the course of a series of observations, amount to more than a much larger but compensating error. The effect of compensating errors is reduced nearly to zero simply by multiplying the number of observations: but cumulative errors should be avoided entirely, or observations made by which they may be corrected.

The uncertainty in the length of a line due to compensating or accidental errors, varies as the square root of the number of units in the line, while the effect of cumulative, or constant errors varies directly as the length. The whole number of cumulative errors remain uncorrected, while only the square root of the compensating errors are uncompensated. For example, if the chain is 0.1 of a link too long, a line 25 chains long will be recorded 2.5 links too short; but if the pin is sometimes set 0.1 of a link beyond the end of the chain, and sometimes the same amount behind, the final error at the end of the line, due to this error is $0.1\sqrt{25}$ or only 0.5 link.

If the head chainman has a fixed habit of always setting the pin beyond the end of the chain, then this becomes a cumulative error, and varies as the distance. This illustrates that in the prosecution of any work, it is desirable that the operator should be cognizant of the nature and importance of every source of error The more thorough and complete his knowledge in this respect, the more readily and accurately will be able to decide what source of error may be wholly neglected, what may be partially provided against, and what must be carefully avoided or eliminated. This knowledge is conducive of both greater accuracy and of economy of time and effort, for the observer then knows what might otherwise have been attended to with considerable care, may be neglected, thus leaving him free to give all his attention to the weakest link in the chain of observations. It enables the observer to correctly proportion his pains to the degree of precision required. A good observer is one who is able to take just care enough to attain the desired accuracy, without wasting time and energy in uselessly perfecting certain parts of the work. He must be able to discover the relative accuracy required in different parts of a complete observation. All this calls for a clear understanding of the causes of error, and the ability to reason out their effect upon the result.

Sources of Error.—The sources of error in chaining are: 1, incorrect length of chain; 2, kinking of the chain and bending of the links; 3, the chain not being drawn straight; 4, unequal tension of the chain; 5, expansion and contraction with changes of temperature; 6, errors of lining the fore chainman; 7, not placing the pin at the end of the chain; 8, drawing the pin by hanging the back handle over it; and 9, such errors as miscounting tallies, or chains, counting from wrong end of chain, making a mistake of 10 in the number of links, reading 18 for 22, 37 for 43, etc. This last class of errors are much too common, but can be obviated by care and thoughtfulness.

1. This is a constant or cumulative error; when one is desirous of attaining the last degree of accuracy, this is the most difficult error to eliminate; even for the degree of accuracy attained in land surveying, it is an important element. In some classes of work, relative accuracy is sufficient, in which case an arbitrary standard may be chosen. In the location of permanent land lines, nothing less than absolute accuracy should be aimed at. Probably the only way to reach a reasonable degree of accuracy is to have the length of a number of standards determined by some competent authority, as the coast survey or an authorized representative.

Some recent experience of the writer will illustrate the difference which exists between supposed true standards. He had occasion to compare a 100 foot steel tape just from the shop of a reputable manufacturer, which was "guaranteed to be true to U. S. standard," with a 20 foot pole said to have been pronounced correct by the "U. S. A. engineers," and also with a standard derived, certainly with inappreciable error, from two 2 ft. steel rules made by the best tool makers in the U. S. The errors of the intercomparisons were inappreciable. The tape was 0.95 of an inch short by the first, by the second it was 0.45 of an inch short. A subsequent comparison at the office of the Mississippi River commission shows the tape to be $0.256 \pm 0.07$ of an inch short. The above differences are practically independent of temperature correction.

After the chain has once been adjusted to the standard, it should be frequently tested. The common chain has 600 wearing surfaces, and if each wears only 0.1 of an inch, the length is increased 6 inches. Mud and ice in the joints have a similar effect in the opposite direction. A tempered steel chain with brazed links, lengthened half an inch by chaining 70 miles. It is nothing uncommon to find chains differing 1, 2 or even 3 inches.

