SHARPLESS & BROWN'S MATHEMATICAL SERIES.

GUMMERE'S

SURVEYING.

REVISED AND ADAPTED TO MODERN METHODS,

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SHARPLESS AND BROWN'S

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GUMMERE'S SURVEYING. Revised by ISAAC SHARPLESS.

PREFACE.

Gummere's Surveying was published in 1814. It was the first book which placed the subject in a systematic shape for the use of students, and the public fully appreciated its merits. The author made frequent revisions during his lifetime, as the numerous editions were called for, and these have enabled it to retain for so long a time the confidence of educators and practical surveyors.

The present edition contains, in addition to the prominent features of the old book, the following:

1. New tables of six-place logarithms and logarithmic sines and tangents, and the adaptation of all examples to them. These tables have been carefully compared with the latest German edition of Schrön's seven-place tables, and are undoubtedly reliable.

2. Surveying with Transit and Theodolite, with examples showing the methods employed in practice.

3. New rules and demonstrations for the calculation of areas and for laying out and dividing land.

4. A large number of minor revisions necessary to secure harmony through the book and to adapt the descriptions to modern methods and usages.

W. & L. E. Gurley, instrument-makers, Troy, N. Y., have furnished electrotypes, thus permitting the insertion of figures of the most approved and modern forms of surveying instruments.

I. S.

Haverford College, 1880.

PREFACE.

The following compilation originated in the belief that our schools are in want of a Treatise on Surveying, adapted to the methods practised in this country, and freed from the defects of the systems now in use. Notwithstanding the importance of the science, and the large number that make it an object of study, it is believed we are not in possession of a treatise on this subject, suited to the wants of the student. The works of Gibson and Jess are the only ones at present in general use; the former, though much the better of the two is deficient in many respects. It may be sufficient here, merely to advert to its want of examples, which renders it entirely unsuitable for a school book. From the latter, the student would in vain expect to become acquainted with the principles of the science, or the *rationale* of any of the rules, necessary in performing the various calculations.*

In order to understand the principles of surveying, a previous knowledge of Geometry is absolutely necessary; and this knowledge will be best acquired from a regular treatise on the subject. In the demonstrations, therefore, throughout this work, the student is supposed to be acquainted with the elements of that science. The references are adapted to Playfair's Geometry but they will in general apply equally well to Simson's translation of Euclid's Elements.

As there are many who wish to obtain a practical knowledge of Surveying, whose leisure may be too limited to admit of their

* Each of these works has lately gone through a new edition, in which considerable additions are stated to have been made. On examination, however, it does not appear, that those additions are such as to supply the deficiencies.

The additions made to Gibson, consist principally of some nautical problems quite foreign to a treatise on Surveying. Those made to Jess, consist of a few extracts from Gibson, in one of which the Pennsylvania method of calculation is introduced, as being quite different from that given by Jess; whereas it is well known to be the method given by that author, and used, as well in the preceding as in the subsequent part of his work.

going through a course of Geometry, the author has adapted his work to this class, by introducing the necessary geometrical definitions and problems, and by giving plain and concise rules, entirely detached from the demonstrations; the latter being placed in the form of notes at the bottom of the page. Each rule is exemplified by one wrought example; and the most of them by several unwrought examples, with the answers annexed.

In the laying out and dividing of land, which forms the most difficult part of surveying, a variety of problems is introduced, adapted to the cases most likely to occur in practice. This part of the subject, however, presents such a great variety of cases, that we should in vain attempt to give rules that would apply to all of them. It cannot therefore be too strongly recommended to every one, who has the opportunity, to make himself well acquainted with Geometry, and also with Algebra, previous to entering on the study of Surveying. Furnished with these useful auxiliaries, and acquainted with the principles of the science, the practitioner will be able to perform, with ease, any thing likely to occur in his practice.

The compiler thinks proper to acknowledge, that in the arrangement of the work, he availed himself of the advice of his learned preceptor and friend, E. Lewis of New-Garden; and that several of the demonstrations were furnished by him.

J. G.

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A

TREATISE ON SURVEYING

OF LOGARITHMS.

Logarithms are a series of numbers so contrived, that by them the work of multiplication is performed by addition, and that of division by subtraction.

If a series of numbers in arithmetical progression be placed as indices, or exponents, to a series of numbers in geometrical progression, the sum or difference of any two of the former, will answer to the product or quotient of the two corresponding terms of the latter. Thus,

0. 1. 2. 3. 4. 5. 6. 7. &c., arith. series, or indices.
1. 2. 4. 8. 16. 32. 64. 128. &c., geom. series.

Now $2+3=5$. also $7-3=4$.
And $4\times8=32$. and $128\div8=16$.

Therefore the arithmetical series, or indices, have the same properties as logarithms; and these properties hold true, whatever may be the ratio of the geometrical series.

There may, therefore, be as many different systems of logarithms, as there can be taken different geometrical series having unity for the first term. But the most convenient system is that in which the ratio of the geometrical series is 10, and this is the one in common use. Thus,

0,	1,	2,	3,	4,	5,	&c.	indices or logar.
1,	10,	100,	1000,	10000,	100000,	&c.	natural numbers.

In this system the log. of 1 is 0*, the log. of 10 is 1, the log. of 100 is 2, &c. Hence it is plain that the log. of any number between 1 and 10 will be expressed by a decimal, the log. of any number between 10 and 100 by 1 and a decimal, the log. of any number between 100 and 1000 by 2 and a decimal, &c.

The numbers, 0, 1, 2, 3, &c., that stand before the decimal part of logarithms are called *indices*, and are always less by unity than the number of figures in the integral part of the corresponding natural number.

The index of the logarithm of a number, consisting in whole, or in part, of integers, is positive; but if the number be a decimal the index is negative, and is marked by a negative sign (—), placed either before or above it. If the first significant figure of the decimal be adjacent to the decimal point, the index is -1, or $\bar{1}$; if there be one cipher between them, the index is -2, or $\bar{2}$; if there be two ciphers between them, the index is -3, or $\bar{3}$, &c.

The decimal parts of the logarithms of numbers, consisting of the same figures and in the same order, are the same, whether the number be integral, fractional, or mixed. This is illustrated as follows:

* In every system the logarithm of 1 is 0.

Number 18960	Logarithm 4.277838
1896	3.277838
189.6	2.277838
18.96	1.277838
1.896	0.277838
.1896	−1.277838
.01896	−2.277838
.001896	−3.277838
.0001896	−4.277838

The method of finding logarithms in the tables, and of multiplying, dividing, &c. by them, is contained in the following problems:

PROBLEM I.

To find the logarithm of a given number.

If the given number consists of *one* or *two* figures only, find it in the column marked *N.* in the first page of the table, and against it, in the next column, marked *log.*, is the logarithm. Thus the log. of 7 will be found 0.845098, and the log. of 85 will be found 1.929419.

But if the given number be either wholly or in part decimal, the index must be changed accordingly. Observing that the index must always be one less than the number of figures in the integral part of the given number; also, when the given number is wholly a decimal, the index is negative, and must be one more than the number of the ciphers between the decimal point and first significant figure on the left hand. Thus the log. of .7 is —1.845098, and the log. of .0085 is —3.929419.

If the given number consists of *three* figures, find it in one of the other pages of the table, in the column marked *N.*, and against it, in the next column, is the decimal part of the logarithm. The index must be placed before it agreeably to the above observation. Thus the log. of 421 is 2.624282, the log. of 4.21 is 0.624282, and the log. of .0421 is —2.624282.

If the given number consists of *four* figures, find the three left hand figures in the column marked *N.* as before, and the remaining, or right hand figure at the top of the table; in the column under this figure, and against the other three, is the decimal part of the logarithm. Thus the log. of 5163 is 3.712902, and the log. of .6387 is —1.805297.

If the given number consists of *five* or *six* figures, find the logarithm of the four left hand figures as before; then take the difference between this logarithm and the next greater in the table. Multiply this difference by the remaining figure or figures of the given number, and cut off one or two figures to the right hand of the product, according as the multiplier consists of one or two figures; then add the remaining figure or figures of the product to the logarithm first taken out of the table, and the sum will be the logarithm required. Thus, let it be required to find the logarithm of 59686; then,

Logarithm of 5968 is - - -	775829
The next greater log. is - -	775902
Difference - - - - -	73
Remaining figure - - -	6
Product - - - - -	43,8
To - - - - - -	775829
Add - - - - - -	44*
Decimal part of the log. - -	775873

* Because 43.8 is nearer 44 than 43.

The natural number consisting of five integers, the index must be 4; therefore the log. of 59686 is 4.775873.

Again, let it be required to find the log. of .0131755; then,

Logarithm of 1317 is - - -	119586
The next greater log. is - -	119915
* Difference - - - - -	329
Remaining figures - - -	55
Product - - - - -	180,95
To - - - - - -	119586
Add - - - - - -	181
Decimal part of the log. - -	119767

As the given number is a decimal, and has one cipher between the decimal point and first significant figure, the index must be —2; therefore the log. of .0131755 is —2.119767.

EXAMPLES.

1. Required the log. of	4.3	*Ans.*	0.633468
2. Required the log. of	7986	*Ans.*	3.902329
3. Required the log. of	.3754	*Ans.*	—1.574494
4. Required the log. of	596.87	*Ans.*	2.775879
5. Required the log. of	785925	*Ans.*	5.895381
6. Required the log. of	6543900	*Ans.*	6.815836
7. Required the log. of	.0027863	*Ans.*	—3.445028

* This difference is given in column D. In the first pages of the table the logarithms vary so rapidly that it is safer in accurate work to make the subtraction as above.

PROBLEM II.

To find the natural number corresponding to a given logarithm.

If *four* figures only be required in the answer, look in the table for the decimal part of the given logarithm, and if it cannot be found exactly, take the one nearest to it, whether greater or less; then the three figures in the first column, marked *N*, which are in a line with the logarithm found, together with the figure at the top of the table directly above it, will form the number required. Observing that, when the index of the given logarithm is positive, the integers in the number found must be one more than the number expressed by the index; but when the index of the given logarithm is negative, the number found will be wholly a decimal, and must have one cipher less—placed between the decimal point and first significant figure on the left hand—than the number expressed by the index. Thus the natural number corresponding to the logarithm 2.902329 is 798.6, the natural number corresponding to the logarithm 3.770557 is 5896, and the natural number corresponding to the logarithm -3.363612 is .00231.

If the exact logarithm be found in the table, and the figures in the number corresponding do not exceed the index by one, annex ciphers to the right hand till they do. Thus the natural number corresponding to the logarithm 6.640680 is 4372000.

If *five* or *six* figures be required in the answer, find in the table the logarithm next less than the given one, and take out the four figures answering to it as before. Subtract this logarithm from the next greater in the table, and also from the given logarithm; to the latter difference annex one or two ciphers, according as five or six figures are required, and divide the number thus produced by the former difference; annex the quotient to the right hand of the four figures already found, and it will give the natural number required.

Thus let it be required to find the natural number corresponding to the logarithm 2.538990 true to *five* figures; then,

Given logarithm -	.538990	
Next less - -	.538951,	the natural number corresponding is 3459.
Diff. with one cipher annexed	390	

Next less log. - -	.538951
Next greater - -	.539076
Difference - -	125

Divide 390 by 125 and the quotient will be 3, which, annexed to the right hand of 3459, the four figures already found, makes 34593; therefore, as the index is 2, the required natural number is 345.93.

Again, let it be required to find the natural number corresponding to the logarithm 4.598590, true to *six* figures; then,

Given logarithm -	.598590	
Next less - -	.598572,	the natural number answering to it is 3968.
Diff. with two ciphers annexed - -	1800	

Next less log. - -	.598572
Next greater - -	.598681
Difference - -	109

Divide 1800 by 109, and the quotient will be 17, which annexed to the right hand of 3968, the four figures already found, makes 396817; therefore, as the index is 4, the required natural number is 39681.7.

EXAMPLES.

1. Required the natural number answering to the logarithm 1.880299. *Ans.* 75.91.

2. Required the natural number answering to the logarithm 5.370817. *Ans.* 234861.

3. Required the natural number answering to the logarithm 3.119770. *Ans.* 1317.56.

4. Required the natural number answering to the logarithm −2.974350. *Ans.* .094265.

PROBLEM III.

To multiply numbers by means of logarithms.

Case 1.—When all the factors are whole or mixed numbers.

RULE.

Add together the logarithms of the factors, and the sum will be the logarithm of the product.

EXAMPLES.

1. Required the product of 84 by 56.

Logarithm of	84	is	1.924279
Do. of	56	is	1.748188
Product 4704		Sum	3.672467

2. Required the continued product of 17.3, 1.907 and 34.

Logarithm of	17.3	is	1.238046
Do.	1.907	is	0.280351
Do.	34.	is	1.531479
Product 1121.71		Sum	3.049876

3. Find by logarithms the product of 76.5 by 5.5. *Ans.* 420.75.

4. Find by logarithms the continued product of 42.35, 1.7364, and 1.76. *Ans.* 129.424.

CASE 2.—When some or all of the factors are decimal numbers.

RULE.

Add the decimal parts of the logarithms as before, and if there be any to carry from the decimal part, add it to the positive index or indices, or else subtract it from the negative.

Then add the indices together, when they are all of the same kind; that is, all positive or all negative; but when they are of different kinds, take the difference between the sums of the positive and negative ones, and prefix the sign of the greater.

Note.—When the index is positive, it is not necessary to place any sign before it; but when it is negative, the sign must not be omitted.

EXAMPLES.

1. Required the continued product of 349.17, 25.43, .93521 and .00576.

Logarithm of	349.17	is	2.543037
Do.	25.43	is	1.405346
Do.	.93521	is	−1.970909
Do.	.00576	is	−3.760422
Product 47.83.		Sum	1.679714

In this example there is 2 to carry from the decimal part of the logarithms, which added to 3, the sum of the affirmative indices, makes 5; from this taking 4, the sum of the negative indices, the remainder is 1, which is the index of the sum of the logarithms, and is affirmative, because the sum of the affirmative indices, together with the number carried, exceed the sum of the negative indices.

2. Required the continued product of .0839, .7536, and .003179.

Logarithm of	.0839	is	-2.923762
Do.	.7536	is	-1.877141
Do.	.003179	is	-3.502291
Product .000201.		Sum	-4.303194

In this example there is 2 to carry from the decimal part of the logarithms, which subtracted from 6, the sum of the negative indices, leaves 4, which is the index of the sum of the logarithms, and is negative, because the sum of the negative indices is the greater.

3. Required the continued product of 13.19, .3765, and .00415. *Ans.* .02061.

4. Required the continued product of 343, 1.794, 5.41, and .019. *Ans.* 63.25.

PROBLEM IV.

To divide numbers by means of Logarithms.

Case 1.—When the dividend and divisoı are both whole or mixed numbers

RULE.

From the logarithm of the dividend subtract the logarithm of the divisor; the remainder will be the logarithm of the quotient.

Note.—When the divisor exceeds the dividend, the question must be wrought by the rule given in the next case.

EXAMPLES.

1. Required the quotient of 3450 divided by 23.

Logarithm of	3450	is	3.537819
Do.	23	is	1.361728
Quotient 150.		Remainder	2.176091

2. Required the quotient of 420.75 divided by 76.5.
Ans. 5.5.

3. Required the quotient of 37.1542 divided by 1.73958. *Ans.* 21.3585.

CASE 2.—When the dividend or divisor, or both of them, are decimal numbers.

RULE.

Subtract the decimal parts of the logarithms as before, and if 1 be borrowed in the left-hand place of the decimal part, add it to the index of the divisor when that index is positive, but subtract it when negative.

Then conceive the sign of the index of the divisor changed from positive to negative, or from negative to positive; and if, when changed, it be of the same name with that of the dividend, add the indices together, but if it be of a different name, take the difference of the indices, and prefix the sign of the greater.

EXAMPLES.

1. Required the quotient of .7591 divided by 32.147.

Logarithm of	.7591	is	-1.880299
Do.	32.147	is	1.507140
Quotient .02361		Remain.	-2.373159

In this example the index of the divisor, with its sign changed, is -1, which added to -1, the index of the dividend, makes -2, for the index of the quotient.

2. Required the quotient of .63153 divided by .00917.

Logarithm of	.63153	is	-1.800394
Do.	.00917	is	-3.962369
Quotient 68.8683.		Remain.	1.838025

In this example there is 1 to carry from the decimal part of the logarithm, which subtracted from -3, the index of the divisor, leaves -2; this, with its sign changed, is $+2$; from which subtracting 1, the index of the dividend, the remainder is 1, and is affirmative, because the affirmative index is the greater.

3. Required the quotient of 13.921 divided by 7965.13.

Logarithm of	13.921	is	1.143670
Do.	7965.13	is	3.901194
Quotient .001748		Remain.	-3.242476

In this example there is 1 to carry from the decimal part of the logarithm, which added to 3, the index of the divisor, makes 4; this, with its sign changed, is -4; from which subtracting 1, the index of the dividend, the remainder is -3.

4. Required the quotient of 79.35 divided by .05178.
Ans. 1532.46.

5. Required the quotient of .5903 divided by .931.
Ans. .63404.

PROBLEM V.

To involve a number to any power; that is, to find the square, cube, &c. of a number logarithmically.

RULE.

Multiply the logarithm of the given number by the index of the power, viz. by 2 for the square, by 3 for the cube, &c., and the product will be the logarithm of the power.

Note.—When the index of the logarithm is negative, if there be any to carry from the decimal part, instead of adding it to the product of the index and multiplier, subtract it, and the remainder will be the index of the logarithm of the power, and will always be negative.

EXAMPLES.

1. Required the square of 317.

Logarithm of 317	is	2.501059
		2
Square 100489		5.002118

2. Required the 5th power of 1.735.

Logarithm of 1.735	is	0.239299
		5
5th power 15.7218		1.196495

3. Required the cube of .08761.

Logarithm of .08761	is	-2.942554
		3
Cube .0006725		-4.827662

4. Required the cube of 7.503. *Ans.* 422.37.

5. Required the 7th power of .32513. *Ans.* .0003841.

PROBLEM VI.

To extract any root of a number logarithmically.

RULE.

Divide the logarithm of the given number by the index of the root—that is, by 2 for the square root, by 3 for the cube root, &c.—and the quotient will be the logarithm of the required root.

Note.—When the index of the logarithm is negative, and does not exactly contain the divisor, increase the index by a number just sufficient to make it exactly divisible by it, and carry the units borrowed, as so many tens, to the left-hand figure of the decimal part; then proceed with the division in the usual manner.

EXAMPLES.

1. Required the cube root of 391.27.

		3)
Logarithm of 391.27	is	2.592477
Cube root 7.314		0.864159

2. Required the square root of .08593.

		2)
Logarithm of .08593	is	−2.934145
Square root .29314		−1.467072

3. Required the cube root of .7596.

		3)
Logarithm of .7596	is	−1.880585
Cube root .9124		−1.960192

4. Required the cube root of .0000613.

		3)
Logarithm of .0000613	is	−5.787460
Cube root .03943		−2.595820

5. Required the square root of 365. *Ans.* 19.105.
6. Required the 5th root of .9563. *Ans.* .9911.
7. Required the 4th root of .00079. *Ans.* .16765.

Of the Arithmetical Complements of Logarithms.

When it is required to subtract several logarithms from others, it will be more convenient to convert the subtraction into an addition, by writing down, instead of the logarithms to be subtracted, what each of them wants of 10.000000, which may readily be done, by writing down what the first figure, on the right hand, wants of 10, and what every other figure wants of 9; this remainder is called the *Arithmetical Complement.* Thus, if the logarithm be 2.53061, its arithmetical complement will be 7.46939. If one or more figures to the right hand be ciphers, write ciphers in their place, and take the first significant figure from 10, and the remaining figures from 9. Thus, if the logarithm be 4.61300, its arithmetical complement will be 5.38700.

In any operation, where the arithmetical complements of logarithms are added to other logarithms, there must be as many tens subtracted from the sum, as there are arithmetical complements used.

As an example, let it be required to divide the product of 76.4 and 35.84 by the product of 473.9 and 4.76.

473.9 - - -	Ar. Co.	7.324313
4.76 - - -	Ar. Co.	9.322393
35.84 - - -	- log.	1.554368
76.4 - - -	- log.	1.883093
Quotient 1.214		0.084167

GEOMETRY.

DEFINITIONS.

1. Geometry is that science wherein the properties of magnitude are considered.

2. A *point* is that which has position, but not magnitude.

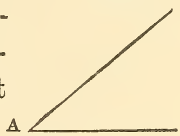
3. A *line* has length but not breadth.

4. A *straight*, or *right line*, is the shortest line that can be drawn between any two points.

5. A *superficies* or *surface* is that which has length and breadth, but not thickness.

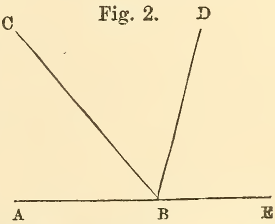
6. A *plane superficies* is that in which any two points being taken, the straight line which joins them lies wholly in that superficies.

Fig. 1.



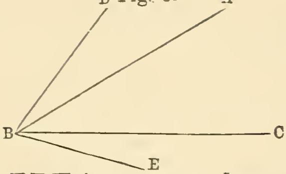
7. A *plane rectilineal angle* is the inclination of two straight lines to one another, which meet together, but are not in the same straight line, as A, Fig. 1.

Fig. 2.



Note.—When several angles are formed about the same point, as at B, Fig. 2, each particular angle is expressed by three letters, whereof the middle letter shows the angular point, and the other two the lines that form the angle; thus, CBD or DBC signifies the angle formed by the lines CB and DB.

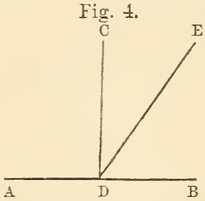
Fig. 3.



8. The *magnitude* of an *angle* depends on the inclination which the lines that form it have to each other, and not on the length of those lines. Thus the angle DBE is greater than the angle ABC, Fig. 3.

9. When a straight line stands on another straight line so as to incline to neither side, but makes the angles on each side equal, then each of those angles is called a *right angle*, and the line which stands on the other is said to be *perpendicular* to it. Thus ADC and BDC are right angles, and the line CD is perpendicular to AB, Fig. 4.

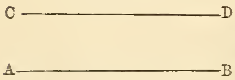
Fig. 4.



10. An *acute* angle is that which is less than a right angle, as BDE, Fig. 4.

11. An *obtuse* angle is that which is greater than a right angle, as ADE, Fig. 4.

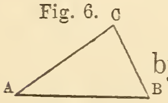
Fig. 5.



12. *Parallel straight lines* are those which are in the same plane, and which, being produced ever so far both ways, do not meet, as AB, CD, Fig. 5.

13. A *figure* is a space bounded by one or more lines.

Fig. 6.



14. A *plane triangle* is a figure bounded by three straight lines, as ABC, Fig. 6.

Fig. 7.

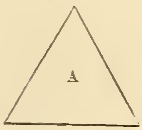
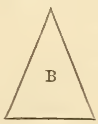


Fig. 8.



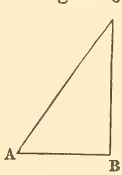
15. An *equilateral* triangle has its three sides equal to each other, as A, Fig. 7.

16. An *isosceles* triangle has only two of its sides equal, as B, Fig. 8.

17. A *scalene* triangle has three unequal sides, as ABC, Fig. 6.

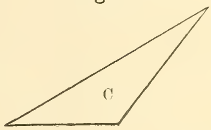
18. A *right angled* triangle has one right angle, as ABC, Fig. 9; in which the side AC, opposite to the right angle, is called the *hypothenuse*.

Fig. 9.



19. An *obtuse angled triangle* has one obtuse angle, as C, Fig. 10.

Fig. 10.



20. An *acute angled triangle* has all its angles acute, as ABC, Fig. 6.

21. Acute and obtuse angled triangles are called *oblique angled triangles*.

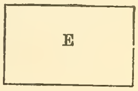
22. Any plane figure bounded by four right lines, is called a *quadrilateral*.

23. Any quadrilateral, whose opposite sides are parallel, is called a *parallelogram*, as D, Fig. 11.

Fig. 11.

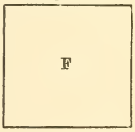
24. A parallelogram, whose angles are all right angles, is called a *rectangle*, as E, Fig. 12.

Fig. 12.



25. A parallelogram whose sides are all equal, and angles right, is called a *square*, as F, Fig. 13.

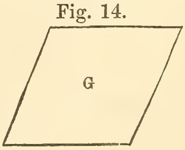
Fig. 13.



26. A *rhomboid* is a parallelogram, whose opposite sides are equal, and angles oblique, as D, Fig. 11.

27. A *rhombus* is a parallelogram, whose sides are all equal and angles oblique, as G, Fig. 14.

Fig. 14.



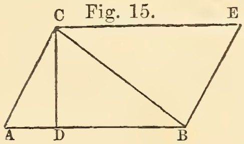
28. Any quadrilateral figure that is not a parallelogram, is called a *trapezium.*

29. A trapezium that has two parallel sides is called a *trapezoid.*

30. A right line joining any two opposite angles of a quadrilateral figure, is called a *diagonal.*

31. That side upon which any parallelogram, or triangle is supposed to stand, is called the *base;* and the perpendicular falling thereon from the opposite angle is called the *altitude* of the parallelogram, or triangle. Thus AD is the base of the parallelogram ABEC, or triangle ABC, and CD is the altitude, Fig. 15.

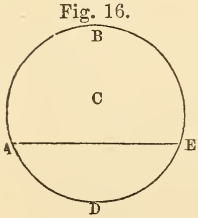
Fig. 15.



32. All plane figures contained by straight lines are called *polygons;* of which those having five sides are called *pentagons;* those having six sides, *hexagons,* and so on.

33. A *regular polygon* is one whose angles, as well as sides, are all equal.

Fig. 16.



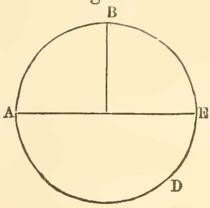
34. A *circle* is a plane figure, bounded by one curve line called the *circumference* or *periphery,* every part of which is equally distant from a certain point within the circle; and this point is called the *centre,* Fig. 16.

35. The *radius* of a circle is a straight line drawn from the centre to the circumference, as CB, Fig. 17.

36. The *diameter* of a circle is a straight line drawn through the centre, and terminated both ways by the

circumference, as AE, Fig. 17. It divides the circle into two equal parts, called *semicircles.*

Fig. 17.



37. A *quadrant* is one quarter of a circle, as ACB, Fig. 17.

Note.—The fourth part of the circumference of a circle is also called a quadrant.

38. A *segment* of a circle is the figure contained by a right line, and the part of the circumference it cuts off: thus AEBA and AEDA are segments of the circle ABED, Fig. 16.

39. An *arc* of a circle is any part of the circumference; as AD or DE, Fig. 17.

40. *Ratio* is a mutual relation between two quantities of the same kind with respect to magnitude.

Note.—A ratio is generally expressed, either by two numbers or by two right lines.

41. When two quantities have the same ratio as two other quantities, the four quantities taken in order are called *proportionals;* and the last is said to be a *fourth proportional* to the other three.

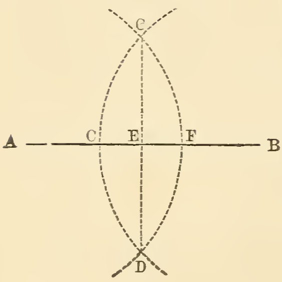
42. When three quantities of the same kind are such that the first has to the second the same ratio which the second has to the third, the third is called a *third proportional* to the first and second, and the second is called a *mean proportional* between the first and third.

GEOMETRICAL PROBLEMS.

PROBLEM I.

To bisect a right line, AB, *Fig.* 18.

Fig. 18.

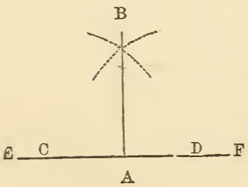


Open the dividers to any distance more than half the line AB, and with one foot in A, describe the arc CFD; with the same opening, and one foot in B, describe the arc CGD, meeting the first arc in C and D; from C to D draw the right line CD, cutting AB in E, which will be equally distant from A and B.

PROBLEM II.

At a given point A, *in a right line* EF, *to erect a perpendicular,* Fig. 19.

Fig. 19.

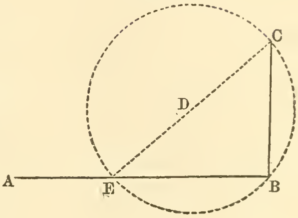


From the point A, lay off on each side, the equal distances AC, AD; from C and D, as centres, with any radius greater than AC or AD, describe two arcs intersecting each other in B; from A to B, draw the line AB, which will be the perpendicular required.

PROBLEM III.

To raise a perpendicular on the end B *of a right line* AB, Fig. 20.

Fig. 20.

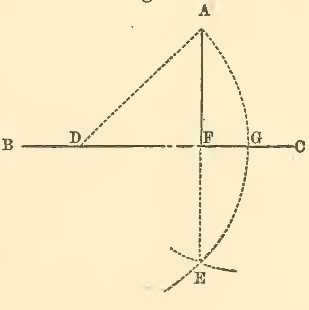


Take any point D not in the line AB, and with the distance from D to B, describe a circle cutting AB in E; from E through D draw the right line EDC, cutting the periphery in C, and join CB, which will be perpendicular to AB.

PROBLEM IV.

To let fall a perpendicular upon a given line BC, *from a given point* A, *without it,* Fig. 21.

Fig. 21.

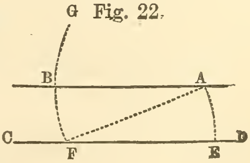


In the line BC take any point D, and with it as a centre and distance DA describe an arc AGE, cutting BC in G; with G as centre, and distance GA, describe an arc cutting AGE in E, and from A to E draw the line AFE; then AF will be perpendicular to AB.

PROBLEM V.

Through a given point A *to draw a right line* AB, *parallel to a given right line* CD, Fig. 22.

Fig. 22.

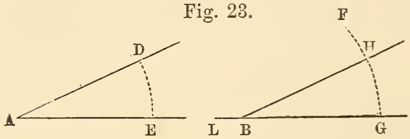


From the point A to any point F, in the line CD, draw the right line AF; with F as a centre and distance FA, describe the arc AE, and with the same distance and centre A describe the arc FB; make FB equal to AE, and through A and B draw the line AB, and it will be parallel to CD.

PROBLEM VI.

At a given point B, *in a given right line* LG, *to make an angle equal to a given angle* A, Fig. 23.

Fig. 23.

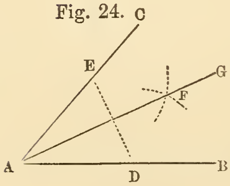


With the centre A and any distance AE, describe the arc DE, and with the same distance and centre B describe the arc FG; make HG equal to DE, and through B and H draw the line BH; then will the angle HBG be equal to the angle A.

PROBLEM VII.

To bisect any right lined angle BAC, Fig. 24.

Fig. 24.



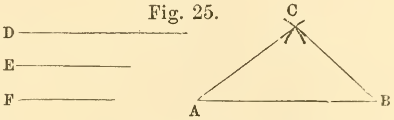
In the lines AB and AC, from the point A, set off equal distances, AD and AE; with the centres D and E and any distance more than half DE, describe two arcs cutting each other in F; from A through F draw the line AG, and it will bisect the angle BAC.

PROBLEM VIII.

To describe a triangle that shall have its sides respectively equal to three right lines, D, E, *and* F, *of which any two must be together greater than the third,* Fig. 25.

Make AB equal to D; with the centre A and distance equal to E, de- describe an arc, and with the centre B and distance equal to F describe another arc, cutting the former in C; draw AC and BC, and ABC is the triangle required.

Fig. 25.

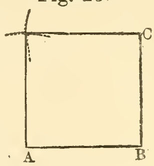


PROBLEM IX.

Upon a given line AB *to describe a square,* Fig. 26.

At the end B of the line AB, by Problem III. erect the perpendicular BC, and make it equal to AB; with A and C as centres, and distance AB or BC, describe two arcs cutting each other in D; draw AD, and CD, then will ABCD be the square required.

Fig. 26

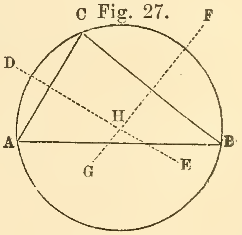


PROBLEM X.

To describe a circle that shall pass through the angular points A, B, *and* C, *of a triangle* ABC, Fig. 27.

By Problem I. bisect any two of the sides, as AC, BC, by the perpendiculars DE and FG; the point H where they intersect each other will be the centre of the circle: with this centre, and the distance from

Fig. 27.

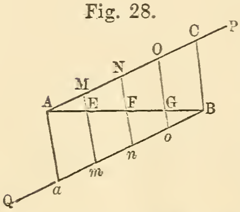


it to either of the points A, B, or C, describe the circle.

PROBLEM XI.

To divide a given right line AB *into any number of equal parts*, Fig. 28.

Fig. 28.



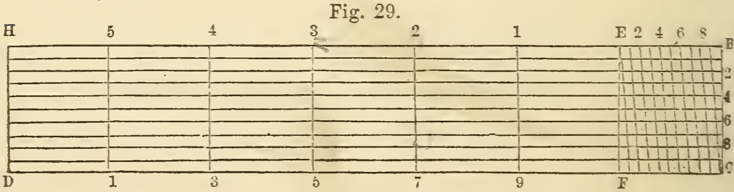
Draw the indefinite right line AP, making an angle with AB, also draw BQ, parallel to AP, in each of which, take as many equal parts AM, MN, &c. *Bo, on*, &c., as the line AB is to be divided into; then draw *Mm, Nn*, &c., intersecting AB in E, F, &c., which will divide the line as required.

PROBLEM XII.

To make a plane diagonal scale, Fig. 29.

Draw eleven lines parallel to, and equidistant from each other; cut them at right angles by the equidistant lines BC; EF; 1, 9; 2, 7; &c. then will BC, &c. be divided into ten equal parts; divide the lines EB, and FC, each into ten equal parts; and from the points of division on the line EB, draw diagonals to the points of division on the line FC: thus join E and the first division on FC, the first division on EB, and the second on FC, &c.

Fig. 29.



Note.—Diagonal scales serve to take off dimensions or numbers of three figures. If the first large divisions be units, the second set of divisions, along EB, will be

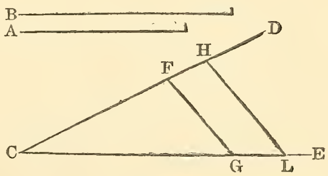
10th parts, and the divisions in the altitude, along BC, will be 100th parts. If HE be tens, EB will be units, and BC will be tenth parts. If HE be hundreds, BE will be tens, and BC units. And so on, each set of divisions being tenth parts of the former ones.

For example, suppose it were required to take off 242 from the scale. Extend the dividers from E to 2 towards H; and with one leg fixed in the point 2, extend the other till it reaches 4 in the line EB; move one leg of the dividers along the line 2, 7, and the other along the line 4, till they come to the line marked 2, in the line BC, and that will give the extent required.

PROBLEM XIII.

To find a third proportional to two given right lines, A and B.

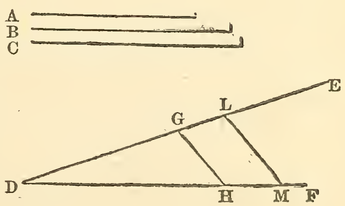
Draw two right lines, CD, CE, containing any angle; make CF equal A, and CG, CH each equal B; join FG, and draw HL parallel to it; then will CL be the third proportional required.



PROBLEM XIV.

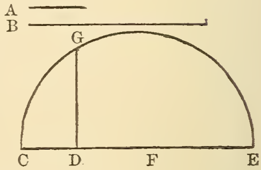
To find a fourth proportional to three given right lines, A, B and C.

Draw two right lines, DE, DF, containing any angle; make DG equal A, DH equal B, and DL equal C; join GH, and draw LM parallel to it; DM will be a fourth proportional to A, B and C.



PROBLEM XV.

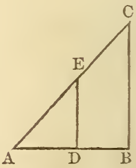
To find a mean proportional between two given right lines, A *and* B.



Draw any right line, CE, and in it take CD equal A, and DE equal B; bisect CE in F, and with the centre F and radius FC or FE describe the semicircle CGE; draw DG perpendicular to CE; then DG will be a mean proportional between A and B.

PROBLEM XVI.

To divide a given right line AB *in two parts in the point* D, *so that* AD *may be to* DB *in the ratio of two given numbers* m *and* n. *For example, let* $m = 3$, *and* $n = 2$.



Draw AC, making any angle with AB; take the number m from any convenient scale of equal parts, and lay it on AC, from A to E, and take the number n from the same scale, and lay it from E to C; join CB and draw ED parallel to it; then AB will be divided as required.

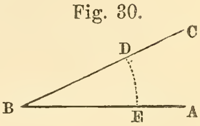
PLANE TRIGONOMETRY.

DEFINITIONS.

1. Plane Trigonometry is the art by which, when any three parts of a plane triangle, except the three angles, are given, the others are determined.

2. The periphery of every circle is supposed to be divided into 360 equal parts, called degrees; each degree into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds, &c.

Fig. 30.



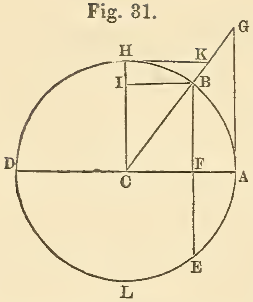
3. The measure of an angle is the arc of a circle, contained between the two lines that form the angle, the angular point being the centre; thus the angle ABC, Fig. 30, is measured by the arc DE, and contains the same number of degrees that the arc does. The measure of a right angle is therefore 90 degrees; for DH, Fig. 31, which measures the right angle DCH, is one-fourth part of the circumference, or 90 degrees.

Note.—The degrees, minutes, seconds, &c., contained in any arc, or angle, are written in this manner, $50° \; 18' \; 35''$; which signifies that the given arc or angle contains 50 degrees, 18 minutes, and 35 seconds.

4. The *complement* of an arc, or of an angle, is what it wants of 90°; and the supplement of an arc, or of an angle, is what it wants of 180°.

5. The *chord* of an arc, is a right line drawn from one extremity of the arc to the other: thus the line BE is the chord of the arc BAE or BDE, Fig. 31.

Fig. 31.



6. The *sine* of an arc, is a right line drawn from one extremity of the arc perpendicular to the diameter which passes through the other extremity: thus BF is the sine of the arc AB or BD, Fig. 31.

7. The *cosine* of an arc, is that part of the diameter which is intercepted between the sine and the centre: thus CF is the cosine of the arc AB, and is equal to BI, the sine of its complement HB, Fig. 31

8. The *versed sine* of an arc, is that part of the diameter which is intercepted between the sine and the arc: thus AF is the versed sine of AB; and DF of DB, Fig. 31.

9. The *tangent* of an arc, is a right line touching the circle in one end of the arc, being perpendicular to the diameter which passes through that end, and is terminated by a right line drawn from the centre through the other end: thus AG is the tangent of the arc AB, Fig. 31.

10. The *secant* of an arc, is the right line drawn from the centre and terminating the tangent; thus CG is the secant of AB, Fig. 31.

11. The *cotangent* of an arc, is the tangent of the complement of that arc; thus HK is the cotangent of AB, Fig. 31.

12. The *cosecant* of an arc, is the secant of the com plement of that arc; thus CK is the cosecant of AB, Fig. 31.

13. The sine, cosine, &c., of an *angle* is the same as the sine, cosine, &c., of the *arc* that measures the angle.

In measuring arcs it is customary to consider A, the right-hand extremity of the horizontal diameter, as the place of beginning, and measure around through 360° in the direction opposite to the motion of the hands of a watch. Thus, AH is the first quadrant, HD the second, DL the third, and LA the fourth.

An angle in the first quadrant is between 0° and 90°; in the second, between 90° and 180°; in the third, between 180° and 270°; in the fourth, between 270° and 360°.

The sine of 0° is 0; as the arc increases through the first quadrant, the sine also increases, till for 90° it is equal to the radius; through the second quadrant the sine diminishes as the arc increases. The cosine, on the other hand, decreases from radius at 0° to 0 at 90°, thence increases to radius at 180°. The tangent increases from 0 at 0° to infinity at 90°, and then decreases to 0 at 180°.

If we suppose radius to be unity, the length of the sine, tangent, &c. may be calculated by a method which it is out of place to explain here. It is evident that the sine and cosine, being never greater than radius, will be represented by a decimal. These decimals, radius being the unit, for every degree and minute of the first quadrant, are given in Table V. Such sines and cosines are known as natural sines and cosines, to distinguish them from the logarithmic sines and cosines. The sine of any angle being equal to the cosine of its complement, a table of natural sines will also be a table of natural cosines. Thus, if we desire the natural cosine of 80°, we may look for the natural sine of 10°. As far as 45° the degrees are given at the top of the page; above 45°, at the bottom.

To obtain from the table the natural sine or cosine of a given angle, find the degrees at the top or bottom of the page, and the minutes in the left-hand or right-hand column; in the column of degrees, and opposite the given minute, is the required sine or cosine.

EXAMPLES.

1. Find the natural sine of 27° 42′. *Ans.* .46484.
2. Find the natural cosine of 46° 52′. *Ans.* .68370.
3. Find the natural cosine of 38°. *Ans.* .78801.
4. Find the natural sine of 30°. *Ans.* .50000.
5. Find the angle whose natural sine is .57742.
   Ans. 35° 16′.
6. Find the angle whose natural cosine is .61120.
   Ans. 52° 19′.

Note.—In the case of the last two examples, take the degrees and minutes corresponding to the sine and cosine nearest the given ones.

Should degrees, minutes and *seconds* be given, find the sine or cosine for the degrees and minutes. Take the difference between this and that corresponding to the next greater minute; divide this difference by 60, and multiply by the given number of seconds; add this to the number first taken out in the case of sines, and subtract in the case of cosines. The result will be the required number.

EXAMPLES.

1. Find the natural sine of 23° 42′ 17″.

Natural sine of 23° 42′ - - -	.40195
Natural sine of 23° 43′ - - -	.40221
Difference - - - -	26

This being the difference corresponding to 1′ or 60″, the difference corresponding to 17″ will be $\frac{17}{60}$ of 26, or 7.

$.40195 + 7 = .40202.$ *Ans.*

2. Find the natural cosine of 23° 42′ 17″.

Ans. .91563.

Note.—We subtract in the case of the cosine, because the cosine decreases as the arc increases.

If it be required to find the degrees, minutes and seconds corresponding to a given sine or cosine, we proceed as in the following examples:

3. Find the degrees, minutes and seconds corresponding to the natural sine .47728.

Next greater sine - - - -	.47741
Next less sine - - - -	.47716
Difference for 60″ - - -	25
Difference for 1″ - - -	.416
Given sine - - - - -	.47728
Next less sine - - - -	.47716
Difference - - - - -	12

.416 being the difference for 1″, 12 will correspond to 12 ÷ .416, or 29. Hence the answer is 28° 30′ 29″.

4. Find the angle corresponding to the natural cosine .40000.

By inspection of table, we find the given cosine falls between the cosines of 66° 25′ and 66° 26′.

Cosine of 66° 25′ - - - -	40008
Cosine of 66° 26′ - - - -	39982
Difference for 60″ - - -	26
Difference for 1″ - - -	.433
Cosine of next less angle - - -	40008
Given cosine - - - - -	40000
Difference - - - - -	8

8 ÷ .433 = 18. *Ans.* 66° 25′ 18″.

OF THE TABLE

OF

LOGARITHMIC OR ARTIFICIAL SINES, TANGENTS, &c.

This table contains the logarithms of the sine, tangent, &c. to every degree and minute of the quadrant, the radius being 10000000000, and consequently its logarithm 10.

Let the radius be supposed to consist of 10000000000 equal parts as above, and let the quadrant be divided into 5400 equal arcs, each of these will therefore contain 1′; and if from the several points of division in the quadrant right lines be drawn perpendicular to the radius, the sine of every minute of the quadrant to the given radius will be exhibited. The lengths of these lines, being computed and arranged in a table, constitute what is usually termed a table of *natural* sines. The logarithms of those numbers taken from a table of logarithms, and properly arranged, form the table of *logarithmic* or *artificial sines.* In like manner the logarithmic tangents and secants are to be understood.

The method by which the sines are computed is too abstruse to be explained in this work, but a familiar exposition of this subject may be seen in *Sharpless's Trigonometry.*

To find, by the table, the sine, tangent, &c. of an arc or angle.

If the degrees in the given angle be less than 45, look for them at the top of the table, and for the minutes, in the left hand column; then in the column marked at the top of the table, sine, tangent, &c. and against the minutes, is the sine, tangent, &c. required. If the degrees are more than 45, look for them at the bottom of the table, and for the minutes, in the right hand column; then in the column marked at the bottom of the table, sine, tangent, &c. and against the minutes, is the sine, tangent, &c. required.

Note.—The sine of an angle and of its supplement being the same, if the given number of degrees be above 90, subtract them from 180°, and find the sine of the remainder.

EXAMPLES.

1. Required the sine of	32° 27′.	*Ans.*	9.729621.
2. Required the tangent of	57° 39′.	*Ans.*	10.198325.
3. What is the cosine of	89° 31′?	*Ans.*	7.926119.
4. What is the sine of	157° 43′?	*Ans.*	9.578853.

To find the degrees and minutes corresponding to a given sine, tangent, &c.

Find, in the table, the nearest logarithm to the given one, and the degrees answering to it will be found at the top of the table, if the name be there, and the minutes on the left hand; but if the name be at the bottom of the table, the degrees must be found at the bottom, and the minutes at the right hand.

EXAMPLES.

1. Required the degrees and minutes in the angle whose sine is 9.643908. *Ans.* 26° 8′.

2. Required the degrees and minutes in the angle whose tangent is 10.474641. *Ans.* 71° 28′.

To find the sine, tangent, &c. of an arc or angle when seconds are required.

Find the sine, tangent, &c. of the degrees and minutes as before. Multiply the corresponding number in column D by the given seconds, and add this to the number first taken out, for sines and tangents, and subtract for cosines and cotangents.

EXAMPLES.

1. Find the sine of 46° 25′ 32″.

Sine of 46° 25′ - - - -	9.859962
Corresponding number in column D	2.00

$$32 \times 2.00 = 64.$$

$$9.859962 + 64 = 9.860026.$$

Note.—The number in column D is the variation in the sine, tangent, &c. for a change of 1″ in the arc. Hence, to multiply by the number of seconds gives the corresponding variation.

2. Find the cotangent of 30° 27′ 48″.
Ans. 10.230488.

3. Find the tangent of 26° 51′ 32″.
Ans. 9.704517.

To find the degrees, minutes and seconds corresponding to a given sine, &c.

Find the two sines, &c. between which the given sine, &c. falls, and take the less of the corresponding minutes. Take the difference between the given sine,

&c. and that opposite to the minutes taken, and divide this difference by the corresponding number in column D for the seconds.

EXAMPLES.

1. Find the angle whose sine is 9.764236.

The next less angle is - - -	35° 31′
Corresponding sine is - - -	9.764131

Difference between this and given sine is 105 (millionths). Number in column D is 2.95. 2.95 being the number corresponding to 1″, 105 will correspond to $\frac{105}{2.95}$ seconds, or 35.6. Hence, the complete angle is 35° 31′ 36″.

2. Find the angle whose cosine is 9.620248.

The tables tell that the true angle is between 65° 20′ and 65° 21′. The cosine corresponding to 65° 20′ is 9.620488. Difference between this and given logarithm is 240.

$$240 \div 4.58 = 52.4.$$

Hence the angle is 65° 20′ 52″.

3. Find the angle whose cosine is 9.764227.

Ans. 54° 28′ 26″.

4. Find the angle whose tangent is 10.876429.

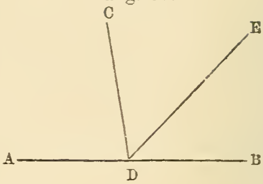
Ans. 82° 25′ 44″.

Remarks on Angles, Triangles, &c.

1. If from a point D in a right line AB, one or more right lines be drawn on the same side of it, the angles thus formed at the point D will be together equal to two

right angles, or 180°; thus ADE + EDB = two right angles, or 180°: also ADC + CDE + EDB = two right angles, or 180°. Fig. 35.

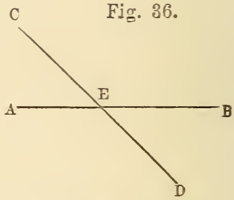
Fig. 35.



2. Since the angles thus formed at the point D, on the other side of AB, would also be equal to two right angles, the sum of all the angles formed about a point is equal to four right angles, or 360°.

3. If two right lines cut one another, the opposite angles will be equal: thus AEC = BED, and AED = CEB, Fig. 36.

Fig. 36.



4. The sum of the three angles of a plane triangle is equal to two rightangles, or 180°.

5. If the sum of two angles of a triangle be sub tracted from 180°, the remainder will be the third angle.

6. If one angle of a triangle be subtracted from 180°, the remainder will be the sum of the other two angles.

7. In right-angled triangles, if one of the acute angles be subtracted from 90°, the remainder will be the other acute angle.

8. The angles at the base of an isosceles triangle are equal to one another.

9. If one side of a triangle be produced, the external angle will be equal to the sum of the two internal and opposite angles: thus the external angle CBD, of the triangle ABC, is equal to the sum of the internal and opposite angles A and C, Fig. 37.

Fig. 37

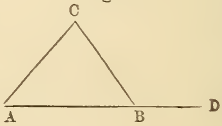
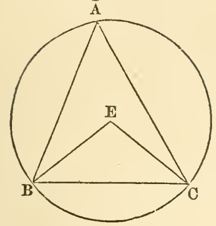
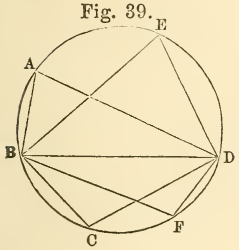


Fig. 38.



10. The angle at the centre of a circle is double of the angle at the circumference, upon the same base, that is upon the same part of the circumference: thus the angle BEC is double of the angle BAC, Fig. 38.

Fig. 39.



11. The angles in the same segment of a circle are equal to one another: thus the angle BAD is equal to the angle BED; also the angle BCD is equal to the angle BFD, Fig. 39.

12. The angle in a semicircle is a right angle: thus the angle ECF, Fig. 45, is a right angle.

13. This mark ′ placed on the sides or in the angles of a triangle, indicates that they are given; and this mark ° placed in the same way, indicates that they are required.

PRACTICAL RULES FOR SOLVING ALL THE CASES OF PLANE TRIGONOMETRY.

CASE 1.

The angles and one side of any plane triangle being given, to find the other sides.

RULE.

As the sine of the angle opposite the given side,
Is to the sine of the angle opposite the required side,
So is the given side,
To the required side.*

---

* DEMONSTRATION. Let ABC, Fig 40, be any plane triangle; take BF= AC, and upon AB let fall the perpendiculars CD and FE, which will be

Note 1.—The proportions in trigonometry are worked by logarithms: thus, from the sum of the logarithm of the second and third terms, subtract the logarithms of the first term, and the remainder will be the logarithm of the fourth term.

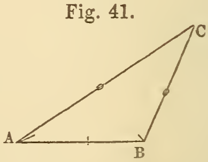
2. The logarithmic sine of a right angle or 90° is 10.00000, being the same as the logarithm of the radius.

EXAMPLES.

1. In the triangle ABC, there are given the angle A $= 32° 15'$, the angle $B = 114° 24'$, and consequently the angle $C = 33° 21'$, and the sides $AB = 98$;* required the sides AC and BC.

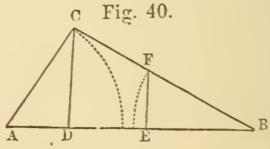
By Construction, Fig. 41.

Fig. 41.



Make AB equal to 98 by a scale of equal parts, and draw AC, making the angle $A = 32° 15'$; also make the angle $B = 114° 24'$, and produce BC, AC, till they meet in C, then is ABC the triangle required; and AC measured by the same scale of equal parts, is 162, and BC is 95.

Fig. 40.



the sines of the angles A and B to the equal radii AC and BF. Now the triangles BDC and BEF being similar, we have CD : FE : : BC : BF : or AC; that is sin. A : sin. B : : BC : AC. In like manner it is proved, that sin. A : sin. C : : BC : AB. When one of the angles is obtuse, the demonstration is the same. Hence it appears, that in any plane triangle, the sides are to one another as the sines of their opposite angles.

* This 98 may express so many feet, or yards, &c., and the other sides will be of the same denomination as the given.

By Calculation.

As sine of the angle C 33° 21′ - -	9.740167
Is to sine of the angle B 114° 24′ -	9.959368
So is AB 98 - - - - -	1.991226
	11.950594
	9.740167
To AC 162.3 - - - - -	2.210427
As sine of C 33° 21′ - - - -	9.740167
Is to sine of A 32° 15′ - - -	9.727228
So is AB 98 - - - - -	1.991226
	11.718454
	9.740167
To BC 95.12 - - - - -	1.978287

This calculation could also be performed by adding the arithmetical complement of the logarithm of the first term to the other logarithms, and subtracting 10 from the result. This is usually the simplest method of working a proportion.

The first proportion would now become—

As sine of C 33° 21′ - - Ar. Co.	0.259833
Is to sine of B 114° 24′ - - -	9.959368
So is AB 98 - - - - -	.991226
To AC 162.3 - - - -	1.210427

2. In the right-angled triangle ABC are given the hypothenuse AC = 480, and the angle A = 53° 8′. To find the base AB and perpendicular BC.

Say, As sine B : sine A :: AC : BC.
90°, 53° 8′, 480, 384.

As sine B : sine C :: AC : AB.
90°, 36° 52′, 480, 288.

The sine of 90° being equal to radius, its logarithm is 10.

3. In the triangle ABC, are given the angle A=79° 23′, the angle B=54° 22′, and the side BC=125; required AC and AB. *Ans.* AC=103.4, and AB=91.87.

4. In a right-angled triangle, there are given the angle A=56° 48′, and the base AB=53.66, to find the perpendicular BC and hypothenuse AC. *Ans.* BC=82 and AC=98.

5. In the right-angled triangle ABC, are given the angle A=39° 10′, and the perpendicular BC=407.37, to find the base AB and hypothenuse AC. *Ans.* AB=500.1, and AC=645.

CASE 2.

Two sides and an angle opposite one of them being given, to find the other angles and side.

RULE.

As the side opposite the given angle,
Is to the other given side,
So is the sine of the angle opposite the former,
To the sine of the angle opposite the latter.*

---

* This is evident from the demonstration of the rule in the preceding case.

Add the angle thus found to the given angle, and subtract their sum from 180°, the remainder will be the third angle.

After finding the angles, the other side may be found by Case 1.

Note.—The angle found by this rule is sometimes ambiguous; for the operation only gives the sine of the angle, not the angle itself; and the sine of every angle is also the sine of its supplement.

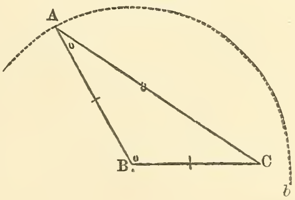
When the side opposite the given angle is equal to, or greater than the other given side, then the angle opposite that other given side is always acute; but when this is not the case, that angle may be either acute or obtuse, and is consequently ambiguous.

EXAMPLES.

1. In the triangle ABC, are given the angle C = 33° 21′, the side AB = .98 and the side BC = .7912; required the angles A and B, and the side BC.

By Construction, Fig. 43.

Fig. 43.



Make BC = .7912 by a scale of equal parts, and draw CA, making the angle C = 33° 21′; with the side AB = .98, in the compasses, taken from the same scale of equal parts, and B as a centre, describe the arc *ab*, cutting AC in the point A, and join BA; then is ABC the triangle required: the side AC, measured by the scale of equal parts, will be 1.54, and the angles A and B, measured by a protractor, will be 26° 21′ and 120° 18′.

Here the arc *ab* cuts AC in one point only, because AB is greater than BC; therefore the angle A is acute, and not ambiguous.

By Calculation.

As AB, .98 - - - - -	-1.991226
Is to BC, .7912 - - - -	-1.898286
So is sine of C, 33° 21′ - - -	9.740167
	9.638453
To sine of A, 26° 21′ - - -	9.647227

To the angle C = 33° 21′ add the angle A = 26° 21′, and the sum is 59° 42′, which, subtracted from 180°, leaves the angle B = 120° 18′.

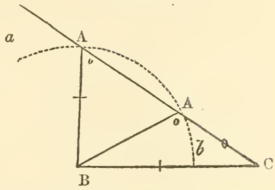
As sine of C, 33° 21′ - - -	9.740167
Is to sine of B, 120° 18′ - - -	9.936210
So is AB, .98 - - - -	-1.991226
	9.927436
To AC, 1.539 - - - -	0.187269

If the side BA had been given of such a length that, when the compass was swung around with B as a centre, it would not cut AC, the problem would have been impossible. No triangle could answer the conditions of the problem. If the arc had touched AC, the triangle would have been right-angled. If it had cut AC in two points, as in next example, either one of the two triangles formed would satisfy the given values. The problem would be said to be ambiguous.

2. In the triangle ABC are given the angle C = 33° 21′, the side BC = 95.12, and the side AB = 60, to find the angles A and B and the side AC.

By Construction, Fig. 44.

Fig. 44.



This is constructed in the same manner as the preceding example; only, AB being shorter than BC, the arc *ab* cuts AC in two points on the same side of BC; hence the angle A may be either acute or obtuse. The side required has also two values, as AC and AC.

By Calculation.

As AB, 60 - - - - -	1.778151
Is to BC, 95.12 - - - -	1.978272
So is sine C, 33° 21′ - - -	9.740167
	11.718440
To sine of A $\left\{\begin{array}{l} 60° \ 38′ \text{ acute} \\ 119 \ 22 \text{ obtuse} \end{array}\right\}$	9.940289

The sum of the angles C and A subtracted from 180° leaves the angle B = 86° 1′ if A be acute, or 27° 17′ if A be obtuse.

To find the side AC answering to the acute value of the angle A.

As sine of C, 33° 21′ - - -	9.740167
Is to sine of B, 86° 1′ - - -	9.998950
So is AB, 60 - - - - -	1.778151
	11.777101
To AC, 108.9 - - - - -	2.036934

To find the side AC, answering to the obtuse value of the angle A.

As sine of C, 33° 21′ - - -	9.740167
Is to sine of B, 27° 17′ - - -	9.661236
So is AB, 60 - - - - -	1.778151
	11.439387
To AC, 50.03 - - - - -	1.699220

3. In a triangle ABC, the side AB is 274, AC 306. and the angle B 78° 13′; required the angles A and C. and the side BC.

Ans. A=40° 33′, C=61° 14′, and BC=203.2.

4. In a right-angled triangle, there are given the hypothenuse AC=272, and the base AB=232; to find the angles A and C, and the perpendicular BC.

Ans. A=31° 28′ C=58° 32′ and BC=142.

5. In a right-angled triangle ABC, the hypothenuse AC is 150, and one side BC 69; required the angles and other side.

Ans. C=62° 37′, A=27° 23′, and AB 133.2.

CASE 3.

Two sides and the included angle being given, to find the other angles and side.

RULE.

Subtract the given angle from 180°, and the remainder will be the sum of the two unknown angles. Then,

As the sum of the two given sides,
Is to their difference;

So is the tangent of half the sum of the two unknown angles,

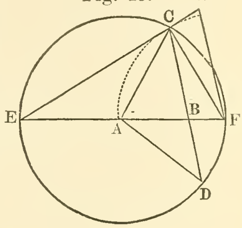
To the tangent of half their difference.*

This half difference of the two unknown angles, added to their half sum, will give the angle opposite the greater of the two given sides, and being subtracted

---

* Demonstration. Let ABC, Fig. 45, be the proposed triangle, having the two given sides AB, AC, including the given angle A. About A as a centre, with AC the greater of the given sides for a radius, describe a circle meeting AB produced in E and F, and BC in D; join DA, EC, and FC, and draw FG parallel to BC, meeting EC produced in G.

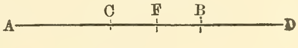
Fig. 45.



The angle EAC (I. 30)† is equal to the sum of the unknown angles ABC, ACB; and the angle EFC at the circumference, is equal to the half of EAC at the centre (III. 16;) therefore EFC is half the sum of the unknown angles; but (I. 30) the angle ABC is equal to the sum of the angles BAD and ADB, or BAD and ACB; therefore FAD is the difference of the unknown angles ABC, ACB; and FCD, at the circumference is the half of that difference; but because of the parallels DC, FG, the angles GFC, FCD are equal; therefore GFC is equal to half the difference of the unknown angles ABC, ACB; but since the angle ECF in a semicircle, is a right angle, EG is perpendicular to CF, and therefore CF being radius, EC, CG are the tangents of the angles EFC, CFG; it is also evident that EB is the sum of the sides BA, AC, and that BF is the difference; therefore since BC, FG are parallel, EB : BF : : EC : CG (V. 2:) that is, the sum of the sides AC, AB, is to their difference, as the tangent of half the sum of the angles ABC, ACB, is to the tangent of half their difference.

To demonstrate the latter part of the rule, let AC and AB, Fig. 46, represent any two magnitudes whatever; in AB produced, take BD equal to AC the less, and bisect AD in E.

Fig. 46.



Then because AE is equal to ED, and AC to BD, CE is equal to EB, therefore AE or ED is half the sum of the given magnitudes AB, AC, and CE, or EB is half their difference; but AB the greater is equal to AE, EB, that is to half the sum added to half the difference; and AC the less, is equal to the excess of AE, half the sum, above CE, half the difference.

† These references are to Sharpless's "Geometry."

from the half sum will give the angle opposite the less given side.

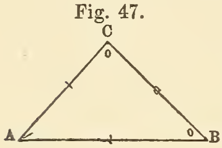
After finding the angles, the other side may be found by Case 1.

EXAMPLES.

1. In the triangle ABC, there are given AB=128, AC=90, and the angle A=48° 12′, to find the angles B and C, and the side BC.

By Construction, Fig. 47.

Fig. 47.



Draw AB=128, and make the angle A=48° 12′; draw AC=90, and join BC. The angle B will measure 44° 37′, the angle C 87° 11′, and the side BC 95.5.

By Calculation.

AB	128		180° 0′
AC	90	Angle A - - - - - -	48 12
Sum	218	Sum of the angles B and C	131 48
Difference	38	Half sum do.	65 54

As the sum of the sides, AB, AC, 218 - -	2.338456
Is to their difference, 38 - - - - - - -	1.579784
So is the tangent of half the sum of the angles B and C, 65° 54′ - - - -	10.349380
	11.929164
To tang. of half their difference, 21° 17′ -	9.590708

Half sum of the angles B and C - -	65° 54′
Add and subtract half their difference -	21 17
Angle C - - - - - - - - - - - -	87 11
Angle B - - - - - - - - - - - -	44 37

To find the side BC.

As sine of B, 44° 37′ - - -	9.846560
Is to sine of A, 48° 12′ - - -	9.872434
So is AC, 90 - - - - -	1.954243
	11.826677
To BC, 95.52 - - - - -	1.980117

2. In a triangle ABC are given AB = 109, BC = 76, and the contained angle B = 101° 30′, to find the other angles and side.

Ans. The angle A = 30° 57′, C = 47° 33′, and the side AC = 144.8.

3. Given, in a right-angled triangle, the base AB = 890 and the perpendicular BC = 787, to find the angles and hypothenuse.

Ans. The angle A = 41° 29′, C = 48° 31′, and the hypothenuse AC = 1188.

This problem may be used to find the distance between two points when separated by a swamp, river or other impassable barrier. Let B and C, Fig. 47, be two such points. Measure AC and AB and the angle A, whence BC may be calculated.

CASE 4.

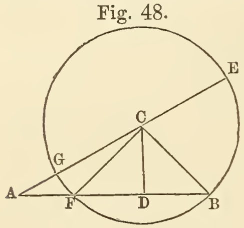
Given the three sides, to find the angles.

RULE 1.

Consider the longest side of the triangle as the base, and on it let fall a perpendicular from the opposite angle. This perpendicular will divide the base into two parts, called segments, and the whole triangle into two right-angled triangles. Then,

As the base, or sum of the segments,
Is to the sum of the other two sides;
So is the difference of those sides,
To the difference of the segments of the base.*

Fig. 48.



* Demonstration. Let ABC, Fig. 48, be a triangle, and CD be perpendicular upon AB About C as a centre, with the less side BC for a radius, describe a circle, meeting AC produced, in G and E, and AB in F. Then it is evident that AE is equal to the sum of the sides AC, BC, and that AG is equal to their difference; also because CD bisects FB (III. 3,) it is plain that AF is the difference of the segments of the base; but $AB \times AF = AE \times AG$ (V. 27 cor.;) therefore AB : AE : : AG : AF (V. 14;) that is, the base, is to the sum of the sides, as the difference of the sides, is to the difference of the segments of the base.

Cor. If AF be considered the base of the triangle AFC, then will CD be a perpendicular on the base produced; AE will be equal to the sum of the sides AC, FC, and AG will be equal to their difference; also AB will be equal to the sum of the segments AD, FD. But by the preceding demonstration, alternately AF : AE : : AG : AB; hence, when the perpendicular falls without the triangle, the base is to the sum of the sides, as the difference of the sides is to the sum of the segments of the base.

A rule might, therefore, be given, making either side of a triangle the base; and such a rule would be rather more convenient, in some cases, than the one above: but then, on account of the perpendicular, sometimes falling within and sometimes without the triangle, it would require two cases, and consequently would be less simple.

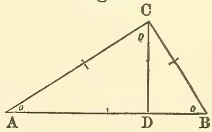
To half the base, add half the difference of the segments, and the sum will be the greater segment; also from half the base, subtract half the difference of the segments, and the remainder will be the less segment.

Then, in each of the two right-angled triangles, there will be known two sides, and an angle opposite to one of them; consequently the other angles may be found by Case 2.

1. In the triangle ABC, are given AB=426, AC=365, and BC=230; required the angles.

By Construction, Fig. 49.

Fig. 49.



Draw AB=426; with AC=365 in the dividers, and one foot in A, describe an arc, and with BC=230, and one foot in B describe another arc, cutting the former in C; join AC, BC, and ABC will be the triangle required. The angles measured by a scale of chords, will be A=32° 39′, B=58° 56′, and C=88° 25′.

By Calculation.

AC - - - - - - - -	365
BC - - - - - - - -	230
Sum - - - - - - - -	595
Difference - - - - - -	135

As the base AB - - - -	- 426	2.629410
Is to the sum of the sides AC, BC	- 595	2.774517
So is the diff. of the sides AC, BC	- 135	2.130334
		4.904851
To the diff. of the segments AD, DB	188.6	2.275441
Half diff. of the segments - -	94.3	
Half base - - - - - -	213.	
Segment AD - - - -	307.3	
Segment BD - - - -	118.7	

The other angles may then be worked out by Case 2.

RULE 2.

Add together the arithmetical complements of the logarithms of the two sides containing the required angle, the logarithm of the half sum of the three sides, and the logarithm of the difference between the half sum and the side opposite the required angle. Then half the sum of these four logarithms will be the logarithmic cosine of half the required angle.*

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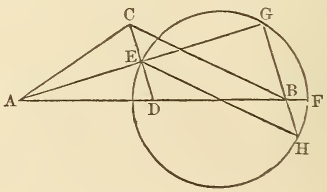
* DEMONSTRATION.—Let ABC be a triangle, of which the side AB is greater than AC; make AD = AC, join DC, bisect it in E, and join AE; draw EH parallel and equal to CB; join HB, and produce it to meet AE produced in G.



Now in the triangles, AED, AEC, all the sides of the one are equal to the sides of the other, each to each; therefore (I. 4) the angle EAD = EAC, and AED = AEC; consequently AED is a right angle.

EXAMPLES.

I. In the triangle ABC are given AB = 426, AC = 365 and BC = 230; required the angle A.

By Calculation.

BC	230		
AC	365	Ar. Co.	7.437707
AB	426	Ar. Co.	7.370590
	2)1021		
Half sum	510.5	log.	2.707996
Difference	280.5	log.	2.447933
			2)19.964226
Cos. ½ A 16° 20′			9.982113
2			
Angle A 32° 40′			

Because EH is equal and parallel to BC, BH is also equal and parallel to EC (I. 31;) now in the triangles EDF and HBF, the angle EFD=BFH, the angle FED=FHB (I. 27) and ED=EC=BH; therefore (I. 24) EF=FH, and FD=FB. Again, the angle HGE=DEA=a right angle; if therefore with the centre F, and radius FE=FH, a circle be described, it will pass through the point G.

Now 2 AF=2 AD+2 DF=AD+AD+DB=AD+AB=AC+AB; therefore AF=½AC+½AB; also FK=½IK=½EH=½BC; hence, by adding equals to equals, AF+FK=½AC+½AB+½BC, or AK=½(AC+AB+BC;) again, AI=AK—IK=½(AC+AB+BC)—BC.

But (Dem. to rule, case 1st.) AD : AE : : sin. AED : sin. ADE : : rad. : cos. EAD (cos. ½BAC.) Also, AB : AG : : sin. AGB : sin. ABG : : rad. : cos. BAG (cos. ½BAC.)

Hence (c. 6) AB × AD : : AG × AE : : rad.^{2} : (cos. ½BAC)^2. But AB × AD=AB × AC, and (cor. 36.3) AG × AE=AK × AI=½(AC+AB+BC) × [½(AC+AB+BC)—BC;] therefore AB × AC: ½(AC+AB+BC) × [½(AC+AB+BC)—BC] : : rad.^{2} : (cos. ½BAC)^2.

Now it is evident, that in working this proportion by logarithms

If the other angles are required, they may be found by Case 1.

2. In a triangle ABC, are given AB = 64, AC = 47, and BC = 34, to find the angle B. *Ans.* Angle B = 45° 38′.

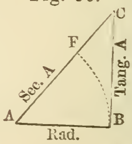
3. In a triangle ABC, are given AC = 88, AB = 108, and BC = 54, to find the angle C. *Ans.* C = 96° 4′.

The preceding rules solve all the cases of plane triangles, both right-angled and oblique. There are, however, other rules, suited to right-angled triangles, which are sometimes more convenient than the general ones. Previous to giving these rules, it will be necessary to make the following

Remarks on right-angled triangles.

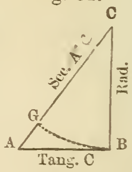
1. ABC, Fig. 50, being a right-angled triangle, make one leg AB radius, that is, with the centre A, and distance AB, describe an arc BF. Then it is evident that the other leg BC represents the tangent of the arc BF, or of the angle A, and the hypothenuse AC the secant of it.

Fig. 50.



2. In like manner, if the leg BC, Fig. 51, be made radius; then the other leg AB will represent the tangent of the arc BG, or angle C, and the hypothenuse AC the secant of it.

Fig. 51.



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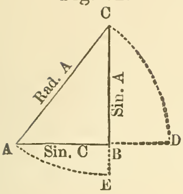
and taking the arithmetical complements of the logarithms of the first term, viz. of the two sides, including the required angle, if we omit the logarithm of the square of radius, which is 20, it is just equivalent to rejecting 20 from the sum of the logarithms, which would otherwise have to be done.

Fig. 52.



3. But if the hypothenuse be made radius, then each leg will represent the sine of its opposite angle; namely, the leg AB, Fig. 52, the sine of the arc AE or angle C, and the leg BC the sine of the arc CD, or angle A, and the cosine of C.

The angles and one side of a right-angled triangle being given, to find the other sides.

RULE.

Call any one of the sides radius, and write upon it the word radius; observe whether the other sides become sines, tangents, or cosines, and write these words on them accordingly. Call the word written upon each side the name of that side. Then,

As the name of the side given,
Is to the name of the side required;
So is the side given,
To the side required.*

Two sides of a right-angled triangle being given, to find the angles and other side.

RULE.

Call either of the given sides radius, and write on them as before. Then,

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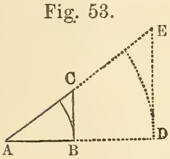
Fig. 53.



* DEMONSTRATION. Let ABC, Fig. 53, be a right-angled triangle; then it is evident that BC is the tangent, and AC the secant of the angle A, to the radius AB. Let AD represent the radius of the tables, and draw DE perpendicular to AD, meeting AC produced in E; then DE is the tangent, and AE the secant of the angle A, to the radius AD. But because of the similar triangles ADE, ABC, AD : DE : : AB : BC; that is, the tabular radius : tabular tangent : : AB : BC. Also AD : AE : : AB : AC; that is, the tabular radius : tabular secant : : AB : AC. These proportions correspond with the rule. When either of the other sides is made radius, the demonstration will be similar.

As the side made radius,
Is to the other given side;
So is radius,
To the name of that other side.*

After finding the angle, the other side is found as in the preceding rule.

EXAMPLES.

1. In a right-angled triangle ABC, are given the base AB=208, and the angle A=35° 16′, to find the hypo thenuse AC and perpendicular BC

By Calculation.

The hypothenuse AC being radius.

As the sine of C, 54° 44′ - - -	9.911942
Is to radius - - - - -	10.000000
So is AB 208 - - - - -	2.318063
	12.318063
To AC 254.8 - - - - -	2.406121

The base AB being radius.

As radius - - - - -	10.000000
Is to the tangent of A, 35° 16′ - -	9.849522
So is AB 208 - - - - -	2.318063
To BC 147.1 - - - - -	2.167585

* This is the converse of the preceding rule.

2. In a right-angled triangle, are given the hypothenuse AC=36.57, and the angle A=27° 46′, to find the base AB, and perpendicular BC.

Ans. Base AB=32.36, and perpendicular BC=17.04

3. In a right-angled triangle, there are given, the perpendicular=193.6, and the angle opposite the base 47° 51′; required the hypothenuse and base.

Ans. Hypothenuse=288.5, and base=213.9

4. Required the angles and hypothenuse of a right-angled triangle, the base of which is 46.72, and perpendicular 57.9.

Ans. { Angle opposite the base 38° 54′, angle opposite the perpendicular 51° 6′, and hypothenuse 74.4.

When two sides of a right-angled triangle are given, the other side may be found by the following rules, without first finding the angles.

1. *When the hypothenuse and one leg are given, to find the other leg.*

RULE.

Subtract the square of the given leg from the square of the hypothenuse; the square root of the remainder will be the leg required.* Or by logarithms thus,

To the logarithm of the sum of the hypothenuse and given side, add the logarithm of their difference; half this sum will be the logarithm of the leg required.

---

* Demonstration. The square of the hypothenuse of a right-angled triangle is equal to the sum of the squares of the sides (I. 42.) Therefore the truth of the first part of each of the rules is evident.

2. *When the two legs are given to find the hypothenuse.*

RULE.

Add together the squares of the two given legs; the square root of the sum will be the hypothenuse.* Or by logarithms thus,

From twice the logarithm of the perpendicular, subtract the logarithm of the base, and add the corresponding natural number to the base; then, half the sum of the logarithms of this sum, and of the base, will be the logarithm of the hypothenuse.

EXAMPLES.

1. The hypothenuse of a right-angled triangle is 272, and the base 232; required the perpendicular.

Calculation by logarithms.

Hypothenuse - - -	272		
Base - - - -	232		
Sum - - - -	504	log.	2.702431
Difference - - -	40	——	1.602060
			2)4.304491
Perpendicular - - -	142	——	2.152245

---

*Put h = the hypothenuse, b = the base, and p = the perpendicular, then (I. 42) $p^2=h^2-b^2=$ (II. 6) $\overline{h+b}\times\overline{h-b}$, or $p=\sqrt{\overline{h+b}\times\overline{h-b}}$, whence, from the nature of logarithms, the latter part of the first rule is evident.

Also $$h^2=b^2+p^2=b\times\overline{b+\frac{p^2}{b}} \text{ or } h=\sqrt{\left(b\times b+\frac{p^2}{b}\right)},$$

which, solved by logarithms, will correspond with the latter part of the second rule.

2. Given the base 186, and the perpendicular 152, to find the hypothenuse.

Calculation by Logarithms.

Perpendicular	152	log.	2.181844	
			2	
			4.363688	
Base - - -	186		2.269513	2.269513
	124.2		2.094175	
	310.2			log. 2.491642
				2)4.761155
Hypothenuse	240.2			log. 2.380577

3. The hypothenuse being given equal 403, and one leg 321; required the other leg. *Ans.* 243.7.

4. What is the hypothenuse of a right-angled triangle, the base of which is 31.04 and perpendicular 27.2?

Ans. 41.27.

The following examples, in which trigonometry is applied to the mensuration of inaccessible distances and heights, will serve to render the student expert in solving the different cases, and also to elucidate its use.

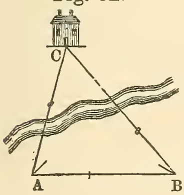


Fig. 54.

THE APPLICATION OF PLANE TRIGONOMETRY TO THE MENSURATION OF DISTANCES AND HEIGHTS.

EXAMPLE 1, Fig. 54.

Being on one side of a river and wanting to know the distance to a house on the other side, I measured 500 yards along the side of the river in a right line AB, and found the two angles* between this line and the object to be CAB$=74°\ 14'$ and CBA$=49°\ 23'$. Required the distance between each station and the object.

Calculation.

The sum of the angles CAB and CBA is $123°\ 37'$, which subtracted from $180°$ leaves the angle ACB$=56°\ 23'$. Then by Case 1;

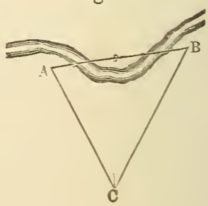
s. ACB	:	s. CBA	: :	AB	:	AC
$56°\ 23'$		$49°\ 23'$		500		455.8

and

s. ACB	:	s. CAB	: :	BA	:	BC
$56°\ 23'$		$74°\ 14'$		500		577.8

EXAMPLE 2, Fig. 55.

Fig. 55.



Suppose I want to know the distance between two places, A and B, accessible at both ends of the line AB, and that I measured AC$=735$ yards, and BC$=840$; also the angle ACB$=55°\ 40'$. What is the distance between A and B?

