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ARITHMETIC

BOOK THREE

BY

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AND

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PREFACE

This book has been prepared to meet the needs of pupils and teachers in Grades VII and VIII, not alone as to subject matter but also as to convenience of arrangement. In deciding upon the latter feature the authors accept the fact that each of the two well-defined types of textbooks in arithmetic has advantages.

With books of the type arranged for the recurrent treatment of topics the market is well supplied. Some of the extreme books of this recurrent form are, it is true, too fragmentary in arrangement to give the pupil that feeling of mastery which is his right and his reward. But excellent books of this type, which avoid its perils and provide all its good features, are now available. There is, however, great need for arithmetics having the topical arrangement but thoroughly modern in spirit and in material. That need this series is written to supply.

The concrete problems are modern in the best sense. Those that seem to be real are real; modern business customs are followed, and the needs of the future citizen are always kept in mind. But in this book is found no idea of considering an example concrete when it treats of topics or contains technicalities that no pupil understands and few teachers are expected to know.

Under each topic there are found an unusually large number of well-graded examples, and at frequent intervals collections of problems without numbers have been inserted as exercises in terse mathematical statement. There are also included numerous sets of exercises that relate to the vocational interests of our people, to the end that pupils may leave school with the real applications of arithmetic to common life clearly in their minds.

To meet a rapidly growing demand, a chapter on algebra has been placed at the end of the book, to be taken or not as circumstances may dictate. In this chapter will be found a brief but clear presentation of that part of algebra that the artisan needs in using the formulas of his trade journals, and that the business man may occasionally demand. This work has been made as concrete as possible, and it properly bears the title of Vocational Algebra.

A few recreations are given in the Appendix, that the lighter side of arithmetic may be seen along with its more serious aspect, and a large amount of drill work has been added for the use of pupils who may be in need of more than is commonly required.

Any corrections or suggestions relating to the work will be thankfully received.

The authors hope that teachers who recognize the strong features of the topical arrangement, who wish for an abundance of well-graded problems requiring thought in their solution, who are sympathetic with the movement to replace the obsolete by the genuine applications of arithmetic to the American life of to-day, who are opposed to certain of the extreme attempts of the present time that are sure to result disastrously to scholarship, — that such teachers will find in this work a sane, modern, and helpful treatment of the subject.

GEORGE WENTWORTH
DAVID EUGENE SMITH

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ARITHMETIC

BOOK THREE

CHAPTER I

PERCENTAGE

1. Per Cent. Another name for hundredths is *per cent.*

Thus 0.01 or $\frac{1}{100}$ is the same as 1 per cent.

That part of arithmetic which treats of per cent is called *percentage.*

2. Symbol for Per Cent. The symbol for per cent is written thus : %.

We may read 0.06 either "6 hundredths" or "6 per cent." In the same way, we may think of 6% either as "6 per cent" or "6 hundredths," although it is read "6 per cent."

The expression 800% means $\frac{800}{100}$ and equals the whole number 8; 225% equals the mixed number 2.25, or $2\frac{1}{4}$; $\frac{1}{2}$% means $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$, and is read either "$\frac{1}{2}$ per cent" or, quite commonly, "$\frac{1}{2}$ of 1%."

3. Relation to Fractions. Since 6% means $\frac{6}{100}$, which equals 0.06, or $\frac{3}{50}$, we see that we may express per cent as a decimal fraction or as a common fraction.

4. Per Cents as Common Fractions. Since $62\frac{1}{2}\% = \dfrac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}$, therefore

To express per cent as a common fraction, write the number indicating the per cent for the numerator and 100 for the denominator, and reduce this fraction to lowest terms.

EXERCISE 1

Reduce to a common fraction, integer, or mixed number:

1. 4%.	**11.** 100%.	**21.** $6\frac{1}{4}$%.	**31.** $3\frac{1}{3}$%.
2. 5%.	**12.** 200%.	**22.** $12\frac{1}{2}$%.	**32.** $6\frac{2}{3}$%.
3. 6%.	**13.** 500%.	**23.** $37\frac{1}{2}$%.	**33.** $33\frac{1}{3}$%.
4. 10%.	**14.** 125%.	**24.** $62\frac{1}{2}$%.	**34.** $66\frac{2}{3}$%.
5. 15%.	**15.** 150%.	**25.** $87\frac{1}{2}$%.	**35.** $16\frac{2}{3}$%.
6. 20%.	**16.** 175%.	**26.** $18\frac{3}{4}$%.	**36.** $83\frac{1}{3}$%.
7. 25%.	**17.** 120%.	**27.** $31\frac{1}{4}$%.	**37.** $112\frac{1}{2}$%.
8. 30%.	**18.** 140%.	**28.** $43\frac{3}{4}$%.	**38.** $133\frac{1}{3}$%.
9. 50%.	**19.** 160%.	**29.** $56\frac{1}{4}$%.	**39.** $166\frac{2}{3}$%.
10. 75%.	**20.** 180%.	**30.** $68\frac{3}{4}$%.	**40.** $187\frac{1}{2}$%.

41. How much is $\frac{1}{25}$ of 75? 4% of 75?

42. How much is $\frac{1}{4}$ of 640? 25% of 640?

43. How much is $\frac{1}{2}$ of 1728? 50% of 1728?

44. How much is $\frac{3}{4}$ of 8440? 75% of 8440?

45. How much is $1\frac{1}{4} \times 864$? 125% of 864?

46. Instead of multiplying 64 by $12\frac{1}{2}$%, what fractional part of 64 may we take? Do this.

47. Instead of multiplying 720 by $37\frac{1}{2}$%, what fractional part of 720 may we take? Do this.

48. Take $87\frac{1}{2}$% of 80 in the shortest way.

49. Take $33\frac{1}{3}$% of 66 in the shortest way.

Find the following:

50. $16\frac{2}{3}$% of 66.	**53.** $112\frac{1}{2}$% of 176.	**56.** $133\frac{1}{3}$% of 729.
51. $16\frac{2}{3}$% of 738.	**54.** $137\frac{1}{2}$% of 256.	**57.** $133\frac{1}{3}$% of 801.
52. $12\frac{1}{2}$% of 736.	**55.** $162\frac{1}{2}$% of 312.	**58.** $166\frac{2}{3}$% of 708.

5. Per Cents as Decimals. Since 25.5% and 0.255 have the same value,

Therefore, *to express as a decimal a number written with the per cent sign, omit the sign and move the decimal point two places to the left.*

Since per cent means hundredths, when we omit the per cent sign we must indicate the hundredths in some other way, as by moving the decimal point two places to the left.

Thus $1\frac{1}{2}\% = 0.01\frac{1}{2}$, or 0.015; $125\% = 1.25$; $0.6\% = 0.006$.

EXERCISE 2

Express as decimals:

1. 6%.	7. $16\frac{2}{3}\%$.	13. 0.9%.	19. 675%.
2. 10%.	8. $33\frac{1}{3}\%$.	14. $0.1\frac{1}{2}\%$.	20. 375%.
3. 25%.	9. $37\frac{1}{2}\%$.	15. $6.2\frac{1}{2}\%$.	21. 100%.
4. 50%.	10. $66\frac{2}{3}\%$.	16. 200%.	22. 1000%.
5. 75%.	11. 0.5%.	17. 150%.	23. 0.005%.
6. $12\frac{1}{2}\%$.	12. 0.7%.	18. 125%.	24. 0.065%.

25. Express $\frac{1}{8}\%$ as a decimal; as a common fraction.

26. Express $\frac{7}{8}\%$ as a decimal; as a common fraction.

27. Express $333\frac{1}{3}\%$ as a decimal; as an improper fraction; as a mixed number.

28. How much is $\frac{1}{100}$ of \$200? 1% of \$200?

29. How much is 0.06 of \$100? 6% of \$100?

30. How much is 0.15 of \$300? 15% of \$300?

31. How much is $1\frac{1}{4} \times \$650$? $1.25 \times \$650$? 125% of \$650?

32. How much is $2.50 \times \$460$? 250% of \$460?

33. If 2% of the pupils of this school are absent to-day, how many are absent out of every 100? out of every 50?

6. Decimals as Per Cents. Since per cent means hundredths, to express a decimal as per cent we have to consider only how many hundredths the decimal represents.

For example, express 0.3 as per cent.

Here $0.3 = 0.30$, which is 30 hundredths, or 30%.

Express 0.375 as per cent.

Here $0.375 = 37.5$ hundredths, or $37\frac{1}{2}$ hundredths. Hence it equals 37.5% or $37\frac{1}{2}\%$.

Express 0.00125 as per cent.

Here $0.00125 = 0.00\frac{125}{1000} = 0.00\frac{1}{8} = \frac{1}{8}\%$.

Express $6.5\frac{1}{2}$ as per cent.

Here $6.5\frac{1}{2} = \frac{655}{100} = 655\%$.

Therefore, *to express a decimal as per cent, write the per cent sign after the number of hundredths.*

EXERCISE 3

Express as per cents:

1. 0.5.	**6.** 0.25.	**11.** 0.125.	**16.** $0.2\frac{1}{2}$.	**21.** $0.3\frac{1}{3}$.
2. 0.7.	**7.** 0.57.	**12.** 0.275.	**17.** $0.5\frac{1}{4}$.	**22.** $0.333\frac{1}{3}$.
3. 0.6.	**8.** 0.34.	**13.** 0.375.	**18.** $0.6\frac{3}{4}$.	**23.** $6.6\frac{2}{3}$.
4. 6.2.	**9.** 0.29.	**14.** 0.758.	**19.** $5.2\frac{1}{8}$.	**24.** 0.00375.
5. 7.5.	**10.** 7.65.	**15.** 9.637.	**20.** $7.3\frac{3}{8}$.	**25.** 0.000125.

26. A pint, being 0.5 qt., is what per cent of a quart?

27. A peck, being 0.25 bu., is what per cent of a bushel?

28. A quart, being 0.125 pk., is what per cent of a bushel?

29. Express 0.007 of a mile as per cent of a mile.

30. What per cent of a dollar is 37¢? is 37 dimes?

31. Express 1 gill as per cent of a gallon.

32. Express 1 quart as per cent of a bushel.

7. Common Fractions as Per Cents. Since per cent means hundredths, to express a common fraction as per cent we have only to reduce it to hundredths.

For example, $\frac{4}{5} = \frac{80}{100}$, and this is the same as 0.80 or 80%. Likewise $\frac{2}{25} = \frac{8}{100} = 8\%$, and $2\frac{1}{2} = \frac{5}{2} = \frac{250}{100} = 250\%$.

Therefore, *to express a common fraction as per cent, reduce it to hundredths, omit the denominator, and write the numerator followed by the per cent sign.*

This is easily done by reducing to a decimal, and writing this in the per cent form (§ 6).

It is often more convenient to express per cents decimally than with the symbol %. Thus, in the case of $25\frac{2}{3}$ we have $25.66\frac{2}{3}$, a more convenient form than $2566\frac{2}{3}\%$.

EXERCISE 4

Express as per cents:

1. $\frac{1}{2}$.	6. $\frac{3}{5}$.	11. $\frac{9}{20}$.	16. $\frac{1}{3}$.	21. $\frac{1}{16}$.	26. $2\frac{1}{4}$.
2. $\frac{1}{4}$.	7. $\frac{4}{5}$.	12. $\frac{1}{25}$.	17. $\frac{2}{3}$.	22. $\frac{5}{16}$.	27. $3\frac{3}{5}$.
3. $\frac{3}{4}$.	8. $\frac{1}{10}$.	13. $\frac{11}{25}$.	18. $\frac{1}{8}$.	23. $\frac{1}{32}$.	28. $7\frac{5}{8}$.
4. $\frac{1}{5}$.	9. $\frac{7}{10}$.	14. $\frac{1}{50}$.	19. $\frac{3}{8}$.	24. $\frac{15}{32}$.	29. $6\frac{1}{3}$.
5. $\frac{2}{5}$.	10. $\frac{1}{20}$.	15. $\frac{37}{50}$.	20. $\frac{5}{8}$.	25. $\frac{5}{24}$.	30. $75\frac{7}{8}$.

31. A foot is what per cent of a yard?

32. An inch is what per cent of a foot?

33. A pint is what per cent of a gallon?

34. What per cent of a gallon is 3 qt.?

35. What per cent of a pound is 4 oz.?

36. What per cent of a pound is 7 oz.?

37. If a man spends $\frac{5}{8}$ of his income, what per cent of his income does he spend?

38. If you have increased $\frac{1}{10}$ in weight during the past year, what per cent have you gained in weight?

8. Terms used in Percentage. The number of which some per cent is to be taken is called the *base*.

The number of hundredths of the base is called the *rate*.

For example, in 25% of \$300, \$300 is the base and 25% is the rate. Sometimes 25 is called the *rate per cent*, 25% being called the *rate*, but these two terms are commonly used to mean the same thing.

The result found by taking a certain per cent of the base is called the *percentage*. Therefore,

The percentage is the product of the base and the rate.

9. Important Per Cents. Certain per cents are used so frequently that their equivalent common fractions should be remembered. These are as follows:

$50\% = \frac{1}{2}$,	$37\frac{1}{2}\% = \frac{3}{8}$,	$16\frac{2}{3}\% = \frac{1}{6}$,	$20\% = \frac{1}{5}$,
$25\% = \frac{1}{4}$,	$62\frac{1}{2}\% = \frac{5}{8}$,	$33\frac{1}{3}\% = \frac{1}{3}$,	$40\% = \frac{2}{5}$,
$12\frac{1}{2}\% = \frac{1}{8}$,	$87\frac{1}{2}\% = \frac{7}{8}$,	$66\frac{2}{3}\% = \frac{2}{3}$,	$60\% = \frac{3}{5}$,
$6\frac{1}{4}\% = \frac{1}{16}$,	$3\frac{1}{8}\% = \frac{1}{32}$,	$83\frac{1}{3}\% = \frac{5}{6}$,	$80\% = \frac{4}{5}$.

To take $87\frac{1}{2}\%$ of 648 is, therefore, the same as to take $\frac{7}{8}$ of 648.

EXERCISE 5

Find:

1. 50% of \$274.
2. 25% of \$372.
3. $12\frac{1}{2}\%$ of \$512.
4. $6\frac{1}{4}\%$ of \$3376.
5. $37\frac{1}{2}\%$ of \$17.76.
6. $62\frac{1}{2}\%$ of \$11.92.
7. $87\frac{1}{2}\%$ of \$34.48.
8. $3\frac{1}{8}\%$ of \$739.20.
9. $16\frac{2}{3}\%$ of \$34.26.
10. $33\frac{1}{3}\%$ of \$77.16.
11. $66\frac{2}{3}\%$ of \$80.07.
12. $83\frac{1}{3}\%$ of \$71.10.
13. 20% of \$735.15.
14. 40% of \$611.25.
15. 60% of \$734.15.
16. $112\frac{1}{2}\%$ of 72.
17. $137\frac{1}{2}\%$ of 96.
18. $116\frac{2}{3}\%$ of 72.
19. $133\frac{1}{3}\%$ of 81.
20. $166\frac{2}{3}\%$ of 84.

10. To find Some Per Cent of a Number. Finding some of the more important per cents of numbers, by reducing to common fractions, has already been considered on page 6. The more general case, that in which common fractions cannot be used to advantage, will now be considered.

For example, required to find $23\frac{3}{4}\%$ of 275.

$$23\tfrac{3}{4}\% = 0.23\tfrac{3}{4}.$$

Multiplying by $0.23\frac{3}{4}$, we have $65.31\frac{1}{4}$.

If we wish, we may write $23\frac{3}{4}\%$ as 0.2375 and then multiply. We may also, if we choose, write the product 65.3125.

$$\begin{array}{r} 275 \\ \underline{.23\frac{3}{4}} \\ 206\frac{1}{4} \\ 825 \\ \underline{550} \\ 65.31\frac{1}{4} \end{array}$$

Therefore, *to find a required per cent of a number, multiply the number by the given rate.*

EXERCISE 6

Find, by using decimals:

1. 22% of 75.	**6.** 26% of 3.4.	**11.** $2\frac{3}{4}\%$ of 8.4.
2. 35% of 86.	**7.** 43% of 6.7.	**12.** 0.5% of 5.7.
3. 27% of 32.	**8.** 39% of 5.3.	**13.** 1.9% of 6.8.
4. 41% of 78.	**9.** 28% of 0.7.	**14.** 235% of 742.
5. 62% of 93.	**10.** 35% of 0.63.	**15.** 622% of 0.72.

Find, by using common fractions:

16. 25% of 1728.	**24.** $66\frac{2}{3}\%$ of 7116.
17. 50% of 6235.	**25.** $87\frac{1}{2}\%$ of 35,360.
18. 20% of 7341.	**26.** $16\frac{2}{3}\%$ of \$13,806.
19. 30% of 2790.	**27.** $37\frac{1}{2}\%$ of 25,040.
20. 75% of 6448.	**28.** $83\frac{1}{3}\%$ of 22,350.
21. $12\frac{1}{2}\%$ of 9616.	**29.** 125% of 948.
22. $6\frac{1}{4}\%$ of 24,328.	**30.** $133\frac{1}{3}\%$ of 729.
23. $33\frac{1}{3}\%$ of 9051.	**31.** $166\frac{2}{3}\%$ of 816.

32. A man borrowed \$175 and had to pay 6% of that sum for the use of it for a year. How much did he pay in addition to the sum borrowed?

33. If a manufacturer sells at a profit of 15% shoes that cost him \$2.33$\frac{1}{3}$ a pair to make, how much is his profit on 1000 pairs of shoes?

34. If a shop manufactures 276 locomotives and sells 75% of them for \$11,125 each and the rest for \$9825 each, how much is received for all?

35. A farmer can have his land poorly plowed for \$3.48 an acre, but it will cost him 66$\frac{2}{3}$% more to have a good job done. How much will it cost to have 60 acres well done?

36. How much butter fat is there in 375 lb. of milk brought to a creamery, the creamery test showing that 3.9% of the weight of the milk is butter fat?

37. A farmer takes 340 lb. of milk to a creamery and is paid 26¢ a pound for the butter fat. The tests show that the butter fat is 3.8% of the weight of this milk. How much does he receive?

38. One cow in a dairy gives 28 lb. of milk a day, which tests 3.1% of butter fat; another gives 24 lb., testing 3.7%. Butter fat being worth 23¢ a pound, which cow is the more profitable, and how much more?

39. A farmer has 75 trees on an acre of woodland, of which he decides to cut 60%. If wood is worth \$5.75 a cord, and he can cut 3 cords from 5 trees, how much will he receive for the wood?

40. A dressmaker bought 37$\frac{1}{2}$ yd. of chiffon velvet at \$4.10 a yard, and received a reduction of 8% for paying cash. She sold the velvet at \$4.25 a yard. How much did she make on the transaction?

11. Given the Percentage and Rate to find the Base. In accordance with the definitions in § 8, finding a number of which a given per cent is known may be spoken of as finding the base when the percentage and rate are given.

In these examples we let 100% of a number equal the number. This is legitimate, because $100\% = \frac{100}{100} = 1$, so that we merely let once the number stand for the number. This is often convenient.

(1) If 15% of a number is 9165, what is the number?

Since	15% of the number = 9165,
therefore	1% of the number = $\frac{1}{15}$ of 9165, or 611,
and	the number = 100×611, or 61,100.

(2) If $33\frac{1}{3}\%$ of a number is 247, what is the number?

Since $33\frac{1}{3}\%$, or $\frac{1}{3}$, of a number is 247, the number is 3×247, or 741.

(3) 275 is $66\frac{2}{3}\%$ more than what number?

Since	100% of the number = the number,
therefore	$66\frac{2}{3}\%$ of the number = the increase,
and	$166\frac{2}{3}\%$, or $\frac{5}{3}$, of the number = 275.
Therefore	$\frac{1}{3}$ of the number = $\frac{1}{5}$ of 275, or 55,
and	the number = 3×55, or 165.

(4) 616 is 12% less than what number?

Since	100% of the number = the number,
therefore	12% of the number = the decrease,
and	88% of the number = 616.
Therefore	1% of the number = $\frac{1}{88}$ of 616, or 7,
and	the number = 100×7, or 700.

In each of these examples it will be noticed that we practically divide the percentage by the rate. Therefore,

The base equals the percentage divided by the rate.

This is also evident because the percentage equals the base multiplied by the rate (§ 10). Therefore we have the product (percentage) and one factor (rate) to find the other factor (base), and this we find by dividing.

EXERCISE 7

Of what numbers are the following the given per cents?

1. 72, 12%.
2. 165, 15%.
3. 252, 21%.
4. 578, 34%.
5. 7999, 25%.
6. 48.72, $33\frac{1}{3}$%.
7. 96.54, $66\frac{2}{3}$%.
8. $114, 6%.
9. $3113, $5\frac{1}{2}$%.
10. 32.86, 106%.
11. 45.10, 110%.
12. 55.55, 125%.

The following numbers are the given per cents more than what numbers?

13. $34\frac{1}{3}$, 3%.
14. 26, 4%.
15. 42, 5%.
16. 259.7, 6%.
17. 342.4, 7%.
18. 459, 8%.
19. 800, 25%.
20. 520, 30%.
21. $1087\frac{1}{2}$, 50%.
22. 9.72, $33\frac{1}{3}$%.
23. 6.37, $16\frac{2}{3}$%.
24. 15.6, $66\frac{2}{3}$%.

The following numbers are the given per cents less than what numbers?

25. 24, 4%.
26. 38, 5%.
27. $31\frac{1}{3}$, 6%.
28. 251.1, 7%.
29. 294.4, 8%.
30. 682.5, 9%.
31. 573, 25%.
32. 140, 50%.
33. 180, 75%.
34. 6.3, $12\frac{1}{2}$%.
35. 20, $37\frac{1}{2}$%.
36. 3.3, $62\frac{1}{2}$%.

37. $435 is 6% of what sum of money?
38. $19.75 is 5% of what sum of money?
39. $38.25 is $4\frac{1}{2}$% of what sum of money?
40. What is the number of which 14.7 is $66\frac{2}{3}$%?
41. What is the sum of which $41.25 is 15%?
42. What is the sum of which $33\frac{1}{3}$% is $2.25?
43. A farmer sold his milk to a factory where he received credit for 1045 lb. of butter fat. If his milk tested 3.8% butter fat, how many pounds of milk did he sell?

12. Given the Base and Percentage to find the Rate. Since the percentage is the product of the base and rate (§ 8), therefore the rate may be found by dividing the percentage by the base. Consider, for example, the following:

(1) What per cent of 5 is 4?

Here 4 is the product of some number and 5. Therefore the required per cent may be found by dividing 4 by 5; and $4 \div 5$, or $\frac{4}{5}$, $= 0.80 = 80\%$.

(2) What per cent of 245 is 29.4?

Here 29.4 is the product of some number and 245. Therefore the required per cent, $29.4 \div 245$, $= 0.12 = 12\%$.

Therefore, *the rate equals the percentage divided by the base.*

This case and the two cases considered in § 10 and § 11 cover all important questions in percentage.

EXERCISE 8

Find what per cent the second number is of the first:

1. 72, 3.6.	**7.** 28.6, 6.006.	**13.** 785.6, 392.8.
2. 97, 6.79.	**8.** 23.4, 10.53.	**14.** 48.36, 8.06.
3. 387, 15.48.	**9.** 33.3, 10.989.	**15.** 542.1, 361.4.
4. 426, 38.34.	**10.** 1671, 50.13.	**16.** 21.36, 8.01.
5. 121, 13.31.	**11.** 1555, 373.2.	**17.** 72.48, 45.3.
6. 29.1, 4.656.	**12.** 2035, 447.7.	**18.** 2.248, 1.967.

19. A dressmaker bought a 20-yard silk dress pattern for \$45 less 20%. She sold it for \$45. What per cent did she gain on what she paid for it?

20. Allowing 8 oz. of Paris green to 50 gal. of water in preparing a spray against leaf-eating insects, and considering a gallon of water to weigh 8.4 lb., the weight of the Paris green is what per cent of the weight of the water?

13. Commercial Discount. A reduction from the list price of an article, from the amount of a bill of goods, or from the amount of a debt is called *commercial discount.*

Commercial discount is also called *trade discount.*

Discounts are usually reckoned at some common fraction of, or at some rate per cent of, the amount from which the discount is made.

The amount of the bill after the discount has been made is called the *net amount,* or the *net price.*

For example, what is the net amount of a bill of $275 after a discount of 20% is made?

Amount of bill	$275
Less 20%, or $\frac{1}{5}$, of $275	55
Net amount	$220

Instead of speaking of "20% off," merchants often speak of "$\frac{1}{5}$ off." So they speak of "$\frac{1}{4}$ off," and "$\frac{1}{3}$ off," but it is not so convenient to use common fractions for such discounts as 12%, 7%, 15%, etc.

EXERCISE 9

Find the net prices of goods billed as follows, with discounts as stated:

1. $27, 6%.	12. $25.50, 6%.	23. $125.50, 4%.
2. $32, 7%.	13. $48.75, 8%.	24. $375.50, 8%.
3. $46, 5%.	14. $62.30, 10%.	25. $426.50, 10%.
4. $75, 8%.	15. $73.40, 10%.	26. $382.60, 10%.
5. $64, 4%.	16. $68.75, 12%.	27. $296.80, 15%.
6. $86, 9%.	17. $86.40, 15%.	28. $492.50, 12%.
7. $68, 10%.	18. $37.90, 10%.	29. $598.20, 15%.
8. $82, 12%.	19. $48.50, 12%.	30. $275.00, 11%.
9. $96, 15%.	20. $52.50, 14%.	31. $826.50, 12%.
10. $252, 8%.	21. $68.20, 15%.	32. $962.20, 15%.
11. $175, 9%.	22. $47.20, 15%.	33. $896.50, 14%.

34. The discount on some goods listed at $460 is $115. What is the rate of discount?

35. The discount at 8% on some goods is $41.60. What is the list price?

36. The discount at 12½% on some goods is $2.05. What is the list price?

37. A bicycle was marked $45, but was sold at 12% discount. What was the net price?

38. A piano was listed at $550, but was sold at 30% discount. What was the selling price?

39. A jobber bought some goods listed at $2450. He was allowed a discount of 22%. What was the net price?

40. A merchant bought 850 yards of muslin at 6¢ a yard, less 18%. How much did it all cost?

41. A merchant bought 975 yards of lawn for $146.25, less 20%. How much did he pay per yard?

42. A grocer bought 36 doz. cans of soup listed at $1.75 a dozen. He was allowed a discount of 25%. What was the net price?

43. A dealer sold some boxes of soap, 60 cakes to the box, for $2.25 a box. He allowed a discount of 20%. How much did he receive per cake?

44. A grocer bought 160 bags of flour, 12¼ lb. to the bag, listed at 4¢ a pound. He was allowed a discount of 15%. What did the flour cost him?

45. A grocer bought 24 doz. pound packages of macaroni for $31.20, less 15%. He sold it for 15¢ per package. How much did he gain in all?

46. After the holidays a man bought some silver goods that had been marked $280 but were now subject to a discount of 12½%. How much did they cost him?

14. Discount Series. The changes in cost of production from time to time, and the variation due to the credit of the buyer, and the quantity of goods purchased, give rise frequently to several discounts known as a discount series.

Goods that are selling at a discount of 20% may be produced at a lower cost owing, for example, to an unusually large cotton crop. An additional discount of 10% may then be allowed. Likewise a person whose credit is exceptional, who buys an especially large quantity, or who pays cash, may be entitled to a still further discount of 5%.

In a discount series the first denotes the discount from the list price, the second denotes the discount from the remainder, and so on.

Thus $100 less 20%, 10%, and 5%, means that 20% is taken from $100, leaving $80; then 10% from $80, leaving $72; then 5% from $72, leaving $68.40. The order is, however, immaterial.

When there is a fraction of a cent in a remainder, the seller usually counts it a whole cent in his favor.

The regular discount is quoted first, and then the special discounts.

Advantages of a discount series are that it saves reprinting elaborate catalogues, allows of easy change, enables the dealer easily to offer different discounts to different customers, and allows for several options to purchasers.

If a bill is due at once it usually bears the words "Terms cash." If due in 60 days without discount, it may bear the marks "N/60" or "Net 60 da." If due in 60 days, but with a special discount of 3% if paid in 10 days, it may read "3/10, N/60," or "3% 10 da., net 60 da."

EXERCISE 10

Given these list prices and discounts, find the net prices:

1. $225, 20% and 6%.
2. $368, 25% and 5%.
3. $480, 12½% and 3%.
4. $350, 15% and 4%.
5. $560, 25% and 10%.
6. $270, 33⅓% and 10%.
7. $480, 25%, 10%, 5%.
8. $560, 25%, 20%, 3%.
9. $729, 33⅓%, 10%, 10%.
10. $280, 25%, 10%, 10%.
11. $2760, 33⅓%, 20%, 10%.
12. $5550, 33⅓%, 25%, 15%.

13. How much will 240 ft. of iron pipe cost if listed at \$1.50 a foot, a discount of 40% and 10% being allowed?

14. How much will a gross of jars of mixed pickles cost at \$6 a dozen, a discount of 20% and 10% being allowed?

15. The list price of 2-pound cartons of prunes is \$24 per 100. A discount of 20% and 10% is allowed. What is the net price of 300 cartons?

16. A book dealer can buy an encyclopedia for \$60 less 20% and 10%, or for the same price less 15% and 15%. Which is the better for him, and how much better?

17. A grocer buys 100 dozen jars of table salt listed at \$1.50 a dozen, a discount of 25% and 8% being allowed. If he sells the salt at 12¢ a jar, how much does he gain?

18. A dry goods dealer buys 960 yd. of silk listed at \$1.50 a yard, a discount of $33\frac{1}{3}$% and 5% being allowed. If he sells it at \$1.50 a yard, how much does he gain?

19. A dealer buys 36 doz. pocket knives listed at \$8.40 a dozen, a discount of 25% and 10% being allowed. If he sells them at 75¢ apiece, how much does he gain?

20. A furniture dealer buys 16 bedroom sets listed at \$36 each, a discount of $16\frac{2}{3}$% and 10% being allowed. If he sells the furniture at \$36 a set, how much does he gain?

21. A hardware dealer bought a shipment of stoves. After deducting a discount of $33\frac{1}{3}$% and 25%, and paying \$9.75 freight and \$2.50 cartage, the stoves cost him \$252.25. What was the list price of the stoves?

22. A dry goods dealer bought a shipment of cloth. After deducting a discount of 25% and 10%, and paying \$8.50 for freight and cartage, the cloth cost him \$410. What was the list price?

15. Bills Discounted. The following is a common form of a bill of goods purchased from a wholesale dealer.

Chicago, March 15, 19.........

Mr. R. D. Scanlan, Springfield, Ill.

Bought of **Lane & Company, Jewelers**
1072 Wabash Avenue

Terms *20%, 10%*

Feb. 10.	*8 doz. spoons No. 487 @ $14.75*	*$118.00*
	7 doz. plated forks No. 511 @ $4.20	*29.40*
		$147.40
	Less 20%, 10%	*41.27*
		$106.13

EXERCISE 11

Make out bills for the following:

1. 36 doz. files at $6.25; 3 doz. saws at $17.50. Discounts 30%, 20%.

2. 960 yd. silk at $1.40; 840 yd. velvet at $1.60. Discounts 25%, 10%.

3. 4 doz. pr. hinges at $4.60; 25 doz. table knives at $8.40. Discounts 20%, 10%.

4. 12 doz. locks at $4.20; 6 doz. mortise locks at $4.60. Discounts 25%, 8%.

5. 360 yd. taffeta at 98¢; 4 gross pompons at $144; 3 doz. pieces braid at $20.40. Discounts 10%, 5%.

6. 60 lb. brads at 6¢; 200 bolts at $4.80 per C; 6 doz. knives at $7.20. Discounts 40%, 20%, 10%.

7. 480 yd. silk at $1.60; 640 yd. lawn at 22¢; 960 yd. taffeta at 96¢. Discounts 10%, 5%, 5%.

16. Single Discount equal to a Series. It is often convenient to find a single discount equal to a series.

For example, find the single discount equal to the discount series 20%, 10%, and 10%.

Let the list price be represented by	100 %	of the list price.
Then deducting 20% of this, or	20 %	of the list price,
we have	80 %	of the list price.
Deducting 10% of this, or	8 %	of the list price,
we have	72 %	of the list price.
Deducting 10% of this, or	7.2%	of the list price,
we have	64.8%	of the list price.

Then 100% − 64.8% = 35.2%, the single discount.

EXERCISE 12

Find the single discount equal to the discount series:

1. 10%, 8%.
2. 15%, 6%.
3. 18%, 3%.
4. 14%, 9%.
5. 20%, 10%.
6. 30%, 10%.
7. 25%, 12½%.
8. 10%, 10%, 10%.
9. 25%, 15%, 10%.
10. 25%, 20%, 10%.
11. 30%, 20%, 10%.
12. 33⅓%, 10%, 5%.
13. 33⅓%, 10%, 10%.
14. 16⅔%, 10%, 10%.
15. 20%, 12½%, 10%.
16. 25%, 12½%, 10%.

17. Compare the single discount equal to the series 10% and 5%, with the sum of 10% and 5% minus the product of 10% and 5%.

18. By the method suggested in Ex. 17, find the single discount equal to the series 20%, 5%.

19. How does the single discount equal to the discount series 20%, 10%, and 5%, compare with that equal to the discount series 10%, 5%, and 20%? Prove it.

17. Gain and Loss. In business transactions gain and loss is often computed as a certain per cent of the cost of the property sold. The following examples may be taken as typical problems:

(1) For how much must a merchant sell goods that cost $275, so as to gain 20%?

If he gains 20% of $275 he gains $\frac{1}{5}$ of $275, or $55. Therefore he must sell the goods for $275 + $55, or $330.

(2) If goods marked $75 are sold at a bargain sale for 15% off, at what price are they sold?

If the goods are sold at 15% off the marked price, they are sold for $75 less 15% of $75, or $75 – $11.25, or $63.75.

(3) If some goods are damaged so that they are sold for $120, or 25% below cost, how much did they cost?

100% of the cost = the cost.
25% of the cost = the loss.
75% of the cost = the selling price = $120.
1% of the cost = $\frac{1}{75}$ of $120.
100% of the cost = $\frac{100}{75}$ (or $\frac{4}{3}$) of $120, or $160.

(4) A dealer sold a hat that cost $2.40 so as to gain 25%. The selling price was 20% less than the marked price. What was the marked price?

He gained 25% (or $\frac{1}{4}$) of $2.40, or $0.60. Therefore the selling price was $2.40 + $0.60, or $3.

100% of the marked price = the marked price.
20% of the marked price = amount taken off.
80% of the marked price = the selling price = $3.
1% of the marked price = $\frac{1}{80}$ of $3.
100% of the marked price = $\frac{100}{80}$ (or $\frac{5}{4}$) of $3, or $3.75.

We may prove this result or any of the preceding results by working backward. Here $3.75 less 20% (or $\frac{1}{5}$) of $3.75 = $3.75 – $0.75 = $3, the selling price. Also $2.40 + 25% of $2.40 = $3, the selling price, and the two results agree.

EXERCISE 13

For how much must a merchant sell goods that he bought at the price here given, so as to gain the per cent specified?

1. $25, 12%.	**6.** $175, $8\frac{1}{2}$%.	**11.** $172.80, $12\frac{1}{2}$%.
2. $36, 14%.	**7.** $190, $9\frac{1}{4}$%.	**12.** $373.20, $16\frac{2}{3}$%.
3. $65, 15%.	**8.** $250, 12%.	**13.** $45,550, $17\frac{1}{2}$%.
4. $72, 18%.	**9.** $375, 15%.	**14.** $27,475, 22%.
5. $85, 25%.	**10.** $735, 18%.	**15.** $75,250, 25%.

If goods marked as here shown are sold at a bargain sale for the given per cent off, find the selling price:

16. $60, 15%.	**19.** $176, $6\frac{1}{4}$%.	**22.** $225.40, 25%.
17. $85, 18%.	**20.** $712, $12\frac{1}{2}$%.	**23.** $27,250, 20%.
18. $72, 22%.	**21.** $972, $16\frac{2}{3}$%.	**24.** $22,400, $17\frac{1}{2}$%.

If goods are sold for the sum specified, which is the given per cent below cost, find the cost:

25. $71.25, 5%.	**28.** $140, 20%.	**31.** $226.20, 13%.
26. $72.16, 12%.	**29.** $207.20, 30%.	**32.** $265.60, 17%.
27. $58.48, 14%.	**30.** $382.50, 15%.	**33.** $358.80, 22%.

34. A merchant sold some goods that cost $175 so as to gain 20%. The selling price was 20% less than the marked price. What was the marked price?

35. A dealer sold some furniture for which he had paid $240, so as to gain $12\frac{1}{2}$%. The selling price was 25% less than the marked price. What was the marked price?

36. A merchant sold some goods at 10% below cost, this being 20% below his marked price. The goods cost $300. What was the marked price?

Given the cost and rate of gain as follows, find the amount of gain:

37. $275, 15%.
38. $365, $12\frac{1}{2}$%.
39. $4723, 14%.
40. $3965, 16%.
41. $7287, 22%.
42. $275.40, 15%.
43. $346.50, 16%.
44. $492.75, 18%.
45. $575.50, 24%.
46. $812.50, 16%.
47. $4275.50, 8%.
48. $2936.40, 12%.
49. $3275.75, 16%.
50. $4206.03, $33\frac{1}{3}$%.
51. $3742.20, $16\frac{2}{3}$%.

Given the cost and rate of loss as follows, find the amount of loss:

52. $275, 6%.
53. $345, 4%.
54. $463, 8%.
55. $362.50, 4%.
56. $427.50, 8%.
57. $732.25, 12%.
58. $4725.25, 8%.
59. $2632.50, 12%.
60. $4170.00, 15%.

61. How much is $33\frac{1}{3}$% profit on goods costing $637.20?

62. A dealer sold a sewing machine at a gain of 25%. It cost him $31.40. Find the selling price.

63. A man bought a horse for $150 and sold it at a loss of 8%. Find the selling price.

64. A merchant bought 85 yd. of silk at $1.25 a yard and sold it at a loss of 8%. How much was his loss?

65. A jeweler bought a dozen clocks at $4.75 apiece and sold them at an advance of 16%. How much was his gain?

66. A man bought a horse for $140 and sold it for $161. What was his gain per cent?

67. A dealer bought a piano for $320 and sold it for $400. What was his gain per cent?

68. A grocer bought 500 lb. of coffee at 32¢ a pound. He sold it at an advance of $6\frac{1}{4}$%. How much did he gain?

69. A dealer sold a piano for $280, thereby losing $12\frac{1}{2}$%. How much did the piano cost him?

70. A wholesale dealer paid \$2.50 per barrel for apples. He sold them to a retailer at a gain of 8%, and the retailer sold them at a gain of $16\frac{2}{3}$%. What was the retail price?

71. A clothier marked a suit of clothes that cost him \$13.20, so as to gain 40%. He discounted the marked price $16\frac{2}{3}$%. What was the actual selling price?

72. A man bought a horse for \$150 and a carriage for \$120. He sold them, gaining 8% on the horse and losing 12% on the carriage. Find his net gain or loss.

73. A farmer received \$160 for some produce. It passed through the hands of a shipper, a commission merchant, and a retailer, who gained respectively $12\frac{1}{2}$%, $3\frac{1}{3}$%, and 20%. What was the retail price?

74. Some goods cost a merchant \$100. At what price must he mark them so that he can take off 20% and still make a profit of 20%?

75. Some goods cost a merchant \$240. At what price must he mark them so that he can take off 20% and still make a profit of 20%?

76. At what per cent above cost must a merchant mark his goods so that he can take off 20% and still make a profit of 20%?

77. At what per cent above cost must a merchant mark his goods so that he can take off 25% and still make a profit of 25%?

78. A man sold two houses for \$9600 each. On one he gained 20% and on the other he lost 20%. Did he gain or lose on the whole transaction, and how much?

79. A man sold two houses for \$9600 each. He sold one at a profit of 25% and the other at a loss of 25%. Did he gain or lose on the whole transaction, and how much?

18. Commission or Brokerage. A sum paid by one person to another for transacting business is called *commission* or *brokerage.*

19. Principal and Agent. The person for whom another transacts business is called the *principal.* The person who transacts the business is called the *agent.*

20. Commission Merchant and Broker. An agent who has actual possession of the goods of his principal is called a *commission merchant.* An agent who makes sales or purchases for his principal without having actual possession of the goods is called a *broker.*

Commission and brokerage are usually reckoned at a certain per cent of the money involved in the transaction. If goods are bought, the commission is a certain per cent of the cost; if goods are sold, the commission is a certain per cent of the amount received; if a collection is made, the commission is a certain per cent of the amount collected.

Brokerage is often a fixed sum per bushel (as of wheat purchased), a fixed sum per 100 bales (as of cotton), a fixed sum per tierce (as of lard), and so on.

In all problems, unless otherwise directed, discard any fraction of a cent less than $\frac{1}{2}$ in a final result; when a fraction of a cent is $\frac{1}{2}$ or more, count it as a full cent.

EXERCISE 14

Find the commissions on sales or purchases of the following amounts, at the rates specified:

1. \$275, 2%.	**6.** \$2750, $1\frac{1}{2}$%.	**11.** \$8000, $\frac{1}{8}$%.
2. \$345, 3%.	**7.** \$3275, $2\frac{1}{2}$%.	**12.** \$7500, $\frac{1}{4}$%.
3. \$675, 4%.	**8.** \$2760, $3\frac{1}{2}$%.	**13.** \$6200, $\frac{1}{2}$%.
4. \$725, 5%.	**9.** \$4175, $4\frac{1}{2}$%.	**14.** \$3520, $\frac{3}{8}$%.
5. \$875, 6%.	**10.** \$6375, $3\frac{1}{3}$%.	**15.** \$4800, $1\frac{1}{8}$%.

Find the net amounts remaining after deducting commissions on sales as follows, at the rates specified:

16. \$4800, $\frac{1}{8}\%$.	**19.** \$2800, $2\frac{1}{4}\%$.	**22.** \$275.50, 4%.
17. \$6400, $\frac{1}{4}\%$.	**20.** \$5600, $2\frac{1}{8}\%$.	**23.** \$612.50, 8%.
18. \$3500, $\frac{1}{5}\%$.	**21.** \$6400, $3\frac{1}{2}\%$.	**24.** \$721.50, 6%.

25. What is the commission at $3\frac{1}{2}\%$ on a sale of potatoes to the amount of \$1270?

26. A broker bought 750 tierces of lard for a customer, charging $2\frac{1}{2}$¢ a tierce. A tierce being 340 lb., how many pounds did he buy, and what was the brokerage?

27. Lard selling at \$7.03 per tierce, how much will 1250 tierces cost? How many pounds are there? What is the brokerage on the purchase at $2\frac{1}{2}$¢ per tierce?

28. A commission merchant sells for a factory 6900 lb. of cheese at 12¢ a pound, charging 5% commission. What sum should he remit to his principal?

29. A commission merchant sold 1700 dozen eggs for a dealer, charging him 5% on the sale, and remitting \$339.15. At what price per dozen did he sell the eggs?

30. A commission merchant sold 160 tubs of butter, 50 lb. to the tub, for \$1920, charging 5% commission. What did the commission amount to per pound?

31. A broker buys 1750 bags of coffee for a customer, at 8.43¢ per pound. There being 130 lb. to the bag and the broker charging \$10 per 250 bags, how many pounds are bought, and what is the total cost, including the brokerage?

32. A broker bought 700 bales of cotton for a customer, at 12.1¢ per pound. Taking the bales as weighing 500 lb. each, and the brokerage at \$5 per hundred bales, find the number of pounds purchased and the total cost including the brokerage.

21. Insurance. An agreement by which, for a lawful consideration, a company agrees to pay an indemnity for loss by fire or other specified cause is called *insurance.*

22. Policy. The written contract of insurance is called a *policy.* The sum agreed to be paid by the terms of an insurance policy is called the *face* of the policy. The consideration, or sum paid for insurance, is called the *premium.*

There are several kinds of insurance. There are companies that insure people's lives (life insurance); that insure against loss by fire (fire insurance), by accidents to ships (marine insurance), by accidents to people (accident insurance), by burglary (burglary insurance), and so on. Those who insure are often called *underwriters.*

23. Fire Insurance. Insurance against loss or damage by fire is called *fire insurance.*

In fire insurance the premium is sometimes stated as a certain sum on $100, and sometimes as a certain rate per cent.

EXERCISE 15

Find the premiums on these policies at the rates specified:

1. $1500, $1\frac{1}{4}$%.	**9.** $12,500, 1%.	**17.** $5200, 2.1%.
2. $1500, $1\frac{1}{2}$%.	**10.** $13,500, 2%.	**18.** $6500, 1.2%.
3. $1750, $1\frac{3}{4}$%.	**11.** $15,000, $1\frac{1}{4}$%.	**19.** $3750, 1.6%.
4. $2250, $2\frac{1}{4}$%.	**12.** $12,750, $1\frac{3}{4}$%.	**20.** $2550, 1.8%.
5. $3500, $1\frac{1}{2}$%.	**13.** $15,250, $1\frac{1}{2}$%.	**21.** $2750, 2.2%.
6. $2600, $2\frac{3}{4}$%.	**14.** $16,500, $2\frac{1}{4}$%.	**22.** $3500, 2.5%.
7. $3800, $2\frac{1}{2}$%.	**15.** $17,250, $2\frac{1}{2}$%.	**23.** $4800, 2.8%.
8. $4500, $1\frac{3}{4}$%.	**16.** $18,750, $2\frac{3}{4}$%.	**24.** $5500, 3.25%.

Find the faces of policies, given the premiums and rates:

25. $65, 1%.	**27.** $19, $\frac{1}{2}$%.	**29.** $19.75, 1%.
26. $48, 2%.	**28.** $24, $\frac{3}{4}$%.	**30.** $41.25, $2\frac{1}{2}$%.

Find the rates, given the faces of policies and premiums:

31. $1800, $18. **33.** $2100, $31.50. **35.** $2800, $56.
32. $1650, $41.25. **34.** $3200, $56. **36.** $3500, $52.50.

Find the premiums on the following policies, given the premiums per $100:

37. $2750, $1.25. **39.** $4800, $1.75. **41.** $15,000, $2.50.
38. $3250, $1.50. **40.** $5600, $2.25. **42.** $27,500, $2.75.

43. At $1.10, what is the premium on a $3750 policy?

44. At $1.30, what is the premium on a $5500 policy?

45. A building worth $4500 is insured for $\frac{2}{3}$ of its value at $1\frac{3}{4}\%$. What is the premium?

46. The premium for insuring some property at $1.75 is $96.25. What is the face of the policy?

47. Some property worth $2700 is insured for $\frac{2}{3}$ of its value, the annual premium being $27. What is the rate?

48. Merchandise worth $3600 is insured for 80% of its value at $1.50 on $100. What is the premium?

49. If a 3-year policy for $2500 costs $60, what is the rate of premium per year?

50. A building is insured at 60¢ per $100, the annual premium being $14.40. If the face of the policy is 80% of the value of the building, what is the value?

51. A dealer insured his stock of goods for 80% of its value at $1.60 per $100. The premium was $57.60. What was the value of the stock?

52. A factory is insured for $7000 in one company and $5000 in another. The factory is partly destroyed by fire, the insurance adjusters fixing the damage at $8000. How much should each company pay?

24. Life Insurance. Insurance whereby some person known as the beneficiary receives a certain sum on the death of the insured, or when the insured reaches a certain age, is called *life insurance.*

The beneficiary may be the estate of the insured, or one or more persons named in the policy.

There are various forms of life insurance, but for the purposes of this arithmetic and of explaining the general principles of the subject, the standard form known as *ordinary life* is sufficient.

The rate is always given as the cost of $1000 worth of insurance, the rate $27.75 meaning that the annual premium on a policy for $1000 is $27.75. This subject is not, therefore, an application of percentage, but is properly considered here in connection with the other insurance problems.

EXERCISE 16

Find the premiums on these policies at the given rates:

1. $5000, $21.75.
2. $7500, $22.65.
3. $3500, $28.45.
4. $4500, $26.36.
5. $7000, $27.75.
6. $4500, $22.35.
7. $3500, $24.65.
8. $12,000, $25.62.
9. $14,000, $26.38.
10. $15,500, $29.46.

11. If a man takes out a policy for $7500 at $26.36 per thousand, how much premium will he pay in 20 payments?

12. A man takes out a $4500 policy in one company at $24.30 per thousand and a $3500 policy in another company at $24.75 per thousand. What do the two premiums together amount to in a year?

13. A man takes out a $6000 policy at $26.65 per thousand. He dies just after eight annual premiums have been paid. How much does the beneficiary receive? How much does he receive in excess of the premiums paid?

25. Taxes. Money levied by a government for its support and for public purposes is called a *tax.*

26. Kinds of Taxes. There are two general kinds of taxes, direct and indirect. A *direct* tax is a tax levied on a person, or on the value of his property or business. An *indirect* tax is a tax levied in other ways, as on goods imported into the country, on tobacco, or on alcoholic liquors.

The expenses of the United States government are, in general, met by indirect taxes, including duties on imports, and excise duties on tobacco and liquor. There is also some revenue from the sale of public lands.

The expenses of the several states, counties, cities, towns, and villages are, in general, met by direct taxes, although some of these corporations levy a few indirect taxes.

27. Poll Tax. A tax levied upon a person is called a *poll tax.*

The old word for head was *poll;* hence this name. In some places the poll tax is no longer collected.

28. Property Tax. A tax levied upon property is called a *property tax.*

This may be upon land and buildings (real estate), or upon money, securities, and movable property generally (personal property).

29. Licenses. A permit to engage in some specified act or to enjoy some special privilege is called a *license.*

For example, there are licenses to run automobiles, to own dogs, to sell liquor, and so on, all of which are forms of taxes.

30. Assessing Taxes. Taxes are usually assessed against the individual by officers called *assessors.* They decide how much each person's property is worth and the rate of tax.

The law provides for methods of reviewing the action of assessors in case it is not considered fair.

The *tax rate* is sometimes expressed as a rate per cent, but more often as the number of mills on a dollar.

31. Collecting Taxes. Taxes are collected by an officer known as the *collector*.

The collector sometimes receives a fixed salary and sometimes a commission on the amount of tax collected.

After the tax collector has collected taxes he pays the money over to an officer known as the *treasurer*, and by him it is paid out for public services according to law.

For example, a tax of $43,600 is to be raised by a town which contains 1200 polls, taxed at $1 each, and which has taxable property assessed at $3,200,000. The town receives from the state $7200 as its share of the railroad tax, and other corporation taxes collected by the state. Find the rate of taxation, and the tax paid by Mr. Brown, if his property is assessed at $5960, if he pays for 1 poll, and if the collector's commission is 1%.

Total tax		$43,600
Deduct poll taxes	$1200	
Amount received from state	7200	
Total to be deducted	$8400	8,400
Net property tax		$35,200

Tax on $1 = $35,200 ÷ 3,200,000 = $0.011.
That is, the tax is 11 mills on $1, or $11 on $1000, or 1.1%.
Therefore Mr. Brown will pay as follows:

Property tax, 1.1% of $5960,	$65.56
Poll tax	1.
Property and poll tax	$66.56
Add the collector's commission, 1% of $66.56,	.67
Total tax, including commission	$67.23

In practice, in a case like that of the above example, the assessors would probably make the tax rate 12 mills, or 1.2%, in order to provide for contingencies such as reduction of assessments and abatement of taxes.

If the assessors make the tax rate a little too high there will be a little larger balance in the treasury at the end of the year.

In the examples in Exercise 17 consider the collector's commission only when told to do so.

32. Tax Table. Assessors usually prepare a table like the following, which is arranged for 5½ mills on a dollar:

TAX TABLE. RATE 5½ MILLS ON $1

	0	1	2	3	4	5	6	7	8	9
0	0000	0055	0110	0165	0220	0275	0330	0385	0440	0495
1	0550	0605	0660	0715	0770	0825	0880	0935	0990	1045
2	1100	1155	1210	1265	1320	1375	1430	1485	1540	1595
3	1650	1705	1760	1815	1870	1925	1980	2035	2090	2145
4	2200	2255	2310	2365	2420	2475	2530	2585	2640	2695
5	2750	2805	2860	2915	2970	3025	3080	3135	3190	3245
6	3300	3355	3410	3465	3520	3575	3630	3685	3740	3795
7	3850	3905	3960	4015	4070	4125	4180	4235	4290	4345
8	4400	4455	4510	4565	4620	4675	4730	4785	4840	4895
9	4950	5005	5060	5115	5170	5225	5280	5335	5390	5445

Here the first figure of the number of dollars assessed is given at the left, and the second one at the top. Thus, at 5½ mills on $1, the tax on $100 is $0.55; on $140, $0.77; on $2500, $13.75.

For example, find by the above table the tax of James Johnson, who pays no poll tax and has his property assessed at $9765, the collector's commission being 1%.

Tax on $9700 (to the right of 9, and under 7)	$53.35	
Tax on $65 (to the right of 6, and under 5)	.36	(.3575)
Property tax	$53.71	
Collector's commission, 1% of $53.71,	.54	
Total tax	$54.25	

EXERCISE 17

Using the above tax table, find the tax on the following amounts, adding the collector's commission of 1%:

1. $250.
2. $850.
3. $900.
4. $4000.
5. $3650.
6. $4550.
7. $7550.
8. $9000.
9. $12,000.
10. $11,500.
11. $11,250.
12. $39,750.

Make out a tax table for the tax rate of 9 mills on a dollar. Then find the tax on the following amounts, adding the collector's commission of 1%:

13. $350.	**17.** $5000.	**21.** $5550.	**25.** $12,500.
14. $550.	**18.** $3550.	**22.** $8000.	**26.** $26,500.
15. $700.	**19.** $4750.	**23.** $13,000.	**27.** $48,750.
16. $3600.	**20.** $3750.	**24.** $35,000.	**28.** $82,350.

29. The tax rate is 11 mills. How much tax must be paid on property assessed at $4500?

30. The tax rate is 13 mills. How much tax must be paid on property assessed at $12,500?

31. If a man's tax is $97.50 and the rate is 13 mills on a dollar, what is the assessed valuation of his property?

32. If a man's property is assessed at $6500, and his tax is $78, what is the rate of tax?

33. If the assessed valuation of the property in a town is $486,500, what must be the tax rate to raise $5838?

34. The tax rate is 14 mills. A man pays a tax of $54.64, of which $2 is poll tax. What is the assessed valuation of his property?

35. If the state tax is 1 mill, the county tax 3½ mills, and the town tax 4 mills, how much tax must a man pay on a house assessed at $5000?

36. A man died, leaving $17,500, of which $17,000 was willed to his children and $500 to his brother. Under the laws of his state there is an inheritance tax of 1% to be paid by children and 4% by brothers or sisters. How much tax must be paid on these bequests?

37. A tax collector receives $49,760.73 in taxes including his commission. How much is his commission at 1¼%?

33. Indirect Taxes. Indirect taxes have already been defined in § 26. One of the most important is the tax on goods imported into this country. This tax is paid to the United States government.

34. Duties or Customs. Taxes levied by the government on imported goods are called *duties* or *customs*.

Duties are collected at customhouses situated in large cities or in important seaports or places on the border between different countries, as between the United States and Canada or Mexico.

35. Kinds of Duty. There are two kinds of duty, specific and ad valorem. A *specific duty* is a definite sum levied on each unit by which an article is measured or weighed. An *ad valorem* duty is a certain per cent on the value of the goods.

36. Tariff. A schedule of duties on different articles, such as are imposed by our Congress, is called a *tariff*.

Goods on which duty is not levied are placed on the *free list*.

EXERCISE 18

Find the ad valorem duty on each of the following invoices of goods at the rate specified:

1. $475, 30%.	**6.** $1500, 25%.	**11.** $12,450, 20%.
2. $625, 35%.	**7.** $1650, 30%.	**12.** $42,375, 30%.
3. $750, 40%.	**8.** $2700, 35%.	**13.** $29,362, 35%.
4. $835, 45%.	**9.** $3650, 40%.	**14.** $32,275, 50%.
5. $975, 60%.	**10.** $7575, 50%.	**15.** $75,295, 60%.

16. Find the specific duty on a gross of articles at 17¢ each.

17. Find the specific duty on 275 articles at $1.25 each.

18. The duty paid on a certain invoice of goods was $4500, at 60% ad valorem. What was the valuation on which the duty was reckoned?

19. Find the specific duty on 756 lb. of a certain material at 55¢ a pound.

20. An importer pays $952.50 ad valorem duty on goods valued at $6350. What is the rate of duty?

21. Ad valorem duty is levied on goods valued at $2750, making the total cost $3850. What is the rate of duty?

22. If the duty at 45% is $3723.75, what is the valuation on which the duty is reckoned?

23. If a duty of $3237.50 ad valorem is levied on goods valued at $9250, what is the rate of duty?

24. The duty on certain farm products is $4 a ton. If a dealer imports 82¾ tons, what is the duty?

25. Some goods cost the importer $10,875, including the duty of 45%. How much was the duty?

26. Some goods cost the importer $3712.50, including the duty of 35%. What was the original cost?

27. The cost of certain goods plus the duty of 33⅓% was $8800. Required to find the original cost.

28. The duty on certain goods is 22¢ a pound, plus 30% ad valorem. If the goods weigh 275 lb. and are worth $360, what is the total duty?

29. The duty on a certain product is 60¢ a pound, plus 45% ad valorem. If the product weighs 168 lb. and is worth $380, what is the total duty?

30. A man imported some goods on which he paid a specific duty of $385. If the duty is 22¢ a pound, how many pounds did he import?

31. A man imported 475 lb. of goods on which he paid a specific duty of 6¢ a pound and an ad valorem duty of 25%. The total duty was $216. What was the valuation on which the duty was reckoned?

EXERCISE 19

Review Problems

Find separately the following per cents:

1. 0.25%, $2\frac{1}{2}$%, 25%, and 250% of 4884.
2. 0.12$\frac{1}{2}$%, $1\frac{1}{4}$%, $12\frac{1}{2}$%, and 125% of 9768.
3. $\frac{1}{3}$%, $3\frac{1}{3}$%, $33\frac{1}{3}$%, $333\frac{1}{3}$%, and 333.33$\frac{1}{3}$% of 7365.
4. $\frac{1}{6}$%, $1\frac{2}{3}$%, $16\frac{2}{3}$%, $166\frac{2}{3}$%, and 166.66$\frac{2}{3}$% of 9732.
5. $\frac{2}{3}$%, $6\frac{2}{3}$%, $66\frac{2}{3}$%, $666\frac{2}{3}$%, and 666.66$\frac{2}{3}$% of 7596.
6. $\frac{3}{4}$%, $7\frac{1}{2}$%, 75%, 750%, and 7500% of 13,748.
7. $\frac{3}{8}$%, $3\frac{3}{4}$%, $37\frac{1}{2}$%, 375%, and 3750% of 11,168.
8. $\frac{5}{8}$%, $6\frac{1}{4}$%, $62\frac{1}{2}$%, 625%, and 6250% of 12,344.
9. $\frac{7}{8}$%, $8\frac{3}{4}$%, $87\frac{1}{2}$%, 875%, and 8750% of 37,072.

10. A man bought a horse for $125 and sold it for $147.50. What per cent did he gain?

11. A man bought a house for $6800 and sold it for $5780. What per cent did he lose?

12. A real estate agent sold a lot for $1775 and received $71 commission. What per cent did he receive?

13. A real estate agent sold a farm, his commission at 3% being $145.50. What was the selling price?

14. A man sold a piece of land for $1320, thereby gaining 10% on the cost. How much did he gain?

15. An agent received $14 for selling some property on a commission of 2%. What was the selling price?

16. A dealer sold some fruit for $12\frac{1}{2}$% less than it cost, thereby losing $70. What was his selling price?

17. An agent sold some property on a commission of 5%. After deducting his commission from the selling price he remitted $2137.50. What was the selling price?

18. The premium for insuring a house at $1\frac{1}{4}\%$ was \$75. What was the face of the policy?

19. A man paid \$42 for insuring his house at $\frac{3}{4}\%$ on 80% of its value. What was its value?

20. A man sold his farm for \$975 less than it cost him, which was a loss of 15%. Find the cost.

21. A merchant bought hats at \$2.75 each, and sold them at \$3.50. Find his gain per cent.

22. A merchant's assets amount to \$4063.50, and this sum will pay 54% of his debts. How much does he owe?

23. A dealer sells cloth at \$2.53 a yard, gaining 15%. What per cent would he gain by selling it at \$2.64 a yard?

24. What was the amount of a bill upon which a collector received \$22.75 for collecting, his commission being $3\frac{1}{2}\%$?

25. A merchant sold cloth at \$1.40 a yard and gained 12%. What did the cloth cost him?

26. A merchant sold cloth that cost him \$1.23 a yard and gained $33\frac{1}{3}\%$. At what price did he sell it?

27. What must be the marked price of a hat that cost \$2.50, so as to make 12% after discounting the price 30%?

28. A man bought a farm at \$62.50 an acre and sold it at \$75 an acre. What per cent did he gain?

29. A workman's wages, amounting to \$1050 a year, was increased 10%. After a year it was decreased 10%. What was it then?

30. Two men own a mill together. The first paid 45% of the purchase price and the second paid the balance, \$9350. What was the cost of the mill?

31. A bank clerk started work at a salary of \$900 a year. The second year his salary was raised 20%, the third year 25%, and the fourth year $33\frac{1}{3}\%$. What was his salary then?

32. A merchant sold a hat for $3.30, thereby gaining 25%. Find the cost.

33. A man sold a horse at a gain of $32.50, which was $33\frac{1}{3}$% of the cost. Find the selling price.

34. At what price must you mark a hat, costing $1.20, so as to discount the price 25% and still make 20%?

35. The premium for insuring a $7000 house for 80% of its value was $70. What was the rate of insurance?

36. Two men own a boat. One paid $132 and the other paid 45% of the cost. How much did the boat cost?

37. What is the per cent of attendance in a school of 35 pupils, if there are 21 days of absence in 5 school weeks?

38. Two men are in partnership. One owns $47\frac{1}{2}$% of the business. If the business is worth $5000, what is the value of the share of each?

39. A merchant's sales in October amounted to $1552.50, which was 9% of the sales for the year. What were the total sales for the year?

40. A dry goods dealer sold a piece of cloth at $1.40 a yard, thereby gaining $16\frac{2}{3}$%. At what price should he have sold it in order to gain 20%?

41. A dry goods dealer sold a piece of cloth and gained $12\frac{1}{2}$%. If he had sold it at 90¢ a yard, he would have gained 25%. What was his selling price?

42. An agent collected rents, charging 3% on all sums collected. The rents in one month amounted to $675. How much did he remit to the owner?

43. A workman in a factory received $1350 the first year. The second year his wages was advanced 10%, and the third year 10%. The fourth year it was reduced 10%. How much did he then receive?

44. The discount from the list price of a sewing machine, at $33\frac{1}{3}\%$, was \$13.50. What was the list price?

45. A man paid \$60 for insuring his \$6000 house at 80% of its value. Find the rate of insurance.

46. An agent collected rents, charging 3% on all sums collected. He remitted to the owner \$271.60. How much rent did he collect?

47. A dry goods dealer sold a piece of cloth at 72¢ a yard, thereby gaining 20%. What would have been his gain per cent if he had sold it at 75¢ a yard?

48. A man was unsuccessful in business and failed, owing \$2376.50. He was able to pay his creditors only 40¢ on a dollar. What was the total loss to his creditors?

49. A man paid \$5500 for a farm. He rents it for \$425 a year and pays \$60.50 for taxes, \$25 for insurance, and \$37 for repairs. What is the per cent of net income?

50. A merchant bought a bill of goods, and paid \$27.49 for freight, \$15 for insurance, and \$4.75 for cartage. These expenditures represented 4% of the purchase price. What was the purchase price? What was the total cost?

51. A sugar factory pays \$4.60 per ton for all beets testing 14% sugar, and 22¢ per ton for each additional 1%. A farmer grows $13\frac{1}{2}$ tons of beets per acre, from which 2.025 tons of sugar are extracted. How much per acre does he receive for his crop?

52. Bordeaux mixture for plant diseases consists of 4 lb. of unslaked lime, 6 lb. of copper sulphate, and 25 gal. of water. Taking the weight of a gallon of water as 8.4 lb., what per cent of the weight of the mixture is the weight of the lime? what per cent is the weight of the copper sulphate? what per cent is the weight of the water?

37. Interest. Money paid for the use of money is called *interest.*

38. Principal. Money loaned on which interest is paid is called the *principal.*

39. Rate of Interest. The rate per cent of the principal paid in interest for one year is called the *rate of interest.*

The sum of the principal and interest is called the *amount.*

40. Illustrative Problems.

(1) Find the interest on \$840 at 5% for 2 yr.

Interest for 1 yr. = 5% of \$840 = \$42.
Interest for 2 yr. = 2 × \$42 = \$84.

(2) Find the interest on \$750 at $5\frac{1}{2}$% for $3\frac{1}{2}$ yr.

Interest for 1 yr. = $5\frac{1}{2}$% of \$750 = \$41.25.
Interest for $3\frac{1}{2}$ yr. = $3\frac{1}{2}$ × \$41.25 = \$144.38.

Therefore, *to find the interest, given the principal, rate, and time, multiply the principal by the rate, and the product by the number that expresses the time in years.*

EXERCISE 20

Given the principal, rate, and time, find the interest:

1. \$275, 5%, 2 yr.
2. \$350, 4%, 3 yr.
3. \$460, 6%, 2 yr.
4. \$525, 5%, 3 yr.
5. \$265, 4%, 2 yr.
6. \$425, 5%, 3 yr.
7. \$635, 6%, 4 yr.
8. \$875, 6%, 2 yr.
9. \$925, 5%, 5 yr.
10. \$975, 4%, 2 yr.
11. \$975, $5\frac{1}{2}$%, 2 yr.
12. \$725, $4\frac{1}{2}$%, 3 yr.
13. \$835, $3\frac{1}{2}$%, 2 yr.
14. \$425.50, 6%, $2\frac{1}{2}$ yr.
15. \$375.75, 6%, $3\frac{1}{2}$ yr.
16. \$420.50, 5%, $2\frac{1}{4}$ yr.
17. \$2450, 4%, $3\frac{3}{4}$ yr.
18. \$3575, 5%, $2\frac{3}{4}$ yr.

41. Interest when Time is expressed in Days. The case considered in § 40 involved time expressed in years. If the time involves days, we may first reduce this to a fraction of a year and proceed as before.

For example, find the interest on $240 for 2 yr. 3 mo. 10 da. at 6%.

$$10 \text{ da.} = \tfrac{1}{3} \text{ mo.}$$

$$3\tfrac{1}{3} \text{ mo.} = \tfrac{10}{3} \text{ of } \tfrac{1}{12} \text{ yr.} = \tfrac{5}{18} \text{ yr.}$$

$$2\tfrac{5}{18} \times 6\% \text{ of } \$240 = \frac{41 \times 6 \times \$240}{18 \times 100} = \$32.80.$$

It is the custom in computing interest, when tables (see § 46) are not used, to take 30 da. to the month, which means 360 da. to the year.

EXERCISE 21

Given the principal, rate, and time, find the interest:

1. $250, 5%, 4 mo. 10 da.
2. $520, 6%, 2 yr. 9 mo. 25 da.
3. $760, 5%, 4 yr. 6 mo. 15 da.
4. $875, 6%, 3 yr. 7 mo. 16 da.
5. $1250, 5%, 1 yr. 10 mo. 5 da.
6. $2500, 5½%, 2 yr. 10 mo. 7 da.
7. $2275, 5½%, 3 yr. 11 mo. 2 da.
8. $3525, 4½%, 1 yr. 11 mo. 3 da.

Given the principal, rate, and time, find the amount:

9. $175, 6%, 6 mo.
10. $225, 6%, 8 mo. 3 da.
11. $625, 5%, 4 mo. 3 da.
12. $1250, 5%, 2 yr. 3 mo. 8 da.
13. $2275, 5½%, 3 yr. 10 mo. 14 da.

42. Problems in Interest. Of the four elements used in computing interest,—principal, rate, time, and interest (or amount, which is only the sum of the principal and interest), —we can find any one if the other three are known.

For, since the interest is the product of three factors (principal, rate, and time), if it is divided by the product of any two of these factors the quotient is the third.

The following examples will illustrate:

(1) What principal will, in 3 yr. 6 mo., yield $52.50 interest at 6%?

Since the interest for $3\frac{1}{2}$ yr. is $52.50,
therefore the interest for 1 yr. is $\$52.50 \div 3\frac{1}{2}$, or $15.

Since 6% of the principal is $15,
therefore 1% of the principal is $2.50,
and 100% of the principal is $250.

(2) What principal will amount to $570 in 2 yr. 4 mo. at 6%?

The amount of $1 for 2 yr. 4 mo. at 6% is $1.14.

Since $1.14 is the amount of $1,
$570 is the amount of $\frac{\$570}{1.14}$, or $500.

(3) In what time will $540 yield $109.80 interest at 6%?

The interest on $540 at 6% for 1 yr. is $32.40.

Since $32.40 is the interest for 1 yr.,
$109.80 is the interest for $\frac{109.80}{32.40}$ yr., or $3\frac{7}{18}$ yr.

$3\frac{7}{18}$ yr. $=$ 3 yr. $4\frac{2}{3}$ mo. $=$ 3 yr. 4 mo. 20 da.

(4) At what rate will $600 yield $81 interest in 3 yr.?

The interest on $600 at 1% for 3 yr. is $18.

Since $18 is the interest at 1%,
$81 is the interest at $\frac{81}{18}$%, or $4\frac{1}{2}$%.

EXERCISE 22

Find the principal that will yield the following interest:

1. $20, in 2 yr. 8 mo., at 6%.
2. $93⅓, in 3 yr. 4 mo., at 5%.
3. $9.59, in 3 yr. 6 mo., at 4%.
4. $40.47, in 1 yr. 7 mo., at 6%.
5. $3.64, in 5 mo. 18 da., at 6%.
6. $45.90, in 4 yr. 3 mo., at 4½%.

Find the principal that will amount to the following:

7. $482.40, in 3 yr. 4 mo., at 6%.
8. $1151.41, in 2 yr. 4 mo., at 4½%.
9. $1210.50, in 2 yr. 5 mo., at 5%.

Find the time, given the principal, rate, and interest:

10. $840, 5%, $147.
11. $625, 4%, $56.25.
12. $350, 6%, $49.
13. $1250, 6%, $187.50.

Find the rate, given the principal, time, and interest:

14. $330, 1 yr. 2 mo., $19.25.
15. $68.50, 3 yr. 6 mo., $9.59.
16. $87.50, 2 yr. 8 mo. 24 da., $14.13.
17. How long will it take $1 to yield $1 interest at 6%?
18. How long will it take $1 to double itself at 5%?
19. What sum must be put at interest at 6% so that it shall amount to $1000 in 16⅔ yr.?
20. What sum must be put at interest at 5% so that it shall amount to $1000 in 20 yr.?

43. Difference between Dates. The time between two dates may be found as here shown. For example, required to find the time from April 16, 1909, to February 5, 1911.

yr.	mo.	da.
1911	2	5
1909	4	16
1	9	19

Since February is the second month of the year, and April the fourth, the work is arranged as here shown. We then subtract as with other compound numbers; $5-16$ is impossible, but $35-16=19$. Proceeding in the usual manner, the result is 1 yr. 9 mo. 19 da. In such cases allow 30 da. to the month.

EXERCISE 23

Find the difference in time between these dates:

1. Oct. 2, 1909, Nov. 15, 1911.
2. Sept. 5, 1909; Dec. 25, 1912.
3. Sept. 7, 1909, July 4, 1913.
4. Dec. 10, 1910, Feb. 5, 1914.
5. Aug. 20, 1909, Jan. 6, 1912.
6. What is the interest on \$375 from July 4, 1910, to May 1, 1913, at 6%?
7. What is the interest on \$650 from June 20, 1910, to April 5, 1913, at 5%?
8. What is the interest on \$825 from May 15, 1910, to Feb. 10, 1914, at $4\frac{1}{2}$%?
9. The interest on \$360 from April 5, 1910, to Feb. 3, 1912, is \$39.48. What is the rate?
10. The interest at 5% on \$480 from March 7, 1910, to a certain other date is \$68. Required to find the second date.
11. Upon what sum is the interest at 6%, from July 4, 1910, to Oct. 4, 1913, equal to \$39?
12. What is the amount of \$650 at interest at 6% from Oct. 27, 1911, to April 12, 1914?

44. Six Per Cent Method. There is a very convenient method of computing the interest at 6%, depending upon the fact that if the rate for 1 yr. is 6%, the rate for 2 mo. is $\frac{1}{6}$ of 6%, or 1%.

Required the interest on $720 for 5 mo. 17 da. at 6%.

The interest for 2 mo. is 1% of $720	=	$7.20
The interest for 2 mo. more	=	7.20
The interest for 1 mo. is $\frac{1}{2}$ of $7.20	=	3.60
The interest for 15 da. is $\frac{1}{2}$ of $3.60	=	1.80
The interest for 1 da. is $\frac{1}{15}$ of $1.80	=	.12
The interest for 1 da. more	=	.12
The interest for 5 mo. 17 da.	=	$20.04

Therefore, *to find the interest for 60 da., move the decimal point two places to the left; for 6 da., three places to the left. From these results find the interest for the required time.*

In most cases in which months and days are involved, this is the simplest of all methods. Thus, to find the interest on $720 for 10 mo. 10 da. at 6%, we have $7.20 for 2mo., 5 × $7.20, or $36, for 10 mo., $\frac{1}{6}$ of $7.20, or $1.20, for 10 da. Hence the total interest is $37.20.

Of all the forms of the six per cent method this is the most easily applied. It is particularly convenient because most bank notes run for 30, 60, or 90 days.

EXERCISE 24

Find the interest at 6% on the following:

1. $240, 4 mo. 6 da.
2. $360, 3 mo. 3 da.
3. $750, 3 mo. 3 da.
4. $475, 3 mo. 6 da.
5. $280, 8 mo. 6 da.
6. $520, 5 mo. 12 da.
7. $450, 2 mo. 15 da.
8. $585, 2 mo. 20 da.
9. $1275, 2 mo. 6 da.
10. $3200, 3 mo. 7 da.
11. $4500, 4 mo. 9 da.
12. $3500, 2 mo. 15 da.
13. $2700, 2 mo. 19 da.
14. $3350, 8 mo. 17 da.
15. $4950, 10 mo. 10 da.
16. $6250, 10 mo. 18 da.

45. Six Per Cent Method for Other Rates. We may use this method to find interest at the other rates in common use.

Thus, since 5 is $\frac{1}{6}$ less than 6, the interest at 5% is $\frac{1}{6}$ less than the interest at 6%.

Similarly, the interest at $4\frac{1}{2}$% is $\frac{1}{4}$ less than the interest at 6%, the interest at 4% is $\frac{1}{3}$ less than the interest at 6%, and so on.

Find the interest on $1240 for 3 mo. 3 da. at 5%.

Interest for 2 mo. at 6%	$12.40
Interest for 1 mo. at 6%	6.20
Interest for 3 da. ($\frac{1}{10}$ mo.) at 6%	.62
Total interest at 6%	$19.22
Deduct $\frac{1}{6}$	3.20
Interest at 5%	$16.02

EXERCISE 25

Find the interest at 5% on the following:

1. $780, 3 mo. 6 da.

2. $860, 2 mo. 12 da.

3. $940, 1 mo. 18 da.

4. $1200, 2 mo. 3 da.

5. $1500, 3 mo. 3 da.

6. $2200, 4 mo. 15 da.

Find the interest at 4% on the following:

7. $250, 2 mo.

8. $320, 2 mo. 6 da.

9. $420, 3 mo. 12 da.