2. The simplest remedy for kinking of the chain and bending of the links is to use a steel tape or wire. A tape or wire obviates the effect of wearing of the links of a chain, as well as having some other advantages.

A steel wire makes an excellent substitute for a chain. The end of the unit may be indicated by a small tube or rod with a hole through it fastened near each end. The distance may count from corresponding ends of the tube or rod or from a V-shaped groove around the rod; for accurate work the former is used, the end being marked on the top of a stake or a board carried for that purpose; for less accurate work, the pin is to be set in the groove. There are several advantages in this way of putting the pin, which will appear on reflection. A handle can be formed by passing the wire through a piece of wood to protect the hand, and then forming it into a loop. The wire should be steel, at least $\frac{1}{8}$ of an inch in diameter. The whole can be made in any shop at a trifling cost.

3. The time required to get the chain straight between pins can be greatly lessened by the fore chainman being careful to walk on line. The error is cumulative.

It is sometimes advocated that, since the chain is seldom, if ever, perfectly straight, it should be made a little longer than the standard, so that the full length of the standard may be laid off each time. The instructions issued by U. S. Land Office to Surveyors General states (report for 1880, page 20, for the first (?) time that "the 66 feet chain shall be 66.06 feet," for the above reason. The French have, or at least had (Gillespie's Land Surveying, page 18, foot-note) a similar practice, the addition being from 1 in 1000 to 1 in 2000. The remedy is at best, very questionable.

4. The effect of a difference in the pull upon the chain or tape is small, and varies as the length of the tape, inversely as the cross-section, and is less for steel than iron. With a steel wire $\frac{1}{8}$ in diameter a difference of 10 pounds will make a difference of one twenty-fifth of an inch. For a chain with bronzed links it would be more owing to the flattening of the rings. It could be determined by trial with a spring balance. It is generally compensating.

5. Only with the extremes of temperature will the expansion and contraction of the tape be appreciable in ordinary work. In accurate work it is one of the chief sources of error, and one very difficult to determine. The temperature of the chain, when the sun is shining, is often quite a good deal higher than that of the atmosphere. For a 100 feet tape, a change of 30° makes a difference in

length of one-fourth of an inch. Generally this error is cumulative.

6. The error due to not setting the fore pin exactly in line is generally overestimated in proportion to the other errors. An undue amount of time is therefore given to lining in the fore chainman. With a 66 feet chain, if the pin is out of line 1 foot, the error is $\frac{1}{10}$ of an inch; the error varies inversely as the length of the chain. It is cumulative. There is no reason why a wire 200 or even 300 feet long should not be used for most of the work where a 66 feet chain is now employed; such a wire would be both more accurate and more speedy.

Vertical inequalities in the ground produce errors similar to those in aligning the fore chainman, and unfortunately in many cases this element limits the degree of accuracy attainable. It is cumulative. It can only be obviated by suspending the tape or wire tween fixed supports, which is complicated and expensive. Supporting the tape by hand and plumbing down is of very little advantage.

7. Many chainmen hold the pin, while setting it, in such a manner that the point enters the ground considerably in front of the end of the chain; when the rear end is brought forward, it is laid on the ground, thus introducing considerable error. The remedy is obvious.

The very general practice of placing the forward handle against one side of the pin, and the back one against the other side, can not be considered very precise. The same side of the pin should be used both times; the length of the chain is then from outside of one handle to the inside of the other.

8. The back handle should not be dropped over the pin until the chain is in position preparatory to lay off the distance, or some device similar to the V-shaped groove, described above, should be used.

Limits of Precision.—It is very desirable that each surveyor should know the uncertainty of his ordinary work; if this could be stated for each method of measuring and for the different kinds of ground it would be very instructive. Each surveyor should learn by repeated trials the amount of labor required to attain a given degree of accuracy. Everybody knows that the labor required increases more rapidly than the degree of precision attained; the more accurate the work the greater the difference.