Calculation.

The angle ACB$=55°\ 40'$, being subtracted from $180°$, leaves $124°\ 20'$; the half of which is $62°\ 10'$. Then by Case 3,

* The angles may be taken with a common surveyor's compass; or more accurately with an instrument called a theodolite.

$$BC+AC : BC-AC :: \text{tangent}\ \frac{CAB+CBA}{2} : \text{tangent}\ \frac{CAB-CBA}{2}$$

1575   105   62° 10′   7° 12′

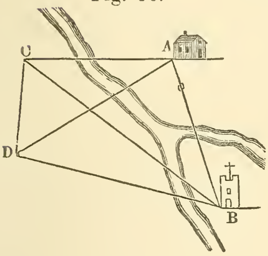
To and from $\frac{CAB+CBA}{2}=62° \ 10'$ add and sub. $\frac{CAB-CBA}{2}=7°$ $12'$, and we shall have $CAB=69° \ 22'$, and $CBA=54° \ 58'$. Then,

$$s.\ ABC : s.\ ACB :: AC : AB$$

54° 58′   55° 40′   735   741.2

EXAMPLE 3, Fig. 56.

Fig. 56.



Wanting to know the distance between two inaccessible objects A and B, I measured a base line $CD=300$ yards: at C the angle BCD was 58° 20′ and ACD 95° 20′; at D the angle CDA was 53° 30′ and CDB 98° 45′. Required the distance AB.

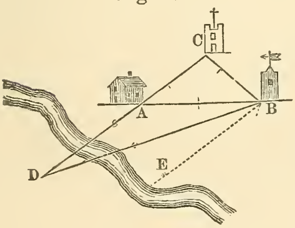
Calculation.

1. In the triangle ACD, are given the angles $ACD=95° \ 20'$, $ADC=53° \ 30'$, and the side $CD=300$, to find $AC=465.98$.

2. In the triangle BCD, are given the angle $BCD=58° \ 20'$, $BDC=98° \ 45'$, and side $CD=300$, to find $BC=761.47$.

3. In the triangle ACB we have now given the angle $ACB=ACD-BCD=37°$, the side $AC=465.98$ and $BC=761.47$, to find $AB=479.8$ yards, the distance required.

Fig. 57.



EXAMPLE 4, Fig. 57.

Being on one side of a river and observing three objects, A, B and C stand on the other side, whose distances apart I knew to be, $AB=3$ miles, AC

=2, and BC=1.8, I took a station D, in a straight line with the objects A and C, being nearer the former, and found the angle ADB=17° 47′. Required my distance from each of the objects.

Construction.

With the three given distances, describe the triangle ABC; from B, draw BE parallel to CA, and draw BD making the angle EBD=17° 47′ (the given angle ADB) and meeting CA produced, in D: then AD, CD and BD will be the distances required.*

Calculation.

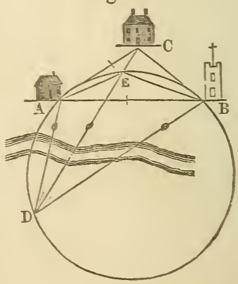
1. In the triangle ABC we have all the sides given, to find the angle C=104° 8′.

2. Subtract the sum of the angles D and C from 180°, the remainder 58° 5′ will be the angle DBC; then in the triangle BCD we know all the angles and the side BC to find DC=5.002 and BD=5.715; therefore DA=DC—AC=3.002.

EXAMPLE 5, Fig. 58.

From a station at D, I perceived three objects, A, B and C, whose distances from each other I knew to be as follows: AB=12 miles, BC =7.2 miles, and AC=8 miles; at D, I took the angle CDB=25° and ADC=19°. Hence it is required to find my distance from each of the objects.

Fig. 58.



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* DEMONSTRATION. By construction, the distances AB, BC and AC are equal to the given distances; also the angle (I. 27) BDC=the angle DBE =the given angle.

Construction.

With the given distances describe the triangle ABC; at B, make the angle EBA=19°=the given angle ADC, and at A, make the angle EAB=25°=the given angle BDC; draw AE, and BE meeting in E, and (by prob. 10,) describe a circle that shall pass through the points A, E and B: join CE and produce it to meet the circle in D, and join AD, BD, then will AD, CD, and BD be the distances required.*

Calculation.

1. In the triangle ABC, all the sides are given, to find the angle BAC=35° 35′.

2. In the triangle AEB, are given all the angles, viz. EAB=25°, EBA=19°, and AEB=136°, and the side AB=12, to find AE=5.624.

---

* Demonstration. The angle ADC standing on the same arc with the angle ABE is equal to it (III. 17.) For the same reason the angle BDC is equal to the angle BAE; but by construction the angles ABE and BAE are equal to the given angles; therefore the angles ADC and BDC are equal to the given angles.

Note.—When the given angles ADC, BDC are respectively equal to the angles ABC, BAC, the point E will fall on the point C, the circle will pass through the points A, C, and B, and the point D may be any where in the arc ADB; consequently, in this case, the situation of the point D, or its distance from each of the objects A, B, C, cannot be determined from the data given.

It may not be improper also to observe, that even when the angle ADB, which is the sum of the given angles, is equal to the sum of the angles ABC, BAC, or which is the same thing, is the supplement of the angle ACB, the circle passes through the points A, C, B; but then the angles ADC, BDC, unless they have been erroneously taken, will be respectively equal to the angles ABC, BAC.

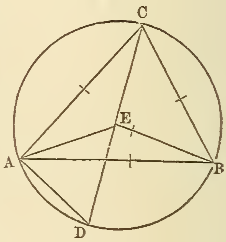
3. In the triangle CAE we have given the side AC =8, AE=5.624, and the angle CAE=BAC—EAB= 10° 35′, to find the angle ACE=22° 41′.

4. In the triangle DAC, all the angles are given, viz. ADC=19°, ACD=22° 41′ and CAD=180°—the sum of the angles ADC and ACD=138° 19′, and the side AC=8, to find AD=9.47 miles, and CD=16.34.

In the triangle ABD, we have the angle ADB=ADC +BDC=44°, the angle BAD=CAD—BAC=102° 44′, and the side AB=12, to find BD=16.85 miles.

EXAMPLE 6, Fig. 59.

Fig. 59.



A person having a triangular field, the sides of which measure AB=50 perches, AC=46 perches, and BC= 40 perches, wishes to have a well dug in it, that shall be equally distant from the corners A, B and C. What must be its distance from each corner, and by what angle from the corner A, may its place be found?

Construction.

With the given sides construct the triangle ABC, and (by Prob. 10.) describe a circle that shall pass through the points A, B, and C; then the centre E of this circle is the required place of the well.

Calculation.

1. In the triangle ABC, all the sides are given, to find the angle ABC=60° 16′.

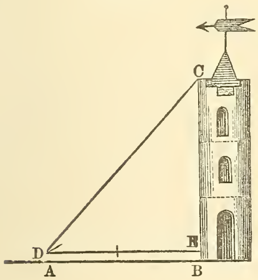
2. Join CE and produce it to meet the circumference in D; also join AE and AD; then the angles ADC, ABC

being angles in the same segment, are equal; also the angle DAC being an angle in a semicircle, is a right-angle: therefore in the right-angled triangle DAC, we have the angle ADC = ABC = 60° 16′, and the side AC, to find CD = 52.98 perches. The half of CD is = 26.49 perches = CE = the distance of the well from each corner.

3. The angle ACD = 90°—ADC = 29° 44; but because AEC is an isosceles triangle, the angle CAE = ACE = 29° 44′ the angle required.

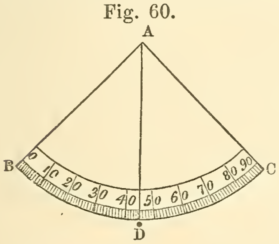
EXAMPLE 7, Fig. 61.

Fig. 61.



Wishing to know the height of a steeple situated on a horizontal plane, I measured 100 feet in a right line from its base, and then took the angle of elevation* of the top, which I found to be 47° 30′, the centre of the quadrant being 5 feet above the ground: required the height of the steeple.

Fig. 60.



* Angles of elevation, or of depression, are usually taken with an instrument called a quadrant, the arc of which is divided into 90 equal parts or degrees, and those, when the instrument is sufficiently large, may be subdivided into halves, quarters, &c. From the centre a plummet is suspended by a fine silk thread. Fig. 60 is a representation of this instrument.

To take an angle of elevation, hold the quadrant in a vertical position, and the degrees being numbered from B towards C, with the eye at C, look along the side CA, moving the quadrant till the top of the object is seen in a range with this side; then the angle BAD made by the plummet with the side BA, will be the angle of elevation required.

A Theodolite or Transit affords a more accurate method, but the quadrant may be made by any one.

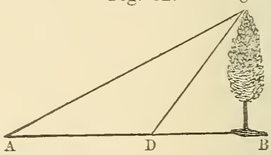
Note.—In finding the height of an object, it is best to take such a position that the observed angle of altitude may be about 45°; for when the observed angle is 45°, a small error committed in taking it, makes the least error in the computed height of the object.

Calculation.

In the right-angled triangle DEC, we have the angle CDE $=47° \ 30'$, and the base DE $=$ AB $=100$ feet, to find CE $=109.13$ feet; to CE add EB $=$ DA $=5$ feet, the height of the quadrant, and it will give BC $=$ 114.13 feet, the required height of the steeple.

EXAMPLE 8, Fig. 62.

Fig. 62.



Wishing to know the height of a tree situated in a bog, at a station D, which appeared to be on a level with the bottom of the tree, I took the angle of elevation BDC $=51° \ 30'$; I then measured DA $=75$ feet in a direct line from the tree, and at A, took the angle of elevation BAC $=26° \ 30'$. Required the height of the tree.

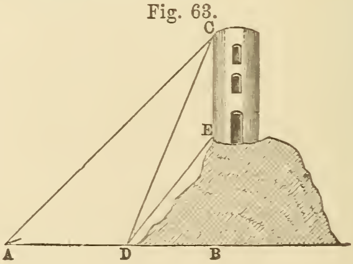
Calculation.

1. Because the exterior angle of a triangle is equal to the sum of the two interior and opposite ones, the angle BDC $=$ DAC $+$ ACD; therefore ACD $=$ BDC $-$ DAC $=25°$: now in the triangle ADC we have DAC $=$ $26° \ 30'$, ACD $=25°$, and AD $=75$, to find DC $=79.18$.

2. In the right-angled triangle DBC are given DC $=$ 79.18, and the angle BDC $=51° \ 30'$ to find BC $=61.97$ feet, the required height of the tree.

EXAMPLE 9, Fig. 63.

Fig. 63.



Wanting to know the height of a tower EC, which stood upon a hill, at A, I took the angle of elevation CAB $=44°$; I then measured AD 134 yards, on level ground, in a straight line towards the tower; at D the angle CDB was 67° 50′ and EDB 51°. Required the height of the tower and also of the hill.

Calculation.

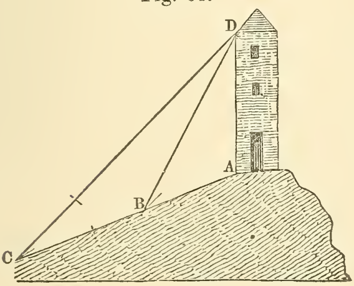
1. In the triangle ADC we have the angle DAC = 44°, the angle ACD = BDC—DAC = 23° 50′, and the side AD, to find DC = 230.4.

2. In the triangle DEC all the angles are given, viz. CDE = BDC—BDE = 16° 50′, DCE = 90°—BDC = 22° 10′, DEC = 180° = the sum of the angles CDE and DCE = 141°, and CD = 230.4, to find CE = 106 yards, the height of the tower.

3. In the right-angled triangle DBC, we have the angle BDC = 67° 50′, and the side DC = 230.4, to find BC = 213.4; then BE = BC—CE = 213.4—106 = 107.4 yards, the height of the hill.

EXAMPLE 10, Fig. 64.

Fig. 64.



An obelisk AD standing on the top of a declivity, I measured from its bottom a distance AB = 40 feet, and then took the angle ABD = 41°; going on in the same direction 60 feet farther to C, I took the angle ACD = 23° 45′: what was the height of the obelisk?

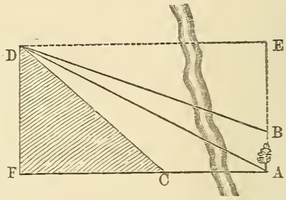
Calculation.

1. In the triangle BCD, we have given the angle BCD = 23° 45′, the angle BDC = ABD—BCD = 17° 15′, and side BC = 60, to find BD = 81.49.

2. In the triangle ABD are given the side AB = 40, BD = 81.49, and the angle ABD = 41°, to find AD = 57.64 feet, the height of the obelisk.

EXAMPLE 11, Fig. 65.

Fig. 65.

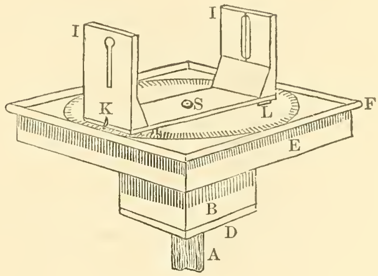


Wanting to know the height of an object on the other side of a river, but which appeared to be on a level with the place where I stood, close by the side of the river; and not having room to go backward on the same plane, on account of the immediate rise of the bank, I placed a mark where I stood, and measured in a direct line from the object, up the hill, whose ascent was so regular that I might account it a right line, to the distance of 132 yards, where I perceived that I was above the level of the top of the object; I there took the angle of depression of the mark by the river's side equal 42°, of the bottom of the object equal 27°, and of its top equal 19°: required the height of the object. *Ans.* 28.64.

Simple Surveying Instrument.

Since many students desire to perform some surveying and trigonometric measuring for themselves, but are unable to procure instruments for taking angles on account of their expensiveness, the following simple instrument is described in detail. The materials for it will cost from one to two dollars, and any careful worker with tools can construct it in a few hours. The results of it will be about as accurate as those of a surveyor's compass.

The board E is fastened to the block B, which turns around the staff A. The staff enters the block far enough to ensure stability. On the board is a graduated card-board circle (this may be had of J. W. Queen & Co., Philadelphia) 8 inches in diameter, graduated to half-degrees, or 13 inches in diameter, graduated to quarter-degrees. This circle is protected by a piece of glass the size of the board, perforated in the centre. The glass is held firmly by the moulding F. On this glass the sights I, I turn around the screw S, pieces of leather, L, L, being glued under each end to give uniform pressure and friction. A narrow slit is sawed down one sight, and a thread is stretched down the wider opening in the other. An index, K, is fitted in one of the sights, and works close to the glass.



The friction of the sights on the glass is so adjusted to that of the block on the staff that either may be moved without disturbing the other.

In taking an angle turn the sights so that the index K will stand over 0 on the circle. Turn the whole block round the staff till the sights point to one object. Then turn the sights around till they point to the other, and the index will read the angle.

In boring the hole in the block, it is better to make it larger than the staff, and partly fill it with perforated cork to secure more uniform friction. The cap D is screwed on to hold the cork and give it firmness.

PRACTICAL QUESTIONS.

1. At 85 feet distance from the bottom of a tower, the angle of its elevation was found to be 52° 30′: required the altitude of the tower. *Ans.* 110.8 feet.

2. To find the distance of an inaccessible object, I measured a line of 73 yards, and at each end of it took

the angle of position of the object and the other end, and found the one to be 90°, and the other 61° 45′: required the distance of the object from each station. *Ans.* 135.9 yards from one, and 154.2 from the other.

3. Wishing to know the distance between two trees C and D, standing in a bog, I measured a base line AB= 339 feet; at A the angle BAD was 100° and BAC 36° 30′; at B the angle ABC was 121° and ABD 49°: required the distance between the trees. *Ans.* 697½ feet.

4. Observing three steeples, A, B and C, in a town at a distance, whose distances asunder are known to be as follows, viz. AB=213, AC=404, and BC=262 yards, I took their angles of position from the place D where I stood, which was nearest the steeple B, and found the angle ADB=13° 30′; and the angle BDC=29° 50′. Required my distance from each of the three steeples. *Ans.* AD=571 yards, BD=389 yards, and CD=514 yards.

5. A May-pole, whose top was broken off by a blast of wind, struck the ground at 15 feet distance from the foot of the pole: what was the height of the whole May-pole, supposing the length of the broken piece to be 39 feet? *Ans.* 75 feet.

6. At a certain place the angle of elevation of an inaccessible tower was 26° 30′; but measuring 75 feet in a direct line towards it, the angle was then found to be 51° 30′: required the height of the tower and its distance from the last station. *Ans.* Height 62 feet, distance 49.

7. From the top of a tower by the sea side, of 143 feet high, I observed that the angle of depression of a ship's

bottom, then at anchor, was 35°; what was its distance from the bottom of the wall? *Ans.* 204.2 feet.

8. There are two columns left standing upright in the ruins of Persepolis; the one is 64 feet above the plane, and the other 50; in a right line between these stands an ancient statue, the head of which is 97 feet from the summit of the higher, and 86 from that of the lower column; and the distance between the lower column and the centre of the statue's base is 76 feet: required the distance between the tops of the columns.

Ans. 157 feet

SURVEYING.

CHAPTER I.

ON THE DIMENSIONS OF A SURVEY.

1. Surveying is the art of measuring, laying out and dividing land.

2. *A Four-Pole Chain* is an instrument used for measuring the boundaries of a survey. It is, as its name imports, 4 poles or 66 feet in length, and is divided into 100 equal parts or links. The length of a link is therefore 7.92 inches.

Note.—A Four-pole Chain is frequently called simply a chain.

3. *A Two-pole Chain* is 2 poles or 33 feet in length, and is usually divided into 50 equal parts or links. When it is thus divided, the links are of the same length as in a four-pole chain; and the measures taken with it are reduced to four-pole chains previous to using them in calculation.

Sometimes the two-pole chain is divided into 40 links; in which case, each two links is the one-tenth of a perch. Measures taken with a two-pole chain, thus divided, are usually expressed in perches and tenths.

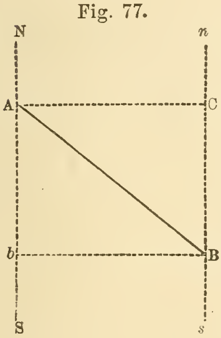
4. The *Distance* of a line in surveying, is its length, estimated in a horizontal direction. It is generally expressed either in chains and links, or in perches and tenths.

5. A *Meridian* or *Meridian Line* is any line that runs due north or south.

Note.—All the meridians passing through any survey of moderate extent may be considered as straight lines, parallel to one another.*

6. The *Bearing* or *Course* of a line, is the angle which it makes, with a meridian passing through one end; and it is reckoned from the North or South Points of the horizon towards the East or West Points.

Fig. 77.



Thus, supposing the line NS, Fig. 77, to be a meridian, and the angle SAB to be 50°; then the bearing of AB from the point A, is 50° to the east of south; which is usually expressed thus: S. 50° E, and read, south, fifty degrees east.

7. The *Reverse Bearing* of a line is the bearing taken from the other end of the line.

Note.—The bearing and the reverse bearing of a line, are angles of the same magnitude,† but lying between

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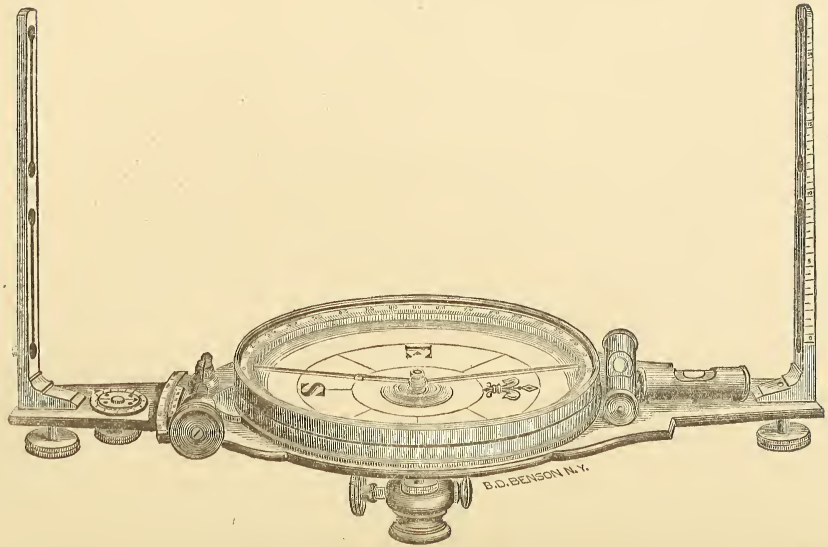
* The meridians are, in reality, curve lines which meet in the north and south poles of the earth. No two of them are therefore exactly parallel; but in usual surveys their deviation from parallelism is so very small, that there is no sensible error in considering them so.

† As the meridians are not exactly parallel, this is not strictly true, except in a few cases; but the difference is too small to be observed in practice. In



SURVEYOR'S COMPASS.

directly opposite points. Thus, if the bearing of AB, from the end A, is S. 50° E., the bearing of the same line from the end B, is N. 50° W.

8. A *Circumferentor* or *Surveyor's Compass*, is an instrument used to take the bearings of lines.

The circumference of its face is divided into degrees, and in some of the larger ones into half degrees, in such manner that two opposite points may be exactly in the direction of the sights with which the instrument is furnished. These points are the north and south points of the instrument. Midway between them, on the circumference, are the east and west points. The degrees are numbered from 0° to 90°, each way from the north and south points to the east and west ones. In the centre of the face is a pin, finely pointed, which supports a Magnetic Needle, moving freely within the instrument. The instrument, when used, is placed on a staff, having a pointed iron at the bottom, and a ball and socket at the top, or on a tripod.

The Chain and Compass are the instruments with which the dimensions of surveys in this country are frequently taken. It is important to have them accurately made. In the selection of a compass, particular attention should be directed to the settling of the needle. If, when the needle has been moved out of its natural position, it settles very soon, it is defective; either its magnetic virtue is weak, or it does not move with sufficient freedom on the pin.

9. The *Difference of Latitude*, or the *Northing* or

---

the latitude of Philadelphia the greatest difference between the bearing and reverse bearing of a line, a mile in length, is only 44″. In higher latitudes the difference is greater.

Southing of a line, is the distance that one end is further north or south than the other end; or it is the distance which is intercepted on a meridian passing through one end, between this end and a perpendicular to the meridian, from the other end.

Thus, if NS, Fig. 77, be a meridian passing through the end A, of the line AB, and B*b* be perpendicular to NS, then is A*b* the difference of latitude or southing of AB.

10. The *Departure* or the *Easting* or *Westing* of a line is the distance that one end is further east or west than the other end; or it is the distance from one end, perpendicular to a meridian passing through the other end.

Thus B*b*, Fig. 77, is the departure or easting of the line AB.

But if *ns* be a meridian, and AC perpendicular to it, and if the bearing of the line be taken from B to A, then is BC the difference of latitude or northing, and AC the departure or westing, of the line AB.

Note.—It is evident from the definitions, that the Distance, Difference of Latitude, and the Departure form the sides of a right-angled triangle; in which, considering the departure as the base, the perpendicular is the difference of latitude, the hypothenuse is the distance, and the angle at the perpendicular is the bearing.

11. The *Meridian Distance* of any station, is its distance from a meridian passing through the first station of the survey, or any other assumed point.

12. The *Traverse Table*, is a table containing the dif-

ferences of latitude and the departures, computed to different courses and distances.

13. The *Area* or *Content* of a tract of land is the *horizontal* surface included within its boundaries, expressed in known measures, as Acres, Roods, and Perches.

14. In going round a tract of land and returning to the place of beginning, it is evident that the whole northing which has been made, must be equal to the southing, and the easting to the westing; or in other words, that the sum of all the northings must be equal to that of the southings, and the sum of the eastings, to that of the westings.

This principle enables us to judge of the accuracy of a survey, when the bearings and distances of all the sides have been taken. If the sums of the computed northings and southings are equal, and also those of the eastings and westings; or, if, though not exactly equal, they are very nearly so, we may conclude that the survey has been correctly made; as very small differences in these sums may be imputed to little, unavoidable errors in taking the bearings and measuring the distances. But when the sum of the northings differs considerably from that of the southings, or that of the eastings from that of the westings, we must infer that an error has been made, too great to be admitted. In this case a re-survey should be taken.

It is a rule with some of our best practical surveyors, that when the difference between the sums of the northings and southings, called the *error in latitude*, or that between the sums of the eastings and westings, called the *error in departure*, exceeds one link for every ten chains in the sum of the distances, a re-survey ought to be made.

When the errors in latitude and departure fall within the limits just mentioned, they should be properly apportioned among the several latitudes* and departures; we shall thus obtain what are called the corrected latitudes and departures. The method of doing this will be given in one of the following problems.

PROBLEM I.

To reduce two-pole chains and links to four-pole chains and links.

RULE.

1. If the number of chains is *even*, divide it by 2, and to the quotient annex the given number of links.

2. If the number of chains is *odd*, divide by 2 as before, for the chains; and for the 1 that is off, add 50 to the given number of links.

EXAMPLES.

1. In 16 two-pole chains and 37 links, how many four-pole chains and links? *Ans.* 8 *ch.* 37 *links*, or 8.37 *ch.*

2. How many four-pole chains and links are there in 17 two-pole chains and 42 links? *Ans.* 8.92 *ch.*

3. How many four-pole chains and links are there in 22 two-pole chains and 7 links? *Ans.* 11.07 *ch.*

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* In order to conciseness of expression, difference of latitude is frequently called simply, latitude.

PROBLEM II.

To reduce two-pole chains and links to perches and hundredths of a perch.

RULE.

Multiply the links by 4, for the hundredths, and the chains by 2, for the perches. If the hundredths exceed 100, set down the excess, and add 1 to the perches.

Note.—This rule supposes the two-pole chain to be divided into 50 links.

EXAMPLES.

1. Reduce 17 two-pole chains and 21 links to perches and hundredths. *Ans.* 34.84 *per.*

2. Reduce 15 two-pole chains and 38 links to perches and hundredths. *Ans.* 31.52 *per.*

3. Reduce 57 two-pole chains and 49 links to perches and hundredths. *Ans.* 115.96 *per.*

PROBLEM III.

To reduce square four-pole chains to acres.

RULE.

Divide by 10, and the quotient will be the acres. If there is a decimal in the quotient, multiply it by 4, for the roods; and the decimal of these by 40, for the perches.

EXAMPLES.

1 Reduce 523.2791 square chains to acres.

$$
\begin{array}{r}
10)523.2791 \\
\hline
52.32791 \\
4 \\
\hline
1.31164 \\
40 \\
\hline
12.46560
\end{array}
\quad \textit{Ans. } 52\ ac.\ 1\ r.\ 12\ p.
$$

2. Reduce 41.9682 square chains to acres.
Ans. 4 *ac.* 0 *r.* 31 *p.*

3 Reduce 132.925 square chains to acres.
Ans. 13 *ac.* 1 *r.* 6.8 *p.*

PROBLEM IV.

To reduce acres, roods and perches to square chains.

RULE.

Divide the perches by 40 and prefix the roods; divide the result by 4 and prefix the acres; then this latter result, multiplied by 10, will give the square chains.

Or reduce the given quantity to perches and divide by 16.

EXAMPLES.

1. Reduce 13 *ac.* 1 *r.* 10 *p.* to square chains

$$
\begin{array}{l}
40)10 \\
\hline
4)1.25 \\
\hline
13.3125 \quad A = 133.125\ sq.\ ch.
\end{array}
$$

2. Reduce 127 *ac.* 3*r.* 23*p.* to square chains.
Ans. 1278.9375 *sq. ch.*

3. Reduce 35 *ac.* 0 *r.* 20 *p.* to square chains.
Ans. 351.25 *sq. ch.*

PROBLEM V.

To find the bearing of a line.

1. Let a stake of six or eight feet in length be set up perpendicularly, at the far end of the line. Set up the compass firmly at the beginning of the line, and, freeing the needle so as to rest on the pivot, adjust it to a horizontal position; the ball and socket admitting a motion for that purpose. This position can be determined with sufficient accuracy, by observing whether, when the compass is turned round, the ends of the needle remain at the same height above the face of the instrument.

2. Turn the compass round so as to bring the south end of it towards the stake at the far end of the line Then applying the eye to the sight at the north end, move the compass gently round till the stake can be seen through the fine slits in both sights, and let it remain in this position.

3. When the needle has settled, observe the number of degrees and parts of a degree, that are intercepted between the south end of the needle and the north or south point of the compass, to whichever it is nearest; which will be the bearing, reckoning it from *that* point, towards the east if the south end of the needle is to the right hand, but towards the west if it is to the left hand

Note 1.—The bearing thus obtained may be, and should be, verified by going to the far end of the line, and from thence taking the bearing to the first end; which, if both bearings are correct will be the reverse of the former.

Note 2.—When there is a fence on the side, or other obstacle in the way, preventing the stake at one end from being seen through the compass sights at the other end, the bearing may be obtained by setting up the compass and stake at small *equal* distances to the right or left, so that the line joining them may be parallel to the side.

Note 3.—The method of obtaining the bearing between two stations when there are obstacles in the way, which also prevent a parallel bearing being readily taken, or when the stations are too distant to be seen from each other, will be noticed in the next chapter.

PROBLEM VI.

To measure the distance of a line.

For convenience in marking the termination of the chain in measuring, *ten* iron pins should be provided, about a foot in length, and terminated at top by a small ring, to which a piece of red flannel or other conspicuous substance should be tied, in order that the pins may be readily found, when set up among high grass or in other situations where they would not otherwise be easily discovered.

Let the person who is to go foremost in carrying the chain, take nine of the pins in his left hand, and one end of the chain and the other pin in his right hand; then he moving on in the direction of the line, let another person take the other end of the chain and hold it at the beginning of the line. When the leader has moved on till the chain is stretched tight, he must set down the pin, perpendicularly, exactly at the end of the chain, the hinder chain-man taking care that the chain is in the direction of the line; which is readily determined by observing whether it is in a range with a stake previously set up at the far end of the line. When the leader has not his end of the chain in the direction of the line, the hinder chain-man can direct him which way to move, by a motion of his left hand. When the distance of one chain or half chain* has been thus determined, the carriers, taking hold of the two ends of the chain, move on till the hinder one comes to the pin which was set up by the other; then the chain being stretched, the person at the fore end of it sets up another pin as before; the hinder chain-man then taking up the pin at his end, they proceed to a third distance of the chain; and so on. When the person at the fore end of the chain has set up all his pins, he still moves on another length of the chain, and then setting his foot on it to keep it in place, he cries "*out*." The hinder chain man then comes forward, and counts to him the ten pins; and he setting up one of them at the end of the chain, again moves on, dragging the chain after him, till he is checked by the hinder chain-man, who, getting the hind end of the chain, applies it as before to the pin set up. The number of outs should be carefully noticed; each *out* being ten chains, when a four-pole chain is used, but only five, when the measuring is done, with a two-pole chain. When arrived at the end of the line, the number of pins, which the one at the fore end of the chain has set up since the last *out*, and the number of links from the last pin to the end of the line, must be carefully noted. From these, and

* When a two-pole chain is used, one length of it may properly be called a half chain

the number of *outs*, the distance measured is readily determined.

All slant or inclined surfaces, as the sides of a hill, should be measured horizontally, and not on the plane or surface of the hill. To effect this, the hind end of the chain, in ascending a hill, should be raised from the ground till it is on a level with the fore end, and, by means of a plummet and line, or when the hill is not very steep, by estimation, should be held perpendicularly above the termination of the preceding chain. In descending a hill, the fore end of the chain should be raised in the same manner, and the plummet being suspended from it will show the commencement of the succeeding chain.

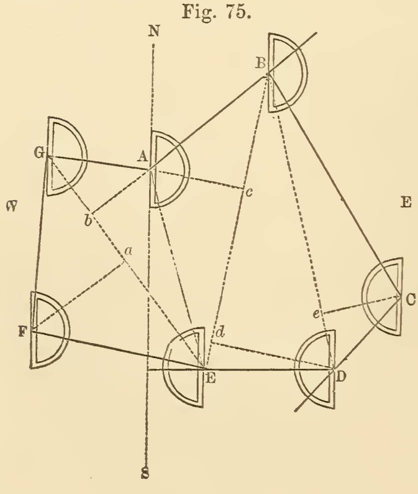
PROBLEM VII.

To protract a Survey, having the bearings and distances of the sides given.

The method of doing this will be best understood by an example. Thus,

Suppose the following field notes to be given, it is required to protract the survey.

Fig. 75.



				Ch.
1.	N.	50°	E.	9.60
2.	S.	32°	E.	16.38
3.	S.	41°	W.	6.30
4.		West		8.43
5.	N.	79°	W.	10.92
6.	N.	5°	E.	11.25
7.	S.	83°	E.	6.48

Method 1st.

Draw NS, Fig. 75, to represent a meridian line; then N standing for the north and

S for the south, the east will be to the right hand, and the west to the left. In NS take any convenient point as A for the place of beginning, and apply the straight edge of the protractor to the line, with the centre to the point A, and the arch turned towards the east, because the first bearing is easterly; then holding the protractor in this position, prick off 50° the first bearing, from the north end, because the bearing is from the north; through this point and the point A, draw the line AB on which lay 9.60 chains, the first distance from A to B. Now apply the centre of the protractor to the point B, with the arch turned toward the east, because the second bearing is easterly, and move it till the line AB produced, cuts the first bearing 50°; the straight edge of the protractor will then be parallel to the meridian NS; hold it in this position, and from the south end prick off the second bearing 32°; draw BC and on it lay the second distance 16.38 chains. Proceed in the same manner at each station, observing always, previous to pricking off the succeeding bearing, to have the arch of the protractor turned easterly or westerly, according to that bearing, and to have its straight edge parallel to the meridian; this last may always be done by applying the centre to the station point, and making the preceding distance line produced if necessary, cut the degrees of the preceding bearing. It may also be done by drawing a straight line through each station, parallel to the first meridian.

When the survey is correct, and the protraction accurately performed, the end of the last distance will fall on the place of beginning.

Case 2.

When angles and distances are given.

When the angles have been taken with the Theodolite or Transit, the following method is simpler:

Beginning at any convenient angle, lay down the first side. Place the flat edge of the protractor on this line, with the centre at its extremity, and prick off the angle between the first and second sides. Through this point and the end of the first side draw a line, and measure off on it the length of the second side, and so go round the tract.

When the survey is correct, and the protraction accurately performed, the last side should terminate at the starting point, and the angle at this point be equal to the first given angle.

Protract the figure of last Case from the following field notes:

Angles.		Distances.	
A	227°	AB	9.60
B	82°	BC	16.38
C	107°	CD	6.30
D	131°	DE	8.43
E	169°	EF	10.92
F	96°	FG	11.25
G	88°	GA	6.48

If the angles are taken, it is usual to take the bearing of one side. This may be used as the first side of the plot, and thus all directions are determined.

PROBLEM VIII.

The bearing of two lines from the same station being given, to find the angle contained between them.

RULE.

When they run from the *same* point of the compass towards the *same* point, *subtract* the less from the greater.

When they run from the *same* point, towards *different* points, *add* them together.

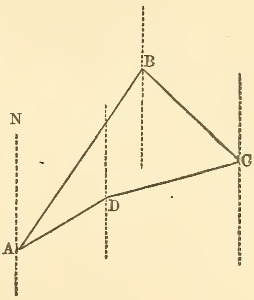
When they run from *different* points, towards the *same* point, *add* them together, and take the *supplement* of the sum.

When they run from *different* points, towards *different* points, *subtract* the less from the greater, and take the *supplement* of the remainder.

Note.—While these rules are useful to some surveyors, it is better, in any case, to draw a figure accurately representing the given lines, when the proper method of obtaining the angle will readily be seen.

EXAMPLES.

Fig. 67.



1. Given the bearing of the line AB, Fig. 67, N. 34° E., and AD, N. 58° E.; required the angle A.

$$\begin{aligned} NAD &= 58° \\ NAB &= 34° \\ \hline \text{Angle } A &= 24° \end{aligned}$$

2. Given the bearing of BA, Fig. 57, S. 34° W., and BC, S. 35° E.; required the angle B. *Ans.* B = 69°.

3. Given the bearing of BC, Fig. 67, S. 35° E., and CD, S. 87° W.; required the angle C. *Ans.* 58°.

4. Given the bearing of DC, Fig. 67, N. 87° E., and DA, S. 58° W.; required the angle D. *Ans.* 151°.

PROBLEM IX.

To change the bearings of the sides of a survey in a corresponding manner, so that any particular one of them may become a Meridian.

RULE.

Subtract the bearing of the side that is to be made a meridian, from those bearings that are between the *same* points that it is, and also from those that are between points directly *opposite* to them. If it is *greater* than either of the bearings from which it is to be subtracted, take the difference, and change E. to W., or W. to E.

Add the bearing of the side which is to be made a meridian, to those bearings which are neither between the same points that it is, nor between the points that are directly opposite to them. If either of the sums exceeds 90°, take the *supplement* and change N. to S., or S. to N.*

Note.—When the bearings of some, or all, of the sides of a survey have been thus changed, and by calculation the changed bearing of another side or line has been

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* The changing of the bearings so as to make a given side become a meridian, may be illustrated by means of a protracted survey. If a protracted survey or plot is held horizontally, with the meridian in a north and south direction, the north end being towards the north, the bearings of the sides of the plot will then correspond with the bearings of the sides of the survey. If then, keeping the paper horizontal, it be turned round till any particular side of the plot has a north and south direction, or becomes a meridian, the bearings of all the other sides of the plot will have been changed by a like quantity. But it is evident, that neither the relation of the different parts of the plot to one another, the area nor the lengths of the sides will have been altered by this change. In any problem of this kind let the student draw a plot, and note the effect of turning each of the sides in the proper direction through the given angle. He can thus easily see what additions or subtractions must be made to find the new bearings.

found, its *true* bearing will be obtained by applying to the changed bearing, the bearing of the side which was made a meridian, in a contrary manner to what is directed in the rule; that is, by adding in the case in which the rule directs to subtract, and by subtracting in the case in which it directs to add.

EXAMPLES.

1. Given the bearings of the sides of a survey as follow; 1st. S. 45½° W.; 2d. N. 50° W.; 3d. North; 4th. N. 85° E.; 5th. S. 47° E.; 6th. S. 20½° W.; and 7th. N. 51¼° W. Required the changed bearings, so that the 5th side may be a meridian.

1st.	S. 45½° W.
	47
	92½
	180
chang. bear.	N. 87½° W.
2d.	N. 50° W.
	47
chang. bear.	N. 3° W.
3d.	N. 0° E.
	47
chang. bear.	N. 47° E.
4th.	N. 85° E.
	47
	132
	180
chang. bear.	S. 48° E.

5th. side, changed bearing, South.

6th. S. $20\frac{1}{2}°$ W.
47

chang. bear. S. $67\frac{1}{2}$

7th. N. 51° W.
47

chang bear. N. $4\frac{1}{4}$ W.

2. Given the following bearings of the sides of a survey; 1st. S. $40\frac{1}{2}°$ E.; 2d. N. 54° E.; 3d. N. $29\frac{1}{4}°$ E.; 4th. N. $28\frac{3}{4}°$ E.; 5th. N. 57° W.; and 6th. S. 47° W.; to find the changed bearings so that the 2d. side may be a meridian. *Ans.* 1st. N. $85\frac{1}{2}°$ E.; 2d. North; 3d. N. $24\frac{3}{4}°$ W.; 4th. N. $25\frac{1}{4}°$ W.; 5th. S. 69° W.; 6th. S. 7° E.

3. Given the bearings as in the 1st. example; viz. 1st. S. $45\frac{1}{2}°$ W.; 2d. N. 50° W.; 3d. North; 4th. N. 85° E.; 5th. S. 47° E.; 6th. S. $20\frac{1}{2}°$ W; 7th. N. $51\frac{1}{4}°$ W.; to find the changed bearings so that the 6th side may be a meridian. *Ans.* 1st. S. 25° W.; 2d. N. $70\frac{1}{2}°$ W.; 3d. N. $20\frac{1}{2}°$ W.; 4th N. $64\frac{1}{2}°$ E.; 5th. S. $67\frac{1}{2}°$ E.; 6th. South; 7th. N. $71\frac{3}{4}°$ W.

PROBLEM X.

Of the bearing, Distance, Difference of Latitude and Departure, any two being given, to find the other two

RULE.

When the bearing and distance are given.

As Rad. : cos. of bearing : : distance : dif. of latitude.
Rad. sin. of bearing : : distance ; departure.

When the bearing and difference of latitude are given.

As cos. of bearing : rad. : : diff. lat. : distance.
Rad. : tang. of bearing : : diff. lat. : departure.

When the bearing and departure are given.

As sin. of bearing : rad. : : departure : distance.
Rad. : cotang. of bearing : : departure : diff. lat.

When the difference of latitude and the departure are given.

As diff. lat. : departure : : rad. : tang. of bearing.
Cos. of bearing : rad. : : diff. lat. : distance.

When the distance and difference of latitude are given.

As distance . diff. lat. : : rad. : cos. of bearing.
Rad. : tang. of bearing : : diff. lat. : departure.

When the distance and departure are given.

As distance : departure : : rad. : sin of bearing.
Rad. : cos. of bearing : : distance : diff. lat.

Note.—It is evident the above proportions are the solutions of a right-angled triangle, having for its sides the distance, difference of latitude, and departure; they may be proved by demonstrations similar to that on page 63.

The following examples may be worked out by the rules. The reason of each rule should be clearly seen before it is applied to the example.

1. Given the bearing of a line, S. 32° 30′ E., and the departure 10.96 ch. to find the distance and difference of latitude. *Ans.* Dist. 20.40 ch.; diff. lat. 17.20 S.

2. Given the distance of a line, running between the north and east, 44 ch. and its difference of latitude 34.43 ch.; to find the bearing and departure.
Ans. Bearing, N. 38° 30′ E.; dep. 27.39 ch. E.

3. The bearing of a line S. 32° 30′ E., and the difference of latitude 17.21 ch. being given, to find the distance and departure. *Ans.* Dist. 20.41 ch.; dep. 10.96 E.

4. Given the difference of latitude of a line 27.92 N., and the departure 5.32 E.; to find the bearing and distance. *Ans.* Bearing, N. 10° 47′ E.; dist. 28.42.

5. The distance of a line, running between the north and west, is 35.35 ch., and its departure 15.08 ch., required the bearing and difference of latitude.
Ans. Bearing N. 25° 15′ W.; diff. lat. 31.97 N.

PROBLEM XI.

To find the difference of latitude and departure corresponding to any given bearing and distance, by means of the Traverse Table.

RULE.

When the distance is any number of whole chains or perches, not exceeding 10.

Find the given bearing at the *top* or *bottom* of the column, according as it is *less* or *more* than 45°. Then against the given distance, found in the column of distances at the side of the table, and under or over the given bearing, is the difference of latitude and departure, which must be taken as marked at the top of the column when the bearing is at the top, but as marked at the bottom when the bearing is at the bottom.

When the distance is any number of chains or perches and decimals of a chain or perch.

Take out the latitude and departure corresponding to each figure, moving the decimal point one place to the right or left, as the number is to the left or right of the unit's place, and add all the results.

EXAMPLES.

1. Given the bearing of a line S. 35¼ E., distance 79 ch.; required the latitude and departure.

	Lat.	Dep.
70.	57.165	40.400
9.	7.3498	5.1943
	64.5148 S.	45.5943 E.

2. Given the bearing N. 56½ W., distance 27.04.

	Lat.	Dep.
20.	11.039	16.678
7.	3.864	5.837
.04	.022	.034
	14.925 N.	22.549 W.

3. Given bearing N. 29¾ E. and distance 29.84.

	Lat.	Dep.
20.	17.364	9.924
9.	7.814	4.466
.8	.695	.397
.04	.035	.020
	25.908 N.	14.807 E.

1. Given the bearing and distance of a line, N. 39½° W. 15.20 ch., to find its difference of latitude and departure. *Ans.* Diff. lat. 11.72 N., and dep. 9.67 W.

2. The bearing and distance of a line are N. 46° E., 27.25 ch.; required its difference of latitude and departure. *Ans.* Diff. lat. 18.93 N. and dep. 19.60 E.

3. The bearing and distance of a line are S. 37¼° W., 137.50 ch.; required its difference of latitude and departure. *Ans.* Diff. lat. 109.45 S., and dep. 83.23 W.

4. Required the difference of latitude and departure of a line, whose bearing and distance are S. 6½° E., 5.60 ch. *Ans.* Diff. lat. 5.56 S., and dep. 0.63 E.

PROBLEM XII.

Given the bearings and distances of all the sides of a tract of land to obtain the corrected latitudes and departures.

RULE.

1. Rule a table as in the annexed example, in the first vertical column of which, place the letters designating the sides, or the numbers denoting the stations at the beginning of each side; in the second column, place the bearings; and, in the third, the distances.

2. Find, by the last problem, the difference of latitude and the departure, corresponding to each side, and place them in the next four columns, under their proper heads of N. or S., E. or W. Add up the northings and southings; and if the sums are not equal, find their difference;

which will be the error of the survey in difference of latitude; which call by the same name as the *least* sum. Proceed in the same manner with the eastings and westings, and find the error in departure. Also add up the column of distances. Then it will be,

As the sum of the distances,
Is to any particular distance,
So is the *error* in latitude or departure
To the *correction* of latitude or departure, corresponding to that distance.

3. Find, by the above proportion, the corrections of latitude and departure corresponding to all the sides calculating them to the nearest two decimal figures, and place them in the next two columns, heading them with the *same names* as the *errors* in latitude and departure. If the sums of these corrections, are not respectively equal to the errors in latitude and departure, which, in consequence of the fractions neglected, will sometimes be the case, alter some of them by a unit in the second decimal figure, to make them so.

4. Apply these corrections to their corresponding differences of latitude and departures, by *adding* when of the *same* name, but by *subtracting* when of *different* names, and the corrected differences of latitude and departures will be obtained; which may be placed in the four succeeding columns.

In these the sums of the northings and southings will be equal, and also those of the eastings and westings.*

* The directions given in the rule, for correcting the errors in difference of latitude and departure, are deduced from the rule given and demonstrated in No. 4, of a periodical work, called the Analyst, by *Nathaniel Bowditch*, A. M., and also by the editor, *Professor Adrain.* The demonstration is too long, and not of a nature for insertion here.

Note 1.—In the proportion for finding the correction of the latitude or departure, the decimal parts of the sum of the distances and of the particular distance may be omitted, taking, in each case, the nearest number of whole chains.

2. The corrections may be frequently estimated with sufficient accuracy without the trouble of working out the proportions.

3. When one or two of the sides are hilly, or when there are other difficulties in the way of obtaining their bearing or distances with accuracy, it is better to allow a considerable part of the errors, on the latitudes and departures corresponding to them, and afterwards to apportion the remaining part among the others.

EXAMPLES.

1. Given the bearings and distances of the sides of a tract of land as follow: 1st. S. 40½° E. 31.80 ch.; 2nd. N. 54° E. 2.08 ch.; 3rd. N. 29¼° E. 2.21 ch.; 4th. N. 28¾° E. 35.35 ch.; 5th. N. 57° W. 21.10 ch.; 6th. S. 47° W. 31.30 ch. Required the corrected differences of latitude and departures.

Sta.	Courses.	Dist. Ch.	N. L.	S. L.	E. D.	W. D.	Cor. S.	Cor. E.	N. L.	S. L.	E. D.	W.D.
1	S. 40½ E.	31.80		24.18	20.65		.03	.05		24.21	20.70	
2	N. 54 E.	2.08	1. 23		1.68		.00	.00	1.23		1.68	
3	N. 29¼ E.	2.21	1.92		1.08		.00	.00	1.92		1.08	
4	N. 28¾ E.	35.35	31.00		17.00		.04	.05	30.96		17.05	
5	N. 57 W.	21.10	11.49			17.69	.02	.03	11.47			17.66
6	S.47. W.	31.30		21.34		22.89	.03	04		21.37		22.85
		123.84	45.64 45.52	45.52	40.41	40.58 40.41	.12	.17	45.58	45.58	40.51	40.51

12 Er. S. .17 Er. E

As 124 : 32 :: .12 : .03	As 124 : 32 : : .17 : .04 or .05
124 : 2 :: .12 : .00	124 : 2 : : .17 : .00
24 : 35 :: .12 : .03 or .04	124 : 35 : : .17 : .05
124 : 21 :: .12 : .02	124 : 21 : : .17 : .03
124 : 31 :: .12 : .03	124 : 31 : : .17 : .04

2. Given the bearings and distances of the sides of a tract of land as follow: 1st. N. 75° E. 13.70 ch.; 2d. N. 20½ E. 10.30 ch.; 3d. East 16.20 ch.; 4th. S. 33½° W. 35.30 ch.; 5th. S. 76 W. 16 ch.; 6th. North 9 ch.; 7th. S. 84° W. 11.60 ch.; 8th. N. 53¼° W. 11.60 ch.; 9th. N. 36¾° E. 19.36 ch.; 10th. N. 22½° E. 14 ch.; 11th. S. 76¾° E. 12 ch.; 12th. S. 15° W. 10.85 ch.; 13th. S. 18° W. 10.62 ch.; to the place of beginning. Required the corrected latitudes and departures.

Ans. 1st. 3.56 N. 13.26 E.; 2d. 9.66 N. 3.62 E.; 3d. 0.02 N. 16.22 E.; 4th. 29.39 S. 19.44 W.; 5th. 3.85 S. 15.50 W.; 6th. 9.01 N. 0.01 E.; 7th. 1.19 S. 11.52 W.; 8th. 6.96 N. 9.27 W.; 9th. 15.54 N. 11.61 E.; 10th. 12.95 N. 5.38 E.; 11th. 2.73 S. 11.70 E.; 12th. 10.46 S. 2.80 W.; 13th. 10.08 S. 3.27 W.

PROBLEM XIII.

To find the difference of latitude and departure corresponding to any given bearing and distance, by means of the table of natural sines and cosines.

RULE.

Find from the table the natural sine of the angle representing the bearing, and multiply it by the distance, this will give the departure; find its cosine, and multiply by the distance, this will give the difference of latitude.*

Note.—This rule will be found convenient when the bearings are given to minutes.

EXAMPLES.

1. Given the bearing and distance of a line N. 28° 42′ E. 26 ch., to find the difference of latitude and departure.

Natural cosine of 28° 42′ = .87715
.87715 × 26. = 22.81 = diff. of lat. N.
Natural sine of 28° 42′ = .48022
.48022 × 26. = 12.49 = dep. E.

2. Given the bearing and distance of a line S. 57° 28′ W. 11.40 ch., to find the difference of latitude and departure.

Ans. Diff. lat. = 6.13 S. Dep. = 9.61 W.

Note.—It is often more convenient to add to the *logarithmic* sine and cosine of the bearing the logarithm of the distance, and find the natural number corresponding.

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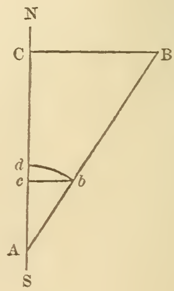
* DEMONSTRATION.—Let NS be a meridian and AB a given line. Then CAB will be its bearing, AC its difference of latitude, and BC its departure. Lay off A*b* = 1, and describe the arc *db*; draw *bc* at right angles to AC. Then *bc* will be the natural sine of A and A*c* the cosine, as given in the table.



Now, Ab or $1 : bc :: AB : BC$,
and Ab or $1 : Ac :: AB : AC$;
whence $BC = AB \sin A$,
and $AC = AB \cos A$,
which agree with the rule.

PROBLEM XIV.

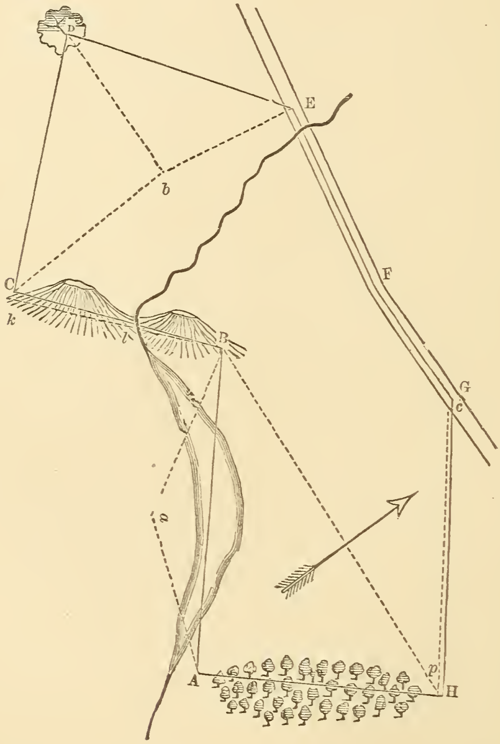
To measure the angles and sides of a tract with a transit or theodolite and chain, and from the angles to calculate the bearings.

Note.—If the student is not familiar with the use of the transit or theodolite, he should now study Chapter VII. Practice with the instruments under a competent instructor is, however, by far the best and easiest way to understand them. The solution of this problem explains the field-work now most commonly used, by those who can command the necessary implements, to find the area of a farm.

In theory the problem of surveying a farm is a very simple one. The instrument must be carried around the tract from corner to corner, and the horizontal angles read, while the chainmen measure the distance from one angle to another. In practice difficulties are frequently met with, owing to inequalities of the ground, the impassable condition of the lines to be chained, or the thickets which prevent sighting from one station to another.

The first step is to determine accurately the positions of the corners. If these are not marked by stones, they must be established by mutual consent of the landowners interested, or by the old deeds.

Let us suppose the accompanying figure to represent a tract to be surveyed. Beginning with A, the instrument is placed directly over the corner stone, with the plumb-bob exactly at the angle. It being impossible to see H on account of an intervening woods, the angle A is left for the present. A pond of water makes it impossible to chain in a direct line to B, so, poles being set up at B and some convenient point as *a*, the angle, B*Aa*, is measured, and the distance A*a* chained. The instrument being conveyed to a, the angle AaB is read. Then by trigonometry the side AB may be calculated. At B the station C cannot be seen, nor can B and C be seen from any intermediate point. A random line, Bk, is therefore run as near as possible in the direction of BC. This is done by pointing the telescope in the supposed direction, while an assistant places a pole as



far along the line as it can be seen, as at l. The angle ABl is now read. The instrument is moved to l, pointed to B, and reversed so as to point in direction of k. This is kept up till the chainmen are at k, in the line DC produced, when the distance kC and the angle DkB are measured. Then in the triangle kBC

the side BC and the angle *k*BC can be calculated, and the true angles ABC, BCD are known. The angle D is inaccessible; hence the point *b* is taken, the angles DC*b*, D*b*C, D*b*E and DE*b* and the sides C*b*, *b*E are measured; whence the angle D and sides CD and DE may be calculated. The stations E, F and G being in the middle of a public road, no difficulty is found in measuring sides and angles. The line GH is covered by a fence which obstructs the sight. A point *c* is therefore taken in the line FG, and a point *p* is taken so that the line *cp* is parallel to GH. The angle F*cp* may now be read. The chaining may also be done along the line *cp*, care being taken to start directly opposite the station G, and not in the line FG. Between H and A there is a thick woods. If not too difficult, it is better to clear a track between the stations, so that one can be seen from the other; or, a random line may be run, and points in the line AH interpolated, to which the instruments may be directed in reading the angles A and H; (the method of performing this is shown on page 111); or, the diagonal BH and the angle ABH may be measured, whence AH and the angles A and H can be calculated. (Other methods of finding the length of lines connecting inaccessible points are given on page 112, &c.) It is well to measure several diagonals as a check on the other work; also angles should be taken, at several stations, to certain well-defined points within the tract, for the same purpose; and a few bearings should be read, to enable us to calculate the remainder. It should be remembered that direct measurements are always preferable to trigonometric calculations when they can be performed.

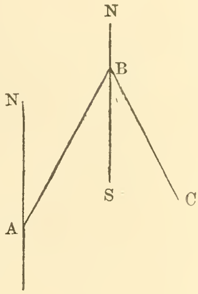
Many other difficulties will arise in practice which an ingenious and thoughtful person will overcome by methods improvised for the occasion. A little practice with a good surveyor is the best means of acquiring facility.

The greatest care should be taken in keeping notes of the work done. No calculations but those necessary to the continuance of the work should be made in the field; hence, it is very important that no confusion should arise in reading the notes made, and some systematic method should be employed to keep them. Every measurement should be noted down as soon as made, and the memory never relied on. Keeping field-notes will be described on page 113.

Having ascertained the various angles and sides of the tract, the next step is to test the accuracy of the work. The interior angles are added, and their sum should equal twice as many right angles as the figure has sides, less four right angles. Should they not be equal, or not within a very few minutes, a remeasurement of the angles becomes necessary, beginning with those in which, from the nature of the intervening ground, an error is most likely to occur. Having these correct, the survey is plotted by Case 2, page 93. Should the plot not close properly, a rechaining of the sides must be performed till the error is found and corrected.

From the angles the bearings may be easily deduced. One of the bearings should be taken in the field with great care if a deed of the property is needed. Or in case the area only is wanted, one of the sides may be assumed to run north and south, and the other bearings may be calculated by addition and subtraction.

If we know the bearing of a line and the angle it makes with the next, the second bearing may easily be determined; thus, let AB run N. 40° E., and the angle ABC be 58° to the right; draw AB, making the angle NAB = 40°. Make ABC = 58°. Then the bearing of BC = SBC = ABC − ABS = 58° − 40° = 18°. If a figure be drawn in this way, the student will readily see how to combine the angles to obtain the bearings.

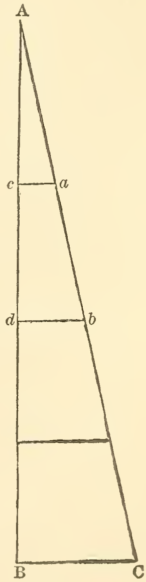


Having the bearings and distances of the sides, the area is found by Problem IX., page 141.

MISCELLANEOUS PROBLEMS.

1. *To interpolate points in a line.*

It is sometimes necessary to find points in a direct line between two stations which are separated from each other by a woods, a hill or other obstacle, preventing the surveyor from seeing one station from the other. This is done by means of a random line. Let AB be the line in which it is desired to find a number of points. Starting from one end A, a line is run as near in the direction desired as can be ascertained. Let this be the line AC. Chain to various points in this line, as *a*, *b*, &c., and to C, and drive stakes. Measure also the distance CB between the ends of the two lines.

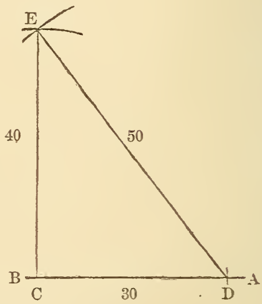


Then say, As AC : A*a* : : CB : *ac*,
and AC : A*b* : : CB : *bd*, &c., &c.

Calculate from these proportions the true length of *ac* and *bd*, and from the stakes at *a* and *b* lay off the calculated distances parallel to CB; *c* and *d* will be points in the line AB.

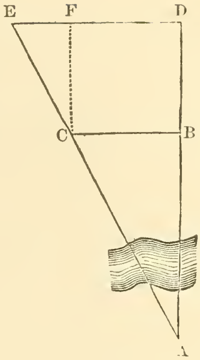
2. *To run a perpendicular to a given line by the use of the chain.*

Let AB be the line, and C a point in it. Lay off CD on AB 30 links. With a radius of 40 links, sweep a circle with C as a centre. With a radius of 50 links, sweep a circle with D as a centre. They cut in E. Then will CE be at right angles to AB, and may be continued to any length. Any numbers in the ratio of 3, 4 and 5 will be the sides of a right-angled triangle.



3. *To measure the distance to an inaccessible point by chain surveying.*

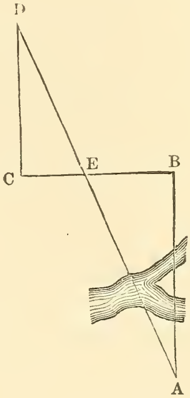
It is frequently necessary to know the distance to a point which cannot be reached, as the distance across a river, to a point out in the ocean, on a mountain, &c. If an instrument for taking angles can be commanded, it is best to measure a base line and angles, and calculate the distance by trigonometry. If not, the following methods are available:



1ST METHOD.—Required the distance AB. Chain BC; run CE and BD directly from A, and chain CE, ED and BD. Then, supposing CF to be drawn, we have

$$ED - CB : DB :: CB : BA,$$

whence BA becomes known.



2D METHOD.—Draw BC at right angles to BA, and CD at right angles to BC; run DE towards A, and chain BE, EC and CD. Then

$$EC : CD :: EB : BA,$$

whence BA becomes known.

Note.—It is not necessary to make the angles B and C right angles if CD be drawn parallel to BA.

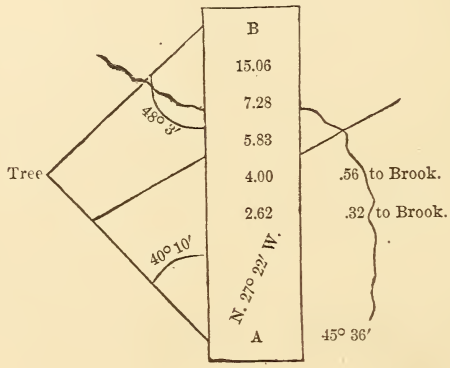
FIELD-NOTES.

It is of great importance to the surveyor that he have a system of keeping a record of his field operations at once simple and full. Almost every one has his own method, which is better for him than any other. If the field-work is performed at a distance from the office, or lasts for several days, the records must be especially full and lucid.

One method is to draw, as accurately as possible, an outline of the tract to be surveyed, and to mark on this map, in the proper places, the figures representing the magnitudes of the lines and angles measured. Another is to place letters at the angles of map, and write out on the margin the various dimensions; thus,

$$AB = 90 \text{ ch.} \quad A = 37°, \&c., \&c.$$

The following plan is adopted by many surveyors: Two lines are drawn, about an inch apart, up and down the middle of the page of the note-book. All measurements in a certain line are kept between these lines, while surrounding measures and notes are kept outside. Thus, the accompanying notes would be interpreted as follows:



It is most convenient to begin at the bottom, for then the right side of the line agrees with the right side of the page. At A the angle with the last side was 45° 36′ to the right. The bearing of the line AB was N. 27° 22′ W. At 2.62 ch. from A there was an offset of .32 ch. to a brook. At 4.00 ch. from A there was an offset of .56 ch. to a brook. At 5.83 ch. from A the side AB crossed a fence, the inclination of the line denoting the angle of the fence with AB, which may be measured if desired. At 7.28 ch. from A the line crossed the brook. At 15.06 ch. from A the other end of the line B was reached. A tree stood to the left of AB. The lines joining this with A and B made angles with AB of 40° 10′ and 48° 3′ respectively.

The student can elaborate this system as much as desired.

CHAPTER II.

On supplying omissions in the dimensions of a survey.

When the bearings and distances of all the sides of a survey are known, except one bearing and one distance, or two bearings, or two distances, these can be obtained by calculation, provided those that are known can be depended on, as sufficiently accurate. This may sometimes be necessary when there are obstacles in the way of obtaining one or two of the bearings or distances; or when, after they have all been taken on the ground, the notes of one or two of them happen to be obliterated. As, however a bearing, or distance thus obtained, must be affected by any error or errors that may have been made in taking the others, it is better, when practicable, to have the bearings and distances of all the sides, as taken on the ground.

PROBLEM I.

The bearings and distances of all the sides of a tract of land, except the bearing and distance of one side, being given, to find these.

RULE.

Find by prob. 11, of the preceding chapter, the differences of latitude and the departures for the sides whose bearings and distances are given, and place them in their proper columns in a table ruled for the purpose: Add up the northings and southings, and taking the difference of their sums, place it opposite the unknown side, in the column whose sum is the least. The sums of the two columns will then be equal. This is called *balancing* the latitudes. Do the same with the eastings and westings. The two numbers inserted to make the latitudes and the departures balance, will be the difference of latitude and the departure of the unknown side; with which its bearing and distance may be found, by prob. 10, of the preceding chapter.

Note 1.—By the application of this rule, the bearing and distance of a line joining two corners or stations, may be found, when there are obstacles in the way which prevent our going directly from one corner to the other, or when one cannot be seen from the other. To do this, let one or two, or more stations, if necessary, be taken out of the line, and take the bearing and distance from the first corner to the first assumed station; from this station to the second; and so on, to the second corner. Then considering these bearings and distances, as the bearings and distances of the sides of a survey, the required bearing and distance of the line may be found by the above rule. The bearing thus found must be *reversed*, in order to have the bearing from the *first* corner to the *second*.

2. In the same way the bearing and distance of a *straight road* to run between two given places, may be found, by taking the several bearings and distances of the old road if there is one; or of lines joining assumed stations and extending from one of the places to the other.

EXAMPLES.

1. The bearings and distances of the side of a tract of land, except the bearing and distance of one side which

are not known, are as in the following field-notes; required the unknown bearing and distance.

		Chains.
1.	S. 45°½ W.	15.16
2.	N. 50° W.	22.10
3.	North	18.83
4.	N. 85° E.	35.65
5.	———	———
6.	S. 20½ W°.	23.80
7	N. 51¼ W°.	26.47

Sta.	Bearings.	Dist.	N.	S.	E.	W.
1	S. 45½ W.	15.16		10.62		10.81
2	N. 50 W.	22.10	14.20			16.93
3	North	18.83	18.83			
4	N. 85 E.	35.65	3.11		35.52	
5				(19.79)	(21.20)	
6	S. 20½ W.	23.80		22.29		8.33
7	N. 51¼ W.	26.47	16.56			20.65
			52.70	52.70	56.72	56.72

As diff. of lat. 19.79 S. Ar. Co. 8.703553
: dep. 21.20 E. - - - - - 1.326336
: : rad. - - - - - - - - - - - 10.000000

: tang. bear. S. 47° E. - - - - - 10.029889

As cos. bear. - - - - - - Ar. Co. 0.166217
: rad. - - - - - - - - - - - - - 10.000000
: : diff. lat. 19.79 - - - - - - 1.296447

: dist. - - 29.02 - - - - - - 1.462664

Ans. S. 47° E. 29.02 ch.

2. Given the bearings and distances of the sides of a tract of land, as follow: 1st. N. 15¾ W 9.40 ch.; 2d N 63¾ E. 10.43 ch.; 3d. S. 49° E. 8.12 ch.; 4th. S. 13½ E. 8.45 ch.; 5th. S. 16¾ E. 6.44 ch.; 6th. Unknown; 7th. N. 60° W. 9.72 ch.; 8th. N. 17¼ E. 7.65 ch.; required the bearing and distance of the 6th. side.

Ans. S. 60° 8′ W. 12.27 ch.

3. One side of a tract of land of which a survey is to be taken, passes through a pond. Two stations are therefore taken on one side of the pond as represented in Fig. 80. The bearings and distances from the first end of the side to the first station, from that to the second, and thence to the other end of the side are; 1st. S. 52° W. 10.70 ch.; 2d. S. 7½° W. 13.92 ch.; and 3d. S. 34¼ E. 9 ch. Required the bearing and distance of the side. *Ans.* S. 10° 33′ W. 28.31 ch.

4. Given the bearings and distances of an old road, running between two places, as follow; 1st. S. 10° E. 92.20 ch.; 2d. S. 15° W. 120.50 ch.; 3d. S. 18½ W. 205. ch.; 4th. S. 71½ E. 68 ch. Required the bearing and distance of a straight road, that shall connect the two places. *Ans.* S. 2° 8′ W. 423.47 ch.

PROBLEM II.

Given all the bearings and distances of the sides of a survey, except the distances of two sides, to find these.

RULE.

By prob. 9, of the preceding chapter, change all the given bearings, in a corresponding manner, so that one of the sides whose bearings only are given, may become a meridian. With the changed bearings and given distances find the corresponding differences of latitude, and the departures. Add up the eastings and westings, and take the difference of their sums, which will be the departure of that unknown side, which is not made a meridian. With this departure and the changed bearing find by prob. 10, of the preceding chapter, the distance and difference of latitude of this side, which place in their proper columns. Now add up the northings and southings, and take their difference, which will be the distance of the side made a meridian.*

EXAMPLES.

Given the following bearings and distances of the sides of a survey; 1st. S. 45½ W. 15.16 ch.; 2d. N. 50° W. 22.10 ch.; 3d. North 18.83 ch.; 4th. N. 85° E. 35.65 ch.; 5th. S. 47° E. dist. unknown; 6th. S. 20½ W. dist. unknown; 7th. N. 51¼ W. 26.47 ch. to the place of beginning. Required the unknown distances.

Sta.	Bearings.	Changed bearings.	Dist.	N.	S.	E.	W.
1	S. 45½° W.	N. 87½° W.	15.16	0.66			15.15
2	N. 50 W.	N. 3 W.	22.10	22.07			1.16
3	North	N. 47 E.	18.83	12.85		13.77	
4	N. 85 E.	S. 48 E.	35.65		23.85	26.49	
5	S. 47 E.	South	(29.02)		(29.02)		
6	S. 20½ W.	S. 67½ W.	(23.80)		(9.11)		(21.99)
7	N. 51¼ W.	N. 4¼ W.	26.47	26.40			1.96
				61.98	61.98	40.26	40.26

* The reason of the rule is obvious. For as the side made a meridian has no departure, the difference of the sums of the departures, must be the departure of the other unknown side. And when the difference of latitude of this side has been found and placed in its proper situation, the difference of the sums of the latitudes must evidently be the difference of latitude of the side made a meridian; or which, in this case, is the same thing, its distance.

As sin. chang. bearing	$67\frac{1}{2}°$	Ar. Co.	0.034385
: rad. - - - - - - - - - - - - -			10.000000
:: dep. - - - - -	21.99	- - -	1.342225
: dist. 6th side - -	23.80	- - -	1.376610

As rad. - - - - - - - - - - - - -			10.000000
: cotang. chang. bearing	$67\frac{1}{2}°$	- - -	9.617224
:: dep. - - - - -	21.99	- - -	1.342225
: diff. lat. 6th side -	9.11	- - -	0.959449

Ans. 5th side 29.02 ch. and 6th side 23.80 ch.

2. Given the bearings and distances of a tract of land as follow: 1st. S. $40\frac{1}{2}$ E. 31.80 ch.; 2d. N. 54° E. dist. unknown; 3d. N. $29\frac{1}{4}$ E. 2.21 ch.; 4th. N. $28\frac{3}{4}$ E. 35.35 ch.; 5th. N. 57° W. dist. unknown; 6th. S. 47° W. 31.30 ch.; to the place of beginning. Required the distances of the 2d. and 5th. sides.

Ans. 2d. side. 2.08 ch. and 5th. side 20.90 ch.

PROBLEM III.

Given the bearings and distances of all the sides of a survey except two; one of which has only its bearing given, and the other, the distance and the points of the compass between which it runs; to find the unknown bearing and distance.

RULE.

As in the last problem, change all the given bearings, so that the side whose bearing only is given, may become a meridian. Find the differences of latitude and the departures, corresponding to the changed bearings and the given distances. Take the difference of the sums of the eastings and westings, which will be the departure of the side whose bearing is not given With the given distance and this departure, find by chap. 1. prob. 10. the changed bearing and difference of latitude, and place them in their proper columns. From the changed bearing, the true bearing may be readily found by note to prob. 9, chap. 1. Lastly, take the difference of the sums of the northings and southings, and it will be the distance of the side, changed to a meridian.

Note.—The changed bearing as found by the rule, must be reckoned from the *north*, or the *south* point of the compass, according as the one, or the other, will render the true bearing when found from it, conformable to the given points. The point from which the changed bearing must be reckoned determines also the column in which the difference of latitude must be placed. Sometimes the changed bearing when reckoned from either north or south, will render the true bearing conformable to the given points. In such cases, there are two different bearings and distances that will answer the conditions of the problem; and we can only know which of them is the right one by previously knowing the required bearing nearly.

EXAMPLES.

1. Given the bearings and distances of a survey as follow: 1st. S. *unknown* W. 15.16 ch.; 2d. N. 50° W. 22.10 ch.; 3d. N. 18.83 ch.; 4th. N. 85° E. 35.65 ch.; 5th. S. 47° E. 29.02 ch.; 6th. S. 20½ W. dist. unknown; 7th. N. 51¼° W. 26.47 ch. Required the unknown bearing and distance.

Sta.	Bearings.	Changed bearings.	Dist.	N.	S.	E.	W.
1	S. (45° 36′) W.	(S. 25° 6′ W.)	15.16		13.73		(6.43)
2	N. 50 W.	N. 70½ W.	22.10	7.37			20.83
3	North	N. 20½ W.	18.83	17.64			6.59
4	N. 85 E.	N. 64½ E.	35.65	15.35		32.18	
5	S. 47 E.	S. 67½ E.	29.02		11.11	26.81	
6	S. 20½ W.	South	(23.81)		(23.81)		
7	N. 51¼ W.	N. 71¾ W.	26.47	8.29			25.14
				48.65	48.65	58.99	58.99

As dist. 1st side 15.16 Ar. Co. 8.819301
: dep. do. 6.43 - - - 0.808211
: : rad. - - - - - - - - - - - - 10.000000

: sin. chang. bear. 25° 6′ - - - - 9.627512

As rad. - - - - - - - - - - - - 10.000000
: cos. chang. bear. 25° 6′ - - - - 9.956921
: : dist. - - - - 15.16 - - - - 1.180699

: diff. lat. - - - 13.73 - - - - 1.137620

Ans. 1st, S. 45° 36′ W.; 6th, 23.81 ch.

2. Given the following bearings and distances of a survey: 1st. S. 40½° E. 31.80 ch.; 2d. N. 54° E. dist. *unknown* 3rd. N. 29¼° E. 2.21 ch.; 4th. N. *unknown* E. 35.35 ch.; 5th. N. 57° W. 20.90 ch.; 6th. S. 47° W. 31.30 ch.; to place of beginning. Required the bearing of the 4th. side and distance of the 2d. side.

Ans. Bearing of 4th. side N. 28¾° E.; dist. of 2d. side, 2.09 ch.

PROBLEM IV.

Given all the bearings and distances of the sides of a tract of land, except the bearings of two sides, to find these bearings.

RULE.

1. Find the difference of latitude and the departure of each side, whose bearing and distance are both given. Take the difference of the sums of the northings and southings of these sides, and also the difference of the sums of the eastings and westings. These differences will be the difference of latitude, and the departure of a line, which, with those sides, would form a closing survey; and which may therefore be called a *closing line.*

2. With the difference of latitude and departure of the closing line, find, by prob. 10. chap. 1, its bearing and distance. Take the closing line and the two sides whose bearings are not given, for the three sides of a triangle, and calculate the angles.

3. To the bearing of the closing line, apply, by addition or subtraction, as the case may require, the angle contained between it, and the side which is the one coming first in the order of the survey; and it will give the bearing of that side. Then to the reverse bearing of that side, apply in a proper manner, the angle contained between the two sides which are sides of the survey, and it will give the bearing of the second of those sides.*

---

* It is easy to see the reason of the rule, by considering that the two sides whose bearings are not given, being made to form with the closing line, the

EXAMPLES.

1. Given the bearings and distances of the sides of a tract of land as follow: 1st. S. *unknown* W. 15.16 ch.; 2d. N. 50° W. 22.10 ch.; 3d. North 18.83 ch.; 4th. N. 85° E. 35.65 ch.; 5th. S. *unknown* E. 29.02 ch.; 6th S 20½° W. 23.80 ch.; and 7th. N. 51¼ W. 26.47. ch. Required the unknown bearings.

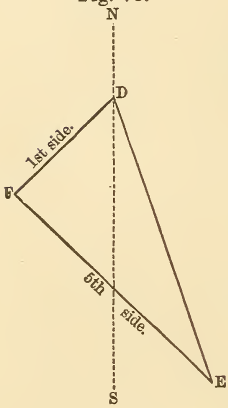
Sta.	Bearings.	Dist.	N.	S.	E.	W.
1	S. W.	15.16				
2	N. 50° W.	22.10	14.20			16.93
3	North	18.83	18.83			
4	N. 85 E.	35.65	3.11		35.52	
5	S. E.	29.02				
6	S. 20½ W.	23.80		22.29		8.33
7	N. 51¼ W.	26.47	16.56			20.65
			52.70	22.29	35.52	45.91
			22.29			35.52
			30.41 S.			10.39 E.

sides of a triangle, the sum or difference of their differences of latitude, will necessarily be equal to the difference of latitude of the closing line; and that, therefore, their differences of latitude will be such as to make the sums of the northings and southings of the whole survey equal; and the same for the departures.

As	diff. lat. - - - -	30.41 S.	Ar. Co.	8.516984
:	dep. - - - - -	10.39 E.		1.016616
: :	rad. - - - - - - - - - - -			10.000000
:	tang. of bear. of clos. line, S. 18° 52′ -			9.533600

As	cos. of bear. of clos. line 18° 52′		Ar. Co.	0.023983
:	rad. - - - - - - - - - - -			10.000000
: :	diff. lat. - - - -	30.41	- - - -	1.483016
:	dist. of clos. line -	32.14	- - - -	1.506999

Fig. 78.



Let DE, Fig. 78, represent the closing line, DF the 1st side of the survey, and FE the 5th side. Then

DE	32.14		
DF	15.16	Ar. Co.	8.819301
FE	29.02	———	8.537313
	2)76.32		
Half sum	38.16	log.	1.581608
Rem.	6.02		— 0.779596
			2)19.717818
Cos.	½ F 43° 44′		
	F 87° 28′		9.858909

As	DE	32.14	Ar. Co.	8.492954
:	FE	29.02		1.462697
: :	sin. F.	87° 28′		9.999575
	sin. D.	64° 26′		9.955226

DE, S. 18° 52′ E.	FD, N. 45° 34′ E.
Angle D 64° 26′	Angle F 87° 28′
1st side, S. 45° 34′ W.	133° 2′
	180° 00′
	5th side, S. 46° 58′ E.