10. $1250, 4 mo.

11. $1575, 3 mo. 5 da.

12. $2500, 2 mo. 12 da.

Find the interest at $5\frac{1}{2}$% on the following:

13. $275, 2 mo.

14. $575, 4 mo. 3 da.

15. $1575, 3 mo.

16. $2550, 3 mo. 8 da.

Find the interest at $4\frac{1}{2}$% on the following:

17. $150, 2 mo.

18. $975, 3 mo. 10 da.

19. $1650, 3 mo.

20. $1850, 4 mo. 5 da.

46. Table of Difference of Time. Bankers always find the difference in time between two dates, and also the interest on any sum, by means of tables. The following is one form of the table of difference of time:

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January . .	365	31	59	90	120	151	181	212	243	273	304	334
February . .	334	365	28	59	89	120	150	181	212	242	273	303
March . . .	306	337	365	31	61	92	122	153	184	214	245	275
April . . .	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August . . .	153	184	212	243	273	304	334	365	31	61	92	122
September .	122	153	181	212	242	273	303	334	365	30	61	91
October . .	92	123	151	182	212	243	273	304	335	365	31	61
November .	61	92	120	151	181	212	242	273	304	334	365	30
December . .	31	62	90	121	151	182	212	243	274	304	335	365

The exact number of days from any day of any month to the corresponding day of any month within a year is found opposite the first month and under the second. For example, from August 7 to December 7 is 122 da.; from November 16 to April 23 is 158 da., since to April 16 it is 151 da., and to April 23 it is 7 da. more, or 158 da. in all.

EXERCISE 26

Find from the table the number of days from:

1. May 7 to Aug. 7.
2. Apr. 6 to Sept. 6.
3. Jan. 5 to Aug. 20.
4. Feb. 7 to Sept. 15.
5. July 4 to Dec. 25.
6. Apr. 15 to Oct. 20.
7. May 20 to Sept. 1.
8. Aug. 15 to Oct. 2.
9. Nov. 16 to July 17.
10. Sept. 20 to Mar. 25.
11. Oct. 21 to Feb. 20.
12. Nov. 25 to Apr. 15.
13. Aug. 27 to Feb. 5.
14. Aug. 19 to Jan. 7.
15. Oct. 7 to Apr. 16.
16. May 13 to Jan. 3.

47. Use of the Table of Time. In computing interest, the table of time (§ 46) is commonly used. We then allow 30 da. to the month and 360 da. to the year.

For example, required the interest on $420 from July 7 to Nov. 4 at 6%.

The difference of time (§ 46) is 123 da. − 3 da., or 120 da.

Interest at 6% for 60 da. (§ 44) = $4.20.
Interest at 6% for 120 da. = 2 × $4.20 = $8.40.

EXERCISE 27

Using the table on p. 44, find the interest at 6% on the following sums from the first date to the second:

1. $250, Apr. 6, Nov. 6.
2. $320, May 4, Nov. 10.
3. $220, Jan. 5, Sept. 2.
4. $625, June 20, Dec. 17.
5. $250, Aug. 16, Feb. 18.
6. $375, Sept. 25, Mar. 24.
7. $525, Oct. 17, Apr. 4.
8. $475, Feb. 4, May 5.
9. $240, Mar. 6, Nov. 1.
10. $560, Apr. 15, Jan. 10.
11. $355, May 1, July 5.
12. $750, Aug. 5, Oct. 1.
13. $800, Sept. 5, Jan. 2.
14. $725, May 7, Oct. 3.

Also on the following at 5%:

15. $125, Sept. 5, Feb. 10.
16. $326, Oct. 10, Apr. 3.
17. $450, July 7, Jan. 20.
18. $540, Aug. 15, Nov. 27.
19. $315, July 24, Apr. 3.
20. $430, Oct. 7, Jan. 16.
21. $585, Nov. 24, Mar. 7.
22. $695, Jan. 16, Aug. 10.
23. $843, Apr. 22, Oct. 7.
24. $925, Mar. 9, June 15.

Also on the following at 4%:

25. $250, Jan. 1, Sept. 15.
26. $375, Mar. 4, Nov. 18.
27. $840, May 7, Aug. 13.
28. $765, Apr. 12, Oct. 4.

48. Interest Tables. The most common interest tables are based on 360 days to the year, although the government and some banks use tables based on 365 days as being more fair, although yielding less interest. The following shows part of a page of an interest table:

3 MONTHS, 6%

Total Days	1000	2000	3000	4000	5000	6000	7000	8000	9000	Days over 3 Months
90	15.00	30.00	45.00	60.00	75.00	90.00	105.00	120.00	135.00	0
91	15.17	30.33	45.50	60.67	75.83	91.00	106.17	121.33	136.50	1
92	15.33	30.67	46.00	61.33	76.67	92.00	107.33	122.67	138.00	2
93	15.50	31.00	46.50	62.00	77.50	93.00	108.50	124.00	139.50	3
94	15.67	31.33	47.00	62.67	78.33	94.00	109.67	125.33	141.00	4
95	15.83	31.67	47.50	63.33	79.17	95.00	110.83	126.67	142.50	5
. . .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. . . .

For example, find from the table the interest on $2750 for 94 da. at 6%.

Interest on $2000 = $31.33
Interest on 700 = 10.97
Interest on 50 = .78
Interest on $2750 = $43.08

EXERCISE 28

Using the above table, find the interest at 6%:

1. $2670, 91 da.
2. $3170, 93 da.
3. $4250, 90 da.
4. $2575, 92 da.
5. $3250, 95 da.
6. $4050, 92 da.
7. $5075, 91 da.
8. $3775, 93 da.
9. $15,750, 90 da.

Using the tables of § 46, § 48, find the interest at 6%:

10. $2300, from Mar. 7 to June 7.
11. $3550, from Nov. 6 to Feb. 9.
12. $4250, from Dec. 15 to Mar. 18.

49. Exact Interest. When interest is computed for exact days, and at 365 days to the year, it is called *exact interest.*

For example, required the exact interest on $1241 from Dec. 21 to Feb. 9 at 5%.

The time, found by the table (§ 46), is 50 da. Then $\frac{50}{365}$ × 5% of $1241 is required. Canceling,

$$\frac{\overset{}{\cancel{50}} \times \cancel{5} \times \$\overset{17}{\cancel{1241}}}{\underset{73}{\cancel{365}} \times \underset{2}{\cancel{100}}} = \$8.50.$$

The common interest is a little greater than the exact interest.

EXERCISE 29

Find the exact interest on the following:

1. $250 for 75 da. at 6%.
2. $275 for 85 da. at 5%.
3. $325 from Apr. 2 to July 1 at 5%.
4. $350 from May 7 to Sept. 9 at 5%.
5. $425 from Jan. 6 to Aug. 4 at 6%.
6. $460 from Feb. 9 to July 9 at 6%.
7. $500 from May 15 to Dec. 16 at 6%.
8. $219 from Aug. 14 to Jan. 11 at 5%.
9. $292 from Sept. 27 to Jan. 25 at 5%.
10. $438 from July 15 to June 15 at 5%.
11. $730 from Aug. 17 to Apr. 14 at 4%.
12. $1460 from May 23 to Mar. 19 at 4%.
13. $1533 from Feb. 1 to Dec. 28 at 5½%.
14. $7300 from Mar. 3 to Oct. 4 at 5½%.
15. $3650 from May 23 to Jan. 23 at 4½%.
16. $1825 from Nov. 19 to June 17 at 4½%.
17. $5475 from Dec. 21 to June 19 at 3½%.

50. Periodic Interest. Simple interest on the principal and on each installment of interest from the time each interest is due until settlement is called *periodic interest.*

Periodic interest in which the installments are due annually is called *annual interest.*

For example, find the interest on $400 for 4 yr. 7 mo. 20 da. at 5%, payable annually.

Simple interest on $400 for 4 yr. 7 mo. 20 da. at 5% = $92.78.

Interest due the 1st year, $20, draws interest 3 yr. 7 mo. 20 da.
Interest due the 2d year, $20, draws interest 2 yr. 7 mo. 20 da.
Interest due the 3d year, $20, draws interest 1 yr. 7 mo. 20 da.
Interest due the 4th year, $20, draws interest 7 mo. 20 da.
Interest upon the interest = interest on $20 for 8 yr. 6 mo. 20 da.

Interest on $20 for 8 yr. 6 mo. 20 da. at 5% = $8.56.
The total interest = $92.78 + $8.56 = $101.34.

The law does not generally allow periodic interest, but in some parts of the country it is common to give notes not only for the principal but for each interest installment, all bearing interest.

EXERCISE 30

Find the interest upon the following, payable annually:

1. $500, 4 yr., 6%.
2. $250, 3 yr., 6%.
3. $475, 5 yr., 5%.
4. $750, 6 yr., 5%.
5. $850, $2\frac{1}{2}$ yr., 6%.
6. $725, $3\frac{1}{2}$ yr., 5%.
7. $650, $2\frac{1}{4}$ yr., 5%.
8. $900, $2\frac{1}{3}$ yr., 5%.
9. $350, 3 yr. 6 mo. 6 da., 6%.
10. $1250, 4 yr. 8 mo. 10 da., $5\frac{1}{2}$%.

Find the amount of the following, interest due annually but remaining unpaid until settlement:

11. $140, 3 yr., 6%.
12. $375, 4 yr., 6%.
13. $1250, $2\frac{3}{4}$ yr., 6%.
14. $3500, $2\frac{2}{3}$ yr., $5\frac{1}{2}$%.

51. Compound Interest. When interest is added to the principal and becomes a part of it at specified intervals, the total interest is called *compound interest.*

In compound interest, the interest draws interest every time it is added to the principal; in periodic interest it draws interest only once.

Interest is compounded annually, semiannually, or quarterly, according to agreement, but it cannot be enforced legally.

Most savings banks allow compound interest at a low rate, the interest usually being compounded quarterly or semiannually. Since the interest is deposited when it becomes due, the problem is merely one of computing simple interest each time.

For example, find the amount of $60 at compound interest for 3 yr. at 5%, the interest being compounded annually. Find also the compound interest.

Interest for 1st year = 5% of $60 = $3. Amount = $63.
Interest for 2d year = 5% of $63 = $3.15. Amount = $66.15.
Interest for 3d year = 5% of $66.15 = $3.31. Amount = $69.46.

$69.46 (compound amt.) − $60 (principal) = $9.46 (compound int.).

If the time were 3½ years, we should find the amount at compound interest for 3 years and add to it the interest upon itself for six months.

If the interest is 5% compounded semiannually for 3 years, it amounts to the same as 2½% compounded annually for 6 years.

EXERCISE 31

Find the amount at 4%, interest compounded annually:

1. $250, 3 yr.	**4.** $650, 5 yr.	**7.** $1500, 1½ yr.
2. $375, 4 yr.	**5.** $875, 6 yr.	**8.** $2650, 3½ yr.
3. $425, 3 yr.	**6.** $1250, 2½ yr.	**9.** $2575, 4¼ yr.

Find the amount at 3%, compounded semiannually:

10. $100, 4 yr.	**13.** $650, 3 yr.	**16.** $2500, 3½ yr.
11. $250, 3 yr.	**14.** $980, 5 yr.	**17.** $1650, 4½ yr.
12. $575, 2 yr.	**15.** $1350, 2½ yr.	**18.** $1700, 4½ yr.

52. Promissory Notes. A written promise to pay a specified sum of money on demand or at a specified time is called a *promissory note,* or simply a *note.*

53. Parties to a Note. The person who signs a note is called the *maker.* The person to whom a note is payable is called the *payee.* The lawful owner of a note is called the *holder.*

A note may be made payable to *bearer,* in which case it is lawful to pay it to any one who presents it to the maker.

It may be made payable to some person named in the note, in which case he or his legal representatives must present it. Such a note is said to be *non-negotiable.*

It may be made payable to the payee, *or order,* in which case the payee may transfer it to another person by writing his own name on the back, thus *indorsing* the note. Such a note is said to be, like one payable to bearer, *negotiable.*

An indorser of a note is responsible for its payment if the maker does not pay it, unless he writes the words "without recourse" above his signature.

54. Form of a Note. A common form of note is as follows:

$375.25. *Boston, Mass., June 1, 1910*

Six months after date, I promise to pay Frank Mills, or order, Three Hundred Seventy-five and $\frac{25}{100}$ Dollars, with interest at 5%, for value received.

James Lang

Here the maker is James Lang, and the payee is Frank Mills.

If the payee writes across the back,

Payable to the order of John B. Brown,
Frank Mills

he becomes an indorser, and John B. Brown becomes the holder.

If Frank Mills writes simply his name across the back, the note becomes payable to bearer, that is, to any one who may have it.

This note is due on Dec. 1, 1910, which is called the *day of maturity.*

The words "with interest," without specifying the rate, carry the legal rate of the state in which the note is made.

EXERCISE 32

Find the day of maturity and amount due, having given:

	Face of Note	*Date of Note*	*Time*	*Rate of Interest*
1.	$350	July 7, 1911	60 da.	6%
2.	$570	Jan. 4, 1910	6 mo.	5%
3.	$460	Feb. 8, 1910	1 yr.	$4\frac{1}{2}$%
4.	$328.45	Mar. 7, 1910	3 mo.	6%
5.	$536.25	Apr. 15, 1911	90 da.	6%
6.	$275.50	May 16, 1910	30 da.	5%

Write notes for the following, and compute the interest:

7.	$480	June 6, 1910	6 mo.	6%
8.	$640	Nov. 9, 1911	4 mo.	5%
9.	$750	Jan. 7, 1910	3 mo.	6%

Find the amount due at maturity on the following notes:

10. $250. CHICAGO, ILL., Oct. 20, 1910

Sixty days after date, I promise to pay George A. Wood, or order, Two Hundred Fifty Dollars, with interest at 5%. Value received.

JAMES SINCLAIR

11. $1250. SPRINGFIELD, MO., Mar. 7, 1910

Ninety days after date, I promise to pay James Goodwin, or order, Twelve Hundred Fifty Dollars, with interest at 6%. Value received.

JAMES HILTON

12. $275.50. CLEVELAND, OHIO, Apr. 6, 1910

One year after date, I promise to pay Francis Warren, or order, Two Hundred Seventy-five and $\frac{50}{100}$ Dollars, with interest at 5%. Value received.

ALFRED GIBSON

55. Partial Payments. Payments of a part of a note are called *partial payments.*

There are several methods of computing interest upon such notes, all recognizing the principle that interest should stop upon the amount paid.

56. Merchants' Rule. This is a convenient rule, although one not generally recognized in law, and is commonly used when settlement is made within a year. It is as follows:

(1) *Find the amount of the note at the date of settlement without regarding payments.*

(2) *Find the amount of each payment with interest from the date of payment to the date of settlement.*

(3) *Subtract the sum of the payment amounts from the total amount.*

For example, a man holds a note of \$380, dated Aug. 16, 1913, on which the following payments are indorsed: Nov. 4, 1913, \$120; Mar. 18, 1914, \$70; May 1, 1914, \$145. Settlement is made June 25, 1914. Find the balance due, interest at 6%.

Time from Aug. 16 to June 25 is 10 mo. 9 da.	Int. on \$1 = \$0.0515.
Time from Nov. 4 to June 25 is 7 mo. 21 da.	Int. on \$1 = \$0.0385.
Time from Mar. 18 to June 25 is 3 mo. 7 da.	Int. on \$1 = \$0.016$\frac{1}{6}$.
Time from May 1 to June 25 is 1 mo. 24 da.	Int. on \$1 = \$0.009.

Int. on \$380 = 380 × \$0.0515 = \$19.57.	Amount is		\$399.57
Int. on \$120 = 120 × \$0.0385 = \$4.62.	Amount is	\$124.62	
Int. on \$70 = 70 × \$0.016$\frac{1}{6}$ = \$1.13.	Amount is	71.13	
Int. on \$145 = 145 × \$0.009 = \$1.31.	Amount is	146.31	
Total payment amounts			342.06
Balance due June 25, 1914			\$57.51

The Merchants' Rule is used by bankers and business men on notes running only a few months, or what are called *short-time notes.*

In the following examples find the difference in time in each case by subtraction of compound numbers, as above, and solve by the Merchants' Rule.

EXERCISE 33

Find the balance due:

1. Note of $230 given May 9, 1910, bearing interest at 6%. Payments: July 1, 1910, $40; Sept. 16, 1910, $75; Jan. 2, 1911, $50. Settlement Feb. 24, 1911.

2. Note of $375 given Mar. 3, 1911, bearing interest at 5%. Payments: May 8, 1911, $30; Aug. 16, 1911, $65; Nov. 20, 1911, $110. Settlement Jan. 1, 1912.

3. Note of $600 given May 16, 1912, bearing interest at 5%. Payments: June 1, $50; July 3, $25; July 10, $50. Settlement Nov. 12, 1912.

4. Note of $1000 given July 18, 1910, bearing interest at 6%. Payments: Sept. 10, $200; Dec. 20, $140; April 21, 1911, $350. Settlement June 2, 1911.

5. Note of $2760 given Oct. 1, 1909, bearing interest at 5%. Payments: Nov. 1, $250; Jan. 1, 1910, $300; Mar. 1, 1910, $750. Settlement July 16, 1910.

6. Note of $1000 given Aug. 4, 1909, bearing interest at 6%. Payments: Sept. 10, $200; Dec. 8, $200; Feb. 2, 1910, $250. Settlement May 3, 1910.

7. Note of $1500 given April 5, 1910, bearing interest at 5%. Payments: June 1, $500; Aug. 2, $300; Dec. 1, $200. Settlement Jan. 4, 1911.

8. Note of $1250 given July 5, 1911, bearing interest at 5½%. Payments: Sept. 1, $275; Dec. 14, $325; Feb. 8, 1912, $280. Settlement May 17, 1912.

9. Note of $80 given Nov. 1, 1912, bearing interest at 6%. Payments: $10 on the first day of each month following. How much interest must be paid with the last payment?

57. United States Rule. The legal rule for partial payments in most of our states is that laid down by the Supreme Court of the United States. It is as follows:

(1) *Find the amount of the principal to the time when the payment, or sum of the payments, is equal to or greater than the interest.*

(2) *From this amount deduct the payment, or sum of the payments.*

(3) *Consider the remainder as a new principal, and proceed as before.*

For example, consider the following promissory note on which partial payments have been made:

$760. BOSTON, MASS., Oct. 16, 1911

Three months after date, I promise to pay Henry W. Street, or order, Seven Hundred Sixty Dollars, with interest at 6%. Value received. JAMES JOHNSON

Payments indorsed on the back of the note: Jan. 10, 1912, $200; Aug. 4, 1914, $75; May 1, 1915, $568. What is due Oct. 7, 1916?

Face of note	$760.00
Int., Oct. 16, 1911, to Jan. 10, 1912, 2 mo. 24 da.	10.64
Amount due Jan. 10, 1912	$770.64
Deduct payment	200.00
New principal	$570.64
Int., Jan. 10, 1912, to Aug. 4, 1914, 2 yr. 6 mo. 24 da., is $87.88, exceeding the payment.	
Int. Jan. 10, 1912, to May 1, 1915, 3 yr. 3 mo. 21 da.	113.27
Amount due May 1, 1915	$683.91
Deduct sum of payments, $75 + $568,	643.00
New principal	$40.91
Int. May 1, 1915, to Oct. 7, 1916, 1 yr. 5 mo. 6 da.	3.52
Amount due Oct. 7, 1916	$44.43

EXERCISE 34

1. Note of $425. Date, Oct. 10, 1910. Interest at 6%. Payments: Mar. 14, 1911, $100; July 8, 1911, $85; Feb. 24, 1912, $125. What was due Oct. 10, 1912?

2. Note of $360. Date, Jan. 10, 1911. Interest at 6%. Payments: Sept. 16, 1911, $75; Oct. 8, 1912, $20; Dec. 2, 1912, $160. What was due Jan. 10, 1913?

3. Note of $350. Date, May 1, 1911. Interest at 6%. Payments: Dec. 25, 1911, $50; June 29, 1912, $5; Aug. 22, 1912, $15; June 4, 1913, $100. What was due Apr. 5, 1914?

4. $375. INDIANAPOLIS, IND., May 20, 1910

Six months after date, I promise to pay Oliver Higgins Three Hundred Seventy-five Dollars, with interest at 5%. Value received. WILLIAM JULIAN

Indorsements: Nov. 20, 1910, $25; Apr. 6, 1911, $75; Aug. 23, 1911, $140. What was due Mar. 12, 1912?

5. $250. OMAHA, NEBR., Aug. 24, 1910

One year after date, I promise to pay, for value received, Ira Hoyt, or order, Two Hundred Fifty Dollars, with interest at 6%. JAMES PATTERSON

Payments: Aug. 24, 1911, $100; Nov. 15, 1911, $40; Feb. 3, 1912, $80; May 10, 1912, $25. Paid in full, Apr. 16, 1913. What was the last payment?

6. $620. AUSTIN, TEX., Nov. 1, 1910

Six months after date, I promise to pay J. R. Roberts, or order, Six Hundred Twenty Dollars, with interest at 6%. Value received. W. J. CAMPBELL

Indorsements: Oct. 6, 1911, $61.07; Mar. 4, 1912, $89.03; Dec. 11, 1912, $107.77; July 23, 1913, $200.50. What was due Oct. 15, 1913?

EXERCISE 35

Review of Interest

1. Find the interest on $725 for 187 da. at 5%.

2. A man has $14,600 at interest at 6%. What is his average daily income from this investment?

3. Find the interest on $475 from Aug. 14, 1913, to Feb. 2, 1915, at 4%.

4. Find the interest on $7550 at 6% for 60 da.; for 30 da.; for 90 da.; for 93 da.

In Exs. 5, 6, 7, find the difference in time by § 43.

5. Find the interest on $660 from Oct. 14 to the following July 1, at 5%.

6. Find the interest on $720 from Aug. 17 to the following April 15, at $5\frac{1}{2}$%.

7. Find the amount of principal and interest on $1440 from Sept. 20 to the following July 1, at 4%.

In Exs. 8, 9, 10, find the difference in time by the table in § 46.

8. Find the interest on $960 from Nov. 16 to the following Aug. 27, at 6%.

9. Find the interest on $7200 from July 20 to the following April 7, at 5%.

10. Find the amount of principal and interest on $420 from June 7 to the following Dec. 1, at 6%.

11. Find the exact interest at 6% on $650 from Sept. 21, 1911, to Jan. 1, 1912.

12. Find the exact interest at 5% on $730 from Oct. 15 to the following Aug. 10.

13. Find the exact interest at 4% on $9125 from Nov. 1 to the following Aug. 5.

14. Find the amount of principal and exact interest on \$1095 from Aug. 9 to the following June 17, at 6%.

15. The interest at 6% for two years on a certain sum is \$93. What is the sum at interest?

16. The interest at 5% for 1 yr. 6 mo. on a certain sum is \$18. What is the sum at interest?

17. The interest at 4% for 2 yr. 3 mo. on a certain sum is \$31.50. What is the sum at interest?

18. What is the sum that will amount, with interest at 6%, to \$517.50 in 2 yr. 6 mo.?

19. What is the sum that will amount, with interest at 5%, to \$793.75 in 1 yr. 2 mo.?

20. What is the sum that will amount, with interest at $5\frac{1}{2}$%, to \$1006.32 in 3 yr. 7 mo. 6 da.?

21. At $3\frac{1}{2}$% interest for a certain time, \$240 amounts to \$281.44. Find the time.

22. At $4\frac{1}{2}$% interest for a certain time, \$850 amounts to \$977.50. Find the time.

23. At $5\frac{1}{2}$% interest for a certain time, \$940 amounts to \$1022.72. Find the time.

24. At $3\frac{1}{2}$% interest for a certain time, \$480 amounts to \$610.90. Find the time.

25. How long will it take \$1200 to earn \$370.15 interest at $5\frac{1}{2}$%?

26. At what rate will \$1500 yield \$84 interest in 1 yr. 10 mo. 12 da.?

27. At what rate will \$5400 yield \$344.25 interest in 1 yr. 5 mo.?

28. At what rate will \$4200 yield \$784 interest in 3 yr. 1 mo. 10 da.?

EXERCISE 36

Problems without Numbers

1. How do you find the discount on the list price of some goods, given the rate of discount?

2. If there is a discount series, how do you find the net price of some goods?

3. If you know the net price of some goods and the single rate of discount, how do you find the list price?

4. If you know the net price of some goods and the two rates of discount in a series, how do you find the list price?

5. If a merchant buys a bill of goods and wishes to mark the goods so as to realize a certain per cent of profit, how does he find the marked price?

6. Explain how a merchant may sell goods at a certain per cent below the marked price and still make a profit.

7. If you know the marked price of some goods and the per cent of profit, how do you find the cost?

8. How do you find a commission merchant's commissions on the sale of produce sent him to sell?

9. If you know the rate and amount of a commission merchant's commission, how do you find the selling price?

10. If you know the price at which a commission merchant sells some goods, and the amount of his commission, how do you find the rate?

11. On what is an insurance premium reckoned? How is the rate stated?

12. If you know the face of an insurance policy, and the premium, how do you find the rate?

13. If you know the premium on an insurance policy, and the rate, how do you find the face of the policy?

14. What is a tax table, and how would you prepare one on the basis of 12 mills on a dollar?

15. If you know the assessed valuation of a man's property, and the rate of taxation, how do you find his tax?

16. If you know the assessed valuation of a man's property, and his tax, how do you find the rate?

17. If you know the tax paid by a man, and the rate of taxation, how do you find the assessed valuation of his property?

18. If you know the amount of tax to be raised and the total assessed valuation, how do you find the tax rate?

19. If you know the number of units of some imported goods, and the rate of specific duty, how do you find the amount of the duty?

20. If you know the value of some imported goods, and the rate of duty ad valorem, how do you find the amount of duty?

21. Given the principal, rate, and time, how do you find the interest?

22. Given the principal, rate, and interest, how do you find the time?

23. Given the principal, interest, and time, how do you find the rate?

24. Given the rate, time, and interest, how do you find the principal?

25. How would you proceed to compute an interest table, given the rate?

26. If you have an example in partial payments, all the dates within a single year, how do you proceed to solve?

27. If you have an example in partial payments extending beyond a year, how do you proceed to solve?

28. How do you express a given per cent as a common fraction?

29. If you are to find a certain fractional part of a number, how do you express this as a per cent?

30. If you have a certain rate per cent expressed with the per cent sign, how do you express it as a decimal fraction?

31. If you have a decimal fraction and wish to express the same number with a per cent sign, how do you proceed?

32. If you know what a given per cent of a number is, how do you find the number?

33. How do you find what per cent one number is of another? Illustrate.

34. A man's salary two years ago was increased a certain per cent. Last year it was decreased the same per cent. Is his salary at present greater than or less than it was two years ago?

35. A merchant marks his goods a certain per cent above cost. Owing to hard times, he sells them at the same per cent below the marked price. Does he sell them for more than or less than cost?

36. Which is the greater, the common interest on a certain sum or the exact interest? Why is this?

37. What are the common methods of finding the difference between two given dates? Which one is the more exact?

38. State the easiest way of finding the interest on a given sum of money for sixty days at 6%.

39. How would you find the compound interest on a given sum of money for a certain number of years at a given rate?

CHAPTER II

RATIO AND PROPORTION

58. Ratio. The relative magnitude of two numbers, as expressed by the fraction which has the first number for the numerator and the second for the denominator, is called the *ratio* of the first number to the second.

The ratio of 2 to 3, or $\frac{2}{3}$, is commonly written 2 : 3.

59. Antecedent and Consequent. The first term of a ratio is called the *antecedent*, and the second term the *consequent.*

For example, $\frac{5 \text{ ft.}}{6 \text{ ft.}} = \frac{5}{6} = \frac{\text{numerator}}{\text{denominator}} = \frac{\text{antecedent}}{\text{consequent}}$.

60. Ratios always Abstract. The two terms of a ratio always being like numbers, as in 2 : 3, \$4 : \$5, 6 ft. : 7 ft., the quotient is always abstract.

Therefore, *a ratio is always abstract, and its terms may be written as abstract numbers.*

Instead of writing $\frac{2 \text{ ft.}}{3 \text{ ft.}}$ we may therefore write simply $\frac{2}{3}$.

It is better to use the fractional form at first, and then to use the other form, 2 : 3.

61. Reduction of Ratios. Since a ratio may be expressed as a fraction, therefore

If the terms of a ratio are both multiplied by or both divided by the same number, the value of the ratio is not altered.

Thus, if both terms of the ratio $2\frac{1}{2} : 3\frac{1}{3}$ are multiplied by 6, the resulting ratio is 15 : 20, and these two ratios are equal.

Again, since $\frac{15}{20} = \frac{3}{4}$, the simplest expression for $2\frac{1}{2} : 3\frac{1}{3}$ is 3 : 4.

62. Separating a Number in a Given Ratio. Required to separate \$63 in the ratio of 3 to 4.

Since there must be \$3 in the first part to every \$4 in the second, there must be \$7 every time two such amounts are taken out. Therefore the first part must contain $\frac{3}{7}$ of the total and the second $\frac{4}{7}$.

But $\frac{3}{7}$ of \$63 is \$27, and $\frac{4}{7}$ of \$63 is \$36.

These are the results, because $\$27 : \$36 = \frac{27}{36} = \frac{3}{4}$.

EXERCISE 37

Express the following ratios in simplest form:

1. $\dfrac{34}{51}$. 3. $\dfrac{46}{115}$. 5. $\dfrac{\frac{4}{5}}{6}$. 7. $\dfrac{13}{3\frac{1}{4}}$. 9. $58:145$.

2. $\dfrac{57}{76}$. 4. $\dfrac{2}{\frac{3}{4}}$. 6. $\dfrac{\frac{1}{2}}{\frac{7}{8}}$. 8. $\dfrac{2\frac{1}{2}}{10}$. 10. $93:124$.

Find the missing term in each of the following ratios:

11. $\dfrac{(?)}{4} = 2$. 13. $\dfrac{(?)}{9} = \dfrac{1}{3}$. 15. $\dfrac{(?)}{10} = \dfrac{3}{5}$. 17. $(?) : 9 = 2$.

12. $\dfrac{6}{(?)} = 2$. 14. $\dfrac{5}{(?)} = \dfrac{1}{4}$. 16. $\dfrac{7}{(?)} = \dfrac{1}{5}$. 18. $7 : (?) = \dfrac{1}{2}$.

Separate the following numbers in the given ratios:

19. $28,\ 3:4$. 22. $31\frac{1}{2},\ 2\frac{1}{2}:5$. 25. $\$78,\ 3:10$.

20. $45,\ 4:5$. 23. $\$72,\ 1:8$. 26. $341,\ 15:16$.

21. $260,\ 5:8$. 24. $\$96,\ 3:5$. 27. $492,\ 5\frac{1}{2}:15$.

28. Two farmers pay \$15 together for some threshing, one having 350 bu. and the other 400 bu. of wheat. What is the share of each?

29. A man leaves an estate of \$13,470, giving \$1 to his widow for every \$2 to his children. How much did the children receive?

EXERCISE 38

Practical Uses of Ratio

1. The ratio of water to solid matter in apples is 423 : 77. How many pounds of water in 250 lb. of apples?

2. In a sack of rough rice, containing 162 lb., the ratio of hulls to cleaned rice is 1 : 4. How many pounds of cleaned rice in a sack?

3. A pineapple grower in Florida received in one year $15 for plants to every $7.50 for fruit. If his total receipts were $3375, how much did he receive for the fruit?

4. In fertilizing land for growing peanuts a Virginia farmer used 30 lb. of cotton-seed meal to 13 lb. of cotton-hull ashes. How many pounds of each in 215 lb. of fertilizer?

5. Two partners invest respectively $2750 and $3500. They divide their profits, $3750, in the ratio of their investments. What is the share of each?

6. A recipe for fertilizer for oats requires $12\frac{1}{2}$ lb. of nitrate of soda to 15 lb. of other ingredients. How many pounds of nitrate of soda in $412\frac{1}{2}$ lb. of fertilizer?

7. A recipe for grafting wax calls for 4 parts of resin to $1\frac{1}{2}$ parts of beeswax and 1 part of linseed oil. How many pounds of resin in 26 lb. of the wax?

8. Two rice planters expended $2240 on an irrigating plant. One owned 80 acres of rice and the other 120 acres. If they paid in the ratio of their acreage, what was the share of each?

9. A New Jersey gardener estimated that the ratio of the cost of planting and caring for asparagus, to the cost of cutting and bunching, was 1 : 2. If the cost of both for his crop was $84, what was the cost of each?

10. Two men rent a pasture for $115. One puts in 9 cattle and the other 14. How much rent should each pay?

11. If the ratio of a man's income to his expenses is 7 : 6, and his expenses are $1800 a year, how much is his income?

12. Taking the ratio of the diagonal to the side of a square as 7 : 5, how long is the diagonal of a square 35 ft. on a side?

13. Taking the ratio of the circumference of a circle to the diameter as 22 : 7, how long is the circumference of a circle whose diameter is 4 ft.?

14. If the ratio of cream to the rest of the milk of a certain quality is 1 : 5, how many pounds of cream in 375 lb. of milk of that quality?

15. If the ratio of butter fat to the rest of the milk of a certain quality is 1 : 24, how many pounds of butter fat in 350 lb. of milk of that quality?

16. If the ratio of solid matter to the rest of the milk of a certain quality is 33 : 217, how many pounds of solid matter in 750 lb. of milk of that quality?

17. Two shippers charter a canal boat for $84 to carry some freight to a certain city. One ships 75 tons and the other 135 tons. How much should each pay?

18. Two partners having the same amount of money in their business divide the profits in the ratio of the time they devote to it. One gives 5 months in the year to the business and the other 7 months, and the profits are $5400. What is the share of each?

19. The ratio of copper sulphate and unslaked lime to water in Bordeaux spraying mixture is 1 : 39. How much copper sulphate and unslaked lime in 160 lb. of the mixture? How much of each, if equal amounts are used?

63. Proportion. An expression of equality between two ratios is called a *proportion.*

Thus $\frac{2}{3} = \frac{10}{15}$ and $2:3 = 10:15$ are proportions, and are read "2 is to 3 as 10 is to 15," or "the ratio of 2 to 3 equals the ratio of 10 to 15."

64. Terms of a Proportion. The first and last numbers of a proportion are called the *extremes*, and the two middle numbers are called the *means*. The extremes and means together are called the *terms* of the proportion.

Thus in the proportion $2:3 = 10:15$, the extremes are 2 and 15, and the means are 3 and 10.

65. Test of a Proportion. In the proportion $\frac{2}{3} = \frac{10}{15}$ we shall still have an equality if we multiply these equals by the same number. If we multiply by 3×15 we have

$$\frac{\cancel{3} \times 15 \times 2}{\cancel{3}} = \frac{3 \times \cancel{15} \times 10}{\cancel{15}}, \text{ or } 15 \times 2 = 3 \times 10.$$

Therefore, *in any proportion the product of the extremes equals the product of the means.*

66. Finding a Missing Term in a Proportion. The product of the extremes equals the product of the means (§ 65).

Therefore, *either extreme equals the product of the means divided by the other extreme ;*

Either mean equals the product of the extremes divided by the other mean.

Hence if three terms are given, the fourth may be found.

(1) Find the missing term of the proportion $(?) : 6 = 9 : 27$.

Dividing the product of the means by the given extreme, we have

$$\frac{9 \times 6}{27} = 2.$$

(2) Find the missing term of the proportion $3 : 7 = (?) : 35$.

Dividing the product of the extremes by the given mean, we have

$$\frac{3 \times 35}{7} = 15.$$

EXERCISE 39

Find the missing term, designated by x or (?):

1. $\frac{x}{12} = \frac{24}{16}$.

2. $\frac{x}{15} = \frac{21}{35}$.

3. $x : 15 = 32 : 6$.

4. $x : 8 = 15 : 20$.

5. $15 : x = 10 : 8$.

6. $55 : x = 22 : 10$.

7. $12 : x = 18 : 10$.

8. $18 : x = 27 : 45$.

9. $12 : 18 = x : 27$.

10. $25 : 15 = x : 21$.

11. $21 : 49 = x : 63$.

12. $18 : 45 = x : 35$.

13. $10 : 45 = 14 : x$.

14. $24 : 54 = 28 : x$.

15. $35 : 49 = 45 : (?)$.

16. $\$5 : \$7 = (?) : 63$.

17. $\$9 : (?) = 189 : 231$.

18. $(?) : \$6 = 85 : 102$.

19. $\$15 : \$17 = 75 : (?)$.

20. 21 ft. : 5 ft. $= 147 : (?)$.

21. 35 ft. : $(?) = 315 : 99$.

22. $2\frac{1}{2} : 7 = 100 : (?)$.

23. $3\frac{1}{3} : 2\frac{1}{2} = (?) : 30$.

24. $12\frac{1}{2} : 16\frac{2}{3} = (?) : 80$.

25. $12\frac{1}{2} : (?) = 33\frac{1}{3} : 160$.

26. $37.5 : (?) = 250 : 80$.

27. $62.5 : 12.5 = 75 : (?)$.

28. $(?) : 66\frac{2}{3} = 90 : 600$.

29. $(?) : 87\frac{1}{2} = 33\frac{1}{3} : 2\frac{1}{3}$.

30. $(?) : 1.6 = 3.1416 : 1$.

31. Show that 29, 435, 17, and 255 are in proportion.

32. Show that 217, 2821, 311, and 3421 cannot be in proportion.

33. What number has the same ratio to 21 that 16 has to 35?

34. If the ratio of 7 to 16 equals the ratio of 63 to some number, what is that number?

35. Show that 401 and 5287 are the extremes and 311 and 6817 are the means of a proportion. Write the proportion in three different ways.

67. Directly Proportional. If one ratio equals another, the terms of the first are said to be *directly proportional* to those of the second.

Since 2 : 3 and 4 : 6 are equal ratios, 2 and 3 are said to be directly proportional to 4 and 6.

In general, the following are among the cases of directly proportional quantities:

Cost of goods is directly proportional to the quantity.
Weight of substances is directly proportional to the volume.
Income from labor is directly proportional to time given.
Distance traveled is directly proportional to time.

68. Inverse Ratio. The result of interchanging the terms of a ratio is called the *inverse ratio* of the given ratio.

Thus 3 : 2 is the inverse of 2 : 3, just as $\frac{3}{2}$ is the inverse of $\frac{2}{3}$.

69. Inversely Proportional. If one ratio equals the inverse of another, the terms of the first are said to be *inversely proportional* to those of the second.

Since 2 : 3 equals the inverse of 6 : 4, 2 and 3 are said to be inversely proportional to 6 and 4; that is, $2 : 3 = 4 : 6$.

As an example of inverse proportion, the time required for some work is inversely proportional to the number of laborers. That is, if we take twice as many workmen, it will take half as long for the work.

70. Commercial Applications. Consider the following problem: If 5 T. of hay cost $87.50, what will 21 T. cost?

Since the ratio of cost is evidently equal to the ratio of quantity, we have

$$\$\,x : \$87.50 = 21 \text{ T.} : 5 \text{ T.}$$

Since we cannot multiply dollars by tons in getting the product of the means, and since we wish to preserve the dollars for the result, we write the second ratio as 21 : 5 (§ 60).

Then (§ 66),

$$\$\,x = \frac{21 \times \$87.50}{5} = \$367.50.$$

We may write the proportion so that the unknown number will come in any term we wish, but the result is the same. Teachers formerly preferred to put it in the fourth term, and this may be done if desired.

If 8 men can do a piece of work in 9 days, how long will it take 12 men to do it?

Since it takes correspondingly less time as we increase the number of workmen, the ratio of the number of days equals the inverse ratio of the number of men.

Therefore x da. : 9 da. = 8 men : 12 men,

or x da. : 9 da. = 8 : 12 (§ 60).

Therefore $x \text{ da.} = \dfrac{8 \times 9 \text{ da.}}{12} = 6 \text{ da.}$

In proportion the words "at the same rate" are always understood.

EXERCISE 40

1. If 75 bu. of wheat cost $72, what will 260 bu. cost?

2. If 36 yd. of cretonne cost $9.72, what will 56 yd. cost?

3. If 42 yd. of ribbon cost $9.24, what will 75 yd. cost?

4. If 18 yd. of braid cost $2.25, what will 62 yd. cost?

5. If $2\frac{1}{2}$ M feet of lumber cost $70, what will 7 M feet cost?

6. If a man earns $16.50 in 6 da., how much will he earn in 25 da.?

7. A manufacturer used 71 yd. of braid on 4 sailor suits. How much would he need for 15 such suits?

8. A merchant bought 640 yd. of muslin for $36.80. How much did 360 yd. cost?

9. For 10 aprons a manufacturer needs 15 yd. of goods. How many yards will he need for 75 aprons?

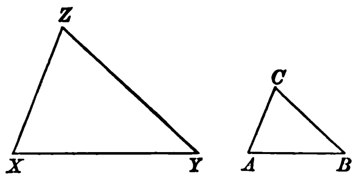
10. How long should it take 16 men to do some work that 52 men can do in 12 da.?

11. If 8 men require 75 da. to do a piece of work, how many men should it take to do it in 40 da.?

12. How far can 36 tons of freight be carried for the money paid for carrying 54 tons 144 miles?

71. Similar Figures. Figures that are of exactly the same shape are called *similar figures.*

For example, these two triangles are similar.



The corresponding sides of similar figures are proportional.

In these figures, $AB:BC = XY:YZ$. Also, $ZX:XY = CA:AB$.

This principle can be used in measuring the height of trees or buildings, as shown in Exs. 3–6 below. It is the principle involved in all map drawing.

EXERCISE 41

1. If $\frac{3}{4}$ in. on a map represents a distance of 375 mi., how many miles will $1\frac{1}{4}$ in. represent?

2. If a distance of 325 mi. is represented on a map by $1\frac{5}{8}$ in., how many inches will represent a distance of 340 mi.?

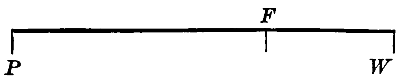
3. If a tree casts a shadow 50 ft. long when a post 4 ft. high casts a shadow 5 ft. long, how high is the tree?

4. If a spire casts a shadow 200 ft. long when a post $4\frac{1}{2}$ ft. high casts a shadow 5 ft. long, how high is the spire?

5. A class wishing to find the height of the school building, one of the boys stood a yard measure upright and another lay on the ground where he could just see the top of the building in line with the top of the stick. The distance from his eye to the foot of the stick was 4 ft. 6 in., and to the building 45 ft. How high was the building?

6. A man whose eye is 5 ft. 6 in. above the ground sights over the top of a 12-foot pole and just sees the top of a tree. If he is 7 ft. from the pole and 63 ft. from the tree, how high is the tree?

72. The Lever. If we have a lever as shown in this illustration, where F represents the fulcrum on which it rests, we may lift a weight placed at W by applying power enough at P. It has been found that



Power and weight are inversely proportional to their distances from the fulcrum.

Thus, if $WF = 8$ in., $PF = 16$ in., and weight $= 7$ lb., we have

$$\text{Power} : 7 \text{ lb.} = 8 \text{ in.} : 16 \text{ in.}$$

Therefore
$$\text{Power} = \frac{8 \times 7 \text{ lb.}}{16} = 3\tfrac{1}{2} \text{ lb.}$$

That is, it would take only $3\frac{1}{2}$ lb. at P to lift 7 lb. at W.

Other simple forms of the lever may be considered if desired.

EXERCISE 42

Given the following distances from the fulcrum, find the power necessary to raise the weight specified:

	PF	*WF*	*Weight*
1.	2 ft.	6 in.	17 oz.
2.	13 in.	5 in.	23 oz.
3.	17 in.	8 in.	12 lb.
4.	32 in.	9 in.	25 lb.
5.	48 in.	8 in.	100 lb.
6.	6 ft.	1 ft.	360 lb.
7.	5 ft.	6 in.	375 lb.
8.	4 ft.	8 in.	900 lb.
9.	5 ft.	4 in.	750 lb.
10.	$5\frac{1}{2}$ ft.	6 in.	1 T.

Given the following distances from the fulcrum, find the greatest weight that can be raised by the power specified:

	PF	*WF*	*Power*
11.	3 ft.	6 in.	10 lb.
12.	4 ft.	4 in.	30 lb.
13.	5 ft.	8 in.	100 lb.
14.	$4\frac{1}{2}$ ft.	6 in.	25 lb.
15.	$5\frac{1}{2}$ ft.	8 in.	125 lb.
16.	$6\frac{1}{2}$ ft.	8 in.	150 lb.

EXERCISE 43

Vocational Problems

1. If a shipment of 5100 lb. of cattle, live weight, sold for $225.42, what would 3500 lb. sell for at the same rate?

2. If a Louisiana farmer paid $75 for 3 T. 1500 lb. of cotton-seed meal for fodder, how much would he have to pay for 5000 lb.?

3. If in 225 lb. of milk there are 8.1 lb. of butter fat, how many pounds of milk will be required to produce 27 lb. of butter fat?

4. A farmer had 26 acres planted to potatoes. The crop from 7 acres amounted to 1260 bu. At the same rate how many bushels did he receive from the whole field?

5. To irrigate a farm at the rate of $\frac{1}{5}$ in. in depth every day requires the flow of 210 gal. an hour through a certain ditch. What flow would be necessary to irrigate it at the rate of 0.3 in. a day?

6. A Wisconsin farmer built a cylindrical silo 20 ft. in diameter and 32 ft. high, and found that it had a capacity of 200 tons. What must be the height of one of the same diameter that it may have a capacity of 125 tons?

7. If a certain kind of bird eats wild berries and also insects that harm fruit trees, and if out of 14 oz. of food eaten by this bird only $1\frac{3}{8}$ oz. is wild berries, what per cent of the bird's food is insects that would harm fruit trees?

8. An Arkansas farmer sprayed an apple tree and the crop numbered 1656 sound apples and 186 diseased ones. From a tree not sprayed he obtained 142 sound apples and 1278 diseased ones. Are these numbers inversely proportional or not?

9. If a farmer needs 108 lb. of seed for a 2-acre lot, how much will he need for $26\frac{1}{2}$ acres?

10. If the cost of 35 ft. of sidewalk is $31.50, what will 75 ft. cost, the width being the same?

11. If 2.8 bbl. of lime are required for 75 sq. yd. of plastering, how many barrels will be needed for 275 sq. yd.?

12. A stretch of railway track runs 437 ft. on the same grade, $6\frac{1}{2}$ in. per 100 ft. What is the total difference in level?

13. A storage warehouse stored 675 bbl. of apples for 90 days at the rate of $6 per 100 bbl. for 30 da. How much was the charge?

14. If a contractor agrees to build a sidewalk 4 ft. 6 in. wide in front of a lot for $36, how much should he charge if it were 5 ft. wide?

15. A man builds a coal bin 6 ft. wide and finds that it holds $11\frac{1}{2}$ tons. How much would it hold if it had been made a foot wider?

16. An ice dealer can gather 2900 tons of ice from a pond when the ice has frozen 12 in. thick. How many tons can he gather when the ice is 15 in. thick?

17. An excavation has been half completed, 35 men having been employed for 16 days. How long should it take to complete the other half if 5 more men are added?

18. Gun metal is composed of 1 part of tin to $5\frac{1}{2}$ parts of copper by weight. How many pounds of tin must be added to 210 lb. 6 oz. of copper to make gun metal?

19. How many pounds of tin are there in 232 lb. 6 oz. of gun metal such as described in Ex. 18?

20. On January 1 a gas meter registers 42,500 cu. ft., and on January 8 it registers 43,400 cu. ft. If this rate is exactly maintained, how much will it register on January 29?

73. Proportional Parts. Just as we have learned (§ 62) how to separate a number into two parts having a given ratio, so we may separate a number into several parts proportional to given numbers.

For example, separate $1230 into parts proportional to 2, 3, and 5.

The whole quantity may be represented by $2 + 3 + 5$, or 10.

Therefore the respective parts are $\frac{1}{5}$, $\frac{3}{10}$, and $\frac{1}{2}$ of $1230. These parts are $246, $369, and $615.

Separate 527 ft. into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$.

Multiply the fractions by 30, the L.C.M. of their denominators. The results are 15, 10, and 6. These numbers may therefore represent the parts, and their sum, 31, will represent the whole.

Therefore the respective parts are $\frac{15}{31}$, $\frac{10}{31}$, and $\frac{6}{31}$ of 527 ft., or 255 ft., 170 ft., and 102 ft.

Two farmers rent a pasture for $2.55. The first puts in 4 cattle for 2 weeks, and the second 3 cattle for 3 weeks. How much should each farmer pay?

Since 4 cattle in 2 weeks eat as much as 8 cattle in 1 week, and 3 cattle in 3 weeks eat as much as 9 cattle in 1 week, one should pay $\frac{8}{17}$ of $2.55, or $1.20, and the other should pay $\frac{9}{17}$ of $2.55, or $1.35.

EXERCISE 44

1. Separate 297 into parts proportional to 7, 9, and 11.

2. Separate 253 into parts proportional to $\frac{1}{6}$, $\frac{1}{2}$, and $\frac{1}{4}$.

3. Divide $2646 between two partners in the ratio of 4 to 5.

4. Divide $3744 among three partners so that their shares shall be proportional to 3, 4, and 5.

5. Four men invested $2700, $3300, $3500, and $4000 respectively in a joint business. They gained $6750. What was each one's share of the profit?

6. Three farmers together pay \$225 for irrigation taxes. The first owns 225 acres, the second 190 acres, and the third 335 acres. What is the tax of each?

7. Three men deliver milk at a creamery. The first delivers 776 lb., the second 938 lb., and the third 870 lb. They receive \$38.76. What is the share of each?

8. Three men pay \$848 for some water power. The first uses 28 horse power, the second 36 horse power, and the third 42 horse power. How much should each pay?

9. Three men rent a yacht for the summer for \$540. The first uses it for 4 weeks, the second 5 weeks, and the third $4\frac{1}{2}$ weeks. How much should each pay?

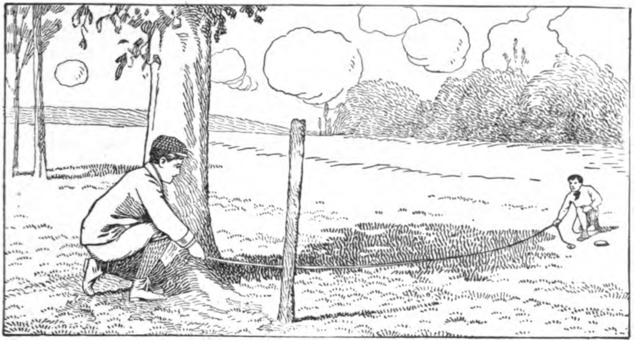
10. Two farmers pasture cattle in a field, the first putting in 24 head for 6 weeks, and the second 36 head for 4 weeks. The bill being \$43.20, what is the share of each?

11. Two hotel proprietors arrange to transport their employees from Florida to New York at the end of the season for \$2793. The first has 65 employees and the second 82. How much should each proprietor pay?

12. Four boys arrange a three weeks' camping trip. The first stays 18 days; the second comes late and stays 12 days; the third is in camp the last two weeks; the fourth stays all through. The total expense is \$32.50. What is the share of each?

13. A contractor employs 16 men 15 days on one job, and 14 men 12 days on another job. The total expense is \$1020. How much should be charged to each job?

14. A manufacturer ships 1200 lb. of goods 300 miles, and 900 lb. 400 miles. If he pays according to weight and distance, and the total bill is \$24, how much does each shipment cost?



EXERCISE 45

Review Problems

1. These boys find that this post is 3 ft. 6 in. high and casts a shadow 3 ft. 2 in. long. They find that the shadow of the tree is 34 ft. 10 in. long at the same time. Required the height of the tree.

2. If a building casts a shadow 75 ft. long when a post 6 ft. 2 in. high casts a shadow 12 ft. 6 in. long, how high is the building?

3. If the railway fare for a journey of 75 mi. is $1.50, what will be the fare for 275 mi.?

4. Two boys weighing respectively 100 lb. and 80 lb. sit 9 ft. apart on the ends of a plank. Not counting the weight of the plank, how far from the heavier boy must the fulcrum be placed so that they will just balance?

5. How much pressure will you have to exert on the handles of a pair of shears just 3 in. from the fulcrum (screw or bolt), in order to exert a pressure of 5 lb. at a point 5 in. from the fulcrum?

EXERCISE 46

Problems without Numbers

1. If you double the consequent of a ratio, what else must you do so as not to change the value of the ratio?

2. How do you proceed to separate a number into parts having a given ratio?

3. How do you ascertain whether four quantities are in proportion?

4. Given the extremes and one mean of a proportion, how do you find the other mean?

5. Given the means and one extreme of a proportion, how do you find the other extreme?

6. If you know all of the terms of a proportion except the first, how is that found?

7. If you know all of the terms of a proportion except the antecedent of the second ratio, how is that found?

8. If you know the weight of two volumes of a substance, and one of the volumes, how do you state the proportion?

9. Knowing the number of men required to do a piece of work in a given number of days, how do you find how many men are required to do it in another given number of days?

10. How may you find the height of your school building by means of shadows and the height of a pole?

11. How may you proceed to find the height of a tree by sighting over the top of a pole of known length?

12. How may you tell where to place the fulcrum under a lever so as to lift a given weight by pressing your full weight on one end of the lever?

13. How do you proceed to separate a number into parts proportional to four given numbers?

CHAPTER III

POWERS AND ROOTS

74. Powers. The result of taking a number any number of times as a factor is called a *power* of the number.

75. Square. The result of taking a number twice as a factor is called the *second power* or *square* of the number.

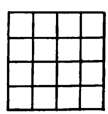
Thus the squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

76. Square Root. One of the two equal factors of a number is called the *square root* of the number.

Thus the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, 100,
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

Some numbers are *perfect squares*. We also speak of the square root of 5 as 2.236 + because the square of 2.236 is nearly 5.

77. Symbols. The square of 5 is written 5^2; the square root of 5 is written $\sqrt{5}$. The symbol $\sqrt{\ }$ is called a *radical sign*. The small figure 2 in 5^2 is called an *exponent*.



78. Square Roots and Areas. If this square is 4 units long, the area is 16 square units. Therefore, considering all the numbers as abstract,

The side of a square equals the square root of its area.

79. Square Roots by Factoring. In the case of perfect squares we may find the square roots if we can factor.

For example, since $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = (2 \times 2 \times 3) \times (2 \times 2 \times 3)$, therefore $\sqrt{144} = 2 \times 2 \times 3 = 12$.

EXERCISE 47

Find the areas of squares whose sides are as follows:

1. 17 ft.	**4.** 2.9 in.	**7.** 0.62 yd.	**10.** $\frac{4}{5}$ mi.
2. 19 ft.	**5.** 3.7 in.	**8.** 1.27 yd.	**11.** $\frac{7}{8}$ in.
3. 23 ft.	**6.** 4.3 in.	**9.** 4.93 yd.	**12.** $\frac{9}{32}$ ft.

By factoring, find the square roots of the following:

13. 196.	**16.** 400.	**19.** 625.	**22.** 900.	**25.** 1225.
14. 225.	**17.** 324.	**20.** 441.	**23.** 729.	**26.** 1089.
15. 256.	**18.** 484.	**21.** 784.	**24.** 1024.	**27.** 1296.

Find the sides of squares whose areas are as follows:

28. 1.44 sq. in.	**31.** 2500 sq. ft.	**34.** 0.09 sq. mi.
29. 1.21 sq. in.	**32.** 3600 sq. yd.	**35.** 0.81 sq. ft.
30. 5.76 sq. in.	**33.** 6400 sq. rd.	**36.** 0.0144 sq. ft.

Find the perimeters of squares whose areas are as follows:

37. 576 sq. in.	**39.** 23.04 sq. in.	**41.** 65.61 sq. ft.
38. 5184 sq. in.	**40.** 40.96 sq. in.	**42.** 110.25 sq. ft.

43. Write and memorize the squares of the numbers from 1 to 25.

44. A square field contains 40 acres. Express the length of the field in rods.

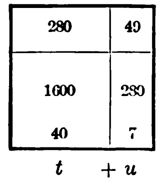
45. A square lot has an area of 169 sq. rd. How many rods of fence will be needed to inclose it?

46. What is the area of the cross section of a square pillar that is $11\frac{7}{8}$ in. on a side?

47. The area of the cross section of a square pillar is 2.89 sq. ft. What is the width of the pillar?

80. Square of the Sum of Two Numbers. Since $47 = 40 + 7$, the square of 47 may be obtained as follows:

$$\begin{array}{rl}
 & \qquad\qquad 40 + 7 \\
 & \qquad\qquad 40 + 7 \\
\hline
 & (40 \times 7) + 7^2 \\
40^2 + & (40 \times 7) \\
\hline
40^2 + 2 \times & (40 \times 7) + 7^2 \\
= 1600 + 2 \times & 280 \qquad + 49 \\
= 1600 + 560 & \qquad\quad + 49 \\
= 2209. &
\end{array}$$



This relationship is seen in the annexed figure where the side of the square is $40 + 7$.

Every number consisting of two or more figures may be regarded as composed of tens and units.

Therefore, *the square of a number contains the square of the tens, plus twice the product of the tens and units, plus the square of the units.*

This is the most important principle in square root, and should be clearly understood, both from the multiplication and from the illustration, before proceeding.

81. Separating into Periods. The first step in extracting the square root of a number is to separate the figures of the number into groups of two figures each, called *periods.*

Since $1 = 1^2$, $100 = 10^2$, $10,000 = 100^2$, and so on, it is evident that the square root of any number between 1 and 100 lies between 1 and 10; of any number between 100 and 10,000 lies between 10 and 100. In other words, the square root of any integral number expressed by *one* or *two* figures is a number of *one* figure; expressed by *three* or *four* figures is a number of *two* figures; and so on.

If, therefore, an integral number is separated into periods of two figures each, from the right to the left, the number of figures in the square root will be equal to the number of the periods of figures. The last period at the left may have one figure or two figures; for example, 22 09, and 7 89 04 81.

82. Extracting the Square Root. The process of extracting the square root of a number not readily factored will now be considered.

For example, required the square root of 2209.

Separating into periods (§ 81), we see that there will be two integral places in the root.

$$\begin{array}{ll} & 22\;09\,(47 \\ & 16 \\ \hline 80 & \;\;6\;09 \\ 87 & \;\;6\;09 \\ \hline \end{array}$$

The first period, 22, contains the square of the tens' number of the root. And since the greatest square in 22 is 16, therefore $\sqrt{16}$, or 4, is the tens' figure of the root.

Subtracting the square of the tens, the remainder contains twice the tens $\times$ the units, plus the square of the units (§ 80). Therefore if we divide by twice the tens (that is, by 80, which is 2×4 tens), we shall find approximately the units. Dividing 609 by 80 (or 60 by 8), we have 7 as the units' figure.

Since twice the tens $\times$ the units, plus the square of the units, is equal to (twice the tens + the units) $\times$ the units, i.e. $2 \times 40 \times 7 + 7^2 = (2 \times 40 + 7) \times 7$, therefore we add 7 to 80 and multiply the sum by 7. The product is 609, thus completing the square of 47.

To check the work, $47^2 = 2209$.

EXERCISE 48

Extract the square root of:

1. 841.	**4.** 1369.	**7.** 3249.	**10.** 8281.
2. 961.	**5.** 1681.	**8.** 3721.	**11.** 9801.
3. 529.	**6.** 1521.	**9.** 1849.	**12.** 6241.

13. A square lot has an area of 6889 sq. ft. How many feet has it on a side?

14. A square piece of linoleum has an area of 37 sq. ft. 1 sq. in. How many inches has it on a side?

15. The bottom of a square box has an area of 484 sq. in. How long is it?

16. A square lot has an area of 6889 sq. ft. How many feet has it on a side?

83. Explanation by Diagram. It sometimes makes the reasoning in square root more clear if we consider a diagram.

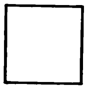


FIG. 1

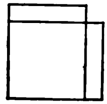


FIG. 2

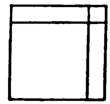


FIG. 3

We have seen (§ 80) a figure showing the square on the lines $40 + 7$. In the explanation in § 82 we first took away from the whole square (as in Fig. 3) the square of the tens, or 1600, which is the square of 40 (represented by Fig. 1).

There then remained the two rectangles and the small square as seen in Fig. 3, the combined area being 609 square units. But these rectangles are as long as Fig. 1, and therefore 40 units long. Dividing their area, plus the area of the small square, by their length, we have approximately their width. (We would get their exact width if it were not for the small square.) This gives the units.

If we now add twice the tens to the units, we shall have the length of the two rectangles and the small square ($80 + 7 = 87$). Multiplying this by the units, the width of the rectangles, we have the area of the two rectangles and the small square ($87 \times 7 = 609$).

We see from the figures that the square is now entirely used; that is, the square of tens (Fig. 1) + twice the product of tens and units (making Fig. 2) + the square of the units (making Fig. 3) uses all the area. Therefore the square root is 47.

84. Square Root of Larger Numbers. The method of § 82 will apply to numbers of more than two periods, by considering *the part of the root already found as so many tens* with respect to the next figure. For example, required the square root of 7,890,481.

	7 89 04 81 (2809
	4
40	3 89
48	3 84
5600	5 04 81
5609	5 04 81

When the third period, 04, is brought down, and the divisor, 56, formed, the next figure of the root is 0, because 560 is not contained in 504. Therefore 0 is placed both in the root and in the divisor, and the next two figures, 81, are brought down.

85. Square Roots of Decimals. If the square root of a number has decimal places, the number itself will have twice as many.

Thus, if 0.25 is the square root of some number, the number will be 0.25^2, or 0.25×0.25, or 0.0625. Hence, if a given number contains a decimal, we separate it into periods of two figures each, beginning at the decimal point and proceeding toward the left for the integral part, and toward the right for the decimal. The last period of the decimal must have two figures, a zero being annexed if necessary.

Extract the square root of 52.2729.

	52.27 29(7.23
	49
140	3 27
142	2 84
1440	43 29
1443	43 29

We see at once that the root will have one integral place. Furthermore, if it is a perfect square it can have only two decimal places, since the square of thousandths would be millionths.

EXERCISE 49

Extract the square root of:

1. 190,969.	**6.** 804,609.	**11.** 1036.84.	**16.** 3.9204.
2. 743,044.	**7.** 194,481.	**12.** 82.2649.	**17.** 462.25.
3. 401,956.	**8.** 173,056.	**13.** 0.063001.	**18.** 0.003969.
4. 758,641.	**9.** 174,724.	**14.** 1.5129.	**19.** 0.182329.
5. 117,649.	**10.** 509,796.	**15.** 2.6244.	**20.** 0.054756.

Find the side of a square whose area is:

21. 12,321 sq. ft. **22.** 8046.09 sq. in.

23. What is the perimeter of a square whose area is 1944.81 sq. in.?

24. The total area of the sides of a certain cube is found to be 355.74 sq. in. What is the length of an edge? What is the volume of the cube?

86. Approximate Square Roots. If a number is not a perfect square, zeros may be annexed and an approximate value of the root found.

For example, required to extract to three places of decimals the square root of 19.

	19.00 00 00 (4.3588+
	16
80	3 00
83	2 49
860	51 00
865	43 25
8700	7 75 00
8708	6 96 64
87160	78 3600

In this example we proceed in the usual way, annexing pairs of zeros for each decimal place in the root. We carry the work to four decimal places, so as to find the nearest approximation for three places. The result is, therefore, 4.359 – ; that is, it is nearer 4.359 than 4.358.

87. Summary of Square Root. We may summarize the process of square root as follows:

Separate the number into periods of two figures each, beginning at the decimal point.

Find the greatest square in the left-hand period and write its root for the first figure of the required root.

Square this root, subtract the result from the left-hand period, and to the remainder annex the next period for a dividend.

For a partial divisor double the root already found, considered as tens, and divide the dividend by it. The quotient (or the quotient diminished slightly if necessary) will be the next figure of the root.

To this partial divisor add this next figure of the root for a complete divisor. Multiply the complete divisor by this next figure of the root, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.

Proceed in this manner until all the periods have been thus annexed. The result will be the square root required.

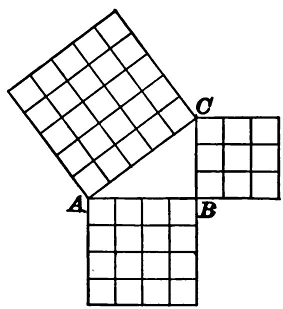
88. Square Roots of Common Fractions. To find the square root of a common fraction we may extract the square roots of the numerator and denominator.

If the denominator is not a perfect square, it is better to multiply both terms of the fraction by such a number as shall make it such, or else to reduce to a decimal and then extract the square root. For example, instead of saying $\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{1.414+}$, which requires us to divide by a long divisor, it is better to say $\sqrt{\frac{1}{2}} = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2} = \frac{1}{2}$ of $1.414+ = 0.707+$, or else to extract the square root of 0.5.

89. Right Triangle. A triangle that contains a right angle is called a *right triangle.* It is sometimes called a *right-angled triangle.*

90. Hypotenuse. In a right triangle the side opposite the right angle is called the *hypotenuse.*

In the triangle ABC the hypotenuse is AC, the base is AB, and the perpendicular (or altitude) is BC.



91. Square on the Hypotenuse. By counting the small squares in this figure we see that the square on AC equals the sum of the squares on AB and BC. It is proved in geometry that this is true for all right triangles.

Therefore, *in a right triangle the square of the hypotenuse equals the sum of the squares of the other two sides.*

The hypotenuse equals the square root of the sum of the squares of the other two sides.

The base or the perpendicular equals the square root of the difference of the squares of the hypotenuse and the other side.

EXERCISE 50

Extract the square root of:

1. $\frac{49}{144}$. **3.** $\frac{225}{361}$. **5.** $\frac{529}{729}$. **7.** $\frac{1369}{1681}$. **9.** $\frac{2209}{2601}$.

2. $\frac{121}{169}$. **4.** $\frac{441}{625}$. **6.** $\frac{961}{1089}$. **8.** $\frac{841}{1849}$. **10.** $\frac{2809}{3025}$.

Extract the square root, to two decimal places:

11. 2. **13.** 11. **15.** 30. **17.** 125. **19.** $\frac{5}{9}$.

12. 7. **14.** 15. **16.** 50. **18.** 650. **20.** $\frac{7}{12}$.

Reduce to fractions with denominators perfect squares and extract the square root, to two decimal places:

21. $\frac{1}{3}$. **22.** $\frac{2}{5}$. **23.** $\frac{2}{3}$. **24.** $\frac{3}{8}$. **25.** $\frac{7}{8}$.

Reduce to decimals and extract the square root, to two decimal places:

26. $\frac{5}{8}$. **27.** $\frac{4}{7}$. **28.** $\frac{3}{5}$. **29.** $\frac{5}{12}$. **30.** $\frac{1}{12}$.

Find the hypotenuse, given the other sides as follows:

31. 39 ft., 52 ft. **32.** 21 ft., 72 ft. **33.** 51 ft., 68 ft.

Also as follows, carrying the results to three decimals:

34. 82 ft., 35 ft. **35.** 31 ft., 23 ft. **36.** 27 ft., 43 ft.

Given the hypotenuse and one side, find the other side:

37. 10 ft., 6 ft. **38.** 17 in., 15 in. **39.** 26 in., 10 in.

Also as follows, carrying the results to three decimals:

40. 15 ft., 6 ft. **41.** 18 ft., 12 ft. **42.** 23 in., 12 in.

Find the diagonal of the square whose side is:

43. 20″. **44.** 32″. **45.** 45″. **46.** 70″. **47.** 75″.

92. Area of a Triangle in Terms of the Sides. It is proved in geometry that

To find the area of any triangle given the sides, multiply half the sum of the sides in succession by the three remainders obtained by subtracting each side separately from the half sum of the sides, and take the square root of this product.

For example, find the area of a triangle whose sides are respectively 5 in., 6 in., and 7 in.

The half sum of the sides is $\frac{1}{2}$ of $(5+6+7)$, or 9. Then $9-5=4$, $9-6=3$, and $9-7=2$. Therefore the area $=\sqrt{9\times4\times3\times2}=\sqrt{216}$ $=14.696+$. That is, the area is 14.696 sq. in.

93. Similar Figures. It is also proved in geometry that

The areas of similar figures are to each other as the squares of their corresponding dimensions.

The corresponding dimensions of similar figures are to each other as the square roots of their areas.

That is, if a side of one square is 5 times as long as a side of another, the area is 25 times as much. If one circle has 16 times the area of another, the radius is 4 times as long.

EXERCISE 51

Find the areas (to two decimal places) of the triangles whose sides are:

1. 5″, 7″, 8″.
2. 6″, 8″, 9″.
3. 4″, 3″, 6″.
4. 9″, 10″, 11″.
5. 12″, 13″, 15″.
6. 17″, 21″, 25″.

7. A triangle has an area of 275 sq. ft. What is the area of a similar triangle whose base is only half as long?

8. A rectangular lot has a front of 160 ft. and is worth $1000. Find the value, at the same rate per square foot, of a similar lot which has twice the front and twice the depth.

94. Finding the Radius. Since the area of a circle equals 3.1416 times the square of the radius, therefore

The radius of a circle equals the square root of the quotient of the area divided by 3.1416.

For example, if the area is 78.54 sq. in., the number of inches in the radius is $\sqrt{\frac{78.54}{3.1416}} = \sqrt{25} = 5$. Therefore the radius is 5 in.

In Exercise 52 carry the square roots to two decimal places.

EXERCISE 52

Find the area of the circle whose radius is:

1. 17″. **2.** 29″. **3.** 37″. **4.** 5.27″. **5.** 6.07′.

Find the area of the circle whose diameter is:

6. 38″. **7.** 46″. **8.** 29″. **9.** 7.54″. **10.** 9.72′.

Find the radius of the circle whose area is:

11. 314.16 sq. in. **12.** 113.0976 sq. in. **13.** 250 sq. in.

14. What must be the diameter of a water pipe in order that the area of a cross section shall be 2 sq. in.?

15. What must be the diameter of a water main in order that the area of a cross section shall be 3 sq. ft.?

16. What must be the diameter of the piston head of an engine in order that the area may be 100 sq. in.?

17. A cylindrical water tank is 20 ft. high and will contain 28,000 cu. ft. of water. What is the diameter?

18. A horse tethered by a rope can graze over 1570.8 sq. ft. of ground. How long is the rope? If the rope was twice as long, over how much ground could he graze?

19. A tinsmith wishes to make some cylindrical gallon cans. They are to be 10 in. high. What must be the area of the base? What radius must he use to draw the circle?

95. Cube. The result of taking a number three times as a factor is called the *third power* or the *cube* of the number.

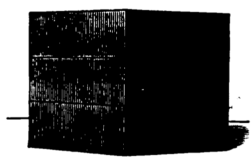
Thus the cubes of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, are 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.

96. Cube Root. One of the three equal factors of a number is called the *cube root* of the number.

Thus the cube roots of 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

97. Symbols. The cube of 5 is written 5^3; the cube root of 125 is written $\sqrt[3]{125}$.

98. Cube Roots and Volumes. If this cube is 3 units on an edge, we can see that there will be 3×3, or 9, cubic units in the front layer of cubes and that there will be 3 of these layers. The volume is therefore $3 \times 3 \times 3$ cubic units, or 27 cubic units.



Therefore, *the volume of a cube equals the cube of the edge.*

The edge of a cube equals the cube root of the volume.

99. Cube Roots by Factoring. In the case of perfect cubes we may find the cube roots if we can factor.

For example, since $1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = (2 \times 2 \times 3) \times (2 \times 2 \times 3) \times (2 \times 2 \times 3)$, therefore $\sqrt[3]{1728} = 2 \times 2 \times 3 = 12$.

EXERCISE 53

Find the value of the following:

1. 37^3.	4. 7.5^3.	7. $\sqrt[3]{1331}$.	10. $\sqrt[3]{5832}$.
2. 49^3.	5. 4.8^3.	8. $\sqrt[3]{3375}$.	11. $\sqrt[3]{9261}$.
3. 59^3.	6. 0.69^3.	9. $\sqrt[3]{4096}$.	12. $\sqrt[3]{10648}$.

13. Find the edge of a cube containing 13,824 cu. in.

100. Cube of the Sum of Two Numbers. Since $47 = 40 + 7$, the cube of 47 may be obtained by multiplying the square of $40 + 7$ (as found in § 80) by $40 + 7$, thus:

$$\begin{array}{rl}
(40+7)^2 = 40^2 + 2 \times (40 \times 7) + 7^2 & \\
40 + 7 & \\
\hline
(40^2 \times 7) + 2 \times (40 \times 7^2) + 7^3 & \text{product by 7} \\
40^3 + 2 \times (40^2 \times 7) + \quad (40 \times 7^2) \quad & \text{product by 40} \\
\hline
40^3 + 3 \times (40^2 \times 7) + 3 \times (40 \times 7^2) + 7^3 &
\end{array}$$

We therefore see that

The cube of any number composed of tens and units contains four parts:

(1) *The cube of the tens.*

(2) *Three times the product of the square of the tens by the units.*

(3) *Three times the product of the tens by the square of the units.*

(4) *The cube of the units.*

101. Separating into Periods. The first step in extracting the cube root of a number is to separate the figures of the number into groups of three figures each, called *periods.*

Since $1 = 1^3$, $1000 = 10^3$, $1{,}000{,}000 = 100^3$, and so on, it follows that the cube root of any integral number between 1 and 1000, that is, the cube root of any integral number that has *one*, *two*, or *three* figures, is a number of *one* figure; that the cube root of any integral number between 1000 and 1,000,000, that is, the cube root of any integral number that has *four*, *five*, or *six* figures, is a number of *two* figures; and so on.

If, therefore, an integral number is separated into periods of three figures each, from right to left, the number of figures in the root will be equal to the number of periods. The last period to the left may consist of one figure, two figures, or three figures.

For example, the cube root of 42,875 will have two integral places, and the cube root of 34,645,976 will have three.

102. Extracting the Cube Root. The process of extracting the cube root of a number not readily factored will now be considered. For example, required the cube root of 42,875.

We see that there will be two integral places in the root (§ 101). The first period, 42, contains the cube of the tens' number. The greatest cube in 42 is 27, and the cube root of 27 is 3. Hence 3 is the tens' figure of the root.

$$
\begin{array}{rr|l}
 & & 42\,875(35 \\
 & & 27 \\
\hline
3 \times 30^2 = & 2700 & 15\,875 \\
3 \times (30 \times 5) = & 450 & \\
5^2 = & 25 & \\
\cline{2-2}
 & 3175 & 15\,875 \\
\cline{3-3}
\end{array}
$$

The remainder, 15,875, resulting from subtracting the cube of the tens, will contain three times the product of the square of the tens by the units + three times the product of the tens by the square of the units + the cube of the units.

Each of these three parts contains the units' number as a factor.

Hence the 15,875 consists of two factors, one of which is the units' number of the root; and the other factor is three times the square of the tens + three times the product of the tens by the units + the square of the units. The largest part of this second factor is three times the square of the tens.

If the 158 hundreds of the remainder is divided by 3×30^2, or 27 hundreds, the quotient will be approximately the units' number of the root. The second factor can now be completed by adding to the 2700 the sum of $3 \times (30 \times 5)$, or 450, and 5^2, or 25.

If this factor, 3175, is now multiplied by 5, the result is 15,875, which completes the cube of 35. There being no remainder, $\sqrt[3]{42875} = 35$.

To check the work, $35^3 = 42,875$.

EXERCISE 54

Extract the cube root of:

1. 2197.	**4.** 13,824.	**7.** 226,981.	**10.** 857,375.
2. 4913.	**5.** 29,791.	**8.** 132,651.	**11.** 884,736.
3. 6859.	**6.** 110,592.	**9.** 373,248.	**12.** 941,192.