The method of least squares shows that in a series of observations affected only by accidental errors, the final error varies as the square root of the number of observations. Now, many of the errors of chaining are cumulative, that is, conspired to make the recorded length too long; hence the method of least squares is not applicable, although it is sometimes so applied. If the error is assumed to vary as the square root of the distance, *i.e.* as the number of times the unit is applied, other things being equal, the longer lines will show apparently the more accurate work; if it is taken proportional to the distance, the longer lines will appear less accurate as between two lines measured under the same conditions, the errors should vary as the square root of the distance. That is, if the difference between two measures is multiplied in a quarter of a mile, it should be twice as much in one mile.

A few words are needed about the difference between real and apparent errors. If a line is twice measured with the same chain by the same men under the same conditions, the difference between the two measurements represents the difference between the accidental errors each time, and does not show the real error in the observed length. Any constant or cumulative error, as an incorrect length of the chain, would be the same in each. This distinction is very important in discussing the error of any series of observations.

The following data is given to assist the surveyor in deciding what he will call accurate work. Unfortunately, there is very little on this subject to be found in engineering literature. If any one having reliable data on this subject will publish it with full details, it will afford an opportunity for some instructive comparisons which may ultimately lead to an increased precision.

Burt (Key to the Solar compass, page 35) found that "the average error in surveying the public lands to be 1 in 500, and sometimes 1 in 220 as between two sets of chainmen; under the most favorable conditions it was 1 in 1600.

The writer's students, in ordinary class work in land surveying, measured lines from 200 to 1000 feet long, with a 66-foot chain, and repeated with a difference of 1 in 15,000 to 1 in 20,000 for the same men; for different men on different days with different chains (compared however with the same standard), the maximum difference was 1 in 800 to 1 in 1000, the average error being 1 in 3000 to 1 in 4000. The ground was favorable, but the work was done with the ordinary expedition of actual practice. Seventy miles of the finished track of the Illinois Central R. R., was measured in 1876 with a 100 foot steel chain, and re-measured in 1885 with a 100-foot steel tape, with a difference, after correcting for the difference of standard and applying a correction for wear of the chain, on the average for the different miles of 1 in 2,360.

On the Atchison, Topeka and Sante Fè R. R., through Western Kansas, the difference between the preliminary survey and the government land survey averaged about 1 in 1000; the difference between the preliminary survey and the location was about 1 in 2500.

The U. S. A. engineers in remeasuring the Union & Central Pacific R. R., found the error of their own work to average 1 in 23,600, the maximum being 1 in 10,000, as determined by retracing distances varying from 2000 to 26,000 feet (See House Ex. Doc., No. 37, 2nd session, 44th Cong., pp. 10, 14, 21, 37, after correcting errors of transcription).

On the U. S. Lake Survey, (See Rept. of Chf. Eng. U. S. A., 1876, Part III, page 9) base lines for topographical surveys were made with a chain 20 meters long, differing from the ordinary chains only in being made of heavier wire and having links 20 inches long; in 15 lines, the maximum error was 1 in 5700, the mean being 1 in 17,500, and the minimum 1 in 41,100.

In connection with the U. S. Lake Survey work, a line 12 miles long was measured on the railroad track with a wire 150 feet long, at the rate of about 10 miles per day, with a difference between two measurements of 1 in 32,000.

The U. S. Coast Survey Report 1882, page 191, contains an account of the preliminary measurement of two base lines with an iron wire $\frac{1}{8}$ of an inch in diameter, 200 feet long, in which the difference, as compared with the measurement of the geodetic base apparatus, was 1 in 30,000 and 1 in 28,000.