2. Given the bearings and distances of the sides of a tract of land as follow: 1st. S. *unknown* E. 31.80 ch.; 2d. N. 54° E. 2.08 ch.; 3d. N. 29¼ E. 2.21 ch.; 4th. N. 28¾° E. 35.35 ch.; 5th. N. 57° W. 20.90 ch.; and 6th. S *unknown* W. 31.30 ch. to the place of beginning. Required the unknown bearings.

Ans. 1st. S. 40° 29′ E.; and 6th. S. 47° W.

CHAPTER III.

Problems for finding the Content of Land.

When the sides of a survey are right lines, and all the bearings and distances are given, the area may be found by a problem that will be given in this chapter. If one or two of the bearings or distances are not known, they may be found by the problems in the last chapter. Although the problem alluded to, is general, and may be applied whatever number of sides there may be, yet there are some particular rules for finding the areas of triangles and quadrilaterals, which are often useful. These, and also rules for finding the areas of circles and ellipses, are given in the first part of the chapter.

When a part of the boundary of a tract of land, is irregular, as is frequently the case, if one or more of the sides are bounded by water, it is sometimes very troublesome to take all the bearings and distances requisite to obtain the area with accuracy. In these cases, it is usual to run one or more straight lines, called *stationary* lines, near to such boundary, and so as to connect the straight sides of the survey. In measuring these stationary lines, perpendicular distances are measured from them, to each bend in the irregular boundary. These perpendicular distances are called *off-sets.* The lengths of the off-sets, and the distance of the foot of each, from the commencement of the stationary line, should be carefully noted in the field book; observing also that such a number of off-sets should be taken, that the part of the irregular boundary intercepted between each adjacent two, may, without material error, be considered a straight line. From these notes, the area or areas of the land contained between the stationary line or lines, and the irregular boundary, may readily be calculated. This area added to the area enclosed by the stationary lines, and straight sides of the survey, when they are on the outside of the stationary lines, or subtracted from it, when on the inside, will give the area of the survey.

In those cases in which water is a boundary of a tract of land, if that water is a brook or rivulet, it is usual to consider a line running through its *middle* as the true boundary; and the off-sets must be measured accordingly. When tide water is the boundary, the land is considered as extending to the line of *low water mark.*

If the bearings of all the corners of a tract of land from two stations, taken either within or out of the tract are given, and also the bearing and distance of these stations from each other, the area may be calculated. It is however necessary, that the two stations should be so taken that they shall not be in a straight line, or very nearly in a straight line, with either of the corners of the land. This method of obtaining the area, though not practically so accurate as where the bearings and distances of the sides are correctly given, may sometimes be found useful.

Some surveyors, in order to calculate the area of a survey, first protract it; then dividing the plot into triangles and trapeziums by lines joining opposite corners, they measure with the scale and dividers the lengths of such lines and perpendiculars as are requisite for calculating the areas of these. The sum of the areas thus obtained, is the area of the survey. When the survey is carefully protracted, and proper attention is given to take the measures with the utmost precision, this method serves to give a near value of the content; but is by no means to be depended on as equally accurate with the general problems mentioned above.

The area of a field or small tract of land, the corners of which can be seen from one another, may readily be found by means of the chain only. To do this, the lengths of the sides must be measured, and also the length of diagonals joining opposite corners, so as to divide the field into triangles. Or instead of the diagonals, the distances from some assumed point within the field, to the several corners, may be used. Having then all the sides of the several triangles, the area of each may be found; and the sum of these areas will be the area of the tract.

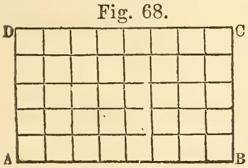
PROBLEM I.

To find the area of a Parallelogram, whether it be a Square, a Rectangle, a Rhombus, or a Rhomboid.

RULE.

Multiply the length by the height or perpendicular breadth, and the product will be the area.*

Fig. 68.



* Demonstration. Let ABCD (Fig. 68) be a rectangle; and let its length AB and CD, and its breadth AD and BC, be each divided into as many equal parts, as are expressed by the number of times they contain the lineal measuring unit; and let all the opposite points of division be connected by right

Note.—Because the length of a square is equal to its height, its area will be found by multiplying the side by itself.

EXAMPLES.

1. Required the area of a square field, a side of which measures 7.29 four-pole chains.

```
        7.29 Ch.
        7.29
        ----
        6561
       1458
      5103
    -------
  10)53.1441  Area 5 A.  1 R.  10 P.
    -------
     5.31441
           4
    --------
     1.25764
          40
    --------
    10.30560
```

2. Required the area of a rectangular field whose length is 13.75 chains, and breadth 9.5 chains.

---

lines. Then, it is evident that these lines divide the rectangle into a number of squares, each equal to the superficial measuring unit; and that the number of these squares is equal to the number of lineal measuring units in the length, as often repeated as there are lineal measuring units in the breadth, or height; that is, equal to the length multiplied by the breadth. But the area is equal to the number of squares or superficial measuring units; and therefore the area of a rectangle is equal to the product of the length and breadth.

Again, a rectangle is equal to any oblique parallelogram of an equal length and perpendicular height (I. 33;) therefore the area of every parallelogram is equal to the product of its length and height.

```
       13.75 Ch.
         9.5
       -----
        6875
      12375
      -------
   10)130.625 Area 13 A. 0 R. 10 P
      -------
      13.0625
            4
      -------
        .2500
           40
      -------
      10.0000
```

3. Required the area of a field, in the form of a rhomboid, whose length AB is 42.5 perches, and perpendicular breadth CD is 32 perches.

```
        42.5 P.
          32
        ----
         850
       1275
       ------
   4|0)136|0.0
       ------
       4)34
       ----
        8 A. 2 R.
```

4. What is the area of a square tract of land whose side measures 176.4 perches? *Ans.* 194 A. 1 R. 36.96 P.

5. What is the area of a rectangular plantation whose length is 52.25 chains, and breadth 38.24 chains?

Ans. 199 A. 3 R. 8.6 P.

6. The length of a field, in the form of a rhombus, measures 16.54 chains, and the perpendicular breadth 12.37 chains: required the area. *Ans.* 20 A. 1 R. 33.6 P.

7. Required the area of a field in the form of a rhomboid, whose length is 21.16 chains, and perpendicular breadth 11.32 chains. *Ans.* 23 A. 3 R. 32.5 P.

PROBLEM II.

To find the area of a triangle when the base and perpendicular height are given.

RULE.

Multiply the base by the perpendicular height, and half the product will be the area.*

EXAMPLES.

1. The base AB of a triangular piece of ground, measures 12.38 chains, and the perpendicular CD 6.78 chains: required the area. Fig. 49.

```
       12.38 Ch.
        6.78
       -----
        9904
       8666
      7428
     -------
    2)83.9364
     -------
   10)41.9682  Area, 4 A. 0 R. 31 P.
     -------
      4.19682
            4
     -------
      .78728
           40
     --------
     31.49120
```

* Demonstration.—A triangle is half a parallelogram of the same base and altitude (I. 35, Cor.).

2. Required the area of a triangular field, one side of which measures 18.37 chains, and the distance from this side to the opposite angle, 13.44 chains.

Ans. 12 A. 1 R. 15 P.

3. What is the area of a triangle whose base is 49 perches and height 34 perches? *Ans.* 5 A. 0 R. 33 P

PROBLEM III.

To find the area of a triangle when two sides and their included angle are given.

RULE.

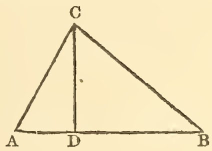
As radius,
Is to the sine of the included angle;
So is the rectangle of the given sides,
To double the area.*

EXAMPLES.

1. In a triangular lot of ground ABC, the side AB measures 64 perches, the side AC 40.5 perches, and their contained angle CAB 30°: required the area. Fig. 49.

---

* DEMONSTRATION.—In the triangle ABC let AB and AC be the given sides, including the given angle A, and let CD be perpendicular on AB. Then by Trig. rad. : sin. A : : AC : CD; but (V. 1) AC : CD : : AC×AB : CD×AB; therefore rad. : sin. A : : AC×AB : CD×AB; but CD×AB is equal to twice the area of the triangle; hence the truth of the rule is evident.



As radius	- - - - - - - - -	10.000000
Is to sin A, 30°	- - - - - - - -	9.698970
So is AB, AC	64 - - - - -	1.806180
	40.5 - - - -	1.607455
		13.112605
To double the area 1296 perches		3.112605

40)648

4)16 8

4 A. 0 R. 8 P.

2. What is the area of a triangle, two sides of which measure 15.36 chains and 11.46 chains respectively, and their included angle 47° 30′? *Ans.* 6 A. 1 R. 38 P.

3. One side of a triangular field bears N. 12° E. dis tance 18.23 chains, and at the same station the other adjacent side bears N. 78° 30′ E. distance 13.84 chains: required the area. *Ans.* 11 A. 2 R. 11 P.

4. Required the area of a triangular piece of ground, one side of which bears N. 82° 30′ W. dist. 19.74 chains, and at the same station the other adjacent side S. 24° 15 E. dist. 17.34 chains. *Ans.* 14 A. 2 R. 8 P

PROBLEM IV.

To find the area of a triangle when one side and the two adjacent angles are given.

RULE.

Subtract the sum of the two given angles from 180°; the remainder will be the angle opposite the given side. Then,

As the rectangle of radius and the sine of the angle opposite the given side,
Is to the rectangle of the sines of the other angles,
So is the square of the given side,
To double the area.*

EXAMPLES.

1. In a triangular field ABC, the side AB measures 76 perches, the angle A 60°, and the angle B 50°: required the area. Fig. 47.

The angle ACB=180° — the sum of the angles A and B,=70°.

As rad. × sin. C,	rad.	Ar. Co.	0.000000
	sin. C, 70°	Ar. Co.	0.027014
: sin. A × sin. B,	sin. A, 60°		9.937531
	sin. B, 50°		9.884254
: : AB^2 = AB × AB,	AB 76		1.880814
	AB 76		1.880814
: double area in perches	4078		3.610427

40)2039

4)50 39

12 A. 2 R. 39 P.

---

* DEMONSTRATION.—Let AB (last Fig.) be the given side of the triangle ABC, and A and B the given angles; also let CD be perpendicular on AB. Then by Trig.

sin. ACB : sin. B : : AB : AC
rad. : sin. A : : AC : CD.

Therefore rad. × sin. ACB : sin. A × sin. B : : AB × AC : CD × AC : : AB : CD : : AB^2 : AB × CD; but AB × CD is equal to double the area of the triangle ABC; therefore rad. × sin. ACB : sin. A × sin. B : : AB^2 : double the area of the triangle ABC.

2. One side of a triangle measures 24.32 chains, and the adjacent angles are 63° and 74°: required the area.
Ans. 37 A. 0 R. 22 P.

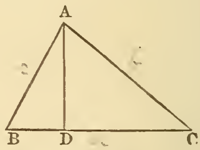
3. What is the area of a triangular field, one side of which is 17.36 chains, and the adjacent angles 37° 30′, and 48° 15′? *Ans.* 6 A. 3 R. 18 P.

PROBLEM V.

To find the area of a triangle when the three sides are given.

RULE.

From half the sum of the three sides subtract each side severally; multiply the half sum and the three remainders continually together, and the square root of the last product will be the area.*



* Let $AB=c$, $BC=a$, and $AC=b$,

then (II. 8) $\quad a^2+c^2=b^2+2a \,.\, BD$;

hence $\quad BD=\dfrac{a^2+c^2-b^2}{2a}$,

and $\quad AD=\sqrt{c^2-BD^2}=\sqrt{c^2-\dfrac{(a^2+c^2-b^2)^2}{4a^2}}=\dfrac{\sqrt{4a^2c^2-(a^2+c^2-b^2)^2}}{2a}$.

Hence, $ABC=\frac{1}{2}BC \,.\, AD=\frac{1}{4}\sqrt{4a^2c^2-(a^2+c^2-b^2)^2}$. The quantity under the radical sign may be resolved into two factors, $[2ac+a^2+c^2-b^2]$ and $[2ac-(a^2+c^2-b^2)]$, and these again into

$$(a+c+b)(a+c-b) \text{ and } (b+a-c)(b+c-a).$$

Hence, the last equation becomes

$$ABC=\sqrt{\left[\frac{a+b+c}{2}\cdot\frac{a+c-b}{2}\cdot\frac{b+a-c}{2}\cdot\frac{b+c-a}{2}\right]};$$

and putting $\quad S=\dfrac{a+b+c}{2}$, $ABC=\sqrt{S(S-b)(S-c)(S-a.)}$

Hence the rule.

EXAMPLES.

1. Required the area of a triangular tract of land whose three sides are 49.00, 50.25 and 25.69 chains.

Half sum	62.47	log.	1.795672
Remainders	13.47		1.129368
	12.22		1.087071
	36.78		1.565612
			2)5.577723
	615 chains		2.788861

61.5 Acres = 61 A. 2 R.

2. What is the area of a triangular field whose sides measure 10.64, 12.28 and 9.00 chains?

Ans. 4 A. 2 R. 26 P.

3. What quantity of land is contained in a triangle, the sides of which are 20, 30 and 40 chains?

Ans. 29 A. 0 R. 7 P.

PROBLEM VI.

To find the area of a trapezium, when one of the diagonals and the two perpendiculars, let fall on it from the opposite angles, are given.

RULE.

Multiply the sum of the perpendiculars by the diagonal, and half the product will be the area.*

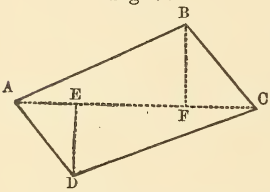
---

* DEMONSTRATION.—The area of the triangle ABC (Fig. 70) $= \frac{AC \times BF}{2}$, and the area of the triangle ADC $= \frac{AC \times DE}{2}$; therefore the sum of these is $\frac{1}{2}(BF + DE)AC$.

Note.—When all the sides and one of the diagonals are given, the trapezium will be divided into two triangles, the area of each of which may be found by the last problem. The sum of these areas will be the area of the trapezium.

EXAMPLES.

Fig. 70.



1. In a field ABCD, in the form of a trapezium, the diagonal AC measures 20.64 chains, the perpendicular BF 6.96 chains, and DE 5.92 chains; required the area. Fig. 70.

2. Required the area of a trapezium whose diagonal measures 16.10 ch. and the perpendiculars 6.80 ch. and 3.40 ch. *Ans.* 8 A. 0 R. 33¾ P.

3. The diagonal of a trapezium is 24 ch. and the perpendiculars are 8.27 ch., and 12.43 ch.; what is the area? *Ans.* 24 A. 3 R. 14 P.

PROBLEM VII.

To find the area of a trapezoid.

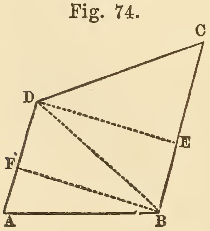
RULE.

Multiply the sum of the parallel sides by their perpendicular distance, and half the product will be the area.*

EXAMPLES.

1. Required the area of a trapezoid ABCD, of which the parallel sides AD, BC measure 6.14 and 9.48 chains,

Fig. 74.



* Demonstration. The trapezoid ABCD, Fig. 74, = the triangle ABD + BDC = (by prob. 2,) $\frac{AD \times BF}{2} + \frac{BC \times DE}{2}$, = (because BF = DE,) $\frac{AD \times BF}{2} + \frac{BC \times BF}{2} = \frac{AD + BC \times BF}{2}$

respectively, and their perpendicular distance BF or DE, 7.80 chains.

$$\begin{array}{r}
\text{Ch.} \\
6.14 \\
9.48 \\
\hline
15.62 \\
7.80 \\
\hline
124960 \\
10934 \\
\hline
2)121.8360 \\
\hline
60.9180
\end{array}$$

60.9180 Ch.=6 A. 0 R. 15 P.

2. The parallel sides of a trapezoid are 12.41 and 8.22 chains, and their perpendicular distance 5.15 chains: required the area. *Ans.* 5 A. 1 R. 10 P.

3. Required the area of a trapezoid whose parallel sides are 11.34 and 18.46 chains, and their perpendicular distance 13.25 chains. *Ans.* 19 A. 2 R. 39 P.

PROBLEM VIII.

*To find the area of a circle or of an ellipse.**

RULE.

Multiply the square of the circle's diameter, or the

---

* If two pins be set upright in a plane, and a thread, the length of which is greater than twice the distance between the pins, having the ends tied together, be put about the pins; and if the point of a pin or pencil applied to the thread, and held so as to keep it uniformly tense, be moved round, till it return to the place from which the motion began; then the point of the pin or pencil will have described on the plane, a curved line called an *Ellipse.*

product of the two diameters of the ellipse, by .7854, for the area.*

Note 1.—If the diameter of a circle be multiplied by 3.1416, the product will be the circumference; also if the circumference be divided by 3.1416, the quotient will be the diameter.

2. If the area of a circle be divided by .7854, the square root of the quotient will be the diameter.

EXAMPLES.

1. How many acres are in a circle a mile in diameter?

$$\begin{array}{r} 1 \text{ mile}=80 \text{ ch.} \\ 80 \\ \hline 6400 \\ .7854 \\ \hline 3141600 \\ 47124 \\ \hline \end{array}$$

5026.5600 Sq. Ch.=502 A. 2 R. 25 P. nearly.

Or by Logarithms.

$$\begin{array}{lr} \text{Square of } 80 \begin{cases} 80 \\ 80 \end{cases} & \begin{array}{r} \text{log. } 1.903090 \\ 1.903090 \end{array} \\ \qquad\qquad .7854 & -1.895091 \\ \hline 5026.56 \text{ Sq. Ch.} & 3.701271 \end{array}$$

2. Required the area of an ellipse, the longer diameter of which measures 5.36 ch. and the shorter 3.28 ch.

---

*The demonstration of this rule in the case of the circle is given in Sharpless's Geometry, Book VI., $.7854=\frac{1}{4}\pi$.

```
      Ch.
     5.36
     3.28
     ----
     4288
    1072
   1608
   -------
   17.5808
     .7854
   -------
    703232
   879040
  1406464
 1230656
 ----------
 13.80796032 Sq. Ch.=1 A. 1 R. 20.9 P.
```

PROBLEM IX.

The bearings and distances of the sides of a tract of land being given, to calculate the area.

RULE.

1. Rule a table and head it as in the annexed example, observing that the letters D. M. D. stand for Double Meridian Distance.

2. Find by prob. 12, chap. 1, the corrected differences of latitude and the departures, corresponding to the several sides, placing them in their proper places in the table.

3. Having drawn a plot of the tract, consider the most western station as the place of beginning. Count all east departures as positive, and all west departures as negative.

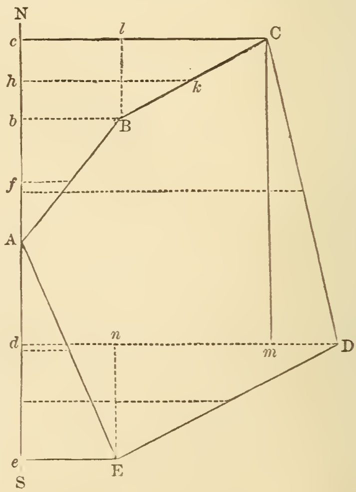
4. Take for the double meridian distance of the first side its corrected departure; take for the double meridian distance of any side the double meridian distance of the preceding side, plus its departure, plus the departure of the side itself.

5. Multiply each of the corrected differences of latitude by its corresponding double meridian distance. Place the product in the column of north or south areas, as the difference of latitude is north or south.

6. Add up the numbers in the columns of areas, take the difference of their sums and divide it by 2; the result will be the area of the survey.*

---

* DEMONSTRATION.—Let ABCDE be a plot of a survey, and let A be the most western corner. Through A draw the meridian NS.



The meridian distance of a side is the distance of its middle point from the meridian. Thus hk is the meridian distance of BC, and its double meridian distance is $2hk = \mathrm{B}b + \mathrm{C}c$.

To prove clause 4 of the rule we proceed as follows:

The double meridian distance of AB is $\mathrm{B}b$, its departure. The double meridian distance of BC is $\mathrm{B}b + \mathrm{C}c = \mathrm{B}b + cl + l\mathrm{C}$, or it is the double meridian distance of AB, plus the departure of AB, plus the departure of BC. The double meridian distance of CD is $\mathrm{C}c + \mathrm{D}d = (\mathrm{C}c + \mathrm{B}b) + l\mathrm{C} + \mathrm{D}m$, which again agrees with the rule. The double meridian distance of DE is $\mathrm{D}d + \mathrm{E}e = (\mathrm{D}d + \mathrm{C}c) + \mathrm{D}m - \mathrm{D}n$, which also agrees with the rule, because $\mathrm{D}n$, being a western departure, is negative; and so on for any number of sides.

To show the meaning of clause 5 of the rule we proceed:

The double area of the triangle $\mathrm{AB}b$ is $\mathrm{A}b \times \mathrm{B}b$, or the difference of latitude of AB multiplied by its double meridian distance. The double area of the trapezoid $\mathrm{B}b\mathrm{C}c$ is $(\mathrm{B}b + \mathrm{C}c) \times bc$, which is the product of the

Note 1.—The word plus in the 4th clause must be understood as having its algebraic meaning. If the quantity to be added be itself negative, as in the case of a west departure, the difference must be taken.

Note 2.—The last double meridian distance should be equal to the last departure; this affords a check to the accuracy of the work.

Note 3.—The eastern station could have been as well chosen as the point of beginning, in which case west departures would have been positive and east negative.

---

difference of latitude of BC and its double meridian distance. Hence the columns of N. areas and S. areas are the double areas of the various triangles and trapezoids between the various sides of the tract and the meridian.

To prove clause 6:

The double area ABCDE $= 2\mathrm{C}c\mathrm{D}d + 2\mathrm{D}d\mathrm{E}e - (2\mathrm{A}b\mathrm{B} + 2\mathrm{B}b\mathrm{C}c + 2\mathrm{A}e\mathrm{E})$; that is, the sum of the south areas minus the sum of the north areas.

The whole may be arranged in tabular form as follows:

Side.	N. Lat.	S. Lat.	D. M. D.	N. Areas.	S. Areas.
AB	A*b*		B*b*	AB*b*	
BC	*bc*		B*b*+C*c*	B*b*C*c*	
CD		*cd*	C*c*+D*d*		C*c*D*d*
DE		*de*	D*d*+E*e*		D*d*E*e*
EA	*e*A		*e*E	AE*e*	

EXAMPLES.

1. Given the bearings and distances, as in the following field-notes, to find the area.

Sta.	Bear.	Dist.	N.	S.	E.	W.	Cor. N.	Cor. W.	N.	S.	E.	W.	D. M. D.	N. Areas.	S. Areas.
1	N. 47 E.	31.30	21.34		22.89		.03	.04	21.37		22.85		22.85	488.3045	
2	S. 57 E.	21.10		11.49	17.69		.02	.03		11.47	17.66		63.36		726.7392
3	S. 28¾ W.	35.35		31.00		17.00	.04	.05		30.96		17.05	63.97		1980.5112
4	S. 29¼ W.	2.21		1.92		1.08				1.92		1.08	45.84		88.0128
5	S. 54 W.	2.08		1.23		1.68				1.23		1.68	43.08		52.9884
6	N. 40½ W.	31.80	24.18			20.65	.03	.05	24.21			20.70	20.70	501.1470	
			45.52	45.64	40.58	40.41	.12	.17	45.58	45.58	40.51	40.51		989.4515	2848.2516
				45.52	40.41										989.4515
			Er. N.	.12	.17 Er. W.										2)1858.8001
															929.40005
															92.940005
															4
															3.760020
															40
															30.40080

Area 92 A. 3 R. 30.4 P.

2. Given the boundaries of a tract of land as follows:

1. N. 23° 45′ W. 6.46 ch.	4. S. 66° 22′ W. 4.895 ch.
2. N. 65° 16′ E. 4.40 ch.	5. N. 24° 2′ W. 2.625 ch.
3. S. 41° 49′ E. 9.68 ch.	6. S. 66° 59′ W. 2.49 ch.

Required the area. *Ans.* 4 A. 2 R. 38.9 P.

Note.—In this case the latitudes and departures should be taken out by Problem XIII., p. 105. When a transit or theodolite is used, angles should be read to minutes. In the remaining examples of this problem the bearings were taken by a compass.

5. Required the area of a tract of land bounded as follows: 1st. S. 62° W. 7.57 ch.; 2d. N. 43½° W. 5.89 ch.; 3d North, 5.82 ch.; 4th. N. 33½° W. 8.83 ch.; 5th. N. 48° E. 4.81 ch.; 6th. N. 12° E. 4.66 ch.; 7th. N. 62½° E. 5.27 ch.; 8th. S. 6½° E. 5.60 ch.; 9th. S. 40½° E. 5.87 ch.; 10th. East, 6.54 ch.; 11th. North, 5.52 ch.; 12th. N. 68¼° E. 3.10 ch.; 13th. S. 30° E. 7.90 ch.; 14th. S. 23° W. 8.80 ch.; 15th. S. 31½° E. 6.42 ch.; 16th. S. 50° W. 8.40 ch.; 17th. N. 44° W. 6.85 ch. to the place of beginning.

Ans. 44 A. 2 R. 22 P.

6. Given the following field-notes to find the area of the survey; also the bearings and distance of the 3d side, which were omitted to be taken on account of obstacles in the way.

		Ch.
1. S. 85¼°	E.	23.30
2. S. 19	E.	31.12
3. ——		——
4. N. 64	W	29.72

		Ch.
5. N. $15\frac{1}{2}°$	W.	22.46
6. N. 58	E.	25.94
7. S. $27\frac{3}{4}$	E.	6.60

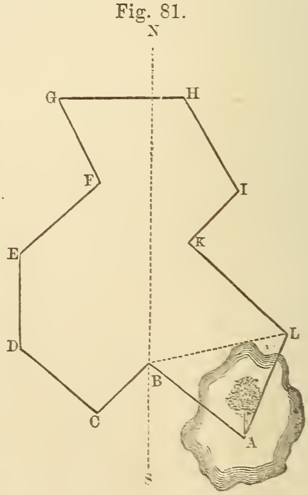
Ans. Area 182 A. 0 R. 21.7 P. and the bearing and distance of the 3d side, S. 66° 23′ W. 28.06 ch.

7. Being furnished with the field-notes of a tract of land, and requested to calculate the area, I found on examining them, that the figures expressing the angles of bearing of the 4th and 5th sides were so defaced as to be illegible: but as the remaining data are sufficient, the area is required. The field-notes are as follows:

		Ch.
1. S. $60\frac{3}{4}°$	W.	10.34
2. N. $27\frac{1}{4}$	W.	17.88
3. N. 51	E.	15.85
4. N. —	E.	9.61
5. S. —	E.	19.18
6. S. $16\frac{3}{4}$	E.	22.21
7. S. $71\frac{1}{2}$	W.	16.66
8. N. $71\frac{1}{4}$	W.	5.76

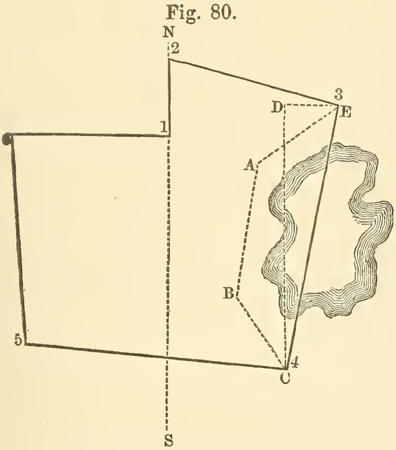
Ans. 81 A. 2 R. 23 P.

Fig. 81.



8. In a survey, represented Fig. 81, the corner at A was inaccessible, occasioned by the overflowing of water; but being a tree, it can be seen from the adjacent corners B and L. I therefore set my instrument at B and took the bearing to A, which I reversed, and set in my field-book as the first bearing. I then proceeded to take the bearings and distances of the several sides to L; and at L, I took the bearing of the side LA. The field-notes being as follows, the length of the sides AB and LA, and the area are required.

Fig. 80.



AB,	N. 51¼°	W.	Ch.
BC,	S. 45½	W.	15.16
CD,	N. 50	W.	22.10
DE,	North,		18.83
EF,	N. 48	E.	22.60
FG,	N. 25½	W.	20.17
GH,	East,		26.57
HI,	S. 30½	E.	22.86
IK,	S. 44	W.	15.04
KL,	S. 47	E.	28.55
LA,	S. 20½	W.	

Ans. AB, 26.47 ch.; LA, 23.81 ch.; and the area 244 A. 3 R. 13 P.

9. In taking a survey of a tract of land bounded by six straight sides, Fig. 80, I was prevented going directly from the 3d to the 4th corner by a pond of water. I therefore set up two stakes near the edge of the pond, and took the bearing and distance from the 3d corner to the first stake, from the first stake to the second, and from the second to the 4th corner, and noted them in my field-book as all belonging to the 3d station of the survey. The field-notes being as follows, the bearing and distance of the 3d side, and the area of the survey are required.

1.	North,		7.81 Ch.
2.	S. 76¼°	E.	18.15
3.	S. 52	W.	10.70
	S. 7½	W.	13.92
	S. 33¼	E.	9.00
4.	N. 84¼	W.	27.12
5.	N. 4½	W.	22.00
6.	East,		16.58

Ans. 3d side, S. 10° 47′ W. 28.42 ch.; and area 80 A. 0 R. 25 P.

PROBLEM X.

To find the area, when off-sets are taken.

RULE.

1. Find by the last problem, the area enclosed by the stationary lines and straight sides of the survey.

2. Subtract the stationary distance of each off-set, from that of the one immediately following; the remainders will be the distances, intercepted on the stationary line, between each two adjacent off-sets. Place these under one another in a column as in the annexed examples. Also take the sums of each two adjacent off-sets, and place them in the next column, so as to correspond with the intercepted distances.

3. Multiply the sum of each two adjacent off-sets by their intercepted distance on the stationary line; then, half the sum of the products will be the area of the off-sets on that line.

4. If there are off-sets on more than one stationary line, proceed in the same manner with the others.

5. When the stationary lines are within the boundary of the survey, add the areas of the off-sets to the area enclosed by the stationary lines and straight sides; but when the stationary lines are without the boundary, subtract the areas of the off-sets.*

---

* Demonstration. Considering the boundary as straight between the ends of each two adjacent off-sets, it is plain that the area contained between the stationary line and boundary will be divided by the off-sets into trapezoids and triangles. Hence the truth of the rule is evident.

EXAMPLE 1, Fig. 82.

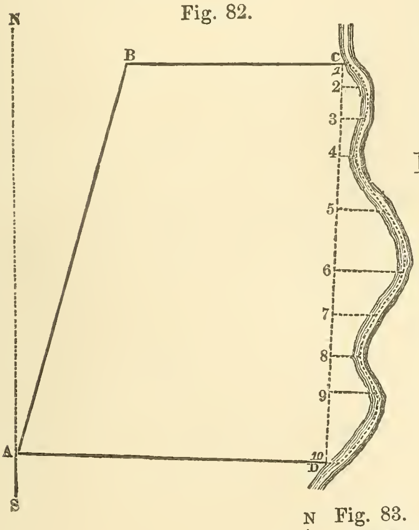
Required the area of a piece of meadow, bounded on one side by a brook; the field-notes being as follows:

1. N. $16\frac{1}{2}°$ E. 14.35 Ch.
2. East, 7.82
3. S. $3\frac{1}{2}$ W. 14.45 Stat. line.
4. N. $86\frac{1}{2}$ W. 11.07

Left-hand off-sets on the stat line.

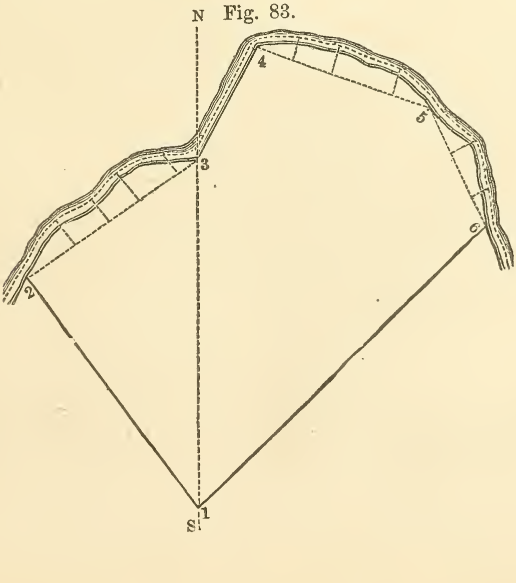
Stat.	Dist.	Off-sets.
No. 1.	0.00 Ch.	0.30 Ch.
2.	0.95	0.84
3.	2.03	0.86
4.	3.28	0.50
5.	5.20	1.80
6.	7.43	2.35
7.	8.98	1.45
8.	10.46	1.08
9.	11.71	1.85
10.	14.45	0.35

Fig. 82.



The area of the part ABCD will be found, by the last problem, to be 13 A. 1 R. 11 P.

Fig. 83.



To find the area of the off-sets.

No.	Sta. Dist. Ch.	Off-sets. Ch.	Intercep. Dist.	Sums of Off-sets.	Products.
1	0.00	0.30			
2	0.95	0.84	0.95	1.14	1.0830
3	2.03	0.86	1.08	1.70	1.8360
4	3.28	0.50	1.25	1.36	1.7000
5	5.20	1.80	1.92	2.30	4.4160
6	7.43	2.35	2.23	4.15	9.2545
7	8.98	1.45	1.55	3.80	5.8900
8	10.46	1.08	1.48	2.53	3.7444
9	11.71	1.85	1.25	2.93	3.6625
10	14.45	0.35	2.74	2.20	6.0280
					2)37.6144
					18.8072 Ch.
					=1 A. 3 R. 21 P.

	A.	R.	P.
Area of ABCD	13	1	11
Do. of off-sets	1	3	21
Whole area	15	0	32

EXAMPLE 2. Fig. 83.

Required the area of a survey from the following field notes.

				Left hand off-sets.					
				1st. Stationary Line.			3d Stat Line.		
				Sta.	Dist.	Off-sets.	Sta.	Dist.	Off-sets.
		Ch.		No.	Ch.	Ch.	No.	Ch.	Ch.
1	N. $36\frac{3}{4}°$ W	30.00		1.	0.00	0.50	1.	0.00	0.55
2.	N. $56\frac{1}{4}$ E.	21.60	stat. line.	2.	6.10	3.40	2.	4.20	2.50
3.	N. $26\frac{1}{2}$ E.	13.44	Do.	3.	10.15	3.10	3.	8.05	3.20
4.	S. $71\frac{1}{2}$ E.	18.96	Do.	4.	14.08	3.96	4.	15.15	2.45
5.	S. $26\frac{1}{2}$ E.	13.46	Do.	5.	19.20	2.70	5.	18.96	0.50
6.	S. 45 W.	42.41		6.	21.60	0.55	4th Stat. Line.		
				2d. Stat. Line.			1.	0.00	0.50
				1.	0.00	0.55	2.	5.12	2.75
				2.	13.44	0.55	3.	10.00	1.90
							4.	13.46	0.70

The area within the stationary lines and straight sides, found by the last problem, is 1152.5381 square chains.

The area of the offsets is	134.4907	"	"
Total area is	1287.0288	"	"

or 128 A. 2 R. 32 P.

EXAMPLE 3.

The following field-notes of the survey of a pond are given, to find its area:

Sides.	Angles.	Right-hand Off-sets. On AB. Dist.	Off-set.	On CD. Dist.	Off-set.
AB = 9.42 ch.	A = 73° 12′	1.00	.35	.34	.41
BC = 2.41 "	B = 88° 41′	1.59	.26	1.00	.00
CD = 8.35 "	C = 112° 25′	2.25	.50	2.15	.68
DA = 5.65 "	D = 85° 38′	4.17	.46	3.75	.00
		4.58	.54	4.40	.00
		7.32	.16	6.50	.25
		7.66	.06	8.00	.00
		8.25	.11		
		8.75	.00		
		9.00	.21		

Stations A, B and C were on the edge of the pond.

Ans. 3 A. 3 R. 37.43 P.

PROBLEM XI.

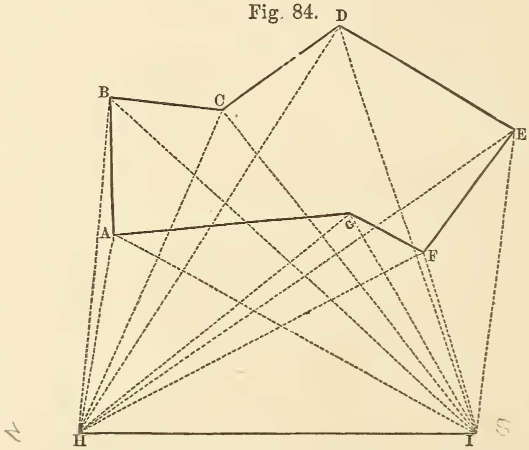
Given the bearing and distance of two stations from each other and the bearings of all the corners of a tract of land from these stations, to find the area of the tract.

The method of doing this will be best explained by an example.

EXAMPLE 1.

Let ABCDEFGA, Fig. 84, represent a field, all the angles of which can be seen from two stations, H and I, without it. The bearing and distance of the stations, and the bearings of all the angles of the field, from each station, being as follow, it is required to find the area.

Fig. 84.



The station H bears from the station I, North, dist. 28. Ch.

	Bearings.		Bearings.
HA	S. 81½° E.	IA	N. 28½° E.
HB	S. 85¾ E.	IB	N. 42¼ E.
HC	S. 68 E.	IC	N. 51½ E.
HD	S. 58¼ E.	ID	N. 71 E.
HE	S. 35½ E.	IE	S. 82½ E.
HF	S. 28½ E.	IF	N. 73½ E.
HG	S. 40 E.	IG	N. 60 E.

Construction.

Draw HI according to the given bearing and distance; and from the points H and I, draw HA. HB. HC, &c.,

and IA, IB, IC, &c. according to the given bearings; then will the intersections A, B, C, &c. of the corresponding bearings HA and IA, HB and IB, HC and IC, &c. be the angular points of the field.

Calculation.

In each of the triangles IHA, IHB, IHC, &c. we have the side IH; and from the bearings of the sides, we have all the angles, to find the sides IA, IB, IC, &c.

Then in each of the triangles, IAB, IBC, ICD, &c. we have two sides, and the included angle; whence the areas may be found by prob. III.

From the sum of the areas of the triangles IAB, IBC, ICD, and IDE, which is equal to the area IABCDEI, subtract the sum of the areas of the triangles IAG, IGF and IFE, which is equal to the area IAGFEI; the remainder will be the area of the field ABCDEFGA.

Note.—In working the proportions for finding the sides IA, IB, &c. it will be unnecessary, when the area only is required, to take out the natural numbers corresponding to the logarithms of those sides; because in the proportions for finding the areas it will be sufficient to know the logarithms of the sides, without knowing their real lengths.

Ans. 21 A. 1 R. 8 P.

EXAMPLE 2.

Being required to calculate the area of a field, the owner of which refuses permission to go on it, I choose two stations, F and G, in the adjacent land, from whence all the angles of the field are visible. FG was measured 20 chains, and the following angles were taken:

GFA = $68\frac{1}{2}°$,	FGA = 23°,
GFB = 24°,	FGB = 114°,
GFC = 38°,	FGC = $98\frac{3}{4}°$,
GFD = 59°,	FGD = $76\frac{1}{2}$,
GFE = $103\frac{1}{2}°$	FGE = 41°.

What is the area? *Ans.* 33 A. 1 R. 7 P.

PROBLEM XII.

To find the area of a survey by protracting it, and dividing the plot into triangles and trapeziums.

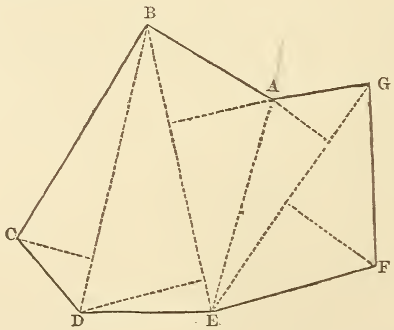
The method of doing this will be understood by the following example:

EXAMPLE 1.

Given the bearings and distances as follows:

1. N. 50° W. 9.60 ch.;
2. S. 32° W. 16.38 ch.;
3. S. 41° E. 6.30 ch.;
4. East 8.43 ch.;
5. N. 79° E. 10.92 ch.;
6. N. 5° W. 11.25 ch.
7. S. 83° W. 6.48 ch.; to the place of beginning.

Required the area.



By drawing lines as in the plot it is divided into two trapeziums, AGFE, AEDB, and a triangle BDC.

Measure the several bases and perpendiculars, on the same scale that was used in the protraction, and find the double areas of the triangle and trapeziums by probs. 2 and 6; the sum of these will be the double area of the survey.

Bases.	Perpens.	
EG 16.68 ×	{ Fa 7.50, Ab 4.71 }	= 203.6628 = 2 AGFE
EB 19.17 ×	{ Ac 5.85, Dd 8.10 }	= 267.4215 = 2 AEDB
BD 19.23 ×	Ce 5.16	= 99.2268 = 2 BDC
		2)570.3111 ch. = 2 ABCDEFG
		285.15555 ch. = 28A. 2R. 2P. = the area required.

Example 2.

The following field-notes are given to protract the survey and find the area.

	Ch.
1. N. 15° 00′ E.	20
2. N. 37° 30′ E.	10
3. East	7.50
4. S. 11° 00′ E.	12.50
5. South	13.50
6. West	10.
7. S. 36° 30′ W.	10.
8. N. 38° 15′ W.	8.50

Ans. 46 A. 2 R. 9 P

CHAPTER IV

LAYING OUT AND DIVIDING LAND.

PROBLEM I.

To lay out a given quantity of land in a square form.

RULE.

Reduce the given quantity to chains or perches, and extract the square root, which will be the length of a side, of the same denomination to which the given quantity is reduced.

EXAMPLES.

1. Required the side of a square that shall contain 9 A. 3 R. 28 P.

```
         40)28 Per.
         ---------
          4)3.7R.
          -------
           9.925 A.=99.25 ch.

       Ch.
      99.25(9.96 ch. the length of a side.
      81
      --
  189)1825
      1701
      ----
 1986)12400
       11916
       -----
         484
```

2. Required the side of a square tract of land that shall contain 325 acres. *Ans.* 57 chains.

PROBLEM II.

To lay out a given quantity of land in a rectangular form, having one side given.

RULE.

Divide the given content by the length of the given side, the quotient will be the length of the required side.

EXAMPLES.

1. It is required to lay out 120 acres in a rectangular form, the length of one side being given, equal 100 perches.

$$
\begin{array}{r}
\text{Acres.} \\
120 \\
4 \\
\hline
480 \\
40 \\
\hline
1{,}00)192{,}00 \\
\hline
\end{array}
$$

192 Per. the length of the other side

2. The length of a rectangular piece of land is 8 chains; what must be its breadth, that the content may be 5 acres?

Ans. 6.25 chains.

PROBLEM III.

To lay out a given quantity of land in a rectangular form, having the length to the breadth in a given ratio.

RULE.

As the less number of the given ratio,
Is to the greater;
So is the given area,
To a fourth term.*

The square root of this fourth term will be the length required. Having the length, the breadth may be found by the preceding problem. Or it may be found in the same manner as the length. Thus,

As the greater number of the given ratio,
Is to the less;
So is the given area,
To a fourth term.

The square root of this fourth term will be the breadth required.

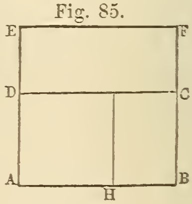
EXAMPLES.

1. It is required to lay out 864 acres in a rectangular form, having the length to the breadth in the ratio of 5 to 3.

---

* DEMONSTRATION. Let ABCD, Fig. 85, be a rectangle, and ABFE and AHGD be squares on the greater and less sides respectively: then (V. I) AD : AE (AB) : : the rectangle AC : square AF. Also AB : AH (AD) : : the rectangle AC : square AG. Hence the truth of the rule is evident.

Fig. 85.



$$864 \text{ A.} = 138240 \text{ P.}$$

$$\text{As } 3 : 5 :: \overset{\text{Sq. P.}}{138240} : \overset{\text{Sq. P.}}{230400}$$

$\sqrt{230400} = 480$ Perches, the length required.

$$\text{As } 5 : 3 :: \overset{\text{Sq. P.}}{138240} : \overset{\text{Sq. P.}}{82944}$$

$\sqrt{82944} = 288$ Perches, the breadth required.

2. It is required to lay out 27 A. 3 R. 20 P. in a rectangular form, having the length to the breadth in the ratio of 9 to 7. *Ans.* Length 75.725 P. Breadth 58.897 P.

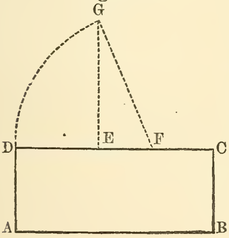
PROBLEM IV.

To lay out a given quantity of land in a rectangular form, having the length to exceed the breadth by a given difference.

RULE.

To the given area, add the square of half the given difference of the sides, and extract the square root of the sum; to this root, add half the given difference for the greater side, and subtract it therefrom for the less.*

Fig. 86.



* Demonstration. Let ABCD, Fig. 86, be a rectangle; in DC let DE be taken equal DA or BC, and let EC be bisected in F; then (II. 6) $DF^2 = DC \times DE + FC^2 = DC \times AD + FC^2 =$ the rectangle AC + the square of half the difference of the sides DC, DA; also $DF + FC = DC$, the greater side, and $DF - FC = DE$ or DA, the less side.

This problem may be neatly constructed thus: take EC equal the given difference of the sides and bisect it in F; make EG perpendicular to EC and equal to the square root of the given area, and with the centre F and radius FG describe the arc DG meeting CE produced in D: make DA perpendicular to DC and equal to DE, and complete the rectangle ABCD which will be the one required. Since (I. 42) $FG^2 = EG^2 + EF^2 =$ the given area + the square of half the given difference of the sides, the truth of the construction is plain from the preceding demonstration.

EXAMPLES.

1. It is required to lay out 47 A. 2 R. 16 P. in a rectangle, of which the length is to exceed the breadth by 80 perches.

2)80 P.	47 A. 2 R. 16 P.=7616 Per.
—	1600
40	——
40	$\sqrt{9216}=96$
——	half diff. add and subtract 40
1600	—
	length 136
	——
	breadth 56

2. It is required to lay out 114 A. 2 R. 33.4 P. in a rectangular form, having the length to exceed the breadth by 15.10 ch. *Ans.* Length 42.25 ch. Breadth 27.15 ch.

PROBLEM V.

To lay out a given quantity of land in the form of a triangle or parallelogram, one side and an adjacent angle being given.

RULE.

For a triangle.

As the rectangle of the given side and sine of the given angle,
Is to twice the given area :
So is radius,
To the other side, adjacent to the given angle.

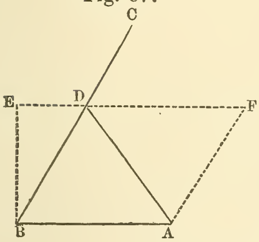
Then having two sides and the included angle given, the other angles and side, if required, may be found by trig. case 3.

For a parallelogram.

As the rectangle of the given side and sine of the given angle,
Is to the given area;
So is radius,
To the other side, adjacent to the given angle.*

EXAMPLES.

Fig. 87.



1. Let AB, BC, Fig. 87, be two sides of a tract of land; the bearing of AB is S. 87½° W. dist. 16.25 ch. and the bearing of BC, N. 27½° E.; it is required to lay off 10 acres by a straight line AD, running from the point A, to the side BC.

Bearing of BA, N. 87½° E.
——— BC, N. 27½ E.

Angle B, 60°

As AB × sin. B	AB 16.25 ch. -	- Ar. Co.	8.789147
	sin. B, 60° - -	- ———	0.062469
: twice the given area 200 sq. ch.		- - -	2.301030
: : rad.		- - -	10.000000
: BD 14.21 ch.		- - -	1.152646

* DEMONSTRATION.—It is demonstrated, prob. 3, chap. 3, Content of Land, that rad. : sin. B : : AB×BD : 2ABD (see Fig. 87); therefore rad.×AB : sin. B×AB : : AB×BD : 2ABD, or sin. B×AB : 2ABD : : rad.×AB : AB×BD : : rad. : BD. Since ABDF is equal to 2ABD, the truth of the rule for the parallelogram is evident.

This problem may be constructed as follows; take AB equal the given side and draw BC making the angle B equal to the given angle; make BE perpendicular to AB, and equal twice the given area of the triangle divided by the given side, or equal the given area of the parallelogram divided by the given side; and parallel to AB, draw EF cutting BC in D, and join DA; then will ABD be the triangle required; or complete the parallelogram ABDF, for the one required. The reason of the construction is plain.

2. Given the side AB, *Fig.* 15, of a parallelogram, equal 20 ch. and the angle A 63° 30′; required the side AC, that the content may be $21\frac{1}{2}$ acres.

As AB × sin A { AB 20 ch. - - -	Ar. Co.	8.698970
sin. A, 63° 30′ - -	———	0.048209
: the given area 215 sq. ch. - - - - - -		2.332438
: : rad. - - - - - - - - - - - - - -		10.000000
: AC 12.01 ch. - - - - - - - - - - -		1.079617

3. Given one side of a triangle, equal 30 perches, an angle adjacent to this side 71° 15′, and the area 2 acres; required the other side adjacent to the given angle.

Ans. 22.53 perches.

4. Given one side of a parallelogram, equal to 32.26 ch., an angle adjacent to this side 83° 30′, and the area 74 acres; required the other side adjacent to the given angle. *Ans.* 23.09 ch.

PROBLEM VI.

The area and base of a triangle being given, to cut off a given part of the area by a line running from the angle opposite the base.

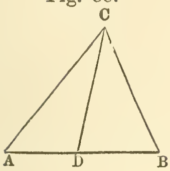
RULE.

As the given area of the triangle,
Is to the area of the part to be cut off;
So is the given base,
To the base corresponding to that area.*

* The truth of the rule is manifest from V. 1, Cor. 2.

EXAMPLES.

Fig. 88.



1. Given the area of the triangle ABC, Fig. 88, equal 650 square perches, and the length of the base AB, 40 perches; it is required to cut off 290 perches towards the angle A, by a line running from the angle C to the base.

	ABC.		ADC.		AB.		AD.
As	650	:	290	: :	40	:	17.85 per.

2. In a triangle ABC, there are given the area 27 A. 1 R. 16 P. and the base AB 35.20 ch., to cut off 10 acres towards the angle B, by a line CD running from the angle C to the base: the part BD of the base is required.

Ans. 12.87 ch.

PROBLEM VII.

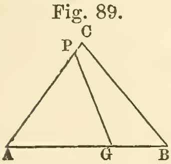
The area and two sides of a triangle being given, to cut off a triangle containing a given area, by a line running from a given point in one of the given sides, and falling on the other.

RULE.

As the given area of the triangle,
Is to the area of the part to be cut off;
So is the rectangle of the given sides,
To a fourth term.

Divide this fourth term by the distance of the given point from the angular point of the two given sides; the quotient will be the distance of the required point from the same angle.*

Fig. 89.



* DEMONSTRATION.—From the demonstration to prob. 3, chap. 3, we have, Fig. 89, rad. : sin. A : : AB × AC : 2ABC, and rad. : sin. A : : AP × AG : 2APG; therefore 2ABC : 2APG : : AB × AC : AP × AG, or ABC : APG : : AB × AC : AP × AG; hence the truth of the rule is manifest.

EXAMPLES.

1. Given the area of the triangle ABC, *Fig.* 89, 5 acres; the side AB 50 perches, the side AC 40 perches, and the distance of a point P from the angle A, 36 perches; it is required to find a point G to which, if a line be drawn from the point P, it shall cut off a triangle APG containing 3 A. 0 R. 20 P.

As the triangle ABC 800 sq. p.	Ar. Co.	7.096910
: the triangle APG 500		2.698970
:: AB × AC { AB 50		1.698970
AC 40		1.602060
: AP × AG		3.096910
AP 36	log.	1.556303
AG 34.72 per.		1.540607

2. Given the area of a triangle ABC, 12 A. 1 R. 23 P. the side AB 20 ch., the side AC 16.25 ch., and the distance of a point P in the side AB, from the angle A 8.50 ch.; it is required to find the distance AG of a point G in the line AC, so that a line drawn from P to G may cut off a triangle APG containing 3 acres. *Ans.* 9.25 ch.

PROBLEM VIII.

The area and base of a triangle being given, to cut off a triangle containing a given area, by a line running parallel to one of the sides.

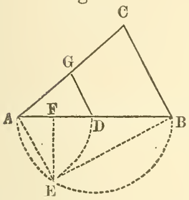
RULE.

As the given area of the triangle,
Is to the area of the triangle to be cut off;
So is the square of the given base,
To the square of the required base.

The square root of the result will be the base of the required triangle.*

EXAMPLES.

Fig. 90.



1. Given the area of the triangle ABC, Fig. 90, 500 square perches, and the base AB 40 perches; it is required to cut off 120 sq. per. towards the angle A, by a line DG running parallel to the side BC.

As the triangle ABC	500	- Ar. Co.	7.301030
: the triangle ADG	120	- - - -	2.079181
: : AB^2 { AB	40	- - - -	1.602060
{ AB	40	- - - -	1.602060
: AD^2 - - - - - - - - - -			2)2.584331
AD 19.6 per. - - - - - - - -			1.292165

2. Given the area of a triangle ABC, 10 acres, and the base AB 25 ch., to find BD a part of the base, so that a line DG running from the point D, parallel to the side AC, may cut off a triangle BDG containing $4\frac{1}{2}$ acres. *Ans.* BD=16.77 ch.†

---

* The truth of this rule is manifest from V. 19.

This problem may be neatly *constructed* as follows: Let ABC, Fig. 90, be the given triangle, and AB the given base; on AB describe the semicircle AEB, and take AF to AB in the ratio of the part to be cut off, to the whole triangle; draw FE perpendicular to AB, meeting the semicircle in E, join AE, and make AD equal to AE; from D draw DG parallel to BC, and the thing is done. For, join EB, and we have, by similar triangles, AB : AE : : AE : AF, or $AB^2 : AE^2 :: AB \cdot AE : AE \cdot AF :: AB : AF$.

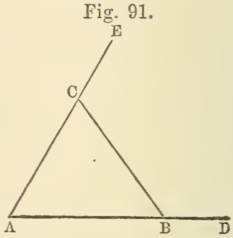
But (V. 19) $AB^2 : AE^2(AD^2) :: ABC : ADG$;

$\therefore$ $AB : AF :: ABC : ADG$.

† If it be required to produce two sides of a given triangle so far that the triangle formed by these sides produced, and a line drawn between them parallel to the third side, may contain a given area, it may be done by the above rule. Thus, Fig. 90. ADG : ABC : : $AD^2 : AB^2$.

PROBLEM IX.

Fig. 91.



The bearings of two adjacent sides AD, AE, Fig. 91, *of a tract of land being given, to cut off a triangle* ABC *containing a given area by a line* BC *running a given course.*

RULE.

From the given bearings of the lines, find the angles A, B, and C; then,

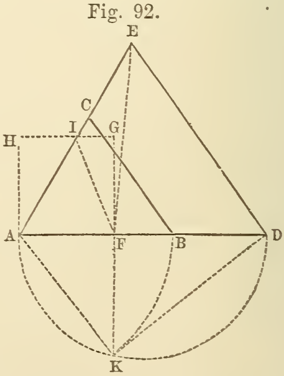
As the rectangle of the sines of the angles A and B,
Is to the rectangle of radius and sine of the angle C;
So is twice the given area,
To the square of the side AB.*

In like manner the other sides may be found; or having found one side, the others may be found by trig. case 1.

EXAMPLES.

1. Let the bearing of AD, Fig. 91, be N. 87° 30′ E. and of AE, N. 27° 30′ E.; it is required to cut off 10 acres by a line BC running N. 38° W.

Fig. 92.



* The truth of this rule is evident from the demonstration to prob. 4, chap. 3.

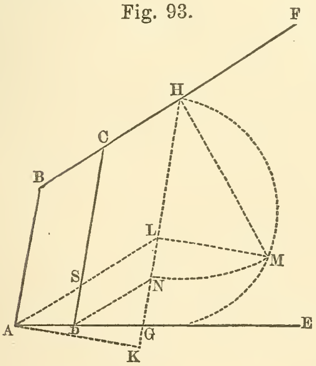
Construction. Draw AD, AE, (Fig. 92,) according to the given bearings, and in AD take AF equal the square root of the given area, and on it describe the square AFGH; make IE = AI, and draw ED, according to the reverse bearing of the division line BC, meeting AD in D; on AD describe a semicircle, and produce GF to meet it in K, join AK and make AB equal to it; draw BC parallel to DE, and ABC will be the triangle required. For join IF, EF and KD; then (III. 21 and V. 8, Cor.) AD : AK (AB) : : AK (AB) : AF; or (last proof) AD : AF : : ADE .: ABC; but (V. 1) AD : AF : : ADE : AFE; therefore ADE : ABC : : ADE : AFE, and consequently ABC = AFE; but because AI = IE, AFE = 2AFI = AFGH; therefore ABC = AFGH = the given area of the triangle.

AD, N. 87° 30′ E.	BA, S. 87° 30′ W.	CA, S. 27° 30′ W.
AE, N. 27 30 E.	BC, N. 38 00 W.	CB, S. 38 00 E.
Angle A, 60 00	125 30	Angle C, 65 30
	180 00	
	Angle B, 54 30	

As sin. A × sin. B,	A 60° 00′	Ar. Co.	0.062469
	B 54 30	———	0.089314
: rad. × sin. C,	C 65 30 - - - -		9.959023
	rad. - - - - - -		10.000000
: : twice the given area, 200 sq. ch. - -			2.301030
: AB^2 - - - - - - - - - - - - -			2)2.411836
AB 16.07 - - - - - - - - - - - -			1.205918

2. Given the bearing of one side of a tract of land, S. 53° 15′ E., and the bearing of an adjacent side taken at the same angle, N. 55° 00′ E., to cut off 4 acres by a line running N. 4° 00′ W.; required the distance on the first side. *Ans.* 9.76 ch.

Fig. 93.



PROBLEM X.

The bearings of three adjacent sides, EA, AB, BF, *Fig.* 93 *or* 94, *of a tract of land, and the length of the middle side* AB, *being given, to cut off a trapezoid* ABCD, *containing a given area, by a line* DC, *parallel to* AB.

RULE.

From the given bearings find the angles A and B; add these together, and take the difference between their sum and 180°, and call it P. Then,

As the product of the sines of A and B,
Is to the product of radius and sine of P;

So is twice the area to be cut off,
To a fourth term.

When the sum of the angles A and B is greater than 180°, add this fourth term to AB^2; but when the sum of these angles is less than 180°, take the difference between this fourth term and AB^2. The square root of the result will be DC. Then,

As the sine of P,
Is to the sine of B,
So is the difference between DC and AB,
To AD.*

---

* DEMONSTRATION.—Produce EA and FB, Fig. 95, to meet in P. Then (V. 19) PDC : PAB : : $CD^2 : AB^2$, or (IV. 10) ABCD : PAB : : $CD^2 - AB^2 : AB^2$, or PAB : ABCD : : $AB^2 : CD^2 - AB^2$, or 2PAB : AB^2 : : 2ABCD : $CD^2 - AB^2$. But by the demonstration to prob. 4, chap. 3, 2PAB : (AB^2) : : sin. A × sin. B : rad. × sin. P. Consequently,

sin. A × sin. B : rad, × sin. P : : 2ABCD : $CD^2 - AB^2$.

Fig. 94.

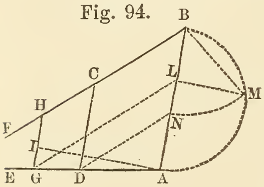
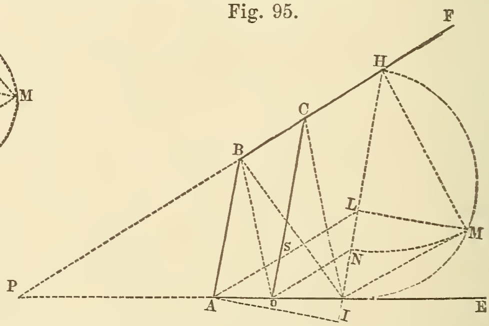


Fig. 95.



Now it is plain that AB^2, added to this 4th term, gives CD^2. A similar demonstration applies when the sum of the angles A and B is less than 180°, as in Fig. 94. The latter part of the rule does not require demonstration.

Construction.—Draw AI perpendicular to AB, and make it equal to the quotient of twice the given area divided by AB. From I, draw IH parallel to AB meeting AE and BF, in G and H, and on GH, describe the semicircle GMH. From A, draw AL parallel to BF; and make LM perpendicular to GH. With the distance HM and centre H, describe the arc MN; and from N, draw ND parallel to AL. Lastly, draw DC parallel to AB, and it will be the division line required. For join BD, BG, and CG, Fig. 95. Then by similar triangles PG : PD : : GH : DC : : GH : HM : : HM : HL : : DC : AB : : PC : PB. Hence (V. 2) CG is parallel to BD; and consequently the triangle BDC is equal to BGD. To each of these, add ABD. Then we have ABCD = ABG. But it is plain from the construction that ABG is equal to the given area. Hence ABCD is equal to the given area.

When the sum of the angles A and B is *less* than 180°, as in Fig. 94,

EXAMPLES.

1. Given the bearing of EA, Fig. 93, West, AB, N. 10° E. dist. 15 ch.; and BF, N. 58° 30′ E. to cut off 10 acres by a line CD, running parallel to AB. Required the length of the division line and the distance AD.

AE, N. 90° E.	BF, N. 58° 30′ E.	A, 80° 0′
AB, N. 10 E.	BA, S. 10 0 W.	B, 131 30
A = 80°	48° 30	211 30
	180 0	180 0
	B = 131 30	P = 31° 30′

As sin. A × sin. B,	A, 80° 00′ Ar. Co.	0.006649
	B, 131 30	0.125544
: rad. × sin. P.	Rad. - - - - - -	10.000000
	P, 31 30 - - -	9.718085
: : twice the given area 200 sq. ch. - - -		2.301030
: fourth term 141.68 - - - - - -		2.151308

$AB^2 = 225.$

$DC = \sqrt{366.68} = 19.15$ ch.

As sin. P,	31° 30′ - - - Ar. Co.	0.281915
: sin. B,	131 30 - - - - - - -	9.874456
: : DC − AB,	4.15 - - - - - - -	0.618048
: AD	5.95 - - - - - - -	0.774419

---

the semicircle must be described on AB; the point L must be determined by drawing GL parallel to FB; and the arc MN must be described with the radius BM and centre B. The other parts of the construction are the same as before.

2. Given the bearings of three adjacent sides of a tract of land and the length of the middle one as follows: 1st. N. 20° W.; 2d. N. 60° 30′ E. dist. 6 ch.; 3d. S. 61° 30′ E.; to cut off a lot containing 2½ acres, by a line parallel to the 2d side. Required the length of the division line and the distance on the 1st side.

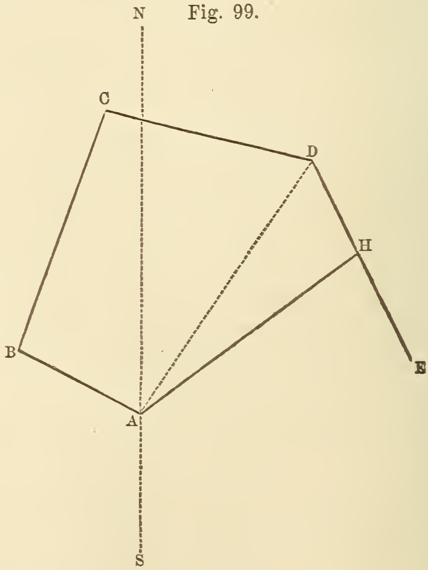
Ans. Division line 8.70 ch.; distance on 1st side 3.45 ch.

3. Given as follows: 1st side N. 31° 15′ W.; 2d N. 58° 45′ E. dist. 13.50 ch.; 3d S. 14° 45′ E.; to cut off 8 acres by a line parallel to the 2d side. The length of the division line and the distance on the 1st side are required. *Ans.* Division line 11.61 ch.; distance on the 1st side 6.38 ch.

PROBLEM XI.

The bearings of several adjacent sides, AB, BC, CD, DE, Fig. 99, *of a tract of land, and the distance of each, except the last, being given, to cut off a given area by a line* AH *running from the angle* A, *and falling on the side* DE.

Fig. 99.



RULE.

Take out the latitudes and departures of the sides whose bearings and distances are known. Find the difference of the sums of the northings and southings, and of the eastings and westings. The remainders will be the latitude and departure of the closing side AD.

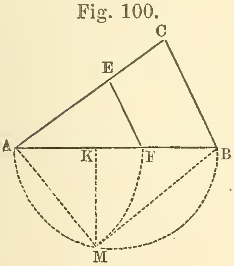
With this latitude and departure calculate the bearing and distance.

Calculate also the area of the figure cut off by this closing line, and subtract it from the given area. The remainder will be the area to be laid off in a triangular form by Problem V., page 160.

The angle DHA applied to the bearing of DE will give the bearing of HA.

PROBLEM XII.

Fig. 100.



The sides AB, BC, CA, Fig. 100, *of a triangular piece of ground being given, to divide it into two parts having a given ratio, by a line* FE, *running parallel to one of the sides as* BC.

RULE.

As the sum of the numbers expressing the ratio of the parts,
Is to that number of the ratio which corresponds to the part to be adjacent to A;
So is the square BC,
To the square of FE.

Then, As BC : AB : : FE : AF.*

---

* DEMONSTRATION. Let m to n be the ratio of the part AFE to the part FECB; then (IV. 9) $m+n : m : :$ ABC : ADE : (V. 19) $BC^2 : FE^2$.

Construction. On AB describe the semicircle AMB, and by Prob. 16 Page 36, divide AB in K, so that AK may be to KB in the given ratio of the part AFE to the part FECB; draw KM perpendicular to AB, meeting the

EXAMPLES.

1. Let AB be 21.26 ch.; BC, 12.76 ch.; and AC, 19.30 ch.; it is required to divide the triangle by the line FE, parallel to BC, so that the part AFE may be to the part FECB as 2 to 3.

$$\text{As } 5 : 2 :: 12.76^2 : FE^2 = 65.12704.$$
$$FE = \sqrt{65.12704} = 8.07.$$
$$\text{As } 12.76 : 21.26 :: 8.07 : AF = 13.45.$$

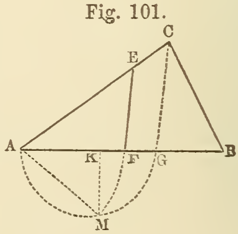
2. The three sides of a triangular piece of land, taken in order, measure 15, 10, and 13 chains respectively; it is required to divide it into two equal parts by a line parallel to the second side. What will be the length of the division line and its distance from the place of beginning, measured on the first side?

Ans. Division line 7.07 ch.; dist. on 1st side 10.61 ch

PROBLEM XIII.

The bearings and distances of the sides AB, BC, CA, Fig. 101, *of a triangular piece of ground being given, to divide it into two parts having a given ratio, by a line* FE, *running a given course.*

Fig. 101.



RULE.

As the product of the sines of F and E,
Is to the product of the sines of B and C;

---

semicircle in M, and with the radius AM and centre A, describe the arc MF. From F draw the division line FE parallel to BC. Since AB : AM (AF) : : AM (AF) : AK, we have (see Demonstration, page 165) AB : AK : : AB^2 : AF^2 : : ABC : AFE. Hence the truth of the construction is evident.

So is the square of BC,
To a fourth term.

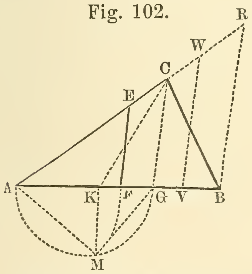
Multiply this fourth term by that number of the ratio which corresponds to the part to be adjacent to the angle A, and divide the product by the sum of the numbers expressing the ratio. The square root of the result will be FE. Then,

As sin. A : sin. E : : FE : AF.*

EXAMPLES.

1. Let the bearing of AB, be S. $82\frac{1}{4}$° E. dist. 14.17 ch.; BC, N. $18\frac{3}{4}$° W. 8.51 ch.; and CA, S. $61\frac{1}{2}$° W. dist. 12.87 ch.; it is required to divide the triangle by the line FE, running N. $14\frac{3}{4}$° E. so that the part AFE may be to the part FECB in the ratio of 2 : 3.

Fig. 102.



* Demonstration.—Draw CG and BR, Fig. 102, parallel to EF; and let VW, also parallel to EF, make the triangle AVW equal to ABC. Then (V. 15, cor.) AB.AC = AV.AW; hence AB : AV : : AW : AC, but (V. 4) AB : AV : : BR : VW, and AW : AC : : VW : CG. ∴ BR . GC = VW^2. Now,

As sin. R (sin. E) : sin. C : : BC : BR,
sin. G (sin. F) : sin. B : : BC : GC.

Hence sin. E × sin. F : sin. B × sin. C : : BC^2 : BR × GC : : BC^2 : VW^2.

Also, $m+n : m$: : ABC : AFE : : AVW : AFE : : (V. 19) VW^2 : FE^2.

Construction.—From C, Fig. 101, draw CG according to the reverse bearing of FE, and on AG describe the semicircle AMG. By prob. 16, page 36, divide AB in K, so that AK may be to KB in the given ratio of the part AFE to the part FECB. Draw KM perpendicular to AB, and with the radius AM and centre A describe the arc MF. From F, and parallel to GC, draw FE, the required division line. For join KC; Fig. 102. (V. 1, cor. 2) KC divides the triangle in the given ratio. Now AC : AE : : AG : AF (AM) : : (V. 8, cor.) AM (AF) : AK; therefore AC : AE : : AF : AK, and hence AC . AK = AE . AF. ∴ (V. 15, cor.) AFE = AKC. Consequently FE divides the triangle in the given ratio.

Angle A=36¼°, B=63½°, C=80¼°, E=46¾° and F=83°.

As sin. E × sin. F,	E, 46° 45′	Ar. Co.	0.137647
	F, 83 00	———	0.003249
: sin. B × sin. C,	B, 63 30	- - - - -	9.951791
	C, 80 15	- - - - -	9.993681
: : BC^2	BC, 8.51	- - - - -	0.929930
	BC, 8.51	- - - - -	0.929930
: fourth term	88.35	- - - - -	1.946228

$$\begin{array}{r} 88.35 \\ 2 \\ \hline 5)176.70 \\ \hline \end{array}$$

$$FE = \sqrt{35.34} = 5.95$$

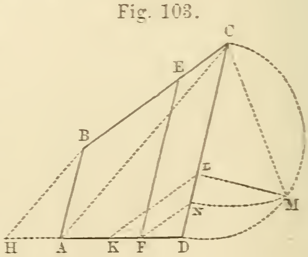
As sin. A, 36° 15′ - - - - - -	Ar. Co.	0.228185
: sin. E, 46° 45′ - - - - - - - - - - -		9.862353
: : FE. 5.95 - - - - - - - - - - - - -		0.774517
: AF, 7.33 - - - - - - - - - - - - -		0.865055

2. The bearings and distances of a triangular piece of land ABC are, AB. S. 69° E. 21.40 ch.; BC, N. 31½ E. 18.66 ch.; and CA, S. 74½ W. 30.85 ch.; and it is required to divide it by a line FE, running due north, so that the part AEF, may be to the part FECB, as 4 to 5. What will be the length of the division line FE, and the distance AF? *Ans.* FE 10.74, and AF 17.40.

PROBLEM XIV.

The bearings and distances of the sides AB, BC, CD, DA, Fig. 103, *of a trapezoidal tract of land being given, to divide it into two parts having a given ratio, by a line* FE, *running parallel to the parallel sides* AB, CD.

Fig. 103.



RULE.

Multiply the square of AB, by that number of the ratio which corresponds to the part, to be adjacent to CD, and the square of CD, by the other number of the ratio. Add the two products together, and divide the sum by the sum of the numbers expressing the ratio. The square root of the quotient will give FE. Then,

As DC—AB : FE—AB : : AD : AF.*

EXAMPLES.

1. Let the bearing of AB be N. 14° E. dist. 10 ch. BC, N. 55¼° E. dist. 18.67 ch.; CD, S. 14° W. dist. 20.98 ch.; and DA, W. 12.70 ch.; it is required to divide the trapezoid into two parts by a line FE, parallel to AB or DC,

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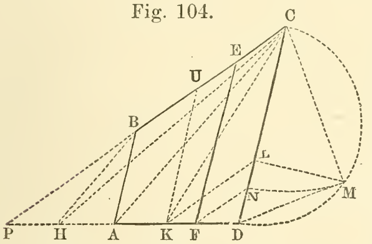
Fig. 104.



* DEMONSTRATION. — Produce DA and CB, Fig. 104, to meet in P. Then (V. 19) PDC : PAB : : $CD^2 : AB^2$, or (IV. 10) ABCD : PAB : : $CD^2-AB^2 : AB^2$, or ABCD : CD^2-AB^2 : : PAB : AB^2. In like manner ABEF : FE^2-AB^2 : : PAB : AB^2. Hence (IV. 15) ABCD : CD^2-AB^2 : : ABEF : FE^2-AB^2, or ABCD : ABEF : : $CD^2-AB^2 : FE^2-AB^2$. Therefore $m+n : m$: : $CD^2-AB^2 : FE^2-AB^2$, or multiplying extremes and means, $\overline{m+n}\,.\,FE^2-\overline{m+n}\,.\,AB^2=m\,.\,CD^2-m\,.\,AB^2$. But $(m+n)\;AB^2=m\,.\,AB^2+n\,.\,AB^2$. Therefore, adding equals to equals, we have $\overline{m+n}\,.\,FE^2=m\,.\,CD^2+n\,.\,AB^2$. Hence the truth of the rule is evident.

Construction.—Join CA, Fig. 103, and parallel to it draw BH, meeting DA produced in H. Divide (prob. 16, page 36) HD in K, so that HK may be to KD in the given ratio of ABEF to FECD, and draw KL parallel to BC. On CD describe the semicircle CMD, and draw LM perpendicular to CD. With the radius CM and centre C, describe the arc MN, and from N draw NF parallel to KL. From F draw the division line FE parallel to AB or CD. For join KC, Fig. 104, and draw KU parallel to AB. Then since BH is parallel to AC, the triangle AHC is equal to ABC; and adding ADC to each, we have CHD = ABCD. Now (V. 4) PC : PE : : CD : FE (CM) : : CM (EF) : CL (UK) : : PF : PK. Therefore (IV. 15) PC : PE : : PF : PK; and hence (V. 15, cor.) the triangle PEF is equal to PCK. Consequently CKD = FECD. But it has been proved that CHD = ABCD; hence taking equals from equals we have HCK = ABEF. But HCK : CKD : : HK : KD : : $m : n$. Therefore ABEF : FECD : : $m : n$.

so that the part ABEF may be to the part FECD as 3 to 2.

$$\begin{array}{r} 2.\ AB^2=200 \\ 3.\ CD^2=1320.4812 \\ \hline 5)1520.4812 \\ \hline \end{array}$$

$$FE=\sqrt{304.0962}=17.44$$

As $10.98 : 7.44 : : 12.70 : AF=8.61$

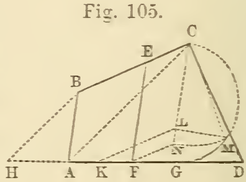
2. The boundaries of a trapezoidal field ABCD are given as follow; viz. AB, N. 80° W. 60 per.; BC, N. 39½° W. 45.5 per.; CD, S. 80° E. 89.4 per.; and DA, South, 30 per.; and it is required to divide it into two equal parts by a line FE parallel to AB or CD. What will be the length of the division line FE, and the distance AF?

Ans. FE 76.13 per., and AF 16.46 per.

PROBLEM XV.

The bearings and distances of the sides AB, BC, CD, DA, Fig. 105, *of any quadrilateral tract of land being given, to divide it into two parts having a given ratio, by a line* FE, *running parallel to one of the sides as* AB *or* CD.

Fig. 105.



RULE.

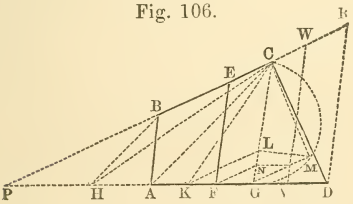
Call the side to which the division line is to be parallel, the *parallel* side; and the one opposite to this, the *opposite* side. From the bearings, find the angles. Take the difference between the sum of the angles adjacent to the parallel side and 180°, and call it P. Then,

As the product of the sines of the angles adjacent to the parallel side,
Is to the product of the sines of the angles adjacent to the opposite side:
So is the square of the opposite side,
To a fourth term.

Multiply this fourth term by that number of the ratio which corresponds to the part to be adjacent to the parallel side, and to the product add the product of the square of the parallel side by the other number of the ratio; and divide the sum by the sum of the numbers expressing the ratio. The square root of the quotient will be the length of the division line FE. Then,

As the sine of P,
Is to the sine of E;
So is the difference between FE and the parallel side,
To the distance of F from the adjacent end of the parallel side.*

Fig. 106.



* Demonstration. — Produce DA and CB, Fig. 106, to meet in P. Draw DR and CG each parallel to AB; and let VW, also parallel to AB, make the triangle PVW equal to PCD. Then (V. 15, cor.) PD : PV : : PW : PC. But (V. 4) PD : PV : : DR : VW, and PW : PC : : VW : CG. Therefore DR : VW : : VW : CG; and hence $DR \times CG = VW^2$. But by trigonometry,

As sin. CRD (sin. B) : sin. C : : CD : DR,
sin. CGD (sin. A) : sin. D : : CD : CG.

Hence sin. A × sin. B : sin. C × sin. D : : $CD^2 : DR \times CG$.
Or, sin. A × sin. B : sin. C × sin. D : : $DC^2 : VW^2$.