13. Find the edge of a cube whose volume is 778,688 cu. in.

14. Find the edge of a cube whose volume is 205,379 cu. in.

103. Explanation by Blocks. It sometimes makes the reasoning in cube root more clear if we consider the building up of a cube by blocks.

We have learned that the cube of $35 = 30^3 + 3 \times (30^2 \times 5) + 3 \times (30 \times 5^2) + 5^3$ (§ 100).

The 30^3 may be represented by a cube whose edge is 30 in. (Fig. 1).

The $3 \times (30^2 \times 5)$ may be represented by three equal rectangular solids, each 30 in. long, 30 in. wide, and 5 in. thick, to be added to three adjacent faces of Fig. 1.

The $3 \times (30 \times 5^2)$ may be represented by three equal rectangular solids, 30 in. long, 5 in. wide, and 5 in. thick, to be added to Fig. 2.

The 5^3 may be represented by the small cube required to complete the cube of Fig. 3.

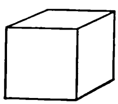


FIG. 1

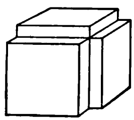


FIG. 2

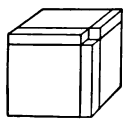


FIG. 3

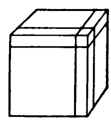


FIG. 4

In extracting the cube root of 42,875, the large cube (Fig. 1) whose edge is 30 in. is first removed.

There remains 42,875 cu. in. – 27,000 cu. in., or 15,875 cu. in.

The greatest part of this is contained in the three rectangular solids, each 30 in. long and 30 in. wide, which are added to Fig. 1.

The thickness of these solids is found by dividing the 15,875 cu. in. by the sum of the three faces, each of which is 30 in. square; that is, by 2700 sq. in. The result is 5 in.

There are also the three rectangular solids which are added to Fig. 2, and which are 30 in. long and 5 in. wide; and a cube which is added to Fig. 3, and which is 5 in. long and 5 in. wide.

Hence the sum of the products of two dimensions of all these solids is

For the larger rectangular solids,	$3 \times (30 \times 30)$ sq. in. =	2700 sq. in.
For the smaller rectangular solids,	$3 \times (30 \times 5)$ sq. in. =	450 sq. in.
For the small cube,	(5×5) sq. in. =	25 sq. in.
		3175 sq. in.

The number 3175 multiplied by the third dimension, 5, gives 5×3175 cu. in., or 15,875 cu. in., as found in § 102.

104. Cube Roots of Larger Numbers. The method of § 102 will apply to numbers of more than two periods, by considering *the part of the root already found as so many tens* with respect to the next figure of the root.

For example, required the cube root of 57,512,456.

		57 512 456 (386
		27
$3 \times 30^2 =$	2700	30 512
$3 \times (30 \times 8) =$	720	
$8^2 =$	64	
	3484	27 872
$3 \times 380^2 =$	433200	2 640 456
$3 \times (380 \times 6) =$	6840	
$6^2 =$	36	
	440076	2 640 456

105. Cube Roots of Decimals. If a cube root has decimal places, the cube will have *three times* as many.

Thus, if 0.11 is the cube root of a number, the number is $0.11 \times 0.11 \times 0.11 = 0.001331$. Hence, if a given number contains a decimal, we separate it into periods of three figures each, beginning at the decimal point and proceeding toward the left for the integral part, and toward the right for the decimal. The last period of the decimal must contain *three* figures, zeros being annexed when necessary.

Extract the cube root of 187.149248.

		187.149 248 (5.72
		125
$3 \times 50^2 =$	7500	62 149
$3 \times (50 \times 7) =$	1050	
$7^2 =$	49	
	8599	60 193
$3 \times 570^2 =$	974700	1 956 248
$3 \times (570 \times 2) =$	3420	
$2^2 =$	4	
	978124	1 956 248

Since there can be only one integral place, the decimal point is placed after the 5.

106. Approximate Cube Roots. If the given number is not a perfect cube, zeros may be annexed, and a value of the root may be found as near to the true value as we please.

Extract the cube root of 1250.6894.

		1 250.689 400(10.77
		1
$3 \times 10^2 =$	300	250

Since 300 is not contained in 250, the next figure of the root is 0.

$3 \times 100^2 =$	30000	250 689
$3 \times (100 \times 7) =$	2100	
$7^2 =$	49	
	32149	225 043
$3 \times 1070^2 =$	3434700	25 646 400
$3 \times (1070 \times 7) =$	22470	
$7^2 =$	49	
	3457219	24 200 533
		1 445 867

107. Cube Roots of Common Fractions. The cube root of a common fraction is found by extracting the cube roots of the numerator and denominator; but, if the denominator is not a perfect cube, it is generally best to reduce the fraction to a decimal, and then extract the root.

EXERCISE 55

Extract the cube root, to two decimal places:

1. 71.296. **3.** 7.1296. **5.** 21.782. **7.** 37.487.

2. 643.25. **4.** 0.75475. **6.** 0.1234. **8.** 81.492.

Extract the cube root of the following fractions:

9. $\frac{8}{729}$. **10.** $\frac{1331}{1728}$. **11.** $\frac{4913}{6859}$. **12.** $\frac{2197}{9261}$. **13.** $\frac{3375}{4096}$

Extract the cube root, to three decimal places:

14. 2. **15.** 3. **16.** 5. **17.** $\frac{1}{4}$. **18.** $\frac{2}{3}$.

108. Summary of Cube Root. We may summarize the process of cube root as follows:

Separate the number into periods of three figures each, beginning at the decimal point.

Find the greatest cube in the left-hand period and write its root for the first figure of the required root.

Cube this root, subtract the result from the left-hand period, and to the remainder annex the next period for a dividend.

For a partial divisor, take three times the square of the root already found, considered as tens, and divide the dividend by it. The quotient (or the quotient diminished slightly if necessary) will be the second figure of the root.

To this partial divisor add three times the product of the first figure of the root, considered as tens, by the second figure, and also the square of the second figure. This sum will be the complete divisor.

Multiply the complete divisor by the second figure of the root, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.

Proceed in this manner until all the periods have been annexed. The result will be the cube root required.

If a given number contains a decimal, the last group on the right of the decimal point must be made to contain three figures, by annexing one zero or two zeros if necessary. In the examples of Exercise 56 carry the result to two decimal places in case the given number is not an exact cube.

109. Similar Solids. Solids that have exactly the same shape are called *similar solids.*

It is proved in geometry that

The volumes of similar solids are to each other as the cubes of corresponding dimensions.

The corresponding dimensions of similar solids are to each other as the cube roots of the volumes.

EXERCISE 56

Extract the cube root of:

1. 1,771,561.	**4.** 47,832,147.	**7.** 4826.809.
2. 1,295,029.	**5.** 11,390,625.	**8.** 0.000912673.
3. 2,048,383.	**6.** 87,528,384.	**9.** 0.114791256.

Find the edges of cubes of the following volumes:

10. 75 cu. in. **11.** 830 cu. ft. **12.** 92.5 cu. in. **13.** $7\frac{1}{2}$ cu. in.

14. Find the diameter of an iron ball that will weigh 27 times as much as an iron ball 2 in. in diameter.

15. The weights of two iron cylinders of the same shape are as 2197 to 4913. Find the ratio of their heights.

16. Find the edge of a cube whose volume is equal to the volume of a rectangular solid $81'' \times 3'' \times 3''$.

17. Find the edge of a cubical cistern that holds as many gallons as a rectangular cistern $12' \times 8' \times 5\frac{1}{3}'$.

18. Find the edge of a cubical cistern that holds as many gallons as a rectangular cistern $18' \times 9' \times 4\frac{1}{2}'$.

19. The edge of a cube is 2 in., and the edge of another is 4 in. How many times the total surface of the smaller is the total surface of the larger? How many times the volume of the smaller is the volume of the larger?

20. The volume of a rectangular solid is 2197 cu. in., and the volume of a similar solid is 17,576 cu. in. How many times a dimension of the smaller solid is the corresponding dimension of the larger?

21. How many spheres 1 inch in diameter are equivalent in volume to a sphere 2 inches in diameter? to a sphere 5 inches in diameter? to a sphere 10 inches in diameter? to a sphere $15\frac{1}{2}$ inches in diameter?

EXERCISE 57

Problems without Numbers

1. How do you find the square of a number? the cube?

2. If you know the side of a square, how do you find the area?

3. If you know the area of a square, how do you find the side?

4. How do you find the square root of a number by factoring? also the cube root by factoring?

5. To what is the square of the sum of two numbers equal? How would you show it?

6. What is the object of separating a number into periods in extracting square root?

7. How do you proceed to extract the square root of an integer that is a perfect square?

8. How do you proceed to extract the square root of a number that contains decimal places?

9. How do you proceed to extract the square root of a common fraction?

10. How do you find the hypotenuse of a right triangle when the other sides are given?

11. How do you find the base of a right triangle, given the hypotenuse and the perpendicular?

12. How do you find the area of a triangle when the three sides are given?

13. What is the relation between the corresponding dimensions of similar figures?

14. Given the area of a circle, how do you find the radius?

15. How do you extract the cube root of a number?

CHAPTER IV

METRIC MEASURES

110. The Metric System. The metric system is now used in several European countries, and in scientific work elsewhere. Since it is on the decimal scale it is much simpler than our system.

111. Meter. The unit of length is the *meter.*

112. Liter. The unit of capacity is the *liter* (pronounced lee′ter), a cube 0.1 of a meter on an edge.

113. Gram. The unit of weight is the *gram.* It is the weight of a cube of water 0.01 of a meter on an edge at the greatest density of water, a little above freezing.

114. The Prefixes. The system is understood as soon as the prefixes are known.

Just as 1 mill $= \frac{1}{1000}$ of a dollar,
so 1 millimeter $= 0.001$ of a meter.
Just as 1 cent $= \frac{1}{100}$ of a dollar,
so 1 centimeter $= 0.01$ of a meter.
Just as decimal means tenths,
so 1 decimeter $= 0.1$ of a meter.

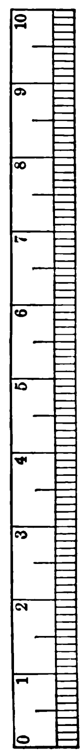
	THE PREFIX	MEANS	AS IN	WHICH MEANS	
From the Greek	myria-	10,000	myriameter	10,000	meters.
	kilo-	1,000	kilogram	1,000	grams.
	hekto-	100	hektoliter	100	liters.
	deka-	10	dekameter	10	meters.
		1		1	
From the Latin	deci-	0.1	decimeter	0.1	of a meter.
	centi-	0.01	centigram	0.01	of a gram.
	milli-	0.001	millimeter	0.001	of a meter.

115. Important Names. As in United States money we seldom speak of anything except dollars and cents, so in metric measures only those measures printed in black letters in the tables of this chapter are in common use.

All the units of the system are derived from the meter. Every compound name is accented on the first syllable ; thus, *mil'limeter.* The teacher should be supplied with a meter stick, a liter, and a cubic centimeter, and these can easily be made in school if necessary. The work should be made as practical as possible by actual measurements.

116. Measures of Length. The table of measures of length is as follows:

A myriameter	=	10,000 meters
A **kilometer** (km.)	=	1,000 meters
A hektometer	=	100 meters
A dekameter	=	10 meters
Meter (m.)		
A decimeter (dm.)	=	0.1 of a meter
A **centimeter** (cm.)	=	0.01 of a meter
A **millimeter** (mm.)	=	0.001 of a meter



10 centimeters = 1 decimeter.

The meter is about 39.37 in., or $3\frac{1}{4}$ ft., or a little over a yard; the kilometer is about 0.62 of a mile. When the metric system was invented in France over a century ago, the meter was intended to be one ten-millionth of the distance on the surface of the earth from the equator to the pole, but it varies slightly from this standard.

The abbreviations in this book are in common use. Some, however, use Km., Dm., dm., for kilometer, dekameter, and decimeter. We do not need abbreviations for the units not commonly used, such as the myriameter.

117. Reduction. Any one of these measures may be expressed in terms of another measure by *simply moving the decimal point to the right or left.*

Thus, as 245¢ = 24.5 d. = \$2.45,

so 2475 mm. = 247.5 cm. = 24.75 dm. = 2.475 m.,

and 35.25 km. = 35,250 m. = 35,250,000 mm.

EXERCISE 58

Express as meters and decimals:

1. 25 km.	**6.** 237 dm.	**11.** 2965 cm.
2. 3.7 km.	**7.** 47.5 dm.	**12.** 48,750 cm.
3. 0.7 km.	**8.** 2.75 dm.	**13.** 29,375 mm.
4. $62\frac{1}{2}$ km.	**9.** 725 cm.	**14.** 96,400 mm.
5. $57\frac{3}{4}$ km.	**10.** 68.5 cm.	**15.** 250,000 mm.

Allowing 39.37 in. to a meter, express as meters:

16. 19.685 in.	**19.** 25 ft.	**22.** 3 ft. 3 in.
17. 78.74 in.	**20.** 4.8 ft.	**23.** 3 ft. 3.37 in.
18. 118.11 in.	**21.** 0.6 ft.	**24.** 29 ft. 6.33 in.

Express as feet:

25. 75 m.	**28.** 35 dm.	**31.** 4500 cm.
26. $48\frac{1}{2}$ m.	**29.** 7.5 dm.	**32.** 6875 mm.
27. $2.7\frac{1}{2}$ m.	**30.** 685 cm.	**33.** 21,475 mm.

Allowing 0.62 mi. to a kilometer, express as miles:

34. 5 km.	**36.** $2\frac{1}{2}$ km.	**38.** 272 km.
35. 75 km.	**37.** 4.75 km.	**39.** 48.9 km.

Express as kilometers:

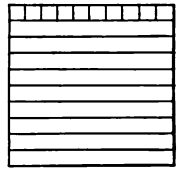
40. 325 mi.	**41.** 42.7 mi.	**42.** 0.75 mi.

43. If sound travels 340.5 m. a second, how many kilometers will it travel in 17 sec.?

44. If the distance from Paris to Brussels is 313 km., how many miles is it?

45. If the distance from Lyons to Marseilles is 218 mi., how many kilometers is it?

118. Square Measure. In square measure the scale is by hundreds instead of by tens. If the square here shown represents a square meter, it is evident that each of the ten equal horizontal bands is one tenth of a square meter, and each of the ten small squares therefore represents $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{100}$, of a square meter. That is, 100 square decimeters = 1 square meter.



The table is as follows:

A square myriameter	=	100,000,000	square meters
A square kilometer (sq. km.)	=	1,000,000	square meters
A square hektometer	=	10,000	square meters
A square dekameter	=	100	square meters
Square meter (sq. m.)			
A square decimeter	=	0.01	of a square meter
A **square centimeter** (sq. cm.)	=	0.0001	of a square meter
A **square millimeter** (sq. mm.)	=	0.000001	of a square meter

119. Land Measure. In the measurement of land the square dekameter is called an *are* (pronounced ăr); the square hektometer is called a hektare (ha.). The hektare equals 2.47 acres, or nearly $2\frac{1}{2}$ acres.

EXERCISE 59

Express as square meters:

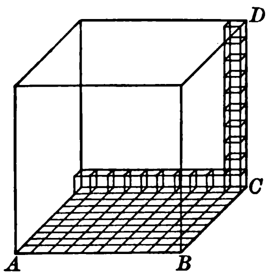
1. 275 sq. km. **4.** 4875 sq. cm. **7.** 123,450 sq. mm.
2. 3.45 sq. km. **5.** 9235 sq. cm. **8.** 2,250,000 sq. mm.
3. 0.75 sq. km. **6.** 64,275 sq. cm. **9.** 37,375,000 sq. mm.

Express as hektares:

10. 10,000 sq. m. **12.** 175,000 sq. m. **14.** 24.7 acres.
11. 45,000 sq. m. **13.** 325,500 sq. m. **15.** 1482 acres.

16. How long is a square field that contains 0.81 ha.?

120. Cubic Measure. In cubic measure the scale is by thousands. If the cube here shown represents a cubic meter, it is evident that each of the ten equal layers represents one tenth of a cubic meter; that each of the rectangular solids, *DC*, represents $\frac{1}{10}$ of $\frac{1}{10}$ of a cubic meter, and that each of the small cubes represents $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{1000}$, of a cubic meter. That is, 1000 cubic decimeters = 1 cubic meter.



The table is as follows:

A cubic hektometer	=	1,000,000 cubic meters	
A cubic dekameter	=	1,000 cubic meters	
Cubic meter (cu. m.)			
A **cubic decimeter** (cu. dm.)	=	0.001	of a cubic meter
A **cubic centimeter** (cu. cm.)	=	0.000001	of a cubic meter
A cubic millimeter (cu. mm.)	=	0.000000001	of a cubic meter

We may also have cubic kilometers and cubic myriameters. The terms above the cubic meter are seldom used.

121. Wood Measure. In measuring wood the cubic meter is called a *stere* (st.). Stere is pronounced stair.

EXERCISE 60

Express as cubic meters:

1. 725.25 st.
2. 2,750,000 cu. dm.
3. 4,625,750 cu. dm.
4. 9,125,000 cu. dm.
5. 8,125,000,000 cu. mm.
6. 9,275,000,000 cu. mm.
7. 25,325,000,000 cu. mm.
8. 37,250,000,000 cu. mm.

9. A pile of wood is 2.5 m. high, 1 m. wide, and 28.7 m. long. How many steres are there?

122. Measures of Capacity. The table of capacity is as follows:

A **hektoliter** (hl.)	=	100 liters
A dekaliter	=	10 liters
Liter (l.)		
A deciliter (dl.)	=	0.1 of a liter
A centiliter (cl.)	=	0.01 of a liter
A milliliter (ml.)	=	0.001 of a liter

A liter is a cubic decimeter. It is practically the same as a quart.

EXERCISE 61

Express as liters:

1. 750 hl.	**3.** 92.75 hl.	**5.** 750 dl.	**7.** 7500 ml.
2. 275 hl.	**4.** 9.375 hl.	**6.** 975 cl.	**8.** 6250 ml.

Express as hektoliters:

9. 3750 l.	**11.** 978.5 l.	**13.** $97\frac{1}{2}$ qt.	**15.** 65 gal.
10. 4875 l.	**12.** 25.75 l.	**14.** $38\frac{3}{4}$ qt.	**16.** $31\frac{1}{2}$ gal.

Express as quarts:

17. 37 hl.	**18.** 9.5 hl.	**19.** 750 dl.	**20.** 8250 cl.

21. How many liters are there in a cubic meter?

22. How many hektoliters of air are there in a room 7 m. long, 6 m. wide, and 3 m. high?

23. A cubical tank in a zoölogical garden is 2.25 m. on an edge. How many liters will it hold?

24. A bin is 11.25 m. long, 4.8 m. wide, and 1.6 m. deep. How many hektoliters will it hold? How many liters?

25. The average depth of a pond is 1.75 m., and its area is 3.5 ha. How many hektoliters of water does it contain?

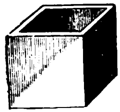
26. The height of a cylindrical tank is 12 m., and the diameter of the base is 9 m. How many liters will it hold?

123. Measures of Weight. The table of weight is as follows:

A metric ton (t.)	=	1,000,000 grams
A quintal (q.)	=	100,000 grams
A myriagram	=	10,000 grams
A kilogram (kg.)	=	1,000 grams
A hektogram	=	100 grams
A dekagram	=	10 grams
Gram (g.)		
A decigram	=	0.1 of a gram
A centigram (cg.)	=	0.01 of a gram
A milligram (mg.)	=	0.001 of a gram

124. Relation of Weight to Capacity.

1 cu. cm. of water weighs 1 g.
1 l. of water weighs 1 kg.
1 cu. m. of water weighs 1 t.


Cubic Centimeter


Gram Weight

The standard units vary slightly from these theoretical units.

A kilogram is about 2.2 lb.; and a metric ton is about 2204.6 lb.

EXERCISE 62

Express as grams:

1. 1.7 kg.	**3.** 0.25 kg.	**5.** 700 cg.	**7.** 7500 mg.
2. 0.9 kg.	**4.** 3.75 kg.	**6.** 950 cg.	**8.** 8125 mg.

Express as milligrams:

9. 7 g.	**11.** 0.27 g.	**13.** 15 kg.	**15.** 75 cg.
10. 8.7 g.	**12.** 4.25 g.	**14.** 2.7 kg.	**16.** 9.8 cg.

Express as kilograms:

17. 27 t.	**18.** 0.5 t.	**19.** 250 g.	**20.** 4400 lb.

Express as pounds:

21. 3 kg.	**22.** 75 kg.	**23.** 300 g.	**24.** 9000 mg.

125. Specific Gravity. The ratio of the weight of a given substance to the weight of an equal volume of water is called the *specific gravity* of the substance.

Thus, if a cubic decimeter of copper weighs 8.9 kg., the specific gravity of copper is 8.9, because 1 cu. dm. = 1 l., and 1 l. of water weighs 1 kg. Therefore copper is 8.9 times as heavy as water.

It is usually easy to find the volume of any substance by immersing it in a vessel full of water and measuring the amount that runs over.

By means of specific gravity the purity of metals and other substances may be determined.

In the case of gases the weight of the gas is usually compared with that of air or hydrogen instead of water.

EXERCISE 63

1. If 2 l. of alcohol weigh 1.58 kg., what is the specific gravity of alcohol?

2. If 5 l. of petroleum weigh $3\frac{1}{2}$ kg., what is the specific gravity of petroleum?

3. If 7 cu. cm. of cork weigh 1.68 g., what is the specific gravity of cork?

4. If 6.5 l. of milk weigh $6.69\frac{1}{2}$ kg., what is the specific gravity of milk?

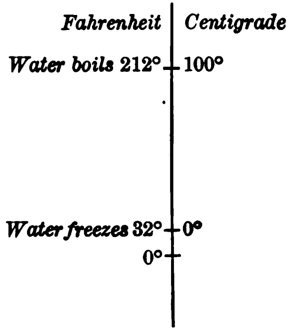
5. The specific gravity of silver is 10.5. Some spoons weighing 371.7 g., thought to be silver, are immersed in a jar full of water and 35.4 cu. cm. of water overflow. Do the spoons stand the specific-gravity test?

6. A body immersed in a liquid always weighs less than it does in air by the weight of the liquid that it displaces. If a piece of gold weighs 9.65 g. in air and 9.15 g. in water, what is its specific gravity?

7. If the specific gravity of steel is 7.8, what is the weight of a cylindrical steel rod 1 m. long and 5 cm. in diameter?

126. Measures of Temperature. The thermometer commonly used in the United States is known as the Fahrenheit. By this thermometer water freezes at 32° and boils at 212°.

The metric system uses the *Centigrade* thermometer. By this thermometer water freezes at 0° and boils at 100°.



127. Reduction from Fahrenheit to Centigrade. If we indicate Fahrenheit by Fah., and Centigrade by C., we have

$$212° - 32° \text{ Fah.} = 180° \text{ Fah. above freezing} = 100° \text{ C.};$$

$$1° \text{ Fah. above freezing} = \tfrac{100}{180}° \text{ C.} = \tfrac{5}{9}° \text{ C.}$$

Therefore 98° Fah. = 98° − 32° = 66° Fah. above freezing, and $66 \times \frac{5}{9}°$ C. = $36\frac{2}{3}°$ C. That is, 98° Fah. = $36\frac{2}{3}°$ C.

128. Reduction from Centigrade to Fahrenheit. Since

$$100° \text{ C.} = 180° \text{ Fah. above freezing,}$$

$$1° \text{ C.} = 1.8° \text{ Fah. above freezing.}$$

Hence $36\frac{2}{3}°$ C. = $36\frac{2}{3} \times 1.8°$ Fah. above freezing = 66° Fah. above freezing = 66° + 32°, or 98°, Fah. above zero.

EXERCISE 64

Express in Centigrade these temperatures Fahrenheit:

1. 59°. **3.** 75°. **5.** 108°. **7.** 150°. **9.** 5°.
2. 42°. **4.** 86°. **6.** 112°. **8.** 200°. **10.** 10°.
11. 10° below zero. **12.** 15° below zero. **13.** 25° below zero.

Express in Fahrenheit these temperatures Centigrade:

14. 40°. **15.** 75°. **16.** 10°. **17.** 150°. **18.** 200°.
19. 10° below zero. **20.** 15° below zero. **21.** 25° below zero.

EXERCISE 65

Review Problems

1. Find the area of a square 1.37 m. on a side.

2. Find the area of a rectangle 3.95 m. by 1.36 m.

3. Find the area of a circle whose radius is 75 cm.

4. Find the area of a circle whose diameter is 324 mm.

5. Find the area of a parallelogram whose base is 2.75 m. and whose altitude is 1.65 m.

6. Find the area of a triangle whose base is 325 cm. and whose altitude is 76 cm.

7. Find the altitude of a parallelogram whose area is 3.75 sq. m. and whose base is 1.875 m.

8. Find the base of a triangle whose area is 9.106 sq. m., and whose altitude is 2.9 m.

9. Find the weight of the water required to fill a tank 2.3 m. long, 0.98 m. wide, and 0.56 m. deep.

10. A liter of air weighs 1.292 g. Find the weight of air in a schoolroom 6.8 m. long, 5.2 m. wide, and 3 m. high.

11. How many liters of water will a tank hold that is 2.7 m. long, 1.4 m. wide, and 0.75 m. deep?

12. If a barrel full of water weighs 123 kg., and when empty weighs 10 kg., how many liters will it hold?

13. What part of a hektare is there in a piece of land 125 m. long and 65 m. wide?

14. If a 40-liter keg of olive oil weighs 40.5 kg., and the empty keg weighs 3.9 kg., what does a liter of oil weigh?

15. In a shop where machines are made for foreign trade an order is received to make some wheels 1.34 m. in diameter. How would the workmen express the diameter in inches?

16. A wheel is 0.85 m. in diameter. Express its circumference in inches.

17. What part of a hektare is there in a rectangular field 75 m. long and 65 m. wide?

18. An importer bought a quantity of goods weighing 347.5 kg. How many pounds did they weigh?

19. Taking the distance from Buffalo to Cleveland as 183 miles, how many kilometers is it?

20. Find the side of a square whose area is 2 ha. (2 decimal places).

21. Find the radius of a circle whose area is 62.832 sq. cm. (3 decimal places).

22. Find the diameter of a circle whose area is 125.664 sq. cm. (3 decimal places).

23. A lady shopping in Paris finds that some cloth costs her 7 francs a meter. Taking a franc as 19.3 cents, how much (to the nearest cent) does it cost a yard?

24. A lady bought some cloth in Paris at $8\frac{1}{2}$ francs a meter. How many dollars did it cost per yard?

25. Find the volume of a cylinder whose altitude is 3.5 m., and whose diameter is 2.2 m.

26. How many metric tons of water will it take to fill a cylindrical tank 13 m. in diameter to a depth of 5 m.?

27. A workman finds that the capacity of a cylinder on an imported engine is given as 11,200 cu. cm. He wishes to find what it is in cubic inches. Taking 1 m. as 39.37 in., express the capacity to 2 decimal places.

28. A mechanic wishes to find the weight of a steel bar that he cannot easily remove from a machine. The bar is 1.3 m. long and has a diameter of 3 cm. The specific gravity of steel being 7.8, what is the weight of the bar?

EXERCISE 66

Problems without Numbers

1. Give the metric table of length.

2. How do you reduce millimeters to meters? kilometers to meters? meters to kilometers?

3. How do you reduce inches to meters? meters to inches? kilometers to miles?

4. Give the metric table of square measure.

5. How do you reduce square millimeters to square meters? square meters to square kilometers?

6. How do you reduce hektares to acres?

7. Give the metric table of cubic measure.

8. How do you reduce cubic meters to cubic centimeters? cubic millimeters to cubic centimeters?

9. If you know the metric dimensions of a pile of wood, how do you find the number of steres?

10. Give the metric table of capacity.

11. How do you reduce liters to hektoliters? hektoliters to liters? liters to quarts?

12. If you know the dimensions of a rectangular tank, how do you find its contents in liters?

13. Give the metric table of weight.

14. How do you reduce milligrams to grams? centigrams to kilograms? grams to kilograms?

15. How do you reduce kilograms to grams? to pounds?

16. How would you find the specific gravity of a rectangular block of stone?

17. How would you find the specific gravity of an irregular piece of copper?

CHAPTER V

STOCKS AND BONDS

129. Corporations. A company composed of a number of persons authorized by law to do business of a certain nature is called a *corporation.*

A *municipal corporation* is a corporation organized for local government, such as a state, a county, a city, or a town.

A corporation organized to transact private business is called a *stock company.*

This is an age of stock companies. Almost all extensive business operations are carried on by such companies. Money is invested more frequently through or in stock companies than in any other way, and it is one of the necessities of business training that we know the general features of such corporations.

130. Capital Stock. The amount contributed by the members of a corporation to carry on the business is called the *capital stock, capital,* or simply *stock* of the company.

One of the equal parts into which the stock of a company is divided is called a *share* of stock.

The most common amount for a share of stock is $100, although shares of $10, $25, $50, and other amounts are issued.

131. Stock Certificates. A statement issued by a corporation specifying the number of shares of stock owned by the holder is called a *stock certificate.*

This certificate is usually signed by the president and either the secretary or the treasurer. Certificates of bank stock are usually signed by the president and the cashier.

A person who owns one or more shares of stock is called a *stockholder.*

132. Organization of Corporations. The stockholders of a company select a few of their number to manage the business. These members are called *directors.* The directors elect the president, secretary, treasurer, and other officers.

133. Dividends. A sum paid to the stockholders out of the earnings of a company is called a *dividend.*

The directors pay the expenses of running a company from all the earnings (the *gross earnings*), and from the remainder (*net earnings*) they usually lay aside a portion (*undivided profits*) for emergencies, and divide the rest among the stockholders as dividends.

Dividends are a certain per cent of the face value of the stock. Thus a 5% dividend on 80 shares of stock of $100 each is $400.

If a company is losing instead of earning money, stockholders are often required to pay *assessments* on their stock to make up the loss, although the company usually borrows money to help in a temporary emergency.

134. Par Value. The face value of stocks is called the *par value.*

The par value of one share (§ 130) is commonly $100, and should be so taken in all examples in this chapter.

135. Market Value. The price at which stocks sell is called their *market value.* If stocks sell for more than their face value, they are said to be *above par*, or at a *premium;* if for less than their face value, they are said to be *below par*, or at a *discount.*

If a company is prosperous and pays higher dividends, say 10%, than the money would earn in other ways, the stock will be sought by investors, and therefore a share of $100 may sell for a larger sum than the par value, perhaps $150 or more. This is because $10 (10% of $100) is $6\frac{2}{3}$% of $150, so that if a man pays $150 for the stock he is still getting $6\frac{2}{3}$% on his investment.

On the other hand, if a company is not prosperous and pays low dividends or none at all, then the stock will be below par.

The market value of stocks varies from day to day and often from minute to minute, depending upon the demand and supply.

136. Kinds of Stock. There are two leading kinds of stock, *preferred* and *common.*

Preferred stock is stock that has a fixed rate of dividend that must be paid before any other dividends can be paid.

The dividend on preferred stock is usually 5% to 7%. Investors who wish good security and a definite rate of income from year to year are inclined to buy preferred stock.

Common stock is stock that is entitled to share in the net earnings of a company after the dividends on the preferred stock have been paid.

If the net earnings of a company are very large, common stock may be worth more than preferred stock. For example, if the preferred stock of a company is entitled to 5%, and if the earnings are such as steadily to pay the common stockholders 10%, the common stock will be worth about twice as much as the preferred.

137. Buying Stock. A purchaser usually buys stock through a *broker,* and the broker obtains it through some other broker who has it for sale. The transaction between the brokers is usually carried on in a place called a *stock exchange.*

A stock exchange is merely a large auction room where brokers having stocks to sell may dispose of them to the highest bidder among the brokers who have orders to buy.

Brokers usually charge *brokerage* of $\frac{1}{8}\%$ for buying and $\frac{1}{8}\%$ for selling stock, computed on the par value. If stock is quoted in the newspaper at $96\frac{1}{2}$, it means that a \$100 share will cost the purchaser $\$96\frac{1}{2} + \$\frac{1}{8}$, or $\$96\frac{5}{8}$. It also means that the man who sells the stock receives $\$96\frac{1}{2} - \$\frac{1}{8}$, or $\$96\frac{3}{8}$, since he also has to pay brokerage.

The only fractions used in stock quotations are halves, quarters, and eighths. Fractional parts of a share are not sold. Stock purchased by speculators is usually in 100-share lots, although investors often purchase smaller amounts.

Stocks form a legitimate and common investment of earnings, in spite of the fact that many persons buy them on credit, hoping to make a profit by some sudden rise in value.

EXERCISE 67

Find the cost of the following stocks, adding $\frac{1}{8}$% brokerage in each case:

1. 100 shares @ $106\frac{3}{8}$.
2. 125 shares @ $112\frac{5}{8}$.
3. 140 shares @ $115\frac{1}{2}$.
4. 275 shares @ $127\frac{1}{4}$.
5. 350 shares @ $142\frac{3}{4}$.
6. 75 shares @ 62.
7. 80 shares @ $75\frac{1}{4}$.
8. 65 shares @ $98\frac{1}{2}$.
9. 125 shares @ $99\frac{1}{8}$.
10. 25 shares @ $127\frac{1}{2}$.

Find the amount received from the sale of the following stocks, deducting $\frac{1}{8}$% brokerage in each case:

11. 75 shares @ 82.
12. 25 shares @ $79\frac{1}{8}$.
13. 50 shares @ $96\frac{7}{8}$.
14. 120 shares @ $98\frac{1}{2}$.
15. 250 shares @ 125.
16. 125 shares @ 110.
17. 150 shares @ $112\frac{1}{2}$.
18. 175 shares @ $121\frac{1}{4}$.
19. 250 shares @ $142\frac{7}{8}$.
20. 275 shares @ par.

21. A man bought 50 shares of stock when it was quoted at $96\frac{7}{8}$, and sold it when quoted at par. Allowing, as usual, $\frac{1}{8}$% brokerage on each transaction, how much did he gain?

22. A man bought 75 shares of stock when the market price was 120, and sold it when the market price was 115. Allowing the usual brokerage, how much did he lose?

23. A man bought 125 shares of stock when the market price was $102\frac{7}{8}$, and sold it when the market price was still the same. Allowing the usual brokerage, how much did he lose on the transaction?

24. A man has $3875 to invest. He decides that he will buy a certain stock quoted at $96\frac{3}{8}$. Allowing the usual brokerage, how many shares can he buy, and how much money will he have left?

138. Newspaper Quotations. In the daily newspapers may be found quotations showing the prices at which leading stocks were sold in the last session of the stock exchange.

The great center for trading in railway stocks is the New York Stock Exchange, on Wall Street. Many large cities have stock exchanges in which local stocks are bought and sold. Stocks of manufacturing and industrial corporations are known as *industrial stocks.*

The following are specimens of newspaper quotations:

American Express Co.	$208\frac{7}{8}$	Iowa Central	$31\frac{5}{8}$
Atchison, Topeka, & S. F.	$101\frac{7}{8}$	Louisville & Nashville	$127\frac{3}{8}$
Baltimore & Ohio	$111\frac{3}{4}$	Missouri Pacific	$68\frac{3}{8}$
Central of N.J.	231	N.Y. Central	$131\frac{1}{8}$
Chicago & Alton	$68\frac{1}{4}$	Northern Pacific	$132\frac{7}{8}$
Illinois Central	$149\frac{7}{8}$	Union Pacific	$184\frac{1}{8}$

In all cases remember to take into account the brokerage of $\frac{1}{8}$%.

EXERCISE 68

Using the above quotations, find the cost of:

1. 150 shares of Iowa Central.
2. 125 shares of Illinois Central.
3. 75 shares of Baltimore & Ohio.
4. 110 shares of Chicago & Alton.
5. 25 shares of American Express Co.
6. 175 shares of Louisville & Nashville.
7. 180 shares of Central of New Jersey.
8. 50 shares of Atchison, Topeka, & Santa Fe.

Find the amount received from the sale of:

9. 25 shares of Missouri Pacific.
10. 35 shares of New York Central.
11. 75 shares of Northern Pacific.

Find the gain or loss in buying 25 shares of the following stocks as quoted on page 113, and selling at the price given below:

12. American Exp., 210.

13. Atchison, $103\frac{1}{2}$.

14. Balt. & Ohio, $109\frac{7}{8}$.

15. Central of N.J., $225\frac{1}{2}$.

16. Chic. & Alton, $68\frac{3}{4}$.

17. Ill. Cent., $151\frac{1}{8}$.

18. Iowa Cent., $33\frac{1}{2}$.

19. Louisv. & N., 129.

20. Missouri Pac., 71.

21. N.Y. Cent., $137\frac{1}{2}$.

22. Northern Pac., 135.

23. Union Pacific, $180\frac{1}{2}$.

24. A man bought 250 shares of American Express at the price quoted on page 113, and sold 100 shares at 207 and the rest at $209\frac{1}{2}$. Did he gain or lose on the lot, and how much?

25. A man bought 250 shares of Baltimore & Ohio at the quotation in Example 14. He kept it a year, receiving meantime a dividend of 6%, and sold it at 112. Money being worth 5%, did he gain or lose, and how much?

26. A man bought 25 shares of New York Central at 138, 125 shares of Iowa Central at 29, and 50 shares of Illinois Central at 150. He sold them at the quotations given on page 113. Did he gain or lose, and how much?

27. A man bought 150 shares of Atchison at $101\frac{7}{8}$. He borrowed the money to pay for it, at 5% interest. He kept the stock 6 months and sold it at the quotation given on page 113, having meantime received a dividend of 3%. Did he gain or lose, and how much?

28. A man bought 75 shares of American Express, 25 shares of Iowa Central, 50 shares of Chicago & Alton, and 150 shares of New York Central, at the quotations given on page 113, and sold them at the quotations given at the top of this page. Did he gain or lose, and how much?

139. Declaring Dividends. If the capital of a company is \$150,000, and the net earnings for six months amount to \$4875, the directors might declare a dividend of $3\frac{1}{4}\%$, since \$4875 is $3\frac{1}{4}\%$ of \$150,000. They will probably, however, declare a dividend not exceeding 3% (\$4500) and carry the balance as undivided profits to the surplus fund to meet an emergency.

EXERCISE 69

1. If the capital is \$125,000, and \$3125 is distributed as dividends, what is the rate of dividend?

2. If the capital is \$75,000, and a dividend of $3\frac{1}{2}\%$ is declared, what is the amount of the dividend?

3. If a company declares a dividend of $4\frac{1}{2}\%$, and this amounts to \$4050, what is the capital?

4. If a company declares a dividend of $5\frac{1}{2}\%$, and this amounts to \$4,152,500, what is the capital?

5. If a company declares a dividend of 4% on its capital stock, how much dividend will a man receive who owns 75 shares?

6. A company having a capital of \$75,000 loses \$1875. If an assessment is made to pay this loss, what per cent of his stock must each stockholder be assessed?

7. The net earnings of a company having a capital of \$150,000 amounted to \$7677.56 in 6 months. The company carried \$927.56 to its surplus fund and divided the remainder in dividends. What was the rate of dividend?

8. A railroad company having a capital of \$178,632,000 earns \$98,369,059 in one year. Its operating expenses are \$75,803,333, and it pays interest on its debt and carries to surplus a total of \$11,847,806, dividing the remainder in dividends. What is the rate of dividend for the year?

140. Rate of Income. If stock paying a 6% dividend is bought at $149\frac{7}{8}$; or 150 including the brokerage, the owner receives \$6 on every \$150 invested. He therefore receives \$6 ÷ \$150, or 4% on his investment.

Always add the brokerage to the market price.

EXERCISE 70

Given the rate of dividend and the market price, find the rate of income on the investment:

1. 5%, $124\frac{7}{8}$.	**7.** $4\frac{1}{2}$%, $112\frac{3}{8}$.	**13.** 5%, $109\frac{7}{8}$.
2. 8%, $199\frac{7}{8}$.	**8.** 8%, $159\frac{7}{8}$.	**14.** 4%, $98\frac{7}{8}$.
3. 7%, $174\frac{7}{8}$.	**9.** 7%, $87\frac{3}{8}$.	**15.** 6%, $120\frac{3}{8}$.
4. 7%, $139\frac{7}{8}$.	**10.** $4\frac{1}{2}$%, $74\frac{7}{8}$.	**16.** 7%, $130\frac{1}{8}$.
5. 9%, $149\frac{7}{8}$.	**11.** 4%, $94\frac{7}{8}$.	**17.** 10%, 190.
6. 10%, $124\frac{7}{8}$.	**12.** $4\frac{1}{2}$%, $82\frac{3}{8}$.	**18.** 15%, $299\frac{7}{8}$.

19. Which will bring the better income, a 5% promissory note or a 6% stock quoted @ $119\frac{7}{8}$?

20. Which will bring the better income, a 5% promissory note or a 7% stock quoted @ 150? How much better?

21. Which will bring the better income, a 5% promissory note or an 8% stock quoted @ $149\frac{7}{8}$? How much better?

22. Which will bring the better income, a 6% stock quoted @ $139\frac{7}{8}$ or a 5% stock quoted @ $119\frac{7}{8}$? How much better?

23. If an investor receives \$6 on each share of stock he owns, and this is 5% on what he paid for the stock, how much did it cost him? What was the quoted market price?

24. If an investor receives 6% on his investment in a stock that pays 9% dividends, how much did the stock cost him? What was the quoted market price?

141. Bonds. A written or printed promise to pay a specified sum at a given time, signed by the maker and often bearing his seal, is called a *bond.*

Bonds are usually secured by *mortgages* on the property of the maker, these being agreements by which the holders of the bonds may sell the property if either the interest or principal is unpaid.

142. Kinds of Bonds. Bonds are called *coupon bonds* if they have small promissory notes (coupons) annexed, which are cut off and presented for payment when the interest is due. They are called *registered bonds* if the owner's name is registered on the books of the company.

143. Illustrative Problems. How much is the income on \$7000 worth of 4% bonds?

The income is 4% of \$7000, or \$280.

Three-year bonds, paying 5%, are quoted at $102\frac{7}{8}$. What rate of income would an investor receive on the cost?

They cost $\$102\frac{7}{8} + \$\frac{1}{8}$, or \$103, for every \$100 face value.

They pay \$5 for every \$100 face value, but at the end of 3 years the investor receives only \$100 instead of \$103, or an average loss of \$1 a year (not counting interest on this amount). Therefore the net income is only \$5 − \$1, or \$4, a year on the \$103 invested.

Then $\$4 \div \$103 = 0.03883+$, the approximate rate of income.

EXERCISE 71

Find the income on the following bonds:

1. 4%, \$9000.
2. 5%, \$7000.
3. 6%, \$6000.
4. 3%, \$8500.
5. 4%, \$6500.
6. $3\frac{1}{2}$%, \$13,500.
7. $4\frac{1}{2}$%, \$17,000.
8. $5\frac{1}{2}$%, \$12,000.
9. $6\frac{1}{2}$%, \$17,000.
10. 6%, due in 2 years, quoted at $101\frac{7}{8}$.
11. 7%, due in 4 years, quoted at $101\frac{7}{8}$.
12. 5%, due in 10 years, quoted at $112\frac{3}{8}$.

EXERCISE 72

Problems without Numbers

1. If you know the newspaper quotation on a certain stock, how do you find the cost of a share?

2. If a certain stock is quoted at a specified amount above par, how do you find the cost of a share?

3. About what would have to be the rate of dividend on a railroad stock to have it at par? to have it below par?

4. If you own some railroad stock and should sell it through a broker and buy it back at the same quoted price, how would you compute your loss?

5. If you own some stock and wish to sell it, and know the market price, how much will you receive for the stock?

6. Knowing the capital and the net earnings of a company, and the amount to be carried to surplus, how do you find the rate of dividend?

7. Knowing the quotation on a certain stock, and the rate of dividend, how do you find the rate of income on the amount invested?

8. How do you find the income on a certain number of bonds of a given par value, the rate being known?

9. How do you find the income on a certain number of bonds of a given par value, the time they have to run and the rate being known?

10. If the rate of interest on some bonds is known, and also the amount of income, how do you find the par value of the bonds?

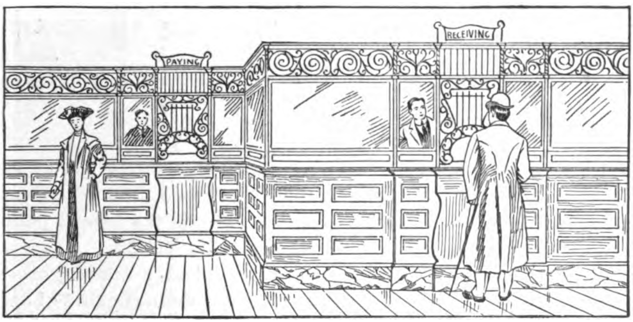
11. What is the difference between preferred stock and common stock? between common stock and a bond? How do you decide which is best for an investment?

CHAPTER VI

BANKING

144. Banks. A corporation authorized by law to receive and loan money and to perform certain other financial operations is called a *bank.*

Formerly there were many private banks, but now banks are usually corporations.



This picture represents the interior of a city bank. The clerk who receives money is called the *receiving teller;* the clerk who pays out money is called the *paying teller.*

145. Savings Banks. The banks where people keep their savings, receiving a low rate of interest, are carefully guarded by the laws of most of the states. These banks are called *savings banks.*

Nearly 9,000,000 persons in the United States have deposits in savings banks. These deposits amount in all to nearly $4,000,000,000.

146. Interest in Savings Banks. Usually savings banks pay interest semiannually, adding this to the principal on deposit. This is, therefore, a case of compound interest. The following is a specimen account at a bank paying 2% every six months (4% a year, payable semiannually), on the smallest balance on deposit during the period.

Date		Deposits		Interest		Payments		Balance	
1910									
July	1	$600	50					$600	50
July	20	75						675	50
Sept.	6					$120		555	50
Dec.	7	60						615	50
Dec.	20					65		550	50
1911									
Jan.	1			$11				561	50
May	9	200						761	50
July	1			11	22			772	72

Here the smallest balance during the first interest period was $550.50. Interest is computed on the dollars only, the cents being neglected. At 2% the interest on $550 is $11. In the second period the smallest balance is $561.50, the interest being $11.22.

Some banks allow interest from the first of each month; others from the first of each quarter; others, as above, from the first of each half year. The interest is computed on the smallest balance on hand between this day and the next interest day, and is usually added to the principal every half year.

In the above example, if the interest period had been 3 months and the interest had been credited every 6 months, then the interest would have been:

First quarter, 1% of $555 = $5.55.
Second quarter, 1% of $550 = 5.50. Credit $11.05.
Third quarter, 1% of $561 = 5.61.
Fourth quarter, 1% of $561 = 5.61. Credit $11.22.

In this case, therefore, the difference in interest would have been only five cents.

EXERCISE 73

In the following, find the balance due on the last date, interest 2% every six months on the smallest balance on deposit during six months preceding Jan. 1 and July 1:

1. Deposits: Jan. 1, 1910, $750; Feb. 3, $425; June 1, $37.50. Payments: Mar. 7, $230; May 6, $26.75. Balance, July 1, 1910.

2. Deposits: Jan. 1, 1910, $675.50; Mar. 2, $923.75; Sept. 5, $327.60. Payments: Feb. 7, $327.40; Oct. 10, $750. Balance, Jan. 1, 1911.

3. Deposits: Dec. 31, 1910, $500; Feb. 2, 1911, $300; Mar. 16, $250; Aug. 1, $50; Oct. 5, $125. Payments: Mar. 6, 1911, $325; May 11, $275; Sept. 12, $25; Nov. 15, $100. Balance, Jan. 1, 1912.

In the following, find the balance due on the last date, interest being 1% every three months on the smallest balance on deposit during the three months preceding Jan. 1, Apr. 1, July 1, and Oct. 1, and all interest being payable on Jan. 1 and July 1.

4. Deposits: Mar. 1, 1910, $200; June 3, $675; July 1, $350. Payments: Mar. 8, $50; May 17, $60; July 8, $50. Balance, Jan. 1, 1911.

5. Deposits: Apr. 1, 1911, $375; May 25, $460; June 14, $380; Aug. 8, $750. Payments: May 3, $75; Sept. 6, $50; Nov. 8, $130; Feb. 1, 1912, $200. Balance, July 1, 1912.

6. A man on July 1 deposited $2500 in a savings bank and left it for 3 years. The rate of interest was 4% a year and the interest was added to the principal semiannually. How much less interest did he receive than he would have received had he loaned the money at 5% simple interest?

147. Commercial Banks. Most banks receive money on deposit and pay it out on written orders, allowing no interest. Such banks are called *commercial banks.*

They are also known as *banks of deposit.* They afford a safe place for keeping money. They make their profit by loaning money. Savings banks are usually allowed to loan only on real estate or certain specified bonds of high grade as security, but commercial banks loan on promissory notes. Commercial banks also transmit money from one place to another, and make collections, as described later.

148. Depositing Money. When a reliable person wishes to open an account with a bank he begins by depositing some money. In depositing money a *deposit slip* is filled out each time, as here shown.

SECOND NATIONAL BANK

SPRINGFIELD, ILL.

Deposited for the account of

J. P. Roberts

Date Feb. 3 1912

Bills	75	
Gold	10	
Silver and small coin	20	
Check on 1st Nat. Bank	126	75
Check on Chicago Nat. Bank	37	60
Total	269	35

The money and deposit slip are then presented to the *receiving teller* or to the *cashier*, together with a bank book in which the teller or cashier enters the amount deposited.

Many banks do not insist on having inserted in a deposit slip the name of the bank on which a check is drawn.

EXERCISE 74

Make out deposit slips for the following deposits, naming some bank in your vicinity:

1. Bills, $478; gold, $20; silver, $12; check on First National Bank, $93.75.

2. Bills, $295; silver, $27; checks on Farmers Bank, $275.50; Second National, $127.62.

3. Bills, $427; gold, $25; silver, $65; checks on Union Trust, $92.50; Manufacturers Bank, $326.75.

4. Bills, $396; gold, $75; silver, etc., $73.75; checks, $72, $40, $127.50, $345.75.

5. Bills, $426; silver, etc., $63.30; checks, $32.75, $16.50, $48.90, $123.45, $63.50.

6. Bills, $275; silver, etc., $50.75; checks, $27.62, $48.35, $62.90, $91.75, $126.30.

7. Bills, $300; silver, etc., $40; checks, $37.75, $52.50, $46.75, $75, $31.90, $43.

8. Bills, $250; silver, etc., $20; checks, $48, $70, $13.62, $19.83, $76.75, $49.32.

9. Bills, $420; gold, $75; silver, etc., $40; checks, $39.50, $42.75, $61.80, $120.35.

10. Bills, $37; checks, $14.75, $35.20, $48, $29.30, $62.75, $81.50, $8.15, $7.80.

11. Checks, $48.50, $16.90, $17.30, $21.25, $48.02, $37.70, $42.72, $30.96, $48.70, $9.20.

12. Checks, $148.62, $7.90, $4.08, $39.70, $62.80, $47, $106.05, $32.75, $81.22, $62.75, $38.40.

13. Bills, $428; gold, $80; silver, etc., $45.75; checks, $272.75, $121.62, $396.80, $92.75, $38.75, $326.42.

149. Drawing Money. When a person opens an account at a bank he is given a *check book.* Each page has one or more *checks* and *stubs.* On the presentation of a check properly filled out, signed, and indorsed the bank will pay the amount specified, if the drawer's account is sufficient, and the holder is properly identified.

Stub	*Check*
No *575*	No. *575* MADISON, WIS., *Feb. 1*, 19*12*
Date *Feb. 1, 1912*	FIRST NATIONAL BANK OF MADISON
To *B. H. Roberts*	Pay to the order of
For *Bal. acct.*	*B. H. Roberts* $*75.75*
Amt. $*75.75*	*Seventy-five and 75/100* ~~~~ Dollars
	Rufus J. Perkins

In this case Rufus J. Perkins is the *drawer* and B. H. Roberts is the *payee.* The stub is kept by the drawer as a memorandum.

A check is usually payable to "self", in which case only the drawer can collect it; or to the order of the payee, in which case he must write his name across the back (*indorse* it); or to the payee or bearer, in which case any one can collect it; or to "cash", in which case any one can collect it. Of these forms, the one to the order of the payee, as above shown, is the most common.

In the above case, if B. H. Roberts draws the money he writes simply his name across the back. This is called a *blank indorsement,* or *indorsement in blank.* If he wishes to use it in paying some other person, for instance, Jacob C. Sturgess, he would write:

Pay to the order of
Jacob C. Sturgess
B. H. Roberts

This is called a *full indorsement,* and only Jacob C. Sturgess could collect it, unless he should indorse it in favor of some one else.

EXERCISE 75

1. Make out a check for $75.60, payable to the order of J. P. Flint. Indorse the check in blank.

2. Make out a check for $62.75, payable to the order of M. G. Thompson. Write the stub. Indorse the check to the order of R. P. Hull.

3. If a man had a balance of $273.75 at the beginning of last month, and has since given checks for $15.63, $28.96, $14.60, $5.75, $12.30, and $38.09, and made no deposits, what is his balance now?

4. If a merchant had a balance of $496.75 on the first of last month, and has since deposited $39.80, $126.70, and $98.90, and given checks for $72.83, $48.65, and $271.96, what is his balance now?

5. If a merchant had a balance of $592.80 on the fifteenth of last month, and has since deposited $98.60, $48.90, $5.70, $19.68, $72.75, $37.63, and $100, and has given checks for $38.40, $125.75, $30.42, and $75, what is his balance now?

6. A merchant's balance on the first of last week was $575.72. He has since deposited $162.75, $380.96, $401.75, $25.60, $75, and $129.80. He has given checks for $272.70, $92.35, $178.62, $325.25, and $68.70. What is his balance now?

7. A merchant's balance on the first of last week was $298.72. He has since deposited $42.80, $121.62, $148.90, $275.26, $72.50, $69.75, $14.02, and $9.70. He has given checks for $38.72, $129.40, $42.60, $25.80, $206.07, and $42.96. He has also paid by check a bill of $82.50 less 10% and 5%, and one of $48.75 less 3%. What is his balance now?

150. Borrowing from Banks. Banks make a business of lending money on short terms to reliable borrowers, taking promissory notes as evidence of the debt. Such a note is made more secure in one of three ways:

1. It is made payable to some responsible person who *indorses* it; that is, he signs his name across the *back*. This makes him responsible to the bank if the maker does not pay it, and if he is promptly notified.
2. It is signed by two or more makers, and is made payable to the bank. Such notes usually read, "We jointly and severally promise to pay," so that the bank may hold all the makers together responsible, or any one of them separately.
3. It is made payable to the order of the bank and is secured by the deposit of stocks, bonds, or other securities. These are called *collateral securities.*

151. Form of a Bank Note. The first of the above forms is as follows:

INDIANAPOLIS, IND., May 15, 1913

Sixty days after date, I promise to pay to the order of

P. M. Stockwell $2000.00

Two Thousand Dollars

at The First National Bank, Indianapolis, Ind.

Value received John Wilson

Due July 14, 1913

This note is due, or it *matures*, on July 14. Formerly three *days of grace* were allowed on a note, so that the above note would mature on July 17, but this custom has been abolished in most states and is not considered in this book.

Bank notes are usually made for 30, 60, or 90 days.

When the time of a note is expressed in months, calendar months are used in computing the date of maturity; when the time is expressed in days, the exact number of days is used.

152. Bank Discount. The note given in § 151 does not mention any interest. This is because banks generally require the interest to be paid in advance.

Interest paid on a note in advance is called *bank discount,* and such a note is said to be *discounted.*

The rate of discount is understood to be by the year, as in the case of interest, unless there is a statement to the contrary.

153. Proceeds. The balance after deducting the discount from the face of a note is called the *proceeds.*

For example, what are the discount and the proceeds on the note given in § 151, the rate being 6%?

Face of note	$2000
Discount on $2000, 60 da., @ 6% .	20
Proceeds	$1980

EXERCISE 76

Find the discount and the proceeds on the following:

1. $250, 30 days, 6%.
2. $575, 30 days, 6%.
3. $625, 60 days, 6%.
4. $1250, 60 days, 5%.
5. $2250, 60 days, 5%.
6. $175, 30 days, 5%.
7. $1275, 60 days, $5\frac{1}{2}$%.
8. $2350, 60 days, $5\frac{1}{2}$%.
9. $275.50, 90 days, 6%.
10. $527.50, 90 days, 6%.

11. Make out a 60-day note for $1250, dated to-day, payable to R. H. Dodge, or order, at some bank in your vicinity. Discount it at 6% and find the proceeds.

12. Make out a 90-day note for $725, dated to-day, payable to P. M. Milton, or order, at some bank in your vicinity. Discount it at 6% and find the proceeds.

13. A cattle dealer needs $1700 for 30 days. He puts up collateral at a bank and borrows this amount. How much discount must he pay, the rate being 5%?

154. Discounting Interest-bearing Notes. It frequently happens that a manufacturer or dealer holds notes of his customers and wishes to get the money on them before they are due. In this case he may indorse the notes and sell them to a bank for the sum due at maturity, less the discount.

For example, a manufacturer has a note of one of his customers for $350, dated October 15 and due 3 months after date, with interest at 5%. He discounts this note at a bank at 6% on Nov. 16. What are the proceeds?

Face of the note	$350.00
Interest for 3 mo., 5%	4.38
Amount due at maturity . . .	$354.38
Discount, 60 da., 6%	3.54
Proceeds	$350.84

Here the time from November 16 to January 15, 60 days, is the *term of discount.*

Bankers compute the discount for the exact number of days in the term of discount. It should be noticed that *interest is computed on the face of the note at the beginning of the term*, *discount on the amount of the note at maturity.*

EXERCISE 77

Find the discount and the proceeds:

	Face	*Date*	*Due*	*Interest*	*Date of Discount*	*Rate of Discount*
1.	$250	Jan. 1	July 1	5%	Jan. 15	6%
2.	$375	Oct. 7	Jan. 7	5%	Oct. 7	6%
3.	$425	Oct. 18	Feb. 18	6%	Oct. 18	6%
4.	$550	Sept. 20	Jan. 20	5%	Nov. 1	6%
5.	$675	Nov. 16	May 16	6%	Dec. 1	5%
6.	$1250	Aug. 7	Feb. 7	5%	Nov. 15	5%
7.	$375.50	May 16	Sept. 16	6%	July 1	6%
8.	$827.25	July 1	Feb. 1	6%	Oct. 1	6%

9. A 60-day note for $1225, without interest, dated June 20, was discounted the day of its date at 6%. Find the discount.

10. A 60-day note for $1225, with interest at 5%, dated June 20, was discounted the day of its date at 6%. Find the discount.

11. A 60-day note for $1225, with interest at 6%, dated June 20, was discounted the day of its date at 5%. Find the discount.

12. A 60-day note for $1225, with interest at 6%, dated June 20, was discounted on July 20 at 6%. Find the discount.

13. A 30-day note for $2500, without interest, dated April 1, was discounted 15 days later at 6%. Find the discount and the proceeds.

14. A 90-day note for $1650, with interest at 5%, is discounted 30 days after date at 6%. Find the proceeds.

15. A 90-day note for $1650, with interest at 6%, is discounted 30 days after date at 5%. Find the proceeds.

16. A 90-day note for $1650, with interest at 6%, is discounted the day of its date at 6%. Find the proceeds.

17. A 90-day note for $1650, without interest, is discounted the day of its date at 6%. Find the proceeds.

18. How much greater are the proceeds on a note for $675 due 30 days hence, than on one for the same amount due 90 days hence, there being no interest in either case, and the rate of discount being 6% ?

19. On which can a man realize the more at a bank where the discount rate is 6%, a 6-months note for $2550 with interest at 6%, or a 4-months note for $2600 without interest? How much more?

155. Drafts. A written order by which one person directs another to pay a certain sum of money is called a *draft.*

A check is therefore a kind of draft. Usually the word *check* is used for an order on a bank, signed by a private individual; *draft* is used for an order other than a check, on one private individual or corporation signed by another. A draft may therefore be made by an individual on a company, by one bank on another, by a company on a private individual or firm, and so on.

156. Commercial Drafts. A draft made by one business house on another to secure the payment of a debt is called a *commercial draft.*

The following is a common form of commercial draft:

CLEVELAND, OHIO, *Aug. 8,* 19*12*

At *thirty* days sight pay to the order of

myself $*975.$ $\frac{00}{}$

Nine Hundred Seventy-five and $\frac{no}{100}$ Dollars

To *Robert J. Clements*
Louisville, Ky. *J. M. Simons*

Accepted Aug. 10, 1912 Robert J. Clements

In this case J. M. Simons has sold Robert J. Clements $975 worth of goods on 30 days' credit. When he sends the bill he also sends this draft. Mr. Clements writes "Accepted, Aug. 10, 1912, Robert J. Clements" across the face, and returns it to Mr. Simons. This becomes thereby a promise to pay $975 "at 30 days sight," that is, on Sept. 9, or 30 days after the acceptance on Aug. 10. Mr. Simons may now discount this at a bank, just as he would a promissory note.

This is coming to be a common way of borrowing money at a bank, replacing the older method by a promissory note. A general name for such evidences of debt as can be sold to banks is *negotiable paper.*

In this draft Mr. Simons is the *drawer*, Mr. Clements the *drawee*, and as the draft is payable to him, Mr. Simons is also the *payee.*

EXERCISE 78

1. A draft dated Aug. 8, at 30 days sight, for \$975, is accepted on Aug. 10, and is discounted on Aug. 16 at 6%. What are the proceeds?

2. A draft dated Sept. 1, at 30 days sight, for \$350, is accepted on Sept. 4, and is discounted on Sept. 7 at 6%. What are the proceeds?

3. A draft dated April 2, at 60 days sight, for \$725, is accepted on April 7, and is discounted on April 12 at 6%. What are the proceeds?

4. A draft dated Oct. 15, at 60 days sight, for \$1225.75, is accepted on Oct. 19, and is discounted on Oct. 25 at 6%. What are the proceeds?

5. A draft dated Feb. 7, at 90 days sight, for \$1675.50, is accepted on Feb. 14, and is discounted on Feb. 22 at 5%. What are the proceeds?

6. A draft dated Aug. 6, at 90 days sight, for \$2170.60, is accepted and discounted on the day of its date. The rate of discount being $5\frac{1}{2}$%, find the proceeds.

7. A manufacturer sells goods listed at \$725.50, discounted at 10% and 5%, and draws on the buyer at 60 days sight. The buyer accepts the draft and it is discounted 10 days later at 6%. Write the draft and find the proceeds.

8. A manufacturer sells goods listed at \$1672.25, discounted at 8% and 3%, and draws on the buyer at 60 days sight. The buyer accepts the draft and it is discounted 5 days later at 5%. Write the draft and find the proceeds.

9. A wholesale house sells goods listed at \$2737.50, discounted at 15%, 8%, and 3%, and draws on the buyer at 90 days sight. The buyer accepts the draft and it is discounted 30 days later at 5%. Write the draft and find the proceeds.

157. Finding the Face. If it is desired to borrow exactly $1000 on a 60-day note that is to be discounted at a bank at 6%, the face of the note must evidently be such a sum as discounted will leave $1000 as the proceeds.

First consider the face of the note as	$1.00
Then the discount, @ 6% for 60 da., is . . .	0.01
Therefore the proceeds are	$0.99

Hence $1000 must be the proceeds of as many times $1 as $1000 ÷ $0.99, or $1010.10.

That is, the face of the note must be $1010.10.

EXERCISE 79

1. What must be the face of a 60-day note in order that when discounted the day of its date at 6% the proceeds shall be $497.50?

2. What must be the face of a 90-day note in order that when discounted the day of its date at 5% the proceeds shall be $1777.50?

3. What must be the face of a 30-day note in order that when discounted the day of its date at 6% the proceeds shall be $500?

4. A manufacturer sold some goods for $1950 less 10% discount. He took the purchaser's note for 60 days for an amount such that when it was discounted at 6% the proceeds just paid the bill. What was the face of the note?

5. A manufacturer sold some goods for $2400 less 10% and 5%. He took the purchaser's note for 30 days for an amount such that when it was discounted at 6% the proceeds just paid the bill. What was the face of the note?

6. A wholesale dealer sold some goods for $3200 less 10% and 3%, due in 90 days. He took the purchaser's note for an amount such that when it was discounted at 6% the proceeds just paid the bill. What was the face of the note?

EXERCISE 80

Review Problems

1. Find the amount of $500, compounded semiannually, at the rate of 4% a year, for 5 years.

2. A man deposits $750 in a savings bank on July 1, 1910. His further deposits are as follows: Aug. 1, 1910, $300; Sept. 15, 1910, $250; Dec. 31, 1910, $1000; June 15, 1911, $500. He draws out as follows: Oct. 12, 1910, $400; June 20, 1911, $375; Aug. 2, 1911, $200. The savings bank pays 1% every three months on the smallest balance for the preceding quarter, on Jan. 1, Apr. 1, July 1, and Oct. 1. Find the balance on Jan. 1, 1912.

3. Make out a deposit slip for the following deposits, naming some bank in your vicinity: bills, $575; gold, $35; silver, $72.25; checks, $48.25, $72.50, $61.30, $227.52, and $86.95. If your balance was $326.42 before, what was it after the deposit?

4. A merchant's balance on the first of the month was $1725.60. He has since deposited $37.50, $296.75, $21.30, $572.32, and $217.65. He has given checks for $128.92, $27.36, $18.92, $78.76, and $273.96. How much is his present balance?

5. Make out a 90-day note for $1250, dated to-day, payable to R. P. Foster, or order, at some bank in your vicinity. Discount it at 6%. If with the proceeds you buy 10 shares of stock when quoted at $108\frac{3}{8}$, how much money will you have left?

6. Which is the greater, the proceeds of a note for $750 for one year, drawing interest at 6%, and discounted on the day of its date at 5%, or the amount of a note for $700 for one year at 6%? How much greater?

EXERCISE 81

Problems without Numbers

1. How is the interest due on the first of the year on a savings-bank account ascertained?

2. Which pays the better interest, if the money is left undisturbed for five years, a 4% promissory note or a savings-bank deposit at 4%? Why?

3. How do you fill out a deposit slip? After entering the items what operation do you perform? How do you make sure of the result?

4. If you know a man's balance in a bank a week ago and his deposits and checks since, how do you proceed to find his balance now?

5. How do you find the discount on a promissory note? How do you find the proceeds?

6. If a note drawing a certain rate of interest is discounted on the day it is made, at the same rate, are the proceeds greater than the face, or equal to it, or less? Why is this?

7. How can a manufacturer discount a claim against a purchaser, the claim not being yet due? How is the discount found?

8. If you know the proceeds and rate of discount, how do you find the face of a note?

9. If you know the face of a note and the proceeds, how do you find the discount? the rate of discount?

10. If a dealer buys some produce and agrees to pay for it in 90 days, with interest at 6%, and the seller takes a note and at once discounts it at 5%, will he receive more than the selling price, or the same amount, or less? Why?

CHAPTER VII

EXCHANGE

158. Paying Bills at a Distance. If a man owes money at a distance, he may pay it in any one of several ways:

(1) He may send or carry the money.

(2) He may send a money order.

(3) He may send his check.

(4) He may send a bank draft.

(5) He may wait until the one whom he owes draws upon him, as mentioned in § 156.

159. Exchange. The payment of money by means of checks, money orders, or drafts is called *exchange.*

160. Paying by Money Order. A man who owes money at a distance may purchase a postal money order at a post office, an express money order at an express office, or a telegraphic money order at a telegraph office.

The cost of postal money orders is as follows: For sums not exceeding $2.50, 3 cents; over $2.50 to $5, 5 cents; over $5 to $10, 8 cents; over $10 to $20, 10 cents; over $20 to $30, 12 cents; over $30 to $40, 15 cents; over $40 to $50, 18 cents; over $50 to $60, 20 cents; over $60 to $75, 25 cents; over $75 to $100, 30 cents. Express money orders cost about the same. Postal money order rates to foreign countries vary with the country and the amount, — from 8 cents for sums not exceeding $10 to some countries, to $1 for sums over $90, but not exceeding $100, to other countries.

The rate for telegraphic money orders is double the cost of a ten-word message, plus 25 cents if the sum does not exceed $25, and plus 1% of the amount of the order for larger sums.

161. Paying by Check. Usually local bills are paid by check. A check sent to pay a bill at a distance is deposited by the payee in the bank where he keeps his account, and this bank sends it to the debtor's bank for collection, often making a slight charge for the trouble.

In some cities the banks agree to charge for collecting all out-of-town checks. This costs the payee of the check from ten cents upward, according to the amount involved.

A bank may guarantee that a check is good. This is done by the cashier writing the word "Certified" across the face, with the date and his signature.

EXERCISE 82

Find the cost of postal money orders for:

1. $1.75.	4. $9.92.	7. $29.88.	10. $47.50.
2. $2.75.	5. $19.20.	8. $31.75.	11. $65.90.
3. $7.50.	6. $25.50.	9. $39.95.	12. $96.75.

13. A man owing $250 sent his check in payment. The creditor deposited the check and the bank charged him 0.1% for collecting. What was the net amount received by the creditor?

14. A manufacturer received checks for $1200, $650, $2250, and $1750. His bank charged him 0.1% for collecting. How much in all did he pay for the collecting of the four checks?

15. A man wishes to send a check to a friend for an amount such that after the bank has deducted 0.1% for collecting there will remain exactly $3246.75. What shall he make the face of the check?

16. A man deposited a check for $1500 in a bank, and the net proceeds after the bank had deducted its charge for collecting was $1497. What was the rate that the bank charged for collecting?

162. Bank Drafts. A check drawn by one bank on another is called a *bank draft*, or simply a *draft.*

Banks usually charge from 0.1% to $\frac{1}{4}$% on the face of a draft to pay for their trouble and expense.

No. *2796*

FIRST NATIONAL BANK OF SALEM

SALEM, OREGON, *April 16,* 19*12*

Pay to the order of ....*Robert Stevens*............$*95.*$\frac{50}{}$

Ninety-five and $\frac{50}{100}$ ~~~~~~~~~~~~~~~~~~~~~~ Dollars

To the Second National Bank,
New York City

D. B. Roberts
Cashier

If Robert Stevens of Salem, Oregon, owed Tiffany & Co. of New York $95.50, he might purchase a draft like the above and indorse it:

Pay to the order of Tiffany & Co., New York
Robert Stevens

He would then send the draft to Tiffany & Co., who would indorse it and deposit it in their bank for collection.

If Robert Stevens had an account at the First National Bank of Salem, he might not be charged any exchange. Otherwise he would probably be charged about 0.1% *premium*, or ten cents in this case.

Sometimes the premiums on drafts are quoted at a number of cents per thousand dollars.

EXERCISE 83

Find the cost of drafts for the following amounts at the premiums stated:

1. $2500, 0.1%.
2. $3750, 0.1%.
3. $4850, 0.1%.
4. $4800, 0.2%.
5. $5600, 0.2%.
6. $250, $\frac{1}{8}$%.
7. $750, $\frac{1}{8}$%.
8. $640, $\frac{1}{4}$%.
9. $1280, $\frac{1}{4}$%.

163. Commercial Drafts. The fifth method mentioned in § 158 is that in which a commercial draft is used. These drafts have already been described (§ 156) and their use in connection with borrowing money has been explained.

For purposes of exchange, if J. M. Simons of Cleveland has sold to Robert J. Clements of Louisville $975 worth of goods on 30 days' credit, he may *draw upon* Mr. Clements by such a draft as that described in § 156. Instead, however, of sending the draft by mail and having it accepted, he may deposit it at once in his bank at Cleveland. This bank will then send it to some bank in Louisville, and that bank will present it to Mr. Clements for acceptance. If accepted it will hold the draft for 30 days, and then collect it and remit the money to Cleveland, charging the Cleveland bank a slight amount for collection, which will be charged finally to Mr. Simons.

EXERCISE 84

1. R. J. Lewis draws on J. P. Greene for $1750. The bank charges 0.1% for collection. What are the proceeds?

2. P. F. Brooks draws on M. R. Cook for $2150. The bank charges $\frac{1}{8}$% for collection. What are the proceeds?

3. M. J. Lange draws on F. P. Dow for $3600. The bank charges $\frac{1}{8}$% for collection. What are the proceeds? Write the draft.

4. F. P. Devoe collects a debt of $1950 against R. J. Stone by drawing upon him for the amount. The proceeds are $1948.05. What is the bank's rate for collection?

5. M. M. Miller draws upon R. D. St. Clair for $2150 and the bank charges $4.30 for making the collection. What rate does the bank charge? Write the draft.

6. R. R. Stewart sells F. M. Nixon 6 dressers @ $15.50, 4 bedsteads @ $12.50, and 4 doz. chairs @ $30 a dozen, all less 10% and 5%. He draws upon him for the amount and the bank charges 25¢ for collecting. What are the proceeds?

164. Variation in Rates. A postal or express money order always costs more than its face, the extra cost constituting a fixed rate of exchange for various amounts. This is not so with drafts, however, for they often can be bought at their face value, and occasionally even below the face value.

If the cost of a draft is more than the face, exchange is said to be *at a premium*; if the cost equals the face, exchange is said to be *at par*; if the cost is less than the face, exchange is said to be *at a discount.*

For small sums, say less than $500, drafts on New York, Chicago, and other commercial centers usually sell at a premium of about 0.1%, and they are usually cashed at par in any place for a customer of a bank; that is, a depositor receives their face value anywhere.

For large sums, like $500,000, the premium is about what it would cost to send the money by express, unless the bank that draws the draft already has such an amount in the bank on which it draws. For example, if a company in St. Louis wishes a draft on New York for $500,000, and if New York banks owe St. Louis banks a large amount just at that time, then the St. Louis bank might be glad to give such a draft, and might even give it for $100 less than the face in order to save the expense and loss of time in shipping the money that New York was just then owing it.

EXERCISE 85

Find the cost of drafts for the following amounts, the premiums per $1000 being given:

1. $27,500, $1.80.
2. $36,500, $2.20.
3. $37,250, $2.40.
4. $48,125, $1.60.
5. $26,500, $1.75.
6. $125,000, $1.85.

Find the cost of drafts for the following amounts at the discounts stated:

7. $48,250, $\frac{1}{10}$%.
8. $75,000, $\frac{1}{8}$%.
9. $64,800, $\frac{1}{4}$%.
10. $327,500, $\frac{1}{20}$%.
11. $775,250, $\frac{1}{20}$%.
12. $1,750,275, $\frac{1}{10}$%.

EXERCISE 86

REVIEW PROBLEMS

Find the cost of the following drafts:

1. $1250, at par.
2. $1750, 0.2% premium.
3. $2760, 0.1% premium.
4. $4500, ¼% discount.
5. $2700, 0.2% discount.
6. $25,750, 0.2% premium.

7. A draft for $27,500 was bought for $27,486.25. Was exchange at a premium or a discount? Find the rate.

8. The Lion Construction Co. of Bloomington, Ind., draws at sight on R. B. James of Memphis, Tenn., for $1760.75 through the First National Bank of Bloomington. Write the draft and find the charge for collection at 0.1%, to the nearest 5 cents.

9. The Acme Manufacturing Co. of Guthrie, Okla., draws at 30 days sight on J. B. Martin of Kansas City, Mo., for $275.60 through the First National Bank of Guthrie. Write the draft and acceptance and find the charge for collection at 0.1%, to the nearest 5 cents.

10. Roberts Bros. of Louisville, Ky., sell to M. R. Freeman of Ashland, Ky., goods to the value of $1257.60, subject to discounts of 10% and 6% if paid within 10 days. Mr. Freeman pays at once by check. Write the check and find the cost of collection at 0.1%, to the nearest 5 cents.

11. Ashland & Co. of Mobile, Ala., sell to F. P. Prentice of Bessemer, Ala., goods to the value of $1425.50, subject to a discount of 6%. They draw for this amount on June 3, at 90 days sight. The draft is accepted on June 5 and discounted at 6% on June 7. If the cost of the collection is 0.1%, to the nearest 5 cents, find the net proceeds realized by Ashland & Co.

