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PRACTICAL ARITHMETIC

BOOK I

PRACTICAL ARITHMETIC

AN INTRODUCTION TO ELEMENTARY MATHEMATICS FOR SCHOLARS BETWEEN THE AGES OF 9 AND 12

BY A. CONSTERDINE, M.A.

AND

S. O. ANDREW, M.A.
Headmaster of Whitgift Grammar School, Croydon

BOOK I

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PREFACE

"*Mesurez, comptez, pesez, comparez. Intéressez l'enfant à ne jamais faire d'efforts insuffisants ou superflus.*" — ROUSSEAU: *Emile.*

THIS course of elementary mathematics is intended for scholars who have learnt the four rules in money; it assumes no more than a knowledge of these rules, and should be begun as soon as possible after such a knowledge has been acquired.

The method adopted throughout the book is that the materials used for arithmetical calculation shall be got for the most part from reasonably accurate measurements made by each scholar for himself, and that these measurements shall be measurements not merely of drawings, but of actual objects selected for the purpose. In following this method a scholar will, of course, learn a good deal besides pure arithmetic. It will be found, as a rule, that a definite sequence of operations is required of him. That part of an object which is for the time under observation is carefully examined and described, then drawn (either in a hand-sketch or to scale), then measured; calculations arising out of the measurements are then made, each arithmetical and geometrical rule being considered as occasion requires, and stated, as often as not, as an algebraical formula. In other words, the several branches of elementary mathematics (measurement, drawing, arithmetic, geometry, algebra) are not merely correlated, but actually fused. It is within the experience of the authors that this treatment of mathematics induces the greatest possible interest on the part of the scholars, and makes them realise that the subject is of great importance in daily life.

All rules which have only an academic interest, and which are not generally used in the ordinary calculations which a scholar has to make on leaving school, have been purposely excluded. Amongst these may be mentioned, L.C.M., G.C.M., complex fractions, recurring decimals, and complicated money problems. Weights and measures which are not in common use have been omitted. For the most part, no subject or rule has been introduced until the actual need for it has arisen.

As far as possible, the course has been arranged on heuristic lines. Each scholar is supposed to engage in individual work. He is therefore thrown on his own resources, and must grapple with the various problems himself. It is only in this way that self-help and initiative can be cultivated.

Before commencing the book a scholar should, as already stated, have become acquainted with the four simple rules of arithmetic, including money. He should then work through the chapters in the order given, beginning with decimals. He should keep a notebook, which should be treated as a diary; his description of the work done should be written in such a way as to leave no doubt that it was done by himself. Each day's record of work should be dated. The notes should be written in ink immediately after each measurement or calculation has been made; fair copies should not be allowed. The "exercises" should be worked on separate sheets of paper.

The authors have endeavoured not only to amalgamate the various branches of mathematics, but to break down the water-tight compartments into which each of the branches is generally divided. The various rules have not, therefore, been introduced and then abandoned; they have been introduced in order that they may be put to constant use. The teacher is, therefore, reminded that there will be little or no need for "revision." When once a rule or operation has been dealt with, its applications will recur again and again throughout the book.

The proofs (or illustrations) of some of the rules are only given in order that the scholars may be led to see that the rules are not mere tricks. It is not intended that the proofs shall be reproduced in examination tests, or even that they shall be remembered. The teacher will, therefore, deal with them as thoroughly as possible when they occur, and push forward.

From what has already been said, it will be understood that it will be neither necessary nor desirable that scholars shall receive separate lessons in Euclid, geometrical drawing, or algebra, while they are working through this book. It will, however, be useful to supplement the lessons here described with exercises on tots, and simple money problems in which speed and mechanical accuracy are required. In order to insure facility and correctness in calculation, the teacher will find it convenient to set frequent "time exercises," in which two or three arithmetical problems (including some of those which are given in these pages) are to be worked in a limited time.

Chapters (or sections) which are confined to geometry should be taken concurrently with the arithmetical exercises at the end of the previous chapter. It will be found advisable to use the exercises for home work, and to take the geometry in school.

All the work which is described in the book can be carried on without any difficulty in an ordinary classroom. The instruments and apparatus required are practically confined to those which are used in elementary geometrical drawing. The various objects required for measurement can be picked up at a very small cost by those teachers who care to put themselves to a little trouble. For those whose leisure is limited, a list of geometrical models, which can be purchased at a reasonable cost, has been drawn up and printed on p. 10.

It may be added that a companion volume for the use of students in evening schools is in course of preparation. The volume is entitled, *The Rudiments of Practical Mathematics;* the authors are Mr A. Consterdine and Mr A. Barnes.

July, 1904.

INSTRUMENTS AND OBJECTS FOR MEASUREMENT

The following materials are required for the course of work outlined in this book :—

Instruments, etc.—The usual instruments used in geometrical drawing, including dividers, pencil compasses, semicircular protractor, and two set squares (45° and 60°); calipers.

Foot-ruler with cm. and mm. on one edge, and inches and tenths on the other, and on the reverse side twelfths of an inch on one edge and sixteenths on the other; metre rule; yard stick.

Squared paper; transparent tracing-paper.

Hinged planes for illustrating projection.

27 inch-cubes for illustrating the calculation of volumes.

Models.—The following wood models for measurement, together with a few of the instruments mentioned above, may be obtained from Messrs Reynolds & Branson, 14 Commercial Street, Leeds. The models are about as large as a cube, with sides of 8 cm.; they are sufficient for a class of thirty scholars.

	£	s.	d.
30 models of each of the following: Cube, square prism, oblong prism, cylinder .	1	10	0
15 models of each of the following: Triangular prism, hexagonal prism, rhomboid prism, cone, pentagonal pyramid	1	0	0
27 cubes with inch sides, accurately made	0	4	6
30 Barrodale's rulers (Victoria Rule, No. 1)	0	12	6
10 iron calipers	0	6	8
5 metre rules	0	5	10
5 yard sticks	0	5	0
1 hinged plane for illustrating projection	0	1	6

Common Objects.—When possible, some of the following objects should be used instead of the models mentioned above. They can easily be got together at a very small cost by teachers who care to put themselves to a little trouble:—

Metal.—Iron washers and wedges. Brass or iron rods and piping (cut up). Large wire nails. Plates of various shapes cut out in iron, tin plate, or brass. Cylindrical and other boxes of tin. Brass and iron rings. Wire of various kinds and thicknesses. Large shot; bullets; ball bearings. Models made in manual instruction workshops.

Stoneware.—Variously shaped tiles for hearths, pavements, walls and roofs (these are extremely convenient). Bricks and faced building stones. Boys' "marbles."

Glass and Earthenware.—Cylindrical vases, jars, and bottles. Thick glass tubing and rods. Letter weights. Spheres from aërated water bottles.

Wood.—Boxes of various shapes and sizes. Floor blocks or planks (cut up). Picture frames and mouldings. Lead pencils and penholders. Brush stales (cut up). Models made in manual instruction workshops.

Cardboard.—Models of geometrical solids, etc., as made in the junior classes of elementary and secondary schools.

Rubber.—Rings, tubing, erasers, solid tyres.

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PRACTICAL ARITHMETIC

CHAPTER I

THE METRE RULE AND DECIMALS

Measurement of Length

1. Your teacher will give you a block of wood called an oblong prism. Examine it carefully, and write down in your note-book :—

(*a*) The number of its faces.
(*b*) The number of its edges.
(*c*) The number of its corners.

Now look at the edges and guess how many of them are the same length. In order to make sure, you may compare the lengths of the edges by using a piece of string. The string should be stretched tightly between the thumbs, and placed first on one edge and then on the others. The results you arrive at should be entered in your note-book.

2. What you have just done tells you that some edges are the same length as some others, but not *how long* they are. If you wish to state how long each of the edges is, you will have to use a **standard of length.** A standard of length is a fixed length which everyone agrees to use in describing lengths. If you wish to state how long an object is, you must say how many times longer or shorter it is than the standard. Different standards are used in different countries. You are now going to examine a standard which is used in France, and for some purposes in England.

The Ruler

3. Take your ruler and look at the edge which is divided into millimetres, centimetres, and decimetres. These words should be pronounced "mil-li-me-ter," "sen-ti-me-ter," and "dessi-me-ter." Notice that all the millimetres are the same length, all the centimetres the same length, and all the decimetres the same length.

Your little finger is probably rather broader than a centimetre. Measure it.

4. Count how many millimetres there are in a centimetre, and how many centimetres in a decimetre.

Your teacher will now pass round some metre rules. Count how many decimetres there are in a metre.

Make a table as follows, and write it down in your note-book :—

— millimetres make 1 centimetre.
— centimetres make 1 decimetre.
— decimetres make 1 metre.

The metre is the standard of length, but it is very convenient to use decimetres, centimetres, or millimetres for lengths of less than a metre.

5. Exercise (oral).

(*a*) How many millimetres are there in :—
1, 2, 7, 9, 10 centimetres?

(*b*) How many centimetres are there in :—
1, 3, 5, 8, 10 decimetres?

(*c*) How many centimetres are there in :—
1, 4, 6, 9, 10 metres?

(*d*) How many millimetres are there in :—
1, 2, 5, 7, 10 metres?

6. Exercise (to be written).

How many times longer is :—

(*a*) 4 decimetres than 4 millimetres?
(*b*) 1 metre than 5 centimetres?
(*c*) 3 metres than 6 millimetres?
(*d*) 5 decimetres than 2 centimetres?
(*e*) 6 centimetres than 3 millimetres?

How many times shorter is :—

(*f*) 1 millimetre than 1 centimetre?
(*g*) 3 millimetres than 9 decimetres?
(*h*) 4 centimetres than 2 decimetres?
(*i*) 5 centimetres than 6 metres?

7. There are 4 farthings in 1 penny and 12 pence in 1 shilling. These facts may also be stated as follows :—

1 farthing is *one-fourth* of 1 penny.
1 penny is *one-twelfth* of 1 shilling.

In the same way it may be said that :—

1 millimetre is *one-tenth* of 1 centimetre.
1 centimetre is *one-tenth* of 1 decimetre.
1 decimetre is *one-tenth* of 1 metre.

8. The following abbreviations should be used :—

mm. for millimetre or millimetres.
cm. for centimetre or centimetres.
dm. for decimetre or decimetres.
m. for metre or metres.

9. Now take the oblong prism again and measure the lengths of its edges in the following manner :— Place the ruler along the edge to be measured, move it so that the mark "0" is exactly at one end of the edge, and from the other end read off the length required in dm. If the length is not an exact number of dm., read off the number of cm. which must be added, and if the length is not an exact number of dm. and cm., read off the number of mm. which must be added.

In making the measurements it is very important that the scale on the ruler should be held quite close to the edge which is being measured, and that those parts of the ruler and of the prism which are being observed should be placed straight in front of you and directly opposite your eyes.

Write down the lengths of the edges of the prism in the following manner:—

Oblong Prism

The four long edges measure (say):—

1 dm. 2 cm. 3 mm.; 1 dm. 2 cm. 3 mm.; 1 dm. 2 cm. 3 mm.; 1 dm. 2 cm. 2 mm.

The four short edges measure (say):—

2 cm. 9 mm.; 2 cm. 9 mm.; 2 cm. 9 mm.; 2 cm. 9 mm.

The remaining four edges measure (say):—

4 cm. 1 mm.; 4 cm. 1 mm.; 4 cm. 0 mm.; 4 cm. 1 mm.

Decimals

10. Let us suppose that one of the edges measures 4 cm. 1 mm. We may for the sake of shortness in writing, place a dot or point, called a **decimal point,** between the figure 4 which represents cm., and the figure 1 which represents mm. The edge would then be said to be 4·1 cm. long. This should be read "four point one centimetres."

The point means that the figure which comes after it represents, not 1 cm., but one-*tenth* of a cm. The figure coming after the point is called a **decimal.**

11. Exercise (oral).

Express in cm. and decimals of a cm.:—

(*a*) 5 cm. 2 mm.	(*d*) 1 cm. 4 mm.
(*b*) 12 cm. 7 mm.	(*e*) 97 cm. 9 mm.
(*c*) 3 cm. 9 mm.	(*f*) 6 cm. 3 mm.

Express the same lengths in mm.

12. Let us suppose that another of the edges of the prism measures 1 dm. 2 cm. 3 mm. This might be written 12·3 cm. (to be read "twelve point three centimetres").

Here the figure 1 represents tens of cm., the figure 2 represents ones of cm., and the figure 3 which follows the point represents *tenths* of a cm.

Take your ruler and convince yourself that 1 dm. 2 cm. 3 mm. is the same length as twelve centimetres and three-tenths of a centimetre, that is, as 12·3 cm.

13. Exercise (to be written).

Express in cm. and decimals of a cm. the following lengths :—

(*a*) 2 dm. 1 cm. 3 mm.
(*b*) 9 dm. 4 cm. 2 mm.
(*c*) 18 dm. 9 cm. 4 mm.
(*d*) 6 dm. 5 cm. 1 mm.
(*e*) 17 dm. 2 cm. 5 mm.
(*f*) 5 dm. 4 cm. 0 mm.
(*g*) The lengths of the edges of the oblong prism which you measured in Section 9.

Express the same lengths in mm.

14. Instead of writing 1 dm. 2 cm. 3 mm. or 12·3 cm., we might have written 1·23 dm. (to be read "one point two three decimetres" but *not* "one point twenty-three decimetres").

Here the figure 1 represents ones of dm., the figure 2 which follows the point represents *tenths* of a dm., and the figure 3 represents *hundredths* of a dm.

How many times longer is a cm. than a mm.? How many times longer is a dm. than a cm.? How many times longer is a dm. than a mm.? You know that 1 mm. is one-tenth of 1 cm.; what part of 1 dm. is 1 mm.?

Look at your ruler and convince yourself that 1 dm. 2 cm. 3 mm. is the same length as one decimetre, two-tenths of a decimetre, and three hundredths of a decimetre, that is, as 1·23 dm.

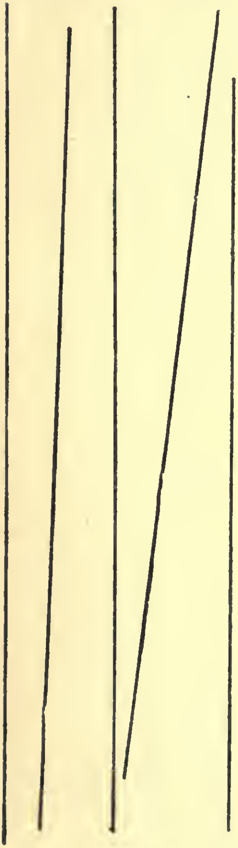


Fig. 1.

15. Exercise (measurement).

(*a*) Measure the length of your pencil and pocket-knife, and the length and breadth of your note-book, text-book, instrument-box, and handkerchief—(1) in dm., cm., and mm.; (2) in dm., and decimals of a dm.

(*b*) Measure the lengths of the lines shown in Fig. 1—(1) in dm., cm., and mm.; (2) in dm., and decimals of a dm.

16. Exercise (to be written).

Express in dm., and decimals of a dm., the following lengths:—

(*a*) 4 dm. 3 cm. 8 mm.
(*b*) 7 dm. 2 cm. 9 mm.
(*c*) 39 dm. 1 cm. 3 mm.
(*d*) 2 dm. 4 cm. 4 mm.
(*e*) 3 dm. 9 cm. 7 mm.
(*f*) 11 dm. 7 cm. 0 mm.

17. Perhaps your teacher's desk measures 1 m. 2 dm. 3 cm. 4 mm. wide. This may be written in four different ways:—

1234 mm. To be read: One thousand two hundred and thirty-four millimetres.
123·4 cm. To be read: One hundred and twenty-three point four centimetres.
12·34 dm. To be read: Twelve point three four decimetres.
1·234 m. To be read: One point two three four metres.

18. Exercise (measurement).

Measure the breadth of a door, the length of a plank in the floor, and the width of your teacher's desk. Express the measurements in four different ways as in the last section.

19. Exercise (to be written).

Express the following lengths in metres and decimals of a metre:—

(*a*) 79·43 dm. (*b*) 475·3 cm. (*c*) 2851 mm.

and the following in cm. and decimals of a cm.:—

(*d*) 85·7 dm. (*e*) 9·384 m. (*f*) 4378·24 dm.

20. The values represented by the figures coming before and after the point are shown below:—

hundreds	tens	ones	(point)			
1	2	3	·	4	5	6
			(point)	tenths	hundredths	thousandths

In the number 111·111, each of the first five figures represents a number ten times larger than the figure which follows it, and each of the last five figures represents a number ten times smaller than the figure which comes before it.

21. Exercise (oral).

Look at the figures marked with a *. State what they represent—(1) in tens, ones, tenths, and so on; (2) in m., dm., cm., or mm. :—

(*a*) $792{\cdot}4\overset{*}{8}1$ m. (*b*) $\overset{*}{4}9{\cdot}7$ cm. (*c*) $2{\cdot}\overset{*}{7}4$ dm.

(*d*) $89{\cdot}4\overset{*}{2}$ mm. (*e*) $\overset{*}{5}55$ m. (*f*) $1{\cdot}49\overset{*}{6}$ m.

Multiplication and Division by 10

22. Multiply by 10 :—

(*a*) 7 m. (*b*) 84 m. (*c*) 79 dm.
(*d*) 434 mm. (*e*) 210 m. (*f*) 52 cm.

Now try to multiply 1·2 dm. by 10. If you have any difficulty, write down the number of cm. in 1·2 dm., then multiply by 10, and then convert the cm. back to dm.

Multiply 2·34 dm. by 10. Here you had better begin by turning the dm. into mm. if you cannot write down the answer at once.

23. Exercise (to be written).

Multiply by 10 :—

(*a*) 431·7 cm. (*d*) 8·643 dm.
(*b*) 942·834 m. (*e*) 1000·1 m.
(*c*) 7·52 cm. (*f*) 0·1 m.

Divide by 10 :—

(*g*) 932·427 m. (*j*) 84·5 dm.
(*h*) 91·36 cm. (*k*) 10·1 m.
(*i*) 9·4 m. (*l*) 0·1 m.

24. Try to write down rules for multiplying and dividing a decimal by 10 and by 100.

25. Exercise (measurement):—

(*a*) If your note-book were ten times larger, what would be its length and its width?

(*b*) What would be the length and the width of a tablecloth if its edges were ten times longer than those of your handkerchief?

(*c*) If your pencil were ten times shorter, what would be its length?

(*d*) Measure the width of a plank in the floor. What is the width of ten planks?

(*e*) Cut a strip of paper exactly 1·27 dm. long. If you cut up the paper into ten equal lengths, what would be the length of each piece?

26. What does the figure 0 represent in:—

(*a*) 190 m.
(*b*) 109 m.
(*c*) 1091 m.
(*d*) 10·9 m.
(*e*) 1·09 m.
(*f*) 1·904 m.
(*g*) 04 m.
(*h*) 0·4 m.
(*i*) 4·0 m.

The figure 0 in such examples as (*g*), (*h*), and (*i*) is usually omitted. Why could it not be omitted in the other examples?

27. Exercise (oral):—

(*a*) Read off:—·3 m., ·03 m., ·33 m., 3·03 m., 3·33 m., 30·03 m.

(*b*) What is the value of each 3 in the above numbers?

(*c*) Arrange in order of magnitude:—·4 dm., ·44 dm., 4 dm., 4·4 dm., ·04 dm., ·36 dm.

(*d*) Express in figures (decimals):—Three-tenths; three hundred; three and a tenth; three and a hundredth; three hundred and a tenth.

(*e*) Multiply by 10 :—
10 m., 1·4 m., 2·09 m., ·3 m., 7·86 m., 5·421 m.

(*f*) Multiply by 100 :—
7·92 m., 4·325 m., 1·1 m., ·8 m., ·01 m., ·67 m.

(*g*) Divide by 10 :—
·1 dm., ·01 dm., 42·7 dm., 3 dm., 129 dm., 1 dm.

(*h*) Divide by 100 :—
20 m., 179·2 m., 14·379 m., 4 m., ·01 m.

The Use of the Hand-Sketch

28. Your teacher will now give you another prism, called a square prism. Write down in your note-book :—

(*a*) The number of its faces.
(*b*) The number of its edges.
(*c*) The number of its corners.

29. You are now going to measure the edges of the square prism and to learn a convenient method of recording their lengths.

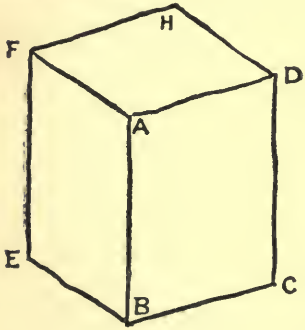


Fig. 2.

In § 9 you recorded the lengths of the edges of the oblong prism in tabular form. Now you are going to learn how to make a hand-sketch of a model and to mark the lengths of the edges on the hand-sketch.

Place the square prism on the desk in front of you and make a hand-sketch, about the same size as the prism. Begin by drawing the upright line A B which is nearest to you. (Fig. 2.)

How many more upright lines can you see? Mark the position of the lower ends C and E of the other upright lines by making dots on the paper; take care that the points C and E are the right distances above the point B and the right distances from the line A B. Draw the lines B E and B C.

Now fix the positions of the points D, F, and H, and complete the drawing.

30. Measure the edges of the square prism in cm.,

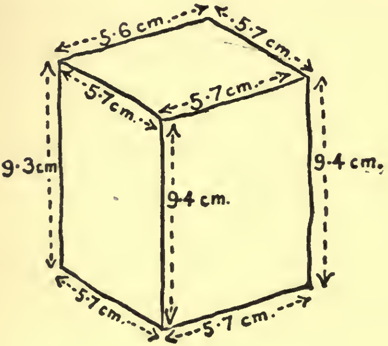


Fig. 3.

and mark the dimensions on your drawing as in Fig. 3.

Addition and Subtraction

31. Add together the lengths of the longest and shortest edges of the prism. In doing this you will have to take care that the tenths are placed under the tenths and the ones (*i.e.* the units) under the ones. Where will you have to place the decimal points?

Find by addition the total length of the long edges, the total length of the short edges, and the total length of all the edges.

32. Exercise (to be written).

Find the sum of the following lengths :—

(*a*) 107·425 m., ·09 m., 17·51 m., 1·4 m.
(*b*) 300 m., ·003 m., 1 m., ·01 m.

Read out and then find the value of :—

(*c*) $3{\cdot}784 + {\cdot}001 + {\cdot}1 + 1 + {\cdot}01$.
(*d*) $42 + {\cdot}042 + {\cdot}3752 + 9{\cdot}1$.

33. Subtract the length of the shortest edge of the prism from that of the longest edge; place point under point before you begin.

34. Exercise (measurement).

(*a*) Express in decimals of a dm. the difference between the length and breadth of your text-book.
(*b*) How much is your note-book longer than your text-book? Express your answer in cm.
(*c*) What is the difference in length between your own pencil and that of your neighbour? Express your result in dm.

35. Exercise (to be written).

Find the difference between :—

(*a*) ·425 m. and 1·535 m.
(*b*) 37 m. and ·37 m.

Read out and then find the value of :—

(*c*) 20·041 m. – 19·042 m.
(*d*) 47·5 m. – 1·305 m.
(*e*) $1{\cdot}01 - {\cdot}2$.
(*f*) $4{\cdot}32 + {\cdot}7 - 3{\cdot}09$.
(*g*) $24{\cdot}97 - 14{\cdot}2 + 7{\cdot}04 - 6{\cdot}3$.

Multiplication and Division

36. Suppose that the four long edges of the square prism are exactly the same length. Find their total length, by multiplying the length of one edge by 4.

37. Suppose that you wish to multiply 20·1234 m. by 4, about what will the answer be? You can see at once that it will be 80 m., and some decimals of a metre. How many figures come before (*i.e.* to the left of) the point in the answer?

If you wish to multiply 20·1234 m. by 5, about what will the answer be? And how many figures will come before the point?

Whenever you multiply a decimal, you should always take great care to put the point in the right place. What would be the effect on your answer if you placed the point too far to the left? And if you placed it too far to the right?

38. Exercise (oral).

Read out and then state the value of :—

(*a*) 1·1 m. × 5.
(*b*) 4·2 m. × 2.
(*c*) 4·1 m. × 4.
(*d*) 2·1 m. × 5.
(*e*) 7·4 m. × 3.
(*f*) 1·1 m. × 10.
(*g*) 1·1 m. × 100.
(*h*) 9·9 cm. × 5.
(*i*) 12·2 cm. × 7.
(*j*) 12·2 cm. × 12.
(*k*) 1·1 mm. × 13.

39. Exercise (to be written).

Multiply by 5 :—

(*a*) 402 m., 40·2 m., 4·02 m., ·402 m.

In each of these cases you can see at once how many figures come before the point in the answer; no rule is necessary to fix the position of the point.

Multiply by 7 :—

(*b*) 791 m., 79·1 m., 7·91 m., ·791 m.

Multiply by 11 :—

(*c*) 999 m., 99·9 m., 9·99 m., ·999 m.

Find the value of :—

(*d*) 4·2 × 29.
(*e*) 101·1 × 8.
(*f*) 56·7 × 5 × 6.
(*g*) 5·4 × 7 × 8.

40. Exercise (oral).

In division the position of the point can be fixed in a similar manner.

Read out and then state the value of :—

(*a*) 10·2 cm. ÷ 2.
(*b*) 14·5 cm. ÷ 5.
(*c*) 3·6 dm. ÷ 3.
(*d*) 12·1 dm. ÷ 11.
(*e*) 29·4 m. ÷ 7.
(*f*) 50·4 dm. ÷ 6.
(*g*) 73·6 m. ÷ 8.
(*h*) 8·1 m. ÷ 9.
(*i*) 10·8 m. ÷ 12.
(*j*) 16·9 m. ÷ 13.
(*k*) 5·6 dm. ÷ 7.

41. Exercise (to be written).

Divide by 5 :—

(*a*) 795 m., 79·5 m., 7·95 m., ·795 m.

Divide by 7 :—

(*b*) 581 m., 58·1 m., 5·81 m., ·581 m.

Divide by 12 :—

(*c*) 148·8 m., 14·88 m., 1·488 m., ·1488 m.

42. In dividing one whole number by another whole number the quotient should when necessary be expressed as a decimal. Thus 9 cm. ÷ 5 = 1 cm. and 4 cm. over; but 4 cm. are equal to 40 mm. and 40 mm. ÷ 5 = 8 mm. The answer is therefore 1 cm. and 8 mm., that is 1·8 cm.

$$\begin{array}{r|l} 5 & 9 \text{ cm.} \\ \hline & 1{\cdot}8 \text{ cm.} \end{array}$$

43. Exercise (to be written).

Divide by 5 :—

(*a*) 571 m.
(*b*) 423 m.
(*c*) 99 m.
(*d*) 877 m.

Divide by 2 :—

(*e*) 681 m., 68·1 m., 6·81 m., ·681 m.

Find the value of :—

(*f*) 489·348 m. ÷ 12. (*i*) ·01 m. ÷ 5.
(*g*) 27·02 dm. ÷ 4. (*j*) (42·7 × 4) ÷ 7.
(*h*) 7694·01 m. ÷ 9. (*k*) (3·141 ÷ 9) × 12.

44. You are now going to deal with a method which is sometimes very useful in shortening the operations of multiplication and division.

Copy the following into your note-book and fill in the required values :—

17 × 48	=	180 ÷ 36	=
(17 × 4) × 12	=	(180 ÷ 4) ÷ 9	=
(17 × 8) × 6	=	(180 ÷ 6) ÷ 6	=

4 and 12 are said to be **a pair of factors** of 48, because their product is 48; 8 and 6 are also a pair of factors of 48 for the same reason. What other pairs of factors of 48 are there?

Besides those given above, what other pairs of factors of 36 are there?

45. Exercise (oral).

Turn into pairs of factors :—

30, 56, 40, 72, 121, 96, 108, 81.

46. Exercise (to be written).

Find the value of :—

(*a*) £179. 18s. $4\frac{1}{2}$d. × 25.
(*b*) £59. 7s. $4\frac{1}{2}$d. ÷ 42.
(*c*) £174·352 × 144.
(*d*) 17424·132 ÷ 132.

An Exercise on the Cube

47. Examine the cube which is given to you and write down in your note-book :—

(*a*) The number of its faces.
(*b*) The number of its edges.
(*c*) The number of its corners.

Make a hand-sketch of it, measure the lengths of its edges in cm., and mark the dimensions on the hand-sketch, according to the directions in § 30.

If the cube has been accurately made, you will have found that all the edges are exactly the same length. Find the total length of the edges by multiplication.

48. Exercises on the cube (to be written).

(*a*) Calculate the total length of the edges of a cube which has edges 6·9 cm. long.

(*b*) What would be the total length of the edges, if each of them were six times as long as in the last exercise?

(*c*) What would be the total length of the edges, if each of them were fifty-six times as long as in exercise (*a*)? Express your answer in metres.

(*d*) The total length of the edges of a cube is 1·356 m. Find the length of one edge both in dm. and cm.

(*e*) If each edge of the cube last mentioned were lengthened by 2·7 cm., what would be the total length of the edges in metres?

(*f*) If each edge were shortened by ·34 dm., what would be the total length of the edges in dm.?

(*g*) What length of wire would be required to make a skeleton cube, with each edge 4·5 cm. long?

Money

49. You will have seen how convenient it is to express lengths in decimals, especially when you have to add, subtract, multiply, or divide them. And it is especially easy to express metric measures in decimals, because a cm. is one-tenth of a dm., a mm.

one-tenth of a cm., and so on. Let us now see how decimals can be applied to money.

What coin is worth one-tenth of a pound? How would you express the value of this coin as the decimal of a pound?

50. Exercise (oral).

Turn into decimals of a pound :—

(*a*) 3 florins.	(*b*) 6 shillings.	(*c*) 9 florins.
(*d*) £1. 8s.	(*e*) £100. 6s.	(*f*) £14. 14s.

Express in £ s. d. :—

(*g*) £1·1.	(*h*) £9·8.	(*i*) £·9.
(*j*) £10·4.	(*k*) £6·7.	(*l*) £4·5.

51. Copy the following into your note-book and fill up the required values in decimals of a decimetre :—

One decimetre	=	dm.
One half of a decimetre	=	dm. (refer to your ruler).
One centimetre	=	dm.
One half of a centimetre	=	dm. (refer to your ruler).

Now copy the following into your note-book and express the required values in decimals of a pound :—

One pound	= £
One half of a pound	= £
One florin	= £
One half of a florin	= £
One shilling	= £

52. Exercise (to be written).

Express in decimals of £1 :—

(*a*) One shilling, two shillings, and so on up to nineteen shillings.
(*b*) £1. 7s.; £17. 9s.; £100. 13s.
(*c*) One guinea, half a sovereign, five guineas.

Express in decimals :—

(*d*) One, five-tenths, one-half.
(*e*) One-tenth, five hundredths, half of one-tenth.

Write in £ s. d. :—

(*f*) £1·5, £2·75, £3·05.
(*g*) £·5, £17·95, £14·65.

What is the meaning of ·1s.? Is there a coin of this value?

What is the meaning of ·1d.? Is there a coin of this value?

Add the following and express your answers in £ s. d. :—

(*h*) £179·4; £·05; £1·75.
(*i*) £1·1; £·15; £·1; £·35.

Multiply the following and express your answers in £ s. d. :—

(*j*) £15·7 by 11.
(*k*) £·05 by 12.
(*l*) £7·15 by 9.
(*m*) £14·95 by 7.

Divide :—

(*n*) £149·4 by 6.
(*o*) £·15 by 3.

Inches

53. Examine the edge of the ruler which is divided into inches and tenths of an inch. How many times bigger is an inch than a centimetre? Is it twice as big? Is it three times as big?

Your thumb-joint is probably about as long as an inch. Measure it.

How many tenths of an inch are there in two inches and in half an inch?

For the sake of shortness the word *inch* is written *in.*

54. Exercise (oral).

(*a*) Read out (using the word "point"):—
4·9 in.; 179·3 in.; ·1 in.; 57·2 in.

(*b*) How many tenths of an inch are there in the above lengths?

(*c*) Express in figures (using the word "point"):—Fourteen inches and eight-tenths; three-tenths of an inch; one inch and a tenth; half an inch.

(*d*) What does the figure 1 represent in the following lengths: 2·1 in.; 1·2 in.; 10·2 in.; 2·01 in.?

55. Exercise (to be written).

How many times longer is:—

(*a*) 1·7 in. than 1 in.?
(*b*) 100 in. than ·1 in.?
(*c*) 1·5 in. than half an inch?
(*d*) 7 in. than ·5 in.

Express in figures (decimals):—

(*e*) Seven hundredths of an inch.
(*f*) Half of one-tenth of an inch.
(*g*) Five hundredths of an inch.
(*h*) One hundredth of an inch.

Question:—

(*i*) Are hundredths of an inch marked on the ruler? Why not?

Approximate Measurements

56. Borrow your neighbour's ruler, measure its breadth, and write down the measurement in inches, tenths of an inch, and halves of one-tenth of an inch.

You ought to have no difficulty in doing this. If the breadth is exactly one inch and two-tenths, you will write "1·2 in." If the breadth is one inch, two-tenths, and half of a tenth, you will write "1·25 in." If it is rather more than 1·25 in., but nearer to 1·25 in. than to 1·3 in., you should still write "1·25 in."; but if it is nearer to 1·3 in. than to 1·25 in., you should write "1·3 in."

If you have done your work properly, your result will be **correct to ·05 in.**; this means that you are quite sure about the inches and about the tenths, and also about the halves of the tenths, but that you are not sure about lengths which are smaller still.

57. Examine the models given to you (triangular and hexagonal prisms). In each case write down :—

(*a*) The number of faces.

(*b*) The number of edges which bound each of the faces.

(*c*) The number of corners.

Can you guess which is the triangular and which the hexagonal prism?

Make a hand-sketch. Measure the edges in inches and decimals of an inch, and let the measurements be correct to ·05 in. Mark the dimensions on the hand-sketch.

58. Exercise on the models (to be written).

(*a*) If the edges of the models were seven times longer, what would be the length of each of them?

(*b*) If the edges of the models were twenty-five times longer, what would be the length of each of them, and what would be their total length?

(*c*) If each of the long edges of the hexagonal prism were shortened by 1·05 in., by how much should each of the short edges be lengthened in order that the total length of all the edges might remain the same?

(*d*) If each of the three long edges of the triangular prism were lengthened by 2·7 in., what should be the sum of the short edges, in order that the total length of all the edges might remain the same?

(*e*) If the model were divided into five equal pieces, what would each of the edges then measure?

(*f*) What would a hexagonal bar of gold, 4·35 dm. long, be worth, if it cost £12 a cm.?

59. Exercises for revision (to be written).

(*a*) Find the value of:—
3·79 in. + 47·85 in. + 2·3 in. + 7 in.

(*b*) Find the value of:—
157·93 in. + ·02 in. − 3·17 in.

(*c*) What is the value of each of the figures in the following lengths? 3·49 in.; 6·402 in.; 11·01 in.; ·0001 in.

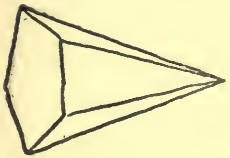
(*d*) Turn into decimals of a £:—
£10. 17s.; £3. 9s.; £6. 7s.

(*e*) Express as decimals:—
17 in. ÷ 2; 15 in. ÷ 6; 6 in. ÷ 12.

(*f*) Multiply 4·72 in. by: 45; 42; 63; 54.

(*g*) Divide 50·688 in. by 64, and 8·91 in. by 99.

(*h*) Add together: 13·1 in.; three-tenths of an inch; 6 in.; 1·05 in.; seven-tenths of an inch.



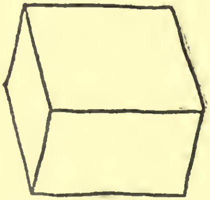


FIG. 4.

(*i*) Fig. 4 shows hand-sketches of two models. Write down the number of their faces, edges, and corners.

Drawing with the Ruler

60. Cut your pencil (HH or H) to a chisel-edge, and sharpen it carefully on sandpaper (No. 0).

Now take your ruler and rule a straight line in your note-book. Take care to hold the pencil upright on the paper and to keep it close to the ruler while you are drawing.

61. Prick two points in your note-book with a pin (use a pin 2 in. long, with a large glass head).

Draw a straight line joining them with your pencil and ruler. The pencil has thickness and the ruler must not therefore be placed so as to cover up the points; before drawing the line make a trial to see whether it will join the points exactly.

62. Draw a straight line 5·7 cm. long in the following manner: Place your ruler on the paper, and mark two points 5·7 cm. apart with a pin; then join the two points.

63. Exercise (drawing).

Draw lines of the following lengths:—

(*a*) 1·24 dm.
(*b*) 7·9 cm.
(*c*) ·8 cm.
(*d*) 3·7 in.
(*e*) ·6 in.
(*f*) 1·45 in.

64. Copy each of the straight lines in Fig. 5 into your note-book.

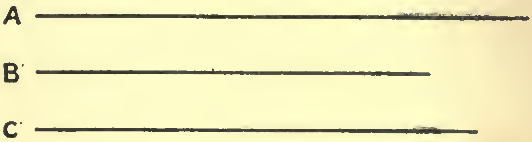


Fig. 5.

To do this, open out the legs of your dividers till they exactly span the length of the line. Remove the dividers, taking care not to alter the distance between the legs, and with them mark two points in your note-book; join the points.

65. Find out which of the straight lines in Fig. 6 is

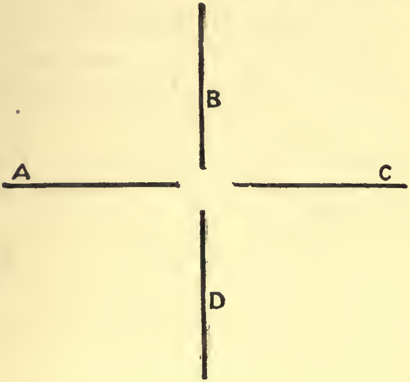


Fig. 6.

the longest, and which the shortest, by applying your dividers.

66. Find out which of the straight lines in Fig. 7 is the longest and which the shortest, by using tracing-paper,

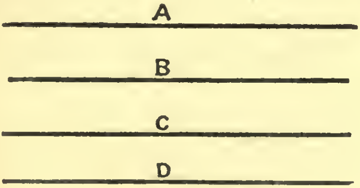


Fig. 7.

in the following manner: Place the tracing-paper over the line A, and prick through it at the ends of the line with a pin; then place the tracing-paper over the other lines, and compare their lengths with the distance between the two pin-holes.

Deal with the lines B, C, and D in the same way.

Accurate Measurements

67. In § 56 you learned how to measure the breadth of a ruler correct to ·05 in. It is fairly easy, however, to make much more accurate measurements of straight lines which are drawn on paper.

Place your ruler on the straight line A and measure its length in inches, tenths of an inch and hundredths of an inch. (Fig. 8.)

You will observe that the line measures two inches, one-tenth of an inch, and part of another tenth. This

Fig. 8.

may be written in figures 2·1 + in., the + sign being used to show that the line is rather longer than 2·1 in.

Now, imagine that the last tenth of an inch is divided into ten equal parts, that is to say, into hundredths of an inch. Guess how many of these parts must be added to 2·1 in. in order that the length of the line may be expressed more accurately. It is quite clear that the line is more than 2·15 in. long. Write down in figures how long you think it really is.

68. Exercise (measurement).

Find the lengths of the straight lines in Fig. 9 in inches, tenths, and hundredths.

Straight and Curved Lines

69. Hitherto you have been dealing with straight lines, but before long you will have to examine curved lines. It is therefore important that you should be able to distinguish between the two.

Find out whether the lines in Fig. 10 are really

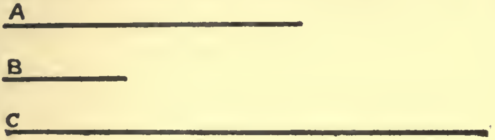


Fig. 9.

straight lines or not, by taking the book in your hands, placing your eye on a level with the page and looking

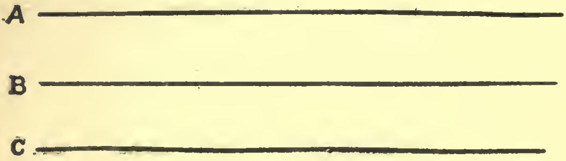


Fig. 10.

along the lines. You will have very little difficulty in seeing whether the lines are straight, or curved.