But by the demonstration to the rule in the last problem, we have

$$\overline{m+n}.\ FE^2 = m.\ VW^2 + n.\ AB^2.$$

Hence the truth of the rule is evident.

Construction. From C, Fig. 105, draw CG parallel to AB, and on it describe the semicircle CMG.

EXAMPLE.

1. Let the bearing of AB be North, 12 ch.; BC, N. 56½° E. 20.78 ch.; CD, S. 33½° E. 22.21 ch.; and DA, S. 80½° W. 30 ch.; it is required to divide the tract into two parts by a line FE, parallel to AB, so that the part ABEF may be to the part FECD as 3 to 5.

Angle A = 80½°, B = 123½°, C = 90°, D = 66°, and P = 24°.

As sin. A × sin. B,	A, 80° 30′	Ar. Co.	0.005997
	B, 123 30	———	0.078893
: sin. C × sin. D,	C, 90 00	- - -	10.000000
	D, 66 00	- - -	9.960730
: : CD^2	CD, 22.21	- - -	1.346549
	CD, 22.21	- - -	1.346549
: fourth term	547.92	- - -	2.738718

$$\begin{array}{r} 547.92 \\ 3 \\ \hline 1643.76 \\ 5AB^2 = 720.00 \\ \hline 8)2363.76 \\ \hline \end{array}$$

$$FE = \sqrt{295.47} = 17.19$$

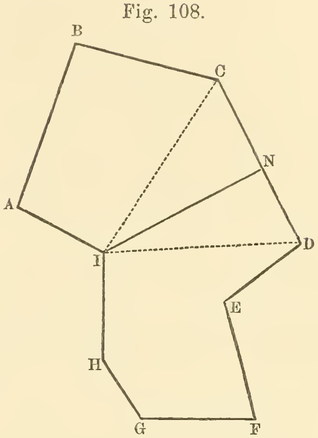
As sin. P, 24° 00′	Ar. Co.	0.390687
: sin. E, 123° 30′	- - -	9.921107
: : FE − AB, 5.19	- - -	0.715167
: AF, 10.64	- - -	1.026961

Join CA, and draw BH parallel to it meeting DA produced in H. Divide HD in K, so that HK may be to KD in the given ratio of the part ABEF to FECD. Draw KL parallel to BC and LM perpendicular to CG. With the radius CM and centre C describe the arc MN. Draw NF parallel to KL and FE parallel to AB. Then will FE be the division line. When the division line is to be parallel to CD, the semicircle must be described on CD, and the line CG need not be drawn. The demonstration of the construction is the same as for the last problem.

PROBLEM XVI.

The boundaries of a tract of land ABCDEFGHIA, Fig. 108, *being given, to divide it into two equal parts by a line* IN *running from the corner* I, *and falling on the opposite side* CD.

Fig. 108.



RULE.

Suppose lines drawn from I, to C and D, and calculate the area of the whole tract.

Take the corrected latitudes and departures of IA, AB, and BC, and by balancing find the latitude and departure of CI; also calculate the area of the part IABCI; from half the area of the whole tract, subtract the area of the part IABCI, the remainder will be the area of the triangle ICNI.

Lay off this area by Problem V., page 160. The angle CNI applied to the bearing of CN will give the bearing of CI.

1. Given the boundaries of a tract of land as follows: *viz.* 1st. S. 35¼° W. 11.20 ch.; 2d. N. 45° W. 24.36 ch.; 3d. N. 15½° E. 10.80 ch.; 4th. S. 77° E. 16 ch.; 5th. N. 87½° E. 21.50 ch.; 6th. S. 60° E. 14.80 ch.; 7th. South, 10.91 ch.; 8th. N. 85° W. 29.28 ch. to the place of beginning; to divide the tract into two equal parts by a line running from the first station and falling on one of the opposite sides; the bearing and distance of the division line are required.

Ans. N. 7° 18′ E. 15.28 ch.

CHAPTER V.

MISCELLANEOUS QUESTIONS.

1. A circular fish-pond is to be dug in a garden, that shall take up just half an acre: what must be the length of the cord that strikes the circle?

Ans. 27.75 yards.

2. Two sides of a triangle are 20 and 40 perches respectively: required the third side, so that the content may be just an acre.

Ans. Either 23.099 or 58.876 perches.

3. In 110 acres of statute measure, in which the pole is 5.5 yards, how many Cheshire acres, where the customary pole is 6 yards; and how many of Ireland, where the pole in use is 7 yards?

Ans. 92 A. 1 R. 29 P. Cheshire; 67 A. 3 R. 25 P. Irish.

4. The ellipse in Grosvenor square, London, measures 840 links the longer way, and 612 the shorter, within the rails; now the wall being 14 inches thick, it is required to find what quantity of ground it encloses, and how much it stands upon.

Ans. It encloses 4 A. 0 R. 6 P. and stands on $1760\frac{1}{2}$ square feet

5. Required the dimensions of an elliptical acre, with the greater and less diameters in the ratio of 3 to 2.

Ans. 17.481 by 11.654 perches.

6. The three sides of a triangular field, containing 6 A. 1 R. 12 P. are in the ratio of the three numbers, 9, 8, 6, respectively: required the sides.

Ans. 59.029, 52.47, and 39.353 perches.

7. In a pentangular field, beginning with the south side and measuring round towards the east, the first or south side is 27.35 ch., the second 31.15 ch., the third 23.70 ch., the fourth 29.25 ch., and the fifth 22.20 ch.; also the diagonal from the first angle to the third is 38 ch., and that from the third to the 5th. 40.10 ch.; required the area of the field. *Ans.* 117 A. 2 R. 39 P.

8. Required the dimensions of an oblong garden, containing three acres, and bounded by 104 perches of pale fence.* *Ans.* 40 P. by 12.

9. How many acres are contained in a square meadow, the diagonal of which is 20 perches longer than either of its sides? *Ans.* 14 A. 2 R. 11 P.

10. A gentleman has a garden 100 feet long and 80 broad, and a gravel walk is to be made of equal width half round it; what must be the width of the walk, so that it may take up just one fourth of the ground.

Ans. 11.8975 feet.

11. A person has a circular yard that is 150 feet in diameter, and wishes a walk of equal width made round it within the fence: required the width of the walk so that it may occupy a fifth part of the ground.

Ans. 7.918 feet.

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* It may not be improper here to observe, that the 2d question, and all those following the 8th, admit of neat geometrical constructions.

12. From a point within a triangular field the sides of which were equal, I measured the distances to the three angles, and found them 12.5, 10, and 7.5 chains, respectively; required the area. *Ans.* 12 A. 1 R 23 P.

13. On examining the field-notes of a lot of ground of which I wished to know the content, I found them as follow: 1st. S. 72° W. 24 per., 2d. North, 38 per., 3d. N. 82½ E. 41 per., 4th. ——, 20 per., 5th. S. 80° E. 11.5 per., 6th. S. 26° W. 22 per., and 7th. ——, 37 per., to the place of beginning. The bearings of the 4th and 7th boundary lines were illegible; but the data remaining being sufficient, the area is required.

Ans. 12 A. 3 R. 2 P.

14. It is required to lay out 4½ acres of land in a triangular form, so that the length of one side may be 15 chains, and the lengths of the other sides in the ratio of 2 to 3; what must be the lengths of those sides?

Ans. 7.7914 and 11.6871 chains; or 29.58536 and 44.37804 chains

15. It is required to lay out five acres of ground in a triangular form to be bounded by 135 perches of fence; the length of one side is to be 50 perches; what must be the lengths of the other sides?

Ans. 33.3785 and 51.6215 perches.

16. The area of a rectangular field is 7½ acres and the length of the diagonal 50 perches: required the sides.

Ans. 30 and 40 perches.

17. In a rectangular tract of land, containing 58 A. 3 R. 8 P. the difference of the lengths of the sides is just equal to the difference between the lengths of the longer side and the diagonal; hence the sides are required.

Ans. 21 and 28 chains.

18. The boundaries of a tract of land are as follow: 1st. N. 14° W. 15.20 ch.; 2d. N. $70\frac{1}{2}$° E. 20.43 ch.; 3d. S. 6° E. 22.79 ch.; 4th. N. $86\frac{1}{2}$° W. 18 to the place of beginning; within the tract there is a spring, the bearing and distance of which, from the 2d corner, is S. 75° E. 7.90 ch. It is required to cut off 10 acres from the west side of this tract by a straight line running through the spring; what must be the distance of the division line from the 1st corner, measured on the fourth side.

Ans. 4.6357 chains.

19. The boundaries of a quadrilateral tract of land are as follow: 1st. N. $35\frac{1}{4}$° E. 23 ch.; 2d. N. $75\frac{1}{2}$° E. 30.50 ch.; 3d. S. $3\frac{1}{4}$° E. 46.49 ch. and 4th. N. $66\frac{1}{4}$° W. 49.64 ch., to the place of beginning. This tract is to be divided into four equal parts by two straight lines, one of which is to run parallel to the 3d side; required the distance of the parallel division line from the first corner, measured on the 4th side; also the bearing of the other division line, and its distance from the same corner, measured on the first side.

Ans. Distance of the parallel division from the first corner 32.50 chains; the bearing of the other, S. 88° 22′ E. and its distance from the same corner 5.99 chains.

CHAPTER VI.

VARIATION OF THE COMPASS.

A MERIDIAN indicated by the magnetic needle is not, in general, a true one; for the needle does not point truly to the north point of the horizon, but varies from it, in some places to the eastward, and in others to the westward.

The angle contained between the true meridian and that indicated by the needle, is called the *variation of the compass.*

The variation is named *east* or *west*, according as the north end of the needle points to the eastward or westward of the true north.

As the variation is different in different places, so also, in the same place, it does not remain the same, but differs sensibly in the course of a few years. Hence, in running a line that was run a number of years previously, the bearing will be found different from what it was at that time; this, together with some difference in compasses, causes many difficulties, and frequently inaccuracies, in tracing old lines.

The easiest way to guard against those difficulties and inaccuracies would be to make and return the surveys according to the true, and not the magnetic bearings. In order to do this, it will be necessary to know the variation of the compass for the place in which the survey is made; and this may readily be found by first tracing a meridian line in the following manner.

To draw a true meridian line by means of the greatest elongation of the pole star.

The pole star is situated about $1\frac{1}{2}°$ from the true pole, and therefore apparently revolves round it, in a small circle, once in about 23 h. 56 m. When at its greatest distance *east* or *west* from the true pole, it is said to be at its greatest east or west *elongation*. It is therefore evident that in the course of one apparent revolution it must be twice at its greatest elongation, once to the east and once to the west.

The following tables exhibit the times, nearly, of the greatest eastern elongations of the pole star for six months of the year, and of the greatest western elongations for the other six months. The other greatest elongations take place in the day time, and are therefore invisible. Some of those inserted in the tables are also invisible; because they occur, either before daylight is gone, in the evening, or after it has returned, in the morning. The most of those in the 3d, 4th, 9th, and 10th months are in this situation.

The time in the tables is reckoned from noon; and therefore when it is less than 12 hours, the greatest elongation takes place in the evening of the same day; but when it exceeds 12 hours, if 12 hours be subtracted from it. the remainder will be the time of greatest elongation in the morning of the following day.

Eastern Elongations.

Day.	April.		May.		June.		July.		August.		Sept.	
	H.	M.	H.	M.	H.	M.	H.	M.	H.	M.	H.	M.
1..............	18	37	16	39	14	37	12	39	10	37	8	36
7..............	18	14	16	16	14	14	12	16	10	14	8	12
13..............	17	50	15	52	13	50	11	52	9	50	7	48
19..............	17	26	15	28	13	26	11	29	9	27	7	25
25..............	17	03	15	05	13	03	11	05	9	03	7	01

Western Elongations.

Day.	October.		Nov.		December.		January.		February.		March.	
	H.	M.	H.	M.	H.	M.	H.	M.	H.	M.	H.	M.
1..............	18	27	16	25	14	28	12	26	10	24	8	30
7..............	18	04	16	02	14	04	12	02	10	00	8	06
13..............	17	40	15	38	13	40	11	39	9	37	7	43
19..............	17	17	15	15	13	17	11	15	9	13	7	19
25..............	16	53	14	51	12	53	10	51	8	49	6	55

In order to determine a true meridian, by the method here used, it is necessary to know the bearing of the pole star, called its *azimuth*, at the time of its greatest elongation. This depends on the latitude of the place, and the distance of the star from the pole. This distance is called the *polar distance* of the star. It is subject to a small annual *diminution*, which is called its annual *precession.* The polar distance of the star on the 1st. of the 1st. month (January) 1830, was 1° 35′ 51″; and its annual precession is 19.3″.

The polar distance may be found for any subsequent time by multiplying 19.3″, by the interval between the 1st. of the year 1830, and the given time, and subtract ing the product from 1° 35′ 51″. Thus, suppose the polar distance of the pole star was required for the 1st of the 7th month (July), 1845. The interval is 15.5 years, and 19.3″×15.5 gives 299.15″=4′ 59″. From 1° 35′ 51″ take 4′ 59″, and we have 1° 30′ 52″ for the polar distance of the star at the time proposed.*

When the polar distance of the pole star is known, its azimuth, at the time of greatest elongation, may be found by the following proportion:

As the cosine of the latitude of the place
Is to radius
So is the sine of the polar distance
To the azimuth.

This azimuth will be *east* or *west*, according as the elongation is *east* or *west*, and consequently its name will be known from the preceding tables.

As an example, let the azimuth of the pole star at Philadelphia, latitude 39° 57′, be required for the 1st of the 7th month (July), 1845. The polar distance found above is 1° 30′ 52″; and it may be taken 1° 31′ without material error.

As	cos. of lat. 39° 57′ - - - - Ar. Co.	0.115428
:	radius - - - - - - - - - - - -	10.000000
: :	sin. of pol. dist. 1° 31′ - - - -	8.422717
:	sin. of azimuth 1° 59′ E. - - -	8.538145

* The polar distance obtained as above is called the *mean* polar distance, and it is sufficiently accurate for our present purpose. To obtain the *true* polar distance two small corrections, called *aberration* and *nutation*, would have to be applied. The true polar distance for January 1st, 1880, was 1° 19′ 24″.

In order to observe the greatest elongation of the pole star, it will be necessary to prepare the following simple apparatus.

Place two posts firmly in the ground, about three feet apart, and nearly east and west from each other; the heights of the posts, which should be the same, may be about two or three feet; on those posts, place a thick board or plank, five or six inches wide, and nail it fast to each of them, taking care that it be level or nearly so; take a piece of board, a foot or eighteen inches long and four or five wide, and near the middle of it fasten a compass-sight perpendicularly; this board is to slide on the horizontal one already mentioned.

Take a stiff pole 18 or 20 feet in length, and fix it in an inclined position, in such a manner that a plumb line suspended from the upper end, may be nearly north, from the middle of the horizontal board, and about ten feet distant from it; the elevation of the pole must be such that the pole star, when viewed through the compass sight placed on the horizontal board, may appear a few inches below its upper end; when in this position the lower end should be fastened in the ground, and the pole should be supported by a couple of crotchets placed near the middle. The plumb should weigh a pound or more, and should swing in a vessel of water, in order to prevent the line being agitated by the motion of the air.

The apparatus being prepared, proceed, about 15 or 20 minutes previous to the time of greatest elongation as indicated by the table, to make the observation as follows: Let an assistant hold a lighted candle near the plumb line, so as to illuminate it and render it distinctly visible; place the small board with the compass-sight attached to it, on the horizontal one, and move it east or west as the case may require, till the pole star, plumb line, and aperture in the compass-sight are all in a direct range. If the star should be deviating to the east, it will leave the plumb-line to the west; and the contrary, if deviating to the west; keep therefore shifting the sight, till the star appears stationary behind the plumb-line; which it will do for several minutes at the time of its greatest elongation, and will then recede from the line on the contrary side from which it did before it became stationary. The compass-sight must not be moved after the star has attained its greatest elongation; but the aperture in it being then in a direct range with the plumb line and star, the board to which the sight is fixed, must be fastened to the one on which it slides, by a small tack passing through each end. This being done let an assistant take a straight stake, with a piece of lighted candle stuck on it, and go north to the distance of 30 or 40 perches; then looking through the compass-sight, direct him to set up the stake perpendicularly, and in such a situation that the candle stuck on the top may appear exactly behind the plumb-line; when thus placed, let it be firmly fixed in the ground. Next, let another straight stake be set up in the same manner near the plumb-line; the remaining part of the work may then be left till morning.

Measure accurately the distance between the two stakes. Then,

As radius,
Is to the tangent of the azimuth;
So is the distance between the stakes, in feet,
To a fourth term, in feet.

Lay off the distance contained in this fourth term from the northerly stake, and perpendicular to a line joining the two stakes; it must be laid off towards the *west* if the azimuth is *east*, but towards the *east* if the azimuth is *west*. Next remove the northerly stake, and set it up at the other extremity of the distance thus laid off; then a straight line joining the two stakes will be a true *meridian* line.

To obtain the variation, set up a compass in the place of the southerly stake, and direct the sights truly to the northerly one; the needle will then point out the variation, which will be *east* or *west*, according as the north end of the needle points to the *east* or *west* of the north point of the compass. The whole process is so simple, that an example is deemed unnecessary.

If the surveyor be provided with a theodolite or transit instrument, the true meridian may be found much more easily and accurately as follows: About half an hour before the time of greatest elongation level the instrument and point it to the star. Follow the star with the vertical spider line until it appears to have no farther side motion, but travels up or down the vertical line. Clamp the telescope firmly, and again examine to see if the star be on the line. Then depress the telescope, and direct an assistant who holds a pole with a light on it if necessary. Drive a stake under the instrument, another under the light. We may now calculate the deviation of this second stake from a meridian through the first as by the last method, or with the transit measure off to the right or left the known polar distance of the north star. The telescope will then point in the line of the true meridian.

If the spider lines cannot be seen, place a piece of white paper nearly in front of the telescope, so as to reflect the light from a lamp down the tube.

The position of the true meridian may also be determined quite accurately as follows: Hold a plumb-line a few feet in front of the telescope, and light up the spider lines as before; make the vertical spider line agree with the plumb-line. Polaris and Alioth (the first star in the handle of the dipper) come on the meridian about the same time; 17 minutes after the plumb-line passes through both stars Polaris is on the meridian. Point the telescope to it at this time, depress it and set a stake. The line from the instrument to the stake will be a true meridian.

To obtain the true bearings of a survey, from the magnetic ones, the variation being given.

This is a simple case of addition or subtraction, which the student can solve in any case.

To find the difference between the present variation, and that at a time when a tract of land was formerly surveyed, in order to trace or run out the original lines.

Go to any part of the premises, where any two adjacent corners are known; and if one can be seen from the other, take its bearing; which compared with that of the same line in the former survey, shows their difference. But if one corner cannot be seen from the other, run the line according to the given bearing, and measure the nearest distance between the line so run, and the corner; then,

As the length of the given line,
Is to the said distance;
So is 57.3 degrees.*
To the difference of variation required.

EXAMPLE.

Suppose it be required to run a line, which some years ago bore N. 45° E. dist. 20 ch. and in running this line by the given bearing, the corner is found 20 links to the left hand; what allowance must be made on each bearing to trace the old lines; and what is the present bearing, by the compass, of this particular line?

	L.		L.		Deg.
As	2000	:	20	: :	57.3
					20
					2000)1146.0(0° 34′

Consequently 34 minutes, or a little more than half a degree, is the allowance required; and the line in question bears N. 44° 26′ E.

Note. The above rule is simple and sufficiently accurate when the distance between the sought corner and

* 57.3 is the radius (nearly) of a circle in such parts as the circumference contains 360

random line, is small. But when this distance is considerable, it will be better to find the angle by trigonometry.

ON LOCAL ATTRACTION.

It is well known that iron or any ferruginous substance attracts the magnetic needle, and consequently when near, will draw it aside from the position in which it would otherwise settle. And as the earth in many places contains, near its surface, substances of this kind, the needle will not unfrequently be attracted from its true direction. The surveyor ought therefore, at each station, to take a back sight to the preceding one; and if he arrive at one at which the compass does not reverse truly, he may conclude, provided no error was committed in taking the bearing at the last station, that at the present one, the needle is affected by some local attraction. In such a case, he should first determine whether any error was committed at the last station, and if none is found, take the difference between the bearing from the last station and the reverse bearing, which will be the local variation of the needle at the present station. This variation must be applied, according to its name, to the bearing of the following station.

If at the first and second station of a survey the compass is found not to reverse truly, the surveyor will be at loss to know which of them is affected by attraction. But by taking another station, either within or without the survey, and taking its bearing from each of those stations, and the bearing of each of those from it, he may, in general, determine at which of them the attraction exists.

Note.—The area of the survey is not affected by the general variation, because it is the same at each station. But where local attraction exists and causes a variation in the position of the needle, as this variation will be different at different stations, it will, unless ascertained, and allowed on the corresponding bearings, materially affect the truth of the survey.

CHAPTER VII.

OF THE THEODOLITE.

A VERTICAL ANGLE is any angle in a plane perpendicular to the horizon. Consequently angles of elevation and depression are vertical angles.

A *Theodolite* is an instrument used to measure both horizontal and vertical angles.

By the aid of a theodolite, surveys may be much more accurately made than with the compass, especially when the tract is large or the ground hilly, or where there is local attraction. Before describing the theodolite, it will be best to notice separately one or two of its appendages.

Of the Spirit Level.

The *Spirit Level* consists principally of a cylindrical glass tube, nearly, but not entirely, filled with alcohol, or some other fluid. The inner part of that side of the tube which, when in use, is to be uppermost, is ground from end to end into a regular curve, having its convexity upwards. In consequence of this curvature in the upper part of the cavity, the *air bubble*, or part not occupied by the fluid, must be at the middle of the tube when the *axis of the level*, that is, the straight line passing lengthwise through its middle, is in a horizontal position. And conversely the axis must be horizontal when the instrument is so placed that the bubble stands at the middle of the tube.

Of the Vernier.

A *Vernier* is a graduated index which serves to subdivide the smallest divisions of a graduated arc or straight line.

Verniers are somewhat different according to the value of the divisions which they are to subdivide and the degree of precision they are designed to give; but the principle of construction is the same in all. The following description applies to the vernier attached to the common theodolite, the limb of which is divided into degrees and half degrees, numbered from 0° to 360°; and the vernier subdivides the half degrees to minutes.

In Fig. 115, AB represents a part of the graduated limb, and CD the vernier; the whole being drawn on an enlarged scale. The extent of the vernier is exactly equal to 29 half degrees, or 870 minutes, on the limb, and is divided into 30 equal parts, numbered from 0 to 30. Each division of the vernier contains therefore 29 minutes. Hence it is evident that when the 0, or *zero* line of the vernier exactly coincides, as in the figure, with a division line on the limb, the first division line of the vernier must be one minute behind the line on the limb first following that with which the zerc of the vernier coincides; the second line of the vernier must be two minutes behind the second on the limb; the third line on the vernier must be three minutes behind the third on the limb; and so on. If then, the vernier were to be moved forward till its first division line coincided with the first on the limb, the zero of the vernier would be one minute past the line on the limb with which it before coincided; if moved till the second lines coincided, the zero of the vernier would be two minutes past the same line; and in like manner for other coincidences. It therefore follows that for any position of the vernier, the number of the division line on it which coincides with one on the limb, must express the number of minutes that the zero of the vernier is past the division line on the limb, next preceding it.

The arc indicated by the vernier index in any position is the arc intercepted in the direction in which the degrees are numbered between the zero line of the limb and the zero line of the vernier. Hence to find the arc, or as it is technically expressed, to *read off* the arc, we must add to the arc expressed by the number of whole degrees intercepted and the odd half degree, if there is one, the number of minutes indicated by the *number* of that division line on the vernier which coincides, or is the nearest to coincidence, with one on the limb. Thus, as the zero of the vernier in Fig. 115 exactly coincides with the division line on the limb, numbered 50, the arc indicated is 50°. In Fig. 116, the zero of the vernier is past the line on the limb denoting 52° 30′, and the 21st division line of the vernier coincides with a line on the limb. The arc indicated is therefore 52° 51′.

Fig. 115.

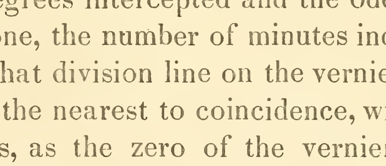
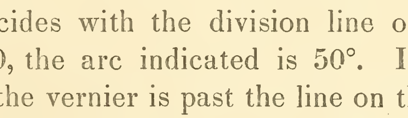


Fig. 116.



To determine the reading of a vernier, applied to a graduated arc in which the value of the least division is more or less than half a degree, we must divide this value by the number of divisions on the vernier, and the quotient will indicate the reading that the vernier is intended to give. In the nautical instrument called a sextant, the value of the least division on the limb is usually 10 minutes, and the vernier is divided into 60 equal parts. Hence 10 minutes, or 600 seconds, divided by 60, gives 10 seconds for the reading of the vernier.

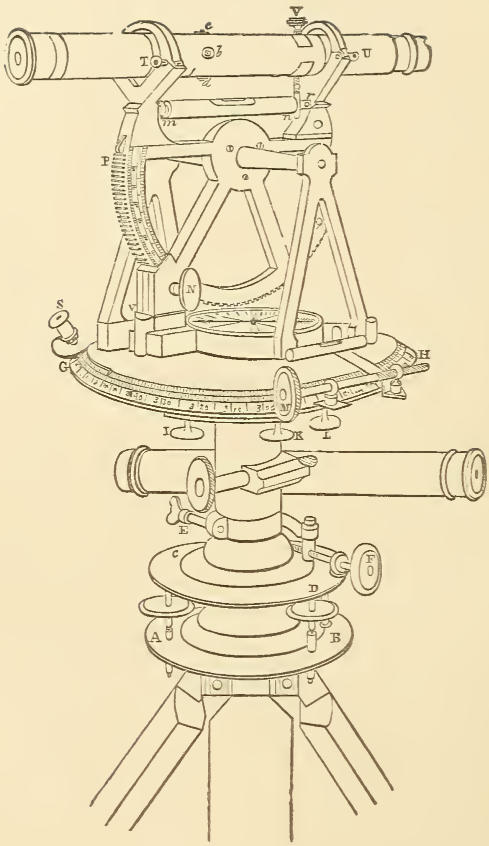
Description of the Theodolite.

The Theodolite is represented in Fig. 109. When in use, the lower circular plate AB is screwed fast to a three-legged stand or tripod, the upper part of which is represented in the figure. The circular plate CD has a hollow axis passing through it at right angles at its centre, and firmly attached to it. The lower part of the axis terminates in a ball, to which it is fastened by a screw. This ball is partly enclosed in a socket projecting from the upper side of the plate AB; the opening in the upper part of the socket being larger than the axis, so as to allow the latter some motion in every direction. In the lower plate AB four screws, called *levelling screws*, are inserted, standing opposite each other in pairs, the tops of which press against the under side of the plate CD. These screws are turned by milled projections or heads, two of which are shown in the figure. By turning them in opposite directions, the position of the plate CD may be changed so that it may be made level, even when the lower plate AB has considerable inclination.

The part GH of the instrument consists of two circular plates in close contact with each other. The lower one is called the *graduated plate*, and its chamfered edge, which is usually made of silver, is called the *limb* of the instrument. To the central part of this plate, a hollow axis is attached, the cavity of which fits to the outside of the axis of the plate CD, so that the former may move easily and steadily about the latter. The lower part of the axis is embraced by a clamping-piece that may be tightened or loosened by the screw E. When the clamp is loose, the graduated plate may be turned round by the hand; but when it is made fast, the axis of the graduated plate, and consequently the plate itself, becomes firmly connected with the plate CD, and also, when the levelling screws are tight, with the stand of the instrument. The graduated plate may however, when thus connected, be moved a small distance either way by turning the screw F, called a *tangent-screw*, which gives it a slow motion. Around the axis of the graduated plate is another hollow axis, with which a telescope, called the *lower* telescope, is connected. When the axis of the graduated plate is clamped fast, this axis and the telescope may be made to revolve by turning the milled head I, and may be secured in any position by the screw K, which clamps the axis to the graduated plate.

The plate immediately above the graduated plate is called the *vernier plate*. It has, at its centre, a solid axis which fits into the cavities of the hollow axes of both the graduated plate and plate CD. When the screw L, which serves to clamp the vernier plate to the graduated plate, is loosened, the former may be turned round by hand; and when this screw is made tight, a slow motion may be given to the vernier plate by the tangent-screw M. The vernier plate has two, or sometimes three, verniers at different parts of its

Fig 109.



edge, which are chamfered for the purpose. When there are only two, they are placed directly opposite to each other. When there are three, they are placed at equal distances around the edge. The microscope S is used to enable the eye more certainly to distinguish the line on the vernier that coincides with one on the limb. Two levels are placed on the vernier plate, at right angles to each other; one of which is shown in the figure.

The frame which supports the vertical semicircle and *upper* telescope with its attached level, is attached to the vernier plate by three screws at equal distances from one another. The immediate supports of this telescope are called *wyes*, or Y's, from their resemblance to the letter Y. The telescope is held in its place by two curved pieces, moveable on joints, which pass over it and are fastened by the pins T and U. When these pins are taken out, the pieces may be turned back and the telescope taken from its place and *reversed*, that is changed end for end.

In the tube of the telescope a flat ring is placed at right angles to its axis, and connected with the tube by four screws, opposite to each other in pairs; the heads of three of which, *b*, *c*, and *d*, are shown. Two spider's lines or very fine wires, at right angles to each other, are attached to the ring, in the directions respectively of each pair of screws, and intersecting each other in the centre of the ring. At each end of the principal tube, and fitting it on the inside, a short moveable tube is inserted. The one at the end to which the eye is applied, and which usually contains several glass lenses, may be moved out or in, by a small pin attached to its lower side, and may thus be so adjusted as to render the spider's lines distinctly visible. The tube at the other end, which contains one lens, may be moved out or in, by turning the milled head V, and may be thus so placed as to render an object to which the telescope is directed, as distinct as its distance permits.

A straight line in the direction in which the intersection of the spider's lines or wires is seen by an eye placed at the eye end of the telescope, is the *line of sight* of the telescope, and is technically called the *line of collimation*. The telescope is said to be *directed* to any point in an object, when it is so placed that the line of collimation passes through the point; or which is the same, when the point is directly behind the intersection of the spider's lines.

The semicircle PQ is called the *vertical limb* of the instrument. That face of it which is not seen in the figure is divided into degrees and half degrees, numbered each way from a line taken as a zero or 0 line; and the arc is read by a vernier numbered as in Fig. 117. The other face has two sets of unequal divisions on it, numbered each way from a zero line. One of these denotes the difference between the real distance of an object to which the upper telescope is directed and its horizontal distance, expressed in 100th parts of the distance. The other denotes the vertical distance of the object, above or below a horizontal line, passing through the instrument, expressed in 100th parts of the horizontal distance. The vertical limb is moved so as to give to the telescope different inclinations to the horizon, by means of the milled head N.

At the bottom of the frame which supports the vertical limb and upper telescope, and directly over the centre of the vernier plate, there is a compass box, containing a magnetic needle. This part of the instrument may be used to take the bearing of an object in the same manner as a common compass.

ADJUSTMENTS OF THE THEODOLITE.

In order that the theodolite may be in a good state for use, the line of collimation of the upper telescope should exactly coincide with the axis of the telescope: the axis of the attached level should be parallel to this line; the axes of the levels on the vernier plate should be parallel to this plate; the line of collimation of the telescope should, when the milled head N is turned. move in a plane perpendicular to the vernier plate; and when this line is brought parallel to the vernier plate, the zero of the vertical limb should coincide with the zero of its vernier. Previous therefore to using the instrument, the different parts should be examined, and *adjusted* if necessary. This may be done by the following methods.

FIRST ADJUSTMENT.

To make the line of collimation coincide with the axis of the telescope.

The instrument being firmly screwed on the tripod and the legs of the latter being sufficiently extended to ensure its remaining fixed in its position, loosen the clamp screw L, and turn the vernier plate till the telescope points to some distant object having on it a small well-defined point. Then, having fastened the screw L, move the telescope by the tangent-screw M, and milled head N, till the line of collimation is directed exactly to the point.

Revolve now the telescope in its Y's, half round, that is, till the level, from being directly below the telescope is directly above it. If the horizontal spider's line still coincides with the point, it requires no adjustment; but if it does not, diminish the distance one-half, by loosening one of the screws *c* and *d* and tightening the other; and then bring the line to coincide with the point by means of the milled head N. Revolve the telescope round to its first position, and if the horizontal line and point do not then coincide, repeat the operation till the coincidence has place in both positions. In a similar manner the vertical line may be adjusted. When both adjustments are complete, the line of collimation should coincide with the same point during a complete revolution of the telescope in its Y's.

SECOND ADJUSTMENT.

To make the axis of the level attached to the upper telescope parallel to the line of collimation.

Turn the vernier plate till the telescope comes directly over two of the levelling screws; and if the telescope is not nearly in a horizontal position, make it so by turning the milled head N. Then turn the levelling screws over which the telescope stands, in opposite directions, till the bubble of the level stands exactly at the middle of the tube; observing to keep the screws firm against the plate CD. When this is done, reverse the telescope in its Y's, and if the bubble does not stand at the middle of the tube, correct *half* of the deviation by one of the screws *m* and *n*, which serve to raise or depress the ends of the level, and then, by the levelling screws, bring the bubble to the middle. Again reverse the telescope and repeat the correction if necessary.

Revolve now the telescope in its Y's so as to bring the level a considerable distance from its proper or lowest position, and if the bubble deviates from the middle of the tube, make the requisite correction by means of two screws, *p* and *q*, which move the end of the level laterally, one of which is shown at *p*. When this part of the adjustment has been so made that the bubble will remain in the middle of the tube while the telescope is revolved either way, the first part of the adjustment should, by again reversing the telescope, be examined, and repeated if necessary.

THIRD ADJUSTMENT.

To make the axes of the levels on the vernier plate parallel to that plate.

Turn the vernier plate till the upper telescope stands over two of the levelling screws; then one of the levels on this plate will be parallel to these two screws, and the other will be parallel to the other two. By means of these levelling screws bring the bubbles of both levels to stand in the middles of their respective tubes. Then move the vernier plate 180°, and if the bubble in either of the levels deviates from the middle, correct one-half of the deviation by one of the screws at its ends, and the other half by the levelling screws, parallel to it; and if the bubble of the other level also deviates from the middle, proceed in the same way to correct the deviation. Repeat the operations till the bubbles of both levels will remain at the middles of their respective tubes during a complete revolution of the vernier plate.

FOURTH ADJUSTMENT

To make the axis about which the vertical limb revolves, parallel to the vernier plate.

Turn the vernier plate till the upper telescope is directed towards a well-defined, elevated point, on a house or other object, not very remote, and having clamped the plate, direct the telescope by means of the tangent-screw M and milled head N, exactly to the point. Then turn the milled head N till the line of collimation coincides with some well-defined point near the ground; or, if none such is found, let an assistant make a suitable mark in the direction of the line of collimation when thus brought nearly horizontal. This being done, reverse the telescope in its Y's; and, proceeding as before, direct it to the elevated point. Then, if by turning the milled head N, the line of collimation is brought to coincide with the lower point or mark, the axis of the vertical limb is parallel to the vernier plate; but if this is not the case, the adjustment must be made by the screws which attach the upper frame to this plate.

FIFTH ADJUSTMENT.

To adjust the vernier of the vertical limb, or determine the correction which should be made, to allow for its deviation from correct adjustment.

Direct the upper telescope to some elevated point and note the angle of elevation indicated by the vernier. Reverse the telescope in its Y's, and again direct it to the same point, and note the angle of elevation If the angles are the same, the vernier is properly adjusted. If they differ, the position of the vernier requires adjustment; and this may be effected by the screw *v*, which is one of those that fasten the upper frame to the vernier plate.

Instead of changing the position of the vernier, we may take half the difference of the two angles of elevation as a *correction*, to be added to all those angles of elevation and subtracted from those of depression, that are taken with the telescope in that position in its Y's, which gave the least of the two angles; but to be applied in a reverse manner when the position of the telescope is that which gave the greater angle.

SIXTH ADJUSTMENT.

To make the axis about which the lower telescope revolves vertically, perpendicular to the axis of the instrument.

Turn the vernier plate till the levels on it are respectively parallel to a pair of opposite levelling screws, and by means of these screws bring the bubbles to stand in the middles of the tubes; the instrument is then levelled, and its axis is perpendicular to the horizon. Suspend a plumb-line of considerable length at a short distance from the instrument, and loosening the clamping screw K, turn the milled head I, till the telescope is directed to the line, and then fasten the clamping-screw. Now, applying the hand to the eye-end of the telescope, move it vertically, and observe whether it continues directed to the line, throughout its whole extent. If it does, the axis is properly adjusted; if it does not, the adjustment may be made by two small screws which move the remote end of the axis vertically.

The lower telescope being used only as a guard to ascertain whether or not the horizontal limb of the instrument remains fixed while the vernier plate is made to revolve, in order to direct the upper telescope to an object, requires no other adjustment than the above, except those for distinct vision of the spider's lines and object.*

In extensive practical operations, the above adjustments should be examined, and corrected if necessary, not only before the commencement, but every day or two during their continuance.

Of the arc of the horizontal limb, corresponding to a given position of the vernier plate.

When there is but one vernier, the arc indicated by it is regarded as the corresponding arc. When there are two or three, it is usual to distinguish them by calling one A, another B, and the third, if there is a third, C, and to consider the arc indicated by the ver-

* When the foregoing adjustments have been completed for the first time, there is another examination which it is well to make. Direct the upper telescope to some well-defined point near the horizon, and note the arcs on the horizontal limb, indicated by the different verniers. Then having directed the lower telescope to the same point, fasten its clamping-screw K; examining first, however, to see that the upper telescope has not changed its position, or if it has, bringing it back by the tangent-screw F. Reverse now the upper telescope in its Y's, and again direct it to the point; examining at the same time to see that the lower telescope has not changed its position, or, if it has, bringing it back by the tangent-screw F. Note the arcs indicated by each of the verniers; and subtract the arc indicated by each vernier in the former position from that in the latter, increasing that of the latter by 360°, if necessary. Add together the remainders, and divide the sum by the number of verniers. If the quotient is 180°, we infer that the axis about which the upper telescope revolves is, as it ought to be, at right angles to the line of collimation of the telescope. If the quotient differs materially from 180°, the instrument is imperfect, except one of the Y's is laterally adjustable. Where this is not the case, the imperfection can only be well remedied by an instrument-maker. If the remainders mentioned above differ much from each other, we infer that the instrument has not been well centered, or not well divided.

nier A, as expressing the position of the vernier plate, at least very nearly ; the readings of the other verniers being used merely as a test, or means of correcting the former. We therefore note the arc indicated by the vernier A. Then note the minutes of the arc indicated by the vernier B; but instead of the number of degrees indicated by it, we take either the number indicated by the vernier A, or this number increased or diminished by a unit, so that the arc set down for the vernier B, may be very nearly the same as that for the vernier A. If there is a third vernier C, we proceed in the same manner. Then the sum of the arcs obtained for each vernier, divided by the number of verniers,* will give the value of the arc corresponding to the given position of the vernier plate, or, as it is sometimes expressed, the *arc indicated by the verniers.* Thus, if we suppose the instrument to have three verniers, and that for a given position of the plate, the reading of the vernier A is 142° 2′, of B 261° 58′, and of C, 22° 3′ ; then, instead of these quantities for B and C, we write for B, 141° 58′, and for C, 142° 3′. Adding together 142° 2′, 141° 58′, and 142° 3′, and dividing the sum by 3, we obtain 142° 1′, for the arc indicated by the verniers.

PROBLEM I.

To measure with the Theodolite the horizontal angula distance of two objects, as seen from a given station.

By means of a plumb-line suspended from the centre of the plate which forms the top of the tripod, set the

* When, as is commonly the case, the number of the degrees of the arc for each vernier is the same, we need only divide the sum of the minutes of the arcs, by the number of verniers, and annex the quotient to that num ber of degrees.

centre of the instrument directly over the station-mark, and then level it, as directed in the sixth adjustment; make the zero of the limb agree with the zero of the vernier; direct the upper telescope to the object which stands to the *left*, when the face is turned towards the angle to be measured. Then, having directed the lower telescope to the same object, and fastened its clamping-screw K, direct the upper telescope to the other object, and note the readings of the verniers; observing first, however, whether there has been any change in the position of the lower telescope; and if there has, bringing it back to the object at the left by means of the tangent-screw F, and again by the tangent-screw M, adjusting the direction of the upper telescope to the object at the right. The verniers will read the angle required.

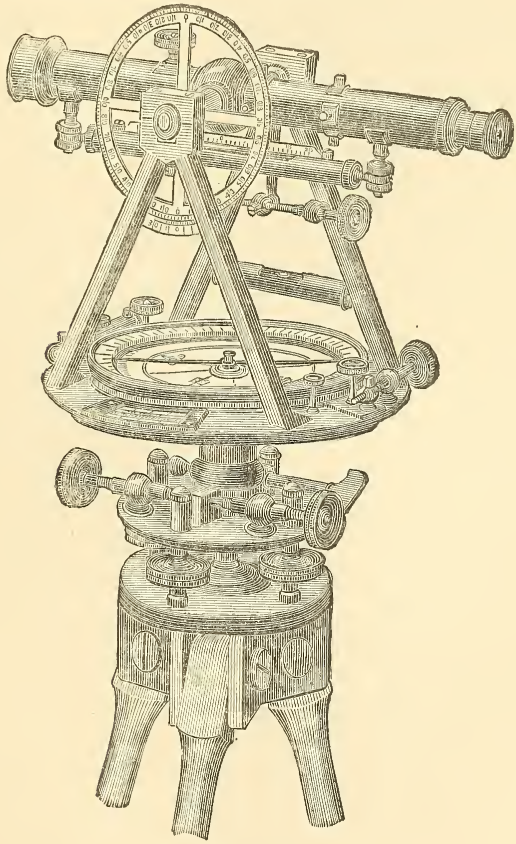
PROBLEM II.

To measure a vertical angle with the Theodolite.

The instrument being placed and levelled, direct the upper telescope to the point, of which the angle of elevation or depression is to be taken. Then, if the vernier of the vertical limb is accurately adjusted, the angle indicated by it will be the angle required. When this is not known to be the case, after having noted the arc indicated by the vernier, move the vertical limb till the telescope is horizontal, as indicated by the bubble of its level standing in the middle of the tube, and note the arc then indicated by the vernier. If in both positions the zero of the limb is on the same side as the zero of the vernier, subtract one arc from the other; but if on different sides, add the two arcs together. The result will be the required angle.

OF THE TRANSIT.

The greater part of the preceding descriptions and adjustments will apply as well to the surveyor's transit as to the theodolite. The main difference between the



two is that the telescope of the transit will turn through 360° in a vertical plane, thereby enabling the surveyor much more easily to reverse. If provided with a graduated vertical circle, it will perform all the problems of the theodolite.

As it can be readily directed both forwards and backwards in the same line, while the telescope of the theodolite must be taken from its supports and reversed, the transit is the much more convenient instrument for surveyors.

The accompanying figure represents one of the most convenient and modern forms of transit instrument.

CHAPTER VIII.

LEVELLING.

The surface of an expanse of tranquil water or any similar surface concentric with it, is called a *Level Surface.* Any points situated in a level surface are said to be on the *same level;* and any line traced in such a surface is called a *line of level.*

If through any place, a level surface be conceived to pass, the distance which another place is from this surface, either above or below, measured on a line perpendicular to it, is called the *difference of level* of the two places.

Levelling is the art of determining the difference or differences of level of two or more places.

In consequence of the globular figure of the earth, a level surface is not, as it appears to be, a plane surface. It is nearly, though not exactly, spherical. In the operations of levelling we may, without sensible error, regard a level surface at any place, as being strictly a spherical surface, with a radius equal to 3956 miles, the mean radius of the earth, or, which is more exact, with a radius equal to 3968 miles;* the centre being

* The earth, if we disregard the inequalities in its surface, is an oblate spheroid. Its polar diameter is 7899 miles, and its equatorial diameter 7925 miles. A level surface is therefore a spheroidal surface. The radius of the spherical surface which most nearly coincides with any small portion of this spheroidal surface, changes slightly with the latitude of the place. For any place in the United States, the greatest error which can occur from

in a straight line conceived to be drawn downwards from the place, perpendicular to the level surface. And for places not very remote from each other, we may regard the spherical surfaces of level passing through them, as having a common centre.

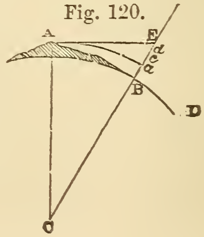
Let A and B, Fig. 120, be two places, not very emote from each other, and C, the common centre of the spherical surfaces of level passing through them. With the centre C and radius CA, describe the arc A*a*. Then will A*a* be a line of level of the place A, and B*a* will be the difference of level of the two places A and B.

The line A*d*, drawn perpendicular to CA, is called a line of *apparent level* of the place A. It is the line of level that would be indicated by an accurately adjusted levelling instrument placed at A. The distance *ad* is called the *correction of the apparent level.* This correction must be *subtracted* from the height B*d* of the apparent level to obtain the height B*a*, of the true level. The correction varies as the square of the distance from the place.* The following table contains the

considering the level surface passing through it, as a spherical surface with a radius equal to the mean radius of the earth, is about $\frac{1}{5}$ of an inch for a distance of two miles. For a spherical surface with a radius 5 or 6 miles greater than the equatorial radius of the earth, the greatest error is about $\frac{1}{10}$ of an inch, at the same distance. It may be further observed, that the greatest error or greatest deviation of the spherical from the spheroidal surface, varies as the square of the distance from the place.

* We have (V. 27), $(2Ca+ad).ad=Ad^2$. But as *ad*, for any distance to which a single sight in levelling is ever extended, is extremely small in comparison with $2Ca$, we may, without sensible error, take $2Ca$ instead of $(2Ca+ad)$. We shall thus have $2Ca.ad=Ad^2$; or $ad=\frac{Ad^2}{2Ca}$. Consequently, as Ca is constant, *ad* varies as the square of the distance A*d* varies.

Fig. 120.



value of the correction in decimal parts of a foot, for each chain of distance, from 1 to 120.

TABLE,

Giving the differences between the true and apparent level, for distances from 1 to 120 chains.

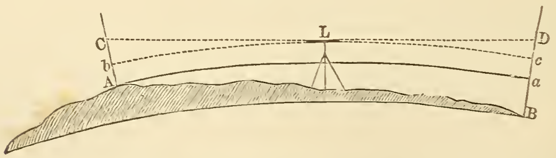
Chains.	Feet.	Chains.	Feet.	Chains.	Feet.	Chains.	Feet.	Chains.	Feet.
1	.000	25	.065	49	.250	73	.554	97	.978
2	.000	26	.070	50	.260	74	.569	98	.998
3	.001	27	.076	51	.270	75	.585	99	1.019
4	.002	28	.082	52	.281	76	.600	100	1.040
5	.003	29	.087	53	.292	77	.616	101	1.060
6	.004	30	.094	54	.303	78	.632	102	1.082
7	.005	31	.100	55	.314	79	.649	103	1.103
8	.007	32	.106	56	.326	80	.665	104	1.124
9	.008	33	.113	57	.338	81	.682	105	1.146
10	.010	34	.120	58	.350	82	.699	106	1.168
11	.013	35	.127	59	.362	83	.716	107	1.190
12	.015	36	.135	60	.374	84	.734	108	1.213
13	.018	37	.142	61	.387	85	.751	109	1.235
14	.020	38	.150	62	.400	86	.769	110	1.258
15	.023	39	.158	63	.413	87	.787	111	1.281
16	.027	40	.166	64	.426	88	.805	112	1.304
17	.030	41	.175	65	.439	89	.823	113	1.327
18	.034	42	.183	66	.453	90	.842	114	1.351
19	.038	43	.192	67	.467	91	.861	115	1.375
20	.042	44	.201	68	.481	92	.880	116	1.399
21	.046	45	.211	69	.495	93	.899	117	1.423
22	.050	46	.220	70	.509	94	.919	118	1.447
23	.055	47	.230	71	.524	95	.938	119	1.472
24	.060	48	.240	72	.539	96	.958	120	1.497

When a ray of light passes obliquely through portions of air of different densities, it becomes bent from a straight line, and enters the eye so as to make the point from which it proceeds appear in a direction slightly different from its true direction. This effect is called *Refraction;* and when the point, or body from which the light proceeds, is on or near the earth, it is called *Terrestrial Refraction.* Terrestrial Refraction generally makes the point appear to be more elevated than it really is. Thus it is not actually the point *d* of the line CE, Fig. 120, that, to an eye at A, appears to be at *d*, but another point *c*, a little below *d*. If therefore we wish to notice the effect of refraction, we must take *ac* instead of *ad*, for the correction of apparent level. In temperate climates, *cd* is, in the usual state of the atmosphere, about $\frac{1}{6}$ of *ad*. Consequently we may obtain the correction of apparent level with allowance for refraction, by diminishing the correction given in the preceding table by a $\frac{1}{6}$ part.

As the effect of refraction in the common operations of levelling is always small, and is subject to considerable variations depending on the state of the air between the object and place of observation, it is commonly disregarded. It would, however, in general be better to allow for it as above, except when the object is but little distant from the place of observation, or when, by the method noticed in the next article, the desired result is obtained independently of the correction of apparent level.

Let CD, Fig. 121, be the line of apparent level indi-

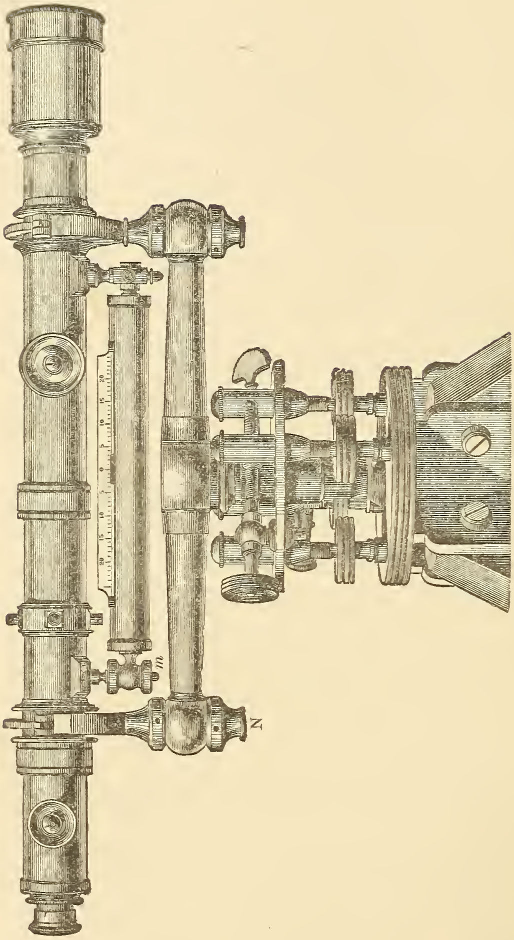
Fig. 121.



cated by a levelling instrument L, placed midway between the places A and B; also let the arc *bc* be the line of true level passing through the instrument, the arc Aa, concentric with the former, the line of true level of the place A, and AC and BD, lines perpendicular to Aa. Then in consequence of the equality of the distances LC and LD, we have bC equal to cD; and therefore as Ab is equal to ac, we have AC equal to aD. Consequently Ba, the difference of level of the places A and B, is obtained by subtracting AC, the height of the apparent level of the instrument, at A, from BD, its height at B. We thus obtain the difference of level of the two places, independently of the correction of apparent level. It is also independent of refraction, as the effect of refraction would be sensibly the same for the two points C and D, and in the same direction. It may be observed that it is not necessary the instrument should be directly between the places. It may be placed in any convenient position on either side of the line joining them, provided its distances from the places are equal, or very nearly so.

OF THE LEVELLING INSTRUMENT.

The *Levelling Instrument*, or *Level* as it is frequently called, is an instrument used to denote the line of apparent level. The instrument is represented on page 218. The tripod and levelling screws are similar to those of the transit. The spirit-level has a screw at one end, which gives it motion vertically, and one at the other end, which gives it motion horizontally. The object glass is moved in and out by a screw, and another screw gives a similar motion to the eye-piece. Thus the focus is adjusted for all distances, and the spider lines



are kept in distinct vision. Clamp and tangent-screws give horizontal motion to the telescope, but there is no opportunity for similar motion in a vertical plane.

FIRST ADJUSTMENT.

To make the line of collimation coincide with the axis of the telescope.

The method of doing this is similar to that employed in the first adjustment of the theodolite.

SECOND ADJUSTMENT.

To make the axis of the level parallel to the line of collimation.

By turning the screw N, bring the bubble of the level to stand at the middle of the tube. Reverse the telescope in its wyes, and, if the bubble does not then stand in the middle, correct one half of the deviation by the screw m, and the other half by the screw N. Again reverse the telescope in its wyes, and repeat the correction if necessary.

Now by revolving the telescope in its wyes, bring the level to some distance on one side of its lowest or proper position, and if the bubble then deviates from the middle, the deviation must be corrected by means of the screws, which move one end of the level laterally, the correction being continued till the bubble will remain at the middle while the telescope is revolved so as to bring the level to a considerable distance on either side of its lowest position. When this has been done, the first part of the adjustment should again be examined and corrected if necessary.*

* A method by which the second adjustment may be made for an instrument in which the telescope is not reversible, will be found in the first of the following problems.

THIRD ADJUSTMENT.

To make the line of collimation at right angles to the axis.

Turn the telescope till it stands directly over two of the levelling-screws, and by means of them bring the bubble to stand at the middle of the tube. Then turn the telescope half round, that is, till it stands over the same screws, but pointing in the opposite direction, and if the bubble does not remain in the middle, correct one half of the deviation by the levelling-screws and the other half by the screw N. Now place the telescope over the other levelling-screws, and proceed in a similar manner. Continue the corrections till the bubble will remain in the middle of the tube during an entire revolution.

These adjustments having been carefully made, the instrument is ready for use. When on the ground it must, in each new position in which it is placed, be *levelled*. This is done by placing the telescope over two of the levelling-screws and by their means bringing the bubble of the level to the middle of the tube, then doing the same with the telescope over the other two, and again over the first two. Then, if the third adjustment has been accurately made, the bubble will stand in the middle of the tube in any position of the telescope.

OF THE LEVELLING STAFF.

A *Levelling Staff* consists of a square or rectangular staff and a small circular or rectangular board, called a *Vane*, which is so attached to the staff as to be moveable along it from end to end. It is used for measuring the height of the line of apparent level passing through the telescope of the levelling instrument, above the place where the staff is placed.

The face of the vane, represented in Fig. 121, is divided into four equal parts by two straight lines intersecting each other at right angles; one line being horizontal, and consequently the other vertical. Two opposite parts of the face are painted white and the other two red; thus the lines and their intersection are easily distinguished even at a considerable distance. A screw, the head of which is shown at *m*, serves to clamp the vane to the staff in any required position.

The staff is composed of two rectangular bars of wood, between six and seven feet long, placed side by side, and forming together a square staff, the breadth of each side of which is about an inch and a quarter. The bars are so connected that one, which is two or three inches the shorter of the two, may be made to slide along the other or principal bar, and thus, when necessary, increase the length of the staff. In order to this, the front or sliding bar has throughout its length, on the side next the other, a projection which is terminated by a brass plate a little wider than the projection and firmly attached to it; and the principal bar has a groove in it to receive the projection and plate of the former. The figure on the next page shows the upper portion of the levelling rod.

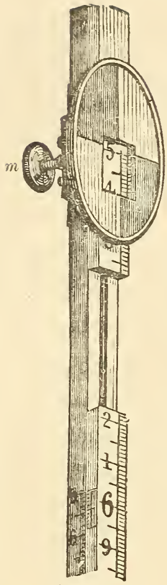


Fig. 121.

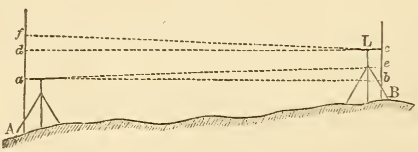
The graduation begins at the bottom, and is to feet and hundredths of a foot. The height to the horizontal mark on the upper vane is the required distance. This is obtained from the divisions on the rod, and a vernier that slides with the vane. To read heights greater than $6\frac{1}{2}$ feet the vane is placed at its highest point, and the bar carrying it is made to slide upwards on the other. A clamp-screw fastens the vane in any required position. Two vanes sometimes slide on the same rod, so that difference of level may be determined.

PROBLEM I.

To test the adjustment of the level.

Select a place where the ground is tolerably level for a distance of 15 or 20 chains, and at each extremity of the distance chosen, as at A and B, Fig. 122, drive

Fig. 122.



a short stake. Set up the level by the stake at A, placing it so that the eye-end of the telescope may be over or nearly over the stake, and level the instrument. Place the levelling-staff on the stake, raising or lowering the vane till its horizontal line is at exactly the same height as the centre of the eye-end of the telescope, and note the height. Now let an assistant take the staff and set it up vertically on the stake at B. Direct the telescope to the middle of the breadth of the staff, and then, by raising or lowering the hand as a signal, direct the staff-bearer to raise or lower the vane, repeating the signal till its centre appears precisely in the direction of the line of collimation, or at least exactly of the same height. When this is the case, by a circular motion of the hand, direct the vane to be clamped, and again sight to it to ascertain that in clamping, its height has not been changed. Note the height of the vane, and subtract from it the correction of apparent level corresponding to the distance between the stakes, taken from the preceding table, or rather this correction diminished by a $\frac{1}{6}$ part, to allow for refraction. Then, if the instrument is accurately adjusted and the observations have been carefully made, the difference between the height of the vane at A, and its corrected height at B, will be the true difference of level of the tops of the two stakes; the higher being that at which the height of the vane is the less. Placing now the instrument at B, proceed in the same manner to find again the difference of level of the stakes. If this difference is the same as the former, the adjustment of the instrument is correct. But if there is any material difference in the two results, we infer that the axis of the level is not parallel to the line of collimation. To make it so, take half the difference of the results obtained, and let the vane, taken in its last position, be *elevated* or *depressed* by that quantity, according as the result obtained at the more elevated of the two stakes, is *less* or *greater* than that obtained at the other.* Then, by the screw N, bring the line of colli-

* Let *ab* be the line of apparent level through the instrument when placed at A, and *ae*, the line of apparent level indicated by the instrument. Then *cd* being the line of apparent level through the instrument when placed at B, the line *cf* making the angle fLd equal to bLe, and consequently *df* equal to *be*, will be the line of apparent level indicated by the instrument

mation to point exactly to the horizontal line of the vane, and with the telescope in that position, bring the bubble again to the middle of the tube by the screw m

Note. In this manner the axis of the level may be made parallel to the line of collimation when the telescope is not reversible, as is the case in some instruments.

EXAMPLE.

Let the height Aa, be 5.295ft.;* Be, 2.063ft.; Bc, 5.527ft.; and Af, 8.935ft.; and the horizontal distance from A to B, 20 chains.

The distance being 20 chains, the correction for apparent level, taken from the table, is 0.042ft. From this deducting $\frac{1}{6}$ of 0.042, we have 0.035ft. for the correction to be subtracted from the heights Be and Af. This gives, for the corrected heights, Be = 2.028ft., and Af = 8.900ft. Hence, taking 2.028 from 5.295, we have 3.227ft. for the true difference of level obtained with the instrument at A; the place B being higher than A. And taking 5.527 from 8.900, we have 3.373ft. for the difference of level obtained with the instrument at B. Half the difference of these results gives 0.053ft. for the value of df, the error due to the error in the

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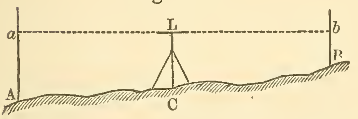
It is therefore evident that the difference of level of the two places obtained with the instrument at A, is less than the true difference by the quantity be, and the difference obtained with the instrument at B, is greater than the true difference, by the equal quantity df. Hence the difference of the two results must be twice df, the error in height produced by the error in the adjustment of the instrument.

* Although the station staff is only divided to hundredths of a foot, we may with tolerable precision estimate the thousandths; and when great accuracy is desired, it is better to do so.

adjustment of the instrument. Hence, as the difference of level given by the instrument at the more elevated place is greater than the other, the vane must be lowered 0.053 ft. from its last height. The adjustment may then be made as directed.

PROBLEM II.

Fig. 123.



To determine the difference of level of two places, A *and* B, Fig. 123, *when they are visible from each other and do not differ in level more than* 8 *or* 10 *feet.*

Place the level in some position C, about equally distant from the places, either in the line joining them or on either side as may be most convenient, and level the instrument. Let the staff-bearer set up the levelling-staff at A, and having sighted to it and obtained the height of the vane at that station, let the staff be removed to B, and do the same. Then the difference of the two heights, without any corrections, will be the difference of level of the places; that place being the higher, at which the height of the vane is the less.

If the surface of the ground between the places is such that when the level is placed at equal or nearly equal distances from them, the line of apparent level of the instrument would pass below one place or too high above the other, as in Fig. 124, it may be placed in any position C, that will permit the sights to be taken to both places. Then, having measured the horizontal distances from A to C, and from C to B, we proceed as above, except that the observed heights A*a* and B*b* must be corrected by the differences between the true and apparent level, as taken from the table:

or, which is generally better, by the tabular quantities diminished by a $\frac{1}{6}$ part. It may, however, be observed that when neither of the distances AC nor BC, exceeds five or six chains, the corrections are so small that they may be generally omitted.

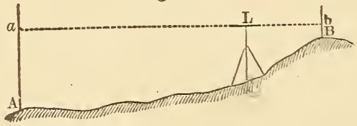
When a valley intervenes between the two places so that there is no suitable intermediate situation for the instrument, it may be placed at one of them; and then the difference of level may be determined as in the preceding problem.

EXAMPLES.

1. Let the observed height A*a*, Fig. 123, be 7.343 ft. and B*b*, 3.635 ft.; then the difference of these, 3.708 ft. is the difference of level of A and B, the place B being higher than A.

2. Let the observed height A*a*, Fig. 124, be 8.457 ft.;

Fig. 124.



B*b*, 1.525 ft.; the distance from A to C, 24.1 ch.; and the distance from C to B, 8.2 ch.

The correction for the distance 24.1 ch., taken from the table and diminished by a $\frac{1}{6}$ part, is 0.050 ft., and for the distance 8.2 ch., it is 0.006 ft. The corrected heights are therefore A*a* 8.407 ft., and B*b* 1.519 ft. Hence the difference of level is 6.888 ft.

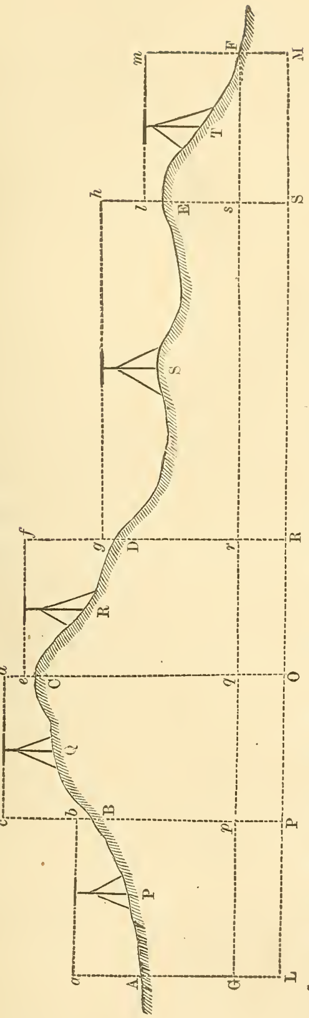


Fig. 125

PROBLEM III.

To find the difference of level of two places not visible from each other, or if visible, differing considerably in level.

Let A and F, Fig. 125, be the two places. Place the level in some position P, that will permit a sight to be taken to A, and also to some other place towards F, at about the same distance from the instrument. Having levelled the instrument, let the staff-bearer set up the staff at A, and when the sight has been taken, let him note the height of the vane as a first *back-sight.* Then let the staff be taken to some station B, about as far from the instrument as that is from the station A, and when the sight has been taken, let the height of the vane be noted as the first *fore-sight.* Next, the staff-bearer remaining at B, take the level to some suitable place Q, beyond B, and proceed to take a back-sight to B, and then a fore-sight to a new station C. In the same manner the operation must be continued from C to D, from D to E, and from E to F; the number of intermediate stations necessary to be taken, depending on the irregularities in the ground and on the difference of level of the given places.

Take the difference between the sum of the back-sights and the sum of the fore-sights, and it will be the difference of level of E and F; the place F being *higher* or *lower* than A, according as the sum of the back-sights is *greater* or *less* than that of the fore-sights.*

Note.—It is not necessary that either the intermediate stations or the places of the instrument should be in the direct line between the given places; and frequently it will be found convenient to deviate considerably from that line. It may further be observed that although it is generally best to take each pair of the sights at equal or nearly equal distances, as the correction for apparent level is thus avoided, and also a slight deviation of the axis of the level from parallelism with the line of collimation will not then sensibly affect the accuracy of the result, yet sometimes in order to diminish the number of stations the sights are taken at unequal distances. When this is done, the distances must be measured and the sights be corrected as directed in such case, in the last problem.

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* Let GF be a line of level through the place F; then AG is the difference of level of the two places A and F. Now we have,

Sum of back-sights,

$$=\mathrm{A}a+\mathrm{Bc}+\mathrm{C}e+\mathrm{D}g+\mathrm{E}l=\mathrm{A}a+bc+\mathrm{B}b+\mathrm{C}e+\mathrm{D}g+\mathrm{E}l \quad \text{Also,}$$

Sum of fore-sights,

$$=\mathrm{B}b+\mathrm{C}d+\mathrm{D}f+\mathrm{E}h+\mathrm{F}m=\mathrm{B}b+\mathrm{C}e+ed+\mathrm{D}g+gf+\mathrm{E}l+lh+\mathrm{F}$$
$$=\mathrm{F}m+lh+gf+ed+\mathrm{B}b+\mathrm{C}e+\mathrm{D}g+\mathrm{E}l=qd+\mathrm{B}b+\mathrm{C}e+\mathrm{D}g+\mathrm{E}l$$
$$=pc+\mathrm{B}b+\mathrm{C}e+\mathrm{D}g+\mathrm{E}l=\mathrm{AG}+\mathrm{A}a+bc+\mathrm{B}b+\mathrm{C}e+\mathrm{D}g+\mathrm{E}l.$$

The difference of these sums is AG, the difference of level of the places A and F.

EXAMPLE.

Let the back-sights and fore-sights, Fig. 125, be as in the following table.

	Back-sights.		Fore-sights.
1.	7.103 ft.		1.566 ft.
2.	9.227		3.178
3.	1.236		9.415
4.	1.610		6.367
5.	2.125		9.910
Sum	21.301	Sum	30.436
			21.301
		Diff.	9.135

Hence the difference of level of A and F is 9.135 ft., and as the sum of the back-sights is less than that of the fore-sights, the place F is lower than A.

CHAPTER IX.

TOPOGRAPHY.

Topography is a branch of surveying,* the object of which is to determine and designate on a map, the various undulations and inequalities in the surface of a particular place, tract of land or district of country. A map in which these inequalities, the courses of streams, and sometimes other circumstances, as the positions and extents of forests, marshes, &c., are designated, is called a *topographical map.*

In addition to the boundaries and content of a tract of land, it is frequently required that the various slopes and irregularities of the surface should be determined and designated, in order to give a more complete view of the ground, and to afford the means for an appropriate location of buildings or works of any kind that may be designed to be erected on it.

If we assume the surface of a tract of land to be intersected by a number of level surfaces or horizontal planes† at equal distances from one another, and transfer all the lines of level in which these planes meet the surface of the ground to an assumed horizontal

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* The term surveying is here used in a more extended sense than as defined in the first chapter.

† In tracts of any moderate extent, the surfaces of level may, for the purpose for which they are here introduced, be regarded as horizontal planes.

plane passing through the lowest point, making them occupy positions on that plane, corresponding with their positions on their respective planes, the variations in the distances of the lines from one another, when thus transferred, will indicate the variations in the inclination of the ground. For as the difference of level from line to line is the same, it is evident that the horizontal distances of the lines, taken in any direction, will diminish as the inclination in that direction increases. Thus in ABCD, Fig. 129, which represents a small tract, of which the length AB is 1000 feet and the breadth AD, 800 feet, the lines 10, 20, 30, &c., represent lines of level in which horizontal planes at the distance of 10 feet from one another, intersect the surface of the ground. The lowest level passes through the point F, at which the stream EF leaves the tract. From F the ground rises more rapidly to the left than to the right, as is indicated by the lines of level being nearer to one another on that side than on the other. In passing from F towards the corner B of the tract, we may observe that the acclivity, which is gentle, increases till we come to the 30 feet level, then diminishes to the 40 feet level; it then again increases, and more rapidly, to the 60 feet level, and lastly slightly diminishes to the 70 feet, or highest level. In descending, the declivity continually diminishes to B. From the point *l* in the side AD, the ground descends moderately towards the corner A, the declivity diminishing till the surface becomes nearly level; and from the stream, towards A, the acclivity, which is slight, diminishes, so that between the two branches of the 30 feet line of level and the corner A, the ground is nearly level.

The distance which should be taken for the distance of the assumed planes from one another, must depend on the extent of the survey, the inequalities in the surface, and the degree of minuteness with which it is required that they should be designated. It may vary from 3 or 4 to 20 or 30 feet, according to circumstances.

The levelling required for a topographical survey may be performed either with a level or theodolite. Where there is considerable ascent or descent in the ground, the latter is the most convenient instrument; and although the results obtained with it are not in general as accurate as those that may be obtained with a good level, they are, when due care is taken, sufficiently so for the object in view. When the theodolite is used, the sight should be taken to a point at the same height above the ground at the station, as the axis about which the upper telescope revolves is above the ground at the place of the instrument. To do this, let an assistant place and clamp the vane of a levelling-staff at that height, or make a mark at the same height on a pole; and when he has taken the staff or pole to the station and set it up vertically, sight to the vane or mark. When this is done, the difference of level between the station and place of the instrument, expressed in 100th parts of the horizontal distance of the two, will be indicated on the vertical limb. It is however better, except when the horizontal distance is quite small, to obtain the difference of level from that distance and the angle of elevation or depression.

PROBLEM I.

Having given the back-sights and fore-sights taken to a number of consecutive stations, or to two or more connected series of stations, to determine the heights of the stations above a line of level or surface of level through the lowest.

1. *When there is but one series of stations.* Assume for the height of the first station above some assumed line of level, any quantity taken at pleasure, observing however to make it sufficiently great for the assumed line of level to be lower than the lowest station, or at least as low. To the assumed height of the first station add the first back-sight, and from the sum subtract the first fore-sight, and the remainder will be the height of the second station above the assumed line of level. With this height and the second back-sight and second fore-sight proceed in like manner to find the height of the third station; and thus on to the last. Now subtract the least of the heights obtained, which must be that of the lowest station, from each of the others, and the remainders will be the heights of the other stations above the line of level passing through the lowest.

2. *When there are two series of stations connected by intervening sights between the first station of the first series and the first of the second series.* Assume for the height of the first station of the first series, a quantity sufficiently great for the assumed surface of level to be below all the stations, or at least as low as

the lowest, and proceed as directed above, to obtain the heights of the other stations of that series above the assumed surface of level. Then commencing again with the assumed height of the first station, proceed in like manner with the sights connecting that with the first station of the second series, and with the sights to the stations in this series, to find the heights of all these stations. When this is done, subtract the least of all the heights obtained from each of the others, and the remainders will be the required heights.

3. In like manner, whatever be the number of the series of stations, the heights of all the stations, above a surface of level passing through the lowest, may be obtained.

Note. If it is required to find the heights of the stations above a line or surface of level at a given distance below the lowest or any other given station, it is easily performed by applying to the heights of the stations above the assumed level, the difference between the height of that station above the assumed level and its height above the given level.

EXAMPLES.

1. Taking to the nearest length of a foot, the back-sights and fore-sights given in the example to the last problem of the preceding chapter, it is required to find the heights of the stations A, B, C, &c. Fig. 125, above a line of level passing through the lowest.

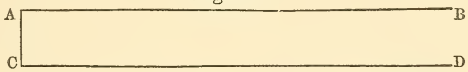
Assume the height AL, of the station A, above an assumed line of level LM to be 15 feet. Then we have

	Feet.			
	15,	height of	A,	above LM,
15 +7.1—1.6=	20.5	"	B,	"
20.5+9.2—3.2=	26.5	"	C,	"
26.5+1.2—9.4=	16.3	"	D,	"
16.3+1.6—6.4=	11.5	"	E,	"
11.5+2.1—9.9=	5.7	"	F,	"

Subtracting 5.7 from each of the above heights, we have the heights above GF, the line of level through F. These are, for A, 9.3ft.; B, 14.8 ft.; C. 20.8 ft.; D, 10.6 ft.; E, 5.8 ft.; and F, 0 ft.

2. Let the back-sights and fore-sights, taken from A to B, Fig. 126, from A to C, and from C to D, be as

Fig. 126.



given below; to find the heights of the stations along AB, AC, and CD, above a surface of level through the lowest station.

AB			AC			CD		
	B-sts.	F-sts.		B-sts.	F-sts.		B-sts.	F-sts.
1	8.7 ft.	3.8 ft.	1	8.5 ft.	5.2 ft.	1	3.1 ft.	8.6 ft.
2	1.1	9.9	2	0.6	9.1	2	2.6	7.7
3	0.7	9.3				3	1.5	9.3
4	2.2	8.0				4	4.2	7.9
5	1.9	7.1				5	3.8	7.3
6	7.4	4.2				6	9.3	2.2
						7	2.8	7.4

By proceeding as directed in the rule, we obtain the following heights of the stations above the surface of level passing through the lowest.

	AB		AC		CD
1	30.8 ft.	1	30.8 ft.	1	25.6 ft.
2	35.7	2	34.1	2	20.1
3	26.9	3	25.6	3	15.0
4	18.3			4	7.2
5	12.5			5	3.5
6	7.3			6	0.0
7	10.5			7	7.1
				8	2.5

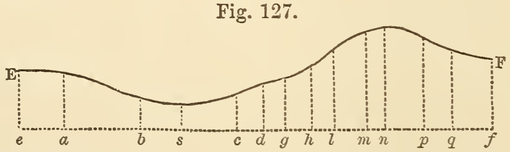
PROBLEM II.

To determine the inequalities in the surface of the ground along a line running in a given direction, and to draw an irregular or curved line to represent them.

Let short stakes be driven at the beginning and end of the line, and at each point along it where there is any material change in the inclination of the ground; and let the horizontal distance of each stake from the beginning of the line, or the distances from stake to stake, be measured. Then level from stake to stake, using intermediate stations whenever the difference of level between any two is too great to permit sights to be taken to both from a single position of the instrument. Find, by the last problem, the heights of the stations, where the stakes are placed, above a line of level passing through the lowest, or above any assumed or given line of level.

Draw a straight line *ef*, Fig. 127, to represent the

Fig. 127.



line of level to which the heights of the stations are referred, and on it make *ea*, *eb*, *es*, *ec*, &c., equal to the distances of the stations from the beginning of the line. From the points *e*, *a*, *b*, *s*, &c., draw lines perpendicular to *ef*, and make them equal to the heights of the respective stations. Through the tops of these perpendiculars, draw the curved line EF, which will be the line required to be drawn.

The line EF is called a *Profile* of the ground in the direction of the given line.

Note. The heights of the perpendiculars are frequently taken from a scale three or four times as great as that used in laying off the horizontal distances. When this is done, the curved line, or profile, as it is still called, indicates more distinctly the lesser changes in the inclination of the ground.

EXAMPLE.

The distances of the stations along a given line, measured from the beginning of the line, and their heights above a given line of level, determined by the last problem, from sights taken on the ground, being as below, the profile of the ground in the direction of the line, obtained by taking the heights from a scale three times as great as that from which the distances are taken, will be that of Fig. 127.

Sta.	Dist.	Ht.	Sta.	Dist	Ht.
1	0ft.	41.1ft.	8	610ft.	41.9ft.
2	90	38.3	9	655	55.0
3	230	22.0	10	725	69.2
4	336str.	15.3	11	770	72.0
5	445	21.1	12	850	69.1
6	505	32.5	13	915	56.5
7	555	36.5	14	1000	50.1

At the fourth station a stream of water crosses the line. and is noted by the letters *str.* placed by the side of the distance.

PROBLEM III.

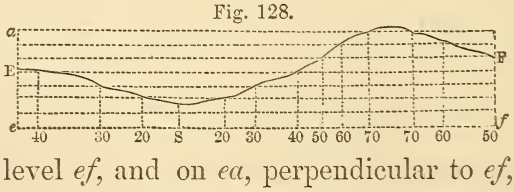
To determine those points along a line running in a given direction, that are at given heights above a given line or surface of level.

Proceed, as directed in the last problem, to find the heights above the given line or surface of level, of those points in the line where there is any material change in the inclination of the ground, and also their distances from the commencement of the line. Observe between which two of the heights obtained, any one of those given falls, and take their difference. Also take the difference between the given height and that one of the two which appertains to the point nearest the beginning of the line. Then, as the first difference : the second : : the horizontal distance between the points : to a fourth term, which added to the distance of the point nearest the beginning of the line, will give the distance of the required point.

Proceed in the same manner with the other given heights.

Or we may draw, by the last problem, a profile of the ground, as EF, Fig. 128, in reference to the given

Fig. 128.



line of level *ef*, and on *ea*, perpendicular to *ef*, set off the given heights, taking them from the scale used in setting off the heights of the stations. Then, through the points in the line *ea*, draw lines parallel to *ef*, as in the figure; and from the points in which these meet the profile EF, draw lines parallel to *ea*. The distances from *e*, at which these last lines meet *ef*, will be the horizontal distances of the required points, from the beginning of the line.

EXAMPLE.

Let the data found on the ground be the same as in the example to the last problem, and let the given heights of the required points be 10, 20, 30, 40, &c. feet.