165. Foreign Exchange. Exchange carried on with foreign countries is called *foreign exchange.*

166. English Money. The table is as follows:

12 pence (d.) = 1 shilling (s. or /) = $0.243 +
20 shillings = 1 pound (£) = $4.8665

We usually think of the penny as about 2¢, the shilling as about 25¢, and the pound as about $5.

167. French Money. The table is as follows:

100 centimes (c.) = 1 franc (fr.) = $0.193

We usually think of the franc as about 20¢, and the 5-franc piece as about $1. This system is used in several countries.

168. German Money. The table is as follows:

100 pfennigs (pf.) = 1 mark (M.) = $0.238

We usually think of 4 pf. as 1¢, and 1 M. as about 25¢.

EXERCISE 87

Express as pence:

1. £7. **2.** £24. **3.** £5 6s. **4.** £7 2s. 4d.

Express as shillings:

5. £27. **6.** £32. **7.** 120d. **8.** 720d.

Express as pounds and shillings:

9. 95s. **10.** 72s. **11.** 127s. **12.** 342s.

Express as francs and decimals:

13. 750 c. **14.** 275 c. **15.** 680 c. **16.** 1250 c.

Express as marks and decimals:

17. 450 pf. **18.** 275 pf. **19.** 1680 pf. **20.** 9275 pf.

169. Bills of Exchange. Foreign drafts are often called *bills of exchange.*

Bills of exchange, like domestic drafts, may be above par (when exchange is at a premium), at par, or below par (when exchange is at a discount).

English exchange, often called Sterling exchange, is quoted at dollars to the pound (4.88 meaning that $4.88 will buy a bill of exchange for £1). If English exchange is above 4.8665, exchange is at a premium; if below, at a discount.

French exchange is quoted either at francs to a dollar (5.15 meaning that $1 will buy a bill of exchange for 5.15 fr.), or at cents to the franc (19.7 meaning that 19.7¢ will buy a draft for 1 fr.).

German exchange is quoted either at cents to 4 marks (98 meaning that a bill of exchange for 4 M. will cost 98¢), or at cents to the mark (24.3 meaning that 24.3¢ will buy a draft for 1 M.).

When necessary to distinguish it from foreign exchange, exchange that is carried on within our own country is called *domestic exchange.*

170. Illustrative Problems. 1. How much will a bill of exchange for £75 6s. cost @ 4.90?

$$\pounds 75\ 6\text{s.} = \pounds 75\tfrac{6}{20} = \pounds 75.3.$$

Since £1 costs $4.90,

therefore £75.3 cost $75.3 \times \$4.90$, or $368.97.

2. How much will a bill of exchange for 26,000 fr. cost @ 5.20?

Since 5.20 fr. cost $1,

therefore 1 fr. costs $\dfrac{\$1}{5.20}$,

and 26,000 fr. cost $26000 \times \dfrac{\$1}{5.20}$, or $5000.

3. How much will a bill of exchange for 1600 M. cost @ $94\frac{1}{4}$?

Since 4 M. cost $\$0.94\frac{1}{4}$,

therefore 1 M. costs $\frac{1}{4}$ of $\$0.94\frac{1}{4}$,

and 1600 M. cost $1600 \times \frac{1}{4}$ of $\$0.94\frac{1}{4}$, or $377.

EXERCISE 88

Find the cost of the following bills of exchange:

1. £7 @ 4.89.
2. £9 @ 4.90.
3. £12 @ 4.87.
4. £25 @ 4.86.
5. £47 @ 4.85.
6. £78 @ 4.92.
7. £2 6s. @ 4.90.
8. £3 8s. @ 4.88.
9. £7 4s. @ 4.86.
10. £12 15s. @ 4.85.
11. 350 fr. @ 5.20.
12. 480 fr. @ 5.23.
13. 575 fr. @ 19.4.
14. 680 fr. @ 19.6.
15. 2975 fr. @ 19.2.
16. 4270 M. @ 98.
17. 2680 M. @ 97.
18. 3275 M. @ 24.1.
19. 4680 M. @ 23.9.
20. 5975 M. @ 23.6.

21. How large a draft on London can be bought for $3359.75 when exchange is 4.91?

22. How large a draft on Paris can be bought for $38.80, when exchange is 19.4?

23. How large a draft on Berlin can be bought for $160.70, when exchange is 95.2?

24. A merchant buys a shipment of Sheffield cutlery amounting to £48 6s. What will a bill of exchange for this amount cost @ 4.91?

25. An importer buys a lot of Lyons silk to the value of 27,375 fr. What will a bill of exchange for this amount cost @ 5.20?

26. A Chicago dealer imports some Nürnberg toys to the value of 2650 M. What do they cost him, exclusive of freight and duty, exchange being 96?

27. Which is better for a man who wishes to pay 1000 fr. in Paris, to buy his bill of exchange from a dealer at 5.20, or from one at 19.2? How much better?

EXERCISE 89

Problems without Numbers

1. What method will you take to pay a debt at a distance? Why? Is it fair to the one you owe?

2. If you purchase a bank draft when exchange is at a premium, how will you find the cost of the draft?

3. If you purchase a bank draft when exchange is at a discount, how will you find the cost of the draft?

4. If a man in another place owes you, state all the steps to be taken in drawing on him for the amount.

5. How will you find the actual value in English money of a sum in our money?

6. How will you find the actual value in our money of a sum in English money?

7. How will you find the actual value in French money of a sum in our money?

8. How will you find the actual value in our money of a sum in German money?

9. How will you find the cost of a draft for a certain number of pounds, when the rate of exchange is given?

10. How will you find the cost of a draft for a certain number of francs, when the rate of exchange is given?

11. How will you find the cost of a draft for a certain number of marks, when the rate of exchange is given?

12. How will you find the face of a draft on London, given the cost and the rate of exchange?

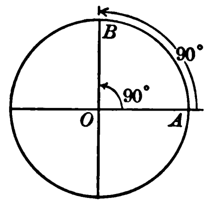
13. How will you find the face of a draft on Paris, given the cost and the quotation in cents?

14. How will you find the face of a draft on Munich, given the cost and the number of cents to a mark?

CHAPTER VIII

PRACTICAL MEASUREMENTS

171. Terms Used. Most of the names used in measuring are already familiar to the class. Some of the more important are repeated here, and a few new names are added.

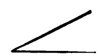


172. Angles. When two straight lines meet they are said to form an *angle*.

An angle of 90° is called a *right angle*.

In the figure the angle AOB is a right angle. When two lines form a right angle they are said to be *perpendicular* to each other.

An angle smaller than a right angle is called an *acute angle*.



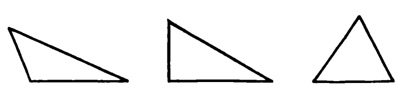
Acute angle

An angle greater than a right angle is called an *obtuse angle*.

Obtuse angle

173. Triangles. A plane figure bounded by three straight lines is called a *triangle*.

A triangle that contains an obtuse angle is called an *obtuse triangle;* a triangle that contains a right angle is called a *right triangle;* a triangle that contains only acute angles is called an *acute triangle*.

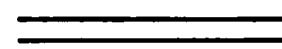


Obtuse Right Acute

The side on which a triangle rests is called the *base*, and the opposite corner is called the *vertex*.

The perpendicular distance from the vertex to the line of the base is called the *altitude* or *height* of the triangle.

174. Parallel Lines. Lines that are everywhere equally distant from each other are called *parallel lines.*



Parallel lines

175. Parallelogram. A plane figure bounded by two pairs of parallel lines is called a *parallelogram.*

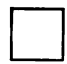
The *base* of a parallelogram is the side on which it rests. The *altitude* is the perpendicular distance from the opposite side to the base. The opposite parallel sides of a parallelogram are equal.

Parallelogram

176. Rectangle. A parallelogram whose angles are right angles is called a *rectangle.*

Rectangle

177. Square. A rectangle whose sides are all equal is called a *square.*



Square

178. Polygon. A plane figure bounded by straight lines is called a *polygon.*

If the sides are all equal and the angles are all equal, the polygon is called a *regular polygon.*

179. Perimeter. The distance around a polygon is called the *perimeter.*

EXERCISE 90

1. What is the perimeter of a triangle that is 59 yd. 15 in. on each side?

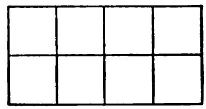
2. What is the perimeter of a parallelogram two of whose sides are respectively 37 ft. 11 in. and 16 ft. 8 in.?

3. What is the perimeter of a triangle whose sides are 37 ft. 7 in., 54 ft. 10 in., and 49 ft. 8 in.?

4. What is the cost of fencing a rectangular field 17 rd. long and 15 rd. wide if the fence costs $67\frac{1}{2}$¢ a yard?

5. How many feet of molding will be needed to go around the four sides of a room 17 ft. 3 in. long and 15 ft. 9 in. wide?

180. Measurement of a Rectangle. If this figure represents a rectangle 4 in. long by 2 in. wide, there are evidently 4 sq. in. in the lower row of squares, and since there are 2 rows, there are 2×4 sq. in., or 8 sq. in., in the rectangle.



If we know the area and one dimension, we may find the other dimension; for, since 8 sq. in. $= 2 \times 4$ sq. in., then 8 sq. in. $\div$ 4 sq. in. $= 2$, the number of units in the other dimension.

Express the length and breadth of a rectangle in the same linear unit; the product of these two numbers will express the area in square units of the same name as the linear unit.

The number of square units in a rectangle divided by the number of linear units in one dimension gives the number of linear units in the other dimension.

This means that the numbers are to be considered as abstract. The complete solutions with concrete numbers are given above.

The expression $4'' \times 7''$ means and is read "4 inches *by* 7 inches."

EXERCISE 91

Find the areas of rectangles with these bases and altitudes:

1. 237 ft., 56 ft.
2. 253 in., 45 in.
3. 375 yd., 92 yd.
4. 348 in., 19 ft.
5. 7 ft. 3 in., 4 ft.
6. 9 ft. 3 in., 7 in.
7. 4 ft. 6 in., 2 ft. 4 in.
8. 12 yd. 2 ft., 7 ft. 6 in.

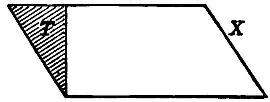
9. How many square rods in a rectangular field 36 rd. long and $17\frac{1}{2}$ rd. wide?

10. How many acres in a rectangular field 224 rd. long and $68\frac{3}{4}$ rd. wide?

11. A rectangular city lot is $229\frac{1}{2}$ ft. long and 45 ft. 4 in. wide. Find the number of square feet in the lot.

12. A rectangular field containing 8.1 A. is 72 rd. long. How wide is it?

181. Measurement of a Parallelogram. If we cut off the triangle T and place it at X, the parallelogram will be changed to a rectangle of the same base and the same altitude.



Therefore, *the area of a parallelogram is equal to the product of the numbers expressing the base and altitude.*

The base and the altitude must be expressed in the same units.

Given the area and the altitude we may find the base, as in the case of a rectangle, and similarly for the area and base to find the altitude.

For example, if the base of a parallelogram is 3 ft. 8 in. and the altitude 1 yd., what is the area?

$$3 \text{ ft. } 8 \text{ in.} = 3\tfrac{2}{3} \text{ ft.}; \; 1 \text{ yd.} = 3 \text{ ft.}$$
$$3 \times 3\tfrac{2}{3} \text{ sq. ft.} = 11 \text{ sq. ft.}$$

If the area of a parallelogram is 15 sq. ft. and the base is 5 ft., required the altitude.

15 sq. ft. ÷ 5 sq. ft. = 3, the number of feet of altitude (§ 180).

EXERCISE 92

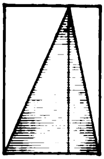
Find the areas of parallelograms with the following bases and altitudes:

1. 259 ft., 97 ft.
2. $42\frac{1}{2}$ ft., $25\frac{1}{2}$ ft.
3. 43 ft., $1\frac{1}{2}$ rd.
4. 37 ft., 9 yd.
5. $45\frac{1}{2}$ ft., 125 in.
6. 248 rd., 200 rd.
7. 38 ft. 8 in., 4 ft. 3 in.
8. 49 ft. 4 in., 9 ft. 9 in.
9. 56 yd. 27 in., 9 yd. 18 in.
10. 72 yd. 16 in., 47 yd. 9 in.

Find the altitudes of parallelograms with the following areas and bases:

11. 1944 sq. ft., 72 ft.
12. 2340 sq. ft., 90 ft.
13. 3456 sq. ft., 36 yd.
14. 80 A., 200 rd.

182. Measurement of a Triangle. We see that this triangle may be cut into two parts, each of which is half of some rectangle, and that therefore the whole triangle is half of a rectangle of the same base and altitude.



Whatever the shape of the triangle, it may, in a similar way, be shown to be half of a parallelogram of the same base and altitude, and therefore (§ 181) equal to half of a rectangle of the same base and altitude.

Therefore, *the area of a triangle equals half the product of the numbers representing the base and altitude.*

EXERCISE 93

Find the areas of triangles with the following bases and altitudes:

1. 67 ft., 38 ft.
2. 85 ft., 35 ft.
3. $72\frac{1}{2}$ ft., 55 ft.
4. 178 rd., 35 rd.
5. 7 ft. 9 in., 27 in.
6. 4 yd. 18 in., 3 ft.
7. 9 ft. 6 in., 4 ft. 8 in.
8. 9 ft. 9 in., 5 ft. 8 in.
9. 14 ft. 8 in., 9 ft. 4 in.
10. 5 yd. 27 in., 3 ft. 4 in.

Find the altitudes of triangles with the following areas and bases:

11. 184 sq. ft., 46 ft.
12. 960 sq. ft., 96 ft.
13. 1188 sq. ft., 108 ft.
14. 2106 sq. in., 78 in.

15. At $6.75 a square foot, what is the value of a triangular city lot with base 96 ft. and altitude 75 ft. 8 in.?

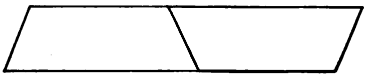
16. What is the area of a triangle with sides all equal, whose perimeter is 36 in. and altitude 10.39 in.?

17. A triangular gable to a house has a base 45 ft. and an altitude 15 ft. 8 in. Find the area of the gable.

183. A Trapezoid. A four-sided plane figure with two parallel sides is called a *trapezoid.*

Trapezoid

184. Area of a Trapezoid. In this figure two equal trapezoids have been placed together, one turned over so that the two make a parallelogram which equals a rectangle whose base is the sum of the two parallel sides.



Therefore, *the area of a trapezoid equals half that of a rectangle with the same altitude and with a base equal to the sum of the two parallel sides.*

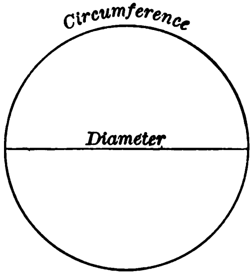
For example, the area of a trapezoid, with altitude 6 in. and parallel sides 10 in. and 15 in., is $\frac{1}{2}$ of $6 \times (15 + 10)$ sq. in., or 75 sq. in.

EXERCISE 94

Find the areas of trapezoids whose altitudes are first given, followed by the two parallel sides:

1. 6″, 9″ and 12″.
2. 8″, 7″ and 13″.
3. 9″, 8″ and 12″.
4. 9″, 7″ and 14″.
5. 8″, 6″ and 13″.
6. 7″, 6″ and 13″.
7. 13″, 19″ and 25″.
8. 14″, 18″ and 23″.
9. 19″, 25″ and 34″.
10. 25″, 37″ and 45″.
11. 29″, 38″ and 46″.
12. 44″, 43″ and 51″.
13. If the area of a trapezoid is 34,500 sq. ft., and the parallel sides are 500 ft. and 650 ft., what is the altitude?
14. If the area of a trapezoid is 44,000 sq. ft., the altitude 160 ft., and one of the parallel sides 450 ft., what is the other parallel side?
15. What is the area of a trapezoid one of whose parallel sides is 172 ft., the altitude being the same length and the other parallel side being twice as long?

185. Circumference of a Circle. If we measure around and across any circular object, we shall find that the circumference is about $3\frac{1}{7}$ times the diameter. It is shown in geometry that 3.1416 is more nearly correct than $3\frac{1}{7}$.



It is sufficient for most ordinary purposes to use $3\frac{1}{7}$ instead of 3.1416.

Thus, if the diameter of a wheel is 7 ft. the circumference is $3\frac{1}{7} \times 7$ ft., or 22 ft. If we use the multiplier 3.1416, the result is 3.1416×7 ft., or 21.9912 ft.

EXERCISE 95

Using $3\frac{1}{7}$, find the circumference, given the diameter:

1. 224 ft.	**3.** 36.4 ft.	**5.** 0.371 in.	**7.** 4340 in.
2. 21.7 ft.	**4.** 2.24 ft.	**6.** 4235 in.	**8.** 52.50 in.

Using $3\frac{1}{7}$, find the circumference, given the radius:

9. 21 in.	**11.** 63 in.	**13.** 63.7 in.	**15.** 81.9 ft.
10. 35 in.	**12.** 280 in.	**14.** 64.4 in.	**16.** 8.47 ft.

Using 3.1416, find the circumference, given the diameter:

17. 25 ft.	**19.** 36 yd.	**21.** 0.63 ft.	**23.** 4.25 ft.
18. 44 ft.	**20.** 4.8 yd.	**22.** 0.79 ft.	**24.** 3.27 in.

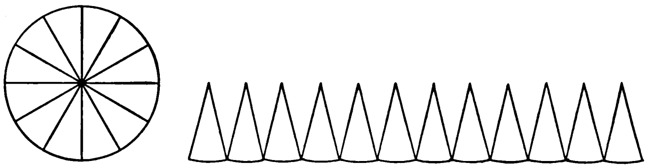
Using 3.1416, find the circumference, given the radius:

25. 28 in.	**27.** 67 in.	**29.** 2.9 yd.	**31.** 0.58 ft.
26. 48 in.	**28.** 4.9 ft.	**30.** 4.8 yd.	**32.** 0.72 ft.

Using 3.1416, find the diameter, given the circumference:

33. 62.832 ft. **34.** 345.576 in. **35.** 502.656 in.

186. Area of a Circle. We may cut a circle as here shown and separate it into a series of figures that are triangles except for their curved bases. If we should call these figures triangles, their altitudes would be the radius of the



circle, and the sum of their bases would be the circumference. The area of each triangle (§ 182) equals half the product of the numbers representing the base and altitude. Therefore their total area equals half the product of the numbers representing the circumference and radius.

In geometry this is proved exactly, but this explanation is enough for our purpose.

Thus, if the radius is 2 in. and the circumference (§ 185) is 12.5664 in., the area is $\frac{1}{2}$ of 2×12.5664 sq. in., or 12.5664 sq. in.

The area of a circle equals half the product of the numbers representing the circumference and radius.

187. Area in Terms of Radius. We have found that

$$\text{area} = \tfrac{1}{2} \times \text{radius} \times \text{circumference (§ 186)}$$

and $$\text{circumference} = 3.1416 \times 2 \times \text{radius (§ 185)};$$

therefore $$\text{area} = \tfrac{1}{2} \times \text{radius} \times 3.1416 \times 2 \times \text{radius},$$

or, canceling, $$\text{area} = 3.1416 \times \text{square of radius}.$$

Therefore, *the area of a circle equals 3.1416 times the square of the radius.*

For example, if the radius is 2 in., the area is 3.1416×4 sq. in., or 12.5664 sq. in.

In Exercise 96 use 3.1416 instead of $3\frac{1}{7}$ (§ 185).

EXERCISE 96

Find the areas of circles with the following radii and circumferences:

1. 3 in., 18.8496 in.	**5.** $7\frac{1}{2}$ in., 47.124 in.
2. 4 in., 25.1328 in.	**6.** 10 in., 62.832 in.
3. $4\frac{1}{2}$ in., 28.2744 in.	**7.** 20 in., 125.664 in.
4. $3\frac{1}{2}$ in., 21.9912 in.	**8.** 30 in., 188.496 in.

Find the areas of circles with the following radii:

9. 1 in.	**12.** 5 in.	**15.** $\frac{1}{2}$ in.	**18.** $\frac{1}{4}$ in.	**21.** $3\frac{1}{2}$ in.
10. 4 in.	**13.** 12 in.	**16.** $\frac{1}{3}$ in.	**19.** $\frac{3}{4}$ in.	**22.** $5\frac{1}{4}$ in.
11. 3 in.	**14.** 25 in.	**17.** 0.1 in.	**20.** $\frac{2}{3}$ in.	**23.** 4.3 in.

24. What is the circumference of a water pipe that is 5 in. in diameter?

25. What is the area of a cross section of a water pipe that is 4 in. in diameter?

26. A water pipe is 6 in. in external diameter. If the iron is $\frac{1}{4}$ in. thick, what is the internal circumference?

27. A school globe is 16 in. in diameter. How long is the equator on the globe?

28. The equator on a school globe is 62.832 in. What is the radius of the globe?

29. What is the circumference of the largest circle that can be drawn in a square 15 ft. on a side?

30. What is the area inclosed between the circle and the sides of the square in Ex. 29?

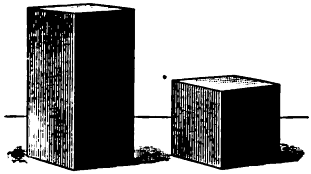
31. What is the diameter of a circular plot of ground of which the circumference is 691.152 ft.? What is the area?

32. A pipe is 4.75 in. in external diameter. If the iron is $\frac{1}{4}$ in. thick, what is the area of an internal cross section?

188. Rectangular Solid. A solid bounded by six rectangles is called a *rectangular solid.*

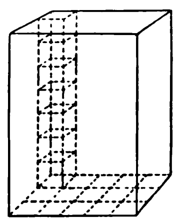
The six rectangles are called the *faces* of the solid, and their sides form the *edges* of the solid.

A rectangular solid has length, breadth, and thickness, and these are called the three *dimensions* of the solid.



189. Cube. A rectangular solid whose faces are squares is called a *cube.*

190. Volume of a Rectangular Solid. If this figure represents a rectangular solid 5 in. long, 3 in. wide, and 7 in. high, it is evident that in the column of cubes shown there are 7 cu. in. It is also evident that on the base we can place 3×5 such columns. Hence the volume is $3 \times 5 \times 7$ cu. in., or 105 cu. in. Hence, if the dimensions are all expressed in the same unit,



The number of units of volume in a rectangular solid equals the product of the numbers representing the three dimensions.

EXERCISE 97

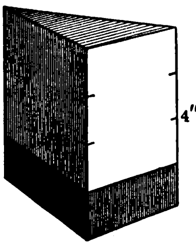
Find the volumes of rectangular solids having the following dimensions:

1. 27″, 29″, 42″.
2. 26″, 32″, 37″.
3. 25″, 26″, 44″.
4. 27″, 29″, 39″.
5. 25′, 39′, 22′.
6. 227′, 16′, $21\frac{1}{2}$′.
7. 224 ft., 26 ft., 9 ft. 9 in.
8. 235 ft., 36 ft., 8 ft. 2 in.
9. 265 ft., 24 ft., 18 ft. 10 in.
10. 7 ft. 6 in., 4 ft., 5 ft. 8 in.
11. 8 ft. 2 in., 3 ft., 5 ft. 9 in.
12. 6 ft. 4 in., 9 ft., 4 ft. 2 in.

191. Prism. A solid bounded by two equal and parallel polygons and by parallelograms is called a *prism.*

The two equal and parallel polygons are called the *bases* of the prism, and the distance between them is called the *altitude.*

192. Volume of the Prism. In this figure it is evident that upon every square inch of the base there can be placed 1 cu. in., reaching 1 in. high. Therefore, if the prism is 4 in. high, the volume is 4 times the number of cubic inches in the shaded part.



Therefore, *the volume of a prism in cubic units equals the number of square units in the base, multiplied by the number of units in the altitude.*

193. Illustrative Problem. Find the volume of a prism with base 8 sq. in. and altitude 3 in.

3×8 cu. in. = 24 cu. in.

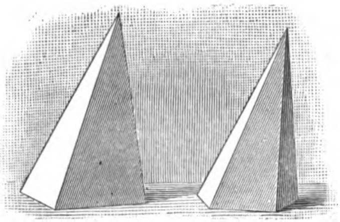
EXERCISE 98

Find the volumes of prisms with the following bases and altitudes:

1. 27 sq. in., 8 in.
2. 38 sq. in., $6\frac{1}{2}$ in.
3. 48 sq. in., $7\frac{3}{8}$ in.
4. 21.6 sq. in., 2.7 in.
5. 42.8 sq. in., 8.9 in.
6. 62.75 sq. in., 12.25 in.
7. 49.9 sq. in., 17.36 in.
8. 57.8 sq. in., 19.35 in.
9. 46.7 sq. in., 27.32 in.
10. 58.3 sq. in., 36.27 in.
11. 28.6 sq. in., 17.5 in.
12. 32.9 sq. in., 18.3 in.
13. 62.7 sq. in., 27.4 in.
14. 43.6 sq. in., 19.8 in.
15. 27.9 sq. in., 16.2 in.
16. 35.4 sq. in., 22.7 in.
17. 73.2 sq. in., 35.8 in.
18. 75.3 sq. in., 27.8 in.
19. 48.9 sq. in., 19.7 in.
20. 56.7 sq. in., 23.4 in.

194. Pyramid. A solid bounded by a polygon and by triangles that all meet in a point is called a *pyramid.*

The polygon is called the *base*, the triangles taken together are called the *lateral surface*, the point in which they meet is called the *vertex* of the pyramid, and the distance from the vertex to the base is called the *altitude.*

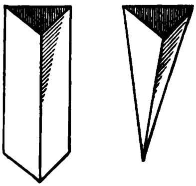


Pyramids

195. Volume of a Pyramid. *The volume of a pyramid equals one third the product of its base and altitude.*

The base and altitude are to be understood as abstract numbers, as in § 192.

The truth of the statement may be seen by taking a hollow pyramid and a hollow prism of the same base and altitude. The amount of water or sand that the pyramid will contain is exactly one third of the amount the prism will contain.



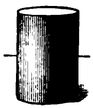
EXERCISE 99

Find the volumes of pyramids with bases and altitudes as follows:

1. 32 sq. in., 9 in.
2. 48 sq. in., 13 in.
3. 64 sq. in., 15 in.
4. 96 sq. in., 22 in.
5. 132 sq. in., 26 in.
6. $8\frac{1}{4}$ sq. in., $2\frac{1}{2}$ in.
7. $9\frac{3}{8}$ sq. in., $3\frac{5}{8}$ in.
8. 8.7 sq. in., 3.1 in.
9. 12.3 sq. in., 4.7 in.
10. 15.75 sq. in., 8.1 in.

11. The Great Pyramid of Egypt has a square base 764 ft. on a side. It is 480 ft. 9 in. high. Assuming its faces smooth, what is its volume?

196. A Cylinder. A solid formed by the revolution of a rectangle about one of its sides is called a *cylinder.*



The two circles that are thus formed are called the *bases*, and the curved surface is called the *lateral surface* of the cylinder.

The distance between the two bases is called the *altitude* of the cylinder.

197. Volume of a Cylinder. We can think of 1 cu. in. on each square inch of the base. Therefore, if the base contains 4 sq. in., and if the cylinder is 1 in. high, the volume will be 4×1 cu. in., or 4 cu. in., and if the cylinder is 5 in. high, the volume will be 5×4 cu. in., or 20 cu. in.

Therefore, to find the volume of a cylinder,

Multiply the number of cubic units corresponding to the number of square units of the base by the number of units of altitude.

That is, if the base has 9 sq. in., and the altitude is 12 in., the volume is 12×9 cu. in., or 108 cu. in.

If the base has a diameter of 2 in., and the altitude is 5 in., then the area of the base is 3.1416×1 sq. in. (§ 187), and the volume is $5 \times 3.1416 \times 1$ cu. in., or 15.708 cu. in.

198. Lateral Area of a Cylinder. We may think of the surface of a cylinder as unrolled, like a paper reaching once around a pencil. In this case it would unfold into a rectangle, one side of which would be the circumference of the base and the other the altitude.

Therefore, to find the lateral area of a cylinder,

Multiply the number of units in the circumference by the number of units in the altitude.

For example, if the circumference is 6 in. and the altitude 4 in., the number of square units of lateral area is 4×6, or 24. We may express the work thus:

$$4 \times 6 \text{ sq. in.} = 24 \text{ sq. in.}$$

EXERCISE 100

Find the volumes of cylinders whose altitudes and areas of base are as follows:

1. 23 in., 37 sq. in.	**4.** 4 ft. 7 in., $9\frac{1}{2}$ sq. in.
2. 27 in., 46 sq. in.	**5.** 4 ft. 3 in., $8\frac{1}{2}$ sq. in.
3. 48 in., 175 sq. in.	**6.** 8 ft. 3 in., 5 sq. ft. 24 sq. in.

Find the altitudes of cylinders whose volumes and areas of base are as follows:

7. 20 cu. ft. 324 cu. in., 8 sq. ft. 72 sq. in.

8. 190 cu. ft. 1305 cu. in., 30 sq. ft. 75 sq. in.

Find the areas of base of cylinders whose volumes and altitudes are as follows:

9. 324 cu. ft., 18 ft.	**11.** 28 cu. ft. 288 cu. in., 2 ft. 2 in.
10. 323 cu. ft., 17 ft.	**12.** 81 cu. ft. 576 cu. in., 5 ft. 4 in.

Find the volumes of cylinders whose altitudes and radii of base are as follows:

13. 27 in., 4 in.	**15.** 32 in., 8 in.	**17.** 2 ft. 6 in., 2 in.
14. 25 in., 6 in.	**16.** 36 in., 9 in.	**18.** 3 ft. 4 in., 1 ft.

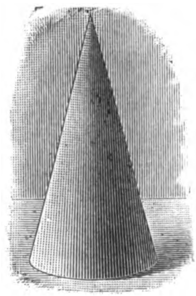
Using $7\frac{1}{2}$ gal. for 1 cu. ft., find the number of gallons in cylindrical tanks of the following diameter and depth of water respectively:

19. 6′, 10′.	**21.** 16′, 14′.	**23.** $48\frac{1}{2}$′, 56′.
20. 15′, 12′.	**22.** 35′, 40′.	**24.** $48\frac{3}{4}$′, $45\frac{1}{2}$′.

Find the number of gallons of water in wells of the following diameter and depth of water respectively:

25. 3 ft., 8 ft.	**27.** 4 ft., 8 ft.	**29.** $3\frac{3}{4}$ ft., $5\frac{1}{4}$ ft.
26. 3 ft., 6 ft.	**28.** $3\frac{1}{2}$ ft., 6 ft.	**30.** $3\frac{3}{4}$ ft., $8\frac{3}{4}$ ft.

199. Cone. A solid bounded by a circle and a curved surface ending in a point is called a *cone*.



Cone

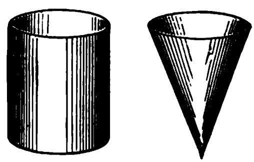
The circle is called the *base*, the curved surface is called the *lateral surface*, the point in which the lateral surface ends is called the *vertex*, and the distance from the vertex to the base is called the *altitude* of the cone.

In the cones here treated the vertex is directly over the center of the base.

200. Volume of a Cone. *The volume of a cone equals one third the product of its base and altitude.*

For if made hollow, as in § 195, the contents of the cone will fill exactly one third of the cylinder of the same base and altitude.

201. Illustrative Problem. Find the volume of a cone whose base has a radius of 2 in., and whose altitude is 3 in.



Area of base $= 3.1416 \times 2^2$ sq. in.
Volume $= \frac{1}{3} \times 3 \times 3.1416 \times 2^2$ cu. in. $= 12.5664$ cu. in.

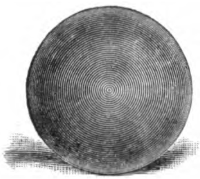
EXERCISE 101

Find the volumes of cones with bases and altitudes as follows:

1. 6 sq. in., 4 in.
2. 15 sq. in., 7 in.
3. 20 sq. in., 9 in.
4. 34 sq. in., 15 in.
5. 68 sq. in., 21 in.
6. $6\frac{3}{4}$ sq. in., $2\frac{1}{2}$ in.
7. $9\frac{7}{8}$ sq. in., $3\frac{3}{4}$ in.
8. $15\frac{3}{4}$ sq. in., $5\frac{3}{8}$ in.
9. 22.7 sq. in., 8.4 in.
10. 35.75 sq. in., 12.3 in.

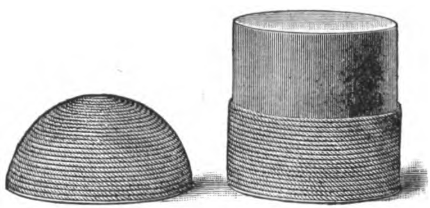
11. What is the volume of a cone whose base is a circle with a radius of 3 in., and whose altitude is 8 in.?

202. Sphere. A solid bounded by a curved surface, all points of which are equally distant from a point within, is called a *sphere*.



The point within from which all points on the surface are equally distant is called the *center*. A straight line from the center to the surface is called a *radius*. A straight line through the center and terminated at each end by the surface is called a *diameter*.

203. Surface of a Sphere. If we wind half of the surface of a sphere with a cord or tape as here shown, and then wind with exactly the same length of the cord or tape the surface of a cylinder having just the same radius, the height of the cylinder equaling the diameter, we find that the cord covers half the curved surface of the cylinder.



Therefore, *the surface of a sphere equals the curved surface of a cylinder of the same radius and height.*

The surface of a sphere also equals 3.1416 times the square of the diameter.

For it is easily seen that the curved surface of a cylinder equals the height times the circumference of the base, and since the height here equals the diameter, therefore

$$\begin{aligned}\text{height} \times \text{circumference} &= \text{diameter} \times 3.1416 \times \text{diameter}\\ &= 3.1416 \times \text{the square of the diameter.}\end{aligned}$$

For practical purposes it is usually sufficient to take $3\frac{1}{7}$ instead of 3.1416 as the multiplier.

If we have given the surface of a sphere, we may find the diameter by dividing the surface by 3.1416, or $3\frac{1}{7}$, and extracting the square root of the quotient.

204. Illustrative Problem. What is the surface of a sphere with a radius of 3 in.?

$$\begin{aligned}\text{Diameter} &= 2 \times 3 \text{ in.} = 6 \text{ in.}\\ \text{Area} &= 3.1416 \times 6^2 \text{ sq. in.}\\ &= 3.1416 \times 36 \text{ sq. in.}\\ &= 113.0976 \text{ sq. in.}\end{aligned}$$

EXERCISE 102

Using $3\frac{1}{7}$ as the multiplier, find the areas of the surfaces of spheres with the following diameters:

1. 14 in.	**4.** 2.1 in.	**7.** $\frac{7}{8}$ in.	**10.** 224 in.
2. 35 in.	**5.** 2.8 in.	**8.** 0.7 in.	**11.** 371 in.
3. 42 in.	**6.** 4.9 in.	**9.** 0.56 in.	**12.** 434 in.

Find the same, using 3.1416 as the multiplier:

13. 7 in.	**15.** 9 in.	**17.** $7\frac{1}{2}$ in.	**19.** 2.2 in.
14. 6 in.	**16.** 4 in.	**18.** $6\frac{1}{4}$ in.	**20.** 3.5 in.

Find the same, using 3.1416, the following being the radii:

21. 1 in.	**23.** $\frac{1}{2}$ in.	**25.** 7 in.	**27.** 2.5 in.
22. 5 in.	**24.** 3 ft.	**26.** 9 in.	**28.** 3.7 in.

29. If the earth is considered a sphere of 4000 mi. radius, what is its surface? (Use $3\frac{1}{7}$.)

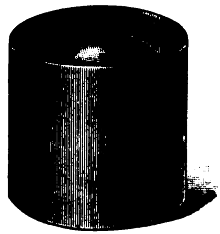
30. How many square inches of flannel will it take to cover a tennis ball 3 in. in diameter? (Use $3\frac{1}{7}$.)

31. If the sun is considered a sphere of 866,500 mi. diameter, what is its surface? (Use $3\frac{1}{7}$.)

32. How many square inches in the surface of a ball 44 in. in diameter? (Use $3\frac{1}{7}$.)

33. What is the diameter of a sphere whose surface is 154 sq. in.? (Use $3\frac{1}{7}$.)

205. Volume of a Sphere. If a sphere is placed in a cylinder of the same diameter and with an altitude equal to the diameter, and the vacant spaces are filled with sand, the sand will be found to fill exactly one third of the cylinder after the sphere is removed.



Therefore, *a sphere is equal to two thirds of a cylinder of the same diameter and altitude.*

The volume of a sphere is $\frac{4}{3} \times$ *3.1416* $\times$ *the cube of the radius.*

For the volume of the cylinder is the product of the altitude and the base, considered as abstract numbers.

The altitude	$= 2 \times$ radius,
and the base	$= 3.1416 \times$ square of radius.
Hence volume of cylinder	$= 2 \times 3.1416 \times$ cube of radius,
and volume of sphere	$= \frac{2}{3}$ of $2 \times 3.1416 \times$ cube of radius
	$= \frac{4}{3} \times 3.1416 \times$ cube of radius.

206. Illustrative Problem. Find the volume of a sphere with a radius of 3 in.

$$\frac{4}{3} \times 3.1416 \times 3^3 \text{ cu. in.} = \frac{4 \times 3.1416 \times 3 \times 3 \times \cancel{3}}{\cancel{3}} \text{ cu. in.}$$
$$= 113.0976 \text{ cu. in.}$$

EXERCISE 103

Find the volumes of spheres with radii as follows:

1. 2 in.	4. 15 in.	7. 2.4 ft.	10. $2\frac{1}{2}$ in.
2. 5 in.	5. 23 in.	8. 4.1 ft.	11. $3\frac{3}{4}$ in.
3. 7 in.	6. 42 in.	9. 3.5 ft.	12. $4\frac{2}{3}$ in.

13. Find the weight of a lead ball 2 in. in diameter, lead being 11.35 times as heavy as water, and a cubic foot of water weighing 1000 oz.

EXERCISE 104

Review Problems

1. Compute the surface of a dome that is half of a sphere, the radius being 20 ft.

2. Find the volume of a sphere 6 in. in diameter and of one 12 in. in diameter.

3. If the moon is considered a sphere of radius 1080 mi., what is its volume?

4. A cornucopia has a circular top $2\frac{7}{8}$ in. across. It is $9\frac{1}{4}$ in. deep. How many cubic inches does it contain?

5. A ball 8 in. in diameter is placed in a cubical box 8 in. on an edge, inside measure. How many cubic inches of the box is not occupied by the ball?

6. A steel rod 4 ft. 8 in. long has a triangular cross section 2 sq. in. in area. If a cubic foot of water weighs 62.5 lb., and steel is 7.83 times as heavy as water, how much does the rod weigh?

7. A funnel in shape of a cone is closed at the bottom. If the circular top is 3 in. across, and the funnel is 4 in. deep (to the vertex), how many cubic inches of water will it hold?

8. A prism has a base of 9.4248 sq. in. and an altitude of 15 in. How does its volume compare with that of a cone whose base has a radius 3 in., and whose altitude is 15 in.?

9. Find the volume of a sphere 1 ft. in diameter, of a cylinder of the same diameter and altitude, and of a cone of the same diameter and altitude. The volume of the sphere is how many times the volume of the cone? The volume of the cylinder is how many times the volume of the cone?

EXERCISE 105

Problems without Numbers

1. Given the base and altitude of a prism, how will you find the volume?

2. Given the volume and base of a prism, how will you find the altitude?

3. Given the base and altitude of a pyramid, how will you find the volume?

4. Given the base and altitude of a cone, how will you find the volume?

5. Given the volume and altitude of a cone, how will you find the base?

6. Given the diameter of a sphere, how will you find the surface?

7. Given the radius of a sphere, how will you find the surface?

8. Given the radius of a sphere, how will you find the volume?

9. Given the diameter of a sphere, how will you find the volume?

10. Given the edge of a cube, how will you find the volume? how will you find the surface?

11. Given the radius of a sphere, how will you find the circumference?

12. Given the length of the earth's equator, how will you find the radius of the earth? Then how will you find the surface of the earth? the volume of the earth?

13. Given the area of the entire surface of a cube, how will you find the area of one face? the length of an edge? the volume of the cube?

CHAPTER IX

VOCATIONAL PROBLEMS

207. Nature of the Problems. You are now nearing the end of your work in arithmetic, and it is necessary to consider some of the miscellaneous problems of business, — the problems that we may all meet in the practical American life of to-day.

EXERCISE 106

Problems of the Merchant

1. Supply the proper numbers in the spaces marked with an asterisk (*) in the following bill:

Houston, Texas, *March 7,* **19***13*

Mr. Robert J. Spencer, Tyler, Texas

Bought of Henderson Bros.

Terms: *6% 30 da., net 60 da.*

March 7	*50*	*gro. Pens*	@ *78¢*	*$39*	
	15	*doz. Notebooks*	@ *$1.70*	*	
	4	*doz. Rulers*	@ *1.32*	*	
	30	*gro. Lead Pencils*	@ *3.35*	*	
				*	
		Less 6%		*	*

Write bills similar to that of Ex. 1, inserting places, dates, and names, and completing each item:

2. 50 M envelopes @ \$2.10, 3 doz. fountain pens @ \$19.50, 8 doz. bottles ink @ 34¢, 5 gross erasers @ \$3.38. Terms: 8% 30 da., net 60 da.

3. 75 bbl. flour @ \$5.23, 63 gal. molasses @ 39¢, 75 gal. fancy sirup @ 58¢, 3 doz. cartons flour @ \$2.64, 260 lb. coffee @ 9.65¢. Discounted @ 6%.

4. 8 doz. hatchets @ \$10.75, 6 doz. pr. hinges @ \$4.35, 5 doz. carpenter's squares @ \$34.50, $\frac{1}{2}$ gross locks @ \$42.50, 8 doz. files @ \$6.25. Discounted @ 40%, 10%, and 5%.

5. 575 yd. cotton flannel @ $38\frac{1}{4}$¢, 684 yd. cotton @ $7\frac{1}{3}$¢, 364 yd. buckram @ $9\frac{1}{2}$¢, 648 yd. sheeting @ $26\frac{3}{4}$¢, 327 yd. Java canvas @ $62\frac{1}{2}$¢. Discounted @ 12%.

Make out a bill in each of the following examples, and write out a sight draft upon the buyer for the amount. Then find the charge for collection at 0.1%, to the nearest 5 cents.

6. L. P. Hall purchased from Myers Bros. 84 gross bone buttons @ $18\frac{1}{2}$¢, 72 doz. pearl buttons @ $9\frac{3}{4}$¢, 694 yd. cambric @ $17\frac{1}{2}$¢, 364 yd. cashmere @ $82\frac{1}{4}$¢, 408 yd. chambray @ $21\frac{3}{4}$¢. Discounted @ 12%.

7. F. J. Peabody purchased from Thompson & Co. 4 gross toothbrushes @ \$9.35, 3 doz. atomizers @ \$19.25, 8 doz. combs @ \$1.95, 4 doz. brushes @ \$18.37, $\frac{1}{2}$ gross nail brushes @ \$27.50. Discounted @ 15% and 5%.

8. M. S. Robbins purchased from the Seacord Carpet Co. 480 yd. matting @ $18\frac{1}{4}$¢, 375 yd. matting @ $19\frac{3}{8}$¢, 284 yd. carpet @ $48\frac{1}{2}$¢, 8 rugs @ \$7.33, 4 doz. door mats @ \$4.75. Discounted at 3% and 1%.

EXERCISE 107

PROBLEMS OF THE MANUFACTURER

1. How much will a manufacturer receive for 6 rockers @ \$6.75, 8 dressers @ \$12.50, and 12 bedsteads @ \$15.75, discounted at 8%?

2. A manufacturer sells the following bill of goods: 4 doz. plates @ \$1.38, 8 doz. cups @ \$2.27, ½ doz. covered dishes @ \$9.50, 12 doz. plates @ \$1.45, ¼ doz. pitchers @ \$6.40. The bill is discounted at 3% and he gains 20% on the goods. Required the cost of manufacture.

3. Iron expands $\frac{3}{16}$ in. to the foot when heated to a red heat. How much longer will a casting be when heated to a red heat, if it is 3 ft. 7 in. long before it is heated?

4. A manufacturing company has a capital of \$275,000, its annual business is 3¼ times its capital, and its annual profits are 7½% of its annual business. How much are its annual profits?

5. A manufacturer sells brass lamps to a dealer at such a price that the dealer can sell them at \$6.60 and make a profit of 32%. The manufacturer himself makes a profit of 25%. What is the cost of manufacture per lamp?

6. On which will a manufacturer make the greater per cent of profit, — an article which it costs \$2.90 to make, and which he sells for \$3.33⅓ less 3%, or one costing \$8.70 to make, which he sells for \$12 less ⅙? How much greater?

7. A manufacturer sold to a retail dealer the following bill of goods: 9 dining sets @ \$62.50, 16 rockers @ \$5.35, 8 sideboards @ \$32.50, 6 card tables @ \$8.75, 4 china closets @ \$17.50. He discounted this bill at 8% and 2%. How much did he receive for the goods?

8. In casting brass hinges the brass shrinks in cooling 1.5625% of its length when melted. How long must be the mold for a hinge that is to be $5\frac{7}{8}$ in. long?

9. The wooden pattern from which an iron casting is made weighs $6\frac{1}{4}$% as much as the iron. If the pattern weighs $32\frac{1}{2}$ lb., how much does the casting weigh?

10. A manufacturer has a water tank that is 8 ft. 3 in. wide, 12 ft. 6 in. long, and 9 ft. 8 in. deep. If a cubic foot of water weighs $62\frac{1}{2}$ lb., what is the weight of the water required to fill the tank?

11. A manufacturer pays a salesman a salary of $1250 a year and a commission of $1\frac{1}{2}$% on all sales, and allows him $1000 a year for expenses. If the salesman sells $172,500 worth of goods, what does the manufacturer pay him, including salary, commissions, and expenses?

12. An iron founder knows that an iron casting will weigh 17 times as much as the wooden pattern. If he has 5 patterns for the parts of a given piece that he wishes to cast, and these weigh 6 lb. 8 oz., 7 lb. 4 oz., 18 lb. 12 oz., 9 lb. 8 oz., and 12 lb. 4 oz., what will the casting weigh?

13. A manufacturer is building a chimney with a circular flue for his boiler room. He has two horizontal boilers, each containing 96 tubes 2 in. in diameter. His chimney must have the same cross-section area as the sum of all the cross-section areas of the tubes of the boilers. Required the diameter of the flue.

14. In a large dyeing establishment there is a tank 8 ft. long, 7 ft. wide, and 6 ft. deep. If the pressure on the end, when the tank is full of water, is equal to half the weight of the water in a rectangular tank whose base is the area of the end of the tank and whose height is the depth of the tank, how much is this pressure?

15. The following is the pay roll of a manufacturer for a week :

PAY ROLL *For the week ending* * 19 *

No.	Name	No. of Hours per Day						Total Time	Wages per Hour	Total Wages	
		M.	T.	W.	T.	F.	S.				
1	J. P. Driscoll	9	9	8	8½	9	9	52½	25¢	$13	12
2	R. L. Bennett	9	8½	9	9	8	9	*	25¢	*	*
3	P. F. Purvis	9	8¼	8	8½	9	9	*	30¢	*	*
4	B. J. Mead	8	9	9	9	8½	8	*	33⅓¢	*	*
5	R. K. King	8	8	9	9	9	9	*	35¢	*	*
6	M. L. Drake	9	9	8	0	9	9	*	27½¢	*	*
		*	*	*	*	*	*	*		*	*

Fill out the spaces marked with an asterisk (*).

Make out pay rolls (inserting names), the numbers, the hours per day, and wages per hour being as follows:

16. No. 1 : 9, 9, 9, 8, 8½, 9, 30¢ ; No. 2 : 8, 8, 8, 9, 9, 8, 27½¢ ; No. 3: 8, 8½, 9, 9, 9, 9, 28¼¢ ; No. 4: 8, 8, 8, 8, 8½, 8, 33⅓¢ ; No. 5 : 8, 9, 8, 8, 8, 8½, 32¢.

17. No. 1 : 9, 9, 9, 9, 9, 9, 37½¢ ; No. 2 : 9, 9, 9, 8¾, 8¾, 8¾, 30¢ ; No. 3: 9, 9, 9, 9, 9, 8¾, 32¢ ; No. 4 : 9, 8, 8¾, 8½, 9, 9, 27¢ ; No. 5: 9, 9, 9, 8½, 8¼, 8, 28¢.

18. No. 1: 8, 8, 8, 8, 8, 8, 32¼¢ ; No. 2 : 8, 8, 7¾, 7½, 7¾, 7¾, 30¢ ; No. 3 : 8, 8, 8, 7¾, 7¾, 7¾, 28¢; No. 4: 7½, 7¼, 7½, 8, 7¾, 7¾, 32¢ ; No. 5: 8, 8, 8, 8, 8, 8, 33⅓¢.

19. No. 1 : 8, 7¾, 7¾, 7¾, 7¾, 7½, 28¢; No. 2: 8, 8, 7, 0, 8, 8, 30¢ ; No. 3 : 8, 8, 8, 7¾, 8, 8, 35¢ ; No. 4 : 8, 8, 8, 7½, 8, 7¾, 32⅜¢ ; No. 5: 8, 8, 8, 8, 7½, 7½, 32⅝¢.

20. No. 1: 8, 8, 7½, 7, 6¾, 8, 31¢ ; No. 2 : 8, 7, 8, 8, 7¾, 8, 31½¢ ; No. 3: 7¾, 7½, 7¼, 7, 6¾, 0, 28¢ ; No. 4: 7, 7, 7½, 7¾, 0, 7, 26¢ ; No. 5 : 8, 8, 8, 8, 8, 8, 36¢.

EXERCISE 110

Workshop Problems

1. How much will 275 cu. in. of steel weigh, if 1 cu. ft. weighs 490 lb.?

2. What is the weight of a circular piece of steel shafting 3 in. in diameter and 20 ft. long, 1 cu. ft. of steel weighing 490 lb.?

3. If the pressure of steam in a boiler is 140 lb. to the square inch, find the pressure on a valve $2\frac{7}{8}$ in. in diameter.

4. A boiler contains 480 smoke tubes, each $2\frac{3}{8}$ in. in internal diameter. Find the total area for draft through them.

5. If a pressure of 120 lb. to the square inch is applied to the circular plunger of a cylinder 2 ft. in diameter, what is the total pressure?

6. The rim of a fly wheel contains 1.56 cu. ft. of cast iron when it is 8 in. wide. If the rim were made a foot wide, how much would it weigh, 1 cu. ft. of cast iron weighing 450 lb.?

7. Out of a rectangular sheet of steel which is 14′ 3″ long, 3′ $3\frac{1}{2}$″ wide, and $\frac{1}{2}$″ thick, a circular plate 13″ in diameter is cut. If 1 cu. ft. of steel weighs 490 lb., what is the weight of the part of the sheet that is left?

8. If the drive wheel of an engine makes 140 revolutions a minute, and the cylinder is 9 in. in diameter and 14 in. long, and is filled with steam twice in each revolution, how many cubic feet of steam are used in a minute?

9. A hollow steel shaft has an external radius of 9″ and an internal radius of 5″. If 1 cu. ft. of steel weighs 490 lb., find the weight of 10 ft. of this shaft.

EXERCISE 111

Problems in Transportation

1. If a freight car is 36 ft. long, 8 ft. 3 in. wide, and 7 ft. 6 in. high, inside measure, how many cubic feet does it contain?

2. In shipping certain goods at the rate of $97\frac{1}{2}$¢ per hundred pounds, there is allowed a discount of 15%. What is the net rate, to the nearest half cent?

3. In shipping certain goods the net rate after deducting 15% from the usual rate for that class is 34¢ per hundred pounds. What is the usual rate?

4. If a wooden coal car can carry 80,000 lb., and a steel car 100,000 lb., the steel car can carry what per cent more than the wooden car? The wooden car can carry what per cent less than the steel car?

5. A car 36 ft. long and 8 ft. 4 in. wide is loaded with wheat to a depth of 4 ft. 6 in. Reckoning a bushel as containing $1\frac{1}{4}$ cu. ft., and a bushel of wheat as weighing 60 lb., how many pounds does the wheat weigh?

6. If the car of Ex. 5 has a capacity of 80,000 lb., to what depth must it be filled with wheat to contain exactly this amount?

7. If a locomotive weighing $115\frac{1}{2}$ tons can exert a pull of $22\frac{1}{4}$% of its weight in starting a train, how many pounds of pull does it exert?

8. If a locomotive and coal tender together weigh 95 tons, and there are seven cars to the train, averaging 21 tons each, and a horizontal pull 0.5% of the total weight of the train is required to maintain the necessary speed on a level track, what is this pull in pounds?

EXERCISE 114

Problems in Farm Mechanics

1. Experiments show that, with a wagon having high wheels, it requires 84.48 lb. pull to move a ton on a dry, level, gravel road, and 30.4% more if the wagon has low wheels. How much power is required for the low-wheeled wagon?

2. Wind pressure increases as the square of the velocity of the wind. That is, when the wind blows 10 mi. an hour, the pressure is four times as much as when the wind blows 5 mi. an hour. When the wind blows 15 mi. an hour the pressure per square foot of area of a windmill is 1.135 lb. What is the pressure per square foot when the wind blows 30 mi. an hour? when the wind blows 40 mi. an hour?

3. The horse power developed by a windmill varies as the cube of the velocity of the wind. If a certain windmill develops $\frac{1}{40}$ of a horse power when the wind blows 5 mi. an hour, how much will it develop when the wind blows 35 mi. an hour?

4. If a man pumping water exerts a pressure of 18 lb. on the pump handle, and his hand is $6\frac{1}{4}$ times as far from the fulcrum as the rod that connects with the piston of the pump is distant from it, what is the lifting force exerted in raising the water?

5. If a 2-inch piston in a pump requires a lifting force of 54 lb., how much will a 3-inch piston require in the same well?

6. If a pump having a $2\frac{1}{2}$-inch cylinder, and working at a certain rate, delivers enough water per hour for 25 cows, a pump having a 3-inch cylinder, working at the same rate, would deliver enough for how many cows?

EXERCISE 115

Problems in Agriculture

1. If 75 lb. of fertilizer furnishes 12 lb. of plant food, what is the per cent of plant food?

2. A farmer paid $5.40 for a bushel of clover seed, but only 60% was good seed. What was the cost of a bushel of good seed?

3. If a cow's average daily ration is 28 lb. of ensilage, 12 lb. of clover hay, 6½ lb. of bran, and 3½ lb. of corn meal, what per cent of the total is each kind of food?

4. A pound of grafting wax contains 6 oz. of resin, 6 oz. of beeswax, and 1⅛ oz. of tallow. Each of these substances is what per cent of the wax?

5. A farmer had 40 A. in oats. He estimated the use of the land as worth $220 a year, and the labor expended as $300. He paid $120 for fertilizers and $18 for seed. Each of these items was what per cent of the total cost?

6. An experiment station found that by delaying the plowing a week in the spring the land lost in moisture at the rate of 9.13 lb. per square foot. How many tons of water were lost to the acre?

7. At 35¢ a rod, what will a ditch cost dug along the two longer sides of an acre of land 8 rd. wide?

8. If the soil is cultivated to the depth of 4 in., how many cubic feet of soil are cultivated to an acre? How much does this weigh, allowing 75 lb. to the cubic foot?

9. A farmer has 20 A. of potatoes. He sprays half the acreage and gets 180 bu. to the acre. The other half is not sprayed, and from this he gets 70 bu. to the acre. If he sells the potatoes at 26¢ a bushel, how much more does he receive from the sprayed half than from the other?

EXERCISE 118

Farm Buildings

1. A silo is built with inside measurements as follows : length, 14 ft.; width, 11 ft.; depth, 36 ft. Allowing 40 cu. ft. to a ton of ensilage, how many tons of ensilage will this silo contain ?

2. The concrete wall and floor of the above silo are 22 in. thick. How many cubic feet of concrete are needed ?

3. If a poultry house should contain 10 cu. ft. of space to every pound of live weight of the fowls, and the fowls average 6 lb. each, how many fowls may be put in a house 36 ft. long, 15 ft. wide, and averaging 10 ft. in height ?

4. How many feet of 1-inch lumber will be required to board up the gables of a barn 32 ft. wide, the peak being 12 ft. above the eaves ?

5. At each end of a barn 32 ft. wide is a brick wall 12 ft. high and 18 in. thick. Allowing 22 bricks to a cubic foot, what will the bricks cost at $7.50 a thousand ?

6. It is desired to build a cylindrical silo 14 ft. in diameter that shall hold 50 tons of ensilage. Allowing 40 cu. ft. to the ton, what must be the height of the silo ? (Take $3\frac{1}{7}$ in finding the area of the base.)

7. Allowing $1\frac{1}{4}$ cu. ft. to the bushel, how many cubic feet must a bin contain to hold 500 bu. ? Give two convenient sets of measures for such a bin.

8. A cylindrical watering trough for a barnyard is to be 8 ft. in diameter and 3 ft. deep inside measure. How many gallons of water will it hold, allowing $7\frac{1}{2}$ gal. to the cubic foot ? (Take $3\frac{1}{7}$ in finding the area of the base.)

9. How many square feet in the inside area of the watering trough of Ex. 8 ?

EXERCISE 119

PROBLEMS ABOUT PLANTING

1. If 1 qt. of corn will plant 230 hills, what will the corn cost for 6210 hills at 25¢ a quart?

2. If 1 qt. of corn will plant 230 hills, and costs 25¢, how many hills can be planted with $4 worth of corn?

3. The beans necessary to plant 11,250 hills cost $26.25, at 35¢ a quart. How many hills will 1 qt. of beans plant?

4. What is the cost of seed for 7000 celery plants at 23¢ an ounce, allowing $\frac{1}{2}$ oz. of seed to 1000 plants?

5. The cost of seed for 2500 cauliflower plants, at $2.30 an ounce, was $5.75. How many plants were there to an ounce of seed?

6. When peas for planting cost $3.50 a bushel it was found that the seed for planting 20 A. cost $105. How many bushels of seed were allowed to an acre?

7. When corn for planting 25 acres costs $12.50, and 8 qt. of seed are allowed to the acre, what is a bushel of seed corn worth?

8. When seed potatoes were worth $1.15 a bushel, a farmer paid $57.50 for enough to plant 5 A. How many bushels did he allow for planting each acre?

9. A farmer plants corn on a strip of land 40 rd. long and $12\frac{1}{2}$ rd. wide. If he allows a peck of seed corn to an acre, what will the seed cost him at $2 a bushel?

10. A farmer, using 8 qt. of seed corn to the acre, planted 6000 hills with $1\frac{1}{4}$ bu. How many hills did he allow to the acre? How many acres did he plant?

11. When 3 bu. of peas at $3.50 a bushel are allowed to plant 2 A., it costs $157.50 for peas to plant a certain field. How many acres are there in the field?

EXERCISE 122

Factory Problems

1. A certain canning factory uses the peas grown on $37\frac{1}{2}$ A. during the season of three weeks, putting up 24,000 cans of peas. At 6 da. to the week, how many acres of peas does it use a day, and how many cans does it put up a day?

2. If a farmer owns $5\frac{1}{2}$ A. of the $37\frac{1}{2}$ A. mentioned in Ex. 1, how many cans are put up from his peas?

3. A workman in the factory puts together 180 cases of 24 cans each a day. How many days will it take him to put together the cases for 24,000 cans, allowing $\frac{4}{9}$ of a day for getting the material ready?

4. If the factory runs from 8 A.M. to 5 P.M., with an hour out at noon, and the total output for one of the weeks is 10,800 cans, what is the average output per hour?

5. To seal 30 cans it takes 8 oz. of solder, of which 20% is lead. If the output of a canning factory is 1500 cans a day, how many pounds of solder are used in a week of 6 da.? How many pounds of this are lead? How many pounds of lead are used in a day?

6. If a laborer in the factory receives $2.25 a day, and has a chance to work for $28\frac{1}{2}$¢ an hour, what is the per cent of increase, allowing 8 hr. to the working day?

7. A certain factory does $276,000 worth of business a year. The capital is $33\frac{1}{3}$% of the annual business, and the dividends amount to 12% of the capital. Required the total amount of the dividends.

8. A certain article sells at retail for $4.80, the retail dealer making a profit of $33\frac{1}{3}$%. The wholesale dealer made a profit of $9\frac{1}{11}$%, and the manufacturer a profit of 10%. What did it cost to manufacture the article?

CHAPTER X

GENERAL REVIEW

208. Nature of the Problems. The problems in this chapter have been selected from recent official examination papers. They show some of the demands made upon the schools and may assist in preparing pupils for examinations.

EXERCISE 123

PROBLEMS IN COMPUTATION WITH INTEGERS

Add, checking the results:

1.	2.	3.	4.	5.
4368	2748	4227	8284	7856
5962	6299	3729	2643	9280
6857	3176	6248	1716	3442
7976	4387	5135	7357	2635
8795	8132	4273	2148	4331

Subtract, checking the results:

6.	7.	8.	9.	10.
8284	9307	9002	4871	3407
1796	2969	3763	2998	1879

Multiply, checking the results:

11.	12.	13.	14.	15.
4879	7963	3148	6779	8764
6273	4072	2909	9283	8297

Divide, checking the results:

16. 92,358 ÷ 233. **17.** 123,540 ÷ 225. **18.** 62,500 ÷ 250.

EXERCISE 124

Problems in Computation with Common Fractions

1. Simplify $\dfrac{2\frac{1}{6} + 2\frac{5}{8} \div \frac{9}{14}}{3 \times 1\frac{3}{4}} + \dfrac{1}{6} \times 6\frac{1}{7}$. (In all such cases perform first the multiplications and divisions as indicated by the symbols, and then the additions and subtractions in the order given.)

2. Simplify $\dfrac{(\frac{7}{20} + \frac{1}{5} \times 1\frac{5}{8}) \div (6 \times \frac{3}{4})}{\frac{3}{1000}}$. (Work first within the parentheses.)

3. Simplify $\dfrac{(\frac{17}{24} - \frac{5}{18}) \div (\frac{17}{18} \times 2\frac{25}{34})}{6\frac{3}{8} - \frac{9}{20} \times \frac{5}{6}}$.

4. Simplify $\dfrac{\frac{3}{8} + \frac{13}{16} \div 1\frac{19}{20}}{2 - 1\frac{13}{22} + \frac{8}{77} \times 1\frac{1}{20}} - \dfrac{19}{36}$.

5. Simplify $\dfrac{\frac{2}{5} + 3\frac{3}{5} \div 4\frac{1}{2}}{4\frac{4}{5} \times \frac{2}{3}}$, and express the result both as a common fraction and as a decimal.

6. Simplify $\dfrac{(\frac{4}{15} + 1\frac{1}{3}) \div (3\frac{1}{9} \times \frac{4}{7})}{1 - \frac{2}{3} \times \frac{3}{20}}$.

7. Simplify $\dfrac{1\frac{4}{5} \times 3\frac{3}{4} \div 6\frac{3}{4}}{\frac{2}{3} \div \frac{7}{12} - \frac{4}{7}}$.

8. Simplify $\dfrac{(\frac{2}{3} + 4\frac{1}{2}) \times (3\frac{1}{2} - \frac{1}{3})}{2\frac{1}{2} \times 3\frac{1}{2}}$.

9. Simplify $\dfrac{4\frac{1}{2} + \frac{2}{3} - (4\frac{1}{2} - \frac{2}{3})}{2 + (3\frac{1}{2} - \frac{1}{8})}$.

10. Simplify $\dfrac{2\frac{1}{2} \times (3 + 4\frac{1}{4})}{3\frac{1}{2} \div (2 + 3\frac{1}{8})}$.

11. Simplify $\dfrac{(\frac{1}{4} + \frac{3}{5}) \times (\frac{3}{4} - \frac{1}{5})}{\frac{3}{10} + \frac{1}{2}}$.

12. Simplify $\dfrac{\frac{1}{6} \times 1\frac{13}{16} + 1\frac{1}{6} \times 6\frac{1}{4} - 1\frac{1}{3} \times 5\frac{4}{9}}{\frac{1}{6} \times 2\frac{5}{6} \times 5\frac{2}{3}}$.

EXERCISE 125

Problems in Computation with Decimals

1. Simplify $\dfrac{3.63 \times 3.5}{8.47 \times 0.5}$.

2. Simplify $\dfrac{1.25 - 1.33\frac{1}{3} \div 1.66\frac{2}{3}}{240 \times .015}$. (See p. 186, Ex. 1.)

3. Simplify $\dfrac{1.75 - 0.27 + 0.36}{2.64 - 2 \times 1.07}$.

Add the following:

4.	5.	6.	7.
2.074	7.265	4.527	5.2793
3.672	8.030	3.298	4.2606
9.42	4.723	1.111	1.1111
8.307	9.481	5.473	4.7207

Subtract:

8.	9.	10.	11.
9.6087	8.1306	21.4273	32.0049
4.7998	5.0987	8.6998	26.8297

Multiply:

12.	13.	14.	15.
5.2346	208.73	29.008	0.07289
3.1416	3.1416	42.606	0.02134

Divide:

16. 53.4072 ÷ 3.1416.
17. 0.062727 ÷ 0.02987.
18. 74.3094 ÷ 4.1283.
19. 0.00037356 ÷ 0.00283.

Divide to three decimal places:

20. 27 ÷ 24.8.
21. 35 ÷ 4.16.
22. 8.4 ÷ 2.96.
23. 5.3 ÷ 0.072.
24. 6.05 ÷ 3.07.
25. 7.92 ÷ 0.063.

EXERCISE 126

Problems in Computation with Compound Numbers

Add:

	ft.	in.		yd.	in.		lb.	oz.
1.	42	7	2.	27	27	3.	15	12
	33	9		32	18		26	9
	26	7		41	26		31	8
	52	11		62	15		42	15
	46	8		33	32		17	13
	75	10		48	12		36	7

Subtract:

	ft.	in.		yd.	in.		lb.	oz.
4.	27	2	5.	127	15	6.	37	6
	18	9		68	27		19	14

Multiply:

7. 26 ft. 4 in.	**8.** 42 yd. 23 in.	**9.** 86 lb. 14 oz.
35	76	$37\frac{1}{2}$

Divide:

10. 111 yd. ÷ 4.

11. 115 ft. 1 in. ÷ 7.

12. 156 lb. 15 oz. ÷ 9.

13. 1482 yd. 8 in. ÷ 40.

14. 222 bu. ÷ 27 bu. 3 pk.

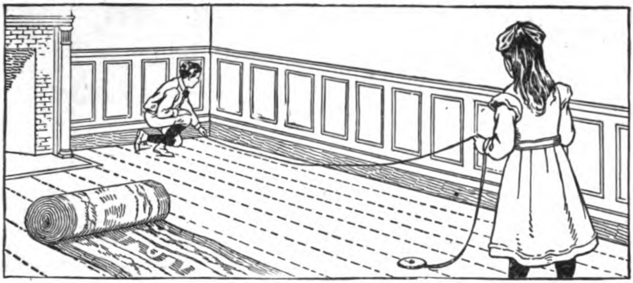
15. 117 ft. 3 in. ÷ 16 ft. 9 in.

16. Reduce 7 mi. 10 rd. 2 yd. 2 ft. 9 in. to inches.

17. Reduce 33 rd. 16 ft. 10 in. to inches; to feet and a decimal; to rods and a decimal.

18. Add 3 mo. 2 wk. 3 da., 5 mo. 1 wk. 6 da., 6 mo. 3 wk. 5 da., and 10 mo. 4 da., allowing 4 weeks to the month.

19. From 1915 yr. 8 mo. 2 da. subtract 1911 yr. 10 mo. 20 da.



EXERCISE 127

PROBLEMS IN PAINTING AND CARPETING

1. This room is 18′ × 15′. How much will it cost at 65¢ a yard for matting 1 yd. wide to cover the floor, no allowances being made?

2. Find the cost at 15¢ a square yard of painting the sides, ends, and tops of 10 freight cars 34′ × 8′ × 7′.

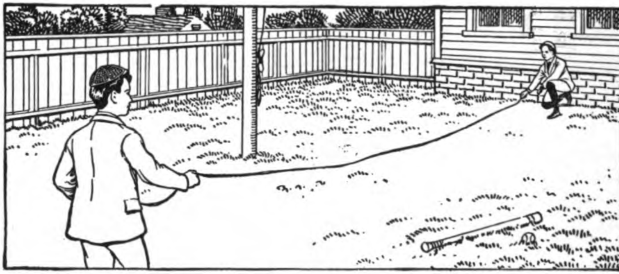
3. How much will it cost at $1.25 a yard to carpet a room 15′ by 12′ with plain carpet $\frac{3}{4}$ yd. wide?

4. How much will it cost at $1.25 a yard to cover the floor of a room 17′ 6″ × 15′ 9″ with 27-inch carpet, allowing 3 yd. for matching patterns?

5. Find the cost at 18¢ a square yard of painting the walls and ceiling of a room 16′ × 14′ × 10$\frac{1}{2}$′, making no allowance for openings.

6. Find the cost of carpeting a room 30′ by 27′ with carpet $\frac{3}{4}$ yd. wide, 3 yd. being allowed for matching, and the carpet costing $2.25 a yard.

7. A painter, estimating the cost of a job, finds there are 925 sq. yd. of surface to be covered. His bid is at the rate of 16¢ per square yard. What is the amount of his bid?



EXERCISE 128

Problems in Practical Measurement

1. These boys are laying out a baseball diamond in the form of a square 90 ft. between bases. How far is it from first base directly across to third base?

2. Find the cost of 20 joists, each 4″ × 6″ and 16′ long, at $32 per M.

3. Find the cost of 40 joists 18′ long, 10″ wide, and 4″ thick @ $27.50 per M.

4. What is the cost of fencing a $2\frac{1}{2}$-acre rectangular lot 25 rd. long @ 45¢ a rod?

5. How many cubic feet in a cistern that holds 54 bbl. of water, allowing 231 cu. in. to a gallon?

6. Find the cost of 20 joists 2″ × 4″ and 12′ long, and 24 boards $1\frac{1}{2}$″ thick, 8″ wide, and 10′ long @ $28 per M.

7. Allowing 2150.4 cu. in. to a bushel, how many tons of coal can be placed in a bin containing 560 cu. ft., if the weight of 1 bu. of coal is 80 lb.?

8. A picket fence incloses a rectangular field 42′ × 30′. The pickets are $2\frac{1}{2}$″ wide and $1\frac{1}{2}$″ apart. Find the cost of the pickets @ $3.25 a hundred.

EXERCISE 129

Miscellaneous Problems

1. An iron pipe $\frac{1}{8}$ in. thick has an internal diameter of 2 in. The pipe is 16 ft. long. Allowing 0.28 lb. to the cubic inch, what is the weight of the pipe? (Use $3\frac{1}{7}$ in the measure of the circle.)

2. If steel rails weighing 80 lb. to the yard are used on a certain double-track road for a distance of 275 mi., how many tons are used, not counting switches?

3. The air is composed of two gases, oxygen and nitrogen. In every cubic foot of air there are 345.6 cu. in. of oxygen. What per cent of the volume of the air is nitrogen?

4. At the temperature at which sound travels 1120 ft. a second, what is the distance of a thundercloud in which lightning is seen 17.4 sec. before the thunder is heard?

5. In a certain blast furnace the casting machine turns out 20 pigs a minute, averaging 110 lb. each. If this machine runs at this rate for 308 da., 16 hr. each day, how many long tons of pig iron will it turn out?

6. The wind pressure in a hurricane has been known to be as great as 49.2 lb. per square foot. In such a storm how many tons pressure on the side of a large office building 104 ft. long and 308 ft. high?

7. A large suspension bridge has four cables, each 1872 ft. long and 1 ft. 2 in. in diameter. In painting these cables it is necessary to know their area. Taking the circumference as $3\frac{1}{7}$ times the diameter, compute this area.

8. An iron tire expands $1\frac{9}{16}\%$ in being heated for shrinking on a wheel. A wooden wheel needs a tire with a diameter of 4 ft. 8 in. How much longer will the tire be when heated? (Use $3\frac{1}{7}$ as in Ex. 7.)

EXERCISE 130

Problems in Profit and Loss

1. Some goods were sold for $1015.30 at a loss of $31\frac{1}{4}\%$. Find the loss.

2. Beans bought @ $1.60 a bushel are sold @ 7¢ a quart. Find the per cent of gain or of loss.

3. A grocer sold tea for $87\frac{1}{2}$¢ a pound, thus gaining $12\frac{1}{2}$¢ a pound. What was his gain per cent?

4. A watch sold for $228 at a loss of 5%. How much would have been gained by selling it at a gain of 5%?

5. Papers bought at the rate of 60 for 50¢ are sold for 1¢ each. Find the per cent of gain and the cost of 75 papers.

6. A box of 150 oranges is bought for $1.40. The oranges are sold at 20¢ per dozen. Find the gain per cent.

7. By selling a farm for $4228 a farmer lost $12\frac{1}{2}\%$. How much would he have gained by selling it for $5040?

8. A boy bought oranges at 20¢ per dozen and sold them at the rate of 2 for 5 cents. Find his gain per cent.

9. A grocer buys 50 bushels of potatoes at 60¢ a bushel and retails them at 20 cents a peck. Find his whole gain and his gain per cent.

10. A grocer sold 4 chests of tea @ $20.08 a chest; on 3 chests he gained $25\frac{1}{2}\%$ and on 1 chest he lost 20%. Find the total cost of the tea.

11. A merchant marked an article so as to gain 15%, but sold it for cash at such a price as to gain $3\frac{1}{2}\%$. What per cent of the marked price did he deduct?

12. A merchant marks goods $12\frac{1}{2}\%$ above cost and sells them 20% below the marked price. Find his per cent of loss. Write the analysis.

EXERCISE 131

Problems Involving Commission

1. An agent sold 325 bu. of wheat @ 80¢ a bushel, charging a commission of 5%. Find the net proceeds.

2. An agent charged \$6.63 for selling 52 bushels of tomatoes @ 85¢ a bushel. Find the rate of his commission.

3. A real estate agent charged \$295.50 for selling a piece of property for \$9850. What was the rate of his commission?

4. The net proceeds of the sale of 244 bbl. of flour is \$1124.23, the commission being 3%. Find the selling price of the flour per barrel.

5. An agent bought for a dealer 240 baskets of peaches @ 75¢ a basket, charging $3\frac{1}{2}$% commission. Find the total cost to the dealer.

6. An agent sold 44 wagons @ \$135 each and paid his principal \$5197.50 as the proceeds of the sale. Find the rate of the agent's commission.

7. A real estate agent charged \$562.50 for selling a piece of property. The rate of commission being $4\frac{1}{2}$%, at what price did he sell the property?

8. A commission merchant sold 45 barrels of apples at \$2.80 a barrel, charging 6¢ a barrel for storage and a commission of $5\frac{1}{2}$%. Find the net proceeds of the sale.

9. A commission merchant sold some produce, charging 3% for his services. He remitted to his principal \$906.95. At what price did he sell the produce?

10. A manufacturer sent his agent \$3177.81 to pay for some leather, and the commission of $\frac{1}{2}$%. How much did the agent invest and how much was his commission?

EXERCISE 132

Problems in Insurance

1. What is the face of the policy when the premium at $\frac{3}{4}\%$ is \$21?

2. A house valued at \$3216 is insured for $\frac{5}{6}$ of its value at $\frac{3}{4}\%$. What premium is paid?

3. A house valued at \$2750 is insured for 80% of its value at $\frac{3}{4}\%$. What premium is paid?

4. A man pays \$105 once in 3 years for insuring his house at $\frac{1}{4}\%$ annually. Find the face of the policy.

5. A house is insured for 80% of its value at 1%, the premium being \$36.80. What is the value of the house?

6. A house valued at \$4400 is insured for 80% of its value, the premium being \$44. What is the rate of insurance?

7. A man has a life insurance policy of \$7500 on which he pays an annual premium of \$28.60 per thousand. How much does he pay in premiums in five years?

