THE PRINCIPLES AND PRACTICE OF SURVEYING

VOLUME II. HIGHER SURVEYING

BY

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PREFACE

This second volume on surveying is chiefly devoted to a consideration of the various methods of conducting topographic and hydrographic surveys. While certain subjects taken up in this book might properly be included in an elementary treatise, the arrangement adopted has the decided advantage of grouping together and treating in a connected manner the different topographic and hydrographic methods.

The subject matter is divided into four parts, namely: The Control of the Survey, Filling in Topographic Details, Hydrographic Surveying and Stream Gauging, and Constructing and Finishing Maps.

In the first part some consideration has been given to Geodetic Surveying, not only to show the methods of controlling the accuracy of the survey, but also to connect the different methods taken up in Parts II and III and to show their relation to the survey as a whole. It is not the intention in any sense to present a treatise on Geodesy, but rather to discuss the subject from the standpoint of the practical surveyor. The matter is therefore limited to what any surveyor without unusual equipment might be expected to apply.

In Part II the methods of making topographic surveys are considered, including the stadia, plane-table, and photographic methods. In the chapter on The Plane Table an attempt has been made to show that this method has advantages which apparently are not appreciated by many surveyors and that its use might be much more general than it is at present. In the description of field methods a comparison has been made of the methods required for maps of different scales. In regard to

Photographic Surveying nothing has been attempted beyond an explanation of the fundamental principles involved in this method. In the chapter on the Relation of Geology to Topography, written by Professor D. W. Johnson, the subject of topography is treated from a point of view that is coming to be more and more appreciated by expert topographers. The importance of geological study to the topographer is now much emphasized by those recognized as authorities. The illustrations for this chapter were drawn by F. E. Matthes, Topographic Inspector, United States Geological Survey, to whom the authors are especially indebted.

The chapter on Hydrographic Surveying treats of the common methods of conducting harbor and river surveys. Some of the up-to-date methods have been explained in detail and are illustrated by several sets of field notes. For valuable suggestions on this subject the authors express their thanks to John R. Burke, formerly Assistant Engineer, Massachusetts Harbor and Land Commission, and to A. J. Ober, Assistant Engineer, United States Engineer Office. Chapter IX, on Stream Gauging, was written by H. K. Barrows, Engineer United States Geological Survey, who for several years has been in charge of the hydrographic work in New York and in New England. Thanks are due also to Professor W. E. Mott of the Massachusetts Institute of Technology for his criticisms of the manuscript of this chapter.

In the last two chapters the common methods of constructing and finishing topographic and hydrographic maps have been described. The details of making conventional signs have been described rather fully and some consideration has been given to the use of symbols on landscape plans. Several illustrations of topographic maps on different scales have been introduced.

The authors desire to acknowledge their indebtedness to all who have aided in the preparation of this book, especially to Professors C. Frank Allen, A. G. Robbins, C. W. Doten, and A. E. Burton of the Massachusetts Institute of Technology for criticisms and valuable suggestions.

The authors wish to express their appreciation of the excellent

work of W. L. Vennard, who prepared the illustrations, and of Miss Edith T. Hosmer, who read all of the proof.

For the cuts and electrotypes which have been loaned for use in this volume acknowledgment has been made in each case under the illustration.

The authors will be grateful for notification of any errors which may be found.

C. B. B.
G. L. H.

Boston, Mass., July, 1908.

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VOLUME II. HIGHER SURVEYING.

PART I.

CONTROL OF THE SURVEY.

CHAPTER I.

TRIANGULATION.

1. **TRIANGULATION.** — In making surveys of large areas it is necessary to establish a few points with great precision in order to insure the accuracy of the survey as a whole. Traverses are not suitable for this purpose, since the sights must in general be short, and the accumulated error in traversing long distances becomes too large. The most accurate and most economical way of locating points for the purpose of controlling the survey is by means of a system of triangulation, i.e., a series of triangles in which a side of one triangle and all of the angles in each triangle are measured; all other lines in the system may then be calculated by trigonometry.

2. **SYSTEMS OF TRIANGULATION.*** — The particular form of the triangulation system adopted will depend upon the purpose of the survey and also upon the character of the country. If a survey extends chiefly in one direction, such as the survey of a river, the most rapid way of covering the distance is by means of a simple chain of triangles, each triangle being made as nearly equilateral as the conditions will permit (Fig. 1). In this system

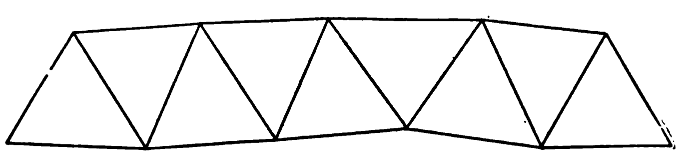

FIG. 1. SIMPLE CHAIN OF TRIANGLES.

the only check on the accuracy of the work is that of the sum of the three measured angles of each triangle.

If the object desired is to survey a broad area rather than a

* For a study of the relative advantages of different systems, see United States Coast and Geodetic Survey Report for 1876, Appendix 20; also Chapter II of Crandall's "Geodesy and Least Squares," published by John Wiley & Sons, New York.

narrow strip, then a system of central polygons may be used (see Fig. 2). Such a system may properly be employed in the

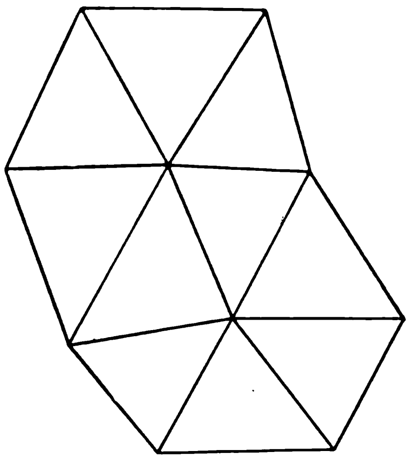

FIG. 2. CENTRAL POLYGONS.

survey of a city or a state. (See Chapter IX, Vol. I.) The lengths of some of the lines in this system may be computed independently by using different sets of triangles, and thus serve as checks on the accuracy of the measurements. The system should not be made more complicated than is necessary to secure proper checks, so that the expense and the difficulties of calculation may be kept within reasonable limits.

The strongest system, i.e., the one having the greatest number of checks in proportion to the number of angles observed, is one consisting of a chain of quadrilaterals (Fig. 3). The chain is

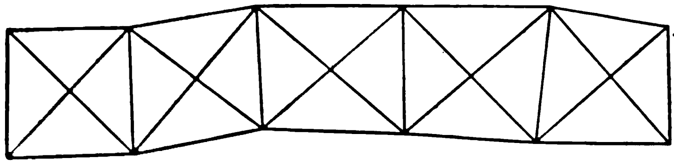

FIG. 3. CHAIN OF QUADRILATERALS.

similar to Fig. 1, with the addition of diagonal lines which are included for the purpose of giving other geometric conditions which should be satisfied by the measurements. In any extensive survey these three systems, — the triangles, the polygons, and the quadrilaterals, — are usually combined according to circumstances.

3. **PRIMARY, SECONDARY, AND TERTIARY TRIANGULATION.** — Triangulation systems covering a large area, or extending over a long distance, such, for example, as that executed by the U. S. Coast and Geodetic Survey, are divided into *primary*, *secondary*, and *tertiary* systems, depending upon the size of the triangles and the purposes for which they are to be used. In the primary system the sights vary from 10 or 15 miles up to about 150 miles, the length being determined by the topography and by the atmospheric conditions. By means of such a system a few well-distributed points, located with great accuracy, form the basis of the later work. From the sides of the primary triangles as bases a secondary system of triangles is laid out, the sides being shorter than those of the primary system, varying usually from 5 to 25 miles in length. The accuracy of the secondary system is slightly inferior to that of the primary. As the lines of this system are often too long to be conveniently used for the detail work, a smaller, or tertiary, system is developed in like manner from the secondary lines. In the tertiary system, the principal use of which is to furnish points for topographic or hydrographic detail, the points are located with regard to convenience in mapping the details rather than for accuracy, extreme precision being a minor consideration because nothing depends upon these points except topographic details. The tertiary triangles may have sides anywhere from half a mile to 5 miles in length.

The above description applies to an extensive survey, where the primary system is planned wholly with reference to accuracy and the tertiary system is laid out with reference to locating details. In comparatively small surveys it may often be advisable to omit the primary, and perhaps the secondary, system, since the tertiary system of triangles gives sufficient control.

4. **BASE-LINE.** — In any system there must be at least one line whose length is known, called the *base-line*. Many surveys consist in extending a new system of triangulation from some triangle side which has already been determined, such, for

example, as a line established by the U. S. Coast and Geodetic Survey. In such a case this line becomes the base for the new system; but if no such line can be obtained, it will be necessary to measure a long line and to connect it by means of triangles with the main system of triangulation. If the triangulation system extends over so long a distance that the accumulated error may become excessive, it is advisable to measure additional bases at intervals to serve as checks on the accuracy of the work.

5. **RECONNOISSANCE FOR TRIANGULATION SCHEME.** — The task of reconnoitering the country and planning the system is one which requires great skill, and the accuracy obtained in the final results depends largely upon the judgment used in laying out the triangulation scheme. The equipment for this work consists of light portable instruments, such as pocket compasses, aneroid barometers, heliotropes, field glasses, etc. All available data, such as sketch maps and barometric heights, should also be carried into the field by the reconnoitering party.

In new country, where little is known of the topography, it will be necessary to visit the highest elevations and to note by means of sketches the prominent mountain peaks and hills that are visible, taking their magnetic bearings, and estimating their distances, noting also which ones are in favorable positions for triangulation stations. After several such points have been examined, a fair idea of the relative position of the different hills or mountains is obtained; and from the bearings or angles measured a sketch map may be made and the triangulation scheme laid out. It should be kept in mind at all times, however, that the accuracy of the result depends largely upon the shape of the triangles. When it can be avoided, angles less than 30 degrees should not be used.

In a country which has been partly covered by triangulation, or which has been roughly surveyed, maps of a greater or less degree of accuracy can usually be obtained. These may be used as a basis for the preliminary scheme, but all lines so laid out should be carefully examined in the field before being finally adopted. Lines passing near the surface of the ground, especially near cities, are to be avoided, if possible, on account of the irregular atmospheric refraction and the delays in observing due to smoke, etc.

The reconnoitering party should also make notes in regard to condition of roads leading to the triangulation stations, and any other information which will be of use in carrying out the subsequent work.

6. Selecting Triangulation Stations. — In selecting stations the most elevated points will naturally be chosen, provided their location is such as to give well-shaped triangles; the selection of low points frequently makes necessary the construction of towers or high signals. Wherever it is practicable, such objects as church spires, standpipes, cupolas, etc. are utilized for the sake of economy. Where such points cannot be occupied with the instrument, however, they should not be used except in the tertiary system.

7. RECONNOISSANCE FOR BASE SITE. — When the positions of the principal stations have been decided upon, the location of the base-line should be chosen. In case of the extension of a system already in existence, the base may be a side of one of the triangles. If a base is to be directly measured, the location must be carefully selected with reference to convenience and accuracy, both in the measurement of the base itself and in its connection with the main triangulation. For convenience it should be on comparatively level ground, although so far as accuracy is concerned it is possible to do good work on steep grades. If better shaped triangles can be obtained by placing the base on rough ground it may be advisable to do this. Narrow gulleys will not be serious obstacles to the measurement, provided they can be spanned with the tape.

The length of the base is usually from one-sixth to one-fourth of that of the sides of the primary triangles. The ends should be located so that the base can be connected with the main scheme by a few well-shaped triangles. This system of triangles connecting the base with the triangle sides is called the *base net*, or the *expansion*. If the triangulation covers a small area, such as a town, the base may be long in comparison with the length of the triangle sides, and no elaborate base net is necessary (see plan of Baltimore triangulation, p. 258, Vol. I).

The ideal expansion is one in which the base-line crosses the triangle side as shown in Fig. 4. By this arrangement the triangles may be made more nearly equilateral. In practice, however, the arrangement of triangles is largely deter-

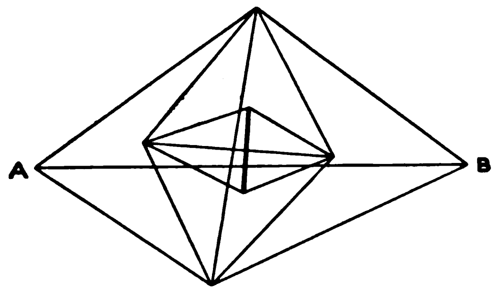

FIG. 4. BASE NET.

mined by the topography, and the ideal expansion can seldom be realized, as illustrated by the Massachusetts Base and the Fire Island Base (Fig. 5), in the U. S. Coast Survey triangulation of New England.

8. MARKING THE STATIONS. — All stations should be marked in such a manner that the points can be identified with certainty and that they are not likely to be disturbed. Triangulation stations are frequently marked by drill-holes. At important stations a copper bolt is usually driven into the hole. Where the point is not on ledge the station may be marked by a stone monument with a drill-hole in the top for a surface mark, and a circular piece of earthenware or stone placed several feet below the surface to be used in resetting the surface mark in case it is disturbed or destroyed.

In addition to the surface and sub-surface marks there should be several witness marks placed at some distance from the station and preferably near boundary walls or fences where the marks will probably not be disturbed. These often consist of drill-holes in ledge, with squares cut around them to distinguish them from the center mark of the station which has a triangle cut

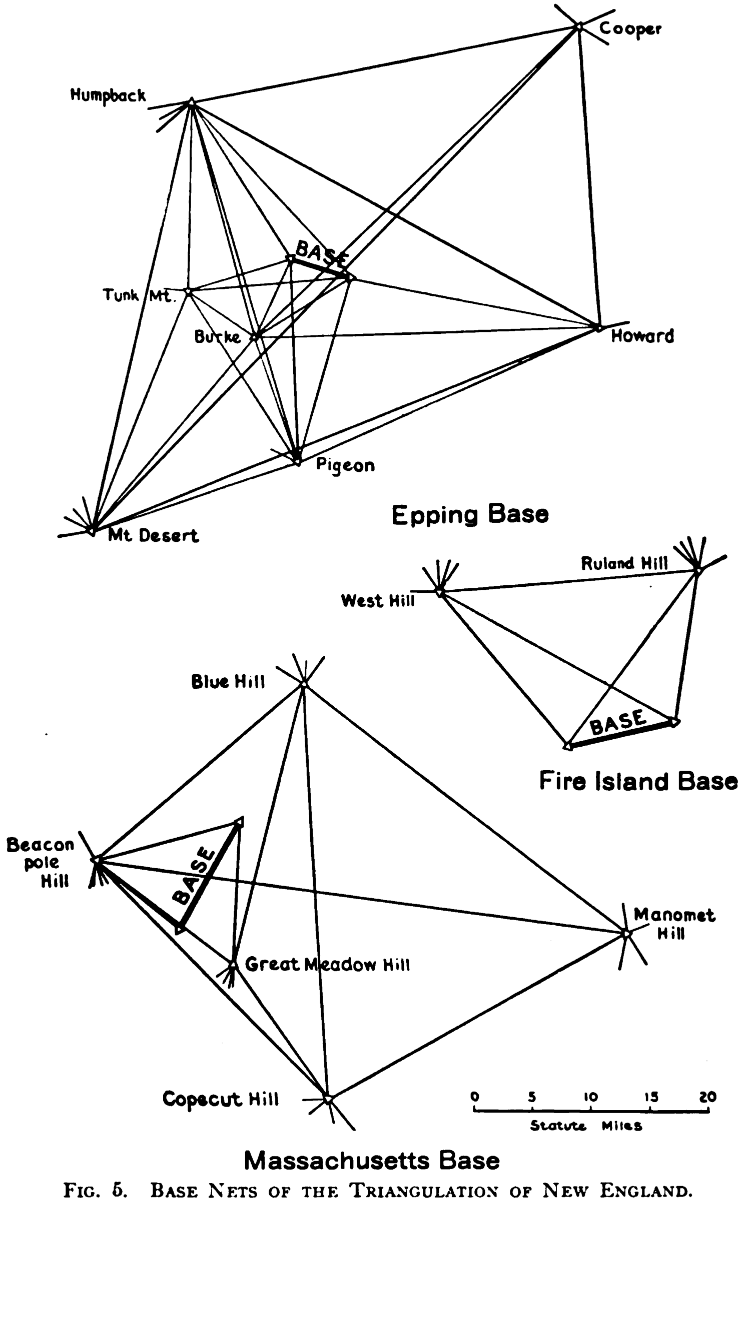

FIG. 5. BASE NETS OF THE TRIANGULATION OF NEW ENGLAND.

around it. The azimuth and distance to each witness mark are determined and recorded.

9. SIGNALS. — The kind of signal used at the triangulation station will depend upon the length of line to be sighted over, the frequency with which the station is to be occupied, the material available for building the signal, the difficulty of transportation, and the nature of the ground at the station.

10. Tripod Signal for Triangulation. — If the station is on a bare summit and the point is to be occupied frequently, as in the case of a primary or a secondary station, the tripod signal (Fig. 6) is the best. This consists of a mast 16 to 24 feet long, made of 4 by 4-inch joist, supported by three legs of about the same size as the mast, the parts being secured by six braces. Three of these braces secure the foot of the mast to the tripod legs; the other three stiffen the tripod. The mast and the tripod legs are fastened together by means of a bolt. The lower end of the mast is usually 7 or 8 feet above the ground so that the instrument used for measuring the angles can be set beneath the signal. The center of the lower end of the mast is carefully placed over the point used to mark the station, and the mast is made vertical by means of either a plumb-line or a transit. The mast may be painted with black and white strips about 2 feet long to aid in identifying the signal and in making pointings. Such a tripod signal may be used for distances up to about 10 or 15 miles, and under favorable atmospheric conditions it can be seen even farther than that.

Before placing the signal in position it will be found convenient to lay off angles of 120 degrees by means of a transit set over the station mark. In this way the positions of the tripod legs may be determined, and at the same time the directions of other stations may be examined so that none of the tripod legs will be on line to interfere with the observations. The distances out from the center mark to the foot of the tripod legs will depend upon the dimensions given to the signal. The dimensions of the signal should be worked out beforehand, on the supposition that the ground is level, and the length of one or two of the tripod legs afterward shortened, if necessary, to fit the ground. The three legs and the mast are laid on the ground and the bolt passed

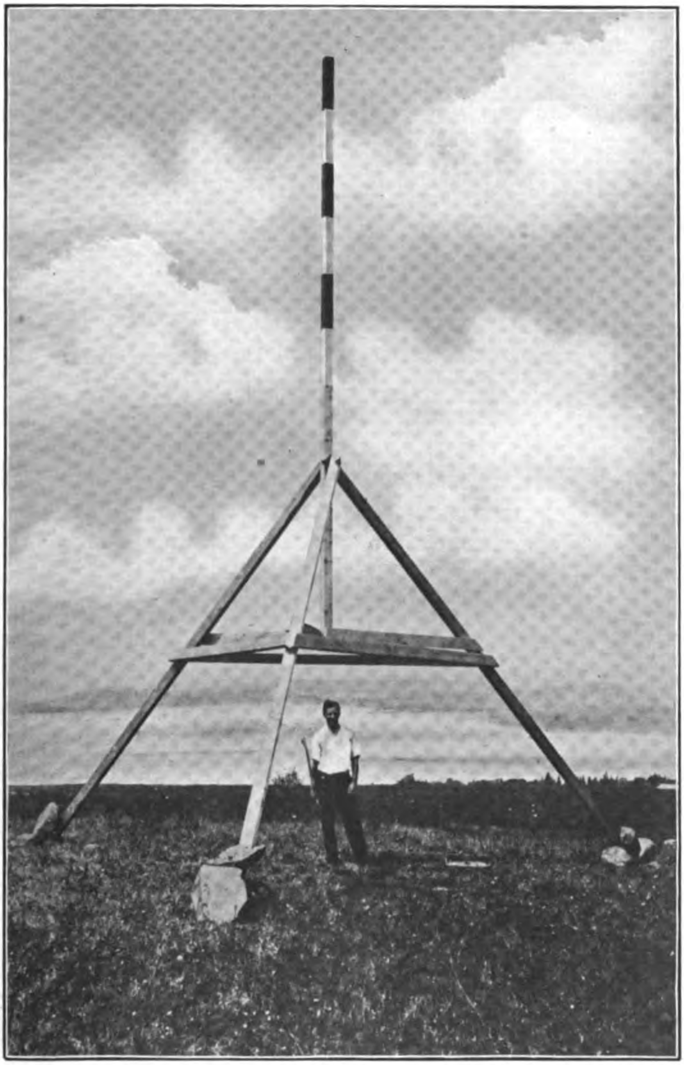

FIG. 6. TRIPOD SIGNAL FOR TRIANGULATION.

through all four at the head of the tripod as shown in Fig. 7, the lower ends of two of the legs being nearly in their final position. Before the tripod is raised the length of the mast should be measured and recorded for future reference. The head of the tripod is then raised, the feet of the two legs being kept in position while the foot of the third one drags on the ground until it is

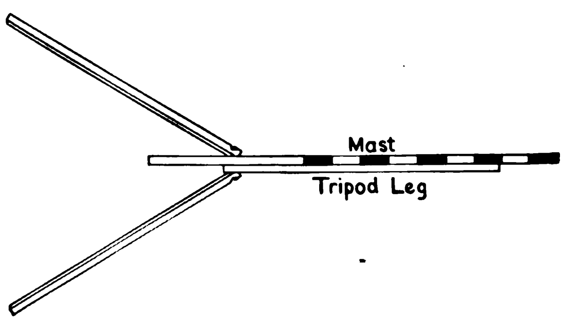

FIG. 7. PLAN OF TRIPOD SIGNAL BEFORE BEING RAISED.

brought into its proper place, the top of the mast still resting on the ground. The mast is then brought into position by means of a rope, which is tied to the foot of the mast when the signal is lying on the ground, and by pulling down on it so as to rotate the mast about the bolt, as shown in Fig. 8. When it is nearly vertical the foot of the mast is secured to one of the legs by means of a short brace. Before the other braces are finally nailed in place the mast is carefully plumbed and centered over the station mark. After the braces are nailed on, it should be tested to see that it is in the correct position. The final plumbing is accomplished by shortening one or two of the tripod legs or by digging away the earth under them.

The tripod should be secured to the ground by some means so that the wind will not move or overturn it, for signals are usually built in exposed places and are subjected to very great strains in high winds. If the station is on ledge the legs may be secured by means of anchor bolts driven into drill-holes in the rock. If the lower ends of the bolts are split and small wedges inserted, the bolts when driven in will spread and hold securely. If the station is on soil or gravel, cross-pieces may be nailed to the tripod legs at the ground and weighted down by piles of stone.

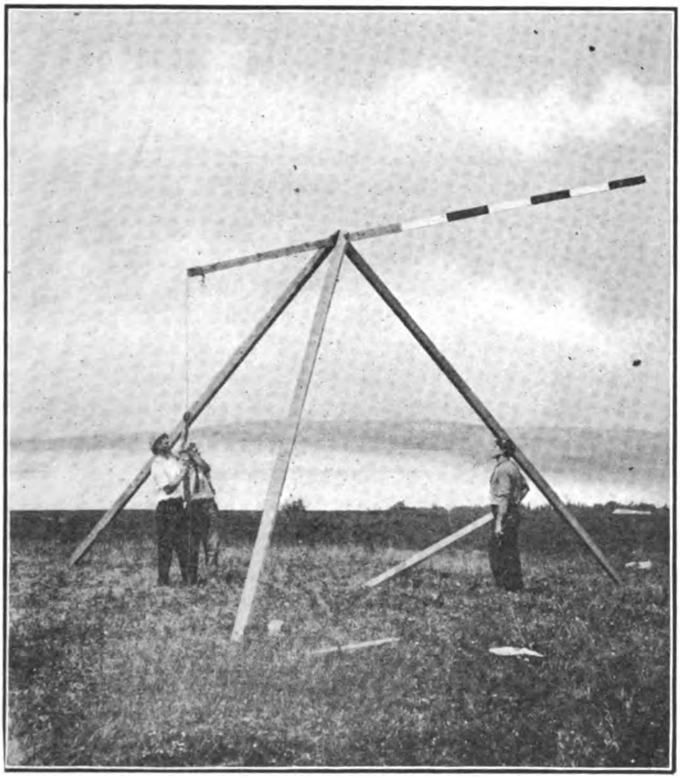

Fig. 8. Raising the Mast of a Tripod Signal.

The 4 by 4-inch mast may be extended upward and also increased in width without greatly adding to the weight by nailing on four 1 by 5-inch boards, making the mast 6 by 6 inches. These strips may be extended above the central piece, thus forming a hollow box which will considerably increase the height with only a slight addition to the weight of the mast.

FIG. 9.

Another device for increasing the width of the signal, used on moderately long sights, is as follows. Alternate black and white cloth squares stretched on frames are nailed to a strip of wood and fastened to the mast on the side toward the observer, as shown in Fig. 9. This can be sighted upon from only one direction at a time and is used only when there is an assistant at the signal, who, upon receiving a message from the observer, places these squares temporarily on the mast, removing them when the angles are finished.

Where lumber is not available a good signal may be made from green poles cut near the station, the mast being bolted to the legs as already described.

11. **Tripod Signal for Topographical Surveying.** — When the signal is to be of a temporary character, as in the case of topographical work, it may be made of green poles and built in the same manner as the triangulation signal, except that all the pieces are nailed together (Fig. 10). The foot of the mast is placed high enough so that the instrument can be set up underneath.

In building this signal the mast is laid on the ground and the top of one of the tripod legs is spiked to it 5 or 6 feet above the butt of the mast, the angle between the two being about 30 degrees. A short brace is then nailed to the foot of the mast and to the tripod leg so as to form a right angle at the foot of the mast. The second leg is raised and held in position while it is spiked to the mast and braced in the same way as the first one. The short braces should all be of the same length. A long brace is then nailed to the two tripod legs at or near the points where the short braces are fastened. This brace should be of such a length that the angle between the short braces is 120 degrees. The signal is then turned over and moved until the butt ends of the two legs rest nearly in the positions they are finally to occupy, the signal resting on three points, namely, the top of the mast and the ends of the two tripod legs. The third leg is then held in position overhead by two men, while it is nailed and braced. Before the signal is raised into position the flags should be tacked to the mast and the dimensions of the signal taken and recorded. The mast may be made more conspicuous by peeling off the bark.

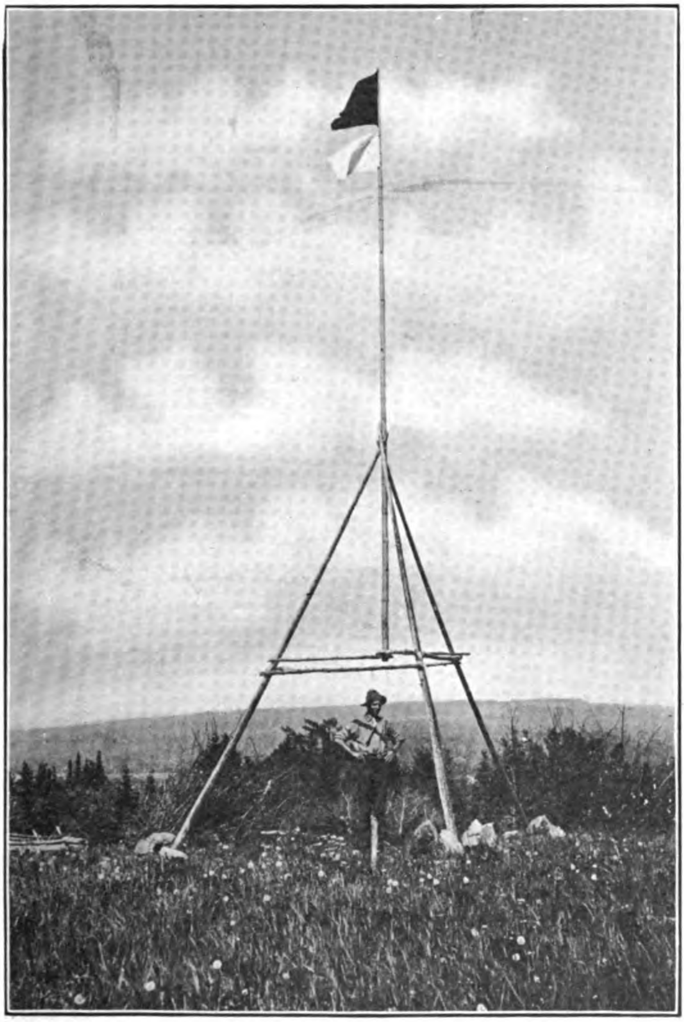

FIG. 10. TRIPOD SIGNAL FOR TOPOGRAPHICAL SURVEYING.

The signal is raised by pushing up on the mast or the tripod, with the aid of poles, until the signal is just balanced on the two legs. While it is in this position one man can reach the third leg and steady the tripod until the rest of the party can assist in lowering it to the ground. The signal is then centered over the point and plumbed, and the tripod should be secured by piling stone on cross-braces at the foot of the tripod.

FIG. 11. BRACED MAST SIGNAL.

12. Braced Mast Signal. — For very short lines, and especially where the station is to be seldom occupied, the mast may be set directly on the surface and held in position by three or four braces as shown in Fig. 11. A flag on the top of the mast is frequently useful in identifying the signal, especially when such objects as white birch trees might easily be mistaken for the mast of the signal. The braces should be secured by stakes driven into the ground and nailed to the tripod legs, or by horizontal pieces weighted with stones. The disadvantage of this kind of signal is that when the station is to be occupied with an instrument the signal must be moved, or else the transit set at one side which necessitates additional calculations. (See Art. 46, p. 44.)

A simple form of signal suitable for short lines is a 3 by 3-inch mast braced by three wire guys from the top. A spike is driven into the center of the foot of the mast and is set into the drill-hole, or in a hole in the top of the stake marking the station, to keep it in place. A black or white band is placed near the top of the mast to sight on, and a flag at the top to identify the signal. This signal can be lifted out of the drill-hole and laid down so that the instrument can be set over the station without detaching the guys. The mast can be quickly put back into position after the angles at the station have been taken.

13. **Guyed Mast Signal.** — When the hill is wooded or the signal is to be set in a low place surrounded by trees, a high signal may be erected by splicing two or three poles and bracing this mast in position by wire guys, several sets being used to secure the mast at different points (Fig. 12). A definite point near the top of the mast is selected as the point to be sighted on, and is marked by black and white cloth bands. A set of guys is secured at this point to keep it from swaying, and this portion of the mast is brought vertically over the station mark. The rest of the pole need not be over the station, and, in fact, it is more convenient to have the foot of the mast a little to one side of the station mark so that the instrument can be set up without disturbing the signal. A flag should be placed at the top to aid in finding the signal when observing on it from other stations.

14. **Observing Towers.** — In flat country towers are necessary on account of the curvature of the earth's surface or of obstructions such as woods. The offset from the tangent to the curve, due to curvature and refraction combined, is about 0.57 feet for

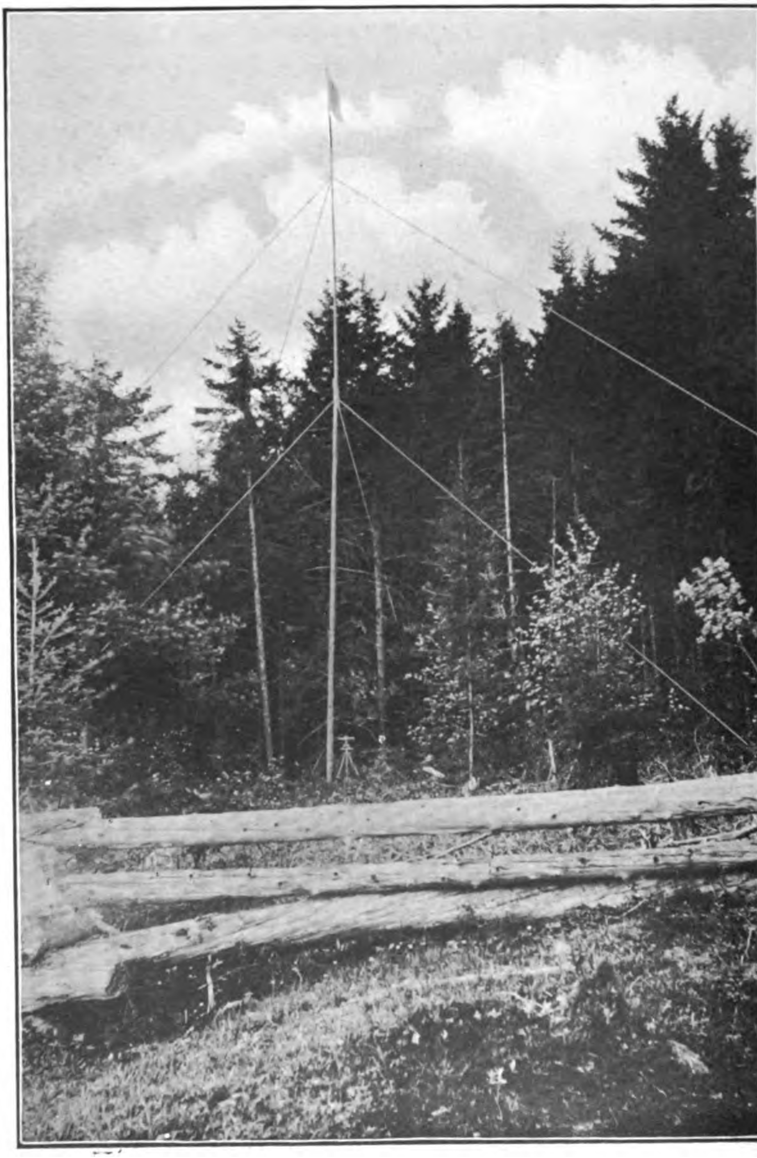

FIG. 12. GUYED MAST.

a point a mile away and varies as the square of the distance. (This offset is given in Table I, p. 387.) Hence in perfectly level country if towers 57 feet high are erected at each end of a line 20 miles long, the line of sight between the tops of the two towers will just clear the intervening surface. In order to avoid the great atmospheric refraction near the surface, as well as obstructions to the line of sight, it would be necessary in practice to make the towers several feet higher than this.

Such a tower usually consists of an inner tripod to support the instrument and an outer stand for the observer (Fig. 13). The two are built **entirely disconnected** so that the observer will not disturb the instrument as he moves about on the observing stand. If the tower is high the outer structure is secured by wire guys.

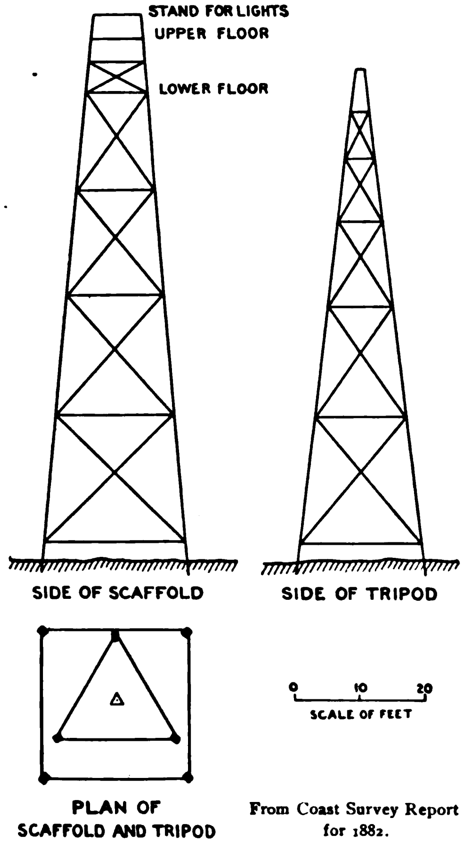

FIG. 13. OBSERVING TOWER.

15. DESCRIPTION OF STATION. — When a station is established or a signal built, a written description of the station and signal should be recorded in the note-book, accompanied by any sketches which will aid in identifying the point. The description of the station should be very complete and include such data as the name of the state, town, railroad station, name of hill, name of property owner, etc.; a sketch should be made indicating the best way to reach the point, and also showing details around the station; the method of marking the point should also be described in detail. Magnetic bearings of the other triangulation stations, as well as prominent objects near by, such as church spires, should be noted. The descriptions of witness marks together with their azimuths and distances should also be given. The dimensions of the signal, and the width of the black and white bands, should be given in the description; also the total height of the signal and the height of the foot of the mast above the center mark, in order that vertical angles taken from other stations to any point on the mast may be reduced to the center mark.

16. HELIOTROPES. — On lines which are much longer than ten or fifteen miles tripod signals are not practicable. In such cases the position of the point is shown by means of sunlight reflected by an instrument called a *heliotrope*, which consists of a plane mirror and some device for pointing the mirror so that the light may be seen at the distant station. Heliotropes are used not only to show the position of the station but also for the purpose of sending messages between the observer and the heliotroper. For these messages a simple code of flashes is improvised, as there will in general be but few messages required in this work, such as increasing and diminishing the light, name of next station to occupy, signal that the work is completed, etc.

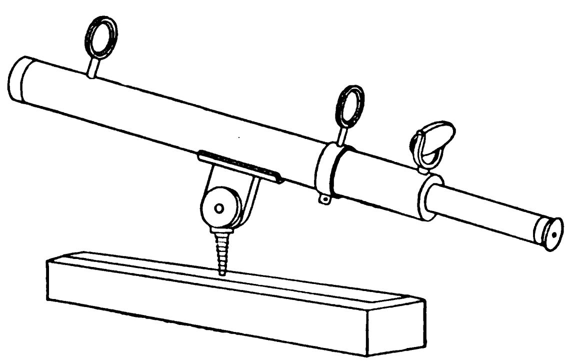

FIG. 14. TELESCOPIC HELIOTROPE.

17. Telescopic Heliotrope. — One common form of heliotrope (Fig. 14) consists of a telescope which has two rings of nearly equal diameter attached to the top of it, with a mirror mounted behind these rings so that it can be moved both in altitude and in azimuth. In using this heliotrope the axis of the rings is pointed in the direction of the station and the mirror turned so that the shadow of the rear ring as seen on the front ring is concentric with the front ring. As the axis of the rings may not be parallel to the line of sight of the telescope there must be some means of pointing this axis accurately toward the distant station. The parallelism of the line of sight of the telescope and the axis of the rings may be tested by sighting the telescope carefully at some nearby point on a vertical surface and then throwing the light through the rings and noting the position of the point with reference to the ring of light. Allowance must be made for the distance between the axis of the rings and the axis of the telescope. The rings may be adjusted or the error may be allowed for simply by estimation. The rings may be pointed, however, with sufficient accuracy without the use of the telescope by marking the centers of the rings with threads, like cross-hairs, and pointing directly by means of these.

18. Improvised Heliotrope. — A serviceable heliotrope may be made by constructing a wooden apparatus similar to that shown in Fig. 15, having two holes bored in the upright pieces.

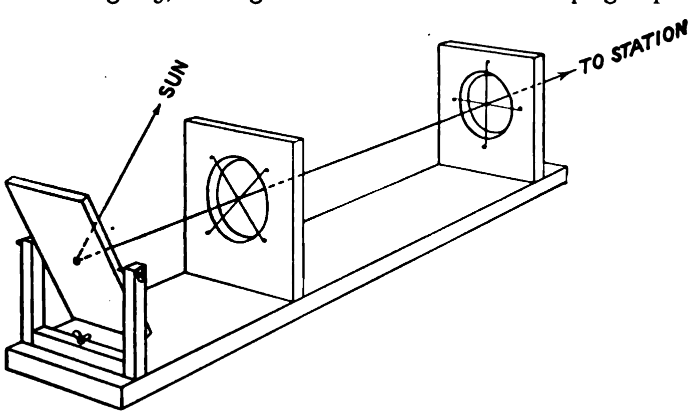

FIG. 15. IMPROVISED HELIOTROPE.

The rear hole should be slightly larger than the front one. Threads are stretched at right angles across the holes to mark their centers. For reflecting the sunlight any ordinary mirror so mounted that it can be moved in altitude and in azimuth will serve the purpose. Such a heliotrope has proved satisfactory on lines 25 miles or more in length.

19. Steinheil Heliotrope.—The Steinheil heliotrope (Fig. 16) is a very convenient instrument on account of its small size

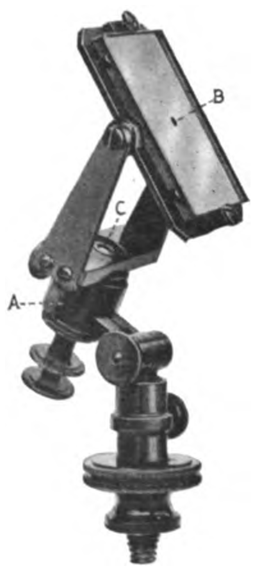

FIG. 16. STEINHEIL HELIOTROPE.

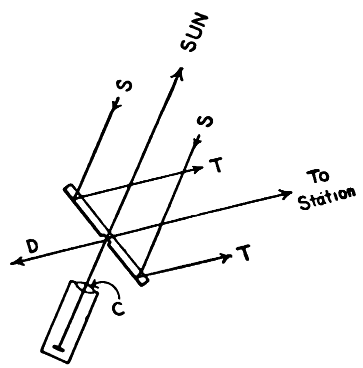

FIG. 17. DIAGRAM OF STEINHEIL HELIOTROPE.

and also because it cannot get out of adjustment. It is particularly useful for reconnoissance work; for the regular triangulation, however, many prefer the more substantial forms of heliotrope. It consists of a mirror, the two faces of which have been made parallel, mounted on a small frame having four different motions. At the center of the mirror a small portion of the silver is removed so that light can pass through the glass. In the frame below the mirror is a tube carrying a small biconvex lens and inside the tube at the focus of the lens is a white surface of chalk or plaster. In operating the instrument the helio-troper turns the tube *A* toward the sun so that sunlight can shine down into the tube. The exact position is shown when the circle of light which shines through the opening in the mirror at *B* just fills a circular opening at *C*. The instrument is clamped in this position. If the heliotroper then turns the mirror so that he can look through the opening *B*, from the back, he will see an image of the sun.* When the mirror is turned so that this image of the sun appears to cover the distant station, then the observer at that station can see the light from the mirror. The reason for this will be apparent from Fig. 17.

The light from the sun *S* is reflected from the silvered surface of the mirror and proceeds to the station as shown at *T*. At the center of the mirror where the silver is removed the light passes down through the lens at *C* and is reflected at the plaster surface back through the lens.† When it strikes the back surface of the mirror it is reflected toward the heliotroper at *D*. Since these rays to *D* are reflected from the same surface as the rays to *T* which reach the distant station, and since the faces of the mirror are parallel, the rays *D* and *T* must be parallel. Hence when the sun's image appears to cover the station the reflected rays must be pointed in that direction.

20. All heliotropes are provided with a second mirror which is used when the angle between the sun and the distant station is so great that satisfactory pointings cannot be made with a single mirror, as is the case when the sun is nearly behind the heliotroper. The second mirror is mounted on an independent support and in such a position that light can be thrown on the first mirror.

The angle of the cone of rays sent from the mirror to the distant station is equal to the sun's apparent diameter, which is about 32 minutes. Hence it is not necessary to point the heliotrope with great precision; if it is pointed a few minutes to one

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* If the lens is a plano-convex (rather than a bi-convex), as is the case in some instruments, there will be a bright reflection from the flat surface of the lens.. This causes a second image of the sun brighter than the first and quite near to it. The heliotroper, however, can always tell which image to use by holding some surface in front of the mirror and noting which image appears in the center of the bright spot on this surface. The other image, which is not used, will ordinarily appear to fall on the surface at some distance from the flash of light.

† The rays reflected from the plaster surface are necessarily parallel to the rays entering the tube, because both in entering and in emerging from the tube these rays are parallel to the line determined by the optical center of the lens and the image on the plaster surface.

side of the true direction sufficient light will still reach the other station.

There should be some means provided for varying the apparent size of the mirror in order that the flash may always appear to the observer like a fine point of light, not like a large blur. This can be effected by placing diaphragms, made of sheet iron or of pasteboard, in front of the mirror, the size of the diaphragm used depending upon the length of the sight.

21. **Signals for Observing at Night.*** — Angle measurements at night have been carried on by observers of the U. S. Coast Survey, using acetylene lamps in place of heliotropes for signals. On account of the favorable condition of the atmosphere and the fact that the light is visible under nearly all weather conditions this method is both economical and accurate. The acetylene light is particularly adapted to this work because it always appears as a bright point and does not appear blurred as is the case with ordinary lights.

22. **INSTRUMENTS FOR MEASURING BASES.** — Various forms of bar apparatus have been used for base-line work of the highest accuracy, and until a few years ago such apparatus furnished the only means of making these measurements.† Experiments have shown, however, that base-lines can be measured with great accuracy by the use of the steel tape if the proper precautions are taken to determine the errors due to various causes, such as the error of the length of the tape on the U. S. Standard, and the variations in length arising from changes in temperature.

23. **U. S. Coast Survey Steel Tape Apparatus.** — The tape apparatus commonly used by the U. S. Coast and Geodetic Survey consists of a 100-meter steel tape of a cross-section of $6.34^{mm} \times 0.47^{mm}$.‡ The tape is supported upon stakes or tripods every 25 meters (sometimes every 10 or 20 meters).

The tension is given to the tape by means of a spring balance

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* See Coast Survey Report for 1903, p. 824.

† For a description of various forms of base apparatus see U. S. Coast and Geodetic Survey reports for 1852, 1856, 1880, 1882, 1892, 1897, and 1901.

‡ In the base-line work on the 98th Meridian in 1900, 50-meter tapes were also used and found to give fully as satisfactory results as the 100-meter tape.

(Fig. 18) which is secured to a vertical bar by a gimbal joint. The latter can be raised or lowered by means of a wheel-nut which works on a screw-thread on the bar. The lower end of the bar is connected by a universal joint to a small platform on which the operator stands. The balance can be moved quickly either vertically or sidewise in order to place the tape in the proper position with respect to line and grade. A counterpoise is added so that the apparatus may be perfectly balanced before the tape is attached.

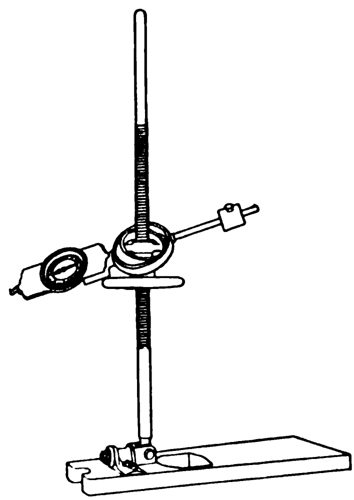

FIG. 18. U. S. COAST AND GEODETIC SURVEY TENSION APPARATUS.

The temperature is determined by means of thermometers tied to the tape. The intermediate supports usually consist of stakes with wire nails driven at the proper heights. These stakes are carefully aligned by means of either a telescope or ordinary field-glasses. The end points of each tape-length are marked by lines scratched on metal plates placed either on heavy stakes or on tripods. The chief difficulty in using such an apparatus is to determine accurately the temperature of the tape. In bright sunlight the temperature indicated by the thermometers is not the same as that of the tape, and for this reason base measurements with such apparatus are usually made at night or in a drizzling rain, because the tape then assumes practically the same temperature as the air.

24. In the spring of 1906 the Coast and Geodetic Survey conducted a series of base-line measurements, using tapes of nickel-steel alloy, called "Invar" tapes. These tapes have a coefficient of expansion of about .000 000 4 per 1 degree C. or roughly $\frac{1}{25}$ that of steel. The advantage of this tape is that comparatively large errors in the temperature determination produce small errors in the computed length. The results of these tests indicate that more accurate measurements can be made with the "Invar" tapes than with steel tapes, although the former have to be handled more carefully than the latter because the alloy is soft and the tape is easily bent. A report on these measurements will be found in the Coast Survey Report for 1907, Appendix No. 4.

25. Massachusetts Institute of Technology Tape Apparatus. — A modified form of the steel tape apparatus described above has been used in an experimental way at the Massachusetts Institute of Technology. This apparatus consists of a 100-meter steel tape supported on stakes every 10 meters, together with a special tension apparatus and a special instrument for measuring the temperature of the tape, which is an electrical appliance known as the *thermophone*, invented by Messrs. Geo. C. Whipple and Henry E. Warren.* The thermophone is a form of Wheatstone's bridge, in which the measuring tape and a German silver wire form two arms of the bridge, while the remainder of the apparatus is enclosed in a small box of convenient size and shape to carry in the field.

Fig. 19 shows a diagram of Wheatstone's bridge. The

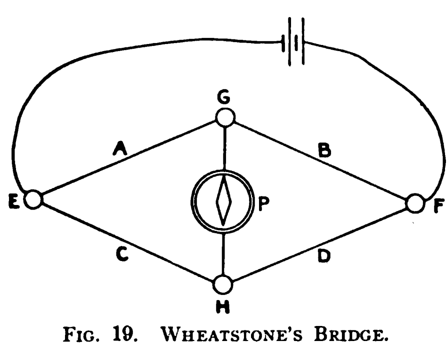

FIG. 19. WHEATSTONE'S BRIDGE.

current flowing from the battery divides at E, a portion passing through A and B, and the remainder through C and D. These four parts, A, B, C, and D, are known as the "arms" of the

* For a description of this instrument by the inventors, see *Technology Quarterly*, 1895, Vol. VIII, p. 125, published by Massachusetts Institute of Technology, Boston, Mass.

bridge. If the ratio of the resistances in A and B equals the ratio in C and D, then G and H are at the same potential, and a galvanometer P placed between G and H will show no deflection.

In the thermophone apparatus the steel tape and the German silver wire form two of the arms A' and B' (Fig. 20), while

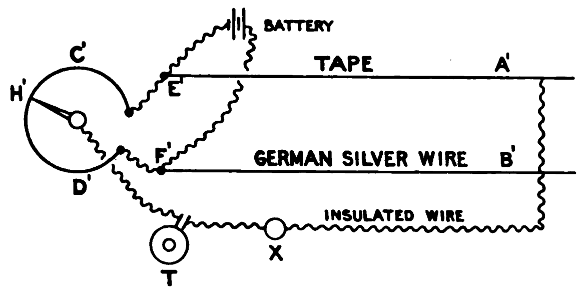

FIG. 20. DIAGRAM OF THERMOPHONE.

the other two arms are the two segments of a piece of metal C' and D' bent into an arc of a circle in contact with a metal bar at H'. This bar can be moved about a central pivot so as to subdivide the arc into the two segments, C' and D', whose resistances shall have any desired ratio. The galvanometer of the ordinary bridge is replaced in this instrument by a circuit breaker X and a telephone T so that any current due to the circuits not being balanced is indicated by a sound in the telephone receiver. The current is sent through the apparatus from a small battery of dry cells in the thermophone box.

The tape, the German silver wire, and an ordinary insulated wire are all connected at the zero end of the tape. The thermophone is at the reading end of the tape and is connected with the tape and the two wires. The tape and the German silver wire are hung in S-shaped hooks covered with rubber tubing for insulation, the German silver wire being suspended a few inches below the tape on the same supporting stakes. The arm which is shown at H' is connected with a pointer which moves around the face of a dial. The face of the dial has a temperature scale and also a scale of temperature corrections for a tape-length, expressed in millimeters.

The process of obtaining the temperature of the tape is as follows. The thermophone is connected with the tape and wires and the batteries switched in so that a current passes through the tape. By listening in the telephone receiver the operator will hear a buzzing sound, unless the resistances happen to be exactly balanced. By moving the pointer around the dial a position can always be found where the sound will cease. The temperature then indicated on the dial is the temperature of the tape, and on the scale opposite the temperature reading is the number of millimeters to be added or subtracted. If the temperature of the tape were to change it would be found necessary to move the pointer to a different position in order to balance the circuits, i.e., the **ratio of the resistances varies with the temperature,** so that this ratio becomes the measure of the temperature of the tape and wire. In the usual arrangement of the thermophone, temperatures can be read to about a tenth of a degree Fahrenheit. The temperature thus found is the mean temperature of the entire tape.

The idea of measuring the temperature by electrical means was first suggested by Mr. George A. Campbell in 1893, and experiments were then made with the ordinary Wheatstone's bridge. Soon after the invention of the thermophone Professor Alfred E. Burton began a series of experiments to determine whether this instrument was suitable for base-line work. The results of these tests indicated that the temperature of the tape could be determined by the thermophone with all the accuracy required in base-line measurements.

The tension apparatus used with this tape, which was designed by Mr. H. C. Bradley in 1891, consists of a metal frame having the form of a cross (Fig. 21), at the center of which is a knife-edge resting in an angle of a small supporting frame clamped to a vertical iron bar. This frame has a joint which allows the knife-edge to be leveled, and by means of the clamp the whole apparatus may be raised or lowered and the knife-edge can be turned squarely across the line. The cross is provided with counterpoises for balancing it when the tape is not attached. The tape is attached to a cord which passes over a groove in the top arm and is fastened underneath to a ratchet wheel for taking up the slack. The weight is attached to another cord which passes over a groove in the horizontal arm. These grooves are

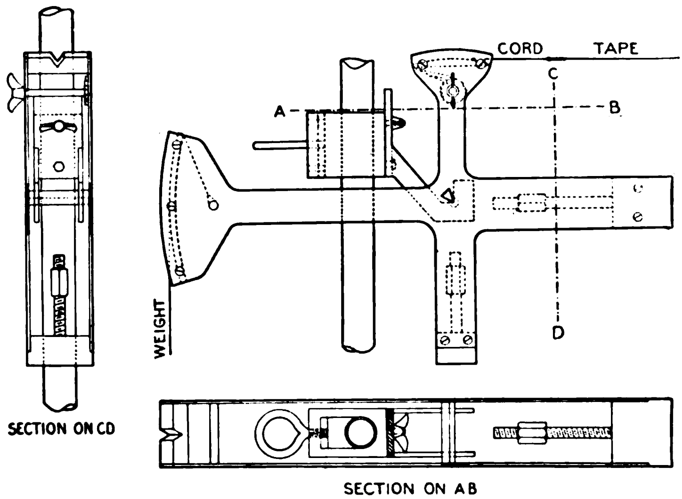

FIG. 21. BRADLEY'S TENSION APPARATUS.

arcs of circles whose centers are at the contact of the knife-edge; hence any change in the length of the tape does not change the length of these two arms and the tension is therefore constant. The lengths of the two arms have a ratio of 2 to 1, so that the tension on the tape is twice the weight applied. The weight is adjusted to give the *normal tension* for this tape when supported every 10 meters. (Art. 32, p. 33.)

Such a tape apparatus as has been described was first constructed at the Massachusetts Institute of Technology as the result of several years' experimenting in base-line measurement at the Summer School of Geodesy and Topography. In 1900 this apparatus was tested on the Alice base-line in Texas by officers of the Coast Survey and was reported upon favorably by Mr. J. F. Hayford, Inspector of Geodetic Work. (See Report of U. S. Coast and Geodetic Survey for 1901.)

26. MEASURING THE BASE.—The process of measuring a base with the tape consists in first clearing out the line and setting measuring stakes in line one tape-length apart. The line is then profiled in order to find the grade of each tape-length. The tape is supported at regular intervals (10 or 20 meters) on small stakes (lined in either by a transit or by a field-glass) by means of hooks hung on nails driven into the sides of the stakes so that the points of support are on a uniform grade. The zero end of the tape is then placed in position and the tension applied. Finally the readings are taken, the scale readings on the tape and the temperature readings being taken simultaneously. As soon as these have been completed the tape is carried forward, set up again, and the process repeated until the end of the line is reached. The intermediate points are marked by means of fine lines scratched on strips of zinc tacked to the tops of the measuring stakes. On all important base-lines the end points should be permanently marked by copper bolts set in stone monuments. The transfer of the end measurements from the tape to the end mark, or *vice versa*, should be made by means of a special apparatus called a *cut-off cylinder*,* or by using a transit in two positions at right angles to each other and transferring the mark vertically downward to the permanent end mark. At least two independent measurements of the length of the base should be made.

The accuracy required in the best base-line work may be set at about one part in 500 000, this being as precise as is necessary to make the base-line measurements consistent with the accuracy which can be obtained in the best angular measurements.

27. CORRECTIONS TO BASE-LINE MEASUREMENTS. — Correction for Slope. — In order to reduce the slope distance to the horizontal distance a correction must be subtracted depending upon the rate of grade. For flat grades it is customary to determine the difference in elevation by leveling. The correction may be conveniently found by the expression $-\frac{h^2}{2L}$, where h is the difference in elevation of the ends of the tape and L is the length of the tape. (See foot-note, Vol. I, p. 13.) If the grade exceeds

* For a description of the cut-off cylinder see U. S. Coast and Geodetic Survey Report for the year 1892, p. 343; also Professional Papers, Corps of Engineers, U. S. Army, No. 24.

about 3 per cent the following more accurate formula for the correction is used, $-\left(\frac{h^2}{2L}+\frac{h^4}{8L^3}\right)$. In case of very steep grades the correction may best be found in terms of the vertical angle α, by the expression $-2L\sin^2\frac{1}{2}\alpha$.

28. Temperature Correction. — The correction for temperature is made by adding to the length of the tape the quantity $100^m \times k \times (t_1^\circ - t_0^\circ)$ where k is the coefficient of expansion of the tape; t_0° is the temperature at which the tape is of standard length and t_1° the observed temperature.

29. Reduction to Sea Level. — In order that the lines of the triangulation shall refer to the surface of the earth at sea level the lines will all require a slight reduction. Since all other lines are derived from the base by calculation, all the lines will be reduced to sea level without further correction if the base itself is first reduced. The approximate formula ordinarily used for this reduction is $-B\frac{h}{R}$, which may be readily deduced from the fact that in a sector of a given angle, arcs are proportional to the radii. B is the measured length, h the altitude above the sea level obtained by leveling, and R the radius of the earth at the point in question. [An average value for $\log R$ (in feet) $= 7.32068$; $\log R$ (in meters) $= 6.80470$.]

30. Correction for Sag. — The tape hangs between the points of support in a curve known as the catenary, and the distance between supports is less than the length of tape by a quantity called the correction for sag. The shortening of the tape due to sag may be found from the expression $\frac{1}{24} \times n \times \frac{w^2}{t^2} \times l^3$ *

where l = the length of a span between points of support
n = the number of spans
w = the weight of a unit length of tape
t = the tension given to the tape

The quantities w and t must be expressed in the same units.

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* For a discussion of the theory of steel tapes see a paper by Professor R. S. Woodward, U. S. Coast and Geodetic Survey Report for 1892, p. 480.

The expression may be put in the form $\frac{L}{24}\left(\frac{wl}{t}\right)^2$, where L is the length of the tape.

This correction may be derived as follows. Since the supports in this work are not far apart the sag of the tape is small and the curve may therefore be safely regarded as a parabola. If we consider the forces acting on a section of the tape at a point half way between supports, and take moments about the point of support we have

$$\frac{wl}{2} \cdot \frac{l}{4} = tv$$

where v is the sag of the middle point of this section of the tape below the supports.

Hence $$v = \frac{wl^2}{8t} \qquad (a)$$

The length of the parabola expressed as a series is

$$P = l\left(1 + \frac{8}{3} \cdot \frac{v^2}{l^2} + \cdots\right) \qquad (b)$$

the remaining terms being negligible.

Substituting in (b) the value of v obtained from (a) and computing $P - l$, the shortening of this section due to sag, we obtain

$$P - l = \frac{l}{24}\left(\frac{wl}{t}\right)^2$$

and, since $nl = L$

$$C_s = \frac{L}{24}\left(\frac{wl}{t}\right)^2$$

where C_s is the correction for sag for the whole tape.

31. Tension. — The increase in the length of the tape, above its unstretched length, when a tension t is applied may be computed by the equation

$$C_p = \frac{Lt}{SE}$$

in which S = the cross-section of the tape and E = the modulus of elasticity of the tape.

The increase in length due to an increase in tension is found by the formula

$$\Delta L = \frac{L \, . \, \Delta t}{S \, . \, E} + \frac{1}{12}\left(\frac{w}{t}\right)^2 n \, . \, l^3 \frac{\Delta t}{t}$$

in which Δt = the increase in tension.

32. Normal Tension. — By equating the expressions for sag and increase in length a value of t can be found at which these two corrections just neutralize each other. Equating and solving for t we have

$$t_n = \sqrt[3]{\frac{SE}{24}(wl)^2}$$

where t_n is the tension at which these corrections are equal, and is called the ***normal tension.*** By far the most accurate way of determining the length of the tape, however, is to compare it in the field with a standard of length which has been determined by means of standard bars, the tape apparatus being used under exactly the same conditions which are to exist in the field, thus eliminating all uncertainties in these corrections.

33. INSTRUMENTS FOR MEASURING HORIZONTAL ANGLES.— There are two types of instrument used in triangulation work for measuring horizontal angles, the *repeating theodolite* and the *direction instrument.* The former is constructed on the same principles as the ordinary transit. The direction instrument is so constructed that the repetition method of measuring angles cannot be used, the directions of the various signals being read on the circle by means of micrometer microscopes. The diameter of the circles of triangulation instruments varies from 6 inches to 30 inches according to the precision with which angles are to be taken. It is found, however, that there is little advantage in using very large circles, those of 10 or 12 inches in diameter giving satisfactory results. Instruments designed for triangulation or astronomical work usually have three leveling screws instead of four as in the ordinary transit. An instrument supported on three points has greater stability than one resting on four points and is less liable to strains due to changes in temperature and to other causes.

34. REPEATING INSTRUMENT.—With the repeating instrument the angle is read on two opposite verniers either to 5 or to 10 seconds, the accuracy of the measurements being increased by repeating the angles. Such an instrument is shown in Fig. 22.

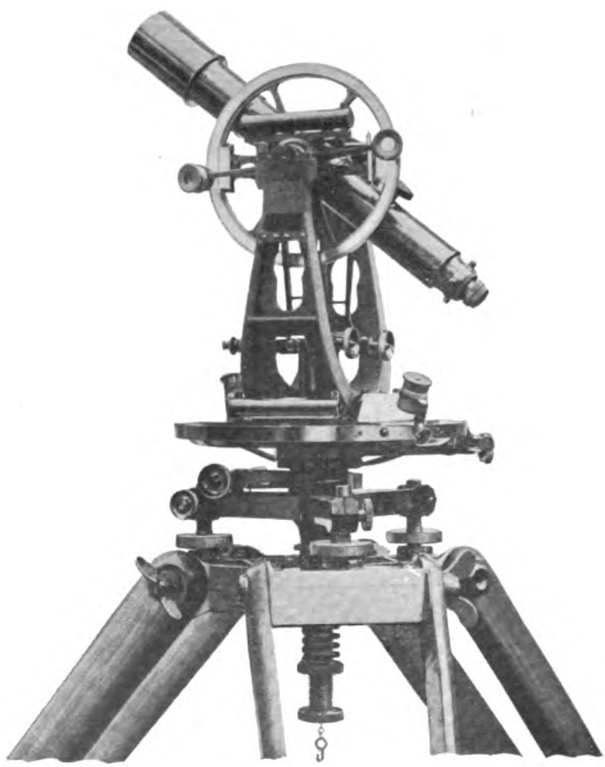

FIG. 22. REPEATING INSTRUMENT.

In all its essential parts it is like the ordinary engineer's transit, excepting that like all other triangulation instruments it has an extra heavy tripod with a broad head. The leveling screws rest in radial slots in the tripod head and the instrument is secured to the tripod by means of a central clamp and spring. A striding

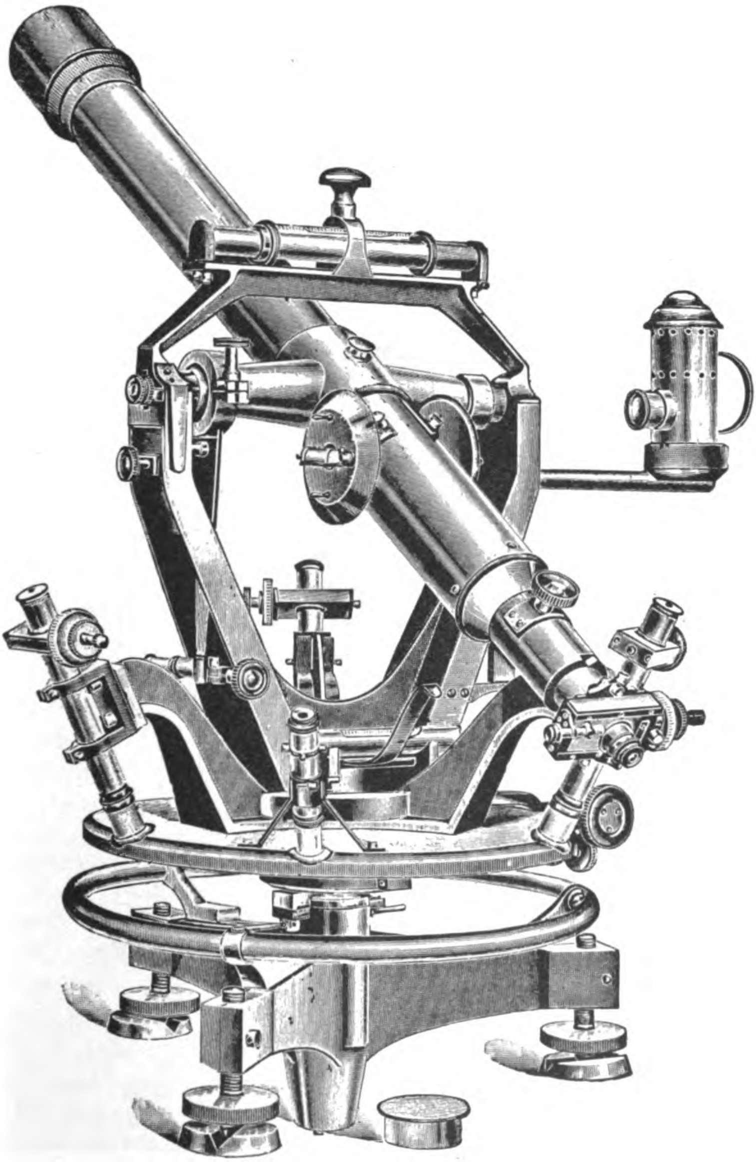

FIG. 23. DIRECTION INSTRUMENT.
(From the catalogue of Bausch & Lomb Optical Co., by permission.)

level is usually provided with the instrument for leveling the horizontal axis in sighting at high altitudes or in astronomical work.

35. DIRECTION INSTRUMENT. — The direction instrument (Fig. 23) has a circle graduated usually to 5-minute divisions. This circle can be turned and clamped in any desired position, independently of the upper portion of the instrument. On the frame supporting the telescope is a set of micrometer microscopes (generally two or three), placed at equal distances apart, by means of which the fractional parts of the 5-minute spaces are read.

36. The Micrometer Microscope. — In the field of each microscope two or more of the graduations of the circle can be seen (Fig. 24). In the focal plane of the eyepiece of each micro-

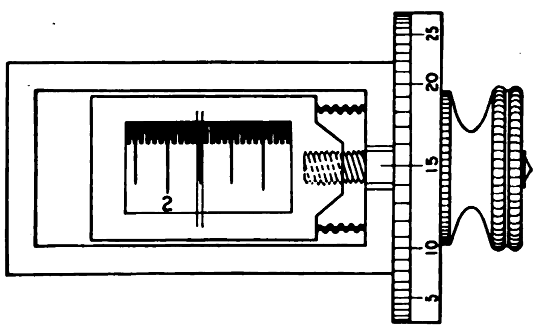

FIG. 24. MICROMETER MICROSCOPE.

scope is a micrometer screw which moves a frame carrying a set of cross-hairs. These are either in the form of an X or else consist of two parallel hairs set far enough apart so that a line of the graduation will not quite fill the space between them. The pitch of the screw and the focal length of the objective of the microscope are so related that some whole number of turns equals one space on the graduated circle. The head of the micrometer is graduated so that angles can be read to seconds or to fractions of a second. In the field of view is a notched scale for counting the whole turns of the screw. In the center of the scale is a deeper notch which represents the zero point of the micrometer readings. For any pointing of the telescope the direction is read by simply turning the micrometer until the cross-hairs are placed symmetrical with respect to the preceding line of the graduation, reading the micrometer screw, and adding this reading to the direct reading of the degrees and minutes on the circle. The other microscopes are read to increase the accuracy and to eliminate eccentricity.

37. **The Run of the Micrometer.** — Since the micrometer screw makes a whole number of turns in moving from one division of the circle to the next, the micrometer should read the same on both the preceding and following graduations. If the microscope is not properly adjusted, then the number of turns of the screw between consecutive divisions of the circle will differ from the true value; the difference is called the *run of the micrometer.* This causes an error which is called the *error of runs.* There are several ways of correcting for this, depending upon the assumption with regard to the cause of these differences.

One method, which is based upon the assumption that the error is largely due to the accidental errors in setting the microscope and in reading the micrometer, is to simply average the readings on the following division and the preceding division.

Another method consists in interpolating a reading between the readings taken on the preceding and following graduations, it being assumed that the error is due to an erroneous value of one turn of the screw. Dividing the length of the space (5′) by the number of revolutions indicated by the forward and backward readings gives the value of one turn of the screw. Multiplying this by the number of turns (and fraction) between the zero of the microscope and the next preceding graduation, the true reading is obtained. The actual work of computing these corrected readings is shortened by the use of tables.*

Direction instruments are generally used on important triangulation such as the primary and secondary work of the U. S. Coast and Geodetic Survey. For work of lesser importance the repeating instrument gives sufficient accuracy, and as every

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* See "Geodetic Astronomy," by J. F. Hayford, p. 206, and Coast Survey Report for 1882, p. 163.

surveyor is familiar with such an instrument it is to be preferred on ordinary work. The chief objection to the repeating instrument is the systematic error due to the action of the clamps and tangent screws, which it is not easy to completely eliminate. This, however, is not a serious matter in any but the most refined measurements.

38. Cross-Hairs. — The cross-hairs used in the telescopes of triangulation instruments are of different forms according to the class of work and the kind of signal used. Repeating instruments which are commonly used on comparatively short lines generally have cross-hairs set in the form of an X, the telescope being set so that the signal pole bisects the angle of the X. The cross-hairs are set by the instrument maker at any angle desired, from 45 degrees to 90 degrees. This form is suitable for sighting at poles and tripod signals. For sighting at very distant objects such as heliotropes, or poles at such distances that they are difficult to see, two vertical cross-hairs set close together are preferred, the space between the cross-hairs being bisected by the signal. The distance between the hairs should be made such that it will subtend an angle of from 10 to 15 seconds, this spacing depending upon the average length of line, the kind of signal, and upon the observer's preference.

39. PREPARATION FOR OCCUPYING THE STATION. — For primary work the instrument should be mounted upon a pier or some other solid support and protected from sun and wind by a tent or by a temporary building. For secondary or tertiary triangulation the instrument may be used upon its tripod and protected by a tent or by an umbrella.

The observer should be provided with a list of azimuths of the stations which he is to sight referred to some conspicuous object as 0 degrees, such as a nearby church spire or any other conspicuous mark. These data should have been taken during the reconnoissance or at some time previous to the occupancy of the station. The observer should have descriptions of the stations he is to occupy and the signals he is to sight. He should also be provided with maps, if any exist, which will aid him in identifying the stations. He should have with him a heliotrope and an assistant to operate it so that he can, upon arriving at the station, establish communication with all the heliotropers at the distant stations. This he is able to do, for he knows the directions of the distant stations from his own station. The flash from the instrument point is often the only means the heliotropers have of locating the observer's station. Signals can frequently be observed through the telescope if the direction is known, at times when it would be very difficult or impossible to find them if nothing were known of their directions. Observations on heliotropes may frequently be made when the hill itself is scarcely visible, provided the observer knows the direction of the distant station accurately enough to send a flash to the heliotroper at that station so that the latter may know the direction in which to send his light. The heliotroper should also be provided with a map of the region showing the scheme of the triangulation. On arriving at the station the heliotroper first identifies by means of his map the point which is occupied by the instrument. He then places his heliotrope on some convenient support anywhere on the line of sight, using a plumb-line for accurately lining it in.

40. MEASURING ANGLES WITH REPEATING INSTRUMENT. — A very common method of measuring an angle with a repeating instrument is to measure, say, six times with the telescope direct, then six times with the telescope reversed; this is called a " set." The first six are measured from left to right and the second six from right to left. Both verniers are read on the first and the sixth readings.

A better result is secured if the instrument is used as follows. In measuring the angles always turn the instrument in a **clockwise direction** (whether the upper or the lower motion is being used). The angle itself is first measured by six repetitions with the telescope direct; then its *explement* (360 degrees − the angle) is measured by six repetitions with the telescope reversed. It is evident that if the error due to the action of the clamps and tangent screws is nearly constant, the mean of the angle as found directly and through its explement will be nearly free from such error. This method of determining an angle by its explement is, however, unnecessarily refined, except for the highest grade of work.

The tangent screws should always be brought up to the desired position by means of a right-hand turn in order to insure proper tension of the opposing spring.

The Coast Survey instructions to observers recommend 5 complete sets on primary work, i.e., 5 on the angle and 5 on the explement; for secondary work, 2 to 4 sets; for tertiary work one set in case an 8-inch circle is used, or 2 to 4 sets with a 7-inch circle. The horizon should be *closed* in all cases, i.e., the series should cover a complete circle; the sum of the measured angles theoretically equals 360°.

The readings may be distributed uniformly over the circle by increasing the initial reading each time by an amount equal to $\frac{360°}{m\,n}$ where m is the number of sets to be taken and n is the number of verniers. In order to distribute the readings over different parts of the vernier the index of the vernier should be set ahead at the beginning of each set by $\frac{1}{m}$ part of the smallest division of the circle. For example, if six sets were to be taken with a two-vernier instrument having the circle graduated to 10-minute spaces, the first setting would be 0° 00′ 00″, the second 30° 01′ 40″, the third 60° 03′ 20″, etc.

41. Measuring Angles with Direction Instrument. — Some line (such as a line to a prominent signal) is chosen as a reference line and the directions of the various signals read in order, beginning with this one. After this series is completed the telescope is reversed, the pivots remaining in the same bearings, and the signals sighted in the reverse order. This completes one "set." The Coast Survey instructions * require 16 sets for primary work. For secondary work from 5 to 10 sets are required; for tertiary, one set. In order to distribute the readings over the circle of the instrument the initial setting on the circle is increased by 15° for each new set, if 16 sets are to be taken, and in order to distribute the micrometer readings uniformly over the screw the setting is increased by about 1′ 10″ each time, the total increase being 15° 01′ 10″, except that after

* See U. S. Coast and Geodetic Survey Report for 1903, Appendix 4, p. 811.

the fourth, eighth, and twelfth sets the increase is 18° 56′ 20″. By this arrangement the readings of the three microscopes are distributed uniformly over the circle at intervals of nearly 4°. The uniform distribution of the settings of the screw renders it practically unnecessary to make a correction for run. Since this setting can be made only approximately it is necessary to take the exact readings of all of the microscopes after the telescope has been carefully pointed.

42. **Precautions in Measuring Angles.** — Great care should be used in securing a steady support for the instrument. If it is necessary to set the tripod on soft ground, heavy pegs should be driven into the ground and holes bored in the tops to receive the points of the tripod legs. The clamp screws controlling the leveling screws should not be loose but should be set up firmly after the instrument has been leveled. The instrument should be protected from wind and sun by a tent or by an umbrella. The observer should be careful not to disturb the instrument or to allow the heat from his body to affect it.

Care should be taken not to observe on a signal whose position is made doubtful by difficult seeing. Where there are two or more signals in the same general direction care should be taken to observe the correct one. If a signal is illuminated on one side only, so that the error of pointing will be appreciable, observations should be delayed or else the error computed and allowed for. This is known as the correction for "phase."

43. **Time for Observing.** — The best seeing is obtained on cloudy days, or on bright days late in the afternoon. Observations made during the middle of the day, however, when the signals appear quite unsteady, have been found to give about as accurate results as those made under apparently better conditions. Observations made very early in the morning are usually unsatisfactory on account of the irregular refraction caused by rising vapors.

44. **Forms of Records.** — The following are forms of records which may be used with the direction instrument and the repeating instrument respectively.

TRIANGULATION NOTES — DIRECTION INSTRUMENT.

Sta., Pitman. Date, June 21, 1907. Observer, A. B. Instr., Wegener •720.

Set No.	Station	Time	Tel.	Micr.	Circle	Circle: Micrometer For'd	Circle: Micrometer Back	Run	Mean	Cor.	Cor. Read.
								″	′ ″		
5	Bald Hill	10h 24m	R	A	152° 00′	01′ 58″	01′ 58″				
				B		02 01	01 59				
						01 59.5	01 58.5	1.0	01 59.0		
	East Mt.	10 27	R	A	181 40	09 34	09 32				
				B		09 48	09 47				
						09 41.0	09 39.5	1.5	49 40.2		

Circle reads to 10 minutes. Micrometer makes 2 revolutions per division Screw divided into 30 parts, giving seconds by estimation.

TRIANGULATION NOTES — REPEATING INSTRUMENT.

Sta., Corey Hill. Date, May 21, 1907. Observer, J. N. B. Instr. B. & B. #1567.

Station	Time	Tel.	Rep.	Ver. A	B	Mean	Angle	Mean
				° ′ ″	″	″	° ′ ″	° ′ ″
Blue Hill to Prospect	3h 20m P.M.	D	0	0 00 00	00	00		
			1	123 28 10	20	15		
			6	*20 49 40	40	40	123 28 16.7	
		R	0	20 49 40	40	40		
			6	0 00 10	10	10	123 28 15.0	123 28 15.8

* Since the angle is over 120 degrees the A vernier has passed 360 degrees twice in the six repetitions. In computing the mean we divide the 720 degrees by 6 mentally and write down 12 –, then divide the 20 degrees by 6, add the whole degrees to 120, and then divide the minutes and seconds. Observe that when six repetitions are used, the remainder, when dividing the degrees by 6, gives the first figure of the minutes, i.e., 20 degrees ÷ 6 = 3 degrees in the mean, plus 2 degrees to be carried to the minutes column giving 20 minutes. Similarly in dividing the minutes by 6 the remainder is the tens place in the seconds.

45. **Reduction of Notes.** — After the angles have been measured the sum of the angles of each triangle should be checked. Before this can be done, however, it is necessary in case of large triangles to compute the *spherical excess*, since the measured angles are spherical angles and their sum will exceed 180 degrees. The amount of the spherical excess depends upon the area of the triangle and also upon the latitudes of the points; it is approximately equal to 1 second for each 75.5 square miles area of the triangle. It may be more accurately computed from the formula

$$e'' = \frac{\text{Area of triangle}}{R^2 \sin 1''}, *$$

$$= \frac{bc \sin A}{2 R^2 \sin 1''} \qquad [1]$$

b and c being sides of the triangle and R the mean radius of the earth. [Log R (in feet) = 7.32068; log R (in meters) = 6.80470.] In order to make allowance for the spheroidal form of the earth we may use the formula

$$e'' = \frac{1}{2 RN \sin 1''} \times bc \sin A \qquad [2]$$

the logarithm of the quantity $\frac{1}{2 RN \sin 1''} = m$ being given in Table II, p. 388, (from U. S. Coast Survey Report for 1894).

If $b = c = 20$ miles, $A = 60°$, and latitude $= 40°$, we find from the last formula, $e = 2''.277$. From the same data the second formula gives $2''.275$, while the rough value (75.5 square miles per $1''$) gives $2''.294$. In computing the spherical excess it is sufficiently accurate to use approximate values of the angles and distances. The sum of the measured angles should equal 180 degrees plus the spherical excess. In small triangles, especially where the angles are measured only to seconds, it is unnecessary to consider the spherical excess.

* This formula depends upon the geometric proposition that in any spherical triangle the spherical excess is to the area of the triangle as one right angle is to the area of the tri-rectangular triangle (or one eighth the surface of the sphere.)

On the work of the U. S. Coast Survey it is required that the errors of closure of the angles of a triangle shall be within the following limits.

Primary triangulation,	3″
Secondary triangulation,	8″
Tertiary triangulation,	15″

The following are averages, for the error of closure, in Coast Survey work.

Primary,	1″
Secondary,	2 to 3″
Tertiary,	4 to 5″

After the sum of the angles has been tested the error is distributed equally among the three angles, if no adjustment is to be made by the Method of Least Squares. The adjusted plane angles are then computed from the adjusted spherical angles by subtracting from each angle one third of the spherical excess. The spherical angles are to be used later in computing the geodetic positions. The plane angles are the ones used in calculating the length of the sides of the triangles, because in any triangle side occurring in practice the result would be practically the same whether computed as a plane or as a spherical triangle.

46. Reduction to Center. — It sometimes happens that the instrument cannot be placed exactly over the station point, but from a point near by, called the *eccentric station*, the angles can be observed. This case may occur when such a point as a church spire or a standpipe is used as a triangulation point. If the angles are taken with exactly the same care that they would be at the center, and if, in addition, the direction and distance of the center mark from the instrument (at the eccentric station) be observed, then the angles at the center can be accurately calculated, provided the approximate distances to the observed signals are known. Even though accurate values of the triangle sides cannot be obtained it is always possible to obtain approximate values, and these are sufficiently accurate for this reduction. In Fig. 25 let C be the station point, E the eccentric station where the angles are observed, and S one of the observed signals. The

direction of each signal is to be corrected for eccentricity and the angles between the signals may be afterward found from these corrected directions as follows. First compute the azimuth of each signal, reckoned, not from the meridian, but from the center *C*. Then for each signal form a triangle *CES*. In this triangle

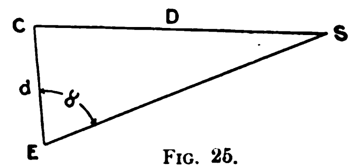


FIG. 25.

$$\frac{\sin S}{\sin E} = \frac{CE}{CS}$$

or approximately
$$S'' = \frac{d \sin \alpha}{D \sin 1''} \qquad [3]$$

In this formula α is the angle *E*, *d* is the distance *CE*, and *D* is the distance *CS*, obtained from the approximate calculation of the triangle sides. This gives *S* in seconds, which is the correction to be added to the azimuth of the line *ES* to obtain the azimuth of *CS*. Careful attention should be paid to the algebraic signs of sin α. After S'' has been calculated and applied to each azimuth the true angles may be found from the differences in the corrected azimuths. Obviously these azimuths have no significance after the reduction has been completed.

The following example illustrates the method of computing the reduction to center.

EXAMPLE OF REDUCTION TO CENTER.

HARPERS △

$d = 1^{m}.342$ log $= 0.12775$

$\log \frac{1}{\sin 1''} = 5.31443$

(Eccentric Station No. 1.) log const. $= 5.44218$

Station	Smith's Cupola	Methodist Church	Cotton's	White Flag	Baldwin
Azimuth	42° 14′ 20″.0	46°01′59″.1	104° 47′ 30″.1	161° 10′ 06″.2	205° 10′ 03″.6
Log sin Azimuth	9.8275	9.8572	9.9853	9.5090	9.6286
Log $\frac{1}{D}$	6.1052	6.1025	6.0640	6.2672	6.0909
Log Const.	5.4422	5.4422	5.4422	5.4422	5.4422
Log S	1.3749	1.4019	1.4015	1.2184	1.1617
S	+23″.7	+25″.2	+31″.0	+16″.5	−14″.5
Azimuth	42° 14′ 43″.7	46°02′24″.3	104° 48′ 01″.1	161° 10′ 22″.7	205° 09′ 49″.1

47. Adjusting the Triangulation. — In large systems of triangulation where it is important to secure all the accuracy which can be obtained from the measurements, the system should be adjusted by the Method of Least Squares, i.e., the errors should be distributed among the measured angles so as to give the most probable values of the angles. On account of the great expense a complete adjustment by Least Squares is not warranted in a triangulation of lesser importance. There are, however, approximate methods of adjusting simple figures, like the quadrilateral, which give fair results with much less labor than the exact adjustment.* In small systems no adjustment is necessary beyond that of distributing the error of closure equally among the three angles of each triangle. If points are being located by a system of rectangular coördinates and the position of any point may be calculated by means of different triangles, an approximate adjustment may be made by plotting the different positions and the directions of the observed lines and selecting a position which seems to best represent the point as indicated by the observations. Greater weight will of course be given to good intersections than to poor ones.

48. Computation of the Triangle Sides. — As an illustration of the computation of the triangle sides take the triangle shown in Fig. 26.

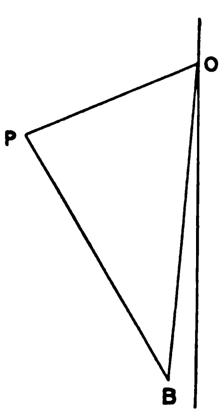


FIG. 26.

The observed angles are

$$
\begin{aligned}
P &= 82^\circ\ 27'\ 27''.9 \\
B &= 35\ \ \ 45\ \ \ 15\ .4 \\
O &= 61\ \ \ 47\ \ \ 18\ .8 \\
\hline
\text{Sum}\quad & 180^\circ\ 00'\ 02''.1
\end{aligned}
$$

The spherical excess is $0''.859$. Hence the error of closure of the triangle is $1''.2$.

* For these methods the student is referred to "Adjustment of Observations," by Wright and Hayford, published by Van Nostrand, New York.

Subtracting $0''.4$ from each angle we obtain for the adjusted spherical angles

$$\begin{aligned} P &= 82^\circ\ 27'\ 27''.5 \\ B &= 35\ \ 45\ \ 15\ .0 \\ O &= 61\ \ 47\ \ 18\ .4 \end{aligned}$$

Sum $180^\circ\ 00'\ 00''.9$

The plane angles are then found by subtracting $0''.3$ from each, giving

$$\begin{aligned} P &= 82^\circ\ 27'\ 27''.2 \\ B &= 35\ \ 45\ \ 14\ .7 \\ O &= 61\ \ 47\ \ 18\ .1 \end{aligned}$$

Sum $180^\circ\ 00'\ 00''.0$

Having given the $\log BP = 4.356\ 4673$ we find the sides BO and PO as follows by the formula $\frac{a}{b} = \frac{\sin A}{\sin B}$. The whole computation is kept in tabular form as shown below.

Stations	Obs'd Angles	Cor'n	Sphr. Ang.	Sph. Exc.	Plane Angles and Dists.	Logarithms
P to B					22723m.08	4.356 4673
O	61° 47′ 18″.8	0″.4	18″.4	0″.3	61° 47′ 18″.1	0.054 9218
P	82 27 27 .9	0 .4	27 .5	0 .3	82 27 27 .2	9.996 2261
B	35 45 15 .4	0 .4	15 .0	0 .3	35 45 14 .7	9.766 6415
O to B					25563.20	4.407 6152
O to P					15067.13	4.178 0306

49. Three-Point Problem.—It sometimes becomes necessary to assume a tertiary point and to locate it accurately without occupying any of the other stations. If three signals can be seen from the proposed station and two angles measured between them, the position of the point may be accurately determined by means of the Three-point Problem. The solution of this problem is as follows.* In Fig. 27, A, B, and C represent the three signals, and O the position of the instrument. A circle is passed through A, C, and O, and the lines OB, AD, and AC are drawn.

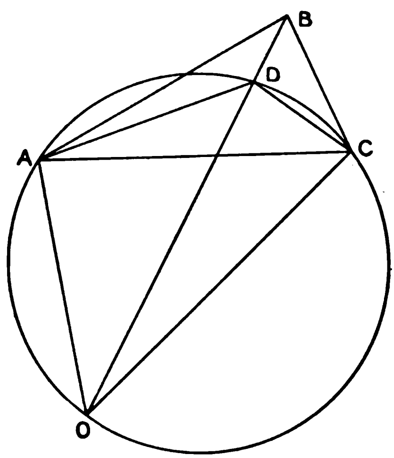


FIG. 27. TRIGONOMETRIC SOLUTION OF THREE-POINT PROBLEM.

Every side and angle of the triangle ABC may be computed from the data of the triangulation.

Let angle $DAC = \alpha$, and $DCA = \beta$

In the triangle ADC one side AC and two angles are known, and AD may be calculated.

In the triangle ADB two sides AD and AB are known, and the angle $BAD = BAC - DAC$. Therefore angle ABD may be computed.

Finally in triangle AOB one side AB and the two adjacent angles are known, from which AO and BO may be found; and in triangle BOC the side CO is readily calculated.

50. ASTRONOMICAL DETERMINATION OF POSITION.— In order that a triangulation net may be accurately located upon the earth's

* Another solution of the Three-point Problem will be found in Chauvenet's "Plane and Spherical Trigonometry," published by J. B. Lippincott Company, Philadelphia, Pa., and still another solution in Appendix 9, p. 184, of the U. S. Coast and Geodetic Survey Report for 1882.

surface it is necessary that the positions of a few points should be accurately determined by astronomical observations for latitude and longitude. The methods of making these observations are fully discussed in works on geodesy and practical astronomy and will not be taken up in detail in this volume.

The latitude is usually determined by observations with a special instrument called the *zenith telescope.* By means of this instrument the latitude is found from the difference in zenith distance of two stars, one of which is north of the zenith and the other south. The stars are so chosen that this difference is small enough to be measured by means of a micrometer. Any change in the inclination of the telescope is measured by means of a sensitive spirit level. By this method latitude can readily be determined within about $0''.05$ of arc, which is equivalent to about 5 feet on the earth's surface.

Longitudes are determined by observing, with a portable astronomical transit instrument, the error of a chronometer on local sidereal time and then comparing the chronometer, by telegraph, with that of another station where similar observations have been made. The difference in time thus found is the difference in longitude. The accuracy obtainable in a longitude determination is slightly less than that of a latitude determination.

From the observed latitudes and longitudes the triangulation net is located on the surface of the spheroid. Since the earth is not a true spheroid but an irregular figure the latitude and longitude of any station as computed from the position of another will be found to disagree with the latitude and longitude as directly observed, the difference between the observed and the computed positions being known as the *station error.* This difference often amounts to 8 or 10 seconds of arc and averages about 3 seconds. For this reason the latitude and longitude of a single point will not be sufficient to accurately locate the triangulation. A large number of points must be observed, so that the station errors are practically eliminated from the mean result and the true positions on the spheroid obtained.

51. Azimuths. — In addition to observations for latitude and longitude of stations it is necessary also to observe the azimuth of several of the triangle sides in order that the triangulation may be correctly oriented. Methods which will apply to all cases except the most precise work are given in Chapter II. For approximate methods see Volume I, Chapter VII.

52. **COMPUTATION OF GEODETIC POSITIONS.*** — After all of the triangle sides are known and the azimuth of some line is determined, the geodetic latitude and longitude of each triangulation station is computed. The calculation of the latitude and longitude is made by means of **differences** rather than by the direct solution. The triangle formed by joining the two stations with the pole, which would be required in the direct solution, is so large that 10 place logarithms are needed to give the required accuracy.

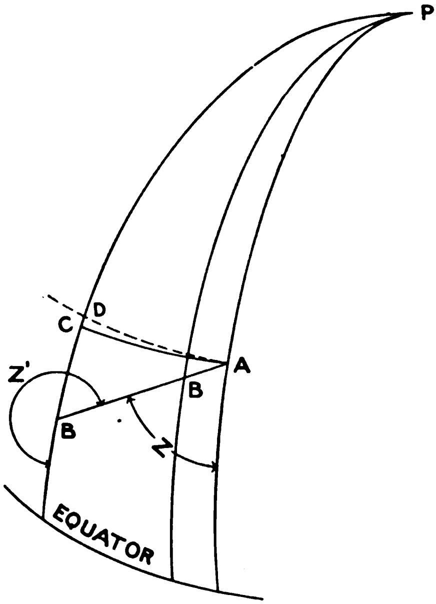


FIG. 28.

In Fig. 28 let A be the position of a point whose latitude and longitude are known; let B be the unknown point whose position is to be found; Z, the azimuth of B from A; x, the distance AB, an arc expressed in circular measure. In the triangle PAB the side PB is evidently a function of x, hence an expression may be obtained for PB by means of Maclaurin's Theorem. Letting $PA = y$, and $PB = y'$

then
$$y' = y + \frac{dy}{dx}x + \frac{d^2y}{dx^2} \cdot \frac{x^2}{2} + \cdots \qquad [4]$$

* For the method of deriving the formulas and computing the constants for computing the geodetic positions, see Appendix 9, Coast Survey Report for 1894; also Crandall's "Geodesy and Least Squares," p. 173.

In the differential triangle PAB', $PB' = y + dy$ and $AB' = dx$; then by trigonometry

$$\cos (y + dy) = \cos y \cos dx + \sin y \sin dx \cos A \qquad [5]$$

By successive differentiation of [5] values may be obtained of the differential coefficients in terms of y and A. If these are substituted in [4] an expression for the difference in latitude of the points A and B is obtained. To put this series in convenient form for computation x is changed from an arc to the corresponding distance in meters (K). The final form of the series is

$$-\,dL = K \cos Z \cdot B + K^2 \sin^2 Z \cdot C + (\delta L)^2 \cdot D - h \,.\, K^2 \sin^2 Z \cdot E \qquad [6]$$

in which K = the distance AB in meters
Z = the azimuth of B from A
L = the latitude of A
dL = the difference in latitude
δL = the sum of the first two terms
and $h = K \cos Z \cdot B$

B, C, D, and E are constants for any given latitude whose logarithms are given in Table III, p. 389. All of these terms are in seconds of arc.

The first term in the series is evidently the distance from B to the foot of the perpendicular AC; the second term is a distance nearly equal to CD, and so on, each term giving a closer approximation to the distance from B to the parallel DA.

The difference in longitude of A and B may be obtained directly from the triangle PAB

$$\sin P = \frac{\sin x}{\cos L'} \sin Z$$

which becomes

$$dM = K \cdot \sin Z \cdot \sec L' \cdot A \qquad [7]$$

in which dM is the difference in longitude in second of arc, and

A is another tabulated quantity. L' in this expression is the latitude of B, as found by equation [6].

The difference in azimuth due to the convergence of the meridians may be found by trigonometry as follows.

$$\tan \tfrac{1}{2} (A + B) = \cot \tfrac{1}{2} dM \frac{\cos \frac{1}{2} (y' - y)}{\cos \frac{1}{2} (y' + y)}$$

This may be put in the form

$$- dZ = dM \sin \tfrac{1}{2} (L + L') \sec \tfrac{1}{2} (dL) \qquad [8]$$

in which $-dZ$ is the correction to the azimuth $180° + Z$ to reduce it to Z'.

The formulas for the complete computation are

$$\begin{aligned}
- dL &= K \cdot \cos Z \cdot B + K^2 \sin^2 Z \cdot C \\
&\quad + (\delta L)^2 \cdot D - h \cdot K^2 \sin^2 Z \cdot E && [6] \\
dM &= K \sin Z \sec L' \cdot A && [7] \\
- dZ &= dM \sin \tfrac{1}{2} (L + L') \sec \tfrac{1}{2} (dL) && [8] \\
L' &= L + dL \\
M' &= M + dM \\
Z' &= Z + dZ + 180°
\end{aligned}$$

The primes in each case refer to the unknown point B.

It is evident that when the azimuth of AB is between 90 degrees and 270 degrees the difference in latitude dL must be added to the latitude of A; but since the algebraic sign of $\cos Z$ is negative for such angles the sign of dL must be negative.

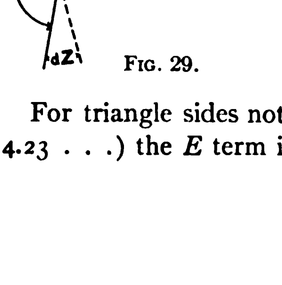


FIG. 29.

From Fig. 29 it is evident that if B is west of A, i.e., if Z is between 0° and 180°, the correction dZ must be subtracted from $180° + Z$ to refer the back-azimuth to the meridian through B.

For triangle sides not exceeding 10 statute miles (or $\log K = 4.23 \ldots$) the E term in the expression for dL may be omitted, and also the factor sec $\frac{1}{2}$ dL in the expression for dZ. The quantity h^2 may be substituted for $(\delta L)^2$ when log K is not greater than 4.93 . . .

When the difference in longitude is very large it is necessary to apply a small correction to allow for the difference between the arc and the sine, an approximation which was made in deducing the formula for dM. This difference is given in Table IV, p. 394. To use this table take out the "difference" opposite log K and also the "difference" opposite log dM, giving each the sign shown at the top of the column. The algebraic sum of the two is to be added to log dM.

On account of the adoption of the letters L, M, and Z to represent the latitude, longitude, and azimuth this problem is commonly known as the "L-M-Z" problem.

53. **The Inverse Problem.** — When the latitude and longitude of two points are given it is possible, by means of these same formulas, to compute the length of the line, K, and the azimuths Z and Z'. For convenience the formulas may be put in the following form.

$$K \cos Z = x = -\frac{1}{B}\left(dL + Cy^2 + D\,(dL)^2 - E\,(dL)\,y^2\right) \quad [9]$$

$$K \sin Z = y = \frac{dM \cos L'}{A} \quad [10]$$

$$\tan Z = \frac{y}{x}\,;\; K = x \sec Z = y \csc Z \quad [11]$$

EXAMPLE OF COMPUTATION OF GEODETIC POSITION.

Referring to the calculations shown on page 47, the adjusted spherical angles are

$$\begin{aligned} P &= 82^\circ\ 27'\ 27''.5 \\ B &= 35\ \ 45\ \ 15\ .0 \\ O &= 61\ \ 47\ \ 18\ .4 \end{aligned}$$

The computation of the position of O from the base-line PB may be put in the following form.

		°	′	″
Z	Prospect to Blue Hill	329	30	20.54
$\angle$	Blue Hill and Observatory	82	27	27.5
Z	Prospect to Observatory	247	02	53.0
dZ			+06	49.5
180°		180		
Z'	Observatory to Prospect	67	09	42.5

L	°	′	″			°	′	″
	42	23	18.831	Prospect	M	71	15	15.333
dL		3	09.976	15067.13* meters	dM		10	07.059
L'	42	26	28.807	Observatory	M'	71	05	08.274

* 9.36 statute miles.

L_m 42° 24′ 53″.8		K	4.178 0306	K^2	8.35606		
		Cos Z	$9.591\ 0189_n$	Sin² Z	9.92836	h^2	4.5594
		B	8.510 6691	C	1.36463	D	2.3908
1st term	−3′ 10″.423	h	$2.279\ 7186_n$		9.64905		6.9502
2d & 3d terms	+ .447		−190.423		+.4457		+.0009
$-dL$	−3′ 09″.976						

K	4.178 0306		
Sin Z	$9.964\ 1805_n$		
A'	8.509 0562	dM	2.78323_n
Sec L'	0.131 9635	Sin L_m	9.82898
	$2.783\ 2308_n$		2.61221_n
dM	″ −607.059	$-dZ$	″ −409.45

It will be observed that in taking out the value of A in computing dM this log is taken out for L', **not** for L. In computing a large number of positions over a limited area it will be convenient to prepare a special table of log $\dfrac{A'}{\cos L}$ for intervals of, say, 10 seconds of latitude and for values of L covering the limits of the survey.