The distances and heights of the stations being the same as for the profile in Fig. 127, we shall obtain a similar profile EF, Fig. 128. Drawing the lines parallel to *ef* at distances from it, equal to 10, 20, 30, &c. feet, we find that the first line above *ef* does not meet the profile EF. Consequently there is no point in the latter so low as 10 feet above the given level. Each of the other parallels meets the line EF in two points. There are therefore two points in the line at the height of 20 feet above the given level; two at the height of 30 feet; and so on to 70 feet. The distances e, 40; *e*, 30; &c., on the line *ef*, are the distances of the required points from the beginning of the line.

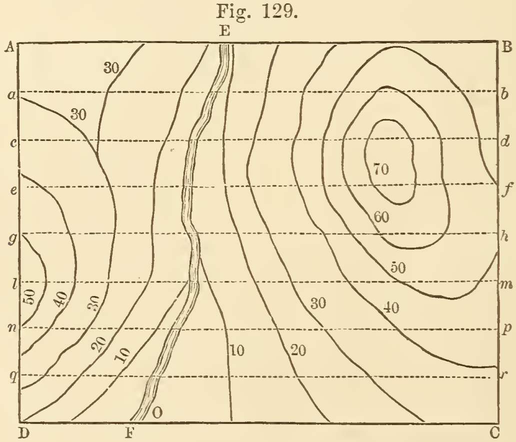
In illustration of the first method of finding the distances of the required points from the beginning of the line, as the given heights of the stations evidently show that there is no point in the line so low as 10 feet above the given level, let it be required to find a point at the height of 20 feet. On examining the given heights of the stations, we perceive that there must be two such points; one between the 3d and

4th stations, and one between the 4th and 5th. For the first of these we take the difference between 15.3 and 22.0, which is 6.7; the difference between 20.0 and 22.0, which is 2.0; and the difference between the distances 230 and 336, which is 106. Hence, as 6.7 : 2.0 : : 106 : 32. Adding therefore 32 to 230, we have 262 feet for the distance of this point.

PROBLEM IV.

To determine the undulations and inequalities of the surface in a tract of land ABCD, Fig. 129, *and to draw a topographical map designating them.*

Fig. 129.

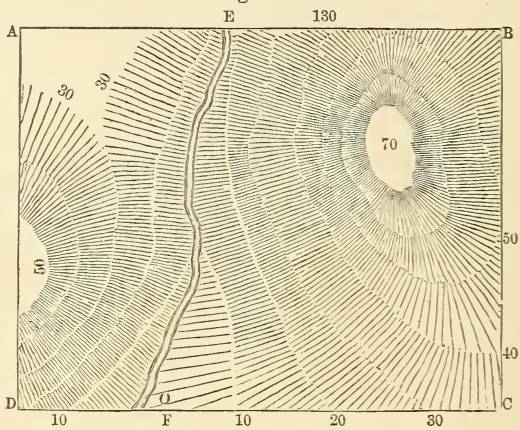


With a compass or theodolite, run a number of lines, *ab*, *cd*, *ef*, &c., across the tract, parallel to one of the sides, as AB; making them nearer together or farther apart, according to the inequalities in the ground and the degree of minuteness with which it is intended to designate them. Drive stakes at the beginnings and ends of the lines AB, *ab*, *cd*, &c., and at all the points along them where there is any material change in the inclination of the ground, and proceed to level from stake to stake along these lines and the line AC; also measure the distances of the stakes from the commencement of the lines, or from one another. Then, by problem I., find the heights of all the stations where the stakes are driven, above a surface of level passing through the lowest station.

Now, having drawn the lines *ab*, *cd*, *ef*, &c., in their proper positions on the map, determine, by the last problem, the points in these lines and the lines AB and DC, that correspond to heights in the lines on the ground, of 10, 20, 30, 40, &c., feet, above the surface of level passing through the lowest station, or to any other heights, increasing by equal differences, that may be deemed expedient. Through each set of points appertaining to the same height, draw a curve line. The curve lines thus drawn will represent lines of level of 10 feet, 20 feet, &c., or of the number of feet of the heights used in obtaining them, whatever that may be.

These lines serve to indicate the changes in the inclination of the ground. But it is usual, instead of drawing them distinctly with ink, to draw them with a pencil only, or faintly with Indian-ink, and then to shade the map by short straight lines, drawn perpendicularly from each curve of higher level, to that of the next lower; the lines being drawn closer together and rather heavier as the distance between the lines of level diminishes. For those parts of the ground that are level, or very nearly so, the shading is omitted. The greater or less darkness of the shading on the different parts of the map, therefore, indicates a greater or less inclination in the ground in those parts; and the omission of the shading in any parts, indicates that in those places the surface is level, or very nearly so In Fig. 130, we have a map of the tract, shaded as described above.

Fig. 130.



Note.—It is not necessary that the lines on which the levels are taken should be run parallel to one another. They may be run making given angles with any given lines of the survey. Sometimes it is desired that the surface of a particular part of the tract should be designated with more minuteness than is important for other remote parts. In this case it will be found convenient to determine the position of some point near the middle of the part which it is desired particularly to designate, and then to run lines from this point, in directions making angles of 20° or 30° with one another.

It may be further remarked, that instead of taking the surface of level through the lowest point, as the plane of reference, we may, if preferred, take that through the highest. The former is however the one generally taken.

The operations to be performed in this problem, so far as observations on the ground and numbers are concerned, are merely repetitions of those which have been exemplified in the preceding problems. An example is not therefore necessary.

CHAPTER X.

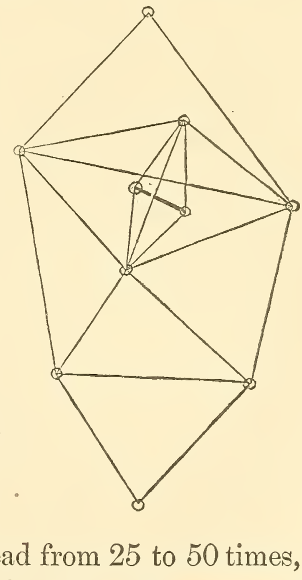
THE COAST SURVEY.

The Coast Survey is an establishment of the United States government to obtain the measurements necessary to a correct map of the country. Similar institutions are in existence in all civilized countries. Part of the work is astronomical, and part is trigonometric. The latter portion only falls within the scope of this treatise.

The method may be briefly described as follows: A base line is carefully measured, from the two ends of which angles are taken to prominent points. Triangles are thus formed, of which one side and the angles are known. The other sides may then be calculated, and will form the base of new triangles. Thus a network of triangles may be established over the country, from which the configuration of the coast line, location of towns, mountains, rivers, &c. may be accurately obtained.

As any error in the length of the base line would be carried through the whole system of triangles and multiplied many times, it is of the utmost importance that it be measured with the greatest possible accuracy. This is done by the use of two measuring bars, each 6 metres (about 20 feet) long, supported near each end by a tripod. One of these being placed in the line to be measured and levelled, the other is carefully placed against its end, aligned and levelled. The first bar is then moved forward, and the process repeated till the end of the line is reached. Care is taken that the pressure of the advance bar does not disturb the position of the other. The bars are compensated for changes of temperature by constructing them of combinations of iron and brass rods, as in the case of the gridiron pendulum. Modern base lines are usually made about 10 kilometres (about 6 miles) in length, and so carefully are all chances for error eliminated, that it is possible to obtain accuracy within $\frac{1}{5}$ of a centimetre, or $\frac{1}{10}$ of an inch.

In triangulating from this base line it is considered desirable to obtain two nearly equilateral triangles, one on each side. The long diagonal of the trapezium thus formed may be calculated by two independent computations, thus obtaining a check to the accuracy of the work. The sides of the triangles are usually from 25 to 60 miles in length, and the stations are located on elevated peaks, easily distinguishable from the surrounding country. The instruments used for reading angles have circles of from 20 to 40 inches in diameter, thus permitting great accuracy in graduation. Each angle is read from 25 to 50 times, and on all parts of the divided limb. The possible error must not exceed one-half a second.



As the triangles are portions of the surface of the earth, they are spherical triangles, and the three angles should be slightly in excess of 180°. The amount of this *spherical excess*, as it is called, may be calculated if the length of the sides be known.

The triangles must also be reduced to the sea level. The stations being at different elevations, incorrect results would be obtained unless a common level were taken for reference. This reduction is easily performed by calculating the amount of convergence of two radii of the earth through the two ends of the line.

The bearings of various lines are then determined by ascertaining the direction of a true meridian by methods similar to those in chapter VI. The latitude of many of the stations is also carefully deduced by astronomical methods. From these the length of a degree of the meridian in different latitudes can be calculated, and the true shape and size of the earth known.

The large triangles thus formed are filled up by measurements of small triangles built on the sides of the large, and the stations may thus be brought as near together as desired, and all prominent points located. From these data maps of the whole country may be constructed.

APPENDIX.

CONTAINING INSTRUCTIONS FOR SURVEYING THE PUBLIC LANDS OF THE UNITED STATES.

BY GEORGE H. HOLLIDAY, A.M.

THE following statement of the mode of making the surveys of public lands of the United States, and the proper method of subdividing, is prepared to meet the wants of landowners and surveyors in those surveys. In most of the leading principles I take pleasure in acknowledging my indebtedness to a pamphlet of instructions prepared, I believe, by the late excellent Surveyor-General for the district of Illinois and Missouri, D. A. Spaulding, Esq., and furnished to applicants at that office, for information.

1. The surveys of the public lands of the United States are based on a line running east and west, called the base line.

2. From certain noted points, as the mouths of principal rivers, are lines run due north, called principal meridians.

3. Townships are bodies of land, six miles square, subdivided into thirty-six parts called sections. The following represents a township with the mode of numbering sections:

NORTH.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

4. Those lines of townships running north and south are called range lines. Rows or tiers of townships running north and south are called ranges. Tiers of townships east and west are called townships, and the lines dividing them are called township lines. Townships are numbered from the base line and the principal meridians, affording an easy method of describing lands. Thus, if the township above be considered township three north of the base line, in range four west of the third principal meridian, the one joining it on the north will be township four range four, the one to the west township three range five, which cannot be mistaken for any other. Sections and parts of sections are described with the same ease and certainty.

5. In running out townships as described hereafter, many errors arise which it would be inconvenient to carry to any considerable distance; to obviate these, generally at intervals of thirty miles, or five townships, a line is run due east and west from the principal meridians, called a standard or correction line. This line also serves to correct the difficulty otherwise arising from the convergence of the meridians.

6. A chain is a measure of four rods. It is more convenient in practice to use a half-chain of two rods, divided into fifty links; each link thus is the one-hundredth part of a chain, and is so expressed and used in taking field-notes and making calculations.

7. In running standard lines, corners are established every 40 chains, for quarter section and section corners; at the end of every sixth mile a township corner is made. These corners are starting points for surveys to the north of this line, and do not apply to any surveys to the south.

8. The mode of laying off a township is as follows:—The south line is a standard line or the north line of a township already surveyed. The east line is a principal meridian or the west line of the township to the east; it only remains to establish the west and north lines. The surveyor therefore proceeds to the south-west corner of the township already established, and runs due north, establishing, as he proceeds, corners at every 40 chains for quarter-section and section corners. These corners were formerly made in timber, by taking two trees as witnesses, by boxing into them, marking with the number of the section, and accurately taking their course and distance; now the regulations require four such witness-trees at section and township corners, and two at quarter-section corners. Formerly in prairie a small mound was thrown up, now one of considerable size is required, in which must be buried stones of size and shape to be described in the field-notes, or a certain quantity of charcoal. At the end of six miles the surveyor establishes his township corner, and the west line is finished. All the corners now established on this line belong to the township lying west, and not to the one now to be run out. The corners for this township are to be made as described hereafter. The surveyor next proceeds to the north-east corner of the township already established and runs west, setting at every 40 chains a temporary post, proceeding until he strikes the west line of the township, and notes down the distance he falls to the north or south of the township corner. If there is any excess or deficiency of six miles in this line, it is all thrown into the last 40 chains, on the west half of the north line of section 6; then if the line run is not a straight line from township corner to corner, he returns, and by offsets from the temporary posts already set he establishes all the corners in a straight line. All the corners on this line are intended for the township north, and not for the one now laid off. In case the township closes on a standard

line, the north-west corner is made at the intersection of the west line with the standard line, whether it be more or less than six miles, and without regard to the township corner already established for the townships north, the surveyor giving in his notes the length of the lines and the distance between his corner and the one on the standard line.

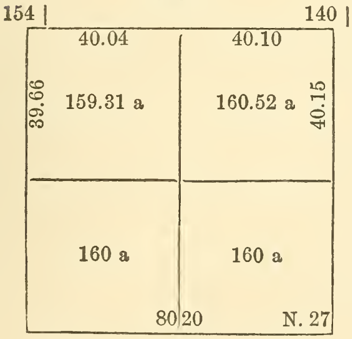
9. The township lines being established, the surveyor proceeds to the subdivision. He commences at the corner to sections 35 and 36 on the south line of the township and runs north, makes a corner at 40 chains, being the quarter-section corner between 35 and 36, and one at 80 chains, corner to 25, 26, 35, 36. Here he turns east, and at 40 chains sets a temporary post, and proceeds in the same line until he comes to the corner to 25 and 36 on range line; he notes the distance and the falling to the north or south of this corner, then returns, and by offsets from the line run establishes the quarter-section corner between sections 25 and 36, at an equal distance from each section corner, and in a straight line with them; then, taking a new start from the corner to 25, 26, 35, 36, proceeds in the same way to run out sections 25, 24, 13, and 12. Then from the corner to sections 1, 2, 11, 12, he runs north 40 chains, establishes the quarter-section corner between 1 and 2, then runs north until he strikes the north line of the township; here he establishes his corner, giving in his notes the length of the line and the distance from the corner already established for the township north. He then returns to the south line of the township, and commencing at corner to sections 34 and 35, runs out another tier of sections in the same manner as the first, and so on until he has completed four tiers of sections. He then returns to corner to sections 31 and 32, and proceeds in the same way with the tier of sections on the east; but at every section corner he also runs west, makes a quarter-section corner at 40 chains, and proceeds in the same line until he strikes the range line, where he makes his corner, giving the falling to the north or south of the corner already established for the township west.

10. It will be perceived that all the section corners are established on lines running north, and these, as well as the quarter-section corners on these lines, are established permanently in

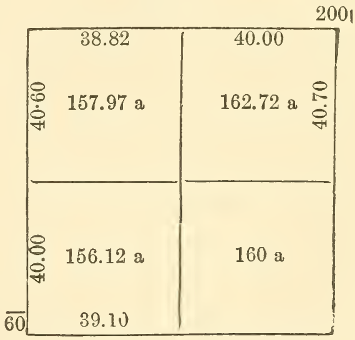
the first instance, and all the quarter-section corners on east and west lines, are corrected to an average distance and in a straight line with the section corners, except those in the western tier of sections, which are established permanently in the first instance.

It will also be seen that all excess or deficiency of measurement is thus thrown into the north half of the sections on the north and the west half of the sections on the west of township; these sections are hence called fractional sections. A complete township with fractions would occupy too much space. Below are three fractional sections, one in the north, one in the northwest, and one in the west of townships; they are taken from the government surveys in township 11 north, range 8 west of third principal meridian:—

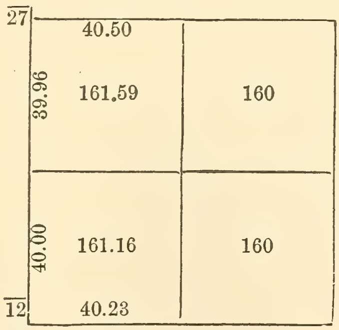
Sec. 3.



Sec. 6.



Sec. 18.



The numbers on the lines represent the length in chains and hundredths; the numbers at offsets on range and township lines represent the difference in links with corners in other townships; the number in the south-east corner of sec. 3 shows the falling north in running the south line.

The general principles of making the public surveys are now before the reader; a knowledge of them is not only important but indispensable to every surveyor.

11. No corners are established in the interior of sections, and as the public lands are sold in tracts of 40, 80, 160, 320, or 640 acres, it devolves upon other surveyors to make the divisions. This at first view would seem a very simple affair; but the difficulties continually arising, and the frequent conflicting surveys by different surveyors, and even clashing surveys by the same, prove the necessity of some well-grounded system being adopted and universally acted upon to insure justice to all parties interested.

12. As the land was originally owned and surveyed by the United States, and conveyed by the corners thus established, it is of the first importance to find these corners before any divisions are attempted. This is frequently a difficult matter. Many witness-trees were killed by boxing; many others die, and even the roots disappear before the land is occupied. In prairie there are many hills that may be mistaken for mounds, the rooting of hogs destroys many more, and there is reason to believe that in many cases, particularly in prairies, corners never were established. In case of the loss of these corners they must be renewed in the same manner in which they should first have been established.

13. In case of loss of corners on a standard line or principal meridian, the line must be followed and measured from one known correct corner to another on this line. This is necessary in order to be assured that both the course and measurement is the same used by the first surveyors; many causes of disagreement in both are frequent, the causes of which it is not here necessary to explain. The missing corner should be placed in a straight line between the original corners, and at an equal or proportional distance.

14. Should any discrepancy, however, appear in either the

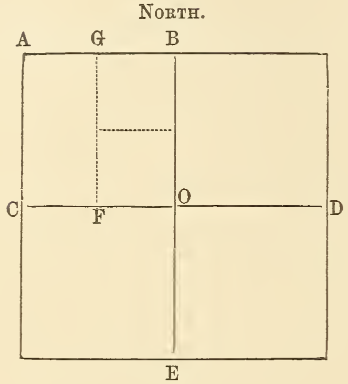
course or length of this line, it will be necessary to examine the section lines intersecting it. The corners on the standard line should be at the points of intersection of the sectional lines originating or terminating in it. The finding of these points as called for by the field-notes, frequently relieves the surveyor of a great deal of difficulty, and throws unexpected light on some of the most complicated cases of difficulty. This remark applies also to all other lines of townships.

15. In running a township corner, the better plan is to commence at the nearest known corner on the range line, and then run north or south past the township corner, until another corner is found, then place the missing corner at an equal or proportional distance as required by the field-notes, and in a straight line; should this, from any circumstances, appear not to have been the point where the original corner was established, take the range of the corners on the township lines to the east and west, and should these unite in one point on the north and south line, the presumption is that that is the true place for the corner, and it should be there established.

16. Section and quarter-section corners on the north and south lines are to be renewed in the same manner, by starting at some corner on these lines, and running past the missing corners until some reliable corner is found, then establishing the missing corner in a straight line and at its proper distance. If the surveyor, in the extension of these lines, strikes a township line, he is to proceed no further on the same course, but the corner here, if missing, must be renewed, as in sec. 13, before any further steps are taken. It is important to re-establish all section corners on north and south lines, because they were originally established in this mode, on the same course and at equal distances, while the lines of sections running east and west are of very unequal lengths and in almost all cases make an angle at section corners. Should any discrepancy, however, arise, these lines should be examined as in establishment of township corners.

17. When all the corners of the original survey are found or renewed, the surveyor is ready to make the subdivisions. and

not before. The following plan will represent the course to be pursued:



Suppose, for instance, a quarter-section is to be surveyed; say the north-west quarter. The corners at A, B, and C, are already established; it only remains to establish the corner at O.

Experience shows that the quarter-section corners on the north and south of sections, required to be made on corrected lines, are much more frequently out of place than those on the east and west, made on the first line run; consequently, as it is desirable to carry errors in the establishment of original corners no farther than possible, sections should be divided by a straight line between the quarter-section corners on the east and west These errors thus never pass the centre of the section. The surveyor should therefore commence at C and run east, set a random stake at 40 chains on his line, continue the same line until he intersects the section line on the east at D, and divides the distance and establishes his corner at O accordingly, without any regard to the corners at B and E.

18. Should a smaller division be necessary, as, for instance, the north-east quarter of the north-west quarter. On the line from C to D a temporary stake must be set at F, as well as O. These, corrected, will be the southern corners of the east half of the quarter-section. A line must be run from A to B, and at an equal distance the corner G established, then a line run

from G to F, and also from O to B, at equal distances from these points, corners are to be established, and the tract is surveyed. If in timber, it only remains to mark the south line, a measurement is unnecessary.

19. The peculiar value of this system consists in never running a line witnout a corner at the starting point and one at the termination. By this means both errors in course and distance are corrected, and if corners are not established in precisely the same place they originally occupied, justice, as far as practicable, is done to all interested parties.

20. Errors in the original surveys often cause a great deal of trouble; those most frequent deserve mention. The measurement was kept by *outs*, each out being the distance one set of pins carries the chain-carriers when the pins are changed to the fore chain-man. Eight outs make the 40 chains, or half-mile, the distance between the original corners. Sometimes the chain-carriers only made seven outs, by mistake, between corners, and sometimes nine. In all cases, however, where the original corners are found they are immovable, however much they may be out of the way. Sometimes a pin was lost, and, consequently, 50 links in every out were lost until the fact was discovered. When chain-carriers are fresh, they carry the chain much tighter than when they are tired; thus a great deal of want of uniformity may be accounted for.

21. Crooked lines are sometimes found. These are frequent in section lines running east and west; the quarter-section corner being made sometimes on a random line without correcting; sometimes the surveyor made his offset in the wrong direction; and, generally in the older surveys, great carelessness was manifested. Sometimes considerable crooks are found in township lines, so as to lead to the opinion that all the corners were made at random. In dealing with these great care must be exercised, and no opinions formed without the most decisive evidence.

22. In the taking of witness-trees there are often considerable discrepancies in the notes. In distance I have often found 10 links, sometimes a chain, more or less than the notes. Sometimes there is reason to believe the distance to one of the trees

was lost, and filled up at random in copying. In bearing there are fewer errors, but more important. In two instances I have found 30° mistaken for 50°. A very frequent error is made by letting the eye rest upon some leading division upon the face of the compass, and counting degrees in the wrong direction. In all cases where there are two trees these are easily rectified; there are four data for determining the corner—the distance and bearing to each tree—when three of these agree, the presumption is that it is correct.

23. The labours of the surveyor, to result in any lasting good, should be spread on record; this is the duty of the county surveyor. But not only does the office fall into the hands of men incompetent to perform the work, but in no instance with which I am acquainted are the records kept in a manner likely ever to settle disputes, or render satisfaction to any intelligent person consulting them. Below is the plan adopted by me, and which should be followed in all records of surveys. It is simple and satisfactory. It will be seen that in all sectional lines and their parallels I make the variations entirely by the scale, noting by that the degree I run on.

FEBRUARY 23, 1854.

For David Holmes, S. E. ¼, N. W. ¼. Sec. 21, T. 10, R. 9. Magnetic var. E. 8° 00′.

Chain-carriers, Riley Hicks and A. B. Carr, sworn.

Chains & Courses.		
		Began at ¼ section corner, between secs. 20 and 21. Stake in original mound; thence
North.		Index 8° 00′.
13	00	Entered brush and young timber, bears E. and W.
27	43	Branch 6 links wide, runs E.
32	47	Left brush, entered Hubbard's field.
40	00	Mound ploughed up, set stake; thence
20	27	Left field, entered hazel.
30	00	Entered edge of prairie, as in field-notes.
40	00	Mound ploughed up between secs. 16 and 17.
80	00	Mound to 8, 9, 16, 17, lost; thence
26	00	Enter barrens, N. E. and S. W. as in field-notes.

Chains & Courses.		
40	00	Mound between 8 and 9 lost, land broken.
47	00	Joe's Creek, 10 links wide, bears S. W.
50	00	Former bed of creek, 5.00 chains farther north than called for by field-notes.
79	85	14 links east of corner to 4, 5, 8, 9, set by me in survey of Jayne's land. Then corrected stake corner to sections 16, 17, 20, 21, thence
East.		Index 8° 00′.
20	00	Set stake.
40	44	6 links south of ¼ sec. corner between sections 16 and 21, S. W. witness standing, N. E. cut down, renewed by marking elm 9 inches diameter, N. 63° E. 49 links.
		Then from this corner
South.		Index 8° 00′.
20	00	Set stake.
41	06	57 links east of corner centre of section, set by me in survey of Carr's land. Corrected this line back, and set stake above at average distance, and took witness black oak, 32 inches diameter. N. 57° E. 9 links.
		Then from corrected stakes, N. W. corner of E. ½, N. W. ¼ this section.
South.		Index 7° 40′.
20	00	Set stake.
40	48	8 links west of corner set by me in survey of Carr's land corrected corner on above line and took witness hickory, 8 inches diameter. S. 5° W. 93 links.

MATHEMATICAL TABLES.

A TABLE

OF THE

LOGARITHMS OF NUMBERS

FROM 1 to 10,000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	26	1.414973	51	1.707570	76	1.880814
2	0.301030	27	1.431364	52	1.716003	77	1.886491
3	0.477121	28	1.447158	53	1.724276	78	1.892095
4	0.602060	29	1.462398	54	1.732394	79	1.897627
5	0.698970	30	1.477121	55	1.740363	80	1.903090
6	0.778151	31	1.491362	56	1.748188	81	1.908485
7	0.845098	32	1.505150	57	1.755875	82	1.913814
8	0.903090	33	1.518514	58	1.763428	83	1.919078
9	0.954243	34	1.531479	59	1.770852	84	1.924279
10	1.000000	35	1.544068	60	1.778151	85	1.929419
11	1.041393	36	1.556303	61	1.785330	86	1.934498
12	1.079181	37	1.568202	62	1.792392	87	1.939519
13	1.113943	38	1.579784	63	1.799341	88	1.944483
14	1.146128	39	1.591065	64	1.806180	89	1.949390
15	1.176091	40	1.602060	65	1.812913	90	1.954243
16	1.204120	41	1.612784	66	1.819544	91	1.959041
17	1.230449	42	1.623249	67	1.826075	92	1.963788
18	1.255273	43	1.633468	68	1.832509	93	1.968483
19	1.278754	44	1.643453	69	1.838849	94	1.973128
20	1.301030	45	1.653213	70	1.845098	95	1.977724
21	1.322219	46	1.662758	71	1.851258	96	1.982271
22	1.342423	47	1.672098	72	1.857332	97	1.986772
23	1.361728	48	1.681241	73	1.863323	98	1.991226
24	1.380211	49	1.690196	74	1.869232	99	1.995635
25	1.397940	50	1.698970	75	1.875061	100	2.000000

N.	0	1	2	3	4	5	6	7	8	9	D.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	8600	9026	9451	9876	^{0}300	^{0}724	^{1}147	^{1}570	^{1}993	^{2}415	424
103	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616	419
104	7033	7451	7868	8284	8700	9116	9532	9947	^{0}361	^{0}775	416
105	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	9384	9789	^{0}195	^{0}600	^{1}004	^{1}408	^{1}812	^{2}216	^{2}619	^{3}021	404
108	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	7426	7825	8223	8620	9017	9414	9811	^{0}207	^{0}602	^{0}998	396
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	389
112	9218	9606	9993	^{0}380	^{0}766	^{1}153	^{1}538	^{1}924	^{2}309	^{2}694	386
113	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	382
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	^{0}320	379
115	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	372
117	8186	8557	8928	9298	9668	^{0}038	^{0}407	^{0}776	^{1}145	^{1}514	369
118	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	079181	9543	9904	^{0}266	^{0}626	^{0}987	^{1}347	^{1}707	^{2}067	^{2}426	360
121	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	9905	^{0}258	^{0}611	^{0}963	^{1}315	^{1}667	^{2}018	^{2}370	^{2}721	^{3}071	351
124	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	6910	7257	7604	7951	8298	8644	8990	9335	9681	^{0}026	346
126	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	340
128	7210	7549	7888	8227	8565	8903	9241	9579	9916	^{0}253	338
129	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	^{0}245	330
132	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	7105	7429	7753	8076	8399	8722	9045	9368	9690	^{0}012	323
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	315
138	9879	^{0}194	^{0}508	^{0}822	^{1}136	^{1}450	^{1}763	^{2}076	^{2}389	^{2}702	314
139	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	9219	9527	9835	^{0}142	^{0}449	^{0}756	^{1}063	^{1}370	^{1}676	^{1}982	307
142	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	8362	8664	8965	9266	9567	9868	^{0}168	^{0}469	^{0}769	^{1}068	301
145	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	6091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
151	8977	9264	9552	9839	^{0}126	^{0}413	^{0}699	^{0}986	^{1}272	^{1}558	287
152	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	^{0}051	281
155	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	8657	8932	9206	9481	9755	^{0}029	^{0}303	^{0}577	^{0}850	^{1}124	274
159	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
160	204120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	9515	9783	^{0}051	^{0}319	^{0}586	^{0}853	^{1}121	^{1}388	^{1}654	^{1}921	267
163	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
166	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
169	7887	8144	8400	8657	8913	9170	9426	9682	9938	^{0}193	256
170	230449	0704	0960	1215	1470	1724	1979	2234	2488	2742	254
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	8046	8297	8548	8799	9049	9299	9550	9800	^{0}050	^{0}300	250
174	240549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
175	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
176	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	7973	8219	8464	8709	8954	9198	9443	9687	9932	^{0}176	245
178	250420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
186	9513	9746	9980	^{0}213	^{0}446	^{0}679	^{0}912	^{1}144	^{1}377	^{1}609	233
187	271842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	8754	8982	9211	9439	9667	9895	^{0}123	^{0}351	^{0}578	^{0}806	228
191	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
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237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
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314	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
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553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
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565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
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615	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
616	9581	9651	9722	9792	9863	9933	0004	0074	0144	0215	70
617	790285	0356	0426	0496	0567	0637	0707	0778	0848	0918	70
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627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
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636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
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695	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
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704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
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775	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
776	9862	9918	9974	^{0}030	^{0}086	^{0}141	^{0}197	^{0}253	^{0}309	^{0}365	56
777	890421	0477	0533	0589	0645	0700	0756	0812	0868	0924	56
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779	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	2095	2150	2206	2262	2317	2373	2429	2484	2540	2595	56
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797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
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805	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
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830	9078	9130	9183	9235	9287	9340	9392	9444	9496	9549	52
831	9601	9653	9706	9758	9810	9862	9914	9967	⁰019	⁰071	52
832	920123	0176	0228	0280	0332	0384	0436	0489	0541	0593	52
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
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835	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
836	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
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843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
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847	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
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850	9419	9470	9521	9572	9623	9674	9725	9776	9827	9879	51
851	9930	9981	⁰032	⁰083	⁰134	⁰185	⁰236	⁰287	⁰338	⁰389	51
852	930440	0491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
854	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
855	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	4498	4549	4599	4650	4700	4751	4801	4852	4902	4953	50
861	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
862	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
863	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
864	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
865	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
868	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
869	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	9519	9569	9619	9669	9719	9769	9819	9869	9918	9968	50
871	940018	0068	0118	0168	0218	0267	0317	0367	0417	0467	50
872	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
873	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
880	944483	4532	4581	4631	4680	4729	4779	4828	4877	4927	49
881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	9390	9439	9488	9536	9585	9634	9683	9731	9780	9829	49
891	9878	9926	9975	^{0}024	^{0}073	^{0}121	^{0}170	^{0}219	^{0}267	^{0}316	49
892	950365	0414	0462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	4243	4291	4339	4387	4435	4484	4532	4580	4628	4677	48
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
905	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	9041	9089	9137	9185	9232	9280	9328	9375	9423	9471	48
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
912	9995	^{0}042	^{0}090	^{0}138	^{0}185	^{0}233	^{0}280	^{0}328	^{0}376	^{0}423	48
913	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
914	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
915	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	47
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
626	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
627	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	8483	8530	8576	8623	8670	8716	8763	8810	8856	8903	47
931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
932	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
933	9882	9928	9975	^{0}021	^{0}068	^{0}114	^{0}161	^{0}207	^{0}254	^{0}300	47
934	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765	46
935	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
940	973128	3174	3220	3266	3313	3359	3405	3451	3497	3543	46
941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	46
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	6772	6817	6861	6906	6951	6996	7040	7085	7130	7175	45
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	9895	9939	9983	0028	0072	0117	0161	0206	0250	0294	44
978	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738	44
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	1226	1270	1315	1359	1403	1448	1492	1536	1580	1625	44
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	5635	5679	5723	5767	5811	5854	5898	5942	5986	6030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N.	0	1	2	3	4	5	6	7	8	9	D.

TABLE

OF

LOGARITHMIC SINES

AND

TANGENTS.

0° LOGARITHMIC

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	Inf. neg.		10.000000	.00	Inf. neg.		Infinite.	60
1	6.463726	5017.17	0		6.463726	5017.17	13.536274	59
2	764756	2934.85	0		764756	2934.85	235244	58
3	6.940847	2082.31	0		6.940847	2082.31	13.059153	57
4	7.065786	1615.17	0		7.065786	1615.17	12.934214	56
5	162696	1319.68	0	.00	162696	1319.69	837304	55
6	241877	1115.78	9.999999	.01	241878	1115.78	758122	54
7	308824	966.53	99		308825	966.54	691175	53
8	366816	852.54	99		366817	852.54	633183	52
9	417968	762.62	99		417970	762.63	582030	51
10	463726	689.88	98		463727	689.88	536273	50
11	7.505118	629.81	9.999998		7.505120	629.81	12.494880	49
12	542906	579.36	97		542909	579.37	457091	48
13	577668	536.41	97		577672	536.42	422328	47
14	609853	499.38	96		609857	499.39	390143	46
15	639816	467.14	96		639820	467.15	360180	45
16	667845	438.81	95		667849	438.82	332151	44
17	694173	413.72	95		694179	413.73	305821	43
18	718997	391.35	94		719003	391.36	280997	42
19	742477	371.27	93		742484	371.28	257516	41
20	764754	353.15	93		764761	353.16	235239	40
21	7.785943	336.72	9.999992		7.785951	336.73	12.214049	39
22	806146	321.75	91		806155	321.76	193845	38
23	825451	308.05	90	.01	825460	308.07	174540	37
24	843934	295.47	89	.02	843944	295.49	156056	36
25	861662	283.88	88		861674	283.90	138326	35
26	878695	273.17	88		878708	273.18	121292	34
27	895085	263.23	87		895099	263.25	104901	33
28	910879	253.99	86		910894	254.01	089106	32
29	926119	245.38	85		926134	245.40	073866	31
30	940842	237.33	83		940858	237.35	059142	30
31	7.955082	229.80	9.999982		7.955100	229.82	12.044900	29
32	968870	222.73	81		968889	222.75	031111	28
33	982233	216.08	80		982253	216.10	017747	27
34	7.995198	209.81	79		7.995219	209.83	12.004781	26
35	8.007787	203.90	77		8.007809	203.92	11.992191	25
36	020021	198.31	76		020045	198.33	979955	24
37	031919	193.02	75		031945	193.05	968055	23
38	043501	188.01	73		043527	188.03	956473	22
39	054781	183.25	72		054809	183.27	945191	21
40	065776	178.72	71		065806	178.75	934194	20
41	8.076500	174.41	9.999969		8.076531	174.44	11.923469	19
42	086965	170.31	68		086997	170.34	913003	18
43	097183	166.39	66	.02	097217	166.42	902783	17
44	107167	162.65	64	.03	107202	162.68	892798	16
45	116926	159.08	63		116963	159.11	883037	15
46	126471	155.66	61		126510	155.68	873490	14
47	135810	152.38	59		135851	152.41	864149	13
48	144953	149.24	58		144996	149.27	855004	12
49	153907	146.22	56		153952	146.25	846048	11
50	162681	143.33	54		162727	143.36	837273	10
51	8.171280	140.54	9.999952		8.171328	140.57	11.828672	9
52	179713	137.86	50		179763	137.90	820237	8
53	187985	135.29	48		188036	135.32	811964	7
54	196102	132.80	46		196156	132.84	803844	6
55	204070	130.41	44	.03	204126	130.44	795874	5
56	211895	128.10	42	.04	211953	128.14	788047	4
57	219581	125.87	40		219641	125.91	780359	3
58	227134	123.72	38		227195	123.76	772805	2
59	234557	121.64	36	.04	234621	121.68	765379	1
60	8.241855		9.999934		8.241921		11.758079	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	89°

1°	SINES AND TANGENTS.							
M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	8.241855	119.63	9.999934	.04	8.241921	119.67	11.758079	60
1	249033	117.68	932		249102	117.72	750898	59
2	256094	115.80	929		256165	115.84	743835	58
3	263042	113.98	927		263115	114.02	736885	57
4	269881	112.21	925		269956	112.25	730044	56
5	276614	110.50	922		276691	110.54	723309	55
6	283243	108.83	920		283323	108.87	716677	54
7	289773	107.22	917		289856	107.26	710144	53
8	296207	105.65	915		296292	105.70	703708	52
9	302546	104.13	912		302634	104.18	697366	51
10	308794	102.66	910		308884	102.70	691116	50
11	8.314954	101.22	9.999907		8.315046	101.26	11.684954	49
12	321027	99.82	905		321122	99.87	678878	48
13	327016	98.47	902	.04	327114	98.51	672886	47
14	332924	97.14	899	.05	333025	97.19	666975	46
15	338753	95.86	897		338856	95.90	661144	45
16	344504	94.60	894		344610	94.65	655390	44
17	350181	93.38	891		350289	93.43	649711	43
18	355783	92.19	888		355895	92.24	644105	42
19	361315	91.03	885		361430	91.08	638570	41
20	366777	89.90	882		366895	89.95	633105	40
21	8.372171	88.80	9.999879		8.372292	88.85	11.627708	39
22	377499	87.72	876		377622	87.77	622378	38
23	382762	86.67	873		382889	86.72	617111	37
24	387962	85.64	870		388092	85.70	611908	36
25	393101	84.64	867		393234	84.69	606766	35
26	398179	83.66	864		398315	83.71	601685	34
27	403199	82.71	861		403338	82.76	596662	33
28	408161	81.77	858		408304	81.82	591696	32
29	413068	80.86	854	.05	413213	80.91	586787	31
30	417919	79.96	851	.06	418068	80.02	581932	30
31	8.422717	79.09	9.999848		8.422869	79.14	11.577131	29
32	427462	78.23	844		427618	78.29	572382	28
33	432156	77.40	841		432315	77.45	567685	27
34	436800	76.57	838		436962	76.63	563038	26
35	441394	75.77	834		441560	75.83	558440	25
36	445941	74.99	831		446110	75.05	553890	24
37	450440	74.22	827		450613	74.28	549387	23
38	454893	73.46	824		455070	73.52	544930	22
39	459301	72.73	820		459481	72.79	540519	21
40	463665	72.00	816		463849	72.06	536151	20
41	8.467985	71.29	9.999813		8.468173	71.35	11.531827	19
42	472263	70.60	809		472454	70.66	527546	18
43	476498	69.91	805		476693	69.98	523307	17
44	480693	69.24	801	.06	480892	69.31	519108	16
45	484848	68.59	797	.07	485050	68.65	514950	15
46	488963	67.94	794		489170	68.01	510830	14
47	493040	67.31	790		493250	67.38	506750	13
48	497078	66.69	786		497293	66.76	502707	12
49	501080	66.08	782		501298	66.15	498702	11
50	505045	65.48	778		505267	65.55	494733	10
51	8.508974	64.89	9.999774		8.509200	64.96	11.490800	9
52	512867	64.32	769		513098	64.39	486902	8
53	516726	63.75	765		516961	63.82	483039	7
54	520551	63.19	761		520790	63.26	479210	6
55	524343	62.64	757		524586	62.72	475414	5
56	528102	62.11	753		528349	62.18	471651	4
57	531828	61.58	748		532080	61.65	467920	3
58	535523	61.06	744		535779	61.13	464221	2
59	539186	60.55	740	.07	539447	60.62	460553	1
60	8.542819		9.999735		8.543084		11.456916	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	88°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	8.542819	60.04	9.999735	.07	8.543084	60.12	11.456916	60
1	46422	59.55	731	.07	46691	59.62	53309	59
2	49995	59.06	726	.07	50268	59.14	49732	58
3	53539	58.58	722	.08	53817	58.66	46183	57
4	57054	58.11	717		57336	58.19	42664	56
5	60540	57.65	713		60828	57.73	39172	55
6	63999	57.19	708		64291	57.27	35709	54
7	67431	56.74	704		67727	56.82	32273	53
8	70836	56.30	699		71137	56.38	28863	52
9	74214	55.87	694		74520	55.95	25480	51
10	77566	55.44	689		77877	55.52	22123	50
11	8.580892	55.02	9.999685		8.581208	55.10	11.418792	49
12	84193	54.60	680		84514	54.68	15486	48
13	87469	54.19	675		87795	54.27	12205	47
14	90721	53.79	670		91051	53.87	08949	46
15	93948	53.39	665		94283	53.47	05717	45
16	8.597152	53.00	660		8.597492	53.08	11.402508	44
17	8.600332	52.61	655		8.600677	52.70	11.399323	43
18	03489	52.23	650	.08	03839	52.32	96161	42
19	06623	51.86	645	.09	06978	51.94	93022	41
20	09734	51.49	640		10094	51.58	89906	40
21	8.612823	51.12	9.999635		8.613189	51.21	11.386811	39
22	15891	50.76	629		16262	50.85	83738	38
23	18937	50.41	624		19313	50.50	80687	37
24	21962	50.06	619		22343	50.15	77657	36
25	24965	49.72	614		25352	49.81	74648	35
26	27948	49.38	608		28340	49.47	71660	34
27	30911	49.04	603		31308	49.13	68692	33
28	33854	48.71	597		34256	48.80	65744	32
29	36776	48.39	592		37184	48.48	62816	31
30	39680	48.06	586		40093	48.16	59907	30
31	8.642563	47.75	9.999581		8.642983	47.84	11.357017	29
32	45428	47.43	575		45853	47.53	54147	28
33	48274	47.12	570		48704	47.22	51296	27
34	51102	46.82	564	.09	51537	46.91	48463	26
35	53911	46.52	558	.10	54352	46.61	45648	25
36	56702	46.22	553		57149	46.31	42851	24
37	59475	45.92	547		59928	46.02	40072	23
38	62230	45.63	541		62689	45.73	37311	22
39	64968	45.35	535		65433	45.44	34567	21
40	67689	45.06	529		68160	45.16	31840	20
41	8.670393	44.79	9.999524		8.670870	44.88	11.329130	19
42	73080	44.51	518		73563	44.61	26437	18
43	75751	44.24	512		76239	44.34	23761	17
44	78405	43.97	506		78900	44.07	21100	16
45	81043	43.70	500		81544	43.80	18456	15
46	83665	43.44	493		84172	43.54	15828	14
47	86272	43.18	487		86784	43.28	13216	13
48	88863	42.92	481		89381	43.03	10619	12
49	91438	42.67	475		91963	42.77	08037	11
50	93998	42.42	469	.10	94529	42.52	05471	10
51	96543	42.17	9.999463	.11	97081	42.28	02919	9
52	8.699073	41.92	456		8.699617	42.03	11.300383	8
53	8.701589	41.68	450		8.702139	41.79	11.297861	7
54	04090	41.44	443		04646	41.55	95354	6
55	06577	41.21	437		07140	41.32	92860	5
56	09049	40.97	431		09618	41.08	90382	4
57	11507	40.74	424		12083	40.85	87917	3
58	13952	40.51	418		14535	40.62	85465	2
59	16383	40.29	411	.11	16972	40.40	83028	1
60	8.718800		9.999404		8.719396		11.280604	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	87°

3° SINES AND TANGENTS.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	8.718800	40.06	9.999404	.11	8.719396	40.17	11.280604	60
1	21204	39.84	9398		21806	39.95	78194	59
2	23595	39.62	9391		24203	39.74	75797	58
3	25972	39.41	9384		26588	39.52	73412	57
4	28337	39.19	9378		28959	39.31	71041	56
5	30688	38.98	9371	.11	31317	39.09	68683	55
6	33027	38.77	9364	.12	33663	38.89	66337	54
7	35354	38.57	9357		35996	38.68	64004	53
8	37667	38.36	9350		38317	38.48	61683	52
9	39969	38.16	9343		40626	38.27	59374	51
10	42259	37.96	9336		42922	38.07	57078	50
11	8.744536	37.76	9.999329		8.745207	37.87	11.254793	49
12	46802	37.56	9322		47479	37.68	52521	48
13	49055	37.37	9315		49740	37.49	50260	47
14	51297	37.17	9308		51989	37.29	48011	46
15	53528	36.98	9301		54227	37.10	45773	45
16	55747	36.79	9294		56453	36.92	43547	44
17	57955	36.61	9286		58668	36.73	41332	43
18	60151	36.42	9279		60872	36.55	39128	42
19	62337	36.24	9272		63065	36.36	36935	41
20	64511	36.06	9265		65246	36.18	34754	40
21	8.766675	35.88	9.999257	.12	8.767417	36.00	11.232583	39
22	68828	35.70	9250	.13	69578	35.83	30422	38
23	70970	35.53	9242		71727	35.65	28273	37
24	73101	35.35	9235		73866	35.48	26134	36
25	75223	35.18	9227		75995	35.31	24005	35
26	77333	35.01	9220		78114	35.14	21886	34
27	79434	34.84	9212		80222	34.97	19778	33
28	81524	34.67	9205		82320	34.80	17680	32
29	83605	34.51	9197		84408	34.64	15592	31
30	85675	34.34	9189		86486	34.47	13514	30
31	8.787736	34.18	9.999181		8.788554	34.31	11.211446	29
32	89787	34.02	9174		90613	34.15	09387	28
33	91828	33.86	9166		92662	33.99	07338	27
34	93859	33.70	9158		94701	33.83	05299	26
35	95881	33.54	9150		96731	33.68	03269	25
36	97894	33.39	9142		8.798752	33.52	11.201248	24
37	8.799897	33.23	9134		8.800763	33.37	11.199237	23
38	8.801892	33.08	9126		02765	33.22	97235	22
39	03876	32.93	9118		04758	33.07	95242	21
40	05852	32.78	9110		06742	32.92	93258	20
41	8.807819	32.63	9.999102	.13	8.808717	32.77	11.191283	19
42	09777	32.49	9094	.14	10683	32.62	89317	18
43	11726	32.34	9086		12641	32.48	87359	17
44	13667	32.19	9077		14589	32.33	85411	16
45	15599	32.05	9069		16529	32.19	83471	15
46	17522	31.91	9061		18461	32.05	81539	14
47	19436	31.77	9053		20384	31.91	79616	13
48	21343	31.63	9044		22298	31.77	77702	12
49	23240	31.49	9036		24205	31.63	75795	11
50	25130	31.35	9027		26103	31.50	73897	10
51	8.827011	31.22	9.999019		8.827992	31.36	11.172008	9
52	28884	31.08	9010		29874	31.23	70126	8
53	30749	30.95	9002		31748	31.09	68252	7
54	32607	30.82	8993		33613	30.96	66387	6
55	34456	30.69	8984		35471	30.83	64529	5
56	36297	30.56	8976	.14	37321	30.70	62679	4
57	38130	30.43	8967	.15	39163	30.57	60837	3
58	39956	30.30	8958	.15	40998	30.45	59002	2
59	41774	30.17	8950	.15	42825	30.32	57175	1
60	8.843585		9.998941		8.844644		11.155356	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	86°

4° LOGARITHMIC

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	8.843585	30.05	9.998941	.15	8.844644	30.19	11.155356	60
1	45387	29.92	932		46455	30.07	53545	59
2	47183	29.80	923		48260	29.95	51740	58
3	48971	29.67	914		50057	29.82	49943	57
4	50751	29.55	905		51846	29.70	48154	56
5	52525	29.43	896		53628	29.58	46372	55
6	54291	29.31	887		55403	29.46	44597	54
7	56049	29.19	878		57171	29.35	42829	53
8	57801	29.08	869		58932	29.23	41068	52
9	59546	28.96	860		60686	29.11	39314	51
10	61283	28.84	851		62433	29.00	37567	50
11	8.863014	28.73	9.998841		8.864173	28.88	11.135827	49
12	64738	28.61	832	.15	65906	28.77	34094	48
13	66455	28.50	823	.16	67632	28.66	32368	47
14	68165	28.39	813		69351	28.54	30649	46
15	69868	28.28	804		71064	28.43	28936	45
16	71565	28.17	795		72770	28.32	27230	44
17	73255	28.06	785		74469	28.21	25531	43
18	74938	27.95	776		76162	28.11	23838	42
19	76615	27.84	766		77849	28.00	22151	41
20	78285	27.73	757		79529	27.89	20471	40
21	8.879949	27.63	9.998747		8.881202	27.79	11.118798	39
22	81607	27.52	738		82869	27.68	17131	38
23	83258	27.42	728		84530	27.58	15470	37
24	84903	27.31	718		86185	27.47	13815	36
25	86542	27.21	708		87833	27.37	12167	35
26	88174	27.11	699		89476	27.27	10524	34
27	89801	27.00	689		91112	27.17	08888	33
28	91421	26.90	679	.16	92742	27.07	07258	32
29	93035	26.80	669	.17	94366	26.97	05634	31
30	94643	26.70	659		95984	26.87	04016	30
31	96245	26.60	9.998649		97596	26.77	02404	29
32	97842	26.51	639		8.899203	26.67	11.100797	28
33	8.899432	26.41	629		8.900803	26.58	11.099197	27
34	8.901017	26.31	619		02398	26.48	97602	26
35	02596	26.22	609		03987	26.38	96013	25
36	04169	26.12	599		05570	26.29	94430	24
37	05736	26.03	589		07147	26.20	92853	23
38	07297	25.93	578		08719	26.10	91281	22
39	08853	25.84	568		10285	26.01	89715	21
40	10404	25.75	558		11846	25.92	88154	20
41	8.911949	25.66	9.998548		8.913401	25.83	11.086599	19
42	13488	25.56	537		14951	25.74	85049	18
43	15022	25.47	527	.17	16495	25.65	83505	17
44	16550	25.38	516	.18	18034	25.56	81966	16
45	18073	25.29	506		19568	25.47	80432	15
46	19591	25.20	495		21096	25.38	78904	14
47	21103	25.12	485		22619	25.30	77381	13
48	22610	25.03	474		24136	25.21	75864	12
49	24112	24.94	464		25649	25.12	74351	11
50	25609	24.86	453		27156	25.03	72844	10
51	8.927100	24.77	9.998442		8.928658	24.95	11.071342	9
52	28587	24.69	431		30155	24.86	69845	8
53	30068	24.60	421		31647	24.78	68353	7
54	31544	24.52	410		33134	24.70	66866	6
55	33015	24.43	399		34616	24.61	65384	5
56	34481	24.35	388		36093	24.53	63907	4
57	35942	24.27	377		37565	24.45	62435	3
58	37398	24.19	366		39032	24.37	60968	2
59	38850	24.11	355	.18	40494	24.29	59506	1
60	8.940296		9.998344		8.941952		11.058048	0
M.	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	85°

5°	SINES AND TANGENTS.							
M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	8.940296	24.03	9.998344	.19	8.941952	24.21	11.058048	60
1	41738	23.94	333		43404	24.13	56596	59
2	43174	23.87	322		44852	24.05	55148	58
3	44606	23.79	311		46295	23.97	53705	57
4	46034	23.71	300		47734	23.90	52266	56
5	47456	23.63	289		49168	23.82	50832	55
6	48874	23.55	277		50597	23.74	49403	54
7	50287	23.48	266		52021	23.66	47979	53
8	51696	23.40	255		53441	23.59	46559	52
9	53100	23.32	243		54856	23.51	45144	51
10	54499	23.25	232		56267	23.44	43733	50
11	8.955894	23.17	9.998220		8.957674	23.37	11.042326	49
12	57284	23.10	209		59075	23.29	40925	48
13	58670	23.02	197		60473	23.22	39527	47
14	60052	22.95	186		61866	23.14	38134	46
15	61429	22.88	174		63255	23.07	36745	45
16	62801	22.80	163		64639	23.00	35361	44
17	64170	22.73	151	.19	66019	22.93	33981	43
18	65534	22.66	139	.20	67394	22.86	32606	42
19	66893	22.59	128		68766	22.79	31234	41
20	68249	22.52	116		70133	22.71	29867	40
21	8.969600	22.45	9.998104		8.971496	22.65	11.028504	39
22	70947	22.38	092		72855	22.57	27145	38
23	72289	22.31	080		74209	22.51	25791	37
24	73628	22.24	068		75560	22.44	24440	36
25	74962	22.17	056		76906	22.37	23094	35
26	76293	22.10	044		78248	22.30	21752	34
27	77619	22.03	032		79586	22.23	20414	33
28	78941	21.97	020		80921	22.17	19079	32
29	80259	21.90	9.998008		82251	22.10	17749	31
30	81573	21.83	9.997996		83577	22.04	16423	30
31	8.982883	21.77	984		8.984899	21.97	11.015101	29
32	84189	21.70	972		86217	21.91	13783	28
33	85491	21.63	959		87532	21.84	12468	27
34	86789	21.57	947	.20	88842	21.78	11158	26
35	88083	21.50	935	.21	90149	21.71	09851	25
36	89374	21.44	922		91451	21.65	08549	24
37	90660	21.38	910		92750	21.58	07250	23
38	91943	21.31	897		94045	21.52	05955	22
39	93222	21.25	885		95337	21.46	04663	21
40	94497	21.19	872		96624	21.40	03376	20
41	8.995768	21.12	9.997860		97908	21.34	02092	19
42	97036	21.06	847		8.999188	21.27	11.000812	18
43	98299	21.00	835		9.000465	21.21	10.999535	17
44	8.999560	20.94	822		01738	21.15	98262	16
45	9.000816	20.88	809		03007	21.09	96993	15
46	02069	20.82	797		04272	21.03	95728	14
47	03318	20.76	784		05534	20.97	94466	13
48	04563	20.70	771		06792	20.91	93208	12
49	05805	20.64	758		08047	20.85	91953	11
50	07044	20.58	745		09298	20.80	90702	10
51	9.008278	20.52	9.997732		9.010546	20.74	10.989454	9
52	09510	20.46	719		11790	20.68	88210	8
53	10737	20.40	706	.21	13031	20.62	86969	7
54	11962	20.34	693	.22	14268	20.56	85732	6
55	13182	20.29	680		15502	20.51	84498	5
56	14400	20.23	667		16732	20.45	83268	4
57	15613	20.17	654		17959	20.40	82041	3
58	16824	20.12	641		19183	20.34	80817	2
59	18031	20.06	628	.22	20403	20.28	79597	1
60	9.019235		9.997614		9.021620		10.978380	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	84°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.019235	20.00	9.997614	.22	9.021620	20.23	10.978380	60
1	20435	19.95	601		22834	20.17	77166	59
2	21632	19.89	588		24044	20.11	75956	58
3	22825	19.84	574		25251	20.06	74749	57
4	24016	19.78	561		26455	20.01	73545	56
5	25203	19.73	547	.22	27655	19.95	72345	55
6	26386	19.67	534	.23	28852	19.90	71148	54
7	27567	19.62	520		30046	19.85	69954	53
8	28744	19.57	507		31237	19.79	68763	52
9	29918	19.51	493		32425	19.74	67575	51
10	31089	19.46	480		33609	19.69	66391	50
11	9.032257	19.41	9.997466		9.034791	19.64	10.965209	49
12	33421	19.36	452		35969	19.58	64031	48
13	34582	19.30	439		37144	19.53	62856	47
14	35741	19.25	425		38316	19.48	61684	46
15	36896	19.20	411		39485	19.43	60515	45
16	38048	19.15	397		40651	19.38	59349	44
17	39197	19.10	383		41813	19.33	58187	43
18	40342	19.05	369		42973	19.28	57027	42
19	41485	18.99	355		44130	19.23	55870	41
20	42625	18.95	341	.23	45284	19.18	54716	40
21	9.043762	18.89	9.997327	.24	9.046434	19.13	10.953566	39
22	44895	18.84	313		47582	19.08	52418	38
23	46026	18.79	299		48727	19.03	51273	37
24	47154	18.75	285		49869	18.98	50131	36
25	48279	18.70	271		51008	18.93	48992	35
26	49400	18.65	257		52144	18.89	47856	34
27	50519	18.60	242		53277	18.84	46723	33
28	51635	18.55	228		54407	18.79	45593	32
29	52749	18.50	214		55535	18.74	44465	31
30	53859	18.45	199		56659	18.70	43341	30
31	9.054966	18.41	9.997185		9.057781	18.65	10.942219	29
32	56071	18.36	170		58900	18.60	41100	28
33	57172	18.31	156		60016	18.55	39984	27
34	58271	18.27	141		61130	18.51	38870	26
35	59367	18.22	127		62240	18.46	37760	25
36	60460	18.17	112		63348	18.42	36652	24
37	61551	18.13	098	.24	64453	18.37	35547	23
38	62639	18.08	083	.25	65556	18.33	34444	22
39	63724	18.04	068		66655	18.28	33345	21
40	64806	17.99	053		67752	18.24	32248	20
41	9.065885	17.94	9.997039		9.068846	18.19	10.931154	19
42	66962	17.90	024		69938	18.15	30062	18
43	68036	17.86	9.997009		71027	18.10	28973	17
44	69107	17.81	9.996994		72113	18.06	27887	16
45	70176	17.77	979		73197	18.02	26803	15
46	71242	17.72	964		74278	17.97	25722	14
47	72305	17.68	949		75356	17.93	24644	13
48	73366	17.63	934		76432	17.89	23568	12
49	74424	17.59	919		77505	17.84	22495	11
50	75480	17.55	904		78576	17.80	21424	10
51	9.076533	17.50	9.996889		9.079644	17.76	10.920356	9
52	77583	17.46	874		80710	17.72	19290	8
53	78631	17.42	858		81773	17.67	18227	7
54	79676	17.38	843		82833	17.63	17167	6
55	80719	17.33	828	.25	83891	17.59	16109	5
56	81759	17.29	812	.26	84947	17.55	15053	4
57	82797	17.25	797		86000	17.51	14000	3
58	83832	17.21	782		87050	17.47	12950	2
59	84864	17.17	766	.26	88098	17.43	11902	1
60	9.085894		9.996751		9.089144		10.910856	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	83°

7° SINES AND TANGENTS.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.085894	17.13	9.996751	.26	9.089144	17.38	10.910856	60
1	86922	17.09	735		90187	17.35	09813	59
2	87947	17.04	720		91228	17.30	08772	58
3	88970	17.00	704		92266	17.27	07734	57
4	89990	16.96	688		93302	17.22	06698	56
5	91008	16.92	673		94335	17.19	05664	55
6	92024	16.88	657		95367	17.15	04633	54
7	93037	16.84	641		96395	17.11	03605	53
8	94047	16.80	625		97422	17.07	02578	52
9	95056	16.76	610		98446	17.03	01554	51
10	96062	16.73	594	.26	99468	16.99	10.900532	50
11	97065	16.68	9.996578	.27	9.100487	16.95	10.899513	49
12	98066	16.65	562		01504	16.91	98496	48
13	9.099065	16.61	546		02519	16.87	97481	47
14	9.100062	16.57	530		03532	16.84	96468	46
15	01056	16.53	514		04542	16.80	95458	45
16	02048	16.49	498		05550	16.76	94450	44
17	03037	16.45	482		06556	16.72	93444	43
18	04025	16.42	465		07559	16.69	92441	42
19	05010	16.38	449		08560	16.65	91440	41
20	05992	16.34	433		09559	16.61	90441	40
21	9.106973	16.30	9.996417		9.110556	16.58	10.889444	39
22	07951	16.27	400		11551	16.54	88449	38
23	08927	16.23	384		12543	16.50	87457	37
24	09901	16.19	368		13533	16.47	86467	36
25	10873	16.16	351		14521	16.43	85479	35
26	11842	16.12	335		15507	16.39	84493	34
27	12809	16.08	318	.27	16491	16.36	83509	33
28	13774	16.05	302	.28	17472	16.32	82528	32
29	14737	16.01	285		18452	16.29	81548	31
30	15698	15.97	269		19429	16.25	80571	30
31	9.116656	15.94	9.996252		9.120404	16.22	10.879596	29
32	17613	15.90	235		21377	16.18	78623	28
33	18567	15.87	219		22348	16.15	77652	27
34	19519	15.83	202		23317	16.11	76683	26
35	20469	15.80	185		24284	16.08	75716	25
36	21417	15.76	168		25249	16.04	74751	24
37	22362	15.73	151		26211	16.01	73789	23
38	23306	15.69	134		27172	15.97	72828	22
39	24248	15.66	117		28130	15.94	71870	21
40	25187	15.62	100	.28	29087	15.91	70913	20
41	9.126125	15.59	9.996083	.29	9.130041	15.87	10.869959	19
42	27060	15.56	066		30994	15.84	69006	18
43	27993	15.52	049		31944	15.81	68056	17
44	28925	15.49	032		32893	15.77	67107	16
45	29854	15.45	9.996015		33839	15.74	66161	15
46	30781	15.42	9.995998		34784	15.71	65216	14
47	31706	15.39	980		35726	15.67	64274	13
48	32630	15.35	963		36667	15.64	63333	12
49	33551	15.32	946		37605	15.61	62395	11
50	34470	15.29	928		38542	15.58	61458	10
51	9.135387	15.25	9.995911		9.139476	15.55	10.860524	9
52	36303	15.22	894		40409	15.51	59591	8
53	37216	15.19	876		41340	15.48	58660	7
54	38128	15.16	859		42269	15.45	57731	6
55	39037	15.12	841		43196	15.42	56804	5
56	39944	15.09	823		44121	15.39	55879	4
57	40850	15.06	806		45044	15.35	54956	3
58	41754	15.03	788		45966	15.32	54034	2
59	42655	15.00	771	.29	46885	15.29	53115	1
60	9.143555		9.995753		9.147803		10.852197	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	82°

8° LOGARITHMIC

M.	Sine.	Diff. 1"	Cosine.	Diff. 1"	Tang.	Diff. 1"	Cotang.	M.
0	9.143555	14.96	9.995753	.30	9.147803	15.26	10.852197	60
1	4453	14.93	735		8718	15.23	1282	59
2	5349	14.90	717		9.149632	15.20	10.850368	58
3	6243	14.87	699		9.150544	15.17	10.849456	57
4	7136	14.84	681		1454	15.14	8546	56
5	8026	14.81	664		2363	15.11	7637	55
6	8915	14.78	646		3269	15.08	6731	54
7	9.149802	14.75	628		4174	15.05	5826	53
8	9.150686	14.72	610		5077	15.02	4923	52
9	1569	14.69	591		5978	14.99	4022	51
10	2451	14.66	573		6877	14.96	3123	50
11	3330	14.63	9.995555		7775	14.93	2225	49
12	4208	14.60	537		8671	14.90	1329	48
13	5083	14.57	519	.30	9.159565	14.87	10.840435	47
14	5957	14.54	501	.31	9.160457	14.84	10.839543	46
15	6830	14.51	482		1347	14.81	8653	45
16	7700	14.48	464		2236	14.78	7764	44
17	8569	14.45	446		3123	14.75	6877	43
18	9.159435	14.42	427		4008	14.73	5992	42
19	9.160301	14.39	409		4892	14.70	5108	41
20	1164	14.36	390		5774	14.67	4226	40
21	2025	14.33	9.995372		6654	14.64	3346	39
22	2885	14.30	353		7532	14.61	2468	38
23	3743	14.27	334		8409	14.58	1591	37
24	4600	14.24	316		9.169284	14.55	10.830716	36
25	5454	14.22	297		9.170157	14.53	10.829843	35
26	6307	14.19	278		1029	14.50	8971	34
27	7159	14.16	260	.31	1899	14.47	8101	33
28	8008	14.13	241	.32	2767	14.44	7233	32
29	8856	14.10	222		3634	14.42	6366	31
30	9.169702	14.07	203		4499	14.39	5501	30
31	9.170547	14.05	9.995184		5362	14.36	4638	29
32	1389	14.02	165		6224	14.33	3776	28
33	2230	13.99	146		7084	14.31	2916	27
34	3070	13.96	127		7942	14.28	2058	26
35	3908	13.94	108		8799	14.25	1201	25
36	4744	13.91	089		9.179655	14.23	10.820345	24
37	5578	13.88	070		9.180508	14.20	10.819492	23
38	6411	13.86	051		1360	14.17	8640	22
39	7242	13.83	032		2211	14.15	7789	21
40	8072	13.80	9.995013		3059	14.12	6941	20
41	8900	13.77	9.994993		3907	14.09	6093	19
42	9.179726	13.74	974		4752	14.07	5248	18
43	9.180551	13.72	955		5597	14.04	4403	17
44	1374	13.69	935	.32	6439	14.02	3561	16
45	2196	13.67	916	.33	7280	13.99	2720	15
46	3016	13.64	896		8120	13.96	1880	14
47	3834	13.61	877		8958	13.94	1042	13
48	4651	13.59	857		9.189794	13.91	10.810206	12
49	5466	13.56	838		9.190629	13.89	10.809371	11
50	6280	13.53	818		1462	13.86	8538	10
51	7092	13.51	9.994798		2294	13.84	7706	9
52	7903	13.48	779		3124	13.81	6876	8
53	8712	13.46	759		3953	13.79	6047	7
54	9.189519	13.43	739		4780	13.76	5220	6
55	9.190325	13.41	719		5606	13.74	4394	5
56	1130	13.38	700		6430	13.71	3570	4
57	1933	13.36	680		7253	13.69	2747	3
58	2734	13.33	660		8074	13.66	1926	2
59	3534	13.30	640	.33	8894	13.64	1106	1
60	9.194332		9.994620		9.199713		10.800287	0
	Cosine.	Diff. 1"	Sine.	Diff. 1"	Cotang.	Diff. 1"	Tang.	81°

9°			SINES AND TANGENTS.					
M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.194332	13.28	9.994620	.33	9.199713	13.61	10.800287	60
1	5129	13.26	600	.33	9.200529	13.59	10.799471	59
2	5925	13.23	580	.33	1345	13.56	8655	58
3	6719	13.21	560	.34	2159	13.54	7841	57
4	7511	13.18	540		2971	13.52	7029	56
5	8302	13.16	519		3782	13.49	6218	55
6	9091	13.13	499		4592	13.47	5408	54
7	9.199879	13.11	479		5400	13.45	4600	53
8	9.200666	13.08	459		6207	13.42	3793	52
9	1451	13.06	438		7013	13.40	2987	51
10	2234	13.04	418		7817	13.38	2183	50
11	3017	13.01	9.994397		8619	13.35	1381	49
12	3797	12.99	377		9.209420	13.33	10.790580	48
13	4577	12.96	357		9.210220	13.31	10.789780	47
14	5354	12.94	336		1018	13.28	8982	46
15	6131	12.92	316		1815	13.26	8185	45
16	6906	12.89	295	.34	2611	13.24	7389	44
17	7679	12.87	274	.35	3405	13.21	6595	43
18	8452	12.85	254		4198	13.19	5802	42
19	9222	12.82	233		4989	13.17	5011	41
20	9.209992	12.80	212		5780	13.15	4220	40
21	9.210760	12.78	9.994191		6568	13.12	3432	39
22	1526	12.75	171		7356	13.10	2644	38
23	2291	12.73	150		8142	13.08	1858	37
24	3055	12.71	129		8926	13.05	1074	36
25	3818	12.68	108		9.219710	13.03	10.780290	35
26	4579	12.66	087		9.220492	13.01	10.779508	34
27	5338	12.64	066		1272	12.99	8728	33
28	6097	12.61	045		2052	12.97	7948	32
29	6854	12.59	024		2830	12.94	7170	31
30	7609	12.57	9.994003		3606	12.92	6394	30
31	8363	12.55	9.993981		4382	12.90	5618	29
32	9116	12.53	960		5156	12.88	4844	28
33	9.219868	12.50	939		5929	12.86	4071	27
34	9.220618	12.48	918	.35	6700	12.84	3300	26
35	1367	12.46	896	.36	7471	12.81	2529	25
36	2115	12.44	875		8239	12.79	1761	24
37	2861	12.42	854		9007	12.77	0993	23
38	3606	12.39	832		9.229773	12.75	10.770227	22
39	4349	12.37	811		9.230539	12.73	10.769461	21
40	5092	12.35	789		1302	12.71	8698	20
41	5833	12.33	9.993768		2065	12.69	7935	19
42	6573	12.31	746		2826	12.67	7174	18
43	7311	12.28	725		3586	12.65	6414	17
44	8048	12.26	703		4345	12.62	5655	16
45	8784	12.24	681		5103	12.60	4897	15
46	9.229518	12.22	660		5859	12.58	4141	14
47	9.230252	12.20	638		6614	12.56	3386	13
48	0984	12.18	616	.36	7368	12.54	2632	12
49	1714	12.16	594	.37	8120	12.52	1880	11
50	2444	12.14	572		8872	12.50	1128	10
51	3172	12.12	9.993550		9.239622	12.48	10.760378	9
52	3899	12.09	528		9.240371	12.46	10.759629	8
53	4625	12.07	506		1118	12.44	8882	7
54	5349	12.05	484		1865	12.42	8135	6
55	6073	12.03	462		2610	12.40	7390	5
56	6795	12.01	440		3354	12.38	6646	4
57	7515	11.99	418		4097	12.36	5903	3
58	8235	11.97	396		4839	12.34	5161	2
59	8953	11.95	374	.37	5579	12.32	4421	1
60	9.239670		9.993351		9.246319		10.753681	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	80°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.239670	11.93	9.993351	.37	9.246319	12.30	10.753681	60
1	9.240386	11.91	329		7057	12.28	2943	59
2	1101	11.89	307		7794	12.26	2206	58
3	1814	11.87	285		8530	12.24	1470	57
4	2526	11.85	262		9264	12.22	0736	56
5	3237	11.83	240	.37	9.249998	12.20	10.750002	55
6	3947	11.81	217	.38	9.250730	12.18	10.749270	54
7	4656	11.79	195		1461	12.17	8539	53
8	5363	11.77	172		2191	12.15	7809	52
9	6069	11.75	149		2920	12.13	7080	51
10	6775	11.73	127		3648	12.11	6352	50
11	7478	11.71	9.993104		4374	12.09	5626	49
12	8181	11.69	081		5100	12.07	4900	48
13	8883	11.67	059		5824	12.05	4176	47
14	9.249583	11.65	036		6547	12.03	3453	46
15	9.250282	11.63	9.993013		7269	12.01	2731	45
16	0980	11.61	9.992990		7990	12.00	2010	44
17	1677	11.59	967		8710	11.98	1290	43
18	2373	11.58	944		9.259429	11.96	10.740571	42
19	3067	11.56	921		9.260146	11.94	10.739854	41
20	3761	11.54	898		0863	11.92	9137	40
21	4453	11.52	9.992875		1578	11.90	8422	39
22	5144	11.50	852	.38	2292	11.89	7708	38
23	5834	11.48	829	.39	3005	11.87	6995	37
24	6523	11.46	806		3717	11.85	6283	36
25	7211	11.44	783		4428	11.83	5572	35
26	7898	11.42	759		5138	11.81	4862	34
27	8583	11.41	736		5847	11.79	4153	33
28	9268	11.39	713		6555	11.78	3445	32
29	9.259951	11.37	690		7261	11.76	2739	31
30	9.260633	11.35	666		7967	11.74	2033	30
31	1314	11.33	9.992643		8671	11.72	1329	29
32	1994	11.31	619		9.269375	11.70	10.730625	28
33	2673	11.30	596		9.270077	11.69	10.729923	27
34	3351	11.28	572		0779	11.67	9221	26
35	4027	11.26	549		1479	11.65	8521	25
36	4703	11.24	525		2178	11.64	7822	24
37	5377	11.22	501	.39	2876	11.62	7124	23
38	6051	11.20	478	.40	3573	11.60	6427	22
39	6723	11.19	454		4269	11.58	5731	21
40	7395	11.17	430		4964	11.57	5036	20
41	8065	11.15	9.992406		5658	11.55	4342	19
42	8734	11.13	382		6351	11.53	3649	18
43	9.269402	11.12	359		7043	11.51	2957	17
44	9.270069	11.10	335		7734	11.50	2266	16
45	0735	11.08	311		8424	11.48	1576	15
46	1400	11.06	287		9113	11.46	0887	14
47	2064	11.05	263		9.279801	11.45	10.720199	13
48	2726	11.03	239		9.280488	11.43	10.719512	12
49	3388	11.01	214		1174	11.41	8826	11
50	4049	10.99	190		1858	11.40	8142	10
51	4708	10.98	9.992166		2542	11.38	7458	9
52	5367	10.96	142	.40	3225	11.36	6775	8
53	6024	10.94	117	.41	3907	11.35	6093	7
54	6681	10.92	093		4588	11.33	5412	6
55	7337	10.91	069		5268	11.31	4732	5
56	7991	10.89	044		5947	11.30	4053	4
57	8644	10.87	9.992020		6624	11.28	3376	3
58	9297	10.86	9.991996		7301	11.26	2699	2
59	9.279948	10.84	971	.41	7977	11.25	2023	1
60	9.280599		9.991947		9.288652		10.711348	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	79°

11°			SINES AND TANGENTS.					
M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.280599	10.82	9.991947	.41	9.288652	11.23	10.711348	60
1	1248	10.81	922		9326	11.22	0674	59
2	1897	10.79	897		9.289999	11.20	10.710001	58
3	2544	10.77	873		9.290671	11.18	10.709329	57
4	3190	10.76	848		1342	11.17	8658	56
5	3836	10.74	823		2013	11.15	7987	55
6	4480	10.72	799	.41	2682	11.14	7318	54
7	5124	10.71	774	.42	3350	11.12	6650	53
8	5766	10.69	749		4017	11.11	5983	52
9	6408	10.67	724		4684	11.09	5316	51
10	7048	10.66	699		5349	11.07	4651	50
11	7687	10.64	9.991674		9.296013	11.06	10.703987	49
12	8326	10.63	649		6677	11.04	3323	48
13	8964	10.61	624		7339	11.03	2661	47
14	9.289600	10.59	599		8001	11.01	1999	46
15	9.290236	10.58	574		8662	11.00	1338	45
16	0870	10.56	549		9322	10.98	0678	44
17	1504	10.54	524		9.299980	10.96	10.700020	43
18	2137	10.53	498		9.300638	10.95	10.699362	42
19	2768	10.51	473		1295	10.93	8705	41
20	3399	10.50	448		1951	10.92	8049	40
21	9.294029	10.48	9.991422		2607	10.90	7393	39
22	4658	10.46	397	.42	3261	10.89	6739	38
23	5286	10.45	372	.43	3914	10.87	6086	37
24	5913	10.43	346		4567	10.86	5433	36
25	6539	10.42	321		5218	10.84	4782	35
26	7164	10.40	295		5869	10.83	4131	34
27	7788	10.39	270		6519	10.81	3481	33
28	8412	10.37	244		7168	10.80	2832	32
29	9034	10.36	218		7816	10.78	2184	31
30	9.299655	10.34	193		8463	10.77	1537	30
31	9.300276	10.32	9.991167		9109	10.75	0891	29
32	0895	10.31	141		9.309754	10.74	10.690246	28
33	1514	10.29	115		9.310398	10.73	10.689602	27
34	2132	10.28	090		1042	10.71	8958	26
35	2748	10.26	064		1685	10.70	8315	25
36	3364	10.25	038		2327	10.68	7673	24
37	3979	10.23	9.991012		2967	10.67	7033	23
38	4593	10.22	9.990986		3608	10.65	6392	22
39	5207	10.20	960	.43	4247	10.64	5753	21
40	5819	10.19	934	.44	4885	10.62	5115	20
41	6430	10.17	9.990908		9.315523	10.61	10.684477	19
42	7041	10.16	882		6159	10.60	3841	18
43	7650	10.14	855		6795	10.58	3205	17
44	8259	10.13	829		7430	10.57	2570	16
45	8867	10.11	803		8064	10.55	1936	15
46	9.309474	10.10	777		8697	10.54	1303	14
47	9.310080	10.08	750		9330	10.53	0670	13
48	0685	10.07	724		9.319961	10.51	10.680039	12
49	1289	10.06	697		9.320592	10.50	10.679408	11
50	1893	10.04	671		1222	10.48	8778	10
51	2495	10.03	9.990644		1851	10.47	8149	9
52	3097	10.01	618		2479	10.45	7521	8
53	3698	10.00	591		3106	10.44	6894	7
54	4297	9.98	565		3733	10.43	6267	6
55	4897	9.97	538	.44	4358	10.41	5642	5
56	5495	9.96	511	.45	4983	10.40	5017	4
57	6092	9.94	485		5607	10.39	4393	3
58	6689	9.93	458		6231	10.37	3769	2
59	7284	9.91	431	.45	6853	10.36	3147	1
60	9.317879		9.990404		9.327475		10.672525	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	78°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.317879	9.90	9.990404	.45	9.327475	10.35	10.672525	60
1	8473	9.88	378		8095	10.33	1905	59
2	9066	9.87	351		8715	10.32	1285	58
3	9.319658	9.86	324		9334	10.30	0666	57
4	9.320249	9.84	297		9.329953	10.29	10.670047	56
5	0840	9.83	270		9.330570	10.28	10.669430	55
6	1430	9.82	243		1187	10.26	8813	54
7	2019	9.80	215		1803	10.25	8197	53
8	2607	9.79	188		2418	10.24	7582	52
9	3194	9.77	161		3033	10.23	6967	51
10	3780	9.76	134	.45	3646	10.21	6354	50
11	9.324366	9.75	9.990107	.46	9.334259	10.20	10.665741	49
12	4950	9.73	079		4871	10.19	5129	48
13	5534	9.72	052		5482	10.17	4518	47
14	6117	9.70	9.990025		6093	10.16	3907	46
15	6700	9.69	9.989997		6702	10.15	3298	45
16	7281	9.68	970		7311	10.13	2689	44
17	7862	9.66	942		7919	10.12	2081	43
18	8442	9.65	915		8527	10.11	1473	42
19	9021	9.64	887		9133	10.10	0867	41
20	9.329599	9.62	860		9.339739	10.08	10.660261	40
21	9.330176	9.61	9.989832		9.340344	10.07	10.659656	39
22	0753	9.60	804		0948	10.06	9052	38
23	1329	9.58	777	.46	1552	10.04	8448	37
24	1903	9.57	749	.47	2155	10.03	7845	36
25	2478	9.56	721		2757	10.02	7243	35
26	3051	9.54	693		3358	10.01	6642	34
27	3624	9.53	665		3958	9.99	6042	33
28	4195	9.52	637		4558	9.98	5442	32
29	4767	9.50	609		5157	9.97	4843	31
30	5337	9.49	582		5755	9.96	4245	30
31	9.335906	9.48	9.989553		9.346353	9.94	10.653647	29
32	6475	9.46	525		6949	9.93	3051	28
33	7043	9.45	497		7545	9.92	2455	27
34	7610	9.44	469		8141	9.91	1859	26
35	8176	9.43	441		8735	9.90	1265	25
36	8742	9.41	413		9329	9.88	0671	24
37	9307	9.40	384		9.349922	9.87	10.650078	23
38	9.339871	9.39	356		9.350514	9.86	10.649486	22
39	9.340434	9.37	328		1106	9.85	8894	21
40	0996	9.36	300		1697	9.83	8303	20
41	1558	9.35	9.989271		2287	9.82	7713	19
42	2119	9.34	243		2876	9.81	7124	18
43	2679	9.32	214		3465	9.80	6535	17
44	3239	9.31	186		4053	9.79	5947	16
45	3797	9.30	157	.47	4640	9.77	5360	15
46	4355	9.29	128	.48	5227	9.76	4773	14
47	4912	9.27	100		5813	9.75	4187	13
48	5469	9.26	071		6398	9.74	3602	12
49	6024	9.25	042		6982	9.73	3018	11
50	6579	9.24	9.989014		7566	9.71	2434	10
51	9.347134	9.22	9.988985		9.358149	9.70	1851	9
52	7687	9.21	956		8731	9.69	1269	8
53	8240	9.20	927		9313	9.68	0687	7
54	8792	9.19	898		9.359893	9.67	10.640107	6
55	9343	9.17	869		9.360474	9.66	10.639526	5
56	9.349893	9.16	840	.48	1053	9.65	8947	4
57	9.350443	9.15	811	.49	1632	9.63	8368	3
58	0992	9.14	782	.49	2210	9.62	7790	2
59	1540	9.13	753	.49	2787	9.61	7213	1
60	9.352088		9.988724		9.363364		10.636636	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	77°