70. If the edge of your ruler is quite straight, you can tell whether the lines in the last section are straight or not, by applying your ruler to them and seeing whether they exactly fit the edge. Do so, and compare your conclusions with the results which you obtained by looking along the lines.

71. Now it is time to find out whether the edges of your ruler are straight.

Take a pin and prick two points in your note-book, about 12 cm. apart. Choose one of the two edges of your ruler, say the one divided into cm., and carefully draw a line joining the points.

Keep your ruler in exactly the same position and turn your note-book round, top to bottom.

Now with the same edge of the ruler, draw another line joining the two points.

If the two lines coincide, the edge of the ruler is

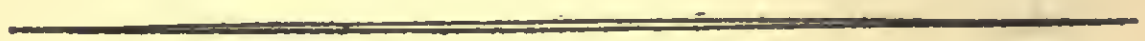


FIG. 11.

straight. If the edge is not quite straight, the two lines will appear as in Fig. 11.

72. Exercises for Revision :—

(*a*) Draw three lines showing the lengths represented by each of the figures in the length 23·4 cm.; and also in the length 3·87 in.

(*b*) Find the value of—
20·001 m. − 1·0002 m. + 120 m.

(*c*) Find the value of :—
·0007 m. − 7000 m. + 70000·7 m. − 70·07 m.

(*d*) If 1·021 is subtracted from a certain number, 17·2 is left. What is the number?

(*e*) A man spends £3·5, £14·2, £·35, £4. What does he spend altogether?

(*f*) What number divided by 1·375 gives a quotient of 23?

(*g*) An iron railing containing 145 uprights equal distances apart, is 676·8 in. long. Make a hand-sketch and find the distance between two uprights?

(*h*) Divide 1081·08 by 108 and 33503·688 by 72.

(*i*) By what number must 49 be multiplied in order to give a product of 7480·34?

(*j*) Turn into decimals of a £ :—
£1. 18s.; £14. 2s.; five shillings.

(*k*) Write in £ s. d. :—
£1·6 ; £1·05 ; £1·75.

(*l*) What number divided by 4·25 gives a quotient of 17?

(*m*) A man walks 142·5 miles in five days, beginning each morning at seven o'clock and stopping at noon. How many miles does he walk in an hour?

(*n*) If I pay 5s. 3d. for 18 yards of ribbon, what is the price of the ribbon per yard?

(*o*) Find the total cost of the following articles :
25 lbs. of tea, at 1s. 5d. per lb.
28 lbs of butter, at 1s. 3d. per lb.
36 lbs. of bacon, at 9d. per lb.
24 lbs. of coffee, at 1s. 8d. per lb.

CHAPTER II

CURVED LINES: USE OF SYMBOLS

The Cylinder

73. Examine the cylinder given to you and write down the number of its faces, edges, and corners. Make a hand-sketch of it.

Mark the two ends of the cylinder so that you will be able to distinguish them, and then measure each of the edges in the three different ways which are given below.

74. First method—

Wrap a long strip of paper tightly round the cylinder, and prick through the overlapping ends of the paper with a pin. Unwrap the paper and measure the distance between the two pin-holes in cm.

75. Second method—

Wind a piece of strong thread once round the cylinder and then cut through the overlapping ends of the thread with your knife. Measure in cm. that part of the thread which was wound round the cylinder.

76. Third method—

Choose one end of the cylinder and make a mark on its edge with your pencil.

Put the cylinder, with the marked end downwards, on a piece of smooth paper.

Place the ruler on the left of the cylinder and adjust them so that the mark on the cylinder touches the mark "0" cm. on the ruler.

Press the cylinder firmly against the ruler and roll it round until the mark on the cylinder touches the ruler again; take care that the cylinder does not slip or slide.

Read off the length of the edge in cm.

Averages and Approximations

77. You have now measured each of the edges of the cylinder three times, and in all probability the results will not be quite alike, though very nearly so.

Add the three results together and then divide by 3; this will give you the **average** result.

Your quotient must be **correct to one place of decimals.** To make sure that it is so, work out the division by 3 (mentally) to two places of decimals. Suppose that the quotient is 18·79 cm. In this case you should write down 18·8 cm. as the answer. If the quotient is 18·73 cm., the answer, correct to one place of decimals, will be 18·7 cm., and so on (see § 56).

The methods which you have used in measuring the edge of the cylinder are not very accurate, and you are of course liable to make mistakes however careful you may be. It would, therefore, be quite useless to give a figure in the second decimal place which represents tenths of a mm. It will be quite proper for you to say that you have found the edge to be, say, 18·8 cm. long, but you cannot truthfully say that you have found it to be 18·79 cm. long. You would require much more accurate instruments and much greater skill to be able to measure to the tenth of a mm.

78. You will observe that the average length is greater than the shortest length and less than the longest length. The average length is more likely to be correct than either the shortest or the longest length.

Mark the average lengths of the edges of the cylinder on the hand-sketch.

79. Measure the height of the cylinder by placing the ruler against it; take care that the edge of the ruler touches the cylinder along its whole height. Make three measurements, find the average height and mark it on the hand-sketch.

By how much is either of the edges of the cylinder longer than its height?

80. Exercise (to be written).

(*a*) A cricketer scores as follows: 144, 0, 29, 0, 7. What is his average score?

(*b*) The attendance of scholars at school on five successive days was: 249, 301, 273, 282, and 255. What was the average daily attendance?

(*c*) If I spend £347 in 365 days, what is my average daily expenditure?

(*d*) Find the value of the following, correct to two places of decimals: $463 \div 15$; $29 \div 6$; $74 \div 23$.

(*e*) Three measurements of the length of one of the edges of a cylinder, gave the following results: 17·4 cm., 17·3 cm., and 17·1 cm. Find the average of the results.

(*f*) Find the average of the following lengths correct to two places of decimals: 7·43 in., 7·45 in., 7·39 in., 7·40 in., 7·42 in., 7·39 in.

(*g*) On seven successive days a barometer stood as follows: 30·14 in., 29·97 in., 29·92 in., 29·99 in., 30·25 in., 30·47 in., 30·34 in. What was the average height during the week?

(*h*) Find the value of the following, correct to three places of decimals: $22 \div 7$; $12{\cdot}5136 \div 3$; $15{\cdot}71 \div 9$.

(*i*) Find the average daily gain of a trader who makes profits of £1. 4s. on Monday, £1. 7s. 9d. on Wednesday, £1. 10s. on Thursday, £1. 3s. 6d. on Friday, and losses of 3s. 9d. on Tuesday and 2s. 6d. on Saturday.

The Circle and its Radius

81. Look again at the faces at the two ends of the cylinder. Each of them is called **a circle**, and each of the edges is called a **circumference.**

Try to draw a circumference, using only your pencil.

Draw another circumference in the following manner: Take a piece of thin string, about 12 cm. long, and tie a loop at each end.

Put a strong pin through one loop and press the pin firmly into your paper.

Hold the pin with the left hand, place a pencil through the other loop and draw a circumference, keeping the string stretched tightly.

Your teacher will now show you how to draw circumferences with the aid of your compasses.

Draw some circumferences for practice, and before you begin take care that the compass pencil has been sharpened to a chisel-edge (see § 60).

82. The method which you have just used in drawing circumferences will enable you to understand a a very important fact about them.

Take your compasses again, open the legs until the distance between them is exactly 4 cm., and then draw the circumference of a circle.

The point in the middle which has been pricked by one leg of the compasses is called the **centre of the circle.**

Draw several straight lines from the centre to the circumference and compare their lengths by using tracing-paper. Each of them is called a **radius** (plural, *radii*). (Fig. 12.)

You will now see that in drawing the circumference of a circle, the distance spanned by the legs of your compasses is a radius of the circle. You will also see that every part of the circumference of a circle is the same distance from the centre of the circle, this distance being the radius of the circle. For this reason, circles are usually described by stating the sizes of their radii; thus, the circumference of the circle you have just drawn is the circumference of a "circle of 4 cm. radius."

Fig. 12. *Radii.*

83. Exercise (drawing).

(*a*) Mark a point A on your paper, and with A as centre draw the circumference of a circle of 3 cm. radius.

(*b*) Mark another point B on your paper, and with B as centre draw the circumference of a circle of 3·7 cm. radius.

(*c*) With C as centre, draw the circumference of a circle of 4·4 cm. radius.

The Diameter of a Circle

84. Now look once more at the ends of your cylinder You will see that the centres of the circles are not marked, and that you cannot therefore describe the circles by stating the lengths of their radii. To get out of this difficulty you will have to learn something more about circles.

Draw the circumference of a circle of 4 cm. radius. Now draw several straight lines from the circumference through the centre to the circumference again, and compare their lengths by means of tracing-paper. Each line so drawn is called a **diameter**. (Fig. 13.)

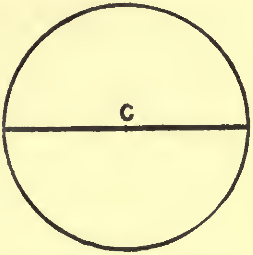


Fig. 13. A diameter.

How many times is a diameter longer than a radius?

85. Exercise (oral).

(*a*) State the lengths of the diameters of circles with the following radii: 7 cm.; 7·5 cm.; 14 cm.; 19·2 cm.; 12·7 cm.; 100 cm.

(*b*) State the lengths of the radii of circles with the following diameters: 5·5 cm.; 7·9 cm.; 13·7 cm.; 30·3 cm.; 27 cm.; 99 cm.

86. Now that you have learnt what diameters are, you will soon discover a fact of great importance about them.

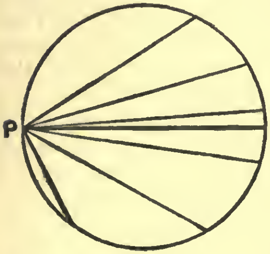


Fig. 14.

Draw the circumference of a circle with a radius of 3 cm., and mark any point, P, on it. From P draw a diameter and also several other lines which are terminated by the circumference, but which do not pass through the centre. (Fig. 14.)

Compare the lengths of all these lines, first by guessing, and second by using your dividers. Which is the longest?

How would you describe in words the longest straight line which can be drawn inside a circle?

87. You will now be able to understand a method for finding the diameters of the circles at the ends of the cylinder.

Your teacher will give you a pair of *calipers*. Notice that they are somewhat similar to your dividers and compasses.

Take the cylinder and open the legs of the calipers so that one of the ends of the cylinder will *just* pass through them. Remember that the diameter is the longest straight line which can be drawn inside a circle, and when, after several trials, you feel quite sure that the distance between the legs of the calipers is a diameter, measure the distance with your ruler.

You can now describe the circles at the ends of the cylinder by stating their radii. Do so, and record in your note-book.

88. Exercise (measurement).

(*a*) In order that you may get some practice in the use of calipers, take the coins and circular discs which are given to you and find their diameters. Record your results in your note-book, and in each case describe the circular faces of the objects by stating their radii.

(*b*) Draw 3 or 4 circles of different sizes and find the lengths of their diameters by using your dividers in the same way as you have been using the calipers.

Bisection of a Straight Line

89. If you are told that a straight line is the radius of a circle, you can draw the circle at once with the aid of your compasses. If, however, you are told that a straight line is the *diameter* of a circle, you will have to find the middle point of the line before you can draw the circle. You might of course find the middle point by measuring the line and dividing by 2, but there is another method which is often more convenient.

Draw a straight line about 3 in. long.

To find its middle point, take your dividers and, after making one or two trials, mark off equal distances from each end of the line—distances which you feel sure are about equal to half the length of the line.

If the two marks coincide, you have of course found the middle point of the line.

If the marks do not quite coincide, as A and B in

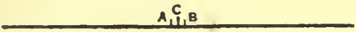


Fig. 15.

Fig. 15, the middle point must, of course, lie halfway between them. Take a pin, or your dividers, and prick off the middle point, C, by very careful guessing.

90. Exercise (drawing).

(*a*) Draw four lines of any convenient lengths in your note-book and **bisect** each of them, that is, divide each into two equal parts by finding its middle point.

(*b*) Draw circles of which the straight lines in (*a*) are diameters.

Diameter and Circumference of a Circle

91. You have now learnt how to measure the circumference, the diameter, and the radius of a circle, and you have found that the diameter of a circle is twice as long as its radius. Now you are going to discover how many times the circumference of a circle is longer than its diameter.

Draw the circumference of a circle having a radius of 3 cm.

Mark off six lengths of 3 cm. round the circumference with your compasses, and join the marks as in Fig. 16.

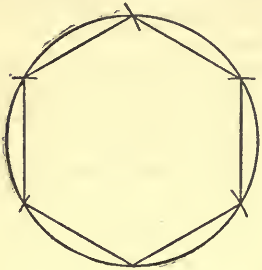


FIG. 16.

What is the total length of the six lines which have been drawn inside the circle?

How many times is this total length greater than a diameter?

Which is the longer, the circumference itself or the total length of the six lines?

About how many times is the circumference longer than a diameter?

92. Let us now try to get a more accurate result by measurement and calculation.

Find the length of the circumference at one end of the cylinder given to you, as in § 74; express the length in mm.

Take your calipers and find the length of the diameter in mm.

Calculate how many times the circumference is longer than the diameter; in dividing, work out the quotient correct to one place of decimals.

Make similar measurements and calculations with any other circular surfaces which are available.*

Tabulate your results. What do you notice about them?

93. You are now going to make some measurements, which will enable you to calculate still more exactly how many times a circumference is longer than its diameter.

* Cylindrical vessels of glass, earthenware and metal, such as jars, boxes, bottles and vases, would be useful for this purpose.

Draw the circumference of a circle, having a radius of exactly 5 cm. Mark a point on the circumference.

Take a strip of tracing-paper, about 35 cm. long and rule two lines on it, a long line and a short line, as shown in Fig. 17.

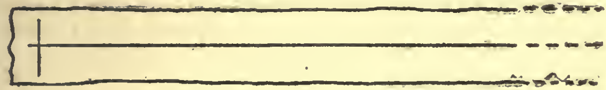


Fig. 17.

Take a sharp pin with a large glass head, and prick it through the tracing-paper where the two lines cross, and on to the mark which you have made on the circumference. The tracing-paper must lie flat on the paper on which the circumference has been drawn.

Now move the tracing-paper so that part of the long line coincides, or nearly coincides, with a small portion (say about 1 cm.) of the circumference, and remove

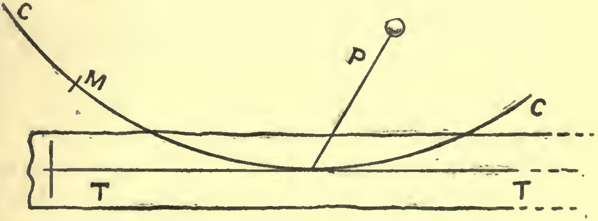


Fig. 18.

CC, circumference of the circle; M, mark on the circumference; P, pin; TT, tracing-paper.

the pin to the further end of the coinciding lines (see Fig. 18).

Work round the circumference gradually in the same way until you come to the mark again, and be careful that the tracing-paper does not slide while you are removing the pin from point to point.

Measure the total distance pricked out on the tracing-paper in cm.; this distance will, of course, be the length of the circumference.

Calculate how many times the circumference is longer than the diameter.

Perform two more experiments in the same way, and then find the average of your three results; work out the average correct to two places of decimals.

94. Exercise (oral).

(*a*) If circles have radii of the following lengths, what are the lengths of their diameters and circumferences: 1 cm.; 2 cm.; 3 cm.; 4 cm.?

(*b*) If circles have diameters of the following lengths, what are the lengths of their radii and circumferences: 1 cm.; 3 cm.; 5 cm.; 7 cm.?

(*c*) If circles have circumferences of the following lengths, what are the lengths of their diameters and radii: 3·14 cm.; 9·42 cm.; 6·28 cm.?

95. Exercise (to be written).

(*a*) Find the lengths of the circumferences of circles having the following diameters: 11 in.; 24 in.; 42 in.; 35 in.; 56 in.

(*b*) A bicycle wheel has a radius of 11 in. What is the length of its circumference?

(*c*) The diameter of a circle is 9 in. Three measurements of its circumference gave the following results: 28·25 in.; 28·30 in.; 28·23 in. Find the average of the results and calculate how many times the circumference is longer than the diameter (to two places of decimals).

(*d*) A cart-wheel with a diameter of one yard goes round 200 times. How far has the cart travelled?

The Use of Symbols

96. You have now found that the length of the circumference of a circle is equal to the length of its diameter multiplied by 3·14. It is often convenient to employ a kind of shorthand to express this fact.

If you suppose that the length of the circumference of a circle is represented by the letter l and the length of the diameter of the circle by the letter d, you can say that:—

$$l = d \times 3{\cdot}14.$$

This statement is called a **formula**, and the letters which it contains, l and d, are called **symbols**. The sign = means "is equal to" or "equals." The formula may therefore be read: "l equals d multiplied by 3·14."

For the sake of shortness, it is usual to omit the sign of multiplication in formulæ. It is also usual to place numbers before symbols. The above formula should therefore be written :—

$$l = 3{\cdot}14d.$$

Read out the formula and state what it means.

97. Now the number 3·14 occurs so often in calculations that it is found convenient to represent it also by a symbol. The symbol which everyone uses is the Greek letter π (pronounced "pi"). You can therefore amend the formula again and write it :—

$$l = \pi d.$$

Read out this formula and then write down in your note-book what it means.

98. You will now see that the formula in the last section is a useful shorthand way of writing down the rule for calculating the length of the circumference of a circle, if the length of the diameter of the circle and the value of π are known. Let us take an example and see how the sum is worked.

"Find the length of the circumference of a circle having a diameter of 5 in." Begin by writing down the formula; then, in place of the symbols on the right-hand side of the formula, write down the numbers which the symbols represent; and work out the sum. Thus :—

$$\begin{aligned} l &= \pi d \\ &= 3{\cdot}14 \times 5 \\ &= 15{\cdot}7 \text{ in.} \end{aligned}$$

99. Exercise (to be written).

Find the lengths of the circumferences of circles having the following diameters :—

(*a*) 22 in. (*b*) 36 in. (*c*) 49 in.

100. In the last few sections you have made many calculations to find the value of π, and in each case you have divided the length of the circumference of a circle by the length of the diameter of the circle. The formula with which you are dealing can therefore be written in another way :—

$$\pi = l \div d.$$

It is usual, however, to write $\frac{l}{d}$ (to be read : l over d) instead of $l \div d$. The formula should therefore be written :—

$$\pi = \frac{l}{d}.$$

Read out this formula and write down what it means.

Find the value of π when $l = 22$ in. and $d = 7$ in.

101. Exercise (oral).

If l and d represent the lengths of the circumference and of the diameter of a circle, and if π represents 3·14, complete the formula:—

(1) $d =$ (using the symbols l and π).
If $l = 6{\cdot}28$ in., what is the value of d?

If r represents the length of the radius of a circle, complete the formulæ (2), (3), and (4):—

(2) $d =$ (using the symbol r).
If $r = 1{\cdot}7$ in., what is the value of d?

(3) $r =$ (using the symbol d).
If $d = 2{\cdot}9$ in., what is the value of r?

(4) $l =$ (using the symbols π and r).

102. Exercise (to be written).

Write in a shorter way:—

(a) $d = 2 \times r$. (c) $d = l \div \pi$.
(b) $r = d \div 2$. (d) $l = 2 \times \pi \times r$.

Write down in words the meaning of each of the above formulæ.

Use the formula $l = 2\pi r$ and find the value of l when:—

(e) $r = 5$ in. (f) $r = 7$ in. (g) $r = 44$ in.

103. Exercise (measurement).

Your teacher will give you an iron washer. Take your calipers and measure the internal and external diameters in mm., and then calculate the lengths of the corresponding circumferences by means of the formula $l = \pi d$.

Alternative Exercise: Find the lengths of the circumferences of the coins, rings, and circular discs which are given to you.

Drawing: Draw some circumferences of circles in your note-book, measure their radii in mm., and calculate the lengths of the circumferences by means of the formula $l = 2\pi r$.

An Exercise on the Cone

104. You are provided with a cone. Write down the number of its faces, edges, and corners, and make a hand-sketch of it.

Take your ruler and measure (in cm.) the "slant height" of the cone, that is the distance between the edge and the corner; in making this measurement take care that the ruler touches the cone along its whole height.

Now take your calipers and measure the "vertical height," that is the distance between the centre of the circle and the corner.

Measure also the diameter with the calipers.

Enter the measurements on the hand-sketch.

105. Exercises on the Cone (to be written).

(*a*) Write down the length of the diameter in mm., and then calculate the length of the circumference.

(*b*) How many times is the vertical height longer, or shorter, than the radius? (Work in mm.).

(*c*) If the radius were increased by 4·7 cm., by how much would the diameter and the circumference be increased?

(*d*) If the diameter were decreased by 2·8 cm., by how much would the radius and the circumference be decreased?

Additional Formulæ

106. Symbols are useful not only in dealing with the circumferences and diameters of circles, but also in dealing with prisms and other models. Let us take one or two examples.

If c represents the length of one of the edges of a cube and L represents the total length of the edges, complete the formula :—

$$L= \quad .$$

Write down the meaning of this formula, and find the value of L when $c=9$ in.

Again, if s represents the length of one of the 8 short edges of a square prism, if p represents the length of one of the 4 long edges, and if L represents the total length of the edges, complete the formula :—

$$L= \quad + \quad .$$

Write down the meaning of this formula, and find the value of L when $s=6\cdot7$ in. and $p=12\cdot3$ in.

107. Exercise (to be written).

(*a*) If D represents the difference between the total length of the short edges and the total length of the long edges of the square prism mentioned in the last section, complete the formula : $D=$

Write down the meaning of the formula, and find the value of D, when $s=6\cdot7$ in. and $p=12\cdot3$ in.

(*b*) Make a formula which could be used in calculating the total length of the edges of an oblong prism.

(*c*) Make a formula which could be used in calculating the total length of the edges of a hexagonal prism.

(*d*) What number does x stand for, if x mm. $=2\cdot7$ cm.?

(*e*) What number does y stand for, if y mm. $=3\cdot45$ dm.?

(*f*) How many shillings are there in : £p; £q; £r?

(*g*) How many cm. are there in : f dm.; g metres?

(*h*) What number does v stand for, if £$v=10$s.?

Brackets

108. Your teacher will give you a square prism.

Let us suppose that a long edge is x cm. long, and that a short edge is y cm. long. What is the total length of a long edge and a short edge? By how much is a long edge greater than a short edge?

Let us now suppose that the short edges are each lengthened by 2 cm. What will be the length of a short edge? By how much will a long edge be greater than a short edge?

109. You will now see how convenient it is to make use of brackets in expressions which contain symbols. In the last paragraph the length of a long edge is x, and the length of a short edge is $y+2$. If D represents the difference between the length of a long and a short edge, we may say that

$$D=x-(y+2).$$

This is another example of a formula. It means that D is equal to $(y+2)$ subtracted from x.

Take your ruler, measure the lengths of a long and a short edge, and then use the formula to calculate the value of D.

110. Let us suppose that x cm. and y cm. represent the lengths of a long and a short edge of a square prism, and that each of the short edges is shortened by 2 cm. What will be the length of a short edge?

Write down a formula showing the difference (D) between the lengths of a long edge and a short edge. Write in words the meaning of the formula.

Find the value of D from the measurements which you made in the last section.

111. It is very often convenient to remove the bracket before calculating the result. In some cases the removal of brackets presents no difficulty, but when a bracket is preceded by a minus sign, you will have to take great care.

Look at the formula in § 109: $D=x-(y+2)$. You have to subtract $(y+2)$ from x. If you subtract y alone and write $x-y$, have you subtracted too much or have you subtracted too little? Would you therefore write $+2$ or -2 after $x-y$? Now write down the whole formula without using a bracket.

Deal with the formula $D=x-(y-2)$ in the same way.

If $a=7$, $b=4$, and $c=2$, verify the following formulæ:—

$$a-(b+c)=a-b-c$$
$$a-(b-c)=a-b+c$$
$$a+(b-c)=a+b-c$$
$$a+(b+c)=a+b+c$$

Now try to write down a rule for removing brackets preceded (1) by a plus sign; (2) by a minus sign.

112. Exercise (to be written).

If $x=4$, $y=3$, and $z=2$, remove the brackets from the following expressions, and then calculate their value:—

(*a*)	$4x+(y+z)$	(*e*)	$4+(x+y+z)$
(*b*)	$4x+(y-z)$	(*f*)	$10-x-(y-z)$
(*c*)	$5y-(x+z)$	(*g*)	$10-(x-y-z)$
(*d*)	$4y-(2x-z)$	(*h*)	$24-(x-z)-y$

113. You are provided with a cube.

Let us suppose that each of the edges is x cm. long. What will be the length of all the edges?

Now suppose that each edge is lengthened by 3 cm. If L represents the length of all the edges, we may say that

$$L=12\ (x+3).$$

This means that L is equal to $(x+3)$ multiplied by 12. Take your ruler, measure the length of a side and find the value of L.

Write down a formula showing what would be the length of all the edges, if each edge were shortened by 3 cm.

114. If $a=5$, $b=4$, and $c=2$, verify the following formulæ:—

$$a(b+c)=ab+ac$$
$$a(b-c)=ab-ac$$

Try to write down a rule for multiplying the sum or the difference of two symbols by another symbol.

115. Exercises (to be written).

(*a*) If the four long edges of a square prism are p in. long, and the eight short edges are q in. long, what is the length of all the edges?

If the long edges are each lengthened by 2 in., and the short edges are each shortened by 3 in., what will be the length of all the edges?

(*b*) In the last exercise, what would be the total length of all the edges, if the long edges were shortened by r in., and the short edges were lengthened by s in.?

(*c*) An oblong prism has edges of three different lengths represented by x cm., y cm., and z cm. What would be the length of all the edges of the prism, if the edges were respectively shortened by a cm., b cm., and c cm.?

(*d*) A cube has an edge of 4·7 in. A second cube has an edge which is 2·3 in. shorter than an edge of the first cube. A third cube has an edge which is seven times shorter than an edge of the second cube. What is the length of all the edges of the third cube?

(*e*) The hand of a watch is 2 cm. long. Through what distance does the end of the hand move in a quarter of an hour?

(*f*) The diameter of a circle is 7 in., and its circumference is 21·99 in. Find the value of π correct to three places of decimals.

(*g*) The diameter of a bicycle track is 100 yards. How far does a man travel who completes seventeen laps?

(*h*) If $a=9$ and $b=8$, find the value of:—

$$a+b;\quad a-b;\quad ab$$

$$\frac{a}{b};\ \frac{b}{a};\ 139-(a-b).$$

(*i*) Write down in tabular form the number of faces, edges, and corners of the following models: cube, triangular prism, cylinder, cricket ball, hexagonal prism, cone.

(*j*) Reduce the following to shillings:—

£21	£x
£21. 3s.	£x. ys.

(*k*) Reduce to pence:—

£19. 3s. 4d. £x. ys. zd.

(*l*) Show that:—

$$10-(5-3) = 10-5+3$$

(*m*) If $x=4{\cdot}75$ and $y=2$, find the value of the following, correct to two places of decimals:—

$$27x+yx-14;\ 3x-(11-5y).$$

(*n*) What is the average of the following, correct to one place of decimals: 17·1, 14·4, 3, 0, 149, 9·8, 237·3?

(*o*) What number added to 2·8 gives 7·17?

(*p*) What number subtracted from 4·37 gives ·06?

(*q*) What number multiplied by 17 gives a product of 74·63?

(*r*) What number when divided by 17 gives a quotient of 4·39?

(*s*) Find the value (correct to three places) of:

$$37{\cdot}1124 \div 9, \quad \frac{6{\cdot}7}{9}, \quad \frac{1}{9}$$

(*t*) Remove the brackets:—

$$2a\,(b+c) - d - (e-f)$$

(*u*) Express in a simpler way by using brackets: $pq+pr+ps$; $ab+bc+ac$.

(*v*) What is the value of: $144 - 2\,(14 - 7)$.

(*w*) Add the following and express your answer (1) in m.; (2) in dm.; (3) in cm.; and (4) in mm.:—

m.	dm.	cm.	mm.
1	4	9	7
3	0	4	2
17	6	3	0
	5	0	9

(*x*) Subtract 7 dm. 4 cm. 9 mm., from 1 m. 6 dm. 3 cm. 4 mm.

(*y*) Find the average scores (correct to two places of decimals) of some of the following cricketers:—

BATTING (1903).

	No. of innings.	Times not out.	Most in an innings.	Total runs.
C. B. Fry . .	40	7	234	2683
Ranjitsinhji . .	41	7	204	1924
A. J. L. Hill .	12	1	150	515
Iremonger . .	31	1	210	1380
Knight . .	46	6	229*	1834
P. Perrin . .	36	4	170	1428
Tyldesley . .	50	6	248	1955
A. C. MacLaren .	52	8	204	1886
W. G. Quaife .	30	4	130	1113
J. Gunn . .	42	3	294	1665
H. K. Foster .	41	3	216	1596
Hirst . . .	44	5	153	1844
C. M. Wells . .	13	4	82*	371

* Signifies "not out."

(z) Find the averages of some of the following bowlers :—

BOWLING (1903).

	Overs.	Maidens.	Runs.	Wickets.
Ringrose . .	195	51	485	36
Mead . . .	971	355	1791	131
Blythe . .	925	292	1953	142
Langford . .	241	72	586	42
Hargreave . .	922	282	1879	134
Hirst (G. H.) .	772	221	1892	127
Rhodes (W.) .	1348	419	2728	189
Hearne (J. T.) .	866	294	1942	127
Moorhouse . .	377	115	870	55

CHAPTER III

INTRODUCTION TO SCALE-DRAWING

Scales

116. Measure the length and breadth of the top of your desk or work-table,* and express the measurements in cm. Record the measurements in your note-book.

117. Let us suppose that the length is 62·4 cm., and the breadth 43 cm.

If you wish to draw the top of the table in your note-book, you will of course be compelled to make the drawing very much smaller than the table. Make a hand-sketch of the top of the table, as you would see it if you were looking down on it from above, and mark the dimensions on it (see § 30).

118. Measure the lengths of the lines in your hand-sketch, and record your results in the following manner :—

(*a*) Line representing length of table in hand-sketch = cm.
(*b*) Real length of table = cm.
(*c*) Line representing breadth of table in hand-sketch = cm.
(*d*) Real breadth of table = cm.

* A drawing-board or other object of suitable size might be used if the work-table is too large.

Calculate how many times (*b*) is longer than (*a*), and how many times (*d*) is longer than (*c*). If you have any difficulty with the division, first turn the cm. into mm. Make a record of the results in this way:—

(*e*) Real length ÷ the length as drawn =
(*f*) Real breadth ÷ the breadth as drawn =

119. The results of the measurements and calculations which you have just made will, in all probability, show you two things—

First, that (*e*) and (*f*) are not whole numbers. Your drawing is not therefore a whole number of times smaller than the table.

Secondly, that (*e*) and (*f*) are not the same. This means that your drawing is not accurate. You have made the length (say) 12·4 times smaller than the length of the table, and you have made the breadth (say) 8·9 times smaller than the breadth of the table. Your drawing is not therefore "in proportion"; you ought to have drawn the breadth smaller or the length bigger. If you had made a correct drawing, *all* the lines in it would have been the same number of times smaller than the corresponding lines on the table.

120. You cannot, of course, be expected to be precisely accurate when you are making a hand-sketch, but with the aid of instruments it is possible to make drawings which are very exact.

Take your ruler and make a ruled drawing which is exactly ten times smaller than the top of the table. If the breadth of the table is 43 cm. you will of course make the breadth in your drawing 4·3 cm. If the length of the table is 62·4 cm., the length in your drawing should be as near 6·25 cm. as you can make it. You cannot draw lines correct to less than ·05 cm., that is half a mm.

121. The drawing which you have just made is called a **drawing to scale.**

In a drawing to scale, the lines are all drawn in exact proportion, and the drawing is always a known number of times smaller (or larger) than the object represented.

Indication of the Scale of a Drawing

122. You may indicate how many times a drawing to scale is smaller (or larger) than the object in several ways.

Turn to your drawing of the top of the table and write under it: "Drawing of a top of a table to a scale of one-tenth." This means that every line in the drawing is one-tenth as long as (or ten times shorter than) the corresponding line on the table.

The same fact may also be expressed in this way: "Drawing of the top of a table to a scale of 1 mm. to 1 cm." This means that a line on the drawing which is x mm. long, represents a line on the table which is x cm. long.

123. Instead of describing the scale in words or figures, it is often convenient to draw the scale.

Here, for example, is a scale (Fig. 19) which will

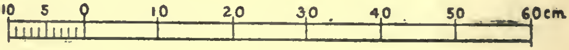


Fig. 19.

enable you to measure your drawing of the top of the table.

You will notice that the distance between two of the larger divisions represents 10 cm., though the real distance is only 1 cm.

The smaller divisions on the left represent cm., though in reality they are only mm.

Observe that the mark "0" is not at the end of the scale. You will see the advantage of this, if you proceed to make use of the scale.

124. Draw a line 57 cm. long to a scale of 1 mm. to 1 cm., in the following manner:—

Take your dividers, place one leg on the mark "50" and stretch out the other leg until it arrives at the seventh division to the left of the mark "0." The distance between the legs of the dividers now represents a length of 50 cm. + 7 cm. = 57 cm. Now draw the line required (see § 64).

125. Whenever you make a drawing to scale, do not forget that it is necessary either to give a description of the scale (§ 122), or to make a drawing of the scale (§ 123).

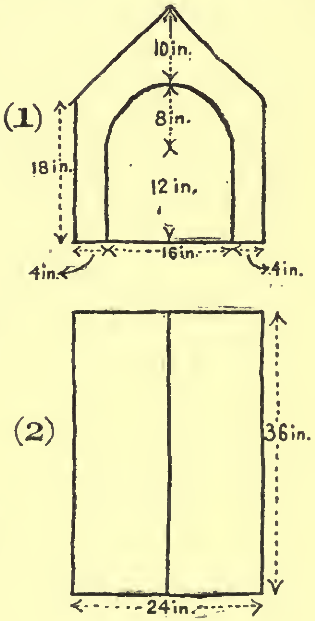


FIG. 20.

126. Exercise.

(*a*) Use the scale given in § 123 and draw lines representing:—

64 cm., 8 cm., 29 cm., 46 cm., 13 cm.

(*b*) Draw the circumference of a circle with a diameter of 54 cm. to a scale of one-tenth.

(*c*) Construct a scale of one-tenth of an inch to an inch, long enough to measure 40 in. (In doing this, let the divisions to the right of the mark "0" represent 10 in. and those to the left 1 in.)

(*d*) Fig. 20 shows hand-sketches of a dog-kennel as you would see it (1) if you were looking at it from the front; (2) if you were looking down on it from above.

Use the scale which you have just constructed, and make two drawings of the kennel to scale, placing (1) immediately above (2) as shown in the hand-sketches.

Varieties of Scales

127. Drawings are frequently made to a scale of much less than one-tenth. All maps are drawings to scale, and, as you know, they are very much smaller than the countries which they represent.

The scale of a map depends upon the purpose for which the map is required. Cycling maps are often drawn to a scale of 1 in. to 2 miles. The maps in your atlas are drawn to a much smaller scale even than this.

Look at a map of England and then at a map of Europe. Notice the scales which are drawn or described in the margins of the maps. In which of the maps is the scale the larger?

128. Exercise.

Find the distances between the following places as accurately as you can:—

(*a*) Manchester and Leeds.
(*b*) London and Dover.
(*c*) Berwick and Land's End.

Measure the lengths of the following rivers and railways by using tracing-paper, as described in § 93:—

(*d*) The Thames.
(*e*) The Tees.
(*f*) The S.W. Railway from London to Exeter.
(*g*) The G.W. Railway from London to Exeter.

129. Sometimes it is necessary to make drawings to scale which are largèr than the objects themselves. Here, for example, is a drawing (Fig. 21) of the Green Fly. You can plainly see the marks on its body and wings, but these marks would hardly have been visible if the drawing had been no larger than the fly.

Measure the length of the fly's body, and the distance between the tips of its wings in mm.

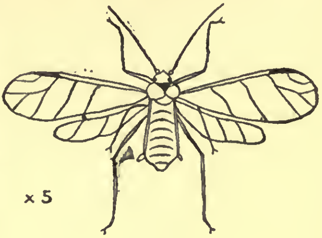


Fig. 21.

130. Drawings of objects which are seen under the microscope are often hundreds of times larger than the objects themselves. The drawing (Fig. 22) represents a small portion of the Green Mould which grows on cheese.

Measure the height of the mould in mm.

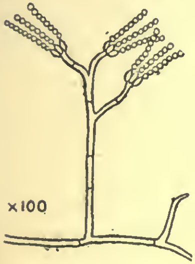


Fig. 22.

131. Exercise.

The lines in Fig. 23 are drawn to a scale of 1 in. to 1 cm. What is the length of the lines which they represent?

Squared Paper

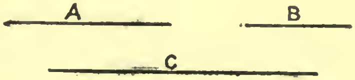


Fig. 23.

132. Examine some squared paper (mm.). Notice that each of the little squares has sides which are 1 mm. long. Do you notice any other squares? If so, what are the lengths of their sides?

Squared paper is very useful in making drawings to scale.

133. Exercises.

(*a*) Turn to your hand-sketch of the top of the table referred to in section 117, read off the dimensions and make a drawing to a scale of one-tenth on squared paper.

Draw lines joining the opposite corners of your drawing, and measure their lengths by using your dividers.

(*b*) Make a similar drawing of the back of your note-book.

(*c*) Get out your atlas and turn to the map of England.

Draw a straight line on squared paper to a scale of 1 mm. to 1 mile, representing the distance between Sheffield and Southampton, and mark on it the points at which the principal rivers cross the line.

(*d*) Let two scholars measure the door of one of the cupboards in the classroom in inches, and write the dimensions on the blackboard.

Take some squared paper (tenth of an inch), and make a drawing of the door to a scale of one-tenth.

Draw lines joining the opposite corners of your drawing, and measure their lengths by using your dividers.

(*e*) Make a drawing to scale of the playground from measurements made by the class.

(*f*) AB is a straight road 6 miles long, between the two towns A and B.

At C there is a cricket ground, at D the road crosses a canal, and at E there is a cotton mill. AC = 1·05 miles, CD = 2·7 miles, EB = 1·6 miles. (Fig. 24.)

Fig. 24.

Draw the road on squared paper to a scale of an inch to a mile, and mark the positions of C, D, and E.

(g) Fig. 25 shows hand-sketches of one of

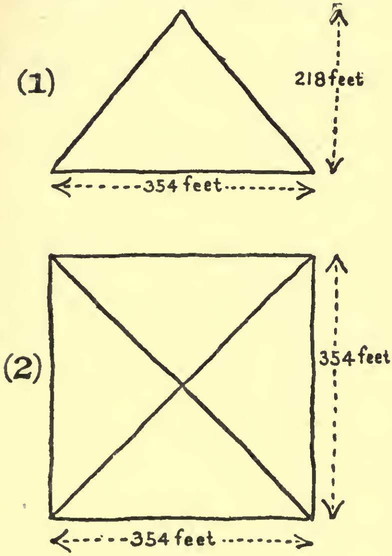


Fig. 25.

the Egyptian Pyramids as you would see it (1) if you were looking at it from the ground; (2) if you were looking down on it from a balloon.

Make two drawings of the pyramid on squared paper to a scale of one hundredth of an inch to one foot, placing (1) immediately above (2), as shown in the hand-sketches.

Plans and Elevations

134. A large cylinder, such as is used in the Art Room, is placed on one of its ends.

Let two scholars measure its height and its diameter in inches (see §§ 79 and 87), and write the dimensions on the blackboard.