		°	′	″
Z	Blue Hill to Prospect	149	35	58.86
$\angle$	Prospect and Observatory	35	45	15.0
Z	Blue Hill to Observatory	185	21	13.9
dZ			1	10 3
180°		180		
Z'	Observatory to Blue Hill	5	22	24.2

	°	′	″			°	′	″
L	42	12	43.941	Blue Hill	M	71	06	52.638
dL		13	44.866	25563.20 meters	dM		01	44.364
L'	42	26	28.807	Observatory	M'	71	05	08.274

L_m 42° 19′ 36″.4		K	4.407 6152	K^2	8.81523		
		Cos Z	9.998 1012$_n$	Sin^2 Z	7.93983	h^2	5.8328
		B	8.510 6827	C	1.36196	D	2.3904
1st term	−13′ 44.896	h	2.916 3991$_n$		8.11702		8.2232
2d & 3d terms	+ .030		−824.896		+ .013		+ .0167
$-dL$	−13′ 44.866						

K	4.407 6152		
Sin Z	8.969 9145$_n$		
A'	8.509 0562	dM	2.01855$_n$
Sec L'	0.131 9635	Sin L_m	9.82824
	2.018 5494$_n$		1.84679$_n$
dM	″ −104.364	$-dZ$	″ −70.27

54. Application of Triangulation to Small Surveys. — The methods which have been described in this chapter are chiefly those which apply to extensive triangulation. In conducting smaller surveys the same general methods will apply, but many of the refinements may profitably be omitted. In such work a repeating instrument is always to be preferred, as it is simpler to use and is always amply accurate for the purpose. An engineer's transit with a 6-inch circle and with verniers reading to 30 or to 20 seconds is suitable for the triangulation here considered. Instead of carrying out the more elaborate programs for measuring angles it will often be sufficient to take but one or two sets of angles, six repetitions being taken with the telescope direct and six with the telescope reversed.

While many of the refinements may profitably be omitted, the work should not be done carelessly, and all of the general precautions in regard to protection and manipulation of the instrument should be carefully observed. The work should be planned so as to provide checks on the accuracy of the results. On account of the triangle sides being short, special attention should be given to centering the instrument over the station, and to centering and plumbing the signals.

In measuring the base-line a steel tape provided with a tension handle or a spring balance should be used. The temperature correction need be only roughly determined. In other words, the base measurement should correspond to a fair grade of city surveying work. Two independent measurements of the base should be made as a check.

The triangulation net may be oriented by observing the azimuth of some line by the method described in Chapter II. (For approximate methods see Vol. I, Chapter VII.)

In surveys of small areas, such as a park system or a small river, it is simpler to use rectangular coördinates rather than latitudes and longitudes. The spherical excess of a triangle may always be neglected in work of this kind. The adjustment of a triangulation by the Method of Least Squares need never be made for the purpose of mapping an area, as the errors of the triangulation can easily be kept so small as not to affect the plotted results. The triangles should always be closed, however, when this is possible, and the error of closure distributed equally among the three measured angles, unless these obviously should have different weights.

55. CONNECTING TRAVERSES WITH TRIANGULATION. — The triangulation system is simply a framework upon which all of the subsequent surveys of details are to depend. The manner of connecting these surveys with the triangulation depends upon the instruments and the methods used for locating the details. Methods which are applicable to topographic and hydrographic work are given in detail in the following chapters.

Certain kinds of surveys, for example a survey to locate town or state boundaries, require that points be located with great accuracy and yet these points are often in such positions that direct location by triangulation is not practicable. In such cases these points must be located by means of traverses run with the transit and tape. These traverses must not only be accurate but must also be tied to the triangulation system so that the relative positions of the located points will be accurately determined.

After the traverses have been connected with the triangulation the accuracy of each one may be tested by computing its error of closure. This may be done in any traverse joining two triangulation points, by computing the difference in latitude and departure for each course of the traverse, the line joining the triangulation points being regarded as the closing line of the traverse (see Fig. 30). If the azimuths of the traverse lines are referred to the meridian through △*A*, then the azimuth *A–B* should be used in computing the difference in latitude and departure for the triangulation line. The sums of the latitudes and departures are found and the error of closure distributed in the usual way, except that the corrections are made only on the traverse lines, the triangulation line remaining unchanged. If desired, the latitudes and departures can be computed by taking the triangulation line as a meridian instead of using the true meridian. The triangulation lines in such surveys are usually short, so that the effect of the earth's curvature on the traverse lines is negligible.

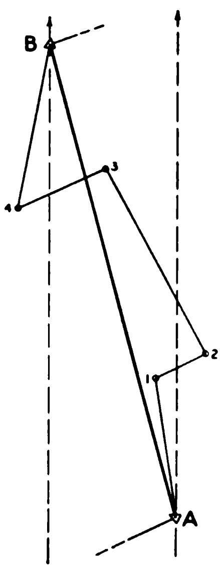


FIG. 30.

In Fig. 30 are shown two triangulation points with a short traverse connecting them. The computation of the latitudes and departures and the distribution of the error of closure are shown below.

NOTES OF TRAVERSE.

Station	Azimuth	Distance (Feet.)
△ *A*	171° 26′ 20″	1321.20
1	243 25 50	524.84
2	152 06 10	1974.50
3	66 15 30	901.08
4	190 44 00	1570.71
△ *A* to △ *B*	165 09 43	4621.3

COMPUTED LATITUDES AND DEPARTURES.

	N. Lat.	S. Lat.	E. Dep.	W. Dep.
△*A*	1306.48			196.68
1	234.75		469.41	
2	1745.04			923.84
3		362.79		824.82
4	1543.23		292.53	
△*B*		4467.20	1183.46	
	4829.50	4829.99	1945.40	1945.34

Error = .49 Error = .06

ADJUSTED LATITUDES AND DEPARTURES.

	Lat.	Dep.
△ *A*	+1306.60	−196.68
1	+ 234.77	+469.40
2	+1745.21	−923.86
3	− 362.76	−824.84
4	+1543.38	+292.52

PROBLEMS

1. Reduction of base-line measurements.

Section	1st. Meas.		2d. Meas.		Diff. in Elev.	Stake	Out of Line
	Scale	Thermophone	Scale	Thermophone			
	mm	mm	mm	mm	ft.		ft.
0 – 1	98.4	+18.4	98.0	+18.7	1.72	0	0.00
1 – 2	90.2	+23.2	96.1	+17.7	3.50	1	0.00
2 – 3	94.8	+20.8	99.2	+16.2	4.35	2	0.06 West
3 – 4	87.1	+14.9	88.1	+13.1	4.92	3	0.11 East
4 – 5	97.0	+18.9	98.2	+17.6	2.01	4	0.21 East
						5	0.10 East

The scale readings are all 1^{cm} too large because the zero point is marked 10 in order to avoid negative readings. The weight is supposed to be adjusted to give the "normal tension."

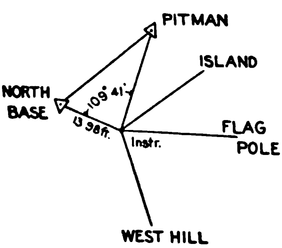
Compute the true length of the line between stakes 0 and 5, assuming the tape to be exactly 100 meters in length.

Result, $500^m.0343$.

2. Reduction to Center.

Angles measured.

Pitman to West Hill .	144°	19′	56″.7
Pitman to Island . . .	64°	05′	20″.0
Pitman to Flag Pole .	105°	06′	07″.6



Distances in feet.

North Base to Pitman	3235.1
North Base to West Hill . . .	8720.5
North Base to Island	10485.0
North Base to Flag Pole . . .	7295.6

Reduce these angles to North Base.

3. Position of pt. L; latitude 42° 26′ 13″.276, longitude 70° 55′ 52″.088. Distance L to N, 3012.0 meters (log = 3.478 8600). Azimuth L to N, 314° 34′ 00″; back-azimuth, 134° 35′ 03″. Position of pt. N, latitude 42° 25′ 04″.764, longitude 70° 54′ 18″.232. Angle at L, 36° 15′ 07″; at N, 63° 44′ 59″; at E, 79° 59′ 57″. Compute the position of E from both lines, LE and NE, and also the back-azimuths of both lines. (E is east of LN.)

4. Position of point B $\begin{cases}\text{lat. } 39° & 13' & 26''.686 \\ \text{long. } 98° & 32' & 30''.506\end{cases}$

Position of point C $\begin{cases}\text{lat. } 38° & 51' & 50''.913 \\ \text{long. } 98° & 29' & 15''.508\end{cases}$

Azimuth B to C 353° 17′ 21″.81 Dist. 40232.35 meters. (log = 4.604 5754)

Back-azimuth 173° 19′ 24″.64

The spherical angles are A 57° 53′ 14″.39
B 62° 23′ 31″.40
C 59° 43′ 17″.93

Compute position of A for both lines and the back-azimuths. (A is east of BC.)

5. A straight line is run due west from a point A in latitude 40° N, for a distance of 10 miles to point B. Compute the distance in feet that point B is due south of a true parallel of latitude through A. See equations [6], [7], and [8] of this chapter and Table III, p. 389. See also Vol. I, Table 6, p. 153.

6. Position of point B $\begin{cases}\text{lat. } 41° & 38' & 11''.330 \\ \text{long. } 98° & 32' & 45''.067\end{cases}$

Position of point C $\begin{cases}\text{lat. } 41° & 27' & 58''.053 \\ \text{long. } 98° & 33' & 34''.309\end{cases}$

Compute the distance BC and the forward and back-azimuths.

CHAPTER II.

ASTRONOMICAL OBSERVATIONS FOR AZIMUTH.

56. **ASTRONOMICAL OBSERVATIONS.** — A complete treatment of astronomical observations for the precise determination of the latitude and longitude* of a station and of the azimuth of a triangulation line belongs properly within the province of geodetic surveying. In this chapter only so much of this subject will be considered as will enable the surveyor to obtain the **astronomical azimuth** of a line with sufficient accuracy for all purposes except for refined geodetic work, and to make such other observations as will be required in obtaining the necessary data such as the approximate latitude, longitude, and time. All of the methods given may be used with the engineer's transit.

57. **DEFINITIONS.** — In the problems of practical astronomy all heavenly bodies are considered as being situated on the surface of a great sphere, whose center is at the center of the earth and whose radius is infinite. This is known as the *celestial sphere.*

Vertical. — A vertical line at any place is the direction of gravity, i.e., the direction assumed by a plumb-line at rest, (ZC, Fig. 31).

Zenith. — The zenith is the point where the vertical, produced upward, pierces the celestial sphere (Z, Fig. 31).

Horizon. — The horizon is the great circle on the celestial sphere cut by a plane (through the earth's center) at right angles to the vertical ($NESW$, Fig. 31).

Axis. — The axis of the celestial sphere is the earth's axis of rotation produced indefinitely ($NP - SP$, Fig. 31).

* For a treatment of these subjects the student is referred to the following works: — Chauvenet, "Spherical and Practical Astronomy," J. B. Lippincott Company. Doolittle, "Practical Astronomy," John Wiley & Sons. Hayford, "Geodetic Astronomy," John Wiley & Sons. Reports of the Superintendent of the U. S. Coast & Geodetic Survey, years 1880 and 1897–98.

Poles. — The poles are the points where the celestial sphere is pierced by the axis (*NP* and *SP*, Fig. 31).

Equator. — The equator is the great circle of the celestial sphere cut by a plane (through the earth's center) perpendicular to the axis (*EDW*, Fig. 31).

Meridian. — The meridian of any observer is the great circle of the celestial sphere which passes through the poles and the observer's zenith (*SZN*, Fig. 31). The intersection of the plane of the meridian with the plane of the horizon is called the *meridian line.*

Ecliptic. — The ecliptic is the great circle of the celestial sphere which the sun appears to describe in its annual (eastward) motion among the stars (*VLA*, Fig. 31). The ecliptic is inclined to the equator by a nearly constant angle which is, at the present time, about 23° 27′.

Equinoxes. — The points of intersection of the equator and the ecliptic are called the equinoxes. The point where the sun crosses the equator when going from south to north (which occurs in March) is called the *vernal equinox.* The other intersection (where the sun crosses the equator in September) is called the *autumnal equinox.* In Fig. 31 *V* is the vernal and *A* the autumnal equinox.

58. SPHERICAL COÖRDINATES. — The positions of the heavenly bodies are defined by means of spherical coördinates, of which there are several systems. The circles of reference in any system are (1) a great circle called the *primary*, and (2) a system of great circles at right angles to the primary, called *secondaries.*

59. The Horizon System. — In the horizon system the circles of reference are the *horizon* and great circles through the zenith called *vertical circles;* the coördinates are called *altitude* and *azimuth.* The *altitude* of a point is its angular distance above (or below) the horizon, measured on a vertical circle through the point. In Fig. 31 the altitude of point *S′* is *BS′*. The complement of the altitude is called the *zenith distance.* The *azimuth* of a point is the arc of the horizon between the meridian and the vertical circle through the point. In Fig. 31 the azimuth of point *S′* is *SWNB.* In astronomical work azimuth is sometimes reckoned from the south point as in surveying, sometimes from the north point, and is generally measured in a clockwise direction.

60. The Equatorial System. — The circles of reference in the equatorial system are the *equator* and great circles through the poles, called *hour circles;* the coördinates are either *declination* and *hour angle* or *declination* and *right ascension.* The *declination* of a point is its angular distance from the equator measured on an hour circle through the point. In Fig. 31 the declination of

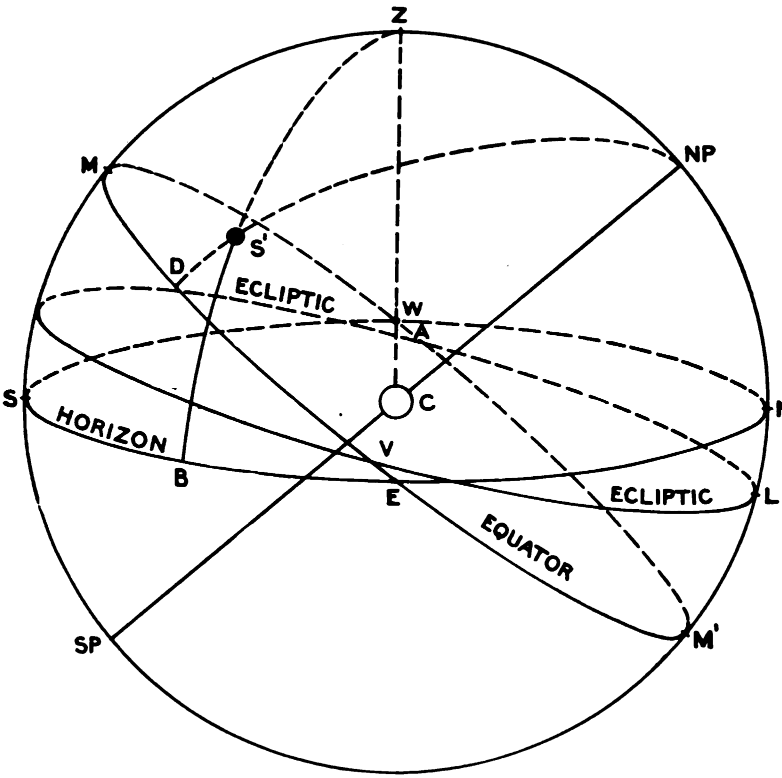


FIG. 31. THE CELESTIAL SPHERE.

point S' is DS'. Points north of the equator are considered as having **plus** declinations and points south of the equator as having **minus** declinations. The complement of the declination is called the *polar distance.*

The *hour angle* of a point is the arc of the equator measured from the meridian **westward** to the hour circle through the point. In Fig. 31 the hour angle of point S' is $MWED$.

The *right ascension* of a point is the arc of the equator measured from the vernal equinox **eastward** to the hour circle through the point. In Fig. 31 the right ascension of point S' is $VM'MD$. For convenience hour angles and right ascensions are usually expressed in hours, minutes, and seconds **of time.**

61. Coördinates of the Observer. — The position of the observer is defined by means of his *latitude* and *longitude*. The latitude is the declination of the observer's zenith. In Fig. 31, MZ is the latitude of the observer. The longitude is the arc of the equator between some primary meridian (usually Greenwich) and the observer's meridian.

62. RELATION BETWEEN ALTITUDE OF POLE AND LATITUDE OF PLACE. — In Fig. 32 the arc PN = arc EZ, since PO is

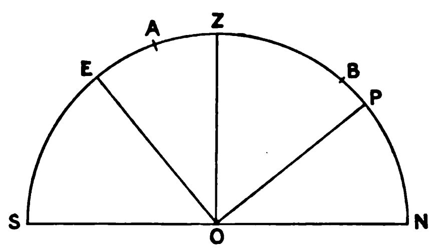


FIG. 32. SECTION OF HEMISPHERE ON PLANE OF MERIDIAN.

perpendicular to EO and NO is perpendicular to ZO. But PN is the altitude of the pole, and EZ is the latitude of the observer. Hence the **altitude of the pole equals the latitude of the observer.**

The relation existing between the altitude of a point on the meridian, the latitude of the observer, and the declination of the point may be seen from Fig. 32.

Let A be the point on the meridian; then

$$EZ = \text{the latitude} = L$$
$$EA = \text{the declination} = D$$
$$SA = \text{the altitude} = h$$
$$ZA = \text{the zenith distance} = Z$$

From the figure

$$ZA = EZ - EA$$

or

$$Z = L - D$$

and

$$h = 90^\circ - (L - D) \qquad [12]$$

If the point is near the pole, as at B, the following relation is convenient.

$$PN = BN - BP$$

or,

$$L = h - p \qquad [13]$$

where p is the polar distance of B, or $90^\circ - D$. If the point B were below the pole the equation would be

$$L = h + p \qquad [14]$$

63. The Astronomical Triangle. — Many of the problems of practical astronomy require the solution of the triangle formed by joining the pole, zenith, and a star (or other heavenly body) by arcs of great circles. By means of this triangle the coördinates of the horizon system may be transformed into those of the equatorial system and *vice versa*.

A special case of the astronomical triangle, and one of great practical importance, is when the star is near the pole, and is at its extreme eastern or western position, called its *greatest elongation*. In Fig. 33 the point S represents a circumpolar star* at western elongation, and S' the same star at eastern elongation. When the star is at S the angle PSZ is 90 degrees because the vertical circle ZS is tangent to the star's diurnal

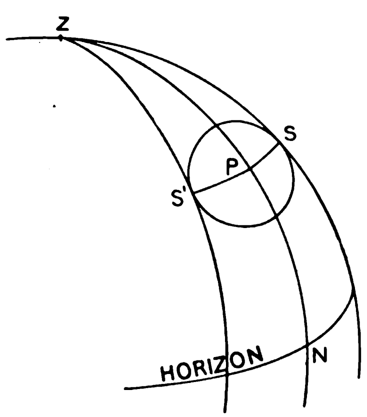


Fig. 33. Circumpolar Star at Elongations.

* A circumpolar star in any given latitude is one which never goes below the horizon; hence its polar distance must be less than the given latitude.

circle at the instant of greatest elongation. The azimuth of the star at elongation is found from the formula

$$\sin Z = \frac{\cos D}{\cos L} \qquad [15]$$

where Z is the azimuth from the north. The hour angle of the star at the instant is found from the relation

$$\cos t = \frac{\tan L}{\tan D} \qquad [16]$$

If the star is not at elongation but its hour angle is known, then its azimuth may be found by the formula

$$\tan Z = \frac{\sin t}{\cos L \tan D - \sin L \cos t} \qquad [17]$$

which may be derived as follows. From the fundamental equation of spherical trigonometry we have

$$\cos h \sin Z = \cos D \sin t \qquad (a)$$

and

$$\cos h \cos Z = \sin D \cos L - \cos D \sin L \cos t \qquad (b)$$

Dividing (a) by (b) we have

$$\tan Z = \frac{\cos D \sin t}{\sin D \cos L - \cos D \sin L \cos t}$$

Dividing both numerator and denominator by $\cos D$ we obtain equation [17] given above.

TIME.

64. DEFINITIONS. — Apparent Motion of the Celestial Sphere. — On account of the earth's rotation on its axis all heavenly bodies appear to revolve in a clockwise direction around the earth and hence to cross the observer's meridian twice each day.

65. Transit. — The instant when a body is on the meridian of an observer is called its *transit* or *culmination*. When it is on the side of the meridian containing the zenith it is called the *upper transit;* when it is on the other side it is called the *lower transit.* Except in the case of circumpolar stars the upper transit is the only one visible to the observer; hence when the transit of a star is mentioned, the upper transit is intended unless otherwise specified.

66. Sidereal Day. — A sidereal day is the interval of time between two successive upper transits of the **vernal equinox** over the same meridian.

67. Sidereal Time. — The sidereal time at a given meridian at any instant is the **hour angle** of the **vernal equinox.** The sidereal time as found from observation on a star is

$$S = R + t \tag{18}$$

where S = the sidereal time

R = the right ascension of the star

t = the hour angle of the star

The relation expressed in [18] is evident from the definitions of these three angles (Art. 60, p. 63). Referring to Fig. 31 we have for the position of point W,

$$MWV = VEM'W + MW$$

which becomes identical with [18] from the definitions.

At the instant of transit we have

$$t = 0$$

hence
$$S = R \tag{19}$$

i.e., **the right ascension of a star equals the sidereal time at the instant when that star is on the meridian.**

68. Solar Day. — A solar day is the interval of time between two successive upper transits of the **sun** over the same meridian.

69. Solar Time. — The solar time at a given meridian at any instant is the **hour angle** of the **sun** at that instant. The apparent angular motion of the sun is not uniform, hence for most purposes we use the time kept by what is known as the *fictitious sun* or the *mean sun*, which is a point assumed to move at a uniform rate along the equator at such a speed as to make one revolution in the same time as the true sun. The time kept by this mean sun is called ***mean solar time.*** The time kept by the real sun is called ***apparent time,*** and is the time shown by a sundial. The difference between mean time and apparent time is called the *equation of time.* It is given in the American Ephemeris and Nautical Almanac for the instant of Greenwich Mean Noon for every day in the year.

70. Astronomical and Civil Time. — In astronomical time the day begins at noon and is divided into 24 hours, numbered successively from 0^h to 24^h. The civil day begins at midnight and is divided into two parts of 12 hours each. From midnight to noon is called A.M., and from noon to midnight is called P.M. The civil day begins 12^h **earlier** than the astronomical day of the same date. For example,

Astr. time May 10, 15^h = Civ. time May 11, 3^h A.M.
Astr. time Jan. 3, 7^h = Civ. time Jan. 3, 7^h P.M.

71. LONGITUDE AND TIME. — The hour angle of the sun at any given meridian is the local (solar) time at that meridian. The hour angle of the sun at Greenwich is the corresponding Greenwich (solar) time. The difference between the two is the *longitude* of the given meridian from Greenwich, expressed in

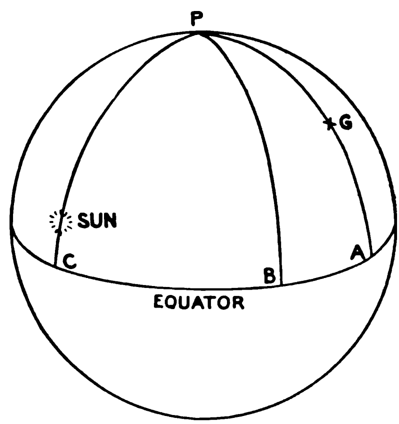


FIG. 34. RELATION BETWEEN LONGITUDE AND TIME.

units of time. In Fig. 34 AC is the hour angle of the sun at Greenwich, or the Greenwich time. BC is the hour angle of the sun at the meridian B, or the local time. The difference, AB, is the longitude of B west of Greenwich.

72. Since $24^h = 360°$, then $15° = 1^h$. Any angle expressed in degrees, minutes, and seconds may be converted into hours, minutes, and seconds by dividing by 15. The following relations will also be found useful.

$$1° = 4^m \text{ and } 1' = 4^s$$

73. **Solar and Sidereal Intervals.** — On account of the earth's motion in its orbit the sun has an apparent eastward motion among the stars. For this reason the sun is retarded in its apparent daily (westward) motion, making the solar day about 4 minutes longer than the sidereal day. It will be seen that the earth makes a little more than one rotation on its axis in a solar day. Since this daily retardation is just enough to bring the sun back to its starting point at the end of one year, there will be just one more sidereal day in the year than there are solar days. The length of the tropical* year is 365.2422 mean solar days, hence

$$366.2422 \text{ sidereal days} = 365.2422 \text{ solar days}$$

or

$$1 \text{ sidereal day} = 0.99726957 \text{ solar days}$$

and

$$1 \text{ solar day} = 1.00273791 \text{ sidereal days}$$

Since the sidereal day is the shorter and is divided into the same number of hours, minutes, and seconds, these units of time are all shorter than the corresponding solar units, so that in any given interval of time there will be more sidereal units than solar units. If a chronometer were regulated to sidereal time and a watch were regulated to mean solar time, the chronometer would run faster than the watch, the gain being about 10^s per hour, or, more nearly, $3^m\ 56^s$ per day. If the two timepieces agreed at a certain date, then they would again agree just one year later, the chronometer having gained exactly one day.

* The tropical year is the interval of time in which the sun apparently makes one revolution about the earth measured from the equinox to the equinox again.

If I_m is a mean solar interval and I_s is the corresponding sidereal interval,

then $I_s = I_m + .00273791 \times I_m$

and $I_m = I_s - .00273043 \times I_s$

These operations are readily performed by the aid of Tables II and III in the Appendix to the Nautical Almanac, in which are given the values of the above corrections for every minute up to 24 hours. Tables V and VI, pp. 395–6 in this volume, may also be used for the same purpose. In using these tables it will be necessary to take out the corrections for the hours, minutes, and seconds separately, and add them together.

74. EXAMPLES. — To reduce $9^h\ 23^m\ 51^s.0$ interval of sidereal time to the equivalent interval of mean solar time we enter Table II, Appendix to Nautical Almanac, and in column headed 9^h and opposite 23^m we find $1^m\ 32^s.234$. In the column for seconds, opposite 51^s, we find $0^s.139$. The sum of these two, $1^m\ 32^s.373$, is the correction to be subtracted from the given sidereal interval, giving $9^h\ 22^m\ 18^s.6$ of mean solar time.

To reduce $7^h\ 10^m$ to sidereal time we find, similarly, from Table III, Nautical Almanac, $1^m\ 10^s.638$, which, added to $7^h\ 10^m$, gives $7^h\ 11^m\ 10^s.6$ of sidereal time.

75. **RELATION BETWEEN SIDEREAL AND MEAN SOLAR TIME.** — The sidereal time at any instant is the hour angle of the vernal equinox. The relation between the sidereal time and mean solar time is

$$S = R_s + t_s \qquad [20]$$

where R_s is the right ascension of the mean sun, and t_s is the hour angle of the mean sun. (See equation [18].) The right ascension in the equation is the right ascension at the given instant, but if the right ascension is taken as that for the **preceding mean noon** * then

$$S = R_s + t_s + C \qquad [21]$$

where C is the correction (from Table III of Appendix to the Ephemeris) to reduce t_s to a sidereal interval. C is the increase

* The sun's right ascension is given in the Nautical Almanac for Greenwich Mean Noon for each day.

in the sun's right ascension during the interval t_s, which reduces the right ascension of the sun to its value at the instant considered.

If it is desired to find the mean time from the sidereal time, then from equation [18],

$$\text{Sidereal interval from noon} = S - R_s$$

or

$$\text{Mean time} = S - R_s - C' \qquad [22]$$

where C' is the correction from Table II of the Appendix to the Ephemeris to reduce $S - R_s$ to a solar interval. C' is the increase in the sun's right ascension in $(S - R_s)$ sidereal hours, and reduces the sun's right ascension to its value at the instant considered.

76. Examples. — To find the sidereal time corresponding to $9^h\ 22^m\ 18^s.60$ on Jan. 7, 1907, at Greenwich. The right ascension of the mean sun at Greenwich Mean Noon is found from the Nautical Almanac to be $19^h\ 03^m\ 36^s.38$. Reducing $9^h\ 22^m\ 18^s.60$ to sidereal time we find from Table III (Appendix to Nautical Almanac) the correction $+\ 1^m\ 32^s.37$.

$$\begin{array}{rl} R_s = & 19^h\ 03^m\ 36^s.38 \\ t_s = & 9\quad 22\quad 18\ .60 \\ C = & \underline{\quad\quad 1\quad 32\ .37} \\ S = & 28^h\ 27^m\ 27^s.35 \\ \text{Sidereal time} = & 4^h\ 27^m\ 27^s.35 \end{array}$$

To find the mean solar time when the sidereal time is $4^h\ 27^m\ 27^s.35$* on the same date.

$$\begin{array}{rl} S = & 28^h\ 27^m\ 27^s.35 \\ R_s = & \underline{19\quad 03\quad 36\ .38} \\ S - R_s = & 9^h\ 23^m\ 50^s.97 \\ C' = & \underline{\quad\quad 1\quad 32\ .37} \\ \text{Mean time} = & 9^h\ 22^m\ 18^s.60 \end{array}$$

The right ascension of the mean sun is given in the Nautical Almanac at Greenwich mean noon for each day. If the right ascension for noon at any other place is desired, we must multiply the change in right ascension in 1 hour ($9^s.8565$) by the number of hours in the longitude and add this to the sun's right ascension

* 24^h may be added to S, when necessary, to make the subtraction possible.

if the place is west of Greenwich, subtract it if the place is east of Greenwich. This correction may be taken directly from Table III in the Appendix to the Nautical Almanac (or Table VI, p. 396, in this volume), since the change in the sun's right ascension during a solar interval is the same as the change from solar to sidereal time. If the sidereal time given above were at a meridian 5 hours west of Greenwich the computation would be as follows.

R_s =	19^h	03^m	36^s	.38
Corr. for 5^h =			49	.28
	19	04	25	.66
S =	28	27	27	.35
	9	23	01	.69
C' =		1	32	.24
Mean Time =	9^h	21^m	29^s	.45

77. STANDARD TIME. — In a country extending over many degrees of longitude, and especially where there is much railroad travel, local time is not convenient, since it would be necessary for a traveler to set his watch frequently and by varying amounts. In the United States, for example, the following system is in use. The country is divided into four time belts known as the Eastern, Central, Mountain, and Pacific time belts. In all parts of the Eastern belt clocks are regulated to the local mean time of the 75° meridian west of Greenwich; in the Central belt the local time of the 90° meridian is used, and so on, the time in each belt being that of a meridian 15° west of the preceding. Hence in the Eastern belt, clocks are 5^h slower than Greenwich; in the Central belt 6^h slower than Greenwich, and so on.

CORRECTIONS TO OBSERVED ALTITUDES.

78. REFRACTION. — Rays of light from celestial bodies are refracted downward upon entering the earth's atmosphere, thus causing these bodies to appear higher above the horizon than they actually are. The amount of the refraction depends upon the apparent altitude of the body observed. The correction for this refraction, always subtractive from an observed altitude, may be taken from Table VII, p. 397, which gives the refraction for temperature 50 degrees F., and 30 inches barometric pressure. The actual refraction correction varies with the temperature and pressure, but in ordinary work it is sufficiently accurate to use the mean values given in the table.

79. **Parallax.** — Parallax is an apparent displacement of a body on the celestial sphere due to the fact that the observer is on the earth's surface instead of at its center. It is inappreciable in case of the stars, but is considerable for bodies in the solar system, being about 9 seconds for the sun and nearly a degree for the moon. The amount of the correction varies as the cosine of the altitude; hence it is zero when the body is in the zenith and a maximum in the horizon. The value of this correction for a body in the horizon, called the ***horizontal parallax***, is simply the angle at the body subtended by the earth's semi-diameter.

80. **DIP.** — The horizon which is visible to an observer at sea is a small circle below the true horizon. When the altitude of a body above the sea horizon is measured, as with the sextant, the true altitude is obtained by subtracting the angle of *dip*. This may be found approximately from the formula

$$\text{Dip (in minutes)} = \sqrt{\text{height of the observer's eye (in feet)}}$$

For example, if the eye of the observer were 64 feet above the surface of the sea, the visible horizon would be nearly 8 minutes of angle below the true horizon.

81. **SEMI-DIAMETER.** — In making observations upon the sun or the moon the angle should be measured to one edge (or limb) of the body and the observation reduced to the center by adding, or subtracting, the apparent semi-diameter. This quantity may be found in the Nautical Almanac.

82. **SUCCESSIVE APPROXIMATION.** — In nearly all astronomical work where several quantities are to be found by observation the value of each unknown depends upon one or more of the others. Hence it will often be necessary to obtain the results by successive approximation. For example, in computing the azimuth of Polaris, or any other circumpolar star, when it is

not at its greatest elongation, it is necessary to know both the latitude of the observer and the hour angle of the star. But in some of the best methods of finding the sidereal time, from which the star's hour angle can be obtained, it is necessary to know the approximate latitude, while in determining the latitude by some methods it is necessary to know the approximate sidereal time. If we can obtain an approximate value of the latitude, then using this value we may compute an approximate value of the sidereal time. From this value of the sidereal time we may obtain an approximate value of the star's hour angle and then recompute the latitude, this second value of the latitude being much more accurate than the first. If desired, the whole process may be repeated, giving still closer results. In this way we may obtain the value of either the latitude or the time with any degree of precision desired. It is always possible, however, to arrange the work so that very few approximations are necessary to give the desired precision.

83. HINTS ON OBSERVING. — In observations of the character here treated the stability of the instrument is of great importance. The support should be firm and care should be taken that the instrument is not disturbed during the observations. If it is necessary to set the tripod in soft ground it will be advisable to drive pegs several inches into the ground and to set the tripod legs in notches cut in the tops of the pegs. In order that the instrument may have time to settle into a stable position and also in order that it may have time to assume the temperature of the surrounding air it should be set in position a half hour or so previous to the time of the intended observations.

Careful attention should be paid to all adjustments of the instrument, especially if it is impossible to eliminate the instrumental errors by the method of observing. But when the best results are desired the observations should be conducted, if possible, in such a way as to eliminate any remaining error, even if the adjustments have been carefully made. In measuring altitudes the index correction should not be neglected, but should be determined each time the instrument is turned in azimuth.

The prismatic eyepiece is a necessary attachment to the transit when high altitudes are to be measured. By screwing this attachment to the eyepiece tube, altitudes as high as 75 degrees may be measured. It should be remembered that the prism inverts the image in the vertical direction but does not affect it in the horizontal direction; i.e., if this prism is attached to a transit having an inverting eyepiece, objects will appear right side up but the left and right sides will remain interchanged.

In making observations at night it is necessary to illuminate the field of view in order to make the cross-hairs visible. This is usually done by means of a ring-shaped diagonal mirror placed in the shade tube in front of the objective. If a lantern is held at one side of the objective the light is reflected into the telescope tube and gives a bright field, against which the cross-hairs are visible. If no such reflector is provided with the instrument a good substitute may be made by fastening a piece of tracing cloth or paper in front of the objective and cutting a hole about half an inch in diameter to admit light from the star. The light used to illuminate the field should be held so that it will not shine directly into the observer's eyes.

OBSERVATIONS FOR TIME.

84. Observations for Time by Transit of a Star across the Meridian. — The simplest way of accurately determining the error of a watch by observation with an engineer's transit is to set the instrument in the plane of the meridian* and to observe the time when some southern star called a *time star* crosses the vertical hair. The star selected for this observation should be near the equator so that its apparent motion is rapid. Stars near the pole move too slowly to permit of an accurate observation. The true time of transit of the star may be calculated as follows. The local sidereal time equals the right ascension of the star observed (see equation [19], p. 67); hence if only the sidereal time were desired the right ascension of the star as shown in the Nautical Almanac would be the sidereal time of the observation; and a comparison of this with the observed watch time

* For approximate methods of determining the direction of the meridian see Vol. I, Chapter VII.

would give the error of the watch on local sidereal time. But if mean time is desired it would be necessary to convert this sidereal time into the corresponding mean solar time, by equation [22] p. 71.

85. EXAMPLE. — Suppose that on April 5, 1902, in longitude $5^h\ 20^m$ West, the star α Hydræ is observed to cross the meridian at $8^h\ 48^m\ 58^s.5$, by an ordinary watch keeping Eastern time. From the Almanac we find that the star's right ascension is $9^h\ 22^m\ 48^s.4$ which is the local sidereal time at the instant of the observation. To reduce this to mean local time we must subtract from it the "right ascension of the *mean sun*" for the preceding local noon. From the Almanac we find that this right ascension for Greenwich noon is $0^h\ 51^m\ 24^s.6$. The increase in this right ascension during the interval between Greenwich noon and local noon is taken from Table III in the Appendix to the Nautical Almanac, which for $5^h\ 20^m$ gives $+\ 52^s.6$. This correction is simply the hourly increase multiplied by the number of hours in the longitude. This gives $0^h\ 52^m\ 17^s.2$ for the corrected right ascension of the sun. The remainder of the computation is as follows.

R. A. star =	9^h	22^m	$48^s.4$
R. A. sun =	0	52	17 .2
	8	30	31 .2
C' =		1	23 .6
Mean local time =	8	29	07 .6
Reduction to 75° Meridian time =		20	00 .0
Eastern time =	8^h	49^m	$07^s.6$
Watch time =	8	48	58 .5
Watch is slow			$9^s.1$

This method presupposes a knowledge of the direction of the meridian and therefore in many cases it could not be used.

86. CHOICE OF METHODS. — In determining the azimuth of a line it is desirable to use a method which will permit of an observation being made at any time and with sufficient precision for the purpose in hand. In order that the star's azimuth may be computed at any time except at elongation it is necessary to know the local sidereal time of the observation, since this is needed in computing the star's hour angle. Since the time of the observation will always be somewhat in error, it is advisable to use only close circumpolar stars (preferably Polaris) for the azimuth observation, so that errors in the time will have the least possible effect on the hour angle. It is also necessary to know the latitude of the observer. In order that the computed azimuth of Polaris shall be correct within about $1''$, it will in general be necessary to know the time within about 1^s and the latitude to the nearest minute of arc. To obtain the latitude and the time with the required accuracy it will usually be necessary to make special observations for these quantities, for latitudes scaled from maps cannot always be relied upon, and the time obtained from any source except direct observations is uncertain, and even if this were reliable an ordinary watch can hardly be depended upon to run accurately for any considerable length of time. Furthermore, the computed local time would depend upon a longitude scaled from a map and hence be doubly uncertain. For these reasons the best results will be obtained when the local time and the latitude are determined at the same time as the azimuth itself.

The simplest way of finding the time would be to observe the transits of several stars across the meridian. But since the direction of the meridian is not known with accuracy it will be necessary to resort to some method which will give the necessary precision without requiring that the instrument be turned into the meridian plane. Below are two methods which fulfill these requirements.

87. Time by Transit of a Star across the Vertical Circle through Polaris.* — Instead of observing the transit of a star across the meridian its transit may be observed across the vertical circle of a circumpolar star, preferably Polaris, and by computing a correction to the star's right ascension the sidereal time can be obtained. The method of observing is as follows. Set up the transit and level it; set the vertical cross-hair on Polaris and note the instant of bisection by a watch; then turn the telescope over about its horizontal axis, **being careful not to disturb its**

* This method is given by Mr. George O. James in the Jour. Assoc. Eng. Soc., Vol. XXXVII, No. 2, and is a modification of that of Professor Frederick H. Seares, given in Bulletin No. 5, Laws Observatory, University of Missouri.

azimuth, and set the vertical arc at the meridian altitude* of some southern star (called a *time-star*) which is about to cross the meridian; note the instant of transit of this star over the vertical cross-hair. The time of transit across the vertical circle through Polaris may differ by as much as 7 or 8 minutes from the time of transit across the meridian, and it will come earlier or later than the meridian passage according to whether Polaris is west or east of the meridian at the time of the observation. The altitude at which the telescope must be set in order to find the star may be found with sufficient accuracy from equation [12], p. 65. If the instrument has a vertical **circle** the altitudes of both Polaris and the time-star should be measured either just before or just after the time observations. It is advisable to eliminate instrumental errors by making a second observation, on a different time-star, with the telescope reversed.

From these two observed times, one on Polaris and one on the time-star, the error of the watch may be computed. Let R and R_0 = the right ascensions of the time-star and Polaris, S and S_0 = the sidereal times of transit, and t and t_0 = the hour angles of the two stars, the subscripts in each case referring to Polaris.

Then $$t = S - R$$

and $$t_0 = S_0 - R_0$$

$$\therefore\ t_0 - t = (R - R_0) - (S - S_0)$$

$S - S_0$ is the observed interval, expressed in sidereal units. If this is observed with an ordinary watch the interval must be reduced to sidereal time by a correction taken from Table III, Appendix to the Nautical Almanac. The equation may then be written in the form

$$t_0 - t = (R - R_0) - (T - T_0) - C \qquad [23]$$

where T and T_0 are the actual watch readings for the two observations, and C is the small correction to reduce $T - T_0$ to sidereal time.

---

* Computed beforehand approximately; see p. 80.

In Fig. 35 let P_0 be the position of Polaris; P, the pole; Z, the zenith; and S, the time-star. Also, let Z and Z_0 be the azimuths (angle P_0ZP); t and t_0 the hour angles; p and p_0 the polar distances; and z and z_0 the zenith distances of the two stars, those marked with subscripts referring to Polaris.

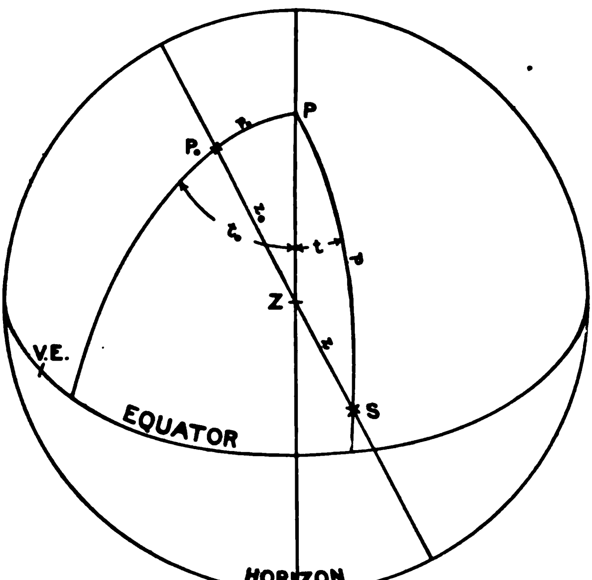


Fig. 35. Projection of the Sphere on the Plane of the Horizon.

In the triangle PP_0Z we have

$$\frac{\sin t_0}{\sin z_0} = \frac{\sin Z_0}{\sin p_0} \qquad [24]$$

and in triangle PZS,

$$\frac{\sin t}{\sin z} = \frac{\sin Z}{\sin p} \qquad [25]$$

Since the azimuth of the instrument has not changed

$$Z = 180° + Z_0$$

and

$$\sin Z_0 = - \sin Z$$

also, since the time-star is not far from the meridian, its zenith distance may be taken as its meridian zenith distance, i.e., $z = L - D$. If the zenith distance of Polaris has not been measured it may be conveniently found by $z_0 = 90° - (L - c)$, where c is the correction given in Table IV of the Appendix to the Nautical Almanac, or it may be taken as $+ p_0 \cos t_0$. By substituting these values in equations [24] and [25], and solving for $\sin t$, we may obtain

$$\sin t = - \sin p_0 \sin t_0 \sec D \sin (L - D) \sec (L - c)$$

Since t and p_0 are always small we may put $t = \sin t$ and $p_0 = \sin p_0$

$$\therefore\ t = - p_0 \sin t_0 \sec D \sin (L - D) \sec (L - c) \qquad [26]$$

or if the altitudes have been directly measured

$$t = - p_0 \sin t_0 \sec D \cos h \sec h_0 \qquad [27]$$

Since t_0 is at first unknown it is necessary to solve by a series of trials. We first obtain an approximate value of t_0 from

$$t_0' = (R - R_0) - (T - T_0) - C \qquad [28]$$

Using this value of t_0' in equation [26] we obtain t', which is an approximate value of t. A corrected value of t_0 is then found by

$$t_0 = t_0' + t' \qquad [29]$$

With this new value of t_0, t may be recomputed. If this second value of t differs much from the first it will be necessary to recompute t_0. We may estimate roughly, in advance, the value of t and add this (algebraically) to t_0', remembering that if t_0 is less than 12^h, t is negative. This will give a closer value of t_0 on the first approximation, and may frequently make it unnecessary to recompute t_0.

The final value of t, the hour angle of the time-star at the instant of observation, is the correction to be added algebraically to the star's right ascension to obtain the true local sidereal time of the observation.

EXAMPLE I.

Lat. 42° 21′ N. Long. $4^h\ 44^m\ 18^s$ W.

Date, May 8, 1906.

Observed time of bisection of Polaris	8^h	35^m	58^s
Observed time of transit of o Virginis	8	39	43
Diff.		3^m	45^s

$$L = 42^\circ\ 21'$$
$$D = +\ 9\ \ 15$$
$$L-D = 33^\circ\ 06'$$

$R =$	12^h	00^m	$26^s.3$
$R_0 =$	1	24	35.4
$R - R_0 =$	10	35	50.9
$T - T_0 =$		3	45.0
$C =$			.6
$t_0' =$	10	32	05.3
$=$	158°	01′.3	
$t' =$	$-$	19′.8	
$t_0' + t' =$	157°	41′.5	

$$p_0 = 71'.85$$
$$\log p_0 = 1.8564$$
$$\log \sec D = 0.0057$$
$$\log \sin (L - D) = 9.7373$$
$$\log \sec (L - c) = 0.1238$$
$$\log \sin t_0' = 9.5732$$
$$\log t' = 1.2964n$$
$$t' = -19'.79$$

New value of $\log \sin t_0' = 9.5793$.

This would increase $\log \sin t_0'$ 61 in units of the 4th place, giving $-20'.07$ for t, or $-80^s.3 = -1^m\ 20^s.3$.

The true sidereal time may now be found by simply subtracting $1^m\ 20^s.3$ from the right ascension of the time star. If the error of the watch on Eastern time is desired the complete reduction is as follows.

$R =$	12^h	00^m	26.3^s
$t =$		-1	20.3
$S =$	11	59	06.0
$R_s =$	3	02	23.6
	8	56	42.4
C', Table II, Appendix to N. A. =		1	27.9
	8	55	14.5
Reduction to 75° Meridian =		15	41.7
Eastern time =	8	39	32.8
Watch reading =	8	39	43.0
Watch fast			$10^s.2$

EXAMPLE 2.

Lat. 42° 02′ N. Long. 72° 21′.2 W.

Date, June 14, 1906.

Observed time bisection of Polaris	8^h	16^m	11^s
Observed time transit of π Hydræ	8	22	53.5
Diff.		6	42.5

$$
\begin{aligned}
R &= 14^h\ 01^m\ 02^s.7 \\
R_0 &= 1\ \ 25\ \ 04.2 \\
R - R_0 &= 12\ \ 35\ \ 58.5 \\
T - T_0 &= 6\ \ 42.5 \\
C &= 1.0 \\
t_0' &= 12\ \ 29\ \ 15.0 \\
&= 187°\ 18'.75 \\
t' &= +\ \ 12.54 \\
t_0' + t' &= 187°\ 31'.3
\end{aligned}
\qquad
\begin{aligned}
p_0 &= 71'.97 \\
\log p_0 &= 1.8572 \\
\log \sec D &= 0.0471 \\
\log \sin (L - D) &= 9.9680 \\
\log \sec (L - c) &= 0.1212 \\
\log \sin t_0' &= 9.1047n \\
\log \sin t' &= 1.0982 \\
t' &= +\ 12.54
\end{aligned}
$$

New log sin t_0 = 9.1170, increase = 123

New value of t = + 12′.90 = + $51^s.6$

$$
\begin{aligned}
R &= 14^h\ 01^m\ 02^s.7 \\
t &= +51.6 \\
S &= 14^h\ 01^m\ 54^s.3
\end{aligned}
$$

88. TIME BY TWO STARS AT EQUAL ALTITUDES. — In this method the time is determined by noting the instant when two stars are at the same altitude, one of the stars being east of the meridian (rising) and the other west (setting).

If the altitude of any star and the corresponding watch time were observed when the star was east of the meridian, and the time again observed, several hours later, when the star had this same altitude, but on the west side of the meridian, then since the apparent motion of the star is uniform the mean of the two observed times would be the watch time of meridian passage of the star. At the instant of meridian passage the sidereal time equals the right ascension of the star; hence the error of the watch may be found by reducing this sidereal time to the corresponding instant of mean time.

While such a determination is very accurate there is the disadvantage that several hours of time would be required to complete the observation. Therefore instead of observing the same star in each case it is more convenient to select two stars having nearly the same declination but differing several hours in right ascension, one being east of the meridian and the other west, and to observe the instant when these two stars have equal altitudes. In this case the sidereal time will not exactly equal the mean of the right ascensions of the two stars but will require a correction for the difference in hour angle due to the difference in declination.

The observation is made by setting the horizontal cross-hair a little above the easterly star (a few minutes before the moment of equal altitudes) and noting the time when the star passes the horizontal hair; the telescope is then turned to the westerly star, **without changing the inclination of the telescope,** and the time noted when this star passes the horizontal hair. The altitude need not be read, because its actual value is not needed, the essential requirement being that the inclination of the telescope shall be the same for both observations.

In Fig. 36 *NESW* represents the horizon, Z the zenith, P the

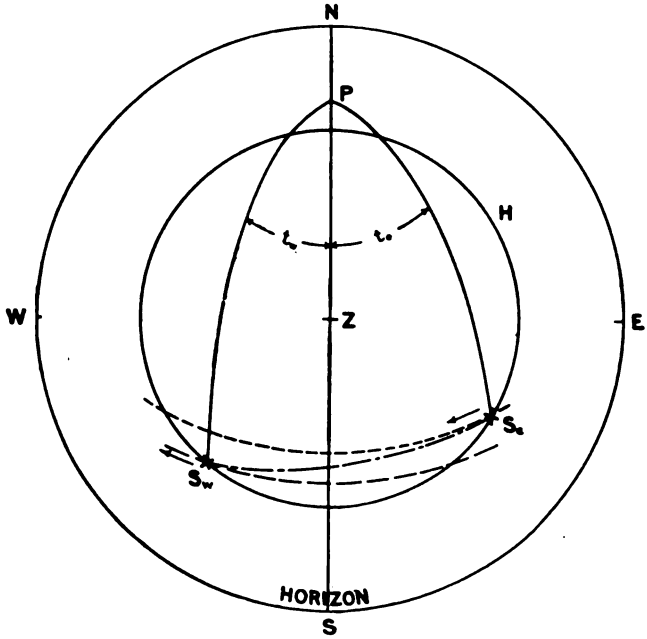


FIG. 36. PROJECTION OF THE SPHERE ON THE PLANE OF THE HORIZON.

pole, S_e the easterly star, and S_w the westerly star. Let t_e and t_w be the hour angles of S_e and S_w respectively, and let $H\ S_e\ S_w$

be a circle parallel to the horizon, i.e., a *circle of equal altitudes.* In the diagram the star S_w is farther from the pole than S_e, i.e., it has a smaller declination and hence has a smaller hour angle, t_w.

The fundamental equation of spherical trigonometry giving the relation between the altitude, the latitude, the declination, and the hour angle is

$$\sin h = \sin D \sin L + \cos D \cos L \cos t \qquad [30]$$

To find the effect upon the hour angle produced by a change in declination we differentiate equation [30] with respect to D and t as variables, obtaining

$$0 = \sin L \cos D \, dD - \cos D \cos L \sin t \, dt - \cos L \cos t \sin D \, dD$$

Solving for dt we obtain

$$dt = dD\left(\frac{\tan L}{\sin t} - \frac{\tan D}{\tan t}\right) \qquad [31]$$

In this formula dt and dD are infinitesimals, dD being a change in declination and dt the resulting change in the hour angle. To apply this formula to the case of two stars at equal altitudes we have from equation [18]

$$\begin{aligned} S &= R_w + t_w \\ S &= R_e - t_e \\ \hline S &= \frac{R_w + R_e}{2} + \frac{t_w - t_e}{2} \end{aligned}$$

i.e., the correction to the mean right ascension of the two stars is half the difference of the hour angles. This half difference of the hour angles is the change produced by half the difference in declination. Since these quantities are small we may substitute $\frac{t_w - t_e}{2} = e$ for the differential dt, and $\frac{1}{2}(D_e - D_w)$ for dD. D may be taken as the mean of D_e and D_w, and t as the mean of the two hour angles. The value of t depends not only upon the difference in the right ascensions, but also upon the time elapsed between the observations and is nearly equal to

$$\frac{1}{2}(R_e - R_w) + \frac{1}{2}(T_w - T_e)$$

when the east star is observed first, and

$$\frac{1}{2}(R_e - R_w) - \frac{1}{2}(T_e - T_w)$$

when the west star is observed first, T_w and T_e being the actual watch readings for the two observations.

Let $\qquad e = $ the correction to $\frac{1}{2}(R_e + R_w)$

$$dt = -15\,e$$

$$d = \frac{D_e - D_w}{2t}\text{*}$$

$$A = -\frac{t}{15 \sin t}$$

$$B = \frac{t}{15 \tan t}$$

Substituting these values, equation [31] then becomes

$$e = A\,d \tan L + B\,d \tan D \qquad [32]$$

which is known as the *equation of equal altitudes.* The factor 15 is introduced to reduce e to seconds of time, $D_e - D_w$ being in seconds of arc. Values of log A and log B will be found in Table VIII, p. 398.

* We may think of the star S as actually moving from its first position at S_e to that of S_w, $\frac{D_e - D_w}{2t}$ corresponding to the *hourly change* in declination.

EXAMPLE 1.

OBSERVATION FOR TIME BY EQUAL ALTITUDES.

Lat. 42° 28′ N. Long. 71° 03′ 45″ W.

Date, December 18, 1904.

STAR	RIGHT ASCENSION	DECLINATION	WATCH
α Orionis (E)	$5^h\ 50^m\ 02^s.6$	+7° 23′ 13″.8	$6^h\ 50^m\ 40^s$
α Aquilæ (W)	19 46 07 .3	+8 37 10 .5	6 41 18 .5
Average	0 48 05 .0	+8 00 12 .2	6 45 59 .2
Difference	10 03 55 .3	−1 13 56 .7	9 21 .5
	9 21 .5		
$2t =$	$9^h\ 54^m\ 33^s.8$		

$$\log (D_e - D_w) = 3.6471n$$
$$\log 2t = 0.9961$$
$$\log d = 2.6510n$$

$\log d = 2.6510n$
$\log A = 9.5354n$
$\log \tan L = 9.9615$
$\log$ 1st term $= 2.1479$
$+ 140^s.6$
$- 5.8$
$e = + 134.8$
$= + 2^m\ 14^s.8$

$\log d = 2.6510n$
$\log B = 8.9672$
$\log \tan D = 9.1480$
$\log$ 2d term $= 0.7662n$
$- 5^s.8$

Average Right Ascension	0^h	48^m	$05^s.0$
e	+	2	14 .8
Sidereal time	0	50	19 .8
Right Ascension of Mean Sun	17	47	27 .1
	7	02	52 .7
Reduction to solar		1	09 .3
	7	01	43 .4
Reduction to 75th Meridian time		15	45 .0
Eastern Standard time	6	45	58 .4
Observed time	6	45	59 .2
Watch fast			0 .8

Strictly speaking the quantity $(T_w - T_e)$ should be reduced to sidereal interval but the effect on the value of log d is usually so small that this correction need not be made.

EXAMPLE 2.

OBSERVATION FOR TIME BY EQUAL ALTITUDES.

Lat. $42° 21'$ N. Long. $4^h 44^m 18^s$ W

STAR	RIGHT ASCENSION	DECLINATION	WATCH
α Ceti (E)	$2^h 57^m 22^s.1$	$+ 3° 43' 69''.1$	$5^h 18^m 00^s$
δ Aquilæ (W)	19 20 43 .6	+ 2 55 44 .0	5 22 13
Averages	23 09 02 .8	+ 3 19 56 .6	5 20 06.5
Differences	7 36 38 .5	$+ 0° 48' 25''.1$	04 13
	4 13 .		
$2t =$	$7^h 40^m 51^s.5$		

$$\log (D_e - D_w) = 3.4632$$
$$\log 2t = 0.8854$$
$$2.5778$$

$\log d = 2.5778$	$\log d = 2.5778$
$\log A = 9.4818n$	$\log B = 9.2107$
$\log \tan L = 9.9598$	$\log \tan D = 8.7652$
$\log$ 1st term $= 2.0194n$	$\log$ 2d term $= 0.5537$
$- 104.6$	$+ 3.6$
3.6	
$e = - 101.0$	

Average Right Ascension	$23^h 09^m 02^s.8$
e	$-$ 1 41 .0
Sidereal time	23 07 21 .8

This, reduced to standard time, shows the watch to be $5^s.0$ fast. (The right ascension of the mean sun at local mean noon is $17^h 30^m 43^s.2$.)

The advantage of the above method is that the absolute value of the altitude is not used, so that errors due to the uncertainty in the refraction and index corrections do not affect the result.

It is well to compute beforehand the approximate time of equal altitudes and to observe the first star a few minutes before this time arrives. In this way the interval between the observations may be kept conveniently small. This interval need not be greater than about 5 minutes.

It is evidently immaterial whether the east star is observed before or after the west star, provided we add $\frac{1}{2}(T_w - T_e)$ in the first case and subtract it in the second. If one of the stars is bright and the other faint it will be well to observe the bright star first. The faint star can be found by estimating the time when it should cross the horizontal hair.

The declinations should not differ by more than about 3 degrees if great precision is desired, but if the difference is as great as 5 degrees the resulting error will not be large enough to have an appreciable effect upon an azimuth observation made with an ordinary engineer's transit. This method is most accurate when the observer is near the equator because an error in the altitude has a small effect on the azimuth; it is practically useless in very high latitudes because the stars move rapidly in azimuth but slowly in altitude.

Equation [31] may be put in the form

$$e = -\frac{1}{2}(D_e - D_w)\left[\frac{\tan L}{\sin t} - \frac{\tan D}{\tan t}\right] \qquad [33]$$

e and $(D_e - D_w)$ both being expressed in seconds of time. In this form the solution may be effected without the tables of log A and log B.*

Table 1, p. 90, contains a list of stars which may be used in observing by this method. All of the stars in this list will be found in the American Ephemeris and Nautical Almanac. A larger number of pairs may be obtained by using also the list given in the Berliner Astronomisches Jahrbuch.

A star chart of convenient size to take into the field, such as Young's "Uranography," † is a great aid in identifying the stars.

89. TIME BY A SINGLE ALTITUDE. — A fair approximation to the sidereal time may be determined by taking the altitude of a star or a planet which is nearly east or west and noting the time corresponding to this altitude. The hour angle of the star may then be computed by the formula

$$\tan \tfrac{1}{2} t = \sqrt{\left(\frac{\cos s \sin (s - h)}{\sin (s - L) \cos (s - p)}\right)} \qquad [34]$$

* This is the solution given by Professor Comstock in "Field Astronomy for Engineers," published by John Wiley & Sons.

† Published by Ginn & Co., Boston.

in which t = the hour angle of the star
h = the altitude of the star
p = the polar distance of the star
L = the observer's latitude

and $$s = \frac{L + h + p}{2}$$

If the star is east of the meridian the computed hour angle should be subtracted from 24^h or else considered as negative. With this hour angle we find the sidereal time from

$$S = R + t$$

To obtain the best results the observation should be made when the star is about due east or west, but not too near the horizon.

EXAMPLE.

Observed altitude of Jupiter (east), January 9, 1907.

Lat. = 42° 18′.0 Long. = 71° 17′.5

Obs. Alt.	= 44° 55′	Time $7^h\ 32^m\ 02^s$	
I. C.	= −1′		
Refrac.	= −1′		
h	= 44° 53′*	D =	+ 23° 18′ 09″.5
		p =	66° 41′ 50″.5

L = 42° 18′.0		
h = 44 53 .0	$s - L$ = 34° 38′.4	
p = 66 41 .8	$s - h$ = 32 03 .4	
152 112 .8	$s - p$ = 10 14 .6	
s = 76 56 .4		

log cos s	= 9.35405	log sin $(s - L)$	= 9.75467
log sin $(s - h)$	= 9.72490	log cos $(s - p)$	= 9.99302
	9.07895		9.74769
	9.74769		
	2)9.33126		
	9.66563		

$\frac{1}{2}t$ = 24° 50′ 48″
t = 49° 41′ 36″
= $3^h\ 18^m\ 46^s.4$ (east)
R. A. = 6 18 59 .8
Sidereal time = $3^h\ 00^m\ 13^s.4$ for watch time $7^h\ 32^m\ 02^s$.

* For the planet Jupiter the parallax correction is negligible. For Mercury and Venus this correction should not be neglected.

TABLE 1.

STARS FOR TIME BY EQUAL ALTITUDES.

Stars	Magnitude	Approximate Sidereal Time of Equal Altitudes	Stars	Magnitude	Approximate Sidereal Time of Equal Altitudes
γ Tauri . . . ε Delphini . .	3.8 4.0	0h 14m	γ¹ Leonis . . . α Arietis . . .	2.5 2.1	6h 12m
γ Tauri . . . α Delphini . .	3.8 3.9	0 25	α Hydræ . . . ε Eridani . . .	2.1 3.7	6 22
γ Tauri . . . ε Pegasi . . .	3.8 2.4	0 46	γ¹ Leonis . . . ε Arietis . . .	2.5 4.6	6 35
β Eridani . . . β Aquarii . . .	2.9 2.9	1 16	ρ Leonis . . . f Tauri . . .	4.0 4.3	7 02
α Tauri . . . α Pegasi . . .	1.0 2.5	1 42	α Leonis . . . γ Tauri. . . .	1.3 3.8	7 15
β Tauri. . . . λ Pegasi . . .	1.8 4.1	2 09	δ Leonis . . . α Tauri . . .	2.7 1.0	7 42
ν Orionis . . . α Pegasi . . .	4.5 2.5	2 31	β Leonis . . . γ Tauri . . .	2.2 3.8	8 00
γ Geminorum . α Pegasi . . .	2.0 2.5	2 43	β Leonis . . . I I Orionis . .	2.2 4.7	8 22
α Leporis . . β Ceti	2.7 2.2	3 01	υ Leonis . . . ε Orionis . . .	4.4 1.8	8 30
α Canis Majoris β Ceti	−1.4 2.2	3 34	η Virginis . . δ Orionis . . .	4.0 2.3	8 51
β Geminorum . α Andromedæ .	1.2 2.1	3 52	γ Virginis . . ε Orionis . . .	2.9 1.8	9 04
β Canis Minoris δ Piscium . .	3.1 4.8	4 00	α Boötis . . . α Tauri. . . .	0.2 1.0	9 16
β Canis Minoris ε Piscium . .	3.1 4.3	4 08	θ Virginis . . δ Orionis . . .	4.6 2.3	9 26
α² Geminorum . β Andromedæ .	1.9 2.2	4 20	α Virginis . . κ Orionis . . .	1.1 2.3	9 34
α Arietis . . . δ Geminorum .	2.1 3.5	4 35	α Serpentis . . α Orionis . . .	2.7 0.9	9 51
α Canis Minoris γ Ceti	0.5 3.6	5 00	β Libræ . . . β Orionis . . .	2.9 0.3	10 12
α Hydræ . . . θ¹ Ceti	2.1 3.6	5 20	α Corona Borealis β Tauri. . . .	2.3 1.8	10 29
ε Leonis . . . β Arietis . . .	3.2 2.8	5 39	α Boötis . . . δ Geminorum .	0.2 3.5	10 47
ε Leonis . . . α Arietis . . .	3.2 2.1	5 50	ρ Boötis . . . α² Geminorum	3.6 1.9	11 00
γ¹ Leonis . . . β Arietis . . .	2.5 2.8	6 02	β Herculis . . η Geminorum .	2.8 3.5	11 20

TABLE 1.—*Continued.*

Stars	Magnitude	Approximate Sidereal Time of Equal Altitudes	Stars	Magnitude	Approximate Sidereal Time of Equal Altitudes
α Serpentis α Canis Minoris	2.7 0.5	11^{h} 35^{m}	α^2 Capricorni α Virginis	3.7 1.1	16^{h} 52^{m}
β Herculis δ Geminorum	2.8 3.5	11 51	β Aquarii θ Virginis	2.9 4.6	17 18
α Serpentis β Cancri	2.7 3.8	12 02	β Aquarii ζ Virginis	2.9 3.6	17 40
α Serpentis ε Hydræ	2.7 3.5	12 11	α Pegasi ε Virginis	2.5 3.1	17 54
β Libræ α Hydræ	2.9 2.1	12 20	λ Pegasi α Boötis	4.1 0.2	18 22
β Herculis γ Cancri	2.8 4.9	12 32	ζ Capricorni δ Scorpii	3.8 2.6	18 36
μ Herculis β Geminorum	3.5 1.2	12 42	λ Aquarii β Libræ	3.8 2.9	18 58
ζ Ophinchi α Hydræ	2.8 2.1	13 02	α Andromedæ ε Boötis	2.1 2.6	19 21
κ Ophinchi ο Leonis	3.4 3.8	13 16	α Andromedæ α Corona Borealis	2.1 2.3	19 45
α^1 Herculis α Leonis	3.2 1.3	13 33	μ Andromedæ δ Boötis	4.0 3.5	19 56
α Ophinchi α Leonis	2.2 1.3	13 47	α Pegasi α Ophinchi	2.5 2.2	20 11
ι Aquilæ α Hydræ	4.0 2.1	13 57	γ Pegasi α Ophinchi	2.8 2.2	20 46
δ Aquilæ ε Hydræ	3.5 3.5	14 08	β Ceti μ Sagittarii	2.2 4.1	21 16
ι Aquilæ δ Crateris	4.0 3.9	14 38	γ Pegasi ζ Aquilæ	2.8 3.1	21 33
α Aquilæ ο Leonis	0.9 3.8	14 44	α Ceti κ Ophinchi	2.6 3.4	22 05
α Aquilæ α Leonis	0.9 1.3	15 01	β Trianguli β Lyræ	3.1 3.6	22 24
η Serpentis η Virginis	3.5 4.0	15 23	β Arietis γ Sagittæ	2.8 3.6	22 50
η Serpentis γ Virginis	3.5 2.9	15 32	α Ceti δ Aquilæ	2.6 3.5	23 08
γ Aquilæ β Leonis	2.8 2.2	15 51	α Ceti β Aquilæ	2.6 3.9	23 29
α Delphini β Leonis	3.9 2.2	16 09	η Tauri γ Sagittæ	3.1 3.6	23 41

OBSERVATIONS FOR LONGITUDE.

90. **METHODS OF DETERMINING LONGITUDE.**— Rough determinations of longitude, sufficiently accurate however for correcting the quantities given in the Ephemeris, may be made by means of a watch or a chronometer. If the error of the watch on the mean solar time of any meridian is obtained by one of the methods described, and the watch then carried to a second meridian and the observations repeated, the difference of the errors at the two places is the difference in longitude, provided the watch has run correctly on mean solar time during the interval between the observations. If the rate of the watch is known, or can be found, it should be allowed for. Unless the rate of the watch is unusually large, however, the resulting error in the longitude will not produce an appreciable error in the results of observations for time or azimuth.

The longitude could be found in exactly the same way if sidereal time were observed except that it would be necessary to use a sidereal chronometer or else allow for the error of the watch on the sidereal rate.

If the Standard time can be obtained accurately at a telegraph station, a comparison of this Standard time with the observed local time will give the difference in longitude between the Standard Meridian and the place.

OBSERVATIONS FOR LATITUDE.

91. **ACCURACY REQUIRED IN THE LATITUDE.** — In order to obtain the time or the azimuth by the methods described in this chapter (except time by transit across the meridian) it is necessary to know the approximate latitude of the place of observation. If an observation can be made on Polaris at culmination, or on the sun at noon, the resulting latitude will be sufficiently accurate for the purpose. (See Vol. I, Arts. 216–7, p. 196.) It is easily seen from equation [15] that in order to obtain the azimuth within about one second it is necessary to know the latitude only to the nearest minute, for places anywhere in the United States. If the point of observation is a triangulation station the latitude will of course be accurately known.

92. **LATITUDE BY ALTITUDE OF POLARIS.** — If the latitude is not known it may be determined conveniently from Polaris when the time observations are made. By means of transits over the vertical circle through Polaris (Art. 87, p. 77), or else by equal altitudes (Art. 88, p. 82), the sidereal time is determined. Just before or just after these time observations a series of altitudes on Polaris is taken, the time being noted at each pointing. The latitude may then be deduced as follows. With an approximate value of the latitude compute the sidereal time and thence the hour angle of the star corresponding to the mean of the observed times. If nothing is known in regard to the latitude the measured altitude may be taken as approximately equal to it. As soon as the approximate value of the hour angle is known a closer value of the latitude may be estimated, and then a closer value for the hour angle computed. The observed altitudes must be corrected for refraction and the mean taken. The latitude may then be computed by the series*

$$L = h - p \cos t + \tfrac{1}{2} \sin 1' \, p_0^2 \sin^2 t \tan h \; . \; . \; . \qquad [35]$$
$$(\log \tfrac{1}{2} \sin 1' = 6.1627 - 10)$$

The polar distance, p, is in minutes of arc. The latitude thus found may be used in the formulas for time and for azimuth.

If the latitude can be found from both southern and circumpolar stars the instrumental errors will be eliminated by taking the mean, for if the altitudes are all too large by a constant amount this will make the latitudes derived from southern stars too small, while those derived from northern stars will be too large.

93. **LATITUDE BY ALTITUDE OF THE TIME-STAR.** — The observation on a southern star when on the meridian would consist in measuring its altitude and computing the latitude by equation [12]. If the line of sight of the transit is in the plane of the meridian the vertical angle is measured when the star crosses the vertical hair. If the direction of the meridian is not even approximately known the star may be followed until its **maxi-**

* The derivation of this formula may be found in Chauvenet's "Spherical and Practical Astronomy," Vol. I, p. 253.

mum altitude is reached, which will be the altitude desired. In such cases as we are now considering it will often be inconvenient to measure the star's meridian altitude. If desired the altitude may be observed at any time within a few minutes of the time of transit, provided the sidereal time of the observation be accurately known, and the maximum altitude may be computed. The path of the star is a curve which becomes horizontal at the meridian. The amount by which the star is below the maximum altitude at any instant may be found by the expression

$$C = 112.5 \times t^2 \times \frac{\cos L \cos D}{\cos h} \sin 1'' \qquad [36]$$

$$\log (112.5 \times \sin 1'') = 6.7367$$

in which t is the interval before or after meridian passage expressed in seconds of time; h is the measured altitude of the star; and D is its declination. The latitude, L, is only approximately known, but since the correction itself is small, an error in the value of L will have but a slight effect on the final result. If desired a second approximation can always be made, giving a closer value for the correction.

Such observations as this may conveniently be made on the southern stars used in determining time by the method of Art. 87, p. 77. The observations should not be made when the star is more than 30^m from the meridian. In the following example all of the observations were made at the same place and on the same date for the purpose of determining the time and the latitude simultaneously.

EXAMPLE.

January 9, 1907.

Observed altitudes of Polaris near upper culmination.

Watch	6^h 49^m 26^s	Alt.	$43°$	$28'.5$	I. C. = − 1′
	51 45			28 .5	
	54 14			28	
	56 45			28	

Observation for time by method of Art. 87, p. 77.

Watch time of bisection of Polaris = 7^h 07^m 20^s
Watch time of transit of γ Ceti = 7 09 02

Observed altitudes of γ Ceti near meridian.

Watch	$7^h\ 10^m\ 00^s$	Alt. $50°\ 33'$	I. C. = $-1'$
	$11\ \ 23$	33.5	
	$13\ \ 30$	33	

Observed altitudes of Polaris.

Watch	$7^h\ 18^m\ 00^s$	Alt. $43°\ 26'$	I. C. = $-1'$
	$19\ \ 15$	26	

Observation for time.

Watch time of bisection of Polaris = $7^h\ 22^m\ 00^s$
Watch time of transit of α Ceti = $7\ \ 27\ \ 42$

Observed altitudes of α Ceti near meridian.

Watch	$7^h\ 28^m\ 30^s$	Alt. $51°\ 26'$	I. C. = $-1'$
	$35\ \ 45$	24	
	$39\ \ 10$	22	
	$40\ \ 37$	21	

From the Nautical Almanac we find that the right ascension of γ Ceti is $2^h\ 38^m\ 28^s.6$; the declination is $+\ 2°\ 50'\ 30''$. The right ascension of Polaris is $1^h\ 25^m\ 49^s.4$; the declination is $+\ 88°\ 48'\ 51''.1$. Since L is not known we may use an approximate value derived from the observation; we know that the pole star is not far from its upper culmination because the sidereal time must be nearly $2^h\ 38^m\ 28^s.6$ and hence the hour angle of Polaris is about 1 hour. (Equation [18].)

Observed altitude	=	$43°\ 28'$
Refraction correction	=	-1
Index correction	=	-1
Polar distance	=	$-\ 1°\ 11'$
Latitude	=	$42°\ 15'$ (approx.)

Also, the term $(L - c)$ may be only approximately determined; but since we have measured the altitude directly it is not necessary to compute $(L - c)$, but the measured altitude $43°\ 28'$ corrected for refraction may be used in computing t from the transit of γ Ceti (Equation [27]). From the above data we find that the local sidereal time corresponding to the watch reading $7^h\ 09^m\ 02^s$ is $2^h\ 37^m\ 14^s.1$.

Reducing the first set of observations of Polaris we find for the mean of the altitudes $43°\ 28'.2$. Correcting for index error and refraction we have $43°\ 26'.2$ as the reduced altitude. The average watch reading is $6^h\ 54^m\ 02^s.5$. To obtain the hour angle of the star we find the watch interval between the transit of γ Ceti and the altitude observations to be $7^h\ 09^m\ 02^s$ — $6^h\ 54^m\ 02^s.5 = 14^m\ 59^s.5$. This interval in sidereal minutes and seconds is $15^m\ 02^s.0$. Subtracting this from $2^h\ 37^m\ 14^s.1$ we obtain $2^h\ 22^m\ 12^s.1$ for the sidereal time of the observation for altitude. By equation [**18**] the hour angle of Polaris is $0^h\ 56^m\ 22^s.7$ or, in arc, $14°\ 05'.7$. The calculation of L is then as follows.

$p_0 = 71'.15$	$\log p_0 = 1.8522$	$\log \text{const.} = 6.1627$
	$\log \cos t = 9.9867$	$\log p_0^2 = 3.7044$
	1.8389	$\log \sin^2 t = 8.7734$
		$\log \tan h = 9.9762$
		8.6167
$-p_0 \cos t = -69'.01$		
		2d term $= +0'.04$

Hence $L = 43°\ 26'.2 - 1°\ 09'.0 = 42°\ 17'.2$ (a).

A recomputation will show no appreciable change in the resulting sidereal time as found from the transits of α and γ Ceti.

From the watch times of the altitudes of γ Ceti we find the hour angle to be as follows.

Red. Alt.	Sid. time	Hour angle $= t$
$50°\ 31'.2$	$2^h\ 38^m\ 12^s.3$	$-0^m\ 16^s.3$
$31\ .7$	$39\ \ 35\ .5$	$+1\ \ 06\ .9$
$31\ .2$	$41\ \ 42\ .8$	$+3\ \ 14\ .2$

For the last altitude we find the correction as follows.

$$\begin{aligned}
\log \cos L &= 9.8691\\
\log \cos D &= 9.9995\\
\log \sec h &= 0.1967\\
\log \text{const} &= 6.7367\\
2 \log t &= 4.5765\\
&\quad 1.3785\\
23''.9 &= 0'.4
\end{aligned}$$

Since this term varies as the square of the number of minutes from the meridian it is evident that the corrections for the other two observations are negligible. Hence the last altitude is $50°\ 31'.6$ and the mean of the three is $50°\ 31'.5$. The latitude is then found as follows.

Meridian altitude	$50°\ 31'.5$
Declination	$2\ \ 50\ .5$
Co-latitude	$47\ \ 41\ .0$
Latitude	$42°\ 19'.0$ (b)

The mean of the two results for latitude (a) and (b) is $42°\ 18'.1$.

OBSERVATIONS FOR AZIMUTH.

94. Observations for Azimuth on a Circumpolar Star at any Hour. — In determining the azimuth of a line of a triangulation the process would consist in first obtaining the angle between an azimuth mark and a star, and from this angle calculating the azimuth of the mark. This azimuth combined with the angle between the mark and the triangulation signal would give the desired azimuth. The method of observing the angle between the mark and the star is similar to that of measuring a horizontal angle in triangulation work, except that since the star is continually changing its azimuth it is necessary to note the time of each pointing upon the star.

95. The Azimuth Mark. — It is not often convenient to use a triangulation point for a mark in such observations; hence a special mark is used which can be sighted on both in daylight and at night. The azimuth mark may consist of a lantern set in a box so that the light may shine through a small aperture which will be visible to the observer. Some sort of target is painted on the side of the box toward the observer for use in the daytime. The aperture for the night observations should be small so that it appears to be a point of light like the star itself, not large so as to give a blurred appearance. The angle subtended by the diameter should be between $1''.0$ and $0''.5$. The following sizes are given by the U. S. Coast and Geodetic Survey.

TABLE 2.

Apertures for Azimuth Mark.

Distance of Mark		Diameter of Aperture	
Kilometers	Statute Miles	Maximum	Minimum
		mm.	mm.
1.5	0.9	7	4
2.0	1.2	10	5
2.5	1.6	12	6
3.	1.9	15	8
4.	2.5	19	10
6.	3.7	29	14
10.	6.2	48	24

The mark must be placed far enough from the instrument so that it will not be necessary to alter the focus in changing from the star to the mark. Any change in the focus may disturb the line of sight. A distance of a mile will ordinarily be sufficient; it will sometimes be necessary, however, to use shorter distances, on account of the difficulty of placing the mark in a good position.

96. CIRCUMPOLARS. — Since the star is changing its azimuth, and also since the latitude and the time are somewhat uncertain, it is advisable to use only very close circumpolars for this observation. In the Nautical Almanac are given the positions of five circumpolars, any one of which is suitable for this observation. These stars (Fig. 37) are α Ursæ Minoris (Polaris), 51 Cephei,

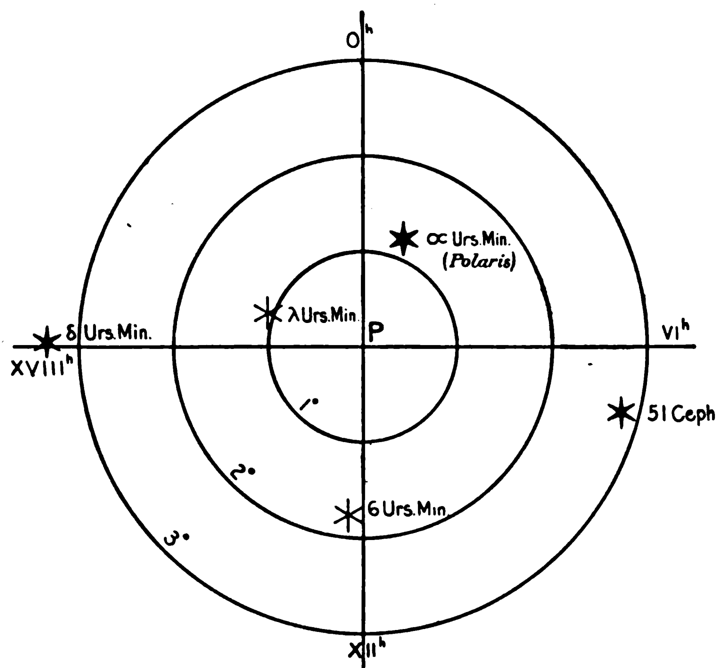


FIG. 37. CIRCUMPOLAR STARS.

6, δ, and λ Ursæ Minoris. Polaris is the only one, however, that is bright enough to be easily seen with the naked eye. By the aid of the diagram and using Polaris as a point of reference it is not difficult to identify the four other fainter stars in a telescope of moderate power.

97. **THE OBSERVATION.** — The details of the observation will vary, depending upon the type of instrument used. With either a direction or a repeating instrument the following precautions are important. Since the star is at a high altitude as compared with the (angular) altitude of ordinary triangulation points, it is of unusual importance to keep the instrument leveled and, by means of a striding level, to measure any small deviation of the horizontal axis from the true horizontal position while the telescope is pointing in the direction of the star. The instrument should be firmly mounted and should be protected from unequal heating of its parts, as from the body of the observer or from the lamp used for illuminating the cross-hairs or for reading the circle. The instrument should be handled with the greatest care, the clamps and tangent screws being used in such a way as to avoid any lateral pressure, which would tend to disturb the instrument. In short, all of the precautions which need to be taken in refined triangulation work will apply here. The angles should be measured as quickly as is consistent with careful and accurate work. The longer time there is between pointings the greater the opportunity for the instrument to shift its position and so introduce errors into the results.

98. **Observation with a Direction Instrument.** — In observing with the direction instrument each set will consist of, say, two pointings on the azimuth mark, with the corresponding circle readings; then three or four pointings on the star, the time being noted at each bisection and the circle read; and finally two more pointings (and circle readings) on the mark. Readings of the striding level should be taken, with the level in both direct and reversed positions, when the telescope is sighted in the direction of the star. The level should be placed in position on the axis some time before it is to be read in order to give it time to settle to the true reading. If the mark is far above or below the horizon, level readings should also be taken when pointing on the mark. The telescope is then reversed in its supports, as in triangulation, and the same process repeated. This completes a single set. The altitude of the star should be read, to the nearest minute, at

the end of each half-set, as it will be needed in the computations. The circle should be shifted between sets so that the readings may be distributed over the whole circumference.

99. Observations with a Repeating Instrument. — If a repeating instrument is used the program may be similar to that frequently used in triangulation, i.e., six repetitions left to right, telescope direct, and six repetitions right to left, telescope reversed. The striding level is read in both positions, at the beginning of the half set and at the end, while the telescope is pointing at the star. At each pointing on the star the time is noted. The circle is read as usual, only at the beginning and the end of the half set. In the first half set all pointings on the star would be made, say, by use of the lower clamp and tangent screw, while all pointings on the mark are made by use of the upper clamp and tangent screw. In the second half set the pointings on the star would be made using the upper clamp and those on the mark using the lower clamp. In order to eliminate as far as possible any constant error of the clamps and tangent screw the plates should always be turned in a clockwise direction and the setting made with the tangent screws in the direction which compresses the spring, so as to insure a positive working of the tangent screw.

100. CALCULATING THE AZIMUTH OF THE STAR. — The azimuth of the star is derived from the equation *

$$\tan Z = \frac{\sin t}{\cos L \tan D - \sin L \cos t} \qquad [17]$$

see Art. 63, p. 66. From this formula it will be seen that the latitude and the star's hour angle must both be known. It will generally be necessary to determine these at the same time as the azimuth by special observations. (See Arts. 87-8 and 92-3.)

101. Curvature Correction. — It would be a long process if we were to calculate the azimuth of the star separately for each pointing made upon it, but it is not necessary to do this provided we make a correction for the curvature of the star's path. Since

* If extreme accuracy is not required in the computed azimuth the following convenient formula may be used. $Z = p \sin t \sec h$.

the star moves slowly and the curvature of the path is slight we may average all of the observed angles and calculate the azimuth corresponding to the mean hour angle; then to this we may apply a correction for curvature by means of the approximate formula* given below.

Let T_1, T_2, T_3 = the observed times; $T_0 = \frac{T_1 + T_2 + \ldots}{n}$ = the mean of the observed times; Z_1, Z_2, etc. = the corresponding azimuths of the star; and Z_0 = the azimuth at the instant T_0.

Then $$\frac{Z_1 + Z_2 + \ldots Z_n}{n} = Z_0 - \tan Z_0 [6.73672] \frac{1}{n} \cdot \Sigma (T - T_0)^2 \quad [37]$$

In this equation [6.73672] is the logarithm of a constant and $\Sigma (T - T_0)^2$ = the sum of the squares of the intervals $(T_1 - T_0)$, $(T_2 - T_0)$, etc. expressed in seconds of time. That is, we may compute the azimuth of the star, Z_0, at the instant corresponding to the mean of the observed times and then correct it by means of the last term of the equation. If an ordinary watch is used the observed intervals $(T_1 - T_0)$, etc., should be reduced to sidereal intervals.

Where extreme precision is not necessary the equation may be put in the form

$$\frac{Z_1 + Z_2 + \ldots Z_n}{n} = Z_0 - \tan Z_0 [0.2930] \frac{1}{n} \Sigma (T - T_0)^2 \quad [38]$$

where the intervals $(T - T_0)$ are in minutes of time and the correction is in seconds of arc. The number in brackets is a logarithm. $(T - T_0)$ may be taken to the nearest minute of time, or the nearest tenth of a minute, according to the precision needed. Since the term itself is often but a few seconds, comparatively large errors in the intervals $(T - T_0)$ have but a small effect on the correction.

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* For the derivation of this formula see Doolittle's "Practical Astronomy," § 306, p. 537, or Hayford's "Geodetic Astronomy," § 194, p. 213.

102. Level Correction. — If the striding level shows any appreciable inclination of the axis the angle may be corrected by the formula

$$C = [(w + w') - (e + e')]\,\frac{d}{4}\tan h$$

where e and w are the scale readings of the east and west ends of the bubble, and e' and w' the same when the level has been reversed; d is the angular value in seconds of one division of the striding level; and h is the altitude of the star, obtained during the observation.

The sign of the correction may be determined from the fact that if the west end of the axis is too high the telescope is turned too far to the left (west) when pointing at the star, and *vice versa.*

If the azimuth mark is placed far above or below the horizon it will be necessary to take readings of the striding level when pointing at the mark and to compute corrections to the circle readings as would be done for the pointings on the star. Ordinarily, however, the mark is not far from the true horizon so that this correction may be neglected provided the plate levels are sensitive and are carefully centered before each set is begun.

103. Diurnal Aberration. — In very exact observations for azimuth, allowance should be made for the effect of diurnal aberration; this is a slight apparent displacement of the star toward the east due to the motion of the observer about the earth's axis. The amount of this displacement depends upon the relation between the velocity of the observer and the velocity of light. The displacement occurs in the direction in which the observer is actually moving, which is always directly toward the east point of his horizon. The expression for the correction for diurnal aberration is

$$0''.319 \times \frac{\cos L \cos Z}{\cos h}$$

In ordinary work it is sufficient to take it as $0''.32$, since $\frac{\cos L \cos Z}{\cos h}$ is always nearly equal to unity.

EXAMPLE.

RECORD OF AZIMUTH OBSERVATION.*

Station, B. C. Sept. 5, 1906.

Object	Pos. of Tel.	No. of Rep.	Times	Circle Readings A	Circle Readings B	Observer, H. Recorder, R.
				I		
Polaris	*D*	6	6h 39m 48s	0° 00′ 00″	50″	
			40 57	(35 18 30)		
			41 39			
			42 29			
			43 13			
Mark	*D*		44 01	211 45 40	50	
						211° 45′ 45″
						− 05
Mark	*R*	6		0 00 00	00	6)211 45 50
			6 50 15			35 17 38.33
			51 16			
			52 52			
			54 26			
			55 37			6)211 31 45
Polaris	*R*		56 55	211 31 50	40	35 15 17.50
				II		
Polaris	*D*	6	7 08 02	0 00 00	00	
			09 08			
			11 34			
			12 47			
			13 52			
			14 52			6)211 16 00
Mark	*D*			211 16 00	00	35 12 40.00
Mark	*R*	6		0 00 00	50	
			7 32 34			
			34 16			210 57 40
			35 56			− 05
			37 15			6)210 57 45
			38 48			35 09 37.50
Polaris	*R*		39 44	210 57 50	30	

* This transit was not provided with a striding level.

TIME OBSERVATIONS.

Sept. 5, 1906.

Time of transit of Polaris	7^h	00^m	02^s
Time of transit of γ^2 Sagittarii	7	04	50
Time of transit of Polaris	7	23	33
Time of transit of λ Sagittarii.	7	26	56

From these observations we find that the sidereal time when the watch read $7^h\ 04^m\ 50^s.9$ is $18^h\ 06^m\ 24^s.2$. (See Art. 87, p. 77). The sidereal times used in the computation of azimuth were obtained by taking the differences in the watch readings and reducing to sidereal intervals by means of Table III in the Nautical Almanac.

DATA.

Station, B. C.; Latitude, 42° 03′ 00″; Longitude 73° 31′ 19″.

	RIGHT ASCENSION.	DECLINATION.
Polaris	$1^h\ 26^m\ 21^s.5$	+ 88° 48′ 12″.9
γ^2 Sagittarii	17 59 48.2	− 30 25 30
λ Sagittarii	18 22 12.3	− 25 28 21

From Table IV, Appendix to the Nautical Almanac for 1906, the correction c is + 26′.6 for the observation on Polaris and γ^2 Sagittarii and + 19′.5 for the observation on Polaris and λ Sagittarii.

COMPUTATION OF AZIMUTH.

	I.	II.
Mean of observed times	$6^h\ 47^m\ 47^s.3$	$7^h\ 24^m\ 04^s.0$
Corresponding sidereal times . . .	17 49 17.8	18 25 40.5
Right ascension of Polaris	1 26 21.5	1 26 21.5
Hour angle at mean of observed times	16 22 56.3	16 59 19.0
t	245° 44′ 05″	254° 49′ 45″
log cos L	9.870732	
log tan D	1.680177	
log cos L tan D	1.550909	
cos L tan D	35.5557	
log sin L	9.82593	9.82593
log cos t	9.61380n	9.41780n
log sin L cos t	9.43973n	9.24373n
sin L cos t	− .2752	− .1753
cos L tan D − sin L cos t	35.8309	35.7310
log sin t	9.959829	9.984595
log denominator	1.554258	1.553045
log tan Z	8.405571	8.431550
Z	1° 27′ 26″.9	1° 32′ 50″.2
Curvature correction	− 1″.8	− 8″.4
Azimuth of star	1° 27′ 25″.1	1° 32′ 41″.8
Mark east of star	35° 16′ 27″.9	35° 11′ 08″.7
Mark east of north	36° 43′ 53″.0	36° 43′ 50″.5
Mark east of A	21° 32′ 00″.6	21° 32′ 00″.6
Azimuth B.C. to A	15° 11′ 52″.4	15° 11′ 49″.9

104. Observation near Elongation. — If the observation is made on Polaris and within half an hour of the time of elongation the azimuth at elongation may be first computed and the azimuth of the star at each pointing reduced to the azimuth at elongation by applying the correction

$$c = 112.5 \times 3600 \times \sin 1'' \times \tan Z_e \times (T - T_0)^2 \qquad [39]$$

where Z_e = the azimuth at elongation, $T - T_0$ = the interval in minutes between the time of elongation and the time of pointing, and c = the correction to the azimuth in seconds of arc. Values of the factor $112.5 \times 3600 \times \sin 1'' \tan Z_e$ may be computed for any given time and place and the correction readily found. In the following table are given values of this factor for different values of Z_e from 1° to 2°.

TABLE 3.

VALUES OF FACTOR $112.5 \times 3600 \times \text{SIN } 1'' \text{ TAN } Z_e$.

Z_e	Factor	Z_e	Factor	Z_e	Factor
1° 00′	.0343	1° 20′	.0457	1° 40′	.0571
01	.0348	21	.0463	41	.0577
02	.0354	22	.0468	42	.0583
03	.0360	23	.0474	43	.0589
04	.0366	24	.0480	44	.0594
05	.0371	25	.0486	45	.0600
06	.0377	26	.0491	46	.0606
07	.0383	27	.0497	47	.0611
08	.0388	28	.0503	48	.0617
09	.0394	29	.0508	49	.0623
10	.0400	30	.0514	50	.0629
11	.0406	31	.0520	51	.0634
12	.0411	32	.0526	52	.0640
13	.0417	33	.0531	53	.0646
14	.0423	34	.0537	54	.0651
15	.0428	35	.0543	55	.0657
16	.0434	36	.0548	56	.0663
17	.0440	37	.0554	57	.0669
18	.0446	38	.0560	58	.0674
19	.0451	39	.0566	59	.0680

EXAMPLE.

April 6, 1904.

Lat. 42° 21′ N. Long. 71° 04′.5 W

An observation was made on Polaris near western elongation by three repetitions of the angle between the star and a meridian mark. The times of pointing on Polaris are $6^h\ 28^m\ 30^s$, $6^h\ 31^m\ 20^s$, and $6^h\ 34^m\ 20^s$; the initial reading of the vernier = 0° 00′; the final reading of the vernier = 4° 51′ 00″.

The right ascension of Polaris = $1^h\ 23^m\ 48^s.3$; declination = + 88° 47′ 43″.6; the right ascension of the mean sun at Greenwich mean noon = $0^h\ 57^m\ 22^s.44$. From formulas [16], [18], and [22], and the above data we find the eastern standard time of western elongation to be $6^h\ 04^m\ 31^s.2$. This gives for the three intervals $(T - T_0)$ $23^m\ 58^s.8$, $26^m\ 48^s.8$, and $29^m\ 48^s.8$. The azimuth of the star as found from formula [15] is 1° 37′ 48″. From Table 3 opposite this azimuth we find the factor .0559. The resulting corrections, formula [39], are 32″, 40″, and 50″. Adding these three corrections to 4° 51′ 00″ and dividing by three (the number of repetitions) we obtain 1° 37′ 41″ as the angle between the star at elongation and the meridian mark. Combining this with the azimuth of the star we find that the meridian mark is 07″ west of the true north.

PROBLEMS.

1. In Example 2, p. 82, the Right Ascension of the Mean Sun for Greenwich Mean Noon is $5^h\ 27^m\ 29^s.48$. What is the error of the watch on Eastern time?

2. Work out the time observations from the transit of γ^2 and λ Sagittarii shown on page 104.

3. Check the result for the error of the watch given in Example 2, p. 87.

4. Work out the sidereal time from transit of γ Ceti in the example on p. 94. Find the error of watch on Standard time using $19^h\ 11^m\ 29^s.5$ for the R. A. Mean Sun at G. M. N. noon and assuming a longitude of 71° 17′ W.

5. Work out also the latitude and time from the second set of observations on Polaris and the observations on α Ceti, p. 95. R. A. of α Ceti = $2^h\ 57^m\ 24^s.8$; Decl., + 3° 43′ 22″.

6. Observed time of transit of δ Capricorni over the vertical circle through Polaris, Oct. 26, 1906. Latitude = 42° 18′.5; longitude = $4^h\ 45^m\ 07^s$. Observed watch time of transit of Polaris = $7^h\ 10^m\ 20^s$; of δ Capricorni = $7^h\ 13^m\ 28^s$, Eastern Time. Declination of Polaris = + 88° 48′ 31″.3; right ascension = $1^h\ 26^m\ 37^s.9$. Declination of δ Capricorni = − 16° 33′ 02″.8; right ascension = $21^h\ 41^m\ 53^s.3$. The right ascension of the Mean Sun at Local Mean Noon = $14^h\ 16^m\ 34^s.6$. Compute the error of the watch on Eastern Time. *Ans.* $32^s.2$ slow.

7. Time observation on May 3, 1907, in latitude 42° 21′.0, longitude $4^h\ 44^m\ 18^s.0$. Observed transit of Polaris = $7^h\ 16^m\ 17^s.0$; of μ Hydræ = $7^h\ 18^m\ 50^s.5$. Decl. of Polaris = + 88° 48′ 28″.3; R. A. = $1^h\ 24^m\ 50^s.2$. Decl. of μ Hydræ = − 16° 21′ 53″.2; R. A. = $10^h\ 21^m\ 36^s.1$. R. A. of Mean Sun at G. M. N. = $2^h\ 40^m\ 56^s.63$. Find the error of the watch. *Ans.* $5^s.1$ fast.

8. Time observation on April 17, 1907, in latitude 42° 21′.0, longitude $4^h\ 44^m\ 18^s.0$. Observed transit of Polaris = $7^h\ 45^m\ 14^s$, of 6 Sextantis, $7^h\ 47^m\ 47^s$. Decl. of Polaris = + 88° 48′ 33″.0; R. A. = $1^h\ 24^m\ 45^s.3$. Decl. of 6 Sextantis = − 3° 48′ 36″; R. A. = $9^h\ 46^m\ 33^s.4$. R. A. of Mean Sun at G. M. N. = $1^h\ 37^m\ 51^s.77$. Find error of watch.

9. Time observation on May 14, 1907, in latitude 42° 21′.0, longitude $4^h\ 44^m\ 18^s.0$. Observed time of transit of Polaris = $8^h\ 00^m\ 50^s$; of β Virginis = $8^h\ 02^m\ 24^s$. Decl. of Polaris = + 88° 48′ 25″.5; R. A. = $1^h\ 24^m\ 55^s.5$. Decl. β Virginis = + 2° 17′.2; R. A. = $11^h\ 45^m\ 51^s.9$. R. A. of Mean Sun at G. M. N. = $3^h\ 24^m\ 18^s.74$.

10. Observation for time by equal altitudes, Dec. 18, 1904.

	R. A.	Decl.	Watch
α Tauri (E)	$4^h\ 30^m\ 29^s.01$	+ 16° 18′ 59″.9	$7^h\ 34^m\ 56^s$
α Pegasi (W)	22 59 61 .12	+ 14 41 43 .7	7 39 45

Lat. = 42° 28′.0; long. = $4^h\ 44^m\ 15^s.0$. R. A. Mean Sun at G. M. N. = $17^h\ 46^m\ 40^s.38$.

Ans. watch $1^s.7$ fast.

11. Time by equal altitudes; Oct. 13, 1906.

	R. A.	Decl.	Watch
ν Ophiuchi (W)	$17^h\ 53^m\ 52^s.15$	− 9° 45′ 34″.6	$7^h\ 13^m\ 49^s$
ι Ceti (E)	0 14 40 .99	− 9 20 25 .7	7 28 25

Lat. = 42° 18′; long. = $4^h\ 45^m\ 06^s.8$. R. A. of Mean Sun at G. M. N. = $13^h\ 24^m\ 32^s.56$.

Ans. Watch $29^s.9$ slow.

12. Time by equal altitudes, Feb. 15, 1902, in latitude 42° 28′, long. $4^h\ 44^m\ 14^s.0$.

	R. A.	Decl.	Watch
α Hydræ (E)	$9^h\ 22^m\ 48^s.7$	− 8° 14′ 18″	$9^h\ 19^m\ 38^s$
β Orionis (W)	5 09 51 .2	− 8 19 08	9 20 56

R. A. of Mean Sun at Local Mean Noon = $21^h\ 39^m\ 00^s.2$. Find the error of the watch on eastern time.

13. Observed equal altitudes of α Boötis and δ Geminorum, May 3, 1907, in lat. 42° 21′.0, long. $4^h\ 44^m\ 18^s.0$.

	R. A.	Decl.	Watch
α Boötis (E)	$14^h\ 11^m\ 26^s.4$	+ 19° 39′ 57″	$7^h\ 45^m\ 54^s.0$
δ Gemin. (W)	7 14 33 .4	+ 22 09 12	7 51 08 .5

R. A. of the Mean Sun at Local Mean Noon = $2^h\ 41^m\ 43^s.3$.

14. Compute the curvature corrections for the observations shown on p. 103.

15. In the example on p. 106 compute the azimuth of the mark by the method of Art. 100.

CHAPTER III.