13° SINES AND TANGENTS.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.352088	9.11	9.988724	.49	9.363364	9.60	10.636636	60
1	2635	9.10	8695		3940	9.59	6060	59
2	3181	9.09	8666		4515	9.58	5485	58
3	3726	9.08	8636		5090	9.57	4910	57
4	4271	9.07	8607		5664	9.55	4336	56
5	4815	9.05	8578		6237	9.54	3763	55
6	5358	9.04	8548		6810	9.53	3190	54
7	5901	9.03	8519		7382	9.52	2618	53
8	6443	9.02	8489		7953	9.51	2047	52
9	6984	9.01	8460		8524	9.50	1476	51
10	7524	8.99	8430		9094	9.49	0906	50
11	8064	8.98	9.988401		9.369663	9.48	10.630337	49
12	8603	8.97	8371		9.370232	9.46	10.629768	48
13	9141	8.96	8342	.49	0799	9.45	9201	47
14	9.359678	8.95	8312	.50	1367	9.44	8633	46
15	9.360215	8.93	8282		1933	9.43	8067	45
16	0752	8.92	8252		2499	9.42	7501	44
17	1287	8.91	8223		3064	9.41	6936	43
18	1822	8.90	8193		3629	9.40	6371	42
19	2356	8.89	8163		4193	9.39	5807	41
20	2889	8.88	8133		4756	9.38	5244	40
21	9.363422	8.87	9.988103		9.375319	9.37	10.624681	39
22	3954	8.85	8073		5881	9.35	4119	38
23	4485	8.84	8043		6442	9.34	3558	37
24	5016	8.83	8013		7003	9.33	2997	36
25	5546	8.82	7983		7563	9.32	2437	35
26	6075	8.81	7953		8122	9.31	1878	34
27	6604	8.80	7922		8681	9.30	1319	33
28	7131	8.79	7892		9239	9.29	0761	32
29	7659	8.78	7862	.50	9797	9.28	0203	31
30	8185	8.76	7832	.51	9.380354	9.27	10.619646	30
31	8711	8.75	9.987801		9.380910	9.26	10.619090	29
32	9236	8.74	7771		1466	9.25	8534	28
33	9.369761	8.73	7740		2020	9.24	7980	27
34	9.370285	8.72	7710		2575	9.23	7425	26
35	0808	8.71	7679		3129	9.22	6871	25
36	1330	8.70	7649		3682	9.21	6318	24
37	1852	8.69	7618		4234	9.20	5766	23
38	2373	8.67	7588		4786	9.19	5214	22
39	2894	8.66	7557		5337	9.18	4663	21
40	3414	8.65	7526		5888	9.17	4112	20
41	9.373933	8.64	9.987496		9.386438	9.15	10.613562	19
42	4452	8.63	7465		6987	9.14	3013	18
43	4970	8.62	7434	.51	7536	9.13	2464	17
44	5487	8.61	7403	.52	8084	9.12	1916	16
45	6003	8.60	7372		8631	9.11	1369	15
46	6519	8.59	7341		9178	9.10	0822	14
47	7035	8.58	7310		9.389724	9.09	10.610276	13
48	7549	8.57	7279		9.390270	9.08	10.609730	12
49	8063	8.56	7248		0815	9.07	9185	11
50	8577	8.54	7217		1360	9.06	8640	10
51	9089	8.53	9.987186		1903	9.05	8097	9
52	9.379601	8.52	7155		2447	9.04	7553	8
53	9.380113	8.51	7124		2989	9.03	7011	7
54	0624	8.50	7092		3531	9.02	6469	6
55	1134	8.49	7061		4073	9.01	5927	5
56	1643	8.48	7030		4614	9.00	5386	4
57	2152	8.47	6998		5154	8.99	4846	3
58	2661	8.46	6967		5694	8.98	4306	2
59	3168	8.45	6936	.52	6233	8.97	3767	1
60	9.383675		9.986904		9.396771		10.603229	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	76°

M.	Sine.	Diff. 1"	Cosine.	Diff. 1"	Tang.	Diff. 1"	Cotang.	M.
0	9.383675	8.44	9.986904	.52	9.396771	8.96	10.603229	60
1	4182	8.43	6873	.53	7309	8.96	2691	59
2	4687	8.42	6841		7846	8.95	2154	58
3	5192	8.41	6809		8383	8.94	1617	57
4	5697	8.40	6778		8919	8.93	1081	56
5	6201	8.39	6746		9455	8.92	0545	55
6	6704	8.38	6714		9.399990	8.91	10.600010	54
7	7207	8.37	6683		9.400524	8.90	10.599476	53
8	7709	8.36	6651		1058	8.89	8942	52
9	8210	8.35	6619		1591	8.88	8409	51
10	8711	8.34	6587		2124	8.87	7876	50
11	9211	8.33	9.986555		2656	8.86	7344	49
12	9.389711	8.32	6523		3187	8.85	6813	48
13	9.390210	8.31	6491		3718	8.84	6282	47
14	0708	8.30	6459		4249	8.83	5751	46
15	1206	8.28	6427		4778	8.82	5222	45
16	1703	8.27	6395	.53	5308	8.81	4692	44
17	2199	8.26	6363	.54	5836	8.80	4164	43
18	2695	8.25	6331		6364	8.79	3636	42
19	3191	8.24	6299		6892	8.78	3108	41
20	3685	8.23	6266		7419	8.77	2581	40
21	9.394179	8.22	9.986234		9.407945	8.76	10.592055	39
22	4673	8.21	6202		8471	8.75	1529	38
23	5166	8.20	6169		8997	8.74	1003	37
24	5658	8.20	6137		9.409521	8.74	10.590479	36
25	6150	8.18	6104		9.410045	8.73	10.589955	35
26	6641	8.17	6072		0569	8.72	9431	34
27	7132	8.17	6039		1092	8.71	8908	33
28	7621	8.16	6007		1615	8.70	8385	32
29	8111	8.15	5974		2137	8.69	7863	31
30	8600	8.14	5942	.54	2658	8.68	7342	30
31	9088	8.13	9.985909	.55	9.413179	8.67	10.586821	29
32	9.399575	8.12	5876		3699	8.66	6301	28
33	9.400062	8.11	5843		4219	8.65	5781	27
34	0549	8.10	5811		4738	8.64	5262	26
35	1035	8.09	5778		5257	8.64	4743	25
36	1520	8.08	5745		5775	8.63	4225	24
37	2005	8.07	5712		6293	8.62	3707	23
38	2489	8.06	5679		6810	8.61	3190	22
39	2972	8.05	5646		7326	8.60	2674	21
40	3455	8.04	5613		7842	8.59	2158	20
41	9.403938	8.03	9.985580		8358	8.58	1642	19
42	4420	8.02	5547		8873	8.57	1127	18
43	4901	8.01	5514		9387	8.56	0613	17
44	5382	8.00	5480		9.419901	8.55	10.580099	16
45	5862	7.99	5447	.55	9.420415	8.55	10.579585	15
46	6341	7.98	5414	.56	0927	8.54	9073	14
47	6820	7.97	5380		1440	8.53	8560	13
48	7299	7.96	5347		1952	8.52	8048	12
49	7777	7.95	5314		2463	8.51	7537	11
50	8254	7.94	5280		2974	8.50	7026	10
51	8731	7.94	9.985247		9.423484	8.49	10.576516	9
52	9207	7.93	5213		3993	8.48	6007	8
53	9.409682	7.92	5180		4503	8.48	5497	7
54	9.410157	7.91	5146		5011	8.47	4989	6
55	0632	7.90	5113		5519	8.46	4481	5
56	1106	7.89	5079		6027	8.45	3973	4
57	1579	7.88	5045		6534	8.44	3466	3
58	2052	7.87	5011		7041	8.43	2959	2
59	2524	7.86	4978	.56	7547	8.43	2453	1
60	9.412996		9.984944		9.428052		10.571948	0
	Cosine.	Diff. 1"	Sine.	Diff. 1"	Cotang.	Diff. 1"	Tang.	75°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.412996	7.85	9.984944	.57	9.428052	8.42	10.571948	60
1	3467	7.84	4910		8558	8.41	1442	59
2	3938	7.83	4876		9062	8.40	0938	58
3	4408	7.83	4842		9.429566	8.39	10.570434	57
4	4878	7.82	4808		9.430070	8.38	10.569930	56
5	5347	7.81	4774		0573	8.38	9427	55
6	5815	7.80	4740		1075	8.37	8925	54
7	6283	7.79	4706		1577	8.36	8423	53
8	6751	7.78	4672		2079	8.35	7921	52
9	7217	7.77	4637		2580	8.34	7420	51
10	7684	7.76	4603		3080	8.33	6920	50
11	9.418150	7.75	9.984569		9.433580	8.32	10.566420	49
12	8615	7.74	4535		4080	8.32	5920	48
13	9079	7.73	4500		4579	8.31	5421	47
14	9.419544	7.73	4466	.57	5078	8.30	4922	46
15	9.420007	7.72	4432	.58	5576	8.29	4424	45
16	0470	7.71	4397		6073	8.28	3927	44
17	0933	7.70	4363		6570	8.28	3430	43
18	1395	7.69	4328		7067	8.27	2933	42
19	1857	7.68	4294		7563	8.26	2437	41
20	2318	7.67	4259		8059	8.25	1941	40
21	9.422778	7.67	9.984224		8554	8.24	1446	39
22	3238	7.66	4190		9048	8.23	0952	38
23	3697	7.65	4155		9.439543	8.23	10.560457	37
24	4156	7.64	4120		9.440036	8.22	10.559964	36
25	4615	7.63	4085		0529	8.21	9471	35
26	5073	7.62	4050		1022	8.20	8978	34
27	5530	7.61	4015		1514	8.19	8486	33
28	5987	7.60	3981		2006	8.19	7994	32
29	6443	7.60	3946		2497	8.18	7503	31
30	6899	7.59	3911		2988	8.17	7012	30
31	9.427354	7.58	9.983875	.58	9.443479	8.16	10.556521	29
32	7809	7.57	3840	.59	3968	8.16	6032	28
33	8263	7.56	3805		4458	8.15	5542	27
34	8717	7.55	3770		4947	8.14	5053	26
35	9170	7.54	3735		5435	8.13	4565	25
36	9.429623	7.54	3700		5923	8.12	4077	24
37	9.430075	7.53	3664		6411	8.12	3589	23
38	0527	7.52	3629		6898	8.11	3102	22
39	0978	7.51	3594		7384	8.10	2616	21
40	1429	7.50	3558		7870	8.09	2130	20
41	9.431879	7.49	9.983523		8356	8.09	1644	19
42	2329	7.49	3487		8841	8.08	1159	18
43	2778	7.48	3452		9326	8.07	0674	17
44	3226	7.47	3416		9.449810	8.06	10.550190	16
45	3675	7.46	3381		9.450294	8.06	10.549706	15
46	4122	7.45	3345		0777	8.05	9223	14
47	4569	7.44	3309	.59	1260	8.04	8740	13
48	5016	7.44	3273	.60	1743	8.03	8257	12
49	5462	7.43	3238		2225	8.02	7775	11
50	5908	7.42	3202		2706	8.02	7294	10
51	9.436353	7.41	9.983166		9.453187	8.01	10.546813	9
52	6798	7.40	3130		3668	8.00	6332	8
53	7242	7.40	3094		4148	7.99	5852	7
54	7686	7.39	3058		4628	7.99	5372	6
55	8129	7.38	3022		5107	7.98	4893	5
56	8572	7.37	2986		5586	7.97	4414	4
57	9014	7.36	2950		6064	7.96	3936	3
58	9456	7.36	2914		6542	7.96	3458	2
59	9.439897	7.35	2878	.60	7019	7.95	2981	1
60	9.440338		9.982842		9.457496		10.542504	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	74°

16° LOGARITHMIC

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.440338	7.34	9.982842	.60	9.457496	7.94	10.542504	60
1	0778	7.33	2805	.60	7973	7.93	2027	59
2	1218	7.32	2769	.61	8449	7.93	1551	58
3	1658	7.31	2733		8925	7.92	1075	57
4	2096	7.31	2696		9400	7.91	0600	56
5	2535	7.30	2660		9.459875	7.90	10.540125	55
6	2973	7.29	2624		9.460349	7.90	10.539651	54
7	3410	7.28	2587		0823	7.89	9177	53
8	3847	7.27	2551		1297	7.88	8703	52
9	4284	7.27	2514		1770	7.88	8230	51
10	4720	7.26	2477		2242	7.87	7758	50
11	9.445155	7.25	9.982441		9.462715	7.86	10.537285	49
12	5590	7.24	2404		3186	7.85	6814	48
13	6025	7.23	2367		3658	7.85	6342	47
14	6459	7.23	2331		4128	7.84	5872	46
15	6893	7.22	2294		4599	7.83	5401	45
16	7326	7.21	2257	.61	5069	7.83	4931	44
17	7759	7.20	2220	.62	5539	7.82	4461	43
18	8191	7.20	2183		6008	7.81	3992	42
19	8623	7.19	2146		6477	7.80	3523	41
20	9054	7.18	2109		6945	7.80	3055	40
21	9485	7.17	9.982072		9.467413	7.79	10.532587	39
22	9.449915	7.16	2035		7880	7.78	2120	38
23	9.450345	7.16	1998		8347	7.78	1653	37
24	0775	7.15	1961		8814	7.77	1186	36
25	1204	7.14	1924		9280	7.76	0720	35
26	1632	7.13	1886		9.469746	7.75	10.530254	34
27	2060	7.13	1849		9.470211	7.75	10.529789	33
28	2488	7.12	1812		0676	7.74	9324	32
29	2915	7.11	1774		1141	7.73	8859	31
30	3342	7.10	1737	.62	1605	7.73	8395	30
31	9.453768	7.10	9.981699	.63	9.472069	7.72	7931	29
32	4194	7.09	1662		2532	7.71	7468	28
33	4619	7.08	1625		2995	7.71	7005	27
34	5044	7.07	1587		3457	7.70	6543	26
35	5469	7.07	1549		3919	7.69	6081	25
36	5893	7.06	1512		4381	7.69	5619	24
37	6316	7.05	1474		4842	7.68	5158	23
38	6739	7.04	1436		5303	7.67	4697	22
39	7162	7.04	1399		5763	7.67	4237	21
40	7584	7.03	1361		6223	7.66	3777	20
41	8006	7.02	9.981323		9.476683	7.65	10.523317	19
42	8427	7.01	1285		7142	7.65	2858	18
43	8848	7.01	1247		7601	7.64	2399	17
44	9268	7.00	1209		8059	7.63	1941	16
45	9.459688	6.99	1171	.63	8517	7.63	1483	15
46	9.460108	6.98	1133	.64	8975	7.62	1025	14
47	0527	6.98	1095		9432	7.61	0568	13
48	0946	6.97	1057		9.479889	7.61	10.520111	12
49	1364	6.96	1019		9.480345	7.60	10.519655	11
50	1782	6.95	0981		0801	7.59	9199	10
51	2199	6.95	9.980942		1257	7.59	8743	9
52	2616	6.94	0904		1712	7.58	8288	8
53	3032	6.93	0866		2167	7.57	7833	7
54	3448	6.93	0827		2621	7.57	7379	6
55	3864	6.92	0789		3075	7.56	6925	5
56	4279	6.91	0750		3529	7.55	6471	4
57	4694	6.90	0712		3982	7.55	6018	3
58	5108	6.90	0673		4435	7.54	5565	2
59	5522	6.89	0635	.64	4887	7.53	5113	1
60	9.465935		9.980596		9.485339		10.514661	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	73°

17°	SINES AND TANGENTS.							
M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.465935	6.88	9.980596	.64	9.485339	7.53	10.514661	60
1	6348	6.88	0558	.64	5791	7.52	4209	59
2	6761	6.87	0519	.65	6242	7.51	3758	58
3	7173	6.86	0480		6693	7.51	3307	57
4	7585	6.85	0442		7143	7.50	2857	56
5	7996	6.85	0403		7593	7.49	2407	55
6	8407	6.84	0364		8043	7.49	1957	54
7	8817	6.83	0325		8492	7.48	1508	53
8	9227	6.83	0286		8941	7.47	1059	52
9	9.469637	6.82	0247		9390	7.47	0610	51
10	9.470046	6.81	0208		9.489838	7.46	10.510162	50
11	0455	6.80	9.980169		9.490286	7.46	10.509714	49
12	0863	6.80	0130		0733	7.45	9267	48
13	1271	6.79	0091		1180	7.44	8820	47
14	1679	6.78	0052		1627	7.44	8373	46
15	2086	6.78	9.980012		2073	7.43	7927	45
16	2492	6.77	9.979973	.65	2519	7.43	7481	44
17	2898	6.76	9934	.66	2965	7.42	7035	43
18	3304	6.76	9895		3410	7.41	6590	42
19	3710	6.75	9855		3854	7.40	6146	41
20	4115	6.74	9816		4299	7.40	5701	40
21	9.474519	6.74	9.979776		9.494743	7.39	10.505257	39
22	4923	6.73	9737		5186	7.39	4814	38
23	5327	6.72	9697		5630	7.38	4370	37
24	5730	6.72	9658		6073	7.37	3927	36
25	6133	6.71	9618		6515	7.37	3485	35
26	6536	6.70	9579		6957	7.36	3043	34
27	6938	6.69	9539		7399	7.36	2601	33
28	7340	6.69	9499		7841	7.35	2159	32
29	7741	6.68	9459		8282	7.34	1718	31
30	8142	6.67	9420		8722	7.34	1278	30
31	8542	6.67	9.979380		9163	7.33	0837	29
32	8942	6.66	9340	.66	9.499603	7.33	10.500397	28
33	9342	6.65	9300	.67	9.500042	7.32	10.499958	27
34	9.479741	6.65	9260		0481	7.31	9519	26
35	9.480140	6.64	9220		0920	7.31	9080	25
36	0539	6.63	9180		1359	7.30	8641	24
37	0937	6.63	9140		1797	7.30	8203	23
38	1334	6.62	9100		2235	7.29	7765	22
39	1731	6.61	9059		2672	7.28	7328	21
40	2128	6.61	9019		3109	7.28	6891	20
41	9.482525	6.60	9.978979		9.503546	7.27	10.496454	19
42	2921	6.59	8939		3982	7.27	6018	18
43	3316	6.59	8898		4418	7.26	5582	17
44	3712	6.58	8858		4854	7.25	5146	16
45	4107	6.57	8817		5289	7.25	4711	15
46	4501	6.57	8777		5724	7.24	4276	14
47	4895	6.56	8737	.67	6159	7.24	3841	13
48	5289	6.55	8696	.68	6593	7.23	3407	12
49	5682	6.55	8655		7027	7.22	2973	11
50	6075	6.54	8615		7460	7.22	2540	10
51	9.486467	6.53	9.978574		7893	7.21	2107	9
52	6860	6.53	8533		8326	7.21	1674	8
53	7251	6.52	8493		8759	7.20	1241	7
54	7643	6.51	8452		9191	7.19	0809	6
55	8034	6.51	8411		9.509622	7.19	10.490378	5
56	8424	6.50	8370		9.510054	7.18	10.489946	4
57	8814	6.50	8329		0485	8.18	9515	3
58	9204	6.49	8288		0916	7.17	9084	2
59	9593	6.48	8247	.68	1346	7.17	8654	1
60	9.489982		9.978206		9.511776		10.488224	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	72°

18°				LOGARITHMIC				
M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.489982	6.48	9.978206	.68	9.511776	7.16	10.488224	60
1	9.490371	6.47	8165		2206	7.16	7794	59
2	0759	6.46	8124	.68	2635	7.15	7365	58
3	1147	6.46	8083	.69	3064	7.14	6936	57
4	1535	6.45	8042		3493	7.14	6507	56
5	1922	6.44	8001		3921	7.13	6079	55
6	2308	6.44	7959		4349	7.13	5651	54
7	2695	6.43	7918		4777	7.12	5223	53
8	3081	6.42	7877		5204	7.12	4796	52
9	3466	6.42	7835		5631	7.11	4369	51
10	3851	6.41	7794		6057	7.10	3943	50
11	9.494236	6.41	9.977752		9.516484	7.10	10.483516	49
12	4621	6.40	7711		6910	7.09	3090	48
13	5005	6.39	7669		7335	7.09	2665	47
14	5388	6.39	7628		7761	7.08	2239	46
15	5772	6.38	7586	.69	8184	7.08	1816	45
16	6154	6.37	7544	.70	8610	7.07	1390	44
17	6537	6.37	7503		9034	7.06	0966	43
18	6919	6.36	7461		9458	7.06	0542	42
19	7301	6.36	7419		9.519882	7.05	10.480118	41
20	7682	6.35	7377		9.520305	7.05	10.479695	40
21	9.498064	6.34	9.977335		0728	7.04	9272	39
22	8444	6.34	7293		1151	7.04	8849	38
23	8825	6.33	7251		1573	7.03	8427	37
24	9204	6.32	7209		1995	7.03	8005	36
25	9584	6.32	7167		2417	7.02	7583	35
26	9.499963	6.31	7125		2838	7.02	7162	34
27	9.500342	6.31	7083		3259	7.01	6741	33
28	0721	6.30	7041		3680	7.01	6320	32
29	1099	6.29	6999		4100	7.00	5900	31
30	1476	6.29	6957		4520	6.99	5480	30
31	9.501854	6.28	9.976914	.70	9.524940	6.99	10.475060	29
32	2231	6.28	6872	.71	5359	6.98	4641	28
33	2607	6.27	6830		5778	6.98	4222	27
34	2984	6.26	6787		6197	6.97	3803	26
35	3360	6.26	6745		6615	6.97	3385	25
36	3735	6.25	6702		7033	6.96	2967	24
37	4110	6.25	6660		7451	6.96	2549	23
38	4485	6.24	6617		7868	6.95	2132	22
39	4860	6.23	6574		8285	6.95	1715	21
40	5234	6.23	6532		8702	6.94	1298	20
41	9.505608	6.22	9.976489		9.529119	6.93	10.470881	19
42	5981	6.22	6446		9535	6.93	0465	18
43	6354	6.21	6404		9.529950	6.93	10.470050	17
44	6727	6.20	6361		9.530366	6.92	10.469634	16
45	7099	6.20	6318		0781	6.91	9219	15
46	7471	6.19	6275	.71	1196	6.91	8804	14
47	7843	6.19	6232	.72	1611	6.90	8389	13
48	8214	6.18	6189		2025	6.90	7975	12
49	8585	6.18	6146		2439	6.89	7561	11
50	8956	6.17	6103		2853	6.89	7147	10
51	9326	6.16	9.976060		9.533266	6.88	10.466734	9
52	9.509696	6.16	6017		3679	6.88	6321	8
53	9.510065	6.15	5974		4092	6.87	5908	7
54	0434	6.15	5930		4504	6.87	5496	6
55	0803	6.14	5887		4916	6.86	5084	5
56	1172	6.13	5844		5328	6.86	4672	4
57	1540	6.13	5800		5739	6.85	4261	3
58	1907	6.12	5757		6150	6.85	3850	2
59	2275	6.12	5714	.72	6561	6.84	3439	1
60	9.512642		9.975670		9.536972		10.463028	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	71°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.512642	6.11	9.975670	.73	9.536972	6.84	10.463028	60
1	3009	6.11	5627		7382	6.83	2618	59
2	3375	6.10	5583		7792	6.83	2208	58
3	3741	6.09	5539		8202	6.82	1798	57
4	4107	6.09	5496		8611	6.82	1389	56
5	4472	6.08	5452		9020	6.81	0980	55
6	4837	6.08	5408		9429	6.81	0571	54
7	5202	6.07	5365		9.539837	6.80	10.460163	53
8	5566	6.07	5321		9.540245	6.80	10.459755	52
9	5930	6.06	5277		0653	6.79	9347	51
10	6294	6.05	5233		1061	6.79	8939	50
11	9.516657	6.05	9.975189		1468	6.78	8532	49
12	7020	6.04	5145		1875	6.78	8125	48
13	7382	6.04	5101		2281	6.77	7719	47
14	7745	6.03	5057		2688	6.77	7312	46
15	8107	6.03	5013	.73	3094	6.76	6906	45
16	8468	6.02	4969	.74	3499	6.76	6501	44
17	8829	6.01	4925		3905	6.75	6095	43
18	9190	6.01	4880		4310	6.75	5690	42
19	9551	6.00	4836		4715	6.74	5285	41
20	9.519911	6.00	4792		5119	6.74	4881	40
21	9.520271	5.99	9.974748		9.545524	6.73	10.454476	39
22	0631	5.99	4703		5928	6.73	4072	38
23	0990	5.98	4659		6331	6.72	3669	37
24	1349	5.98	4614		6735	6.72	3265	36
25	1707	5.97	4570		7138	6.71	2862	35
26	2066	5.96	4525		7540	6.71	2460	34
27	2424	5.96	4481		7943	6.70	2057	33
28	2781	5.95	4436		8345	6.70	1655	32
29	3138	5.95	4391	.74	8747	6.69	1253	31
30	3495	5.94	4347	.75	9149	6.69	0851	30
31	9.523852	5.94	9.974302		9550	6.68	0450	29
32	4208	5.93	4257		9.549951	6.68	10.450049	28
33	4564	5.93	4212		9.550352	6.67	10.449648	27
34	4920	5.92	4167		0752	6.67	9248	26
35	5275	5.91	4122		1153	6.66	8847	25
36	5630	5.91	4077		1552	6.66	8448	24
37	5984	5.90	4032		1952	6.65	8048	23
38	6339	5.90	3987		2351	6.65	7649	22
39	6693	5.89	3942		2750	6.65	7250	21
40	7046	5.89	3897		3149	6.64	6851	20
41	9.527400	5.88	9.973852		9.553548	6.64	10.446452	19
42	7753	5.88	3807		3946	6.63	6054	18
43	8105	5.87	3761	.75	4344	6.63	5656	17
44	8458	5.87	3716	.76	4741	6.62	5259	16
45	8810	5.86	3671		5139	6.62	4861	15
46	9161	5.86	3625		5536	6.61	4464	14
47	9513	5.85	3580		5933	6.61	4067	13
48	9.529864	5.85	3535		6329	6.60	3671	12
49	9.530215	5.84	3489		6725	6.60	3275	11
50	0565	5.84	3444		7121	6.59	2879	10
51	0915	5.83	9.973398		9.557517	6.59	10.442483	9
52	1265	5.82	3352		7913	6.59	2087	8
53	1614	5.82	3307		8308	6.58	1692	7
54	1963	5.81	3261		8703	6.58	1297	6
55	2312	5.81	3215		9097	6.57	0903	5
56	2661	5.80	3169		9491	6.57	0509	4
57	3009	5.80	3124		9.559885	6.56	10.440115	3
58	3357	5.79	3078	.76	9.560279	6.56	10.439721	2
59	3704	5.79	3032	.77	0673	6.55	9327	1
60	9.534052		9.972986		9.561066		10.438934	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	70°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.534052	5.78	9.972986	.77	9.561066	6.55	10.438934	60
1	4399	5.78	2940		1459	6.54	8541	59
2	4745	5.77	2894		1851	6.54	8149	58
3	5092	5.77	2848		2244	6.53	7756	57
4	5438	5.76	2802		2636	6.53	7364	56
5	5783	5.76	2755		3028	6.53	6972	55
6	6129	5.75	2709		3419	6.52	6581	54
7	6474	5.74	2663		3811	6.52	6189	53
8	6818	5.74	2617		4202	6.51	5798	52
9	7163	5.73	2570		4593	6.51	5407	51
10	7507	5.73	2524		4983	6.50	5017	50
11	9.537851	5.72	9.972478	.77	9.565373	6.50	10.434627	49
12	8194	5.72	2431	.78	5763	6.49	4237	48
13	8538	5.71	2385		6153	6.49	3847	47
14	8880	5.71	2338		6542	6.49	3458	46
15	9223	5.70	2291		6932	6.48	3068	45
16	9565	5.70	2245		7320	6.48	2680	44
17	9.539907	5.69	2198		7709	6.47	2291	43
18	9.540249	5.69	2151		8098	6.47	1902	42
19	0590	5.68	2105		8486	6.46	1514	41
20	0931	5.68	2058		8873	6.46	1127	40
21	1272	5.67	9.972011		9261	6.45	0739	39
22	1613	5.67	1964		9.569648	6.45	10.430352	38
23	1953	5.66	1917		9.570035	6.45	10.429965	37
24	2293	5.66	1870		0422	6.44	9578	36
25	2632	5.65	1823		0809	6.44	9191	35
26	2971	5.65	1776	.78	1195	6.43	8805	34
27	3310	5.64	1729	.79	1581	6.43	8419	33
28	3649	5.64	1682		1967	6.42	8033	32
29	3987	5.63	1635		2352	6.42	7648	31
30	4325	5.63	1588		2738	6.42	7262	30
31	9.544663	5.62	9.971540		9.573123	6.41	10.426877	29
32	5000	5.62	1493		3507	6.41	6493	28
33	5338	5.61	1446		3892	6.40	6108	27
34	5674	5.61	1398		4276	6.40	5724	26
35	6011	5.60	1351		4660	6.39	5340	25
36	6347	5.60	1303		5044	6.39	4956	24
37	6683	5.59	1256		5427	6.39	4573	23
38	7019	5.59	1208		5810	6.38	4190	22
39	7354	5.58	1161		6193	6.38	3807	21
40	7689	5.58	1113	.79	6576	6.37	3424	20
41	8024	5.57	9.971066	.80	9.576959	6.37	10.423041	19
42	8359	5.57	1018		7341	6.36	2659	18
43	8693	5.56	0970		7723	6.36	2277	17
44	9027	5.56	0922		8104	6.36	1896	16
45	9360	5.55	0874		8486	6.35	1514	15
46	9.549693	5.55	0827		8867	6.35	1133	14
47	9.550026	5.54	0779		9248	6.34	0752	13
48	0359	5.54	0731		9.579629	6.34	10.420371	12
49	0692	5.53	0683		9.580009	6.34	10.419991	11
50	1024	5.53	0635		0389	6.33	9611	10
51	1356	5.52	9.970586		0769	6.33	9231	9
52	1687	5.52	0538		1149	6.32	8851	8
53	2018	5.52	0490		1528	6.32	8472	7
54	2349	5.51	0442		1907	6.32	8093	6
55	2680	5.51	0394	.80	2286	6.31	7714	5
56	3010	5.50	0345	.81	2665	6.31	7335	4
57	3341	5.50	0297		3044	6.30	6956	3
58	3670	5.49	0249		3422	6.30	6578	2
59	4000	5.49	0200	.81	3800	6.29	6200	1
60	9.554329		9.970152		9.584177		10.415823	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	69°

21° SINES AND TANGENTS.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.554329	5.48	9.970152	.81	9.584177	6.29	10.415823	60
1	4658	5.48	0103		4555	6.29	5445	59
2	4987	5.47	0055		4932	6.28	5068	58
3	5315	5.47	9.970006		5309	6.28	4691	57
4	5643	5.46	9.969957		5686	6.27	4314	56
5	5971	5.46	9909		6062	6.27	3938	55
6	6299	5.45	9860		6439	6.27	3561	54
7	6626	5.45	9811		6815	6.26	3185	53
8	6953	5.44	9762		7190	6.26	2810	52
9	7280	5.44	9714		7566	6.25	2434	51
10	7606	5.43	9665	.81	7941	6.25	2059	50
11	9.557932	5.43	9.969616	.82	8316	6.25	1684	49
12	8258	5.43	9567		8691	6.24	1309	48
13	8583	5.42	9518		9066	6.24	0934	47
14	8909	5.42	9469		9440	6.23	0560	46
15	9234	5.41	9420		9.589814	6.23	10.410186	45
16	9558	5.41	9370		9.590188	6.23	10.409812	44
17	9.559883	5.40	9321		0562	6.22	9438	43
18	9.560207	5.40	9272		0935	6.22	9065	42
19	0531	5.39	9223		1308	6.22	8692	41
20	0855	5.39	9173		1681	6.21	8319	40
21	1178	5.38	9.969124		2054	6.21	7946	39
22	1501	5.38	9075		2426	6.20	7574	38
23	1824	5.37	9025		2799	6.20	7201	37
24	2146	5.37	8976	.82	3171	6.20	6829	36
25	2468	5.36	8926	.83	3542	6.19	6458	35
26	2790	5.36	8877		3914	6.19	6086	34
27	3112	5.36	8827		4285	6.18	5715	33
28	3433	5.35	8777		4656	6.18	5344	32
29	3755	5.35	8728		5027	6.18	4973	31
30	4075	5.34	8678		5398	6.17	4602	30
31	9.564396	5.34	9.968628		9.595768	6.17	10.404232	29
32	4716	5.33	8578		6138	6.16	3862	28
33	5036	5.33	8528		6508	6.16	3492	27
34	5356	5.32	8479		6878	6.16	3122	26
35	5676	5.32	8429		7247	6.15	2753	25
36	5995	5.31	8379		7616	6.15	2384	24
37	6314	5.31	8329		7985	6.15	2015	23
38	6632	5.31	8278	.83	8354	6.14	1646	22
39	6951	5.30	8228	.84	8722	6.14	1278	21
40	7269	5.30	8178		9091	6.13	0909	20
41	9.567587	5.29	9.968128		9459	6.13	0541	19
42	7904	5.29	8078		9.599827	6.13	10.400173	18
43	8222	5.28	8027		9.600194	6.12	10.399806	17
44	8539	5.28	7977		0562	6.12	9438	16
45	8856	5.28	7927		0929	6.11	9071	15
46	9172	5.27	7876		1296	6.11	8704	14
47	9488	5.27	7826		1663	6.11	8337	13
48	9.569804	5.26	7775		2029	6.10	7971	12
49	9.570120	5.26	7725		2395	6.10	7605	11
50	0435	5.25	7674		2761	6.10	7239	10
51	0751	5.25	9.967624		9.603127	6.09	10.396873	9
52	1066	5.24	7573	.84	3493	6.09	6507	8
53	1380	5.24	7522	.85	3858	6.09	6142	7
54	1695	5.23	7471		4223	6.08	5777	6
55	2009	5.23	7421		4588	6.08	5412	5
56	2323	5.23	7370		4953	6.07	5047	4
57	2636	5.22	7319		5317	6.07	4683	3
58	2950	5.22	7268		5682	6.07	4318	2
59	3263	5.21	7217	.85	6046	6.06	3954	1
60	9.573575		9.967166		9.606410		10.393590	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	68°

22° LOGARITHMIC

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.573575	5.21	9.967166	.85	9.606410	6.06	10.393590	60
1	3888	5.20	7115		6773	6.06	3227	59
2	4200	5.20	7064		7137	6.05	2863	58
3	4512	5.19	7013		7500	6.05	2500	57
4	4824	5.19	6961		7863	6.04	2137	56
5	5136	5.19	6910		8225	6.04	1775	55
6	5447	5.18	6859		8588	6.04	1412	54
7	5758	5.18	6808	.85	8950	6.03	1050	53
8	6069	5.17	6756	.86	9312	6.03	0688	52
9	6379	5.17	6705		9.609674	6.03	10.390326	51
10	6689	5.16	6653		9.610036	6.02	10.389964	50
11	9.576999	5.16	9.966602		0397	6.02	9603	49
12	7309	5.16	6550		0759	6.02	9241	48
13	7618	5.15	6499		1120	6.01	8880	47
14	7927	5.15	6447		1480	6.01	8520	46
15	8236	5.14	6395		1841	6.01	8159	45
16	8545	5.14	6344		2201	6.00	7799	44
17	8853	5.13	6292		2561	6.00	7439	43
18	9162	5.13	6240		2921	6.00	7079	42
19	9470	5.13	6188		3281	5.99	6719	41
20	9.579777	5.12	6136	.86	3641	5.99	6359	40
21	9.580085	5.12	9.966085	.87	9.614000	5.98	10.386000	39
22	0392	5.11	6033		4359	5.98	5641	38
23	0699	5.11	5981		4718	5.98	5282	37
24	1005	5.11	5929		5077	5.97	4923	36
25	1312	5.10	5876		5435	5.97	4565	35
26	1618	5.10	5824		5793	5.97	4207	34
27	1924	5.09	5772		6151	5.96	3849	33
28	2229	5.09	5720		6509	5.96	3491	32
29	2535	5.09	5668		6867	5.96	3133	31
30	2840	5.08	5615		7224	5.95	2776	30
31	9.583145	5.08	9.965563		9.617582	5.95	10.382418	29
32	3449	5.07	5511		7939	5.95	2061	28
33	3754	5.07	5458		8295	5.94	1705	27
34	4058	5.06	5406	.87	8652	5.94	1348	26
35	4361	5.06	5353	.88	9008	5.94	0992	25
36	4665	5.06	5301		9364	5.93	0636	24
37	4968	5.05	5248		9.619720	5.93	10.380280	23
38	5272	5.05	5195		9.620076	5.93	10.379924	22
39	5574	5.04	5143		0432	5.92	9568	21
40	5877	5.04	5090		0787	5.92	9213	20
41	9.586179	5.03	9.965037		1142	5.92	8858	19
42	6482	5.03	4984		1497	5.91	8503	18
43	6783	5.03	4931		1852	5.91	8148	17
44	7085	5.02	4879		2207	5.90	7793	16
45	7386	5.02	4826		2561	5.90	7439	15
46	7688	5.01	4773		2915	5.90	7085	14
47	7989	5.01	4720	.88	3269	5.89	6731	13
48	8289	5.01	4666	.89	3623	5.89	6377	12
49	8590	5.00	4613		3976	5.89	6024	11
50	8890	5.00	4560		4330	5.88	5670	10
51	9190	4.99	9.964507		9.624683	5.88	10.375317	9
52	9489	4.99	4454		5036	5.88	4964	8
53	9.589789	4.99	4400		5388	5.87	4612	7
54	9.590088	4.98	4347		5741	5.87	4259	6
55	0387	4.98	4294		6093	5.87	3907	5
56	0686	4.97	4240		6445	5.86	3555	4
57	0984	4.97	4187		6797	5.86	3203	3
58	1282	4.97	4133		7149	5.86	2851	2
59	1580	4.96	4080	.89	7501	5.85	2499	1
60	9.591878		9.964026		9.627852		10.372148	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	67°

23° SINES AND TANGENTS.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.591878	4.96	9.964026	.89	9.627852	5.85	10.372148	60
1	2176	4.95	3972	.89	8203	5.85	1797	59
2	2473	4.95	3919	.89	8554	5.85	1446	58
3	2770	4.95	3865	.90	8905	5.84	1095	57
4	3067	4.94	3811		9255	5.84	0745	56
5	3363	4.94	3757		9606	5.83	0394	55
6	3659	4.93	3704		9.629956	5.83	10.370044	54
7	3955	4.93	3650		9.630306	5.83	10.369694	53
8	4251	4.93	3596		0656	5.83	9344	52
9	4547	4.92	3542		1005	5.82	8995	51
10	4842	4.92	3488		1355	5.82	8645	50
11	9.595137	4.91	9.963434		1704	5.82	8296	49
12	5432	4.91	3379		2053	5.81	7947	48
13	5727	4.91	3325		2402	8.81	7598	47
14	6021	4.90	3271		2750	5.81	7250	46
15	6315	4.90	3217		3099	5.80	6901	45
16	6609	4.89	3163	.90	3447	5.80	6553	44
17	6903	4.89	3108	.91	3795	5.80	6205	43
18	7196	4.89	3054		4143	5.79	5857	42
19	7490	4.88	2999		4490	5.79	5510	41
20	7783	4.88	2945		4838	5.79	5162	40
21	9.598075	4.87	9.962890		9.635185	5.78	10.364815	39
22	8368	4.87	2836		5532	5.78	4468	38
23	8660	4.87	2781		5879	5.78	4121	37
24	8952	4.86	2727		6226	5.77	3774	36
25	9244	4.86	2672		6572	5.77	3428	35
26	9536	4.85	2617		6919	5.77	3081	34
27	9.599827	4.85	2562		7265	5.77	2735	33
28	9.600118	4.85	2508		7611	5.76	2389	32
29	0409	4.84	2453	.91	7956	5.76	2044	31
30	0700	4.84	2398	.92	8302	5.76	1698	30
31	0990	4.84	9.962343		8647	5.75	1353	29
32	1280	4.83	2288		8992	5.75	1008	28
33	1570	4.83	2233		9337	5.75	0663	27
34	1860	4.82	2178		9.639682	5.74	10.360318	26
35	2150	4.82	2123		9.640027	5.74	10.359973	25
36	2439	4.82	2067		0371	5.74	9629	24
37	2728	4.81	2012		0716	5.73	9284	23
38	3017	4.81	1957		1060	5.73	8940	22
39	3305	4.81	1902		1404	5.73	8596	21
40	3594	4.80	1846		1747	5.72	8253	20
41	9.603882	4.80	9.961791		2091	5.72	7909	19
42	4170	4.79	1735		2434	5.72	7566	18
43	4457	4.79	1680	.92	2777	5.72	7223	17
44	4745	4.79	1624	.93	3120	5.71	6880	16
45	5032	4.78	1569		3463	5.71	6537	15
46	5319	4.78	1513		3806	5.71	6194	14
47	5606	4.78	1458		4148	5.70	5852	13
48	5892	4.77	1402		4490	5.70	5510	12
49	6179	4.77	1346		4832	5.70	5168	11
50	6465	4.76	1290		5174	5.69	4826	10
51	9.606751	4.76	9.961235		9.645516	5.69	10.354484	9
52	7036	4.76	1179		5857	5.69	4143	8
53	7322	4.75	1123		6199	5.69	3801	7
54	7607	4.75	1067		6540	5.68	3460	6
55	7892	4.74	1011		6881	5.68	3119	5
56	8177	4.74	0955		7222	5.68	2778	4
57	8461	4.74	0899	.93	7562	5.67	2438	3
58	8745	4.73	0843	.94	7903	5.67	2097	2
59	9029	4.73	0786	.94	8243	5.67	1757	1
60	9.609313		9.960730		9.648583		10.351417	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	66°

24° LOGARITHMIC

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.609313	4.73	9.960730	.94	9.648583	5.66	10.351417	60
1	9597	4.72	0674		8923	5.66	1077	59
2	9.609880	4.72	0618		9263	5.66	0737	58
3	9.610164	4.72	0561		9602	5.66	0398	57
4	0447	4.71	0505		9.649942	5.65	10.350058	56
5	0729	4.71	0448		9.650281	5.65	10.349719	55
6	1012	4.70	0392		0620	5.65	9380	54
7	1294	4.70	0335		0959	5.64	9041	53
8	1576	4.70	0279		1297	5.64	8703	52
9	1858	4.69	0222		1636	5.64	8364	51
10	2140	4.69	0165	.94	1974	5.63	8026	50
11	9.612421	4.69	0109	.95	9.652312	5.63	10.347688	49
12	2702	4.68	9.960052		2650	5.63	7350	48
13	2983	4.68	9.959995		2988	5.63	7012	47
14	3264	4.67	9938		3326	5.62	6674	46
15	3545	4.67	9882		3663	5.62	6337	45
16	3825	4.67	9825		4000	5.62	6000	44
17	4105	4.66	9768		4337	5.61	5663	43
18	4385	4.66	9711		4674	5.61	5326	42
19	4665	4.66	9654		5011	5.61	4989	41
20	4944	4.65	9596		5348	5.61	4652	40
21	9.615223	4.65	9.959539		9.655684	5.60	10.344316	39
22	5502	4.65	9482		6020	5.60	3980	38
23	5781	4.64	9425		6356	5.60	3644	37
24	6060	4.64	9368	.95	6692	5.59	3308	36
25	6338	4.64	9310	.96	7028	5.59	2972	35
26	6616	4.63	9253		7364	5.59	2636	34
27	6894	4.63	9195		7699	5.59	2301	33
28	7172	4.62	9138		8034	5.58	1966	32
29	7450	4.62	9081		8369	5.58	1631	31
30	7727	4.62	9023		8704	5.58	1296	30
31	9.618004	4.61	9.958965		9039	5.58	0961	29
32	8281	4.61	8908		9373	5.57	0627	28
33	8558	4.61	8850		9.659708	5.57	10.340292	27
34	8834	4.60	8792		9.660042	5.57	10.339958	26
35	9110	4.60	8734		0376	5.57	9624	25
36	9386	4.60	8677		0710	5.56	9290	24
37	9662	4.59	8619		1043	5.56	8957	23
38	9.619938	4.59	8561	.96	1377	5.56	8623	22
39	9.620213	4.59	8503	.97	1710	5.55	8290	21
40	0488	4.58	8445		2043	5.55	7957	20
41	0763	4.58	9.958387		9.662376	5.55	10.337624	19
42	1038	4.57	8329		2709	5.54	7291	18
43	1313	4.57	8271		3042	5.54	6958	17
44	1587	4.57	8213		3375	5.54	6625	16
45	1861	4.56	8154		3707	5.54	6293	15
46	2135	4.56	8096		4039	5.53	5961	14
47	2409	4.56	8038		4371	5.53	5629	13
48	2682	4.55	7979		4703	5.53	5297	12
49	2956	4.55	7921		5035	5.53	4965	11
50	3229	4.55	7863		5366	5.52	4634	10
51	9.623502	4.54	9.957804	.97	9.665698	5.52	10.334302	9
52	3774	4.54	7746	.98	6029	5.52	3971	8
53	4047	4.54	7687		6360	5.51	3640	7
54	4319	4.53	7628		6691	5.51	3309	6
55	4591	4.53	7570		7021	5.51	2979	5
56	4863	4.53	7511		7352	5.51	2648	4
57	5135	4.52	7452		7682	5.50	2318	3
58	5406	4.52	7393		8013	5.50	1987	2
59	5677	4.52	7335	.98	8343	5.50	1657	1
60	9.625948		9.957276		9.668673		10.331327	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	65°

25° SINES AND TANGENTS.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.625948	4.51	9.957276	.98	9.668673	5.50	10.331327	60
1	6219	4.51	7217		9002	5.49	0998	59
2	6490	4.51	7158		9332	5.49	0668	58
3	6760	4.50	7099		9661	5.49	0339	57
4	7030	4.50	7040		9.669991	5.48	10.330009	56
5	7300	4.50	6981	.98	9.670320	5.48	10.329680	55
6	7570	4.49	6921	.99	0649	5.48	9351	54
7	7840	4.49	6862		0977	5.48	9023	53
8	8109	4.49	6803		1306	5.47	8694	52
9	8378	4.48	6744		1635	5.47	8365	51
10	8647	4.48	6684		1963	5.47	8037	50
11	8916	4.47	9.956625		9.672291	5.47	10.327709	49
12	9185	4.47	6566		2619	5.46	7381	48
13	9453	4.47	6506		2947	5.46	7053	47
14	9721	4.46	6447		3274	5.46	6726	46
15	9.629989	4.46	6387		3602	5.46	6398	45
16	9.630257	4.46	6327		3929	5.45	6071	44
17	0524	4.46	6268	.99	4257	5.45	5743	43
18	0792	4.45	6208	1.00	4584	5.45	5416	42
19	1059	4.45	6148		4911	5.44	5089	41
20	1326	4.45	6089		5237	5.44	4763	40
21	9.631593	4.44	9.956029		9.675564	5.44	10.324436	39
22	1859	4.44	5969		5890	5.44	4110	38
23	2125	4.44	5909		6216	5.43	3784	37
24	2392	4.43	5849		6543	5.43	3457	36
25	2658	4.43	5789		6869	5.43	3131	35
26	2923	4.43	5729		7194	5.43	2806	34
27	3189	4.42	5669		7520	5.42	2480	33
28	3454	4.42	5609		7846	5.42	2154	32
29	3719	4.42	5548		8171	5.42	1829	31
30	3984	4.41	5488	1.00	8496	5.42	1504	30
31	9.634249	4.41	9.955428	1.01	8821	5.41	1179	29
32	4514	4.40	5368		9146	5.41	0854	28
33	4778	4.40	5307		9471	5.41	0529	27
34	5042	4.40	5247		9.679795	5.41	10.320205	26
35	5306	4.39	5186		9.680120	5.40	10.319880	25
36	5570	4.39	5126		0444	5.40	9556	24
37	5834	4.39	5065		0768	5.40	9232	23
38	6097	4.39	5005		1092	5.40	8908	22
39	6360	4.38	4944		1416	5.39	8584	21
40	6623	4.38	4883		1740	5.39	8260	20
41	9.636886	4.37	9.954823		9.682063	5.39	10.317937	19
42	7148	4.37	4762		2387	5.39	7613	18
43	7411	4.37	4701		2710	5.38	7290	17
44	7673	4.37	4640		3033	5.38	6967	16
45	7935	4.36	4579	1.01	3356	5.38	6644	15
46	8197	4.36	4518	1.02	3679	5.38	6321	14
47	8458	4.36	4457		4001	5.37	5999	13
48	8720	4.35	4396		4324	5.37	5676	12
49	8981	4.35	4335		4646	5.37	5354	11
50	9242	4.35	4274		4968	5.37	5032	10
51	9503	4.34	9.954213		9.685290	5.36	10.314710	9
52	9.639764	4.34	4152		5612	5.36	4388	8
53	9.640024	4.34	4090		5934	5.36	4066	7
54	0284	4.33	4029		6255	5.36	3745	6
55	0544	4.33	3968		6577	5.35	3423	5
56	0804	4.33	3906		6898	5.35	3102	4
57	1064	4.32	3845		7219	5.35	2781	3
58	1324	4.32	3783	1.02	7540	5.35	2460	2
59	1583	4.32	3722	1.03	7861	5.34	2139	1
60	9.641842		9.953660		9.688182		10.311818	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	64°

26° LOGARITHMIC

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.641842	4.31	9.953660	1.03	9.688182	5.34	10.311818	60
1	2101	4.31	3599		8502	5.34	1498	59
2	2360	4.31	3537		8823	5.34	1177	58
3	2618	4.30	3475		9143	5.33	0857	57
4	2877	4.30	3413		9463	5.33	0537	56
5	3135	4.30	3352		9.689783	5.33	10.310217	55
6	3393	4.30	3290		9.690103	5.33	10.309897	54
7	3650	4.29	3228		0423	5.33	9577	53
8	3908	4.29	3166		0742	5.32	9258	52
9	4165	4.29	3104		1062	5.32	8938	51
10	4423	4.28	3042	1.03	1381	5.32	8619	50
11	9.644680	4.28	9.952980	1.04	9.691700	5.31	10.308300	49
12	4936	4.28	2918		2019	5.31	7981	48
13	5193	4.27	2855		2338	5.31	7662	47
14	5450	4.27	2793		2656	5.31	7344	46
15	5706	4.27	2731		2975	5.31	7025	45
16	5962	4.26	2669		3293	5.30	6707	44
17	6218	4.26	2606		3612	5.30	6388	43
18	6474	4.26	2544		3930	5.30	6070	42
19	6729	4.26	2481		4248	5.30	5752	41
20	6984	4.25	2419		4566	5.29	5434	40
21	9.647240	4.25	9.952356		9.694883	5.29	10.305117	39
22	7494	4.24	2294		5201	5.29	4799	38
23	7749	4.24	2231	1.04	5518	5.29	4482	37
24	8004	4.24	2168	1.05	5836	5.29	4164	36
25	8258	4.24	2106		6153	5.28	3847	35
26	8512	4.23	2043		6470	5.28	3530	34
27	8766	4.23	1980		6787	5.28	3213	33
28	9020	4.23	1917		7103	5.28	2897	32
29	9274	4.22	1854		7420	5.27	2580	31
30	9527	4.22	1791		7736	5.27	2264	30
31	9.649781	4.22	9.951728		9.698053	5.27	10.301947	29
32	9.650034	4.22	1665		8369	5.27	1631	28
33	0287	4.21	1602		8685	5.26	1315	27
34	0539	4.21	1539		9001	5.26	0999	26
35	0792	4.21	1476		9316	5.26	0684	25
36	1044	4.20	1412	1.05	9632	5.26	0368	24
37	1297	4.20	1349	1.06	9.699947	5.26	10.300053	23
38	1549	4.20	1286		9.700263	5.25	10.299737	22
39	1800	4.19	1222		0578	5.25	9422	21
40	2052	4.19	1159		0893	5.25	9107	20
41	9.652304	4.19	9.951096		1208	5.24	8792	19
42	2555	4.18	1032		1523	5.24	8477	18
43	2806	4.18	0968		1837	5.24	8163	17
44	3057	4.18	0905		2152	5.24	7848	16
45	3308	4.18	0841		2466	5.24	7534	15
46	3558	4.17	0778		2781	5.23	7219	14
47	3808	4.17	0714		3095	5.23	6905	13
48	4059	4.17	0650		3409	5.23	6591	12
49	4309	4.16	0586	1.06	3722	5.23	6278	11
50	4558	4.16	0522	1.07	4036	5.22	5964	10
51	9.654808	4.16	9.950458		9.704350	5.22	10.295650	9
52	5058	4.16	0394		4663	5.22	5337	8
53	5307	4.15	0330		4976	5.22	5024	7
54	5556	4.15	0266		5290	5.22	4710	6
55	5805	4.15	0202		5603	5.21	4397	5
56	6054	4.14	0138		5916	5.21	4084	4
57	6302	4.14	0074		6228	5.21	3772	3
58	6551	4.14	9.950010		6541	5.21	3459	2
59	6799	4.13	9.949945	1.07	6854	5.21	3146	1
60	9.657047		9.949881		9.707166		10.292834	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	63°

27° SINES AND TANGENTS.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.657047	4.13	9.949881	1.07	9.707166	5.20	10.292834	60
1	7295	4.13	9816	1.07	7478	5.20	2522	59
2	7542	4.12	9752	1.07	7790	5.20	2210	58
3	7790	4.12	9688	1.08	8102	5.20	1898	57
4	8037	4.12	9623		8414	5.19	1586	56
5	8284	4.12	9558		8726	5.19	1274	55
6	8531	4.11	9494		9037	5.19	0963	54
7	8778	4.11	9429		9349	5.19	0651	53
8	9025	4.11	9364		9660	5.19	0340	52
9	9271	4.10	9300		9.709971	5.18	10.290029	51
10	9517	4.10	9235		9.710282	5.18	10.289718	50
11	9.659763	4.10	9.949170		0593	5.18	9407	49
12	9.660009	4.09	9105		0904	5.18	9096	48
13	0255	4.09	9040		1215	5.18	8785	47
14	0501	4.09	8975		1525	5.17	8475	46
15	0746	4.09	8910		1836	5.17	8164	45
16	0991	4.08	8845	1.08	2146	5.17	7854	44
17	1236	4.08	8780	1.09	2456	5.17	7544	43
18	1481	4.08	8715		2766	5.16	7234	42
19	1726	4.07	8650		3076	5.16	6924	41
20	1970	4.07	8584		3386	5.16	6614	40
21	9.662214	4.07	9.948519		9.713696	5.16	10.286304	39
22	2459	4.07	8454		4005	5.16	5995	38
23	2703	4.06	8388		4314	5.15	5686	37
24	2946	4.06	8323		4624	5.15	5376	36
25	3190	4.06	8257		4933	5.15	5067	35
26	3433	4.05	8192		5242	5.15	4758	34
27	3677	4.05	8126		5551	5.14	4449	33
28	3920	4.05	8060	1.09	5860	5.14	4140	32
29	4163	4.05	7995	1.10	6168	5.14	3832	31
30	4406	4.04	7929		6477	5.14	3523	30
31	9.664648	4.04	9.947863		9.716785	5.14	10.283215	29
32	4891	4.04	7797		7093	5.13	2907	28
33	5133	4.03	7731		7401	5.13	2599	27
34	5375	4.03	7665		7709	5.13	2291	26
35	5617	4.03	7600		8017	5.13	1983	25
36	5859	4.02	7533		8325	5.13	1675	24
37	6100	4.02	7467		8633	5.12	1367	23
38	6342	4.02	7401		8940	5.12	1060	22
39	6583	4.02	7335		9248	5.12	0752	21
40	6824	4.01	7269		9555	5.12	0445	20
41	9.667065	4.01	9.947203	1.10	9.719862	5.12	10.280138	19
42	7305	4.01	7136	1.11	9.720169	5.11	10.279831	18
43	7546	4.01	7070		0476	5.11	9524	17
44	7786	4.00	7004		0783	5.11	9217	16
45	8027	4.00	6937		1089	5.11	8911	15
46	8267	4.00	6871		1396	5.11	8604	14
47	8506	3.99	6804		1702	5.10	8298	13
48	8746	3.99	6738		2009	5.10	7991	12
49	8986	3.99	6671		2315	5.10	7685	11
50	9225	3.99	6604		2621	5.10	7379	10
51	9464	3.98	9.946538		9.722927	5.10	10.277073	9
52	9703	3.98	6471		3232	5.09	6768	8
53	9.669942	3.98	6404		3538	5.09	6462	7
54	9.670181	3.97	6337	1.11	3844	5.09	6156	6
55	0419	3.97	6270	1.12	4149	5.09	5851	5
56	0658	3.97	6203		4454	5.09	5546	4
57	0896	3.97	6136		4760	5.08	5240	3
58	1134	3.96	6069		5065	5.08	4935	2
59	1372	3.96	6002	1.12	5370	5.08	4630	1
60	9.671609		9.945935		9.725674		10.274326	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	62°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.671609	3.96	9.945935	1.12	9.725674	5.08	10.274326	60
1	1847	3.95	5868		5979	5.08	4021	59
2	2084	3.95	5800		6284	5.07	3716	58
3	2321	3.95	5733		6588	5.07	3412	57
4	2558	3.95	5666		6892	5.07	3108	56
5	2795	3.94	5598		7197	5.07	2803	55
6	3032	3.94	5531	1.12	7501	5.07	2499	54
7	3268	3.94	5464	1.13	7805	5.06	2195	53
8	3505	3.94	5396		8109	5.06	1891	52
9	3741	3.93	5328		8412	5.06	1588	51
10	3977	3.93	5261		8716	5.06	1284	50
11	9.674213	3.93	9.945193		9020	5.06	0980	49
12	4448	3.92	5125		9323	5.05	0677	48
13	4684	3.92	5058		9626	5.05	0374	47
14	4919	3.92	4990		9.729929	5.05	10.270071	46
15	5155	3.92	4922		9.730233	5.05	10.269767	45
16	5390	3.91	4854		0535	5.05	9465	44
17	5624	3.91	4786		0838	5.04	9162	43
18	5859	3.91	4718		1141	5.04	8859	42
19	6094	3.91	4650	1.13	1444	5.04	8556	41
20	6328	3.90	4582	1.14	1746	5.04	8254	40
21	9.676562	3.90	9.944514		9.732048	5.04	10.267952	39
22	6796	3.90	4446		2351	5.03	7649	38
23	7030	3.90	4377		2653	5.03	7347	37
24	7264	3.89	4309		2955	5.03	7045	36
25	7498	3.89	4241		3257	5.03	6743	35
26	7731	3.89	4172		3558	5.03	6442	34
27	7964	3.88	4104		3860	5.02	6140	33
28	8197	3.88	4036		4162	5.02	5838	32
29	8430	3.88	3967		4463	5.02	5537	31
30	8663	3.88	3899		4764	5.02	5236	30
31	9.678895	3.87	9.943830		9.735066	5.02	10.264934	29
32	9128	3.87	3761	1.14	5367	5.02	4633	28
33	9360	3.87	3693	1.15	5668	5.01	4332	27
34	9592	3.87	3624		5969	5.01	4031	26
35	9.679824	3.86	3555		6269	5.01	3731	25
36	9.680056	3.86	3486		6570	5.01	3430	24
37	0288	3.86	3417		6871	5.01	3129	23
38	0519	3.85	3348		7171	5.00	2829	22
39	0750	3.85	3279		7471	5.00	2529	21
40	0982	3.85	3210		7771	5.00	2229	20
41	9.681213	3.85	9.943141		9.738071	5.00	10.261929	19
42	1443	3.84	3072		8371	5.00	1629	18
43	1674	3.84	3003		8671	4.99	1329	17
44	1905	3.84	2934		8971	4.99	1029	16
45	2135	3.84	2864	1.15	9271	4.99	0729	15
46	2365	3.83	2795	1.16	9570	4.99	0430	14
47	2595	3.83	2726		9.739870	4.99	10.260130	13
48	2825	3.83	2656		9.740169	4.99	10.259831	12
49	3055	3.83	2587		0468	4.98	9532	11
50	3284	3.82	2517		0767	4.98	9233	10
51	9.683514	3.82	9.942448		1066	4.98	8934	9
52	3743	3.82	2378		1365	4.98	8635	8
53	3972	3.82	2308		1664	4.98	8336	7
54	4201	3.81	2239		1962	4.97	8038	6
55	4430	3.81	2169		2261	4.97	7739	5
56	4658	3.81	2099		2559	4.97	7441	4
57	4887	3.80	2029		2858	4.97	7142	3
58	5115	3.80	1959	1.16	3156	4.97	6844	2
59	5343	3.80	1889	1.17	3454	4.97	6546	1
60	9.685571		9.941819		9.743752		10.256248	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	61°

29° SINES AND TANGENTS.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.685571	3.80	9.941819	1.17	9.743752	4.96	10.256248	60
1	5799	3.79	1749		4050	4.96	5950	59
2	6027	3.79	1679		4348	4.96	5652	58
3	6254	3.79	1609		4645	4.96	5355	57
4	6482	3.79	1539		4943	4.96	5057	56
5	6709	3.78	1469		5240	4.95	4760	55
6	6936	3.78	1398		5538	4.95	4462	54
7	7163	3.78	1328		5835	4.95	4165	53
8	7389	3.78	1258		6132	4.95	3868	52
9	7616	3.77	1187		6429	4.95	3571	51
10	7843	3.77	1117	1.17	6726	4.95	3274	50
11	9.688069	3.77	9.941046	1.18	9.747023	4.94	10.252977	49
12	8295	3.77	0975		7319	4.94	2681	48
13	8521	3.76	0905		7616	4.94	2384	47
14	8747	3.76	0834		7913	4.94	2087	46
15	8972	3.76	0763		8209	4.94	1791	45
16	9198	3.76	0693		8505	4.93	1495	44
17	9423	3.75	0622		8801	4.93	1199	43
18	9648	3.75	0551		9097	4.93	0903	42
19	9.689873	3.75	0480		9393	4.93	0607	41
20	9.690098	3.75	0409		9689	4.93	0311	40
21	0323	3.74	9.940338		9.749985	4.93	10.250015	39
22	0548	3.74	0267		9.750281	4.93	10.249719	38
23	0772	3.74	0196	1.18	0576	4.92	9424	37
24	0996	3.74	0125	1.19	0872	4.92	9128	36
25	1220	3.73	9.940054		1167	4.92	8833	35
26	1444	3.73	9.939982		1462	4.92	8538	34
27	1668	3.73	9911		1757	4.92	8243	33
28	1892	3.73	9840		2052	4.91	7948	32
29	2115	3.72	9768		2347	4.91	7653	31
30	2339	3.72	9697		2642	4.91	7358	30
31	9.692562	3.72	9.939625		9.752937	4.91	10.247063	29
32	2785	3.71	9554		3231	4.91	6769	28
33	3008	3.71	9482		3526	4.91	6474	27
34	3231	3.71	9410		3820	4.90	6180	26
35	3453	3.71	9339	1.19	4115	4.90	5885	25
36	3676	3.70	9267	1.20	4409	4.90	5591	24
37	3898	3.70	9195		4703	4.90	5297	23
38	4120	3.70	9123		4997	4.90	5003	22
39	4342	3.70	9052		5291	4.90	4709	21
40	4564	3.69	8980		5585	4.89	4415	20
41	9.694786	3.69	9.938908		9.755878	4.89	10.244122	19
42	5007	3.69	8836		6172	4.89	3828	18
43	5229	3.69	8763		6465	4.89	3535	17
44	5450	3.68	8691		6759	4.89	3241	16
45	5671	3.68	8619		7052	4.89	2948	15
46	5892	3.68	8547		7345	4.88	2655	14
47	6113	3.68	8475	1.20	7638	4.88	2362	13
48	6334	3.67	8402	1.21	7931	4.88	2069	12
49	6554	3.67	8330		8224	4.88	1776	11
50	6775	3.67	8258		8517	4.88	1483	10
51	9.696995	3.67	9.938185		9.758810	4.88	10.241190	9
52	7215	3.66	8113		9102	4.87	0898	8
53	7435	3.66	8040		9395	4.87	0605	7
54	7654	3.66	7967		9687	4.87	0313	6
55	7874	3.66	7895		9.759979	4.87	10.240021	5
56	8094	3.65	7822		9.760272	4.87	10.239728	4
57	8313	3.65	7749		0564	4.87	9436	3
58	8532	3.65	7676		0856	4.86	9144	2
59	8751	3.65	7604	1.21	1148	4.86	8852	1
60	9.698970		9.937531		9.761439		10.238561	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	60°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.698970	3.64	9.937531	1.21	9.761439	4.86	10.238561	60
1	9189	3.64	7458	1.22	1731	4.86	8269	59
2	9407	3.64	7385		2023	4.86	7977	58
3	9626	3.64	7312		2314	4.86	7686	57
4	9.699844	3.63	7238		2606	4.85	7394	56
5	9.700062	3.63	7165		2897	4.85	7103	55
6	0280	3.63	7092		3188	4.85	6812	54
7	0498	3.63	7019		3479	4.85	6521	53
8	0716	3.63	6946		3770	4.85	6230	52
9	0933	3.62	6872		4061	4.85	5939	51
10	1151	3.62	6799		4352	4.84	5648	50
11	9.701368	3.62	9.936725	1.22	9.764643	4.84	10.235357	49
12	1585	3.62	6652	1.23	4933	4.84	5067	48
13	1802	3.61	6578		5224	4.84	4776	47
14	2019	3.61	6505		5514	4.84	4486	46
15	2236	3.61	6431		5805	4.84	4195	45
16	2452	3.61	6357		6095	4.84	3905	44
17	2669	3.60	6284		6385	4.83	3615	43
18	2885	3.60	6210		6675	4.83	3325	42
19	3101	3.60	6136		6965	4.83	3035	41
20	3317	3.60	6062		7255	4.83	2745	40
21	9.703533	3.59	9.935988		9.767545	4.83	10.232455	39
22	3749	3.59	5914		7834	4.83	2166	38
23	3964	3.59	5840	1.23	8124	4.82	1876	37
24	4179	3.59	5766	1.24	8414	4.82	1586	36
25	4395	3.59	5692		8703	4.82	1297	35
26	4610	3.58	5618		8992	4.82	1008	34
27	4825	3.58	5543		9281	4.82	0719	33
28	5040	3.58	5469		9571	4.82	0429	32
29	5254	3.58	5395		9.769860	4.81	10.230140	31
30	5469	3.57	5320		9.770148	4.81	10.229852	30
31	9.705683	3.57	9.935246		0437	4.81	9563	29
32	5898	3.57	5171		0726	4.81	9274	28
33	6112	3.57	5097		1015	4.81	8985	27
34	6326	3.56	5022		1303	4.81	8697	26
35	6539	3.56	4948		1592	4.81	8408	25
36	6753	3.56	4873	1.24	1880	4.80	8120	24
37	6967	3.56	4798	1.25	2168	4.80	7832	23
38	7180	3.55	4723		2457	4.80	7543	22
39	7393	3.55	4649		2745	4.80	7255	21
40	7606	3.55	4574		3033	4.80	6967	20
41	9.707819	3.55	9.934499		9.773321	4.80	10.226679	19
42	8032	3.54	4424		3608	4.79	6392	18
43	8245	3.54	4349		3896	4.79	6104	17
44	8458	3.54	4274		4184	4.79	5816	16
45	8670	3.54	4199		4471	4.79	5529	15
46	8882	3.53	4123		4759	4.79	5241	14
47	9094	3.53	4048		5046	4.79	4954	13
48	9306	3.53	3973	1.25	5333	4.79	4667	12
49	9518	3.53	3898	1.26	5621	4.78	4379	11
50	9730	3.53	3822		5908	4.78	4092	10
51	9.709941	3.52	9.933747		9.776195	4.78	10.223805	9
52	9.710153	3.52	3671		6482	4.78	3518	8
53	0364	3.52	3596		6768	4.78	3232	7
54	0575	3.52	3520		7055	4.78	2945	6
55	0786	3.51	3445		7342	4.78	2658	5
56	0997	3.51	3369		7628	4.77	2372	4
57	1208	3.51	3293		7915	4.77	2085	3
58	1419	3.51	3217		8201	4.77	1799	2
59	1629	3.50	3141	1.26	8488	4.77	1512	1
60	9.711839		9.933066		9.778774		10.221226	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	59°

31° SINES AND TANGENTS.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.711839	3.50	9.933066	1.26	9.778774	4.77	10.221226	60
1	2050	3.50	2990	1.27	9060	4.77	0940	59
2	2260	3.50	2914		9346	4.76	0654	58
3	2469	3.49	2838		9632	4.76	0368	57
4	2679	3.49	2762		9.779918	4.76	10.220082	56
5	2889	3.49	2685		9.780203	4.76	10.219797	55
6	3098	3.49	2609		0489	4.76	9511	54
7	3308	3.49	2533		0775	4.76	9225	53
8	3517	3.48	2457		1060	4.76	8940	52
9	3726	3.48	2380		1346	4.75	8654	51
10	3935	3.48	2304		1631	4.75	8369	50
11	9.714144	3.48	9.932228		9.781916	4.75	10.218084	49
12	4352	3.47	2151	1.27	2201	4.75	7799	48
13	4561	3.47	2075	1.28	2486	4.75	7514	47
14	4769	3.47	1998		2771	4.75	7229	46
15	4978	3.47	1921		3056	4.75	6944	45
16	5186	3.47	1845		3341	4.75	6659	44
17	5394	3.46	1768		3626	4.74	6374	43
18	5602	3.46	1691		3910	4.74	6090	42
19	5809	3.46	1614		4195	4.74	5805	41
20	6017	3.46	1537		4479	4.74	5521	40
21	9.716224	3.45	9.931460		9.784764	4.74	10.215236	39
22	6432	3.45	1383		5048	4.74	4952	38
23	6639	3.45	1306	1.28	5332	4.73	4668	37
24	6846	3.45	1229	1.29	5616	4.73	4384	36
25	7053	3.45	1152		5900	4.73	4100	35
26	7259	3.44	1075		6184	4.73	3816	34
27	7466	3.44	0998		6468	4.73	3532	33
28	7673	3.44	0921		6752	4.73	3248	32
29	7879	3.44	0843		7036	4.73	2964	31
30	8085	3.43	0766		7319	4.72	2681	30
31	9.718291	3.43	9.930688		9.787603	4.72	10.212397	29
32	8497	3.43	0611		7886	4.72	2114	28
33	8703	3.43	0533		8170	4.72	1830	27
34	8909	3.43	0456		8453	4.72	1547	26
35	9114	3.42	0378	1.29	8736	4.72	1264	25
36	9320	3.42	0300	1.30	9019	4.72	0981	24
37	9525	3.42	0223		9302	4.71	0698	23
38	9730	3.42	0145		9585	4.71	0415	22
39	9.719935	3.41	9.930067		9.789868	4.71	10.210132	21
40	9.720140	3.41	9.929989		9.790151	4.71	10.209849	20
41	0345	3.41	9911		0434	4.71	9566	19
42	0549	3.41	9833		0716	4.71	9284	18
43	0754	3.40	9755		0999	4.71	9001	17
44	0958	3.40	9677		1281	4.71	8719	16
45	1162	3.40	9599		1563	4.70	8437	15
46	1366	3.40	9521		1846	4.70	8154	14
47	1570	3.40	9442	1.30	2128	4.70	7872	13
48	1774	3.39	9364	1.31	2410	4.70	7590	12
49	1978	3.39	9286		2692	4.70	7308	11
50	2181	3.39	9207		2974	4.70	7026	10
51	9.722385	3.39	9.929129		9.793256	4.70	10.206744	9
52	2588	3.39	9050		3538	4.69	6462	8
53	2791	3.38	8972		3819	4.69	6181	7
54	2994	3.38	8893		4101	4.69	5899	6
55	3197	3.38	8815		4383	4.69	5617	5
56	3400	3.38	8736		4664	4.69	5336	4
57	3603	3.37	8657		4946	4.69	5054	3
58	3805	3.37	8578		5227	4.69	4773	2
59	4007	3.37	8499	1.31	5508	4.68	4492	1
60	9.724210		9.928420		9.795789		10.204211	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	58°

32° LOGARITHMIC

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.724210	3.37	9.928420	1.32	9.795789	4.68	10.204211	60
1	4412	3.37	8342		6070	4.68	3930	59
2	4614	3.36	8263		6351	4.68	3649	58
3	4816	3.36	8183		6632	4.68	3368	57
4	5017	3.36	8104		6913	4.68	3087	56
5	5219	3.36	8025		7194	4.68	2806	55
6	5420	3.35	7946		7476	4.68	2524	54
7	5622	3.35	7867		7755	4.68	2245	53
8	5823	3.35	7787		8036	4.67	1964	52
9	6024	3.35	7708		8316	4.67	1684	51
10	6225	3.35	7629		8596	4.67	1404	50
11	9.726426	3.34	9.927549	1.32	8877	4.67	1123	49
12	6626	3.34	7470	1.33	9157	4.67	0843	48
13	6827	3.34	7390		9437	4.67	0563	47
14	7027	3.34	7310		9717	4.67	0283	46
15	7228	3.34	7231		9.799997	4.66	10.200003	45
16	7428	3.33	7151		9.800277	4.66	10.199723	44
17	7628	3.33	7071		0557	4.66	9443	43
18	7828	3.33	6991		0836	4.66	9164	42
19	8027	3.33	6911		1116	4.66	8884	41
20	8227	3.33	6831		1396	4.66	8604	40
21	9.728427	3.32	9.926751		1675	4.66	8325	39
22	8626	3.32	6671		1955	4.66	8045	38
23	8825	3.32	6591	1.33	2234	4.65	7766	37
24	9024	3.32	6511	1.34	2513	4.65	7487	36
25	9223	3.31	6431		2792	4.65	7208	35
26	9422	3.31	6351		3072	4.65	6928	34
27	9621	3.31	6270		3351	4.65	6649	33
28	9.729820	3.31	6190		3630	4.65	6370	32
29	9.730018	3.30	6110		3909	4.65	6091	31
30	0216	3.30	6029		4187	4.65	5813	30
31	0415	3.30	9.925949		9.804466	4.64	10.195534	29
32	0613	3.30	5868		4745	4.64	5255	28
33	0811	3.30	5788		5023	4.64	4977	27
34	1009	3.29	5707		5302	4.64	4698	26
35	1206	3.29	5626	1.34	5580	4.64	4420	25
36	1404	3.29	5545	1.35	5859	4.64	4141	24
37	1602	3.29	5465		6137	4.64	3863	23
38	1799	3.29	5384		6415	4.63	3585	22
39	1996	3.28	5303		6693	4.63	3307	21
40	2193	3.28	5222		6971	4.63	3029	20
41	9.732390	3.28	9.925141		9.807249	4.63	10.192751	19
42	2587	3.28	5060		7527	4.63	2473	18
43	2784	3.28	4979		7805	4.63	2195	17
44	2980	3.27	4897		8083	4.63	1917	16
45	3177	3.27	4816	1.35	8361	4.63	1639	15
46	3373	3.27	4735	1.36	8638	4.62	1362	14
47	3569	3.27	4654		8916	4.62	1084	13
48	3765	3.27	4572		9193	4.62	0807	12
49	3961	3.26	4491		9471	4.62	0529	11
50	4157	3.26	4409		9.809748	4.62	10.190252	10
51	9.734353	3.26	9.924328		9.810025	4.62	10.189975	9
52	4549	3.26	4246		0302	4.62	9698	8
53	4744	3.25	4164		0580	4.62	9420	7
54	4939	3.25	4083		0857	4.62	9143	6
55	5135	3.25	4001		1134	4.61	8866	5
56	5330	3.25	3919		1410	4.61	8590	4
57	5525	3.25	3837	1.36	1687	4.61	8313	3
58	5719	3.24	3755	1.37	1964	4.61	8036	2
59	5914	3.24	3673	1.37	2241	4.61	7759	1
60	9.736109		9.923591		9.812517		10.187483	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	57°