Take some squared paper and make a front view and a top view of the cylinder to a scale of 1 cm. to 1 in.; draw the front view first and the top view immediately beneath it.

The top view is called the **plan**, and the front view is called the **elevation.**

135. Look at the drawings in Fig. 26. A is a sketch of a house. B is the plan of the house; it shows only the roof, chimneys, and the tops of the bay windows. C is the **front elevation**, and D is a **side elevation.**

What other elevations should be drawn in order to provide a complete description of the exterior of the house?

Why does a smaller number of elevations suffice in the case of the cylinder?

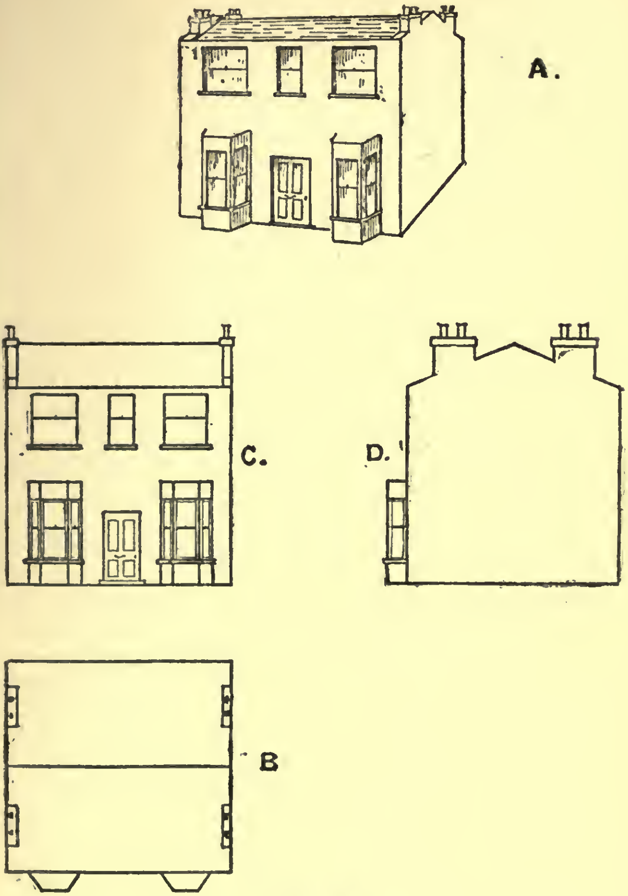


FIG. 26.

CHAPTER IV

ENGLISH MEASURES OF LENGTH, AND VULGAR FRACTIONS

Meaning of a Vulgar Fraction

136. Take a piece of string and divide it into two equal parts by folding and cutting it.

Again divide each of these parts into two equal parts.

Repeat the operation a third time.

Into how many equal parts have you now divided the string?

137. A part of any whole is called a *fraction* of the whole. If the whole is divided into two *equal* parts, each part (or fraction) is called a half; if the whole is divided into three equal parts, each part (or fraction) is called a third; if the whole is divided into four equal parts, each part (or fraction) is called a fourth (or a quarter), and so on.

What fraction of the whole string is each piece of string which you have cut?

138. Exercise (oral).

(*a*) What fraction of a penny is $\frac{1}{2}$d., and $\frac{1}{4}$d.?
(*b*) What fraction of an hour is five minutes, and ten minutes?

(*c*) What fraction of a week is one day?
(*d*) What fraction of a year is one month?
(*e*) What fraction of an hour is 12 minutes, and three minutes?
(*f*) What fraction of a shilling is 2d.?
(*g*) If a thing is divided into sixteen equal parts, what fraction is each of these parts?
(*h*) Take a slip of paper with a straight edge; fold it into halves, quarters, and eighths.
(*i*) If a thing is divided into x equal parts, what fraction is each of these parts?

139. If you divided a thing into ten equal parts, each part would be a tenth, and three of these parts would make a fraction of three-tenths.

What fractions of the whole string are 2, 3, 4, 5, 6, 7, and 8 of the small pieces into which you have cut it?

140. Write down in figures one farthing.

What you have written is a fraction, or, as it is sometimes called to distinguish it from a Decimal Fraction, a **Vulgar Fraction.**

Write in figures a halfpenny, and three farthings. What do the figures 2, 4, 1, 3 mean as you have written them?

141. The lower figure in a Vulgar Fraction is called the **Denominator**, that is, the Namer, because it names the part which is taken, *e.g.*, a fourth or a half. The upper figure in a Vulgar Fraction is called the **Numerator**, that is, the Numberer, because it tells what number of parts is taken, *e.g.*, one or three.

142. Exercise (oral).

(*a*) In the fractions $\frac{3}{4}$, $\frac{5}{6}$, $\frac{11}{16}$, $\frac{7}{10}$, $\frac{19}{24}$, $\frac{1}{12}$, which are the numerators?
(*b*) Which are the denominators?
(*c*) What part of the whole is named by each of these denominators?

(*d*) Read off the fractions in (1).
(*e*) What is the meaning of the word *farthing*?

Equivalent Fractions

143. Exercise (to be written).

(*a*) Write down all the fractions with 16 as denominator, from one-sixteenth to sixteen-sixteenths.
(*b*) Do the same with eighths, quarters, halves, tenths and twelfths.
(*c*) What fraction is each of the lines A, B, C, D of the line above it? (Fig. 27.)

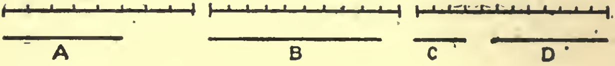


FIG. 27.

144. "Two farthings," that is, "two-fourths of a penny," might be written $\frac{2}{4}$d., but it is usually written $\frac{1}{2}$d. Similarly, $\frac{2}{4}$ of anything is usually written $\frac{1}{2}$. Let us see why this is.

Look at the several edges of your ruler. One edge is marked with sixteenths of an inch, and there are larger divisions showing halves, quarters, and eighths. Another edge is marked with tenths of an inch: what larger divisions are there? A third edge is marked with twelfths of an inch; what larger divisions are there?

145. Fig. 28 shows drawings to an enlarged scale of an inch and the several divisions of an inch as seen on your ruler.

Carefully inspect the figure, and write down in tabular form all the fractions which are equal to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{4}$, and 1.

146. You observe that $\frac{1}{2}$ has the same value as $\frac{2}{4}$ or $\frac{4}{8}$. What must you do to the numerator and denominator of $\frac{1}{2}$ in order to make it $\frac{2}{4}$ or $\frac{4}{8}$? Ask the same question about each of the fractions in the table, and its equivalents. Complete the following statement, and

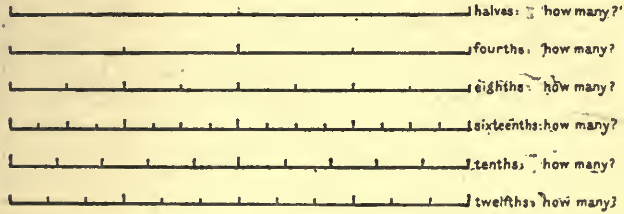


FIG. 28.

enter it in your note-book: "The value of a fraction remains the same when . . ."

What must you do to the fraction $\frac{4}{8}$ in order to make it $\frac{1}{2}$?

Now make the rule you have written in your note-book more complete.

147. Exercise (to be written).

(*a*) Express each of the following fractions in another way: $\frac{3}{8}$, $\frac{10}{12}$, $\frac{4}{5}$, $\frac{10}{16}$, $\frac{6}{10}$, $\frac{5}{6}$.

(*b*) Express each of the following in two other ways: $\frac{6}{8}$, $\frac{8}{12}$, $\frac{4}{16}$.

(*c*) Draw lines, from the enlarged scale in § 145, of $\frac{7}{8}$ in., $\frac{11}{12}$ in., $\frac{9}{10}$ in.

(*d*) Draw lines with your ruler of $\frac{7}{8}$ in., $\frac{11}{12}$ in., $\frac{9}{10}$ in.

(*e*) Now use your dividers or compasses to find what the scale is.

148. Exercise (oral).

(*a*) How many eighths are there in $\frac{1}{2}$, in $\frac{1}{4}$, in $\frac{3}{4}$?

(*b*) What fractions are equal to $\frac{1}{3}$, $\frac{1}{6}$, $\frac{2}{3}$, $\frac{2}{5}$? Are the fractions here mentioned anywhere marked on your ruler?

(*c*) How many quarters are there in $\frac{4}{16}$, in $\frac{12}{16}$, in $\frac{1}{2}$?

149. We now come back to the question why $\frac{2}{4}$ is commonly written $\frac{1}{2}$. You have seen that $\frac{2}{4}$ has the *same value* as $\frac{1}{2}$, so that it does not really matter whether you write $\frac{2}{4}$ or $\frac{1}{2}$. But you write $\frac{1}{2}$ rather than $\frac{2}{4}$, because the former is the simplest form of the fraction. A fraction is in its simplest form (or in its **lowest terms**) when there is no whole number which will divide both numerator and denominator.

150. Exercise (to be written).

(*a*) Reduce to their lowest terms, $\frac{10}{16}$, $\frac{4}{12}$, $\frac{12}{16}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{6}{6}$.

(*b*) Verify your results by referring to the ruler.

(*c*) What would you write instead of $\frac{8}{8}$, $\frac{10}{10}$, $\frac{16}{16}$? and why?

(*d*) Write down all the fractions from one-eighth to eight-eighths, and reduce each to its lowest terms.

(*e*) Do the same with twelfths, tenths, and sixteenths.

(*f*) Reduce to their lowest terms:—

$$\frac{3}{3x},\ \frac{x}{5x},\ \frac{x}{x},\ (x \text{ being any number}).$$

Mixed Numbers

151. When you write $2\frac{1}{4}$d., you mean, not two farthings, but twopence farthing—that is, two *and* a quarter pence. Similarly, $2\frac{1}{4}$ in. means two *and* a quarter inches.

Read out $2\frac{1}{3}$ in., $3\frac{3}{16}$ in., $3\frac{1}{7}$, $10\frac{3}{4}$d., $1\frac{5}{12}$ in., $6\frac{2}{3}$.

These numbers, made up of wholes and fractions, are called **mixed numbers**.

152. You are given an oblong prism.

Measure several edges of the model in inches and sixteenths; probably some of the edges do not measure an exact number of sixteenths. You might, therefore, express the length of an edge as $2\frac{5}{16}+$ in., meaning $2\frac{5}{16}$ in. and a bit more.

But, if necessary, you can express the measurement more accurately. Imagine a sixteenth to be divided into two parts; what fraction of an inch would each part be?

You can imagine *each* sixteenth to be so divided, though the divisions are not marked on your ruler.

153. Exercise (requiring the use of the square prism).

(*a*) Make a hand-sketch of the square prism. With the edge of the ruler marked with sixteenths measure each edge of the model as accurately as you can; in measuring, make use of imaginary divisions. Mark these measurements on the hand-sketch.

(*b*) Make a plan and elevation of the model to a scale of $\frac{1}{1}$.

154. Again, you can imagine each tenth or twelfth to be divided into two parts, or even into four parts. What fraction of an inch would each of these parts be?

Fractional Scales

155. You will now find it easy, with the help of your ruler, to draw fractional scales. Let us suppose that you have to draw a scale of $\frac{1}{4}$ to measure inches and eighths.

Look at the sixteenths edge of the ruler. Each quarter-inch must represent 1 inch, each sixteenth must represent $\frac{1}{4}$ inch. You must represent eighths of an inch by dividing the quarter-inch spaces on the scale into two. Do this by careful guessing.

In this way you will get a scale like the following:—

FIG. 29.

Draw to the above scale lines of $4\frac{1}{2}$ in., $3\frac{1}{4}$ in., $10\frac{1}{8}$ in., $11\frac{7}{8}$ in.

156. Exercise (requiring the use of the oblong prism).

(*a*) Make a hand-sketch of the prism. With the twelfths edge of the ruler measure each edge of the model as accurately as you can in inches and fractions of an inch. Mark the measurements on the hand-sketch.

(*b*) Using the twelfths edge of the ruler, draw a scale of $\frac{1}{2}$ to measure inches and twelfths.

(*c*) Draw a plan and elevation of the model to this scale. Make use of imaginary divisions on the scale.

(*d*) Using the twelfths edge of your ruler, draw a scale of $\frac{1}{3}$ to measure inches and eighths.

(*e*) Draw to a scale of $\frac{1}{3}$ lines of $5\frac{1}{8}$ in., $3\frac{11}{16}$ in., and $10\frac{7}{8}$ in.

Addition and Subtraction of Fractions

157. You are now to learn how to add fractions together. 3 apples and 1 apple make 4 apples; in the same way, 3 sixteenths and 1 sixteenth make 4 sixteenths.

You write this down as follows :—

$$\tfrac{3}{16} + \tfrac{1}{16} = \tfrac{4}{16} \text{ (or } \tfrac{1}{4}).$$

In the same way :—

$$\tfrac{3}{16} - \tfrac{1}{16} = \tfrac{2}{16} \text{ (or } \tfrac{1}{8}).$$

158. Exercise (oral).

(*a*) To $\frac{3}{8}$ add $\frac{5}{8}$; to $\frac{1}{12}$ add $\frac{5}{12}$; to $\frac{3}{10}$ add $\frac{1}{10}$.

(*b*) From $\frac{5}{8}$ take $\frac{3}{8}$; from $\frac{5}{12}$ take $\frac{1}{12}$; from $\frac{3}{10}$ take $\frac{1}{10}$.

(*c*) Simplify

$\frac{3}{16} + \frac{5}{16}$, $\frac{9}{20} - \frac{7}{20}$, and $\frac{5}{24} + \frac{11}{24} - \frac{7}{24}$.

(*d*) Add together $\frac{1}{8}$ and $\frac{3}{8}$,
$\frac{3}{16}$ and $\frac{9}{16}$,
$\frac{7}{20}$ and $\frac{17}{20}$.

159. You probably noticed that in the last sum you got a fraction with the numerator greater than the denominator. Such a fraction is called an **improper fraction.**

160. You have seen that

$$\tfrac{20}{20} = 1; \quad \therefore \tfrac{24}{20} = \tfrac{20}{20} + \tfrac{4}{20} = 1\tfrac{4}{20} \text{ (or } 1\tfrac{1}{5}).$$

Similarly, $2\frac{3}{16} = \frac{32}{16} + \frac{3}{16} = \frac{35}{16}$.

It is usual to write a quantity as a mixed number rather than as an improper fraction; but there are problems, as we shall see, in which we must make use of improper fractions.

161. Exercise (oral).

(*a*) Express as mixed numbers: $\frac{17}{16}$, $\frac{27}{12}$, $\frac{9}{2}$, $\frac{31}{4}$.

(*b*) Express as improper fractions: $1\frac{7}{8}$, $2\frac{5}{12}$, $3\frac{1}{3}$, $10\frac{7}{8}$.

(*c*) Which of the following numbers are greater than 1? $\frac{9}{12}$, $\frac{9}{8}$, $\frac{10}{2}$, $\frac{10}{10}$?

162. Exercise (to be written).

(*a*) Write down six improper fractions, and opposite each the mixed number equivalent to it.

(*b*) Write down six mixed numbers, and opposite each the improper fraction equivalent to it.

(*c*) Add together $\frac{11}{12}$ and $\frac{7}{12}$, $\frac{9}{16}$ and $\frac{7}{16}$, $\frac{19}{20}$ and $\frac{17}{20}$ and $\frac{13}{20}$.

(*d*) Simplify $\frac{11}{12} + \frac{7}{12} + \frac{5}{12} + \frac{1}{12} - \frac{3}{12}$.

163. You ought now to be able to make a rule for converting mixed numbers into improper fractions, and *vice versâ*.

Enter the rule in your note-book.

164. When you have to do an addition or subtraction sum with mixed numbers, deal with the whole numbers first, thus:—

$$3\tfrac{3}{16} + 2\tfrac{9}{16} = 5 + \tfrac{3}{16} + \tfrac{9}{16}$$
$$= 5\tfrac{12}{16}$$
$$= 5\tfrac{3}{4}.$$

Again, $$7\tfrac{11}{12} - 4\tfrac{7}{12} = 3 + \tfrac{11}{12} - \tfrac{7}{12}$$
$$= 3\tfrac{4}{12}$$
$$= 3\tfrac{1}{3}.$$

165. Exercise (to be written).

(*a*) Add together $5\frac{1}{8}$ and $7\frac{3}{8}$, $9\frac{7}{20}$ and $1\frac{11}{20}$, $6\frac{5}{12}$ and $\frac{1}{12}$.

(*b*) Simplify

$$1\tfrac{1}{16} + 2\tfrac{3}{16} + 3\tfrac{5}{16} + 4\tfrac{7}{16} + 5\tfrac{9}{16} - 6\tfrac{11}{16}.$$

(*c*) A square prism is $4\frac{9}{24}$ in. broad, and $8\frac{13}{24}$ in. high. Find the length of a long and a short edge together.

(*d*) Find the difference between a long and a short edge.

166. Exercise (oral).

(*a*) Read out the formula : $\frac{a}{x} + \frac{b}{x} = \frac{a+b}{x}$.

What does it mean, *a*, *b*, etc., being any numbers?

(*b*) Read out the formula :—

$\frac{a}{x} + \frac{b}{x} - \frac{c}{x} = \frac{a+b-c}{x}$. What does it mean?

(*c*) Simplify $\frac{1}{x} + \frac{3}{x} + \frac{5}{x}$.

(*d*) What is the value of the fraction, if $x = 12$?

167. You have learnt that $1\frac{17}{16}$ has the same value as $2\frac{1}{16}$, but that it is usual to write $2\frac{1}{16}$. Let us see when it would be convenient to write $1\frac{17}{16}$ instead of $2\frac{1}{16}$:—

$$3\tfrac{1}{16} - 1\tfrac{11}{16} = 2\tfrac{1}{16} - \tfrac{11}{16}.$$

We cannot, however, subtract $\frac{11}{16}$ from $\frac{1}{16}$.

But since $2\frac{1}{16} = 1\frac{17}{16}$,

$$\begin{aligned} 3\tfrac{1}{16} - 1\tfrac{11}{16} &= 2\tfrac{1}{16} - \tfrac{11}{16}. \\ &= 1\tfrac{17}{16} - \tfrac{11}{16} \\ &= 1\tfrac{6}{16} \\ &= 1\tfrac{3}{8}. \end{aligned}$$

168. Exercise (requiring the use of models).

(*a*) Subtract $3\frac{7}{12}$ from $5\frac{5}{12}$, $1\frac{7}{8}$ from $2\frac{3}{8}$, $6\frac{3}{10}$ from $10\frac{1}{10}$.

(*b*) Simplify

$2\frac{7}{20} + 3\frac{9}{20} - 4\frac{19}{20}$, and $30 + \frac{1}{24} - 20\frac{11}{24}$.

(*c*) A cube is $14\frac{1}{16}$ in. high. What is the length of each edge of a cube which is $\frac{13}{16}$ in. shorter each way?

(*d*) Measure in inches and sixteenths the long and the short edges of the hexagonal prism, the rhomboid prism, and the pentagonal pyramid, and in each case find the difference between them.

Common Denominators

169. Hitherto you have added together the same kind of things—that is, you have added sixteenths to sixteenths, twelfths to twelfths, and so forth.

How would you add together unlike things, *e.g.*, $\frac{3}{8}$ and $\frac{1}{16}$? They do not make either four-eighths or four-sixteenths, any more than 3 apples and 1 orange make either 4 apples or 4 oranges. Before you can add them together, you must make them *like* things—that is, you must make the denominator the same in both.

170. Now you have learnt that $\frac{3}{8} = \frac{6}{16}$

$$\therefore \quad \frac{3}{8} + \frac{1}{16} = \frac{6}{16} + \frac{1}{16} = \frac{7}{16}.$$

Similarly, $$\frac{3}{8} - \frac{1}{16} = \frac{6}{16} - \frac{1}{16} = \frac{5}{16}.$$

171. Exercise (to be written).

(*a*) Find the value of $\frac{3}{4}+\frac{1}{8}$, $2\frac{1}{2}+\frac{1}{10}$, $\frac{1}{12}+\frac{1}{24}$, $\frac{1}{3}+\frac{1}{6}$.

Verify your results by referring to your ruler.

(*b*) Add together $\frac{3}{4}$, $2\frac{1}{2}$, $\frac{3}{8}$, $5\frac{1}{16}$.

(*c*) Subtract $\frac{1}{12}$ from $\frac{1}{4}$, $\frac{1}{10}$ from $2\frac{1}{2}$, $\frac{1}{8}$ from $\frac{3}{4}$, $\frac{1}{16}$ from $9\frac{1}{4}$.

Verify your results by referring to your ruler.

(*d*) Simplify $8 + 2\frac{3}{4} - 5\frac{5}{8}$, $\frac{1}{4}+\frac{1}{16}-\frac{1}{8}$, $\frac{7}{10}-\frac{2}{5}+3\frac{1}{2}$, $\frac{11}{12}-\frac{5}{6}+\frac{1}{12}$.

(*e*) Set off in order along a straight line $\frac{7}{8}$ in., $\frac{3}{4}$ in., $1\frac{5}{16}$ in., $\frac{3}{16}$ in. Add these quantities together, and verify the result by *measuring* the whole distance set off.

(*f*) Do the same with $1\frac{1}{12}$ in., $\frac{1}{2}$ in., $\frac{1}{4}$ in., $\frac{5}{6}$ in.

(*g*) Find, in inches and fractions of an inch, the total length of the edges of each of the models which you measured in §§ 153-156.

(*h*) If I give $\frac{1}{4}$ of 24s. to A, and $\frac{1}{12}$ each to B and C, how many shillings have I given?

(*i*) What fraction of the whole is left? How many shillings is it?

172. Use your ruler to find the value of $\frac{1}{2}+\frac{1}{3}$. How would you write $\frac{1}{2}$ and $\frac{1}{3}$ with a common, that is to say, with the same, denominator?

Find a common denominator for $\frac{1}{8}$ and $\frac{1}{10}$, $\frac{1}{8}$ and $\frac{1}{12}$, $\frac{1}{10}$ and $\frac{1}{12}$, $\frac{1}{4}$ and $\frac{1}{12}$.

173. Exercise (oral).

(*a*) Add together $\frac{2}{3}$ and $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{9}{12}$, $\frac{9}{10}$ and $\frac{11}{12}$, $\frac{3}{8}$ and $\frac{7}{12}$.

(*b*) Which is the greater fraction? $\frac{2}{3}$ or $\frac{3}{4}$, $\frac{5}{8}$ or $\frac{9}{12}$, $\frac{9}{10}$ or $\frac{11}{12}$ or $\frac{7}{8}$, $\frac{3}{8}$ or $\frac{1}{4}$?

(*c*) Which of the following fractions could you subtract from $\frac{3}{4}$? $\frac{7}{8}$, $\frac{7}{10}$, $\frac{7}{12}$?

174. You will now be able to subtract $5\frac{5}{6}$ from $7\frac{3}{8}$:—

$$\begin{aligned} 7\tfrac{3}{8} - 5\tfrac{5}{6} &= 2\tfrac{9}{24} - \tfrac{20}{24} \\ &= 1\tfrac{33}{24} - \tfrac{20}{24} \\ &= 1\tfrac{13}{24}. \end{aligned}$$

175. Exercise (to be written).

(*a*) $2\frac{3}{4}$ in. is cut off a stick $21\frac{1}{6}$ in. long. What length is left?

(*b*) Simplify $\frac{1}{2}+\frac{5}{6}$, $\frac{3}{4}+\frac{1}{12}$, $1\frac{1}{2}+\frac{3}{10}$. Verify by your ruler.

(*c*) Find the value of $1\frac{1}{2}-\frac{7}{8}$, $7\frac{1}{10}-1\frac{9}{10}$, $10\frac{5}{16}-4\frac{7}{12}$, $21\frac{5}{12}-19\frac{9}{10}$.

(*d*) Simplify $\dfrac{1}{a}+\dfrac{1}{b}$ and $\dfrac{1}{a}-\dfrac{1}{b}$

(*e*) Simplify $\frac{1}{12}-\frac{3}{10}+\frac{7}{16}$, $\frac{5}{6}-\frac{1}{8}-\frac{1}{10}$, $\frac{2}{3}-\frac{3}{4}+1\frac{1}{2}$, $7\frac{1}{3}-9\frac{4}{5}+2\frac{1}{2}$.

(*f*) From a line 2 in. long is cut off $1\frac{1}{4}$ in., and then $\frac{1}{5}$ in. How much is left? Verify by drawing and measuring.

Multiplication of Fractions

176. Cut a slip of paper exactly 3 in. long. Fold it twice; then find by measuring the value of "$\frac{1}{4}$ of 3."

$\frac{1}{4} \times 3$ means "three times one-fourth," just as 4×3 means "three times four." How much is "three times one shilling?" How much is "three times one-fourth?" How much is $\frac{1}{4} \times 3$?

You know that 3×4 has the same value as 4×3.

Similarly, $3 \times \frac{1}{4}$ has the same value as $\frac{1}{4} \times 3$. How much is that?

You are now able to see that $\frac{1}{4}$ of 3, $\frac{1}{4} \times 3$, and $3 \times \frac{1}{4}$ have all the same value. What is that value?

How much is 3×4? And how much is $\frac{3}{4} \times 4$? Is it the same as $\frac{3}{4}$ of 4? Verify by your ruler.

177. Exercise (oral).

(*a*) How much is $\frac{1}{4} \times 2$, $\frac{1}{3}$ of 2, $2 \times \frac{1}{10}$, $\frac{1}{4}$ of 5, $8 \times \frac{1}{3}$?

(*b*) Express each of these quantities in two other ways.

(*c*) Find the value of $\frac{3}{4}$ of 3, $\frac{7}{8} \times 4$, $4 \times \frac{3}{16}$, $\frac{3}{10}$ of 5.

178. You ought now to be able to make a rule for multiplying a fraction by a whole number, or a whole number by a fraction. Write the rule in your note-book.

179. You will remember that it does not alter the value of a fraction if you divide numerator and denominator by the same number. Dividing in this way will often simplify your working, thus:—

$$\frac{5}{12} \times 9 = \frac{5 \times \overset{3}{\cancel{9}}}{\underset{4}{\cancel{12}}}$$

$$= \frac{15}{4} \text{ (or } 3\tfrac{3}{4}\text{)}$$

This process is called **cancelling**.

180. Exercise (to be written).

(*a*) Find the value of $\frac{7}{12} \times 48$, $\frac{11}{24}$ of 72, $\frac{4}{5}$ of 35, $4 \times \frac{15}{16}$.

(*b*) Multiply 45s. by $\frac{4}{5}$, 9d. by $\frac{5}{6}$, £1 by $\frac{7}{10}$.

(*c*) How many minutes is $\frac{9}{10}$ of 3 hours? How many hours is $\frac{5}{6}$ of a day?

(*d*) Find the value of $\frac{7}{8}$ of £5. 10s., $\frac{5}{12}$ of 3s. 6d., $\frac{4}{7}$ of 2 guineas.

(*e*) How much is £$3\frac{3}{4}$ + $3\frac{3}{4}$s. + $3\frac{3}{4}$d.?

181. Exercise (requiring the use of models).

(*a*) Measure an edge of the cube in inches and fractions of an inch; taking all the edges as equal, find the total length of all the edges.

(*b*) Do the same with all the long, and with all the short, edges of the square prism.

182. You have now learnt how to multiply a fraction by a whole number. It is somewhat harder to understand how to multiply a fraction by a fraction.

Examine your ruler along the edge divided into twelfths. From it find out the following values *in twelfths*, and enter them in your note-book, thus:—

$$\tfrac{1}{6} \text{ of } \tfrac{1}{2} \text{ (or } \tfrac{1}{2} \times \tfrac{1}{6}) =$$
$$\tfrac{1}{3} \text{ of } \tfrac{1}{4} \text{ (or } \tfrac{1}{4} \times \tfrac{1}{3}) =$$
$$\tfrac{2}{3} \text{ of } \tfrac{1}{4} \text{ (or } \tfrac{1}{4} \times \tfrac{2}{3}) =$$
$$\tfrac{1}{3} \text{ of } \tfrac{3}{4} \text{ (or } \tfrac{3}{4} \times \tfrac{1}{3}) =$$
$$\tfrac{2}{3} \text{ of } \tfrac{3}{4} \text{ (or } \tfrac{3}{4} \times \tfrac{2}{3}) =$$

Carefully inspect your results, and try to make a rule for multiplying one fraction by another. Enter the rule in your note-book.

183. Exercise (oral).

(*a*) Multiply $\frac{1}{5}$ by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

(*b*) What is $\frac{1}{4}$ of $\frac{1}{4}$, of $\frac{1}{2}$, of $\frac{3}{4}$, of $\frac{3}{5}$, of $\frac{3}{8}$?

(*c*) Simplify $\frac{5}{8} \times \frac{1}{2}$, $\frac{5}{8} \times \frac{3}{4}$, $\frac{2}{3} \times \frac{3}{8}$.

(*d*) Simplify $\frac{1}{2}$ of $\frac{1}{4}$ of 4, $\frac{2}{3}$ of $\frac{3}{4} \times 10$, $6 \times \frac{1}{10} \times \frac{3}{4}$.

(*e*) Find the value of $\frac{1}{3}$ of $\frac{1}{2}$ in., $\frac{1}{4}$ of $\frac{2}{3}$ in., $\frac{3}{4}$ of $\frac{3}{4}$ in. Verify by your ruler.

(*f*) Read the formula, $\frac{a}{b}\left(\frac{c}{d}\right) = \frac{ac}{bd}$. What does it mean?

184. Exercise (to be written).

Simplify :—

(*a*) $\frac{7}{12} \times \frac{3}{5} \times \frac{5}{8}$.

(*b*) $\frac{9}{10}$ of $\frac{2}{3}$ of $\frac{1}{2}$.

(*c*) $\frac{3}{4} \times \frac{3}{8} \times \frac{4}{5} \times 20$.

(*d*) $\frac{9}{16}$ of $4 \times \frac{3}{24}$ of $\frac{2}{3}$.

(*e*) $(\frac{7}{20} \times \frac{4}{7}) - \frac{1}{8}$.

(*f*) $14\frac{5}{8} - (\frac{3}{4}$ of $\frac{5}{12})$.

(*g*) $\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d}$.

Multiplication of Mixed Numbers

185. When you have to multiply a mixed number by a fraction or by another mixed number, you must reduce the mixed numbers to improper fractions before multiplying.

$$12\tfrac{1}{6} \times \frac{3}{4} = \frac{73}{\underset{2}{\cancel{6}}} \times \frac{\cancel{3}}{4}$$

$$= \frac{73}{8}$$

$$= 9\tfrac{1}{8}$$

186. Exercise (requiring the use of models).

(*a*) Simplify $9\frac{3}{4} \times \frac{2}{5}$, $\frac{7}{8} \times 2\frac{1}{10}$, $2\frac{1}{2} \times 2\frac{1}{2}$.

(*b*) What would be the length of each edge of the cube in a drawing to the scale $\frac{3}{4}$?

(*c*) What would be the length of each edge of the square prism in a drawing to the scale $\frac{2}{3}$?

(*d*) What would be the length of each edge of the rhomboid prism in a drawing to the scale $\frac{1}{4}$?

(*e*) If the hexagonal prism were made $3\frac{3}{4}$ times greater, what would be the length of each of its edges?

(*f*) What would be the total length of the edges?

Division of Fractions

187. To divide a fraction by a whole number is quite a simple matter. $\frac{3}{4} \div 2$ is the same as $\frac{1}{2}$ of $\frac{3}{4}$, just as $3 \div 2$ is the same as $\frac{1}{2}$ of 3. You have already learnt that $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{4} \times \frac{1}{2}$. You conclude, therefore, that $\frac{3}{4} \div 2 = \frac{3}{4} \times \frac{1}{2}$.

188. Exercise (oral).

(*a*) What is the value of $\frac{1}{3} \div 2$, $\frac{1}{4} \div 3$, $\frac{1}{5} \div 4$, $\frac{1}{6} \div 5$?

(*b*) Simplify $\frac{3}{4} \div 4$, $\frac{2}{3} \div 8$, $\frac{5}{6} \div 2$, $\frac{7}{8} \div 5$.

You can verify your answers by your ruler.

189. When dividing a whole number by a fraction you must think what you are really doing. $1 \div \frac{1}{4}$ means "the number of times that 1 contains $\frac{1}{4}$," that is, 4. In ordinary division the quotient is always less than the dividend; and you may think it singular that the quotient should ever be greater than the dividend. But it is always so when the divider is a fraction less than 1, *e.g.*,

$$5 \div 2 = 2\tfrac{1}{2},$$
$$\text{but } 5 \div \tfrac{1}{2} = 10,$$
$$\text{and } 5 \div \tfrac{1}{3} = 15.$$

If you have any difficulty in understanding this, refer to your ruler. How much is $5 \div \frac{1}{4}$, $5 \div \frac{1}{8}$, $5 \div \frac{1}{10}$, $5 \div \frac{1}{12}$?

190. Exercise (oral).

(*a*) How many times does 2 contain $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{10}, \frac{1}{12}, \frac{1}{16}$?

(*b*) Simplify $3 \div \frac{1}{8}, 4 \div \frac{1}{10}, 2 \div \frac{1}{16}, 7 \div \frac{1}{5}, 10 \div \frac{1}{12}$.

191. It is somewhat harder to understand how to divide a fraction by a fraction. Let us take an example: $\frac{3}{4} \div \frac{1}{3}$. Remember that this means "how many times is $\frac{1}{3}$ contained in $\frac{3}{4}$?" You will find no difficulty in answering this question if you first bring the fractions to a common denominator, so that you can easily compare them.

$\frac{3}{4} \div \frac{1}{3}$ is the same as $\frac{9}{12} \div \frac{4}{12}$. To ask "how many times is four *twelfths* contained in nine *twelfths*" is the same thing as to ask "how many times is 4 *inches* contained in 9 *inches?*" How many times is that? Express your answer as an improper fraction.

Find in the same way the values of the following, and write them in your note-book :—

$$\frac{1}{2} \div \frac{1}{5} =$$
$$\frac{2}{5} \div \frac{3}{4} =$$
$$\frac{3}{4} \div \frac{2}{3} =$$
$$\frac{1}{3} \div \frac{1}{4} =$$

Now make a rule for dividing a whole number or a fraction by a fraction.

192. Exercise (oral).

(*a*) How much is $\frac{1}{2} \div \frac{1}{4}, \frac{3}{4} \div \frac{1}{8}, \frac{3}{4} \div \frac{1}{4}, \frac{1}{2} \div \frac{1}{12}$?

(*b*) Simplify $\frac{2}{3} \div \frac{3}{4}, \frac{3}{4} \div \frac{3}{5}, \frac{3}{4} \div \frac{1}{2}, \frac{7}{8} \div \frac{1}{2}, \frac{4}{5} \div \frac{3}{5}$.

a, b, x, y, etc., represent any numbers.

(*c*) Complete the formula, $\frac{a}{b} \div \frac{x}{y} =$

(*d*) Simplify $\frac{x}{y} \div \frac{z}{y}$.

(*e*) Simplify $\left(\frac{1}{x} \times \frac{1}{y}\right) \div \frac{1}{z}$.

(*f*) What is the value of this fraction if $x=2$, $y=3$, $z=4$?

193. When you have to deal with mixed numbers, you will, of course, remember to reduce them to improper fractions before working.

194. Exercise (to be written).

(*a*) Simplify $3\frac{1}{8} \div 3$, $5\frac{1}{4} \div 2\frac{1}{4}$, $6 \div 1\frac{3}{4}$.
(*b*) Divide $2\frac{1}{2}$ by $\frac{3}{4}$, $5\frac{1}{4}$ by $\frac{3}{8}$, $3\frac{1}{2}$ by $1\frac{3}{8}$, $9\frac{1}{4}$ by $4\frac{5}{8}$.
(*c*) The total length of the edges of a cube is $92\frac{1}{4}$ in. What is the length of one edge?

195. Exercise (requiring the use of the oblong prism).

(*a*) Make a hand-sketch of the model, and mark on it the measurements.
(*b*) Make a plan and elevation of the model, scale $\frac{3}{4}$.
(*c*) What would be the lengths of the edges if the model were $2\frac{3}{4}$ times larger?
(*d*) What would be their lengths if the model were only $\frac{5}{6}$ as large?

Comparison of Vulgar Fractions and Decimals

196. It sometimes happens that you have to compare a decimal with a vulgar fraction. For example, you may be asked which is the longer, ·12 in. or $\frac{1}{12}$ in. You must therefore find out how to convert vulgar fractions into decimals, and *vice versâ*.

How would you write down as vulgar fractions, three-tenths, seven-tenths, nine-tenths?

How would you write down as decimals, three-tenths, seven-tenths, nine-tenths?

You will of course see that $\frac{3}{10}$, $\frac{7}{10}$, $\frac{9}{10}$, have just the same value as ·3, ·7, ·9, respectively.

Can you make a rule for turning vulgar fractions into decimals, and for turning decimals into vulgar fractions? Write it in your note-book.

197. Write down both as vulgar fractions and as decimals, three-hundredths, seven-hundredths, and nine-hundredths.

·39 means three-tenths and nine-hundredths, that is, $\frac{3}{10} + \frac{9}{100}$, or $\frac{30}{100} + \frac{9}{100}$, or $\frac{39}{100}$.

Similarly, $\frac{63}{100} = \frac{60}{100} + \frac{3}{100}$, or $\frac{6}{10} + \frac{3}{100}$, or ·63.

Does your rule apply to these cases? If not, express it so that it does apply.

198. Exercise (to be written).

(*a*) Write as vulgar fractions, ·3, ·63, ·03, ·603, ·063, ·007.

(*b*) Write as vulgar fractions, 3·3, 10·3, 10·03, 16·63, 70·1, 700·007.

(*c*) Express as vulgar fractions in their lowest terms, ·5, ·25, ·24, 5·75, 50·025, ·404.

(*d*) Write as decimals, $\frac{3}{10}$, $\frac{3}{100}$, $\frac{33}{100}$, $33\frac{3}{100}$, $7\frac{7}{10}$, $11\frac{11}{1000}$.

199. The vulgar fractions which you have just dealt with have 10 or 100 or 1000 for their denominator.

You have now to find a rule for converting *any* vulgar fraction into a decimal. You have already seen, and you can easily verify, that $\frac{1}{8}$ in. has the same value as 1 in. $\div$ 8, that $\frac{5}{8}$ in. has the same value as 5 in. $\div$ 8, and so forth; and you have already discovered how to find the value of $5 \div 8$ in decimals. What decimal therefore is equal in value to $\frac{5}{8}$? State a general rule.

200. Exercise (to be written).

(*a*) Write down (in their lowest terms) all the fractional parts of a whole divided into eighths, and opposite each the decimal equivalent to it.

(*b*) Measure the edges of the triangular prism, first in inches and sixteenths, then in inches and decimals of an inch.

(c) Find by addition the total length of the edges in both measurements. Then compare the totals, after converting the fractions of the first total into decimals. If the totals do not quite agree, can you suggest a reason?

(d) Express as decimals the series of fractions, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{1}{16}, \frac{1}{20}$.

(e) Express as decimals, $3\frac{3}{4}$, $7\frac{1}{20}$, $4\frac{3}{40}$, $100\frac{49}{50}$, $98\frac{15}{16}$.

(f) Express as vulgar fractions, 9·32, 18·012, 100·01.

Some Common Fractions

201. You will have noticed that in the decimal of $\frac{1}{3}$ the figure 3 comes in every place, and the decimal never ends. This decimal is commonly written $\cdot\dot{3}$; the dot over the figure 3 denotes that it recurs (or keeps on coming), and is used simply to save the trouble of writing all the 3's. Such a decimal is called a recurring decimal. What other recurring decimals have you noticed in the above exercises?

202. Exercise (oral).

(a) Read off $\cdot\dot{3}$, $\cdot\dot{6}$, $\cdot1\dot{6}$, $\cdot\dot{1}$.

(b) What vulgar fractions do these recurring decimals represent?