PRECISE, TRIGONOMETRIC, AND BAROMETRIC LEVELING.

105. Precise spirit leveling is used for establishing a few elevations with great precision for the purpose of furnishing an accurate control of all of the elevations in the region. Trigonometric leveling, which is carried on simultaneously with the triangulation, furnishes a rapid means of determining elevations of the triangulation stations, giving a number of well-distributed points for use in the topographic work. Barometric leveling yields but rough results as compared with the two preceding methods, but is very useful in reconnoissance work and in filling in details on small scale maps.

106. PRECISE LEVELING. — Precise spirit leveling differs from ordinary leveling in that certain refinements are introduced into the methods used and into the construction of the instruments. Its relation to ordinary leveling is that of a general control of the accuracy of the leveling work, just as triangulation furnishes a control of the surveys of the details. Lines of precise levels have been run by the U. S. Coast and Geodetic Survey, the U. S. Geological Survey, the Mississippi River Survey, and other Government surveys, and cover at present (1908) the greater part of the eastern half of the United States, as well as some portions of the West. Bench marks have been established at frequent intervals along these lines of levels, the elevations and descriptions of which may be obtained from the published reports.

107. SOURCES OF ERROR. — Some of the chief sources of error recognized in leveling work are

1. Gradual settling of the instrument when set up on soft ground.

2. Unequal expansion or contraction of the different parts of the instrument due to changes of temperature.

3. Irregular refraction of the air near the surface of the ground.

4. Unequal lengths of backsights and foresights (errors of adjustment not completely eliminated).

5. Selecting poor turning points.

6. Rod not held plumb.

7. Bubble not in the center of the tube at the instant of sighting.

In precise leveling the observations are carried out in such a manner as to eliminate these errors as completely as possible. Errors due to the settling of the tripod are eliminated by taking the sights in such an order that if the settling of the instrument raises one line of levels it will depress the other. For example, if A and A' are two backsights and B and B' two foresights, taken from the same set-up, then if A is read first and B second, any settling of the tripod will make B too small and hence the computed elevation of the T. P. will be too high. If B' is read next and A' last, then the backsight A' is too small and the elevation of the T. P. will be too low. If it be assumed that the amount of settling is the same in the two cases, then the mean of the two resulting elevations of the T. P. is free from this error On some surveys it is customary to run double-rodded lines (see Vol. I, Art. 222, p. 201) and to take the readings in the order indicated above. A double-rodded line has the advantage that a comparison of the two lines of levels gives an indication of the precision of the work, which is necessary when the levels are not run in a circuit.

The shorter the time elapsing between the reading on the backsight and that on the foresight the smaller will be the error due to temperature changes or to settling of tripod; therefore that form of level which allows of quickest readings also secures the greatest precision. In all cases the instrument should be shielded from the sun's rays and from the wind, either by an umbrella or by a special shield; furthermore the observer should not allow the heat of his body to affect the instrument. Some instruments are made so that all parts which have to be touched, such as the leveling screws or the focusing screws, are made of ivory or some other material which does not readily conduct heat.

Errors due to refraction of the air may be partly avoided if the instrument is set high above the ground by giving the tripod legs less spread than in ordinary leveling. This makes the instrument less stable, but if the reflection of the bubble can be seen in a mirror at the instant of sighting there is less need of a steady instrument than when the observer must rely upon the bubble's remaining central while he takes the sight, as is the case in ordinary leveling. If a tripod with long legs is used it is possible to gain height without loss of stability. Another means of increasing the accuracy is to make the observations during the middle of the day rather than in the early morning or late afternoon, because at that time of the day the refraction is less variable.

Errors arising from unequal backsights and foresights may be avoided by providing the telescope with stadia hairs for reading the distance to the rod at each sight, so as to keep the backsight and foresight distance equal. (See Chapter IV.) By this means the notes will show at all times whether the sums of the foresights and backsights are equal. The difference between the two can be kept as small as desired. Artificial turning points (plates or pins) are generally used so that it will not be necessary to take turning points of an unsatisfactory character, as is frequently the case in ordinary leveling, in attempting to make the foresights and backsights equal.

The rod is always made plumb by means of a spirit level, which is either fastened permanently to the rod or held against its edge. (See Vol. I, Fig. 42, p. 82.) The adjustment of the rod-level should be tested at frequent intervals.

Errors arising from the telescope bubble not being in the center of the tube at the instant of sighting are avoided by so arranging the instrument that the observer can see the bubble while he is looking through the telescope. This is accomplished in various ways but usually by means of a mirror in which the observer can see with one eye the reflection of the bubble and its scale at the same time that he sees the rod with the other eye. A micrometer screw is placed under the eyepiece so that the telescope can be given a slight motion in the vertical plane in order to center the bubble.

108. Precise Leveling Instruments. — The characteristic features of a precise level are an inverting telescope of high power and good definition, provided with a vertical and also three horizontal hairs; and a sensitive spirit level of uniform curvature having a mirror so adjusted that the observer can see the bubble while he is looking at the rod. The instrument rests on three leveling screws as this gives greater stability than four. (See Art. 33, p. 33.) There are two spirit levels at right angles to each other, or else a circular level, to be used for the approximate leveling. Some of these instruments are of the wye type and some are of the dumpy type. (See Vol. I, Art. 99, p. 77.) Following are brief descriptions of some of the instruments which have been used in this country.

109. The Kern Level. — This instrument, manufactured by Kern in Aarau, Switzerland, is of the wye-level type and has a striding level resting on the collars near the wye supports. Above the striding level is a mirror turning about a horizontal axis, so that the image of the bubble is reflected to the eye of the observer. For the rough leveling it has a circular level attached to the supporting frame of the instrument instead of two levels at right angles to each other. The telescope can be given a slight motion in the vertical plane by means of a micrometer screw under the wye at the eye end, the pivot being at the opposite wye. The magnifying power of the telescope is about 50 diameters. This level has been used by the Lake Survey and by the U. S. Corps of Engineers.

110. The Stampfer Level. — The Stampfer level was used by the U. S. Coast Survey from 1877 to about 1899. It is of the wye-level type, having a striding level and a micrometer like the Kern level. The instrument as used by the Coast Survey did not have a mirror above the striding level, as the method of making the observations did not require its use. The telescope had a magnifying power of about 37. (See Coast Survey Report for 1879, p. 202.)

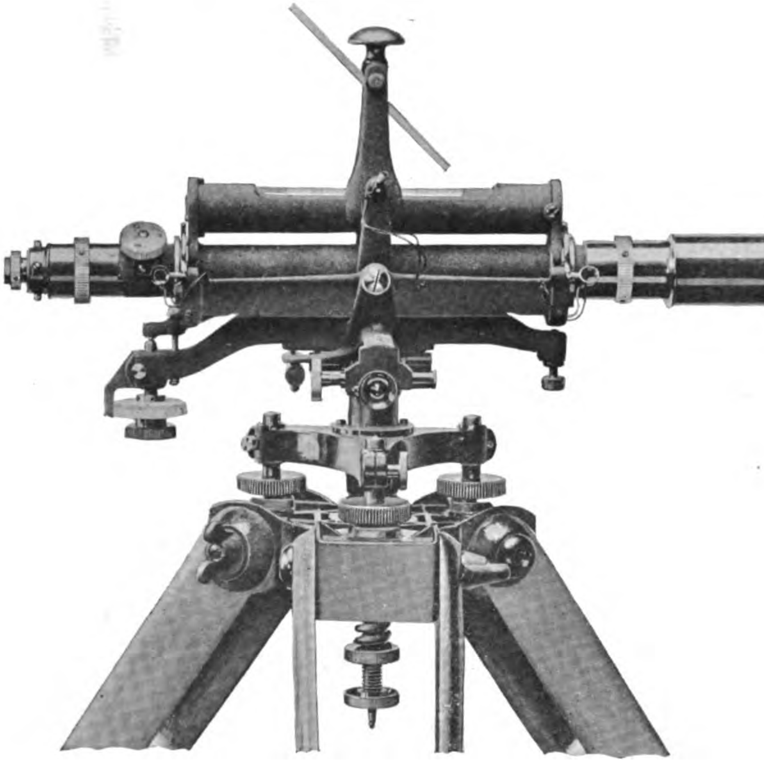

FIG. 38. THE MENDENHALL PRECISE LEVEL.
(From a Photograph loaned by C. L. Berger & Sons.)

111. THE MENDENHALL LEVEL. — The Mendenhall level differs from the preceding chiefly in having the pivot placed near the center so that releveling the instrument does not alter its height (see Fig. 38). It also has all of the screw heads made of a non-conducting material to avoid heating the metal by the hand. The magnifying power of the telescope is 50. (See Proc. Amer. Soc. Civ. Engrs., Vol. 26, p. 784.)

112. THE U. S. COAST SURVEY LEVEL. — This level differs radically from those just described in that it is of the dumpy-

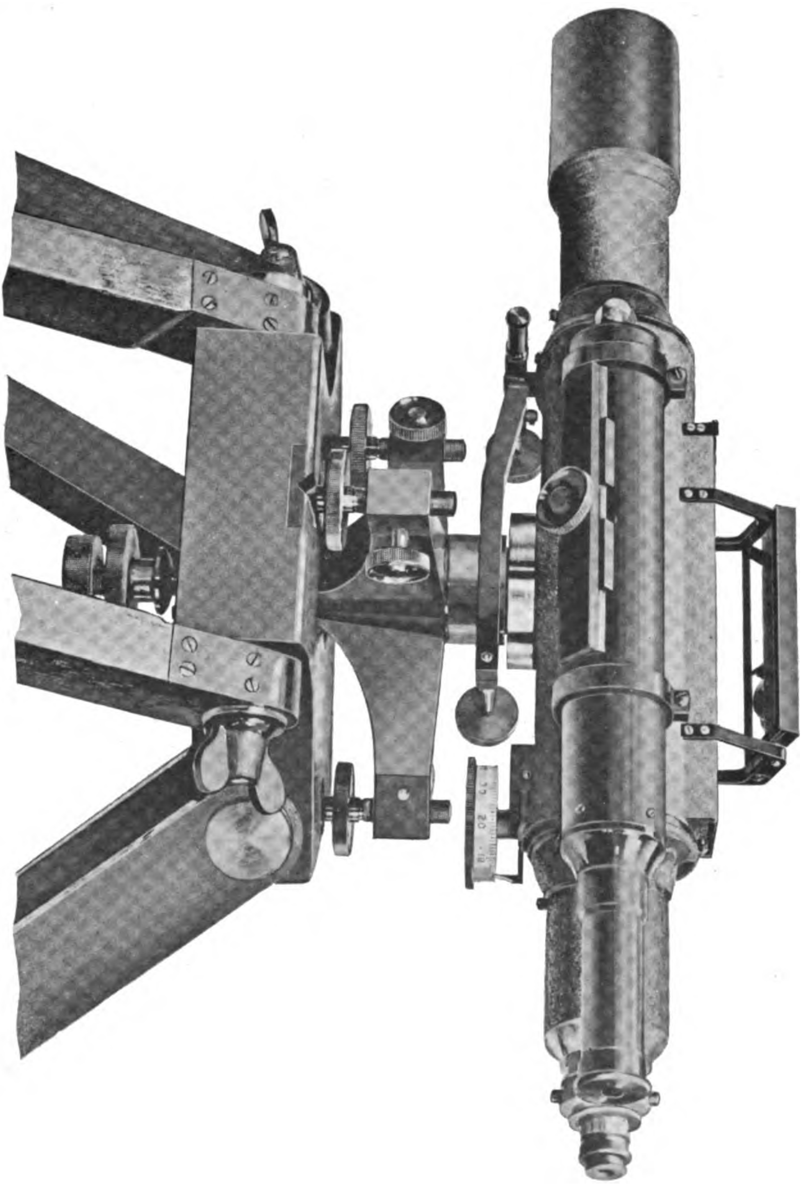

FIG. 39. THE U. S. COAST SURVEY PRECISE LEVEL.
(From a Photograph loaned by the U. S. Coast and Geodetic Survey.)

level type (see Fig. 39). The telescope is made of iron-nickel alloy which has a very small coefficient of expansion. The bubble is placed as near the line of sight as possible, so that the parallelism of the two will not be disturbed by local temperature changes. The arrangement for watching the bubble is more elaborate than in other instruments. A second tube is set at one side of the telescope and carries a set of prisms so arranged that the observer sees the bubble with the left eye and the rod with the right eye as though he were looking through a binocular telescope. The settings are made with a micrometer screw under the eyepiece. This arrangement enables the observer to take the readings very rapidly so that greater speed can be made in the leveling. The telescope has stadia hairs; the magnifying power is about 43 diameters. (For a complete description of this instrument see Coast Survey reports for 1900, p. 521, and 1903, p. 200.)

113. PRECISE LEVELING RODS. — In the early precise leveling work target rods were chiefly used. They were made of long strips of pine wood treated so as to be impervious to moisture. The targets, which were of the bisection pattern (see Vol. I, Fig. 41, p. 80) were raised and lowered by means of chains or tapes running over pulleys. The foot of the rod carried a metal shoe rounded at the bottom.

In recent work the tendency is toward the use of the self-reading rod exclusively (see Fig. 40). These rods are usually not extensible, and in section are either in the form of a + or a T. They are treated with paraffin to prevent them from absorbing moisture. Since they are always used with inverting telescopes the figures on the rod are made upside down so that they will appear erect in the field of the telescope. The U. S. Coast Survey rod has a metric scale the smallest division of which is the centimeter. The rod is in the form of a cross (in section) the divisions being marked in black and white on the edge of the cross. Metal plugs are inserted at regular intervals for detecting changes in the length of the rod. A thermometer is attached for reading the temperature. The Molitor rod has a metric scale in which the smallest division is 2 millimeters. It is of a T section and carries a circular level for plumbing and a device

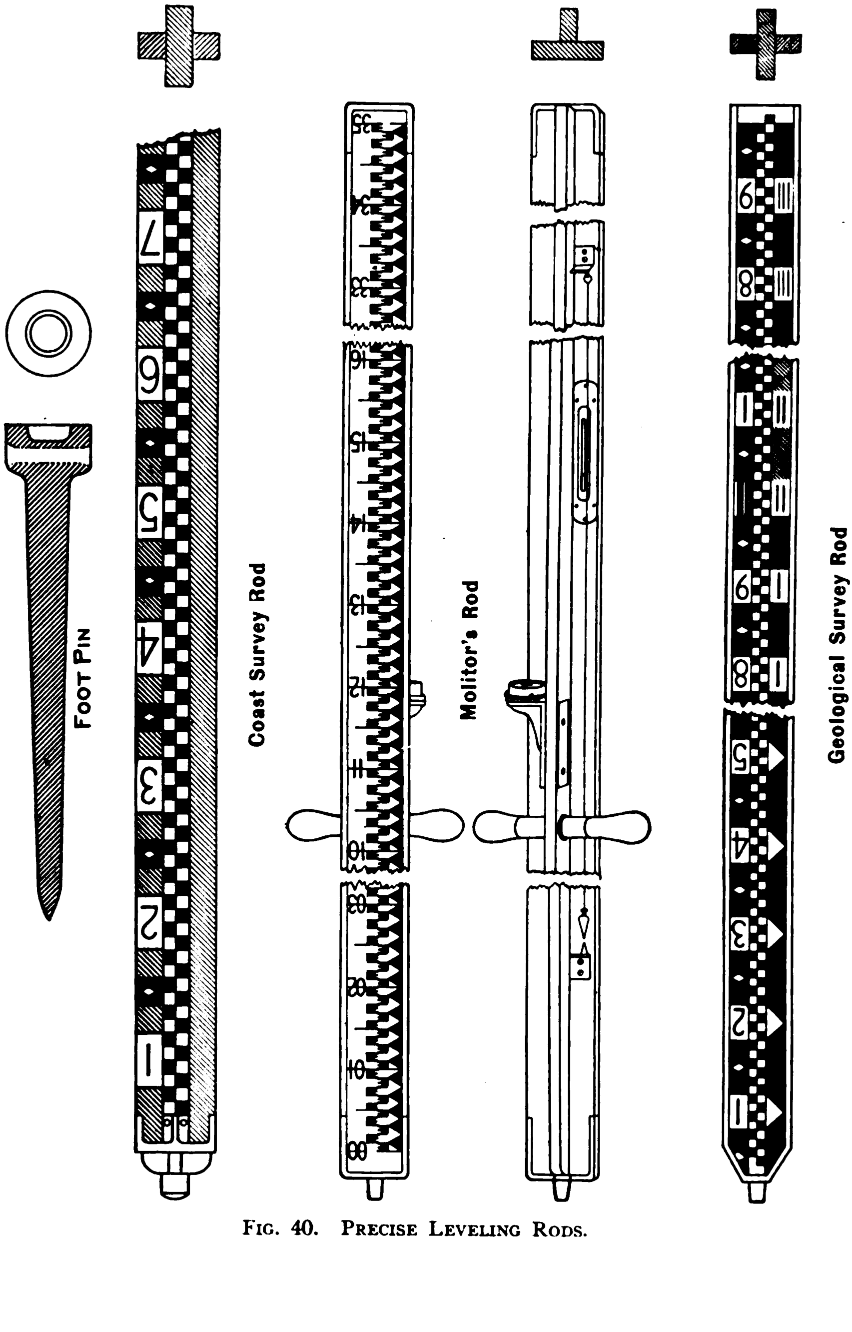

Fig. 40. Precise Leveling Rods.

for adjusting the level by means of a plumb-line. The U. S. Geological Survey rod, devised by Mr. E. M. Douglas, is in the form of a cross (in section). The unit of the scale is the yard, the smallest division being one hundredth of a yard.

Foot pins are used as turning points, the one shown in Fig. 40 being the type used by the Coast Survey. A hole is drilled through the top of the pin to admit a quarter-inch rope, used in pulling the pin out of the ground.

114. PRECISE LEVELING METHODS.* — The particular method employed in precise leveling work depends upon the type of instrument used, upon the rod, and the conditions under which the work is carried on. The following general remarks apply to all precise leveling work. While being carried from station to station the instrument should be covered to protect it from the sun. The micrometer should be unscrewed so that the telescope will not bear on the point of the screw. The nuts holding the tripod legs should be loose; the central clamp should be tight if the instrument is carried on the tripod. When the instrument is set up the three tripod nuts are tightened, and the tension on the central clamp screw should be released so that the instrument is held on the tripod by its own weight. The instrument should be protected from the sun and wind by an umbrella or by a special shield.

In using the instrument the bubbles of the small levels should be brought to a central position by means of the leveling screws, the telescope then turned toward the rod, and the large bubble centered by means of the micrometer screw. The rod-reading is then taken, the manner of doing this depending upon the instrument and the methods to be used. The distance to rod is determined by the readings of the upper and lower hairs. The different rod-readings must agree within certain specified limits according to instructions, and the instrument must not be moved until the levelman is satisfied that the readings are within the required limits of accuracy.

Special care should be used by the rodmen in holding the rods

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* For a valuable paper by Mr. D. A. Molitor on Precise Leveling Methods see Trans. Am. Soc. C. E., vol. XLV, p. 1.

plumb. In a strong wind it is sometimes necessary to steady the rod by means of two wooden braces. Rod levels should be examined frequently and kept in good adjustment. The temperature should be read at frequent intervals.

In the following articles are given some of the details of the special methods employed by different Government surveys.

115. U. S. Engineers' Method. — In the method employed by the U. S. Engineer Corps, where the Kern level is used, the bubble is not centered exactly, but its position is read just before and again just after the rod-readings are taken, and a correction is made for the inclination of the line of sight. The self-reading rod is used and readings are taken with all three horizontal hairs. The longest sight allowed is 100 meters. The greatest allowable difference between the distances to the foresight and to the backsight is 10 meters. The adjustment of the instrument is tested daily. The striding level is tested, by reversals, and adjusted until correct within two divisions of the scale. The line of sight is tested in the usual manner by rotating the telescope in the wyes, all three of the hairs being read with the telescope direct and reversed. If the means of the readings in the two positions differ by more than 2.5^{mm} in a distance of 50^{m} the line of sight is adjusted; if less than this amount, a correction is applied to the rod-readings to allow for this difference.

116. U. S. Coast Survey Methods. — (*a*) **Old Method.** — In the old Coast Survey method, where the Stampfer level was used, the bubble was brought nearly to the center of the tube and the target moved until it was approximately on the cross-hair and clamped. The inclination of the line of sight was then measured by means of the micrometer screw. The bubble of the striding level was centered, first in the direct position, then in the reversed position, and the micrometer read in each case. The mean of these two gave the micrometer reading for the level line. The target was then bisected with the cross-hair and the micrometer again read. The stadia distance was also determined. The difference between the two micrometer readings, converted into angle, is the inclination of the line of sight from the instrument to the target. From this angle and the stadia distance a correction to the rod-reading could be computed. In order to eliminate errors in the position of the cross-hairs the same operation was repeated after the telescope had been rotated 180 degrees (about the telescope axis) in the wye supports. This method of observing was slow and the computations elaborate, so that it has been entirely superseded by the following method.

117. (*b*) NEW METHOD. — With the level described in Art. 112 (see U. S. Coast and Geodetic Survey Report for 1903, p. 189) the observations are made by bringing the bubble exactly to the center of the tube by means of the micrometer screw under the eyepiece, and by reading the three hairs on the self-reading rod, the position of the bubble being examined by the observer at the instant of sighting. Two self-reading rods are used, each rod being used alternately for a foresight and a backsight; the same rod is held on a T.P. for both the backsight and foresight. This allows rapid handling of the level between turning points with a corresponding increase in precision by reducing those errors likely to result from changes in condition of atmosphere and temperature or settling of tripod. At odd numbered instrument stations the backsight is taken before the foresight; at even numbered stations the foresight is taken first. The rod thermometers are read at each rod station. The distances between bench marks are broken up into sections of 1 or 2 kilometers, and each section is leveled forward and backward, the forward and backward measurements being made, if possible, under different atmospheric conditions. If the measurements on a section do not check within $4^{mm}\sqrt{\text{kilometers}}$, both measures are repeated until they do check within this amount. The lengths of the foresights and backsights are derived from the readings of the upper and lower hairs and the total difference in the sums of foresights and backsights is not allowed to become greater than 20 meters. The difference allowable on any one set-up is 10 meters. The maximum length of sight allowable is 150 meters, which is practicable only under favorable conditions.

The adjustment of the instrument is tested once each day. This test is made by taking a foresight of ordinary length, then turning the instrument in the opposite direction and taking a backsight on a rod only 10^m distant. The instrument is next moved to a point within 10^m of the front rod; readings are then taken on the front rod and on the back rod. From these readings a constant C is computed, which is the ratio of the correction for any rod-reading to the corresponding rod interval. This constant is computed by the equation

$$C = \frac{(\text{sum of near rod-readings}) - (\text{sum of distant rod-readings})}{(\text{sum of distant rod intervals}) - (\text{sum of near rod intervals})}$$

If C exceeds .010 the instrument must be adjusted. After the adjustment is made the new value of C must be determined. The adjustments are made by moving the level vial rather than the cross-hair ring. The following example of a determination of C has been taken from the Coast Survey Report for 1903.

DETERMINATION OF C. 8.20 A.M., AUGUST 28, 1900.

(Left-hand page)				(Right-hand page)			
Number of Station	Thread Reading, backsight	Mean	Thread Interval	Rod	Thread Reading, foresight	Mean	Thread Interval
	1515		13		0357		105
A	1528	1528.3	14	W	0462	0461.7	104
	1542		27		0566		209
	2252		105		1276		
B	2357	2357.0	105	W	1288	1288.3	12
	2462		210		1301		13
		0461.7	419			1528.3	25
		2818.7	52			2816.6	
Corr. for curv. and ref.		−0.8	367			2817.9	
		2817.9			367)− 1.3(−0.004 = C		

In computing the difference in elevation between any two bench marks the following corrections are applied. 1. Correction for curvature and refraction. 2. Correction for error in the adjustment of the level (C times the difference in sums of foresight and backsight distances). 3. Correction for length of rod at standard temperature. 4. Correction for actual temperature of rod.

The form of notes for precise leveling, taken from the same report, is given below.

SPIRIT LEVELING.

(Left-hand page)

Date : August 29, 1900.
Sun : *C*. Forward. ~~Backward.~~
(Strike out one word.)

(Right-hand page)

From B.M. : 68. To B.M. : *G*
Wind : S.T.

No. of Station	Thread Reading, backsight	Mean	Thread Interval	Sum of Intervals	Rod and Temp.	Thread Reading foresight	Mean	Thread Interval	Sum of Interval
43	0674		99		V	2683		99	
	0773	0773.0	99		38	2782	2782.3	100	
	0872		198			2882		199	
	0925		106		W	2415		103	
44	1031	1030.3	104		35	2518	2518.0	103	
	1135		210	408		2621		206	405
	0484		98		V	2510		96	
45	0582	0582.3	99		35	2606	2606.0	96	
	0681		197	605		2702		192	597
	0398		97		W	2859		96	
46	0495	0495.0	97		34	2955	2954.7	95	
	0592		194	799		3050		191	788
	1027		26		V	1006		29	
47	1053	1053.3	27		34	1035	1034.7	28	
	1080		53	852		1063		57	845
							11895.7		
		3933.9					−7961.8		

2 : 25 P.M.

118. U. S. GEOLOGICAL SURVEY METHODS. — In the earlier work of the Geological Survey target rods were chiefly used and the lines were double-rodded, the two sets of levels being distinguished by having two targets of different colors on each rod. The bubble was centered when each sight was taken. The instrumental adjustments were tested daily.

In the recent work of the Geological Survey the prism level, like the one now used by the Coast Survey, has been adopted.

The rods employed are the self-reading **yard** rods shown in Fig. 40. Bench marks are established every mile. Lines are run in circuits when it is practicable, and the greatest error of closure allowed is 0.04 ft. $\sqrt{\text{Miles}}$. The instrument is tested every day as described in Art. 117. The maximum sight allowed is 350 feet; foresights and backsights are equalized as nearly as possible, a continuous record being kept of the stadia intervals.

119. **ACCURACY REQUIRED.** — An idea of the accuracy expected in precise leveling may be obtained from the following allowable errors of closure, K being the length of the line of levels in kilometers and D the distance in miles. On the old Coast Survey work the greatest allowable error was $5^{mm}\sqrt{2\,K}$; on the U. S. Lake Survey an error of $10^{mm}\sqrt{K}$ was allowed; on the Mississippi River Survey, $5^{mm}\sqrt{K}$; on the new Coast Survey levels, $4^{mm}\sqrt{K}$; and on the Geological Survey .04 ft. $\sqrt{D}$. These are the greatest errors that will be accepted without requiring the line to be re-run; the results actually reached, however, fall well within these limits.

120. **PRECISE LEVELING WITH AN ORDINARY LEVEL.**— Very fair results may be obtained by applying precise leveling methods to lines run with the ordinary dumpy level, provided the telescope is of sufficient magnifying power and the spirit level is correspondingly sensitive. In order to be certain that the bubble is centered at the instant of sighting it is well to have an assistant (the recorder for example) keep the bubble centered while the observer is sighting. The self-reading rod will be found preferable in this work, and if the telescope is provided with stadia hairs all three of these should be read. Reading all three hairs not only increases the precision but also furnishes a check against a wrong reading and affords at the same time a measure of the distance to the rod so that the sums of the foresights and backsights may be kept equal. The ordinary Philadelphia rod, reading to hundredths, may be used in such work, provided the sights are not long. On long sights the .01 ft. space is rather small for estimating tenths (.001 ft.) since the cross-hair nearly covers the space. The centimeter or the one-hundredth part of a yard is a better division for long sights.

A high grade of leveling has been done on the Barge Canal Survey of New York with an ordinary wye level of good construction. Two target rods provided with spirit levels were used and lines were run forward and backward or else in circuits. The instrument was shielded by an umbrella when the readings were taken. The limit allowed for maximum sight was 300 feet; backsights and foresights were equalized. The adjustment of the level was tested at least once a day and corrected whenever necessary. Steel pegs similar to the one shown in Fig. 40 were used for turning points. The error of closure allowed was but 0.02 ft. $\sqrt{\text{Miles}}$, which indicates as great accuracy as that required in precise level work.

A further illustration of a high grade of leveling done with the ordinary instrument is found in the levels run on the Catskill Aqueduct Line, in New York.* In this work an inverting dumpy level was used; the rods were of the non-extensible self-reading pattern, similar in many respects to the Molitor rod (Fig. 40) except that the graduations are in feet, the smallest divisions being .02 ft. Three horizontal hairs were used in making the readings. These were spaced but half the distance of the ordinary stadia hairs, as this was found to be a more convenient interval. The accuracy required in closing circuits was .02 ft. $\sqrt{\text{Miles}}$, the actual results falling well within this limit.

The best work obtained with good wye and dumpy levels seems to indicate that the required accuracy of precise leveling may be reached with such instruments, but that the same accuracy is reached at less cost if the precise leveling instruments are used. With the Coast Survey prism level results of the required accuracy are obtained with great rapidity and consequently at low cost. It is probable that lines of levels of inferior accuracy could be run more cheaply with such an instrument than with the common leveling instruments.

121. DATUM. — The datum generally used for elevations on land is *mean sea level*, although in some localities mean high water

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* This work is described by Mr. M. E. Zipser in the "Engineering News," Vol. LIX, p. 186, Feb. 20, 1908, under the title "Bench Level Operations on the Catskill Aqueduct Line."

has been used. Mean sea level has the advantage of being nearly the same in all places and is therefore the most convenient datum for leveling operations. (See Vol. I, Art. 237, p. 211.)

TRIGONOMETRIC LEVELING.

122. Trigonometric Leveling. — The elevations of triangulation stations are usually established by observing the vertical angles between stations and computing the differences in elevation trigonometrically. These angles are taken at the time the station is occupied for the purpose of measuring the horizontal angles. The elevations of certain points in the survey, for example the ends of the base-line, are established by direct leveling from tide water or from a known bench mark. From the elevations of these points the elevations of the triangulation stations may be found by means of the differences in height derived from the vertical angles and the lengths of the triangle sides.

In measuring the vertical angles the instrument should be placed, if possible, over the center mark of the station; if it has to be placed to one side its position should be located by azimuth and distance from the center mark. In either case its height above the station should be measured. A definite point on the signal at the distant station should be selected for sighting the cross-hair when measuring the vertical angle. From the known dimensions of the distant signal the height of the point sighted above the station mark may be obtained (see Art. 15, p. 19).

In the most exact work the vertical angles are measured with a special vertical circle instrument; this may be either a repeating instrument or a direction instrument read by microscopes. In work of a less precise character fair results can be obtained by using the vertical circle of a theodolite in which the verniers read to 20″ or to 10″. With this instrument only single measurements of an angle can be made. The best result which such an instrument can give will be obtained by taking the average of several measurements, half of them with the telescope direct and the other half with the telescope reversed. Attention should of course be given to the index correction.

The chief difficulty in obtaining accurate results in this kind of work arises from the uncertainty of the angle of refraction, i.e., the angular deviation of the line of sight on account of the refraction of the air. This angle not only varies with the temperature and the atmospheric pressure but also varies with the locality. It may be nearly eliminated by taking simultaneous observations between two stations, so that the atmospheric conditions for the two observations may be assumed to be the same. If the two observations are made at different times, or if an angle is taken at only one of the stations, then the mean value of the refraction correction must be used in computing the difference in elevation, which introduces a very uncertain factor into the results. The best time to observe vertical angles is during the middle of the day, as the refraction is then much less variable than in the morning or evening.

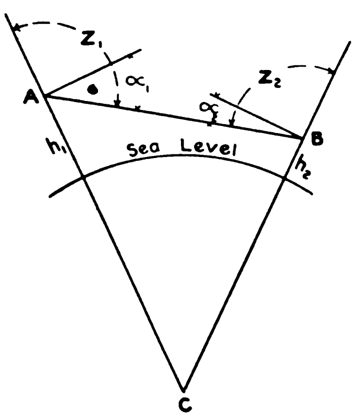

FIG. 41. SIMULTANEOUS OBSERVATIONS.

123. SIMULTANEOUS OBSERVATIONS.— Let A and B, Fig. 41, be two stations at which the vertical angles α_1 and α_2 are observed at the same instant. In the following discussion angles of elevation are considered positive and angles of depression negative. Let $\alpha_1 = 90° - Z_1$ and $\alpha_2 = 90° - Z_2$, where Z_1 and Z_2 are the zenith distances. Let the elevations of the instruments be h_1 and h_2, and K the arc subtended by the angle C, or approximately AB.

In the triangle ACB,

$$\frac{1}{2}(Z_1 + Z_2) = 90° + \frac{1}{2}C$$

and
$$\tan\frac{1}{2}C \tan\frac{1}{2}(Z_1 - Z_2) = \frac{(h_1 + R) - (h_2 + R)}{(h_1 + R) + (h_2 + R)} \qquad \text{(a)}$$

By expanding $\tan \frac{1}{2} C$ in series we have

$$\tan \frac{1}{2} C = \frac{1}{2} C + \frac{\left(\frac{1}{2} C\right)^3}{3} + \ldots$$

$$= \frac{1}{2} C + \frac{C^3}{24} + \ldots$$

Also $$C = \frac{K}{R}$$

Substituting these values in (a) and simplifying, we have

$$h_1 - h_2 = K \tan \frac{1}{2} (Z_1 - Z_2) \left(1 + \frac{h_1 + h_2}{2R} + \frac{K^2}{12 R^2}\right)$$

or, since $Z_1 = 90° - \alpha_1$ and $Z_2 = 90° - \alpha_2$

$$h_1 - h_2 = K \tan \frac{1}{2} (\alpha_2 - \alpha_1) \left(1 + \frac{h_1 + h_2}{2R} + \frac{K^2}{12 R^2}\right) \quad [40]$$

Since the refraction makes each station appear higher than it really is, each observed zenith distance will be too small. Formula [40] is evidently based on the assumption that the atmospheric conditions are the same for both observations and therefore that the effect of refraction is the same, and hence that the difference of the **observed** zenith distances is the same as the difference of the **true** zenith distances. It is evident that while this assumption holds true for simultaneous observations it is not true for observations made at different times.

The value of R used in this formula should, strictly speaking, be taken for the latitude of the place of observation and for the azimuth of the sight, since the radius of curvature is different for different latitudes on the spheroid and for different sections through the same point, the radius of curvature of the meridian being shorter than that of a section taken at right angles to the meridian.* An average value of R sufficiently accurate for such lines as occur in tertiary triangulation is given below.

$$\log R \text{ (in feet)} = 7.32068$$
$$\log R \text{ (in meters)} = 6.80470$$

124. REFRACTION COEFFICIENT. — The coefficient of refraction is the ratio of the angle of refraction to the angle at the center of the earth between the stations. If r = the angle of refraction, C = the angle at the center of the earth, and m = the coefficient of refraction, then

$$m = \frac{r}{C}$$

hence $$r = mC$$

125. OBSERVATIONS AT ONE STATION ONLY. — If the observation is made at one station only, the value of m must be known in order to compute the difference in height, and it should pref-

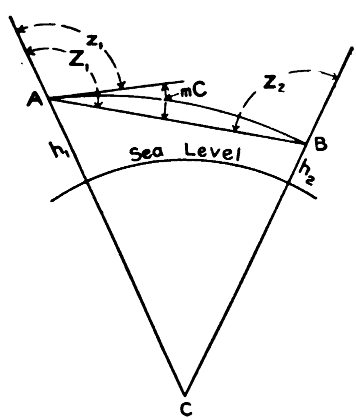

FIG. 42.

erably be found from observations made in the same locality. In Fig. 42 Z_1 and Z_2 are the true zenith distances, z_1 is an observed

* For exact values of R for different latitudes and different azimuths see U. S. Coast Survey Report for 1882, p. 180.

zenith distance, the actual line of sight being the curve AB. The angle of refraction is marked mC in the figure.

Then $$Z_1 = z_1 + mC$$

and $$Z_2 = 180° - z_1 - mC + C$$

Therefore $$\tfrac{1}{2}(Z_2 - Z_1) = 90° - (z_1 + mC - \tfrac{1}{2}C)$$

and $$\tan\tfrac{1}{2}(Z_2 - Z_1) = \cot(z_1 + mC - \tfrac{1}{2}C)$$

Hence by a similar process to that followed in deriving equation [40] we find that

$$h_2 - h_1 = K\cot(z_1 + mC - \tfrac{1}{2}C)\left\{1 + \frac{h_1 + h_2}{2R} + \frac{K^2}{12R^2}\right\}$$

$\frac{K}{R}$ being the circular measure of the angle C. If $z_1 = 90° - \alpha_1$ where α_1 is the observed angle of elevation, being positive when the angle is above the horizon and negative below, then

$$h_2 - h_1 = K\tan\left\{\alpha_1 + (\tfrac{1}{2} - m)\frac{K}{R\sin 1''}\right\}\left\{1 + \frac{h_1 + h_2}{2R} + \frac{K^2}{12R^2}\right\} \quad [41]$$

The angle $\frac{K}{R}$ is divided by $\sin 1''$ in order to reduce it to seconds of arc. In solving for $h_2 - h_1$, approximate values are first obtained by omitting the last factor; these approximate values are then substituted in the last factor and corrected values computed.

126. Value of m. — The value of m as found from a large number of observations is given in reports of the Coast Survey as follows.

Lines crossing the sea	.078
Between primary stations (high elevation)	.071
In the interior of the country, about	.065

In his "Geodesy," Clarke gives for the value of m

For rays crossing the sea	.0809
For rays not crossing the sea	.0750

For approximate results it is sufficiently accurate to use an average value of m, but for precise work the value should be derived from observations made in the particular locality in question.

The following are examples of trigonometric leveling taken from the Coast Survey Report for 1882.

EXAMPLE 1. SIMULTANEOUS OBSERVATIONS.

The distance from Santa Cruz to Mt. Bache $= 23931^{m}.6$

Vertical Angle at Santa Cruz $= +\ 2°\ 24'\ 58''.94$
Vertical Angle at Mt. Bache $= -\ 2°\ 35'\ 34''.20$
Elevation of Santa Cruz $= 108^{m}.87$
$\frac{1}{2}(\alpha_2 - \alpha_1) = 2°\ 30'\ 16''.57$

$$\begin{aligned} \log K &= 4.37897 \\ \log \tan \tfrac{1}{2}(\alpha_2 - \alpha_1) &= 8.64089 \\ \hline & \ 3.01986 \end{aligned}$$

$$\begin{aligned} h_1 &= 1155.67 \\ h_2 &= 108.87 \\ \hline & \ 1264.54 \\ \frac{h_1 + h_2}{2} &= 632.27 \\ \log &= 2.8009 \\ \log R &= 6.8047 \\ \hline & \ 5.9962 \end{aligned}$$

$$\begin{aligned} dh &= 1046^{m}.8 \\ h_2 &= 108\ .87 \\ \hline \text{(approx.) } h_1 &= 1155\ .67 \end{aligned}$$

$$\frac{h_1 + h_2}{2R} = .000099$$

$$\begin{aligned} \log K^2 &= 8.7579 \\ \log 12\,R^2 &= 14.6886 \\ \hline & \ 4.0693 - 10 \end{aligned}$$

$$\frac{K^2}{12\,R^2} = .000001$$

$$\begin{aligned} & 1. \\ + & \ .000099 \\ + & \ .000001 \\ \hline & 1.000100 \\ \log = & \ 0.00004 \\ & 3.01986 \\ \hline & 3.01990 \end{aligned}$$

$$\begin{aligned} \text{Santa Cruz} &= 108^{m}.87 \\ h_1 - h_2 &= 1046\ .9 \\ \hline \text{Mt. Bache}\ & 1155\ .77 \end{aligned}$$

$h_1 - h_2 = 1046.9$ meters.

Example 2. Observation at One Station.

Vertical Angle at Farmington to Mt. Blue, $+2^\circ\,52'\,41''.2$. Distance $= 15519^{\mathrm{m}}$. $m = 0.071$. Instrument $2^{\mathrm{m}}.2$ above ground. Point sighted $4^{\mathrm{m}}.4$ above ground. Elevation of Farmington $= 181^{\mathrm{m}}.20$.

$$
\begin{array}{rl}
 & .500 \\
m = & .071 \\
(\tfrac{1}{2} - m) = & 0.429 \\
\log R = & 6.8047 \\
\log \sin 1'' = & 4.6856 \\
\hline
 & 1.4903
\end{array}
\qquad
\begin{array}{rl}
\log = & 9.6325 \\
\log K = & 4.1909 \\
\operatorname{colog} R \sin 1'' = & 8.5097 \\
\hline
 & 2.3331 \\
 & 215''.3 \\
 & 3'\,35''.3
\end{array}
\qquad
\begin{array}{rl}
\alpha_1 = & +2^\circ\,52'\,41''.2 \\
 & 3\;\;35\;.3 \\
\hline
 & +2^\circ\,56'\,16''.5 \\
\log \tan = & 8.71030 \\
\log K = & 4.19086 \\
\hline
 & 2.90116 \\
dh = & 796^{\mathrm{m}}.45 \\
 & 181\;.20 \\
\hline
 & 977\;.65 \\
 & 2\;.20 \\
\hline
\text{Elev. Mt. Blue (approx.)} = & 975\;.45
\end{array}
$$

$$
\begin{array}{lll}
 & 4^{\mathrm{m}}.4 & 975.45 \\
 & 2\;.2 & 181.20 \\
\hline
\text{Reduction to ground.} & 2\;.2 & 1156.65
\end{array}
$$

$$
\begin{array}{rl}
\dfrac{h_1 + h_2}{2} = & 578.33 \\
\log \dfrac{h_1 + h_2}{2} = & 2.7622 \\
\log R = & 6.8047 \\
\hline
 & 5.9575 \\
\dfrac{h_1 + h_2}{2R} = & .000009074 \\
\log 1.0001 = & 0.00004 \\
 & 2.90116 \\
\hline
 & 2.90120 \\
 & 796^{\mathrm{m}}.53 \\
\text{Reduction to ground} & 2\;.20 \\
\hline
\text{Hence} \quad h_2 - h_1 = & 794\;.33 \text{ meters.}
\end{array}
\qquad
\begin{array}{rl}
\text{Farmington} & 181^{\mathrm{m}}.20 \\
h_2 - h_1 & 794\;.33 \\
\hline
\text{Mt. Blue} & 975\;.53
\end{array}
$$

127. Rough Computations. — A Rough determination of the difference in height may be found by multiplying the horizontal distance to the station by the tangent of the vertical angle and applying a correction for curvature and refraction

(Table I, p. 387). The curvature and refraction may also be found by the formula *

$$h = \frac{K^2}{1.7426} \qquad [42]$$

where K is the distance in miles and h is the correction in feet.

This method of leveling is the one used in determining differences in elevation by means of the stadia or plane table, except that for very short sights or for rough work the refraction correction is omitted.

BAROMETRIC LEVELING.†

128. THE BAROMETER. — The barometer is an instrument for measuring variations in the pressure of the air. Since this pressure varies with the height above the sea-level the barometer may be used as a means of measuring differences in altitude, an inch in the height of the mercury column corresponding to a difference of about 900 feet in altitude. The atmospheric pressure varies also with changes of temperature and humidity, so that it is necessary in measuring difference in altitude with the barometer to determine the amount of these variations and to make proper allowance for them.

There are two kinds of barometer used in surveying work, (1) the *mercurial barometer* and (2) the *aneroid barometer*. The former is usually of the type known as the *Fortin*, as this is the most portable form. Aneroid barometers differ considerably in size and in the details of construction, but are all based on the same general principle. The aneroid is a very convenient instrument on account of its small size and its sensitiveness to slight variations of pressure; it is liable, however, to become deranged if subjected to great ranges of pressure or if roughly handled. It is also subject to errors due to temperature changes.

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* See Coast Survey Report for 1882, p. 154.

† For a more complete treatment of this subject the student is referred to "The Use of the Barometer;" by R. S. Williamson, published by D. Van Nostrand, New York. See also Trans. of the Am. Soc. of Civ. Eng., Vol. I, p. 277, for an article by Gen. Theo. G. Ellis on the Aneroid Barometer; and the U. S. Coast and Geodetic Survey Report for 1881, App. 10.

Barometric leveling is at best only approximate, but it is often extremely useful in such work as reconnoissance or in contouring on very small scale maps. The barometer does not give absolute heights, but may be used to measure differences in elevation. A barometer left at a station will often show large variations of pressure due to changes in the state of the atmosphere, so that a reading of the scale of the instrument is a function not only of the altitude but also of the atmospheric condition and of the errors peculiar to the instrument.

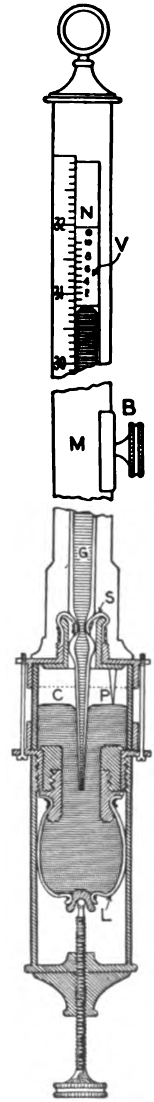


FIG. 43. FORTIN'S BAROMETER.

129. THE MERCURY BAROMETER. — The mercury barometer (Fig. 43) consists of a glass tube *G*, a little over 30 inches long, which contains a column of mercury. The lower end of the tube is connected with a cistern *C* which is filled with mercury. The glass tube is enclosed in a metal case *M* which is open front and back near the top (at *N*) so that the top of the mercury column can be seen. On the side of the case is a scale of inches and a vernier *V* for reading the height of the column. The upper end of the tube is closed and the air exhausted so as to form a vacuum above the mercury. The column is sustained by the pressure of the air on the mercury in the cistern. The lower end of the cistern *L* is made of soft leather. Underneath is a screw which supports the leather case of the cistern so that by turning the screw the mercury column can be raised or lowered. The tube is connected with the cistern by means of a piece of chamois skin *S* so that no mercury can escape, but air can go through the pores of the chamois skin and thus transmit pressure to the mercury in the cistern. When the barometer is to be read the screw at the bottom is turned until the mercury surface in the cistern is just tangent to an ivory pointer *P* which marks the zero end of the scale. The height of the column is read by means of the vernier on a piece of metal which may be raised and lowered in the case by means of the screw *B*. A thermometer (not shown in Fig. 43) is secured to the glass tube with its bulb in the mercury column for reading the temperature of the mercury.

130. THE USE OF THE MERCURY BAROMETER. — When the barometer is to be transported the screw at the bottom is turned until the mercury rises to the top of the tube. The screw should not be turned hard enough to strain the connections at the cistern, but sufficiently to insure the contact of the mercury with the top end of the tube. If the tube is held in a vertical position and is given a slight jolt, a sharp metallic click will be heard, due to the mercury striking against the upper end of the tube. If a very slight vertical movement of the tube produces a sharp click it indicates that the screw should be set up a little tighter, but the screw should not be turned so hard that no click can be heard. The barometer is then placed in its carrying case. It must be carried in an **inverted** position, and as the weight of the mercury is sufficient to break the glass or to loosen the joints in the cistern the instrument must be handled carefully. When a reading is to be made the barometer is taken out of the case and hung upright from a tripod or other support (see Fig. 44). The screw is loosened until the mercury surface is below the ivory point. It is then raised until the ivory point and its reflection in the mercury surface are just in

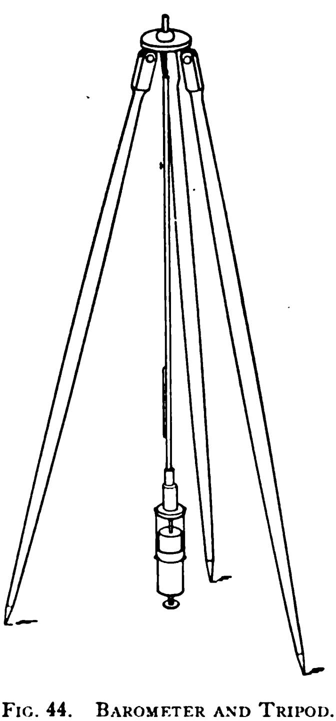


FIG. 44. BAROMETER AND TRIPOD.

contact. The vernier is then raised or lowered until its lower edge is just in contact (apparently) with the top of the mercury column. This contact can be observed by holding a piece of white paper behind the mercury column and noting when the light just disappears at the highest point of the mercury surface. The vernier is then read. Several readings should be taken as a check, the surface in the cistern being lowered and again brought into contact with the ivory point each time. The temperature of the air and of the mercury are both read.

131. THE ANEROID BAROMETER. — The aneroid barometer consists of a hollow corrugated metal box A (Fig. 45) from which

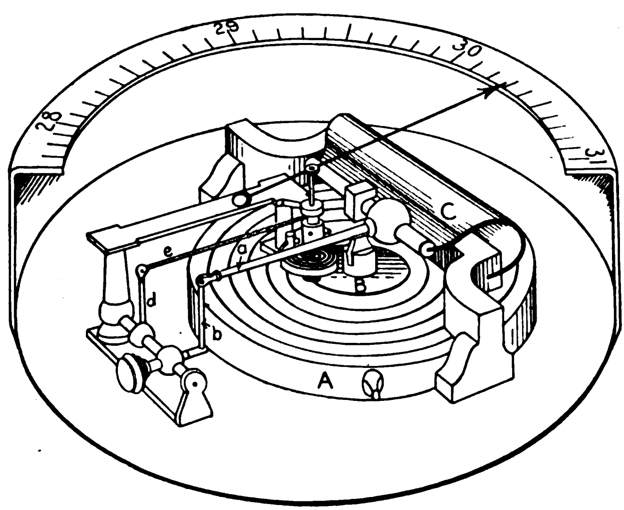


FIG. 45. MECHANISM OF THE ANEROID BAROMETER.

the air has been partially exhausted and which is so thin that it will change its form when the air pressure changes. The movement of the top of the box B is communicated to the spring C and then through levers a, b, and d, thus moving the link e, at the end of which is a chain wound around the shaft on which the pointer is fastened.

On the face of the dial (Fig. 46) are two scales, the inner one corresponding to inches of the mercury column and the outer to feet of altitude. This scale is constructed by comparing the aneroid with a standard mercurial barometer in an air chamber under different pressures. On most aneroids the zero of the altitude scale is placed at 31 inches of the scale corresponding

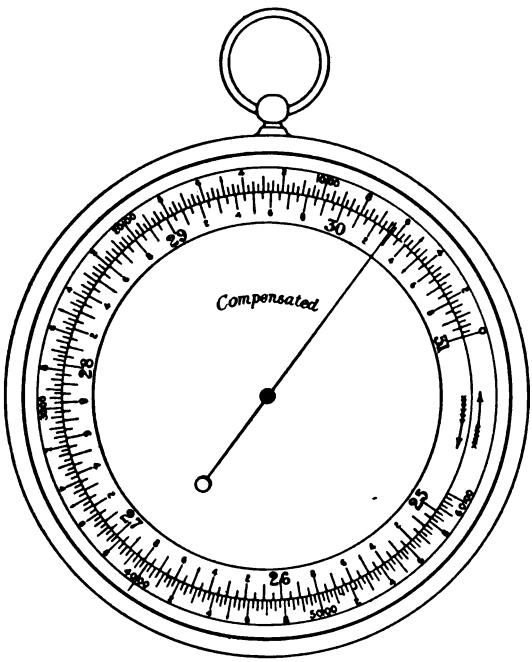


FIG. 46. FACE OF ANEROID BAROMETER.

to the mercury column. The outer scale should **not** be movable with reference to the inner scale, since the number of feet of altitude corresponding to one inch of mercury is different in different parts of the scale. On the back of the aneroid is an adjusting screw which regulates the pointing of the needle by altering the form of the corrugated box. This is used in adjusting the aneroid when comparing it with a standard mercury barometer. Most aneroids are marked "compensated," which means that the mechanism is so arranged, by the use of different metals, that changes of temperature of the instrument will not affect the reading of the needle. A thermometer is often placed on the dial which is used by the maker in adjusting the instrument for compensation. This compensation, however, is seldom found to be perfect.

132. **THE USE OF THE ANEROID BAROMETER.** — The aneroid should be handled carefully in order to avoid disturbing the delicate mechanism and thus changing the relation between the reading of the pointer and the form of the metal box. When the instrument is to be read the case should be tapped lightly to be sure that the chain has not stuck and that the instrument has adjusted itself to the changed pressure. The barometer should not be heated either by the sun's rays or by the body. It should stand a few minutes before it is read so as to allow it to come to the true reading. It may be held either horizontal or vertical, but should be held in the **same position at all stations,** as the reading is usually not the same in the two positions. Experience in using the aneroid indicates that as accurate results may be obtained from small aneroids as from larger ones.

133. **CALCULATING THE HEIGHT.** — **Laplace's Formula.** The difference in elevation may be calculated by formula or by table, or it may be found directly by means of the readings of the altitude scale if an aneroid is used.

There are several formulas for reducing the barometric readings to difference in elevation. One that has been very generally adopted is the formula given by Laplace, the principal term of which is

$$D = 60158.6 \times (\log h - \log H) \qquad [43]$$

where h is the reading in inches at the lower station and H the reading in inches at the higher station. This gives the difference in elevation in feet, uncorrected for the temperature of the air, change in gravity, etc.

Tables have been computed, based upon this formula or an equivalent one, giving values of $(60158.6 \log H)$ for different heights of the mercury column. The difference of the two tabular numbers corresponding to h and H is the value of D, the uncorrected difference in height. The best of these tables is the one computed by Guyot, a portion of which is given in Table IX., p. 400.

134. Corrections. — In order to obtain correctly the difference in elevation between two stations it is necessary to apply the following corrections to the height obtained by formula [43].

135. 1. THE AIR TEMPERATURE. — Since the height of the column of mercury sustained by the air is different at different air temperatures it is necessary to apply to the observed difference in elevation a corresponding correction. It is assumed that the mean of the observed temperatures at the two stations represents the mean temperature of the air along the line. The correction is made by multiplying the value of D in formula [43] by the factor

$$1 + \frac{t_a' + t_a - 64^\circ}{900} \qquad [43a]$$

where t_a and t_a' are the observed temperatures in Fahrenheit degrees. This air temperature must of course be taken with a thermometer which is not attached to the barometer.

136. 2. TEMPERATURE OF THE MERCURY. — If a mercurial barometer is used, a correction due to the difference in temperature of the mercury at the two stations must be applied. Since the mercury column will expand and contract with changes of temperature the observed heights of the column at the two stations will not correspond if the temperatures are different. This is allowed for by reducing the observed height of the column at the upper station to the height corresponding to the temperature of the mercury at the lower station. If h' is the observed height at the upper station and H is the height at the upper station reduced to the temperature at the lower station,

then $$H = h'\left(1 + 0.00008967\ (t_m - t_m')\right) \qquad [43b]$$

where t_m = the temperature of the mercury at the lower station and t_m' = the temperature of the mercury at the upper station.

137. 3. VARIATION IN GRAVITY. — The variation in gravity due either to difference in altitude or to the latitude affects the difference in the pressure, and theoretically these corrections

should be applied to the difference in elevation. The correction for latitude to reduce the barometer reading to latitude 45° (for which Table IX is computed) is

$$D \times .0026 \cos 2\,L \qquad [43c]$$

where D is an approximate value of the difference in elevation and L is the latitude. The correction for the decrease of gravity due to altitude is

$$D\left(\frac{D + 52\,252}{20\,886\,860} + \frac{H}{10\,443\,430}\right) \qquad [43d]$$

These corrections, however, are too small to affect ordinary barometric work. In high altitudes and for latitudes which differ much from 45° these corrections should be applied. In ordinary practice the temperature corrections are the only ones that are of great importance. The formula as it would ordinarily be used is

$$D = 60\,158.6\,(\log h - \log H)\left(1 + \frac{t_a' + t_a - 64^\circ}{900}\right) \qquad [44]$$

where H is height of the mercury column at the upper station corrected for temperature of the mercury.

138. Airy's Formula. — A convenient formula for reducing barometric observations is that given by Airy, namely,

$$D = 62\,759\,(\log h - \log H)\left(1 + \frac{t' + t - 100^\circ}{1000}\right) \qquad [45]$$

As Airy's Tables, based on this formula, are the ones generally used in graduating the altitude scales on aneroids, the following is a convenient rule when the heights are read in feet directly from the altitude scale. Take the difference in the two altitude readings and correct it for air temperature by increasing it by its $\frac{1}{1000}$ part for every degree which the sum of the temperatures exceeds 100° F.

139. Rough Calculations. — A very convenient formula for making rough calculations without tables is that given by Schuckburg,

$$D = 55\,000\,\frac{h - H}{h + H}\ \text{plus}\ \frac{1}{500}\,D\left\{\begin{array}{l}\text{for each degree of the mean}\\ \text{temperature above } 55^\circ \text{ F.}\end{array}\right\} \qquad [46]$$

where H and h are in inches.

To insure the best results the mercurial barometer should be used, because it is not subject to as serious derangements as the aneroid. In the hands of an expert observer results within 2 or 3 feet have been obtained, but it will require unusual care to secure results much closer than 5 or 6 feet. With the aneroid, however, great care must be used to obtain results within 5 or 10 feet, assuming that the aneroid agrees with the standard barometer before and after the observations. It is very seldom that an aneroid will stay in adjustment for any great length of time, and the observer should be continually on the lookout for a sudden change in the adjustment.

140. METHOD OF MEASURING DIFFERENCE IN ELEVATION. — 1. BY TWO BAROMETERS. In this method the variation of the atmospheric pressure at some fixed station whose elevation is known is directly observed by means of one of the barometers kept at that station, and it is assumed that the variation in pressure at other stations is the same as that observed at the fixed station. If the stations are not far apart this will in general be nearly true. The barometer which is left at the first station is read at short intervals, often enough to determine accurately a curve of pressure variation for the day, or for the interval of time covered by the altitude observations. This stationary barometer should, if possible, be a mercury rather than an aneroid, as the former is more reliable. The time of the readings and the temperature of the air and of the mercury should be noted. The second (moving) barometer, which, on account of its portability, is usually an aneroid, is read at the beginning of the observations, at the first station, and the air temperature is also noted by means of a detached thermometer placed in the shade. The time should also be recorded. Care should be taken to place both barometers at the same level. The difference between the readings of the two barometers is an *index correction* to be applied to the readings of the moving barometer to reduce them to the readings of the fixed barometer. The barometer is then carried to the second station and the pressure, temperature, and time read again. Both the barometer and the detached thermometer should be kept in the shade. As many other stations may be included as desired. If a long stay is made at any station the readings should be taken upon arriving and just before leaving. This will check the variation curve given by the stationary barometer. It is well to make the time of traveling between stations as short as possible. The pressure, time, and temperature are read again at the first station upon the return. The change in pressure due to weather conditions may now be estimated and allowed for. This is done by interpolating a reading of the stationary barometer corresponding to the instant at which the moving barometer was read at the distant station. The difference between this interpolated reading of the fixed barometer and the corresponding reading of the moving barometer while at the distant station, corrected for index error, is due to difference in elevation. This difference must be corrected for the temperature of the air as described in Art. 135. The result is the difference in elevation of the two points. It is well to make a second observation at important stations on the return trip, thereby obtaining two independent determinations of the difference in height. The following is a set of notes showing a convenient method of keeping the records and computing the results when two barometers are to be used.

Record of Barometer Readings at Fixed Station.

December 6, 1905.

J. Jones.

Station	Time	Mercury Barometer	Attached Therm.	Air Therm.	
		(Inches)			
Field Office at P.C.R.R. Depot	10.58 A.M.	30.720	44° F.	40° F.	30.720 30.695 .025 fall, 10.58 A.M. to 12 M.
	11.30	30.705	44°	41°	
	12.00 M.	30.695	44°	43°	
	12.30 P.M.	30.685	46°	45°	30.695 30.680 .015 fall, 12 M. to 12.45 P.M.
	12.45	30.680	47°	48°	

RECORD OF BAROMETER READINGS OF MOVING BAROMETER.

DECEMBER 6, 1905.

L. BROWN.

Station	Time	Aneroid Barometer	Air Temp.
		(Inches)	
B.M. at P.C.R.R. Depot	10.58 A.M.	30.57 30.58	40° F.
Prospect △	12.00 M.	30.05	36°
B.M. at P.C.R.R. Depot	12.43 P.M.	30.54	48°

COMPUTATION OF DIFFERENCE IN ELEVATION.

30.57	30.54
30.58	30.05
30.575 = mean	30.49
.025	.015 Correction for change in pressure.
30.550	30.505 Aneroid reading at lower station reduced to reading at 12 M.

From Table IX, p. 400.

(Inches)		(Inches)		
30.550	29 178	30.54	29 169	t = 43°
30.050	28 746	30.05	28 746	t' = 36°
Diff.	432		423	79°
Temp. corr.	7		7	64°
Diff. in Elev.	439 ft.		430 ft.	15°

Mean Diff. in Elev. 435 ft.

$$\frac{15}{900} \times 432 = 7.2 \text{ ft.}$$

The above computations show a discrepancy of nine feet between the difference in elevation as computed from the first and second and that computed from the second and last readings of the aneroid barometer, which is as close an agreement as could be expected from two aneroid observations.

If there has been no record kept at the fixed station the computation of the difference in elevation from the aneroid readings alone would be as follows.

30.575 Mean of first readings at P.C.R.R.
30.54 Last reading at P.C.R.R.
———
.035 Diff.

12 : 00	12 : 43	$\frac{62}{105} \times .035$	30.575	
10 : 58	10 : 58		.021	
62 min.	105 min.		30.554	Probable reading at P.C.R.R. at 12 M.

$$\begin{array}{r} 44^\circ \\ 36^\circ \\ \hline 80^\circ \\ 64^\circ \\ \hline 16^\circ \end{array}$$

$$\frac{16}{900} \times 435 = 8$$

From Table IX, p. 400.

$$\begin{array}{lr} 30.554 & 29\ 181 \\ 30.05 & 28\ 746 \\ \hline & 435 \\ \text{Temp. Corr.} & 8 \\ \hline & 443 \text{ ft.} \end{array}$$

Where the elevations of several stations are to be determined by the use of only one barometer the following method may be employed.

141. 2. BY ONE BAROMETER. — If only one barometer can be used the variations in pressure may be found **roughly** by waiting a short time at each station and noting the change in the reading during this interval and also the time corresponding to each reading. Before leaving the first station the readings are taken as described in Art. 140. Readings are taken upon arrival at and departure from each station. The first of the two readings at any station should not be recorded until the readings indicate that the barometer has adjusted itself to the changed pressure. The variation observed during these waits will aid in estimating the amount of the change due to weather conditions. Upon returning to the starting point the pressure, the temperature, and the time are again read. The total change in pressure due to weather conditions is given by the first and last readings. Whether or not the change has been uniform may be judged by the rate of change observed at the various intermediate stations. If desired a curve may be constructed which will be approximately the curve given by a barometer if one had been left at the first station. On this curve the only points which are known are the first and last, but the observations at the various stations give the slope of the curve, i.e., the rate of change, at different times of the day. From this curve readings may be interpolated corresponding to the various observations at other stations.

142. PRECAUTIONS IN USE OF BAROMETER. — Upon arriving at any station the reading of the barometer should be watched until the instrument settles down to a fixed reading, showing that it has adjusted itself to the changed pressure. For good results the barometer should be used in clear weather when the pressure is probably constant. Since the altitude scale for an aneroid is graduated from the inch scale by the use of an empirical formula there is always a slight uncertainty in using this scale, so that some prefer to always read the inch scale and determine the difference in elevation by one of the formulas given above. The error in the height determined by a barometer is approximately constant within the range for which the instrument is designed, so that the percentage error is much smaller for large differences in elevation than it is for small differences.

PROBLEMS.

1 Trigonometric Leveling. — Instrument at South Base. H. I. = 4.8 feet. Vertical angle to △ Hutton (top of mast) = + 2° 08′ 18″.7. Distance = 3643.3 feet. Height of Signal at Hutton = 32.7 feet. Elevation of ground at South Base = 77.8 feet. (m = .071.) Compute the elevation of the ground at △ Hutton by method of Art. 125. Check by method of Art. 127.

Ans. 186.1 ft.

2. Barometric leveling.

Station	Time	Aneroid	Air Temp.
		(Inches)	
Sleeper's	7h 00m A.M.	29.65	65° F.
Mt. Cardigan	9h 00m A.M.	28.75	68°
Mt. Cardigan	3h 00m P.M.	28.69	78°
Sleeper's	5h 30m P.M.	29.53	76°

3. Barometric leveling.

Station	Time	Aneroid	Air Temp.
		(Inches)	
Hooker's Stable	$8^h\ 15^m$ A.M.	29.57	56° F.
Dwelley	$9^h\ 00^m$ A.M.	29.37	72°
Dwelley	$10^h\ 30^m$ A.M.	29.38	70°
Hooker's Stable	$12^h\ 30^m$ P.M.	29.58	70°

4. Barometric leveling.

Station	Time	Mercury Barometer	Att. Ther.	Air Ther.
		(Inches)		
Neponset River	$10^h\ 10^m$ A.M.	30.580	52°.5 F.	54°.0 F.
Blue Hill	$11^h\ 10^m$ A.M.	29.90	49°.5	51°.0
Neponset River	$11^h\ 55^m$ A.M.	30.540	54°.0	54°.0

5. Barometric leveling.

Station	Time	Aneroid	Air Temp.
		(Inches)	
Steamboat Wharf	$9^h\ 15^m$ A.M.	28.64	61° F.
Saddleback Mtn.	$2^h\ 30^m$ P.M.	25.87	42°
Steamboat Wharf	$7^h\ 00^m$ P.M.	28.58	56°

6. Barometric leveling.

Station	Time	Aneroid	Temp.
		(Inches)	
Ravine House	8.20 A.M.	28.59	12°.3 F.
Durand Ridge	1.15 P.M.	25.34	−1°.5
Ravine House	4.25 P.M.	28.75	10°.0

Ravine House is 1285 feet above sea-level.

7. Barometric leveling.

Station	Time	Aneroid	Air Temp.
		(Inches)	
Wonaloncet	8^h A.M.	29.20	36° F.
Whiteface	1^h P.M.	25.92	2°
Wonaloncet	$4^h\ 30^m$ P.M.	29.26	12°

PART II.

FILLING IN TOPOGRAPHIC DETAILS.

PART II.

FILLING IN TOPOGRAPHIC DETAILS.

CHAPTER IV.

THE STADIA METHOD.*

143. Stadia Method of Measuring Distances. — The stadia method of locating points is one in which distances are measured by observing through the telescope of a transit the space, on a graduated rod, included between two horizontal hairs called *stadia hairs.* If the rod is held at different distances from the telescope different intervals on the rod are included between the stadia hairs, the spaces on the rod being proportional to the distances from the rod to the instrument, so that the intercepted space is the measure of the distance to the rod.

The stadia method of locating points furnishes a rapid means of filling in the details of a topographic or a hydrographic map and gives results of sufficient accuracy except for maps of very large scale. This method, used either with the transit or with the plane table, is the one chiefly employed in this country on the government topographical surveys, and has practically replaced the older and slower methods on all topographical work on small scale maps. Owing to the fact that in making a stadia measurement the intervening country does not have to be traversed, as is necessary when making a tape measurement, distances can be taken across ravines and water surfaces, and over rough as readily as over smooth ground. This gives the stadia method a great advantage over the tape in point of speed. Another advantage of this method over that of chaining is that the errors of stadia measurements are compensating while those of chaining are cumulative. (Vol. I, Art. 23, p. 14.) Furthermore, the accuracy of the stadia measurements is not diminished

* The word *telemeter* is sometimes used instead of *stadia.*

in rough country, so that this method is frequently, under favorable conditions, as accurate as chaining. Heretofore the stadia method has been almost exclusively applied to topographical work, but it can be applied with great advantage to other kinds of surveys. The determination of areas, for example, often demands no greater precision than can be obtained by the stadia method.

144. **INSTRUMENTS.** — The only equipment needed for this work in addition to the ordinary engineer's transit is a set of stadia hairs in the telescope, and some form of graduated rod on which distances may be read from the instrument. The two stadia hairs are usually placed on the cross-hair diaphragm parallel to and equidistant from the horizontal cross-hair. In some instruments they are so arranged that the distance between them is adjustable. For exact work the fixed hairs are to be preferred, as they do not get out of adjustment and the instrument maker can readily set them at the desired distance apart with sufficient accuracy for ordinary work. Sometimes the stadia hairs and the cross-hairs are placed on separate diaphragms or in different planes, so that when the eyepiece is focused on the ordinary cross-hairs the stadia hairs are invisible, and *vice versa;* these are called *disappearing* stadia hairs. There should be diagonal (X) cross-hairs in the same plane as the stadia hairs to define the center line of sight so that horizontal and vertical angles can be measured without throwing the stadia hairs out of focus. This arrangement of cross-hairs is open to the objection that when it is desired to read half a stadia interval, as is explained later, it cannot be done conveniently or accurately.

The telescope of a transit intended for stadia work should have a magnifying power of from 20 to 30 diameters, and should give a clear, well-defined image. It is desirable, although not necessary, that the instrument should be provided with a compass needle. Stadia instruments should have a vertical arc suitable for measuring vertical angles to the nearest minute. In some kinds of work it is convenient to have a complete vertical circle so that vertical angles can be read both on foresights and on backsights. Since vertical angles play such an important part in this work it will prove to be a great saving in time to have the vernier and spirit level mounted on a separate arm, so that the index correction can be made zero each time a vertical angle is read. (See Vol. I, Art. 67, p. 54, and Fig. 57, p. 193 of this volume.)

145. **Stadia Rods.** — There are many patterns of stadia rods both as regards the construction of the rod itself and the style of diagram used to mark the graduations. The diagrams for ordinary work should be simple, so that long distances may be quickly read. Complicated diagrams are to be avoided except where all of the sights are short and where greater precision is desired than is usually required in stadia work. The rods shown in Fig. 47 are all in common use.

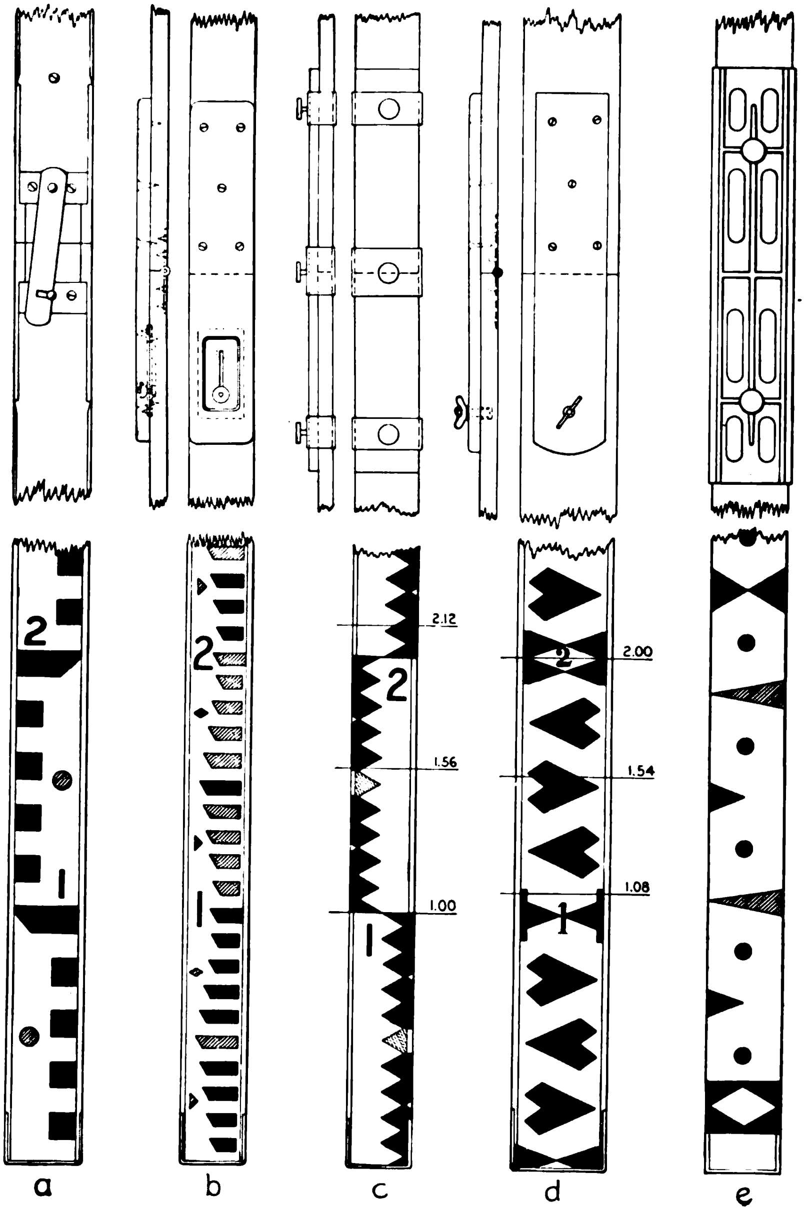
In these rods the diagrams are made so that the .05 ft. or .10 ft. spaces can be easily distinguished and the hundredths of a foot estimated. Rods (*a*), (*b*),* and (*c*) are used where the distances to be measured are not over, say, 800 feet; while a rod marked like (*d*) is serviceable for long distances as well as short ones. The metric rod (*e*) is one of the patterns used by the U. S. Coast Survey for long distances. The numbers marked on the face of the stadia rod are sometimes made 1.0 feet or .15 feet high, the lines of the numbers themselves being .01 feet or .02 feet in thickness.

The rods on which these graduations are painted consist of wooden strips from 3 to 5 inches wide and 10 to 15 feet in length. For convenience in carrying the rods they are usually made in two sections joined by hinges, but in some cases the two parts are separate. Above each of the rods shown in Fig. 47 is a type of hinge or clamp; these do not necessarily apply, however, to the particular rods above which they are shown in the figure.

Sometimes the scale is painted on a strip of flexible material which may be fastened temporarily to a board while in use.

Rods are sometimes graduated to correspond with the stadia interval of a particular instrument, but this is not always convenient, since the rod cannot be used except with the instrument for which it was designed. The common practice is to graduate the rod to feet and tenths, and to have the stadia hairs set to

* This rod was designed by Pierce and Barnes, Surveyors, Boston, Mass.



(Cross-hatched portions are painted red.)

FIG. 47. STADIA RODS.

agree as nearly as possible with this graduation. If very precise results are sought, rod intervals may be taken at known distances and a set of corrections determined for the instrument.

146. PRINCIPLE OF THE STADIA. — The fundamental principle upon which the stadia method depends is the simple geometric proposition that in two similar triangles homologous sides are proportional. Suppose that the telescope is level and is sighted at a vertical rod, then a certain space on the rod will apparently be intercepted between the stadia hairs, the length of this space on the rod depending upon the distance from the instrument to the rod.

In Fig. 48 let L be the optical center * of the objective, a and b the stadia hairs, and A and B the points on the rod where the

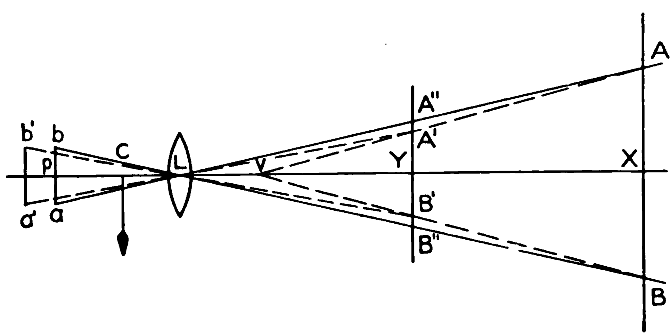


FIG. 48.

stadia hairs appear to cut it. AB is then the intercepted space when the rod is at the distance LX. If the rod were moved to the position Y, where the distance is one-half LX, then the intercepted space $A''B''$ would be one half AB. That is, **the space on the rod is proportional to the distance.** Let $Lp = f_1$

* Strictly speaking the lines representing the rays of light should not be drawn straight through the optical center as in the figure but should be bent at the surfaces of the lens. As a result the vertices of the two triangles ALB and aLb do not fall at L but fall at two points L' and L'' inside the lens. The same ratio will be seen to exist between the sides of these triangles ($AL'B$ and $aL''b$) as exists in case of the triangles discussed in the text. The error introduced by neglecting the distance $L'L''$ is entirely inappreciable in stadia measurements.

and $LX = f_2$ (these distances being known as *conjugate foci*); also let $AB = s$, and $ab = i$.

Then from the similar triangles,

$$f_2 : f_1 = s : i \qquad [47]$$

When the rod is moved from X to Y it becomes necessary to alter the focus of the objective, i.e., the distance between the objective and the cross-hairs must be increased. In some telescopes this is accomplished by moving the tube containing the eyepiece and the cross-hairs, and in others by moving the objective. In the figure let L remain fixed and suppose the cross-hairs to move to the position $a'b'$. If the rod is now viewed through the telescope the cross-hairs will not appear to strike at A'' and B'' but at A' and B'. The lines AA' and BB' continued will meet at some point V on the optical axis, in front of L.

In determining the position of this point it is necessary to make use of the "Law of Lenses," namely,

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F} \qquad [48]$$

in which F is the *focal length* of the objective, i.e., the distance from the optical center to the cross-hairs when the telescope is focused on a distant object. (Vol. I, Art. 46, p. 35.) Solving equations [47] and [48] simultaneously,

$$f_2 = \frac{F}{i}\, s + F \qquad [49]$$

i.e., the distance LX is made up of two parts, — the variable distance VX, or $\frac{F}{i}\, s$, and the constant LV, or F. Since equation [49] was derived from a general case, LV is a constant for all stadia readings made with the same instrument, and the distance VX is a variable depending upon the rod interval. Hence all stadia distances (from the objective) are obtained by multiplying the space on the rod by a constant, $\frac{F}{i}$, and adding to this result the focal length, F. The distance desired, however, is from the center of the instrument to the rod. This is found by adding to the above result the distance CL from the objective to the center of the instrument, which will be called c. The complete expression for the distance is then

$$\text{Distance} = \frac{F}{i} s + (F + c) \qquad [50]$$

147. Stadia Constants. — In equation [50] the ratio $\frac{F}{i}$ is a constant for any given instrument in which the stadia hairs are not adjustable. The quantity $(F + c)$ is practically a constant for any given instrument. F is always strictly a constant, and c is also a constant provided the telescope is focused by moving the eyepiece; if the focusing is done by moving the objective, c will not vary more than a hundredth of a foot in ordinary stadia sights, a negligible quantity in stadia measurements. The distance between the cross-hairs is generally made equal to one one-hundredth part of the focal length, so that the distance (VX) may be found by multiplying the space on the rod by 100, i.e., every hundredth of a foot on the rod corresponds to a foot in distance. In practice it is customary to read the rod interval to the nearest hundredth of a foot only, so that the distance is obtained to the nearest foot.

If the stadia hairs are not set at exactly this interval the error may be determined by measuring a base-line with a steel tape and taking several readings on a rod held at two different distances, say 100 feet and 600 feet. In order to obtain these rod intervals accurately it is advisable to use a leveling rod (like the New York rod) with two targets, the lower hair being set on the lower target and the upper target being set opposite the upper hair. When the rod intervals have been carefully determined at both of the distances, the constant may be found by substituting these values and the measured distances in equation [50], thus forming two equations of the same form in which $\frac{F}{i}$ and $(F + c)$ are the only unknowns. Solving these two equations simultaneously will give an accurate value for the constant $\frac{F}{i}$.

The determination of $\frac{F}{i}$ should not be made to depend on one set of measurements; a sufficient number of readings should be taken to eliminate errors of reading. It is advisable to determine this constant under conditions which approximate those existing in the fieldwork.*

EXAMPLE OF DETERMINATION OF $\frac{F}{i}$. — At a distance of 604.9 feet the following rod intervals were taken.

6.059
6.057
6.058

With the same instrument and with the rod 65.0 feet away the following readings were taken.

0.637
0.637
0.638

Taking the mean in each case and substituting in equation [50] we obtain

$$604.9 = \frac{F}{i} \times 6.058 + (F + c)$$

$$65.0 = \frac{F}{i} \times 0.637 + (F + c)$$

The simultaneous solution of these gives $\frac{F}{i} = 99.59$

This is not a satisfactory method for finding $(F + c)$, since errors in the readings cause a comparatively large error in the result. The constant $(F + c)$ may, however, be directly measured with all the accuracy needed. F is the distance from the objective to the cross-hairs when the objective is focused for a

* For a valuable paper, by Professor L. S. Smith, on the effect of differential refraction on the constant $\frac{F}{i}$ see Bulletin of the University of Wisconsin, Vol. I, No. 5.

distant object, and c is the distance from the objective to the center of the instrument when focused for an average length of sight.

If it is desired to obtain $\frac{F}{i}$ from a single set of readings it is possible to substitute the measured value of $(F + c)$ in either one of the above equations and obtain $\frac{F}{i}$ from that equation alone.

148. THE CONSTANT $(F + c)$. — The constant $(F + c)$ is different for different instruments as shown by the following table, which contains the values of this constant as determined for four instruments of common types.

	F.	c.	$F + c$.
Ordinary engineer's transit (erecting)	0.60 ft.	0.43 ft.	1.03 ft.
Large transit (inverting)	0.84	0.45	1.29
Mountain transit (inverting)	0.57	0.32	0.89
Plane-table alidade (inverting)*	1.21	0.61	1.82

Although the constant $(F + c)$ varies from about 0.75 to about 1.35 in different transits it is customary and sufficiently accurate to regard it as 1 foot, since the distances are read to the nearest foot only. In the case of plane-table alidades, however, this constant is often nearly 2 feet.

149. **FORMULAS FOR INCLINED SIGHTS.** — In practice it is customary to hold the rod **plumb** rather than perpendicular to the line of sight, because the former position can be readily and accurately judged, while it is not easy to determine when the rod is perpendicular to the line of sight. On inclined sights, when the rod is plumb, the vertical and horizontal distances evidently cannot be found by solving a simple right triangle.

* On account of the way in which the alidade is used in plane-table work c actually varies several tenths of a foot but this variation is of no consequence in surveying with the plane table.

In Fig. 49 let AB be the intercept on the rod when it is held vertical, $A'B'$ the intercept when the rod is perpendicular to the

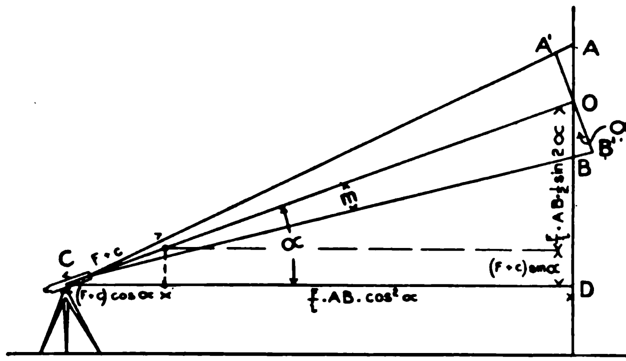


FIG. 49.

line of sight, i.e., $A'B'$ is perpendicular to CO. * In the triangle AOA', $\angle O = \angle \alpha$, the measured vertical angle; $\angle A' = 90° + m$; and $\angle A = 90° - (\alpha + m)$, m being half the angle between the stadia hairs. In the triangle BOB', $\angle O = \angle \alpha$; $\angle B' = 90° - m$; and $\angle B = 90° - (\alpha - m)$.

Then $$\frac{AO}{A'O} = \frac{\sin (90° + m)}{\sin \{90° - (\alpha + m)\}}$$

and $$\frac{BO}{B'O} = \frac{\sin (90° - m)}{\sin \{90° - (\alpha - m)\}}$$

$$AO + OB = AB = \tfrac{1}{2} A'B' \left\{ \frac{\cos m}{\cos (\alpha + m)} + \frac{\cos m}{\cos (\alpha - m)} \right\}$$

from which may be obtained

$$A'B' = AB \cos \alpha - AB \frac{\sin^2 \alpha}{\cos \alpha} \tan^2 m$$

The value of the second term is very small; for $AB = 15.00$ feet, $\alpha = 45°$, and $m = \tan^{-1} 0.005$, this term is only 0.0002 feet, and hence it may always be neglected. In other words it is

* This demonstration is nearly the same as that given by Mr. George J. Specht in "Topographical Surveying," published by Van Nostrand Company, New York.

always sufficiently accurate to regard the angles A' and B' as right angles.

$$\therefore\ A'B' = AB \cos \alpha$$

The difference in elevation between the center of the instrument and the point O on the rod is derived as follows.

$$\begin{aligned} DO &= CO \sin \alpha \\ &= \left\{ \frac{F}{i} A'B' + (F + c) \right\} \sin \alpha \\ &= \frac{F}{i} AB \sin \alpha \cos \alpha + (F + c) \sin \alpha \\ &= \frac{F}{i} AB \,.\, \tfrac{1}{2} \sin 2\alpha + (F + c) \sin \alpha \end{aligned} \qquad [51]$$

For the horizontal distance from the transit point to the rod we have

$$\begin{aligned} CD &= CO \cos \alpha \\ &= \left\{ \frac{F}{i} A'B' + (F + c) \right\} \cos \alpha \\ &= \frac{F}{i} AB \cos^2 \alpha + (F + c) \cos \alpha \end{aligned} \qquad [52]$$

150. Most of the stadia tables, diagrams, and stadia slide rules in common use are based upon these two formulas, [51] and [52]. Table X, p. 404, contains the value of $\frac{F}{i} \,.\, AB \,.\, \frac{1}{2} \sin 2\alpha$ for values of α varying from 0 degrees to 30 degrees, and for a rod interval (AB) of one foot. $\frac{F}{i}$ is taken as 100, the usual ratio. The vertical height for any rod interval can be obtained by multiplying the tabular number by that rod interval. The table of horizontal corrections (Table X) contains the number of feet to be subtracted from the distance read, $100 \times AB$, and is given for different distances up to 1000 feet and for angles from 0 degrees to 29 degrees. It is ordinarily possible to interpolate mentally and to obtain the correction to the nearest foot, which is sufficiently accurate for most stadia work.

To allow for the constant $(F + c)$ when obtaining the vertical height the term $(F+c)\sin\alpha$ should be added to the result found from the table. Strictly speaking this term should be computed separately, but the error is almost always negligible if the constant $(F+c)$ is first added to the distance $100 \times AB$, which is equivalent to adding $\frac{F+c}{100}$ to the rod interval, and then the vertical height obtained from the table as before. In allowing for this constant when determining the horizontal distance the term $(F+c)\cos^2\alpha$ is always so nearly equal to $(F+c)$ that the true horizontal distance may be found by adding $(F+c)$ to $100 \times AB$ and then subtracting the horizontal correction.

The error resulting from these approximations may be seen from the following examples. Suppose the constant $(F+c)$ is one foot and the vertical angle is 30 degrees, then the value of $(F+c)\sin\alpha$ is 0.500 feet, which would be the true correction. The value of $(F+c) \cdot \frac{1}{2}\sin 2\alpha$, the approximate correction, is 0.433 feet, i.e., the error in height is about 0.07 feet for this extreme case. Except where great accuracy in leveling is required, even this error of .07 feet would not be important. For ordinary angles, say 5 degrees, the error is less than .001 feet; for 10 degrees it is less than .003 feet.

The error in the horizontal distance for $\alpha = 30$ degrees and $(F+c) =$ one foot is but .012 feet, a negligible quantity in ordinary stadia measurements. For alidades of plane tables these errors will be nearly twice those given above, since the constant $(F+c)$ is often nearly 2 feet. It is evident therefore that it is nearly always safe to make this approximation.

151. On most stadia work the vertical angles are not large, so that the resulting horizontal corrections amount to only a few feet. In the following table are given certain angles and distances for which the horizontal correction is just one foot.

Angle	Distance	Horizontal Correction
2°	821	−1.0
3	365	−1.0
4	205	−1.0
5	132	−1.0
6	92	−1.0

For the angles and distances in this table the **minus** horizontal correction is nearly equal to the **plus** correction $(F + c)$, which is generally about one foot. On most topographical work the distances are not required nearer than one or two feet, hence the horizontal correction may usually be neglected for vertical angles of less than 3 degrees. For long distances the horizontal corrections for angles of less than 3 degrees may amount to more than one foot, but the uncertainty in the reading for such distances is much greater than the horizontal correction itself, so that it is usually safe to omit the horizontal correction in these cases. The constant $(F + c)$ is usually omitted in topographical work. If the horizontal correction is neglected, as suggested above, the resulting error will be small, since the two corrections always tend to neutralize each other.

152. **FIELDWORK.** — In surveying by the stadia method points are located by means of (1) the azimuth, (2) the angle of elevation or depression, and (3) the distance. When the survey does not involve the determination of elevations the vertical angles are simply read close enough for computing the horizontal correction to the distances read. But when a topographical survey is made, the determination of elevations becomes necessary and all vertical angles must be read closer, usually to the nearest minute. The azimuths and distances are read in both kinds of surveys with a precision that is consistent with the accuracy required in the final result.

Traverse lines which form the control of the survey are run out either by means of the transit and tape or by stadia, according to the accuracy demanded. Where a number of transit points are to be distributed over a large area this can be most rapidly done by laying out a small system of triangulation, in which a few of the lines are measured with the tape and the lengths of all of the lines in the system are computed; those lines which were measured will serve as checks on the accuracy of the work. In a triangulation system of this nature the angles need not be read closer than to the nearest minute and the distances measured to the nearest tenth of a foot, both of which are precise enough for the control of any ordinary stadia survey. Where the survey is controlled by such a triangulation or by tape traverses the stadia work is confined to filling in details.

Where topographic work is to be done by stadia for a small scale map it is customary to run out the traverses by the stadia alone, as this will afford a sufficient degree of accuracy. (See Art. 156, p. 163.) In making detail plans on a large scale, such as those required by a landscape architect, the work may best be done by establishing a few controlling points by transit and tape traverses and using the stadia for details only. In running traverses with the tape for any purpose the stadia readings furnish a most valuable check upon the distances.

From the traverse line as a control, points taken for the purpose of locating details are determined by angles and stadia distances. These observations are commonly called "side shots." The accuracy with which these measurements are taken need not be so great as that of the traverse measurements, because any error in the measurements will affect only a single point, whereas an error in the traverse line will be carried through the rest of the traverse. The side shots are usually numbered consecutively in the note-book.

In locating points as above described the readings are conveniently taken in the following order. The vertical hair is first set on the rod and the upper plate clamped; the distance is then read by setting one stadia hair on a whole foot-mark and reading the position of the other stadia hair; finally the middle horizontal hair is set on the point on the rod to which the vertical angle is to be taken. After making this setting the transitman signals the rodman to proceed to the next point and in the meantime he reads the azimuth and the vertical angle.

153. Azimuth Angles. — Azimuths are usually reckoned from the south point through the west up to 360 degrees in accordance with geodetic practice. If the true azimuth of any line is known, or if observations for the true meridian have been made, all the azimuths of the survey may be referred to the meridian. If the direction of the true meridian is not known an initial azimuth may be taken from the magnetic bearing of some line and all azimuths referred to the magnetic meridian. The latter method has the advantage that all azimuths may be directly checked (roughly) by reading magnetic bearings in addition to the azimuth indicated by the vernier. If for any reason the magnetic meridian cannot be used, any direction may be arbitrarily assumed as a meridian and all azimuths referred to this direction. In case a transit and tape traverse has been previously run it is often convenient to assume one of the transit lines as the 0° line of azimuths, a new reference line for the azimuths being taken at each set-up if desired.

In running traverses by stadia it is necessary to carry forward the direction of the meridian from one transit point to the next; there are two methods in common use, each of which has its advantages. Suppose that the work at point A is completed, all of the azimuth angles about A having been referred to the meridian which was chosen as the 0° direction, and that the transit is to be moved to a new station B. Before leaving A the transit point B is located from A by its distance and azimuth. The transit is then set up at B and the azimuth of any line (BC) is determined in either of the following ways.

(1) Backsight on A with the telescope inverted, the horizontal circle remaining at the same reading it had at A (the azimuth of line AB); clamp the lower plate, turn the telescope into its direct position, and, loosening the upper plate, turn toward C. The circle will then read an angle which will be the azimuth of BC referred to the same meridian as the azimuth of AB. It is evident that this method does not eliminate any error that may exist in the line of collimation, so that the error in the azimuth will accumulate. The advantage of this method is the rapidity with which the instrument can be oriented.

(2) Add 180 degrees to the azimuth of AB, set this off on the plate, and sight on A with the telescope direct. Sight the telescope toward C, and the angle read will be the azimuth of BC. The disadvantages of this method as compared with the former are that time is consumed in setting the circle at each new set-up of the instrument, and that there is an opportunity for mistakes in calculating and in making the setting on the vernier.

154. **Vertical Angles.** — When vertical angles are to be taken the middle horizontal cross-hair is sighted at a point on the rod whose distance above the foot of the rod is equal to the distance

from the center of the transit to the ground (or the stake) beneath. This distance is known as the *height of instrument* (H.I.); it is **not** the same as the H.I. used in ordinary leveling, which is the height of the instrument above the datum plane. If the cross-hair is sighted at this H.I. point on the rod, it is evident that the line of sight is parallel to the line from the transit point to the foot of the rod, and that the difference in elevation between the center of the instrument and the H.I. on the rod is the same as the difference in elevation between the point under the transit and the foot of rod.

155. **Distances.** — The distance is read by setting one of the stadia hairs on a whole foot-mark and counting up the feet and tenths between the stadia hairs, the hundredths of a foot being estimated. If a Philadelphia leveling rod is used and the distances are short, the hundredths of a foot may be read directly. Great care should be taken not to mistake the middle horizontal cross-hair for one of the stadia hairs. This mistake is likely to occur when the telescope is of high power, so that the stadia hairs appear to be far apart in the field of view and consequently the eye does not readily see all three hairs at once. In counting the number of feet in the rod interval between the stadia hairs great care should be taken to obtain this interval correctly. It can be checked by reading the interval between the middle hair and a stadia hair and observing if this is approximately one half of the whole interval.

It is customary in reading the distance to set the lower stadia hair on that foot-mark which will bring the middle cross-hair in the vicinity of the H.I. In finding the horizontal correction to apply to the distance read it is customary to use the vertical angle taken on the H.I. Theoretically a slight error is introduced by taking the distance reading with a stadia hair on a whole foot-mark and then making the vertical angle setting on the H.I., thus taking a distance and an angle which do not exactly correspond. The middle hair, however, need not be more than half a foot above or below the H.I. when the distance is read, and in this case it is easy to show that the error can never be a serious one. For example, with a rod interval of 5.00 feet and a vertical angle of 20 degrees the corresponding reading on a rod held perpendicular to the line of sight is 4.6985. If it is assumed that it is necessary to move the cross-hair over one foot in order to set on the H.I., the effect on the vertical angle then amounts to about 0° 06′ 28″. The interval on a vertical rod held at the same place, for an angle of 20° 06′ 28″, would be 5.0036. The error in the distance would therefore be less than 0.4 feet. This indicates that for the angles and distances occurring in practice no special attention need be paid to this error, since the angles will generally be less than 20 degrees. For very large angles and great changes in rod-reading, however, it may become appreciable.

Whenever a portion of the rod is obscured, by leaves for instance, or when the distance is so great that the two stadia hairs do not both fall on the rod at the same time, an approximate value of the reading may be obtained by reading first the interval between the upper and middle hairs and then the interval between the middle and lower hairs, and taking the sum of the two readings. If the two spaces are known to be exactly equal it will be sufficient to take one reading and double it, but it should never be assumed that the two are equal. This method gives good results for small vertical angles, but when the vertical angle is large the change in the angle $\left(0° \ 17' \ 11'' \text{ for the ordinary value of } \frac{F}{i}\right)$ may introduce an appreciable error. If the rod interval is 2.30 feet (or 230 feet distance) and the vertical angle = 30 degrees, the error in the distance will amount to 0.7 feet.

It is important that the rod should be held plumb when the distance is being read, as any inclination of the rod will evidently introduce an error into the observed distance. While this error may not be a serious one for level sights, it becomes large on steeply inclined sights. In some classes of stadia work it is desirable to plumb the rod by means of a rod level whenever highly inclined sights are taken.

156. Stadia Traverses. — In a stadia traverse the instrument is set at the first station and the telescope set on the meridian (or reference line) with the vernier reading 0°. The position of the second station is located by reading the distance, azimuth, and vertical angle. In determining the azimuth of the line it is well for the rodman to show the narrow edge of his rod as a foresight, so that the transitman can make a more exact setting of the vertical cross-hair. The transit is then moved to the second station and placed in position by backsighting on the first point as explained in Art. 153; at the same time the vertical angle and the distance are again read, thus checking the distance between the two points and also the difference in elevation if stadia levels are also being taken. In reading the distances both ways there is also an opportunity to guard against an inaccurate reading due to poor illumination of the rod. By sighting both directions with the telescope erect the index error of the vertical circle is eliminated; this process is similar in principle to the peg method of testing a level. (See Vol. I, Art. 128, p. 91.)

157. Checks on the Traverse. — In running a closed stadia traverse the azimuths may be checked by redetermining the azimuth of the first line from the last and noting whether this value checks the azimuth of the first line as determined at the beginning of the traverse. If these differ by less than 5 minutes of angle the result will be sufficiently accurate for most topographical purposes. The azimuths may be roughly checked at any point by reading the magnetic bearings of the lines. Where there are triangulation points connected with the survey the known azimuths of these triangle sides will furnish a complete check.

If very long sights occur in the traverse it is desirable to measure the distance both by taking the sight for the full length of the line and by setting the instrument at a point about midway along the line and measuring the stadia distances and the vertical angles to both the backward and forward stations. The sum of the two corrected distances will usually be more accurate than the single reading from one end of the line to the other. This single reading, however, will furnish a check on the distance. It will be observed that the constant $(F + c)$ enters twice where the distance is measured by an intermediate set-up.

A useful method of checking a long stadia traverse consists in occasionally sighting on some distant object, if there is one that can be frequently seen, such as a church spire or a triangulation signal. This checks both the azimuths and the distances. (See Vol. I, Arts. 172–5, pp. 157–8.)

In running stadia traverses for several miles where no other check on the azimuths can be obtained it is advisable to make observations on the sun for azimuth, (see Vol. I, Art. 214, p. 193). Such observations can be quickly taken and if made in the morning or afternoon, when the sun is not very near the meridian, will give the azimuth within about one minute of arc, which is as accurate as is required for this purpose.

158. **Precision of the Measurements.** — The precision required in any traverse will depend upon the purpose of the survey. If it is a survey for determining the approximate area of a piece of property the distances must be read to the nearest foot or half-foot and checked by backsight readings as already explained. The angles of the traverse are usually recorded to the nearest minute, which is evidently closer than is consistent with distances which are correct to the nearest foot only. This practice of recording the distances to the nearest foot and the traverse angles to the nearest minute is also followed in traverses for topographical work. Under good conditions there will be no difficulty in obtaining the distance by stadia methods within about one part in 500. On long traverses, on account of the compensation of errors in stadia distances, an accuracy as high as one in 1000 has been reached.