33° SINES AND TANGENTS.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.736109	3.24	9.923591	1.37	9.812517	4.61	10.187483	60
1	6303	3.24	3509		2794	4.61	7206	59
2	6498	3.24	3427		3070	4.61	6930	58
3	6692	3.23	3345		3347	4.60	6653	57
4	6886	3.23	3263		3623	4.60	6377	56
5	7080	3.23	3181		3899	4.60	6101	55
6	7274	3.23	3098		4176	4.60	5824	54
7	7467	3.23	3016		4452	4.60	5548	53
8	7661	3.22	2933		4728	4.60	5272	52
9	7855	3.22	2851	1.37	5004	4.60	4996	51
10	8048	3.22	2768	1.38	5280	4.60	4720	50
11	9.738241	3.22	9.922686		9.815555	4.59	10.184445	49
12	8434	3.22	2603		5831	4.59	4169	48
13	8627	3.21	2520		6107	4.59	3893	47
14	8820	3.21	2438		6382	4.59	3618	46
15	9013	3.21	2355		6658	4.59	3342	45
16	9206	3.21	2272		6933	4.59	3067	44
17	9398	3.21	2189		7209	4.59	2791	43
18	9590	3.20	2106		7484	4.59	2516	42
19	9783	3.20	2023		7759	4.59	2241	41
20	9.739975	3.20	1940	1.38	8035	4.58	1965	40
21	9.740167	3.20	9.921857	1.39	9.818310	4.58	10.181690	39
22	0359	3.20	1774		8585	4.58	1415	38
23	0550	3.19	1691		8860	4.58	1140	37
24	0742	3.19	1607		9135	4.58	0865	36
25	0934	3.19	1524		9410	4.58	0590	35
26	1125	3.19	1441		9684	4.58	0316	34
27	1316	3.19	1357		9.819959	4.58	10.180041	33
28	1508	3.18	1274		9.820234	4.58	10.179766	32
29	1699	3.18	1190		0508	4.57	9492	31
30	1889	3.18	1107		0783	4.57	9217	30
31	9.742080	3.18	9.921023	1.39	1057	4.57	8943	29
32	2271	3.18	0939	1.40	1332	4.57	8668	28
33	2462	3.17	0856		1606	4.57	8394	27
34	2652	3.17	0772		1880	4.57	8120	26
35	2842	3.17	0688		2154	4.57	7846	25
36	3033	3.17	0604		2429	4.57	7571	24
37	3223	3.17	0520		2703	4.57	7297	23
38	3413	3.16	0436		2977	4.56	7023	22
39	3602	3.16	0352		3251	4.56	6749	21
40	3792	3.16	0268		3524	4.56	6476	20
41	9.743982	3.16	0184		9.823798	4.56	10.176202	19
42	4171	3.16	0099		4072	4.56	5928	18
43	4361	3.15	9.920015	1.40	4345	4.56	5655	17
44	4550	3.15	9.919931	1.41	4619	4.56	5381	16
45	4739	3.15	9846		4893	4.56	5107	15
46	4928	3.15	9762		5166	4.56	4834	14
47	5117	3.15	9677		5439	4.55	4561	13
48	5306	3.14	9593		5713	4.55	4287	12
49	5494	3.14	9508		5986	4.55	4014	11
50	5683	3.14	9424		6259	4.55	3741	10
51	9.745871	3.14	9.919339		9.826532	4.55	10.173468	9
52	6060	3.14	9254		6805	4.55	3195	8
53	6248	3.13	9169		7078	4.55	2922	7
54	6436	3.13	9085	1.41	7351	4.55	2649	6
55	6624	3.13	9000	1.42	7624	4.55	2376	5
56	6812	3.13	8915		7897	4.54	2103	4
57	6999	3.13	8830		8170	4.54	1830	3
58	7187	3.12	8745		8442	4.54	1558	2
59	7374	3.12	8659	1.42	8715	4.54	1285	1
60	9.747562		9.918574		9.828987		10.171013	0
M.	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	56°

M.	Sine.	Diff. 1"	Cosine.	Diff. 1"	Tang.	Diff. 1"	Cotang.	M.
0	9.747562	3.12	9.918574	1.42	9.828987	4.54	10.171013	60
1	7749	3.12	8489		9260	4.54	0740	59
2	7936	3.12	8404		9532	4.54	0468	58
3	8123	3.11	8318		9.829805	4.54	10.170195	57
4	8310	3.11	8232		9.830077	4.54	10.169923	56
5	8497	3.11	8147	1.42	0349	4.53	9651	55
6	8683	3.11	8062	1.43	0621	4.53	9379	54
7	8870	3.11	7976		0893	4.53	9107	53
8	9056	3.10	7891		1165	4.53	8835	52
9	9243	3.10	7805		1437	4.53	8563	51
10	9429	3.10	7719		1709	4.53	8291	50
11	9615	3.10	9.917634		9.831981	4.53	10.168019	49
12	9801	3.10	7548		2253	4.53	7747	48
13	9.740987	3.09	7462		2525	4.53	7475	47
14	9.750172	3.09	7376		2796	4.53	7204	46
15	0358	3.09	7290		3068	4.52	6932	45
16	0543	3.09	7204	1.43	3339	4.52	6661	44
17	0729	3.09	7118	1.44	3611	4.52	6389	43
18	0914	3.08	7032		3882	4.52	6118	42
19	1099	3.08	6946		4154	4.52	5846	41
20	1284	3.08	6859		4425	4.52	5575	40
21	9.751469	3.08	9.916773		9.834696	4.52	10.165304	39
22	1654	3.08	6687		4967	4.52	5033	38
23	1839	3.08	6600		5238	4.52	4762	37
24	2023	3.07	6514		5509	4.52	4491	36
25	2208	3.07	6427		5780	4.51	4220	35
26	2392	3.07	6341		6051	4.51	3949	34
27	2576	3.07	6254	1.44	6322	4.51	3678	33
28	2760	3.07	6167	1.45	6593	4.51	3407	32
29	2944	3.06	6081		6864	4.51	3136	31
30	3128	3.06	5994		7134	4.51	2866	30
31	9.753312	3.06	9.915907		9.837405	4.51	10.162595	29
32	3495	3.06	5820		7675	4.51	2325	28
33	3679	3.06	5733		7946	4.51	2054	27
34	3862	3.05	5646		8216	4.50	1784	26
35	4046	3.05	5559		8487	4.50	1513	25
36	4229	3.05	5472		8757	4.50	1243	24
37	4412	3.05	5385		9027	4.50	0973	23
38	4595	3.05	5297		9297	4.50	0703	22
39	4778	3.04	5210	1.45	9568	4.50	0432	21
40	4960	3.04	5123	1.46	9.839838	4.50	10.160162	20
41	9.755143	3.04	9.915035		9.840108	4.50	10.159892	19
42	5326	3.04	4948		0378	4.50	9622	18
43	5508	3.04	4860		0648	4.50	9352	17
44	5690	3.04	4773		0917	4.49	9083	16
45	5872	3.03	4685		1187	4.49	8813	15
46	6054	3.03	4598		1457	4.49	8543	14
47	6236	3.03	4510		1727	4.49	8273	13
48	6418	3.03	4422		1996	4.49	8004	12
49	6600	3.03	4334	1.46	2266	4.49	7734	11
50	6782	3.02	4246	1.47	2535	4.49	7465	10
51	9.756963	3.02	9.914158		9.842805	4.49	10.157195	9
52	7144	3.02	4070		3074	4.49	6926	8
53	7326	3.02	3982		3343	4.49	6657	7
54	7507	3.02	3894		3612	4.49	6388	6
55	7688	3.01	3806		3882	4.48	6118	5
56	7869	3.01	3718		4151	4.48	5849	4
57	8050	3.01	3630		4420	4.48	5580	3
58	8230	3.01	3541		4689	4.48	5311	2
59	8411	3.01	3453	1.47	4958	4.48	5042	1
60	9.758591		9.913365		9.845227		10.154773	0
	Cosine.	Diff. 1"	Sine.	Diff. 1"	Cotang.	Diff. 1"	Tang.	55°

35° SINES AND TANGENTS.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.758591	3.01	9.913365	1.47	9.845227	4.48	10.154773	60
1	8772	3.00	3276	1.47	5496	4.48	4504	59
2	8952	3.00	3187	1.48	5764	4.48	4236	58
3	9132	3.00	3099		6033	4.48	3967	57
4	9312	3.00	3010		6302	4.48	3698	56
5	9492	3.00	2922		6570	4.47	3430	55
6	9672	2.99	2833		6839	4.47	3161	54
7	9.759852	2.99	2744		7107	4.47	2893	53
8	9.760031	2.99	2655		7376	4.47	2624	52
9	0211	2.99	2566		7644	4.47	2356	51
10	0390	2.99	2477		7913	4.47	2087	50
11	0569	2.98	9.912388	1.48	9.848181	4.47	10.151819	49
12	0748	2.98	2299	1.49	8449	4.47	1551	48
13	0927	2.98	2210		8717	4.47	1283	47
14	1106	2.98	2121		8986	4.47	1014	46
15	1285	2.98	2031		9254	4.47	0746	45
16	1464	2.98	1942		9522	4.47	0478	44
17	1642	2.97	1853		9.849790	4.46	10.150210	43
18	1821	2.97	1763		9.850057	4.46	10.149943	42
19	1999	2.97	1674		0325	4.46	9675	41
20	2177	2.97	1584		0593	4.46	9407	40
21	9.762356	2.97	9.911495		0861	4.46	9139	39
22	2534	2.96	1405	1.49	1129	4.46	8871	38
23	2712	2.96	1315	1.50	1396	4.46	8604	37
24	2889	2.96	1226		1664	4.46	8336	36
25	3067	2.96	1136		1931	4.46	8069	35
26	3245	2.96	1046		2199	4.46	7801	34
27	3422	2.96	0956		2466	4.46	7534	33
28	3600	2.95	0866		2733	4.45	7267	32
29	3777	2.95	0776		3001	4.45	6999	31
30	3954	2.95	0686		3268	4.45	6732	30
31	9.764131	2.95	9.910596		9.853535	4.45	10.146465	29
32	4308	2.95	0506	1.50	3802	4.45	6198	28
33	4485	2.94	0415	1.51	4069	4.45	5931	27
34	4662	2.94	0325		4336	4.45	5664	26
35	4838	2.94	0235		4603	4.45	5397	25
36	5015	2.94	0144		4870	4.45	5130	24
37	5191	2.94	9.910054		5137	4.45	4863	23
38	5367	2.94	9.909963		5404	4.45	4596	22
39	5544	2.93	9873		5671	4.44	4329	21
40	5720	2.93	9782		5938	4.44	4062	20
41	9.765896	2.93	9691		9.856204	4.44	10.143796	19
42	6072	2.93	9601		6471	4.44	3529	18
43	6247	2.93	9510		6737	4.44	3263	17
44	6423	2.93	9419	1.51	7004	4.44	2996	16
45	6598	2.92	9328	1.52	7270	4.44	2730	15
46	6774	2.92	9237		7537	4.44	2463	14
47	6949	2.92	9146		7803	4.44	2197	13
48	7124	2.92	9055		8069	4.44	1931	12
49	7300	2.92	8964		8336	4.44	1664	11
50	7475	2.91	8873		8602	4.43	1398	10
51	9.767649	2.91	9.908781		8868	4.43	1132	9
52	7824	2.91	8690		9134	4.43	0866	8
53	7999	2.91	8599		9400	4.43	0600	7
54	8173	2.91	8507	1.52	9666	4.43	0334	6
55	8348	2.90	8416	1.53	9.859932	4.43	10.140068	5
56	8522	2.90	8324		9.860198	4.43	10.139802	4
57	8697	2.90	8233		0464	4.43	9536	3
58	8871	2.90	8141		0730	4.43	9270	2
59	9045	2.90	8049	1.53	0995	4.43	9005	1
60	9.769219		9.907958		9.861261		10.138739	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	54°

M.	Sine.	Diff. 1"	Cosine.	Diff. 1"	Tang.	Diff. 1"	Cotang.	M.
0	9.769219	2.90	9.907958	1.53	9.861261	4.43	10.138739	60
1	9393	2.89	7866		1527	4.43	8473	59
2	9566	2.89	7774		1792	4.42	8208	58
3	9740	2.89	7682		2058	4.42	7942	57
4	9.769913	2.89	7590		2323	4.42	7677	56
5	9.770087	2.89	7498		2589	4.42	7411	55
6	0260	2.88	7406	1.53	2854	4.42	7146	54
7	0433	2.88	7314	1.54	3119	4.42	6881	53
8	0606	2.88	7222		3385	4.42	6615	52
9	0779	2.88	7129		3650	4.42	6350	51
10	0952	2.88	7037		3915	4.42	6085	50
11	9.771125	2.88	9.906945		9.864180	4.42	10.135820	49
12	1298	2.88	6852		4445	4.42	5555	48
13	1470	2.87	6760		4710	4.42	5290	47
14	1643	2.87	6667		4975	4.41	5025	46
15	1815	2.87	6575		5240	4.41	4760	45
16	1987	2.87	6482	1.54	5505	4.41	4495	44
17	2159	2.87	6389	1.55	5770	4.41	4230	43
18	2331	2.86	6296		6035	4.41	3965	42
19	2503	2.86	6204		6300	4.41	3700	41
20	2675	2.86	6111		6564	4.41	3436	40
21	9.772847	2.86	9.906018		9.866829	4.41	10.133171	39
22	3018	2.86	5925		7094	4.41	2906	38
23	3190	2.86	5832		7358	4.41	2642	37
24	3361	2.85	5739		7623	4.41	2377	36
25	3533	2.85	5645		7887	4.41	2113	35
26	3704	2.85	5552		8152	4.40	1848	34
27	3875	2.85	5459	1.55	8416	4.40	1584	33
28	4046	2.85	5366	1.56	8680	4.40	1320	32
29	4217	2.85	5272		8945	4.40	1055	31
30	4388	2.84	5179		9209	4.40	0791	30
31	9.774558	2.84	9.905085		9473	4.40	0527	29
32	4729	2.84	4992		9.869737	4.40	10.130263	28
33	4899	2.84	4898		9.870001	4.40	10.129999	27
34	5070	2.84	4804		0265	4.40	9735	26
35	5240	2.84	4711		0529	4.40	9471	25
36	5410	2.83	4617		0793	4.40	9207	24
37	5580	2.83	4523	1.56	1057	4.40	8943	23
38	5750	2.83	4429	1.57	1321	4.40	8679	22
39	5920	2.83	4335		1585	4.40	8415	21
40	6090	2.83	4241		1849	4.39	8151	20
41	9.776259	2.83	9.904147		9.872112	4.39	10.127888	19
42	6429	2.82	4053		2376	4.39	7624	18
43	6598	2.82	3959		2640	4.39	7360	17
44	6768	2.82	3864		2903	4.39	7097	16
45	6937	2.82	3770		3167	4.39	6833	15
46	7106	2.82	3676		3430	4.39	6570	14
47	7275	2.81	3581		3694	4.39	6306	13
48	7444	2.81	3487	1.57	3957	4.39	6043	12
49	7613	2.81	3392	1.58	4220	4.39	5780	11
50	7781	2.81	3298		4484	4.39	5516	10
51	9.777950	2.81	9.903203		9.874747	4.39	10.125253	9
52	8119	2.81	3108		5010	4.39	4990	8
53	8287	2.80	3014		5273	4.38	4727	7
54	8455	2.80	2919		5537	4.38	4463	6
55	8624	2.80	2824		5800	4.38	4200	5
56	8792	2.80	2729		6063	4.38	3937	4
57	8960	2.80	2634		6326	4.38	3674	3
58	9128	2.80	2539	1.58	6589	4.38	3411	2
59	9295	2.79	2444	1.59	6852	4.38	3148	1
60	9.779463		9.902349		9.877114		10.122886	0
	Cosine.	Diff. 1"	Sine.	Diff. 1"	Cotang.	Diff. 1"	Tang.	53°

37° SINES AND TANGENTS.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.779463	2.79	9.902349	1.59	9.877114	4.38	10.122886	60
1	9631	2.79	2253		7377	4.38	2623	59
2	9798	2.79	2158		7640	4.38	2360	58
3	9966	2.79	2063		7903	4.38	2097	57
4	9.780133	2.79	1967		8165	4.38	1835	56
5	0300	2.78	1872		8428	4.38	1572	55
6	0467	2.78	1776		8691	4.38	1309	54
7	0634	2.78	1681		8953	4.37	1047	53
8	0801	2.78	1585		9216	4.37	0784	52
9	0968	2.78	1490	1.59	9478	4.37	0522	51
10	1134	2.78	1394	1.60	9.879741	4.37	10.120259	50
11	9.781301	2.77	9.901298		9.880003	4.37	10.119997	49
12	1468	2.77	1202		0265	4.37	9735	48
13	1634	2.77	1106		0528	4.37	9472	47
14	1800	2.77	1010		0790	4.37	9210	46
15	1966	2.77	0914		1052	4.37	8948	45
16	2132	2.77	0818		1314	4.37	8686	44
17	2298	2.76	0722		1576	4.37	8424	43
18	2464	2.76	0626		1839	4.37	8161	42
19	2630	2.76	0529	1.60	2101	4.37	7899	41
20	2796	2.76	0433	1.61	2363	4.36	7637	40
21	9.782961	2.76	0337		9.882625	4.36	10.117375	39
22	3127	2.76	0240		2887	4.36	7113	38
23	3292	2.75	0144		3148	4.36	6852	37
24	3458	2.75	9.900047		3410	4.36	6590	36
25	3623	2.75	9.899951		3672	4.36	6328	35
26	3788	2.75	9854		3934	4.36	6066	34
27	3953	2.75	9757		4196	4.36	5804	33
28	4118	2.74	9660		4457	4.36	5543	32
29	4282	2.74	9564	1.61	4719	4.36	5281	31
30	4447	2.74	9467	1.62	4980	4.36	5020	30
31	9.784612	2.74	9.899370		9.885242	4.36	10.114758	29
32	4776	2.74	9273		5504	4.36	4496	28
33	4941	2.74	9176		5765	4.36	4235	27
34	5105	2.74	9078		6026	4.36	3974	26
35	5269	2.73	8981		6288	4.36	3712	25
36	5433	2.73	8884		6549	4.35	3451	24
37	5597	2.73	8787		6811	4.35	3189	23
38	5761	2.73	8689		7072	4.35	2928	22
39	5925	2.73	8592	1.62	7333	4.35	2667	21
40	6089	2.73	8494	1.63	7594	4.35	2406	20
41	9.786252	2.72	9.898397		9.887855	4.35	10.112145	19
42	6416	2.72	8299		8116	4.35	1884	18
43	6579	2.72	8202		8378	4.35	1622	17
44	6742	2.72	8104		8639	4.35	1361	16
45	6906	2.72	8006		8900	4.35	1100	15
46	7069	2.72	7908		9161	4.35	0839	14
47	7232	2.71	7810		9421	4.35	0579	13
48	7395	2.71	7712		9682	4.35	0318	12
49	7557	2.71	7614		9.889943	4.35	10.110057	11
50	7720	2.71	7516	1.63	9.890204	4.34	10.109796	10
51	9.787883	2.71	9.897418	1.64	0465	4.34	9535	9
52	8045	2.71	7320		0725	4.34	9275	8
53	8208	2.71	7222		0986	4.34	9014	7
54	8370	2.70	7123		1247	4.34	8753	6
55	8532	2.70	7025		1507	4.34	8493	5
56	8694	2.70	6926		1768	4.34	8232	4
57	8856	2.70	6828		2028	4.34	7972	3
58	9018	2.70	6729		2289	4.34	7711	2
59	9180	2.70	6631	1.64	2549	4.34	7451	1
60	9.789342		9.896532		9.892810		10.107190	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	52°

38° LOGARITHMIC

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.789342	2.69	9.896532	1.64	9.892810	4.34	10.107190	60
1	9504	2.69	6433	1.65	3070	4.34	6930	59
2	9665	2.69	6335		3331	4.34	6669	58
3	9827	2.69	6236		3591	4.34	6409	57
4	9.789988	2.69	6137		3851	4.34	6149	56
5	9.790149	2.69	6038		4111	4.34	5889	55
6	0310	2.68	5939		4371	4.34	5629	54
7	0471	2.68	5840		4632	4.33	5368	53
8	0632	2.68	5741		4892	4.33	5108	52
9	0793	2.68	5641		5152	4.33	4848	51
10	0954	2.68	5542	1.65	5412	4.33	4588	50
11	9.791115	2.68	9.895443	1.66	9.895672	4.33	10.104328	49
12	1275	2.67	5343		5932	4.33	4068	48
13	1436	2.67	5244		6192	4.33	3808	47
14	1596	2.67	5145		6452	4.33	3548	46
15	1757	2.67	5045		6712	4.33	3288	45
16	1917	2.67	4945		6971	4.33	3029	44
17	2077	2.67	4846		7231	4.33	2769	43
18	2237	2.66	4746		7491	4.33	2509	42
19	2397	2.66	4646		7751	4.33	2249	41
20	2557	2.66	4546	1.66	8010	4.33	1990	40
21	9.792716	2.66	9.894446	1.67	9.898270	4.33	10.101730	39
22	2876	2.66	4346		8530	4.33	1470	38
23	3035	2.66	4246		8789	4.32	1211	37
24	3195	2.66	4146		9049	4.32	0951	36
25	3354	2.65	4046		9308	4.32	0692	35
26	3514	2.65	3946		9568	4.32	0432	34
27	3673	2.65	3846		9.899827	4.32	10.100173	33
28	3832	2.65	3745		9.900087	4.32	10.099913	32
29	3991	2.65	3645		0346	4.32	9654	31
30	4150	2.64	3544	1.67	0605	4.32	9395	30
31	9.794308	2.64	9.893444	1.68	0864	4.32	9136	29
32	4467	2.64	3343		1124	4.32	8876	28
33	4626	2.64	3243		1383	4.32	8617	27
34	4784	2.64	3142		1642	4.32	8358	26
35	4942	2.64	3041		1901	4.32	8099	25
36	5101	2.64	2940		2160	4.32	7840	24
37	5259	2.63	2839		2420	4.32	7580	23
38	5417	2.63	2739		2679	4.32	7321	22
39	5575	2.63	2638		2938	4.32	7062	21
40	5733	2.63	2536	1.68	3197	4.31	6803	20
41	9.795891	2.63	9.892435	1.69	9.903456	4.31	10.096544	19
42	6049	2.63	2334		3714	4.31	6286	18
43	6206	2.63	2233		3973	4.31	6027	17
44	6364	2.62	2132		4232	4.31	5768	16
45	6521	2.62	2030		4491	4.31	5509	15
46	6679	2.62	1929		4750	4.31	5250	14
47	6836	2.62	1827		5008	4.31	4992	13
48	6993	2.62	1726		5267	4.31	4733	12
49	7150	2.62	1624	1.69	5526	4.31	4474	11
50	7307	2.61	1523	1.70	5785	4.31	4215	10
51	9.797464	2.61	9.891421		9.906043	4.31	10.093957	9
52	7621	2.61	1319		6302	4.31	3698	8
53	7777	2.61	1217		6560	4.31	3440	7
54	7934	2.61	1115		6819	4.31	3181	6
55	8091	2.61	1013		7077	4.31	2923	5
56	8247	2.61	0911		7336	4.31	2664	4
57	8403	2.60	0809		7594	4.31	2406	3
58	8560	2.60	0707		7853	4.31	2147	2
59	8716	2.60	0605	1.70	8111	4.30	1889	1
60	9.798872		9.890503		9.908369		10.091631	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	51°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.798872	2.60	9.890503	1.70	9.908369	4.30	10.091631	60
1	9028	2.60	0400	1.71	8628	4.30	1372	59
2	9184	2.60	0298		8886	4.30	1114	58
3	9339	2.59	0195		9144	4.30	0856	57
4	9495	2.59	9.890093		9402	4.30	0598	56
5	9651	2.59	9.889990		9660	4.30	0340	55
6	9806	2.59	9888		9.909918	4.30	10.090082	54
7	9.799962	2.59	9785		9.910177	4.30	10.089823	53
8	9.800117	2.59	9682		0435	4.30	9565	52
9	0272	2.58	9579		0693	4.30	9307	51
10	0427	2.58	9477	1.71	0951	4.30	9049	50
11	0582	2.58	9.889374	1.72	1209	4.30	8791	49
12	0737	2.58	9271		1467	4.30	8533	48
13	0892	2.58	9168		1725	4.30	8275	47
14	1047	2.58	9064		1982	4.30	8018	46
15	1201	2.58	8961		2240	4.30	7760	45
16	1356	2.57	8858		2498	4.30	7502	44
17	1511	2.57	8755		2756	4.30	7244	43
18	1665	2.57	8651		3014	4.29	6986	42
19	1819	2.57	8548	1.72	3271	4.29	6729	41
20	1973	2.57	8444	1.73	3529	4.29	6471	40
21	9.302128	2.57	9.888341		9.913787	4.29	10.086213	39
22	2282	2.56	8237		4044	4.29	5956	38
23	2436	2.56	8134		4302	4.29	5698	37
24	2589	2.56	8030		4560	4.29	5440	36
25	2743	2.56	7926		4817	4.29	5183	35
26	2897	2.56	7822		5075	4.29	4925	34
27	3050	2.56	7718		5332	4.29	4668	33
28	3204	2.56	7614		5590	4.29	4410	32
29	3357	2.55	7510	1.73	5847	4.29	4153	31
30	3511	2.55	7406	1.74	6104	4.29	3896	30
31	9.803664	2.55	9.887302		9.916362	4.29	10.083638	29
32	3817	2.55	7198		6619	4.29	3381	28
33	3970	2.55	7093		6877	4.29	3123	27
34	4123	2.55	6989		7134	4.29	2866	26
35	4276	2.54	6885		7391	4.29	2609	25
36	4428	2.54	6780		7648	4.29	2352	24
37	4581	2.54	6676		7906	4.29	2094	23
38	4734	2.54	6571		8163	4.28	1837	22
39	4886	2.54	6466	1.74	8420	4.28	1580	21
40	5039	2.54	6362	1.75	8677	4.28	1323	20
41	9.805191	2.54	9.886257		8934	4.28	1066	19
42	5343	2.53	6152		9191	4.28	0809	18
43	5495	2.53	6047		9448	4.28	0552	17
44	5647	2.53	5942		9705	4.28	0295	16
45	5799	2.53	5837		9.919962	4.28	10.080038	15
46	5951	2.53	5732		9.920219	4.28	10.079781	14
47	6103	2.53	5627		0476	4.28	9524	13
48	6254	2.53	5522		0733	4.28	9267	12
49	6406	2.52	5416	1.75	0990	4.28	9010	11
50	6557	2.52	5311	1.76	1247	4.28	8753	10
51	9.806709	2.52	9.885205		1503	4.28	8497	9
52	6860	2.52	5100		1760	4.28	8240	8
53	7011	2.52	4994		2017	4.28	7983	7
54	7163	2.52	4889		2274	4.28	7726	6
55	7314	2.52	4783		2530	4.28	7470	5
56	7465	2.51	4677		2787	4.28	7213	4
57	7615	2.51	4572	1.76	3044	4.28	6956	3
58	7766	2.51	4466	1.77	3300	4.28	6700	2
59	7917	2.51	4360	1.77	3557	4.27	6443	1
60	9.808067		9.884254		9.923814		10.076186	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	50°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.808067	2.51	9.884254	1.77	9.923814	4.28	10.076186	60
1	8218	2.51	4148		4070	4.27	5930	59
2	8368	2.51	4042		4327	4.27	5673	58
3	8519	2.50	3936		4583	4.27	5417	57
4	8669	2.50	3829		4840	4.27	5160	56
5	8819	2.50	3723		5096	4.27	4904	55
6	8969	2.50	3617		5352	4.27	4648	54
7	9119	2.50	3510		5609	4.27	4391	53
8	9269	2.50	3404	1.77	5865	4.27	4135	52
9	9419	2.49	3297	1.78	6122	4.27	3878	51
10	9569	2.49	3191		6378	4.27	3622	50
11	9718	2.49	9.883084		9.926634	4.27	10.073366	49
12	9.809868	2.49	2977		6890	4.27	3110	48
13	9.810017	2.49	2871		7147	4.27	2853	47
14	0167	2.49	2764		7403	4.27	2597	46
15	0316	2.48	2657		7659	4.27	2341	45
16	0465	2.48	2550		7915	4.27	2085	44
17	0614	2.48	2443	1.78	8171	4.27	1829	43
18	0763	2.48	2336	1.79	8427	4.27	1573	42
19	0912	2.48	2229		8684	4.27	1316	41
20	1061	2.48	2121		8940	4.27	1060	40
21	9.811210	2.48	9.882014		9196	4.27	0804	39
22	1358	2.48	1907		9452	4.27	0548	38
23	1507	2.47	1799		9708	4.27	0292	37
24	1655	2.47	1692		9.929964	4.27	10.070036	36
25	1804	2.47	1584		9.930220	4.26	10.069780	35
26	1952	2.47	1477		0475	4.26	9525	34
27	2100	2.47	1369	1.79	0731	4.26	9269	33
28	2248	2.47	1261	1.80	0987	4.26	9013	32
29	2396	2.46	1153		1243	4.26	8757	31
30	2544	2.46	1046		1499	4.26	8501	30
31	9.812692	2.46	9.880938		9.931755	4.26	10.068245	29
32	2840	2.46	0830		2010	4.26	7990	28
33	2988	2.46	0722		2266	4.26	7734	27
34	3135	2.46	0613		2522	4.26	7478	26
35	3283	2.46	0505		2778	4.26	7222	25
36	3430	2.46	0397	1.80	3033	4.26	6967	24
37	3578	2.45	0289	1.81	3289	4.26	6711	23
38	3725	2.45	0180		3545	4.26	6455	22
39	3872	2.45	9.880072		3800	4.26	6200	21
40	4019	2.45	9.879963		4056	4.26	5944	20
41	9.814166	2.45	9855		9.934311	4.26	10.065689	19
42	4313	2.45	9746		4567	4.26	5433	18
43	4460	2.44	9637		4822	4.26	5178	17
44	4607	2.44	9529		5078	4.26	4922	16
45	4753	2.44	9420		5333	4.26	4667	15
46	4900	2.44	9311	1.81	5589	4.26	4411	14
47	5046	2.44	9202	1.82	5844	4.26	4156	13
48	5193	2.44	9093		6100	4.26	3900	12
49	5339	2.44	8984		6355	4.26	3645	11
50	5485	2.43	8875		6611	4.26	3389	10
51	9.815632	2.43	9.878766		9.936866	4.25	10.063134	9
52	5778	2.43	8656		7121	4.25	2879	8
53	5924	2.43	8547		7377	4.25	2623	7
54	6069	2.43	8438		7632	4.25	2368	6
55	6215	2.43	8328	1.82	7887	4.25	2113	5
56	6361	2.43	8219	1.83	8142	4.25	1858	4
57	6507	2.42	8109		8398	4.25	1602	3
58	6652	2.42	7999		8653	4.25	1347	2
59	6798	2.42	7890	1.83	8908	4.25	1092	1
60	9.816943		9.877780		9.939163		10.060837	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	49°

41° SINES AND TANGENTS.

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.816943	2.42	9.877780	1.83	9.939163	4.25	10.060837	60
1	7088	2.42	7670		9418	4.25	0582	59
2	7233	2.42	7560		9673	4.25	0327	58
3	7379	2.42	7450		9.939928	4.25	10.060072	57
4	7524	2.42	7340	1.83	9.940183	4.25	10.059817	56
5	7668	2.41	7230	1.84	0439	4.25	9561	55
6	7813	2.41	7120		0694	4.25	9306	54
7	7958	2.41	7010		0949	4.25	9051	53
8	8103	2.41	6899		1204	4.25	8796	52
9	8247	2.41	6789		1459	4.25	8541	51
10	8392	2.41	6678		1713	4.25	8287	50
11	9.818536	2.40	9.876568		9.941968	4.25	10.058032	49
12	8681	2.40	6457		2223	4.25	7777	48
13	8825	2.40	6347	1.84	2478	4.25	7522	47
14	8969	2.40	6236	1.85	2733	4.25	7267	46
15	9113	2.40	6125		2988	4.25	7012	45
16	9257	2.40	6014		3243	4.25	6757	44
17	9401	2.40	5904		3498	4.25	6502	43
18	9545	2.40	5793		3752	4.25	6248	42
19	9689	2.39	5682		4007	4.25	5993	41
20	9832	2.39	5571		4262	4.25	5738	40
21	9.819976	2.39	9.875459		9.944517	4.25	10.055483	39
22	9.820120	2.39	5348		4771	4.24	5229	38
23	0263	2.39	5237	1.85	5026	4.24	4974	37
24	0406	2.39	5126	1.86	5281	4.24	4719	36
25	0550	2.38	5014		5535	4.34	4465	35
26	0693	2.38	4903		5790	4.24	4210	34
27	0836	2.38	4791		6045	4.24	3955	33
28	0979	2.38	4680		6299	4.24	3701	32
29	1122	2.38	4568		6554	4.24	3446	31
30	1265	2.38	4456		6808	4.24	3192	30
31	9.821407	2.38	9.874344	1.86	9.947063	4.24	10.052937	29
32	1550	2.38	4232	1.87	7318	4.24	2682	28
33	1693	2.37	4121		7572	4.24	2428	27
34	1835	2.37	4009		7827	4.24	2173	26
35	1977	2.37	3896		8081	4.24	1919	25
36	2120	2.37	3784		8335	4.24	1665	24
37	2262	2.37	3672		8590	4.24	1410	23
38	2404	2.37	3560		8844	4.24	1156	22
39	2546	2.37	3448		9099	4.24	0901	21
40	2688	2.36	3335		9353	4.24	0647	20
41	9.822830	2.36	9.873223	1.87	9608	4.24	0392	19
42	2972	2.36	3110	1.88	9.949862	4.24	10.050138	18
43	3114	2.36	2998		9.950116	4.24	10.049884	17
44	3255	2.36	2885		0371	4.24	9629	16
45	3397	2.36	2772		0625	4.24	9375	15
46	3539	2.36	2659		0879	4.24	9121	14
47	3680	2.35	2547		1133	4.24	8867	13
48	3821	2.35	2434		1388	4.24	8612	12
49	3963	2.35	2321		1642	4.24	8358	11
50	4104	2.35	2208	1.88	1896	4.24	8104	10
51	9.824245	2.35	9.872095	1.89	9.952150	4.24	10.047850	9
52	4386	2.35	1981		2405	4.24	7595	8
53	4527	2.35	1868		2659	4.24	7341	7
54	4668	2.34	1755		2913	4.24	7087	6
55	4808	2.34	1641		3167	4.23	6833	5
56	4949	2.34	1528		3421	4.23	6579	4
57	5090	2.34	1414		3675	4.23	6325	3
58	5230	2.34	1301		3929	4.23	6071	2
59	5371	2.34	1187	1.89	4183	4.23	5817	1
60	9.825511		9.871073		9.954437		10.045563	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	48°

42°				LOGARITHMIC				
M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.825511	2.34	9.871073	1.90	9.954437	4.23	10.045563	60
1	5651	2.33	0960		4691	4.23	5309	59
2	5791	2.33	0846		4946	4.23	5054	58
3	5931	2.33	0732		5200	4.23	4800	57
4	6071	2.33	0618		5454	4.23	4546	56
5	6211	2.33	0504		5708	4.23	4292	55
6	6351	2.33	0390		5961	4.23	4039	54
7	6491	2.33	0276		6215	4.23	3785	53
8	6631	2.33	0161	1.90	6469	4.23	3531	52
9	6770	2.32	9.870047	1.91	6723	4.23	3277	51
10	6910	2.32	9.869933		6977	4.23	3023	50
11	9.827049	2.32	9818		9.957231	4.23	10.042769	49
12	7189	2.32	9704		7485	4.23	2515	48
13	7328	2.32	9589		7739	4.23	2261	47
14	7467	2.32	9474		7993	4.23	2007	46
15	7606	2.32	9360		8247	4.23	1753	45
16	7745	2.32	9245		8500	4.23	1500	44
17	7884	2.31	9130	1.91	8754	4.23	1246	43
18	8023	2.31	9015	1.92	9008	4.23	0992	42
19	8162	2.31	8900		9262	4.23	0738	41
20	8301	2.31	8785		9516	4.23	0484	40
21	9.828439	2.31	9.868670		9.959769	4.23	10.040231	39
22	8578	2.31	8555		9.960023	4.23	10.039977	38
23	8716	2.31	8440		0277	4.23	9723	37
24	8855	2.30	8324		0530	4.23	9470	36
25	8993	2.30	8209		0784	4.23	9216	35
26	9131	2.30	8093	.92	1038	4.23	8962	34
27	9269	2.30	7978	1.93	1292	4.23	8708	33
28	9407	2.30	7862		1545	4.23	8455	32
29	9545	2.30	7747		1799	4.23	8201	31
30	9683	2.30	7631		2052	4.23	7948	30
31	9821	2.29	9.867515		9.962306	4.23	10.037694	29
32	9.829959	2.29	7399		2560	4.23	7440	28
33	9.830097	2.29	7283		2813	4.23	7187	27
34	0234	2.29	7167		3067	4.23	6933	26
35	0372	2.29	7051	1.93	3320	4.23	6680	25
36	0509	2.29	6935	1.94	3574	4.23	6426	24
37	0646	2.29	6819		3828	4.23	6172	23
38	0784	2.29	6703		4081	4.23	5919	22
39	0921	2.28	6586		4335	4.23	5665	21
40	1058	2.28	6470		4588	4.22	5412	20
41	9.831195	2.28	9.866353		9.964842	4.22	10.035158	19
42	1332	2.28	6237		5095	4.22	4905	18
43	1469	2.28	6120	1.94	5349	4.22	4651	17
44	1606	2.28	6004	1.95	5602	4.22	4398	16
45	1742	2.28	5887		5855	4.22	4145	15
46	1879	2.28	5770		6109	4.22	3891	14
47	2015	2.27	5653		6362	4.22	3638	13
48	2152	2.27	5536		6616	4.22	3384	12
49	2288	2.27	5419		6869	4.22	3131	11
50	2425	2.27	5302		7123	4.22	2877	10
51	9.832561	2.27	9.865185		9.967376	4.22	10.032624	9
52	2697	2.27	5068		7629	4.22	2371	8
53	2833	2.27	4950	1.95	7883	4.22	2117	7
54	2969	2.26	4833	1.96	8136	4.22	1864	6
55	3105	2.26	4716		8389	4.22	1611	5
56	3241	2.26	4598		8643	4.22	1357	4
57	3377	2.26	4481		8896	4.22	1104	3
58	3512	2.26	4363		9149	4.22	0851	2
59	3648	2.26	4245	1.96	9403	4.22	0597	1
60	9.833783		9.864127		9.969656		10.030344	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	47°

43° SINES AND TANGENTS.

M.	Sine.	Diff. 1"	Cosine.	Diff. 1"	Tang.	Diff. 1"	Cotang.	M.
0	9.833783	2.26	9.864127	1.96	9.969656	4.22	10.030344	60
1	3919	2.25	4010	1.96	9909	4.22	0091	59
2	4054	2.25	3892	1.97	9.970162	4.22	10.029838	58
3	4189	2.25	3774		0416	4.22	9584	57
4	4325	2.25	3656		0669	4.22	9331	56
5	4460	2.25	3538		0922	4.22	9078	55
6	4595	2.25	3419		1175	4.22	8825	54
7	4730	2.25	3301		1429	4.22	8571	53
8	4865	2.25	3183		1682	4.22	8318	52
9	4999	2.24	3064	1.97	1935	4.22	8065	51
10	5134	2.24	2946	1.98	2188	4.22	7812	50
11	9.835269	2.24	9.862827		9.972441	4.22	10.027559	49
12	5403	2.24	2709		2695	4.22	7305	48
13	5538	2.24	2590		2948	4.22	7052	47
14	5672	2.24	2471		3201	4.22	6799	46
15	5807	2.24	2353		3454	4.22	6546	45
16	5941	2.24	2234		3707	4.22	6293	44
17	6075	2.23	2115		3960	4.22	6040	43
18	6209	2.23	1996		4213	4.22	5787	42
19	6343	2.23	1877	1.98	4466	4.22	5534	41
20	6477	2.23	1758	1.99	4720	4.22	5280	40
21	9.836611	2.23	9.861638		9.974973	4.22	10.025027	39
22	6745	2.23	1519		5226	4.22	4774	38
23	6878	2.23	1400		5479	4.22	4521	37
24	7012	2.22	1280		5732	4.22	4268	36
25	7146	2.22	1161		5985	4.22	4015	35
26	7279	2.22	1041		6238	4.22	3762	34
27	7412	2.22	0922		6491	4.22	3509	33
28	7546	2.22	0802	1.99	6744	4.22	3256	32
29	7679	2.22	0682	2.00	6997	4.22	3003	31
30	7812	2.22	0562		7250	4.22	2750	30
31	9.837945	2.22	0442		9.977503	4.22	10.022497	29
32	8078	2.21	0322		7756	4.22	2244	28
33	8211	2.21	0202		8009	4.22	1991	27
34	8344	2.21	9.860082		8262	4.22	1738	26
35	8477	2.21	9.859962		8515	4.22	1485	25
36	8610	2.21	9842	2.00	8768	4.22	1232	24
37	8742	2.21	9721	2.01	9021	4.22	0979	23
38	8875	2.21	9601		9274	4.22	0726	22
39	9007	2.21	9480		9527	4.22	0473	21
40	9140	2.20	9360		9.979780	4.22	10.020220	20
41	9272	2.20	9239		9.980033	4.22	10.019967	19
42	9404	2.20	9119		0286	4.22	9714	18
43	9536	2.20	8998		0538	4.22	9462	17
44	9668	2.20	8877	2.01	0791	4.21	9209	16
45	9800	2.20	8756	2.02	1044	4.21	8956	15
46	9.839932	2.20	8635		1297	4.21	8703	14
47	9.840064	2.19	8514		1550	4.21	8450	13
48	0196	2.19	8393		1803	4.21	8197	12
49	0328	2.19	8272		2056	4.21	7944	11
50	0459	2.19	8151		2309	4.21	7691	10
51	0591	2.19	9.858029		9.982562	4.21	10.017438	9
52	0722	2.19	7908		2814	4.21	7186	8
53	0854	2.19	7786	2.02	3067	4.21	6933	7
54	0985	2.19	7665	2.03	3320	4.21	6680	6
55	1116	2.19	7543		3573	4.21	6427	5
56	1247	2.18	7422		3826	4.21	6174	4
57	1378	2.18	7300		4079	4.21	5921	3
58	1509	2.18	7178		4332	4.21	5668	2
59	1640	2.18	7056	2.03	4584	4.21	5416	1
60	9.841771		9.856934		9.984837		10.015163	0
	Cosine.	Diff. 1"	Sine.	Diff. 1"	Cotang.	Diff. 1"	Tang.	46°

M.	Sine.	Diff. 1″	Cosine.	Diff. 1″	Tang.	Diff. 1″	Cotang.	M.
0	9.841771	2.18	9.856934	2.03	9.984837	4.21	10.015163	60
1	1902	2.18	6812	2.03	5090	4.21	4910	59
2	2033	2.18	6690	2.04	5343	4.21	4657	58
3	2163	2.17	6568		5596	4.21	4404	57
4	2294	2.17	6446		5848	4.21	4152	56
5	2424	2.17	6323		6101	4.21	3899	55
6	2555	2.17	6201		6354	4.21	3646	54
7	2685	2.17	6078		6607	4.21	3393	53
8	2815	2.17	5956		6860	4.21	3140	52
9	2946	2.17	5833	2.04	7112	4.21	2888	51
10	3076	2.17	5711	2.05	7365	4.21	2635	50
11	9.843206	2.16	9.855588		9.987618	4.21	10.012382	49
12	3336	2.16	5465		7871	4.21	2129	48
13	3466	2.16	5342		8123	4.21	1877	47
14	3595	2.16	5219		8376	4.21	1624	46
15	3725	2.16	5096		8629	4.21	1371	45
16	3855	2.16	4973		8882	4.21	1118	44
17	3984	2.16	4850		9134	4.21	0866	43
18	4114	2.16	4727	2.05	9387	4.21	0613	42
19	4243	2.15	4603	2.06	9640	4.21	0360	41
20	4372	2.15	4480		9.989893	4.21	10.010107	40
21	9.844502	2.15	9.854356		9.990145	4.21	10.009855	39
22	4631	2.15	4233		0398	4.21	9602	38
23	4760	2.15	4109		0651	4.21	9349	37
24	4889	2.15	3986		0903	4.21	9097	36
25	5018	2.15	3862		1156	4.21	8844	35
26	5147	2.15	3738	2.06	1409	4.21	8591	34
27	5276	2.14	3614	2.07	1662	4.21	8338	33
28	5405	2.14	3490		1914	4.21	8086	32
29	5533	2.14	3366		2167	4.21	7833	31
30	5662	2.14	3242		2420	4.21	7580	30
31	9.845790	2.14	9.853118		9.992672	4.21	10.007328	29
32	5919	2.14	2994		2925	4.21	7075	28
33	6047	2.14	2869		3178	4.21	6822	27
34	6175	2.14	2745		3431	4.21	6569	26
35	6304	2.14	2620	2.07	3683	4.21	6317	25
36	6432	2.13	2496	2.08	3936	4.21	6064	24
37	6560	2.13	2371		4189	4.21	5811	23
38	6688	2.13	2247		4441	4.21	5559	22
39	6816	2.13	2122		4694	4.21	5306	21
40	6944	2.13	1997		4947	4.21	5053	20
41	9.847071	2.13	9.851872		9.995199	4.21	10.004801	19
42	7199	2.13	1747		5452	4.21	4548	18
43	7327	2.13	1622	2.08	5705	4.21	4295	17
44	7454	2.12	1497	2.09	5957	4.21	4043	16
45	7582	2.12	1372		6210	4.21	3790	15
46	7709	2.12	1246		6463	4.21	3537	14
47	7836	2.12	1121		6715	4.21	3285	13
48	7964	2.12	0996		6968	4.21	3032	12
49	8091	2.12	0870		7221	4.21	2779	11
50	8218	2.12	0745		7473	4.21	2527	10
51	9.848345	2.12	9.850619	2.09	9.997726	4.21	10.002274	9
52	8472	2.11	0493	2.10	7979	4.21	2021	8
53	8599	2.11	0368		8231	4.21	1769	7
54	8726	2.11	0242		8484	4.21	1516	6
55	8852	2.11	9.850116		8737	4.21	1263	5
56	8979	2.11	9.849990		8989	4.21	1011	4
57	9106	2.11	9864		9242	4.21	0758	3
58	9232	2.11	9738		9495	4.21	0505	2
59	9359	2.11	9611	2.10	9.999747	4.21	0253	1
60	9.849485		9.849485		10.000000		10.000000	0
	Cosine.	Diff. 1″	Sine.	Diff. 1″	Cotang.	Diff. 1″	Tang.	45°

PROPORTIONAL

D.	1	2	3	4	5	6	7	8	9
44	4	9	13	18	22	26	31	35	40
45	5	9	14	18	23	27	32	36	41
46	5	9	14	18	23	28	32	37	41
47	5	9	14	19	24	28	33	38	42
48	5	10	14	19	24	29	34	38	43
49	5	10	15	20	25	29	34	39	44
50	5	10	15	20	25	30	35	40	45
51	5	10	15	20	26	31	36	41	46
52	5	10	16	21	26	31	36	42	47
53	5	11	16	21	27	32	37	42	48
54	5	11	16	22	27	32	38	43	49
55	6	11	17	22	28	33	39	44	50
56	6	11	17	22	28	34	39	45	50
57	6	11	17	23	29	34	40	46	51
58	6	12	17	23	29	35	41	46	52
59	6	12	18	24	30	35	41	47	53
60	6	12	18	24	30	36	42	48	54
61	6	12	18	24	31	37	43	49	55
62	6	12	19	25	31	37	43	50	56
63	6	13	19	25	32	38	44	50	57
64	6	13	19	26	32	38	45	51	58
65	7	13	20	26	33	39	46	52	59
66	7	13	20	26	33	40	46	53	59
67	7	13	20	27	34	40	47	54	60
68	7	14	20	27	34	41	48	54	61
69	7	14	21	28	35	41	48	55	62
70	7	14	21	28	35	42	49	56	63
71	7	14	21	28	36	43	50	57	64
72	7	14	22	29	36	43	50	58	65
73	7	15	22	29	37	44	51	58	66
74	7	15	22	30	37	44	52	59	67
75	8	15	23	30	38	45	53	60	68
76	8	15	23	30	38	46	53	61	68
77	8	15	23	31	39	46	54	62	69
78	8	16	23	31	39	47	55	62	70
80	8	16	24	32	40	48	56	64	72
82	8	16	25	33	41	49	57	66	74
84	8	17	25	34	42	50	59	67	76
86	9	17	26	34	43	52	60	69	77
88	9	18	26	35	44	53	62	70	79
90	9	18	27	36	45	54	63	72	81
92	9	18	28	37	46	55	64	74	83
94	9	19	28	38	47	56	66	75	85
96	10	19	29	38	48	58	67	77	86
98	10	20	29	39	49	59	69	78	88
100	10	20	30	40	50	60	70	80	90
104	10	21	31	42	52	62	73	83	94
108	11	22	32	43	54	65	76	86	97
112	11	22	34	45	56	67	78	90	101
116	12	23	35	46	58	70	81	93	104
120	12	24	36	48	60	72	84	96	108
124	12	25	37	50	62	74	87	99	112
128	13	26	38	51	64	77	90	102	115
132	13	26	40	53	66	79	92	106	119
136	14	27	41	54	68	82	95	109	122
140	14	28	42	56	70	84	98	112	126
144	14	29	43	58	72	86	101	115	130
148	15	30	44	59	74	89	104	118	133
152	15	30	46	61	76	91	106	122	137
156	16	31	47	62	78	94	109	125	140

PARTS.									
D.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
160	16	32	48	64	80	96	112	128	144
164	16	33	49	66	82	98	115	131	148
168	17	34	50	67	84	101	118	134	151
172	17	34	52	69	86	103	120	138	155
176	18	35	53	70	88	106	123	141	158
180	18	36	54	72	90	108	126	144	162
184	18	37	55	74	92	110	129	147	166
188	19	38	56	75	94	113	132	150	169
192	19	38	58	77	96	115	134	154	173
196	20	39	59	78	98	118	137	157	176
200	20	40	60	80	100	120	140	160	180
204	20	41	61	82	102	122	143	163	184
208	21	42	62	83	104	125	146	166	187
212	21	42	64	85	106	127	148	170	191
216	22	43	65	86	108	130	151	173	194
220	22	44	66	88	110	132	154	176	198
224	22	45	67	90	112	134	157	179	202
228	23	46	68	91	114	137	160	182	205
232	23	46	70	93	116	139	162	186	209
236	24	47	71	94	118	142	165	189	212
240	24	48	72	96	120	144	168	192	216
244	24	49	73	98	122	146	171	195	220
248	25	50	74	99	124	149	174	198	223
252	25	50	76	101	126	151	176	202	227
256	26	51	77	102	128	154	179	205	230
260	26	52	78	104	130	156	182	208	234
264	26	53	79	106	132	158	185	211	238
268	27	54	80	107	134	161	188	214	241
272	27	54	82	109	136	163	190	218	245
276	28	55	83	110	138	166	193	221	248
280	28	56	84	112	140	168	196	224	252
284	28	57	85	114	142	170	199	227	256
288	29	58	86	115	144	173	202	230	259
292	29	58	88	117	146	175	204	234	263
296	30	59	89	118	148	178	207	237	266
300	30	60	90	120	150	180	210	240	270
304	30	61	91	122	152	182	213	243	274
308	31	62	92	123	154	185	216	246	277
312	31	62	94	125	156	187	218	250	281
316	32	63	95	126	158	190	221	253	284
320	32	64	96	128	160	192	224	256	288
324	32	65	97	130	162	194	227	259	292
328	33	66	98	131	164	197	230	262	295
332	33	66	100	133	166	199	232	266	299
336	34	67	101	134	168	202	235	269	302
340	34	68	102	136	170	204	238	272	306
344	34	69	103	138	172	206	241	275	310
348	35	70	104	139	174	209	244	278	313
352	35	70	106	141	176	211	246	282	317
356	36	71	107	142	178	214	249	285	320
360	36	72	108	144	180	216	252	288	324
364	36	73	109	146	182	218	255	291	328
368	37	74	110	147	184	221	258	294	331
372	37	74	112	149	186	223	260	298	335
376	38	75	113	150	188	226	263	301	338
380	38	76	114	152	190	228	266	304	342
384	38	77	115	154	192	230	269	307	346
388	39	78	116	155	194	233	272	310	349
392	39	78	118	157	196	235	274	314	353
396	40	79	119	158	198	238	277	317	356

TRAVERSE TABLE;

OR,

TABLE OF

Differences of Latitude and Departures.

LATITUDES AND DEPARTURES.

D.	¼ Deg.		½ Deg.		¾ Deg.		1 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	1.0000	.0044	1.0000	.0087	.9999	.0131	.9998	.0175	1
2	2.0000	.0087	1.9999	.0175	1.9998	.0262	1.9997	.0349	2
3	3.0000	.0131	2.9999	.0262	2.9997	.0393	2.9995	.0524	3
4	4.0000	.0175	3.9998	.0349	3.9997	.0524	3.9994	.0698	4
5	5.0000	.0218	4.9998	.0436	4.9996	.0654	4.9992	.0873	5
6	5.9999	.0262	5.9998	.0524	5.9995	.0785	5.9991	.1047	6
7	6.9999	.0305	6.9997	.0611	6.9994	.0916	6.9989	.1222	7
8	7.9999	.0349	7.9997	.0698	7.9993	.1047	7.9988	.1396	8
9	8.9999	.0393	8.9997	.0785	8.9992	.1178	8.9986	.1571	9
10	9.9999	.0436	9.9996	.0873	9.9991	.1309	9.9985	.1745	10
	89¾ Deg.		89½ Deg.		89¼ Deg.		89 Deg.		
	1¼ Deg.		1½ Deg.		1¾ Deg.		2 Deg.		
1	.9998	.0218	.9997	.0262	.9995	.0305	.9994	.0349	1
2	1.9995	.0436	1.9993	.0524	1.9991	.0611	1.9988	.0698	2
3	2.9993	.0654	2.9990	.0785	2.9986	.0916	2.9982	.1047	3
4	3.9990	.0873	3.9986	.1047	3.9981	.1222	3.9976	.1396	4
5	4.9988	.1091	4.9983	.1309	4.9977	.1527	4.9970	.1745	5
6	5.9986	.1309	5.9979	.1571	5.9972	.1832	5.9963	.2094	6
7	6.9983	.1527	6.9976	.1832	6.9967	.2138	6.9957	.2443	7
8	7.9981	.1745	7.9973	.2094	7.9963	.2443	7.9951	.2792	8
9	8.9979	.1963	8.9969	.2356	8.9958	.2748	8.9945	.3141	9
10	9.9976	.2181	9.9966	.2618	9.9953	.3054	9.9939	.3490	10
	88¾ Deg.		88½ Deg.		88¼ Deg.		88 Deg.		
	2¼ Deg.		2½ Deg.		2¾ Deg.		3 Deg.		
1	.9992	.0393	.9990	.0436	.9988	.0480	.9986	.0523	1
2	1.9985	.0785	1.9981	.0872	1.9977	.0960	1.9973	.1047	2
3	2.9977	.1178	2.9971	.1308	2.9965	.1439	2.9959	.1570	3
4	3.9969	.1570	3.9962	.1745	3.9954	.1919	3.9945	.2093	4
5	4.9961	.1963	4.9952	.2181	4.9942	.2399	4.9931	.2617	5
6	5.9954	.2356	5.9943	.2617	5.9931	.2879	5.9918	.3140	6
7	6.9946	.2748	6.9933	.3053	6.9919	.3358	6.9904	.3664	7
8	7.9938	.3140	7.9924	.3490	7.9908	.3838	7.9890	.4187	8
9	8.9931	.3533	8.9914	.3926	8.9896	.4318	8.9877	.4710	9
10	9.9913	.3926	9.9905	.4362	9.9885	.4798	9.9863	.5234	10
	87¾ Deg.		87½ Deg.		87¼ Deg.		87 Deg.		
	3¼ Deg.		3½ Deg.		3¾ Deg.		4 Deg.		
1	.9984	.0567	.9981	.0610	.9979	.0654	.9976	.0698	1
2	1.9968	.1134	1.9963	.1221	1.9957	.1308	1.9951	.1395	2
3	2.9952	.1701	2.9944	.1831	2.9936	.1962	2.9927	.2093	3
4	3.9936	.2268	3.9925	.2442	3.9914	.2616	3.9903	.2790	4
5	4.9920	.2835	4.9907	.3052	4.9893	.3270	4.9878	.3488	5
6	5.9904	.3402	5.9888	.3663	5.9872	.3924	5.9854	.4185	6
7	6.9887	.3968	6.9869	.4273	6.9850	.4578	6.9829	.4883	7
8	7.9871	.4535	7.9851	.4884	7.9829	.5232	7.9805	.5581	8
9	8.9855	.5102	8.9832	.5494	8.9807	.5886	8.9781	.6278	9
10	9.9839	.5669	9.9813	.6105	9.9786	.6540	9.9756	.6976	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	86¾ Deg.		86½ Deg.		86¼ Deg.		86 Deg.		

LATITUDES AND DEPARTURES.

D.	4¼ Deg. Lat.	4¼ Deg. Dep.	4½ Deg. Lat.	4½ Deg. Dep.	4¾ Deg. Lat.	4¾ Deg. Dep.	5 Deg. Lat.	5 Deg. Dep.	D.
1	.9973	.0741	.9969	.0785	.9966	.0828	.9962	.0872	1
2	1.9945	.1482	1.9938	.1569	1.9931	.1656	1.9924	.1743	2
3	2.9918	.2223	2.9908	.2354	2.9897	.2484	2.9886	.2615	3
4	3.9890	.2964	3.9877	.3138	3.9863	.3312	3.9848	.3486	4
5	4.9863	.3705	4.9846	.3923	4.9828	.4140	4.9810	.4358	5
6	5.9835	.4447	5.9815	.4708	5.9794	.4968	5.9772	.5229	6
7	6.9808	.5188	6.9784	.5492	6.9760	.5797	6.9734	.6101	7
8	7.9780	.5929	7.9753	.6277	7.9725	.6625	7.9696	.6972	8
9	8.9753	.6670	8.9723	.7061	8.9691	.7453	8.9658	.7844	9
10	9.9725	.7411	9.9692	.7846	9.9657	.8281	9.9619	.8716	10
	85¾ Deg.		85½ Deg.		85¼ Deg.		85 Deg.		
	5¼ Deg.		5½ Deg.		5¾ Deg.		6 Deg.		
1	.9958	.0915	.9954	.0958	.9950	.1002	.9945	.1045	1
2	1.9916	.1830	1.9908	.1917	1.9899	.2004	1.9890	.2091	2
3	2.9874	.2745	2.9862	.2875	2.9849	.3006	2.9836	.3136	3
4	3.9832	.3660	3.9816	.3834	3.9799	.4008	3.9781	.4181	4
5	4.9790	.4575	4.9770	.4792	4.9748	.5009	4.9726	.5226	5
6	5.9748	.5490	5.9724	.5751	5.9698	.6011	5.9671	.6272	6
7	6.9706	.6405	6.9678	.6709	6.9648	.7013	6.9617	.7317	7
8	7.9664	.7320	7.9632	.7668	7.9597	.8015	7.9562	.8362	8
9	8.9622	.8235	8.9586	.8626	8.9547	.9017	8.9507	.9408	9
10	9.9580	.9150	9.9540	.9585	9.9497	1.0019	9.9452	1.0453	10
	84¾ Deg.		84½ Deg.		84¼ Deg.		84 Deg.		
	6¼ Deg.		6½ Deg.		6¾ Deg.		7 Deg.		
1	.9941	.1089	.9936	.1132	.9931	.1175	.9925	.1219	1
2	1.9881	.2177	1.9871	.2264	1.9861	.2351	1.9851	.2437	2
3	2.9822	.3266	2.9807	.3396	2.9792	.3526	2.9776	.3656	3
4	3.9762	.4355	3.9743	.4528	3.9723	.4701	3.9702	.4875	4
5	4.9703	.5443	4.9679	.5660	4.9653	.5877	4.9627	.6093	5
6	5.9643	.6532	5.9614	.6792	5.9584	.7052	5.9553	.7312	6
7	6.9584	.7621	6.9550	.7924	6.9515	.8228	6.9478	.8531	7
8	7.9524	.8709	7.9486	.9056	7.9445	.9403	7.9404	.9750	8
9	8.9465	.9798	8.9421	1.0188	8.9376	1.0578	8.9329	1.0968	9
10	9.9406	1.0887	9.9357	1.1320	9.9307	1.1754	9.9255	1.2187	10
	83¾ Deg.		83½ Deg.		83¼ Deg.		83 Deg.		
	7¼ Deg.		7½ Deg.		7¾ Deg.		8 Deg.		
1	.9920	.1262	.9914	.1305	.9909	.1349	.9903	.1392	1
2	1.9840	.2524	1.9829	.2611	1.9817	.2697	1.9805	.2783	2
3	2.9760	.3786	2.9743	.3916	2.9726	.4046	2.9708	.4175	3
4	3.9680	.5048	3.9658	.5221	3.9635	.5394	3.9611	.5567	4
5	4.9600	.6310	4.9572	.6526	4.9543	.6743	4.9513	.6959	5
6	5.9520	.7572	5.9487	.7832	5.9452	.8091	5.9416	.8350	6
7	6.9440	.8834	6.9401	.9137	6.9361	.9440	6.9319	.9742	7
8	7.9360	1.0096	7.9316	1.0442	7.9269	1.0788	7.9221	1.1134	8
9	8.9280	1.1358	8.9230	1.1747	8.9178	1.2137	8.9124	1.2526	9
10	9.9200	1.2620	9.9144	1.3053	9.9087	1.3485	9.9027	1.3917	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	82¾ Deg.		82½ Deg.		82¼ Deg.		82 Deg.		

LATITUDES AND DEPARTURES.

D.	8¼ Deg.		8½ Deg.		8¾ Deg.		9 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.9897	.1435	.9890	.1478	.9884	.1521	.9877	.1564	1
2	1.9793	.2870	1.9780	.2956	1.9767	.3042	1.9754	.3129	2
3	2.9690	.4305	2.9670	.4434	2.9651	.4564	2.9631	.4693	3
4	3.9586	.5740	3.9561	.5912	3.9534	.6085	3.9508	.6257	4
5	4.9483	.7175	4.9451	.7390	4.9418	.7606	4.9384	.7822	5
6	5.9379	.8610	5.9341	.8869	5.9302	.9127	5.9261	.9386	6
7	6.9276	1.0044	6.9231	1.0347	6.9185	1.0649	6.9138	1.0950	7
8	7.9172	1.1479	7.9121	1.1825	7.9069	1.2170	7.9015	1.2515	8
9	8.9069	1.2914	8.9011	1.3303	8.8953	1.3691	8.8892	1.4079	9
10	9.8965	1.4349	9.8902	1.4781	9.8836	1.5212	9.8769	1.5643	10
	81¾ Deg.		81½ Deg.		81¼ Deg.		81 Deg.		

	9¼ Deg.		9½ Deg.		9¾ Deg.		10 Deg.		
1	.9870	.1607	.9863	.1650	.9856	.1693	.9848	.1736	1
2	1.9740	.3215	1.9726	.3301	1.9711	.3387	1.9696	.3473	2
3	2.9610	.4822	2.9589	.4951	2.9567	.5080	2.9544	.5209	3
4	3.9480	.6430	3.9451	.6602	3.9422	.6774	3.9392	.6946	4
5	4.9350	.8037	4.9314	.8252	4.9278	.8467	4.9240	.8682	5
6	5.9220	.9645	5.9177	.9903	5.9133	1.0161	5.9088	1.0419	6
7	6.9090	1.1252	6.9040	1.1553	6.8989	1.1854	6.8937	1.2155	7
8	7.8960	1.2859	7.8903	1.3204	7.8844	1.3548	7.8785	1.3892	8
9	8.8830	1.4467	8.8766	1.4854	8.8700	1.5241	8.8633	1.5628	9
10	9.8700	1.6074	9.8629	1.6505	9.8556	1.6935	9.8481	1.7365	10
	80¾ Deg.		80½ Deg.		80¼ Deg.		80 Deg.		

	10¼ Deg.		10½ Deg.		10¾ Deg.		11 Deg.		
1	.9840	.1779	.9833	.1822	.9825	.1865	.9816	.1908	1
2	1.9681	.3559	1.9665	.3645	1.9649	.3730	1.9633	.3816	2
3	2.9521	.5338	2.9498	.5467	2.9474	.5596	2.9449	.5724	3
4	3.9362	.7118	3.9330	.7289	3.9298	.7461	3.9265	.7632	4
5	4.9202	.8897	4.9163	.9112	4.9123	.9326	4.9081	.9540	5
6	5.9042	1.0677	5.8995	1.0934	5.8947	1.1191	5.8898	1.1449	6
7	6.8883	1.2456	6.8828	1.2756	6.8772	1.3057	6.8714	1.3357	7
8	7.8723	1.4235	7.8660	1.4579	7.8596	1.4922	7.8530	1.5265	8
9	8.8564	1.6015	8.8493	1.6401	8.8421	1.6787	8.8346	1.7173	9
10	9.8404	1.7794	9.8325	1.8224	9.8245	1.8652	9.8163	1.9081	10
	79¾ Deg.		79½ Deg.		79¼ Deg.		79 Deg.		

	11¼ Deg.		11½ Deg.		11¾ Deg.		12 Deg.		
1	.9808	.1951	.9799	.1994	.9790	.2036	.9781	.2079	1
2	1.9616	.3902	1.9598	.3987	1.9581	.4073	1.9563	.4158	2
3	2.9424	.5853	2.9398	.5981	2.9371	.6109	2.9344	.6237	3
4	3.9231	.7804	3.9197	.7975	3.9162	.8146	3.9126	.8316	4
5	4.9039	.9755	4.8996	.9968	4.8952	1.0182	4.8907	1.0396	5
6	5.8847	1.1705	5.8795	1.1962	5.8743	1.2219	5.8689	1.2475	6
7	6.8655	1.3656	6.8595	1.3956	6.8533	1.4255	6.8470	1.4554	7
8	7.8463	1.5607	7.8394	1.5949	7.8324	1.6291	7.8252	1.6633	8
9	8.8271	1.7558	8.8193	1.7943	8.8114	1.8328	8.8033	1.8712	9
10	9.8079	1.9509	9.7992	1.9937	9.7905	2.0364	9.7815	2.0791	10
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
D.	78¾ Deg.		78½ Deg.		78¼ Deg.		78 Deg.		D.

LATITUDES AND DEPARTURES.

D.	12¼ Deg. Lat.	12¼ Deg. Dep.	12½ Deg. Lat.	12½ Deg. Dep.	12¾ Deg. Lat.	12¾ Deg. Dep.	13 Deg. Lat.	13 Deg. Dep.	D.
1	.9772	.2122	.9763	.2164	.9753	.2207	.9744	.2250	1
2	1.9545	.4244	1.9526	.4329	1.9507	.4414	1.9487	.4499	2
3	2.9317	.6365	2.9289	.6493	2.9260	.6621	2.9231	.6749	3
4	3.9089	.8487	3.9052	.8658	3.9014	.8828	3.8975	.8998	4
5	4.8862	1.0609	4.8815	1.0822	4.8767	1.1035	4.8719	1.1248	5
6	5.8634	1.2731	5.8578	1.2986	5.8521	1.3242	5.8462	1.3497	6
7	6.8406	1.4852	6.8341	1.5151	6.8274	1.5449	6.8206	1.5747	7
8	7.8178	1.6974	7.8104	1.7315	7.8027	1.7656	7.7950	1.7996	8
9	8.7951	1.9096	8.7867	1.9480	8.7781	1.9863	8.7693	2.0246	9
10	9.7723	2.1218	9.7630	2.1644	9.7534	2.2070	9.7437	2.2495	10
	77¾ Deg.		77½ Deg.		77¼ Deg.		77 Deg.		
	13¼ Deg.		13½ Deg.		13¾ Deg.		14 Deg.		
1	.9734	.2292	.9724	.2334	.9713	.2377	.9703	.2419	1
2	1.9468	.4584	1.9447	.4669	1.9427	.4754	1.9406	.4838	2
3	2.9201	.6876	2.9171	.7003	2.9140	.7131	2.9109	.7258	3
4	3.8935	.9168	3.8895	.9338	3.8854	.9507	3.8812	.9677	4
5	4.8669	1.1460	4.8618	1.1672	4.8567	1.1884	4.8515	1.2096	5
6	5.8403	1.3752	5.8342	1.4007	5.8281	1.4261	5.8218	1.4515	6
7	6.8137	1.6044	6.8066	1.6341	6.7994	1.6638	6.7921	1.6935	7
8	7.7870	1.8336	7.7790	1.8676	7.7707	1.9015	7.7624	1.9354	8
9	8.7604	2.0628	8.7513	2.1010	8.7421	2.1392	8.7327	2.1773	9
10	9.7338	2.2920	9.7237	2.3345	9.7134	2.3769	9.7030	2.4192	10
	76¾ Deg.		76½ Deg.		76¼ Deg.		76 Deg.		
	14¼ Deg.		14½ Deg.		14¾ Deg.		15 Deg.		
1	.9692	.2462	.9681	.2504	.9670	.2546	.9659	.2588	1
2	1.9385	.4923	1.9363	.5008	1.9341	.5092	1.9319	.5176	2
3	2.9077	.7385	2.9044	.7511	2.9011	.7638	2.8978	.7765	3
4	3.8769	.9846	3.8726	1.0015	3.8682	1.0184	3.8637	1.0353	4
5	4.8462	1.2308	4.8407	1.2519	4.8352	1.2730	4.8296	1.2941	5
6	5.8154	1.4769	5.8089	1.5023	5.8023	1.5276	5.7956	1.5529	6
7	6.7846	1.7231	6.7770	1.7527	6.7693	1.7822	6.7615	1.8117	7
8	7.7538	1.9692	7.7452	2.0030	7.7364	2.0368	7.7274	2.0706	8
9	8.7231	2.2154	8.7133	2.2534	8.7034	2.2914	8.6933	2.3294	9
10	9.6923	2.4615	9.6815	2.5038	9.6705	2.5460	9.6593	2.5882	10
	75¾ Deg.		75½ Deg.		75¼ Deg.		75 Deg.		
	15¼ Deg.		15½ Deg.		15¾ Deg.		16 Deg.		
1	.9648	.2630	.9636	.2672	.9625	.2714	.9613	.2756	1
2	1.9296	.5261	1.9273	.5345	1.9249	.5429	1.9225	.5513	2
3	2.8944	.7891	2.8909	.8017	2.8874	.8143	2.8838	.8269	3
4	3.8591	1.0521	3.8545	1.0690	3.8498	1.0858	3.8450	1.1025	4
5	4.8239	1.3152	4.8182	1.3362	4.8123	1.3572	4.8063	1.3782	5
6	5.7887	1.5782	5.7818	1.6034	5.7747	1.6286	5.7676	1.6538	6
7	6.7535	1.8412	6.7454	1.8707	6.7372	1.9001	6.7288	1.9295	7
8	7.7183	2.1042	7.7090	2.1379	7.6996	2.1715	7.6901	2.2051	8
9	8.6831	2.3673	8.6727	2.4051	8.6621	2.4430	8.6514	2.4807	9
10	9.6479	2.6303	9.6363	2.6724	9.6246	2.7144	9.6126	2.7564	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	74¾ Deg.		74½ Deg.		74¼ Deg.		74 Deg.		

D.	16¼ Deg.		16½ Deg.		16¾ Deg.		17 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.9600	.2798	.9588	.2840	.9576	.2882	.9563	.2924	1
2	1.9201	.5597	1.9176	.5680	1.9151	.5764	1.9126	.5847	2
3	2.8801	.8395	2.8765	.8520	2.8727	.8646	2.8689	.8771	3
4	3.8402	1.1193	3.8353	1.1361	3.8303	1.1528	3.8252	1.1695	4
5	4.8002	1.3991	4.7941	1.4201	4.7879	1.4410	4.7815	1.4619	5
6	5.7603	1.6790	5.7529	1.7041	5.7454	1.7292	5.7378	1.7542	6
7	6.7203	1.9588	6.7117	1.9881	6.7030	2.0174	6.6941	2.0466	7
8	7.6804	2.2386	7.6706	2.2721	7.6606	2.3056	7.6504	2.3390	8
9	8.6404	2.5185	8.6294	2.5561	8.6181	2.5938	8.6067	2.6313	9
10	9.6005	2.7983	9.5882	2.8402	9.5757	2.8820	9.5630	2.9237	10
	73¾ Deg.		73½ Deg.		73¼ Deg.		73 Deg.		

D.	17¼ Deg.		17½ Deg.		17¾ Deg.		18 Deg.		D.
1	.9550	.2965	.9537	.3007	.9524	.3049	.9511	.3090	1
2	1.9100	.5931	1.9074	.6014	1.9048	.6097	1.9021	.6180	2
3	2.8651	.8896	2.8612	.9021	2.8572	.9146	2.8532	.9271	3
4	3.8201	1.1862	3.8149	1.2028	3.8096	1.2195	3.8042	1.2361	4
5	4.7751	1.4827	4.7686	1.5035	4.7620	1.5243	4.7553	1.5451	5
6	5.7301	1.7792	5.7223	1.8042	5.7144	1.8292	5.7063	1.8541	6
7	6.6851	2.0758	6.6760	2.1049	6.6668	2.1341	6.6574	2.1631	7
8	7.6402	2.3723	7.6297	2.4056	7.6192	2.4389	7.6085	2.4721	8
9	8.5952	2.6689	8.5835	2.7064	8.5716	2.7438	8.5595	2.7812	9
10	9.5502	2.9654	9.5372	3.0071	9.5240	3.0486	9.5106	3.0902	10
	72¾ Deg.		72½ Deg.		72¼ Deg.		72 Deg.		

D.	18¼ Deg.		18½ Deg.		18¾ Deg.		19 Deg.		D.
1	.9497	.3132	.9483	.3173	.9469	.3214	.9455	.3256	1
2	1.8994	.6263	1.8966	.6346	1.8939	.6429	1.8910	.6511	2
3	2.8491	.9395	2.8450	.9519	2.8408	.9643	2.8366	.9767	3
4	3.7988	1.2527	3.7933	1.2692	3.7877	1.2858	3.7821	1.3023	4
5	4.7485	1.5658	4.7416	1.5865	4.7347	1.6072	4.7276	1.6278	5
6	5.6982	1.8790	5.6899	1.9038	5.6816	1.9286	5.6731	1.9534	6
7	6.6479	2.1921	6.6383	2.2211	6.6285	2.2501	6.6186	2.2790	7
8	7.5976	2.5053	7.5866	2.5384	7.5754	2.5715	7.5641	2.6045	8
9	8.5473	2.8185	8.5349	2.8557	8.5224	2.8930	8.5097	2.9301	9
10	9.4970	3.1316	9.4832	3.1730	9.4693	3.2144	9.4552	3.2557	10
	71¾ Deg.		71½ Deg.		71¼ Deg.		71 Deg.		

D.	19¼ Deg.		19½ Deg.		19¾ Deg.		20 Deg.		D.
1	.9441	.3297	.9426	.3338	.9412	.3379	.9397	.3420	1
2	1.8882	.6594	1.8853	.6676	1.8824	.6758	1.8794	.6840	2
3	2.8323	.9891	2.8279	1.0014	2.8235	1.0138	2.8191	1.0261	3
4	3.7764	1.3188	3.7706	1.3352	3.7647	1.3517	3.7588	1.3681	4
5	4.7204	1.6485	4.7132	1.6690	4.7059	1.6896	4.6985	1.7101	5
6	5.6645	1.9781	5.6558	2.0028	5.6471	2.0275	5.6382	2.0521	6
7	6.6086	2.3078	6.5985	2.3366	6.5882	2.3654	6.5778	2.3941	7
8	7.5527	2.6375	7.5411	2.6705	7.5294	2.7033	7.5175	2.7362	8
9	8.4968	2.9672	8.4838	3.0043	8.4706	3.0413	8.4572	3.0782	9
10	9.4409	3.2969	9.4264	3.3381	9.4118	3.3792	9.3969	3.4202	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	70¾ Deg.		70½ Deg.		70¼ Deg.		70 Deg.		

LATITUDES AND DEPARTURES.