203. You should know by heart the decimal equivalents of the most common fractions. Express the following as decimals: $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{6}, \frac{1}{8}, \frac{3}{8}$.

It is well to know these equivalents, because it is sometimes easier to work with a decimal, and sometimes with a vulgar fraction, *e.g.*—

$$104 \times \cdot75 = \tfrac{3}{4} \text{ of } 104 = 3 \times 26 = 78.$$

You have learnt that $\pi = 3{\cdot}14$. Now $3{\cdot}14 = 3\,\frac{7}{50}$, which is very near $3\frac{7}{49}$ (or $3\frac{1}{7}$).

You will sometimes find it easier to take π as $3\frac{1}{7}$.

204. Exercise (to be written).

(*a*) How much is $880 \times \cdot 125$, $404 \times \cdot 25$, $336 \times \cdot 1\dot{6}$?

(*b*) A wheel has a radius of 14 feet, what is its circumference?

(*c*) Simplify

$$\cdot 5 \text{ of } £264 + \cdot 625 \text{ of } £448 - \cdot\dot{3} \text{ of } £399.$$

(*d*) Which is the greater, $3\frac{7}{50}$ or $3\frac{7}{49}$, and by how much?

(*e*) Find the value of $3 \div \cdot 75$, $3 \div \cdot 125$, $3 \div \cdot 375$, $22 \div 2\frac{1}{5}$.

(*f*) A bicycle wheel covers 5 ft. 6 in. in one revolution, what is its diameter?

205. The vulgar fractions which you commonly use are those marked on your ruler. But, of course, you may have *any* number as the denominator of a vulgar fraction. If you take a whole with 31 parts (*e.g.* a month of 31 days), then one part (in this case one day) is $\frac{1}{31}$ of the whole.

If you try to convert fractions like $\frac{1}{7}$, $\frac{1}{11}$, $\frac{1}{13}$, $\frac{1}{31}$ into decimals, you will find the decimals very awkward numbers; in fact, it is much more convenient to keep them, and to work with them, as vulgar fractions.

206. Exercise (oral).

(*a*) What fraction is 3 days of a week?

(*b*) What fraction is 1 day of a leap-year February?

(*c*) What fraction is 1 week of a leap-year February?

(*d*) What fraction is 5 days of a year?

(*e*) What fraction is 40 yards of a quarter of a mile.

(*f*) What fraction is 3 days of a fortnight?

Longer Measures

207. Let us now examine some longer measures. Look at your ruler; how many inches are there in a foot?

Why do you suppose it is called a foot?

208. Exercise (to be written).

(*a*) Make a hand-sketch of the top of your desk, mark on it the length of each edge in feet, inches, and fractions of an inch.
(*b*) What is the total length of the edges?
(*c*) Reduce 2 yds. 2 ft. 2 in. to inches.
(*d*) Draw a plan of the desk top, scale $\frac{1}{8}$.
(*e*) Guess how many feet the door is broad, and how many feet high. Let one boy measure it, and write the measurements on the board, in feet and inches.
Draw a plan of the door, scale $\frac{1}{40}$.

209. Look at your yard-stick. How many feet are there in a yard?

How many inches are there in a yard?
Make a table of the measures, thus:—

—— inches make 1 foot.
—— feet, or —— inches make 1 yard.

210. Exercise (to be written).

(*a*) Guess how many yards the room is long, and how many broad. Let one boy measure it, and write the measurements on the board. Make a hand-sketch of the room, and mark on it the measurements in yards, feet, and inches. Express these measurements in feet and inches, and again in inches. Draw a plan of the room, scale $\frac{1}{8}$ in. to 1 ft.
(*b*) What fraction of a foot is 1 in., 2 in., 3 in., 4 in., 6 in., 8 in., 9 in.?

(*c*) Express these fractions as decimals of a foot.

(*d*) Express them as fractions and as decimals of a yard.

(*e*) What fraction and what decimal of a yard is 1 ft., $\frac{1}{2}$ ft., $1\frac{1}{2}$ ft., 2 ft., $2\frac{1}{2}$ ft.?

(*f*) Express 20 in., 18 in., 16 in., 30 in., 15 in., 32 in., as fractions of a yard.

(*g*) Add together (by fractions) $3\frac{5}{6}$ yd., $8\frac{1}{3}$ yd., $2\frac{3}{4}$ yd., $1\frac{11}{12}$ yd.

(*h*) Express each of these lengths in yards, feet, and inches.

Now add these together, and verify your result by comparing the result arrived at by adding the fractions.

(*i*) Find the price of 14 in. of silver wire at 1s. 6d. a yard.

(*j*) What distance does a bicycle wheel, 21 in. high, cover in 6 revolutions?

(*k*) Find the average height of four brothers, whose heights are 5 ft. 5 in., 5 ft. $3\frac{1}{2}$ in., 5 ft. 1 in., 4 ft. $11\frac{1}{2}$ in.

211. You may have noticed that two yard-sticks are not always exactly the same length. This is because one or the other of them is not accurately made; and you may have wondered how it can be decided which yard is the correct one. This can be decided by a comparison with the *Imperial Standard Yard*, which is kept in a wall in the House of Parliament. It is a solid square bar of bronze 38 in. long, and the yard is marked by two fine lines cut on two gold studs let into the bar, one near each end. You are no doubt aware that there are inspectors whose duty it is to see that the measures used by tradesmen are correct.

212. A distance is often measured by hundreds of yards. Let the scholars measure with a tape a distance of 100 yds. in the school grounds, so as to realise how much it is.

Let them guess other convenient distances about the school—the length of the buildings, the width and the length of the playground, etc., and make plans of these to several scales, from measurements supplied to them.

Miles and Kilometres

213. 1760 yds. make a mile. This is the measure used for measuring long distances. The scholars should get some clear idea of what a mile is. For this purpose let a town map on a fairly large scale be used, and distances of a mile familiar to several groups of pupils be ascertained from it.

214. Exercise (oral).

(*a*) In what time can you walk a mile?
(*b*) How many yards are there in $\frac{1}{2}$ ml., $\frac{1}{4}$ ml., $\frac{1}{8}$ ml.,* $\frac{1}{11}$ ml.?
(*c*) What fraction of a mile is 10 yds., 11 yds., 110 yds., 1100 yds., 330 yds.?
(*d*) How many feet are there in a mile?

215. Exercise (to be written).

(*a*) How many curbstones, each 2 ft. 9 in. long, are required for a road $\frac{1}{2}$ ml. long?
(*b*) What is their cost at $9\frac{1}{2}$d. a foot?
(*c*) On a racing track there are $5\frac{1}{2}$ laps to the mile. What is the distance round?
(*d*) If the track is circular, what is its diameter?

216. Put a ruler-edge marked with cm., and a ruler-edge marked with inches, side by side. Note that 2 in. are equal to 5 cm. and a little more, say 5 + cm.

* $\frac{1}{8}$ ml. is called a furlong.

217. Exercise (oral).

(*a*) How many cm., roughly, would you expect to find in 4 in., 6 in., 9 in., 1 ft?
(*b*) How many inches in 30 cm., 10 m., 15 cm.?
(*c*) Verify results in both cases by rulers.

Now measure 10 in. *quite exactly* in cm. Enter the measurement in your note-book.

218. Exercise (to be written).

(*a*) Calculate how many times an inch is longer than a cm.
(*b*) Calculate how many cm. are there in a foot? Verify by the ruler.
(*c*) How many cm. are there in a yard?
(*d*) By how many cm. is a yard shorter than a metre?
(*e*) By how many inches is a metre longer than a yard?
(*f*) How many times is a foot longer than a dm.?
(*g*) Find from your ruler how many inches are equal to 10 cm.
(*h*) What decimal of an inch is 1 cm.?

219. Exercise (to be written).

Take the cube, the cylinder, the square prism, and the oblong prism. Measure the edges of each in cm., and then in inches. In each case verify your measurements by converting cm. into inches by means of the value discovered in the last exercise.

220. A kilometre is 1000 metres: "kilometre" is usually written km.

221. Exercise (to be written).

(*a*) How many yards are there in a kilometre?

(*b*) By how many yards is the kilometre longer or shorter than a mile?

(*c*) How many miles are there in 8 km., approximately?

(*d*) What fraction, therefore, of a mile is a kilometre?

CHAPTER V

MONEY AND OTHER PROBLEMS

222. What fractions of 1s. are 1d., $1\frac{1}{2}$d., 2d., 3d., 4d., 6d.?

What fractions of £1 are 1s., 2s., 2s. 6d., 4s., 5s., 10s., 3s. 4d., 6s. 8d.?

When one quantity is contained an exact number of times in another, it is said to be an **aliquot part** of that other. Thus, 1d., $1\frac{1}{2}$d., 2d., etc., are aliquot parts of 1s.; 1s., 2s., 2s. 6d., etc., are aliquot parts of £1.

223. 36 articles at 1s. cost 36s. What is the cost of 36 articles at 3d.?

Since 3d. is $\frac{1}{4}$ of 1s., 36 articles at 3d. cost $\frac{1}{4}$ as much as 36 articles at 1s. That is, they cost $\frac{36\text{s.}}{4}$, or 9s.

In the same way, since 64 articles at £1 cost £64, and since 2s. 6d. is an $\frac{1}{8}$ of £1, 64 articles at 2s. 6d. cost $\frac{£64}{8}$, or £8.

224. Exercise (oral).

(*a*) Find the cost of:—

66 articles at 2d.	72 articles at $1\frac{1}{2}$d.
82 articles at 6d.	120 articles at 1d.
99 articles at 4d.	160 articles at 3d.

(*b*) Find the cost of :—

55 articles at 4s.
120 articles at 3s. 4d.
96 articles at 2s. 6d.
102 articles at 10s.
110 articles at 2s.
39 articles at 6s. 8d.

225. How much in shillings and pence are the following fractions of £? $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{12}$, $\frac{1}{15}$, $\frac{1}{16}$. These aliquot parts are worth remembering. Write them in your note-book, and learn them off.

226. Exercise (oral).

What is the value of :—

45 brushes at 1s. 4d.
624 vases at 3s. 4d.
160 balls at 1s. 3d.
150 mats at 1s. 4d.?

227. The two following rules are worth remembering.

(*a*) If 1 thing costs 2d., a dozen cost 2s.; if 1 thing costs $2\frac{1}{2}$d. (two and a half pence), a dozen cost $2\frac{1}{2}$s. (or 2s. 6d.); and, generally, if 1 thing costs x pence, a dozen cost x shillings.

(*b*) If 1 thing costs 2s., a score cost £2; if 1 thing cost 2s. 6d. ($2\frac{1}{2}$s.), a score cost £2. 10s. (£$2\frac{1}{2}$); and, generally, if 1 thing costs x shillings, a score cost x pounds.

228. Exercise (oral).

(*a*) Find the cost of a dozen articles at 5d., at $3\frac{1}{2}$d., at $9\frac{1}{4}$d., at $11\frac{3}{4}$d.

(*b*) Find the cost of 2, and of 3, dozen articles at the same prices.

(*c*) Find the cost of 20 lbs. at 2s. 6d., 17s. 3d., 12s. 9d., 4s. 2d., 2s. $1\frac{1}{2}$d.

229. The following is the form in which a tradesman makes out his bills.

Bought of Cairns & Son.

May 1, 1904.

Mr A. Smith.

	£	s.	d.
28 lbs. brown sugar, at $1\frac{1}{2}$d. .		3	6
12 lbs. biscuits, at $7\frac{1}{2}$d. . .		7	6
4 lbs. raisins, at 9d. . .		3	0
		14	0

Note.—At this point the scholars may begin Chapter VI., and use the following exercises for home work in arithmetic.

230. Exercise (to be written).

(*a*) Find, by a short method of working, the cost of:—

111 watches at 6s. 8d.
384 caps at 1s. 3d.
1 score lbs. at 2s. 4d.
7 score lbs. at 2s. 4d.
6 dozen glasses at $7\frac{1}{2}$d.
330 baskets at 1s. 4d.

(*b*) Make out bills for the following purchases:—

(1) 12 lbs. beef at $10\frac{1}{2}$d., 10 lbs. mutton at 9d., $9\frac{1}{4}$ lbs. lamb at 1s., 6 lbs. brawn at 11d.

(2) 5 tons household coal at 18s., 3 tons slack at 13s. 4d., $3\frac{1}{2}$ tons selected nibs at 21s.

(3) 6 score flasks at 1s. 4d., 9 balances at 46s. 8d., 100 yds. tubing at 9d., 1 dozen filters at 5s.

(4) 1 gross service caps at 2s. 6d., 90 water-bottles at 3s. 4d., 90 bandoliers at 4s., 160 putties at 1s. 3d.

(5) 10 gross exercise books at 18s. a gross, 4 gross drawing books at 25s. a gross, 9 dozen pencils at $1\frac{1}{2}$d., 20 wall-maps at 15s. 9d.

231. General Exercise.

(*a*) On a piece of squared paper draw an oblong containing five rows of four small squares. Mark this figure so as to show that $\frac{4}{12}=\frac{1}{3}$, $\frac{3}{12}=\frac{1}{4}$, $\frac{8}{12}=\frac{2}{3}$.

(*b*) Draw a figure on squared paper to prove that $\frac{4}{20}=\frac{1}{5}$, $\frac{5}{20}=\frac{1}{4}$, $\frac{10}{20}=\frac{1}{2}$, $\frac{15}{20}=\frac{3}{4}$.

(*c*) Express with a common denominator :—

(1) $\frac{5}{6}$, $\frac{8}{9}$, $\frac{11}{12}$.

(2) $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$.

(3) $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.

(*d*) Arrange the above groups of fractions in order of magnitude.

(*e*) Express as wholes or as mixed numbers, $\frac{31}{6}$, $\frac{18}{18}$, $\frac{98}{7}$, $\frac{31}{16}$, $\frac{132}{12}$, $\frac{111}{12}$.

(*f*) Express as improper fractions, $9\frac{5}{6}$, $18\frac{7}{12}$, $20\frac{11}{16}$, $4\frac{3}{8}$, $200\frac{19}{20}$, $30\frac{3}{10}$.

(*g*) Simplify $9\frac{7}{20}+9\frac{4}{5}$, $10\frac{1}{8}-1\frac{3}{16}$, $4\frac{1}{2}\div 2\frac{2}{5}$, $3\frac{1}{7}\times 6\frac{2}{3}$.

(*h*) How many m. and cm. are there in 10 yds. 1 ft. 10 in.?

(*i*) A man makes £420 in $3\frac{1}{2}$ years. What is his annual income?

(*j*) The difference between $12\frac{1}{12}$ and another number is $3\frac{3}{4}$. What is the other number?

(*k*) Find the value of $\frac{1}{6}$ of £2 + $\frac{2}{3}$ of 2s. + ·75 of 2d.

(*l*) Reduce 1 yd. 2 ft. 11 in. to inches.
(*m*) Reduce $4\frac{13}{24}$ yds. to inches.
(*n*) What fraction of 6 yds. 2 ft. 3 in. is 4 yds. 1 ft. 6 in.?
(*o*) Express 22 ft. $7\frac{1}{2}$ in. in yards and fractions of a yard.
(*p*) The sum of two numbers is $13\frac{3}{16}$: if one is $12\frac{1}{4}$, what is the other?
(*q*) Express in shillings and pence $\frac{1}{7}$ of 3 gns. + $\frac{1}{30}$ of 2s. 6d. + $\cdot\dot{3}$ of 5s.
(*r*) A man is given 30 pages to print. What fraction of the whole has he done after printing $6\frac{2}{3}$ pages?
(*s*) Simplify $\cdot 875$ of 10s. − $\frac{5}{12}$ of £1 + $\cdot\dot{6}$ of 2s.
(*t*) A boy gains 75 marks out of 90. What fraction is that?
(*u*) How much is $\frac{11}{12}$ of £36, 16s.?
(*v*) My share of a legacy is $\frac{6}{7}$. The remainder is £1200. How much do I get?
(*w*) How much is $\frac{1}{5}-\frac{1}{6}$? Verify by taking these fractions of £1.
(*x*) A man spends $\frac{7}{8}$ of his money and has 2s. 4d. left. What had he at first?
(*y*) A man goes $\frac{2}{3}$ of a journey by train, $\frac{1}{4}$ by tram, and the remainder, $2\frac{1}{2}$ mls., on foot. Calculate the whole distance.
(*z*) What must be added to $\frac{1}{4}$ to make $\frac{5}{8}$?
(*aa*) A man spends $\frac{5}{6}$ of his money and loses $\frac{1}{4}$ of the remainder. He then buys a ticket at 1s. 2d., and has 4d. left. What had he to begin with? How much did he spend, and how much did he lose?
(*bb*) What must be taken from $10\frac{1}{16}$ to leave $\frac{3}{4}$?
(*cc*) A square park has sides 2000 yds. long. Calculate how far it is round the park in miles and fractions of a mile.

(*dd*) Calculate the distance in kilometres correct to the nearest kilometre.

(*ee*) The total length of the edges of a cube is z in. What is the length of each edge?

(*ff*) A square prism is x in. high and y in. broad. What is the total length of all its edges?

CHAPTER VI

RECTANGULAR FIGURES AND THEIR AREAS: SQUARE MEASURE

Plane Surfaces

232. Look at the faces of the models given to you (square prism and cylinder). Some of the surfaces are what is commonly called flat, and some are what is commonly called rounded or curved.

233. A perfectly flat surface is called a **plane.** The following is a method of testing whether a surface is a plane. Take the square prism in one hand and your ruler in the other, and lay the ruler on its edge across one of the faces of the prism; notice whether the edge of the ruler lies close to the surface without leaving a space. Apply the ruler again in the same way at several places on the surface, and in several directions; again notice whether the edge of the ruler everywhere touches the surface. If there is no space between the surface and the edge of the ruler, wherever and in whatever direction you apply it, then the surface is a plane.

234. It is important that you should test the surface in several directions and not in one direction only. Test the rounded surface of the cylinder. You will find that in one direction the ruler can be applied so as to touch the surface all along. Is the surface therefore a plane?

Test the other surfaces of the cylinder.

You are given some other models to test. Record in your note-book which models have all their surfaces plane and which have not. Keep all the models by you.

Test your desk and the surface of this page.

Names of Plane Figures

235. Examine once more the faces of the square prism and the cylinder. You will notice that some of these faces are bounded by straight lines, and some by curved lines.

Surfaces bounded by straight lines are called **rectilinear** figures. Which of your models have faces which are rectilinear figures, and which have faces which are not rectilinear figures?

236. Now you are to examine rectilinear figures more particularly.

Look at some of the faces of the hexagonal prism, and notice that not all have the same number of sides. Count the sides of each face, and record them in your note-book, thus:—

Hexagonal Prism

No. of 3-sided faces . .	...
No. of 4-sided faces . .	...
No. of 5-sided faces . .	...
No. of 6-sided faces . .	...
Total No. of faces . .	

Check the total by counting the faces of the model. Do the same with the pentagonal pyramid.

237. A three-sided rectilinear figure is called a **triangle.** (Triangular is the adjective of "triangle.")

A four-sided rectilinear figure is called a **quadrilateral.** (Quadrilateral is also the adjective.)

A five-sided rectilinear figure is called a **pentagon.** (Pentagonal is the adjective of "pentagon.")

A six-sided rectilinear figure is called a **hexagon.** (Hexagonal is the adjective of "hexagon.")

238. Of figures which are not rectilinear, the only one which you need consider is the **circle.** As you have already discovered by experiment, a circle is a plane figure bounded by a curved line, every point on which is the same distance from a point within the figure, called the centre. What do you call the line which bounds the circle?

239. You are given a hexagonal prism, a pentagonal pyramid, a cylinder, a triangular prism, and a cone. Take these models in order. Observe the number of faces on each model, and the number of sides in each face, and record in your note-book, thus:—

Name of Model

No. of triangles . . .	...
No. of quadrilaterals . .	...
No. of pentagons . . .	...
No. of hexagons . . .	...
No. of circles . . .	...
No. of surfaces not plane . .	...
Total No. of faces . .	

Check the total by counting the faces of the model.

240. Exercise (drawing).

(*a*) Draw with a ruler three triangles, three quadrilaterals, three pentagons, three hexagons.

(*b*) Try to draw a one-sided figure and a two-sided figure. Are they rectilinear figures?

(*c*) Draw a rectilinear figure with more sides than six.

241. Examine the rectilinear figures which you have just drawn. They have "corners" as well as sides.

Count the number of corners in each figure, and record them in your note-book, as follows:—

	Name of Figure.	No. of Sides.	No. of Corners.
1			
2			
etc.			

242. You see that from the number of sides in any figure you can at once arrive at the number of corners, and *vice versâ.* It does not really matter whether you describe a figure by saying that it has so many sides, or by saying that it has so many corners. The word "triangle" means "three-cornered"; the word "quadrilateral" means "four-sided"; the word "pentagon," again, means "five-cornered."

How many corners would you expect to find in an eight-sided figure? How many sides in a ten-cornered figure?

What plane figure has no corners?

Equal-sided Figures

243. You have now learnt how to describe a rectilinear figure by the number of its sides or corners. But there are still many things to find out about it.

You are given the square prism, the hexagonal prism, and the pentagonal pyramid.

Examine a quadrilateral face of the hexagonal prism. Are all its sides equal to each other? Are any of them equal to each other? Look at a hexagonal face. Are the sides of the hexagon equal to each other?

If you cannot guess, or if you do not remember, for you have already measured them many times, measure them again.

Look at the pentagonal pyramid. Are all the sides of each triangle equal? are any of them equal? Are the sides of the pentagon equal?

Which quadrilaterals on the square prism have all their sides equal?

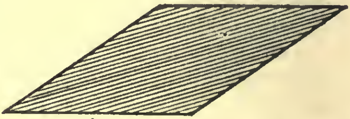
244. Your examination of these models will have taught you that some rectilinear figures are equal-sided.

Let us see if it is a sufficient description of a figure to say that it is equal-sided.

Here (Fig. 30) (1) is the equal-sided quadrilateral, which you see on the square prism; and

(2) by the side of it is another quadrilateral, whose sides are equal to each other, and also equal to the sides of (1).

(1)



(2)

FIG. 30.

Angles in Figures

245. You see that the figures are different. How would you describe the difference between them?

They differ in the space at each corner between the pairs of sides. This space between the sides is called an **angle**.

246. You will better understand the nature of an angle from the following experiment:—

Take your dividers (or compasses) and close them. There is no angle between the legs of the dividers.

Hold one leg firmly and somewhat open the other, as in Fig. 31.

There is now an angle between the legs.

Let the leg which has moved continue to move in the same direction. The angle increases as the leg moves.

You ought now to have no difficulty in judging which of two angles is the greater.

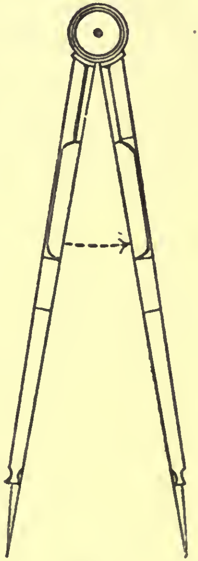


Fig. 31.

247. You are given a cube, a hexagonal prism, and a pentagonal pyramid.

Take the cube and the hexagonal prism, and find an angle in a face of the one equal to an angle in a face of the other, by laying the angles together.

Now take the hexagonal prism alone and guess which angles of the hexagon are equal. Verify your guess by drawing round one angle on a sheet of paper and then applying the other angles to the drawing.

In the same way discover whether all the angles of the pentagon on the pentagonal prism are equal to each other.

Regular Figures

248. You will now have seen that in some rectilinear figures all the angles are equal to each other. And you have already seen (§ 243), that in some rectilinear figures all the *sides* are equal to each other.

A figure which has all its sides equal and all its angles equal is called a **regular** figure.

Is your hexagon a regular hexagon? Is your pentagon a regular pentagon? Record in your note-book.

A regular quadrilateral has a special name, **square.** Can you find any squares on your models? Say on which models, and how many there are on each? Record in your note-book.

A regular triangle is called an **equilateral** triangle. Are any of the triangles on your models equilateral?

Drawing of Angles

249. Now you will have no difficulty in drawing an angle. Prick or mark a point (A) on your paper, and *from* this point draw two straight lines (AB, AC). These two lines form an angle. (Fig. 32.)

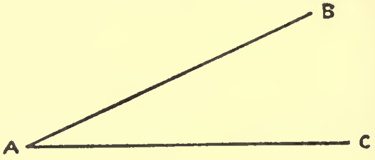
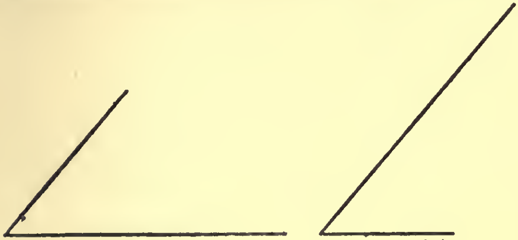


Fig. 32.

The point A is called the **vertex,** and the lines AB, AC, the **arms,** of the angle which has been drawn.

250. You will, of course, bear in mind that the size of an angle has nothing whatever to do with the length of the arms, but depends wholly on the amount of "opening" between the arms, however long or short they may be. The angles here drawn (Fig. 33) (1) and

(2), are equal to each other, despite the difference of length in their arms. Convince yourself of this by using tracing-paper.



(1) (2)

Fig. 33.

251. Exercise (drawing).

(*a*) Draw several angles, of different sizes, with your ruler.

(*b*) Try to draw with your ruler two angles equal to each other.

(*c*) Draw an angle equal to another by using tracing-paper, as follows: Lay the tracing-paper over the angle to be copied, and make pricks with a pin at

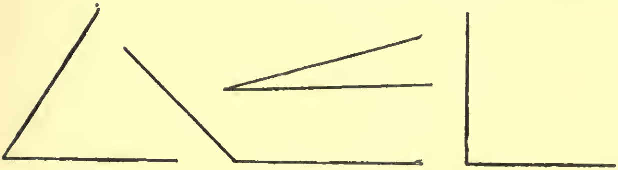


Fig. 34.

the vertex and on each of the arms. Remove the tracing-paper elsewhere on your page, prick marks through the pin-holes, and complete the angle by drawing. Remember which mark is the vertex.

(*d*) Copy the angles in Fig. 34 in the same way.

Naming of Angles

252. It is often convenient to distinguish angles by naming them. You name an angle either by means of a letter placed at the vertex, inside or outside (Fig. 35),

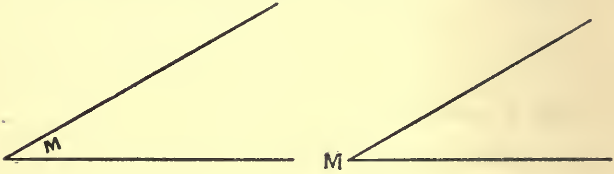


FIG. 35.

or by means of letters placed at the vertex, and at the ends of the arms as in Fig. 36.

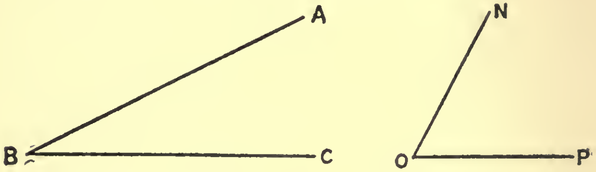


FIG. 36.

In naming an angle in the latter way you can start from the end of either arm, but the letter of the vertex must come in the middle, thus: either ABC or CBA, either NOP or PON.

253. Look at the cross lines in Fig. 37. How many angles do they make?

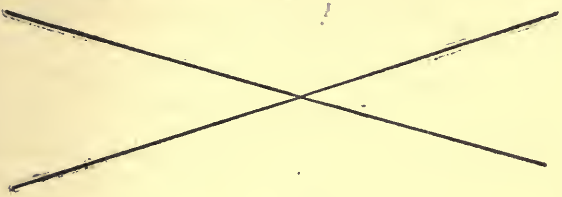


Fig. 37.

Give the angles letters to distinguish them. Which of the angles seem to be equal? Test them by means of tracing-paper.

The equal angles are called **opposite** angles.

Two angles which lie together with one arm common, are called **adjacent** angles. What *pairs* of adjacent angles are there in the figure?

254. Exercise (drawing).

(*a*) Draw three sets of cross lines with your ruler. Name the angles thus formed, and test by means of tracing-paper whether the opposite angles are equal. Record your results.

(*b*) Which pairs of angles are adjacent angles?

(*c*) Draw three pairs of adjacent angles with your ruler and name them.

The Standard Angle

255. How would you describe the size of any of the angles which you have just drawn?

It would be very inconvenient to carry about a copy of any angle which you have to speak about. If you wish to describe the length of a line, you say how many inches or how many cm. it is long. In other words, you make use of a standard of length. In the same way, if you wish to describe the size of an angle, you must make use of a **standard angle**, and say how many times the angle you are describing is larger or smaller than the standard.

256. Look at the angles at each corner of your note-book, or of a door, or of your desk-top. Are they all the same size? You will find the same angle all over a sheet of squared paper. This is an angle which is often chosen as a standard angle: it is called a **right angle.** "Right" in this sense simply means "upright."

257. Find a right angle on the cube. Take two cubes, find what you think are two right angles, one on each model, and make sure that they are *equal*. Then lay them side by side as in Fig. 38.

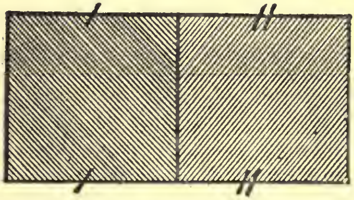


Fig. 38.

What do you notice about the two pairs of arms marked / and //?

Each pair of arms ought to be in the same straight line. Test with the edge of your ruler whether they are in the same straight line. If they are not, the angles are not right angles; if they are, the angles are right angles.

258. Take your compasses once more. Open the

Fig. 39.

arms until they form a right angle. Then open them further until they form an angle equal to two right angles. The two arms will then be in a straight line; and the angle formed by them is called a **straight angle.** (Fig. 39.)

Test of a Right Angle

259. Whenever the outer arms of two *equal* adjacent angles lie in the same straight line or, in other words, form a straight angle, *each* of the two equal angles is a right angle.

Use this test in order to discover other right angles on your models.

Take a piece of paper with a straight edge, and make a right angle by folding.

260. If you continued moving one leg of your compasses until it had turned right round and met the other leg again, it would have turned through all the space round a point. How many straight angles would it have turned through?

Since two right angles make a straight angle, four right angles ought to make two straight angles, *i.e.*, they ought to occupy all the space round a point. Test whether they do so, by placing right angles on four cubes together at a point.

Make four right angles round a point by folding and refolding a piece of paper.

261. Which angles on your set-squares do you think are right angles? Test them in the following way.

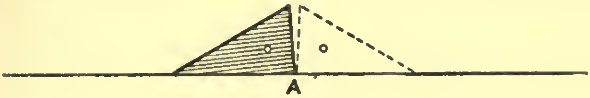


Fig. 40.

Draw a straight line, and on it mark a point A. Lay the set-square, as in Fig. 40, with one arm of the right angle along the straight line, and the angular point at A. Through A draw a straight line along the other arm of the right angle. Turn the set-square over, and by a similar manipulation draw another straight line through A very carefully. This straight line ought to coincide with the one already drawn. If it does not, the set-square angle is not a right angle. How do you know that?

262. When two lines contain a right angle, either is said to be **at right angles** to the other, or **perpendicular** to the other.

With your ruler, try to draw a line perpendicular to another. Test it with your set-square.

Drawing a Perpendicular

263. The best way to draw a perpendicular is with a set-square.

Suppose you have to draw a perpendicular to AB. Proceed as follows :—

Arrange your ruler with the long edge of the set-

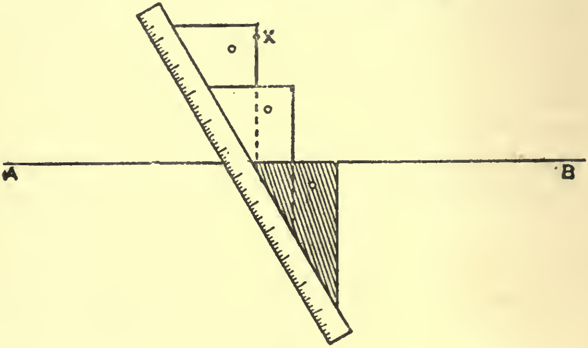


Fig. 41.

square placed against it, so that one arm of the right angle of the set-square lies exactly on AB. Then the other arm of the right angle is perpendicular to AB, and wherever you move your set-square along the ruler, this arm is still perpendicular to AB. (Fig. 41.)

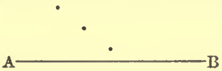
If you have to draw a perpendicular to AB through a point *x*, you will, of course, move the set-square until the upright arm *nearly* covers the point *x*, and then draw a straight line through *x*.

264. Exercise (drawing).

(*a*) Draw several perpendiculars in your note-book, until you can draw them easily.

(*b*) On a line AB mark points *x, y, z*. Draw perpendiculars from them.

(*c*) Mark 3 points outside AB thus :—



Draw perpendiculars through these.

Rectangular Figures

265. You have now learnt what a right angle is, how to test it, and how to draw it. Look again at the square prism, and pick out some faces in which all the angles are right angles.

Such figures are called rectangular (which means "right-angled"). Notice carefully how many sides each rectangular figure has. Is the number of sides

Fig. 42.

Fig. 43.

always the same? Complete the following statement and write it in your note-book: "A rectangular figure is always a . . . ."

There are two kinds of rectangular figures. One has all its sides equal. What is it called? The other has not all its sides equal. It is called a **rectangle** or an **oblong.** (See Figs. 42 and 43.)

On which of the models do you find rectangles, and how many? Measure the sides of some of these rectangles. Which sides of a rectangle are equal?

On which models do you find squares, and how many?

266. Exercise (drawing).

(*a*) Draw some rectangles, several sizes and shapes, with your set-squares.* Mark the length of the sides on them.

(*b*) Draw some squares, several sizes, with your set-squares.*

267. The total length of the sides of a plane figure is called its perimeter. You will find it convenient to remember this word and its meaning.

268. Exercise (to be written).

(*a*) A rectangle is $4\frac{3}{8}$ in. long, and $2\frac{11}{12}$ in. high. What is its perimeter?

(*b*) A square flower-bed is 2 yd. 2 ft. $7\frac{1}{2}$ in. each way. Calculate the length of a border to go round it.

(*c*) A rectangular field is 3080 yd. by 1100 yd. Find its perimeter in miles and fractions of a mile.

(*d*) The edges of a paper which ought to be square are 19·1 cm., 18·85 cm., 18·9 cm., and 19 cm. Find the average length of an edge.

(*e*) If one side of a square is x in. long, what is its perimeter?

* After drawing two sides of a rectangle, or of a square, with the set-square, you will find it easier to complete the drawing with compasses. You already have three corners; try to get the fourth by drawing parts of circles which cut each other.

(*f*) If one side of a rectangle is x in. long, and another side y in. long, what is the perimeter in inches?

(*g*) What would be the perimeter if each side were lengthened by $1\frac{3}{16}$ in.?

(*h*) Draw to scale the page of a book 18 cm. long and 12 cm. broad. Find out how far it is across the page from corner to corner.

Areas of Rectangular Figures

269. It is now time to see how we can measure the **area** of a figure, that is to say, the amount of surface which it contains.

We will begin with those faces of the models which are rectangular figures.

You are provided with a cube and a hexagonal prism.

Compare any face of the cube with one of the rectangular faces of the hexagon. Guess which is the larger. You can, if you like, lay the two faces one on the other, so as to help you in judging. Perhaps one is so much larger that it is easy to judge.

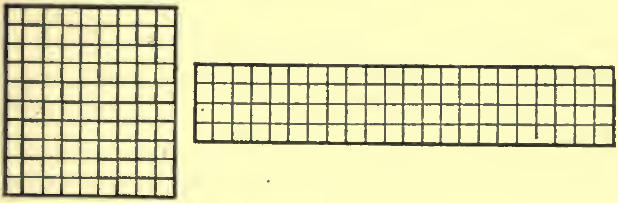


Fig. 44.

Now guess which of the figures shown in Fig. 44 is the larger.

270. You notice that each figure is divided into little squares. All these squares are exactly the same size. Find out which figure is the larger by counting the number of little squares in each. Was your guess right? Whether right or wrong, you will recognise the need of having a standard to measure by.

Standard of Area

271. In measuring these figures by the little squares you *have* been using a kind of standard. It has enabled you to compare the areas of the figures, but it is not the standard which is commonly used.

The standard commonly used is the **square inch** or the **square centimetre**. (Fig. 45.)

Square Inch. Square Centimetre.

FIG. 45.

272. Each edge of the square inch is an inch long, and each edge of the square centimetre is a centimetre long.

It is important to remember that each of these standard areas is a *square*, for it would be possible to have an area, bounded by lines one inch (or one cm.) long, much smaller than the square inch (or the square cm.).

Measure the edges of the two figures in Fig. 46. Are the edges the same length in both? Measure the areas by counting the number of small squares in each. Along the sides of the right-hand figure you will have to count fractions of squares. Count fractions of more than half a square as one square, and neglect

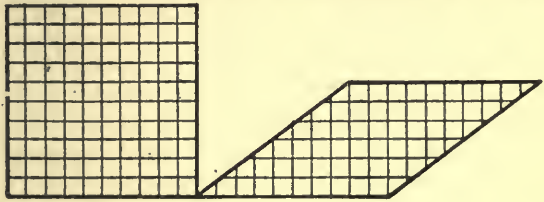


FIG. 46.

those which are less than half a square. How many times is the square inch bigger?

273. Examine a piece of squared paper (millimetre). Each of the smallest squares is a square mm. Find a square cm. on the squared paper.* How many square mm. are there in a square cm.?

274. Now look at the rectangles drawn on mm. paper (shown in Fig. 47).

275. Take rectangle No. 1. Measure (or read off) the lengths of the sides in cm.; then find the area by counting up the square cm. Do the same with the other rectangles. Record these measurements in your note-book thus:—

	Long Side in cm.	Short Side in cm.	Area in sq. cm.
No. 1.			
No. 2.			
etc.			

* Sometimes each square cm. is divided into quarters. Scholars should beware of taking these as square cm.

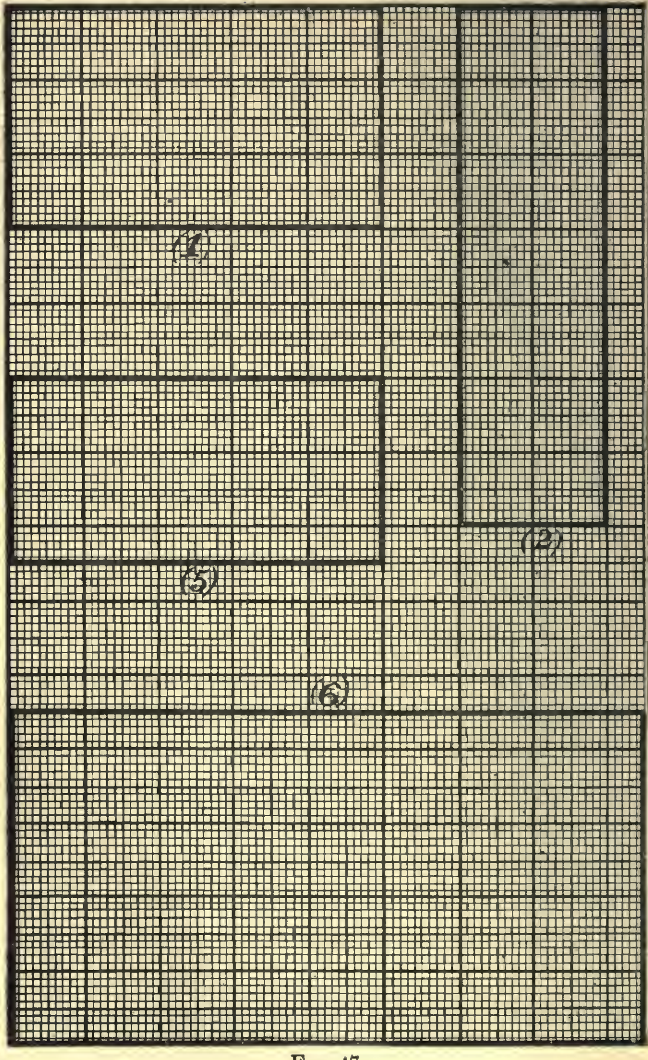


FIG. 47.