In taking side shots for locating topographic details the angles should be taken with a precision which is consistent with the precision of the measures of distance and with the accuracy of the plotting. The distances will often need to be read as closely as the method permits with the ordinary form of rod, i.e., within about one foot. At a distance of about 60 feet the nearest 1° of azimuth corresponds to the nearest foot in distance; at about 340 feet the nearest 10′ of azimuth corresponds to the nearest foot of distance; and at 680 feet the nearest 5′ of azimuth corresponds to the nearest foot in distance. At distances greater than 700 or 800 feet it is not usually necessary or possible to read distances to the nearest foot, so it is safe to say that the azimuth need never be read closer than the nearest 5′ on side shots. Moreover the side shots are generally plotted with a protractor, and 5′ is about the limit of accuracy for plotting angles by this method when the ordinary form of protractor is used.

It will usually be necessary to take the vertical angles to the nearest minute in order to obtain elevations with the required accuracy. On a 340-foot sight the nearest 1′ of vertical angle corresponds to the nearest tenth of a foot in elevation; on a 1000-foot sight 1′ corresponds to 0.3 feet. This indicates that on ordinary work vertical angles should be read to the nearest minute. Where the elevations are required to a tenth of a foot, long distances are avoided because the vertical angle cannot be taken with sufficient precision, but where the elevations are not required closer than to the nearest foot, longer distances may be used, although it will still be necessary to read the vertical angles to the nearest minute. For approximate elevations and on short sights the vertical angles frequently need not be read closer than to 5 or 10 minutes.

Where it is required to locate a certain side shot with greater precision than is possible by a single stadia measurement it can be done by taking another azimuth and distance from some other transit point where the two lines to the desired point will form a good intersection. The intersection of the two lines as determined by their azimuths will give a more precise location of the point than stadia distances, provided the traverse stations have been located by tape measurements. The stadia distances to the point may be used to check its position.

159. REMARKS ON FIELDWORK. — Under ordinary conditions a party of three men is sufficient for stadia work, the note-keeper, transitman, and rodman; but where the rodman has to walk long distances between shots or where the traveling is difficult it is often an advantage to have two rodmen. So far as the transitman is concerned there is little difficulty in taking the readings rapidly enough to keep up with two rodmen, but if the note-keeper is taking full notes in regard to the topography it is probable that under ordinary conditions he will so limit the speed of the party that one rodman will be sufficient.

It sometimes happens that the rodman is so far away that directions have to be given to him by signals. A very simple and practical way to do this is by making use of a whistle, some simple code having been devised for the purpose. In determining the H.I. at a new set-up it is convenient to take this measurement with a pocket tape, as this will obviate the necessity of bringing the rod up to the transit. Since the H.I. is usually a little less than 5 feet it is convenient to have a red mark painted at the five-foot point on the rod to aid in quickly finding the H.I. in making settings. Some surveyors place a rubber band or a piece of red cloth on the rod so that it can be moved, at each set-up, to the new H.I. This is open to the objections that the band is likely to be moved accidentally and that the rodman may forget to change it at the new set-up; but it has the advantage of saving time in sighting the middle hair.

It is sometimes impossible to see the H.I. point on the rod; in such cases the vertical angle is taken at some whole number of feet above or below the H.I. and is so recorded in the notes (see Fig. 51), this correction being applied when the differences in elevation are computed. The transitman should keep in mind constantly the H.I. for the set-up he is using, not only for the purpose stated above, but also to check the position of the red marker, which may have been accidentally moved from the proper H.I. reading.

In running stadia traverses the plate is likely to get out of level; this will introduce only a slight error in the azimuths but a considerable error in the readings of the vertical circle, requiring large index corrections. For convenience the plate should be releveled, but this should not be done unless precautions are taken to prevent introducing additional error into the azimuth. This can be accomplished by noting some foresight in line with the vertical hair, releveling the instrument and then resetting, if necessary, on the foresight by means of the **lower** tangent screw.

160. **STADIA NOTES.** — To the beginner the taking of good stadia notes presents great difficulties. The general instructions with regard to transit notes apply equally well to notes for stadia surveys. (See Vol. I, Art. 148, p. 112.) A large amount of sketching or description of details is required in order to convey sufficient information to enable the draftsman, who may

not be familiar with the locality, to plot the results. Furthermore it is necessary to locate a large number of points in order to be sure of sufficient data to correctly sketch the details. If the map can be plotted soon after the fieldwork is done and by the person who made the survey, and especially if the map can be afterward taken into the field and the sketching there completed, then much greater accuracy in regard to details can be secured; this field sketching on the map is important and should be done whenever it is practicable.

The notes are usually kept in the forms shown in Figs. 50 and 51. Fig. 50 is a double page of stadia notes for "flat topography," i.e., where no elevations are required; and Fig. 51 is an example of notes for a topographical survey requiring elevations. In both cases the traverse has been run by stadia measurements at the same time that the details were taken. In locating buildings by stadia it is customary to take the dimensions of the buildings with a tape or with the rod. These dimensions are included in the notes, and to distinguish them from the numbers of the side shots it is well to mark the stadia shots in some way, such as that shown in the notes of Fig. 50, where circles are drawn around the numbers designating the side shots. In this set of notes the vertical angles have been recorded only where they were large enough to require a correction to the measured distance; and for the small angles here observed it is close enough to record them to the nearest half-degree. Evidently there is no necessity in this case for indicating whether the vertical angles are + or −.

When only a few dimensions appear on the sketch, as in Fig. 51, the numbers of the side shots may be written without any distinguishing mark. The points occupied with the instrument are numbered consecutively, and are distinguished by a ⊡, the usual symbol for stadia station. The side shots are numbered consecutively, either throughout the whole survey or through each day's work. The vertical angles are marked + or − to indicate whether the angles are elevation or depression. If the middle cross-hair is sighted on any point other than the H.I. it should be noted immediately below the vertical angle; if this note is put on the opposite page it is liable to be overlooked when the elevations are calculated. Columns are provided for the differences in elevation and for the actual elevations. Some surveyors prefer to have a separate column for the H.I., marked "middle cross-hair" (M.C.H.). If many of the vertical angles are large it may be well to have a column for the reduced distances. Ordinarily, however, it is sufficient to write the horizontal correction immediately under (or to one side of) the distance when the notes are worked up in the office and to subtract it, mentally, when the point is plotted. In Fig. 51 the first five columns only are entered in the field; the last two columns are worked out in the office before plotting. It will be well, however, to compute the elevations of the instrument points while in the field in order to check these governing points as the fieldwork progresses and to correct any discrepancies which may be found. In Fig. 51 the elevations that were computed in the field have been written in these notes with inclined figures, while the office computations are here represented by erect figures, the distinction being made in this set of notes simply for the benefit of the student.

By referring to Fig. 51 it will be seen that the elevation of ⊡ *T* has been computed from the elevation of BM_{12} which was established by direct leveling. Before leaving ⊡ *T* the elevation of ⊡ *P* was determined by stadia leveling and checked when the backsight on ⊡ *T* was taken. Similarly the elevation of ⊡ *Q* was determined by stadia leveling when the instrument was at ⊡ *P*, but when the instrument was set up at ⊡ *Q* a level rod-reading was obtained on ⊡ *P* and the elevation of ⊡ *Q* determined as 91.7; and since this method is more accurate than that of stadia leveling the value 91.7 was used as the elevation of ⊡ *Q*, the value 91.5 being simply a check on it. For convenience in computing the elevations of the side shots the elevations of the various station points are included in little rectangles in these notes, so that they may be readily identified.

It will be noticed that the vertical angle for point 17 was taken at 14.5 feet on the rod. As the rod in this case was not over 12 feet in length, the rodman obtained this reading by holding the rod on his knee, a process which is allowable for an isolated side shot where a precise determination is not required. From the

Survey of Main St.						
Sta.	Dist.	Az. ∠	Magnetic Bear.	Vert ∠		
⊼ at ⊡A12	701	0° on ⊡11	S 35° E	4° 30′		
18	111	337° 10′				
19	80	326° 00′				
20	35	146° 50′				
21	21	118° 30′				
22	78	40° 30′				
23	100	58° 25′				
24	92	12° 45′				
25	117	202° 10′				
26	210	190° 00′				
27	116	192° 35′				
28	260	184° 20′				
29	214	153° 10′				
30	232	155° 15′				
31	430	175° 05′		C of Cross-road W. side Main St.		
32	469	173° 55′				
33	157_{-2}	226° 30′		6° 30′		
34	138_{-4}	251° 00′		9° 30′		
35	129_{-2}	282° 00′		8° +		
⊡A13	503_{-5}	171° 32′	N 43½° W	6° −		
⊼ at ⊡A13	502_{-5}	0° on ⊡12	S 43¼° E	6° −		
36	297_{-3}	323° 30′		6° −		
37	241	323° 30′				
38	100_{-2}	306° 30′		7° 30′		
39	93	220° 10′				
40	91	233° 50′				
41	70	258° 00′				

FIG. 50. STADIA NOTES FOR "FLAT TOPOGRAPHY." (See page opposite.)

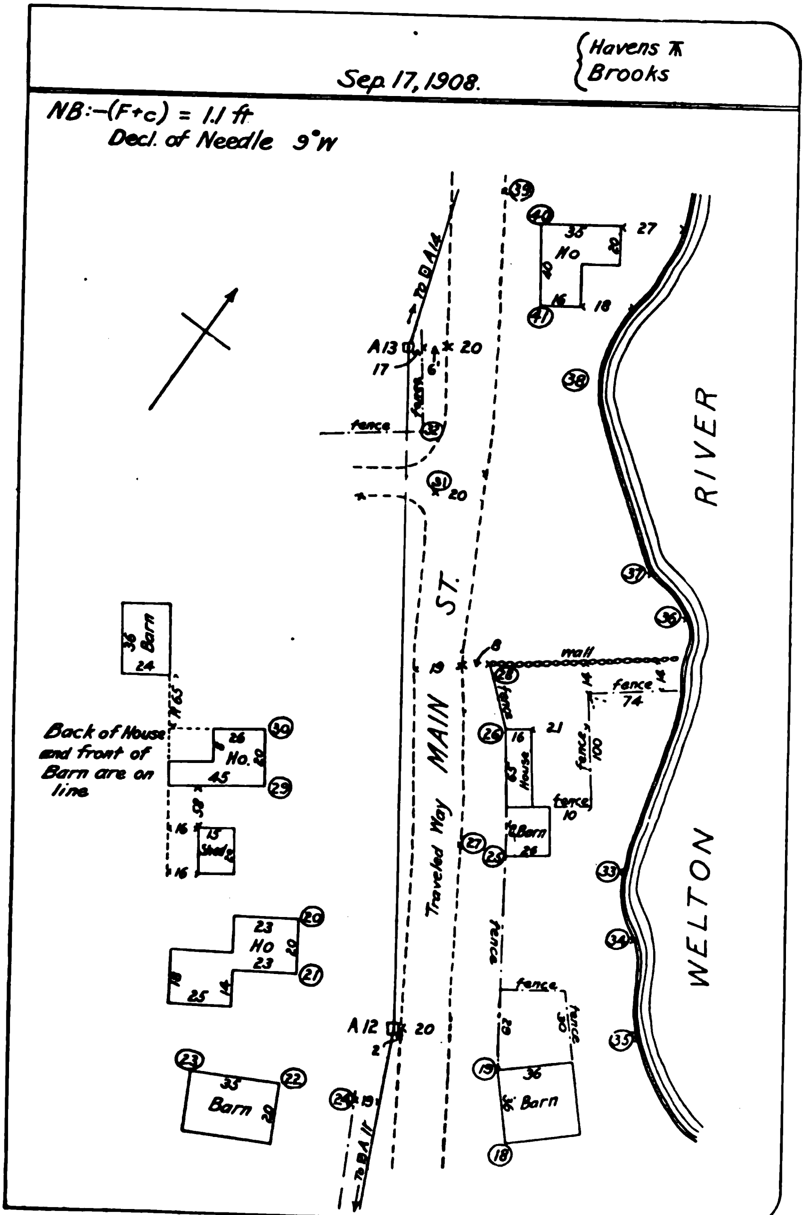


Fig. 50. Stadia Notes for "Flat Topography." (See page opposite.)

Preliminary Survey for Highway
Westwood to Lincoln.

Sta.	Dist.	Az. Ang.	Bear.	Vert. Ang.	Diff. El.	Elev.
⊼ at ⊡ T,	O° on Mag.	South,	BS. on ⊡ R.	H.I. = 4.7		$\boxed{82.7}$
⊡ R	421	222°36′	N 42½° E	−0°49′	−6.0	76.7
1 B.M.$_{12}$	53_{-4}	277°00′		−16°54′	−14.8	72.64
2	163_{-2}	274°20′		−6°41′	−18.9	63.8
3	106	326°30′		−1°08′	−2.1	80.6
⊡ P	458	335°37′	S 24¼° E	+0°14′	+1.9	84.6
⊼ at ⊡ P	B.S. on ⊡ T			H.I. = 4.7		$\boxed{84.6}$
⊡ T	457	155°37′	N 24° W	−0°15′	−2.0	82.6
4	221	157°10′		−1°18′	−5.0	79.6
5	101	168°00′		−2°35′	−4.6	80.0
6	42	334°00′		−4°50′	−3.6	81.0
7	209_{-1}	328°50′		−4°33′ on 10.7	−22.6	62.8
8	175	314°15′		−1°18′ on 9.7	−9.0	75.6
9	209_{-1}	290°00′		−4°09′ on 10.7	−21.2	63.4
10	90_{-2}	222°-10′		−8°44′ on 10.7	−19.7	64.9
11	$111 \times 2_{-1}$	174°-10′		−4°20′ on 7.7	−20.8	63.8
12	294_{-2}	205°30′		−4°20′	−22.2	62.4
⊡ Q	415	287°24	S 72¼° E	+0°54′	+6.9	91.5
⊼ at ⊡ Q	B.S. on ⊡ P			H.I. = 4.5		$\boxed{91.7}$
⊡ P	415	107°24′	N 72½° W	0°00′ on 11.6	−7.1	84.6
13	406_{-2}	166°10′		−4°00′	−28.9	62.8
14	260_{-3}	126°15′		−6°25′	−29.0	62.7
15	135_{-3}	128°20′		−8°50′ on 3.5	−19.7	71.0
16	27	246°00′		0°00′ on 5.5	−1.0	90.7
17	130_{-2}	215°15′		−6°28′ on 14.5	−24.6	66.1
18	158_{-2}	176°00′		−5°46′ on 10.5	−22.0	68.7
19	206_{-2}	192°50′		−5°15′	−19.7	71.0
⊡ R	450+451	160°04′	N 20° W	−0°58′	−15.2	76.5

FIG. 51. STADIA NOTES OF TOPOGRAPHICAL SURVEY. (See page opposite.)

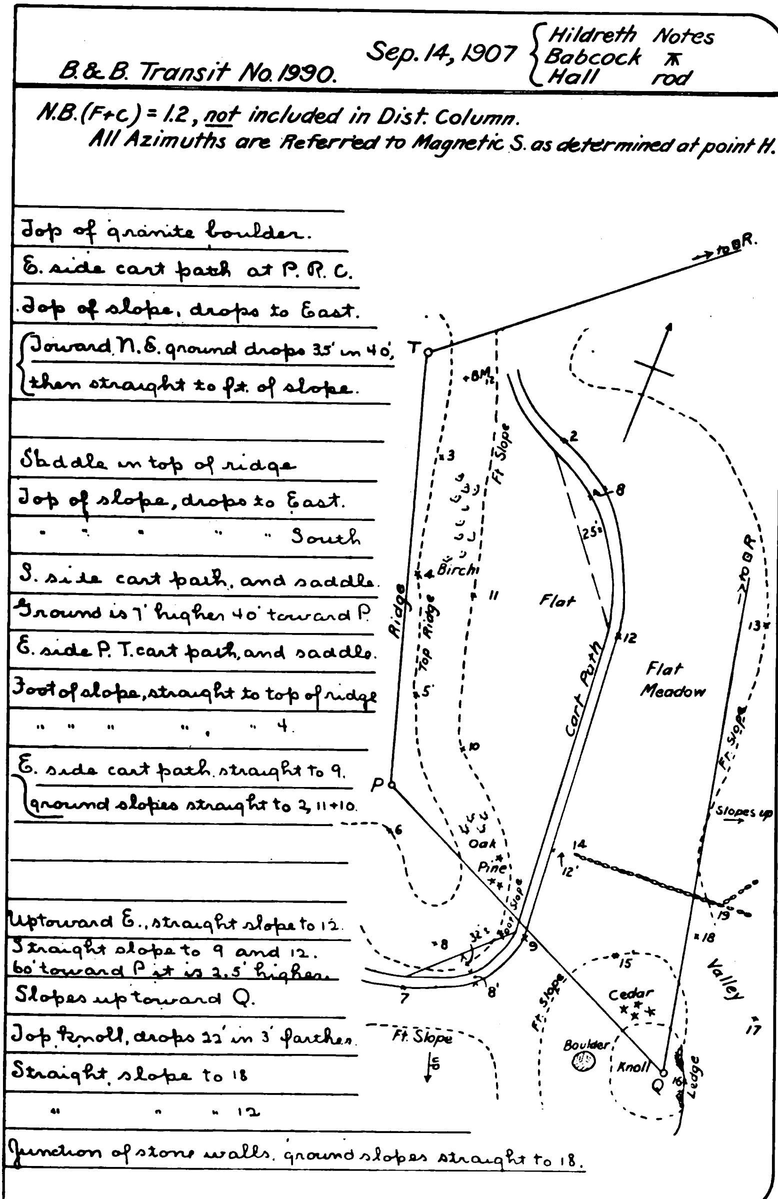


FIG. 51. STADIA NOTES OF TOPOGRAPHICAL SURVEY. (See page opposite.)

notes it will be seen that the forward and reverse bearings were taken as a check on the azimuths of the traverse lines. The azimuths of the traverse are recorded to the nearest minute, while the side shots were taken to the nearest 5 minutes only, which is close enough for most topographic details. All of the azimuths in this particular case were measured from the magnetic south, but the note-keeper took the precaution to indicate in his notes the station used as a backsight (B.S.), so as to aid in correcting errors if it is found in the office that the azimuth of any of the traverse lines is in error. The magnetic bearings will indicate any large error, which should be corrected in the field before it is carried forward into any of the subsequent work. At point 11, owing to obstructions, the distance had to be obtained by a half-interval reading on the rod. Again at ⊡ *R* the half-interval had to be used, but since this distance was along the traverse line each half-interval was read and recorded as shown, instead of relying on the middle hair's being exactly halfway between the other two hairs. This assumption was allowable, however, in the case of the side shot No. 11.

The notes and descriptions on the right-hand pages of both of these sets of notes are essentially the same as for any other transit notes and need no special description.

It is often convenient to have the rodman keep a set of sketches and side notes. He, of course, is in the best position to judge the slope of the ground and to note many other details. It is a good plan for the rodman to keep the same numbers of the shots that the note-keeper has, and to record these on his sketch. He signals or calls every fifth or every tenth shot to the note-keeper in order to be sure that their numbers correspond. The information on the rodman's sketches may afterwards be transferred to the note-book or to the map.

161. METHODS OF REDUCING THE NOTES. — The computation of the differences in elevation and of the horizontal corrections by means of the formulas is so laborious that it is generally done by means of a table, diagram, or stadia slide rule. Tables are used to some extent in reducing stadia notes, but the work is much more rapidly performed by means of diagrams or by a stadia slide rule.

162. Use of Stadia Tables. — To illustrate the use of Table X it is assumed that the following readings have been taken.

Station	Observed Rod Intervals	Observed Vert. Angle	Computed Diff. El.	Computed Hor. Dist.
1	311	$+\ 3^\circ\ 54'$	+21.18	311
2	91	$-\ 25^\circ\ 28'$	−35.71	75
3	240	$+\ 0^\circ\ 39'$	+ 2.74	241

$(F + c) = 1.0$ foot.

The rod intervals are recorded here as distances (100 × rod interval), which is in accordance with common practice, rather than as actual rod intervals. The constant $(F + c)$ has not been included in the recorded distances in the above example.

To obtain the difference in elevation for Station 1 from the Table of Vertical Heights (Table X), in the column headed 3 degrees and opposite 54 minutes, 6.79 feet is found as the difference in elevation for a 1-foot rod interval, i.e., a 100-foot distance. By adding $(F + c)$ to the distance 311 the corrected distance 312 is obtained. Then 6.79 feet (the difference for 100 feet) multiplied by 3.12 gives 21.18 feet as the difference in elevation between the center of the instrument and the point on the rod where the middle cross-hair was sighted when taking the vertical angle. This multiplication may be performed with an ordinary slide rule. Beneath the Table of Vertical Heights is given the Table of Horizontal Corrections. The horizontal correction for 311 feet and 3 degrees 54 minutes is found by interpolating between 3 degrees and 4 degrees and also between 300 feet and 400 feet, the result being 1.4 feet. The horizontal distance, to the nearest foot, is then $311 + 1 - 1 = 311$.

At Station 2 the difference in elevation is $.92 \times 38.82 = 35.71$. The horizontal distance is $91 + 1 - 17 = 75$ feet. At Station 3 the vertical height for 0 degrees 39 minutes and for 100 feet is taken from the 0 degree column by interpolation between 38 and 40 minutes and is 1.14, the difference in elevation for the rod interval 2.40 being 2.74 feet. The horizontal correction is only 0.1 and is therefore neglected. Suppose that at Station 3 the vertical angle $+ 0° 39'$ had been taken at a point on the rod 4 feet **below** the H.I., the difference in elevation would then be $+ 2.74 + 4.00 = + 6.74$. Had the sight been taken on the rod 4 feet **above** the H.I. the difference in elevation would be $+ 2.74 - 4.00 = - 1.26$.

A convenient way of reducing stadia readings when a table of horizontal corrections only is available consists in first subtracting the correction from the distance read, to obtain the horizontal distance, and then computing the difference in elevation by multiplying the horizontal distance by the natural tangent of the vertical angle. The ordinary slide rule may be used for this operation.

In ordinary stadia work the distance is read to only 3 significant figures, and since the resulting vertical height can be accurate to only 3 figures the ordinary 10-inch slide rule is sufficiently precise for these computations. It is not customary to carry the elevations closer than to the nearest tenth of a foot.

163. Stadia Reduction Diagrams. — There are several forms of stadia diagram, each of which has its special advantages and disadvantages. They are usually designed either for office or for field use, the latter being much more compact than the former. Some of these diagrams may be obtained through publishers but surveyors often construct their own stadia reduction diagrams according to the size and arrangement desired. For convenience these are sometimes plotted on cross-section paper, but for the best results the entire diagram should be carefully constructed to scale. One of the simplest forms of diagram for office work is here represented, in order to show the principle of construction.

In Fig. 52 the right-hand portion is a Diagram of Differences in Elevation and the left-hand portion is a Diagram of Horizontal Corrections. The scale on the lower side of the rectangle represents the distances read; on the right-hand side is a scale of vertical heights; the sloping lines represent vertical angles. On the right-hand vertical line, which corresponds to a distance of 1000 feet, vertical heights are plotted. These are obtained by multiplying by 10 the vertical heights from Table X corresponding to any desired angles, say 1 degree, 2 degrees, etc. These plotted points are joined by straight lines to the center o. From the first term in formula [51], p. 157, it will be seen that the differences in elevation vary directly as the distances; hence it is evident that these inclined lines cut off on any vertical line the difference in elevation corresponding to that distance. In a similar manner a diagram giving values of the horizontal correction may be constructed as shown in Fig. 52. In practice

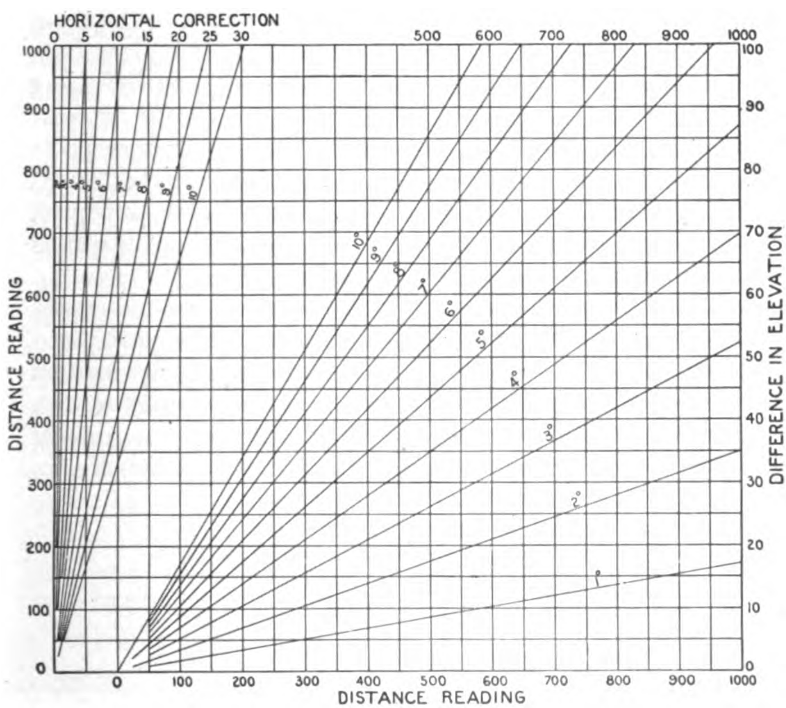


FIG. 52. STADIA REDUCTION DIAGRAM.

it would be necessary to draw lines for every 5 minutes or every 10 minutes of the vertical angle, so that little interpolation would be necessary.

To obtain the difference in elevation from the diagram, find a vertical line corresponding to the distance read and on this line note the point where it is crossed by the inclined line corresponding to the vertical angle. The horizontal line passing through this point represents the difference in elevation. The usual approximation of first adding the constant $(F + c)$ to the distance is made before taking the vertical height from the diagram. The horizontal correction is found in a similar manner from the Horizontal Correction Diagram.

164. Stadia Slide Rules. — The most rapid means of reducing stadia readings is by the use of a slide rule which has, in addition to the ordinary scale of numbers (logarithms of the distances), two scales especially constructed for stadia work, one consisting of values of $\log \cos^2 \alpha$ and the other of $\log \frac{1}{2} \sin 2\alpha$ for different values of α. On some rules the values of α range from 0° 34′ to 45°, on others from 0° 03′ to 45°. In some forms the horizontal distance is read directly; in others the horizontal correction $1 - \cos^2 \alpha$, or $\sin^2 \alpha$, is given. A 10-inch slide rule gives results sufficiently accurate for all ordinary purposes.

Fig. 53 represents the face of a stadia slide rule manufactured by Kern & Co., Aarau, Switzerland, which is very compact and convenient for stadia reductions.* The log distances are marked on the **rule**; the $\log \frac{1}{2} \sin 2\alpha$ is on the **slide** and is marked with the degrees of the vertical angle α; on the **runner** is a scale of $\log \cos^2 \alpha$, which is used in obtaining the horizontal distances.

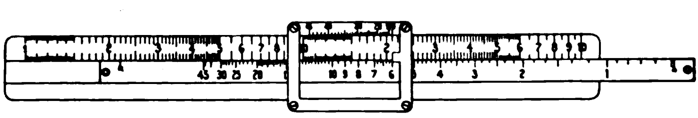


FIG. 53. THE KERN STADIA SLIDE RULE.

In Fig. 53 the rule is set for an observed distance of 210 feet and a vertical angle of 6 degrees. The difference in elevation is read opposite the star at the end of the slide, the reading being

* These rules are made for a circumference divided into 360 degrees or into 400 degrees, as desired.

21.8. There are three stars on the slide, any one of which will give the true reading provided this star is not off the end of the rule. On the **runner** the 0° mark is set opposite 210 on the rule, and under the 6° mark on the **runner** is found the reduced horizontal distance, 208. On the runner is a little projection the edge of which is opposite the zero-point of the runner scale. When this index is set at the proper distance reading, the zero of the horizontal scale is then in the correct position. It will be noticed that the scale on the slide is arranged so as to increase from right to left, which is contrary to the usual arrangement of a slide rule. On the lower side of the Kern rule is a scale (not shown) giving curvature and refraction corrections in metric units.

There are several other types of stadia slide rule in common use. The one invented by Mr. B. H. Colby, of St. Louis, is so arranged that differences in elevation can be found in the same unit as that used in reading the distance, or in a different unit, according to which index point is used. This is convenient where distances are taken in meters and elevations are expressed in feet, as is done on some topographic surveys.

The slide rule designed by Mr. G. H. Matthes and used on the U.S. Geological Survey is similar to the Kern rule described above, but contains also a scale of log tangents, a scale of curvature corrections in feet, and a scale of distances expressed in miles. The scale of vertical angles on this rule extends to 0° 03′.

The slide rule designed by Mr. W. L. Webb is cylindrical in form and differs from the others in that it gives horizontal corrections instead of horizontal distances.

The most convenient means of reducing stadia notes is to use a stadia slide rule for differences in elevation and a condensed table like that given on p. 180 for horizontal corrections. If desired, such a table can be made in the form of a small blue-print and carried into the field. In Table 4 it will be found that for long distances and large vertical angles the interpolation will not always give the correction to the nearest foot. These errors of interpolation are, however, always small as compared with the errors in the observed distances.

TABLE 4. — HORIZONTAL CORRECTION.

Vertical Angle	Distances Read									
	100	200	300	400	500	600	700	800	900	1000
2°	......	......	......	......	1	1	1	1	1	1
3	......	1	1	1	1	2	2	2	2	3
4	.....	1	1	2	2	3	3	4	4	5
5	1	2	2	3	3	5	5	6	7	8
6	1	2	3	4	5	7	8	9	10	11
7	1	3	4	6	7	9	10	12	13	15
8	2	4	6	8	10	12	14	15	17	19
9	2	5	7	10	12	15	17	20	22	24
10	3	6	9	12	15	18	21	24	27	30
11	4	7	11	15	18	22	25	29	33	36
12	4	9	13	17	22	26	30	35	39	43
13	5	10	15	20	25	30	35	40	46	51
14	6	12	18	23	29	35	41	47	53	59
15	7	13	20	27	33	40	47	54	60	67
16	8	15	23	30	38	46	53	61	68	76
17	9	17	26	34	43	51	60	68	77	85
18	10	19	29	38	48	57	67	76	86	95
19	11	21	32	52	43	64	74	85	95	106
20	12	23	35	47	58	70	82	94	105	117

165. METHODS OF PLOTTING STADIA NOTES. — Stadia notes are usually plotted by means of a circular protractor and a scale. If the main traverse is a transit and tape survey, or if the scale of the map is such that a protractor would not be sufficiently accurate, the traverse may be plotted by some more accurate method and the side shots put in afterwards by the protractor and scale. In general, however, any measurements taken by stadia may be plotted with sufficient accuracy by means of the protractor. (See Vol. I, Chap. XV, for methods of plotting traverses.)

In setting the protractor in position for plotting it should be centered with care and turned to the proper azimuth as defined by a 0° line drawn through the point and extending each way beyond the circumference of the protractor. It is not safe to trust to a line extending only one way, because the center of a protractor is usually marked in such a way that it is difficult to place it exactly over the transit point on the plan. Many protractors which are accurately graduated have the center point carelessly marked. The most accurate way to use a protractor is to draw two lines at right angles to each other, one of them being the meridian or reference line. These lines may be drawn at right angles as explained in Vol. I, Art. 451, p. 402. The protractor is then *oriented*, i.e., turned in the proper direction by means of the cardinal points on the circumference of the circle, without regard to the position of the center mark on the protractor.

The usual process is to place the protractor in position and plot all of the azimuths first, marking each by a light dot or a short radial line in the proper azimuth and writing opposite the mark the number of the shot. This work can be conveniently done by two persons, one reading the azimuths and the numbers while the other plots the angles. When all of the azimuths have been plotted the protractor is removed and the distances are scaled off, the proper elevation being written opposite each point. Sometimes the plotted position of the point is indicated by a dot enclosed by a small circle, the height being written at one side. Another way, which is convenient when the plotted points are close together, is to write the whole number of feet of the elevation to the left of the point and the tenths to the right, the plotted point itself serving as the decimal point.

If many points are to be located from one transit station a convenient arrangement for the plotting is as follows. Take a paper protractor of 10 or 12 inches diameter and cut out the central portion as shown in Fig. 54. The protractor must be placed in position by means of two long lines at right angles to each other, since the center of the circle is not available. To the zero end of an engineer's scale attach a small piece of metal bent so as to hold to the under side of the scale as shown in Fig. 54. Bore a small hole in the metal at the front edge of the scale. The metal piece is to be slid along until this hole is opposite the zero point of the scale. This may be most accurately done by first laying off on the map some convenient distance from the plotted position of the transit point, the metal piece being off the scale. Then replace the metal piece on the scale, center the hole over the transit point, and insert a small needle. The scale is slid along through the metal until it reads this distance which was laid off. The metal piece

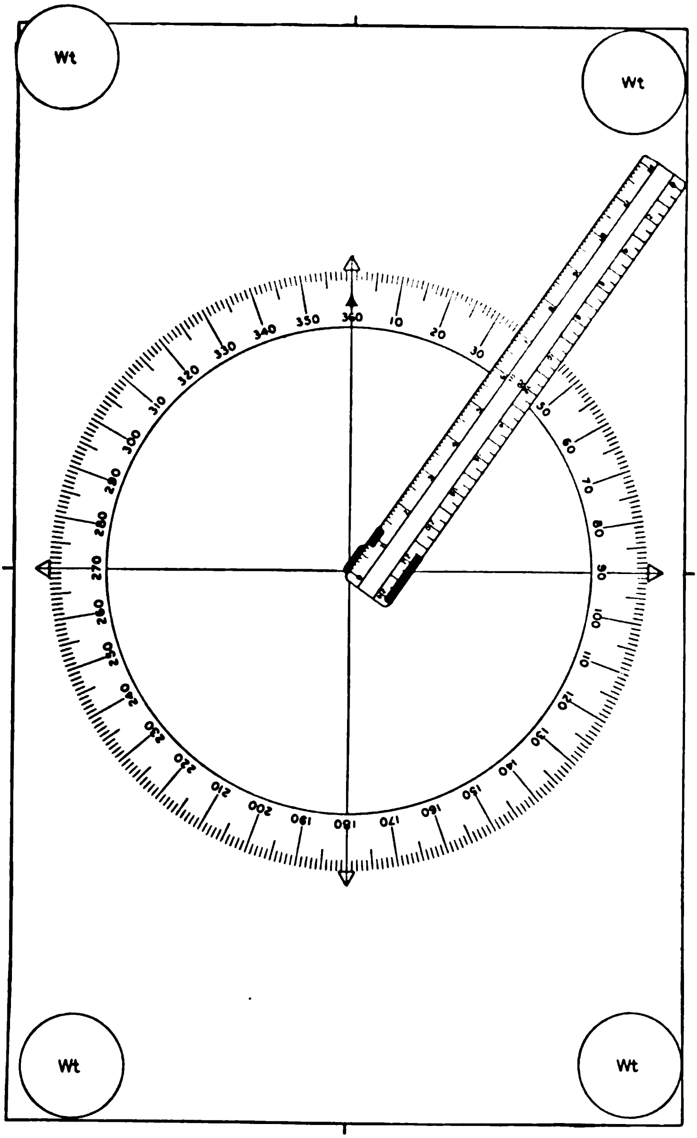


FIG. 54. PROTRACTOR FOR PLOTTING STADIA NOTES.

should fit tightly enough so that the scale cannot readily slip through it. The edge of scale will then be on a radial line, its zero point being at the center. The position of the zero point of the scale should be tested occasionally by re-scaling the distance first laid off. The edge of the scale should be tested to see that it is straight. In plotting a point the scale is turned to read the proper azimuth, and the point is then marked by pressing a needle point into the paper or by making a pencil dot at the proper reading on the scale. By this method no time is wasted in marking azimuths and in placing and removing the scale, both of which are required by the previous process. If only a few shots are to be taken from one transit point this method is not so rapid as the former, as it takes longer to set the protractor in position before plotting. Protractors are manufactured the general arrangement of which is similar to the one just described, the scale being pivoted at its zero point, which is at the center of an annular protractor.

166. "**Stepping**" **Method.** — When the vertical angle is small, as it usually is on long distances, many approximate methods may safely be used to economize time in the field or to avoid calculation. The following method of finding differences in elevation has been practiced by the U. S. Geological Survey, especially in connection with the plane table, where it is necessary to make the calculations at once in order to utilize the results in the sketching. In this method it is assumed that varying the vertical angle has no appreciable effect on the length of the intercepted space on the rod; in other words, that if the rod were lengthened out until a level sight could be taken upon it, the same stadia interval would be read near the top of the rod as near the bottom, which is nearly true for small angles.

In determining the difference in elevation by this method the observer first reads the distance as usual on the rod; he then levels the telescope, which of course brings the middle horizontal cross-hair on the true horizon. Then, if the rod is below the instrument, he notes a point in the landscape which is covered by the lower stadia hair. The upper hair is next set on this point and a new point noted in line with the lower hair. The process is continued until the rod is reached. If the middle hair is somewhere on the rod then a whole number of spaces has been stepped off between the level line through the transit and the point on the rod where the middle hair strikes. Suppose that the rod interval is 5.00 feet (i.e., dist. = 500 feet + $(F + c)$), and that 4 stadia intervals have been stepped off, bringing the middle hair to 9.10 on the rod. Then the foot of the rod is $5.00 \times 4 + 9.10 = 29.10$ feet below the center of the instrument. If the middle cross-hair is not on the rod one of the stadia hairs must strike the rod, and it is only necessary to allow for half an interval in addition to the whole number stepped off in order to obtain the result.

167. BEAMAN'S STADIA ARC. — Another method of simplifying the calculation of elevations in the field consists in using only those vertical angles for which the differences in elevation are simple multiples of the rod interval, i.e., the angles for which the function $\frac{1}{2} \sin 2\alpha$ is a simple multiple (see equation [51], p. 157). These angles for multiples 1 to 20 are given in Table 5. The rod interval is first read as usual; the vertical angle is then examined and the telescope inclined until the vernier reads the nearest angle given in the table, and the corresponding reading of the middle cross-hair is noted.

TABLE 5. — VERTICAL ANGLES FOR WHICH THE DIFFERENCE IN ELEVATION IS A SIMPLE MULTIPLE OF THE ROD INTERVAL.

Multiple	Vertical Angle	Multiple	Vertical Angle
	° ′ ″		° ′ ″
1	0 34 23	11	6 21 16
2	1 08 46	12	6 56 36
3	1 43 12	13	7 32 06
4	2 17 39	14	8 07 48
5	2 52 10	15	8 43 44
6	3 26 46	16	9 19 53
7	4 01 26	17	9 56 18
8	4 36 12	18	10 33 00
9	5 11 06	19	11 10 01
10	5 46 07	20	11 47 21

The intercept on the rod multiplied by the corresponding number in Table 5 gives the difference in elevation between the center of the instrument and the point where the middle cross-hair strikes the rod. Combining the rod-reading of the horizontal cross-hair with the difference in elevation just computed gives at once the difference in elevation between the center of the instrument and the point where the rod is held. It should be noted that in this method the differences in elevation are computed with reference to the center of the instrument and not to the point on the ground under the instrument.

The above principle is made use of in a very simple and ingenious device which may be attached to the vertical arc of a transit or to the alidade of a plane table. This attachment (Fig. 55) is

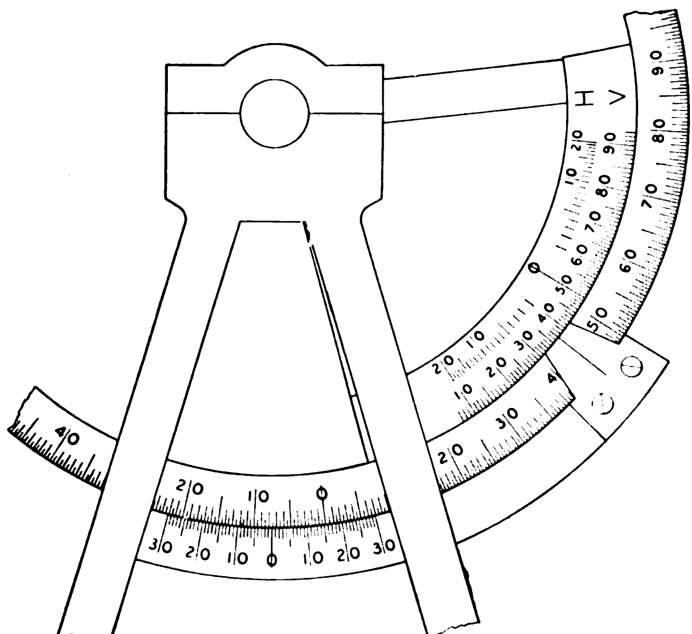


FIG. 55. THE BEAMAN STADIA ARC ATTACHMENT.

known as the *Beaman Stadia Arc* and was invented by Mr. William M. Beaman, Topographic Inspector, United States Geological Survey. On the attached arc are graduations which correspond to the vertical angles for which the multiples are given in Table 5. The zero graduation of this arc is marked 50 instead of 0, and consequently to obtain the multiple 50 must be subtracted from the reading. The advantage of this numbering of the graduations is that the direct reading of the arc will show whether the angle is **plus** or **minus.** In using an instrument provided with this attachment the observer reads the distance, turns the tangent screw of the telescope until the index line is opposite some line on the attached arc, and then notes the rod-reading of the middle cross-hair. The number of the graduation on the arc and the observed rod-reading are recorded in the note-book. The difference of elevation between the center of the instrument and the reading of the horizontal cross-hair on the rod is then computed by multiplying the rod interval by the reading of the arc **minus** 50. The elevation of the point on the ground will be equal to the elevation of the center of the instrument plus or minus the calculated difference minus the rod-reading. For example, by referring to Fig. 55 it will be seen that the stadia arc is set on 37, which corresponds to a multiple of -13. Suppose that the rod interval is 2.50 feet, that the rod-reading is 7.8 feet, and that the elevation of the center of the instrument is 207.4 feet, the elevation of the foot of the rod is then $207.4 - 13 \times 2.5 - 7.8 = 167.1$ feet. It is evident that the elevation can be obtained by this method with very little calculation. Careful attention must be paid to the index correction either by leveling the telescope and setting the index at 50 before taking the readings or else by means of the vernier level described in Art. 144, p. 148.

On the same arc is a scale for applying the horizontal correction. These graduations give the ratio of the horizontal correction to the distance read, expressed as a percentage. For example, in Fig. 55 the index is opposite the second line, showing that the horizontal correction is 2 per cent of the distance.

168. STADIA LEVELING. — In hilly country it is often more economical, and quite accurate enough for topographical purposes, to run levels by the stadia method rather than by direct leveling. Leveling by stadia is similar to ordinary leveling, excepting that instead of obtaining the difference in elevation by means of two level rod-readings it is obtained by a combination of two stadia distances and two vertical angles taken to the rod when held on the two stations whose difference in elevation is desired.

In stadia leveling the position of the instrument is chosen, as in ordinary leveling, with reference to the distance to the backsight and to the foresight points, because any error in the adjustments of the line of sight, or the level bubble, will be eliminated just as in ordinary leveling. But since the precision of stadia leveling never can equal that of direct leveling the matter of equalizing the aggregate of the foresights and backsights is not of quite so much importance in the former as it is in the latter.

In stadia leveling the transit is set up at a convenient place (preferably not over 400 to 500 feet from the bench mark) and the stadia distance is read. The middle cross-hair is then made to coincide with the nearest foot-mark on the rod, and the vertical angle is read and recorded together with the rod-reading of the middle cross-hair. The index error of the vertical arc is then obtained and recorded, provided it has not already been eliminated by use of the vernier level. The rod is then carried to the T.P., the stadia distance measured, and, if possible, the vertical angle taken on the **same** foot-mark on the rod as was used at the previous point. If it is not possible to sight on the same foot-mark some other point on the rod is sighted and the proper record of it made in the note-book.

By the use of stadia tables the difference in elevation between the point sighted on the rod and the center of the instrument can be calculated for both the foresight and the backsight readings. Neither the diagrams nor slide-rules give close enough results for the better class of this work. The constant $(F + c)$ should not be neglected in the computation. For small vertical angles sufficiently accurate results are obtained if $(F + c)$ is added to the distance read. (Art. 150, p. 157.) A combination of these two differences in elevation will give the difference in elevation between the B.M. and T.P. if the two readings of the middle cross-hair were the same. If it was found necessary when taking the T.P. rod-reading to use a different foot-mark from that used on the B.M. the difference in rod-reading should be applied as a correction to the difference in elevation. The line of levels is carried along as in ordinary leveling, B.M.'s being established as frequently as desired.

Stadia levels can be run with an accuracy of about 0.5 feet per mile, which represents the best that can be expected in such work. While the errors of a single reading are large, it is only on account of the compensation of errors that the above accuracy can be

Rough Survey of Portion of Webster Mill Property
Malvern, Conn.

Sta.	Dist.	Magnetic Bearing	Vert. ∠	Diff. El.	Elev.	
⊼ at ⊡2	H.I. = 4.5					
1	370	S 31¼° W	−1° 10′		792.5	
3	219	N 21° E	+2° 12′			
⊼ at ⊡4	H.I. = 4.2					
3	165	S 51° W	−4° 00′			
5	401	N 59° E	+3° 19′			
⊼ at ⊡6	H.I. = 4.0					
5	275	S 60½° W	−2° 51′			
7	100	N 71¾° E	+3° 16′			at Brook
⊼ at ⊡8	H.I. = 5.0					
7	259	S 45° W	−3° 01′			
9	200	N 35½° W	−6° 02′			
10	315	N 76° W				
11	501	N 59° W				
12	550	N 48¼° W				
13	350	N 37½° W				
14	260	N 28° W				
15	310	N 35½° E	+1° 26′			
⊼ at ⊡16	H.I. = 4.6					
15	206	S 11½° W	−2° 06′			

FIG. 56. NOTES OF ROUGH STADIA TRAVERSE. (See page opposite.)

reached. This degree of accuracy applies to work where the vertical angles do not exceed about 3 degrees. As the vertical angles increase the accuracy decreases rapidly.

169. Stadia for Rough Surveys. — If a rapid survey of inferior accuracy is required the reading of the compass needle may be substituted for the azimuth reading. In this case only

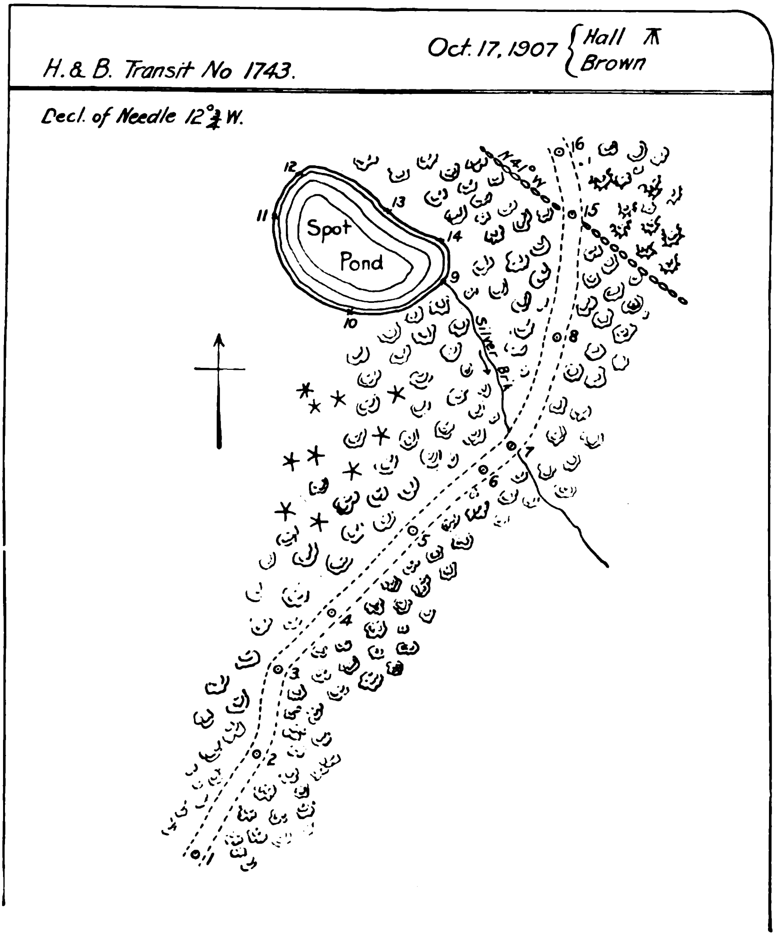


Fig. 56. Notes of Rough Stadia Traverse. (See page opposite.)

the alternate stations need be occupied with the transit, the back bearing and distance being used to locate the instrument point from the preceding point, as illustrated in the notes of Fig. 56.

In this way levels may be carried with the usual accuracy, but the accuracy of the horizontal location is no greater than that of compass surveying. Such traverses are subject to considerable error in any place where the magnetic needle is affected by local attraction.

170. STADIA FOR PRECISE WORK. — By taking special precautions it is possible to do extremely accurate work with the stadia. If measurements need to be taken across ravines, canyons, etc., where chaining would be practically impossible and triangulation would be slow, the stadia may be used very successfully. Instead of using the ordinary self-reading rod two targets of the "bisection" pattern are fastened to the back of a rod. The upper target is fixed and the lower one movable. In measuring a distance the upper stadia hair is set on the upper target and the lower target is moved by the rodman until it is opposite the lower stadia hair. The distance between the targets is then measured with a steel tape. By taking several measurements in this manner a precision of $\frac{1}{5000}$, or sometimes greater, can be reached. Special attection should be paid to the corrections and to all approximations ordinarily made in stadia work. The stadia interval $\left(\text{constant } \frac{F}{i}\right)$ should be carefully determined by several observations taken under the same atmospheric conditions as those existing when the measurements are taken. If possible such measurements should be made with the instrument protected from the sun and the targets well illuminated by the sunlight. If the line of sight passes close to the ground the refraction may make great precision impossible, but fortunately this method is most applicable to ravines where there is the least trouble from this source. In order to obtain the greatest accuracy it is necessary to pay especial attention to plumbing the rod and to bracing it so that it will remain steady.

CHAPTER V.

THE PLANE-TABLE METHOD.

171. THE PLANE-TABLE METHOD.* — The plane table is an instrument by means of which points are located in the field directly on the map by graphical methods, the map being fastened to a table top supported by a tripod, from which the instrument derives its name.

From a mathematical standpoint the plane table is by no means an exact instrument, for there are theoretical errors introduced by the method of using the table; but from the topographer's standpoint it is a most useful instrument. The accuracy of a map is usually limited by the accuracy of the plotting rather than by that of the field measurements, hence if points can be located with the plane table as accurately as distances can be scaled from the map this method is well adapted to topographic work. Furthermore the method of plotting which is used with the plane table is superior to most of those used in the office. All of the errors due to defects in the methods used in plane-table work become inappreciable on small scale maps. The method is adapted to large scale maps also, provided special precautions are taken to insure accuracy. Since, however, in this work the map must be taken into the field and consequently exposed to the weather it is also necessary to take special care to prevent the map from becoming so distorted that its accuracy is lost, and if such precautions are not taken the resulting map may be seriously in error.

The plane table is the only surveying instrument admitting of a rapid solution of the Three-point Problem in the field; this makes it practicable to locate stations independently of each other so that errors cannot accumulate as they do in traversing.

The most important advantage of the plane-table method over

* See U. S. Coast Survey Report for 1905, "A Treatise on the Plane Table," by D. B. Wainwright.

other topographic methods, however, is that all of the sketching is done in the field, where the topographer can see the form of the ground that he is mapping. He can sketch details at once in their proper position, without burdening his memory and without making elaborate notes. For this reason the details may be accurately sketched from a much smaller number of located points than would be required, for instance, by the transit and stadia method. Furthermore no time need be spent in taking unnecessary data, since the topographer can decide at once what points are needed. Every topographic map consists of a sketch controlled by a number of accurately located points. The greater the number of such points the greater the accuracy of the map and also the greater the cost. When the details can be sketched in the field and all points can be taken in the positions where they are most needed the map will evidently require a smaller number of points than where the sketch is made up from descriptions or from fragmentary sketches in a note-book.

The plane-table method has the disadvantage, however, of requiring more time for the fieldwork than other methods, and it is also more dependent upon favorable weather than a method where the map is not exposed. But taking into account both the field and office work the plane table will prove to be more economical than the transit and stadia for work in open country, and at the same time the results obtained will be sufficiently accurate for most topographic work. The plane table cannot be used to good advantage in heavily wooded country, since the amount of topography visible from any one point is so limited that very little sketching can be done at one set-up.

The impression is quite general that the plane table can be properly used only on topographic work where the map is on a small scale. Its use, however, need not be so restricted. In surveying wharves and docks and in locating buildings the Harbor and Land Commission of Massachusetts have used the plane table more than any other instrument. Where a plan on a large scale is required showing much detail, as for example the plan of a cemetery or survey of the grounds of a public institution, the plane-table method will frequently prove to be more economical than the common methods of stadia or tape surveying.

172. **The Instruments.** — The plane table itself consists of a board usually about 24″ × 30″ mounted on a tripod. In the most elaborate instruments the table is provided with three leveling screws and with a clamp and a slow-motion screw as in Fig. 57. While this makes it possible to level the table exactly and to

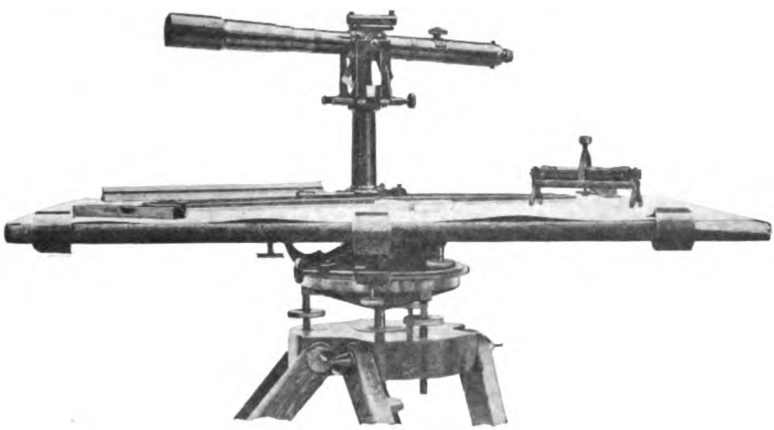


Fig. 57. The Plane Table.

turn it accurately in any desired direction, it has the disadvantage of adding weight to the outfit. A lighter form of tripod,

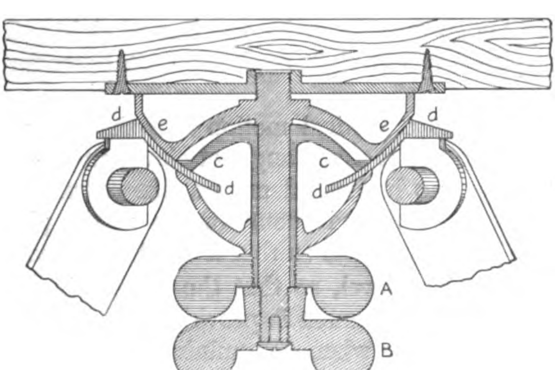


Fig. 58. Johnson Plane-Table Head.

devised by Mr. W. D. Johnson of the U. S. Geological Survey, is shown in Fig. 58. The motions for leveling and for turning horizontally are combined in a very compact form. Although this arrangement is not so delicate as the former it answers the ordinary requirements of plane-table work. The upper clamp *A* controls the leveling of the table and the lower clamp *B* controls the azimuth motion. When the instrument is to be set up both clamps are loosened. The table is brought into a level position and the clamp *A* tightened. This forces the level-cup *c* against the tripod head *d*, the friction holding it in this position. The table is then turned to the desired azimuth and the azimuth clamp *B* tightened, which forces the azimuth cup *e* against the tripod head *d*, thus preventing the table from turning in azimuth. The nut *B* acts as a check-nut on *A*. In small plane tables intended only for rough work the leveling arrangement is omitted entirely and the weight thus reduced to a minimum. With these tables the leveling is done by moving the tripod legs.

173. ALIDADE.* — The instrumental part of the outfit, called the *alidade*, consists essentially of a telescope mounted on a horiontal axis resting in wye supports like a transit, these wyes being connected with a metal column which is rigidly attached to a metal base. The base is a flat piece of brass or aluminum, usually about 18 inches long, having both edges made perfectly straight and parallel to the plane of motion of the telescope. In some alidades a portion of the base is cut away so that there is also a straight-edge directly under the line of sight. The base carries two spirit levels at right angles to each other for leveling the table.

The telescope has a vertical motion like that of the transit, but it is limited to about a 30° angle of elevation or depression. The telescope has no horizontal motion like a transit, but the whole alidade can be moved about on top of the plane table. The telescope is usually made so that it can be rotated about its own axis as in a wye level, so that the cross-hair adjustment can be tested. The instrument is provided with a striding level for leveling the telescope when determining the index correction or when using the alidade for direct leveling. Some instruments have the vernier of the vertical arc mounted on an arm which

* While this term properly applies to the upper motion of surveying instruments like the transit, it is applied chiefly to the instrument used with the plane table.

carries a level and is moved independently of the telescope by means of a tangent screw (see Fig. 57), so that the index correction may be reduced to zero before a vertical angle is read. (See Vol. I, Art. 67, p. 54. This attachment can be used much more rapidly than the striding level, and there is no danger of applying the index correction the wrong way. The index correction is of unusual importance on a plane table, since on account of the flexibility of the table and the weight of the alidade this correction is often large and is quite variable, especially when the alidade must be placed near the edge of the board. The telescope of the alidade should be of fairly high power and should be provided with the ordinary horizontal and vertical cross-hairs and with stadia hairs as in the transit. In the alidades used by the U. S. Coast Survey the diaphragms consist of lines ruled on glass instead of spider threads; and besides the three usual horizontal lines other stadia lines are ruled, enabling the topographer to read one-fourth or one-eighth of the whole stadia interval when long shots are to be taken.

174. Accessories to the Plane Table. — When a plane table is to be used for large scale maps it is sometimes provided with a device for plumbing over the station point. It is seldom necessary to use such an attachment, however, as on even the largest scale maps the table can be set over the point by sighting in with an ordinary plumb-line. A small accessory to the plane table which is often useful is the *declinatoire*, an oblong box carrying a magnetic needle. The long edges of the box are made parallel to the line through the 0° points of the graduated arcs at the ends of the box. This is used in turning the table approximately into position with reference to the magnetic meridian as explained in Art. 185, p. 199.

The plane-table sheet is fastened to the table by means of U-shaped metal clamps. In using large sheets the width of the sheet is made equal to the length of the board, and the ends of the map are rolled under the sides of the table and held in place by the metal clamps. Some tables have thumb screws placed at intervals along the edge of the board so that the paper can be held in place by passing the screws through holes in the paper. The surface of the board around the screw hole is countersunk so that the screw head will not project above the surface of the paper and thus interfere with the movement of the alidade.

For locating topographic details the stadia method is generally used in connection with the plane table, and since the readings must be reduced in the field it is convenient to use the stadia slide rule for this purpose (Art. 164, p. 178). It is important in this work to keep the weight of the outfit as light as possible, and consequently large diagrams or stadia tables ordinarily used in the office are not convenient for use on plane-table work. For reducing stadia readings in the field the topographers of the U. S. Coast Survey use a special form of slide rule called the *hypsograph.** It is circular in form and the scales are so arranged as to give heights in **feet** when the distance is taken in **meters.**

175. **STADIA RODS FOR PLANE TABLES.** — The same kinds of stadia rod are used with the plane table as with the transit. (See description of the common forms of rod, Art. 145, p. 149). On the U. S. Coast Survey it is customary to graduate a special rod for each instrument. The graduations are determined in the following manner. The alidade is mounted on a stand at one end of a 100-meter base and a rod with two targets is placed at the other end of the base. One stadia hair is set on one of the targets and the other target is moved until the other stadia hair bisects it. The interval between the targets is measured and laid off on the stadia rod. This space is then subdivided and the same spacing used in graduating the remainder of the rod. Hence it will be seen that for this rod at a distance of just 100 meters there will be no correction for $(F + c)$. At all other distances there will be a small error introduced unless a correction is applied, but this error will not ordinarily be large enough to affect the results of topographic work on small scale maps.

176. **ADJUSTMENTS OF THE ALIDADE.** — The following adjustments should be tested occasionally and should be corrected if found in error. Small errors in the adjustment of the alidade

* For a complete description of this instrument see U. S. Coast Survey Report for 1902, Appendix 4.

have an inappreciable effect on the accuracy of the plane-table sheet and it is therefore unnecessary to make these adjustments with the same refinement as would be done in a transit.

177. Testing the Straight-Edge. — The edges of the base, or rule of the alidade, should be straight. This may be tested by drawing a fine pencil line the full length of one edge, and then reversing the alidade end-for-end and placing the same edge again on the line. The edge should coincide with the line throughout its length. It should be made true if found to be seriously in error.

178. Adjusting the Levels. — The levels attached to the base may be tested by the usual method of reversing each level end-for-end. This may be done either on the table-top or, better, upon some more solid foundation where a level surface can be obtained. If this test is made on the plane table itself the alidade should be reversed by lifting it from the table and turning it end for end and placing it exactly on the same line, **not** by turning the table itself upon the vertical axis. The adjustment is made by bringing each bubble half-way back to the central position by means of its adjustment screw.

179. Adjustment of the Striding Level. — If the telescope is provided with a striding level the bubble is adjusted by placing the striding level on the rings on the telescope and bringing the bubble to the middle of the tube by means of the vertical tangent screw. The striding level is then turned end for end and placed again on the rings. If the bubble moves from the central position it is moved half-way back by means of the adjusting screw.

180. Adjustment of the Vernier Level. — If the instrument is provided with a vernier level this may be adjusted by first leveling the telescope by means of the striding level, turning the vernier to read zero, and then centering the vernier bubble by means of its adjusting screw. If there is no striding level the vernier level must be tested by the Peg Method (Vol. I, Art. 128, p. 91). After the level line has been established by the Peg Method the telescope is sighted along this line, the vernier is set at 0°, and the bubble brought to the center by means of the adjusting screw.

181. Testing the Line of Sight. — Since the telescope in most alidades is leveled by means of a striding level resting on two rings of the telescope tube it is important that the line of sight should be parallel to the axis of the rings. This adjustment is tested by rotating the telescope about its own axis in the collar which supports it, the test being made precisely as with a wye level (see Vol. I, Art. 121, p. 89). If a vernier level only is used this adjustment is not necessary.

182. LOCATING POINTS BY INTERSECTION. — In order to begin a survey with a plane table it will in general be necessary to have on the map two plotted points corresponding to two points on the ground, the distance between which is known and at least one of which can be occupied with the plane table. The simplest method of locating points by means of the plane table without measuring any distance is as follows. The base-line *ab* is plotted on the plane-table sheet, representing, to some scale, the measured base *AB*. In Fig. 59 *A* and *B* represent the ends

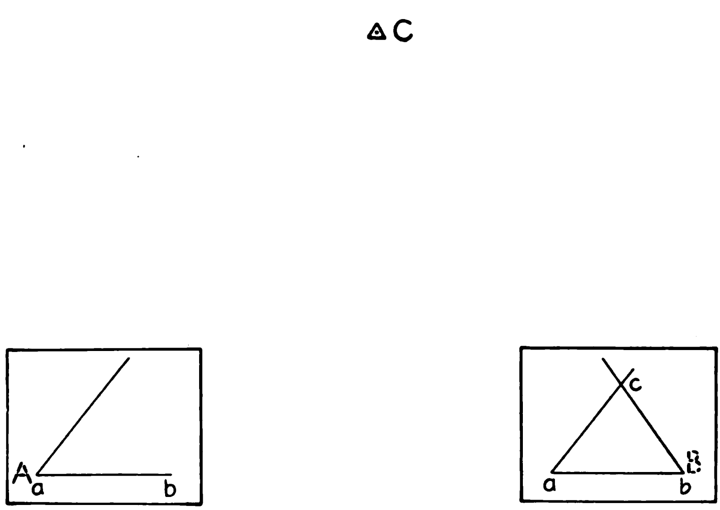


FIG. 59. INTERSECTION METHOD.

of the base on the ground, and the rectangles drawn around these two points represent the plane table. The table is set over one end of the base-line so that *a* on the map is vertically above *A* on the ground and the table is leveled; then one edge of the alidade is placed along the base-line *ab* drawn on the map, the table is turned until the telescope sights the signal *B*, and the horizontal motion clamped. The line *ab* is now parallel to *AB* and the table is said to be ***oriented.*** The alidade is placed so that the straight-edge passes through *a*, the telescope is sighted to some signal *C*, and an indefinite line drawn toward *C*. The point *c* on the map (representing *C*) is somewhere on this line. If the table is now moved to *B* (*b* being vertically over *B*) and the process of orienting the table and sighting toward *C* is repeated, the point *c* is located on another indefinite line through *b*; hence it is at the intersection of these two lines *ac* and *bc*. The triangle *abc* is similar to the triangle *ABC* and each line on the map is parallel to the corresponding line on the ground. In a similar manner any number of points may be located. This is called the method of ***intersection.***

183. Locating Points by Direction and Distance.—The simplest way of locating points by the plane table is by obtaining the direction with the alidade and measuring the distance by stadia or, in special cases, by means of the tape. This is the method most commonly used for filling in the details of a plane-table survey after the table has been oriented.

184. Triangulation for Control of Plane-Table Work. — In extensive surveys the accuracy of the map is usually controlled by a system of triangulation. The computed positions of the triangulation points are plotted on the plane-table sheet and the details of the map are put in by graphical triangulation and by stadia measurements.

In laying out plane-table sheets care should be taken that there is a sufficient number of well-determined triangulation points on the sheet and that these are intervisible; also that their positions are such that they will be of the greatest value in filling in the topography.

If plane-table work is being done on large scale maps the control may consist of either a transit and tape traverse or a system of triangulation like that described in Arts. 54 and 152, pp. 55 and 159.

185. Graphical Triangulation. — The first operation in beginning the fieldwork is to establish, by intersection, a number of points which will later on serve as plane-table stations when filling in topography. The plane table is set up at some plotted triangulation point and oriented by sighting at some other signal which has been plotted on the sheet, the line between these stations serving as a base-line. In order to be certain that the plotting is correct and also to observe whether the paper has become distorted since the points were plotted, other points represented on the sheet should be sighted; the rule of the alidade should always pass through the plotted position of the station occupied by the table. If the alidade has a straight-edge placed vertically beneath the telescope this should be used when the scale of the map is very large.

After the orientation has been satisfactorily checked, lines should be drawn to all conspicuous objects which are likely to be useful as signals in the later work. It is a poor plan to begin a plane-table survey before a sufficient number of points has been determined, and although unnecessary points should not be located it is far better to have too many than too few. Some of these points will be cupolas or church spires, flag-poles, etc., which cannot be occupied with the plane table, while others will be temporary signals erected at points which will subsequently be used as plane-table stations. For drawing these intersection lines the pencil should be kept sharp and should be held at the same inclination the whole length of the line. After all such points have been sighted the table may be taken to another triangulation point and lines drawn in a similar manner, intersecting those already drawn, thus locating the signals. In this graphical triangulation it is important that the point should be located by good intersections. As it is difficult to judge from the first station whether the lines from other stations will give good intersections it will be well to sight all points which will probably be of use as signals, although it will often be found that some of the points sighted from the first station are wholly invisible from other stations. Wherever it is possible the located points should be checked by additional intersection lines, even if the intersections in this case are not favorable. The importance of such checks can hardly be overestimated, since the topography of a large area may depend wholly upon one or two such intersection points and a mistake in the position of one of them may necessitate repeating a large amount of work. In "cutting in" these points great care should be taken not to disturb the table, and the orientation should be occasionally checked by sighting again on the base-line; this check should always be made in any case just before leaving the station. On account of the expansion or contraction of the paper, which it is difficult to allow for, it is important that all of this graphical triangulation should be done before proceeding to put in any of the details.

Every plotted point should be marked by a small needle hole and carefully preserved. The intersection lines should be interrupted so as not to be drawn through the station point occupied by the table, as the exact position of the latter would soon be covered up and lost. It is important that these lines should be determined by the full length of the alidade rule; the direction can be fixed by drawing a short line near each end of the rule, so that when these lines are again used for orienting the table the orientation may be as accurate as possible.

While the plane table is at a triangulation station and the orientation is known to be correct it is well to place the declinatoire on the table and draw a pencil line along the edge of the box to represent the magnetic meridian. This will prove to be of great use in the later work. It is advisable to repeat this operation at different triangulation stations in order to detect any local attraction or changes in the magnetic declination.

Points located by intersection may be occupied with the plane table and used like any triangulation points. When such a point is occupied the table should be oriented by sighting on one of the triangulation points from which this one was located and its plotted position checked by sighting on other triangulation points.

186. It is frequently necessary to carry the plane-table work beyond the limits of the triangulation system. In this case the controlling points may be located wholly by graphical triangulation; and with care in the orientation and pointing and with attention to good intersections the final results may be made nearly as accurate as when the points are located by the original triangulation, provided the survey is not carried far from the original base.

187. LOCATING POINTS BY RESECTION. — It sometimes happens that it is desired to locate a plane-table station from a base only one end of which can be occupied with the table. In such a case we may proceed as follows. In Fig. 60 A and B represent

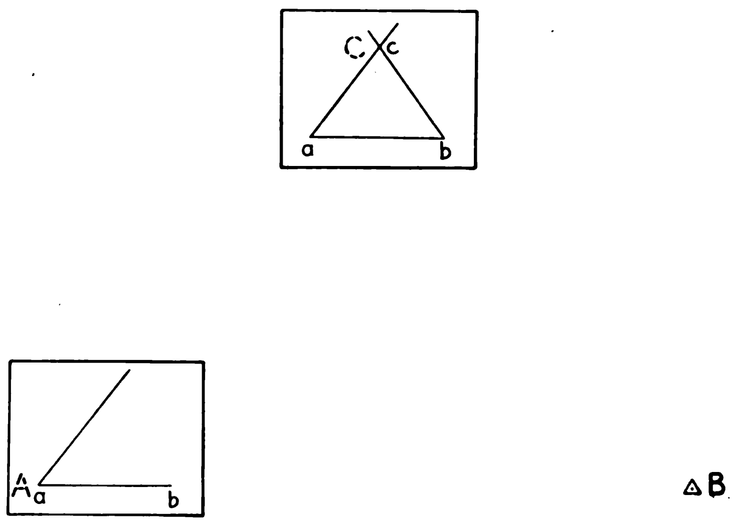


FIG. 60. RESECTION METHOD.

the points on the ground at the ends of the base-line, C is the signal which is to be located, and ab represents the base-line plotted on the plane-table sheet. Set up at A, the end of the base which is accessible, and orient the table. Then, centering * the alidade on a, draw an indefinite line toward C as for intersection. This line should be drawn the full length of the alidade. The table is then taken to C and oriented by means of the indefinite line just drawn. Since the position of c, on the indefinite line, is not known it is necessary to estimate its position and

* By *centering* the alidade is meant placing it so that some part of the straight-edge passes through the plotted point. Unless the scale of the map is very large the straight-edge used is the one at the side of the base and is not the one exactly under the telescope. This error, however, is usually negligible.

to use this point in placing the table over the point C. If the alidade is now centered on b and sighted toward B, a *resection* line may be drawn, and this line will cut the first indefinite line, thus locating the point c desired. Thus without going to station B the same line has been drawn on the map that would have been obtained by an intersection from B.

It is evident that this method may be advantageous even if B could be occupied, for the point C has been located without taking the time required to go to station B. The position of c found by this method should be checked if possible by resection lines from other points whose positions are known to be correct.

188. THE THREE-POINT PROBLEM. — One of the great advantages of the plane table is that it may be set up at any place where three triangulation points (plotted on the sheet) can be seen and the position of this plane-table station located on the sheet simply by observations from this point. By this method each plane-table point is located from the triangulation stations independently of the other plane-table points, which gives it a great advantage over traversing, where each point is located from the preceding station. The position of the plane table is found by means of the so-called *Three-point Problem*, which, in this case, is an application of the principle of resection.

The Three-point Problem may be solved by trigonometry as shown in Art. 49, p. 47; mechanically, by the three-arm protractor (Art. 279, p. 307); or by geometry (graphically), which is the method chiefly used in plane-table surveying (see also Art. 279, p. 307).

The two graphical solutions chiefly used in plane-table surveying are known as (1) *Lehmann's* method, or the *Triangle-of-Error* method; and (2) *Bessel's* method of the *Inscribed Quadrilateral.* The former is a trial method, but it is the more rapid of the two for ordinary work and is used far more in practice than the latter method. Bessel's method gives an exact solution and consequently requires less experience than the former. It has the disadvantage, however, that in certain positions of the signals a part of the required geometric construction falls outside the limits of the plane-table sheet, in which case the solution is not practicable.

189. Lehmann's Method. — If three signals A, B, and C (Fig. 61) have their plotted positions at a, b, and c, and if the table

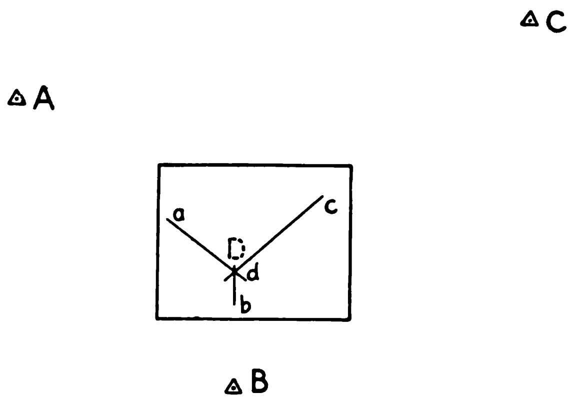


FIG. 61. THREE-POINT PROBLEM.

be set up at D and **oriented correctly,** the resection lines drawn from a, b, and c will all pass through d, the plotted position of D. Since there is no means of accurately orienting the table, the position of d being unknown at the start, the table must be

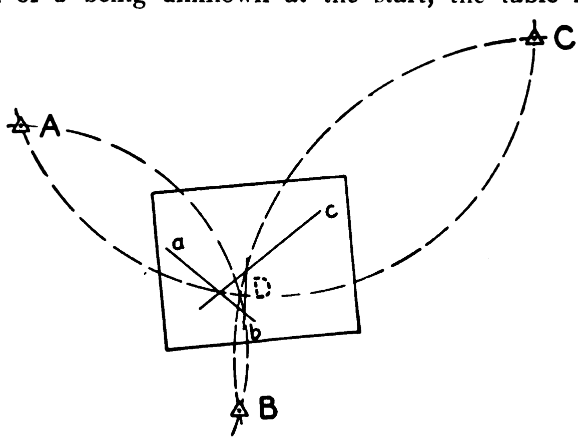


FIG. 62. TRIANGLE OF ERROR.

oriented approximately by estimation. If the plane table is not oriented exactly the three resection lines will not ordinarily pass through a common point but will form a triangle known as the *triangle of error* (Fig. 62). The size of this triangle in any case depends upon the angular error in the orientation of the table. From this triangle of error the true position of *d* may be estimated, and by a second trial a new triangle of error may be obtained which is smaller than the former. By successive trials this triangle may be made so small that it is almost a point. In practice very few trials are necessary, the triangle often being reduced to a point in the second trial, so that the method is in reality a rapid one.

If several points have already been located on the sheet, or if some topography has been filled in, the topographer, in attempting to orient the table approximately, can often make use of *ranges* which happen to pass near the table. For example, if the position of some previously located plane-table station or some building happens to be nearly in line with a triangulation station then by setting the alidade on this line on the sheet and turning the table until the triangulation signal is sighted the table may be put at once into approximately the correct position. The magnetic needle can also be used to advantage in orienting the table for this first trial, provided the direction of the magnetic meridian has already been drawn on the sheet.

In estimating the correct position of the point on the map, after the first triangle of error has been drawn, the following geometric relations will be found useful.

1. If the table is inside the triangle *ABC* the point *d* is inside the triangle of error and *vice versa.*

2. If a circle be passed through *a*, *b*, and the intersection of the two resection lines from *a* and *b* it will also pass through the true position of *d*, for the angle made by any two resection lines from *a* and *b* is the angle *ADB*. Similarly a circle through *a*, *c*, and the intersection of the resection lines from *a* and *c* will pass through *d*. Hence the intersection of these *position circles* will give the position of the point sought. A corresponding circle through *b* and *c* should pass through the intersection of these two circles which is the true position of *d* (Fig. 62). In practice these circles are not actually constructed but their positions are estimated by eye or are sketched in roughly as an aid in judging the true position of *d*.

3. The distance of d from any resection line is proportional to the distance of the table from the signal from which that line

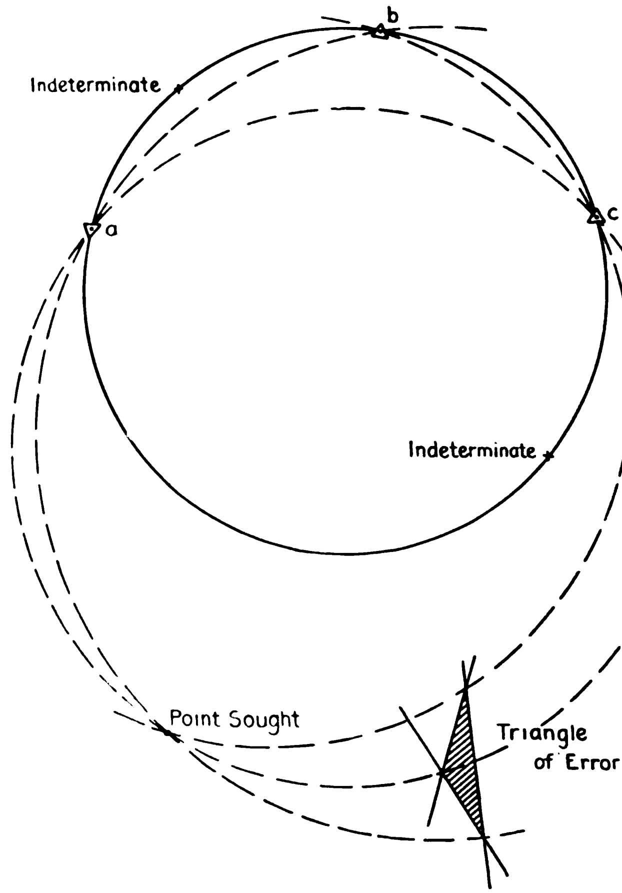


FIG. 63. UNFAVORABLE POSITION OF SIGNALS.

was drawn, i.e., if the table is nearer to B than to the other points, then d will be nearer to the resection line drawn through b than to the others (Fig. 62). That this is true may be seen by observing that when turning the table into the correct position the resection line from a distant signal will move a greater distance at the triangle of error than the resection line from a nearer signal.

190. Positions of the Signals. — The relative position of the three signals with reference to the point sought has an important influence on the precision with which the point can be determined on the map, since the precision depends upon the angles at which the three position circles intersect. If the table is **inside the triangle** *ABC*, as in Fig. 62, the position circles give good intersections and this position of the table is the **most favorable** one for an accurate location. If the table is outside this triangle there are certain positions of the signals which are not favorable. This condition occurs when the angles subtended by the sides of the triangle formed by the signals are small and the middle signal is the farthest from the plane-table point, an example of which is shown in Fig. 63. If the plane-table point were outside the trangle but within the circle passing through all three of the stations its determination would constantly grow stronger as the plane-table point approached the middle signal. If, for example, the point sought happened to be near the center of the circle passing through the three stations its determination would be much stronger than the case shown in Fig. 63, but

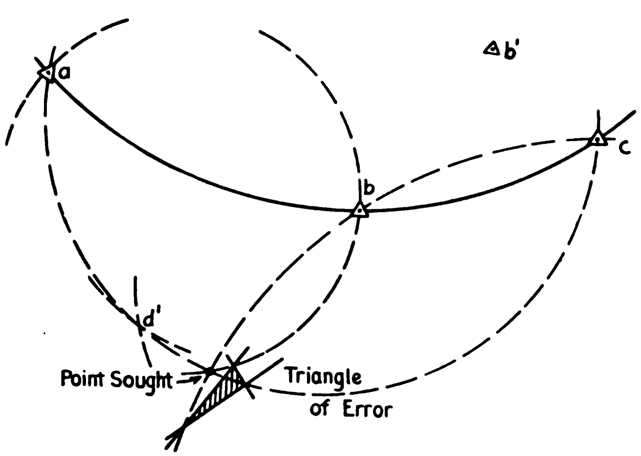


FIG. 64. FAVORABLE POSITION OF SIGNALS.

would not be as strong as when located inside the triangle. If, on the other hand, the middle signal is nearer the plane-table point than the other two signals, as in Fig. 64, the determination is strong, the strength increasing as the plane-table point approaches the middle signal. In Fig. 64 the position of the "point sought" is determined by the position circles (shown by dash lines), which in this case make good intersections. Had the middle signal been at b' instead of b it is evident that the triangle of error would have been identical with the one here drawn, but the position circles would have intersected at d', making very flat angles and affording a weak determination of the point. If d is on the line ab or nearly on the line, its position will be accurately determined provided the resection line from c does not cut the line ab at an acute angle. This also applies to the case where d may be on or nearly on ab produced, provided ab is long as compared with the distance from d to the nearer of the two stations a and b. When a, b, and c are in a straight line, d will be accurately determined if not too far from the line.

In case the table is on the circumference of a circle through A, B, and C the point d will be on the circumference through a, b, and c (Fig. 63) and its position is **indeterminate,** i.e., the three resection lines will pass through a common point no matter whether the table is oriented or not. It will be seen that in this case the three circles of position mentioned above will all coincide. Before the resection lines are drawn for the purpose of locating a new station D it is important that the positions of the signals should be examined to see that D **is not on or near the circle through** A, B, **and** C.

191. **Bessel's Method.** — Bessel's method, although not as frequently used as Lehmann's, will sometimes be found useful. It consists in constructing a quadrilateral having the four vertices in the circumference of a circle. In order to orient the table by this method three signals, A, B, and C, must be plotted on the sheet and must be visible from the table. The four vertices of the quadrilateral are the **extreme** signals A and C (B is assumed to be the **middle** signal), the point sought, and a construction point. In Fig. 65 let B represent the middle signal as seen from the table which is set up at the point D. It is desired to determine the position of D on the plane-table sheet. To construct the quadrilateral, place the alidade on ca, turn the table about its vertical axis until the line ca points toward the signal A (the

a end of the line being toward *A*), and clamp the table. Center the alidade on *c*, sight the middle signal *B*, and draw a line; this line will have the direction *ce*, since the angle *ace* has been laid off equal to *BDA*. Then set the alidade on *ac*, turn the table so

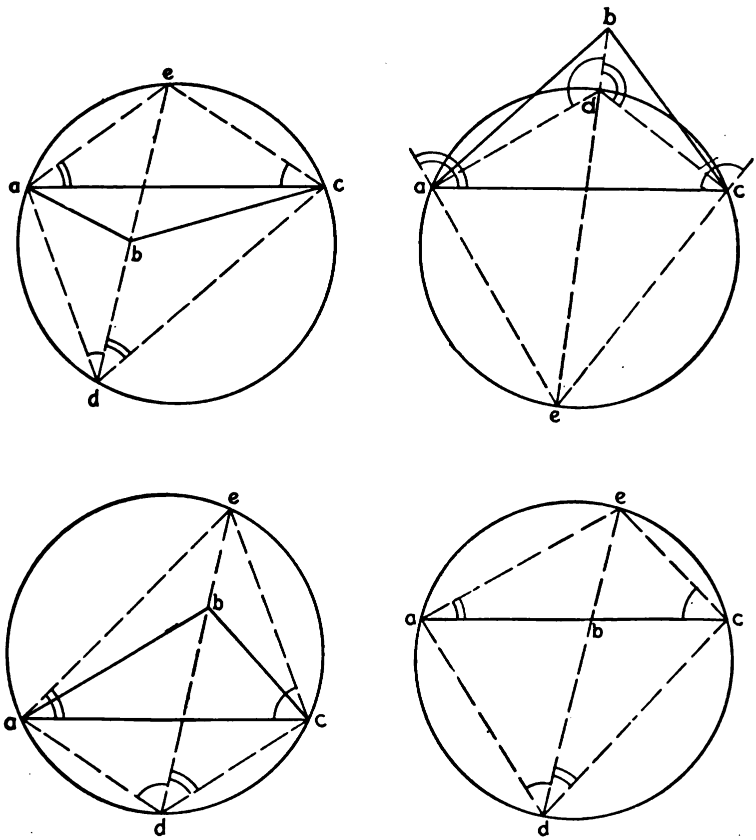


FIG. 65. BESSEL'S METHOD OF THE INSCRIBED QUADRILATERAL.

as to direct *c* toward the signal *C*, and clamp. Center on *a* and again draw toward *B*, obtaining the line *ae*; this lays off the angle *cae* equal to *CDB*. The point *e* is at the intersection of these two lines. The point sought, *d*, is somewhere on the line *be*. This is seen by constructing the circle and noting that angle *ade*

= angle *ace*, which was constructed equal to *ADB;* therefore *d*, *b*, and *e* are in the same straight line. Place the alidade on *be*, sight at the signal *B*, and clamp. The table is now oriented; by resecting from *A* and intersecting the line *be* the position of *d* may be found. A resection line from *C* should also pass through the same point.

To show that this process gives the desired point suppose *d* to be the intersection of the resection lines from *b* and *c*, and *d'* the intersection of the lines from *a* and *b*. By construction, the angle *eac* = angle *edc* ; hence the point *d* is on the circumference of the circle *aec*. Similarly *d'* is on the circumference of the same circle. Since *d* and *d'* are on the same circumference they must coincide. There is only one point where angles subtended by the lines *AB* and *BC* equal the angles *ad'b* and *bdc* respectively; therefore point *d* must be the position sought.

If the table is on the circumference of the circle through the three signals its position is indeterminate (see Art. 190).

192. THE TWO-POINT PROBLEM. — It sometimes happens that only two triangulation points are available and that neither of these can be occupied with the table, for example two cupolas or church spires whose positions are plotted on the sheet. In this case it is possible to set up at a third point, to orient the table and to proceed with the survey; there will be no check, however, on the accuracy of the work. The method consists essentially in assuming some temporary base-line (of unknown length) and two points on the sheet to represent this base-line; then by intersection from this base-line the two signals are located on the sheet. The line joining the two points thus located is parallel to the line between the two signals, but it will not be parallel to its plotted position unless the table happens to be correctly oriented; the angle between these two plotted positions is the angular error of the orientation. In Fig. 66 *a* and *b* are plotted positions of *A* and *B*, the two signals. Assume a convenient base-line *CD*, set up at *D*, and orient the table by estimation. By resecting from *A* and *B* the point *d* is obtained on the sheet, which may temporarily be taken to represent *D*. Center the alidade on *d*, sight point *C*, and draw an indefinite line *dc*. Proceed to point *C*, set the table at *C*, and orient by means of the line *cd*, sighting the telescope at *D*. The table is now parallel to its former position at *D*. By resecting from *B* point *c* is obtained as the plotted position of *C*. Now from *c* draw a line toward *A*. This gives *a′* as the plotted position of *A*, whereas *a* was the position

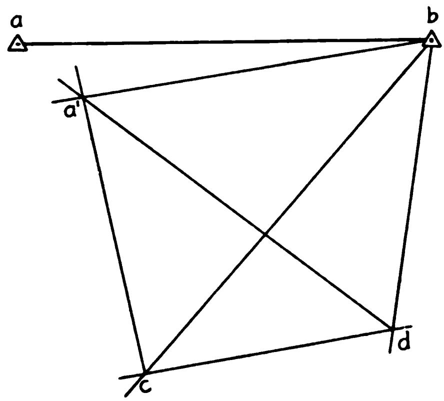


FIG. 66. TWO-POINT PROBLEM.

originally plotted to represent *A*. The angle *aba′* is the error of orientation. The table may now be turned so that *ab* is parallel to *AB* by placing the alidade on *a′b* and noting some distant object in line, then setting the alidade on *ab* and turning the table until the same object is again sighted. The table is now correctly oriented and *ab* is parallel to *AB*. By resecting from *A* and *B* the true position of *C* is found.

193. **ELEVATION OF THE INSTRUMENT.** — In working on scales of 40 to 100 feet to the inch the table is set up at points of known elevation or else near enough to bench-marks so that by direct levels (sometimes by stadia levels) readings can be taken for determining the height of the alidade. On small scale maps, however, the table is often set up at a point which is unknown in regard to both position and elevation, and in such cases it will be necessary to determine the height by trigonometric leveling, measuring the vertical angles to signals whose heights are known. (See Art. 127, p. 129.)

If the position is determined by the Three-point Problem, as is commonly the case on small-scale work, there will of course be three signals visible from the table, and ordinarily the elevations of all of these will be known. After the plotted position of the plane-table station has been found, vertical angles are taken to all of these signals. The horizontal cross-hair must be set on a definite part of the signal, so that the elevation of the point sighted will be known. The index correction should be determined or eliminated. It is well to eliminate errors of adjustment of the cross-hair by rotating the telescope 180 degrees in its bearings and again observing the vertical angle in this position. The distances of the signals from the table may be scaled from the map with sufficient accuracy for computing the elevations. By multiplying each scaled distance by the natural tangent of the vertical angle the difference in elevation between the alidade and the point sighted is obtained. This difference must be corrected for curvature and refraction, the amount of this correction being 0.57 feet multiplied by the square of the distance in miles.* Values of this correction are given in Table I, p. 387. The algebraic sign of the correction may be determined by reference to Fig. 67.

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* The curvature and refraction correction may be computed by the formula

$$h = \frac{K^2}{1.7426}$$

in which h is in feet and K in miles. For plane-table work on scales of $\frac{1}{5000}$ or $\frac{1}{10000}$ the curvature and refraction correction will frequently have to be computed for short distances, say less than 2 miles. The above formula may be modified as follows. Letting k = the distance expressed in units of 1000 feet, then

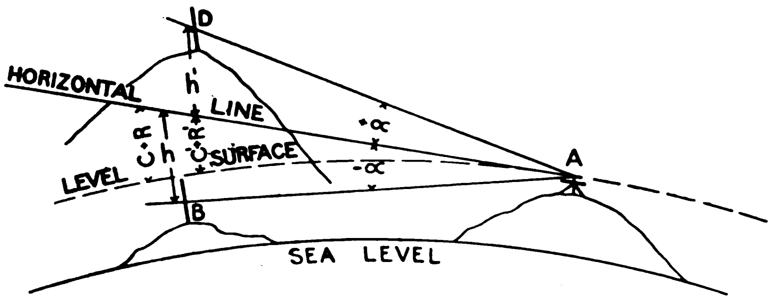
$$h = \frac{k^2}{48.58}$$

or

$$= .02 \times k^2 \text{ (nearly)}$$

This formula will give the correction about 3 per cent too small, but it is sufficiently accurate for correcting short lines when computing the height of the plane table. For example, if the distance is 8000 feet, then $k = 8$; $k^2 = 64$; and $h = 1.28$ feet. The more accurate formula gives 1.32 feet. In using this formula the distance is scaled from the plane-table sheet. Only approximate distances need be used, since the correction itself is small, especially as compared with the errors in elevation due to errors in the vertical angles.

The following example is an illustration of this computation. The table is set at an unknown point X and the position found by



Diff. Elev. of A and $B = -h + (C + R)$
Diff. Elev. of A and $D = +h' + (C' + R')$

FIG. 67. CORRECTION FOR CURVATURE AND REFRACTION.

means of the triangle-of-error method, resecting from the signals A, B, and C.

Vertical angles are observed as follows.

To A, top of mast,	$+ 7° 12'$	Index correction	$- 1'$
B, bolt of tripod,	$+ 2° 34'$	" "	$+ 5'$
C, top of mast,	$- 0° 55'$	" "	$0'$

Scaled distances,

$$XA = 1710 \text{ ft.}$$
$$XB = 7440 \text{ "}$$
$$XC = 11800 \text{ "}$$

Elevations of stations.	Height of point sighted (above station).
A, 911.4	A, 28.7
D, 1054.7	B, 15.6
C, 507.0	C, 30.6

The differences in elevation are

$$A, \quad 1710 \times \tan 7° 11' = 215.5$$
$$B, \quad 7440 \times \tan 2° 39' = 344.3$$
$$C, \quad 11800 \times \tan 0° 55' = 188.8$$

The curvature corrections (Table I, p. 387) are

$$A, + 0.1$$
$$B, + 1.1$$
$$C, - 2.8$$

Corrected differences,

A, 215.6
B, 345.4
C, 186.0

Elevation from A =	$911.4 + 28.7 - 215.6$ =	724.5
" " B =	$1054.7 + 15.6 - 345.4$ =	724.9
" " C =	$507.0 + 30.6 + 186.0$ =	723.6
	Mean =	724.3
	Height of alidade above the ground =	4.2
	Elevation of ground =	720.1

194. FIELD METHODS. — The equipment for plane-table work consists of the table and tripod, alidade, stadia rod, and plane-table sheet, beside the small accessories such as the scale, hard (6 H) pencil, and the declinatoire. The slide rule is convenient for obtaining differences in elevation; a condensed table of horizontal corrections will often be found more convenient than the slide rule for calculating the distances. This table may be of compact form like that shown in Art. 164, p. 180. A curvature and refraction table will also be needed if the height of the alidade is to be determined by trigonometric leveling. An umbrella is generally used to cut off the glare of the sun on the paper, and a waterproof cover of some sort should be carried to protect the map in case of showers. It is also well to carry manila paper to lay over the parts of the map which are not being used, to keep the plane-table sheet clean.

The general method pursued in conducting the survey will depend chiefly upon the scale of the map. The following three methods will indicate the procedure for different scale maps.

195. 1. If scales of 20 to 50 feet to the inch are used, such for example as might be needed for landscape architects' purposes, the plane-table stations should not be located by means of the plane table itself but points should be determined by a transit and tape traverse and plotted on the map before the plane-table work is begun. The table is then set up only at these points and the details located by the stadia. Elevations of several points should be determined by direct leveling, to serve as bench-marks. In using such large scales as these it is important that the plotted point on the sheet should be carefully plumbed over the point on the ground. This may be done by means of a special plumbing device provided with the plane-table outfit, or, with sufficient accuracy, by sighting the point in by an ordinary plumb-bob, or simply by dropping a small stone from the under side of the table at a point estimated to be under the plotted point. Before this can be done accurately the table must be roughly oriented. After the plane table has been placed over the point it should be carefully oriented; the position of the plotted point is then tested by means of the plumb-line. When points are to be located as accurately as the scale permits the stadia should not be used, since errors of one or two feet are comparatively large on the scales here considered. The tape should be used to measure the distances from the table to corners of buildings and other points whose location is important; or such points may be located by intersection, provided the angles of intersection are favorable. Lines of an indefinite character, such as outlines of shrubbery, paths, etc., may be located by stadia. The contours on such a map should be accurately drawn, since they may be used for calculating earthwork. They should not be sketched in so as to generalize the topography, as would be done on small scale maps. If the contour interval is small (1 or 2 feet) the contours may be put in by direct leveling with the alidade rather than by vertical angles, and also by locating points that are exactly on the contour lines so that no interpolation will be necessary. Sometimes the elevations of points on the contours are determined by means of the wye level and the points afterward located on the sheet by stadia measurements with the alidade. In this way a greater difference in elevation can be covered without the necessity of moving the table, because by taking a turning point occasionally the leveling can be carried on when the point is too high or too low to be reached by direct leveling with the alidade.

196. 2. On scales of $\frac{1}{5000}$ (about 1 foot to a mile) to $\frac{1}{10000}$ (about 0.5 foot to a mile) graphical triangulation is of sufficient accuracy for locating plane-table points, and the details may be safely put in by the stadia method. On such small scale maps there are usually plotted a few triangulation points, or else the ends of some measured base-line. Plane-table stations are located graphically, the elevations of these stations are deter-

mined by trigonometric leveling, and the details are all put in by the ordinary stadia method. When the table is set up at a plotted triangulation station, as would often be the case when beginning a survey, it is oriented as usual by sighting other triangulation signals. It is not necessary to center the plotted point over the station except by a rough estimate, because on these scales an error of 2 or 3 feet is inappreciable. In filling in topography on such maps it is customary to select plane-table points from which a good view of the ground in the immediate vicinity can be obtained and to locate these station points by the Three-point Problem, generally by the triangle-of-error method. Points are not taken exactly on the contours as on large-scale maps, but the elevations of controlling points are found and the contours sketched in by interpolation.

In locating points on the map by stadia it is not necessary to draw radiating lines in the directions of the various shots. It is sufficiently accurate to lay the scale close to the edge of the alidade, to place the pencil against the alidade opposite the correct reading on the scale, and to mark the point on the map by a dot. In some alidades the scale is graduated on the edge of the rule. In using such an instrument the distances may be taken off by dividers and transferred to the map.

197. 3. On scales of $\frac{1}{30000}$ or smaller, such as those employed in making government maps, the work usually consists almost entirely of graphical triangulation, although a combination of graphical triangulation and stadia is sometimes employed. Where great detail is not required the stadia may be omitted, in which case the summits are located by intersection lines from the triangulation points and elevations may be found by means of vertical angles and scaled distances. After many such points have been located the rest of the work consists almost entirely of sketching. The sketching is sometimes supplemented by additional elevations determined with the aneroid barometer (see Art. 140, p. 138) and occasionally by stadia traverses or by surveys made with the traverse plane table (see Art. 203, p. 221). When an intersection line is drawn to an unknown point the outline of the hill is sometimes sketched on the sheet in approximately the right position, either in the form of a profile of the hill which

serves as a memorandum to be used later or in the form of contours representing the shape of the hill as well as it can be judged from this one point of view; when two or three lines have been drawn to the hill from different points the sketches are assembled and the contours completed. On such maps the contours are drawn to show the general form of the ground, so that the topography is necessarily generalized and the allowable error in elevations is somewhat greater than on large-scale maps. Some topographers make the rule that a contour must not be in error by so much as half a contour interval.

From the preceding it will be seen that one of the advantages of the plane table is that it may be set up at any place where the topographer can obtain a good view of the ground in the locality to be mapped; the position can easily be located on the map, provided three located signals can be seen. For this reason each point is entirely independent of the preceding or the following points, so that errors do not accumulate as they do in traverses.

In filling in the details the topographer generally proceeds from station to station, completing the map as he goes along and following out some definite plan of work so as to economize time. If wooded country is encountered the plane table is at a disadvantage. It may be used, however, even when no triangulation points are visible and when but little of the ground can be seen from any one station, for traverses can be run with the plane table, using the alidade to obtain the angles.

In running traverses with the table it is oriented at each station by a backsight on the preceding station, and the next station in advance is located by a stadia shot. Care should be taken in this work to draw long lines so that the direction is well defined on the sheet, as this determines the accuracy of the orientation on the next set-up. On large-scale maps the center line of the alidade should be used. The declinatoire can be used to good advantage for checking the orientation. The direction of the magnetic needle is always uncertain, but the fact that its errors are irregular and not cumulative like the errors of the traverse makes it valuable in spite of its defects. The elevations are carried along exactly as on an ordinary stadia traverse. A traverse run by the plane table should be checked if possible by connecting with triangulation points, as such a traverse is at best only approximate.