8. A man has a life insurance policy on which he pays \$201 a year. The premiums are at the rate of \$26.80 per thousand. What is the face of the policy?

9. A man who pays an annual premium of $2\frac{1}{2}\%$ on a life insurance policy for \$1500 dies after 18 payments. How much more did his heirs receive than he paid?

10. A man insures his stock of goods for \$6000, this being 80% of their value. The rate is \$1.15 per thousand. What is the value of the property and the annual premium?

11. A man pays an annual premium of \$29.60 a thousand on a life insurance policy. If the total annual premium is \$244.20, what is the face of the policy?

EXERCISE 133

PROBLEMS IN TAXES

1. The assessed valuation of the property of a certain town is $1,872,500, and the tax levied is $14,043.75. Find the rate of tax on $1000.

2. A tax of $22,400 is raised in a certain school district to pay for a new school building. A resident who is assessed $5600 pays a tax of $62.72. What is the total valuation?

3. In a certain school district assessed at $80,000 a tax of $1200 is raised. Mr. B's assessed valuation is $6840. Find the rate of taxation and Mr. B's tax.

4. The assessed valuation of a city is $65,489,000; the amount to be raised by tax is $229,211.50. Find the tax on property assessed at $12,400.

5. The assessed valuation of a certain school district is $2,559,000; the amount to be raised by tax is $11,515.50. Find the tax on a business block assessed at $85,000.

6. The assessed valuation of a certain school district is $2,345,000; the amount to be raised by tax is $10,552.50. Find the tax on property assessed at $4500.

7. The assessed valuation of a certain city is $48,250,000; the amount to be raised by tax is $337,750. Find the tax on a piece of property assessed at $75,000.

8. If the tax rate is 11 mills, what must a man pay who has a farm assessed at $4750, a village residence assessed at $3500, and personal property assessed at $1500?

9. A tax of $16,500 is raised in a certain town as its share of the cost of a bridge. If the assessed valuation of the town is $2,750,000, what is the rate of taxation for this purpose?

EXERCISE 134

Problems in Trade Discount

1. Find the net amount of a bill for \$880 subject to discounts of $2\frac{1}{2}\%$, 5%, and 10%.

2. On a bill of \$620 commercial discounts of 5% and 5% are allowed. What is the net price?

3. Find the net cost of a piano marked \$450, with trade discounts of 3%, 8%, and 10%.

4. What is the list price of an article whose net cost is \$6.15 after discounts of 20% and 25% are deducted?

5. What per cent of profit will a man make by paying \$17.10 for an article, with discounts of 20%, 10%, and 5% from the list price, if he sells it at the list price?

6. Furniture is bought at trade discounts of 5% and 16% from the list price, and the net cost is \$518.70. Find the list price of the furniture.

7. Find the net amount of a bill of \$470, the trade discounts being 20% and 5%. What single rate of discount is equivalent to these discounts?

8. What single discount is equivalent to a discount series of 40%, 20%, 10%, and 5%?

9. Which is the better for the purchaser, and how much better: two successive trade discounts of 25% and 10% on a bill of \$500, or a single discount of $33\frac{1}{3}\%$?

10. Which is the better for the seller, and how much better: trade discounts of 20%, 12%, and 3%, on a bill of \$800, or a single discount of $31\frac{3}{4}\%$?

11. Which is the better for the purchaser, and how much better: trade discounts of 10%, 10%, and 10%, on a bill of \$625.75, or a single discount of $27\frac{1}{10}\%$?

EXERCISE 135

Problems in Interest and Discount

1. Find the simple interest on $920 at 4¼% for 2 yr.

2. Find the simple interest on $685.70 at 5½% for 1 yr. 7 mo. 20 da.

3. Find the simple interest on $653.25 @ 3½% from Sept. 28, 1910, to June 18, 1913.

4. Find the simple interest on $786 at 5½% from Dec. 29, 1910, to Oct. 2, 1911.

5. How long must $650.82 be on interest at 5% to amount to $761.44?

6. Find the interest at 5½% on a note of $500 dated Oct. 22, 1910, and paid June 7, 1912.

7. A 60-day note, without interest, is discounted at a bank 13 days after date at 6%. The proceeds are $734.70. What is the face of the note?

8. A 90-day note for $376.85 with interest at 5% is due March 30. Find the amount due and the date of the note.

9. A 60-day note for $630 without interest, dated Dec. 27, is discounted at a bank on Jan. 16, at 6%. Find the discount.

10. The proceeds of a 90-day note, without interest, discounted 32 days before it is due @ 6%, are $574.42. Find the face of the note.

11. Find the proceeds of a 60-day note for $420, without interest, dated Aug. 29, 1910, and discounted at a bank five days later at 6%.

12. A note of $600 for 6 months without interest, dated Dec. 4, is discounted at a bank on Dec. 29 at 6%. Find the discount and the proceeds.

EXERCISE 136

Problems in Stocks and Bonds

1. Find the cost of 36 shares of N. Y. Central stock at $130\frac{3}{4}$, brokerage $\frac{1}{8}\%$.

2. What will be the total cost to the purchaser of 96 shares of stock at $126\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

3. What per cent is realized from an investment in $4\frac{1}{2}\%$ stock at $124\frac{7}{8}$, brokerage $\frac{1}{8}\%$?

4. Find the number of shares of stock that can be bought for $5160.75 @ $122\frac{3}{4}$, brokerage $\frac{1}{8}\%$.

5. What dividend is a stock paying when the rate on the investment is $4\frac{4}{49}\%$, if the stock was bought for $110\frac{1}{8}$?

6. A man bought stock at 97 and sold at 102, brokerage in each case $\frac{1}{8}\%$. Find the net gain per share.

7. What would be the cost of an investment in U.S. 4's (United States 4% bonds) @ $131\frac{1}{4}$, brokerage $\frac{1}{8}\%$, to secure an annual income of $720?

8. A man sold through his broker 156 shares of stock @ $3\frac{1}{2}\%$ discount, paying a brokerage of $\frac{1}{8}\%$. Find the brokerage and the net proceeds of the sale.

9. How much better rate of income will be received from an investment in 6% stock at $137\frac{3}{8}$ than in 4% stock at $109\frac{7}{8}$, brokerage in each case being $\frac{1}{8}\%$?

10. A man paid $2550 for some 5% railway stock at $127\frac{3}{8}$, brokerage $\frac{1}{8}\%$. How many shares did he buy, and what was his annual income from the stock?

11. A man bought 50 shares of stock at $120\frac{1}{4}$, and 6 months later a $3\frac{1}{2}\%$ dividend was declared; he then sold the stock at 118, brokerage in each case $\frac{1}{8}\%$. Find his net gain or loss, money being worth 5%.

EXERCISE 137

PROBLEMS IN RATIO, PROPORTION, AND ANALYSIS

1. Find the fourth term of the proportion $6\frac{2}{5} : 3\frac{1}{3} :: 1\frac{3}{5}$; that is, $6\frac{2}{5} : 3\frac{1}{3} = 1\frac{3}{5} : (?)$. (The use of the symbol :: for = is nearly obsolete.)

2. The means of a proportion are $3\frac{1}{2}$ and $\frac{5}{21}$. One of the extremes is $\frac{5}{12}$. Write the proportion.

3. A prize of \$60 was divided between two persons in the ratio of $\frac{2}{3}$ to $\frac{5}{6}$. How much was the share of each?

4. If 18 men can do a piece of work in 24 days, in how many days should 27 men do the work? Solve by (*a*) analysis; (*b*) proportion.

5. X can do a piece of work in $3\frac{1}{2}$ days, Y in 3 days, and Z in $2\frac{3}{4}$ days. How many days will it take them to do it if they all work together?

6. In 20 days 36 men can do a piece of work. Find how many men it would take to do the work in 15 days. Solve by proportion.

7. A map is drawn so that 1 in. represents 0.8 mi. on the surface of the earth. How many acres of land are represented by a portion of the map 3 in. square?

8. A tank can be filled by a pipe in $3\frac{1}{2}$ hours and can be emptied by another pipe in 5 hours. If the tank is empty and both pipes are open, how long will it take to fill the tank, the flow in the pipes being constant?

9. Two stockmen, A and B, maintain a common drinking trough and share the annual cost of repairs in proportion to the number of cattle owned by each. A has 1253 head of cattle and B has 2747. If the annual cost of repairs is \$80, how much should each contribute?

EXERCISE 138

Problems in the Metric System

1. A meter is 39.37 in. long. How many rods in a kilometer?

2. Find the weight, in kilograms, of the water in a tank 3 m. long, 2 m. wide, and 1.6 m. deep.

3. How many blocks 2 dm. long and 12 cm. wide will be required to pave a court 14 m. by 10.8 m.?

4. A tank containing 5400 l. of water is 4 m. long and 1.8 m. wide. Find the depth of water in the tank.

5. A cistern 2.5 m. by 3.6 m. contains 14 kiloliters of water. How deep is the water?

6. Find the cost of the milk in a vat 1.5 m. long, 8 dm. wide, and 4 dm. deep, @ 4¢ a liter.

7. A wood pile containing 13.86 steres is 5.5 m. long and 1.8 m. wide. Find the height of the pile.

8. A circular park contains 78.54 ha. Find the distance from the center to the circumference.

9. A tank is 5.4 dm. wide, 2.5 dm. deep, and 1.75 m. long. Find the weight, in kilograms, of the water it contains when full of water.

10. Sea water is 2.8% salt, and is 1.025 times as heavy as fresh water. How many kilograms of salt can be obtained from 100 cu. m. of sea water?

11. Find the weight in kilograms of a bar of iron 2 m. long, having a rectangular cross section 8 cm. by 12 cm., iron being 7.8 times as heavy as water.

12. In a circular reservoir 18 m. in diameter rainfall increased the depth of the water 17 cm. How many liters of water were added to the reservoir?

EXERCISE 139

Problems Involving Square Root

1. Find the square root of 119,716.

2. Find the square root of 170,569.

3. Find to three decimal places the square root of 6.

4. Find to three decimal places the square root of 30.

5. Determine the number of rods around a square field, the diagonal of which is 380 rd.

6. The diagonal of a square is 15 ft. Find to two decimal places the length of one side.

7. The side of a square is 15 ft. Find to two decimal places the length of the diagonal.

8. The area of a square is 37 sq. ft. Find to two decimal places the length of one side.

9. Separate 15,876 into prime factors, and from these factors determine the square root of the number.

10. The area of a circle is 113.0976 sq. in. Find the length of the radius.

11. The area of a circle is 153.9384 sq. ft. Find the length of the radius.

12. The foot of a 37-foot ladder is 12 ft. from the wall of a building against which the top rests. How high does the ladder reach on the wall?

13. How far from the wall of a house must the foot of a 50-foot ladder be placed in order that the top of the ladder may touch a window sill 48 ft. from the ground?

14. A rope stretched from the top of a 28-foot pole just reaches the ground 21 ft. from the foot of the pole. Assuming the rope to be straight, how long is it?

EXERCISE 140

MISCELLANEOUS PROBLEMS

1. If 48 yd. of cloth costing 3s. 6d. a yard are sold for £10 4s., what is the per cent of gain?

2. What will it cost for rails for 5 mi. of double track, if the rails cost $28 a ton and weigh 100 lb. per yard?

3. A and B together have $153. If $\frac{3}{4}$ of A's money is equal to $\frac{2}{3}$ of B's, how much has each?

4. A horse tied to a stake can reach the grass 30 ft. in any direction from the stake. Over how many square feet of land can the horse graze?

5. Allowing 1000 shingles to 120 sq. ft., find how many thousand shingles will cover a pitched roof each side of which is 58 ft. long and 24 ft. wide.

6. A corner lot has 96 ft. front and is 180 ft. deep. Find the cost of laying a 4-foot stone walk on the front and side, at 18¢ per square foot.

7. Find in inches the depth of a cylindrical tank 5 ft. in diameter that has the same capacity as a rectangular cistern 8 ft. square and 6 ft. deep.

8. A boat travels 15 mi. down stream in $2\frac{1}{2}$ hr. The boat's rate of travel in still water is $4\frac{1}{2}$ mi. an hour. In what time can the boat return?

9. A man has $\frac{5}{8}$ of his property invested in a farm, $\frac{1}{6}$ in railway stock, and the remainder, $1500, is deposited in a bank. Find the value of the farm and of the railway stock.

10. Two men start from points 33 mi. apart and walk towards each other, the first at the rate of $4\frac{1}{2}$ mi. per hour, and the second at the rate of $3\frac{3}{4}$ mi. per hour. How far from where the second man started will they meet?

11. The amount of water that flows over Niagara Falls averages 264,000 cu. ft. a second. Find the average number of gallons carried over the falls in a second.

12. The diagonal of a square is 20 ft. Find to two decimal places the length of one side.

13. If $\frac{1}{8}$ in. on a map represents an actual distance of $35\frac{1}{2}$ mi., what distance on the map represents 28.4 mi.?

14. How many loads of gravel averaging 1 cu. yd. will be required to grade $2\frac{3}{4}$ mi. of road, the gravel to be laid 14 ft. wide and 6 in. deep?

15. If a dealer pays $9737.50 for some wool, including the purchasing agent's commission of $2\frac{1}{2}\%$, how much is the amount of the commission?

16. The sugar contained in the sugar beet is $6\frac{1}{4}\%$ of the weight of the beet. How many pounds of beets will be required to produce $122\frac{1}{2}$ lb. of sugar?

17. How many boards 12 ft. long will be required to build a straight fence 4 boards high about a rectangular field 40 rd. wide containing 20 acres?

18. Find the circumference of a circle whose diameter is 22.6 in., using 3.1416 as the ratio of circumference to diameter; also using $3\frac{1}{7}$; also using the ratio 355 : 113.

19. If 15 burners, each consuming 6 cu. ft. of gas per hour, are used on an average 4 hr. per day for 365 days, what will be the gas bill at 80¢ per thousand?

20. Flint glass contains by weight 72% sand, 14% soda, 12% lime, and 2% alumina. Find the number of pounds of each of these substances in 150 lb. of flint glass.

21. A bell weighing 1250 lb. is composed of 77% copper, 21% tin, and the rest zinc. How many pounds of each metal does the bell contain?

22. Bronze contains 9 parts of copper by weight to 2 parts of tin. How many pounds of tin in a bronze statue that weighs 2662 lb.?

23. The time from one new moon to the next is 29 da. 12 hr. 44 min. 3 sec. Express this in days and the decimal fraction of a day.

24. Find the difference in the number of acres in two fields, one of which contains 75 sq. rd., and the other of which is 75 rd. square.

25. Find to the nearest pound the weight of the air in a schoolroom $30' \times 16' \times 12'$, if water is 770 times as heavy as air and weighs 1000 oz. per cubic foot.

26. If a certain gun metal is composed of 84% copper and the rest tin, how many pounds of tin will be needed with 58.8 lb. of copper to make the gun metal?

27. A man, computing the cost of 54 articles, multiplied by mistake the cost of each by 45 instead of 54, and obtained $121.50. What was the cost of the 54 articles?

28. A company with a capital of $350,000 earned net $28,250 last year. It paid a 7% dividend and carried the balance to surplus. How much did it carry to surplus?

29. A gardener finds that a bin 8 ft. long, 3 ft. 6 in. wide, and 5 ft. deep holds 112 bu. How many bushels in a bin that is 25% longer, 25% narrower, and has the same depth?

30. Find the cost of a stair carpet at $1.35 a yard for a straight flight of 19 steps 10 in. wide and 8 in. rise, allowing 15 in. extra for the bottom of the stairs and the same for the top.

31. Water is flowing into a rectangular tank whose base is 6 ft. 8 in. long and 4 ft. 6 in. wide, at the rate of 2 cu. ft. in 3 min. How long will it take the water to fill it to a depth of 6 ft.?

32. Find the cost of 325 boards 16 ft. long, 4 in. wide, and $\frac{7}{8}$ in. thick at $27 per M.

33. Find the number of dry quarts a cylindrical measure $16\frac{1}{2}$ in. in diameter and 1 ft. high will hold.

34. At what rate will $1250 yield $112.50 interest in 2 yr.? at what rate will it yield $206.25 interest in 3 yr.?

35. What fraction of the year 1912 is the time from January 20 to March 15, including both of these days?

36. On seven successive days at noon the thermometer indicated 66°, 72°, 61°, 68°, 73°, 62°, 65°. What was the average for the week?

37. On seven successive days the barometer indicated 29.22 in., 29.18 in., 29.65 in., 30.02 in., 30.12 in., 30.18 in., and 30.21 in. What was the average for the week?

38. A cubic inch of iron weighs $4\frac{3}{8}$ oz. Find the weight in pounds of an iron bar 1 in. square at the end and 1 yd. long; 1 in. square at the end and 15 ft. long.

39. The driving wheels of a locomotive are 6 ft. 8 in. in diameter. How many revolutions a minute does each wheel make when the locomotive is traveling 48 mi. an hour?

40. When water is heated from 32° to 212° it expands $\frac{1}{24}$ in volume. If a cubic foot of water at the freezing point weighs $62\frac{1}{2}$ lb., what does a cubic foot of boiling water weigh?

41. A man bought a bill of goods in Paris to the value of 2600 fr. He paid 40% ad valorem duty and $17.65 freight, and sent a draft for the purchase price when exchange was 19.4. What was the total cost?

42. A pole is broken by the wind 12 ft. above the ground. The two parts hang together, but the upper part is bent over and touches the ground 12 ft. from the base. How long was the pole originally? (Two decimals.)

43. If it takes 1 bbl. of lime and $\frac{5}{8}$ cu. yd. of sand to make mortar for 1000 bricks, and if $22\frac{1}{2}$ bricks are allowed per cubic foot of wall, how much lime and how much sand are needed for a wall $28' \times 6' \times 1'\,6''$?

44. The diagonal of one face of a cube is $\sqrt{450}$ in. Find the number of square inches in the entire surface of the cube and the number of cubic inches in the volume.

45. If a horse is tethered by a 40-foot rope, the length of the largest circumference he can walk around is how many times what it would be if he were tethered by a 20-foot rope?

46. In Ex. 45 the horse can graze over how many times as much ground in the first case as in the second?

47. One man can do a piece of work in 8 days and another man can do it in 10 days. How many days will it take the two together to do the work?

48. If X can build a wall in 18 days, Y in 15 days, and Z in 12 days, how many days will it take all three working together to build the wall?

49. If X, Y, and Z working together can dig a ditch in 24 hr., and X and Z working together can dig it in 32 hr., how many hours will it take Y alone to dig the ditch?

50. A water tank can be filled by three pipes, running separately, in 4 hours, 5 hours, and 6 hours respectively. In how many hours will the empty tank be filled by all three running together?

51. If the eye is 8 in. above the surface of a lake it can see about 1 mi.; if it is $2^2 \times 8$ in. above the surface, 2 mi.; if it is $3^2 \times 8$ in. above the surface, 3 mi.; and so on, the height being approximately 8 in. multiplied by the square of the distance in miles. About how far can a man see in one direction, if his eye is 36 ft. above the surface of the sea? (Two decimals.)

52. How many steps of 2 ft. 9 in. each would you have to take a minute in order to walk at the rate of 3 mi. an hour?

53. A machine does some work in 8 hr. If its capacity is increased $14\frac{2}{7}\%$, how long will it take to do the work?

54. Of three partners, the first is entitled to $37\frac{1}{2}\%$ and the second to $43\frac{3}{4}\%$ of the profits. If they make \$4800, what is the share of each of the three?

55. A certain alloy of metal is composed of $17\frac{1}{2}\%$ tin and the rest copper. How many pounds of tin are needed with $742\frac{1}{2}$ lb. of copper to make the alloy?

56. Two cans have a capacity of 5 pt. and 8 pt. respectively. How could you measure out 14 pt. into a third can by using these alone?

57. A dealer marked a set of furniture 25% above cost. He then sold it at 10% less than the marked price, and made a profit of \$5. What did the furniture cost him?

58. A piece of carpet 27 in. wide weighs 4 lb. to the yard. What is the weight of carpet necessary to cover a room 18 ft. wide and 24 ft. long?

59. If a gallon of water weighs $8\frac{1}{4}$ lb. and a cubic foot of water weighs 1000 oz., how many gallons are there in a cubic foot of water?

60. What is the area of the square inscribed in a circle whose radius is $3\sqrt{2}$ in., so that the corners lie on the circumference? of the square circumscribed about the circle so that its sides just touch the circumference?

61. The area of a circle with radius 3 in. is 28.2744 sq. in. Required the area of a circle with radius 6 in.

62. The equatorial diameter of the earth is estimated as 20,926,202 ft., and the polar diameter as 20,854,895 ft. Express each in miles and find the difference.

63. A man computing the cost of 56 articles made the mistake of multiplying by 65 instead of 56, and obtained \$487.50. What was the correct cost of the lot?

64. Two masts of a ship are 52 ft. apart, one being 80 ft. high and the other 70 ft. Supposing them perpendicular to the deck, how far is it from the top of one mast directly to the top of the other? (Two decimal places.)

65. In closing out a line of goods a merchant asked 25% less than the cost of a certain article, but at an auction he succeeded in getting $33\frac{1}{3}$% more than this asking price. Did he gain or lose on the cost, and what per cent?

66. Water is flowing into a tank whose base measure is 4′ 2″ by 3′ 6″, at the rate of 1 cu. ft. every 90 sec. How long will it take to fill the tank to a depth of 1′ 6″?

67. A 12-inch gun can fire a shell weighing 850 lb. every 30 sec. At this rate how many pounds of shell could a battleship fire from four 12-inch guns in 2 min. 30 sec.?

68. A train leaves a city at 10.45 A.M. and reaches another city $127\frac{1}{2}$ mi. distant at 5 minutes past 2 P.M. Allowing 20 minutes for stops, what is the rate per hour?

69. A farmer finds that a bin 8 ft. long, 3 ft. 6 in. wide, and 5 ft. deep holds 112 bu. How many bushels in a bin that is half as long, 6 in. wider, and 1 ft. less in depth?

70. If 12 men can lay the asphalt blocks in a certain length of a city street in 16 da., how many men must be added to the force, to complete the work 4 da. quicker?

71. A village has a water tank 42 ft. in diameter. In 3 hr. 45 min., when no water is being pumped in, the water is lowered 3 ft. How many gallons are drawn out on an average per hour? (Use $7\frac{1}{2}$ gal. to 1 cu. ft.)

72. The rates of the express and local trains are 45.6 mi. an hour and 28.4 mi. an hour respectively. What time is saved by taking the express train for a distance of 161.88 mi.?

73. From a tank full of water $\frac{3}{5}$ of the water was drawn off. The tank then lacked 70 gal. of being half full. What is the capacity of the tank?

74. A tank that holds $166\frac{1}{2}$ gal. is $\frac{2}{5}$ full of water. How long will it take a pipe supplying water at the rate of 3.7 gal. a minute to fill the rest of the tank?

75. A man who owned some shares of stock in a mill sold $\frac{1}{3}$ of his shares to A, $\frac{1}{5}$ to B, and $\frac{1}{6}$ to C. He then had 90 shares left. How many shares had he at first?

76. A boy lost $\frac{1}{6}$ of his kite string in a tree and 30 ft. of it in some telegraph wires. He then had $\frac{3}{4}$ of it left. How long was it at first?

77. How many bricks will be required for the walls of a house 42 ft. long, 28 ft. front, and 18 ft. high, deducting 3 doors 7 ft. 6 in. by 4 ft., and 9 windows 5 ft. by 3 ft., the walls to be 1 ft. thick, allowing 22 bricks to 1 cu. ft.?

78. The sugar contained in the sugar beet is $6\frac{1}{4}\%$ of the weight of the beet. In a recent year we produced in the United States 967,223,000 lb. of sugar beets. How many pounds of sugar did this represent?

79. A man has a salary of \$1750 a year. He pays 15% of his salary for rent, 25% for groceries, 12% for meats, 13% for clothing, and \$112.50 for other expenses. How much does he spend for each of these items, and what per cent of his salary does he save?

80. How many tons of coal can be placed in a bin 16 ft. 6 in. long, 7 ft. wide, and 6 ft. deep, allowing 33 cu. ft. to the ton?

81. The flow of water over Niagara Falls averages 264,000 cu. ft. a second. Suppose your recitation room is 25 ft. long, 20 ft. wide, and 10 ft. high. The water that flows over the Falls in the 2 min. that it takes you to solve this problem is sufficient to fill how many such rooms?

82. If a map is drawn to a scale of $\frac{1}{1000000}$, what part of an inch on the map represents 1 mi. on the earth's surface? How many miles are represented by 1 in. on the map?

83. A man spent 12% of his money for a lot and 49% for a house. He then had $3510 left. How much did he spend for the lot? How much for the house?

84. How many loads (cubic yards) of gravel will be required to grade $3\frac{1}{2}$ mi. of road, the gravel to be laid 12 ft. wide and 6 in. deep?

85. How many cubic feet of water flow under a bridge every hour if the stream is 240 ft. wide, averages 4 ft. 8 in. deep, and flows at the rate of 2 mi. an hour?

86. If the consumption of sugar in the United States averages 81.17 lb. for each person, how many tons did we consume when our population was 90,000,000?

87. The sugar in the sugar beet being $6\frac{1}{4}$% of the weight of the beet, how many pounds of sugar will be obtained from $3\frac{1}{2}$ A. of beets, producing 400 bu. per acre?

88. A silver dollar weighs $412\frac{1}{2}$ grains. How many tons avoirdupois will a million silver dollars weigh?

89. A tinsmith wishes to make a cubical tin can that shall hold exactly a liquid quart. A quart contains $57\frac{3}{4}$ cu. in. Find the edge of the cube to the nearest tenth of an inch.

90. How many boards 15 ft. long will be required to build a fence 5 boards high about a rectangular field 75 rd. long and 45 rd. wide?

91. Find the difference in acres between two fields, one containing 100 sq. rd. and the other being 100 rd. square.

92. Find the cost of a stair carpet at $1.50 a yard, for a straight flight of 16 steps 12 in. wide and 8 in. rise, allowing 20 in. extra for the bottom of the stairs and the same, besides the 12 in., for the top.

93. Flint glass contains by weight 72% sand, 14% soda, 12% lime, and 2% alumina. Find the number of pounds of each of these substances in 650 lb. of flint glass.

94. A bell weighing 875 lb. is composed by weight of 77% copper, 2% zinc, and the rest tin. How many pounds of each metal does the bell contain?

95. Gun metal is composed by weight of 84% copper and the rest tin. How many pounds of copper must be put with 112 lb. of tin to make gun metal?

96. Bronze is composed by weight of 18% tin and the rest copper. How many pounds of tin must be put with 1066 lb. of copper to make this quality of bronze?

97. The driving wheels of a locomotive have a diameter of 6 ft. 5 in. How many revolutions a minute will each wheel make when the locomotive is traveling at the rate of 50 mi. an hour? (Take $3\frac{1}{7}$ as the ratio of the circumference to the diameter.)

98. The front wheels of a wagon are 3 ft. 6 in. in diameter and the rear wheels are 4 ft. 1 in. How many more revolutions will a front wheel make than a rear wheel in going a mile?

99. It is required to make a rectangular tank that shall contain exactly 1 gal. (231 cu. in.) Required to know the three dimensions such that they shall each be in exact inches.

100. A cubic inch of iron weighs $4\frac{4}{9}$ oz. Find the weight in pounds of an iron bar 2 in. square at the end and 12 ft. long.

101. An emery wheel may safely have a grinding speed of 5500 ft. a minute on the circumference. If the wheel has a diameter of $10\frac{1}{2}$ in., how many revolutions must it make a minute to have this speed? (Take $3\frac{1}{7}$ as the ratio of the circumference to the diameter.)

102. A brush polishing wheel may safely have a speed of 11,000 ft. a minute on the circumference. If the wheel has a diameter of 9.1 in., how many revolutions must it make a minute to have this speed? (Take $3\frac{1}{7}$ as in Ex. 101.)

103. In a machine shop it is necessary to know the equivalents, to 4 decimal places, of the common fractions most frequently used. Express $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$, $\frac{11}{16}$, $\frac{13}{16}$, $\frac{15}{16}$ to the nearest ten-thousandth.

104. How many square feet of sheet copper will be required to make a rectangular tank 5 ft. long, $2\frac{1}{2}$ ft. wide, and $1\frac{3}{4}$ ft. deep, allowing 10% extra for waste?

105. If sheet copper weighs 12 lb. per square foot and costs 26¢ a pound, what will be the cost of the copper required to line a rectangular tank 3 ft. long, 2 ft. wide, and 18 in. deep, allowing 10% extra for waste? What will be the weight of the tank?

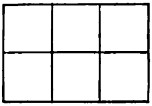
106. A gallon contains 231 cu. in. What is the diameter of an oil tank 7 ft. long that contains 2592 gal.? (Take $3\frac{1}{7}$ as in Ex. 101.)

107. One ship fired a signal to another, which was answered as soon as it was heard. The first ship heard the answering gun $18\frac{1}{4}$ sec. after the first gun was fired. What was the distance between the ships, sound traveling, at the temperature then observed, 1142 ft. a second?

CHAPTER XI

VOCATIONAL ALGEBRA

209. Algebraic Forms. In arithmetic we have learned that the area of a rectangle equals the product of its base and height. We express this more briefly in *algebraic* form as follows:



$$a = bh.$$

Here we have let a stand for the word *area*, b for the word *base*, and h for the word *height*. When we write bh it indicates the *product* of b and h.

In arithmetic 25 means $20 + 5$, but in algebra bh means $b \times h$.

When two letters are written side by side in algebra, with no sign between them, the product of their numerical values is to be taken.

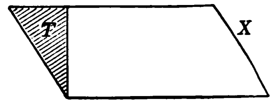
Thus if $b = 4$ and $h = 2$, then $a = bh = 4 \times 2 = 8$. In writing algebraic forms it is not customary to express denominations, like feet or inches. When we speak of the product of two lines, this is a brief way of expressing the product of their *numerical values*.

EXERCISE 141

Given $a = bh$, find the value of a when:

1. $b = 5$, $h = 3$.
2. $b = 7$, $h = 2\frac{1}{2}$.
3. $b = 9$, $h = 4\frac{1}{3}$.
4. $b = 12$, $h = 5\frac{3}{4}$.
5. $b = 27.4$, $h = 5.4$.
6. $b = 52.6$, $h = 7.9$.
7. $b = 125\frac{3}{4}$, $h = 26\frac{1}{4}$.
8. $b = 137\frac{1}{4}$, $h = 52\frac{1}{2}$.
9. $b = 325.75$, $h = 124.25$.
10. $b = 438.25$, $h = 226.75$.

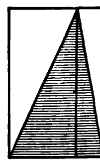
210. Parallelogram. We have learned that the area of a parallelogram equals the product of its base and height. We may therefore express this in algebraic form thus: $a = bh$.



For the triangle T may be cut off and placed at X, the parallelogram then becoming a rectangle.

If $b = 5$ and $h = 3\frac{1}{2}$, then $a = bh = 5 \times 3\frac{1}{2} = 17\frac{1}{2}$. If b and h represent inches, then a represents square inches.

211. Triangle. We have learned that the area of a triangle equals half the product of its base and height. This is expressed in algebraic form thus: $a = \frac{1}{2}bh$.



For the triangle may be cut as here shown so that it is seen to be half of the rectangle of base b and height h.

If $b = 7$ and $h = 10\frac{1}{2}$, then $a = \frac{1}{2}bh = \frac{1}{2}$ of $7 \times 10\frac{1}{2} = 36\frac{3}{4}$.

EXERCISE 142

Given $a = bh$, find the value of a when:

1. $b = 7.2$, $h = 3.4$.
2. $b = 8.9$, $h = 4.7$.
3. $b = 127\frac{3}{4}$, $h = 27.4$.
4. $b = 22.75$, $h = 14\frac{3}{4}$.

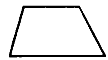
Given $a = \frac{1}{2}bh$, find the value of a when:

5. $b = 42$, $h = 25$.
6. $b = 36$, $h = 19$.
7. $b = 56$, $h = 27$.
8. $b = 23.8$, $h = 4.75$.
9. $b = 41.2$, $h = 19.65$.
10. $b = 33.3$, $h = 24.50$.

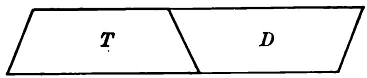
11. A playground is l feet long and w feet wide. Find the area in square feet. What is the number of square feet when $l = 124$ and $w = 62\frac{1}{2}$?

12. A triangular space at the end of a house, under the roof, is f feet wide and h feet high. What is the area in square feet?

212. Trapezoid. We have learned that the area of a trapezoid equals half the product of the sum of the two parallel sides multiplied by the height.



For an equal trapezoid may be turned over and put down by the side of the first one, as here shown. The whole figure, or twice the trapezoid, then equals a parallelogram whose base is the sum of the parallel sides of the trapezoid. The trapezoid is therefore half this parallelogram.



We indicate this in algebraic form as follows:

$$a = \tfrac{1}{2}(b + b')h,$$

where b and b' ("b prime") are the two parallel sides, usually called the *bases*, and h is the height.

The parentheses show that b and b' are to be added before being multiplied by h. The operations indicated within parentheses are always performed first. Thus, if $b = 6$, $b' = 5$, and $h = 4$, we have $a = \frac{1}{2}(b + b')h = \frac{1}{2}(6 + 5) \times 4 = \frac{1}{2}$ of $11 \times 4 = 22$.

EXERCISE 143

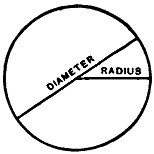
Given $a = \frac{1}{2}(b + b')h$, find the value of a when:

1. $b = 6,\ b' = 4,\ h = 3.$
2. $b = 9,\ b' = 5,\ h = 7.$
3. $b = 11,\ b' = 8,\ h = 6.$
4. $b = 13,\ b' = 7,\ h = 7.$
5. $b = 19,\ b' = 9,\ h = 9.$
6. $b = 21,\ b' = 9.5,\ h = 7.2.$
7. $b = 32,\ b' = 16,\ h = 19.$
8. $b = 28,\ b' = 9.8,\ h = 8.6.$
9. $b = 4.9,\ b' = 3.4,\ h = 1\frac{1}{2}.$
10. $b = 4\frac{3}{4},\ b' = 3\frac{1}{4},\ h = 2\frac{1}{2}.$

11. The lower base of a trapezoid is 10 in. and the altitude and upper base are each 5 in. Find the area.

12. A field is in the form of a trapezoid, with bases x and y, and with altitude z. What is the area of the field? How many square rods are there in the field if $x = 30$ rods, $y = 26$ rods, and $z = 24$ rods?

213. Circumference. We have learned that the circumference of a circle equals $3.1416 \times$ the diameter. In mathematics the number 3.1416, which is nearly $3\frac{1}{7}$, is represented by the Greek letter π (pronounced *pī*). We may therefore express this law as follows:



$$c = \pi d,$$

where c stands for circumference, d for diameter, and π for $3\frac{1}{7}$ or 3.1416, nearly.

Since the diameter equals twice the radius, we may write $2r$ for d, and have $c = \pi \times 2r$, or $c = 2\pi r$.

Thus if $d = 7$, and we take $3\frac{1}{7}$ as the value of π, we have

$$c = \pi d = 3\tfrac{1}{7} \times 7 = 22.$$

If $r = 5$, we have $c = 2\pi r = 2 \times 3\frac{1}{7} \times 5 = 31\frac{3}{7}$.

EXERCISE 144

Given $c = \pi d = 2\pi r$, and taking $\pi = 3\frac{1}{7}$, find c when:

1. $d = 14$.	**4.** $d = 3.5$.	**7.** $r = 7$.	**10.** $r = 6.3$.
2. $d = 21$.	**5.** $d = 4.9$.	**8.** $r = 3.5$.	**11.** $r = 9.1$.
3. $d = 28$.	**6.** $d = 7.7$.	**9.** $r = 5.6$.	**12.** $r = 10\frac{1}{2}$.

Taking $\pi = 3.1416$, find c when:

13. $d = 10$.	**15.** $d = 50$.	**17.** $r = 30$.	**19.** $r = 2\frac{1}{2}$.
14. $d = 20$.	**16.** $d = 25$.	**18.** $r = 40$.	**20.** $r = 7.5$.

21. A workman measures the diameter of a steel shaft and finds it to be $3\frac{7}{16}$ in. What is the circumference?

22. If you describe a circle with a radius of 12 in., how many inches will there be in the circumference?

23. If a globe has a diameter of 14 in., what is the circumference?

214. Area of a Circle. We have learned that

1. The area of a circle equals half the product of the circumference and radius.

2. The area of a circle equals 3.1416 times the square of the radius.

We may express the first of these laws as follows:

$$a = \tfrac{1}{2}cr.$$

To express the second of these laws, since $c = 2\pi r$ (§ 213) we may put $2\pi r$ for c, and then we have $a = \frac{1}{2}$ of $2\pi rr$, or, writing r^2 ("r square") for rr,

$$a = \pi r^2.$$

Thus if $r = 10$, we have $a = \pi r^2 = 3.1416 \times 10^2 = 314.16$.

EXERCISE 145

Given $a = \frac{1}{2}cr$, find a when:

1. $r = 5$, $c = 31.416$.
2. $r = 10$, $c = 62.832$.
3. $r = 20$, $c = 125.664$.
4. $r = 2\frac{1}{2}$, $c = 15.708$.
5. $r = 25$, $c = 157.08$.
6. $r = 50$, $c = 314.16$.

Given $a = \pi r^2$, and taking $\pi = 3\frac{1}{7}$, find a when:

7. $r = 7$.
8. $r = 3\frac{1}{2}$.
9. $r = 14$.
10. $r = 2.1$.
11. $r = 2.8$.
12. $r = 35$.
13. $r = 4.9$.
14. $r = 7.7$.

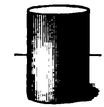
Given $a = \pi r^2$, and taking $\pi = 3.1416$, find a when:

15. $r = 5$.
16. $r = 10$.
17. $r = 20$.
18. $r = 50$.

19. How many square feet in the area of a circle whose radius is 2 ft.? (Take $\pi = 3.1416$.)

20. How many square feet in the circular base of a water tank whose radius is 15 ft.? (Take $\pi = 3.1416$; then take $\pi = 3\frac{1}{7}$, and find the difference in the results.)

215. Cylinder. We have found that the volume of a cylinder equals the product of its base and height; that is, $v = bh$, where v stands for volume, b for base, and h for height.



But since $b = \pi r^2$ (§ 214), we may put πr^2 for b, and then we have

$$v = \pi r^2 h.$$

216. Formula. We therefore see that algebra enables us to put a long rule of arithmetic into a brief form. An algebraic form of this kind is called a *formula*.

When we find the value of an algebraic expression by putting numbers in place of the letters, we are said to *substitute* numbers for the letters and to *evaluate* the formula.

EXERCISE 146

Express the following statements in formulas:

1. The lateral area of a cylinder equals the product of the circumference and height. (Use l for lateral area, c for circumference, and h for height.)

2. The lateral area of a cylinder equals 3.1416 times the diameter multiplied by the height.

3. The lateral area of a cylinder equals twice 3.1416 times the radius multiplied by the height. (Use r for radius, and in general in a formula use the initial letter for a word.)

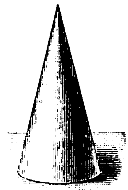
4. The volume of a prism equals the product of the base and height.

5. The volume of a pyramid equals one third the product of the base and height.

6. The volume of a cone equals one third the product of 3.1416 times the square of the radius of the base multiplied by the height.

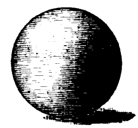
7. The formula for the lateral surface of a cone in terms of the circumference of the base and k the distance from the vertex to the circumference of the base is $l = \frac{1}{2}kc$. Write this as an ordinary sentence.

8. The formula for the lateral surface of a cone in terms of the radius of the base and the distance from the vertex to the circumference of the base is $l = \pi kr$. Write this as an ordinary sentence.



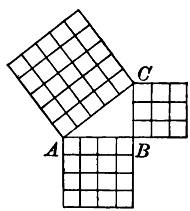
9. The formula for the surface of a sphere is $s = 4\pi r^2$. Write this as an ordinary sentence.

10. The formula for the volume of a sphere is $v = \frac{4}{3}\pi r^3$, where r^3 ("r cube") equals rrr. Write this as an ordinary sentence.



11. The square on the hypotenuse of a right triangle has been shown to be equal to the sum of the squares on the other two sides. Letting $AB = x$, $BC = y$, and $AC = z$, we have $z^2 = x^2 + y^2$, or $z = \sqrt{x^2 + y^2}$. Suppose $x = 4$ and $y = 3$, find the value of z.

(The expression $\sqrt{x^2 + y^2}$ means that we are to square x, and to square y, then to add these results, and then to extract the square root of the sum.)



12. Given $z = \sqrt{x^2 + y^2}$, find the value of z when $x = 24$ and $y = 32$.

13. In the same formula find the value of z when $x = 36$ and $y = 27$.

14. In the same formula find the value of z when $x = 66$ and $y = 88$; when $x = 36$ and $y = 48$.

15. In the same formula find the value of z when $x = 56$ and $y = 42$; when $x = 8\frac{2}{3}$, and $y = 11\frac{1}{3}$.

Evaluate the following by substituting the given values:

16. $l = dh$, where $d = 16.4$ and $h = 7$.

17. $l = ch$, where $c = 17.6$ and $h = 19.3$.

18. $z = \sqrt{x^2 + y^2}$, where $x = 33$ and $y = 44$.

19. $v = bh$, where $b = 147.6$ and $h = 78.4$.

20. $v = \frac{1}{3} bh$, where $b = 275.4$ and $h = 124.2$.

21. $v = \frac{1}{3} \pi r^2 h$, where $r = 9$, $h = 7$, and $\pi = 3\frac{1}{7}$.

22. $l = \frac{1}{2} dc$, where $d = 10$ and $c = 31.416$.

23. $l = \pi kc$, where $k = 7$, $c = 12$, and $\pi = 3\frac{1}{7}$.

24. $l = \pi kr$, where $k = 5$, $r = 7$, and $\pi = 3\frac{1}{7}$.

25. $s = 4 \pi r^2$, where $r = 10$ and $\pi = 3.1416$.

26. $v = \frac{4}{3} \pi r^3$, where $r = 3$ and $\pi = 3.1416$.

27. If an automobile has a constant velocity of 10 mi. an hour, how far will it go in 4 hr.? Write a formula for d, the distance it will go in t hr. at v mi. per hour.

28. What is the volume of a box 9 in. long, 8 in. wide, and 6 in. deep? Write a formula for v, the volume of a box l in. long, w in. wide, and d in. deep.

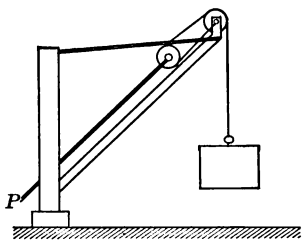
29. The formula for the radius in terms of the circumference is $r = \frac{c}{2\pi}$. Write this as an ordinary sentence.

30. If it is c in. around an iron pipe, the diameter is $\frac{c}{\pi}$ in.; that is, $d = \frac{c}{\pi}$. Write this as an ordinary sentence.

31. The circumference of a water pipe is 22 in. Using the formula in Ex. 30, and taking $3\frac{1}{7}$ as the value of π, find the diameter.

32. The circumference of a water tank is 44 ft. Using the formula of Ex. 29, and taking $3\frac{1}{7}$ as the value of π, find the radius.

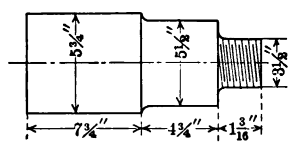
33. A man who runs a stationary engine for hoisting iron reads in a book about engines that if s is the area of the outside shell of his boiler and h is the heating surface, then $h = \frac{2}{3}s$. What does this mean? What is the value of h if $s = 120\frac{5}{16}$ sq. ft.?



34. A foreman of a shop has a hoist like this for lifting heavy weights. He reads in a trade journal that in order to lift a weight w, the power p must be such that $p = 2w$. What power must his engine apply to the cable so as to lift a weight of 15,000 lb.? What weight could be lifted with a power of 75.4 T.?

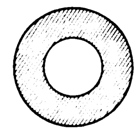
35. If a cubic inch of steel weighs 0.28 lb., what will be the weight of a steel cylinder d in. in diameter and l in. long, it being known that the volume is $v = \frac{1}{4}\pi d^2 l$? What will it be if $d = 1$, $l = 7$, and $\pi = 3\frac{1}{7}$?

36. A machinist is making a "crank pin" (a kind of bolt) for an engine, according to this drawing. He considers it as weighing the same as three steel cylinders having the diameters and lengths in inches as here shown, where $7\frac{3}{4}''$ means $7\frac{3}{4}$ in. He has this formula for the weight (w) of a steel cylinder where d is the diameter and l is the length: $w = 0.07\pi d^2 l$. Taking $\pi = 3\frac{1}{7}$, find the weight of the pin.



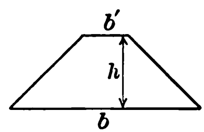
37. The volume of a cylinder being represented by $v = \pi r^2 h$, what will the water in a cylindrical water tank weigh, when $r = 9$ ft., $h = 10$ ft., $\pi = 3\frac{1}{7}$, and the weight of 1 cu. ft. of water is $62\frac{1}{2}$ lb.?

38. This figure represents the cross section of a water pipe. If the radius of the outer circle is r and that of the inner circle is r', then the area of the shaded portion (the iron) is found by this formula: $a = \pi(r^2 - r'^2)$. Find the value of a if $r = 4$ and $r' = 3$. (Use $\pi = 3\frac{1}{7}$.)



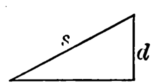
39. The weight of a hollow steel pillar whose length is l, and the radii of whose outer and inner circles are r and r', is $483\, l\pi(r^2 - r'^2)$ pounds, where l, r, and r' are expressed in feet. Find the weight when $l = 12$, $r = \frac{1}{3}$, $r' = \frac{1}{6}$, and $\pi = 3\frac{1}{7}$.

40. If this represents the cross section of a railway embankment l ft. long, h ft. high, b ft. wide at the bottom, and b' ft. wide at the top, the number of cubic feet in the embankment is represented by $v = \frac{1}{2}\, lh(b + b')$. Find the volume if $l = 300$, $h = 8$, $b = 60$, and $b' = 28$.

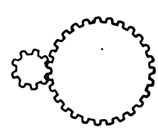


41. A cast-iron bar is l ft. long, and its cross section is a square e in. on a side. Give a formula for its volume in cubic inches; also in cubic feet. If $l = 6$ and $e = 2$, what will the bar weigh, allowing 0.27 lb. to the cubic inch?

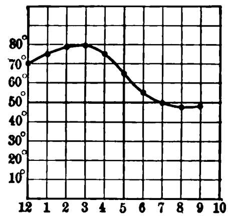
42. If a barrel weighing w lb. is rolled up an incline s ft. long, to a point d ft. high, there is exerted a power of $\frac{wd}{s}$; that is, $p = \frac{wd}{s}\cdot$ How much power must be used to roll a 200-pound barrel up a 10-foot incline to a height of 4 ft.?



43. Two cogwheels, one having 9 cogs and the other 27, are fitted together. How many times will the smaller wheel turn for each turn of the larger? How many times, if the larger has a cogs and the smaller b cogs?



217. Curve Tracing. In this figure the successive hours of the day from noon to 9 P.M. are represented on a horizontal line, and the temperatures are represented by points on the vertical lines. It shows that at noon the temperature was 70°, at 1 P.M. 75°, at 2 P.M. 78°, at 3 P.M. 80°, and so on. If we connect these points by a curve, this line gives us a picture of the change in temperature for nine hours.



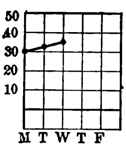
EXERCISE 147

Rule some paper and trace curves to show the following:

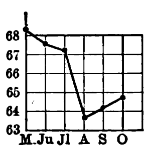
1. The temperature on a certain day varied as follows: 6 A.M. 50°, 7 A.M. 52°, 8 A.M. 55°, 9 A.M. 65°, 10 A.M. 68°, 11 A.M. 70°, noon 75°, 1 P.M. 76°, 2 P.M. 77°, 3 P.M. 77°, 4 P.M. 75°, 5 P.M. 73°, 6 P.M. 68°.

2. The temperature on a certain day varied as follows: midnight 48°, 3 A.M. 45°, 6 A.M. 49°, 9 A.M. 60°, noon 75°, 3 P.M. 80°, 6 P.M. 70°, 9 P.M. 60°, midnight 50°.

3. The attendance in a certain eighth-grade class was 30 on Monday, 32 on Tuesday, 35 on Wednesday, 30 on Thursday, and 28 on Friday. Part of the line is here shown. Trace the entire line.



4. The per cent of games won to games played up to the first day of May, June, and so on to December, by a certain team, was 68.4%, 67.6%, 67.2%, 63.7%, 64.2%, 64.7%, 65.5%, 67%. Trace the entire line of which a portion is here shown.



5. The pressure on the steam gauge of a boiler varies as follows: 6 A.M. 50 lb., 7 A.M. 112 lb., 8 A.M..122 lb., 9 A.M. 120 lb., 10 A.M. 116 lb., 11 A.M. 120 lb., noon 114 lb., 12.30 P.M. 110 lb., 1 P.M. 115 lb., 2 P.M. 120 lb., 3 P.M. 116 lb., 4 P.M. 118 lb., 5 P.M. 114 lb., 6 P.M. 30 lb.

6. A man's income for ten consecutive years, stated in hundreds of dollars, was as follows: 9.7, 10.1, 10.5, 12, 13.5, 15, 15.2, 15.7, 17, 20.

7. A boy's height from the age of 5 to the age of 15, stated in inches, varied as follows: 5 yr., 42; 6 yr., 44; 7 yr., 46; 8 yr., 49; 9 yr., 52; 10 yr., 54; 11 yr., 56; 12 yr., 58; 13 yr., 61; 14 yr., 63; 15 yr., 68.

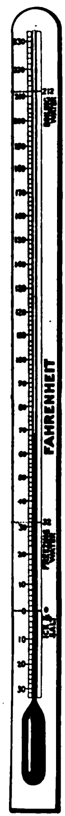
8. A girl's height from the age of 5 to the age of 15, stated in inches, varied as follows: 5 yr., 42; 6 yr., 44; 7 yr., 45; 8 yr., 48; 9 yr., 50; 10 yr., 52; 11 yr., 54; 12 yr., 58; 13 yr., 60; 14 yr., 63; 15 yr., 65.

9. The population of the United States, in millions, for various years was as follows: 1820, 10; 1830, 13; 1840, 17; 1850, 23; 1860, 31; 1870, 39; 1880, 50; 1890, 63; 1900, 76; 1910, 91. In such a case it is convenient to use 0.1 in. to represent 10 million.

10. The population of the United States at a recent census, arranged according to age, in millions, was as follows: 10 years of age, 1.7; 20 years, 1.5; 30 years, 1.5; 40 years, 1.2; 50 years, 0.9; 60 years, 0.5; 70 years, 0.3; 80 years, 0.09; 90 years, 0.01. In such a case it is convenient to use 1 in. to represent 1 million, merely estimating the small fractions 0.09 and 0.01.

11. The capital of the national banks of the United States, in hundred millions of dollars, has varied as follows: 1880, 545; 1890, 625; 1900, 608; 1910, 940. (Use 1 in. to represent 100 billion, 545 million being about $\frac{1}{2}$ in.)

218. Numbers below Zero. Sometimes the temperature goes below zero. When it is necessary to distinguish between temperature below zero and temperature above zero, we write 10° above zero, $+10°$; and 10° below zero, $-10°$.



If the temperature is 20° above zero and it decreases 15°, it is then 5° above zero, or $+5°$. If it decreases 5° more, it is then 0°. If it decreases 5° more, it is 5° below zero, or $-5°$. If it decreases 20° more, it is 25° below zero, or $-25°$.

We therefore find a new meaning for the signs + and −. They not only indicate addition and subtraction (signs of *operation*), but they tell on which side of zero a number is (signs of *quality*).

219. Positive Numbers. The ordinary numbers which we use in arithmetic are called *positive numbers.*

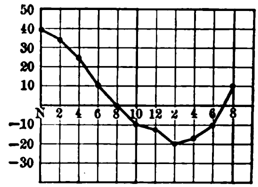
Thus 3°, 3 in., $\frac{3}{4}$, $\sqrt{3}$, are all positive numbers. If we wish to make this fact emphatic, we may write them thus: $+3°$, $+3$ in., $+\frac{3}{4}$, $+\sqrt{3}$, but otherwise the + sign is unnecessary here. The expression $+3$ is read "positive 3" or "plus 3."

220. Negative Numbers. Numbers on the other side of zero from positive numbers are called *negative numbers.*

Thus $-3°$ is a negative number. If distance upwards, above the earth's surface, is called positive, distance below the surface may be called negative, so that we may have $+10$ ft. and -10 ft. The expression -3 is read "negative 3" or "minus 3."

If the temperature is 20° below zero, or $-20°$, and it increases 5°, it is then 15° below zero, or $-15°$. If it increases 10° more, it is 5° below zero, or $-5°$. If it increases 5° more, it is then zero, or 0°. If it increases 5° more, it is 5° above zero, or $+5°$.

221. Curve Tracing with Negative Numbers. If the temperature in Chicago, on a winter's day, varies from + 40° at noon to − 20° at 2 o'clock the next morning, as here shown, the curve tells us that it was below zero from 8 P.M. until about 7 the next morning.



We represent negative numbers below the horizontal line, which indicates zero.

EXERCISE 148

Trace the curves to show the following variations in temperature:

1. Noon 32°, 2 P.M. 40°, 4 P.M. 30°, 6 P.M. 28°, 8 P.M. 20°, 10 P.M. 5°, midnight −10°, 2 A.M. −12°, 4 A.M. −5°, 6 A.M. 0°, 8 A.M. 10°, 10 A.M. 30°, noon 35°.

2. Noon 45°, 2 P.M. 40°, 4 P.M. 38°, 6 P.M. 20°, 8 P.M. 0°, 10 P.M. −5°, midnight −6°, 2 A.M. −5°, 4 A.M. 0°, 6 A.M. 2°, 8 A.M. 2°, 10 A.M. 20°, noon 50°.

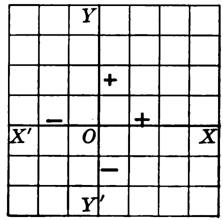
3. Noon 0°, 2 P.M. 5°, 4 P.M. 20°, 6 P.M. 15°, 8 P.M. 0°, 10 P.M. −2°, midnight −10°, 2 A.M. −8°, 4 A.M. −2°, 6 A.M. 10°, 8 A.M. 30°, 10 A.M. 32°, noon 40°.

4. Noon 10°, 1 P.M. 12°, 2 P.M. 14°, 3 P.M. 10°, 4 P.M. 0°, 5 P.M. −2°, 6 P.M. −5°, 7 P.M. −3°, 8 P.M. 0°, 9 P.M. −5°, 10 P.M. −12°, 11 P.M. −8°, midnight 0°.

5. Noon 5°, 1 P.M. 8°, 2 P.M. 10°, 3 P.M. 5°, 4 P.M. −5°, 5 P.M. −7°, 6 P.M. −8°, 7 P.M. −9°, 8 P.M. −12°, 9 P.M. −15°, 10 P.M. −20°, 11 P.M. −21°, midnight −20°.

6. Noon −2°, 1 P.M. 0°, 2 P.M. 5°, 3 P.M. 0°, 4 P.M. −5°, 5 P.M. −6°, 6 P.M. −9°, 7 P.M. −10°, 8 P.M. −15°, 9 P.M. −16°, 10 P.M. −20°, 11 P.M. −15°, midnight −10°.

222. Other Uses of Negative Numbers. If we call some special point on a line zero (0), we usually call distances to the right positive and distances to the left negative, just as we call distances up (as on the thermometer) positive and distances down negative. But because we usually call weight positive, we speak of the weight of a balloon (which pulls upward) negative.



These are some illustrations of negative numbers:

If a man is worth \$1000, we may say that he has + \$1000; but if he is \$1000 in debt, we may say that he is worth − \$1000.

If we call latitude north of the equator positive, we may call latitude south of the equator negative.

If we call longitude west of Greenwich positive, we may call longitude east of Greenwich negative.

If we call the motion of a piston rod of an engine positive when it is to the right, we should call the motion negative when it is to the left.

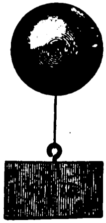
If we call steam pressure positive, we might speak of a vacuum as having negative pressure.

If we call distance above the earth's surface positive, we should call distance below the earth's surface negative.

We therefore see that negative numbers are just as real as positive numbers, for the temperature is just as real when the thermometer indicates that it is below zero as it is when the mercury rises above zero, and a man's debts are just as real as his capital.

In ancient times people used only whole numbers (integers). Other kinds of numbers were invented as they became necessary, and these are sometimes called artificial numbers. Artificial numbers like $\frac{3}{4}$, $\sqrt{3}$, and -3 all have their uses, as we have seen, not only in algebra but also in practical life.

223. Adding Negative Numbers. If we tie to a 10-pound weight a toy balloon that pulls upward 1 pound, what will the two together weigh?



To add a positive number to a negative number, take their numerical difference and prefix the sign of the numerically greater number.

Thus $+10$ lb. and -1 lb. are $+9$ lb.;
$+10$ lb. and -10 lb. are 0 lb.;
$+10$ lb. and -15 lb. are -5 lb.

That is, if a 10-pound weight is tied to a balloon that pulls upward 15 lb. (or weighs -15 lb.), the two would weigh -5 lb. We also see that -10 lb. and -5 lb. are -15 lb.

EXERCISE 149

1. What is the combined weight of $+25$ lb. and -5 lb.? of $+25$ lb. and -25 lb.? of -25 lb. and $+5$ lb.?

2. What is the combined weight of 30 lb. and -60 lb.? of 40 lb. and -45 lb.? of 100 lb. and -75 lb.? of -100 lb. and -75 lb.? of -100 lb. and -100 lb.?

3. A freight engine is switching in front of a station. If it runs 400 ft. to the right of the station ($+400$ ft.) and then backs 525 ft. (-525 ft.), how many feet is it from the station? (Add 400 and -525.)

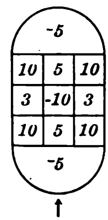
4. In drilling a well the drill is raised $+8$ ft. above the surface of the ground. It is then dropped 15 ft. (-15 ft.). Where is it then with respect to the surface? (Add 8 and -15. A negative distance above the surface means distance below the surface.)

5. A boy is fishing in deep water with a line 20 ft. long. If the tip of the pole is $+6$ ft. above the water, how far is the sinker from the surface of the water, if it is 3 ft. from the hook? (Add 6 and -20 and 3.)

6. A man who was $80 in debt paid $30. How much was he then in debt? Suppose he earns $50 more, how much is he then worth? (Add – 80 and 30. Then add 50.)

7. A man who was worth $3500 lost $1750 and then earned $900. How much was he then worth? (Add 3500, – 1750, and 900.)

8. A man who was $350 in debt contracted another debt of $200. He then earned $1000. How much was he then worth?



9. A game is played by throwing bean bags in the direction of the arrow. Suppose the score stands – 5, 3, 10, 10, 0, – 10, 5, 10, 10, how much is the total score?

10. If this board without any weights at the ends just balances, and if I put 5 lb. at one end and 8 lb. at the other end, how much must I add to the 5 lb. to make it balance? Instead of adding to the 5 lb., how much must I add to the 8 lb. to make it balance?

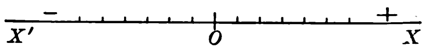
11. A boat that runs 16 mi. an hour in still water is going against a stream flowing 4 mi. an hour. What is the rate at which the boat will travel? (16 mi. and – 4 mi. are how many miles?)

12. If a mine is opened 300 ft. above the base of a mountain and a shaft is sunk 700 ft., how much is the base of the shaft above or below the base of the mountain?

Add the following:

13.	14.	15.	16.	17.
75	– 60	– 30	– 90	– 35
– 80	20	– 40	– 72	– 75
10	30	70	– 86	23

224. Subtracting Negative Numbers. If the temperature is $-10°$ at midnight and $+40°$ at noon, the difference in temperature is evidently $50°$, for the mercury must rise $10°$ to reach $0°$, and $40°$ more to reach $+40°$. Likewise, in this figure the difference between -2 and $+4$ is evidently 6; for a point must move 2 spaces to get from -2 to 0, and 4 more to reach $+4$.



To subtract a negative number we may obtain the same result by adding a positive number with the same numerical value.

That is, $4-(-2)=4+(+2)=6$. Likewise, $-4-(-2)=-4+2=-2$. We also see that $-4-2=-6$, as in § 223.

EXERCISE 150

1. How much difference in price is there in selling a horse \$25 below cost or \$30 above cost?

2. The temperature on one January morning in Denver was $+9°$, and the next day it was $-5°$. What was the difference in temperature?

3. If there is a house for every number, how many houses would you pass in going from 42 East Washington Street to 15 West Washington Street, including both these houses?

4. Jefferson Street is 5 blocks east of Adams Street, and Monroe Street is 8 blocks west of Adams Street. Monroe Street is how many blocks west of Jefferson Street?

Subtract the following:

5. -3 from 0. **7.** -9 from 17. **9.** -39.4 from 82.7.

6. -5 from 12. **8.** -7 from 15. **10.** $-48\frac{2}{3}$ from $15\frac{1}{3}$.

225. Multiplying and Dividing Negative Numbers. We multiply and divide negative numbers just as we multiply and divide positive numbers. If a man has $-$ \$5 (is \$5 in debt), he will have $-$ \$10 if he is twice as much in debt.

That is
$$2 \times (-\$5) = -\$10,$$
$$(-\$10) \div 2 = -\$5.$$

EXERCISE 151

1. If one balloon pulls up 300 lb. (weighs $-$ 300 lb.), what will be the upward pull of 3 such balloons? Represent the result as a negative number.

2. If the thermometer indicates $-4°$, what is the temperature when it indicates half as much below zero?

3. If a carrier pigeon can fly 40 mi. an hour in still air, at what rate will it fly against a 20-mile wind? against a wind that blows twice as fast? three times as fast?

4. A checker of bales of cotton allows 500 lb. to the bale. He finds one bale 18 lb. short, a second bale twice as much short, and a third bale half as much short in weight. Express these shortages in algebraic language.

5. A man's debts amounted last year to \$375. The year before they were 4 times as much. This year he has paid his debts and has \$925 in the bank. What is the difference between his financial standing two years ago and now?

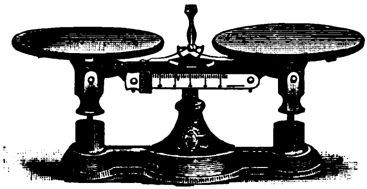
Multiply the following:

6.	7.	8.	9.	10.	11.
-25	-47	-39	-57	-63	-77
3	6	8	21	18	33

Divide the following:

12. $-125 \div 25$. 13. $-121 \div 11$. 14. $-49.3 \div 1.7$.

226. The Equation. If we take a pair of scales, they will exactly balance if they are empty. If we add 3 oz. to each side, they will still balance. If we multiply each of these 3 oz. by 6, they will still balance. If we divide the weights by 2, they will still balance. If we tie to each side a balloon that pulls up 10 lb., they will still balance.



An expression of equality between two quantities is called an *equation.*

For example, $x + 5 = 7$. Here we see that $x = 2$.

227. The Principles of the Equation. We have seen that

(1) *If equals are added to equals, the results are equal.*

(2) *If equals are subtracted from equals, the results are equal.*

(3) *If equals are multiplied by equals, the results are equal.*

(4) *If equals are divided by equals, the results are equal.*

228. Solving an Equation by Subtracting. If we have

$$x + 5 = 7,$$

we may subtract 5 from these equals (§ 227), and then

$$x = 7 - 5 = 2.$$

EXERCISE 152

Solve the following:

1. $x + 6 = 10$.
2. $x + 9 = 15$.
3. $x + 7 = 32$.
4. $x + 12 = 70$.
5. $x + 15 = 69$.
6. $x + 27 = 52$.
7. $x + 29 = 101$.
8. $x + 37 = 105$.
9. $x + 48.5 = 100$.
10. $x + 62\frac{3}{4} = 127$.

229. Solving an Equation by Dividing. If we have the equation

$$3x = 27,$$

we may divide these equals by 3 (§ 227), and obtain

$$x = 9.$$

If we have the equation

$$3x + 7 = 34,$$

we may first subtract 7 from these equals, and obtain

$$3x = 27.$$

We may then divide by 3, and we find the result,

$$x = 9.$$

EXERCISE 153

Solve the following:

1. $3x = 30.$
2. $3x = 72.$
3. $5x = 95.$
4. $6x = 96.$
5. $7x = 91.$
6. $8x = 104.$
7. $9x = 153.$
8. $17x = 187.$
9. $23x = 483.$
10. $3.8x = 34.2.$
11. $28.6x = 20.02.$
12. $16\frac{2}{3}x = 316\frac{2}{3}.$
13. $14.4x = 100.8.$
14. $27.9x = 251.1.$
15. $1254x = 7524.$
16. $3x + 5 = 38.$
17. $3x + 7 = 106.$
18. $4x + 1 = 121.$
19. $5x + 6 = 131.$
20. $6x + 9 = 339.$
21. $7x + 8 = 106.$
22. $9x + 11 = 191.$
23. $12x + 27 = 171$
24. $17x + 13 = 200.$
25. $2.9x + 29 = 58.$
26. $326x + 9 = 987.$
27. $2.3\frac{1}{2}x + 5 = 240.$
28. $142x + 7 = 291.$
29. $62.4x + 50.4 = 300.$
30. $444x + 10 = 1342.$

230. Solving an Equation by Adding. If we have the equation

$$4x - 7 = 57,$$

we may add 7 to these equals. Then because $4x - 7 + 7$ is the same as $4x$, we have

$$4x = 64,$$

whence $x = 16$, by dividing by 4.

231. Solving an Equation by Multiplying. If we have the equation

$$\frac{2x}{7} = 12,$$

we may divide these equals by 2, and obtain

$$\frac{x}{7} = 6,$$

whence $x = 42$, by multiplying by 7.

EXERCISE 154

Solve by adding:

1. $x - 5 = 35.$
2. $4x - 9 = 31.$
3. $5x - 6 = 69.$
4. $6x - 3 = 93.$
5. $8x - 7 = 97.$
6. $12x - 6 = 150.$
7. $15x - 12 = 363.$
8. $2.7x - 0.8 = 15.4.$

Solve by multiplying:

9. $\frac{x}{9} = 15.$
10. $\frac{x}{12} = 11.$
11. $\frac{x}{27} = 22.$
12. $\frac{x}{3.5} = 4.8.$
13. $\frac{2x}{9} = 36.$
14. $\frac{4x}{7} = 64.$
15. $\frac{7x}{9} = 63.$
16. $\frac{2.4x}{5} = 16.8.$

232. Letters used in Solving Problems. One of the chief uses of letters in algebra is in solving problems. A problem in arithmetic can often be more clearly solved in this way than by numbers alone.

For example, I am thinking of a number. When it is multiplied by 3, and 5 is added to the result, the sum is 38. What is the number?

Solution using a letter:

If I am thinking of n, then

$3n$ is 3 times the number,

$3n + 5$ is 5 added to $3n$,

and $3n + 5 = 38$, as stated in the problem.

Then $3n = 33$, by subtracting 5 from equals,

and $n = 11$, by dividing these equals by 3.

Check or Proof. $3 \times 11 + 5 = 38$.

Solution without using letters:

Because 3 times the number added to 5 equals 38, therefore if 5 is taken away from 38 there remains 3 times the number. But $38 - 5 = 33$. Therefore 33 is 3 times the number. Therefore once the number is $\frac{1}{3}$ of 33, or 11.

The solutions compared:

$3n + 5 = 38$.	38
Subtracting 5,	5
$3n = 33$.	3)33
Therefore $n = 11$.	11

We therefore see that the two solutions are the same, but that the letters make the reasoning clearer.

Therefore in solving a problem by algebra,

(1) *Write a letter for the number sought.*

(2) *Use this letter in the statement of the problem.*

(3) *This will give an equation, as shown above.*

(4) *Solve this equation.*

EXERCISE 155

Represent the following algebraically:

1. 2 times some number.
2. The sum of 2 times some number and 7.
3. The sum of 4 times some number and 5.
4. The sum of 7 times some number and $2\frac{1}{2}$.
5. A number multiplied by 16, with 9 taken away.
6. The sum of some number and 10, divided by 7.
7. The sum of some number and 15, divided by 25.
8. The sum of 5 times some number and 6.
9. 3 times some number equals 21.
10. The sum of 2 times some number and 7 equals 17.
11. The sum of 5 times some number and 6 equals 36.
12. 5 times some number, less 21, equals 29.

Solve the following:

13. What number multiplied by 13 equals 247?
14. What number added to 39 equals 121?
15. What number diminished by $27\frac{3}{4}$ equals $15\frac{1}{8}$?
16. What number divided by 5 equals 17?
17. What number multiplied by $\frac{2}{3}$ equals 22?
18. What number added to $\frac{2}{3}$ equals $17\frac{1}{3}$?
19. What number diminished by $\frac{2}{3}$ equals $16\frac{2}{3}$?
20. What number divided by $\frac{2}{3}$ equals 12?
21. Twice what number is 1102?
22. A number is multiplied by 2, and 5 is taken from the product. The remainder is 55. What is the number?
23. A number is multiplied by 5, and 7 is added to the product. The sum is 82. What is the number?

24. The length of a schoolroom is 11 ft. more than the width. The length is 32 ft. What is the width?

25. A ball team has played 20 games. Out of these it has lost 9. How many has it won?

26. A ball team has lost 7 games and won twice this number, less 3. How many games has it played?

27. Our ball team played with the seventh-grade team. Our score was 3 more than twice theirs. Ours was 11. What was theirs?

28. Our ball team played with the Washington School team. Their score was 14 less than 3 times ours. Theirs was 7. What was ours?

29. A table is to be made such that the width is 6 in. more than half the length. The width is 48 in. What is the length?

30. A box is to be made that lacks 2 in. of being twice as long as it is wide. It is to be 17 in. long. How wide is it to be?

31. A steam hoist is so made that it can raise a weight that is twice the power used. What weight can be raised if the power is 1675 lb.?

32. In Ex. 31 what power is necessary to raise a weight of 2250 lb.?

33. There is a certain lever by which a man can lift 3.8 times his own weight. If he can lift 665 lb., what is his weight?

34. There is a certain lever by which a man can lift 5 times his weight. If he can lift 794 lb., what is his weight?

35. A man has a lever by which he can lift $4\frac{3}{5}$ times his own weight, lacking 7 lb. He can lift 683 lb. What is his weight?

233. Use of the Letter x. While we may represent a number by the initial letter, as n for number, d for dollars, and f for feet, or by any other letters, *it is customary to represent by the letter x a number that is to be found.*

In the equations already solved, the number to be found has been represented both by x and by the initial letter. Hereafter, in general, x will be used, except where initials are more convenient in formulas.

EXERCISE 156

Find the value of x:

1. $9x = 315$.
2. $8x + 3 = 139$.
3. $7x + 9 = 142$.
4. $9x + 11 = 200$.
5. $11x + 13 = 134$.
6. $19x + 7 = 140$.
7. $31x + 12 = 260$.
8. $51x + 45 = 300$.
9. $73x - 11 = 354$.
10. $81x - 52 = 758$.

Make up problems for the following, writing them out as on page 236, and solving:

11. $2x + 8 = 30$.
12. $3x + 7 = 28$.
13. $5x + 12 = 72$.
14. $7x + 15 = 85$.
15. $9x + 23 = 212$.
16. $37x + 29 = 140$.
17. $4x - 17 = 23$.
18. $6x - 34 = 32$.
19. $8x - 51 = 357$.
20. $10x - 73 = 927$.
21. $13x - 69 = 100$.
22. $273x - 111 = 1800$.

23. If to twice a certain number I add 57, the result is 171. What is the number?

24. If from 5 times a certain number I take 57, the result is 228. What is the number?

25. If to 73 I add 6 times a certain number, the result is 181. What is the number?

234. Addition. We add quantities in algebra in much the same way that we add denominate numbers.

In algebra $2 \cdot 3$ means 2×3, and $a \cdot b$ means $a \times b$, or ab.

If a letter is written alone, the number 1 is understood before it. Thus x is the same as $1x$.

For example, study these sums:

2 ft.	$\frac{2}{15}$	$2f$	$2 \cdot 5$	$2\,abc$	$3x$
3 ft.	$\frac{3}{15}$	$3f$	$3 \cdot 5$	$3\,abc$	$-2x$
7 ft.	$\frac{7}{15}$	$7f$	$7 \cdot 5$	$7\,abc$	x
12 ft.	$\frac{12}{15}$	$12f$	$12 \cdot 5$	$12\,abc$	$2x$

Also study the following:

2 ft. 6 in.	$2f + 6i$	$2x + 6y$
8 ft. 2 in.	$8f + 2i$	$8x - 2y$
10 ft. 8 in.	$10f + 8i$	$10x + 4y$

EXERCISE 157

1.
3 ft. 2 in.
5 ft. 3 in.

2.
$3f + 2i$
$5f + 3i$

3.
$3 \cdot 6 + 2 \cdot 4$
$5 \cdot 6 + 3 \cdot 4$

4.
$7x + 8y$
$9x + 3y$

5.
$2a + 3b + 4c$
$5a + 2b + 6c$

6.
$4a + 3b + 9c$
$2a + 2b - 3c$

7.
$7a + 2b + c$
$3a + 8b - 9c$

8.
$14a + 18b + 10c$
$3a - b + 7c$

9.
$x + y + z$
$x + y - z$

10.
$2x + 3y + 5z$
$3x + 7y + 8z$

11.
$5x + 2y + z$
$7x + 3y + z$

12.
$2x + 8y - 7z$
$5x - y + 6z$

235. Equations involving Addition. If we have the equation $3x + 8x = 132$, we proceed as follows:

$$3x + 8x = 132.$$
$$11x = 132, \text{ by adding.}$$
$$x = 12, \text{ by dividing by 11.}$$

EXERCISE 158

Solve the following:

1. $2x + 3x = 75.$
2. $3x + 4x = 84.$
3. $3x + 5x = 96.$
4. $7x + 4x = 143.$
5. $8x + 6x = 154.$
6. $7x + 9x = 176.$
7. $9x + 4x = 169.$
8. $10x + 3x = 117.$
9. $12x + 5x = 306.$
10. $15x + 7x = 462.$
11. $x + x = 24.$
12. $x + 2x = 72.$
13. $x + 9x = 300.$
14. $x + 13x = 308.$
15. $x + \frac{1}{2}x = 165.$
16. $x + \frac{1}{4}x = 165.$
17. $x + \frac{3}{4}x = 154.$
18. $x + 0.5x = 195.$
19. $2x + 2\frac{1}{2}x = 165.$
20. $3x + 5\frac{1}{8}x = 275.$

21. If to twice a certain number I add 7 times that number, the result is 261. What is the number?

22. If to 3 times a certain weight I add 15 times that weight, the result is 720 lb. What is the weight?

23. If to $2\frac{1}{2}$ times a certain weight I add $3\frac{1}{8}$ times that weight, the result is 225 lb. What is the weight?

24. If to 7 times a certain number I add 6 times the number, and then add 5 times the number, the result is 126. What is the number?

25. If to 15 times a certain quantity of milk a dealer adds 6 times that quantity, he has 147 gallons. What is the quantity?