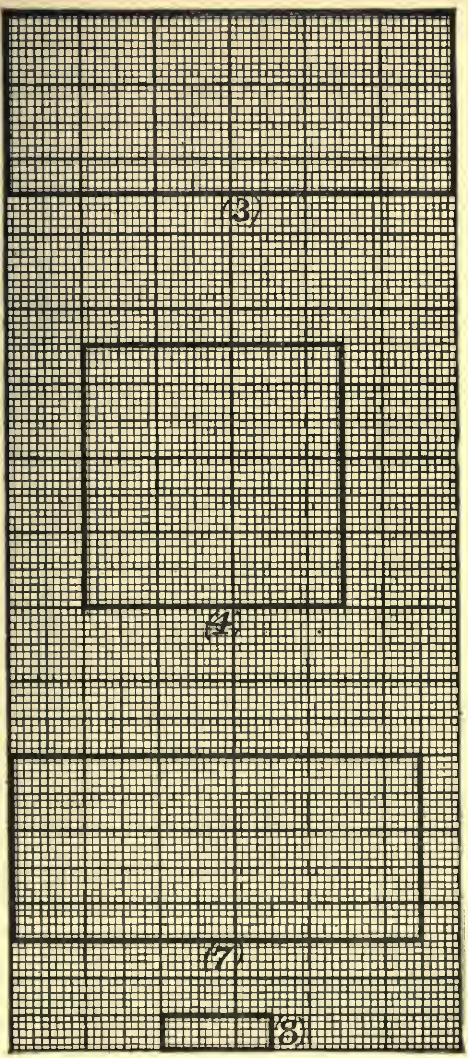


Fig. 47.

Inspect your results carefully and try to discover a rule for *calculating* the area of a rectangle from the lengths of the sides.

Write down the rule in your note-book.

Write down a rule for calculating the area of a square from the length of a side.

276. You describe a rectangle by saying that it is (say) $3\frac{1}{2}$ in. long and $2\frac{1}{2}$ in. high (or broad). You can express this more shortly by saying that it is $3\frac{1}{2}$ in. by $2\frac{1}{2}$ in. This is usually written $3\frac{1}{2}$ in. × $2\frac{1}{2}$ in.

You can describe a square by saying that it is (say) 9 in. square. This means that each side is 9 in. long. You must carefully distinguish "9 in. square" from "9 square inches." When you speak of a square of 9 sq. in., you mean that its *area* is 9 square inches. What is the area of a square with 9 in. sides?

277. Exercise (oral).

(*a*) Calculate the areas of rectangles—
3 cm. × 7 cm., 9 in. × 7 in., 11 cm. × 12 cm.

(*b*) Calculate the areas of rectangles—
$2\frac{1}{2}$ in. × 4 in., $3\frac{1}{4}$ in. × 4 in., $9\frac{1}{2}$ in. × 2 in.

(*c*) Calculate the areas of rectangles—
3·5 cm. × 2 cm., 2·5 cm. × 8 cm., 2·2 cm. × 4 cm.

(*d*) Calculate the areas of squares with 4 in., 7 in., and 9 in. sides.

278. Exercise (requiring the use of models).

Draw hand-sketches of the square prism, the oblong prism, and the hexagonal prism, marking the measurements in inches and fractions. Calculate the area in square in. of each face of the square prism and the oblong prism, and of each rectangular face of the hexagonal prism.

279. Exercise (to be written).

(*a*) What are the areas of rectangles—
$4\frac{1}{2}$ in. × $3\frac{1}{2}$ in. and $\frac{1}{2}$ in. × $3\frac{1}{2}$ in.?
Verify by drawing on squared paper.

(*b*) Calculate the areas of rectangles—
$14\frac{1}{8}$ in. × $3\frac{1}{5}$ in. and $5\frac{2}{3}$ in. × $5\frac{3}{8}$ in.

(*c*) Calculate the areas of squares with sides of $2\frac{1}{2}$ in. and $1\frac{1}{2}$ in. Verify by drawing on squared paper.

(*d*) Calculate the areas of squares with sides of $2\frac{1}{8}$ in. and $2\frac{1}{10}$ in.

(*e*) A piece of paper is $3\frac{1}{5}$ in. square. What is its area?

(*f*) Draw a square containing 9 sq. in. How many inches square is it?

(*g*) Draw two rectangles each containing 9 sq. in. Mark the lengths of their sides.

(*h*) How many stamps, each 3 cm. × 2 cm. would exactly fill a page 25 cm. × 29·76 cm.? You are to suppose that the stamps can be cut up.

(*i*) How many $\frac{3}{4}$ in. squares of glass would be required to fill a rectangular space of mosaic 18 in. × $10\frac{1}{2}$ in.?

(*j*) Out of a piece of paper $7\frac{3}{4}$ in. square I cut a rectangle $4\frac{1}{2}$ in. × $3\frac{1}{2}$ in. How many square in. are left?

(*k*) Draw to a scale of $\frac{1}{6}$ a rectangle, $19\frac{3}{4}$ in. × $11\frac{1}{4}$ in.

Multiplication of Decimals

280. You are given a square prism. Measure correct to a mm. the edges of one of the long faces, and mark the measurements on a hand-sketch.

What kind of figure is this long face?

You have now to find its area. In order to do so you will have to multiply a decimal by a decimal.

Proceed as follows: suppose you have found that the edges are 3·2 cm. and 6·9 cm.

$$\begin{array}{r} 3{\cdot}2 \\ 6{\cdot}9 \\ \hline 19{\cdot}2 \\ 2{\cdot}88 \\ \hline 22{\cdot}08 \\ \hline\hline \end{array}$$

Begin by multiplying by the 6, fixing the decimal point as you have already learnt to do. $3{\cdot}2 \times 6 = 19{\cdot}2$. Now multiply by the 9. In writing down the product, begin one place to the *right* of the last figure of the first line. Place the decimal point immediately under the point of the first line. And add.

Take care to check the position of the decimal point in your answers by the test described in § 37; since $3 \times 7 = 21$, the point must come after the second figure.

281. The 8 in the found answer signifies eight hundredths * of a square cm. These are so small that you need not take account of *one or two* of them, even if you are assured that the measurements of the sides are quite exact. You can therefore write 22·1 sq. cm. instead of 22·08 sq. cm.

282. Of course, if you are *given* the measurements 3·2 cm. and 6·9 cm. you must take them as correct. But you know that measurements which you yourself make often are only approximations, and that 3·1 and 6·8 cm. might possibly be nearer to the true measurements. See what a difference this would make in the product:—

$$\begin{array}{r} 3{\cdot}1 \\ 6{\cdot}8 \\ \hline 18{\cdot}6 \\ 2{\cdot}48 \\ \hline 21{\cdot}08 \\ \hline\hline \end{array} \text{ sq. cm.}$$

There is a difference, not of hundredths of a sq. cm., but of a whole sq. cm. between this product and the

* How much is $\frac{1}{100}$ of a square cm.? Remember that ·1 sq. cm. does not mean 1 sq. mm.

first. So that even if you measure the sides of the rectangle *very* carefully, a calculation of the area correct to a sq. cm. is exact enough for any use to which you can fairly put it. For 22·08 sq. cm., therefore, write 22 sq. cm., if you are dealing with your own measurements.

283. Exercise (oral).

(*a*) How many sq. mm. are there in 22·08 sq. cm., and in 22·1 sq. cm.?
(*b*) By how many sq. mm. is 22·08 sq. cm. larger or smaller than 22·1 sq. cm.?
(*c*) Read out the following correct to one place of decimals: 33·16 sq. cm., 3·89 sq. cm., 9·52 sq. cm., 11·07 sq. cm., 11·05 sq. cm.
(*d*) Read them out correct to a sq. cm. and to a sq. mm.

284. Exercise (to be written).

(*a*) Calculate the areas of rectangles 3·7 cm. × 9·1 cm., 4·6 cm. × 10·3 cm., and ·9 cm. × 2·5 cm.
(*b*) Calculate the areas of squares 1·2 in., 2·4 in., and 3·8 in. square.
(*c*) Calculate the areas of squares with sides of 3·3 cm., 4·4 cm., and 5·5 cm.
(*d*) Verify your results in (*c*) by drawing on mm. paper.

285. Exercise (requiring the use of models).

(*a*) Calculate the area in sq. cm. of each face of the square prism. What degree of correctness will you try to reach?
(*b*) Calculate the area in sq. cm. of each rectangular face of the hexagonal prism.

Degrees of Accuracy

286. You have learnt to measure the edges of a model in inches correct to ·05 in. You ought to be able to find the area of the faces in square inches correct to ·5 sq. in., if your measurements of the edges are carefully made.

In multiplying, follow the rule given in § 280, thus:—

```
 13·65
  9·35
------
122·85
  4·095
   ·6825
-------
127·6   (say 127·5 sq. in.)
```

You need not add up all the figures to the right of the first column after the decimal point. You must remember, however, to add up mentally the column in the second place (8, 9, 5), in order to carry the 2 to the first column.

287. Exercise (oral).

(*a*) Read out, correct to ·5, 13·459, 3·295, 6·109, 3·791, ·956.

(*b*) Find, correct to ·5, the sum and the difference of 4·388 and 9·019.

(*c*) Calculate, correct to ·5, $39{\cdot}37 \times 21{\cdot}93$, $68{\cdot}41 \times 22{\cdot}7$, and $100{\cdot}9 \times 10{\cdot}03$.

288. Exercise.

You are given a rhomboid prism and an oblong prism.

Make a hand-sketch of each model, marking the measurements of the edges in inches.

Calculate in square inches the area of the rectangular faces of the models. Do your best to get your result correct to ·5 sq. in.

Formulæ

289. Denoting the base of a rectangle—that is, the side on which it seems to stand—by b, and the height of the rectangle by h, and the area by A_r, complete the formula, $A_r =$

Write the formula in your note-book.

290. Exercise (to be written).

(*a*) If one side of a rectangle is x in., and the other y in., long, what is the area?
(*b*) If the first side is increased by $2\frac{1}{2}$ in., what is then the area?
(*c*) Verify your answer by taking x as 5, and y as 7.

291. You have learnt to calculate the area of a rectangle by multiplying the length of the long side by that of the short side; if you were told the area and the length of *one* side, how would you calculate the length of the other side?

Express your rule as a formula, thus:—

$$h_r = \ldots, \quad b_r = \ldots$$

292. Exercise (oral).

(*a*) The area of a rectangle is 72 cm.; the base is 9 cm. What is the height?
(*b*) The area of a rectangle is 21·5 in.; the height is 5 in. What is the base?
(*c*) Express in words the formula $b_r = \frac{A}{h}$.

Division of Decimals

293. In working out one of these problems, you may have to divide a decimal by a decimal. Proceed as follows :—

$$139{\cdot}42 \div 7{\cdot}25$$

```
         19·23 = Quotient
  7·25)139·42
        72·5
        66·92
        65·25
         1·670
         1·450
           220
```

First divide mentally by the whole number 7, so as to see where the decimal point will come in the quotient. If you were dividing 139 by 7 (short division), you would write the first figure of the quotient, 1, under the 3. In this case, place it *over* the 3. This will help you; for then the decimal point of the quotient will come over the decimal point of the dividend, just as in short division it comes under it. Complete the sum as in long division.

294. Exercise (to be written).

(*a*) Divide, correct to ·05, 331·9 by 23·9, by 239·45, and by 92·25.

(*b*) Find the height of a rectangle of which the area is 529·4 sq. cm., and the base 69·25 cm.

(*c*) The area of a rectangle is $39\frac{15}{16}$ sq. in.; the height is $11\frac{5}{8}$ in. Calculate the base.

(*d*) In § 217 you found that 1 inch was rather more than 2·5 cm. Call it 2·5 cm. Find, by calculation, how many sq. cm. there are in a square inch.

(*e*) How many sq. cm. are there in 32 sq. in., 25 sq. in., 4 sq. in., $7\frac{1}{2}$ sq. in.?

(*f*) How many sq. in. are there in 25 sq. cm., 30 sq. cm., 12·5 sq. cm., 1 sq. cm.?

(*g*) Take the cube. Find the area of a face in sq. cm. correct to a sq. cm.

(*h*) Convert the sq. cm. into sq. in.

(*i*) Now measure the edges in inches, and find the area in square inches, correct to one place of decimals. Does this agree with your former result? If it does not, can you suggest a reason?

295. $\frac{32 \cdot 4}{\cdot 04} = \frac{3240}{4}$, for if you multiply numerator and denominator by the same number (*e.g.* 100) the value of the fraction is not altered.

When you have to divide by a decimal in which there is no whole number, multiply both dividend and divider by a number which will give you a whole number in the divider. Then divide as you have already learnt to do, *e.g.*—

$$4 \cdot 17 \div \cdot 139 = \frac{4 \cdot 17 \times 1000}{\cdot 139 \times 1000} = \frac{4170}{139} = 30.$$

296. Exercise (to be written).

(*a*) Divide 32·4 by ·19, by ·91, and by ·091.

(*b*) The area of a passage is 72·65 sq. m.; the breadth is ·95 m. Find the length.

(*c*) Find the height of an oblong, of which the area is 3·15 sq. in., and the base $3\frac{1}{2}$ in.

Squares and Square Roots

297. 3×3 is often written 3^2 (to be read "three squared"). In the same way you should write x^2, not xx.

Denoting the side of a square by s and the area by A_s, complete the formula, $A_s =$

Write the formula in your note-book.

The number 9 is said to be the "square" of the number 3.

298. Exercise (oral).

(*a*) Find the squares of 6, 11, 12, $\frac{5}{2}$, $2\frac{1}{4}$, $1\frac{1}{4}$, 1·1, 1·2.
(*b*) Find the value of 3^2, 9^2, 10^2, 1^2.
(*c*) If $x=5$ and $y=8$, what is the value of x^2, xy, y^2?
(*d*) Write a definition of the "square of a number."

299. As 9 is called the "square" of 3, so, conversely, 3 is called the "square root" of 9, which is written $\sqrt{9}$ (to be read "root nine").

Exercise (oral).

(*a*) Express in words, $5^2=25$, and $\sqrt{25}=5$.
(*b*) What is the square root of 4, of 1, of x^2, of a^2?
(*c*) If A is the area of a square, and s its side, complete the formula, $s_s=$
(*d*) What is the square root of $\frac{9}{4}$, of $\frac{49}{16}$, of $6\frac{1}{4}$, of $5\frac{1}{16}$, of 1·44, and of 1·21?

300. Write in your note-book in a table the squares of all the numbers from 1 to 13, thus:—

$1^2 =$
$2^2 =$ etc.

From the squares you can of course work backwards, and read off the square roots. Learn this table.

301. Exercise (to be written).

(*a*) Find the value of $\sqrt{121}$, $\sqrt{169}$, $\sqrt{5\frac{19}{25}}$.
(*b*) Calculate the length of the side of a square whose area is $1\frac{69}{100}$ sq. in.
(*c*) Verify your result by drawing on squared paper.

(*d*) Calculate the length of the side of a square equal in area to a rectangle with sides of x in. and y in.

(*e*) Verify by taking x as 5 in. and y as $6\frac{1}{20}$ in.

Larger Square Measures

302. Let us now consider some larger square measures. A square inch, as we saw, is a square with sides of 1 inch. Similarly, a square foot is a square with sides of 1 foot; and a square yard is a square with sides of 1 yard. How many square inches are there in a square foot? How many square feet are there in a square yard?

303. Exercise (to be written).

(*a*) Draw a plan of a square foot (scale $\frac{1}{4}$), divided into square inches. Count the number of square inches. Does the number agree with your previous calculation?

(*b*) How many square yards are there in 29 sq. ft., in 3 sq. ft., in 10 sq. ft., in 1 sq. ft.?

(*c*) How many square feet are there in 72 sq. in., in 468 sq. in., in 108 sq. in.?

(*d*) Calculate the number of sq. cm. in a square foot. (1 in. = 2·54 cm.)

(*e*) What is the area of a desk-top 2 ft. × $17\frac{1}{2}$ in.? *

(*f*) Find the area of your desk-top in square feet.

(*g*) Find the area of your class-room (1) in sq. yd.; (2) in sq. m.

* Remember that 2 ft. × 20 in. makes neither 40 square in. nor 40 square ft. The dimensions multiplied must be of *the same kind*. 2 ft. × 20 in. may be expressed as 24 in. × 20 in. (=480 square in.), or as 2 ft. × $1\frac{2}{3}$ ft. (=$3\frac{1}{3}$ square ft., which is just the same as 480 square in.).

(*h*) A bowling alley is $21\frac{1}{2}$ yd. long, and its area is $612\frac{3}{4}$ sq. ft. Find its breadth in feet and inches.

304. The square mile is used for measuring large areas, *e.g.* a country or a large island.

The acre is used for measuring areas which, though fairly large, are too small to be measured in square miles, *e.g.*, a field, a park, or a large square. An acre contains 4840 square yards; this would make a square with sides of about $69\frac{1}{2}$ yards.

Calculate the number of square yards in a square mile, and then calculate the number of acres in a square mile.

(In order that the scholars may realise what an acre is, an acre should be staked out in the playground. If this cannot be done, they should be referred to some open space which is about an acre in extent.)

305. Write out in your note-book the following tables of square measures:—

(*a*)

	—— sq. in.	=	1 sq. ft.
—— sq. ft. or	—— sq. in.	=	1 sq. yd.
	—— sq. yd.	=	1 acre
	—— acres	=	1 sq. ml.

(*b*)

	—— sq. mm.	=	1 sq. cm.
	—— sq. cm.	=	1 sq. dm.
—— sq. dm. or	—— sq. cm.	=	1 sq. m.

306. Exercise (to be written).

(*a*) How many acres are there in a park 1400 yd. × 520 yd.?

(*b*) The greatest breadth of a country is 363 ml., and the greatest length $102\frac{1}{2}$ ml. What is its greatest possible area?

(*c*) The actual area is found to be only 27,500 sq. ml. Why is this?

(*d*) Take a map of England and examine the scale. How many square miles does a square inch on the map represent?

(*e*) A field is 360 yd. × 234 yd.; find its area in acres to the nearest quarter of an acre.

(*f*) Draw a plan of the park, scale $\frac{1}{3000}$.

(*g*) How many yards long and how many broad might a rectangular field be which contains $3\frac{1}{4}$ acres?

(*h*) A field of $7\frac{1}{2}$ acres is broken up into plots 11 yd. square. How many plots are there?

(*i*) What is the value of the field at $2\frac{3}{4}$d. a square yard?

(*j*) On a map a square inch represents 100 sq. ml. How many sq. miles will it represent if the scale is reduced by a half?

Vertical and Horizontal

307. You have learnt how to test whether a line is perpendicular to a line. But you have not yet learnt that a line may be perpendicular to a plane. If you observe a steamboat from behind, the funnel appears as in Fig. 48, that is, it appears to be perpendicular to the line of the deck. But if you see the steamboat

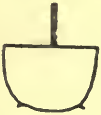


FIG. 48.

FIG. 49.

from the side, the funnel appears, as in Fig. 49, no longer perpendicular to the line of the deck.

A line is perpendicular to a plane only when it appears to be perpendicular from every point of view. Try to set up your pencil on a table, perpendicular to the plane of the table.

A *plane, x,* is perpendicular to a plane, *y,* when a line can be drawn in plane *x* perpendicular to plane *y*.

Your teacher will illustrate this for you by means of a hinged plane.

308. Look out in your dictionary the meaning of the word "horizon." You have probably observed the horizon out at sea. If you confine your attention to a small stretch of it, it seems to go from right to left, or from left to right, in a straight line, without sloping upwards or downwards. Such a line is called a horizontal line. If you place a match on still water, its position is horizontal, and the surface of the water is a horizontal plane. Mention some other horizontal planes.

A line at right angles to a horizontal plane is a vertical line. Try to find some vertical lines in your class-room.

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PRACTICAL ARITHMETIC

BOOK II

PRACTICAL ARITHMETIC

AN INTRODUCTION TO ELEMENTARY MATHEMATICS FOR SCHOLARS BETWEEN THE AGES OF 9 AND 12

BY A. CONSTERDINE, M.A.

AND

S. O. ANDREW, M.A.

Headmaster of Whitgift Grammar School, Croydon

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CHAPTER VII

THE MEASUREMENT OF ANGLES

Acute and Obtuse Angles

309. In Chapter VI. you learnt something about right angles. Now you are going to deal with angles which are not right angles.

Your teacher will give you a hexagonal prism. Look at the angles of the hexagonal faces and guess whether they are larger or smaller than right angles. Verify your guess by applying a set-square.

Find out in the same way which of the angles of

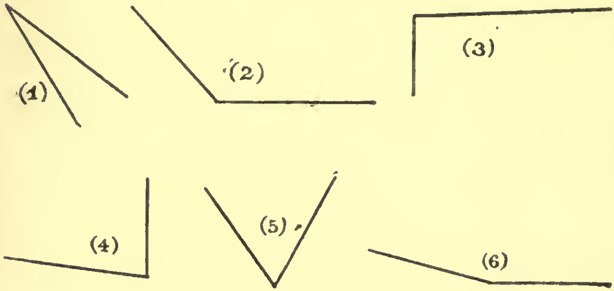


Fig. 50.

your set-squares are larger, and which are smaller than right angles.

All angles which are smaller than a right angle are called **acute angles**; all those which are larger than a right angle, are called **obtuse angles**.

310. Exercise.

(*a*) Draw some obtuse and some acute angles.
(*b*) Which of the angles in Fig. 50 are obtuse and which acute?

The Right Angle as a Standard

311. Let us now use the right angle as a standard angle in order to measure the sizes of some common angles and groups of angles.

You have already learnt that the angles of a regular hexagon are equal to one another. Now try to dis-

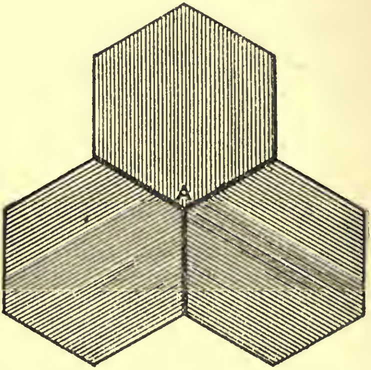


Fig. 51.

cover how many times each of them is greater than a right angle, in the following manner: Arrange three hexagonal prisms as shown in Fig. 51, and notice that three of the angles exactly fit together round a point (see point A in the figure).

How many right angles would in the same way fill up the space round a point?

How many times, then, is each of the angles of a regular hexagon greater than a right angle?

312. We will now examine the angles in some triangles. Look at your set-squares, and pick out the one which contains two equal angles. Are the equal angles acute or obtuse?

Find out how many times each of the equal angles is larger or smaller than a right angle. In doing this, you may borrow one or more similar set-squares from your neighbours, and place them together, so that the angles in question will form one or more right angles, as in the preceding section.

313. Take your other set-square. How many of its angles are equal? One of the angles is a right angle. Are the two other angles obtuse or acute?

Choose the smaller angle, and find out what fraction it is of a right angle.

Do the same with the larger angle.

How many times is the large angle greater than the small one?

314. Draw three triangles of different shapes and sizes on some stiff paper. Cut them out with a pair of scissors.

Mark the three angles of one of them with your pencil; cut them off and fit them together, as shown in Fig. 52.

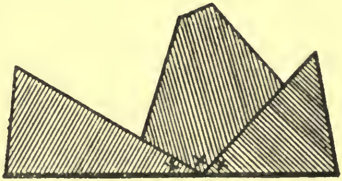


FIG. 52.

Treat the other triangles in the same way.

How many right angles are the three angles of a triangle equal to?

315. How many right angles are the four angles of a square, or of a rectangle equal to?

Draw three quadrilaterals of different shapes and sizes, and cut them out. Mark their angles, cut them off, and fit them together round a point. What do you observe, and what conclusion do you come to?

Need for a Smaller Standard Angle

316. You will now see that the right angle is a convenient standard for measuring certain large angles and groups of angles. But there are a good many angles which are not easily measured by means of the right angle.

Take some tracing-paper, and try to measure the

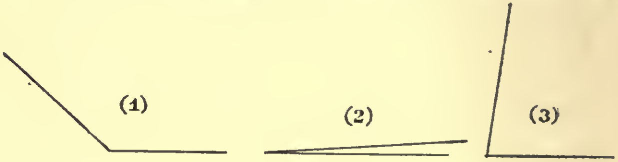


FIG. 53.

angles in Fig. 53 in right angles and fractions of a right angle.

No doubt you find this difficult. As a matter of fact, the angles measure $1\frac{24}{45}$, $\frac{2}{45}$, and $\frac{41}{45}$ of a right angle. These fractions are, of course, very clumsy; it is as though you measured the arms of the angles in yards instead of inches.

It is very easy to see that a standard much smaller than a right angle would be very convenient.

The Protractor

317. Take out your protractor, which is an instrument for measuring angles. Notice that it is shaped like a half-circle. The straight edge is a diameter.

The vertex of an angle to be measured is placed at the middle point of the diameter, and one of the arms of the angle is placed along the diameter in such a way that the other arm cuts the circumference.

The circumference is divided into 180 equal divisions. There are, therefore, 180 divisions between the arms of a straight angle, and 90 between the arms of a right angle.

One of these divisions, called a **degree**, is the standard angle. A right angle contains 90 degrees (to be written, 90°), and a straight angle contains 180°.

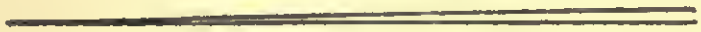


Fig. 54.

Fig. 54 is a drawing of the standard angle, that is, of an angle of 1°.

318. Exercise (measurement and drawing).

(*a*) Measure in degrees the sizes of the angles of :—

(1) A hexagonal prism.
(2) A pentagonal pyramid.
(3) Your set-squares.

(*b*) Measure the angles given in Fig. 53 in degrees.

(*c*) Take your protractor and draw angles of 75°, 15°, 30°, 97°, 179°, 60°, 180°, 120°.

319. Exercise (oral).

(*a*) How many degrees are there in $1\frac{1}{2}$ right angles ; half a straight angle ; one-third of a right angle ; $1\frac{1}{3}$ right angles ; one-sixth of a right angle ; two-thirds of a right angle ?

(*b*) What fraction of a right angle is 60°, 45°, 120°, 15°, 30°?

(*c*) Add together the angles in each of the

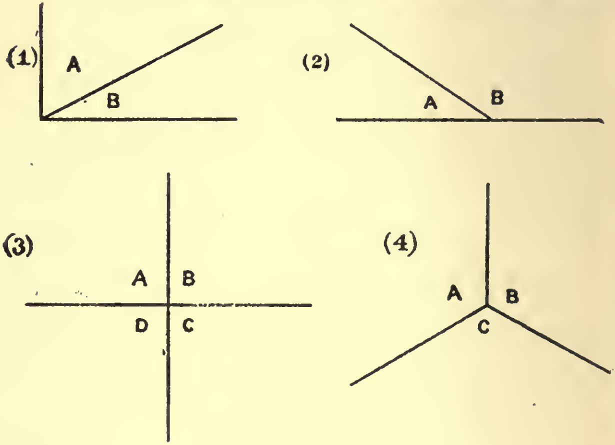


Fig. 55.

figures in Fig. 55, and express your results in degrees.

(*d*) How many degrees are there in 4 right angles?

(*e*) Through how many degrees does the minute-hand of a watch turn in half an hour, in 20 minutes, in 5 minutes, in 1 minute?

(*f*) What is the size in degrees of the sum of all the angles of a quadrilateral, and of a triangle?

(*g*) What is the size in degrees of each of the angles in a square, and in an equilateral triangle?

(*h*) One angle of a triangle is a right angle, and the other angles are equal to one another. What is the size of each of the angles in degrees?

Angles and Arcs

320. You will have observed that you have been measuring angles by using the divisions on the circumference of your protractor. Let us see whether this is a trustworthy method.

Draw the circumference of a circle with a radius of 3 cm. Mark two points A and B on the circumference, and join the points to O, the centre of the circle.

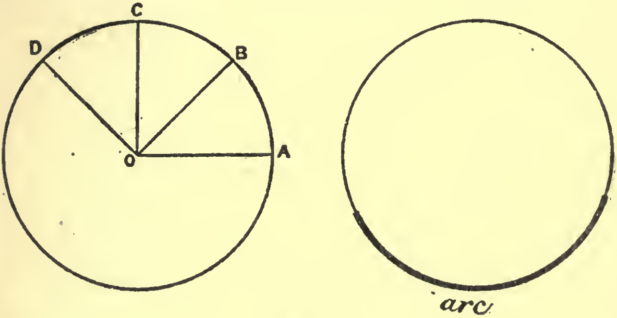


Fig. 56.

Any part of a circumference, like AB, is called an **arc**, and an angle like AOB at the centre of the circle is said to be **subtended** by the arc AB.

321. Now mark off some more arcs equal in length to AB in the following manner: Draw another circumference of a circle of 3 cm. radius on tracing-paper. Place it on the top of the first circumference, and put a pin through the two centres. The two circumferences will then coincide.

Prick off the arc AB, and then prick off equal arcs, like BC and CD, by rotating the tracing-paper. Remove the tracing-paper, and join CO and DO.

322. You have now marked off equal arcs on the circumference of a circle, and you have drawn the angles which these arcs subtend at the centre of the circle.

Are these angles equal to one another? Replace the tracing-paper and find out.

From these experiments are you able to conclude that the method of measuring angles by your protractor is trustworthy? If so, why?

323. Exercise (oral).

(*a*) Look at the figure in § 320. The arc AC is twice as big as the arc CD. How many times is the angle AOC greater than the angle COD?

(*b*) If an arc of a circle is 1 in. long, and subtends an angle of 15° at the centre of the circle, what will be the size of the angle subtended by an arc of 3 in.?

(*c*) What fraction of the circumference of a circle is included between the arms of a right angle whose vertex is at the centre of the circle?

(*d*) And what fraction is cut off by the arms of an angle of 60°?

324. Exercise (to be written).

(*a*) An angle of 45° is drawn with its vertex at the centre of a circle having a radius of 1 yd. How many inches long is the arc which subtends the angle?

(*b*) The circumference of a circle is 12 times as big as an arc of the same circle. How many degrees does the angle subtended by the arc at the centre of the circle contain?

(*c*) An angle has its vertex at the centre of a circle having a radius *r*. The length of the arc subtending the angle is *r*. What is the size of the angle in degrees?

Large and Small Protractors

325. It is very important to understand that equal angles are subtended by equal arcs only when the arcs are parts of the *same* or of *equal* circumferences.

Look at Fig. 57. You will see that the same angle AOB is subtended by two arcs AB and CD, which are parts of unequal circumferences. The arc AB is obviously longer than the arc CD.

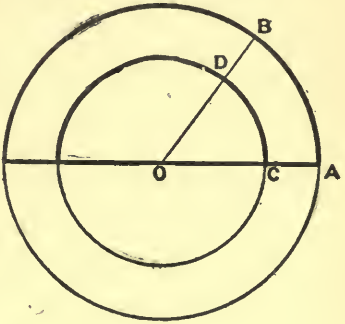


FIG. 57.

Some protractors are bigger than others, as you have perhaps observed. Yet they all measure angles with a standard which is the same. The bigger the protractor, the bigger is its circumference, and the bigger the circumference, the bigger are the arcs into which the circumference is divided (look at the figure again).

Arcs and Chords

326. Draw the circumference of a circle (Fig. 58) and mark off equal arcs AB, BC, and CD in the manner described in § 321.

Now draw the straight lines AB, BC, and CD. These lines are chords. A **chord** is a straight line drawn in a circle and terminated at each end by the circumference.

You have made the arcs AB, BC, and CD equal to one another. Take your dividers and find out whether the corresponding chords are also equal to one another.

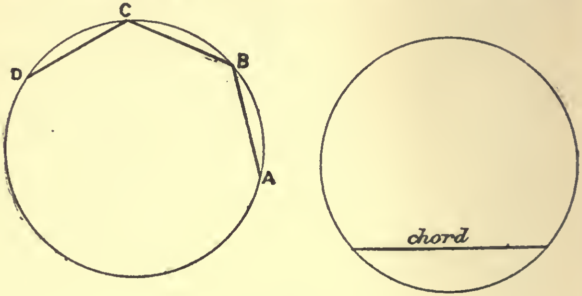


Fig. 58.

Suppose that instead of drawing equal arcs you had used your dividers and ruler and drawn equal chords,

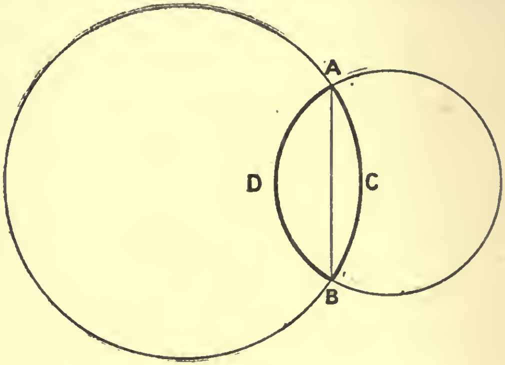


Fig. 59.

what would have been the sizes of the arcs cut off by the chords?

You will now see that you can mark off equal arcs on a circle without going to the trouble of using tracing-paper.

327. Look at Fig. 59. You will see that the straight line AB is a chord of two unequal circles, and that the corresponding arcs ACB and ADB are by no means equal.

It is therefore very important for you to remember that equal chords do not cut off equal arcs of circles unless the circles are the same size—that is, unless they have equal radii. You will understand how important this statement is when you come to the next section, in which you will learn how to copy an angle by using your compasses.

The Construction of Angles

328. Copy the angle ABC (Fig. 60) in the following manner:—With centre B, and any convenient radius, draw an arc as shown in the figure.

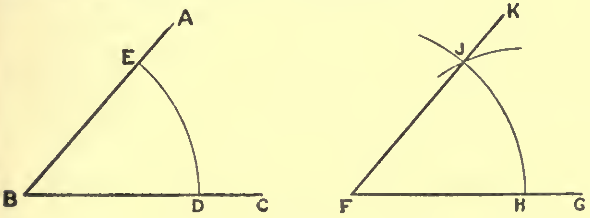


Fig. 60.

Draw a straight line FG, and with F as centre, draw an arc having the *same radius* as the first arc.

Take your compasses, and from H set off a distance HJ equal to DE.

If you were to draw the chords DE and HJ, they would of course be equal to one another. The corresponding arcs DE and HJ are therefore equal to one another as well.

Draw a straight line through F and J.
The angle KFG is equal to the angle ABC. Why?

329. Exercise (drawing).

Draw some angles in your note-book and make copies of them without using your protractor or tracing-paper.

330. You ought now to have very little difficulty in drawing angles which are two or more times as big as a given angle.

Draw an angle ABC in your note-book, and then try to draw an angle DEF which is twice as big as ABC. In doing this you will have to arrange (1) that the arcs subtending the two angles shall have the same radius, and (2) that the arc subtending DEF shall be twice as long as the arc subtending ABC.

331. Exercise (drawing).

(*a*) Draw angles which are twice as big as those in Fig. 61.

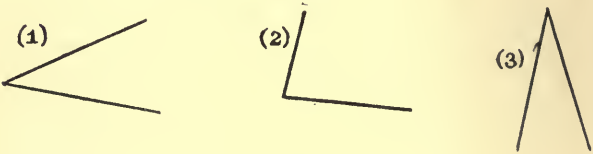


Fig. 61.

(*b*) Draw angles which are three times as big as those in Fig. 62.

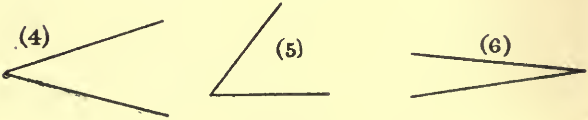


Fig. 62.

The Bisection of an Angle

332. Let us now learn how to bisect an arc and an angle. We will begin with an arc.

Draw an arc on some tracing-paper. Fold the paper and find the middle point of the arc.

Draw another arc in your note-book and bisect it in the same way as you bisect a straight line (§ 89).

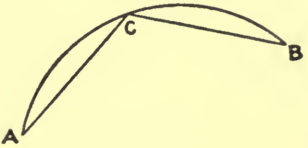


Fig. 63.

Suppose that by this method you find C (Fig. 63) to be the middle point of the arc. To test whether the arc is really bisected at the point C, draw the chords CA and CB, and compare their lengths by using your dividers or compasses. If the chords are equal the arcs will also be equal (see § 326).

333. Now draw an angle on some tracing-paper. Then fold the paper in such a way as to bisect the angle.

Draw an angle in your note-book and draw also an arc between the arms of the angle. Now try to bisect the angle without using your protractor or tracing-paper.

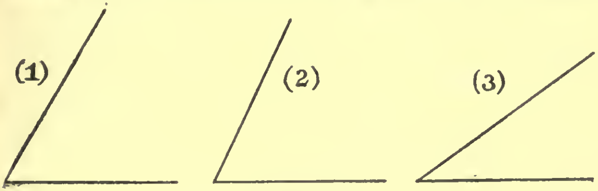


Fig. 64.

334. Exercise.

(*a*) Bisect the angles in Fig. 64 by using your compasses and ruler.

(*b*) The angles in (*a*) are $\frac{2}{3}$ of a right angle, $\frac{13}{18}$ of a right angle, and $\frac{2}{5}$ of a right angle. What will be the sizes of their halves in degrees? Verify your answers by using your protractor.

FIG. 65.

(*c*) Divide the angle in Fig. 65 into four equal parts.

(*d*) The angle in (*c*) is $1\frac{4}{5}$ of a right angle. What are the sizes of its quarters in degrees? Verify your answer by using your protractor.

The Construction of Certain Common Angles

335. You will now be able to draw angles of certain sizes without using your protractor or tracing-paper.

Draw the circumference of a circle and a radius OA. Begin at A and use your compasses to mark off lengths equal to the radius round the circumference. Notice that the radius goes *exactly* six times round the circumference.

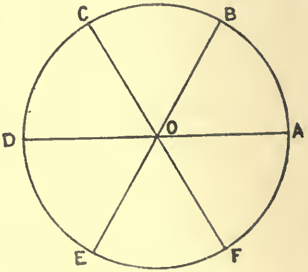


FIG. 66.

Join OB, OC, OD, OE, and OF.

If you drew the chords AB, BC, CD, etc., they would of course be equal. Why? The corresponding arcs and the angles which these arcs subtend at the centre of the circle are therefore also equal. Why?

State the size of the equal angles in degrees without using your protractor.

336. Now try to draw a single angle of 60°.

Begin by drawing a straight line AB. Remember what you discovered in the last section and then draw an angle BAC equal to 60°.

337. In drawing other angles with your compasses (and ruler) you will have to start with an angle of 60°.

To draw a right angle you will have to take one angle of 60° and half of another angle of 60°.

Now try to draw a right angle in the shortest possible way. In doing this you will not find it necessary to actually draw an angle of 60°.

338. Exercise (drawing).

Draw, by construction, angles of:—

(*a*) 120°	(*d*) 135°
(*b*) 45°	(*e*) 15°
(*c*) 30°	(*f*) 150°.

Measuring Angles in Space

339. You have already learnt that an angle, *e.g.* AEB, at the centre of a circle is said to be subtended by a chord, *e.g.* AB (Fig. 67). Now imagine that your eye is at point E, and that AB is the breadth of one of the windows in the class-room. To measure the angle AEB, which represents the angle subtended at your eye by the breadth of the window, you will require a special instrument.

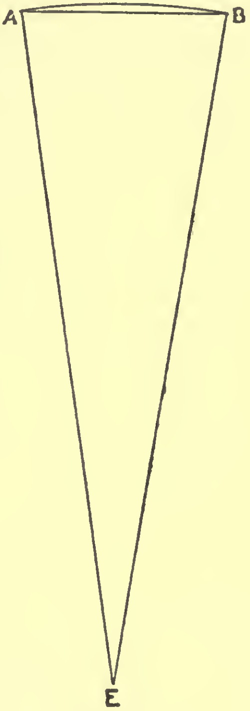


Fig. 67.