The area covered by such a traverse can be greatly extended, with sufficient accuracy for small-scale topography, by the use of the hand level (Vol. I, Art. 307, p. 278). After points have been located by the stadia as far as possible an assistant can use one of these points as a bench mark and run out an approximate profile in some known direction, obtaining the elevations by the hand level and pacing the distances, and then transfer this information to the plane-table sheet.

199. SKETCHING CONTOURS. — The method to be pursued in locating contours will depend upon the scale of the map and the character of the ground. One method is to select points which are on or near the contour, choosing a sufficient number to give the shape of the contour. If the scale is very large and the contour interval small, say one or two feet, the points should be **exactly** on the contour. On a slope it is often convenient to locate points on two or three adjacent contours at one time, completing this group of contours before another set is begun.

On smaller scales the points may be taken **approximately** on the contour, the slope on either side estimated, and the contour sketched accordingly. This method is fairly accurate because the located points are near the contours, so that an error in estimating the slope has little effect on the plotted position of the contour. It has the advantage also that when the topographer is in a position which makes it difficult for him to judge the slope the rodman can easily estimate the slope and call it to him.

Another method, applicable to intermediate scales where the contour interval is 5 feet or more, is to select points along the ridges, along the valleys, on knolls, and in depressions, thus determining directly the two systems of lines upon which the shape of the contours mainly depends. Points should be so selected that slopes are nearly straight between consecutive points. The best positions for obtaining a good view of the ground are on the ridges and knolls.

On small scales where much of the detail is sketched in the topographer finds it necessary to judge long slopes. In selecting plane-table points, therefore, the topographer should endeavor to find positions which command a good view of the surrounding country, and which at the same time afford a good opportunity to correctly judge the slope of the ground. The summits and the valleys are not so well adapted to this purpose as a position midway between, because from the latter position a better general view may be had of the area to be covered from that plane-table station. Frequently much of the detail is hidden from the topographer, in which case assistance may be rendered by the rodman, who is in a position to judge the slopes around the points located and to note many details not visible from the plane table. In some cases it will be economical to have another assistant who devotes the whole of his time to making notes and sketches, thus aiding the topographer in covering a large area from a single station.

Sketching contours on small scales where only a few points are accurately determined requires great skill on the part of the topographer. He must not only be able to accurately judge the slopes from the appearance of the country, but he must be familiar with the characteristics of contour maps of different topographic forms. The topographic features of any region bear an intimate relation to the geological formation, any given geological feature producing contours having certain characteristics. A knowledge of this relation will be of great assistance to the topographer. (See Chapter VII for a discussion of the Relation of Geology to Topography.)

200. THE DATUM PLANE. — The datum for topography is sometimes taken as mean sea-level and sometimes as mean high water. The former has the advantage of being more nearly the same for all places, whereas mean high water differs many feet in different parts of the same coast. Mean sea-level can also be more readily determined from a limited number of tidal observations. (See Vol. I, Art. 237, p. 211, for the method of establishing a datum by observations.*)

201. PLANE-TABLE PAPER. — A source of inaccuracy in all plane-table work is the change in the dimensions of the paper due to changes in the hygrometric conditions. As plane-table sheets

* See U.S. Coast Survey Report for 1897, pp. 315–320, 480–489, also report for 1853, pp. 94–96.

are exposed to the weather more than ordinary maps these changes are often quite large. The paper not only changes its dimensions but it expands and contracts unequally in different directions, so that it is not easy to allow for the change in scale. The only precautions that can be taken against this are to use mounted paper which has been exposed to such changes for a long time and has become "seasoned," and to protect the paper from moisture when in the field. Seasoned paper is much less liable to distortion than fresh paper. Paper which has been kept rolled or in a drawer is not seasoned as well as that which has been left free to the action of the moisture in the air. Sometimes double mounted paper is used, in which the two pieces of cloth are laid with the warp at right angles so as to prevent unequal changes in the paper.

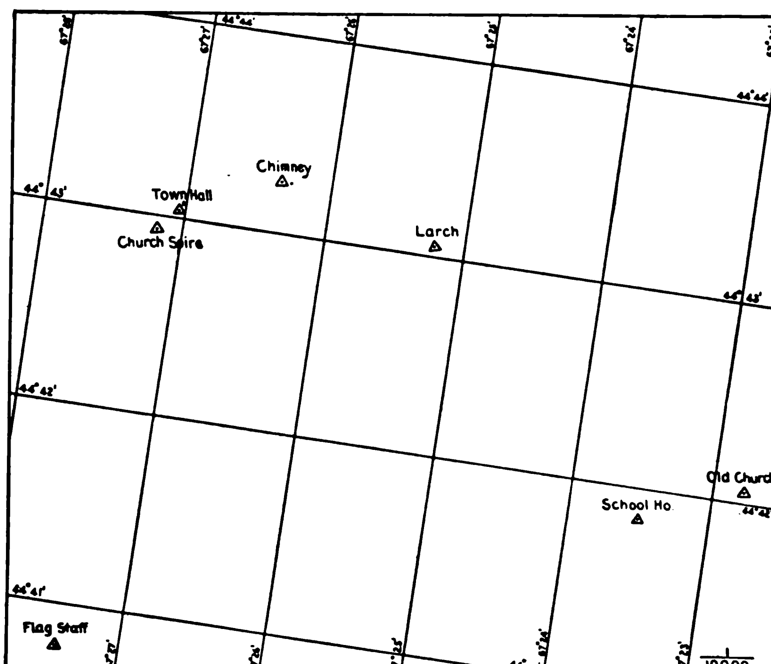


FIG. 68. PLANE-TABLE SHEET READY FOR FIELDWORK.

202. PREPARING PLANE-TABLE SHEETS FOR FIELDWORK.—Plane-table sheets on small scales, especially those which form a part of a large survey, are usually plotted on some form of *projection*, the triangulation stations being plotted in their proper latitudes and longitudes. (See description of the different projections, Chapter X, p. 345.) The *polyconic projection* is used almost entirely for large areas. For very small areas the polyconic can scarcely be distinguished from the *rectangular* projection, and the latter may often be substituted for the former. Fig. 68 represents a plane-table sheet ready for the field originally plotted on a scale of $\frac{1}{10000}$ and reduced to approximately $\frac{1}{72000}$. The triangulation points shown on the sheet have been located and their latitudes and longitudes calculated; these form the basis of the plane-table work.

203. The Traverse Plane Table. — The traverse plane table (Fig. 69) consists of a board about 15 inches square mounted

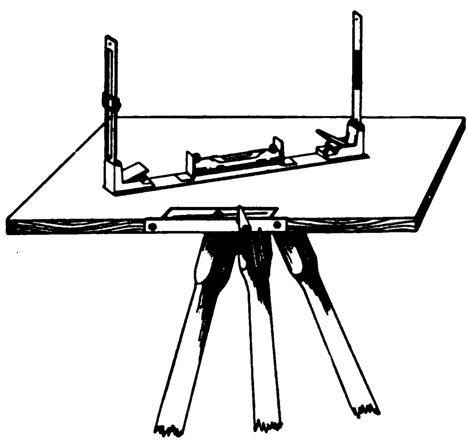


Fig. 69. The Traverse Plane Table.

on a light tripod and having a small declinatoire set in at one edge, the top of which is flush with the top of the table. The alidade consists of a rule with two vertical sights like those used on the surveyor's compass. This table, when used for traversing, is oriented entirely by means of the magnetic needle. The table is set up at **alternate stations** only, the unoccupied traverse stations consisting of natural objects occurring along the line of the traverse, such as chimneys, lone trees, etc. This instrument is used by the U. S. Geological Survey in traversing highways on scales of about $\frac{1}{30000}$, the results of these traverses being afterward transferred to the large plane-table sheets. The outfit may be carried along in a carriage and the distances measured by counting the revolutions of one of the wheels. The length of foresights or backsights is plotted by a specially constructed scale with which revolutions of the wheel can be plotted to the proper scale for the map. The positions of objects which are located far from the highway are determined by intersection. Details near the highway are sketched as the work proceeds. Triangulation points are cut in whenever possible so that the sheet may be correctly oriented when joined to the large map. An approximate profile of the route is obtained from readings of the aneroid barometer.

204. **ROUGH SURVEYS WITH PLANE TABLE.** — It is obvious that a complete map may be made with a plane table without measuring any distances whatever, provided it is not necessary to know the exact scale of the map. A distance may be assumed on the paper representing some base-line (of unknown length) on the ground, and other points may be located by intersection and resection. This procedure is often very useful in reconnoissance or exploration surveys. The scale of the map may be found at any time during the survey by measuring the distance between any two points which can be identified. With the light plane-table outfit and an aneroid barometer one man can make a very fair map of a large tract of country by using these rough methods.

PROBLEMS.

Compute the elevation of the ground at the plane-table station in each of the following sets of observations.

1. Elevation of Plane Table.

Signal	Vert. Ang.	Dist.	El. of Sta.	Ht. of pt. sighted above △
		ft.	ft.	ft.
Flag in Tree.	+ 3° 37′	3545	top 327.4	
Ch. Spire ...	+ 1° 31′	2328	ball 167.7	
Hooper	+ 5° 00′	1447	dr. h. 197.4	32.7
		Ht. of Al. 4.2		Ans. 99.1 ft.

2. Elevation of Plane Table.

Robinson....	+ 2° 13′.5	4810	stake 270.7	45.2
Hubbard....	+ 7° 31′	2533	dr. h. 433.4	30.6
Pitman......	+ 10° 08′	2135	dr. h. 469.5	41.1
		Ht. of Al. 4.2		Ans. 124.8 ft.

3. Elevation of Plane Table.

Hotel.......	− 2° 25′	3775	ball 212.0	
Flag Pole ...	− 1° 10′	7073	ball 227.9	
Pitman......	+ 3° 24′	2362	dr. h. 469.5	41.1
		Ht. of Al. 3.5		Ans. 367.2 ft.

4. Elevation of Plane Table.

Robinson....	+ 0° 30′	5879	270.7	45.2
Hubbard....	+ 3° 48′	2989	433.4	30.6
Pitman	+ 14° 15′	952	469.5	41.1
		Ht. of Al. 4.1		

5. Elevation of Plane Table.

Pitman	+ 3° 52′	1529	dr. h. 469.5	41.1
Hotel.......	− 2° 33′	4338	212.0	
Flag Pole ...	− 1° 23′	7419	227.9	
		Ht. of Al. 3.6		

6. Elevation of Plane Table.

Flag Pole...	− 1° 07′	8327	227.9	
Robinson....	− 0° 39′	8400	st. 270.7	45.2
Pitman	− 12° 46′	542	dr. h. 469.5	41.1
		Ht. of Al. 3.8		

CHAPTER VI

PHOTOGRAPHIC SURVEYING.*

205. **PHOTOGRAPHIC SURVEYING.** — In this chapter only the fundamental principles of photographic surveying will be considered and no attempt will be made to describe the details of special methods.

Photography applied to surveying furnishes one of the most rapid means of locating topographic details, and in some countries it has been used extensively for making small-scale maps. The advantages of the method are that the fieldwork at any station may be completed in a very short time and that everything which can be seen from a station is permanently recorded by means of the photographs. The reduction in the time required to occupy a station is obtained, however, at the expense of the office work; the long time required to work up the results is one of the disadvantages of this method. Furthermore, the accuracy of photographic surveying is inferior to that of the methods previously described. This method is therefore particularly applicable to cases where the time available for fieldwork is quite limited, or where great detail is not required, as on a small-scale map. In Alaska, for instance, the photographic method has proved valuable, because on account of the continued cloudiness on high mountain peaks there would be great delays in measuring the angles of a triangulation system and in taking the topography by the ordinary methods. By the photographic method a few minutes of clear weather may be all that is needed for obtaining the data at one station. In the surveys made by the Canadian Government the photographic method is extensively used; the published maps are on a scale of $\frac{1}{120000}$, the contour interval being 100 feet.

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* For a very complete treatment of the subject, including the history of its development, see "Photographic Methods and Instruments," by J. A. Flemer, published by John Wiley & Sons.

206. Principles of Photographic Surveying. — A photograph is a perspective of the landscape represented and furnishes the means of determining graphically the angles at the camera station between points on the photograph. Points may be located on the map by the intersection of lines drawn from the plotted positions of the camera stations, the method of locating the points being essentially that of graphical triangulation. The general principle upon which the method of photographic sur-

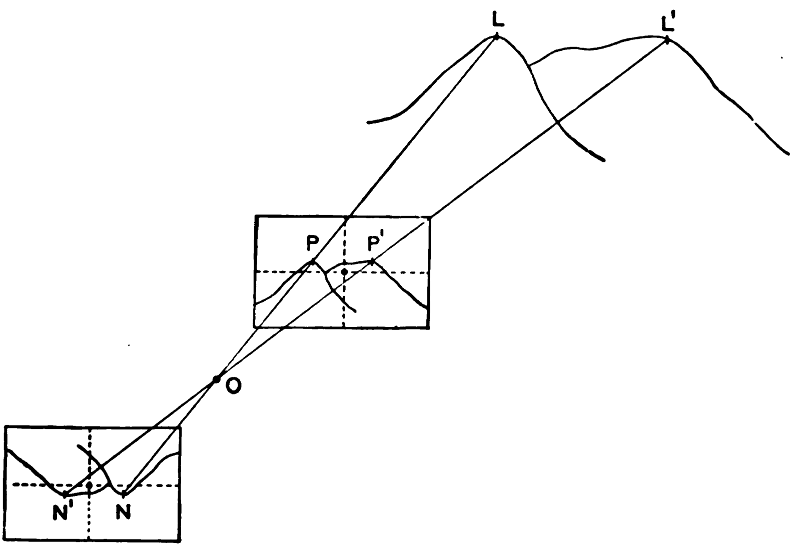

Fig. 70.

veying is based may be understood by reference to Fig. 70. The point O represents the optical center of the camera lens, through which rays of light are passing (in straight lines) from the landscape LL' to the negative NN'. The plane PP' is the *picture plane* and represents a photographic print made from this negative and placed in front of the lens at a distance equal to f, its focal length. The distance of the negative from O is also equal to f. The center of the print, the center of the negative, and point O are all on the same straight line, and the negative and print are vertical. Since the negative is an inverted image of the landscape, and since the picture is printed from the negative by direct contact, the print is an erect image of the landscape and of the same size as the negative. If the eye were placed at O every point on the picture would appear to cover the corresponding point in the landscape; therefore the angle between any two points on the print is the same as the angle between the corresponding points in the landscape. The print, then, when held the proper distance (focal distance) in front of the eye, furnishes a means of measuring the angles between the photographed points.

Strictly speaking the vertex of these angles is not the optical center, as shown in Fig. 70, but is one of the *principal points*, or *nodal points*, of the lens; the general principle, however, is not in any way altered by the assumption that the line LON is straight, since the line emerges from the lens parallel to its original direction. If the lens were replaced by a pinhole in the front of the camera box, photographs could be taken, and the same general principles of perspective would hold true as in the case where a lens is used.

207. **THE SURVEYING CAMERA.**—Cameras used for surveying are of various types, some being ordinary cameras with a few attachments added, while others are a combination of plane table and camera or theodolite and camera. Fig. 71 shows a camera designed especially for surveying work. The lower part of this instrument is so arranged that the camera can be removed and a transit set in its place on the tripod. Whatever form of camera is used, it should have a ground glass plate at the back of the box, so arranged that it can be placed in the same position as that to be occupied by the sensitized plate; by this means the camera can be focused and the image of the landscape may be examined before a plate is exposed. Spirit levels should be provided for leveling the instrument whenever a picture is taken. Cameras intended for this work are commonly made of fixed focus, so that the plate shall always be at the same distance from the lens when the exposure is made. If the focus is not fixed the distance from the lens to the plate must be determined by some means every time a photograph is taken; this latter method has the disadvantages that additional time is consumed in the field

and that error is liable to be introduced in this determination. The outfit is often taken over very rough country where it is subjected to severe shocks; it is desirable that there should be few adjustable parts which can become deranged. At the back of the box where the plate holders are inserted there should be a frame having a rectangular opening; the plate, when exposed,

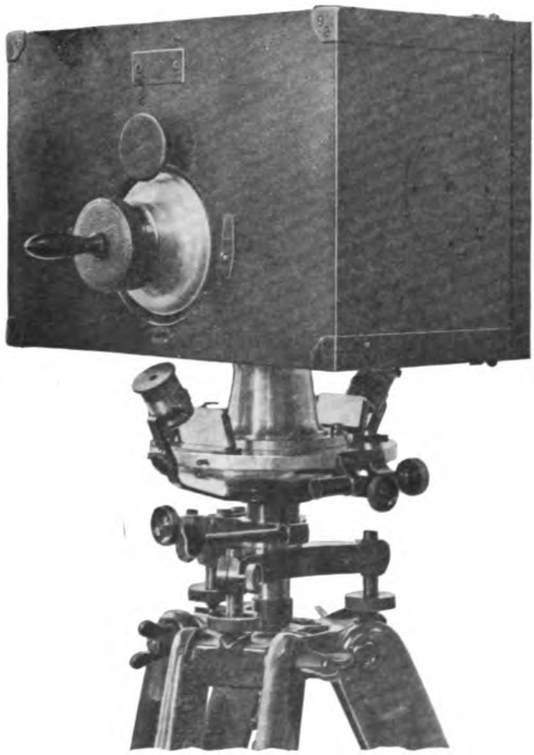

FIG. 71. SURVEYING CAMERA.

(From a photograph loaned by C. L. Berger & Sons.)

should be in contact with this frame so that the exposed portion of the plate is a perfect rectangle. In order to mark certain reference points on the plate a number of notches are cut in the edges of the frame; these will be photographed on each plate when it is exposed. Two of these notches, at the sides, mark the position of the *horizon line;* two others mark the position of a vertical line known as the *principal line.* The remaining notches mark the focal length of the lens, as will be shown later. The horizon line and the principal line intersect at the *principal point* of the plate, which is the point where the optical axis of the lens should intersect the plate. The horizon line is the line of intersection of the picture plane and the horizontal plane passing through the optical center of the lens. The principal line is the intersection of the picture plane and a vertical plane through the optical center. This vertical plane is known as the *principal plane.*

208. ADJUSTMENTS OF THE CAMERA.—The particular method used in adjusting the camera will depend upon its construction. The following adjustments apply to a camera which can be placed on the tripod in two positions at right angles to each other, so that the longer dimension of the plate may be either horizontal or vertical. In this type of camera there are four spirit levels, two for each position of the camera on the tripod.

209. Adjustment of the Bubble which is Perpendicular to the Ground Glass. — The bubble which is perpendicular to the plane of the ground glass should be so adjusted that when the plate is vertical the bubble is central, or else a reading of the bubble should be noted which corresponds to the vertical position of the plate and this position should be marked in some way on the glass tube. To determine the vertical position of the ground glass, set up a leveling instrument in front of and at the same height as the camera and find some distant point which is on the horizontal cross-hair. Turn the level around on its vertical axis until it points toward the camera. Level the camera and place the ground glass in position. Turn the camera so that the reflection of the distant object from the back surface of the ground glass can be seen through the telescope of the leveling instrument. If the ground glass is vertical the image of this object will appear somewhere on the horizontal cross-hair of the leveling instrument; if it is not the camera must be tilted until this condition is fulfilled. Then the level tube may be adjusted; if it is not adjustable, the scale readings of the ends of the bubble may be noted and preferably marked in some way on the tube.

The camera is then placed on the tripod so that the long dimension of the plate is vertical, and then the other level which is perpendicular to the plate is adjusted in a similar manner.

210. Determining the Position of the Principal Point. — After the bubbles are adjusted so that the plate can be made vertical, a transit or a leveling instrument is set up and leveled, and two well-defined points on the horizon are found which, when photographed on one plate, will come near its opposite edges. The camera is set at the same height as the leveling instrument, the level bubble which is perpendicular to the ground glass being centered, and a plate is then exposed. A line drawn on the negative through the photographs of these points determines a horizontal line. The camera is now placed on the tripod at right angles to its first position and the operation repeated, two new points at the same height as the camera being selected, if necessary, to bring them within the limits of the plate. A line through the points on the second negative is a horizontal line for this second position. The line on the second negative may now be transferred to the first one by means of measurements made along the edges of the exposed portion of the plate. The intersection of these two lines, both of which are now on the same negative, will determine the position of the principal point. Since the bubbles which are parallel to the plate have not been adjusted these two lines are not necessarily at right angles to each other.

In case it is impossible to find distinct points exactly on the true horizon, any well-defined points (preferably near the horizon) may be chosen and vertical angles may be measured to these points. The vertical distance on the print from these points to the horizon line may be computed by use of the azimuths, vertical angles, and the focal distance. If these vertical distances are laid off on the print they will give a series of points which will be on the true horizon.

211. Determining the Positions of Horizon and Principal Lines. — Now that the principal point has been determined the horizontal and principal lines may be laid out by means of measurements along the edges of the opening at the back of the box. The distances of the principal point from the edges of the exposed part of the negative are measured, and these distances are transferred to the edges of the rectangular frame so that when points on opposite sides of the rectangle are joined by straight lines these two lines will be parallel to the sides of the rectangle and will intersect at the principal point. These four points on the edges of the frame should be marked by notches so cut that they will be photographed on each plate. The horizon and principal lines may then be drawn on any print by simply joining these opposite notches by straight lines.

212. Adjustment of the Bubble which is Parallel to the Ground Glass. — The level which is parallel to the ground glass should be so adjusted as to be central when the horizon line is truly horizontal, or else a reading on the level bubble should be found which corresponds to this horizontal position.

This may be done by tipping the camera until the bubble is at one end of the tube and then taking a photograph of the points which have already been used in fixing the horizon line. This will show the horizon line inclined to the actual horizon. The camera is then inclined until the bubble is at the other end of its tube, and another plate is exposed. This second plate will also show the horizon line inclined to the true horizon, but in the opposite direction. The exact position of the level bubble should be noted in each case. After measuring on the edges of the plate the distance from each notch to the horizon as determined by the photographed points, simple interpolation between the bubble readings and between the measured distances will show what the bubble should read when the horizon line defined by the notches is level. If the level is adjustable the bubble should be made central while the camera is in the correct position. During the above adjustment the plate should be kept vertical by means of the bubble which is perpendicular to the plate.

213. Determining the Focal Length of the Lens. — The focal length is usually given by the instrument maker, but it may be found in the field by either of the following methods.

214. First Method.* — Set a transit over the camera station and measure the angle AOB between two distant points A and B. In Fig. 72. let $AOB = \alpha + \beta = \omega$. Expose a plate and measure on the negative the distances x and y from the points a and b to the principal line at c. Let $f = Oc$. Then

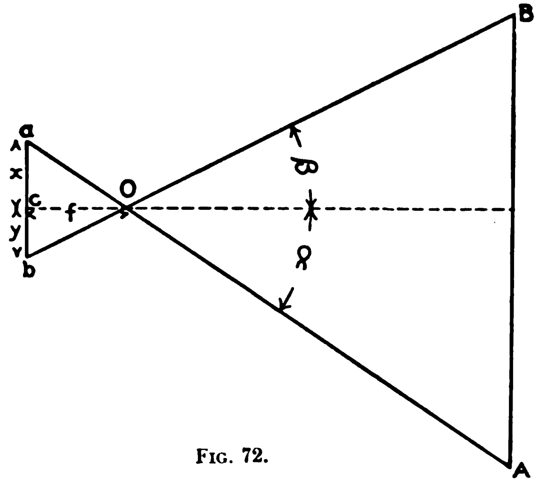

Fig. 72.

$$\tan \alpha = \frac{x}{f}$$

$$\tan \beta = \frac{y}{f}$$

and $$\tan \alpha \tan \beta = \frac{xy}{f^2}$$

$$\tan(\alpha + \beta) = \tan \omega = \frac{\frac{x}{f} + \frac{y}{f}}{1 - \frac{xy}{f^2}}$$

hence $$f^2 - \frac{x + y}{\tan \omega} \cdot f - xy = 0$$

Solving this equation gives

$$f = \frac{x + y}{2 \tan \omega} + \sqrt{\frac{(x + y)^2}{4 \tan^2 \omega} + xy} \qquad [53]$$

* This is the method given by Capt. E. Deville, Surveyor General of Dominion Lands, in "Photographic Surveying," published by the Government Printing Bureau at Ottawa.

EXAMPLE.

Distance of 1st point from principal line = 3.026 inches
Distance of 2d point from principal line = 2.736 inches
$x + y$ = 5.762 inches

Angle between 1st and 2d point = 39° 43′

log 2 = 0.30103	log $(x + y)$ = 0.76057
log tan ω = 9.91945	log denom. = 0.22048
log denom. = 0.22048	0.54009
	1st term = 3.468 inches
	$\log \frac{(x+y)^2}{4 \tan^2 \omega}$ = 1.08018
$\frac{(x+y)^2}{4 \tan^2 \omega}$ = 12.028	
xy = 8.279	
20.307	log = 1.30771
	½ log = .65386
	2d term = 4.507
	1st term = 3.468
	f = 7.975 inches

215. SECOND METHOD. — Set up the camera and level it; set two poles, or select two well-defined points, A and B, Fig. 73,

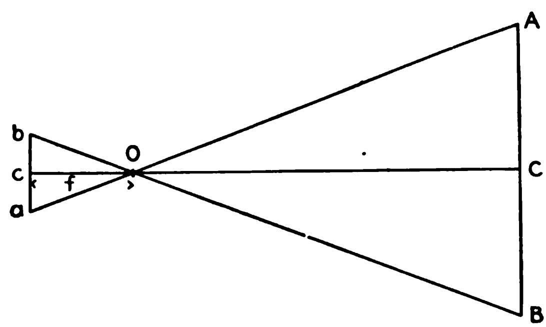

FIG. 73.

on the same level as the camera, the two being equally distant from the camera. The angle AOB should not be very acute if an accurate determination of the focal length is desired. Measure the distances AB, cC, and ab. The first two should be measured with a tape and the third scaled from a negative.

Then

$$CO + cO = cC$$

and

$$CO : AB = cO : ab$$

From these equations is found the value of cO, or f. The poles should be far enough away so that the focus is practically the same as it will be for the surveying work.

In order to detect distortion in the photographic prints, distances equal to $\frac{1}{2} f$ and $\frac{1}{4} f$ respectively are laid off on the long and short sides of the rectangular opening at the back of the camera and marked by notches similar to those used in marking the horizon and principal lines. In this way distances of $\frac{1}{2} f$ and $\frac{1}{4} f$ are shown on all prints and any change in the dimensions of the print may be measured and allowed for.

216. CONDUCTING A PHOTOGRAPHIC SURVEY. — The general method of conducting a photographic survey is to establish first a number of triangulation stations by the usual methods and then to occupy these stations with the camera, taking as many views from each point as will be of use in mapping the country. The triangulation may be executed with the ordinary transit, since great precision is not required. The transit is usually carried along with the rest of the outfit and is used not only for locating camera stations but also serves the purpose of locating the views, as will be explained later. Secondary camera stations should be located by the camera itself only where it is practically impossible to locate these stations by triangulation. If at any time it is desired to locate a camera station without occupying other triangulation stations with the transit, this, of course, can always be done by means of the Three-point Problem, provided three suitable triangulation stations can be seen from the camera station.

The accuracy of the survey will depend to a considerable extent upon the selection of such camera stations as will afford good intersections. In this work it will be necessary to establish a much larger number of triangulation points than would be needed for other kinds of work, because the accuracy of the resulting map depends upon the selection of camera stations which will give good intersections at the prominent topographic features. A camera station may sometimes be located for the purpose of obtaining a single photograph. The triangulation scheme for a photographic survey will therefore be of a different character from an ordinary triangulation, the highest peaks, which would naturally be selected for an ordinary triangulation, sometimes being avoided entirely in this work. Such high peaks are likely to be covered with clouds much of the time, so that progress is delayed; furthermore it may be impossible to obtain desirable views from such elevated points. To definitely locate a point it is necessary that it should be shown on photographs taken from at least two different camera stations; but it is desirable that all important points appear on views taken from three or more stations, in order that the accuracy of their location may be checked.

It is often advantageous to set up the camera at some eccentric point near the triangulation station, in which case the location of the eccentric station with reference to the triangulation point must be determined by means of its azimuth and distance.

A rough sketch should be made at every exposure, showing the limits of the photograph, a few prominent points, and any points to which angles are measured. One or more horizontal and vertical angles should be taken to well-defined points which appear in the photographs. The horizontal angles serve to locate camera stations and also to orient the picture when plotting the map; these angles will also furnish a check on the focal length and on the position of the principal line. The vertical angles afford a means of determining the elevations of points and a check on the position of the horizon line. If the camera is provided with an azimuth circle, this may be read at each exposure and the pictures oriented on the drawing by means of these circle readings.

If no angles are measured at the camera station when the photographs are taken, at least one of the plates must include the photograph of a triangulation point in order that the pictures may be oriented on the map, and it will be advantageous in any case to have one or more triangulation points appear in each photograph.

The pictures should overlap far enough so that one or more conspicuous points are common to the two adjacent plates. In order that the pictures may be oriented on the map independently there should be a triangulation station in each photograph, but any print which does not contain a triangulation signal can still be oriented, provided there is, near the edge of this print, some point which also appears on the edge of the adjoining print.

217. **PLOTTING.** — The first step to be taken in preparing the map is to plot the triangulation stations. Before any of the details can be plotted it is necessary to draw on the map the position of the *picture trace*, or *ground line*, for each photograph taken. This trace is the intersection of the picture plane with the plane of the map when the print is held vertical and at a

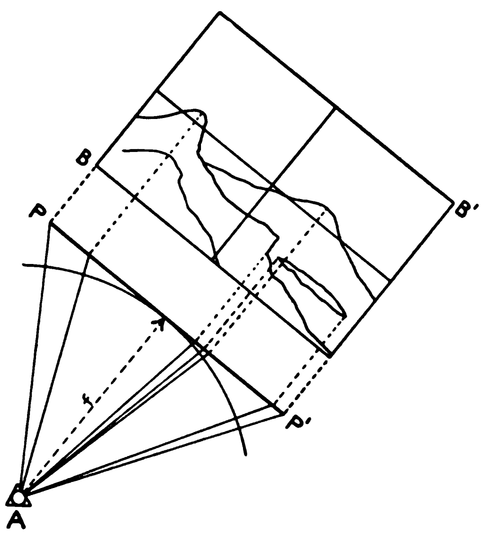

FIG. 74.

distance from the camera station equal to the focal length of the lens, and in such a position that every point on the print has its proper azimuth from the camera station. It is evident that when the print is in this position the principal plane is perpendicular to the print and that the picture trace is tangent to a circle of radius f drawn about the camera station as a center. In Fig. 74, A represents the plotted position of the camera station; BB' is a photograph (lying on the map) taken from A, and PP' is the picture trace. If the print is held in a vertical position on the picture trace the azimuth of every point on the print as seen from the plotted position of the camera station will be the same as it is in the field. The process of locating the picture trace is called *orienting the picture*, and it should be done accurately because all subsequent plotting from this picture will depend upon this location of the trace.

218. Orienting the Picture Trace. — In order that the picture trace may be oriented it is necessary to know the position of the horizon and principal lines and also the focal length of the lens. In addition to this the position of the camera station must be known and the print must contain the photograph of at least one point whose direction from the camera station has been determined.

In Fig. 75 suppose that a base-line AB has been plotted in the position ab. Let the focal length f equal bc, sometimes

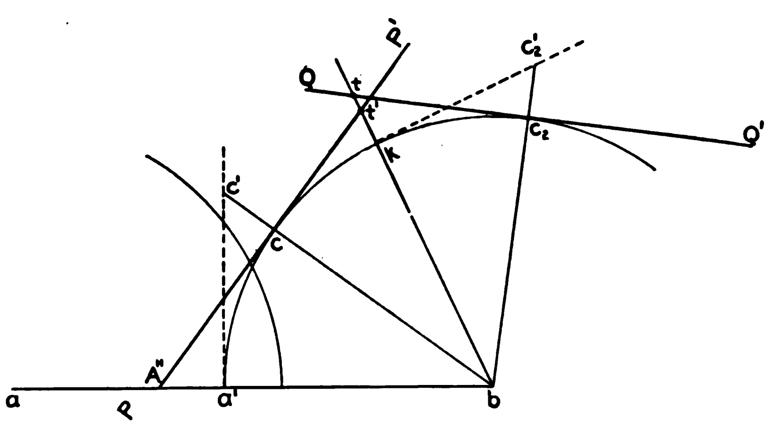

FIG. 75.

called the *distance line*. About a and b as centers, draw circles of radius f. If different cameras have been used at the two stations the circles must each have a radius equal to the value of f for the camera used. The focal length laid off must be the **actual length,** no matter what scale is chosen for the map.

Let us suppose that the first photograph taken from station B includes the picture of A and let Fig. 76 be the print so taken. Any point on the picture may be projected down on to the picture trace by means of a vertical line, and so far as the horizontal location of points is concerned it may be considered that they lie on the picture trace. When the picture is placed in position the principal point c on the trace (Fig. 75) must be on

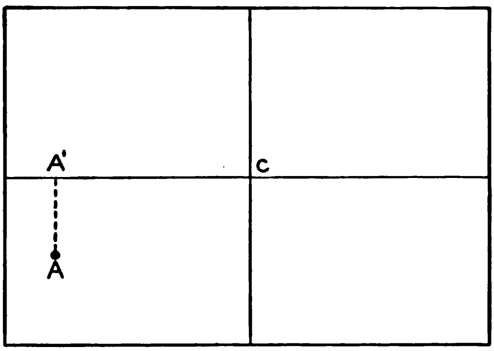

FIG. 76.

the circumference of the circle about b, and the photographed position of A must be somewhere on the line ba, produced if necessary. To find the position of the picture trace, erect $a'c'$ perpendicular to ba', scale from the print the distance cA' (Fig. 76), and lay it off on this perpendicular; this gives point c'. Draw a line from c' to the center b. Point c, where the line cuts the circle, is the center of the picture trace. Through c draw a line perpendicular to bc; this is the picture trace. Point A'' is the position on the picture trace PcP' corresponding to the position of A on the ground. PcP' is the picture trace desired, because it is perpendicular to the principal plane bc at point c; and c is the principal point because $cA'' = c'a'$ (Fig. 75) $= cA'$ as scaled from the print (Fig. 76).

A second picture trace may be located from this one if there is on the right side of this first picture some point which is also on the left side of the next picture. The geometric construction for locating the second photograph is similar to that used in orienting the first. In Fig. 75 T is a point which appears on two adjacent prints, PP' and QQ'. The trace QQ' is determined on the map by means of point t', which is the position of T on the trace PP'. Where the line tb cuts the circle at k erect a perpendicular kc_2', the distance kc_2' being obtained from the second photograph just as $a'c'$ was obtained from the print shown in Fig. 76. The line $c_2'b$ cuts the circle at c_2, which is the principal point of the second picture trace. The line bt cuts QQ' at t, which is the position of T on the second trace.

In case the focal length of the lens is not known, or if it is desired to verify its length, the picture trace may be located by the following method, provided that there are several points shown in the print which have been connected with triangulation points by measured horizontal angles. On the straight edge of a strip of paper mark a point representing the trace of the principal line and from this point lay off the distance from the principal line to each of the points to which angles have been measured. Draw radial lines on the plan from the camera station toward the points sighted with the transit, using the measured angles to obtain the direction of the points. The strip of paper may now be laid on the plan and moved about until each point on the paper lies on the corresponding radial line on the plan. This position of the edge of the paper is the true position of the trace, and the distance from the plotted camera station to the principal point on the strip is the focal length, or distance line. The distance line should, of course, be perpendicular to the trace. In order that the focal length may be determined accurately by this method two of the points sighted should lie near opposite edges of the print.

Since the prints are liable to become distorted, measurements which must be obtained with great accuracy, such as those for obtaining the focal length, should be scaled directly from the negative. For locating details it will be sufficiently accurate to scale from the print and to make allowance for the distortion as shown by the notches on the edge of the print.

219. Locating Points on the Plan. — Any point on the first picture may be plotted on the line PcP' (Fig. 75) by scaling on the print the horizontal distance from the principal line and by laying off this distance from c on the trace PcP'. By drawing lines from b to the points thus plotted on the picture trace a set of radiating lines is obtained, showing the directions of these points as seen from B, and the location of these points on the map will be somewhere on the corresponding radial lines.

By means of a photograph from A containing these same points the trace of this photograph may be oriented on the circle about a in the same manner as that described for the point b. (See Fig. 77.) This locates these same points on another set

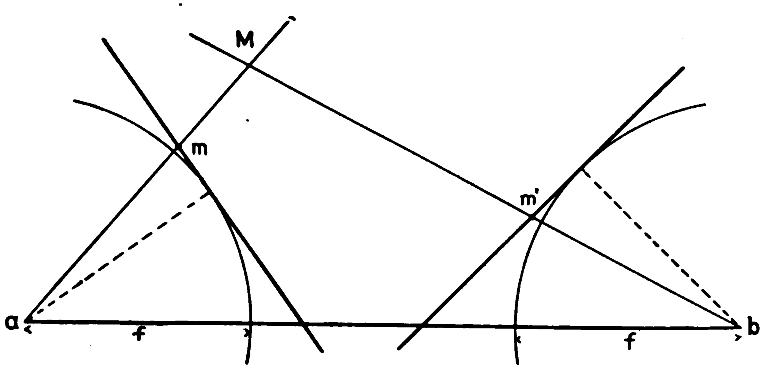

FIG. 77.

of radiating lines drawn from a, and the intersection of corresponding lines from a and b, such as am and bm', locates the point, M, on the plan.

Where there are many points to be plotted from one base-line a convenient method is to first transfer to the straight edge of a strip of paper the principal point of one of the prints and to lay off from this point the distances to all the other points to be plotted. This paper can be fastened in the proper position on the plan to serve as the picture trace. A similar strip is constructed for each photograph taken, and fastened in its proper place on the map. Points may then be located on the plan by fastening two fine threads to needles stuck in the end points of the base-line, passing these threads through corresponding points on the traces, and marking the intersection on the plan.

From the points so located the details may be sketched, the topography being judged from the appearance of the prints. There is no limit to the number of points that may be located, provided these points can be positively identified on the prints. Points which can be identified should be marked with the same numbers on the different prints for convenience in plotting.

220. Determining Elevations from the Photographs. — Differences in elevation may be found graphically or by computation, the former being the more common method. The distance of a point above or below the horizon line may be scaled from the print; this vertical distance divided by the horizontal distance from the plotted camera station to the point on the trace is the natural tangent of the angle of elevation or depression. It is necessary that the actual horizontal distance from the camera to the point should be known before the elevation of the point can be determined. This horizontal distance may be scaled from the map after the point has been located by the method already described. The difference in elevation between the camera and the point may be found by simple proportion as follows.

Fig. 78 shows the plate with the horizon and principal lines drawn upon it. A is the point whose elevation above the camera station O is desired. Fig. 79 is a portion of the map, O is the lens, and PP' is the trace of this picture on the map. In both Figs. 78 and 79, c is the center of the print, B is the point vertically under A, and OB is the horizontal distance from O to A. If in Fig. 79 a perpendicular BA' is laid off at B, this perpendicular being equal to the distance AB scaled from the print, then the angle $A'OB$ is the true angle of elevation of A above a horizontal plane through O. The plotted position of A will lie on the line OB on the plan. If a is assumed to be the plotted position of A and aa' is drawn perpendicular to OB, and if aa' is measured with the same scale as that used in laying out the base-line, the result will be the actual difference in elevation between O and A. The distance aa' could of course be computed by proportion if desired, the data being the actual distances OB and BA in inches (OB being scaled from the map and BA from the print) and the distance Oa in feet as scaled from the map.

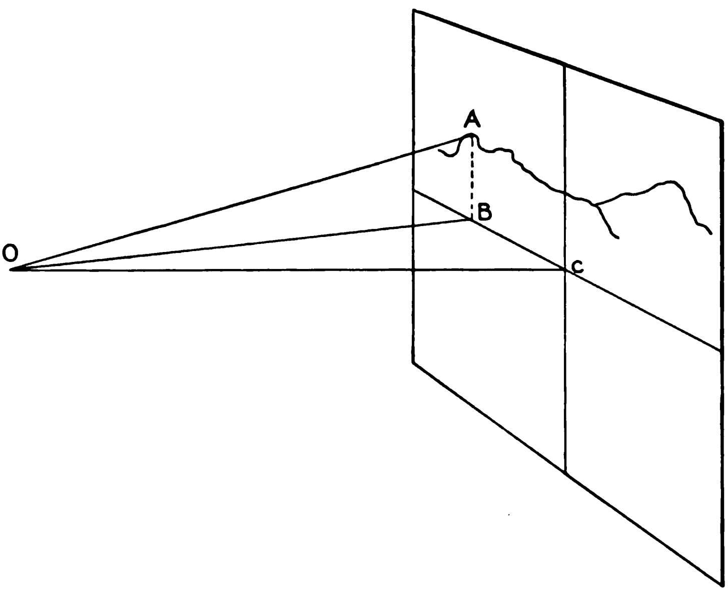

FIG. 78.

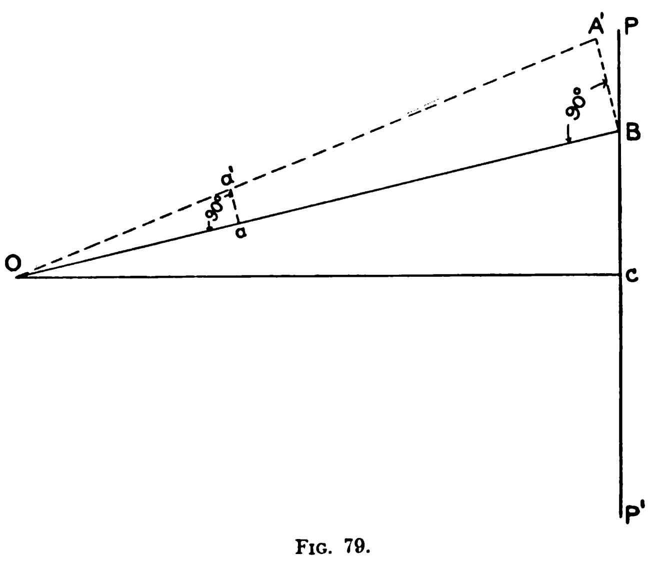

FIG. 79.

FIG. 80a.

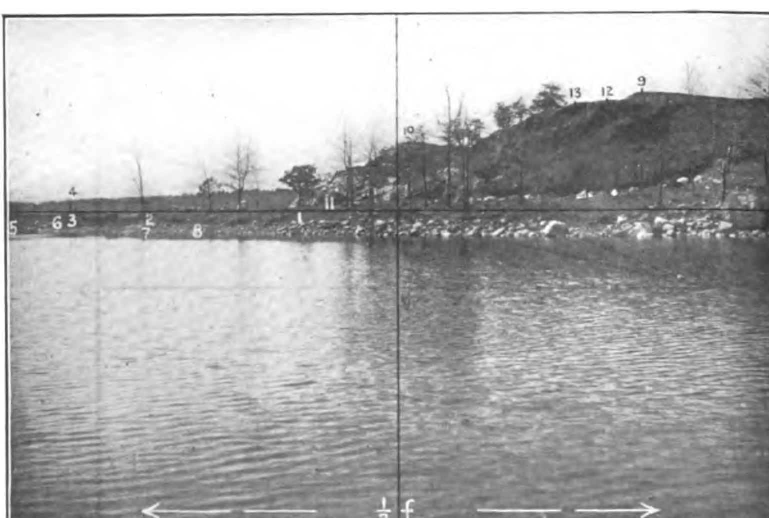

FIG. 80b.

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221. Contours. — Contours may be sketched by obtaining the elevations of controlling points and then judging the variations in slope between the points by the appearance of the photographs. Sometimes the contours are first sketched on the photographs by making use of the elevations already determined, and then these contours are drawn on the map. Since there is a large amount of sketching involved, and since such a map must be generalized to a great extent, a knowledge of the geological formation of the country will be of great assistance in determining the characteristics of the contours. (See Chapter VII.)

As a further illustration of the method of locating points, determining elevations, and sketching contours the plan shown in Fig. 81 has been made from the photographs shown in Figs. 80a and 80b, which were taken from the ends of a short base-line. In this example the picture traces have been oriented by means of the triangulation point T, which was cut in with an ordinary transit from the ends of the base-line. The construction lines used for orienting the prints and locating the points are shown as dotted lines; the construction for finding the elevation of point 9 above S. Base is given in the separate diagram to the right. The focal length of the camera lens used in drawing the circles was determined by the method of Art. 214. The numbers of the points on the picture traces correspond with the numbers of the same points on the plan, so that the details of the construction may be followed. Several points which could be easily identified were first located and their elevations determined graphically; five-foot contours were then drawn by interpolating between these elevations and judging the spacing of the contours from the slopes shown in the photographs.

PROBLEMS.

1. The distances from two points on a photographic print to the principal line are 2.860 inches and 2.160 inches. The angle between the points measured with a transit is $34° 53'$. Determine the focal length of the lens.

2. With the same camera as in Problem 1 the following determination of f was made; x = 1.104 inches, y = 2.836 inches, and angle $\omega = 27° 23'$. (See Fig. 72, p. 231.) Compute the value of f.

CHAPTER VII.

THE RELATION OF GEOLOGY TO TOPOGRAPHY.*

222. The Value of a Knowledge of Geology to the Topographer. — The features of the earth's surface are determined by geologic processes working upon geologic structures. There is little need of emphasizing the intimate relation existing between the geology of a region and the topographic forms there represented. The engineer engaged in mapping topographic forms soon perceives that districts having the same geologic history have certain characteristic features in common, whereas districts of diverse geologic histories may be distinguished by differences in surface form. Contour maps show peculiar resemblances or peculiar differences, according as the regions mapped have developed along the same or different geological lines.

There is a close relation between geology and topography, and the maker of topographic maps should have a clear appreciation of this relation and a thorough knowledge of the types of topographic forms produced by the various geologic processes and structures. It is true that the engineer whose work is restricted to the mapping of small areas can afford to dispense with a knowledge of geology, for such a knowledge would hardly prove of practical benefit in making maps of city blocks or country farms. The engineer who reproduces large areas of the earth's surface, however, should have some understanding of the conditions under which that surface was formed.

The necessity for such knowledge on the part of the map maker becomes apparent when one considers the method of map making described in the preceding chapters. If it were possible for the engineer carefully to survey every contour line, locating every point along every line with accuracy, the map would be

* This chapter was written by Professor Douglas Wilson Johnson of Harvard University. The drawings were made by Mr. François E. Matthes, of the United States Geological Survey.

accurate whether the engineer had a knowledge of geology or not. One can make a fair copy of a thing that he does not understand, especially if he reproduces it on the same scale as the original. No map was ever made in this manner, for no map is a copy of the region it represents. Neither time nor money is available for such a method of map making. All maps are on a smaller scale than nature, and hence all require the omission of details, and a broad generalization of the features which are retained. In his effort to do the best that is practicable under the limitations imposed upon him the engineer will be greatly aided if he understands the things he has to generalize. One could hardly expect that a satisfactory abstract of a botanical report would be written by a man who had never studied botany. Neither can one expect that the most satisfactory map-abstract of land forms will be sketched by an engineer who knows nothing about land forms.

In the practice of contour mapping, a number of points are located with a fair degree of accuracy, and the intervening areas are sketched. It is the introduction of the element of sketching that makes it necessary for the topographer to acquire a knowledge of land forms, if he would produce the most satisfactory map. In the first place, as has already been pointed out (p. 219),

"Sketching contours on small scales where only a few points are accurately determined requires great skill on the part of the topographer. He must not only be able to accurately judge the slopes from the appearance of the country, but he must be familiar with the characteristics of contour maps of different topographic forms."

The topographer who is familiar with the kind of slopes which characterize regions of alpine glaciation, for example, and who knows the general characteristics of maps of such regions, will have a distinct advantage over the topographer who knows nothing of land forms, if both are assigned to work in a district formerly dissected by alpine glaciers. The man of broader training will know what to expect as to relations between different slopes and the behavior of streams, and in drawing his generalized sketches will have constantly in mind the essential relations which a truthful map should represent. His practiced eye will quickly detect features less readily observed by an untrained eye. Other things being equal, he can in a given time detect and represent on his map more of the essential elements of the topography than can his companion of more limited knowledge. A map of a given degree of accuracy can be produced by him more quickly than by his companion. The increased efficiency of the man of broad training makes possible a saving in time as well as a higher standard of work.

In the second place, it is not possible to represent on a small-scale map all the features that one recognizes in a region. In sketching, the topographer must decide how much to put in and how much to leave out. In most landscapes there are many small or minute features which are not critically important, and others, also small in size, which give a key to the whole topography. The intelligent topographer will select the most significant features for his sketch, omitting the unessential details. The topographer who is ignorant of the relation of geology to topography will have no means of determining what is significant and what is not. It is not merely a question of representing the biggest things and omitting the small things. On hills of the same altitude some changes in slope are critically important, while others are not. The topographer must decide which to represent, if he finds it impracticable to take equal account of all. Only the intelligent topographer can make an intelligent selection.

The truth of the principles outlined above is attested by those who have had wide experience in topographic work. Henry Gannett, for many years the director of the topographic branch of the United States Geological Survey, is the author of a "Manual of Topographic Methods," from which the following paragraphs are quoted.

"Sketching is artistic work. The power of seeing topographic forms in their proper shapes and proportions and of faithfully transferring these impressions to paper is one of the most difficult acquirements to obtain. The difficulty is increased by the necessity of expressing form by means of continuous contour lines at fixed intervals. This work involves knowledge of the elements of structural geology and good judgment in applying them.

"Whatever its scale may be, every map is a representation reduced from nature, and consequently there is more or less generalization. It is therefore impossible to make any map an accurate, faithful picture of the country it represents. Moreover, the smaller the scale the higher must be the degree of generalization, and the further must the map necessarily depart from the original.

"Now, it is in this matter of generalization that the judgment of the topographer is most severely tested. He must be able to take a broad as well as a detailed view of the country, he must understand the meaning of its broad features, and then must be able to interpret details in the light of those features. Only such a man is competent to make just generalizations and to decide what details should be omitted and what preserved, and, where details are omitted, what to put in their places in order to bring out the dominant features." *

In an address before the American Society of Civil Engineers, Professor John C. Branner, who has employed many topographers, emphasized the relation of geology to topography. The following quotations make clear the conclusions he reached as a result of his experience.

" 'We see, according to the light that is within us.' One cannot picture a subject he has not studied. However skilled a draftsman or artist may be in the technique of his art, unless he understands the animal or plant he has to draw, he cannot make a correct picture of it. In topographic representation this is equally true, and it is the more important because a large part of every map must be sketched in, and this sketching cannot be done properly unless he who does it knows what ought to be there. Unless the topographer knows what to look for he doesn't find it, or he finds only a part of it. This statement is based on no small amount of experience of this fact. It has been the author's duty to employ many topographers, and all his experience of their work has but confirmed this opinion.

" It is of the utmost importance to the topographer that he should know what kind of topography to expect, and, to this end, the more he knows of the materials in which topography is cast, and of the agencies that shape it, the clearer will be his insight, the less the waste of time and energy will be, and the truer will be his representation of the relief." †

Facts of field experience might be multiplied to show the need of a knowledge of land forms on the part of the topographer. The contour maps made by the topographic branch of the United States Geological Survey are prepared as a base for the geological maps of the country. Since one of the chief objects of making the map is to aid in the representation of geological facts, it is evident that the map must correctly express the relation of the topography to the geology. If the geologist finds, when he goes into the field, that the topographer who preceded him has not produced a map which shows this relation correctly, he may be compelled to postpone his work until the map is corrected. A number of areas have had to be remapped because the intimate relation of geology and topography was not properly portrayed on the maps first made.

* Bulletin of the United States Geological Survey, 307, p. 84.

† Proceedings of the American Society of Civil Engineers, 1897, pp. 474-5.

The need of a knowledge of land forms as a guide to mapping is likewise appreciated by the practical topographer at work in the field. Constantly dealing with the features of nature's handiwork, he comes to wonder how those features were produced. He observes certain evidence that definite laws are in operation, and finds in different regions certain similarities of topographic form. From such observations it is but a short step to the desire for definite rules regarding topographic relations, and as Branner remarks,

"There is a constant demand, among those who have not devoted much time to a study of the subject, for short and simple empirical rules for topography."

Indeed, such rules have been coined by the topographer, based on his experience in mapping one or more regions. Tested in other regions the rules prove of no value, or even misleading; for they are based on limited observations in regions of some one geological structure or similar geological history. The coining of such rules, however, proves two interesting facts: first, that there is a demand on the part of practical topographers for a knowledge of the laws of land forms; and second, that there is danger in substituting empirical rules for a broad knowledge of land forms in general. As Branner has summed up the matter,

"There is not, and cannot be, a fixed rule for all topographic forms, and . . . in order to understand topography one must understand geology."

In closing this part of our consideration I wish to quote the words of a well-known topographer and inspector in the topographic branch of the United States Geological Survey, Mr. François E. Matthes. Writing in "Science," he says:

"Most topographic maps give little more than an imperfect, incomplete picture of the relief. Others again are overburdened with unnecessary, irrelevant details. Some actually amount to misrepresentations, even though they be the product of sincere and painstaking effort. The topographer is to-day and always has been more or less uncertain as to the matter of detail. Both in the selection of scale and contour interval, and in the actual field sketching he is at a loss to decide which of the smaller topographic units he must show, and which he must leave out. He needs, in short, criteria to guide him where to draw the line." *

* Science, New Series, Vol. XXVII, p. 146.

Some consideration may properly be given to the use to be made of topographic maps. Reference has already been made to their employment as a base for geologic mapping, and it is clear that for this purpose the maps must properly represent the relation of the topography to the geology. In technical and scientific enterprises of many kinds, whether governmental, state, or private, contour maps which accurately represent the topography of the regions under investigation are absolutely necessary. The importance of accurate maps in such work is too evident to require special emphasis. Contour maps are now being used in increasing numbers in our schools and colleges as important aids to the study of geology and physical geography. Lists of the best maps representing land forms of different types have been prepared for the use of the schools by teachers of geography, and by the Government. Here, also, the maps must represent the physiographic features of the lands with fidelity, if the study of the maps is to be successful. Professional students of land forms make constant use of good contour maps, but find broadly generalized, mechanically sketched maps practically worthless. This class of students, however, is relatively small. But no matter what use is made of a map, accuracy is a most essential element of its value. The engineer planning a reservoir, or laying out a railway route, may not be interested in the geological history of the region, but he does want a map representing the **facts** of present form. It is as an aid to seeing the facts quickly and representing them accurately that a knowledge of land forms is of the most practical value to the topographer.

A word of caution may well be added. Just as no amount of technical skill will enable the topographer to profitably dispense with a knowledge of land forms, so no amount of geological knowledge can take the place of technical skill. A geologist may make a map, but unless he be a topographer as well his map will abound in technical errors. So, too, a topographer may make a map free from technical errors, but unless he has had a geological training he cannot produce the most expressive representation of land forms. An unusual degree of technical skill may enable the topographer with no geological training to produce a map of high standard. Good maps have been thus produced. But this fact does not argue against the need of geological training by the topographer. Had that technical skill been combined with a broad knowledge of land forms it is only fair to expect that a still more expressive and sympathetic map could have been produced, and that a saving in time and money could have been effected by the better equipped man. The ideal topographer is one who combines high technical ability with a broad knowledge of the earth's surface features.

There is danger, also, that the topographer with a slight knowledge of land forms may **think** he sees things which do not exist, and thus be influenced in his sketching to make erroneous representations. As in other branches "a little knowledge is a dangerous thing." The danger is a real one, but the remedy lies not in abstaining from all knowledge, but rather in gaining a broader and more thorough knowledge.

Attention may now be given to certain general points which should be considered by the maker of contour maps; and also to a few examples of maps which do or do not express the facts of form in the region mapped. Brief consideration is given to the relation of geologic structure and geologic history to the forms in question. In order to emphasize the different effects obtainable from contours alone, streams and other features are omitted.

223. **MAP EXPRESSION.** — One of the most frequent difficulties with contour maps is of a general rather than of a specific character. The maps are marked by broadly generalized contours with uniformly graceful curves, producing an effect of indefiniteness which is often described by saying that the map "lacks expression." It seldom, if ever, happens that a portion of the earth's surface is so devoid of expression as such a map would indicate. Even where a land form is maturely dissected by broadly open ravines, and the interstream areas are well rounded, there are usually changes in slope which are critically important, and which give "character" to the country. These relations are apparent to the topographer trained to recognize them and to appreciate their significance, but are lost to one who sees nothing in the landscape but a series of meaningless hills and valleys. Fig. 82 represents the contours of a map lacking in expression.

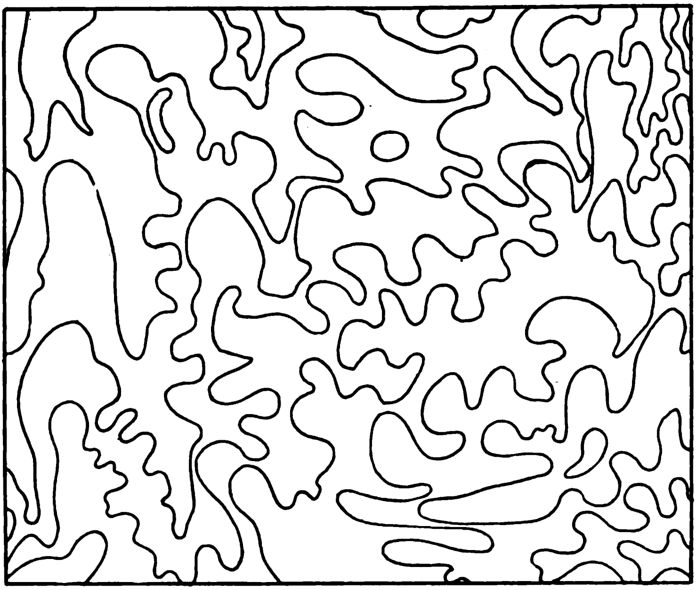

FIG. 82. AN EXPRESSIONLESS CONTOUR MAP.

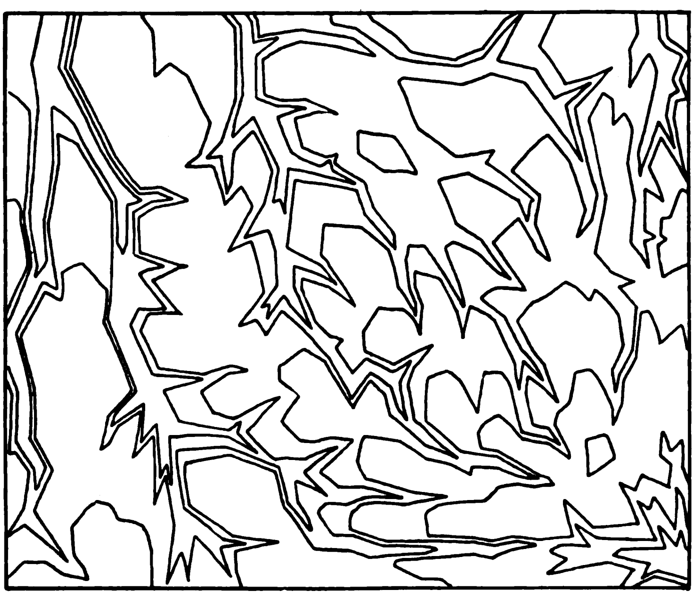

FIG. 83. CONTOUR MAP SHOWING UNNATURAL ANGULARITY.

In general, nature fashions the surface of the earth on irregularly curving lines, not on regular curves, nor on straight lines and sharp angles. As indicated in the preceding paragraph, a map which is characterized by broadly rounded, evenly curved contours may safely be regarded as inaccurate. So, also, a map in which the contours are straight for long distances, and then bend in sharp angles (Fig. 83), may safely be considered as not representing the true character of the topography. Some degree of angularity may be expected where the drainage of the region is joint-controlled or fault-controlled. Such angularity is usually quite local, however, and not nearly so pronounced as a map like Fig. 83 would lead one to suppose.

224. **SELECTION OF SCALE AND CONTOUR INTERVAL.** — "The value of a map as a means of representing land forms depends upon two factors: selection of scale and contour interval, and ability on the part of the map maker to express topographic character." We have already seen that the ability of a topographer properly to represent topographic character depends in part upon his knowledge of the relation of geology to topography. It is also true that the selection of a proper scale and contour interval is in part dependent upon a thorough knowledge of land forms. It too often happens that the principal factors taken into consideration in determining the scale and contour interval for a map of a given region are the cost per square unit, the funds available, the degree of cultural development of the region to be mapped, etc. Such factors may necessarily control the selection in many cases, but if the best results are to be secured it is essential that definite physiographic criteria should be considered in this connection. The object of the topographer should be to produce a map that will show the essential topographic features of the region as economically as possible. If the representation of land forms alone is considered, the cost may be excessive. If economy alone is considered, the map may fail to properly represent the region. The topographer must understand the essential features of the region so thoroughly that he can intelligently decide what is the smallest scale and largest contour interval which will truthfully represent these features. Such an understanding must be based on a knowledge of the relation of

geology to topography. The triangular facets which terminate the spurs along the face of a fault-block mountain range are critically important points in that type of mountain topography. Too small a scale and too large a contour interval would prevent even an intelligent topographer from showing these facets. Too large a scale and too small a contour interval would show a great mass of other details not essentially important, and would thus lead to a wasteful expenditure of time and money. An intelligent selection will permit the map to show essential features and yet come within the bounds of economy.

225. **Texture of Topography.** — A study of land forms reveals the fact that their dissection by streams may be accomplished in either of two ways: by streams whose branches are few in number, each branch draining a relatively large portion of the country; or by streams which branch indefinitely, each of the numberless small branches having a very small drainage basin. In the one case, the interstream areas are large, the contours have few reëntrants, and the topography as a whole appears large featured, or "coarse textured." In the other case, the interstream areas are exceedingly small, due to the indefinite ramification of the drainage lines, the contours have many reëntrants, and the topography is said to be "fine textured."

Coarse and fine texture do not occur indiscriminately over the earth's surface, but are controlled in their development by certain definite factors, among which may be mentioned: the character of the country rock, the character of the climate, the age of the land form, and the number of erosion cycles through which it has passed. A region of fine-grained, unconsolidated rocks, which has a climate so arid that there is little vegetation to form a protecting cover, will possess a fine texture during youth and early maturity, although it may lose some of its fineness in later stages. A region of resistant rocks, in a humid climate, will probably maintain a coarse texture throughout its erosion cycle, since drainage channels are not readily eroded by the smaller branch streams when the rocks are resistant and protected by vegetation. In like manner other factors help to determine the texture of the topography in any given region.

The topographer who sees no special significance in minor branch ravines, who is anxious to complete the map with the smallest possible expenditure of time and money, and who finds it possible to locate with accuracy the position of only the larger drainage lines, may be tempted to omit entirely the minor branches, and produce a map on which the contours are broadly rounded and generalized, giving a coarse textured topography as in Fig. 84. The better trained topographer, realizing the full significance of the minor facts of form, although limited like the other as to available time and money, can nevertheless sketch

FIG. 84. COARSE TEXTURED TOPOGRAPHY.
(Same Area as Fig. 85, poorly mapped.)

FIG. 85. FINE TEXTURED TOPOGRAPHY.

a more expressive representation of the areas between the larger drainage lines, thus showing the fact that the region is fine textured (Fig. 85). Even if the limitations imposed on the better topographer make it absolutely impossible for him to determine the number and location of the smaller branch ravines, his map shows the fact of their existence, is a more truthful representation of the kind of land form mapped, and is more valuable to any one who appeals to the map for information about the region represented.

226. ENLARGEMENT AND REDUCTION.—The apparent texture of a region is misrepresented by the enlargement of a contour map. If a map is made by the topographer on the scale $\frac{1}{250000}$, and then enlarged to the scale $\frac{1}{62500}$, the effect is to produce a coarser appearing texture than that indicated by the original map. The topographer who is making a map on a given scale determines the amount of detail which he can profitably represent on that scale, and makes his map accordingly. He will not be able to show as much detail on a scale of $\frac{1}{250000}$ as on a scale of $\frac{1}{62500}$, nor is he expected to do so. But if he works on a scale of $\frac{1}{62500}$, and shows only such detail as might be expected on a map with the scale $\frac{1}{250000}$, he is open to criticism; because a large scale map with important details lacking suggests poor mapping. Enlarging a map from the small scale on which the work was well done to a larger scale on which better work is supposed to be done results in injustice to the topographer, and misleads the one who uses the map.

227. **INDEX FORMS.** — Associated with the surface features produced by any geological history are usually found certain forms so characteristic of that history that they serve as a key to the whole physiographic development of the region. These forms may be minor features of the landscape so far as size or prominence is concerned, but they are of prime importance as guides to the significance of the larger features represented. Inasmuch as these forms serve as an index of the history of the region, and aid us in interpreting the meaning of all the associated topographic features, it has been suggested by an experienced topographer that we call them "index forms."

Triangular facets terminating the spurs of a mountain range along a straight base-line, regardless of rock structure, point strongly to a fault-block origin for the range, and under these circumstances may be regarded as index forms of block mountain structure. Circular rows of "hogbacks" are index forms of dome mountain structure, and enable us to distinguish between this type of mountain and a volcano. Cirques, U-shaped valleys and hanging lateral valleys are index forms of glaciation, and serve to differentiate a glaciated region from one of normal stream dissection. The list of index forms might be prolonged indefinitely, but these examples will suffice to make clear the meaning of the term as understood by topographers.

It is essential to good map making that the topographer should be able to recognize the different types of index forms when he encounters them in the field, and that he should fully appreciate their significance. Not only will this insure his representing on the map those particular features which might otherwise be slighted in his sketching, but it will open his eyes to important field relations which would otherwise escape him. He is thus

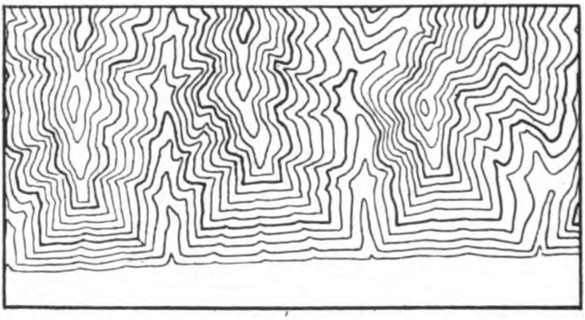

FIG. 86. CONTOUR MAP SHOWING TRIANGULAR FACETS.

able to prepare a map which tells a truthful and consistent story, a map unencumbered by unnecessary details, yet lacking nothing essentially important.

The difference between maps which make clear the index

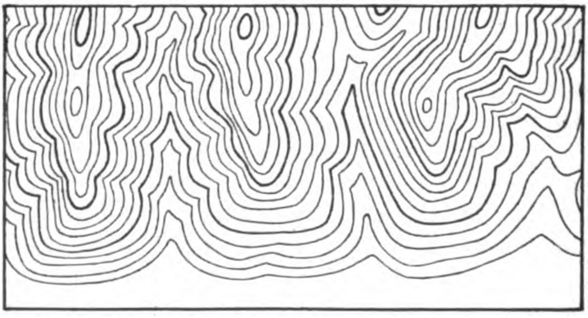

FIG. 87. SAME REGION AS FIG. 86, POORLY MAPPED.

forms of a region and those which do not may be seen in the accompanying illustrations. Figs. 86 and 87 represent two sketches of a portion of the dissected fault face of a block mountain. One map makes plain the existence of good triangular facets terminating the mountain spurs along the straight base-line of the range, while the other map most effectually obscures these index forms.

The general form of a dome mountain readily distinguishes it from the block or folded type, but does not distinguish it from a volcano; for a maturely dissected dome mountain and a maturely dissected volcano may both have radial drainage, a rudely circular outline, and radially disposed ridges highest toward the center of the mass. The index forms which enable one to discriminate between the two are the hogbacks which encircle the dome mountain. When these are present the field observer

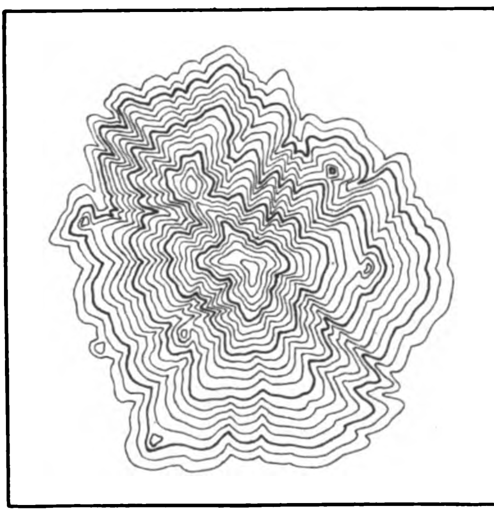

FIG. 88. SUBMATURELY DISSECTED VOLCANO.

can readily make the discrimination, even without a detailed study of the geology of the region. Where they are represented on a contour map the discrimination is as readily and almost, if not quite, as surely made by one who studies the map. If for any reason the hogbacks are not represented on the map, the student may remain in doubt as to whether the form is a volcano, a dome mountain with hogbacks worn very low or covered by alluvial fan deposits, or a dome mountain with good hogbacks

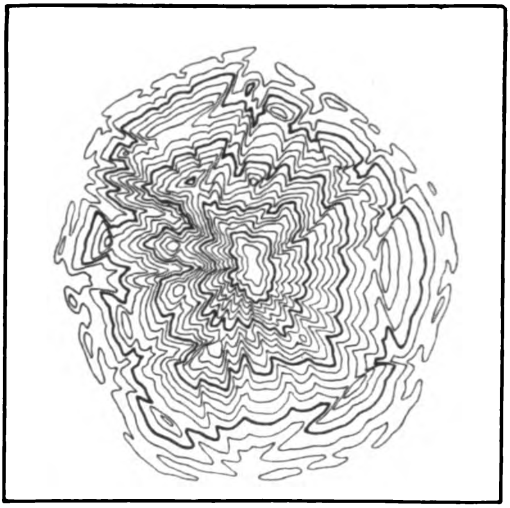

FIG. 89. DOME MOUNTAIN WITH ENCIRCLING HOGBACKS.

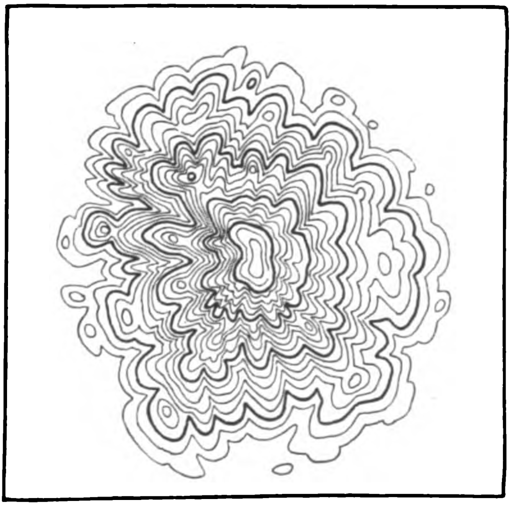

FIG. 90. THE SAME DOME MOUNTAIN AS IN FIG. 89, POORLY MAPPED.

so poorly represented by the topographer that the existence of the hogbacks is concealed. If the hogbacks are really present, the desirability of the topographer's being able to recognize them, to appreciate their significance, and to record them on the map, is evident. In the accompanying figures the first sketch, Fig. 88, represents a sub-maturely dissected volcano, the second sketch, Fig. 89, a dome mountain with encircling hogbacks, the third, Fig. 90, the same dome mountain poorly mapped. It will appear that the poor sketch of the dome mountain might as well represent a volcano, so far as any certain evidence of the index forms of dome mountain structure is concerned.

The index forms of glaciation are so numerous and varied that we may profitably limit our consideration to those of alpine glaciation. Among the most prominent index forms of alpine glaciation are cirques, U-shaped valleys, hanging lateral valleys, alpine peaks, and lakes. A maturely dissected mountain range which has been glaciated is apt to show all these index forms. A maturely dissected mountain range which has never suffered glaciation shows none of them. On any good contour map of a mountain range it is easy to determine whether or not the range has been glaciated to any appreciable extent. Some maps are so generalized and imperfect, however, that the extensive glaciation of the region mapped is scarcely more than hinted at. There is usually a hint in the shape of one of the index forms of glaciation, i.e. lakes. The topographer recognizes a lake when he sees it, even if he does not recognize a cirque, U-shaped valley, hanging valley, or alpine type of peak. Accordingly the lakes in a glaciated region are usually represented if of any size, while the other features may be effectually obscured. From the presence of the lakes the existence of glacial forms in a region has been detected by a study of maps which in other respects suggested normal stream erosion only. The accompanying map, Fig. 91, indicates that the region represented has been greatly modified by alpine glaciers, for the principal index forms of alpine glaciation, except the lakes, are sharply brought out by the contours. In contrast to this figure, another map of the same region, Fig. 92, is so far generalized that the hanging lateral valleys are lost, the alpine character of the mountain peaks is obscured, the U-shaped

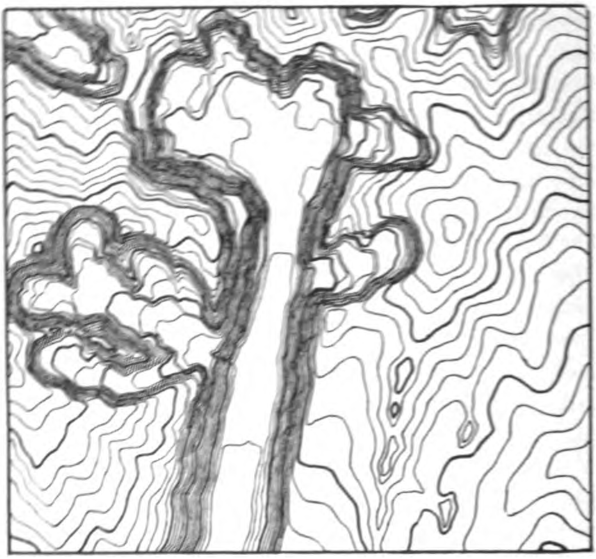

FIG. 91. CONTOUR MAP SHOWING INDEX FORMS OF ALPINE GLACIATION.

FIG. 92. SAME REGION AS FIG. 91, POORLY MAPPED.

cross-profile of the main valley is poorly shown, while several of the cirques have lost their most essential characteristics. The first sketch makes clear the kind of land forms to be found in the region. The second does not.

Alluvial fans are often formed on a large scale, as, for example, about the bases of mountain ranges in the arid districts of the west. Each fan has its apex at the mouth of the canyon from which the alluvial material was derived. If the canyons are numerous and fairly close together, the different fans coalesce to form an extensive alluvial plain sloping gently away from the mountain. A careful examination of the plain, however, reveals the character of its origin, for the convex slopes of individual fans can be detected. This feature is most evident near the base of the range, but may be observed far down the slope where the individuality of the several fans is more nearly lost.

A truthful contour map makes clear the origin of such a plain. Near the mountain base the individual fans are indicated by a distinct scalloping of the contours, while farther out the scalloping becomes less pronounced. The curves are convex forward opposite the canyon mouths, showing the source from which the alluvial material of each fan came. The origin and significance of the relations are apparent to the trained topographer. But the slopes are often so gentle and the fans so extensive that one unfamiliar with land forms might easily fail to grasp the critically important points, and represent the plain as of uniform slope, or as gently undulating without any special relation between the higher areas and the canyon mouths. On such a map the true form of the plain is not apparent. The straight contour lines suggest slopes smoothed by marine action, while indefinitely undulating contours suggest low divides left by stream erosion rather than alluvial fans built up by stream deposition. The three sketches of Figs. 93, 94, and 95, all of the same region, show some possible differences between different maps of this type of topography.

228. PHYSIOGRAPHY. — The foregoing illustrations are taken at random from the many types of land forms with which the topographer should be familiar. His range of knowledge should be sufficiently broad to include the principal topics treated in that branch of geological science known as Physiography, or Topographic Geology. He must understand the structures involved in the formation of plains and plateaus, and the different

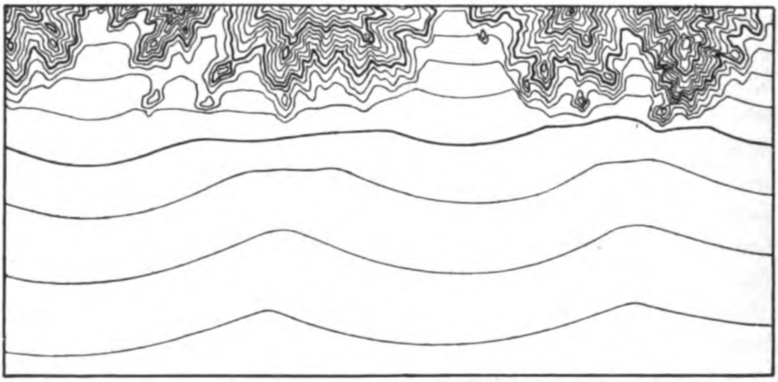

FIG. 93. CONTOUR MAP OF ALLUVIAL FANS.

classes of mountains, volcanoes, and constructional depressions, and must have a clear idea of the appearance of such land forms

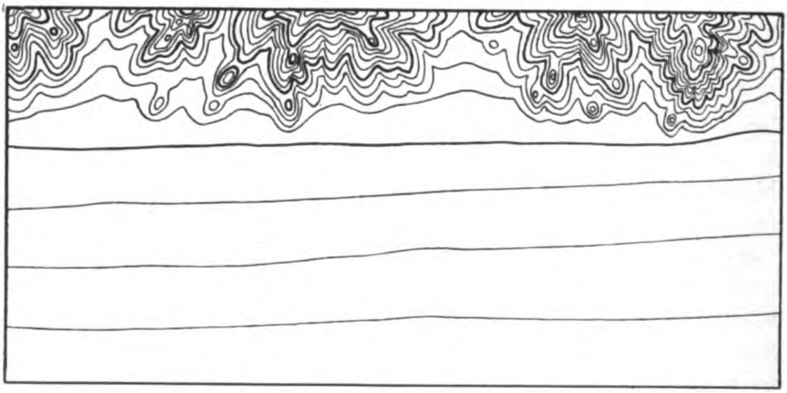

FIG. 94. SAME REGION AS FIG. 93, POORLY MAPPED.

in their initial stages, before weathering, stream erosion, and other destructional forces have altered their appearance. The activities of weathering agencies, and of rivers, glaciers, waves, and winds must be studied, in order to secure an understanding of the destructional forms they produce. These destructional forms themselves must be studied and described, in the field when possible, on the basis of good maps and photographs when they alone are available, until the topographer knows the appearance of these forms and the types of contours which represent them. The study of destructional forms due to river action should include valleys and gorges in different stages of development; alluvial fans and plains, flood plains, and terraces; entrenched

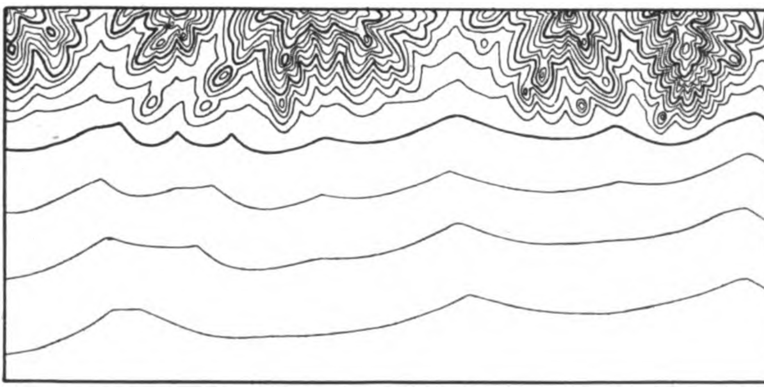

FIG. 95. SAME REGION AS FIG. 93, POORLY MAPPED.

meandering valleys, longitudinal and transverse valleys, uplifted and dissected peneplains; and other forms, both large and small; directly due to stream erosion on various structures in various stages of an erosion cycle. In the study of glaciation should be included cirques, U-shaped valleys, hanging lateral valleys, alpine peaks, and lakes; terminal, medial, and lateral moraines; outwash plains, delta plains, eskers, kames, and drumlins. Wave-cut cliffs and benches, barrier reefs and lagoons, and the various types of bars and beaches should be among the shore forms studied. Sand dunes and their associated features should be considered in connection with the study of wind-made forms. To these general features produced by the destructional forces should be added those forms which occur only in connection with the dissection of certain constructional forms; such are the concentric lines of hogbacks and annular drainage lines of dome mountains, the anticlinal valleys and synclinal ridges of folded mountains, and the cuestas and inner lowlands of coastal plains. The trained topographer, familiar with these and similar forms, possesses that combination of engineering skill and geological knowledge which is essential to the best map making.

PART III.

HYDROGRAPHIC SURVEYING AND STREAM GAUGING.

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CHAPTER VIII.

HYDROGRAPHIC SURVEYING.

229. DEFINITION. — Hydrographic surveying is the term applied to the processes used in surveying any body of water. In the case of oceans or lakes this may include the determination of shore lines, soundings, characteristics of the bottom, location of buoys, etc.; the survey of a river may also include the determination of the velocity and characteristics of the flow. In its broad sense the term may be applied to the survey of drainage areas and proposed reservoirs for the storage of water.

SHORE LINE SURVEYS.

230. SHORE LINE AND STREAM SURVEYS.—The shore line of a body of water may be surveyed by running a transit and tape traverse at a convenient distance from the water and locating the shore by offsets measured with a tape from the traverse line at all points where there is a noticeable change in the direction of the shore line.

Surveys of such irregular lines, however, are often made by first establishing instrument points by traverse or by triangulation and filling in the details by the Stadia or the Plane-Table Method (Chapters IV and V), as the work can be done more rapidly and economically by these methods. If a traverse is used for the control it may be run out by means of the transit and tape, or it may be made a stadia traverse and run at the same time that the side shots are taken to locate the details.

In surveying a river whose width is too great for accurate stadia measurements it is necessary to run traverse lines on both banks and to locate each shore line by side shots from its traverse line. As a check on the survey it is well to connect the traverse lines on each side of the river by occasionally measuring angles to transit points in the traverse on the opposite bank. But where the river is not over half a mile wide the opposite bank can be located, within 5 to 10 feet under favorable conditions, by stadia distances from the transit line, or by occasional intersections from adjacent transit points.

It is frequently advisable to make no attempt to run a traverse which will follow very closely every turn of the shore or river; small auxiliary traverses composed of short courses may occasionally be run around the arm of a lake or the bend of a river while the main traverse shoots directly across it. The stadia method has great advantages over the transit and tape for locating a shore line, because the distances to inaccessible points across the water or over the brush along the shore can be readily measured by this method. If the survey covers a long stretch of shore line, and especially if great accuracy is required, it should be based on a system of triangulation executed with whatever accuracy the work itself demands (see Chapter I).

231. SHORE LINES OF HARBORS, LAKES, AND RIVERS. — Unless a body of water is long and narrow, and conditions along the shore are favorable for a survey by traverse, it will usually be found desirable to use a system of triangulation as a basis of the survey, supplementing this with stadia and plane-table work. An excellent method which gives accurate results is first to run out a system of triangulation with the transit and then to plot the stations on a plane-table sheet, using the plane table to fill in the details.

In planning a system of triangulation where stations occur on opposite banks of a body of water it must be possible to see between successive stations on one shore or the other in order to complete the system. For example, in Fig. 96 it is evident that unless a sight can be taken from 5 to 7 or from 6 to 8 the computation or plotting of the system cannot extend farther than line 5 — 6. It is obvious in Fig. 97 also that the system stops on line 4 — 5 unless a sight can be taken from 4 to 6 or from 3 to 6.

It is not usually economical to locate shore lines of lakes by traversing around them, owing to the difficulties presented by natural obstructions, such as woods. It may be necessary, however, to supplement the triangulation and plane-table work by side traverses, in order to secure the necessary details in bays or irregular inlets.

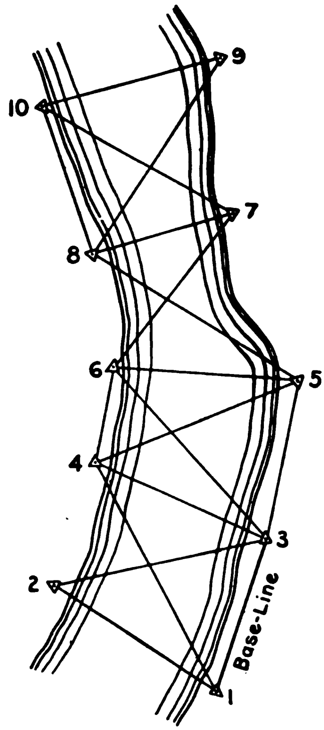

Fig. 96.

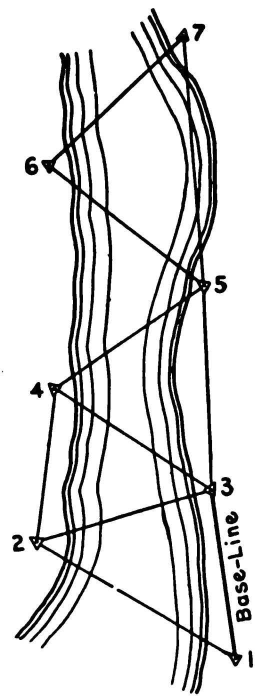

Fig. 97.

232. The following device for extending triangulation along a shore line has been used by the U. S. Lake Survey. A series of triangles is established along the shore, the base of each triangle being a line between points on land, while the vertex of each triangle is a movable point on the water. In Fig. 98 the points A, B, C, and D are instrument stations on shore; the length of AB is known and the positions of C and D are desired. The point E represents some signal on a boat which can be seen from the stations A, B, and C on the shore. With instruments at A, B, and C the point E (the signal on the boat) is located by angles taken simultaneously, the observations being made according to some definite program; for example, the angles may be taken at the instant that the steam appears when the whistle is blown. The three angles so taken constitute a set, and several of these sets of angles are usually taken to secure the required accuracy. From

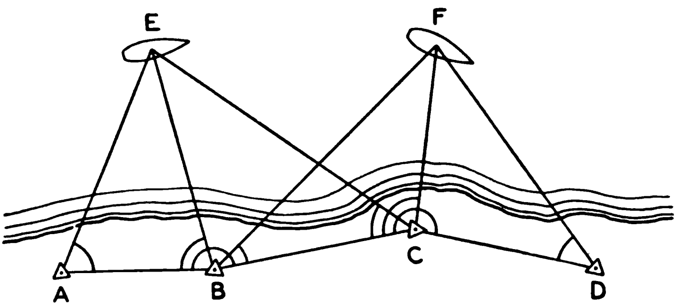

FIG. 98.

each set of angles the length of BE may be computed from that of AB, and BC may be computed from BE. In this way an independent value for the length of BC is obtained for each set of observations. In a similar manner CD may be computed from BC by means of the new position of the boat at F. If the work is carried on in winter, targets set on the ice may be used instead of a boat. Points can be located in this way with sufficient accuracy for secondary or tertiary triangulation.

233. **River and Lake Surveys in Winter.** — Some surveyors prefer to make river and lake surveys during the winter season, when the ice can be utilized for both triangulation and traverses. It is doubtful whether on the whole this is more economical than summer work, owing to the unfavorable weather conditions, prevalence of snow on banks, difficulty of securing permanent points, etc. It is true, however, that soundings may be made more rapidly and cheaply through the ice than in open water (Art. 276, p. 305).

234. **Ocean Shore Lines.** — The shore line of the ocean or any other body of water which is affected by tidal action is usually taken as the line of average high tide. When this line is to be located with precision (as when the shore line defines a boundary)

it is impossible to judge with sufficient accuracy the position of the high water line from the deposited refuse or weather marks on the rocks. It becomes necessary in such cases therefore to determine by tidal observations (see Vol. I, Art. 237, p. 211) the elevation of mean high water and to locate points on the shore at that elevation. A line connecting these points is the shore line at mean high water.

These points may be determined just as any single contour line is run out, by a leveling party working with, or just in advance of, a transit party, the latter locating the points which have been marked by the leveling party. Such work can also be readily carried on by one stadia party, and this is the method most commonly employed. In using this method the transitman levels the telescope, and the rodman moves up or down the bank until the proper rod-reading is obtained; the rod is then located by an azimuth and stadia distance, both of which are recorded in the note-book.

In order to avoid locating unnecessary points the rodman chooses only those points where the shore line changes its direction. The levels should be checked occasionally on bench marks previously established; large errors in level, however, may be detected by noticing the position of each located point with respect to the water surface or to the line of deposited refuse.

For maps on a scale of $\frac{1}{1000}$ or smaller it is usually possible to run in the shore line with sufficient accuracy by stadia, judging the position of the high water line from the appearance of the drift along the shore; where the slope of the shore is very flat, however, elevations may be required. It is always necessary to bear in mind the scale of the map to be produced, in order that useless refinements of measurements and observations may be avoided and a saving in both time and cost effected without sacrificing accuracy.

235. Contour Surveys of River Banks. — Surveys made for the purpose of investigating water resources frequently include the topography of the banks of a river, the fall of the water surface between successive points, etc., in addition to the location of the shore lines. In such cases the general methods of procedure are like those previously described for locating shore lines (Art. 231,

p. 268), but in addition a line of levels must be carried along as a basis for the determination of contours. Levels are taken on the water surface at the head and foot of rapids and wherever it is necessary to determine changes in slope of the water surface. The places where these level readings are taken should be located with reference to the transit line. The bank contours may be sketched in, using the transit line and the levels as a basis, or if greater accuracy is desired they may be located by stadia or by plane table. In a river survey it is often sufficient to determine contours on the bank for an interval of 5 or 10 feet, while water surface contours should be shown for differences of, say, one foot.

236. Contour Surveys of the Shores of Lakes and Ponds. — Surveys of the shores of lakes are chiefly required in projects involving a change of pond level, usually where it is desired to ascertain the effects of raising the water level to obtain a greater pondage. Such a survey will be essentially a shore line survey as described in Art. 231, p. 268, with enough additional work by plane table or stadia to give contours covering any probable change in pond level. Frequently the only contour that is run out in addition to the shore line is the proposed high water line. Usually the plane table suffices in this work, but occasionally a spur traverse must be run with a transit to cover flowage of a low area, this added information being plotted on the plane-table sheet from the transit notes. This work may be done roughly by sketching in the contours, using the water surface as a datum for elevations and taking distances from the shore line for locating the contours. In following out this method it may be necessary to set up a gauge somewhere on the pond, to be used in correcting for any fluctuations in the level of the water surface.

237. DRAINAGE AREAS AND STORAGE BASINS. — Surveys to determine drainage areas or the capacity of storage basins before filling with water are usually topographical surveys which, if made with the transit and tape, are carried on as has been described in Vol. I, Chapter X; or, if made by the stadia or plane-table methods, they are conducted as described in Chapter IV or V of this volume.

After the contours have been plotted the drainage areas can be readily sketched (see Vol. I, Art. 299, p. 273). Also the shore line of the proposed reservoir can be sketched on the topographic map as soon as the elevation of the spillway of the dam has been determined.

Usually it is found that existing maps of the country, e.g., the topographical maps of the U. S. Geological Survey or of some state survey, are sufficiently accurate for the purpose of studying the drainage area, but for construction purposes and for accurately determining the capacity of the reservoir it is necessary to run out the shore line and to take any measurements needed in computing the volume of water which the reservoir will hold.

The elevation of the spillway, having been decided upon, determines the height of the water surface in the reservoir, and the survey of the shore line of the reservoir then becomes merely the problem of determining the position of a single contour line on the ground. This can be rapidly and satisfactorily done by means of the stadia as explained in Art. 234, p. 270.

The capacity of the reservoir can be calculated from data taken in various forms. For instance, a topographical survey of the entire area of the reservoir may be made and the volume calculated by the End Area Method by using the areas of the successive contour planes as bases of vertical prismoids, as is described in Vol. I, Art. 375, p. 344. Or the reservoir may be cross-sectioned into horizontal squares or rectangles as described in Vol. I, Art. 227, p. 206, and the volume calculated by the Borrow Pit Method, as shown in Vol. I, Art. 373, p. 242. Still another method is to take vertical sections across the reservoir site and compute the volume by the End Area Method, using each vertical cross-section as a base of a horizontal prismoid. The areas of these vertical sections (or of the horizontal contour planes in the first case) may be readily determined after being plotted to scale, by the use of the planimeter which is described in Appendix B of Volume I; or this area may be determined approximately by placing over the drawing a piece of tracing linen which has been divided into small squares and counting the number of whole squares and estimating the fractional parts of squares contained in the area.

238. APPROXIMATE SURVEYS OF LAKES OR WIDE RIVERS. — An approximate survey of a lake or a wide river may be rapidly made with a small traverse table (Art. 203, p. 221) by measuring a base-line at the beginning of the survey and then carrying on the location of other points on the shores by intersections, virtually extending a system of graphical triangulation. Details of the shore line may be sketched in, and also the approximate position of contours near shore if desired, by using a hand level and by pacing the distances (Art. 307, p. 278, Vol. I). A good check on this graphical triangulation may be obtained by occasionally measuring another base-line with the tape where convenient; or, where it is impossible to measure a long line on shore, a base may be obtained indirectly as follows. A short base-line, say 100 to 300 feet long, is measured off in some suitable place along the shore, so as to form a right angle with a line to a third point across the pond, this distance across being the line which it is desired to use as a base; this distance may be computed after measuring with a sextant * the small angle at the third point subtended by the measured distance. If this angle is small it may be desirable to correct the computed distance for the position of the vertex. (See Art. 250.) Two men with a small sextant, traverse table, tape, and hand level can make such surveys very quickly and procure valuable results for preliminary investigations.

Natural objects along the shores, such as prominent trees, buildings, etc., can often be used for intersection points, so that few, if any, additional signals will be required. A scale of about 2000 feet to an inch is suitable for work of this nature.

THE SEXTANT.

239. GENERAL DESCRIPTION. — In addition to the instruments employed in land surveying, the *Sextant* (Fig. 99) is frequently used in hydrographic surveying. It is an instrument which, unlike the transit, is adapted to measuring angles in any plane. The frame of the instrument is in the form of a sector whose arc is 60 degrees, or the sixth part of a circle, from which its name is derived. It is constructed, however, in such a way that angles as

* If a small transit is used instead of a sextant, it may be found convenient to measure the base-lines across the water surface by stadia. By taking the mean of several readings the distance across a pond half a mile wide can be measured within about 5 feet.

large as 120 degrees can be measured. Owing to the fact that it can be used by an observer who is on a moving object, such as a boat, it is especially valuable for hydrographic work. It is employed not only for taking angles from a boat in locating soundings but is also in common use for making astronomical observations which are necessary in determining the latitude, longitude, and time at sea.

The frame *ABI* (Fig. 99) is usually of brass, on the under side of which is attached the wooden handle *D*. The ***index arm IE***

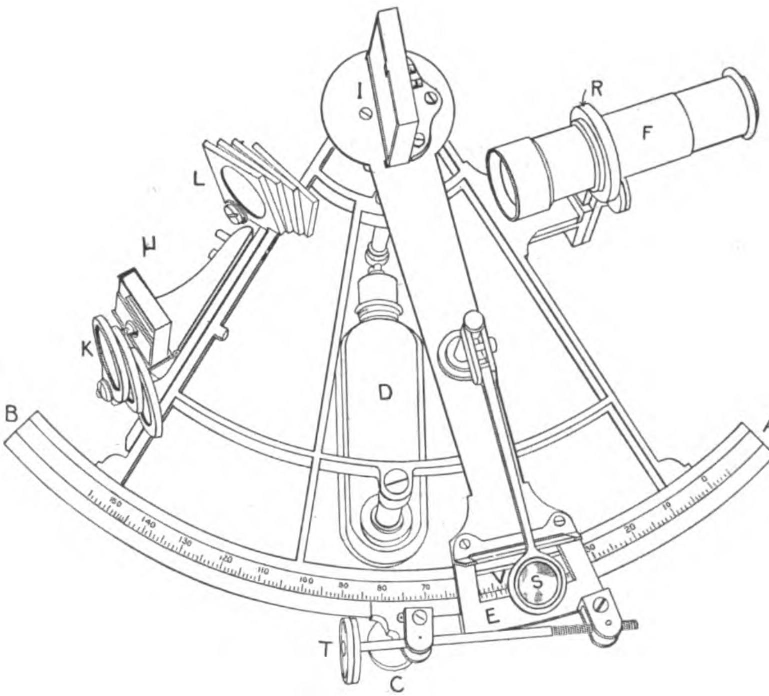

FIG. 99. THE SEXTANT.

is pivoted at *I*, the center of the arc *AB*, and this arm can be swung around *I* as a center so that the vernier *V* which is fastened to this arm can pass from *A* to *B* on the arc (or limb) and can be set at any position on the arc by means of the clamp *C* and tangent screw T. At I is a plane glass mirror called the ***index glass;*** it is attached rigidly to the index arm and perpendicular to the plane of the sextant, its reflecting surface being over the pivot about which the index arm revolves. Rigidly attached and perpendicular to the frame of the sextant is the plane ***horizon glass*** H, the upper half of which is transparent, while the lower half is a mirror; this glass is set so that its plane is parallel to the index glass when the vernier is set at 0°. The telescope is at F; it is screwed into the ring R, which has a motion perpendicular to the plane of the sextant so that the distance from the telescope to the plane of the arc can be varied. At S is a magnifying glass for reading the vernier. K and L are colored glasses which are hinged so that they can be swung around the pivot into the path of the rays of light to protect the eye of the observer in making observations on the sun. There are three short metal legs on the under side of the frame of the sextant on which the instrument may rest when not in use.

The limb AB is graduated into spaces which are really half-degrees, but on account of the construction of the instrument **each of these is marked as a whole degree** so that the scale has an extent of 120 degrees.* The graduations are so subdivided that the angles can be read in most instruments to 10 seconds; in some of the smaller instruments the vernier reads only to half-minutes.

In the ordinary sextant the arm IE is from 5 to 8 inches in length. A pocket sextant having an arm about 2 inches long is very convenient for reconnoissance surveys and for filling in the details of more accurate surveys.

Most sextants are provided with both a low power and a high power telescope (usually with an inverting eyepiece) and with a plain tube. Some instruments have telescopes with large objectives for night observations. For some kinds of work, however, such as locating soundings, the sextant is used without the telescope.

240. The Quadrant. — The *Quadrant* is an instrument similar in every respect to the sextant, except that its limb is an eighth

* The scale is usually extended several degrees beyond 120 in order that the vernier may be read when the angle is about 120 degrees. It also extends a degree or two beyond the 0° mark so that the index error may be determined.

(not a quarter) of a circumference, and it is therefore capable of measuring angles up to 90 degrees. To be consistent with the term "sextant" this instrument should have been called an "octant." The quadrant is very little used at the present time.