236. Equations involving Per Cents. The equation is very useful in percentage. For example: After adding 15% to the cost of a horse a dealer sold it for \$184. What did the horse cost the dealer?

Let x represent the *number* of dollars in the cost.

Then $x + 0.15x =$ the selling price.

But \$184 = the selling price.

Therefore $x + 0.15x = 184$,

or, by adding, $1.15x = 184$.

Therefore $x = 160$, by dividing by 1.15.

Therefore the horse cost \$160. *Ans.*

We may check this result by adding 15% of \$160 to \$160, and we have \$160 + \$24 = \$184, the selling price.

EXERCISE 159

1. What number increased by 10% of itself equals 176?
2. What number increased by 25% of itself equals 200?
3. What number increased by $33\frac{1}{3}$% of itself equals 200?
4. What number increased by $12\frac{1}{2}$% of itself equals 189?
5. What number increased by $66\frac{2}{3}$% of itself equals 275?
6. A boy weighs 60 lb. now, which is 20% more than he weighed four years ago. What did he weigh then?
7. A certain sum increased by 8% of itself is \$135. What is the sum?
8. A certain sum plus 6% interest for one year amounts to \$265. What is the sum?
9. A certain school has 10% more pupils than it had last year. It now has 132 pupils. How many pupils did it have last year?
10. A merchant saved \$2497.50 last year, which was 11% more than he saved the year before. How much did he save the year before?

Solve the following:

11. $1.25x = 13.75$.

12. $1.40x = 26.60$.

13. $1.75x = 29.75$.

14. $1.12\frac{1}{2}x = 540$.

15. $1.16\frac{2}{3}x = 777$.

16. $x + 0.25x = 13.75$.

17. $x + 0.40x = 2100$.

18. $x + 0.75x = 8470$.

19. $x + 12\frac{1}{2}\% \, x = 720$.

20. $x + 33\frac{1}{3}\% \, x = 164$.

21. A dealer sold some damaged furniture at 15% below cost. He sold it for $204. What did it cost him?

22. A dealer sold some furniture at 15% above cost. He sold it for $276. What did it cost him?

23. A man's expenses in one year were cut down 17% the next year, when they were $1660. What were they the first year?

24. The steam pressure on a boiler was 126 lb. per square inch at 10 A.M., which was 5% more than it was at 9 A.M. What was the pressure at 9 A.M.?

25. A flywheel of an engine is making 91 revolutions a minute, an increase of $8\frac{1}{3}$% over its speed an hour ago. What was its speed an hour ago?

26. A poultry raiser has 371 hens, which is 6% more than he had last month. How many did he have last month?

27. A year's interest on a certain sum at 6% is $45. What is the sum?

28. The population of a village is now 4950, an increase of $12\frac{1}{2}$% over the population ten years ago. What was it ten years ago?

29. The number of pupils in a school is 147, an increase of 5% on the number last year. What was it last year?

30. A man sold a lot of Texas ponies for $5664, thus making a profit of 18%. What did the ponies cost him?

237. Subtraction. We subtract quantities in algebra in much the same way that we subtract denominate numbers. For example, study these cases of subtraction :

$$\begin{array}{r} 21 \text{ ft.} \\ 4 \text{ ft.} \\ \hline 17 \text{ ft.} \end{array} \qquad \begin{array}{c} \frac{21}{25} \\ \frac{4}{25} \\ \hline \frac{17}{25} \end{array} \qquad \begin{array}{r} 21\,a \\ 4\,a \\ \hline 17\,a \end{array} \qquad \begin{array}{r} 21\,ax \\ 4\,ax \\ \hline 17\,ax \end{array} \qquad \begin{array}{r} 21 \cdot 3 \\ 4 \cdot 3 \\ \hline 17 \cdot 3 \end{array}$$

We have found (§ 224) that to subtract a negative number we may change the sign and add. Thus :

$$\begin{array}{r} 4 \\ -2 \\ \hline 6 \end{array} \qquad \begin{array}{r} 4\,x \\ -2\,x \\ \hline 6\,x \end{array} \qquad \begin{array}{r} 4\,ax \\ -2\,ax \\ \hline 6\,ax \end{array} \qquad \begin{array}{r} 5\,abc \\ -abc \\ \hline 6\,abc \end{array} \qquad \begin{array}{r} -3\,abx \\ -7\,abx \\ \hline 4\,abx \end{array}$$

We may also have a negative remainder, as in the subtraction of $10\,x$ from $3\,x$.

If the temperature drops 10° from 3°, the thermometer registers $-7°$. If it drops 3° from $-7°$, it registers $-10°$.

$$\begin{array}{r} 3\,x \\ 10\,x \\ \hline -7\,x \end{array}$$

EXERCISE 160

1. $$\begin{array}{r} 13 \text{ ft. } 7 \text{ in.} \\ 6 \text{ ft. } 3 \text{ in.} \\ \hline \end{array}$$

2. $$\begin{array}{r} 13\,x + 7\,y \\ 6\,x + 3\,y \\ \hline \end{array}$$

3. $$\begin{array}{r} 15\,x + 9\,y \\ 7\,x + y \\ \hline \end{array}$$

4. $$\begin{array}{r} 27 \cdot 2 + 16 \cdot 3 \\ 12 \cdot 2 + 18 \cdot 3 \\ \hline \end{array}$$

5. $$\begin{array}{r} 6\,ax \\ 4\,ax \\ \hline \end{array}$$

6. $$\begin{array}{r} 6\,ax \\ -4\,ax \\ \hline \end{array}$$

7. $$\begin{array}{r} 9\,axy \\ 10\,axy \\ \hline \end{array}$$

8. $$\begin{array}{r} -12\,abc \\ -4\,abc \\ \hline \end{array}$$

9. $$\begin{array}{r} 2\,x + 3\,y \\ x - 2\,y \\ \hline \end{array}$$

10. $$\begin{array}{r} 4\,x + 7\,y \\ 5\,x - 2\,y \\ \hline \end{array}$$

11. $$\begin{array}{r} 5\,xy + 3\,z \\ 2\,xy - 3\,z \\ \hline \end{array}$$

12. $$\begin{array}{r} 17\,xy - 12\,z \\ 20\,xy - 16\,z \\ \hline \end{array}$$

238. Equations involving Subtraction. If we have the equation $7x - 2x = 235$, we proceed as follows:

$$7x - 2x = 235.$$
$$5x = 235, \text{ by subtracting.}$$
$$x = 47, \text{ by dividing by } 5.$$

EXERCISE 161

Solve the following:

1. $9x - x = 72.$
2. $8x - 2x = 354.$
3. $7x - 5x = 238.$
4. $13x - 5x = 184.$
5. $17x - 6x = 242.$
6. $19x - 5x = 294.$
7. $23x - 7x = 336.$
8. $25x - 9x = 672.$
9. $27x - 8x = 228.$
10. $31x - 4x = 324.$
11. $2x - x = 3\frac{1}{2}.$
12. $2x - \frac{1}{2}x = 63.$
13. $3x - \frac{1}{4}x = 88.$
14. $3x - \frac{3}{4}x = 81.$
15. $5x - 1\frac{1}{2}x = 84.$
16. $7x - 3\frac{1}{4}x = 150.$
17. $2\frac{1}{2}x - \frac{1}{4}x = -126.$
18. $3\frac{1}{4}x - \frac{3}{4}x = -120.$
19. $2.7x - 1.3x = 29.4.$
20. $3.9x - 2.8x = 10.01.$

21. If from 9 times a certain number I take 6 times the number, the result is 321. What is the number?

22. If from 16 times a certain weight I take 9 times the weight, the result is 161. What is the weight?

23. If from 17 times a certain number I take 8 times the number, the result is 333. What is the number?

24. If from 23 times a certain quantity of grain there is taken 15 times the quantity, the result is 248 bu. What is the quantity?

25. If from 26 times a certain quantity of oil there is taken 17 times that quantity, there remains 603 gal. What is the quantity?

239. Equations involving Per Cents. A man sold a house for \$4200, thereby losing $12\frac{1}{2}\%$ on the cost. What did it cost him?

Let x represent the *number* of dollars in the cost.

Then	$x - 0.12\frac{1}{2}x =$ the selling price.
But	$\$4200 =$ the selling price.
Therefore	$x - 0.12\frac{1}{2}x = 4200.$
Subtracting,	$0.87\frac{1}{2}x = 4200.$
Dividing by $0.87\frac{1}{2}$,	$x = 4800.$

Therefore the house cost him \$4800. *Ans.*

Check. $\$4800 - 12\frac{1}{2}\%$ of $\$4800 = \$4800 - \$600 = \$4200.$

EXERCISE 162

1. What number decreased by 8% of itself equals 184?

2. What number decreased by 25% of itself equals 360?

3. What number decreased by $33\frac{1}{3}\%$ of itself equals 160?

4. A piece of cloth after shrinking $1\frac{1}{2}\%$ of its length is 157.6 yd. long. How long was it before shrinking?

5. A stock of goods after being damaged $66\frac{2}{3}\%$ by fire is worth \$1600. What was it worth before the fire?

6. From a barrel $12\frac{1}{2}\%$ of the vinegar leaked out, leaving 28 gallons. How many gallons of vinegar were there at first?

7. An agent collected some money for a man, and after deducting his commission of 5% he sent to the man \$522.50. How much did he collect?

8. An agent collected some money for a man, and after deducting his commission of $2\frac{1}{2}\%$ he remitted \$1072.50. How much did he collect?

9. From an oil tank 17% of the oil was drawn off, leaving 10,541 gallons. How many gallons of oil were there in the tank at first?

240. Multiplication. We multiply quantities in algebra in much the same way that we multiply denominate numbers. For example, study these cases of multiplication:

$$\begin{array}{r} 3\text{ ft. } 2\text{ in.} \\ 4 \\ \hline 12\text{ ft. } 8\text{ in.} \end{array} \qquad \begin{array}{r} 3f+2i \\ 4 \\ \hline 12f+8i \end{array} \qquad \begin{array}{r} 3x+2y \\ 4 \\ \hline 12x+8y \end{array}$$

The multiplier may also contain a letter, thus:

$$\begin{array}{r} 3x+2y \\ a \\ \hline 3ax+2ay \end{array} \qquad \begin{array}{r} 3x+2y \\ 4a \\ \hline 12ax+8ay \end{array} \qquad \begin{array}{r} 3x-2y \\ 4a \\ \hline 12ax-8ay \end{array}$$

We indicate the multiplication of $a+b$ by x thus: $x(a+b)$.

EXERCISE 163

Multiply:

1.	6.	11.
$\begin{array}{r} 3a+2b \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 52x+37y \\ 2a \\ \hline \end{array}$	$\begin{array}{r} x+y+z \\ 3 \\ \hline \end{array}$
2.	**7.**	**12.**
$\begin{array}{r} 5a-3b \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 49x-56y \\ 12 \\ \hline \end{array}$	$\begin{array}{r} x-2y+z \\ 4a \\ \hline \end{array}$
3.	**8.**	**13.**
$\begin{array}{r} 27x+15y \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 59x+47y \\ 23a \\ \hline \end{array}$	$\begin{array}{r} 3x-4y-5z \\ 15ab \\ \hline \end{array}$
4.	**9.**	**14.**
$\begin{array}{r} 26p+12q \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 76x-37y \\ 31a \\ \hline \end{array}$	$\begin{array}{r} xy+yz+1 \\ a \\ \hline \end{array}$
5.	**10.**	**15.**
$\begin{array}{r} 37m+15n \\ 12 \\ \hline \end{array}$	$\begin{array}{r} 37x-96y \\ 23m \\ \hline \end{array}$	$\begin{array}{r} ab+cd+ef \\ x \\ \hline \end{array}$

241. Factors. Quantities which are multiplied together are called the *factors* of the product.

The factors of 6 are 2 and 3; the factors of $5ab$ are 5, a, and b.

242. Square. If two factors are equal, their product is called a *square.*

The product 2×2 may be written 2^2, called the square of 2.

243. Power. The product arising from taking a quantity a certain number of times as a factor is called a *power.*

Thus $2 \times 2 \times 2 = 2^3$, or 8. Likewise a^4 is the fourth power of a, and equals $aaaa$. Hence $a^4 \div a = a^3$.

244. Exponent. In the quantity a^4, 4 is called the *exponent* of a, and indicates the power to which a is raised.

In a quantity like a or x, the exponent 1 is understood.

245. Coefficient. A numerical factor written before a letter is called the *coefficient* of the letter.

Thus 2 is the coefficient of a in the quantity $2a$. In a quantity like x, a, or m^2, the coefficient 1 is understood.

246. Algebraic Expression. A letter, or a collection of letters or of letters and numbers representing operations ($+$, $-$, $\times$, $\div$, etc.), is called an *algebraic expression.*

247. Term. An algebraic expression containing neither the $+$ nor the $-$ sign of operation is called a *term.*

The terms of $2a^2 - 3ab$ are $2a^2$ and $3ab$. The *literal factor* of the term $3ab$ is ab, and the *numerical factor* is 3.

248. Algebraic Expressions classified according to Terms. An algebraic expression of one term is called a *monomial;* of two terms, a *binomial;* of three terms, a *trinomial;* of several terms, a *polynomial.*

Thus $4a^2b$ is a monomial; $2 + 3ax$ is a binomial.

EXERCISE 164

Write the factors of:

1. $2\pi r$.	3. πd.	5. lh.	7. $2a^2bc$.
2. πr^2.	4. $\pi r^2 h$.	6. lbh.	8. $3a^2b^3c$.

Write the coefficients of the literal part in the following:

9. x^2.	10. $3x^4$.	11. $5abc$.	12. $\frac{1}{2}xyz$.

Write the exponents of r in the following:

13. πr^2.	14. $2\pi r$.	15. $\frac{4}{3}\pi r^3$.	16. $\pi r^2 h$.

17. Is $2xy$ a monomial, a binomial, or a trinomial?

18. Is $2 + xy$ a monomial or a binomial?

19. What kind of an expression, as to number of terms, is $2 + x + y$?

20. If you wish to reduce $\frac{4}{6}$ to lowest terms, what common factor do you cancel from numerator and denominator?

21. If you wish to reduce $\frac{3\pi r}{9}$ to lowest terms, what factor do you cancel?

22. If you wish to reduce $\frac{ax}{ay}$ to lowest terms, what factor do you cancel?

Reduce to lowest terms:

23. $\frac{6}{8}$.	25. $\frac{ab}{axy}$.	27. $\frac{\pi r^2}{\pi}$.	29. $\frac{x^3}{ax}$.
24. $\frac{12}{16}$.	26. $\frac{a^2bc}{a^2bx}$.	28. $\frac{a^2}{a}$.	30. $\frac{x^3}{ax^2}$.

31. If you wish to reduce $\frac{a(x+y)}{b(x+y)}$ to lowest terms, what factor do you cancel?

32. If you wish to reduce $\frac{a(x+y+z)}{2a(x+y+z)}$ to lowest terms, what factors do you cancel?

249. Multiplication involving Powers. If two factors have the same letter, we shall have in the product a power greater than the first power.

For example, study these cases of multiplication:

$$\begin{array}{r} a \\ a \\ \hline a^2 \end{array} \qquad \begin{array}{r} ab \\ a \\ \hline a^2b \end{array} \qquad \begin{array}{r} -a^2 \\ a \\ \hline -a^3 \end{array} \qquad \begin{array}{r} -a^2b \\ a \\ \hline -a^3b \end{array} \qquad \begin{array}{r} \pi r^2 \\ r \\ \hline \pi r^3 \end{array} \qquad \begin{array}{r} 4\,\pi r^2 \\ \frac{1}{3}r \\ \hline \frac{4}{3}\,\pi r^3 \end{array}$$

$$\begin{array}{r} a + b \\ a \\ \hline a^2 + ab \end{array} \qquad \begin{array}{r} a^2 + 2\,ab \\ ab \\ \hline a^3b + 2\,a^2b^2 \end{array} \qquad \begin{array}{r} a^2 - 2\,ab \\ 3\,ab \\ \hline 3\,a^3b - 6\,a^2b^2 \end{array}$$

EXERCISE 165

Multiply:

1.	6.	11.
$\begin{array}{r} a^2 \\ a^2 \\ \hline \end{array}$	$\begin{array}{r} a + b \\ b \\ \hline \end{array}$	$\begin{array}{r} a + b + c \\ a \\ \hline \end{array}$
2.	**7.**	**12.**
$\begin{array}{r} a^2x \\ a^2 \\ \hline \end{array}$	$\begin{array}{r} a^2 + b^2 \\ a^2 \\ \hline \end{array}$	$\begin{array}{r} a^2 + 2\,a + 1 \\ a^2 \\ \hline \end{array}$
3.	**8.**	**13.**
$\begin{array}{r} 3\,a^3x \\ 2\,a^2 \\ \hline \end{array}$	$\begin{array}{r} a^2 - b^2 \\ 2\,a \\ \hline \end{array}$	$\begin{array}{r} x^2 - 2\,xy + y^2 \\ xy \\ \hline \end{array}$
4.	**9.**	**14.**
$\begin{array}{r} 5\,\pi r^2 \\ 2\,r \\ \hline \end{array}$	$\begin{array}{r} a^2 - 3\,b^2 \\ 4\,a^2 \\ \hline \end{array}$	$\begin{array}{r} 3\,x^2 - 4\,xy - 7\,y^2 \\ 2\,xy \\ \hline \end{array}$
5.	**10.**	**15.**
$\begin{array}{r} 27\,a^2bc \\ 12\,ab \\ \hline \end{array}$	$\begin{array}{r} a^3 - 3\,b^3 \\ 7\,a^2 \\ \hline \end{array}$	$\begin{array}{r} 7\,x^2 - 5\,xy - 3\,y^2 \\ 9\,xy \\ \hline \end{array}$

250. Multiplying a Fraction. We multiply fractions in algebra in the same way as in arithmetic. For example:

$$2 \times \frac{2}{5} = \frac{4}{5}. \qquad \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}.$$

$$2 \times \frac{a}{b} = \frac{2a}{b}. \qquad \frac{2}{3} \times \frac{a}{b} = \frac{2a}{3b}.$$

$$2 \times \frac{3}{8} = \frac{3}{4}. \qquad \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}.$$

$$2 \times \frac{a}{2b} = \frac{a}{b}. \qquad \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}.$$

$$a \times \frac{x}{ab} = \frac{x}{b}. \qquad \frac{a}{b} \times \frac{2b^2}{3a} = \frac{2b}{3}.$$

EXERCISE 166

Multiply:

1. $\frac{3}{5}$ by 7.
2. $\frac{3}{5}$ by a.
3. $\frac{3}{5}$ by 5.
4. $\frac{3}{5}$ by $5a$.
5. $\frac{3}{5}$ by $15a$.
6. $\frac{a}{b}$ by a.
7. $\frac{a}{b}$ by b.
8. $\frac{a}{b}$ by ab.
9. $\frac{a}{b}$ by $\frac{b}{a}$.
10. $\frac{a}{2b}$ by 4.
11. $\frac{a}{2b}$ by b.
12. $\frac{a}{2b}$ by $2b$.
13. $\frac{a}{2b}$ by $6b^2$.
14. $\frac{3a}{2b}$ by $\frac{4b}{a}$.
15. $\frac{a^2}{b^2}$ by $3a$.
16. $\frac{a^2}{b^2}$ by $3ab$.
17. $\frac{a^2}{b^2}$ by $\frac{b^2}{a}$.
18. $\frac{a^2}{b^2}$ by $\frac{b}{a}$.
19. $\frac{a+b}{b}$ by b.
20. $\frac{a+b}{2}$ by 2.
21. $\frac{a+b}{2b}$ by $4b$.
22. $\frac{a+b}{2b}$ by c.
23. $\frac{a+b}{2bc}$ by c.
24. $\frac{a-b}{2bc}$ by $2bc$.
25. $\frac{2a+b}{3}$ by $2a$.
26. $\frac{3a-b}{c}$ by $4a$.
27. $\frac{s^2+h^2}{2h}$ by $2h$.

251. The Formula as an Equation. A formula may be thought of as an equation, and one of the chief uses of algebra is the solution of equations of this nature.

For example, if the rate of interest is r, the interest for 1 yr. on p dollars is rp. Since this is the interest for 1 yr., for t years it is trp. Therefore

$$i = trp.$$

Here i stands for interest, t for time, r for rate, and p for principal.

If from this formula we wish to find t, we divide both sides of the equation by rp. Then

$$\frac{i}{rp} = t.$$

Likewise we may find r or p, thus: $r = \frac{i}{tp}$, and $p = \frac{i}{tr}$.

EXERCISE 167

Given $i = trp$, find:

1. i, when $t = 2$, $r = 6\%$, $p = \$500$.
2. t, when $i = \$20$, $r = 4\%$, $p = \$250$.
3. r, when $i = \$52.50$, $t = 3$, $p = \$350$.
4. p, when $i = \$135$, $t = 5$, $r = 6\%$.
5. What principal, put at interest at 6%, will produce \$135 in 3 years?
6. How long will it take \$1250 to produce \$250 interest at 5%? at $2\frac{1}{2}\%$?
7. How long will it take \$2750 to produce \$1155 interest at 6%? at 3%? at $4\frac{1}{2}\%$?
8. The formula for the amount (sum of principal and interest) is $a = p(1 + tr)$. From this formula deduce the formula for p.

9. Given the formula for the circumference of a circle, $c = 2\pi r$, find the formula for r.

10. If the circumference of a circle is 47.124 in., what is the radius? (Take $\pi = 3.1416$.)

11. Given the formula for the area of a circle, $a = \pi r^2$, find the formula for r^2; also the formula for r.

12. If the area of a circle is 154 sq. in., what is the radius? (Take $\pi = 3\frac{1}{7}$.)

13. Given the formula for the surface of a sphere, $s = 4\pi r^2$, find the formula for r^2; also the formula for r.

14. Given the formula for the volume of a sphere, $v = \frac{4}{3}\pi r^3$, find the formula for r^3; also the formula for r.

15. Given the formula for the volume of a cylinder, $v = h\pi r^2$, find the formula for h.

16. If the volume of a cylinder is 308 cu. in., and the radius is 7 in., what is the height? (Take $\pi = 3\frac{1}{7}$.)

17. If the volume of a cylinder is 1540 cu. in., and the height is 10 in., what is the radius? (Take $\pi = 3\frac{1}{7}$.)

18. Given the formula for the volume of a cone, $v = \frac{1}{3}h\pi r^2$, find the formula for h.

19. From the same formula, $v = \frac{1}{3}h\pi r^2$, find the formula for r^2; also the formula for r.

20. Given the formula for the area of a great circle of a sphere, $a = \pi r^2$, and the formula for the surface of a sphere, $s = 4\pi r^2$, find the ratio of a to s.

21. Given the formula for the volume of a cylinder, $v = h\pi r^2$, find the volume when $h = 2r$.

22. From Ex. 21 find the volume of a cylinder that is 2 in. high and 2 in. in diameter.

23. From the formula $v = h\pi r^2$ find the volume of a cylindrical shaft 6 ft. long and 4 in. in diameter.

24. If a train travels 80 mi. in 2 hr., what is its rate per hour? If it travels the distance d mi. in the time t hr., what is its rate per hour?

25. From the formula $r = \frac{d}{t}$ find r when $d = 80$, $t = 2$; when $d = 98$, $t = 2$; when $d = 134$, $t = 3$.

26. From the formula of Ex. 25 find the formula for d. Find the value of d when $r = 39$, $t = 4$.

27. From the formula of Ex. 25 find the formula for t. (First multiply by t and then divide by r.)

28. From the formula of Ex. 25 find the value of t when $r = 42$, $d = 105$.

29. It was shown on page 70 that in a lever the power (p) and the weight (w) are inversely proportional to their distances (a and b) from the fulcrum; that is, $\frac{p}{w} = \frac{b}{a}\cdot$ From this equation find the formula for p.

30. From the equation of Ex. 29 find the value of p when $w = 25$, $b = 7$, and $a = 5$.

31. From the equation of Ex. 29 find the value of w when $p = 150$, $b = 15$, and $a = 12$.

32. Given the formula for the area of a triangle, $a = \frac{1}{2}bh$, find the formula for b; also the formula for h.

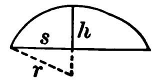
33. Given the formula for the area of a trapezoid, $a = \frac{1}{2}(b + b')h$, find the formula for h.

34. Given the formula of Ex. 33, find the value of h when $a = 32$, $b = 10$, and $b' = 6$.

35. Given the formula of Ex. 33, let $a = 66$, $b = 12$, $h = 6$, and substitute these values in the formula. Then find the value of b'.

36. In the same way as in Ex. 35 find the value of b when $a = 35$, $b' = 8$, and $h = 5$.

37. Carpenters have a formula for finding the radius of a circle used in constructing an arch of height h, the span being $2s$, as follows: $r = \frac{s^2 + h^2}{2h}$. Find the value of r when $s = 4$ and $h = 3$.



38. Using the formula given in Ex. 37, find the value of r when $s = 8$ and $h = 2$.

39. Using the formula given in Ex. 37, find the value of s when $r = 8\frac{1}{3}$ and $h = 6$.

40. The area of an equilateral triangle, each of whose sides is s, is expressed by the formula $a = \frac{1}{4}s^2\sqrt{3}$. Find the value of a to two decimal places when $s = 2$.

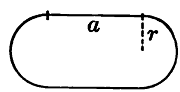
41. In the formula $a = \frac{1}{4}s^2\sqrt{3}$ multiply both of these equals by 4 and then divide both by $\sqrt{3}$.

42. The volume of a rectangular solid that is l long, w wide, and h high, is expressed by the formula $v = lwh$. Find the value of l in terms of v, w, and h.

43. A machinist finds in the *Scientific American* this formula: $p = r(2 + \pi)$. He wishes from this to obtain a formula for r. What is the formula?

44. A carpenter finds in a book this formula: $h^2 = a^2 + b^2$. He wishes from this to obtain a formula for a^2. What is the formula?

45. The formula for the distance around a running track with semicircular ends is $d = 2a + 2\pi r$. From this find a formula for r, so we may know the radius to be used in drawing the circles.



46. From the formula of Ex. 45 find the value of d when $a = 300$ and $r = 50$.

47. From the formula of Ex. 45 find the value of a when $d = 1114.16$ and $r = 50$. (Take $\pi = 3.1416$.)

252. Division. We divide polynomials in much the same way that we divide compound numbers. For example, study these cases of division:

$$\begin{array}{r} 2\overline{)14\text{ ft. }8\text{ in.}} \\ 7\text{ ft. }4\text{ in.} \end{array} \qquad \begin{array}{r} 2\overline{)14f+8i} \\ 7f+4i \end{array} \qquad \begin{array}{r} 2\overline{)14\cdot 5+8\cdot 3} \\ 7\cdot 5+4\cdot 3 \end{array}$$

$$\begin{array}{r} x\overline{)14x^2+8xy} \\ 14x\ +8y \end{array} \qquad \begin{array}{r} 2x\overline{)14x^2+8xy} \\ 7x\ +4y \end{array} \qquad \begin{array}{r} 2x\overline{)14x^2-8xy} \\ 7x\ -4y \end{array}$$

EXERCISE 168

1. $3\overline{)27\text{ ft. }9\text{ in.}}$

2. $3\overline{)27x+9y}$

3. $3x\overline{)27x^2+9xy}$

4. $3x\overline{)27xy+9xz}$

5. $3xy\overline{)27x^2y-9xy}$

6. $3x^2\overline{)27x^2-9x^2y}$

7. $3y^2\overline{)27x^2y^2-9y^2}$

8. $3\overline{)27xyz-9z^2}$

9. $3\overline{)15a-27b}$

10. $3a\overline{)15a^2-27ab}$

11. $25a\overline{)25a^2+125a}$

12. $19a\overline{)57ab-38a^2}$

13. $3x^2\overline{)84x^2-27x^2y}$

14. $4xy\overline{)64x^2y^2-28xy}$

15. $7x^2\overline{)84x^2y^2-133x^2}$

16. $3abc\overline{)111a^2bc-9ab^2c}$

17. $x\overline{)2x+3xy+xz}$

18. $x\overline{)3x-4xy-xz}$

19. $x\overline{)5x^2-7xy+xz}$

20. $x^2\overline{)7x^2+8x^3+9x^4}$

21. $2x\overline{)2x^2+4x^3+6x^4}$

22. $3x\overline{)6x^2+9xy-12x}$

23. $5x^2\overline{)5x^4+10x^3-5x^2}$

24. $7x^3\overline{)14x^6-7x^5+7x^3}$

253. Factoring. To factor a polynomial is to separate it into factors that themselves cannot be further separated into factors.

For example, to factor $ax + ay$ we see that a is a factor of each term, and that $ax + ay$ divided by a equals $x + y$. Therefore

$$ax + ay = a(x + y),$$

where the parentheses show that $x + y$ is all to be multiplied by a. Hence the factors of $ax + ay$ are a and $x + y$.

Study also the following:

$$axy + axz = ax(y + z).$$
$$2a^2x - 4a^2y = 2a^2(x - 2y).$$
$$a^2b^3 + a^3b^2 = a^2b^2(b + a).$$

EXERCISE 169

Factor:

1. $ax - ay.$
2. $\pi a - \pi b.$
3. $2\pi a + 2\pi b.$
4. $\pi d - \pi d'.$
5. $\pi r^2a - \pi r^2b.$
6. $axy + axz.$
7. $ax^2y - axz.$
8. $\pi r^2x - \pi r^2y.$
9. $mnx^2 + mny^2.$
10. $2ax^2y - 4a^2xy^2.$
11. $3ax^2y + 9ax^2z.$
12. $7a^2b^2c^2 + 14abc.$
13. $25a^2b + 35ab^2.$
14. $125x^2y^2z^2 + 75xyz.$
15. $111abcd + 99a^2c^2.$
16. $p + prt.$
17. $p - prt.$
18. $\pi hr^2 + \pi kr^2.$
19. $\pi ma^2 + \pi nb^2.$
20. $\frac{1}{2}bh - \frac{1}{2}bk.$
21. $abc + 3a^2b^2c^2.$
22. $3a^2b + 3ab^2.$
23. $3a^2b - 3ab^2.$
24. $a^3 + 3a^2b + 3ab^2.$
25. $a^3 - 3a^2b + 3ab^2.$
26. $x^3 + 3x^2y - 3xy^2.$
27. $a^3b^3 + a^2b^2 + ab.$
28. $a^3b^3c^3 + a^2b^2c^2.$
29. $27a^3b^3 + 9a^2b^2.$
30. $123x^2y^2 + 321xy.$

254. Unknown Quantity. A letter in an equation, for which a value is to be found, is called an *unknown quantity.*

Thus in the equation $3x + 19 = 49$, the unknown quantity is x.

255. Root. The value of an unknown quantity in an equation is called a *root* of the equation.

In the equation $3x + 19 = 49$, the root is 10.

256. Members of an Equation. The two sides of an equation, separated by the sign of equality, are called the *members* of the equation.

The one to the left is the *first member;* the one to the right is the *second member.* The members are often called the *sides* of the equation.

257. Transpose. In the equation $2x = x + 7$ we may subtract x from both members and we have $x = 7$. We then say that we have *transposed* x.

Likewise if we have $7x - 3 = 46$, we may *add* 3 to both members and we have $7x = 49$. We then say that we have *transposed* -3.

We therefore see that *we may transpose a term from one member of an equation to the other by changing its sign.*

258. Degree. The number of literal factors in a term is called its *degree.*

Thus $4a^2$ and $\frac{1}{2}ab$ are both of the second degree, for $4a^2$ contains the two literal factors a and a, and $\frac{1}{2}ab$ contains the two literal factors a and b.

As already stated, a term like x has the exponent 1 understood; that is, $x = x^1$. It is said to be of the first degree.

The term $5ax$ is of the second degree, considering both a and x, but we also speak of it as of the *first degree in* x.

259. Simple Equation. An equation of which the terms in the unknown quantity are all of the first degree is called *a simple equation.*

A simple equation is also called an *equation of the first degree.*

260. Solving a Simple Equation. We have solved many simple equations in this chapter, and have found that we proceed as follows:

(1) *Transpose all terms involving x to one member, and all other terms to the other member.*

(2) *Unite the terms in the members.*

(3) *Divide both members by the coefficient of x.*

Thus, given

$$5x - 7 = 2x + 20.$$

$5x = 2x + 27$, by adding 7 (transposing -7).

$3x = 27$, by subtracting $2x$ (transposing $2x$).

$x = 9$, by dividing by 3.

EXERCISE 170

Solve the following:

1. $3x + x = 64.$
2. $5x + 4x = 81.$
3. $9x - 3x = 78.$
4. $12x - 7x = 95.$
5. $17x - 9x = 96.$
6. $15x + 7 = 52.$
7. $13x + 9 = 100.$
8. $19x + 8 = 179.$
9. $23x - 7 = 108.$
10. $27x - 3 = 267.$
11. $4x + 7 = 3x + 64.$
12. $4x - 7 = 3x + 72.$
13. $7x + 3 = 3x + 67.$
14. $8x + 5 = 5x + 116.$
15. $9x - 4 = 3x + 68.$
16. $11x - 9 = 5x + 117.$
17. $24x - 7 = 9x + 158.$
18. $27 - 3x = 68 - 4x.$
19. $42 - 3x = 48 - 9x.$
20. $77 - 8x = 91 - 15x.$

21. The interest on a note of $\$p$, the rate being r and the number of years t, is represented by the formula $i = rtp$. Find the value of p in terms of i, r, and t.

22. The amount of a note is equal to the principal plus the interest. It is represented by the formula $a = p + rtp$. Factor the second member.

23. In the formula $a = p + rtp$, after factoring the second member find the value of p. What does p equal when $a = \$224$, $r = 6\%$, and $t = 2$ yr.?

24. In the formula $a = p + rtp$ find the value of t.

25. How long will it take \$325 and interest to amount to \$373.75 at 5%?

26. In the formula $a = p + rtp$ find the value of r.

27. At what rate of interest will \$275 produce \$49.50 in 3 yr.?

28. A man lost 11% of his capital and then had \$6675. How much had he at first?

29. A man gained 11% on his capital and then had \$8325. How much had he at first?

30. A dealer sold some goods for \$2170, thus making a profit of 24%. What did the goods cost him?

31. A collection agency charges 4% for collecting a debt, and remits \$1200. How much did it collect?

32. A bank charges 0.1% exchange on a draft. The entire cost of draft and exchange is \$1751.75. What is the face of the draft?

33. A piece of cloth lost 20% of its length in shrinking, and was then 72 yards long. How long was it originally?

34. In a machine shop the machine work on each casting of a certain kind was \$0.67. The machine work on a lot of castings cost \$871.75, which included an extra charge of \$17.50 for freight. How many castings were there?

35. The total weight of a 12-wheel passenger coach with the equipment for lighting it by electricity is 107,910 lb., of which $\frac{1}{33}$ is due to the electric-light equipment. What was the weight on each pair of wheels before the electric-light equipment was added?

36. On a certain road it is known that a passenger coach weighs 90% as much as a sleeping car, that the pay car weighs 88% as much as a passenger coach, and that the pay car weighs 39.6 tons. What does a sleeping car weigh?

37. A freight engine took on 7000 lb. of coal, and after $5\frac{1}{2}$ hr. it had 400 lb. left. How much did it use per hour?

38. A boy working in a shop from 7.30 A.M. to 5.15 P.M., with 45 minutes off for lunch, earned $1.35, which included an extra 10¢ for going on an errand at noon. What were his regular wages per working hour?

39. To make 14 steel frames for locomotive tenders a shop needed 57,800 lb. of steel beams (called "channels"), in which was included 120 lb. allowance for waste. Each tender used 4 channels, each 25 ft. 9 in. long. What was the weight of the channels per foot?

40. A passenger coach with passengers and baggage weighs 95,900 lb. The passengers weigh 5% of the weight of the coach, and the baggage weighs 1400 lb. Allowing 150 lb. as the average weight of the passengers, how many passengers are there?

41. An axle of a car was turned in a lathe and lost $6\frac{1}{4}$% of its weight. It then lacked $32\frac{1}{2}$ lb. of weighing just 1000 lb. What was its weight before it was turned?

42. How many revolutions per minute must be made by a wheel 10 in. in diameter so that the rim shall be traveling 523.6 ft. per minute? (Use 3.1416 for π.)

43. What is the length of a steel bar with a square cross section 2 in. wide, that weighs 245 lb.? (Steel weighs 490 lb. per cubic foot.)

44. A copper pipe 24 in. in diameter weighs 774.4 lb. It is made from sheet copper weighing 15.4 lb. per square foot. What is the length of the pipe? (Use $\frac{22}{7}$ for π.)

261. Common Problems of Algebra. Besides the problems taken from daily life in the shops and in mercantile houses there are many interesting problems that are easily solved by algebra and that usually make up much of the algebraic work of the school. The following is an example:

Find a number whose seventh part minus its eleventh part equals 4.

Let $\quad x =$ the number.

Then $\quad \frac{x}{7} - \frac{x}{11} = 4.$

If we multiply the first fraction by 7, it will become an integer, x. If we multiply the second fraction by 11, it also becomes an integer. We therefore *clear of fractions* by multiplying both members of the equation by 7×11, and we have

$$11x - 7x = 308,$$

whence $\quad 4x = 308,$

and $\quad x = 77.$

EXERCISE 171

1. Find a number whose third part plus 9 equals 25.

2. Find a number whose fifth part minus 1 equals 29.

3. Find a number such that if 7 is added to $\frac{2}{3}$ of it, the sum is 35.

4. Find a number whose fourth part minus its fifth part equals 2.

5. Find a number such that $\frac{5}{7}$ of it less $\frac{3}{8}$ of it equals 19.

6. What number increased by $\frac{4}{5}$ of itself equals 99?

7. Half the remainder found by subtracting 14 from a certain number equals a fourth of the sum of the number and 14. What is the number?

8. Find a number whose half, third, and fourth parts added together equals 39.

9. After selling $\frac{1}{2}$ of his farm, and then $\frac{1}{8}$ of what was left, a man still had 140 acres. How many acres had he at first?

10. There is a certain number such that its fourth part added to its fifth part equals one less than its half. What is the number?

11. The sum of two numbers is 90, and one is 80% of the other. What are the numbers?

12. The sum of two numbers is 100, and the less is $\frac{2}{3}$ the greater. What are the numbers?

13. If 220 is divided by one less than a certain number, the result is 55. What is the number?

14. Find three consecutive numbers whose sum is 87. (Represent the numbers by $x - 1$, x, and $x + 1$.)

15. A tree 60 feet high is broken so that the part broken off is 4 times the length of the part left standing. What is the length of each part?

16. A tree 84 ft. high is broken so that the part broken off is five times as long as the part left standing. What is the length of each part?

17. Distribute $36 among four men so that the first two shall each receive twice as much as each of the second two.

18. If we add 24 to a certain number, the sum is as much above 80 as the number is below 80. What is the number?

19. A farmer bought 18 sheep. If he had bought 3 more for the same money, each sheep would have cost him $1 less. How much did he pay for each sheep?

20. Two men buy an automobile for $2500. One pays $100 more than twice what the other pays. If they use the automobile in proportion to what they pay, how many days should each use it out of every 25 days?

262. Easy Equations involving Two Unknown Quantities. In solving a problem it is sometimes convenient to employ two unknown quantities.

1. Find two numbers whose sum is 12 and whose difference is 6.

Let $x =$ the larger number,

and $y =$ the smaller number.

Then $x + y = 12$, since their sum is 12, (1)

and $x - y = 6$, since their difference is 6. (2)

Adding (1) and (2), $2x = 18$.

Dividing by 2, $x = 9$.

Substituting 9 for x in (1),

$$9 + y = 12.$$

Transposing 9, $y = 3$.

Therefore the two numbers required are 9 and 3.

2. Find two numbers such that three times the first plus twice the second is 20, and five times the first minus the second is 3.

Let $x =$ the first number,

and $y =$ the second number.

Then from the statement of the problem

$$3x + 2y = 20, \qquad (1)$$

and $$5x - y = 3. \qquad (2)$$

We see that if we multiply both members of (2) by 2, we can add (1) to this result and the y's will disappear. Therefore, multiplying (2) by 2, $10x - 2y = 6$. (3)

Adding (1) and (3), $13x = 26$.

Dividing by 13, $x = 2$.

Substituting 2 for x in (1),

$$6 + 2y = 20.$$

Transposing 6, $2y = 14$.

Dividing by 2, $y = 7$.

Therefore the two numbers required are 2 and 7.

EXERCISE 172

1. The sum of two numbers is 88, and one number is 20 more than the other. What are the numbers?

2. The sum of the ages of a father and son is 84 yr., and the father is twice as old as the son. How old is the son?

3. The distance around a certain rectangular field is 78 rd., and the length is 2.9 times the width. What is the width? the length?

4. A dealer bought 25 dozen oranges for $9.25. For a part he paid 40 cents a dozen and for the remainder 35 cents a dozen. How many dozen of each did he buy?

5. A boy is three times as old as his sister. A year ago he was four times as old as she, and three years hence he will be twice as old as she. How old are they now?

6. The age of one boy is $2\frac{1}{2}$ times that of another, and the sum of their ages is 21 yr. Find the age of each.

7. One number is four times as large as another. If I take the smaller number from 12 and the larger from 21, the remainders are equal. What are the numbers?

8. The sum of the ages of a father and son is 50 yr. If the age of the son is doubled, the result is 17 yr. less than the age of the father. What is the age of each?

9. In a class of 29 pupils there are 3 more girls than boys. How many are there of each?

10. At an election there were two candidates, and 3478 votes were cast. The successful candidate had a majority of 436. How many votes were cast for each?

11. One man has twice as much money as another; but if the second gives the first $10, the first will then have $3\frac{1}{2}$ times as much as the second. How much has each?

APPENDIX

263. Subjects Treated. There are certain subjects in arithmetic which some schools prefer to treat in Grades V and VI, but which others find it advantageous to consider in Grades VII and VIII. There are legitimate reasons for this variation, certain topics correlating with the rest of the curriculum better in Grade VI in one school and in Grade VII in another school. There are certain other subjects that are valuable for review and drill in all of the grades, but which one school finds it necessary to emphasize by an unusual amount of work, and which a teacher often finds it advantageous to use with one class but not with another. There are still other topics that are generally omitted, and yet which may profitably be taken in case the time allows. All topics of this nature find their proper place in an Appendix, to be used at the discretion of the teacher.

The subjects treated are: (1) Short Methods. These have already been given in Book Two of this series, but may profitably be reviewed in Grades VII and VIII. (2) The related and important topic of Casting out Nines. (3) Longitude and Time. This topic is usually studied in Grade VI, where it correlates well with the work in geography; but in case it is found better adapted to Grade VII it may be taken there. (4) Progressions and Compound Interest, topics particularly useful if a little work in algebra has been covered. (5) Account Sales and the Settlement of Accounts. (6) Additional Drill Problems. (7) Arithmetical Recreations. (8) Tables for Reference. (9) Spanish Land Measures.

264. Short Methods. A few of the short methods that have practical value will now be reviewed.

265. Short Methods of Addition. The most helpful suggestion in rapid addition is to add by groups instead of by single figures.

For example, in this column the pupil should see 19 when he glances at the three lowest figures; he should see two eights, or 16, in the next three, and 9 in the two top figures. He should therefore think "19, 35, 44," or some similar adding of groups.

Probably he will at first find it easier to group by twos and think "15, 24, 35, 44."

Professional computers often add two columns at once, but this is not recommended for pupils in school, where absolute accuracy is more important than speed.

7
2
8
3
5
4
6
9

When there are two or more columns of figures in the addends, professional computers usually write down the sum of each column. In this way an error may be found more easily when the work is checked by adding in the direction opposite to that of the original addition. The immediate discovery of such errors is the most important means of saving time.

It is often necessary to add horizontally as well as vertically.

EXERCISE 173

Add rapidly without copying:

1. 25, 50, 30, 70, 25, 10, 90, 45, 15, 40, 25, 75.
2. 35, 45, 20, 60, 40, 50, 75, 25, 35, 15, 50, 75, 25.
3. 50, 25, 75, 50, 30, 70, 20, 80, 10, 90, 30, 30, 30, 10.
4. 27, 23, 50, 36, 64, 18, 32, 56, 24, 20, 74, 26, 87, 13.
5. 15, 25, 17, 23, 36, 41, 82, 39, 28, 32, 62, 81, 73, 48.
6. $1.75, $2.50, $3.25, $4.50, $5.20, $6.80, $10.50, $2.
7. $1.50, $3.75, $2.25, $4.50, $6.60, $3.40, $2, $5.
8. $2.75, $3.25, $4.10, $5.40, $2.50, $4.25, $7, $3.
9. $4.20, $2.60, $5.30, $4.20, $2.60, $3.80, $10.75.

Add as rapidly as possible, trying always to add by groups of two or three figures. Check each result.

10.	13.	16.	19.	22.	25.
26	97	29	428	$34.78	$121.42
43	60	91	607	62.48	137.60
28	89	72	320	71.92	89.48
91	34	30	556	38.46	125.50
72	42	47	707	50.09	38.75
54	80	66	62	77.05	192.50
81	75	55	491	82.91	65.82
24	66	43	377	4.80	48.75

11.	14.	17.	20.	23.	26.
47	84	22	572	$75.50	$263.08
63	60	89	681	26.75	148.75
96	32	72	708	8.93	192.68
34	98	35	320	14.98	635.35
81	77	60	572	6.37	870.08
29	63	81	647	24.42	428.70
88	49	92	998	69.70	642.35
72	92	37	123	8.84	408.77

12.	15.	18.	21.	24.	27.
29	35	29	547	$18.92	$221.62
68	42	68	629	17.86	342.98
73	87	73	372	23.48	476.35
40	98	52	588	52.96	192.09
96	63	31	793	78.32	778.38
85	47	48	874	91.46	658.02
77	72	77	645	87.00	523.65
42	31	65	880	9.85	807.91
50	60	75	300	10.00	500.00

266. Rapid Horizontal Addition. As already shown on page 266, it is frequently necessary in business to add the numbers when written horizontally. A few additional types of this kind are given, to be used when drill work of this nature appears to be necessary.

EXERCISE 174

Add the following without copying in columns:

1. 48, 62, 68, 42, 54, 56, 65, 45, 61, 49, 50, 27, 33, 45.
2. 26, 84, 34, 76, 59, 51, 37, 73, 82, 28, 68, 42, 34, 76.
3. 29, 81, 30, 70, 42, 68, 28, 32, 39, 71, 61, 49, 57, 53.
4. 39, 61, 82, 18, 61, 39, 53, 47, 42, 58, 37, 63, 72, 28.
5. 34, 26, 59, 36, 82, 96, 54, 30, 48, 72, 81, 96, 34, 23.
6. 37, 56, 48, 65, 62, 34, 59, 53, 22, 78, 61, 82, 50, 96.
7. 97, 42, 20, 86, 82, 31, 57, 75, 68, 37, 64, 48, 53, 90.
8. 22, 30, 88, 34, 57, 46, 55, 98, 99, 72, 66, 54, 60, 77.
9. 19, 55, 91, 73, 80, 28, 68, 64, 54, 48, 37, 82, 32, 46.
10. 24, 36, 95, 81, 27, 64, 79, 83, 40, 17, 39, 19, 20, 86.
11. 37, 42, 81, 93, 87, 49, 80, 17, 45, 36, 81, 92, 77, 60.
12. 30, 20, 75, 85, 90, 27, 96, 19, 48, 32, 57, 86, 41, 32.
13. 68, 30, 43, 77, 26, 32, 86, 34, 82, 95, 96, 87, 99, 80.
14. 99, 58, 57, 26, 89, 36, 48, 47, 79, 15, 38, 37, 69, 28.
15. 12, 56, 89, 77, 52, 23, 19, 67, 36, 34, 40, 78, 45, 90.
16. 28, 40, 84, 62, 16, 17, 28, 95, 39, 39, 56, 51, 47, 73.
17. 50, 94, 98, 65, 31, 64, 61, 42, 76, 53, 72, 75, 87, 83.
18. 23, 67, 90, 63, 81, 78, 34, 72, 89, 54, 45, 45, 34, 56.
19. 16, 50, 83, 46, 27, 61, 94, 57, 38, 86, 35, 49, 72, 68.
20. 33, 40, 28, 96, 81, 73, 54, 62, 91, 43, 70, 92, 60, 55.
21. 0.40, 0.32, 0.26, 1.84, 0.96, 0.53, 1.70, 2.40, 3.55.

267. Short Methods of Subtraction. There is only one short method in subtraction that is of much practical value.

This one will be found to be particularly helpful. In a case like 40,000 – 17,652 we may think of the places of the minuend as 3, 9, 9, 9, 10 and subtract from left to right.

3	9	9	9	10
1	7	6	5	2
2	2	3	4	8

EXERCISE 175

Subtract as rapidly as possible, beginning at the left when this can easily be done:

1.	**7.**	**13.**	**19.**
7000	50,000	$100.00	$250.
2763	26,492	75.62	75.65
2.	**8.**	**14.**	**20.**
6000	60,000	$300.00	$350.
4296	31,429	128.64	82.50
3.	**9.**	**15.**	**21.**
5000	40,000	$500.00	$640.
1297	17,792	275.55	121.75
4.	**10.**	**16.**	**22.**
8000	70,000	$700.00	$480.
2776	20,078	428.50	135.25
5.	**11.**	**17.**	**23.**
9000	65,000	$900.00	$530.
4326	21,572	230.20	192.25
6.	**12.**	**18.**	**24.**
8000	80,000	$800.00	$320.
7080	27,600	206.00	175.75

268. Short Methods of Multiplication. There are several short methods of multiplication that are valuable, particularly because certain multipliers are so often used.

Such multipliers are 10, 100, 1000, and other powers of 10; 5, 25, 125, and $12\frac{1}{2}$; $16\frac{2}{3}$, $33\frac{1}{3}$, and $66\frac{2}{3}$; 9, 11, and 75.

269. Aliquot Parts. A number that will exactly divide another number is called an *aliquot part* of that number.

Thus 5, 10, 25, $12\frac{1}{2}$, $16\frac{2}{3}$, $33\frac{1}{3}$, and so on, are aliquot parts of 100, and they include some of the most important multipliers mentioned above.

The most important aliquot parts of 100 are as follows:

$50 = \frac{1}{2}$ of 100	$33\frac{1}{3} = \frac{1}{3}$ of 100	$20 = \frac{1}{5}$ of 100
$25 = \frac{1}{4}$ of 100	$16\frac{2}{3} = \frac{1}{6}$ of 100	$10 = \frac{1}{10}$ of 100
$12\frac{1}{2} = \frac{1}{8}$ of 100	$8\frac{1}{3} = \frac{1}{12}$ of 100	$5 = \frac{1}{20}$ of 100
$6\frac{1}{4} = \frac{1}{16}$ of 100	$4\frac{1}{6} = \frac{1}{24}$ of 100	$2 = \frac{1}{50}$ of 100

Other important fractional parts of 100, but not aliquot parts, are as follows:

$75 = \frac{3}{4}$ of 100	$62\frac{1}{2} = \frac{5}{8}$ of 100	$66\frac{2}{3} = \frac{2}{3}$ of 100
$37\frac{1}{2} = \frac{3}{8}$ of 100	$87\frac{1}{2} = \frac{7}{8}$ of 100	$83\frac{1}{3} = \frac{5}{6}$ of 100

270. Multiplication by Aliquot Parts. We may simplify multiplication by 10, 100, and 1000, and by their aliquot parts, as follows:

(1) *To multiply by 10, move the decimal point one place to the right, annexing zero if necessary.*

Thus $10 \times 46.75 = 467.5$, and $10 \times 75\frac{1}{2} = 10 \times 75.5 = 755$.

(2) *To multiply by 100, move the decimal point two places to the right, annexing zeros if necessary.*

Thus $100 \times 0.275 = 27.5$, $100 \times 6.5 = 650$, and $100 \times 72 = 7200$.

(3) *To multiply by 1000, move the decimal point three places to the right, annexing zeros if necessary.*

Thus $1000 \times 0.02365 = 23.65$, and $1000 \times 147 = 147,000$.

(4) *To multiply by 50, multiply by 100 and divide by 2.*
For $50 = \frac{100}{2}$; therefore $50 \times 84 = \frac{100}{2} \times 84 = \frac{8400}{2} = 4200$.

(5) *To multiply by 25, multiply by 100 and divide by 4.*
For $25 = \frac{100}{4}$; therefore $25 \times 84 = \frac{100}{4} \times 84 = \frac{8400}{4} = 2100$.

(6) *To multiply by $12\frac{1}{2}$, multiply by 100 and divide by 8.*
For $12\frac{1}{2} = \frac{100}{8}$; therefore $12\frac{1}{2} \times 64 = \frac{100}{8} \times 64 = 800$.

(7) *To multiply by 125, multiply by 1000 and divide by 8.*
For $125 = \frac{1000}{8}$; therefore $125 \times 72 = \frac{1000}{8} \times 72 = 9000$.

(8) *To multiply by $33\frac{1}{3}$, multiply by 100 and divide by 3.*
For $33\frac{1}{3} = \frac{100}{3}$; therefore $33\frac{1}{3} \times 69 = \frac{6900}{3} = 2300$.

(9) *To multiply by $16\frac{2}{3}$, multiply by 100 and divide by 6.*
For $16\frac{2}{3} = \frac{100}{6}$; therefore $16\frac{2}{3} \times 84 = \frac{8400}{6} = 1400$.

(10) *To multiply by 5, multiply by 10 and divide by 2.*
For $5 = \frac{10}{2}$; therefore $5 \times 0.42 = \frac{4.2}{2} = 2.1$.

EXERCISE 176

Multiply:

1. 10 × \$0.25.
2. 10 × \$8251.
3. 10 × \$6.25.
4. 10 × \$5.50.
5. 100 × \$0.37.
6. 100 × \$2.87½.
7. 1000 × \$0.25.
8. 1000 × \$6.37½.
9. 50 × \$64,224.
10. 50 × \$752.20.
11. 50 × \$861.55.
12. 25 × \$144.
13. 25 × \$640.
14. 25 × \$325.
15. 675 × \$25.
16. 25 × \$25.50.
17. 25 × \$75.25.
18. 12½ × \$6400.
19. 12½ × \$4880.
20. 125 × \$640.
21. 125 × \$6400.
22. 125 × \$2800.
23. 33⅓ × 36.
24. 33⅓ × 48.
25. 33⅓ × 144.
26. 33⅓ × \$312.30.
27. 16⅔ × 12,630.
28. 16⅔ × \$636.30.
29. 5 × 47,664.80.
30. 5 × \$210,650.
31. 125 × \$84,480.
32. 125 × \$844.80.
33. 16⅔ × 4,800,000.

34. What will 25 books cost at $1.20 each?

35. What will 25 wagons cost at $116 each?

36. What will 50 cows cost at $54 a head?

37. What will 100 books cost at 62½¢ each?

38. What will 25 tons of hay cost at $12.40 a ton?

39. What will 12½ yards of cloth cost at 35¢ a yard?

40. What will 25 dozen pencils cost at $0.48 a dozen?

41. What will 48 office desks cost at $25 each?

42. At $5 an acre, what is a tract of 14,264 acres of mountain land worth?

43. How much will a man earn in 36 weeks at $25 a week? at $33⅓ a week?

44. How much will a dozen tables weigh at 25 pounds each? at 50 pounds each?

45. How much will 615 men earn in a week if each earns $12½? if each earns $16⅔?

46. How much must a dealer pay for 336 easy-chairs at $12½ each? at $16⅔ each? at $6¼ each?

47. A dealer buys 144 boxes of soap, each weighing 12½ pounds. What is the total weight?

48. At $33⅓ a month, how much rent will a man pay for his house in a year? in six months?

49 At $33⅓ a week, how much will a salesman receive if he works 48 weeks? if he works 52 weeks?

50. If a man earns $16⅔ a week, how much will he earn in 36 weeks? in 48 weeks? in 52 weeks?

51. Multiply 1728 by 1¼; 172.8 by 12½; 17.28 by 125. How do the results differ? Write the reason for this.

52. Multiply 1728 by 3⅓; 172.8 by 33⅓; 17.28 by 333⅓. Compare the results and explain, as in Ex. 51.

271. Multiplication by Fractional Parts of 100. We may simplify multiplication by fractional parts of 100 as follows:

(1) *To multiply by 75, multiply by 100 and take $\frac{3}{4}$ of the product.*

For $75 = \frac{3}{4}$ of 100; therefore $75 \times \$64.40 = \frac{3}{4}$ of $\$6440 = \$4830$.

(2) *To multiply by $37\frac{1}{2}$, multiply by 100 and take $\frac{3}{8}$ of the product.*

For $37\frac{1}{2}$ is $\frac{3}{8}$ of 100; therefore $37\frac{1}{2} \times \$72 = \frac{3}{8}$ of $\$7200 = \$2700$.

(3) *To multiply by $62\frac{1}{2}$, multiply by 100 and take $\frac{5}{8}$ of the product.*

For $62\frac{1}{2} = \frac{5}{8}$ of 100; therefore $62\frac{1}{2} \times 640 = \frac{5}{8}$ of $64{,}000 = 40{,}000$.

(4) *To multiply by $87\frac{1}{2}$, multiply by 100 and take $\frac{7}{8}$ of the product.*

For $87\frac{1}{2} = \frac{7}{8}$ of 100; therefore $87\frac{1}{2} \times 96 = \frac{7}{8}$ of $9600 = 8400$.

(5) *To multiply by $66\frac{2}{3}$, multiply by 100 and take $\frac{2}{3}$ of the product.*

For $66\frac{2}{3} = \frac{2}{3}$ of 100; therefore $66\frac{2}{3} \times 144 = \frac{2}{3}$ of $14{,}400 = 9600$.

272. Multiplication by Certain Decimals. From § 270 and § 271 the following short methods are evident:

(1) *To multiply by 0.1, move the decimal point one place to the left; by 0.01, move the decimal point two places to the left; by 0.001, move the decimal point three places to the left.*

(2) *To multiply by 0.5, take half of the multiplicand.*

(3) *To multiply by 0.25, take a fourth of the multiplicand.*

(4) *To multiply by $0.12\frac{1}{2}$, take an eighth of the multiplicand.*

(5) *To multiply by $0.33\frac{1}{3}$, take a third of the multiplicand.*

(6) *To multiply by $0.16\frac{2}{3}$, take a sixth of the multiplicand.*

(7) *To multiply by 0.75, take three fourths of the multiplicand.*

Other similar rules will easily be found by the student.

EXERCISE 177

Multiply:

1. 75×684.	17. 0.75 of 6884.	33. $744 \times 0.87\frac{1}{2}$.
2. 75×816.	18. $37\frac{1}{2} \times 424$.	34. $66\frac{2}{3} \times 3366$.
3. 75×936.	19. $37\frac{1}{2} \times 1776$.	35. $66\frac{2}{3} \times 2886$.
4. $75 \times \$1.48$.	20. $736 \times \$0.37\frac{1}{2}$.	36. $66\frac{2}{3} \times 288.6$.
5. $75 \times \$3.20$.	21. $656 \times \$0.37\frac{1}{2}$.	37. $66\frac{2}{3} \times 28.86$.
6. $75 \times 24{,}616$.	22. $568 \times \$0.62\frac{1}{2}$.	38. 0.5 of 6432.
7. $248 \times \$0.75$.	23. $736 \times \$0.62\frac{1}{2}$.	39. 0.5 of 1338.
8. $432 \times \$0.75$.	24. $824 \times \$0.87\frac{1}{2}$.	40. 0.5 of 7952.
9. 0.75 of 2436.	25. $592 \times \$0.87\frac{1}{2}$.	41. 2952×0.05.
10. 3574×0.05.	26. $368 \times \$0.12\frac{1}{2}$.	42. 248×0.05.
11. 0.25 of 536.	27. $336 \times \$0.33\frac{1}{3}$.	43. 24.8×0.5.
12. 0.25 of 5.36.	28. $297 \times \$0.33\frac{1}{3}$.	44. 2.48×0.5.
13. 0.25 of 0.536.	29. $372 \times \$0.66\frac{2}{3}$.	45. 24×7.5.
14. $704 \times \$0.25$.	30. $492 \times \$0.66\frac{2}{3}$.	46. 2.4×7.5.
15. $172 \times \$0.25$.	31. $558 \times \$0.16\frac{2}{3}$.	47. 2.4×0.75.
16. $776 \times \$0.12\frac{1}{2}$.	32. $174 \times \$0.16\frac{2}{3}$.	48. $0.12\frac{1}{2} \times 0.16$.

49. How much will 75 cows cost at $48 each?

50. How much will 75 cows cost at $55 each?

51. How much will 72 horses cost at $75 each?

52. How much will 88 horses cost at $125 each?

53. How much will 288 whalebones cost at 25¢ each?

54. How much will 275 packing boxes cost at 62½¢ each?

55. How much will 484 yards of silk cost at 75¢ a yard?

56. How much will 364 yards of cloth cost at 75¢ a yard?

57. How much will 624 yards of jute burlap cost at 37½¢ a yard?

273. Multiplication by Numbers near 10, 100, and so on. Since $9 = 10 - 1$, we may multiply easily by 9 by annexing a zero, which multiplies by 10, and then subtracting the given multiplicand from the product by 10.

$$\begin{aligned} 10 \times 7286 &= 72860 \\ 1 \times 7286 &= \underline{7286} \\ 9 \times 7286 &= 65574 \end{aligned}$$

Thus 9×7286 is evidently the same as $10 \times 7286 - 7286$, as here shown. It is evidently shorter to subtract 7286 from 72,860 than to multiply 7286 by 9.

From this reasoning and similar courses of reasoning we have the following short processes:

(1) *To multiply by 9, multiply by 10 and subtract the given multiplicand.*

Thus $9 \times \$374.62 = \$3746.20 - \$374.62 = \$3371.58$.

(2) *To multiply by 11, multiply by 10 and add the given multiplicand.*

Thus $11 \times \$374.62 = \$3746.20 + \$374.62 = \$4120.82$.

(3) *To multiply by 99, multiply by 100 and subtract the given multiplicand.*

Thus $99 \times 4368 = 436{,}800 - 4368 = 432{,}432$.

(4) *To multiply by 98, multiply by 100 and subtract twice the given multiplicand.*

Thus $98 \times \$435.75 = \$43{,}575 - 2 \times \$435.75 = \$43{,}575 - \$871.50 = \$42{,}703.50$.

To multiply by 999, 998, and like numbers, proceed in a manner similar to the above.

EXERCISE 178

Multiply:

1. $9 \times \$724.56$.	5. $99 \times \$128.72$.	9. $999 \times \$43.75$.
2. $9 \times \$875.93$.	6. $99 \times \$426.45$.	10. $998 \times \$62.86$.
3. $11 \times \$682.97$.	7. $98 \times \$327.47$.	11. $1001 \times \$27.63$.
4. $11 \times \$493.82$.	8. $98 \times \$462.09$.	12. $1002 \times \$324.75$.

274. Short Methods of Division. There are certain divisors that are used so often that it becomes convenient to know short methods of dividing by them.

The principal ones are 10, 100, 1000, and other powers of 10; 5, 25, 125, $12\frac{1}{2}$, and $33\frac{1}{3}$.

Since division is the inverse of multiplication, we easily derive the following from the cases already considered:

(1) *To divide by 10, move the decimal point one place to the left; by 100, two places; by 1000, three places; and so on.*

Thus $4936 \div 10 = 493.6$; $\$4876 \div 100 = \$48.76$; $29.5 \div 1000 = 0.0295$. This has already been explained in studying decimal fractions.

(2) *To divide by 5, multiply by 2 and divide by 10.*

For to divide by 5 is to multiply by $\frac{1}{5}$ or $\frac{2}{10}$. To divide 32,305 by 5 we may multiply by 2, obtaining 64,610, and cut off the zero, obtaining 6461, a process somewhat easier than actual division.

(3) *To divide by 25, multiply by 4 and divide by 100.*

For $\frac{1}{25} = \frac{4}{100}$. It is much easier to multiply by 4 and move the decimal point two places than to divide by 25.

(4) *To divide by 125, multiply by 8 and divide by 1000.*

Why is this true?

(5) *To divide by $33\frac{1}{3}$, multiply by 3 and divide by 100.*

Why is this true?

EXERCISE 179

Divide:

1. $\$7463 \div 10$.
2. $\$2896 \div 100$.
3. $\$4960 \div 1000$.
4. $\$2215.50 \div 5$.
5. $\$3122.35 \div 5$.
6. $\$172,232 \div \$5$.
7. $\$9125 \div \$25$.
8. $12,325 \div 25$.
9. $31,325 \div 125$.
10. $62,750 \div 125$.
11. $\$1525 \div 33\frac{1}{3}$.
12. $\$3275 \div 33\frac{1}{3}$.

13. At 25¢ a yard, how many yards of gingham can a dealer buy for $124?

14. At 25¢ a yard, how many yards of percaline can a dealer buy for $160?

15. At 25¢ a dozen, how many dozen spools of darning cotton can a dealer buy for $180?

16. At $33\frac{1}{3}$¢ a yard, how many yards of canvas can a dealer buy for $135?

17. At $33\frac{1}{3}$¢ a yard, how many yards of jute burlap can a dealer buy for $165?

18. At 3.33\frac{1}{3}$ a day, how many days must a man work in order to earn $130?

19. At $3\frac{1}{3}$¢ a yard, how many yards of cheese cloth can a dealer buy for $115?

20. At $2\frac{1}{2}$¢ a skein, how many skeins of cotton must a manufacturer sell in order to receive $125.75?

21. At $1.25 a box, how many boxes of fancy candies can a dealer buy for $180?

22. At $125 each, how many horses can a dealer buy for $1750? for $1375?

23. At $5 each, how many rocking-chairs must a factory sell in order to receive $3375?

24. A manufacturer sells $3750 worth of typewriters at $50 each. How many does he sell?

25. How long will it take an express train to go 362.5 miles at the rate of 50 miles an hour?

26. How long will it take a freight train to go $166\frac{2}{3}$ miles at the rate of 25 miles an hour?

27. A manufacturer sells $1810 worth of silk at $1.25 a yard. How many yards does he sell?

275. Check by Casting out Nines. One of the most valuable checks upon computations, particularly in multiplication and division, is the one known as Casting out Nines, and this will now be explained.

276. Excess of Nines. The remainder after dividing any integer by 9 is called the *excess of nines* in the integer.

Thus the excess of nines in 11 is 2, because $11 \div 9 = 1$, with 2 remainder. The excess of nines in 48 is 3, the excess in 85 is 4, the excess in 108 is 0.

277. Finding the Excess of Nines. Instead of dividing by 9 to find the excess there is a much shorter method. Consider, for example, the number 2348.

We may separate this into $2000 + 300 + 40 + 8$.

$$\begin{aligned} 2000 &= 2 \times 999 + 2 \\ 300 &= 3 \times 99 + 3 \\ 40 &= 4 \times 9 + 4 \\ 8 &= \phantom{4 \times 999 +{}} 8 \\ \hline 2348 &= \text{a multiple of } 9, \ + 2 + 3 + 4 + 8 \end{aligned}$$

That is, 2348 contains 9 exactly except for the sum of its digits, 2, 3, 4, 8.

Therefore, *the excess of nines in any number is equal to the excess in the sum of the digits.*

278. Casting out Nines. In finding the remainder from dividing the sum of the digits by 9, that is, in casting out the nines, we may of course omit the nines, or any two or three digits which we see at a glance will make 9.

Thus in casting out the nines from 1,926,754, we see at once that $1 + 2 + 6 = 9$ and $5 + 4 = 9$, and the single 7 will be the remainder. So in 254,786, we reject 5, 4, and 2, 7, and add $8 + 6$; from the sum we reject 9 and have 5 left.

279. Check in Multiplication. *The excess of nines in a product equals the excess in the product of the excesses in the factors.*

Check the product of 61×47 by casting out nines.

The remainder after the nines are cast out:

$$\begin{array}{llllll} \text{From} & 47 & \text{is} & & 2 & \multirow{2}{*}{} \\ \text{From} & \underline{61} & \text{is} & & \underline{7} & \end{array}\left.\vphantom{\begin{array}{l}2\\7\end{array}}\right\}\text{Multiply}$$

$$\text{From } 2867 \text{ is } \mathbf{5}. \qquad \text{From } 14 \text{ is } \mathbf{5}.$$

The product of the two *numbers* has an excess 5, and the product of the two *excesses* has 5 remaining after the nines are cast out. Therefore the work may be assumed to be correct. The check does not, however, discover errors that are independent of the sum of the digits.

A convenient arrangement of work is as follows:

$$\begin{array}{r} 257 \\ \underline{84} \\ 1028 \\ \underline{2056} \\ 21588 \end{array} \qquad \begin{array}{ccc} & 6 & \\ 5 & \times & 3 \\ & 6 & \end{array}$$

In the two angles at the sides of the cross write 5 and 3, the excesses in the factors. At the top write the excess in the product of these excesses, 6 being the excess in 3×5, or 15. At the bottom write the excess in the product, 6 being the excess in 21,588. The top and bottom numbers in the cross should then agree.

280. Check in Division. *The excess of nines in a dividend equals the excess in the product of the excesses in the divisor and quotient, plus the excess in the remainder.*

For, if $1,348,708 \div 498$ is 2708 with remainder 124, then $1,348,708 = 2708 \times 498 + 124$. Now 8, the excess in 2708, times 3, the excess in 498, is 24; and 6, the excess in 24, plus 7, the excess in 124, is 13, or simply 4. Since the excess in 1,348,708 is 4, the work may be assumed to be correct.

EXERCISE 180

Multiply, and check by casting out nines:

1. 75×628.	3. 27×691.	5. 246×489.
2. 34×872.	4. 36×987.	6. 492×572.

Divide, and check by casting out nines:

7. $144 \div 12$.	9. $1331 \div 11$.	11. $1342 \div 21$.
8. $1524 \div 12$.	10. $2714 \div 15$.	12. $27,634 \div 128$.

281. Longitude and Time. One class of problems that arises when we think of the change in time as we travel or telegraph east or west relates to longitude and time.

282. Meridian. The shortest line on the earth's surface passing from one pole to the other is called a *meridian.*

283. Longitude. The distance in circular measure east or west of a certain meridian agreed upon is called *longitude.*

284. Prime Meridian. A meridian agreed upon as the one from which longitude is reckoned is called a *prime meridian.*

The prime meridian now used by most of the maritime nations of the world is that which passes through the National Observatory at Greenwich, England.

A place 75° 30′ 20″ west of this meridian is said to be in 75° 30′ 20″ west longitude. A place 15° 25′ east of this meridian is said to be in 15° 25′ east longitude.

Since 180° is halfway around the earth from the prime meridian there can be no more than 180° E. (that is, east longitude) or 180° W. (that is, west longitude).

285. Difference in Longitude. When two places are both east or both west of the prime meridian, the difference in longitude is evidently found by subtracting one longitude from the other.

When one place is east and the other is west of the prime meridian, the difference in longitude is evidently found by adding the two longitudes.

This is clearly seen by looking at a map or globe of the world.

Since the earth is spherical, there are always two differences in longitude between any two places, and we may go from one longitude to the other by traveling either east or west. If, therefore, the difference is more than 180°, we may subtract this from 360° to get the smaller difference. Thus the difference between 100° E. and 100° W. is 200° or 160°, depending on which way we think of going from one longitude to the other. It is usually better to take the smaller number, although occasionally it is more convenient to use the larger.

286. Relation of Longitude to Time. Since the earth makes one complete revolution upon its axis in 24 hr., a point on the surface passes through 360° in 24 hr. That is,

360° corresponds to 24 hr.
1° corresponds to $\frac{1}{360}$ of 24 hr., or $\frac{1}{15}$ hr., or 4 min.
1′ corresponds to $\frac{1}{60}$ of 4 min., or $\frac{1}{15}$ min., or 4 sec.
1″ corresponds to $\frac{1}{60}$ of 4 sec., or $\frac{1}{15}$ sec.

Hence we have the following correspondence:

Longitude to Time		*Time to Longitude*	
360° . . .	24 hr.	24 hr. . . .	360°
1° . . .	$\frac{1}{15}$ hr. = 4 min.	1 hr. . . .	15°
1′ . . .	$\frac{1}{15}$ min. = 4 sec.	1 min. . . .	$\frac{1}{4}$° = 15′
1″ . . .	$\frac{1}{15}$ sec.	1 sec. . . .	$\frac{1}{4}$′ = 15″

287. Finding Difference in Time. Given the difference in longitude, the difference in time may be found.

For example, two ships at sea are 25° 24′ 15″ of longitude apart. What is the difference in time?

Looking at the above table, we find the difference in time corresponding to 1°, 1′, and 1″, and then we obtain the following:

$$25 \times 4 \text{ min.} = 100 \text{ min.} = 1 \text{ hr. } 40 \text{ min.}$$
$$24 \times 4 \text{ sec.} = 96 \text{ sec.} = 1 \text{ min. } 36 \text{ sec.}$$
$$15 \times \tfrac{1}{15} \text{ sec.} = 1 \text{ sec.}$$

Therefore the difference in time is 1 hr. 41 min. 37 sec.

Therefore, *given the difference in longitude to find the difference in time, multiply 4 minutes of time by the number of degrees, 4 seconds of time by the number of minutes of longitude, and $\frac{1}{15}$ of a second of time by the number of seconds of longitude, and add the products.*

288. Later to the East, Earlier to the West. Because the earth turns from the west to the east, the sun appears to go from east to west. It is therefore 1 hr. later 15° east of here, and 1 hr. earlier 15° west.

EXERCISE 181

Find the difference in time between two places, given the following differences in longitude:

1. $15° 15'$.	6. $25° 15'$.	11. $5° 10' 15''$.
2. $15' 15''$.	7. $32° 30'$.	12. $7° 25' 30''$.
3. $30° 15'$.	8. $48° 45'$.	13. $28° 42' 30''$.
4. $45° 30'$.	9. $127° 15''$.	14. $62° 27' 45''$.
5. $60° 30''$.	10. $168° 30''$.	15. $128° 6' 7\frac{1}{2}''$.

16. Two ships are $32° 40' 30''$ of longitude apart. What is the difference in time?

17. Two observatories are $42° 27' 45''$ of longitude apart. What is the difference in time?

18. A ship in 15° W. sends a wireless telegram at noon. If the telegram reaches Greenwich immediately, at what time does it arrive at Greenwich?

19. A ship in $58° 17' 20''$ W. receives at noon a wireless telegram from one in $43° 2' 5''$ W. When was it sent?

20. Italy uses the time of 15° E., and Indiana that of 90° W. When it is noon in Italy, what time is it in Indiana?

21. Constantinople uses for railway purposes the time of 30° E., and New England uses that of 75° W. When it is 1 P.M. in Constantinople, what time is it in New England?

22. A ship at sea carries a chronometer that shows Greenwich time. A ship is in longitude $37° 42' 30''$ W. at noon. What is the time by the Greenwich chronometer?

23. A traveler sailing from New York has his watch set by 75th meridian time. Two days later he finds that the ship is in longitude 55° W. Is his watch faster or slower than ship time, and how much?

289. Finding Difference in Longitude. If the difference in time between two ships is 2 hr. 17 min. 22 sec., what is the difference in longitude?

Consulting the table (§ 286), we find the difference in longitude corresponding to 1 hr., 1 min., and 1 sec., and then obtain:

$$2 \times 15^\circ = 30^\circ$$
$$17 \times 15' = 255' = 4^\circ\,15'$$
$$22 \times 15'' = 330'' = 5'\,30''$$

Therefore the difference in longitude is 34° 20′ 30″.

Given the difference in time to find the difference in longitude, multiply 15°, 15′, 15″ respectively by the number of hours, minutes, and seconds of time, and add the products.