340. Take a piece of thick cardboard, about 18 in. square, and prepare it as shown in Fig. 68.

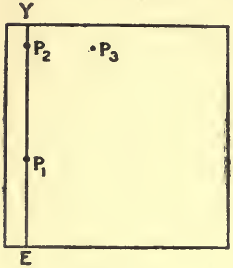


FIG. 68.

E, position of eye; EY, eye-line drawn on the cardboard; P_1, P_2, P_3, pins with large glass beads pricked into the cardboard at right angles.

Hold the cardboard in a horizontal position and arrange it so that the left-hand side of the window is in the same straight line as the pins P_1 and P_2, the eye being placed at E. Keep the eye and the cardboard quite steady and move P_3 so that it comes exactly between E and the right-hand side of the window. Draw a line from P_3 to E, and measure the angle P_3EP_2 with your protractor.

Find in the same way the angles subtended at your eye by several other objects in the room.

Find the angular height of the sun above the horizon at noon.*

341. Exercise (to be written).

(*a*) A line joining the two towns A and B subtends an angle of 37° at the point O. $OA = 7\frac{1}{2}$ miles and $OB = 5\frac{7}{8}$ miles. What is the distance between the two towns?

(*b*) The earth revolves in 24 hours. Through what angle does a point on it pass in 12 hours, in 8 hours, in 3 hours, and in 1 hour?

(*c*) What angle is subtended at the centre of a circle by a chord which is equal to the radius?

(*d*) The arc of a circle cut off by two radii

* This should be done once a month for a year, and the results recorded in the form of a graph on squared paper.

containing an angle of 77° is $1\frac{7}{12}$ in. What is the size of the arc of the same circle which is cut off by two radii at an angle of 89°. Express the result in decimals (correct to two places).

(*e*) An arc ABC is $1\frac{15}{16}$ times as big as an arc DEF. If ABC = 17 in., find the length of DEF in inches and decimals (correct to two places).

(*f*) Through what angle does each of the hands of a clock turn :—

Between 1 and 2 o'clock.
Between 1.12 and 1.32.
Between 2.25 and 3.10.
Between 4.45 and 5.15.

(*g*) Two ships leave a port at the same time. One sails due west and the other south-west. On the following day at noon the first has sailed 280 miles, and the second 320 miles. How far are the ships apart?

(*h*) A dog runs $\frac{1}{2}$ mile in a straight line from a house to a tree. He then turns at right angles to the left, and runs 1 mile to a village. He then turns through 60° to the right, and runs 1 mile to a bridge. Finally he turns to the left through an angle of 45°, and runs $\frac{3}{4}$ mile to a windmill. How far is he from his starting-point?

(*i*) Draw a right angle with arms of 2 in. and $2\frac{2}{3}$ in. What is the distance between the ends of the arms?

(*j*) Fig. 69 shows a regular pentagon drawn inside a circle. Calculate the sizes of the angles subtended at the centre of the circle by the sides of the pentagon.

Verify your answer by using your protractor.

(*k*) Draw a circle, and then draw a regular pentagon inside it.

(*l*) Draw a circle having a radius of $1\frac{7}{12}$ in., and in it draw a chord $1\frac{1}{8}$ in. long. Measure the angle subtended by the chord at the centre of the circle.

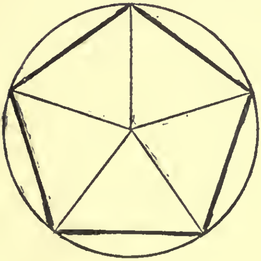


Fig. 69.

(*m*) Two straight lines cross one another, and make an angle of 15°. What are the sizes of the other angles in degrees?

(*n*) Draw a rectangle with sides of 5 cm. and 4 cm., and join two opposite corners. Measure all the angles of the figure in degrees.

(*o*) Draw an angle of 95° having arms of 3·7 cm. and 4·5 cm., and find the distance between the ends of the arms.

(*p*) If a circle has a radius of 7 in., and if $\pi = \frac{22}{7}$, calculate how many radii can be drawn so that each adjacent pair of them cut off an arc of 4 in.

(*q*) A circle has a diameter of 1 ft. What will be the length of the arcs cut off by these angles at the centre: 60°, 45°, 120°, 135°?

CHAPTER VIII

THE MEASUREMENT OF NON-RECTANGULAR AREAS

Parallel Lines

342. You are provided with a rectangular prism. Measure the distance between the opposite sides of any one of the faces, first at one end of the face, and then at the other. Are the opposite sides the same distance apart?

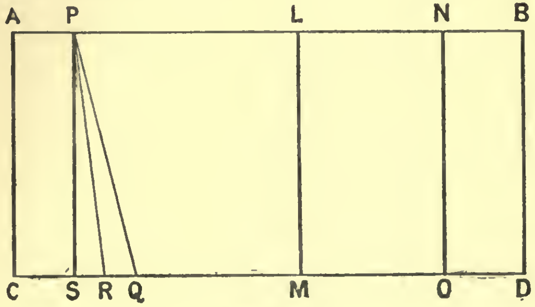


Fig. 70.

343. Draw a rectangle, ABCD (Fig. 70), with great care, and mark a point, P, on one of its sides.

Now try to draw the shortest possible straight line between P and CD. In doing this, draw several lines like PQ, PR; draw also PS at right angles to CD. Compare the lengths of these lines by using your dividers or tracing-paper.

How would you describe the shortest straight line which can be drawn from a point to a line?

Draw LM and NO at right angles to CD, and compare their lengths with those of PS, AC, and BD. Are the lines AB and CD the same distance apart?

Two lines which are the same distance apart along their whole lengths are called **parallel lines**.

344. Look at the windows and the doors of the class-room, and at your note-book, your desk, and other objects in the room. Mention any pairs of parallel lines which you can see.

Your teacher will give you a rhomboid prism and a pyramid. Look for pairs of parallel lines, and for pairs of lines which are not parallel.

345. To draw a line parallel to AB (Fig. 71), you must adopt a method which will ensure that the line

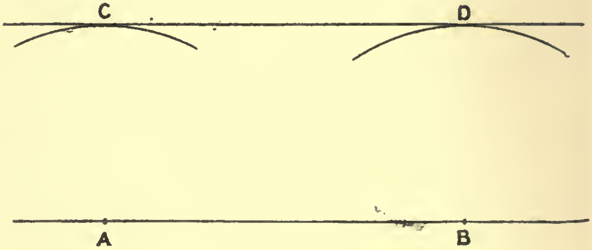


Fig. 71.

shall be the same distance from AB along its whole length.

With A as centre, and any convenient radius, draw an arc. With B as centre, and with the same radius, draw another arc. Draw a line just touching both of the arcs. This line will be parallel to AB. Why?

346. The best way of drawing parallel lines is by means of set-squares, thus: Draw a straight line, AB, and arrange your set-squares as shown in Fig. 72.

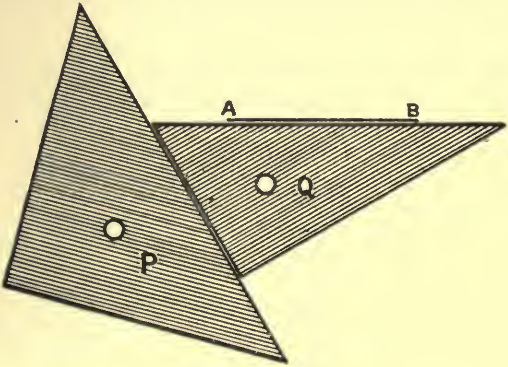


Fig. 72.

Hold P firmly with the left hand, and slide Q along it. Lines drawn along the upper edge of Q will be parallel to AB.

Draw several pairs of parallel lines for practice.

347. Let us see why the above method of drawing gives parallel lines.

Draw two parallel lines by the method of arcs (see § 345), and draw also a cross line (Fig. 73).

The two angles marked *a*, on the same side of the cross line, are called **corresponding angles**. So are the two angles marked *b*. Mark the corresponding angles on the other side of the cross line.

Take your protractor or some tracing-paper, and compare the sizes of the corresponding angles. What do you notice?

Complete the following statement: "When a straight line crosses two parallel lines, the corresponding angles . . ."

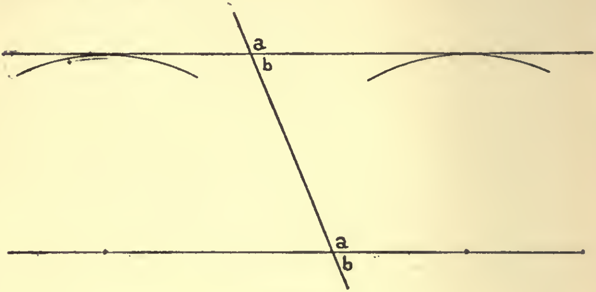


Fig. 73.

If two lines are crossed by a third line, and you find that the corresponding angles are equal, what would you conclude about the two lines?

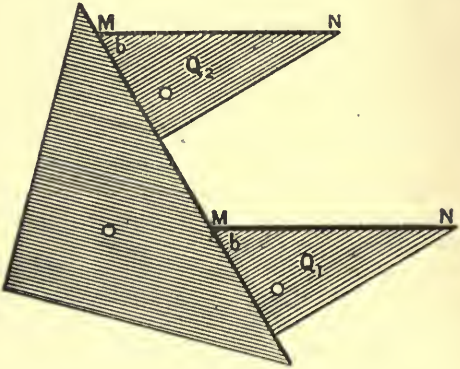


Fig. 74.

348. Now compare Fig. 74 with Figs. 72 and 73.

The set-square Q_1 is moved to Q_2. Are the corresponding angles marked *b* equal to one another? Are the two lines marked MN parallel lines?

Turn to § 263. Can you now explain the method which is used for drawing a line at right angles to another line?

Parallelograms

349. Draw two parallel lines, AB and CD, and then two other parallel lines AC and BD, crossing the first two (Fig. 75).

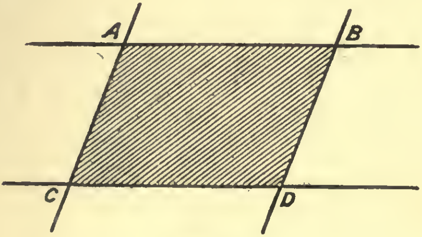


FIG. 75.

You have now made a quadrilateral ABDC with its opposite sides parallel. All figures of this kind are called **parallelograms**.

350. A parallelogram which has its four sides equal

A Rhombus. A Rhomboid.

FIG. 76.

is called a **rhombus**. A parallelogram in which the four sides are not equal is called a **rhomboid**.

351. Draw three or four parallelograms, and compare the lengths of their opposite sides. Record your results in your note-book.

If the opposite sides of a quadrilateral are equal, what would you call the figure?

Take your protractor, or some tracing-paper, and compare the sizes of the opposite angles of the parallelograms which you have just drawn. Record your results.

If the opposite angles of a quadrilateral are equal, what would you call the figure?

352. You are now going to learn how to draw a parallelogram whose dimensions are given. Draw a parallelogram having sides of 3 cm. and 2 cm., and an angle of 60°, in the following manner:—

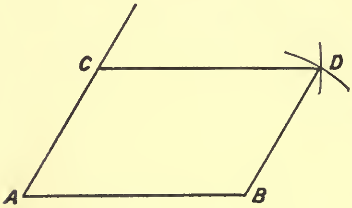


Fig. 77.

Draw AB 3 cm. long. At A make an angle, BAC, of 60°. Cut off AC equal to 2 cm. With centre B, and a radius equal to AC, draw an arc. With centre C, and a radius equal to AB, draw another arc, cutting the first one. Join BD and CD.

You have now drawn a figure with its opposite sides equal. How do you know this? Read through the last section again, and then state whether you think that the opposite sides are parallel. Test them. Is the figure you have drawn a parallelogram?

353. Exercise (drawing).

(*a*) Draw a parallelogram, with sides of 3·5 cm. and 2·5 cm., and an angle of 90°. Can a rectangle be called a parallelogram?

(*b*) Draw a parallelogram, with each of its sides 4·2 cm. long and an angle of 90°. Can a square be called a parallelogram?

Draw parallelograms with the following dimensions:—

(*c*) sides, 4 cm. and 3 cm.; one angle, 60°.
(*d*) sides, 2 in. and $1\frac{3}{8}$ in.; one angle, 45°.
(*e*) a rhombus, with sides of 4·7 cm.; one angle, 30°.

Draw the following parallelograms to scale:—

(*f*) sides, 1·129 m. and ·935 m.; one angle, 45°; scale, $\frac{1}{10}$.
(*g*) sides, 7 yd. 2 ft. and 5 yd. 1 ft. 6 in.; one angle, 120°; scale, $\frac{1}{8}$ in. = 1 ft.

354. Exercise (to be written).

(*a*) A parallelogram has the following dimensions: perimeter, 14·11 in.; one side, 3·74 in. Find the lengths of the other sides correct to one place of decimals.
(*b*) A parallelogram has one side 39·77 in. long, and its perimeter is 4·37 times as long as this side. Find the lengths of all the sides correct to two places of decimals.
(*c*) One side of a parallelogram is $5\frac{3}{8}$ in., and the perimeter is $13\frac{7}{16}$ in. What are the lengths of the other sides?
(*d*) The perimeter of a parallelogram is $17\frac{3}{4}$ in., and the total length of two opposite sides is $\frac{3}{4}$ of the perimeter. Find the lengths of the sides correct to one place of decimals.
(*e*) Express the sum of the angles of a parallelogram in degrees (see § 315).
(*f*) The sum of two opposite angles of a

parallelogram is 78°. What are the sizes of the other angles?

(g) If one angle of a parallelogram measures 79°, what are the sizes of the other angles?

Area of a Parallelogram

355. Draw a parallelogram, ABCD, on squared paper (mm.), as in Fig. 78. Then draw the rectangle, EBCF.

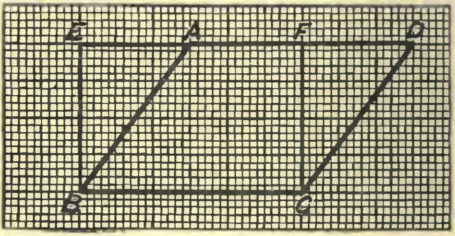


Fig. 78.

On the left the rectangle overlaps the parallelogram by the triangular area, EBA, and on the right the parallelogram overlaps the rectangle by the triangular area, FCD. The figure, ABCF, belongs both to the rectangle and the parallelogram.

Take a pair of scissors, and cut out the figure, EBCD. Now cut off the triangle, FCD, and place it on the top of the triangle, EBA. What do you notice?

What does this experiment tell you about the areas of the parallelogram and the rectangle?

356. The area of the rectangle, EBCF, can be found, as you know, by multiplying BC by CF.

The area of the parallelogram can, therefore, be found in the same way by multiplying BC, which is one of its sides, by CF, which is the perpendicular distance between that side and the opposite side.

CF is called the **altitude** of the parallelogram corresponding to the side BC. How many altitudes of different lengths has a parallelogram?

If s represents the length of a side of a parallelogram, a its corresponding altitude, and A_p its area, complete the formula given below :—

$$A_p =$$

357. Exercise (requiring the use of models).

(*a*) You are provided with a rhomboid prism. Make a hand-sketch of it. Measure the edges of the prism in inches (and decimals of an inch), and insert the dimensions on the hand-sketch.

Draw a plan of one of the parallelograms to a suitable scale, and insert the dimensions of the angles, sides, and altitudes.

Calculate the areas of the rectangular faces of the prism.

Calculate also the area of the parallelogram whose plan you have drawn. Here you will be able to take the average of two results derived from the two altitudes.

(*b*) Work similar exercises with any other objects, such as tiles, which have faces shaped like parallelograms.

358. Exercise (calculations).

(*a* to *g*) Calculate the areas of the parallelograms whose dimensions are given in § 353. In each case your answer must be the average of two results derived from the two altitudes.

(*h*) A church window contains 225 panes, each shaped like a rhombus with sides of 3 in. and an angle of 60°. Find the area of the window in square feet.

(*i*) Fig. 79 shows hand-sketches of a rectangle

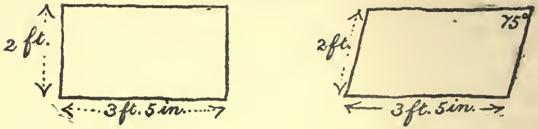


FIG. 79.

and a parallelogram. By how much is the area of one of them greater than the area of the other.

(*j*) The four parallelograms in the hand-sketch in Fig. 80 have one side of 2 cm. and

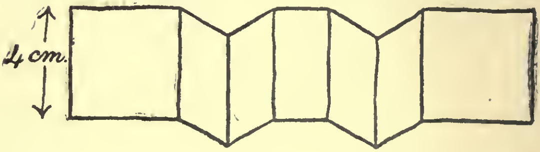


FIG. 80.

one angle of 60°. The rectangle has one side which is half as long as a side of one of the two squares. How many times is the combined area of the rectangular figures greater than that of the non-rectangular figures?

Triangles

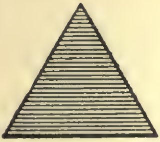
359. Make hand-sketches of one of the triangular faces of each of the following: (*a*) pentagonal pyramid; (*b*) triangular tile*; (*c*) set-square, with angle of 60°.

Measure the sides and the angles, and mark the dimensions on the hand-sketches.

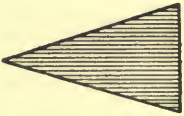
Notice that there are different kinds of triangles.

An **equilateral triangle** has all its sides equal. Which of its angles are also equal?

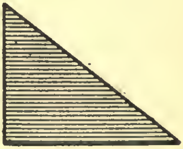
An **isosceles triangle** has two of its sides equal. How many of its angles are also equal, and how would you describe their position with reference to the equal sides?



Equilateral Triangle.



Isosceles Triangle.



Right Triangle.

FIG. 81.

A **right triangle** contains a right angle. Could it contain two right angles?

Which of your set-squares is both right and isosceles?

360. *Definitions.*

The **base** of a triangle is the side on which for the moment you conceive it to stand. Any one of the sides may be regarded as the base.

In a right triangle the side opposite the right angle is called the **hypotenuse** (Fig. 82).

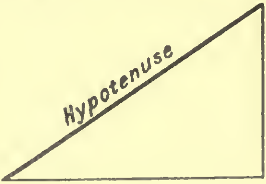


FIG. 82.

The **altitude** of a triangle is the perpendicular distance between a vertex and the opposite side (Fig. 83). Sometimes a side has to be

* Equilateral triangle.

lengthened before the corresponding altitude can be drawn. How many altitudes has a triangle? Which are the altitudes of a right triangle?

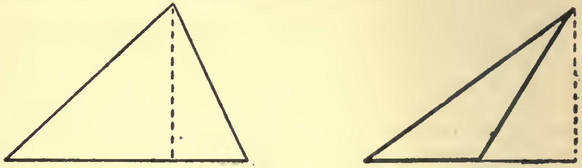


FIG. 83.

361. Exercise (drawing).

(*a*) Draw an equilateral triangle with sides of 3 cm.

(*b*) Draw a right triangle in which the arms of the right angle are 3·4 cm. and 2·7 cm. Measure the length of the hypotenuse.

(*c*) Draw three isosceles triangles, the equal sides in each being 2·2 in. Measure the angles at the bases.

(*d*) On a base of 4·2 cm. draw an isosceles triangle, the equal sides being 2·5 cm. Draw the three altitudes.

(*e*) Draw a triangle with a base of 3·4 cm., and having angles (at the base) of 60° and 30°. What is the size of the remaining angle?

(*f*) Draw a triangle with sides of 2 cm., 3 cm., and 4 cm.

(*g*) The base of a triangle is 4·1 cm., and its altitude is 3·7 cm. If one side is 7·4 cm. long, what is the length of the third side?

362. Try to draw a triangle with sides of 2 cm., 3 cm., and 6 cm.

Have you been able to do it? If not, what have you learnt about the sides of a triangle?

363. Try to draw a triangle with angles of 90°, 60°, and 45°.

Have you been able to do it? If not, what have you learnt about the angles of a triangle? (See § 314.)

364. Exercise (to be written).

(*a*) The perimeter of a right isosceles triangle is $16\frac{3}{16}$ in., and each of the equal sides is $4\frac{3}{4}$ in. What is the length of the hypotenuse?

(*b*) One angle of a triangle is 147°, and the other angles are equal to one another. What is their size?

(*c*) In a right triangle one angle is 30°. What is the size of the other angle?

(*d*) The angle at the vertex of an isosceles triangle is 15°. What are the sizes of the angles at the base?

(*e*) Which is the longest side in a right triangle?

(*f*) The perimeter of an equilateral triangle is $4\frac{7}{8}$ in. What is the length of each side? What is the size of the angles in degrees?

365. Exercise (drawing).

(*a*) AB is a high chimney. At C there is an observer on the ground. AC = 57 yd. The angle subtended at C by AB = 48°. Draw the triangle ABC on squared paper to a scale of 1 mm. to 1 yd., and read off the height of the chimney (Fig. 84).

(*b*) Draw to scale an equilateral triangle with sides of 5 yd. 0 ft. 9 in.

(*c*) Draw an isosceles triangle with the equal sides 2·13 in. long, the angle between them being 30°, and measure the sizes of the remaining side and angles.

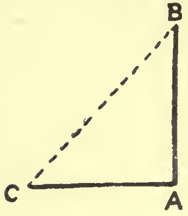


FIG. 84.

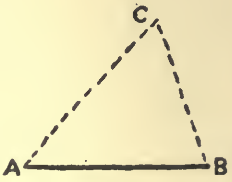


FIG. 85.

(*d*) A triangular field has sides of 157 yd., 223 yd., and 311 yd. Draw it to any convenient scale, and make a record of the lengths of the altitudes.

(*e*) A and B are two houses on the sea-shore. AB = 749 yd. C is a lighthouse out at sea. The angle CAB = 47°. The angle ABC = 72°. Find the distance of the lighthouse from the shore (Fig. 85).

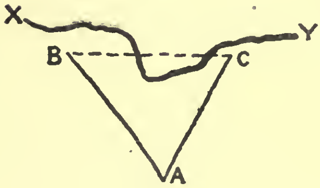


FIG. 86.

(*f*) XY is a wide river. Find the distance BC if AB = 543 yd., AC = 493 yd., and angle BAC = 63°. (Fig. 86.)

(*g*) A triangular window has sides of 1 ft. 7 in., 11 in., and 1 ft. 7 in. Draw it to scale. Draw also the altitudes of the triangle, and measure their lengths. Measure the angles of the triangle.

(*h*) The distance between Leeds and Manchester is 37 miles, between Manchester and Sheffield 33 miles, and between Sheffield and Leeds 28 miles. Draw a map to a scale of 1 inch to 10 miles, showing the relative positions of these towns.

(*i*) An equilateral triangle has a side of $\frac{5}{16}$ in. Make a drawing of it to scale, and measure the lengths of its altitudes.

(*j*) ABC is an isosceles right triangle. ABC is a right angle. AB = 14·3 cm. Draw the triangle to a scale of one-third. Measure all its sides and angles.

Area of a Triangle

366. Draw a parallelogram of any convenient size on stiff paper.

Draw a line joining two opposite corners. This line is called a **diagonal**. How many more diagonals could you draw?

Take your scissors and cut out the parallelogram. Then cut the parallelogram in two along the diagonal.

You have now got two triangles. Place one on the

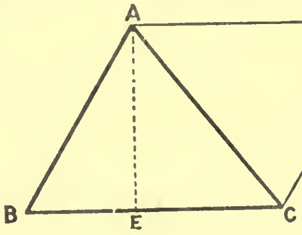


Fig. 87.

top of the other and compare their sides, angles, and areas.

You will see at once that a diagonal divides a parallelogram into two triangles which are exactly alike in all respects.

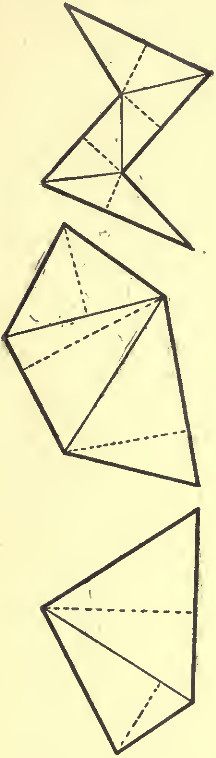


FIG. 88.

367. Draw a triangle ABC (Fig. 87). From A draw AD parallel to BC, and from C draw CD parallel to AB. Then, ABCD is a parallelogram. Why?

From A draw AE at right angles to BC. You will see that AE is the altitude both of the triangle and the parallelogram. [Note that AE is the altitude corresponding to the base BC.]

Now, in § 356 you discovered that the area of a parallelogram may be expressed by the formula—

$$A_p = sa.$$

But you have already shown that a diagonal, like AC, divides a parallelogram, like ABCD, into two triangles which are equal in area. If A_t represents the area of a triangle, you will therefore be able to complete the following formula—

$$A_t =$$

368. The areas of figures which are bounded by more than three straight lines can be found by dividing them into triangles and measuring the areas of the triangles. (Fig. 88.)

369. Exercise (requiring the use of objects).

(*a*) You are provided with a triangular tile.

Make a hand-sketch of it. Measure the edges in inches (and decimals of an inch), and insert the dimensions on the hand-sketch.

Draw a plan of one of the triangular faces to scale, and insert the dimensions of the angles, sides, and altitudes.

Calculate the areas of the rectangular faces of the tile.

Calculate also the area of the triangle whose plan you have drawn. Your answer should be the average of three results derived from the three altitudes.

(*b*) Work similar exercises with a triangular prism, and with each of your set-squares.

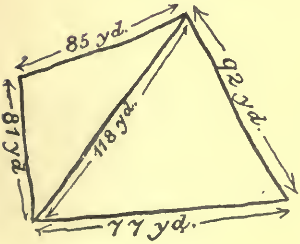


Fig. 89.

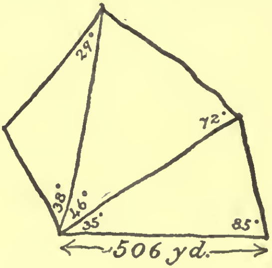


Fig. 90.

370. Exercise (calculations).

(1) Calculate the areas of the triangles whose dimensions are given in § 365. Each answer should be the average of three results, and the respective answers should be expressed as follows :—

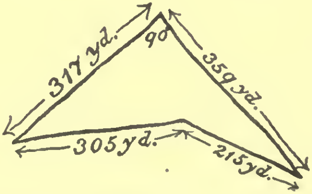


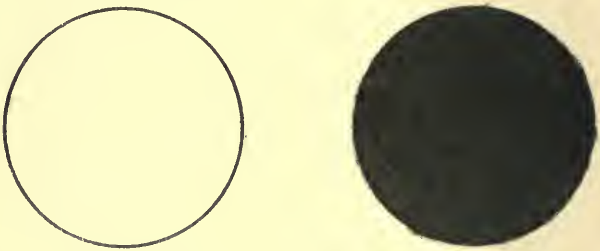
Fig. 91.

(b) In square yards and decimals of a square yard.
(c) In square inches and decimals.
(d) In acres and decimals.
(g) In square inches.
(i) As the fraction of a square inch.
(j) In square decimetres.

(2) Figs. 89, 90, and 91 are hand-sketches of fields. Draw them to suitable scales, and calculate their areas. Express your results in acres and square yards, and find the value of the fields if an acre is worth £100.

The Circle

371. You have already learnt what a circle is, and what a circumference is. It is important that you should not confuse them. Look at Fig. 92.



Circumference. Circle.

Fig. 92.

372. Certain parts of circles have special names (Fig. 93).

A sector is bounded by any two radii and the arc between them.

Write an accurate description of a semicircle and of a quadrant.

373. Exercise (drawing).

(*a*) Draw semicircles with radii of 3·7 cm., 2·3 in., and 4 cm.

(*b*) Draw quadrants with radii of 3·1 in., 4·2 cm., and 2·3 cm.

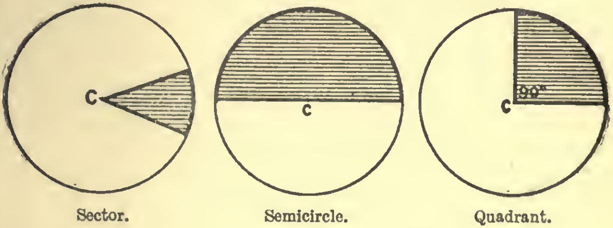


Fig. 93.

(*c*) Draw a sector with a radius of 3 cm., and an angle of 60°.

(*d*) Draw a sector with a radius of 4 cm., and an angle of 87°.

(*e*) Draw a sector with a radius of 3·7 cm., and an angle of 20°.

374. Let us now see how we can compare the sizes of two unequal sectors in the same circle.

Draw a circle with a radius of 4 cm., and mark off two equal arcs by using tracing-paper as described in § 321. Join the ends of the arcs to the centre of the circle.

You have now made two sectors. Use the tracing-paper again in order to compare their angles and areas.

If one sector of a circle has an arc which is twice as long as that of another sector, what do you know about the areas of the two sectors?

If one sector of a circle has an angle which is three times as large as that of another sector, what do you know about the areas of the two sectors?

375. Exercise (oral).

(*a*) Into how many quadrants may a circle be divided? And into how many semicircles? And sectors?

(*b*) How many times is the area of a circle bigger than the area of one of its semicircles? And of one of its quadrants?

(*c*) In the same circle, how many times is the area of a quadrant greater than the area of a sector with an angle of 45°? And of a sector with an angle of 30°? And with an angle of 60°?

(*d*) How many times is 360° greater than 180°, 90°, 60°, 27°?

(*e*) How many times is the area of a circle greater than the areas of those of its sectors which have angles of: 90°, 60°, 30°, 50°, 17°, 123°?

Area of the Circle

376. You are now going to learn how to find the area of a circle.

Look at the diagram in Fig. 94. If r represents the length of the radius of the circle, how would you express the area of one of the little squares? And how would you express the area of the big square?

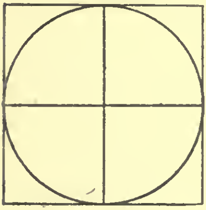


Fig. 94.

The big square is rather larger than the circle. Guess how many times the area of the circle is greater than that of one of the little squares.

377. Draw the diagram in the last section on squared paper (mm.), and let the circle have a radius of 3 cm. Find out (by counting the little squares) how many times the area of the circle is greater than that of the square on the radius, that is, than r^2.

If A_c represents the area of a circle, and r its radius, complete the formula—

$$A_c =$$

378. The formula in the last paragraph may be illustrated in the following manner: Draw a circle with a radius of 2 in. on some thin paper. Cut out

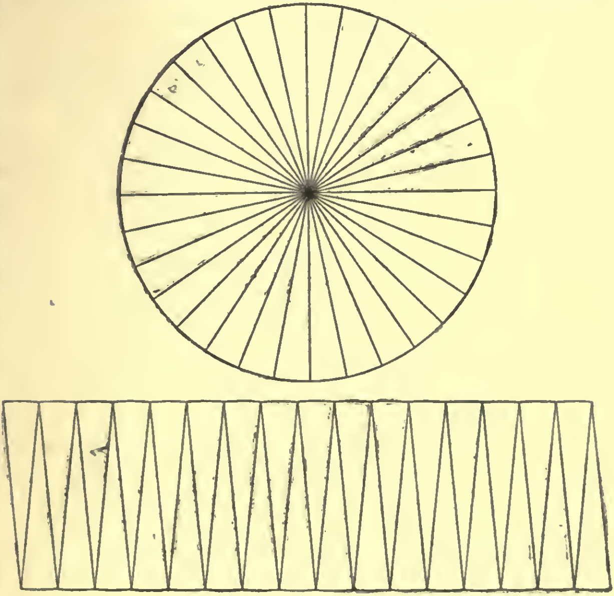


Fig. 95.

the circle with a pair of scissors, and fold it four times, so as to make 16 equal sectors. Cut out the sectors, and arrange them as shown in Fig. 95. (*Note.*—There are 32 sectors in Fig. 95.)

You will now see that the sectors make up a figure which is almost exactly a rectangle. The area of the rectangle can, of course, be found by multiplying its length by its breadth. In this case the length of the rectangle is half the length of the circumference of the circle, that is $\frac{2\pi r}{2}$, or πr. The breadth of the rectangle is obviously equal to r. The area of the rectangle (and also of the circle) is therefore πr^2.

379. Exercise (oral).

(*a*) State the areas of the circles which have radii of 5 in., 8 cm., and 11 in. Let your answers contain the symbol π.

(*b*) Take π as being 3·1, and state the areas of the circles having radii of 2 cm., 3 in., and 4 cm.

(*c*) How would you express the area of a circle which has a radius of x? And of a circle whose diameter is y?

(*d*) How would you express the length of the circumference of a circle whose area is πz^2? And the area of a circle whose circumference is $2\pi y$?

(*e*) If a circle has a radius of r, what is the area of one of its semicircles? Of one of its quadrants? And of those of its sectors which have angles of 60°, 27°, 15°?

380. Exercise (requiring the use of objects).

Find the areas of the surfaces of: An iron washer, a shilling, a rubber ring, and a metal sector.

381. Exercise (to be written).

(*a*) Find the area of a circle with a radius of 17·7 in.; let your answer be correct to one place of decimals.

(*b*) Find the area of a circle with a radius of $1\frac{3}{4}$ in.; take π as being $\frac{22}{7}$.

(*c*) What is the radius of a circle which has an area of 200·96 square miles?

(*d*) Find the angle and the area of a sector having the following dimensions:—
Radius $\frac{1}{2}$ in., arc $\frac{\pi}{6}$ in. Let your answer contain the symbol π.

(*e*) Fig. 96 is a hand-sketch of a church door. Find its area, π being $3\frac{1}{7}$.

(*f*) The graduated circle of a mariner's compass has a diameter of 3·83 dm. Find the area of a half quadrant.

(*g*) What is the length of the circumference of a circle which has an area of 2·5434 square miles?

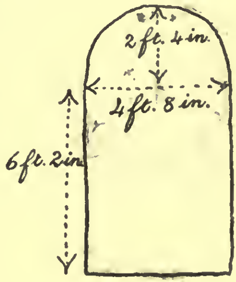


Fig. 96.

Surfaces of the Cylinder and Cone

382. You are provided with a cylinder. Make a hand-sketch of it, and insert the dimensions of its diameter and height in inches and decimals.

Calculate the area of one of the circular faces.

383. Can you suggest a method for calculating the area of the curved surface? If not, perform the following experiments, which will give you some help:—

(*a*) Roll the cylinder once round on a piece of paper, in such a way that a point on

d

one of its edges starts on the paper, moves round, and comes back again on to the paper. Draw the boundary lines of the area which the curved surface has covered.*

(*b*) Cut out a piece of paper which will exactly fit round the curved surface.

If h represents the height of a cylinder, and r represents its radius, write down the formula for the area of the curved surface.

Find out how many times the area of the curved surface is larger or smaller than that of one of the circular surfaces.

384. Make a hand-sketch of the cone given to you. Measure the diameter, the slant height, and the vertical height, and insert the dimensions on the hand-sketch.

Calculate the areas of the two surfaces. If you have any difficulty with the curved surface, perform experiments exactly similar to those described in the last section.

If h represents the slant height of a cone, and if r represents its radius, write down the formula for the area of the curved surface.

385. Exercise (requiring the use of models).

Find the areas of the curved surfaces of: A lead pencil (uncut), a jam pot, an iron washer, the conical end of a lead pencil which has been sharpened in a machine.

386. Exercise (to be written).

(*a*) If both the height and the radius of the base of a cylinder are r inches long, how many times is the area of the curved

* The teacher might illustrate this with a cyclostyle roller.

surface greater than that of one of the ends?

(*b*) A garden roller has a diameter of 2 ft., and a breadth of 3 ft. How many times will it rotate in covering a tennis court 78 ft. by 27 ft.?

(*c*) A rolling-pin is $11\frac{1}{16}$ in. long, and has a diameter of $1\frac{3}{4}$ in. Find the areas of its surfaces, taking π as $\frac{22}{7}$.

(*d*) Find the area of the curved surface of a cone which has a slant height of 28 in., and a diameter of $9\frac{3}{4}$ in.

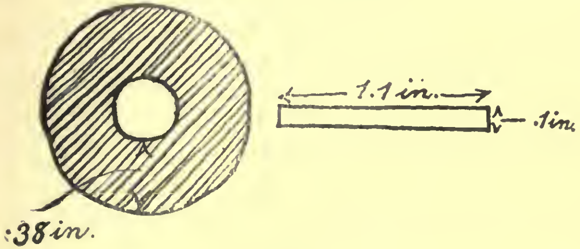


FIG. 97.

(*e*) Find the total area of the surfaces of the washer (Fig. 97).

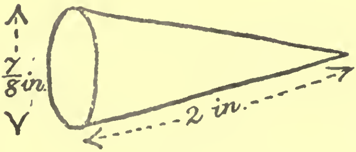


FIG. 98.

(*f*) Fig. 98 is a hand-sketch of a candle extinguisher. Find the area of its surface.

Exercise for Revision

387. *Note.*—In working these exercises, scholars should take care :—

(*a*) To calculate average results when possible.
(*b*) To express results correct to a reasonable number of decimal places.
(*c*) To make hand-sketches and ruled drawings when necessary.
(*d*) To use suitable scales when ruled drawings are made.

388. Exercise (requiring the use of objects and models).

(*a*) Find the total area of the faces of the pentagonal pyramid.
(*b*) Find the area of one side of your protractor.
(*c*) Calculate the areas of all the faces of your two set-squares (taking the holes into account).
(*d*) Find the areas of the three faces of a penny.
(*e*) Find the area of the triangular flap of an envelope.
(*f*) Find the areas of the internal and external surfaces of a piece of iron pipe or glass tube.

389. Exercise (to be written).

(*a*) Find the area of a parallelogram having the following dimensions : sides, ·07 in. and ·1 in. ; one angle, 45°.
(*b*) On a base of 2 in. draw an isosceles triangle of which the equal sides are 3 in. Show that the altitude bisects the base, and find the area of the triangle.

(c) Find the total length of the sides of a figure which has the following dimensions: $4\frac{7}{18}$ in., $3\frac{5}{6}$ in., $3\frac{8}{9}$ in., $4\frac{2}{3}$ in., $5\frac{1}{12}$ in.

(d) Find the cost of carpeting a circular floor with a radius of 17·4 yd., if the carpet costs 1s. 1d. per sq. ft.

(e) A quadrant of a circle has an arc which is $1\frac{1}{2}$ in. long. Find the area of the quadrant.

(f) A round ruler is 1 ft. 3 in. long and its diameter is ·1 ft. Find the areas of its surfaces.

(g) What would be the cost of carpeting this staircase if carpet costs 5s. 6d. per linear yard? (Fig. 99.)

FIG. 99.

(h) Draw a circle with a radius of 3 cm. and inside it draw a regular hexagon. Find the area of the hexagon by dividing it into triangles.

(i) Show that each of the altitudes of an equilateral triangle divides the triangle into two equal parts.

(j) What is the area of a semicircle, if its curved side measures $5{\cdot}42\,\pi$ in.?

(k) Find the area of the kite. (Fig. 100.)

(l) Draw a rhombus of any convenient size and shape. Draw the two diagonals and measure the angles which they make at the point O where they cross. Measure also the lengths of the lines joining O to the corners of the rhombus;

what do you notice? If x and y represent the lengths of the diagonals, what is the area of the rhombus?

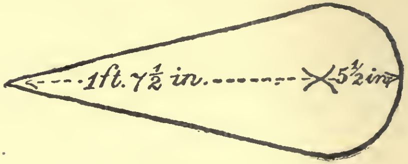


FIG. 100.

(*m*) A rectangle has one side 3 in. long. Find the length of the other side, if the rectangle is equal in area to a circle with a diameter of 5 in.