## TO DETERMINE A TRUE MERIDIAN.

In connection with the article on "*Local Attraction in Land Surveying*" in ENGINEERING NEWS of Oct. 31, 1885, the writer has received several requests for his opinion as to the best method of determining a true meridian, and as in the past he has occasionally received similar requests, he therefore ventures to offer the following:

POLARIS AND MIZAR METHOD.—For this time of year (December) this is about the most convenient method, and is sufficient accurate for compass work. The principle of the method is the same as that described in the text-books as the Alioth and Polaris Method. Mizar is the second star, counting from the fore end, in the handle of the Great Dipper; Alioth is the third one; Polaris is the pole star. When Polaris and Mizar are in a vertical line, they are also very nearly on a meridian. (see Fig. 1.)

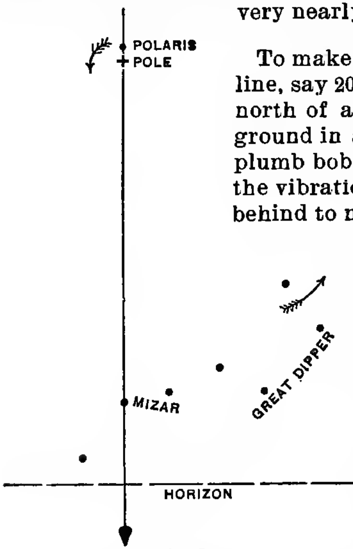

Fig. 1

To make the observation, suspend a plumb line, say 20 feet long, about the same distance north of a board fixed horizontally near the ground in an east and west direction; let the plumb bob swing in a pail of water to check the vibrations; hang a lantern overhead and behind to make the line visible. Watch and wait until Polaris and Mizar are exactly covered by the plumb line; see Figure 1.; when the stars are in the same vertical plane, mark a point on the board in range with the stars and plumb line. On the first of January, 1886, this line will lie about 2.7 seconds west of the true meridian: the line will move *west* about 26″ per year for several years. The whole apparatus may be left undisturbed until morning, when the line can be permanently marked; or it may be marked at the time of observation by having an assistant hold a light, seen through a small aperture in a board, which is moved according to signals until it is in line with the plumb line and the point marked on the board.

On the 1st of January, Polaris and Mizar are in the same vertical about 6:30 p. m.; the time for any day before that can be found by subtracting four minutes for each day, and for any day after January 1st, add a like amount. In the summer and early fall these stars come into the required position at an inconvenient hour, and can not be used at all in the spring on account of coming in the desired position in daytime. At these time the same method may be applied in observing a different pair, Polaris and δ Cassiopeia. (See Fig. 2.)

The constellation Cassiopeia is on the opposite side of the pole from the Great Dipper; it is on the meridian under the pole about 6.30 June 1st, and four minutes earlier each day later. The relative position of the stars at the time of observation is shown in Fig. 2. The line determined by observing these stars June 1, 1886, will be within 1″ or 2″ of a true meridian and will move *east* about 25″ per year.

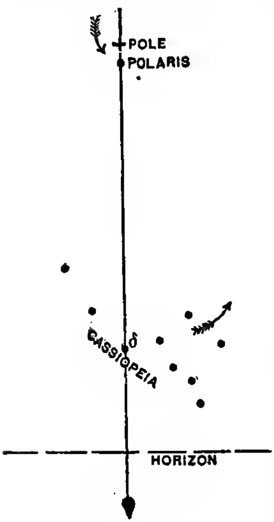

Fig. 2.

Neither of these pairs can be observed from June 1st to August 1st, and from the latter date until November 1st at inconvenient hours; at such times one of the following methods must be used:

(In passing, it may be well to caution the surveyor against following the text-books in the matter of the accuracy of the Polaris-Alioth method; it is sometimes stated that when Polaris and Alioth are in the same vertical, they are "very nearly in a meridian." and it is also stated more accurately that "Polaris is precisely on a meridian seventeen minutes after the two are in the same vertical." The facts are that the two stars were 12′20″ east of a true meridian January 1, 1886, and Polaris came to a true meridian about twenty-eight minutes thereafter; these quantities increase from year to year.)