EXERCISE 182

Find the difference in longitude, given the following differences in time:

1. 2 hr. 3 min.
2. 4 hr. 7 min.
3. 6 hr. 9 min.
4. 10 hr. 4 sec.
5. 11 hr. 8 sec.
6. 4 hr. 20 min.
7. 5 hr. 25 min.
8. 7 hr. 6 min. 8 sec.
9. 9 hr. 35 min. 40 sec.
10. 11 hr. 58 min. 48 sec.

11. When it is noon by the Greenwich chronometer it is 2 P.M. where a ship is. What is the ship's longitude?

12. When it is 3 P.M. by the Greenwich chronometer it is noon where a ship is. What is the ship's longitude?

13. The captain of another ship finds that when it is noon where the ship is it is 2 P.M. by the Greenwich chronometer. What is the longitude of the ship?

14. An ocean traveler whose watch is set by Greenwich time finds that when it is noon by ship time it is 1.45 P.M. by his watch. What is his longitude?

290. Standard Time. For practical purposes most of the civilized world is now divided into sections, and all places in the same section use the time of a certain meridian in that section. Generally this meridian is 15°, 30°, 45°, or other multiple of 15° east or west of Greenwich.

Thus England uses the time of 0° (Greenwich meridian); Germany, Switzerland, Italy, and central Europe in general use that of 15° E.; Japan uses that of 135° E.; and Australia, on account of its size, is divided into different sections.

A system in which all places in a section of territory near some chosen meridian use the time of that meridian is called a system of *standard time.*

The following is the standard-time map of the United States:

The irregular boundaries of the sections are caused by the desire to have the division lines pass through important railway centers.

When it is noon by Eastern Time (75° W.) it is 11 A.M. by Central Time (90° W.), 10 A.M. by Mountain Time (105° W.), 9 A.M. by Pacific Time (120° W.), and 5 P.M. by Greenwich (0°) time.

In Exercise 183 every example refers to standard time.

EXERCISE 183

1. When it is noon in Wisconsin, what time is it in Wyoming? in California? in New York? in Missouri?

2. When it is 11 A.M. in Minnesota, what time is it in Colorado? in eastern Nebraska? in Vermont? in England?

3. When it is 1 P.M. in Montana, what time is it in Iowa? in Indiana? in Seattle? in Boston? in London?

4. When it is 2.30 P.M. in Utah, what time is it in southern Idaho? in Arkansas? in New Hampshire?

5. When it is 9.45 A.M. in San Francisco, what time is it in eastern Oklahoma? in Louisiana? in Florida? in Alabama? in Maine?

6. When it is 11.45 P.M. in Los Angeles, what time is it in western Tennessee and Kentucky? in Mississippi? in Virginia? in Connecticut? in England?

7. At 9 P.M. a telegram is sent from London to San Francisco. Allowing an hour for transmission and delivery, at what time will it reach its destination?

8. At 2 A.M. on January 1 a telegram is sent from Constantinople (time longitude 30° E.) to Portland, Oregon. Allowing an hour for transmission, on what day and at what hour will it reach its destination?

9. If an event happens in Japan (135° E.) at 10 P.M. and the news is telegraphed without delay, at what time will it reach Italy? England? New York? San Francisco?

10. The longitude of San Francisco is $122° 25' 40\frac{4}{5}''$ W. What is the difference between local (the real meridian) time and standard time? Which is the faster?

11. The longitude of New York is $73° 58' 25\frac{1}{2}''$ W. What is the difference between local time and standard time? Which is the faster?

EXERCISE 184

1. The longitude of Cincinnati is 84° 26′ W. and that of Berlin is 13° 23′ 45″ E. Find the difference in local time.

2. The longitude of Boston is 71° 4′ W. and that of Rochester is 77° 51′ W. Find the difference in local time.

3. The longitude of St. Paul is 93° 5′ W. and that of St. Petersburg is 30° 18′ E. Find the difference in local time.

4. The longitude of Constantinople is 28° 59′ E. and that of Rome is 12° 29′ E. Find the difference in local time.

5. New York is 74° 0′ 3″ west longitude and Chicago is 87° 38′ west; how much sooner does the sun rise at New York than at Chicago?

6. When it is noon at Greenwich it is 5 hr. 20 min. 59.4 sec. P.M., local time, at Madras, India. What is the longitude of Madras?

7. When it is noon at Portland, Me., local time, it is 4 hr. 41 min. 1.2 sec. P.M. at Greenwich. What is the longitude of Portland?

8. When it is noon at Greenwich it is 6 hr. 12 min. 48 sec. A.M., local time, at Nashville, Tenn. What is the longitude of Nashville?

9. When it is noon at the Cape Hatteras lighthouse, local time, it is 5 hr. 2 min. 5 sec. P.M. at Greenwich. What is the longitude of the Cape Hatteras lighthouse?

10. When it is noon at Greenwich it is 6 hr. 2 min. 22.2 sec. A.M., local time, at Madison, Wis. What is the difference between local and standard time at Madison?

11. On a certain day a sea captain found his Greenwich chronometer had lost 2 hr. 45 min. 30 sec., compared with the sun. Find his longitude at the time of the observation. Explain fully.

291. Arithmetical Progression. A series of numbers that increase or decrease by a common difference is called an *arithmetical progression.*

Thus the numbers 6, 9, 12, 15 form an arithmetical progression with the common difference 3. The numbers of a progression are called its *terms.*

292. Finding a Required Term. In the arithmetical progression

1st	2d	3d	4th	5th	6th
4,	7,	10,	13,	16,	19,

it is evident that any term, as for example the 6th, is found by adding to the first term the product of the common difference by a number 1 less than the number of the term. In this case, the 6th term is $4 + 3 \times 5$, or 19.

In an expression like $4 + 3 \times 5$ the multiplication is performed before the addition. If the addition were to be performed first, the expression would be written $(4 + 3) \times 5$.

If the series is a decreasing series, we subtract from the first term the product of the common difference by a number 1 less than the number of the term. Thus, the 12th term of the series 50, 47, 44, 41, etc., is $50 - 3 \times 11$, or 17.

Therefore, *to find any required term of an arithmetical progression, multiply the common difference by a number that is 1 less than the number of the required term. Add this product to the first term if the series is an increasing series; subtract it from the first term if the series is a decreasing series.*

EXERCISE 185

1. Find the 5th term of 4, 9, 14, etc.
2. Find the 10th term of 3, 8, 13, etc.
3. Find the 12th term of 5, 8, 11, etc.
4. Find the 6th term of 63, 58, 53, etc.
5. Find the 7th term of 100, 92, 84, etc.

293. Finding the Sum. Taking seven terms of the series, 1, 3, 5, etc., the sum is indicated by

$$1 + 3 + 5 + 7 + 9 + 11 + 13$$

In reverse order it is $13 + 11 + 9 + 7 + 5 + 3 + 1$

Hence, *twice the sum* is $14 + 14 + 14 + 14 + 14 + 14 + 14$

Hence, *once the sum* is $\frac{1}{2}$ of 7×14, or 49.

Therefore, *to find the sum of an arithmetical progression, take half the product of the number of terms by the sum of the first and last terms.*

Thus the sum of nine terms of an arithmetical progression whose first term is 3 and last term 41, is $\frac{1}{2}$ of $9 \times (3 + 41)$, or 198.

EXERCISE 186

Given the first and last terms, and the number of terms, find the sum:

	First	*Last*	*No.*		*First*	*Last*	*No.*
1.	3	30	10	**7.**	11	41	11
2.	5	45	11	**8.**	12	56	11
3.	7	40	12	**9.**	15	70	12
4.	4	39	8	**10.**	45	13	9
5.	6	30	13	**11.**	57	21	7
6.	7	35	15	**12.**	49	35	15

Given the first term, common difference, and number of terms, find the last term (§ 292) and then the sum:

	First	*Diff.*	*No.*		*First*	*Diff.*	*No.*
13.	3	3	8	**17.**	5	7	6
14.	7	2	9	**18.**	7	8	10
15.	5	9	7	**19.**	2	3	20
16.	1	6	6	**20.**	1	4	15

Find the sum of:

21. 1, 4, 7, to 8 terms.
22. 2, 5, 8, to 9 terms.
23. 4, 6, 8, to 12 terms.
24. 5, 8, 11, to 10 terms.
25. 7, 8, 9, to 25 terms.
26. 11, 16, 21, to 7 terms.
27. 14, 19, 24, to 6 terms.
28. 23, 20, 17, to 7 terms.

29. What is the sum of the first 25 numbers, beginning with 2?

30. What is the sum of the first 50 odd numbers, beginning with 1?

31. What is the sum of the first 20 numbers that are divisible by 3, beginning with 3?

32. How many times in a day does a clock strike that strikes the hours only?

33. Supposing a body to fall through space $16\frac{1}{12}$ ft. in the first second of its fall, and in each succeeding second $32\frac{1}{6}$ ft. more than in the second just before, how far will it fall in the 6th second? how far in 6 seconds?

34. From the preceding example, how far will a body fall in the 10th second of its fall? in 10 seconds?

35. In a potato race 30 potatoes are placed in a straight line 3 ft. distant from one another. A boy starting from a basket in the line 3 ft. from the first potato is required to pick them up one by one, beginning with the nearest one, and carry them to the basket. To finish the race, how far must the boy run?

36. Solve the preceding example on the supposition that there are 100 potatoes.

37. Show that if the first term is a, the last term l, the common difference d, the number of terms n, and the sum of the terms s, then $l = a + (n - 1)d$, and $s = \frac{1}{2}n(a + l)$.

294. Geometric Progression. A series in which each term after the first is obtained by multiplying the preceding term by a constant multiplier is called a *geometric progression.*

The constant multiplier is called the *ratio* of the progression.

Thus the numbers 2, 6, 18, 54, form a geometric progression, the ratio being 3. The numbers 27, 9, 3, 1, $\frac{1}{3}$, also form a geometric progression, the ratio being $\frac{1}{3}$.

295. Finding a Required Term. In the geometric progression

1st	2d	3d	4th	5th
2,	6,	18,	54,	162,

it is evident that any term, as for example the 5th, is the product of the first term by the power of the ratio that is 1 less than the number of the term.

That is, the 2d term is 2×3; the 3d term is 2×3^2; the 4th term is 2×3^3; the 5th term is 2×3^4; and so on.

Therefore, *to find any required term of a geometric progression, raise the ratio to a power 1 less than the number of the required term, and multiply this by the first term.*

EXERCISE 187

Given the first term and ratio to find the term whose number is indicated:

	First	*Ratio*	*No.*		*First*	*Ratio*	*No.*
1.	1	2	6	**7.**	64	$\frac{1}{2}$	6
2.	2	4	5	**8.**	81	$\frac{1}{3}$	4
3.	5	3	7	**9.**	128	$\frac{1}{2}$	4
4.	7	5	5	**10.**	243	$\frac{1}{3}$	4
5.	6	6	6	**11.**	256	$\frac{1}{4}$	3
6.	7	4	5	**12.**	729	$\frac{1}{9}$	3

13. Write a formula for s, the sum of the terms, given $a =$ the first term, $r =$ the ratio, and $n =$ the number of terms.

296. Finding the Sum. Taking five terms of the series 2, 6, 18, etc., the sum is indicated by $2 + 6 + 18 + 54 + 162$. If we multiply this sum by the ratio 3, we have:

$$\begin{array}{ll} 3 \times \text{the sum is} & \phantom{2 + {}}6 + 18 + 54 + 162 + 486 \\ \underline{1} \times \text{the sum is} & \underline{2 + 6 + 18 + 54 + 162\phantom{{}+486}} \\ 2 \times \text{the sum is} & \phantom{2+6+18+54+{}}486 - 2, \quad \text{by subtracting.} \end{array}$$

Hence the sum is $\dfrac{486 - 2}{2}$.

The numerator of this fraction is the difference between the first term and the product of the last term by the ratio; the denominator is the difference between the ratio and 1.

Therefore, *to find the sum of a geometric progression, multiply the last term by the ratio and subtract the first term from the product. Then divide the remainder by the ratio minus 1.*

If the ratio is less than 1, simply reverse the progression. That is, to find the sum of the series 128, 64, 32, 16, where the ratio is $\frac{1}{2}$, find the sum of the series 16, 32, 64, 128, where the ratio is 2.

EXERCISE 188

Given the first and last terms and the ratio, find the sum:

	First	*Last*	*Ratio*		*First*	*Last*	*Ratio*
1.	2	128	2	**6.**	5	625	5
2.	1	243	3	**7.**	3	108	6
3.	1	256	2	**8.**	2	162	3
4.	3	192	4	**9.**	32	1	$\frac{1}{2}$
5.	4	108	3	**10.**	729	81	$\frac{1}{3}$

Find the sum of:

11. 3, 6, 12, to 8 terms.

12. 1, 5, 25, to 7 terms.

13. 4, 8, 16, to 6 terms.

14. 405, 135, 45, to 5 terms.

297. Geometric Progression and Compound Interest. To what sum will $1000 amount, at 6% compound interest, in 4 yr.?

Original principal	$1000	
Amount at end of 1st year	$1060,	or $1.06 \times$ $1000
Amount at end of 2d year	$1123.60,	or $1.06 \times$ $1060
Amount at end of 3d year	$1191.02,	or $1.06 \times$ $1123.60
Amount at end of 4th year	$1262.48,	or $1.06 \times$ $1191.02

The first term is $1000, the ratio 1.06, and the fifth term $1262.48.

To what sum will $250 amount, in 5 years, at 4% compound interest?

By § 295 the *sixth* term equals

$1.04^5 \times \$250$, or $1.21665 \times \$250$, or $304.16.

298. Compound Interest Table. The table on page 293 shows the amount of $1 at compound interest at various rates. The compound interest on $1 is found by subtracting 1 from the proper number shown in the table.

(1) What principal will in 10 yr., at 6% compound interest, yield $1898.04 interest?

The interest on $1 for 10 yr. at 6% is $0.79085 (see the table).

Since $0.79085 is the interest on $1,

$1898.04 is the interest on $\frac{\$1898.04}{0.79085}$, or $2400.

(2) In what time will $1600, at $4\frac{1}{2}$% compound interest, yield $1000 interest?

Since $1600 yields $1000, $1 will yield $\frac{1}{1600}$ of $1000, or $0.625, in the same time, and $1 will amount to $1.625. By the table, $1 will, in 11 yr. at $4\frac{1}{2}$%, amount to $1.62285, and in 12 yr. to $1.69588. Hence the required time is a little more than 11 yr.

(3) At what rate, compound interest, will $1500 yield $1201.41 interest in 15 yr.?

Since $1500 yields $1201.41 interest in 15 yr., $1 in 15 yr. will yield $\frac{1}{1500}$ of $1201.41, or $0.80094, and $1 will amount in 15 yr. to $1.80094. In the table, opposite 15 yr., we find in the 4% column the amount of $1 is $1.80094. Therefore the rate is 4%.

Table showing the amount of $1 at compound interest for:

Year	2 Per Cent	2½ Per Cent	3 Per Cent	3½ Per Cent	4 Per Cent
1	1.02000	1.02500	1.03000	1.03500	1.04000
2	1.04040	1.05063	1.06090	1.07123	1.08160
3	1.06121	1.07689	1.09273	1.10872	1.12486
4	1.08243	1.10381	1.12551	1.14752	1.16986
5	1.10408	1.13141	1.15927	1.18769	1.21665
6	1.12616	1.15969	1.19405	1.22926	1.26532
7	1.14869	1.18869	1.22987	1.27228	1.31593
8	1.17166	1.21840	1.26677	1.31681	1.36857
9	1.19509	1.24886	1.30477	1.36290	1.42331
10	1.21899	1.28009	1.34392	1.41060	1.48024
11	1.24337	1.31209	1.38423	1.45997	1.53945
12	1.26824	1.34489	1.42576	1.51107	1.60103
13	1.29361	1.37851	1.46853	1.56396	1.66507
14	1.31948	1.41297	1.51259	1.61870	1.73168
15	1.34587	1.44830	1.55797	1.67535	1.80094
16	1.37279	1.48451	1.60471	1.73399	1.87298
17	1.40024	1.52162	1.65285	1.79468	1.94790
18	1.42825	1.55966	1.70243	1.85749	2.02582
19	1.45681	1.59865	1.75351	1.92250	2.10685
20	1.48595	1.63862	1.80611	1.98979	2.19112

Year	4½ Per Cent	5 Per Cent	5½ Per Cent	6 Per Cent	7 Per Cent
1	1.04500	1.05000	1.05500	1.06000	1.07000
2	1.09203	1.10250	1.11303	1.12360	1.14490
3	1.14117	1.15763	1.17424	1.19102	1.22504
4	1.19252	1.21551	1.23882	1.26248	1.31080
5	1.24618	1.27628	1.30696	1.33823	1.40255
6	1.30226	1.34010	1.37884	1.41852	1.50073
7	1.36086	1.40710	1.45468	1.50363	1.60578
8	1.42210	1.47746	1.53469	1.59385	1.71819
9	1.48610	1.55133	1.61909	1.68948	1.83846
10	1.55297	1.62889	1.70814	1.79085	1.96715
11	1.62285	1.71034	1.80209	1.89830	2.10485
12	1.69588	1.79586	1.90121	2.01220	2.25219
13	1.77220	1.88565	2.00577	2.13293	2.40985
14	1.85194	1.97993	2.11609	2.26090	2.57853
15	1.93528	2.07893	2.23248	2.39656	2.75903
16	2.02237	2.18287	2.35526	2.54035	2.95216
17	2.11338	2.29202	2.48480	2.69277	3.15882
18	2.20848	2.40662	2.62147	2.85434	3.37993
19	2.30786	2.52695	2.76565	3.02560	3.61653
20	2.41171	2.65330	2.91776	3.20714	3.86968

EXERCISE 189

1. Find the compound interest at 6% for 5 yr. on $287.

2. Find the compound interest at $3\frac{1}{2}$% for 12 yr. 6 mo. on $686.70.

3. Find the amount of $960 for 9 yr. at 5%, interest compounded semiannually.

4. Find the amount of $1216 for 5 yr. at 8%, interest compounded quarterly.

5. What principal will in 14 yr. at $5\frac{1}{2}$% compound interest amount to $1880.78?

6. At what rate compound interest will $2200 yield $4749.40 interest in 17 yr.?

7. In what time will $1800 at 5% compound interest amount to $3000?

8. In what time will $962.44 yield $1080.72 interest at 6% compound interest?

9. What principal will in 20 yr. at 4% compound interest yield $2500 interest?

10. At what rate compound interest will $462.50 yield $277.98 interest in 12 yr.?

11. What principal will in 10 yr. at 6% amount to $3612.22, interest being compounded semiannually?

12. In what time at 5% will $1250 amount to $2000, interest being compounded semiannually?

13. At what rate per annum will $500 amount to $779.83 in 9 yr., interest being compounded semiannually?

14. At what rate compound interest will $1500 amount to $2229.15 in 9 yr.?

15. At what rate compound interest will $2500 amount to $3620.75 in 15 yr.?

299. Account. A statement of related income and expenses is called an *account.*

1914					*1914*				
Aug.	*3*	*Balance*	*$125*	*50*	*Aug.*	*3*	*Rent*	*$75*	
	4	*T. B. Sullivan*	*26*	*75*		*5*	*Grocery bill*	*18*	*42*
	5	*R. S. Peters*	*87*	*50*		*10*	*Balance*	*146*	*33*
			$239	*75*				*$239*	*75*
Aug.	*10*	*Balance*	*$146*	*33*					

This illustrates a cash account. The receipts are placed on the left or *debit* side, and the payments on the right or *credit* side. The balance is found by subtracting the expenses ($75 + $18.42) from the receipts ($239.75). This balance is then carried over to the left side for the beginning of the new week.

300. Account Sales. A statement rendered by a commission merchant to the one who employs him, showing the items of a transaction, is called an *account sales.*

New York, Aug. 8, 1914

Sale of Merchandise for Account of

R. P. Sanders, Ithaca, N.Y.

By HUNTLEY and ROBERTS, 4062 E. Canal St.

Aug.	*4*	*300 bbl. King apples $3.20*	*$960*			
		75 bbl. Greenings 2.90	*217*	*50*		
					$1177	*50*
		CHARGES				
		Drayage	*6*	*75*		
		Commission, 2%	*23*	*55*	*30*	*30*
		Remitted herewith			*$1147*	*20*

This shows that Huntley and Roberts have sold $1177.50 worth of apples for R. P. Sanders, charging drayage and commission and remitting the balance.

EXERCISE 190

Make out accounts, inserting dates, names, and items, and find the balance, given the following:

1. Receipts: Balance, $127.30, $75, $46.50, $21.75. Payments: $50, $26.30, $14.90.

2. Receipts: Balance, $1275.60, $128.75, $62.50, $27.80. Payments: $125, $37.60, $29.75, $48.25.

3. Receipts: Balance, $265.35, $28.90, $47.60, $85. Payments: $275, $10, $3.60, $1.75, $1.35, $2.80.

4. Receipts: Balance, $1526.30, $27.42, $428.75, $125.75. Payments: $250, $175, $26.75, $135.50, $12.75.

5. Receipts: Balance, $1275.50, $248.75, $432.50, $18.75. Payments: $360, $245, $31.80, $129.75, $15.50.

Make out account sales, inserting dates and names, and find the balance remitted:

6. Sold: 400 bbl. Eagle flour @ $5.90, 350 bbl. Acme flour @ $6.20. Charges: Freight, $17.20; cartage, $16.50; commission, 4%.

7. Sold: 125 crates eggs @ $6, 2500 lb. cheese @ 12½¢. Charges: Freight and drayage, $21.75; commission, 4%.

8. Sold: 300 bbl. King apples @ $3.25, 400 bbl. greenings @ $2.80, 250 bbl. Baldwins @ $3.10. Charges: Freight, $31.50; drayage, $18.25; commission, 5%.

9. Sold: 6000 lb. creamery butter @ 26¢, 3000 lb. cheese @ 13¢, 200 crates eggs @ $6.75. Charges: Freight and drayage, $32.75; commission, 5%.

10. Sold: 1200 bbl. Sun flour @ $4.80, 600 bbl. Rose flour @ $5.40. Charges: Freight and drayage, $42.60; storage @ 5¢ per barrel; commission, 5%.

301. Settlement of Accounts. Find the balance due on the following account, Sept. 10, 1914, computing interest at 6% on each item from its date to the day of settlement, and reckoning the time in days :

1914		DR.	INT.	1914		CR.	INT.
June 29.	To Mdse.	$250	$3.04	July 3.	By Cash.	$200	$2.30
July 13.	To Mdse.	400	3.93	July 17.	By Cash.	125	1.15
July 27.	To Mdse.	500	3.75	July 31.	By Cash.	350	2.39
				Sept. 10.	By bal. acct.	475	
Settled Sept. 10, '14.				Sept. 10.	By bal. int.		4.88
		$1150	$10.72			$1150	$10.72

Hence, the cash balance is $475 + $4.88, or $479.88.

When the balance of account and the balance of interest fall on *opposite* sides, the cash balance is their *difference.*

EXERCISE 191

Find the cash balance, Sept. 10, 1914, of the following accounts, reckoning interest at 6% :

1.

1914		Dr.	1914		Cr.
May 13.	To Mdse.	$250.00	May 27.	By Cash.	$200.00
May 28.	To Mdse.	610.00	June 22.	By Cash.	500.00
June 17.	To Mdse.	300.00	June 29.	By Cash.	400.00

2.

1914		Dr.	1914		Cr.
Mar. 7.	To Mdse.	$350.00	Apr. 3.	By Cash.	$150.00
Apr. 10.	To Mdse.	98.50	May 2.	By Cash.	150.00
May 25.	To Mdse.	300.00	June 4.	By Cash.	200.00

3.

1914		Dr.	1914		Cr.
May 8.	To Mdse.	$250.00	June 22.	By Cash.	$200.00
June 5.	To Mdse.	670.00	July 20.	By Cash.	500.00
July 3.	To Mdse.	200.00	Aug. 19.	By Cash.	300.00

302. Drill Problems. Teachers occasionally wish more drill problems than a text-book supplies. An opportunity is offered in the following exercise to select from 15,800 problems in the fundamental operations, but teachers are advised to omit the entire exercise unless there is need for special attention to some particular deficiency. The problems are arranged as follows:

1. Add each of the 10 numbers in the first column of I to each of the lettered numbers of the second column, thus giving 100 examples.

For example, (7) + (c) = 10,125 + 27 = 10,152.

2. In the same way, subtract, giving 100 examples.
3. In the same way, multiply, giving 100 examples.
4. In the same way, divide, giving 100 examples.
5. Proceed in the same way for II, giving 400 more examples, or 800 on this page. In the same way III affords 2500 different examples, and so on, making a total of 15,800 different examples.

EXERCISE 192

I		II	
1. 2025.	a. 9.	**1.** 1176.	a. 4.
2. 8100.	b. 15.	**2.** 4704.	b. 6.
3. 6075.	c. 27.	**3.** 2352.	c. 7.
4. 16,200.	d. 45.	**4.** 5880.	d. 8.
5. 12,150.	e. 25.	**5.** 3528.	e. 21.
6. 18,225.	f. 75.	**6.** 9408.	f. 49.
7. 10,125.	g. 81.	**7.** 8232.	g. 98.
8. 14,175.	h. 225.	**8.** 10,584.	h. 147.
9. 22,275.	i. 135.	**9.** 12,936.	i. 196.
10. 20,250.	j. 675.	**10.** 11,760.	j. 392.

Add, subtract, multiply, and divide, using each number of the first column with each lettered number of the second column:

III

1. 8640.	a. 2.
2. 5184.	b. 8.
3. 3456.	c. 4.
4. 6912.	d. 6.
5. 17,280.	e. 9.
6. 34,560.	f. 3.
7. 51,840.	g. 16.
8. 55,296.	h. 18.
9. 58,752.	i. 24.
10. 65,664.	j. 27.
11. 10,368.	k. 32.
12. 12,096.	l. 96.
13. 32,832.	m. 12.
14. 13,824.	n. 36.
15. 15,552.	o. 72.
16. 29,376.	p. 54.
17. 20,736.	q. 48.
18. 15,552.	r. 108.
19. 69,120.	s. 216.
20. 25,920.	t. 288.
21. 27,648.	u. 144.
22. 414,720.	v. 864.
23. 103,680.	w. 432.
24. 829,440.	x. 192.
25. 207,360.	y. 1728.

IV

1. 10,368.	a. 16.
2. 20,736.	b. 32.
3. 51,840.	c. 27.
4. 41,472.	d. 81.
5. 62,208.	e. 54.
6. 31,104.	f. 36.
7. 72,576.	g. 64.
8. 93,312.	h. 48.
9. 134,784.	i. 96.
10. 155,520.	j. 162.
11. 176,256.	k. 108.
12. 103,680.	l. 192.
13. 145,152.	m. 144.
14. 165,888.	n. 288.
15. 114,048.	o. 324.
16. 196,992.	p. 648.
17. 186,624.	q. 576.
18. 124,416.	r. 384.
19. 228,096.	s. 1728.
20. 207,360.	t. 1296.
21. 259,200.	u. 2592.
22. 238,464.	v. 1152.
23. 269,568.	w. 5184.
24. 248,832.	x. 3456.
25. 217,728.	y. 10,368.

Add, subtract, multiply, and divide, using each number of the first column with each lettered number of the second column:

V

1.	64.68.	a.	0.4.
2.	582.12.	b.	0.8.
3.	12.936.	c.	0.6.
4.	258.72.	d.	2.4.
5.	7.7616.	e.	20.
6.	905.52.	f.	0.07.
7.	3880.8.	g.	1.10.
8.	194.04.	h.	7.7.
9.	106.424.	i.	5.5.
10.	84.084.	j.	26.4.
11.	3.234.	k.	4.9.
12.	129.36.	l.	0.35.
13.	13.5828.	m.	15.
14.	4.5276.	n.	8.8.
15.	12.2892.	o.	0.66.
16.	0.135828.	p.	30.
17.	51.744.	q.	4.4.
18.	97.020.	r.	1.65.
19.	711.48.	s.	38.5.
20.	0.122892.	t.	24.5.
21.	1164.24.	u.	1.05.
22.	5.8212.	v.	0.22.
23.	103.488.	w.	5.39.
24.	0.9702.	x.	0.49.
25.	0.6468.	y.	0.12.

VI

1.	189.728.	a.	0.4.
2.	3794.56.	b.	0.8.
3.	5.69184.	c.	1.6.
4.	75891.2.	d.	3.2.
5.	94.864.	e.	1.4.
6.	113.8368.	f.	2.8.
7.	13.28096.	g.	0.56.
8.	1517.824.	h.	0.11.
9.	17.07552.	i.	1.21.
10.	18.9728.	j.	0.77.
11.	2087.008.	k.	0.14.
12.	227.6736.	l.	0.49.
13.	246.6464.	m.	8.8.
14.	2656.192.	n.	3.52.
15.	2845.92.	o.	1.76.
16.	303.5648.	p.	6.16.
17.	474.32.	q.	5.39.
18.	3794.56.	r.	0.98.
19.	455.3472.	s.	24.2.
20.	3415.104.	t.	8.47.
21.	41740.16.	u.	11.2.
22.	3225.376.	v.	6.05.
23.	36.04832.	w.	61.6.
24.	3984.288.	x.	2.24.
25.	4363.744.	y.	48.4.

Add, subtract, multiply, and divide, using each number of the first column with each lettered number of the second column:

VII

1. $\frac{5}{8}$.	a. $\frac{1}{2}$.
2. $\frac{7}{8}$.	b. $\frac{1}{3}$.
3. $\frac{5}{6}$.	c. $\frac{1}{4}$.
4. $\frac{3}{4}$.	d. $\frac{1}{5}$.
5. $\frac{3}{5}$.	e. $\frac{1}{6}$.
6. $\frac{4}{7}$.	f. $\frac{1}{8}$.
7. $\frac{5}{7}$.	g. $\frac{2}{5}$.
8. $\frac{6}{7}$.	h. $\frac{3}{8}$.
9. $\frac{5}{9}$.	i. $\frac{1}{9}$.
10. $\frac{7}{9}$.	j. $\frac{2}{9}$.
11. $\frac{8}{9}$.	k. $\frac{4}{9}$.
12. $\frac{7}{10}$.	l. $\frac{1}{12}$.
13. $\frac{9}{10}$.	m. $\frac{5}{12}$.
14. $\frac{6}{11}$.	n. $\frac{1}{16}$.
15. $\frac{7}{12}$.	o. $\frac{3}{16}$.
16. $\frac{7}{16}$.	p. $\frac{5}{16}$.
17. $\frac{9}{16}$.	q. $\frac{7}{16}$.
18. $\frac{13}{16}$.	r. $\frac{1}{24}$.
19. $\frac{15}{16}$.	s. $\frac{5}{24}$.
20. $\frac{17}{32}$.	t. $\frac{7}{24}$.
21. $\frac{23}{24}$.	u. $\frac{11}{24}$.
22. $\frac{19}{24}$.	v. $\frac{1}{32}$.
23. $\frac{17}{48}$.	w. $\frac{3}{32}$.
24. $\frac{39}{48}$.	x. $\frac{5}{32}$.
25. $\frac{51}{100}$.	y. $\frac{5}{64}$.

VIII

1. $4\frac{1}{2}$.	a. 2.
2. $3\frac{1}{4}$.	b. 3.
3. $3\frac{3}{4}$.	c. $1\frac{1}{8}$.
4. $4\frac{1}{8}$.	d. $1\frac{3}{8}$.
5. $5\frac{3}{8}$.	e. $2\frac{5}{8}$.
6. $3\frac{5}{8}$.	f. $2\frac{7}{8}$.
7. $4\frac{7}{8}$.	g. $1\frac{1}{5}$.
8. $3\frac{1}{16}$.	h. $2\frac{3}{5}$.
9. $4\frac{3}{16}$.	i. $1\frac{4}{5}$.
10. $3\frac{5}{16}$.	j. $2\frac{1}{6}$.
11. $3\frac{7}{16}$.	k. $1\frac{5}{6}$.
12. $4\frac{9}{16}$.	l. $1\frac{1}{10}$.
13. $3\frac{11}{16}$.	m. $2\frac{3}{10}$.
14. $4\frac{15}{16}$.	n. $1\frac{1}{16}$.
15. $3\frac{1}{24}$.	o. $1\frac{3}{16}$.
16. $3\frac{5}{24}$.	p. $2\frac{5}{16}$.
17. $4\frac{11}{24}$.	q. $1\frac{9}{16}$.
18. $3\frac{1}{32}$.	r. $1\frac{1}{24}$.
19. $3\frac{3}{32}$.	s. $2\frac{5}{24}$.
20. $4\frac{5}{32}$.	t. $2\frac{7}{24}$.
21. $3\frac{9}{32}$.	u. $1\frac{1}{32}$.
22. $3\frac{1}{48}$.	v. $1\frac{5}{32}$.
23. $4\frac{5}{48}$.	w. $2\frac{1}{48}$.
24. $3\frac{7}{48}$.	x. $1\frac{1}{64}$.
25. $3\frac{1}{10}$.	y. $2\frac{3}{100}$.

303. Arithmetical Recreations. Most of the following problems have had a long and interesting history. They are part of the inheritance of the school, and they show some of the amusing features of arithmetic. They claim to have no practical bearing, and they should be omitted unless taken in the spirit intended, as mere recreations showing the lighter side of the subject.

EXERCISE 193

1. Multiply 142,857 by 7.

2. Multiply 37 by 3; by 6; by 9; by 12; by 15; by 18; by 21.

3. How can you write one hundred with Arabic figures without using zeros?

4. How can you write an even number, using only the odd digits?

5. If a goose weighs 10 lb. and half of its own weight, what is the weight of the goose?

6. How many days would it take to cut a piece of cloth 50 yd. long into 1-yard lengths, one yard being cut off daily?

7. If a post in a pond stands $\frac{1}{3}$ in the ground, $\frac{1}{2}$ in the water, and 10 feet above the water, how long is it?

8. A bottle and its stopper cost $1.10, and the bottle cost $1 more than the stopper. What did the stopper cost?

9. A new steel coal car and its contents weigh 139,700 lb., and the contents weighs 60,300 lb. more than the car. What does the car weigh?

10. Two men purpose to cross the Rocky Mountains. X can walk 13 miles a day and Y only 10 miles a day. Therefore Y starts 9 days ahead. How long will it take X to overtake Y?

11. A snail climbing up a post 20 ft. high ascends 5 ft. every day and slips down 4 ft. every night. How long will it take the snail to reach the top of the post?

12. A visitor to a class said, "Good morning, you 100 pupils." One of them replied, "We are not 100, but our number, and the number again, and its half, and its fourth, and one more, make 100." How many were there?

13. A man borrowed as much money as he had and then spent \$10. He again borrowed as much money as he then had and again spent \$10. A third time he borrowed as much money as he then had and then spent \$10. He then had nothing left. How much had he at first?

14. One man said to another, "If you give me \$1, I shall have as much as you." The other replied, "If you give me \$1, I shall have twice as much as you." How much did each have?

15. In several parts of the world twenty-four-hour time is used. Assuming the clock to strike the hours from 1 to 24, how many strokes would such a clock make in a day? How many, if it also struck each half hour?

16. Two Arabs met at meal time, one having 4 loaves and the other 3. Just as they were beginning to eat, a third Arab, without any loaves, joined them, and they all ate equally. The third Arab then produced 7 pence and said, "Divide this equitably." How much did each receive?

17. A Persian fable relates that a father left to his three sons 17 camels, the first son to have half, the second a third, and the youngest a ninth. They could not divide them, and so a wise man loaned them another camel, and they divided the 18, the first taking 9, the second 6, and the youngest 2. This made 17, and they returned the one that was borrowed. Was the division right?

18. It is said that the inventor of the game of chess was asked by the king to name his reward. He asked for a single grain of wheat to place on the first square, 2 on the second, 4 on the third, 8 on the fourth, and so on, doubling each time, until the 64 squares were completed. How many grains of wheat did the inventor ask?

19. A woman dropped a basket of eggs and broke them all Some one asked her how many she had, and she said she did not know, but that when she put them in the basket by twos there remained one egg; when she took them out by threes there also remained one egg; and when she took them out again, five at a time, there remained none. What is the smallest number that she could have had?

20. A thief leaving an orchard met three other thieves, who threatened to expose him unless he gave them a share of his apples. To the first he gave half, who then returned 12 to him. To the second he gave half of the remainder, who then returned 7. To the third he gave half of what he then had, 4 being returned. He then had 20 apples. How many did he steal?

21. Two men are employed by a manufacturing company, one beginning with a salary of $1000 a year, with a yearly increase of $200, and the other at a salary of $1000 a year, with a half-yearly increase of $50. In each case the salary was paid every six months. Which had the larger income?

22. The first of two bottles is half full of water and the second is half full of vinegar. From the first bottle a tablespoonful of water is taken and poured into the second bottle. A tablespoonful of the mixture is then taken from the second bottle and poured into the first bottle. Is the quantity of water removed from the first bottle greater or less than the quantity of vinegar removed from the second bottle?

304. Tables of Denominate Numbers. The following tables are here brought together for convenience in reference. Certain tables of equivalents have been added, and these will frequently be found useful in making reductions and in avoiding long multiplications.

Table of Length

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

			ft.	in.
		yd.	1 =	12
	rd.	1 =	3 =	36
mi.	1 =	$5\frac{1}{2}$ =	$16\frac{1}{2}$ =	198
1 =	320 =	1760 =	5280 =	63,360

A *hand* (4 in.) is used in measuring the height of horses; a *fathom* (6 ft.) and *cable length* (120 fathoms) in measuring depths of water; a *knot* (nautical mile, 1.152 common or statute miles, or 6080.27 ft.) in measuring distances at sea. Carpenters, mechanics, and others usually write 2′ 6″ for 2 ft. 6 in.

Table of Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

				sq. ft.	sq. in.
			sq. yd.	1 =	144
		sq. rd.	1 =	9 =	1296
	A.	1 =	$30\frac{1}{4}$ =	$272\frac{1}{4}$ =	39,204
sq. mi.	1 =	160 =	4840 =	43,560 =	6,272,640
1 =	640 =	102,400 =	3,097,600 =	27,878,400 =	4,014,489,600

Table of Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

		cu. ft.	cu. in.
cu. yd.		1 =	1728
1 =		27 =	46,656

A perch of stone or masonry is usually 1 rd. long, 1 ft. high, and 1½ ft. thick, and contains 24¾ cu. ft., but this varies in different parts of the country. A cubic yard of earth is considered a load.

Table of Avoirdupois Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

	lb.	oz.
T.	1 =	16
1 =	2000 =	32,000

100 lb. is sometimes called a *hundredweight* (cwt.). The ton of 2000 lb. is sometimes called the *short ton*, there being an old ton of 2240 lb., known as the *long ton*, which is used in the customhouse and in some wholesale transactions in mining products. There is also the *long hundredweight* of 112 lb., but it is rarely used except in English trade.

Table of Troy Weight

24 grains (gr.) = 1 pennyweight (pwt. or dwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

The avoirdupois pound contains 7000 gr., the troy pound 5760 gr. For the *carat*, see page 170, Book Two.

Table of Apothecaries' Weight

20 grains (gr.) = 1 scruple (sc. or ℈)
3 scruples = 1 dram (dr. or ʒ)
8 drams = 1 ounce (oz. or ℥)
12 ounces = 1 pound (lb.)

Table of Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
$31\frac{1}{2}$ gallons = 1 barrel (bbl.)
63 gallons = 1 hogshead (hhd.)

				pt.	gi.
			qt.	1 =	4
		gal.	1 =	2 =	8
	bbl.	1 =	4 =	8 =	32
	1 =	$31\frac{1}{2}$ =	126 =	252 =	1008

A gallon contains 231 cubic inches.

Casks holding from 28 gal. to 43 gal. are called barrels, and casks holding from 54 gal. to 63 gal. are called hogsheads. Whenever barrels or hogsheads are used as *measures*, a barrel means $31\frac{1}{2}$ gallons, and a hogshead means 63 gallons. A barrel contains about $4\frac{1}{5}$ cu. ft.

Apothecaries and physicians use the following table of liquid measure, but it need not be learned:

60 minims (♏) = 1 dram (♏ lx. or f. ʒ)
8 fluid drams = 1 ounce (fl. drm. viij. or f. ℥)
16 ounces = 1 pint (fl. oz. xvj. or O)
8 pints = 1 gallon (cong.)

Table of Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

			qt.	pt.
		pk.	1 =	2
	bu.	1 =	8 =	16
	1 =	4 =	32 =	64

A bushel contains 2150.42 cu. in., or about $1\frac{1}{4}$ cu. ft. A dry quart contains 67.2 cu.in., while the liquid quart contains only 57.75 cu. in. In measuring grain, seeds, and small fruits the measure must be *even* full. In measuring apples, potatoes, and other large articles the measure must be *heaping* full.

Table of Surveyors' Linear Measure

7.92 inches (in.) = 1 link (l.)
25 links = 1 rod (rd.)
100 links = 1 chain (ch.)
80 chains = 1 mile (mi.)

			l.	in.
		rd.	1 =	7.92
	ch.	1 =	25 =	198
mi.	1 =	4 =	100 =	792
1 =	80 =	320 =	8000 =	63,360

Table of Surveyors' Square Measure

16 square rods (sq. rd.) = 1 square chain (sq. ch.)
10 square chains = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
1 square mile = 1 section (sec.)
36 sections = 1 township (T.)

		sq. ch.	sq. rd.
	A.	1 =	16
sq. mi.	1 =	10 =	160
1 =	640 =	6400 =	102,400

Table of Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
365 days = 1 common year (yr.)
366 days = 1 leap year

				min.	sec.
			hr.	1 =	60
		da.	1 =	60 =	3600
	wk.	1 =	24 =	1440 =	86,400
	1 =	7 =	168 =	10,080 =	604,800
yr. mo. 1 = 12 =		365 =	8760 =	525,600 =	31,536,000
		366 =	8784 =	527,040 =	31,622,400

305. Metric System Equivalents.

Length

Meter	= 39.370 in.	Inch	= 0.0254 m.
	= 3.281 ft.	Foot	= 0.3048 m.
	= 1.093 yd.	Yard	= 0.9144 m.
	= about 1.1 yd.		= about 0.9 m.
Millimeter	= 0.039 in.	Inch	= 25.400 mm.
Centimeter	= 0.3937 in.		= 2.540 cm.
Kilometer	= 0.62137 mi.	Mile	= 1.60935 km.
	= about $\frac{3}{5}$ mi.		= about 1.6 km.

Surface

Sq. meter	= 10.764 sq. ft.	Sq. foot	= 0.0929 sq. m.
	= 1.1960 sq. yd.	Sq. yard	= 0.8361 sq. m.
Hektare	= 2.471 A.	Acre	= 0.4047 ha.
	= about $2\frac{1}{2}$ A.		= about $\frac{2}{5}$ ha.
Sq. kilometer	= 0.3861 sq. mi.	Sq. mile	= 2.59 sq. km.

Volume

Cu. centimeter	= 0.061 cu. in.	Cu. inch	= 16.3872 cu. cm.
Cu. meter	= 35.314 cu. ft.	Cu. foot	= 0.0283 cu. m.
	= 1.3079 cu. yd.	Cu. yard	= 0.7646 cu. m.
Stere	= 0.2759 cd.	Cord	= 3.6245 st.

Capacity

Liter	= 1.05668 liq. qt.	Liq. quart	= 0.94636 l.
	= 0.9081 dry qt.	Dry quart	= 1.1012 l.
	= 0.26417 gal.	Gallon	= 3.78543 l.
	= 0.11351 pk.	Peck	= 8.80982 l.
Hektoliter	= 2.83774 bu.	Bushel	= 0.35239 hl.

Weight

Gram	= 15.4324 gr.	Grain	= 0.0648 g.
	= about $15\frac{1}{2}$ gr.		= about $\frac{3}{50}$ g.
	= 0.03527 oz. av.	Ounce av.	$\doteq$ 28.3495 g.
Kilogram	= 2.20462 lb. av.	Pound av.	= 0.45359 kg.
	= about $2\frac{1}{5}$ lb.		= 0.00045 t.
Met. ton.	= 2204.62 lb. av.	Short ton	= 0.90718 t.

306. Legal Weights in Pounds per Bushel.

	Apples	Barley	Beans	Beets	Buck-wheat	Shelled Corn	Flaxseed	Oats	Onions	Peas	Potatoes	Sweet Potatoes	Rye	Wheat
U.S. customs .		48			42		56	32		60	60		56	60
Ala.		47	60			56		32		60		55	56	60
Ariz.		45	55					32					56	60
Ark.	50	48	60		52	56	56	32	57	60	60	50	56	60
Cal.		50			40			32					54	60
Colo.		48	60		52			32	57		60		56	60
Conn.	48	48	60	60	48		55	32	52	60	60	54	56	60
Del.														60
D.C.											60			
Fla.	48	48	60			56		32	56			60	56	60
Ga.		47	60		52	56	56	32	57	60		55	56	60
Hawaii		48						32					56	60
Idaho	45	48			42		56	36			60		56	60
Ill.		48	60		52	56	56	32	57			50	56	60
Ind.		48	60		50	56		32	48		60	55	56	60
Iowa	48	48	60		52	56	56	32	57		60	46	56	60
Kans.	48	48	60		50		56	32	57		60	50	56	60
Ky.		47	60		56	56	56	32	57	60	60	55	56	60
La.		48											56	60
Me.	44	48	60	60	48			32	52	60	60		50	60
Md.								26			56			
Mass.	48	48	60		48	50	55	32	52	60	60	54	56	60
Mich.	48	48	60		48	56	56	32	54	60		56	56	60
Minn.	50	48	60	50	50	56		32	52	60		55	56	60
Miss.		48	60		48	56	56	32	57	60		60	56	60
Mo.	48	48	60		52	56	56	32	57	60		56	56	60

Legal Weights in Pounds per Bushel (*Continued*).

	Apples	Barley	Beans	Beets	Buck-wheat	Shelled Corn	Flaxseed	Oats	Onions	Peas	Potatoes	Sweet Potatoes	Rye	Wheat
Mont.	45	48	60	50	52	56	56	32	57	60	60		56	60
Nebr.		48	60		52	56	56	32	57	60		50	56	60
Nev.														
N.H.			62					32		60	60		56	60
N.J.	50	48	60		50		55	30	57	60		54	56	60
N.Mex.														
N.Y.	48	48	60		48		55	32	57	60		54	56	60
N.C.		48			50		55	32		60			56	60
N.Dak.	50	48	60	60	42	56	56	32	52	60		46	56	60
Ohio	50	48	60	56	50	56	56	32	55	60		50	56	60
Okla.		48	60	60	42	56	56	32	52	60		46	56	60
Oreg.	45	46			42			32			60		56	60
Pa.		47			48			32	50		56		56	60
R.I.	48	48	60	50	48	56	56	32	50	60		54	56	60
S.C.														
S.Dak.		48	60	60	42	56	56	32	52	60		46	56	60
Tenn.	50	48	60	50	50	56	56	32	56	60		50	56	60
Tex.	45	48	60		42	56	56	32	57			55	56	60
Utah														
Vt.	46	48	62	60	48			32	52	60	60		56	60
Va.		48	60		52	56	56	30	57	60		56	56	60
Wash.	45	48			42		56	32			60		56	60
W.Va.		48	60		52		56	32			60		56	60
Wis.	50	48	60	50	50		56	32	57	60		54	56	60
Wyo.														

307. Spanish Land Measures. In those sections of the United States that formerly belonged to Mexico, the Spanish measures are still used in land surveying.

The *vara* is taken as the unit of measure.

In Mexico the length of the vara is 32.9927 in., in California 33 in., in Texas $33\frac{1}{3}$ in. In the problems below its length will be taken as $33\frac{1}{3}$ in., as in Texas. In California it should be taken as 33 in.

A *labor* is a square each side of which is 1000 varas.

A *square league* is a square each side of which is 5000 varas.

308. Table of Length.

1 vara = $33\frac{1}{3}$ inches
3 varas = 100 inches
36 varas = 100 feet
108 varas = 100 yards
1900.8 varas = 1 mile

309. Table of Area.

1,000,000 square varas = 1 labor = 177.136 acres
25 labors = 1 square league = 4428.4 acres
5645.376 square varas = 1 acre

EXERCISE 194

Change to yards:

1. 540 varas. **2.** 936 varas. **3.** 257 varas.

Change to feet:

4. 864 varas. **5.** 324 varas. **6.** 750 varas.

Change to miles and decimals of a mile:

7. 3801.6 varas. **8.** 8553.6 varas. **9.** 22,809.6 varas.

Change to labors or square varas:

10. 3542.72 acres. **11.** 10,000 sq. ft. **12.** 1 sq. mi.

DEFINITIONS OF COMMON TERMS

Abstract Number. A number used without reference to any particular unit.

Addends. Numbers added.

Addition. The operation of finding a number equal to two or more numbers taken together.

Agent. A person who transacts business for another.

Aliquot Part. An exact divisor of another number.

Amount. The sum of principal and interest.

Analysis. Any process of reasoning by which we prove a mathematical truth.

Annual Interest. Periodic interest in which installments are due annually.

Antecedent. The first term of a ratio.

Area. The number of square units in a surface.

Arithmetic. The science that treats of numbers and the methods of using them.

Arithmetical Progression. A series of numbers that increase or decrease by a common difference.

Assets. All the property of an estate, individual, or corporation.

Bank. A corporation authorized by law to receive and loan money and to perform certain other financial operations.

Bank Discount. Interest paid in advance on a note.

Base. The number of which some per cent is to be taken.

Bond. A written or printed promise to pay a specified sum at a given time, signed by the maker and often bearing his seal.

Broker. An agent who makes sales or purchases for his principal without having actual possession of the goods.

Brokerage. A sum paid a broker for transacting business.

Cancellation. The process of shortening work in division by removing or canceling equal factors from both dividend and divisor.

Capital Stock. The amount contributed by the members of a corporation to carry on the business.

Check. Any method tending to verify an operation. A written order on a bank to pay money.

Commercial Discount. A reduction from the list price of an article, from the amount of a bill of goods, or from the amount of a debt.

Commercial Draft. A draft made by one business house on another to secure the payment of a debt.

Commission. A sum paid an agent for transacting business.

Commission Merchant. An agent who has actual possession of goods to be sold for his principal.

Common Denominator. A denominator common to two or more fractions.

Common Divisor. The same as Common Measure.

Common Fraction. A fraction expressed by two numbers, one under the other with a line between them.

Common Measure. A number that exactly divides each of two or more numbers.

Common Multiple. A multiple of two or more numbers.

Complex Fraction. A fraction that has a fraction in one or both of its terms.

Composite Number. A number that is not a prime number.

Compound Interest. The total interest when interest is added to the principal and becomes a part of it at specified intervals.

Compound Number. A denominate number involving two or more different units.

Concrete Number. If the unit of a number is named, the number together with the unit is called a *concrete number.*

Consequent. The second term of a ratio.

Consignee. The person to whom goods are sent.

Consignor. The person who sends goods to another.

Corporation. A company composed of a number of persons authorized by law to do business of a certain nature.

Creditor. A person to whom money is due.

Cube. The third power of a number.

Cube Root. One of the three equal factors of a number.

Customs. Taxes imposed by law on merchandise imported.

Debtor. A person who owes money to another.

Decimal Fraction. A fraction whose denominator is not written, but is some power of 10.

Decimal Point. The period written at the left of tenths.

Decimal Scale. The common mode of representing numbers, each order having a value ten times that of the order to the right.

Denominate Number. A concrete number expressing measure.

Denominator. The number below the line in a common fraction.

Difference. The result found in subtraction.

Discount. Allowance made for the payment of money before it becomes due. Also the difference between the market value and the face value when the market value is *below* the face value. A reduction from the list price of goods (trade discount).

Discount Series. A series of trade discounts.

Dividend. In division, the given number which is equal to the product of a given factor (called divisor) and a required factor (called quotient). In business, the sum paid on each share of stock from the profits of the business.

Divisible Number. A number is *divisible* by another when it is exactly divisible, that is, divisible without a remainder.

Division. The process by which, when a product and one of two factors are given, the other factor is found.

Divisor. The number by which a given dividend is divided.

Draft. A written order directing one person to pay a specified sum of money to another.

Drawee of a Draft. The person to whose order the sum of money named in the draft is to be paid.

Drawer of a Draft. The person who signs the draft.

Duties. The same as customs.

Even Number. A number that is exactly divisible by 2.

Exact Divisor. A number that divides another without a remainder.

Exact Interest. Interest computed for exact days, and at 365 days to the year.

Exchange. A system of paying debts, due to persons living at a distance, without actually sending the money.

Exponent. A small figure placed at the right of a number to show how many times the number is taken as a factor.

Extremes. The first and last numbers of a proportion.

Factors. Integral numbers whose product is a given number.

Figures. Symbols used to represent numbers in the common system of notation. Also diagrams used to represent geometric forms.

Fraction. A number that shows what part or what number of parts of a unit is taken.

Geometric Progression. A series of numbers in which each term after the first is obtained by multiplying the preceding term by a constant multiplier.

Greatest Common Divisor. The same as Greatest Common Measure.

Greatest Common Measure. The greatest number that exactly divides each of two or more numbers.

Improper Fraction. A fraction whose numerator is not less than the denominator.

Indorse. To write across the back of a note, draft, or check.

Insurance. An agreement by which, for a lawful consideration, a company agrees to pay an indemnity for loss by specified cause.

Integers. Numbers applied to whole units.

Interest. Money paid for the use of money.

Inverse Ratio. The result of interchanging the terms of a ratio.

Least Common Multiple. The smallest common multiple of two or more numbers.

License. A permit to engage in some specified act or to enjoy some special privilege.

Like Numbers. Numbers applied to the same unit.

Long Division. The method of dividing in which the processes are written in full.

Longitude. The distance in circular measure east or west of a certain meridian agreed upon.

Lowest Terms. If the terms of a fraction are prime to each other, the fraction is expressed in *lowest terms.*

Maker of a Note. The person who signs a note.

Market Value. The price at which goods or stocks sell.

Maturity of a Note. The date at which a note legally becomes due.

Means. The two middle numbers of a proportion.

Meridian. The shortest line on the earth's surface passing from one pole to the other.

Minuend. The number from which the subtrahend is taken.

Mixed Number. A number composed of an integer and a fraction taken together.

Multiple. The product of two or more integers is called a *multiple* of each.

Multiplicand. A number to be multiplied.

Multiplication. The process of taking a number as many times as there are units in another number.

Multiplier. A number by which another is multiplied.

Net Price. The amount of a bill after the discount is made.

Net Proceeds. The money that remains of the money received for property after all expenses and discounts are paid.

Notation. A system of expressing numbers by symbols.

Note. A written promise to pay a specified sum of money on demand or at a specified time.

Number. That which shows how many times a unit is taken, or what part of a unit is taken.

Numerals. The characters used in any system of notation.

Numeration. The reading of numbers.

Numerator. The number above the line in a common fraction.

Odd Number. A number that is not an even number.

Order of Units. Each successive place in a number that may be occupied by a figure.

Par Value. The face value of stocks.

Partial Payments. Payments of a part of a note.

Payee. The person to whom a note, check, or draft is payable.

Per Cent. Another name for hundredths.

Percentage. The part of arithmetic treating of per cent. The result found by taking a certain per cent of the base.

Perimeter. The distance around a polygon.

Period. A group of three figures.

Policy. A written contract of insurance.

Poll Tax. A tax levied upon a person.

Power. The result of taking a number any number of times as a factor.

Premium. Money paid for insurance. Also the excess of market value above par value.

Prime Factor. A factor that is a prime number.

Prime Number. A number that has no factors.

Prime to Each Other. Two integers that have no common measure, always excepting 1.

Principal. The person for whom another transacts business. Money loaned on which interest is paid.

Proceeds. The balance after deducting the discount from the face of a note.

Product. The result found in multiplication.

Promissory Note. A written promise to pay a specified sum of money on demand or at a specified time.

Proper Fraction. A fraction whose numerator is less than the denominator.

Property Tax. A tax levied upon property.

Proportion. An expression of equality between two ratios.

Quotient. The number found by division.

Rate of Interest. The rate per cent of the principal paid in interest for one year.

Rate Per Cent. Rate in hundredths.

Ratio. The relative magnitude of two numbers, as expressed by the fraction which has the first number for the numerator and the second for the denominator.

Reciprocal of a Fraction. The fraction inverted.

Reduction. The process of changing the unit of a number without changing the value.

Remainder. The part of the dividend that is left in division.

Root. One of the equal factors of a number.

Security. Property used to guarantee the payment of any debt.

Short Division. The process of dividing in which the operations of multiplying and subtracting are performed mentally.

Similar Decimals. Decimals that have the same number of decimal places.

Similar Fractions. Fractions with the same denominator.

Simple Fraction. A fraction whose terms are both integers.

Specific Gravity. The ratio of the weight of a given substance to the weight of an equal volume of water.

Square. The result of taking a number twice as a factor.

Square Root. One of the two equal factors of a number.

Stock. Capital invested in business.

Subtraction. The process of taking one number from another, or of finding what number must be added to one of two numbers to make the other.

Subtrahend. The number taken away in subtraction.

Sum. The result obtained in addition.

Taxes. Money levied by a government for its support and for public purposes.

Terms of a Fraction. The numerator and denominator.

Terms of a Proportion. The extremes and means.

Unitary Analysis. An analysis that proceeds first to the unit and then to the required number of units.

Unit Fraction. A fraction whose numerator is 1.

Units. The standards by which we count or measure.

Volume. The number of cubic units a body contains.

INDEX

ANSWERS

Exercise 1. Page 2

1. $\frac{1}{25}$.
2. $\frac{1}{20}$.
3. $\frac{3}{50}$.
4. $\frac{1}{10}$.
5. $\frac{3}{20}$.
6. $\frac{1}{5}$.
7. $\frac{1}{4}$.
8. $\frac{3}{10}$.
9. $\frac{1}{2}$.
10. $\frac{3}{4}$.
11. 1.
12. 2.
13. 5.
14. $1\frac{1}{4}$.
15. $1\frac{1}{2}$.
16. $1\frac{3}{4}$.
17. $1\frac{1}{5}$.
18. $1\frac{2}{5}$.
19. $1\frac{3}{5}$.
20. $1\frac{4}{5}$.
21. $\frac{1}{16}$.
22. $\frac{1}{8}$.
23. $\frac{3}{8}$.
24. $\frac{5}{8}$.
25. $\frac{7}{8}$.
26. $\frac{3}{16}$.
27. $\frac{5}{16}$.
28. $\frac{7}{16}$.
29. $\frac{9}{16}$.
30. $\frac{11}{16}$.
31. $\frac{1}{30}$.
32. $\frac{1}{15}$.
33. $\frac{1}{3}$.
34. $\frac{2}{3}$.
35. $\frac{1}{6}$.
36. $\frac{5}{6}$.
37. $1\frac{1}{8}$.
38. $1\frac{1}{3}$.
39. $1\frac{2}{3}$.
40. $1\frac{7}{8}$.
41. 3; 3.
42. 160; 160.
43. 864; 864.
44. 6330; 6330.
45. 1080; 1080.
46. $\frac{1}{8}$; 8.
47. $\frac{3}{8}$; 270.
48. 70.
49. 22.
50. 11.
51. 123.
52. 92.
53. 198.
54. 352.
55. 507.
56. 972.
57. 1068.
58. 1180.

Exercise 2. Page 3

1. 0.06.
2. 0.10.
3. 0.25.
4. 0.50.
5. 0.75.
6. 0.125.
7. $0.16\frac{2}{3}$.
8. $0.33\frac{1}{3}$.
9. 0.375.
10. $0.66\frac{2}{3}$.
11. 0.005.
12. 0.007.
13. 0.009.
14. 0.0015.
15. 0.0625.
16. 2.
17. 1.5.
18. 1.25.
19. 6.75.
20. 3.75.
21. 1.
22. 10.
23. 0.00005.
24. 0.00065.
25. 0.00125; $\frac{1}{800}$.
26. 0.00875; $\frac{7}{800}$.
27. $3.33\frac{1}{3}$; $\frac{10}{3}$; $3\frac{1}{3}$.
28. \$2; \$2.
29. \$6; \$6.
30. \$45; \$45.
31. \$812.50; \$812.50; \$812.50.
32. \$1150; \$1150.
33. 2; 1.

Exercise 3. Page 4

1. 50%.
2. 70%.
3. 60%.
4. 620%.
5. 750%.
6. 25%.
7. 57%.
8. 34%.
9. 29%.
10. 765%.
11. 12.5%.
12. 27.5%.
13. 37.5%.
14. 75.8%.
15. 963.7%.
16. 25%.
17. 52.5%.
18. 67.5%.
19. 521.25%.
20. 733.75%.
21. $33\frac{1}{3}$%.
22. $33\frac{1}{3}$%.
23. $666\frac{2}{3}$%.
24. $\frac{3}{8}$%.
25. $\frac{1}{80}$%.
26. 50%.
27. 25%.
28. $3\frac{1}{8}$%.
29. 0.7%.
30. 37%; 370%.
31. $3\frac{1}{8}$%.
32. $3\frac{1}{8}$%.

Exercise 4. Page 5

1. 50%.
2. 25%.
3. 75%.
4. 20%.
5. 40%.
6. 60%.
7. 80%.
8. 10%.
9. 70%.
10. 5%.
11. 45%.
12. 4%.
13. 44%.
14. 2%.
15. 74%.
16. $33\frac{1}{3}\%$.
17. $66\frac{2}{3}\%$.
18. $12\frac{1}{2}\%$.
19. $37\frac{1}{2}\%$.
20. $62\frac{1}{2}\%$.
21. $6\frac{1}{4}\%$.
22. $31\frac{1}{4}\%$.
23. $3\frac{1}{8}\%$.
24. $46\frac{7}{8}\%$.
25. $20\frac{5}{6}\%$.
26. 225%.
27. 360%.
28. $762\frac{1}{2}\%$.
29. $633\frac{1}{3}\%$.
30. $7587\frac{1}{2}\%$.
31. $33\frac{1}{3}\%$.
32. $8\frac{1}{3}\%$.
33. $12\frac{1}{2}\%$.
34. 75%.
35. 25%.
36. $43\frac{3}{4}\%$.
37. $83\frac{1}{3}\%$.
38. 10%.

Exercise 5. Page 6

1. \$137.
2. \$93.
3. \$64.
4. \$211.
5. \$6.66.
6. \$7.45.
7. \$30.17.
8. \$23.10.
9. \$5.71.
10. \$25.72.
11. \$53.38.
12. \$59.25.
13. \$147.03.
14. \$244.50.
15. \$440.49.
16. 81.
17. 132.
18. 84.
19. 108.
20. 140.

Exercise 6. Page 7

1. 16.5.
2. 30.1.
3. 8.64.
4. 31.98.
5. 57.66.
6. 0.884.
7. 2.881.
8. 2.067.
9. 0.196.
10. 0.2205.
11. 0.231.
12. 0.0285.
13. 0.1292.
14. 1743.7.
15. 4.4784.
16. 432.
17. $3117\frac{1}{2}$.
18. $1468\frac{1}{5}$.
19. 837.
20. 4836.
21. 1202.
22. $1520\frac{1}{2}$.
23. 3017.
24. 4744.
25. 30,940.
26. \$2301.
27. 9390.
28. 18,625.
29. 1185.
30. 972.
31. 1360.
32. \$10.50.
33. \$350.
34. \$2,980,800.
35. \$348.
36. 14.625 lb.
37. \$3.36.
38. Second ; 0.46¢ per day.
39. \$155.25.
40. \$17.93.

Exercise 7. Page 10

1. 600.
2. 1100.
3. 1200.
4. 1700.
5. 31,996.
6. 146.16.
7. 144.81.
8. \$1900.
9. \$56,600.
10. 31.
11. 41.
12. 44.44.
13. $33\frac{1}{3}$.
14. 25.
15. 40.
16. 245.
17. 320.
18. 425.
19. 640.
20. 400.
21. 725.
22. 7.29.
23. 5.46.
24. 9.36.
25. 25.
26. 40.
27. $33\frac{1}{3}$.
28. 270.
29. 320.
30. 750.
31. 764.
32. 280.
33. 720.
34. 7.2.
35. 32.
36. 8.8.
37. \$7250.
38. \$395.
39. \$850.
40. 22.05.
41. \$275.
42. \$6.75.
43. 27,500.

Exercise 8. Page 11

1. 5%.	5. 11%.	9. 33%.	13. 50%.	17. $62\frac{1}{2}$%.
2. 7%.	6. 16%.	10. 3%.	14. $16\frac{2}{3}$%.	18. $87\frac{1}{2}$%.
3. 4%.	7. 21%.	11. 24%.	15. $66\frac{2}{3}$%.	19. 25%.
4. 9%.	8. 45%.	12. 22%.	16. $37\frac{1}{2}$%.	20. $\frac{5}{12}$%.

Exercise 9. Page 12

1. $25.38.	13. $44.85.	25. $383.85.	37. $39.60.
2. $29.76.	14. $56.07.	26. $344.34.	38. $385.
3. $43.70.	15. $66.06.	27. $252.28.	39. $1911.
4. $69.	16. $60.50.	28. $433.40.	40. $41.82.
5. $61.44.	17. $73.44.	29. $508.47.	41. 12¢.
6. $78.26.	18. $34.11.	30. $244.75.	42. $47.25.
7. $61.20.	19. $42.68.	31. $727.82.	43. 3¢.
8. $72.16.	20. $45.15.	32. $817.87.	44. $66.64.
9. $81.60.	21. $57.97.	33. $770.99.	45. $16.68.
10. $231.84.	22. $40.12.	34. 25%.	46. $245.
11. $159.25.	23. $120.48.	35. $520.	
12. $23.97.	24. $345.46.	36. $16.40.	

Exercise 10. Page 14

1. $169.20.	9. $393.66.	17. $40.50.
2. $262.20.	10. $170.10.	18. $528.
3. $407.40.	11. $1324.80.	19. $119.88.
4. $285.60.	12. $2358.75.	20. $144.
5. $378.	13. $194.40.	21. $480.
6. $162.	14. $51.84.	22. $594.81.
7. $307.80.	15. $51.84.	
8. $325.92.	16. 20% and 10% by 15¢.	

Exercise 11. Page 16

1. $155.40.	3. $164.45.	5. $846.45.	7. $1486.75.
2. $1814.40.	4. $53.82.	6. $24.37.	

Exercise 12. Page 17

1. 17.2%.	**6.** 37%.	**11.** 49.6%.	**16.** $40\frac{15}{16}$%.
2. 20.1%.	**7.** $34\frac{3}{8}$%.	**12.** 43%.	**17.** They are equal.
3. 20.46%.	**8.** 27.1%.	**13.** 46%.	**18.** 24%.
4. 21.74%.	**9.** 42.625%.	**14.** $32\frac{1}{2}$%.	**19.** They are equal.
5. 28%.	**10.** 46%.	**15.** 37%.	

Exercise 13. Page 19

1. \$28.	**21.** \$810.	**41.** \$1603.14.	**61.** \$212.40.
2. \$41.04.	**22.** \$169.05.	**42.** \$41.31.	**62.** \$39.25.
3. \$74.75.	**23.** \$21,800.	**43.** \$55.44.	**63.** \$138.
4. \$84.96.	**24.** \$18,480.	**44.** \$88.70.	**64.** \$8.50.
5. \$106.25.	**25.** \$75.	**45.** \$138.12.	**65.** \$9.12.
6. \$189.88.	**26.** \$82.	**46.** \$130.	**66.** 15%.
7. \$207.58.	**27.** \$68.	**47.** \$342.04.	**67.** 25%.
8. \$280.	**28.** \$175.	**48.** \$352.37.	**68.** \$10.
9. \$431.25.	**29.** \$296.	**49.** \$524.12.	**69.** \$320.
10. \$867.30.	**30.** \$450.	**50.** \$1402.01.	**70.** \$3.15.
11. \$194.40.	**31.** \$260.	**51.** \$623.70.	**71.** \$15.40.
12. \$435.40.	**32.** \$320.	**52.** \$16.50.	**72.** \$2.40, loss.
13. \$53,521.25.	**33.** \$460.	**53.** \$13.80.	**73.** \$223.20.
14. \$33,519.50.	**34.** \$262.50.	**54.** \$37.04.	**74.** \$150.
15. \$94,062.50.	**35.** \$360.	**55.** \$14.50.	**75.** \$360.
16. \$51.	**36.** \$337.50.	**56.** \$34.20.	**76.** 50%.
17. \$69.70.	**37.** \$41.25.	**57.** \$87.87.	**77.** $66\frac{2}{3}$%.
18. \$56.16.	**38.** \$45.63.	**58.** \$378.02.	**78.** \$800, loss.
19. \$165.	**39.** \$661.22.	**59.** \$315.90.	**79.** \$1280, loss.
20. \$623.	**40.** \$634.40.	**60.** \$625.50.	

Exercise 14. Page 22

1. \$5.50.	**7.** \$81.88.	**13.** \$31.	**19.** \$2737.
2. \$10.35.	**8.** \$96.60.	**14.** \$13.20.	**20.** \$5481.
3. \$27.	**9.** \$187.88.	**15.** \$54.	**21.** \$6176.
4. \$36.25.	**10.** \$212.50.	**16.** \$4794.	**22.** \$264.48.
5. \$52.50.	**11.** \$10.	**17.** \$6384.	**23.** \$563.50.
6. \$41.25.	**12.** \$18.75.	**18.** \$3493.	**24.** \$678.21.

25. \$44.45.
26. 255,000; \$18.75.
27. \$8787.50; 425,000; \$31.25.
28. \$786.60.
29. 21¢.
30. $1\frac{1}{5}$¢.
31. 227,500; \$19,248.25.
32. 350,000; \$42,385.

Exercise 15. Page 24

1. \$18.75.
2. \$22.50.
3. \$30.63.
4. \$50.63.
5. \$52.50.
6. \$71.50.
7. \$95.
8. \$78.75.
9. \$125.
10. \$270.
11. \$187.50.
12. \$223.13.
13. \$228.75.
14. \$371.25.
15. \$431.25.
16. \$515.63.
17. \$109.20.
18. \$78.
19. \$60.
20. \$45.90.
21. \$60.50.
22. \$87.50.
23. \$134.40.
24. \$178.75.
25. \$6500.
26. \$2400.
27. \$3800.
28. \$3200.
29. \$1975.
30. \$1650.
31. 1%.
32. $2\frac{1}{2}$%.
33. $1\frac{1}{2}$%.
34. $1\frac{3}{4}$%.
35. 2%.
36. $1\frac{1}{2}$%.
37. \$34.38.
38. \$48.75.
39. \$84.
40. \$126.
41. \$375.
42. \$756.25.
43. \$41.25.
44. \$71.50.
45. \$52.50.
46. \$5500.
47. $1\frac{1}{2}$%.
48. \$43.20.
49. $\frac{4}{5}$%.
50. \$3000.
51. \$4500.
52. \$4666.67; \$3333.33.

Exercise 16. Page 26

1. \$108.75.
2. \$169.88.
3. \$99.58.
4. \$118.62.
5. \$194.25.
6. \$100.58.
7. \$86.28.
8. \$307.44.
9. \$369.32.
10. \$456.63.
11. \$3954.
12. \$195.98.
13. \$6000; \$4720.80.

Exercise 17. Page 29

1. \$1.39.
2. \$4.73.
3. \$5.
4. \$22.22.
5. \$20.28.
6. \$25.28.
7. \$41.95.
8. \$50.
9. \$66.66.
10. \$63.88.
11. \$62.50.
12. \$220.82.
13. \$3.18.
14. \$5.
15. \$6.36.
16. \$32.72.
17. \$45.45.
18. \$32.27.
19. \$43.18.
20. \$34.09.
21. \$50.45.
22. \$72.72.
23. \$118.17.
24. \$318.15.
25. \$113.63.
26. \$240.89.
27. \$443.14.
28. \$748.56.
29. \$49.50.
30. \$162.50.
31. \$7500.
32. 12 mills.
33. 12 mills.
34. \$3760.
35. \$42.50.
36. \$190.
37. \$614.33.

Exercise 18. Page 31

1. $142.50.
2. $218.75.
3. $300.
4. $375.75.
5. $585.
6. $375.
7. $495.
8. $945.
9. $1460.
10. $3787.50.
11. $2490.
12. $12,712.50.
13. $10,276.70.
14. $16,137.50.
15. $45,177.
16. $24.48.
17. $343.75.
18. $7500.
19. $415.80.
20. 15%.
21. 40%.
22. $8275.
23. 35%.
24. $331.
25. $3375.
26. $2750.
27. $6600.
28. $168.50.
29. $271.80.
30. 1750.
31. $750.

Exercise 19. Page 33

1. 12.21 ; 122.1 ; 1221 ; 12,210.
2. 12.21 ; 122.1 ; 1221 ; 12,210.
3. 24.55 ; 245.5 ; 2455 ; 24,550 ; 24,550.
4. 16.22 ; 162.2 ; 1622 ; 16,220 ; 16,220.
5. 50.64 ; 506.4 ; 5064 ; 50,640 ; 50,640.
6. 103.11 ; 1031.1 ; 10,311 ; 103,110 ; 1,031,100.
7. 41.88 ; 418.8 ; 4188 ; 41,880 ; 418,800.
8. 77.15 ; 771.5 ; 7715 ; 77,150 ; 771,500.
9. 324.38 ; 3243.8 ; 32,438 ; 324,380 ; 3,243,800.
10. 18%.
11. 15%.
12. 4%.
13. $4850.
14. $120.
15. $700.
16. $490.
17. $2250.
18. $6000.
19. $7000.
20. $6500.
21. $27\frac{3}{11}$%.
22. $7525.
23. 20%.
24. $650.
25. $1.25.
26. $1.64.
27. $4.
28. 20%.
29. $1039.50.
30. $17,000.
31. $1800.
32. $2.64.
33. $130.
34. $1.92.
35. $1\frac{1}{4}$%.
36. $240.
37. 97.6%.
38. $2375 ; $2625.
39. $17,250.
40. $1.44.
41. 81¢.
42. $654.75.
43. $1470.15.
44. $40.50.
45. $1\frac{1}{4}$%.
46. $280.
47. 25%.
48. $1425.90.
49. $5\frac{1}{2}$%.
50. $1181 ; $1228.24.
51. $65.07.
52. $1.81\frac{9}{11}$% ; $2.72\frac{8}{11}$% ; $95.45\frac{5}{11}$%.

Exercise 20. Page 37

1. $27.50.
2. $42.
3. $55.20.
4. $78.75.
5. $21.20.
6. $63.75.
7. $152.40.
8. $105.
9. $231.25.
10. $78.
11. $107.25.
12. $97.88.
13. $58.45.
14. $63.83.
15. $78.91.
16. $47.31.
17. $367.50.
18. $491.56.

Exercise 21. Page 38

1. $4.51.
2. $87.97.
3. $172.58.
4. $190.46.
5. $115.45.
6. $392.26.
7. $490.77.
8. $305.35.
9. $180.25.
10. $234.11.
11. $635.68.
12. $1392.01.
13. $2759.51.

Exercise 22. Page 40

1. $125.
2. $560.
3. $68.50.
4. $426.
5. $130.
6. $240.
7. $402.
8. $1042.
9. $1080.
10. 3 yr. 6 mo.
11. 2 yr. 3 mo.
12. 2 yr. 4 mo.
13. 2 yr. 6 mo.
14. 5%.
15. 4%.
16. 6%.
17. $16\frac{2}{3}$ yr.
18. 20 yr.
19. $500.
20. $500.

Exercise 23. Page 41

1. 2 yr. 1 mo. 13 da.
2. 3 yr. 3 mo. 20 da.
3. 3 yr. 9 mo. 27 da.
4. 3 yr. 1 mo. 25 da.
5. 2 yr. 4 mo. 16 da.
6. $63.56.
7. $90.73.
8. $138.70.
9. 6%.
10. Jan. 7, 1913.
11. $200.
12. $745.88.