(*n*) Draw figures showing that a straight line can be drawn dividing any triangle into two right triangles.

(*o*) Find the radius of a circle with an area of 153·86 sq. yds. ($\pi = 3{\cdot}14$).

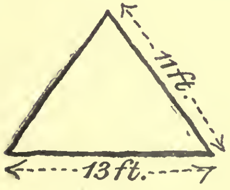


FIG. 101.

(*p*) What would be the cost of the canvas which would be required in making this conical tent, if the price of the canvas is 9d. per sq. yd.? (Fig. 101.)

(*q*) Draw a rhombus with sides of 4·7 cm. and an angle of 86°. Calculate its area.

(*r*) An isosceles triangle has a base of 4·7 in. and a perimeter of 16·1 in. What are the lengths of the equal sides?

(*s*) A cylinder has a diameter of 2·7 in. What is its height, if the area of the curved surface is 118·692 sq. in.?

(*t*) Find r, when $\pi r^2 = 452{\cdot}16$.

(*u*) Find the area of the pennon. (Fig. 102.)

(*v*) Find the area of a fan which opens out into a sector of 125°. Radius = 9·7 in.

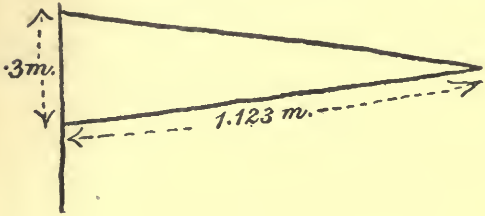


Fig. 102.

(*w*) A piece of wire trellis-work consists of rhombi each with sides of 3·1 in. and an angle of 60°. How many rhombi are there in 25 sq. yd.?

(*x*) Find the area of the curved surface

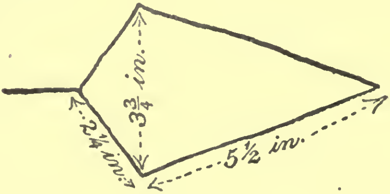


Fig. 103.

of a conical oil-can which has a slant height of 3·4 in. and a diameter of 2·7 in.

(*y*) Find the area of the bricklayer's trowel. (Fig. 103.)

(*z*) "In the midst of a meadow, well stored with grass
I engag'd just one acre to tether my ass;
What length must the cord be, that he, feeding all round,
May not graze less or more than an acre of ground?"

[From NESBIT's *Mensuration*, 1848.]

CHAPTER IX

PROPORTION AND SIMPLE GRAPHS

Ratio

390. You have sometimes had to calculate how many times one quantity is greater than another. You have found, for example, that 1 in. is $2\frac{1}{2}$ (or $\frac{5}{2}$) times greater than 1 cm.

Instead of saying that 1 in. is $\frac{5}{2}$ times greater than 1 cm., you can use another expression, and say that the **ratio** of 1 in. to 1 cm. is $\frac{5}{2}$ (read "5 over 2," or "as 5 to 2"). This means that if 1 cm. were divided into 2 equal parts, 1 inch would contain 5 of these parts. Conversely, you would say that the ratio of 1 cm. to 1 in. is $\frac{2}{5}$.

It would not be wrong to say that the ratio of 1 in. to 1 cm. is $\frac{10}{4}$. For if the cm. were divided into 4 equal parts (instead of 2), the inch would, of course, contain 10 of those parts (instead of 5). But it is found more convenient to express a ratio, as you have learnt to express a fraction, in its lowest terms. You speak, therefore, of the ratio $\frac{5}{2}$ rather than of the ratio $\frac{10}{4}$.

391. Exercise (oral).

(*a*) What is the ratio of 1 ft. 9 in. to 1 yd.?
(*b*) What is the ratio of 2 yd. to 28 in.?

(*c*) What is the ratio of 1000 yd. to 1 mile?
(*d*) What is the ratio of 2 km. to 300 m.?
(*e*) What is the ratio of A's income (£1400) to B's (£800)?
(*f*) What is the ratio of the population of England (42,000,000) to that of Germany (56,000,000)?

392. You must not express either term of a ratio as a vulgar fraction. The ratio of $3\frac{1}{2}$ in. to 5 in.

$$= \frac{3\frac{1}{2}}{5} = \frac{3\frac{1}{2} \times 2}{5 \times 2} = \frac{7}{10}.$$

For in a ratio, as in a vulgar fraction, the value is not changed if you multiply or divide both terms by the same number.

393. Exercise (to be written).

(*a*) What is the ratio of 6 in. to $5\frac{1}{2}$ in., and to $4\frac{1}{2}$ in.?
(*b*) What is the ratio of £2. 10s. to £6. 5s., and to £7. 5s.?
(*c*) If a circumference is $3\frac{1}{7}$ times greater than the diameter, what is the ratio of the diameter to the circumference?
(*d*) And what is the ratio of the circumference to the radius?

394. If, however, you are working with decimals, you will find it an advantage to express a ratio with one of the terms as 1. In a circle 12 cm. in diameter the circumference is 37·7 cm., and the ratio of circumference to diameter

$$\left(\frac{37{\cdot}7}{12}\right) = \frac{37{\cdot}7 \div 12}{12 \div 12} = \frac{3{\cdot}14}{1}.$$

Similarly, in a circle of 3·5 cm. diameter, the ratio of circumference to diameter

$$= \frac{11}{3{\cdot}5} = \frac{11 \div 3{\cdot}5}{3{\cdot}5 \div 3{\cdot}5} = \frac{3{\cdot}14}{1}.$$

You see that this way of expressing it enables you to judge at once whether the ratio is the same in both calculations.

395. Exercise (to be written).

(*a*) In each of the figures in Fig. 104 measure in cm. the lines AB, BC, and find the ratio of AB to BC.

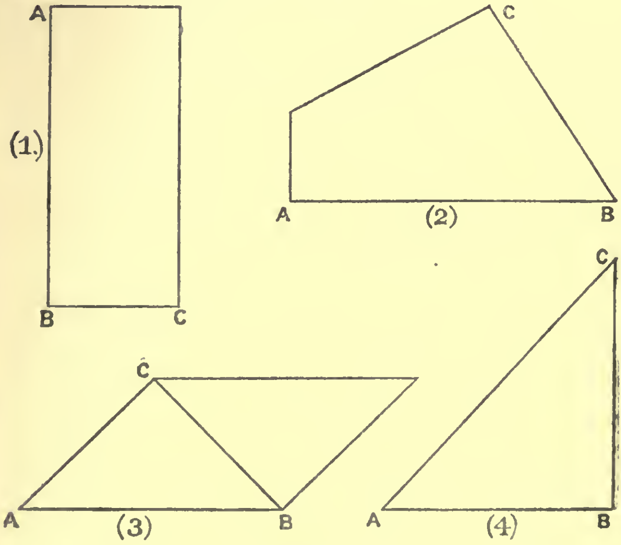


FIG. 104.

(*b*) It is found by measurement that 1·2 km. = ·7445 ml., 1·4 km. = ·871 ml., and 1·6 km. = ·9914 ml. Find the ratio of 1 km. to 1 ml., taking an average of the three measurements.

Constant Ratio

396. Fig. 105 is an equilateral triangle with sides of 1 in.

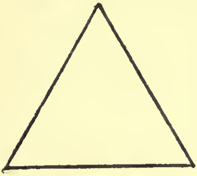


Fig. 105.

Measure the altitude as accurately as you can in decimals of an inch.

What is the ratio of the altitude to the side?

Here are drawn three other equilateral triangles with their altitudes. (Fig. 106.)

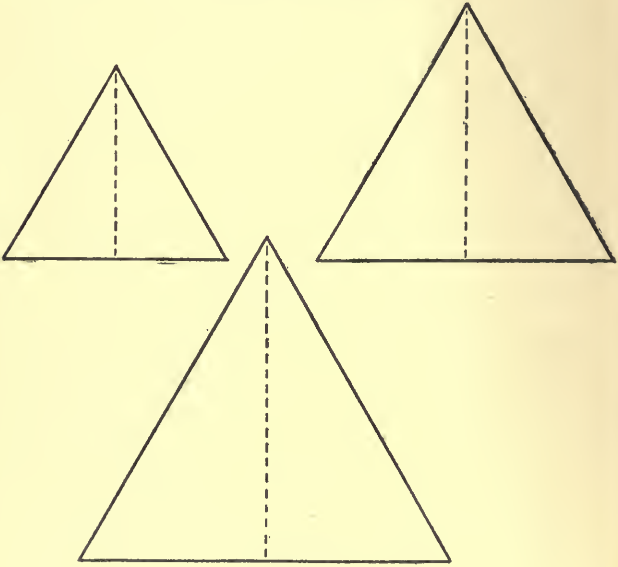


Fig. 106.

Measure carefully in cm. a side and the altitude of each triangle, and record in your note-book as follows :—

	Length of Side.	Length of Altitude.	Ratio of Altitude to Side.
No. 1.			
No. 2.			
No. 3.			

397. Is the ratio the same, or approximately the same, in each triangle? Is it the same as that which you found in the last section?

When two quantities—for example, the lengths of an altitude and of a side—become greater or smaller together, but the ratio between them remains the same, the ratio is said to be **constant.**

You have already found that the ratio of a circumference to its diameter is constant, and you have also calculated what that ratio is.

Find the constant ratio of the altitude to a side of an equilateral triangle by taking an average of the ratios which you have calculated.

398. Suppose the constant ratio is $\frac{\cdot 86}{1}$. With this ratio you can easily calculate the altitude of *any* equilateral triangle, if you know the length of a side.

E.g.—

When the side is 1 in., the altitude is ·86 in.

∴ when the side is 2·3 in., the altitude is ·86 in. × 2·3,
= 1·978 (or 1·98) in.

399. Exercise (to be written).

(*a*) Find, by calculation, the altitude of an equilateral triangle with 8-in. sides.

(*b*) Find also the area of this triangle.

(*c*) Find in sq. ft. the area of an equilateral triangular flower-bed with sides of 2 yds.

(*d*) Draw a square and a diagonal of it. Find the ratio of a diagonal to a side.

(*e*) Draw two other squares, and find out whether the ratio of diagonal to side is constant. If so, what is it?

(*f*) Find out whether the ratio of the height of a hexagon to a side is constant, and if so, what it is.

(*g*) What is the height of a hexagon with 10 cm. sides?

(*h*) Measure the length of a side of the hexagon on your hexagonal prism. Calculate the height of the hexagon, and verify by measurement.

(*i*) A quadrangle has sides 31 yd. 2 ft. 4 in. long. Calculate the length of a path laid diagonally across it. See Ex. (*e*).

(*j*) Draw the quadrangle to a convenient scale, and verify your calculation by measurement.

Unitary Method

400. Constant ratios are always meeting you in all kinds of problems. If 1 book costs 1s., you know that 2 of the same books cost 2s., 3 cost 3s., and so forth. Here the ratio between two numbers, the number of shillings paid and the number of books bought, is constant. If 1 book cost 1s., and yet two books cost 2s. 6d., and 3 books, perhaps, 2s. 9d., you could not reckon the price; you would have no constant ratio to work upon.

If you know the price of 1 thing, it is easy to find the price of 63; you have only to multiply the price of

1 by 63. But if you are told the price of 14 things, and are asked to find the price of 63, that is rather harder.

Yet you will find it easy if you make your calculation in two steps, thus :—

The price of 14 things is 16d.

$\therefore$ the price of 1 thing is $\frac{16}{14}$d.

$\therefore$ the price of 63 things is $\frac{16}{14}$d. $\times$ 63.

$$= \frac{\overset{8}{\cancel{16}} \times \overset{9}{\cancel{63}}}{\underset{2}{\cancel{14}}}\text{d.}$$

$= 72$d.

$= 6$s.

In these problems do not work out your fractions as you go on, but keep them to the end; then cancel them.

This method of working in two steps is called the **unitary method.**

401. Exercise (to be written).

(*a*) Oranges are 20 a shilling. What do 50 cost?

(*b*) What do 35 cost?

(*c*) How many can you buy for 3s. 6d.?

(*d*) How many for 2s. 3d.?

(*e*) 20 bananas cost 1s. 3d. What do 48 cost?

(*f*) What do 67 cost?

(*g*) How many can you buy for 2s.? And for 2s. 3d.?

(*h*) How many buns, at 9 for 6d., can you get for 10d.?

(*i*) $3\frac{1}{4}$ yd. of ribbon cost 1s. $7\frac{1}{2}$d. What is the cost of $2\frac{1}{2}$ yd.?

(*j*) 2 sq. yd. 4 sq. ft. of brick tiling costs 6s. 5d. What would $9\frac{1}{2}$ sq. yd. cost?

Proportion

402. When two ratios are equal, the four terms of the ratios are said to be **in proportion**. For example, since $\frac{6}{9} = \frac{10}{15}$, the numbers 6, 9, 10, 15 are in proportion. This is sometimes expressed, "6 is to 9 as 10 is to 15," or "6 bears to 9 the same ratio as 10 bears to 15."

You must take great care to arrange the terms in the right order. It would be wrong to say that 6, 9, 15, 10 are in proportion. This would mean that $\frac{6}{9} = \frac{15}{10}$; which, of course, is false.

403. Let us look once more at the problem dealt with in §400. "14 things cost 16d., what do 63 cost?"

You will now see that this problem can be stated as one of proportion, viz.: "What number bears the same ratio to 63 as 16 bears to 14?"*

If we denote the unknown number by the symbol x, then

$$\frac{x}{63} = \frac{16}{14}.$$

Read this out; it is called an **equation.**

An Equation

404. If you multiply two equal quantities by the

* In order to get the terms of a proportion in the right order, scholars should always be required to *describe* the terms of the proportion, thus: "The number of pence to be paid is to the number of things 63, as the number of pence 16 is to the number of things 14."

same number, the products are equal. For example, if $a = b$, then

$$4a = 4b$$

and

$$ax = bx.$$

Let us apply this simple rule to our equation—

$$\frac{x}{63} = \frac{16}{14}.$$

Multiply both sides of the equation by the same number, 63. Then

$$\frac{x}{63} \times 63 = \frac{16}{14} \times 63,$$

that is,

$$x = \frac{\overset{8}{\cancel{16}} \times \overset{9}{\cancel{63}}}{\underset{\cancel{2}}{\cancel{14}}}$$

or,

$$x = 72.$$

We have found, by working the equation, that the unknown number which we want is 72. 72 what? 72 *pence*, or 6 shillings.

405. Here is another problem: "If oranges are 20 a shilling (12d.), how many can be bought for 3s. 6d. (or 42d.)?"

In other words, what number bears the same ratio to 42 as 20 bears to 12? (See footnote, § 403.)

Let x be the unknown number. Then

$$\frac{x}{42} = \frac{20}{12}.$$

Multiply both sides by 42.

$$\frac{x}{42} \times 42 = \frac{20}{12} \times 42,$$

that is,

$$x = \frac{\overset{10}{\cancel{20}} \times \overset{7}{\cancel{42}}}{\underset{\cancel{2}}{\cancel{12}}}$$

$$= 70.$$

The answer is therefore "70 *oranges*."

406. Another problem : "The sides of a rectangle are 9·2 cm. and 7·6 cm. If the longer side is lengthened by 4 cm., how long must the shorter side be made so as to remain in proportion?"

Let x = the number of cm. in the lengthened shorter side. Then

$$\frac{x}{13{\cdot}2} = \frac{7{\cdot}6}{9{\cdot}2}$$

$$\therefore \quad \frac{x}{13{\cdot}2} \times 13{\cdot}2 = \frac{7{\cdot}6}{9{\cdot}2} \times 13{\cdot}2$$

that is,
$$x = \frac{7{\cdot}6 \times 13{\cdot}2}{9{\cdot}2}$$
$$= 10{\cdot}9.$$

The shorter side must therefore be made 10·9 cm. long.

407. Exercise (to be written). (See footnote, § 403.)

(*a*) Work out all the examples in Exercise 401 by equations.

(*b*) The first three terms in a proportion are 3·6, 4·5, 6·3. Find the fourth.

(*c*) The shadow of a flag-pole 39 ft. high is 58 ft. 6 in. long. Find the height of a man whose shadow at the same moment is 9 ft. long.

(*d*) The interest of a sum of money for 1 year is £13. 13s. 9d. Calculate the interest for 111 days.

(*e*) A square prism is 13·8 cm. high and 9·2 cm. broad. What is the breadth of a prism of the same shape which is 21·1 cm. high?

The Graph of Proportion

408. You are now to learn a simple way of representing proportion by a drawing or graph. (Fig. 107.)

On squared paper (tenth-inch) draw a line OX along one of the heavier lines. Set off OD 1·5 in. long, and from D draw DG *perpendicular* to OX, and (say) ·75 in. long. Then the ratio of OD to DG is $\frac{2}{1}$.

Now, take two other lengths in the same ratio, *e.g.*, 2·2 in. and 1·1.

Set off OE 2·2 in. long, and from E draw EH *perpendicular* to OX, and 1·1 in. long. The lines OD, DG, OE, EH are, of course, in proportion, for $\frac{OD}{DG} = \frac{OE}{EH}$.

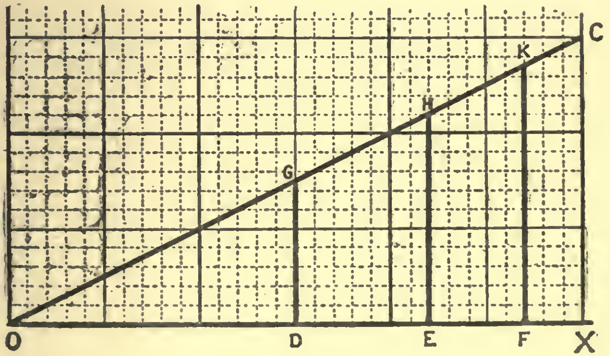


FIG. 107.

Join GH, and produce it both ways.

You have now to notice two things: first, that the line GH produced passes through O; and secondly, that if you take any point F on OX, the lines OF and FK (FK being perpendicular to OX) are in the same ratio as OD and DG. Assure yourself that this is so by measuring OF and FK, and also by taking other points along OX.

409. You will now see that the line OC enables you to draw a large number of pairs of lines which vary in the same ratio.

Goods and prices are two quantities which vary in the same ratio.

Suppose each little division along OX to represent 1 lb., and each little division in the lines perpendicular to OX to represent 1 penny.

Then the lines OD, DG can be read to mean that 15 lbs. cost $7\frac{1}{2}$d.; the lines OE, EH can be read to mean that 22 lbs. cost 11d.; and so forth. You could count off any number of lbs., and their cost, if the graph were large enough. The graph, in fact, can be used as a sort of ready reckoner.

Drawing the Graph

410. It would clearly be more convenient if you could mark the number of lbs. represented by each

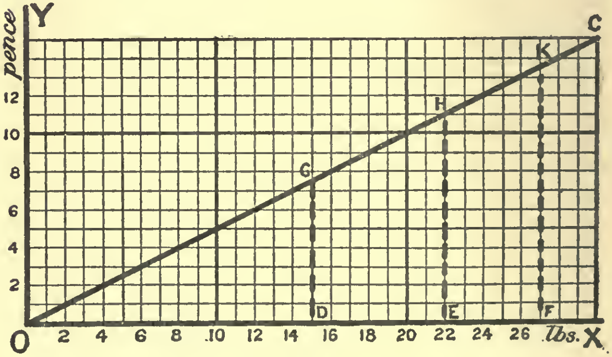


Fig. 108.

division along OX so as to save the labour of counting. But even then you would still have to count the divisions along the lines perpendicular to OX.

Let us see if we cannot mend this defect.

Draw OY (Fig. 108) at right angles to OX, and mark at convenient distances the number of pence represented by the divisions along OY.

You will notice that by following the horizontal lines

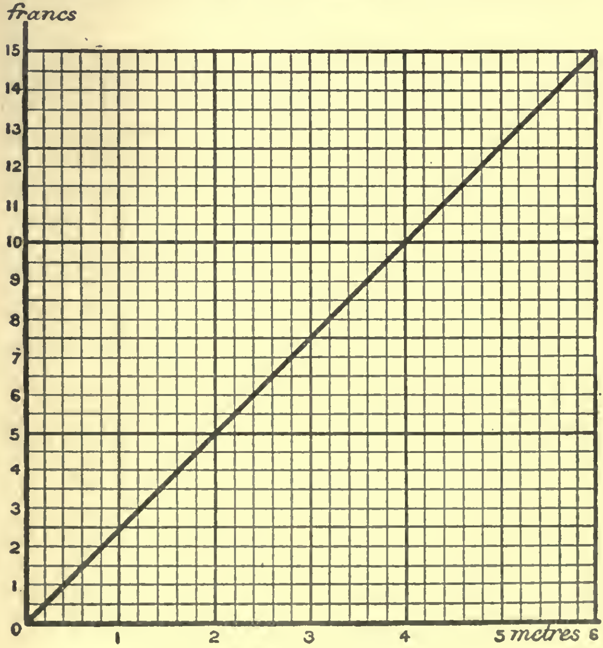


Fig. 109.

with your eye, you can easily judge the length along OY of the lines DG, EH, FK, etc.

Now, therefore, you can read off the quantities and their prices without the labour of counting. Read off some quantities and prices for practice.

411. Exercise.

(*a*) Draw graphs which will enable you to read off the answers to problems (*a*) to (*h*) in Exercise 401.

(*b*) On the graph (Fig. 109) read off the prices of these quantities of silk :—

2 m., 4 m., 5 m., 6 m., $3\frac{1}{2}$ m.
1·4 m., 2·7 m., 4·8 m., 5·9 m., 3·1 m.
60 cm., 240 cm., 470 cm., 9 dm., 5 dm.

412. In drawing a graph you will often find it convenient to let the terms of the ratio with which you are dealing be represented by *equal* lengths along the axes (as OX and OY are called). Thus, in the graph drawn in the last exercise, 1 m. costs $2\frac{1}{2}$ francs ; and the distances along the axes representing these quantities are made equal.

Take care to mark the quantities along each axis, and to state what the quantities represent (metres, yards, shillings, and so forth).

413. Exercise.

(*a*) The interest on £100 is £5 a year ; the interest on £200 is £10 a year, and so forth.

On squared paper draw a graph which will enable you to read off the interest of sums of money up to £1000.

(*b*) From your graph read off the interest on these sums of money—

£200, £400, £500, £900.
£320, £540, £680, £860.
£250, £950, £750.

(*c*) Confirm your results by calculation.

(*d*) What sums of money yield as interest—
£24, £36, £39, £47,
£13. 10s., £18. 10s., £49. 10s., £31. 10s.?

(*e*) Confirm your results by calculation.

A Kind of Proportion

414. Problems like the following are a little different from those which you have had, and require care. "9 men do a piece of work in 12 hours; how long will 8 men take?"

Ask yourself the question whether they will take a longer or a shorter time.

9 men take 12 hrs.

∴ 1 man takes 9×12 hrs.

∴ 8 men take $\frac{9 \times 12}{8}$ hrs. $= 13\frac{1}{2}$ hrs.

415. Exercise (to be written).

(*a*) 5 men mow a field in 14 hrs. How long will 7 men take?

(*b*) 5 men mow a field in 15 hrs. How many men will mow it in 5 hrs.?

(*c*) A stack of hay feeds 6 horses for 3 weeks. How long will it feed 14 horses?

(*d*) How many horses would eat it up in a fortnight?

(*e*) Running $18\frac{1}{2}$ knots, a ship does a voyage in 33 days. How long will she take running $16\frac{1}{2}$ knots?

(*f*) A garrison of 8250 men is provisioned for 37 days. If the garrison is reinforced by 1000 men, how long will the food last?

CHAPTER X

SOLIDS AND CUBIC MEASURE: PLAN AND ELEVATION

416. Hitherto you have considered the properties of surfaces, or plane figures, and you have learnt how to find the area of some of them.

Surfaces have only two dimensions; they have length and breadth, but no thickness.

Your models, however, have three dimensions—length, breadth, and thickness: such objects are called **solids.**

You are now going to learn something about the properties of a solid, and, especially, how to find the **volume** of a solid—that is, its bulk, or the amount of space it contains.

Among your models there are two principal kinds of solids—prisms and pyramids. You have already handled them a good deal. Now you are to examine them more particularly.

Prisms

417. Look at the hexagonal prism which your teacher will give you. You have already discovered that the two ends (called the **bases**) are equal to each other; notice further that they are parallel. This means that any two corresponding points on the two bases are the same distance apart. Test this with your calipers.

Notice, further, the other faces (called the **sides** of the prism), formed by joining the edges of the two bases. What kind of figures are they?

Write in your note-book what you now know about the bases and the sides of the prism.

Test the other prisms, and see whether they have the same properties. If they have, note the fact.

418. Prisms are distinguished from each other by their *bases*. If the base is a triangle, the prism is called a triangular prism; and so with the rest—square prism, oblong prism, pentagonal prism, and the like.

The cube is only a special kind of square prism. What do you notice about the faces of the cube? Which are its bases?

Take the prisms in order. Count in each the number of corners, of edges, and of faces, and record in your note-book as follows:—

	Name.	No. of Sides in a Base.	No. of Corners.	No. of Edges.	No. of Faces.
1					
2					
etc.					

You notice that the number of corners is always a multiple of 2 (that is, it can be divided by 2). Why is this?

The number of edges is always a multiple of 3. Why is this?

The cylinder is a kind of prism. How does it differ from the other prisms?

419. Exercise (to be written).

(*a*) Denoting the number of sides in a base by n, the number of corners by a, the number of edges by e, and the number of faces by f, write down in terms of n, formulæ—

for the number of corners } in
for the number of edges } any
for the number of faces } prism.

(*b*) Express each of these formulæ in words.

(*c*) How many corners, how many edges, and how many faces would you expect to find in—

(1) An octagonal (*i.e.*, an 8-sided) prism?
(2) A 20-sided prism?

420. You have already learnt what is meant when it is said that one plane is at right angles to another.

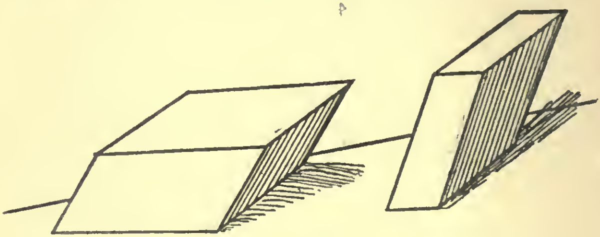


FIG. 110.

Test with a try-square (or with a set-square) whether the sides of your prisms are at right angles to the bases. Such prisms are called **right** prisms. Test whether your cylinder is a right prism.

There are prisms other than right prisms, of which examples are shown in Fig. 110, but you will not deal with these.

421. Take the prisms in order. Write down the name of each in your note-book, and underneath make a hand-sketch of it. In making a hand-sketch you have hitherto drawn only those edges which you could see. You must now draw (in dotted lines) the edges which you cannot see. Your teacher may be able to show you a gauze or a skeleton model, which will enable you to realise better the position of the unseen edges. Notice, among other things, that the edges further away from you appear to be smaller than edges of the same length

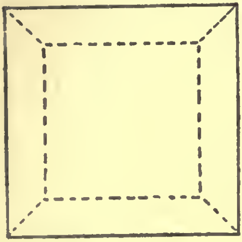


FIG. 111.

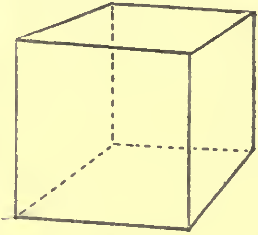


FIG. 112.

which are nearer to you, *e.g.*, the cube appears like Fig. 111 or Fig. 112.

Such a drawing is called a **perspective** drawing.

Cardboard Modelling

422. At this point the scholars may conveniently begin Woodwork. If they are not to do woodwork, they may still get very fair constructive ideas of the several solids by making cardboard models of them. For this purpose, the following developments are here given. A **development** (or a **net**, as it is sometimes called) of a solid is a drawing of its surface, so unfolded that all the faces are brought into the same plane (Fig. 113).

423. Exercise (cardboard modelling).

Make models in cardboard of the four prisms whose developments are given below, having edges equal to those of your wooden models.

Begin by drawing the development in pencil on the

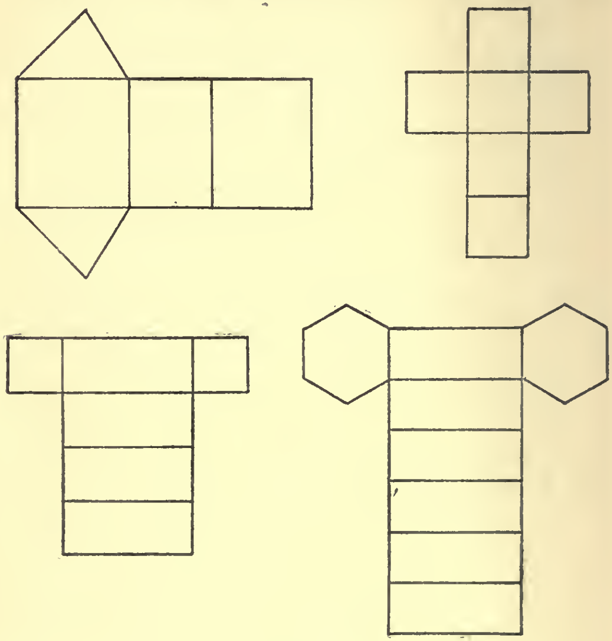


Fig. 113.

cardboard. Then cut the outer lines of the development quite through with a penknife, and cut the inner lines half-way through. The outer edges may then be brought together and fixed with gummed paper or cloth. It will add to the neatness of your model if you cover *all* the edges with strips of gummed paper or cloth.

Volume of a Rectangular Prism

424. You are now to find out how to measure the volume of a prism. If you will look at the cone and the cube, you will easily guess which has the greater volume. Now take the oblong prism and the square prism, lay them against each other, and try to guess which has the greater volume. You will probably find this much harder.

And if you were further asked to say how great is the volume of either, you would be puzzled.

In fact, before you can measure any volume you must have a **standard of volume**.

Let us see what the standard is.

425. Your teacher will give out some small cubes, one to each scholar. Examine your cube, and measure each edge carefully.

How long is each edge?

This cube is called a **cubic inch**, and is one of the standards used for measuring volumes. Each face of it is a square inch. The other standard commonly used is the **cubic centimetre**, that is, a cube with edges of 1 cm.

All the cubes now given to you are cubic inches.

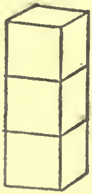


FIG. 114.

426. Your teacher will take three of the cubes, and arrange them as in Fig. 114.

What kind of solid is this?

What is the area of its base? What is its height? What is its volume? In stating the volume, take care to state that it is x *cubic* inches. It is wrong to say that it is x *inches;* and it would, of course, be quite absurd to say that it is x *square* inches.

Record the following measurements in your notebook :—

	Area of Base in sq. in.	Height of Solid in in.	Volume of Solid in cub. in.
No. 1.			

Now your teacher will arrange six of the cubes as in Fig. 115.

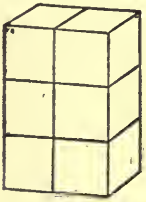


FIG. 115.

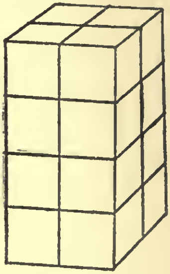


FIG. 116.

What kind of solid is this? What is its volume? its height? the area of its base?

Record the measurements as before.

Now he will arrange sixteen of the cubes as in Fig. 116.

What kind of solid is this? What is its volume? its height? the area of its base?

Record the measurements as before.

427. Examine your records carefully, and try to make a rule for calculating the volume of a rectangular prism.

Express your rule as a formula, using V for volume.

428. Exercise (oral).

(*a*) The base of a prism contains 10 sq. in., and its height is 6 in. What is its volume?

(*b*) Find the volume of a chimney 4 ft. square, whose height is 50 ft.

(*c*) What is the volume of a box 3 ft. × 4 ft. × $1\frac{1}{2}$ ft.?

(*d*) A book has pages 9 in. × 6 in., and it is 1 in. thick. Find its volume.

(*e*) Find the volume of a cube with edges of 10 cm.

(*f*) Three edges of an oblong prism are 2·5 cm., 4 cm., and 5·5 cm. Calculate its volume.

429. Exercise (to be written).

(*a*) Find the volume of a piece of stone 3 in. × $2\frac{1}{2}$ ft. × 6 ft.*

(*b*) The bottom of a box is 3 ft. × 18 in., and its depth is 3 ft. Find the volume of the box.

(*c*) How many cubic inches are there in a cubic foot? how many cubic feet in a cubic yard?

(*d*) The dimensions of a brick are 3 in. × $4\frac{1}{2}$ in. × 9 in. Find its volume both in c. in. and in c. ft.

(*e*) How many bricks would fill the box mentioned in (*b*)?

(*f*) If there are 2·54 cm. in 1 in., find the number of c. cm. in 1 c. in., correct to one place of decimals.

430. If you were told the volume of a prism, and its height, how would you calculate the area of the base?

* Take care with your quantities. (See § 303.)

Express your rule as a formula.

If you were told the volume of a prism, and the dimensions of its base, how would you calculate the height of the prism?

Express your rule as a formula.

431. Exercise (oral).

(*a*) The volume of a prism is 25 c. in., and its height is 4 in. Find the area of the base.

(*b*) The volume of a prism is 100 c. in., and the area of its base is 16 sq. in. Find the height of the prism.

(*c*) The volume of a square prism is 20 c. cm., its height is 5 cm. How long are the edges of the base?

(*d*) Calculate the lengths of all the edges of a square prism whose volume is 30 c. in., and whose base has one edge of 3 in.

(*e*) How long is each edge of a cube whose volume is 27 c. cm.?

Approximate Working

432. A prism is 3·25 cm. × 6·5 cm. × 4·35 cm. The volume is therefore 3·25 × 6·5 × 4·35 c. cm.

```
 3·25
 6·5
─────
19·50
 1·625
──────
21·125 (Area of base in sq. cm.)
 4·35
──────
84·500
 6·3375
 1·05625
────────
91·89 (Volume in c. cm.)
═════
```

433. In this product the figure in the third place of decimals would represent cubic millimetres. Now, a c. mm. is smaller than a pin-head ; so that, *even if the measurements are given to you as quite exact,* and even if the product is, therefore, quite exact also, you can ignore the third figure in the decimals.

You can, therefore, shorten your working by adding only the first two columns of the decimals, but you must not forget to "carry" from the third column.

434. You know, however, that any measurements which you make yourself are never quite exact. For instance, if the above measurements had been your own, you might have been doubtful whether the height was nearer to 4·3 cm. or to 4·35 cm. If the height really was 4·3 cm., and not 4·35 cm., the last line of figures, 1·05625, would have to be omitted, and the total would then be diminished by more than a whole c. cm.

Whenever, therefore, you are calculating from your own measurements, even if you have measured very carefully, you cannot hope to calculate the volume more accurately than to the nearest c. cm.

435. Exercise (requiring the use of models).

(*a*) Take in order the cube, the square prism, and the oblong prism; make a hand-sketch of each, marking such measurements as are necessary to enable you to calculate the volume; then calculate the volume in c. cm.

(*b*) Measure the cube in inches, and calculate its volume in c. in.

(*c*) Reduce these c. in. to c. cm., and compare with the previous result.

(*d*) If the height of the oblong prism were increased to 15 cm., by how much must the edges of the base be increased so as to remain in proportion?

(*e*) What is the height of a prism whose volume is 203·5 c. cm., and whose base is 6·5 cm. × 4·35 cm.?

(*f*) Find the area of the base of a prism $3\frac{7}{16}$ in. high, whose volume is $32\frac{1}{12}$ c. in.

Plan and Elevation

436. You have seen that in a perspective drawing some parts of the solid appear smaller than they really are, and the angles often appear quite untrue.

It would clearly be very convenient to be able to represent the parts of a solid in their true shape and proportion, so that by applying the scale you could measure accurately any part of the solid which you wished to measure.

437. You have already learnt how to draw very simple plans and elevations. A plan is a top view of anything with the lines drawn to scale, and an elevation is a front view of anything with the lines drawn to scale.

438. Your teacher will show you a hinged plane. When the two planes are set at right angles to each other, the upright plane is called the **vertical** plane, and the other the **horizontal** plane. Do you remember what these words mean?

The line where the two planes meet is called the **ground line**, or the **xy line.**

439. Let a large model (say a square prism) be placed on the horizontal plane in such a way as to lie close to the vertical plane, as in Fig. 117.

Look at the model from above, and by means of long knitting needles, mark the points on the horizontal plane which lie *immediately beneath* all the corners of the model which you can see. Join these points by chalk lines. You have now a *plan* of the model.

The lines represented by the knitting needles are called **projection lines.** It is very important that they should be perpendicular to the plane. You have

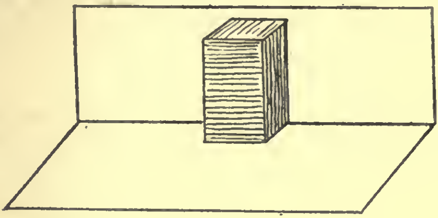


Fig. 117.

already learnt how to judge whether a line is perpendicular to a plane.

440. Now do the same with the model as seen from the front, and you will get an *elevation* of the model drawn on the vertical plane.

Remove the model, and let the vertical plane turn back until it is in the same plane as the horizontal plane, as in Fig. 118.

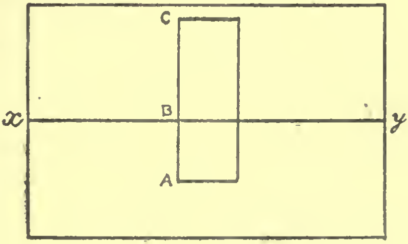


Fig. 118.

441. You have now a plan and an elevation in the same drawing. You have to notice—

(*a*) That the elevation of any point shown in the plan lies on a line (the projection line) drawn through that point *at right angles* to the *xy* line. Thus A is the plan both of an upper and of a lower corner of the model; B is the elevation of the lower corner, C the elevation of the upper corner, and ABC is at right angles to *xy*.

(*b*) That the lines in the drawing can be measured by scale. If the drawing given above were the plan and elevation of a square prism to a scale of $\frac{1}{8}$, what would be the real dimensions of the prism?

442. Now let a large hexagonal prism be placed on the hinged plane, with one edge of the base 3 inches from the *xy* line, and parallel to it.

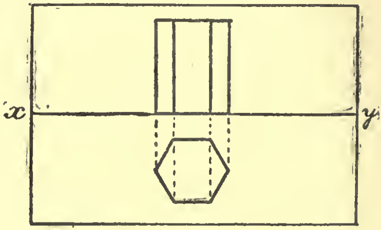


Fig. 119.

The plan and elevation of the prism will appear as in Fig. 119.

You see again that the elevation can be got from the plan, or the plan from the elevation, by drawing the projection lines.

The scale being $\frac{1}{20}$, calculate the linear dimensions of the hexagonal prism represented above, and describe its position.

Your teacher will show you other experiments with the square prism in different positions, and also with a pyramid and other solids.

443. Exercise (drawing, etc.).

(*a*) Draw a plan and elevation of the cube, of the square prism, and of the rectangular prism, each with one edge on the ground line: scale $\frac{1}{2}$.

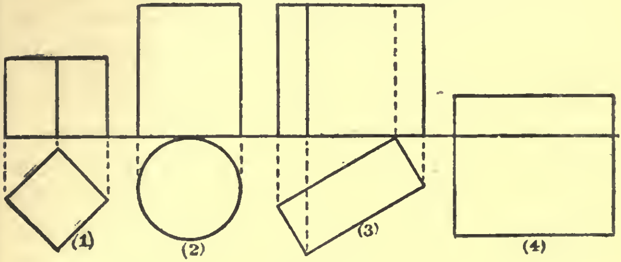


FIG. 120.

(*b*) Interpret the drawings in Fig. 120, that is—

(1.) State the kind of solid represented by each figure.

(2.) Give the linear dimensions of the solid, if the scale is $\frac{1}{8}$.

(3.) Describe the position of the solid.