241. **PRINCIPLE OF THE SEXTANT.**—In Fig. 100 the index glass is at *I*, the horizon glass at *H*, and the eye at *O*. *AB* represents

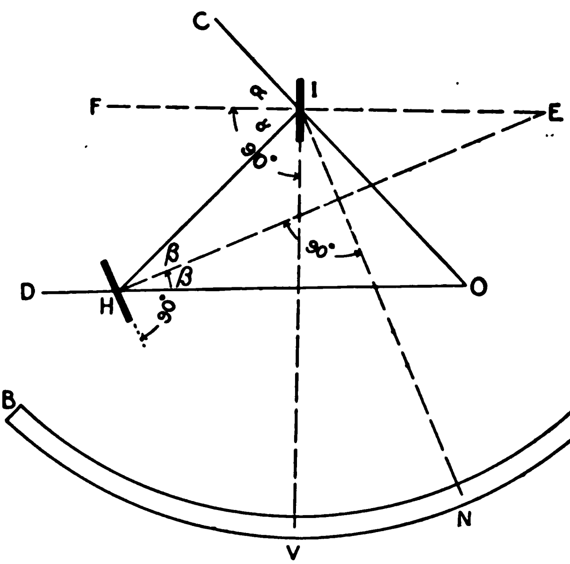

FIG. 100. PRINCIPLE OF THE SEXTANT.

the arc. Suppose the angle to be measured is between two flag-poles, *C* and *D*. A ray of light coming from *D* passes through the upper (transparent) part of the horizon glass *H* to the eye at *O*. A ray of light from *C* strikes the silvered index glass *I* and is reflected along *IH*; the lower part of the horizon glass (silvered portion) then reflects the ray along *HO*, so that both objects *D* and *C* can be seen at the same instant, the point *D* appearing through the transparent part of *H* to be in line with, or practically to coincide with, the point *C* as seen in the silvered portion of *H*. If the arm *VI* is moved the line *IH* will move until it is off the horizon glass; it is evident therefore that there is but one position of the arm VI where IH will intersect the horizon glass at a point in the line OD. Bringing the image of C (by moving the arm) so that it appears on the horizon glass in coincidence with the point D constitutes an observation with the sextant. When this is done the angle COD is read; it is represented by the arc NV as shown below.

The theory of the sextant is based upon the simple physical law that when a ray of light is reflected from a plane mirror the angles of incidence and reflection are equal. In Fig. 100 the two angles marked α are equal, and also the angles marked β.

In the triangle OIH

$$\text{angle } O = CIH - IHO = 2\,\alpha - 2\,\beta = 2\,(\alpha - \beta)$$

In the triangle EIH

$$\text{angle } E = FIH - IHE = \alpha - \beta$$

$$\therefore \text{Angle } O = 2 \text{ angle } E$$

If IN represents the position of the index arm when N is at 0°, then, since IN is parallel to the horizon glass, the angle E equals VIN. But VIN is the angle passed over by the vernier from the 0° mark, and as the half-degree spaces on the circle AB are marked as whole degrees the angle O is read directly from the arc AB.

242. ADJUSTMENTS OF THE SEXTANT. — To Make the Index Glass Perpendicular to the Plane of the Sextant. — Set the arm at a reading near the middle of the arc, then look into the index glass and observe if the image of the arc as seen in the index glass appears to form a continuous arc with that portion of the limb itself which is seen directly. This will be the case if the index glass is perpendicular to the plane of the sextant. If the arc appears to be bent at the edge of the mirror the index glass is not perpendicular to the plane of the sextant; if the image appears above the arc the mirror is leaning forward; if below the arc the mirror leans backward. It should be adjusted by loosening the screws on the back side at the base of the index glass, sliding a thin piece of paper between the frame of the index glass and the arm, and then tightening the screws. This process should be repeated until the adjustment is perfected.

243. To Make the Horizon Glass Perpendicular to the Plane of the Sextant. — With the sextant held in a horizontal plane, sight on a distant horizontal line such as the sea horizon or the roof of a building. If the reflected image of this horizontal line as seen in the mirrored portion of the horizon glass does not coincide with the image as seen through the clear portion, then the horizon glass should be tipped forward or backward by means of the adjusting screw on the back. Some instruments have no adjusting screw, in which case the holding screws are loosened and a slip of paper is pushed in under the frame of the horizon glass and the screws again tightened.

244. To Make the Horizon Glass Parallel to the Index Glass When the Vernier reads 0°. — After the two previous adjustments have been made set the vernier exactly on 0°, and while looking through the telescope and the transparent portion of the horizon glass at some well-defined point, such as a distant church steeple or a star, observe if the doubly reflected image of this same point in the lower portion of the horizon glass coincides with the direct image. If it does not, the horizon glass must be adjusted by means of adjusting screws at its base so that it will stand this test.

245. Index Correction. — In some instruments there is no provision for making the preceding adjustment of the horizon glass, in which case the two images of the same point are brought into exact coincidence and the vernier is then read, giving what is called the *index correction*, which is to be applied to every angle that is read. This cannot be accurately obtained unless the index and horizon glasses have both been made perpendicular to the plane of the sextant. Even though the horizon glass can be adjusted for index error this error is frequently so small that it is not worth while to actually make the adjustment, but the error is determined and applied as a correction whenever the required precision of the results demands it.

If it is necessary to obtain the index correction accurately it may be determined by means of an observation on the sun. The sun and its reflected image are brought so that they are just tangent to each other, one appearing directly beside the other, and the vernier is read. The index arm is then moved so that one image moves past the other and becomes tangent to it on the other side, and the vernier is again read. One of these readings will fall on the arc while the other will fall beyond the 0° point. Half the difference of the two vernier readings is the **index correction,** which is **plus** if the reading beyond 0° is the greater. It will be found that the contacts can be judged much more accurately if different colored shades are used for the two images, and that the contacts can be made more exact if one of the telescopes is used.

Care must be taken to observe whether this index correction is **plus** or **minus** and to apply it correctly to the measured angles.

246. **To Make the Line of Sight of the Telescope Parallel to the Plane of the Arc.** — The telescope usually contains two horizontal and two vertical hairs forming a small square; the images of the objects between which an angle is to be measured are brought into the center of this square.

Set the sextant on a table, sight through the telescope, and mark a point 20 or 30 feet away which shall appear in the center of the square formed by the cross-hairs. Then take two small pieces of wood, which are practically equal in height to the height of the center of the telescope above the plane of the sextant, and place one of these pieces of wood on top of the arc near the zero end and the other piece near the other end of the arc. Sight across the tops of these wooden sights toward the mark already made when looking through the telescope. The line of sight across the tops of the pieces of wood should coincide nearly with the mark. A difference of half an inch at a distance of 20 feet will make an error of only about a second in the angle measured with the sextant, so that great precision in this adjustment is unnecessary. Instead of pieces of wood two pencils may be used in the same manner, allowance being made, at the point marked, for the distance that the tops of the pencils are below the center of the telescope. When the error in this adjustment is large enough to require correction it is done by means of the screws in the collar holding the telescope tube. This adjustment is entirely independent of any of the other adjustments.

247. One of the weak points in the design of the sextant is the fact that there is no method of eliminating any eccentricity of

the circle or irregular graduations of the arc. The only way such errors can be discovered is by measuring with the sextant angles whose true values are known.

248. Reflecting Circle. — The reflecting circle is an instrument similar to a sextant but having a full circle. With such an instrument errors due to eccentricity can be eliminated by reading both verniers as with a transit; but errors due to irregular graduations cannot be eliminated, because with this instrument every angle must be measured from the zero of the arc, and the principle of repetition cannot be applied.

249. USE OF THE SEXTANT. — To measure an angle with the sextant, hold the instrument by its handle in the right hand and turn it so that the plane of the sextant coincides with the plane through the two objects to be observed, with the telescope on the upper side of the sextant if the angle is horizontal, or on the left-hand side in the case of a vertical angle. Without changing the plane of the sextant twist it in the hand so as to turn the telescope toward the left-hand object and observe it through the upper (transparent) portion of the horizon glass. Then, holding the instrument as steady as possible, turn the index arm with the left hand until the other object appears in the silvered portion of the horizon glass opposite the first point. Bring the second point exactly opposite the first one by means of the clamp and tangent screw of the index arm. The coincidence of the images should be tested by twisting the instrument a little so as to make the reflected image move back and forth across the direct image. Read the vernier and apply the index correction as explained in Art. 245, p. 279.

Under ordinary conditions the above method of using the sextant is the most convenient. It sometimes happens, however, that the right-hand object is so faint that its reflected image cannot be readily seen, in which case the instrument is turned upside down and held in the left hand, a sight is taken directly at the right-hand object, and the left-hand object is seen as a reflected image; the index arm is operated in this case with the right hand.

Occasionally the angle to be measured is over 120 degrees; in this case the angle is measured in two parts, from one of the objects to some intermediate point **in the same plane,** and then from this intermediate point to the second object.

In making astronomical observations it is frequently necessary to measure with a sextant the vertical angle between the sea horizon and the sun, or some star. When this is done a correction must be applied for the dip of the True Horizon below the Apparent Horizon, as explained in Art. 80, p. 73. In making solar observations the sun's image is dropped down by moving the index arm until the sun's lower (or upper) limb is tangent to the horizon, and the vernier is read. This angle is then corrected for refraction (Table VII), dip, and the semi-diameter of the sun.*

250. PRECAUTIONS IN THE USE OF THE SEXTANT. — It should be borne in mind that the angle measured with a sextant is **not** the horizontal angle between the two points, as is the case when measured with a transit, but is **an angle lying in the plane defined by the two objects and the eye of the observer.**

Furthermore the **vertex of this angle** (O, Fig. 100) **is not a fixed point.** It is evident from the figure, since the positions of I and H do not change, that as the angle diminishes in size the point O must move farther away from the instrument, and that for very small angles it may be at a considerable distance back of the observer. For angles taken in astronomical observations where the objects sighted are at a great distance the assumption that O is at the same place as the observer does not introduce any appreciable error, but where the instrument is used to locate details such as soundings it is not good practice to measure between points that will give small angles, not only because the vertex of the angle is not at the observer, but also because small angles give poor intersections. The sextant is therefore not used in work where the position of the vertex of the angle must be known with any great degree of accuracy. With a sextant of the ordinary size the vertex of a 20° angle will be about 5 inches back of the index glass; with a 5° angle it is about 2 feet; and with a 1° angle it is about 10 feet. These approximate values will illustrate the importance of keeping this point constantly in mind in all work with this instrument.

* See American Ephemeris and Nautical Almanac.

251. Use of the Artificial Horizon. — In astronomical observations the altitude of a celestial body is often desired. When the sea horizon is not visible an artificial horizon may be substituted, if the observation is to be made on land. A basin of mercury is usually employed for this purpose. The angle measured is that between the object and its image reflected from the artificial horizon, and it is obtained by observing through the transparent portion of the horizon glass the image reflected from the mercury surface and bringing the doubly reflected image of the object into coincidence with the image seen in the artificial horizon. Half the observed angle is the required altitude. In Fig. 101 the eye is at O, the artificial horizon at M, and the distant

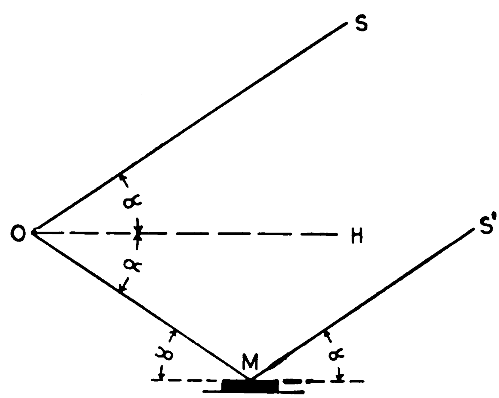

Fig. 101. Artificial Horizon.

object at S; OH is a horizontal line. The lines SO and $S'M$ are assumed to be parallel, which will be practically true when the object observed is a celestial body. It is evident that all the angles marked α in this figure are equal. The angle measured is $SOM = 2\,\alpha$; the required angle is $SOH = \alpha$.

In measuring altitudes of near objects by means of the artificial horizon it will be necessary to hold the sextant as close to the mercury surface as possible, so that the lines $S'M$ and SO (Fig. 101) will be practically parallel. It must also be remembered that the surface of the mercury determines a plane at the elevation of the artificial horizon and not at sea-level.

SUBAQUEOUS SURVEYS.

252. **SUBMERGED AREAS.** — The determination of the topography of the bottom of a lake, harbor, or other body of water is one of the common problems in hydrographic surveying. In connection with such surveys the character of the material composing the bottom is often desired. Surveys of this kind are made for a variety of purposes, such as to prepare charts for navigation, to determine where material shall be dredged and where such dredged material may be dumped, or to measure the quantity removed. They are also made for the purpose of discovering what changes are taking place in the bed of a river, canal, or harbor due to dynamic agencies, or to acquire data for projecting wharves, sea-walls, breakwaters, levees, dikes, etc.

This work is usually done by first establishing certain points on shore (by triangulation or traverse) to which the hydrographic survey may be referred, and then measuring, usually from a boat, the depth of the water at various points and determining the positions of these points. The measurements of depth are called *soundings*. Since the subaqueous surface is not visible it is evident that for a given degree of accuracy a great many more points must be located to obtain the shape of the surface than would be necessary in an ordinary topographical survey of equal area.

The points on shore to which the hydrographic survey is referred should be so chosen that they will be in clear view from the water surface. Such prominent objects as church spires, windmills, lighthouses, flag-poles, and the like are serviceable, provided they are near enough to the shore to come within the boundaries of the sheet when plotted on the map.* The shore line and some of the adjacent topography is usually desired; this is obtained as explained in Arts. 231, 235–6, pp. 268, 271–2.

253. **INSTRUMENTS USED.** — Besides the ordinary surveyor's transit, tape, and lining-poles a sounding-pole or lead-line and a boat with its necessary equipment will be required. In many

* Sometimes it is necessary to locate important points which when plotted may fall beyond the limits of the map. In this case the point may be plotted on a piece of paper fastened to the drawing table, the plotted position of these points being used only while the drawing is being made.

sounding operations the sextant, signals, buoys, and gauges for recording the height of the water surface are used.

254. **The Boat.** — It often happens that the surveyor has little or no choice regarding the type of boat, but is obliged to make use of whatever he can hire in the vicinity. Where any choice is possible a round-bottomed boat, like a dinghy or a Whitehall boat should be selected, as it is less affected by wind and current, and can be more easily kept on range than one with a flat bottom. Flat-bottomed boats, however, are more serviceable in shoal water and where the water is not rough. Such a boat with a wide beam and high sides will be found to be less "cranky" than a round-bottomed boat and particularly convenient when landing on flat beaches. The leadsman stands in the bow while taking the soundings, and for this reason the boat should be fairly stiff. Sometimes a platform is built in the bow of the boat on which the leadsman stands.

A power boat, provided it can be run at a slow speed, is better in many respects than a rowboat. It can be kept exactly on a range with little difficulty even if the wind is blowing and the current strong. Its speed is more nearly uniform, and it is much stiffer and steadier. A gasolene power boat can be used to good advantage where the water is free from grass and the ranges are long. The speed can be regulated by a drag consisting of a 4″ × 8″ timber 8 to 10 feet long (or other size to suit the power of the boat) with a rope attached to each end. One rope is fastened to a cleat on the boat and when running free the other rope is loosened so that the drag tows freely through the water. When it is desired to sound, the loose rope is pulled in until the drag takes a position crosswise to the direction of the boat. By varying the angle that the timber makes with the direction of the boat the speed can be regulated to suit the conditions. If a power boat is used a skiff will also be needed for landing along shore unless there are wharves in the vicinity.

255. **Sounding-Pole and Lead-Line.** — *Sounding-poles* are made similar to an ordinary self-reading leveling rod. They are usually from 12 to 20 feet long, 1 to 2 inches square in section, and with an iron or lead shoe of sufficient area to prevent the pole from sinking into the mud or sand. The shoe is sometimes

provided with a cup-shaped cavity in the bottom which enables samples of material to be collected. This cavity should be smeared with some such material as tallow or soap so that the soil will adhere to the metal shoe. The pole is graduated to feet and tenths, and it will be found convenient in reading the sounding to have these graduations on two opposite faces. If, however, the scale is painted on but one side of the rod the leadsman can, by using proper care, handle the rod so that all the wear will come on the back side.

256. The *Lead-line* consists of a long chain or line at the end of which is attached a lead weight. Lines of hemp or cotton are in common use, but on account of changes in their length due to shrinking and stretching many surveyors prefer to use a chain. Some prefer a steel chain with brazed links, which is very light and strong. The principal use of a line is where the water is deep. If a line is used it should be stretched before it is graduated, so that it will not change in length appreciably while in use. One method of stretching is to wind the line tightly around a circular post and after both ends are securely fastened, to wet it thoroughly. It will shrink and tighten around the post, but when it dries out it will be slack. The slack is taken up by rewinding and it is wet again. This operation should be repeated until very little stretching occurs. The line is then thoroughly wet again and the feet are marked by cloth or leather tags, the zero of the graduations being the bottom of the lead weight. Every fifth foot-mark should be indicated by a tag of different color from the other foot-marks, and every tenth foot-mark by a tag so marked that the leadsman can tell at a glance which ten-foot mark it is. A leather tag with points, similar to the points on the tag of a surveyor's chain, forms a good marker for the ten-foot divisions. If a line is used it must be tested frequently by comparing it with a tape, and, if necessary, the foot-marks changed from time to time to insure accurate results. But when the variation in length is small the error in the lead-line may be recorded in the notes and applied to the soundings when they are worked up in the office. The line should be soaked in water for an hour or two before its length is tested or before the soundings are taken.

Many surveyors consider that for ordinary purposes a brass sash-chain is the most satisfactory kind of lead-line. Chains of this type are found to cause very little trouble by kinking. The chief advantage of the chain is that its length is practically constant so that when once marked it may be used a long time without requiring corrections. In marking it is best to avoid metal tags or wire fasteners, as they are likely to injure the leadsman's hands. For this reason leather or cloth markings are preferable, and although they may wear off quickly they can be easily replaced. Narrow strips of cloth of various colors are sometimes used for tags, a simple system of distinguishing marks being readily improvised. Where chains are used it is necessary to test them occasionally and to readjust the tags, because the wear which takes place in the links sometimes causes an appreciable lengthening if the chain is 25 feet or more in length.

The weights used with a lead-line vary from 3 to 20 pounds, depending upon the depth of water and strength of current. Where there is not much current a 6 to 10-pound weight will suffice for depths up to about 40 feet. A good form of lead is one with a circular or rectangular cross-section, about three times as long as its greatest diameter, and tapering slightly from the bottom upward; in the top is fastened an eye to which the line is attached (see Fig. 102). A cavity may be made in the bottom of the lead to collect samples of the material of the submerged surface, as explained above for the sounding-pole. Other forms of leads are made with various devices for collecting samples.

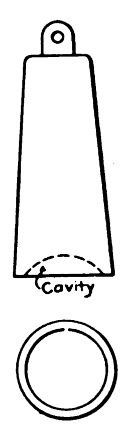

Fig. 102.

257. Signals and Buoys. — Shore signals, if used to mark the ranges on which soundings are to be taken, may be either common $\frac{7}{8}'' \times 2''$ scantling or poles with iron shoes driven into the ground. On the poles are fastened flags which are visible for a long distance. The color of the flags used will depend upon the background and the distance they must be seen. A white flag is clearly visible against any dark background; a red flag is clear against anything except a medium dark background;

and a black flag is the best when the background is very light. The visibility of a flag may be affected by many different conditions, the manner in which the sunlight strikes it being the most important. A shore signal may sometimes consist of a $2'' \times 4''$ or $4'' \times 4''$ mast properly braced at the bottom, with a flag made of cloth nailed to two short horizontal pieces of lath. The ordinary tripod signal employed in triangulation work may also be used (see Arts. 10–12, pp. 10–17). Where there are many signals, such as those used to mark range lines, they are sometimes distinguished by Roman numerals made of short strips of wood which are fastened crosswise to the mast. For short ranges where a flag of any color will be visible a system consisting of a number of colors is often used to identify the signals where several different ranges are in use at one time.

Conditions may exist where one of the signals on each range will have to be in the water, and in such cases if the water is deep it may be necessary to use floats, properly moored and provided with a mast to which the flag is attached. When floats are used allowance will have to be made for changes in position due to the action of currents, wind, and tide. Where there is no tidal action a buoy may be anchored by three guys so that it will remain in one position. Sometimes it is found to be so difficult to set shore ranges that it is advisable to put all the ranges in the water. Ordinary scantling $\frac{7}{8}'' \times 2''$ in section may be used for this purpose. These are cut 3 feet longer than the depth of the water; a weight is attached to one end of the stick and a flag to the other, and then it is placed in the proper position in the water, the weight resting on the bottom. Unless the water is quiet it may be necessary to steady these sticks by guys.

258. The Tide Gauge. — The tide gauge usually consists of a board painted white, with a scale of feet and tenths painted on it in black and secured in an upright position in the water. The elevation of the zero point of the gauge should be connected by a line of levels with some bench mark on shore. Care should be taken in setting the gauge to have it extend low enough for extreme low tides and high enough for readings at very high tides.

Gauge readings are taken at regular intervals of time during the work so that a sounding whose time has been taken may be referred to mean low water or to some other desirable datum plane. (See Fig. 105, p. 295, for form of notes.)

Where a tide gauge must be used on an open coast subject to the action of waves it is necessary to use a gauge that is unaffected by this action. A simple form (Fig. 103) consists of a long wooden box in which an empty bottle, having a graduated wooden rod fastened tightly in its mouth, floats up and down with the tide. The box has a wooden bottom and top and has several

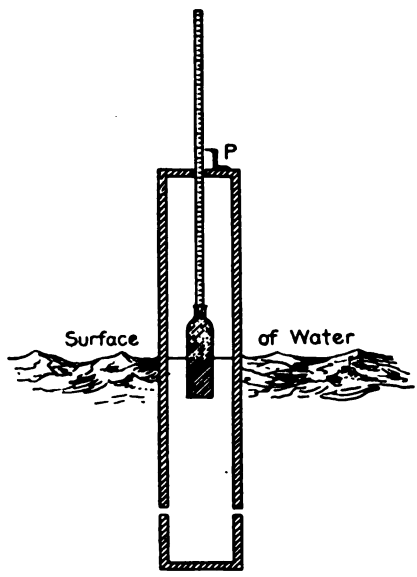

FIG. 103. TIDE GAUGE FOR ROUGH WATER.

holes bored low down in its sides to admit the water. The graduated rod moves freely up and down through a hole in the top of the box as the bottle rises or falls, and the scale is read at the pointer *P*. The graduations on the rod should read from top to bottom. The elevation of *P* must of course be referred to permanent shore points, and the box must be fastened to some stationary object such as a wharf or a pile.

Another form of tide gauge for use in rough water is illustrated in Vol. I, Art. 238, p. 212.

259. River and Lake Gauges. — In taking soundings or in measuring the discharge of rivers it is necessary to erect gauges at convenient places, e.g., on bridge piers or abutments, and to have them read at regular intervals. When the gauge is fastened to a bridge pier it is usually found that the eddying of the water at the pier makes it difficult to obtain an accurate reading of the gauge. This effect can be largely done away with by attaching, lengthwise to the staff, wing pieces making an angle of about 30 degrees with the face of the staff and flaring toward the back of the staff. These wing pieces prevent the eddies and make the water surface in front of the gauge smooth.

Sometimes it is impossible to find a good place for a vertical staff, and in such cases an inclined staff may be fastened to the

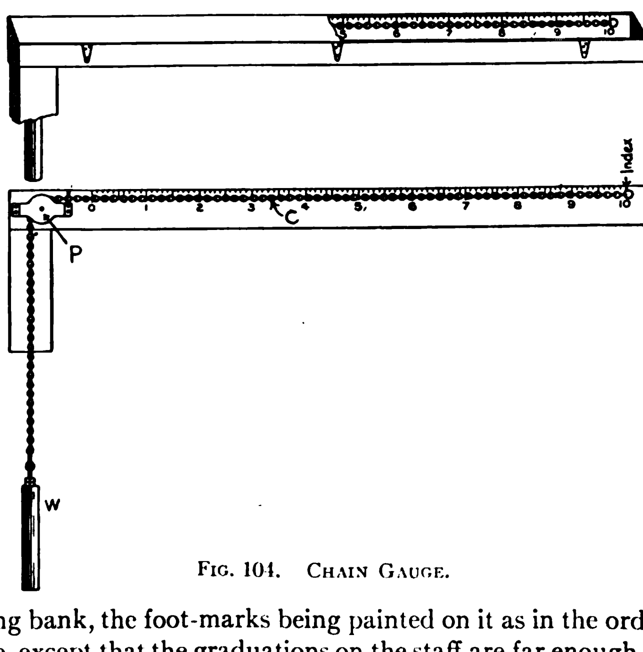

FIG. 104. CHAIN GAUGE.

sloping bank, the foot-marks being painted on it as in the ordinary gauge, except that the graduations on the staff are far enough apart so that the **vertical** distance between them is the same as on a vertical staff.

A good type of gauge to use where drift logs or ice might destroy a staff gauge is the chain gauge illustrated in Fig. 104. This gauge is used extensively by the U. S. Geological Survey. It consists of a board 10 to 14 feet long fastened in a horizontal position so that one end is over the water. At this end is attached a pulley *P* over which passes a chain *C* having a 12-pound weight *W* at its lower end. The ring at the other end of the chain is hung over a small hook at the right-hand end of the board when the gauge is not in use. When in use this ring is taken from the hook and the chain is allowed to pass over the pulley until the bottom of the weight just touches the water surface. The position of the index point of the chain is then read on the scale which is painted on the horizontal board. As the length of the chain may change somewhat it should be compared occasionally with a steel tape and corrected by means of an adjusting screw in the top of the weight. The gauge readings should also be referred to some permanent bench mark near by and tested by leveling occasionally to detect any settling of the gauge as a whole. The elevation of the bottom of the weight when the gauge reads zero should be used for connecting the gauge readings with the datum of the bench mark.

As these river gauges are located at intervals along the shore their zero points are not at the same elevation. They are connected however by leveling, so that the water level at the various gauges may be referred to the same datum as the map of the surrounding country. The slope of the river may then be determined from simultaneous gauge readings at different points along the river. On account of the slope of a river it is impracticable to refer soundings taken in the river to a datum **plane.** It is customary, therefore, to refer them to the surface of the water when the river is at a certain stage, — high water, mean water, or low water stage, the soundings being the depth of the river bottom below the water surface at this particular stage. As the soundings may have to be taken when the river is not at the assumed stage it will be necessary before taking them to first determine from a series of observations (taken daily or oftener at each gauging station) the various gauge readings corresponding to the assumed stage of the river; for example, at the upper gauge a reading of 2.3 may be found to represent the mean water stage, while 1.7 is the reading for mean water at the next lower gauge, 2.1 at the next staff, and so on.

For lake surveys the datum to which the soundings are referred is often the lowest recorded stage of the water. In this case the frequency with which the gauge should be read will depend upon the various conditions which influence the level of the water surface.

260. **ORGANIZATION OF SOUNDING PARTY.** — The organization of the sounding party will depend upon the method used in locating the soundings. In any event there will be in the boat the **chief of party**, the **recorder**, the **leadsman**, the **oarsman**, and possibly a **signalman**. This comprises the entire complement of the boat party provided the soundings are located by angles taken from shore, in which event the transitman and assistants (if needed) will remain on land. If, however, the angles are to be taken from the boat the instrumentmen who will use the sextants will also be in the boat. Where the work is to be done in tidal water a **gauge reader** may also be needed.

261. The **recorder** writes in his note-book the depths as they are called off by the leadsman and also the times when the soundings are made. He also notifies the leadsman by calling out "Sound" about 5 seconds before each sounding is desired. The leadsman takes the sounding as quickly as possible, usually within two or three seconds of the desired time. In the hydrographic work of the Corps of Engineers, U. S. A., where nearly all of the soundings are located by two angles taken with transits on shore, soundings are usually taken at 15, 20, or 30-second intervals, and a location made each minute by the instrumentmen at the instant the signal is given by the recorder or signalman in the boat. The chief of party usually acts as signalman and directs the work in the boat, and sees that the boat is kept on the ranges (if any are used) and that the boat is so propelled as to properly cover the area to be sounded. The signal is given by holding up a flag for about 10 seconds and dropping it suddenly the instant the sounding is taken, at which moment the transitmen on shore take angles to the leadsman or to his hand if visible. In the work of the U. S. Engineers white, red, and sometimes black flags are used for signalling. Both the recorder's and the instrumentmen's notes should show the colors of the signal as well as the time for each located sounding, thus giving

two means of identifying the angles and the corresponding soundings. Usually from one to five signals are given in succession with a white flag, and then from one to five with a red one, no particular attention being given to the order of color changes. This double check is of particular value where the lines of soundings are long. In tidal waters the time is required also as a means of reducing the soundings to low water or to whatever other datum is used.

Where the soundings are taken with a view to obtaining every slight change in the slope of the bottom, instead of taking the soundings at a given interval of time, it is desirable in order to catch any change in slope to take them as frequently as the leadsman can conveniently handle the pole. In this method the boat is usually rowed on a range and the instrumentmen on shore "cut in" only those soundings that are designated by the signalman in the boat. The time in this case is recorded to the nearest second. As a rule every sixth or eighth sounding is located.

262. The **instrumentmen,** if on shore, should set their watches to agree with that of the recorder each day before the sounding operations begin and compare them at the close of the work. Each man reads the azimuth angle from some known point on shore to the sounding pole or to the leadsman at the instant the signal is given from the boat, and records the angle, the time the angle was taken, and the color of the signal given (if the color system is used). He should occasionally check his instrument setting by sighting again on the known point. The transitmen and recorder should compare notes at the close of each day to discover any discrepancies in the recorded times and to number the corresponding simultaneous observations for convenience in the plotting. If the angles are measured from the boat with the sextant they are called off to the recorder who enters them in his note-book.

263. The **leadsman** stands in the bow of the boat, and he inclines the foot of the pole forward at such an angle (depending upon the speed of the boat) that when he plunges it into the water it will reach the bottom just opposite him and will be vertical when he reads it. The leadsman calls out the reading in feet and tenths to the recorder and gives the character of the bottom if desired. In deep water, where the lead-line is used, he will cast the lead ahead a proper distance, letting the line pay out freely at first but tightening his hold on the line so that when the lead reaches the bottom the line is taut and is practically vertical. The feet are read by observing the tag on the line which is just above the water surface, the tenths of a foot being estimated. In withdrawing the lead-line he coils it in such a manner that it will pay out without becoming tangled when again cast. Evidently the leadsman's duty requires the use of good judgment and not a little skill and labor.

264. Where the scope and method of the work require it, a **signalman** is employed to wave a flag or other signal when directed to do so by the **recorder.** Just before the sounding is taken the recorder calls out "Ready"; the signalman then holds the flag vertical so that it may be seen by the instrumentmen on shore. At the instant the sounding is taken the flag is dropped and the transitmen obtain the angles to the sounding pole. The recorder himself sometimes gives this signal when a special man for this purpose is not employed.

The **boat crew** will comprise one or more experienced oarsmen and sometimes an extra man to steer the boat and keep it on the range. For ordinary work in moderately calm water two oarsmen are sufficient.

If signals are erected on shore for ranges the **signalman** or **assistant** shifts them from one range to the next as required. Usually several sets of signals are erected at one time to mark a number of ranges, and the changing of signals is done by the boat party. Occasionally no ranges are erected, but instead the signalman directs the boat on approximately parallel courses, the spacing of the lines being paced off along the shore, and the parallel directions being estimated by eye, guided somewhat by the trend of the shores or by the directions of distant objects.

265. In tidal waters it is necessary to carry on a series of gauge readings while the sounding operations are going on. The **gauge reader's** duty is to read the gauge at regular intervals, recording the time of the gauge reading so that the soundings can be reduced to a given level. Usually readings at 15-minute intervals

will be sufficient, unless the rise and fall is more than a foot per hour. The gauge height at any intermediate time can readily be obtained by interpolation. Where it can be done it is best to employ a man to read the tide gauge and to record the readings at 5-minute intervals. A convenient form for these notes is

POINT MERIDETH HARBOR. — TIDE GAUGE READINGS.

David Ray, Observer, Sept. 11, 1906.

Time	Gauge Reading	Correct Tide	Remarks
8.00	2.8	2.2	Fresh N. W. wind.
.15	2.7	2.1	
.30	2.6	2.0	
.45	2.4	1.8	
9.00	2.1	1.5	
.15	1.8	1.2	
.30	1.5	0.9	
.45	1.2	0.6	Gauge set on N. E. Pile of Thompson's Wharf. Bottom of fender cap of wharf reads 12.85 on gauge. Referred to B.M. No. 1 on plan of 1903, correction for gauge −0.6.
10.00	1.1	0.5	
.15	0.9	0.3	
.30	0.7	0.1	
.45	0.6	0.0	
11.00	0.4	−0.2	
.15	0.3	−0.3	
.30	0.3	−0.3	
.45	0.3	−0.3	

FIG. 105. TIDE GAUGE READER'S NOTES.

shown in Fig. 105. The direction and force of the wind should also be recorded, especially when the tide gauge is not located close to the place where the soundings are being taken. It is well to locate the tide gauge as near to the sounding operations as convenient, and it is extremely important that the gauge should be located in the same tidal basin as the soundings. In the "Remarks" in the above notes it will be seen that the gauge reading of the bottom of the fender cap is recorded. By referring to such a record the position of the gauge can be readily checked at any time should there be any suspicion that it had been knocked out of place and not replaced at exactly the point where it was originally fastened.

Under some conditions the tide gauge can be read by one of the boat's crew by means of a field-glass or by the shore assistant or transitmen if the gauge is near enough to any of them.

266. METHODS OF LOCATING SOUNDINGS. — There are six general methods of locating soundings, as follows.

(1) The boat is rowed on a range at a uniform rate of speed and the soundings are located by time intervals.

(2) The boat is rowed on a range line and the positions of the soundings are "cut in" by a transit angle taken on shore or by an angle taken with a sextant from the boat.

(3) The boat may or may not be rowed on any definite range and its position is located by angles taken simultaneously by two transits on shore or by angles taken simultaneously to shore points from the boat by means of two sextants.

(4) The positions of the soundings are located by the stadia method.

(5) The positions of the soundings are defined by the intersection of fixed ranges.

(6) A wire or line is stretched across a stream from shore to shore and soundings are taken at different points along this wire and located by measured distances from one end of it.

267. Locating Soundings by Time Interval. — In this method each range is determined by two signals on shore or by a signal on shore and a buoy near the shore. In the latter case the boat is rowed from the shore toward the buoy, the oarsmen being careful to take a steady stroke so that when the boat passes the buoy it is running at a uniform speed, which is kept as uniform as possible along the entire range. The time (minutes and seconds) when the boat passed the buoy is recorded and the soundings are taken and recorded at regular intervals for the entire length of the range, together with the time each sounding was taken. In plotting the soundings the speed of the boat is assumed to be uniform, so that the distances along the range may be made proportional to the time consumed in traveling that distance. Evidently this method is only applicable when the soundings are taken between points whose distances apart are known, such as between the two banks of a river or from one shore to a buoy the location of which has been fixed by some other method. The short distance between the shore and the first buoy can be sounded out by estimating the distances along the range.

This method of locating soundings may be used where only rough results are required. It is particularly applicable to sounding over short distances, as on ranges across a river where the current is not strong or in the still waters of a pond.

268. Locating a Sounding by a Range and an Angle from Shore. — In still water where there is no difficulty in keeping the boat in any desired position soundings may be conveniently located by keeping the boat on a range line marked by two objects on shore, such as range poles, and then "cutting in" the position of the leadsman by means of a transit angle taken from shore at the instant the sounding is made (see Fig. 106). Where the

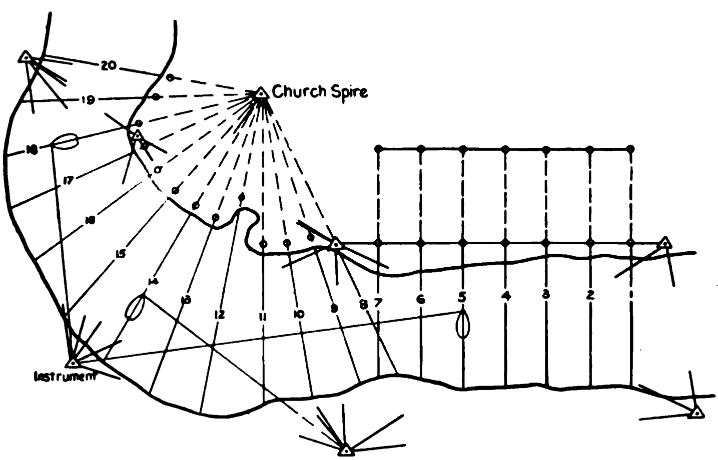

FIG. 106. LOCATING SOUNDINGS BY RANGE LINE AND ANGLE FROM SHORE.

ranges are marked by poles one of these may be set in the water or a float may be anchored and used to define one end of the line if it is not convenient to place both poles on land.

Evidently the positions of all of the shore signals defining these ranges must be accurately known so that they can be plotted on the plan before the positions of the soundings are plotted. These signals are located by triangulation, by direct tape measurements, or by stadia, the last being a very common method. It will be noticed in Fig. 106 that the ranges in the right-hand series are parallel, necessitating the changing of two shore signals for each range, while on the left-hand end is a fan-shaped system in which only the one signal near shore has to be changed. A fan-shaped system will therefore be preferable, except where the range lines diverge so rapidly as to leave too great an interval between them near the further shore. This may be avoided if the back range point can be chosen at a long distance from the shore. In any case the range lines must be so chosen that they will afford a base long enough to be accurately projected across the water and so as to cover the entire area as closely as is desired. When the fan-shaped ranges are used it is advisable on account of their divergence at the further shore to take a series of soundings on other ranges running crosswise. This is particularly desirable where the radial lines are very far apart. In fact it is always a good plan to run at least a few lines crosswise to the first series, as it helps to verify the work and also to fill in the gaps.

269. Fieldwork and Notes. — In this method of locating soundings the transit point should be so chosen that the intersection of the lines of sight will cut the range lines as nearly at right angles as possible. The vernier of the plate is set at 0°, or if preferred at the proper magnetic or true azimuth, and the telescope sighted at some signal or object on shore whose location is known. The telescope is then turned in the direction of the boat, ready for measuring angles. The transitman keeps the vertical cross-hair of his instrument on the sounding-chain, as nearly as he can, and follows it as the boat moves along the range line. At a signal from the boat the transitman clamps the instrument (sometimes leaves it loose) as nearly as possible on the angle to the sounding line, and reads and records both the angle and the time, as shown in the notes in Fig. 107. If colored flags are used by the signalman in the boat, the color of the flag shown is also recorded. At the end of the day the transitman's notes and the notes of the soundings are compared, and the observations are numbered to correspond. Occasionally the transitman will fail to measure an angle when a certain sounding is taken. This should be discovered when the notes are numbered and compared, since both parties have recorded the times of their observations.

POINT MERIDETH. — SOUNDING LOCATIONS.

K. and E. ⊼, at S.B. "A." L. Hayes, Sept. 11, 1906.

Time	Flag	Angles		No.	
0° on S.B. "B"					
h m s					
8-15-20 Begin	B	32°	$15'$	1	
....	W	34	00	2	
....	W	35	45	3	
....	B	37	05	4	
....	W	38	45	5	
....	W	40	20	6	
....	W	?		7	Yacht in way.
....	R	43	30	8	
....	W	44	40	9	
....	W	46	05	10	
....	W	47	25	11	
....	W	48	35	12	
8-27-10 End	R & W	49	40	13	
8-35-00 Begin	W	50	50	14	
....	W	53	00	15	
....	B	54	05	16	

FIG. 107. TRANSITMAN'S NOTES.

On account of the rapidity with which the angles must be read the transitman will usually leave the upper plate of the instrument unclamped and simply set by hand. A good transitman can easily take two angles per minute and record them, together with the time; with an assistant to record he can take five or six angles per minute if necessary. The minutes in the angle are sometimes read by estimation instead of by the vernier, as it is usually sufficiently accurate for this kind of work if the angles are correct to the nearest 5 minutes.

In the notes shown in Fig. 108 the soundings were not taken at any regular time interval, as is done by some surveyors. In this method it is not usually practicable to record the time of each angle, the signals being given too rapidly, but it is sufficient to take the time at the beginning and at the end of each short range and at a few intermediate points on long ranges, leaving to the recorder in the boat the duty of noting the time when each signal was given. Where the angles are taken on a particular time interval, as illustrated in Fig. 109, p. 302, the time may be recorded for each angle. After the soundings on two or three range lines have been taken the transitman should check his work by sighting again on the signal and seeing that the vernier reads the azimuth originally set. The recorder and the transitman should check up their notes each half-day if possible, but in any case at the end of each day.

It will be observed in the following notes (Fig. 108) that the recorder in the boat has recorded the depth of each sounding and also the times of those soundings which are located.

POINT MERIDETH HARBOR. — SOUNDINGS.

Sept. 11, 1906.

H. G. Wells, in charge. D. Ray, gauge.
John Smith, recorder. J. Sidley, leadsman.
L. Hayes, at △ S. B. "A."

Time	Flag	Sounding	Tide	Reduced Sounding	No.	Remarks
Line No. 1. Running South.						Fresh N. W. wind.
h m s 8-15-20 Begin	B	3.0	2.1	0.9	1	125′ ± from shore.
....	......	3.5	....	1.4	......	
....	......	3.7	....	1.6	......	
....	......	3.7	....	1.6	......	
....	W	4.3	....	2.2	2	
....	......	5.7	....	3.6	......	
....	......	5.7	....	3.6	......	
....	......	7.9	....	5.8	......	
8-17-10	W	7.5	....	5.4	3	
....	......	8.6	2.1	6.5	......	
Etc.						
8-26	W	34.3	2.0	32.3	12	
....	......	34.5	....	32.3	......	
....	......	34.5	....	32.3	......	
8-27-10 End	R & W	33.5	....	31.3	13	Near W. end of breakwater.
Line No. 2, Running North.						
8-35-00 Begin	W	34.7	2.0	32.7	14	Near W. end of breakwater.

FIG. 108. RECORDER'S NOTES.

The observed times are used to reduce the soundings to the datum, and also to identify the angles taken by the transitman. The letters in the second column indicate which soundings were located by transit angles and the color of flags used; and it will be seen that these correspond to the transitman's notes in Fig. 107. The reduced gauge readings, which are shown in the column headed "Tide," are copied from the gauge reader's note-book; the tide for any time may be obtained by interpolation between the observations taken. The first three columns are the only ones that are filled in during the fieldwork. The sixth column, headed "No.," is filled in when the transitman's notes are compared with those of the recorder at the end of the day.

In plotting notes like these where the soundings are quite close together the points which were "cut in" are located on the plan and the intermediate readings are interpolated between them; the soundings are assumed to be equally spaced between the ones which were located.

It will sometimes be found more convenient to take the located soundings exactly one or two minutes apart and to take the intermediate soundings at equal intervals of time, as shown in Fig. 109.

270. Locating a Sounding by a Range and an Angle from the Boat. — The method of locating a sounding by a range and an angle from the boat is like the one just described, with the exception that the angle instead of being taken from the shore is measured with a sextant from the boat. It is not as common a method as the previous one because it increases the office work, and the only advantage so far as the fieldwork is concerned is that the instrumentman is in the boat with the chief of party, who can therefore direct the work to better advantage. But since it will probably be necessary to use a transit in laying out the ranges the soundings may as well be located with the same instrument and the office work thus simplified.

271. Locating a Sounding by Two Angles from Shore. — Where it is impossible to keep the boat always accurately on range or where it is not convenient to establish ranges, the position of soundings may be located by two angles taken simultaneously from shore. In this method two transits are used, each being set up at previously determined triangulation points or points on the shore traverse, or else at selected instrument points which can later be tied to the triangulation system. These points should be so chosen that the transit lines locating the soundings will cross **as nearly at right angles as is practicable.** The instruments are both run as described in Art. 269, p. 298, the angles being taken **simultaneously** on signal from the boat. When the boat is a considerable distance away it is customary to take the angle to the leadsman or to the portion of the boat in which he is standing rather than to set exactly on the sounding-pole. Evidently for a given degree of precision this approximation is as applicable for short as for long distances.

POINT MERIDETH HARBOR. — SOUNDING LOCATIONS.

K and E. ⊼ at S.B. "A." Frank Jones, Sept. 11, 1906.

Time	Flag	Angle	Number	Remarks
0° on Lantern Staff on Breakwater. L. Hayes ⊼ occupying S.B. "B."				
		134° 17′		to S.B. "B"
8.53 Begin	W	7° 56′	1	Line No. 7, South.
.54	R	10 14	2	
.55	W	12 47	3	
.56	W	15 15	4	
.57	W	17 46	5	
.58	R	19 48	6	
.59	W	21 41	7	
9.00	R & W	23 28	8	R & W check watch, 9.00 A.M.
.01	W	24 55	9	
.02	R	26 19	10	
.03	W	27 33	11	
Etc.				
.10 End.	W	34 35	18	
9.20 Begin	W	36 13	19	Line No. 8, North.
.21	W	35 01	20	
.22	R	34 03	21	
.23	W	33 51	22	
.24	W	33 00	23	
.25	?	?	24	Schooner in way.
.26	R	31 31	25	

FIG. 109. TRANSITMAN'S NOTES.

This method affords a quick way of obtaining the necessary data for such a survey. In the U. S. Engineer office this method, together with the use of approximate ranges, is the one commonly employed. The soundings are taken on ranges determined by pacing off nearly equal distances along the shore and determining by eye the parallel courses. The soundings are taken at equal intervals of time and the recorder's notes are kept practically as shown in Fig. 108, except that the time of each sounding, say one every 15 seconds, is recorded in the first column. Each of the transitmen keeps a set of notes similar to those shown in Fig. 109.

272. Locating a Sounding by Two Angles from a Boat. — A common method of locating soundings when ranges are not used is by taking two angles simultaneously from the boat to three signals, or any previously determined points, *A*, *B*, and *C* on shore (see Fig. 110). This is an application of the **Three point**

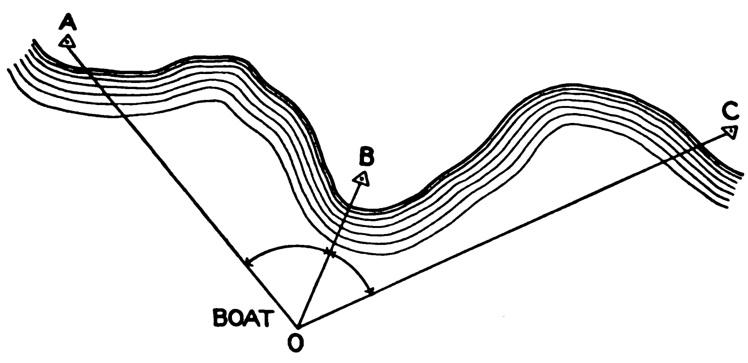

FIG. 110. LOCATING SOUNDINGS BY TWO SEXTANT ANGLES FROM BOAT.

Problem which is frequently used in plane-table work. (See Art. 49, p. 47, and Art. 188, p. 203.) It is essential that the signals or points should be fixed stations; such points as buoys or floats will not give satisfactory locations.

In measuring angles from a boat it is necessary to employ some instrument which does not require a steady support like the transit. For this reason the angles are usually taken with two sextants. These two angles are sufficient to locate the position of the sounding, except in the one case where the boat happens to be on the circumference of the circle passing through the three signals between which the angles are measured. There are some positions of the three signals with reference to the boat which will give poor locations of the soundings; these are explained in Art. 190, p. 207. When the sextant is employed for this purpose it will be found difficult to measure the angles with sufficient rapidity unless its telescope is discarded and the observer sights through the ring which holds the telescope. Under ordinary conditions the error that can enter into an angle measured in this manner is not over 1 minute, which is far within the accuracy desired for locating soundings. The angles are usually read without the use of a reading glass to the nearest minute (sometimes to 5 minutes only).

In some kinds of work this method is less frequently employed than the "range and angle" method or the "two angles from shore" method, because it often happens that the two angles taken with the sextant do not give good intersections; this is especially true when the soundings extend far from shore. Furthermore, if the signals happen to be high above the shore, and consequently not at the level of the boat, the angle measured will be enough different from the horizontal angle to introduce serious error into some parts of the work.

273. Locating Soundings by Stadia. — As the stadia method is admirably adapted to measuring distances to inaccessible points, and as its results are within the limit of accuracy required for most sounding work, it would appear at first thought to be most useful for locating soundings. This would be quite true if it were not for the fact that both the instrument and the rod must be at rest if an accurate observation is to be made. (See Chapter IV.)

The transit may be set up on shore and the stadia rod carried in the boat. The rod if properly painted and of sufficient length may serve the double purpose of a sounding-rod and a stadia rod. At the instant the sounding is taken the transitman observes the interval on the rod, and then records it. The corresponding azimuth angle can be read while the boat is proceeding along its course. The transit should be set up at a point which will be as nearly at the shore level as possible, so that there will be no necessity for noting the vertical angles. It will simplify his work if the transitman sets up his instrument at the end of a range on which the boat is being rowed, for in this case he will have only the rod interval and the time to observe. It will sometimes facilitate matters if a regular sounding-pole is used for the soundings and a separate stadia rod is used for the transit sights. If the boat is rather heavy and the water calm it may be possible to read the stadia rod when held vertical with its base resting on the bottom of the boat.

It is evident that this method is applicable only in shallow waters. The "range and angle" method is fully as accurate as the stadia method, but where only a few isolated soundings are to be taken the latter method is to be preferred because it requires less fieldwork.

274. Locating Soundings by the Intersection of Fixed Ranges. — In cases where it is necessary to take soundings at various periods of time at some given point, fixed ranges are located on shore so that their lines will intersect at angles as near 90 degrees as is practicable. The boat will then proceed to the several intersections of these ranges and soundings may be taken as often as desired. This method is especially employed, for example, in making observations at different times to determine whether the bottom of a channel in a given place is filling or scouring, or to determine how much material has been removed by dredging.

275. Locating Soundings by Distances along a Wire Stretched across a Stream. — In sounding out the cross-section of a canal or narrow river in connection with stream gauging measurements a wire or rope is sometimes stretched taut from one shore to the other with tags fastened to it at equal intervals, and the soundings are taken at these points. This method may be used to advantage in connection with dredging work, the soundings being taken before and after the dredging has been done. (See Art. 282, p. 310.) This is obviously the most accurate as well as the most expensive of the methods that have been described.

276. Soundings through Ice. — Soundings through ice can be rapidly made by boring holes with an ice auger, say 1½″ in diameter, and using a long narrow weight with a chain and markers. The location of soundings on the ice is also readily accomplished by ranges or by stadia unless the weather conditions are very severe. This is not only a very rapid and economical but also an accurate method of obtaining soundings.

277. REDUCING THE SOUNDINGS TO DATUM. — In tidal waters the datum is determined by a series of observations, as explained in Vol. I, Art. 237, p. 211. It is the general practice to use Mean Low Water as the datum, and to reduce all soundings by subtracting (algebraically) from each sounding the corresponding adjusted gauge reading. Besides this reduction, account should be taken of the effect of wind and current, which may be sufficient to make it necessary to apply a correction for errors due to these causes. Furthermore the correction on account of erroneous length of lead-line should not be neglected unless it is small enough to cause no appreciable error.

On river or lake work the soundings will be reduced to the datum with reference to which the gauges have been set, as explained in Art. 259, p. 290.

278. PLOTTING THE SOUNDINGS. — Where the soundings have been located by ranges and angles taken from points on shore, the plotting of the position of the soundings is so simple that it requires no explanation other than to mention that the angles are all laid off on the map with a protractor; a paper protractor will suffice for most work of this sort.

Several methods have been devised for rapidly plotting the positions of soundings which have been located by the various field methods; most of these require that the plan shall be on some transparent material such as tracing cloth. Where for example the soundings are located by angles from two ends of a base-line, this base-line is plotted on the plan which is made on tracing cloth. Paper protractors with the radial lines drawn on them and properly marked may be made and kept in the office for general use. One of these protractors should be drawn in black ink on firm white paper and another in red ink on very transparent tracing paper. In plotting the soundings by this method the plan is placed over the black protractor so that one end of the base-line is exactly over the center of the protractor. The other (transparent) protractor is then slid in under the plan and above the black protractor until its center is exactly under the other end of the base-line. Both protractors are, of course, placed so that their zero lines coincide with the base-line or whatever zero line was used by the respective transitmen in the field; and where the black radial line representing the angle read from one end of the base-line cuts the red line representing the corresponding angle at the other end of the base-line is the location of that sounding.

Another method of plotting the soundings upon the original plan without the use of transparent paper as in the method just explained is to use two annular protractors printed on firm paper with graduations on the outer circumference. These are properly set at the respective transit points and the soundings plotted by the intersection of two threads or straight-edges pivoted at the centers and swung to the corresponding angles of each sounding. Two men, one for each protractor, may plot the located soundings very rapidly by this method by first numbering the locations faintly in pencil and afterward putting on the depths in ink. The size of protractors required will depend upon the size of the sheet used, but for most work a diameter of 14 inches will be found sufficient. Occasionally a location may come on one or perhaps both the protractor rings; if so it may be pricked through and numbered after the protractor is removed.

279. The Three-point Problem. — Where the soundings are located by sextant angles taken from the boat their positions may be quickly plotted as follows. On a piece of tracing cloth lay off the measured angles *AOB* and *BOC* (Fig. 110). Lay the tracing cloth down on the map and move it about until the three lines *AO*, *BO*, and *CO* pass through the corresponding signals *A*, *B*, and *C* as plotted on the map. The point *O* may then be pricked through to indicate the position of the sounding. A more rapid method of plotting the soundings is to use an instrument called a *three-arm protractor* (Fig. 111). In this instrument the two arms *C* and *B* are movable, while the middle arm *A* is fixed so that its beveled edge is at the 0° mark of the circle. This circle is graduated from 0° each way to 360°. The two movable arms are set at the two angles which were measured simultaneously with the sextants. The protractor is then laid on the plan and moved about until the beveled edges of the three arms pass through the plotted positions of the three shore signals. When the instrument is in this position its center locates the position of the sounding on the plan, which may be marked by a needle point. Only one position of the center point can be found from which the

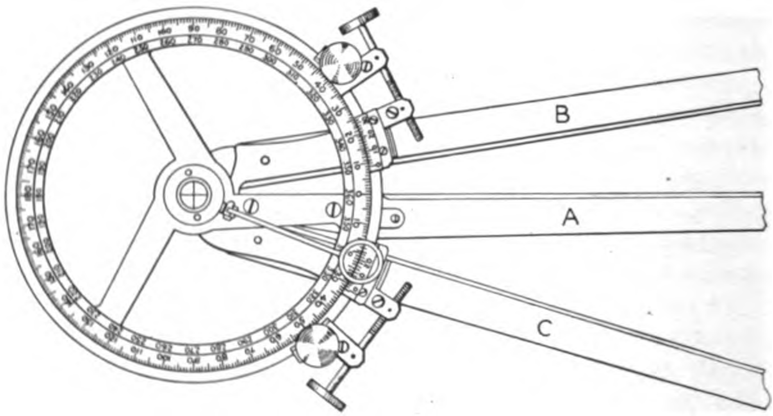

FIG. 111. THREE-ARM PROTRACTOR.

three beveled edges pass through the plotted signals, except in the case where the three shore points lie in the circumference of a circle which also passes through the center point. (See Art. 190, p. 207.) If the sounding is near this circle the location of the center point cannot be determined with sufficient accuracy.

The three-arm protractor can be tested for eccentricity as follows. Draw two fine lines exactly perpendicular to each other, lay the center of the protractor over the intersection of these lines, and see if the 90° points on the arc coincide with the lines. Several angles may be laid off on the same piece of paper by the method of tangents or chords (see Vol. I, Arts. 455–9, pp. 405–9) and the intermediate graduations of the arc compared with these lines to test the regularity of the graduations.

280. Geometric Solution of Three-point Problem. — While the geometric solution of the Three-point Problem is seldom used for plotting, on account of the rapidity with which the methods just mentioned can be applied, still its demonstration will doubtless give the student a better understanding of the problem.

In Fig. 112 A, B, and C represent the three shore signals. Let the two angles read from the boat be α and β. At both extremities of the line AB lay off an angle equal to $90° - \beta$, the sides of which if extended will intersect at P; and at both

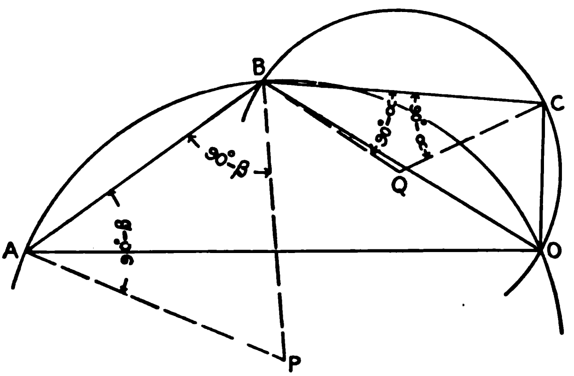

FIG. 112. GEOMETRIC SOLUTION OF THREE-POINT PROBLEM.

extremities of BC lay off an angle equal to $90° - \alpha$, thus obtaining point Q. Then with P as a center describe an arc passing through A and B, and with Q as a center describe an arc through B and C. Where these two arcs intersect, at O, is the position of the boat.

$$\text{Angle } P = 180° - 2\,(90° - \alpha) = 2\,\alpha$$

Similarly $$Q = 2\,\beta$$

But $$AOB = \frac{APB}{2} = \alpha$$

And $$BOC = \frac{BQC}{2} = \beta$$

The angle β determines the circle ABO, which is the locus of the point O. In like manner the angle α determines the circle BCO as the locus of O, and the intersection of these two circles (at O) is the only point (except B) satisfying both conditions.

The trigonometric solution of the Three-point Problem will be found in Art. 49, p. 47.

281. HYDROGRAPHIC MAPS.—Hydrographic maps are usually constructed for some special purpose and certain conventional methods of representation are used which are peculiar to this kind of map, a sample of which is shown in Fig. 162.

The method of finishing such a map is described in detail in Chapter XI.

282. MEASUREMENT OF DREDGED MATERIAL. — When a harbor or channel is to be dredged the basis of such work is a hydrographic map which has been prepared as described in the previous articles. This map gives the information needed in deciding just where the dredging shall be done and also in estimating the amount of material which must be removed. For dredging the material the contractor is usually paid by the cubic yard; the quantity may be obtained by **measurement in place** or by **scow measurement.**

When the **measurement-in-place** method is used soundings are taken before and again after the dredging work is done, and the volume of the material which has been removed is computed either by the **Borrow-Pit Method** or by the **End Area Method** of cross-sections, as may be the more convenient. Both of these methods are briefly described in Vol. I, Chapter XII.

If the quantity dredged is determined by scow measurement each pocket of the scow is carefully measured and its capacity computed. When the scows have been loaded with the dredged material the surveyor or inspector makes a note of the number of full pockets, or if they are not full he measures the distance down from the top of the coaming of the scow to where he estimates that the material in a pocket would come if it were leveled off. The volume of this small rectangular prism is calculated and deducted from the capacity of the corresponding pocket. For each scow in use tables giving the quantity in each pocket at various distances below the coaming are usually prepared for convenience. These scow measurements should be taken just before the tow starts for the dumping ground. When scows remain moored for a day or so before being towed to the dumping ground some of the material in the pockets leaks out through the bottom doors if they are not tightly closed, and such material may find its way back again into the dredged portion of the channel if the scows are moored near the work. In the case of a deck scow where the material is piled on the deck any practical and convenient method may be used to determine the volume of earth, the measurements taken depending upon the shape of the pile.

When the material dredged is rock the amount taken out can be calculated by obtaining its weight, and this is arrived at by taking measurements to determine the displacement of the scow before and after loading. The determination of the quantity dredged is more frequently made by the measurement-in-place method, the soundings being taken at cross-section points (spaced as close as deemed necessary) before and after the dredging is done.

283. **Measurement of Surface Currents.** — In certain engineering problems it is important to determine the direction and velocity of the tidal currents in a harbor at various points and at various stages of the tide. This may be done by setting off floats from points whose locations are known or can be determined, and locating these floats from time to time by sextant angles taken between signals on shore. The float used for this purpose should set deep enough in the water to be carried along by the current and not appreciably affected by the wind. A satisfactory float can be made by using a piece of joist about two feet long, weighted with lead at one end so that the top projects but little above the surface of the water. (See Fig. 113.) In order to make the float visible at a distance it should have a small flag of some bright color attached to it, but this flag should not be larger than is absolutely necessary, as a strong wind might carry the float out of its true course. Floats should be numbered and a proper record made in the note-book, showing the position of each float and the time when it was located. In locating a float the boat is pulled up alongside of it and sextant angles taken just as in locating soundings. Notes as to the stage of the tide and the direction and

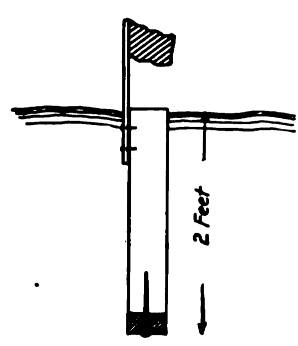

Fig. 113.

velocity of wind should be made in connection with the float observations. The results of these measurements may be plotted so as to show the path taken by each float, which, together with the observed times, will give its velocity. For convenience in studying the currents the observations at different stages of tide may be worked up on separate plots.

CHAPTER IX.

MEASUREMENT OF THE FLOW OF WATER IN OPEN CHANNELS. * †

284. **DEFINITIONS.**—The volume of water flowing in a stream, called the *discharge*, or in some cases called *run-off*, is usually expressed as a rate of flow and in one of the following units.

Second-feet is the unit of flow most commonly used. It is an abbreviation of cubic feet per second and is the quantity of water flowing in a stream one foot wide, one foot deep, and with an average velocity of one foot per second.

Gallons per minute and *gallons per day* are common units of flow in connection with pumping and city water supplies.

The miner's inch, a common unit of flow used by miners and irrigators in the West, is the quantity of water that passes through an orifice one inch square under a *head* (depth of water above the orifice), which varies in the different states, being defined by statute. The California miner's inch, which is probably the most common, is equivalent to $\frac{1}{40}$ of a second-foot. (See Vol. I, Art. 341, p. 310.)

Second-feet per square mile is the ratio of the discharge at the point considered to the tributary drainage area of the stream above that point, the discharge being expressed in second-feet and the drainage area in square miles. It assumes that the run-off is uniformly distributed, both as regards time and drainage area, which of course is not true. It is a convenient unit for comparing the run-off of different drainage areas.

Besides the above-mentioned units the discharge of a stream is frequently expressed as an actual quantity of water, the following being common terms of this nature.

Run-off in inches is the depth, in inches, to which the drainage

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* This chapter was written by Mr. H. K. Barrows, Engineer U. S. Geological Survey.

† For a more complete treatment of this subject see "River Discharge," by Hoyt and Grover, published by John Wiley & Sons.

area would be covered if all the water flowing from it in a given period were conserved and uniformly distributed over the surface. It is used for comparing run-off with rainfall, the latter being the average precipitation for the period as determined by rain gauge observations, and usually expressed as depth in inches. Run-off in inches is also a convenient unit in computations regarding storage of water in natural or artificial reservoirs.

An *acre-foot* is the quantity of water required to cover an acre to the depth of one foot; it is equivalent to 43,560 cubic feet. One second-foot flowing for 24 hours will deliver approximately 2 acre-feet, or in other words, one second-foot is approximately one acre-inch per hour. This unit is commonly used in connection with storage for irrigation work.

285. THEORY OF STREAM FLOW. — *Steady* flow in a channel is said to occur when the same quantity of water per unit of time passes through each cross-section. While at times the flow in a channel may be said to be nearly or quite steady, it is evident that the natural regimen of stream flow is one of continual change and variation owing to the irregular manner in which water is supplied from adjacent drainage areas and by tributaries. Moreover, many rivers are now controlled by dams, where water is used intermittently, thus causing still further variation in flow. For practical purposes, however, at a given section of river and for short periods of time the flow will usually be found to be nearly steady or else to vary with sufficient regularity to permit of the determination of its average amount.

Uniform flow is a special case of steady flow and would occur if all water cross-sections were alike, so that with the same quantity of water passing each cross-section per second the mean velocity in each section would be the same. This would be the case with a conduit or canal of constant size and slope and whose supply does not vary.

If the stream bed were perfectly smooth and frictionless the water, under constant supply, would tend to flow with accelerated motion under the action of the force of gravity, but owing to resistances primarily caused by the roughness of the bed and banks this tendency to accelerate is overcome, and the result is approximately steady flow. The resistances include not only surface friction but also the effect of eddies and "boiling" of the water.

286. The Chezy Formula for Velocity of Flow. — The relation between velocity, cross-sectional area, and slope of a stream for different discharges is a complex one. The formula for velocity in most general use is that deduced by Chezy, an empirical formula assuming a condition of uniform flow; it is

$$V = C\sqrt{rs}\,* \qquad [54]$$

In this formula V = mean velocity of flow in the cross-section considered, this being equal to the discharge divided by the cross-sectional area, the usual units being discharge in second-feet, area in square feet, and mean velocity in feet per second.

The term r is the *hydraulic mean depth*, or, as more often called, the *hydraulic radius*, and is the ratio of the water cross-section to the wetted perimeter (the portion of the cross-sectional perimeter wet by the flowing water). If the area is expressed in square feet and the wetted perimeter in feet, r will be in feet. For ordinary rivers where the width is many times the depth, it is evident that the hydraulic radius is approximately equal to the average depth, hence the term *hydraulic mean depth*.

The term s is the slope of the water surface in a longitudinal direction and is expressed as the ratio of the fall to the length in which that fall occurs. Both the fall and the length should be expressed in the same unit.

C is a coefficient which varies with the physical characteristics of the stream bed, with the hydraulic radius, and with the slope. Kutter's formula (deduced in 1869 by Ganguillet and Kutter † from a study of many gaugings of flow) is an empirical formula intended to give values of C in the Chezy formula for different values of r, s, and roughness of bed n. It is as follows (for English units).

$$C = \frac{41.65 + \frac{1.811}{n} + \frac{0.00281}{s}}{1 + \left(41.65 + \frac{0.00281}{s}\right)\frac{n}{\sqrt{r}}} \qquad [55]$$

* For a discussion of this formula see Merriman's Hydraulics, p. 269.

† See Hering and Trautwine's translation of "Flow in Open Channels," by Ganguillet and Kutter, published by John Wiley & Sons, N.Y.

In this formula n is an abstract number whose value depends only upon the roughness of the channel. The following are the values of n assigned by Kutter to different surfaces.

For a lining of pure cement plaster; planed timber; glazed surface in perfect order, $n = 0.010$.

For unplaned timber in good order; plaster of sand and cement; clean iron and steel surfaces, $n = 0.012$.

For brickwork; well-dressed stonework; iron; cement and terra cotta in perfect condition, $n = 0.015$.

For canals in earth or gravel, straight, well trimmed, in perfect order, and lined with a film of sediment, $n = 0.020$.

For rough rubble; brickwork in bad order; good earth canals in perfect order, $n = 0.0225$.

For canals in earth in average condition, free from stones and weeds; rivers above the average in regimen, alignment, and uniformity of cross-section and slope, and free from detritus, $n = 0.025$.

For canals in rather bad order, having stones or weeds occasionally; rivers in fair order and regimen, $n = 0.030$.

For canals in very bad order, having stones and weeds in large quantities; rivers rather below the average in order and regimen, $n = 0.035$.

The use of Kutter's formula (numerically) involves much labor in computation but the various tables and diagrams based upon it which have been constructed permit of obtaining results more rapidly.

Table XI, p. 407, gives values of C for different values of n, r, and s for use in the Chezy formula.

It will be seen by analysis of the Kutter formula that for a given slope and hydraulic radius the mean velocity of a stream is nearly proportional to $\frac{1}{n}$; that is, the velocity is inversely proportional to the roughness of the stream bed.

It must be remembered that the Chezy formula presupposes a condition of **uniform** flow, — a condition which is rarely or never attained in the case of ordinary streams. Therefore the use of this formula must be limited to short stretches of river, with constant slope, in which there is little variation in cross-section and condition of bed. For artificial channels where area, slope, etc. can be regulated, and uniform flow prevails, the Chezy formula is in general use in the shape of tables and diagrams; the Swan and Horton "Hydraulic Diagrams for the Discharge of Conduits and Canals" and Church's "Diagrams of Mean Velocity in Open Channels" are both in convenient form.

287. **THE GENERAL NATURE OF MEASUREMENTS OF STREAM FLOW.** — The flow of a river is primarily a function of the rainfall upon its drainage area, and is therefore subject to fluctuations. To procure an accurate knowledge of this flow at all times requires frequent and systematic observations and is much more difficult and expensive than the measurement of the amount of water flowing at a given time. With the exception of a few records of the continuous flow of rivers kept by private corporations this work has been left entirely for the national government to do, and for several years annual reports of stream flow at various points on important rivers have been issued in the form of "Water Supply and Irrigation Papers" by the U. S. Geological Survey. This information serves as a basis for the work of the private engineer, who can, by carrying on measurements of flow for a short time at the locality in which he is interested, secure sufficient data to enable him to estimate discharge by comparing his records with the longer-timed Government records taken at some point in that vicinity.

The general subject of determining flow may be divided as follows.

(1) Determination of flow at a given time, or during a short period of time.

(2) Determination of flow during a considerable period of time, covering seasonal and annual fluctuations.

The instruments and methods used are the same up to a certain point in both of these classes of work, the additional essential in the case of long time records of flow being the recording of the variations in river height by means of some form of gauge.

288. INSTRUMENTS FOR MEASURING DIFFERENCES IN LEVEL OF WATER. — The **Hook Gauge** (Fig. 114) is the most accurate instrument for measuring difference in level of water. It consists essentially of a hook with a needle point fastened to a vertical movable scale, which by means of a vernier can be read very closely, often to $\frac{1}{1000}$ feet. The hook is lowered just beneath the surface of the water and then slowly raised by a micrometer screw at the top until the point just touches the surface. Very accurate determinations of water level can be made in this way, the gauge being securely clamped to some firm support and the elevation of the zero of the scale determined by careful leveling. The hook gauge is frequently used to determine the depth of water flowing over a weir (Art. 308, p. 327), in which case it is first necessary to determine the gauge reading for the crest of the weir, or the reading when the water is just level with the crest. This is called the "*zero gauge reading,*" which is subtracted from other gauge readings to obtain the depth of water flowing over the crest.

FIG. 114. HOOK GAUGE.

289. The **Staff Gauge** in its various modifications is used most generally for noting changes in water level. In its simplest form this is a smooth board, from 4 to 10 inches wide, set vertically in the water, attached to some stable support, as a pier or wall, and graduated by painted lines, or preferably by metal staples, so that it can be easily read, the distance apart of the units of graduation depending upon the precision desired in gauge readings. For ordinary observations of river height a gauge of this kind should be graduated to feet and tenths of feet, and the observer should read to the nearest tenth of a foot. For more careful work the tenths are subdivided to half-tenths, permitting the easy estimation of hundredths; where greater precision is desired every hundredth is shown. More permanent forms of staff gauges, made of cast iron or of enameled metal

strips, are used where observations are continued over long periods of time. (See Vol. I, Art. 238, p. 212.)

290. The **Chain Gauge,** described in Art. 259, p. 290, is largely used for observations of river height, especially by the U. S. Geological Survey. It is really a modification of the staff gauge and has the great advantage that when not in use it is safe from blows by any floating object such as ice and logs. Its disadvantage is the tendency of the chain to stretch and the necessity of frequent adjustment and correction in length.

291. **Float Gauges** are frequently used at power stations where it is desirable to have gauge readings made inside the building. A hollow air-tight copper cylinder floats in a pipe of slightly larger diameter set vertically in the water. The float is connected by a wire or chain over suitable pulleys, and a marker attached to the wire moves over a scale where the readings are made.

292. **Automatic Gauges** are sometimes used where a **continuous** record of water level is desired. These are of various kinds, a common form consisting essentially of a drum upon which is fastened a sheet of record paper, and the drum made to revolve by clock-work at a regular rate, the record of water level being transferred by a recording pencil or pen, which is in turn connected by a suitable reducing device with a float which moves up and down with changes in water level.

293. **Piezometers.** — Heights of water surface may also be measured with refinement by a *piezometer*, or *manometer*, consisting of a vertical glass tube with graduated scale, the tube being connected with the flowing water by means of a pipe. The *piezometer float gauge* is a modification of the piezometer in which a small float and index is used in the vertical tube to enable exact readings on the scale to be made. In either case the connecting tube should enter the channel of flow exactly at right angles to the direction of flow and be cut off flush with the side of the channel.

294. **Plumb-bob.** — A steel tape and plumb-bob, passing over a pulley or rounded surface and referred to a horizontal scale, is another device sometimes used for careful measurement of fluctuations in water level.

295. INSTRUMENTS USED FOR MEASURING THE VELOCITY OF FLOWING WATER. — Floats. — The velocity of water may be measured **directly** by determining the velocity of a body, such as a float, which moves along with the water. A simple form of **surface** float, i.e., one which is intended to move on or near the surface of a river, consists of a corked bottle with a flag in the top and weighted at the bottom.

296. The *tube* float is a copper or tin tube weighted at its lower end so that it will stand vertical in the water. It is made long enough to reach nearly to the bottom of the channel and to project a few inches above the water surface. As each float should fulfil these conditions it is evident that a number of tubes of different lengths will be required for different depths of water. *Rod floats* made of wood similar in shape to the tube floats are also used.

297. Use of Floats. — Surface floats are used for approximate determinations of velocity, as for example in freshet flow where no other method can be used, owing to floating ice, etc., or in making a reconnoissance where only an approximate value for the flow is desired.

298. Tube and Rod Floats are best adapted for use in canals or artificial channels, where the depth of water is fairly constant. They have the advantage of being simple in form and construction, of interfering little with the velocity of the water, and of being little affected by silt, floating ice, etc. On the other hand, they are affected somewhat by wind, cannot be used in deep streams or where the bed is rough and irregular, and are expensive to operate.

299. Current Meters. — The velocity of flow may also be measured **indirectly** by means of a *current meter*, which consists essentially of a wheel with cups or vanes so constructed that the impact of flowing water will cause the wheel to revolve, the number of revolutions being indicated by some counting device. Current meters may be divided into two classes:

(1) Those in which the revolving part turns about a vertical axis.

(2) Those in which it turns about a horizontal axis.

The former class is illustrated by the Price meter (see Fig. 115) and the latter class by the Haskell and Fteley meters (Figs. 117 and 118, pp. 323–4).

300. The Price Meter. — The *Price* meter is in most general use and has been largely developed and used by the

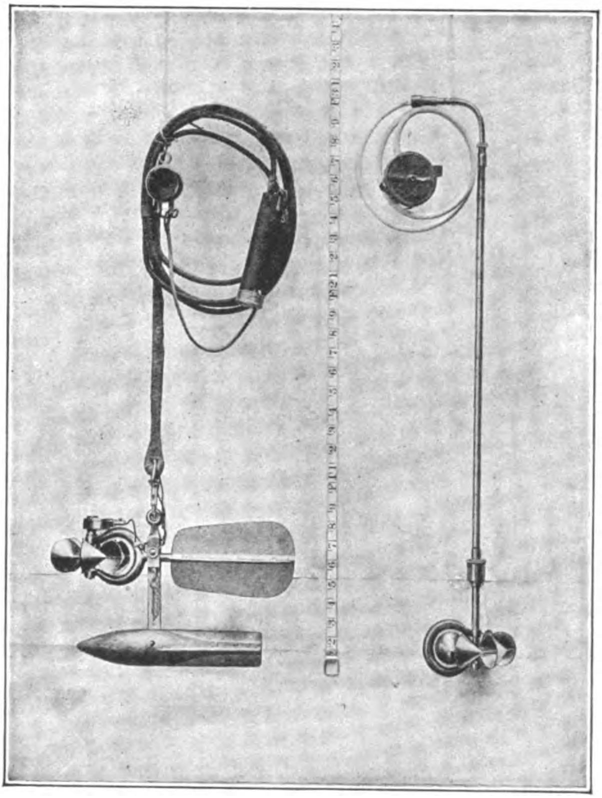

FIG. 115. PRICE CURRENT METER
(From U. S. Geological Survey.)

U. S. Geological Survey. It is made in two sizes and consists of a wheel (Fig. 116) made with cups fastened to a vertical shaft and turning upon a sharp steel point in a conical bearing which is capable of adjustment at *e*. The wheel and shaft are carried by a yoke *Y*, to which is attached a vane *V*, a stem *S*, at the bottom of which a weight is attached, and trunnion *T*, to which last is hooked the cable supporting the meter, and which is usually composed of two insulated electric wires.

The current meter may also be suspended by a single wire, using the ground and water to complete the circuit needed to operate the recording apparatus, which must be of the telephone receiver type in order to be successfully used. The wire is

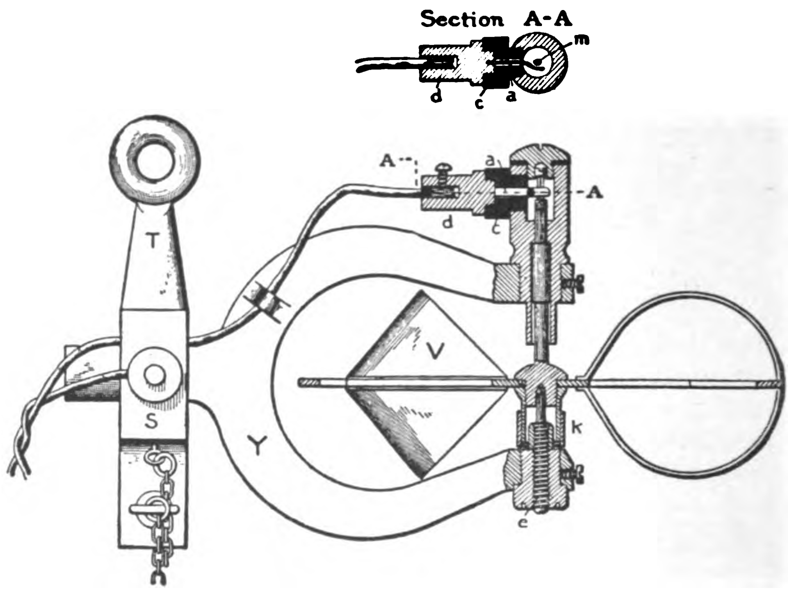

FIG. 116. DETAILS OF PRICE CURRENT METER.

insulated from the body of the meter but connected with the spring in the head of the meter which makes and breaks the circuit. The other end of the wire passes in circuit through the battery cell and telephone receiver and is connected with a conductor leading to the ground, such as a bridge iron, the station cable, or an extra wire stretched along the bridge. This method of operating the current meter is especially applicable in streams of great depth and high velocity, as a comparatively small steel wire (piano wire) can be used and offers little resistance to the current.

While the upper end of the wheel shaft is held in place by a bearing surface the weight of the wheel is carried upon the lower point, and in order that no shock may come upon this point while the meter is being carried from place to place a sleeve nut *k* is arranged to be screwed down against the frame, thereby lifting the shaft and bringing the bearing off the point.

The device for indicating revolutions of the meter wheel is shown in the cross-section of "*A–A*." One of the small wires from the cable enters the middle binding post at *d*, extends through the hard rubber circular insulating nipple *c*, and terminates in a slender platinum spring *a*. The top end of the shaft of the meter wheel at *m* is slightly bent, so as to give it an eccentric motion as the wheel revolves, thus making and breaking the electric circuit with each revolution.

A small compact form of battery is employed, which is arranged in a case with a form of "buzzer," or sounder, and fastened to the upper end of the wire cable supporting the meter, the contact wires being properly connected with the poles of the battery. The revolutions of the meter, as indicated by the "buzzer," are counted by the observer.

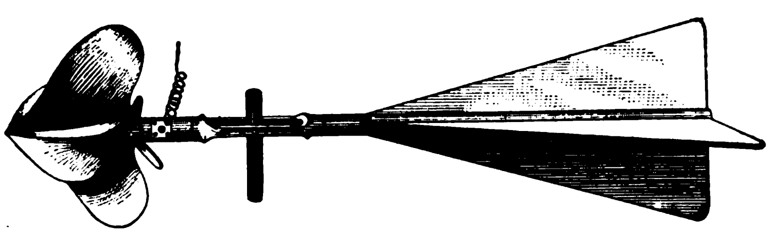

FIG. 117. HASKELL CURRENT METER.
(From U. S. Geological Survey.)

301. THE HASKELL METER. — The *Haskell* current meter (Fig. 117) is of the screw propeller type, with horizontal axis of rotation, arranged for electrically recording revolutions in a manner similar to that of the Price meter.

302. THE FTELEY METER. — The *Fteley* current meter (Fig. 118) has a wheel made up of helicoidal blades, which rotates upon a horizontal axis. The older form is arranged with a mechanical recorder operated by gear wheels and has to be lifted from the water and read after each velocity observation.

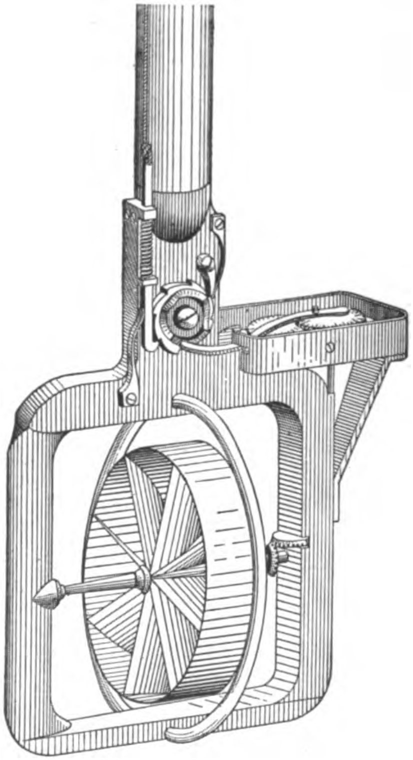

FIG. 118. FTELEY CURRENT METER.
(From U. S. Geological Survey.)

This meter is also arranged with an electrical recording device, which, however, involves considerably more friction in the contact parts than does that used with the Price meter. The Fteley meter is used wholly by means of a rod, thus limiting its scope in depth and velocity, whereas the Price and Haskell meters may be operated by either rod or cable.

303. Rating Current Meters. — The relation between the revolutions of the meter wheel in a unit of time and the velocity of flow is determined by *rating* the meter, i.e., by moving it along in still water at various speeds and determining the relation between revolutions and velocity. A car to which the meter is attached, either by cable or rod, is commonly used for this purpose, being pushed along a track close to the water's edge.

The values observed are plotted in the form of a curve (Fig. 119), using velocity in feet per second as abscissas and revolutions per

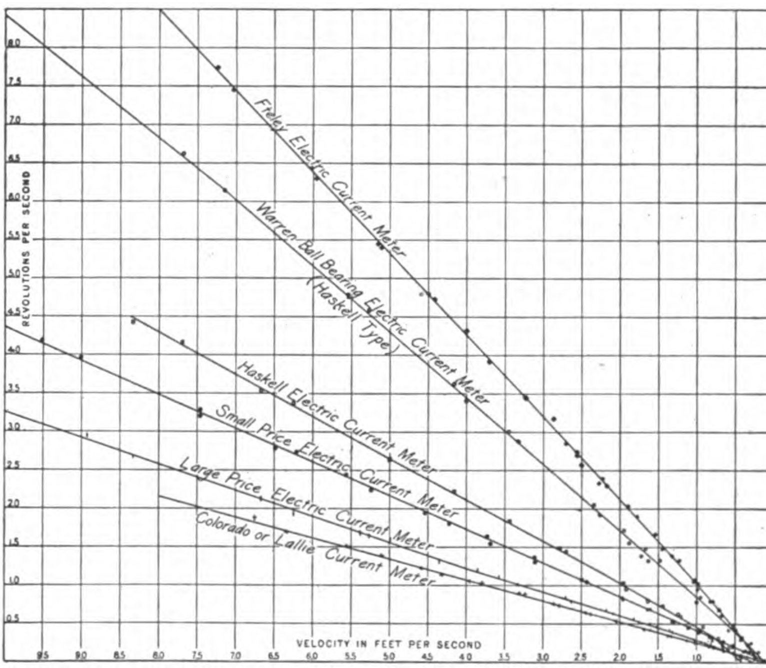

FIG. 119. METER RATING CURVES.

second as ordinates. This curve is usually a straight line except near the origin, where, with meters of the Haskell and Fteley type, it is curved slightly downward owing to the effect of friction of the bearings. The curve for the Price meters is usually a straight line throughout. A meter rating table is made from the plotted curve for convenience in use.

Where a suitable rating apparatus as above is not available a current meter may be rated by propelling a boat through a stretch of still water, or by moving the meter through the water by the side of a wharf.

304. Use of Current Meters. — The current meter is, in general, the most satisfactory instrument for measuring stream flow, since velocities can be accurately taken at nearly all points in the cross-section of the stream, if desired, and over a sufficient length of time to obviate the effect of pulsations in flow. It cannot be used where weeds, grass, or floating ice occur, but on the other hand it furnishes the only means of measuring velocity of flow under ice cover. Current meters must be frequently rated, however, and used with considerable care.

305. METHODS OF MEASURING STREAM FLOW. — There are three distinct methods of determining the flow in open channels, as follows.

(1) By measurement of slope and cross-section and the computation of flow by the Chezy formula.

(2) By means of a weir or dam, observing the head of water on the crest and computing the flow.

(3) By measurements of the velocity of current and the area of the cross-section, their product giving the discharge.

306. SLOPE METHOD OF MEASURING STREAM FLOW. — The slope method of measuring stream flow requires a straight stretch of river of uniform slope and cross-section. The difference in level of the water surface at two cross-sections, as far apart as practicable, must be carefully determined. This will require the use of hook gauges (Art. 288, p. 318) for the best results, and observations for a given section should be made at each shore and their mean taken and used as the elevation of the water surface at that section. The water surface near the middle of a stream is often at a slightly higher level than that at the sides, but if care is taken in selecting a stretch of river in which the distribution of flow is approximately the same at all cross-sections no error of consequence will be involved in slope measurements by locating the hook gauges near the shores.

The hook gauges are connected by careful lines of levels, and a sufficient number of cross-sections of the stream obtained to give an average value of the area and of the hydraulic radius as explained in Art. 286, p. 315. Assuming a proper value of n for the given conditions, the discharge is then computed by the Chezy formula, using tables or diagram.

307. Limitation and Use. — The results obtained by the slope method are as a rule only approximate, principally owing to the difficulty in securing favorable conditions of flow, in measuring the slope, and in assigning a proper value of n in Kutter's formula.

When other methods are not convenient this method is of value in estimating approximately the flood discharge of a stream, frequently by use of high-water marks left after a flood.

308. Weir Method of Measuring Stream Flow. — A *weir* is a notch in the top of the vertical side of a vessel or reservoir through which water flows. This notch is usually rectangular, with a horizontal lower edge called the *crest*. The upstream edge of the crest of a weir is usually made exactly rectangular and the crest itself thin, so that the water in flowing out will be completely contracted, touching the crest only in the line of its inner edge. In this form the weir is known as a *thin-edged* or *standard weir*, and provides one of the most accurate methods of measuring the flow of water. Where the vertical edges of the notch are at some

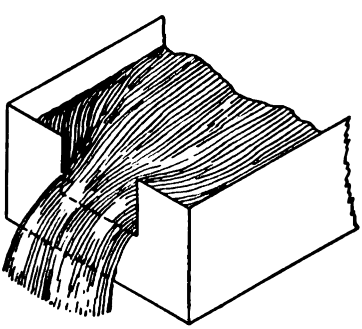

Fig. 120. Weir with End Contractions.

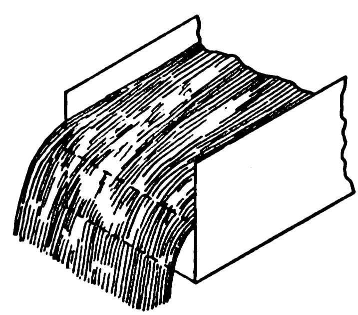

Fig. 121. Weir without End Contractions.

distance from the sides of the channel of approach, so that the particles of water in passing around the ends of the notch move in a curved path, the weir is said to be one *with end contractions* (Fig. 120). In this case the length of the sheet of water passing over the weir is less than the length of the weir. Where the edges of the notch coincide with the sides of the feeding canal, so that the particles of water at the sides pass over without deflection from the vertical planes in which they move, the weir is said to be one *without end contractions* or *with end contractions suppressed* (Fig. 121).

The head of water flowing over the crest of the weir is determined by means of a hook gauge (Art. 288, p. 318) or by a scale set sufficiently far up stream to avoid the surface curve formed by the water as it passes over the weir crest (usually 5 or 6 feet) and referred to the crest level. The flow is computed by formula.

309. Weir Formulas. — There are numerous weir formulas, but those of Hamilton Smith given below are convenient in application.

$$Q = cbH^{\frac{3}{2}} \qquad [56]$$

where Q = discharge in second-feet,
c = a constant based upon various experiments,
b = length of crest of weir in feet,
H = head on crest of weir in feet.

The value of c is taken from the Tables XII and XIII, pp. 408–9, separate tables being given for suppressed and contracted weirs.

The Francis, the Fteley and Stearns, and the Bazin formulas for weirs are all in common use.*

Where the channel above the weir is narrow the *velocity of approach* may have to be considered in weir computations. This is the mean velocity of the water in the channel at the section where the hook gauge is placed, and tends to increase the discharge.

In the Smith formulas H is **increased** as follows to allow for velocity of approach.

$H_v = H + 1.4\,h$, for contracted weirs. $H_v = H + 1\frac{1}{3}h$, for suppressed weirs. In these formulas H_v is the head on the crest

* See Water Supply and Irrigation Paper No. 200, U.S. Geological Survey for Weir Experiments, Coefficients, and Formulas.

of the weir corrected for velocity of approach, this being used instead of H in formula [56].

The term $h = \frac{V^2}{2g} = \frac{\left(\frac{Q}{a}\right)^2}{2g}$, where a is the cross-sectional area of *approach channel* in square feet at the hook gauge, V is the mean velocity in this cross-section in feet per second, and g is the acceleration due to gravity (usually considered as 32.16 feet per second per second).

In making this correction V can first be computed neglecting velocity of approach, and this approximate value used in computing h, a step which usually involves only slight error. In refined calculations h should be recomputed, using the new value for V, and Q calculated anew.

310. Submerged Weirs. — In a *submerged* weir the water on the downstream side stands above the crest level. Such a weir sometimes has to be utilized in measurements of flow in connection with works of river improvement, canals, etc., or may be necessary in situations where the loss of head will preclude the use of an ordinary weir.

Let H be the head above the crest measured by the hook gauge in the usual manner on the upstream side of the weir, and H' be the head above the crest, of the water on the downstream side of the weir, measured by a second hook gauge. If the level of the water on the downstream side of the weir is lower than the crest, then the flow is the same as for an ordinary weir provided there is full access of air beneath the sheet of water. As the water level on the downstream side rises, the contraction of the weir crest is suppressed as soon as H' has any appreciable value, and the discharge is therefore increased; but as H' is further increased the discharge is diminished on account of the effect of backwater on the downstream side.

There are several formulas in use for computing the flow over submerged weirs, all based upon experiments which have been almost entirely confined to weirs without end contractions. Herschel's * formula, based upon experiments made by Francis

* Transactions Am. Soc. C.E., 1885, Vol. XIV., p. 194.

and by Fteley and Stearns, is in convenient form. The observed head H is first multiplied by a number n which depends upon the ratio of H' to H and the discharge is obtained from the formula

$$Q = 3.33\, b\, (nH)^{\frac{3}{2}} \qquad [57]$$

Values for n may be found in Water Supply Paper 200, p. 140, or in Merriman's Hydraulics, Table 27.

311. Construction of Weirs. — On small streams a weir can readily be made of planks and timbers, taking care to shut off any flow of water underneath or at the sides, by driving sheathing. The crest can be made of a plank, with square edge upstream and with a chamfered edge on the downstream side, so that the width of crest will not be more than $\frac{1}{4}$ inch. For a more permanent crest a steel angle iron may be used. One leg of the angle iron is set vertically, its edge planed to a true right angle and then chamfered in a similar way as described above. In any case care must be taken to set the crest **exactly level.**

Where considerable fluctuation in flow occurs it may be desirable to have the crest in two levels such that the smaller quantities can be carried by the low part of the weir, which is made quite short so that the depth of flow will not be too shallow. This is necessary on account of the relatively large percentage error involved in measuring low heads. To illustrate, an error in measured head of 0.01 feet would mean an error of 15 per cent in flow, where the total head on the crest is 0.1 feet, and only 1.5 per cent where the total head is one foot.

There are other precautions necessary in weir measurements. The depth of water on the upstream side of the weir crest should be considerable, so as to diminish the velocity of approach and insure complete contraction of water flowing over the crest. According to Hamilton Smith it should not be less than twice the head of water on the crest.

The overfalling sheet of water, or *nappe*, should be perfectly aërated, this being insured by having a good fall below the crest down stream and a free entrance of air at the ends of the weir.

Unless it is intended that end contractions be suppressed the width of the channel of approach should be sufficient to provide complete end contractions, and the distance from the end of the weir to the side of the channel should be at least 2 H.

Weirs are expensive to construct and maintain on streams of any considerable size, for they have to be constructed like any dam of the overfall type and in addition require considerable attention to keep the crest level and in good condition.

312. Use of Dams as Weirs. — A dam can frequently be utilized as a weir in measuring the flow of a stream, if the physical characteristics of the dam are suitable and the manner of use of water around and through it will permit.

The essentials as regards the dam are (1) sufficient height so that back water below the dam will not interfere with free fall over it, (2) little or no leakage, (3) level crest, (4) crest and cross-section of some form for which the coefficient c is known in the usual weir formula $Q = cbH^{\frac{3}{2}}$.

Numerous experiments have been made to determine values of C in this formula for different types of broad crested dams, or weirs. The most recent compilation of data regarding C is in Water Supply Paper No. 200, U. S. Geological Survey. As there noted, the formula $Q = 2.64\ bH^{\frac{3}{2}}$ is applicable to broad crested weirs or dams, whose crests exceed 2 feet in width and having heads from 0.5 feet up to 1.5 or 2 times the breadth of the weir crest.

If water is used through or around the dam its amount must be measured either by weir or by current meter, or, where turbines of standard make are installed, the quantity may be estimated from the head, gate opening, and speed.*

313. "VELOCITY" METHOD OF MEASURING STREAM FLOW. — General Requirements. — The discharge through a certain section of the stream at a given time is the product of two factors, viz., the mean velocity and the area of the cross-section. In the velocity method of measuring stream flow these two factors are determined separately, the area by means of soundings (Art. 275, p. 305) and the velocity either by floats or current meters.

The essential requirements in channel conditions for securing good determinations of velocity are much the same, whether these

* See Water Supply Paper No. 180, published by the U. S. Geological Survey.

are made by current meter or by floats. The channel should be straight, both above and below the portion of the river where the gaugings are made, and the bed of the stream fairly even in cross-section and free from bowlders, so that cross currents, "boiling" of water, etc. will not occur.

314. Variation of Velocity in a Given Cross-Section. — The variation of velocity in the cross-section of a stream flowing in an open channel follows in general fairly well-defined laws which are little affected by changes in stage. This makes it possible to determine the mean velocity of flow at a given time by a comparatively small number of observations of velocity, provided these are properly distributed in the cross-section.

In any given vertical section the velocity varies at different depths, and also the mean velocity in vertical sections varies with the distance of the sections from the banks of the stream.

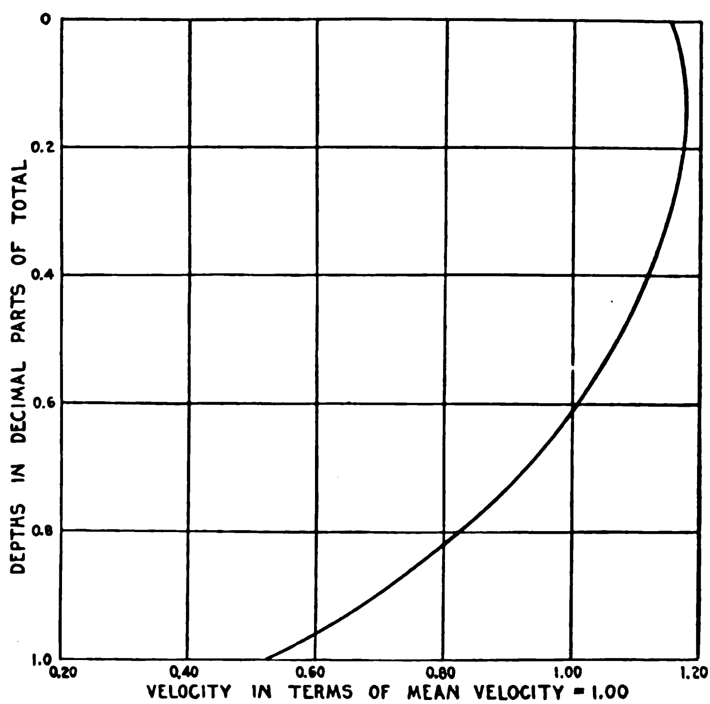

FIG 122. TYPICAL VERTICAL VELOCITY CURVE.

315. 1. Velocity in the Vertical. — If the velocities at different depths in a vertical section a short distance from the stream banks are plotted as abscissas and depths as ordinates it will be found that the typical *vertical velocity curve* (Fig. 122) is approximately that of a parabola with a horizontal axis. It will vary in form, however, with variation in total depth, mean velocity, condition of bed of channel, effect of wind, distance from the banks, etc. Usually the maximum velocity is at a point 0.1 to 0.3 of the total depth below the surface. The surface velocity is somewhat less than the maximum, the mean velocity is at about 0.6 the total depth, and the minimum velocity at the bottom. From experiment it has been found that the mean of the velocities at 0.2 and 0.8 of the total depth is quite closely the mean velocity in the vertical.

316. 2. Velocity from Bank to Bank. — If the mean velocities in the verticals at different distances out from either shore are plotted as ordinates, and the corresponding distances from the shore as abscissas, a *horizontal mean velocity curve* is obtained. The form of this curve will vary largely with the cross-section. Usually the maximum velocity will be at or near the middle of the stream and the minimum velocity at the banks. If piers or other obstructions occur in the section the curve may be quite complex. It is not practicable to find any one vertical section in the stream whose mean velocity will be the mean velocity for the entire cross-section.

317. USE OF FLOATS IN DETERMINING VELOCITY AND DISCHARGE. — The kinds of floats in common use have been described in Arts. 295–6, p. 320. In measuring velocity by means of surface floats record is made of the time taken by the float to pass over a selected stretch of a river, say, from 50 to 200 feet or more in length.