METHOD BY EQUAL SHADOWS OF THE SUN: "On the south side of any level surface, erect an upright staff, shown in horizontal projection, at *S*, Fig. 3. Two or three hours before noon, mark the extremity, *A*, of its shadow. Describe an arc of a circle with *S*, the foot of the staff, for a center, and *SA*, the distance to the extremity of the shadow, for a radius. About as many hours after noon as it had been before noon when the first mark was made, watch for the moment when the end of the shadow touches the arc at another point, *B*. Bisect the arc *AB* at *N*. Draw *SN*, and it will be the meridian."

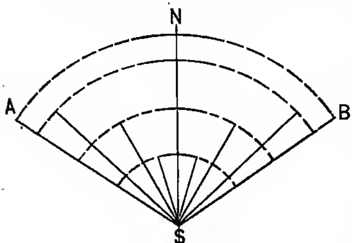

Fig. 3

"For greater accuracy, describe several arcs before hand, mark the point in which each of them is touched by the shadow, bisect each and adopt the average of all. The shadow will be better defined if a piece of tin with a hole through it be placed at the top of the staff, as a bright spot will thus be substituted for the less definite shadow; nor need the staff be vertical, if from its summit a plumb line be dropped to the ground, and the point which this strikes be adopted as the center of the arcs."

This method is strictly correct, June 21st and December 21st; on March 21st, if the observations are made within two hours of noon, the line thus determined is about $2\frac{1}{2}'$ west of a true meridian, on September 21st, same amount to the east, and proportional for intermediate dates.

This is a very simple method, can be applied at a convenient hour, and with moderate care will give results sufficiently accurate for the best compass work. Experience of the writer's students shows that a meridian can be determined within 2 minutes by making three or four pairs of observations and taking the mean.

METHOD BY ELONGATION OF POLARIS.—The Pole Star is about 1 degree 20 minutes from the pole of the heavens, and describes a circle about it once in a little less than 24 hours. The star then apparently moves from the meridian for nearly 6 hours, and then returns to the meridian; when at its greatest angular distance from the meridian, east or west, it is said to be at its greatest eastern or western elongation. One of the very best methods of determining the meridian is by observing Polaris at elongation; when the star is in this position it seems only to move up and down, and neither to the right nor left, consequently the conditions are such as not only to allow an accurate observation but also to give time to repeat the observation.

The angle, or azimuth of elongation will vary with the latitude, and also with the year, owing to a change in the position of the star. Within the United States it varies from $1\frac{1}{2}$ degrees to 2 degrees. Tables are often given which show the azimuth of elongation, but they are too voluminous to be quoted here; besides it takes only a moment to compute it. The formula is

$$\text{sine of azimuth} = \frac{\text{sine of Polar distance}}{\text{cosine of latitude}}$$

The polar distance of Polaris was 1 degree 17 minutes 57 seconds for January 1, 1886, and decreases 19 seconds per year. A change of 20 seconds in the polar distance of Polaris will make only about 0.4 seconds in the angle of elongation, and a difference of 1 degree in the latitude in the southern part of the United States will make only about 0.8 seconds difference in the azimuth, and in the extreme northern part about 2.4 seconds; therefore it will not be difficult to compute the azimuth of elongation as accurately as even a transit will read.

The Polar distance of Polaris for any year can be determined sufficiently accurate for many years to come by applying the above annual variation; or it could doubtless be obtained by sending an addressed postal card to the Naval Observatory, Washington, D. C.

To make the observation, arrange the plumb line and other apparatus as in the Alioth-Mizar Method; a little before the time of elongation, mark a point by a pin, knife, etc., in line with Polaris and the plumb line. If the star departs from the plumb line move the point in the opposite way, and keep moving one way as the star goes the other, until the star attains its greatest elongation, when it will continue behind the plumb-line for several minutes; then it will recede from the line in the direction contrary to its motion before its becomes stationary.