D.	20¼ Deg. Lat.	Dep.	20½ Deg. Lat.	Dep.	20¾ Deg. Lat.	Dep.	21 Deg. Lat.	Dep.	D.
1	.9382	.3461	.9367	.3502	.9351	.3543	.9336	.3584	1
2	1.8764	.6922	1.8733	.7004	1.8703	.7086	1.8672	.7167	2
3	2.8146	1.0384	2.8100	1.0506	2.8054	1.0629	2.8007	1.0751	3
4	3.7528	1.3845	3.7467	1.4008	3.7405	1.4172	3.7343	1.4335	4
5	4.6910	1.7306	4.6834	1.7510	4.6757	1.7715	4.6679	1.7918	5
6	5.6291	2.0767	5.6200	2.1012	5.6108	2.1257	5.6015	2.1502	6
7	6.5673	2.4228	6.5567	2.4515	6.5459	2.4800	6.5351	2.5086	7
8	7.5055	2.7689	7.4934	2.8017	7.4811	2.8343	7.4686	2.8669	8
9	8.4437	3.1151	8.4300	3.1519	8.4162	3.1886	8.4022	3.2253	9
10	9.3819	3.4612	9.3667	3.5021	9.3514	3.5429	9.3358	3.5837	10
	69¾ Deg.		69½ Deg.		69¼ Deg.		69 Deg.		
	21¼ Deg.		21½ Deg.		21¾ Deg.		22 Deg.		
1	.9320	.3624	.9304	.3665	.9288	.3706	.9272	.3746	1
2	1.8640	.7249	1.8608	.7330	1.8576	.7411	1.8544	.7492	2
3	2.7960	1.0873	2.7913	1.0995	2.7864	1.1117	2.7816	1.1238	3
4	3.7280	1.4498	3.7217	1.4660	3.7152	1.4822	3.7087	1.4984	4
5	4.6600	1.8122	4.6521	1.8325	4.6440	1.8528	4.6359	1.8730	5
6	5.5920	2.1746	5.5825	2.1990	5.5729	2.2233	5.5631	2.2476	6
7	6.5241	2.5371	6.5129	2.5655	6.5017	2.5939	6.4903	2.6222	7
8	7.4561	2.8995	7.4433	2.9320	7.4305	2.9645	7.4175	2.9969	8
9	8.3881	3.2619	8.3738	3.2985	8.3593	3.3350	8.3447	3.3715	9
10	9.3201	3.6244	9.3042	3.6650	9.2881	3.7056	9.2718	3.7461	10
	68¾ Deg.		68½ Deg.		68¼ Deg.		68 Deg.		
	22¼ Deg.		22½ Deg.		22¾ Deg.		23 Deg.		
1	.9255	.3786	.9239	.3827	.9222	.3867	.9205	.3907	1
2	1.8511	.7573	1.8478	.7654	1.8444	.7734	1.8410	.7815	2
3	2.7766	1.1359	2.7716	1.1481	2.7666	1.1601	2.7615	1.1722	3
4	3.7022	1.5146	3.6955	1.5307	3.6888	1.5468	3.6820	1.5629	4
5	4.6277	1.8932	4.6194	1.9134	4.6110	1.9336	4.6025	1.9537	5
6	5.5532	2.2719	5.5433	2.2961	5.5332	2.3203	5.5230	2.3444	6
7	6.4788	2.6505	6.4672	2.6788	6.4554	2.7070	6.4435	2.7351	7
8	7.4043	3.0292	7.3910	3.0615	7.3776	3.0937	7.3640	3.1258	8
9	8.3299	3.4078	8.3149	3.4442	8.2998	3.4804	8.2845	3.5166	9
10	9.2554	3.7865	9.2388	3.8268	9.2220	3.8671	9.2050	3.9073	10
	67¾ Deg.		67½ Deg.		67¼ Deg.		67 Deg.		
	23¼ Deg.		23½ Deg.		23¾ Deg.		24 Deg.		
1	.9188	.3947	.9171	.3987	.9153	.4027	.9135	.4067	1
2	1.8376	.7895	1.8341	.7975	1.8306	.8055	1.8271	.8135	2
3	2.7564	1.1842	2.7512	1.1962	2.7459	1.2082	2.7406	1.2202	3
4	3.6752	1.5790	3.6682	1.5950	3.6612	1.6110	3.6542	1.6269	4
5	4.5940	1.9737	4.5853	1.9937	4.5766	2.0137	4.5677	2.0337	5
6	5.5127	2.3685	5.5024	2.3925	5.4919	2.4165	5.4813	2.4404	6
7	6.4315	2.7632	6.4194	2.7912	6.4072	2.8192	6.3948	2.8472	7
8	7.3503	3.1580	7.3365	3.1900	7.3225	3.2220	7.3084	3.2539	8
9	8.2691	3.5527	8.2535	3.5887	8.2378	3.6247	8.2219	3.6606	9
10	9.1879	3.9474	9.1706	3.9875	9.1531	4.0275	9.1355	4.0674	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	66¾ Deg.		66½ Deg.		66¼ Deg.		66 Deg.		

LATITUDES AND DEPARTURES.

D.	24¼ Deg.		24½ Deg.		24¾ Deg.		25 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.9118	.4107	.9100	.4147	.9081	.4187	.9063	.4226	1
2	1.8235	.8214	1.8199	.8294	1.8163	.8373	1.8126	.8452	2
3	2.7353	1.2322	2.7299	1.2441	2.7244	1.2560	2.7189	1.2679	3
4	3.6470	1.6429	3.6398	1.6588	3.6326	1.6746	3.6252	1.6905	4
5	4.5588	2.0536	4.5498	2.0735	4.5407	2.0933	4.5315	2.1131	5
6	5.4706	2.4643	5.4598	2.4882	5.4489	2.5120	5.4378	2.5357	6
7	6.3823	2.8750	6.3697	2.9029	6.3570	2.9306	6.3442	2.9583	7
8	7.2941	3.2858	7.2797	3.3175	7.2651	3.3493	7.2505	3.3809	8
9	8.2059	3.6965	8.1897	3.7322	8.1733	3.7679	8.1568	3.8036	9
10	9.1176	4.1072	9.0996	4.1469	9.0814	4.1866	9.0631	4.2262	10
	65¾ Deg.		65½ Deg.		65¼ Deg.		65 Deg.		
	25¼ Deg.		25½ Deg.		25¾ Deg.		26 Deg.		
1	.9045	.4266	.9026	.4305	.9007	.4344	.8988	.4384	1
2	1.8089	.8531	1.8052	.8610	1.8014	.8689	1.7976	.8767	2
3	2.7134	1.2797	2.7078	1.2915	2.7021	1.3033	2.6964	1.3151	3
4	3.6178	1.7063	3.6103	1.7220	3.6028	1.7378	3.5952	1.7535	4
5	4.5223	2.1328	4.5129	2.1526	4.5035	2.1722	4.4940	2.1919	5
6	5.4267	2.5594	5.4155	2.5831	5.4042	2.6067	5.3928	2.6302	6
7	6.3312	2.9860	6.3181	3.0136	6.3049	3.0411	6.2916	3.0686	7
8	7.2356	3.4125	7.2207	3.4441	7.2056	3.4756	7.1904	3.5070	8
9	8.1401	3.8391	8.1233	3.8746	8.1063	3.9100	8.0891	3.9453	9
10	9.0446	4.2657	9.0259	4.3051	9.0070	4.3445	8.9879	4.3837	10
	64¾ Deg.		64½ Deg.		64¼ Deg.		64 Deg.		
	26¼ Deg.		26½ Deg.		26¾ Deg.		27 Deg.		
1	.8969	.4423	.8949	.4462	.8930	.4501	.8910	.4540	1
2	1.7937	.8846	1.7899	.8924	1.7860	.9002	1.7820	.9080	2
3	2.6906	1.3269	2.6848	1.3386	2.6789	1.3503	2.6730	1.3620	3
4	3.5875	1.7692	3.5797	1.7848	3.5719	1.8004	3.5640	1.8160	4
5	4.4844	2.2114	4.4747	2.2310	4.4649	2.2505	4.4550	2.2700	5
6	5.3812	2.6537	5.3696	2.6772	5.3579	2.7006	5.3460	2.7239	6
7	6.2781	3.0960	6.2645	3.1234	6.2509	3.1507	6.2370	3.1779	7
8	7.1750	3.5383	7.1595	3.5696	7.1438	3.6008	7.1281	3.6319	8
9	8.0719	3.9806	8.0544	4.0158	8.0368	4.0509	8.0191	4.0859	9
10	8.9687	4.4229	8.9493	4.4620	8.9298	4.5010	8.9101	4.5399	10
	63¾ Deg.		63½ Deg.		63¼ Deg.		63 Deg.		
	27¼ Deg.		27½ Deg.		27¾ Deg.		28 Deg.		
1	.8890	.4579	.8870	.4617	.8850	.4656	.8829	.4695	1
2	1.7780	.9157	1.7740	.9235	1.7700	.9312	1.7659	.9389	2
3	2.6671	1.3736	2.6610	1.3852	2.6550	1.3968	2.6488	1.4084	3
4	3.5561	1.8315	3.5480	1.8470	3.5400	1.8625	3.5318	1.8779	4
5	4.4451	2.2894	4.4351	2.3087	4.4249	2.3281	4.4147	2.3474	5
6	5.3341	2.7472	5.3221	2.7705	5.3099	2.7937	5.2977	2.8168	6
7	6.2231	3.2051	6.2091	3.2322	6.1949	3.2593	6.1806	3.2863	7
8	7.1121	3.6630	7.0961	3.6940	7.0799	3.7249	7.0636	3.7558	8
9	8.0012	4.1209	7.9831	4.1557	7.9649	4.1905	7.9465	4.2252	9
10	8.8902	4.5787	8.8701	4.6175	8.8499	4.6561	8.8295	4.6947	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	62¾ Deg.		62½ Deg.		62¼ Deg.		62 Deg.		

LATITUDES AND DEPARTURES.

D.	28¼ Deg.		28½ Deg.		28¾ Deg.		29 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.8809	.4733	.8788	.4772	.8767	.4810	.8746	.4848	1
2	1.7618	.9466	1.7576	.9543	1.7535	.9620	1.7492	.9696	2
3	2.6427	1.4200	2.6365	1.4315	2.6302	1.4430	2.6239	1.4544	3
4	3.5236	1.8933	3.5153	1.9086	3.5069	1.9240	3.4985	1.9392	4
5	4.4045	2.3666	4.3941	2.3858	4.3836	2.4049	4.3731	2.4240	5
6	5.2853	2.8399	5.2729	2.8630	5.2604	2.8859	5.2477	2.9089	6
7	6.1662	3.3132	6.1517	3.3401	6.1371	3.3669	6.1223	3.3937	7
8	7.0471	3.7866	7.0305	3.8173	7.0138	3.8479	6.9970	3.8785	8
9	7.9280	4.2599	7.9094	4.2944	7.8905	4.3289	7.8716	4.3633	9
10	8.8089	4.7332	8.7882	4.7716	8.7673	4.8099	8.7462	4.8481	10
	61¾ Deg.		61½ Deg.		61¼ Deg.		61 Deg.		
	29¼ Deg.		29½ Deg.		29¾ Deg.		30 Deg.		
1	.8725	.4886	.8704	.4924	.8682	.4962	.8660	.5000	1
2	1.7450	.9772	1.7407	.9848	1.7364	.9924	1.7321	1.0000	2
3	2.6175	1.4659	2.6111	1.4773	2.6046	1.4886	2.5981	1.5000	3
4	3.4900	1.9545	3.4814	1.9697	3.4728	1.9849	3.4641	2.0000	4
5	4.3625	2.4431	4.3518	2.4621	4.3410	2.4811	4.3301	2.5000	5
6	5.2350	2.9317	5.2221	2.9545	5.2092	2.9773	5.1962	3.0000	6
7	6.1075	3.4203	6.0925	3.4470	6.0774	3.4735	6.0622	3.5000	7
8	6.9800	3.9090	6.9628	3.9394	6.9456	3.9697	6.9282	4.0000	8
9	7.8525	4.3976	7.8332	4.4318	7.8138	4.4659	7.7942	4.5000	9
10	8.7250	4.8862	8.7036	4.9242	8.6820	4.9622	8.6603	5.0000	10
	60¾ Deg.		60½ Deg.		60¼ Deg.		60 Deg.		
	30¼ Deg.		30½ Deg.		30¾ Deg.		31 Deg.		
1	.8638	.5038	.8616	.5075	.8594	.5113	.8572	.5150	1
2	1.7277	1.0075	1.7233	1.0151	1.7188	1.0226	1.7143	1.0301	2
3	2.5915	1.5113	2.5849	1.5226	2.5782	1.5339	2.5715	1.5451	3
4	3.4553	2.0151	3.4465	2.0302	3.4376	2.0452	3.4287	2.0602	4
5	4.3192	2.5189	4.3081	2.5377	4.2970	2.5565	4.2858	2.5752	5
6	5.1830	3.0226	5.1698	3.0452	5.1564	3.0678	5.1430	3.0902	6
7	6.0468	3.5264	6.0314	3.5528	6.0158	3.5791	6.0002	3.6053	7
8	6.9107	4.0302	6.8930	4.0603	6.8753	4.0903	6.8573	4.1203	8
9	7.7745	4.5340	7.7547	4.5678	7.7347	4.6016	7.7145	4.6353	9
10	8.6384	5.0377	8.6163	5.0754	8.5941	5.1129	8.5717	5.1504	10
	59¾ Deg.		59½ Deg.		59¼ Deg.		59 Deg.		
	31¼ Deg.		31½ Deg.		31¾ Deg.		32 Deg.		
1	.8549	.5188	.8526	.5225	.8504	.5262	.8480	.5299	1
2	1.7098	1.0375	1.7053	1.0450	1.7007	1.0524	1.6961	1.0598	2
3	2.5647	1.5563	2.5579	1.5675	2.5511	1.5786	2.5441	1.5898	3
4	3.4196	2.0751	3.4106	2.0900	3.4014	2.1049	3.3922	2.1197	4
5	4.2746	2.5939	4.2632	2.6125	4.2518	2.6311	4.2402	2.6496	5
6	5.1295	3.1126	5.1158	3.1350	5.1021	3.1573	5.0883	3.1795	6
7	5.9844	3.6314	5.9685	3.6575	5.9525	3.6835	5.9363	3.7094	7
8	6.8393	4.1502	6.8211	4.1800	6.8028	4.2097	6.7844	4.2394	8
9	7.6942	4.6690	7.6738	4.7025	7.6532	4.7359	7.6324	4.7693	9
10	8.5491	5.1877	8.5264	5.2250	8.5035	5.2621	8.4805	5.2992	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	58¾ Deg.		58½ Deg.		58¼ Deg.		58 Deg.		

LATITUDES AND DEPARTURES.

D.	32¼ Deg. Lat.	32¼ Deg. Dep.	32½ Deg. Lat.	32½ Deg. Dep.	32¾ Deg. Lat.	32¾ Deg. Dep.	33 Deg. Lat.	33 Deg. Dep.	D.
1	.8457	.5336	.8434	.5373	.8410	.5410	.8387	.5446	1
2	1.6915	1.0672	1.6868	1.0746	1.6821	1.0819	1.6773	1.0893	2
3	2.5372	1.6008	2.5302	1.6119	2.5231	1.6229	2.5160	1.6339	3
4	3.3829	2.1345	3.3736	2.1492	3.3642	2.1639	3.3547	2.1786	4
5	4.2286	2.6681	4.2170	2.6865	4.2052	2.7049	4.1934	2.7232	5
6	5.0744	3.2017	5.0603	3.2238	5.0462	3.2458	5.0320	3.2678	6
7	5.9201	3.7353	5.9037	3.7611	5.8873	3.7868	5.8707	3.8125	7
8	6.7658	4.2689	6.7471	4.2984	6.7283	4.3278	6.7094	4.3571	8
9	7.6116	4.8025	7.5905	4.8357	7.5694	4.8688	7.5480	4.9018	9
10	8.4573	5.3361	8.4339	5.3730	8.4104	5.4097	8.3867	5.4464	10
	57¾ Deg.		57½ Deg.		57¼ Deg.		57 Deg.		
	33¼ Deg.		33½ Deg.		33¾ Deg.		34 Deg.		
1	.8363	.5483	.8339	.5519	.8315	.5556	.8290	.5592	1
2	1.6726	1.0966	1.6678	1.1039	1.6629	1.1111	1.6581	1.1184	2
3	2.5089	1.6449	2.5017	1.6558	2.4944	1.6667	2.4871	1.6776	3
4	3.3451	2.1932	3.3355	2.2077	3.3259	2.2223	3.3162	2.2368	4
5	4.1814	2.7415	4.1694	2.7597	4.1573	2.7779	4.1452	2.7960	5
6	5.0177	3.2898	5.0033	3.3116	4.9888	3.3334	4.9742	3.3552	6
7	5.8540	3.8381	5.8372	3.8636	5.8203	3.8890	5.8033	3.9144	7
8	6.6903	4.3863	6.6711	4.4155	6.6518	4.4446	6.6323	4.4735	8
9	7.5266	4.9346	7.5050	4.9674	7.4832	5.0001	7.4613	5.0327	9
10	8.3629	5.4829	8.3389	5.5194	8.3147	5.5557	8.2904	5.5919	10
	56¾ Deg.		56½ Deg.		56¼ Deg.		56 Deg.		
	34¼ Deg.		34½ Deg.		34¾ Deg.		35 Deg.		
1	.8266	.5628	.8241	.5664	.8216	.5700	.8192	.5736	1
2	1.6532	1.1256	1.6483	1.1328	1.6433	1.1400	1.6383	1.1472	2
3	2.4798	1.6884	2.4724	1.6992	2.4649	1.7100	2.4575	1.7207	3
4	3.3064	2.2512	3.2965	2.2656	3.2866	2.2800	3.2766	2.2943	4
5	4.1329	2.8140	4.1206	2.8320	4.1082	2.8500	4.0958	2.8679	5
6	4.9595	3.3768	4.9448	3.3984	4.9299	3.4200	4.9149	3.4415	6
7	5.7861	3.9396	5.7689	3.9648	5.7515	3.9900	5.7341	4.0150	7
8	6.6127	4.5024	6.5930	4.5312	6.5732	4.5600	6.5532	4.5886	8
9	7.4393	5.0652	7.4171	5.0977	7.3948	5.1300	7.3724	5.1622	9
10	8.2659	5.6280	8.2413	5.6641	8.2165	5.7000	8.1915	5.7358	10
	55¾ Deg.		55½ Deg.		55¼ Deg.		55 Deg.		
	35¼ Deg.		35½ Deg.		35¾ Deg.		36 Deg.		
1	.8166	.5771	.8141	.5807	.8116	.5842	.8090	.5878	1
2	1.6333	1.1543	1.6282	1.1614	1.6231	1.1685	1.6180	1.1756	2
3	2.4499	1.7314	2.4423	1.7421	2.4347	1.7527	2.4271	1.7634	3
4	3.2666	2.3086	3.2565	2.3228	3.2463	2.3370	3.2361	2.3511	4
5	4.0832	2.8857	4.0706	2.9035	4.0579	2.9212	4.0451	2.9389	5
6	4.8998	3.4629	4.8847	3.4842	4.8694	3.5055	4.8541	3.5267	6
7	5.7165	4.0400	5.6988	4.0649	5.6810	4.0897	5.6631	4.1145	7
8	6.5331	4.6172	6.5129	4.6456	6.4926	4.6740	6.4721	4.7023	8
9	7.3498	5.1943	7.3270	5.2263	7.3042	5.2582	7.2812	5.2901	9
10	8.1664	5.7715	8.1412	5.8070	8.1157	5.8425	8.0902	5.8779	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	54¾ Deg.		54½ Deg.		54¼ Deg.		54 Deg.		

LATITUDES AND DEPARTURES.

D.	36¼ Deg.		36½ Deg.		36¾ Deg.		37 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.8064	.5913	.8039	.5948	.8013	.5983	.7986	.6018	1
2	1.6129	1.1826	1.6077	1.1896	1.6025	1.1966	1.5973	1.2036	2
3	2.4193	1.7739	2.4116	1.7845	2.4038	1.7950	2.3959	1.8054	3
4	3.2258	2.3652	3.2154	2.3793	3.2050	2.3933	3.1945	2.4073	4
5	4.0322	2.9565	4.0193	2.9741	4.0063	2.9916	3.9932	3.0091	5
6	4.8387	3.5479	4.8231	3.5689	4.8075	3.5899	4.7918	3.6109	6
7	5.6451	4.1392	5.6270	4.1638	5.6088	4.1883	5.5904	4.2127	7
8	6.4516	4.7305	6.4309	4.7586	6.4100	4.7866	6.3891	4.8145	8
9	7.2580	5.3218	7.2347	5.3534	7.2113	5.3849	7.1877	5.4163	9
10	8.0644	5.9131	8.0386	5.9482	8.0125	5.9832	7.9864	6.0181	10
	53¾ Deg.		53½ Deg.		53¼ Deg.		53 Deg.		
	37¼ Deg.		37½ Deg.		37¾ Deg.		38 Deg.		
1	.7960	.6053	.7934	.6088	.7907	.6122	.7880	.6157	1
2	1.5920	1.2106	1.5867	1.2175	1.5814	1.2244	1.5760	1.2313	2
3	2.3880	1.8159	2.3801	1.8263	2.3721	1.8367	2.3640	1.8470	3
4	3.1840	2.4212	3.1734	2.4350	3.1628	2.4489	3.1520	2.4626	4
5	3.9800	3.0265	3.9668	3.0438	3.9534	3.0611	3.9401	3.0783	5
6	4.7760	3.6318	4.7601	3.6526	4.7441	3.6733	4.7281	3.6940	6
7	5.5720	4.2371	5.5535	4.2613	5.5348	4.2855	5.5161	4.3096	7
8	6.3680	4.8424	6.3468	4.8701	6.3255	4.8977	6.3041	4.9253	8
9	7.1640	5.4476	7.1402	5.4789	7.1162	5.5100	7.0921	5.5410	9
10	7.9600	6.0529	7.9335	6.0876	7.9069	6.1222	7.8801	6.1566	10
	52¾ Deg.		52½ Deg.		52¼ Deg.		52 Deg.		
	38¼ Deg.		38½ Deg.		38¾ Deg.		39 Deg.		
1	.7853	.6191	.7826	.6225	.7799	.6259	.7771	.6293	1
2	1.5706	1.2382	1.5652	1.2450	1.5598	1.2518	1.5543	1.2586	2
3	2.3560	1.8573	2.3478	1.8675	2.3397	1.8778	2.3314	1.8880	3
4	3.1413	2.4764	3.1304	2.4901	3.1195	2.5037	3.1086	2.5173	4
5	3.9266	3.0955	3.9130	3.1126	3.8994	3.1296	3.8857	3.1466	5
6	4.7119	3.7146	4.6956	3.7351	4.6793	3.7555	4.6629	3.7759	6
7	5.4972	4.3337	5.4783	4.3576	5.4592	4.3815	5.4400	4.4052	7
8	6.2825	4.9528	6.2609	4.9801	6.2391	5.0074	6.2172	5.0346	8
9	7.0679	5.5718	7.0435	5.6026	7.0190	5.6333	6.9943	5.6639	9
10	7.8532	6.1909	7.8261	6.2251	7.7988	6.2592	7.7715	6.2932	10
	51¾ Deg.		51½ Deg.		51¼ Deg.		51 Deg.		
	39¼ Deg.		39½ Deg.		39¾ Deg.		40 Deg.		
1	.7744	.6327	.7716	.6361	.7688	.6394	.7660	.6428	1
2	1.5488	1.2654	1.5432	1.2722	1.5377	1.2789	1.5321	1.2856	2
3	2.3232	1.8981	2.3149	1.9082	2.3065	1.9183	2.2981	1.9284	3
4	3.0976	2.5308	3.0865	2.5443	3.0754	2.5578	3.0642	2.5712	4
5	3.8720	3.1635	3.8581	3.1804	3.8442	3.1972	3.8302	3.2139	5
6	4.6464	3.7962	4.6297	3.8165	4.6131	3.8366	4.5963	3.8567	6
7	5.4207	4.4289	5.4014	4.4525	5.3819	4.4761	5.3623	4.4995	7
8	6.1951	5.0616	6.1730	5.0886	6.1507	5.1155	6.1284	5.1423	8
9	6.9695	5.6943	6.9446	5.7247	6.9196	5.7550	6.8944	5.7851	9
10	7.7439	6.3271	7.7162	6.3608	7.6884	6.3944	7.6604	6.4279	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	50¾ Deg.		50½ Deg.		50¼ Deg.		50 Deg.		

LATITUDES AND DEPARTURES.

D.	40¼ Deg.		40½ Deg.		40¾ Deg.		41 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.7632	.6461	.7604	.6494	.7576	.6528	.7547	.6561	1
2	1.5265	1.2922	1.5208	1.2989	1.5151	1.3055	1.5094	1.3121	2
3	2.2897	1.9384	2.2812	1.9483	2.2727	1.9583	2.2641	1.9682	3
4	3.0529	2.5845	3.0416	2.5978	3.0303	2.6110	3.0188	2.6242	4
5	3.8162	3.2306	3.8020	3.2472	3.7878	3.2638	3.7735	3.2803	5
6	4.5794	3.8767	4.5624	3.8967	4.5454	3.9166	4.5283	3.9364	6
7	5.3426	4.5229	5.3228	4.5461	5.3030	4.5693	5.2830	4.5924	7
8	6.1059	5.1690	6.0832	5.1956	6.0605	5.2221	6.0377	5.2485	8
9	6.8691	5.8151	6.8437	5.8450	6.8181	5.8748	6.7924	5.9045	9
10	7.6323	6.4612	7.6041	6.4945	7.5756	6.5276	7.5471	6.5606	10
	49¾ Deg.		49½ Deg.		49¼ Deg.		49 Deg.		
	41¼ Deg.		41½ Deg.		41¾ Deg.		42 Deg.		
1	.7518	.6593	.7490	.6626	.7461	.6659	.7431	.6691	1
2	1.5037	1.3187	1.4979	1.3252	1.4921	1.3318	1.4863	1.3383	2
3	2.2555	1.9780	2.2469	1.9879	2.2382	1.9976	2.2294	2.0074	3
4	3.0074	2.6374	2.9958	2.6505	2.9842	2.6635	2.9726	2.6765	4
5	3.7592	3.2967	3.7448	3.3131	3.7303	3.3294	3.7157	3.3457	5
6	4.5110	3.9561	4.4937	3.9757	4.4763	3.9953	4.4589	4.0148	6
7	5.2629	4.6154	5.2427	4.6383	5.2224	4.6612	5.2020	4.6839	7
8	6.0147	5.2748	5.9916	5.3010	5.9685	5.3271	5.9452	5.3530	8
9	6.7666	5.9341	6.7406	5.9636	6.7145	5.9929	6.6883	6.0222	9
10	7.5184	6.5935	7.4896	6.6262	7.4606	6.6588	7.4314	6.6913	10
	48¾ Deg.		48½ Deg.		48¼ Deg.		48 Deg.		
	42¼ Deg.		42½ Deg.		42¾ Deg.		43 Deg.		
1	.7402	.6724	.7373	.6756	.7343	.6788	.7314	.6820	1
2	1.4804	1.3447	1.4746	1.3512	1.4686	1.3576	1.4627	1.3640	2
3	2.2207	2.0171	2.2118	2.0268	2.2030	2.0364	2.1941	2.0460	3
4	2.9609	2.6895	2.9491	2.7024	2.9373	2.7152	2.9254	2.7280	4
5	3.7011	3.3618	3.6864	3.3780	3.6716	3.3940	3.6568	3.4100	5
6	4.4413	4.0342	4.4237	4.0535	4.4059	4.0728	4.3881	4.0920	6
7	5.1815	4.7066	5.1609	4.7291	5.1403	4.7516	5.1195	4.7740	7
8	5.9217	5.3789	5.8982	5.4047	5.8746	5.4304	5.8508	5.4560	8
9	6.6620	6.0513	6.6355	6.0803	6.6089	6.1092	6.5822	6.1380	9
10	7.4022	6.7237	7.3728	6.7559	7.3432	6.7880	7.3135	6.8200	10
	47¾ Deg.		47½ Deg.		47¼ Deg.		47 Deg.		
	43¼ Deg.		43½ Deg.		43¾ Deg.		44 Deg.		
1	.7284	.6852	.7254	.6884	.7224	.6915	.7193	.6947	1
2	1.4567	1.3704	1.4507	1.3767	1.4447	1.3830	1.4387	1.3893	2
3	2.1851	2.0555	2.1761	2.0651	2.1671	2.0745	2.1580	2.0840	3
4	2.9135	2.7407	2.9015	2.7534	2.8895	2.7661	2.8774	2.7786	4
5	3.6419	3.4259	3.6269	3.4418	3.6118	3.4576	3.5967	3.4733	5
6	4.3702	4.1111	4.3522	4.1301	4.3342	4.1491	4.3160	4.1680	6
7	5.0986	4.7963	5.0776	4.8185	5.0565	4.8406	5.0354	4.8626	7
8	5.8270	5.4815	5.8030	5.5068	5.7789	5.5321	5.7547	5.5573	8
9	6.5553	6.1666	6.5284	6.1952	6.5013	6.2236	6.4741	6.2519	9
10	7.2837	6.8518	7.2537	6.8835	7.2236	6.9151	7.1934	6.9466	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	46¾ Deg.		46½ Deg.		46¼ Deg.		46 Deg.		

LATITUDES AND DEPARTURES.

D.	44¼ Deg.		44½ Deg.		44¾ Deg.		45 Deg.		D.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	.7163	.6978	.7133	.7009	.7102	.7040	.7071	.7071	1
2	1.4326	1.3956	1.4265	1.4018	1.4204	1.4080	1.4142	1.4142	2
3	2.1489	2.0934	2.1398	2.1027	2.1306	2.1120	2.1213	2.1213	3
4	2.8652	2.7912	2.8530	2.8036	2.8407	2.8161	2.8284	2.8284	4
5	3.5815	3.4890	3.5663	3.5045	3.5509	3.5201	3.5355	3.5355	5
6	4.2978	4.1867	4.2795	4.2055	4.2611	4.2241	4.2426	4.2426	6
7	5.0141	4.8845	4.9928	4.9064	4.9713	4.9281	4.9497	4.9497	7
8	5.7304	5.5823	5.7060	5.6073	5.6815	5.6321	5.6569	5.6569	8
9	6.4467	6.2801	6.4193	6.3082	6.3917	6.3361	6.3640	6.3640	9
10	7.1630	6.9779	7.1325	7.0091	7.1019	7.0401	7.0711	7.0711	10
D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.
	45¾ Deg.		45½ Deg.		45¼ Deg.		45 Deg.		

TABLE OF USEFUL NUMBERS.

Logarithms.

Ratio of circumference to diameter π = 3.1415926536..........................0.4971499

Area of circle to radius 1 = ... " .... "

Surface of sphere to diameter 1 = ... " .... "

Area of circle to diameter 1 = ... .7853981634......................—1.8950899

Base of Napierian Logarithms = ...2.7182818285.......................... .4342945

Modulus of common " = ... .4342944819.......................—1.6377843

Equatorial radius of the earth, in feet = 20923599.98..........................7.3206364

Polar " " " = 20853657.16..........................7.3191823

Length of seconds pendulum, in London, in inches = 39.13929.

" " " Paris " = 39.1285.

" " " New York " = 39.1012.

U. S. standard gallon contains 231 c. in., or 58372.175 grains = 8.338882 lbs. avoirdupois of water at 39.8° Fahr.

U. S. standard bushel contains 2150.42 c. in., or 77.627413 lbs. av. of water at 39.8° Fahr.

British imperial gallon contains 277.274 c. in., = 1.2003 wine gallons of 231 c. in.

French metre = 39.37079 in. = 3.28089917 feet.

" toise = 6.39459252 feet.

" are = 100 sq. metres = 1076.4299 sq. ft.

" hectare = 100 ares = 2.471143 acres = 107642.9936 sq. ft.

" litre = 1 cubic decimeter = 61.02705 c. in. = .26418637 gallons of 231 c. in.

" hectolitre = 100 litres = 26.418637 gallons.

1 pound avoirdupois = 7000 grs. = 1.215277 pounds Troy.

1 " Troy = 5760 grs. = .822857 pounds avoir.

1 gramme = 15.442 grains.

1 kilogramme = 1000 grammes = 15442 grs. = 2.20607 lbs. avoir.

Tropical year = 365 d. 5 h. 45 m. 47.588 sec.

TABLE

OF

NATURAL SINES AND COSINES.

NATURAL SINES AND COSINES.

′	0°		1°		2°		3°		4°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	00000	Unit.	01745	99985	03490	99939	05234	99863	06976	99756	60
1	00029	Unit.	01774	99984	03519	99938	05263	99861	07005	99754	59
2	00058	Unit.	01803	99984	03548	99937	05292	99860	07034	99752	58
3	00087	Unit.	01832	99983	03577	99936	05321	99858	07063	99750	57
4	00116	Unit.	01862	99983	03606	99935	05350	99857	07092	99748	56
5	00145	Unit.	01891	99982	03635	99934	05379	99855	07121	99746	55
6	00175	Unit.	01920	99982	03664	99933	05408	99854	07150	99744	54
7	00204	Unit.	01949	99981	03693	99932	05437	99852	07179	99742	53
8	00233	Unit.	01978	99980	03723	99931	05466	99851	07208	99740	52
9	00262	Unit.	02007	99980	03752	99930	05495	99849	07237	99738	51
10	00291	Unit.	02036	99979	03781	99929	05524	99847	07266	99736	50
11	00320	99999	02065	99979	03810	99927	05553	99846	07295	99734	49
12	00349	99999	02094	99978	03839	99926	05582	99844	07324	99731	48
13	00378	99999	02123	99977	03868	99925	05611	99842	07353	99729	47
14	00407	99999	02152	99977	03897	99924	05640	99841	07382	99727	46
15	00436	99999	02181	99976	03926	99923	05669	99839	07411	99725	45
16	00465	99999	02211	99976	03955	99922	05698	99838	07440	99723	44
17	00495	99999	02240	99975	03984	99921	05727	99836	07469	99721	43
18	00524	99999	02269	99974	04013	99919	05756	99834	07498	99719	42
19	00553	99998	02298	99974	04042	99918	05785	99833	07527	99716	41
20	00582	99998	02327	99973	04071	99917	05814	99831	07556	99714	40
21	00611	99998	02356	99972	04100	99916	05844	99829	07585	99712	39
22	00640	99998	02385	99972	04129	99915	05873	99827	07614	99710	38
23	00669	99998	02414	99971	04159	99913	05902	99826	07643	99708	37
24	00698	99998	02443	99970	04188	99912	05931	99824	07672	99705	36
25	00727	99997	02472	99969	04217	99911	05960	99822	07701	99703	35
26	00756	99997	02501	99969	04246	99910	05989	99821	07730	99701	34
27	00785	99997	02530	99968	04275	99909	06018	99819	07759	99699	33
28	00814	99997	02560	99967	04304	99907	06047	99817	07788	99696	32
29	00844	99996	02589	99966	04333	99906	06076	99815	07817	99694	31
30	00873	99996	02618	99966	04362	99905	06105	99813	07846	99692	30
31	00902	99996	02647	99965	04391	99904	06134	99812	07875	99689	29
32	00931	99996	02676	99964	04420	99902	06163	99810	07904	99687	28
33	00960	99995	02705	99963	04449	99901	06192	99808	07933	99685	27
34	00989	99995	02734	99963	04478	99900	06221	99806	07962	99683	26
35	01018	99995	02763	99962	04507	99898	06250	99804	07991	99680	25
36	01047	99995	02792	99961	04536	99897	06279	99803	08020	99678	24
37	01076	99994	02821	99960	04565	99896	06308	99801	08049	99676	23
38	01105	99994	02850	99959	04594	99894	06337	99799	08078	99673	22
39	01134	99994	02879	99959	04623	99893	06366	99797	08107	99671	21
40	01164	99993	02908	99958	04653	99892	06395	99795	08136	99668	20
41	01193	99993	02938	99957	04682	99890	06424	99793	08165	99666	19
42	01222	99993	02967	99956	04711	99889	06453	99792	08194	99664	18
43	01251	99992	02996	99955	04740	99888	06482	99790	08223	99661	17
44	01280	99992	03025	99954	04769	99886	06511	99788	08252	99659	16
45	01309	99991	03054	99953	04798	99885	06540	99786	08281	99657	15
46	01338	99991	03083	99952	04827	99883	06569	99784	08310	99654	14
47	01367	99991	03112	99952	04856	99882	06598	99782	08339	99652	13
48	01396	99990	03141	99951	04885	99881	06627	99780	08368	99649	12
49	01425	99990	03170	99950	04914	99879	06656	99778	08397	99647	11
50	01454	99989	03199	99949	04943	99878	06685	99776	08426	99644	10
51	01483	99989	03228	99948	04972	99876	06714	99774	08455	99642	9
52	01513	99989	03257	99947	05001	99875	06743	99772	08484	99639	8
53	01542	99988	03286	99946	05030	99873	06773	99770	08513	99637	7
54	01571	99988	03316	99945	05059	99872	06802	99768	08542	99635	6
55	01600	99987	03345	99944	05088	99870	06831	99766	08571	99632	5
56	01629	99987	03374	99943	05117	99869	06860	99764	08600	99630	4
57	01658	99986	03403	99942	05146	99867	06889	99762	08629	99627	3
58	01687	99986	03432	99941	05175	99866	06918	99760	08658	99625	2
59	01716	99985	03461	99940	05205	99864	06947	99758	08687	99622	1
60	01745	99985	03490	99939	05234	99863	06976	99756	08716	99619	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	89°		88°		87°		86°		85°		′

NATURAL SINES AND COSINES.

′	5°		6°		7°		8°		9°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	08716	99619	10453	99452	12187	99255	13917	99027	15643	98769	60
1	08745	99617	10482	99449	12216	99251	13946	99023	15672	98764	59
2	08774	99614	10511	99446	12245	99248	13975	99019	15701	98760	58
3	08803	99612	10540	99443	12274	99244	14004	99015	15730	98755	57
4	08831	99609	10569	99440	12302	99240	14033	99011	15758	98751	56
5	08860	99607	10597	99437	12331	99237	14061	99006	15787	98746	55
6	08889	99604	10626	99434	12360	99233	14090	99002	15816	98741	54
7	08918	99602	10655	99431	12389	99230	14119	98998	15845	98737	53
8	08947	99599	10684	99428	12418	99226	14148	98994	15873	98732	52
9	08976	99596	10713	99424	12447	99222	14177	98990	15902	98728	51
10	09005	99594	10742	99421	12476	99219	14205	98986	15931	98723	50
11	09034	99591	10771	99418	12504	99215	14234	98982	15959	98718	49
12	09063	99588	10800	99415	12533	99211	14263	98978	15988	98714	48
13	09092	99586	10829	99412	12562	99208	14292	98973	16017	98709	47
14	09121	99583	10858	99409	12591	99204	14320	98969	16046	98704	46
15	09150	99580	10887	99406	12620	99200	14349	98965	16074	98700	45
16	09179	99578	10916	99402	12649	99197	14378	98961	16103	98695	44
17	09208	99575	10945	99399	12678	99193	14407	98957	16132	98690	43
18	09237	99572	10973	99396	12706	99189	14436	98953	16160	98686	42
19	09266	99570	11002	99393	12735	99186	14464	98948	16189	98681	41
20	09295	99567	11031	99390	12764	99182	14493	98944	16218	98676	40
21	09324	99564	11060	99386	12793	99178	14522	98940	16246	98671	39
22	09353	99562	11089	99383	12822	99175	14551	98936	16275	98667	38
23	09382	99559	11118	99380	12851	99171	14580	98931	16304	98662	37
24	09411	99556	11147	99377	12880	99167	14608	98927	16333	98657	36
25	09440	99553	11176	99374	12908	99163	14637	98923	16361	98652	35
26	09469	99551	11205	99370	12937	99160	14666	98919	16390	98648	34
27	09498	99548	11234	99367	12966	99156	14695	98914	16419	98643	33
28	09527	99545	11263	99364	12995	99152	14723	98910	16447	98638	32
29	09556	99542	11291	99360	13024	99148	14752	98906	16476	98633	31
30	09585	99540	11320	99357	13053	99144	14781	98902	16505	98629	30
31	09614	99537	11349	99354	13081	99141	14810	98897	16533	98624	29
32	09642	99534	11378	99351	13110	99137	14838	98893	16562	98619	28
33	09671	99531	11407	99347	13139	99133	14867	98889	16591	98614	27
34	09700	99528	11436	99344	13168	99129	14896	98884	16620	98609	26
35	09729	99526	11465	99341	13197	99125	14925	98880	16648	98604	25
36	09758	99523	11494	99337	13226	99122	14954	98876	16677	98600	24
37	09787	99520	11523	99334	13254	99118	14982	98871	16706	98595	23
38	09816	99517	11552	99331	13283	99114	15011	98867	16734	98590	22
39	09845	99514	11580	99327	13312	99110	15040	98863	16763	98585	21
40	09874	99511	11609	99324	13341	99106	15069	98858	16792	98580	20
41	09903	99508	11638	99320	13370	99102	15097	98854	16820	98575	19
42	09932	99506	11667	99317	13399	99098	15126	98849	16849	98570	18
43	09961	99503	11696	99314	13427	99094	15155	98845	16878	98565	17
44	09990	99500	11725	99310	13456	99091	15184	98841	16906	98561	16
45	10019	99497	11754	99307	13485	99087	15212	98836	16935	98556	15
46	10048	99494	11783	99303	13514	99083	15241	98832	16964	98551	14
47	10077	99491	11812	99300	13543	99079	15270	98827	16992	98546	13
48	10106	99488	11840	99297	13572	99075	15299	98823	17021	98541	12
49	10135	99485	11869	99293	13600	99071	15327	98818	17050	98536	11
50	10164	99482	11898	99290	13629	99067	15356	98814	17078	98531	10
51	10192	99479	11927	99286	13658	99063	15385	98809	17107	98526	9
52	10221	99476	11956	99283	13687	99059	15414	98805	17136	98521	8
53	10250	99473	11985	99279	13716	99055	15442	98800	17164	98516	7
54	10279	99470	12014	99276	13744	99051	15471	98796	17193	98511	6
55	10308	99467	12043	99272	13773	99047	15500	98791	17222	98506	5
56	10337	99464	12071	99269	13802	99043	15529	98787	17250	98501	4
57	10366	99461	12100	99265	13831	99039	15557	98782	17279	98496	3
58	10395	99458	12129	99262	13860	99035	15586	98778	17308	98491	2
59	10424	99455	12158	99258	13889	99031	15615	98773	17336	98486	1
60	10453	99452	12187	99255	13917	99027	15643	98769	17365	98481	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	84°		83°		82°		81°		80°		′

NATURAL SINES AND COSINES.

′	10°		11°		12°		13°		14°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	17365	98481	19081	98163	20791	97815	22495	97437	24192	97030	60
1	17393	98476	19109	98157	20820	97809	22523	97430	24220	97023	59
2	17422	98471	19138	98152	20848	97803	22552	97424	24249	97015	58
3	17451	98466	19167	98146	20877	97797	22580	97417	24277	97008	57
4	17479	98461	19195	98140	20905	97791	22608	97411	24305	97001	56
5	17508	98455	19224	98135	20933	97784	22637	97404	24333	96994	55
6	17537	98450	19252	98129	20962	97778	22665	97398	24362	96987	54
7	17565	98445	19281	98124	20990	97772	22693	97391	24390	96980	53
8	17594	98440	19309	98118	21019	97766	22722	97384	24418	96973	52
9	17623	98435	19338	98112	21047	97760	22750	97378	24446	96966	51
10	17651	98430	19366	98107	21076	97754	22778	97371	24474	96959	50
11	17680	98425	19395	98101	21104	97748	22807	97365	24503	96952	49
12	17708	98420	19423	98096	21132	97742	22835	97358	24531	96945	48
13	17737	98414	19452	98090	21161	97735	22863	97351	24559	96937	47
14	17766	98409	19481	98084	21189	97729	22892	97345	24587	96930	46
15	17794	98404	19509	98079	21218	97723	22920	97338	24615	96923	45
16	17823	98399	19538	98073	21246	97717	22948	97331	24644	96916	44
17	17852	98394	19566	98067	21275	97711	22977	97325	24672	96909	43
18	17880	98389	19595	98061	21303	97705	23005	97318	24700	96902	42
19	17909	98383	19623	98056	21331	97698	23033	97311	24728	96894	41
20	17937	98378	19652	98050	21360	97692	23062	97304	24756	96887	40
21	17966	98373	19680	98044	21388	97686	23090	97298	24784	96880	39
22	17995	98368	19709	98039	21417	97680	23118	97291	24813	96873	38
23	18023	98362	19737	98033	21445	97673	23146	97284	24841	96866	37
24	18052	98357	19766	98027	21474	97667	23175	97278	24869	96858	36
25	18081	98352	19794	98021	21502	97661	23203	97271	24897	96851	35
26	18109	98347	19823	98016	21530	97655	23231	97264	24925	96844	34
27	18138	98341	19851	98010	21559	97648	23260	97257	24954	96837	33
28	18166	98336	19880	98004	21587	97642	23288	97251	24982	96829	32
29	18195	98331	19908	97998	21616	97636	23316	97244	25010	96822	31
30	18224	98325	19937	97992	21644	97630	23345	97237	25038	96815	30
31	18252	98320	19965	97987	21672	97623	23373	97230	25066	96807	29
32	18281	98315	19994	97981	21701	97617	23401	97223	25094	96800	28
33	18309	98310	20022	97975	21729	97611	23429	97217	25122	96793	27
34	18338	98304	20051	97969	21758	97604	23458	97210	25151	96786	26
35	18367	98299	20079	97963	21786	97598	23486	97203	25179	96778	25
36	18395	98294	20108	97958	21814	97592	23514	97196	25207	96771	24
37	18424	98288	20136	97952	21843	97585	23542	97189	25235	96764	23
38	18452	98283	20165	97946	21871	97579	23571	97182	25263	96756	22
39	18481	98277	20193	97940	21899	97573	23599	97176	25291	96749	21
40	18509	98272	20222	97934	21928	97566	23627	97169	25320	96742	20
41	18538	98267	20250	97928	21956	97560	23656	97162	25348	96734	19
42	18567	98261	20279	97922	21985	97553	23684	97155	25376	96727	18
43	18595	98256	20307	97916	22013	97547	23712	97148	25404	96719	17
44	18624	98250	20336	97910	22041	97541	23740	97141	25432	96712	16
45	18652	98245	20364	97905	22070	97534	23769	97134	25460	96705	15
46	18681	98240	20393	97899	22098	97528	23797	97127	25488	96697	14
47	18710	98234	20421	97893	22126	97521	23825	97120	25516	96690	13
48	18738	98229	20450	97887	22155	97515	23853	97113	25545	96682	12
49	18767	98223	20478	97881	22183	97508	23882	97106	25573	96675	11
50	18795	98218	20507	97875	22212	97502	23910	97100	25601	96667	10
51	18824	98212	20535	97869	22240	97496	23938	97093	25629	96660	9
52	18852	98207	20563	97863	22268	97489	23966	97086	25657	96653	8
53	18881	98201	20592	97857	22297	97483	23995	97079	25685	96645	7
54	18910	98196	20620	97851	22325	97476	24023	97072	25713	96638	6
55	18938	98190	20649	97845	22353	97470	24051	97065	25741	96630	5
56	18967	98185	20677	97839	22382	97463	24079	97058	25769	96623	4
57	18995	98179	20706	97833	22410	97457	24108	97051	25798	96615	3
58	19024	98174	20734	97827	22438	97450	24136	97044	25826	96608	2
59	19052	98168	20763	97821	22467	97444	24164	97037	25854	96600	1
60	19081	98163	20791	97815	22495	97437	24192	97030	25882	96593	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine	Sine.	Cosine.	Sine.	
′	79°		78°		77°		76°		75°		′

NATURAL SINES AND COSINES.

′	15°		16°		17°		18°		19°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	25882	96593	27564	96126	29237	95630	30902	95106	32557	94552	60
1	25910	96585	27592	96118	29265	95622	30929	95097	32584	94542	59
2	25938	96578	27620	96110	29293	95613	30957	95088	32612	94533	58
3	25966	96570	27648	96102	29321	95605	30985	95079	32639	94523	57
4	25994	96562	27676	96094	29348	95596	31012	95070	32667	94514	56
5	26022	96555	27704	96086	29376	95588	31040	95061	32694	94504	55
6	26050	96547	27731	96078	29404	95579	31068	95052	32722	94495	54
7	26079	96540	27759	96070	29432	95571	31095	95043	32749	94485	53
8	26107	96532	27787	96062	29460	95562	31123	95033	32777	94476	52
9	26135	96524	27815	96054	29487	95554	31151	95024	32804	94466	51
10	26163	96517	27843	96046	29515	95545	31178	95015	32832	94457	50
11	26191	96509	27871	96037	29543	95536	31206	95006	32859	94447	49
12	26219	96502	27899	96029	29571	95528	31233	94997	32887	94438	48
13	26247	96494	27927	96021	29599	95519	31261	94988	32914	94428	47
14	26275	96486	27955	96013	29626	95511	31289	94979	32942	94418	46
15	26303	96479	27983	96005	29654	95502	31316	94970	32969	94409	45
16	26331	96471	28011	95997	29682	95493	31344	94961	32997	94399	44
17	26359	96463	28039	95989	29710	95485	31372	94952	33024	94390	43
18	26387	96456	28067	95981	29737	95476	31399	94943	33051	94380	42
19	26415	96448	28095	95972	29765	95467	31427	94933	33079	94370	41
20	26443	96440	28123	95964	29793	95459	31454	94924	33106	94361	40
21	26471	96433	28150	95956	29821	95450	31482	94915	33134	94351	39
22	26500	96425	28178	95948	29849	95441	31510	94906	33161	94342	38
23	26528	96417	28206	95940	29876	95433	31537	94897	33189	94332	37
24	26556	96410	28234	95931	29904	95424	31565	94888	33216	94322	36
25	26584	96402	28262	95923	29932	95415	31593	94878	33244	94313	35
26	26612	96394	28290	95915	29960	95407	31620	94869	33271	94303	34
27	26640	96386	28318	95907	29987	95398	31648	94860	33298	94293	33
28	26668	96379	28346	95898	30015	95389	31675	94851	33326	94284	32
29	26696	96371	28374	95890	30043	95380	31703	94842	33353	94274	31
30	26724	96363	28402	95882	30071	95372	31730	94832	33381	94264	30
31	26752	96355	28429	95874	30098	95363	31758	94823	33408	94254	29
32	26780	96347	28457	95865	30126	95354	31786	94814	33436	94245	28
33	26808	96340	28485	95857	30154	95345	31813	94805	33463	94235	27
34	26836	96332	28513	95849	30182	95337	31841	94795	33490	94225	26
35	26864	96324	28541	95841	30209	95328	31868	94786	33518	94215	25
36	26892	96316	28569	95832	30237	95319	31896	94777	33545	94206	24
37	26920	96308	28597	95824	30265	95310	31923	94768	33573	94196	23
38	26948	96301	28625	95816	30292	95301	31951	94758	33600	94186	22
39	26976	96293	28652	95807	30320	95293	31979	94749	33627	94176	21
40	27004	96285	28680	95799	30348	95284	32006	94740	33655	94167	20
41	27032	96277	28708	95791	30376	95275	32034	94730	33682	94157	19
42	27060	96269	28736	95782	30403	95266	32061	94721	33710	94147	18
43	27088	96261	28764	95774	30431	95257	32089	94712	33737	94137	17
44	27116	96253	28792	95766	30459	95248	32116	94702	33764	94127	16
45	27144	96246	28820	95757	30486	95240	32144	94693	33792	94118	15
46	27172	96238	28847	95749	30514	95231	32171	94684	33819	94108	14
47	27200	96230	28875	95740	30542	95222	32199	94674	33846	94098	13
48	27228	96222	28903	95732	30570	95213	32227	94665	33874	94088	12
49	27256	96214	28931	95724	30597	95204	32254	94656	33901	94078	11
50	27284	96206	28959	95715	30625	95195	32282	94646	33929	94068	10
51	27312	96198	28987	95707	30653	95186	32309	94637	33956	94058	9
52	27340	96190	29015	95698	30680	95177	32337	94627	33983	94049	8
53	27368	96182	29042	95690	30708	95168	32364	94618	34011	94039	7
54	27396	96174	29070	95681	30736	95159	32392	94609	34038	94029	6
55	27424	96166	29098	95673	30763	95150	32419	94599	34065	94019	5
56	27452	96158	29126	95664	30791	95142	32447	94590	34093	94009	4
57	27480	96150	29154	95656	30819	95133	32474	94580	34120	93999	3
58	27508	96142	29182	95647	30846	95124	32502	94571	34147	93989	2
59	27536	96134	29209	95639	30874	95115	32529	94561	34175	93979	1
60	27564	96126	29237	95630	30902	95106	32557	94552	34202	93969	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine	Cosine.	Sine.	
′	74°		73°		72°		71°		70°		′

NATURAL SINES AND COSINES.

′	20°		21°		22°		23°		24°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	34202	93969	35837	93358	37461	92718	39073	92050	40674	91355	60
1	34229	93959	35864	93348	37488	92707	39100	92039	40700	91343	59
2	34257	93949	35891	93337	37515	92697	39127	92028	40727	91331	58
3	34284	93939	35918	93327	37542	92686	39153	92016	40753	91319	57
4	34311	93929	35945	93316	37569	92675	39180	92005	40780	91307	56
5	34339	93919	35973	93306	37595	92664	39207	91994	40806	91295	55
6	34366	93909	36000	93295	37622	92653	39234	91982	40833	91283	54
7	34393	93899	36027	93285	37649	92642	39260	91971	40860	91272	53
8	34421	93889	36054	93274	37676	92631	39287	91959	40886	91260	52
9	34448	93879	36081	93264	37703	92620	39314	91948	40913	91248	51
10	34475	93869	36108	93253	37730	92609	39341	91936	40939	91236	50
11	34503	93859	36135	93243	37757	92598	39367	91925	40966	91224	49
12	34530	93849	36162	93232	37784	92587	39394	91914	40992	91212	48
13	34557	93839	36190	93222	37811	92576	39421	91902	41019	91200	47
14	34584	93829	36217	93211	37838	92565	39448	91891	41045	91188	46
15	34612	93819	36244	93201	37865	92554	39474	91879	41072	91176	45
16	34639	93809	36271	93190	37892	92543	39501	91868	41098	91164	44
17	34666	93799	36298	93180	37919	92532	39528	91856	41125	91152	43
18	34694	93789	36325	93169	37946	92521	39555	91845	41151	91140	42
19	34721	93779	36352	93159	37973	92510	39581	91833	41178	91128	41
20	34748	93769	36379	93148	37999	92499	39608	91822	41204	91116	40
21	34775	93759	36406	93137	38026	92488	39635	91810	41231	91104	39
22	34803	93748	36434	93127	38053	92477	39661	91799	41257	91092	38
23	34830	93738	36461	93116	38080	92466	39688	91787	41284	91080	37
24	34857	93728	36488	93106	38107	92455	39715	91775	41310	91068	36
25	34884	93718	36515	93095	38134	92444	39741	91764	41337	91056	35
26	34912	93708	36542	93084	38161	92432	39768	91752	41363	91044	34
27	34939	93698	36569	93074	38188	92421	39795	91741	41390	91032	33
28	34966	93688	36596	93063	38215	92410	39822	91729	41416	91020	32
29	34993	93677	36623	93052	38241	92399	39848	91718	41443	91008	31
30	35021	93667	36650	93042	38268	92388	39875	91706	41469	90996	30
31	35048	93657	36677	93031	38295	92377	39902	91694	41496	90984	29
32	35075	93647	36704	93020	38322	92366	39928	91683	41522	90972	28
33	35102	93637	36731	93010	38349	92355	39955	91671	41549	90960	27
34	35130	93626	36758	92999	38376	92343	39982	91660	41575	90948	26
35	35157	93616	36785	92988	38403	92332	40008	91648	41602	90936	25
36	35184	93606	36812	92978	38430	92321	40035	91636	41628	90924	24
37	35211	93596	36839	92967	38456	92310	40062	91625	41655	90911	23
38	35239	93585	36867	92956	38483	92299	40088	91613	41681	90899	22
39	35266	93575	36894	92945	38510	92287	40115	91601	41707	90887	21
40	35293	93565	36921	92935	38537	92276	40141	91590	41734	90875	20
41	35320	93555	36948	92924	38564	92265	40168	91578	41760	90863	19
42	35347	93544	36975	92913	38591	92254	40195	91566	41787	90851	18
43	35375	93534	37002	92902	38617	92243	40221	91555	41813	90839	17
44	35402	93524	37029	92892	38644	92231	40248	91543	41840	90826	16
45	35429	93514	37056	92881	38671	92220	40275	91531	41866	90814	15
46	35456	93503	37083	92870	38698	92209	40301	91519	41892	90802	14
47	35484	93493	37110	92859	38725	92198	40328	91508	41919	90790	13
48	35511	93483	37137	92849	38752	92186	40355	91496	41945	90778	12
49	35538	93472	37164	92838	38778	92175	40381	91484	41972	90766	11
50	35565	93462	37191	92827	38805	92164	40408	91472	41998	90753	10
51	35592	93452	37218	92816	38832	92152	40434	91461	42024	90741	9
52	35619	93441	37245	92805	38859	92141	40461	91449	42051	90729	8
53	35647	93431	37272	92794	38886	92130	40488	91437	42077	90717	7
54	35674	93420	37299	92784	38912	92119	40514	91425	42104	90704	6
55	35701	93410	37326	92773	38939	92107	40541	91414	42130	90692	5
56	35728	93400	37353	92762	38966	92096	40567	91402	42156	90680	4
57	35755	93389	37380	92751	38993	92085	40594	91390	42183	90668	3
58	35782	93379	37407	92740	39020	92073	40621	91378	42209	90655	2
59	35810	93368	37434	92729	39046	92062	40647	91366	42235	90643	1
60	35837	93358	37461	92718	39073	92050	40674	91355	42262	90631	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine	Sine.	Cosine.	Sine.	
′	69°		68°		67°		66°		65°		′

NATURAL SINES AND COSINES.

′	25°		26°		27°		28°		29°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	60
1	42288	90618	43863	89867	45425	89087	46973	88281	48506	87448	59
2	42315	90606	43889	89854	45451	89074	46999	88267	48532	87434	58
3	42341	90594	43916	89841	45477	89061	47024	88254	48557	87420	57
4	42367	90582	43942	89828	45503	89048	47050	88240	48583	87406	56
5	42394	90569	43968	89816	45529	89035	47076	88226	48608	87391	55
6	42420	90557	43994	89803	45554	89021	47101	88213	48634	87377	54
7	42446	90545	44020	89790	45580	89008	47127	88199	48659	87363	53
8	42473	90532	44046	89777	45606	88995	47153	88185	48684	87349	52
9	42499	90520	44072	89764	45632	88981	47178	88172	48710	87335	51
10	42525	90507	44098	89752	45658	88968	47204	88158	48735	87321	50
11	42552	90495	44124	89739	45684	88955	47229	88144	48761	87306	49
12	42578	90483	44151	89726	45710	88942	47255	88130	48786	87292	48
13	42604	90470	44177	89713	45736	88928	47281	88117	48811	87278	47
14	42631	90458	44203	89700	45762	88915	47306	88103	48837	87264	46
15	42657	90446	44229	89687	45787	88902	47332	88089	48862	87250	45
16	42683	90433	44255	89674	45813	88888	47358	88075	48888	87235	44
17	42709	90421	44281	89662	45839	88875	47383	88062	48913	87221	43
18	42736	90408	44307	89649	45865	88862	47409	88048	48938	87207	42
19	42762	90396	44333	89636	45891	88848	47434	88034	48964	87193	41
20	42788	90383	44359	89623	45917	88835	47460	88020	48989	87178	40
21	42815	90371	44385	89610	45942	88822	47486	88006	49014	87164	39
22	42841	90358	44411	89597	45968	88808	47511	87993	49040	87150	38
23	42867	90346	44437	89584	45994	88795	47537	87979	49065	87136	37
24	42894	90334	44464	89571	46020	88782	47562	87965	49090	87121	36
25	42920	90321	44490	89558	46046	88768	47588	87951	49116	87107	35
26	42946	90309	44516	89545	46072	88755	47614	87937	49141	87093	34
27	42972	90296	44542	89532	46097	88741	47639	87923	49166	87079	33
28	42999	90284	44568	89519	46123	88728	47665	87909	49192	87064	32
29	43025	90271	44594	89506	46149	88715	47690	87896	49217	87050	31
30	43051	90259	44620	89493	46175	88701	47716	87882	49242	87036	30
31	43077	90246	44646	89480	46201	88688	47741	87868	49268	87021	29
32	43104	90233	44672	89467	46226	88674	47767	87854	49293	87007	28
33	43130	90221	44698	89454	46252	88661	47793	87840	49318	86993	27
34	43156	90208	44724	89441	46278	88647	47818	87826	49344	86978	26
35	43182	90196	44750	89428	46304	88634	47844	87812	49369	86964	25
36	43209	90183	44776	89415	46330	88620	47869	87798	49394	86949	24
37	43235	90171	44802	89402	46355	88607	47895	87784	49419	86935	23
38	43261	90158	44828	89389	46381	88593	47920	87770	49445	86921	22
39	43287	90146	44854	89376	46407	88580	47946	87756	49470	86906	21
40	43313	90133	44880	89363	46433	88566	47971	87743	49495	86892	20
41	43340	90120	44906	89350	46458	88553	47997	87729	49521	86878	19
42	43366	90108	44932	89337	46484	88539	48022	87715	49546	86863	18
43	43392	90095	44958	89324	46510	88526	48048	87701	49571	86849	17
44	43418	90082	44984	89311	46536	88512	48073	87687	49596	86834	16
45	43445	90070	45010	89298	46561	88499	48099	87673	49622	86820	15
46	43471	90057	45036	89285	46587	88485	48124	87659	49647	86805	14
47	43497	90045	45062	89272	46613	88472	48150	87645	49672	86791	13
48	43523	90032	45088	89259	46639	88458	48175	87631	49697	86777	12
49	43549	90019	45114	89245	46664	88445	48201	87617	49723	86762	11
50	43575	90007	45140	89232	46690	88431	48226	87603	49748	86748	10
51	43602	89994	45166	89219	46716	88417	48252	87589	49773	86733	9
52	43628	89981	45192	89206	46742	88404	48277	87575	49798	86719	8
53	43654	89968	45218	89193	46767	88390	48303	87561	49824	86704	7
54	43680	89956	45243	89180	46793	88377	48328	87546	49849	86690	6
55	43706	89943	45269	89167	46819	88363	48354	87532	49874	86675	5
56	43733	89930	45295	89153	46844	88349	48379	87518	49899	86661	4
57	43759	89918	45321	89140	46870	88336	48405	87504	49924	86646	3
58	43785	89905	45347	89127	46896	88322	48430	87490	49950	86632	2
59	43811	89892	45373	89114	46921	88308	48456	87476	49975	86617	1
60	43837	89879	45399	89101	46947	88295	48481	87462	50000	86603	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	64°		63°		62°		61°		60°		′

NATURAL SINES AND COSINES.

′	30° Sine.	30° Cosine.	31° Sine.	31° Cosine.	32° Sine.	32° Cosine.	33° Sine.	33° Cosine.	34° Sine.	34° Cosine.	′
0	50000	86603	51504	85717	52992	84805	54464	83867	55919	82904	60
1	50025	86588	51529	85702	53017	84789	54488	83851	55943	82887	59
2	50050	86573	51554	85687	53041	84774	54513	83835	55968	82871	58
3	50076	86559	51579	85672	53066	84759	54537	83819	55992	82855	57
4	50101	86544	51604	85657	53091	84743	54561	83804	56016	82839	56
5	50126	86530	51628	85642	53115	84728	54586	83788	56040	82822	55
6	50151	86515	51653	85627	53140	84712	54610	83772	56064	82806	54
7	50176	86501	51678	85612	53164	84697	54635	83756	56088	82790	53
8	50201	86486	51703	85597	53189	84681	54659	83740	56112	82773	52
9	50227	86471	51728	85582	53214	84666	54683	83724	56136	82757	51
10	50252	86457	51753	85567	53238	84650	54708	83708	56160	82741	50
11	50277	86442	51778	85551	53263	84635	54732	83692	56184	82724	49
12	50302	86427	51803	85536	53288	84619	54756	83676	56208	82708	48
13	50327	86413	51828	85521	53312	84604	54781	83660	56232	82692	47
14	50352	86398	51852	85506	53337	84588	54805	83645	56256	82675	46
15	50377	86384	51877	85491	53361	84573	54829	83629	56280	82659	45
16	50403	86369	51902	85476	53386	84557	54854	83613	56305	82643	44
17	50428	86354	51927	85461	53411	84542	54878	83597	56329	82626	43
18	50453	86340	51952	85446	53435	84526	54902	83581	56353	82610	42
19	50478	86325	51977	85431	53460	84511	54927	83565	56377	82593	41
20	50503	86310	52002	85416	53484	84495	54951	83549	56401	82577	40
21	50528	86295	52026	85401	53509	84480	54975	83533	56425	82561	39
22	50553	86281	52051	85385	53534	84464	54999	83517	56449	82544	38
23	50578	86266	52076	85370	53558	84448	55024	83501	56473	82528	37
24	50603	86251	52101	85355	53583	84433	55048	83485	56497	82511	36
25	50628	86237	52126	85340	53607	84417	55072	83469	56521	82495	35
26	50654	86222	52151	85325	53632	84402	55097	83453	56545	82478	34
27	50679	86207	52175	85310	53656	84386	55121	83437	56569	82462	33
28	50704	86192	52200	85294	53681	84370	55145	83421	56593	82446	32
29	50729	86178	52225	85279	53705	84355	55169	83405	56617	82429	31
30	50754	86163	52250	85264	53730	84339	55194	83389	56641	82413	30
31	50779	86148	52275	85249	53754	84324	55218	83373	56665	82396	29
32	50804	86133	52299	85234	53779	84308	55242	83356	56689	82380	28
33	50829	86119	52324	85218	53804	84292	55266	83340	56713	82363	27
34	50854	86104	52349	85203	53828	84277	55291	83324	56736	82347	26
35	50879	86089	52374	85188	53853	84261	55315	83308	56760	82330	25
36	50904	86074	52399	85173	53877	84245	55339	83292	56784	82314	24
37	50929	86059	52423	85157	53902	84230	55363	83276	56808	82297	23
38	50954	86045	52448	85142	53926	84214	55388	83260	56832	82281	22
39	50979	86030	52473	85127	53951	84198	55412	83244	56856	82264	21
40	51004	86015	52498	85112	53975	84182	55436	83228	56880	82248	20
41	51029	86000	52522	85096	54000	84167	55460	83212	56904	82231	19
42	51054	85985	52547	85081	54024	84151	55484	83195	56928	82214	18
43	51079	85970	52572	85066	54049	84135	55509	83179	56952	82198	17
44	51104	85956	52597	85051	54073	84120	55533	83163	56976	82181	16
45	51129	85941	52621	85035	54097	84104	55557	83147	57000	82165	15
46	51154	85926	52646	85020	54122	84088	55581	83131	57024	82148	14
47	51179	85911	52671	85005	54146	84072	55605	83115	57047	82132	13
48	51204	85896	52696	84989	54171	84057	55630	83098	57071	82115	12
49	51229	85881	52720	84974	54195	84041	55654	83082	57095	82098	11
50	51254	85866	52745	84959	54220	84025	55678	83066	57119	82082	10
51	51279	85851	52770	84943	54244	84009	55702	83050	57143	82065	9
52	51304	85836	52794	84928	54269	83994	55726	83034	57167	82048	8
53	51329	85821	52819	84913	54293	83978	55750	83017	57191	82032	7
54	51354	85806	52844	84897	54317	83962	55775	83001	57215	82015	6
55	51379	85792	52869	84882	54342	83946	55799	82985	57238	81999	5
56	51404	85777	52893	84866	54366	83930	55823	82969	57262	81982	4
57	51429	85762	52918	84851	54391	83915	55847	82953	57286	81965	3
58	51454	85747	52943	84836	54415	83899	55871	82936	57310	81949	2
59	51479	85732	52967	84820	54440	83883	55895	82920	57334	81932	1
60	51504	85717	52992	84805	54464	83867	55919	82904	57358	81915	0
′	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	′
	59°		58°		57°		56°		55°		

NATURAL SINES AND COSINES.

′	35°		36°		37°		38°		39°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	57358	81915	58779	80902	60182	79864	61566	78801	62932	77715	60
1	57381	81899	58802	80885	60205	79846	61589	78783	62955	77696	59
2	57405	81882	58826	80867	60228	79829	61612	78765	62977	77678	58
3	57429	81865	58849	80850	60251	79811	61635	78747	63000	77660	57
4	57453	81848	58873	80833	60274	79793	61658	78729	63022	77641	56
5	57477	81832	58896	80816	60298	79776	61681	78711	63045	77623	55
6	57501	81815	58920	80799	60321	79758	61704	78694	63068	77605	54
7	57524	81798	58943	80782	60344	79741	61726	78676	63090	77586	53
8	57548	81782	58967	80765	60367	79723	61749	78658	63113	77568	52
9	57572	81765	58990	80748	60390	79706	61772	78640	63135	77550	51
10	57596	81748	59014	80730	60414	79688	61795	78622	63158	77531	50
11	57619	81731	59037	89713	60437	79671	61818	78604	63180	77513	49
12	57643	81714	59061	80696	60460	79653	61841	78586	63203	77494	48
13	57667	81698	59084	80679	60483	79635	61864	78568	63225	77476	47
14	57691	81681	59108	80662	60506	79618	61887	78550	63248	77458	46
15	57715	81664	59131	80644	60529	79600	61909	78532	63271	77439	45
16	57738	81647	59154	80627	60553	79583	61932	78514	63293	77421	44
17	57762	81631	59178	80610	60576	79565	61955	78496	63316	77402	43
18	57786	81614	59201	80593	60599	79547	61978	78478	63338	77384	42
19	57810	81597	59225	80576	60622	79530	62001	78460	63361	77366	41
20	57833	81580	59248	80558	60645	79512	62024	78442	63383	77347	40
21	57857	81563	59272	80541	60668	79494	62046	78424	63406	77329	39
22	57881	81546	59295	80524	60691	79477	62069	78405	63428	77310	38
23	57904	81530	59318	80507	60714	79459	62092	78387	63451	77292	37
24	57928	81513	59342	80489	60738	79441	62115	78369	63473	77273	36
25	57952	81496	59365	80472	60761	79424	62138	78351	63496	77255	35
26	57976	81479	59389	80455	60784	79406	62160	78333	63518	77236	34
27	57999	81462	59412	80438	60807	79388	62183	78315	63540	77218	33
28	58023	81445	59436	80420	60830	79371	62206	78297	63563	77199	32
29	58047	81428	59459	80403	60853	79353	62229	78279	63585	77181	31
30	58070	81412	59482	80386	60876	79335	62251	78261	63608	77162	30
31	58094	81395	59506	80368	60899	79318	62274	78243	63630	77144	29
32	58118	81378	59529	80351	60922	79300	62297	78225	63653	77125	28
33	58141	81361	59552	80334	60945	79282	62320	78206	63675	77107	27
34	58165	81344	59576	80316	60968	79264	62342	78188	63698	77088	26
35	58189	81327	59599	80299	60991	79247	62365	78170	63720	77070	25
36	58212	81310	59622	80282	61015	79229	62388	78152	63742	77051	24
37	58236	81293	59646	80264	61038	79211	62411	78134	63765	77033	23
38	58260	81276	59669	80247	61061	79193	62433	78116	63787	77014	22
39	58283	81259	59693	80230	61084	79176	62456	78098	63810	76996	21
40	58307	81242	59716	80212	61107	79158	62479	78079	63832	76977	20
41	58330	81225	59739	80195	61130	79140	62502	78061	63854	76959	19
42	58354	81208	59763	80178	61153	79122	62524	78043	63877	76940	18
43	58378	81191	59786	80160	61176	79105	62547	78025	63899	76921	17
44	58401	81174	59809	80143	61199	79087	62570	78007	63922	76903	16
45	58425	81157	59832	80125	61222	79069	62592	77988	63944	76884	15
46	58449	81140	59856	80108	61245	79051	62615	77970	63966	76866	14
47	58472	81123	59879	80091	61268	79033	62638	77952	63989	76847	13
48	58496	81106	59902	80073	61291	79016	62660	77934	64011	76828	12
49	58519	81089	59926	80056	61314	78998	62683	77916	64033	76810	11
50	58543	81072	59949	80038	61337	78980	62706	77897	64056	76791	10
51	58567	81055	59972	80021	61360	78962	62728	77879	64078	76772	9
52	58590	81038	59995	80003	61383	78944	62751	77861	64100	76754	8
53	58614	81021	60019	79986	61406	78926	62774	77843	64123	76735	7
54	58637	81004	60042	79968	61429	78908	62796	77824	64145	76717	6
55	58661	80987	60065	79951	61451	78891	62819	77806	64167	76698	5
56	58684	80970	60089	79934	61474	78873	62842	77788	64190	76679	4
57	58708	80953	60112	79916	61497	78855	62864	77769	64212	76661	3
58	58731	80936	60135	79899	61520	78837	62887	77751	64234	76642	2
59	58755	80919	60158	79881	61543	78819	62909	77733	64256	76623	1
60	58779	80902	60182	79864	61566	78801	62932	77715	64279	76604	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	54°		53°		52°		51°		50°		′

NATURAL SINES AND COSINES.

′	40°		41°		42°		43°		44°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	64279	76604	65606	75471	66913	74314	68200	73135	69466	71934	60
1	64301	76586	65628	75452	66935	74295	68221	73116	69487	71914	59
2	64323	76567	65650	75433	66956	74276	68242	73096	69508	71894	58
3	64346	76548	65672	75414	66978	74256	68264	73076	69529	71873	57
4	64368	76530	65694	75395	66999	74237	68285	73056	69549	71853	56
5	64390	76511	65716	75375	67021	74217	68306	73036	69570	71833	55
6	64412	76492	65738	75356	67043	74198	68327	73016	69591	71813	54
7	64435	76473	65759	75337	67064	74178	68349	72996	69612	71792	53
8	64457	76455	65781	75318	67086	74159	68370	72976	69633	71772	52
9	64479	76436	65803	75299	67107	74139	68391	72957	69654	71752	51
10	64501	76417	65825	75280	67129	74120	68412	72937	69675	71732	50
11	64524	76398	65847	75261	67151	74100	68434	72917	69696	71711	49
12	64546	76380	65869	75241	67172	74080	68455	72897	69717	71691	48
13	64568	76361	65891	75222	67194	74061	68476	72877	69737	71671	47
14	64590	76342	65913	75203	67215	74041	68497	72857	69758	71650	46
15	64612	76323	65935	75184	67237	74022	68518	72837	69779	71630	45
16	64635	76304	65956	75165	67258	74002	68539	72817	69800	71610	44
17	64657	76286	65978	75146	67280	73983	68561	72797	69821	71590	43
18	64679	76267	66000	75126	67301	73963	68582	72777	69842	71569	42
19	64701	76248	66022	75107	67323	73944	68603	72757	69862	71549	41
20	64723	76229	66044	75088	67344	73924	68624	72737	69883	71529	40
21	64746	76210	66066	75069	67366	73904	68645	72717	69904	71508	39
22	64768	76192	66088	75050	67387	73885	68666	72697	69925	71488	38
23	64790	76173	66109	75030	67409	73865	68688	72677	69946	71468	37
24	64812	76154	66131	75011	67430	73846	68709	72657	69966	71447	36
25	64834	76135	66153	74992	67452	73826	68730	72637	69987	71427	35
26	64856	76116	66175	74973	67473	73806	68751	72617	70008	71407	34
27	64878	76097	66197	74953	67495	73787	68772	72597	70029	71386	33
28	64901	76078	66218	74934	67516	73767	68793	72577	70049	71366	32
29	64923	76059	66240	74915	67538	73747	68814	72557	70070	71345	31
30	64945	76041	66262	74896	67559	73728	68835	72537	70091	71325	30
31	64967	76022	66284	74876	67580	73708	68857	72517	70112	71305	29
32	64989	76003	66306	74857	67602	73688	68878	72497	70132	71284	28
33	65011	75984	66327	74838	67623	73669	68899	72477	70153	71264	27
34	65033	75965	66349	74818	67645	73649	68920	72457	70174	71243	26
35	65055	75946	66371	74799	67666	73629	68941	72437	70195	71223	25
36	65077	75927	66393	74780	67688	73610	68962	72417	70215	71203	24
37	65100	75908	66414	74760	67709	73590	68983	72397	70236	71182	23
38	65122	75889	66436	74741	67730	73570	69004	72377	70257	71162	22
39	65144	75870	66458	74722	67752	73551	69025	72357	70277	71141	21
40	65166	75851	66480	74703	67773	73531	69046	72337	70298	71121	20
41	65188	75832	66501	74683	67795	73511	69067	72317	70319	71100	19
42	65210	75813	66523	74664	67816	73491	69088	72297	70339	71080	18
43	65232	75794	66545	74644	67837	73472	69109	72277	70360	71059	17
44	65254	75775	66566	74625	67859	73452	69130	72257	70381	71039	16
45	65276	75756	66588	74606	67880	73432	69151	72236	70401	71019	15
46	65298	75738	66610	74586	67901	73413	69172	72216	70422	70998	14
47	65320	75719	66632	74567	67923	73393	69193	72196	70443	70978	13
48	65342	75700	66653	74548	67944	73373	69214	72176	70463	70957	12
49	65364	75680	66675	74528	67965	73353	69235	72156	70484	70937	11
50	65386	75661	66697	74509	67987	73333	69256	72136	70505	70916	10
51	65408	75642	66718	74489	68008	73314	69277	72116	70525	70896	9
52	65430	75623	66740	74470	68029	73294	69298	72095	70546	70875	8
53	65452	75604	66762	74451	68051	73274	69319	72075	70567	70855	7
54	65474	75585	66783	74431	68072	73254	69340	72055	70587	70834	6
55	65496	75566	66805	74412	68093	73234	69361	72035	70608	70813	5
56	65518	75547	66827	74392	68115	73215	69382	72015	70628	70793	4
57	65540	75528	66848	74373	68136	73195	69403	71995	70649	70772	3
58	65562	75509	66870	74353	68157	73175	69424	71974	70670	70752	2
59	65584	75490	66891	74334	68179	73155	69445	71954	70690	70731	1
60	65606	75471	66913	74314	68200	73135	69466	71934	70711	70711	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	49°		48°		47°		46°		45°		′