A sufficient number of such velocity determinations is made at different points across the width of the stream to determine the mean velocity of the whole section. By plotting as abscissas the mean position of the floats, as indicated by the distances from the bank, and as ordinates their average velocity, a curve showing the variation in velocity between the banks can be obtained, and from this the mean velocity of the whole cross-section may be determined. The product of this mean velocity and the average cross-sectional area is the discharge. Since the surface velocity is usually greater than the mean velocity it is necessary to multiply the observed results by a coefficient (usually about 0.85) to reduce the observed velocity to the mean velocity in the vertical. (See form of vertical velocity curve, Fig. 122, p. 332.)

Tube or rod floats are intended to give directly the mean velocity in the vertical. The velocities obtained with them, however, must be corrected slightly owing to the fact that they do not move through that portion of the water near the bottom of the channel, nor at exactly the same velocity as the water itself, so that the true mean velocity is somewhat less than that measured by the tube float.

Francis deduced the following empirical formula for use in such cases.

$$V_m = V_r\left(1.012 - 0.116\sqrt{\frac{d'}{d}}\right) \qquad [58]$$

where V_m is the mean velocity desired; V_r is the mean velocity of the tube float; d is the total depth of the stream; d' is the depth of the water below the rod. In using this formula, however, d' must be small in comparison with d.

318. CURRENT METER DETERMINATIONS OF VELOCITY AND DISCHARGE. — Appliances Used. — The use of a current meter in obtaining velocity of stream flow requires some means for placing the meter at any point in the stream cross-section. Where available, a bridge can be used for this purpose, and on account of the height of bridge floor above the water this usually requires that the current meter be suspended by cable. If no bridge is available a boat or canoe may be used, the boat being held in any given position by means of stay lines stretched across the stream at bow and stern. (See Fig. 123.) The bow is pointed upstream and the meter either suspended by a cable from an outrigger and pulley, or fastened to a rod which is held vertically in front of the bow of the boat. For more accurate results than can be obtained by the use of a boat a wire cable stretched across the stream with a box or car running on the cable for the observer can be used. Where the stream is not too deep excellent results may be obtained by wading and holding the meter fastened to a rod. In any case it is necessary to lay off, transverse to the axis of the stream, stations spaced at equal distances apart, the interval depending upon the width of the stream, and beginning with some suitable reference point on the bank. These stations may be marked with paint or chalk if on a bridge, or by a tagged

FIG. 123. BOAT STATION.
(From U. S. Geological Survey.)

line where a boat or cable is used or when measurements are made by wading. Either a gauge should be read at the beginning and end of the work or the distance measured to the water surface from some fairly permanent reference point, so as to determine the stage of the river at which the gauging is made. Soundings can usually be made with the meter by graduating the rod used for supporting it, or, in case the meter is suspended by a cable, by using a cloth tape and reference mark on the bridge, boat, or car, taking these readings usually to the nearest 0.1 of a foot. The distance apart of these soundings and their accompanying velocity observations will depend upon the width of the stream, varying from 2 feet or less, with very narrow streams, to 25 feet or more, where very wide.

319. Methods for Velocity Observations in the Vertical. — There are three general methods in use for determining the mean velocity in a vertical section.

(1) Multiple Point
(2) Single Point
(3) Integration

In the MULTIPLE-POINT METHOD the meter is held at two or more points in the vertical, an observation being made at each point by counting the number of revolutions of the meter for a given number of seconds, usually for 50 or 25 seconds, repeating this over an additional time period of the same amount to serve as a check and to further eliminate the effect of pulsations. There are several multiple point methods in use; the most accurate, called the *vertical velocity curve method*, consists in taking a sufficient number of observations in the vertical to enable the vertical curve to be plotted, from which the mean velocity is obtained by scaling or by planimeter. A quicker method, called the 0.2 *and* 0.8 *method*, usually accurate within a small percentage, is to observe the velocity at 0.2 and 0.8 total depth respectively, the mean of these being the mean velocity in the vertical. This is the method now in most general use by the U. S. Geological Survey.

320. In the SINGLE-POINT METHOD the meter is held at the depth of the thread of mean velocity, or at some depth for which the coefficient for reducing to mean velocity has been determined. The most common method of this kind is the 0.6 *depth method*, this being approximately the depth of the thread of mean velocity, and gives the mean velocity with sufficient accuracy in most cases.

In another single point method called the *sub-surface method*, the meter is held just below the surface, usually about 1 foot, and a coefficient, usually about 0.90 is applied, to obtain the mean velocity. This is a very useful method where the current is so swift as to interfere with holding the meter at a greater depth, and is much used during high-water stages.

321. In the INTEGRATION METHOD the meter is moved at a slow, uniform speed, from the surface to the bottom and back again to the surface, noting the number of revolutions and the time taken in the operation. It is not a convenient method for use, although fairly accurate if the meter is moved slowly up and down. The Price meter is not as well adapted to this method as are the other forms, owing to the fact that a vertical movement of the meter through the water, whether up or down, will increase the number of revolutions of the wheel and thus give observed velocities which are too large.

322. COMPUTATION OF DISCHARGE. — In computing the discharge from the observed velocities the cross-section is considered as divided into vertical strips, each sounding and its corresponding vertical velocity observation having been taken at the middle verticals of the strips. The area of a strip is substantially equal to the observed depth at its middle point multiplied by the width. The area of a given strip, multiplied by the observed velocity at its middle vertical, gives the discharge through the strip. The summation of these elementary quantities gives the total discharge. Fig. 125 is a typical set of current meter notes and computations. It will be seen that the velocity observations were taken at 0.2 and 0.8 of the total depth except at the banks. Fig. 124

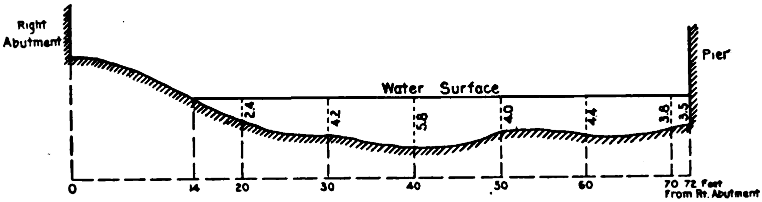

FIG. 124. CROSS-SECTION OF RIVER, ILLUSTRATING THE CURRENT METER MEASUREMENTS AND COMPUTATIONS GIVEN IN FIG. 125.

is a cross-section of the river where these measurements were taken.

GAUGING MADE AUG. 24, 1907, BY J. L. BROWN ON WHITE RIVER AT SHARON, STATE OF VERMONT, GAUGE HEIGHT IN FEET: BEGINNING, 3.58; END, 3.62; MEAN, 3.60; METER NO. 100; TOTAL AREA, 230 SQ. FT.; MEAN VELOCITY, 1.27; DISCHARGE, 292.

Distance from Initial Point	Depth	Observations				Velocity Computations			Computations of Area of Section		Discharge of Section	Remarks on condition of channel, wind, equipment, gauge, boat, cable, methods, accuracy. Use cross-section pages in back of book if necessary
		Depth of Observa.	Time in Seconds	Revolutions		Revolutions per Second	Velocity per Second	Mean Velocity per Second	Width	Area		
14	0.0	....	....	....	....	......	None	....	3	0.0	......	2.15—3.30
20	2.4	0.5	50 seconds	18	17	0.35	0.84	0.75	8	19.2	14.4	Length of
....	....	1.9		13	14	0.27	0.66	....	..	......	......	gauge chain
30	4.2	0.8		27	27	0.54	1.28	1.17	10	42.0	49.1	checked
....	....	3.4		21	23	0.44	1.06	....	..	......	......	and found
40	5.8	1.2		32	33	0.65	1.53	1.49	10	58.0	86.4	to be 21.78
....	....	4.6		31	30	0.61	1.45	....	..	......	......	feet, cor-
50	4.0	0.8		32	33	0.65	1.53	1.40	10	40.0	56.0	rected to
....	....	3.2		26	27	0.53	1.27	....	..	......	......	21.75 feet.
60	4.4	0.9		28	29	0.57	1.35	1.28	10	44.0	56.3	
....	....	3.5		26	25	0.51	1.21	....	..	......	......	Good gauging
70	3.8	0.8		25	25	0.50	1.19	1.17	6	22.8	26.7	
....	....	3.0		24	24	0.48	1.15	....	..	......	......	
72	3.5	....	....	....	....	about	same	1.0	1	3.5	3.5	
									58	229.5	292.4	

Computed by J. L. B.
Checked by W. H. S.

FIG. 125. CURRENT METER NOTES AND COMPUTATIONS.

323. Measurements of Flow of Ice-Covered Streams. — The use of weirs or dams in measuring the flow of streams in winter is frequently impossible unless the crest can be kept clear of ice. Measurements of flow by means of the current meter can usually be made through holes cut in the ice, except where accumulations of anchor and needle ice prevent. The methods suitable for current meter velocity observations under ice cover are:

(1) By taking vertical velocity curves.

(2) By the 0.2 and 0.8 method, which gives practically as good results for ice cover as for open section.

(3) By observations at 0.5 depth (i.e., below bottom of ice), and the application of a coefficient of 0.88, this usually giving the mean velocity within a few per cent.

324. Measurement of Flow in Artificial Channels. — In the case of canals, flumes, etc. any of the methods previously described for streams may be used where the flow is not affected by draft through wheels or gates. For example, where water is being drawn from the lower end of a canal or head bay, through water wheels, the flow is no longer like that in an ordinary channel, the distribution of velocity in the vertical and horizontal being quite different. Float measurements may still be made, but the shorter current meter methods will not in general give good results. Under such conditions the best method of obtaining velocity by current meter is to use the vertical velocity curve method or integration method.

325. Methods of Estimating Stream Flow During a Period of Time. — Estimates of stream flow covering a period of time may be made

(1) By means of weirs or dams, observing daily the gauge heights on the crest.

(2) By the velocity method, using either current meter or floats for occasional measurements of flow at different stages, and some form of gauge for obtaining a record of daily stage of the river and to which the discharge measurements can be referred.

The gauge heights for daily stage in any of the above methods should be taken in sufficient number to give an average determination for the day. Usually a reading in the morning and at night will suffice, but where large fluctuations occur on account of the pondage of water in dams etc. more readings may be necessary.

If the **velocity method** be used great care must be taken in locating the gauge so as to obtain good results. The bed and banks of the stream at and near the gauge should be fairly permanent in character, besides complying with conditions previously noted for good velocity measurements. The mean velocity at low stages should not be much less than 0.50 feet per second for good current meter work, and the gauge should be sensitive to changes in flow. Any back water influence from streams or dams below is fatal to the correct estimation of discharge. The gauge must be always kept at the same height, and should frequently be checked up by a level, using some permanent bench mark as a reference point (see Art. 259, p. 290). The results of the separate discharge measurements are plotted, using discharge in second-feet as abscissas and gauge heights in feet as ordinates. This enables one to draw a *station rating curve*, from which the discharge corresponding to a given gauge height can be taken off.

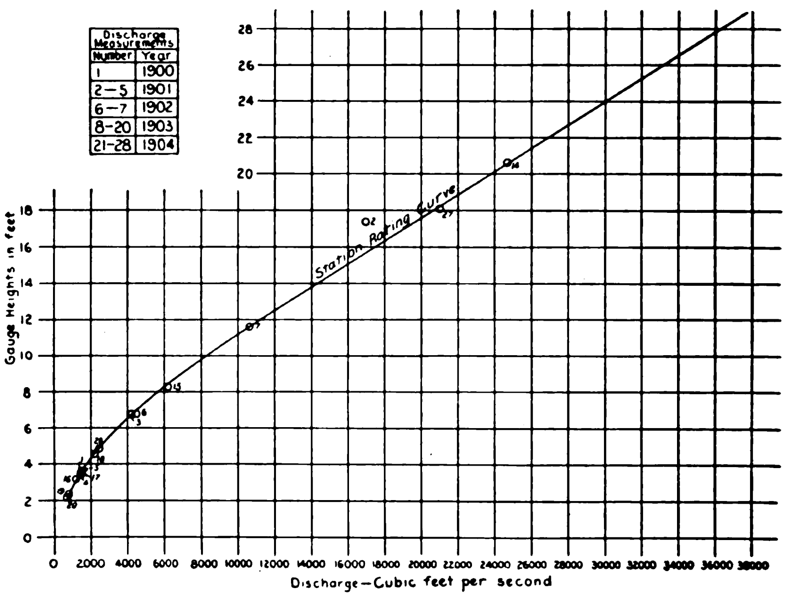

FIG. 126. STATION RATING CURVE.

Fig. 126 shows such a station rating curve, which is usually parabolic in its lower portion and a tangent in the upper part.

Using the daily records of gauge heights and the station rating curve (or a rating table constructed from it) the discharge may be obtained for any desired period of time.

326. Streams with Shifting Beds. — Where a river bed changes rapidly in condition, owing to the unstable character of materials comprising it, or owing to frequent floods, the use of the velocity method of estimating flow requires frequent discharge measurements. If changes are frequent, gaugings may have to be made every few days. Where changes are slower, and perhaps only during floods, satisfactory results may be obtained with a smaller number of current meter measurements. Under such conditions evidently the station rating curve will be continually changing in form. The weir method of estimate, where feasible, is much preferable on such streams.

327. **Estimates of Flow in Winter.** — Continuous measurements of flow through the winter months, where cold weather prevails, are frequently impossible. Probably the most satisfactory method for obtaining winter records of flow is to use a dam or weir, taking care to keep the ice cut away from the crest.

Where most of the flow during winter is through wheels it can be estimated as previously noted. Under conditions of continuous ice cover and freedom from anchor and needle ice, the velocity method, using current meter and gauge, is applicable, although a different rating curve, based upon measurements of flow under ice cover, must be developed for winter discharge.

328. **COMPARATIVE VALUE OF WEIR METHOD AND VELOCITY METHOD IN MEASURING AND ESTIMATING FLOW.** — A standard weir furnishes the most accurate method of measuring the flow of small streams. The expense of such a weir, however, increases very rapidly as the width of stream increases, and in most cases prohibits its use on streams of greater than 10 or 15 square miles drainage area. Of course this does not apply to existing dams, which, if suited for measuring flow, can be utilized on any stream.

Above the limit of the economical use of weirs the velocity method, using current meters, is generally the best. In fact, on small streams the current meter can frequently be used to advantage where the expense of a weir is prohibitive.

329. **USE OF MEASUREMENTS AND ESTIMATES OF FLOW.** — Single measurements of flow of rivers are not, as a rule, of value in determining the regimen of flow, unless at extreme stages, such as very high water or extreme drought. It is these extremes of stage that especially affect the design and operation of water power plants and use of water. Detached measurements may

be needed, however, for such purposes as determining leakage through a dam, division of water between different users, efficiency of water wheels, etc.

Estimates of flow of rivers, to be of conclusive value in estimating water power and water supply, must be carried over several years and embrace as widely varying conditions of flow as possible. The stream flow records of the U. S. Geological Survey will in many cases provide estimates of flow extending over several years' time at one or more points in the drainage area in question. Usually it will be desirable to carry on measurements of flow and gauge readings for a few months or more at the point considered, so as to ascertain the relative yield in second-feet per square mile at this point and at the localities where longer time records are available. Then advantage can be taken of the longer time records to estimate the probable yield at the point in question during this same period of years.

PART IV.

CONSTRUCTING AND FINISHING MAPS.

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CONSTRUCTING AND FINISHING MAPS.

CHAPTER X.

MAP PROJECTIONS.

330. **MAP PROJECTIONS.*** — A map is a representation of any portion of the surface of the earth, on a plane surface, for the purpose of showing on a convenient scale the relative positions of points and natural features on the earth. Any map showing topographic details is essentially a sketch, and its accuracy is controlled by certain points whose positions have been accurately determined. The greater the number of such points the greater is the accuracy of the sketch as a whole. Since on a survey extending over a large area the positions of the points of control are usually defined by means of spherical coördinates (latitude and longitude) it is customary to show meridians and parallels on the map and to plot the positions of these controlling points from their latitudes and longitudes, the rest of the map being filled in with reference to these controlling points.

Any representation of a portion of a spherical surface on a plane is necessarily distorted, the amount of the distortion depending upon the area mapped. One of the first problems in map making, then, is to find some mode of projection which will make it possible to show a portion of the earth's surface on a plane with the minimum amount of distortion.

Various forms of projection have been devised, each one suited to some special purpose. Some of these projections are purely geometric, while others are arbitrary. On very small areas the distortion is small and the different kinds of projection give

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* For a discussion of the relative accuracy of the different projections and historical notes see the Coast Survey Report for 1880, Appendix 15, p. 287.

nearly the same results, but for very large areas, such as that of a continent or a hemisphere, the different projections give widely different results.

These map projections may be divided into two general classes: first, those which are true projections or perspectives; and second, those which are constructed by developing a cone or a cylinder on a plane surface. Many different kinds of projection have been devised, but the most common systems are enumerated below.

The chief projections of the first class are

1. Orthographic.
2. Stereographic.
3. Gnomonic.

Of the second class the following may be mentioned.

1. Rectangular.
2. Mercator's.
3. Simple Conic.
4. Bonne's.
5. Polyconic.

331. **Orthographic Projection.** — In orthographic projection the eye is supposed to be looking along a line which is perpendicular to the plane of the map. This is the ordinary system of projection used in architectural and engineering drawings, where objects are shown in plan, and front and side elevations. In map making this method is used chiefly in representing hemispheres. This projection shows the map greatly contracted near the edges, as will be seen by an examination of Fig. 127. Such a projection could be used to show the details of those regions only which are near the middle of the map.

332. **Stereographic Projection.** — In stereographic projection the eye is assumed to be on the surface of the sphere at the pole of a great circle, whose plane is the plane of projection, and in the opposite hemisphere from that which is to be mapped. The points on the hemisphere to be mapped are projected on this plane by straight lines drawn from these points to the position of the eye. This projection is used to represent a hemisphere where

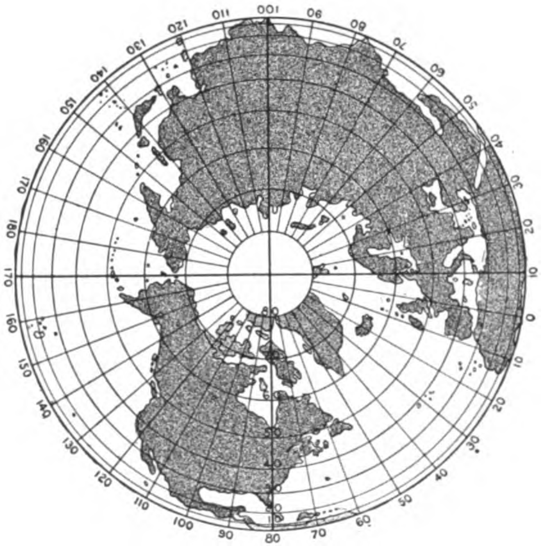


FIG. 127. ORTHOGRAPHIC PROJECTION ON THE PLANE OF THE EQUATOR.

FIG. 128. STEREOGRAPHIC PROJECTION ON THE PLANE OF THE EQUATOR.

it is desired to show details near the edge of the map, since the area near the edge is greatly expanded as compared with the central portion, as will be seen by comparing the length of 10 degrees of latitude near the equator (Fig. 128) with 10 degrees near the pole.

333. **Gnomonic Projection.** — In the gnomonic projection the area to be mapped is projected on a plane tangent to the sphere, and the eye is supposed to be at the center of the sphere. The characteristic of this projection is that all great circles appear on the map as straight lines, since great circles are projected on the map by planes passing through the position of the eye. This projection is much used in constructing charts for "great circle sailing," the shortest route * between points on the globe being shown as a straight line.

The simplest chart to construct on this projection is the "polar chart," in which the plane of projection is tangent to the earth at the pole. All parallels of latitude then appear on the chart as circles, whose centers are at the pole. The radius of any circle is equal to R cot L where R is the radius of the sphere, and L is the latitude of the parallel. The length of a degree of latitude becomes greater as the latitude itself decreases, this spacing increasing so rapidly toward the equator that the polar chart cannot be conveniently extended to the tropics. The meridians all appear on the chart as straight lines radiating from the pole. In using this chart to obtain the position of a great circle it is only necessary to draw a straight line between the two points in question and this will be the great circle desired. The *vertex*, or point where the track comes nearest the pole, may be determined at once from the chart. The latitudes and longitudes of any points, and the bearing of any portion of the line, may be taken directly from this chart. Fig. 129 shows a chart constructed on a plane tangent to the equator in longitude 80 degrees west. The parallels on this chart are not circular curves as in the polar chart. It will be seen that the meridians are necessarily straight on all charts constructed by the gnomonic projection.

* On the assumption that the earth is a sphere the great circle is the shortest route between two points.

334. Rectangular Projection. — Rectangular projection consists in drawing meridians and parallels on the area to be represented, both being shown as straight lines and uniformly spaced. A vertical line near the middle of the map is chosen as a central meridian and this line is subdivided into parts which are proportional to the lengths of the degrees of latitude on the earth's surface (see Table XIV, p. 410). Lines are drawn through these

FIG. 129. GNOMONIC PROJECTION ON PLANE TANGENT AT THE EQUATOR.

points at right angles to the central meridian, and the degrees of longitude are laid off on these lines. The relative length of a degree of latitude and a degree of longitude is taken as that corresponding to the latitude of the middle of the map, i.e., the length of a degree of longitude **in any part of the map** equals the length of a degree of latitude multiplied by the cosine of the **middle latitude** of the area mapped. Such a projection is expanded at the top, i.e., at the north in the northern hemisphere, and contracted at the bottom. The relation between the degrees of latitude and longitude on the rectangular projection will be found so close, however, to that which exists on the sphere that this projection is often sufficiently accurate for making a projection on a small area, such as that covered by a plane-table sheet. (See Fig. 130.)

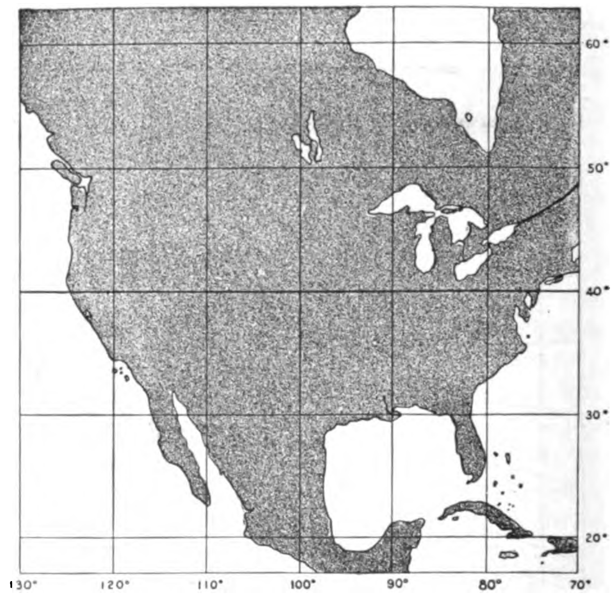


FIG. 130. RECTANGULAR PROJECTION.

335. Projection with Converging Meridians. — A modification of the rectangular projection consists in selecting two parallels, one near the top and one near the bottom of the map, and subdividing these into degrees of longitude corresponding to the latitudes of these two parallels. A degree of longitude on one of these parallels equals a degree of latitude times the cosine of the latitude of that parallel. These corresponding points of subdivision are then joined by straight lines representing the meridians. These meridians all converge at approximately the true angle on the sphere. Only two of the parallels, however, are graduated exactly in the right proportion. The parallels of latitude are all shown as straight lines. (See Fig. 131.) This projection is suitable for plane-table sheets covering ordinary areas.

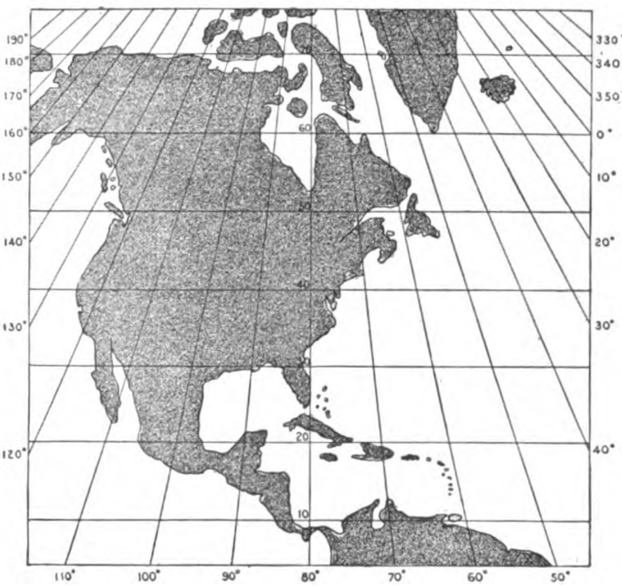


FIG. 131. PROJECTION WITH CONVERGING MERIDIANS.

336. The Mercator Projection. — The Mercator projection (Fig. 132) is a modification of the simple cylindrical projection. The latter is constructed by projecting the meridians and parallels onto the surface of a cylinder which is tangent to the earth at the equator, and then developing this cylinder on the map. The meridians and parallels all appear as straight lines; the meridians are equally spaced, while the distances between the parallels increase toward the poles. In the Mercator chart, which is much used in navigation, the parallels are so spaced that the ratio of the length of a degree of latitude to that of a degree of longitude in any part of the map is the same as the ratio existing at the corresponding point on the sphere. Hence the bearing of one point from another as shown by this chart is that course which a vessel would have to steer continuously in order to sail from one point to the other. A straight line on the chart cuts all of the meridians at the same angle. This path on the earth's surface which corresponds to a straight line on the Mercator chart is a curve known as the *loxodrome.* The distance between any two points on the earth is not correctly shown by this chart, and to obtain the true distance between them these two points

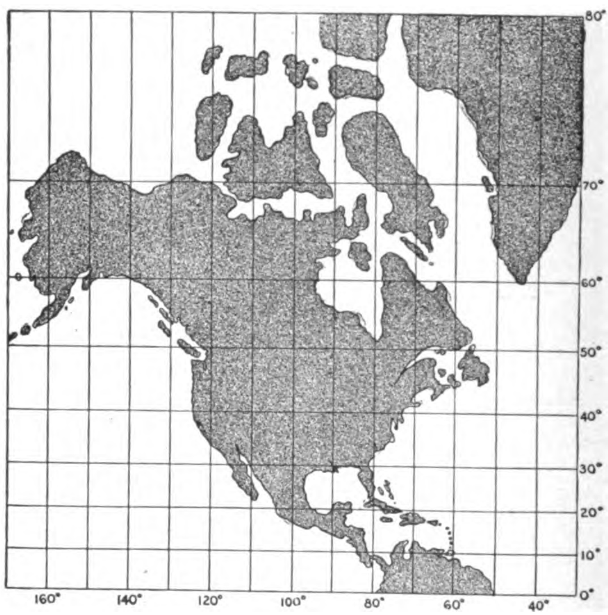


FIG. 132. MERCATOR'S PROJECTION.

must be transferred to some other kind of chart (such as the polyconic chart) and the distance scaled, or else the distance must be calculated by the proper formulas. On account of the varying length of the degrees of latitude all areas near the pole are greatly expanded, while those at the equator are not distorted.

337. Conic Projection. — In the simple conic projection a cone is conceived to be tangent to the middle parallel of the map, the apex of the cone being therefore in the earth's axis produced. This cone is developed on the plane of the map as follows. A vertical line is chosen as the central meridian of the map and a point on this line is chosen as the middle latitude. The radius of this middle parallel of latitude is then laid off on the side toward the pole, giving the position of the apex of the cone on the central meridian, which is the center of a series of circles representing the parallels of latitude. This radius equals N cot L, where N is the normal * and L is the latitude of this parallel. A

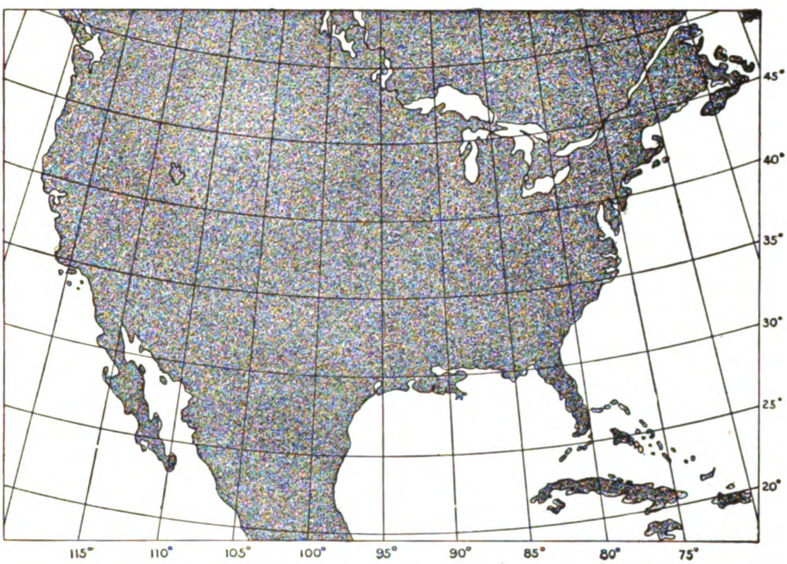


FIG. 133. SIMPLE CONIC PROJECTION.

circle is then drawn representing the middle parallel of latitude. Beginning at the middle parallel, distances are laid off on the central meridian which are proportional to the degrees of latitude on the earth's surface. The middle parallel itself is also subdivided into degrees of longitude which are proportional to the degrees of longitude on the earth's surface for this particular latitude. The meridians are all shown as straight lines drawn

* The normal is the distance from the parallel of latitude to the axis of the spheroid, measured along a vertical line. If the earth may be considered as a sphere, then the radius of the circle representing the parallel is R cot L, where R is the radius of the sphere.

from the apex of the cone to the points laid off on the middle parallel. The remaining parallels of latitude are circles through the points laid off on the central meridian, the center in each case being the apex of the cone. The parallels and meridians will therefore intersect at right angles in all parts of the map. The length of the degree, however, on all parallels except the middle one, is evidently slightly in error. (See Fig. 133.) The distortion in this projection is so slight that it becomes appreciable only on very large areas.

A modification of this projection which is sometimes used consists in assuming a cone whose surface intersects that of the sphere near the middie portion of the map. This will be found to produce slightly less distortion than where the tangent cone is used.

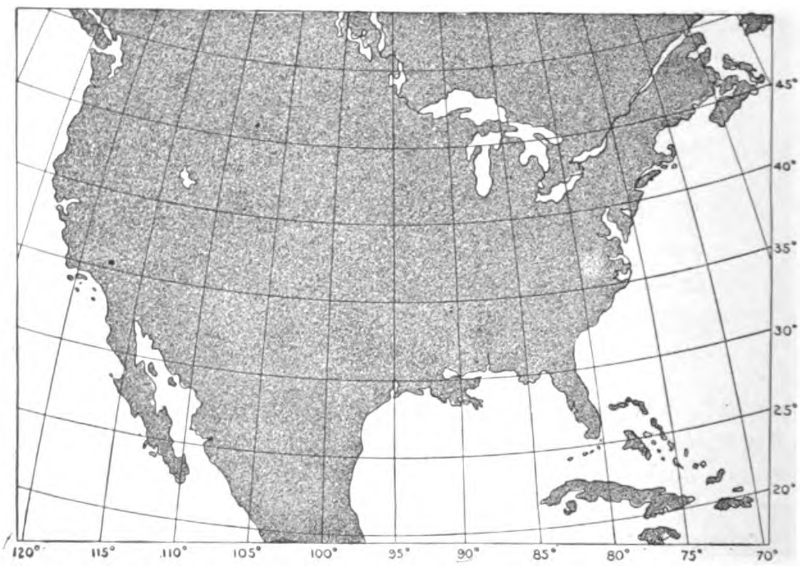


FIG. 134. BONNE'S PROJECTION.

338. Bonne's Projection. — Bonne's projection (Fig. 134) is a modification of the simple conic projection. Each of the concentric parallels of latitude is divided into degrees of longitude proportional to those on the sphere. The parallels are shown as circles, the radius being equal to $R \cot L$ (when the earth is regarded as a sphere) as in the conic projection. Hence the central meridian and every parallel of latitude is divided as on the sphere. The distortion is but slight and distances on the map can be scaled quite accurately. This projection is chiefly used in France.

339. **Polyconic Projection.** — In the polyconic projection the surface is developed on a series of cones, a different cone being used for each parallel of latitude. Each parallel, then, is developed independently on a cone whose apex is somewhere in the prolongation of the earth's axis. In this form of projection the degrees of latitude are laid off their true lengths on the central meridian, but it will be seen later that they are slightly too large near the east and west edges of the map, because the circles representing the parallels are not concentric as in the conic and in Bonne's projection. The angles of intersection of the meridians and parallels are always nearly true right angles. In fact, on a map of an area as large as that of the United States the unaided eye cannot easily detect the errors in these angles. An investigation of the errors in this projection shows that there is very little distortion and that distances can be scaled accurately enough to satisfy all of the requirements of a map. The polyconic projection was first used by the U. S. Coast and Geodetic Survey and has been adopted by nearly all of the Government surveys for certain kinds of maps.

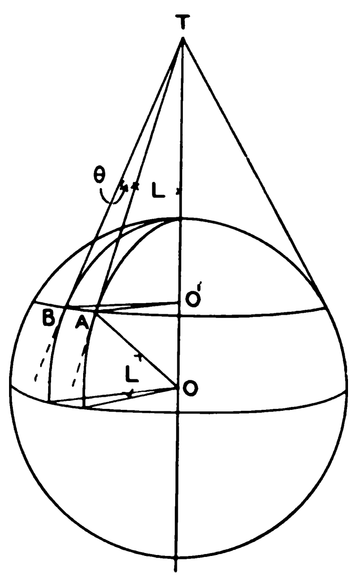


FIG. 135.

In each of the cones used in this projection the angle between the axis of the cone and an element will be seen from Fig. 135 to equal the latitude L of the parallel in question. The side of the tangent cone, TA, is found by the equation

$$TA = N \cot L$$

where N is the length of the normal and L is the latitude. In a sphere N would of course equal the radius AO. TA is the radius of the circle representing the developed parallel. If θ is the angle at T between two points AB on the parallel and the difference in longitude of A and B is dM then

$$\theta = dM \sin L$$

(see Vol. I, p. 154, and Art. 52, p. 50, of this volume).

Since the radius of curvature of the meridians is long it is not convenient to construct these circles by compass; they are therefore usually constructed by plotting the intersections of the meridians and parallels by means of their rectangular coördinates.

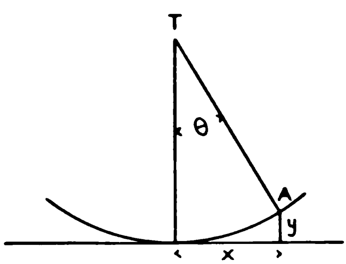


FIG. 136.

In Fig. 136 A represents the intersection of a meridian and a parallel. In order to compute the coördinates we have

$$\begin{aligned} x &= TA \sin \theta \\ &= N \cot L \sin \theta \\ &= N \cot L \sin (dM \sin L) \end{aligned} \qquad [59]$$

and

$$\begin{aligned} y &= TA \text{ vers } \theta \\ &= \frac{x \text{ vers } \theta}{\sin \theta} = x \tan \tfrac{1}{2}\theta \\ &= x \tan \tfrac{1}{2} (dM \sin L) \end{aligned} \qquad [60]$$

Tables have been computed from formulas [59] and [60] giving the coördinates in meters for different latitudes and for different distances east or west from the central meridian of the map. Tables XVI and XVII, pp. 412 and 414, giving these coördinates for a limited area, are extracted from a larger table in the Coast Survey Report for 1884, Appendix No. 6, p. 135.

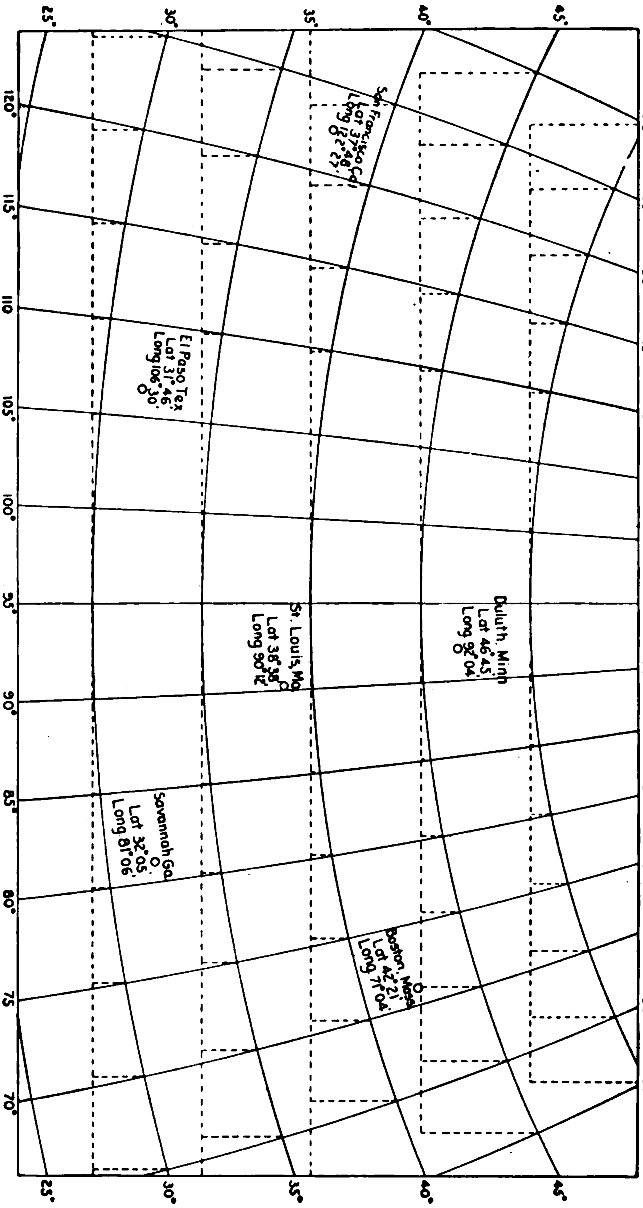


FIG. 137. POLYCONIC PROJECTION OF AN AREA INCLUDING THE UNITED STATES.

In laying out a map by the polyconic projection a central meridian is first drawn and the true distances between the parallels laid off (see Tables XIV and XV, pp. 410 and 411). Lines are then drawn at right angles to this central meridian through the points laid off. The abscissas of the intersections of meridians and parallels desired are then laid off on these perpendiculars and the ordinates are measured off at right angles on the side toward the pole. (See Fig. 137.) After these points of intersection have all been determined smooth curves are drawn through them representing meridians and parallels. Since the parallels are true circles on the map it will sometimes be convenient to draw these circles by means of *railroad curves* of proper radii.

The interval between consecutive meridians and parallels should be small enough so that the inclosed areas are practically rectangular. This not only facilitates the plotting of triangulation points but insures the accuracy of the map. In finishing the map it may be found convenient to omit some of the lines used in the plotting, in order that the details may not be confused by so large a number of lines passing through them. The interval should be the same in all parts of the map, i.e., if a 5-minute interval is adopted every 5-minute parallel and every 5-minute meridian should be shown. Portions of these lines may be omitted, if necessary, in order to avoid confusion at certain points, or to avoid drawing a line through a title or a note. In some maps only the intersections of meridians and parallels are preserved by means of very short lines which do not interfere with the detail shown on the map, and from which the original lines can be reproduced at any time.

In constructing a polyconic projection for small areas, such as for plane-table sheets, the meridians and parallels will be found almost straight, so that in many cases the rectangular projection or that described in Art. 335 can be substituted. The plane-table sheet shown in Fig. 68, p. 220, is a reproduction of a sheet which was originally drawn on a scale of $\frac{1}{10000}$; an examination of the tables used for plotting the polyconic projection will show that on this scale ($\frac{1}{10000}$) the projection which falls within the limits of a plane-table sheet is practically rectangular.

CHAPTER XI.

PLOTTING AND FINISHING TOPOGRAPHIC AND HYDROGRAPHIC MAPS.

340. IN this chapter the common methods of constructing and finishing topographic and hydrographic maps are briefly considered. The process may be divided into three parts.

1. The laying out of the projection, to show the position of the map on the earth's surface and to control the accuracy of the plotting.

2. The plotting, which includes the plotting of the points of control and the topographic points, and the sketching in pencil of all the details.

3. The inking of all lines, the tinting, and the lettering.

The various methods of constructing projections have been considered in Chapter X.

341. **PLOTTING THE TRIANGULATION.** — The triangulation points are plotted on the map by measurements from the meridians and parallels which have been laid out by one of the methods described in Chapter X. The linear distance of each triangulation station from the nearest meridian and from the nearest parallel is first computed and these two distances are laid out in the same way as when plotting a traverse point by the method of rectangular coördinates. For example, if a triangulation station has a latitude of 42° 12′ 43″.94 and a longitude of 71° 06′ 52″.64 and the meridians and parallels have been drawn on the map for every minute of latitude and longitude, it would only be necessary to compute the number of feet or meters in the seconds of the latitude and longitude and to plot the point by means of these distances. The distance of any point from the meridian and from the parallel may be found by means of tables given in Appendix No. 6 of the Coast Survey Report for 1884. In these tables are given the lengths of arcs of the meridian and

of the parallel, expressed in meters, for different latitudes. In these tables it will be found that for latitude 42° 13′,

$$43''.94 \text{ of latitude} = 1355.7 \text{ meters}$$
and
$$52''.64 \text{ of longitude} = 1207.4 \text{ meters}$$

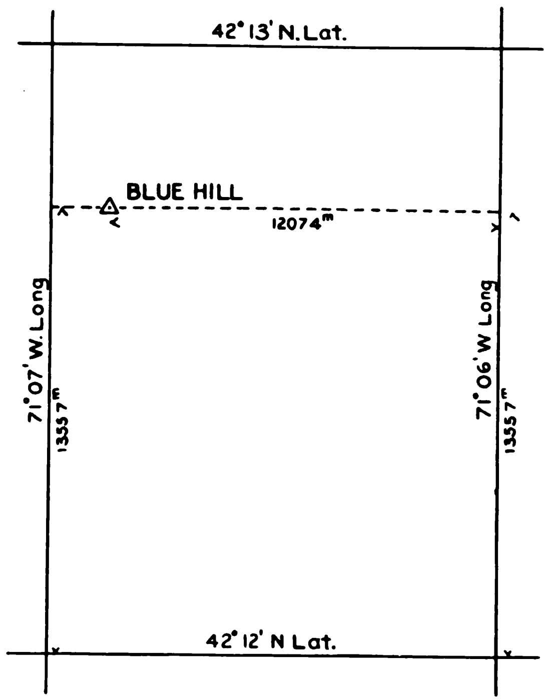


FIG. 138. PLOTTING A TRIANGULATION POINT.

The necessary measurements for plotting the triangulation point are indicated in Fig. 138. If desired the distances in meters for 16″.06 and 07″.36 could have been found and the point plotted from the nearer meridian and parallel. The positions of these stations on the map are then checked by scaling the lengths of the triangle sides. The interval between consecutive meridians and parallels should be such that the figure enclosed can safely be treated as a rectangle when plotting the triangulation points.

342. **Rectangular Coördinates.** — In city surveys or other surveys of limited extent where all points are to be referred to a pair of rectangular coördinate axes instead of to the actual meridians and parallels some meridian near the middle of the area to be mapped is chosen as the axis of Y, and all directions are referred to this primary meridian. All of the other so-called "meridians" are not true north lines but are parallel to this primary meridian line. As the axis of X is a straight line through the initial point perpendicular to the axis Y it is evident that neither the axis of X nor any of the lines parallel to it is a true east and west line. The positions of the triangulation points are plotted by means of their known distances from the two axes, just as the triangulation points of larger surveys are plotted by their distances from the actual meridians and parallels.

343. **Plotting Details on the Map.** — All points on the map, whether instrument stations or minor details, should be plotted with reference to the triangulation points, which are already on the map, in order that the accuracy of the map as a whole may be properly controlled. All of the triangulation stations should therefore be plotted before proceeding to plot any of the details. When traverses have been run they may, if desired, be plotted on the final map directly from the field notes or by means of coördinates. In laying out these traverses they should begin at one of the triangulation points even though the fieldwork did not start at any triangulation point, and the plotting of all traverses should be checked by closing on a triangulation point, or, in case the traverse does not close on a triangulation point, by calculating the coördinates of the end point. If the accuracy of a traverse is checked by closing on a plotted triangulation point this also checks the fieldwork, whereas checking by calculated coördinates checks only the plotting. If traverses have been run by means of deflection angles, as is often the case, it will be advisable to calculate the azimuths by means of these deflection angles for the purpose of plotting. The points on the traverse may be laid out from calculated coördinates if the traverse has already been calculated and adjusted to the triangulation (see Art. 55, p. 56). The details may be filled in by means of a protractor and scale. None of the details should be plotted, however, until the traverse

has been found to check. For methods of plotting a transit and tape survey see Vol. I, Chapter XV; for methods of plotting stadia traverses see Art. 165, p. 180 of this volume.

The transit points may be plotted directly from the notes by the usual methods, and if the position of any triangulation point (with which the traverse is connected) as located by the plotting of the traverse fails to coincide with the established position of the triangulation station, this error must be distributed through the traverse by shifting the transit points on the map in such a way as to make the traverse close and to alter the lengths and azimuths of the lines as little as possible; the position of a triangulation point should never be changed simply because a traverse fails to check its location. If the error of closure, however, is so large as to indicate a mistake in the work rather than an accumulated error, the mistake should be discovered and corrected before proceeding. If there are several transit stations close together on the map and there are only a few shots from each station it will be convenient to plot these azimuths by orienting the protractor on the nearest meridian shown on the map, plotting these azimuths at that point, and then transferring them to the proper station by means of a parallel ruler or by triangles. Where a large number of side shots have been taken at any station they can best be plotted by methods explained in Art. 165, p. 180.

344. TRANSFERRING PLANE-TABLE SHEETS OR OTHER MAPS TO THE LARGE MAP. — It is frequently necessary to assemble several small maps or field sheets on one large map. In this case the triangulation is first plotted on the large sheet; the details may then be transferred by means of the pantograph (Vol. I, Art. 423, p. 385) or by a system of squares which are drawn in pencil on each map to the proper scale. Since the squares may be drawn as small as desired the latter method may be made quite accurate. If it is desired to avoid marking the original plan squares may be drawn on tracing paper and laid over the original map when the drawing is transferred. Where a large area is to be covered the pantograph method will probably be the more rapid of the two. If only a small area is to be transferred so much time will be required to set up and adjust the pantograph that the method of transferring by squares may be preferable. In using either method the plotted positions of the triangulation points should be taken as reference points. If the pantograph is used it should be so adjusted that when the tracing point moves from one triangulation station to another on the original sheet the pencil point will move between the corresponding plotted points on the map.

345. **Finishing Plane-Table Sheets.** — Plane-table sheets, whether they are to be used for photographic reproduction, for making tracings, or for transferring to large maps, should be finished in ink to preserve the original sheet. If they are to be used for photographic reproduction they should be finished in black or some other color which will photograph clearly; blue should not be used for such work, as it will not photograph. If the original sheets are to be used as the finished map the conventional colors for contours and for water should be used. Practice differs as to some of the details of finishing maps, but the standards set by the Government surveys are chiefly followed where they are applicable.

When the plane-table sheets are to be used as a basis for one of the photographic processes of reproduction the map should be finished in every respect so as to give the appearance required in the printed copy. If, however, an engraving is to be made the appearance of the plane-table sheet is of secondary importance, since the neatness of the final copy will depend upon the work of the engraver; but so far as accuracy is concerned it should be remembered that he follows exactly the lines on the plane-table sheet.

The meridians and parallels should be numbered with the degrees and minutes of latitude and longitude. The positions of triangulation points should be preserved by means of a dot with a red triangle around it. Points whose positions were determined graphically during the plane-table work, such as cupolas, flag-poles, etc., may be indicated by a small red circle.

Lines should be sharp and clear in order to preserve the accuracy; in all cases the original lines should be closely followed. Roads should be inked in with parallel lines unless the width obviously varies. The conventional signs for stone walls, fences, indefinite lines, etc. are to be used if the scale is large enough to permit it. Houses should have a clear outline, with sharp corners; the conventional signs for vegetation should be used if the scale warrants such details.

346. FINISHING TOPOGRAPHIC MAPS. — The method of finishing the map will depend upon whether the original map is to be used as the final product or for making duplicates by means of photographic reproduction.

If the map is to be used for engineering purposes it should be finished with regard to preserving the accuracy of the surveys and the positions of the instrument points rather than to its general appearance. The lettering should not be ornamental and should be used in place of conventional signs except where the latter are clearer. Meridians and parallels, or coördinate lines, should be carefully preserved as a check on the change in scale due to shrinkage of the paper. If the map is for general (public) use it will need to be somewhat more ornamental than when it is to be used simply for engineering studies, and especial attention should be paid to making clear everything of a technical character which would not be readily understood by one who is unfamiliar with maps.

It is common practice to make the field sheets on a larger scale than that intended for the published map; in many cases the field sheets are made twice the size of the final map. The weight of lines and size of letters used on the office map will be governed by the amount of the intended reduction. It requires considerable experience to properly prepare a map for reduction, partly because the appearance of such a map is so different from that of the final map to which the draftsman is accustomed. The only way for the beginner to accomplish good results in this work is to first determine what the sizes of the lines and letters are to be on the final map and then to lay out the lines and letters exactly twice, three times, etc. larger so that they will reduce to the proper size when photographed down. Such a drawing will not look like ordinary drawings; the lines and letters will appear bold and the letters spaced too far apart and too far from the lines to which they refer. After a little practice the draftsman will acquire the ability to judge the appearance of the final plan from that of the orginal. The tendency of beginners in making drawings for reduction is to crowd the letters, to make them too small, and to place them too close to the lines.

A topographic map should be finished in such a manner that it will convey the desired information and can be readily interpreted. Although the extensive use of different colors on maps is not to be recommended it is sometimes necessary, in order to distinguish readily between land and water surfaces, to use at least one color beside black. In some cases water surfaces are shown in blue and in other cases the land is shown by a flat tint of yellow. When a tint is used on any map it should be a thin wash, so that there will be just enough color to show the desired distinction; deep colors are too conspicuous and injure the appearance of the map. As instances of the use of tints on maps the following may be mentioned. Some of the maps prepared by the U. S. Lake Survey show the land as a flat tint of yellow and the water as a bluish-green tint, different shades being used to represent different depths; the deepest shade represents the shoalest water and very deep water is indicated by white. On some of the maps of the U. S. Army Engineer Corps only one color is used; the water is shown in different shades of blue, the shoal water being shown by dark blue and deepest water by white. On the latter maps there is often so much detail shown on the land that it is all represented in black, because a tint in this case would be of no advantage in conveying information.

347. Scales. — A scale should always be shown on a map, both for convenience in scaling distances and for detecting errors due to changes in the dimensions of the paper. If, as it is assumed, the paper expands or contracts in exactly the same proportion in all its parts and in all directions, and if a scale is drawn on the map when the plotting is begun, this scale will always give correct distances. Since, however, maps of large size seldom do change so uniformly a complete elimination of such errors can only be effected by observing the changes in the spacing between consecutive meridians or parallels, or between lines of a rectangular coördinate system, and by making due allowance for these changes when scaling distances.

For convenience the scale is often shown on a map in different units of measurement, e.g., on many of the charts issued by the Government scales are shown giving distances in statute miles and in kilometers, and in some cases in nautical miles. Not infrequently two scales are given on the same map, one showing miles, half-miles, quarter-miles, etc., and the other showing distances in feet.

The most common form of scale consists of two parallel horizontal lines, drawn close together, with vertical lines of subdivision drawn between them, and spaced so as to give the desired units, the alternate spaces being inked in black. These spaces represent some large unit (such as a mile, a kilometer, or a thousand feet), except the one at the extreme left end, which is subdivided into tenths of a unit or other convenient fraction; these small spaces are also shown in alternate black and white like the main divisions of the scale. The vertical lines of subdivision are numbered for convenience in taking distances off the scale. A convenient arrangement of the scale is to mark the zero point at the right end of this subdivided space so that all spaces to the right of zero are long and those to the left are short spaces. In taking a distance from the scale with a pair of dividers, for instance, one point of the dividers is placed at the vertical line of the scale marked with the desired number of miles, feet, or other unit; keeping the right-hand point of the dividers in this position the left-hand point may be set at the division giving the desired decimal or other fraction. In this form of scale it is necessary to estimate the fractional parts of the smallest space.

It is customary to state the scale of the map even if this is also shown by the scales just described. This may be done by giving the number of feet or miles to one inch, or it may be stated as a fraction whose numerator is unity, for example $\frac{1}{12000}$ (1,000 feet to one inch). This method has the advantage that distances can be taken from the map in any desired unit of measurement without reference to the particular unit which was used in constructing the map, so that a person who is accustomed for example to the metric system only could take off distances in meters, with a metric scale, from a map which has been made with a foot scale. It is desirable that the scale adopted should be one that gives a simple fraction such as $\frac{1}{1000}$, $\frac{1}{10000}$, $\frac{1}{100000}$, rather than such numbers as $\frac{1}{960}$, $\frac{1}{1800}$, $\frac{1}{4800}$, which correspond to 80 feet, 150 feet, and 400 feet to one inch respectively. Such scales as $\frac{1}{1000}$, $\frac{1}{10000}$, etc. are especially convenient when using a metric scale, because the metric scale has a decimal subdivision. The arbitrary selection of a scale, however, is not always practicable, and in many cases this matter of choosing a simple scale would be of minor importance.

348. Diagonal Scale. — If a scale is desired in which it is unnecessary to estimate the fractional part of the small spaces, so that no uncertainty shall be introduced from this source, a diagonal scale may be constructed. Such a scale might be required in laying out a map projection, which of course must be done with great accuracy. The diagonal scale is constructed by drawing a series of equidistant horizontal lines and subdividing them by vertical lines as described above for the ordinary scale, except as regards the subdivision at the left-hand end of the scale, from which the fractional distances are obtained. (See

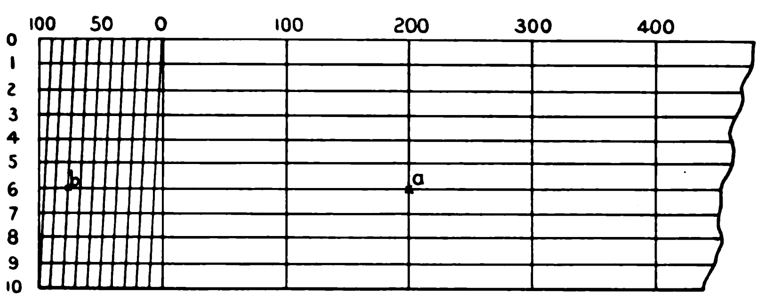


FIG. 139. DIAGONAL SCALE.

Fig. 139.) The number of horizontal lines must be one greater than the number of subdivisions of the smallest space shown, i.e., if a decimal subdivision is to be used eleven lines will be required. The large space on the left of the zero point is subdivided on the upper and lower lines into the required number of spaces, in this case ten. Diagonal lines are then drawn, each point on the upper line being joined to the point imme-

diately to the left of it on the lower line, so that each of these lines is the diagonal of a rectangle whose sides are respectively the distance between the upper and lower lines and one of these small spaces on the upper (or lower) line. In the diagonal scale shown in Fig. 139 the large spaces are hundreds, the small divisions tens, and units can be obtained by means of the diagonal, since this diagonal line divides the space into proportional parts. Each of these diagonal lines moves one unit farther from the zero line in descending from one horizontal line to the next. To obtain a distance by means of this scale first take off the distance on the top line with a pair of dividers, taking the hundreds on the right of zero and the tens on the left; then move the dividers down to the horizontal line numbered in a column at the left end of the scale with the last figure (units) and open out the dividers until the left-hand point is on the next diagonal line to the left, the right-hand point being on the same vertical line as before. For example, 276 would be obtained by spacing off the distance *ab*, Fig. 139.

It is evident that this scale must be constructed with extreme care in order to give results of greater accuracy than those obtained by the ordinary scale, for the errors of estimating fractional parts of a division are not large.

349. Conventional Signs for Topographic Maps.* — Topographic conventional signs are used to represent the form of the surface and such physical features as roads, buildings, cultivated fields, forest growth, rivers, etc. The conventional signs which have come into general use are those which have been adopted and extensively used by the Government surveys. For the most part such symbols have been devised as will suggest by their shape the plan of the objects represented and at the same time be readily recognized and easily drawn.

The kinds of conventional signs used on any map have an intimate relation to the purpose for which the map is made. European maps, for example, have been made chiefly for military purposes, and the topographic features which are impor-

* For a complete description of the conventional signs used on topographic maps see "Topographical Drawing and Sketching" by Lieut. Henry A. Reed, published by John Wiley & Sons.

tant in military operations are prominently represented on these maps. In Switzerland, on the other hand, many of the maps have been prepared for the benefit of tourists, and such features as roads and footpaths are given in great detail. In the United States the Geological Survey maps have been prepared primarily as a basis for a geological study of the country, while the Coast Survey maps are chiefly for the benefit of navigators. The character of the conventional signs used on these maps consequently varies according to the purposes for which the maps were made.

For convenience these topographic signs may be divided into three general classes: those adapted to small-scale maps, those for maps of intermediate scale ($\frac{1}{5000}$ to $\frac{1}{10000}$), and those for large-scale maps such as landscape architects' plans. On small-scale maps the conventional signs necessarily represent the object much larger than it would be if drawn to scale, whereas on large-scale maps the object may in many cases be drawn to the actual scale of the map.

350. **Conventional Signs for Small and Intermediate-Scale Maps.** — The conventional signs which are used by the Coast Survey are well adapted to small-scale maps. The conventional signs for maps of intermediate scale vary but little from those used on small-scale maps except as to size. On very small scales, however, such as those published by the U. S. Geological Survey, it is impracticable to represent vegetation except by a flat tint. In Figs. 157 to 159 will be found specimen maps of the U. S. Coast and Geodetic Survey, the U. S. Geological Survey, and a portion of a plane-table survey which was made on a scale of $\frac{1}{5000}$.

Most of these topographic signs are executed free hand and require not a little skill to obtain good results. The appearance of a map depends largely upon the execution of the topographic signs. To learn to draw these symbols accurately and with skill requires a careful study of the details of making each kind of sign, some of the more important of which will be described.

351. Grass. — The symbol for grass land, or cleared land, is intended to imitate the appearance of tufts of grass. (See

Fig. 140.) These tufts are always made with their bases parallel to the bottom of the map and are distributed over the area in such a way as not to form rows or to have the appearance of regularity in size, and yet so as to have the appearance of uniformity when viewed as a whole. To accomplish this result it will be well to draw a few isolated signs in different parts of the area to be covered and then to fill in between these with additional tufts, so that the symbols will be evenly distributed over the area. To indicate that it is not cultivated growth it is well to fill in some of the larger spaces with a few small, incomplete tufts as shown in Fig. 140.

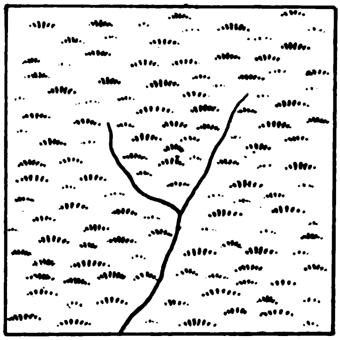


FIG. 140. GRASS.

The individual sign is drawn as follows. It is composed of five to seven short lines (see Fig. 141) all apparently radiating from a point a little below the base of the symbol, the tops of these lines forming a curve. These are made by beginning at the left-hand side with a dot and increasing the length of the lines up to the middle one, which is vertical, and then diminishing the length of line and ending with a dot. These lines should all be made with a down stroke of the pen and may be slightly curved, as shown in Fig. 141.

FIG. 141.

The size of the individual tufts will depend upon the scale of the map; a common fault of beginners is to make the symbol too large. It is a mistake to cover large areas with this symbol for grass because it has a tendency to make the area appear level on account of the general parallelism of the individual symbols, so that in some cases the relief indicated by the contours may be hidden. For this reason some draftsmen prefer to use the grass symbol only to represent flat cleared land, such as meadows, and where a large hilly area is to be shown as grass to represent this by a flat, light-green tint and to letter the word "grass" on the area.

352. SALT AND FRESH MARSHES. — A salt marsh is represented by fine horizontal lines equally spaced and ruled parallel to the bottom of the map, with occasional grass signs on the

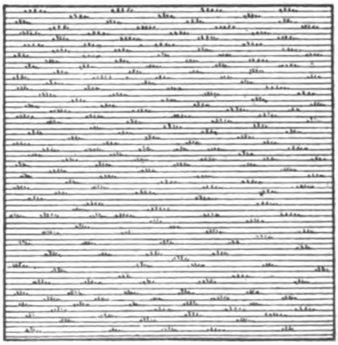


FIG. 142. SALT MARSH.

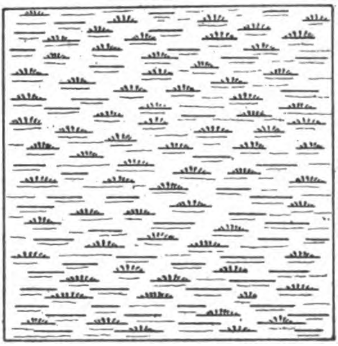


FIG. 143. FRESH MARSH.

horizontal lines. (See Fig. 142.) The lines are commonly placed about $\frac{1}{30}$ inch apart for small or intermediate scale maps. For a fresh marsh, or swamp, the horizontal lines are short and somewhat heavier, and on many of these lines tufts of grass are represented. The intermediate spaces are then filled in by very light horizontal free-hand lines. (See Fig. 143.)

353. CULTIVATED LAND.—This symbol is shown by alternate rows of short dashes and dots representing furrows in ploughed land. (See Fig. 144.) In drawing this symbol a straight-edge and a right-line pen should be used for the dashes, but an ordinary fine-pointed pen will be required in making the dots. It will be well to draw the dash lines first and to fill in the dotted lines afterward. The dashes should be so drawn that the spaces between them will not be opposite to those in the adjacent lines on either

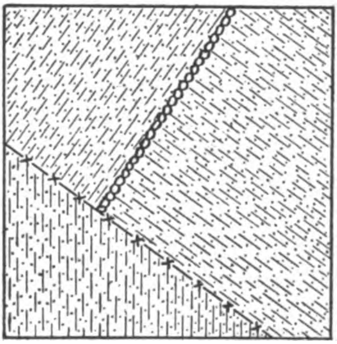


FIG. 144. CULTIVATED LAND.

side. The dots should be fine and close together. It is customary to draw the lines parallel to one of the sides of the area but preferably not parallel to the bottom of the map because horizontal lines are often used in symbols for water, such as marsh, swamp, tidal flats, etc. The conventional sign on adjoining cultivated fields should not be parallel.

354. SAND AND GRAVEL. — Sand is represented by dots evenly distributed over the area and close together. In representing a shore line a row of heavy dots closely spaced is first drawn. (See Fig. 145.) A row of lighter dots is next drawn, these being placed opposite the spaces between the dots of the first row. The third row is shown smaller, the dots being lighter than those of the second but the same spacing between the rows being observed. No attempt is made to draw more than these three rows in this manner, but light dots are distributed over the remaining area, the spaces being wider than those along the shore.

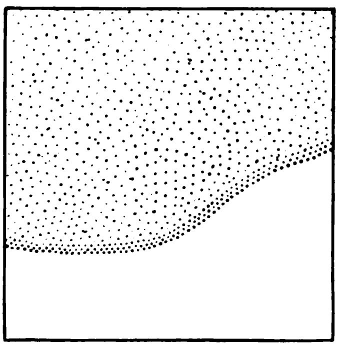


FIG. 145. SAND.

Great care must be exercised to make the spaces equal and the dots round, not rough and jagged. To avoid a streaked appearance the dots should not be made in straight rows. Some draftsmen make them in fan-shaped clusters, being careful to change their direction frequently.

355. WATER-LINING. — Water surfaces are indicated either by a flat wash of a blue tint or by water-lining. In the latter method a series of fine lines is drawn parallel to the shore line, the spaces between the lines being gradually increased as the distance from the shore increases. (See Fig. 146.) To execute good water-lining the first line should be drawn at a distance from the shore equal to about the width of the line, and every deviation in the shore line should be carefully followed. The next line should be so drawn as to be everywhere at exactly the same distance from the preceding line, the space

between these two lines being a little greater than the space between the shore and the first line. The third line will be at a slightly greater spacing but parallel to the second line throughout its length, and so on, the interval between the successive lines being gradually increased and the weight of the line being gradually decreased. The change in spacing between the lines should be so gradual that no one place can be found where the spacing suddenly increases. In order that all the water-lining in the different bodies of water shown on a map may appear uniform it will be well to draw all of the water-lines which go next to the shore-lines throughout the map and then draw the second lines of all of the water-lining, and so on. In this way the intervals between the lines can be accurately judged and the weights of the lines can be kept uniform.

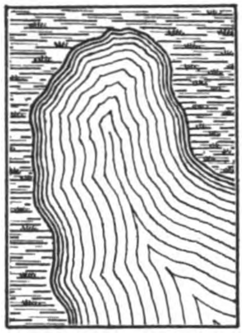


FIG. 146. WATER-LINING.

On the U. S. Coast Survey maps, where it is important to show soundings and the character of the bottom, the water-lining is omitted; the high water line is usually shown as a full line and the low water line is indicated by the conventional sign for sand, ledge, etc.

356. TREES. — In representing trees distinction is generally made between deciduous and evergreen by using different symbols, and a further distinction is sometimes made between oak and other deciduous trees. These distinctions originated in the requirements of military topography.

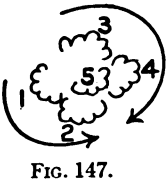


FIG. 147.

The sign for the round-leaf deciduous trees, which applies to all except oak, is shown in detail in Fig. 147. Each stroke of the pen is numbered in the order in which the strokes are made, and the arrow shows the direction in which the pen is moved in each case. It should give the effect of a plan of a tree with a slight shading on the lower right-hand side. These little symbols are scattered rather irregularly over the area if wild growth is to be represented. (See Fig. 148.) In all conventional signs wild growth is distinguished from cultivated growth by giving the symbol the appearance of irregularity as contrasted with the regularity of cultivation. For example, trees representing an orchard are placed in rows and have a smoother and rounder outline than trees in a forest.

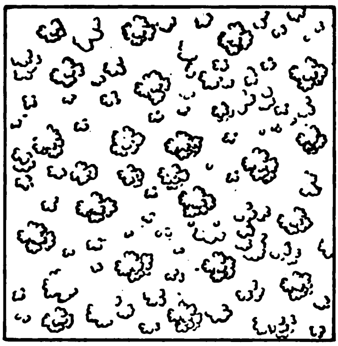


FIG. 148. ROUND LEAF.

The symbol for an evergreen tree is composed of 5 or 6 short radiating lines as shown in Fig. 149, the lines being of uniform width. These symbols should be drawn on the area in varying sizes and with different weights of lines. (See Fig. 150.) The tendency of beginners is to make these symbols in rows, and in order

FIG. 149.

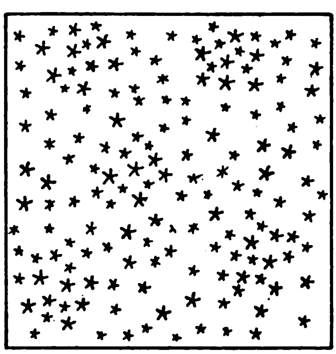


FIG. 150. EVERGREEN.

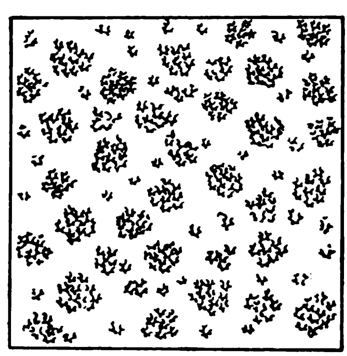


FIG. 151. OAK.

to avoid this it is well to first make groups of trees in different parts of the area and then to fill the intervening spaces with smaller symbols drawn with a lighter stroke. It is only by such a systematic arrangement that the desired effect of irregularity can be obtained.

The symbol for oak trees is illustrated in Fig. 151. Each individual symbol is made without raising the pen from the paper. These symbols are made in clusters of five or six so as to form a unit about the same size as the sign for roundleaf. It will be noticed that the strokes made in drawing the sign are similar to those for the roundleaf symbol except that the scallops are concave outward instead of inward.

Orchards are represented by rows of the deciduous tree symbols (roundleaf), the individual symbols being carefully spaced and the rows parallel to a side of the field. (See Fig. 152.)

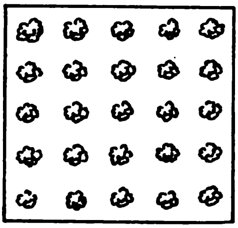


FIG. 152. ORCHARD.

Other topographic signs which require no special explanation are shown in Fig. 153.

357. **Conventional Signs for Large-Scale Maps.** — On maps of large scale the conventional symbols used are often similar to those used for small-scale maps, but the positions of the individual trees are located and the symbols are executed with more detail. These symbols are used on landscape maps which are arranged to produce a certain effect such as is shown in Fig. 161.

Landscape plans made for office use only often have symbols which are less ornamental than the usual ones and which can be more rapidly made. For example, on these office maps the stems of the trees are sometimes represented by black dots with the diameter marked in inches beside them. The approximate extent of the foliage of the trees may also be shown to scale by dash circles. The limits of plantings are shown by dash lines, and the botanical names of the shrubs are lettered on the area.

If the landscape plan is finished on tracing cloth water color tints are not satisfactory, because the tracing cloth will not lie flat after it has once been wet. A fair result may be obtained by making the ink drawing on the glossy side of the cloth and putting the tint on the reverse side with colored crayons. In using tints great care should be taken to make them all light and uniform. The general appearance of the map will depend largely on a proper harmonizing of the colors used; the different

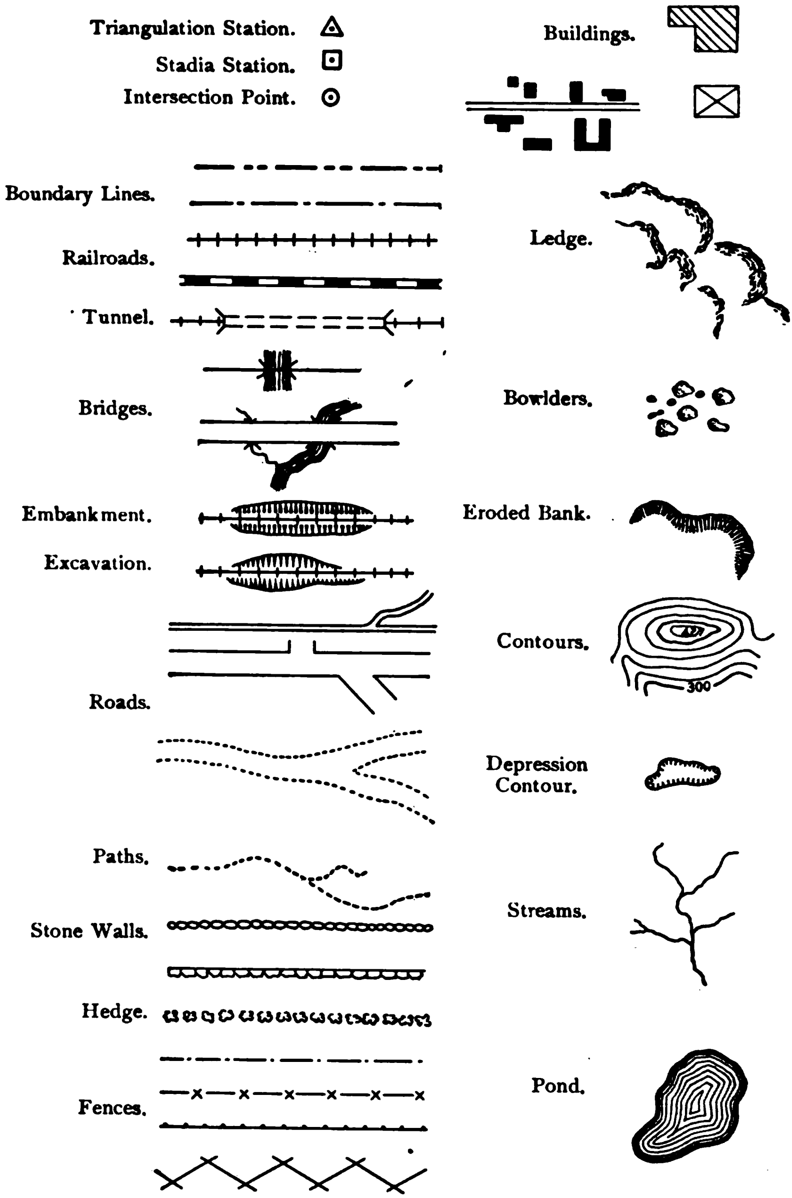


FIG. 153. CONVENTIONAL SIGNS FOR TOPOGRAPHIC MAPS.

tints can be varied slightly so as to produce a result that is pleasing to the eye.

Fig. 160 is a portion of a contour map on a scale of 60 feet to one inch, prepared for landscape architects' studies. It shows the contours, outlines of trees, roads, fences, and buildings.*

358. COLORED TOPOGRAPHIC SYMBOLS. — Flat tints of different colors are sometimes employed to represent different topographic signs. These, however, are not much used in this country except on maps of large scale. Unless the paper has been mounted, it will be necessary to stretch it before attempting to lay a tint over a large area. The method of applying tints is explained in most books on drafting. The following are the usual colors for such work.

Water....................	Flat tint of Prussian blue or indigo.
Sand.....................	Flat tint of yellow ochre.
Cultivated land...........	Flat tint of burnt sienna with parallel lines of a darker shade of the same tint ruled across it.
Grass....................	Flat tint of Hooker's green, No. 2.
Trees....................	Flat tint of a mixture of Prussian blue and gamboge.
Rock....................	Flat tint of sepia.
Roads....................	Flat tint of yellow ochre.
Buildings, wood..........	Yellow.
Buildings, brick..........	Crimson lake.
Contours................	Burnt Sienna (on ordinary plan). Crimson Lake (on tinted surface).

Gamboge should never be used on tracing cloth as it spreads through the cloth and ruins the tracing.

It is a common fault of beginners to apply too dark a tint. The shade of the wash should depend largely upon the area to be tinted; the smaller the area the deeper should be the tint, to give a proper effect.

359. **Representation of Relief.** — There are two general systems of representing the form of the surface of the ground, by shading and by contour lines. The first system may consist of horizontal curves or of hachure lines, which are lines showing the direction of steepest slope. In any shading system the object is to show the steepness of slope and the location of summits, valleys, etc. in such a manner that the map may be easily read. In the contour system the shape of the ground is represented by

* See Appendix A for specifications.

curves of equal elevation, but in addition to showing the form of the surface, contours show the actual elevations of points on the map. If the contour interval is taken sufficiently small the contour system really becomes a shading system and shows slopes by means of the depth of color and at the same time does not affect the accuracy of the elevations. The shading system gives a result which is pleasing to the eye and easily interpreted, but it does not give the exact information conveyed by the contour system. For this reason contours are chiefly used in this country in making topographic maps. Hachures are commonly used, however, for special purposes, such as representing eroded banks, etc. In some maps the contour and hachure systems are combined.

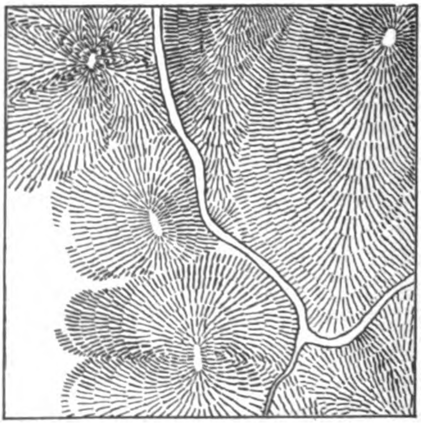


FIG. 154. HACHURES.

Fig. 154 is a portion of a map in which the topography is shown by hachures. In drawing hachure lines, contours are first sketched in lightly in pencil; then the hachures are put in by single strokes of the pen, each line being drawn at right angles to the contour, the hachures being equally spaced. The steepness of the slope is shown by the thickness of the line, a heavy line indicating a steep slope.

Contour lines are usually represented in burnt sienna or in black. They are drawn either with a contour pen (swivel pen) or with a fine-pointed common pen like Gillott's No. 303. The contour lines should be very fine so that they will not obscure the rest of the drawing. For ease in counting the lines it is customary to make every fifth line heavier than the others and to mark the elevations of some of these contours so that the elevation of any point may be quickly determined. Contours are generally shown for some whole number of feet above the datum, e.g., every 10 feet, every 20 feet, etc. (See Fig. 158.)

360. **Hydrographic Maps.** — The general method of finishing a hydrographic map is similar to that used for a topographic map. Such maps show, in addition to the shore line, the topography along the shores and of the submerged portions. Just how much of the topography of the surrounding country will be required depends upon the use which is to be made of the map. If, for example, it is prepared for a study of a wharf project the location of streets and buildings in the vicinity should appear on the plan. A hydrographic chart for the purposes of navigation will include merely a sufficient amount of topography to show any landmarks which may be of use to the navigator, such as lighthouses, church spires, or other conspicuous objects. It is well to leave plotted on the finished map a number of the transit stations, so that the survey can be easily connected with other maps or adjacent surveys.

Where the soundings are represented they are usually given in feet and tenths and lettered in black ink, the number representing the depth of the water below the datum. Where the datum is mean low water, as is customary, those soundings which are below the datum are lettered in black, while those above the datum are shown in some other color. The figures are usually written so that the decimal point is at the exact position of the sounding. On the finished map it will not be necessary in most cases to show all of the soundings; only enough of them are lettered to give all necessary information.

By means of the plotted soundings contour lines are sketched in the same manner as the contour lines on any topographic map, the contour interval depending upon the amount of detail required. In some of the charts of the U. S. Coast and Geodetic Survey the depths are represented to the nearest quarter of a foot up to about 24 feet (4 fathoms) and beyond this depth they are recorded in fathoms; the contours are usually drawn for every 6-foot interval. On the navigation chart of a small river the soundings should be recorded in feet and tenths, and contours every 3 or 6 feet will suffice. For dredging work in a harbor the soundings are usually recorded in feet and tenths and the contours are sketched with one, three, or six-foot intervals. A map of this kind is shown in Fig. 162.

In finishing hydrographic maps the high-water line should be the heaviest line on the map, the original pencil line being carefully followed; the low-water line should be the next heaviest line on the map. The conventional signs for sand, ledge, etc. are used in representing the low-water line. Swamps are not limited by any definite line drawn on the map but the area is covered by the conventional symbol; this symbol should, however, be made to follow very closely the limiting line sketched on the original sheet. Signs for marsh, grass, etc. should always be parallel to the bottom of the map. Lighthouses, buoys, etc., located on the chart should be shown by their conventional signs or, if preferred, by lettering the name of the object. The color and number of buoys should be indicated.

Subaqueous contours are usually represented as dot-and-dash lines, the shallowest contour having one dot between the dashes, the next contour in depth having two dots between the dashes, the next three dots, and so on. Where the contour interval is one fathom, the number of dots between the dashes shows the depth in fathoms at any contour. Any intermediate contours are represented by a continuous row of dots.

Maps of rivers should have, in addition to the data already mentioned, an arrow showing the direction of the current, and sometimes the kind of material forming the bottom is lettered at the proper place on the map.

361. Conventional Signs for Hydrographic Maps. — The conventional signs shown in Fig. 155 are in common use on hydrographic maps.

362. LETTERING. — In all lettering on topographic or hydrographic maps simple rather than elaborate and ornamental styles are preferable. If a plan is to be published for general

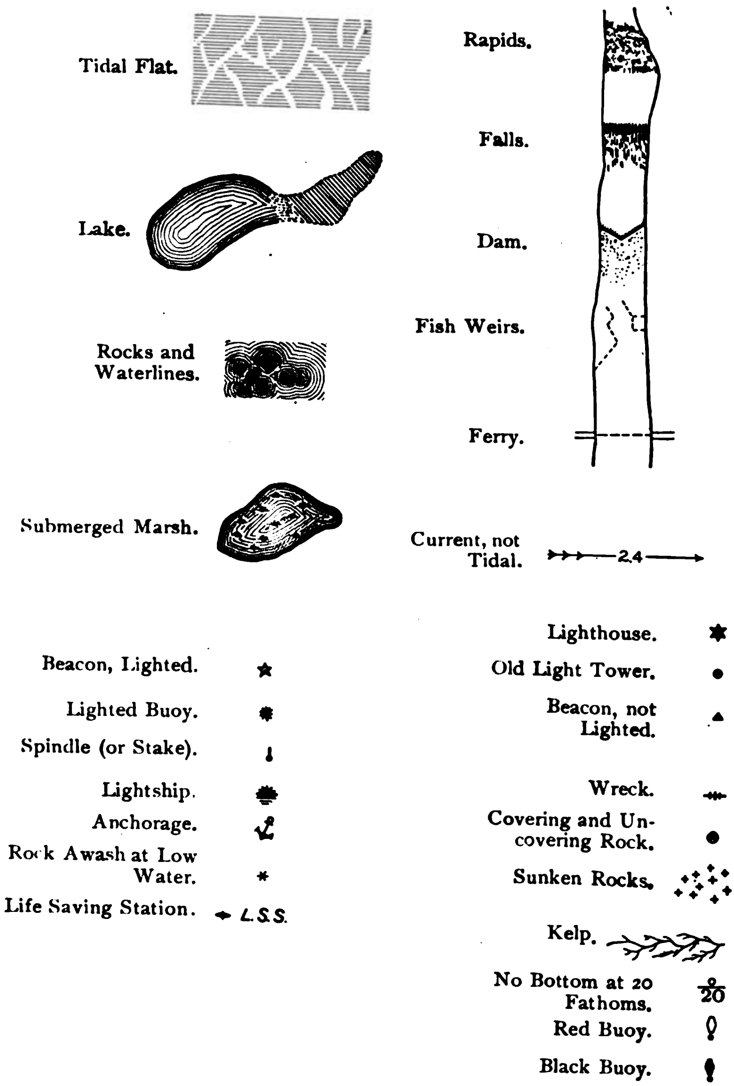


FIG. 156. CONVENTIONAL SIGNS FOR HYDROGRAPHIC MAPS.

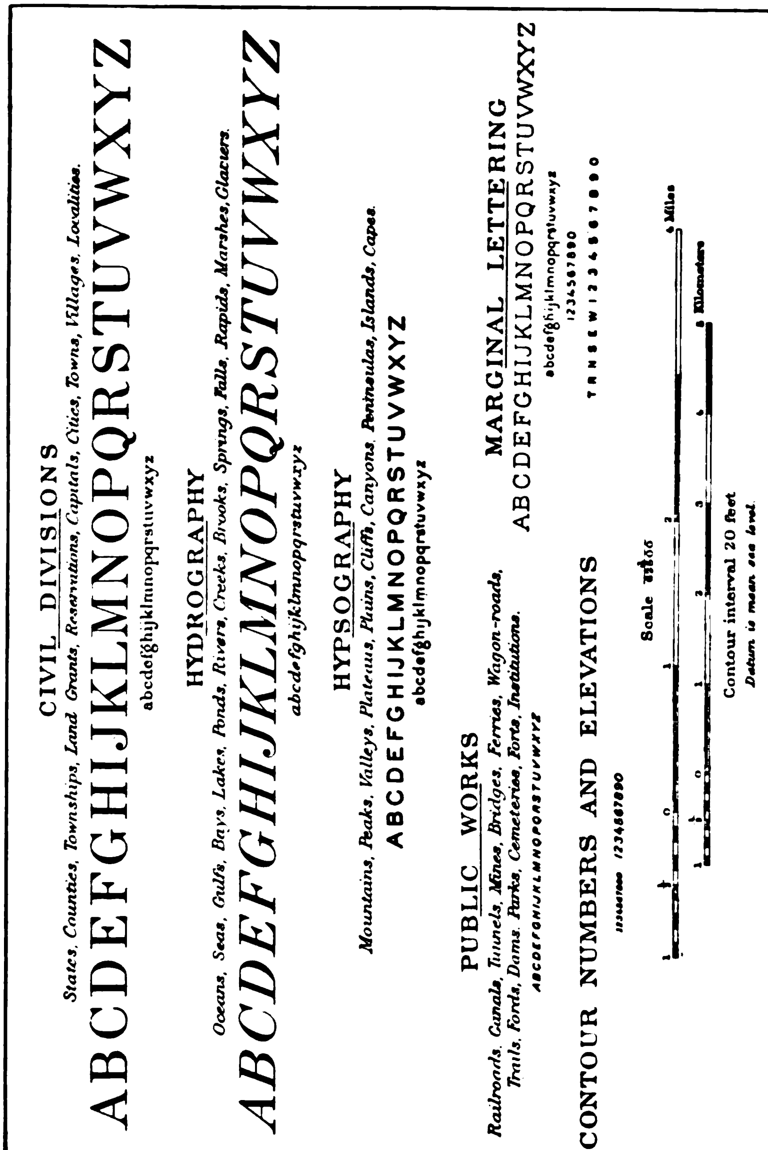


FIG. 158. LETTERS USED ON U. S. GEOLOGICAL SURVEY MAPS.

use the title should as a rule be in vertical Roman letters, or at least the principal line of the title should be in Roman letters, the other letters being vertical Gothic. It is bad taste to put both vertical and inclined letters in the same title. For office use a plainer letter, such as the Reinhardt style, may be adopted. On the Coast Survey and the Geological Survey all words on the map referring to land are in vertical letters and those referring to water are in inclined letters. (See Vol. I, pp. 424–9.) The size of letters to be used on a map depends upon the scale and the importance of the object described. In the title for a topographic map the most important word (usually the locality) is represented by the largest letters, and the size of the other lines of the title will be in proportion to the importance of the information conveyed.

Fig. 156 shows the styles of letters used on the U. S. Geological Survey maps.

363. Border Line.—Title.—Meridian.— Every finished topographic or hydrographic map should have a suitable title, a border line, a meridian line, a scale, and a note giving any other needful data, such as the datum used, the contour interval, unit for soundings, etc. (See Vol. I, Arts. 473–8, pp. 422–8.) The title, note, and meridian line should be placed in such positions on the map that the drawing as a whole will look well balanced. These positions will of course depend upon the spaces around the drawing which are available for this purpose. The weight of line used for the meridian, scale, etc. should be consistent with the general style of the map.

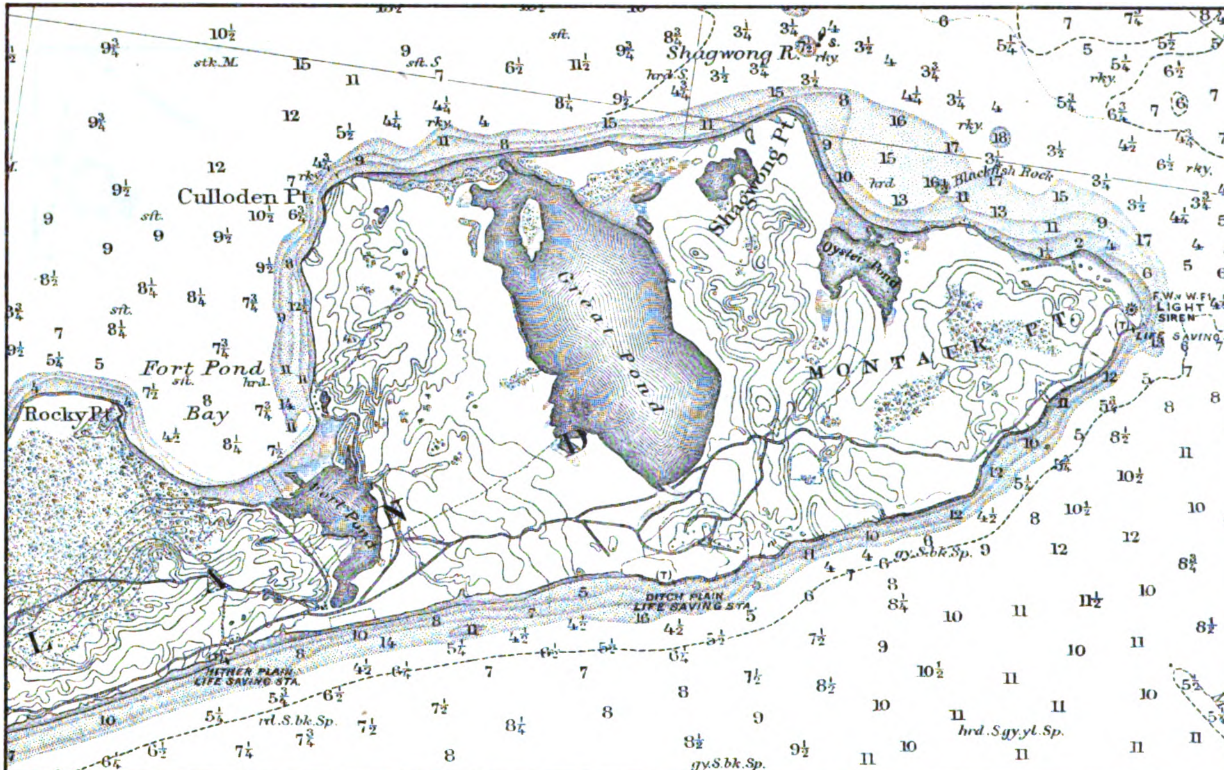


FIG. 157. PORTION OF THE COAST CHART NO. 114.

(By permission of U. S. Coast and Geodetic Survey.)

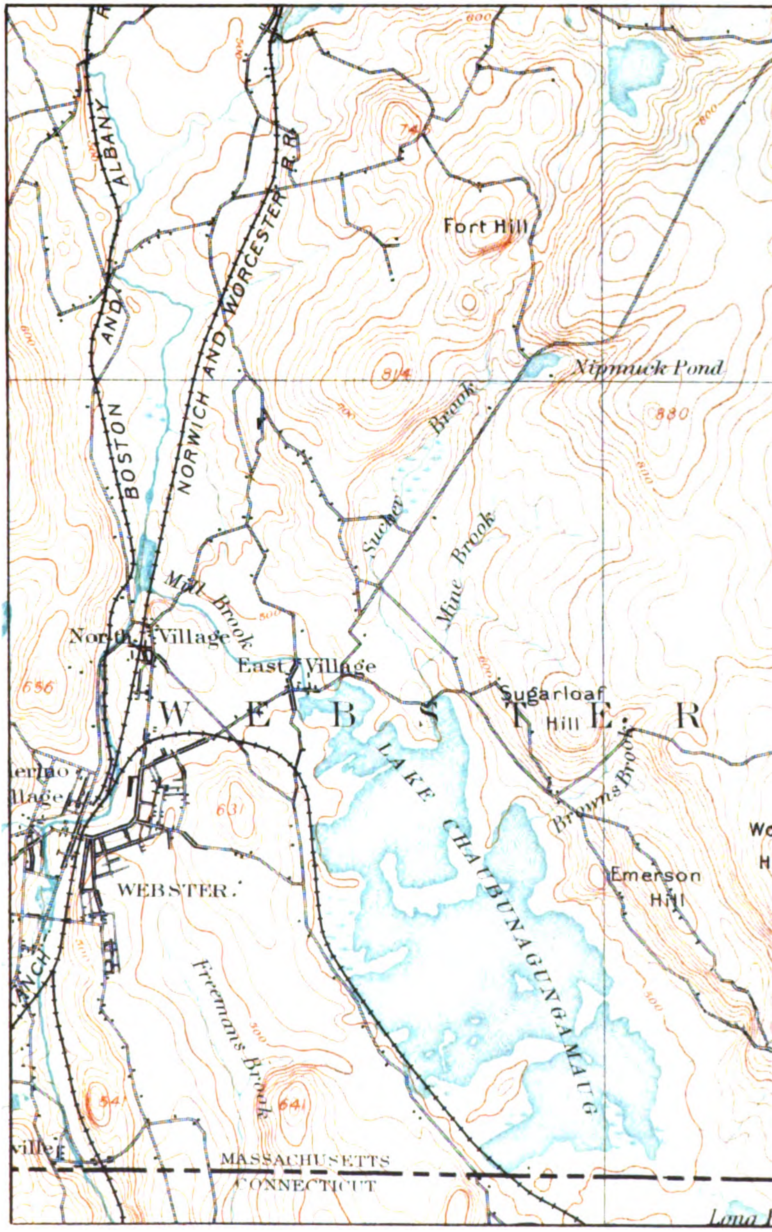


Fig. 158. Portion of Webster, Mass. Sheet. Scale $\frac{1}{62,500}$

(By Permission of U. S. Geological Survey)

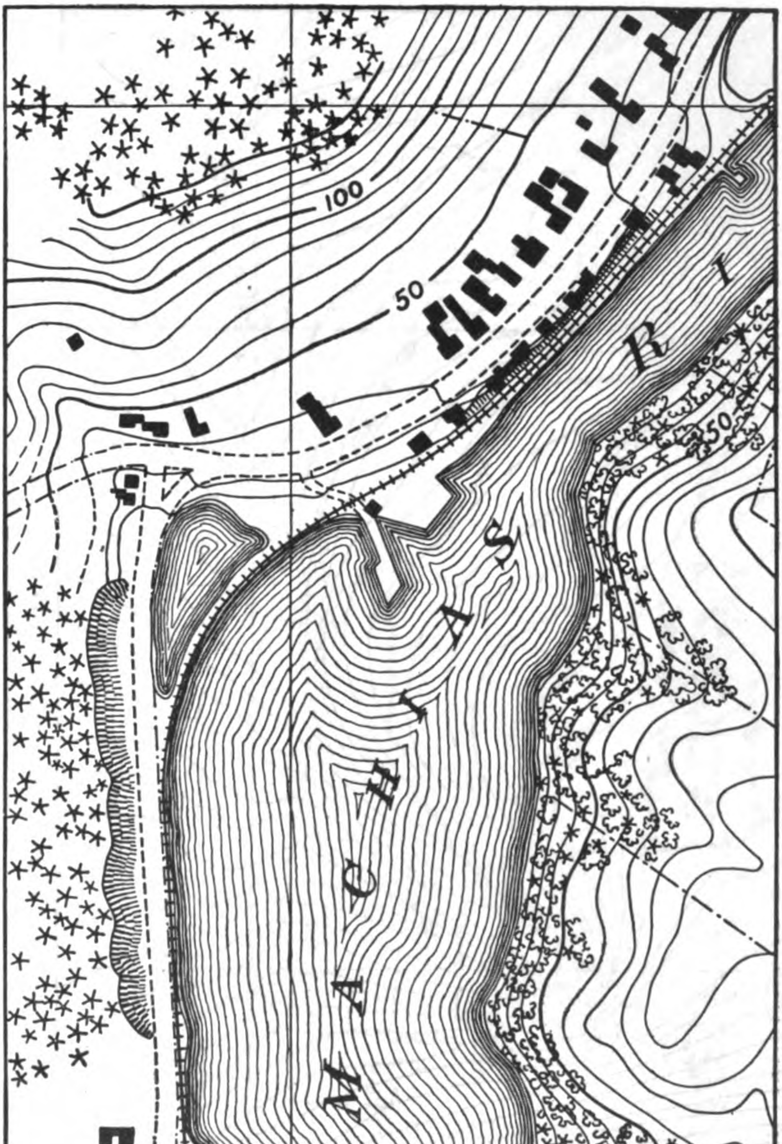


FIG. 159. — PORTION OF A PLANE-TABLE SURVEY. Scale $\frac{1}{5000}$.
(By permission of Professor A. G. Robbins.)

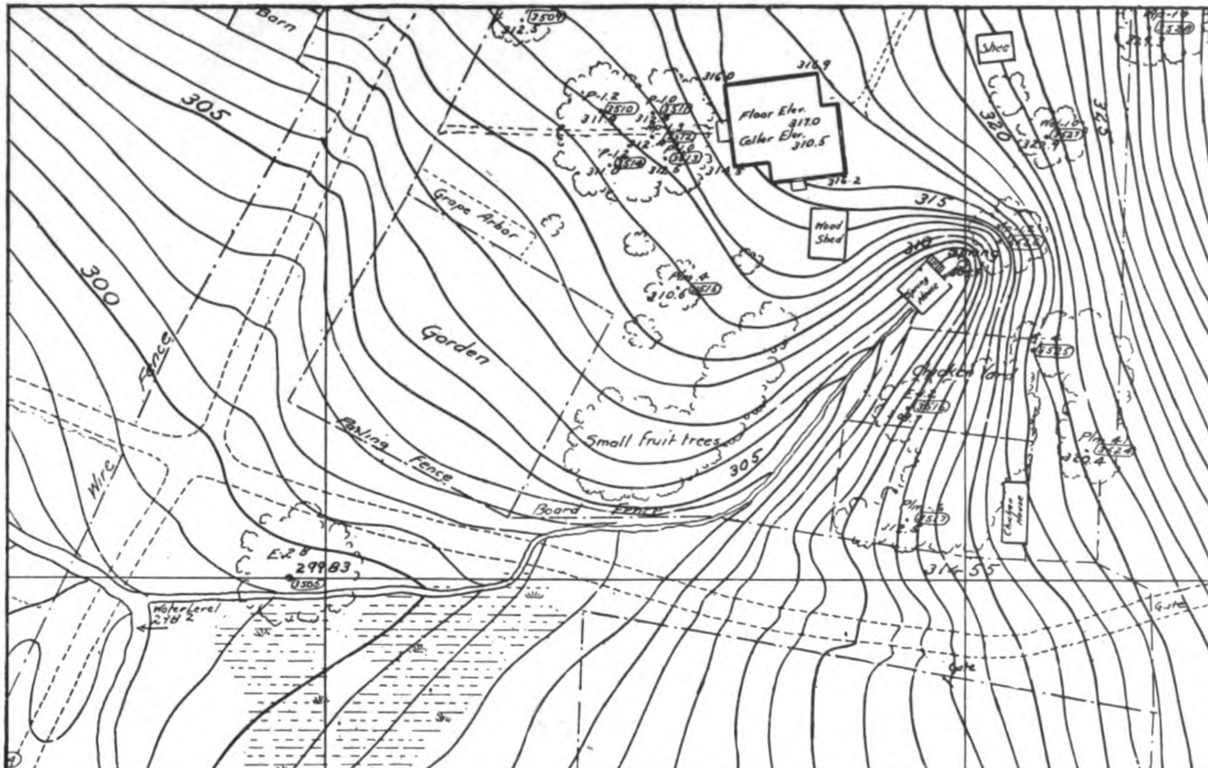
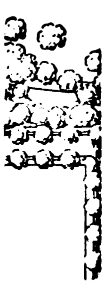
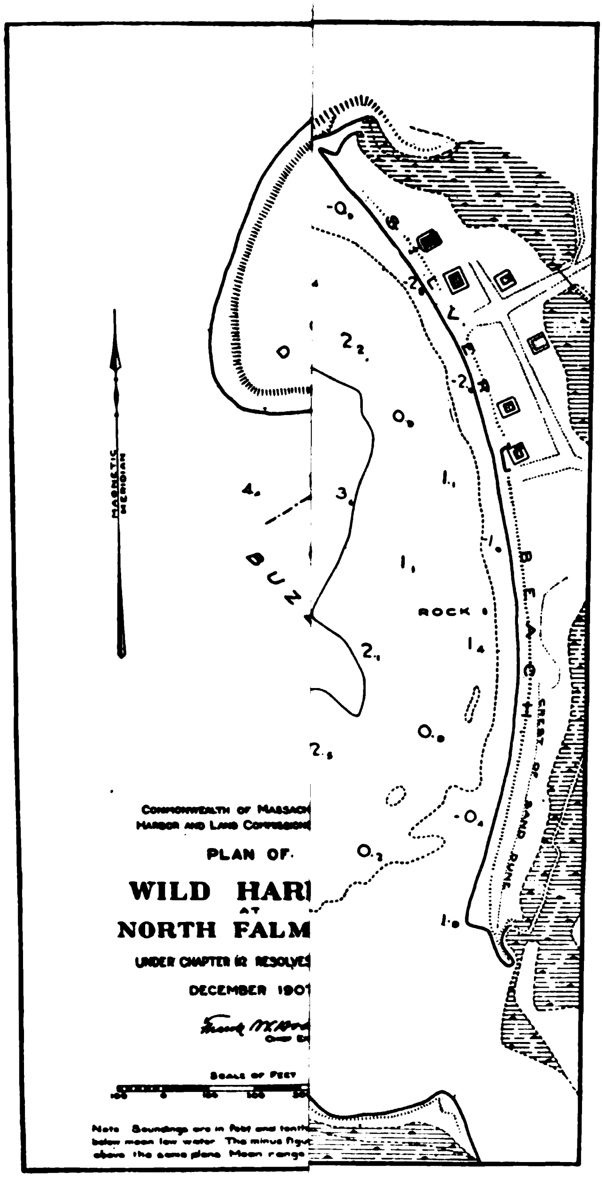


FIG. 160. PORTION OF TOPOGRAPHICAL MAP FOR LANDSCAPE ARCHITECTS' STUDIES. Scale 1 inch = 60 feet.
(By permission of G. H. Taylor.)