N.B.—Dotted lines *in a figure* indicate edges that cannot be seen.

(*c*) Complete the drawings in Fig. 121, making the plan from the elevation, or *vice versâ*: scale $\frac{1}{8}$.

(1) is a square prism 12 in. high.

(2) is a cube, with one corner on the ground line.

(3) is an oblong prism 4 in. high.

Volumes of other Prisms

444. You have found that the formula for the volume of a rectangular prism is $V_P = Ah$. Now take the rhomboid prism. Let us see whether the formula is true for this also.

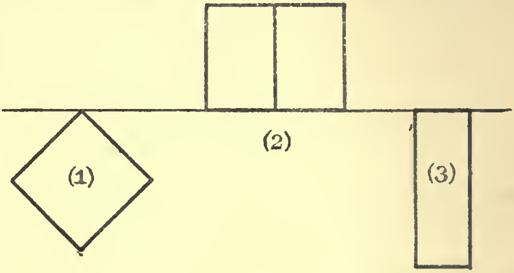


FIG. 121.

You learnt in § 355 how to make a rhomboid into a rectangle by cutting off a triangular piece from one end and removing it to the other.

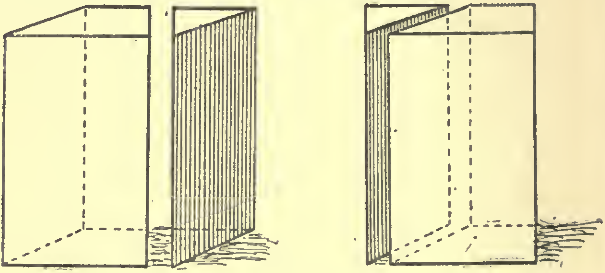


FIG. 122.

So you can imagine a rhomboid prism changed into a rectangular prism by cutting a triangular prism from one end and removing it to the other, as in Fig. 122.

Can you now say whether the formula $V_P = Ah$ is true for the rhomboid prism?

445. Exercise (requiring the use of the rhomboid prism).

(*a*) Make a hand-sketch of the model, marking on it in inches and fractions of an inch all measurements necessary to calculate the volume.

(*b*) Calculate the volume in c. in. and fractions of a c. in.

(*c*) Draw a plan and elevation of the model: scale $\frac{1}{3}$.

446. The volumes of other right prisms can be calculated in the same way.

447. Exercise (requiring the use of the triangular prism).

(*a*) Calculate the volume of your triangular prism in c. in.

(*b*) Draw a plan and elevation of it with one edge parallel to the ground line, and 1 in. from it: scale $\frac{2}{3}$.

(*c*) If the prism were made 1 in. higher, by how much must you increase the other edges to keep them in proportion?

(*d*) The base of a prism is a 5 in. equilateral triangle. Its height is also 5 in. Calculate its volume.

448. Exercise (requiring the use of the cylinder).

(*a*) Calculate the volume of your cylinder in c. cm.

(*b*) Draw a plan and elevation of it, touching the ground line: scale $\frac{3}{4}$.

(*c*) The diameter of a cylinder is 4 cm., and its volume is 102·38 c. cm. What is its height?

(*d*) A cylindrical tank is 12 ft. in diameter and $14\frac{7}{12}$ ft. high. Calculate its volume.

(*e*) Draw a plan and elevation of it: scale $\frac{1}{8}$ in. to 1 ft.

(*f*) If the diameter were only 6 ft., what would then be the volume of the cylinder? And what would be the ratio of this volume to the volume of the 12 ft. cylinder?

Pyramids

449. Examine the pyramid given to you by your teacher. You will have no difficulty in recognising the base, and you will notice that the sides of the pyramid are formed by lines joining the corners of the base to a point without it. This point is called the **apex** of the pyramid.

State as accurately as you can what kind of figures the sides of the pyramid are.

450. Pyramids are distinguished from each other as prisms are, by their *bases.*

Thus, there are triangular pyramids, square pyramids, and so forth.

What kind of pyramid is the one which you have?

Fig. 123 shows the developments of two other pyramids.

Of what kinds of pyramid are these the developments?

A circular pyramid is called a cone.

451. Exercise (cardboard modelling).

Make cardboard models of a triangular and of a square pyramid, using the developments and the dimensions given above.

Follow the instructions given in § 423.

452. Take your three pyramids in order, make a hand-sketch of each in your note-book, and write the name of it underneath. Indicate the unseen edges by dotted lines.

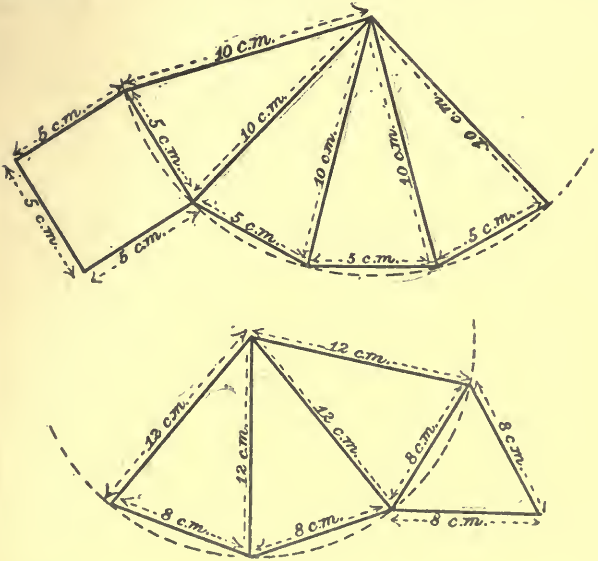


Fig. 123.

453. Count in each pyramid the number of corners, of edges, and of faces, and record in your note-book as follows :—

	Name.	No. of Sides in Base.	No. of Corners.	No. of Edges.	No. of Faces.
1					
2					
etc.					

454. Exercise (to be written).

(*a*) Denoting the number of sides in a base by *n*, the number of corners by *a*, the number of edges by *e*, and the number of faces by *f*, write down in terms of *n*, formulæ

for the number of corners }
for the number of edges } in *any*
for the number of faces } pyramid.

(*b*) Express in words the meaning of each of these formulæ.

(*c*) How many corners, how many edges, and how many faces are there in an octagonal pyramid?

455. Exercise.

(*a*) Interpret the drawing shown in Fig. 124, that is—

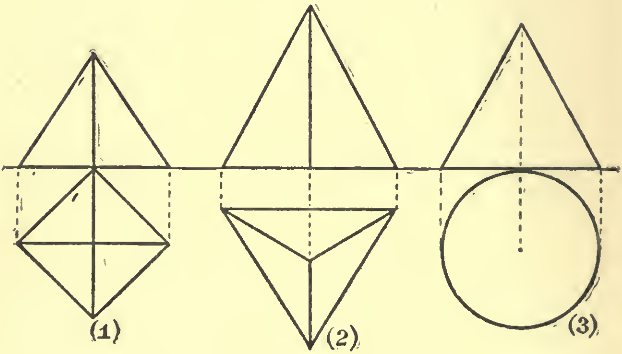


Fig. 124.

(1) State accurately the kind of solid represented by each figure.

(2) State the linear dimensions of the solid, *i.e.*, its height and the dimensions of the base: scale $\frac{1}{8}$.
(3) Describe the position of the solid.

Volume of a Pyramid

456. The volume of any pyramid is one-third of the volume of a prism of the same height standing on the same base. Why this is so is too difficult for you to understand at present. Remember that by the height of a pyramid is meant the vertical height.

How would you use your calipers to find the height of a pyramid?

457. Exercise (oral).

(*a*) The height of a pyramid is 8 in., and the area of its basc 6 sq. in. What is its volume?
(*b*) Find the volume of a square pyramid 10 in. high, when an edge of its base is 3 in.
(*c*) If the volume of a pyramid is 30 c. cm., and the area of its base 12 sq. cm., find the height of the pyramid.

458. Exercise (requiring the use of models).

(*a*) Calculate in c. in. the volume of the square pyramid which you have made.
(*b*) Draw a plan and elevation of the pyramid, with one edge on the ground line: scale $\frac{1}{2}$.
(*c*) Calculate in c. cm. the volume of the cone.
(*d*) Draw a plan and elevation of it, full size.
(*e*) If each side of the base of your square pyramid were lengthencd by 2 in., how long would you make the other edges of the pyramid in order to keep them in proportion?

(*f*) Suppose you wished to make a cone of the same shape as your wooden model, but having the diameter of the base 4 cm. greater, how high would you make the cone?

(*g*) The volume of a square pyramid is 16·05 c. in., and its height 5·35 in. How long is an edge of its base?

(*h*) The volume of a cone is 153·67 c. cm.; the diameter of its base is 6·55 cm. Find its height correct to one place of decimals.

(*i*) Denoting the volume of a pyramid by V_p, complete the formula,

$$V_p =$$

The Sphere

459. You frequently meet with another solid of regular shape, which is neither prism nor pyramid, and that is the **sphere.** (Fig. 125.)

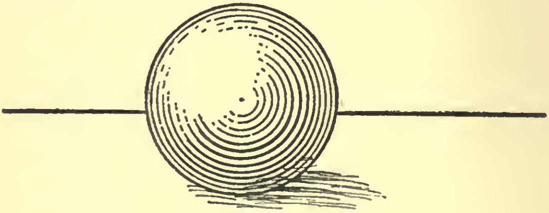


Fig. 125.

You have examples of the sphere in a marble, a cricket-ball, and an orange. These objects are generally described as round, but if you wish to describe their shape accurately, you must call them **spherical,** for you might describe a penny or a saucer as round.

What other spherical objects do you know?

460. In a perfect sphere all the diameters, that is, all the imaginary lines drawn through the centre from surface to surface in whichever direction, are exactly equal to each other. In other words, from whatever point you look at the sphere, its outline is always a perfect circle.

461. Many spheres are not quite perfect. You will find, for instance, that one diameter of a cricket-ball is rather longer than another. This is because it is hard to make a perfect sphere. Still, "spherical" is the best description of the shape of a cricket-ball; it is as nearly as possible spherical.

462. The sphere has no edges. You describe its dimensions by stating the length of its diameter.

In measuring the diameter great care must be taken. With a little practice you will be able to mark the ends of an imaginary diameter with your finger and thumb. Measure this diameter with your calipers; to ensure accuracy, always take an average of three different diameters.

463. Exercise (requiring the use of objects).

(*a*) Measure the diameter of a large taw in cm., correct to one place of decimals.

(*b*) Measure the diameter of your cricket-ball (1) in inches, (2) in cm.

(*c*) Test your measurements by converting inches into cm.

464. The volume of a sphere is two-thirds of the volume of a cylinder having the same diameter and the same height. Why this is so you cannot understand at present. But you have learnt how to find the volume of a cylinder; and you will easily remember that the volume of a cone is one-third, and the volume of a sphere two-thirds, of the volume of the corresponding cylinder.

465. Exercise (to be written).

(*a*) Find the volume of the large taw given to you, correct to one c. cm.

(*b*) Draw a plan and elevation of the taw, full size.

(*c*) Why are the plan and the elevation of a sphere *always* represented by the same figure? What is that figure?

(*d*) Find the volume of your cricket-ball, correct to one c. cm.

(*e*) The inside diameter of an Association football is 20 cm. How many c. cm. of air will it hold?

(*f*) Write down in terms of r, a formula for the volume of a cylinder whose height is equal to the diameter of its base.

(*g*) Write down formulæ (1) for the volume of a cone having the same base and the same height, (2) for the volume of a sphere having the same diameter.

(*h*) A billiard ball is 2 in. in diameter. Find its volume.

(*i*) A sphere and a cube will both just fit into a box which is 10 in. each way. What are their volumes?

(*j*) Calculate the ratio of the volume of the cube to that of the sphere?

(*k*) Calculate the ratio of a cubic foot to the volume of a sphere 1 foot in diameter. Is it the same as the ratio you found in (*j*)?

Larger Cubic Measures

466. The measures of volume most commonly used are the cubic inch and the cubic centimetre. Builders, however, measure masonry in cubic feet, and navvies measure earth and rock in cubic yards. Mention is rarely made of any measure greater than the cubic yard.

Write in your note-book tables of cubic measure, as follows :—

(*a*) —— c. in. make 1 c. ft.
—— c. ft., or —— c. in. make 1 c. yd.

(*b*) —— c. mm. make 1 c. cm.
—— c. cm. make 1 c. dm.
—— c. dm. make 1 c.m.

467. Exercise (to be written).

(*a*) Find in c. ft. the volume of a wall 120 yd. long, 6 ft. high, and $13\frac{1}{2}$ in. thick.*

(*b*) How many bricks would be required to make the wall, one-twelfth of the space being occupied by mortar?

(*c*) A trench is dug 84 yd. long, 1 ft. 9 in. wide, and $2\frac{1}{2}$ ft. deep. How many c. yd. of earth are removed?

(*d*) What is the cost of removing it at $10\frac{1}{2}$d. a c. yd.?

(*e*) To make a circular mine-shaft 10 ft. in diameter, 15,700 c. yd. of earth are excavated. What is the depth of the shaft?

(*f*) The Great Pyramid is 480 ft. high and has a base 760 ft. square. How many c. yd. does it contain?

468. Exercise (General).

(*a*) How many cubic feet of air are there in a room 42 ft. long, $26\frac{1}{2}$ ft. wide, and $18\frac{3}{4}$ ft. high?

(*b*) What is the cost of 29,500 c. ft. of gas at 2s. 6d. a thousand?

(*c*) A cylindrical tank is 7 ft. in diameter and 8 ft. 2 in. high. Find its volume.

* See § 301.

(*d*) How many gallons of water will it hold? 1 gallon = 277 c. in.

(*e*) How many sq. ft. of lead are required to make such a tank? You are to assume that it has no lid.

(*f*) Calculate the cost of the material, if lead is $11\frac{1}{4}$d. a sq. ft.

(*g*) A square pyramid has each side of its base 6 in. long and is 6 in. high. Calculate its volume.

(*h*) What is the volume of a square pyramid whose dimensions each way are 3 in. instead of 6 in.?

(*i*) What is the ratio of the latter volume to the former?

(*j*) How many c. in. of lead are required to make 1 ft. of water-piping of 1 in. internal, and $1\frac{1}{4}$ in. external, diameter?

ANSWERS

The numbers refer to the sections

CHAPTER I

6. (*a*) 100 (*b*) 20 (*c*) 500
(*d*) 25 (*e*) 20 (*f*) 10
(*g*) 300 (*h*) 5 (*i*) 120

32. (*a*) 126·425 m. (*b*) 301·013 m.
(*c*) 4·895 (*d*) 51·5172

35. (*a*) 1·11 m. (*b*) 36·63 m. (*c*) ·999 m.
(*d*) 46·195 m. (*e*) ·81 (*f*) 1·93
(*g*) 11·51

39. (*d*) 121·8 (*e*) 808·8 (*f*) 1701 (*g*) 302·4

43. (*a*) 114·2 m. (*b*) 84·6 m.
(*c*) 19·8 m. (*d*) 175·4 m.
(*f*) 40·779 m. (*g*) 6·755 dm.
(*h*) 854·89 m. (*i*) ·002 m.
(*j*) 24·4 (*k*) 4·188

46. (*a*) £4497. 19s. $4\frac{1}{2}$d. (*b*) £1. 8s. $3\frac{1}{4}$d.
(*c*) £25106·688 (*d*) 132·001

48. (*a*) 82·8 cm. (*b*) 496·8 cm.
(*c*) 46·368 m. (*d*) 1·13 dm., or 11·3 cm.
(*e*) 1·68 m. (*f*) 9·48 dm.
(*g*) 54 cm.

52. (*a*) £·05; £·1; £·15, etc.
(*b*) £1·35; £17·45; £100·65
(*c*) £1·05; £·5; £5·25
(*d*) 1·0; ·5; ·5
(*e*) ·1; ·05; ·05
(*f*) £1. 10s.; £2. 15s.; £3. 1s.
(*g*) 10s.; £17. 19s.; £14. 13s.
(*h*) £181. 4s. (*i*) £1. 14s. (*j*) £172. 14s.
(*k*) 12s. (*l*) £64. 7s. (*m*) £104. 13s.
(*n*) £24. 18s. (*o*) 9s.

55. (*a*) 1·7 (*b*) 1000 (*c*) 3 (*d*) 14
(*e*) ·07 in. (*f*) ·05 in. (*g*) ·05 in. (*h*) ·01 in.

59. (*a*) 60·94 in. (*b*) 154·78 in.
(*d*) £10·85; £3·45; £6·35
(*e*) 8·5 in.; 2·5 in.; ·5 in.
(*f*) 212·4 in.; 198·24 in.; 297·36 in.; 254·88 in.
(*g*) ·792 in.; ·09 in. (*h*) 21·15 in.
(*i*) 6 faces, 10 edges, 6 corners; 6 faces, 12 edges, 8 corners

72. (*b*) 139·0008 (*c*) 62930·6367 m.
(*d*) 18·221 (*e*) £22. 1s.
(*f*) 31·625 (*g*) 4·7 in.
(*h*) 10·01 ; 465·329 (*i*) 152·66
(*j*) £1·9 ; £14·1 ; £·25
(*k*) £1. 12s. ; £1. 1s. ; £1. 15s.
(*l*) 72·25 (*m*) 5·7 miles
(*n*) $3\frac{1}{2}$d. (*o*) £6. 17s. 5d.

CHAPTER II

80. (*a*) 36 (*b*) 272
(*c*) £·951 (*d*) 30·87 ; 4·83 ; 3·22
(*e*) 17·3 cm. (*f*) 7·41 in.
(*g*) 30·15 in. (*h*) 3·143; 4·171; 1·745
(*i*) 16s. 6d.

95. (*a*) 34·54 in. ; 75·36 in. ; 131·88 in. ; 109·9 in. ; 175·84 in.
(*b*) 69·08 in. (*c*) 28·26 in. ; 3·14 times
(*d*) 628 yd.

99. (*a*) 69·08 in. (*b*) 113·04 in. (*c*) 153·86 in.

102. (*a*) $d=2r.$ (*b*) $r=\frac{d}{2}$ (*c*) $d=\frac{l}{\pi}$
(*d*) $l=2\pi r$ (*e*) 31·4 in. (*f*) 43·96 in.
(*g*) 276·32 in.

107. (*a*) $D = 8s - 4p$; 4·4 in.
(*b*) $L = 4a + 4b + 4c$ (*c*) $L = 12f + 6g$
(*d*) 27 (*e*) 345
(*f*) $20p$ shillings; $20q$ shillings; $20r$ shillings
(*g*) $10f$ cm.; $100g$ cm. (*h*) ·5

112. (*a*) 21 (*b*) 17 (*c*) 9
(*d*) 6 (*e*) 13 (*f*) 5
(*g*) 11 (*h*) 19

115. (*a*) $[4p + 8q]$ inches; $[4p + 8q - 16]$ inches
(*b*) $4[p - r + 2q + 2s]$ inches
(*c*) $4[x - a + y - b + z - c]$ centimetres
(*d*) 4·11 in. (*e*) 3·14 cm.
(*f*) 3·141 (*g*) 53·38 yd.
(*h*) 17; 1; 72; 1·125; ·888; 138
(*j*) 420 shillings; $20x$ shillings; 423 shillings; $[20x + y]$ shillings
(*k*) 4600 pence; $[240x + 12y + z]$ pence
(*m*) 123·75; 13·25 (*n*) 61·5
(*o*) 4·37 (*p*) 4·31 (*q*) 4·39
(*r*) 74·63 (*s*) 4·124; ·744; ·111
(*t*) $2ab + 2ac - d - e + f$
(*u*) $p[q + r + s]$; $b[a + c] + ac$
(*v*) 130 (*w*) 22·678 m. (*x*) ·885 m.
(*y*) Fry, 81·3; Ranjitsinhji, 56·6; Hill, 46·8; Iremonger, 46·0; Knight, 45·8; Perrin, 44·6; Tyldesley, 44·4; MacLaren, 42·9; Quaife, 42·8; Gunn, 42·7; Foster, 42·0; Hirst, 47·3; Wells, 41·2

(*z*) Ringrose, 13·5; Mead, 13·7; Blythe, 13·8; Langford, 14·0; Hargreave, 14·0; Rhodes, 14·4; Hearne, 15·3; Moorhouse, 15·8

CHAPTER IV

147. (*a*) $\frac{6}{16}, \frac{5}{6}, \frac{8}{10}, \frac{5}{8}, \frac{3}{5}, \frac{10}{12}$
(*b*) $\frac{3}{4}$ and $\frac{12}{16}$, $\frac{4}{6}$ and $\frac{2}{3}$, $\frac{2}{8}$ and $\frac{1}{4}$
(*e*) $2\frac{2}{5}$

150. (*a*) $\frac{5}{6}, \frac{1}{3}, \frac{3}{4}, \frac{1}{3}, \frac{1}{2}, 1$
(*c*) 1
(*d*) $\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}, 1$
(*e*) $\frac{1}{12}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{5}{12}, \frac{1}{2}, \frac{7}{12}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{11}{12}, 1$;
$\frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}, 1$;
$\frac{1}{16}, \frac{1}{8}, \frac{3}{16}, \frac{1}{4}, \frac{5}{16}, \frac{3}{8}, \frac{7}{16}, \frac{1}{2}, \frac{9}{16}, \frac{5}{8}, \frac{11}{16}, \frac{3}{4}, \frac{13}{16}, \frac{7}{8}, \frac{15}{16}, 1$
(*f*) $\frac{1}{x}, \frac{1}{5}, 1.$

162. (*c*) $1\frac{1}{2}, 1, 2\frac{9}{20}$
(*d*) $1\frac{3}{4}$

165. (*a*) $12\frac{1}{2}, 10\frac{9}{10}, 6\frac{1}{2}$
(*b*) $9\frac{7}{8}$
(*c*) $12\frac{11}{12}$
(*d*) $4\frac{1}{6}$

171. (*a*) $\frac{7}{8}, 2\frac{3}{5}, \frac{1}{8}, \frac{1}{2}$
(*b*) $8\frac{11}{16}$
(*c*) $\frac{1}{6}, 2\frac{2}{5}, \frac{5}{8}, 9\frac{3}{16}$
(*d*) $5\frac{1}{8}, \frac{3}{16}, 3\frac{4}{5}, \frac{1}{6}$
(*e*) $3\frac{1}{8}$
(*f*) $2\frac{2}{3}$
(*h*) 10s.
(*i*) $\frac{7}{12}$, 14s.

175. (*a*) $18\frac{5}{12}$ in. (*b*) $1\frac{1}{3}, \frac{5}{6}, 1\frac{4}{5}$

(*c*) $\frac{5}{8}, 5\frac{1}{5}, 5\frac{35}{48}, 1\frac{31}{60}$ (*d*) $\frac{a+b}{ab}, \frac{b-a}{ab}$

(*e*) $\frac{53}{240}, \frac{73}{120}, \frac{15}{12}, \frac{1}{30}$ (*f*) $\frac{11}{20}$

180. (*a*) 28, 33, 28, $3\frac{3}{4}$ (*b*) 36s., $7\frac{1}{2}$d., 14s.

(*c*) 162 min., 20 hrs.

(*d*) £4. 16s. 3d.; 1s. $5\frac{1}{2}$d.; 24s.

(*e*) £3. 19s. $0\frac{3}{4}$d.

184. (*a*) $\frac{7}{32}$ (*b*) $\frac{3}{10}$ (*c*) $4\frac{1}{2}$ (*d*) $\frac{3}{16}$

(*e*) $\frac{3}{40}$ (*f*) $14\frac{15}{48}$ (*g*) $\frac{a}{d}$

194. (*a*) $1\frac{1}{24}, 2\frac{1}{3}, 3\frac{3}{7}$ (*b*) $3\frac{1}{3}$, 14, $2\frac{6}{11}$, 2 (*c*) $7\frac{33}{48}$

198. (*a*) $\frac{3}{10}, \frac{63}{100}, \frac{3}{100}, \frac{603}{1000}, \frac{63}{1000}, \frac{7}{1000}$

(*b*) $3\frac{3}{10}, 10\frac{3}{10}, 10\frac{3}{100}, 16\frac{63}{100}, 70\frac{1}{10}, 700\frac{7}{1000}$

(*c*) $\frac{1}{2}, \frac{1}{4}, \frac{6}{25}, 5\frac{3}{4}, 50\frac{1}{40}, \frac{101}{250}$

(*d*) ·3, ·03, ·33, 33·03, 7·7, 11·011

200. (*a*) $\frac{1}{8}$ = ·125, $\frac{1}{4}$ = ·25, $\frac{3}{8}$ = ·375, $\frac{1}{2}$ = ·5, $\frac{5}{8}$ = ·625, $\frac{3}{4}$ = ·75, $\frac{7}{8}$ = ·875

(*d*) ·5, ·333, ·25, ·2, ·166, ·142, ·125, ·111, ·1, ·062, ·05

(*e*) 3·75, 7·05, 4·075, 100·98, 98·94

(*f*) $9\frac{8}{25}, 18\frac{3}{250}, 100\frac{1}{100}$

204. (*a*) 110, 101, 56 (*b*) 88 ft. (*c*) £545

(*d*) $3\frac{7}{49}$, by $\frac{1}{350}$ (*e*) 4, 24, 8, 10 (*f*) 21 in.

208. (*c*) 98 in.

210. (*b*) $\frac{1}{12}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

(*c*) $\cdot 08, \cdot 1\dot{6}, \cdot 25, \cdot\dot{3}, \cdot 5, \cdot\dot{6}, \cdot 75$

(*d*) $\frac{1}{36}, \frac{1}{18}, \frac{1}{12}, \frac{1}{9}, \frac{1}{6}, \frac{2}{9}, \frac{1}{4}$

$\cdot 029, \cdot 0\dot{5}, \cdot 08, \cdot\dot{1}, \cdot 1\dot{6}, \cdot\dot{2}, \cdot 25$

(*e*) $\frac{1}{3}, \frac{1}{6}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}$

$\cdot\dot{3}, \cdot 1\dot{6}, \cdot 5, \cdot\dot{6}, \cdot 8\dot{3}$

(*f*) $\frac{5}{9}, \frac{1}{2}, \frac{4}{9}, \frac{5}{12}, \frac{5}{6}, \frac{8}{9}$ (*g*) $16\frac{5}{6}$ yd.

(*h*) 16 yd. 2 ft. 6 in. (*i*) 7d.

(*j*) 11 yd. (*k*) 5 ft. $2\frac{1}{4}$ in.

215. (*a*) 960 (*b*) £114

(*c*) 320 yd (*d*) $101\frac{9}{11}$ yd.

218. (*a*) 2·54 (*b*) 30·48 cm. by calculation

(*c*) 91·4 cm. (*d*) 8·5 cm. (*e*) 3·37 in.

(*f*) 3·05 times (*g*) 3·94 in. (*h*) ·394

221. (*a*) 1094 yd. approximately

(*b*) 666 yd. shorter (*c*) 5 ml. (*d*) $\frac{5}{8}$ ml.

CHAPTER V

230. (*a*) £37; £24; £2. 6s. 8d.; £16. 6s. 8d.; £2. 5s.; £22

(*b*) (1) £1. 12s. 9d.; (2) £10. 3s. 6d.; (3) £37. 15s.; (4) £61; (5) £30. 8s. 6d.

231. (*c*) (1) $\frac{30}{36}, \frac{32}{36}, \frac{33}{36}$; (2) $\frac{18}{24}, \frac{15}{24}, \frac{14}{24}$; (3) $\frac{40}{60}, \frac{45}{60}, \frac{48}{60}$

(*d*) (1) $\frac{11}{12}, \frac{8}{9}, \frac{5}{6}$; (2) $\frac{3}{4}, \frac{5}{8}, \frac{7}{12}$; (3) $\frac{4}{5}, \frac{3}{4}, \frac{2}{3}$

(*e*) $5\frac{1}{6}$, 1, 14, $1\frac{15}{16}$, 11, $9\frac{1}{4}$

(*f*) $\frac{59}{6}$, $\frac{223}{12}$, $\frac{331}{16}$, $\frac{35}{8}$, $\frac{4019}{20}$, $\frac{305}{10}$
(*g*) $19\frac{3}{20}$, $8\frac{15}{16}$, $1\frac{7}{8}$, $20\frac{20}{21}$
(*h*) 970·3 cm., by calculation
(*i*) £120 (*j*) $15\frac{5}{6}$ (*k*) 8s. $1\frac{1}{2}$d.
(*l*) 71 in. (*m*) $163\frac{1}{2}$ in. (*n*) $\frac{2}{3}$
(*o*) $7\frac{13}{24}$ (*p*) $\frac{15}{16}$ (*q*) 10s. 9d.
(*r*) $\frac{2}{9}$ (*s*) 8s. 5d. (*t*) $\frac{5}{6}$
(*u*) £33. 14s. 8d. (*v*) £7200 (*w*) $\frac{1}{30}$
(*x*) 18s. 8d. (*y*) 30 ml. (*z*) $\frac{7}{12}$
(*aa*) 12s., 10s., 6d. (*bb*) $9\frac{5}{16}$ (*cc*) $4\frac{6}{11}$
(*dd*) 7 (*ee*) $\frac{z}{12}$ (*ff*) $4x+8y$ in.

CHAPTER VI

268. (*a*) $14\frac{7}{12}$ (*b*) 11 yd. 1 ft. 6 in.
(*c*) $4\frac{3}{4}$ ml. (*d*) 18·96 cm.
(*e*) $4x$ (*f*) $2(x+y)$ in.
(*g*) $2(x+y+2\frac{3}{8})$ in.

279. (*a*) $15\frac{3}{4}$ sq. in., $1\frac{3}{4}$ sq. in.
(*b*) $45\frac{1}{5}$ sq. in., $30\frac{11}{24}$ sq. in.
(*c*) $6\frac{1}{4}$ sq. in., $2\frac{1}{4}$ sq. in.
(*d*) $4\frac{33}{64}$ sq. in., $4\frac{41}{100}$ sq. in.
(*e*) $10\frac{6}{25}$ sq. in. (*f*) 3 in.
(*g*) *e.g.* 4 and $2\frac{1}{4}$, 2 and $4\frac{1}{2}$
(*h*) 124 (*i*) 252 (*j*) $44\frac{5}{16}$ sq. in.

284. (*a*) 33·67 sq. cm., 47·38 sq. cm., 2·25 sq. cm.
(*b*) 1·44 sq. in., 5·76 sq. in., 14·44 sq. in.
(*c*) 10·89 sq. cm., 19·36 sq. cm., 30·25 sq. cm.

290. (*a*) xy sq. in. (*b*) $y(x+\frac{5}{2})$ sq. in.
(*c*) $52\frac{1}{2}$ sq. in. $=7(5+\frac{5}{2})$ sq. in.

294. (*a*) 13·9, 1·5, 3·6 (*b*) 7·65 cm.
(*c*) $3\frac{3}{8}$ in. (*d*) 6·25 sq. cm.
(*e*) 200 sq. cm., 187·5 sq. cm., 25 sq. cm., 46·87 sq. cm.
(*f*) 4 sq. in., 4·8 sq. in., 2 sq. in., ·16 sq. in.

296. (*a*) 170·5, 3·56, 35·6
(*b*) 76·5 m. (*c*) ·9 in.

301. (*a*) 11, 13, $2\frac{3}{5}$ (*b*) 1·3 sq. in. (*d*) $\sqrt{xy}$ in.

303. (*b*) $3\frac{2}{9}$ sq. yd., $\frac{1}{3}$ sq. yd., $1\frac{1}{9}$ sq. yd., $\frac{1}{9}$ sq. yd.
(*c*) $\frac{1}{2}$ sq. ft., $3\frac{1}{4}$ sq. ft., $\frac{3}{4}$ sq. ft.
(*d*) 928·8 sq. cm. (*e*) 420 sq. in.
(*h*) 9 ft. 6 in.

306. (*a*) $150\frac{50}{121}$ acres (*b*) $37207\frac{1}{2}$ sq. ml.
(*e*) $17\frac{1}{2}$ acres (*h*) 300 plots
(*i*) £415. 18s. 9d. (*j*) 400 sq. ml.

CHAPTER VII

324. (*a*) 28·26 in. (*b*) 30° (*c*) 57°

341. (*a*) $4\frac{9}{16}$ miles (*b*) 180°; 120°; 45°; 15°
(*c*) 60° (*d*) 1·83 in. (*e*) 8·77 in.
(*f*) big hand, 360°, 120°, 270°, 180°; little hand, 30°, 10°, $22\frac{1}{2}$°, 15°
(*g*) 232 miles (*h*) 2·72 miles
(*i*) $3\frac{1}{3}$ in. (*j*) 72°
(*l*) 41° (*m*) 165°; 15°; 165°
(*n*) 90°; $51\frac{1}{2}$°; $38\frac{1}{2}$° (*o*) 6·1 cm. (*p*) 11
(*q*) 6·28 in.; 4·71 in.; 12·56 in.; 14·13 in.

CHAPTER VIII

354. (*a*) 3·3 in.; 3·3 in.; 3·7 in.
(*b*) 47·13 in.; 47·13 in.; 39·77 in.
(*c*) $1\frac{11}{32}$ in.; $1\frac{11}{32}$ in.; $5\frac{3}{8}$ in.
(*d*) 6·7 in.; 2·2 in. (*e*) 360°
(*f*) 141° (*g*) 79°; 101°

358. (*a*) 8·75 sq. cm. (*b*) 17·64 sq. cm.
(*c*) 10·4 sq. cm. (*d*) 1·9 sq. in.
(*e*) 11 sq. cm. (*f*) ·75 sq. m.
(*g*) 36·51 sq. yd. (*h*) 12·2 sq. ft.
(*i*) The rectangle is the greater by ·23 sq. ft.
(*j*) 1·4

361. (*b*) 4·3 cm. (*e*) 90° (*g*) 4·4 cm.

364. (*a*) $6\frac{11}{16}$ in. (*b*) $16\frac{1}{2}°$
(*c*) 60° (*d*) $82\frac{1}{2}°$
(*e*) The hypotenuse (*f*) 1·625 in.; 60°

365. (*a*) 64 yd. (*c*) 1·12 in.; 75°
(*d*) 212 yd.; 149 yd.; 106 yd.
(*e*) 600 yd. (*f*) 541 yd.
(*g*) Altitudes: $10\frac{1}{2}$ in.; 1 ft. 6 in.; $10\frac{1}{2}$ in.
Angles: $73\frac{1}{2}°$; $73\frac{1}{2}°$; 33°
(*i*) ·27 in.
(*j*) 14·3 cm.; 20·2 cm.; 90°; 45°; 45°

370. (1) (*b*) 11·93 sq. yd.; (*c*) 1·1 sq. in.; (*d*) 3·4 acres; (*g*) 99 sq. in.; (*i*) $\frac{1}{25}$ sq. in.; (*j*) 1·02 sq. dm.
(2) Fig. 89: 1 ac. 2150 sq. yd.; £144. Fig. 90: 58 ac.; £5800. Fig. 91: 6 ac. 3200 sq. yd.; £670

381. (*a*) 1046·5 sq. in.; (*b*) 9·625 sq. in.; (*c*) 8 miles; (*d*) 60°; $\frac{\pi}{24}$ sq. in.; (*e*) $37\frac{1}{3}$ sq. ft.; (*f*) 1·44 sq. dm.; (*g*) ·9 sq. ml.

386. (*a*) Twice (*b*) 112 times
(*c*) Each flat surface = 2·4 sq. in.; curved surface = 60·8 sq. in.
(*d*) 429 sq. in. (*e*) 2·17 sq. in.
(*f*) 2·75 sq. in.

389. (*a*) ·005 sq. in. (*b*) 2·8 sq. in.
(*c*) $21\frac{31}{36}$ in. (*d*) £463. 9s.
(*e*) ·72 sq. in.
(*f*) ·785 sq. ft.; 3·925 sq. ft.
(*g*) £5. 16s. 8d. (*h*) 23·4 sq. cm.
(*j*) 46·1 sq. in. (*k*) 154·8 sq. in.
(*l*) $\frac{1}{2}xy$ (*m*) 6·54 in.
(*o*) 7 yd. (*p*) 18s. 9d.
(*q*) 4·1 sq. cm. (*r*) 5·7 in.
(*s*) 14 in. (*t*) 12
(*u*) 16·7 sq. dm. (*v*) 102·6 sq. in.
(*w*) 3893 (*x*) 14·4 sq. in.
(*y*) 12 sq. in. (*z*) 39·3 yd.

CHAPTER IX

393. (*a*) $\frac{12}{11}$, $\frac{4}{3}$ (*b*) $\frac{2}{5}$, $\frac{10}{29}$ (*c*) $\frac{7}{22}$ (*d*) $\frac{44}{7}$

395. (*b*) $\frac{1}{1·611}$

399. (*a*) 6·88 in. (*b*) 27·5 sq. in.
(*c*) $15\frac{1}{2}$ sq. ft. (*d*) $\frac{1·41}{1}$
(*f*) $\frac{1·72}{1}$ (*g*) 17·2 cm.
(*i*) 44 yd. 2 ft. 5 in.

401. (*a*) 2s. 6d. (*b*) 1s. 9d. (*c*) 70
(*d*) 45 (*e*) 3s. (*f*) 4s. $2\frac{1}{4}$d.
(*g*) 32, 36 (*h*) 15
(*i*) 1s. 3d. (*j*) £1. 4s. $11\frac{1}{4}$d.

407. (*b*) 7·875 (*c*) 6 ft.
(*d*) £4. 3s. 3d. (*e*) 14·07 cm.

413. (*b*) £10, £20, £25, £45
£16, £27, £34, £43
£12. 10s., £47. 10s., £37. 10s.
(*d*) £480, £720, £780, £940
£270, £370, £990, £630

415. (*a*) 10 hrs. (*b*) 15 men (*c*) 9 days
(*d*) 9 horses (*e*) 37 days (*f*) 33 days

CHAPTER X

419. (*a*) $d=2n$ $e=3n$ $f=n+2$
(*c*) 16, 24, 10; 40, 60, 22

429. (*a*) $3\frac{3}{4}$ c. ft. (*b*) $13\frac{1}{2}$ c. ft.
(*c*) 1728 c. in.; 27 c. ft.
(*d*) $121\frac{1}{2}$ c. in.; $\frac{9}{128}$ c. ft.
(*e*) 192 bricks (*f*) 16·4 c. cm.

435. (*e*) 7·15 cm. (approx.) (*f*) $9\frac{1}{3}$ sq. in.

447. (*d*) $53\frac{3}{4}$ c. in.

448. (*c*) 8·15 cm. (*d*) 1650 c. ft. (*f*) $412\frac{1}{2}$ c. ft.

454. (*a*) $a=n+1$, $c=2n$, $f=n+1$
(*c*) 9, 18, 9

458. (*g*) 3 in. (*h*) 12·8 cm. (*i*) $V_p=\frac{Ah}{3}$

465. (*e*) 4186·6 c. cm. (say 4187)
(*f*) $2\pi r^3$ (*g*) $\frac{2\pi r^3}{3}$, $\frac{4\pi r^3}{3}$
(*h*) 4·19 c. in. (say 4·2)
(*i*) 1000 c. in., $523\cdot\dot{3}$ c. in.
(*j*) $\frac{1}{\cdot 52\dot{3}}$ (*k*) $\frac{1}{\cdot 523}$

467. (*a*) 2430 c. ft. (*b*) 31,680 bricks
(*c*) $40\frac{5}{6}$ c. yd. (*d*) £1. 15s. $8\frac{3}{4}$d.
(*e*) 1800 yd. (*f*) 3,422,815 c. yd. (approx.)

468. (*a*) $20868\frac{3}{4}$ c. ft. (*b*) £3. 13s. 9d.
(*c*) $314\frac{5}{12}$ c. ft. (*d*) 1961 gals.
(*e*) $218\frac{1}{6}$ sq. ft. (*f*) £10. 4s. $6\frac{1}{2}$d.
(*g*) 72 c. in. (*h*) 9 c. in.
(*i*) $\frac{1}{2}$ (*N.B.* not $\frac{1}{3}$) (*j*) 5·3 c. in.

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