Exercise 24. Page 42

1. $5.04.
2. $5.58.
3. $11.63.
4. $7.60.
5. $11.48.
6. $14.04.
7. $5.63.
8. $7.80.
9. $14.03.
10. $51.73.
11. $96.75.
12. $43.75.
13. $35.55.
14. $143.49.
15. $255.75.
16. $331.25.

Exercise 25. Page 43

1. $10.40.
2. $8.60.
3. $6.27.
4. $10.50.
5. $19.38.
6. $41.25.
7. $1.67.
8. $2.35.
9. $4.76.
10. $16.67.
11. $16.63.
12. $20.
13. $2.52.
14. $10.81.
15. $21.66.
16. $38.18.
17. $1.13.
18. $12.19.
19. $18.56.
20. $28.91.

Exercise 26. Page 44

1. 92.
2. 153.
3. 227.
4. 220.
5. 174.
6. 188.
7. 104.
8. 48.
9. 243.
10. 186.
11. 122.
12. 141.
13. 162.
14. 141.
15. 191.
16. 235.

Exercise 27. Page 45

1. $8.92.
2. $10.13.
3. $8.80.
4. $18.75.
5. $7.75.
6. $11.25.
7. $14.79.
8. $7.13.
9. $9.60.
10. $25.20.
11. $3.85.
12. $7.13.
13. $15.87.
14. $18.
15. $2.74.
16. $7.92.
17. $12.31.
18. $7.80.
19. $11.07.
20. $6.03.
21. $8.37.
22. $19.88.
23. $19.67.
24. $12.59.
25. $7.14.
26. $10.79.
27. $9.15.
28. $14.88.

Exercise 28. Page 46

1. $40.49.
2. $49.14.
3. $63.75.
4. $39.49.
5. $51.46.
6. $62.10.
7. $76.97.
8. $58.51.
9. $236.25.
10. $35.27.
11. $56.21.
12. $65.88.

Exercise 29. Page 47

1. $3.08.
2. $3.20.
3. $4.01.
4. $5.99.
5. $14.67.
6. $11.34.
7. $17.67.
8. $4.50.
9. $4.80.
10. $20.10.
11. $19.20.
12. $48.
13. $76.23.
14. $236.50.
15. $110.25.
16. $47.25.
17. $94.50.

Exercise 30. Page 48

1. $130.80.
2. $47.70.
3. $130.63.
4. $253.13.
5. $133.62.
6. $135.04.
7. $75.57.
8. $108.75.
9. $79.58.
10. $355.93.
11. $166.71.
12. $473.10.
13. $1467.50.
14. $4038.03.

Exercise 31. Page 49

1. $281.22.
2. $438.69.
3. $478.07.
4. $790.83.
5. $1107.16.
6. $1379.04.
7. $1591.20.
8. $3040.51.
9. $3042.50.
10. $112.65.
11. $273.36.
12. $610.29.
13. $710.74.
14. $1137.32.
15. $1454.32.
16. $2774.60.
17. $1886.59.
18. $1943.75.

Exercise 32. Page 51

1. Sept. 5, 1911; $353.50.
2. July 4, 1910; $584.25.
3. Feb. 8, 1911; $480.70.
4. June 7, 1910; $333.38.
5. July 14, 1911; $544.29.
6. June 15, 1910; $276.65.
7. $14.40.
8. $10.67.
9. $11.25.
10. $252.08.
11. $1268.75.
12. $289.28.

Exercise 33. Page 53

1. $71.97.
2. $182.70.
3. $487.25.
4. $347.43.
5. $1538.21.
6. $378.44.
7. $534.
8. $406.98.
9. $1.80.

Exercise 34. Page 55

1. $147.33.
2. $142.43.
3. $231.48.
4. $161.04.
5. $17.80.
6. $254.40.

Exercise 35. Page 56

1. $18.83.
2. $2.40.
3. $28.08.
4. $75.50; $37.75; $113.25; $117.03.
5. $23.56.
6. $26.18.
7. $1484.96.
8. $45.44.
9. $261.
10. $432.39.
11. $10.90.
12. $29.90.
13. $277.
14. $1151.16.
15. $775.
16. $240.
17. $350.
18. $450.
19. $750.
20. $840.
21. 4 yr. 11 mo. 6 da.
22. 3 yr. 4 mo.
23. 1 yr. 7 mo. 6 da.
24. 7 yr. 9 mo. 15 da.
25. 5 yr. 7 mo. 9 da.
26. 3%.
27. $4\frac{1}{2}$%.
28. 6%.

Exercise 37. Page 62

1. $\frac{2}{3}$.
2. $\frac{3}{4}$.
3. $\frac{2}{5}$.
4. $\frac{8}{3}$.
5. $\frac{2}{15}$.
6. $\frac{4}{7}$.
7. 4.
8. $\frac{1}{4}$.
9. $\frac{2}{5}$.
10. $\frac{3}{4}$.
11. 8.
12. 3.
13. 3.
14. 20.
15. 6.
16. 35.
17. 18.
18. 14.
19. 12; 16.
20. 20; 25.
21. 100; 160.
22. $10\frac{1}{2}$; 21.
23. $8; $64.
24. $36; $60.
25. $18; $60.
26. 165; 176.
27. 132; 360.
28. $7; $8.
29. $8980.

Exercise 38. Page 63

1. $211\frac{1}{2}$.
2. 129.6.
3. \$1125.
4. 150 lb.; 65 lb.
5. \$1650; \$2100.
6. $187\frac{1}{2}$.
7. 16.
8. \$896; \$1344.
9. \$28; \$56.
10. \$45; \$70.
11. \$2100.
12. 49 ft.
13. $12\frac{4}{7}$ ft.
14. $62\frac{1}{2}$.
15. 14.
16. 99.
17. \$30; \$54.
18. \$2250; \$3150.
19. 4 lb.; 2 lb.

Exercise 39. Page 66

1. 18.
2. 9.
3. 80.
4. 6.
5. 12.
6. 25.
7. $6\frac{2}{3}$.
8. 30.
9. 18.
10. 35.
11. 27.
12. 14.
13. 63.
14. 63.
15. 63.
16. 45.
17. \$11.
18. \$5.
19. 85.
20. 35.
21. 11 ft.
22. 280.
23. 40.
24. 60.
25. 60.
26. 12.
27. 15.
28. 10.
29. 1250.
30. 5.02656.
33. $9\frac{3}{5}$.
34. 144.
35. $401:311 = 6817:5287$; $5287:6817 = 311:401$; $401:6817 = 311:5287$.

Exercise 40. Page 68

1. \$249.60.
2. \$15.12.
3. \$16.50.
4. \$7.75.
5. \$196.
6. \$68.75.
7. $266\frac{1}{4}$ yd.
8. \$20.70.
9. $112\frac{1}{2}$.
10. 39 da.
11. 15.
12. 216 mi.

Exercise 41. Page 69

1. 625.
2. 1.7.
3. 40 ft.
4. 180 ft.
5. 30 ft.
6. 64 ft.

Exercise 42. Page 70

1. $4\frac{1}{4}$ oz.
2. $8\frac{11}{13}$ oz.
3. $5\frac{11}{17}$ lb.
4. $7\frac{1}{32}$ lb.
5. $16\frac{2}{3}$ lb.
6. 60 lb.
7. $37\frac{1}{2}$ lb.
8. 150 lb.
9. 50 lb.
10. $181\frac{9}{11}$ lb.
11. 60 lb.
12. 360 lb.
13. 750 lb.
14. 225 lb.
15. $1031\frac{1}{4}$ lb.
16. $1462\frac{1}{2}$ lb.

Exercise 43. Page 71

1. \$154.70.	**6.** 20 ft.	**11.** $10\frac{4}{15}$.	**16.** 3625.
2. \$50.	**7.** 90%.	**12.** 28.405 in.	**17.** 14 da.
3. 750.	**8.** No.	**13.** \$121.50.	**18.** $38\frac{1}{4}$.
4. 4680.	**9.** 1431 lb.	**14.** \$40.	**19.** $35\frac{3}{4}$.
5. 315 gal.	**10.** \$67.50.	**15.** $13\frac{5}{12}$ T.	**20.** 46,100 cu. ft.

Exercise 44. Page 73

1. 77 ; 99 ; 121.	**8.** \$224 ; \$288 ; \$336.
2. 46 ; 138 ; 69.	**9.** \$160 ; \$200 ; \$180.
3. \$1176 ; \$1470.	**10.** \$21.60 ; \$21.60.
4. \$936 ; \$1248 ; \$1560.	**11.** \$1235 ; \$1558.
5. \$1350 ; \$1650 ; \$1750 ; \$2000.	**12.** \$9 ; \$6 ; \$7 ; \$10.50.
6. \$67.50 ; \$57 ; \$100.50.	**13.** \$600 ; \$420.
7. \$11.64 ; \$14.07 ; \$13.05.	**14.** \$12 ; \$12.

Exercise 45. Page 75

1. 38 ft. 6 in. **2.** 37 ft. **3.** \$5.50. **4.** 4 ft. **5.** $8\frac{1}{8}$ lb.

Exercise 47. Page 78

1. 289 sq. ft.	**13.** 14.	**25.** 35.	**37.** 96 in.
2. 361 sq. ft.	**14.** 15.	**26.** 33.	**38.** 288 in.
3. 529 sq. ft.	**15.** 16.	**27.** 36.	**39.** 19.2 in.
4. 8.41 sq. in.	**16.** 20.	**28.** 1.2 in.	**40.** 25.6 in.
5. 13.69 sq. in.	**17.** 18.	**29.** 1.1 in.	**41.** 32.4 ft.
6. 18.49 sq. in.	**18.** 22.	**30.** 2.4 in.	**42.** 42 ft.
7. 0.3844 sq. yd.	**19.** 25.	**31.** 50 ft.	**44.** 80 rd.
8. 1.6129 sq. yd.	**20.** 21.	**32.** 60 yd.	**45.** 52.
9. 24.3049 sq. yd.	**21.** 28.	**33.** 80 rd.	**46.** $141\frac{1}{64}$ sq. in
10. 0.64 sq. mi.	**22.** 30.	**34.** 0.3 mi.	**47.** 1.7 ft.
11. $\frac{49}{64}$ sq. in.	**23.** 27.	**35.** 0.9 ft.	
12. $\frac{81}{1024}$ sq. ft.	**24.** 32.	**36.** 0.12 ft.	

Exercise 48. Page 80

1. 29.	**5.** 41.	**9.** 43.	**13.** 83.
2. 31.	**6.** 39.	**10.** 91.	**14.** 73.
3. 23.	**7.** 57.	**11.** 99.	**15.** 22 in.
4. 37.	**8.** 61.	**12.** 79.	**16.** 87.

Exercise 49. Page 82

1. 437.	7. 441.	13. 0.251.	19. 0.427.
2. 862.	8. 416.	14. 1.23.	20. 0.234.
3. 634.	9. 418.	15. 1.62.	21. 111 ft.
4. 871.	10. 714.	16. 1.98.	22. 89.7 in.
5. 343.	11. 32.2.	17. 21.5.	23. 176.4 in.
6. 897.	12. 9.07.	18. 0.063.	24. 7.7 in.; 456.533 cu. in.

Exercise 50. Page 85

1. $\frac{7}{12}$.	13. 3.32.	25. 0.94.	37. 8 ft.
2. $\frac{11}{13}$.	14. 3.87.	26. 0.79.	38. 8 in.
3. $\frac{15}{19}$.	15. 5.48.	27. 0.76.	39. 24 in.
4. $\frac{21}{25}$.	16. 7.07.	28. 0.77.	40. 13.748 ft.
5. $\frac{23}{27}$.	17. 11.18.	29. 0.65.	41. 13.416 ft.
6. $\frac{31}{33}$.	18. 25.50.	30. 0.96.	42. 19.621 in.
7. $\frac{37}{41}$.	19. 0.75.	31. 65 ft.	43. 28.284 in.
8. $\frac{29}{43}$.	20. 0.76.	32. 75 ft.	44. 45.255 in.
9. $\frac{47}{51}$.	21. 0.58.	33. 85 ft.	45. 63.640 in.
10. $\frac{53}{55}$.	22. 0.63.	34. 89.157 ft.	46. 98.995 in.
11. 1.41.	23. 0.82.	35. 38.601 ft.	47. 106.066 in.
12. 2.65.	24. 0.61.	36. 50.774 ft.	

Exercise 51. Page 86

1. 17.32 sq. in.	3. 5.33 sq. in.	5. 74.83 sq. in.	7. $68\frac{3}{4}$ sq. ft.
2. 23.53 sq. in.	4. 42.43 sq. in.	6. 176.56 sq. in.	8. $4000.

Exercise 52. Page 87

1. 907.9224 sq. in.	8. 660.5214 sq. in.	15. 1.95 ft.
2. 2642.0856 sq. in.	9. 44.65124664 sq. in.	16. 11.28 in.
3. 4300.8504 sq. in.	10. 74.20333536 sq. ft.	17. 42.22 ft.
4. 87.25134264 sq. in.	11. 10 in.	18. 22.36 ft.; 6283.2 sq. ft.
5. 115.75193784 sq. ft.	12. 6 in.	19. 23.1 sq. in.; 2.71 in.
6. 1134.1176 sq. in.	13. 8.92 in.	
7. 1661.9064 sq. in.	14. 1.60 in.	

Exercise 53. Page 88

1. 50,653.	**3.** 205,379.	**5.** 110.592.	**7.** 11.	**9.** 16.	**11.** 21.
2. 117,649.	**4.** 421.875.	**6.** 0.328509.	**8.** 15.	**10.** 18.	**12.** 22.
					13. 24 in.

Exercise 54. Page 90

1. 13.	**3.** 19.	**5.** 31.	**7.** 61.	**9.** 72.	**11.** 96.	**13.** 92 in.
2. 17.	**4.** 24.	**6.** 48.	**8.** 51.	**10.** 95.	**12.** 98.	**14.** 59 in.

Exercise 55. Page 93

1. 4.15.	**4.** 0.91.	**7.** 3.35.	**10.** $\frac{11}{12}$.	**13.** $\frac{15}{16}$.	**16.** 1.710.
2. 8.63.	**5.** 2.79.	**8.** 4.34.	**11.** $\frac{17}{19}$.	**14.** 1.260.	**17.** 0.630.
3. 1.92.	**6.** 0.50.	**9.** $\frac{2}{9}$.	**12.** $\frac{18}{21}$.	**15.** 1.442.	**18.** 0.874.

Exercise 56. Page 95

1. 121.	**5.** 225.	**9.** 0.486.	**13.** 1.96 in.	**17.** 8 ft.
2. 109.	**6.** 444.	**10.** 4.22 in.	**14.** 6 in.	**18.** 9 ft.
3. 127.	**7.** 16.9.	**11.** 9.40 ft.	**15.** 13:17.	**19.** 4; 8.
4. 363.	**8.** 0.097.	**12.** 4.52 in.	**16.** 9 in.	**20.** 2.

21. 8; 125; 1000; 3723.875.

Exercise 58. Page 99

1. 25,000 m.	**12.** 487.5 m.	**23.** 1 m.	**34.** 3.1 mi.
2. 3700 m.	**13.** 29.375 m.	**24.** 9 m.	**35.** 46.5 mi.
3. 700 m.	**14.** 96.4 m.	**25.** 246.06 ft.	**36.** 1.55 mi.
4. 62,500 m.	**15.** 250 m.	**26.** 159.12 ft.	**37.** 2.945 mi.
5. 57,750 m.	**16.** 0.5 m.	**27.** 9.02 ft.	**38.** 168.64 mi.
6. 23.7 m.	**17.** 2 m.	**28.** 11.48 ft.	**39.** 30.318 mi.
7. 4.75 m.	**18.** 3 m.	**29.** 2.46 ft.	**40.** 524.19 km.
8. 0.275 m.	**19.** 7.62 m.	**30.** 22.47 ft.	**41.** 68.87 km.
9. 7.25 m.	**20.** 1.46 m.	**31.** 147.64 ft.	**42.** 1.21 km.
10. 0.685 m.	**21.** 0.18 m.	**32.** 22.56 ft.	**43.** 5.7885.
11. 29.65 m.	**22.** 0.99 m.	**33.** 70.46 ft.	**44.** 194.06.
			45. 351.61.

Exercise 59. Page 100

1. 275,000,000 sq. m.	**6.** 6.4275 sq. m.	**11.** 4.5 ha.
2. 3,450,000 sq. m.	**7.** 0.12345 sq. m.	**12.** 17.5 ha.
3. 750,000 sq. m.	**8.** 2.25 sq. m.	**13.** 32.55 ha.
4. 0.4875 sq. m.	**9.** 37.375 sq. m.	**14.** 10 ha.
5. 0.9235 sq. m.	**10.** 1 ha.	**15.** 600 ha.
		16. 90 m.

Exercise 60. Page 101

1. 725.25 cu. m.	**4.** 9125 cu. m.	**7.** 25.325 cu. m.
2. 2750 cu. m.	**5.** 8.125 cu. m.	**8.** 37.25 cu. m.
3. 4625.75 cu. m.	**6.** 9.275 cu. m.	**9.** 71.75.

Exercise 61. Page 102

1. 75,000 l.	**8.** 6.25 l.	**15.** 2.6 hl.	**22.** 1260.
2. 27,500 l.	**9.** 37.5 hl.	**16.** 1.26 hl.	**23.** 11,390.625.
3. 9275 l.	**10.** 48.75 hl.	**17.** 3700 qt.	**24.** 864; 86,400.
4. 937.5 l.	**11.** 9.785 hl.	**18.** 950 qt.	**25.** 612,500.
5. 75 l.	**12.** 0.2575 hl.	**19.** 75 qt.	**26.** 763,408.8.
6. 9.75 l.	**13.** 0.975 hl.	**20.** 82.5 qt.	
7. 7.5 l.	**14.** 0.3875 hl.	**21.** 1000.	

Exercise 62. Page 103

1. 1700 g.	**7.** 7.5 g.	**13.** 15,000,000 mg.	**19.** 0.25 kg.
2. 900 g.	**8.** 8.125 g.	**14.** 2,700,000 mg.	**20.** 2000 kg.
3. 250 g.	**9.** 7000 mg.	**15.** 750 mg.	**21.** 6.6 lb.
4. 3750 g.	**10.** 8700 mg.	**16.** 98 mg.	**22.** 165 lb.
5. 7 g.	**11.** 270 mg.	**17.** 27,000 kg.	**23.** 0.66 lb.
6. 9.5 g.	**12.** 4250 mg.	**18.** 500 kg.	**24.** 0.0198 lb.

Exercise 63. Page 104

1. 0.79. **2.** 0.7. **3.** 0.24. **4.** 1.03. **5.** Yes. **6.** 19.3. **7.** 15.3153 kg.

Exercise 64. Page 105

1. 15°.
2. $5\frac{5}{9}^\circ$.
3. $23\frac{8}{9}^\circ$.
4. 30°.
5. $42\frac{2}{9}^\circ$.
6. $44\frac{4}{9}^\circ$.
7. $65\frac{5}{9}^\circ$.
8. $93\frac{1}{3}^\circ$.
9. 15° below zero.
10. $12\frac{2}{9}^\circ$ below zero.
11. $23\frac{1}{3}^\circ$ below zero.
12. $26\frac{1}{9}^\circ$ below zero.
13. $31\frac{2}{3}^\circ$ below zero.
14. 104°.
15. 167°.
16. 50°.
17. 302°.
18. 392°.
19. 14°.
20. 5°.
21. 13° below zero.

Exercise 65. Page 106

1. 1.8769 sq. m.
2. 5.372 sq. m.
3. 17,671.5 sq. cm.
4. 82,448.1504 sq. mm.
5. 4.5375 sq. m.
6. 12,350 sq. cm.
7. 2 m.
8. 6.28 m.
9. 1262.24 kg.
10. 137.05536 kg.
11. 2835.
12. 113.
13. 0.8125.
14. 0.915 kg.
15. 52.7558 in.
16. 105.132 in.
17. 0.4875.
18. 764.5.
19. 295.16.
20. 141.42 m.
21. 4.472 cm.
22. 12.649 cm.
23. $1.24.
24. $1.50.
25. 13,304.676 l.
26. 663.663.
27. 683.46 cu. in.
28. 7.168 kg.

Exercise 67. Page 112

1. $10,650.
2. $14,093.75.
3. $16,187.50.
4. $35,028.13.
5. $50,006.25.
6. $4659.38.
7. $6030.
8. $6410.63.
9. $12,406.25.
10. $3190.63.
11. $6140.63.
12. $1975.
13. $4837.50.
14. $11,805.
15. $31,218.75.
16. $13,734.38.
17. $16,856.25.
18. $21,196.88.
19. $35,687.50.
20. $27,465.63.
21. $143.75.
22. $393.75.
23. $31.26.
24. 40; $15.

Exercise 68. Page 113

1. $4762.50.
2. $18,750.
3. $8390.63.
4. $7521.25.
5. $5225.
6. $22,312.50.
7. $41,602.50.
8. $5100.
9. $1706.25.
10. $4585.
11. $9956.25.
12. $21.87, gain.
13. $34.37, gain.
14. $53.13, loss.
15. $143.75, loss.
16. $6.25, gain.
17. $25, gain.
18. $40.62, gain.
19. $34.37, gain.
20. $59.37, gain.
21. $153.12, gain.
22. $46.87, gain.
23. $96.88, loss.
24. $156.25, loss.
25. $593.75, gain.
26. $99.99, gain.
27. $30, gain.
28. $1037.49, gain.

Exercise 69. Page 115

1. $2\frac{1}{2}\%$.
2. $2625.
3. $90,000.
4. $75,500,000.
5. $300.
6. $2\frac{1}{2}\%$.
7. $4\frac{1}{2}\%$.
8. 6%.

Exercise 70. Page 116

1. 4%.
2. 4%.
3. 4%.
4. 5%.
5. 6%.
6. 8%.
7. 4%.
8. 5%.
9. 8%.
10. 6%.
11. $4\frac{4}{19}\%$.
12. $5\frac{5}{11}\%$.
13. $4\frac{6}{11}\%$.
14. 4.04%.
15. 4.98%.
16. 5.37%.
17. 5.26%.
18. 5%.
19. The same.
20. The 5% note; 0.34%.
21. The 8% stock; $\frac{1}{3}\%$.
22. The 6% stock; 0.12%.
23. $120; $119\frac{7}{8}$.
24. $150; $149\frac{7}{8}$.

Exercise 71. Page 117

1. $360.
2. $350.
3. $360.
4. $255.
5. $260.
6. $472.50.
7. $765.
8. $660.
9. $1105.
10. 4.90%.
11. 6.37%.
12. $3\frac{1}{3}\%$.

Exercise 73. Page 121

1. $970.75.
2. $873.53.
3. $518.18.
4. $1087.31.
5. $1572.65.
6. $59.60.

Exercise 74. Page 123

1. $603.75.
2. $725.12.
3. $936.25.
4. $1130.
5. $774.40.
6. $682.67.
7. $626.90.
8. $547.52.
9. $799.40.
10. $324.45.
11. $321.25.
12. $631.27.
13. $1802.84.

Exercise 75. Page 125

3. $158.42. **4.** $368.71. **5.** $706.49. **6.** $813.96. **7.** $449.89.

Exercise 76. Page 127

1. $1.25; $248.75.
2. $2.88; $572.12.
3. $6.25; $618.75.
4. $10.42; $1239.58.
5. $18.75; $2231.25.
6. $0.73; $174.27.
7. $11.69; $1263.31.
8. $21.54; $2328.46.
9. $4.13; $271.37.
10. $7.91; $519.59.
11. $1237.50.
12. $714.12.
13. $7.08.

Exercise 77. Page 128

1. $7.13; $249.12.
2. $5.82; $373.81.
3. $8.89; $424.61.
4. $7.46; $551.71.
5. $16.03; $679.22.
6. $14.95; $1266.30.
7. $4.92; $378.09.
8. $17.55; $838.65.
9. $12.25.
10. $12.35.
11. $10.31.
12. $6.19.
13. $6.25; $2493.75.
14. $1653.92.
15. $1660.79.
16. $1649.63.
17. $1625.25.
18. $6.75.
19. On the 4-months note; $0.30.

Exercise 78. Page 131

1. $971.10.
2. $348.42.
3. $718.35.
4. $1214.72.
5. $1656.42.
6. $2140.75.
7. $615.13.
8. $1480.92.
9. $2059.20.

Exercise 79. Page 132

1. $502.53.
2. $1800.
3. $502.51.
4. $1772.73.
5. $2062.31.
6. $2836.14.

Exercise 80. Page 133

1. $609.49.
2. $1917.86.
3. $1505.19.
4. $2343.20.
5. $146.25.
6. Proceeds of the $750 note; $13.25.

Exercise 82. Page 136

1. 3¢.
2. 5¢.
3. 8¢.
4. 8¢.
5. 10¢.
6. 12¢.
7. 12¢.
8. 15¢.
9. 15¢.
10. 18¢.
11. 25¢.
12. 30¢.
13. $249.75.
14. $5.85.
15. $3250.
16. 0.2%.

Exercise 83. Page 137

1. $2502.50.
2. $3753.75.
3. $4854.85.
4. $4809.60.
5. $5611.20.
6. $250.31.
7. $751.50.
8. $641.60.
9. $1283.20.

Exercise 84. Page 138

1. $1748.25.
2. $2145.70.
3. $3595.50.
4. 0.1%.
5. 0.2%.
6. $224.62.

Exercise 85. Page 139

1. $27,549.50.	**4.** $48,202.	**7.** $48,201.75.	**10.** $327,336.25.
2. $36,580.30.	**5.** $26,546.38.	**8.** $74,850.	**11.** $774,862.38.
3. $37,339.40.	**6.** $125,231.25.	**9.** $64,638.	**12.** $1,748,524.73.

Exercise 86. Page 140

1. $1250.	**4.** $4488.75.	**7.** Discount; $\frac{1}{20}$%.	**10.** $1.05.
2. $1753.50.	**5.** $2694.60.	**8.** $1.75.	**11.** $1319.02.
3. $2762.76.	**6.** $25,801.50.	**9.** $0.30.	

Exercise 87. Page 141

1. 1680d.	**5.** 540s.	**9.** £4 15s.	**13.** 7.50 fr.	**17.** 4.50 M.
2. 5760d.	**6.** 640s.	**10.** £3 12s.	**14.** 2.75 fr.	**18.** 2.75 M.
3. 1272d.	**7.** 10s.	**11.** £6 7s.	**15.** 6.80 fr.	**19.** 16.80 M.
4. 1708d.	**8.** 60s.	**12.** £17 2s.	**16.** 12.50 fr.	**20.** 92.75 M.

Exercise 88. Page 143

1. $34.23.	**8.** $16.59.	**15.** $571.20.	**22.** 200 fr.
2. $44.10.	**9.** $34.99.	**16.** $1046.15.	**23.** 675.21 M.
3. $58.44.	**10.** $61.84.	**17.** $649.90.	**24.** $237.15.
4. $121.50.	**11.** $67.31.	**18.** $789.28.	**25.** $5264.42.
5. $227.95.	**12.** $91.78.	**19.** $1118.52.	**26.** $636.
6. $383.76.	**13.** $111.55.	**20.** $1410.10.	**27.** Exchange at
7. $11.27.	**14.** $133.28.	**21.** £684 5s. 4d.	19.2; $0.31.

Exercise 90. Page 146

1. 178 yd. 9 in.	**3.** 142 ft. 1 in.	**5.** 66.
2. 109 ft. 2 in.	**4.** $237.60.	

Exercise 91. Page 147

1. 13,272 sq. ft.	**5.** 29 sq. ft.	**9.** 630.
2. 11,385 sq. in.	**6.** 777 sq. in.	**10.** 96¼.
3. 34,500 sq. yd.	**7.** 1512 sq. in.	**11.** 10,404.
4. 551 sq. ft.	**8.** 285 sq. ft.	**12.** 18 rd.

Exercise 92. Page 148

1. 25,123 sq. ft.
2. $1083\frac{3}{4}$ sq. ft.
3. $1064\frac{1}{4}$ sq. ft.
4. 111 sq. yd.
5. 68,250 sq. in.
6. 49,600 sq. rd.
7. $164\frac{1}{3}$ sq. ft.
8. 481 sq. ft.
9. $539\frac{1}{8}$ sq. yd.
10. 3423 sq. yd.
11. 27 ft.
12. 26 ft.
13. 32 ft.
14. 64 rd.

Exercise 93. Page 149

1. 1273 sq. ft.
2. $1487\frac{1}{2}$ sq. ft.
3. $1993\frac{3}{4}$ sq. ft.
4. 3115 sq. rd.
5. $1255\frac{1}{2}$ sq. in.
6. $2\frac{1}{4}$ sq. yd.
7. $22\frac{1}{6}$ sq. ft.
8. $27\frac{5}{8}$ sq. ft.
9. $68\frac{4}{9}$ sq. ft.
10. $28\frac{3}{4}$ sq. ft.
11. 8 ft.
12. 20 ft.
13. 22 ft.
14. 54 in.
15. $24,516.
16. 62.34 sq. in.
17. $352\frac{1}{2}$ sq. ft.

Exercise 94. Page 150

1. 63 sq. in.
2. 80 sq. in.
3. 90 sq. in.
4. $94\frac{1}{2}$ sq. in.
5. 76 sq. in.
6. $66\frac{1}{2}$ sq. in.
7. 286 sq. in.
8. 287 sq. in.
9. $560\frac{1}{2}$ sq. in.
10. 1025 sq. in.
11. 1218 sq. in.
12. 2068 sq. in.
13. 60 ft.
14. 100 ft.
15. 44,376 sq. ft.

Exercise 95. Page 151

1. 704 ft.
2. 68.2 ft.
3. 114.4 ft.
4. 7.04 ft.
5. 1.166 in.
6. 13,310 in.
7. 13,640 in.
8. 165 in.
9. 132 in.
10. 220 in.
11. 396 in.
12. 1760 in.
13. 400.4 in.
14. 404.8 in.
15. 514.8 ft.
16. 53.24 ft.
17. 78.54 ft.
18. 138.2304 ft.
19. 113.0976 yd.
20. 15.07968 yd.
21. 1.979208 ft.
22. 2.481864 ft.
23. 13.3518 ft.
24. 10.273032 in.
25. 175.9296 in.
26. 301.5936 in.
27. 420.9744 in.
28. 30.78768 ft.
29. 18.22128 yd.
30. 30.15936 yd.
31. 3.644256 ft.
32. 4.523904 ft.
33. 20 ft.
34. 110 in.
35. 160 in.

Exercise 96. Page 153

1. 28.2744 sq. in.
2. 50.2656 sq. in.
3. 63.6174 sq. in.
4. 38.4846 sq. in.
5. 176.715 sq. in.
6. 314.16 sq. in.
7. 1256.64 sq. in.
8. 2827.44 sq. in.
9. 3.1416 sq. in.
10. 50.2656 sq. in.
11. 28.2744 sq. in.
12. 78.54 sq. in.
13. 452.3904 sq. in.
14. 1963.5 sq. in.
15. 0.7854 sq. in.
16. $0.3490\frac{2}{3}$ sq. in.
17. 0.031416 sq. in.
18. 0.19635 sq. in.

19. 1.76715 sq. in.
20. 1.3962$\frac{2}{3}$ sq. in.
21. 38.4846 sq. in.
22. 86.59035 sq. in.
23. 58.088184 sq. in.
24. 15.708 in.
25. 12.5664 sq. in.
26. 17.2788 in.
27. 50.2656 in.
28. 10 in.
29. 47.124 ft.
30. 48.285 sq. ft.
31. 220 ft.; 38,013.36 sq. ft.
32. 14.1862875 sq. in.

Exercise 97. Page 154

1. 32,886 cu. in.
2. 30,784 cu. in.
3. 28,600 cu. in.
4. 30,537 cu. in.
5. 21,450 cu. ft.
6. 78,088 cu. ft.
7. 56,784 cu. ft.
8. 69,090 cu. ft.
9. 119,780 cu. ft.
10. 170 cu. ft.
11. 140$\frac{7}{8}$ cu. ft.
12. 237$\frac{1}{2}$ cu. ft.

Exercise 98. Page 155

1. 216 cu. in.
2. 247 cu. in.
3. 354 cu. in.
4. 58.32 cu. in.
5. 380.92 cu. in.
6. 768.6875 cu. in.
7. 866.264 cu. in.
8. 1118.43 cu. in.
9. 1275.844 cu. in.
10. 2114.541 cu. in.
11. 500.5 cu. in.
12. 602.07 cu. in.
13. 1717.98 cu. in.
14. 863.28 cu. in.
15. 451.98 cu. in.
16. 803.58 cu. in.
17. 2620.56 cu. in.
18. 2093.34 cu. in.
19. 963.33 cu. in.
20. 1326.78 cu. in.

Exercise 99. Page 156

1. 96 cu. in.
2. 208 cu. in.
3. 320 cu. in.
4. 704 cu. in.
5. 1144 cu. in.
6. 6$\frac{7}{8}$ cu. in.
7. 11$\frac{21}{64}$ cu. in.
8. 8.99 cu. in.
9. 19.27 cu. in.
10. 42.525 cu. in.
11. 93,537,284 cu. ft.

Exercise 100. Page 158

1. 851 cu. in.
2. 1242 cu. in.
3. 8400 cu. in.
4. 522$\frac{1}{2}$ cu. in.
5. 433$\frac{1}{2}$ cu. in.
6. 73,656 cu. in.
7. 2 ft. 4$\frac{1}{2}$ in.
8. 6 ft. 3 in.
9. 18 sq. ft.
10. 19 sq. ft.
11. 13 sq. ft.
12. 15 sq. ft. 36 sq. in.
13. 1357.1712 cu. in.
14. 2827.44 cu. in.
15. 6433.9968 cu. in.
16. 9160.9056 cu. in.
17. 376.992 cu. in.
18. 10.472 cu. ft.
19. 2120.58.
20. 15,904.35.
21. 21,111.552.
22. 288,634.5.
23. 775,932.003.
24. 636,960.933984375.
25. 424.116.
26. 318.087.
27. 753.984.
28. 432.95175.
29. 434.8845703125.
30. 724.8076171875.

Exercise 101. Page 159

1. 8 cu. in.
2. 35 cu. in.
3. 60 cu. in.
4. 170 cu. in.
5. 476 cu. in.
6. $5\frac{5}{8}$ cu. in.
7. $12\frac{11}{32}$ cu. in.
8. $28\frac{7}{32}$ cu. in.
9. 63.56 cu. in.
10. 146.575 cu. in.
11. 75.3984 cu. in.

Exercise 102. Page 161

1. 616 sq. in.
2. 3850 sq. in.
3. 5544 sq. in.
4. 13.86 sq. in.
5. 24.64 sq. in.
6. 75.46 sq. in.
7. $2\frac{13}{32}$ sq. in.
8. 1.54 sq. in.
9. 0.9856 sq. in.
10. 157,696 sq. in.
11. 432,586 sq. in.
12. 591,976 sq. in.
13. 153.9384 sq. in.
14. 113.0976 sq. in.
15. 254.4696 sq. in.
16. 50.2656 sq. in.
17. 176.715 sq. in.
18. 122.71875 sq. in.
19. 15.205344 sq. in.
20. 38.4846 sq. in.
21. 12.5664 sq. in.
22. 314.16 sq. in.
23. 3.1416 sq. in.
24. 113.0976 sq. ft.
25. 615.7536 sq. in.
26. 1017.8784 sq. in.
27. 78.54 sq. in.
28. 172.034016 sq. in.
29. $201{,}142{,}857\frac{1}{7}$ sq. mi.
30. $28\frac{2}{7}$.
31. $2{,}359{,}727{,}071{,}428\frac{4}{7}$ sq. mi.
32. $6084\frac{4}{7}$.
33. 7 in.

Exercise 103. Page 162

1. 33.5104 cu. in.
2. 523.6 cu. in.
3. 1436.7584 cu. in.
4. 14,137.2 cu. in.
5. 50,965.1296 cu. in.
6. 310,339.8144 cu. in.
7. 57.9059712 cu. ft.
8. 288.6962848 cu. ft.
9. 179.5948 cu. ft.
10. 65.45 cu. in.
11. 220.89375 cu. in.
12. 425.7062 cu. in.
13. 27.5132 oz.

Exercise 104. Page 163

1. 2513.28 sq. ft.
2. 113.0976 cu. in.; 904.7808 cu. in.
3. 5,276,681,625.6 cu. mi.
4. 20.0165.
5. 243.9168.
6. $31\frac{23}{32}$ lb.
7. 9.4248.
8. The same.
9. 0.5236 cu. ft.; 0.7854 cu. ft.; 0.2618 cu. ft.; twice; three times.

Exercise 106. Page 165

1. $160.06.
2. $168.47.
3. $463.73.
4. $182.55.
5. $600.51.
6. $468.28; $0.45.
7. $159.87; $0.15.
8. $361.78; $0.35.

Exercise 107. Page 167

1. \$303.14.
2. \$38.34.
3. $\frac{43}{64}$ in.
4. \$67,031.25.
5. \$4.
6. The second; $3\frac{18}{29}$%.
7. \$929.19.
8. $5\frac{61}{63}$ in.
9. 520 lb.
10. $62{,}304\frac{11}{16}$ lb.
11. \$4837.50.
12. 922 lb. 4 oz.
13. 27.713 in.
14. 7875 lb.
15. \$89.23.
16. \$76.34.
17. \$82.02.
18. \$73.37.
19. \$72.06.
20. \$65.61.

Exercise 108. Page 170

1. 103.
2. \$46.30.
3. \$5180.70.
4. \$85.51.

Exercise 109. Page 171

1. \$635.60.
2. \$73.01.
3. 70.88%.
4. \$80.
5. $7\frac{1}{2}$ ft.

Exercise 110. Page 172

1. 77.98 lb.
2. 481.0575 lb.
3. 908.855 lb.
4. 2126.4705 sq. in.
5. 54,286.848 lb.
6. 1053 lb.
7. 938.850 lb.
8. 144.31725.
9. 1496.656 lb.

Exercise 111. Page 173

1. $2227\frac{1}{2}$.
2. \$0.83.
3. 40¢.
4. 25%; 20%.
5. 64,800.
6. $5\frac{5}{9}$ ft.
7. 51,397.5.
8. 2420.

Exercise 112. Page 174

1. 5 ft.
2. 14.32 ft.
3. 7.3304 ft.
4. 73.304 sq. ft.
5. 245.4375 sq. ft.
6. 1143.04.

Exercise 113. Page 175

1. 23.562 ft.
2. 5236 ft.
3. 1884.96 ft.
4. 1200.
5. 14.927 in.
6. 36.319 in.
7. 68,706.792 lb.
8. 1847.2608 sq. ft.
9. 17.
10. 750.

Exercise 114. Page 176

1. 110.16 lb.
2. 4.54 lb.; 8.071 lb.
3. $8\frac{23}{40}$ H. P.
4. $112\frac{1}{2}$ lb.
5. $121\frac{1}{2}$ lb.
6. 36.

Exercise 115. Page 177

1. 16%.
2. \$9.
3. Ensilage, 56%; clover hay, 24%; bran, 13%; corn meal, 7%.
4. Resin, $37\frac{1}{2}$%; beeswax, $37\frac{1}{2}$%; tallow, $7\frac{7}{24}$%.
5. Land, $33\frac{2}{5}$%; labor, $45\frac{3}{5}$%; fertilizers, $18\frac{6}{25}$%; seed, $2\frac{37}{50}$%.
6. 198.8514.
7. \$14.
8. 14,520; $544\frac{1}{2}$ T.
9. \$286.

Exercise 116. Page 178

1. 500.
2. $714\frac{6}{21}$ lb.
3. 2080 gal.
4. 1.032.
5. 251.748 lb.
6. 3400; \$102.
7. 1724 sq. ft.
8. 3302.4.
9. \$1684.80.
10. 4232.8125 lb.

Exercise 117. Page 179

1. 1102.5.
2. \$346.08.
3. 16,800.
4. The second; 96¢.
5. \$19.45.
6. 185.625.
7. 36.

Exercise 118. Page 180

1. $138\frac{3}{5}$.
2. 3784.
3. 90.
4. 384.
5. \$190.08.
6. 13 ft.
7. 625.
8. $1131\frac{3}{7}$.
9. $125\frac{5}{7}$.

Exercise 119. Page 181

1. \$6.75.
2. 3680.
3. 150.
4. 81¢.
5. 1000.
6. $1\frac{1}{2}$.
7. \$2.
8. 10.
9. \$1.56.
10. 1200; 5.
11. 30.

Exercise 120. Page 182

1. 22.68 lb.
2. 224.64 lb.
3. 5 ft.
4. $1\frac{1}{8}$ in.
5. 633.49 lb.
6. 19.32 lb.; 24.9%.
7. 24.
8. \$2.69.
9. 965.92 lb.

Exercise 121. Page 183

1. \$1850.
2. \$1850.
3. 90%.
4. \$800.
5. \$535.50.
6. \$28.30.
7. \$1650.
8. 20%.
9. The former; \$40.

Exercise 122. Page 184

1. $2\frac{1}{12}$; $1333\frac{1}{3}$.
2. 3520.
3. 6.
4. 225 cans.
5. 150; 30; 5.
6. $1\frac{1}{8}$%.
7. \$11,040.
8. \$3.

Exercise 123. Page 185

1. 33,958.
2. 24,742.
3. 23,612.
4. 22,148.
5. 27,544.
6. 6488.
7. 6338.
8. 5239.
9. 1873.
10. 1528.
11. 30,605,967.
12. 32,425,336.
13. 9,157,532.
14. 62,929,457.
15. 72,714,908.
16. $396\frac{90}{233}$.
17. $549\frac{15}{225}$.
18. 250.

Exercise 124. Page 186

1. $2\frac{3}{14}$.
2. 50.
3. $\frac{1}{36}$.
4. 1.
5. $\frac{3}{8}$; 0.375.
6. 1.
7. $1\frac{3}{4}$.
8. $1\frac{274}{315}$.
9. $\frac{32}{125}$.
10. $26\frac{121}{224}$.
11. $\frac{187}{320}$.
12. $\frac{1}{8}$.

Exercise 125. Page 187

1. 3.
2. $\frac{1}{8}$.
3. 2.
4. 23.473.
5. 29.499.
6. 14.409.
7. 15.3717.
8. 4.8089.
9. 3.0319.
10. 12.7275.
11. 5.1752.
12. 16.44501936.
13. 655.746168.
14. 1235.914848.
15. 0.0015554726.
16. 17.
17. 2.1.
18. 18.
19. 0.132.
20. 1.089.
21. 8.413.
22. 2.838.
23. 73.611.
24. 1.971.
25. 125.714.

Exercise 126. Page 188

1. 278 ft. 4 in.
2. 246 yd. 22 in.
3. 171 lb.
4. 8 ft. 5 in.
5. 58 yd. 24 in.
6. 17 lb. 8 oz.
7. 921 ft. 8 in.
8. 3240 yd. 20 in.
9. 3257 lb. 13 oz.
10. 27 yd. 2 ft. 3 in.
11. 16 ft. $5\frac{2}{7}$ in.
12. 17 lb. 7 oz.
13. 37 yd. 2 in.
14. 8.
15. 7.
16. 445,605 in.
17. 6736 in.; $561.33\frac{1}{3}$ ft.; 34.0202 rd.
18. 26 mo. 4 da.
19. 3 yr. 9 mo. 12 da.

Exercise 127. Page 189

1. $19.50.
2. $143.40.
3. $37.50.
4. $54.79.
5. $17.10.
6. $276.75.
7. $148.

Exercise 128. Page 190

1. 127.279 ft.
2. $20.48.
3. $66.
4. $36.90.
5. $227\frac{25}{64}$.
6. $11.20.
7. 18.
8. $14.04.

Exercise 129. Page 191

1. 44.88 lb.	3. 80%.	5. 290,400.	7. $3050\frac{2}{3}$ sq. yd.
2. 19,360.	4. $3\frac{88}{55}$ mi.	6. 787.9872.	8. $2\frac{3}{4}$ in.

Exercise 130. Page 192

1. $461.50.	4. $12.	7. $208.	10. $73.10.
2. 40%, gain.	5. 20%; $62\frac{1}{2}$¢.	8. 50%.	11. 10%.
3. $16\frac{2}{3}$%.	6. $78\frac{4}{7}$%.	9. $10; $33\frac{1}{3}$%.	12. 10%.

Exercise 131. Page 193

1. $247.	3. 3%.	5. $186.30.	7. $12,500.	9. $935.
2. 15%.	4. $4.75.	6. $12\frac{1}{2}$%.	8. $116.37.	10. $3162; $15.81.

Exercise 132. Page 194

1. $2800.	3. $16.50.	5. $4600.	7. $1072.50.	9. $825.
2. $20.10.	4. $14,000.	6. $1\frac{1}{4}$%.	8. $7500.	10. $7500; $6.90.
				11. $8250.

Exercise 133. Page 195

1. $7.50.	4. $43.40.	7. $525.
2. $2,000,000.	5. $382.50.	8. $107.25.
3. 15 mills; $102.60.	6. $20.25.	9. 6 mills.

Exercise 134. Page 196

1. $733.59.	5. 46.199%.	9. The single discount; $4.17.
2. $559.55.	6. $650.	10. The discount series; $0.30.
3. $361.42.	7. $357.20; 24%.	11. The same.
4. $10.25.	8. 58.96%.	

Exercise 135. Page 197

1. $78.20.	5. 3 yr. 4 mo. 24 da.	9. $4.20.
2. $61.81.	6. $44.92.	10. $577.50.
3. $62.43.	7. $740.50.	11. $416.15.
4. $33.26.	8. $381.56; Dec. 30.	12. $15.70; $584.30.

Exercise 136. Page 198

1. \$4711.50.
2. \$12,180.
3. $3\frac{3}{5}\%$.
4. 42.
5. $4\frac{1}{2}\%$.
6. \$4.75.
7. \$23,647.50.
8. \$19.50; \$15,034.50.
9. $\frac{8}{11}\%$.
10. 20; \$100.
11. \$100.47, loss.

Exercise 137. Page 199

1. $\frac{5}{8}$.
2. $\frac{5}{12} : \frac{5}{21} = 3\frac{1}{2} : 2$.
3. \$26.67; \$33.33.
4. 16.
5. $1\frac{4}{227}$.
6. 48.
7. 3686.4.
8. $11\frac{2}{3}$ hr.
9. \$25.06; \$54.94.

Exercise 138. Page 200

1. $198\frac{88}{99}$.
2. 9600 kg.
3. 6300.
4. 0.75 m.
5. $1\frac{5}{9}$ m.
6. \$19.20.
7. 1.4 m.
8. 500 m.
9. 236.25 kg.
10. 2870.
11. 149.76 kg.
12. 43,259.832.

Exercise 139. Page 201

1. 346.
2. 413.
3. 2.449.
4. 5.477.
5. 1074.8.
6. 10.61 ft.
7. 21.21 ft.
8. 6.08 ft.
9. 126.
10. 6 in.
11. 7 ft.
12. 35 ft.
13. 14 ft.
14. 35 ft.

Exercise 140. Page 202

1. $21\frac{3}{7}\%$.
2. \$49,280.
3. A, \$72; B, \$81.
4. 2827.44.
5. $23\frac{1}{5}$.
6. \$201.60.
7. 234.68 in.
8. 5 hr.
9. \$4500; \$1200.
10. 15 mi.
11. $1,974,857\frac{1}{7}$.
12. 14.14 ft.
13. 0.7 in.
14. $3764\frac{4}{9}$.
15. \$237.50.
16. 1960.
17. 1320.
18. 71.00016 in.; $71.028571\frac{1}{7}$ in.; 71 in.
19. \$105.12.
20. Sand, 108; soda, 21; lime, 18; alumina, 3.
21. Copper, 962.5; tin, 262.5; zinc, 25.
22. 484.
23. 29.5305903 da.
24. $34\frac{11}{16}$.
25. 468 lb.
26. 11.2.
27. \$145.80.
28. \$3750.
29. 105.
30. \$13.95.
31. $4\frac{1}{2}$ hr.
32. \$46.80.
33. 38.1831.
34. $4\frac{1}{2}\%$; $5\frac{1}{2}\%$.
35. $\frac{28}{183}$.
36. $66\frac{2}{7}°$.
37. $29.79\frac{5}{7}$ in.
38. 10 lb.; 50 lb.

39. 201.68.
40. 60 lb.
41. \$723.81.
42. 28.97 ft.
43. 5.67 bbl.; 3.54 cu. yd.
44. 1350; 3375.
45. 2.
46. 4.
47. $4\frac{4}{9}$.
48. $4\frac{32}{37}$.
49. 96.
50. $1\frac{23}{37}$.
51. 7.35 mi.
52. 96.
53. 7 hr.
54. \$1800; \$2100; \$900.
55. $157\frac{1}{2}$.
56. Pour in 8 pt. once, and 3 pt. twice.
57. \$40.
58. 256 lb.
59. $7\frac{19}{33}$.
60. 36 sq. in.; 72 sq. in.
61. 113.0976 sq. in.
62. 3963.296; 3949.791; 13.505.
63. \$420.
64. 52.95 ft.
65. 0.
66. 32 min. $48\frac{3}{4}$ sec.
67. 17,000.
68. $42\frac{1}{2}$ mi.
69. $51\frac{1}{5}$.
70. 4.
71. 8312.6736.
72. 2 hr. 9 min.
73. 700 gal.
74. 27 min.
75. 300.
76. 360 ft.
77. 48,906.
78. $60,451,437\frac{1}{2}$.
79. Rent, \$262.50; groceries, \$437.50; meats, \$210; clothing, \$227.50; saves $28\frac{4}{7}\%$.
80. 21.
81. 6336.
82. 0.6336; 1.5783.
83. \$1080; \$4410.
84. 4107.
85. 11,827,200.
86. 3,652,650.
87. 5250.
88. $29.46\frac{3}{7}$.
89. 3.9 in.
90. 1320.
91. $61\frac{7}{8}$ A.
92. \$15.
93. Sand, 468; soda, 91; lime, 78; alumina, 13.
94. Copper, 673.75; zinc, 17.5; tin, 183.75.
95. 588.
96. 234.
97. $218\frac{2}{11}$.
98. 68.6.
99. 11; 7; 3.
100. 160 lb.
101. 2000.
102. $4615\frac{5}{13}$.
103. 0.5000; 0.2500; 0.7500; 0.1250; 0.3750; 0.6250; 0.0625; 0.1875; 0.3125; 0.4375; 0.5625; 0.6875; 0.8125; 0.9375.
104. $42\frac{5}{8}$.
105. \$72.07; $277\frac{1}{5}$ lb.
106. 7.937 ft.
107. 1.9736 mi.

Exercise 141. Page 213

1. 15.
2. $17\frac{1}{2}$.
3. 39.
4. 69.
5. 147.96.
6. 415.54.
7. $3300\frac{15}{16}$.
8. $7205\frac{5}{8}$.
9. 40,474.4375.
10. 99,373.1875.

Exercise 142. Page 214

1. 24.48.
2. 41.83.
3. 3500.35.
4. 335.5625.
5. 525.
6. 342.
7. 756.
8. 56.525.
9. 404.79.
10. 407.925.
11. lw; 7750.
12. $\frac{1}{2}fh$.

Exercise 143. Page 215

1. 15.	**3.** 57.	**5.** 126.	**7.** 456.	**9.** 6.225.	**11.** $37\frac{1}{2}$ sq. in.
2. 49.	**4.** 70.	**6.** 109.8.	**8.** 162.54.	**10.** 10.	**12.** $\frac{1}{2}(x+y)z$; 672.

Exercise 144. Page 216

1. 44.	**6.** 24.2.	**11.** 57.2.	**16.** 78.54.	**21.** 10.79925 in.
2. 66.	**7.** 44.	**12.** 66.	**17.** 188.496.	**22.** 75.3984.
3. 88.	**8.** 22.	**13.** 31.416.	**18.** 251.328.	**23.** 43.9824 in.
4. 11.	**9.** 35.2.	**14.** 62.832.	**19.** 15.708.	
5. 15.4.	**10.** 39.6.	**15.** 157.08.	**20.** 47.124.	

Exercise 145. Page 217

1. 78.54.	**5.** 1963.5.	**9.** 616.	**13.** 75.46.	**17.** 1256.64.
2. 314.16.	**6.** 7854.	**10.** 13.86.	**14.** 186.34.	**18.** 7854.
3. 1256.64.	**7.** 154.	**11.** 24.64.	**15.** 78.54.	**19.** 12.5664.
4. 19.635.	**8.** $38\frac{1}{2}$.	**12.** 3850.	**16.** 314.16.	**20.** 706.86; $707\frac{1}{7}$; $0.28\frac{2}{7}$.

Exercise 146. Page 218

1. $l = ch$.	**19.** 11,571.84.	**34.** 30,000 lb.; 37.7 T.
2. $l = \pi dh$.	**20.** 11,401.56.	**35.** $0.219912\,d^2l$ lb.; 1.54 lb.
3. $l = 2\,\pi rh$.	**21.** 594.	**36.** 91.183125 lb.
4. $v = bh$.	**22.** 157.08.	**37.** $159{,}107\frac{1}{7}$ lb.
5. $v = \frac{1}{3}\,bh$.	**23.** 264.	**38.** 22.
6. $v = \frac{1}{3}\,\pi r^2h$.	**24.** 110.	**39.** 1518 lb.
11. 5.	**25.** 1256.64	**40.** 105,600 cu. ft.
12. 40.	**26.** 113.0976.	**41.** $v = 12\,le^2$; $v = \frac{1}{144}\,le^2$; 77.76 lb.
13. 45.	**27.** 40 mi.; $d = tv$.	**42.** 80 lb.
14. 110; 60.	**28.** 432 cu. in.; $v = lwd$.	**43.** 3; $\frac{a}{b}$.
15. 70; 14.	**31.** 7 in.	
16. 114.8.	**32.** 7 ft.	
17. 339.68.	**33.** $80\frac{5}{8}$ sq. ft.	
18. 55.		

Exercise 149. Page 228

8. \$450.	**9.** 33.	**10.** 3 lb.; -3 lb.

Exercise 150. Page 230

1. \$55.	**3.** 57.	**5.** 3.	**7.** 26.	**9.** 122.1.
2. 14°.	**4.** 13.	**6.** 17.	**8.** 22.	**10.** 64.

Exercise 151. Page 231

1. – 900 lb.	**3.** 20 mi.; 0 mi.; – 20 mi.	**5.** \$2425.
2. – 2°.	**4.** 500 – 18; 500 – 36; 500 – 9.	

Exercise 152. Page 232

1. 4.	**3.** 25.	**5.** 54.	**7.** 72.	**9.** 51.5.
2. 6.	**4.** 58.	**6.** 25.	**8.** 68.	**10.** $64\frac{1}{4}$.

Exercise 153. Page 233

1. 10.	**6.** 13.	**11.** 0.7.	**16.** 11.	**21.** 14.	**26.** 3.
2. 24.	**7.** 17.	**12.** 19.	**17.** 33.	**22.** 20.	**27.** 100.
3. 19.	**8.** 11.	**13.** 7.	**18.** 30.	**23.** 12.	**28.** 2.
4. 16.	**9.** 21.	**14.** 9.	**19.** 25.	**24.** 11.	**29.** 4.
5. 13.	**10.** 9.	**15.** 6.	**20.** 55.	**25.** 10.	**30.** 3.

Exercise 154. Page 234

1. 40.	**4.** 16.	**7.** 25.	**10.** 132.	**13.** 162.	**16.** 35.
2. 10.	**5.** 13.	**8.** 6.	**11.** 594.	**14.** 112.	
3. 15.	**6.** 13.	**9.** 135.	**12.** 16.8.	**15.** 81.	

Exercise 155. Page 236

13. 19.	**17.** 33.	**21.** 551.	**25.** 11.	**29.** 84 in.	**33.** 175 lb.
14. 82.	**18.** $16\frac{2}{3}$.	**22.** 30.	**26.** 18.	**30.** $9\frac{1}{2}$ in.	**34.** $158\frac{4}{5}$ lb.
15. $42\frac{7}{8}$.	**19.** $17\frac{1}{3}$.	**23.** 15.	**27.** 4.	**31.** 3350 lb.	**35.** 150 lb.
16. 85.	**20.** 8.	**24.** 21 ft.	**28.** 7.	**32.** 1125 lb.	

Exercise 156. Page 238

1. 35.	**6.** 7.	**11.** 12.	**16.** 3.	**21.** 13.
2. 17.	**7.** 8.	**12.** 7.	**17.** 10.	**22.** 7.
3. 19.	**8.** 5.	**13.** 12.	**18.** 11.	**23.** 57.
4. 21.	**9.** 5.	**14.** 10.	**19.** 51.	**24.** 57.
5. 11.	**10.** 10.	**15.** 21.	**20.** 100.	**25.** 18.

Exercise 157. Page 239

1. 8 ft. 5 in.
2. $8f + 5i$.
3. $8 \cdot 6 + 5 \cdot 4$.
4. $16x + 11y$.
5. $7a + 5b + 10c$.
6. $6a + 5b + 6c$.
7. $10a + 10b - 8c$.
8. $17a + 17b + 17c$.
9. $2x + 2y$.
10. $5x + 10y + 13z$.
11. $12x + 5y + 2z$.
12. $7x + 7y - z$.

Exercise 158. Page 240

1. 15.
2. 12.
3. 12.
4. 13.
5. 11.
6. 11.
7. 13.
8. 9.
9. 18.
10. 21.
11. 12.
12. 24.
13. 30.
14. 22.
15. 110.
16. 132.
17. 88.
18. 130.
19. $36\frac{3}{8}$.
20. 33.
21. 29.
22. 40 lb.
23. 40 lb.
24. 7.
25. 7 gal.

Exercise 159. Page 241

1. 160.
2. 160.
3. 150.
4. 168.
5. 165.
6. 50 lb.
7. $125.
8. $250.
9. 120.
10. $2250.
11. 11.
12. 19.
13. 17.
14. 480.
15. 666.
16. 11.
17. 1500.
18. 4840.
19. 640.
20. 123.
21. $240.
22. $240.
23. $2000.
24. 120 lb.
25. 84 revolutions.
26. 350.
27. $750.
28. 4400.
29. 140.
30. $4800.

Exercise 160. Page 243

1. 7 ft. 4 in.
2. $7x + 4y$.
3. $8x + 8y$.
4. $15 \cdot 2 - 2 \cdot 3$.
5. $2ax$.
6. $10ax$.
7. $-axy$.
8. $-8abc$.
9. $x + 5y$.
10. $-x + 9y$.
11. $3xy + 6z$.
12. $-3xy + 4z$.

Exercise 161. Page 244

1. 9.
2. 59.
3. 119.
4. 23.
5. 22.
6. 21.
7. 21.
8. 42.
9. 12.
10. 12.
11. $3\frac{1}{3}$.
12. 42.
13. 32.
14. 36.
15. 24.
16. 40.
17. -56.
18. -48.
19. 21.
20. 9.1.
21. 107.
22. 23.
23. 37.
24. 31 bu.
25. 67 gal.

Exercise 162. Page 245

1. 200. 2. 480. 3. 240. 4. 160 yd. 5. $4800. 6. 32. 7. $550. 8. $1100. 9. 12,700.

Exercise 163. Page 246

1. $15a + 10b$.
2. $30a - 18b$.
3. $189x + 105y$.
4. $208p + 96q$.
5. $444m + 180n$.
6. $104ax + 74ay$.
7. $588x - 672y$.
8. $1357ax + 1081ay$.
9. $2356ax - 1147ay$.
10. $851mx - 2208my$.
11. $3x + 3y + 3z$.
12. $4ax - 8ay + 4az$.
13. $45abx - 60aby - 75abz$.
14. $axy + ayz + a$.
15. $abx + cdx + efx$.

Exercise 164. Page 248

25. $\frac{b}{xy}$. 26. $\frac{c}{x}$. 27. r^2. 28. a. 29. $\frac{x^2}{a}$. 30. $\frac{x}{a}$. 31. $x + y$. 32. $a(x + y + z)$.

Exercise 165. Page 249

1. a^4.
2. a^4x.
3. $6a^5x$.
4. $10\pi r^3$.
5. $324a^3b^2c$.
6. $ab + b^2$.
7. $a^4 + a^2b^2$.
8. $2a^3 - 2ab^2$.
9. $4a^4 - 12a^2b^2$.
10. $7a^5 - 21a^2b^3$.
11. $a^2 + ab + ac$.
12. $a^4 + 2a^3 + a^2$.
13. $x^3y - 2x^2y^2 + xy^3$.
14. $6x^3y - 8x^2y^2 - 14xy^3$.
15. $63x^3y - 45x^2y^2 - 27xy^3$.

Exercise 166. Page 250

2. $\frac{3a}{5}$.
4. $3a$.
5. $9a$.
6. $\frac{a^2}{b}$.
7. a.
8. a^2.
9. 1.
10. $\frac{2a}{b}$.
11. $\frac{a}{2}$.
12. a.
13. $3ab$.
14. 6.
15. $\frac{3a^3}{b^2}$.
16. $\frac{3a^3}{b}$.
17. a.
18. $\frac{a}{b}$.
19. $a + b$.
20. $a + b$.
21. $2a + 2b$.
22. $\frac{ac + bc}{2b}$.
23. $\frac{a + b}{2b}$.
24. $a - b$.
25. $\frac{4a^2 + 2ab}{3}$.
26. $\frac{12a^2 - 4ab}{c}$.
27. $s^2 + h^2$.

Exercise 167. Page 251

1. $60.
2. 2 yr.
3. 5%.
4. $450.
5. $750.
6. 4 yr.; 8 yr.
7. 7 yr.; 14 yr.; $9\frac{1}{3}$ yr.
8. $p = \frac{a}{1 + tr}$.
9. $r = \frac{c}{2\pi}$.
10. 7.5 in.
11. $r^2 = \frac{a}{\pi}$; $r = \sqrt{\frac{a}{\pi}}$.
12. 7 in.

13. $r^2 = \frac{s}{4\pi}$; $r = \frac{1}{2}\sqrt{\frac{s}{\pi}}$.

14. $r^3 = \frac{3v}{4\pi}$; $r = \sqrt[3]{\frac{3v}{4\pi}}$.

15. $h = \frac{v}{\pi r^2}$.

16. 2 in.

17. 7 in.

18. $h = \frac{3v}{\pi r^2}$.

19. $r^2 = \frac{3v}{h\pi}$; $r = \sqrt{\frac{3v}{h\pi}}$.

20. $\frac{1}{4}$.

21. $2\pi r^3$.

22. $6\frac{2}{7}$ cu. in.

23. $905\frac{1}{7}$ cu. in.

24. 40 mi.; $\frac{d}{t}$ mi.

25. 40; 49; $44\frac{2}{3}$.

26. $d = rt$; 156.

27. $t = \frac{d}{r}$.

28. $2\frac{1}{2}$.

29. $p = \frac{wb}{a}$.

30. 35.

31. 120.

32. $b = \frac{2a}{h}$; $h = \frac{2a}{b}$.

33. $h = \frac{2a}{b + b'}$.

34. 4.

35. 10.

36. 6.

37. $4\frac{1}{6}$.

38. 17.

39. 8.

40. 1.73.

41. $\frac{4a}{\sqrt{3}} = s^2$.

42. $l = \frac{v}{wh}$.

43. $r = \frac{p}{2 + \pi}$.

44. $a^2 = h^2 - b^2$.

45. $r = \frac{1}{2}\left(\frac{d - 2a}{\pi}\right)$.

46. $914\frac{2}{7}$.

47. 400.

Exercise 168. Page 255

2. $9x + 3y$.
3. $9x + 3y$.
4. $9y + 3z$.
5. $9x - 3$.
6. $9 - 3y$.
7. $9x^2 - 3$.
8. $9xyz - 3z^2$.
9. $5a - 9b$.
10. $5a - 9b$.
11. $a + 5$.
12. $3b - 2a$.
13. $28 - 9y$.
14. $16xy - 7$.
15. $12y^2 - 19$.
16. $37a - 3b$.
17. $2 + 3y + z$.
18. $3 - 4y - z$.
19. $5x - 7y + z$.
20. $7 + 8x + 9x^2$.
21. $x + 2x^2 + 3x^3$.
22. $2x + 3y - 4$.
23. $x^2 + 2x - 1$.
24. $2x^3 - x^2 + 1$.

Exercise 169. Page 256

1. $a(x - y)$.
2. $\pi(a - b)$.
3. $2\pi(a + b)$.
4. $\pi(d - d')$.
5. $\pi r^2(a - b)$.
6. $ax(y + z)$.
7. $ax(xy - z)$.
8. $\pi r^2(x - y)$.
9. $mn(x^2 + y^2)$.
10. $2axy(x - 2ay)$.
11. $3ax^2(y + 3z)$.
12. $7abc(abc + 2)$.
13. $5ab(5a + 7b)$.
14. $25xyz(5xyz + 3)$.
15. $3ac(37bd + 33ac)$.
16. $p(1 + rt)$.
17. $p(1 - rt)$.
18. $\pi r^2(h + k)$.
19. $\pi(ma^2 + nb^2)$.
20. $\frac{1}{2}b(h - k)$.
21. $abc(1 + 3abc)$.
22. $3ab(a + b)$.
23. $3ab(a - b)$.
24. $a(a^2 + 3ab + 3b^2)$.
25. $a(a^2 - 3ab + 3b^2)$.
26. $x(x^2 + 3xy - 3y^2)$.
27. $ab(a^2b^2 + ab + 1)$.
28. $a^2b^2c^2(abc + 1)$.
29. $9a^2b^2(3ab + 1)$.
30. $3xy(41xy + 107)$.

Exercise 170. Page 258

1. 16.
2. 9.
3. 13.
4. 19.
5. 12.
6. 3.
7. 7.
8. 9.
9. 5.
10. 10.
11. 57.
12. 79.
13. 16.
14. 37.
15. 12.
16. 21.
17. 11.
18. 41.
19. 1.
20. 2.
21. $\frac{i}{rt}$.
22. $p(1+rt)$.
23. $\frac{a}{1+rt}$; $200.
24. $\frac{a-p}{rp}$.
25. 3 yr.
26. $\frac{a-p}{tp}$.
27. 6%.
28. $7500.
29. $7500.
30. $1750.
31. $1250.
32. $1750.
33. 90 yd.
34. 1275.
35. 17,440 lb.
36. 50 T.
37. 1200 lb.
38. $13\frac{8}{9}$¢.
39. 40 lb.
40. 30.
41. 1032 lb.
42. 200.
43. 18 ft.
44. 8 ft.

Exercise 171. Page 261

1. 48.
2. 150.
3. 42.
4. 40.
5. 56.
6. 55.
7. 42.
8. 36.
9. 320.
10. 20.
11. 50; 40.
12. 60; 40.
13. 5.
14. 28; 29; 30.
15. 12 ft.; 48 ft.
16. 14 ft.; 70 ft.
17. $12; $12; $6; $6.
18. 68.
19. $7.
20. The first, 17; the other, 8.

Exercise 172. Page 264

1. 54; 34.
2. 28 yr.
3. 10 rd.; 29 rd.
4. 10 doz. at 40¢; 15 doz. at 35¢.
5. Boy, 9 yr.; girl, 3 yr.
6. 15 yr.; 6 yr.
7. 12; 3.
8. Father, 39 yr.; son, 11 yr.
9. 16 girls; 13 boys.
10. 1957; 1521.
11. $60; $30.

Exercise 176. Page 271

6. $287.50.
7. $250.
8. $6375.
9. $3,211,200.
10. $37,610.
11. $43,077.50.
12. $3600.
13. $16,000.
14. $8125.
15. $16,875.
16. $637.50.
17. $1881.25.
18. $80,000.
19. $61,000.
20. $80,000.
21. $800,000.
22. $350,000.
23. 1200.
24. 1600.
25. 4800.
26. $10,410.
27. 210,500.
28. $10,605.
29. 238,324.
30. $1,053,250.
31. $10,560,000.
32. $105,600.
33. 80,000,000.
34. $30.
35. $2900.
36. $2700.
37. $62.50.
38. $310.
39. $4.38.
40. $12.
41. $1200.

42. $71,320.
43. $900; $1200.
44. 300 lb.; 600 lb.
45. $7687.50; $10,250.
46. $4200; $5600; $2100.
47. 1800 lb.
48. $400; $200.
49. $1600; $1733.33.
50. $600; $800; $866.67.
51. 2160; 2160; 2160.
52. 5760; 5760; 5760.

Exercise 177. Page 274

1. 51,300.
2. 61,200.
3. 70,200.
4. $111.
5. $240.
6. 1,846,200.
7. $186.
8. $324.
9. 1827.
10. 178.7.
11. 134.
12. 1.34.
13. 0.134.
14. $176.
15. $43.
16. $97.
17. 5163.
18. 15,900.
19. 66,600.
20. $276.
21. $246.
22. $355.
23. $460.
24. $721.
25. $518.
26. $46.
27. $112.
28. $99.
29. $248.
30. $328.
31. $93.
32. $29.
33. 651.
34. 224,400.
35. 192,400.
36. 19,240.
37. 1924.
38. 3216.
39. 669.
40. 3976.
41. 147.6.
42. 12.4.
43. 12.4.
44. 1.24.
45. 180.
46. 18.
47. 1.8.
48. 0.02.
49. $3600.
50. $4125.
51. $5400.
52. $11,000.
53. $72.
54. $171.88.
55. $363.
56. $273.
57. $234.

Exercise 178. Page 275

1. $6521.04.
2. $7883.37.
3. $7512.67.
4. $5432.02.
5. $12,743.28.
6. $42,218.55.
7. $32,092.06.
8. $45,284.82.
9. $43,706.25.
10. $62,734.28.
11. $27,657.63.
12. $325,399.50.

Exercise 179. Page 276

4. $443.10.
5. $624.47.
6. 34,446.4.
7. 365.
8. 493.
9. 250.6.
10. 502.
11. $45.75.
12. $98.25.
13. 496.
14. 640.
15. 720.
16. 405.
17. 495.
18. 39..
19. 3450.
20. 5030.
21. 144.
22. 14; 11.
23. 675.
24. 75.
25. $7\frac{1}{4}$ hr.
26. $6\frac{2}{3}$ hr.
27. 1448.

Exercise 180. Page 279

1. 47,100.
2. 29,648.
3. 18,657.
4. 35,532.
5. 120,294.
6. 281,424.
7. 12.
8. 127.
9. 121.
10. $180\frac{14}{15}$.
11. $63\frac{19}{21}$.
12. $215\frac{114}{128}$.

Exercise 181. Page 282

1. 1 hr. 1 min.	**6.** 1 hr. 41 min.	**11.** 20 min. 41 sec.
2. 1 min. 1 sec.	**7.** 2 hr. 10 min.	**12.** 29 min. 42 sec.
3. 2 hr. 1 min.	**8.** 3 hr. 15 min.	**13.** 1 hr. 54 min. 50 sec.
4. 3 hr. 2 min.	**9.** 8 hr. 28 min. 1 sec.	**14.** 4 hr. 9 min. 51 sec.
5. 4 hr. 2 sec.	**10.** 11 hr. 12 min. 2 sec.	**15.** 8 hr. 32 min. $24\frac{1}{2}$ sec.

16. 2 hr. 10 min. 42 sec.	**20.** 5 A.M.
17. 2 hr. 49 min. 51 sec.	**21.** 6 A.M.
18. 1 P.M.	**22.** 2 hr. 30 min. 50 sec. P.M.
19. 1 hr. 1 min. 1 sec. P.M.	**23.** 1 hr. 20 min. slower.

Exercise 182. Page 283

1. 30° 45′.	**5.** 165° 2′.	**9.** 143° 55′.	**13.** 30° W.
2. 61° 45′.	**6.** 65°.	**10.** 179° 42′.	**14.** 26° 15′ W.
3. 92° 15′.	**7.** 81° 15′.	**11.** 30° E.	
4. 150° 1′.	**8.** 106° 32′.	**12.** 45° W.	

Exercise 183. Page 285

1. 11 A.M.; 10 A.M.; 1 P.M.; 12 M.
2. 10 A.M.; 11 A.M.; 12 M.; 5 P.M.
3. 2 P.M.; 2 P.M.; 12 M.; 3 P.M.; 8 P.M.
4. 2.30 P.M.; 3.30 P.M.; 4.30 P.M.
5. 11.45 A.M.; 11.45 A.M.; 11.45 A.M.; 11.45 A.M.; 12.45 P.M.
6. 1.45 A.M.; 1.45 A.M.; 2.45 A.M.; 2.45 A.M.; 7.45 A.M.
7. 2 P.M.
8. December 31, 5 P.M.
9. 2 P.M.; 1 P.M.; 8 A.M.; 5 A.M.
10. 9 min. $42\frac{18}{25}$ sec.; standard time.
11. 4 min. $6\frac{3}{10}$ sec.; local time.

Exercise 184. Page 286

1. 6 hr. 31 min. 19 sec.	**5.** 54 min. $31\frac{4}{5}$ sec.	**9.** 75° 31′ 15″ W.
2. 27 min. 8 sec.	**6.** 80° 14′ 51″ E.	**10.** 2 min. 22.2 sec.
3. 8 hr. 13 min. 32 sec.	**7.** 70° 15′ 18″ W.	**11.** 41° 22′ 30″ E.
4. 1 hr. 6 min.	**8.** 86° 48′ W.	

Exercise 185. Page 287

1. 24.	**2.** 48.	**3.** 38.	**4.** 38.	**5.** 52.

Exercise 186. Page 288

1. 165.	9. 510.	17. 40; 135.	25. 475.	33. $176\frac{11}{12}$ ft.;
2. 275.	10. 261.	18. 79; 430.	26. 182.	579 ft.
3. 282.	11. 273.	19. 59; 610.	27. 159.	34. $305\frac{7}{12}$ ft.;
4. 172.	12. 630.	20. 57; 435.	28. 98.	$1608\frac{1}{3}$ ft.
5. 234.	13. 24; 108.	21. 92.	29. 350.	35. 2790 ft.
6. 315.	14. 23; 135.	22. 126.	30. 2500.	36. 30,300 ft.
7. 286.	15. 59; 224.	23. 180.	31. 630.	
8. 374.	16. 31; 96.	24. 185.	32. 156.	

Exercise 187. Page 290

1. 32.	5. 46,656.	9. 16.	13. $s = \frac{a(r^n - 1)}{r - 1}$.
2. 512.	6. 1792.	10. 9.	
3. 3645.	7. 2.	11. 16.	
4. 4375.	8. 3.	12. 9.	

Exercise 188. Page 291

1. 254.	5. 160.	9. 63.	13. 252.
2. 364.	6. 780.	10. 1053.	14. 605.
3. 511.	7. 129.	11. 765.	
4. 255.	8. 242.	12. 19,531.	

Exercise 189. Page 294

1. \$97.07.	5. \$888.80.	9. \$2098.86.	13. 5%.
2. \$369.11.	6. 7%.	10. 4%.	14. $4\frac{1}{2}$%.
3. \$1497.27.	7. $10\frac{1}{2}$ yr.	11. \$2000.	15. $2\frac{1}{2}$%.
4. \$1806.92.	8. 13 yr.	12. $9\frac{1}{2}$ yr.	

Exercise 190. Page 296

1. \$179.35.	3. \$132.35.	5. \$1193.45.	7. \$998.25.	9. \$3102.25.
2. \$1254.05.	4. \$1508.22.	6. \$4315.10.	8. \$2676.75.	10. \$8417.40.

Exercise 191. Page 297

1. \$64.86.	2. \$256.27.	3. \$130.24.

Exercise 194. Page 312

1. 500 yd.	4. 2400 ft.	7. 2 mi.	10. 20 labors.
2. $866\frac{2}{3}$ yd.	5. 900 ft.	8. 4.5 mi.	11. 1296 square varas.
3. 237.96 yd.	6. $2083\frac{1}{3}$ ft.	9. 12 mi.	12. 3,613,040.64 square varas.

8	1	6
3	5	7
4	9	2