MAGNETIC MERIDIAN
ROCK
CREST OF SAND DUNE
PLAN OF
WILD HAR
AT
NORTH FALM
SCALE OF FEET

TABLES.

TABLE I. CORRECTION FOR EARTH'S CURVATURE AND REFRACTION.

(Art. 14, p. 17; Art. 117, p. 119; Art. 193, p. 212.)

Feet.	Feet.	Feet.	Feet.	Miles.	Feet.	Miles.	Feet.
100	.000	1000	.02	1	0.6	11	69.4
200	.001	2000	.08	2	2.3	12	82.7
300	.002	3000	.18	3	5.2	13	97.0
400	.003	4000	.33	4	9.2	14	112.5
500	.005	5000	.51	5	14.4	15	129.1
600	.007	6000	.74	6	20.6	16	146.9
700	.010	7000	1.01	7	28.1	17	165.8
800	.013	8000	1.32	8	36.7	18	185.9
900	.017	9000	1.67	9	46.4	19	207.2
1000	.020	10000	2.06	10	57.4	20	229.5

Miles.	Feet.	Miles.	Feet.	Miles.	Feet.	Miles.	Feet.
21	253.1	31	551.4	41	964.7	51	1492.5
22	277.7	32	587.6	42	1012.2	52	1551.6
23	303.6	33	624.9	43	1061.0	53	1611.9
24	330.5	34	663.3	44	1111.0	54	1673.3
25	358.6	35	703.0	45	1162.0	55	1735.8
26	388.0	36	743.7	46	1214.2	56	1799.6
27	418.3	37	785.6	47	1267.7	57	1864.4
28	449.9	38	828.6	48	1322.1	58	1930.4
29	482.6	39	872.8	49	1377.7	59	1997.5
30	516.4	40	918.1	50	1434.6	60	2065.8

TABLE III. Continued.

Lat.	Log *A*	Log *B*	Log *C*	Log *D*	Log *E*
° ′					
25 00	8.509 4639	8.511 8881	1 .07457	2 .2759	5 .8300
10	4606	8783	1 .07785	2 .2780	5 .8326
20	4573	8684	1 .08111	2 .2801	5 .8352
30	4540	8584	1 .08435	2 .2822	5 .8379
40	4507	8484	1 .08758	2 .2842	5 .8405
50	4473	8383	1 .09080	2 .2862	5 .8431
26 00	4439	8283	1 .09400	2 .2882	5 .8458
10	4406	8181	1 .09718	2 .2902	5 .8485
20	4372	8079	1 .10036	2 .2922	5 .8512
30	4337	7977	1 .10351	2 .2941	5 .8539
40	4303	7874	1 .10666	2 .2960	5 .8566
50	4269	7771	1 .10979	2 .2978	5 .8593
27 00	4234	7667	1 .11290	2 .2997	5 .8620
10	4200	7563	1 .11600	2 .3015	5 .8647
20	4165	7458	1 .11909	2 .3033	5 .8675
30	4130	7353	1 .12217	2 .3051	5 .8702
40	4094	7248	1 .12523	2 .3069	5 .8730
50	4059	7142	1 .12829	2 .3086	5 .8757
28 00	4024	7036	1 .13132	2 .3104	5 .8785
10	3988	6929	1 .13435	2 .3121	5 .8813
20	3952	6822	1 .13737	2 .3137	5 .8841
30	3917	6714	1 .14037	2 .3154	5 .8870
40	3881	6607	1 .14337	2 .3170	5 .8898
50	3845	6498	1 .14635	2 .3187	5 .8926
29 00	3808	6389	1 .14932	2 .3203	5 .8955
10	3772	6280	1 .15228	2 .3218	5 .8983
20	3735	6171	1 .15522	2 .3234	5 .9012
30	3699	6061	1 .15816	2 .3249	5 .9041
40	3662	5950	1 .16109	2 .3264	5 .9069
50	3625	5840	1 .16401	2 .3279	5 .9098
30 00	3588	5729	1 .16692	2 .3294	5 .9127
10	3551	5617	1 .16981	2 .3309	5 .9157
20	3514	5505	1 .17270	2 .3323	5 .9186
30	3476	5393	1 .17558	2 .3337	5 .9215
40	3439	5281	1 .17845	2 .3351	5 .9245
50	3401	5168	1 .18131	2 .3365	5 .9274
31 00	3363	5054	1 .18416	2 .3379	5 .9304
10	3325	4941	1 .18700	2 .3392	5 .9334
20	3287	4827	1 .18983	2 .3405	5 .9363
30	3249	4713	1 .19266	2 .3418	5 .9393
40	3211	4598	1 .19548	2 .3431	5 .9423
50	3173	4483	1 .19828	2 .3444	5 .9453
60	8.509 3134	8.511 4368	1 .20108	2 .3456	5 .9484

TABLE III. Continued.

Lat.	Log *A*	Log *B*	Log *C*	Log *D*	Log *E*
° ′					
32 00	8.509 3134	8.511 4368	1 .20108	2 .3456	5 .9484
10	3096	4252	1 .20387	2 .3469	5 .9514
20	3057	4136	1 .20666	2 .3481	5 .9544
30	3018	4020	1 .20944	2 .3493	5 .9575
40	2980	3903	1 .21220	2 .3504	5 .9605
50	2940	3786	1 .21496	2 .3516	5 .9636
33 00	2901	3669	1 .21772	2 .3527	5 .9667
10	2862	3551	1 .22047	2 .3539	5 .9698
20	2823	3433	1 .22321	2 .3550	5 .9729
30	2784	3315	1 .22594	2 .3561	5 .9760
40	2744	3197	1 .22866	2 .3571	5 .9791
50	2704	3078	1 .23138	2 .3582	5 .9822
34 00	2665	2959	1 .23409	2 .3592	5 .9853
10	2625	2840	1 .23680	2 .3602	5 .9885
20	2585	2720	1 .23950	2 .3612	5 .9916
30	2545	2600	1 .24219	2 .3622	5 .9948
40	2505	2480	1 .24488	2 .3632	5 .9980
50	2465	2360	1 .24756	2 .3642	6 .0011
35 00	2425	2239	1 .25024	2 .3651	6 .0043
10	2384	2118	1 .25291	2 .3660	6 .0075
20	2344	1997	1 .25557	2 .3669	6 .0107
30	2304	1875	1 .25823	2 .3678	6 .0140
40	2263	1754	1 .26088	2 .3687	6 .0172
50	2222	1632	1 .26353	2 .3695	6 .0204
36 00	2182	1510	1 .26617	2 .3704	6 .0237
10	2141	1387	1 .26881	2 .3712	6 .0269
20	2100	1265	1 .27145	2 .3720	6 .0302
30	2059	1142	1 .27407	2 .3728	6 .0334
40	2018	1019	1 .27670	2 .3735	6 .0367
50	1977	0895	1 .27932	2 .3743	6 .0400
37 00	1936	0772	1 .28193	2 .3750	6 .0433
10	1895	0648	1 .28454	2 .3758	6 .0466
20	1853	0524	1 .28715	2 .3765	6 .0499
30	1812	0400	1 .28975	2 .3772	6 .0533
40	1771	0276	1 .29234	2 .3779	6 .0566
50	1729	0151	1 .29494	2 .3785	6 .0600
38 00	1687	8.511 0027	1 .29753	2 .3792	6 .0633
10	1646	8.510 9902	1 .30011	2 .3798	6 .0667
20	1604	9777	1 .30269	2 .3804	6 .0701
30	1562	9652	1 .30527	2 .3810	6 .0734
40	1521	9526	1 .30785	2 .3816	6 .0768
50	1479	9401	1 .31042	2 .3822	6 .0802
60	8.509 1437	8.510 9275	1 .31299	2.3827	6 .0836

TABLE III. Continued.

Lat.	Log *A*	Log *B*	Log *C*	Log *D*	Log *E*
° ′					
39 00	8.509 1437	8.510 9275	1.31299	2.3827	6.0836
10	1395	9149	1.31555	2.3832	6.0871
20	1353	9023	1.31811	2.3838	6.0905
30	1311	8897	1.32067	2.3843	6.0939
40	1269	8771	1.32323	2.3848	6.0974
50	1227	8644	1.32578	2.3852	6.1008
40 00	1184	8517	1.32833	2.3857	6.1043
10	1142	8391	1.33088	2.3861	6.1078
20	1100	8264	1.33342	2.3866	6.1113
30	1057	8137	1.33596	2.3870	6.1148
40	1015	8010	1.33850	2.3874	6.1183
50	0973	7883	1.34109	2.3878	6.1218
41 00	0930	7755	1.34358	2.3882	6.1253
10	0888	7628	1.34611	2.3885	6.1289
20	0845	7500	1.34864	2.3889	6.1324
30	0803	7373	1.35117	2.3892	6.1360
40	0760	7245	1.35370	2.3895	6.1395
50	0718	7117	1.35623	2.3898	6.1431
42 00	0675	6989	1.35875	2.3901	6.1467
10	0632	6861	1.36127	2.3903	6.1503
20	0590	6733	1.36379	2.3906	6.1539
30	0547	6605	1.36631	2.3908	6.1575
40	0504	6477	1.36883	2.3910	6.1612
50	0461	6348	1.37135	2.3913	6.1648
43 00	0419	6220	1.37386	2.3914	6.1684
10	0376	6092	1.37638	2.3916	6.1721
20	0333	5963	1.37889	2.3918	6.1758
30	0290	5835	1.38141	2.3919	6.1795
40	0247	5706	1.38392	2.3921	6.1831
50	0204	5578	1.38643	2.3922	6.1868
44 00	0162	5449	1.38894	2.3923	6.1905
10	0119	5320	1.39145	2.3924	6.1943
20	0076	5192	1.39396	2.3925	6.1980
30	8.509 0033	5063	1.39648	2.3925	6.2017
40	8.508 9990	4935	1.39898	2.3926	6.2055
50	9947	4806	1.40149	2.3926	6.2092
45 00	9904	4677	1.40400	2.3926	6.2130
10	9861	4548	1.40651	2.3926	6.2168
20	9818	4420	1.40902	2.3926	6.2206
30	9776	4291	1.41153	2.3926	6.2244
40	9733	4162	1.41404	2.3925	6.2283
50	9689	4034	1.41655	2.3925	6.2321
60	8.508 9647	8.510 3905	1.41906	2.3924	6.2359

TABLE III. Continued.

Lat. ° ′	Log A	Log B	Log C	Log D	Log E
46 00	8.508 9647	8.510 3905	1.41906	2.3924	6.2359
10	9604	3776	1.42157	2.3923	6.2398
20	9561	3648	1.42409	2.3922	6.2436
30	9518	3519	1.42660	2.3921	6.2475
40	9475	3391	1.42911	2.3920	6.2514
50	9433	3262	1.43163	2.3918	6.2553
47 00	9390	3134	1.43414	2.3917	6.2592
10	9347	3005	1.43666	2.3915	6.2632
20	9304	2877	1.43917	2.3913	6.2671
30	9261	2749	1.44169	2.3911	6.2710
40	9219	2621	1.44421	2.3909	6.2750
50	9176	2493	1.44673	2.3906	6.2790
48 00	9133	2364	1.44926	2.3904	6.2830
10	9091	2236	1.45178	2.3901	6.2870
20	9048	2108	1.45431	2.3898	6.2910
30	9005	1981	1.45683	2.3895	6.2950
40	8963	1853	1.45937	2.3892	6.2990
50	8920	1725	1.46190	2.3889	6.3031
49 00	8878	1598	1.46443	2.3886	6.3071
10	8835	1470	1.46696	2.3882	6.3112
20	8793	1343	1.46950	2.3878	6.3153
30	8750	1216	1.47204	2.3875	6.3194
40	8708	1088	1.47459	2.3871	6.3235
50	8666	0962	1.47713	2.3866	6.3276
50 00	8623	0835	1.47968	2.3862	6.3318
10	8581	0708	1.48223	2.3858	6.3359
20	8539	0581	1.48478	2.3853	6.3401
30	8497	0455	1.48734	2.3848	6.3443
40	8455	0328	1.48989	2.3843	6.3485
50	8413	0202	1.49246	2.3838	6.3527
51 00	8371	8.510 0076	1.49502	2.3833	6.3569
10	8329	8.509 9950	1.49759	2.3828	6.3612
20	8287	9825	1.50016	2.3822	6.3654
30	8245	9699	1.50273	2.3817	6.3697
40	8203	9574	1.50531	2.3811	6.3740
50	8161	9448	1.50789	2.3805	6.3782
52 00	8120	9323	1.51048	2.3799	6.3826
10	8078	9198	1.51307	2.3792	6.3869
20	8036	9074	1.51566	2.3786	6.3912
30	7995	8949	1.51826	2.3779	6.3956
40	7953	8825	1.52086	2.3773	6.4000
50	7912	8701	1.52347	2.3766	6.4043
53 00	7871	8577	1.52608	2.3759	6.4088
10	7829	8453	1.52869	2.3751	6.4132
20	7788	8329	1.53131	2.3744	6.4176
30	7747	8206	1.53393	2.3736	6.4221
40	7706	8083	1.53656	2.3729	6.4265
50	7665	7960	1.53919	2.3721	6.4310
60	8.508 7624	8.509 7838	1.54183	2.3713	6.4355

TABLE IV. CORRECTION TO LONGITUDE FOR DIFFERENCE BETWEEN ARC AND SINE.

(Art. 52, p. 53.)

log K (−)	log difference.	log dM (+)	log K (−)	log difference.	log dM (+)
3.876	0.000 0001	2.385	4.871	0.000 0098	3.380
4.026	02	2.535	4.882	103	3.391
4.114	03	2.623	4.892	108	3.401
4.177	04	2.686	4.903	114	3.412
4.225	05	2.734	4.913	119	3.422
4.265	06	2.774	4.922	124	3.431
4.298	07	2.807	4.932	130	3.441
4.327	08	2.836	4.941	136	3.450
4.353	09	2.862	4.950	142	3.459
4.376	10	2.885	4.959	147	3.468
4.396	11	2.905	4.968	153	3.477
4.415	12	2.924	4.976	160	3.485
4.433	13	2.942	4.985	166	3.494
4.449	14	2.958	4.993	172	3.502
4.464	15	2.973	5.002	179	3.511
4.478	16	2.987	5.010	186	3.519
4.491	17	3.000	5.017	192	3.526
4.503	18	3.012	5.025	199	3.534
4.526	20	3.035	5.033	206	3.542
4.548	23	3.057	5.040	213	3.549
4.570	25	3.079	5.047	221	3.556
4.591	27	3.100	5.054	228	3.563
4.612	30	3.121	5.062	236	3.571
4.631	33	3.140	5.068	243	3.577
4.649	36	3.158	5.075	251	3.584
4.667	39	3.176	5.082	259	3.591
4.684	42	3.193	5.088	267	3.597
4.701	45	3.210	5.095	275	3.604
4.716	48	3.225	5.102	284	3.611
4.732	52	3.241	5.108	292	3.617
4.746	56	3.255	5.114	300	3.623
4.761	59	3.270	5.120	309	3.629
4.774	63	3.283	5.126	318	3.635
4.788	67	3.297	5.132	327	3.641
4.801	71	3.310	5.138	336	3.647
4.813	75	3.322	5.144	345	3.653
4.825	80	3.334	5.150	354	3.659
4.834	84	3.343	5.156	364	3.665
4.849	89	3.358	5.161	373	3.670
4.860	94	3.369	5.167	383	3.676

TABLE V. FOR CONVERTING SIDEREAL INTO MEAN SOLAR TIME.

(Increase in Sun's Right Ascension in Sidereal h. m. s.)

(Art 73, p. 70.)

Mean Time = Sidereal Time − C'.

Sid. Hrs.	Corr.	Sid. Min.	Corr.	Sid. Min.	Corr.	Sid. Sec.	Corr.	Sid. Sec.	Corr.
	m s		s		s		s		s
1	0 9.830	1	0.164	31	5.079	1	0.003	31	0.085
2	0 19.659	2	0.328	32	5.242	2	0.005	32	0.087
3	0 29.489	3	0.491	33	5.406	3	0.008	33	0.090
4	0 39.318	4	0.655	34	5.570	4	0.011	34	0.093
5	0 49.148	5	0.819	35	5.734	5	0.014	35	0.096
6	0 58.977	6	0.983	36	5.898	6	0.016	36	0.098
7	1 8.807	7	1.147	37	6.062	7	0.019	37	0.101
8	1 18.636	8	1.311	38	6.225	8	0.022	38	0.104
9	1 28.466	9	1.474	39	6.389	9	0.025	39	0.106
10	1 38.296	10	1.638	40	6.553	10	0.027	40	0.109
11	1 48.125	11	1.802	41	6.717	11	0.030	41	0.112
12	1 57.955	12	1.966	42	6.881	12	0.033	42	0.115
13	2 7.784	13	2.130	43	7.045	13	0.035	43	0.117
14	2 17.614	14	2.294	44	7.208	14	0.038	44	0.120
15	2 27.443	15	2.457	45	7.372	15	0.041	45	0.123
16	2 37.273	16	2.621	46	7.536	16	0.044	46	0.126
17	2 47.102	17	2.785	47	7.700	17	0.046	47	0.128
18	2 56.932	18	2.949	48	7.864	18	0.049	48	0.131
19	3 6.762	19	3.113	49	8.027	19	0.052	49	0.134
20	3 16.591	20	3.277	50	8.191	20	0.055	50	0.137
21	3 26.421	21	3.440	51	8.355	21	0.057	51	0.139
22	3 36.250	22	3.604	52	8.519	22	0.060	52	0.142
23	3 46.080	23	3.768	53	8.683	23	0.063	53	0.145
24	3 55.909	24	3.932	54	8.847	24	0.066	54	0.147
		25	4.096	55	9.010	25	0.068	55	0.150
		26	4.259	56	9.174	26	0.071	56	0.153
		27	4.423	57	9.338	27	0.074	57	0.156
		28	4.587	58	9.502	28	0.076	58	0.158
		29	4.751	59	9.666	29	0.079	59	0.161
		30	4.915	60	9.830	30	0.082	60	0.164

TABLE VI. FOR CONVERTING MEAN SOLAR INTO SIDEREAL TIME.

(Increase in Sun's Right Ascension in Solar h. m. s.)

(Art. 73, p. 70.)

Sidereal Time = Mean Time + C.

Mean Hrs.	Corr.	Mean Min.	Corr.	Mean Min.	Corr.	Mean Sec.	Corr.	Mean Sec.	Corr.
	m s		s		s		s		s
1	0 9.856	1	0.164	31	5.093	1	0.003	31	0.085
2	0 19.713	2	0.329	32	5.257	2	0.005	32	0.088
3	0 29.569	3	0.493	33	5.421	3	0.008	33	0.090
4	0 39.426	4	0.657	34	5.585	4	0.011	34	0.093
5	0 49.282	5	0.821	35	5.750	5	0.014	35	0.096
6	0 59.139	6	0.986	36	5.914	6	0.016	36	0.099
7	1 8.995	7	1.150	37	6.078	7	0.019	37	0.101
8	1 18.852	8	1.314	38	6.242	8	0.022	38	0.104
9	1 28.708	9	1.478	39	6.407	9	0.025	39	0.107
10	1 38.565	10	1.643	40	6.571	10	0.027	40	0.110
11	1 48.421	11	1.807	41	6.735	11	0.030	41	0.112
12	1 58.278	12	1.971	42	6.900	12	0.033	42	0.115
13	2 8.134	13	2.136	43	7.064	13	0.036	43	0.118
14	2 17.991	14	2.300	44	7.228	14	0.038	44	0.120
15	2 27.847	15	2.464	45	7.392	15	0.041	45	0.123
16	2 37.704	16	2.628	46	7.557	16	0.044	46	0.126
17	2 47.560	17	2.793	47	7.721	17	0.047	47	0.129
18	2 57.417	18	2.957	48	7.885	18	0.049	48	0.131
19	3 7.273	19	3.121	49	8.049	19	0.052	49	0.134
20	3 17.129	20	3.285	50	8.214	20	0.055	50	0.137
21	3 26.986	21	3.450	51	8.378	21	0.057	51	0.140
22	3 36.842	22	3.614	52	8.542	22	0.060	52	0.142
23	3 46.699	23	3.778	53	8.707	23	0.063	53	0.145
24	3 56.555	24	3.943	54	8.871	24	0.066	54	0.148
		25	4.107	55	9.035	25	0.068	55	0.151
		26	4.271	56	9.199	26	0.071	56	0.153
		27	4.435	57	9.364	27	0.074	57	0.156
		28	4.600	58	9.528	28	0.077	58	0.160
		29	4.764	59	9.692	29	0.079	59	0.162
		30	4.928	60	9.856	30	0.082	60	0.164

TABLE VII. MEAN REFRACTION CORRECTION.

(Art. 78, p. 73.)

Barometric Pressure 30 Inches. Temperature 50° F.

Apparent Altitude	Mean Refraction	Apparent Altitude	Mean Refraction
0°	36′ 29″	17°	3′ 08″
1	24 54	18	2 58
2	18 26	19	2 48
3	14 25	20	2 39
4	11 44	25	2 04
5	9 52	30	1 41
6	8 28	35	1 23
7	7 24	40	1 09
8	6 33	45	0 58
9	5 53	50	0 49
10	5 19	55	0 41
11	4 51	60	0 34
12	4 28	65	0 27
13	4 07	70	0 21
14	3 50	75	0 16
15	3 34	80	0 10
16	3 20	85	0 05
		90	0 00

TABLE VIII. LOG A AND LOG B (FOR COMPUTING EQUATION OF EQUAL ALTITUDES).

(Art. 88, p. 85.)

(Log A is − and Log B is +)

$2t$	3 h		4 h		5 h		6 h		7 h	
	Log A	Log B	Log A	Log B	Log A	Log B	Log A	Log B	Log A	Log B
m.										
0	9.4172	9.3828	9.4260	9.3635	9.4374	9.3369	9.4515	9.3010	9.4685	9.2530
1	.4173	.3825	.4261	.3631	.4376	.3364	.4518	.3003	.4688	.2520
2	.4174	.3822	.4263	.3627	.4378	.3358	.4521	.2996	.4691	.2511
3	.4175	.3820	.4265	.3624	.4380	.3353	.4523	.2989	.4694	.2502
4	.4177	.3817	.4266	.3620	.4383	.3348	.4526	.2982	.4697	.2492
5	9.4178	9.3814	9.4268	9.3616	9.4385	9.3343	9.4528	9.2975	9.4701	9.2483
6	.4179	.3811	.4270	.3612	.4387	.3337	.4531	.2968	.4704	.2473
7	.4181	.3809	.4272	.3608	.4389	.3332	.4534	.2961	.4707	.2463
8	.4182	.3806	.4273	.3604	.4391	.3327	.4536	.2954	.4710	.2454
9	.4183	.3803	.4275	.3600	.4393	.3321	.4539	.2947	.4713	.2444
10	9.4184	9.3800	9.4277	9.3596	9.4396	9.3316	9.4542	9.2940	9.4716	9.2434
11	.4186	.3797	.4279	.3592	.4398	.3311	.4544	.2932	.4719	.2425
12	.4187	.3794	.4280	.3588	.4400	.3305	.4547	.2925	.4723	.2415
13	.4188	.3792	.4282	.3584	.4402	.3300	.4550	.2918	.4726	.2405
14	.4190	.3789	.4284	.3580	.4405	.3294	.4552	.2911	.4729	.2395
15	9.4191	9.3786	9.4286	9.3576	9.4407	9.3289	9.4555	9.2903	9.4732	9.2385
16	.4193	.3783	.4288	.3572	.4409	.3283	.4558	.2896	.4735	.2375
17	.4194	.3780	.4289	.3568	.4411	.3278	.4561	.2888	.4738	.2365
18	.4195	.3777	.4291	.3564	.4414	.3272	.4563	.2881	.4742	.2355
19	.4197	.3774	.4293	.3559	.4416	.3266	.4566	.2873	.4745	.2344
20	9.4198	9.3771	9.4295	9.3555	9.4418	9.3261	9.4569	9.2866	9.4748	9.2334
21	.4199	.3768	.4297	.3551	.4420	.3255	.4572	.2858	.4751	.2324
22	.4201	.3765	.4299	.3547	.4423	.3249	.4574	.2850	.4755	.2313
23	.4202	.3762	.4300	.3542	.4425	.3244	.4577	.2843	.4758	.2303
24	.4204	.3759	.4302	.3538	.4427	.3238	.4580	.2835	.4761	.2292
25	9.4205	9.3756	9.4304	9.3534	9.4430	9.3232	9.4583	9.2827	9.4764	9.2282
26	.4207	.3752	.4306	.3530	.4432	.3226	.4585	.2819	.4768	.2271
27	.4208	.3749	.4308	.3525	.4434	.3220	.4588	.2812	.4771	.2261
28	.4209	.3746	.4310	.3521	.4437	.3214	.4591	.2804	.4774	.2250
29	.4211	.3743	.4312	.3516	.4439	.3208	.4594	.2796	.4778	.2239
30	9.4212	9.3740	9.4314	9.3512	9.4441	9.3203	9.4597	9.2788	9.4781	9.2228
31	.4214	.3737	.4315	.3508	.4444	.3197	.4600	.2780	.4784	.2217
32	.4215	.3733	.4317	.3503	.4446	.3191	.4602	.2772	.4788	.2206
33	.4217	.3730	.4319	.3499	.4448	.3185	.4605	.2764	.4791	.2195
34	.4218	.3727	.4321	.3494	.4451	.3178	.4608	.2756	.4794	.2184
35	9.4220	9.3723	9.4323	9.3490	9.4453	9.3172	9.4611	9.2747	9.4798	9.2173
36	.4221	.3720	.4325	.3485	.4456	.3166	.4614	.2739	.4801	.2162
37	.4223	.3717	.4327	.3480	.4458	.3160	.4617	.2731	.4804	.2151
38	.4224	.3713	.4329	.3476	.4460	.3154	.4620	.2723	.4808	.2140
39	.4226	.3710	.4331	.3471	.4463	.3148	.4622	.2714	.4811	.2128
40	9.4227	9.3707	9.4333	9.3467	9.4465	9.3142	9.4625	9.2706	9.4815	9.2117
41	.4229	.3703	.4335	.3462	.4468	.3135	.4628	.2698	.4818	.2105
42	.4231	.3700	.4337	.3457	.4470	.3129	.4631	.2689	.4821	.2094
43	.4232	.3696	.4339	.3453	.4473	.3123	.4634	.2681	.4825	.2082
44	.4234	.3693	.4341	.3448	.4475	.3116	.4637	.2672	.4828	.2070
45	9.4235	9.3690	9.4343	9.3443	9.4477	9.3110	9.4640	9.2664	9.4832	9.2059
46	.4237	.3686	.4345	.3438	.4480	.3103	.4643	.2655	.4835	.2047
47	.4238	.3683	.4347	.3433	.4482	.3097	.4646	.2646	.4839	.2035
48	.4240	.3679	.4349	.3429	.4485	.3091	.4649	.2638	.4842	.2023
49	.4242	.3675	.4351	.3424	.4487	.3084	.4652	.2629	.4846	.2011
50	9.4243	9.3672	9.4353	9.3419	9.4490	9.3078	9.4655	9.2620	9.4849	9.1999
51	.4245	.3668	.4355	.3414	.4492	.3071	.4658	.2611	.4853	.1987
52	.4246	.3665	.4357	.3409	.4494	.3064	.4661	.2602	.4856	.1974
53	.4248	.3661	.4359	.3404	.4497	.3058	.4664	.2593	.4860	.1962
54	.4250	.3657	.4361	.3399	.4500	.3051	.4667	.2584	.4863	.1950
55	9.4251	9.3654	9.4363	9.3394	9.4503	9.3044	9.4670	9.2575	9.4867	9.1937
56	.4253	.3650	.4366	.3389	.4505	.3038	.4673	.2566	.4870	.1925
57	.4255	.3646	.4368	.3384	.4508	.3031	.4676	.2557	.4874	.1912
58	.4256	.3643	.4370	.3379	.4510	.3024	.4679	.2548	.4877	.1900
59	.4258	.3639	.4372	.3374	.4513	.3017	.4682	.2539	.4881	.1887
60	9.4260	9.3635	9.4374	9.3369	9.4515	9.3010	9.4685	9.2530	9.4884	9.1874

TABLE VIII. LOG A AND LOG B (FOR COMPUTING EQUATION OF EQUAL ALTITUDES).

(Art. 88, p. 85.)

(Log A is $-$ and Log B is $+$)

$2t$	8 h		9 h		10 h		11 h		12 h	
	Log A	Log B	Log A	Log B	Log A	Log B	Log A	Log B	Log A	Log B
m.										
0	9.4884	9.1874	9.5115	9.0943	9.5379	8.9509	9.5680	8.6837	9.6021	Inf.
1	.4888	.1861	.5119	.0925	.5384	.9478	.5685	.6770	.6027	6.9603
2	.4892	.1848	.5123	.0906	.5389	.9447	.5691	.6701	.6033	7.2431
3	.4895	.1835	.5127	.0887	.5393	.9416	.5696	.6632	.6039	.4198
4	.4899	.1822	.5132	.0867	.5398	.9384	.5701	.6560	.6045	.5453
5	9.4902	9.1809	9.5136	9.0848	9.5403	8.9352	9.5707	8.6488	9.6051	7.6428
6	.4906	.1796	.5140	.0828	.5408	.9320	.5712	.6414	.6057	.7226
7	.4910	.1782	.5144	.0809	.5412	.9287	.5718	.6339	.6063	.7902
8	.4913	.1769	.5148	.0789	.5417	.9254	.5723	.6262	.6069	.8488
9	.4917	.1756	.5153	.0769	.5422	.9221	.5728	.6183	.6075	.9005
10	9.4921	9.1742	9.5157	9.0749	9.5427	8.9187	9.5734	8.6103	9.6082	7.9469
11	.4924	.1728	.5161	.0729	.5432	.9153	.5739	.6021	.6088	.9889
12	.4928	.1715	.5165	.0708	.5436	.9118	.5745	.5937	.6094	8.0273
13	.4932	.1701	.5169	.0688	.5441	.9083	.5750	.5852	.6100	.0627
14	.4935	.1687	.5174	.0667	.5446	.9048	.5756	.5764	.6106	.0955
15	9.4939	9.1673	9.5178	9.0646	9.5451	8.9013	9.5761	8.5674	9.6112	8.1260
16	.4943	.1659	.5182	.0625	.5456	.8977	.5767	.5583	.6119	.1547
17	.4946	.1645	.5186	.0604	.5461	.8940	.5772	.5488	.6125	.1816
18	.4950	.1630	.5191	.0583	.5466	.8903	.5778	.5392	.6131	.2071
19	.4954	.1616	.5195	.0561	.5470	.8866	.5783	.5293	.6137	.2312
20	9.4958	9.1602	9.5199	9.0540	9.5475	8.8829	9.5789	8.5192	9.6144	8.2541
21	.4961	.1587	.5204	.0518	.5480	.8791	.5794	.5088	.6150	.2759
22	.4965	.1573	.5208	.0496	.5485	.8752	.5800	.4981	.6156	.2967
23	.4969	.1558	.5212	.0474	.5490	.8713	.5806	.4871	.6163	.3166
24	.4973	.1543	.5217	.0452	.5495	.8674	.5811	.4758	.6169	.3357
25	9.4977	9.1528	9.5221	9.0429	9.5500	8.8634	9.5817	8.4641	9.6175	8.3540
26	.4980	.1513	.5225	.0406	.5505	.8594	.5822	.4521	.6182	.3717
27	.4984	.1498	.5230	.0383	.5510	.8553	.5828	.4397	.6188	.3887
28	.4988	.1483	.5234	.0360	.5515	.8512	.5834	.4270	.6194	.4051
29	.4992	.1468	.5238	.0337	.5520	.8470	.5839	.4138	.6201	.4210
30	9.4996	9.1453	9.5243	9.0314	9.5525	8.8427	9.5845	8.4001	9.6207	8.4363
31	.5000	.1437	.5247	.0290	.5530	.8384	.5851	.3860	.6214	.4512
32	.5003	.1422	.5252	.0266	.5535	.8341	.5856	.3713	.6220	.4657
33	.5007	.1406	.5256	.0242	.5540	.8297	.5862	.3561	.6226	.4796
34	.5011	.1390	.5261	.0218	.5545	.8253	.5868	.3403	.6233	.4932
35	9.5015	9.1375	9.5265	9.0194	9.5550	8.8208	9.5874	8.3239	9.6239	8.5064
36	.5019	.1359	.5269	.0169	.5555	.8162	.5879	.3067	.6246	.5192
37	.5023	.1343	.5274	.0144	.5560	.8115	.5885	.2888	.6252	.5318
38	.5027	.1327	.5278	.0119	.5565	.8068	.5891	.2701	.6259	.5440
39	.5031	.1310	.5283	.0094	.5570	.8020	.5897	.2505	.6265	.5559
40	9.5035	9.1294	9.5287	9.0069	9.5576	8.7972	9.5902	8.2299	9.6272	8.5675
41	.5038	.1278	.5292	.0043	.5581	.7923	.5908	.2082	.6279	.5788
42	.5042	.1261	.5296	.0017	.5586	.7873	.5914	.1853	.6285	.5899
43	.5046	.1244	.5301	8.9991	.5591	.7823	.5920	.1611	.6292	.6008
44	.5050	.1228	.5305	.9965	.5596	.7772	.5926	.1354	.6298	.6114
45	9.5054	9.1211	9.5310	8.9938	9.5601	8.7720	9.5931	8.1080	9.6305	8.6218
46	.5058	.1194	.5315	.9911	.5606	.7668	.5937	.0786	.6311	.6320
47	.5062	.1177	.5319	.9884	.5612	.7614	.5943	.0470	.6318	.6419
48	.5066	.1159	.5324	.9857	.5617	.7560	.5949	.0128	.6325	.6517
49	.5070	.1142	.5328	.9830	.5622	.7505	.5955	7.9756	.6331	.6613
50	9.5074	9.1125	9.5333	8.9802	9.5627	8.7449	9.5961	7.9348	9.6338	8.6707
51	.5078	.1107	.5337	.9774	.5632	.7392	.5967	.8897	.6345	.6799
52	.5082	.1089	.5342	.9745	.5638	.7335	.5973	.8391	.6351	.6890
53	.5086	.1072	.5347	.9717	.5643	.7276	.5979	.7817	.6358	.6979
54	.5091	.1054	.5351	.9688	.5648	.7217	.5985	.7154	.6365	.7067
55	9.5095	9.1036	9.5356	8.9659	9.5654	8.7156	9.5991	7.6368	9.6372	8.7153
56	.5099	.1017	.5361	.9630	.5659	.7094	.5997	.5405	.6378	.7237
57	.5103	.0999	.5365	.9600	.5664	.7032	.6003	.4162	.6385	.7321
58	.5107	.0981	.5370	.9570	.5669	.6968	.6009	.2407	.6392	.7402
59	.5111	.0962	.5375	.9540	.5675	.6903	.6015	6.9591	.6399	.7483
60	9.5115	9.0943	9.5379	8.9509	9.5680	8.6837	9.6021	Inf.	9.6406	8.7563

TABLE IX. FOR DETERMINING DIFFERENCE IN ELEVATION BY THE BAROMETER.

(Art. 133, p. 136.)

$$D = 60{,}158.58 \times \log H.$$

(Extracted from Smithsonian Miscellaneous Contributions.)

Barometer in Inches	Hundredth of an Inch										Diff. for .01
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	
	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet
14.5	9,708	9,726	9,744	9,762	9,780	9,798	9,816	9,834	9,851	9,869	18
14.6	9,887	9,905	9,923	9,941	9,959	9,977	9,994	10,012	10,030	10,048	18
14.7	10,066	10,083	10,101	10,119	10,137	10,154	10,172	10,190	10,207	10,225	17
14.8	10,243	10,260	10,278	10,296	10,313	10,331	10,349	10,366	10,384	10,401	18
14.9	10,419	10,436	10,454	10,471	10,489	10,506	10,524	10,541	10,559	10,576	17
15.0	10,593	10,611	10,628	10,646	10,663	10,680	10,698	10,715	10,732	10,750	17
15.1	10,767	10,784	10,802	10,819	10,836	10,853	10,871	10,888	10,905	10,922	17
15.2	10,939	10,957	10,974	10,991	11,008	11,025	11,042	11,059	11,076	11,094	17
15.3	11,111	11,128	11,145	11,162	11,179	11,196	11,213	11,230	11,247	11,264	18
15.4	11,281	11,298	11,315	11,332	11,349	11,366	11,382	11,399	11,416	11,433	17
15.5	11,450	11,467	11,484	11,500	11,517	11,534	11,551	11,568	11,584	11,601	17
15.6	11,618	11,635	11,651	11,668	11,685	11,702	11,718	11,735	11,752	11,768	17
15.7	11,785	11,802	11,818	11,835	11,851	11,868	11,885	11,901	11,918	11,934	17
15.8	11,951	11,967	11,983	12,000	12,017	12,033	12,050	12,066	12,083	12,099	16
15.9	12,116	12,132	12,148	12,165	12,181	12,198	12,214	12,230	12,247	12,263	17
16.0	12,280	12,296	12,312	12,329	12,345	12,361	12,377	12,394	12,410	12,426	16
16.1	12,442	12,459	12,475	12,491	12,507	12,523	12,540	12,556	12,572	12,588	16
16.2	12,604	12,620	12,636	12,653	12,669	12,685	12,701	12,717	12,733	12,749	16
16.3	12,765	12,781	12,797	12,813	12,829	12,845	12,861	12,877	12,893	12,909	16
16.4	12,925	12,941	12,957	12,973	12,988	13,004	13,020	13,036	13,052	13,068	16
16.5	13,084	13,099	13,115	13,131	13,147	13,163	13,178	13,194	13,210	13,226	16
16.6	13,242	13,257	13,273	13,289	13,304	13,320	13,336	13,352	13,367	13,383	16
16.7	13,398	13,414	13,430	13,445	13,461	13,476	13,492	13,508	13,523	13,539	15
16.8	13,554	13,570	13,585	13,601	13,616	13,632	13,647	13,663	13,678	13,694	16
16.9	13,709	13,725	13,740	13,756	13,771	13,787	13,802	13,817	13,833	13,848	16
17.0	13,864	13,879	13,894	13,910	13,925	13,940	13,956	13,971	13,986	14,002	15
17.1	14,017	14,032	14,047	14,063	14,078	14,093	14,108	14,124	14,139	14,154	15
17.2	14,169	14,184	14,199	14,215	14,230	14,245	14,260	14,275	14,290	14,306	15
17.3	14,321	14,336	14,351	14,366	14,381	14,396	14,411	14,426	14,441	14,456	15
17.4	14,471	14,486	14,501	14,516	14,531	14,546	14,561	14,576	14,591	14,606	15
17.5	14,621	14,636	14,651	14,665	14,681	14,695	14,710	14,725	14,740	14,755	14
17.6	14,770	14,785	14,799	14,814	14,829	14,844	14,859	14,874	14,888	14,903	15
17.7	14,918	14,933	14,947	14,962	14,977	14,992	15,006	15,021	15,036	15,050	15
17.8	15,065	15,080	15,094	15,109	15,124	15,138	15,153	15,168	15,122	15,197	14
17.9	15,211	15,226	15,241	15,255	15,270	15,284	15,299	15,313	15,328	15,342	15
18.0	15,357	15,371	15,386	15,400	15,415	15,429	15,444	15,458	15,473	15,487	14
18.1	15,502	15,516	15,530	15,545	15,559	15,574	15,588	15,602	15,617	15,631	15
18.2	15,646	15,660	15,674	15,689	15,703	15,717	15,732	15,746	15,760	15,774	14
18.3	15,789	15,803	15,817	15,831	15,846	15,860	15,874	15,888	15,903	15,917	14
18.4	15,931	15,945	15,959	15,974	15,988	16,002	16,016	16,030	16,044	16,059	14

TABLE IX. Continued.

Barometer in Inches	Hundredth of an Inch										Diff. for .01
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	
	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet
18.5	16,073	16,087	16,101	16,115	16,129	16,143	16,157	16,171	16,185	16,200	14
18.6	16,214	16,228	16,242	16,256	16,270	16,284	16,298	16,312	16,326	16,340	14
18.7	16,354	16,368	16,382	16,395	16,409	16,423	16,437	16,451	16,465	16,479	14
18.8	16,493	16,507	16,521	16,535	16,549	16,562	16,576	16,590	16,604	16,618	13
18.9	16,632	16,645	16,659	16,673	16,687	16,701	16,714	16,728	16,742	16,756	14
19.0	16,769	16,783	16,797	16,811	16,824	16,838	16,852	16,866	16,879	16,893	14
19.1	16,907	16,920	16,934	16,948	16,961	16,975	16,989	17,002	17,016	17,029	14
19.2	17,043	17,057	17,070	17,084	17,097	17,111	17,125	17,138	17,152	17,165	14
19.3	17,179	17,192	17,206	17,219	17,233	17,246	17,260	17,273	17,287	17,300	13
19.4	17,314	17,327	17,341	17,354	17,368	17,381	17,394	17,408	17,421	17,435	13
19.5	17,448	17,461	17,475	17,488	17,502	17,516	17,528	17,542	17,555	17,568	14
19.6	17,582	17,595	17,608	17,622	17,635	17,648	17,662	17,675	17,688	17,701	13
19.7	17,715	17,728	17,741	17,754	17,768	17,781	17,794	17,807	17,821	17,834	13
19.8	17,847	17,860	17,873	17,887	17,900	17,913	17,926	17,939	17,952	17,965	13
19.9	17,979	17,992	18,005	18,018	18,031	18,044	18,057	18,070	18,083	18,096	13
20.0	18,110	18,123	18,136	18,149	18,162	18,175	18,188	18,201	18,214	18,227	13
20.1	18,240	18,253	18,266	18,279	18,292	18,305	18,318	18,331	18,344	18,357	13
20.2	18,370	18,383	18,395	18,408	18,421	18,434	18,447	18,460	18,472	18,486	13
20.3	18,499	18,511	18,524	18,537	18,550	18,563	18,576	18,589	18,601	18,614	13
20.4	18,627	18,640	18,653	18,665	18,678	18,691	18,704	18,716	18,729	18,742	13
20.5	18,755	18,767	18,780	18,792	18,806	18,818	18,831	18,844	18,856	18,869	12
20.6	18,882	18,895	18,907	18,920	18,933	18,945	18,958	18,971	18,983	18,996	12
20.7	19,008	19,021	19,034	19,046	19,059	19,071	19,084	19,097	19,109	19,122	12
20.8	19,134	19,147	19,159	19,172	19,184	19,197	19,210	19,222	19,235	19,247	13
20.9	19,260	19,272	19,285	19,297	19,310	19,322	19,334	19,347	19,359	19,372	12
21.0	19,384	19,397	19,409	19,422	19,434	19,446	19,459	19,471	19,484	19,496	12
21.1	19,508	19,521	19,533	19,546	19,558	19,570	19,590	19,595	19,607	19,620	12
21.2	19,632	19,644	19,657	19,669	19,681	19,694	19,706	19,718	19,730	19,743	13
21.3	19,755	19,767	19,779	19,792	19,804	19,816	19,828	19,841	19,853	19,865	12
21.4	19,877	19,890	19,902	19,914	19,926	19,938	19,950	19,963	19,975	19,987	12
21.5	19,999	20,011	20,023	20,036	20,048	20,060	20,072	20,084	20,096	20,108	12
21.6	20,120	20,132	20,144	20,157	20,169	20,181	20,193	20,205	20,217	20,229	12
21.7	20,241	20,253	20,265	20,277	20,289	20,301	20,313	20,325	20,337	20,349	12
21.8	20,361	20,373	20,385	20,397	20,409	20,421	20,433	20,445	20,457	20,469	12
21.9	20,481	20,493	20,505	20,516	20,528	20,540	20,552	20,564	20,576	20,588	12
22.0	20,600	20,612	20,623	20,636	20,647	20,659	20,671	20,683	20,695	20,706	12
22.1	20,718	20,732	20,742	20,754	20,765	20,777	20,789	20,802	20,813	20,824	12
22.2	20,836	20,848	20,860	20,871	20,883	20,894	20,907	20,918	20,930	20,942	11
22.3	20,954	20,965	20,977	20,989	21,000	21,012	21,024	21,035	21,047	21,059	12
22.4	21,071	21,082	21,094	21,105	21,117	21,129	21,140	21,152	21,164	21,175	12

TABLE IX. Continued.

Barometer in Inches	Hundredth of an Inch .00	.01	.02	.03	.04	.05	.06	.07	.08	.09	Diff. for .01
	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet
22.5	21,187	21,199	21,210	21,222	21,233	21,245	21,256	21,268	21,280	21,291	12
22.6	21,303	21,314	21,326	21,337	21,349	21,360	21,372	21,384	21,395	21,407	11
22.7	21,418	21,430	21,441	21,453	21,464	21,466	21,487	21,499	21,510	21,521	12
22.8	21,533	21,545	21,556	21,577	21,579	21,590	21,602	21,613	21,624	21,636	11
22.9	21,647	21,659	21,670	21,681	21,693	21,704	21,716	21,727	21,738	21,750	11
23.0	21,761	21,772	21,784	21,795	21,806	21,818	21,829	21,840	21,852	21,863	12
23.1	21,874	21,886	21,897	21,908	21,920	21,931	21,942	21,953	21,965	21,977	11
23.2	21,987	21,999	22,010	22,021	22,032	22,044	22,055	22,066	22,077	22,088	12
23.3	22,100	22,111	22,122	22,133	22,145	22,156	22,167	22,178	22,189	22,200	11
23.4	22,212	22,223	22,234	22,245	22,256	22,267	22,278	22,290	22,301	22,312	11
23.5	22,323	22,334	22,345	22,356	22,367	22,378	22,390	22,401	22,412	22,423	11
23.6	22,434	22,445	22,456	22,467	22,478	22,489	22,500	22,511	22,522	22,533	11
23.7	22,544	22,555	22,566	22,577	22,588	22,599	22,610	22,621	22,632	22,643	11
23.8	22,654	22,665	22,676	22,687	22,698	22,709	22,720	22,731	22,742	22,753	11
23.9	22,764	22,775	22,786	22,797	22,808	22,818	22,829	22,840	22,851	22,862	10
24.0	22,873	22,884	22,895	22,906	22,917	22,927	22,938	22,949	22,960	22,971	10
24.1	22,982	22,993	23,003	23,014	23,025	23,036	23,047	23,058	23,068	23,079	11
24.2	23,090	23,101	23,111	23,122	23,133	23,144	23,155	23,165	23,176	23,187	11
24.3	23,198	23,208	23,219	23,230	23,241	23,251	23,262	23,273	23,283	23,294	10
24.4	23,305	23,316	23,326	23,337	23,348	23,358	23,369	23,380	23,390	23,401	10
24.5	23,412	23,422	23,433	23,444	23,454	23,465	23,476	23,486	23,497	23,507	11
24.6	23,518	23,529	23,539	23,550	23,561	23,571	23,582	23,592	23,603	23,614	10
24.7	23,624	23,635	23,645	23,656	33,666	23,677	23,688	23,698	23,709	23,719	11
24.8	23,730	23,740	23,751	23,761	23,772	23,782	23,793	23,803	23,814	23,824	10
24.9	23,835	23,845	23,856	23,866	23,877	23,887	23,898	23,908	23,919	23,929	10
25.0	23,940	23,950	23,960	23,971	23,981	23,992	24,002	24,013	24,023	24,033	11
25.1	24,044	24,054	24,065	24,075	24,085	24,096	24,106	24,117	24,127	24,137	11
25.2	24,148	24,158	24,168	24,179	24,189	24,199	24,210	24,220	24,230	24,241	10
25.3	24,251	24,261	24,272	24,282	24,292	24,303	24,313	24,323	24,333	24,344	11
25.4	24,354	24,365	24,375	24,385	24,395	24,406	24,416	24,426	24,436	24,447	11
25.5	24,457	24,467	24,477	24,488	24,498	24,508	24,518	24,528	24,539	24,549	10
25.6	24,559	24,569	24,580	24,590	24,600	24,610	24,620	24,630	24,641	24,651	10
25.7	24,661	24,671	24,681	24,691	24,702	24,712	24,722	24,732	24,742	24,752	10
25.8	24,762	24,773	24,783	24,793	24,803	24,813	24,823	24,833	24,843	24,853	10
25.9	24,864	24,874	24,884	24,894	24,904	24,914	24,921	24,934	24,944	24,944	10
26.0	24,964	24,974	24,984	24,994	25,004	25,014	25,024	25,034	25,045	25,055	11
26.1	25,065	25,075	25,085	25,095	25,105	25,115	25,125	25,135	25,144	25,154	10
26.2	25,164	25,174	25,184	25,194	25,204	25,214	25,224	25,234	25,244	25,254	10
26.3	25,264	25,274	25,284	25,294	25,304	25,314	25,323	25,333	25,343	25,353	10
26.4	25,363	25,373	25,383	25,393	25,403	25,412	25,422	25,432	25,442	25,452	9

TABLE IX. Continued.

Barometer in Inches	Hundredth of an Inch										Diff. for .01
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	
	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet
26.5	25,462	25,472	25,482	25,491	25,501	25,511	25,521	25,531	25,541	25,550	10
26.6	25,560	25,570	25,580	25,590	25,600	25,609	25,619	25,629	25,639	25,649	9
26.7	25,658	25,668	25,678	25,688	25,697	25,707	25,717	25,727	25,736	25,746	10
26.8	25,755	25,766	25,775	25,785	25,795	25,805	25,814	25,824	25,834	25,844	10
26.9	25,853	25,863	25,873	25,882	25,892	25,902	25,911	25,921	25,931	25,941	10
27.0	25,950	25,960	25,970	25,979	25,989	25,999	26,008	26,018	26,028	26,037	10
27.1	26,047	26,057	26,066	26,076	26,085	26,095	26,105	26,114	26,124	26,133	10
27.2	26,143	26,153	26,162	26,172	26,181	26,191	26,201	26,210	26,220	26,229	10
27.3	26,239	26,248	26,258	26,268	26,277	26,287	26,296	26,306	26,315	26,325	10
27.4	26,334	26,344	26,354	26,363	26,372	26,382	26,392	26,401	26,411	26,420	10
27.5	26,430	26,439	26,449	26,458	26,468	26,477	26,487	26,496	26,506	26,515	9
27.6	26,524	26,534	26,543	26,553	26,562	26,572	26,581	26,591	26,600	26,610	10
27.7	26,619	26,628	26,638	26,647	26,657	26,666	26,676	26,685	26,694	26,704	9
27.8	26,713	26,723	26,732	26,741	26,751	26,760	26,770	26,779	26,788	26,798	9
27.9	26,807	26,816	26,826	26,835	26,844	26,854	26,863	26,872	26,882	26,891	10
28.0	26,900	26,910	26,919	26,928	26,938	26,947	26,956	26,966	26,975	26,984	9
28.1	26,994	27,003	27,012	27,022	27,031	27,040	27,049	27,060	27,068	27,077	9
28.2	27,086	27,096	27,105	27,114	27,123	27,133	27,142	27,151	27,160	27,170	10
28.3	27,179	27,188	27,197	27,207	27,216	27,225	27,234	27,243	27,253	27,262	9
28.4	27,271	27,280	27,289	27,299	27,308	27,317	27,326	27,335	27,345	27,354	9
28.5	27,362	27,372	27,381	27,390	27,400	27,409	27,418	27,427	27,436	27,445	9
28.6	27,454	27,464	27,473	27,482	27,491	27,500	27,509	27,518	27,527	27,537	9
28.7	27,545	27,555	27,564	27,573	27,582	27,591	27,600	27,609	27,618	27,627	9
28.8	27,637	27,646	27,655	27,664	27,673	27,682	27,691	27,700	27,709	27,718	9
28.9	27,727	27,736	27,745	27,754	27,763	27,772	27,781	27,790	27,799	27,808	9
29.0	27,817	27,826	27,835	27,844	27,853	27,862	27,871	27,880	27,889	27,898	9
29.1	27,907	27,916	27,925	27,934	27,943	27,952	27,961	27,970	27,979	27,988	9
29.2	27,997	28,006	28,015	28,024	28,032	28,041	28,050	28,059	28,068	28,077	9
29.3	28,086	28,095	28,104	28,113	28,122	28,131	28,140	28,148	28,157	28,166	9
29.4	28,175	28,184	28,193	28,202	28,211	28,220	28,228	28,237	28,246	28,255	9
29.5	28,264	28,273	28,282	28,290	28,299	28,308	28,317	28,326	28,335	28,343	9
29.6	28,352	28,361	28,370	28,379	28,388	28,396	28,405	28,414	28,423	28,432	8
29.7	28,440	28,449	28,458	28,467	28,475	28,484	28,493	28,502	28,511	28,519	9
29.8	28,528	28,537	28,546	28,554	28,563	28,572	28,581	28,589	28,598	28,607	9
29.9	28,616	28,624	28,633	28,642	28,651	28,659	28,668	28,677	28,685	28,694	8
30.0	28,703	28,712	28,720	28,729	28,738	28,746	28,755	28,764	28,773	28,781	8
30.1	28,790	28,799	28,807	28,816	28,825	28,833	28,842	28,851	28,859	28,868	8
30.2	28,877	28,885	28,894	28,903	28,911	28,920	28,928	28,937	28,946	28,954	9
30.3	28,963	28,972	28,980	28,989	28,997	29,006	29,015	29,023	29,032	29,040	9
30.4	29,049	29,058	29,066	29,075	29,083	29,092	29,100	29,109	29,118	29,126	9
30.5	29,135	29,143	29,152	29,160	29,169	29,178	29,186	29,195	29,203	29,212	9
30.6	29,220	29,229	29,237	29,246	29,254	29,263	29,272	29,280	29,289	29,297	9
30.7	29,306	29,314	29,323	29,331	29,340	29,348	29,357	29,365	29,374	29,382	8
30.8	29,391	29,399	29,408	29,416	29,424	29,433	29,441	29,450	29,458	29,467	8
30.9	29,475	29,484	29,492	29,501	29,509	29,518	29,526	29,534	29,543	29,551	8

TABLE X. STADIA REDUCTIONS.

(Art. 150, p. 157.)

VERTICAL HEIGHTS.

Minutes	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
0...	0.00	1.74	3.49	5.23	6.96	8.68	10.40	12.10	13.78	15.45
2...	0.06	1.80	3.55	5.28	7.02	8.74	10.45	12.15	13.84	15.51
4...	0.12	1.86	3.60	5.34	7.07	8.80	10.51	12.21	13.89	15.56
6...	0.17	1.92	3.66	5.40	7.13	8.85	10.57	12.26	13.95	15.62
8...	0.23	1.98	3.72	5.46	7.19	8.91	10.62	12.32	14.01	15.67
10...	0.29	2.04	3.78	5.52	7.25	8.97	10.68	12.38	14.06	15.73
12...	0.35	2.09	3.84	5.57	7.30	9.03	10.74	12.43	14.12	15.78
14...	0.41	2.15	3.90	5.63	7.36	9.08	10.79	12.49	14.17	15.84
16...	0.47	2.21	3.95	5.69	7.42	9.14	10.85	12.55	14.23	15.89
18...	0.52	2.27	4.01	5.75	7.48	9.20	10.91	12.60	14.28	15.95
20...	0.58	2.33	4.07	5.80	7.53	9.25	10.96	12.66	14.34	16.00
22...	0.64	2.38	4.13	5.86	7.59	9.31	11.02	12.72	14.40	16.06
24...	0.70	2.44	4.18	5.92	7.65	9.37	11.08	12.77	14.45	16.11
26...	0.76	2.50	4.24	5.98	7.71	9.43	11.13	12.83	14.51	16.17
28...	0.81	2.56	4.30	6.04	7.76	9.48	11.19	12.88	14.56	16 22
30...	0.87	2.62	4.36	6.09	7.82	9.54	11.25	12.94	14.62	16.28
32...	0.93	2.67	4.42	6.15	7.88	9.60	11.30	13.00	14.67	16.33
34...	0.99	2.73	4.48	6.21	7.94	9.65	11.36	13.05	14.73	16.39
36...	1.05	2.79	4.53	6.27	7.99	9.71	11.42	13.11	14.79	16.44
38...	1.11	2.85	4.59	6.33	8.05	9.77	11.47	13.17	14.84	16.50
40...	1.16	2.91	4.65	6.38	8.11	9.83	11.53	13.22	14.90	16.55
42...	1.22	2.97	4.71	6.44	8.17	9.88	11.59	13.28	14.95	16.61
44...	1.28	3.02	4.76	6.50	8.22	9.94	11.64	13.33	15.01	16.66
46...	1.34	3.08	4.82	6.56	8.28	10.00	11.70	13.39	15.06	16.72
48...	1.40	3.14	4.88	6.61	8.34	10.05	11.76	13.45	15.12	16.77
50...	1.45	3.20	4.94	6.67	8.40	10.11	11.81	13.50	15.17	16.83
52...	1.51	3.26	4.99	6.73	8.45	10.17	11.87	13.56	15.23	16.88
54...	1.57	3.31	5.05	6.79	8.51	10.22	11.93	13.61	15.28	16.94
56...	1.63	3.37	5.11	6.84	8.57	10.28	11.98	13.67	15.34	16.99
58...	1.69	3.43	5.17	6.90	8.63	10.34	12.04	13.73	15.40	17.05
60...	1.74	3.49	5.23	6.96	8.68	10.40	12.10	13.78	15.45	17.10

HORIZONTAL CORRECTIONS

Dist.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
100..	0.0	0.0	0.1	0.3	0.5	0.8	1.1	1.5	1.9	2.5
200..	0.0	0.1	0.2	0.5	1.0	1.5	2.2	3.0	3.9	4.9
300..	0.0	0.1	0.4	0.8	1.5	2.3	3.3	4.5	5.8	7.4
400..	0.0	0.1	0.5	1.1	2.0	3.0	4.4	6.0	7.8	9.8
500..	0.0	0.2	0.6	1.4	2.5	3.8	5.5	7.5	9.7	12.3
600..	0.0	0.2	0.7	1.6	2.9	4.6	6.5	8.9	11.6	14.7
700..	0.0	0.2	0.8	1.9	3.4	5.3	7.6	10.4	13.6	17.2
800..	0.0	0.2	1.0	2.2	3.9	6.1	8.7	11.9	15.5	19.6
900..	6.0	0.3	1.1	2.4	4.4	6.8	9.8	13.4	17.5	22.1
1000..	0.0	0.3	1.2	2.7	4.9	7.6	10.9	14.9	19.4	24.5

TABLE X. STADIA REDUCTIONS.

VERTICAL HEIGHTS.

Minutes	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°
0...	17.10	18.73	20.34	21.92	23.47	25.00	26.50	27.96	29.39	30.78
2...	17.16	18.78	20.39	21.97	23.52	25.05	26.55	28.01	29.44	30.83
4...	17.21	18.84	20.44	22.02	23.58	25.10	26.59	28.06	29.48	30.87
6...	17.26	18.89	20.50	22.08	23.63	25.15	26.64	28.10	29.53	30.92
8...	17.32	18.95	20.55	22.13	23.68	25.20	26.69	28.15	29.58	30.97
10...	17.37	19.00	20.60	22.18	23.73	25.25	26.74	28.20	29.62	31.01
12...	17.43	19.05	20.66	22.23	23.78	25.30	26.79	28.25	29.67	31.06
14...	17.48	19.11	20.71	22.28	23.83	25.35	26.84	28.30	29.72	31.10
16...	17.54	19.16	20.76	22.34	23.88	25.40	26.89	28.34	29.76	31.15
18...	17.59	19.21	20.81	22.39	23.93	25.45	26.94	28.39	29.81	31.19
20...	17.65	19.27	20.87	22.44	23.99	25.50	26.99	28.44	29.86	31.24
22...	17.70	19.32	20.92	22.49	24.04	25.55	27.04	28.49	29.90	31.28
24...	17.76	19.38	20.97	22.54	24.09	25.60	27.09	28.54	29.95	31.33
26..	17.81	19.43	21.03	22.60	24.14	25.65	27.13	28.58	30.00	31.38
28...	17.86	19.48	21.08	22.65	24.19	25.70	27.18	28.63	30.04	31.42
30...	17.92	19.54	21.13	22.70	24.24	25.75	27.23	28.68	30.09	31.47
32...	17.97	19.59	21.18	22.75	24.29	25.80	27.28	28.73	30.14	31.51
34...	18.03	19.64	21.24	22.80	24.34	25.85	27.33	28.77	30.19	31.56
36...	18.08	19.70	21.29	22.85	24.39	25.90	27.38	28.82	30.23	31.60
38...	18.14	19.75	21.34	22.91	24.44	25.95	27.43	28.87	30.28	31.65
40...	18.19	19.80	21.39	22.96	24.49	26.00	27.48	28.92	30.32	31.69
42...	18.24	19.86	21.45	23.01	24.55	26.05	27.52	28.96	30.37	31.74
44...	18.30	19.91	21.50	23.06	24.60	26.10	27.57	29.01	30.41	31.78
46...	18.35	19.96	21.55	23.11	24.65	26.15	27.62	29.06	30.46	31.83
48...	18.41	20.02	21.60	23.16	24.70	26.20	27.67	29.11	30.51	31.87
50...	18.46	20.07	21.66	23.22	24.75	26.25	27.72	29.15	30.55	31.92
52...	18.51	20.12	21.71	23.27	24.80	26.30	27.77	29.20	30.60	31.96
54. .	18.57	20.18	21.76	23.32	24.85	26.35	27.81	29.25	30.65	32.01
56...	18.62	20.23	21.81	23.37	24.90	26.40	27.86	29.30	30.69	32.05
58...	18.68	20.28	21.87	23.42	24.95	26.45	27.91	29.34	30.74	32.09
60...	18.73	20.34	21.92	23.47	25.00	26.50	27 96	29.39	30.78	32.14

HORIZONTAL CORRECTIONS

Dist.	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°
100..	3.0	3.6	4.3	5.1	5.9	6.7	7.6	8.5	9 5	10.6
200..	6.0	7.3	8.6	10.1	11.7	13.4	15.2	17.1	19.1	21.2
300..	9.1	10.9	13.0	15.2	17.6	20.1	22.8	25.6	28.6	31.8
400..	12.1	14.6	17.3	20.2	23.4	26.8	30.4	34.2	38.2	52.4
500..	15.1	18.2	21.6	25.3	29.3	33.5	38.0	42.7	47.7	43.0
600..	18.1	21.8	25.9	30.4	35.1	40.2	45.6	51.3	57.3	63.6
700..	21.1	25.5	30.2	35.4	41.0	46.9	53.2	59.8	66.8	74.2
800..	24.2	29.1	34.6	40.5	46.8	53.6	60.8	68.4	76.4	84.8
900..	27.2	32.8	38.9	45.5	52.7	60.3	68.4	76.9	85.9	95.4
1000..	30.2	36.4	43.2	50.6	58.5	67.0	76.0	85.5	95 5	106.0

TABLE X. STADIA REDUCTIONS.

VERTICAL HEIGHTS.

Minutes	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°
0...	32.1[illegible]	33.46	34.73	35.97	37.16	38.30	39.40	40.45	41.45	42.40
2...	32.1[illegible]	33.50	34.77	36.01	37.20	38.34	39.44	40.49	41.48	42.43
4...	32.23	33.54	34.82	36.05	37.23	38.38	39.47	40.52	41.52	42.46
6...	32.27	33.59	34.86	36.09	37.27	38.41	39.51	40.55	41.55	42.49
8...	32.32	33.63	34.90	36.13	37.31	38.45	39.54	40.59	41.58	42.53
10...	32.36	33.67	34.94	36.17	37.35	38.49	39.58	40.62	41.61	42.56
12...	32.41	33.72	34.98	36.21	37.39	38.53	39.61	40.66	41.65	42.59
14...	32.45	33.76	35.02	36.25	37.43	38.56	39.65	40.69	41.68	42.62
16...	32.49	33.80	35.07	36.29	37.47	38.60	39.69	40.72	41.71	42.65
18...	32.54	33.84	35.11	36.33	37.51	38.64	39.72	40.76	41.74	42.68
20...	32.58	33.89	35.15	36.37	37.54	38.67	39.76	40.79	41.77	42.71
22...	32.63	33.93	35.19	36.41	37.58	38.71	39.79	40.82	41.81	42.74
24...	32.67	33.97	35.23	36.45	37.62	38.75	39.83	40.86	41.84	42.77
26...	32.72	34.01	35.27	36.49	37.66	38.78	39.86	40.89	41.87	42.80
28...	32.76	34.06	35.31	36.53	37.70	38.82	39.90	40.92	41.90	42.83
30...	32.80	34.10	35.36	36.57	37.74	38.86	39.93	40.96	41.93	42.86
32...	32.85	34.14	35.40	36.61	37.77	38.89	39.97	40.99	41.97	42.89
34...	32.89	34.18	35.44	36.65	37.81	38.93	40.00	41.02	42.00	42.92
36...	32.93	34.23	35.48	36.69	37.85	38.97	40.04	41.06	42.03	42.95
38...	32.98	34.27	35.52	36.73	37.89	39.00	40.07	41.09	42.06	42.98
40...	33.02	34.31	35.56	36.77	37.93	39.04	40.11	41.12	42.09	43.01
42...	33.07	34.35	35.60	36.80	37.96	39.08	40.14	41.16	42.12	43.04
44...	33.11	34.40	35.64	36.84	38.00	39.11	40.18	41.19	42.15	43.07
46...	33.15	34.44	35.68	36.88	38.04	39.15	40.21	41.22	42.19	43.10
48...	33.20	34.48	35.72	36.92	38.08	39.18	40.24	41.26	42.22	43.13
50...	33.24	34.52	35.76	36.96	38.11	39.22	40.28	41.29	42.25	43.16
52...	33.28	34.57	35.80	37.00	38.15	39.26	40.31	41.32	42.28	43.18
54...	33.33	34.61	35.85	37.04	38.19	39.29	40.35	41.35	42.31	43.21
56...	33.37	34.65	35.89	37.08	38.23	39.33	40.38	41.39	42.34	43.24
58...	33.41	34.69	35.93	37.12	38.26	39.36	40.42	41.42	42.37	43.27
60...	33.46	34.73	35.97	37.16	38.30	39.40	40.45	41.45	42.40	43.30

HORIZONTAL CORRECTIONS

Dist.	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°
100..	11.7	12.8	14.0	15.3	16.5	17.9	19.2	20.6	22.0	23.5
200..	23.4	25.7	28.1	30.5	33.1	35.7	38.4	41.2	44.1	47.0
300..	35.1	38.5	42.1	45.8	49.6	53.6	57.7	61.8	66.1	70.5
400..	46.8	51.4	56.1	61.1	66.2	71.4	76.9	82.4	88.2	94.0
500..	58.5	64.2	70.2	76.4	82.7	89.3	96.1	103.1	110.2	117.5
600..	70.2	77.0	84.2	91.6	99.2	107.2	115.3	123.7	132.2	141.0
700..	81.9	89.9	98.2	106.9	115.8	125.0	134.5	144.3	154.3	164.5
800..	93.6	102.7	112.2	122.2	132.3	142.9	153.8	164.9	176.3	188.0
900..	105.3	115.6	126.3	137.4	148.9	160.7	173.0	185.5	198.4	211.5
1000..	117.0	128.4	140.3	152.7	165.4	178.6	192.2	206.1	220.4	235.0

TABLE XI. VALUES OF C FOR USE IN THE CHEZY FORMULA

$$V = C\sqrt{rs}$$

(Art. 286, p. 315.)

Slope	r	n .020	n .025	n .030	n .035	n .040	n .045	n .050	n .055	n .060
.0001	3.28	91	73	60	52	46	40	36	33	30
	10	111	92	78	69	62	55	50	46	42
	20	122	102	89	79	71	65	60	55	51
	50	134	114	100	91	83	76	71	67	63
	100	140	121	108	98	91	84	79	74	70
.0002	10	108	89	76	67	60	53	49	45	41
	20	117	98	85	76	68	61	57	53	49
	50	126	108	94	85	78	71	66	62	58
	100	131	113	99	90	83	77	72	68	64
.0004	10	107	88	75	66	59	53	48	44	41
	20	115	96	83	73	66	60	55	51	48
	50	123	104	91	82	75	68	63	59	56
	100	127	108	96	87	80	73	68	64	61
.0010	10	105	87	74	65	58	52	47	44	40
	20	113	94	81	72	65	59	54	50	47
	50	120	101	89	79	72	66	61	57	54
	100	124	105	94	85	77	71	66	62	59
.010	10	105	86	74	65	58	51	47	43	40
	20	112	93	80	71	64	58	53	49	46
	50	119	100	87	78	71	65	60	56	53
	100	122	104	91	82	75	69	65	61	58

NOTE. — For r = 3.28 feet, n constant, C is constant for all values of slope. For slopes greater than 0.01, or fall of 52.8 feet per mile, C remains nearly constant.

From "River Discharge," by Hoyt and Grover.

TABLE XII. HAMILTON SMITH'S COEFFICIENTS FOR WEIRS WITH CONTRACTION SUPPRESSED AT BOTH ENDS, FOR USE IN THE FORMULA $Q = cbH^{\frac{3}{2}}$.

(Art. 309, p. 328.)

H = head in feet	b = length of weir, in feet								
	19	15	10	7	5	4	3*	2*	0.66*
0.1	3.515	3.515	3.520	3.520	3.526	.....	.....	.....	3.611
.15	3.440	3.445	3.445	3.451	3.451	3.461	3.472	3.488	3.542
.2	3.397	3.403	3.408	3.408	3.413	3.429	3.435	3.450	3.510
.25	3.371	3.376	3.381	3.386	3.392	3.403	3.413	3.429	3.494
.3	3.349	3.354	3.360	3.365	3.376	3.386	3.403	3.418	3.483
.4	3.322	3.328	3.333	3.344	3.360	3.371	3.386	3.403	3.478
.5	3.312	3.317	3.322	3.338	3.354	3.371	3.386	3.408	3.478
.6	3.306	3.312	3.317	3.333	3.354	3.371	3.392	3.413	3.483
.7	3.306	3.312	3.317	3.338	3.360	3.376	3.397	3.424	3.494
.8	3.306	3.317	3.322	3.344	3.365	3.386	3.408	3.441	3.510
.9	3.312	3.317	3.328	3.354	3.375	3.397	3.418	3.451	.....
1.0	3.312	3.322	3.338	3.360	3.386	3.408	3.429	3.467	.....
1.1	3.317	3.328	3.344	3.371	3.397	3.419	3.445	.....	.....
1.2	3.317	3.333	3.349	3.381	3.403	3.429	3.456	.....	.....
1.3	3.322	3.338	3.360	3.386	3.413	3.440	3.467	.....	.....
1.4	3.328	3.344	3.365	3.392	3.424	3.445	.....	.....	.....
1.5	3.328	3.344	3.371	3.403	3.429	3.456	.....	.....	.....
1.6	3.333	3.349	3.376	3.408	3.435	3.461	.....	.....	.....
1.7	3.333	3.349	3.381	3.413	.....	.....	.....	.....	.....
2.0	.....	.....	.....	.....	.....	.....	.....	.....	.....

* Approximate.

TABLE XIII. HAMILTON SMITH'S COEFFICIENTS FOR WEIRS WITH TWO COMPLETE END CONTRACTIONS, FOR USE IN THE FORMULA $Q = cbH^{\frac{3}{2}}$.

(Art. 309, p. 328.)

H = head in feet	b = length of weir, in feet										
	0.66	1*	2	2.6	3	4	5	7	10	15	19
0.1	3.381	3.419	3.456	3.478	3.488	3.494	3.494	3.499	3.504	3.504	3.510
.15	3.312	3.344	3.392	3.408	3.413	3.419	3.424	3.424	3.429	3.435	3.435
.2	3.269	3.306	3.349	3.365	3.371	3.376	3.376	3.381	3.386	3.392	3.392
.25	3.237	3.274	3.322	3.333	3.338	3.344	3.349	3.354	3.360	3.360	3.365
.3	3.215	3.253	3.296	3.306	3.312	3.322	3.322	3.333	3.338	3.338	3.344
.4	3.183	3.215	3.258	3.274	3.280	3.285	3.290	3.301	3.306	3.312	3.317
.5	3.156	3.189	3.237	3.247	3.253	3.264	3.269	3.280	3.290	3.295	3.301
.6	3.140	3.172	3.215	3.231	3.237	3.247	3.253	3.269	3.280	3.285	3.290
.7	3.130	3.156	3.199	3.210	3.226	3.231	3.242	3.258	3.274	3.280	3.285
.8	.....	.....	3.183	3.199	3.215	3.221	3.231	3.247	3.269	3.274	3.280
.9	.....	.....	3.167	3.189	3.199	3.210	3.226	3.242	3.258	3.269	3.274
1.0	.....	.....	3.156	3.172	3.183	3.199	3.215	3.231	3.253	3.264	3.269
1.1	.....	.....	3.140	3.162	3.172	3.189	3.205	3.226	3.242	3.258	3.264
1.2	.....	.....	3.130	3.151	3.162	3.178	3.194	3.215	3.237	3.253	3.264
1.3	.....	.....	3.114	3.135	3.151	3.167	3.199	3.205	3.231	3.247	3 258
1.4	.....	.....	3.103	3.124	3.140	3.156	3.178	3.199	3.221	3.242	3.258
1.5	.....	.....	.....	3.114	3.130	3.151	3.167	3.189	3.215	3.237	3.253
1.6	.....	.....	.....	3.103	3.114	3.140	3.162	3.183	3.210	3.231	3.247
1.7	.....	.....	.....	.....	.....	.....	.....	3.178	3.205	3.226	3.247
2.0	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

* Approximate.

TABLE XIV. LENGTHS OF DEGREES OF THE MERIDIAN.*

(Art. 339, p. 355.)

Latitude	Meters	Latitude	Meters	Latitude	Meters
0°	110 567.2	30°	110 848.5	60°	111 414.5
1	567.6	31	865.7	61	431.5
2	568.6	32	883.2	62	448.2
3	570.3	33	901.1	63	464.4
4	572.7	34	919.2	64	480.3
5	575.8	35	937.6	65	495.7
6	579.5	36	956.2	66	510.7
7	583.9	37	975.1	67	525.3
8	589.0	38	110 994.1	68	539.3
9	594.7	39	111 013.3	69	552.9
10	601.1	40	032.7	70	565.9
11	608.1	41	052.2	71	578.4
12	615.8	42	071.7	72	590.4
13	624.1	43	091.4	73	601.8
14	633.0	44	111.1	74	612.7
15	642.5	45	130.9	75	622.9
16	652.6	46	150.6	76	632.6
17	663.3	47	170.4	77	641.6
18	674.5	48	190.1	78	650.0
19	686.3	49	209.7	79	657.8
20	698.7	50	229.3	80	664.9
21	711.6	51	248.7	81	671.4
22	725.0	52	268.0	82	677.2
23	738.8	53	287.1	83	682.4
24	753.2	54	306.0	84	686.9
25	768.0	55	324.8	85	690.7
26	783.3	56	343.3	86	693.8
27	799.0	57	361.5	87	696.2
28	815.1	58	379.5	88	697.9
29	831.6	59	397.2	89	699.0
30	110 848.5	60	111 414.5	90	111 699.3

* These lengths of a degree of the meridian extend 0° 30′ north and 0° 30′ south of the given latitude.

TABLE XV. — MERIDIONAL DISTANCE IN METERS FROM WHOLE DEGREE PARALLEL.

(Art. 339, p. 355.)

Lat.	Minutes from Whole Degree Parallel									
	1′	2′	3′	4′	5′	6′	7′	8′	9′	10′
25°	1846.1	3692.3	5538.4	7384.6	9230.7	11076.9	12923.0	14769.2	16615.4	18461.5
26	1846.4	3692.8	5539.2	7385.6	9232.0	11078.4	12924.8	14771.2	16617.7	18464.1
27	1846.7	3693.3	5540.0	7386.6	9233.3	11080.0	12926.7	14773.3	16620.0	18466.7
28	1846.9	3693.8	5540.8	7387.7	9234.6	11081.6	12928.5	14775.5	16622.5	18469.4
29	1847.2	3694.4	5541.6	7388.8	9236.0	11083.2	12930.5	14777.7	16624.9	18472.2
30	1847.5	3695.0	5542.4	7389.9	9237.4	11084.9	12932.4	14779.9	16627 4	18475.0
31	1847.8	3695.5	5543.3	7391.1	9238.9	11086.7	12934.4	14782.2	16630.0	18477.9
32	1848.1	3696.1	5544.2	7392.3	9240.3	11088.4	12936.5	14784.6	16632.7	18480.8
33	1848.4	3696.7	5545.1	7393.4	9241.8	11090.2	12938.6	14787.0	16635.4	18483 8
34	1848.7	3697.3	5546.0	7394.6	9243.3	11092.0	12940.7	14789.4	16638.1	18486.8
35	1849.0	3697.9	5546.9	7395.9	9244.9	11093.9	12942.8	14791.8	16640.8	18489.9
36	1849.3	3698.5	5547.8	7397.1	9246.4	11095.7	12945.0	14794.3	16643.6	18493.0
37	1849.6	3699.2	5548.8	7398.4	9248.0	11097.6	12947.2	14796.8	16646.5	18496.1
38	1849.9	3699.8	5549.7	7399.6	9249.6	11099.5	12949.4	14799.4	16649.3	18499.3
39	1850.2	3700.5	5550.7	7400.9	9251.2	11101.4	12951.7	14801.9	16652.2	18502.5
40	1850.5	3701.1	5551.7	7402.2	9252.8	11103.4	12953.9	14804.5	16655.1	18505.7
41	1850.9	3701.7	5552.6	7403.5	9254.4	11105.3	12956.2	14807.1	16658.0	18509.0
42	1851.2	3702.4	5553.6	7404.8	9256.0	11107.3	12958.5	14809.7	16661.0	18512.2
43	1851.5	3703.1	5554.6	7406.1	9257.7	11109.2	12960.8	14812.4	16663.9	18515.5
44	1851.9	3703.7	5555.6	7407.4	9259.3	11111.2	12963.1	14815.0	16666.9	18518.8
45	1852.2	3704.4	5556.6	7408.8	9261.0	11113.2	12965.4	14817.6	16669.9	18522.1
46	1852.5	3705.0	5557.6	7410.1	9262.6	11115.2	12967.7	14820.3	16672.8	18525.4
47	1852.8	3705.7	5558.5	7411.4	9264.3	11117.1	12970.0	14822.9	16675.8	18528.7
48	1853.2	3706.3	5559.5	7412.7	9265.9	11119.1	12972.3	14825.5	16678.7	18531.9
49	1853.5	3707.0	5560.5	7414.0	9267.5	11121.1	12974.6	14828.1	16681.7	18535.2
50	1853.8	3707.7	5561.5	7415.3	9269.2	11123.0	12976.9	14830.7	16684.6	18538.5

TABLE XVI. COÖRDINATES OF CURVATURE.

(Art. 339, p. 355.)

Long.	26° X	26° Y	27° X	27° Y	28° X	28° Y	29° X	29° Y
1′	1668.7	0.1	1654.3	0.1	1639.4	0.1	1624.0	0.1
2	3337.3	0.4	3308.5	0.4	3278.8	0.4	3248.0	0.5
3	5006.0	1.0	4962.8	1.0	4918.2	1.0	4872.0	1.0
4	6674.6	1.7	6617.1	1.7	6557.6	1.8	6496.1	1.8
5	8343.3	2.7	8271.4	2.7	8197.0	2.8	8120.1	2.9
6	10011.9	3.8	9925.7	3.9	9836.4	4.0	9744.1	4.1
7	11680.6	5.2	11579.9	5.4	11475.7	5.5	11368.1	5.6
8	13349.2	6.8	13234.2	7.0	13115.1	7.2	12992.1	7.3
9	15017.9	8.6	14888.5	8.8	14754.5	9.1	14616.1	9.3
10	16686.6	10.6	16542.8	10.9	16393.9	11.2	16240.1	11.5

Long.	30° X	30° Y	31° X	31° Y	32° X	32° Y	33° X	33° Y
1′	1608.1	0.1	1591.8	0.1	1574.9	0.1	1557.6	0.1
2	3216.3	0.5	3183.5	0.5	3149.8	0.5	3115.2	0.5
3	4824.4	1.1	4775.3	1.1	4724.8	1.1	4672.8	1.1
4	6432.6	1.9	6367.1	1.9	6299.7	1.9	6230.3	2.0
5	8040.7	2.9	7958.9	3.0	7874.6	3.0	7787.9	3.1
6	9648.8	4.2	9550.6	4.3	9449.5	4.4	9345.5	4.4
7	11257.0	5.7	11142.4	5.8	11024.4	6.0	10903.1	6.0
8	12865.1	7.5	12734.2	7.6	12599.4	7.8	12460.7	7.9
9	14473.2	9.5	14325.9	9.7	14174.3	9.8	14018.3	10.0
10	16081.4	11.7	15917.7	11.9	15749.2	12.1	15575.9	12.3

Long.	34° X	34° Y	35° X	35° Y	36° X	36° Y	37° X	37° Y
1′	1539.8	0.1	1521.5	0.1	1502.8	0.1	1483.6	0.1
2	3079.6	0.5	3043.0	0.5	3005.5	0.5	2967.1	0.5
3	4619.3	1.1	4564.5	1.1	4508.3	1.2	4450.7	1.2
4	6159.1	2.0	6086.0	2.0	6011.1	2.1	5934.2	2.1
5	7698.9	3.1	7607.5	3.2	7513.8	3.2	7417.8	3.3
6	9238.7	4.5	9129.0	4.6	9016.6	4.6	8901.4	4.7
7	10778.5	6.1	10650.5	6.2	10519.3	6.3	10384.9	6.4
8	12318.3	8.0	12172.0	8.1	12022.1	8.2	11868.5	8.3
9	13858.0	10.1	13693.5	10.3	13524.8	10.4	13352.1	10.5
10	15397.9	12.5	15215.0	12.7	15027.6	12.8	14835.6	13.0

TABLE XVI. COÖRDINATES OF CURVATURE.

(Art. 339, p. 355.)

Long.	Latitudes. 38°		39°		40°		41°	
	X	Y	X	Y	X	Y	X	Y
1′	1463.9	0.1	1443.8	0.1	1423.3	0.1	1402.3	0.1
2	2927.8	0.5	2887.6	0.5	2846.5	0.5	2804.6	0.5
3	4391.7	1.2	4331.4	1.2	4269.8	1.2	4206.9	1.2
4	5855.6	2.1	5775.2	2.1	5693.0	2.1	5609.2	2.1
5	7319.6	3.3	7219.0	3.3	7116.3	3.3	7011.5	3.3
6	8783.5	4.7	8662.9	4.8	8539.6	4.8	8413.7	4.8
7	10247.4	6.4	10106.7	6.5	9962.8	6.5	9816.0	6.6
8	11711.3	8.4	11550.5	8.5	11386.1	8.5	11218.3	8.6
9	13175.2	10.6	12994.3	10.7	12809.3	10.8	12620.6	10.8
10	14639.1	13.1	14438.1	13.2	14232.6	13.3	14022.9	13.4

Long.	42°		43°		44°		45°	
1′	1380.9	0.1	1359.1	0.1	1336.8	0.1	1314.1	0.1
2	2761.8	0.5	2718.1	0.5	2673.6	0.5	2628.3	0.5
3	4142.7	1.2	4077.2	1.2	4010.4	1.2	3942.5	1.2
4	5523.5	2.2	5436.2	2.2	5347.2	2.2	5256.6	2.2
5	6904.4	3.4	6795.3	3.4	6684.0	3.4	6570.8	3.4
6	8285.3	4.8	8154.3	4.9	8020.8	4.9	7884.9	4.9
7	9666.2	6.6	9513.4	6.6	9357.7	6.6	9199.1	6.6
8	11047.1	8.6	10872.4	8.6	10694.5	8.6	10513.2	8.6
9	12428.0	10.9	12231.5	10.9	12031.3	10.9	11827.4	10.9
10	13808.8	13.4	13590.5	13.5	13368.1	13.5	13141.5	13.5

Long.	46°		47°		48°		49°	
1′	1291.1	0.1	1267.6	0.1	1243.8	0.1	1219.6	0.1
2	2582.2	0.5	2535.3	0.5	2487.6	0.5	2439.1	0.5
3	3873.3	1.2	3802.9	1.2	3731.4	1.2	3658.7	1.2
4	5164.4	2.2	5070.5	2.2	4975.2	2.1	4878.3	2.1
5	6455.5	3.4	6338.2	3.4	6219.0	3.3	6097.9	3.3
6	7746.6	4.9	7605.8	4.8	7462.8	4.8	7317.5	4.8
7	9037.6	6.6	8873.5	6.6	8706.6	6.6	8537.0	6.6
8	10328.7	8.6	10141.1	8.6	9950.4	8.6	9756.6	8.6
9	11619.8	10.9	11408.7	10.9	11194.2	10.9	10976.2	10.8
10	12910.9	13.5	12676.4	13.5	12437.9	13.4	12195.8	13.4

TABLE XVII. COÖRDINATES OF CURVATURE.

(Art. 339, p. 355.)

Long.	Latitudes.					
	25°		30°		35°	
	X	Y	X	Y	X	Y
5°	504 645	9 307	482 288	10 523	456 261	11 421
10	1 008 603	37 215	963 658	42 074	911 379	45 656
15	1 511 190	83 685	1 443 193	94 591	1 364 214	102 619
20	2 011 722	148 656	1 919 982	167 977	1 813 632	182 168
25	2 509 518	232 038	2 393 116	262 089	2 258 507	284 102
30	3 003 900	333 718	2 861 694	376 749	2 697 724	408 168

Long.	40°		45°		50°	
5°	426 757	11 972	393 996	12 160	358 224	11 978
10	852 171	47 852	786 492	48 594	714 847	47 859
15	1 274 904	107 525	1 175 994	109 162	1 068 277	107 482
20	1 693 628	190 805	1 561 019	193 635	1 416 934	190 581
25	2 107 023	297 430	1 940 103	301 690	1 759 262	296 785
30	2 513 790	427 063	2 311 802	432 918	2 093 731	425 619

Greek Alphabet.

LETTERS	NAME
Α, α,	Alpha
Β, β,	Beta
Γ, γ,	Gamma
Δ, δ,	Delta
Ε, ε,	Epsilon
Ζ, ζ,	Zeta
Η, η,	Eta
Θ, θ,	Theta
Ι, ι,	Iota
Κ, κ,	Kappa
Λ, λ,	Lambda
Μ, μ,	Mu
Ν, ν,	Nu
Ξ, ξ,	Xi
Ο, ο,	Omicron
Π, π,	Pi
Ρ, ρ,	Rho
Σ, σ, ς,	Sigma
Τ, τ,	Tau
Υ, υ,	Upsilon
Φ, φ,	Phi
Χ, χ,	Chi
Ψ, ψ,	Psi
Ω, ω,	Omega

APPENDIX

APPENDIX A.

The following specifications are introduced to give the student more definite information as to what is required in topographical surveys for landscape architects' studies. These specifications are not intended to be included in every contract, but portions of them may be cut out or other specifications added to suit the requirements of any special case.

SPECIFICATIONS FOR TOPOGRAPHICAL MAP.

ON SCALE OF 40 FEET TO AN INCH.*

1. PURPOSE OF THE MAP.

The purpose of the map is to aid the landscape architect in designing proposed improvements. Therefore it is important to show on it many objects which would receive little or no consideration in designing purely engineering works. One class of such objects, of no value in themselves, such as old fence lines and the wild growths along them, old buildings, cart roads, trails, limits of cultivated land, dense masses of brushes or coppice, limits of marshy ground as indicated by the kind of vegetation, etc., are needed (*a*) to aid in identifying on the map one's position on the ground or to rapidly and approximately locate on the ground certain lines drawn on a sunprint of the map. Another class of objects, such as good trees, ledges, etc., are needed both for the above reason (*a*), and also because (*b*) the objects may be worth preserving as features of the landscape so that proposed improvements, such as drives and walks, grading and planting, may have to be adjusted to them. For the same reasons contours should show, approximately, minor irregularities of the surface, such as hollows, summits, cuts and embankments along roads and watercourses, accumulations of earth along the lower margins of old fields, little gullies, and ridges and terraces.

2. CROSS SECTIONING.

Unless otherwise directed, cross section lines 200 feet apart are to be surveyed on the ground with the transit and tape and points located thereon 100 feet apart at the corners of a system of 100-foot squares, and the remaining corners of the cross section squares are to be located with the stadia or by pac-

* The authors are indebted to Olmsted Brothers, Brookline, Mass., for permission to print the above specifications.

ing and sighting, and all corners are be to marked, except in roads or where impracticable by strong stakes of heart of red cedar, or cypress, or heart of locust one and three-fourths inches square and eighteen inches long, with their tops one inch above the surface. Where it will not interfere with the use of the land, as in pastures and woodland, an additional sawed stake is to be strongly set at each cross section stake so as to show two feet above the ground. The cross section lines and stakes are to be lettered and numbered, and each cross section stake, after it has been driven, is to have nailed upon its top with two one-inch hard brass wire nails a brass label of No. 28 hard sheet brass, one and one-half inches by one inch, upon which are to be stamped with proper hard steel dies or punches, letters and numbers one-quarter of an inch high, to be placed so as to read in the proper direction, and the corresponding letters and numbers are to be indicated on the map. The cross section system is to be thoroughly referenced to permanent points or objects so that the system can be accurately and readily reproduced after stakes have disappeared.

3. Instrument Stations and Traverse Lines.

If cross section squares are not required the instrument stations and points 100 feet apart on all base lines. and traverse lines, also triangulation, plane-table, stadia, and transit stations, are to be staked as above, and all such stakes and corresponding points on the map are to be lettered and numbered and labelled as above. When points to be marked come on ledges or large boulders, drill holes or V cuts witnessed by stones are to be substituted for stakes.

4. Boundaries.

Locate boundaries both as indicated by existing landmarks and as called for by recorded deeds. Wherever these boundaries do not agree, the surveyor is to stake and show on the map the boundary which he advises should be adopted. Where there are slight angles in a boundary, the turning point is to be indicated on the map by a dot in a small circle.

5. Boundary and Other Roads.

Show by full lines all existing railroads, roads and private rights-of-way bounding upon, leading to or within the property. Indicate travelled ways, and trees, turf planting strips, curbs, walks, and street railways, lamp or telegraph posts, fire hydrants, man-hole covers, catch-basins, and sewer, water and gas pipes, if any, in these roads. Indicate width of travelled ways, sidewalks and planting strips by figures if established by public authorities. Show by broken lines all proposed public roads and private rights-of-way bordering on, leading to, or within the property.

6. INTERIOR FEATURES.

Survey and indicate on the map all interior roads, drives, cart tracks, wood roads, walks, trails, hydrants, water, gas and sewer pipes, cultivated areas, fences, walls, pits, quarries, ledges, large boulders, ditches, culverts, catch-basins, watercourses, ponds, swamps, springs, wells, cisterns, and outlines of all buildings, showing outside doors, steps, cellar bulkheads, verandas, terraces, etc., belonging to them. Give minimum dimensions of all culverts.

7. TREES AND FOLIAGE.

Indicate on the map by sketchy lines the overhang of branches of isolated trees, important trees in the midst of woods and masses of trees and of high bushes. Locate accurately and show on the map by strong dots all isolated trees over three inches in diameter of trunk, and all the principal trees along fence lines, along the margin of bodies of wood, and the best of the trees in the midst of woods. Unless otherwise directed the kind of every tree so located is to be indicated on the map by letters forming abbreviations of their common names. The following abbreviations for the commoner sorts of trees are recommended:

Apple	Ap.	Locust	Lo.
Arbor vitæ	Arb.	Magnolia	Mg.
Ash	A.	Norway Maple	N. Mp.
Basswood	Bass	Peach	Pch.
Beech	B.	Pear	Pr.
Birch	Br.	Poplar	Pop.
Catalpa	Cat.	Red Maple	R. Mp.
Cedar	Ced.	Red Oak	R. Ok.
Cherry	Chr.	Silver Maple	Si. Mp.
Chestnut	C.	Spruce	Sp.
Elm	E.	Sugar Maple	Su. Mp.
Hard pine	H. P.	Sweet Gum	S. G.
Hemlock	H.	Sycamore	Syc.
Hickorynut	Hic.	Thorn	Thr.
Hornbeam	Horn.	Tulip	Tp.
Horse Chestnut	H. C.	White Oak	W. Ok.
Ironwood	Irw.	White Pine	W. P.
Live Oak	L. Ok.	Willow	W.

The diameter of the trunk of each tree four feet above ground is to be indicated in feet and tenths. In woods state the predominating kinds of trees and average height of the taller trees. Unless otherwise directed, each tree located is to have affixed, horizontally, to its south side, five feet above the ground, by a hard brass wire nail one and one-half inches long, a hard brass label one-thirtieth of an inch thick, three-fourths of an inch wide, and three inches long, with a number four-tenths of an inch high stamped upon it, the last digit to be at least one inch from the nail hole to allow for growth of tree. The nail hole should be near righthand end so label can be held by left hand while nail is being driven. A memorandum book is to be furnished by the surveyor in case the trees are thus numbered giving the number of each tree, its common or botanical name, and the diameter of its trunk four feet above the ground, this list to be preceded by a key containing the various names used and followed by the abbreviation adopted for each.

8. CONTOURS.

Show contour lines for each foot of elevation, except on very steep, high banks, where they may be for every five feet of elevation, and except on the face of cliffs and irregular ledges, where they may be omitted. The contours are to show all irregularities of surface, steep banks along roadsides, and gravel pits, mounds, dumps, washouts and the like. The contours are to be numbered on the map at frequent intervals. If the contours do not clearly show any well marked forms of the ground, such as low, artificial banks or terraces, these should be indicated by additional signs of some sort. The office plotting of contours by the method of interpolation is less satisfactory than the locating of numerous points on the contours and the plotting of the contours and other features in the field at the time of locating points.

9. ELEVATIONS.

Give figures of elevation to the nearest tenth of a foot along center line of all roads bounding upon, leading to, and within the property with sufficient frequency to indicate gradients or changes in gradient. Give figures of elevation of normal surface of water in ponds, brooks, and swamps. Give figures of elevation of summits, or salient points on ledges, and of bowl-shaped depressions. Give figures of elevation of the top of the first floor and of the basement or cellar floor of all buildings, and of the ground at their entrances, and at foot of outside flights of steps. Give elevation at top and bottom of all other flights of steps. Give figures of elevation of bottom at each end of every culvert, on top of retaining walls and dams, at intervals on rails of railroads, especially at grade crossings, and at overhead or under crossing bridges and over culverts. Where contours are more than 200 feet apart, give figures of elevation to nearest tenth of a foot at every cross section stake, when these are called for. Give figure of elevation of ground on uphill side of base of trunk of each tree shown on plan.

10. DATUM.

The city or town datum is to be used when practicable. In case some other arbitrary datum is assumed, it should be so low that minus figures of elevation will not be needed. In case the property is upon or near the seashore or a lake or large river, and a city datum is not used, the datum should be the low water elevation of the sea, or the normal elevation of the lake or river.

11. BENCH MARKS.

Suitable bench marks are to be selected at least one in each 1,000 foot square, using a spike near base of a tree or fence post, if nothing better is at hand, and these bench marks are to be indicated on the map by the letters B. M., followed by figures giving the elevation in feet and hundredths.

12. ACCURACY.

In measuring cross section squares and setting cross section stakes, or in locating and setting stakes for plane-table, stadia, transit or triangulation stations, the error, from one corner or station or stake to the next, is not to exceed one-half of one per cent in distances of 500 feet or less, and is not to exceed one-quarter of one per cent in distances of 500 feet to 1000 feet, and is not to exceed one-tenth of one per cent in distances of 1000 feet to 5000 feet. In locating boundaries, bound marks, streets, buildings, retaining walls, and other objects having formal or definite shape, the error in reference to adjoining cross section stakes is not to exceed one foot. The error in locating trees, dry-boulder walls, ditches, brooks, ponds, swamps and springs, ledges, large boulders, quarries, gravel pits, and washouts is not to exceed two feet. The error in surveying elevations is not to exceed one-tenth foot for all bench marks, buildings, walls, tracks, culverts, macadamized drives, and other definite constructions, and is not to exceed five-tenths foot for ground at cross section stakes, for summits and hollows, and for water surfaces. The error in contours is not to exceed that which would normally result from carefully estimating by the eye their difference from what they would be if the slope were straight from one cross section stake to the next, or, in case there are no cross section stakes, between points of elevation equally numerous but irregularly located.

13. DRAUGHTING.

The map is to be a tracing drawn on the rough side of tracing cloth. All lines, including contours, figures and lettering, are to be drawn with black ink; every fifth contour is to be drawn with a heavier line. The contours are to be numbered on the map at frequent intervals. All lines are to be strong enough, and all lettering and figures simple enough and large enough, to show well on sunprints, or when reduced by photolithography to one-third scale. No